ESSAYS ON FINANCIAL CONTRACTING IN MACROECONOMICS

By

Kyle P. Dempsey

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The dissertation is approved by the following members of the Final Oral Committee:

Professor D. Corbae, Professor, Economics and Finance

Professor M. Ready, Professor, Finance

Professor E. Quintin, Associate Professor, Real Estate

Professor N. Williams, Professor, Economics

Professor A. Seshadri, Professor, Economics

Abstract

The first chapter studies the effects of capital requirements on banks when firms can borrow from both bank and non-bank lenders. Banks fund loans with insured deposits and must maintain a minimum capital to asset ratio; non-banks do not. Capital requirements resolve a risk-shifting externality, inducing banks to monitor borrowers, mitigating default risk and reducing bank failures. Although raising capital requirements reduces default on bank loans, aggregate loan default responds non-monotonically as borrowers substitute into non-bank finance. At a low capital requirement, an incentive effect makes bank lending safer: tightening the capital requirement induces banks to monitor more, reducing default on their loans. At higher capital requirements, though, a substitution effect takes over: bank loans become scarce, borrowers substitute into riskier, unmonitored non-bank finance, and aggregate default rises.

The second chapter proposes a theory of unsecured consumer credit where: borrowers have the option to default; defaulters are not exogenously excluded from future borrowing; there is free entry of lenders; and lenders cannot collude to punish defaulters. Limited credit through higher interest rates following default arises from lenders optimal response to limited information about borrowers types. Lenders learn from an individuals borrowing and repayment behavior about his type, encapsulating his reputation for not defaulting in a credit score. My coauthors and I take the theory to data by matching key data moments such as the overall delinquency rate. We use the model to quantify the value of reputation in the credit market, and compare static and dynamic default costs.

The third chapter explores empirically how lines of credit extended by banks to firms can amplify financial shocks. The two-sided run banks experienced in 2008 deepened the financial crisis and slowed the subsequent recovery. I demonstrate that banks typically finance credit line drawdowns by expanding their balance sheets (mostly through non-deposit debt), and that these drawdowns adversely affect net interest margins. These standard effects did not hold during the recent financial crisis, however. I show that banks responded to increased funding costs by expanding less to meet credit line commitments, and that there were more adverse effects on profitability.

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Chapter 1

Safer Banks, Riskier Economy? Capital Requirements with Non-Bank Finance

1.1 Introduction

Excessive risk taking in the banking sector played a major role in the financial crisis of 2008. The pronounced and persistent downturn following the crisis has spurred policymakers to consider more stringent regulations designed to improve financial stability. One such proposal is raising capital requirements for commercial banks, which impose that banks' capital must be no lower than a given fraction of total risk-weighted assets. This mitigates the extent to which banks can finance lending with borrowed money, thereby forcing banks to internalize downside risks and promoting stronger capital buffers against economic downturns. While higher capital requirements may decrease risk in the banking sector, they may also limit the amount of credit banks can supply to borrowers, potentially lowering investment and output.[1] Balancing decreased risk with reduced investment is the key challenge regulators face when setting bank capital requirements.

In this paper, I study the effects of changing capital requirements when lending can occur

[1]This will be the case as long as equity is privately costly to banks. For an excellent discussion of the social and private costs to banks, see Admati et al. (2013).

outside the traditional banking sector. The modern financial system includes a large body of alternative lending institutions, such as investment banks, finance companies, hedge funds, and public debt markets. This "non-bank" sector has grown drastically since the 1980s, comprising more than two thirds of total lending in the U.S. economy.[2] In addition, non-banks are not subject to the same regulatory capital regime as traditional banks.

As capital regulations curtail bank lending, this non-bank sector will step in and fill the gap, at least partially.[3] How does substitution out of bank lending induced by capital requirements jointly impact aggregate risk and investment? How do these effects impact aggregate welfare? Does the welfare-maximizing capital requirement change in the presence of a non-bank sector? If so, in what direction and by how much?

I explore these questions in the context of a novel general equilibrium model with both bank and non-bank lenders and two-sided moral hazard. For my purposes, the two most critical differences between banks and non-banks are: (i) banks finance themselves with insured deposits, while non-banks do not; and (ii) banks are able to better monitor borrowers to mitigate default risk. I show that including the non-bank sector alters both sides of the key tradeoff between increased safety and decreased lending facing regulators who set capital requirements. My framework endogenizes the level of aggregate default in the underlying assets in the economy as a function of the composition of total lending across the safer, monitored bank sector and the riskier, unmonitored non-bank sector. As this composition changes with the capital requirement, so too does the level of aggregate default and volatility.

In the model, capital requirements make bank lending safer by mitigating banks' moral hazard: they resolve the risk-shifting externality associated with limited liability and deposit

[2]Focusing on business debt, Figure 1.1 shows that bank loans comprise only a modest portion of firms' total debt (13% as of 2015), and that this proportion has declined over time (from a peak of 35% in 1982). Non-banks comprise a large and growing portion of other types of lending, too, such as mortgages.

[3]For example, in a detailed study of firms' financing choices in the wake of the financial crisis, Adrian et al. (2012) find evidence that a shock to the supply of bank credit induces a corresponding increase in firms' demand for non-bank credit. Even though total issuance of credit by the firms in their sample drops, new bank loan issuances decrease by 75% while new bond issuances increase by 50%.

insurance, inducing banks to make safer loans. Once the capital requirement binds tightly enough, banks monitor more, and the default rates on their loans decrease. This "incentive effect" makes the economy safer as banks increasingly use their monitoring technology: bank loans are safer and banks themselves default less. This benefit may be undone in the aggregate, however, by a "substitution effect." As non-bank lending rises to meet loan demand no longer serviced by the banking sector, the share of non-bank lending increases. Since non-bank lenders do not resolve the firm side moral hazard problem, aggregate default rises. At high enough capital requirements, aggregate default may even exceed the unregulated level. In order to set capital requirements from a macroprudential perspective, then, regulators must appropriately balance these incentive and substitution effects.

The model features three types of agents: firms, banks, and households. Households are the non-bank lenders in the model.[4] Firms produce output via a risky investment technology and face a moral hazard problem. Effort is costly: firms must choose whether to "work" or "shirk" on their projects. If firms work, their projects succeed for sure; if they shirk, they may fail, but they also obtain a private benefit. The risk of failure associated with shirking is higher in adverse states of the world, defined by the level of the single aggregate shock. Firms finance their investments from their own internal capital and from funds borrowed from either banks or households.

I distinguish between bank and non-bank lenders in two key ways. First, banks tend to *monitor* borrowers more closely than other types of lenders.[5] In the spirit of Diamond (1984) and Holmstrom and Tirole (1997), I assume that banks can monitor firms at a cost, mitigating

[4]This assumption is to keep the model parsimonious. Alternatively, one could allow for a different type of intermediary, and allow the household to allocate its capital between both intermediaries. This assumption makes corporate bond markets the most natural interpretation of non-bank finance in the model.

[5]For analysis of why non-banks (individuals, bondholders, etc.) may not monitor as much, see Park (2000) or the survey by Shleifer and Vishny (1997). A simple explanation appeals to a "tragedy of the commons." Because banks take large stakes in the firms to which they lend, they internalize all the benefits of monitoring them. On the other hand, bondholders take relatively small stakes, and so the cost of monitoring outweighs the (private) benefit.

the moral hazard problem by lowering the opportunity cost of effort. This makes shirking less attractive to firms, thereby decreasing the fraction of firms that shirk and ultimately reducing default risk.[6] Households cannot monitor, and therefore cannot exert any influence over firms' default risk. Second, banks finance their lending with *insured deposits*; non-banks do not. With deposit insurance, banks' funding costs do not adjust in response to the riskiness of bank lending; that is, households do not demand a risk premium commensurate with the default probability of the bank, knowing they will be repaid in full no matter what.

These two features not only distinguish between banks and non-banks, but also motivate capital requirements in the model. First, because banks choose how intensively to monitor borrowers, they can directly impact the riskiness of their loans. Second, the combination of deposit insurance with limited liability convexifies bank payoffs. This delivers incentives for banks to seek risk, which in this case means economizing on lending costs by monitoring less. Capital requirements serve to weaken the resulting risk-shifting incentive, inducing banks to monitor more.[7]

Banks maximize profits by choosing how much to lend, how closely to monitor the firms to whom they lend, and how many deposits to take in from households. Monitoring has a higher payoff ex post in adverse states with high default risk. If banks fail in these states anyway and receive a payoff of zero regardless of how negative their returns are due to limited liability protection, banks may under-invest in monitoring in order to lend more. Capital requirements encourage banks to monitor more because they cap lending, reducing the profitability of the strategy of over-lending and failing in adverse states in order to profit more in high states.

As bank loan supply decreases with tighter capital requirements, though, equilibrium price

[6]Other theories about the differences between bank debt and other types of debt include flexibility to renegotiate in times of financial distress (Bolton and Freixas (2000); Crouzet (2015a,b)) and the ability to form relationships and acquire reputation (Diamond (1991); Rajan (1992)).

[7]This motivation mirrors a major driver of capital requirements in the real world. For example, Sheila Bair, the head of the FDIC, stated in 2007 that "Without proper capital regulation, [...] governments and deposit insurers end up holding the bag, bearing much of the risk and cost of failure."

adjustment makes bank lending less attractive, and firms shift to borrowing from households. Since households cannot monitor, then, the economy may in fact become riskier even though *bank lending* has become safer and *total lending* remains about the same.

After establishing these core results in theory and establishing existence of equilibrium, I calibrate the model to match key aggregates about lending and risk across bank and non-bank sectors in the economy. Using this calibrated model, I map out how key economic aggregates and welfare evolve in equilibrium as the capital requirement is increased. For very low levels of the capital requirement, aggregate risk is high because banks prefer to lend as much as possible, monitor very little, and fail in the bad state of the world in order to increase profits in the good state. Aggregate and bank default rates are 2.1% and 1.5%, respectively. Capital requirements have no effect on risk or lending until they become large enough to resolve this externality.

Once the capital requirement becomes large enough to prohibit this high-lending, low-monitoring policy, default rates on bank loans drop by 22.5% to about 1.2%, because banks begin to monitor more, internalizing the riskiness of the bad state of the world. The aggregate default rate, though, is roughly unchanged as substitution into unmonitored direct finance undoes the improvements in risk in the banking sector in the aggregate. Beyond this point, though, aggregate default strictly increases with the capital requirement: even though bank lending becomes safer and safer, substitution into unmonitored direct lending implies that aggregate default increases through a composition effect.

In addition, I compare my baseline model with lending by both banks and households to an alternative model in which firms can only borrow from banks. I find that the welfare-maximizing capital requirement in the baseline model is 4.5%, while the welfare-maximizing capital requirement with bank lending only is 1.6%. The intuition behind this result is as follows. In both models, increasing capital requirements induces banks to lend less and monitor more. This decreases the default risk and also the total investment of firms who borrow from banks. In the model with banks only, this decrease in bank lending simply disappears from the economy: no other lending institution can pick up the slack. Therefore, increasing the capital requirement imposes a large cost in terms of foregone investment and production, even though it makes bank lending safer. With both bank and non-bank lending, however, the bulk of this decline in investment is offset by the household sector. Therefore, the marginal cost of increasing the regulatory burden on banks is much lower. On the other hand, since the benefits of increasing capital requirements occur only when banks take into account both states of the world, and this occurs at a higher level with both types of financing, the welfare-maximizing capital requirement is higher.

Ultimately, this paper provides a framework with which to analyze the effects of capital requirements in the context of a modern financial system. Though there is a reason to impose capital regulations on banks with or without a non-bank lending sector, I demonstrate that the aggregate impacts of raising capital requirements can differ drastically from the banking sector-specific effects. In particular, "over-regulation" of banks can completely undo the positive effects of reducing risk by incentivizing substitution into other types of finance. This margin may be a critical one for policymakers seeking to protect the economy against downside risk.

1.1.1 Related literature

I contribute most directly to the literature on banking regulation and firm borrowing choices, which can be divided into three main groups: (i) models of bank regulations with banks only; (ii) models with banks and non-banks with no reason for regulation; and (iii) models with banks and non-banks with reason for regulation. I also build on insights from empirical literature which studies spillover effects from changes in bank capital requirements.

The first group (e.g. Van den Heuvel (2008); Corbae and D'Erasmo (2014)) models the impact of capital requirements on the commercial banking sector with banks as the only source of external financing for borrowers. Therefore, while the results of these models help assess the effect of regulations on the banking industry, they fail to assess the economy-wide impact of such regulations. Relative to these papers, I explicitly model firms' choice of lender, thereby allowing substitution by borrowers and a richer set of equilibrium responses to changes in capital requirements.

The second group (e.g. Crouzet (2015a,b); De Fiore and Uhlig (2011, 2015)) explores firms' incentives to issue bank and non-bank debt and the transmission of financial shocks to investment. These models, however, do not analyze the incentives of different types of lenders, and therefore do not allow for lenders to behave in a socially inefficient way. Thus, models in this group cannot be used to directly analyze the aggregate effects of bank regulations on bank behavior, and therefore economic outcomes.[8] I fill this gap in my model by allowing banks to make decisions which impact the level of risk in the economy. Since banks may not internalize all these risks, there is a role for regulation. Reining in risk in the banking sector, though, can in general have consequences in other sectors of the economy: in this way I take macroprudential approach to regulation.

Finally, models in the third group (e.g. Begenau(2016) and Plantin (2015)) actually explore the role of bank capital regulation in environments with alternative, unregulated intermediaries. These models focus almost exclusively on the liability side of intermediaries' balance sheets: that is, they focus on how banks' risk-taking can adversely impact the holders of bank liabilities by limiting valuable liquidity. In this class of models, though, risks are taken to be exogenous and banks choose a level of exposure. My framework allows for a deeper analysis of risk by explicitly modeling how the interactions between lenders and borrowers determine the riskiness of the underlying assets in the economy.

[8] It is worth noting that this same criticism applies to foundational works on the differential roles of banks and other lenders, such as Diamond (1984) and Holmstrom and Tirole (1997).

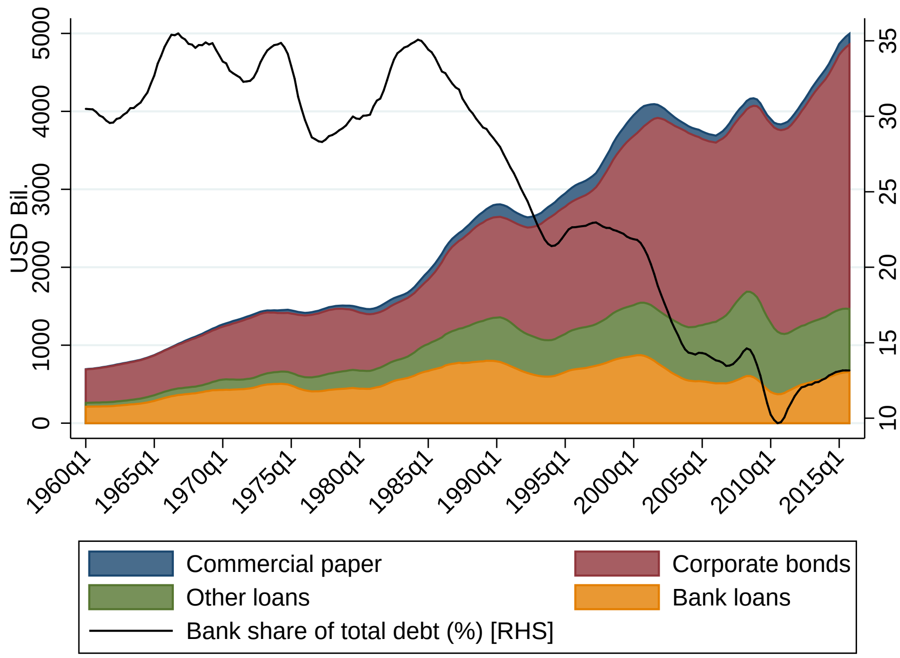
Furthermore, my work complements a growing body of empirical work (Aiyar et al. (2014b,a, 2016); Uluc et al. (2015); Jimenez et al. (2015)) In the presence of this possible substitution into non-bank lending, Aiyar et al. (2014a) identify three conditions necessary to insure that capital requirements regulate aggregate credit in the economy: (i) bank equity must be relatively costly; (ii) capital requirements must "bind" (i.e. they must impact banks' lending choices); and (iii) other sources of credit must not fully offset the change in bank credit. I draw on these insights in my analysis and contribute by adding a fourth point. Specifically, even though other sources do almost completely offset the change in credit quantity, I show that there can still be a change in the associated credit risk.

1.1.2 Roadmap

The rest of this paper is organized as follows. In Section 1.2, I present data on business debt composition, its affect on investment, and the different risks in bank and non-bank finance. Then, in Section 1.3, I present the environment of the baseline model. I analyze the equilibrium of the model in Section 1.4. I calibrate the model in Section 1.5. Then, using the calibrated model, in Section 1.6, I explore the effects of changing capital requirements with and without direct lending by households. Section 1.7 concludes. All proofs can be found in Appendix A.1, and descriptions of data, definitions of model moments, and computational details can be found in Appendix A.2.

1.2 Empirical Motivation: Debt, Investment, and Risk in the Bank and Non-Bank Sectors

Regulators tasked with setting capital requirements must balance a reduction of risk in the banking sector with decreased bank lending to and and investment by firms. This paper studies how substitution by firms between borrowing from banks and obtaining credit from



Notes: This figure is constructed using quarterly data from the Flow of Funds of the Financial Accounts of the United States, 1960Q1 through 2015Q4. Each shaded region represents the total volume of the specified type of debt for non-financial corporate firms. Bank loans are defined as loans from traditional depository institutions, while other loans are comprised of loans from non-bank financial firms.

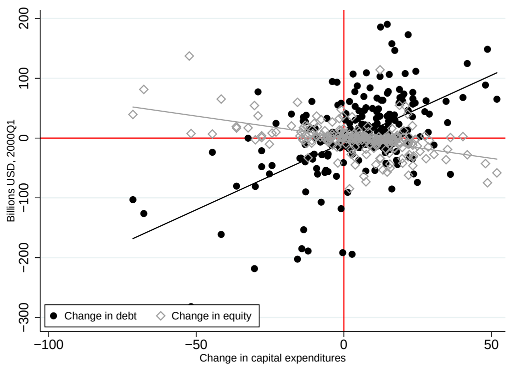
Figure 1.1: Debt growth for non-financial business in the United States

other sources affects this key tradeoff. To inform this analysis, in this section I present three stylized facts on (i) the composition of business debt; (ii) the relationship between debt, debt types, and investment; and (iii) the risk profiles of bank and non-bank debt. These facts inform the key themes in the model studied in the remainder of the paper.

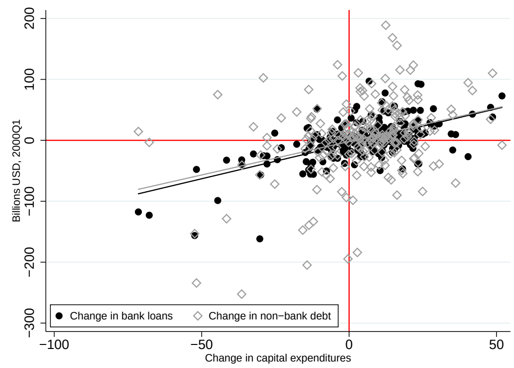
Fact 1.1 *Bank loans comprise less than one third of total business debt.*

Figure 1.1 plots composition of business debt in the United States over the last half century. I focus on corporate firms (as opposed to noncorporate firms) for consistency with the model I will present in Section 1.3.[9] Specifically, I consider only firms who have access to both bank financing and public debt markets, since firms in my model all have access to both. In

[9]The grouping between corporate and non-corporate firms comes from the Flow of Funds. In general, corporate firms are larger and have credit ratings; non-corporates are smaller and unrated.



(a) Capital structure



(b) Debt structure

Notes: All panels are constructed using quarterly data from the Flow of Funds of the Financial Accounts of the United States, 1960Q1 through 2015Q4. In each panel, the vertical axis measures the relevant sector's change in the given financing type (debt / equity / bank debt / non-bank debt).

Figure 1.2: Corporate firms' capital structure, debt structure, and investment

addition, for the purposes of this paper, I define bank debt as "Bank loans" in Figure 1.1 and non-bank debt as the sum of "Other loans," "Corporate bonds," and "Commercial paper."[10] Two features of the data are immediately clear. First, business debt of all types has boomed. Second, non-bank debt has experienced more pronounced growth than bank debt, suggesting that inclusion of this side of the market is critical in constructing a realistic description of the economy's aggregate credit supply. Corporate bonds, by far the most important non-bank source of debt financing for firms, represent more than half of total business debt; other loans (from financial institutions besides traditional deposit-taking banks) make up just under a quarter of the total.

Fact 1.2 *Capital expenditures move in line with both bank and non-bank debt financing.*

[10]Note that commercial paper could be excluded from this definition, since firms often issue it solely for liquidity purposes, which I do not consider in this paper. Since the volume of commercial paper issuance is small relative to the other three categories in Figure 1.1, though, this exclusion has little impact.

Figure 1.1 demonstrates that firms often turn to non-banks for debt financing. How, though, does the composition of external financing impact real outcomes? I explore this question in Figure 1.2 focusing on investment. Figure 1.2a plots firms' total change in debt (black) and change in equity (gray) versus total investment as measured by capital expenditures. Figure 1.2b performs an analogous breakdown, isolating debt types – bank and non-bank debt. Panel 1.2a immediately demonstrates the relative importance of debt for driving investment in the aggregate. Capital expenditures increase almost one-for-one with debt, while equity growth is essentially neutral. Panel 1.2b pushes this analysis further into types of debt, dividing firms' liabilities into bank and non-bank debt. Investment increases with changes in *both* bank loans and corporate bonds, but less so for each type. This pattern indicates substitutability between the two types of debt, a critical margin I will explore throughout the remainder of the paper.

Fact 1.3 *Non-bank debt has higher default risk than bank loans.*

To this point, I have focused on the composition of debt and its impact on total investment. Next I link these effects to overall risk in the economy, precisely the types of risks which capital regulations for banks are designed to rein in. Giesecke et al. (2015) show that default rates on corporate bonds tend to be much higher than on bank loans (1.5% vs. 0.5% in the time series average), particularly in economic downturns (late 1980s and early 2000s in the sample period). In analyzing the aggregate impact of substituting corporate bonds (direct financing) for bank loans (bank financing), this is a potentially critical effect: if a regulation designed to make bank lending safer has the additional effect of pushing lending into a sector that is intrinsically less safe, the regulation may in fact be less effective.[11]

[11] Does this "push" actually occur in the real world? A growing body of empirical work documents the effects of increasing capital requirements on certain banks on lending by unaffected banks and other types of lenders. Using a policy experiment on micro-prudential bank regulations in the United Kingdom, Aiyar et al. (2014b,a, 2016) find that more than one third of the decline in lending associated with tightening capital requirements on certain banks is offset by unregulated foreign banks alone. Conducting a similar study for Spain, Jiménez et al., (forthcoming) find similar results. Furthermore, these authors document an interesting asymmetry: in good times, firms readily substitute away from constrained banks into other types of financing, while in bad times

In total, the facts documented in this section suggest that non-bank debt comprises a large part of total debt and drives a significant amount of investment. At the same time, though, non-bank debt tends to be riskier, suggesting a tradeoff between the amount of credit and the risks associated with it. The remainder of this paper studies how capital requirements interact with these debt dynamics.

1.3 Model Environment

In this section, I describe the basic features of the model. I leave a detailed discussion of each assumption until Section 1.4.5, once I have presented the full model. Since my focus is on the long run effects of capital regulations, I consider a static model.

1.3.1 Preliminaries

There is one period and a single good used for both consumption and investment: I call this good capital throughout. All consumption occurs at the end of the period. I assume there is a storage technology which can be used to preserve capital from the beginning of the period to the end with no net return.[12] All agents behave competitively, and prices adjust to clear the relevant markets in equilibrium. There are two aggregate states: an up state in which default risk is low, and a down state in which default risk is high (see below for specifics).

1.3.2 Firms

A continuum of identical, risk-neutral firms have initial capital k_F and a risky production technology. This technology turns an investment of I units (comprised of internal capital and

this substitution is limited. Though these European-based studies do not find varying degrees of evidence of substitution into non-bank finance, for the United States Adrian et al. (2012) find significant substitution into non-bank finance.

[12]The purpose of this storage technology is to bound the deposit rate in a realistic way despite household risk aversion. Since banks and firms are risk neutral, I do not explicitly discuss the storage technology in their decisions, since it will generically go unused.

any borrowing, b) into $f(I)$ units of the good in the case of success and 0 in the case of failure. I assume that the function $f(\cdot)$ is strictly increasing and strictly concave. Firms can borrow additional capital from either banks or households (described below), or simply invest only their own internal capital.

After contracting and making their initial investments, firms realize a random variable, $x \sim G(x)$, i.i.d. across firms, which denotes the private benefit they can obtain by shirking on their respective projects.[13] Taking x and the expected returns of the project into account, firms must then choose whether to exert high effort ("work") or low effort ("shirk"). Firms who work succeed with certainty, producing output $f(I)$ regardless of the aggregate state. Firms who shirk succeed with probability p, where

$$p = \begin{cases} p_h & \text{with probability } \psi \\ p_l & \text{with probability } 1-\psi \end{cases}$$

The realization of p – the only aggregate shock in the model – occurs after the firm chooses whether to work or shirk. I assume that $1 \geq p_h \geq p_l \geq 0$. For convenience, I define $\overline{p} = \psi p_h + (1-\psi)p_l$ to be the expected probability of success when shirking *before* p is realized. Firms receive the private benefit x from shirking, but shirking costs m: the level of m depends on the extent to which the firm is monitored, described below. Firms cannot commit to working or shirking, and the private benefit is private information and cannot be shared with other agents.

Firms behave competitively, taking all prices as given. If a firm chooses to demand a loan, it must then determine the size of the loan, $b \geq 0$, to demand from its lender. Finally, when choosing between the two lenders and self-financing the project, each firm receives an additive, idiosyncratic preference shock $\epsilon = \{\epsilon_H, \epsilon_B, \epsilon_O\}$, which shifts the value from household (H),

[13] All that is needed is for a law of large numbers to hold for both x and ϵ. The model works if firms are heterogeneous ex ante with respect to x, as long as there is residual uncertainty over the realization of x after contracting.

bank (B), and self (O, for outside option) financing. I assume that this shock is drawn from a type-one extreme value distribution with scale parameter α.

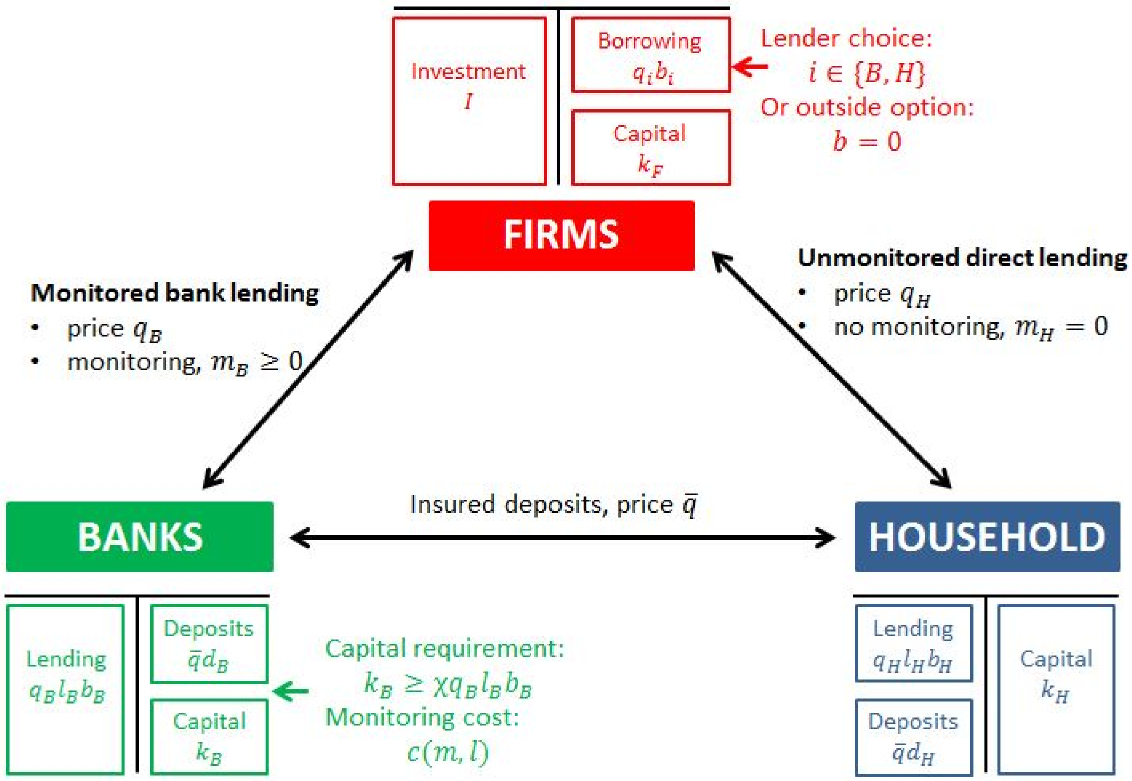
1.3.3 Banks

A continuum of identical, risk-neutral banks have initial capital k_B and a monitoring technology which increases firms' cost of shirking. Specifically, at a cost $c(m, \ell)$, banks can lend ℓ to firms and set these firms' cost of shirking to m. I assume this cost function is increasing and convex in all its arguments, and I assume that the cross partial, $c_{m\ell}$, is strictly positive: that is, increasing monitoring costs more when lending more. Banks choose how much to monitor, m_B, how much to lend, ℓ_B, and initial borrowing (deposits demanded) from households, d_B. Banks behave competitively, taking the risk-free price $\overline{q}$ and the bank loan price q_B as given.

In addition, I assume that banks are subject to a capital requirement. Specifically, their capital must be at least a fraction χ of their total lending. Banks can borrow at the risk-free discount price $\overline{q}$, protected by deposit insurance. Even if the bank cannot repay depositors in a given state of the world, this insurance is implemented via a lump-sum, state contingent tax $T(p)$ on households. Lastly, I assume that banks have limited liability: their payoff may not be less than 0 in any state p, creating the potential for default by banks.

1.3.4 Household

The representative household has initial capital k_H and is risk averse, with a strictly increasing and strictly concave utility function $U(C)$ over consumption $C \geq 0$. The household can invest in storage, a_H, bank deposits, d_H, and direct loans to firms, ℓ_H. Critically, unlike banks households cannot raise the cost of shirking to firms: that is, for firms borrowing from the household, m_H is always equal to 0. The household takes the risk-free price $\overline{q}$ and the direct loan price, q_H, as given.



Notes: This figure depicts the balance sheets of all three types of agents in the benchmark model described in Section 1.3, as well as the lending channels between them and the relevant constraints they face.

Figure 1.3: Summary of model structure

1.3.5 Timing

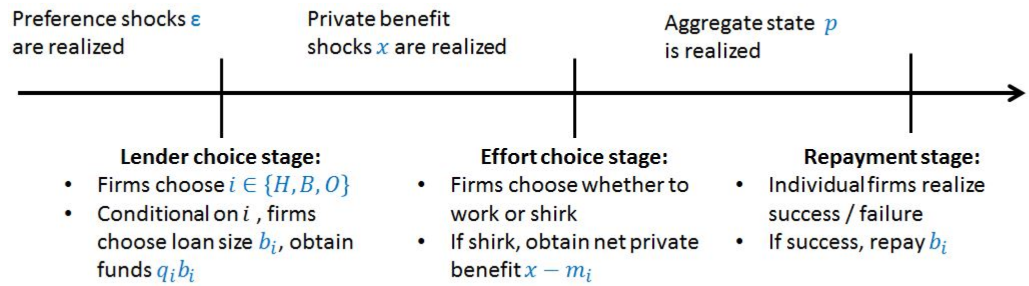
The timing of the model is as follows:

1. Taking prices $q = (\bar{q}, q_B, q_H)$ as given,

   (a) After receiving their preference shocks $\{\epsilon_i\}$, firms choose from borrowing from bank, borrowing from HH, or self-financing. If a firm borrows from the bank (household) it also chooses the loan size, b_B (b_H).

   (b) Taking b_B as given, banks choose (d_B, ℓ_B, m_B).

   (c) Taking b_H as given, HH chooses (a_H, d_H, ℓ_H).



Notes: This figure displays the timeline of the firm problem described in Section 1.4.1 and all its phases from the beginning of the period until the end.

Figure 1.4: Timeline of the firm problem

2. Firms realize the private shirking benefit x and choose to work or shirk, factoring in the lender-dependent shirking cost m_i.

3. The aggregate state p is realized.

4. Firms' success / failure is realized, repayments are made, and consumption occurs.

1.4 Equilibrium

1.4.1 Firm problem

Firms take the lending terms (q_i, m_i) for $i \in \{H, B\}$ as given. I analyze the firm problem backwards, beginning with the value of working or shirking conditional on financing source and amount of borrowing ("effort choice stage"), then analyzing the choice of financing source ("lender choice stage"). A detailed description of the timeline of the firm problem and all its relevant stages can be found in Figure 1.4.

Repayment stage

A firm which borrowed b from lender i, invested I, and was successful receives a payoff of $f(I) - b$, the full proceeds of the project after repaying their lender.[14] Firms who fail are unable to pay back their lenders, and receive a payoff of 0. Firms who have chosen to self-finance receive all project proceeds if successful.[15]

Effort choice stage

Conditional on lender i, investment I, and borrowing b, firms must decide to work or shirk having learned their private shirking benefit x but not yet the aggregate shock p. Since working firms succeed with probability one in all states p, the return from working is simply the net proceeds after repaying the lender:

$$\tilde{V}_i^W(I, b) = f(I) - b \tag{1.1}$$

The return from shirking incorporates the net private benefit, $x - m_i$, plus the net investment proceeds adjusted for the reduced probability of success:

$$\tilde{V}_i^S(I, b, x) = \overline{p}[f(I) - b] + x - m_i. \tag{1.2}$$

Note that the first term in the expression on the right hand side of (1.2) must be weighted by the *expected* success probability associated with shirking, since this decision is made before the realization of p.[16]

Finally, the firm chooses to work if and only if $\tilde{V}_i^W(I, b) \geq \tilde{V}_i^S(I, b, x)$, or

$$x \leq \overline{x}(m_i) \equiv \underbrace{(1 - \overline{p})(f(I) - b)}_{\text{lost proceeds}} + \underbrace{m_i}_{\text{shirking cost}} . \tag{1.3}$$

[14] Note that given the risk neutrality of firms, and the presence of the storage technology which caps all prices at 1, it is trivial to show that borrowing will only occur after firms have invested all of their own capital into the productive technology.

[15] This assumption assumes that there is no recovery in the case of failure and default. This ingredient is qualitatively unimportant, but may play a role quantitatively.

[16] That is, $\tilde{V}_i^S(I, b, x) = \psi \left[p_h(f(I) - b) + x - m_i\right] + (1 - \psi) \left[p_l(f(I) - b) + x - m_i\right]$, yielding (1.2).

The ex ante probability of a firm working is then $G(\overline{x}(m_i))$. The first term on the right-hand side of (1.3) represents the expected lost proceeds from shirking, while the second term is the cost of shirking imposed by lender i. Higher values of both increase $\overline{x}(m_i)$, thereby decreasing the likelihood that the firm shirks. Banks can impact this likelihood through monitoring (increasing $m_B > 0$), while households cannot (restricted to $m_H = 0$).

Lender choice stage

Before the draw of x firms must choose (i) how much to borrow from each type of lender, b_i, and (ii) which lender i to choose. I present each of these decision problems in succession. Since the firm cannot commit to working or shirking, firms' expected value from borrowing b_i from lender i and investing an amount I_i weights the values from working and shirking by the relative likelihood that the firm will take each of these actions:[17]

$$\begin{aligned}\tilde{V}_i(I,b) &= \mathbb{E}_x[\max\{\tilde{V}_i^W(I,b), \tilde{V}_i^S(I,b,x)\}] \\ &= \tilde{V}_i^W(I,b)G(\overline{x}(m_i)) + \int_{\overline{x}(m_i)}^{\infty} \tilde{V}_i^S(I,b,x)dG(x) \end{aligned} \tag{1.4}$$

This can be decomposed into the following useful form:

$$\tilde{V}_i(I,b) = \underbrace{P(m_i,\overline{p})[f(I)-b]}_{\text{expected returns from project with lender } i} + \underbrace{\int_{\overline{x}(m_i)}^{\infty}(x-m_i)dG(x),}_{\text{expected net benefit of shirking with lender } i} \tag{1.5}$$

where

$$P(m_i,p) = \underbrace{G(\overline{x}(m_i))}_{\text{mass of successful working firms}} + \underbrace{p(1-G(\overline{x}(m_i)))}_{\text{mass of successful shirking firms}}, \tag{1.6}$$

the ex post aggregate success rate of all firms who borrow from source i when the realized aggregate state is p. Note that $\mathbb{E}_p[P(m_i,p)] = P(m_i,\overline{p})$, yielding the expression in (1.5).

