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MAKING MAPS FROM PHOTOGRAPHS FOR GEOLOGIC USE.

Introduction. The coming into use of photographs taken from aircraft has opened new fields to the geologist, not only in interpretaion of land forms but in making accurate maps. While teaching mapping at the University of Wisconsin the writer had extensive experience in teaching the making of maps from both air and ground photographs, as well as interpretation of such photographs. One of the difficulties is that so much of the published literature is so complex that it is hard to understand. Either mathematics ~~is~~ are very complex or the entire derivation of many formulas is omitted. In fact it has been suggested that many authors tried to find "hard ways to do easy things." Very few ariticles or books on the use of photographs were found to be enlightening to beginners and in fact most were confusing to young students; In many the accuracy demanded proved far beyond both the accuracy of the available photographs and the possible accuracy of drafting. It is true that the making of maps of large areas is best left to specialists but the geologist finds much help in using photographs either (1) to fill in detail on existing maps or (2) to make accuracte maps of relatively small areas. The specialist can afford to buy and to use complex expensive labor-saving apparatus but when only occasional use of photography is needed it is best to avoid such helps and the following discussion is devoted to general principles which do ~~not entail such expensive help~~. The understanding of principles is vital to all methods of work rather than the manipulation of complex apparatus.

General principles of use of photography The use of photographic methods of surveying is demanded (1) when the time of travel is so great that time at a given location must be limited, (2) when another point of view than those affor ded from the earths surface is desirable. For the second condition is is evident that the use of aircraft is essential. We will here considermappinh from (1) ground photographs, and (2) air photographs. ~~In both catagories~~ The axis of the camera may be either horiaontal, inclined, or vertical(in air photographs). The differing geometiric relations of grund and photograph in these differing conditions is hard to explain but must be understood.

B asic idea of cameras. A ll cameras have the same basic principle. a convex lens forms a real image on either a photographic plate or film. The lens must be placed at the right distance from the plate or film by various methods, and the amount of light which enters the lens regulated by either or both (1) a shutter, or (2) a diaphram within or just inside the lens. The box enclosing lens and image must be light tight. Although the above requirements are simple it must be realized that the construction of an accurate lens which gives sharp focus for the different colors of sunlight and which makes all parts of the image equally definite is by no means simple. We need not here discuss either the methods of compensating for these factors nor the details of camera construction but simply remark that camers for mapping are much more expensive than those ordinarily used for other purposes and that many lenses are not perfect in the accuracy of the images. Cameras for mapping take only objects far enough from the camera that devices for changing the focus are unimportant for it is always what is termed "infinity." Distance from lens to image is focal length. Plates are no longer used extensively in aircraft. Many air cameras are automatic and use large rolls of film.

Negatives and positives. The image taken on the photograph is turned 180 degrees from the position of the original ground. See Fig. 1 In black and white photography the density of the photograph is reversed from the original subject making a negative. The negative is converted to a positive either by printing or by further steps in development. Color picture are developed into positives. From the figure, it is clear that photographs are a means of recording both vertical and horizontal angles of objects in the view thes angles being measured from the axis of the camera. The angle is not measured in degrees and minutes but by a triangle with focal length on one side and distance from the center of the picture as another side of a right triangle.

Now if a positive were made on a transparent base it can easily be placed in front of an observors eye at such distance that objects in the photograph will coincide with the original view. This distance is the focal length of the camera

Ground survey. In order to make accurate maps from phtographs it is necessary to have points on the ground located both in elevation and horizontal location by some kind of ground survey using conventional surveying methods. There is no need to discuss either the requisite number f of points to be determined nor the methods of surveying which depened to a large extent on the size of the area which is to be mapped.

Haze problem. Photography of distant objects like those below an airplane involves the problem of haze. Haze is due to dust and water vapor in the atmosphere which result in dispersal of the shorter waves of light, the violet and blue. This causes the blue color of distant objects as is noted in many p ace names. There are two ways of reducing the blurring of a photograph by haze. First, we can somewhat reduce the exposure and second, we can shut out the shorter wave lengths of light by appropirate colored glass called filters. the extent to which the second alternative can be employed depends upon the kind of plate or film which is being used. Older photographic materials were sensitive up throughye llow in the spectrum of sunlight. They would not take a filter deeper color than medium yellow. Modern films and plates are sensitive to orange red and in some cases into the infra red or invisible light rays. With these material orange, and red filters may be used giving much clearer picutures than were ever obtaind in early days. The use of infra red involves some difficulty in obtaining exact focus for one cannot focus when the image is invisible. Color films have now been made which require much shorter exposures than any black and white material of a few years ago. These films are not suited to use with filtersalthough they yield good results in very clear weather

Ground Photograph surveying.

Introduction. Before air photographs were available considerable surveying was done by means of photographs taken from the ground. The method had great advantage where travel was difficult and clear weather limited ~~in occurence~~ as in the mountains of the west coast. Local conditions still make it a valuable method. It is essentially planetable surveying with the angles previously registed on the pictures. Both horizontal and inclined photographs could ~~may~~ be used but the following discussion is limited to the former

Field photography. Horizontal photographs for use in photo surveying must be taken with the camera level which makes the horizon parallel to the edge of the picture. Leveling may be done either with level vials attached to the camera or by placing the camera on a levelled plane table Every photograph must include recognizable points whose location and elevation had previously been determined by a ground survey. If this is done no measurment of the direction of the camera is needed. An essential is to mark the position of the camera axis on the picture. This is accomplished by markers called collimation marks. These marks can be inserted in almost any kind of camera by ~~with~~ accurate shop work. They must be close to the plane of the film in order to register clearly. Care must be taken to determine the proper exposure, to use a filter and to take overlapping pictures of all of the view from a given station. One of the major difficulties is to be certain that all of the land within the area to be mapped is covered by at least two pictures. If there is much forest this could not be accomplished. The photographic stations may be either located by ground survey or included in each picture there must be three points whose location and elevation are known.

Refering to Figure 2 it is noted that equal horizontal angles from the direction of the camera axis lie on straight lines. If we are familiar with a surveyors transit this relation is clear. If the instrument is clamped on its vertical axis at a given angle from that of the picture and if the telescope is ~~then~~ moved around its horizontal axis it is clear that the line of sight passes through a vertical plane represe ted on the picture by a straight line normal to the ~~h~~ horizon line. If we free the vertical axis and ~~set~~ clamp the telescope on its horizontal axis at a given angle of elevation or depression and giving it horizontally the line of sight ~~rotation on the vertical axis~~ will describe the surface of a cone. On the picture the lines of given vertical angles are curves which represent the intersection of these cones with the plane of the photograph. If we desire to be technical it is evident that the distance of a point from the horizon or plane of the axis of the picture is not proportioned directly to the angle in degrees but instead to its tangent on account of the fact that the photograph is a plane normal to the cz mera axis.

Platting of points. In order to make a map from ground photographs it is desirable that the focal length of the camera be known or determined If the location on the map of the camera station is known the map direction of the axis of each picture is readily located by platting the position of the right triangle alongside this exis which also touches another point of known location. Refering to Figure 3 locations of both A and B were known for the distance between these two st ations was a base line on which the entire survey ~~is~~ was based It is laid out to map scale and in proper direction and the location of line through the axis of photo ~~A~~ A O is determined by the right triangle A , AO, D' using the focal length for one side and distance OD' for the other measured on the photograph in a direction parallel to the line of the horizon. In this case it is assumed that the map direction of D is known In the same way the axis of Photo B may be laid out using the triangle B, O", D" Note that these picture planes are normal to the lines of axis. no ¶

~~It can be seen that~~ If the focal length of the camera has not been measured by the maker
it can be found by
~~from~~ laying out the rays and measuring distances O-D' and O"-D" no ¶
The next problem is to find the map location of point C which was unknown. The ~~draw~~
are of the lines at C
map rays to C determined from distances O-C' and O"-C" The intersection then gives the desired map location. The similarity to planetable mapping is evident and the chief cautions are (1) not the take the location of intersection of lines not to the same point, and (2) to try to obtain more than two rays to the same point for the intersection of only two rays is not entirely reliable. If the picutres were not all taken from points of known location the focal length of the camera must be known. The map location of the camera station is then obtained by drawing map rays to three known locations shown in the picture. This work is best done on tracing paper or celluloid. The transparent sheet is then shifted until all the three rays pass through the ~~map~~
on the map
location of the points. Then the map location of the camera station can be pricked through onto the map. This is often called the "tracing paper solution of the three point problem." This method is difficult in the field with the plane table but is convenient in the drafting room. Following the methods outlined above a map of a considerable area can be built up thus greatly shortening the time spent in the field but increasing the time in the drafting room.

Elevation finding. Figure 4 shows the method of finding differences of elevation from photographs by graphic solution of the familiar planetable method. Difference of
horizontal or map
elevation = distance X tangent of vertical angle. The map distances are scaled from the map and the vertical angles found from a construction similar to that used for horizontal angles. Note that the distances from the horizon line are scaled normal to that line parallel to the plane of the camera axis. If some points have known elevations the elevations of all points can be computed. ~~The~~ Low angles make elevation determinations of rather low accuracy.

Stereoscopic examination. It is possible to obtain a much better idea of relative distances in a pair of duplicate photographs by stereoscopic examination than can be obtained by the eye at any location or from single photographs. Most stereoscopic
which
cameras have the two lenses placed at eye distance. This is entirely too small.

The methods of stereoscopic examination are fully described under the head of vertical air photographs. In order to compare with those the distance between the camera stations should be a large fraction of the horizontal distances, ~~which are to be compared~~ ~~and~~ never less than 50 feet unlesss the local topography around the camera station will not allow it. On account of the lack of exact knowledge of either the scale or mean distance of objects in a horizontal photograph it is diffcult to apply the formulas ~~which apply~~ for to vertial air photographs. However stereoscopic examination should be a great help in sketching in details between points which were located by intersections.

Limitations of ground photograph surveying. The accuracy of mapping by ground photographs depends upon (1) scarcity of forest cover which impeeds views and (2) the taking of enough photographs so as to obtain sufficent intersecting lines at angles of more than 30 degrees as well as three or more rays to the same point. The method is limited to nearly treeless terrane where travel is reasonably easy and the camera stations are on smooth enough spots to permit of adequate stereoscopic side stations

AIR PHOTOGRAPHY.

Introduction. The term air photography is preferred by many to the older term aerial photography for all phtographs taken from aircraft. Photographhs were taken from balloons and kites long ago but the development of the airplane allowed the art to come into its own and to cover sonsiderable areas A photograph from the air is the only possible way to see a hill from all sides at once and from a distance such that the effect of perspective which plagues the ground surveyor ~~to be~~ ~~chiefly~~ ~~eleiminated~~ is minimized. Many obsticals such as haze, static electricity in the camera, irregular alitude, errors in course speed variation, and control of the plane had to be overcome before present day pictures were possible. We will not here discuss the problems of field photography but turn at once to the use of the finished pictures and leave the field work to specialists.

Kinds of photographs. In flying the camera axis can be (1) directed as near as possible vertical, or (2) inclined . If the angle from vertical is small the picture is termed a low oblique; if large a high oblique. Low obliques should if possible show the horizon . Some methods took both vertical and low obliques ~~simultaneously~~ from the same location by using three camers which took simultanoeus exposures. This was termed the trimetrogon Others used special camers which took exposures in different directions. These methods are under constant development and will not be discussed in detail.

Geometry of vertical photographs. The writer has seen few so-called verticals which were exactly true to that name. Furthermore, plotting of a line of successive photographs and slight scale difference demonstrates that the pilot had difficulty in maintaining either a straight course or constant altitude. Turbulence of the air is worst over rough terrance and ~~the~~ the contact between land and a large body of water and it may well prove that automatic control may never eintirely overcome the errors due to turbulence. If we could obtain precise verticallity of the camera axis and level land werepphotgraphed at uniforn alitiude we would then have an exact an exact map of the land and fitting together the pictures into a mosaic would be a correct map indtead of an approximation.

We will neglect for the present the effects of slight tilt of the camera axis in distorting the picture.Figure 5 shows how the difference in elevation of the ground should affect the accurzcy of a perfectly vertical photogrsph.The top of hill B is not moved in the picture wheras that of hills C and A ~~is~~ are distorted outward from the center ~~and~~ That this displacement is not a ~~small amount~~ negligible is obvious from the fact that the common photographs cover a width of somewhat over two miles when tsken at an altitude of about 14,000 feet (See Figure 9) Distortion is outward from the point where the vertical line touches the ground is radial from this so-called nadir point.

Stereoscopic vision. The displacement of photographic images by reason of difference in relief is also illustrated in Figure 6. It is not simply a nusiance as it is in making mosaics or tother maps which show only plan relations in a single plane for it is the cause of stereoscopic vision in which r lief is made plain to the eyes and enables the quantitative measurment of differences in elvation of the ground. The basic cause of depth preception is the fact that any elevation is looked at from a different angle with each eye thus giving the impression of depth (Figure 7) It is obvious that to see this effect the observer must have normal vision in both eyes Some persons cannot see stereoscopiczlly because they see only with one eye at a time.

Stereoscopic vision does not absolutely demand ~~require~~ the use of any instrument to separate the vision ~~visitor~~ of each eye from that of the other. but ~~It~~ it does require two pictures taken from different points of view. Some persons find that when the pictures are properly placed a ~~paper or~~ sheet of paper or carboard held vertically between the two pictures is enough to cause the eyes to have spearate vision. Many can dispense with such an aid ~~and get the desired result~~. Several methods of practice to attain this result have been suggested but the writer has found that the simplest and most reliable is to relax or "daydream" until the eyes see the same object in two different positions. When the eyes are in this condition attention is devoted to the two pictures placed with common points somewhat less than 2.5 inches apart. Three pictures will be seen of whi h the center one is in relief. It is often necessary to shift the pictures until perfect relief is obtained in the center one. Not all, probably only a minority of people, ever attain this result which is most convenient when examining stereoscopic pictures in the field and in any case it is of little or no value in mapping. It does not injure the eyes.

<u>Stereoscopes</u>. Several devices are used to attain stereoscopic vision without trouble and over long periods of time. Common types are (1) two convex lenses at eye distance apart on a suitable stand which permits focusing on the pictures, (2) a combination of mirrors with lenses or prisms including formerly miniature binocular telescopes arrznged to secure either direct or enlarged view ~~vision~~ of the photographs, one with each eye. The lens type of stereoscope has the disadvantage of obtaining in relief only a part of the common area of the photographs. Devices with mirrors or prisms can obtain all of the area common to the two pairs of the <u>stereopair</u> and hence may prove much sperior if a large area is to be examined although some of them do not show the relief in as prominent a manner as does the lens stereoscope. With the lens type there may be "blind spots" which czn never be seen in relief no matter how the photographs are overlapped or trimmed. (3) Photographs of a pair may be printed one in red and the other in green or blue. When this pair is examined through suitable colored glasses the relief effect is obtained. ~~This device is sometimes modified to use cross polarized light~~

relief of

no II

The method of using colors so that each eye sees only one of the p itures is called an <u>anaglyph</u>. They have not come into common use ~~probably~~ in part because of the cost and the fact that if the glasses are lost the picturs appear, only a blur.

(4) Projection devices which throw the pair of photographs onto a board or screen. Either two colors ~~with glasses~~ or cross polarized light with glasses may be used to separate the pictures to the observor. Polarized light requires the use of <u>polaroid</u> glasses.

<u>Definitions.</u> The center of a photograph where the camera <u>optic axis</u> intersects the ground is called the <u>principal point</u>. This point is found by drawing straight lines between the <u>collimation points</u> on the edges or corners of the prints. The <u>plumb</u> or <u>nadir</u> point is where the vertical line through the lens of the camera intersects the picture. Since the exact tilt is rarely known it is commonly assumed that this coincides with the principal point. The marking of photographs must inclue the transferred principal points of adjacent pictures in the <u>flight</u> or <u>run</u>. Many authorities recommend setting the adjacent photographs under a stereoscope when the prijcipal point of <u>one</u> photgraph appears superimposed on the adjacent one and is readily marked. Unfortunately Stereoscopic visiton is obtained threough considerable angle of tolerance and ~~so~~ practice has demonstrated that the above method is suject to considerable error. The writer has found that the principal points may be located by distance from nearby landmarks such as field corners which can be seen on both photographs. Since the distances are small any error in scale due to change of elevation of the plane is unimportant. This method is cannot be ~~unuseab~~ used e either in continuous land with no local marks or over water.

A line is drawn between the principal point of the central photograph and the transferred points to follow what is assumed to be the <u>line of flight</u>. In older pictures this is rarely a perfectly straight line. When photographs are properly set up for steeoscopic examination this line of flight of two adjacent photographs must be parallel to the axis of the stereoscope. No attention has thus far been given to the tilt which moves the nadir points a short distance from the principal points. The line of flight must then be drawn through the nadir points if known.

Height finding Figure 5 demonstrates that elevation differences ~~vary with~~ are shown by the position apart of the images of different points in a steeopair.. Parralax is defined as the apparent movement of a object in reference to a plane of reference which is due to the real movement of the observor. The movement of the air plane is real and the apparent movement is that of the same object shown in two successive pictures. The above principles may be used to obtain a quanitative vale of the difference in elevation of the object with reference to some plane of elevation. If the stereopair is properly set up along the line of flight parallax differences can be measured with an ordinary scale whose position should be parallel to the line between ~~xx~~ principal or nadir points. However, the small distances which must be measured make it preferable to do this work under the magnification afforded by a stereoscope. It is easy to see that the movement of the plane, B, (air base) is in the same proportion to the elevation of the plane above the hilltop above ~~datum~~ ~~hilltop~~ (H-h) as ~~differen~~ ~~in~~ elevation of the hill dh is to the difference in parallax ~~dp~~ c The last figure is the apparent movement of the same point measured ~~from the~~ on the two pictures. (in map scale) Solving this to obtain a usable value for ~~dh~~ c we must realize that B the movement of the plane between successive exposures cannot be measured directly. All that can be measured is the distance (b) between the principal point of one photo and ~~its~~ (its) transferred position on the other member of the pair. ~~(b)~~ It is easy to see that this latter determination of the photo base, b is related to the original length in the air in the ratio f: H. b= B f/ H By substitution in (H-h) ; ~~dh~~ c: B ; dp it is clear that ~~dh~~ B = b.H/f.(H-h) and dh = bH/(H-h)/f Although simple to derive by similar triangles it is obvious that this formula is not usable for neither f (focal length of camera) nor H (altitude of the plane) is always avilable and h is the final answer! Figure 8 shows a ~~more~~ practical solution. We will assume that both pictures are truly vertical and were taken at the same elevation and that dh (height of the hill) is amall compared to H. We can then ignore the figure (H-h). The apparent movement of the hilltop between the two pictures is ~~x~~ c. We will set up two values for ~~x~~ c which can be equated and thus eliminate it from further consideration. The total distance of the movement of the hilltop in the pictures dp is the sum of the

out

two para llax displacements in each picture. Now dp : c:: f;H by the scale ratio which was explained earlier. From this we can readily see that c = dp.H/ f
By similar triangles dh ;c;; H; B whence c= dh.B/H The two values of c may be put equal to each other and we have dp.H /f = dh.B.H What we need is the difference of elevation dh in terms of difference in parallx, dp . Solving the above equation for dh we get dh = dp.H^2/ f.B In this we can substitue the relation that b, which can be measured on the photographs, is equal to ~~B.f/H~~ or B = b.~~f/H~~ H/f ~~This gives dh = H^2 f/f.B~~
This gives dh = dpH^{2}f / f.H Next the fs and one H cancel out and we have dh= dp.H/ b. dp will be taken as unity for we desire to find how many feet of vertical distance there are for a unit of parallx displacement. However, we do not always have the value for H so that we can further substitute the value of H in relation to easily measured quanities . The easiest way in country which ~~is~~ was divided into sections by the Land Survey is to measure the length of a North-South mile on the ground This should be averaged for not all surveying was accurate and there is some variation due to altitude of the plane and ground. Theoreticaly, the distance on the photograph m which represents a mile is in the ratio f/H to the original distance on the ground g.
Hdnce H = g.f/m Since f is not always known to the user of the photographs we can solve the equation f= H,m/ g using for the value of H H = dh.b/dp provided we already know the value of dh from measurment of parallx difference of a hill of known height. Leaving the value of f for the present we find that dh = dp.g.f/ b/m
The distances measured on the phtographs are b and m which are readily determined by an ordinary scale showing decimal divisions of inches. The next thing is to fix upon the proper units of mesurment. Our maps will genraly be with English units, feet, inches, and miles. Scales on most stereoscopic mapping devices for determining parallax differences are divided into millimeters What we need is number of feet per millimeter of parallx and hence 1:12:25.4 = 1:5280:12: 25.4: f . 25.4/ b in in .25.4 . m in in .25.4 Cancellation of convereion factors simplfies this to dh (feet) = 5280:12: 25.4 / b": 25.4 . m": 25.4 Since s very popular camera used for most of the photographs taken in the United States had a focal length, f, of 8.25 inches we can solve the above by further simplification to dh (feet) per millimeter of parallax = 1715/ b.m (both in inches)

Figure 10 is a diagram showing the values of dh in feet per millimeter for different values of m, and b in inches. Most photographs taken in this country were intended to be at a scale of 1;20,000 so that the photo distance of a ~~standard~~ mile of 5280 feet~~. If the scale is exact this~~ should be slightly over 3.16 inches, on the photograph but the pictures are rarely exact. Lines are drawn for three different values of m and intermediate vzlues may be estimated. It must be remembered that the limit of accuracy is not ~~on~~ of this diagram but lies in the precision with which parallax differences can be measured. The test of this is how closely do separate determinations agree which depends upon the quality of the photography. It is uncommon to have sucessive determinations agree closer than about 1/20 millimeter. Hence a fairly rough determination of the value of dh is all that is reasonable to expect. Measurments are made in two different ways on instruments designed for this purpose. Some use a <u>micrometer</u> or revolving scale and others a <u>rotary</u> guage. The latter is easier to read but should always be read when ~~turning~~ tightening the screw ~~tighter~~ against the spring. The points ot measurments over each photograph of the pair are dots on glass, often in a slightly elevated central area. Despite small electric bulbs to illuminate the pictures these dots are hard to see on a dark background and if the stereopair is left set up and not dusted off reularly the glasses soon wear to ~~a~~ ground glass, ~~condition~~. Dots on culloid which although soft is reasily replaced were tried. Just why the measurments are in millimeters where other measurments are feet or inches is not clear.

<u>Effects of tilt</u>. Slight tilt is apparent in the majority of air photographs which were intended to be vertials. Figure 11 shows some principles of tilt. The up side of a photograph is too small ~~large~~ and contains the principal point. The plumb or nadir point lies on the down side which is too large ~~small~~. The <u>axis of tilt</u> lies half way between the principal point and nadir point the point on a line joining these two and crossing the axis of tilt is termed the <u>isocenter</u> or point of no distortion. The axis of tilt may trend in any direction. Since the tilt affects the scale of the photograph it changes parallax measurments so that points of the same elevation on one side of the line of flight do not check with others at the same elevationon the opposite side.

A nother phenomenon is that parallax measurment between points of the same elevation differes in different parts of the stereopair. Still more easily observed is failure of ~~pointsxofxthexsame~~ locations of the same point in the two picture to maintain a line parallel to the line of flight. If ground distances are known tilted photographs do not display the same scale at the same elevation everywhere in the area common to the two pictures of a stereopair. All these phenomena proved existance of tilt although they are only qualitative and prove neither the direction nor amount of tilt. These relations are shown in Fig.13 where the total tilt is shown as two components one transverse to line of flight and the other normal to it.

<u>Correction of tilt</u>. The beginner may well ask how, if tilt is so prevalent and causes so many errors are we going to use air photographs to make accurate maps? The answere lies in the <u>radial line method</u> which is illustrated in Figure 14. The basis of this idea is that directions from the isocenter to any oint in the photographs are less afected by tilt than are distances. Note that this principal is opposite to that of the legal method of correcting old ground land surveys where distances hold over recorded directions. To obtain accurate results by drawing radial lines on an overlay above the photographs it is essential to have some lines included whose true length has been found by a ground survey. Were this not done the entire map would be at the scale of the first photograph. Unfortunately the location of the ~~nadir point~~ isocenter is rarely known and it is generally assumed to be at the principal point. The radial lines on one overlay must include points which are shown in adjacent pictures. Some of the side points must be recognized in three successive photographs for the method of locating points is exactly the same as ordinary planetable surveying and it is essential for accuracy to have as many points located by the intersection of three lines as possible.Note that it is exactly as if a planetable had been set up ~~directly~~ below the plane at every exposure but has the advantage that shots can be taken to many points which could not be seen on the ground. It is also essential that points on adjacent flights be included so that the flights may be joined together. Several conditions are included in Figure14. In some cases the points were not shown in the adjacent flight, and in others the spacing of photographs

vsries from the intended plan.

Correction for differences in elevation of ground. Figure 9 showed how points are displaced due to difference of elevation of the ground which is radial along lines drawn from the nadir point. This point is rarely known precisely so that we must assume it to be coincident with the principal point. Fortunately this is of comparatively little importance with the scales used for making most maps.

Quanitative measurment. Lack of knowledge of actual tilt makes quanitative determination of ~~either~~ displacements due either to elevation or to tilt almost impossible. Figure 9 showed the computation for elevation displacements, d for elevation above datum of h. Figure 12 demonstrates the same for tilt. Here the tilted photograph ~~with~~ isocenter at i is solid line and the equivalent vertical picture is dotted, The displacement is related to angle B between a line to the point from the lens and the line to the isocenter, The angle of tilt is t. The vertical displacement ~~either~~ db' or ad = ia (or ib) sin t. Horizontal displacement is vertical displacement tan B. Now tan B = ia (or ib) / f so that substitution shows that d = ia^2 (or ib^2) sin t /f Displacemnts are radial from the isocenter. The importance is that displacements increase with the square of the distance from the isocenter so that mapping from the edges of photographs is relatively inaccurate Displacement in opposite directions ~~in~~ on the up and down sides of the photograph.

Practical use of radial line control. Students were taught to use tracing paper overlays in making radial line construction over a group of pictures in two adjacent flights. This was done to teach principles but if the area to be mapped is large this simple method becomes cumbersome. It s better to make the templets, as the overlays are called, out of carboard with round holes at the principal points and slots along the radial lines centered on the other points. Small metal buttons with holes in the centers are inserted and the whole assembled and shaken down. The points of intersection of lines are thus determined and failure of any individual templet to settle smoothly into place is evidence of either a mistake in laying out the lines or of abnormal tilt. Such a construction by what is termed the slotted templet method is cumbersome and wasteful of cardboard. As a substitute slotted metal strips can be used which are clamped on the central botton by a screw. Map locations are pricked through in the same way. Today most mapping is done with some form of projection device which throws ~~an image of~~ the stereoscopic image onto a board. The projector is tilted to compensate for tilt until ~~it~~ the ~~relief~~ image in relief agrees with several known elevations within ~~the map~~ each steeopair. Such devices were not used at the University of Wisconsin because of their expense and it was desired to not get students used to such apparatus ~~devices~~ and hence unable to work without it ~~them~~. Another method of correction of elevations for tilt was to draw contours on an overlay showing the variation in values of dh across each stereopair. Published examples appear to show that the more known elevations in the area the more complicated the contour pattern which appears to show that much of the discrepancy is due to errors in photography including shrinkage of the prints

<u>Exact determinations of tilt.</u> Some (prehaps many) have tried methods of computing the exact amount and direction of the tilt of air photograph verticals. A method which appeared to offer possibilities is that of Rihn. He tired to compute the scales of the sides of a triangle on the photograph which had been surveyed by ground methods using the formula photo distance in inches / ground distance in feet ~~using different units~~. The difficulty appears to be to find the point on the side of the triangle where the scale attains the mean value, the ~~so called~~ scale point. Rihn used an arbitrary construction to attain this point. The trouble is that ~~almost any line~~ two ~~one or more~~ sides of the triangle side crosses the axis of tilt. Now as has been shown the relative relation of photo to ground distane changes sign at the line of tilt being shorter on one side and longer on the other than it should be if the picture was truly vertical. This destroys all possibility of success.

<u>Drafting the map.</u> A map is most easily compiled at a scale close to the average scale of vertical photographs . Only part of the map will be the exact scale especially after radial line corrections of locationhave been made. Students commonly try to ~~trace~~ work with gtracing paper directly on the photographs. This method leaves lines on the photogrsphs wherever the pencil was used For this reason optical devices have been made to permit drawing on a sheet of paper which is not over the photographs. These devices were not used for the same reason that other mechanisms were not employed, namely to get students into habits

of depending too greatly on such helps. For the desired scale of the final map it seemed preferable to either shift a tracing slightly with change of positions or to sketch details with these controls in mind. Much detail from trom the picutres czn abe sketched with the stereocomparitor which has a drawing pencil attached. Those instruments used by the writer all had the great disadvantage that this pencil was put on the back where it is ~~proved~~ difficult to reach the map to sketch on it. On some it was The writer ~~necessary to~~ moved the drawing attachment to the right. Another fault of most instruments was that a heavy stereocomparitor was attached by friction to a light drafting arm in order to keep it always parallel. The writer devised a double parallel rule attachment which proved much more rugged and dependable. This is now used by some manufacturers.

Oblique photographs.

Introduction. Oblique photographs have the very great advantage of including much more land surface in each than is possible with verticals. This minimizes flying. They have the disadvantage that some of the land is hidden behind hills and woods and that they display perspective with distant objects much reduced in size compared with those in the foreground. This effect is most prominent in high ~~high~~ obliques which approach horizontal photographs. It is possible either by geometric construction or through optical means to change many obliques (rectify) into verticals. ~~Obviously obliques are favored by the cost factor for they require much less flying to cover an area than do verticals. and the requirements of this flying are not as strict.~~

Perspective geometry of photographs Figure 15 shows the comparative perspective geometry of horizontal, oblique and vertial photographs. Displacements are measured by showing the intersection of lines of certain vertical and horizontal angles upon the photograph. Such angles are similar to those observed with either a telescopic alidade or a transit. As previously demonstrated the horizontal photograph has lines which approximate a grid whose center is determined by the axis of the camera. A vertical photograph has a spiderweb pattern also with the center fixed by the axis of the camera. . This axis is intended to be vertical. In an oblique it is an advantage to show the horizon. This is a low oblique. It is transition between vertical and horizontal pictures and lines of equal horizontal

~~as~~ angles radiate from a point below the picutre to the same spacing on the horizon as they have in a horizontal photograph.

Definitions. Figure 16 is a vertical section through an oblique photograph ~~which~~ which passes through both lens and center of the photograph. (the principal plane) This section is on the principal plane The vertical line through the lens reaches the nadir point The line to the true horizon is at 90 degrees from the vertical. Since the true horizon is above the apparent horizon in the photograph it does not show in the picture. The apparent horizon is not always visible because of either topography or poor photographic conditions. The angle between it and the true horizon which is higher because of the curviture of the earth is appro ximately in minutes the sqyare root of the elevation of the plane in feet. Hence the elevation is essentizl to mapping from an ablique. The line marked focal length is the optic axis of the camera and is normal to the plane of the photograph. Figure 17 is a ground plan or map drawn in a horizontal plane ~~through the lens.~~ through the level of the lens. Figure 18 is a plan showing relations in the plane of the picture. It is essential to realize that to get true directions on a map of any point sought the angles must be those in a horizontal plane. ~~THE ANGLES MUST BE THOSE OF A HORIZONTAL PLANE.~~ They are angles from the plumb or nadir-lens line. Now it is possible to compute the point from which a true map direction ray can be drawn through the point sought. Points for this purpose will differ for different parts of the picture. This point was illustrated in teaching by a three dimenional model of ~~the~~ an oblique picture showing the relation of true map rays to different planes. The angle of depression is readily determined from Figure 16. A construction like that of this figure also ~~determines~~ locates the distance of the nadir point on the photograph from the lens ~~to scale of~~ on the photograph. In order to locate this point on the map three true direction rays to points of known position are determined, platted on tracing paper and the three point problem is solved as previously described for horizontal or ground photos. A vital fact which must be realized is that the intercepts for given angles on different planes are all the same on the horizon line. All that is necessary to draw map rays is to find (1) the intercepts on the horizon and (2) the position of the vertical line to the nadir point.

<u>Drawing from nadir point</u>. The position on the map of the nadir point below the lens of the camera which took the oblique photo is useful as the center from which to draw map rays controlled by intersepts on the horizon line. If it is not awkwardly distatnt this simple method of obtaing such true rays was widely practices on trimetrogon wing photographs. A mechanical device where There were ~~the~~ two constructions, (1) in the photo plane, (2) on the map were connected by a single bar which ran on the horizon so that the offsets of horizon distance were the same. The two arms differed in length (1) written distance on photograph from horizon to nadir point and (2) distance on map or horizontal plane from horizon line to lens position. Exact values for these distances may either be computed or scaled from construction like Figures 17 and 18. When the map rays have been determined the overlapping photographs of the trimetrogon system allowed the laying out of radial line control which covered a wide area and tied together an entire flight.

<u>Crones solution</u> In many obliques the nadir point is very distant on the photo ~~map~~ from position of the horizon line. In this case it is more convenient to use ~~fall back upon~~ Crone's graphic construction. This will ~~is to~~ find the map position of the vertical line at the level of the point sought, A, ~~south~~ as shown in Figure 16 or the left side of Figure 19. In the latter figure it is evident that a line to A, B, the point sought to the vertical line is equal to D, L, or AC We must find this length on ~~to scale of~~ the photograph so as to get the direction of the ray. Construct the triangle HOL of transparent plastic like celluloid. Measure the distance DL from the left part of the diagram. Construct the horizon line in the right hand diagram. Construct a line parallel to the horizon through the point sought, A. Set an arc with radius AD Find point B by shifting the triange LOH along the principal line of the right diagram until side HO extended is the same distance below the horizon as D, that is the arc is tangent to both horizon and principal line at D Now the point L which is vertically above N is the center from which a map ray may be drawn through A, (line L A) For the same camera and same angle of depression the same transparent triangle may be used.

<u>Elevztion determination.</u> Elevations cannOt be determined accurately in a single Oblique photograph. Where there are overlapping pictures, as with the trmetorogon system, it is possible to obtain reasonable determinations of objects not too far from the camera. The following method was used by one of the groups working in the southwest Pacific in World War II but no derivation of the formulas was given in their publication. The basic principles, with the writers derivation, are shown in Figuree 20 and 21. Let m = a point whose elevation above datum is desired. The base line at the bottom of the photographs is the line where the oblique is at the same scale as the vertical which was taken at the same time.. Next we must recognize that parallel lines on the ground in a given direction all come together on the horizon at a <u>vanishing point.</u> The horizon line must then be marked on two adjacent pictures both of which show the point m. Two vanishing points V_1 and V2 at distance apart equal to B, the distance the plane traveled between exposrues to the scale of the vertical picture Referring to the left side of Figure 20 which is a drawing in the photo plane, we may project the position of m onto the base line. The parallax motion is d which may be measured in inches on the common line or <u>isoline.</u> In these diagrams the elevation base is taken as this line. The right part of Figure 20 is a xsection of one of the photos on the principal plane. This shows that the actual elevation of m above datum is E but due to inclination of the picture this corresponds to E' in the photo plane. Now the displacement of point m may be divided into d_1 and d_2 the part on each picutre of the overlapping pair. B shows the distance the air plane moved between exposures to photo sde which may be divided into D_1 and D_2 B is also the distance betwen the two vanishing points. Next it is clear that $d_1 = E' (D_1 + d_1)/ IH'$ where I is on the isoline. $d_2 = E' (D_2 + d_2)/IH'$ Hence $d = E' (B + d)/ IH'$ Next E = E' cos angle of depression, which is shown at the right in Fig. 20 H = IH' cos same angle. Hence E' ; IH' ;; d; (d + B) and E/ H = d / (d + B) The first expression is proved by similar triangles and in the second equation the cosine of the angle of depression cancels out so that our final result is E = h.d / (B + d) In Figure 21 the method of determination is shown. Here a datum plane of elevatioh was established by a point Y whose elevation was known.

This spot is close to the principal plane. A well marked point near or on the horizon and visible in both photographs is selected and the ray to it through Y is drawn. These points are V_1 snd V_2 Thi is the pointV_1. Extended to the base line this ray intersects at O which is the base point for all measurments. Similar rays to V_1 and V_2 are drawn from the from nadir points, N and N' Both extend to the isoline. The principles of perspective show that these rays are parallel on the map for they go to the same vanishing point. Using an inch scale measurments are takenfrom O to intersections of oth r rays with the isoline. B, the air base is equal to the distance from the transferred at O on right photo and the line from nadir point N to same vanishing point V_2 in right photo. It is equal to V_1-V_2 d is the difference in readings fo rays through m in the two photos . H is the sea level alitude of the plane above :point Y If m is lower than Y the formula becomes B-d Note that B and d are measured in inches and H and E in feet.

Smith's method of finding vertical angles. The method illustrated by Figure 22 is the method proposed by H. T. U. Smith. It may prove confusing because more than one plane is shown on the same diagram. The solid lines show the outline of an ablique photograph with true horizon at ha' The nadir point is at n, the principal point at O. In the left part the veritcal section through lens and principal plane is shown. l is the focal length L is the lens position with lines 2 and 3 at 90 degrees h L O is the angle of depression of the pricipal point. a is the point sought Line 5 is drawn through this from nadir point n to horizon at a' Using n as a center line, 4 is an arc with radius nL Now if a right triangle shown in double lines is so placed that the right angle is on arc 4 at L' and the sides through a' and n the angle a' L' a is the true angle of depression of a. One side of this angle is line 7 defined as aL' This angle can be measured with a protractor. By making the triangle ~~shown in double line~~s of transparent material it can be combined with the protractorThe method works best if the angle of depression is fairly large.

Elevation finding. When the ~~point sought and the plane locaton of the photograph~~ have both been ocated on the map, the elvation of plane is easily computed The formula is elevation = distance times tangent of angle. ~~The same idea was used to get elevation of plane.~~ Distances were scaled on the map.

Canadian grid system of rectification. The early mapping of north-central Canada covered a region of rather low relief with many lakes. Directions were, therefore little affected by local differences of elevation and it was decided to employ oblique pictures so as to ecomonize on flying. The basic idea was to draw a system of squares on the ground as they appear in the photograph. Requisite data are (1) focal length of the camera and (2) altitude of the plane at the time of exp osure. Control was supplied by ground surveys which located definite objects which could be recognized in the photographs. Figures 23 and 24 show methods used in constructing the perspective grid which is done on transparent material and is usable with all photographs taken by the same camera at same elevation. Grids must be prepared for several different elevations close to that which it was intended to use in the field for such data is derived from altimeter readings which are subject to error through weather changes. Turning to Figure 23 we see that two scales must be employed for different parts: (1) the size of the picture and focal length ~~and scale of elevation~~ must be full size of the original and (2) the ground with its grid and the elevation scale must be on the desired map scale. First mark the apparent horizon on the picture, compute the position of the true horizon from available altitude of the plane as explained previously and plot it. Find the ~~pri~~ principle point of the photograph and measure the distance below the true horizon. Lay out at right angles the picture plane and the axis of the camera. Draw horizon line at right angles to the vertical line below the lens position. Carry picture plane line across the ground line, which is normal to the elevation vertical, at a distance to map scale which gives the alitude at the time of exposure. Starting at this intersection lay out on the ground line the intersections of the sides of the squares of the grid. Most of the laboratory work at the University of Wisconsin used mile squares. Some advise starting these at the point where the camera axis extended cuts the ground line. Draw lines from these points to the lens taking care to mark the positions where they interersect the picture plane. Omit those outside the picture limits and those so far distant from the camera that drawing from them on the photograph would be inaccurate. Turn next to Figure 24 which is to construct the lines which converge away from the camera in the direction of view. Construct lines for true horizon and vertical line as before but remember that we are here looking zt the picutre in the direction that the czmera was pointed. Next turn attention to the ground or junction line where the surface of the land cuts the plane of the picture in Figure 23. On this line both map and picture are are the same scale. Then lay out both ways from the principal plane the squares to the same scale as was used in Figure 23. Conncet each mark for the division between squares to the vanishing point on the true horizon at its intersection of true horizon with the principal plane. This will give us a series of cn converging lines, each of which shows the effect of perspective on the sides of the squares. Then transfer the transverse divisions from Figure 23 to the line of the principal plane. Drawing at either ground line or true horizon Through each mark construct lines parallel to the junction line thus making up the squares in perspective Care should be taken not to show the squares too far away where distance and at atmospheric conditions make mapping of detail very ~~crude~~ poor. Also omit squares outside the limits of the photograph. The perspective squares are now laid over the picture and the distance between points located by ground survey compared with that indicated by the squares. If it not correct a diagram for a slightly differnent plane altitude must be tried. When the correct grid has been found the details of the lanscape may be drawn on true squares of the map In making the map it is not seential to pay any attention to equality of scale. The map squares may be either larger or smaller than those of the grid on the junction line.

Rich's method. Rich's method of rectification of an oblique picture is illustrated in Figure 25. Like the Canadian grid method it is applicable only to nearly level ground where local differences of elevation cause no important distortion of directions. In Figure 25 we have a section on the principal plane of the photograph. This ~~pone~~ photograph was taken from a bluff and not from a plane so that the alitude is small. All features of the picture and are full original size. The elevation and map parts are at the deisred map scale as with the Canadian system. The locations of points in the photograph are determined by intersecting lines which are based upon two vanishing points. We know that all lines to a given vanishing point are parallel on tne map. Here three points were used, two for each side of the map with the central one common to both halves, The map position of these parallel lines was fixed at the points where they cross the ground line Figure 26 shows how a map was constructed from the low oblique phtograph. The outside lines of the triangle V_1 V_2 L are ture map direction rays for they pass through the nadir point of figure 25 (not shown there). Each point was located at the intersection of lines, one of them parallel to the line HL, the principal line and passing through the intersection of the corresponding ray on the photograph to the ground line. The other ray uses one or the other of the vanishig points V_1 and V_2 and the line is drawn parallel to the side lines through the intersection of rays with the ground line The two side vanishing points must be far enough away from the principal plane to give good intersections. In this photograph a section line could be identified by a row of trees and was used to give the north line.

<u>Summary</u>. The writer has endeavored to showw that the preparation of a base map from photographs taken either on the ground or from the air requires neither great experience nor skill, simply care in drawing and in following auimple directions. Methods are easiest with vertical air photographs. Oblique and ground photographs are distored by perspective but have the advantage that a single oblique of fairly level ground or a shoreline can be made into a map. To obtain photographs taken from the air the U. S. Geological Survey publishes a map of coverage in the United States including how to obtain copies. It must also be noted that besides furnishing a basemap air photographs are invaluable in deliniating the boundaries of both soil and geological formations by their representation not only of topography but also of differences in moisture content of the soil and mantle rock from which much valuable information may be deduced. If no photographs are avialable a geologist can take obliques with a hand camera from a light plane avoiding such low angles of depression that much of the country is hidden

It is assumed that the student was already familiar with the principles of plane table surveying

Photographic map-making for geologists.

Introduction. The discussion following is a resumed (with some improvements) of material formerly taught to geolgists at the University of Wisconsin covering use of photographs in making base maps. Much of the published literature on this subject is so complex that it is hard to understand. A common fault of writers is to omit ~~entirel~~y the derivation of formulas. It has been remarked that much of ~~it~~ this literature consists of "hard ways to do easy things." For this reason many references are omitted, for their reading proved more confusing than enlightening to beginners. The accuracy demanded by many methods is far beyond both the accuracy of available photogrpahs. ~~as well as beyond the~~ and possible accuracy of drawing. An effort was made to avoid use of complex instruments and to concentrate on principles which when understood

~~Kinx~~ When photographic methods are necessary. Surveying from photographs is necessary (a) when available time for surveying is limited by time of travel or by short duration of good weather or (b) when show the value of expensive instruments pictures are taken from aircraft. In mountains the ~~time necessary to climb peeks as well as the relative proportion of clear and~~ The best condition prevails to ~~stormyweather combine to make recording of views in photographs a essential.~~

Kinds of photographs. Photographs may be taken either from the ground or from the air. ~~with~~ The axis of the camera ~~either~~ may be vertical, horizontal, or inclined. Obviously the first position is possible only from the air.

