# Essays in Macro-Finance

by

Wentao Zhou

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The dissertation is approved by the following members of the Final Oral Committee:

Kenneth D. West, Professor, Economics
Kim J. Ruhl, Professor, Economics
Charles Engel, Professor, Economics
Oliver Levine, Associate Professor, Finance

*To my family.*

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# Abstract

This dissertation consists of three chapters investigating the role of financial frictions in transmitting macroeconomic shocks and its implications for stabilization policies. Each chapter employs both empirical and quantitative macroeconomic methods.

The first chapter studies both empirically and theoretically how macro uncertainty shocks affect the real economy via a firm balance sheet channel and highlights its novel policy implications. I document that following an increase in macro uncertainty, firm-level capital stock and outstanding debt fall while cash holdings increase, and such capital drop and cash buildup is more pronounced among ex-ante more indebted firms. I develop a quantitative heterogeneous firm model with financial frictions to illustrate the mechanism. In the model, firms fear liquidity shortages for debt repayments, thereby trading off capital investment for less debt burden and more cash holdings as heightened uncertainty creates greater downside risk. Cash buildup is strong, especially among more indebted firms, as cash preserves internal funds for both future debt repayment and growth opportunities triggered by increased uncertainty. A calibrated model featuring the transmission mechanism reproduces the observed impacts of macro uncertainty shocks at both micro and macro levels. Quantitative experiments suggest that conventional stimulus policies, like investment tax credits, yield only modest effects in counteracting the adverse impact of uncertainty shocks. In contrast, credit interventions, such as debt relief, can strongly and effectively stabilize uncertainty-driven recessions.

The second chapter studies the macroeconomic implications of debt covenants in a dynamic general equilibrium model that features long-term defaultable debt. In our model, the ex-post penalty associated with covenant violations aligns shareholders' incentives with lenders' interests in the face of default risk, thereby mitigating ex-ante debt dilution and debt overhang. We show that this mechanism has significant macroeconomic effects: (1). it reduces the counter-cyclical variation in firm leverage, default risk, and credit spreads, substantially lowering aggregate volatility; (2). it alleviates the debt overhang problem and thus boosts capital accumulation, resulting in

higher wages, output, and consumption. Our results, therefore, challenge the existing literature where debt covenants, modeled as distortionary borrowing constraints in models without default risk, amplify volatility and distort output. Moreover, we show that the calibrated economy with the level of covenant tightness observed in the U.S. approximates the constrained efficient allocation in which a social planner maximizes the values of both equity and debt claims.

The third chapter studies how financial frictions influence the transmission of monetary policy. Contrary to the financial accelerator effects on fixed capital investment in the literature, this chapter shows both empirically and theoretically that financial frictions dampen the effects of monetary policy shocks on inventory investment. Using firm-level data combined with externally identified monetary policy shocks, I first show that following contractionary monetary policy shocks, more financially constrained firms cut much fewer inventories than their less financially constrained counterparts despite similar effects of monetary policy shocks on their sales. To explain the empirical patterns, I build a dynamic New Keynesian general equilibrium model in which firms face demand uncertainty and financial frictions and thus manage inventory to avoid stock-outs and cash flow shortfalls. When contractionary monetary policy shocks lower households' demand for goods and thus firms' expected sales and revenues, more financially constrained firms slash their goods' prices and put more inventories on the shelves to increase operating cash flows, thereby avoiding costly external financing. My calibrated model successfully replicates a wide set of data features: pro-cyclical inventories and sales, counter-cyclical inventory-to-sales ratio and markups, and heterogeneous responses across differently financially constrained firms. Counterfactual exercises show that the aggregate effect of monetary policy is smaller in a more financially constrained economy through the inventory channel.

# Chapter 1

# The Firm Balance Sheet Channel of Uncertainty Shocks

## 1.1 Introduction

Macroeconomic uncertainty rises sharply after extreme events, such as the 9/11 terrorist attacks, the collapse of Lehman Brothers, and the recent COVID-19 pandemic. High macro uncertainty is recessionary, generating sharp and protracted drops in aggregate output.[1] It also leads to weak monetary policy transmission, contributing to slow recoveries from economic downturns.[2] The adverse effects of elevated macro uncertainty have made it important to understand (i). how macro uncertainty shocks transmit to the real economy, and (ii). how to stabilize uncertainty-driven recessions.

At the micro level, elevated macro uncertainty is followed by balance sheet adjustments across firms. Using firm-level data, I document that following an increase in uncertainty, firm-level capital stock and outstanding debt fall while cash holdings increase. In the cross-section, such capital drop and cash buildup are more pronounced among more indebted firms. The empirical patterns point to the importance of firms' cash demand and their *ex-ante* financial conditions in shaping their responses to uncer-

[1]An extensive literature has provided ample empirical evidence of the negative impact of heightened uncertainty on real economic activities. Seminal examples include Bloom (2009), Gilchrist et al. (2014), and Jurado et al. (2015)

[2]A growing literature has shown that monetary stimulus is less effective at stimulating output during periods of high uncertainty both empirically (see, e.g., Castelnuovo and Pellegrino 2018 and Aastveit et al. 2017) and theoretically (See, e.g., Vavra 2014, Baley and Blanco 2019, and Fang 2020).

tainty shocks while existing theories of uncertainty transmission abstract from these forces. [3]

In this paper, I show that accounting for corporate cash demand reveals a new transmission mechanism of uncertainty shocks. The new mechanism not only provides a unified explanation for the observed capital, cash, and debt dynamics following macro uncertainty shocks but also has novel policy implications. I first formalize the mechanism by constructing a quantitative heterogeneous firm model that features empirically consistent corporate borrowing and saving behavior. I then study policy counterfactuals in a calibrated model that reproduces the observed impacts of uncertainty shocks at both micro and macro levels.

The central feature of the mechanism is the interaction between elevated uncertainty and firms' precautionary behavior driven by financial frictions. My model captures two forces at play. On the one hand, an increase in uncertainty creates a greater downside risk, leading firms to cut capital investment in favor of lower debt burdens and higher savings to avoid costly liquidity shortages in the face of debt repayment. On the other hand, the higher uncertainty also triggers a greater upside potential, further spurring corporate savings. This occurs because firms use cash as their marginal source of funding when facing credit and equity market frictions and therefore save cash for future investment opportunities. The two forces thereby stimulate corporate savings while dampening borrowing, leading to large investment drops. Responses to these forces also depend on firms' *ex-ante* balance sheet conditions: more indebted firms build up more cash for large stocks of outstanding debt while waiting for potential investment opportunities. I refer to the transmission of uncertainty shocks through such joint capital, cash, and debt adjustments as *the firm balance sheet channel of uncertainty shocks*.

The balance sheet channel has two important lessons for policy. On one hand, heightened uncertainty elevates corporate cash demand, thereby reducing firms' responsiveness to investment stimulus policies. On the other hand, uncertainty-induced cash buildup and deleveraging could be mitigated through credit interventions. I simulate an uncertainty-driven recession in the quantitative model and show that conventional

[3]Existing transmission mechanisms include the 'real-options' channel driven by non-convex capital costs (See, e.g., Bloom (2009) and Bloom et al. (2018)), and the 'credit spreads' channel emphasizing higher borrowing costs driven by high uncertainty (See, e.g. Gilchrist et al. (2014) and Arellano et al. (2012)). In both frameworks, firms do not hold cash, and corporate saving motives and financial conditions have no role in uncertainty transmission.

stimulus policies, like investment tax credits, yield only modest effects in counteracting the adverse impact of uncertainty shocks, while credit interventions, such as debt relief, can strongly and effectively stabilize the recession. These results shed novel light on the policy responses to uncertainty shocks and recent debates on credit interventions as stabilization tools.

The paper begins by presenting new empirical evidence on firms' balance sheet adjustments following macro uncertainty shocks. Exploiting a panel local projection approach that combines COMPUSTAT firm-level data with the Macro Uncertainty Index of Jurado et al. (2015), I show that an increase in macroeconomic uncertainty is followed by declines in physical capital and outstanding debt alongside a cash buildup. A one-standard-deviation increase (4.5 %) in the index predicts a 0.8% lower capital stock, 2.0% lower outstanding debt, and 2.2% higher cash holdings on average across U.S. public firms six quarters after the initial shock. Such balance sheet changes reflect significant changes in firms' investment, borrowing, and saving decisions in response to elevated macro uncertainty. The results are robust to a wide set of firm and aggregate controls.

Importantly, *ex-ante* balance sheet conditions yield heterogeneous responses of capital and cash to uncertainty shocks across firms. A firm that is one standard deviation (measured by net leverage ratio) more indebted than its industry average has a larger decline in its capital stock and a larger increase in cash holdings. The heterogeneous responses also hold when using the within-firm variation over time, suggesting that firms respond more to shocks when they are more indebted. These results are robust to controlling for other firm-level heterogeneity, such as investment opportunities (proxied by Tobin's q and firm size), cash flows, debt maturity, and business cycle sensitivities.

To further confirm the empirical findings, I conduct an event study exploiting the 9/11 terrorist attacks as a plausibly exogenous increase in macro uncertainty (e.g., Bloom 2009; Kim and Kung 2017). I find that observed firm behavior around the 9/11 terrorist attacks aligns with the baseline results.

To illustrate the underlying mechanisms behind the data patterns and study policy counterfactuals, I incorporate corporate cash choice into a conventional heterogeneous firm model with capital and debt choices under financial frictions. In the model, firms invest in physical capital, borrow via long-term debt, and save using cash, subject to idiosyncratic productivity shocks, corporate taxation, financial frictions, and exogenous entry and exit. Firms borrow to enjoy the tax benefits of debt and to finance investment.

As in conventional models, firms face collateral constraints imposed by creditors and equity issuance costs. Financial frictions, along with exogenous entry and exit, enable the model to generate an endogenous and non-trivial mass of firms that differ in capital, cash, and debt.

The key innovation of the model is that it breaks the prevalent 'net debt' assumption in the macro-finance literature by introducing empirically consistent corporate borrowing and saving behavior.[4] Instead of being either savers or borrowers, firms in this model hold cash while having outstanding debt. The model relies on two additional financial frictions relative to conventional models. First, liquidity shortages when facing debt repayments incur cash flow penalties in the model.[5] The liquidity shortage penalty increases the marginal costs of borrowing, lowering firms' demand for debt. On the other hand, it motivates firms to hold cash in preparation for future debt repayment. Second, firms face debt issuance costs when they have outstanding debt that has not matured yet in the model. These costs capture the restrictive covenants existing creditors impose in a reduced-form way. The debt issuance frictions, along with conventional collateral constraints and equity issuance costs, motivate firms to hold cash for future investment opportunities. Specifically, when an investment opportunity is realized, cash holdings enable firms to finance investment internally without paying financing costs. Thus, firms hold cash as their marginal source of funding. I estimate the model parameters that govern firms' financial behavior to match firm-level moments on profitability, leverage ratio, cash ratio, and equity financing.

The key theoretical contribution of the paper is to show how elevated macro uncertainty, interacting with the frictions shaping corporate borrowing and saving behavior, leads to joint capital, cash, and debt adjustments across firms. The model captures two forces at work. On the one hand, elevated uncertainty implies a higher likelihood of low productivity tomorrow, resulting in a higher risk of liquidity shortages. This larger downside risk motivates firms to cut capital investment in favor of lower debt burdens and higher savings. On the other hand, elevated uncertainty also implies a larger chance of drawing high productivity, motivating firms to expand. Cash holdings play a unique role in such episodes: cash preserves internal funds for both debt repayment

[4] Jeenas (2019) introduces frictions in debt issuance, which makes cash-holding the marginal source of funding for firms, and studies its implication for monetary transmission. This paper features an additional cash-holding motive, saving for debt repayment, and finds both cash-holding motives are essential for understanding corporate cash choices in response to uncertainty shocks.

[5] This type of costly liquidity shortfall captures real-world difficulties that firms face in dealing with customers, employees, and strategic partners during liquidity distress.

and investment opportunities, addressing not only the downside risk but also the upside potential triggered by higher uncertainty. As a result, the greater upside potential triggered by heightened uncertainty further spurs corporate savings, especially for firms with larger stocks of outstanding debt.

I study the stationary equilibrium of a calibrated model and validate the model by showing its ability to reproduce observed investment, saving, and borrowing behavior, which are not targeted in the calibration. First, the calibrated model generates an endogenous mass of firms that differ in their balance sheet conditions, reproducing the observed cross-sectional variation in leverage and cash holdings in the data. Second, firms in the calibrated model save cash for future debt repayment, and the saving motive is stronger for more indebted firms. I show that in both the data and the model, conditional on investment opportunities, more indebted firms are associated with less capital investment and new borrowing but larger cash growth. Third, firms hold cash as their marginal source of funding in the model. Therefore, they draw down their cash reserves to fund capital investment when growth opportunities arise. I show that, in both the data and the model, idiosyncratic productivity growth is positively correlated with capital investment and debt growth, while negatively correlated with cash growth. Further, I show that models without the costly liquidity shortages or debt issuance frictions highlighted in this paper fail to reproduce these empirical patterns.

I then study the responses of the model economy to an unexpected uncertainty shock and show that the calibrated model reproduces the observed impacts of macro uncertainty shocks at both firm and aggregate levels. As standard in the literature, I compute the perfect foresight transition path of the economy to a mean-preserving spread in the aggregate productivity distribution. In the calibrated model, a sharp surge in macro uncertainty leads to significant balance sheet adjustments across firms that align well with the data patterns documented in the empirical exercises. It also generates a sharp and protracted drop in aggregate output along with a spike in the cross-sectional dispersion of sales growth, consistent with the empirical findings in the literature. Notably, existing uncertainty literature takes the observed increase in the dispersion of sales growth following uncertainty shocks as exogenous. Understanding the variable is important since it has been used to characterize periods of high uncertainty. The balance sheet channel suggests that the increased dispersion reflects heterogeneous responses to elevated uncertainty across differently indebted firms.

Finally, I exploit the calibrated model to illustrate the novel policy implications of the balance sheet channel. Two findings stand out. Stimulus policy, such as investment tax credits, that simulates aggregate output by 0.5% on impact during normal times can barely drive up aggregate output following an uncertainty shock. The weak effect of stimulus policy in uncertainty-driven recessions arises since heightened uncertainty motivates firms to hoard cash, thereby depressing firms' use of cash for policy-induced capital investments. In sharp contrast, the debt relief program that stimulates aggregate output by 0.5% on impact during normal times can drive up aggregate output by 1.5% following uncertainty shocks. The net present value of the debt relief program also exceeds one when implemented following uncertainty shocks. By reducing corporate deleveraging and cash buildup in response to elevated macro uncertainty, debt relief mitigates the balance sheet transmission of uncertainty shocks, thereby strongly and effectively stabilizing uncertainty-driven recessions.

My policy experiments provide novel insights into the recent debate on credit interventions as stabilization tools. First, existing work abstracts from the elevated uncertainty during recessions and argues credit interventions as ineffective stabilization tools. This paper contributes to the growing literature by highlighting the ability of credit interventions to counteract the adverse impact of uncertainty shocks. Second, I also show that credit interventions yield much weaker effects in a TFP-driven recession than in an uncertainty-driven recession, which suggests the nature of the recessions in shaping policy impacts. Third, I show that corporate cash choice is important for understanding the effects of credit interventions. A counterfactual simulation that fails to capture cash buildup following elevated uncertainty underestimates the stimulative effects of debt relief by more than 30%.

### 1.1.1 Literature and Contributions

The paper fits into a new research agenda discussed in Brunnermeier and Krishnamurthy (2020), which aims to integrate firm-level corporate financing considerations in quantitative macroeconomic models to study the macroeconomic implications of corporate financial decisions. In particular, this paper joins and contributes to four strands of literature in macroeconomics and finance.

**Financial frictions and aggregate shocks.** The paper contributes to a large macro-finance literature that studies the role of financial frictions and corporate financial decisions in transmitting and amplifying aggregate shocks. Seminal examples include

Bernanke et al. (1999), Cooley and Quadrini (2006),Khan and Thomas (2013), Gomes et al. (2016), Crouzet et al. (2016), Jungherr and Schott (2019), and Ottonello and Winberry (2020). This paper studies the role of corporate financial considerations in transmitting uncertainty shocks. Gilchrist et al. (2014) and Arellano et al. (2019) highlight corporate deleveraging in propagating uncertainty shocks. Like most macro-finance models, they abstract from corporate cash choice.[6]I build a model with joint capital, cash, and debt choices, which generates both deleveraging and cash buildup observed in the data. The paper highlights the importance of accounting for corporate cash behavior in understanding the transmission of uncertainty shocks and the stabilizing effects of credit interventions in an uncertainty-driven recession.

**Impact of uncertainty shocks.** This paper contributes to an extensive literature in macroeconomics and finance that studies the impact of heightened uncertainty on firm behavior. On the empirical front, I first provide new empirical evidence on the effects of uncertainty shocks on firms' capital, cash, and debt choices. My results echo some existing findings in the literature: a negative investment-uncertainty relationship (Leahy and Whited (1996), Bloom et al. (2007), Gulen and Ion (2016), Kim and Kung (2017), Kermani and Ma (2020)), a positive cash-uncertainty relationship (Opler et al. (1999), Bates et al. (2009), Gao et al. (2017), Smietanka et al. (2018), a negative debt-uncertainty relationship (Rashid (2013), Gilchrist et al. (2014). This paper also adds to the empirical literature by documenting heterogeneous responses of capital and cash to elevated uncertainty across differently indebted firms.

On the theoretical front, this paper illustrates how corporate saving motives and ex-ante financial conditions determine firms' responses to uncertainty shocks. The mechanism differs significantly from existing transmission mechanisms. The 'real-options' effect of uncertainty shocks emphasizes the delays in capital investment driven by non-convex investment technology (see, e.g. Bloom 2009, Bloom et al. 2018). Another strand of the theoretical literature emphasizes the positive effect of uncertainty shocks on credit spreads, which leads firms to cut investment and employees (see, e.g. Gilchrist et al. (2014) and Arellano et al. (2019)). Alfaro et al. 2019 incorporates corporate cash choice into a 'real-options' framework. They show that adding cash choice amplifies the classical 'real-options' effects. The driving force of their models is still the 'real-options' effect. As they show in the paper, once the 'real-options' effect is shut down, heightened

[6]Alfaro et al. (2019) shows that financial frictions amplify the 'real options' effects of uncertainty shocks on capital investment in a model with capital and cash choice. The core mechanism of their model is still the 'real options' effects of uncertainty shocks.

uncertainty does not affect capital investment. I see my mechanism as a third one in the literature, and my mechanism is motivated by different aspects of the data.

**Empirical and theoretical corporate finance.** The paper joins and builds on empirical and theoretical corporate finance literature. First, a large empirical corporate finance literature studies corporate cash-holding motives and finds empirical evidence on the role of cash holding in overcoming both financial distress and financing frictions. Some prominent examples include Opler et al. (1999), Faulkender and Wang (2006), Bates et al. (2009). This paper incorporates the two cash-holding motives into a structural model and shows their roles in rationalizing firm responses to uncertainty shocks. Second, the model framework builds upon existing dynamic corporate finance models, for example, Hennessy and Whited (2005), Titman and Tsyplakov (2007), Gamba and Triantis (2008), Riddick and Whited (2009), Eisfeldt and Muir (2016), Chen et al. (2021), Gomes and Schmid (2021) and Gao et al. (2021). I contribute to the literature by showing both empirically and theoretically how indebtedness shapes firms' choices between physical capital and cash holding, a trade-off faced by firms that have received relatively little attention in the literature so far.

**Recessions and Credit interventions.** Large-scale fiscal support for the corporate sector during the recent Covid crisis sparked a rapidly growing literature studying the efficacy of credit interventions using quantitative macro models, for example, Ebsim et al. (2020), Elenev et al. (2022), Crouzet and Tourre (2021), and Guntin (2022). Existing work focuses on recessions driven by first-moment shocks, such as a negative productivity shock, have very modest effects in recessions driven by aggregate productivity shocks. This paper instead emphasizes second-moment shocks in recessions and shows that credit interventions are powerful in stabilizing aggregate output drops driven by high uncertainty. This result sheds novel insights on the design of stabilization policies in recessions since heightened uncertainty has been a key feature of U.S. recessions ignored by existing work. Furthermore, I highlight the importance of models used to analyze policy impacts. Models that fail to capture observed cash buildup substantially underestimate output responses to debt relief programs.

The rest of the paper is organized as follows. Section 1.2 provides empirical evidence. Section 1.3 develops a quantitative heterogeneous-firm model with financial frictions. Section 1.4 discusses model calibration. Section 1.5 presents model mechanics and model validation. Section 1.6 studies the transmission mechanism of uncertainty shocks in the model. Section 1.7 examines the policy implications of the model. Section 1.8 concludes.

## 1.2 Empirical Evidence

In this section, I provide new empirical evidence on firm-level responses to heightened uncertainty. The empirical analysis highlights two key empirical patterns:

1. Following a macro uncertainty shock, physical capital and outstanding debt fall while cash builds up.

2. Firm indebtedness predicts heterogeneous asset choices in response to uncertainty shocks. The declines in physical capital and the buildup of cash holding are much more pronounced among *ex-ante* more indebted firms.

In Section 1.2.2, I exploit a Jordà (2005)-style local projection approach with firm-quarter data to estimate dynamic firm-level responses to changes in Macro Uncertainty Index by Jurado et al. (2015). In Section 1.2.3 and 1.2.4, I show that the baseline results hold both across and within firms and are robust to a wide set of controls and specifications.

### 1.2.1 Data

**Measure of aggregate uncertainty.** I employ the Macro Uncertainty Index developed by Jurado et al. (2015) as the baseline measure of macroeconomic uncertainty faced by U.S. firms, which captures forecast volatility of major macroeconomic variables implied by a large-scale time-series model. I take the quarterly average of their 1 month-ahead macroeconomic uncertainty index and use it as a proxy for quarterly macroeconomic uncertainty. Uncertainty shocks, or changes in aggregate uncertainty, are measured as the log growth of the index.

**Firm-level variables.** I draw firm-quarter observations from Compustat Quarterly. Compustat is ideal for this study: First, it contains rich balance sheet information, which allows me to study firms' financial behavior and measure firms' financial positions. Second, it includes detailed information on firms' sales and cash flows. This is important to a study that examines the effects of uncertainty (second-moment) on firm behavior, in which controlling for changes in first-moment variables, i.e. investment opportunities becomes essential. To the best of my knowledge, Compustat is the only U.S. dataset that satisfies these requirements. The sample period is 1990q1 to 2018q4, which avoids changes in accounting rules in the late 1990s and in 2019. Firms in the financial (SIC code 6000-6999), utilities (SIC code 4900-4949), and government-regulated industries

(SIC code > 9000) are excluded since the study focuses on non-financial corporate business. The key dependent variables include firm-level growth in physical capital, cash holding, and total outstanding debt. I also construct widely used firm-level control variables such as Tobin's Q, Sales Growth, Firm Size, Cash Flows, and Debt Maturity. All variables are deflated by the 2012 GDP deflator. Sample selection and variable construction follow standard practices in the literature, which is detailed in Appendix 1.9. Table 1.10 presents summary statistics of key firm-level variables.

**Firm indebtedness.** Firm indebtedness is defined as the net leverage of firms, total outstanding debt of firms *minus* their cash holding and then scaled by their total assets. To capture cross-sectional variation in indebtedness in each quarter, I standardize each of the firm-quarter observations of indebtedness for a firm $i$ in quarter $t$ by its industry-level average and standard deviation in quarter $t$. Therefore, the firm-level indebtedness measure used in the following regressions captures how one firm is more or less indebted than its industry average in each quarter. As documented by Kim and Kung (2017) and Gulen and Ion (2016), the impact of uncertainty varies across industries that feature different levels of capital irreversibility. Since the levels of indebtedness also vary across industries, the heterogeneous effects driven by differences in indebtedness might be simply driven by firms that operate in certain industries that feature both high indebtedness and high sensitivity to uncertainty shocks. The use of the 'within-industry cross-sectional variation' in indebtedness addresses this concern.

### 1.2.2 Firm-Level Responses to Uncertainty Shocks

**Baseline local projection.** I employ a Panel Local Projection empirical specification to estimate both the average responses to uncertainty shocks across all sample firms, as well as heterogeneous responses across differently indebted firms:

$$\underbrace{\Delta_h \log(y_{i,t+h})}_{\text{Cumulative growth}} = \alpha_{i,h} + \alpha_{fq,h} + \Big( \underbrace{\boldsymbol{\beta_h}}_{\text{Average}} + \underbrace{\gamma_h}_{\text{Heterogeneous}} \text{Indebtedness}_{i,t-1} \Big) \cdot \underbrace{\Delta \log \sigma_t}_{\text{Uncertainty Shock}} \tag{1.1}$$

$$+\eta_h \text{Indebtedness}_{i,t-1} + \Gamma_h' \underbrace{\mathbf{Z}_{i,t-1}}_{\text{Firm controls}} + \sum_{l=0}^{4} \Lambda_{l,h}' \underbrace{\mathbf{Y}_{t-l}}_{\text{Macro controls}} + \mu_{i,t+h}$$

$$\forall i, h = 0, 1, 2, 3, ..., 12$$

where $h \geq 1$ denotes the horizon at which the impact is being estimated, $\Delta_h \log(y_{i,t+h}) = \log(y_{i,t+h}) - \log(y_{i,t})$ is the cumulative growth in firm-level outcomes over horizon $h$. $\Delta \log \sigma_t$ denotes the growth in the Macro Uncertainty Index in quarter $t$. The coefficient of interest $\beta_h$, therefore, captures average growth in dependent variables across firms at quarter $t + h$ following a change in the Macro Uncertainty Index at quarter $t$. $\text{Indebtedness}_{i,t}$ measures how many standard deviations of firm $i$'s net leverage at $t$ is away from its industry average. The industry is defined as 1-digit SIC level. Hence, the coefficient of interest $\boldsymbol{\gamma_h}$ captures differences in firm growth at quarter $t+h$ among firms with differential indebtedness following a change in Macro Uncertainty Index at quarter $t$. If firm indebtedness affects how firms react to uncertainty shocks, then $\boldsymbol{\gamma_h}$ should be statistically significantly different from zero. Firm fixed effects $\alpha_{i,h}$ are included to absorb unobserved permanent differences across firms. Fiscal-quarter dummy $\alpha_{fq,h}$ is included to absorb the impact of differences in fiscal-quarter across firms on firm behavior. I cluster standard errors in two ways to account for correlation within firms and within quarters.

One common concern in estimating the effects of aggregate uncertainty is that changes in firm behavior following a rise in aggregate uncertainty might be driven by changes in other macroeconomic conditions that are correlated with changes in uncertainty. Recent literature has shown that uncertainty is counter-cyclical, and large rises in uncertainty tend to occur in recessions, see e.g. Bloom et al. (2018). To mitigate these concerns, I control both current and lagged macroeconomic variables $\sum_{l=0}^{4} \Lambda'_{l,h} \mathbf{Y}_{t-l}$, including real GDP growth rate, inflation rate, real federal funds rate, and credit spreads to absorb the effects of confounding macroeconomic forces on firm behavior. In addition, I include a vector of firm-level variables $\mathbf{Z}_{i,t-1}$ to control for cross-sectional differences in investment opportunities and financial conditions at the firm level: Tobin's Q, Sales Growth, Firm size, Cash Flows, and Debt Maturity, which are widely used in the empirical literature.

**Baseline results.** Figure 1.1 plots both average and heterogeneous responses of (a) physical capital, (b) cash holding, and (c) outstanding debt to a one-standard-deviation growth in Macro Uncertainty Index. Figure 1.1 shows that following a one-standard-deviation growth (4.5 %) in the Macro Uncertainty Index, average firm-level physical capital drops, cash holding grows, and outstanding debt falls. The average responses are statistically significant at the 5% significance level and persist for more than three years, with the peak appearing two years after the shock. The estimated average responses echo previous findings in the literature.

I find that variation in *ex-ante* firm indebtedness foreshadows a statistically significant shift in firms' asset choices following heightened uncertainty. Panel (A) and (B) of Figure 1.1 show that following a one-standard-deviation growth (4.5 %) in the Macro Uncertainty Index, the decline in physical capital is around 0.5% larger and the buildup of cash is around 1.5% larger for firms that are one-standard-deviation more indebted than their industry averages. Moreover, Panel (C) of Figure 1.1 shows that there is no statistically significant difference in debt growth across differently indebted firms. Taken together, instead of cutting more debt, *ex-ante* more indebted firms respond to heightened uncertainty by reallocating more of their assets towards cash holding. A key takeaway of the empirical analysis is that cash build-up is a salient feature of firm responses to an elevated uncertainty in the data, while conventional macro-finance models typically abstract from corporate cash choice when firms have outstanding debt.

### 1.2.3 Heterogeneous Responses by Firm indebtedness

**Extended local projection.** To mitigate concerns on the differential responses driven by variation in firm indebtedness, I estimate the following specification:

$$\Delta_h \log(y_{i,t+h}) = \alpha_{i,h} + \alpha_{fq,h} + \alpha_{s,t,h} + \underbrace{\gamma_h \text{Indebtedness}_{i,t-1} \cdot \Delta \log \sigma_t}_{\text{Heterogeneous responses}} + \beta_h \text{Indebtedness}_{i,t-1} \tag{1.2}$$

$$+ \underbrace{\boldsymbol{\Psi}'_h \mathbf{Z}_{i,t-1} \cdot \Delta \log \sigma_t + \Gamma'_h \mathbf{Z}_{i,t-1}}_{\text{Firm controls}} + \underbrace{\eta_h \text{Indebtedness}_{i,t-1} \cdot \Delta \log GDP_t}_{\text{Cyclical sensitivity}} + \mu_{i,t+h}$$

$$\forall i, h = 0, 1, 2, 3, ..., 12$$

where $h \geq 1$ denotes the horizon at which the impact is being estimated, $\frac{1}{h}\Delta_h \log(y_{i,t+h}) = \log(\frac{y_{i,t+h}}{y_{i,t}})$ is the average cumulative growth in firm-level outcomes over horizon $h$. $\Delta \log \sigma_t$ measures log growth in Macro Uncertainty Index at quarter t, and $\Delta \log GDP_t$ measures real GDP growth at quarter t. $\alpha_{i,h}$ indicate firm fixed effects. Fiscal-quarter dummy $\alpha_{fq,h}$ is included to absorb the impact of differences in fiscal-quarter across firms on firm behavior. Since the focus is heterogeneous responses across firms, I include industry-by-quarter fixed effects $\alpha_{s,t,h}$ to absorb differences in how broad industries are exposed to aggregate shocks. The industry is defined at 1-digit SIC level. $\text{Indebtedness}_{i,t-1}$ measures how many standard deviations of firm $i$'s net leverage at $t-1$ is away from its industry average at quarter $t-1$. $Z_{i,t-1}$ indicates a vector of firm-level control variables. The main coefficients of interest $\boldsymbol{\gamma_h}$ capture heterogeneous responses to changes in the Macroeconomic Uncertainty Index driven by pre-shock variation in corporate indebtedness across firms.

Regression (1.2) addresses two major concerns on the heterogeneous responses to uncertainty shocks driven by variation in firm indebtedness. First, Firm indebtedness is endogenous and might vary systematically with other dimensions of firms. For example, more indebted firms might simply have fewer investment opportunities, and thus the heterogeneous responses across differently indebted firms might be driven by cross-sectional variation in investment opportunities. To mitigate this type of concern, I interact $\boldsymbol{\Delta \log \sigma_t}$ with **Firm controls** that have been found to be important drivers of firms' investment and financial behavior: Tobin's Q, Sales Growth, Firm Size, Cash flows, and Debt Maturity. Hence, the extended specification also allows firms' responses to differ along other dimensions of firms. The second type of concern is that more indebted firms might be more sensitive to fluctuations in business cycles, which are negatively correlated with aggregate uncertainty. To mitigate this concern, I include an interaction term $\text{Indebtedness}_{\boldsymbol{i,t-1}} \cdot \boldsymbol{\Delta \log GDP_t}$ to absorb potential heterogeneity in cyclical sensitivity across firms with differential indebtedness.

Figure 1.2 shows that the baseline results are robust to including heterogeneous responses along other dimensions of firms and heterogeneous cyclical sensitivity across firms. Consistent with the baseline results, the response of capital growth is more negative, and the response of cash growth is more positive for *ex-ante* more indebted firms.

### 1.2.4 Additional Empirical Results

I conclude the empirical analysis with additional results and robustness exercises.

**Within-firm variation.** The baseline results suggest that cross-sectional variation in firm indebtedness predicts differential responses to uncertainty shocks. In Appendix 1.9.4, I show that similar patterns emerge when using within-firm variation in indebtedness over time. I compute the deviation of firm's net leverage from its unconditional firm-specific average, and interact it with uncertainty shocks. Figure 1.11 shows that the responses of physical capital and cash holding to changes in the Macro Uncertainty Index are also stronger when firms are more indebted than their own average levels. These results provide additional evidence on the role of firm indebtedness in shaping firm responses to uncertainty shocks.

**Event study: 9/11 terrorist attacks.** To further confirm the interpretation of the empirical findings, I conduct an event study that follows the uncertainty literature exploiting 9/11 terrorist attacks as a plausibly exogenous increase in aggregate uncertainty (e.g. Bloom (2009); Kim and Kung (2017)). Appendix 1.9.5 details the empirical design. I find that the firm behavior observed around the 9/11 terrorist attacks accords well with the baseline results. Panel A of Figure 1.12 shows that the post-9/11 period features statistically significant declines in physical capital and outstanding debt, as well as a large buildup in cash holding on average across firms. Panel B of Figure 1.12 shows that differences in lagged indebtedness predict differential asset choices in the post-911 period.

## 1.3 Quantitative Model

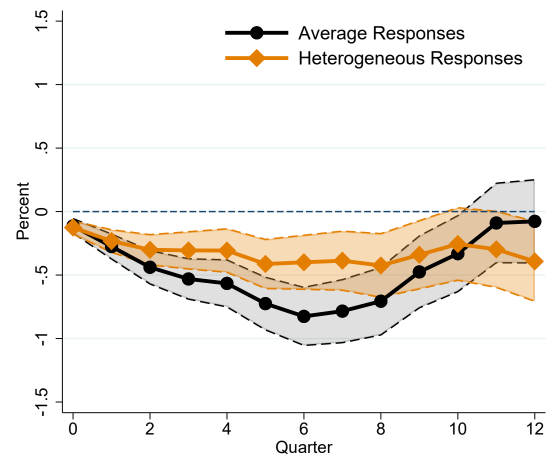
In this section, I develop a quantitative heterogeneous-firm model in which firms make optimal capital, cash, and debt choices in the presence of financial market frictions. Relative to the existing heterogeneous firm model with financial frictions, the model features empirically consistent corporate borrowing and saving behavior. To do so, my model incorporates two additional financial frictions motivated by corporate finance literature : (i). costly liquidity shortages in the face of debt repayment; (ii). debt issuance frictions when firms have an outstanding debt that has not matured. In this section, I first describe the details of the model in a stationary industry equilibrium. I study the perfect transition path of the economy to uncertainty shocks in Section 1.6,.

### 1.3.1 Environment

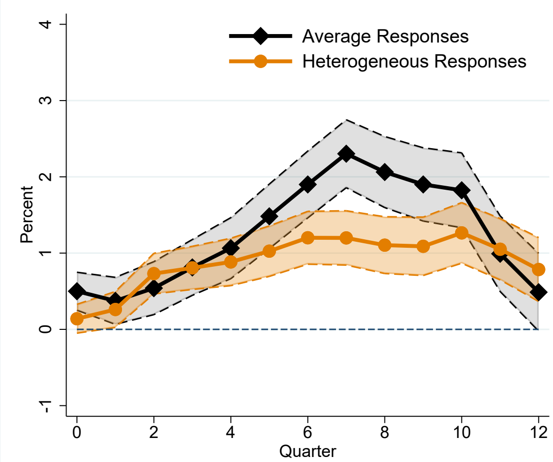
Time is discrete and the horizon is infinite. The economy is populated by a continuum of heterogeneous firms that make optimal investment and financial decisions in the presence of idiosyncratic productivity shocks and financial market frictions. Firms produce a homogeneous good in a competitive market and sell their products at a price of 1. They hire labor in the labor market at a wage rate $W$ determined by labor market clearing.

There is a representative household that has preferences over the final consumption

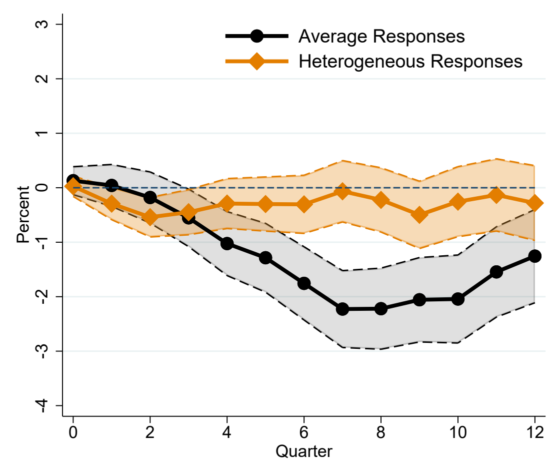
**(a) Physical capital**




**(b) Cash holding**




**(c) Outstanding debt**




**Notes:** the figure plots both the average and heterogeneous responses of (a) physical capital, (b) Cash holding, and (c) outstanding debt to a one-standard-deviation growth in Macro Uncertainty Index by Jurado et al. (2015) at quarter $t$. The heterogeneous responses are driven by cross-sectional variation in indebtedness at quarter $t-1$. Indebtedness$_{i,t-1}$ measures how many standard deviations of firm $i$'s net leverage at $t-1$ is away from its industry average at quarter $t-1$. Point estimates and 95% confidence intervals for $\beta_h$ and $\gamma_h$ are plotted. Standard errors are two-way clustered at both firm and time levels. The sample period is from 1990Q1 to 2018Q4.