[17] In equation (1.4), I use the notation $\mathbb{E}_x(\cdot)$ for the expectation operator to denote that the expectation is taken over realizations of the private shirking benefit x. Implicitly, the expectation is also over the aggregate state p, but this is captured in expressions (1.2) and (1.3). Throughout, a subscript on the $\mathbb{E}(\cdot)$ operator denotes the variable with respect to which the expectation is taken.

Furthermore, note that although I only explicitly write m_i and p as the arguments to $P(\cdot)$, this function also depends on prices, capital, and borrowing and investment choices.

In order to choose the amount to borrow, then, the firm solves

$$V_i(k) = \max_{I,b\geq 0} \tilde{V}_i(I,b) \text{ subject to } I \leq q_i b + k \tag{1.7}$$

Lemma 1.1 presents the solution to problem (1.7).

Lemma 1.1 ***Optimal loan size.*** *Firms' optimal loan size from lender i, b_i^*, satisfies*

$$f'(k + q_i b_i^*) = 1/q_i. \tag{1.8}$$

Despite the fact that borrowing affects the relative likelihoods of working and shirking, and therefore the likelihood of receiving the private benefit $x - m_i$, then, the problem (1.7) has the standard optimality condition (1.8).[18]

At the beginning of the period, firms choose between the two sources of external financing and self-financing according to:

$$v_F(k) = \max_{i\in\{H,B,O\}} V_i(k) + \epsilon_i, \tag{1.9}$$

where the value of self-financing is simply $V_O(k) = \tilde{V}_O(k,0)$, i.e. $I_O = k$.[19] Under the assumption that ϵ_i are i.i.d. type one extreme value with scale parameter $1/\alpha$, appealing to standard results in the discrete choice econometrics literature (see, for example, McFadden (1973); Rust (1987)), the share of firms choosing each alternative i is:

$$L_i(k) = \frac{\exp\{\alpha V_i(k)\}}{\exp\{\alpha V_H(k)\} + \exp\{\alpha V_B(k)\} + \exp\{\alpha V_O(k)\}}. \tag{1.10}$$

Total loan demand from source $i \in \{H,B\}$, then, is simply $L_i b_i$.[20]

[18] This is a convenient offshoot of the fact that the private benefits are additive and do not scale with the size of investment. In addition, this allows the borrowing decision to be separate from the level of monitoring chosen by lenders, easing the solution of the model.

[19] More specifically, the threshold for working or shirking under self-financing is $\overline{x}_O = (1-\overline{p})f(k)$, and so $V_O(k) = [G((1-\overline{p})f(k)) + \overline{p}(1 - G((1-\overline{p})f(k)))]f(k) + \int_{(1-\overline{p})f(k)}^{\infty} x dG(x)$.

[20] Furthermore, the value function in (1.9) has the convenient closed form

$$v_F(k) = \frac{\gamma_E}{\alpha} + \frac{1}{\alpha}\ln\left(\exp\{\alpha V_H(k)\} + \exp\{\alpha V_B(k)\} + \exp\{\alpha V_O(k)\}\right),$$

The lender-specific preference shocks have three properties worth noting. First, it is always the case that $\sum_i L_i(k) = 1$ and $L_i(k) > 0$ for all i. Second, the share of firms choosing financing type i is increasing in the value that choice i delivers to the firm: that is, $\partial L_i / \partial V_i > 0$. This result has the obvious corollary that firms' modal financing choice delivers the highest value: that is, $\arg\max_i L_i(k) = \arg\max V_i(k)$. Third, lowering the variance of the preference shocks (i.e. increasing α) implies that the highest value actions are taken by a higher share of firms, while lower value actions are taken by a smaller share of firms.[21]

1.4.2 Bank problem

Banks choose lending, ℓ_B, monitoring, m_B, and deposits to demand d_B to maximize their expected profits:

$$\begin{aligned} v_B(k) \quad &= \quad \max_{\ell_B, y_B, d_B \geq 0} \mathbb{E}_p \left[\max \left\{ P(m_B, p)\ell_B b_B - d_B, 0 \right\} \right] \\ \text{subject to:} \quad & \quad c(m_B, q_B \ell_B b_B) \leq k + \overline{q} d_B \\ & \quad \chi q_B \ell_B b_B \leq k \end{aligned} \tag{1.11}$$

Note that the objective function reflects banks' limited liability with the interior max function inside the expectation operator. The first term inside this max operator is the total proceeds from lending, net of deposit repayments. Recall that banks take the loan size, b_B (determined by (1.8)), as given and choose the mass of firms to which they will lend, ℓ_B.

The first constraint in problem (1.11) – the budget constraint – states that total internal (k) and external ($\overline{q} d_B$) bank funding must cover the total cost of lending, which includes both the cash outlay for investment ($q_B \ell_B b_B$), and the expenditure on monitoring (m_B). The

where $\gamma_E = 0.577216...$ is a constant.

[21] To see this, observe that

$$\frac{\partial L_i}{\partial \alpha} = \frac{\exp\{\alpha V_i\} \sum_j \exp\{\alpha V_j\}(V_i - V_j)}{\left(\sum_j \exp\{\alpha V_j\}\right)^2}$$

which is positive for the highest V_i choice, negative for the lowest V_i choice, and in general indeterminate for choices which deliver values between these extremes due to the weighting in the sum function in the numerator.

second constraint is a simple form of a capital requirement: it states that the total amount lent, $q_B \ell_B b_B$ cannot exceed total bank capital by more than a factor of $1/\chi$, for $\chi \geq 0$.[22] Lastly, note that to implement deposit insurance in the case when the bank cannot repay its depositors completely (i.e. the case when limited liability binds), the government must levy a tax

$$T(p) = \max\{d_B - P_B(m_B, p)\ell_B b_B, 0\}, \tag{1.12}$$

which is exactly the shortfall of loan repayments relative to the required repayments on deposits.

Bank policies: limited liability and the motivation for capital requirements

In this section, I discuss the two key features of the bank problem – limited liability and capital requirements – in a "partial equilibrium" setting, taking prices as given and considering banks' optimal policies. Under deposit insurance, this partial equilibrium approach yields valid insight into the properties of the full general equilibrium because not all prices (specifically $\overline{q}$) adjust in response to bank decisions.

The first order conditions of the bank problem are:[23]

$$P(m_B, p^*) = \frac{q_B}{\overline{q}} c_\ell(m_B, q_B \ell_B b_B) + \mu \chi q_B b_B \tag{1.13}$$

$$P_m(m_B, p^*) = \frac{1}{\overline{q}} c_m(m_B, q_B \ell_B b_B), \tag{1.14}$$

where F_i is the partial derivative of the function F with respect to its ith argument, p^* is the expectation of the aggregate state over states which the bank takes into account only, and $\mu \geq 0$ is the multiplier on the capital requirement. That is, if limited liability binds in the

[22] If $\chi = 0$, then since $k > 0$ this constraint is satisfied for all feasible lending choices, and there is no restriction.

[23] The max operator implies a potential point of non-differentiability in the objective function. I deal with this issue in detail in the Appendix, but here provide the first order conditions "away from the kink" to build intuition.

down state, then the bank does not account for the impact of its choices in this state, and $p^* = p_h$. Otherwise, the bank cares about both the up and the down states, and $p^* = \overline{p}$.

The loan optimality condition (1.13) equates the marginal benefit of lending (the expected success rate on a project across the states in which the bank receives a positive payoff) with the marginal cost of lending, plus an additional term that is positive only when the capital requirement is binding. The monitoring optimality condition (1.14) also equates margins: the marginal increase in success probability (and therefore payoff) on the left hand side with the marginal cost of monitoring on the right.

Analysis of conditions (1.13) and (1.14) delivers insight into how limited liability impacts bank incentives. Regardless of whether limited liability binds in the down state, the right-hand side of (1.13) is unchanged. The left-hand side, though, changes: it is equal to $P(m_B, \overline{p})$ ($P(m_B, p_h)$) if limited liability is slack (binds). Examining (1.6) immediately reveals that $P_p(m, p) > 0$. Therefore, the left-hand side of (1.13) is greater if limited liability binds in the down state. This implies that the marginal cost of lending must be higher, or the capital requirement must bind more tightly. In either case, this implies that the bank lends more for a given set of prices when it does not internalize the down state.

Next consider the monitoring condition (1.14). Since $P_m(m, p) = (1 - p)g(\overline{x}(m))$, then $P_{mp}(m, p) = -g(\overline{x}(m)) \leq 0$ and the left-hand side of (1.14) is decreasing in p. Intuitively, in better states of the world, monitoring is less valuable ex post since the default risk of shirking firms is lower. Since the right-hand side is invariant to p, then, when limited liability binds, the marginal cost of monitoring must be lower: that is, monitoring must be lower. Together these results imply that when limited liability binds, banks will tend to eschew monitoring in favor of increased lending.

The above discussion describes the motivation for capital requirements. The following lemma summarizes their impact.

Lemma 1.2 ***Bank policies and capital requirements.*** *Bank lending, ℓ_B^*, is weakly decreasing in the capital requirement χ. Bank monitoring, m_B^*, is weakly increasing in χ.*

This result underlies a key mechanism in the model. If the capital requirement is slack, then the multiplier μ in (1.13) is equal to 0, and the marginal return is equal to the marginal cost of lending. If the capital requirement binds, however, then $\mu > 0$, and the marginal return must *exceed* the marginal cost of lending. Since reducing m_B lowers both the right and left sides of (1.13), and reducing ℓ_B only lowers the right, it is in general optimal to decrease lending in response to an increase in the capital requirement. As lending is reduced, though, the right hand side of (1.14) falls since $c_{m\ell} > 0$. Then, in order for this condition to hold, monitoring must increase. These effects in turn deliver the result from Lemma 1.2.

This partial equilibrium result sets the stage for the core insights of the full general equilibrium model. An increase in bank capital requirements induces banks to lend less and monitor more. This makes bank lending safer, since fewer firms who borrow from banks end up shirking, and therefore fewer firms fail in every possible state. Total bank lending, however, declines as bank loan interest rates rise, creating an incentive for firms to increasingly seek loans from households who do not monitor.

1.4.3 Household problem

The household solves a simple, standard portfolio problem, maximizing expected utility by choosing storage, a_H, deposits, d_H, and direct loans to firms, ℓ_H, subject to beginning and end of period budget constraints:

$$\begin{aligned} v_H(k) \quad &= \quad \max_{a_H, d_H, \ell_H \geq 0} \mathbb{E}_p[U(C)] \\ \text{subject to:} \quad & C \leq a_H + d_H + P(0,p)\ell_H b_H - T(p) \text{ for all } p \qquad (1.15) \\ & a_H + \overline{q} d_H + q_H \ell_H b_H \leq k \end{aligned}$$

The first constraint reflects that end of period consumption comes from the sum of return on deposits and the realized returns of direct loans of all types, which depend on the aggregate state. The second constraint states that the household's total investment outlay cannot exceed its initial capital.

Since deposit insurance implies that deposits (like storage) are repaid in full in all states of the world, as long as $\overline{q} < 1$ storage is strictly dominated and $a_H^* = 0$. If $\overline{q} = 1$, then the household is indifferent between storage and deposits. Therefore, throughout the remainder of the analysis, I focus solely on the tradeoff between deposits and direct loans to firms.

At an interior solution (with both d_H and $\ell_H > 0$), then, the household's portfolio satisfies

$$\frac{q_H}{\overline{q}} = \frac{\mathbb{E}_p \left[U'(C)P(0,p)\right]}{\left[U'(C)\right]}. \tag{1.16}$$

Since $P(0,p) \leq 1$, equation (1.16) implies that $q_H \leq \overline{q}$. The severity of the gap between the direct loan price and the deposit price measures the riskiness of direct lending given the risk averse preferences of households.

1.4.4 Equilibrium definition and existence

Definition 1.1 ***Baseline model equilibrium.*** *An equilibrium in this model is a list of: firm lender share and loan amount choices, L_i^* and b_i^* for $i \in \{H, B, O\}$; bank choices of deposits, d_B^*, monitoring, m_B^* , and loan supply, ℓ_B^*; household choices of storage, a_H^*, deposits, d_H^*, and direct loans to firms, ℓ_H^*; bank and direct lending prices, q_B^* and q_H^*; a deposit price $\overline{q}^*$; and a lump-sum tax $T^*(p)$ for all p such that: firms' lender choices solve (1.9), firms' loan sizes solve (1.7), and direct and bank values are given by (1.4); bank choices solve (1.11), taking b_B^* as given; household choices solve (1.15), taking b_H^* and $T^*(p)$ as given; direct and bank lending* *and bank deposit markets clear:*

$$L_B^* = \ell_B^* \tag{1.17}$$

$$L_H^* = \ell_H^* \tag{1.18}$$

$$d_B^* = d_H^*, \tag{1.19}$$

where L_i^* is given by (1.10); and the tax $T^*(p)$ is given by (1.12).

The equilibrium definition 1.1 is mostly standard, but the market clearing conditions warrant comment. Because firms choose b_i^* on their own, and lenders take this as given, conditions (1.17) and (1.18) are only in terms of masses of firms, L_i^* and ℓ_i^*.

Proposition 1.1 *Under the assumptions stated in Section 1.3, an equilibrium exists.*

The intuition behind this existence result is as follows. Both firm and household policies are smooth and monotone in prices. On the firm side, the loan amount b_i varies smoothly with q_i. It is straightforward to show that $\partial b_i / \partial q_i > 0$, and therefore $V_i(k)$ is also increasing in q_i. Then, from (1.10), it follows that firm demand is continuously (weakly) increasing in q_i. The household policies for investment in direct lending (since $\overline{q} \leq 1$, and at an interior solution where $\ell_H > 0$) obey (1.16), which implies continuously decreasing direct loan supply as a function of q_H. Similarly, decreasing $\overline{q}$ makes deposits more attractive and direct lending less attractive, also in a smooth and monotone fashion.

The only potential discontinuity in the model arises from bank optimal policies. Because of limited liability, there are two sets of "potentially optimal" policies: one in which limited liability binds in the down state and one in which it does not.[24] Since monitoring is more valuable ex post in the down state, banks will monitor more if they factor in this state. If they do not, they will choose to lend more and monitor less in order to economize on the cost of

[24]Indeed, in analyzing the properties of the bank optimal policies I consider two subproblems based on variants of limited liability in Lemmas A.1, A.2, and A.3 in Appendix A.1.

lending and seek higher net returns in the high p state. For certain prices, the first set delivers higher value; for others, the second set do. By continuity, then, there are price combinations in which banks are indifferent between the two sets of policies. To avoid the problems of existence which can arise if bank policies jump around this indifference point, I simply allow banks to randomize between the sets of policies.

1.4.5 Discussion of assumptions

Costly monitoring

This is the key friction in the model. The notion of costly monitoring as a key function of banks dates back to Diamond (1984), and my formulation echoes key features of Holmstrom and Tirole (1997). If monitoring were costless, then all lending could seamlessly flow through banks. Banks would monitor all firms enough to fully eliminate default risk, nullifying any opportunity for bank risk taking and obviating the capital requirement. Since monitoring is costly, though, banks must optimally choose how much to monitor borrowers, trading off the benefit of deterring shirking with the increased upfront cost of lending. Monitoring proves more valuable ex post in more adverse (low p) states. Thus, banks who don't internalize these adverse states – because limited liability binds and deposit insurance prevents banks' borrowing costs from responding to increased risk – will in general have incentive to engage in too little monitoring.

I assume a very stark difference in monitoring cost between households and banks. By assuming that households cannot monitor, I essentially assume that their monitoring cost is infinite. While this assumption simplifies the analysis, it is not essential, and the same intuition I have laid out applies in a model where banks have a more modest cost advantage in monitoring borrowers.[25]

[25] For example, given the form of the bank lending cost function I use in the quantitative analysis in equation (1.21), I could assume that the household may lend to firms with this same cost function and $\rho_1^{\text{HH}} = \rho_1^{\text{bank}}$ and

Deposit insurance and limited liability

I assume that bank deposits are insured in order to guarantee that banks do not internalize the effects of their increased risk-taking through equilibrium price adjustment of $\overline{q}$. Deposit prices reflect only the relative demand for riskless assets by households and for bank loans by firms. Together with limited liability, which convexifies the bank payoff function, this assumption easily delivers risk-shifting by banks: banks gamble on payoffs in the good state at the cost of increased exposure to severely negative payoffs in the down state. This creates a role for capital regulation, taking deposit insurance as given.[26]

Preference shocks

There are two firm preference shocks in the model: shocks to firms' private benefits of shirking, and shocks to firms' value of choosing a specific lender.

Private benefit of shirking. I model the benefits and costs of shirking as simple additive shifters to the firms' payoff. This specification draws in spirit from a long line of literature in theoretical corporate finance (beginning with Holmstrom and Tirole (1997)), with one key modification: *shirking actually still occurs in equilibrium.*[27] This is critical because it allows monitoring choices by the bank to influence the actual underlying risk of investment, rather than simply the quantity. It is this model feature which creates a motive for banks to take risk, and therefore allows the model to be meaningfully used to analyze regulations designed to rein in bank risk taking.

Lender-specific firm preference shocks. The presence of the ϵ shocks in firms' choice

$\rho_2^{\text{HH}} > \rho_2^{\text{bank}}$.

[26] From a modeling standpoint, the benefits of deposit insurance can be micro-founded by incorporating liquidity demand by the household and creating the possibility of runs on banks. I leave these dynamics out of the current model and instead take the presence of deposit insurance as a model primitive.

[27] In Holmstrom and Tirole (1997), the private benefit affects the right-hand side of an incentive compatibility constraint, which must be satisfied in any optimal contract. Therefore, it is never violated in the equilibrium of their model.

of financing source serves two purposes. First, they smooth out demand for each type of lending. Without these shocks, loan demand for source i would be either 0 or a strictly positive number. For direct lending, however, this sharp discontinuity in demand creates issues of non-existence of equilibrium given the continuity in the direct lending supply function dictated by the household optimality conditions under a smooth, concave utility function. Second, these shocks allow me to map the model to the data more easily. Looking across the cross-section of firms, no firms completely specialize in direct debt, even though certain classes of firms do specialize completely in bank debt.[28]

1.5 Estimation and Model Fit

To consider counterfactual changes to capital requirements, the mapping of the model to the data must be consistent with key empirical facts about (i) firm debt composition, risk, and profitability; (ii) bank capital structure and loan risk; and (iii) household supply of risky loans and th riskiness of these loans. In this section, I present this mapping to the data. I first describe the target moments for the calibration. Then, I present the parameters which deliver these moments. Finally, I discuss the fit of the model. Detailed definitions of moments and descriptions of data sources can be found in Appendix A.2.

1.5.1 Moments

I divide the targeted moments for calibration into three categories corresponding to the three main types of agents in the model: firms, banks, and households.

On the firm side of the model, the main target is the bank share of total debt, defined as the fraction of total borrowing by firms which is supplied by banks. The bank share is critical since it represents the portion of total lending whose riskiness is mitigated by monitoring. As

[28] For a detailed analysis of this feature of the data, see Rauh and Sufi (2010) and Colla et al. (2013).

	Moment	Source	Data	Model
Firm	Bank share of total business debt (%)	QFR	23.7	29.3
	Firm leverage	QFR	0.26	0.30
	Firm return on capital (%)	QFR	39.6	33.6
	Standard deviation of output (%)	NIPA	1.62	1.05
Bank	Default rate on bank loans (%)	Call reports	0.50	1.54
	Monitoring to total expense (%)	Call reports	1.02	1.13
	Bank leverage	Call reports	0.92	0.95
	Bank loan return (%)	Call reports	3.06	3.56
	Net interest margin (%)	Call reports	2.98	3.49
HH	Default rate on corporate bonds (%)	Giesecke et al. (2015)	1.52	2.29
	Household riskless / total investment (%)	Flow of funds	20.6	30.2
	Household assets / bank assets	Flow of funds	3.77	3.21

Notes: Descriptions of data used in computation of moments can be found in Appendix A.2. Variable definitions in the data and in the model can be found in Tables A.1 and A.2,respectively. The model moments are computed for the values of the parameters reported in Table 1.2.

Table 1.1: Model moments targeted for estimation of baseline model

this fraction changes with the capital requirement, so will the economy's aggregate risk profile. Since all firms in my model are able to borrow both directly and from banks, I must insure that the firms I look to match in the data also have this dual access. Therefore, I follow Crouzet (2015a) and focus only on the largest firms, those with assets of at least $250 million in the Quarterly Financial Report of Manufacturing, Mining and Trade firms (QFR). In order to be broadly consistent with firms' capital structure and to discipline prices, I target firms' average leverage ratio and their overall return on capital. Finally, in order to make sure that my model has overall risk commensurate with the overall US economy, I also target the volatility of output, using aggregated GDP from the National Income and Product Accounts.

All data for the targeted bank moments are taken from the FDIC's Consolidated Reports of Condition and Income ("Call reports") for commercial banks in the United States. Since all banks lend only to firms in the model, I calculate and target the default rate and loan return for C&I loans only. In order to get a sense of the expenses incurred by banks beyond the actual outlays of funds, I also target the ratio of monitoring expense to total expense, proxied by the ratio of salaries to total expenses. This is critical to insure that the cost function in my model captures an appropriate mix of lending and monitoring. Finally, as in the case of firms described above, the last three moments in Table 1.1 target banks' overall capital structure (as measured by average leverage), return on capital, and net interest margin (the spread between banks' lending rates and borrowing rates).

Similar to the bank side, I target the default rate on household lending, measured by the default rate on corporate bonds. Since reliable data on corporate bond defaults is scarce, I pull these figures directly from the detailed study of Giesecke et al. (2015). I include the share of riskless assets in the household portfolio in order to pin down the risk which households are willing to take. Finally, I target the ratio of household to bank assets to insure that capital levels are consistent.

1.5.2 Functional forms, parameters, and identification

It is necessary to assign functional forms to several key model objects. I assume that the process for the private benefit of shirking, x, is lognormal with mean μ_x and variance σ_x; that is, $\log(x) \sim \Phi(\mu_x, \sigma_x)$. The idiosyncratic preference shocks for firms' choices of lender, ϵ, are drawn from an extreme value distribution with scale parameter $1/\alpha$. The representative household has standard CRRA preferences with coefficient γ; that is, $u(c) = c^{1-\gamma}/(1-\gamma)$, for $\gamma > 1$. In addition, firms have the production function

$$f(I) = \theta_1 I^{\theta_2}, \tag{1.20}$$

where θ_1 is a scale factor and $\theta_2 \in (0, 1)$ represents the degree of decreasing returns to scale. Finally, I assume that banks' monitoring and lending cost function takes the form

$$c(m, \ell) = (1 + \ell)^{\rho_1 + \rho_2 m} - 1, \tag{1.21}$$

	Parameter	Notation	Value	Notes
Estimated	firm productivity / TFP	θ_1	1.40	$f(I) = \theta_1 I^{\theta_2}$
	firm production returns to scale	θ_2	0.88	
	mean of private benefit distribution	μ_x	-1.86	$\log(x) \sim \Phi(\mu_x, \sigma_x)$
	s.d. of private benefit distribution	σ_x	0.48	
	high state shirking success prob.	p_h	0.90	$\psi = \Pr(p = p_h)$
	low state shirking success prob.	p_l	0.49	$1 - \psi = \Pr(p = p_l)$
	bank cost function: loans	ρ_1	1.01	$c(m, \ell) =$
	bank cost function: monitoring	ρ_2	0.15	$(1 + \ell)^{\rho_1 + \rho_2 m} - 1$
	firm value shock variance	α	58.1	$\epsilon \sim T1EV(1/\alpha)$
	bank capital	k_B	0.01	
	firm capital	k_F	1.59	
	HH capital	k_H	0.93	
Selected	CRRA for HH utility	γ	3.00	macro standard
	baseline capital requirement (%)	χ	4.00	Basel II minimum
	prob. of high state	ψ	0.96	$1 - \Pr(\text{enter rec.})$

Notes: "Estimated" parameters are determined via Simulated Method of Moments (SMM), and "Selected" parameters are chosen outside the model. The rightmost column contains all relevant functional assumptions for the quantitative analysis.

Table 1.2: Parameterization of baseline model

where $\rho_1 \geq 1$ and $\rho_2 \geq 0$.[29] Since the bank cost function (1.21) is unique to my framework, its form and its properties warrant some discussion. First, observe that if $\rho_1 = 1$, then banks have the option of a simple linear investment cost: with no monitoring ($m = 0$), $c(0, \ell) = \ell$. A value of $\rho_1 > 1$ implies that lending costs for banks are convex, regardless of monitoring. This feature captures real world costs of scaling up lending activities and insures that the bank problem has a finite solution. The expenses on monitoring m further convexify the total cost of lending to the bank, increasingly so for higher ρ_2.

In total, there are 15 parameters to specify. I divide the parameters into two classes: those I estimate explicitly to match the moments in Table 1.1 using a standard SMM routine,[30] and those I select based on standard values in the literature or simple, outside the model

[29]Many other functional forms are equally valid: what is most critical is convexity in and complementarity between lending and monitoring.

[30]For details of the algorithm used to compute the model embedded in SMM, see Appendix A.2.

calculations. A summary of these parameters and their values can be found in Table 1.2.

Estimated parameters

The top half of Table 1.2 shows the ten parameters that I have calibrated to match key targets in Table 1.1. The parameters of the productivity process, θ_1 and θ_2, determine the optimal investment scale (given prices) according to the firm optimality condition (1.8), and so these parameters map into total firm leverage. The estimated function $f(I)$ from (1.20) is depicted in Figure A.2 in the appendix. The *level* of default risk in the model, conditional on lender, is dictated in large part by the mean and standard deviation of the private benefit of shirking process, μ_x and σ_x. Note that the private benefit x is in fixed units (i.e. does not scale with the amount invested or borrowed). The implied distribution is represented in Figure A.3 in the appendix. Then, the relative *difference* between default risk for bank and household loans is a function of banks' comparative advantage in monitoring and the extent to which it is exercised. This depends critically on the parameters of the bank cost function from (1.21), ρ_1 and ρ_2. This cost function is shown for various levels of monitoring over lending and vice versa in Figure A.4 in the appendix. Both these parameters also critically effect the bank's loan return, and the ratio of non-interest to total expenses. The amount of default in the model is also driven by the success rate of shirkers in each state and the likelihood of each state.

Given that I estimate a static model, it is critical to have reasonable estimates of the amount of capital in the economy and its distribution across sectors. These variables link critically to bank and firm leverage, respectively. These values reflect that banks in reality have only a small share of the total capital in the economy, relative to the amount of investment that flows through them.

Parameters selected outside the model

The bottom portion of Table 1.2 contains parameters that I have selected outside the model (i.e. not explicitly estimated via SMM). The value of $\gamma = 3$ for the CRRA coefficient of households is standard in the macro literature. The probability of the low p state, $\psi = 0.96$, was chosen to match the probability of entering a recession in the post-WW2 United States.[31] The baseline capital requirement of $\chi = 4.0\%$ comes from the specification under the Federal Reserve (as guided by Basel II) for banks to be considered "adequately capitalized."[32]

1.5.3 Model fit

The results in Table 1.1 suggest that this model is able to match aggregate features of the economy which pin down the relative sizes and default risk of bank and non-bank lending to firms. Specifically, the bank share of total debt is consistent with the manufacturing firms in the QFR. In addition, while default rates on both bank loans and corporate bonds exceed their counterparts in the data by about a percentage point, I capture closely the relative difference between these default rates, which preserves the validity of the directional changes in default I find as firms substitute between the types of financing. Firms in my model are slightly more levered as their counterparts in the data, but this could be reflective of some of the abstractions in my model: in particular, I assume no default costs or equity issuance. Banks maintain high leverage as observed in the data. In terms of interest rates, banks' loan returns are consistent with the return on C&I loans in the call report data. Ultimately, though, the calibrated model is sufficiently close to the chosen targets to provide a reasonable laboratory in which to conduct counterfactuals. I proceed to these experiments in the next section.

[31] That is, $q = \sum_{t=1945Q1}^{2015Q4} \mathbf{1}[rec_{t+1} = 1, rec_t = 0]$, where rec_t is an indicator equal to one if the economy is in recession in period t and $\mathbf{1}(\cdot)$ is an indicator function equal to one if all events within are true.

[32] Under the current capital regulatory regime, a bank holding company must have a Tier 1 capital ratio / Tier 1 + Tier 2 capital ratio / leverage ratio of at least 4% / 8% / 4% in order to be considered adequately capitalized. In order to be considered well-capitalized, the analogous figures must be 6% / 10% / 5%. For further details, see, for example, Corbae and D'Erasmo (2014).

1.6 The Effects of Increasing Capital Requirements with and without Direct Lending

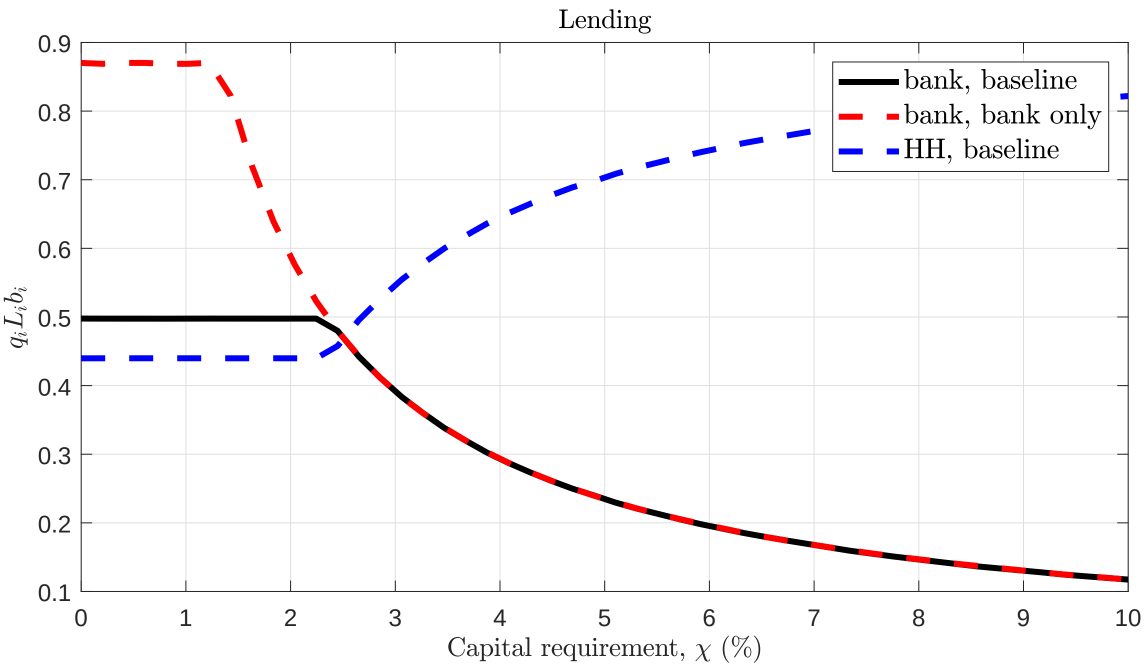
In this section, I explore the effects of changing capital requirements in both the baseline model described in Section 1.4 and a variant of this model in which the direct lending channel is shut down. A full description of the model without direct lending is available in Appendix A.1.2. For each value of χ in all the analysis which follows, I solve for the resulting equilibrium and compute the desired moments. I first consider lending, then risk, and then combine these two effects to consider total welfare.

1.6.1 Lending, output, and investment

Figure 1.5 shows the total lending of each type for both models across the range of capital requirements. In both models, very low capital requirements do not bind, and so there is an interval near zero where the equilibrium of the model does not change. As the capital requirement increases to the point where it begins to bind (just over 2%), bank lending in both models decreases.

In the baseline model, direct lending increases (solid blue line) once the capital requirement curtails bank lending. In fact, the pickup in direct lending *offsets* the decline in bank lending relative to lower levels of the capital requirement.[33] In contrast, in the model with banks only no other sector can rise to meet the demand. Therefore, the capital requirement on banks acts as an effective ceiling on total investment in the economy. This imposes a steep and sudden dropoff in total lending, and therefore also in investment and output.

[33] This result of almost complete offsetting is stark and bears comment. Since all firms in my model have "access" to both types of financing, there are no strict barriers to obtaining direct finance as bank finance becomes scarce. If firms faced costs of substitution – say, getting rated or listed on an exchange –, or if the "firms" were other risky assets like mortgages, this effect could be dampened considerably.



Notes: This figure is constructed using the parameters reported in Table 1.2, with the exception that the capital requirement χ is changed to vary in the range represented on the x-axis in the figure. "Baseline" refers to the model studied in Sections 1.3 and 1.4, while "bank only" refers to the model variant in which direct lending to firms by households is ruled out.

Figure 1.5: Lending across capital requirements

1.6.2 Aggregate and sectoral default rates

Figure 1.5 demonstrates that substitution into direct finance picks up as binding capital requirements limit bank lending. How does this substitution impact aggregate risk in the economy? Figures 1.6 and 1.7 address this question by showing the evolution of aggregate risk and its components over the range of capital requirements.

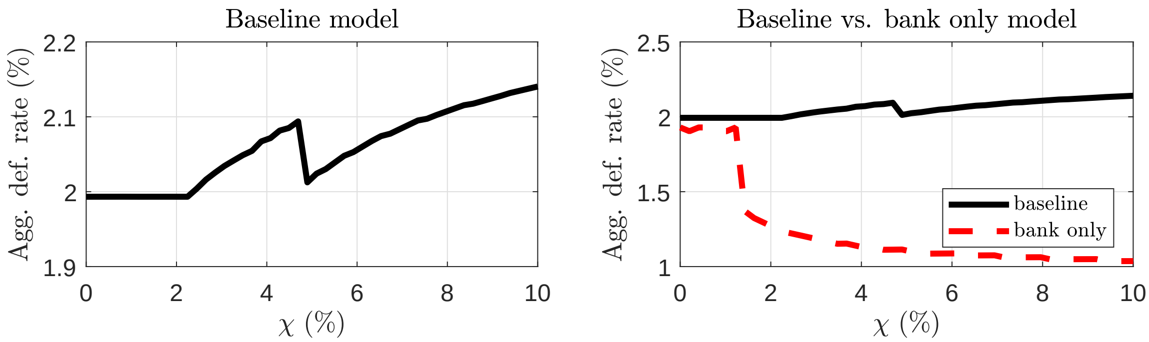
The left panel of Figure 1.6 shows the aggregate default rate for the baseline model only. We can divide the evolution of aggregate default in to four regions. First, to the left of about 2.2%, the capital requirement is non-binding. Banks fail in the bad state of the world, and monitor little, but capture a high share of total lending. Since bank lending is still monitored and therefore safer than direct lending, aggregate default is low. Second, between about 2.2%

and 4.4%, the capital requirement binds, but not tightly enough to make the strategy of over-lending and failing in the bad state not viable. Therefore, even though bank incentives are improving and bank loans default less (bottom left panel, Figure 1.7), substitution into direct finance drives up the aggregate default rate.

In the third region, at about a 4.5% capital requirement, we see an immediate and stark drop in aggregate default. Lending is capped by the capital requirement to the point where the strategy of over-lending, under-monitoring, and failing in the bad state is no longer attractive, consistent with the results in Lemma 1.2. Thus, bank and aggregate default rates drop immediately as banks increase their monitoring to protect themselves in the down state. As the capital requirement is increased further, though, we enter the fourth region to the right of 4.5%, in which the aggregate default rate rises as monitored bank lending is replaced with unmonitored direct lending. On one hand, as bank monitoring increases, the default rate on bank loans decreases.

In this sense, the capital requirement has its intended effect: bank lending becomes monotonically safer as χ is raised. These effects are balanced, though, by general equilibrium substitution into riskier, unmonitored direct financing. Moreover, due to equilibrium price adjustment (see the section below), the default rate on this direct lending decreases as well. Still, though, since households do not monitor, the level of the default rate on direct loans is higher than the default rate on bank loans. Then, since the composition of total lending tilts to direct lending, aggregate default increases.