Lenses. Cameras consist of a convex lens which throws a "real" image on a film. The construction of a lens which gives an ~~correct~~ image ~~all in the same plane and~~ which ~~shows all~~ focuses all colors of ~~the~~ visible light in one plane, and shows little distortion of form, is not at all simple. Exact measurments show that there are many minor errors ~~from~~ in lens construction. In our discussion these defects ~~have to be~~ are ignored. It must be realized that the actual image on the film ~~of what is shown by the camera~~ is turned 180 degress from the original on the ground. ~~film.~~ Development ~~then produces~~ yields a negative. ~~If~~ This negative is converted to a positive by printing. If a print were transparent and ~~and~~ held in front of an observers eye at the same distance that the film was from the center of the lens it ~~should~~ must coincide very closely with the original view. This assumption is used in most demonstrations and needs no further proof. The distance of the film from the lens is known as the focal length of the camera when it was focused for objects at a considerable distance (100 feet or more). It is therefore apparent that aside from minor errors due to lens and printing defects a ~~view~~ picture taken with a camera ~~furnishes~~ is a record of the view from which it is possible to find both vertical and horizontal angles of objects ~~refered to~~ from the axis of the camera. Finding of angles enables one to locate objects seem in more than one photograph by intersection of rays jus t as is done with the planetable. provided the camera axis was horizontal Differences of elevation can be obtained from angle of slope and distance as with a telescopic alidade

Ground photo surveying.

Introduction. Surveying by photographs taken from the ground is not now very ~~so~~ common since it is possible to obtain photographs from the air. Local conditions of terrane and weather may, however, make it (still) desirable. Although both horizontal and inclined photographs can be taken the former are ~~is~~ easier to understand and are ~~was~~ more commonly used for mapping. They ~~It~~ alone are ~~is~~ discussed here.

Taking the photographs. It is essential that the camera ~~should~~ be level, ~~that is the axis of the lens be level, and~~ The horizon ~~as~~ shown in the photographs must ~~is~~ also be level and parallel to the edge of the negative positive. It is desirable that the photographs show not only the vertical plane through the axis but also ~~as well as~~ the horizontal plane of the horizon. This is done by small markers which register on the film called collimation marks. These can be inserted in almost any kind of camera. Leveling of the camera may either be accomplished by attaching it to a tripod with a leveling head or placing it on a levelled planetable board. Blurring of the distance in photographs by haze is avoided by using a filter. If the black and white film is sensitive to red a red filter is best. If not, a yellow filter will reduce the effect of atmospheric haze for that is mainly in the blue ~~ray~~ light. Infra red film and filter are best. Color film was not commonly used. (Thereafter

Ground control. In order to do accurate ~~effective~~ mapping on a known scale it is necessary that camera locations and recognizable points within the photographs be located by a ground survey. The more such points are located the greater the accuracy. Elevations as well as locations are ~~is~~ needed. It helps to locate the camera stations although this could be dispensed with if three points with one of the above from each were located.

Registration of angles. A little study of Figure 1 shows that since the film is normal to the camera axis ~~the~~ horizontal angles ~~of~~ from the direction of the axis are recorded not in direct proportion to distance from the central plane but to ~~as~~ the tangent of the angle. Vertical angles are ~~further~~ distorted by the greater distance of the film from the center of the lens toward the sides of the photograph. All objects at a given horizontal angle lie in the same vertical line but points of the same vertical angle lie in a gentle curve as shown in Figure 2. Imagine that you moved a telescopic alidade or transit up and down and then swung it around horizontally on the level table and recorded the path of the line of sight on a vertical wall

Map locations. The focal length of the camera should be known. If not, it can be found if an angle subtended at the camera station between two recognizable points is compared with the distance ~~it~~ which is represented ~~by~~ it on the photograph. If the Locations of all camera stations ~~should be~~ are not known, ~~although~~ it is possible to find such by the angles subtended by three points on the ground, ~~shown~~ in the photograph, ~~and~~ whose map locations are known, ~~have been found~~ (tracing paper solution). The angles are laid out on tracing paper and fitted to the locations on the map just as can be done with the three point problem on the planetable.

The direction on the map of the axis of a photograph is then laid out either from direct observation in the field or by finding ~~the~~ its angle from a point in the photograph of known compass direction. Every point to be located on the map must be shown in two or more photographs taken from different camera stations. When the directions of picture axes have been platted the picture planes are laid out normal to each of these at a distance equal to the focal length of the camera from the map position of each station. These can be thought of as the map locations of the photographs when held in front of the observor at each station at the proper distance to just cover the original views. Now we are ready to plat rays from the camera stations measuring angles in all cases parallel to the horizon, (from the direction of the vertical plane through the axis, ~~or~~ the principal plane.) When this is done for different photographs points are located at the intersections of rays just as with intersections on the planetable. Care must be taken to use correct identifications of lines so that the right intersection is used of the many formed on the map. When enough intersections have been determined features shown on the photograph may be sketched in.

Elevation finding. Fig. 4 shows a graphic method of finding differences of elevation from the vertical angles recorded on the photograph. One must consider the fact that the distance to the photograph increases towards the sides so that the scaled difference above or below the horizon line must take this into consideration as shown. It is evident that the greater the vertical angle the greater the accuracy of this method of solution. It can be used not only to find elevations with respect to the camera station but ~~also~~ can also be used to find the elevation of that station from a point of known elevation.

Stereoscopic examination. If duplicate photographs are taken of the same view from points at least 50 feet apart the two pictures can be examined under a stereoscope which gives a vastly enhanced preception of distance ~~depth~~ compared to that of ordinary eye vision. Such "depth preception" greatly increases the accuracy of sketching contours and other features. Methods of stereoscopic elevation-finding described under vertical air photographs could also be used to determine the horizontal distances to points which were not located by intersection. Generally enough points will have both position and elevation determined that contours can be sketched with reasonable accuracy.

Limitations of method. The method of mapping from ground photographs is impossible ~~limited~~ in areas of dense ~~abundant~~ forest or brush ~~growth~~. It is hard to tell when all ~~are~~ of an area has been photographed in at least two views whose directions cross one another at a big enough angle to make intersections reliable. The method is best adapted to nearly treeless terrane where atmospheric conditions permit good photography and travel is reasonably easy to ~~the~~ commanding camera stations, and these cover ~~all~~ the landscape ~~adequately~~ without any large concealed areas.

Air photography.

Introduction. The term air photography is preferred by many to the older term aerial photography for ~~applied to~~ photographs taken from aircraft. Athough photographs were ~~have been~~ taken from balloons and kites many years ago ~~for a log time~~ it was not until the development of the airplane that such an art came into its own. Photographs taken from considerable alitudes either may be directed (a) as nearly vertical as possible ~~that is~~ (verticals) or (b) ~~and those~~ with camera axis inclined. ~~or~~ (obliques). Obliques are divided into low ~~or~~ (near verticals) and high which show the horizon in the view. Some methods of surveying combine verticals and obliques. Many obsticals had to be overcome in order to secure satisfactory air photographs. The amount of air with included dust and water vapor that lies between camera and ground, static electricity which affected films, altitude, and course control of the plane are a few of these. Haze is overcome by flying only in good weather and by using a filter. Some photographs are now taken with the invisible infra-red rays which give maximum penetration of haze, because these rays ~~they~~ are less distorted than is visible light

Geometry of vertical photographs. Few intended vertial photographs are ~~truly~~ exactly vertical. It is very difficult to fly a straight course at the same altitude without some deviation from verticallity. Automatic control with a gyroscope is impracticable because of the weight of the ~~automatic~~ big roll film camera used for such a purpose. Were two conditions met the photographs would be true maps of the land they show. These are: (a) level land and (b) exact verticallity. ~~Details of~~ Tilt will be considered below. Figure 5 shows that hilltops directly under the camera are not distored in position whereas those towards the sides of the picture are displaced outward from the center of the photograph. A straight line passing over a hill is distorted near the edge of a photograph. Figures 6 and 7 show more of this phenomenon and how the difference of location of points in sucessive pictures enables one to see the country in relief when looking at one picutre with each eye.

Stereoscopes. Several devices are possible ~~of use~~ to attain the above end. These instruments are called stereoscopes. Common types are (a) two convex lenses at eye distance apart on a stand ~~at eye distance from one another~~, one over each photograph (b) a combination of mirrors or prisms dividing the lines of vision of the two eyes sometimes combined with miniature telescopes to secure an enlarged view. The mirror type enables the observer to see all of the area common to two overlapping photographs at once. With type (a) there are apt to be "blind spots" which cannot be seen in relief even if the edges of the prints are trimmed. (c) printing of one photograph in red and the next in blue or green. When the two colored picutres are superimposed in printing the result is called an anaglyph. To the ~~unaided~~ naked eye it is a blur but when proper colored glasses are worn one color is seen with each eye and the result is stereosocopic vision. Anaglyphs have never come into common use in part due to their cost, (d) projection devices which project sucessive pictures onto a drawing board in alternate red and green or blue. When proper glasses are worn then a relief model apears to stand on ~~rise from~~ the board. Some devices use cross polarized light which can be viewed through polaroid glasses.

~~polarized light~~

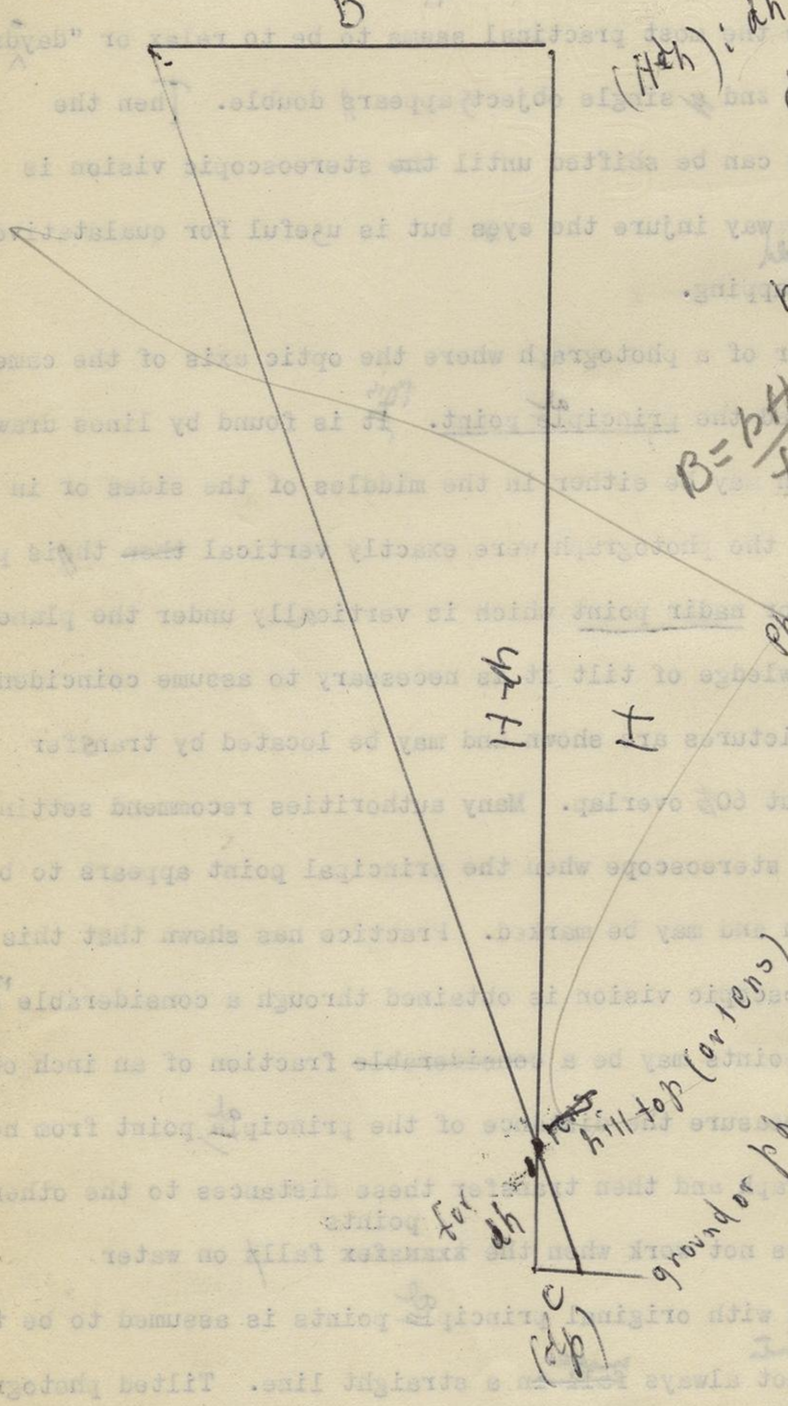
Besides the use of intruments it is possible to see two photographs of the same area taken from different points in relief without any aids. This ability can only be attained by practice and all persons cannot ~~acquire~~ attain or acquire it. Various methods of practice have been suggested. Of these the most practical seems to be to relax or "daydream" until binocular vision is lost and ~~a~~ single objects appear double. Then the ~~highlights of the~~ two pictures can be shifted until ~~the~~ stereoscopic vision is obtained. It does not in any way injure the eyes but is useful for qualatative examination only especially in the field and not for mapping.

Definitions. The center of a photograph where the optic axis of the camera intersects the picture is called the principal point. This ~~It~~ is found by lines drawn between collimation marks which may be either in the middles of the sides or in the four corners of the photograph. If the photograph were exactly vertical ~~then~~ this point would coincide with the plumb or nadir point which is vertically under the plane at the time of exposure. Lacking knowledge of tilt it is necessary to assume coincidence. Principal points of adjacent pictures are shown and may be located by transfer for there is intended to be about 60% overlap. Many authorities recommend setting the adjacent photographs up under a stereoscope when the principal point appears to be located on the other photograph and may be marked. Practice has shown that this method is not ~~always~~ reliable. Stereoscopic vision is obtained through a considerable "angle of tolerance" and the transferred points may be a ~~considerable~~ fraction of an inch out of place. A better method is to measure the distance of the principal point from nearby definite features on the photograph and then transfer these distances to the other photograph. Of course this does not work when the ~~transfer~~ points fall on water.

The line joining transferred with original principal points is assumed to be the line of flight and successive principal points ~~three points~~ do not always make ~~fall in~~ a straight line. Tilted photographs have the plumb point separated from the principal point. As will be shown below the axis of tilt lies half way between these two points and is normal to the line joining them. A point on this line half way between ~~in line with~~ the two points is termed the isocenter.

Height finding. It is evident from Figure 5 ~~5~~ that the position of points at different elevations differs in sucessive photographs of the same area.

Fig 8



$(H-h) : dh :: B : \cancel{c}\ dp$

$dh = \frac{B}{c \cdot (H-h)}$

$b \text{ on photo} = \frac{B \cdot f}{H} = b$

$dp = \frac{C \cdot f}{H}$

$\cancel{\frac{C \cdot f}{H}} = dp$

$B = \frac{bH}{f}$

Photo scale $= \frac{f}{H}$

$b : (H-h) :: dh : dp$

$\cancel{H} = \frac{\cancel{(H-h)\,dh}}{\cancel{dp}}$

$dh = \frac{dp \cdot b}{(H-dh)}$

$(H-h) : dh :: \frac{bH}{f} : dp$

$dh = \frac{\cancel{dp}\, b \cdot dp \cdot H}{f \cdot (H-h)}$

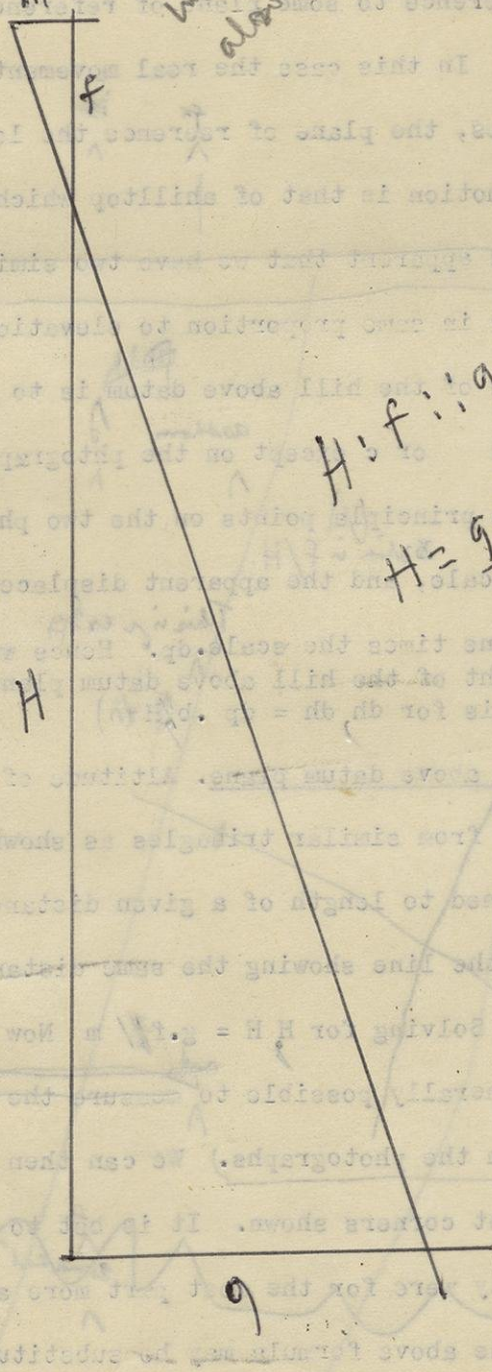
It is possible to apply the principles of parallax to obtain a quanitative measurment of this in terms of difference in elevation. Parallax is defined as the apparent movement of an object ~~taken~~ in reference to some plane of reference which is due to the real movement of the observer. In this case the real movement is the motion of the plane between sucessive exposures, the plane of reference is the lower ground shown in the picture, and the apparent motion is that of a hilltop which rises above this plane.

~~Referring to Fig. 8 it is apparent that we have two similar triangles.~~ Then the movement of the plane B is in same proportion to elevation above the hilltop (H-dh) as the ratio of the height of the hill above datum is to its apparent displacement, c. We cannot measure either B or c except as shown on the photographs. ~~Exis~~ The average of the distance between sucessive principal points on the two photographs (b) is the real distance in the ~~axe~~ air reduced by ~~times the~~ scale, and the apparent displacement is that which could have been observed from the plane times the scale. dp. This is c.f/H. Hence we may write b: (H-h) :: dh : dp ~~where~~ where dh is the height of the hill above datum plane. and H is altitude of the plane above the "plane of reference". Solving this for dh, dh = dp . b/H (h)

Elevation of the plane above datum plane. Altitude of the plane from which the photographs were taken is found from similar triangles as shown in Figure 9. Here altitude, H, is proportioned to length of a given distance on the ground, g, as focal length of camera, f, is to the line showing the same distance in a photograph, m. We may write H: g:: f: m. Solving for H, H = g.f / m Now in country covered by the U. S. Land Survey it is generally possible to ~~measure the length of a mile and~~ to identify section corners on the photographs. We can then measure the average length of a mile between the different corners shown. It is best to take the north-south sides of sections since they were for the most part more accurately surveyed than the east-west distances. The above formula may be substituted for H in the parallax equation (neglecting dh) and we find that dh = dp.g.f / b.m Now dp is taken as a unit of the scale used to measure parallax difference, g is the length of a mile, 5280 feet, and f is for many cameras 8.25 inches. ~~If parallax is measured in millimeters then dh (feet per mm) = 1. 5280' .8.25" / b" .m" mm~~ ~~To change m to inches multiply by 25.4 so that the different units balance one another and we have the final formula dh = 1715 / b".m" Figure 10 shows the values of dh for different~~ values of m and for

Fig h K

include also 9B formerly text



$$H : f :: g : m$$

$$H = \frac{g \cdot f}{m}$$

It is clear that we must assume (Fig 8) (1) that the picures are exactly vertical with nadir points coincident with principal points, and (2) taken at same elevation above the average ground surface or plane of reference and that the ehight of objects on the ground such as a hill of height dh is a small amount comparted to the elevation of the plane, H. Then we can ignore for simplicity the elevation of the plane above the hilltop, H - dh The picu~~t~~res were taken after the plane flew the distance B which is impossible to measure in the air. This flight caused the apparent movement of the hilltop by distance x, which also cannot be measured. The problem then is to set up two different values for x in terms of known q uantities, equate the two, and thus eliminate x from furthr consider tion. We will first make it clear that the total distance the hilltop moves on the phtographs is dp which is the sum of d_1 and d_2, also that distances in the photograph are related to the distances they show on the ground in the ratio f:H This is known as the scale ratio. From the above it is clear that dp : x :: f: H Whence x + dp . H /f

Also by similar trianges it is obvious that dh : x :: H : B whence x = dh .B/H

We can equate the two values for x and then dp.H/f= dh.B/H

If we want to find dh, a difference of elevation in terms of dh, difference in parallax solution of the above yields dh = dp.H^2 /f.B Since b, the distance between successive nadir points on the photos b = B.f/H, B = bH/f By substitution, to use quanities which we can easily measure, dh= dp.H /b. Although this gets rid of f which is not always known to the user of the photographs, we will go forward another step and express H in terms of ~~ixxx~~ a known ground distance, m g and m, the length of this distance in the photograph. Hence H = g.f/m and then by substitution dh = dp, g. f/ b.m The simplest thing to measure is the length of a mile in the phtographs if section corners can be identified, Now the easiest thing to measure is the amount of elevstion change represented by a millimeter of parallax difference. The focal length of the camera must be known or ~~could~~ can be found vy working the above equation backwards with a known value of dh. The distances b and g are readily measured with a scale of ~~in~~ inches nad decimal divisions thereof. Need in to balance the equation by using similar units of measurements in both

$dh = \frac{dp\,H^2}{f(B)}$

$B = b\frac{f}{H}$

numerator and denomenator. Turning to units in the equation we desire first to change all to millimeters. 1.12 . 25.4 = 1.5280.12.5.4 .8.25.25.4 / b" .25.4 . m" .25.4 Cancelation and transposition simplifies this to dh' = 5280.8.5.25.4/b" .m" . 12 . 25.4 or dh in feet = 1715/ b".m"

Figure 10 is a diagram showing values of dh for different values of b and m. This is assuming f = 8.25" and the photo scale is close to 1:20000 and the standard ground distance is one mile. Its use simplifies computations greatly as no observations are necessary other than measurment of both b and m with a scale divided to hundreths of inches. m is preferably measured along north-south section lines which are more precise than east-west distances on account of the method of surveying the land net.

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~~different lengths of a mile on the photographs~~. For other lengths interpolation suffices. for it is not possible to measure dp closer than about ~~1/20~~ .05 millimeter. The value of dh is for the mot part somewhat over 200 feet per millimeter on photographs taken with a focal length of 8.25 inches at scale about 1:20,000. If we want to be very particular we could call H the altitude of the plane above datum plane in which the expression would be H-dh. Computation shows that at ordinary altitudes of flight a 500 foot hill would make little difference in the result so it can be neglected as unimportant. If we take 14000 (H) ~~m~~ and reduce this to 13500 it would change the value of dh from 220 feet per millimeter to 213 feet which is well within the limit of probable error (with b = 2.5") ~~andx~~ We now have a "multiplying factor" by which measurments of parallax difference can be converted to elevation difference. It is given in millimeters since most instruments use that unit in measuring parallax. (as measurements on the photograph are in inches)

Scale errors. We have seen how alititude H affects scale of the photographs. The trouble is that it is very difficult to keep the same altitude in flying and hence no two pictures are exactly the same scale. To compensate for this it is absolutely necessary to have some distance between recognizable points shown in the photographs actually measured on the ground by ordinary surveying methods. ~~Points between which distances are determined must be recognizable in the photographs.~~ The principle applied to correct for scale deviations is equivalent to planetabling from the air. Directions are ~~held to be~~ more accurate than ~~are~~ distances scaled ~~from~~ on the photographs. Directions ~~must~~ should be measured from the isocenter ~~principal~~ point ~~for the isocenter~~ but since that is ~~generally~~ unknown. They must be taken from the principal point. To apply this in practice we may make tracings of each photograph with lines from the principal point to points which can be seen in other photographs and in those of the adjacent flight. ~~As many~~ Points near the edge of photographs which also appear in two other photographs of the same flight must be included. The direction and apparent distance of the next principal point of adjacent photographs of same flight must also be shown as shown in Fig. 1. Some of the points to which lines are drawn must be points ~~tx~~ between which distances were measured. If this is not done the resulting map will be to the scale of the first photgrzph used. Next these tracings must be superimposed paying attention to having three lines intersect at a point in as many places as possible ~~and the~~ These points ~~in~~ should agree with adjacent flights. This process of locating points by intersections is exactly the same as planetable surveying by intersections wit h the ~~plane~~ table stations at the plumb points below the places from which the photographs were taken. The great advantage over surface surveying, however, is that we can intersect points which would not be visible from the locations of the table on the ground. The same precautions of intersections of (a) more than 30 degrees and (b) no certain location without at least three rays which intersect at a point must be observed. The ~~system~~ method is called the radial line system. Note that there ~~is an~~ are ~~two~~ three inherent errors. ~~First~~ (a) the rays ~~should~~ ought theoretically be drawn from the isocenter whose location is unknown, ~~and second,~~ (b) ~~no attention is paid to tilt.~~ ~~or (c) to elevation displacement~~ A little computation shows that neither makes an important error on the scale ~~to~~ at which most photographs are taken. The method compensates for elevation distortion although theoretically such should be corrected by lines from the plumb point whose location is also unknown. Again the error is small if we ~~consider~~ assume that the principal point of the photograph coincides with both plumb point and isocenter with a small tilt angle.

Making the adjustment. Making the necessary adjustments for the radial line system with tracing overlays is troublesome and time-consuming. If a big job is undertaken other methods must be used. The overlays are called templets and can be made either of (a) cardboard or (b) slotted metal arms. In both the intersected points are slots in which buttons with center hole in them can move freely and make an automatic adjustment. The principal points are holes which ~~remain~~ are put in correct relation to the slots around them. When adjustment is complete the locations of both holes and buttons are pricked onto the map. It is best, however, for beginners to use tracings and not lean too heavily on devices which would save time for a large map but are not essential to the basic idea. on a small job

Instruments. All measurments of parallax must be made under a stereoscope. Although one can make approximate measurments with an ordinary scale accuracy demands both magnification of the photograph and use of a micrometer measuring device. The expense of a projecting device is not justified for small jobs and again it is best to not learn to depend solely upon elaborate intstruments. Two different types of instruments are in use which are called stereocomparators; that is they are primarily instruments for measuring parallax differences. One uses a lens stereoscope, another a mirror instrument which enables the user to map a larger area at once. One uses a circular gage to make measurments, another a micrometer scale. The circular gage is easier to read. Although it is not essential most instruments are aranged on a drafting machine to stay always parallel and have attached pencils to drew a ma p as they are moved across the phtotgraphs. The sketches made with such ~~instrum~~ instruments must then be adjusted by eye to fit the map made by the radial line method. Most instruments have the pencil located in a very ~~poor~~ awkward position behind the moving stereoscope which makes it hard to add details by eye sketching. Another fault is the use of a drafting machine which is too light for the heavy stereoscope. A double prarallel rule designed by the writer ~~would be~~ is much more sturdy and some now use it

Settingup photographs. To prepare a pair of adjacent photographs ~~or a~~ (stereopair) for mapping, it must be set up with the line joining the principal points of the two pictures parallel to the axis of the stereoscope. This is generally accomplished by a ~~strizght line~~ straight line rule on (front ~~side~~ or back of) the instrument. The two pictures are then stapled or taped into position. The measuring device consits of two dots which must be made to coincide with the two images of the same point. A motion at right angles to the line of flight is provided and must be used for this purpose. ~~Directions along~~ The line of flight ~~are~~ is called ~~along~~ the x axis, ~~so~~ and this sideways motion ~~is called~~ the y motion. Vertical differences are ~~on~~ the the z axis. When viewed together the two dots merge into one. A great trouble with beginners is that the eyes make the dots merge as long as attention is directed to them. More experienced observers see the dots as one but it appears to float above(or below) the surface of the ~~visual relief image.~~ ground A surer way for beginners is to use one eye at a time and set the left dot to a given definite point, then set the right dot in the same way to the other image of the same point and read the scale. Some scales can be set to 0 Then the dots are set on another point and the reading noted. The difference is the parallax difference which is multiplied by the proper factor to obtain vertical difference in feet. More than one independent setting of the dots is needed before an assured value is found. If one of the points is a known elevation from a ground survey then a large number of spot elevations can be established. provided tilt does not interfere

Drawing contours. When all points have been platted with elevations it is time to start drawing contours. The problem of sketching lines of equal elevation is the same as that met with in the field with ordinary surface surveys. Before accepting any location it must be checked in more than one photo of the same area wherever possible. Never draw contours where you cannot see the ground in at least one picture. Try to make the drawing an real interpretation of the ground form and not a mechanical interpolation between known points. Note that if two views are taken from each camera station with camera on same level and separated by 50 feet or more ground distance these can be used in an ordinary stereoscope thus getting a better idea of topographic form than can be observed either with the unaided eye or on a single photograph. All that is needed for a ground photographic survey is an ordinary camera properly equipped to take distant landscapes. The camera must be levelled for every exposure. Panchromatic film with a red filter is best. Some form of ground control must also be surveyed to find position and elevation of as many camera stations as poss ible. Use of photography on the ground is still a good method for surveying some areas.

End

To set the pictures to obtain stereovision those for the lens stereoscope should be have common points not over 2.45 inches apart. A mirror stereosope requires about 6.1 to 6.3 inches between common points. Beginners often ~~If you~~ loose stereovision ~~it will~~ commonly ~~be~~ because of failure of the drafting arm ~~machine~~, ~~and~~ not motion of the pictures. Tracing of streams, roads, and other features of the map should be done with left dot only since that does not move in relation to the pencil. The sketch will be easier to trace if you acdentuate the different features with colored pencil, say black for man-made features, blue for water and some other color for contours. ~~if drawn.~~

Contouring. The ability to sketch accurate contours by eye ~~alone~~ appears to be rare. Most users of air photographs ~~will~~ need to either (a) determine many elevation points or (b) set the dots to the proper distance for a given contour elevation and then follow around the same level watching that the dots do not separate when attention is directed to the ground and not to them. Projection devices use a stand to carry the pencil with a small light above whose elevation can be changed to fit a given contour. With this device it is possible to ~~actually~~ trace contours very accurately. Practice and the ability to show ~~render~~ topography in contours are essential to obtain satisfactory maps.

Effect of tilt on accuracy of elevations. Figure 11 shows how a square on the ground is distorted in the photograph as a result of tilt of the camera axis. Note than on one side of the axis of tilt points are displaced inward toward the center ~~toward the isocenter~~ and on the other side the displacement is outward. Figure 12 demonstrates that the amount of displacement from true position normal to the axis of tilt in the tilted photograph increases approximately as the square of the distance from the axis of tilt. Figure 13 demonstrates how ~~displax~~ tilt may be divided into two components, right angles to the line of flight and parallel to it. It is ~~the effect of~~ tilt which affects parallax difference measurment parallel to the line of flight ~~which~~ is ~~what that~~ important in elevation finding ~~that interests us~~.

Detection of tilt. The diagrams may look ~~fairly~~ simple but we must remember that in practice the location of the axis of tilt and plumb or nadir point are unknown. Tilt is noted (a) when parallax measuremnts in the x direction vary for the same elevation in different parts of the stereopair, (b) when you must use the y movement of the right hand dot ~~constantly~~ in moving across the photographs. (c) when scale at same elevation varies in different parts of the same photographs. The first two are most commonly noted

V Correction of tilt, error. Many attempts have been made to correct errors in elevation due to tilt. No direct and fully satisfactory method of finding tilt has ever been discovered Only ~~one~~ ~~such~~ one method based on scale differences was tried but unreliable or not very satisfactory ~~is here given in brief~~. Other methods involve so much of a chance of error in drafting that they are of little practical importance. With projection methods the relief model is made to coincide with at least three known elevations within a photograph. The accuracy of tilt of thd projecting lantern to compensate for tilt of the camera is then dependant on the acuteness of vision of the operator. One method of minimizing the effect of the component of tilt normal to the line of flight is to find the total parallax (distance apart) of a known elevation point on one side of the pair (shown in both photographs) ~~Keeping the dots set to this distance~~ this point is fastened on one photograph with a needle driven through it and another ~~poix~~ known point on the other side made to ~~coincide~~ agree with ~~it~~ this distance, after making allowance for any difference of elevation between the two points. Theoretically this method of turning one of the photographs should correct mainly for error in transferring principal points It can not in any ~~no~~ way compensate for the component along the line of flight. The best answer seems to be not to depend upon parallax differences if the distance ~~apart~~ on the photograph is too great and to confine all measurments to the region near to the principal point where tilt distortion is at a minimum. Formulas have been worked out by which relative tilt of photographs is accurately determined but since the actual tilt of neither one is known the formulas will not be repeated.

stet part

rewrite ~~Rihn's method. The method of~~ scale differences suggested by Rhin is ~~illustrated in Figure 14.~~ to have modified from that Steps are:

(1) select three points preferably around the principal point whose true position and elevations are known

(2) Set your datum plane at the lower point. Compute difference of elevation of points

(3) Compute elevgation of plane(see above) $H = \frac{fg}{m}$ where f = focal length in inches, g = a distance on ground in feet and m = length of this distance on photo in inches

(4) Compute elevation displacements assuming here that principal point is close to plumb point. Displacements, d, are all outward on these lines and found from the formula Elevation of plane, H : height, h, above datum :: ~~displacement d~~ : distance of the point on photograph from principal point, R ~~d:~~ displacement, d Solving for d, d = ~~hd~~ hR / H Result will be in units in which D is measured on the photograph, here inches.

out

(5) mark corrected position of each points on photograph thus bringing them all in to the same plane.

(6) Measure on the photograph the distance between the three points , P;
Look up on map or ground survey the (Call sides of trinagle h, m, and l.)
(7) ~~Comparexthisxwith~~ actual measurment of same distances on ground or G.

(8) Calculate scale of each side of the triangle by formula S ~~f~~ = P/G

(Note this is reciprocal of ordinary method of measuring scale which is G/P)

The scales of the sides are given as Sh, Sm and Sl. designating highest, medium and lowest scale

new, rewrite

(9) Find the principal point O and draw perpindiculars to it from each of the three sides

Measure distances on photograph of L, M. and H1 ~~H~~. Lay off these distances from opposite ends of triangle on side they were measured giving the scale points h, m, l. If the perpindicular should fall not on the side of the triange but on an extension of it then lay off an extension at the other end so that the scalepoint is on it & the midpoint of the ~~triange side or scale~~ datum line ~~is midway~~ is midway between the foot of the perpindicular and the scale point.

(10) Find a point a on line hl from formula ha = hl(Sh-Sm)/ (Sh-Sl) or la = hl $\left(\frac{Sm-Sl}{Sh-Sl}\right)$ why?

(11) Draw ma which is supposed to be a "line of constant scale". ~~althoh~~ ~~Theoretically~~ Only the axis of tilt has ~~a constant scale or~~ the true scale. ~~Scale on this line is not necess arily true scale.~~ Drop a perpindicular oc to ma from the principal point and another form h or l whichever dist ance is longer.

or ds = $\frac{(Sm-Sl)}{ld}$

(12) Find the rate of change in scale dS at o from formula dS = (Sh-Sm)/hb when ld is normal to am

(13) Find scale at O, So, from S= Sm+ (Oc) (dS) when o is on same side of ma as h or S_o= Sm-(oc)(dS) when o is on same side of ma as l.

(14) Angle of tilt, t, is sin t = f.dS/So where f = focal length of camera

(15) Locate plumb or nadir point, n and isocenter, i, on the line oc on the side toward the higher scale on = f.tan t and oi = f (tan t/2) (sin and tan of small angles are nearly identical)
n is on side of O toward the high scale Sh. i is half way from O to n.

(16) If tilt is large repeat all steps but substitute n for o in step 5

Then Sl= So = (ol)(dS) in Step 14 and H = f/Si in Step 3 In Step 4 d= (h .R)(f± x sin t)/ (H.f) which is + on nadir side and — on principal point side x = perpindicuular distance from image point to axis of tilt.

Insert on p. 12

(9) Assume that the scales apply to the mid points of the three sides of the triangle. This is not Rihn's method which was to erect perpindiculars from each side to the principal point. However since the change in scale along any line is directly related to distance this method has the advantage of simplicity.

~~(10)x~~ (10) Draw a new triangle through these points and find the location of the same scale value, m on the side opposite to that point. This is solved by simple proportion just as if the scale values were elevations, ~~andxnot~~

(11) Connect this point to the location of Sm. Now we have a line on which the scale is constant and this line is parallel to the axis of tilt.

(12) The next step is to find the tilt angle, t and this ~~will~~ should enable us to locate n the plumb or nadir point on a line normal to this one which passes through the principal point, o. We must translate scale into angle of tilt to do this.

The fundamental step is that scale = $\frac{f + x \sin t}{H}$ where t is angle of tilt, f the focal length ~~and~~ H the alitude of the plane and, x = distance of any given point in a straight line on principal plane from the isocenter. Let us take two random points on the principal plane and call their scales Sa and Sb. $Sa = \frac{f + xa \sin t}{H}$ and $Sb = \frac{f + xb \sin t}{H}$ Check this with Figure 12 where x = ia or ib. Now subtact the two scales $Sa - Sb = \left(\frac{f + xa \sin t}{H}\right) - \left(\frac{f + xb \sin t}{H}\right)$

multiplying both sides by H, $H(Sa - Sb) = \sin t\,(xa - xb)$

Now let $dS = \frac{Sa - Sb}{xa - xb}$ where dS is the rate of change in scale per unit of distance on the photograph and it is clear that $\sin t = H.dS$ Let Si = scale at isocenter and consider that $Si = f/H$ and hence $H = f/Si$, ~~and~~ make a substitution to

$\sin t = f.dS/Si$ Now we do not know the location of I, the isocenter as yet so for the present will have to use So the scale at the nearby principal point.

In practice the determination of dS should be on a line normal to the line we have found to show constant scale, that is a normal passing through o. f must be known for the camera used. So can be computed from its distance from the line of constant scale.

(13) a study of Figure 12 shows that the plumb point and isocenter must lie on the ~~same~~ side of this line through o toward the highest scale.

(14) Compute sin t and for a small angle sine and tangent are not much different

out 15

(15) Find the value in inches of on and oi by multiplying f by tangent of t

(16) Plot these locations which can be used in radial ~~aj~~ line drawing.

The weak point in above computation is the correct determination of dS becaue the

~~dii~~ distance between the exact points at which scale is known is not easily found. Neither Rihn's method nor that here proposed is entirely accurate and reliable. Results for tilt are for the most part much too great. A possible way out is to halve the value found on the basis that using the midpoints halves the real distances but there seems no valid reason for this. It would be better if we could take s hort scale lines for instance the length of ~~x~~ 40.5 in different parts of a photograph corrected for altitude ~~so that~~ the distances between these could be found. The method has not yet been tried but offers distinct possibiliites

Oblique photographs.

Introduction. Oblique photographs have the very great advantage of including more land surface in the pictures with less flying than is the case with verticals. Degree of obliquity is measured by departure from verticallity. Low obliques not far from vertical can be treated much in the same way as true verticals. They can be projected as verticals and can be rectified by optical means High obliques include the horizon in the field of view. Such require special treatment to make maps. Requirements for flying to take high obliques are by no means as rigorous as for verticals.

Perspective geometry of ~~obliques~~ photographs. The transformation of photographs into maps is a problem in solid geometry. We may designate the position of objects by means of vertical and horizontal angles from the place at which the picture was taken. These are what you would measure if you could have set up a telescopic alidade or a transit at the point of exposure. In the following figure (Figure 15) lines of equal angles are shown for horizontal oblique and vertical photographs. The horizontal photograph previously discussed has lines which approximate a grid whoese center is the exis of the camera. In a vertical photograph we have a spider web pattern with its center vertically below the camera at the time of exposure. In a high oblique it is a great help to be able to see the horizon for then departure from a level plane is easy to find. However, both weather and topography may prvent the horizon from being seen.

Horizon. We must now define what is meant by horizon. The true horizon is s plane normal to a vertical line through the place of exposure. The apparent horizon which is visible in many photographs is lower than the true horizon because of curviture of the earth which is slightly offset by atmospheric refraction. The angle in minutes between the two is ~~placed as~~ approximates the square root of the elevation of the plane in feet. To find true horizon we must ~~then~~ know elevation.

Importance of the plumb point. All lines of equal horizontal angles converge to a point vertically below the camera This is the location of the vertical angle of 90 degrees.

Figs 16 and 17 show the projection of objects in a high oblique into both vertial plane through the center of the photograph and a horizontal plane through the lens of the camera. Note that this horizontal plane of fig. 17 which corresponds to a map. Fig. 18 shows relations in the plane of the photograph including its extension to include the plumb point which is not in the picture.

Problem of directions. Our problem in making a map from an oblique is to find directions brom the plumb line or nadir location of the plane to points in the picture. When this is done a radial line plat may be made to tie one photograph to another and to find map locations by intersecting rays like shots with a plane table. There are ~~ts~~ two possible ways to find map directions. One is to draw rays from the location of the plumb point to the true horizon. These will cut the same intercepts on the horizon line and the corresponding true rays draw n in a horizontal plane through the lens. The other is to find the relations in a ~~pl~~ horizontal plane through the particular point which is to be located. It is clear as shown in Figs 19 and 20 that if we can find the distance to picture scale of the plumb line at any level then a ray from it to the point in the picture which is to be located can be drawn. Choice between the two methods is largely a matter of convenience.

Angle of depression. The angle by which the camera axis is depressed below the horizon is often called the angle of tilt but since we have defined that as an angle from vertical another term will be employed. Figure 16 shows how this is determined. After fining the apparent horizon and the elevation of the plane camera the angle is computed and platted. Knowing the focal length of the camera the position of its axis is laid out and the angle measured. From this angle of depression below the true horizon the position of the plumb point is easily found.

Locating plumb point on map. If we can draw three rays each to three points in the photograph whose map locations are known either from a ground survey or other photographs the tracing paper solution of the familiar three point porblem will locate the map position vertically below the plane position from which the photograph was taken.

Crone's graphic solution. The second method of finding a point from which a true ray may be drawn in an oblique is often more convenient than the offset on the horizon which was standard practice with the trimetrogon photographs taken during World War II. Abrams rectoblique plotter did this mechanically. This is the case where the angle of depression is small and the distance in the picture plane to the plumb point is inconveniently long. To find the radius in the plane in which the point sought is located shown in Figs 19 and 20 as PN'. We must consider in Fig. 19 the triangle HP'p, here shaded. The radius to be used in drawing is here projected up to the horizon or horizontal line of Fig. 19 where it is shown as LP'. HP' is in the photo plane and is easily measured once the horizon has been marked. Fig. 20 is in the plane of the photograph and shows how distance PN' or P'L can be found graphically. Two triangles are shown in Fig. 20 One is the same as HCL but is here designated as hcl Note that the size of this triangle is constant for a given photograph so that it can be made of celluloid or other transparent material and used repeatedly. Laid over this triangle is triangle hp'p which is the same as HP'P which is shaded in Fig. 19 When an arc is laid out with center at p and a radius equal to PP' or pp' with sides tangent to true horizon and to the trace of the vertical (principal) line through the principal point of the photograph as shown in Figure 20 then it is clear that we have found again distance PN' or p'l and have the center from which ~~a~~ true direction rays can be drawn for any point on line p'p of Fig 20 or P of Figure 19. In practice we do not need to superimpose the two triangles but simply strike the arc with proper radius from point sought to true horizon ~~eit~~ either from a point on hc or its extension. Since the true rays are drawn from different points it is necessary to transfer them to a single point of origin for making a radial line plot.