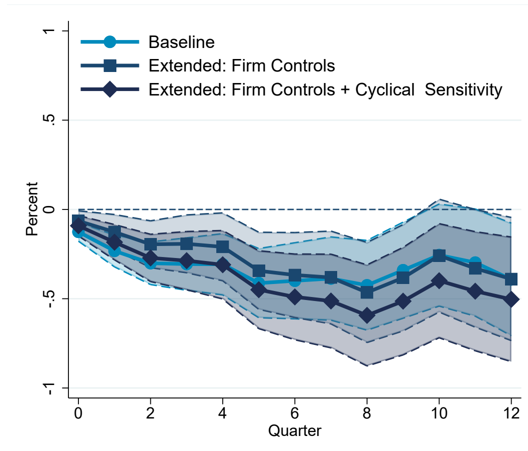
**Figure 1.1:** Baseline Local Projection: Firm-Level Responses to Uncertainty Shocks

good and supplies labor according to

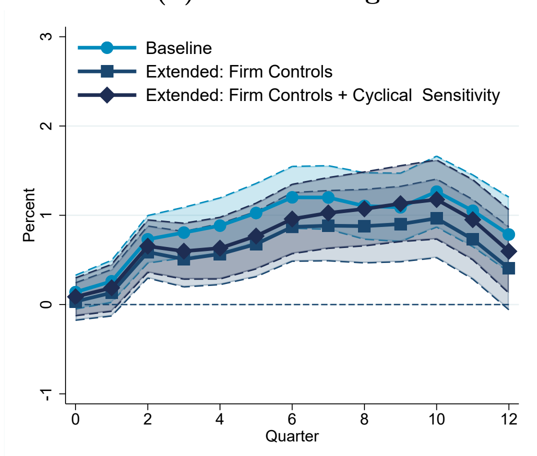
$$L^s(W) = \psi W^{\zeta}, \tag{1.3}$$

where $\psi > 0$ denotes the disutility of working, and $\zeta > 0$ is the labor supply elasticity.

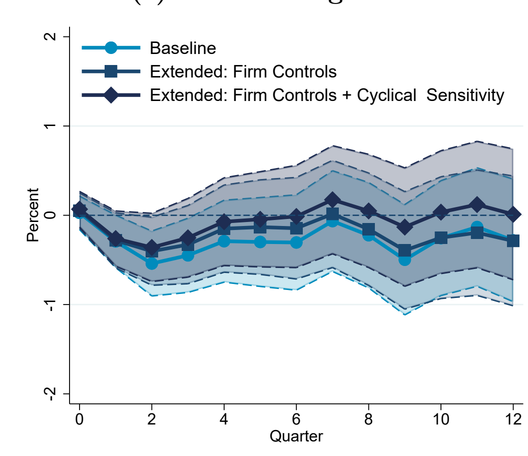
**(a) Physical capital**




**(b) Cash holding**




**(c) Outstanding debt**




**Notes:** this figure plots both the heterogeneous responses of (a) physical capital, (b) cash holding, and (c) outstanding debt to a one-standard-deviation growth in Macro Uncertainty Index by Jurado et al. (2015) at quarter $t$. The heterogeneous responses are driven by cross-sectional variation in indebtedness at quarter $t-1$. Indebtedness$_{i,t-1}$ measures how many standard deviations of firm $i$'s net leverage at $t-1$ is away from its industry average at quarter $t-1$. I interact $\boldsymbol{\Delta \log \sigma_t}$ with **Firm controls** that have been found to be important drivers of firms' investment and financial behavior: Tobin's Q, Sales Growth, Firm Size, Cash flows and Debt Maturity. Hence, the extended specification also allows firms' responses to differ along other dimensions of firms. I also include an interaction term Indebtedness$_{\boldsymbol{i,t-1}} \cdot \boldsymbol{\Delta \log GDP_t}$ to absorb potential heterogeneity in cyclical sensitivity across firms with differential indebtedness. Point estimates and 95% confidence intervals for $\beta_h$ and $\gamma_h$ are plotted. Standard errors are two-way clustered at both firm and time levels. The sample period is from 1990Q1 to 2018Q4.

**Figure 1.2:** Extended Local Projection: Heterogeneous Responses by Firm indebtedness

There is also a mass of risk-neutral/deep-pocketed financial intermediaries who provide financial services.

I first study a stationary equilibrium in which there is no aggregate shock and all aggregate variables are constant. I then study the perfect foresight transition path in response to unexpected uncertainty shocks. I drop subscripts for a firm $i$ and period $t$, and adopt the recursive timing convention, except in parts where such choice may jeopardize the clarity of exposition.

### 1.3.2 Firm's Setup

Firms are **risk-neutral** and discount the future at an exogenous risk-free interest rate $r$. Firms have access to the same production and financing technologies. In each period, each firm's risk-neutral manager maximizes the expected present value of dividends to equity holders by choosing capital, cash, and debt.

**Technology.** Each firm combines physical capital $k$ and labor $l$ to produce a homogeneous good $y$ using a decreasing return to scale production technology. Firm production is subject to idiosyncratic productivity shocks $z$. The production function is as follows:

$$y = z^{1-\nu} k^{\alpha} l^{\nu}, \alpha + \nu < 1 \tag{1.4}$$

$\alpha$ is the value-added share of capital, and $\nu$ is the value-added share of labor.

**Productivity.** Firm-specific productivity shock $z_{it}$ evolves according to

$$\log(z_{i,t+1}) = \mu_t + \rho \log(z_{it}) + \sigma_t \epsilon_{i,t+1} \tag{1.5}$$

where the innovations $\epsilon_{i,t+1} \sim N(0,1)$ are independent across firms. $\mu_t$ denotes an adjustment to the level of firms' productivity. $\sigma_t$ denotes the volatility of the innovations.

Equation (1.5) implies that the level of volatility $\sigma_t$ today determines the distribution of next-period idiosyncratic productivity $z'(\sigma_t)$. Thus, from the perspective of firms in the model, high volatility $\sigma_t$ today indicates a more widely spread distribution of tomorrow's idiosyncratic productivity. That is, during a period of high volatility $\sigma_t$, firms are more likely to draw both high and low idiosyncratic productivity, a scenario where firms face both higher downside risk (due to a higher probability of bad productivity shock) and larger growth potential (due to higher probability of good productivity shock). As in Gilchrist et al. (2014), the volatility term $\sigma_t$ is common across firms and thus an increase in $\sigma_t$ affects all firms and hence captures "uncertainty" shock in the aggregate sense.

Importantly, $\mu_t$ is chosen to be $\frac{-\sigma_t^2}{2}$ , and consequently the conditional mean of firms' productivity $E[\log(z_{i,t+1})|\log(z_{i,t}), \sigma_t]$ is not affected by the level of volatility $\sigma_t$. In other words, changes in $\sigma_t$ do not affect the expected aggregate productivity of the economy. Therefore, a change in $\sigma_t$ is also considered as a "Second-moment Shock" or a "Dispersion Shock". In this paper, I first solve for a stationary equilibrium by fixing $\sigma_t$ at $\sigma_L$ to study firms' optimal investment and financial decisions. I then study a perfect foresight transition path in response to unexpected jumps in $\sigma_t$, i.e. uncertainty shocks.

**Operating profits.** Physical capital $k$ is owned by firms and chosen one period before. After the realization of idiosyncratic productivity $z$ each period, firms hire labor from a competitive labor market at a wage rate $W$ to maximize their operating profits. As in Gilchrist et al. (2014) and Xiao (2018), firms also pay operating costs each period. To account for the fact that bigger firms tend to incur larger operating costs, these costs are scaled by firms' existing stock of physical capital. Thus, a firm with physical capital $k$ will pay operating costs $f_o k$. [7] Firms' per-period operating profits are therefore given by the solution to the following static profit-maximization problem:

$$\begin{aligned}\pi(z,k;W) &= \max_l \{z^{1-\nu}k^\alpha l^\nu - f_o k - Wl\} \\ &= (1-\nu)\left(\frac{\nu}{W}\right)^{\frac{\nu}{1-\nu}} z k^{\frac{\alpha}{1-\nu}} \\ &= z\psi(W)k^\gamma - f_o k\end{aligned}$$

where $W$ denotes the (real) wage and

$$\gamma = \frac{\alpha}{1-\nu} \qquad \text{and} \qquad \psi(W) = (1-\nu)\left(\frac{\nu}{W}\right)^{\frac{\nu}{1-\nu}}$$

This setup ensures that the firm's profit function is linear in its productivity, as in Gilchrist et al. (2014). The detailed solution to the problem is shown in Appendix 1.10.1.

**Asset structure.** Firms own physical capital, which depreciates at a constant rate $\delta > 0$. Each period firms have an opportunity to choose their next period's capital stock,

[7] The operating cost $f_o$ helps to match the average operating profits of firms in the data, which further affects average cash held by firms, as in Xiao (2018).

$k'$. The law of motion for firms' capital stock is given by

$$k' = (1 - \delta)k + i \tag{1.6}$$

where $i$ denotes the net capital (dis)investment of firms.

In addition to holding physical capital $k$, firms can also save in liquid assets $c$ at an exogenous risk-free rate $r$. I interchangeably refer to liquid assets as "cash" throughout the paper.

**Entry and Exit.** As in Khan and Thomas (2013), firms are forced to exit the economy after production with a fixed probability $\pi^e$, This assumption precludes all firms from overcoming the financial frictions in the steady state of the economy, which leads to an unrealistic and uninteresting firm distribution. The exit shock is i.i.d across firms and time. Equity holders of exiting firms receive the residual firm value, i.e. book value of total assets net of all debt obligations. Exiting firms are then replaced by entrants such that there is always a unit mass of firms. Entrants' problems are discussed in greater detail in Section 1.3.5. Firms that survive the exit shocks choose next-period physical capital, cash holding, and outstanding debt and enter the next period with entrants.

### 1.3.3 Sources of Funds and Financial frictions

Firms can finance their assets and operation through three different sources of funds: internal liquidity, debt, and outside equity. Firms enter the period with their physical capital $k$, liquid assets holding $c$, and outstanding debt $b$.

**Internal liquidity.** Each period, after production and tax, the internal liquidity available to the firms includes after-tax operating profits, liquid assets holding, and tax rebates:

$$\underbrace{l(z,k,c,b)}_{\text{Internal liquidity}} = (1-\tau)\underbrace{\pi(z,k)}_{\text{Operating profits}} + \underbrace{[1+(1-\tau)r]c}_{\text{Liquid assets}} + \underbrace{\tau(rb+\delta k)}_{\text{Tax rebates}} \tag{1.7}$$

where $\tau$ denotes the corporate tax rate. Note that interest income $rc$ from corporate cash savings are taxed, and interest expenses $rb$ and depreciation $\delta k$ are tax-deductible.

**Debt financing.** Firms in the model take on debt to finance their asset choices or to enjoy the tax shield of debt. Risk-neutral deep-pocket lenders impose a collateral constraint ensuring that the outstanding debt obligation is not larger than the value of the capital stock, and thus debt service only requires a coupon rate equal to the risk-free rate $r$. Consequently, firms' choice of next-period debt $b'$ must satisfy the borrowing constraint:

$$\underbrace{(1+r)b'}_{\text{debt obligation}} \leq \theta \underbrace{(1-\delta)k'}_{\text{collateral value}}, 0 < \theta < 1 \tag{1.8}$$

where $\theta$ denotes the pledgeability of physical capital.

**Debt maturity and adjustment.** As in Gomes and Schmid (2021) and Chen et al. (2021), corporate debt is modeled as long-term debt that matures randomly with a given probability $\lambda$. Specifically, with probability $\lambda$, the firm's outstanding debt matures, and with probability $1-\lambda$, the firm's outstanding debt continues, and firms only repay coupon payments. The expected debt maturity is, therefore, $\frac{1}{\lambda}$. This setup keeps the model tractable while allowing the model to generate a more realistic debt maturity process: (i). The average maturity of outstanding debt of non-financial firms is significantly longer than one period. (ii). Firms pay coupon payments before maturity and repay the principal at maturity.

When existing debt matures, firms cannot take on new debt until they have repaid their debt obligations in full,[8] and importantly, firms are in liquidity shortfalls if internal liquidity is insufficient to meet their maturing debt obligations.[9] The liquidity gap for debt repayment is given by:

$$\underbrace{m}_{\text{Liquidity gap}} = \underbrace{l(z,k,c,b)}_{\text{Internal liquidity}} - \underbrace{(1+r)b}_{\text{Maturing debt obligations}}$$

where $m < 0$ indicates the event of liquidity shortfalls. When liquidity shortfalls arise, a penalty is triggered. Specifically, firms suffer a cash flow penalty proportional to their liquidity gap, as in Titman and Tsyplakov (2007). In these scenarios, firms' cash flows after taking into account costly liquidity shortfalls can be written as:

$$m - s \cdot |m| \cdot \mathbf{1}_{m<0} \tag{1.9}$$

where $s$ is the parameter that governs the costs of liquidity shortfalls. Firms can then finance their liquidity shortfalls by disinvestment or new debt/equity issuance.

[8]This timing convention follows Hennessy and Whited (2005), Titman and Tsyplakov (2007), Gamba and Triantis (2008), and so on.

[9]Hennessy and Whited (2005) abstracts from corporate cash holding, and thus firms are considered in liquidity shortfalls as long as firms' realized operating profits are insufficient to cover their debt burdens. This paper models cash holding explicitly and highlights firms' liquidity management.

Prior to debt maturity, indebted firms can adjust their outstanding debt, that is, debt is callable at par.[10] Firms can lower their debt level without any costs while increasing debt level entails issuance costs proportional to additional debt issued $\eta$. The debt adjustment of firms with non-maturing debt can be summarized as follows:

$$\underbrace{R(b,b')}_{\text{Debt adjustment}} = \begin{cases} b' - (1+r)b - (1-\eta)\left|b' - b\right|, & \text{if } b' > b \\ b' - (1+r)b, & \text{if } b' < b \end{cases} \tag{1.10}$$

The debt issuance costs capture the difficulties of issuing new debt when firms have debt outstanding. For example, restrictive covenants and seniority rules in debt contracts make new debt issuance especially costly.

**Equity financing.** Firms' choices of next-period physical capital $k'$, liquid assets holding $c'$, and outstanding debt $b'$, together with their internal liquidity $l(z,k,c,b)$ and undepreciated capital stock $(1-\delta)k$, determine firms' cash flows to their equity holders $d$. When $d \geq 0$, it represents dividend payout to the equity holders. When $d < 0$, firms issue new equity. Following Hennessy and Whited (2007) and Eisfeldt and Muir (2016), the equity issuance cost is as follows:

$$\Phi(d) = \mathbf{1}_{d<0} \cdot \left(\kappa_0 + \frac{\kappa_1}{2} d^2\right) \tag{1.11}$$

where $\kappa_0$ captures the fixed costs of equity issuance and $\kappa_1$ captures the variable costs of equity issuance. The equity issuance costs reflect agency problems in financial markets.

### 1.3.4 Timing

The timing of events within each period is as follows:

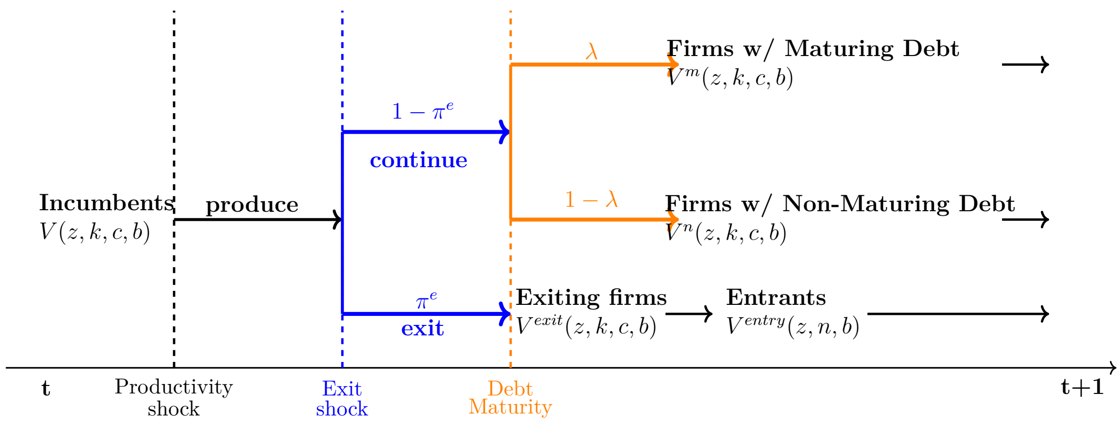
1. Firms enter the period with physical capital $k$, liquid assets holding $c$, and outstanding debt $b$. After observing their idiosyncratic productivity $z$, firms hire labor to maximize their current operating profits. Firms also observe aggregate uncertainty $\sigma_t$ and thus form beliefs about tomorrow's idiosyncratic productivity.

[10] Conceptually, debt restructuring, as in Goldstein et al. (2001), requires firms to call back all of their outstanding debt first and then issue new debt at the desired level. Therefore, there is always only one vintage of debt from the firm's most recent restructuring.

2. After production, exit shocks realize. $\pi^e$ fraction of firms that are hit by exit shocks exit the economy permanently. $(1-\pi^e)$ fraction of incumbent firms continue to the next stage.

3. With probability $\lambda$, firm's outstanding debt $b$ matures. Continuing firms with maturing debt repay their debt first, then choose next-period capital $k'$/cash $c'$ /new debt $b'$. Continuing firms with non-maturing debt can choose next-period capital $k'$, cash $c'$, and debt $R(b, b')$.

4. Potential entrants replace exiting firms and solve entrants' problems. They then enter the next period with continuing firms.




### 1.3.5 Firms' Problems

I now characterize firms' problems recursively in detail.

**Begin-the-period firm value.** Let $V(z, k, c, b)$ represent the expected discounted value of a firm that enters the period with productivity $z$, physical capital $k$, liquid assets holding $c$, and outstanding debt $b$ before it learns whether it will exit and whether its outstanding debt will mature.

$$\underbrace{V(z,k,c,b)}_{\text{Begin-the-period Firm Value}} = \pi^e \underbrace{V^{exit}(z,k,c,b)}_{\text{Value of Exiting Firms}} + (1-\pi^e)\Big[\underbrace{\lambda V^m(z,k,c,b) + (1-\lambda)V^n(z,k,c,b)}_{\text{Value of Continuing Firms}}\Big] \tag{1.12}$$

**Value of existing firms.** Equity holders of exiting firms receive the residual firm value, i.e. book value of total assets net of all debt obligations. Therefore, the value of exiting firm is defined as follows:

$$\underbrace{V^{exit}(z,k,c,b)}_{\text{Value of Exiting Firms}} = \underbrace{l(z,k,c,b) + (1-\delta)k}_{\text{Asset value}} - \underbrace{(1+r)b}_{\text{debt value}} \tag{1.13}$$

**Value of continuing firms w/ maturing debt.** Conditional on survival, firms with maturing debt today need to pay off their maturing debt obligations. As discussed in Section 1.3.3, when their internal liquidity is insufficient to cover debt repayment, they suffer a reduction in their cash flows. They then choose next period's capital $k'$, cash $c'$, and new debt $b'$ to maximize:

$$V^m(z,k,c,b) = \max_{k',c',b'} d - \Phi(d) + \frac{1}{1+r} E_{z'|z}[V(z',k',c',b')] \tag{1.14}$$

subject to

$$\begin{aligned}
\text{[Liquidity gap]:} \quad & m = \underbrace{l(z,k,c,b)}_{\text{Internal liquidity}} - \underbrace{(1+r)b}_{\text{Maturing debt obligations}} \\
\text{[Dividend flow]:} \quad & d = m - \underbrace{\mathbf{s} \cdot |m| \cdot \mathbf{1}_{m<0}}_{\textbf{Liquidity shortfalls}} - \underbrace{[k' - (1-\delta)k]}_{\text{Investment}} - \underbrace{c'}_{\text{Cash}} + \underbrace{b'}_{\text{new debt}} \\
\text{[Borrowing constraint]:} \quad & (1+r)b' \leq \theta(1-\delta)k', 0 < \theta < 1 \\
\text{[Equity issuance costs]:} \quad & \Phi(d) = \mathbf{1}_{d<0} \cdot \left(\kappa_0 + \frac{\kappa_1}{2} d^2\right)
\end{aligned}$$

**Value of continuing firms w/ non-maturing debt.** Continuing firms with non-maturing debt can choose next-period capital $k'$, cash $c'$, and debt $R(b,b')$ to maximize:

$$V^n(z,k,c,b) = \max_{k',c',b'} d - \Phi(d) + \frac{1}{1+r} E_{z'|z}[V(z',k',c',b')] \tag{1.15}$$

subject to

$$\text{[Dividend flow]:} \quad d = \underbrace{l(z,k,c,b)}_{\text{Internal liquidity}} + \underbrace{R(b,b')}_{\textbf{Debt adjustment}} - \underbrace{[k' - (1-\delta)k]}_{\text{Investment}} - \underbrace{c'}_{\text{Cash}}$$

$$\text{[Debt adjustment]:} \quad R(b,b') = \begin{cases} (1-\boldsymbol{\eta})(b'-b) - rb, & \text{if } b' > b \\ b' - (1+r)b, & \text{if } b' < b \end{cases}$$

$$\text{[Borrowing constraint]:} \quad (1+r)b' \leq \theta(1-\delta)k', 0 < \theta < 1$$

$$\text{[Equity issuance costs]:} \quad \Phi(d) = \mathbf{1}_{d<0} \cdot \left(\kappa_0 + \frac{\kappa_1}{2}d^2\right)$$

**Value of entrants.** Every period, entrants will replace exiting firms. Entrants draw an initial realization of the idiosyncratic shock $z$ from the long-run invariant distribution implied by Equation (1.5), denoted by $\mu^{Entry}(z)$. They know about their initial asset size $n_0$, and initial leverage ratio $\frac{b_0}{n_0}$, and then choose their next period's asset portfolio $k'$ and $c'$:

$$V^{entry}(z, n_0, b_0) = \max_{c',k'} \beta E_{z'}[V(z', k', c', b_0)] \tag{1.16}$$

$$k' + c' = n_0$$

where $n_0$ and $b_0$ will be calibrated to match the size of entrants and the average entrant's leverage ratio in the data.

### 1.3.6 Equilibrium

**Firm distribution.** I begin by defining $\mu(z,k,c,b)$ as the cross-sectional distribution of firms over idiosyncratic productivity $z$, physical capital $k$, liquid assets holding $c$, and outstanding debt $b$. The evolution of the distribution of firms $\mu_{t+1}(z,k,c,b)$ is given by

$$\begin{aligned} &\mu_{t+1}(z',k',c',b') = \\ &(1-\pi_e)\Bigg[\int\int_{z'} \underbrace{\lambda \mathbf{1}\{\hat{k}_t^m(z,k,c,b) = k'\} \times \mathbf{1}\{\hat{c}_t^m(z,k,c,b) = c'\} \times \mathbf{1}\{\hat{b}_t^m(z,k,c,b) = b'\}}_{\text{transition of continuing firms with maturing debt}} \\ &+ \underbrace{(1-\lambda)\mathbf{1}\{\hat{k}_t^n(z,k,c,b) = k'\} \times \mathbf{1}\{\hat{c}_t^n(z,k,c,b) = c'\} \times \mathbf{1}\{\hat{b}_t^n(z,k,c,b) = b'\}}_{\text{transition of continuing firms with non-maturing debt}} dF(z'|z)d\mu_t(z,k,c,b)\Bigg] \\ &+ \pi_e\Bigg[\int\int_{z'} \underbrace{\mathbf{1}\{\hat{k}_t^o(z,n_0,b_0) = k'\} \times \mathbf{1}\{\hat{c}_t^o(z,n_0,b_0) = c'\} \times \mathbf{1}\{\hat{b}_t^o(z,n_0,b_0) = b_0\}}_{\text{transition of entry firms}} dF(z'|z)d\mu^{Entry}(z)\Bigg] \end{aligned} \tag{1.17}$$

where $\{\hat{k}^m, \hat{c}^m, \hat{b}^m\}$ denote the policy functions of firms with maturing debt, $\{\hat{k}^n, \hat{c}^n, \hat{b}^n\}$ denote the policy functions of firms with non-maturing debt, and $\{\hat{k}^o, \hat{c}^o, \hat{b}^o\}$ denote the policy functions of entrants.

**Aggregation.** Given the firm distribution $\mu_t(z, k, c, b)$, I aggregate firm-level variables to aggregate variables. The aggregate output and aggregate labor demand are given by:

$$Y_t = \int y_t(z, k, c, b) d\mu_t(z, k, c, b) \quad \text{and} \quad L_t^d = \int n_t(z, k, c, b) d\mu_t(z, k, c, b)$$

Similarly, other variables can be aggregated, such as aggregate capital stock, liquid assets, and outstanding debt.

**Equilibrium definition.** *A stationary industry equilibrium in this economy consists of (i). aggregate prices: wage $W$ and interest rate $r$, (ii). firm value functions$\{V, V^m, V^n, V^{entry}, V^{exit}\}$ ,*
*related firms policy functions, (iii). firm distribution $\mu(z, k, c, b)$, and a measure of entrants $\mu^{entry}(z)$ such that*

1. *Given $W$ and $r$, $V(z, k, c, b), V^e(z, k, c, b), V^m(z, k, c, b), V^n(z, k, c, b)$ solve the continuing firms' problems (1.14) - (1.15) with related policy functions.*

2. *Given $r$, $\tilde{V}(z, n_0, b_0)$ solve the entrants' problem (1.16) with related policy functions.*

3. *The labor market clears:*

$$L_t^d = \int n_t(z, k, c, b) d\mu_t(z, k, c, b) = L^s(W) = \psi W^{\zeta}, \forall t$$

4. *The distribution of firms satisfies (1.17). In a steady state, the distribution's law of motion is a fixed point.*

### 1.3.7 Optimal Firm Policies

In this subsection, I analyze firms' optimal investment and financial policies in detail, tracing their costs and benefits. For illustration purposes, I assume firms' value functions are differentiable.[11] For simplicity, I set the exogenous exit probability $\pi^e = 0$ in this section. Details on the analytical derivations below can be found in Appendix 1.10.1.

[11]Firms' value functions are not everywhere due to equity issuance costs, liquidity shortfalls, and debt issuance costs.

**Optimal payout policy.** In an economy without equity issuance costs, the marginal value of cash flows to shareholders is always equal to one. On the other hand, the marginal value of firms' cash flows to shareholders might be greater than one in the presence of costly external finance. The first-order condition for dividends reveals the marginal value of firms' cash flows to shareholders in the model :

$$\Lambda(d) = \begin{cases} 1, & \text{if } d \geq 0 \\ 1 + \kappa_1 |d|, & \text{if } d < 0 \end{cases} \tag{1.18}$$

Formally, the marginal value of firms' cash flows to shareholders equals one when firms payout dividends $d \geq 0$, while it becomes larger than one when firms issue new equity $d < 0$ due to the equity issuance costs. In the model, an additional unit of internal funds might help firms avoid and reduce costly external finance. Thus firms can benefit from liquidity management in anticipation of future funding needs.

**Optimal cash policy.** Corporate cash holding allows firms to transfer internal resources across time and states where the marginal values of firms' cash flows to shareholders differ. The condition for optimal cash holding is as follows:

$$\overbrace{\Lambda(d) \cdot \mathbf{1}}^{\text{marginal cost of cash}} \geq \overbrace{\frac{1}{1+r} E_{z'|z} \left[ \Lambda(d') \underbrace{[1 + (1-\tau)r]}_{\text{increase in internal liquidity}} (1 + \lambda \cdot s \cdot \mathbf{1}_{m'<0}) \right]}^{\text{marginal benefit of cash}} \tag{1.19}$$

The left-hand side of Equation (1.19) represents the firm's marginal cost of carrying one additional dollar of cash into the subsequent period. When firm payouts dividends, this cost is the marginal value of foregone dividends, which equals one. When firm issues equity, one additional unit of cash saving implies an additional unit of equity issuance, and thus its marginal cost is given by one plus the equity issuance costs. The right-hand side of Equation (1.19) represents the firm's marginal benefit of holding cash. Carrying one more unit of cash leads to an increase in next-period internal liquidity by $1 + (1 - \tau)r$ and thus increase dividends payout or reduces the amount of costly equity issuance. When a firm faces a liquidity shortfall for debt repayment $m' < 0$, the increased internal liquidity further helps to reduce the amount of liquidity penalty firms need to cover.

Cash holding helps firms to avoid and reduce equity issuance costs in anticipation of both good and bad productivity shocks: (i). when firms are hit by good productivity shocks and thus have high investment needs, cash holding allows firms to fund investment without tapping frictional financial markets. (ii). when firms are hit by bad productivity shocks and thus generate low operating profits, cash holding allows firms to avoid and reduce liquidity shortfalls for debt repayment. These two cash-holding motives are consistent with the empirical patterns documented by a large empirical corporate finance literature.

**Optimal investment policy.** In the model with financing frictions, liquidity management is intimately intertwined with firms' capital investment decisions. The optimality condition pertaining to firms' investment policies is given by:

$$\overbrace{\Lambda(d)\cdot 1}^{\text{marginal cost of capital}} = \overbrace{\mu_b\theta(1-\delta) + \frac{1}{1+r}E_{z'|z}\left[\Lambda(d')\left[\underbrace{\left[(1-\tau)\frac{\partial\pi(z,k)}{\partial k} + \tau\delta\right]}_{\text{increase in internal liquidity}}(1+\lambda\cdot s\cdot \mathbf{1}_{m'<0}) + (1-\delta)\right]\right]}^{\text{marginal benefit of capital}} \tag{1.20}$$

The left-hand side of Equation (1.20) represents the marginal cost of capital investment. Similar to cash saving, investing in one more unit of physical capital today reduces current dividends or increases equity issuance, which is valued at the marginal value of current cash flows to shareholders $\Lambda(d)$. The right-hand side of Equation (1.20) represents the marginal benefit of capital investment, which has several components. Specifically, investing today builds up collateral and thus relaxes a firm's borrowing constraint (first component), and increases future internal liquidity (second component) and capital stock (third component). $\mu_b$ indicates the shadow value of collateral constraint.

**Optimal debt policy.** Increasing leverage, on the one hand, allows the firm to improve its current cash flows to shareholders, reflecting either increased dividends or lower equity issuance. On the other hand, it increases the firm's debt service tomorrow, thereby raising the likelihood that it will have to issue costly new shares in the future. The first-order conditions with respect to debt choice $b'$ for firms with maturing and non-maturing debt are as follows:

$$\overbrace{\Lambda(d)\cdot 1-\mu_b}^{\text{marginal benefit of debt}} = \overbrace{\frac{1}{1+r}E_{z'|z}\left[\Lambda(d')\left[[1+(1-\tau)r](1+\lambda\cdot s\cdot\mathbf{1}_{m'<0}) - (1-\lambda)\cdot\eta\cdot\mathbf{1}_{b''>b'}\right]\right]}^{\text{marginal cost of debt}} \tag{1.21}$$

$$\overbrace{\Lambda(d)\cdot(1-\eta)-\mu_b}^{\text{marginal benefit of debt}}=\overbrace{\frac{1}{1+r}E_{z'|z}\left[\Lambda(d')\left[[1+(1-\tau)r](1+\lambda\cdot s\cdot\mathbf{1}_{m'<0})-(1-\lambda)\cdot\eta\cdot\mathbf{1}_{b''>b'}\right]\right]}^{\text{marginal cost of debt}} \tag{1.22}$$

Equation (1.21) represents the first-order condition with respect to debt for firms with maturing debt obligations. Equation (1.22) represents the first-order condition with respect to debt for firms with non-maturing debt. The left-hand side of Equation (1.21) and Equation (1.22) is the marginal benefit of one more unit of borrowing today which increases firms' current cash flows to shareholders while reducing firms' future debt capacity. Since firms with non-maturing debt face debt issuance frictions, the proceeds from their debt issuance are reduced by the proportional issuance costs $\eta$. The right-hand side of Equation (1.21) and Equation (1.22) represent the marginal costs of outstanding debt firms face. Servicing an additional unit of debt tomorrow reduces next-period cash flows to shareholders, especially when a firm's internal liquidity is insufficient to meet its maturing debt obligations.

## 1.4 Calibration

This section describes the calibration strategy. The model is calibrated at a quarterly frequency. There are two groups of parameters. The first group consists of externally set parameters, which are either standard parameters in the literature or parameters that have a natural data counterpart. The second group of parameters is calibrated internally to minimize the difference between model-simulated moments and their empirical counterparts. Details on model simulation are described in Appendix 1.10.2.

### 1.4.1 Externally Set Parameters

Panel A of Table 2.1 displays the values for fixed parameters and their sources.

**Technology and productivity.** Capital share $\alpha$ is set to $\alpha = 0.30$, and capital depreciates at rate $\delta = 0.025$ quarterly. Return-to-scale is set to $\chi = 0.85$. These parameter choices are fairly standard in the literature. As suggested by Foster et al. (2008), the persistence of firm-specific productivity is set to $\rho_z = 0.90$. Following Bloom et al. (2018), I set the low uncertainty state as $\sigma_L$ as 0.51.

**Institutions.** Parameters in this group have natural data counterparts, which capture features of the U.S. economy outside the model. The quarterly risk-free interest rate

**Table 1.1:** Externally Set Parameters

| Parameter | Description | Value | Source/Targets |
|---|---|---|---|
| | | (a). **Technology** | |
| $\alpha$ | Capital share | 0.30 | Gilchrist et al. (2014) |
| $\chi$ | Decreasing returns-to-scale | 0.85 | Gilchrist et al. (2014) |
| $\delta$ | Depreciation rate | 0.025 | Standard |
| | | (b). **Productivity** | |
| $\rho_z$ | Persistence | 0.90 | Foster et al. (2008) |
| $\sigma_z$ | Volatility | 0.051 | Bloom et al. (2018) |
| | | (c).**Institutions** | |
| $r_f$ | Risk-free interest rate | 0.0121 | $\beta = 0.988 = 1/(1+r)$ |
| $\tau$ | Effective corporate tax rate | 0.20 | CBO (2017) |
| $\pi^e$ | Exogenous exit rate | 0.025 | Annual exit rate=0.10 (BED) |
| $\lambda$ | Debt maturity | 0.07 | Maturity $\frac{1}{\lambda}$= 3.5 years |
| $\theta$ | Pledgeability | 0.71 | $P_{95}(Leverage)$ |

is chosen to be $r = 0.121$, which implies the subjective discount factor $\beta = 0.988$. As reported by Congressional Budget Office in 2017, the marginal effective corporate tax rate is 0.20. Following the survey of Business Employment Dynamics, the quarterly firm exit rate is $\pi^e = 0.025$, which implies an average 10-year corporate duration, in line with Khan and Thomas (2013).

**Debt maturity.** Expected maturity of debt is set to be 3.5 years, which implies that $\lambda = 0.07$. The average maturity of outstanding debt for samples of U.S. public firms calculated by empirical literature varies from 3 years to 4 years.

**Assets pledgeability.** I set the assets pledgeability $\theta$ to 0.71, which corresponds to the 95th percentile of the leverage distribution calculated using my sample. This parameter value helps the model generate a realistic leverage distribution. The value is slightly lower than the average recovery rate of corporate loans and bonds reported by Moody's Ultimate Recovery Database, 0.75, which is used in Begenau and Salomao (2019).

### 1.4.2 Internally Calibrated Parameters.

Panel B of Table 2.4 displays the values for internally calibrated parameters as well as the calibration targets. I use 8 empirical moments to estimate 7 parameters using Simulated Methods of Moments. This choice produces an overidentified model by one degree of freedom. Appendix 1.10.3 details how the empirical targets are computed from a firm-quarter panel and their model counterparts. I also discuss the empirical targets used in the literature. While every targeted moment is simultaneously affected by all parameters, in what follows I provide some intuition for their identification. Table 2.4 displays the values for internally calibrated parameters and shows that the model matches the targeted moments reasonably well.

**Table 1.2:** Internal Calibrated Parameters and Model Fit

| Parameter | Data | Value | Targets | Data | Model |
|---|---|---|---|---|---|
| | | (a). **Financial Frictions** | | | |
| $s$ | Liquidity penalty | 0.51 | Mean leverage ratio | 0.26 | 0.27 |
| $\eta$ | Debt issuance costs | 0.09 | SD leverage ratio | 0.15 | 0.15 |
| | | | Mean cash-to-asset ratio | 0.10 | 0.10 |
| $f_o$ | Production costs | 0.09 | Mean operating income-to-assets | 0.10 | 0.11 |
| $\kappa_0$ | Fixed equity issuance cost | 0.02 | Fraction of net equity issuer | 0.05 | 0.04 |
| $\kappa_1$ | Convex equity issuance cost | 0.21 | Mean equity-issuance-to-assets | 0.13 | 0.14 |
| | | (b). **Firm Life Cycle** | | | |
| $n_0$ | Entrant' assets | 0.34 | Entrants' Relative Size | 0.23 | 0.24 |
| $b_0$ | Entrant' debt | 0.24 | Entrants' Debt/Assets | 0.45 | 0.47 |

**Financial frictions.** The first set of parameters governs the financial behaviors of firms, therefore they are calibrated to match key financial ratios. As shown in 1.3.7, the expected marginal costs of debt is directly affected by liquidity penalty $s$. Since the costs of liquidity shortfalls grow as liquidity penalty $s$ increases, the average leverage ratio decreases. It also shapes the cross-sectional difference in leverage ratio: when liquidity penalty $s$ is low, all firms, regardless of their productivity, will use debt to take advantage of its tax benefits, implying a small standard deviation of leverage ratio across firms. In the presence of debt issuance frictions, corporate cash is used as the marginal source of funding for firms, therefore, the average cash-to-assets ratio increases in debt issuance costs. I set the operating costs $f_o$ that firms pay after production to reproduce the average EBITDA-to-assets ratio of firms, which is the empirical counterpart of firms' operating profits in the model. Fixed equity issuance cost $\kappa_0$ and convex equity issuance cost $\kappa_1$ directly affect firms' equity issuance behavior in the model. Fixed equity issuance cost $\kappa_0$ is calibrated to reproduce the average fraction of firms that issue (net) equity across quarters. The convex equity issuance cost $\kappa_1$ is calibrated to match the average size of equity issuance (equity issuance over total firm assets).

**Entrants.** Two salient empirical patterns on entrants documented by firm dynamic literature is that entrants are smaller in size than the incumbents while entrants tend to have a higher leverage ratio.[12] Therefore, the entrants' size and leverage ratio in the model are calibrated to reproduce these empirical patterns. Specifically, I calibrate entrants' total asset $n_0$ by targeting an entrant's size of 0.23 relative to the average firm's size in the economy, as in Begenau and Salomao (2019). Entrants' debt $b_0$ is targeted to match the average firm-level leverage of 0.45 at age 0-2. Note that the model period is one quarter, while the statistics reported in the literature are calculated using annual data. Hence, I aggregate the simulated data to annual frequency appropriately before computing the simulated moments to make sure they are indeed comparable to data moments.