How does this response differ in the economy with banks only? The top right panel of Figure 1.6 plots the aggregate default rates for both the baseline and bank only models on the same scale. Without a direct lending market, aggregate and bank loan default rates coincide. As the capital requirement increases, banks monitor more, just as in the baseline model. This implies that, once again, the regulation has its intended effect. In fact, capital requirements



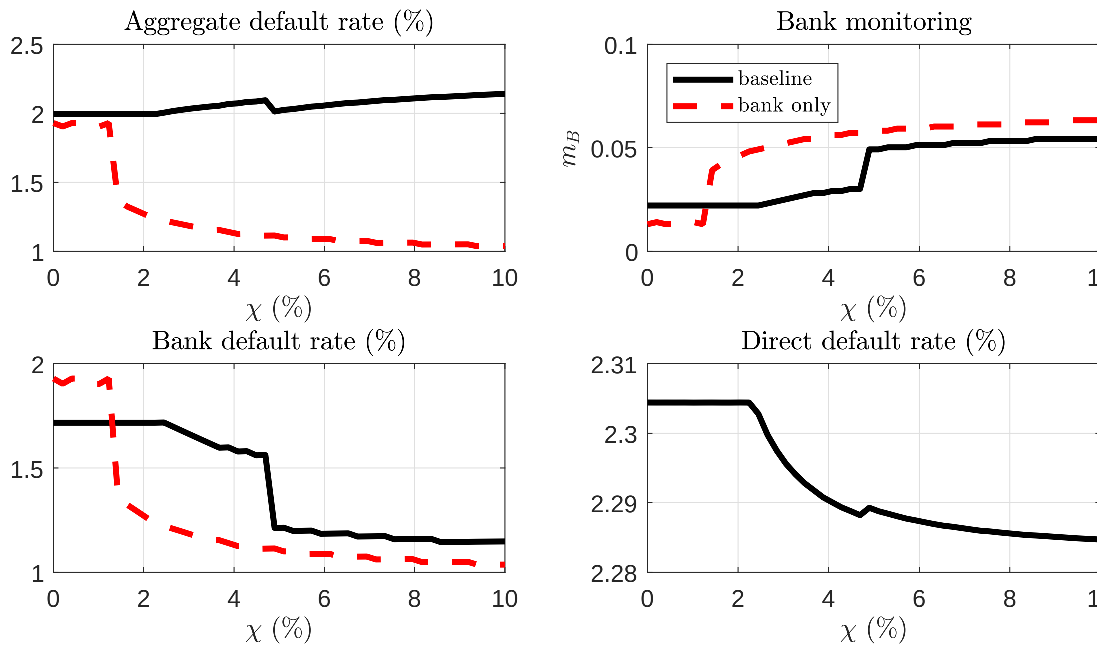
Notes: This figure is constructed using the parameters reported in Table 1.2, with the exception that the capital requirement χ is changed to vary in the range represented on the x-axis in the figure. "Baseline" refers to the model studied in Sections 1.3 and 1.4, while "bank only" refers to the model variant in which direct lending to firms by households is ruled out. Aggregate default is computed as the lending-share-weighted average of default rates on bank, direct, and self financing by firms.

Figure 1.6: Aggregate default rate across capital requirements: baseline versus bank only models

are even *more* effective at inducing banks to monitor in this version of the model. Examining the bank monitoring optimality condition (1.14) reveals that the effective marginal cost of monitoring is lower for each level of the capital requirement in the model with only banks since $\overline{q}$ rises more steeply than in the baseline model (see Figure 1.8 and the discussion below). As banks monitor borrowing firms more, fewer firms shirk and lending becomes safer, reflected in falling default rates. Finally, without the substitution effect present in the baseline model, the decline in default rate in the aggregate is roughly eight times larger.

1.6.3 Interest rates and taxes

Since I present a general equilibrium model, and all agents behave competitively and take prices as given, understanding how prices adjust in response to changes in the capital requirement is critical. Figure 1.8 plots the evolution of all three prices in the model (presented as net interest rates to ease the discussion), as well as the down state tax rate, $T(p_l)$. The first and most straightforward price effect is that interest rates on bank loans rise, as the capital requirement induces a mechanical inward shift of the bank loan supply function. As the bank loan supply

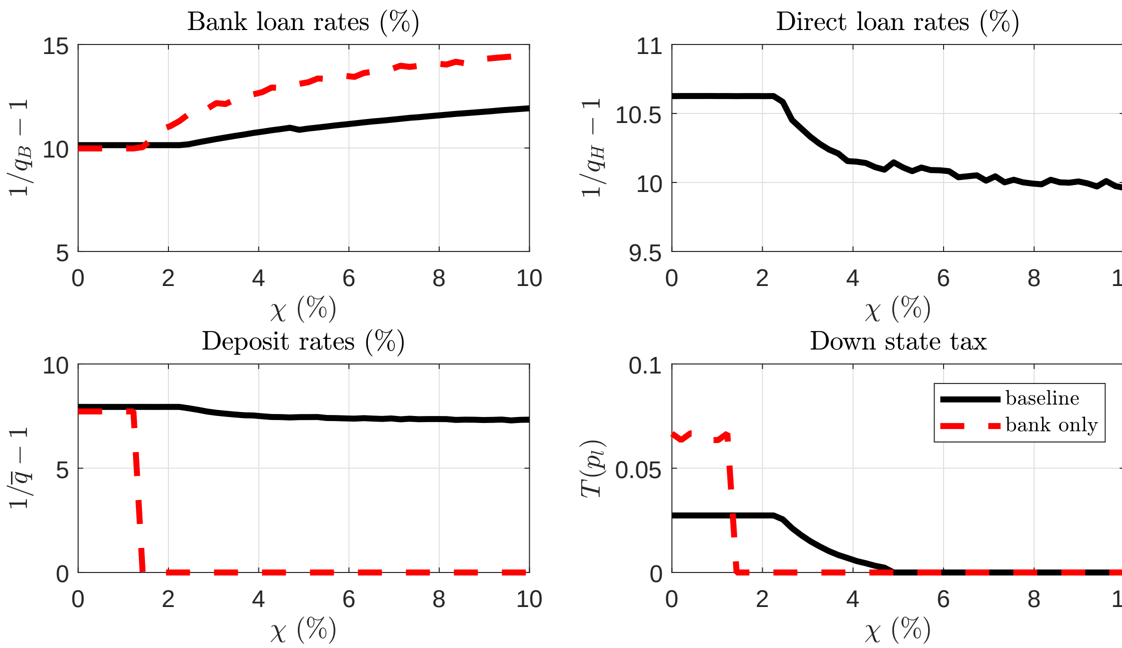


Notes: This figure is constructed using the parameters reported in Table 1.2, with the exception that the capital requirement χ is changed to vary in the range represented on the x-axis in the figure. "Baseline" refers to the model studied in Sections 1.3 and 1.4, while "bank only" refers to the model variant in which direct lending to firms by households is ruled out. Aggregate default is computed as the lending-share-weighted average of default rates on bank, direct, and self financing by firms.

Figure 1.7: Default rates and monitoring across capital requirements

function shifts inward, banks also demand fewer deposits. Therefore, the deposit rate increases, making deposits much less attractive to the household. Thus, even though firms' demand for direct loans increases in response to the new scarcity of bank loans, households' direct loan supply increases so much that the direct loan rate actually falls.

Prices evolves quite differently in response to increased capital requirements in the model with banks only. When bank lending declines in this model, households cannot soak up loan demand and offset the resulting loss in investment by banked firms. Given this inability to substitute, firms' bank loan demand is much more inelastic than in the baseline. Thus, the bank loan rate must rise rapidly in order for the market to clear. In addition, since the household cannot invest in direct lending, the supply of deposits in this bank only model is highly inelastic: the household optimally invests $d_H^* = k_H$ as long as $\overline{q} < 1$. Therefore, as



Notes: This figure is constructed using the parameters reported in Table 1.2, with the exception that the capital requirement χ is changed to vary in the range represented on the x-axis in the figure. "Baseline" refers to the model studied in Sections 1.3 and 1.4, while "bank only" refers to the model variant in which direct lending to firms by households is ruled out. The reported interested rate associated with a price q is simply $1/q - 1$.

Figure 1.8: Interest rates and taxes across capital requirements

soon as bank loan supply decreases to the point where banks' total external financing demand, $c(m_B, q_B \ell_B b_B) - k_B$, is below k_H, the deposit rate shoots immediately up to 0. At this point, households do no better than the storage technology: any investment they make returns one unit per unit invested.

1.6.4 Welfare and additional metrics

Throughout this paper, I have argued that the inclusion of a non-bank lending sector alters the aggregate effects of changing capital requirements on banks. I have explored the impacts on risk, investment, and prices. Ultimately, though, we would like to know: are agents better off at *different* capital requirements in a world with banks and other types of lenders, compared to a world with banks only? To answer this question, in this section I solve numerically for the welfare-maximizing capital requirement in each of these models. For each level of the capital requirement, I compute the welfare criterion

$$W(\chi) = \omega_B v_B(k_B) + \omega_H v_H(k_H) + \omega_F v_F(k_F), \tag{1.22}$$

where the weight $\omega_i = k_i/(k_B + k_H + k_F)$ corresponds to agent i's share of the economy's total capital. I find that the welfare-maximizing capital requirement in the baseline model is 4.5%, compared to 1.6% in the analogous model with banks only.

In both the baseline model and the variant with banks as the only lenders, maximizing welfare using a capital requirement involves balancing the costs of decreasing bank lending with the benefits of mitigating economy-wide risk. In the baseline, the first order costs of decreasing bank lending are low: households can readily offset the decline in bank lending. This is precisely what we observe in Figure 1.5. Therefore, raising bank capital requirements benefits the economy as a whole, making banks safer without much loss of investment and output. As capital requirements become very stringent, however, the shift in loan composition toward unmonitored direct lending induces such a high level of risk that total welfare begins to decline as the initial improvement in risk is undone, as shown in Figure 1.7. In contrast, in the model with banks only, the first order cost of the decline in bank lending is very high: no one in the economy provides an alternative source of credit to productive firms. Therefore, the benefit of decreasing risk pales in comparison to the cost of lost investment. In total, these forces deliver the result that the welfare-maximizing capital requirement with both banks and direct lending is significantly higher than the one with banks only.

To this point, I have focused my discussion around risk and investment on default rates and total lending, respectively. The model, though, permits analysis of a wide variety of additional risk and investment metrics. In this section I explore these metrics in the context of a further explanation of the gap in the welfare-maximizing capital requirement.

Table 1.3 contains a fuller set of model moments under each of these two categories. Each

	Model	Baseline			Banks only	
	Capital requirement, χ (%)	4.0	4.5	1.6	4.5	1.6
	Description	Basel	χ_1^*	χ_2^*	χ_1^*	χ_2^*
Risk	Bank default rate (%)	1.54	1.24	1.75	1.16	1.93
	Aggregate default rate (%)	2.19	2.01	2.00	1.16	1.93
	Output volatility (%)	1.05	1.03	1.03	1.66	2.08
	HH risk premium (%)	10.2	10.6	10.1	-	-
	Firm leverage	0.30	0.31	0.31	0.15	0.27
	Bank leverage	0.96	0.96	0.97	0.98	0.99
Investment	Aggregate investment (I/K)	1.00	1.00	1.00	0.76	0.92
	Mean output ($\mathbb{E}(Y)/K$)	1.23	1.23	1.23	0.97	1.14
	Bank share of total lending (%)	29.3	36.3	51.8	100	100

Notes: "Basel" refers to the minimum capital requirement to be in compliance with the Basel II Accord. The values of $\chi = \chi_1^*$ and $\chi = \chi_2^*$ maximize welfare in the baseline and bank only models, respectively. Definitions of model moments can be found in Appendix A.2.

Table 1.3: Additional moments and welfare comparison: baseline versus bank only models

column represents one of five models. In the first column, I present the baseline model, at the targeted Basel II minimum capital requirement. The next two columns present moments from the baseline model, and the final two present moments from the model with banks only. For each of the baseline and bank only models, I show how the economy behaves at two levels of the capital requirement. The first level, $\chi_1^* = 4.5\%$, maximizes welfare in the baseline model; the second, $\chi_2^* = 1.6\%$, maximizes welfare in the model with banks only.

For the baseline model, output volatility, risk premia, and measures of leverage make it clear that the increase in capital requirements from χ_2^* to χ_1^* is not very impactful. Volatility goes down by just two basis point (bp), and the risk premium on direct debt by ten bps. Firm and bank leverage are also virtually unchanged. Furthermore, on the investment side, the impacts are not really seen until χ increases beyond χ_2^*, closer to the level observed in the data.

Considering the bank only model, though, one immediately sees why the drivers of welfare are so different. While risk metrics improve across the board as the capital requirement increases from χ_1^* to χ_2^* – default rates and volatility fall –, the costs on the output and investment side are stark. Perhaps most notably, even though all capital in the economy is invested into the productive technology in the baseline model over the range of capital requirements shown in Table 1.3, at χ_2^* only 76% of the capital stock goes into production, as households increasingly store their goods due to the lack of attractive investments. Furthermore, going from χ_1^* to χ_2^* involves a steep decline in expected output to capital, from 1.14 to 0.97. Clearly, these losses in investment and output outweigh all the beneficial decreases in risk and volatility.

1.7 Conclusion

In this paper, I have presented a tractable framework to analyze the effects of banking regulations when borrowers can obtain financing from non-bank lenders. Critically, this framework features (i) a choice between lenders for firms; (ii) a distinction between these lenders which creates a unique role for banks (monitoring); and (iii) bank risk-taking incentives, and therefore a role for bank regulation. I have used this framework to map out the impacts of increasing bank capital requirements. Capital requirements resolve the risk-shifting externality associated with the pairing of deposit insurance with limited liability. When capital requirements increase, banks decrease the amount they lend, but also increase the intensity with which they monitor borrowers. This leaves the portion of the total lending in the economy accounted for by banks safer and smaller. However, the reduction in lending by banks is offset by an increase in direct lending by households. Since this latter form of lending is unmonitored, aggregate default risk in the economy can increase, even if the impacts on investment and expected output are minimal. The welfare-maximizing capital requirement is just high enough that banks take into account payoffs in both states of the world, and mitigate risk accordingly.

Using the calibrated model, I solve for and compare equilibria across a range of capital requirements. I find that this welfare-maximizing capital requirement on banks is *higher* when one accounts for substitution by firms into direct lending, relative to the one from a model with banks only. The intuition behind this critical result is as follows. In a model with banks only, the loss of output associated with decreased bank lending is so severe that reining in risk-taking by banks by setting a very stringent capital requirement is too costly. With both banks and direct lending, however, this first order effect of loss of output is mitigated almost completely by an increase in direct lending.

I have highlighted the tradeoff between making one lending sector of the economy safer, while potentially pushing lending into another sector. For simplicity of exposition of the core ideas, this paper has several limitations which leave room for future research. For example, the model presented above is static, but it will be critical going forward to account for the *dynamic* costs and benefits. Does the increased risk associated with higher capital requirements and more direct lending amplify economic downturns? How might firms' and households' saving and investment policies change when we consider capital accumulation? These additional effects may be critical. Furthermore, my work considers a single type of firm. The data show that different types of firms use very different mixes of bank and non-bank financing,[34] and so the impact of increasing capital requirements on banks could vary widely in the cross-section of firms. Lastly, the model does not allow for equity issuance by banks (i.e. I assume that it is infinitely costly). In response to a binding capital requirement, though, banks may be able to maintain some additional lending by issuing equity. I leave these and other questions for future research.

[34]See , for example, Becker and Ivashina (2014); Colla et al. (2013); Faulkender and Petersen (2006); Houston and James (1996); Rauh and Sufi (2010).

Chapter 2

A Theory of Credit Scoring and the Competitive Pricing of Default Risk (with Satyajit Chatterjee, Dean Corbae, and José-Víctor Ríos-Rull)

2.1 Introduction

It is well known that lenders use credit scores to regulate the extension of consumer credit. People with high scores are offered credit on more favorable terms. People who default on their loans experience a decline in their scores and lose access to credit on favorable terms as a result. People who run up debt also experience a decline in their credit scores and have to pay higher interest rates on new loans. While credit scores play an important role in the allocation of consumer credit, credit scoring has not been adequately integrated into the theoretical literature on consumption smoothing and asset pricing. This paper attempts to remedy this gap.

We propose a theory of unsecured consumer credit where: (i) borrowers have the legal option to default; (ii) defaulters are not exogenously excluded from future borrowing; (iii) there is free entry of lenders; and (iv) lenders cannot collude to punish defaulters. We use the framework to try to understand why households typically face limited credit or credit at higher interest rates following default, and how these terms evolve over time. We show that such outcomes arise from the lender's optimal response to limited information about the agent's type. The lender learns from an individual's borrowing and repayment behavior about his type and encapsulates his reputation for not defaulting in a credit score.

Beginning with the work of Athreya (2002), there has been a growing number of papers that have tried to understand bankruptcy data using quantitative, heterogeneous agent models (for example, Chatterjee et al. (2007); Livshits et al. (2007)). For simplicity, these models have assumed that an individual is exogenously excluded from borrowing while a bankruptcy remains on his credit record. This exclusion restriction is often modeled as a Markov process and calibrated so that on average the household is excluded for 10 years, after which the Fair Credit Reporting Act requires that it be stricken from the household's record. This assumption is roughly consistent with the findings by Musto (2004) who documents the following important facts: (i) households with low credit ratings face very limited credit lines (averaging around $215 prior to and $600 following the removal of a bankruptcy flag); (ii) for households with medium and high credit ratings, their average credit lines were a little over $800 and $2000 respectively prior to the year their bankruptcy flag was removed from their record; and (iii) for households with high and medium credit ratings, their average credit lines jumped nearly doubled to $2,810 and $4,578 in the year that the bankruptcy flag was removed from their record.[1]

While this exogenous exclusion restriction is broadly consistent with the empirical facts, a fundamental question remains. Since a Chapter 7 filer is ineligible for a subsequent Chapter 7 discharge for 6 years (and at worst forced into a subsequent Chapter 13 repayment schedule), why don't we see more lending to those who declare bankruptcy? If lenders believe that

[1] These numbers are actually drawn from Table III, panel A of Musto (2004).

the Chapter 7 bankruptcy signals something relatively permanent about the household's unobservable characteristics, then it may be optimal for lenders to limit future credit. But if the circumstances surrounding bankruptcy are temporary (like a transitory, adverse income shock), those individuals who have just shed their previous obligations may be a good future credit risk. Competitive lenders use current repayment and bankruptcy status to try to infer an individual's future likelihood of default in order to correctly price loans. There is virtually no existing work embedding this inference problem into a quantitative, dynamic model.

Given commitment frictions, it's important for a lender to assess the probability that a borrower will fail to pay back – that is, assess the risk of default. In the U.S., lenders use *credit scores* as an index of the risk of default. The credit scores most commonly used are produced by a single company, the Fair Isaac and Company, and are known as FICO scores.[2] These scores range between 300 and 850, where a higher score signals a lower probability of default. Scores under 620, which account for roughly one quarter of the population with scores, are called '"subprime." There is ample empirical evidence that households with subprime credit scores are more likely to default. Figure 2.2 provides one such example. As discipline on our theory, we require our framework to match key credit market facts like the frequency of delinquency across the distribution of credit scores depicted in Figure 2.2.

There is now a fairly substantial literature (beginning with Kehoe and Levine (1993)) on how and to what extent borrowing can occur when agents cannot commit to pay back. This literature typically assumes that a default triggers permanent exclusion from credit markets. A challenge for this literature is to specify a structure with free entry of lenders and where lenders cannot collude to punish defaulters that can make quantitative sense of the characterization of a competitive unsecured consumer credit market with on-the-equilibrium-path default offered in the previous paragraphs. This paper take steps toward meeting this challenge.[3] We consider

[2]Over 75% of mortgage lenders and 80% of the largest financial institutions use FICO scores in their evaluation and approvals process for credit applications.

[3]In Chatterjee et al. (2008) we show that credit can be supported even in a finite horizon model where trigger

an environment with a continuum of infinitely-lived agents who at any point in time may be one of a finite number of types that affect their preferences. An agent's type is drawn independently from others and follows a persistent process. Importantly, a person's type is unobservable to the lender.[4] As in the discrete choice literature (McFadden (1973); Rust (1987)) the agent is also subject to a purely transitory shock to his preferences.

These people interact with competitive financial intermediaries that can borrow in the international credit market at some fixed risk-free rate and make one-period loans to individuals at an interest rate that reflects that person's risk of default.[5] Because differences in preferences bear on the willingness of each type of agent to default, intermediaries must form some assessment of a person's type which is an input into his credit score. We model this assessment as a Bayesian inference problem: intermediaries use the recorded history of a person's actions in the credit market to update their prior probability of his or her type and then charge an interest rate that is appropriate for that posterior. The fundamental inference problem for the lender is to assess whether a borrower or a defaulter is chronically "risky" or just experiencing a transitory preference shock. A rational expectations equilibrium requires that a lender's perceived probability of an agent's default must equal the objective probability implied by the agent's decision rule. Incorporating this equilibrium Bayesian credit scoring function into a dynamic incomplete markets model is the main technical challenge of our paper. The adoption of a discrete choice framework, which implies that all feasible actions are taken with positive probability, greatly simplifies the model by ruling out zeros in the denominator of the Bayesian posterior and speeding up the computation of an equilibrium.

This is possibly the simplest environment one could imagine that could make sense of the observed connection between credit history and the terms of credit. Suppose it turns out that,

strategies cannot support credit.

[4]Ausubel (1999) documents adverse selection in the credit market both with respect to observable and unobservable household characteristics.

[5]Our earlier paper Chatterjee et al. (2007) shows that there is not a big gain to relaxing the fixed risk-free rate assumption.

in equilibrium, one type of person, say type g, has a lower probability of default. Then, under competition, the price of a discount bond (of any size) could be expected to be positively related to the probability of a person being of type g. Further, default will lower the *posterior* probability of being of type g because type g people default less frequently. This provides the basis for a theory why people with high scores are offered credit on more favorable terms. This would explain the fact that people with high scores are offered credit on more favorable terms.

We model the pricing of unsecured consumer loans in a similar way as our predecessor paper Chatterjee et al. (2007). As in that paper, all one-period loans are viewed as discount bonds and the price of these bonds depend on the size of the bond. the price of a one-period bond can depend on certain observable household characteristics like their earnings. The price of a bond cannot, however, depend on unobservable characteristics like their preference type. Instead we assume that the bond depends on the agent's probability of repayment, which is encapsulated via a credit scorecard.[6] The probability of repayment depends on the posterior probability of a person being of a given type *conditional* on selling that particular sized bond. This is necessary because the different types will not have the same probability of default for any given sized bond and a person's asset choice is potentially informative about the person's type.[7] With this asset market structure, competition implies that the expected rate of return on each type of bond is equal to the (exogenous) risk-free rate. Our equilibrium concept is closest to a signaling game.

2.1.1 Roadmap

The paper is organized as follows. Section 2.2 describes our benchmark economy with private information. Section 2.3 describes the equilibrium problems faced by our agents. Section 2.4

[6] Livshits et al. (2016) document the increasing use of credit scorecards and provide a theory of why this might happen.

[7] Athreya et al. (2012) also consider a signaling model but assume anonymity so that past asset market choices encapsulated in a type score cannot be used as a prior when calculating posteriors associated with current asset market choices.

describes our estimation procedure, and in particular includes a discussion on the import of the discrete choice shocks for our computational analysis and estimation in Section 2.4.2. Section 2.5 studies the properties of the benchmark model (Section 2.5.1), and also compares this to a model in which there is no private information about the agent's persistent type (Section 2.5.2). Section 2.6 presents additional quantitative analysis for the benchmark economy: specifically, we estimate the value to having a good reputation (Section 2.6.1), and compare the static and dynamic costs of default (Section 2.6.2). Section 2.7 concludes. All the details of the model's computation are contained in Appendix B.

2.2 Environment

There is a unit measure of infinitely lived individuals. The persistent component of an individual's earnings, denoted $e_t \in \mathcal{E} = \{e_1, e_2, ..., e_E\} \subset \mathbb{R}_{++}$, is exogenously drawn from a finite state Markov process $Q^e(e_{t+1}|e_t)$. The purely transitory component of an individual's earnings, denoted $z_t \in \mathcal{Z} = \{z_1, z_2, \ldots, z_Z\} \subset \mathbb{R}$, is exogenously drawn from probability distribution $H(z_t)$. All earnings shocks are independent across individuals.

Preferences are additively separable over time. In each period t, the individual orders consumption using a bounded, increasing, concave utility function $u(c_t)$. Period t preferences are subject to an additive unobservable shock ϵ_t drawn i.i.d. across time and individuals from a Type 1 extreme value distribution $G(\epsilon_t)$. Agents discount utility at rate $\beta_t \in \mathcal{B} = \{\beta_1, \beta_2, ..., \beta_B\} \subset [0, 1)$ exogenously drawn from a finite state Markov process $Q^\beta(\beta_{t+1}|\beta_t)$. As part of our normalization, the reward function in period t is given by $(1 - \beta_t)u(c_t)$.

Consumption depends on the individual's earnings draw and asset market choices. At time t individuals can borrow $a_{t+1} \in \mathcal{A} \subset \mathbb{R}_{--}$ or save $a_{t+1} \in \mathcal{A}_+ \subset \mathbb{R}_+$ at discount price q_t determined in a competitive market with risk free net interest rate $r \geq 0$. Let $\mathcal{A} = \mathcal{A}_{--} \cup \mathcal{A}_+$ be a finite set which includes 0 with A elements. At the beginning of any period, if an agent holds debt (i.e. $a_t < 0$), he can choose whether or not to default $d_t \in \{0,1\}$. If he defaults (i.e. $d_t = 1$), then he cannot borrow or save (i.e. $a_{t+1} = 0$), and his earnings become $(1-\eta)\cdot(e_t+z_t)$ where $\eta \in [0,1)$ in that period *only*. We assume that $\min\{\mathcal{E}\} + \min\{\mathcal{Z}\} > \min\{\mathcal{A}\}$.[8]

Intermediaries can observe individuals' earnings and asset market behavior (i.e. e_t, z_t, and a_{t+1}), but cannot observe their preferences (i.e. ϵ_t and β_t). Since ϵ_t is i.i.d. over time and individuals, there is nothing to be learned by trying to infer that part of an individual's state. However, since β_t is drawn from a persistent Markov process, there is something to be learned. We will call β_t an agent's type. In that case, the fraction of individuals of each type is given by the stationary distribution implied by Q^{β}. We denote the creditor's assessment of an individual's type at the beginning of period t before any actions are taken as $s_t = (s_t(\beta_1), ..., s_t(\beta_B)) \in \mathcal{S} = [0,1]^B$.[9]

Importantly, note that there is no further punishment to default except possibly loss of reputation since an individual's credit market behavior affects creditors' assessment of their unobservable type. The timing in any given period is as follows:

1. Individuals begin period t with state vector (β_t, e_t, a_t, s_t).
2. Individuals receive a transitory shock z_t and an action-specific preference shock $\epsilon_t^{(d_t,a_{t+1})}$ for every feasible action (d_t, a_{t+1}).
3. Given a menu of prices q_t, agents choose $d_t \in \{0,1\}$ if $a_t < 0$; if $d_t = 0$, they choose a_{t+1}.
4. Based on each individual's actions, intermediaries revise their assessments of an individual's type, updating s_t to s_{t+1} via Bayes' rule.
5. Beginning of next period realizations of β_{t+1} and e_{t+1} are drawn from $Q^{\beta}(\cdot|\beta_t)$ and $Q^e(\cdot|e_t)$ respectively.

[8] This has the convenient implication that all debt choices are feasible for non-negative prices for all agents, regardless of their state.

[9] Of course the framework is rich enough to add more unobservables. For instance, if the persistent component of earnings are unobservable, then $s_t = (s_t(\beta_1, e_1), ..., s_t(\beta_B, e_E)) \in \mathcal{S} = [0,1]^{B\cdot E}$.

2.3 Equilibrium

2.3.1 Individuals' problem

Let a current value x_t be denoted x and next period's variable x_{t+1} be denoted x'. After the realization of shocks, an individual begins the period in the following state $(\epsilon, \beta, e, z, a, s)$ with only (e, z, a, s) observable to financial intermediaries.

Let

$$\mathcal{Y} = \{(d, a') : (d, a') \in (0 \times \mathcal{A}) \text{ or } (d, a') = (1, 0)\}$$

be the set of all possible default and asset choices.[10] Let Y be the cardinality of the choice set $\mathcal{Y}$ $(Y = A + 1)$.

Each individual takes as given

- the price function $q(a', p) : \{\mathcal{A}_{--} \times [0,1]\} \cup \{\mathcal{A}_{+} \times \{1\}\} \to [0, 1/(1+r)]$ where p is the repayment probability of an individual who chooses asset position a'.[11]

- the credit scorecard function $p(a', s^{'}, \gamma^{'}) : \mathcal{A}_{--} \times \mathcal{S} \times \mathcal{S} \to [0,1]$, which determines the probability that an individual with scorecard $(s^{'}, \gamma^{'}) = (\psi^{(d,a')}, \Gamma^{(0,a')})$ repays debt $a' < 0$.

- the type scoring function $\psi^{(d,a')}(e, z, a, s) : \mathcal{E} \times \mathcal{Z} \times \mathcal{A} \times \mathcal{S} \times \mathcal{Y} \to \mathcal{S}$, which is a function that performs Bayesian updating of an individual's type based on all observables, including the current type score and the action taken.

- the repayment scoring function $\Gamma^{(0,a')}(e, s') : \mathcal{E} \times \mathcal{S} \times \mathcal{A}_{--} \to \mathcal{S}$, which is a function that evaluates the likelihood of future repayment for each possible future type across observable future states.

For ease of notation, we will denote the set of functions $\{q(\cdot), p(\cdot), \psi(\cdot), \Gamma(\cdot)\}$ by f.

[10] We have assumed that if a person declares bankruptcy (i.e. $d = 1$), then they cannot borrow or save in that period only ($a' = 0$).

[11] Note that we consider only the asset choice a' when $d = 0$ here since default in the model is assumed to be full: when $d = 1$, there is no repayment by the individual to the financial intermediary.

Definition 2.1 ***Feasible set:*** *Given observable state* (e, z, a, s) *and a set of equilibrium functions* f *the set of feasible actions is a finite set* $\mathcal{F}(e, z, a, s|f) \subseteq \mathcal{Y}$ *that contains all actions such that consumption,* $c^{(d,a')}$*, satisfies:*

$$c^{(d,a')} = \begin{cases} e + z + a - q(a', p(a', \psi^{(0,a')}(e, a, s))) \cdot a' \geq 0 & \textit{for } d = 0, a' < 0 \\ e + z + a - q(a', 1)a' \geq 0 & \textit{for } d = 0, a' \geq 0 \\ (e + z) \cdot (1 - \eta) & \textit{for } d = 1, a' = 0 \end{cases} \tag{2.1}$$

Given an individual's state and the functions f, the current-period return for an individual choosing any feasible action $(d, a') \in \mathcal{F}$ is $u\left(c^{(d,a')}\right)$, where $u(\cdot)$ is bounded, increasing, and concave. Denote by $V(\epsilon, \beta, e, z, a, s|f) : \mathbb{R}^Y \times \mathcal{B} \times \mathcal{E} \times \mathcal{Z} \times \mathcal{A} \times \mathcal{S} \to \mathbb{R}$ the value function of an individual in state $(\epsilon, \beta, e, z, a, s)$ who has the unobservable Y-dimensional action-specific utility shock vector $\epsilon = (\epsilon^{(d,a')})_{(d,a')\in\mathcal{Y}}$ drawn each period from distribution $G(\epsilon)$. Following the discrete choice literature, $G(\epsilon)$ is assumed to be a type 1 extreme value distribution with scale parameter $1/\alpha$. An individual's recursive decision problem is then given by

$$V(\epsilon, \beta, e, z, a, s|f) = \max_{(d,a')\in\mathcal{F}(e,a,s|f)} v^{(d,a')}(\beta, e, z, a, s|f) + \epsilon^{(d,a')} \tag{2.2}$$

where the conditional value function is given by

$$\begin{aligned} v^{(d,a')}(\beta, e, z, a, s|f) &= (1 - \beta)u\left(c^{(d,a')}\right) \\ &+\beta \cdot \sum_{(\beta',e',z')\in\mathcal{B}\times\mathcal{E}\times\mathcal{Z}} Q^{\beta}(\beta'|\beta)Q^{e}(e'|e)H(z')W(\beta', e', z', a', s'|f) \end{aligned} \tag{2.3}$$

is the value associated with a specific action (d, a') and $W(\cdot)$ integrates the value function over transitory preference shocks: that is,

$$W(\beta, e, z, a, s|f) = \int V(\epsilon, \beta, e, z, a, s|f)dG(\epsilon).$$

Let $\sigma^{(d,a')}(\beta, e, z, a, s|f)$ be the probability that the individual in state (β, e, z, a, s) chooses action (d, a'). As in **?** we have:

$$\sigma^{(d,a')}(\beta, e, z, a, s|f) = \frac{\exp\left\{\alpha \cdot v^{(d,a')}(\beta, e, z, a, s|f)\right\}}{\sum_{(\hat{d},\hat{a}')\in\mathcal{F}(e,z,a,s|f)} \exp\left\{\alpha \cdot v^{(\hat{d},\hat{a}')}(\beta, e, z, a, s|f)\right\}} \tag{2.4}$$

and

$$W(\beta, e, z, a, s|f) = \frac{1}{\alpha} \ln\left(\sum_{(d,a')\in\mathcal{F}(e,z,a,s|f)} \exp\left\{\alpha \cdot v^{(d,a')}(\beta, e, z, a, s|f)\right\}\right). \tag{2.5}$$

2.3.2 Intermediaries' problem

Competitive intermediaries with deep pockets have access to an international credit market where they can borrow or lend at the risk-free interest rate $r \geq 0$. An intermediary also incurs a proportional cost $\iota \geq 0$ when making loans to individuals. Any given intermediary takes the price function $q(a', p)$ as given. The profit $\pi(a', p)$ on a financial contract of type (a', p) is:

$$\pi(a', p) = \begin{cases} \frac{p\cdot(-a')}{1+r+\iota} - q(a', p) \cdot (-a') & \text{if } a' < 0 \\ q(a', 1) \cdot a' - \frac{a'}{1+r} & \text{if } a' \geq 0 \end{cases} \tag{2.6}$$

Let $\mathbb{B}(\mathcal{A} \times [0, 1])$ be the Borel sets of $\mathcal{A} \times [0, 1]$. Let $\mathcal{M}$ be the set of all measures defined on the measurable space $(\mathcal{A} \times [0, 1], \mathbb{B}(\mathcal{A} \times [0, 1]))$. For $m \in \mathcal{M}$, $m(a', p)$ is the measure of financial contracts of type $(a', p) \in \mathbb{B}(\mathcal{A} \times [0, 1])$ sold by a financial intermediary. The decision problem of a financial intermediary is:

$$\max_{m\in\mathcal{M}} \int \pi(y, p)\, dm(y, p). \tag{2.7}$$

Given perfect competition in financial intermediation and constant returns to scale in the lending technology, the bank problem (2.7) together with the profit function (2.6) implies that prices perfectly compensate for risk, leaving lenders to break even on any loan they extend:

$$q(a', p) = \begin{cases} \frac{p}{1+r+\iota} & \text{if } a' < 0 \\ \frac{1}{1+r} & \text{if } a' \geq 0 \end{cases} \tag{2.8}$$

To assess an individual's probability (p) of repaying a debt *tomorrow* in order to price debt *today*, an intermediary must solve an inference problem since neither the persistent β nor the transitory ϵ are observable. The probability of repayment tomorrow on a loan of size a' taken out by an individual in observable state (e, z, a, s) can be separated into two steps:

1. Assess the probability that an individual in state (e, z, a, s) who takes action (d, a') today will be of unobservable type β' tomorrow. This step is performed by the type scoring function, $\psi(\cdot)$.

2. For each possible future unobservable type β', compute the individual's probability of future repayment conditional on being that type and transitions over observable characteristics. This step is performed by the repayment scoring function $\Gamma(\cdot)$.

After performing these two steps, computing the repayment probability is simply a matter of computing a weighted sum over future types. Since the information about the individual that informs the pricing of debt can be summarized entirely by these functions, we refer to the pair of functions (ψ, Γ) as the *credit scorecard* used by intermediaries.

Type scoring

An intermediary must assign a likelihood of an individual being of a given type β tomorrow based on (i) the individual's observables, (e, z, a, s), and (ii) her choice of action (d, a'). We assume intermediaries uses Bayes' rule to infer an individual's probability of being type $s' = (s(\beta'_1), ..., s(\beta'_B))$ tomorrow given by the type scoring function

$$\psi^{(d,a')}(e, z, a, s) = \left(\psi^{(d,a')}_{\beta'_1}(e, z, a, s), ..., \psi^{(d,a')}_{\beta'_B}(e, z, a, s)\right).$$

For each possible value of $\beta' \in \mathcal{B}$, the intermediary assigns probability

$$\psi^{(d,a')}_{\beta'}(e, z, a, s) = \sum_{\beta} Q^{\beta}(\beta'|\beta) \cdot \frac{\sigma^{(d,a')}(\beta, e, z, a, s) \cdot s(\beta)}{\sum_{\hat{\beta}} \left[\sigma^{(d,a')}(\hat{\beta}, e, z, a, s) \cdot s(\hat{\beta})\right]} \tag{2.9}$$

to an individual of type β with observable state (e, z, a, s) and action (d, a') being of type β' tomorrow.[12] The first term in the sum in (2.9) is the probability of transitioning to the particular β' from a given β today. The numerator of the second term is the probability that an agent with a given β today and observable state (e, z, a, s) chooses an action (d, a') today, weighted by the currently assessed probability that the agent in question actually has this β. Finally, the denominator of the second term is a scaling term to aggregate over all possible current values of β. Note that $\psi^{(\cdot)}(\cdot)$ is a B-vector-valued function as specified above and satisfies $\sum_{\beta' \in \mathcal{B}} \psi_{\beta'}^{(d,a')}(e, z, a, s) = 1$ for all $(e, z, a, s, (d, a')) \in \mathcal{E} \times \mathcal{Z} \times \mathcal{A} \times \mathcal{S} \times \mathcal{Y}$.