Elevation determinations. Points are displaced in oblique photographs by differences of elevation. ~~It is often stated that~~ Elevations cannot be found in single oblique ~~such~~ photographs. The following method was used in mapping the southwest Pacific during World War II. It depends upon overlapping obliques taken from sucessive locations of verticals, the trimetrogon method of mapping. The basic principle is illustrated in Figures 20 and 21, ~~22, 23~~ The derivation is by the writer for it was not given in the ~~origi~~ original publication. Let m be a point whose elevation is desired. It is shown in both obliques. Its elevation is E feet above a datum plane. The base line in all three figures is the isoline where the oblique photograph is the same scale as the vertical taken at ~~the~~ the same time and whose scale is the map scale. Parallel lines on the ground converge to vanishing points V_1 and V_2 as shown in Fig. 21. The two sets of parallel lines show only one position for m but when projected to the base line there is a parallax displacement equal to d. Figure ~~22~~ 20 left shows a section through the principal plane of one photograph showing the elevation of the plane above datum as ~~H~~ E' and the relation of actual elevation of m, E, to E' the apparent elevation in plane of the photograph. We can consider the parallax displacement of m on the isoline as the sum of $d_1 + d_2$ The distance that the plane moved between exposures is the air base B which is the sum of $D_1 + D_2$ in Figure 21. B is equal to distance $(V_1 - V_2)$ ~~between~~ vanishing points on true horizon line $d_1 = E'(D_1 + d_1)/IH'$ where I ~~is~~ is on the isoline or baseline. $d_2 = E'(D_2 + d_2)/IH'$ Hence $d = E'(B + d)/IH'$. $E = E' \cos$ ~~angle~~ the angle of depression which is ~~found~~ shown in Figure 22. $H' = IH' \cos$ the same angle Hence $E' : IH' :: d : (d+B)$ and ~~E'~~ $E/H' = d/(d + B)$ The ~~right hand~~ first expression is proved by similarity of triangles In the second ~~proportion~~ equation the cosine of angle of depression cancels out so that our final result is $E = H.d/(B+d)$.

In Figure ~~23~~ 21 the practical use is shown. Here a datum plane of elevation was established by a point Y whose elevation was known. ~~It is near the principal line of one photograph.~~ A well-marked point ~~X~~ on or near the horizon which can be found in both photographs was then marked, ~~X~~ V_1. The line ~~YX~~ YV_1 was drawn on both photographs. Extended to the isoline this ray intersects at C forming a base point for measurment. The location of N and N' which ~~is~~ are the plumb points of the photographs is also shown and lines are drawn from ~~it~~ them to ~~X~~ V_1 and V_2 in ~~both~~ the two photographs respectively. Another ray is drawn through m in both photographs. Now since these rays meet at the horizon vanishing points V_1 and V_2 ~~near X~~ they must be parallel on the map. ~~In the example given the reading along the isoline was 10.00 inches at a C~~ The air base is equal to the distance $V_1 - V_2$ or $N - N'$ ~~from the transferred~~

~~base line at O in right photograph and the line from nadir point~~, ~~N, to same vanishing point, V in right photograph.~~ d = difference in readings to m ~~of rays~~ in each photograph H = difference of sea level altitude of plane and that of the ~~bse~~ a elevation point Y. If m is lower than Y then the formula given above is changed to B-d. B and d are given in inches and H and E in feet. The point measured need not be close to the isoline or to the datum point

Smiths Method of finding vertical angles. The following, which is shown in Figure ~~24~~ 22, is from H. T. U. Smith's method. ~~It~~ This may be confusing for ~~the attempt is made to~~ it shows more than one plane in the same figure. The outline of an oblique photograph is shown in solid lines with the true horizon at ha' The nadir point is shown at n, the principal point at O p. In the left part the ~~vArtical~~ vertical section through the principal line and lens position is shown. Line 1 is the focal length, o the lens position, lines 2 and 3 are at 90 degrees. The angle hop is the angle of depression. a is a "point sought", Line 5 is drawn through this to the true horizon at a' Using n as center line 4 is an arc with radius on Now if a right triangle is so placed that the right angle is on arc 4 at o" and sides through a' and n the line 7 is defined from o" to a. The angle a' o" ~~x~~ a is then the true vertical angle from lens to a for the right part of the diagram shows a vertical section along the plumb line and through a . This angle can be measured with a protractor which could easily be combined with the transparent right triangle, of which only the outside edge is shown by double lines. The method works best if the angle of depression is considerable.

Elevation finding. Elevations can be found when points seen in two oblique photographs have been located on a map. The solution requires measurment of both true vertical angle and map distance. Figure 25 shows a graphic solution which avoids use of formulas. Angles could also be found from the methods of Figures 16 and 17 but it is then necessary to make diagrams for every point sought because ~~for~~ the angle varies with distance from the principal line. Map distance was obtained from intersecting rays. The solution is exactly like planetable surveying where distances are obtained by intersection and vertical angles measured with the alidade. In fact several persons have made

Photo Cop 1946

instruments like a miniature alidade which can be sighted on the picture which is set up at correct distance from the center of the instrument at correct angle to horizontal. The directions of map rays are obtained from a straight edge attached to the axis of the alidade below in the map plane. Once distance and angle are found to a point of known elevation the sea level elevation of the plane is ~~known~~ can be found and from ~~it~~ this other unknown points may be measured.

Perspective methods of rectification. If oblique photographs show fairly level country it is possible to use perspective adjustment to rectify them into a correct map. Two methods have been used (a) the Canadian grid system and (b) Riche's method of parallel lines. These methods can also be used to map shorelines of constant altitude.

Canadian grid system. The system extensively used in Canada for small scale mapping of fairly level country is one of making the outlines of rectangles on the ground as they should appear from the altitude and location at which the picture was taken. Data must include altitude and focal length of the camera. It is advisable to have prominent points on the ground located by a ground survey. Figures ~~26~~ 23 and ~~27~~ 24 show how to construct the grid. In Figure ~~26~~ 23 we will use two scales (1) the full size of the photograph and camera and (2) the scale of the map for altitude and the ground. First mark apparent horizon on the photograph and compute position of true horizon and mark it. Find the principal point and measure how far this is below the true horizon. Lay out camera axis and photograph plane at right angles. Extend the camera axis until it intersects the ground . The left side is platted to map scale to show the altitude. ~~Newxstartingxatxthexx~~ Extend the plane of photograph until it intersects the ground line . Measure distance of true horizon to ground intersection. Next start at this intersection and lay out equal distances, for instance miles to ground scale starting at this point. You could start this where the camera axis intersects the ground but this would make distances greater in lower part of the picture. Draw lines from these points to lens position.

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marking intersections with plane of photograph. Copy these intersections along with position of both true horizon and ground line on a strip of paper

Turning to Figure ~~27~~ 24 which is ~~in~~ drawn in the plane of the picture the principal line is drawn and extended to the vanishing point on the true horizon. The ground line corresponds to distance from true horizon found in Figure 23 and is at right angles to the principal line. Transfer the intersection of lines at equal map distance found in Figure ~~26~~ 23 starting at ground intersection line on which may lie outside the border of the photograph. Through each draw a line parallel to the ground line and you will have a true perspective appreance of rectangles as soon as you connect points of equal map distance on ground line to the vanishing point where the principal line intersects the true horizon. Reported altitude of the plane may be adjusted if the grid does not show correct distances on the ground between recognizable locations. Grids are prepared on a transparent base for slightly different altitudes and the best fit obtained by trial and error. Corrections may be obtained from other photographs which show the same area from a different angle of approach. Do not try to draw too far in the background where the rectangles become very small.

<u>Army method</u>. The U. S. Army manuals show another method of rectification of obliques which involves the recognition of several points in the photograph whose map locations are known. If desired vanishing points with converging lines can be used. A grid is constructed on the photograph which enables one to sketch in details on a map of the same area. The method is best adapted to adding details to an existing map rather than to map a previously unknown region.

<u>Rich's method</u>. Rich's method of using perspective to rectify an oblique photograph into a map involves the use of parallel lines and vanishing points. In Figure 25 we have a vertical section through the principal line of a photograph. The photograph plane and focal length are full size and the ground line and elevaion to map scale just as with the Canadian grid method. Fig. ~~30~~ 26 shows a combination of picture plane and map plane. We choose 3 vanishing points remembering that

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true map rays to each will be parallel to a line from them to the nadir point By the use of two or more vanishing points these true rays will intersect showing the map positions of the points desired. The parallel lines are each drawn through the places where the photograph rays cross the ground line at map scale. Like the Canadian grid the method will only work ~~for~~ on reasonablely level ground. The persepective methods require only single photographs and a minimum of ground control and hence are useful reconnaisance tools.

Summary. ~~We have~~ The writer has endeavored to show that preparation of a basemap from air photographs requires ~~no very~~ neither great experinece or skill but simply care in drawing and following of simple methods. It is easiest with vertical photographs but obliques which can be taken with an ordinary hand camera can be used provided there is some ground control. The greatest attention must be devoted to radial line adjustment with is applicable both to vertical and oblique photographs. For elevation-finding stereoscopic measurments with verticals are easiest to use but some measurments are possible with obliques provided they are not taken at such a low angle of depression that much of the country is concealed. Even if vegetation and mantle rock conceals much geology maps made from the air are incomparably better than ~~old~~ by ground surveys. If vegetaion and mantle rock allow it many geologic boundaries can be placed on a map without ground tracing. Differences in soil are often discernable because soils contain different amounts of moisture

(15) Find the value in inches of on and oi by multiplying f by tangent of t

(16) Plat these locations which can be used in radial ~~agm~~line drawing.

The weak point in above computation is the correct determination of dS becaue the ~~dii~~ distance between the points at which scale is known is not easily found. Neither Rihn's method nor that here proposed is accurate and reliable. Results are for the most part much too great. A possible way out is to halve the value found on the basis that using the midpoints halves the real distances but there seems no valid reason for this.

for instance the length of a 40

It would be better if we could take s hort scale lines in different parts of a photograph so that the distances between these could be found. The method has not yet been tried but offers distinct possibiliites

Insert on p. 12

(9) Assume that the scales apply to the mid points of the three sides of the triangle. This is not Rihn's method which was to erect perpindiculars from each side to the principal point. However since the change in scale along any line is directly related to distance this method has the advantage of simplicity.

~~xiiax~~ (10) Draw a new tirnagle through these points and find the location of the same scale value, m, on the side opposite to that point. This is solved by simple proportion just as if the scale values were elevations. ~~xxdxxxk~~

(11) Conncet this point to the location of Sm. Now we have a line on which the scale is constant and this line is parallel to the axis of tilt.

(12) The next step is to find the tilt angle, t, and this ~~will~~ should enable us to locate n the plumb or nadir point on a line normal to this one which passes through the principal point, o. We must translate scale into angle of tilt to do this.

The fundamental step is that scale = $\frac{f + x \sin t}{H}$ where t is angle of tilt, f the focal length ~~and~~ H the alitude of the plane and x = distance of any given point in a straight line on principal plane from the isocenter. Let us take two random points on the principal plane and call their scales Sa and Sb. $Sa = \frac{f + xa \sin t}{H}$ and $Sb = \frac{f + xb \sin t}{H}$ Check this with Figure 12 where x = ia or ib. Now subtact the two scales $Sa - Sb = \frac{f + xa \sin t}{H} - \frac{f + xb \sin t}{H}$

multiplying both sides by H, $H(Sa-Sb) = \sin t\,(xa-xb)$

Now let $dS = \frac{Sa-Sb}{xa-xb}$ where dS is the rate of change in scale per unit of distance on the photograph and it is clear that $\sin t = H.dS$ Let Si = scale at isocenter and consider that $Si = f/H$ and hence $H = f/Si$ and make a substitution to $\sin t = f.dS/Si$ Now we do not know the location of I, the isocenter as yet so for the present will have to use So the scale at the nearby principal point.

In practice the determination of dS should be on a line normal to the line we have found to show constant scale, that is a normal passing through o. f must be known for the camera used. So can be computed from its distance from the line of constant scale.

(13) a study of Figure 12 shows that the plumb point and isocenter must lie on the ~~xxxx~~side of this line through o toward the highest scale.

(14) Compute sin t ~~and~~ (for a small angle sine and tangent are not much different)

The following example is taken from photograph 1072 flight 16, Sauk Co., Wis. taken 19 Sept. 1937

Datum is point C elevation 954. Point A is 61 feet higher and point B only 9 feet higher. Measuring from point ~~o~~ R is 4.20 " for A and 3.35" for B. Displacement is .02" for A toward o and is negliable or absent for the other two points. Line AD measures 9500 ' on ground hence the scale Sh = 673

Line Bd measures 12275' on map and hence Sm = 658 Line AB measures 7950' and Sl = 637 AD = 6.40" on photograph after correction, line BD = ~~5.25~~ 8.05" and line AB = 5.05"

m, l, and h are all found as specified in Step 9. Note that m is on medium scale line. Next to find ha scale hl which is 3.20" Multiply this by (Sh-Sm)/ (Sh-Sl) This is ~~657~~(673-658)/ ~~7~~ (673-637) which is 15/36 making the result ~~is~~ 1.33 "

The alternative computation is; (658-637)/ (673-637) of 21/36 Multiplying by 3.20 the result is 1.87" which is laid off from other end of line h'. We will take the average between these two locations is the true ~~to~~ point to connect with m. This line is normal to direction of tilt or parallel to axis of tilt. Next find dS from formula

dS = (Sh - Sm)/ hb or (Sm; Sl)/ ld hb = 1.85" as scaled and ld = 2.3"

15/ 1.85 = 8.1 and 21/ 2.3 =9.12 Mean of which is 8.61

Find angle of tilt from sin t = focal length times dS / So where So = Sm + (oc) (dS)

oc = .05" hence So = 658 +.05 times 8.61 = .43 sin t = 8.25 times ~~658x43~~

8.61 / So where ~~xx~~ So = 658 + .43 = 658.43 sin t = ~~x001x~~ .01 F

Distance from princip al point o to nadir point n = f sin t = 8.25 times ~~x001~~ .01 = ~~.0206~~ .0825

= ~~.089"~~ ~~.08~~ ". Isocenter is half way between these points. The nadir point is toward the highest scale

<u>Conclsusion</u>. Unless one follows through the above lengthy computaiton before drawing radial lines the best solution to the tilt problem is to work rather close to the principal point and avoid basing differences of elevation on known elevations more than a fraction of an inch distant on the photograph

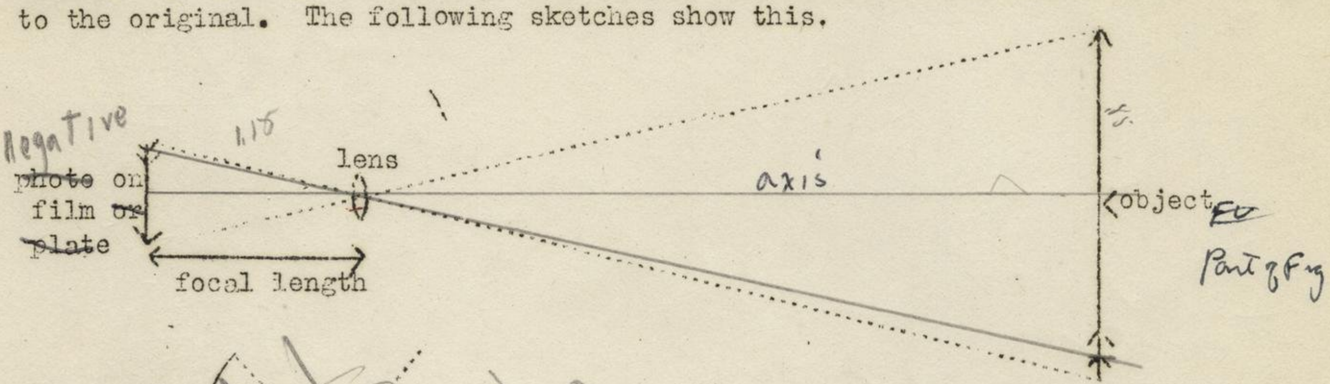
NEGATIVE
POSITIVE

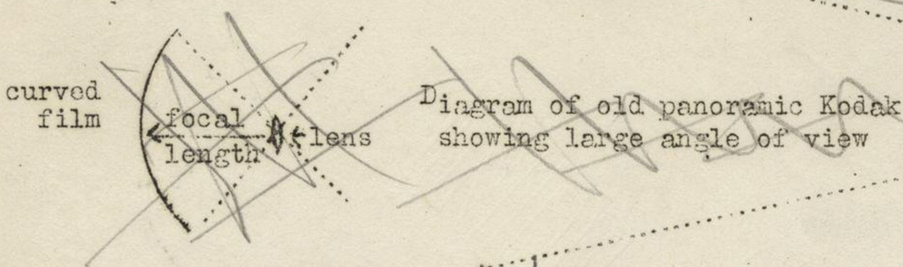
GEOLOGY 11
MAPPING
Problem 15, Edition, 1941.

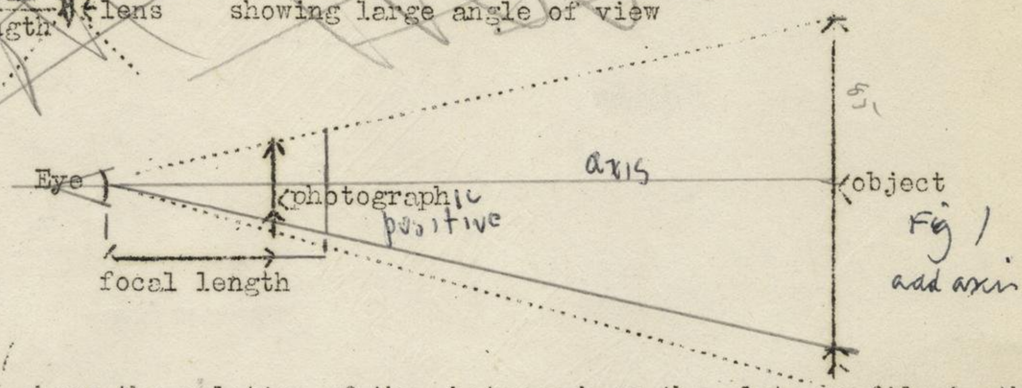
Object: To draw a map from photographs taken from the ground.

Material: Hard pencil, eraser, scale giving tenths or other decimal divisions of inches, protractor, paper either plain or cross section.

Method: It is first necessary to grasp the mathematical relation of a photograph to the original. The following sketches show this.







The first sketch shows the relation of the photograph on the plate or film to the original, or object, provided an ordinary camera was used and placed with axis level. The sketch applies to either a vertical or a horizontal plane. Note definition of <u>focal length</u>. It is essential to grasp the following ideas. (a) Every photograph with the axis horizontal shows a portion of the ground which is shaped like a triangle. (b) In a photograph all objects which have the same horizontal angle, away from the vertical plane through the axis line lie in the same straight line on the picture. (c) All points on the level of the camera are in a horizontal line called the <u>horizon line</u>. (d) In a photograph taken with horizontal axis, both vertical angles are shown by distances from the horizon line or vertical plane which are proportional to the <u>tangent</u> of that angle. (An exception to the last statement is the old Panoramic Kodak; in this instrument, illustrated in the middle sketch, the film was curved in a semi-circle to secure a larger angle in each picture than is possible with an ordinary camera. Its horizontal angles were <u>directly</u> proportioned to distances on the photograph. All points with the same horizontal angle lie in same vertical line. All points with same vertical angle lie along a line which curves farther away from the horizon line with increasing distance from center vertical line with ordinary camera (straight lines with Panoramic Kodak - sketch, next page.) The lowest sketch illustrates another fundamental idea, namely that if the photograph were transparent and held in front of the human eye at a distance equal to the focal length of the camera it would just match with the original if the observer were on the same spot as the picture was taken from.

Laying out the map. Two different kinds of photographs are available: (a) some sets of views taken in Alaska with the old Panoramic Kodak, and (b) some photographs taken at Devis Lake with an ordinary camera. Note carefully which you have. Look over the available data which includes the distance apart of some of the camera stations and the compass bearings of some of these and other lines. Without such auxiliary data no map could be made. In practice ground control is made by either transit or plane table. Some elevations are also supplied. Points shown in more than one photograph are in some cases marked in the pictures. The first step is to lay out the skeleton of the map (control points). Look over the photographs and decide what part of the paper you should start on.. Plat compass directions of other camera stations first. Now at every station lay off a line for the vertical axial plane of the camera for each picture. To locate this line accurately extend a line to another station near to this plane. On one side of a sheet of paper mark the focal length; on adjacent side of same paper, at right angles to first side, mark the distance on picture between the other station and the axial plane measuring same in direction parallel to horizon line. Now fit the triangle to the extended line in proper relation to bring line at right angles to axial plane at distance from picture station equal to focal length. See diagram below. Now extend this line and call it the picture plane. It is where you would put a transparent picture to just cover the original view with your eye at station. To obtain angle of rays to any object in a picture, measure horizontal distance from vertical axial plane and plat on picture plane of the map. Join this point to picture station and you have same result as if you had sighted an alidade on a plane table. Obviously, the intersection of such lines from different stations gives locations just as do intersections on the planetable. It constitutes indoor planetabling. The advantage is that time spent in the field is greatly reduced but time in the office is much increased. Certain climates make this desirable.

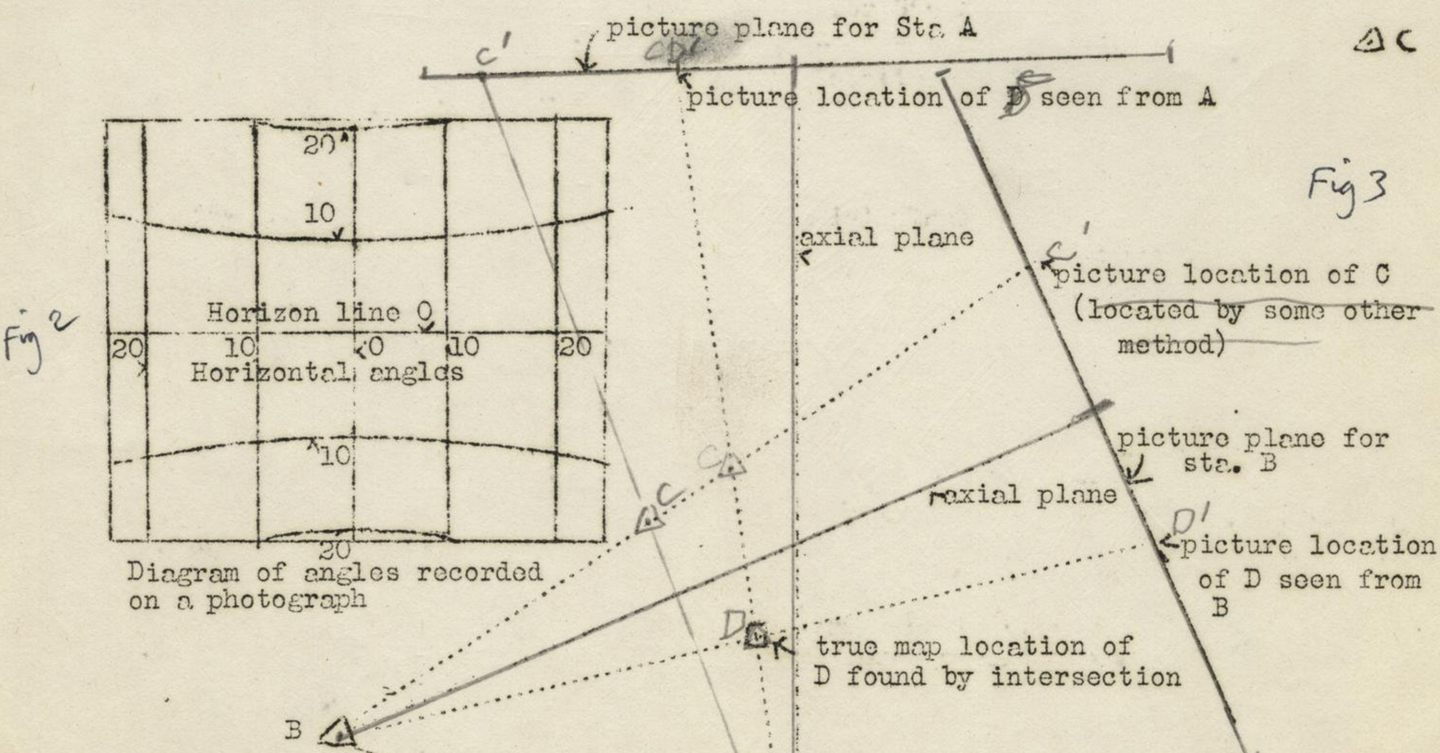


Diagram of angles recorded on a photograph

Finding elevations. The elevations of some of the camera stations are given, otherwise no map could be made. After you have located enough points to enable you to make a good map it is time to find difference of elevation of these points as compared with known places. The most practical way of doing this is to solve the problem of finding elevations from known distance and known angle. The distances can now be taken from the map, working from a picture where the elevtion of camera is known. The angles can be determined from the picture. To do this a right triangle is solved. One side of this, say the horizontal base, is the distance from the location of the station on the map to the platted position of this point in question on the picture plane. The other side, at right angles to the above, (the vertical side) is the picture distance of the point in question above or below the horizon line shown on the photograph. The third side or hypothenuse joins the last point with the other end of the horizontal side. The angle at the last point is the actual vertical angle which would have been measured had you sighted the point in question from the camera station. It is not desirable to measure this angle in degrees. The problem can be solved graphically. On the horizontal line of the triangle plat the map distance of the point in questio n as transferred with dividers or otherwise. Use the same scale as used for the map to scale vertical distance above or below this point to the hypothenuse. This is the difference of elevation and should be properly applied to the elevation of the camera station to get desired elevation. Note that with an ordinary camera the distance from the station to the picture plane is not constant but increases toward the sides of the picture. It is therefore necessary to use several triangles for each picture. In the case of the Panoramic Kodak the focal length was constant. This instrument is now obsolete.

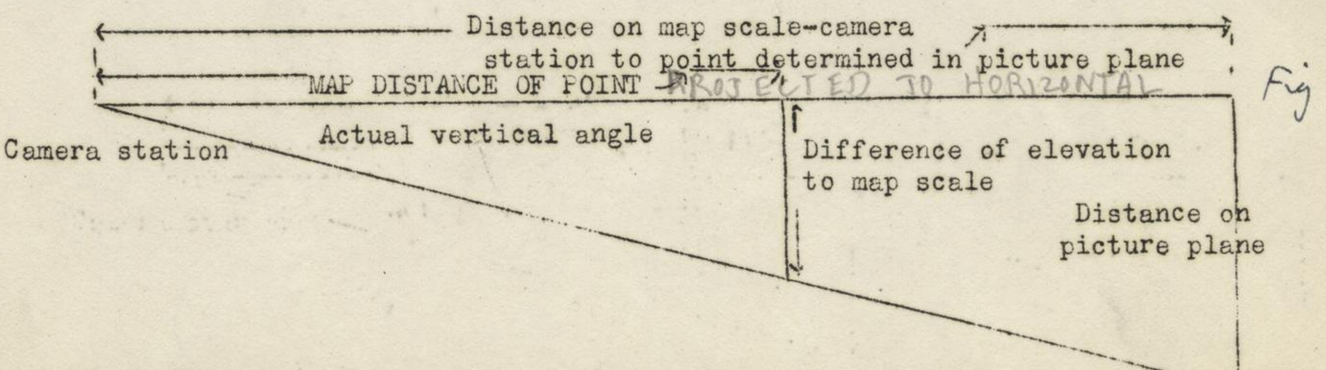
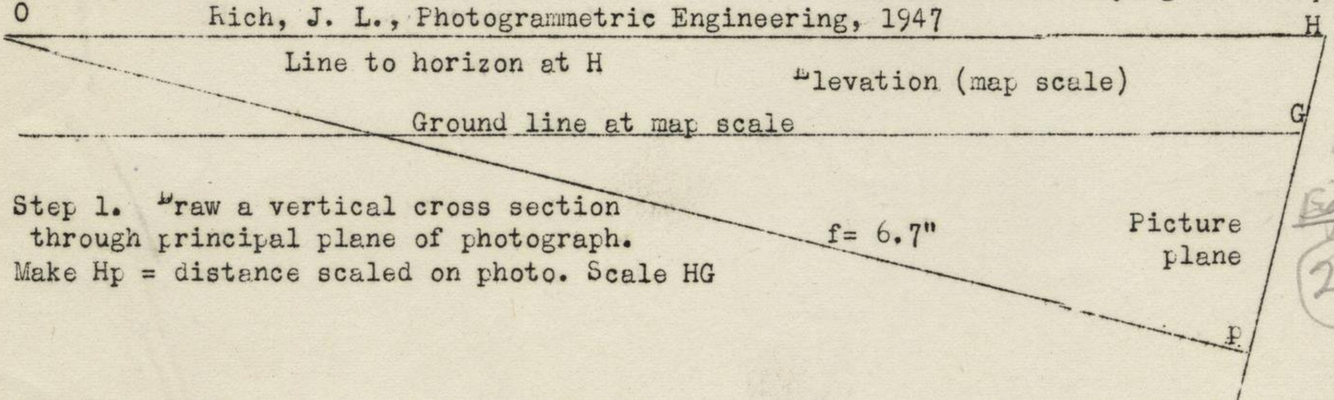


Fig 4

Note that line to picture plane is not always the same but depends upoint relation of point determined from central vertical plane.

Supplement to Problem 17, edition of 1949 Rich's method of rectifying an oblique.

Rich, J. L., Photogrammetric Engineering, 1947



Step l. Draw a vertical cross section
through principal plane of photograph.
Make Hp = distance scaled on photo. Scale HG

24

~~Eq 26~~

25

GEOLOGY 11
MAPPING
Problem 17, edition 1943 (old17 now combined with 18)

Object: To draw a map form oblique aerial photographs

Material: Hard pencil, eraser, scale, magnifier, triangles, tracing paper, one or more photographs, maps of area shown giving control data, white paper

Method: Photographs may be taken from the air with the axis of the camera tilted. If the tilt is not far from vertical the phtos are called "low obliques"; if the angle is not far from horizontal they are "high obliques". Note that the reference is to vertical and not to horizontal in this. Low obliques can be changed to verticals by use of a special printer. This is regular practice with multilens camers which take both verticals and obliques at the same time. High obliques cannot be treated thus but are often used in reconnaisance mapping because less flying is needed to cover a given area and the requirements for flying are not so exacting.

POINT SOUGHT
PROJECTED

Geometry of aerial photos. The transformation of aerial photographs into maps is a problem in solid geometry. Study the following closely as several published papers contain errors. Every photograph is a record of horizontal and vertical angles which could have been determined if an alidade and planetable had been at the place of exposure. From a given spot in space all points having the same horizontal angle (equal to a single ray on planetable) lie in same vertical plane. All objects which have same vertical angle lie on surface of a cone whose apex is at point of observation. ~~Try this with a telescopic alidade. As you sight at points along one ray you move the telescope up and down in a vertical plane.~~ If you ~~set it to say 10 degrees above the horizon and roatate it to different directions from the planetable you have described the surface of a cone.~~ The three diagrams of Fig. 1 show the differences between pictures taken with axis of camera horizontal, with it vertical, and with it inclined at an angle between these two extremes.

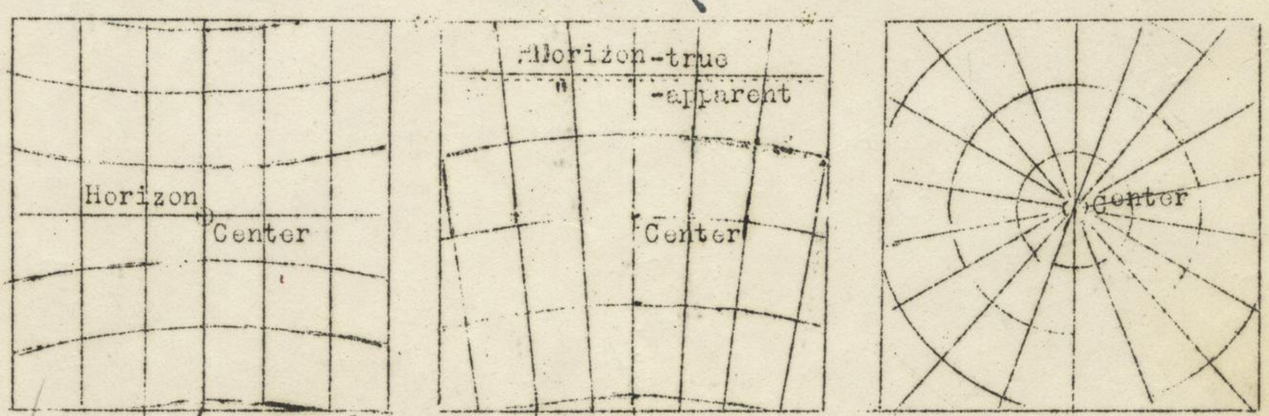


FIG. 15

Lines of equal angles in horizontal photo ~~See Problem 15.~~

Lines of equal angles in oblique photo

Lines of equal angles in vertical photo ~~See Problem 16~~

Note that the oblique is transitional between the other two extremes. The ground included is shaped like a triangle with one end cut off.
It is highly desirable that the horizon show in an oblique but this is not always possible because of weather conditions or topography. The line on the level of the plane is called the true horizon. It is found above the apparent horizon at an angle in minutes almost exactly equal to the square root of the elevation of the plane in feet. Center of picture is found in same way as with verticals. In practice the camera may also be rotated sideways around its axis but this has no effect on geometry of photo.

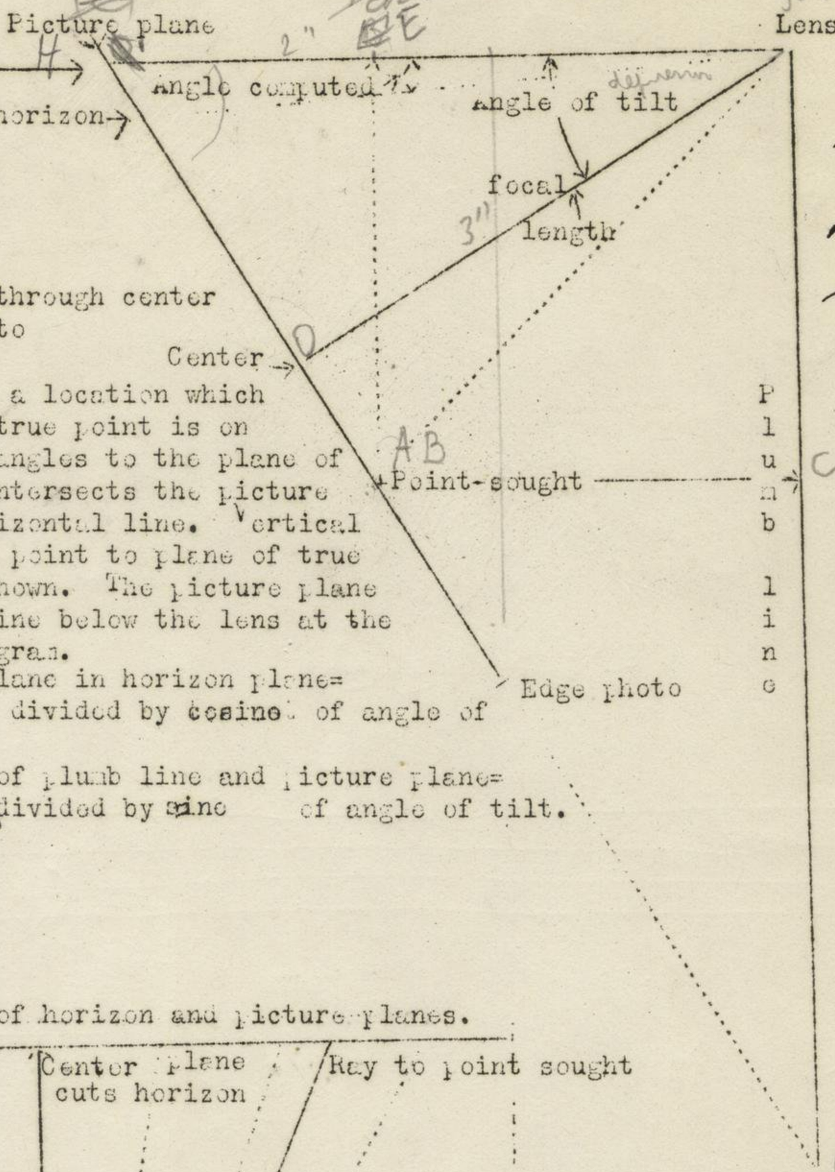


FIG. 2 16

Vertical section through center of an oblique photo

"Point sought" is a location which is desired. The true point is on a plane at right angles to the plane of the section and intersects the picture plane along a horizontal line. Vertical projection of the point to plane of true horizon is also shown. The picture plane meets the plumb line below the lens at the bottom of the diagram.

Lens to picture plane in horizon plane= focal length divided by cosine of angle of tilt.

Lens to junction of plumb line and picture plane= focal length divided by sine of angle of tilt.

FIG. 3 17

Intersection of horizon and picture planes.

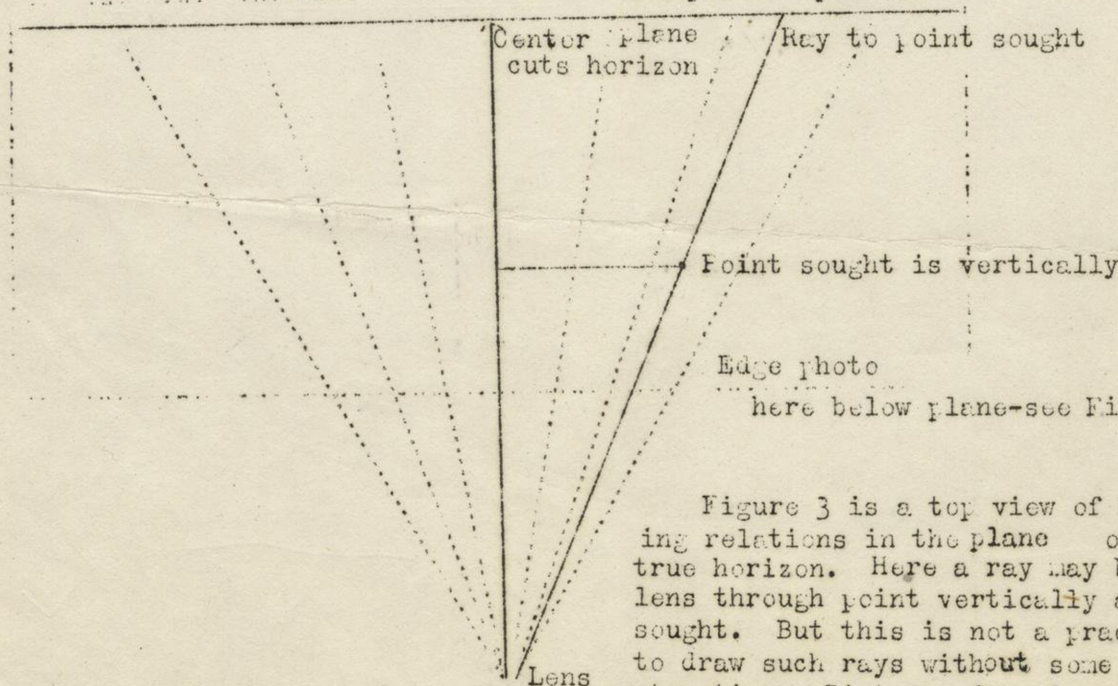


Figure 3 is a top view of Figure 2 showing relations in the plane of lens and true horizon. Here a ray may be drawn from lens through point vertically above the point sought. But this is not a practicable way to draw such rays without some other construction. Distance from lens to intersection of true horizon and picture plane = focal length divided by cosine of angle of tilt. Note that true angle of depression to point sought may be found from this diagram combined with Fig. 2 as explained later.

depression

The following principles must first be understood. (1) As a reference point is required for angles those measured on the map will be taken from the intersection of a vertical plane through the lens and center of the photo with the plane of lens and horizon, (2) All horizontal angles to be shown on a map must be measured either in the horizon plane or a horizontal plane parallel to it, (3) The vertical angle from lens to point sought must be measured in a vertical plane passing through both of these points. It is evident that we cannot choose a point in the plane of the photo and use it to draw map rays to points shown if such points are at different angles below the horizon. The following diagram shows some of the principles.

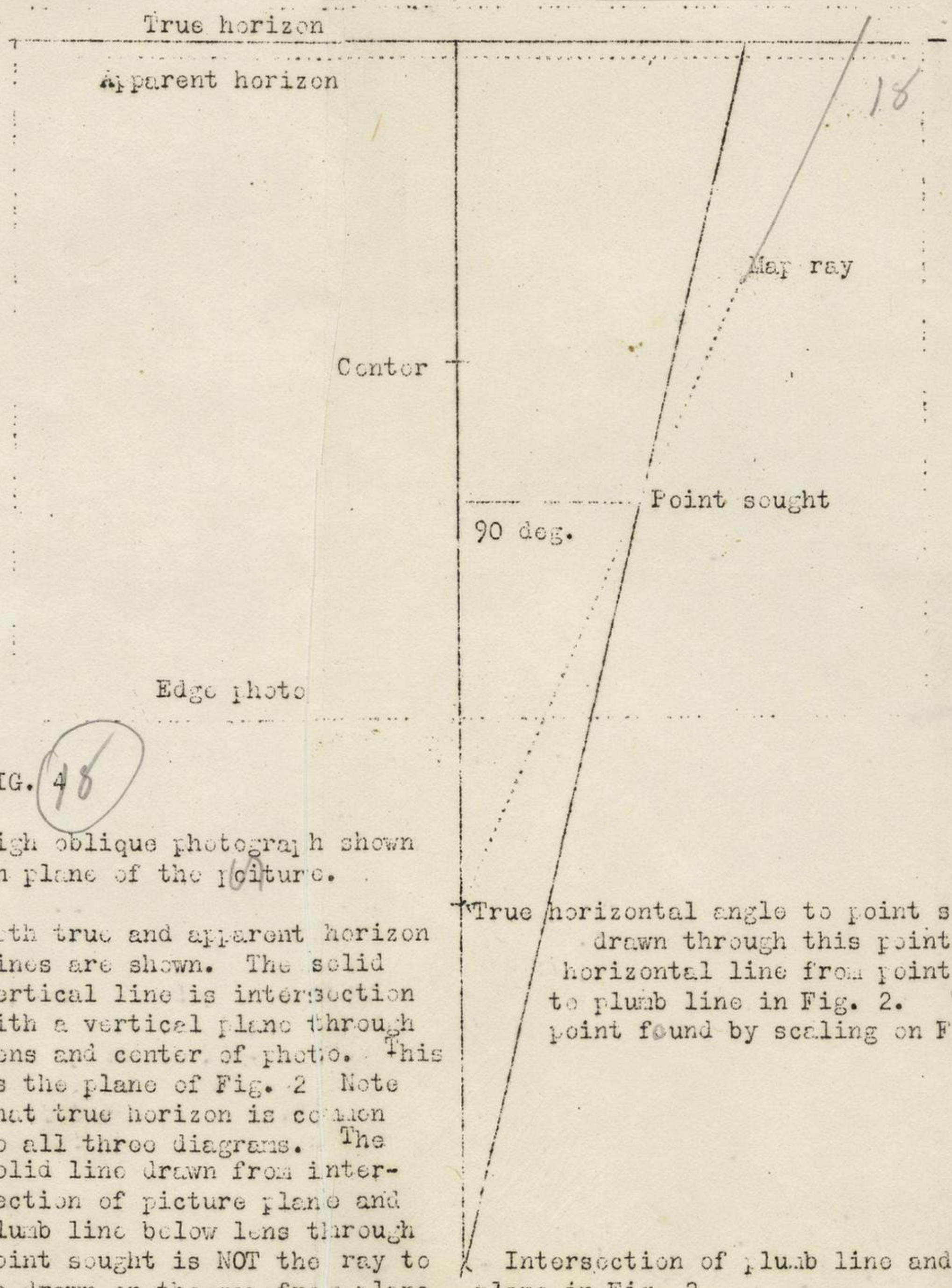


FIG. 4 (18)

High oblique photograph shown in plane of the picture.