### 1.4.3 Relation to Capital Structure Theory

This subsection discusses the implications of the baseline calibration for firms' financing choices in the model. First, the calibrated model features larger frictions in the equity market than the credit market as well as tax advantage of debt, and thus firms prioritize debt financing over equity financing. Second, the existence of debt issuance frictions implies that corporate internal liquidity is the cheapest source of funding. As a result, firms in the model hold cash holding for potential future growth opportunities. Taken together, financing behavior in the calibrated model closely follows the *Pecking Order Theory*: when a firm finances an investment opportunity, firms prefer internal financing to external financing. In terms of external financing, firms prefer to use debt over equity.

## 1.5 Firm Behavior in Steady State

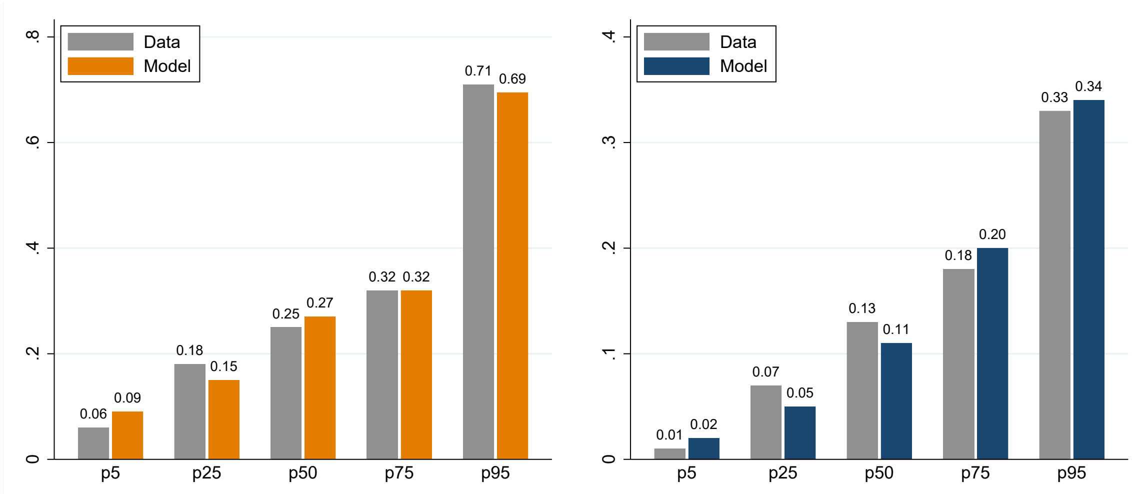
Before testing the ability of the calibrated model to replicate the observed firm-level transmission of uncertainty shocks, in this section, I show that the steady state of the calibrated model generates salient cross-sectional heterogeneity in firms' balance sheets and dynamic investment, saving, borrowing behavior consistent with the data, which validates the model mechanisms. Importantly, the steady-state firm behavior sheds light on how firms behave in the presence of financial frictions, which helps to understand firms' responses to elevated uncertainty.

[12]A recent empirical study can be seen in Kochen (2022).

### 1.5.1 Cross-Sectional Implications

**Balance-sheet heterogeneity.** Figure 1.3 shows the unconditional distribution of leverage ratio and cash ratio in the model and the data. The calibrated model generates empirically-plausible cross-sectional variation in firm balance sheets, which are not directly targeted in the calibration. In the model, firms experience different paths of productivity realization and debt maturity, and thereby choose different stocks of physical capital, cash holding, and outstanding debt.

**(a) Leverage ratio** **(b) Cash ratio**




**Notes:** This figure compares the 5 percentile, 25 percentile, 50 percentile, 75 percentile, and 95 percentile of leverage ratio distribution (panel a) and liquidity ratio distribution (panel b) from Compustat panel and simulated panel.

**Figure 1.3:** Non-Targeted Cross-Sectional Moments: Data versus Model

**Life-cycle patterns.** Both corporate finance and firm dynamics literature has documented life-cycle patterns of firms' real and financial behavior.

1. Real behavior: younger firms are smaller, more profitable, and experience larger growth in output.

2. Financial behavior, younger firms tend to have a larger leverage ratio, lower cash ratio, and lower dividend ratio.

As shown in Table 1.3, the model does a good job of reproducing these empirical patterns. In the model, due to the financing frictions, small entrant firms build up their assets slowly. When firms are young, they are far from their optimal production scales and thus borrow to invest in physical capital. As they approach their optimal scales, they rely less on external financing, save in liquid assets, and pay out dividends. Furthermore, consistent with empirical literature, firm age is important in understanding firm heterogeneity: in the model, firm age can explain around 16% variation in firm size and around 10% variation in profitability, leverage ratio, and cash ratio.

**Table 1.3:** Life-Cycle Patterns of Firms in the Model

| | (1)<br>Firm Size | (2)<br>Profitability | (3)<br>Output Growth | (4)<br>Leverage ratio | (5)<br>Cash ratio | (6)<br>Dividend ratio |
|---|---|---|---|---|---|---|
| Age | 0.0393*** | -0.0041*** | -0.0055*** | -0.0060*** | 0.0029*** | 0.0074*** |
| | (0.0001) | (0.0000) | (0.0000) | (0.0000) | (0.0000) | (0.0001) |
| R-Squared | 0.161 | 0.111 | 0.075 | 0.124 | 0.102 | 0.009 |

**Notes:** This table reports the estimated relationship between firm age and firms' real and financial behavior using univariate OLS and simulated panel.

## 1.5.2 Dynamic Investment and Financial Behavior

In this subsection, I show that the full-fledged model reproduces non-targeted investment and financial behavior consistent with those observed in the data. I discuss the role of key model ingredients in shaping firm behavior and illustrate how alternative models fail to reproduce salient features of the data.

### 1.5.2.1 Firm Behavior and Firm Characteristics

Model-implied policy functions. To understand the key forces that drive firm behavior in the model, I estimate model-implied policy functions using a model-simulated firm panel, which characterizes firms' optimal decisions based on the states of the firms. As in Bazdresch et al. (2018), I transform the actual state and control variables of the model into widely-used variables in the empirical literature, which allows me to directly compare model predictions and observed data patterns. Using both Compustat and model-simulated data, I run the following fixed-effect panel regressions:

$$\Delta \ln y_{i,t+1} = \alpha_i + \alpha_{s,t} + \alpha_{fq,t} + \beta_1 \textbf{Tobin's Q}_{i,t} + \beta_2 \textbf{Size}_{i,t} + \beta_3 \textbf{Indebtedness}_{i,t} + \epsilon_{i,t} \quad (1.23)$$

**Table 1.4:** Firm Characteristics and Firm Behavior: Data Versus Model

| $\Delta \ln y_{i,t+1}$ | $\Delta$Capital$_{i,t+1}$ | | $\Delta$Cash$_{i,t+1}$ | | $\Delta$Debt$_{i,t+1}$ | |
|---|---|---|---|---|---|---|
| | **Data** | **Model** | **Data** | **Model** | **Data** | **Model** |
| **Indebtedness$_{i,t}$** | **-0.023*** ** | **-0.027*** ** | **0.122*** ** | **0.110*** ** | **-0.080*** ** | **-0.060*** ** |
| | (0.001) | (0.000) | (0.003) | (0.001) | (0.003) | (0.001) |
| Tobin's $Q_{i,t}$ | 0.022*** | 0.056*** | 0.038*** | 0.008*** | 0.013*** | 0.033*** |
| | (0.000) | (0.000) | (0.001) | (0.001) | (0.002) | (0.000) |
| Firm Size$_{i,t}$ | -0.003*** | -0.012*** | -0.043*** | -0.051*** | -0.015*** | -0.044*** |
| | (0.001) | (0.000) | (0.002) | (0.001) | (0.002) | (0.001) |
| Firm FE | ✓ | — | ✓ | — | ✓ | — |
| Sector-Quarter FE | ✓ | — | ✓ | — | ✓ | — |
| $R^2$ | 0.098 | 0.784 | 0.055 | 0.045 | 0.054 | 0.144 |

**Notes:** This table reports the estimated relationship between firm behavior and firm indebtedness using Compustat data and model-simulated data. $*, **$, and $***$ represent results significant at the $10\%, 5\%$, and $1\%$ levels, respectively.

For Compustat sample, I control for firm fixed effects $\alpha_t$, fiscal-quarter dummy $\alpha_{fq,t}$, and industry-quarter fixed effects $\alpha_{s,t}$ to absorb permanent heterogeneity across firms, fiscal-quarter effects, and impact of aggregate shocks that do not exist in the stationary equilibrium of the model. Standard errors are two-way clustered to account for correlation within firms and within quarters in regressions using Compustat data. Table **??** details the construction of the firm characteristics variables. Note that I standardize firm $i$' indebtedness$_{i,t}$ (net leverage) using its 1-digit industry average and standard deviation. Table 1.4 reports the estimated relation between firm characteristics and the firm's capital investment, cash growth, and debt growth.

**Tobin's Q.** First, firms in the model differ in their idiosyncratic productivity. All else equal, more productivity firms have higher Tobin's Q. With decreasing-to-return technology, higher productivity also implies a higher optimal scale of production. Therefore, firms with higher Tobin's Q have large investment demand and hence invest and borrow more. The larger growth in debt today means a larger debt burden tomorrow, and thus these firms also save more. The model predicts a positive relationship between Tobin's Q and capital investment, cash growth, and debt growth, consistent with the data pattern shown in Table 1.4.

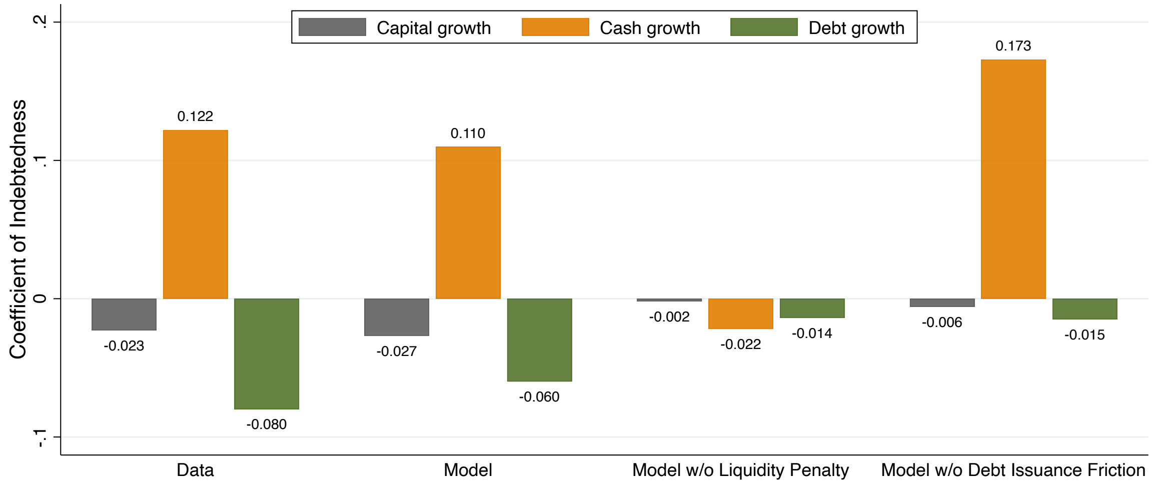
**Firm Size.** Conditional on productivity and outstanding debt, larger firms in the model tend to be closer to their optimal capital level, leading to lower investment demand.

The larger size of firms also means larger internal funds and, therefore, smaller demand for external finance. Therefore, larger firms invest and borrow less, and, consequently, save less. Both in the data and the model, conditional on firm indebtedness and Tobin's Q, Firm Size is negatively correlated with growth in capital, cash, and debt.

**Indebtedness.** In the model, everything else equal, firms with more outstanding debt today are closer to the collateral constraints and have more pre-existing debt burdens compared to their less indebted counterparts. Since liquid assets holding can reduce the likelihood of incurring the liquidity penalty when running out of internal liquidity for maturing debt obligations, more indebted firms, therefore, have larger cash demand for future debt repayment. The smaller borrowing capacity and larger cash demand among more indebted firms lead to lower debt borrowing and higher cash saving today, which results in less capital investment. As shown in Table 1.4, both in the data and the model, conditional on Firm Size and Tobin's Q, one-standard-deviation higher indebtedness is associated with smaller capital investment, larger cash growth, and smaller debt growth.

**Role of model ingredients.** To illustrate the role of model ingredients in driving the impact of firm indebtedness on firms' investment and financial choices, I run the same regressions while using simulated data from alternative models without liquidity penalty ($s = 0$) or debt issuance frictions ($\eta = 0$). Appendix 1.13 reports the full estimation results. Note that models without liquidity penalty or credit frictions still exhibit the positive effects of Tobin's Q and negative effects of Firm Size on firm behavior, since these effects are mostly driven by productivity heterogeneity and decreasing-to-scale technology. Figure 1.4 compares the estimated relationship between firm indebtedness and firm behavior using Compustat data and simulated data from different models.

The liquidity penalty has two effects in the full-fledged model. First, it motivates firms to save in liquid assets to avoid liquidity shortfalls for debt repayment, leading to a positive relation between firm indebtedness and liquid assets growth. In a model without liquidity penalty, firms can repay their maturing debt using new debt/disinvestment/new equity without any additional cash flow penalty, which substantially reduces firms' cash demand. More indebted firms in this case borrow less due to their smaller debt capacity and hence have fewer funds for liquid assets holding, resulting in a negative relation between firm indebtedness and liquid assets growth. As shown in Figure 1.4, in a model without liquidity penalty, indebtedness is negatively associated with liquid assets growth, which is in sharp contrast to the positive association observed in the data and the full-fledged model. Second, as shown in Equation (1.21) and (1.22), liquidity penalty




**Notes:** This figure plots the estimated relationship between firm behavior and firm indebtedness using Compustat data and model-simulated data, conditional on Tobin's Q and Firm Size.

**Figure 1.4:** Firm Indebtedness and Firm Behavior: Data versus Model

increases the expected marginal costs of debt, leading to a much stronger negative relation between firm indebtedness and debt growth in the full-fledged model relative to a model without liquidity penalty. The higher demand for cash and larger costs of debt triggered by liquidity penalty induce less capital investment among firms in the full-fledged model, resulting in a stronger negative relation between firm indebtedness and capital investment, relative to that of a model without liquidity penalty.

In addition to liquidity penalty, firms with non-maturing debt in the full-fledged model also face proportional debt issuance costs that directly reduce the marginal benefits of borrowing. The lower marginal benefits of borrowing make more indebted firms in the full-fledged model borrow, invest, and save less, which on the one hand, amplifies the negative effects of firm indebtedness on capital investment and borrowing, and on the other hand, dampens the positive effects of firm indebtedness on cash saving. As shown in Figure 1.4, the full-fledged model shows a stronger negative correlation between firm indebtedness and capital investment and debt growth while a weaker positive association between indebtedness and liquid assets growth, relative to a model without debt issuance frictions.

#### 1.5.2.2 Cash as Marginal Source of Funding

In the presence of uncertainty and financial market frictions, firms save in cash holding for future growth opportunities. The idea is simple: when a good productivity shock is realized, cash holding allows firms to fund capital investment internally and thus avoid incurring the transaction costs associated with new security issuance in the financial market. In this subsection, I show that the model is able to reproduce these empirical patterns.

**Firm responses to productivity shocks.** I examine how firms respond to growth in their firm-level TFP by running the following regression using both Compustat data and simulated data :

$$\Delta \ln y_{i,t+1} = \alpha_i + \alpha_{s,t} + \alpha_{fq,t} + \beta \Delta \ln \mathrm{TFP}_{i,t} + \Gamma' X_{i,t} + \epsilon_{i,t} \tag{1.24}$$

where $\Delta \ln \mathrm{TFP}_{i,t}$ denotes measured firm-level productivity growth. Appendix 1.9.3 discusses the construction of firm-level productivity using Compustat Quarterly. $X_{i,t}$ denotes a vector of control variables that include Indebtedness, Tobin's Q, and Firm Size. For the Compustat sample, I control for firm fixed effects $\alpha_t$, fiscal-year dummy $\alpha_{fq,t}$, and industry-year fixed effects $\alpha_{s,t}$ to absorb permanent heterogeneity across firms, fiscal-year effects, and impact of aggregate shocks that do not exist in the stationary equilibrium of the model. Standard errors are two-way clustered to account for correlation within firms and within quarters in regressions using Compustat data.

**Table 1.5:** Firm Responses to Idiosyncratic Productivity Growth: Data versus Model

| | **Data** | | | **Model** | | |
|---|---|---|---|---|---|---|
| $\Delta \ln y_{i,t+1}$: | $\Delta\mathrm{Capital}_{i,t+1}$ | $\Delta\mathrm{Cash}_{i,t+1}$ | $\Delta\mathrm{Debt}_{i,t+1}$ | $\Delta\mathrm{Capital}_{i,t+1}$ | $\Delta\mathrm{Cash}_{i,t+1}$ | $\Delta\mathrm{Debt}_{i,t+1}$ |
| $\Delta \ln \mathrm{TFP}_{i,t}$ | 0.27*** | **-0.15*** ** | 0.26*** | 0.849*** | **-0.955*** ** | 0.381*** |
| | (0.001) | (0.005) | (0.003) | (0.002) | (0.021) | (0.012) |
| Firm Controls | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Firm FE | ✓ | ✓ | ✓ | — | — | — |
| Sector-Quarter FE | ✓ | ✓ | ✓ | —- | — | — |
| $R^2$ | 0.176 | 0.080 | 0.084 | 0.896 | 0.112 | 0.171 |

**Notes:** This table reports estimated firm responses to an idiosyncratic productivity growth using Compustat data and model-simulated data. $*, **$, and $***$ represent results significant at the $10\%, 5\%$, and $1\%$ levels, respectively.

**Table 1.6:** Firm Responses to Idiosyncratic Productivity Growth: Alternative Models

| | **Model w/o liquidity penalty** | | | **Model w/o debt issuance frictions** | | |
|---|---|---|---|---|---|---|
| $\Delta \ln y_{i,t+1}$: | $\Delta$Capital$_{i,t+1}$ | $\Delta$Cash$_{i,t+1}$ | $\Delta$Debt$_{i,t+1}$ | $\Delta$Capital$_{i,t+1}$ | $\Delta$Cash$_{i,t+1}$ | $\Delta$Debt$_{i,t+1}$ |
| $\Delta \ln \text{TFP}_{i,t}$ | 0.890*** | -0.347*** | 0.538*** | 0.803*** | **1.439*** ** | 0.859*** |
| | (0.002) | (0.012) | (0.004) | (0.003) | (0.019) | (0.008) |
| Firm Controls | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $R^2$ | 0.903 | 0.201 | 0.684 | 0.857 | 0.334 | 0.376 |

**Notes:** This table reports estimated firm responses to an idiosyncratic productivity growth using simulated data from alternative models. $*, **$, and $***$ represent results significant at the $10\%, 5\%$, and $1\%$ levels, respectively.

Table 1.5 shows that both in the data and the model, firm-level productivity growth is positively correlated with capital investment and debt growth while negatively correlated with liquid assets growth. Firms in the model save in liquid assets and use them as the marginal funding source, allowing them to save on transaction costs in the financial markets. In contrast, in Table 1.6, firm-level productivity growth is positively correlated with liquid assets growth in a model without debt issuance frictions. This occurs because firms with larger productivity growth borrow more, and therefore they also save more for future debt repayment.

## 1.6 The Model-implied Transmission of Uncertainty Shocks

In this section, I first show the calibrated model reproduces both firm-level and aggregate-level impacts of uncertainty shocks observed in the data and then inspect the transmission mechanisms. In Section 1.6.1, I show that the calibrated model accounts well for the observed firm-level response to uncertainty shocks documented in Section 1.2. In Section 1.6.2, I inspect the transmission mechanisms by shutting down key model ingredients emphasized in Section 1.5. In Section 1.6.3, I show the calibrated model also reproduces the macro-level impacts of uncertainty shocks documented in the literature and provides novel interpretations of the data patterns.

The economy is initially in a steady state and unexpectedly receives a shock to the distribution of productivity shocks. Specially, all firms in the economy suddenly receive a jump in the dispersion of their productivity shocks $\sigma_t = \sigma_H$, which reverts back to $\sigma_L$ according to $\sigma_{t+1} = 0.5\ \sigma_t$. The jump in the dispersion is common across firms, and note that the expected productivity is always kept constant. I calibrate the initial jump to induce a 2.5% drop in aggregate output on impact. [13]

### 1.6.1 Firm-level Responses to Uncertainty Shocks in the Model

I study the transmission of uncertainty shocks within the model by estimating firm-level responses to uncertainty shocks using a simulated panel. I compute the perfect foresight transition path of the economy as it converges back to a steady state. I simulate a panel of 10,000 firms and estimate the following specification using data from one year before the initial shock to two years after the initial shock:

$$\Delta \ln y_{i,t+1} = \alpha + \Big(\boldsymbol{\beta} + \boldsymbol{\gamma}\text{Indebtedness}_{i,t}\Big) \cdot \Delta \log \sigma_t + \eta\text{Indebtedness}_{i,t} \qquad (1.25)$$

$$+\boldsymbol{\Psi'}\mathbf{Z}_{i,t} \cdot \Delta \log \sigma_t + \Gamma'\mathbf{Z}_{i,t} + \mu_{i,t}$$

where $\Delta \log(\sigma_t)$ measures the log deviation of $\sigma_t$ from the steady-state level $\sigma_L$. A high $\log(\sigma_t)$ implies a more widely spread distribution of next-period idiosyncratic productivity shocks compared to the steady state level. Indebtedness$_{i,t}$ measures how many standard deviations firm $i$'s net leverage is away from industry mean. $\mathbf{Z}_{i,t}$ is a vector of the control variable that captures firms' growth opportunity in the context of the model: Tobin's Q and Firm Size. Note that Equation (1.25) indeed resembles the empirical specification Equation (1.2) since there is no permanent unobserved heterogeneity across firms, fiscal-quarter differences, and other confounding macro shocks in the model environment.

Table 1.7 reports the estimated firm-level responses to uncertainty shocks using simulated data. The full-fledged model does a good job of reproducing the observed firm-level responses to uncertainty shocks in the data: an increase in aggregate uncertainty facing firms is followed by capital investment drops, cash buildups, and deleveraging. In the cross-section, the decline in capital investment and the increase in cash holding are much more pronounced among more indebted firms.

[13] The choice of 2.5% decrease in aggregate output driven by uncertainty shocks on impact follows Bloom et al. (2018). The persistence of the shock 0.5 is standard in the literature on MIT shocks.

**Table 1.7:** Model-Implied Transmission of Uncertainty Shocks

| $\Delta \ln y_{i,t+1} \times 100$ : | $\Delta$Capital$_{i,t+1}$ | $\Delta$Cash$_{i,t+1}$ | $\Delta$Debt$_{i,t+1}$ |
|---|---|---|---|
| $\Delta \log \sigma_{t+1}$ | -0.214*** | 0.753*** | -0.193*** |
| | (0.016) | (0.026) | (0.069) |
| $\Delta \log \sigma_{t+1} \times$ Indebtedness$_{i,t}$ | -0.280*** | 0.257*** | 0.086 |
| | (0.025) | (0.039) | (0.103) |
| R-Squared | 0.796 | 0.069 | 0.158 |
| Firm Controls$_{i,t}$ | ✓ | ✓ | ✓ |
| $\Delta \log \sigma_{t+1} \times$ Z$_{i,t}$ | ✓ | ✓ | ✓ |

**Notes:** This table reports estimated firm responses to uncertainty shocks using simulated data from the full-fledged model. $*, **$, and $* * *$ represent results significant at the $10\%, 5\%$, and $1\%$ levels, respectively. $\Delta \log(\sigma_t)$ measures the log deviation of $\sigma_t$ from the steady-state level $\sigma_L$. Indebtedness$_{i,t}$ measures how many standard deviations firm $i$'s net leverage is away from mean. Firm control variables include Indebtedness$_{i,t}$ and $Z_{i,t}$. $Z_{i,t}$ includes Tobin's Q and Firm Size, which captures firms' growth opportunity in the context of the model.

Two forces are at work: on the one hand, an increase in uncertainty implies a higher probability of having a bad productivity shock, which generates a higher likelihood of liquidity shortfalls due to lower operating profits. This motivates firms to deleverage and hold more cash to avoid potential liquidity penalties in the face of debt repayment. On the other hand, an increase in uncertainty also implies a larger chance of drawing a good productivity shock. This upside potential amplifies firms' saving motives while counteracting the deleveraging pressure since deleveraging shrinks firms' internal funds for future expansion. That is, cash holding plays a unique role in the face of the two forces: it preserves internal funds for both future debt repayment and investment opportunities, thereby addressing both the downside risk and upside potential triggered by uncertainty shocks: it allows firms to avoid potential liquidity penalties for debt repayment and also enables firms to preserve enough internal funds for growth potential. In the face of the two forces triggered by a higher uncertainty, firms re-structure their balance sheets by deleveraging and accumulating cash holding, both of which divert firms' internal funds away from capital investment. *Ex-ante* more indebted firms choose to accumulate more cash holding since they face a higher likelihood of liquidity shortfalls due to large stocks of outstanding debt.

### 1.6.2 Inspecting the Transmission Mechanisms

To better understand each of the two underlying forces at play, I conduct three experiments to explore the transmission of uncertainty shocks in alternative setups. I find that liquidity penalty is the key to generating the negative impact of uncertainty shocks on capital and debt, while frictions in debt issuance are the key to generating the salient liquidity buildup in response to uncertainty shocks. Further, the severity of debt issuance frictions shapes how differently indebted firms react to heightened uncertainty.

**Role of liquidity penalty.** I first shut down the liquidity penalty by setting $s = 0$. In a model without liquidity penalty, firms can repay their maturing debt using new debt/disinvestment/new equity without any additional costs. Consequently, firms have no concern over the elevated likelihood of liquidity shortfalls, that is, the deleveraging pressure is muted. As shown in Panel (A) of Table 1.8, uncertainty shocks, in this case, have no statistically significant effects on firms' outstanding debt. Since firms do not need to trade off capital investment for cash holding to reduce the likelihood of incurring liquidity penalty, the strong precautionary saving motive triggered by the elevated uncertainty motivates firms to increase both their capital investment and cash holding today in an effort to generate large internal liquidity for future investment. As a result, both capital investment and cash holding rise following uncertainty shocks.[14] To sum up, this model predicts a positive effect of heightened uncertainty on capital investment and an insignificant effect on debt, contradicting the empirical findings.

**Role of debt issuance frictions.** I shut down the debt issuance frictions by setting $\eta = 0$. In this case, firms can issue debt without any additional costs when a good productivity shock realizes, thereby eliminating firms' precautionary saving motives for growth opportunities. As a result, when uncertainty rises, the firms are only concerned about the larger downside risk caused by an elevated uncertainty, and thus they deleverage to prevent liquidity shortfalls. The decrease in firms' debt obligations also reduces their cash demand for debt repayment, and therefore firms in this model also decrease their cash holding in response to heightened uncertainty. As shown in Panel (B) of Table 1.8, cash holding drops following uncertainty shocks in this model, contradicting the salient buildup of corporate liquidity observed in the data.

[14] Note that the average increase in cash holding across firms, in this case, is completely driven by a decrease in dividend payout due to better growth potential, while in the full-fledged model, cash buildup is also partly driven by capital investment cut for future debt repayment. As shown in Section 1.5.2, liquidity penalty is the key to generating a trade-off between capital and cash for debt repayment.

Degrees of debt issuance frictions. To further understand how frictions in debt issuance shape firm responses to uncertainty shocks by governing firms' precautionary saving motives, I experiment with two different levels of debt issuance costs relative to the baseline calibration. As shown in Table 1.9, when debt issuance costs are 50% lower than the baseline level, in response to uncertainty shocks, more indebted firms also deleverage more relative to their less indebted counterparts. This occurs since reduced debt issuance frictions mitigate firms' precautionary saving motives. Specifically, more indebted firms, in this case, can deleverage first and issue new debt cheaply to fund capital investment if a good productivity shock realizes. By contrast, when debt issuance costs are at the baseline level or 50% higher than the baseline level, issuing new debt is especially costly, and thus more indebted firms choose to hold more cash to reduce their higher likelihood of liquidity shortfalls rather than cut more debt. To sum up, the severity of debt issuance frictions plays a key role in shaping the heterogeneous responses to uncertainty shocks across differently indebted firms.

**Table 1.8:** Model-Implied Transmission of Uncertainty Shocks: Alternative Models

| | **(A) Model w/o liquidity penalty** | | | **(B) Model w/o debt issuance frictions** | | |
|---|---|---|---|---|---|---|
| $\Delta \log y_{i,t+1} \times 100$ : | $\Delta$Capital$_{i,t+1}$ | $\Delta$Cash$_{i,t+1}$ | $\Delta$Debt$_{i,t+1}$ | $\Delta$Capital$_{i,t+1}$ | $\Delta$Cash$_{i,t+1}$ | $\Delta$Debt$_{i,t+1}$ |
| $\Delta \log \sigma_{t+1}$ | 0.033** | 0.239*** | -0.018 | -0.389*** | -2.426*** | -5.447*** |
| | (0.016) | (0.008) | (0.022) | (0.017) | (0.158) | (0.152) |
| Firm Controls$_{i,t}$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $R^2$ | 0.727 | 0.084 | 0.589 | 0.716 | 0.059 | 0.086 |

**Notes:** This table reports estimated firm responses to uncertainty shocks using simulated data from alternative models. $*$, $**$, and $***$ represent results significant at the 10%, 5%, and 1% levels, respectively. $\Delta \log(\sigma_t)$ measures the log deviation of $\sigma_t$ from the steady-state level $\sigma_L$. Firm control variables include Indebtedness, Tobin's Q and Firm Size.

**Table 1.9:** Debt Issuance Frictions and Firm Responses to Uncertainty Shocks

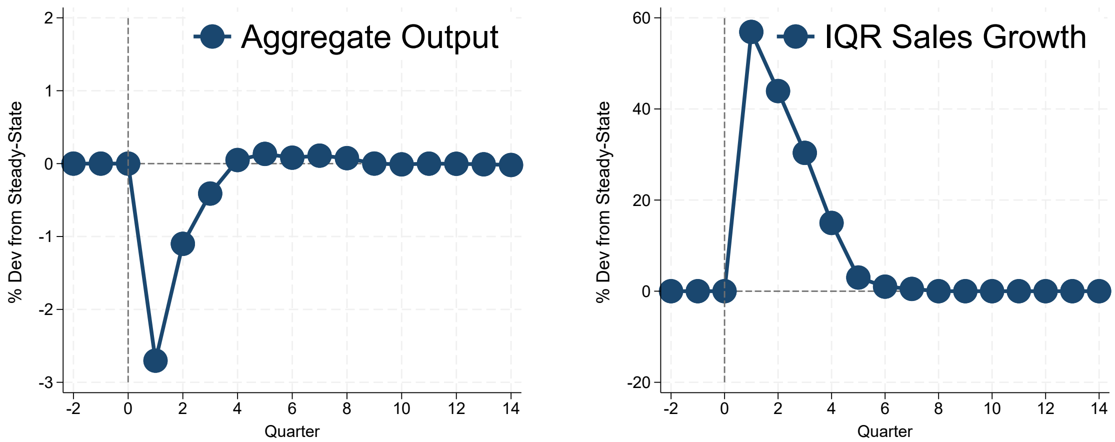
| | **Low Debt Issuance Frictions** $= 0.5 \cdot \eta_{\textbf{baseline}}$ | | | **High Debt Issuance Frictions** $= 1.5 \cdot \eta_{\textbf{baseline}}$ | | |
|---|---|---|---|---|---|---|
| $\Delta \ln y_{i,t+1} \times 100$ : | $\Delta\text{Capital}_{i,t+1}$ | $\Delta\text{Cash}_{i,t+1}$ | $\Delta\text{Debt}_{i,t+1}$ | $\Delta\text{Capital}_{i,t+1}$ | $\Delta\text{Cash}_{i,t+1}$ | $\Delta\text{Debt}_{i,t+1}$ |
| $\Delta \log \sigma_{t+1}$ | -0.205*** | 0.813*** | -0.187** | -0.260*** | 0.775*** | -0.261** |
| | (0.028) | (0.036) | (0.094) | (0.030) | (0.042) | (0.119) |
| $\Delta \log \sigma_{t+1} \times \text{Indebtedness}_{i,t}$ | -0.342*** | 0.201*** | **-0.213**** | -0.314*** | 0.468*** | **0.209** |
| | (0.027) | (0.035) | (0.091) | (0.032) | (0.045) | (0.127) |
| R-Squared | 0.725 | 0.115 | 0.182 | 0.675 | 0.091 | 0.102 |
| Firm $\text{Controls}_{i,t}$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $\Delta \log \sigma_{t+1} \times Z_{i,t}$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |

**Notes:** This table reports estimated firm responses to uncertainty shocks using simulated data from the full-fledged model. $*, **$, and $***$ represent results significant at the $10\%, 5\%$, and $1\%$ levels, respectively. $\Delta \log(\sigma_t)$ measures the log deviation of $\sigma_t$ from the steady-state level $\sigma_L$. $\text{Indebtedness}_{i,t}$ measures how many standard deviations firm $i$'s net leverage is away from mean. Firm control variables include $\text{Indebtedness}_{i,t}$ and $Z_{i,t}$. $Z_{i,t}$ includes Tobin's Q and Firm Size, which captures firms' growth opportunity in the context of the model.

### 1.6.3 Aggregate Impacts of Macro Uncertainty Shocks in the Model

The previous sections showed that the calibrated model accounts well for the observed firm-level balance sheet transmission of uncertainty shocks. In this subsection, I show that the calibrated model also reproduces the aggregate impacts of macro uncertainty shocks. As shown in Figure 1.5, a macro uncertainty shock in the calibrated model generates a sharp and protracted drop in aggregate output along with a spike in the cross-sectional dispersion of sales growth, consistent with the empirical findings in the literature.

Notably, existing uncertainty literature takes the observed increase in the dispersion of sales growth during periods of high uncertainty as exogenous. The balance sheet channel suggests the increased dispersion of sales growth is an endogenous response of the economy to elevated macro uncertainty due to heterogeneous adjustments across differently indebted firms. Understanding the increased dispersion of sales growth is important since the variable has been used widely to identify and characterize periods of high macro uncertainty in the empirical literature and to calibrate the size of macro uncertainty shocks in structural models.




**Notes:** This figure plots the responses of aggregate output and cross-sectional dispersion of sales growth to uncertainty shocks. As standard in the literature, cross-sectional dispersion is measured by the Interquartile range.

**Figure 1.5:** The Macro Impact of Uncertainty Shocks

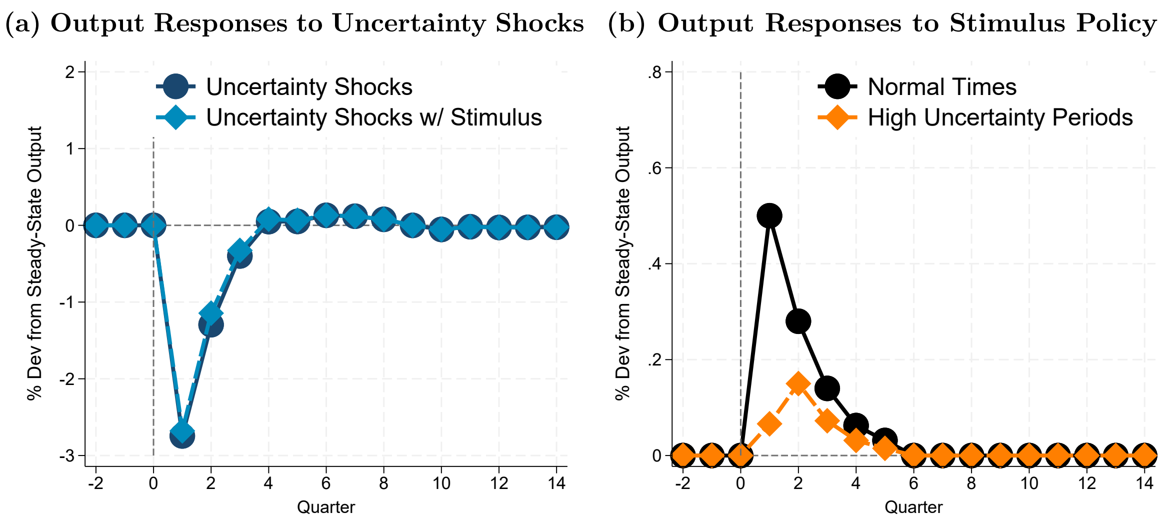
## 1.7 Policy Implications of the Balance Sheet Channel

In this section, I discuss the novel policy implications of the balance sheet channel of uncertainty shocks. In Section 1.7.1, I show that investment stimulus policies, like investment tax credit, yield modest effects in counteracting the adverse impact of uncertainty shocks. In Section 1.7.2, I show that credit interventions, like debt relief, are powerful in stabilizing uncertainty-driven recessions. The reason behind the results is that high uncertainty dampens the effects of investment stimulus policies while amplifying the effects of credit interventions. In Section 1.7.3, I study two extensions that further illustrate how the nature of recessions and corporate cash choices shape policy impacts.

### 1.7.1 Stimulus Policies in Uncertainty-driven Recessions

Research in macroeconomics found that monetary stimulus has weak effects in recessions, advocating more fiscal or credit policies to stabilize the economy in recessions (see, e.g., Tenreyro and Thwaites 2016, Fang (2020)). In this subsection, I examine whether a common investment stimulus policy in the United States, the investment tax credit, can help to counteract the aggregate output drops driven by uncertainty shocks. The idea is simple: investment tax credits lower the effective marginal costs of investment, and thus, it might stimulate aggregate investments and then aggregate output.

**Investment tax credit** In this policy, a $x_t$ fraction of investment costs can be rebated back to firms as tax credits when firms make the investments. Therefore, the effective investment cost for $I$ amount of capital investment is $(1 - \tau x_t)I$ with the policy and $x_t = 0$ in steady state. I consider an unexpected and temporary policy with $x_t = 0.07$ at time 0, which fades at a rate of 0.5. I calibrate the initial size of program $x_t = 0.07$ to generate a 0.5% increase in aggregate output during normal times at impact.