Repayment scoring

Given today's observables and actions, an intermediary can also compute the likelihood of repayment tomorrow for each possible type β'. As discussed in the preceding section, the type scoring function $\psi(\cdot)$ maps today's observable state and actions into tomorrow's type scores, s'. Since tomorrow's asset position is also known (because it is part of the observed choice (d, a')), the only remaining components of the individual's state tomorrow relevant for tomorrow's default decision are β' and e'. Earnings are governed by an exogenous transition, and so an individual's expected repayment behavior can be computed from this transition and today's earnings. Putting these pieces together formally, we can compute the probability that an agent repays a debt tomorrow for every possible type:

$$\Gamma_{\beta'}^{(0,a')}(e, s') = \sum_{(e',z') \in \mathcal{E} \times \mathcal{Z}} Q^e(e'|e) \cdot H(z') \cdot \left[1 - \sigma^{(1,0)}\left(\beta', e', z', a', s'\right)\right] \tag{2.10}$$

The first term captures the exogenous transition over future earnings conditional on the persistent component of today's earnings. The second term is an agent's probability of repayment

[12] It is worth noting a critical distinction here. The type score s and s', which is updated according the $\psi(\cdot)$ function, reflects an individual's *assessed* probability of being a given type. The *objective* probability of being a given type in the current period (unconditionally) is given by the stationary distribution implied by $Q^\beta(\cdot)$; conditional on today's type, the objective probability of being a given type β' tomorrow is given by $Q^\beta(\beta'|\beta)$.

tomorrow conditional on type and the observable future state. Notice that, like $\psi(\cdot)$, $\Gamma(\cdot)$ is a B-vector-valued function; that is,

$$\Gamma^{(0,a')}(e,s') = \left(\Gamma^{(0,a')}_{\beta'_1}(e,s'), ..., \Gamma^{(0,a')}_{\beta'_B}(e,s')\right).$$

However, $\Gamma(\cdot)$ does sum to 1 across β'. Lastly, notice that $\Gamma(\cdot)$ only requires knowledge of today's variables which have bearing on tomorrow's state. For instance, a is not an argument to $\Gamma(\cdot)$ because the relevant asset position is the updated a'; on the other hand, e is required because it informs the likelihood of each of tomorrow's possible persistent earnings realizations.

Scorecard

Given observable state (e, z, a, s), the type scoring $s' = \psi^{(0,a')}(e, z, a, s)$ in (2.9) and repayment scoring $\gamma' = \Gamma^{(0,a')}_{\beta'}(e, s')$ in (2.10), we obtain the probability of repayment the intermediary uses for pricing debt simply by aggregating over types:

$$p(a', \psi, \Gamma) = \sum_{\beta' \in \mathcal{B}} \psi^{(0,a')}_{\beta'}(e, z, a, s) \cdot \Gamma^{(0,a')}_{\beta'}\left(e, \psi^{(0,a')}(e, z, a, s)\right). \tag{2.11}$$

2.3.3 Evolution

The probability that an individual in state (β, e, z, a, s) transits to state (β', e', z', a', s') given a set of functions f is:

$$T^*(\beta', e', z', a', s'|\beta, e, z, a, s, f) = \sigma^{(d,a')}(\beta, e, z, a, s|f) \cdot \chi_{[s'=\psi^{(d,a')}(e,z,a,s)]} \cdot Q^{\beta}(\beta'|\beta) \cdot Q^{e}(e'|e) \cdot H(z'),$$

where the first term reflects the probability that an individual in state (β, e, z, a, s) chooses the asset position a', the second is an indicator ensuring that the type score is governed by the equilibrium $\psi(\cdot)$ function, and the last three terms account for the exogenous transitions over discount factors and earnings.

Let $\mu(\beta, e, z, a, s|f)$ be the measure of individuals in state (β, e, z, a, s) today for a given set of equilibrium functions f. Then, the population evolves according to

$$\mu'(\beta', e', z', a', s'|f) = \sum_{(\beta,e,z,a,s)\in\mathcal{B}\times\mathcal{E}\times\mathcal{Z}\times\mathcal{A}\times\mathcal{S}} T^*(\beta', e', z', a', s'|\beta, e, z, a, s, f) \cdot \mu(\beta, e, z, a, s|f). \tag{2.12}$$

A stationary distribution in this model is a fixed point $\mu^*(\cdot)$ of (2.12).

Remark: Note that although the stationary distribution is critical for computing cross-sectional moments used to map the model to the data in later sections of the paper, none of the other equilibrium objects – the set of functions f, the value function $V(\cdot)$ or the decision rule $\sigma^{(\cdot)}(\cdot)$ – take $\mu(\cdot)$ as an argument. This simplifies the model and eases the computational burden, but is not necessary. Other specifications in which knowledge of the distribution is required are possible, but we do not consider these in the benchmark case.

2.3.4 Equilibrium definition

We can now give the definition of a stationary recursive competitive equilibrium.

Definition 2.2 ***Stationary recursive competitive equilibrium:*** *A stationary recursive competitive equilibrium is a pricing function $q^*(a', p)$, a credit scorecard function $p^*(a', s', \gamma')$, a type scoring function $\psi^{*\,(d,a')}(e, z, a, s)$, a repayment scoring function $\Gamma^{*\,(0,a')}(e, s')$, quantal response function $\sigma^*(\beta, e, z, a, s|f^*)$, and a steady state distribution over individual states $\mu^*(\beta, e, z, a, s|f^*)$ such that:*

- $\sigma^*(\beta, e, z, a, s|f^*)$ satisfies (2.4),
- $q^*(a', p)$ is such that $\pi(a'^*) = 0$ in (2.6) for all (a', p),
- $p = p^*(a', s', \gamma')$ satisfies (2.11),
- $s' = \psi^{*\,(d,a')}(e, z, a, s)$ satisfies(2.9),

- $\gamma' = \Gamma^{*\,(0,a')}(e, s')$ satisfies (2.10), and

- $\mu^*(\beta, e, z, a, s|f^*)$ solves (2.12) for T^*.

2.4 Estimation

2.4.1 Parameters and moments

We calibrate the earnings process outside the model using estimates from Floden and Lindé (2001), Table 4. In particular, the variance of the log of the transitory component of earnings reported in Floden and Lindé (2001) is 0.0421. We approximate this process by a three-point uniform distribution on support $\mathcal{Z} = \{-z, 0, z\}$, where $z = \sqrt{\frac{3}{2} \cdot 0.0421} = 0.18$. The persistent component of earnings is an AR(1) in logs, with autocorrelation of 0.9136 and innovation variance of 0.0426. We approximate this process by a 3-state Markov process using the method developed by Adda and Cooper (2003). The resulting support, $\mathcal{E}$, and transitions, $Q^e(e'|e)$, are given in Table 2.1b.

Aside from the earnings process, there are 9 parameters that must be chosen. In addition to these, we must also set appropriate grids for the remaining state variables, (β, e, a, s), which are relevant for an individual's decision problem. Details on parameters are contained in Tables 2.1a; similar information on grid is contained in the Appendix, Table B.1.

We divide the set of parameters into two groups: those we calibrate, and those we choose based on previous studies. The calibrated parameters include: (i) the extreme value scale parameter, α; (ii/iii) the two switching probabilities for each β type, $Q^\beta(L|H)$ and $Q^\beta(H|L)$; (iv) the discount factor for the low type, β_L; and (v) the exogenous default cost, η. We use a two-state type process because this allows us to collapse the generally vector-valued state variable s and equilibrium functions $\psi(\cdot)$ and $\Gamma(\cdot)$ into a scalar.[13] Subsection 2.4.2 below

[13]That is, we now have $s(\beta_L) = 1 - s(\beta_H)$, and likewise for $\psi(\cdot)$ and $\Gamma(\cdot)$.

discusses the scale parameter in detail. Intuitively, the nature of the discount factor type process is important in the model: the farther away the two types are, the less important should we expect type scoring to be. This is because if the types have very fundamentally different preferences, their actions should be sufficiently different so that the intermediary can get a relatively clean read on a given agent's type based on their behavior over time. Put differently, the benefit of acting like the other type (in terms of reputation and the associated prices) will be small relative to the cost of taking these actions, and so the types will separate more.[14] Similarly, the more persistent is the type transition process (i.e. the lower are $Q^{\beta}(L|H)$ and $Q^{\beta}(H|L)$), the greater is the benefit to the intermediary of inferring correctly an agent's type, since this assessment is unlikely to be wrong based on a transition of actual type. Finally, the exogenous default cost η is used to mitigate the extent of default in the model, and to study the relative efficacy of static and dynamic punishments (Section 2.6.2).

The parameters we select rather than calibrate are: (i) the discount factor of the high β type, β_H; (ii), the coefficient of relative risk aversion, ν; (iii) the risk-free rate r; and (iv) the cost of intermediation, ι. The high β is standard for models with a period length of one year. The CRRA, which indexes the curvature of the flow utility function $u(c) = c^{1-\nu}/(1-\nu)$, is standard in the macro literature. A risk-free rate of 3% is consistent with the observed average for a one-year time horizon, and intermediation costs of 2% roughly reflect fixed costs in operating a bank.

We calibrate these parameters to a set of five moments drawn from Chatterjee et al. (2007) and Chatterjee and Eyigungor (2015). These moments, presented in Table 2.2 below, are: (i) the economy-wide default rate; (ii) the average interest rate paid in the economy; (iii) median assets to median income; (iv) the fraction of households in debt; and (v) the aggregate debt to earnings ratio. For details on the computation of these moments within the model, please see

[14] Note that under the extreme value shocks perfect separation is not possible because all feasible actions are chosen with positive probability.

Appendix B.3. We have chosen these targets for two reasons. First, since the fact that they are standard in the literature allows for simple comparison across studies. Second, they provide convenient metrics for the size and riskiness of unsecured credit markets, without constraining our model to directly match key facts about credit scores and prices, allowing these moments to serve as validation for our model.

For a given set of parameters, we can compute the model analogs of the moments presented in Table 2.2, applying Simulated Method of Moments (SMM).[15] We require that the aggregate line of the benchmark model match the data as closely as possible: the other figures in the table are presented for discussion and comparison. The benchmark model delivers a tight fit to the data in the aggregate, with the notable exception of median net worth to median income.[16]

2.4.2 Details of extreme value preference shocks

In relation to the standard macroeconomics and finance literature, one of the key modifications in our model is the addition of the additive, action-specific, extreme value preference shocks. Although these shocks are assumed to be mean zero, we have allowed the variance to be general in formulating the model through the scale parameter α.[17] How does behavior in the model change with this variance? Following Train (2002), a simple calculation shows that

[15]Let M^D be the 5-vector of data moments, and let $M(x)$ be the analogous vector of model moments implied by the set of parameters $x = (\alpha, Q^{\beta}(L|H), Q^{\beta}(H|L), \beta_L, \eta)$. The estimation problem, then, is simply

$$\hat{x} = \arg\min_{x} (M^D - M(x))' W (M^D - M(x)), \tag{2.13}$$

where W is an appropriately chosen positive semi-definite weighting matrix. In our computations, we set $W = I_5$. Because each computation of our model is quite costly and the equilibrium objects are highly nonlinear, we use the derivative-free, least squares minimization routine developed in Zhang et al. (2010).

[16]This is to be expected, since ours is a net worth model in which agents cannot simultaneously hold both assets and debt.

[17]This amounts to setting the location parameter of the extreme value distribution equal to 0. Since our model contains no outside option, and only the $difference$ between the shocks associated with each pair of actions matters for the determination of choice probabilities, this normalization has no impact on behavior in the model.

$\partial\sigma^{(d,a')}(\beta,e,z,a,s)/\partial\alpha$ takes the sign of

$$\sum_{(\tilde{d},\tilde{a}')\in\mathcal{F}(e,z,a,s|f)} \left[v^{(d,a')}(\beta,e,z,a,s) - v^{(\tilde{d},\tilde{a}')}(\beta,e,z,a,s)\right] \cdot \exp\left\{\alpha\cdot\left(v^{(d,a')}(\beta,e,z,a,s) + v^{(\tilde{d},\tilde{a}')}(\beta,e,z,a,s)\right)\right\}$$

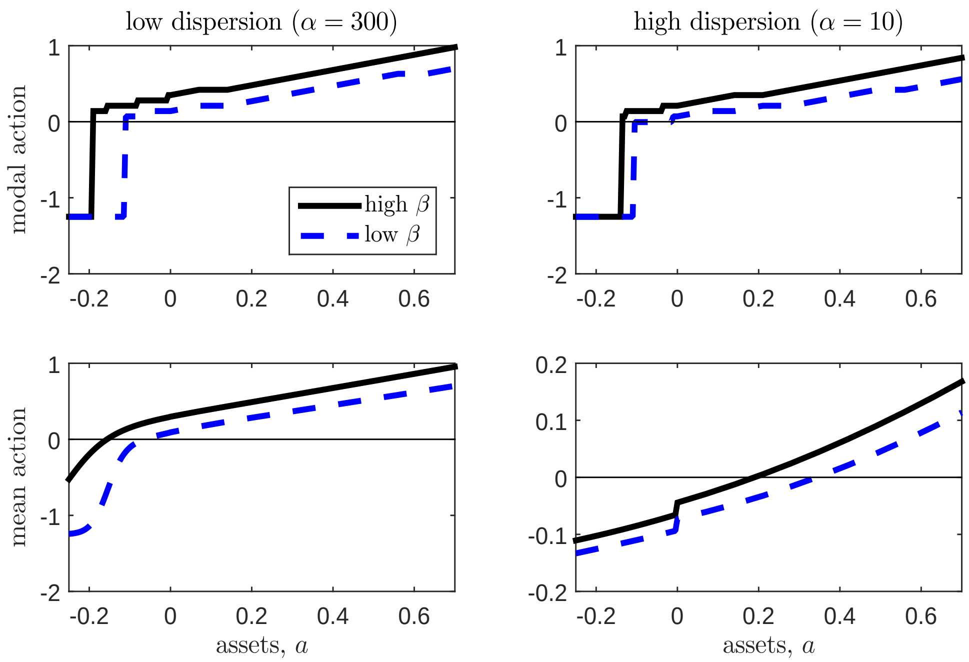
Furthermore, examining equation (2.4) reveals that

$$\arg\max_{(d,a')\in\mathcal{F}(e,z,a,s|f)} \sigma^{(d,a')}(\beta,e,z,a,s) = \arg\max_{(d,a')\in\mathcal{F}(e,z,a,s|f)} v^{(d,a')}(\beta,e,z,a,s),$$

so that the action which delivers the highest total utility *before* the extreme value shock is chosen with the highest probability. Combining these two pieces of information, we see that as α increases, the probability of choosing the action with the highest conditional value increases relative to all other feasible actions. Put differently, as α increases, more and more weight is placed on the modal action, and the mean action converges to the modal action. For actions that are "suboptimal" in the sense that they deliver lower conditional value than the modal action, the change in weight depends on (i) the degree to which the action is suboptimal relative to other actions and (ii) the total value of these actions. This can have a meaningful impact on the mean action taken, if not the mode, which can effect prices and type scores significantly.

Figure 2.1 demonstrates the impact of changing α on decisions in the model. The top two panels present the *modal* decisions for each β across all a for $e = 1, z = 0$ in the full information case of the model, discussed in Section 2.5.2 below. That is, we plot the a' level that corresponds to $\arg\max_{(d,a')}\sigma^{(d,a')}(\beta,1,0,a)$. Note that the bottom of the asset grid is -0.25, and we represent the default decision $(d,a') = (1,0)$ by $a' = -1.25$ on the graph. The bottom two panels depict the *mean* decision $E(a') = \sum_{a'\in\mathcal{Y}} a'\cdot\sigma^{(d,a')}(\beta,1,0,a)$ for the same subset of the state space. The left panels are for a high value of the scale parameter (low dispersion), $\alpha = 300$; the right panels are for a lower value of $\alpha = 10$ (high dispersion).

Immediately, we see that there are two key changes to observed decisions in the model resulting from changing α. First, as the bottom two panels reveal, the mean action is much



Notes: These figures are example figures which represent agents' equilibrium decision rules at the specified state under the parameters in Table 2.1a, only with the value of the extreme value scale parameter, α, changed. Modal actions have the highest value in the $\sigma(\cdot)$ function for that state, while mean actions weight over all choices given all σ values.

Figure 2.1: Impact of extreme value preference shocks

closer to the modal action for the higher value of α; this graphically confirms the intuition discussed above, since the modal action is chosen with higher probability. Second, we can observe that the modal action itself is not invariant to α. It is crucial to note that this is an *equilibrium property* of our model, whereas the first point would remain true even if we solved only the decision problem in partial equilibrium, taking the set of equilibrium functions f – and therefore prices – as given.

To flesh out this idea, consider the example of the high β type default decision (the solid black line in Figure 2.1). With $\alpha = 300$, the modal action is default for the high β type only at the very bottom of the asset grid. When $\alpha = 10$, though, she defaults (in the mode) for every asset position lower than about -0.11. How might this come about? Since higher α translates to lower weight placed on actions that yield less than the conditional value, the agent rarely defaults on debts in the model with high α. However, as the agent goes deeper into debt, the benefit of defaulting relative to other feasible actions increases as the required repayment increases, and so default begins to look more attractive: this effect is present in both cases in Figure 2.1. Only for the lower α, though, is the weight placed on the suboptimal default action sufficiently high to actually impact prices. This price effect makes defaulting relatively more attractive than choosing other (still feasible) debt levels, and so defaulting becomes the modal action for sufficiently large debts.

2.5 Model Properties

2.5.1 Benchmark equilibrium

In this section, we present the key properties of the benchmark model presented in Section 2.3 and calibrated in Section 2.4.

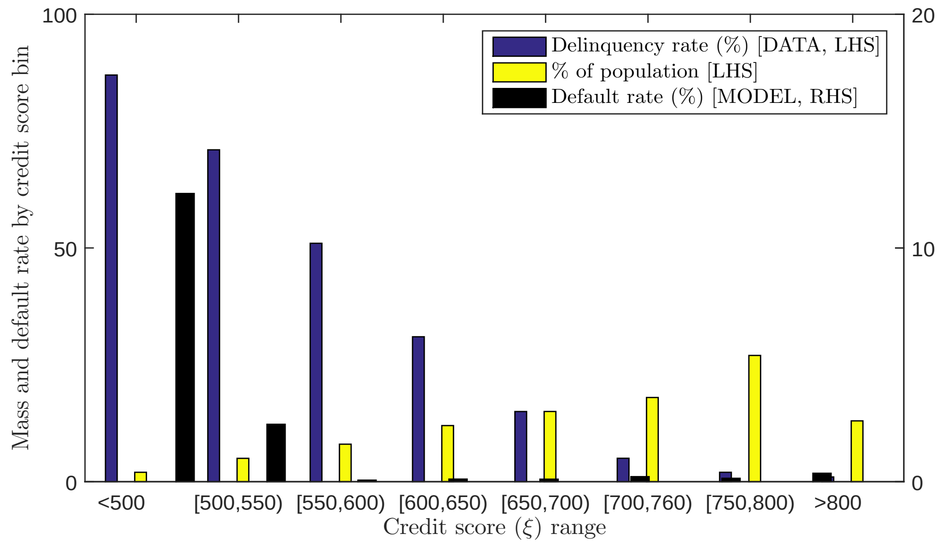
Perhaps the most important point to consider is the behavior of the different types in the model. Looking at Table 2.2, we see that high β types: (i) default at a much lower rate (0.39%, compared to 0.61% for low β types); (ii) face higher interest rates on average (10.06% vs. 9.92%); (iii) hold more assets and take on debt less frequently (high median net worth to median income, half as high a fraction in debt as low types); and (iv) take on smaller debts when they do go into debt. At first glance, all these properties appear consistent with intuition, except for the interest rates. How can agents who default less often face higher interest rates on average in an environment with perfectly competitive, risk neutral pricing? The answer lies in a selection effect: for a given debt level choice, high β types tend to face more favorable terms, but they tend to choose deeper debt levels whose increased risk demands a higher interest rate. Note that these debts are larger only in the absolute sense, since higher β types tend to take out less debt relative to their income (last column).

While these core moments help underscore the differences in behavior between the two types in the benchmark model, it is difficult to tease out the critical effects of reputation by considering only the stationary equilibrium of the benchmark economy. Therefore, in the remaining sections of the paper, we use the estimated model to compute moments, conduct simulations, and run counterfactual experiments which can more directly address this question.

Credit scoring: data vs. model

In the introduction, we motivated our analysis of credit scoring by appealing to the stark trend of delinquency profiles by credit score subgroup in Figure 2.2. Since we have not calibrated our model to anything to do with credit scores directly, a good test of the model is to compute credit scores, divide the population into ranges, and compute the default rate within each credit score range in order to compare the profiles of each credit score subgroup in our model to those in the real world. The result of this analysis is presented in Figure 2.2 below.

In order to construct Figure 2.2, we must first compute the analog of a credit score in our model. While the technical details of this procedure are contained in Appendix B.3, it is worthwhile to motivate the definition and outline the computational procedure here. In the real world, consumers have a credit score which, in principle, summarizes all the knowable, relevant information about the consumer which affects their probability of repayment. In general, the consumer then takes on credit at a rate jointly determined by (i) this credit score and (ii) the size of the loan they wish to take out. In our model, there is a vector of prices, one for each possible action the agent can take, which also depends on all the knowable, relevant information about the agent: namely, e, z, a, and s. Since these prices are action-specific, the appropriate construction of a credit score must integrate out over possible actions a given agent in observable state (e, z, a, s) can take using knowledge of (i) the equilibrium choice probabilities $\sigma(\cdot)$ and (ii) the stationary distribution of agents in the population, $\mu(\cdot)$.



Notes: Delinquency data are provided by Ronel Elul of the Federal Reserve Bank of Philadelphia. The percent of the population in each credit score bracket in the data is taken as given, and the model analog brackets are computed by defining the same percentile ranges. The default rate in the model is then computed as the average within these ranges.

Figure 2.2: Distribution of credit scores, default and delinquency

Performing this calculation gives the average probability of default in the next period *before* a given action is chosen. An analogous procedure can give the probability of default in the next n periods.

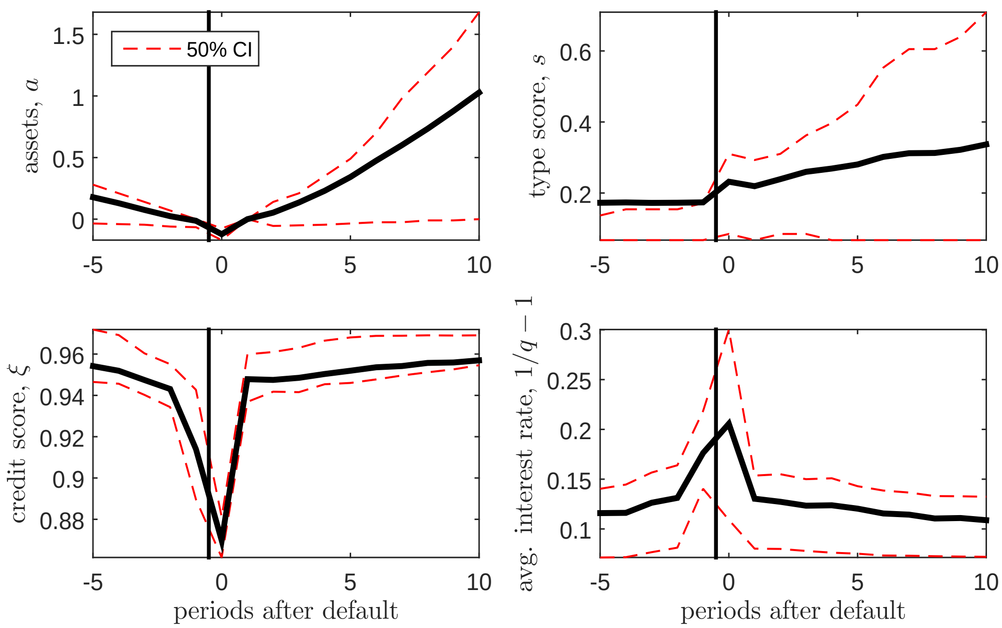
Upon examining Figure 2.2, we immediately see that the distribution of default rates over the different credit score brackets in our model *very* closely resembles those in the real world presented in Figure 2.2. Figure 2.2 is constructed by taking the population fractions within each credit score bracket as given in order to define the relevant score thresholds, and then computing the default rate between these thresholds. We see that default is very common (about 90\%) for the lowest credit scores, and becomes less and less common as credit score increases, until it disappears at the top end of the distribution. One interesting divergence between the two figures is that our model produces more default on the low end of the credit score range than what we observe in the real world, and less default at the high end of the range. This is directly attributable to the fact that, in our model, the high end of the credit score range is occupied exclusively by individuals who have $a > 0$. Since our model features only a *net worth* savings decision – i.e. agents cannot simultaneously have a stock of assets ($a > 0$) and take on debts ($a < 0$) – these agents by construction cannot default. This is an important difference between the environment of our model and the one in the real world, where having positive assets (say, a savings account) and debts (say, a mortgage) are very common. The reasons for this are outside the scope of the current study.

What happens before and after default?

In the introduction, we alluded to the fact that individuals tend to face significantly less favorable terms in the wake of defaulting. Is this the case in our model? If so, why? In this section, we use the estimated model to simulate a panel of individuals over a large number of periods in order to isolate trends in key state variables and pricing terms leading up to and following default events.[18]

Each panel of Figure 2.3 depicts a relevant state or pricing variable of an individual who chooses to default in the period indexed as 0. The solid black lines represent the mean, and the dotted red lines are the inter-quartile range. It is most natural to read this figure from left to right and top to bottom. First, in the top left we see that agents naturally tend to decline into debt, default on a relatively large debt, then immediately begin to save out of debt. Interestingly, the dispersion in wealth post-default is quite wide. Next, in the top right, we see that agents tend to be assigned very low type scores (i.e. high probability of having $\beta = \beta_L$) while declining into debt. Then, as they begin to save out of debt in the wake of their default, their reputation recovers, and they are assigned higher and higher probabilities of having a

[18]Specifically, we construct a pseudo-panel with $N = 5,000$ individuals for $T = 1,000$ periods. Then, we drop the first 100 periods, and collect all the default events with 5 periods preceding and 10 periods trailing. All summary statistics are then computed off of this sub-sample with even weighting for all observations. Note that since we simulate from the stationary distribution of the model, a default in period t is the same as a default in period t'.

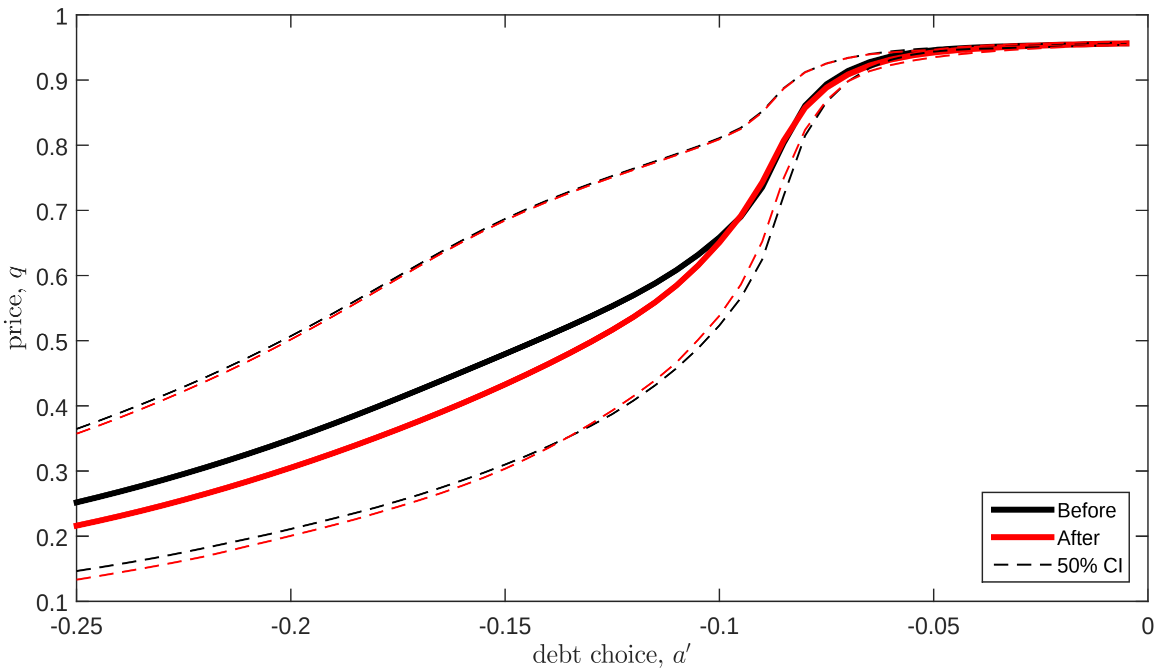


Notes: These results come from a simulation of a panel of $N = 5,000$ individuals for $T = 1,000$ periods from the model under the parameters in Table 2.1a. The black line is the sample mean response (over all agents who default), while the upper and lower dashed red lines correspond to the 75th and 25th percentiles of this distribution, respectively.

Figure 2.3: Behavior of key state variables before and after default

high β. Third, agents credit scores (bottom left) begin to decline as they accumulate debt, and then plummet in the period in which they default. This is consistent with the findings of Musto (2004). Then, as agents shed debt, they immediately begin to sharply improve their credit score. It follows logically that the first order effect of a default in our model is actually positive on net because of debt forgiveness: even if your reputation declines, shedding debt lowers your subsequent default risk. Finally, the bottom right panel plots the average (net) interest rate agents face in taking out debt before and after default. As the asset position and reputation weaken in the lead-up to a default, the typical terms of credit worsen in kind. This trend then spikes in the period after default (i.e. it would be very costly to go immediately back into debt after defaulting), and then settles back to normal in subsequent periods.

Figure 2.4 builds on this last panel of Figure 2.3. Rather than simply showing the average



Notes: These results come from a simulation of a panel of $N = 5,000$ individuals for $T = 1,000$ periods from the model under the parameters in Table 2.1a. The black line shows the average price menu defaulters face in the period before default, while the red line shows the analogous figure for the period after default. Dashed lines above and below correspond to the 75th and 25th percentiles, respectively.

Figure 2.4: Price menu for debt before and after default

interest rate from *chosen* actions, which includes the effect of transitory preference shocks, this figure depicts the entire price menu an agent faces on average before (black) and after (red) default. This allows us to see not only the selected interest rates, but also – critically – the pricing terms which drive those decisions. Most notable is the fact that terms worsen in the wake of default most severely for higher levels of debt. This completes the logical circle which extends back to the top two panels of Figure 2.3: as prices for larger debt levels worsen in the wake of default, agents are disincentivized from going deep into debt and tend to save.

2.5.2 Equilibrium with observable types

In order to understand the effect of private information in our model, we consider a case where β is observable while ϵ remains unobservable as in standard discrete choice models like **?**. In this case, the relevant state for an individual is simply (β, e, z, a), since the observability of type obviates type scoring. That is, intermediaries need not *assess* types since they can actually *see* them. Therefore, the set of equilibrium functions is simply $f^{FI} = \{p(\cdot), q(\cdot)\}$, since (i) type scores – $\psi(\cdot)$ – are no longer necessary and (ii) repayment probabilities are given entirely by repayment scores, and so $p(\cdot) = \Gamma(\cdot)$. This case offers a desirable comparison, since it isolates the effects of pricing without being clouded by the inference problem introduced in the full model.

We first compare the model's performance to our targeted set of moments. The results of this comparison are presented in Table 2.2. Note that we do not recalibrate the model for the full information case: rather, we simply compute the full information model for the parameters given in Table 2.1. Relative to the benchmark case, in the full information model: (i) agents default less frequently (0.45%, compared to 0.53% in the benchmark); (ii) average interest rates are much higher (by about 2 percentage points); (iii) median assets are a slightly higher fraction of income (2.20 vs. 2.13), while slightly fewer households take on debt (7.98% vs 8.24%); and (iv) the size of debt choices relative to income are about the same.

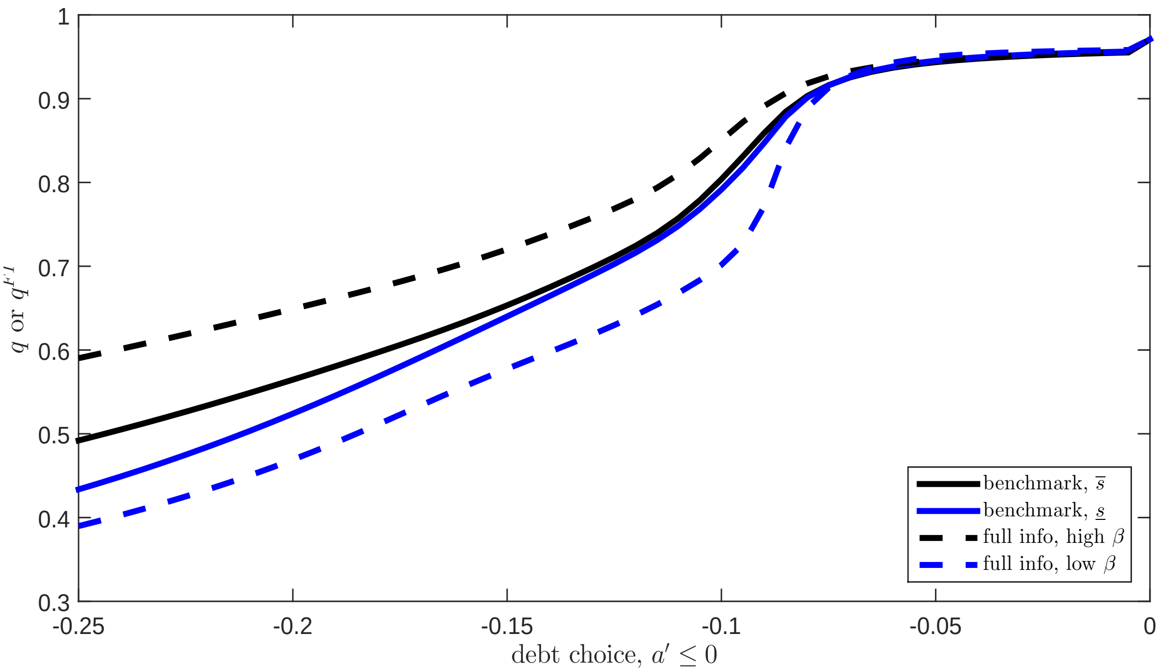
The best way to understand how private information influences behavior in the model is to compare the differences in the behavior between types under the benchmark case and the full information case. Table 2.2 shows that default is much less dispersed across types under full information, suggesting that when high β types do not stand to suffer a reputational loss (their true β is known), the incentive not to default is lowered moderately. Conversely, low β types default less (0.50% vs 0.61%) because they obtain on average less debt (debt to income of 0.72% vs. 0.77%), and take on debt less frequently (9.86% vs. 10.22%). Average interest rates rise across the board in the full information model.

Finally, we complete the discussion of the full information model's properties by presenting the full price schedules for all debt choices in Figure 2.5. The black lines are for the "best" types in the full information and benchmark models: in the full information model (dotted) these are simply high β types, while in the benchmark model (solid) these are the agents with a current type score at the highest possible level, $s = \overline{s}$. Likewise, the blue lines are for the worst types: low β in the full information case, $s = \underline{s}$ in the benchmark. Much like in the pre- and post-default image of Figure 2.4, prices worsen more and more for worse types (assessed or actual) for deeper and deeper choices of a'. Perhaps most interesting, though is the fact that price dispersion is much greater in the full information model than in the benchmark. That is, the solid lines in the figure are much closer than the dashed. This suggests that the presence of private information in the benchmark model limits the extent to which intermediaries can effectively price discriminate based on type. Furthermore, prices are "less extreme" in the benchmark model in the sense that agents with good reputation face less favorable terms than those known to have high β in the full information case; likewise, agents with bad reputation face more favorable terms than those known to have low β.

Welfare analysis

How much more consumption per period must an agent receive in the benchmark economy to be indifferent with the full information economy? In order to answer this question, we can use a procedure analogous to the consumption equivalent. Given benchmark and full information expected values, $W(\beta, e, z, a, s)$ and $W^{FI}(\beta, e, z, a)$, respectively, we can compute the measure

$$\lambda(\beta, e, z, a, s) = \left[\frac{W^{FI}(\beta, e, z, a)}{W(\beta, e, z, as)}\right]^{\frac{1}{1-\nu}}, \tag{2.14}$$



Notes: This figure shows the average price menu faced by high (black) and low (blue) reputation agents in the benchmark (solid, details in Sections 2.2 and 2.3) and full information (dashed, details in Section 2.5.2) models. For the benchmark model, a high (low) reputation corresponds to a high (low) s, whereas in the full information model, a high (low) reputation corresponds to a high (low) observed β.

Figure 2.5: Prices of debt by reputation (type) in benchmark (full information) model

where ν is the coefficient of relative risk aversion.[19] Note that a positive value of λ implies that agents are better off in the full information economy than in the benchmark economy with private information.

The results of computing this measure over the stationary distribution and for different subsets of the population are depicted below in Table 2.6a.[20] Immediately, we see that agents much prefer the full information economy on the whole, with an average value of 0.038%

[19]It is important for this analysis to integrate out the transitory preference shocks. Therefore, we use $W(\cdot)$ instead of $V(\cdot)$ in the analysis. Expression (2.14) for λ is obtained by solving

$$W^{FI}(\beta, e, z, a) = \mathbb{E}\left[\sum_{t=0}^{\infty} \beta_t^t u\left(c_t(\beta, e, z, a, s) \cdot (1 + \lambda(\beta, e, z, a, s)\right)\right],$$

where $c_t(\beta, e, z, a, s)$ is the consumption implied by agents' optimal policies in the benchmark model.