Both true and apparent horizon lines are shown. The solid vertical line is intersection with a vertical plane through lens and center of photo. This is the plane of Fig. 2 Note that true horizon is common to all three diagrams. The solid line drawn from intersection of picture plane and plumb line below lens through point sought is NOT the ray to be drawn on the map from plane location to point sought.

Although the solid ray above is not correct for a map ray the true angle away from center line of photo can be found by transfering the distance between the two rays on the true horizon line to Fig. 3

However, this solution is not always possible. The lower the angle of tilt below the horizon plane the longer becomes the vertical line through center of photo to intersection with plumb line below lens. This distance would be infinity with a picture taken with horizontal axis. A better solution is to figure out the distance from point sought in the photo plane to place from which true horizontal angle may be drawn to get a map ray. First the point sought may be connected with center line by a line parllel to the horizon. Then Fig. 2 can be used to scale the correct radius for this particular point as done in Fig. 4. Crone's graphical solution is much easier and is explained below.

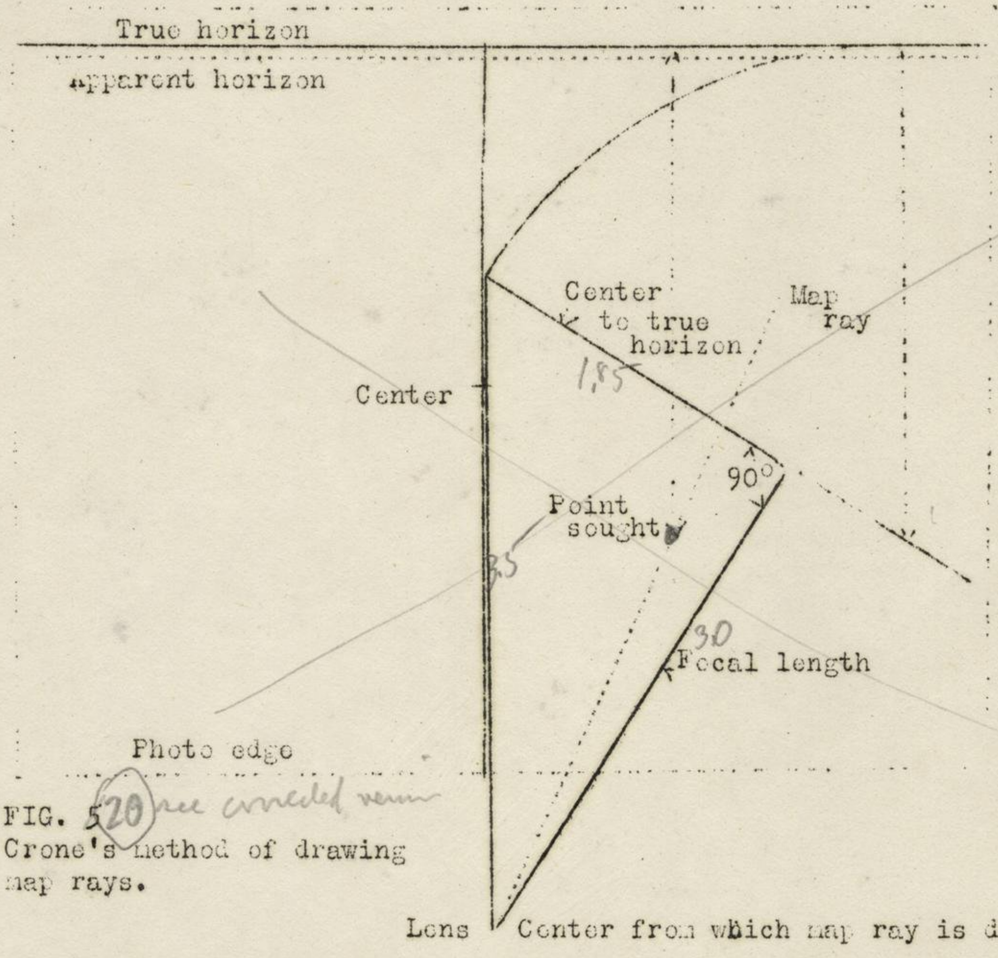


FIG. 5 20 see corrected version
Crone's method of drawing map rays.

void

Crone's graphical solution. Crone's method invloves drawing a right triangle; one side (here at right) is equal to focal length 30; the side at right angles (here upper right) equals distance Of photo center 1,85 from true horizon; third side equals distance of horizon line from lens 3,5 as in Fig. 2. Note this is the same triangle as above camera axis (focal length) in Fig. 2. This triangle should be laid out on tracing celluloid. The top side may be extended when necessary. Lay the triangle so that the long side is along the line showing intersection with picture plane of a vertical plane through center, that is the line on which Fig. 2 is drawn. 2,6
Next scale distance in picture plane of point sought from true horizon. = ~~2,5~~
Strike a n arc whose center lies on the upper side of triangle(here extended) and passes both through center line of photo and just touches true horizon. The apex of the triangle is now in proper place to draw a map ray through point sought.

Other solutions. Other solutions of this problem have been used which include both graphical and mecanical methods. Several plotting instruments have been devised which somewhat resemble telescopic alidades.

With the instruments the picture is set up in proper relation to map, that is tilted just as taken. (See Fig.2) Then sights are taken on it with a telescope or other similar device. The machine either measures angles in degrees or plats rays on the drawing board.

Locating position of plane. The map position of the plane at instant the photograph was taken may be found as soon as three rays to objects whose map locations are already known can be drawn. These can be platted on tracing paper and the sheet of paper shifted until each ray passes through the known location. Point from which rays were drawn can then be pricked through onto the map and is the point vertically below the plane. Once this location has been found rays to unknown points can be drawn. When other pictures give rays to the same points locations are secured by intersection just as in Problem 15 or with a planetable.

Vertical angles. In order to find elevation of the plane (any record furnished by the photographer was based on aneroid readings) we must measure the vertical angles to points of known elevation whose location is also known on the map. The angle must be found in the vertical plane passing through lens and point sought. No quick and easy solution seems to have been published. It is necessary to draw both Figs. 2 and 3. Then the distance from lens to a point vertically above the point sought and the vertical distance of point sought below horizon plane can be found. Knowing these two sides of a right triangle the angle sought can be found graphically as in Fig. 6

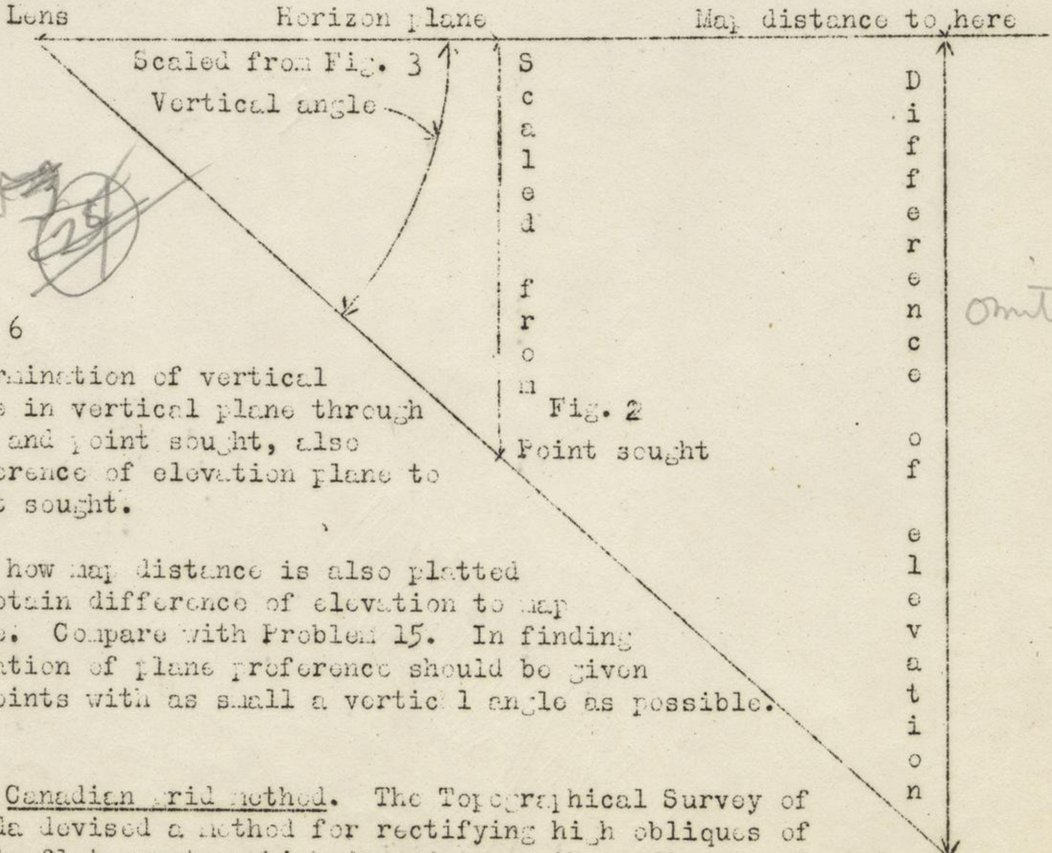


FIG. 6

Determination of vertical angle in vertical plane through lens and point sought, also difference of elevation plane to point sought.

Note how map distance is also platted to obtain difference of elevation to map scale. Compare with Problem 15. In finding elevation of plane preference should be given to points with as small a vertic l angle as possible.

Omit

Canadian grid method. The Topographical Survey of Canada devised a method for rectifying high obliques of nearly flat country which depends upon the ordinary principles of perspective. In Fig. 7 the elevation of the plane is shown to map scale but the picture plane is shown at actual focal length distance in front of lens position. Picture plane is extended to meet the ground. Along line of intersection with ground picture and map scales are the same. Axis of camera is also prolonged to meet the ground. Equal intervals on ground are laid out to map scale both beyond and back from place where

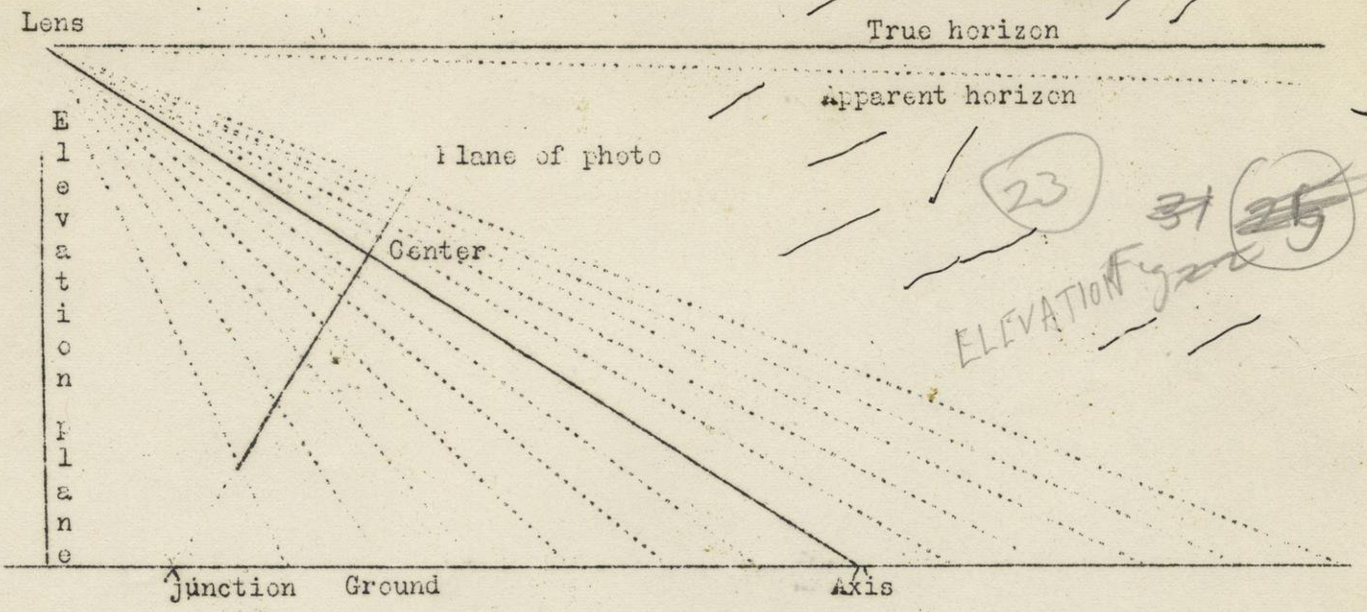
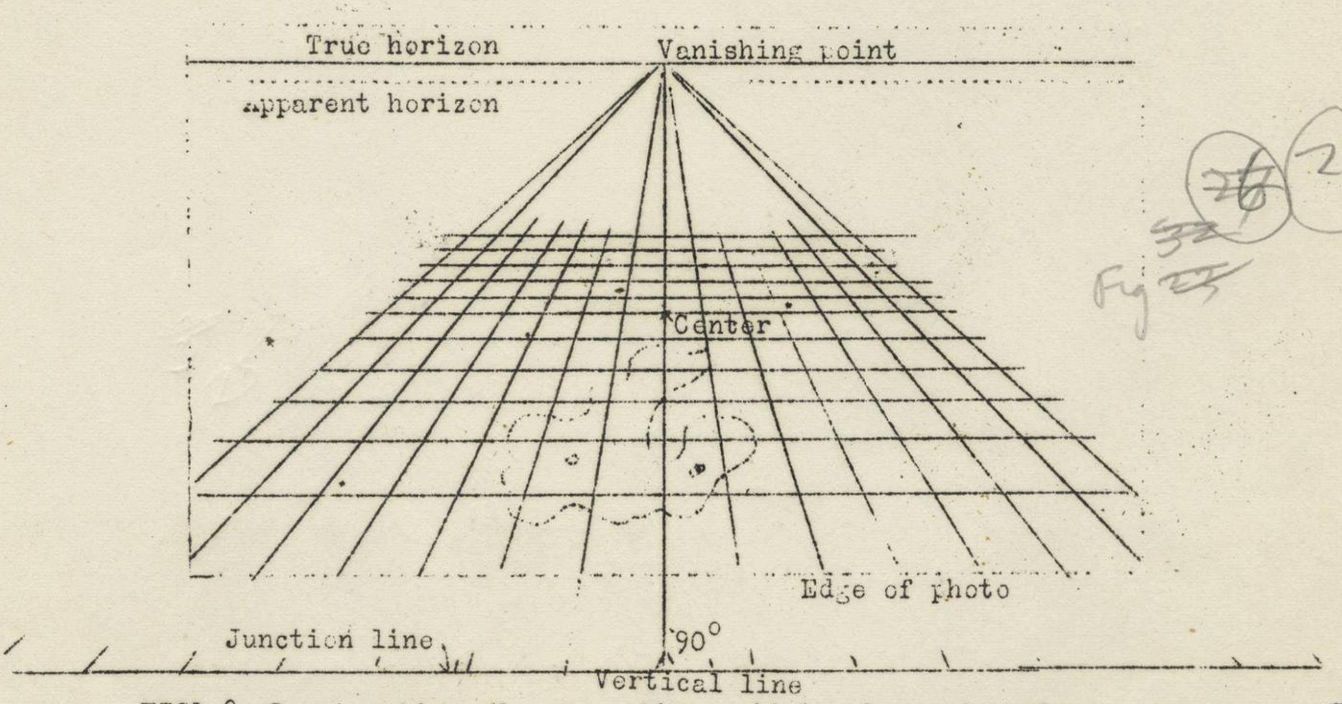


FIG. 7 Vertical section through axis of camera



FIG? 8 Construction of perspective grid in plane of photo.

the axis of the camera intersects the level of the ground (fig. 7). Next connect each of these points to the lens position as shown by dotted lines. In Fig. 8 the plane of the photograph is shown. The line marked "junction" in fig. 7 is drawn and intervals equal to those shown on the ground in Fig. 7 are laid off on both sides of the vertical line through the center of the photo. The intervals determined above are than laid off on this line. Lines parallel to the true horizon are drawn through these points. Then lines from the points on the junction line are extended to onverge at a <u>vanishing point</u> at the intersection of the vertical line and the true horizon. Each figure formed by crossing of these lines corresponds to a square on the ground. To lay out this figure we must have both focal length of camera and alitude at time picture was taken. It is best, however, to check the grid and see if it shows known points on the ground in correct relation. Such known or control points are shown by dots above. Draw the squares one inch to a side and then sketch true outline of lake.

U. S. Army method. The U. S. Army manuals describe a method of drawing maps from obliques which involves finding four or more points in the picture whose map locations are known. The diagrams below show how this is made use of.. Care should be taken not to draw too many triangles so that they will be confusing. With only four known points the principles of perspective are used and two vanishing points are found. The work has to be done on a sheet of tracing paper considerably larger than the photograph.

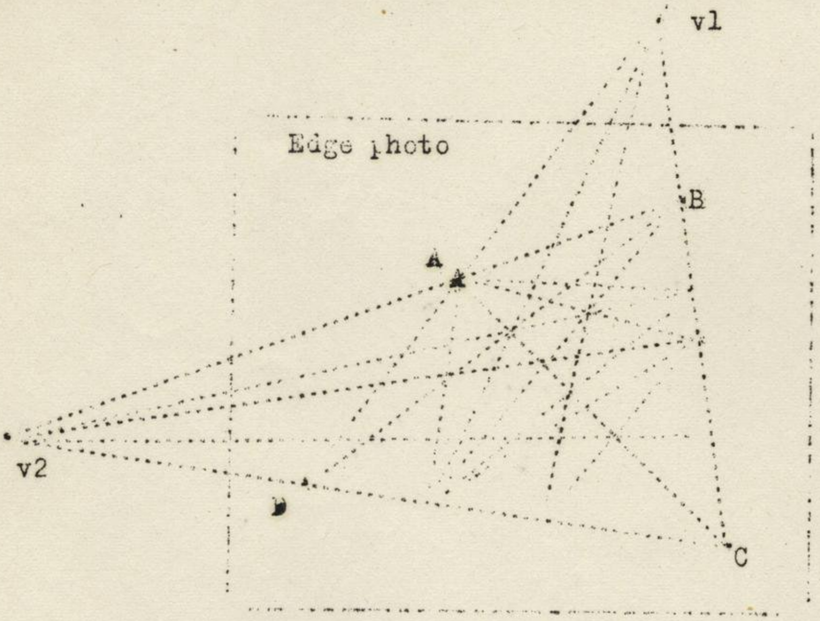
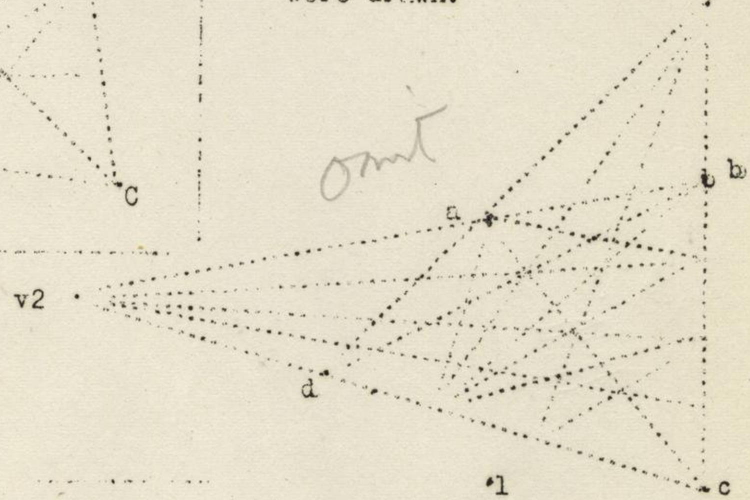
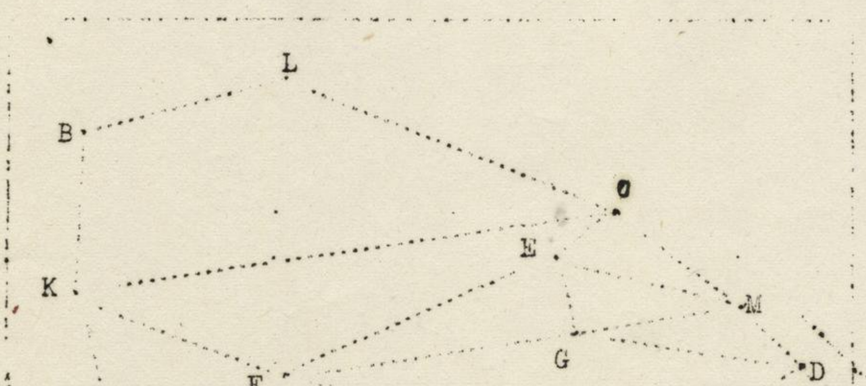


FIG. 8 Army method of drawing map from oblique photo with four known points. Note two vanishing points and how triangles were drawn. The points are shown in correct relation below and the same triangles were drawn.



Omit

Reduce Fig ~~33~~ 28



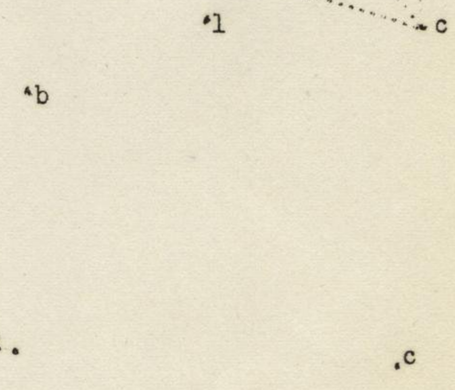


FIG. 9 Army method where more than four known points are present. Draw more triangles on the photo and then draw all the triangles on map shown at right. What kind of country can this method be used for? What advantage over other methods has it?

References. See list at end of Problem 16. Also Trorey, L. G., Survey by high obliques. The Canadian plotter and Crone's graphical solution: Geographical Journal, vol. 100, pp. 57-64, 1942

GEOLOGY 11

MAPPING
Problem 16 and 16a, Edition 1950

Object: To draw a map from aerial photographs.

Material: Hard pencil, eraser, scale, stereoscope with parallel arm (stereocomparagraph), magnifier, triangle, tracing paper (about $8\frac{1}{2}$ x 11"), photos, map with elevation data.

Method: Photographs taken from the air may be divided into (a) obliques with camera axis tilted downward and (b) verticals taken with camera axis as nearly vertical as possible. Obliques are taken up in Problem 17. Read TM 5-230, pp. 188-249.

Vertical Pictures: Every vertical picture is a complete and accurate map of the area shown, provided that (a) the camera axis is exactly vertical and (b) the land is essentially a plane. The scale of this map is fixed by the elevation at which the picture was taken in relation to the focal length of the camera. In actual practice, however, it is very difficult to meet these conditions precisely. Obviously if the land is not flat, the tops of hills are shown at a larger scale than are the lowlands. Check the scale of your photographs by measuring several known distances. If you do not get the same result in different parts of the area at the same elevation, the camera was tilted. A line on the land surface which is straight but runs up and down hill is not straight in the picture. The amount of curve due to differences of elevation increases radially away from the center of the photograph as shown in the following figures:

FIG. 1

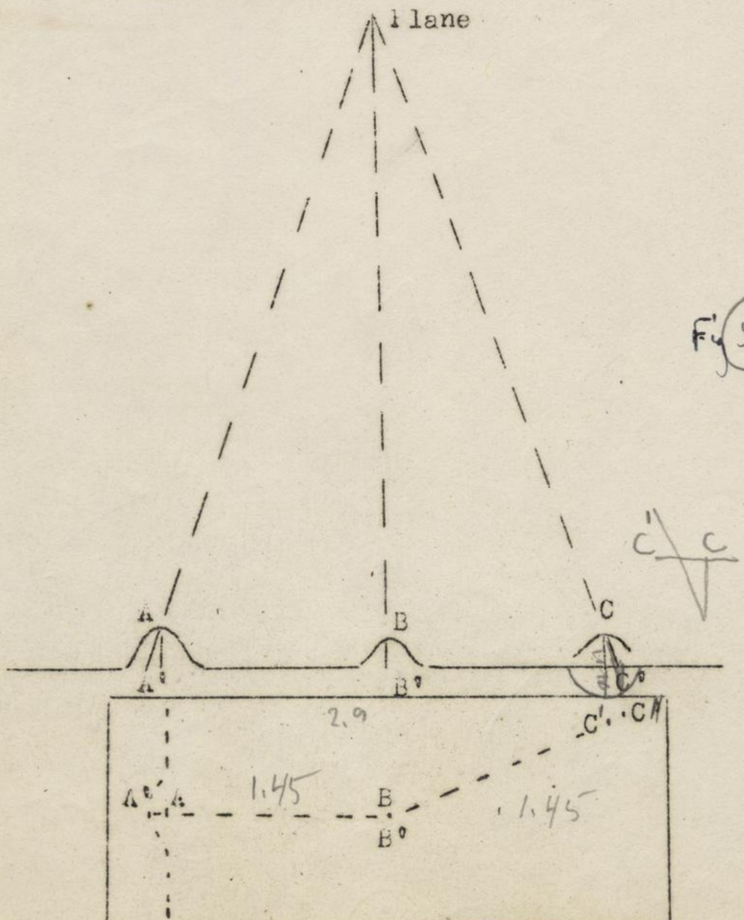
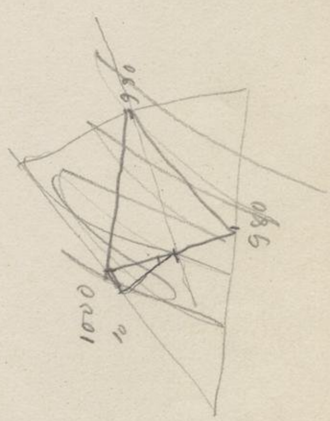


Fig 5



Note distortion of straight line across hill at A and that C' is displaced from C along a radial line from center of photograph.

Three hills rise from a plain. The one vertically below the camera has its top B shown on the picture in the same place as if it had been at plain level B'. But the hills near the sides of the picture have their tops A and C. displaced outward from the center of the view to A' and C'. The distortion of straight lines across each hill is also shown.

Stereoscopic effect. Photographs are taken in the air at horizontal distances such that each overlaps the adjacent ones by 60 per cent or more. Were the land all flat you could not tell any difference in one picture from another of the same spot, but where there is relief the relative positions of high and low points differ in the overlapping pictures.

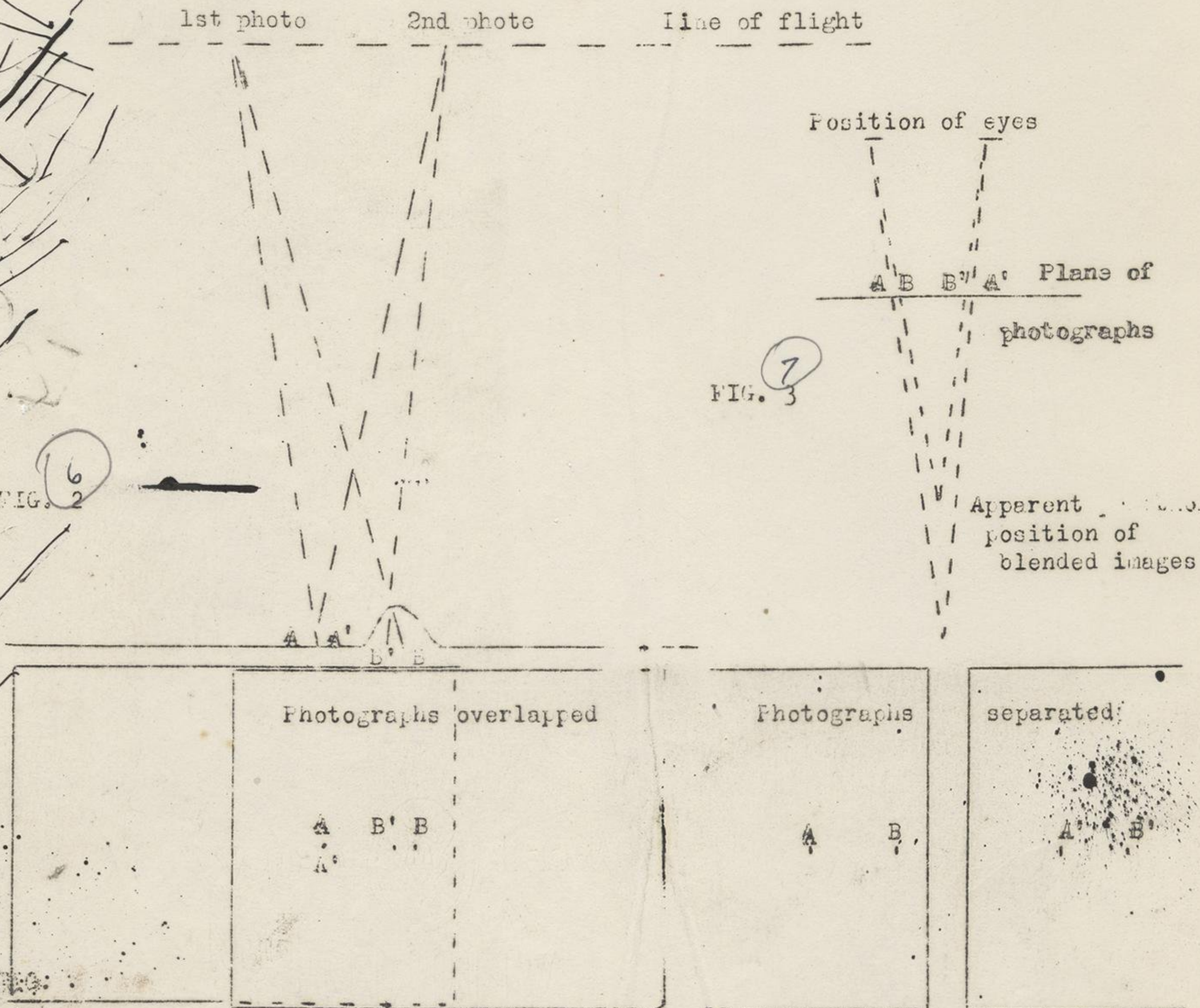


FIG. 2 6

FIG. 3 7

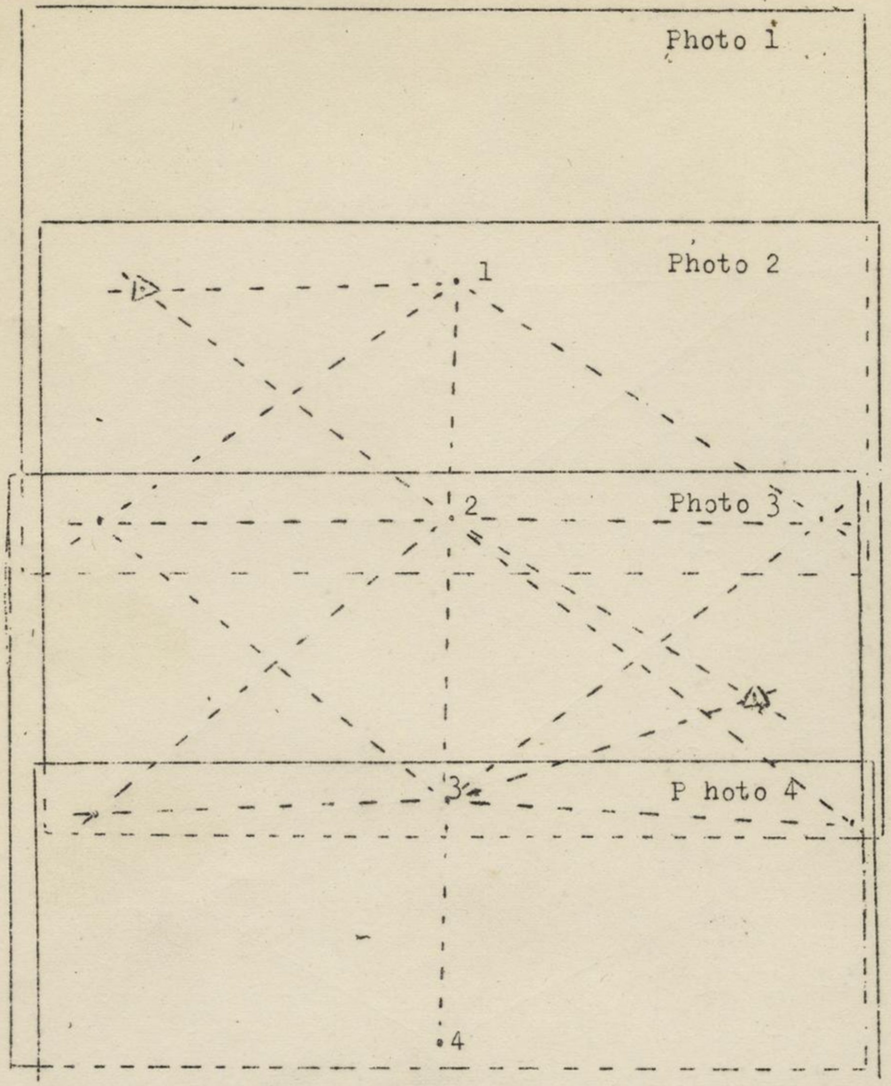
Now place a pair of adjacent photos so that the common areas are next to one another. Shift one or both until you see some of the highlights double. Make these coincide and the fused image will look like a relief model. Keep looking as if you were in the plane at a great distance from ground until you can get the effect. The mirrors or lenses serve only to separate the eyes so that each sees only one picture. A similar result may be obtained with two magnifiers or by holding a paper between the eyes. In these cases the common points in the pictures must be eye distance (about 2½") apart. It is possible to dispense with the paper. Put the common points of two pictures about 2½ inches apart. Slowly raise the pictures into line of sight with eyes relaxed (daydreaming). This does not injure the eyes but considerable practice is needed. The closer the eyes come to the picture the better the stereoscopic effect. Note that two other images, one on either side of that in relief, are seen. These do no harm to the eyes. Shift photos until you get relief effect.

Map location of points. NOTE: Actual construction done only for problem 16a but all must know the principle.

The rays drawn from the center of the photograph (which was vertically under the plane and is found by intersecting lines either between marks in the margins or the corners) to objects seen in the picture represent true angles just as though you had set up a plane table there and sighted those points through an alidade. The map location of this center or principal point may be found by passing the rays to three recognizable points in the picture which had previously been located on the ground through the plotted map position of those points. This is merely solving the "three point problem". This orients the photograph in respect to the map control. Now rays may be drawn on tracing paper from the picture center to unknown points, mainly hill tops. Mark on each ray apparent position of point using a small definite dot for each but extend lines. Next comes the problem of correctly locating the next overlapping photograph. Find its center as before. Locate this point on the first photograph. If the centers are both at the same elevation then the distance between them is the correct map distance. But if the elevations differ a correction must be applied. Draw the line between the two centers on your map. Use this line to orient the tracing over the second picture. Shift along this line until rays from center of second picture to known points already correctly located pass through the map locations. One such intersection would be enough to give map location of the second center but two properly located off to the sides are much better. Note similarity to plane table method of location of unknown point. You may also find correctly located points such as section corners in the second picture. See TM 5-230 pp. 188-212.

Show on your final map to proper scale the locations of section corners as given by U. S. Land Survey. These will be your control points. Draw radial lines from centers of each photograph to these on your tracings. When agreement on intersections has been reached prick through proper map locations onto your final sheet. The chief error in such "plane tabling from the air" is tilt of the photographs. But compare it with error in doing the same work on the ground!

FIG. 4 Radial line adjustment



This shows method of tying together photos in the same flight. There are two control points shown, here marked with triangles. These have been shown on final map underneath the tracing paper.sheets with radial lines. Note the points which are common to three photos as these are very important. The same points also show in adjoining flights and serve to connect the two flights. Center of Photo 4 is also shown as in preparation to extend the assembley. Note that line of flight is not always straight. See how use of slotted templets would help a big job but take too much time to prepare for a small one. See following directions for how to mark centers of photos (principal points).

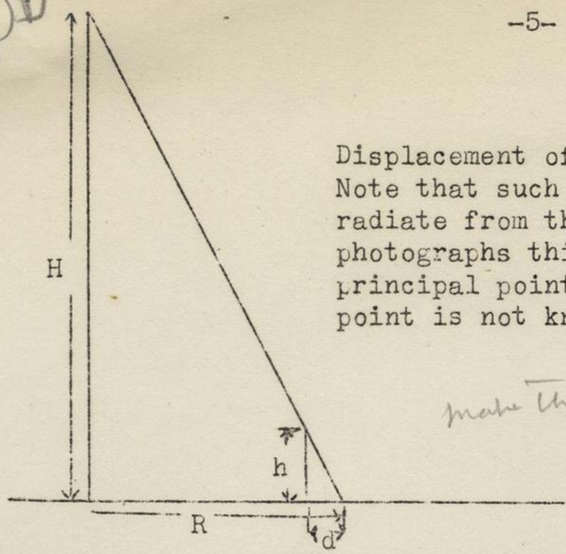
Be sure to understand that directions hold over distances and that all lines to a certain location must intersect at same point.

In problem 16a the photos will cover two adjacent flights. Flights are commonly made going either north or south with an overlap of about 30%. After arranging the tracing paper sheets (templets) you will soon realize the advantage of the slotted templet system. This was at first done with cardboard sheets cut with a special punch along the radial lines. These sheets conceal the basemap and are thrown away when it is finished. Hence metal arms which may be clamped into propr relation to one another have displaced cardboard. No templet need be made for problem 16 since it would add little to accuracy of results but a width of only two miles should be mapped to avoid distortion due to relief and tilt which affects the sides of the photos most.

Distortion due to relief. Displacement of points due to difference of elevation above or below the normal or datum plane is along lines which radiate outward from the point vertically below the camera. It is commonly assumed that this plumb or nadir point coincides with the center or principal point. The actual displacement depends upon the proportion: altitude of plane, H: difference of elevation, h:: radial photo distance of point, R: displacement from true position, d. In solving this note that each pair of terms must be in same units, for instance if H = 14000 feet, and h = 200 feet, then R and d must be expressed either both in inches or both in millimeters. In this case if R = 100 mm. then d = 1.43 mm. and if h is above the datum plane the true location of the point must be moved along the radial line toward the principal point by this amount. See figure 5 on next page.

$d'' : h' :: R'' : H'$ $d'' = \frac{h'R''}{H'}$ where R is distance on photo from nadir point in inches and H in feet

FIG. ~~3~~ 1 D



Displacement of a point in photograph due to relief. Note that such dsiplacements are along lines which radiate from the plumb or nadir point. For most photographs this is not far enough from the center or principal point to make a material error if the nadir point is not known.

make this to show altitude determination

Setting up photographs. Even if you find your photos already set up on a board you should understand just how this was done. Look over the books of instructions. The "principal points" are all marked with red circles. Note the marks in margins or corners which were used to locate these. If you are using the Abrams instrument you should mark the line between the principal points with a sharp hard pencil. Turn the y dial (motion across line of flight) to read 0. Place instrument over a sheet of white paper and turn up the right hand marker about 4 complete turns. Then adjust left hand marker on its y axis until the two dots blend as one when you look at them through the lenses. If you wear glasses you may have to leave them off while working. Choose place on the board where the parallel arm will allow the instrument to cover the common part of the pair of photos. Lay down the left photo and turn until the line joining the centers (principal points) is on the line of the two dot markers. Staple down left corners of this photo. (Directions say use tape but this may cause more injury than do staples.) Now set the right photo over the left so that when left marker dot is on principal point of left photo the right dot is on principal point of the right photo and the two lines joining the principal points coincide. Another way to set up photos is to use front edge of the instrument as a straight edge; this is more rapid but less accurate. Next look through the lenses and see that you get good stereoscopic effect without eye strain. You may have to shift the right photo keeping it in same alignment. When satisfactory, staple down temporarily and flap over the edges to see that there is no "blind spot". Another shift may remove such, but you must not adjust without leaving enough space on the x motion to allow of moving the right dot by turning to left to allow of placing it on lowest points on the photos. Distance between common points must not exceed 2.45". Now practice reading from the dots. Whenever you look at the dots they will blend into one (unless too far apart). Practice looking at the ground at the same time; then when the dots blend into one it will appear to touch the surface. At first it is best to blink from one eye to the other to see that each dot is set at the same object in the two photographs. First try this on road intersections and other definite points. When you have the dots coinciding make the dial read 0 by turning its edge. Study map of area to find a road intersection near one edge of the area whose elevation is known. 0 dial on it. Now shift to other side of the area and see if you still have the dots coincide on an object at nearly or exactly the same elevation. If they do not one picture is tilted with respect to the other. Effect of tilt is largely, although not wholly, eliminated by following Abrams method as described on pp. 16-17 of their manual. Stick a needle through check point on one side; then turn around this axis until you get same parallax reading at the other check point. Then restaple. A tolerance of about 0.05 mm. is allowed in this measurement, otherwise it must be repeated. Always check back on first point again.

If you are using the Fairchild instrument you will find it easier to set up the photographs. Lock the parallel arm. Then line up left photo with the common principal points under the scale at the back. Staple down at once. Then measure map base or distance between principal points in millimeters. The corresponding points on right photo are then lined up in same way along same straight line and its map base is measured. Note that with this type of instrument you do not overlap the photos. To get proper distance for stereo vision do the same as with the other type. Place white paper below the dots and then make them blend by use of the large adjusting knob underneath the center. Now remove paper and shift right photo along the line of orientation until you get good relief effect. Be sure the micrometer is about the center of its scale when you fasten the right photo. Now pick out points of known elevation on the two sides of the flight and make the same adjustment as with the Abrams. Note that although the right dot can be moved for y parallax there is no means of measuring this motion. Distance between common points is about 6.20" with this machine.

Measuring elevations. After the photographs have been finially set and you have good stereo vision begin to practice measuring elevations. As suggested above you can best do this at first by blinking from one eye to the other and thus locating both dots on corresponding mark in each photo.