**Notes:** Panels (A) plots the aggregate output responses to an uncertainty shock with and without investment tax credits. Panels (B) plots the aggregate output responses to the policy intervention during normal times and periods of high uncertainty. Appendix 1.10.3 details the computation of aggregate impulse response functions.

**Figure 1.6:** Uncertainty-driven Recessions and Stimulus Policy

**Stimulus policies in uncertainty-driven recessions.** Panel (A) of Figure 1.6 plots the impact of uncertainty shocks on aggregate output with and without investment tax credit. Both uncertainty shocks and policy interventions happen at time 0 and fade at a rate of 0.5. The key finding is that investment tax credit has very limited effects in counteracting the adverse impact of uncertainty. To see this clearly, Panel (B) of Figure 1.6 plots the output responses to the investment tax credit program during normal times and periods of high uncertainty. The orange line is simply the percentage difference between the two aggregate output responses in Panel (A). Panel (B) shows that the stimulative impact of the same investment tax credit program is much weaker during periods of high uncertainty than normal times. The reason is that heightened uncertainty motivates firms to hoard cash, thereby depressing firms' use of cash for policy-induced capital investments. In Appendix 1.10.7, I show that aggregate cash goes down by more than 8% in response to the policy during normal times, while aggregate cash only goes down by 3% in response to the policy during periods of high uncertainty. The impact of high uncertainty on policy effect can also be seen in the shapes of impulse response functions. During normal times, the positive effects of the policy decrease with policy intensity. During periods of high uncertainty, as uncertainty decreases, firms start to respond to the investment stimulus, leading to a larger output response at time 2 than at time 1, even though the policy intensity is cut by half.

### 1.7.2 Credit Interventions in Uncertainty-driven Recessions

The recent COVID-19 pandemic has seen aggressive credit interventions provided to corporate sectors around the world. The Paycheck Protection Program (PPP) in the U.S. provided corporate businesses with more than $800 billion in the form of debt relief and forgivable loans. In this subsection, I study the stabilizing effects of credit interventions in uncertainty-driven recessions.

**Credit interventions.** I focus on two credit inventions during the recent COVID crisis. I consider one-time policy interventions in the model simulation, which resembles the implementation of loan programs observed in 2020. As shown in Cho et al. (2022), the distribution of PPP loans is untargeted and prioritizes speedy loan disbursement.

1. Debt relief programs: a fraction of each firm's outstanding debt is unexpectedly written off at time 0.

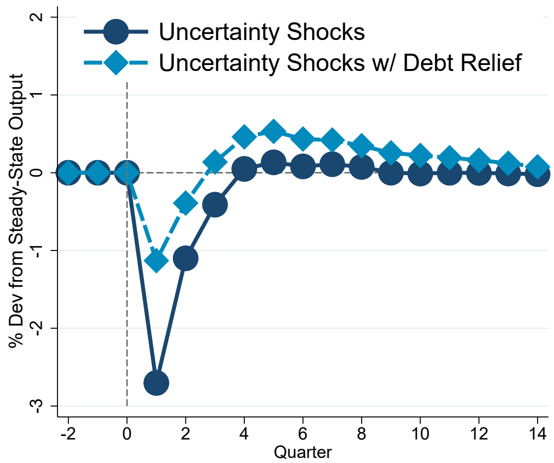
2. Cash grant programs: each firm unexpectedly receives a cash grant at time 0, which equals a fraction of the steady-state wage bills they pay.

I calibrate the size of each program to generate a 0.5% increase in aggregate output during normal times at impact. Note that the one-time credit interventions have persistent effects on output since credit interventions directly increase firms' net worth, and thus, firms become less financially constrained for the rest of their lifecycle.

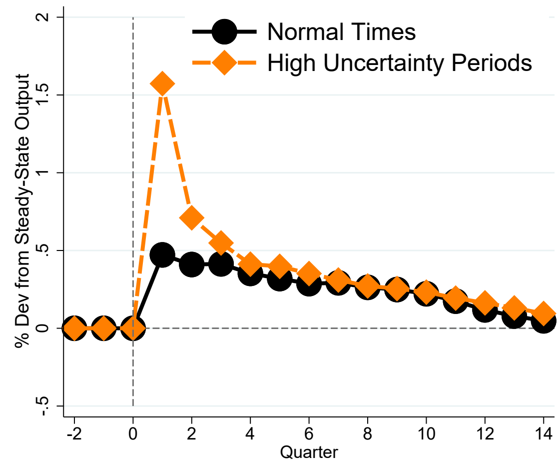
**Credit interventions in uncertainty-driven recessions.** Panel (A) and (C) of Figure 1.7 plot the impact of uncertainty shocks on aggregate output with and without credit interventions. Panel (B) and (D) of Figure 1.7 plot the output responses to policy interventions during normal times and periods of high uncertainty. Three patterns emerge. First, credit interventions substantially mitigate the negative effects of uncertainty shocks. With debt relief or cash grant programs, aggregate output drops by only 1% or 1.5%, respectively. The workings of the policies are simple: debt relief and cash grant programs directly reduce firms' need to reduce debt and hoard cash in response to heightened uncertainty, thereby mitigating the decrease in capital investment and the negative effects of uncertainty shocks. Second, as shown in Panel (B) and (D) of Figure 1.7, output responses to credit interventions are much larger during periods of high uncertainty than during normal times. The debt relief and cash grants programs that stimulate aggregate output by 0.5% during normal times can drive up aggregate output in an uncertainty-driven recession by 1.5% and 1.0%, respectively. This is because credit interventions during periods of high uncertainty not only increase firms' net worth but also directly alleviate the balance sheet adjustments of firms in response to heightened uncertainty. Third, the response of aggregate output to debt relief is stronger than its response to cash grants during periods of high uncertainty, despite similar effects during normal times. This occurs since debt relief programs reduce firms' debt burdens, and this, in turn, lowers firms' cash demand for debt repayment, thereby freeing up firms' cash holding for more capital investment. The effect of debt relief programs on firms' cash choices is critical for the strong stabilizing effects of debt relief in uncertainty-driven recessions. In Section 1.7.3, I show that a counterfactual simulation failing to capture the observed cash buildup underestimates output response to debt relief by more than 30%.

**Policy effectiveness.** To gauge the effectiveness of credit interventions, I compute the present value of all the output gains using the discount factor and then divide it by the total fiscal cost of the program, which measures the expected output gain per unit of fiscal costs. Figure 1.8 plots the effectiveness of both programs during normal times and uncertainty-driven recessions. Since output responses increase during periods of high uncertainty, the estimated output gain per dollar rises from 0.74 to 1.13 for debt relief programs. It also goes up from 0.64 to 0.85 for cash grants programs. The debt relief program is more effective since it not only directly reduces firms' debt burdens but also indirectly lowers firms' cash demand, freeing up firms' internal funds for more capital investment, as discussed before. The difference in effectiveness between the two interventions becomes especially pronounced, as output response to debt relief rises substantially following uncertainty shocks.

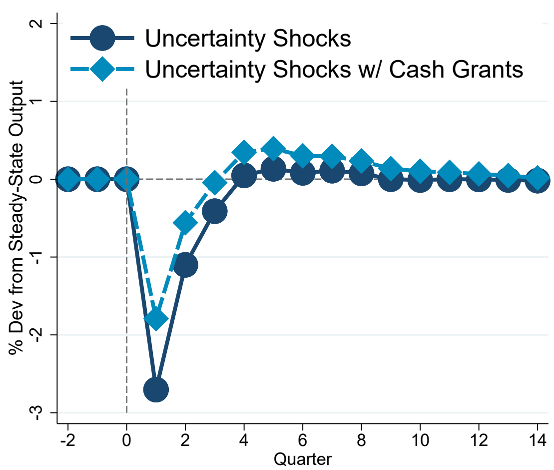
**(a) Output Responses to Uncertainty Shocks**



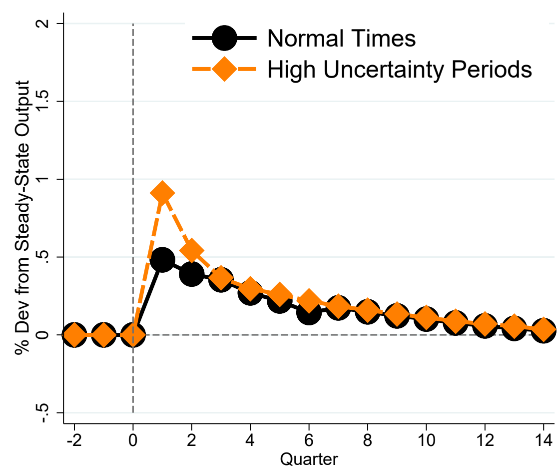

**(b) Output Responses to Debt Relief**




**(c) Output Responses to Uncertainty Shocks**




**(d) Output Responses to Debt Relief**

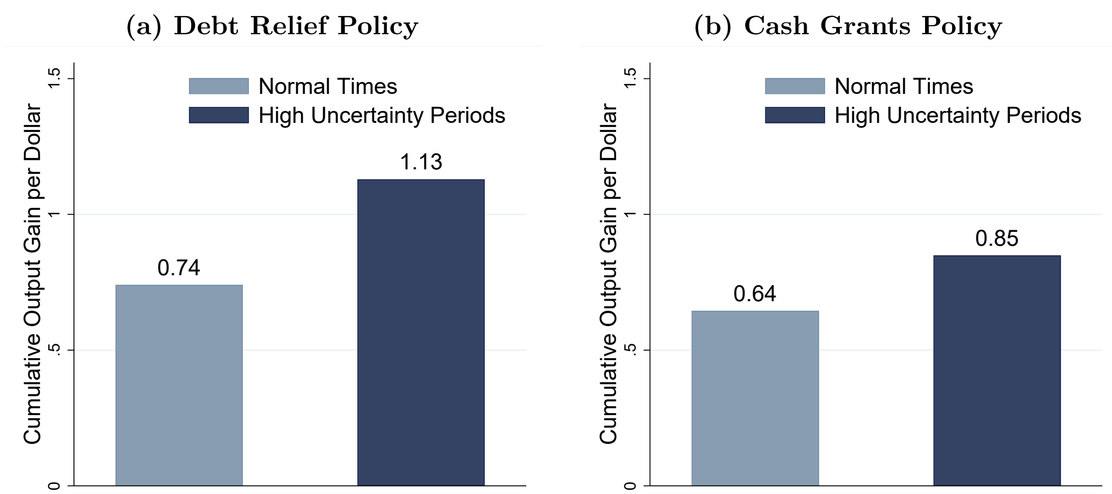



**Notes:** Panels (A) and (C) plot the aggregate output responses to an uncertainty shock with and without credit interventions. Panels (B) and (D) plot the aggregate output responses to policy interventions during normal times and periods of high uncertainty. Appendix 1.10.3 details the computation of aggregate impulse response functions.

**Figure 1.7:** Uncertainty-driven Recessions and Credit Interventions

### 1.7.3 Extentions

The unprecedented credit interventions during the recent COVID crisis sparked a rapidly growing literature that evaluates the effects of credit interventions using quantitative models. The previous section adds to the literature by demonstrating the strong effects of credit interventions in uncertainty-driven recessions. This subsection adds to the




**Notes:** This figure plots the effectiveness of credit interventions during normal times and uncertainty-driven recessions. To gauge the effectiveness of credit interventions, I compute the present value of all the output gains using the discount factor and then divide it by the total fiscal cost of the program, which measures the expected output gain per unit of fiscal costs.

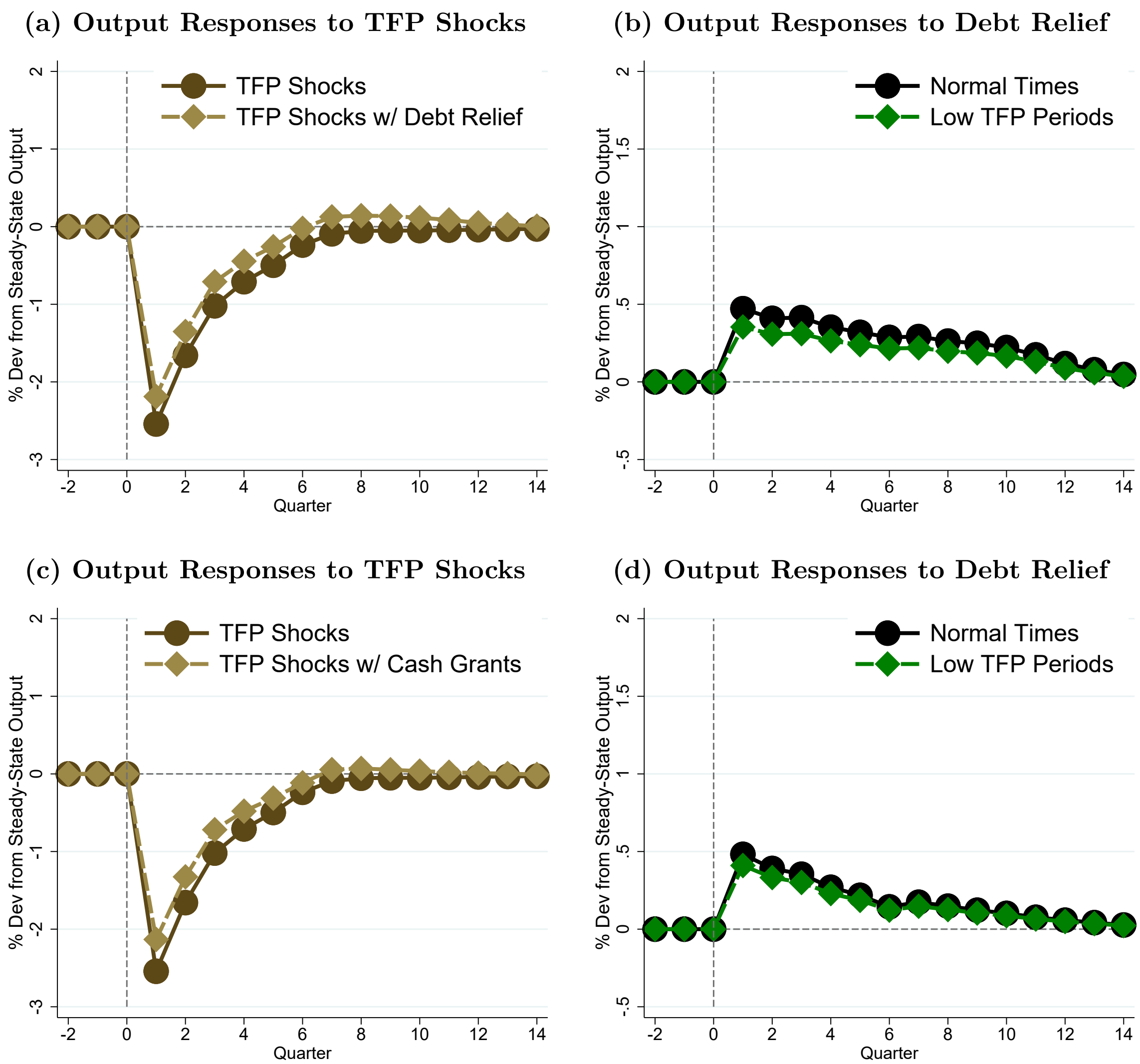
**Figure 1.8:** Policy Effectiveness of Credit Interventions

growing literature in two ways. First, I highlight the importance of corporate cash choice in shaping policy impacts by showing that a counterfactual simulation failing to capture the observed cash buildup underestimates the stabilizing effects of debt relief in uncertainty-driven recessions. Second, I highlight the nature of the recessions in shaping policy impacts by showing that credit interventions have modest effects in a recession driven by negative productivity shocks.

**Credit interventions in TFP-driven recessions.** Panel (A) and (C) of Figure 1.9 plot aggregate output responses to negative productivity shocks with and without credit interventions. Panel (B) and (D) of Figure 1.9 show that output responses to the interventions in TFP-driven recessions turn out to be slightly smaller than those in normal times. This is because lower aggregate productivity reduces firms' investment demand and financial needs, thereby mitigating the role of credit interventions in relaxing firms' financial constraints.[15] In sharp contrast, credit interventions directly mitigate the balance sheet transmissions of uncertainty shocks, thereby strongly stabilizing aggregate output drops in an uncertainty-driven recession. This result echoes Crouzet and Tourre (2021), where they find that credit interventions have larger stabilizing effects in a

[15] In Appendix 1.10.8, I show that output responses to credit interventions are slightly larger during booms when investment demand is high and firms have larger financial needs.

TFP-driven recession accompanied by financial market disruptions. Therefore, the nature of the recession plays an important role in shaping policy effectiveness.

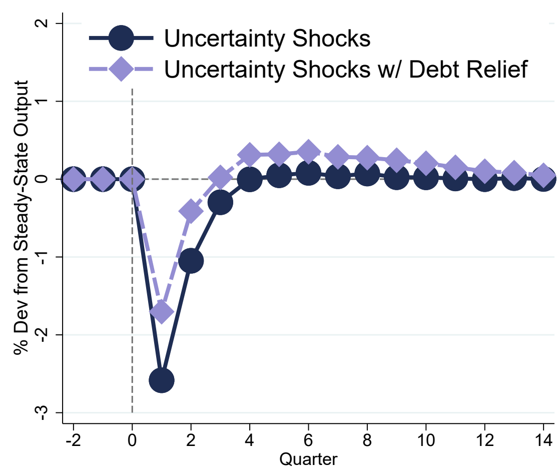



**Notes:** Panels (A) and (C) plot the aggregate output responses to a negative aggregate TFP shock with and without credit interventions. Panels (B) and (D) plot the aggregate output responses to policy interventions during normal times and periods of low aggregate productivity. Appendix 1.10.3 details the computation of aggregate impulse response functions.

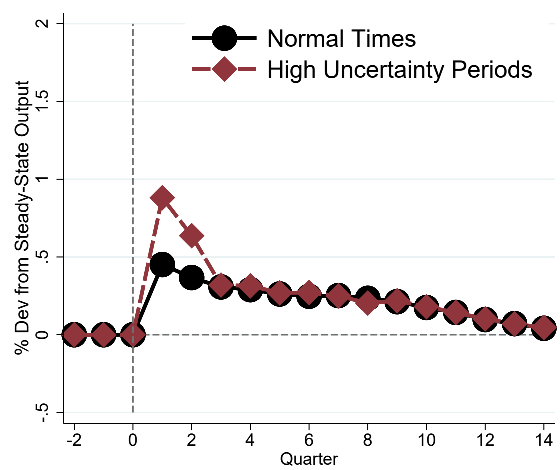
**Figure 1.9:** TFP-driven Recessions and Credit Interventions

**Implications of Corporate Cash Choice.** Conventional macro-finance models typically treat firms' cash holding as 'net debt'. In this class of models, firms either borrow or save, but not both. To illustrate the importance of modeling firms' cash choices for understanding the effects of debt relief, I shut down the debt issuance frictions by setting $\eta = 0$, which eliminates cash buildup in response to heightened uncertainty as shown in Section 1.6. In this case, the negative effects of uncertainty shocks are completely due to the deleveraging of firms. The initial jump in uncertainty and the sizes of policy interventions are re-calibrated to ensure compatibility with the baseline simulation. Panel (A) and (C) of Figure 1.10 plot the impact of uncertainty shocks on aggregate output with and without debt relief. Panel (B) and (D) of Figure 1.10 plot the output responses to debt relief during normal times and periods of high uncertainty. The effects of debt relief policy are much weaker in this counterfactual simulation without cash buildup: estimated output response to the debt relief program following uncertainty shocks is around 1.0 % upon impact in contrast to the 1.5% in the baseline simulation. In this case, the effects of debt relief programs in mitigating firms' cash buildup are missing, leading to an underestimation of policy effects.

**(a) Output Responses to Uncertainty Shocks**




**(b) Output Responses to Debt Relief**




**Notes:** Panels (A) and (C) plot aggregate output responses to uncertainty shocks with and without credit interventions. Panels (B) and (D) plot the output responses to policy interventions during normal times and periods of high uncertainty. Note that in the model without frictions in debt issuance ($\eta = 0$), Appendix 1.10.3 details the computation of aggregate impulse response functions.

**Figure 1.10:** Credit Interventions in a Counterfactual Economy

## 1.8 Concluding Remarks

The Federal Open Market Committee has repeatedly underlined uncertainty as a key factor in US recessions. Understanding how uncertainty shocks transmit to the real economy is key to the design of stabilization policies.

In this paper, I show both empirically and theoretically how elevated macro uncertainty affects the real economy through the balance sheet adjustments of firms. My model mechanism illustrates how corporate saving motives and ex-ante financial conditions determine firms' responses to uncertainty shocks, which differs significantly from existing transmission mechanisms. Importantly, the new mechanism provides a unified explanation for the observed capital, cash, and debt dynamics following macro uncertainty shocks and has novel policy implications. Exploiting a calibrated model that reproduces both firm-level and aggregate impacts of uncertainty shocks, I show that conventional stimulus policies have modest effects in counteracting the adverse effects of uncertainty shocks, while credit interventions, in particular debt relief, strongly and effectively stabilize uncertainty-driven recessions.

My model abstracts from other uncertainty transmission mechanisms in order to demonstrate the ability of the new mechanism to reproduce data patterns. An interesting question to ask is which transmission channel is the most important channel in determining the aggregate effect of uncertainty shock. Answering this question requires a model with additional real and financial frictions, which substantially increases the computational burden. I leave the task for future research. Moreover, my model can be used to understand the transmission of other macroeconomic shocks. For example, to what extent do corporate cash holdings buffer financial shocks? Answering such questions needs a quantitative model that captures empirically consistent corporate cash behavior, like the one presented in this paper.

## 1.9 Data Appendix

### 1.9.1 Macro Time Series Data

For the macro data, I use data from the Federal Reserve Bank of St. Louis (FRED) for the United States.

| Definition | Code | Unit |
|---|---|---|
| GDP | GDP | Billions of Dollars |
| GDP Deflator | GDPDEF | Index 2012=100 |
| Effective Federal Funds Rate | FEDFUNDS | Percentage |
| Credit Spread | BAA10Y | Percentage |

The aggregate variables used in the panel local projection include Real GDP Growth measured as the log growth of real GDP, Inflation Rate measured as the log difference in GDP deflator, Real Federal Funds Rate measured as the difference between Effective Federal Funds Rate and Inflation Rate, and Credit Spread.

### 1.9.2 Firm-level Data

This subsection describes the firm-level variables used in the empirical analysis of the paper, based on quarterly Compustat data. The variable definition and sample selection follow standard practices in the literature, for example, Kim and Kung (2017), and Ottonello and Winberry (2020).

**Variable Construction:** All variables are deflated by the 2012 GDP deflator.

1. *Capital Investment*: defined as $\Delta \log(k_{i,t+1})$, where $k_{i,t+1}$ denotes the capital stock of firm I at the end of period t. For each firm, we set the first value of $k_{i,t+1}$ to be the level of Gross Plant, Property, and Equipment $PPEGTQ$ in the first period in which this variable is reported in Compustat. From this period onwards, I compute the evolution of $k_{i,t+1}$ using the changes of Net Plant, Property, and Equipment $PPENTQ$, which is a measure of net investment with significantly more observations than $PPEGTQ$.

2. *Leverage Ratio*: measured as Total Debt divided by Total Assets $ATQ$, with Total Debt computed as the sum of Debt in Current Liabilities and Total Long-Term Debt $(DLCQ + DLTTQ)$.

3. *Cash Ratio*: measured as the ratio of Cash and Short-term Investments $CHEQ$ to Total Assets $ATQ$.

4. *Net Leverage*: measured as the ratio of Total Debt minus Cash and Short-term Investments $CHEQ$ to Total Assets $ATQ$.

5. *Firm Size*: measured as the log of Total Assets $ATQ$.

6. *Tobin's Q*: is defined as follows:

$$Tobin's\ Q = \frac{ATQ + CSHOQ \times PRCCQ - CEQQ}{ATQ}$$

where $CSHOQ$ is the number of Common Shares Outstanding, $PRCCQ$ is the Share Price (Close), $CEQQ$ is Common/Ordinary Equity - Total, and $ATQ$ is Total Assets.

7. *Real Sales Growth*: measured asthe year-on-year growth in quarterly sales $SALEQ$.

8. *Cash Flows*: measured as the sum of Income before Extraordinary Items $IBQ$ and Depreciation and Amortization $DPQ$ divided by lagged Total Assets $ATQ$.

9. *Debt Maturity*: measured as (1 – Debt Maturing within a Year $DD1$) / Debt in Current Liabilities and Total Long-Term Debt ($DLCQ + DLTTQ$).

10. *(Net) Equity Issuance*: measured as ($SSTKQ - PRSTKCQ$), where $SSTKQ$ is the quarterly Sale of Common and Preferred Stock, constructed based on the Compustat reported Year-to-date Sale of Common and Preferred Stock $SSTKY$; $PRSTKCQ$ is the quarterly Purchase of Common and Preferred Stock, constructed based on the Compustat reported Year-to-date Purchase of Common and Preferred Stock $PRSTKCY$. I normalize these quarterly net issuances by lagged Total Assets $ATQ$, as in Hennessy and Whited (2007).

**Panel Local Projection:** The sample covers the period from 1990Q1 to 2018Q4 at a quarterly frequency.

1. I exclude firms in finance (SIC codes 6000-6999), utility (SIC codes 4900-4949), and government-related sectors (SIC codes 9000-9999).

2. I exclude firms that are not incorporated in the United States.

3. I exclude firm-quarter observations with negative values for non-negative accounting items.

4. I exclude firm-observations with net property, plant, and equipment of less than \$1M and total assets of less than \$3M. This eliminates extremely small firms that might be very sensitive to aggregate shocks. These only account for less than 1% of total firm-quarter observations.

5. I include firm-quarter observations from firms that are observed for at least 20 quarters during the sample period (a reasonably long time dimension is required for firm-level fixed effects and within the estimator).

6. I winsorize observations of all variables at the top and bottom 1% of the distribution to exclude extreme observations, e.g., those driven by mergers and acquisitions.

**Table 1.10:** Summary Statistics of Key Firm-level Variables

| | Mean | S.D. | P25 | P50 | P75 |
|---|---|---|---|---|---|
| $\Delta log(Capital_{i,t})$ | 0.01 | 0.10 | -0.02 | -0.00 | 0.03 |
| $\Delta log(Cash_{i,t})$ | 0.02 | 0.69 | -0.24 | -0.00 | 0.24 |
| $\Delta log(Debt_{i,t})$ | 0.01 | 0.35 | -0.06 | -0.00 | 0.05 |
| $\Delta_8 log(Capital_{i,t+8})$ | 0.08 | 0.45 | -0.13 | 0.04 | 0.27 |
| $\Delta_8 log(Cash_{i,t+8})$ | 0.12 | 1.15 | -0.47 | 0.09 | 0.66 |
| $\Delta_8 log(Debt_{i,t+8})$ | 0.13 | 1.06 | -0.26 | 0.03 | 0.48 |
| Tobin's Q | 1.81 | 1.29 | 1.08 | 1.42 | 2.04 |
| Firm Size | 6.12 | 2.11 | 4.55 | 6.12 | 7.60 |
| Sales Growth | 0.02 | 0.24 | -0.06 | 0.02 | 0.10 |
| Cash flows | 0.01 | 0.05 | 0.01 | 0.02 | 0.03 |

**Notes:** this table presents summary statistics of key firm-level variables. The sample period is 1990q1 to 2018q4. All variables are winsorized at the 1% level to eliminate outliers.

### 1.9.3 Measured Firm-level Productivity

I assume that the production function at the firm level is Cobb-Douglas and allow the parameters of the production function to be industry-specific:

$$y_{i,j,t} = z_{i,j,t} k_{i,j,t}^{\alpha} n_{i,j,t}^{\nu}$$

Since data on employment is not available in the Compustat Quarterly, I rewrite the production function based on the optimal static choice of labor in the model:

$$y_{i,j,t} = z_{i,j,t} \psi(W_t) k_{i,j,t}^{\gamma}$$

where $y_{i,j,t}$ is sales, $z_{i,j,t}$ is firm-level productivity, $\psi(W_t)$ is a time-specific term related to equilibrium wage, and $k_{i,j,t}$ is capital stock.

Within each 1-digit SIC industry, I then estimate firm-level productivity as the residual of the following equation:

$$\ln(y_{i,t}) = \alpha_i + \alpha_t + \alpha_k \ln(k_{i,t-1}) + v_{i,t}$$

where $y_{i,t}$ is firm sales in quarter t, $k_{i,t-1}$ is the firm's physical capital stock at the beginning of period t, $\alpha_i$ is a firm fixed effect, and $\alpha_t$ is a time fixed effect.

$\hat{v}_{i,t}$ therefore denotes the estimated log productivity $ln(z_{i,t})$ of firm $i$ in quarter $t$. The year-on-year firm-level productivity growth used in the regressions is then $\Delta lnTFP_{i,t} = ln(z_{i,t}) - ln(z_{i,t-4})$.

### 1.9.4 Within-firm Variation in Indebtedness

I examine whether within-firm variation in firm indebtedness predicts heterogeneous responses to uncertainty shocks by estimating the following specification:

$$\Delta_h \log(y_{i,t+h}) = \alpha_{i,h} + \alpha_{fq,h} + \alpha_{s,t,h} + \underbrace{\boldsymbol{\gamma_h}\Big(D_{i,t-1} - \bar{D}_i\Big) \cdot \Delta \log \sigma_t}_{\text{Heterogeneous responses}} + \beta_h\Big(D_{i,t-1} - \bar{D}_i\Big) \tag{1.26}$$

$$+\boldsymbol{\Psi'_h}\Big(\mathbf{Z}_{i,t-1} - \bar{Z}_i\Big) \cdot \Delta \log \sigma_t + \Gamma'_h\Big(\mathbf{Z}_{i,t-1} - \bar{Z}_i\Big) + \boldsymbol{\eta_h}\Big(D_{i,t-1} - \bar{D}_i\Big) \cdot \Delta \log GDP_t + \mu_{i,t+h}$$

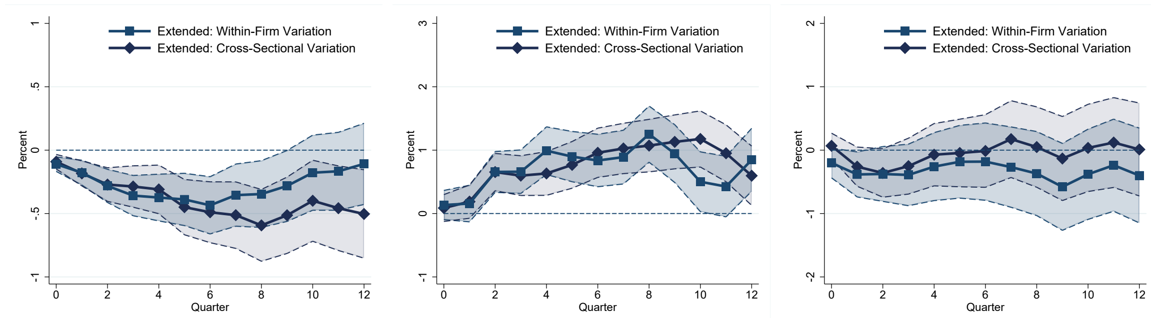
$$\forall i, h = 0, 1, 2, 3, ..., 12$$

The Equation 1.26 differs from Equation 1.2 by using within-firm variation in firm characteristics. Specifically, $\Big(D_{i,t-1} - \bar{D}_i\Big)$ is the deviation of firm i's net leverage from its unconditional firm-specific average, and $Z_{i,t-1}$ is a vector of control variables all in deviation from their respective firm-specific averages. Figure 1.11 shows that the responses of physical capital and liquid assets holding to changes in the Macro Uncertainty Index are also stronger when firms are more indebted than their own average levels. These results provide additional evidence of the role of firm indebtedness in shaping firm responses to uncertainty shocks.

### 1.9.5 Event Study: 9/11 Terrorist Attacks

As in Kim and Kung (2017), I exploit the 9/11 terrorist attacks as an event study to study changes in firm behavior before and after large uncertainty events. Using the 9/11 terrorist attacks to study the effects of heightened aggregate uncertainty on firm behavior has several advantages: First, the terrorists' attacks on U.S. soil in September 2001 were exogenous to the U.S. economy and struck as a total surprise. Second, the event induces a significant increase in economic uncertainty. For example, Macro Uncertainty Index by Jurado et al. (2015) increases by 5.5%, the largest single-quarter change before

(a) **Physical capital** (b) **Liquid assets holding** (c) **Outstanding debt**




**Notes:** the figure plots both the average and heterogeneous responses of (a) physical capital, (b) liquid assets holding, and (c) outstanding debt to a one-standard-deviation growth in Macro Uncertainty Index by Jurado et al. (2015) at quarter $t$. The heterogeneous responses are driven by cross-sectional variation and within-firm variation in indebtedness at quarter $t-1$. Point estimates and 95% confidence intervals for $\beta_h$ and $\gamma_h$ are plotted. Standard errors are two-way clustered at both firm and time levels. The sample period is from 1990Q1 to 2018Q4.

**Figure 1.11:** Heterogeneous Responses by Within-firm Variation in Indebtedness

the Great Recession. The jump in the VIX index in September 2001 is more than 1.65 standard deviation above the mean, as shown in Bloom et al. (2018). The Federal Open Market Committee (FOMC) also stated in October 2001 that "the events of September 11 produced a marked increase in uncertainty". Third, compared with other events that result in a rise in the uncertainty of a similar magnitude, this political event appears to be relatively unconfounded by changes in other macroeconomic factors. For example, the 2007-2009 financial crisis is a period with both high macroeconomic uncertainty and financial sector disruption, therefore, it is hard to disentangle what factors drive the changes in firm behavior.

To examine the average changes in firm behavior across firms around the 9/11 terrorist attacks, I estimate a simple fixed effects regression:

$$\log(y_{i,t}) = \alpha_i + \alpha_{fq} + \sum_t \beta_t \text{Quarter}_t + \epsilon_{i,t} \tag{1.27}$$

$$\forall t \in \{2001q1, ..., 2002q2\} \backslash \{2000q4\}$$

To explore how the impact of firm indebtedness on firm behavior varies over the event window, I estimate the following regression:

$$\log(y_{it}) = \alpha_i + \alpha_{s,t} + \alpha_{fq} + \sum_t \gamma_t \text{Indebtednes}_{i,t-1} \cdot \text{Quarter}_t + \beta \text{Indebtednes}_{i,t-1} \tag{1.28}$$

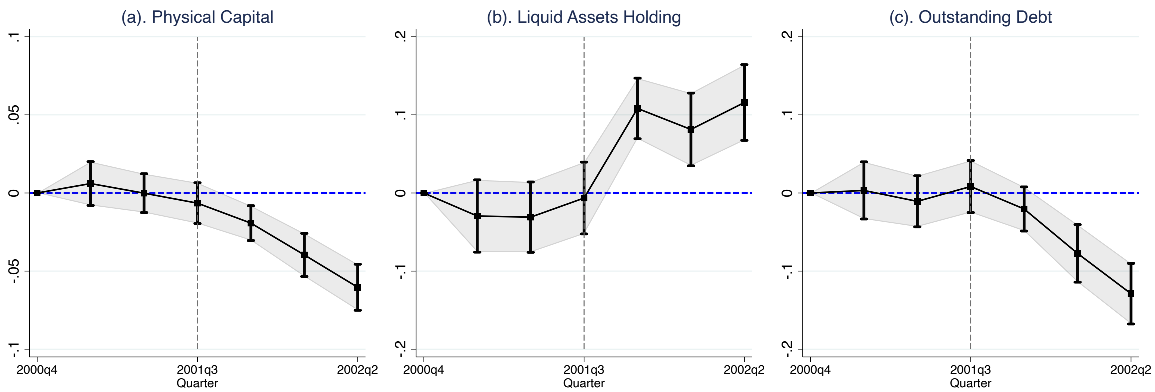
$$+\Gamma'\mathbf{X}_{i,t-1} + \sum_t \Lambda'_t \mathbf{X}_{i,t-1} \cdot \text{Quarter}_t + \epsilon_{i,t}$$
$$\forall t \in \{2001q1, ..., 2002q2\} \backslash \{2000q4\}$$

where $\text{Quarter}_t$ is a quarter dummy for the time period from 2000q4 to 2002q2, with 2000q4 taken as the omitted category. $\alpha_i$ indicates firm fixed effects that absorb permanent differences in the levels of dependent variables across firms. Fiscal-quarter dummy $\alpha_{fq}$ is included to absorb the impact of the difference in fiscal-quarter across firms on firm behavior. $\alpha_{s,t}$ represents the industry-by-quarter fixed effects that absorb differences in how broad industries are exposed to aggregate shocks. The industry is defined as 1-digit SIC level. $\text{Indebtedness}_{i,t-1}$ measures how many standard deviations away firm $i$'s net leverage is from its industry average in quarter $t-1$. As discussed earlier, differences in indebtedness might correlate with other factors that affect firm behavior. I control for a vector of widely used control variables $\mathbf{X}_{i,t-1}$ that include Tobin's Q, Sales growth, and Cash flows, and allow their effects on firm behavior also vary over time by interacting these variables with a quarter dummy. Standard errors are clustered by both firm and quarter. Since the goal is to capture within-firm changes in firm behavior before and after the event, firms that enter and exit the sample during the event window are excluded. Finally, $\boldsymbol{\beta_t}$ capture 'within-firm' changes in firm behavior over time relative to the base period 2000q4. $\boldsymbol{\gamma_t}$ captures the time-varying relation between indebtedness and changes in dependent variables over the event window.