[20]Throughout, all aggregation is done with respect to the stationary distribution of the model, $\mu(\cdot)$.

for λ. High β agents prefer the full information setup more than low β agents (0.063% vs. 0.021%) because they stand to benefit from the reputation boost they'd receive if their types were revealed. Notably, low β types with bad reputation who are in debt actually $prefer$ the benchmark economy with private information. This is because these agents represent the small subset who both use the credit market and stand to gain a better reputation when information is removed from the economy. Finally, in Figure 2.6b we plot the average value of λ across the asset space and the entire range of type scores, corroborating the evidence in the table above.

2.6 Further Quantitative Analysis

2.6.1 Measuring the value of reputation

In order to measure reputation, we define for each state (β, e, z, a, s) a number $\tau(\beta, e, z, a, s)$ such that for all type scores $s \geq \underline{s}$, where $\underline{s}$ is the lowest possible type score (equal to the probability of transitioning from β_L to β_H),[21]

$$W(\beta, e, z, a, s) = W(\beta, e, z, a + \tau(\beta, e, z, a, s), \underline{s}). \tag{2.15}$$

In this sense, $\tau(\beta, e, z, a, s)$ measures how much an agent in state (β, e, z, a, s) would be willing to give up in terms of assets in the current period in order to avoid being re-assigned *today* to the lowest possible type score. We can gain insight into the relative size of reputation by integrating these τ values over the stationary distribution, i.e. by looking at

$$\overline{\tau} = \sum_{\beta,e,z,a,s} \tau(\beta, e, z, a, s)\mu(\beta, e, z, a, s), \tag{2.16}$$

or by looking at how $\tau(\cdot)$ across possible states.[22] In our model, integrated over the stationary distribution, we obtain a value of $\tau^* = 0.015\%$; for different subsets of the population, the

[21] Note: this exercise would have to be generalized to the case of vector-valued s (i.e. more than 2 types) carefully.

[22] Note that $\tau(\cdot)$ has the property that $\tau(\beta, e, z, a, \underline{s}) = 0$, since agents already in the lowest score can't be made to pay to avoid it.

results are presented in Table 2.7a and Figure 2.7b.

Looking at this table, we see immediately that the average household requires compensation in order to accept the lowest possible reputation. As expected, this compensation tends to be higher for high types than for low types (0.020% vs. 0.011%). Moreover, the value of τ tends to be much higher (9.2 times on average) for agents in debt, for whom reputation immediately and tangibly affects consumption through the equilibrium debt pricing schedule. This effect is particularly severe for agents in debt who have the best possible reputation: for example, low β types who are incorrectly assessed with $s = \overline{s}$ require compensation of 0.847% of median earnings to have their assessed type reset to the lowest level. On the other hand, agents who save require very little compensation to have their reputations lowered. Since saving always occurs at the risk-free rate in the model, they both receive no immediately adverse price impacts and tend to recover their reputations quickly by saving. Figure 2.7b fleshes out these ideas, plotting τ for each type over various slices of the state space. Most notable in this figure is the top left panel, which isolates agents currently in debt. Agents very deeply in debt can require up to 3.5% of median earnings as compensation to have their reputation set to $\underline{s}$.

2.6.2 Dynamic and static costs of default

In the spirit of the empirical findings of Albanesi and Nosal (2015), who examine the quantitative impact of a significant bankruptcy reform that went into effect in 2005 in the United States, in this section we examine how changes in the static cost of default, η, impact the dynamic value of reputation, τ. To this end, Table 2.3 contains two sets of results: the first with the benchmark calibrated value of $\eta = 9.8\%$, and the second with no static income loss, $\eta = 0$. For the second model, we maintain all other parameters at the values indicated in Table 2.1.

The first two moments in Table 2.3, which (loosely) measure the riskiness of the credit market, increase each by a factor of about 5 when we do away with the income loss associated with default. The last three moments, which (again, loosely) measure the size of the credit market in the model economy, change significantly less. In particular, the fraction of households in debt decreases from 8.24% to 6.69%, although the size of the average debt relative to income increases from 0.64% to 0.82%. Most interestingly, though, the value of τ increases by a factor of 10.5 when we go from the model with positive η to the one with $\eta = 0$. This suggests that having a good reputation becomes much, much more value the lower is the static incentive of agents to repay.

2.7 Conclusion

In this paper, we presented a model of unsecured consumer credit with an endogenous default decision and no exogenous exclusion from credit markets in the event of a default. In an environment in which households are subject to both persistent and transitory preference shocks, lenders have to solve an inference problem in order to properly assess agents' type. Given these assessments, lenders price debt in order to break even in expectation given the households' endogenously determined default risk, given perfect competition. In equilibrium, assessed default risk must be consistent with the actual choices of borrowers.

From a methodological perspective, we contribute to the extant literature in two primary ways. First, we scrap the assumption of exogenous stochastic exclusion from credit markets following a default event, and instead endogenize the dynamic punishment for default through prices. To do so, we must solve for the optimal pricing function on the lender side, which necessarily incorporates dynamic assessments of an individual's underlying type. In our model, this type captures the propensity to default. In executing this first contribution, we deliver our second contribution: the adoption of techniques from the discrete choice literature. Given the necessity of Bayesian updating for assessing the probability of an agent being of a given type, this inclusion has the desirable property that all feasible actions are chosen with positive probability by all agents, imposing good behavior on the type scoring function. Additionally, these techniques ease computation and therefore estimation of the model.

Given the calibrated version of the model described above, we explore the model's main properties and run a series of quantitative experiments designed to assess: (i) the effect of private information; (ii) the value of reputation; and (iii) the efficacy of static versus dynamic punishments in deterring default and sustaining credit in an environment with limited commitment. We find that agents would on average need to have their consumption increased by about 0.03% per period in the benchmark economy in order to be indifferent between this economy and one with full information about type. Furthermore, agents would require a non-trivial amount of compensation – about 0.015% of median earnings – in order to "lose their reputation," or be assigned to the lowest possible type score. Finally, we find evidence of significant substitutability between static and dynamic punishment for default. In the benchmark model with a relatively high income loss in the event of a default, the value of maintaining a good reputation is relatively low: that is, most of the deterrence from default is achieved by the static component of the punishment. In a world with no income loss from default, however, the value of maintaining a good reputation is significantly higher.

Parameter	Notation	Value
Calibrated		
Low type discount factor	β_L	0.89
Low β to high β transition probability	$Q^\beta(\beta'_H \mid \beta_L)$	0.05
High β to low β transition probability	$Q^\beta(\beta'_L \mid \beta_H)$	0.11
Exogenous default cost	η	9.8%
Extreme value scale parameter	α	183.3
Selected		
High type discount factor	β_H	0.97
Coefficient of relative risk aversion	ν	3
Risk-free rate	r	3.0%
Intermediation cost	ι	1.0%
Earnings	$e, z, Q^e(\cdot \mid \cdot), H(\cdot)$	See Table 2.1b

(a) Model parameters

Persistent

	$e'_1 = 0.575$	$e'_2 = 1.000$	$e'_3 = 1.740$
$e_1 = 0.575$	0.818	0.178	0.004
$e_2 = 1.000$	0.178	0.643	0.178
$e_3 = 1.740$	0.004	0.178	0.818

Transitory

level	$z_1 = -0.18$	$z_2 = 0$	$z_3 = 0.18$
probability	1/3	1/3	1/3

(b) Detail: persistent component of earnings

Notes: The parameter estimates reported in the section of Table 2.1a labeled "Calibrated" are the result of the SMM procedure described in Section 2.4.1. Those in the section labeled "Selected" are chosen outside the model. The persistent earnings process is calibrated outside the model using the estimates of Floden and Linde (2004) for a 3 point grid; the transitory component of the process is chosen externally.

Table 2.1: Parameterization of benchmark credit scoring model

	Default rate (%)	Average interest rate (%)	Med. net worth to med. income	Fraction HH in debt (%)	Average debt to income (%)
Data					
aggregate	0.54%	11.35%	1.28	6.73%	0.67%
Benchmark					
aggregate	0.53	9.98	2.13	8.24	0.64
β_H	0.39	10.06	2.80	5.24	0.44
β_L	0.61	9.92	1.76	10.22	0.77
Full information					
aggregate	0.45	11.61	2.20	7.98	0.61
β_H	0.42	12.94	2.92	5.02	0.45
β_L	0.50	10.77	1.83	9.86	0.72

Notes: The moments in this table are the moments targeted in the estimation procedure described in Section 2.4.1. "Data" moments come from the data sources described in the main text; "Benchmark" moments correspond to equilibrium values from the models described in Sections 2.2 and 2.3. "Full information" moments come from a version of the model with observable β's, described in Section 2.5.2. Both benchmark and full information moments are computed with the parameter estimates in Table 2.1a.

Table 2.2: Model moments: data, benchmark, and full information

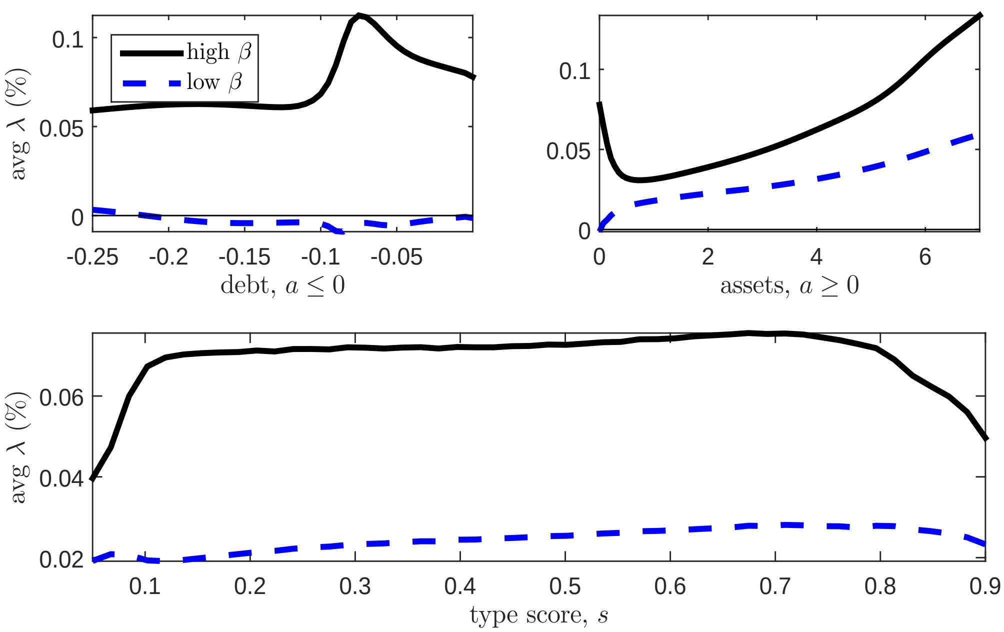
Moment	**Data**	**Model**	
		$\eta = 9.8\%$	$\eta = 0.0\%$
Targets			
Default rate (%)	0.54	0.53	2.63
Average interest rate (%)	11.35	9.98	57.73
Median net worth to median income	1.28	2.13	2.20
Fraction of households in debt	6.73	8.24	6.69
Average debt to income ratio	0.67	0.64	0.82
Other			
$\overline{\tau}$ (%)	-	0.02	0.21

Notes: The moments reported in this table are computed at equilibrium for the parameter values in Table 2.1a, with the only variation being the indicated static cost η in each column. The reported $\overline{\tau}$ is averaged across the entire distribution of agents in the economy.

Table 2.3: Dynamic and static costs of default

$\overline{\lambda}$ (%)	**aggregate**	**in debt** $(a < 0)$			**saving** $(a \geq 0)$		
		total	**worst rep.**	**best rep.**	**total**	**worst rep.**	**best rep.**
aggregate	0.038	0.016	0.020	0.048	0.040	0.895	0.049
β_H	0.063	0.052	0.076	0.165	0.062	1.729	0.000
β_L	0.021	0.014	-0.003	0.029	0.024	0.845	0.000

(a) By sub-population



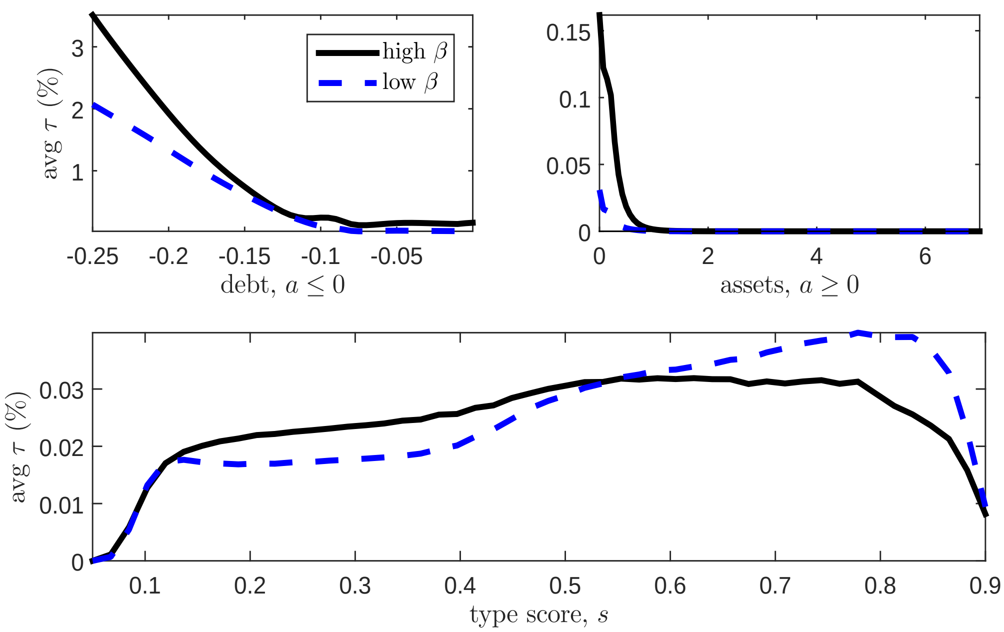
(b) Across assets and type scores

Notes: The results in panel 2.6a are aggregated over the indicated sub population in the column for each subpopulation in the row. “Worst reputation” corresponds to s at the lowest point on the grid, while ”best reputation” corresponds to s at the highest point on the grid. Panel 2.6b displays the same information in visual form. All aggregation is over the equilibrium stationary distribution μ.

Figure 2.6: Welfare analysis: benchmark versus full information

$\overline{\tau}$ (%)	**aggregate**	**in debt** $(a < 0)$ total	worst rep.	best rep.	**saving** $(a \geq 0)$ total	worst rep.	best rep.
aggregate	0.015	0.139	0.000	0.613	0.004	0.000	0.006
β_H	0.020	0.252	0.000	0.586	0.007	0.000	0.006
β_L	0.011	0.101	0.000	0.847	0.002	0.000	0.006

(a) By sub-population



(b) Across assets and type scores

Notes: The results in panel 2.7a are aggregated over the indicated sub population in the column for each subpopulation in the row. "Worst reputation" corresponds to s at the lowest point on the grid, while "best reputation" corresponds to s at the highest point on the grid. Panel 2.7b displays the same information in visual form. All aggregation is over the equilibrium stationary distribution μ.

Figure 2.7: Reputation analysis: asset value of avoiding bad reputation

Chapter 3

Credit Lines and the Amplification of Financial Shocks

3.1 Introduction

The financial crisis of 2008 has drawn significant attention to how the fragility of banks' funding sources can have sizeable impacts on the real economy. Many studies have duly highlighted disruptions to short-term borrowing markets for banks, such as the repo market, which precipitated the financial crisis in late 2008.[1] These disruptions made it difficult for banks to finance lending and other operations, and therefore exerted strong contractionary pressures on banks' balance sheets from the liability side. Many researchers have correctly identified this effect as a modern form of a bank run.

Less research has been devoted to the dynamics of the asset side of banks' balance sheets during the financial crisis. Several studies (such as Ivashina and Scharfstein (2010) and Acharya and Mora (2015)) have pointed out that, in response to indications of stress and fragility within the financial system, many firms drew heavily on their existing lines of credit with banks. This exerted expansionary pressure on the asset side of banks' balance sheets. Therefore, the banking sector effectively experienced a "two-sided bank run," as the stress on runnable liabilities was in fact accompanied by an increased demand for pre-committed loans in the

[1] For a detailed discussion, see, for example, Acharya et al. (2013b); Brunnermeier (2009); Gorton and Metrick (2012).

form of credit lines.

These simultaneous effects suggest new questions for the macro-finance literature which has grown in response to the questions posed in the wake of the financial crisis. How much did drawdowns on credit lines deepen the financial crisis of 2008? How much did the increased lending burden on banks during this crisis slow the recovery from the recession associated with the crisis? In order to assess these questions at the macro level, we first need to know much more about the nature of credit lines in the banking industry. To this end, in this paper I analyze empirically banks' responses to increased drawdowns on lines of credit. Specifically, I ask two main questions. First, how do banks fund drawdowns of credit lines? Second, how does meeting lending obligations associated with credit line drawdowns impact banks' profitability?

Credit lines are contractual arrangements whereby a bank specifies to a firm that it may draw up to a maximum amount of funds at a pre-specified interest rate (or spread over a benchmark rate) at any point within a given time period. These facilities stand in contrast to the usually-studied term loans, in which the bank supplies the amount of funding at the initiation of the contract. Although many practitioners consider lending in the form of a credit line rather than a term loan "business as usual," the timing of the actual funding of the loan is critical. Under a credit line, the bank promises to deliver funds to the firms at any point within the period, but does not immediately fund these loans. Therefore, if there is a stress event in between the initiation of the credit line contract and the point at which the firm draws on the credit line and demands funds from the bank, banks may be compelled to make loans that are unprofitable or excessively costly to fund *ex post*.

Given the potential for such time inconsistency in banks' willingness to lend, then, why do they enter into these credit line contracts? Kashyap et al. (1993) and Gatev and Strahan (2006) use an equilibrium argument to argue that banks are efficient providers of insurance to firms in the form of credit lines. Specifically, they show that when firms impose stress on banks due to increased drawdowns on credit lines, banks typically experience an inflow of cheap funds (mostly deposits) through an equilibrium "flight to safety" effect associated with the aggregate shock that imposed the original stress on firms. While this argument holds for normal recessions, I argue that this classic link broke down during the recent financial crisis. That is, as firms saw their borrowing costs increase and accordingly turned to their backup lines with banks, banks did not see a reduction in their borrowing costs. In fact, the opposite occurred, and so banks were hit with increased stress on both sides of their balance sheet.

This paper is devoted to documenting and analyzing these effects empirically. I begin by documenting key trends in the aggregate about credit line lending at commercial banks in the United States using data from the Call Reports. I show that, after a large runup in credit line issuance in the 1990s, recessions have been associated with large draws by firms on existing lines of credit. This effect was particularly pronounced in the recent financial crisis. Given this "reintermediation" effect, it is natural to ask how banks' funding responds in the aggregate during recessions. Here, I document a divergence between the two most recent recessions. In 2001, deposits flowed into banks and banks were able to maintain a stable funding mix before, during, and after the downturn. In 2008, on the other hand, banks did not see these immediate inflows, and had to lean even more heavily on other sources of financing, precisely when the markets for such financing were disrupted.

Motivated by this aggregate evidence, I then leverage the micro-level panel data in the call reports to analyze banks's response to credit line drawdowns. In order to overcome data limitations and endogeneity concerns, I adopt an "event study" framework to analyze banks' responses to "large drawdown events," defined as instances in which the ratio of unused credit lines to the sum of unused credit lines plus total loans decreases significantly within a period.[2]

I find striking results in favor of the posited two-sided run mechanism using this framework.

[2]For robustness, I also include an alternative specification which does not mimic an event study. My results from this alternative specification corroborate the main reported findings.

First, banks who experience significant credit line drawdowns tend to expand their balance sheets: that is, they respond by issuing new liabilities to offset their new assets, rather than shedding or selling off existing assets. Second, they finance these new loans mostly with non-deposit debt. Critically, this effect exposes them to fragility when drawdowns on credit lines occur alongside disruptions in short-term borrowing markets. Third, banks' net interest margins generally decline in response to increases in drawdowns.

When I examine the analogous results during the period of the financial crisis of 2008, however, I find important departures. Banks still turn to non-deposit debt to finance credit line draws, but they expand their balance sheets by less, indicating that they also are forced to shed assets, possibly exposing them to fire sale effects. Furthermore, banks' net interest margin tends to decline after a large drawdown episode in the crisis: this indicates the possibility of persistent effects through the recovery phase of the severe downturn.

3.1.1 Related literature

Many studies document empirically the role which a run in or collapse of wholesale funding markets for banks played in the sudden crisis of 2008 (see, for example: Acharya et al. (2013b); Gorton and Metrick (2012); Brunnermeier (2009)). Broadly, these studies demonstrate that uncertainty about the quality of assets originated by banks (particularly, sub-prime home loans) and packaged into securitized products like mortgage-backed securities (MBS) and asset-backed commercial paper (ABCP) caused liquidity in the markets for these assets to dry up. As a result, they were less valuable as collateral in repo financing, and as a result banks faced increased costs of borrowing at the height of the crisis.

As funding became more expensive for banks due to the "run on repo," though, banks experienced another crucial – though far less discussed – type of stress. Specifically, as documented in great detail in Ivashina and Scharfstein (2010), firms with pre-existing lines of credit with banks increasingly drew down on these credit lines in defense against the uncertainty pervading traditional funding markets for non-financial firms, such as the corporate bond and commercial paper markets. Therefore, precisely as funding became more expensive, banks had to fund a large amount of C&I lending based on prior commitments. Banks thus experienced a "double run" (Ippolito et al. (2015)), simultaneously bearing contractionary forces on the liability side of the balance sheet and expansionary forces on the asset side.

In typical crises, general equilibrium effects mitigate the severity of a double run. In particular, adverse shocks to credit markets encourage savers to fly to safety, flooding the banking system with cheap deposit financing precisely when the onus of credit line drawdowns is large (Kashyap et al. (1993); Gatev and Strahan (2006)). When credit conditions deteriorate *and* savers face uncertainty about bank safety, though, this standard equilibrium effect can break down as documented in Acharya and Mora (2015) – this is the focus of the present paper.

Many theories exist as to why firms use lines of credit: Acharya et al. (2013a, 2014); Boot et al. (1987); Holmstrom and Tirole (1998) are good examples. Furthermore, a long literature dating back several decades seeks to determine firms' choices in using credit lines over cash to manage liquidity (Sufi (2009); for crisis periods in particular, see Cornett et al. (2011); Campello et al. (2011)) and the determinants of credit line contract terms (Boot et al. (1987, 1993); Thakor (2005)). An excellent review of these issues is presented in Demiroglu and James (2011). Finally, there is large and growing evidence that firms use credit lines to finance real activity, rather than simply to manage liquidity in the very short term: see, for example, the study by Berrospide and Meisenzahl (2015).

3.1.2 Roadmap

The rest of the paper is organized as follows. Section 3.2 presents the basic data used in the paper and documents key facts about credit lines, drawdowns, and bank funding in normal times, recessions, and financial crises. Section 3.3 explores banks' responses to surges in credit line drawdowns using an event study framework. Section 3.4 concludes. Additional results complementary to those in Section 3.3 can be found in Appendix C.

3.2 Data and Stylized Facts

This section presents analysis of both macro- and micro-level trends in credit line drawdowns and bank funding in the years leading up to the and following the 2008 financial crisis. Unless otherwise noted, all data are for a sample of commercial banks from the Consolidated Reports of Condition and Income ("Call Reports"), maintained by the FDIC. For a detailed description of the data cleaning process and sample selection, see Section 3.2.1.

Section 3.2.2 contains summary statistics on the sample of banks used for the analysis in this section. In Section 3.2.3, I use the sample to present aggregate trends in bank credit line lending and capital structure. These trends and observations motivate the micro-level analysis of Section 3.3.

3.2.1 Data: Call Reports

Sample selection

The Call Reports are maintained and updated quarterly by the Federal Deposit Insurance Corporation (FDIC) and contain detailed balance sheet and income statement data for banks operating in the United States. Data are assembled at the holding company level. All the statistics reported in the paper from the call reports come from a data set I have cleaned and subsetted in the following ways: (i) include only banks in the 50 states and Washington, D.C.; (ii) include only commercial bank holding companies; (iii) delete observations with zero, negative, or missing assets, loans, or unused commitments; (iv) remove observations with spurious percent changes in the unused commitment ratio (discussed below); and (v) delete

Step	Step name	#obs. dropped	#obs. remaining
0	*#obs. in raw data*	-	*828,654*
1	drop if outside 50 states + D.C.	1,617	827,037
2	drop if not commercial bank BHC	84,765	742,272
3	drop if missing assets or loans	1,923	740,349
4	drop if missing unused commitments	49,141	691,208
5	drop if $\Delta UCR_{it} < -1$ or missing	42,739	648,469
6	drop if $< N_i = 25$ consecutive observations	121,578	526,891
7	*#obs. in final sample*	-	*526,891*

Notes: Step 1 corresponds to RSSD9210 between 0 and 57. Step 2 corresponds to RSSD9048 = 200 and RSSD9331 = 1. Step 3 corresponds to RCFD2170 or RCFD1400 zero, negative, or missing. Step 4 corresponds to total unused commitments (sum of RCFD3814, RCFD3815, RCFD3816, RCFD3817, RCFD3818, RCFD6550) zero, negative, or missing. Step 5 corresponds to the percent change in total unused commitment ratio (see (3.1)) less than -1 or missing. Step 6 eliminates all observations of bank i if bank i does not have at least 25 consecutive observations. The final sample is an unbalanced panel of 5,427 banks over 84 quarters.

Table 3.1: Data cleaning: from raw data to final sample

banks with less than 25 consecutive observations over the sample period of 1990Q1 through 2010Q4.[3] The resulting sample contains 526,891 bank-quarter observations, and this data cleaning process is summarized in Table 3.1.

Limitations and measurement

The focus of the present paper is on credit lines and how firms drawing on these credit lines impact banks. With this in mind, an important limitation of the Call Reports is that they only allow researchers to distinguish between on-balance-sheet "loans" and off-balance-sheet "unused commitments." In particular, one cannot disentangle a loan that comes from a credit line being drawn from a simple newly originated term loan.

To see this, consider Figure 3.1 below. The bank's balance sheet at date 0 consists of securities (S) and loans (L) on the asset side, and deposits (D_1), non-deposit debt (D_2), and equity (E) on the liability side. In addition, there is an amount of unused credit lines X, which are off-balance sheet. Suppose that, between dates 0 and 1, firms use a fraction ω of

[3] I select this as the sample period because there is limited availability of reliable data on loan commitments in the call reports before 1990.

Balance sheet BEFORE drawdowns (Date 0)

Assets	Liabilities
Securities (S)	Deposits (D_1)
Loans ($\tilde{L}_0 = L$)	Other Debt (D_2)
	Equity (E)

Off BS
Unused Credit Lines ($\tilde{X}_0 = X$)

$UCR_0 = \frac{X}{X+L}$

Balance sheet AFTER drawdowns (Date 1)

Assets	Liabilities
Securities (S)	Deposits (D_1)
Loans and Used Credit Lines ($\tilde{L}_1 = L + \omega X$)	Other Debt (D_2')
	Equity (E)

Off BS
Unused Credit Lines ($\tilde{X}_1 = (1-\omega)X$)

$UCR_1 = \frac{(1-\omega)X}{(1-\omega)X+[\omega X+L]} = (1-\omega)UCR_0$

Notes: This figure represents the changes to a standard, stylized bank balance sheet brought about by a drawdown on a line of credit. Assuming no new loan originations, when a fraction ω of off-balance-sheet loan commitments X are drawn, the asset side of the balance sheet is expanded by the amount ωX. Since these new loans, which were previously just commitments, must be funded, by the balance sheet identity either (i) the liability side must also expand by ωX; (ii) the asset side must contract by an amount ωX (by shedding other assets); or (iii) a combination of (i) and (ii). This figure illustrates the simple case where the loans are funded with non-deposit debt. "Off BS" refers to items off the bank balance sheet.

Figure 3.1: Illustration of credit line drawdowns on bank balance sheets

their unused credit lines. Then, at date 1, the banks' balance sheet shows loans of $L + \omega X$, and unused credit lines of $(1-\omega)X$, *assuming that there is no new loan issuance or retirement*, i.e. that L does not change. Using the call report data, the researcher can only observe that loans are equal to $\tilde{L}_0 = L$ and $\tilde{L}_1 = L + \omega X$ and commitments are equal to $\tilde{X}_0 = X$ and $\tilde{X}_1 = (1-\omega)X$ at dates 0 and 1, respectively.

The ideal data source would allow me to directly observe L_t, X_t, and ω_t directly, not simply $\tilde{L}_t$ and $\tilde{X}_t$. In order to measure changes in credit line usage, then, I must construct a metric that bypasses this issue. To this end, I define the unused commitment ratio for bank i at time t (UCR_{it}) as the ratio of unused commitments to the sum of unused commitments plus total

Size group	Assets ($M)	Loans ($M)	Leverage	Deposit share	UCR
Bottom 50%	49.5	30.1	0.89	0.96	0.10
	(24.7)	(18.4)	(0.04)	(0.06)	(0.07)
51st - 75th percentile	135.2	85.9	0.90	0.95	0.12
	(44.0)	(38.7)	(0.03)	(0.06)	(0.07)
76th - 90th percentile	294.3	191.4	0.91	0.93	0.14
	(106.8)	(91.3)	(0.03)	(0.08)	(0.07)
91st - 95th percentile	666.0	436.7	0.91	0.90	0.17
	(217.0)	(190.2)	(0.03)	(0.10)	(0.08)
96th - 99th percentile	2,765.4	1,753.4	0.91	0.85	0.20
	(2,041.7)	(1,382.6)	(0.04)	(0.13)	(0.10)
Top 1%	62,317.0	34,826.0	0.90	0.75	0.31
	(140,420.3)	(69,482.4)	(0.04)	(0.16)	(0.14)

Notes: Leverage is the ratio of total liabilities to total assets. Deposit share is the share of deposits relative to total liabilities. The unused commitment ratio (UCR, see (3.1)) is defined as the ratio of total unused commitments (excluding credit cards) to the sum of total unused commitments (excluding credit cards) and total loans. The main numbers are within group means, and the numbers in parentheses are within group standard deviations. Size divisions come from asset percentile divisions.

Table 3.2: Summary statistics for sample of banks

loans:

$$UCR_{it} = \frac{\text{unused commitments}_{it}}{\text{unused commitments}_{it} + \text{total loans}_{it}} = \frac{\tilde{X}_{it}}{\tilde{X}_{it} + \tilde{L}_{it}}. \tag{3.1}$$

This measure is increasing in unused commitments and decreasing in total loans. Therefore, fixing total loans and the amount of credit lines, UCR decreases when firms draw on their credit lines.

3.2.2 Summary statistics

Table 3.2 shows selected summary statistics about bank size, funding, and commitment lending, grouped across the bank size distribution by assets. As has been well-documented, the bank size distribution has an extremely fat right tail, with the top 1% of banks on average being much, much larger than the bottom 50% of banks. Leverage is relatively constant and high across the entire bank size distribution, between 89% and 91% on average for each size group.

The last two columns in Table 3.2 show key differentiating features between banks in terms

Size group	Commitment type Credit card	Residential real estate	Secured CRE	Unsecured CRE	Other
Bottom 50%	0.05	0.07	0.18	0.01	0.68
51st - 75th percentile	0.06	0.12	0.24	0.01	0.56
76th - 90th percentile	0.06	0.15	0.26	0.01	0.52
91st - 95th percentile	0.08	0.17	0.23	0.01	0.51
96th - 99th percentile	0.11	0.16	0.19	0.01	0.53
Top 1%	0.16	0.13	0.07	0.01	0.62

Notes: This table reports the share of total unused commitments comprised of each type of commitment in each column for each of the bank sizes in the rows. Size divisions come from asset percentile divisions.

Table 3.3: Breakdown of bank credit lines by type

of funding and the extent to which they take on large off-balance sheet lending exposures via commitment lending. First, smaller banks almost exclusively use simple deposits for their debt financing (96% of total debt), while larger banks tend to use significant amounts of other types of debt, with deposits only accounting for 75% of total debt. This suggests that larger banks rely on more fragile, "runnable" sorts of external financing, typically repo markets and unsecured debt issuance.

Second, larger banks also tend to have large off-balance sheet commitment balances as measured by UCR. The smallest banks maintain commitments of less than 10% of their total lending, while the analogous figure for the largest banks is above 30%.[4] This feature of the data has several possible explanations. First, large banks may simply have the scale to provide the large credit lines to large firms who require them. Second, since large banks tend to have access to more cheap sources of funding than small banks, they may be the efficient providers of backup lines of credit which firms use as insurance against credit market shocks.

Table 3.3 breaks down bank loan commitments by type of lending across the size distribution of banks. In each cell, the reported figure represents the share of total *unused* commitments

[4]The UCR measure reported in Table 3.2 excludes lines of credit associated with credit cards. Since credit cards are largely used as means of payment and have very different risk and liquidity profiles from other types of loans, I follow the standard in this literature and exclude them from my core measure.

that are unused commitments of the given loan type. Most notably, more than half of all unused loan commitments fall into the category of "Other" lending, which is comprised mostly of standard credit line facilities with commercial and industrial firms. For smaller banks, credit card lending comprise less than 10% of total unused commitments, while this figure is about 16% for larger banks. This reflects the considerable concentration and value of name recognition in the credit card industry. Real estate lending comprises the rest, and these facilities have similar purposes and usage patterns to a standard C&I facility.

3.2.3 Aggregate trends and stylized facts

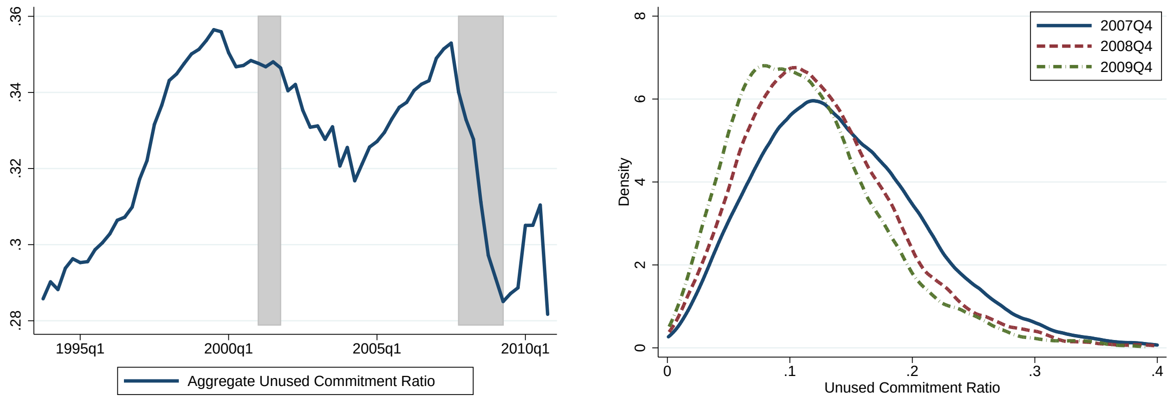
In this section, I present a series of aggregate trends in credit line drawdowns and bank funding. The evidence presented in this section is intended to be suggestive of the potential for real, macroeconomic impacts which can occur when banks experience significant drawdowns of off-balance-sheet credit lines, particularly in recessions in financial crisis. The analysis is presented as a series of stylized facts, accompanied by more detailed discussion.

Fact 3.1 *Commercial banks experience increases in drawdowns on credit lines during and following recessions. This effect was particularly large in the financial crisis of 2008.*

Figure 3.2 depicts time series trends in the aggregate UCR[5] over the course of the sample, as well as cross-sectional trends in individual banks' UCR around the time of the crisis. In Figure 3.2a, we see immediately the sharp drop in aggregate UCR during the most recent recession, with a peak to trough drop by nearly a fifth. Since new loan originations were very low during the crisis, this is evidence that firms drew heavily on their existing lines of credit during their times of stress.[6] Note that the tick down in UCR around the recession of 2001

[5] Aggregated moments for this variable, and throughout, are simply computed by summing all the relevant component variables over all banks within a given period to compute the aggregate analog, then computing the relevant ratio.

[6] For a more micro-level discussion of this effect, see Ivashina and Scharfstein (2010).



(a) Time series

(b) Distribution around the crisis

Notes: The unused commitment ratio is defined as the ratio of total unused commitments (excluding credit cards) to the sum of total unused commitments (excluding credit cards) and total loans. The data for this figure come from the Call Reports, and the density estimates come from a standard Epanechnikov kernel function.