Parallax equation

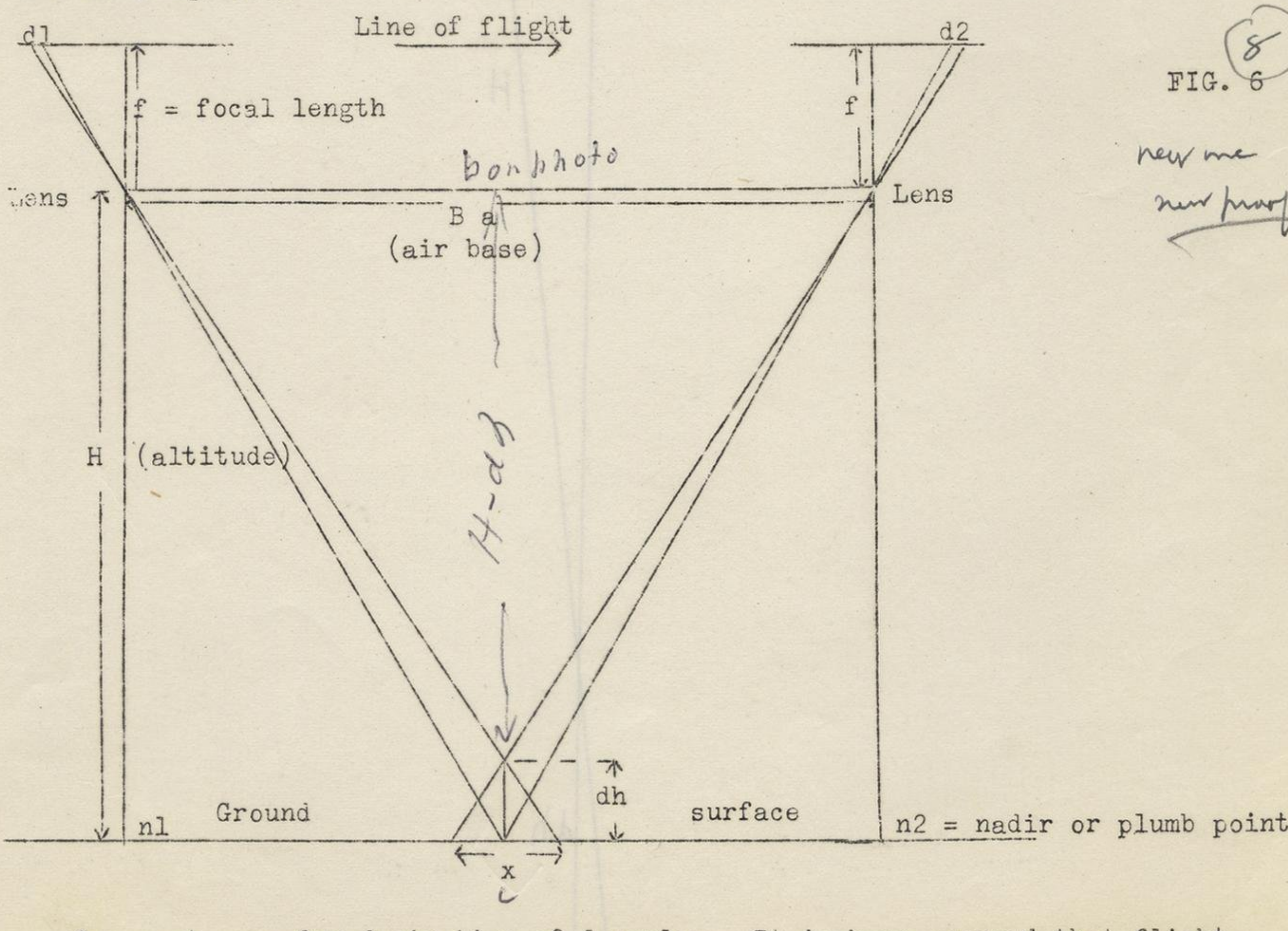
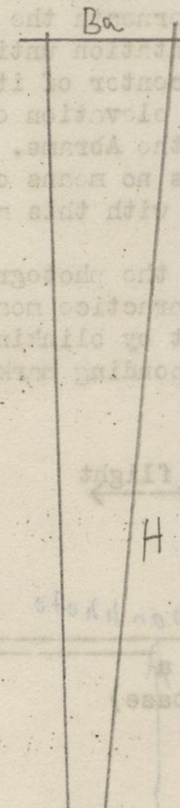


FIG. 6 (8)

new one
new proof

See next page for derivation of formulas. It is here assumed that flight was exactly level, that axis of camera was exactly vertical in both photos, and that dh is small compared to H. On the two photographs Ba is represented by b, the map or photo base which is commonly assumed to be the distance between one principal point and the next. This distance is shown on both members of the stereo pair of photographs.


Ba
H
dh

difference in parallax = $d_1 + d_2 = dp$

scale ratio = f/H = focal length/altitude

dp : x :: f : H, by scale ratio, whence x = dp.H/f (strictly speaking (H-dh) but dh is so small it may commonly be neglected)

also
dh : x :: H : Ba, by similar triangles, whence x = dh.Ba/H

equating to eliminate x,

dp.H/f = dh.B/H, hence dh = dp.H^2/f.Ba

p used in two ways
change p to m

but b on photo = Ba . scale ratio or=Ba.f/H

hence by substitution dh = dp.H/~~t~~ b

next substitute for H its value g.f/~~p~~ m where g = a ground distance, and ~~p~~ m its length in the photo and f = focal length

Measure ~~p~~ m on photos, also b usually in ",

hence dh = dp.g.f/b.~~p~~ m

$dh' = \frac{dp^{mm}\, g' \cdot f''}{b'' \cdot m''} = \frac{1 \cdot 5280 \cdot 12 \cdot 8.25}{b'' \cdot m^{mm} \times 25.4}$

Assume dp = 1 mm., g = 5280 ft., and f = 8.25 in.
In order to balance different units of measurement either p or b must be in mm.
To change mm to inches multiply by 25.4

$\frac{1715}{b'' m''}$

8.25 x 25.4 x 5280 x 12 x 25.4

Hence dh' = dp^{mm} f" g' / b" .p^{mm} or dh' - 1715/b".~~p~~ m"

$\frac{1 \times 5280 \cdot 12 \times 25.4 \cdot 8.25 \times 25.4}{b'' \cdot 25.4 \cdot m'' 25.4 \times 12 \times 25.4}$

(This derivation is required of all students.)

<u>Finding the altitude</u>. Even if you had the altitude at which the photographs were supposed to have been taken it may not be correct, for it was based on barometric determination. Most of these photographs were taken with the K-3B camera which had a focal length of $8\frac{1}{4}$ inches. The scale of the photographs is expressed thus:

$$\text{Scale} = \frac{\text{focal length}}{\text{altitude}} = \frac{\text{photo distance}}{\text{ground distance}} = \frac{f}{H} = \frac{\cancel{p}\ m}{g}$$

$\frac{1 \cdot 5280 \cdot 8.25}{b'' \times m''\ 25.4}$

Note that these ratios demand that both terms be in the same units.
Solve for altitude and:

$$H\ (\text{feet}) = \frac{\text{ground distance (ft)} \cdot f(\text{in})}{\text{photo distance (in)}} = \frac{g.f}{\cancel{p}\ m}$$

$= \frac{5280 \times 8.25}{b \cdot m \cdot 25.4} = \frac{1715}{b\ m}$

Note that we can use two different units because there is only one term of each above and below the line so that the correction factor cancels.
We could equally well use millimeters for both focal length and photo distance.
Example: f= $8\frac{1}{4}$ in.; g = 5280 ft., and p = 3.1 in.
Substituting these values above altitude, H = 14000 ft.
Study your photos in comparison with a map on which known horizontal distances are shown and compute flying altitude for each. The scale of the two photos may not be exactly the same.

1 inch = 25.40 mm. 1 mm. = .0394 in.

$1 = \frac{5280 \cdot 12 \times 25.4 \cdot 8.25 \cdot 25.4}{25.4 \cdot b\ 25.4 \cdot m \times 25.4}$

Computation of differences of elevation. The formula demonstrated above relates difference in parallax on the photographs to difference of elevation, altitude, and map base. The latter is the average distance between principal points of a pair of photos. It may be expressed either in inches or millimeters. We can also solve the formula for difference in elevation for unit difference in parallax.

$$\text{difference of parallax} = \frac{\text{diff. in elev. . map base}}{\text{altitude}} \qquad dp = \frac{dh.b}{H} \quad (1)$$

Solving for dh,

$$dh = \frac{dp.H}{b} \quad (2) \qquad \text{Substitute for H,} \quad H = \frac{g.f}{p_m} \quad \text{then } dh = \frac{dp.g.f}{b.p_m} \quad (3)$$

$$\text{If } dp = 1^{mm},\ {}^{M}g = 5280',\ f = 8.25'' \text{ then } \quad dh = \frac{1715}{b''.p''_m} \quad (4)$$

(This form gives either feet per inch of parallax or feet per millimeter. Taking H = 14000 feet, and b = 2.5 inches the result of (3) is 5600 feet per inch or 220 feet per millimeter. Note that again different units may be used provided that the conversion factors cancel out. Check these computations for your particular pair of photos. You can measure b in millimeters if desired. Next you must realize that altitude, H, is that above the particular spot on the photo which you are measuring. For instance if there were a 500 foot hill in the photo described above H would be reduced to 13500 feet changing the results to 5400 feet/in and 213 ft/mm respectively. Although there are tables which may be used to compensate for this in TM 5-230 a simpler solution is to figure the value of dh for the average H over the photo. This may be used to determine differences of elevation such as heighth of hills above known low places. Using (4) if b = 2.5" and p = 3.1" then dh = 222 ft/mm. or 5650 ft/in. Compute dh for your photos. You can also get results by measuring a known difference of elevation on the photo.

Note that measurements uphill register on the Abrams machine by motion of the pointer in the + direction. On the Fairchild instrument the readings on the scale increase with elevation, that is increase toward the left. Compare this with what causes hilltops to appear closer under the stereoscope (FIG. 3). When you measure you may have to use the y motion to make dots coincide. This is a result of tilt (see later). Tilt results in change of scale of the photos so that corresponding points are displaced in relative distance from the line of flight or line of orientation. In the Abrams instrument use the motion of right dot only. Left dot is moved only when first setting up the photos. Study the scale on the Fairchild machine. Whole millimeters are marked on lower side of line except for those which are numbered where line is carried across. Half millimeters are marked on upper side. Since the micrometer head reads only to 50 and measures hundredths of millimeters you must add 50. to its reading whenever a half point is passed. Continue practice on reading spot elevations. Base your parallax difference on elevations on roads, etc., on low ground not too far away on the photo. If you try to measure over long horizontal distances the effect of even slight tilt will make results inaccurate. On the home-made instrument parallax is measured in thousandths of inches; divisions on side are 25/1000", on head of drum 1/1000".

Drawing the map. Each of the machines is provided with some kind of pencil. See that yours is sharp and not too hard. If you can obtain only faint lines go over them with a softer pencil. Colored pencils (provided when possible) are good - for this. Use blue for streams, red for roads and houses, and green for timber.

Use a sheet of bond paper which shows a watermark when held to light. If drawing board is rough put a sheet of poorer paper below. Fasten down in position so you can map the area common to the two photographs remembering that you have to reverse the photos with all but the Fairchild. In tracing roads, streams and boundaries of dense timber use left hand dot only. Locate dwellings which at most farms are close the road and more or less concealed by shade trees.

Do not map all farm buildings. You will have trouble in following some streams because of trees, also in telling what portions of some streams are now part of the actual channel. Dot all uncertain parts. Show streams which do not carry water all the time by dash-dot lines. Last locate spot elevations. Compare with the old topo. sheets! All students will make a map based on common area of one pair of photos.

Contouring. The choice of two methods of drawing contours is possible. All books of directions advise setting the distance apart of the dots to that for a given level and then tracing around the contour. This involves finding the difference, dp, of parallax for a given contour interval. Tilt of the photos makes it necessary to shift the setting for different parts of the map in a very complex manner. Keeping the fused dots together is also difficult at first. After practice they should appear to just touch the surface while looking at the topography and not concentrating on the dots. You can always make the dots blend if you look too hard at them alone. The other method is to measure spot elevations and then sketch the contours from them checking with the stereoscope. Combination of the two methods is suggested. Try to trace the 100 foot contours first. Blink from one eye to another to see that dots rest on same ground point. Change reading in different points of map in response to spot elevation data. Then fill in 50 foot contours between by sketching. Compare your results with the old map which was sketched from the ground! Last locate section corners using the old map as a guide. From these locate the lines about ¼ mile apart which subdivide the sections into 40 acre tracts. Draw all those on your paper making section lines solid black and the other lines dotted black. Also draw a line exactly a mile long according to the scale you found. Your map may be presented either (a) in pencil on original paper or (b) traced in ink on tracing paper. If the latter also hand in your sketch. First maps cover only the area seen on both photographs of the pair. Contour interval , 50 feet. Roads may be left as single lines but make branch roads dotted. Show also all dwellings (not barns), streams and swamps as far as possible.

FOR 4 CREDIT STUDENTS
Problem 16a

Material: Aside from that for problem 16, you need 10 sheets of tracing paper 7" x 9" plus a sheet of tracing paper about 18" x 18".

Introduction: 4 credit students will do Problem 16a, map of 10 vertical photos located in two flights, a portion at least of which must be contoured and all completed to show roads, dwellings, streams, woodland, swamps, section and forty lines. The tracing paper about 7 x 9 inches will be required for the radial line plot and larger sheet for final map. First, make the radial line tracings taking lines from principal point of each photo to all section corners, other principal points, and to such points common to the two flights as necessary. Draw fine accurate lines and mark the picture locations definitely although lines should extend half an inch beyond each. Mark photo number in same position as on original. Next lay these tracings over one another to get true locations by intersections. Make lines between principal points coincide. Assume that N-S township lines are straight. Apply same criteria to accuracy of location as with planetable. When you have a satisfactory adjustment, disregarding the apparent locations on photo, tape or staple your assembly together. Then lay it over the master sheet and prick through the revised locations. Remove overlay and mark each prick point with cross for section corner, circle for principal points and other locations. Place photo number at each principal point.

Draw in the section lines on master tracing. Next subdivide each section into 40's using solid lines for sections and dotted for subdivisions. You now have an accurate framework to which sketches and tracings of each photo may be fitted. Last use the stereocomparagraph to make a contour sketch of <u>not less</u> than common area of two stereopairs one in each flight. Adjust contours between the two flights as seems best. Do as much contouring as time permits; if you cannot contour more than minimum finish rest of area as a flat map. <u>Submit your sketches</u> and <u>overlay</u> with the final map. This may be completed either in pencil or ink. Use green crayon for woodland.

Effect of tilt on vertical aerial photographs.

<u>Introduction</u>. As noted above, few photographs taken from planes and intended to be true verticals are such in fact. Efforts to control verticality by levels, views of horizon, gyroscopes, etc., have failed although it may prove practicable to indicate the plumb point by a spot of light regulated by a gyroscope. Fortunately the angle of tilt is rarely over 3 degrees from vertical and can safely be neglected for radial line construction unless the topography is very rugged with high relief. A tilted photo has the same perspective as a very low oblique as explained in Problem 17. Note diagrams on following page. These show that the side which is tilted below the position of an equivalent vertical picture includes a smaller area than it should and is expanded; the converse is true on the other side, that toward which the principal point is displaced from the plumb or nadir point. Midway between the locations of these two points is a line where the equivalent vertical photo intersects the actual picture. Only along this line which is perpendicular to the principal line is scale true.

11

Fig. 7A See next page

Distortion of a square when tilted along a diagonal line.
True shape borken line; shape in photo shown as solid line

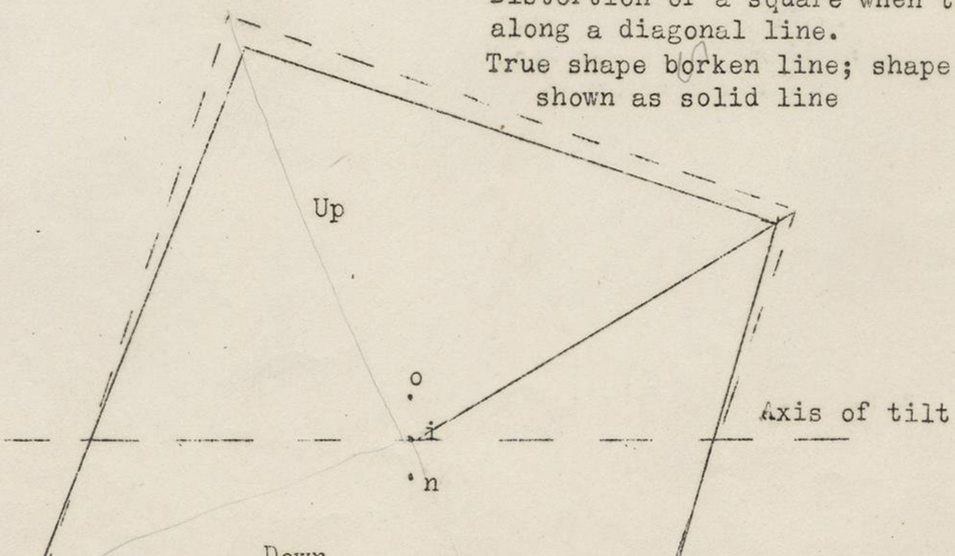
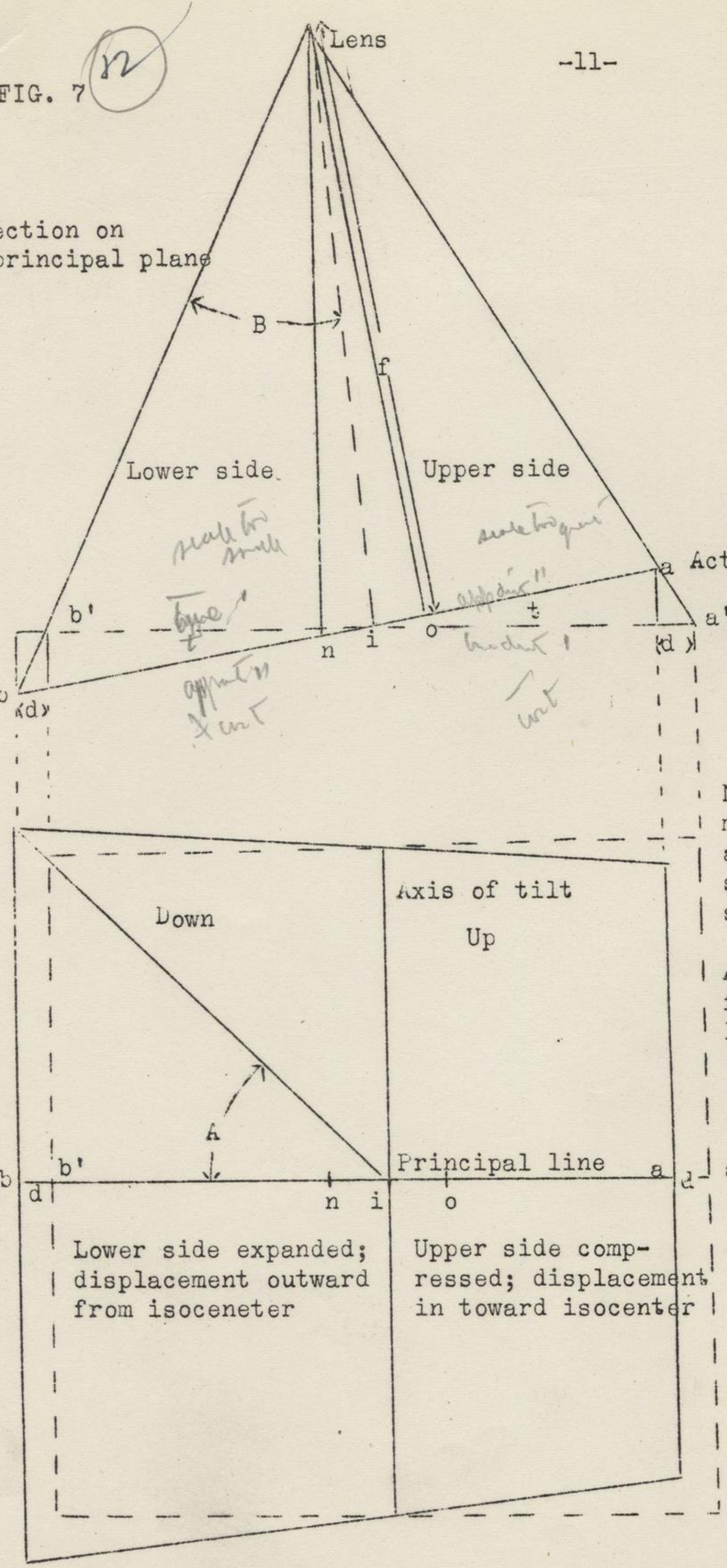


FIG. 7 (82)



Geometry of a tilted photo
O = principal point
n = nadir or plumb point
i = isocenter or center of distortion
d = displacement
t = angle of tilt
f = focal length
B = angle between line to a point and line to isocenter

Actual photograph

a' Equivalent vertical (same f) or line parallel ground.

Note that the tilted photograph makes a rectangle on the ground appear too big on lower or down side and the converse on upper side (here to right).

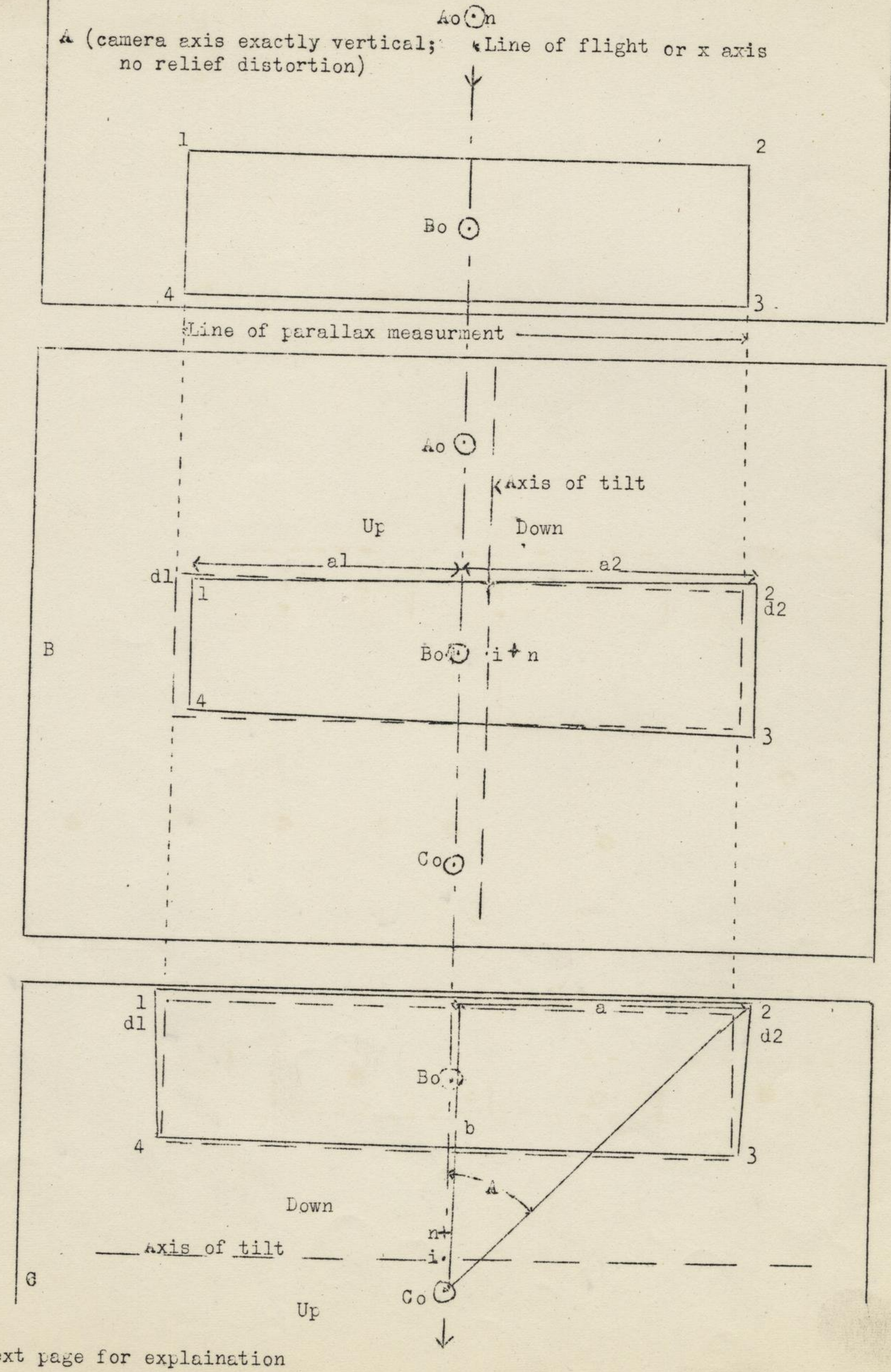
A = angle between a radial line from isocenter and the principal line.

Derivation: (required for 4 credit students)
Vertical displacement of point a' is ia (or ib) sin t
Horizontal displacement = vert. displacement tan B Since
tan B = ia (or ib) / f then
$d = ia^2 (\text{or } ib^2) \sin t / f$ and
$\sin t = d.f/(ia^2)$ whence distance no = f sin t = $d.f^2/(ia^2)$

See Fig. 7A on last page for effect of distortion when axis of tilt is diagonal to sides of a square.

Displacement along a line radial from isocenter = d/cos A
Displacement along line perpendicular to principal line = d tan A
Note that displacements are proportional to square of distance from axis of tilt but are along lines radial from isocenter. Displacements due to relief are radial from plumb (nadir) point. Therefore, there is no single place on a photograph from which both tilt and relief can be corrected. Fortunately, error due to this fact is small in most cases when the tilt does not exceed 3 degrees. In country with high relief, measured in thousands of feet, it is necessary to find the plumb or nadir point for radial line construction. Note carefully the distortion of a sq. due to tilt. If axis of tilt diagonal, square is "skewed".

FIG. 13



See next page for explaination

Figure 8 shows results of tilt of camera axis in successive photographs, A, B, C. Centers (principal points) are designated Ao, Bo, Co. It is assumed that A is exactly vertical with the plumb or nadir point coincident with Ao. The rectangle 1, 2, 3, 4 is correctly shown. Centers of adjacent photos are shown in every case. When set up for stereoscopic examination these are aligned along the line of flight or x axis which is horizontal in the stereoscope. Distances perpendicular to the line of flight are called along the y axis. Photo B is assumed to be tilted along an axis parallel to the line of flight. As explained in Fig. 7 the rectangle is then distorted into the shape shown by solid lines (its true shape is shown by broken lines). n = nadir point, i = isocenter or intersection of axis of tilt with a plane passing through both camera axis and plumb line. Note that points 1, 2, 3, 4 are displaced along lines which radiate from i (see Fig. 7). Distance of 1 in y direction from line of flight is a1, of 2 = a2 Displacement of 1 from a line through 1 in A is d1 and of 2 = d2 both measured in y direction. Such measurements are made with y dial on Abrams instrument; the Fairchild has no graduations on its y motion. Photo C is assumed to be tilted along a direction perpendicular to the x axis (along y axis). Distortion is shown by same method. Displacements from lines parallel line of flight, d1 and d2, are measured in same way. y distance of 2 from line of flight = a and x distance of 2 from Co = b. Angle A is determined by its tangent a/b. Note than tilt is demonstrated when you have to use to y motion to put dots on corresponding points in two photos. Cases of tilt along axes in other directions than those shown by be resolved into components along x and y axes. Displacements due to relief are assumed to be absent.

Correction of parallax errors due to tilt. When a pair of overlapping photos is set up for steroscopic examination the average value of dh must be found first. Known elevations should be present along both sides of the common area and nearly opposite one another. Elevations of 1, 2, 3, 4 should be known. Dots are set on 1 and 1 in photo B; the effect of difference of elevation of 1 and 2 is computed; the x dial is reset and checked on 2 in A and 2 in B. Photo B is then loosened and turned until this distance checks. (Abrams method). This adjustment will (unless the error was due only to faulty determination of centers) serve only to correct readings in a relatively narrow area between the two correction points, i. e. an area extending in the y direction. A different photo position would be needed for points 3 and 4. Hence it is well to use tape on right photo. Error due to the component of tilt on y axis is not affected by this method.

Measurment of relative tilt. Fig. 7 shows how it is possible to compute the displacement of the nadir point in both x and y directions with proper signs. In Fig. 8 displacement left (below) line of flight is - and above (or right) is +. Displacement of n above or left of principal point is - and below or right is +. Photo B shows a + displacement in y direction. Such a displacement is called Dy and is equal to f·sin t. Since sin t = d.f/a^2 it remains only to find a. This must be the average of the two a's on B because we do not know the position of the axis of tilt. This average value corresponds to ia or ib of Fig. 7 and hence d = (d1 + d2)/2. By substitution Dn = f^2(d1+ d2) / $2a^2$ Note that since both d1 and d2 are to right in B of Fig. 8 a positive value will attach to result. If the displacements had been the other way the result would be - indicating a nadir point below (left of) the x axis. In photo C the tilt is along a transverse axis; d1 and d2 are both measured along the y axis as before but one is - and the other is +. In this case we must use the average of d1 -d2 multiplied by the tangent of the angle A that is by a/b. Now to obtain this average value for d we must recall that the displacements are opposite in sign and that we must therefore use their algebraic difference. Thus $d = (d_1 - d_2)a/2b$. Substituting and cancelling a above and below the line it appears that $Dx = f^2(d_1 - d_2)/2ab$. In the case shown the algebraic sum of the y displacements is negative and the nadir point is displaced to the left or backward on the line of flight. In this manner the x and y displacement components may be found and thus the relative relation to the photo at the left (earlier in the series). But since we have no knowledge of the tilt of that photograph the solution remains relative until the missing data is supplied. Then relative tilts may be carried forward by algebraic differences. This method is described by Van Camp in "Manual of Photogrammetry," pp. 290-302. Possibly we could make a true drawing of an area based on a ground survey. corrected to relief distortion, and platted to photo scalle

The following method is adapted from one by BAGLEY, pp. 180-192, modified to use a rectangle (or square) formed by one or more sections of land. Because most aerial photographs were taken by flying along a N-S section line it would be best to use a pair of adjacent sections east and west of the line flight. It is essential to have the principal point well inside the area used. True dimensions and directions of the sides should be checked by a ground survey with planetable or otherwise. Elevations of each corner must also be known. Also focal length of camera in inches. Steps in construction follow.

(1) Locate both on map and photograph the principal point using radial line construction with former.

(2) Find the average scale of the photograph from a section line near the principal point and compute elevation, H, by usual method. $H = g.f/p$, where g is a ground distance in feet and p the same on photo in inches.

(3) Compute displacement d of all corners to each plane passing through elevation of the lowest corner. $d = h.R/H$, where h = difference in feet elevation of point considered above lowest corner, and R its distance from principal point in inches. Move the high points in toward the principal point as the isocenter is as yet unknown. See Fig. 5

(4) Scale, in inches, the four sides of the quadrilateral as shown on corrected photograph (notediagram); also distance of each corner from the principal point. Also obtain corresponding ground distance in feet from map and field data. See Fig. 9

(5) Revise the figure for altitude $H = (L^o)$ from the fact that $L^o:A^o:: f:ao'$ whence $L^{o}{}' = f''.A\ o' / ad''$ approximately. Use mean value from four solutions.

(6) The four triangles embracing lines from L (position of plane) to each corner of the figure can now be drawn to map scale (preferably larger than photo scale), Include corresponding lines in the photograph. Remember that plane of photo is normal to line (L^o) to principal point. It will now be noted that since f is the same in all triangles the lines representing the plane of the photograph will not be parallel to the ground. See Fig. 10

(7) Now we have the data to construct triangles for each of the four exterior sides of the pyramid making the ground line straight. It is not normal to ? line $L^{o}{}'$. See Fig. 11

(8) The next step is to pass vertical planes through each apex of these triangles. The intersection of these planes with the outside of the pyramid are normal to the ground line and each will pass through the line (Ln) from L to the nadir point. See Fig. 11

(9) Next use the map and draw on it lines normal to each side at proper distances from each corner as shown on base of each triangle. These lines should all intersect at the nadir point, n. See Fig. 12

(10) Transfer this nadir point to the photograph by tracing paper solution of the three point problem. Location of isocenter and axis of tilt are now known. Angle of tilt, t, may be computed since tan t = n'o'/f.

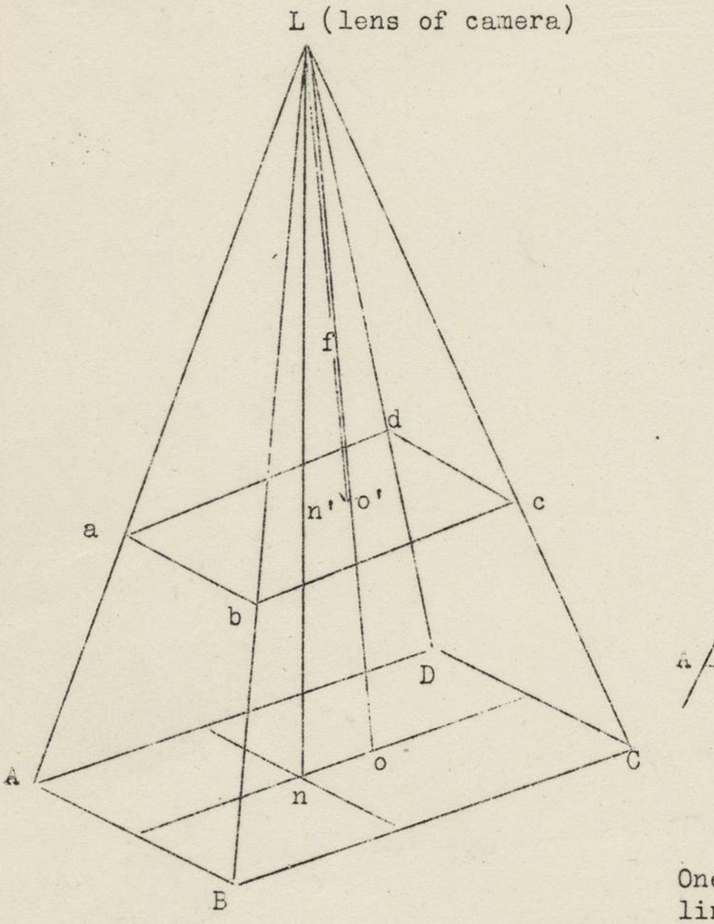


Fig. 9

Pyramid over two adjacent sections of land including the principal point. abcd = tilted photograph ABCD = map on scale larger than that of photo. Because the ground or map distances are horizontal the photodistances must be first corrected for differences of elevation. o = principal point on ground, o' = principal point of photo on line normal to plane of photograph. n = nadir or plumb point on ground and n' = nadir point on photograph.

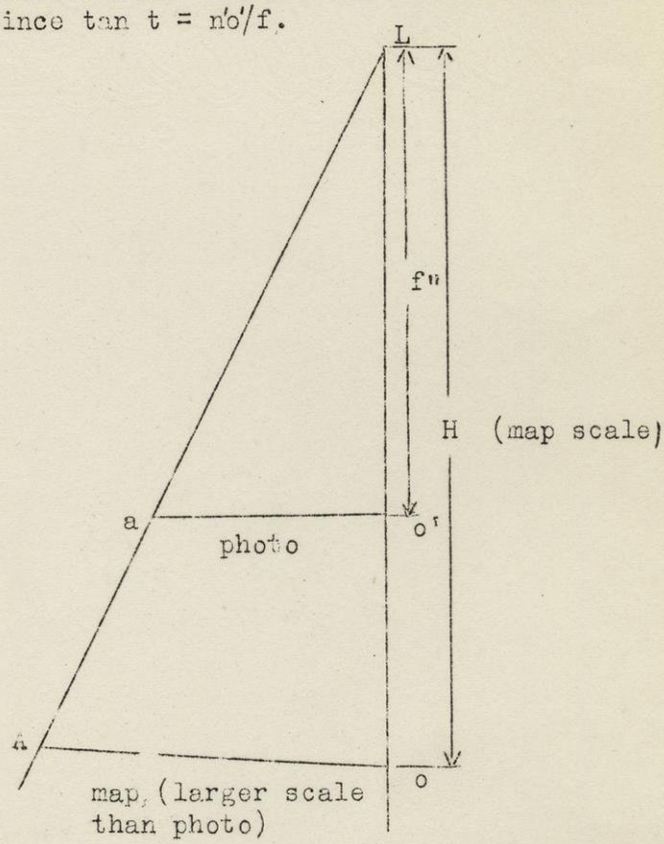


Fig. 10

One of a series of four triangles from line Lo out to each of the four corners of the area on the ground. f = focal length of camera H = altitude of plane at map scale. In order to fit it the true ground distances at map scale it is necessary to make them inclined which indicates tilt of the photograph Construction must include all of the four triangles.

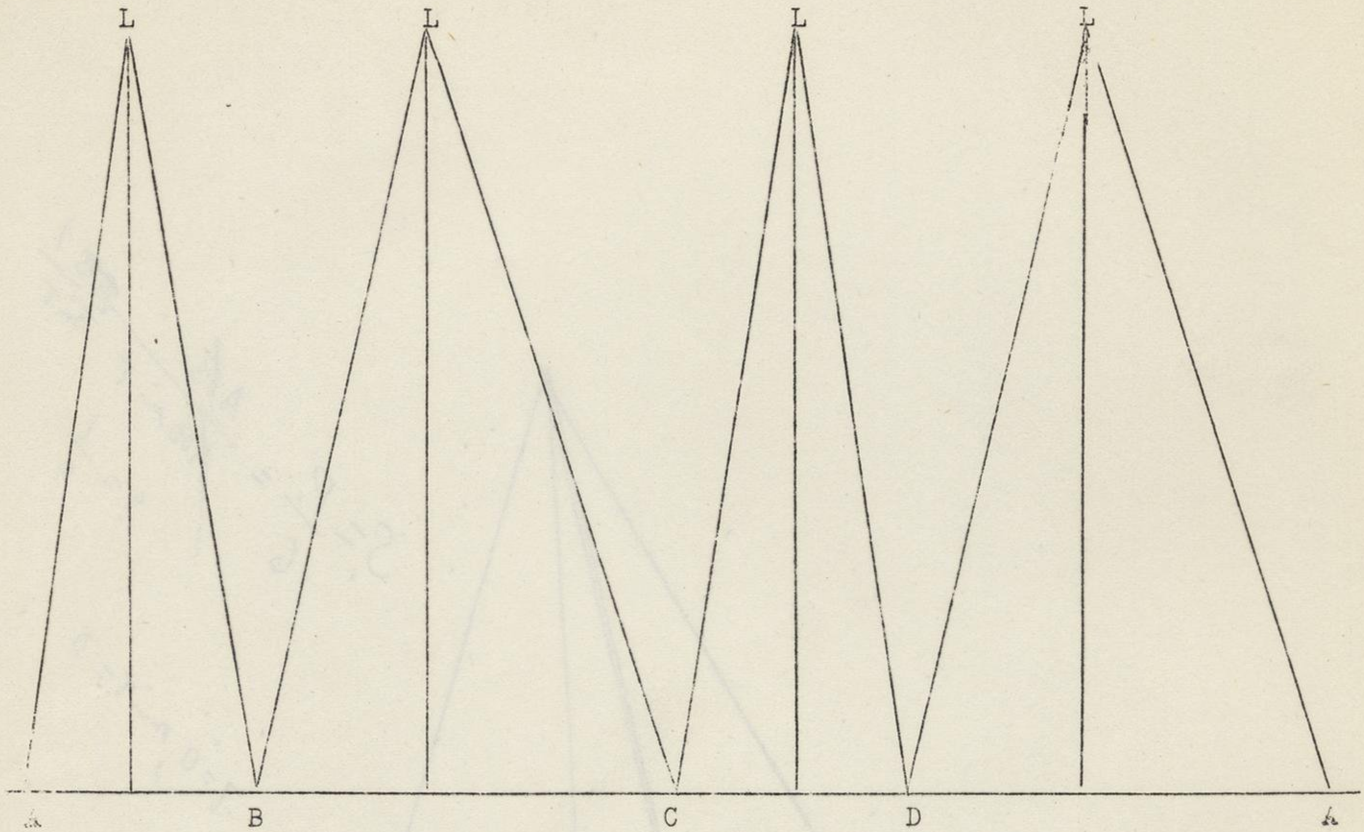


Fig. 11
Four sides of the pyramid spread out with base or ground level as a straight line. Sies of the triangles scaled from four construction figures like Fig. 10 The lines normal to the base line through the apex of each trinagle at L are the intersections of vertical planes with the sides of the pyramid of Fig. 9 Each of these planes must pass through the plumb line below the lens position and hence the nadir point on the map. These planes are not shown on Fig. 9 except where they intersect the ground or map plane.

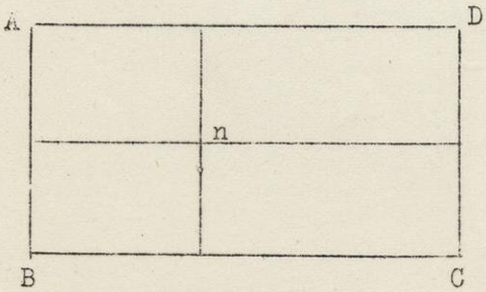
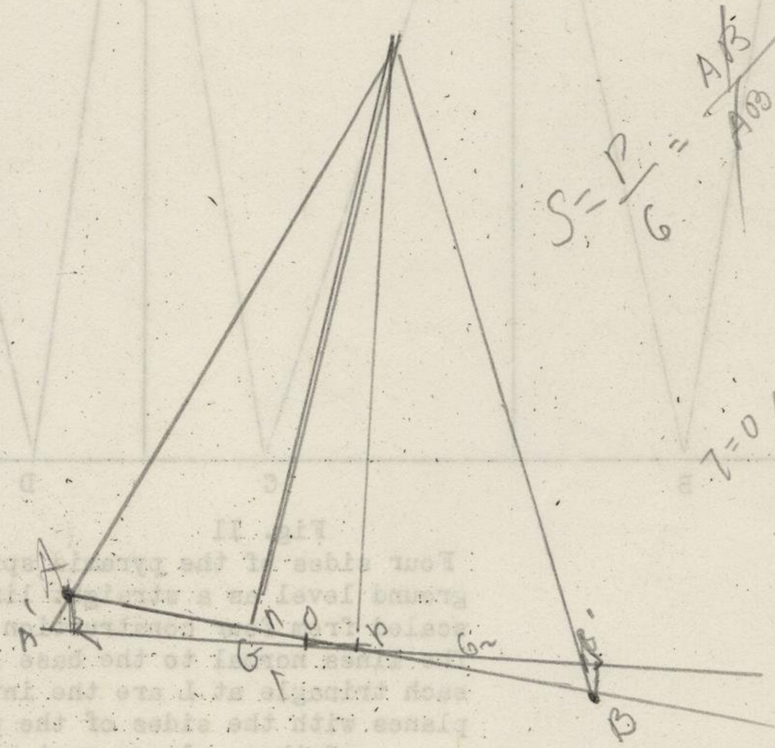


Fig. 12
Map to scale of the area shown in Fig. 9 showing how the nadir point n is located by the interestion of vertical planes through L This position may be then transferred to the photograph by the tracing paper solution of the three point peoblem, here actually four points.

This lengthy solution is justified only for photographs at start and end of a flight and then generally in rather rough country. Location of the nadir point aids in drawing radial lines. It does not aid in tilt correction where displacements are radial from the isocenter.



$S = \frac{P}{G} = \frac{AB}{AB \sin t} = \frac{\cancel{AB}\,1}{\sin t}$

$S = \frac{1}{0}$

$t = 0 \quad \sin t = 0$

$p = AB$

$G_1 = AO \sin t$

$G_2 = BO \sin t$

$G = AB \sin t$

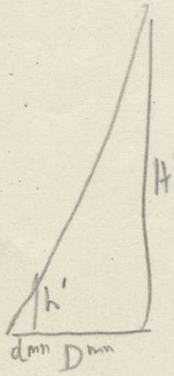
Rihn's method, "Manual of photogrammetry", 1944, pp. 274-289.

Introduction. Rihn's method is based upon three points, preferably surrounding the principal point, whose elevations and true distances apart in a horizontal plane have been determined by a ground survey. It is first necessary to find the location of each point in a plane at level of the lowest of them. Find the principal point. Measure distance to each control point. Compute displacement by formula d = (diff. elev.) x (radial photo distance)/(elevation of plane). A point 200 feet above datum and 100 mm. from principal point is displaced with photos taken at 14000 feet (200 x 100)/14000 = 1.43 mm. outward. Therefore the location to be used in correcting tilt must be moved by this amount toward the principal point since the isocenter is as yet unknown.