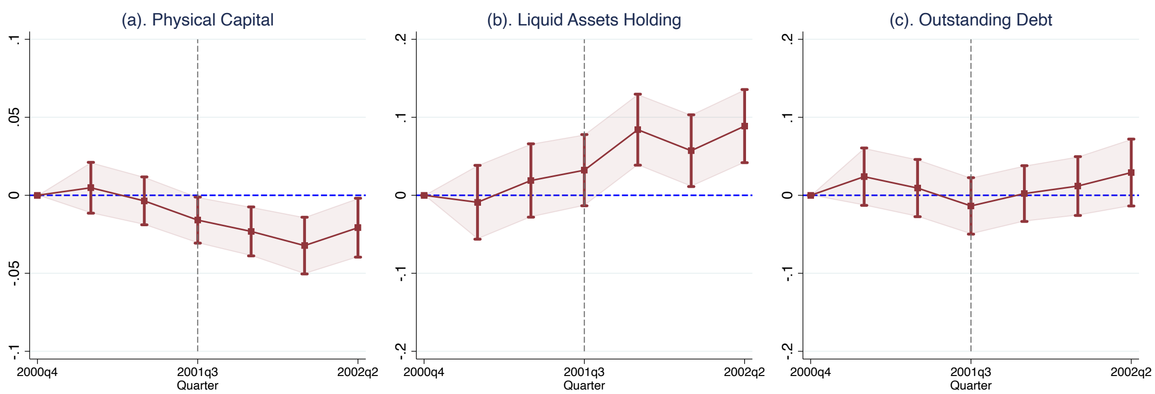
Panel A of Figure 1.12 plots the estimated average firm-level growth in physical capital, liquid assets holding, and outstanding debt from 2000q4 to 2002q2, along with a 95% confidence interval. The Post-9/11 period features statistically significant declines in physical capital and outstanding debt, while a large buildup in liquid assets holding across firms. The average dynamics following the 9/11 terrorist attacks are consistent with the baseline results.

Panel B of Figure 1.12 plots the estimated time-varying relation between firm indebtedness and firm-level changes in (a) physical capital, (b) liquid assets holding, and (c) outstanding debt from 2000q4 to 2002q2, along with 95% confidence interval. Notably, after the third quarter of 2001, higher indebtedness at $t-1$ foreshadowed statistically significant a larger decline in physical capital and a larger growth in liquid assets holdings. Moreover, differences in lagged indebtedness do not predict differences in debt growth across differently indebted firms after the event. Taken together, during periods of high uncertainty, high indebtedness is mainly associated with a larger shift in

**Panel A**. Average Firm Growth in Capital, Cash, and Debt




**Panel B**. Time-Varying Effects of Firm Indebtedness on Firm Choices of Capital, Cash, and Debt




***Notes:*** Panel A reports estimated average firm-level growth in (a) physical capital, (b) liquid assets holding, and (c) outstanding debt from 2000q4 to 2002q2, along with 95% confidence interval. Panel B reports estimated time-varying relation between firm indebtedness and firm-level changes in (a) physical capital, (b) liquid assets holding, and (c) outstanding debt from 2000q4 to 2002q2, along with a 95% confidence interval.

**Figure 1.12:** Firm Behavior around 9/11 Terrorist Attacks

firms' asset choice, consistent with the more direct evidence based on local projection discussed in Section 1.2.2.

## 1.10 Model Appendix

### 1.10.1 Model Details

**Static Labor Choice and Operating Profits.** Given productivity $z$, capital stock $k$, and Wage $W$, firms solve the following static profit-maximization problem:

$$\pi(z,k;W) = \max_n \{z^{1-\nu}k^{\alpha}n^{\nu} - f_o k - Wn\}$$

Optimal labor choice is given by

$$n^*(z,k;W) = \left(\frac{\nu}{W}\right)^{\frac{1}{1-\nu}} zk^{\frac{\alpha}{1-\nu}}$$

Therefore, the production of the firm is given by

$$y^*(z,k;W) = \left(\frac{\nu}{W}\right)^{\frac{\nu}{1-\nu}} zk^{\frac{\alpha}{1-\nu}}$$

Operating profits is given by

$$\begin{aligned}\pi(z,k;W) &= (1-\nu)\left(\frac{\nu}{W}\right)^{\frac{\nu}{1-\nu}} zk^{\frac{\alpha}{1-\nu}} \\ &= z\psi(W)k^{\gamma} - f_o k\end{aligned}$$

where $W$ denotes the (real) wage and

$$\gamma = \frac{\alpha}{1-\nu} \qquad \text{and} \qquad \psi(W) = (1-\nu)\left(\frac{\nu}{W}\right)^{\frac{\nu}{1-\nu}}$$

$\alpha$ is the value-added share of capital, and $\nu$ is the value-added share of labor. This set-up ensures that the firm's profit function is linear in its productivity, as in Gilchrist et al. (2014).

**Optimality Conditions** First-order condition with respect to dividends is as follows:

$$\Lambda(d) = \begin{cases} 1, & \text{if } d \geq 0 \\ 1 + \kappa_1 |d|, & \text{if } d < 0 \end{cases} \tag{1.29}$$

**Step 1**: using the envelop theorem, I obtain the marginal value of cash, capital, and debt for firms with non-maturing debt:

$$\frac{\partial V^m(z,k,c,b)}{\partial c} = \Lambda(d)\Bigg[[1+(1-\tau)r](1+s\cdot\mathbf{1}_{m<0})\Bigg] \tag{1.30}$$

$$\frac{\partial V^m(z,k,c,b)}{\partial k} = \Lambda(d)\Bigg[\Bigg[(1-\tau)\frac{\partial \pi(z,k)}{\partial k}+\tau\delta\Bigg](1+s\cdot\mathbf{1}_{m<0})+(1-\delta)\Bigg] \tag{1.31}$$

$$\frac{\partial V^m(z,k,c,b)}{\partial b} = -\Lambda(d)\Bigg[[1+(1-\tau)r](1+s\cdot\mathbf{1}_{m<0})\Bigg] \tag{1.32}$$

**Step 2**: using the envelop theorem, I obtain the marginal value of cash, capital and debt for firms with non-maturing debt:

$$\frac{\partial V^n(z,k,c,b)}{\partial c} = \Lambda(d)[1+(1-\tau)r] \tag{1.33}$$

$$\frac{\partial V^n(z,k,c,b)}{\partial k} = \Lambda(d)\Bigg[(1-\tau)\frac{\partial \pi(z,k)}{\partial k}+\tau\delta+(1-\delta)\Bigg] \tag{1.34}$$

$$\frac{\partial V^n(z,k,c,b)}{\partial b} = -\Lambda(d)\Bigg[[1+(1-\tau)r]-\eta\cdot\mathbf{1}_{b'>b}\Bigg] \tag{1.35}$$

**Step 3**: first-order conditions with respect to cash choice $c'$ and capital choice $k'$ are the same for firms with maturing and non-maturing debt:

$$FOC[c'] : \Lambda(d)\cdot\mathbf{1} \geq \frac{1}{1+r}E_{z'|z}\Bigg[\lambda\frac{\partial V^m(z',k',c',b')}{\partial c'}+(1-\lambda)\frac{\partial V^n(z',k',c',b')}{\partial c'}\Bigg] \tag{1.36}$$

$$FOC[k'] : \Lambda(d)\cdot\mathbf{1} = \frac{\mu_b\theta(1-\delta)}{1+r}+\frac{1}{1+r}E_{z'|z}\Bigg[\lambda\frac{\partial V^m(z',k',c',b')}{\partial k'}+(1-\lambda)\frac{\partial V^n(z',k',c',b')}{\partial k'}\Bigg] \tag{1.37}$$

**Step 4**: first-order conditions with respect to debt choice $b'$ for firms with maturing debt:

$$FOC[b'] : \Lambda(d)\cdot\mathbf{1}-\mu_b = \frac{1}{1+r}E_{z'|z}\Bigg[\lambda\frac{\partial V^m(z',k',c',b')}{\partial k'}+(1-\lambda)\frac{\partial V^n(z',k',c',b')}{\partial k'}\Bigg] \tag{1.38}$$

$$FOC[b'] : \Lambda(d) \cdot (\mathbf{1} - \eta) - \mu_b = \frac{1}{1+r} E_{z'|z} \left[ \lambda \frac{\partial V^m(z', k', c', b')}{\partial k'} + (1-\lambda) \frac{\partial V^n(z', k', c', b')}{\partial k'} \right] \tag{1.39}$$

**Step 5**: plugging the envelope conditions (B2)-(B7) into the first-order conditions (B8)-(B11), I obtain Euler equations (1.19)-(1.22) for cash, capital, and debt in the main text.

### 1.10.2 Model Computation

**Stationary Equilibrium.** I first assume the economy is at steady-state with normal volatility. In the stationary equilibrium, there is no aggregate shock, so $r = 1/\beta - 1$ and I solve for equilibrium wage to clear the labor market. The algorithm is as follows:

**Step 1**: Guess an equilibrium wage $W^{old}$.

**Step 2**: Solve the firm's problem using Value Function Iteration.

**Step 3**: Using the policy functions and distributions, compute aggregate quantities.

**Step 4**: Using the labor market clearing condition, compute the *Excessive Demand* $\epsilon = L^s - L^d$ by taking the difference between labor demand and labor supply. *STOP* if $\max|\epsilon| < 10^{-5}$.

**Step 5**: Update the wage with a given weight and return to Step 2.

**Transition Dynamics.** The key assumption of the transition dynamics is that after a sufficiently long time, the economy will converge back to its original stationary equilibrium after any temporary and unexpected (MIT) shocks. The solution algorithm here is outlined as follows:

**Step 1**: Fix a sufficient long transition period $t = 1$ to $t = T$ (say 200), at which point we assume the economy has reached steady state.

**Step 2**: Generate an initial jump in volatility $\sigma_t$ and assume the shock follows $\sigma_{t+1} = \rho\sigma_t$ with $\rho = 0.5$.

**Step 3**: Guess a time-series of aggregate prices $\{W_t\}$ of length T.

**Step 4**: **Backward Induction**: solve the value functions (and policy functions) backwards from $t = T - 1, ...1$ setting value functions at time T as the steady-state value functions. This yields value functions and policy functions along the transition path from $t = 1$ to $t = T - 1$ .

**Step 5**: **Forward Simulation**: starting from the steady state distribution, simulate the distribution forward from $t = 1, ..., T$ using the policy functions and idiosyncratic productivity Markov transition matrix. This yields firm distributions along the transition path from $t = 1$ to $t = T - 1$.

**Step 6**: Using the policy functions and distributions, compute aggregate quantities.

**Step 7**: Using the labor market clearing condition, compute the *Excessive Demand* $\epsilon_t = L_t^s - L_t^d$ by taking the difference between labor demand and labor supply.

**Step 8**: *STOP* if $\max|\epsilon_t| < 10^{-5}$.

**Step 9**: Update $(\{W_t\}_{t=1}^T)^{New} = v\epsilon_t + (1-v)(\{W_t\}_{t=1}^T)^{Old}$, and GO TO step 4. $v$ is chose to be 0.5.

### 1.10.3 Model Simulation

I simulate this economy for 200 quarters until they converge to the steady-state distribution. Then I keep simulating this economy for an additional 300 quarters which is used for the calculation of moments. Finally, I keep simulating the economy starting from the quarter 500 forwards with the transitional policy functions and aggregate prices until the economy converges back to the steady state at the quarter 700.

**Simulated Methods of Moments** The SMM proceeds as follows: The simulated data vector $y_i(\beta)$ depends on a vector of structural parameter $\beta$. The goal is to estimate $\beta$ by matching a set of simulated moments, denoted as $h(y_i(\beta))$, with the set of actual data moments $h(x_i)$, where $x_i$ is an i.i.d. data vector. Define

$$g_n(\beta) = \frac{1}{n}\sum_{i=1}^{n}\Big[h(x_i) - h(y_i(\beta))\Big]$$

The simulated moment estimator of $\beta$ is then defined as the solution to the minimization of

$$\hat{\beta} = \arg\min_{\beta} g_n(\beta)' W g_n(\beta)$$

The optimal parameter estimate $\beta$ is obtained by searching over the parameter space using the simulated annealing algorithm.

**Mapping Model to Data.** Table below details the mapping between model variables to Compustat Variables.

**Aggregate Impulse Response Functions.** I compute perfect-foresight transition path following unexpected uncertainty shocks or both unexpected uncertainty shocks and policy interventions. Following Koop et al. (1996), aggregate impulse response

**Table 1.11:** Mapping Model to Data

| Variable | Construction | |
|---|---|---|
| | Data | Model |
| Tobin's Q | (ATQ + PRCCQ × CSHOQ - CEQQ ) / ATQ | $\frac{V_{t-1}(z,k,c,b)}{k+c}$ |
| Firm Size | log(ATQ) | $\log(k+c)$ |
| Leverage ratio | (DLTTQ+DLCQ)/ATQ | $\frac{b}{k+c}$ |
| Net leverage ratio | (DLTTQ+DLCQ-CHEQ)/ATQ | $\frac{b-c}{k+c}$ |
| Cash ratio | CHEQ/ATQ | $\frac{c}{k+c}$ |
| Dividends ratio | DVY/ATQ | $\frac{d}{k+c}$ |
| Equity-issuance-to-assets | (SSTKY - PRSTKY) /ATQ | $\frac{e}{k+c}$ |

**Notes**: Variables ending in Y in Compustat are stated as year-to-date. I convert them into quarterly frequency by subtracting the past quarter from the current observation for all but the rest quarter of the firm.

functions are computed using "Simulation Differencing":

$$\hat{X}_t^{shock} = 100 \log\left(\frac{X_t^{\ shock}}{X_t^{no\ shock}}\right) \qquad \hat{X}_t^{shock,policy} = 100 \log\left(\frac{X_t^{shock,\ policy}}{X_t^{no\ shock}}\right)$$

where $\hat{X}_t^{shock}$ denotes the aggregate impact of uncertainty shocks. $\hat{X}_t^{shock,policy}$ denotes the aggregate impact of uncertainty shocks with policy interventions. To evaluate whether the effectiveness of the credit policies differs during normal times and periods of high uncertainty, I compute the effects of policies as follows:

$$\hat{X}_t^{policy} = 100 \log\left(\frac{X_t^{\ policy}}{X_t^{no\ shock}}\right) \qquad \hat{X}_t^{policy,shock} = 100 \log\left(\frac{X_t^{shock,\ policy}}{X_t^{shock}}\right)$$

where $\hat{X}_t^{shock}$ denotes the aggregate effects of policy interventions during normal times, $\hat{X}_t^{shock}$ denotes the aggregate effects of policy interventions with uncertainty shocks.

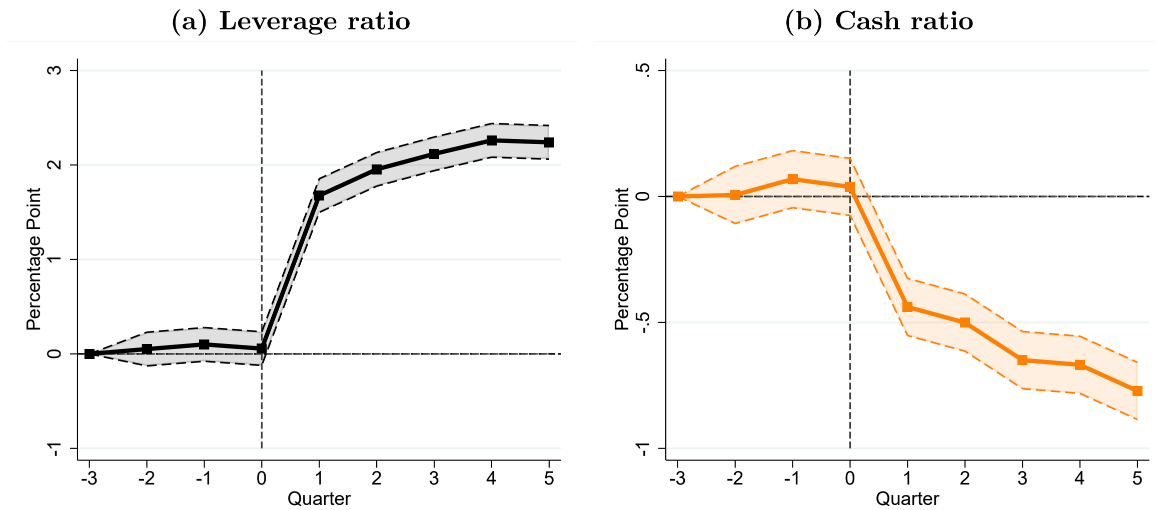
### 1.10.4 Debt issuance frictions and financial behavior

Recent empirical literature shows that strengthened creditor rights protection by law leads to a smaller number of restrictive covenants and more favorable contracting terms (e.g. looser covenants) in debt contracts, e.g. Mann (2018) and Ghanbari (2019). Gao et al. (2021) further shows that the passage of the laws that enhance creditor rights is followed by an increase in leverage ratio and a decrease in cash ratio. Motivated by the empirical evidence, I test whether firm responses to a reduction in debt issuance costs are consistent with the data patterns. Note that debt issuance costs in the model serve as a reduced-form way to capture various types of frictions in debt issuance. Specifically, I simulate a Randomized Controlled Trial research design where half of the simulated firms are randomly selected as a treated group. At time 0, treated firms unexpectedly enjoy reduced debt issuance costs ($\eta = 0.5\eta_{\text{baseline}}$) and thereafter. I keep simulated data 3 quarters before and five quarters after the event and then run the following difference-in-difference specification:

$$y_{i,t} = \alpha + \sum_{t \geq -3}^{t \leq 5} \boldsymbol{\beta_t} \text{Treated}_i \times \text{Quarter}_t + \Gamma' X_{i,t} + \epsilon_{i,t} \tag{1.40}$$

where $\text{Treated}_i$ equals one if firm $i$ belongs to the treated group that will face lower debt issuance costs after Quarter 0. $\text{Quarter}_t$ denotes the periods before and after the experiment. $X_{i,t}$ denotes a vector of control variables, including Indebtedness, Tobin's Q, and Firm Size.

Figure 1.13 shows that treated firms respond to the reduced debt issuance frictions by increasing leverage ratio and decreasing cash ratio, similar to the empirical patterns documented in Gao et al. (2021). In the model, lower debt issuance costs increase the marginal benefits of debt, motivating firms to borrow more. In the meantime, reduced debt issuance frictions mean that treated firms can cheaply borrow from credit markets when an investment opportunity is realized, thereby reducing firms' precautionary saving motives and generating a cut in cash holding.




**Notes:** This table reports estimated firm responses to a reduction in debt issuance costs. Point estimates and 95% confidence level are plotted.

**Figure 1.13:** Reduced Debt Issuance Frictions and Changes in Financial Policies

### 1.10.5 Firm Behavior and Firm Characteristics

This subsection shows firms' investment, saving, and borrowing behavior in alternative setups. Notably, models without liquidity penalties generate a negative relationship between firm indebtedness and cash growth, which is inconsistent with the data.

### 1.10.6 Net-Debt Models

As in the baseline model, frictions in debt issuance also govern firms' cash demand in response to uncertainty shocks in the net-debt models. Figure 1.14 plots output responses to the same uncertainty shocks in the net-debt model with different levels of debt issuance cost $\eta$ when $\eta = 0$, firms' precautionary saving motives are muted. As in the baseline model with $\eta = 0$, the drops in aggregate output in this model are purely driven by firm deleveraging in response to uncertainty shocks. When $\eta > 0$, firms have incentives to generate internal liquidity through capital investment, which counteracts the deleveraging pressure caused by uncertainty shocks and thereby generates smaller output drops. I calibrate $\eta = \eta*$ to match the net leverage ratio as in the baseline model. The net-debt model predicts an overshoot in output in the medium run. When $\eta = 0.5\eta*$, firms' precautionary saving motives are weaker, and thus the output overshoot is less

**Table 1.12:** Firm Behavior and Firm Characteristics: Alternative Models

| | **Model w/o liquidity penalty** | | |
|---|---|---|---|
| $\Delta \ln y_{i,t+1}$: | $\Delta$Capital$_{i,t+1}$ | $\Delta$Cash$_{i,t+1}$ | $\Delta$Debt$_{i,t+1}$ |
| **Indebtedness**$_{i,t}$ | -0.002*** | **-0.022*****  | -0.014*** |
| | (0.000) | (0.000) | (0.000) |
| Tobin's $Q_{i,t}$ | 0.022*** | 0.018*** | 0.023*** |
| | (0.000) | (0.000) | (0.000) |
| Size$_{i,t}$ | -0.079*** | -0.087*** | -0.070*** |
| | (0.000) | (0.001) | (0.000) |
| R-Squared | 0.726 | 0.116 | 0.594 |

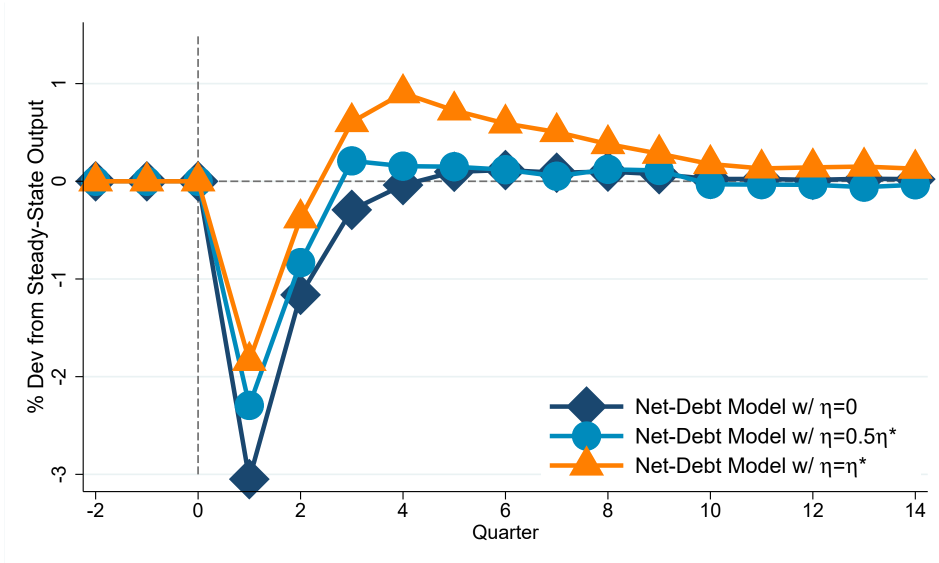
**Notes**: The table reports the estimated relationship between firm behavior and firm characteristics using simulated data from alternative models. $*, **$, and $***$ represent results significant at the $10\%, 5\%$, and $1\%$ levels, respectively.

**Table 1.13:** Firm Behavior and Firm Characteristics: Alternative Models

| | **Model w/o debt issuance frictions** | | |
|---|---|---|---|
| $\Delta \ln y_{i,t+1}$: | $\Delta$Capital$_{i,t+1}$ | $\Delta$Cash$_{i,t+1}$ | $\Delta$Debt$_{i,t+1}$ |
| **Indebtedness**$_{i,t}$ | -0.006*** | 0.173*** | -0.015*** |
| | (0.000) | (0.001) | (0.000) |
| Tobin's $Q_{i,t}$ | 0.052*** | 0.026*** | 0.036*** |
| | (0.000) | (0.000) | (0.000) |
| Size$_{i,t}$ | -0.024*** | -0.040*** | -0.033*** |
| | (0.000) | (0.002) | (0.001) |
| R-Squared | 0.754 | 0.123 | 0.279 |

**Notes**: The table reports the estimated relationship between firm behavior and firm characteristics using simulated data from alternative models. $*, **$, and $***$ represent results significant at the $10\%, 5\%$, and $1\%$ levels, respectively.

pronounced. However, this calibration also predicts a higher leverage ratio and a lower cash ratio relative to the baseline model.




**Notes:** Figure 1.14 plots output responses to the same uncertainty shocks in the net-debt model with different levels of debt issuance cost $\eta$.

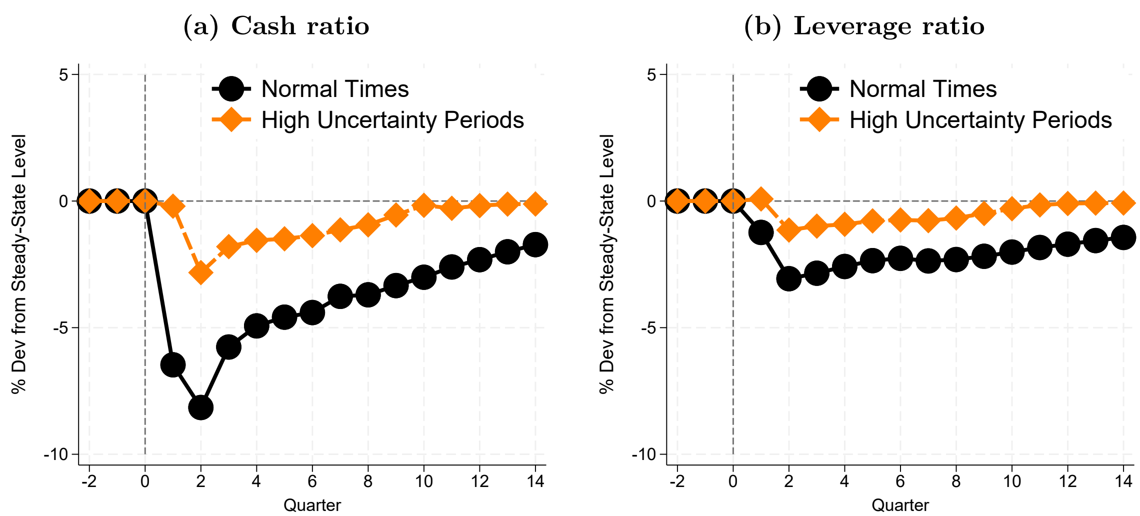
**Figure 1.14:** Output Responses to Uncertainty Shocks

### 1.10.7 Firm Responses to Investment Tax Credits

This subsection shows how firms finance capital investment induced by investment tax credits. Importantly, Panel (A) of Figure 1.15 shows that firms draw down cash to finance capital investment, and such behavior is significantly mitigated during periods of high uncertainty. Besides, Investment tax credits lower the costs of investment and thus reduce firms' reliance on credit. This leads to a drop in leverage ratio, as shown in Panel (B) of Figure 1.15.

### 1.10.8 TFP-driven Booms and Credit Interventions

The estimated output responses to credit interventions in TFP-driven booms are slightly larger than in normal times. This occurs because positive productivity shocks increase firms' investment demand and financial needs, thereby amplifying the role of credit interventions in relaxing firms' financial constraints. This is in contrast to the weaker effects of credit interventions during TFP-driven recessions, where investment demand becomes lower than the steady-state level.




**Notes:** This figure plots the responses of aggregate cash ratio and leverage ratio to investment tax credits during normal times and periods of high uncertainty.

**Figure 1.15:** Firm Responses to Investment Tax Credits

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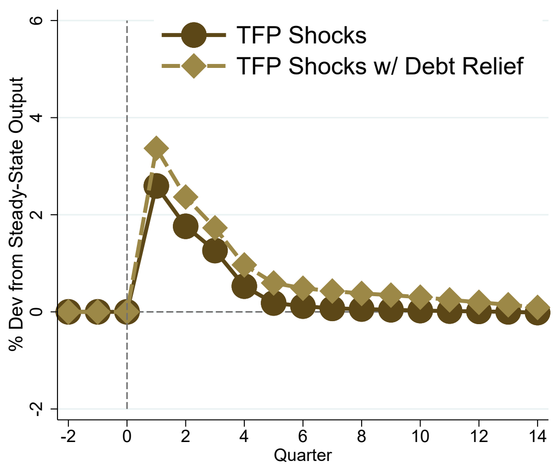
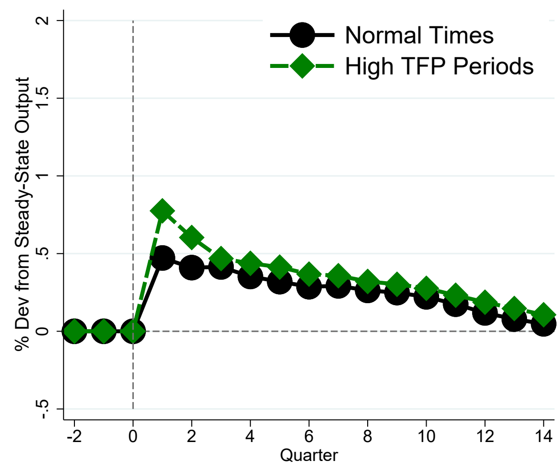
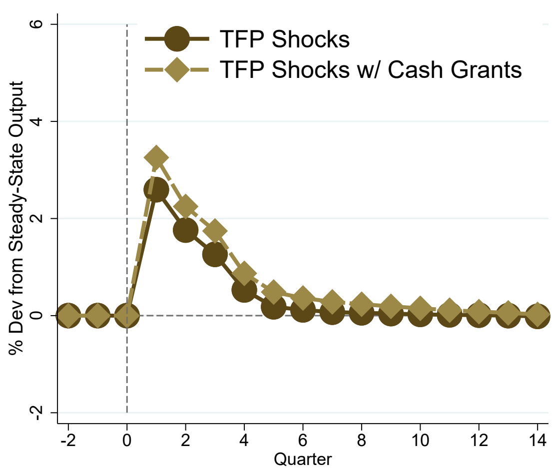
**(a) Output Responses to TFP Shocks**



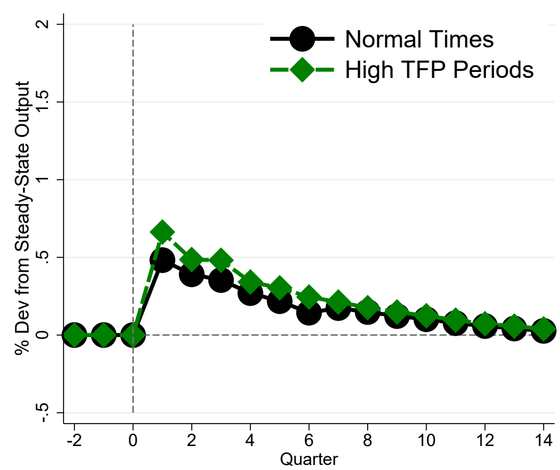

**(b) Output Responses to Debt Relief**




(c) Output Responses to TFP Shocks




**(d) Output Responses to Cash Grants**




**Notes:** Panels (A) and (C) plot aggregate output responses to positive productivity shocks with and without credit interventions. Panels (B) and (D) plot the output responses to policy interventions during normal times and periods of high productivity. Appendix 1.10.3 details the computation of aggregate impulse response functions.

**Figure 1.16:** TFP-driven Booms and Credit Interventions

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# Chapter 2

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# Debt Dilution, Debt Covenants, and Macroeconomic Fluctuations

*by* Min Fang and Wentao Zhou

## 2.1 Introduction

Financial frictions have long been recognized as an essential component of macroeconomic fluctuations. An extensive literature has studied how one friction, the inability to commit to debt repayment, affects firm and macroeconomic dynamics (Bernanke et al., 1999; Cooley and Quadrini, 2001; Albuquerque and Hopenhayn, 2004; Arellano et al., 2019; Jungherr and Schott, 2022).[1] Commitment is particularly important as most corporate debt is long-term. Conditional on not defaulting today, in many models firms are unable to commit to not diluting the value of existing debt through future borrowing. If creditors cannot intervene in such cases, severe macroeconomic consequences result, especially during economic downturns.

In reality, creditors do act. To prevent dilution of existing debt, most creditors usually set covenants enforcing a maximum debt- or (interest)-to-earnings ratio for borrowing firms *(debt-to-earnings ratio, for short).* As documented by Lian and Ma (2021), 80% of U.S. corporate borrowing by non-financial firms (by value) utilizes debt covenants. These debt covenants have a long history spanning millennia. One of the

[1]There is also an extensive literature, e.g., Bernanke and Gertler (1995) and Kiyotaki and Moore (1997) among many others, focusing on how collateral addresses shareholder expropriations of lenders' wealth and the business cycle consequences. We focus, however, on the defaultable debt framework.

earliest recorded instances of debt covenants is found in the Code of Hammurabi, a set of ancient Babylonian laws from around 1754 BCE.[2]

In this paper, we revisit the macroeconomic effects of debt in the presence of debt covenants. We build a business cycle model of production, long-term firm debt with covenants, and costly default based on the framework of Jungherr and Schott (2022). Holding financial conditions constant, we show that introducing debt covenants into a real business cycle model reduces business cycle volatility. Debt covenants also significantly ease the severity of debt overhang and thus boost capital accumulation, output, and consumption.

The key mechanism is that the ex-post penalty associated with covenant violations aligns shareholders' incentives with lenders' interests in the face of default risk, thereby mitigating ex-ante debt dilution and debt overhang. Without debt covenants, firms have a strong incentive to dilute existing debt during economic downturns as shareholders are unwilling to reduce debt and correspondingly default risk since the benefits would mostly accrue to creditors. This worsens financial conditions and amplifies recessions. This debt dilution mechanism is well-documented in Jungherr and Schott (2022) for corporate debt and in Hatchondo et al. (2016) for sovereign debt.

With debt covenants, an intermediate layer of covenant violation, also known as "technical default", is introduced between repaying and costly full default. Now firms cannot easily dilute existing debt due to covenant penalties but are instead incentivized to remain financially healthy by keeping a debt-to-earnings ratio relatively low. Covenant violation still occurs frequently since "technical default" is much less costly than a true default. Less debt dilution improves financial conditions, reflected in higher debt prices and lower credit spreads, and mitigates recessions.

Furthermore, debt covenants boost capital accumulation and therefore the long-run levels of output and consumption. Without debt covenants, the debt dilution incentives facing shareholders create severe debt overhang problems since creditors ask for a higher credit spread, even when the firm is financially healthy. This dissuades profitable investments since earnings from such projects would largely go to debt holders. With debt covenants, reduced debt dilution mitigates the debt overhang problem. Creditors accept lower rates, especially for financially healthy firms, inducing investment as

[2]These laws contained provisions related to lending and borrowing, including the terms and conditions under which loans were made. While not referred to as "covenants" in the modern sense, these provisions served a similar purpose by outlining the obligations and consequences for borrowers who failed to meet them.

shareholders now keep most earnings from new projects. Therefore, holding financial constant, debt covenants increase long-run economic performance.

We quantitatively take our baseline business cycle model with both debt dilution and debt covenants to U.S. data and compare it to business cycle models with only debt dilution and with neither. For the model with only debt dilution, we recalibrate the model to the same leverage and credit spread moments for a fair comparison. For the model with neither, we assume a constrained efficiency model in which a social planner maximizes both shareholder and creditor value. The constrained efficiency model completely resolves the debt dilution problem and consequently, debt covenants play no role. The constrained efficiency model has less leverage, lower credit spreads, and higher output, capital, and consumption.

Comparing all three models, we quantitatively show that debt covenants mitigate debt dilution by reducing desired leverage, increasing debt prices, and reducing credit spreads conditional on the outstanding debt level. More importantly, mitigation of debt dilution significantly reduces the countercyclical movements in leverage following a negative productivity shock. In response to a negative 2% TFP shock, the baseline model has a leverage spike of 4.2% instead of 7% in the model without debt covenants. Consequently, output and capital dropped by 8% and 6.8% instead of by 9.5% and 9.2% without debt covenants. In contrast, the constrained efficiency model has the smallest recessionary effects, as expected, without dilution problems. The presence of debt covenants helps stabilize the amplified recessions caused by debt dilution. We find that debt covenants also reduce business cycle asymmetry.

Finally, we show the long-run effects on economic performance with debt covenants. The model with debt covenants generates higher output, capital, and consumption than the model without debt covenants due to reduced financial frictions between firms and creditors. Although both models have the same mean leverage and credit spread, the model with debt covenants can maintain a 5% larger capital stock, 2% higher output, and 1.5% higher consumption.

### 2.1.1 Literature

Our paper contributes to two strands of literature. First, this paper is related to the broader literature of studies on financial frictions and their implications for the aggregate economy. In most work in this literature, debt dilution is not an issue since debt contracts are only short-term.[3] However, the literature shows that most corporate debt is long-term debt, which makes debt dilution a serious commitment issue between shareholders and creditors. As documented in Adrian et al. (2013), the average term to maturity is three to four years for bank loans and more than eight years for corporate bonds. Recent macroeconomic literature (Gomes et al., 2016; Jungherr and Schott, 2021, 2022; Jungherr et al., 2022; Deng and Fang, 2022; Poeschl, 2023) shows that long-term debt commitment issues matter for macroeconomic dynamics and monetary policy.[4] The paper we build on, Jungherr and Schott (2022), shows that debt dilution generates countercyclical leverage, amplified output volatility, and prolonged recessions. We show that the existence of debt covenants partially resolves the issue of debt dilution, reduces the volatility of leverage and output, and shortens recessions. Meanwhile, reduced debt dilution due to debt covenants also mitigates the debt overhang problem and thus boosts capital accumulation, output, and consumption.

The second strand of literature investigates debt covenants in finance and macroeconomics. A large finance literature has empirically and clearly documented the existence of debt covenants and the effects of covenant violations on firm-level outcomes (Chava and Roberts, 2008; Roberts and Sufi, 2009; Nini et al., 2012; Roberts, 2015; Falato and Liang, 2016; Chodorow-Reich and Falato, 2022; Ersahin et al., 2021). More recently, macro-finance literature has started to focus on the micro and macro effects of debt covenants outside of covenant violations (earning-based borrowing constraints) both empirically and theoretically (Lian and Ma, 2021; Adler, 2020; Greenwald, 2019; Drechsel, 2023; Öztürk, 2022).[5] These five papers provide rich empirical evidence and quantitative implications, with Lian and Ma (2021) and Adler (2020) focusing more on empirical findings, and Drechsel (2023), Greenwald (2019), and Öztürk (2022) focusing more on quantitative implications.

These papers model debt covenants as earning-based borrowing constraints in which

[3]Including many influential papers such as Kiyotaki and Moore (1997), Bernanke et al. (1999), Cooley and Quadrini (2001), Albuquerque and Hopenhayn (2004), Jermann and Quadrini (2012), Khan and Thomas (2013), Gilchrist and Zakrajšek (2012), and Arellano et al. (2019).

[4]There is a large sovereign default literature studying debt dilution and commitment, including Arellano and Ramanarayanan (2012), Chatterjee and Eyigungor (2012), Hatchondo et al. (2016), and Aguiar et al. (2019). The most closely related to us is Hatchondo et al. (2016), which shows that if debt covenants existed for a sovereign, sovereign debt dilution could be largely resolved with corresponding welfare gains. Literature in corporate finance, e.g., Admati et al. (2018) and DeMarzo and He (2021), also considers such commitment issues. Literature in financial intermediation, e.g., Corhay and Tong (2021), examines commitment issues (profit shifting) between firms and banks due to inflation shocks.