Figure 3.2: Trends in unused commitment ratio

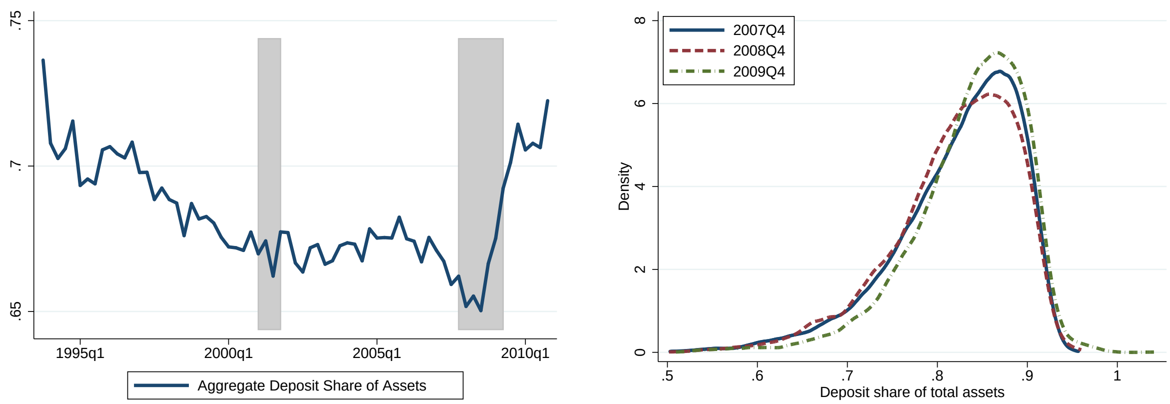
was much less (it was a much milder recession), and that during the 1990s there was a secular increase in the amount of credit lines extended.

Digging down into the crisis period around 2008, Figure 3.2b shows that the downturn in UCR in the aggregate during the crisis was not driven by extreme instances of stress, but rather by a banking industry wide trend. The distribution of UCR shifted increasingly leftward in the time around the crisis, as virtually all banks experienced increased calls to fulfill their lending commitments.

Fact 3.2 *During recessions, commercial banks usually experience inflows of deposits and cheap debt. The financial crisis of 2008, however, did not share this feature.*

Figure 3.3 plots the aggregate share of total bank funding comprised of deposits.[7] First, it is clear that there was a secular trend in the 1990s away from the traditional deposit financing model by commercial banks, as the share of assets financed by deposits fell from a peak of 74% to just over 66% before the 2001 recession. Given this lower level of deposit financing,

[7]Here and throughout, deposits include checking deposits, small time deposits, and large time deposits.



(a) Time series

(b) Distribution around the crisis

Notes: The deposit share of total assets is defined as the ratio of total deposits to total assets. The data for this figure come from the Call Reports, and the density estimates come from a standard Epanechnikov kernel function.

Figure 3.3: Trends in deposit share of total debt financing

though, the different dynamics in the recessions of 2001 and 2008 are striking. During the 2001 recession, the deposit share of total assets was unchanged. Given the increase in drawdowns on credit lines documented in Figure 3.2a, this suggests that banks saw sufficient inflows of deposits to maintain an approximately constant funding mix.[8]

During the 2008 financial crisis, though, this relationship did not hold. The deposit share of total assets decreased below 65%, before recovering even beyond pre-crisis levels by the end of the recession. In order to fund the credit line drawdowns, then, banks were forced to turn to other sources of debt financing. As has been well documented, though (see, for example, Gorton and Metrick (2012)), the markets for these sources of financing were precisely the ones experiencing the most disruption during the financial crisis.[9]

[8]Not shown in this section is the fact that bank leverage, both in the aggregate and at the micro level, changes very little over the business cycle. Therefore, given the preservation of leverage ratios, changes in debt funding mix are the critical variable of interest.

[9]In addition, Acharya and Mora (2015) document that banks with large commitment exposures were required to increase their deposit rates at the outset of the crisis.

Together, then, Figures 3.2 and 3.3 provide evidence of a “two-sided bank run.” on the one hand, firms drew down on pre-existing lines of credit, increasing the lending (and therefore funding) burden of the banking industry as a whole. All else equal, this had an expansionary effect on banks’ balance sheets. At the same time, though, as investors lost confidence in banks, banks faced great difficulty in rolling over short term borrowing facilities and attracting new deposits. This had a countervailing contractionary effect on banks’ balance sheets. The tension between these two effects exacerbates and amplifies the existing stress on the financial system. With this in mind, I link these effects more rigorously in the next section, turning to study in more detail banks’ responses to increases in drawdowns on credit lines.

3.3 Empirical Analysis

Motivated by the micro-level and aggregate trends in the previous section, I now use an event study framework to study banks’ responses to episodes of significant stress induced by surges in drawdowns on credit lines. Section 3.3.1 explains the main empirical specification for the analysis. Section 3.3.2 presents the core results, divided into two sections. First, in Section 3.3.2, I document how banks fund increased drawdowns on lines of credit. Then, in Section 3.3.2, I analyze the impacts of drawdowns on profitability measures, such as loan returns and funding costs. Section 3.3.3 checks the main results for robustness by considering an alternative empirical specification.

3.3.1 Empirical specification

In order to explore how banks respond to large increases in drawdowns on credit lines by their clients, I use an “event study” framework. The “event” of interest is what I refer to throughout as a “large drawdown episode” (LDE), an instance in which a bank’s UCR drops at least $N\%$

in a given period. Specifically, I define a $N\%$ LDE for bank i at time t via the criterion

$$\chi_{it}(N) = 1 \iff \Delta UCR_{it} < -N\%,$$

where N is a percentage threshold. For the baseline analysis in what follows, I set $N = 36.5\%$, which makes the LDEs correspond to the top 5% drops in UCR in the whole sample over the whole period.

With this structure in mind, I consider the following empirical specification:

$$y_{it} = \alpha_i + \eta_t + \sum_{s \in \mathcal{S}} \beta_s \chi_{i,t+s}(N) + \Gamma X_{it} + \epsilon_{it}, \tag{3.2}$$

where: (i) y_{it} is a dependent variable for bank i in quarter t; (ii) α_i is a bank fixed effect; (iii) η_t is a quarter fixed effect, designed to soak up variation in macro-level conditions; (iv) β_s is the coefficient on the effect of a drawdown event ($\chi_{it} = 1$) s periods ahead, for s in the treatment window $\mathcal{S} = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$; (v) X_{it} is a vector of control variables, with coefficients Γ; and (v) ϵ_{it} is an error term. The bank fixed effects, α_i, are included to control for any idiosyncratic, bank-level tendencies pertaining to credit line financing. The quarter fixed effects, η_t, are designed to account for analogous macro level effects associated with a given period. Finally, the vector of controls, X_{it}, accounts for financing and lending behavior around the time of the LDE above and beyond what is captured in the mean bank-level effect, α_i. For example, X_{it} includes lags of the banks' unused commitment ratio, leverage, and "credit line exposure," which I define as the ratio of total unused commitments (net of credit cards) to total assets.

I choose this specification for my baseline analysis for several reasons. First, since the focus of this paper is on banks' responses to downturns during recessions, financial crises, and other times of extreme stress, I want to focus on severe situations. Second, focusing on stark, sudden drops in UCR alleviates some (but not all) of the original concerns associated with the UCR measure (see Section 3.2.1 for a discussion). In particular, shifts in UCR attributable to drawdowns in credit lines will tend to be larger than shifts in UCR attributable to issuance or retirement of standard term loans. This is simply because changes in unused credit lines effect both the numerator and the denominator in (3.1), whereas changes in term loans impact only the denominator. This speaks to the value of considering the tails of the distribution of changes in UCR. Third, if we assume that banks face convex costs of external financing – particularly under stress – then small changes in UCR are unlikely to have a meaningful impact. Fourth, it stands to reason that the impact of changes in UCR will be asymmetric around 0. Since credit line cancellations are relatively rare, sudden *increases* in UCR are likely attributable to cuts in total lending or balance sheet size, which likely stems from stress not associated with credit line draws.

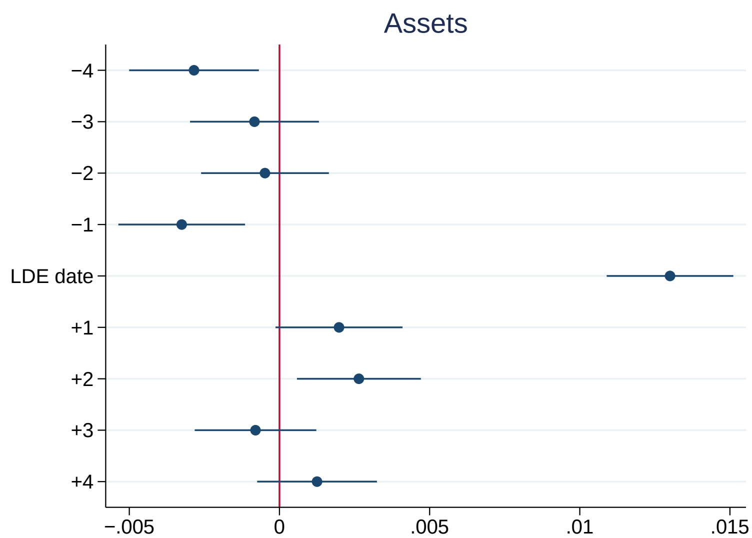
3.3.2 Results

This section contains the results of estimating equation (3.2) on the sample of banks discussed in Section 3.2.1. Section 3.3.2 contains results on how drawdowns affect banks' funding decisions and balance sheet size, while Section 3.3.2 demonstrates the impact of drawdowns on banks' profitability.

Funding of drawdowns

First, Figure 3.4 shows how LDEs impact the size of a bank, as measured by total assets. In this specification of Equation (3.2), the dependent variable y_{it} is $\Delta Assets_{it}/Assets_{it-1}$. The pattern that emerges around the date of a significant amount of credit line draws is stark: these events are overwhelmingly associated with balance sheet expansions. Referring back to Figure 3.1, this suggests that banks typically respond to increased drawdowns by increasing their total size, and therefore their total funding base, rather than shedding existing assets.

Furthermore, the fact that β_s is not significantly different than 0 for any other date besides



Notes: The results reported in this figure are the estimated coefficients $\hat{\beta}_s$ for $s \in \mathcal{S}$ from regression Equation (3.2), where $y_{it} = (Assets_{it} - Assets_{it-1})/Assets_{it-1}$. The central dot reports the point estimate, and the line around it reports the 95% confidence interval around that point estimate.

Figure 3.4: Balance sheet size before and after large drawdown events

the date of impact suggests two things. First, looking at the dates *before* the LDE, it is clear that banks do not anticipate LDEs. That is, it is apparent in the sample that banks do not "prepare" for LDEs by shedding assets in anticipation of the new loans likely to come on their balance sheets. Critically, this indicates that we can safely think of LDEs as largely "surprise" events.[10] Second, looking now at the dates *after* the LDE, we see that banks do not re-adjust balance sheet size after these events.

Given that Figure 3.4 shows that banks respond to LDEs by expanding rather than shedding existing assets, the next logical question is: how do banks finance these new assets? The answer to this question is presented in the various panels of Figure 3.5. In the top two panels 3.5a and 3.5b, I consider the responses of the two main components of banks' capital structure, debt and equity, respectively. In these panels, the dependent variable y_{it} is $\Delta Debt_{it}/Debt_{it-1}$ and

[10]Not only does this fact shape thinking about drawdowns in terms of models, it also alleviates some endogeneity concerns around the regression specification (3.2). Since the "events" themselves appear to be plausibly exogenous, despite in some sense being derived from changes in choice variables of the bank, the regression is well-specified.

$\Delta Equity_{it}/Equity_{it-1}$, respectively. First, both sources of financing increase. The response on the debt side, however, is roughly twice as large as the response in equity. This suggests that banks finance the increased drawdowns by increasing debt and equity in a manner designed to leave their total leverage basically unchanged.[11]

Next, in panels 3.5c and 3.5d of Figure 3.5, I break down debt into deposits and non-deposit debt, respectively. The results here are striking. While both types of debt respond to a LDE, the response on the non-deposit debt side is about 4 times larger. This poses an interesting challenge to the classic line of thinking presented in Kashyap et al. (1993) and Gatev and Strahan (2006), whose reasoning posits that banks naturally receive large deposit inflows precisely in the periods of stress in which firms draw extensively on credit lines. Even if this is the case, banks at impact seemingly prefer to finance with other types of debt.

Profitability: loan returns and funding costs

The analysis in the previous subsection suggests that (i) banks respond to credit line drawdowns by expanding their balance sheets; (ii) banks mostly finance these balance sheet expansions with debt (rather than equity), largely preserving their leverage ratios; and (iii) within debt, banks tend to prefer to finance in response using non-deposit debt. In this section, I explore the impacts of LDEs on measures of bank profitability, given these choices about funding.

Figure 3.6 shows the response of the two core components of the banks' major profitability metric, net interest margin: the return on loans and the cost of deposits. In Figure 3.6a, we see that loan return experiences only a slightly negative shock at the impact of a LDE. This suggests that credit line drawdowns do not increase expected returns on lending, despite the potential stress they create in bank borrowing.

Figure 3.6b looks at the bank funding cost side of net interest margin. The shape of the

[11] Indeed, running the regression (3.2) with the change in leverage as the dependent variable shows no impact on or around the LDE date.

response of the cost of deposits around the LDE date displays an interesting pattern.[12] In the periods leading up to the LDE date, the cost of deposits gradually increases, spiking most at impact. In the periods after impact, though, the effect essentially reverses itself. This suggests that the increased strain on a bank associated with a sudden burst in required funding first increases borrowing costs, then decreases them.[13] On the whole, then, since loan returns are unaffected and funding costs increases, the net effect is that net interest margin declines leading up to and upon impact, and then gradually recovers thereafter, driven by normalization in costs of funding.

Ultimately, the results for funding and profitability suggest some interesting nuance to the arguments of Kashyap et al. (1993) and Gatev and Strahan (2006). In particular, it appears that *over time*, flight to safety effects drive down the costs of funding, mitigating the dynamic costs to banks of credit line drawdowns. *At impact*, however, the stress can be large and hard to offset immediately, suggesting differential responses between downturns originating from the financial sector and those originating outside it.

3.3.3 Robustness and changes during the 2008 financial crisis

The analysis in this section to this point has leveraged the event study style framework described in Section 3.3.1. It is natural to question, however, the robustness of the findings discussed in Section 3.3.2 to an alternative (and perhaps more standard or traditional) regression framework. To this end, in Tables 3.4 and 3.5 I present estimates of the following regression:

$$y_{it} = \alpha_i + \eta_t + \beta_1 \Delta UCR_{it} \times (Crisis_t = 0) + \beta_2 \Delta UCR_{it} \times (Crisis_t = 1) + \Gamma X_{it} + \epsilon_{it}, \quad (3.3)$$

[12] The results for the cost of funding, more generally, are quite similar.

[13] Or, as Acharya and Mora (2015) suggest, in order to attract the required additional funding, banks are required to offer higher rates of return. The net results are the same.

where y_{it}, α_i, η_t, X_{it} and ϵ_{it} are the same as in (3.2), ΔUCR_{it} is the quarter-over-quarter change in UCR, and $Crisis_t$ is a dummy variable equal to 1 for the quarters of 2007Q4 through 2009Q4, and 0 otherwise. Given the standard time effect η_t, the variable $Crisis_t$ interacted with ΔUCR_{it} uniquely captures the differential impact of changes in UCR during the financial crisis as opposed to during the rest of the sample period. Note that a negative sign on the coefficients β_1 and β_2 in specification (3.3) has the same interpretation as a positive sign on the coefficient β in specification (3.2). This is because a value of $\chi_{it} = 1$ is associated with a negative value of ΔUCR_{it}.

This specification has two important departures from the specification in (3.2). First, and most simply, this specification departs from the event study framework and considers the full range of changes in ΔUCR_{it} for all bank-quarters in the sample. While this precludes us from considering the lead-lag effects as in the previous analysis, it allows us to consider the average effect of changes in UCR without selecting large stress events. Second, this formulation allows for a very simple analysis of how responses changed during the stress period of the financial crisis through the interaction term. The differences between the coefficients β_1 and β_2, therefore, provide insight into the differential impacts of credit line drawdowns in normal times versus during the financial crisis.

The results in the first rows of Tables 3.4 and 3.5 suggest caution about the results presented in the event study framework in Section 3.3.2. For example, the insignificant coefficient on asset growth reported in column (1) of Table 3.4 appears to run counter to the effect reported in Figure 3.4. The same caution holds for the results in Columns (2) through (5).

Interestingly, the support for the results pertaining to profitability is stronger. The impact on loan return (column (7)) remains significantly different from 0, but the upward pressure on cost of deposits (column (8)) is insignificant. Therefore, the aggregated effect on net interest margin (NIM, column (6)) is significant and negative.

	(1) Assets	(2) Debt	(3) Equity	(4) Deposits	(5) Non-Dep. Debt
$\%\Delta UCR_{it}$ (normal)	0.0000120	0.0000113	0.0000108	-0.0000233	-0.0000637
	(0.0000182)	(0.0000275)	(0.0000333)	(0.0000191)	(0.000686)
$\%\Delta UCR_{it}$ (crisis)	0.0000172	0.0000139	0.00000819	0.0000443	-0.000134
	(0.0000549)	(0.0000828)	(0.000100)	(0.0000574)	(0.00207)
CL exposure (lag)	0.0964***	0.0917***	0.0112	0.0835***	-0.378
	(0.00602)	(0.00908)	(0.0110)	(0.00629)	(0.227)
Leverage (lag)	-0.676***	-1.225***	0.460***	-1.034***	-5.085***
	(0.00864)	(0.0130)	(0.0158)	(0.00902)	(0.325)
UCR (lag)	0.0114	0.0161	0.0426**	0.00400	2.192***
	(0.00733)	(0.0111)	(0.0134)	(0.00766)	(0.276)
Observations	518104	518104	518103	518104	518091
R^2	0.020	0.022	0.006	0.034	0.002

Notes: Standard errors in parentheses, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. The figures reported in this table reflect estimates from the general specification in Equation (3.3). The dependent variable is the percentage growth rate of the stated variable.

Table 3.4: Robustness for funding results

Since the first set of results largely confirm the findings from the event study framework, perhaps the most interesting application of this alternative specification comes from the analysis of the interaction term with the crisis period in specification (3.3). These results are reported in the second rows of Tables 3.4 and 3.5. Notably, in column (1) we see a significant positive coefficient for asset growth. This implies that, during the crisis, banks respond to increases in credit line drawdowns by actually *shrinking* their balance sheets on average. The other changes in funding are insignificant. Next, on the profitability side, we find a strongly positive impact on the cost of deposits from increased credit line drawdowns. This suggest that while increased drawdowns do not impose stress in normal times, during a financial crisis the impact can be quite different.

	(1) Net interest margin	(2) Loan return	(3) Cost of deposits
$\%\Delta UCR_{it}$ (normal)	0.000301***	0.000301***	0.000000135
	(0.0000799)	(0.0000799)	(0.000000342)
$\%\Delta UCR_{it}$ (crisis)	-0.000150	-0.000151	-0.00000148
	(0.000240)	(0.000240)	(0.00000103)
CL exposure (lag)	-0.179***	-0.175***	0.00407***
	(0.0273)	(0.0273)	(0.000117)
Leverage (lag)	-0.0183	0.00168	0.0202***
	(0.0432)	(0.0432)	(0.000185)
UCR (lag)	0.259***	0.249***	-0.00987***
	(0.0333)	(0.0333)	(0.000143)
Observations	500526	500528	500526
R^2	0.000	0.000	0.826

Notes: Standard errors in parentheses, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. The figures reported in this table reflect estimates from the general specification in Equation (3.3). The dependent variable is simply the level of the relevant rate.

Table 3.5: Robustness for profitability results

3.4 Conclusion

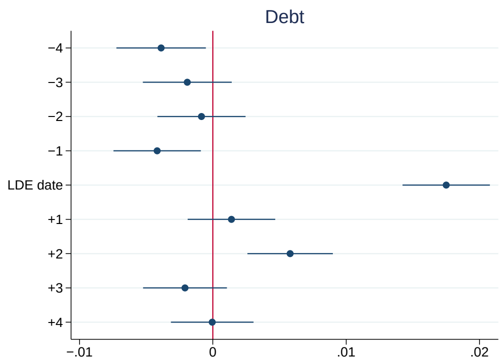
This paper studies how banks respond to sudden increases in drawdowns on credit lines they have extended to their customers. The analysis is intended to help fill some of the gaps in our understanding of the recent financial crisis of 2008. In particular, in studies of the crisis, much attention has been paid to the severe disruptions in bank funding markets (such as repo and commercial paper markets), which exerted contractionary pressure on the liability side of banks' balance sheets. Much less attention has been paid to the asset side of banks' balance sheets.

In this paper, I document (consistently with other authors) that the freezes in traditional bank funding markets during the financial crisis were accompanied by an increased lending burden on banks associated with increased drawdowns on pre-existing lines of credit. While existing theories of credit line financing suggest that banks can efficiently supply backup financing or insurance to firms during times of stress which disrupt standard credit markets, I posit that this efficiency breaks down in the context of a financial crisis in which banks experience difficulty in attracting new funds to lend.

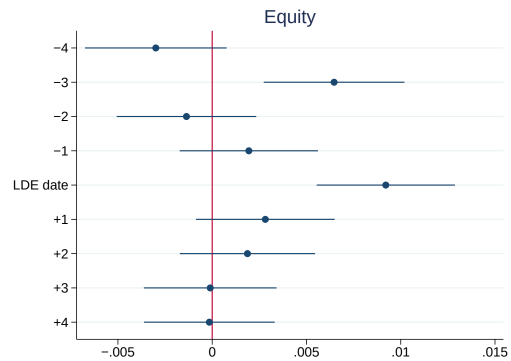
As a step towards understanding these effects, I explore how banks tend to respond to increases in drawdowns on credit lines, and how this response changed during the financial crisis. I find that on average: (i) banks respond to credit line drawdowns by expanding their balance sheets; (ii) banks mostly finance these balance sheet expansions with debt (rather than equity), largely preserving their leverage ratios; (iii) banks tend to use more non-deposit debt than deposits to meet credit line draws; (iv) loan returns are mostly unaffected by credit line drawdowns; and (v) banks' funding costs tend to increase in response to increases in credit line drawdowns, then normalize in subsequent periods.

The first set of results, which pertain to the funding of banks, suggest that banks expose themselves to the possibility of "two-sided runs" in which firms can demand funds on their credit lines precisely when bank funding is becoming scarcer and more expensive due to fears of the stability of the banking sector. These results speak to how credit line drawdowns can amplify the *depth* of a financial crisis by exacerbating stress in the banking sector. The second set of results show that increased credit line drawdowns have a negative impact on banks' profitability by lowering their net interest margins, primarily through an increase in funding costs (rather than through a decrease in loan returns). This results suggests that credit line drawdowns can amplify the *length* of a financial crisis by diminishing banks' profitability and making it harder for them to recover from a shock.

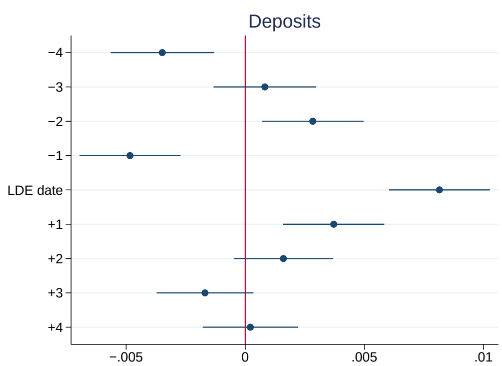
These combined effects indicate the possibility for future research into quantifying, at the macro level, just how much credit line drawdowns amplified the depth and duration of the recent financial crisis. Understanding these effects, then, leaves room for more detailed analysis into how policy and optimal contracting can be used to mitigate some of these perverse effects.



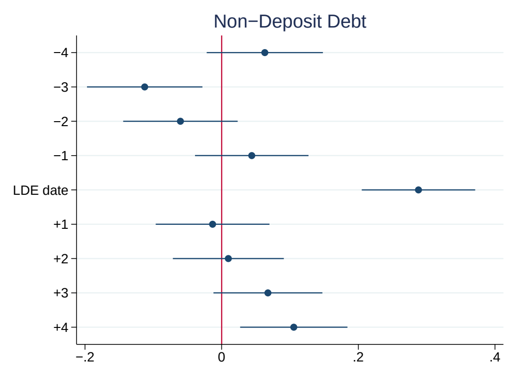
(a) Debt



(b) Equity



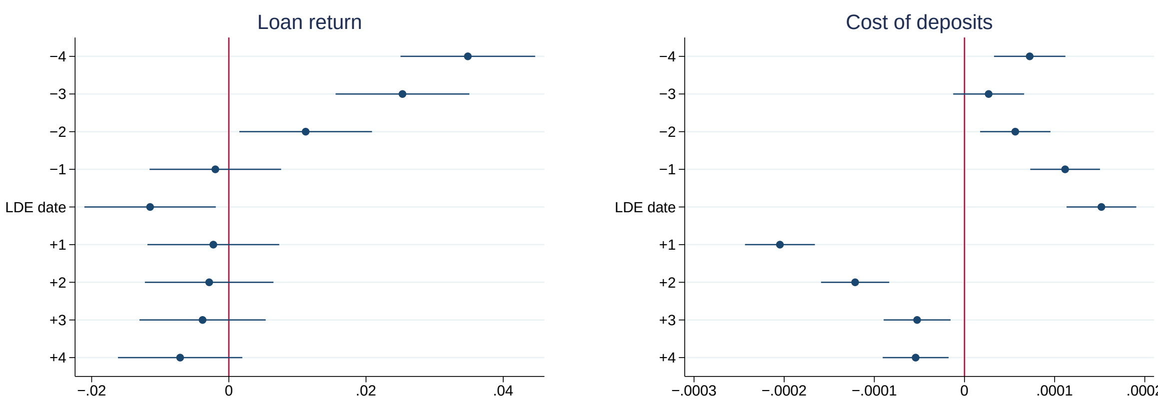
(c) Deposits



(d) Non-deposit debt

Notes: The results reported in this figure are the estimated coefficients $\hat{\beta}_s$ for $s \in \mathcal{S}$ from regression Equation (3.2), where $y_{it} = (Debt_{it} - Debt_{it-1})/Debt_{it-1}$, $(Equity_{it} - Equity_{it-1})/Equity_{it-1}$, $(Deposits_{it} - Deposits_{it-1})/Deposits_{it-1}$, and $((Debt_{it} - Deposits_{it}) - (Debt_{it-1} - Deposits_{it-1}))/(Debt_{it-1} - Deposits_{it-1})$ for panels 3.5a, 3.5b, 3.5c, and 3.5d, respectively. The central dot reports the point estimate, and the line around it reports the 95% confidence interval around that point estimate.

Figure 3.5: Funding changes before and after large drawdown events



(a) Loan return

(b) Cost of deposits

Notes: The results reported in this figure are the estimated coefficients $\hat{\beta}_s$ for $s \in \mathcal{S}$ from regression Equation (3.2) for the levels of the indicated dependent variable. The central dot reports the point estimate, and the line around it reports the 95% confidence interval around that point estimate.

Figure 3.6: Returns and costs before and after large drawdown events

Appendix A

Chapter 1 Appendix

A.1 Model Appendix

A.1.1 Proofs

Proof of Lemma 1.1

The value associated with borrowing from lender $i \in \{H, B\}$ can be expressed as

$$V_i(k) = \max_{I,b} P(m_i, \overline{p})(f(I) - b) + \int_{\overline{x}(m_i)}^{\infty} (x - m_i)dG(x) \text{ subject to } I \leq q_i b + k. \tag{A.1}$$

Since $q_i \leq 1$ and firms are risk neutral, borrowing only occurs if investment exceeds internal capital: I focus only on this case (when $b > 0$). In this case, the budget constraint must bind, and so the problem can be expressed as a single, unconstrained optimization in terms of the loan size, b. The first order condition of this problem is

$$\begin{aligned} P(m_i, \overline{p})(q_i f'(q_i b + k) - 1) + \frac{\partial P(m_i, \overline{p})}{\partial b}(f(q_i b + k) - b) \\ -\overline{x}(m_i) g(\overline{x}(m_i)) \frac{\partial \overline{x}(m_i)}{\partial b} + m_i g(\overline{x}(m_i)) \frac{\partial \overline{x}(m_i)}{\partial b} &= 0, \end{aligned} \tag{A.2}$$

where $g(\cdot)$ is the p.d.f. of x. The partial derivative terms in this expression can be computed as:

$$\begin{aligned} \frac{\partial P(m_i, \overline{p})}{\partial b} &= (1 - \overline{p}) g(\overline{x}(m_i)) \frac{\partial \overline{x}(m_i)}{\partial b} \\ \frac{\partial \overline{x}(m_i)}{\partial b} &= (1 - \overline{p})(q_i f'(q_i b + k) - 1). \end{aligned}$$

Plugging the first expression into (A.2) yields

$$P(m_i,\overline{p})(q_i f'(q_i b+k)-1)+\frac{\partial \overline{x}(m_i)}{\partial b} g(\overline{x}(m_i))\left[(1-\overline{p})(f(q_i b+k)-b)+m_i-\overline{x}(m_i)\right]=0. \quad \text{(A.3)}$$

Recalling equation (1.3), however, the term in brackets is equal to zero. Therefore, as long as $P(m_i,\overline{p})>0$, the optimality condition reduces to (1.8). *QED*.

Proof of Lemma 1.2

To prove this claim, I need not take a stance on whether limited liability binds for the bank. Therefore, consider the optimality conditions (1.13) and (1.14). If the capital requirement is slack, then $\mu=0$ and varying the level of χ has no impact. Therefore, for the remainder of this proof I consider only the case in which the capital requirement binds.

We proceed using a two-dimensional version of the Implicit Function Theorem. Define

$$\begin{aligned} F_1(m_B,\ell_B|\chi) &= P(m_B,p)-\frac{q_B}{\overline{q}}c_\ell(m_B,q_B\ell_B b_B)-\mu\chi q_B b_B \\ F_2(m_B,\ell_B|\chi) &= P_1(m_B,p)-\frac{1}{\overline{q}}c_m(m_B,q_B\ell_B b_B) \end{aligned}$$

Then, the optimality conditions of the firm reduce to $F(m_B,\ell_B|\chi)=(F_1(m_B,\ell_B|\chi),F_2(m_B,\ell_B|\chi))=0$. Then, using the Implicit Function Theorem, we find that

$$\begin{aligned} \frac{\partial \ell_B^*}{\partial \chi} &= -\det\begin{bmatrix} \frac{\partial F_1}{\partial \chi} & \frac{\partial F_1}{\partial m_B} \\ \frac{\partial F_2}{\partial \chi} & \frac{\partial F_2}{\partial m_B}\end{bmatrix} \Big/ \det\begin{bmatrix} \frac{\partial F_1}{\partial \ell_B} & \frac{\partial F_1}{\partial m_B} \\ \frac{\partial F_2}{\partial \ell_B} & \frac{\partial F_2}{\partial m_B}\end{bmatrix} \\ \frac{\partial m_B^*}{\partial \chi} &= -\det\begin{bmatrix} \frac{\partial F_1}{\partial \ell_B} & \frac{\partial F_1}{\partial \chi} \\ \frac{\partial F_2}{\partial \ell_B} & \frac{\partial F_2}{\partial \chi}\end{bmatrix} \Big/ \det\begin{bmatrix} \frac{\partial F_1}{\partial \ell_B} & \frac{\partial F_1}{\partial m_B} \\ \frac{\partial F_2}{\partial \ell_B} & \frac{\partial F_2}{\partial m_B}\end{bmatrix}, \end{aligned}$$

and so

$$\frac{\partial \ell_B^*}{\partial \chi} = -\frac{\partial F_1}{\partial \chi}\frac{\partial F_2}{\partial m_B} \Big/ \left[\frac{\partial F_1}{\partial m_B}\frac{\partial F_2}{\partial \ell_B}-\frac{\partial F_1}{\partial \ell_B}\frac{\partial F_2}{\partial m_B}\right] \quad \text{(A.4)}$$

$$\frac{\partial m_B^*}{\partial \chi} = \frac{\partial F_1}{\partial \chi}\frac{\partial F_2}{\partial \ell_B} \Big/ \left[\frac{\partial F_1}{\partial m_B}\frac{\partial F_2}{\partial \ell_B}-\frac{\partial F_1}{\partial \ell_B}\frac{\partial F_2}{\partial m_B}\right], \quad \text{(A.5)}$$

where we have applied the fact that we know that $\frac{\partial F_2}{\partial \chi} = 0$. Plugging in the relevant partial derivatives reveals that the denominator in both (A.4) and (A.5) is equal to

$$D = -\frac{q_B b_B}{\overline{q}} \left[q_B c_{\ell\ell} \left(P_{11} - \frac{1}{\overline{q}} c_{mm} \right) - c_{m\ell} \left(P_1 - \frac{q_B}{\overline{q}} c_{m\ell} \right) \right]$$

where I have suppressed the arguments to the cost function and $\overline{x}$ to ease notation. Furthermore, the second order conditions of problem (1.11) imply that

$$S = -\frac{q_B b_B}{\overline{q}} \left[q_B c_{\ell\ell} \left(P_{11} - \frac{1}{\overline{q}} c_{mm} \right) - \frac{q_B}{\overline{q}} c_{m\ell}^2 \right] > 0,$$

so that $D - S > 0$. Since $S > 0$, then $D > 0$, and so the denominator in (A.4) and (A.5) must be strictly positive.

Then, equation (A.4) can be expressed as

$$\frac{\partial \ell_B^*}{\partial \chi} = \frac{\mu q_B b_B \left(P_{11} - \frac{1}{\overline{q}} c_{mm} \right)}{D} = -\frac{\mu \overline{q} \left(P_{11} - \frac{1}{\overline{q}} c_{mm} \right)}{q_B c_{\ell\ell} \left(P_{11} - \frac{1}{\overline{q}} c_{mm} \right) - c_{m\ell} \left(P_1 - \frac{q_B}{\overline{q}} c_{m\ell} \right)} \leq 0.$$

Once more, comparing the expressions for D and S above reveals that the denominator of this last expression must be negative, and so the sign of the derivative of bank loan supply with respect to the capital requirement takes the sign of $P_{11} - \frac{1}{\overline{q}} c_{mm}$. The second term in this expression is negative since $c_{mm} > 0$ by assumption. The first term is equal to $(1 - \overline{p}) g'(\overline{x})$, which must be negative at an optimal policy.

Finally, equation (A.5) can be computed as

$$\frac{\partial m_B^*}{\partial \chi} = \frac{\mu q_B^2 b_B^2 c_{m\ell}}{\overline{q} D} \geq 0,$$

since $c_{m\ell} > 0$ by assumption. Note that whenever the capital requirement is binding so that $\mu > 0$, this inequality is strict. QED.

Proof of Proposition 1.1

Analyzing the bank problem. The presence of the limited liability in the bank problem creates the potential for discontinuities and "jumps" in bank optimal policies which make demonstrating existence of equilibrium difficult. Therefore, the first step in the existence proof is to deal with these jumps. To this end, I show in Lemmas A.1 and A.2 that the bank problem (1.11) can be analyzed using two subproblems. Then, I augment the problem by allowing individual banks to randomize over policies. Throughout this section, I explicitly make the argument to the value functions the price vector q, to make clear the fixed point argument at the end of the proof.

Lemma A.1 ***Two subproblems.*** *Define the state-specific payoff function of the bank for a given policy to be*

$$\tilde{v}_B(y_B, p|q) = P(m_B, p)\ell_B b_B - d_B,$$

where $y_B = (m_B, \ell_B, d_B)$*, and define the following constraint correspondence, using the budget and capital requirement constraints from (1.11):*

$$\Gamma_B(q) = \left\{ y_B = (m_B, \ell_B, d_B) \in \mathbb{R}^3_+ | c(m_B, q_B\ell_B b_B) \leq k + \overline{q} d_B \text{ and } \chi q_B \ell_B b_B \leq k \right\}. \tag{A.6}$$

Then, define

$$\begin{aligned} \Gamma^1_B(q) &= \{y_B \in \Gamma_B(q) | \tilde{v}_B(y_B, p_l) \geq 0\} \\ \Gamma^2_B(q) &= \{y_B \in \Gamma_B(q) | \tilde{v}_B(y_B, p_l) \leq 0\} \end{aligned}$$

the following two subproblems:

$$v^1_B(q) = \max_{y_B \in \Gamma^1_B(q)} \psi \tilde{v}_B(y_B, p_h|q) + (1-\psi)\tilde{v}_B(y_B, p_l) \tag{A.7}$$

$$v^2_B(q) = \max_{y_B \in \Gamma^2_B(q)} \psi \tilde{v}_B(y_B, p_h|q) \tag{A.8}$$

Then, for $i \in \{1, 2\}$*,* $v^i_B(q)$ *is continuous and the optimal policy* $y^{i*}_B(q)$ *is non-empty, compact-valued, and upper hemi-continuous.*

Proof: Problem (A.7) solves the bank problem imposing that limited liability does not bind in the low state, while problem (A.8) assumes that it binds. The constraint correspondence

(A.6) is clearly non-empty, compact- and convex-valued and continuous. In addition, the constraints $\tilde{v}_B(y_B, p_l) \geq 0$ and $\tilde{v}_B(y_B, p_l) \leq 0$ which augment Γ_B into Γ_B^1 and Γ_B^2, define closed and bounded regions in $\mathbb{R}_+^3$ and preserve continuity. Therefore, Γ_B^1 and Γ_B^2 are also non-empty, compact-valued, and upper hemi-continuous. In addition, since $\tilde{v}_B$ is continuous in q for all p, the objective functions in both (A.7) and (A.8) are continuous. Therefore, each of these subproblems satisfies all the conditions of the Theorem of the Maximum, and the result follows. *QED*.