Method.

(1) Select the three points. (2) Set datum plane at lowest of them.
(3) Compute elevation of plane from scale = focal length / elevation.
(4) Compute displacements. (5) Locate position of each point in datum plane.
(6) Measure in inches or mm. length of each side of triangle = P.
(7) Find the actual length of each side in feet or meters = G.
(8) Calculate scale of each side from formula S = P/G. The different scales of each side are shown as Sh, Sm and Sl as ratios.

A, B, D = the three control points
y = plumb or nadir point
i = isocenter
O = principal point



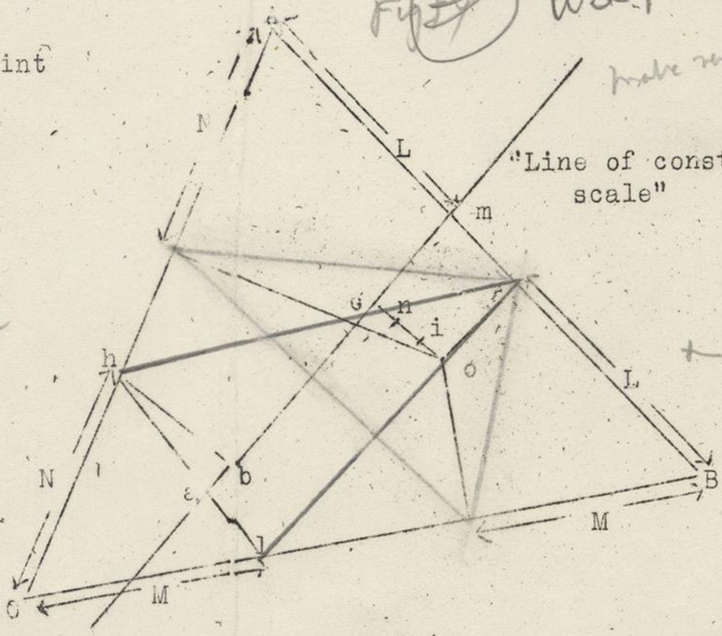


Fig 14
No use?
omit? 14

FIG. 13

(9) After finding O (principal point) drop perpendiculars from it to all sides of the triangle. Measure distances L, M, N. Now lay off the same distances from the opposite corners of each side of the triangle. These will give "scale points" h, m, l. If the perpendicular falls on an extension of the side of triangle then lay off an extension at other end. In every case the midpoint of the datum line is midway between the foot of the perpendicular and the "scale point".
(10) Find a point "a" on line hl from formula ha = hl(Sh-Sm)/(Sh-Sl) where S with subscripts refers to the scale at the three "scale points".
(11) Draw ma, the "line of constant scale", and drop a perpendicular oc to ma from the principal point and another from either h or l whichever is longer.
(12) Compute rate of change in scale, dS from formula dS = (Sh-Sm)/hb.
(13) Find the scale at o from S_o = Sm + (oc)(dS) when o is on same side of ma as h or S_o = Sm - (oc)(dS) when o is on same side of ma as l.
(14) Find the angle of tilt, t, from formula $\sin t = \frac{f.dS}{S_o}$ where f = focal length.

(15) The plumb or nadir point, n, is on the line oc on the side of o toward the higher scale on = f.tan t and oi = f tan(t/2) which places it almost half way between n and o.

(16) If the tilt proves large repeat all steps substituting n for o in Step 5. Then Si = So + (oi)(dS) in Step 14 and H = f/(Si) in Step 3. In Step 4 d = (h.R)(f±x sin t)/(H.f) + on nadir point side, - on principal point side, and x = perpendicular distance from image point to axis of tilt. If h is less than 150 feet omit this correction.

Example of a computation:

Point	Elevation	h, ft.	R, in.	d, in.
A	692	171	3.90	.05
B	809	288	2.91	.06
D	521	0	--	.00

h = elevation above datum
R = distance from principal pt. before correction
d = correction for relief

Line	G, ft.	P, in.	S = P/G	Scale Line
AB	9574	6.21	648.65	Sm
BD	8894	5.74	645.42	Sl
DA	10398	6.80	653.98	Sh

Sh-Sm 653.98 648.65 5.33

hl = 2.78" hence al = (2.78) (323)/(8.56) = 1.05"
hb = 1.42" (scaled)
dS = (653.98 - 648.65)/1.42 = 3.75 per inch
oa = 0.52" read
So = 648.65 - (0.52 x 3.75) = 646.70
sin t = (8.27 x 3.75)/646.32 = .04796 t = 2 45' f = 8.27
on = 8.27 x 0.04798 = 0.397 in.
oi = 0.20"

Sh = 653.98 645.42 8.56

Sm = 648.65 Sl 645.42 3.23

Summary. A common way to allow for errors in parallax measurement due to tilt is to draw a graph of error as described in T. M. 5-230, pp. 253-254 and Bagley, pp. 198-199. This method of changing the set of the points for error in parallax is applicable only when following contours. It has often been observed that the more known elevations the more complex the graph, which seems to indicate that distortion of the photograph in developing and printing is an important item. The only real answer to the tilt problem lies in the use of one of the plotting machines such as the "multiplex" projection method where the position of the photograph is shifted until it satisfies the ground control. For less expensive machines the best answer lies in the Abrams method plus limitation of the area used to the center of each photograph. This greatly reduces errors but obviously the more known elevations there are the better. Elevation differences should always be measured from known elevations as close to the point concerned as possible. No computations of absolute or relative tilt will be required in Problem 16a but you must understand the principles involved in order to answer questions in the final examination. See specifications for final map given at start of Problem 16a. Be sure you include scale and nadir point.

Supplement to Problem 17, edition, 1950

Proof of Crones method.

The method of getting true map rays from map location of photo where camera axis was inclined can be solved by more than one method as explained before. The fact that displacements on the horizon line or line common to both photograph and map is the one used by the U. S. A. F. The Abrams "Rectoblique Plotter" is a mechanical means of doing this. However, when the angle of depression is low in a high oblique photo the lines from horizon to nadir point are very long and another method is better, i.e. Crones. This method is briefly explained above but is proved and corrected below. In Fig. 13 it is clear that we must obtain on the photo the radius PN' from which to draw true map rays to all points which lie the same distance (in the photo) below the horizon. By projecting this radius to the horizon line O ~~HL~~ it is equal to P/L. Attention must then be directed to the triangle HPP' which is here shaded with vertical lines. The distance HP' is the excess over the length of the desired line which is P/L. Fig. ~~14~~ 19 is drawn in the plane of the photo. Here are two triangles. One is the same as triangle HCL of Fig. 3 here designated hcl and shaded with horizontal lines. This is in practice made of celluloid or other transparent material. An arc with radius equal to HP of Fig. 13 is drawn with center on prolongation of line hc; this arc is tangent to horizon line and to principal line HN' (drawing in Fig. 5 is incorrect). Next we must superimpose triangle HPP' here shown as hpp' and shaded as before so that it has the same relation as before being simply turned face up. It is then clear that the side hp' cuts off the same amount of the side hl as it does in Fig. 13, and that point p' is on the horizontal line through P. The distance below, PN' is then the same as PN' is Fig. 13 and locates the point on the plumb line from which true maps rays are drawn to all locations on this horizontal line.

O'PA

O'P

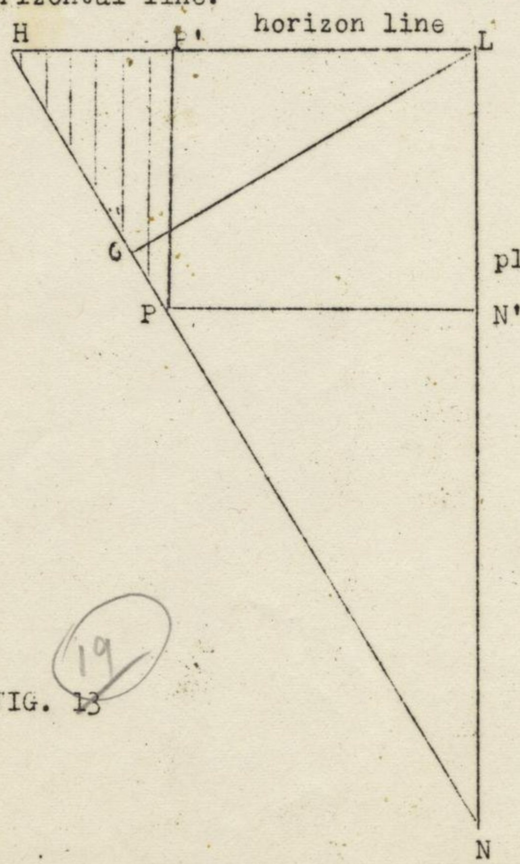


FIG. ~~13~~

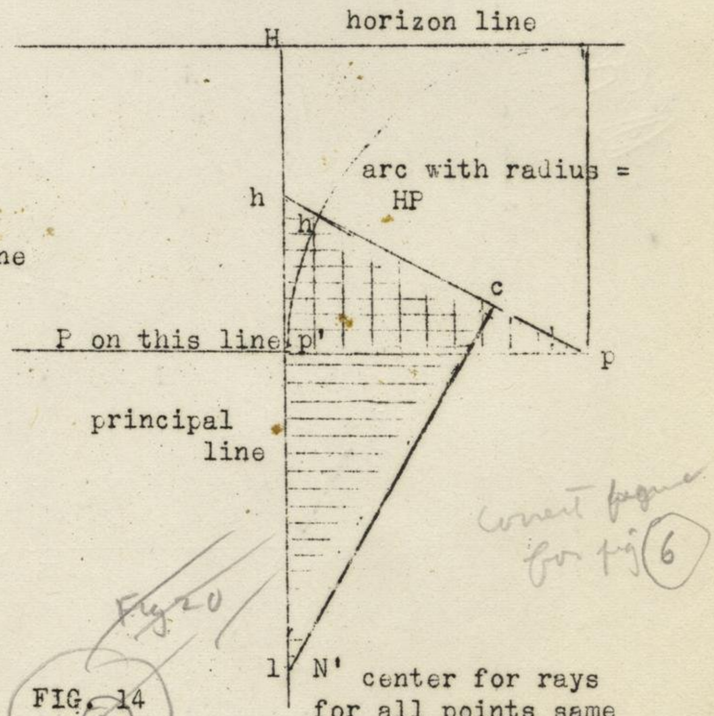


FIG. 14

Differences of elevation in oblique photos by U. S. A. F. method

The method outlined below was used with the obliques taken along with verticals in the "trimetrogon" method used in the southwest Pacific area. It works only when there are overlapping obliques all taken with essentially the same angle of depression. See Fig. 15 on following page.

FIG. 15

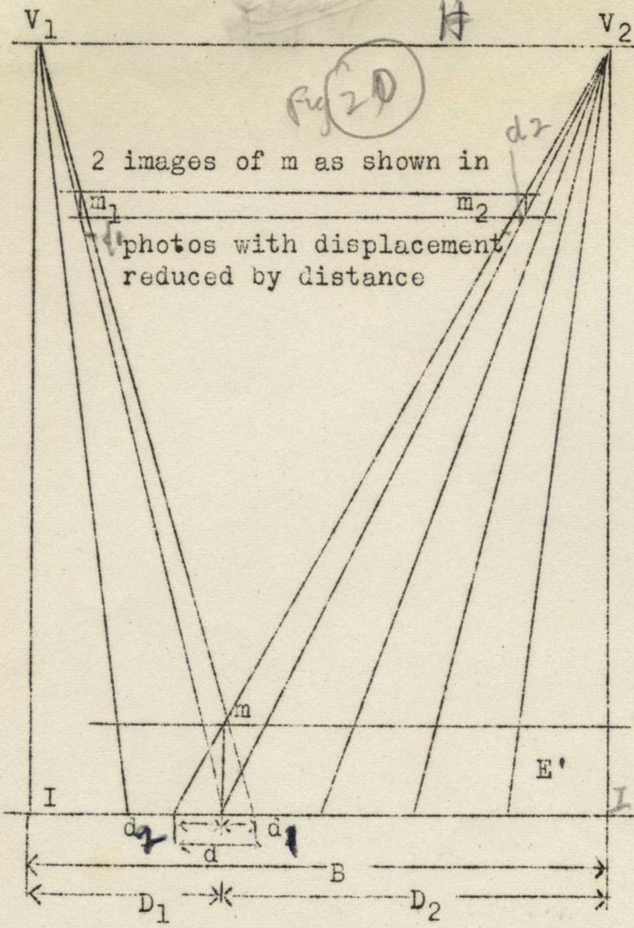
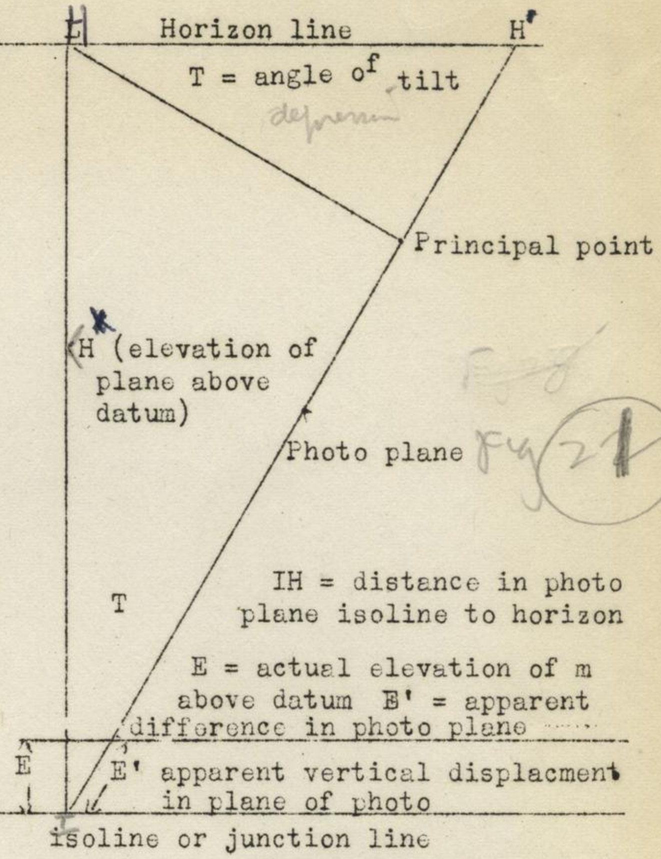


Diagram in photo plane showing converging line to vanishing point of each photograph

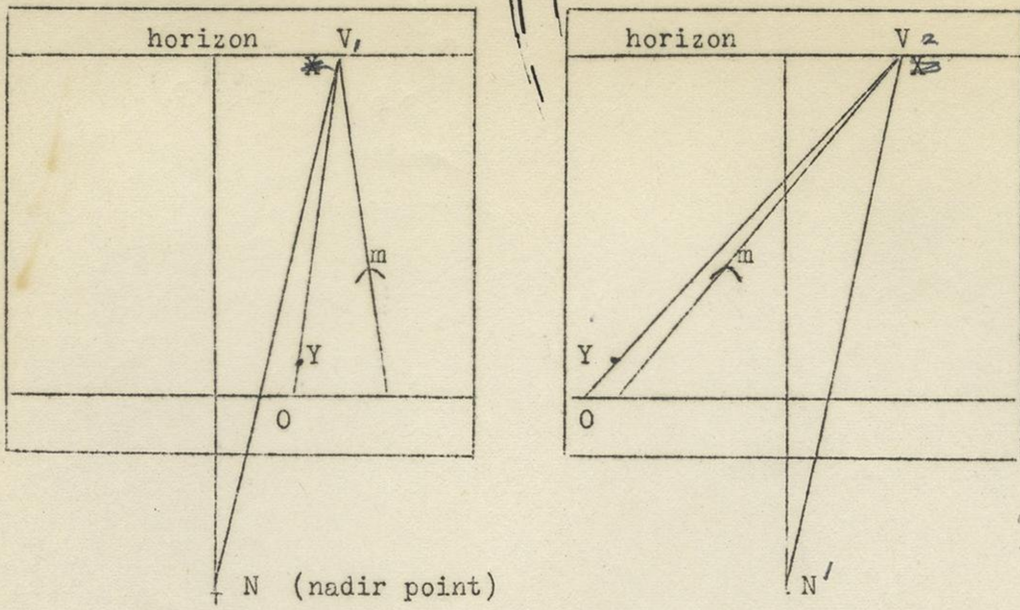
FIG. 16



Section on principal plane of one photograph showing relation of E' to E and IH to H (elevation of plane above datum).

Let m be a point whose elevation is E feet above datum plane. Base of perspective line in Fig. 15 is the isoline or ground line where photo is equal to map scale. Note that although perspective lines show only one position for m the positions on isoline indicate an apparent or parallax displacment of m here denoted as d. This is the sum of $d_1 + d_2$ total distance of the air base, B is the sum of $D_1 + D_2$ Note that B is equal to V_1V_2 Now $d_1 = E' (D_1 + d_1)/IH$ and $d_2 = E' (D_2 + d_2)/IH$ Hence $d = E' (B + d)/ IH$ Also $E = E' \cos T$ and $H = IH \cos T$ Hence $E' : IH :: d : (d + B)$ and $E/H = d/(d + B)$
The right hand expression of the first proportion is proved by similar triangles. Solving the second proportion where cos T has been cancelled out gives the result that $E = H \cdot d / (B + d)$

For practical application refer to Fig. 17. Here a datum plane was established by a point Y of known elevation which is near principal plane of one photo. A well marked point X near to the horizon and visible in both photos is then selected and a ray to it through Y is drawn. As extended to the isoline this ray intersects at O which is the base point for all measurments. Similar rays are drawn to the same vanishing point on the true horizon which pass through m and through the plumb or nadir point N. Both are extened to the isoline. Next the same rays are drawn to the same vanishing point on the adjacent photo. The principles of perspective demonstrate that these rays are essentially parallel on the map to those of the first photo. Using an inch scale measurments are taken from the intersection of the base line through X and Y to intersections of the other rays on the isoline. In this the reading of O is taken as equal to 10.00 inches. B, the air base is equal to the distance from the transferred



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base line at O in right photo and the line from nadir point, N to same vanishing point, V in right photo. d is the difference in readings of rays in each photo to the point sought, m. H is the difference of sea level altitude of plane and that of the base point Y. If m is lower than Y then the formula given above is changed to B-d. B and d are given in inches, H and E in feet. This method is very briefly described in a publication by Base Map Plant No. 1, G. H. Q, AFPAC, 1945.

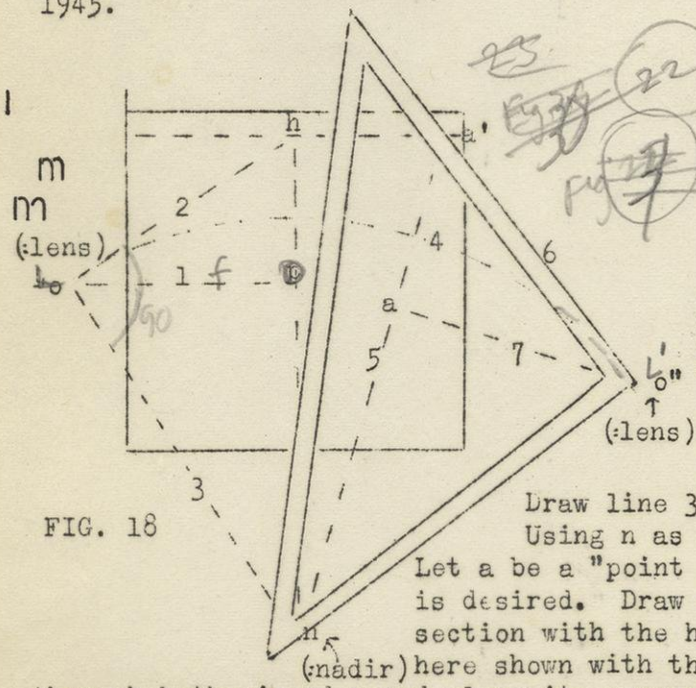


FIG. 18

(22)

R. T. U. Smith's method of obtaining true vertical angles from oblique photos, from Photogrammetric Engineering, 1946 In Fig. 18 the plane of an oblique photo is shown with the construction lines dotted.

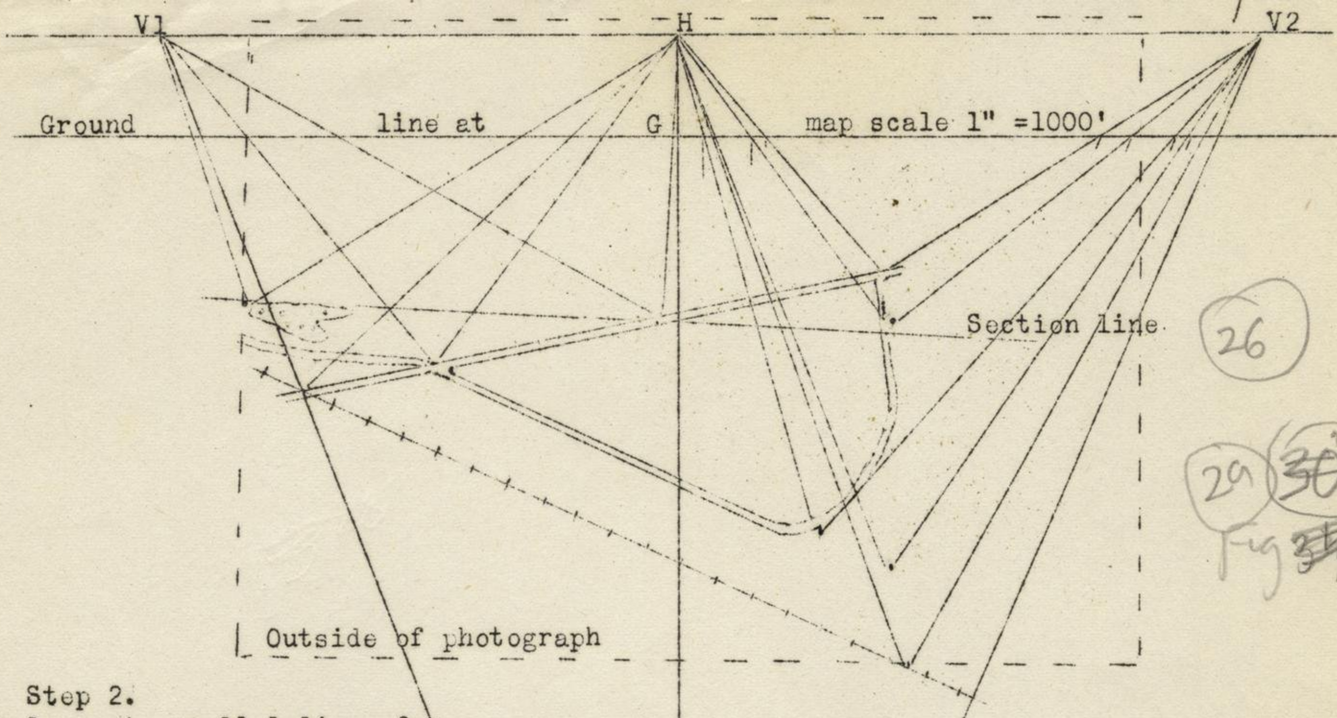
Draw line nh through principal point p. Draw line 1 from p to o, the location of the lens as shown by tilting the vertical plane through the principal point down into the photo plane. op is then equal to the focal length of the camera, f. Draw line 3 from n to o making a right angle at o. Using n as a center strike arc 4 through ~~point o"~~. Let a be a "point sought" for which the angle of depression is desired. Draw line 5 from n through a to its intersection with the horizon at a' Take a right triangle 6 here shown with the double solid linesand pass its sides through both a' and n and place its corner on ~~line~~ arc 4. This determines point o" which is the same in space as o but is here turned into the photo plane. In other words the triangle na'o" is a representation of a vertical plane through lens point O and point sought a. The line o" a (line 7) is then drawn and the desired vertical angle a o"a' can easily be measured with a protractor. Smith suggests that the right angle and protractor be combined. To find elevation of a on the map its horizontal distance from the map location of the photo must be found by other methods. Compare with method shown in Fig. 6. Smith's method works best with photographs where focal length is short and angle of tilt is considerable.

depression

End of Problem 17.

$E = E' \cot T \quad d = \frac{E'(B+d)}{TH}$

$E' = E \cot T = \frac{E'(B+D)}{TL} = \frac{E(B+d)}{TL}$

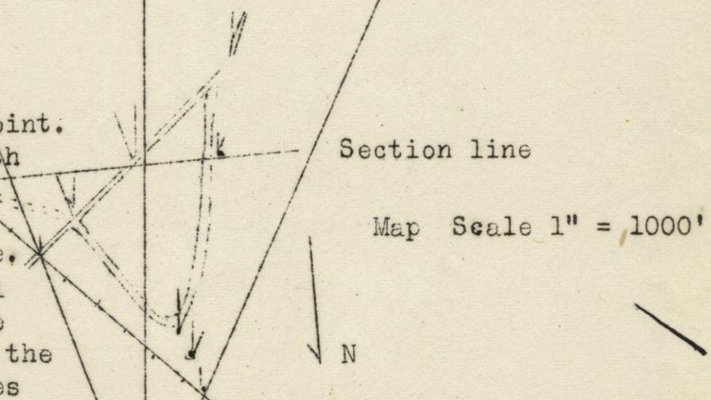


26

29 ~~30~~

Fig ~~3~~

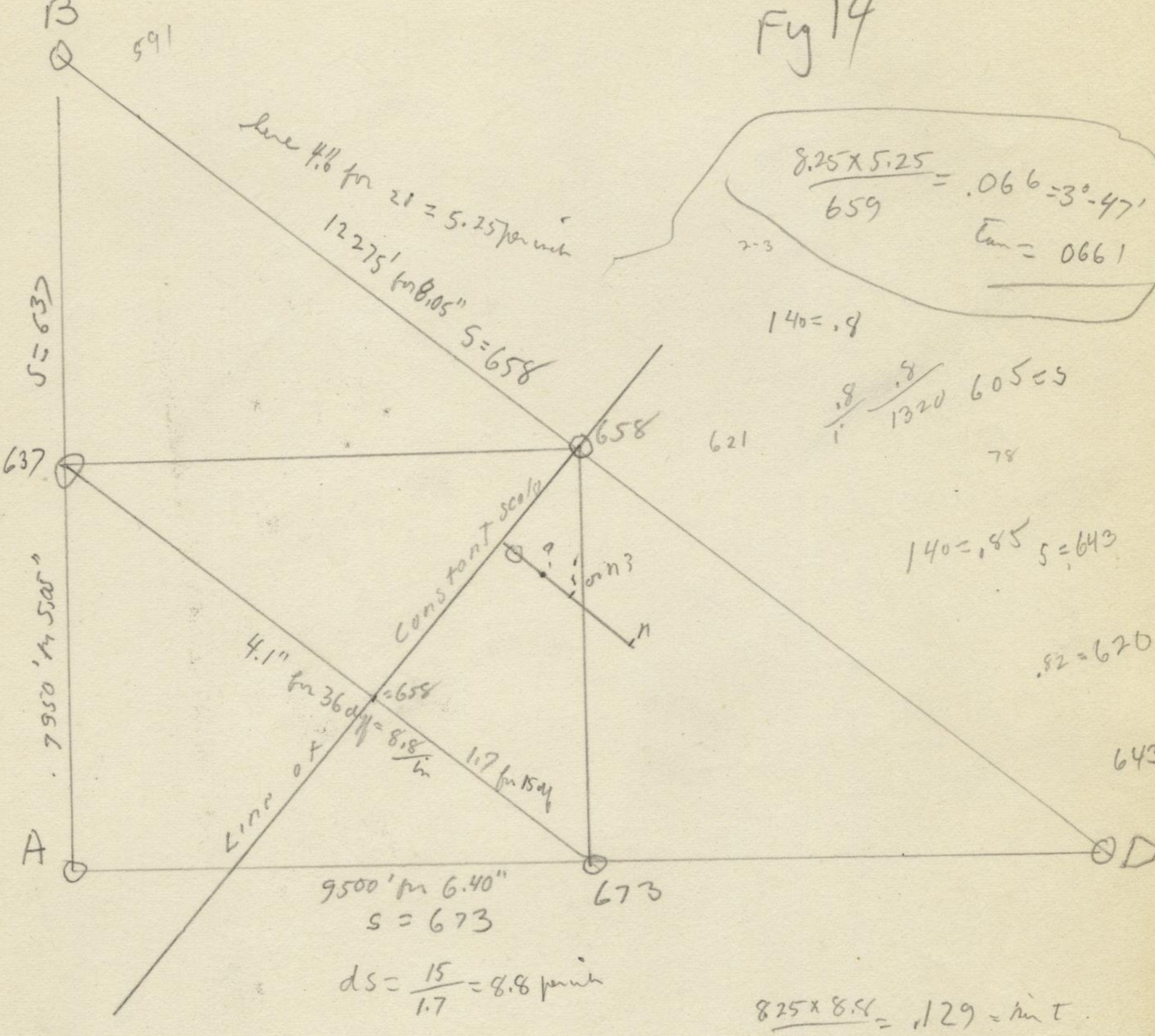
Step 2.
Lay out parallel lines for horizon and ground line at proper distance apart for map scale. Lay out line HO with scaled length. Choose two vanishing points, V1 and V2 at any convenient locations. H is also used as a vanishing point. Connect all vanishing points with O. Now V1 - V2 - O represent the map plane. Distances along the ground line are to map scale. By the theory of perspective all lines which converge to a single vanishing point are parallel on the ground. Therefore, we draw lines parallel to V1-O, H-O, and V2-O each of which passes through the point of intersection of a line to the vanishing point where it crosses the ground line. Draw such lines to at least two different vanishing points from each point it is desired to locate. Intersection of two or more lines drawn parallel in the map plane gives the desired locations to map scale.



Caution: Like the Canadian grid system Rich's method applies only to nearly flat country or to points at very nearly the same elevation.

my method

Fig 14



$$\frac{8.25 \times 5.25}{659} = .066 = 3°\text{-}47'$$

tan = 0661

2-3

140 = .8

$\frac{.8}{1}$ $\frac{.8}{1320}$ 605 = s

78

140 = .85 s = 643

.82 = 620

643

$ds = \frac{15}{1.7} = 8.8$ per inch

$$\frac{8.25 \times 8.8}{659} = .129 = \sin T$$

7° - 25'

tan = .1302

divide by 2

$$\frac{8.25 \times 4.4}{659} = .055 = \sin T \quad 3°\text{-}10' \quad \tan = 0553$$

825 X .0553 = .456"

3.05 for 36 dif = 11.8 per in

half = 5.09 gives 0635 = 3.38

tan 0638

$\frac{15}{36} =$

15 : 36 :: x : 4.1

A-D Sh=673 $\begin{cases} 6.40'' = P \\ 9500' = G \end{cases}$

B-D

B-D Sm=658 $\begin{cases} 8.05'' \\ 12275' \end{cases}$

use size of diagr

AB S1=637 $\begin{cases} \frac{5.05''}{7950'} \end{cases}$

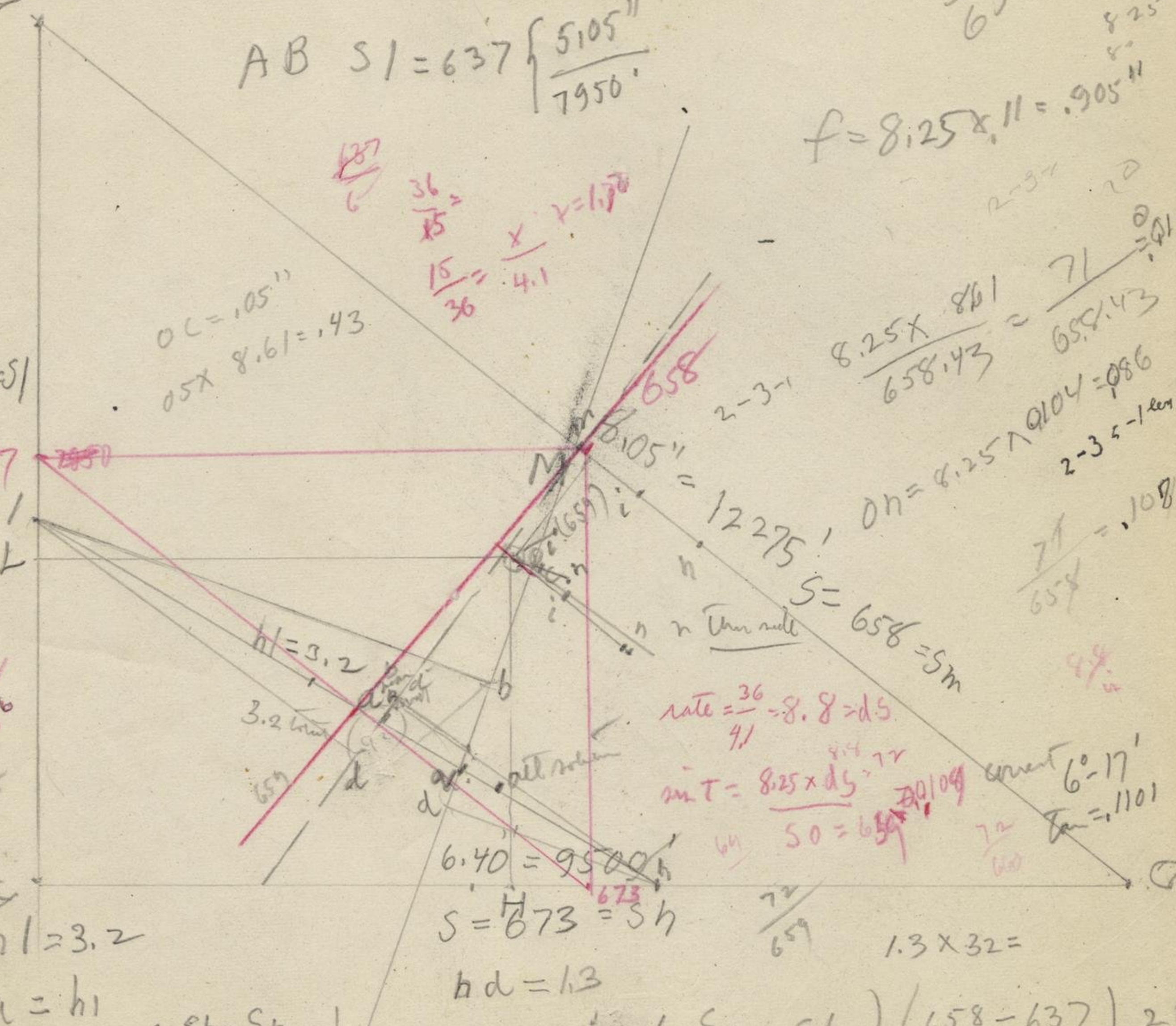
$\frac{710}{658} = .1.04$

$f = 8.25 \times .11 = .905''$

$\frac{8.25 \times .861}{658.43} = \frac{71}{658.43} = .0104$

on = 8.25 × .0104 = .086

S=637=S1
5.05''
7950



$\frac{36}{4.1} = 8.75$

hl = 3.2

hd = 1.3

1.3 × 32 =

ha = hl

$3.2\left(\frac{Sh-Sm}{Sh-S1}\right)$

or la = 3.2 $\left(\frac{Sm-S1}{Sh-S1}\right)\left(\frac{658-637}{673-637}\right) = \frac{21}{36} \times 3.2 = 1.87$

$3.2\left(\frac{673-658}{673-637}\right) = \frac{1.5}{36} \times 3.2 = 1.33$

hb = ~~2.8~~ 1.85

or ed = 2.3

$ds = \frac{Sh - Sm}{lb(2.8)}$ hb

or $ds = \frac{(Sm - S1)}{lb} = \frac{658-637}{2.8} = \frac{21}{2.8} =$

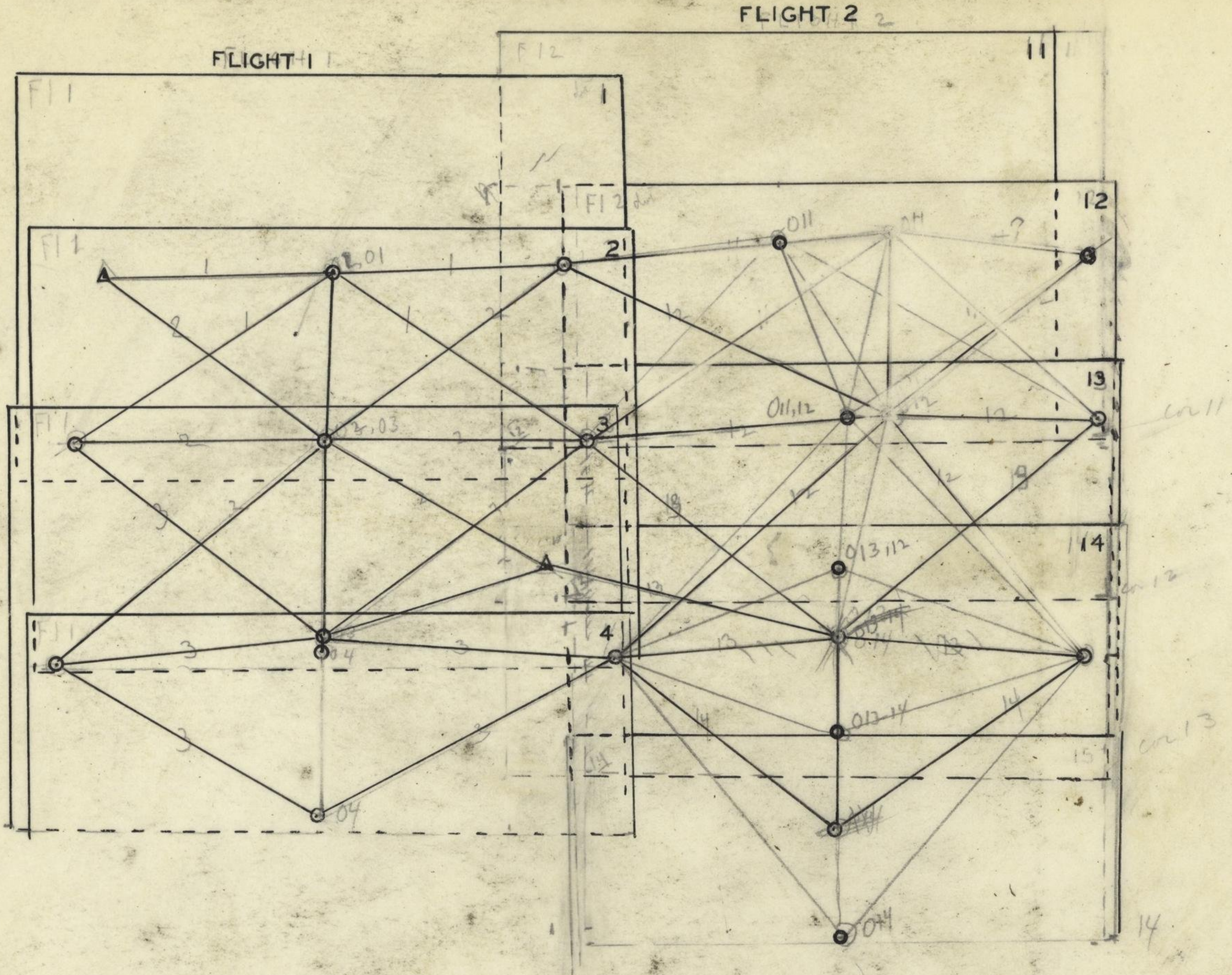
$= \frac{673-658}{2.8} = \frac{15}{2.8} = 5.35$

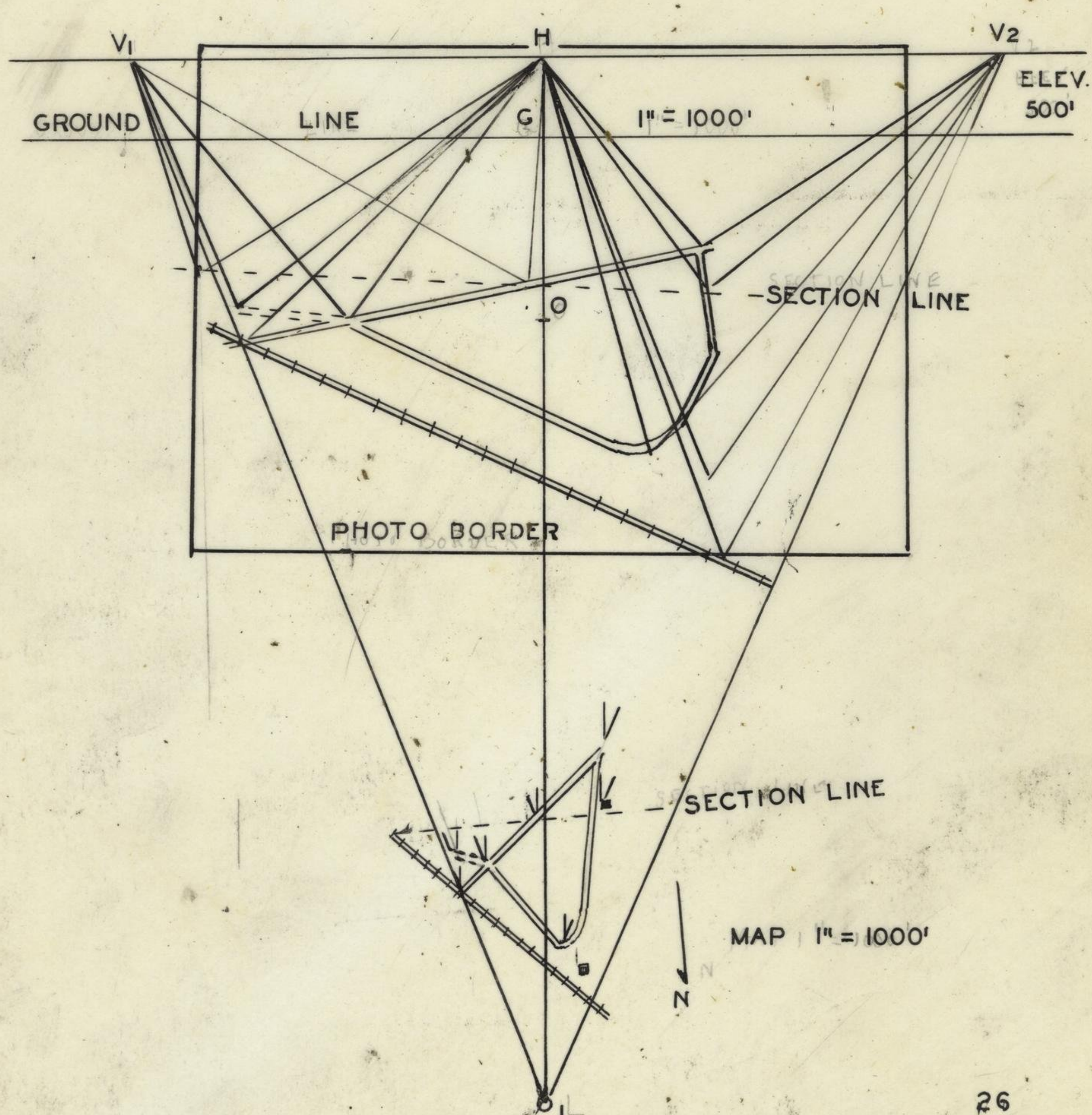
$\sin t = \frac{8.25 \times 5.35}{650} = .0442 \quad t = 2°\text{-}32'$

So = Sm − (oc.ds)