[5]There are other important recent works on debt covenants, including Nikolov et al. (2019), Xiang (2023), Gamba and Mao (2020), Davydenko et al. (2020), and Arnold and Westermann (2023).

corporate debt is short-term, so there are no debt dilution or debt overhang issues for creditors. Lian and Ma (2021) shows that cash flows in the form of operating earnings can directly relax borrowing constraints and consequently make firms less vulnerable to collateral prices. Adler (2020) focuses on the precautionary effect of covenants that leads to a reduction in aggregate investment. Drechsel (2023) shows that earning-based borrowing constraints amplify the transmission of supply shocks to output. Greenwald (2019) shows that interest coverage covenants amplify monetary policy due to changes in coverage limits, while Öztürk (2022) focuses on changes in collateral value and shows that asset-based borrowers are more responsive than cash flow-based borrowers to monetary policy. Our paper focuses on the essential role of debt covenants in preventing debt dilution and easing debt overhang and, therefore, improving economic performance in terms of both reduced business cycle volatility and increased long-run level of output.

The rest of the paper is organized as follows. Section 2.2 exhibits the full model. Section **??** takes the model to the data. Section 2.3 illustrates the mechanism and properties of the model. Finally, Section 2.4 concludes.

## 2.2 The Model

We build a dynamic open economy business cycle model with firm production and financing following the long-term debt model in Jungherr and Schott (2022). We extend their framework in two ways. First, we add debt covenants. Second, we modify their capital quality shocks to match both observed covenant violations and credit spread in the data. The international risk-free rate $r$ is fixed as in Arellano et al. (2019).

### 2.2.1 Firm Setup

**Production and Earning** A firm $i$ uses capital $k_{it}$ and labor $l_{it}$ to produce output according to $y_{it} = A_t(k_{it}^{\psi} l_{it}^{1-\psi})^{\zeta}$, where $\psi, \zeta \in (0,1)$. The natural logarithm of aggregate revenue productivity $A_t$ follows an AR(1) process and is realized at the beginning of period $t$. Firm then choose labor $l_{it}$ to maximize profits

$$\pi(k_{it}; A_t, w_t) = \max_{l_{it}} y_{it} - w_t l_{it} \tag{2.1}$$

**Idiosyncratic Shocks** Firms face idiosyncratic risk, which might drive down firms' values and then lead to defaults. There are two sources of idiosyncratic risk for firms:

cash flow shocks and capital quality shocks. The cash flow shock directly affects firms' earnings, and thus a firm's earnings before interest, taxes, depreciation, and amortization (EBITDA) are then given by

$$\text{EBITDA}_{it} = \pi_{it} - \epsilon_{it} \tag{2.2}$$

where $\epsilon_{it}$ is i.i.d and follows a standard normal distribution $F(\epsilon)$ with $\sigma_{\epsilon}^{n}$. This setup allows our model to capture the existence of negative earnings in the data.

Firms also face capital quality shocks $z_{it}$ that affect their capital stock, such as unforeseen changes in consumer demand that affect the value of their capital or unforeseen natural disasters that affect the amount of their capital. Each period after production, capital quality shock realizes and the end-of-period firm-specific capital stock is given by:

$$\text{End-of-period capital stock} = (1-\delta)k_{it} - z_{it}k_{it} \tag{2.3}$$

where the capital quality shocks $z_{it}$ i.i.d and follows a standard normal distribution $\Phi(z)$ with $\sigma_z$.

**Firm Financing** The firm can finance capital with equity and long-term debt. We model long-term debt using the computationally tractable specification from Leland (1994). A long-term bond issued at the end of period $t-1$ is a promise to pay a fixed coupon payment $c$ and a fraction $\gamma \in (0,1)$ of the principal in period $t$, implying total debt payments of $(c+\gamma)b_{it-1}$.

In period $t$, fraction $1-\gamma$ of the bond remains outstanding. Payments decay geometrically over time. The number of long-term bonds chosen by the firm in period $t$ is $b_{it}$. The firm can issue equity freely at no cost but with a lower bound $e_{it} \geq -\underline{e}$ where $\underline{e} > 0$ means firms cannot finance through equity Ponzi games. The firm also has limited liability. Shareholders are free to default and hand over the firm's assets to creditors for liquidation. In this case, a fixed fraction $\xi$ of firm assets is lost during the fire sale of assets or debt restructuring.

**Debt Covenants** Long-term debt has earnings-to-debt covenants as in the data. A firm breaches its earnings-to-debt covenant if

$$\frac{\text{EBITDA}_{it}}{b_{it}} \leq \eta \tag{2.4}$$

where $\eta$ is the contracted violation threshold. When a debt is violated, the creditor punishes the firm with a financial penalty of $\chi b_{it}$, which is proportional to its current debt position. Such a reduced-form penalty summarizes various forms of debt covenant violation penalties that are usually proportional to the size of the debt, including renegotiation of higher interest rates, up-front fees in exchange for waiving the covenant violation, or immediate repayment of the debt.[6]

### 2.2.2 Shareholder's Problem

Each period, after production and realization of idiosyncratic shocks, a firm's net worth is given by:

$$n_{it} = (1-\tau)(\underbrace{\pi_{it} - \epsilon_{it}}_{\text{EBITDA}_{it}}) + \underbrace{(1-\delta)k_{it} + \tau\delta k_{it} - z_{it}k_{it}}_{\text{capital stock}} - \underbrace{[(1-\tau)c + \gamma + \chi \cdot \mathbf{1}_{\{\epsilon > \epsilon^c\}} b_{it}]}_{\text{debt burden}} - f \tag{2.5}$$

where $\tau$ denotes corporate tax, $f$ is a fixed operation cost, $\epsilon_{it}^c$ denotes the cutoff value of cash flow shock that triggers covenant violation:

$$\epsilon_{it}^c = \pi_{it} - b_{it}\eta \tag{2.6}$$

Denote the aggregate state of the economy as $S_t$, firm net worth as $n(z_{it}, k_{it}, b_{it}, \epsilon_{it}, z_{it})$ , and firm continuation value as $V((1-\gamma)b_t, S_t)$. Firms will default if

$$n\Big(k_{it}, b_{it}, \epsilon_{it}, z_{it}^d(\epsilon_{it}), S_t\Big) + E_{S_t|S_{t-1}} V(b_{it}, S_t) < 0 \tag{2.7}$$

Given each cash flow shock $\epsilon_{it}$, we know that there exists a cutoff value of capital quality shock $z_{it}^d(\epsilon_{it})$ such that firm will default, therefore, $z_{it}^d(\epsilon_{it})$ is defined as follows:

$$n\Big(k_{it}, b_{it}, \epsilon_{it}, z_{it}^d(\epsilon_{it}), S_t\Big) + E_{S_t|S_{t-1}} V(b_{it}, S_t) = 0 \tag{2.8}$$

[6]As Greenwald (2019) *Background: Debt Covenants* describes: "In practice, lenders typically do not demand full repayment upon violation, but instead renegotiate the terms of the loan, often extracting some concession in the form of a higher interest rate or up-front fee in exchange for waiving the covenant violation." We also intentionally choose not to model the penalty as "repayment acceleration" which would shorten the maturity $\lambda$ when covenants are violated since the shortening the maturity would mechanically reduce debt dilution issues. Jiang and Xu (2019) provides direct empirical evidence that paying covenant amendment fees after covenant violations is a common practice. They also show that there is a significantly positive real value added by creditors taking explicit actions intervening in the operation of borrowers in covenant violations.

The continuation value can be expressed as:

$$V\Big(b_{it}, S_t\Big) = \max_{k_{i,t+1}, b_{i,t+1}} -e_{i,t+1} + \frac{1}{1+r}\int_{-\infty}^{\infty}\left[\int_{-\infty}^{z^d_{i,t+1}(\epsilon_{i,t+1})} n_{i,t+1} + E_{S_{t+1}|S_t} V\Big(b_{i,t+1}, S_{i,t+1}\Big) d\Phi(z)\right] dF(\epsilon) \tag{2.9}$$

where $e_{i,t+1}$ denotes firm's equity issuance and capital choice is determined by debt and equity issuance as follows:

$$k_{i,t+1} = Q(k_{i,t+1}, b_{i,t+1}, S_t)[b_{i,t+1} - (1-\gamma)b_{it}] + e_{i,t+1} \tag{2.10}$$

$Q(k_{i,t+1}, b_{i,t+1}, S_{t+1})$ is firm's bond price, and firm capital and debt choices will determine their covenant violation cutoffs $\epsilon^c_{i,t+1}$ and default cutoffs $z^d_{i,t+1}(\epsilon_{i,t+1})$ as in equations (2.7) and (2.8).

### 2.2.3 Creditor Problem

Competitive creditors break even in expectations. Like shareholders, they discount cash flow at the international risk-free rate $r$. In case of default, the liquidation value of the firm is

$$n^d_{i,t+1} = (1-\xi)\Bigg[(1-\tau)(\underbrace{\pi_{i,t+1} - \epsilon_{i,t+1}}_{\text{EBITDA}_{it}}) + \underbrace{(1-\delta)k_{i,t+1} + \tau\delta k_{i,t+1} - z_{it}k_{i,t+1}}_{\text{capital stock}} - f\Bigg] \tag{2.11}$$

where $\xi$ reflects the proportional liquidation cost of the firm as a whole. The bond price also depends on the future market value of long-term debt:

$$\begin{aligned} Q(k_{i,t+1}, b_{i,t+1}, S_t) = \frac{1}{1+r}\int_{-\infty}^{\infty}\Bigg[ & \underbrace{\int_{z^d_{i,t+1}(\epsilon_{i,t+1})}^{\infty} \frac{n^d_{i,t+1}}{b_{i,t+1}} d\Phi(\epsilon_{it})}_{\text{default}} + \\ & \underbrace{\int_{-\infty}^{z^d_{i,t+1}(\epsilon_{i,t+1})} (\gamma + c) + (1-\gamma)E_{S_{t+1}|S_t} Q(k_{i,t+2}, b_{i,t+2}, S_{t+1}) d\Phi(z)}_{\text{non-default}}\Bigg] dF(\epsilon) \end{aligned} \tag{2.12}$$

where $Q(k_{i,t+2}, b_{i,t+2}, S_{t+1})$ denotes firm's bond price given their capital and debt choices at period $t+2$.

### 2.2.4 Firm Policy and Aggregation

In equilibrium, a firm maximizes shareholder value (2.9) subject to the equilibrium bond price functions (2.12), net worth accumulation equations (2.5), covenant violation equation (2.8), default cutoff equation (2.7). As in Klein, Krusell, and Rios-Rull (2008), we restrict attention to the Markov Perfect equilibrium, i.e. we consider policy rules which are functions of the payoff-relevant state variables. The time-consistent policy is a fixed point in which the future firm policy coincides with today's firm policy.

**Firm Policy** To solve for the equilibrium firm policy, we compute the continuation value term $V(b, S)$ recursively. We define a policy vector $\phi(b_{it}, S_t) = \{k_{i,t+1}, b_{i,t+1}\}$ which solves

$$V\Big(b_{it}, S_t\Big) = \max_{\phi(b_{it},S_t)} -e_{i,t+1} + \frac{1}{1+r}\int_{-\infty}^{\infty}\left[\int_{-\infty}^{z^d_{i,t+1}(\epsilon_{i,t+1})} n_{i,t+1} + E_{S_{t+1}|S_t}V\Big(b_{i,t+1}, S_{i,t+1}\Big)d\Phi(z)\right]dF(\epsilon) \tag{2.13}$$

subjects to

$$\begin{aligned}
k_{i,t+1} &= Q(k_{i,t+1}, b_{i,t+1}, S_t)[b_{i,t+1} - (1-\gamma)b_{it}] + e_{i,t+1} \\
n_{i,t+1} &= (1-\tau)(\pi_{i,t+1} - \epsilon_{i,t+1}) + (1-\delta)k_{i,t+1} + \tau\delta k_{i,t+1} - z_{i,t+1}k_{i,t+1} \\
&\qquad\qquad - [(1-\tau)c + \gamma + \chi\cdot\mathbf{1}_{\{\epsilon_{i,t+1}>\epsilon^c_{i,t+1}\}}b_{i,t+1}] - f \\
\epsilon^c_{i,t+1} &= \pi_{i,t+1} - b_{i,t+1}\eta \\
n_{i,t+1} &+ E_{S_{t+1}|S_t}V\Big(b_{i,t+1}, S_{i,t+1}\Big) = 0 \\
Q(k_{i,t+1}, b_{i,t+1}, S_t) &= \frac{1}{1+r}\int_{-\infty}^{\infty}\left[\int_{z^d_{i,t+1}(\epsilon_{i,t+1})}^{\infty}\frac{n^d_{i,t+1}}{b_{i,t+1}}d\Phi(\epsilon_{it}) + \right. \\
&\qquad\left.\int_{-\infty}^{z^d_{i,t+1}(\epsilon_{i,t+1})}(\gamma + c) + (1-\gamma)E_{S_{t+1}|S_t}Q(k_{i,t+2}, b_{i,t+2}, S_{t+1})d\Phi(z)\right]dF(\epsilon) \\
n^d_{i,t+1} &= (1-\xi)\Big[(1-\tau)(\pi_{i,t+1} - \epsilon_{i,t+1}) + (1-\delta)k_{i,t+1} + \tau\delta k_{i,t+1} - z_{it}k_{i,t+1} - f\Big]
\end{aligned}$$

Note that firm outcomes differ ex-post because of the i.i.d. shocks. However, because there are no equity adjustment costs, past earnings do not affect the current optimal firm policy $\phi(b_{it}, S_t)$.

**Aggregation** We ensure all firms in the economy are ex-ante identical with three assumptions. First, we assume a constant unit mass of firms. Second, we presume defaulting firms exit the economy and are replaced by precisely the same amount of new entrants. Third, entering firms pay an entry cost financed by long-term debt $b$ that matches the debt of incumbent firms. These assumptions imply the model aggregates exactly such that the aggregate quantities equal firm-level quantities $\{b = B, l = L, k = K\}$.

### 2.2.5 Households and Equilibrium

**Households** We close the general equilibrium model by introducing a representative domestic household. The household works, consumes, and invests its savings at the international risk-free rate $r$. Government revenue from taxation is paid out to the household as a lump-sum transfer. We assume GHH preferences over consumption C and labor L. Period utility is, therefore, $u(C_t - \frac{L_t^{1+\theta}}{1+\theta})$, which yields a labor supply curve $w_t = L_t^{\theta}$. Period consumption $C_t$ equals the total output $Y_t$ minus capital depreciation (aggregate investment).

**Equilibrium** We define the equilibrium of a dynamic open economy business cycle model by a given international risk-free rate $r$ and an endogenous wage $w$. The aggregate state, $S$, of the economy is sufficiently summarized by the aggregate productivity $z'$ and the aggregate stock of existing debt $B$: $S = (z', B)$.

**Definition 2.1** (Recursive Competitive Equilibrium)**.** The recursive competitive equilibrium consists of (1) a policy vector $\phi(b, S) = \{\tilde{e}, k, b, \epsilon^c, \epsilon^d\}$, bond price $Q$, and a value function $V(b, S)$; (2) a wage function $w = L^{\theta}$; (3) a stochastic aggregate law of motion $S' = F(S)$; and (4) aggregate quantities equalling firm-level quantities $\{b = B, l = L, k = K\}$ such that

1. $\phi(b, S)$, $Q$, and $V(b, S)$ solve the firm problem (2.13).

2. The labor market clears $L = l(b, S)$.

3. Total output $Y = y - f - \frac{\xi}{1-\xi} \int_{-\infty}^{\epsilon^d} \frac{n^d}{b} d\Phi(\epsilon') - \chi \int_{\epsilon_d}^{\epsilon^c} b d\Phi(\epsilon')$.

### 2.2.6 Constrained Efficiency

The baseline model is inefficient because of firms' inability to commit to future actions. They excessively dilute debt as part of the costs is placed externally on the existing creditors, who bear increased default risk from new debt issuance. Though debt covenants do help align incentives and keep default risks low, firms still often violate covenants.

To compare efficiency between models, we construct a constrained efficiency model as a benchmark.

We assume a social planner maximizes the total value of individual firms, which is the sum of all equity and all outstanding debt, both existing and newly issued. The planner is still subject to the same lack of commitment and faces the same set of constraints. The planner solves:

$$W(b_{it-1}, S_{t-1}) = \max_{\phi(b_{it-1}, S_{t-1})} -\tilde{e}_{it-1} + b_{it-1}Q_{it-1} + \frac{1}{1+r}E_{S_t|S_{t-1}}\left[\int_{\epsilon_{it}^d}^{\infty}\Big(n_{it} + V(b_{it}, S_t)\Big)d\Phi(\epsilon_{it})\right] \tag{2.14}$$

subject to the same set of constraints as in the firm problem (2.13).

### 2.2.7 Solution Method

We follow Jungherr and Schott (2022) and Hatchondo et al. (2016) to solve our model. We solve for the equilibrium of a finite-horizon economy with sufficient periods to ensure the value functions and bond prices for the first and second periods are very close. We then use the first-period equilibrium functions as the infinite-horizon economy equilibrium functions.

### 2.2.8 Calibration of the Baseline Model

Our parameterization of the baseline model proceeds in two steps. First, we externally fix a set of parameters to match standard macroeconomic targets in the open economy business cycle models (Arellano et al., 2019; Jungherr and Schott, 2022). Second, given these fixed parameter values, we choose the remaining fitted parameters to match moments in the US data.

**Fixed Parameters** Table 2.1 lists the externally fixed parameters. We first pick parameters for the general environment faced by firms. The model is set at a yearly frequency, so we choose a discount factor of 0.97. Correspondingly, the international risk-free rate equals $1/\beta - 1 = 0.0309$. The inverted Frisch elasticity is set to 0.25 as in King and Rebelo (1999), and the corporate tax rate is taken to be 40% as in Gomes et al. (2016), which suggests that $\tau$ should be capturing all benefits of using debt rather than equity. We then choose the production technology parameters. Firms face a capital

**Table 2.1:** Externally Fixed Parameters

| Parameter | Description | Value | Source/Targets |
|---|---|---|---|
| | | (a).**General Environment** | |
| $\beta$ | Discount factor | 0.97 | Annual frequency |
| $r$ | International risk-free rate | 0.0309 | $r = 1/\beta - 1$ |
| $\theta$ | Inverted Frisch elasticity | 0.25 | King and Rebelo (1999) |
| $\tau$ | Corporate tax rate | 0.40 | Gomes et al. (2016) |
| | | (b).**Production Technology** | |
| $\psi$ | Capital share | 0.33 | Standard as in Bloom et al. (2018) |
| $\zeta$ | Decreasing returns-to-scale | 0.75 | Standard as in Bloom et al. (2018) |
| $\delta$ | Depreciation rate | 0.10 | Annual rate of 10% |
| $\rho_z$ | Persistence | 0.909 | Standard as in Khan and Thomas (2008) |
| | | (c).**Financial Market** | |
| $\gamma$ | Debt repayment rate | 0.1284 | Maturity $1/\gamma = 6.47$ years |
| $\tau$ | Debt coupon | 0.0309 | Debt coupon= $r$ |
| $\eta$ | Debt-to-earnings ratio threshold | 0.25 | Market threshold as in Lian and Ma (2021) |
| $\xi$ | Default cost | 0.469 | Liquidation cost as in Kermani and Ma (2023) |

share $\psi$ of 0.33, a decreasing returns-to-scale parameter $\zeta$ of 0.75, and an annual capital depreciation of 0.1. We choose the persistence of the aggregate TFP shock $\rho_z$ to be 0.0909 as in Khan and Thomas (2008) and would later fit the volatility $\sigma_z$ to match the business cycle output volatility in the U.S. GDP data.

Finally, we choose the parameters capturing the financial market. The debt repayment rate $\gamma$ is set to 0.1284 to match an average debt maturity of 6.47 years as suggested by Gilchrist and Zakrajšek (2012). The debt coupon $c$ is chosen to be the same as the risk-free rate $r$. Finally, we choose the debt-to-earnings ratio threshold $\eta$ to be 0.22 to match a standard characteristic of U.S. debt covenants as documented for the majority of U.S. debt documented in Lian and Ma (2021) and the default cost $\xi$ to be 0.469 to match the bankruptcy liquidation cost estimated in Kermani and Ma (2023).

**Table 2.2:** Internally Fitted Parameters and Model Fit

| Param. | Description | Value | Targets | Data | Model |
|---|---|---|---|---|---|
| $f$ | Fixed operation cost | 0.151 | Leverage ratio | 33% | 33% |
| $\chi$ | Covenant violation cost | 0.015 | Covenant violation rate | 23% | 23% |
| $\sigma_z$ | Productivity shock vol. | 0.006 | Volatility of U.S. GDP | 2.8% | 2.8% |
| $\sigma_\epsilon^n$ | Cash flow shock vol. | 0.326 | Frequency of negative EBIT | 18% | 18% |
| $\sigma_\epsilon^f$ | Capital quality shock vol. | 0.795 | Credit spread | 2.0% | 2.0% |

**Fitted Parameters** Table 2.4 lists the parameters that we internally fitted and the corresponding moments they matched in the data. Though the fitted parameters are jointly determined, they are closely tied to specific moments. We first choose the productivity shock volatility $\sigma_z = 0.0006$ to generate GDP volatility of 2.8%. We then choose the capital quality shock volatility $\sigma_\epsilon^n = 0.293$ and the fixed operation cost $f = 0.169$ to match the average leverage of about 34% roughly and a 15% frequency of negative EBITDA. We target a default rate of about 3.2% with the future capital quality shock volatility $\sigma_\epsilon^f = 0.782$.[7] Finally, we match an annual covenant violation rate of 18% by choosing the covenant violation cost $\chi = 0.015$.

### 2.2.9 Calibration of Alternative Models

To illustrate the properties of the baseline model, we compare our baseline model with covenants, the COV model, to the model without covenants, the NoCOV model, and the constrained efficiency model, the CEE model. The fitted parameters and model fits are displayed in Table 2.3 below. In the CEE model, since the social planner maximizes the total value of the firm, creditors do not punish firms with covenant violation costs. The debt dilution issue is also fully resolved by the social planner. We observe lower leverage ratios, GDP volatility, and credit spreads relative to the baseline COV model with debt dilution.

**Table 2.3:** Alternative Models: Fitted Parameters and Model Fit

| Description<br>Calibration | Param. | COV<br>(baseline) | NoCOV<br>(same mom.) | NoCOV<br>(no recali.) | CEE<br>(no recali.) |
|---|---|---|---|---|---|
| Fixed operation cost | $f$ | 0.151 | 0.164 | 0.151 | 0.151 |
| Covenant violation cost | $\chi$ | 0.015 | n.a. | n.a. | n.a. |
| Productivity shock vol. | $\sigma_z$ | 0.006 | 0.006 | 0.006 | 0.006 |
| Cash flow shock vol. | $\sigma_\epsilon^n$ | 0.326 | 0.652 | 0.326 | 0.326 |
| Capital quality shock vol. | $\sigma_\epsilon^f$ | 0.795 | 0.652 | 0.795 | 0.795 |
| **Moments** | **Data** | | | | |
| Leverage ratio | 33% | 33% | 33% | 37% | 27% |
| Covenant violation rate | 23% | 23% | n.a. | n.a. | n.a. |
| U.S. volatility of GDP | 2.8% | 2.8% | 3.1% | 2.8% | 2.6% |
| Frequency of negative EBITA | 18% | 18% | n.a. | n.a. | n.a. |
| Credit Spread | 2.0% | 2.0% | 2.0% | 2.2% | 1.8% |

For the NoCOV models, we focus on the recalibrated one. Alternative models are only comparable to the baseline COV model when indebtedness and credit risks across

[7] The mean annual default rate of 3.2% is taken from the survey by Dun and Bradstreet (www.dnb.com).

both economies are similar. Therefore, we recalibrated a NoCOV model to match the leverage ratio and credit spread in the baseline COV model. Without debt covenants, the NoCOV economy suffers from more volatile output. Removing debt covenants also reveals properties of the baseline COV model. To this end, we also show a NoCOV model which simply removes debt covenants without recalibration. Such an economy has higher leverage and credit spreads than the baseline COV model, indicating the role of covenants in equilibrium financial conditions. These secondary results are located in the appendix.

## 2.3 Quantitative Results

We now quantitatively simulate the model and examine the key roles of debt covenants: 1) debt dilution mitigation, 2) business cycle stabilization, 3) reduction of business cycle asymmetry, and 4) improved long-run economic outcomes.

### 2.3.1 Debt Dilution Mitigation

We first show how introducing debt covenants reduces debt dilution. Without debt covenants, shareholders reoptimize debt every period conditional on their existing debt. Taking on any new debt dilutes the value of existing debt by reducing the claim incumbent borrowers have on earnings and assets. More indebted shareholders are further incentivized to dilute as this implies a larger value transfer from existing creditors to shareholders.

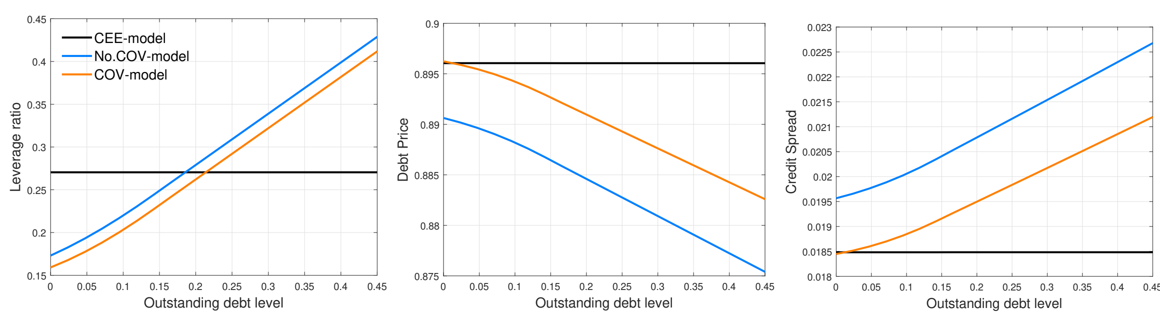



**Figure 2.1:** Debt Dilution Mitigation with Debt Covenants

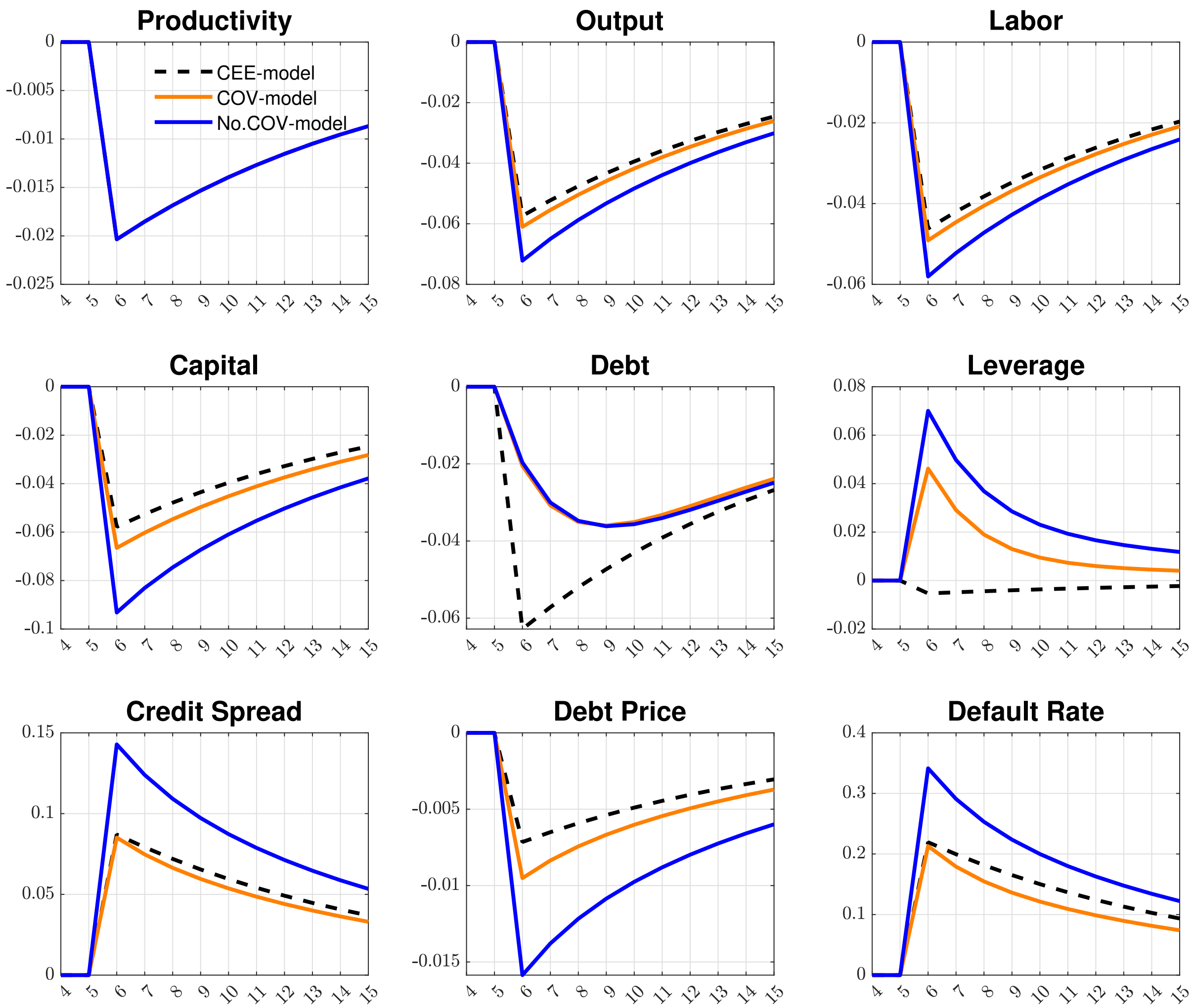
Figure 2.1 shows how the model with debt covenants prevents debt dilution in equilibrium. To better display the mechanism, we compare the baseline COV model with alternative models: the constrained efficiency CEE model and the NoCOV model without debt covenants. Figure 2.1(a) shows the leverage policy ($b'/k'$) conditional on existing debt ($b$). In the CEE model, leverage policy is independent of the existing debt since the social planner optimizes the total value of shareholders and creditors, eradicating debt dilution. In both the NoCOV and COV models, firms choose more leverage when carrying existing debt as shareholders are incentivized to issue new debt to dilute existing debt. The COV model partially mitigates such incentives through covenant violation penalties.

Similar patterns are observed in Figure 2.1(b) and 2.1(c) for debt prices and credit spreads, respectively. In the CEE model, the debt price and credit spread are unaffected by the level of existing debt since the social planner optimizes the total value of shareholders and creditors. In both the NoCOV and COV models, more debt increases default risk and, therefore, decreases debt prices and increases credit spreads. Due to the existence of covenant violation penalties, which mitigate the debt dilution incentives of shareholders in the COV model, creditors are willing to provide a better debt price – equivalently, a lower credit spread – given the same stock of debt.

### 2.3.2 Business Cycle Stabilization

We now show how debt covenants partially mitigate the amplified and prolonged responses of output and capital due to debt dilution following a negative productivity shock in Figure 2.2. We again compare the baseline COV model with debt dilution and debt covenants to alternative models: the constrained efficiency CEE model without debt dilution and the NoCOV model with debt dilution but without debt covenants.

**Without Debt Dilution** We first demonstrate the effects of a negative 2% productivity shock $Z_t$ in the CEE model, i.e., without debt dilution incentives. The social planner maximizes the combined value of shareholders and creditors and reacts to a negative productivity shock by reducing investment and labor demand, which results in declining output, labor, and capital (sub-figures 2 to 4). As shown above, the social planner internalizes potential default costs, which accrue to the holders of existing debt, leading to reductions in outstanding debt to reduce default risks. This results in a sharp decline in debt and even a counterfactual procyclical leverage response (sub-figures 5 and 6). An increased credit spread, decreased debt price, and increased default risk remain, which are beyond the control of the social planner due to the bad shock.




Notes: CEE model: Black dashed lines show impulse response functions in the constrained efficiency model without debt dilution. NoCOV model: Blue solid lines show impulse response functions in the model with debt dilution but without debt covenants. COV model: Orange solid lines show impulse response functions in the model with debt dilution and debt covenants.

**Figure 2.2:** Impulse Response Functions to a -2% TFP Shock

**With Debt Dilution/Without Debt Covenants** Given how the model without debt dilution performs, we now demonstrate the effects of strong debt dilution incentives in the NoCOV model compared to the CEE model. Such effects are well-documented and thoroughly discussed in Jungherr and Schott (2022) as *slow debt, deep recession, and slow recovery.* In the NoCOV model, firms again react to a negative productivity shock by reducing investment and labor demand, which results in the initial decline of output, labor, and capital. The key difference from the CEE model is the financial decisions as now shareholders do not internalize the increased default risks from carrying debt.

Firms choose to dilute existing debt by embracing more leverage and higher default risk. Their optimal leverage policy leads to debt falling more slowly than capital (*slow debt*). The slow deleveraging process leads to higher default risk, which in turn increases the cost of capital through higher credit spreads and further discourages investment and capital formation (and further increases leverage and default risk). Such an accelerating feedback loop between default risk and capital leads to deeper negative responses in capital and output compared to the CEE model without debt dilution (*deep recession*). Finally, the slow-moving debt choices create an extended period of high credit spreads relative to the CEE model without debt dilution. Output and capital remain further from their unconditional means for longer compared to the CEE model (*slow recovery*). These results show that debt dilution amplifies and prolongs a productivity recession.

**With Debt Dilution and Debt Covenants** Understanding how debt dilution amplifies and prolongs recessions caused by negative productivity shocks, we demonstrate the effects of debt covenants in the COV model compared to both the NoCOV model and the CEE model. As in both models above, COV model firms also react to a negative productivity shock by reducing investment and labor demand, again initially decreasing output, labor, and capital. Firms in the COV model also want to dilute existing debt by taking on more leverage and higher default risk. Ideally, they would choose enough leverage to maximize current tax benefits but transfer the potential losses due to default costs to debt holders. However, creditors are aware of shareholders' incentives and debt covenants are formalized in lending contracts. Shareholder optimization now internalizes the potential penalties of an increased debt-to-earnings ratio hitting the covenant violation threshold. Such debt covenants lead to a less aggressive optimal leverage policy response to the initial capital drop following the negative productivity shock by breaking the accelerating feedback loop between default risk and capital.

To avoid debt covenant violation costs, firms adopt relatively lower leverage ratios even though the potential gain via tax benefits from diluting existing debt is high. Meanwhile, since debt covenants are always triggered before default, the motivation to maintain a lower leverage ratio and, simultaneously, a lower debt-to-earnings ratio, also lowers default risk. This decreases the credit spread and increases debt prices relative to the NoCOV model. The slow debt, deep recession, and slow recovery effects are less severe in the COV model. Output and capital decline less compared to the NoCOV model and recovery from the recession is faster. However, the COV model economy still suffers from some debt dilution since debt covenants are only a partial solution, especially when firms are far from the violation threshold. Therefore, all debt dilution effects still exist but are mitigated.

As a result, the effects are quantitatively significant. In response to a negative 2% TFP shock, the baseline model has a leverage spike of 4.2% instead of 7% in the model without debt covenants. Consequently, output and capital dropped by 8% and 6.8% instead of 9.5% and 9.2% without debt covenants. In contrast, the constrained efficiency model has the smallest recessionary effects.

### 2.3.3 Business Cycle Asymmetry Reduction

This section compares the peaks from a +2% TFP shock to the troughs from a -2% TFP shock. We show that the existence of debt covenants also reduces business cycle asymmetry. To better display the mechanism, we again compare the baseline COV model with debt dilution and debt covenants to alternative models: the constrained efficiency CEE model without debt dilution and the NoCOV model with debt dilution but without debt covenants.

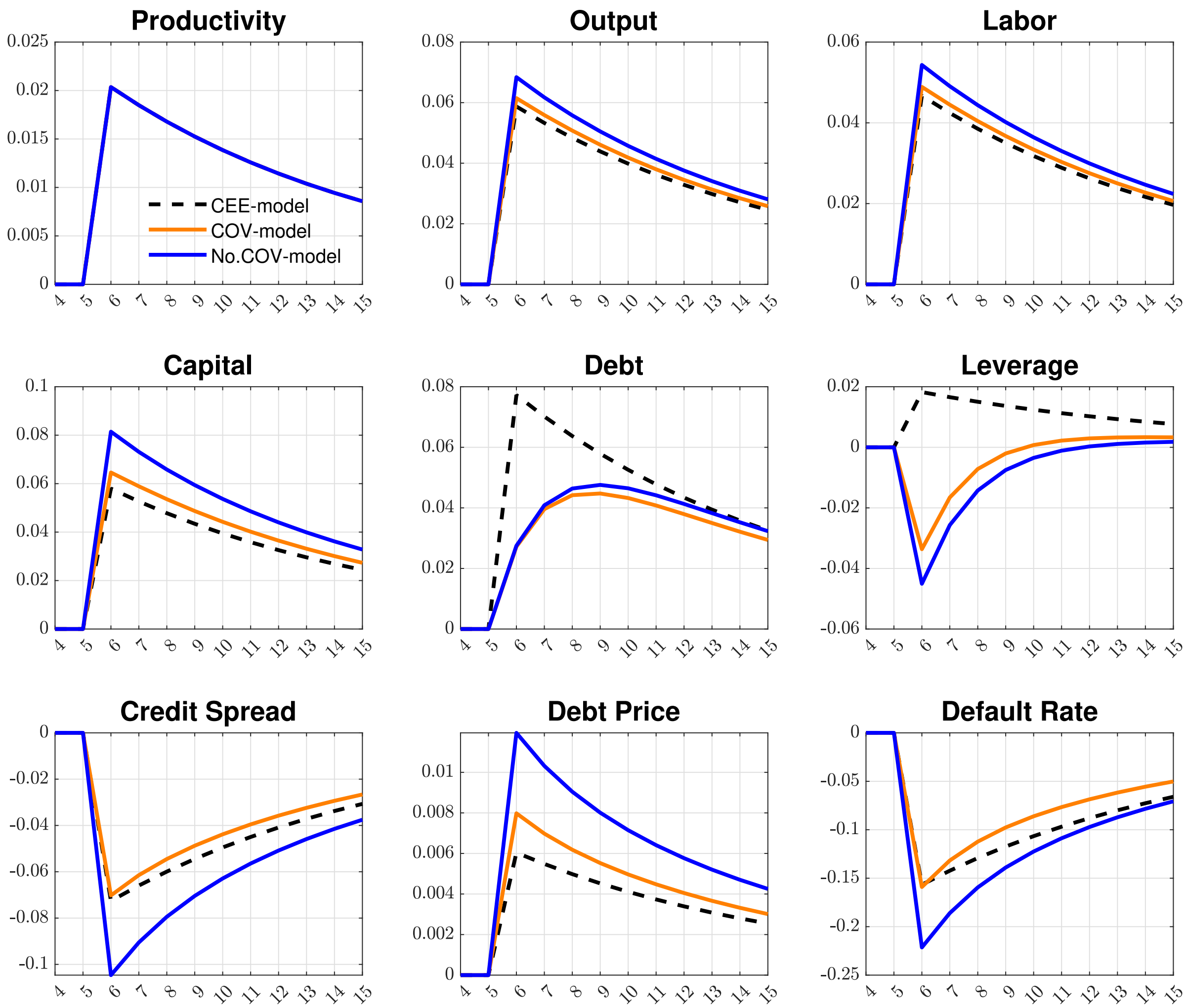
**Table 2.4:** Asymmetry in the Peak Responses to TFP Shocks: *(Recession/Boom-100%)*

| Model | Output | Capital | Leverage | Credit Spread | Debt Price | Default Rate |
|---|---|---|---|---|---|---|
| CEE. | 0% | 0% | -84% | 14% | 20% | 46% |
| Cov. | 0% | 4% | 40% | 14% | 20% | 46% |
| No.Cov. | 7% | 14% | 56% | 36% | 29% | 55% |

Notes: This table calculates the asymmetries in the peak responses to TFP shocks across three models. The peak responses are displayed in Figures 2.2 and 2.3. CEE model: the constrained efficiency model without debt dilution. NoCOV model: the model with debt dilution but without debt covenants. COV model: the model with debt dilution and debt covenants.