The subproblems from the preceding lemma were constructed based on whether or not limited liability would bind in the down state: in (A.7), it was assumed to be slack, whereas in (A.8) it was assumed to bind. The next lemma shows that the value function from the original bank problem can be recovered from the two subproblems analyzed in Lemma A.1.

Lemma A.2 ***Recovering the original problem from the subproblems.*** *The bank's value function v_B from (1.11) can be expressed as*

$$v_B(q) = \max\left\{v_B^1(q), v_B^2(q)\right\}. \tag{A.9}$$

Proof: By definition, any solution to either problem (A.7) or problem (A.8) satisfies the constraints of the original problem (1.11). What remains to show is that the objective functions coincide. If limited liability is slack in all states, then

$$\begin{aligned} \mathbb{E}\left[\max\left\{P(m_B, p)\ell_B b_B - d_B, 0\right\}\right] &= \mathbb{E}\left[P(m_B, p)\ell_B b_B - d_B\right] \\ &= \psi\tilde{v}_B(y_B, p_h|q) + (1-\psi)\tilde{v}_B(y_B, p_h|q) \end{aligned}$$

which is the exact objective function from (A.7). Likewise, if limited liability binds in the down state,

$$\mathbb{E}\left[\max\left\{P(m_B, p)\ell_B b_B - d_B, 0\right\}\right] = \psi\tilde{v}_B(y_B, p_h|q) + (1-\psi)\cdot 0,$$

which is the expression in (A.8). Then, since both solutions satisfy both the objective and the constraints of the original problem in the case in which they deliver the highest value (see the possible scenarios discussed in the proof of Lemma A.3 below), the one with the highest value must be the solution to the original problem. *QED*.

While both subproblems from Lemma A.1 are "well-behaved," and Lemma A.2 shows that we can use them to find the solution to the original bank problem, in order to establish the properties of the aggregate bank optimal policies considered below, we need to make sure that banks' optimal decisions do not switch back and forth suddenly between the two sets of policies. This is indeed not the case, and this result is formalized in the following lemma.

Lemma A.3 ***Indifference points.*** *For every $\overline{q}$, there is at most a single value of q_B such that $v_B^1(q) = v_B^2(q)$. Define this value of q_B to be $\overline{q}_B(\overline{q})$. Then, for all $q_B < \overline{q}_B(\overline{q})$, $v_B(q) = v_B^2(q)$, and for all $q_B > \overline{q}_B(\overline{q})$, $v_B(q) = v_B^1(q)$.*

Proof: I consider four cases which involve different scenarios for whether or not the limited liability consistency constraints bind or not. Let $\zeta^1 \geq 0$ and $\zeta^2 \geq 0$ be the Lagrange multipliers on the constraints $\tilde{v}_B(y_B, p_l|q) \geq 0$ in problem (A.7) and $\tilde{v}_B(y_B, p_l|q) \leq 0$ in problem (A.8), respectively. Then we have four possible cases:

1. $\zeta^1 > 0, \zeta^2 = 0$. In this case, the constraint for problem (A.7) binds, and so $\tilde{v}_B(y_B^{1*}, p_l|q) = 0$. Then, the objective function is the same as in problem (A.8), but the constraint set includes an additional constraint. Therefore, $v_B^2(q) > v_B^1(q)$, where the inequality is strict because the constraint strictly binds (i.e. ζ^1 is strictly positive).

2. $\zeta^1 = 0, \zeta^2 > 0$. In this case, the constraint for problem (A.8) binds, and so $\tilde{v}_B(y_B^{2*}, p_l|q) = 0$. Then, since the constraint set $\Gamma_B^1(q)$ from problem (A.7) allows $\tilde{v}_B(y_B, p_l|q)$ to adjust freely into positive territory, it must be the case that $v_B^1(q) > v_B^2(q)$. Again, the inequality is strict because ζ^2 is strictly positive.

3. $\zeta^1 = 0, \zeta^2 = 0$. In this case, then both constraints are slack and can be ignored, effectively reducing the constraint sets to $\Gamma_B^1(q) = \Gamma_B^2(q) = \Gamma_B(q)$. Fix $\overline{q} \in (0, 1]$. Then, if either $v_B^1(q)$ or $v_B^2(q)$ is strictly higher for all q_B, there are no indifference points. Therefore,

assume that there exists $\overline{q}_B(\overline{q})$ such that $v_B^1(q) = v_B^2(q)$. Then, it must be the case that at these prices

$$\begin{aligned} & \psi\left[P(m_B^{1*}(q), p_h)\ell_B^{1*}(q)b_B - d_B^{1*}(q)\right] + (1-\psi)\left[P(m_B^{1*}(q), p_l)\ell_B^{1*}(q)b_B - d_B^{1*}(q)\right] \\ = \; & \psi\left[P(m_B^{2*}(q), p_h)\ell_B^{2*}(q)b_B - d_B^{2*}(q)\right] \end{aligned}$$

Applying the Envelope Theorem, we know that

$$\begin{aligned} \frac{\partial v_B^1(q)}{\partial q_B} &= -\frac{\ell_B^{1*}b_B}{\overline{q}} c_\ell(m_B^{1*}, q_B\ell_B^{1*}b_B) - \mu^1\chi\ell_B^{1*}b_B \\ \frac{\partial v_B^2(q)}{\partial q_B} &= -\psi\frac{\ell_B^{2*}b_B}{\overline{q}} c_\ell(m_B^{2*}, q_B\ell_B^{2*}b_B) - \mu^2\chi\ell_B^{2*}b_B, \end{aligned}$$

and so

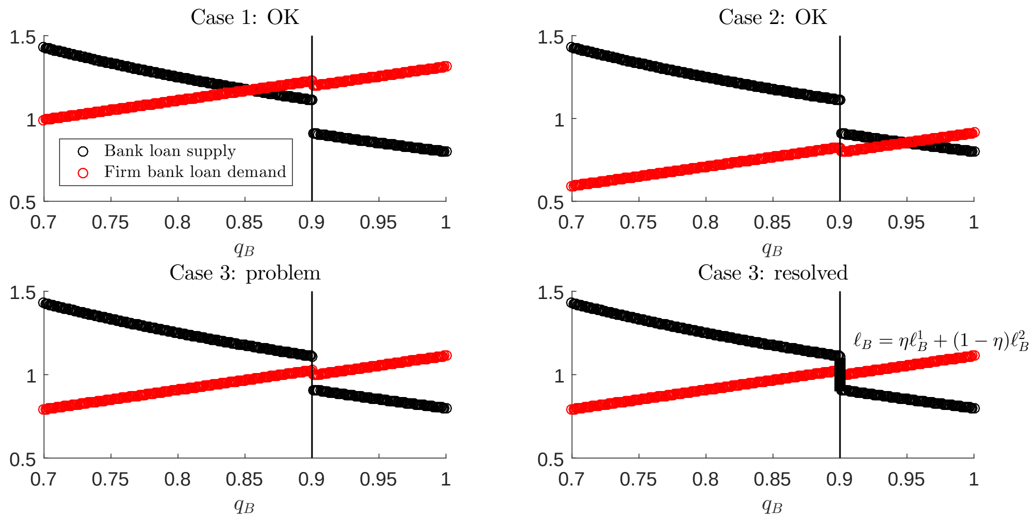
$$\begin{aligned} \frac{\partial v_B^1(q)}{\partial q_B} - \frac{\partial v_B^2(q)}{\partial q_B} &= b_B\left[-\ell_B^{1*}\left(\frac{c_\ell(m_B^{1*}, q_B\ell_B^{1*}b_B)}{\overline{q}} + \mu^1\chi\right) + \ell_B^{2*}\left(\frac{c_\ell(m_B^{2*}, q_B\ell_B^{2*}b_B)}{\overline{q}} + \mu^2\chi\right)\right] \\ &= \frac{b_B}{q_B}\left[P(m_B^{2*}, p_h)\ell_B^{2*} - P(m_B^{1*}, \overline{p})\ell_B^{1*}\right] > 0, \end{aligned}$$

where the second line plugs in the first order condition for loans (1.13) for each case. Since the value associated with the policies from the subproblem (A.7) rises faster in q_B than the corresponding value for subproblem (A.8), and the value increases are monotone, it follows that there is at most one indifference point between the two. If this indifference point $\overline{q}_B(\overline{q})$ exists, then, it is such that the policies from (A.8) are chosen for $q_B < \overline{q}_B(\overline{q})$ while the policies from (A.7) are chosen for $q_B > \overline{q}_B(\overline{q})$.

4. $\zeta^1 > 0, \zeta^2 > 0$. This is a knife-edge case which can be safely ignored.

In cases 1 and 2, an indifference point between the two problems (in the sense that $v_B^1(q) = v_B^2(q)$) is ruled out. In case 3, I have demonstrated that there is at most one indifference point. Case 4 is irrelevant (though a logic similar to that applied to case 3 could be applied). Since these cases exhaust all possibilities, this completes the proof. QED.

Convexifying policies using bank mixed strategies. The transition between optimal policy regimes for the bank outlined in Lemma A.3 establishes that bank optimal policies may



Notes: This figure illustrates the potential for discontinuity in the bank problem and how randomization resolves it. The top two panels depict instances in which the discontinuity poses no problem. The bottom left panel shows the case in which bank loan supply always exceeds firm loan demand at low q_B, and vice versa for high q_B. The bottom right figure shows how randomization resolves this case.

Figure A.1: Illustration of potential discontinuity in the bank problem

have a non-convexity around the point of indifference. Several instances of this situation are illustrated in Figure A.1. If this non-convexity induces, for example, the bank loan supply function to "skip over" the aggregate bank loan demand function, then, there can be problems for equilibrium existence (case 3 in the bottom two panels). Note that this non-convexity does not imply nonexistence of equilibrium: as long as demand crosses supply on either side of the jump, there is no problem (cases 1 and 2 in the top two panels).

At $\overline{q}_B(\overline{q})$, all banks are indifferent between operating under policies $y_B^{1*}(q)$ and $y_B^{2*}(q)$; that is, $v_B^1(q) = v_B^2(q)$. Elsewhere, one policy gives strictly greater value. Therefore, if we allow the bank to use mixed strategies, we can "connect" the end points of the jump. Specifically, define the mixing probability η used by banks as the optimal weight placed on policies associated with problem (A.7) (with the corresponding fraction $1-\eta$ on problem (A.8)). Then,

$$\eta^*(q) = \begin{cases} 1 & \text{if } v_B^1(q) > v_B^2(q) \\ [0,1] & \text{if } v_B^1(q) = v_B^2(q) \\ 0 & \text{if } v_B^1(q) < v_B^2(q) \end{cases} \tag{A.10}$$

the optimal randomization correspondence. Note that this implies that the tax in state p is now

$$T(p,\eta) = \eta \max\left\{d_B^1 - P_B(m_B^1, p)\ell_B^1 b_B, 0\right\} + (1-\eta)\max\left\{d_B^2 - P_B(m_B^2, p)\ell_B^2 b_B, 0\right\} \tag{A.11}$$

I assume that firms choose to go to bank finance and are then randomly assigned to a bank. Define the value of a firm who goes to bank finance and is matched with a bank using policy $j \in \{1,2\}$ by $V_B^j(k)$. Then, we can replace V_B from the main text by

$$V_B(k) = \eta V_B^1(k) + (1-\eta)V_B^2(k), \tag{A.12}$$

where $\eta \in \eta^*(q)$. Note that firms demand the same size loan regardless of who they are paired with, since this choice is independent of m (which may vary across policy 1 and policy 2 banks). Then, aggregate loan demand for each type of loan, $L_i b_i$, is still governed by (1.10), with the V_B term in L_i replaced by (A.12).

Since deposits are fully insured, households are indifferent between depositing at banks operating under either policy. We obtain the following modified equilibrium definition:

Definition A.1 ***Equilibrium with bank mixed strategies.*** *An equilibrium allowing for bank mixed strategies is a set of: firm lender share and loan amount choices, L_i^* and b_i^* for $i \in \{H,B,O\}$; bank choices y_B^{1*}, y_B^{2*}, and η^*; household choices of storage, a_H^*, deposits, d_H^*, and direct loans to firms, ℓ_H^*; bank and direct lending prices, q_B^* and q_H^*; a deposit price $\overline{q}^*$; and a lump-sum tax $T^*(p,\eta)$ for all p such that: firms' lender choices solve (1.9), firms' loan sizes solve (1.7), and direct and bank values are given by (1.4) (with (A.12) taking the place* *for firms' value of going to the bank); bank choices solve (A.7) and (A.8), with mixing fraction* $\eta \in \eta^*(q)$*, given by (A.10), taking* b_B^* *as given; household choices solve (1.15), taking* b_H^* *and* $T^*(p,\eta)$ *as given; direct and bank lending and bank deposit markets clear:*

$$L_B^* = \eta^* \ell_B^{1*} + (1-\eta^*)\ell_B^{2*} \quad \text{(A.13)}$$

$$L_H^* = \ell_H^* \quad \text{(A.14)}$$

$$\eta^* d_B^{1*} + (1-\eta^*) d_B^{2*} = d_H^*, \quad \text{(A.15)}$$

where L_i^* *is given by (1.10); and the tax* $T^*(p,\eta)$ *is given by (A.11).*

The remainder of this proof is devoted to proving existence of an equilibrium of this form.

Excess demand correspondence. Define the aggregate excess demand mapping

$$\mathcal{Z}(q) = \left(z_H(q), z_B(q), \overline{z}(q)\right), \quad \text{(A.16)}$$

where each component correspondence represents the excess demand in one of the three sub-markets:

$$z_H(q) = (L_H^*(q) - \ell_H^*(q))b_H^*(q) \quad \text{(A.17)}$$

$$z_B(q) = (L_B^*(q) - \ell_B^*(q))b_B^*(q) \quad \text{(A.18)}$$

$$\overline{z}(q) = d_B^*(q) - d_H^*(q) \quad \text{(A.19)}$$

Lemma A.4 ***Properties of the excess demand correspondence.*** *The excess demand correspondence* $\mathcal{Z}(q)$ *is non-empty, compact- and convex-valued, and upper hemi-continuous.*

Proof: To see this, I show that each term in each component of the excess demand function satisfies these properties. Then, since each component satisfies the required properties, and each component of the excess demand function $\mathcal{Z}$ is just a sum of components, the result will follow.

Households. First, observe that $q \in [0,1]^3 \subset \mathbb{R}^3$. Then, we can define the constraint set to be

$$\Gamma_H(q) = \left\{ y_H = (a_H, d_H, \ell_H) \in \mathbb{R}^3_+ | a_H + \overline{q} d_H + q_H \ell_H b_H \leq k_H \right\}.$$

This correspondence is convex-valued, compact-valued, and continuous. Then, defining

$$f_H(y_H|q) = \mathbb{E}[U(a_H + d_H + P(0,p)\ell_H b_H - T(p))],$$

so that $f : [0,1]^3 \times \mathbb{R}^3_+ \to \mathbb{R}$, the household's problem is $\max_{y_H \in \Gamma_H(q)} f_H(y_H|q)$, where $f(\cdot)$ is continuous. Then, the Theorem of the Maximum implies that the optimal policy correspondence $y^*_H(q)$ is nonempty, compact-valued, and upper hemi-continuous.

Firms. This result is obtained by examining equations (1.7) and (1.10) in succession. Define $y_{Fi} = (b_i, I_i)$ to be the firm's choices conditional on a choice of lender i. The constraints on the firm problem can be expressed via the correspondence

$$\Gamma_{Fi}(q) = \left\{ y_{Fi} \in \mathbb{R}^2_+ | I_i \leq q_i b_i + k_F \right\},$$

which is compact-valued, convex-valued, and continuous. In addition, define

$$f_{Fi}(y_{Fi}|q) = P(m_i, \overline{p})[f(I) - b] + \int_{\overline{x}(m_i)}^{\infty} (x - m_i) dG(x),$$

which is continuous in prices (so long as the density function $g(\cdot)$ of the private shirking benefit x is continuous as assumed). Then, (1.7) becomes $V_i(q) = \max_{y_{Fi} \in \Gamma_{Fi}(q)} f_{Fi}(y_{Fi}|q)$, and the Theorem of the Maximum implies that $y^*_{Fi}(q)$ is non-empty, compact-valued, and upper hemi-continuous, and $V_i(\cdot)$ is continuous.

Since $V_i(\cdot)$ is continuous in prices, and examining (1.10) reveals that L_i is continuous in V_j for all $i, j \in \{H, B, O\}$, it follows immediately that $L^*_i(q)$ is continuous in q.

Banks. On the bank side, we consider the *aggregate* policy correspondence. Lemma A.1 implies that both policies, $\ell^{1*}_B(q)$ and $\ell^{2*}_B(q)$ are non-empty, compact-valued and upper hemi-continuous. What remains, then, is to analyze the aggregate bank loan supply function, given by the right-hand side of (A.13) for $\eta^* \in \eta^*(q)$ defined by (A.10). Lemma A.3 implies that there are three possible cases. First, $\eta^*(q) = 1$ everywhere: then, Lemma A.1 delivers the result because $\ell_B^*(q) = \ell_B^{1*}(q)$. Second, $\eta^*(q) = 0$ everywhere: then Lemma A.1 delivers the result because $\ell_B^*(q) = \ell_B^{2*}(q)$. The third and final case arises at the single value of $\overline{q}_B$. In this case, any $\eta \in [0,1]$ is consistent with bank optimality because $v_B^1(q) = v_B^2(q)$, and therefore the optimal policy correspondence is the convex combination of two non-empty, compact-valued and upper hemi-continuous correspondences. QED.

Lemma A.4 shows that the excess demand function in (A.16) has the standard properties to use a fixed point equilibrium existence argument. The next three lemmas complete this argument.

For small $\epsilon > 0$,[1] define the set

$$\mathcal{Q}^\epsilon = \left\{ q \in [0,1]^3 \mid \overline{q} \in [\epsilon, 1], q_B \in [\tilde{q}_B + \epsilon, 1], q_H \in [\tilde{q}_H + \epsilon, 1] \right\}, \tag{A.20}$$

where $\tilde{q}_i$ for $i \in \{H, B\}$ is the solution to $f'(k_F) = 1/\tilde{q}_i$, i.e. the price just at which the non-negativity constraint on b binds. All prices lower than this are irrelevant since demand must be 0. Further, define the tatonnement price adjustment operator

$$T^\epsilon(q) = \max_{q \in \mathcal{Q}^\epsilon} q_H z_H + q_B z_B + \overline{qz} \tag{A.21}$$

Lemma A.5 ***Fixed point of*** T^ϵ***.*** *The operator T^ϵ has a fixed point q^* in $\mathcal{Q}^\epsilon$ for all $\epsilon \in (0,1)$.*

Proof: I have previously shown that the excess demand correspondence $\mathcal{Z}(\cdot)$ is non-empty, compact-valued and upper hemi-continuous. In addition, standard arguments following Hildenbrand and Kirman (1988) imply that for strictly positive ϵ, T^ϵ is non-empty, convex-valued and upper hemi-continuous. Therefore, Kakutani's fixed point theorem implies that T^ϵ has a fixed point $q^* \in \mathcal{Q}^\epsilon$. QED.

[1] Of course, ϵ in this definition is not associated with the additive preference shocks from the model.

Lemma A.5 demonstrates that the T^ϵ operator from (A.21) has a fixed point. The remaining lemmas are devoted to demonstrating that this fixed point is an equilibrium. Before proceeding, though, it is useful to first establish several properties of the optimal policies of all agents. These are easily verified and therefore are stated here without proof: (i) firm loan type i demand is increasing in q_i and decreasing in q_j for $j \neq i$; (ii) bank loan supply is decreasing in q_B and increasing in $\overline{q}$; (iii) bank deposit demand is increasing in $\overline{q}$ and decreasing in q_B; (iv) HH loan supply is decreasing in q_H and increasing in $\overline{q}$; and (v) HH deposit supply is decreasing in $\overline{q}$ and increasing in q_H. Having established these (intuitive) properties, we can proceed to analyze the fixed point from Lemma A.5.

Lemma A.6 ***Two conditions for fixed point to be an equilibrium.*** *If either (i)* $q^* \in$ *int*$(\mathcal{Q}^\epsilon)$ *or (ii)* $q_B^* \in (\tilde{q}_B + \epsilon, 1)$, $q_H^* \in (\tilde{q}_H^* + \epsilon, 1)$, *and* $\overline{q} = 1$, *then* $0 \in \mathcal{Z}(q^*)$. *That is, if either condition (i) or (ii) holds, then an equilibrium exists.*

Proof: I consider each case in turn. First, consider the case when $q^* \in \text{int}(\mathcal{Q}^\epsilon)$, and suppose to the contrary that $0 \notin \mathcal{Z}(q^*)$. This implies that at least one of z_B, z_H, and $\overline{z}$ is not equal to zero. If $z_B > 0$, then $L_B b_B > \ell_B b_B$. Given the results above, though, a higher q_B will increase this difference, and so by the definition of T^ϵ, $q_B = 1$ eventually, which contradicts the assumption that $q^* \in \text{int}(\mathcal{Q}^\epsilon)$. The opposite analysis rules out the case that $z_B < 0$: eventually, we would have $q_B = \tilde{q}_B + \epsilon \notin \text{int}(\mathcal{Q}^\epsilon)$, a contradiction. The same analysis resolves both cases for household lending. Finally, consider the case in which $\overline{z} > 0$. Then, it is possible to increase $\overline{q}$ until $\overline{q} = 1$, a contradiction. The reverse case reveals a contradiction with convergence of $\overline{q}$ to ϵ. This proves the first part of the claim.

Second, consider the case when $\overline{q} = 1$. The logic from the household and bank loan prices from the first case above still applies. What remains, therefore, is to show that $\overline{q} = 1$ is consistent with zero excess demand. If $\overline{q} = 1$, given deposit insurance, households are indifferent between storage and depositing at the bank. Therefore, $y_H^*(q|\overline{q} = 1)$ includes any

d_H^* and a_H^* such that $d_H^* + a_H^* = k_H - q_H \ell_H^* b_H$. In particular, $d_H^* = 0$ is included in the set of optimal policies. For $\overline{q} = 1$ to obtain, it must be the case that $d_B^* < d_H^*$ for $\overline{q} < 1$. Since $d_H^* = 0$ is in the optimal policy set of the household, though, and $d_B^* > 0$, there must be some value of d_H^* in the optimal policy correspondence such that $d_B^* = d_H^*$, by continuity. *QED*.

Lemma A.7 ***Fixed point is an equilibrium.*** *The fixed point q^* from Lemma A.5 satisfies either (i) $q^* \in int(\mathcal{Q}^\epsilon)$ or (ii) $q_B^* \in (\tilde{q}_B + \epsilon, 1)$, $q_H^* \in (\tilde{q}_H^* + \epsilon, 1)$, and $\overline{q} = 1$.*

Proof: In both cases (i) and (ii), we have that $q_i^* \in (\tilde{q}_i^* + \epsilon, 1)$ for $i \in \{B, H\}$. We know that $\lim_{q_i \to 0} \ell_i = +\infty$, and so there exists a strictly positive price below which excess demand $z_i < 0$. Moreover, it is clear that $\lim_{q_i \to \overline{q}} \ell_i^* = 0$, since banks or HH would rather invest in deposits, and so excess demand must turn positive as q_i increases: that is, $z_i > 0$. Since $\overline{q} \leq 1$, there must exist $\hat{q}_i < \overline{q}$ such that excess demand crosses 0.

Having established the desired results for the two loan markets, I turn lastly to the deposit market. It is clear from the household problem that $\lim_{\overline{q} \to 0} d_H(q) = +\infty$, and so there exists $\hat{\overline{q}}$ such that excess demand for deposits is strictly negative whenever $\overline{q} \leq \hat{\overline{q}}$. Furthermore, as discussed in the proof of Lemma A.6, $\overline{z}(q) \geq 0$ when $\overline{q} = 1$. Therefore, the operator T^ϵ sends the deposit price in the interval $(\epsilon, 1]$. *QED*.

Define $\epsilon = 1/n$ for $n = 1, 2, ...$ Then we can take ϵ arbitrarily low while preserving the properties of T^ϵ. Lemma A.5 demonstrated that T^ϵ from (A.21) mapping prices from $\mathcal{Q}^\epsilon$ from (A.20) into itself has a fixed point. Lemma A.6 showed sufficient conditions for the fixed point of this operator to be an equilibrium. Finally, Lemma A.7 showed that indeed the fixed point must have these sufficient conditions, completing the proof. *QED*.

A.1.2 Model with banks only

In order to "shut down" the direct lending market, we must essentially constrain $\ell_H = 0$ in the household's portfolio problem, and "artificially" set $q_H = 0$. [2] Then, firms effectively choose between borrowing from the bank and self-financing the project. Therefore, instead of (1.10), we now have

$$L_B(k) = \frac{\exp\{\alpha V_B(k)\}}{\exp\{\alpha V_B(k)\} + \exp\{\alpha V_O(k)\}} \tag{A.22}$$

Then, since HH cannot invest directly in firms, the HH beginning and end of period budget constraints are simply

$$C = a_H + d_H - T(p) \text{ for all } p \tag{A.23}$$

$$k_H = a_H + \overline{q} d_H. \tag{A.24}$$

The portfolio problem for the HH, then, has a very simple solution: if $\overline{q} < 1$, then $d_H^* = k_H$ and $a_H^* = 0$; otherwise, if $\overline{q} = 1$, then the HH is indifferent between storage and bank deposits, and any $(a_H, d_H) \in [0, k_H]^2$ such that $a_H + d_H = k_H$ is a solution. The bank problem is completely unchanged.

Definition A.2 ***Equilibrium with bank lending only***. *An equilibrium in the model with only bank lending is a list of: firm lender share and loan amount choices, L_i^* and b_i^* for $i \in \{B, O\}$; bank choices of deposits, d_B^*, monitoring, m_B^* , and loan supply ℓ_B^*; household choices of storage, a_H^*, and deposits, d_H^*; a bank loan price, q_B^*; a deposit price, $\overline{q}^*$; and a tax rate $T^*(p)$, such that: firm choices solve (1.9), where bank values are given by (1.4); bank choices solve (1.11); household choices solve (1.15) subject to the modified budget constraints (A.23) and (A.24); bank lending and bank deposit markets clear according to (1.17) and (1.19), respectively, where L_i^* is given by (A.22) and the tax rate is given by (1.12).*

[2] Although the ϵ shocks still imply that $L_H > 0$, for $k_F > 0$ given our assumptions on $f(\cdot)$ it must be the case that $b_H = 0$, and so this market is shut down.

A.2 Data Appendix

A.2.1 Data sources

I use data from three primary sources. In this section I provide brief descriptions of each of these data sources, highlighting the advantages and limitations of each and pointing out what specific data I obtain from each source. All nominal variables, regardless of data source, are deflated by the CPI.

Flow of funds (FF)

The Flow of Funds is a quarterly data release from the Federal Reserve Board of Governors. It contains detailed accounting of the flows and levels of financial assets in the United States, broken down by sector and instrument, allowing me to identify the composition of external debt for firms and banks at the aggregate level. The data I draw from this report come from two main sectors: Domestic Nonfinancial Corporate Business (Schedules F.103 and L.103) and U.S. Chartered Depository Institutions (Schedules F.111 and L.111). I focus on corporate business (excluding noncorporate) for consistency of comparison with my model: since firms in this category have access to both bank loans and corporate bond markets, I want to compare only to the sector of the real economy with the same access. Also, I focus on private, deposit-taking banks to abstract from other types of financial institutions who may perform different monitoring functions, finance their operations differently, or be subject to alternative regulatory regimes. In addition, this allows for consistency with the bank data I obtain from the call reports (described below).

Quarterly Financial Report for Manufacturing, Mining and Trade (QFR)

The QFR contains quarterly information on the financial position and profits of firms in the manufacturing (mining, wholesale and retail trade) industry with domestic assets of at least

\$250K (\$50M). The data come from a sample survey, and individual firms are not identifiable. Rather, all responding firms are grouped into size and industry classes. I focus only on the largest two size brackets ($assetsizecode$ = 9 or 10, for total assets of at least \$250M) for the same reasons as I focus on corporate firms in the FF data above. The key advantage of the QFR data is its detailed breakdown of the debt structures of firms which are not available in other widely used data sources (such as Compustat, below).

Consolidated Reports of Condition and Income (Call reports)

The call reports are maintained and updated quarterly by the Federal Deposit Insurance Corporation (FDIC) and contain detailed balance sheet and income statement data for banks operating in the U.S. Data are assembled at the holding company level. All the statistics reported in the paper from the call reports come from a data set I have cleaned and subsetted in the following ways: (i) include only commercial bank holding companies (charter type, RSSD9048 = 200, entity type, RSSD9331 = 1); (ii) include only banks in the 50 states and Washington, D.C. (RSSD9210 between 0 and 57); (iii) delete observations with missing or 0 assets (RCFD2170 missing or 0); (iv) delete banks with less than 10 consecutive observations over the sample period of 1984Q1:2011Q4. Throughout the paper, since my focus is on lending to businesses, the default rates and loan returns are specific to the commercial and industrial (C&I) loan portfolios of the banks in the sample. All moments used in the paper which are specific to the banking sector come from the call report.

A.2.2 Definitions of model moments

In Table A.1 below, I provide the definitions of the key moments in the data used in the estimation procedure discussed in Section 1.5. The model counterpart is in Table A.2, which contains the model definition of each moment.

	Moment	Source	Data definition
Firm	Bank share of business debt	QFR	bank debt / total debt
	Firm leverage	QFR	total firm liabilities / total firm assets
	Firm return on capital	QFR	$\frac{\text{net interest before taxes + accumulated depreciation}}{\text{net property plant and equipment}}$
	Output volatility	NIPA	standard deviation of cyclical component of HP-filtered log real GDP
Bank	Default rate, bank loans	CR	(C&I chargeoffs - C&I recoveries) / C&I loans
	Monitoring to total expense	CR	salaries / total expenses
	Bank leverage	CR	total bank liabilities / total assets
	Bank loan return	CR	$\frac{1+\frac{\text{interest income on C\&I loans}}{\text{total C\&I loans}}}{1+\text{inflation rate}} - 1$
	Net interest margin	CR	loan return - cost of deposits
HH	Default rate, corp. bonds	GFSS	corp. bonds defaulted / total corporate bonds
	Riskless / total HH assets	FF	(deposits + treasuries) / total HH fin. assets
	HH / bank assets	FF	HH total fin. assets / bank total fin. assets

Notes: This table shows the definitions of model moments as calculated in the data. The data sources in the "Source" column are: QFR – Quarterly Financial Reports for Manufacturing, Mining, and Trade Firms; NIPA – National Income and Product Accounts; CR – Call Reports; FF – Flow of Funds; GFSS – Giesecke et al. (2015).

Table A.1: Moment definitions: data

For all agents, I define leverage as total liabilities to total assets. Using the QFR data, bank debt is the sum of short term bank debt, long term bank debt maturing in less than one year, and long term bank debt maturing in more than one year. I compute the standard deviation of output as the standard deviation of the HP-filtered log real GDP series with a smoothing parameter of 1600 (given the quarterly data from the National Income and Product Accounts (NIPA)). For the return on firm capital, I add back in depreciation since in my static model, all capital consumed in the production process is used for consumption at the end of the period.

On the bank side, the default rate is for C&I loans only, and is computed as the total C&I dollars charged off, less total C&I dollars recovered, divided by total C&I loans. To measure the relative importance of monitoring in banks' cost structures, I use the rough proxy

	Moment	Model definition
Firm	Bank share of business debt (%)	$\frac{q_B L_B b_B}{q_B L_B b_B + q_H L_H b_H}$
	Firm leverage	$L_B \frac{q_B L_B b_B}{q_B L_B b_B + k_F} + L_H \frac{q_H L_H b_H}{q_H L_H b_H + k_F}$
	Firm return on capital (%)	$L_B r_B^K + L_H r_H^K + L_O r_O^K$
	Output volatility (%)	$\text{Var}(\log(Y(p))$
Bank	Default rate, bank loans (%)	$(1-\overline{p})(1-G(\overline{x}(m_B)))$
	Monitoring to total expense (%)	$\frac{c(m_B, q_B \ell_B b_B) - c(0, q_B \ell_B b_B)}{c(m_B, q_B \ell_B b_B) - k_B}$
	Bank leverage	$\frac{\overline{q} d_B}{\overline{q} d_B + k_B}$
	Bank loan return (%)	$\frac{P(m_B, \overline{p}) \ell_B b_B}{c(m_B, q_B \ell_B b_B)} - 1$
	Net interest margin (%)	$1/q_B - 1/\overline{q}$
HH	Default rate, corporate bonds (%)	$(1-\overline{p})(1-G(\overline{x}(0)))$
	Riskless / total HH assets	$\overline{q} d_H / k_H$
	HH assets to bank assets	$k_H / q_B \ell_B b_B$

Notes: This table shows the definitions of model moments as calculated at equilibrium values in the model. The notation follows that used in Chapter 1.

Table A.2: Moment definitions: model

of salaries to total expenses. This measure is far from perfect – monitoring involves not only salaries, but infrastructure expenditures, and not all salaries are devoted to monitoring –, but the quantitative results are robust to various measures. Bank loan returns adjust for inflation, and the net interest margin nets out the cost of funding.

On the household side, I proxy for the default rate on non-bank loans using the default rate on corporate bonds from Giesecke et al. (2015). I focus only on corporate bonds for two reasons. First, data availability on loan returns for other non-bank financial institutions is scarce. Second, corporate bonds map most clearly into the unmonitored lending of my model, since there is a well known "tragedy of the commons" under which no individual bondholder monitors borrowers. Riskless household assets include only deposits and US Treasuries.

Next, I consider the model calculations for the moments used in the estimation. A few of the entries in Table A.2 warrant comment. First, firm leverage is computed as the weighted average of the leverage of firms who borrow from banks, those who borrow from households, and those who self-finance (and therefore have leverage of 0). Firms' expected return on capital for lender i is given by

$$r_i^K = P(m_i, \overline{p}) \frac{f(I_i) - b_i}{k_F} - 1. \quad \text{(A.25)}$$

Total output in the economy in state p is given by

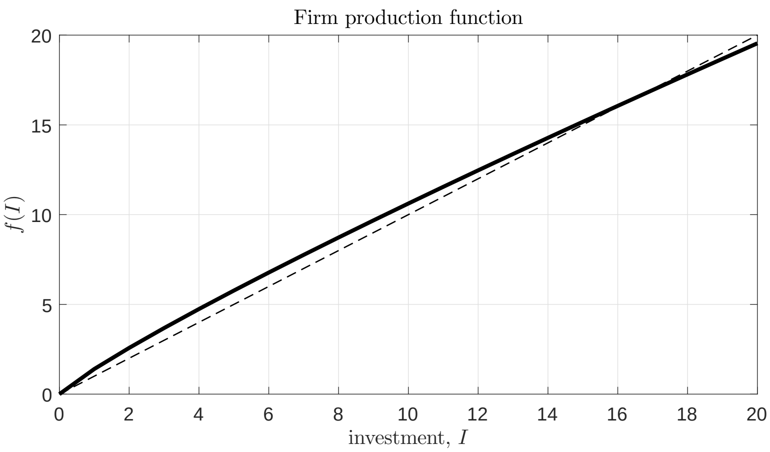
$$Y(p) = \sum_i L_i P(m_i, p) f(I_i). \quad \text{(A.26)}$$

Output volatility is measured as the standard deviation of the log of $Y(p)$. In addition, default rates (for both bank and direct loans) are computed as the ex ante expected or average default rate, the product of the fraction of firms shirking, $1 - G(\overline{x}(m_i))$, and the expected failure rate of these firms, $\psi(1 - p_h) + (1 - \psi)(1 - p_l) = 1 - \overline{p}$.

My measurement of bank expenses in the model is designed to create a simple division between interest expenses (the cost of *lending only*, net of internal funding) and non-interest expenses (the cost of *monitoring only*. Total expense (the denominator) is the total cost of lending, $c(m_B, q_B \ell_B b_B)$, less the savings from investing internal capital, k_B. The cost associated with monitoring only, is defined as the difference between the cost of lending and monitoring at the selected level, $c(m_B, q_B \ell_B b_B)$, and the cost of lending the chosen amount without monitoring at all, $c(0, q_B \ell_B b_B)$.