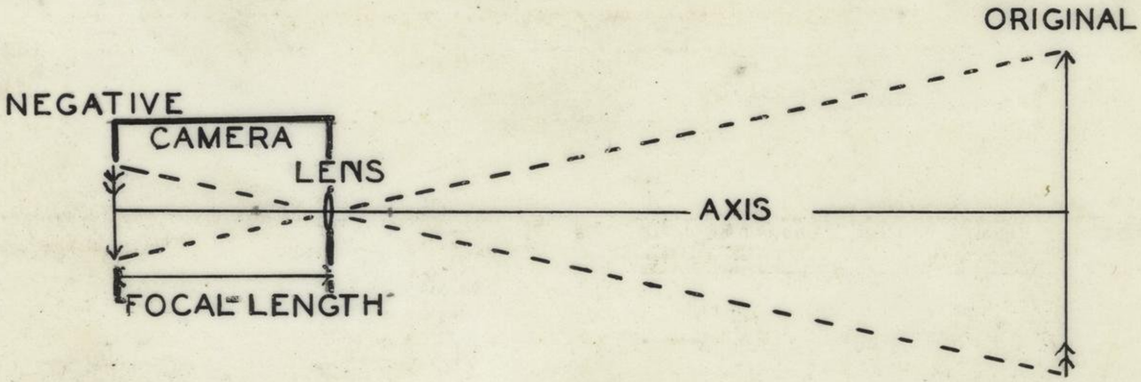
= 658 − .15 × 5.35 = 658 − 8 = 650 = 5.0

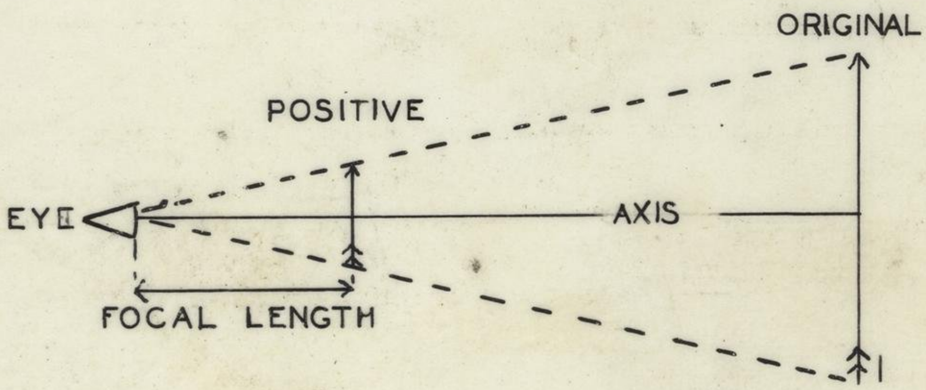
oh = 8.25 × .0442 = .364''

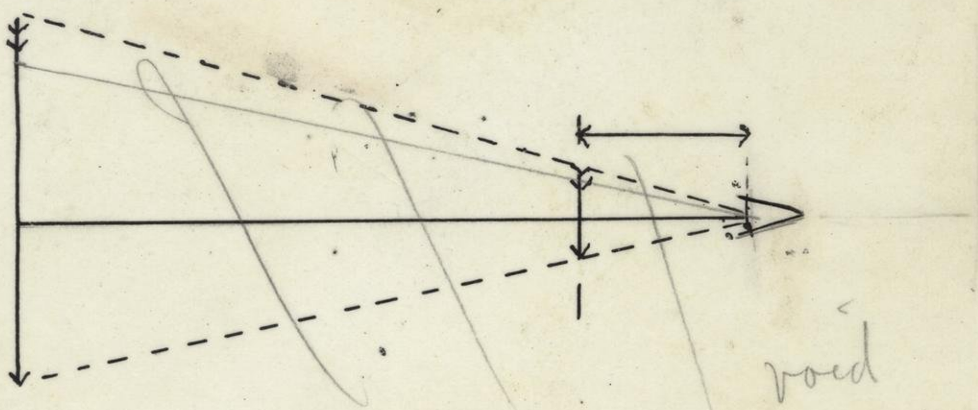

FLIGHT 1
FLIGHT 2
F1 1
F1 2
1
2
3
4
11
12
13
14
O1
O3,O3
O4
O11
O11,12
O13,12
O12-14
O14

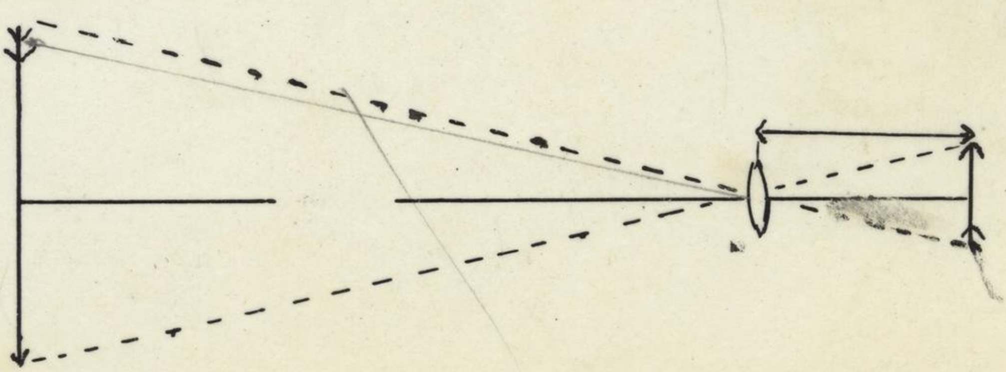


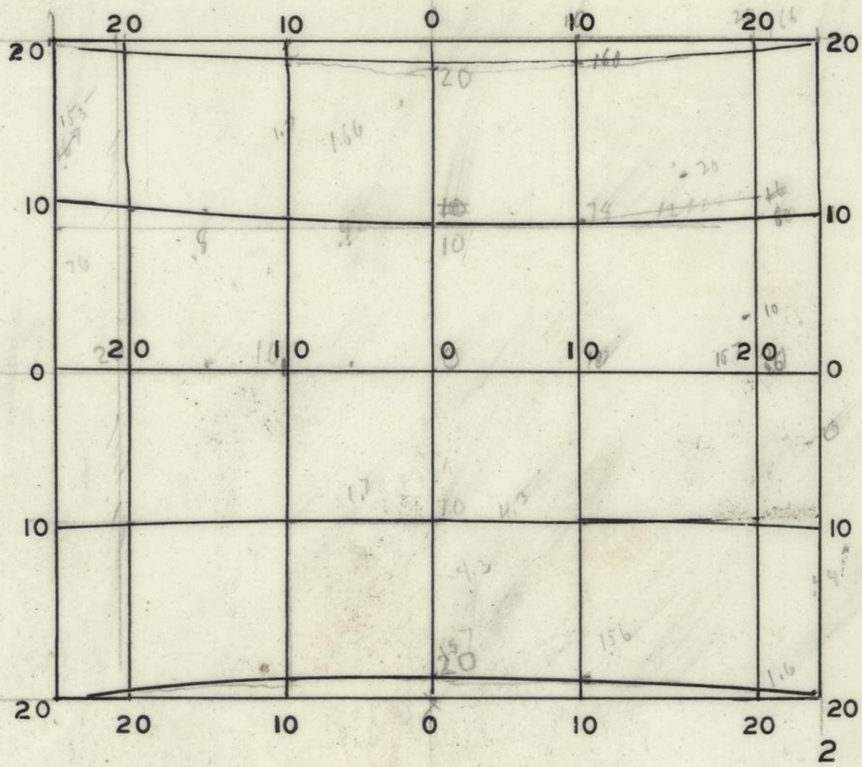
V1
H
V2
GROUND
LINE
G
1" = 1000'
ELEV.
500'
O
SECTION LINE
PHOTO BORDER
SECTION LINE
MAP 1" = 1000'
N
L

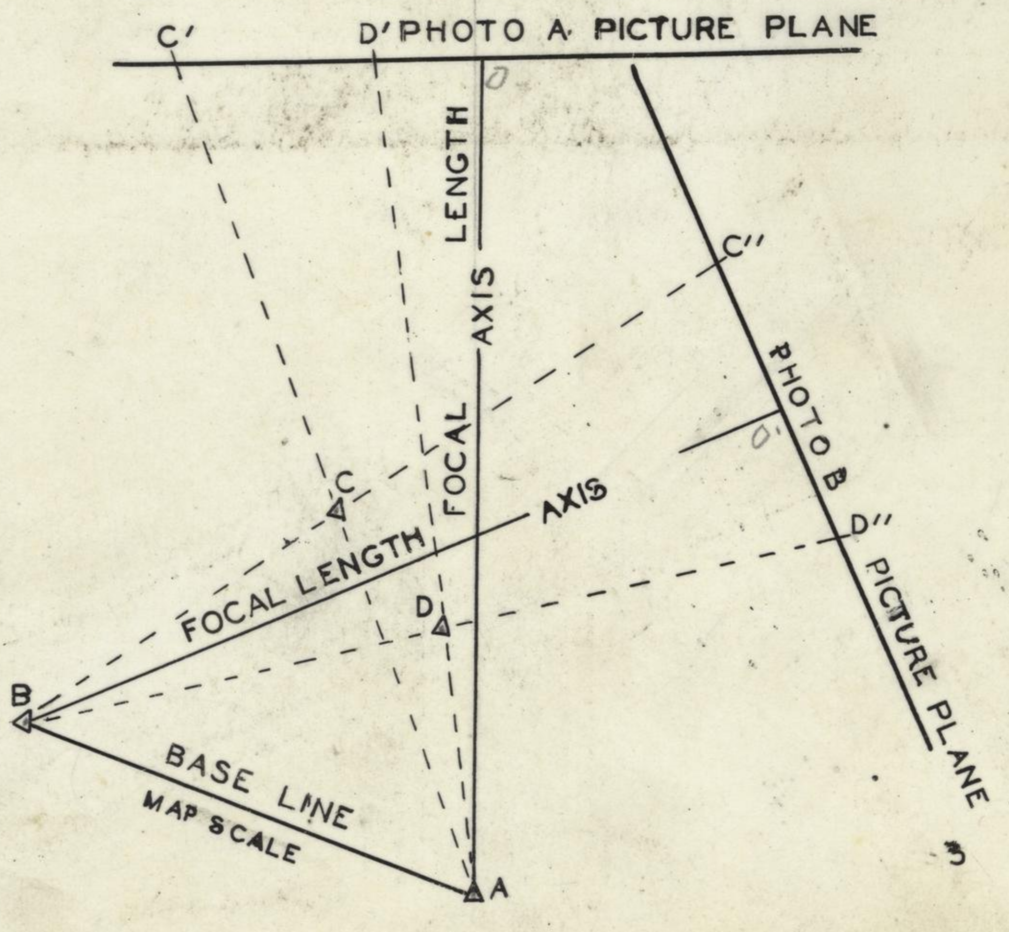

ORIGINAL
NEGATIVE
CAMERA
LENS
AXIS
FOCAL LENGTH

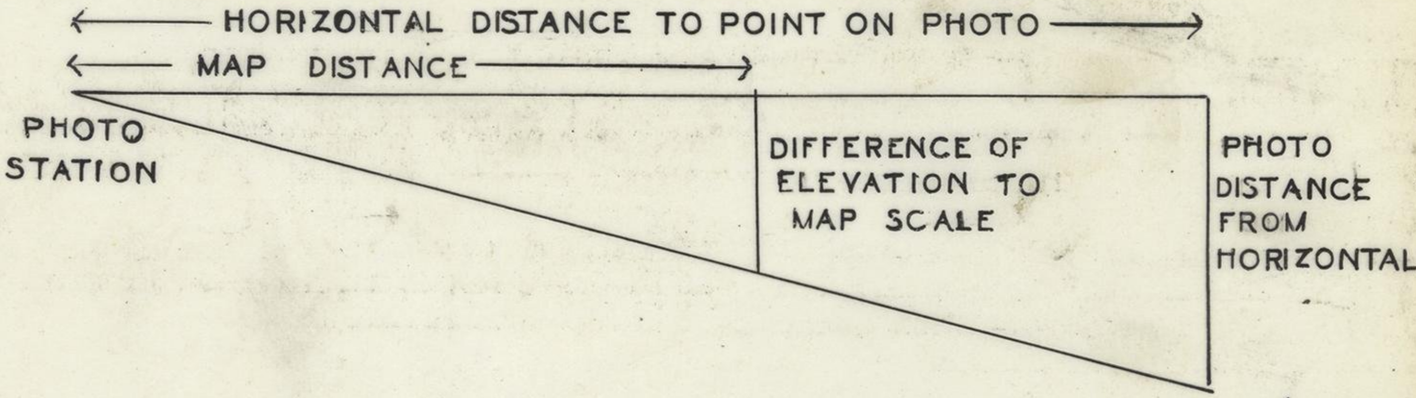

ORIGINAL
POSITIVE
EYE
AXIS
FOCAL LENGTH


void

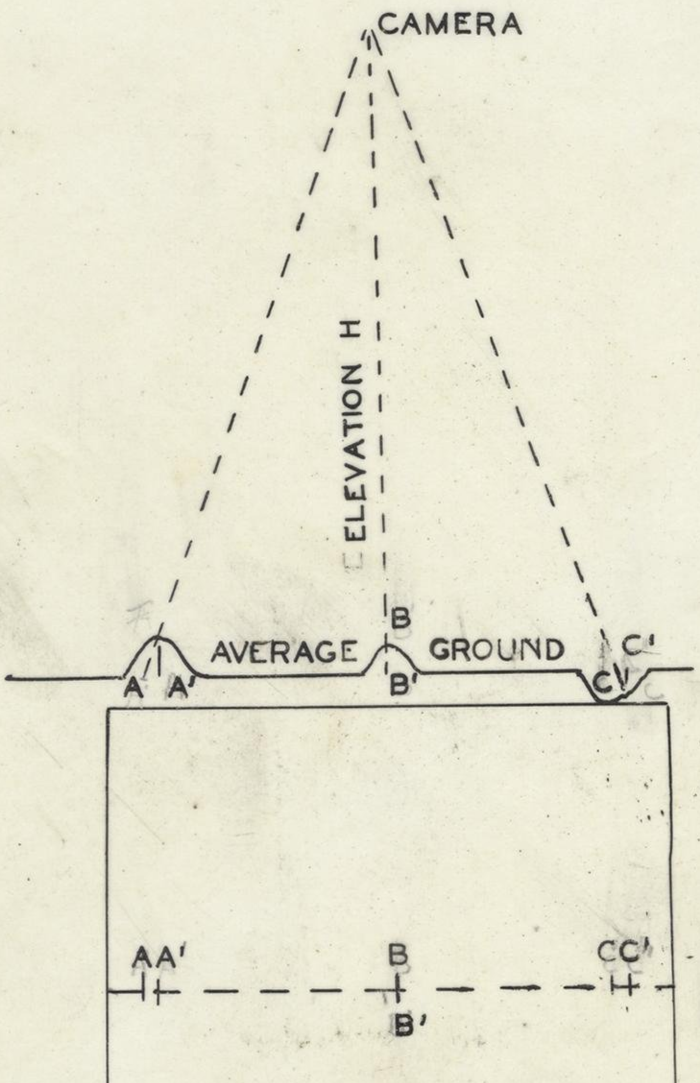



20 10 0 10 20
20 10 0 10 20
20 10 0 10 20
2


C'
D'
PHOTO A PICTURE PLANE
FOCAL LENGTH AXIS
C''
PHOTO B PICTURE PLANE
D''
C
D
FOCAL LENGTH AXIS
B
BASE LINE
MAP SCALE
A


HORIZONTAL DISTANCE TO POINT ON PHOTO
MAP DISTANCE
PHOTO STATION
DIFFERENCE OF ELEVATION TO MAP SCALE
PHOTO DISTANCE FROM HORIZONTAL

4


CAMERA
ELEVATION H
A
A'
AVERAGE
B
B'
GROUND
C
C'
AA'
B
B'
CC'

5

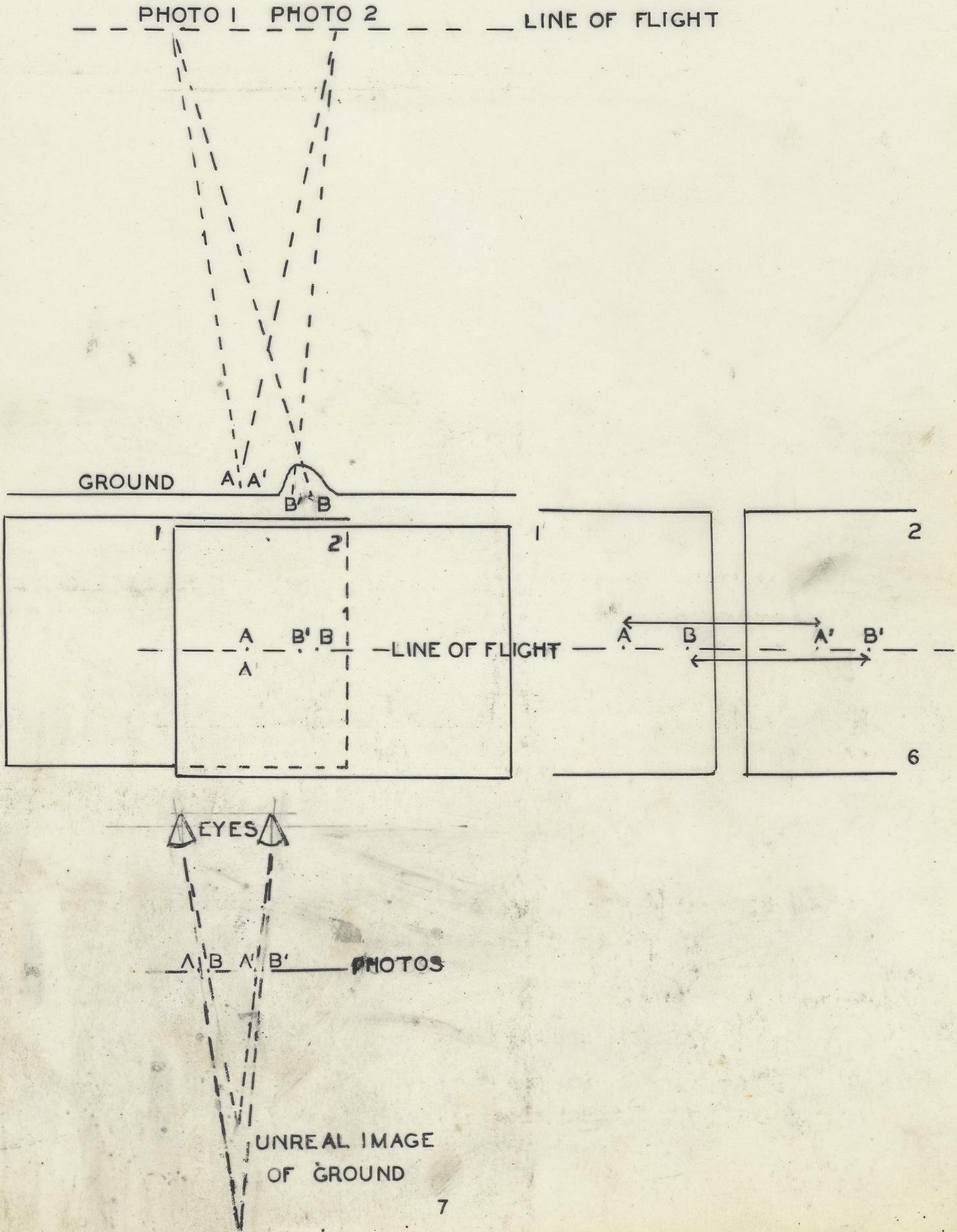
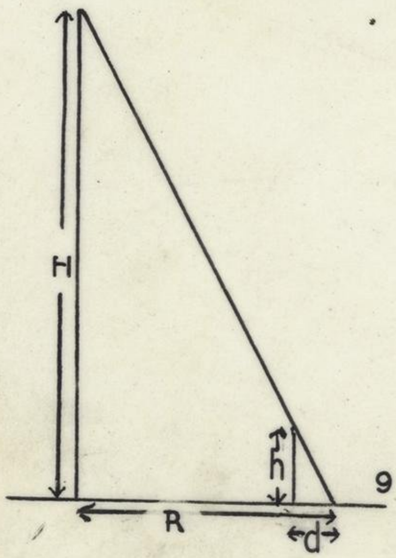
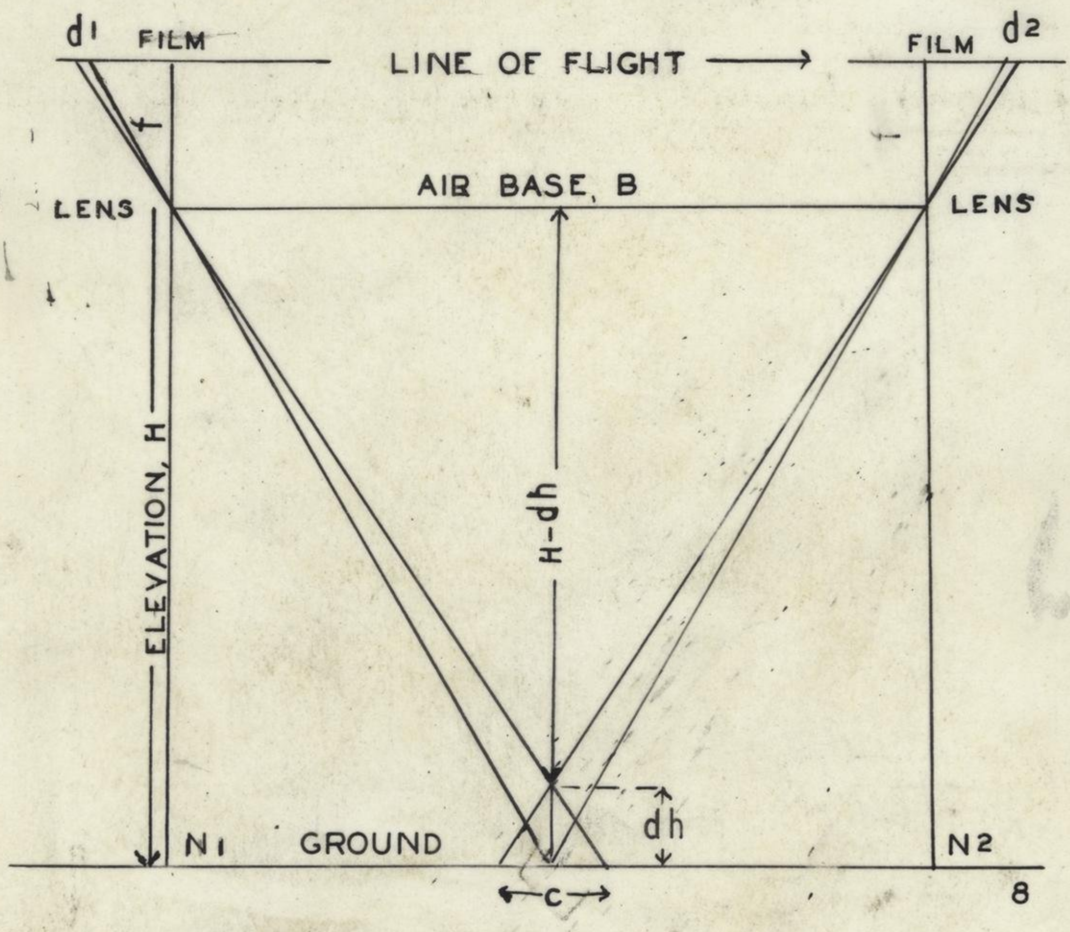
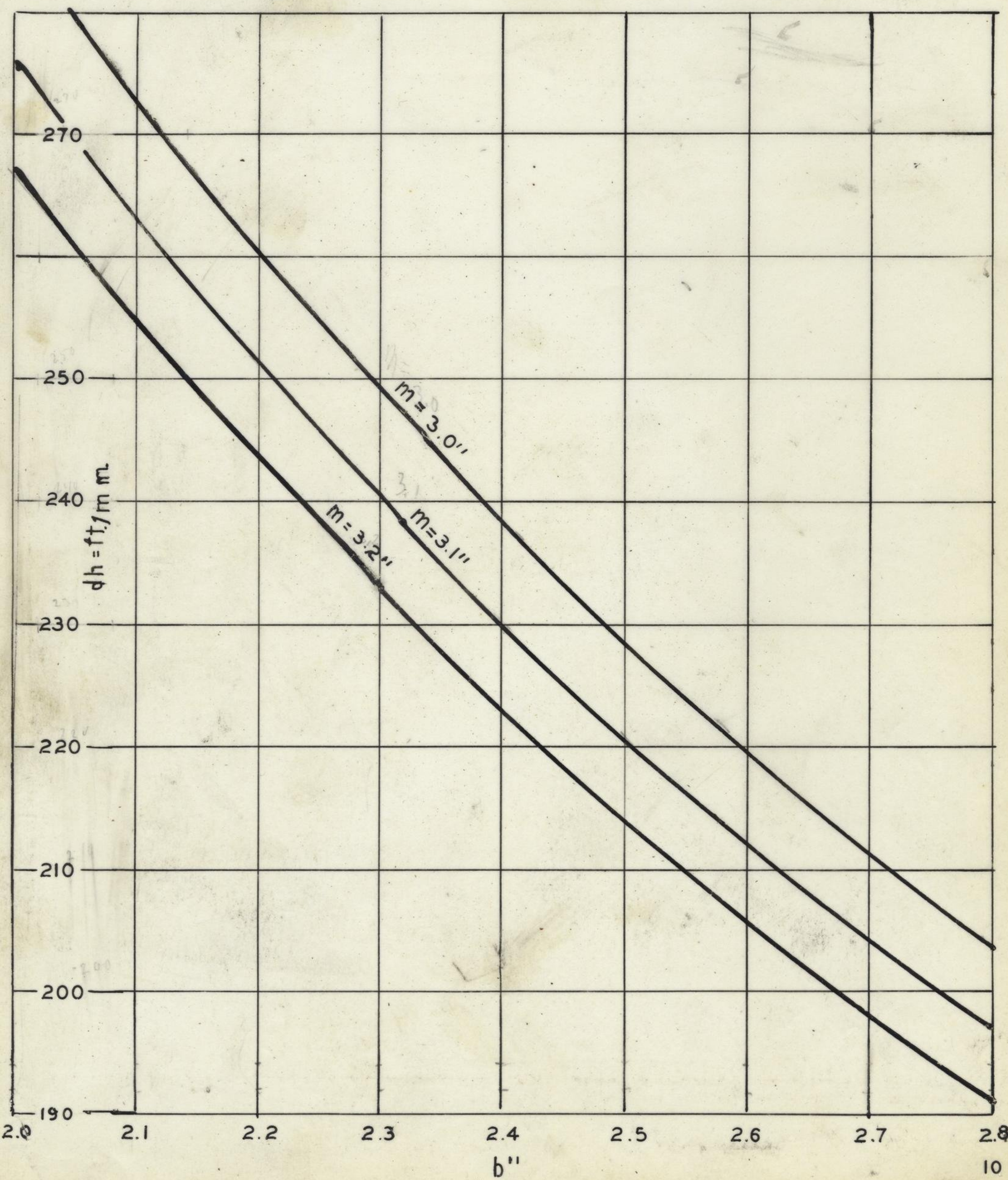


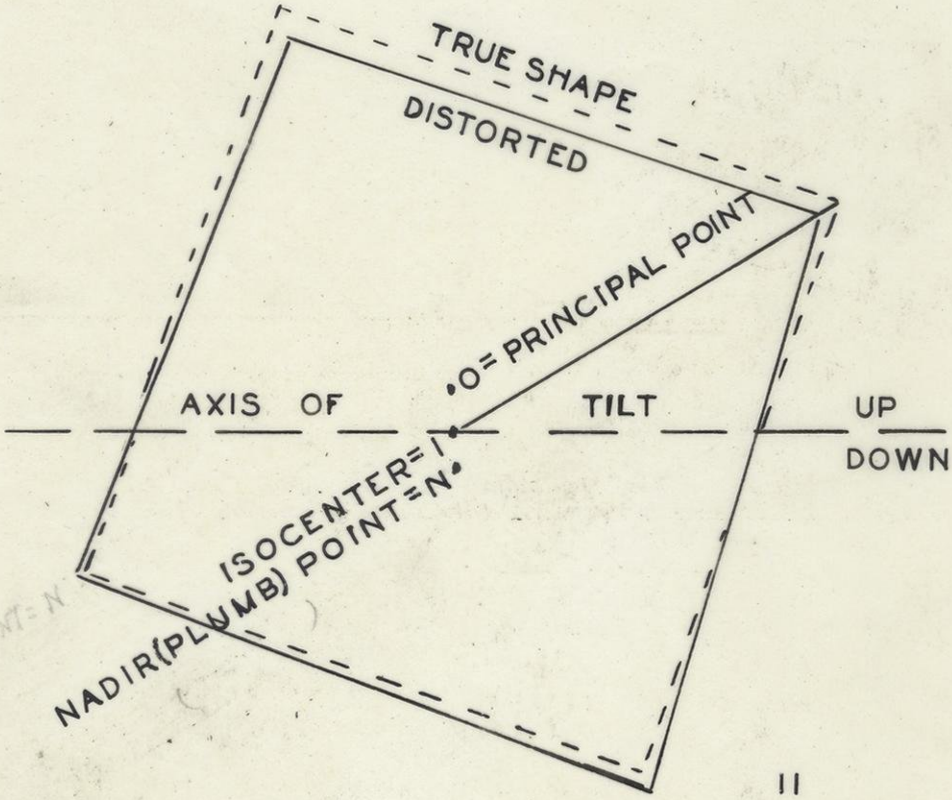
PHOTO 1
PHOTO 2
LINE OF FLIGHT
GROUND
A
A'
B'
B
1
2
A
B'
B
A'
LINE OF FLIGHT
1
2
A
B
A'
B'
6
EYES
A B A' B'
PHOTOS
UNREAL IMAGE
OF GROUND

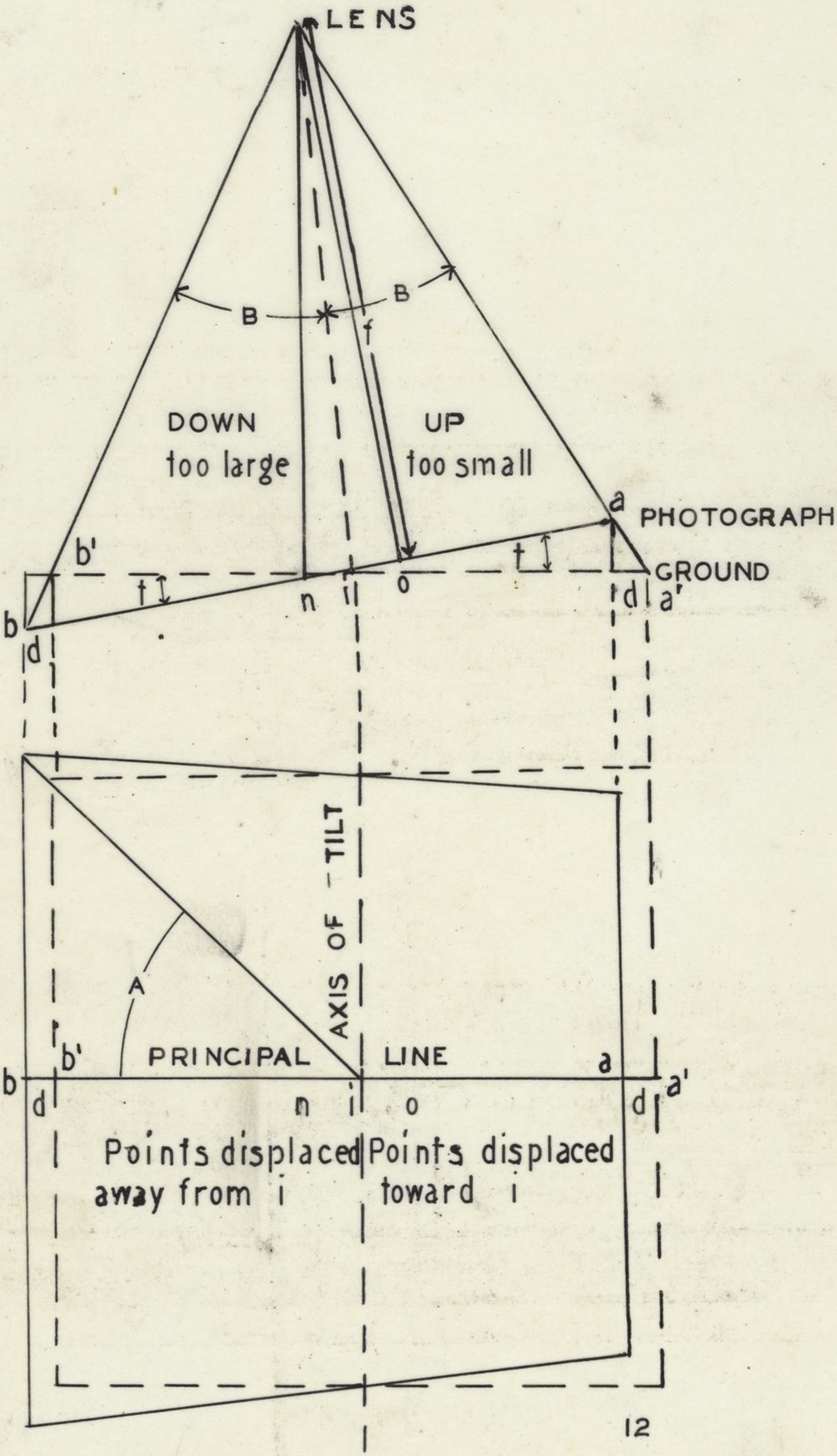

H
h
R
d
9

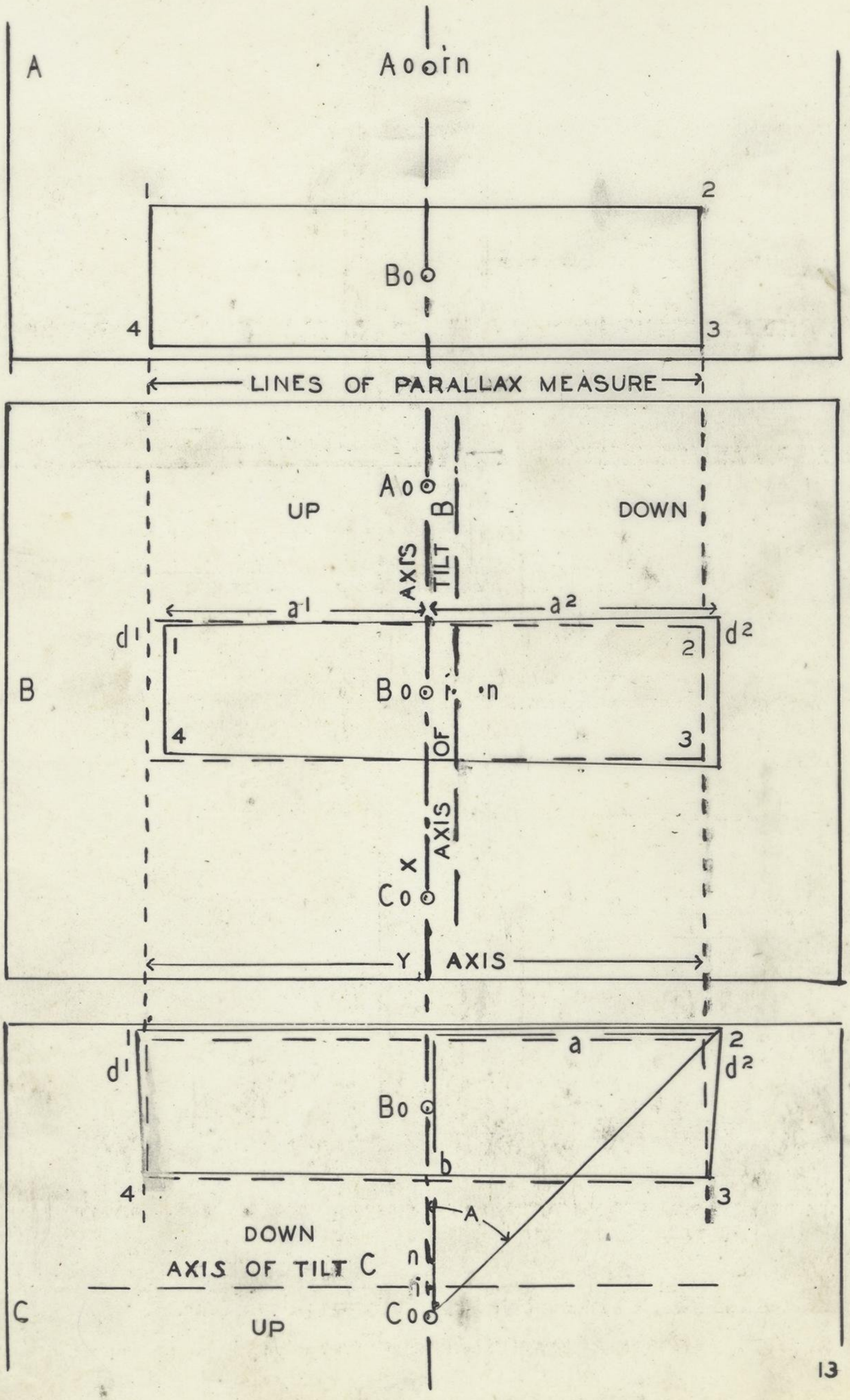

d1
FILM
LINE OF FLIGHT
FILM
d2
f
LENS
AIR BASE, B
LENS
ELEVATION, H
H-dh
dh
N1
GROUND
N2
c
8

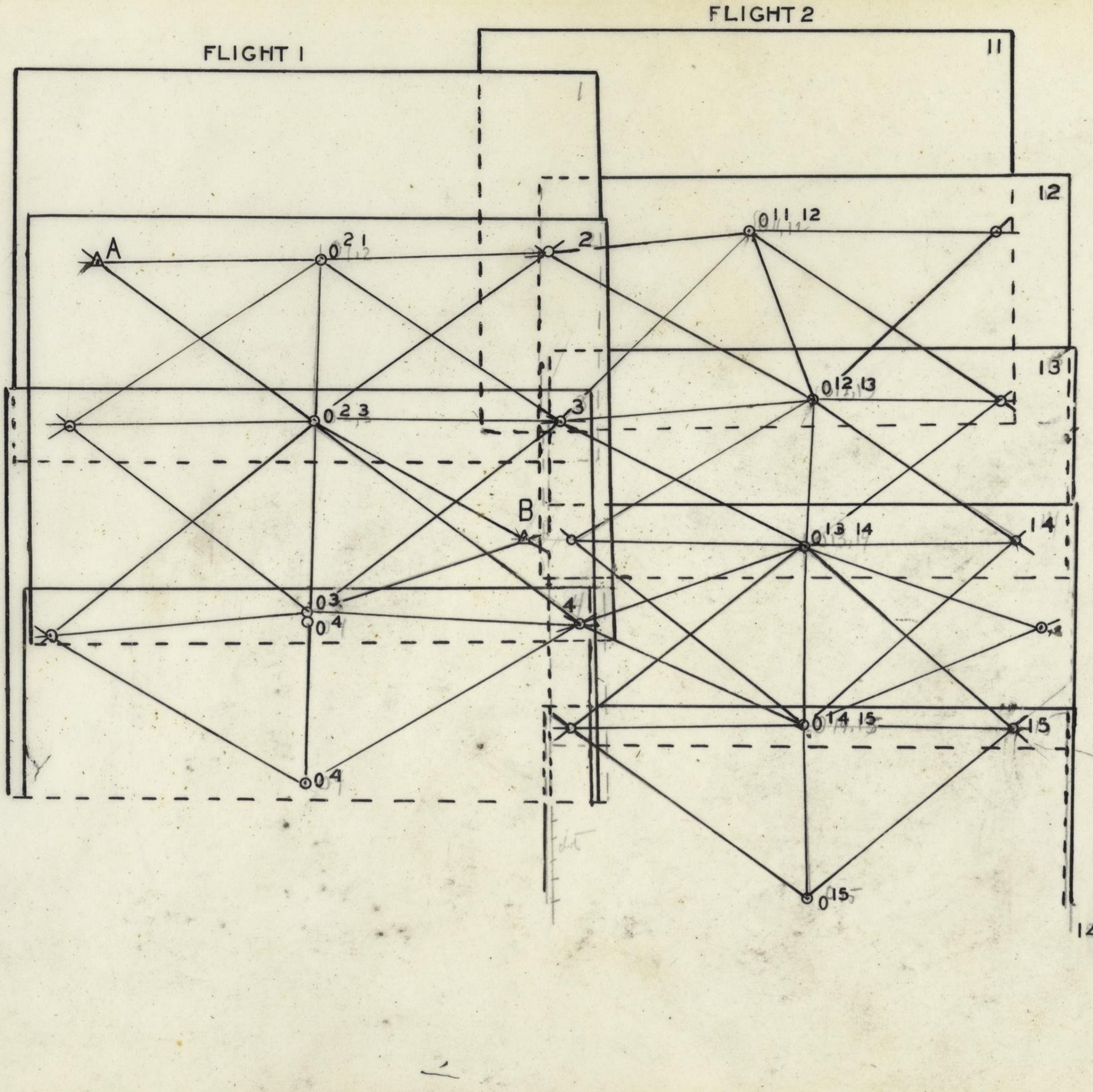


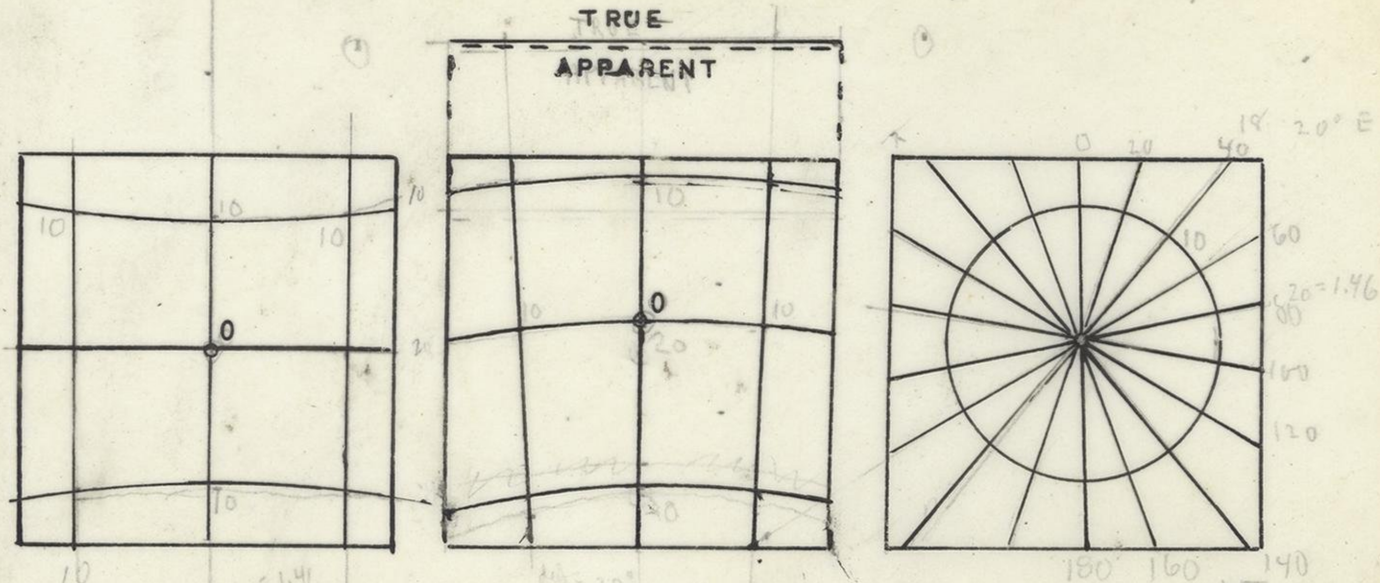
270
250
240
230
220
210
200
190
2.0
2.1
2.2
2.3
2.4
2.5
2.6
2.7
2.8
b''
dh = ft./m m.
m = 3.0''
m = 3.1''
m = 3.2''