We plot the impulse responses to a 2% positive TFP shock in Figure 2.3. To provide a more intuitive comparison, we calculate the difference in the peak responses between the absolute value of peak responses and present these in Table 2.4. For example, the peak difference of the NoCOV model output during the boom is +9.15%, and while the bust is -9.80%, the asymmetry is $\frac{9.80\%}{9.15\%} - 100\% = 7\%$. We calculate asymmetry across all three models for real and financial variables.

In the CEE model, both output and capital responses are symmetric since the social planner maximizes the total value of both shareholders and creditors so there is no debt




Notes: CEE model: Black dashed lines show impulse response functions in the constrained efficiency model without debt dilution. NoCOV model: Blue solid lines show impulse response functions in the model with debt dilution but without debt covenants. COV model: Orange solid lines show impulse response functions in the model with debt dilution and debt covenants.

**Figure 2.3:** Impulse Response Functions to a +2% TFP Shock

dilution to amplify bad shocks. The business cycle remains asymmetric in financial variables since default risk is still high during the bust. The NoCOV model shows the largest asymmetry since the amplification effects during the bust are much stronger than during the boom. The COV model reduces asymmetries between the bust and the boom.

### 2.3.4 Long-run Level Effects

Finally, we discuss the long-run effects of debt dilution and debt covenants on the macroeconomy. We again compare the baseline COV model with debt dilution and debt covenants to alternative models: the constrained efficiency CEE model without debt dilution and the NoCOV model with debt dilution but without debt covenants.

**Table 2.5:** Long-run Effects of Debt Covenants

| Model | **Output** | | **Capital** | | **Consumption** | |
|---|---|---|---|---|---|---|
| | Mean | S.D. | Mean | S.D. | Mean | S.D. |
| CEE model | 0.659 | 0.026 | 1.115 | 0.042 | 0.548 | 0.024 |
| COV model | 0.653 | 0.028 | 1.089 | 0.050 | 0.544 | 0.027 |
| NoCOV model | 0.639 | 0.031 | 1.036 | 0.062 | 0.536 | 0.031 |

Notes: This table calculates the mean and standard deviation of output, capital, and consumption in each model. CEE model: the constrained efficiency model without debt dilution. NoCOV model: the model with debt dilution but without debt covenants. COV model: the model with debt dilution and debt covenants.

Without debt covenants, debt dilution creates severe debt overhang problems since creditors ask for a higher credit spread even when firms are financially healthy. This dissuades firms from profitable investments since earnings from new projects largely accrue to debt holders. Debt covenants reduce debt dilution and mitigate the debt overhang problem. Creditors accept a lower credit spread, especially for financially healthy firms. These firms then undertake more profitable investments because the shareholders capture more of the earnings from new projects. Therefore, holding financial conditions (leverage and credit spread) constant, debt covenants increase economic performance in the long run.

Beyond low volatility, the CEE model has the highest output, capital, and consumption since the model economy suffers the least from debt frictions when the social planner takes control of firms. The NoCOV model, on the other hand, has the lowest output, capital, and consumption because it suffers the most from the debt friction between firm owners and creditors. To avoid existing debt being diluted, creditors reduce lending to firms, resulting in a low capital equilibrium m, implying low output and consumption. As usual, the levels of all real variables in the COV model are between those of the CEE and NoCOV models. Quantitatively, the model with debt covenants can maintain a 5% larger capital stock, 2% higher output, and 1.5% higher consumption. This indicates that debt covenants not only reduce business cycle fluctuations but also improve long-run economic performance. Notably, our calibrated economy with the level of covenant tightness observed in the U.S. approximates the constrained efficient allocation in which a social planner maximizes the values of both equity and debt claims.

## 2.4 Conclusion

Debt covenants are pervasive in corporate debt contracts. This paper develops a dynamic general equilibrium model with long-term defaultable debt to study the macroeconomic implications of debt covenants. In our model, the ex-post penalty associated with covenant violations aligns shareholders' incentives with lenders' interests in the face of default risk, thereby mitigating ex-ante debt dilution and debt overhang. We show that this mechanism has significant macroeconomic effects: (1). it reduces the countercyclical variation in firm leverage, default risk, and credit spreads, substantially lowering aggregate volatility; (2). it alleviates the debt overhang problem and thus boosts capital accumulation, resulting in higher wages, output, and consumption. Our results, therefore, challenge the existing literature where debt covenants, modeled as distortionary borrowing constraints in models without default risk, amplify volatility and distort output. Moreover, we show that the calibrated economy with the level of covenant tightness observed in the U.S. approximates the constrained efficient allocation in which a social planner maximizes the values of both equity and debt claims. Therefore, debt covenants not only help creditors maintain the value of their debt but also help stabilize the macroeconomic fluctuations and boost long-run economic performance.

## 2.5 Appendix

### 2.5.1 Additional Quantitative Results

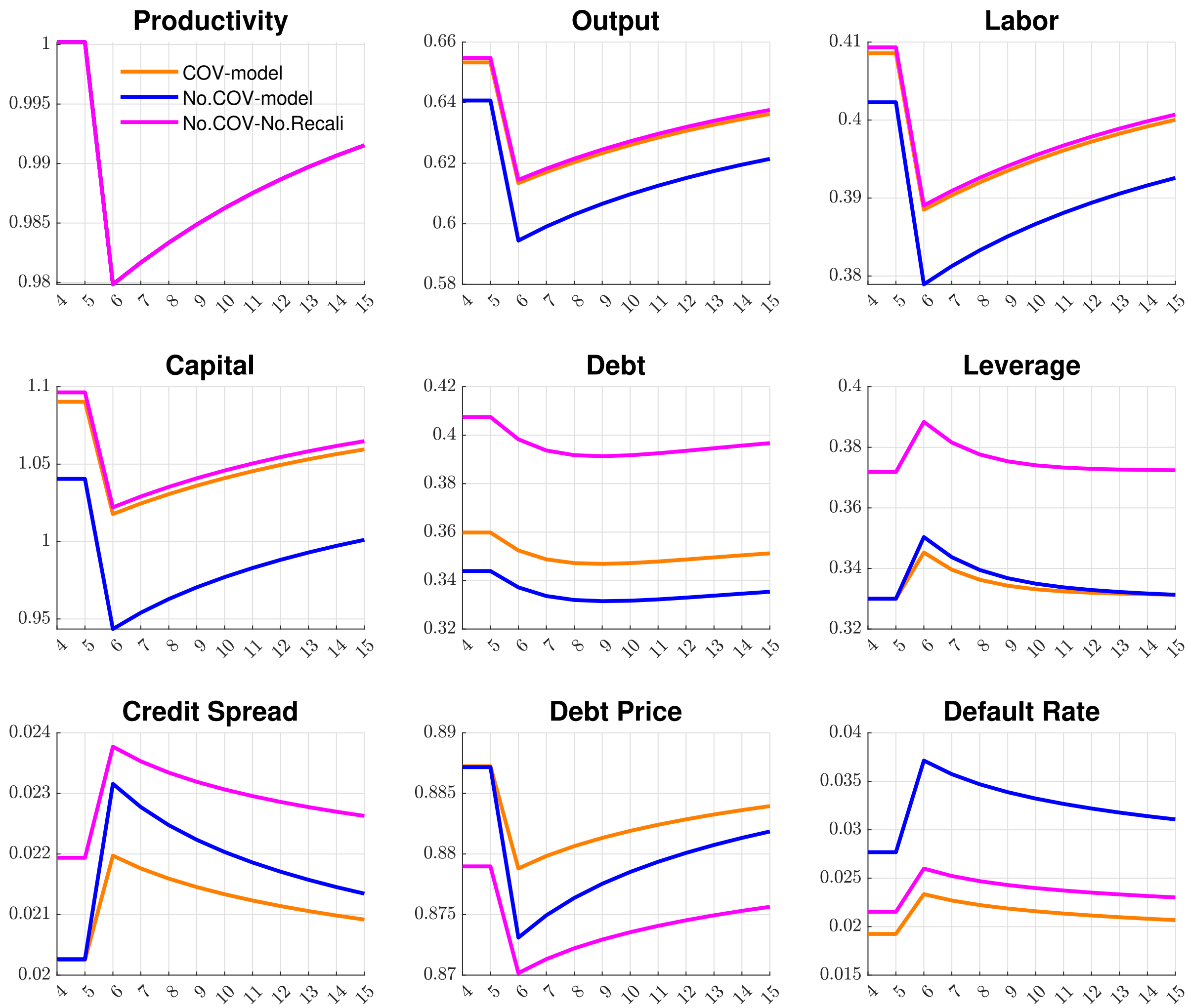
In this section, we demonstrate additional quantitative results focusing on the non-recalibrated NoCOV model without debt covenants. As we have emphasized in Section 2.2.9, a comparison between the baseline COV model and the non-recalibrated NoCOV model without debt covenants is not economically meaningful. The NoCOV model would only be comparable to the baseline COV model when the indebtedness and credit risk of both economies are similar and, more importantly, match the data. Still, we hope this comparison illustrates the mechanism of the baseline COV model. As we have already shown in Table 2.3, the non-recalibrated NoCOV model without debt covenants features a higher leverage ratio (37% vs. 33%) and a higher credit spread (2.2% vs. 2.0%) compared to the baseline COV model. Therefore, directly removing debt covenants yields a counterfactual overborrowing equilibrium and significantly increases debt and credit risk in the economy.

**Business Cycle Stabilization** Figures 2.4 and 2.5 show the impulse responses to a negative 2% TFP shock in levels and percentages, respectively. The non-recalibrated NoCOV model without debt covenants shows similar responses in real variables but has more debt and leverage, plus higher spreads and default rates. In terms of percentages, the debt changes are larger.

**Business Cycle Asymmetry Reduction** Figures 2.6 and 2.7 show the impulse responses to a positive 2% TFP shock in levels and percentages, respectively. The non-recalibrated NoCOV model without debt covenants shows similar responses in real variables but has higher levels of debt, leverage, spread, and default rate. In terms of percentage, the reactions in debt changes are deeper.

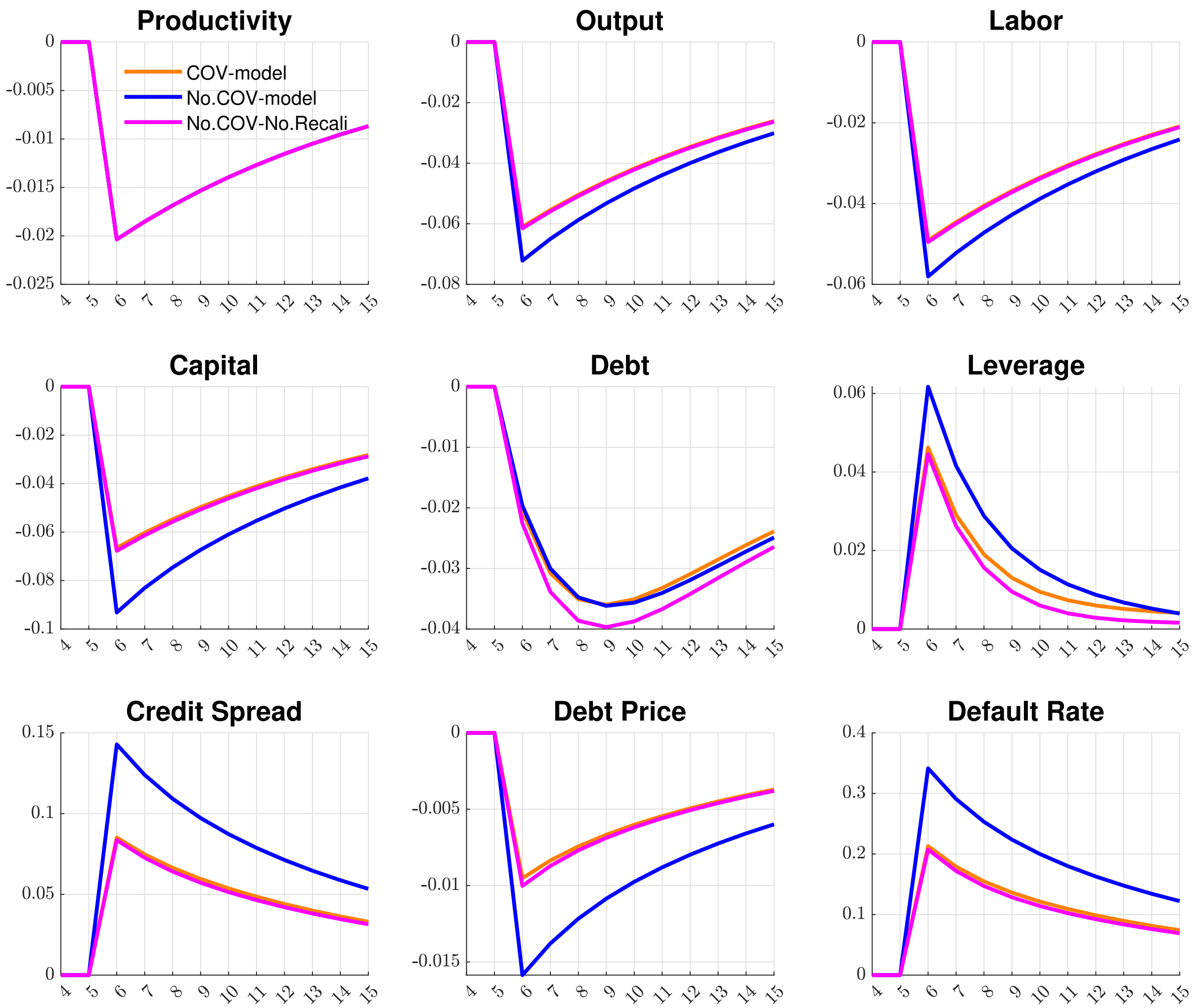
**Long-run Level Effects** Table 2.6 contrasts the long-term effects of debt covenants including the non-recalibrated NoCOV model without debt covenants. The equilibrium output, capital, and consumption of the non-recalibrated NoCOV model are almost identical to the baseline COV model with debt covenants.

**Overall Evaluation** The counterfactual non-recalibrated NoCOV model exhibits similar levels and volatility of real economic variables, including output, capital, and consumption, to our baseline COV model. However, it features counterfactually high leverage (debt), credit spreads, and default risk. Considering that default is costly, the counterfactual non-recalibrated NoCOV model is also worse than the baseline COV model. Again, a fair comparison to the counterfactual non-recalibrated NoCOV model would be recalibrating the baseline COV model to the same counterfactual financial conditions (same higher level of leverage and credit spread). In such a case, we would exactly reproduce the same findings in the main text.



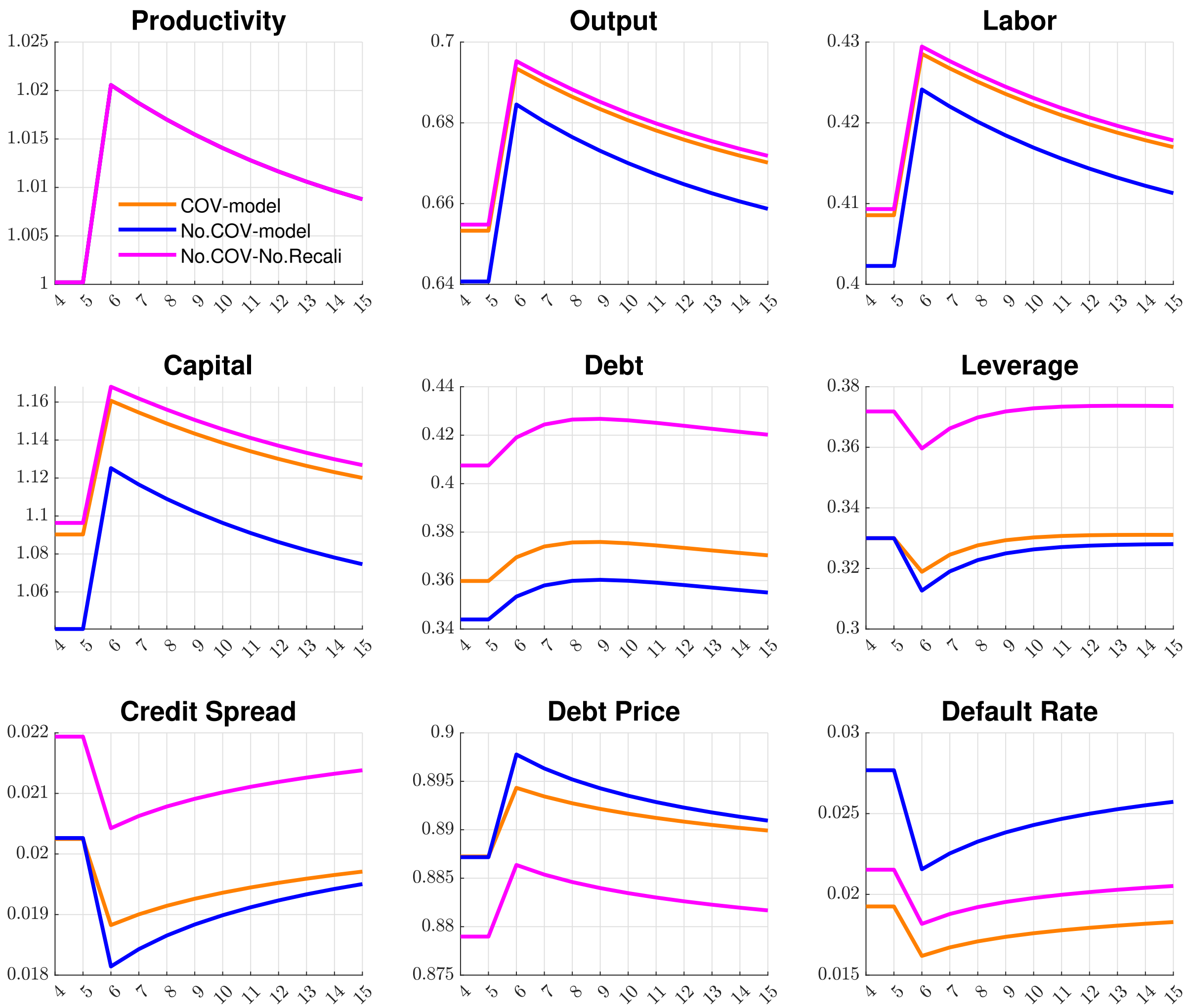

Notes: COV model: Orange solid lines show impulse response functions in the model with debt dilution and debt covenants. NoCOV model: Blue solid lines show impulse response functions in the model with debt dilution but without debt covenants. NoCOV-NoRecali model: Pink solid lines show impulse response functions in the model with debt dilution but without debt covenants that are not recalibrated.

**Figure 2.4:** Impulse Response Functions to a -2% TFP Shock (In Levels)



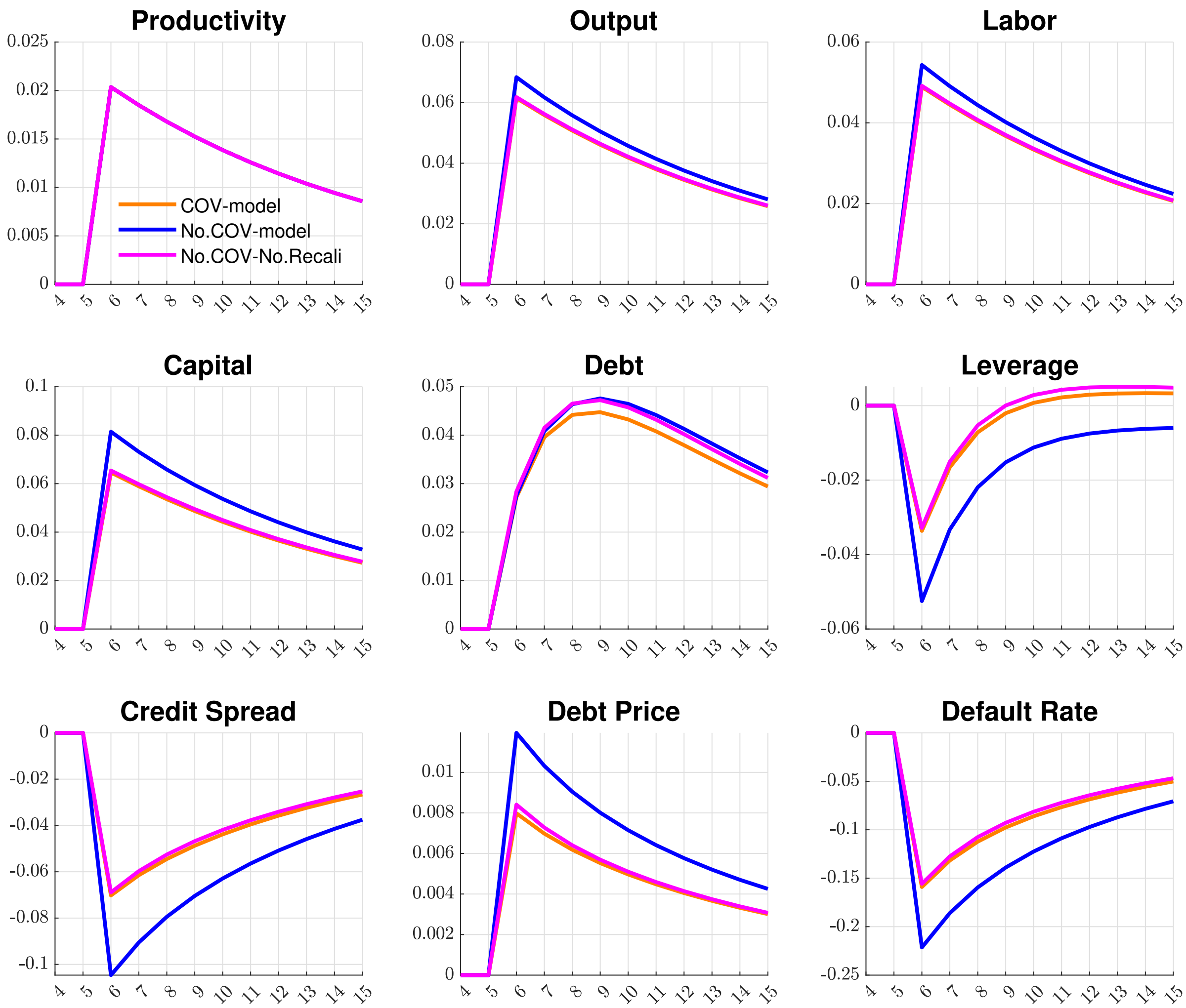

Notes: COV model: Orange solid lines show impulse response functions in the model with debt dilution and debt covenants. NoCOV model: Blue solid lines show impulse response functions in the model with debt dilution but without debt covenants. NoCOV-NoRecali model: Pink solid lines show impulse response functions in the model with debt dilution but without debt covenants that are not recalibrated.

**Figure 2.5:** Impulse Response Functions to a -2% TFP Shock (In Percentage)




Notes: COV model: Orange solid lines show impulse response functions in the model with debt dilution and debt covenants. NoCOV model: Blue solid lines show impulse response functions in the model with debt dilution but without debt covenants. NoCOV-NoRecali model: Pink solid lines show impulse response functions in the model with debt dilution but without debt covenants that are not recalibrated.

**Figure 2.6:** Impulse Response Functions to a +2% TFP Shock (In Levels)




Notes: COV model: Orange solid lines show impulse response functions in the model with debt dilution and debt covenants. NoCOV model: Blue solid lines show impulse response functions in the model with debt dilution but without debt covenants. NoCOV-NoRecali model: Pink solid lines show impulse response functions in the model with debt dilution but without debt covenants that are not recalibrated.

**Figure 2.7:** Impulse Response Functions to a +2% TFP Shock (In Percentage)

**Table 2.6:** Long-run Effects of Debt Covenants

| Model | **Output** | | **Capital** | | **Consumption** | |
|---|---|---|---|---|---|---|
| | Mean | S.D. | Mean | S.D. | Mean | S.D. |
| CEE model | 0.659 | 0.026 | 1.115 | 0.042 | 0.548 | 0.024 |
| COV model | 0.653 | 0.028 | 1.089 | 0.050 | 0.544 | 0.027 |
| NoCOV model | 0.639 | 0.031 | 1.036 | 0.062 | 0.536 | 0.031 |
| NoCOV-NoRecali model | 0.654 | 0.028 | 1.095 | 0.051 | 0.545 | 0.027 |

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# Chapter 3

# Financial Frictions and The Inventory Channel of Monetary Policy

## 3.1 Introduction

How do financial frictions influence the transmission of monetary policy? Contrary to the amplification effects of financial frictions mostly found in the literature, this paper shows both empirically and theoretically that financial frictions dampen the effects of monetary policy shocks on inventory investment. Using firm-level data combined with externally identified monetary policy shocks, I first show that following contractionary monetary policy shocks, more financially constrained firms cut much fewer inventories than their less financially constrained counterparts despite similar effects of monetary policy shocks on their sales. Motivated by this evidence, I build a dynamic New Keynesian general equilibrium model where firms manage inventory under idiosyncratic uncertainty and financial frictions and study the transmission of monetary policy shock to firm sales and inventories.

In the model, when contractionary monetary policy shocks lower households' demand for goods and thus firms' expected sales and revenues, more financially constrained firms slash their goods' prices and put more inventories on the shelves to increase operating cash flows, thereby reducing costly external financing. My calibrated model successfully replicates a wide set of data features: pro-cyclical inventories and sales, counter-cyclical inventory-to-sales ratio and markups, as well as heterogeneous responses across differently financially constrained firms. Counterfactual exercises show that the aggregate effect of monetary policy is smaller in a more financially constrained economy through the inventory channel.

My empirical exercises combine high-frequency identified monetary policy shocks and quarterly Compustat data to estimate the effects of monetary policy shocks on firm sales and inventories and how the effects differ across differently financially constrained firms. Following Gilchrist et al. (2017), a firm with higher operating leverage is considered more financially constrained. The intuition is that, everything else being equal, a firm with higher operating leverage pays more fixed production costs and thus depends more on external financing. Therefore, these firms are more likely to be affected by financial frictions.

I find that firms' sales and inventories fall while the inventory-to-sales ratio rises in response to a contractionary monetary policy shock, consistent with Kryvtsov and Midrigan (2012), which uses industry-level data. More importantly, I find that the drop in inventories is much smaller for firms with high operating leverage than their counterparts despite similar drops in sales across firms, leading to a much larger increase in inventory-to-sales ratio. I show that the heterogeneous effects driven by operating leverage are robust to controlling for a wide set of firm-level controls.

To interpret the empirical patterns documented above and explore how financial frictions affect firms' responses, I build a two-sector New Keynesian model where intermediate goods producers in the two sectors have different levels of operating leverage and thus depend differently on external financing. The model is built upon a stock-out avoidance inventory model. As discussed in Kryvtsov and Midrigan (2012), this class of inventory model succeeds in generating pro-cyclical sales and inventories and counter-cyclical inventory-to-sales ratio in response to monetary policy shock. Besides, to make financial frictions important for firms' decisions, I incorporate idiosyncratic productivity uncertainty and costly external financing, similar to Gilchrist et al. (2017), into the baseline inventory model. In this setting, firms' marginal costs of production and goods' prices will not only be affected by wage payments but also by firms' cash flow conditions, or in other words, firms' shadow value of their internal funds. Monetary policy affects firms' cash flows by affecting household demand. For firms that pay high fixed operating costs, the shadow value of their internal funds is thus more sensitive to cash flow changes.

The key mechanism driving the heterogeneous responses is that contractionary monetary policy reduces household demand, which reduces firms' expected revenues. Firms that pay high fixed costs are, therefore, more likely to face a liquidity shortage. As a result, they have a smaller decrease in their marginal costs and a larger increase in their shadow value of internal funds. Price rigidity prevents them from slashing their goods' prices, thereby leading to a larger rise in their markups relative to their low fixed-cost counterparts who have weaker motives to cut prices. Higher markups then induce firms paying high costs to choose more on-shelf goods relative to their expected demand because the profit lost by failing to make a sale is greater when markups are higher. On the other hand, a smaller decrease in marginal costs and a huge rise in markups largely counteract the effects of a larger increase in the shadow value of internal funds. Therefore, the actual output prices chosen are similar between high-cost firms and low-cost firms. Consequently, actual final sales are similar across firms.

Finally, I use the model to examine how financial frictions affect the aggregate effects of monetary policy shocks via the inventory channel. I study the effects of the same contractionary monetary policy shock in two economies with different levels of operating leverage. I find that the monetary shock generates an approximately 50% smaller change in the inventory stock in the more financially constrained economy. Therefore, unlike the conventional wisdom that financial frictions amplify the effects of monetary policy on capital investment, I show that financial frictions dampen the effects of monetary policy on inventory investment.

## 3.2 Literature

This paper relates and contributes to four strands of literature. First, this paper joins the rising literature on how micro-level heterogeneity affects the transmission of monetary policy to the real economy. To date, this literature has focused on how household heterogeneity affects the transmission of monetary policy to consumption, for example, Wong (2019) and Kaplan et al. (2018) and how financial heterogeneity affects the transmission to transmission to fixed capital investment, including floating rate debt (Ippolito et al. (2018)), leverage (Ottonello and Winberry (2018)), liquidity (Jeenas (2019)), and financial constraints proxied by firm ages (Cloyne et al. (2018)). This paper instead explores the implication of firm-level heterogeneity in operating leverage on the transmission of monetary policy to sales and inventories. My empirical work suggests that heterogeneity in operating leverage does differentiate firms' decisions in response to monetary policy, and more importantly, high operating leverage might affect the aggregate effects of a contractionary monetary policy shock by inducing firms to cut fewer inventories.

Second, this paper contributes to the literature that studies how monetary policy affects inventories. Early work by Gertler and Gilchrist (1994) and Kashyap et al. (1994) document that firms cut inventories in response to a contractionary monetary shock. Besides, they find firms with worse credit conditions tend to cut more inventories. For example, Gertler and Gilchrist (1994) finds that small firms have a much lower inventory-to-sales ratio than large firms after contractionary monetary shocks, because it becomes much harder for them to raise short-term debt after monetary tightening. In this paper, I show firms with high operating leverage instead has a higher inventory-to-sales ratio in response to a contractionary shock, and this effect is robust to various measures of firms' financial conditions.

Third, this paper relates to a large literature that studies inventory dynamics in macroeconomic episodes. Previous studies mainly focus on inventory behaviors in business cycles, and the role of inventories in aggregate fluctuations, for example, Ramey and West (1999), Bils and Kahn (2000),Khan and Thomas (2007), and Wen (2011). Kryvtsov and Midrigan (2012) discusses inventory dynamics in response to monetary policy shocks. They find inventory-to-sales ratio is counter-cyclical in response to monetary policy shocks by using industry-level data, and builds a stock-out avoidance inventory model to explain the counter-cyclical inventory-to-sales ratio. Using firm-level data, I find similar patterns, moreover, I find cross-sectional differences in inventory dynamics due to operating leverage. Recent work by Crouzet and Oh (2016) examines inventory dynamics in response to news shocks.

Finally, the key mechanism in the paper highlights the role of counter-cyclical markup in the transmission of monetary policy shock. There has been a long-run debate on whether cost rigidity or counter-cyclical markups explain the real effects of monetary policy shocks. For example, Christiano et al. (2005) argues that wage rigidity explains the real effects of monetary policy shocks on many macroeconomic variables, while Kryvtsov and Midrigan (2012) shows that only a model with counter-cyclical markups is able to reproduce the counter-cyclical inventory-to-sales ratio in the data. In this paper, I show heterogeneous counter-cyclical markups across firms are able to generate the cross-sectional differences in how firms' sales and inventories respond to monetary shocks.

**Road Map** The rest of the paper is structured as follows: Section 3.3 discusses data, empirical strategies, and empirical findings. Section 3.4 develops a two-sector New Keynesian model with heterogeneous operating leverage. Section 3.5 discusses the model mechanisms and simulates the model to reproduce the empirical patterns. Section 3.6 evaluates how operating leverage affects the aggregate effects of monetary policy. Section 3.7 concludes.

## 3.3 Empirics

### 3.3.1 Data

#### Monetary Policy Shocks

Following recent monetary policy literature, I consider the changes in the three month ahead monthly fed future rates (FF4) within a 30-minute window around FOMC press release as the exogenous surprises in the monetary policy. This measure of monetary policy surprises are considered as unexpected monetary policy shocks to private agents. I obtained the monthly monetary shock data by running the programs in Gertler and Karadi (2015)'s online replication file. The monthly shock data begins in January 1990, when the Fed Funds futures market opened, and ends in December 2007, before the Zero Lower Bound period, therefore the focus of this paper is on the transmission of conventional monetary policy. In order to merge the shocks with the quarterly firm-level data, I sum up the monthly shocks within a quarter to yield a measure of quarterly shocks, as in Wong (2019) and Jeenas (2019). I ended up having 72 quarterly monetary policy shocks for 72 quarters from 1990Q1 to 2007Q4. The descriptive statistics are exactly the same as those reported in Jeenas (2019).

**Table 3.1:** Summary Statistics of Monetary Policy Shocks

| Mean | S.D. | Min | Median | Max | Num |
|---|---|---|---|---|---|
| -0.0446 | 0.1104 | -0.5460 | -0.0085 | 0.1700 | 72 |

#### Firm-level Variables

Our sample combines monetary policy shocks with firm-level variables from quarterly Compustat data from 1990Q1 to 2007Q12. We exclude firms from financial, services, and government-related industries. The sample selection approach and firm characteristics variables used are standard in the literature, so I relegate the description to Appendix A. Moreover, I will be estimating IRFs over 20 quarters, so I restrict the sample to only include firms with observations for at least 20 quarters.

### 3.3.2 Heterogeneous Effects by Operating Leverage

In this section, I aim to estimate the heterogeneous effects of a contractionary monetary policy shock on firms' sales and inventory decisions. To jointly study firms' sales and inventories, I follow Gertler and Gilchrist (1994), Bils and Kahn (2000), and Kryvtsov and Midrigan (2012) to examine how firms adjust their inventory-to-sales ratio in response to an unexpected increase in interest rate. I begin by estimating the heterogeneous effects of monetary policy shocks on inventory-to-sales ratio due to operating leverage using the following baseline regression:

$$IS_{i,t} = \alpha_i + \alpha_{s,t} + \beta x_{i,t-1} \cdot \epsilon_t^M + \Gamma' Z_{i,t-1} + \mu_{i,t} \tag{3.1}$$

where $\alpha_i$ is a firm fixed effect, $IS_{i,t}$ is the dependent variable, inventory-to-sales ratio. $\alpha_{s,t}$ is a sector-by-quarter fixed effect, $\epsilon_t^M$ is the monetary policy shock. $x_{i,t-1}$ is the firm characteristics we are interested in. $Z_{i,t-1}$ includes $x_{i,t-1}$ and firm-level controls: sales growth, inventory growth, log sales, and log asset size. Our main coefficient of interest is $\beta$, which measures how $IS_{i,t}$ responds to monetary policy shocks depends on the firm's characteristics $x_{i,t-1}$. firm fixed effects $\alpha_i$ capture permanent differences in inventory-to-sales ratio across firms. Sector-by-quarter fixed effects $\alpha_{s,t}$ capture differences in how different sectors are exposed to aggregate shocks. Standard errors are two-way clustered by firms and quarters. This specification estimates how firms with different $x_{i,t-1}$ at quarter $t-1$ adjust their inventory-to-sales ratio $IS_{i,t}$ in response to an unexpected monetary policy shock at quarter t. In all regressions in this paper, we focus on a 1 sd contractionary monetary policy shock, which amounts to a 11 basis point increase in federal funds rate.

Table 3.2 reports the results from estimating the variants of regression (3.1). First, as shown in column (4), a 1 sd contractionary monetary policy shock increases firm-level inventory-to-sales ratio by 0.6% on average, which suggest a counter-cyclical inventory-to-sales ratio in response to monetary policy shocks. This result is consistent with Kryvtsov and Midrigan (2012), which uses industry-level data. More importantly, differences in the firms' operating leverage generate both economically and statistically significant differences in firms' responses to a contractionary monetary policy shock. As shown in all columns in Table 3.2, a difference in SGA ratio (operating leverage) of 10 percentage point between two firms before the shock is associated with a 0.2% difference in inventory-to-sales ratio in response to the shock, which is around one third of the average effect (0.6%) of monetary policy shock on inventory-to-sales ratio.