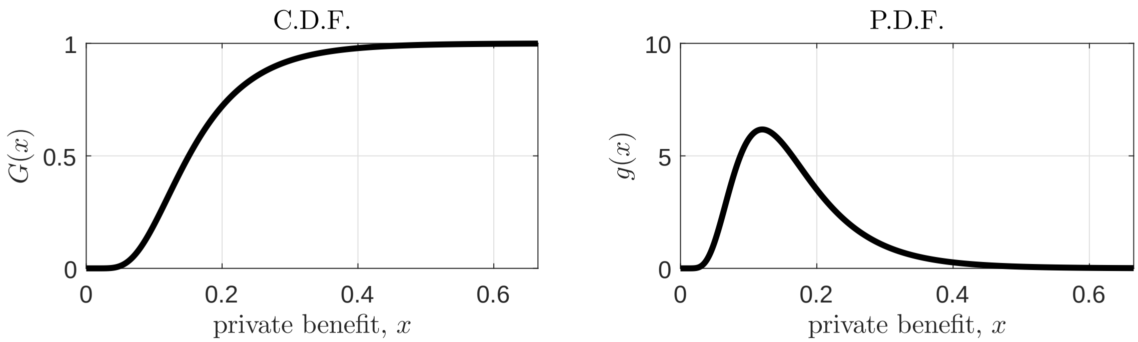
A.2.3 Model technologies

Figures A.2, A.3, and A.4 depict the parameterized technologies underlying the model: the firm production function $f(I)$, the distribution $G(x)$ of the private shirking benefit x, and the bank lending and monitoring cost function $c(m, \ell)$. The production function implies an optimal scale of about 5.69. The distribution of private benefits implies a mean of $\mathbb{E}(x) = 0.17$, with variance of 8.62%. Given the equilibrium borrowing in the model, this is equal to about 15% of the loan size.



Notes: The production function takes the form from equation (1.20), with scale and curvature parameters θ_1 and θ_2 from Table 1.2.

Figure A.2: Firm production function



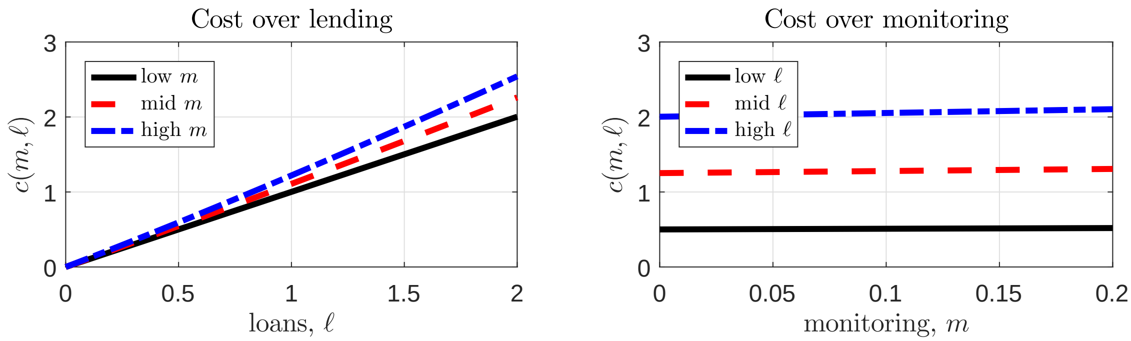
Notes: The private benefit distribution is lognormal, with parameters μ_x and σ_x from Table 1.2.

Figure A.3: Firm private shirking benefit distribution

A.2.4 Computational algorithm

The computational algorithm employed to solve this model uses a grid search to find the bank and household optimal policies nested within price updating based on excess demand functions in each of three markets: bank loans, direct loans, and deposits. In this section, I lay out the key steps of the algorithm in detail. Note that the estimation of the model involves embedding the steps outlined below within a standard SMM routine.

1. Fix the set of parameters in Table 1.2. Create grids for bank loans (ℓ_B), bank monitoring



Notes: The cost function takes the form from equation (1.21), with loan and monitoring curvature parameters ρ_1 and ρ_2 from Table 1.2.

Figure A.4: Bank lending cost function

(m_B), and household direct lending.

2. Compute the value of firms' outside option, $V_O(k_F)$, according to (1.7).

3. Make an initial guess of the price vector, q^0.

4. Taking these prices as given,

   (a) use the loan size optimality condition (1.8) to compute the size of the loan demanded by firms, b_i^*, for $i \in \{B, H\}$.

   (b) Solve the bank problem, taking b_B^* as given:

      i. For each pair of points on the grids for ℓ_B and m_B,

         A. Use the bank budget constraint in (1.11) and current prices to compute the amount of deposits the bank must take in to implement this policy, $d_B(\ell_B, m_B)$.[3]

         B. Compute the total value associated with each choice using the objective function in (1.11).

[3] In principle, the bank may want to take in no deposits and even lend less than its own internal capital, k_B. I allow for this in the algorithm by allowing the bank to invest in the storage technology if lending is sufficiently unfavorable. Generically, this case does not arise.

ii. Find the maximal value and the associated optimal policies, ℓ_B^*, m_B^*, and $d_B^* = d_B(\ell_B^*, m_B^*)$.

(c) Solve the household problem, taking b_H^* as given:

i. For each possible choice of ℓ_H,

A. Use the household budget constraint in (1.15) and current prices to compute the amount of deposits the household will supply at this policy, $d_H(\ell_H)$.[4]

B. Compute the total value associated with each choice using the objective function in (1.15).

ii. Find the maximal value and the associated optimal policies, ℓ_H^* and $d_H^* = d_H(\ell_H^*)$.

(d) Compute firm values from each type of lender, $V_i(k_F)$, according to (1.7), taking m_B as given for V_B. Then, use (1.10) to compute the shares choosing each lender, $L_i(k)$.

5. Compute the excess demand function associated with the current prices, $\mathcal{Z}(q^0)$ from (A.16).

6. If the convergence metric $\sup\{|z_B(q^0)|, |z_H(q^0)|, |\overline{z}(q^0)|\}$ is within the tolerance defined at the outset, an equilibrium has been found. Go to step 9. Otherwise, proceed to steps 7 and 8.

7. Update prices according to:

$$q^1 = \begin{bmatrix} q_B^1 \\ q_H^1 \\ \overline{q}^1 \end{bmatrix} = \begin{bmatrix} (1-\nu_1)q_B^0 + \nu_1(q_B^0 - \nu_2 z_B(q^0)) \\ (1-\nu_1)q_H^0 + \nu_1(q_H^0 - \nu_2 z_H(q^0)) \\ (1-\nu_1)\overline{q}^0 + \nu_1(\overline{q}^0 - \nu_2 \overline{z}(q^0)) \end{bmatrix},$$

[4] If $\overline{q} = 1$, the household is indifferent between deposits and the storage technology. In this case, the algorithm reflects this indifference via an updated market clearing, requiring only that $d_H + a_H \geq d_B$. If $\overline{q} < 1$, storage is strictly dominated.

where ν_1 and ν_2 are parameters which govern the speed and smoothness of convergence. Both parameters decline deterministically from iteration to iteration, so that prices adjust more in early iterations and less in later iterations as the algorithm gets closer and closer to convergence.

8. Set $q^0 = q^1$ and go back to step 4.

9. Use the equilibrium prices and quantities to compute desired model moments.

Appendix B

Chapter 2 Appendix

B.1 Computational algorithm

In this section, we describe the algorithm used to compute the benchmark model presented in this paper. Note that the model is calibrated by using the procedure below to solve the model for a given set of parameters, and then updating these parameters to minimize the distance between the model moments and the data moments.

1. Set parameters and tolerances for convergence and create grids for (β, e, z, a, s).[1] Denote the length of these grids by $n_\beta, n_e, n_z, n_a,$ and n_s respectively. The parameters should include the exogenous transition matrices $Q^\beta(\beta'|\beta)$ and $Q^e(e'|e)$. Further, note that although the variable s is in principle continuous, its values should be discretized as finely as possible in the range $[Q^\beta_{LH}, 1 - Q^\beta_{HL}]$ to minimize interpolation costs (details below).

2. Initialize the following equilibrium objects with sensible initial conditions:[2]

   (a) $W(\beta, e, z, a, s) = 0$ for all β, e, z, a, s.

[1]Though the algorithm is presented here with the separated e and z components of the earnings process for consistency with the text, the code condenses these two states into one. As long as both are observable, this simplification is completely acceptable and without loss of generality.

[2]Note that throughout this computational algorithm description, we will employ the mathematical notation from the paper. However, in terms of coding, we have

$$p(a', s', \gamma') = p(a', \psi^{(d,a')}(e, z, a, s), \Gamma^{(0,a')}(e, \psi^{(d,a')}(e, z, a, s))) = \tilde{p}(e, z, a, s, a'),$$

and likewise for the $q(\cdot)$ and $\Gamma(\cdot)$ functions.

(b) $\psi^{(d,a')}(e,z,a,s) = \frac{Q^{\beta}_{HL}}{Q^{\beta}_{HL}+Q^{\beta}_{LH}}$ for all $e, z, a, s, (d, a')$, so that there is no information initially on types other than the stationary distribution implied by $Q^{\beta}(\cdot)$.

(c) $\Gamma^{(0,a')}(e, s') = 1$ for all e, a', s', so that the initial assumption is that all types repay all the time.

(d) $p(a', s', \gamma') = 1$ for all a', s', γ'.

(e) $q(a', p) = \frac{1}{1+r}$ for all a', so that initial pricing is consistent with the assumption on p.

(f) $\mu(\beta, e, z, a, s) = \frac{1}{n_\beta \cdot n_e \cdot n_z \cdot n_a \cdot n_s}$ for all β, e, z, a, s so that the initial guess of the distribution of agents is uniform over the state space.

3. Taking the current guess of the equilibrium functions $f_0 = \{\psi_0, \Gamma_0, q_0, p_0\}$ as given, enter the equilibrium computation loop:

   (a) Solve for the expected value function $W_1(\cdot|f_0)$ taking as given $W_0(\cdot|f_0)$:

      i. Assess budget feasibility for the current schedule of prices, finding the set of feasible actions $\mathcal{F}(e, z, a, s|f_0)$.

      ii. For each value of β, e, z, a, s on their respective grids, compute the conditional value associated with each action $(d, a') \in \mathcal{F}(e, z, a, s|f_0)$, $v_1^{(d,a')}(\beta, e, z, a, s|f_0)$, according to (2.3), with $W(\cdot) = W_0(\cdot)$.

         A. Note that the mapping from $s \to s'$ implied by $\psi_0(\cdot)$ will not, in general, yield a value ofs' on the grid constructed for s in step 1. In this case, it is necessary to find two adjacent grid points s'_i and s'_j such that $s'_i \leq \psi^{(d,a')}(e, z, a, s) \leq s'_j$, and assign a continuation value

$$W(\beta', e', z', a', \psi^{(d,a')}(e, z, a, s)) = \omega W(\beta', e', z', a', s'_i) + (1-\omega) W(\beta', e', z', a', s'_j),$$

where

$$\omega = \frac{s'_j - \psi^{(d,a')}(e,z,a,s)}{s'_j - s'_i}.$$

iii. Having looped over all feasible actions, aggregate the conditional values $v_1^{(d,a')}(\cdot|f_0)$ into the new expected value function $W_1(\cdot|f_0)$ according to (2.2).

iv. Assess value function convergence in terms of the sup norm metric,

$$dist = \sup_{\beta,e,z,a,s} |W_1(\beta,e,z,a,s|f_0) - W_0(\beta,e,z,a,s|f_0)$$

If $dist < tol$, go to 3.b; if $dist \geq tol$, set $W_0(\cdot|f_0) = W_1(\cdot|f_0)$ and go back to 3.a.ii.

(b) Compute the decision probabilities $\sigma_1(\cdot|f_0)$ implied by $W_1(\cdot|f_0)$ according to (2.4).

(c) Given the decision probabilities $\sigma_1(\cdot|f_0)$, compute the new set of equilibrium functions, $f_1 = \{\psi_1, \Gamma_1, p_1, q_1\}$:

i. Compute $\psi_1(\cdot)$ according to (2.9).

ii. Compute $\Gamma_1(\cdot)$ according to (2.10).

iii. Compute $p_1(\cdot)$ according to (2.11).

A. Computing $p(\cdot)$ involves looking at the decision rule $\sigma(\cdot)$ across possible states *tomorrow*, and so interpolation on $s' = \psi^{(d,a')}(e,z,a,s)$ is required here, also, as in step 3.a.ii.A.

iv. Compute $q_1(\cdot)$ according to (2.8).

(d) Assess equilibrium function convergence in terms of the sup norm metric

$$dist = \max\{\sup|\psi_1(\cdot) - \psi_0(\cdot)|, \sup|\Gamma_1(\cdot) - \Gamma_0(\cdot)|, \sup|p_1(\cdot) - p_0(\cdot)|, \sup|q_1(\cdot) - q_0(\cdot)|\}$$

If $dist < tol$, then proceed to step 4; otherwise, set $f_0 = f_1$ and go back to the beginning of step 3.

4. Compute the stationary distribution associated with the equilibrium behavior and the equilibrium functions computed in step 3.

   (a) For each state β, e, z, a, s, compute $\mu_1(\beta, e, z, a, s)$ according to (2.12), with $\mu(\cdot) = \mu_0(\cdot)$.

      i. Computing $p(\cdot)$ involves looking at the decision rule $\sigma(\cdot)$ across possible states *tomorrow*, and so interpolation on $s' = \psi^{(d,a')}(e, z, a, s)$ is required here, also, as in step 3.c.iii.A in the equilibrium function iteration procedure described above.

   (b) Assess convergence based on the sup norm metric $dist = \sup|\mu_1(\cdot) - \mu_0(\cdot)|$. If $dist < tol$, go to step 5; otherwise, set $\mu_0 = \mu_1$ and go back to 4.a.

5. Compute any desired moments.

B.2 Grids

Table B.1 presents the key grid used in the computational analysis. Note in particular that the asset and type score grids are quite dense in order to insure convergence, while in contrast the earnings and type grids are coarse in order to ease the computational burden and simplify the analysis.

B.3 Model moment definitions

Default rate. The default rate is computed as the total fraction of the population who defaults within a given period. The probability of a given state is given by $\mu(\cdot)$, and the probability of default given a state is $\sigma^{(1,0)}(\cdot)$, and so the *aggregate* default rate is

$$\text{aggregate default rate} = \sum_{\beta,e,z,a,s} \sigma^{(1,0)}(\beta, e, z, a, s) \cdot \mu(\beta, e, z, a, s). \tag{B.1}$$

Variable	Notation	No. points	Range / Values	Notes
Discount factor	β	2	$\{0.97, 0.89\}$	2-point support makes Bayesian functions scalar-valued.
Earnings (persistent)	e	3	$\{0.58, 1.00, 1.74\}$	See Section 2.4.
Earnings (transitory)	z	3	$\{-0.18, 0.00, 0.18\}$	See Section 2.4.
Assets	a	150	$[-0.25, 8.00]$	50 points in neg. region, 100 in pos. Density close to 0 is critical.
Type score	s	50	$[0.05, 0.89]$	bounded below by low β to high β transition, above by high to low.

Notes: The entries in this table describe the 5 discrete grids used in the quantitative analysis in Chapter 2.

Table B.1: Grids used in computational analysis

By type, we have $\sum_{e,z,a,s} \sigma^{(1,0)}(\beta, e, z, a, s) \cdot \mu(\beta, e, z, a, s) / \sum_{\hat{e},\hat{z},\hat{a},\hat{s}} \mu(\beta, \hat{e}, \hat{z}, \hat{a}, \hat{s})$.

Fraction in debt. This is simply the fraction of of the population with $a < 0$ in a given period, and so

$$\text{fraction in debt } = \sum_{\beta,e,z,s} \sum_{a<0} \mu(\beta, e, z, a, s). \tag{B.2}$$

By type, the analogous figure is $\sum_{e,z,s} \sum_{a<0} \mu(\beta, e, z, a, s) / \sum_{\hat{e},\hat{z},\hat{s},\hat{a}} \mu(\beta, \hat{e}, \hat{z}, \hat{a}, \hat{s})$.

Debt to income. Income in the model is given by the sum of earnings (persistent and transitory) and net interest on assets. That is, income $= e + z + (1/q(a', p) - 1) \cdot a$. Therefore, debt to income is computed as the weighted average of the ratio of assets, a, to income conditional on a being negative:

$$\text{debt to earnings} = \sum_{\beta,e,a<0,s} \frac{a}{e + z + (1/q(\cdot) - 1) \cdot a} \cdot \frac{\mu(\beta, e, z, a, s)}{\sum_{\hat{\beta},\hat{e},\hat{z},\hat{a}<0,\hat{s}} \mu(\hat{\beta}, \hat{e}, \hat{z}, \hat{a}, \hat{s})}, \tag{B.3}$$

where it is understood that

$$q(\cdot) = q\left(a', p\left(a', \psi^{(0,a')}(e, z, a, s), \Gamma^{(0,a')}(e, \psi^{(0,a')}(e, z, a, s))\right)\right).$$

Average interest rate. The average interest paid (or received) by the agents in the economy is the weighted average of the interest rates paid, $1/q - 1$, over the stationary distribution and decision probabilities.

$$\text{average interest rate} = \sum_{e,z,a,s,a'} \left(\frac{1}{q(\cdot)} - 1\right) \cdot \sum_{\beta} \sigma^{(0,a')}(\beta, e, z, a, s) \cdot \frac{\mu(\beta, e, z, a, s)}{\sum_{\hat{\beta}} \mu(\hat{\beta}, e, z, a, s)} \quad \text{(B.4)}$$

B.3.1 Credit scores

The goal of this section is to map the key equilibrium objects of the model into credit scores which reflect the key features of credit scores that we observe in the real world. It is important to note, however, that these credit scores are secondary moments in our model, and not key drivers of the pricing of debt. This is because a credit score must necessarily aggregate over *possible future actions*, as will be made clear below.

The basic idea of a credit score is to measure an agent's probability of a default event within a certain period of time, given today's observables. We can compute these probabilities for windows of $n = 1, ..., N$ periods ahead, where N is an arbitrary finite number greater than or equal to 1. In this sense, we can compute an N-vector of credit scores based on the observable state (e, z, a, s), $\xi(e, z, a, s)$, such that

$$\xi(e, z, a, s) = (\xi_1(e, z, a, s), ..., \xi_N(e, z, a, s)),$$

where $\xi_n(e, z, a, s)$ represents the probability of a default event within n periods.

How can we compute these scores? The $p(\cdot)$ function computes the probability of repayment next period on a given action for a given state today; $\sigma(\cdot)$ indicates the probability of each of these actions, and we can weight out the unobservable parts of the state relevant for $\sigma(\cdot)$ (i.e. β) using the stationary distribution $\mu(\cdot)$. Let's begin with the 1-period credit score:

$$\begin{aligned} \xi_1(e, z, a, s) \quad = \quad & \sum_{(d,a') \in \mathcal{Y}} \Bigg[p\left(a', \psi^{(d,a')}(e, z, a, s), \Gamma^{(d,a')}\left(e, \psi^{(d,a')}(e, z, a, s)\right)\right) \\ & \cdot \sum_{\beta \in \mathcal{B}} \sigma^{(d,a')}(\beta, e, z, a, s) \cdot \frac{\mu(\beta, e, z, a, s)}{\sum_{\hat{\beta} \in \mathcal{B}} \mu(\hat{\beta}, e, z, a, s)} \Bigg] \end{aligned}$$

The first term captures the probability that an agent in state (e, z, a, s) today (period t) choosing a' will default tomorrow (period $t + n = t + 1$). The second two terms capture the probability that an agent in state (e, z, a, s) will choose action a': $\sigma^{(d,a')}(\beta, e, z, a, s)$ is the probability that an agent with full state $(\beta, e, z, , a, s)$ will choose a', and $\frac{\mu(\beta,e,z,a,s)}{\sum_{\hat{\beta}} \mu(\hat{\beta},e,z,a,s)}$ is the share of β-types in the sub-population of agents in state (e, z, a, s). Multiplying these terms and summing over β gives the desired action weight. To ease notation in what follows, define the *observable* decision rule $\tilde{\sigma}(\cdot)$ by

$$\tilde{\sigma}^{(d,a')}(e, z, a, s) = \sum_{\beta \in \mathcal{B}} \sigma^{(d,a')}(\beta, e, z, a, s) \cdot \frac{\mu(\beta, e, z, a, s)}{\sum_{\hat{\beta} \in \mathcal{B}} \mu(\hat{\beta}, z, e, a, s)}$$

so that

$$z_1(e, z, a, s) = \sum_{(d,a') \in \mathcal{Y}} p\left(a', \psi^{(d,a')}(e, z, a, s), \Gamma^{(d,a')}\left(e, \psi^{(d,a')}(e, z, a, s)\right)\right) \cdot \tilde{\sigma}^{(d,a')}(e, z, a, s). \tag{B.5}$$

This definition is particularly useful given the stationarity of the distribution $\mu(\cdot)$.

We can perform an analogous procedure for subsequent periods, and there turns out to be a nice recursive formulation of the n-period score. In words, $\xi_1(e, z, a, s)$ represents the assessed probability that an agent in state (e, z, a, s) in period t repays his debt (whatever that turns out to be) in period $t+1$; in probability notation, $\xi_1(e, z, a, s) = \Pr(\text{repay in } t+1 | e, z, a, s \text{ in } t)$, and likewise $\xi_n(e, z, a, s) = \Pr(\text{repay in } t + n | e, z, a, s \text{ in } t)$.

Starting with $n = 2$, we have

$$\begin{aligned}
\xi_2(e, z, a, s) &= \Pr(\text{repay in } t + 2 | e, z, a, s \text{ in } t) \\
&= \sum_{e',z',a',s'} \Big(\Pr(\text{repay in } t + 2 | e', z', a', s' \text{ in } t + 1) \\
&\quad \cdot \Pr(e', z', a', s' \text{ in } t + 1 | e, z, a, s \text{ in } t) \Big) \\
&= \sum_{e',z',a',s'} \xi_1(e', z', a', s') \times \Pr(e', z', a', s' \text{ in } t + 1 | e, z, a, s \text{ in } t).
\end{aligned}$$

As the expression above makes clear, once we have computed the one period ahead score across all states, all that remains is to compute the conditional probability of transitioning states, which is given by $\Pr(e', z', a', s' \text{ in } t+1 | e, z, a, s \text{ in } t) \equiv Q(e', z', a', s' | e, z, a, s)$

$$Q(e', z', a', s' | e, z, a, s) = Q^e(e'|e) \cdot H(z') \cdot \tilde{\sigma}^{(d,a')}(e, z, a, s) \cdot \chi_{[s'=\psi^{(d,a')}(e,z,a,s)]}, \tag{B.6}$$

where χ is an indicator function. Since s' is exactly pinned down by $\psi(\cdot)$ in equilibrium, we need only sum over e', z', and a', yielding

$$\xi_2(e, z, a, s) = \sum_{(e', z', (d,a')) \in \mathcal{E} \times \mathcal{Z} \times \mathcal{Y}} \xi_1 \left(e', z', a', \psi^{(d,a')}(e, z, a, s) \right) \cdot Q^e(e'|e) \cdot H(z') \cdot \tilde{\sigma}^{(d,a')}(e, z, a, s).$$

Repeating this procedure, we find that

$$\xi_{n+1}(e, z, a, s) = \sum_{(e', z', (d,a')) \in \mathcal{E} \times \mathcal{Z} \times \mathcal{Y}} \xi_n \left(e', z', a', \psi^{(d,a')}(e, z, a, s) \right) \cdot Q^e(e'|e) \cdot H(z') \cdot \tilde{\sigma}^{(d,a')}(e, z, a, s), \tag{B.7}$$

for all $n = 1, ..., N-1$, and so we have computed the entire range of credit scores across possible time horizons.

B.3.2 Credit score transitions

Individuals naturally move through the range of possible credit scores over time: indeed, this is precisely what we observe in the Equifax data. The goal of this section is to map the transition matrix of agents through credit scores. That is, for each $n = 1, ..., N$, we would like to find a transition matrix $Q_n^{\xi}(\xi_n' | \xi_n)$ that gives the probability of transiting from credit score ξ_n today to credit score ξ_n' tomorrow. Once we have computed this, we can aggregate into a transition matrix $Q_n^X(X_n' | X_n)$, which gives the probability of transitioning from a given credit score *group* $X_n = \{\xi_n^1, ..., \xi_n^J\}$ today to a different group $X_n' = \{\xi_n^{1\prime}, ..., \xi_n^{J\prime}\}$ tomorrow.

Every possible state $(e, z, a, s) \in \mathcal{E} \times \mathcal{Z} \times \mathcal{A} \times \mathcal{S}$ maps into a credit score (N-vector) $\xi = \xi(e, z, a, s)$; therefore, the transition over credit scores is governed entirely by the transition over states, $(e,z,a,s) \to (e',z',a',s')$. However, the mapping is not one-to-one (for example, multiple states can have certain repayment for a credit score of $\xi_1 = 1$), and so the score transitions must sum over states that have the same credit score.

Let ξ_n and ξ_n' be arbitrary n-period credit scores for today and tomorrow respectively. The probability of transitioning from $\xi_n \to \xi_n'$ is the probability of transitioning from any state (e,z,a,s) today such that $\xi_n = \xi_n(e,z,a,s)$ to any state (e',z',a',s') tomorrow such that $\xi_n' = \xi_n'(e',z',a',s')$. Define

$$\mathcal{X}(\xi_n) = \{(e,z,a,s) \in \mathcal{E} \times \mathcal{Z} \times \mathcal{A} \times \mathcal{S} | \xi_n(e,z,a,s) = \xi_n\} \tag{B.8}$$

to be the set of all states that map into a given credit score. Then, the probability of transitioning from ξ_n to ξ_n' is simply the double sum[3]

$$Q_n^{\xi}(\xi_n'|\xi_n) = \sum_{(e,z,a,s)\in\mathcal{X}(\xi_n)} \sum_{(e',z',a',s')\in\mathcal{X}(\xi_n')} Q(e',z',a',s'|e,z,a,s) \cdot \frac{\sum_\beta \mu(\beta,e,a,s)}{\sum_{\beta,(e,z,a,s)\in\mathcal{X}(\xi_n)} \mu(\beta,e,z,a,s)}, \tag{B.9}$$

where \$$Q(e',z',a',s'|e,z,a,s)$ is given by (B.6) and the second term weights the relative size of each particular state within the group of states that yield the given current score.

Having defined the score-by-score transitions in the preceding section, we can now extend the procedure to "score brackets" or transitions over ranges of credit scores. Define a score bracket X_n to be a collection of J credit scores $\{\xi_n^1, ..., \xi_n^J\}$.[4] Then, we can define the set of all states that map into a credit score in bracket X_n to be

$$\mathcal{X}(X_n) = \bigcup_{j=1}^{J} \mathcal{X}(\xi_n^j), \tag{B.10}$$

[3]NB: Though we use discrete grids to compute the model, this sum notation is not technically mathematically correct because s is, in principle, a continuous variable.

[4]Again, here we run into the mathematical detail of continuous vs. discrete variables: in principle, the space of credit scores could be continuous, and then this procedure we outline would not be exhausted. Computationally, though, it is not an issue.

where $\mathcal{X}(\xi_n)$ is given by (B.8). Then, the transition from score bracket $X_n \to X_n'$ is simply

$$Q_n^X(X_n'|X_n) = \sum_{(e,z,a,s)\in\mathcal{X}(X_n)} \sum_{(e',z',a',s')\in\mathcal{X}(X_n')} Q(e',z',a',s'|e,z,a,s)\cdot\frac{\sum_\beta \mu(\beta,e,z,a,s)}{\sum_{\beta,(e,z,a,s)\in\mathcal{Z}(Z_n)} \mu(\beta,e,z,a,s)}, \tag{B.11}$$

where again $Q(e',z',a',s'|e,z,a,s)$ is given by (B.6) and the second term weights the relative size of each particular state within the group of states that yield scores within the given current score range.

Appendix C

Chapter 3 Appendix

This appendix simply provides more detailed results for the event study analysis contained in Chapter 3. Tables C.1 and C.3 contain the numerical results underlying Figures 3.4, 3.5, and 3.6. Notably, these results also contain the coefficient for the control variables X_{it}, as well as specific significance levels and standard errors.

Tables C.2 and C.4 contain the analogous information, but subsetted to include only observations during the financial crisis surrounding 2008. These tables are meant to facilitate comparison with the robustness analysis from Section 3.3.3. The responses are similar for the most part, though different in magnitude.[1] For example, the response at impact of assets is 25% smaller during crisis times, suggesting that the same events were not associated with such balance sheet expansions during crises as in normal times. We see similar patterns emerging for all of the key variables about funding in Tables C.1 and C.2, with the exception of the sub-types of debt: these responses are stronger. Comparing the profitability results across Tables C.3 and C.4, a similar pattern emerges: the responses on both loan returns and cost of deposits – and therefore on net interest margin – are more muted.

[1] In addition, the estimates are not as tight, since there the existence of fewer observations in the panel yields larger standard errors.

	(1) Assets	(2) Debt	(3) Equity	(4) Deposits	(5) Non-Dep. Debt
$\chi_{i,t-4}$	-0.00284**	-0.00388*	-0.00299	-0.00348**	0.0631
	(0.00110)	(0.00171)	(0.00192)	(0.00111)	(0.0434)
$\chi_{i,t-3}$	-0.000832	-0.00192	0.00647***	0.000823	-0.113**
	(0.00109)	(0.00170)	(0.00190)	(0.00110)	(0.0431)
$\chi_{i,t-2}$	-0.000482	-0.000852	-0.00136	0.00283**	-0.0603
	(0.00109)	(0.00169)	(0.00189)	(0.00109)	(0.0427)
$\chi_{i,t-1}$	-0.00326**	-0.00417*	0.00194	-0.00484***	0.0440
	(0.00108)	(0.00167)	(0.00187)	(0.00108)	(0.0424)
χ_{it}	0.0130***	0.0175***	0.00921***	0.00815***	0.288***
	(0.00108)	(0.00167)	(0.00187)	(0.00108)	(0.0423)
$\chi_{i,t+1}$	0.00198	0.00140	0.00282	0.00372***	-0.0133
	(0.00108)	(0.00168)	(0.00188)	(0.00108)	(0.0425)
$\chi_{i,t+2}$	0.00265*	0.00580***	0.00187	0.00160	0.00979
	(0.00105)	(0.00163)	(0.00183)	(0.00106)	(0.0414)
$\chi_{i,t+3}$	-0.000797	-0.00209	-0.000105	-0.00169	0.0677
	(0.00103)	(0.00160)	(0.00180)	(0.00104)	(0.0407)
$\chi_{i,t+4}$	0.00125	-0.0000459	-0.000151	0.000215	0.106**
	(0.00102)	(0.00158)	(0.00177)	(0.00102)	(0.0400)
CL exposure (lag)	0.112***	0.109***	0.00898	0.109***	-0.311
	(0.00684)	(0.0106)	(0.0119)	(0.00687)	(0.269)
Leverage (lag)	-0.543***	-1.008***	0.678***	-0.639***	-3.163***
	(0.0117)	(0.0182)	(0.0204)	(0.0118)	(0.462)
UCR (lag)	-0.0341***	-0.0365**	0.0119	-0.0596***	1.955***
	(0.00845)	(0.0131)	(0.0147)	(0.00848)	(0.332)
Observations	456595	456595	456595	456595	456587
R^2	0.012	0.011	0.007	0.016	0.001

Notes: Standard errors in parentheses, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. The figures reported in this table reflect estimates from the general specification in Equation (3.2). The dependent variable is simply the level of the relevant rate.

Table C.1: Funding coefficient estimates: whole sample

	(1) Assets	(2) Debt	(3) Equity	(4) Deposits	(5) Non-Dep. Debt
$\chi_{i,t-4}$	0.00265	0.00247	0.00586	-0.000423	0.227***
	(0.00273)	(0.00283)	(0.00467)	(0.00305)	(0.0643)
$\chi_{i,t-3}$	0.000772	0.00293	-0.00501	0.00409	-0.136*
	(0.00287)	(0.00297)	(0.00491)	(0.00321)	(0.0676)
$\chi_{i,t-2}$	0.00285	0.00527	-0.00978	0.00742*	-0.0981
	(0.00299)	(0.00309)	(0.00510)	(0.00334)	(0.0704)
$\chi_{i,t-1}$	0.00565	0.00793*	0.00221	0.00559	0.0784
	(0.00305)	(0.00315)	(0.00521)	(0.00341)	(0.0718)
χ_{it}	0.00988**	0.0124***	-0.00158	0.00901**	0.344***
	(0.00310)	(0.00320)	(0.00529)	(0.00346)	(0.0729)
$\chi_{i,t+1}$	0.00701*	0.00725*	-0.0000582	0.00449	0.0861
	(0.00320)	(0.00330)	(0.00546)	(0.00357)	(0.0752)
$\chi_{i,t+2}$	0.00433	0.00486	-0.000346	0.00393	-0.162*
	(0.00308)	(0.00318)	(0.00526)	(0.00344)	(0.0725)
$\chi_{i,t+3}$	0.000602	0.00193	-0.000711	0.000570	0.0747
	(0.00308)	(0.00318)	(0.00526)	(0.00344)	(0.0724)
$\chi_{i,t+4}$	0.00112	0.00155	-0.00173	-0.00164	0.217**
	(0.00301)	(0.00311)	(0.00514)	(0.00336)	(0.0709)
CL exposure (lag)	0.111***	0.115***	0.0626**	0.0434***	-0.294
	(0.0117)	(0.0121)	(0.0200)	(0.0131)	(0.275)
Leverage (lag)	-0.791***	-1.411***	3.102***	-1.174***	-13.95***
	(0.0455)	(0.0470)	(0.0777)	(0.0508)	(1.071)
UCR (lag)	-0.0276	-0.0440	0.0811*	-0.0789**	3.229***
	(0.0231)	(0.0239)	(0.0395)	(0.0258)	(0.544)
Observations	41979	41979	41979	41979	41979
R^2	0.061	0.073	0.067	0.067	0.013

Notes: Standard errors in parentheses, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. The figures reported in this table reflect estimates from the general specification in Equation (3.2). The dependent variable is simply the level of the relevant rate.

Table C.2: Funding coefficient estimates: crisis

	(1) Net interest margin	(2) Loan return	(3) Cost of deposits
$\chi_{i,t-4}$	0.0348***	0.0348***	0.0000723***
	(0.00501)	(0.00501)	(0.0000202)
$\chi_{i,t-3}$	0.0253***	0.0253***	0.0000268
	(0.00498)	(0.00498)	(0.0000201)
$\chi_{i,t-2}$	0.0111*	0.0112*	0.0000564**
	(0.00493)	(0.00493)	(0.0000199)
$\chi_{i,t-1}$	-0.00208	-0.00196	0.000112***
	(0.00489)	(0.00489)	(0.0000197)
χ_{it}	-0.0116*	-0.0115*	0.000152***
	(0.00489)	(0.00489)	(0.0000197)
$\chi_{i,t+1}$	-0.00205	-0.00226	-0.000205***
	(0.00490)	(0.00490)	(0.0000198)
$\chi_{i,t+2}$	-0.00274	-0.00287	-0.000121***
	(0.00478)	(0.00478)	(0.0000193)
$\chi_{i,t+3}$	-0.00378	-0.00383	-0.0000525**
	(0.00469)	(0.00469)	(0.0000189)
$\chi_{i,t+4}$	-0.00705	-0.00710	-0.0000542**
	(0.00462)	(0.00462)	(0.0000186)
CL exposure (lag)	-0.204***	-0.200***	0.00388***
	(0.0311)	(0.0311)	(0.000125)
Leverage (lag)	0.0565	0.0713	0.0148***
	(0.0534)	(0.0534)	(0.000215)
UCR (lag)	0.300***	0.291***	-0.00895***
	(0.0384)	(0.0384)	(0.000155)
Observations	456595	456595	456595
R^2	0.001	0.001	0.817

Notes: Standard errors in parentheses, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. The figures reported in this table reflect estimates from the general specification in Equation (3.2). The dependent variable is simply the level of the relevant rate.

Table C.3: Profitability coefficient estimates: whole sample

	(1) Net interest margin	(2) Loan return	(3) Cost of deposits
$\chi_{i,t-4}$	0.000231*	0.000225*	-0.00000589
	(0.000105)	(0.0000962)	(0.0000658)
$\chi_{i,t-3}$	0.000279*	0.000262**	-0.0000171
	(0.000111)	(0.000101)	(0.0000691)
$\chi_{i,t-2}$	0.000221	0.000195	-0.0000265
	(0.000115)	(0.000105)	(0.0000719)
$\chi_{i,t-1}$	-0.000128	-0.000136	-0.00000826
	(0.000117)	(0.000107)	(0.0000734)
χ_{it}	-0.000535***	-0.000531***	0.00000401
	(0.000119)	(0.000109)	(0.0000745)
$\chi_{i,t+1}$	0.0000184	-0.0000154	-0.0000338
	(0.000123)	(0.000113)	(0.0000769)
$\chi_{i,t+2}$	-0.000103	-0.0000832	0.0000196
	(0.000119)	(0.000108)	(0.0000741)
$\chi_{i,t+3}$	0.0000826	0.0000801	-0.00000248
	(0.000118)	(0.000108)	(0.0000740)
$\chi_{i,t+4}$	0.0000641	0.000112	0.0000478
	(0.000116)	(0.000106)	(0.0000724)
CL exposure (lag)	0.000160	-0.000185	-0.000345
	(0.000450)	(0.000411)	(0.000281)
Leverage (lag)	-0.0168***	-0.00644***	0.0103***
	(0.00175)	(0.00160)	(0.00109)
UCR (lag)	0.0111***	0.0116***	0.000494
	(0.000890)	(0.000814)	(0.000556)
Observations	41979	41979	41979
R^2	0.025	0.708	0.849

Notes: Standard errors in parentheses, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. The figures reported in this table reflect estimates from the general specification in Equation (3.2). The dependent variable is simply the level of the relevant rate.

Table C.4: Profitability coefficient estimates: crisis

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