TRUE SHAPE
DISTORTED
O= PRINCIPAL POINT
AXIS OF TILT
UP
DOWN
ISOCENTER= I
NADIR(PLUMB) POINT=N

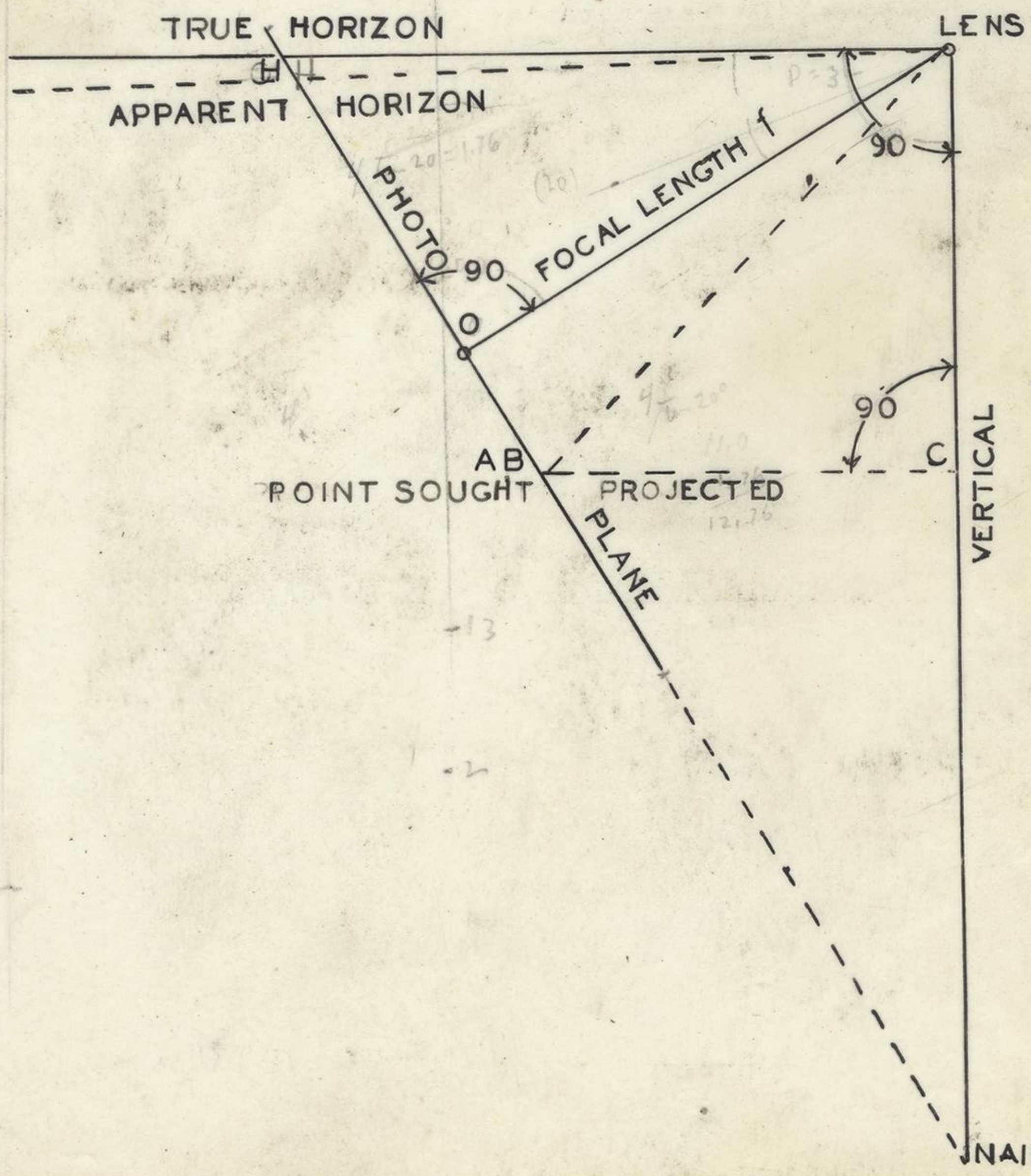

LENS
B
B
f
DOWN
too large
UP
too small
a
PHOTOGRAPH
b'
t
t
GROUND
b
n
o
d
a'
d
AXIS OF TILT
A
PRINCIPAL
LINE
b
b'
a
a'
d
n
i
o
d
Points displaced
away from i
Points displaced
toward i


A
Aoo n
1
2
Bo
4
3
LINES OF PARALLAX MEASURE
Ao
UP
DOWN
AXIS TILT B
a1
a2
d1
d2
1
2
B
Bo n
4
3
OF
AXIS
X
Co
Y AXIS
1
2
a
d1
d2
Bo
b
4
3
A
DOWN
AXIS OF TILT C
n
C
Co
UP

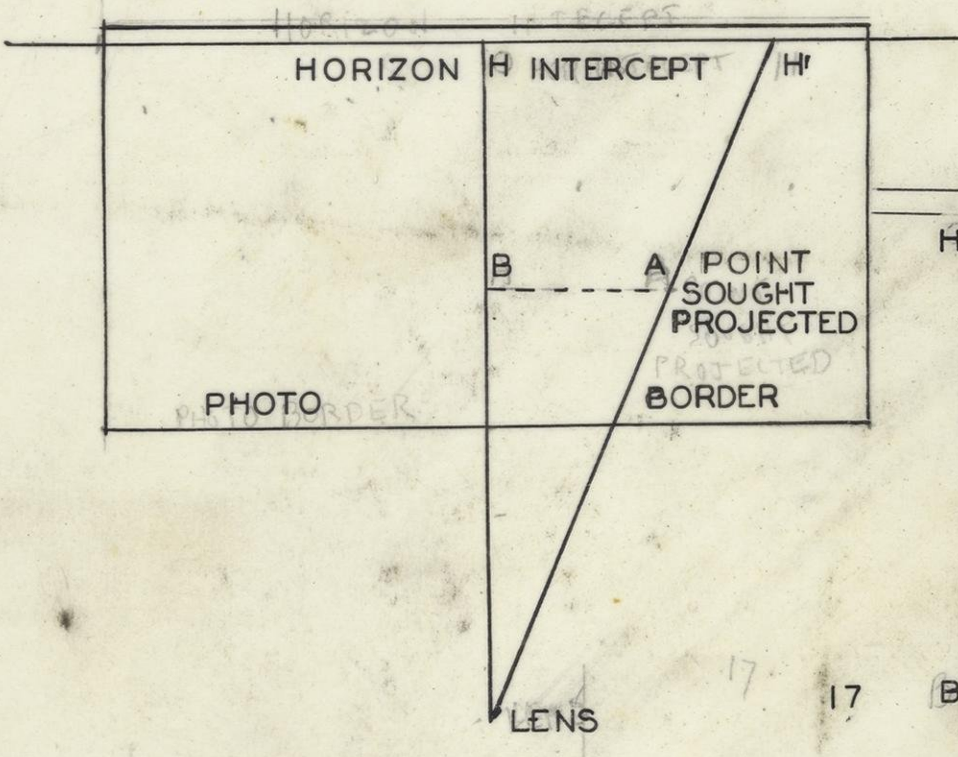

FLIGHT 1
FLIGHT 2
11
12
13
14
15
A
B
2
3
4
0 2 1
0 2 3
0 3
0 4
0 4
0 11 12
0 12 13
0 13 14
0 14 15
0 15
14


TRUE
APPARENT
0
0
0
20
40
180
160
140

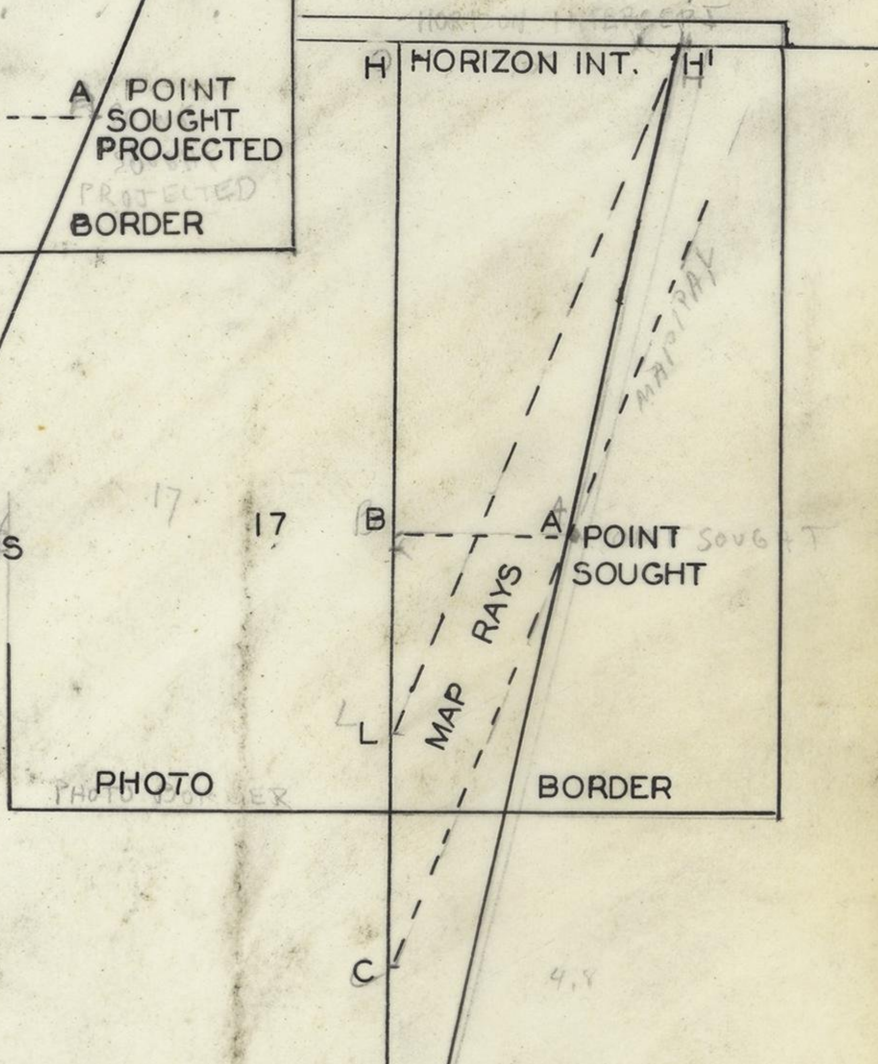
15


TRUE HORIZON
APPARENT HORIZON
LENS
90
PHOTO
PLANE
FOCAL LENGTH f
90
O
90
AB
C
POINT SOUGHT
PROJECTED
VERTICAL
NAIDIR

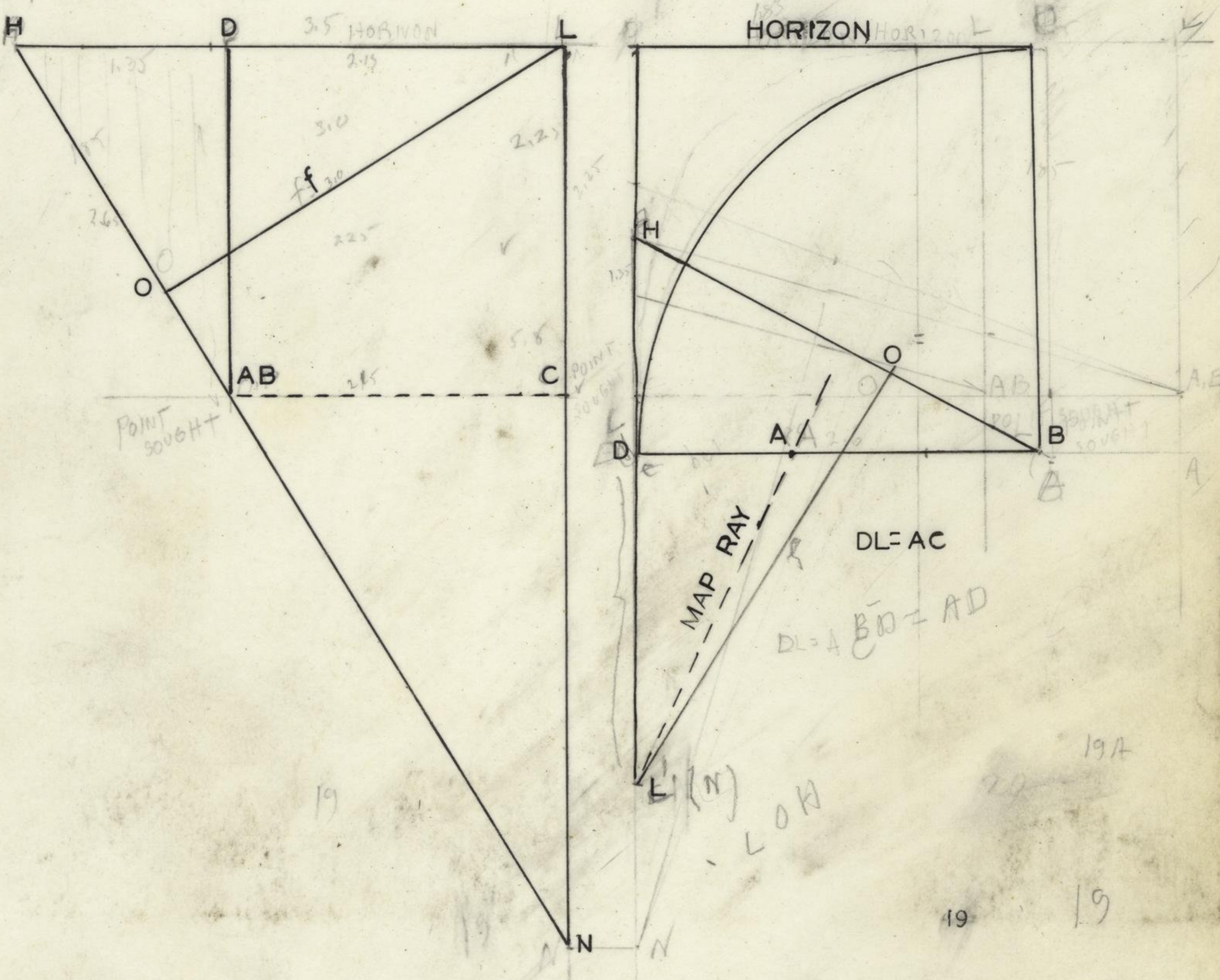
16

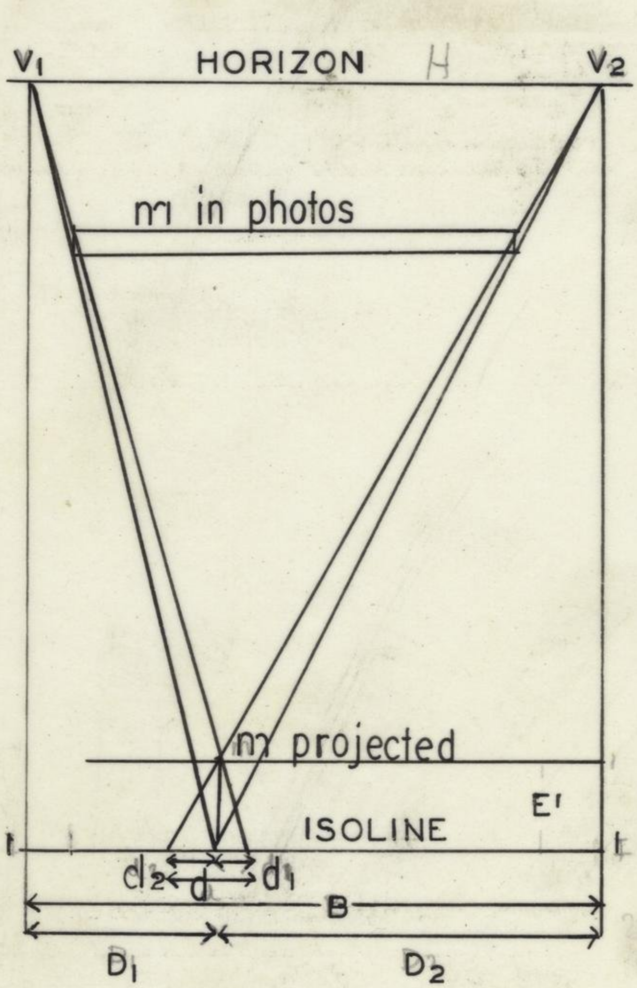

HORIZON
H
INTERCEPT
H'
B
A
POINT
SOUGHT
PROJECTED
PHOTO
BORDER
LENS

17

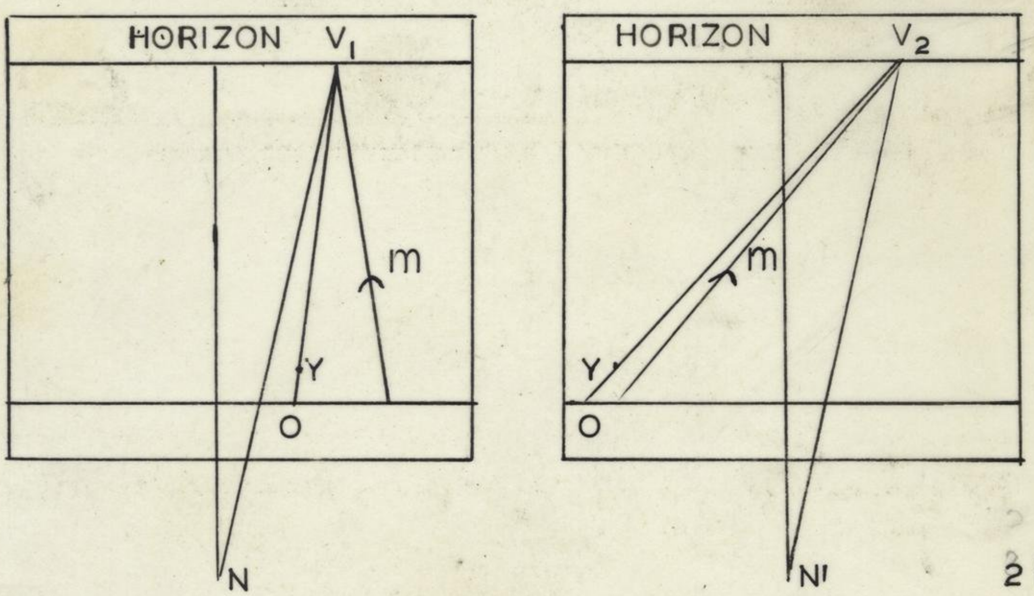

H
HORIZON INT.
H'
B
A
POINT
SOUGHT
MAP RAYS
MAP RAYS
L
PHOTO
BORDER
C
N
NADER OF FIG. 16

18

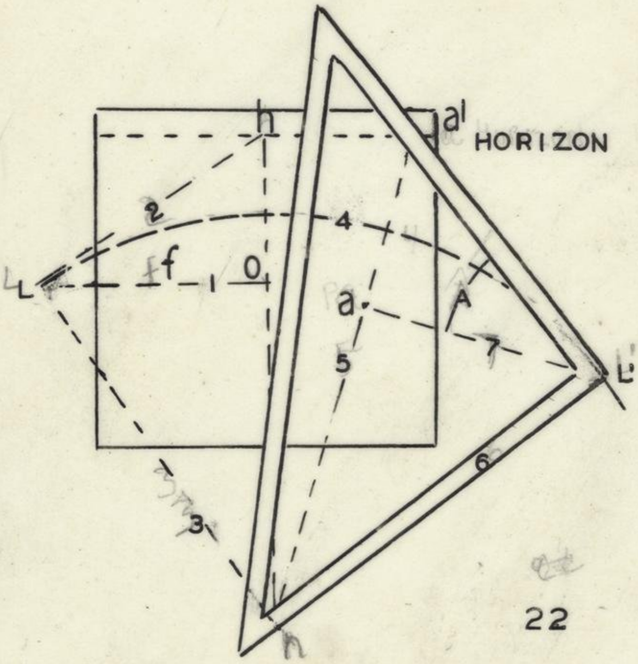

H
D
L
HORIZON
f
O
AB
C
POINT SOUGHT
N
H
O
A
B
D
L
MAP RAY
DL= AC
DL= AC
19
19A
19


V1
HORIZON
H
V2
m in photos
m projected
E'
ISOLINE
I
d2
d
d1
B
D1
D2
H
L
HORIZON
H'
ANGLE OF
DEPRESSION
f
O
H
E
E'
I

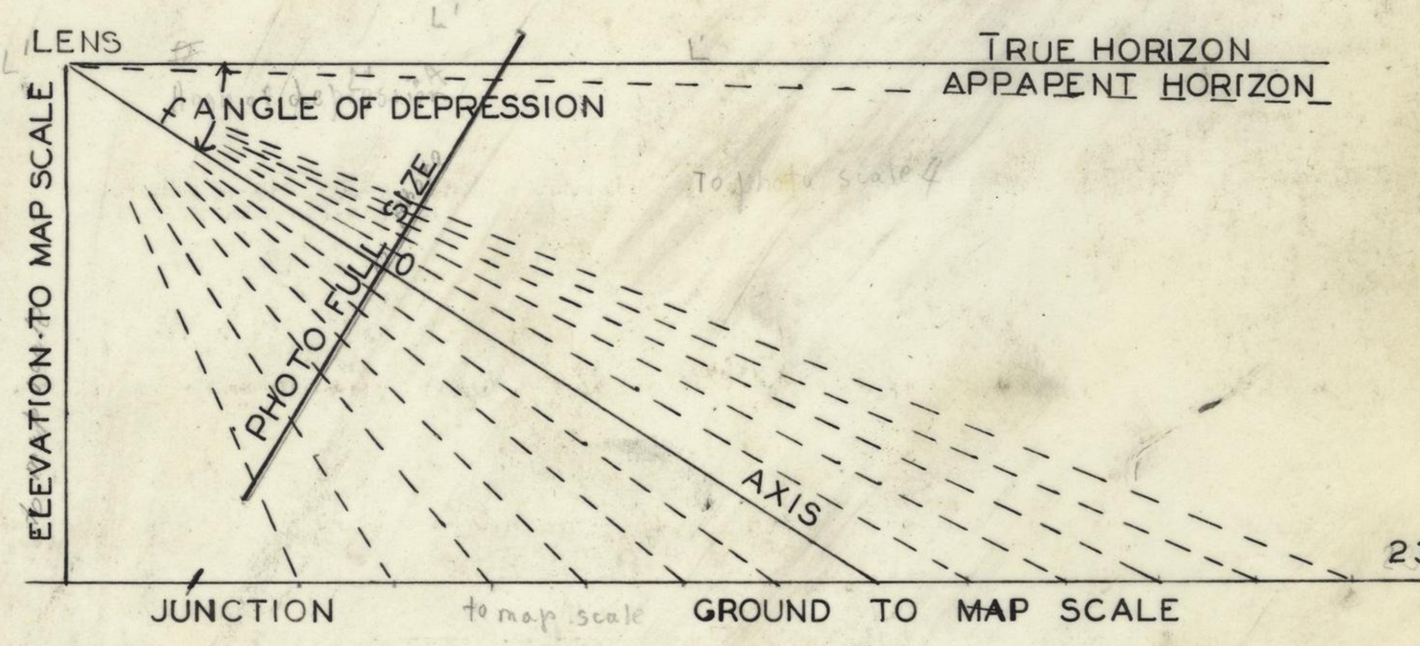
20


HORIZON
V1
m
Y
O
N
HORIZON
V2
m
Y
O
N'

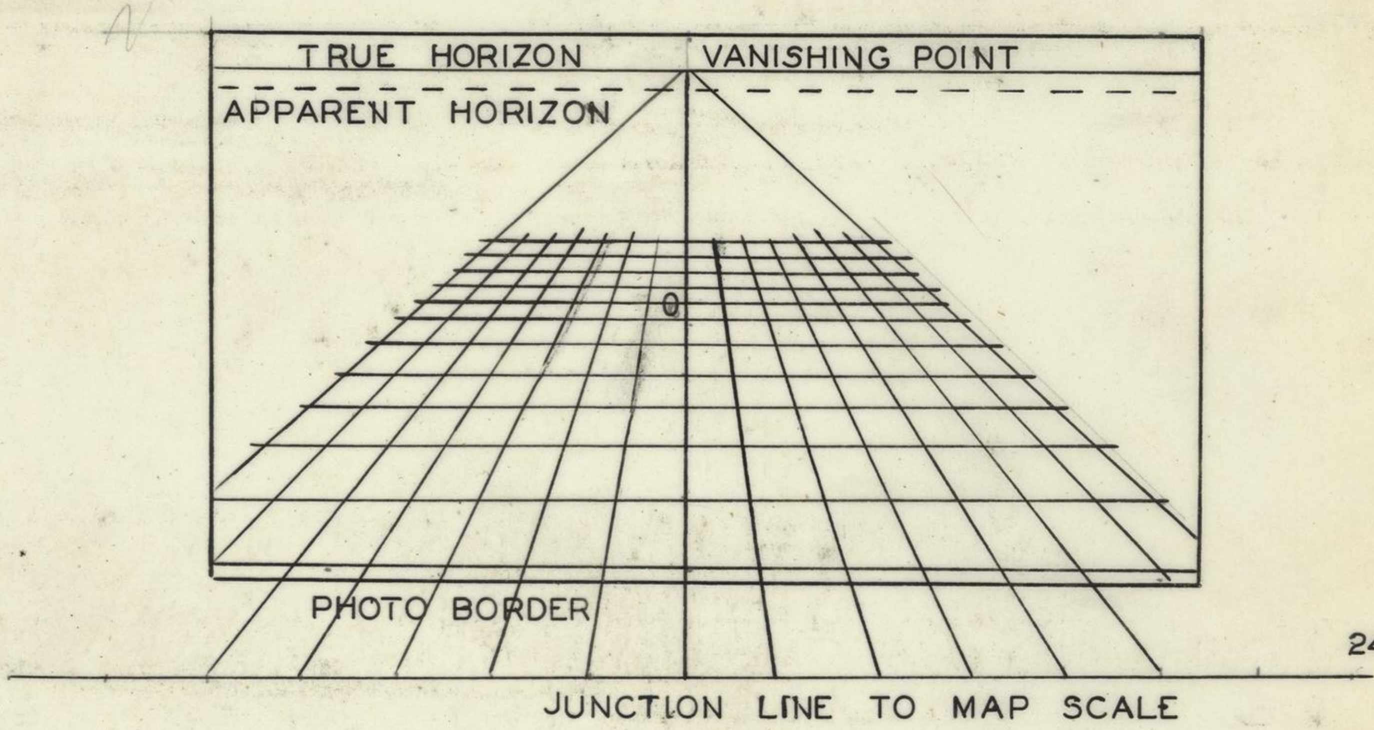
21


h
a'
HORIZON
L
f
0
1
2
3
4
5
6
7
a
A
L'

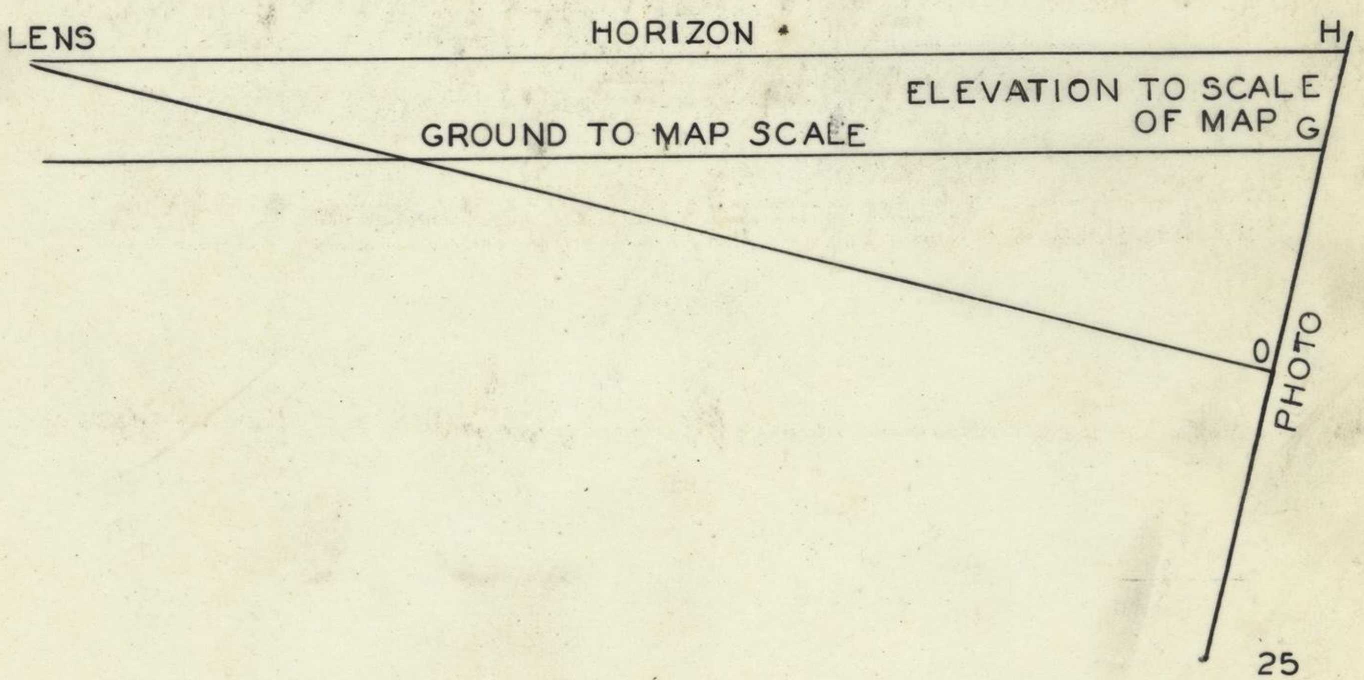
22


LENS
TRUE HORIZON
APPARENT HORIZON
ANGLE OF DEPRESSION
PHOTO FULL SIZE
O
AXIS
ELEVATION TO MAP SCALE
JUNCTION
to map scale
GROUND TO MAP SCALE

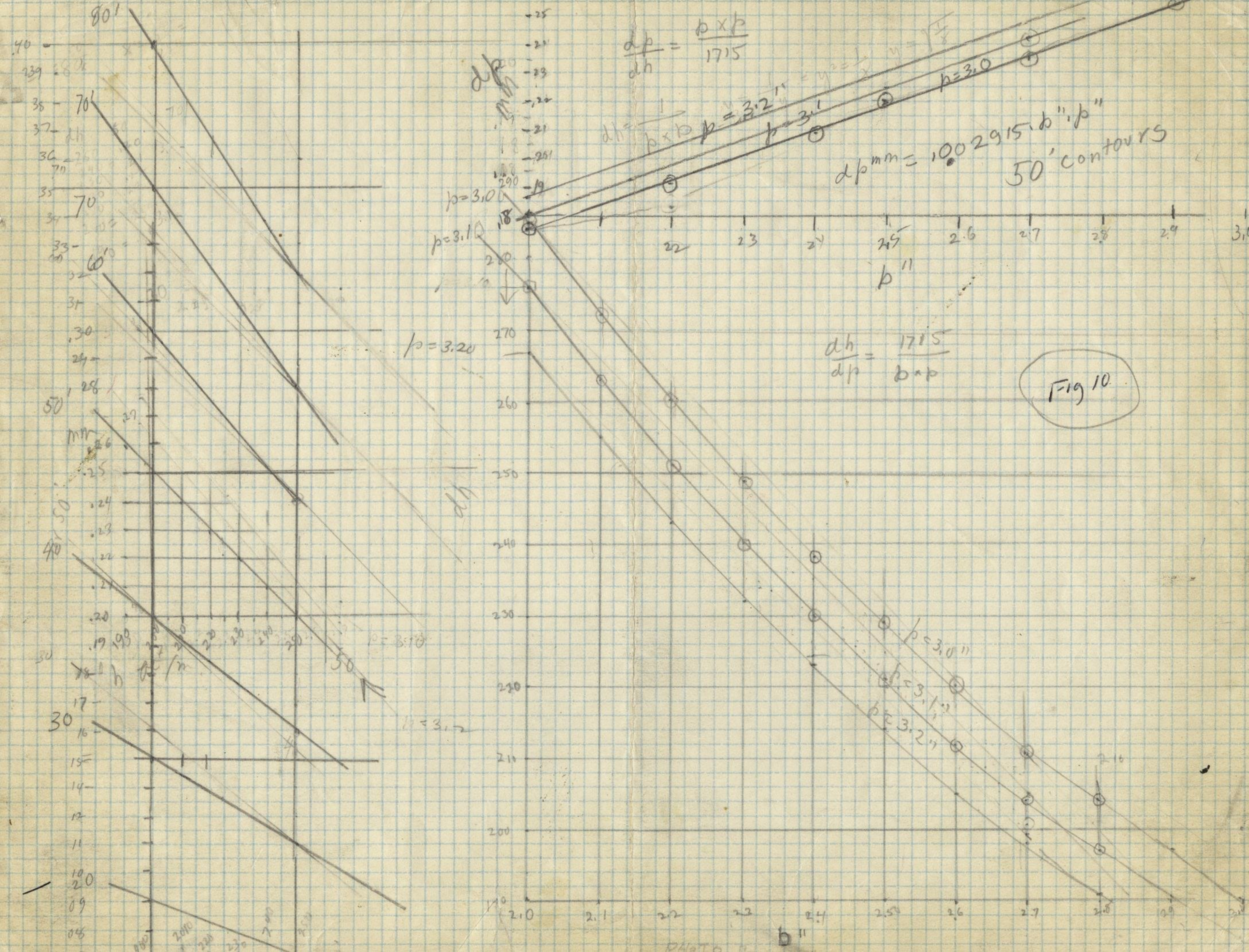
23


TRUE HORIZON
VANISHING POINT
APPARENT HORIZON
0
PHOTO BORDER
JUNCTION LINE TO MAP SCALE

24

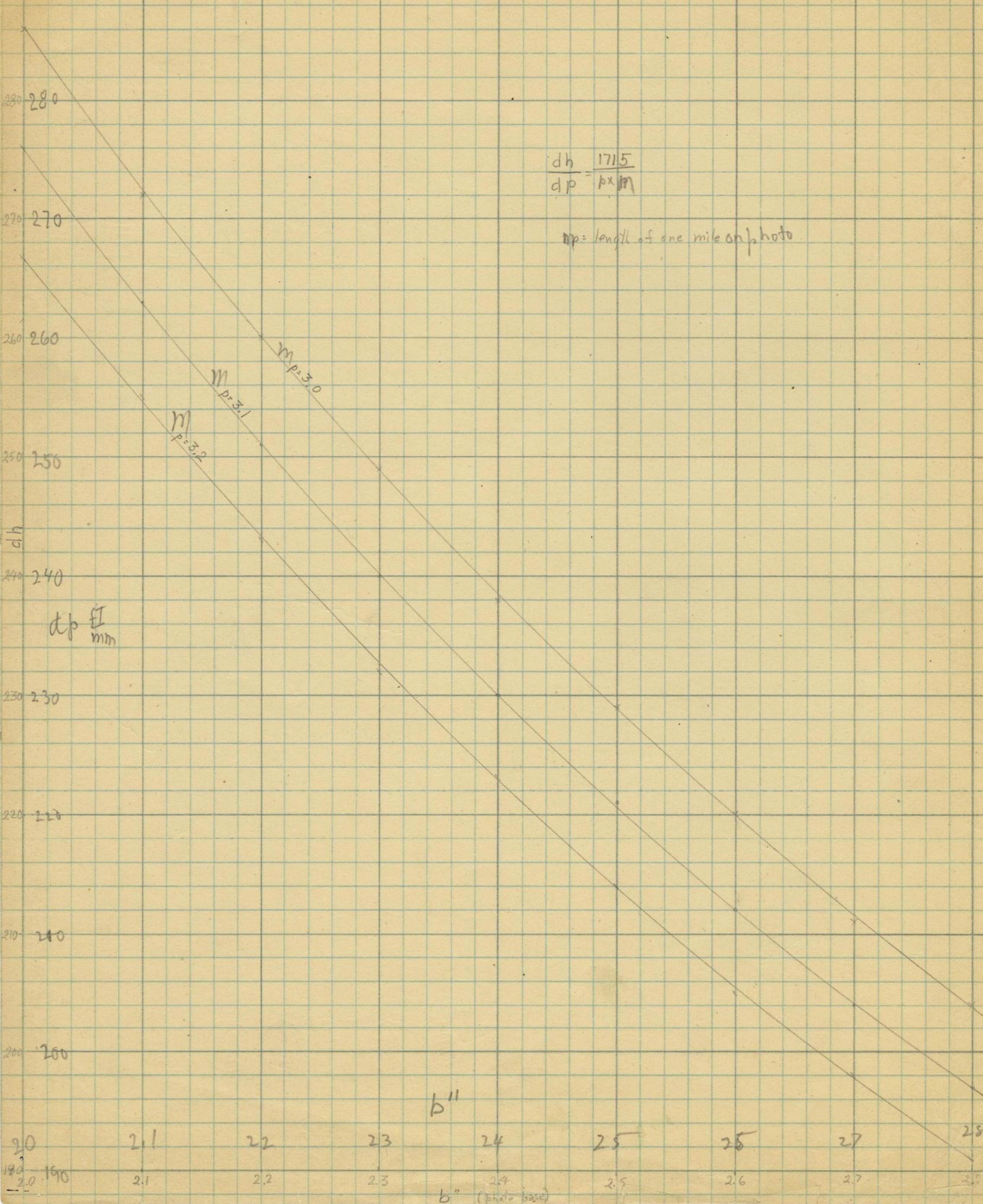

LENS
HORIZON
H
ELEVATION TO SCALE
OF MAP
G
GROUND TO MAP SCALE
0
PHOTO

25

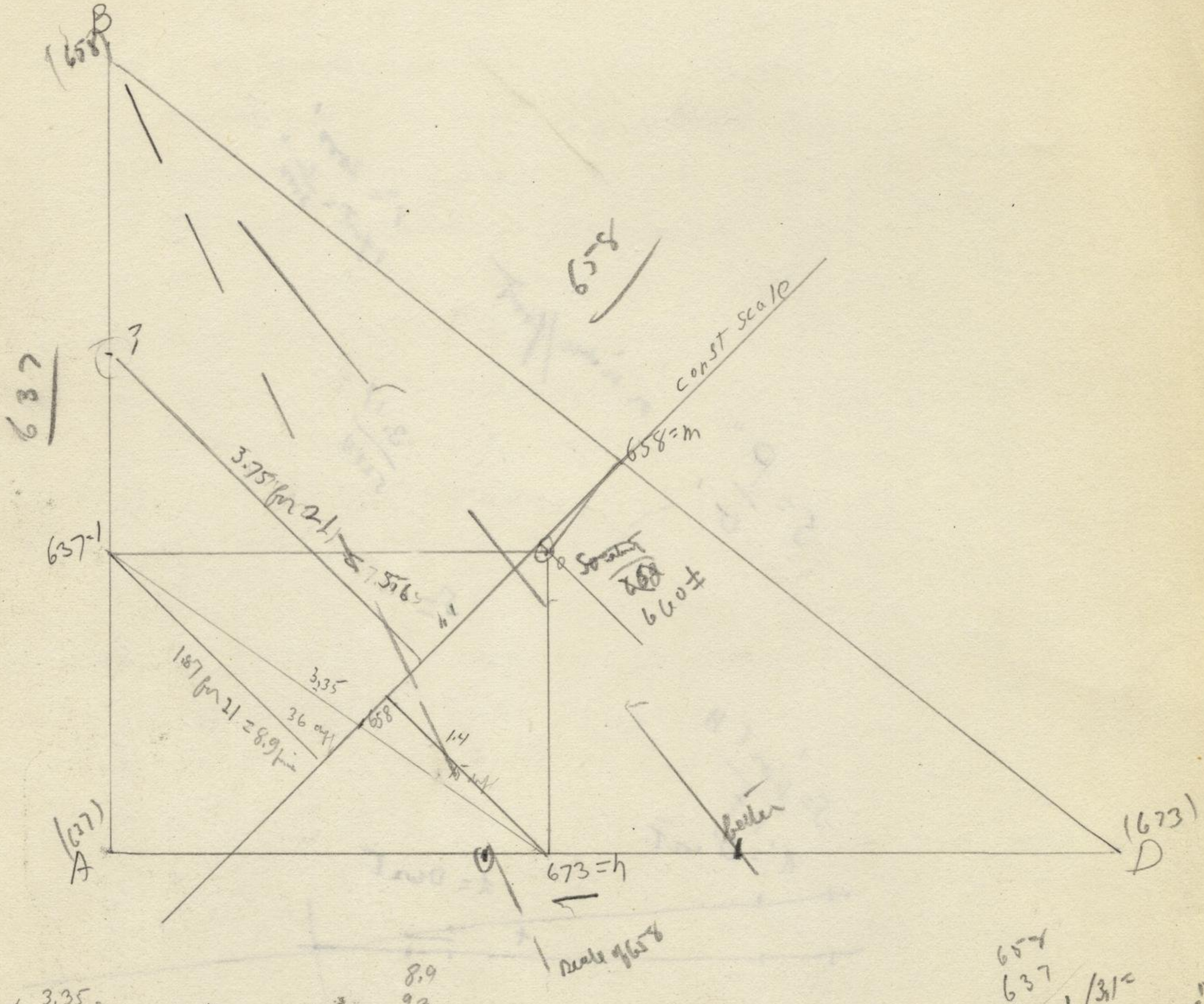


dp/dh = p×p/1715
dp^mm = 1.002915 b" p"
50' contours
dh/dp = 1715/b×p
p = 3.2"
p = 3.1"
p = 3.0
p = 3.0"
p = 3.1"
p = 3.2"
p = 3.20
p = 3.10
b"

Fig 10


dh/dp = 1715/px m
mp = length of one mile on photo
mp = 3.0
mp = 3.1
mp = 3.2
dh
dp ft/mm
280
270
260
250
240
230
220
210
200
190
180
b''
2.0
2.1
2.2
2.3
2.4
2.5
2.6
2.7
2.8
b'' (photo base)

Rihns method



$\frac{15}{36}$ of 3.35 =

1.4" for 15 dif

~~10.7~~ d's
9.3

8.9
93
182
av 9.1

658
637
21

673
637
36

673
658
15

658
637
21/3.1 =

21 change for 2.15"
1" per ~~9.7~~ 6.78

3.35 for 36

$6.37 \frac{15}{36} = 2.35$

$S = \frac{P''}{G'}$ or inches/feet

$1'' = 2000'$

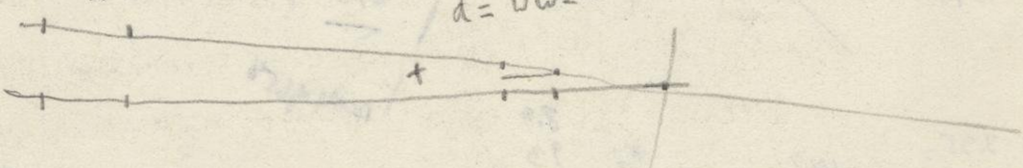
$1 \text{ foot} = \frac{1}{2000}''$

$\frac{3.17}{5280}$

$S = \frac{D' \cos T\, H}{D f}$

$d' = D' \cos T$

$d = D \cos T$



f = 6

Black Earth quarry.

Make a contour sketch of this photograph. Scale 1 inch to 200 feet. Figures represent elevations determined by leveling. How can you get distances? Do you need additional information? The camera was held 3 feet above the ground.

f = 5.0"