**Table 3.2:** Heterogeneous Effects of Contractinary Monetary Policy by Operating Leverage

| Dep. Variable<br>IS ratio | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| SGA ratio$_{t-1}$ $\times$ $\epsilon_t^m$ | 0.020*** | 0.018*** | 0.024*** | 0.021*** |
| | (2.723) | (2.785) | (3.005) | (2.772) |
| $\epsilon_t^m$ | | | | 0.006*** |
| | | | | (2.772) |
| Observations | 188,338 | 211,125 | 188,338 | 188,338 |
| R-squared | 0.637 | 0.616 | 0.162 | 0.632 |
| Firm Controls | Yes | No | Yes | Yes |
| Firm FE | Yes | Yes | No | Yes |
| Sector-Quarter FE | Yes | Yes | Yes | No |

Notes: results from estimating Equation 3.1

$$IS_{i,t} = \alpha_i + \alpha_{s,t} + \beta x_{i,t-1} \cdot \epsilon_t^M + \Gamma' Z_{i,t-1} + \mu_{i,t}$$

where $IS_{i,t}$ is the inventory-to-sales ratio (IS ratio), $\alpha_i$ is a firm fixed effect, $\alpha_{s,t}$ is a sector-by-quarter fixed effect, $x_{i,t-1}$ are firm characteristics we are interested in, $\epsilon_t^M$ is a 1 sd contractionary monetary policy shock (federal funds rate goes up by around 11bp), and $Z_{i,t-1}$ is a vector of firm-level controls containing $x_{i,t-1}$,sales growth, inventory growth, log sales, and log asset size. Standard errors are two-way clustered by firms and quarter. High SGA ratio means high operating leverage.

One potential concern with above regressions is that the heterogeneous effects due to operating leverage might be just picking up the effects caused by other firm-level heterogeneity. Therefore, I add another interaction between monetary policy shock and firm-level variable into regression (3.2) to examine the robustness of above estimates. As shown in columns (1) to (8) in Table 3.3, the heterogeneous effects of monetary policy by operating leverage are robust to a variety of firm-level heterogeneity that have been found to generate differential firm behaviors in previous literature, including financial leverage, liquidity, cash flow, firm size, asset growth, and tangibility.

Besides, I also find firms with 10% higher financial leverage will have about 0.1% lower inventory-to-sales ratio in response to a contractionary monetary policy shock, as shown in column (1) and column (7) of Table 3.3. This result is consistent with the the role of financial frictions in monetary policy transmission, as argued in Gertler and Gilchrist (1994) and a large literature on financial accelerator. Firms with high financial leverage will have a higher increase in default risk after contractionary monetary policy, which make them harder to finance their inventories. Therefore, compared with their low financial leverage counterparts, they tend to have a lower inventory-to-sales ratio after unexpected interest rate rise.

One problem with the firm-level data is that firms in the sample only report the total inventory stock they are holding, therefore I cannot distinguish between input (raw material and work-in-progress) and output (finished-goods) inventories. In this paper, I am primarily interested in how monetary policy affects output inventories, which belong to total industrial output of that period. I tackle this problem by estimating the effects of monetary policy shocks on inventory-to-sales ratio using only firms from retail and wholesale industries. Since most of the inventories held by retail and wholesale firms are finished-goods inventories, the responses in inventory-to-sales ratio of those firms should shed lights on how firms in general adjust their output inventories relative to their sales in response to monetary policy shocks.

Table 3.4 reports the heterogeneous effects of contractionary monetary policy by firm characteristics using sample from retail and wholesale industries. In this case, the heterogeneous effects by operating leverage are statistically significant and even economically stronger than the baseline case. A difference in SGA ratio (operating leverage) of 10 percentage point between two firms before the shock is associated with a 0.35% difference in inventory-to-sales ratio in response to the contractionary shock. And the estimates are robust to controlling for a variety of firm-level heterogeneity. Indeed, as shown in Table 3.4, other firm-level heterogeneity do not matter for firms' inventory-to-sales decisions in response to unexpected changes in policy interest rate.

In summary, firms increase their inventory-to-sales ratio in response to a contractionary monetary policy shock. What's more, firms with high operating leverage adjust to an even higher inventory-to-sales ratio relative to their low operating leverage counterparts. These heterogeneous effects are robust to several other firm-level heterogeneity that have been found important in generating heterogeneous firm behaviors in the literature.

**Table 3.3:** Heterogeneous Effects of a Contractionary MP Shock (Full Sample)

| Dep. Variable IS ratio | (1) | (2) | (3) | (4) | (5) | (6) | (7) |
|---|---|---|---|---|---|---|---|
| SGA ratio$_{t-1}$ × $\epsilon_t^m$ | 0.020*** | 0.018** | 0.023*** | 0.021*** | 0.021*** | 0.019** | 0.018** |
| | (2.653) | (2.272) | (2.758) | (2.729) | (2.814) | (2.435) | (2.350) |
| Leverage$_{t-1}$ × $\epsilon_t^m$ | -0.012*** | | | | | | -0.010** |
| | (-2.767) | | | | | | (-2.239) |
| Liquidity$_{t-1}$ × $\epsilon_t^m$ | | 0.017* | | | | | |
| | | (1.776) | | | | | |
| Cash Flow$_{t-1}$ × $\epsilon_t^m$ | | | 0.044 | | | | |
| | | | (1.512) | | | | |
| Log Size$_{t-1}$ × $\epsilon_t^m$ | | | | 0.001 | | | |
| | | | | (0.899) | | | |
| Asset Growth$_{t-1}$ × $\epsilon_t^m$ | | | | | -0.000 | | |
| | | | | | (-0.035) | | |
| Tangiblity$_{t-1}$ × $\epsilon_t^m$ | | | | | | 0.023*** | 0.021*** |
| | | | | | | (4.300) | (3.962) |
| Observations | 188,338 | 188,338 | 188,330 | 188,338 | 188,136 | 188,338 | 188,338 |
| R-squared | 0.632 | 0.635 | 0.632 | 0.632 | 0.625 | 0.632 | 0.632 |
| Firm Controls | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
| Firm FE | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
| Sector-Quarter FE | Yes | Yes | Yes | Yes | Yes | Yes | Yes |

Notes: results from estimating Equation 3.1

$$IS_{i,t} = \alpha_i + \alpha_{s,t} + \beta_1 x_{i,t-1} \cdot \epsilon_t^M + \beta_2 y_{i,t-1} \cdot \epsilon_t^M + \Gamma' Z_{i,t-1} + \mu_{i,t}$$

where $IS_{i,t}$ is the inventory-to-sales ratio (IS ratio), $\alpha_i$ is a firm fixed effect, $\alpha_{s,t}$ is a sector-by-quarter fixed effect, $x_{i,t-1}$ and $y_{i,t-1}$ are firm characteristics we are interested in, $\epsilon_t^M$ is a 1 sd contractionary monetary policy shock (federal funds rate goes up by around 11bp), and $Z_{i,t-1}$ is a vector of firm-level controls containing $x_{i,t-1}$,sales growth, inventory growth, log sales, and log asset size. Standard errors are two-way clustered by firms and quarter. High SGA ratio means high operating leverage. Leverage stands for a firm's book leverage (Total Debt over Total Asset).

### 3.3.3 Heterogeous IRFs by Operating Leverage

The above linear panel regressions assume a linear relation between operating leverage and the effect of monetary policy shocks on firms' decisions. However, the influences of operating leverage on firms' decisions are more often found to be non-linear in previous literature (e.g. Gourio and Rudanko (2014)). Besides, above regressions only estimate the effects of monetary policy at impact, while the dynamic effects of monetary policy are also of great interests. Therefore, this section exploits a local projection approach to study the dynamic effects of monetary policy shocks on firms' inventory-to-sales ratio, growth of sales and growth of inventory stocks. In order to capture the heterogeneous

**Table 3.4:** Heterogeneous Effects of a Contractionary MP Shock (Retail and Wholesale)

| Dep. Variable IS ratio | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|
| SGA ratio$_{t-1}$ $\epsilon_t^m$ | 0.036** | 0.036** | 0.037** | 0.038** | 0.036** | 0.035** |
| | (2.125) | (2.101) | (2.171) | (2.029) | (2.124) | (2.024) |
| Leverage$_{t-1}$ $\times$ $\epsilon_t^m$ | -0.001 | | | | | |
| | (-0.173) | | | | | |
| Liquidity$_{t-1}$ $\times$ $\epsilon_t^m$ | | 0.003 | | | | |
| | | (0.191) | | | | |
| Cash Flow$_{t-1}$ $\times$ $\epsilon_t^m$ | | | 0.025 | | | |
| | | | (0.671) | | | |
| Log Size$_{t-1}$ $\times$ $\epsilon_t^m$ | | | | 0.000 | | |
| | | | | (0.139) | | |
| Asset Growth$_{t-1}$ $\times$ $\epsilon_t^m$ | | | | | -0.002 | |
| | | | | | (-0.093) | |
| Tangibility$_{t-1}$ $\times$ $\epsilon_t^m$ | | | | | | 0.008 |
| | | | | | | (1.430) |
| Observations | 36,641 | 36,641 | 36,641 | 36,641 | 36,631 | 36,641 |
| R-squared | 0.751 | 0.751 | 0.752 | 0.751 | 0.751 | 0.751 |
| Firm Controls | Yes | Yes | Yes | Yes | Yes | Yes |
| Firm FE | Yes | Yes | Yes | Yes | Yes | Yes |
| Sector-Quarter FE | Yes | Yes | Yes | Yes | Yes | Yes |

Notes: results from estimating Equation 3.1

$$IS_{i,t} = \alpha_i + \alpha_{s,t} + \beta_1 x_{i,t-1} \cdot \epsilon_t^M + \beta_2 y_{i,t-1} \cdot \epsilon_t^M + \Gamma' Z_{i,t-1} + \mu_{i,t}$$

where $IS_{i,t}$ is the inventory-to-sales ratio (IS ratio), $\alpha_i$ is a firm fixed effect, $\alpha_{s,t}$ is a sector-by-quarter fixed effect, $x_{i,t-1}$ and $y_{i,t-1}$ are firm characteristics we are interested in, $\epsilon_t^M$ is a 1 sd contractionary monetary policy shock (federal funds rate goes up by around 11bp), and $Z_{i,t-1}$ is a vector of firm-level controls containing $x_{i,t-1}$,sales growth, inventory growth, log sales, and log asset size. Standard errors are two-way clustered by firms and quarter. High SGA ratio means high operating leverage. Leverage stands for a firm's book leverage (Total Debt over Total Asset).

effects by operating leverage, I sort firms into two groups based on whether their operating leverage (SGA ratio) in quarter t are below or above the median of 2-digit SIC industry they belong to. This sorting criteria tries to avoid the heterogeneity in operating leverage at the industry level. For example, the average operating leverage of Chemical products industry is significantly higher than that of coal mining industry, as shown in **??**. Therefore, the differences in the effects of monetary policy are not driven by differences in operating leverage at the industry-level. I estimate the Impulse Response Functions by using the following specification, which is in spirit of Jordà (2005) and similar to Cloyne et al. (2018).

$$y_{i,t+h} = \alpha_{i,h} + \sum_{g=1}^{G} \beta_h^g \cdot I[x_{i,t-1} \in g] \cdot \epsilon_t^M + \sum_{g=1}^{G} \gamma_h^g \cdot I[x_{i,t-1} \in g] + \mu_{i,t+h} \tag{3.2}$$

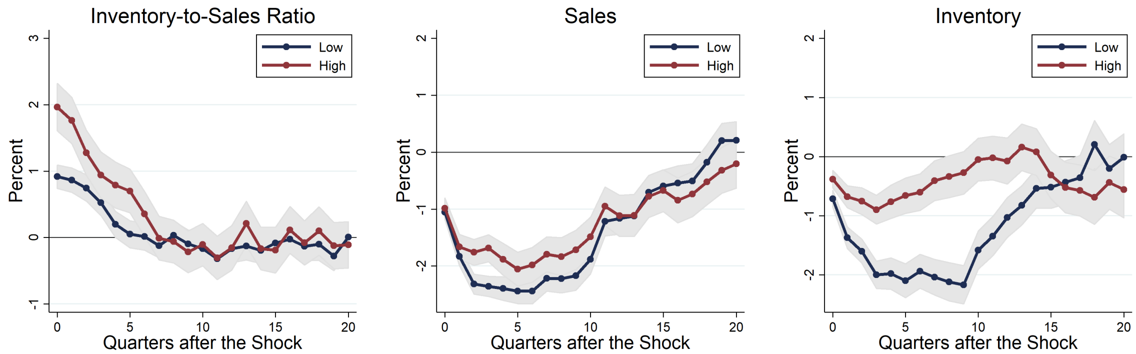
where $t$ refers to quarters and $h \geq 0$ indexes the forecast horizon. $\alpha_{i,h}$ is a firm fixed effect. $\beta_h^g$ captures the cumulative effects of monetary policy shock $\epsilon_t^M$ on $y_{i,t+h}$ of a particular group of firms whose firm characteristics $x_{i,t-1}$ falls into a particular "bin" of the distribution. $g$ indexes the bin that firms fall into. Therefore, $\beta_h^g$ gives us the impulse response functions of a particular group of firms with similar characteristics over h horizons in response to a monetary policy shock at quarter t. As discussed in the literature, high frequency identified monetary $\epsilon_t^M$ is uncorrelated with other macro variables and also unexpected by private agents, thus we do not need to control for any macro-level or firm-level variables.

The dependent variables include inventory-to-sales ratio, log growth of sales ($\Delta logSALES_t$), and log growth of inventory stocks ($\Delta logINVENTORY_t$). I split firms into two groups based on whether their $x_{i,t-1}$ (SGA ratio) is below or above the median of 2-digit SIC industry they belong to. The group of firms with above median operating leverage is denoted as High group, while firms with below median operating leverage belong to the Low group.

I estimate the IRFs of High group and Low group by estimating regression (3.2). As shown in Figure 3.1, high operating leverage firms tend to have a higher inventory-to-sales ratio in response to a contractionary shock. They also tend to experience a relatively lower decrease in sales, though the difference between these two groups might be not significant. More importantly, compared with the low operating leverage counterparts, high operating leverage firms cut significantly fewer inventories in response to a monetary tightening. The responses in inventory-to-sales ratio, sales, and inventory of both groups are statistically significant and persist for almost 2 years after the shock.

### 3.3.4 Summary

Taken together, these empirical results show the influences of operating leverage on the transmission of monetary policy to sales and inventories. High operating leverage firms adjust to a higher inventory-to-sales ratio by cutting much fewer inventories relative to




Notes: the grey areas in the graphs stand for the 95% confidence intervals.

**Figure 3.1:** IRFs to Contractionary Monetary Policy by Operating Leverage

their low operating leverage counterparts in response to a contractionary monetary policy. Since a contractionary monetary policy shock usually aims to dampen the economy, the smaller decrease in inventories caused by high operating leverage counteracts the strength of a contractionary monetary policy shock.

## 3.4 A Two-sector New Keynesian Model with Heterogeneous Operating Leverage

To rationalize the empirical results documented in the empirical analysis and highlight the channels through which monetary policy affects firms' decisions, I build a two-sector New Keynesian DSGE model where the intermediate firms in these two sectors pay different levels of fixed operating costs, that is, they differ in their operating leverage. This section sets up the model economy, describes, and solves the problems of households, final goods producers, and intermediate goods producers in the economy. Besides, since sales, inventories, and inventory-to-sales ratio are variable of interests in this paper, the baseline model here is a stock-out avoidance inventory model, which largely follows Kryvtsov and Midrigan (2012). As discussed in Kryvtsov and Midrigan (2012), this class of inventory model succeeds in generating counter-cyclical inventory-to-sales ratio, pro-cyclical sales, and pro-cyclical inventories in response to monetary policy shocks, which is consistent with the empirical results in this paper. Therefore, the goal of my model is to show how operating leverage (fixed operating costs) differentiates firms' sales and inventories decisions in response to monetary policy shocks.

The economy is populated by a large number of infinitely lived households consuming a composite good that are made up from final goods produced by two sectors. Within each sector, they are two types of firms: perfectly competitive final goods producers and monopolistically competitive intermediate goods producers. Intermediate goods producers have to pay a fixed operating cost for operation in this economy. Within the same sector, all intermediate goods producers pay the same level of fixed operating cost, however, one sector has a high fixed operating cost and the other has a relatively lower fixed operating cost. I therefore use subscript H to denote objects in high-cost sector, and subscript L to denote objects in low-cost sector.

Besides, households supply monopolistically competitive labor services to intermediate goods producers and receive wage payments and dividends (probably negative) from intermediate goods producers. Intermediate goods producers post prices and choose how many on-shelf goods that are ready to sell to final goods producers in their own sector. The on-shelf goods of intermediate goods contain undepreciated inventories they have on hands from last period and intermediate goods they choose to produce today. The final goods producers in each sector buy intermediate goods and then produce the sector-level final goods and post prices to households.

Besides, two features make the model a New Keynesian model. First, households value real cash balances, which is similar to Christiano et al. (2005). Second, I follow Gilchrist et al. (2017) to assume that intermediate goods producers are facing adjustment costs of changing nominal prices and households are facing adjustment costs of changing nominal wages. The adjustment costs are of Rotemberg (1982)-type. These set-ups give rise to the real effects of a monetary policy shock in the model.

The economy features three types of shocks: aggregate shocks to the money supply, idiosyncratic productivity shocks, and idiosyncratic demand shocks. I describe idiosyncratic shocks later in the intermediate goods producers' problem. I follow Kryvtsov and Midrigan (2012) to assume that supply of money follows a random-walk process of the following form:

$$\log M_t = \log M_{t-1} + g_m \tag{3.3}$$

where $g_m$ is monetary growth, a normally distributed i.i.d random variable with mean 0 and standard deviation $\sigma_m$.

### 3.4.1 Households

The model economy contains a continuum of identical, infinitely lived households indexed by $j \in [0,1]$. Households consume a composite good made up from the final goods produced by two sectors. They supply a differentiated labor input to intermediate firms in both sectors and act as monopolistically competitive labor unions in the labor market. Households own all the intermediate firms and receive dividend payments. Plus, I follow the monetary policy literature to assume that households get utility from holding real cash balances, and face costs of changing nominal wages. Household j in the economy maximizes the discounted expected utility:

$$E_t \sum_{t=0}^{\infty} \beta^t \left[ \ln(c_{jt} - hc_{j,t-1}) - \omega \frac{l_{jt}^{1+\eta}}{1+\eta} + \theta_m ln\left(\frac{M_{jt}}{P_t}\right) \right] \tag{3.4}$$

subjects to a nominal budget constraint:

$$P_t c_{jt} + \frac{B_{jt}}{R_t} + M_{jt} \leq B_{j,t-1} + M_{j,t-1} + W_{jt} l_{jt} - \Phi(W_{jt}) + D_{jt} \tag{3.5}$$

Here, $P_t$ and $c_t$ denote the aggregate price and consumption. Household enters the period with a stock of nominal bond $B_{j,t-1}$ and a stock of money $M_{j,t-1}$, they then choose how much cash to hold $M_{jt}$ and how many nominal bonds to buy at a nominal price $\frac{1}{R_t}$ today. The household also receives labor income $W_{jt} l_{jt}$ for supplying labor $l_{jt}$, and are subject to an adjustment cost of changing nominal wages $\Phi(W_{jt})$. Plus, the household receives dividend payments $D_{jt}$ from intermediate firms.

The demand for household j's labor is given by:

$$l_{jt} = \left(\frac{W_{jt}}{W_t}\right)^{-\theta_w} n_t$$

where $W_{jt}$ denotes the nominal wage demand of household j,$W_t$ is an aggregate wage, $\theta_w$ is the labor demand wage elasticity, and $n_t$ is the aggregate labor demand by all intermediate goods producers in the economy. Individual households take $n_t$ and $W_t$ as given. For wage rigidity, I follow Gilchrist et al. (2017) so that households face a convex adjustment cost of changing nominal wage of Rotemberg (1982)-type:

$$\Phi(W_{jt}) = \frac{\psi_w}{2}\left(\frac{W_{jt}}{W_{j,t-1}} - 1\right)^2 n_t$$

Since all households make exactly the same decisions, I throw away subscript j such that above optimization problem yields the following FOCs:

Optimal bond-holding decisions are described by the intertemporal Euler equation:

$$1 = E_t\beta\frac{\Lambda_{t+1}}{\Lambda_t}\frac{R_t}{\Pi_{t+1}} \tag{3.6}$$

where $\Pi_{t+1} = \frac{P_{t+1}}{P_t}$ denotes the aggregate inflation rate, and $\Lambda_t$ denotes the marginal utility of consumption given by:

$$\Lambda_t = \frac{1}{c_t - hc_{t-1}} - E_t\frac{\beta h}{c_{t+1} - hc_t} \tag{3.7}$$

Optimal cash-holding decision:

$$\theta_m\left(\frac{M_t}{P_t}\right)^{-1} = \frac{1 - R_t}{R_t}\Lambda_t \tag{3.8}$$

The intra-temporal trade-off between consumption and labor is given by:

$$\omega l_t^\eta\frac{P_t}{\Lambda_t} = \frac{\theta_w - 1}{\theta_w}W_t + \frac{\psi_w}{\theta_w}\left[\left(\frac{W_t}{W_{t-1}} - 1\right)\frac{W_t}{W_{t-1}}\frac{n_t}{l_t} - \frac{1}{R_t}\left(\frac{W_{t+1}}{W_t} - 1\right)\frac{W_{t+1}}{W_t}\frac{n_{t+1}}{l_t}\right] \tag{3.9}$$

To facilitate understanding, consider the case without habit and nominal wage adjustment costs ,that is, $h = 0$ and $\psi_w = 0$, Equation 3.9 reduces to :

$$\omega l_t^\eta c_t = \frac{\theta_w - 1}{\theta_w}\frac{W_t}{P_t}$$

where labor supply $l_t$ is increasing in wage markup $\frac{\theta_w - 1}{\theta_w}$ and real wage $\frac{W_t}{P_t}$, and is decreasing in aggregate consumption $c_t$ and labor disutility $\omega$.

### 3.4.2 Production

The production side of the economy contains two different goods-producing sectors. In each sector, there are a continuum of perfectly competitive final good producers buying intermediate goods from monopolistically competitve intermediate goods producers and then producing final goods of the sector. Intermediate goods producers. Intermediate goods producers have to pay a fixed operating cost for operation in this economy. Within the same sector, all intermediate goods producers pay the same level of fixed operating cost, however, one sector has a high fixed operating cost $f_H$, and the other has a relatively lower fixed operating cost $f_L$, in other words, $f_H > f_L$. I therefore use subscript H to denote objects in high-cost sector, and subscript L to denote objects in low-cost sector.

The composite goods consumed by households is given by:

$$c_t = \left[\omega_H^{\frac{1}{\zeta}}(c_t^H)^{1-\frac{1}{\zeta}} + \omega_L^{\frac{1}{\zeta}}(c_t^L)^{1-\frac{1}{\zeta}}\right]^{\frac{\zeta}{\zeta-1}} \tag{3.10}$$

where $\omega_H$ and $\omega_L$ are the size of the sectors, $\zeta$ is the elasticity of substitution between sector final goods $c_t^H$ and $c_t^L$. The price of the composite good is therefore as follows:

$$P_t = \left[\omega_H(P_t^H)^{1-\zeta} + \omega_L(P_t^L)^{1-\zeta}\right]^{\frac{1}{1-\zeta}} \tag{3.11}$$

Final goods producers and intermediate goods producers in these two sectors face exactly the same problems. The only difference is the fixed operating costs the intermediate goods producers pay, which are exogenously given parameters. Thus, I stop distinguishing high-cost sector and low-cost sector in describing firms' problems to save notation.

#### 3.4.2.1 Final goods producers

The problem of final goods producers follows Kryvtsov and Midrigan (2012) exactly. In each sector, a unit mass of identical and perfectly competitive final good producers produce final goods of the sector by combining varieties sold by intermediate goods producers according to its production technology:

$$c_t = \left(\int_0^1 v_{it}^{\frac{1}{\theta}} q_{it}^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}} \tag{3.12}$$

where $q_{it}$ is the amount of variety produced by producer i, $v_{it}$ is a vareity-specific demand shock which is an i.i.d lognormal random variable with CDF denoted as F(v). Final goods producers observe these demand shocks when making decisions, while intermediate goods producers do not. This gives rise the motive for intermediate goods producers to hold finish-goods inventories as on-shelf goods that are ready to sell, which we will discuss in details later. $\theta$ is the elasticity of substitution between varieties in the sector. Besides, the amount of variety $q_{it}$ the final goods producers can purchase cannot exceed the amount of on-shelf goods chosen by intermediate goods producers, that is,

$$q_{it} \leq a_{it}, \forall i \tag{3.13}$$

The problem of a final goods producer is therefore

$$\max_{q_{it}} P_t c_t - \int_0^1 P_{it}^M q_{it} di \tag{3.14}$$

subject to constraints (3.12) and (3.13).

Cost minimization of the final goods producers imply that the demand function for each intermediate goods is as follows:

$$q_{it} = min\left\{ v_{it} \left( \frac{P_{it}^M}{P_t} \right)^{-\theta} c_t, a_{it} \right\} \tag{3.15}$$

And the price of sector final goods is given by:

$$P_t = \left( \int_0^1 v_{it} [P_{it}^M + \lambda_{it}]^{1-\theta} di \right)^{\frac{1}{1-\theta}} \tag{3.16}$$

where $\lambda_{it}$ is the multiplier on constraint (3.13). This is also another feature of the stock-out avoidance inventory model. In equilibrium, for intermediate goods that stock out (constraint (3.13) is binding), final goods producers evaluate those intermediate goods by their shadow values expressed as below:

$$P_{it}^M + \lambda_{it} = \left( \frac{a_{it}}{v_{it} P_t^{\theta} c_t} \right)^{-\frac{1}{\theta}} \tag{3.17}$$

while for intermediate goods that do not stock out, $\lambda_{it} = 0$.

#### 3.4.2.2 Intermediate goods producers

In each sector, there are a continuum of monopolistically competitive intermediate goods producers producing a differentiated variety of goods indexed by $i \in [0,1]$. As mentioned before, intermediate goods producers face two types of uncertainty in the economy: the first is idiosyncratic demand shock, which is the key feature of stock-out avoidance inventory models. Besides, I follow Gilchrist et al. (2017) to assume intermediate goods producers are also facing idiosyncratic productivity shocks when making decisions, which gives rise to a *Financially-adjusted Marginal Costs*, similar to that in Gilchrist et al. (2017). Moreover, when intermediate goods producer are making goods-pricing and on-shelf goods decisions, they do not know about their idiosyncratic demand and productivity shocks. Therefore, instead of maximizing actual profits based on their actual sales and actual marginal costs like models without those uncertainties, intermediate goods producers in this model make optimal pricing and on-shelf goods decisions to maximize expected dividend payments based on expected sales and expected marginal costs, which I will discuss in details below.

**Idiosyncratic Demand Shocks and On-shelf Goods**

The defining feature of stock-avoidance inventory model is that firms face demand uncertainty, that is, prior to knowing their idiosyncratic demand shock $v_{it}$, they need to post prices to final goods producers and determinate how many on-shelf goods to put for sales. Therefore, based on their demand function Equation 3.15, they have an expected sale as follows:

$$s_{it} = \int_{v\in\Omega(v)} min\left\{ v\left(\frac{P_{it}^M}{P_t}\right)^{-\theta} c_t, a_{it}\right\} dF(v) \tag{3.18}$$

where $\log v \overset{i.i.d}{\sim} N(0, \sigma_v^2)$ Since their sales are limited by their on-shelf goods and idiosyncratic demand shocks are unknown at the time of decisions, it is always optimal for intermediate goods producers to hold inventories, as long as they don't depreciate too much. Therefore, on-shelf goods consists of undepreciated stock of inventories from previous period $(1-\delta)invt_{i,t-1}$ and current production $y_{it}$. Unsold on-shelf goods are then become inventories. Equation 3.35 and Equation 3.20 summarize the law of motion of on-shelf goods and inventories.

$$a_{it} = (1-\delta)inv_{i,t-1} + y_{it} \tag{3.19}$$

$$inv_{it} = a_{it} - s_{it} \tag{3.20}$$

**Idiosyncratic Cost Shocks and Costly External Funds**

The production technology of the intermediate goods producers is given by

$$y_{it} = \left(\frac{h_{it}}{z_{it}}\right)^{\alpha}; \quad 0 < \alpha \leq 1 \tag{3.21}$$

where $z_{it}$ is an i.i.d adverse idiosyncratic productivity shock—that is, an idiosyncratic cost shock—distributed as $\log z_{it} \overset{i.i.d}{\sim} N(-0.5\sigma_z^2, \sigma_z^2)$ with the associated CDF denoted by F(z).

A key feature here is that I follow Gilchrist et al. (2017) to assume that firms do not know about their idiosyncratic cost shocks when they make pricing and on-shelf goods decisions. Therefore, instead of making their decisions based on realized marginal costs like most of the models in the literature, they make decisions based on an expected marginal costs. After the cost shocks realize, they then hire labor to carry out the production plan they make ex-ante.

This type of timing assumption creates possibility of an ex-post cash flow shortage. That is, after the cost shock realize, some firms (those hit by low productivity shocks) will find that their expected revenues ($P_{it}^M s_{it}$) may be too low to cover the total costs of production, since they choose to produce too much ex-ante. In this case, the firm must raise external funds, which incurs a dilution cost. Formally, for every dollar of external funds raised from households, firm only receives $1 - \varphi$ dollar to cover the portion of production costs that exceed revenues. Therefore, after idiosyncratic cost shocks realize, some firms may pay non-negative dividends, that is, $D_{it} \geq 0$, while other firms might have to issue equity to pay for production costs, that is, $D_{it} < 0$.

Above setups have two major implications. First, the actual marginal costs of production and labor hired by each intermediate goods producers are idiosyncratic, which depend on their realized idiosyncratic cost shocks. Accordingly, firms' expected flow-of-funds also depend on the realized idiosyncratic cost shocks through their idiosyncratic labor decisions, which can be expressed as follows:

$$0 = P_{it}^M s_{it} - W_t h_{it} - P_t f - D_{it} + \varphi min\{0, D_{it}\} \tag{3.22}$$

This further implies that the actual shadow value of firms' operating cash flows (internal funds) depend on their realized idiosyncratic cost shocks as well. However, recall intermediate goods producers' prices and on-shelf goods decisions are made before idiosyncratic cost shocks realize, therefore, their ex-ante prices and on-shelf goods decision are made based on expectations on their marginal costs and shadow value of operating cash flows (internal funds) with respect to i.i.d idiosyncratic cost shocks.

**Firm's Problem**

Each monopolistically competitive intermediate goods producers in this economy $i \in [0,1]$ maximizes the discounted sum of their expected dividends:

$$\max_{h_{it}, D_{it}} E_t \sum_{t=0}^{\infty} Q_{0,t} D_{it} \tag{3.23}$$

where $Q_{0,t}$ is nominal stochastic discount factor of households which equals to $\frac{1}{R_{0,t}}$, and $D_{it}$ denotes the nominal dividend payout when it is positive, or equity issuance when negative. Taking aggregate prices $W_t, P_t$ as given, intermediate goods producers choose their optimal prices and on-shelf goods. The constraints of the problem include production technology Equation 3.21, expected flows-of-funds Equation 3.22, law of motion for on-shelf goods Equation 3.35, and law of motion for inventories Equation 3.20.

The Lagrangian is as follows:

$$\begin{aligned}\mathcal{L} = E_0 \sum_{t=0}^{\infty} Q_{0,t} \Bigg\{ D_{it} + \gamma_{it} \Big[ P_{it}^M s_{it} - W_t h_{it} - P_t f - D_{it} + \varphi min\{0, D_{it}\} \Big] \\ + mc_{it} \Bigg[ \left( \frac{h_{it}}{z_{it}} \right)^{\alpha} - y_{it} \Bigg] \\ + \kappa_{it} \Big[ (1-\delta) inv_{i,t-1} + y_{it} - a_{it} \Big] \\ + \xi_{it} \Bigg( a_{it} - s_{it} - inv_{it} \Bigg) \Bigg\}.\end{aligned} \tag{3.24}$$

The optimal decisions can be summarized by the following first-order conditions:

$$\gamma_{it} = \begin{cases} 1, & \text{if} \quad D_{it} \geq 0 \\ \frac{1}{1-\varphi}, & \text{if} \quad D_{it} < 0 \end{cases} \tag{3.25}$$

$$mc_{it} = z_{it}\gamma_{it}\left(\frac{W_t y_{it}^{\frac{1-\alpha}{\alpha}}}{\alpha}\right) \tag{3.26}$$

$$\kappa_{it} = E_t^z[\gamma_{it}]P_{it}^M \frac{\partial s_{it}}{\partial a_{it}} + \left(1 - \frac{\partial s_{it}}{\partial a_{it}}\right)\xi_{it} \tag{3.27}$$

$$P_{it}^M = \frac{1}{1 + \frac{s_{it}}{P_{it}^M \frac{\partial s_{it}}{\partial P_{it}^M}}} \frac{\xi_{it}}{E_t^z[\gamma_{it}]} \tag{3.28}$$

Equation 3.25 and Equation 3.26 are formed based on firms' realized idiosyncratic productivity due to the timing assumption that firms' dividends and labor hiring decisions are made after idiosyncratic productivity shock $z_{it}$ realize. Equation 3.27 and Equation 3.28 implicitly show that firms' prices and on-shelf goods decisions are made prior to the realization of the idiosyncratic productivity shocks. Accordingly, these two involve firms' expected shadow value of internal funds $E_t^z[\gamma_{it}]$ and expected marginal costs $E_t^z[mc_{it}]$, where the expectations are formed using all aggregate information up to time t.

Besides, $\kappa_{it}$ is the shadow value of one more unit of on-shelf goods today, and $\xi_{it}$ is the shadow value of one more unit of unsold inventory (end-of-period inventory). Notice that the only way to have one more unit of on-shelf goods today is to produce one more unit of intermediate goods today, which implies that $\kappa_{it}$ should equal to firms' expected marginal costs as follows:

$$\kappa_{it} = E_t^z[mc_{it}] \tag{3.29}$$

Also, having one more unit of end-of-period inventory saves the costs of producing one more unit of goods tomorrow. Since inventories depreciate, the value of having one more unit of end-of-period inventory is therefore as follows:

$$\xi_{it} = (1-\delta)E_t[Q_{t,t+1}E_{t+1}^z[\kappa_{i,t+1}]] = (1-\delta)E_t[Q_{t,t+1}E_{t+1}^z[mc_{i,t+1}]] \tag{3.30}$$

**Envelope Conditions:** For optimal prices and on-shelf good decisions, the expected sales can be re-written as below:

$$s_{it} = \int_0^{v_{it}^*} vdF(v) \left( \frac{P_{it}^M}{P_t} \right)^{-\theta} c_t + (1 - F(v_{it}^*))a_{it} \tag{3.31}$$

where $v_{it}^*$ is the cut-off demand level implicitly determined by the optimal prices and on-shelf goods, which is expressed below:

$$v_{it}^* = \frac{a_{it}}{\left( \frac{P_{it}^M}{P_t} \right)^{-\theta} c_t} \tag{3.32}$$

Therefore, it is easy to see that when the realized idiosyncratic demand shock $v_{it}$ is greater than $v_{it}^*$, the intermediate goods producer stocks out, thus the realized sales is limited by the on-shelf goods $a_{it}$ chosen ex-ante.

The envelope conditions are thus given by:

$$\frac{\partial s_{it}}{\partial a_{it}} = 1 - F(v_{it}^*) \tag{3.33}$$

$$\frac{\partial s_{it}}{\partial P_{it}^M} = -\frac{\theta}{P_{it}^M} \int_0^{v_{it}^*} vdF(v) \left( \frac{P_{it}^M}{P_t} \right)^{-\theta} c_t \tag{3.34}$$

Finally, I can plug the envelop conditions and shadow values into the first-order conditions Equation 3.27 and Equation 3.28. Therefore, the optimal on-shelf goods and prices decisions satisfy the following first-order conditions:

$$\frac{\partial s_{it}}{\partial a_{it}} = 1 - F(v_{it}^*) = \frac{1 - r_{it}^I}{\mu_{it} - r_{it}^I} \tag{3.35}$$

$$P_{it}^M = \frac{\theta}{\theta - 1 - \frac{v_{it}^*[1-F(v_{it}^*)]}{\int_0^{v_{it}^*} vdF(v)}} \frac{(1-\delta)E_t[Q_{t,t+1}E_{t+1}^z[mc_{i,t+1}]]}{E_t^z[\gamma_{it}]} \tag{3.36}$$

where

$$\mu_{it} = \frac{P_{it}^M}{\frac{E_t^z[mc_{it}]}{E_t^z[\gamma_{it}]}} \tag{3.37}$$

$$r_{it}^I = \frac{(1-\delta)E_t[Q_{t,t+1}E_{t+1}^z[mc_{i,t+1}]]}{E_t^z[mc_{it}]} \tag{3.38}$$

$\mu_{it}$ denotes firms' markup of period t. $r^I_{it}$ denotes firms' return to inter-temporal production substitution: the benefit from producing one more unit of intermediate goods today and stored it as inventory (instead of producing this much tomorrow).

As discussed before, when the realized idiosyncratic demand shock $v_{it}$ is greater than $v^*_{it}$, the intermediate goods producer will thus stock out in equilibrium. Therefore, $1 - F(v^*_{it})$ denotes the firm's stock-out probability. According to Equation 3.32, when household demand $c_t$ goes down, $v^*_{it}$ will goes up, which lowers firms' stock-out probability $1 - F(v^*_{it})$. Taken together, firms' stock-out probability $1 - F(v^*_{it})$, or in other words, $v^*_{it}$ is determined by households' demand $c_t$, firms' return to inter-temporal production substitution $r^I_{it}$, and firms' markup $\mu_{it}$.

Besides, Equation 3.36 shows the optimal prices of intermediate goods are determined by endogenous markup, expected shadow value of producing one more unit of unsold inventory, and expected shadow value of internal funds. The key difference between this model and a standard stock-out avoidance inventory model is that optimal prices in this model are also affected by expected shadow value of internal funds. That is, when intermediate goods producers' expected shadow value of internal funds is high, firms want to decrease prices to facilitate sales, thereby increasing expected operating cash flows and avoiding costly external funds. This relationship can be seen in Equation 3.36.

One thing is worth noting before moving on: Equation 3.36 is the optimal flexible prices of intermediate goods producers. I also assume that intermediate goods producers face adjustment costs of changing nominal prices of Rotemberg (1982)-type, the first-order condition with price adjustment costs is derived in the Appendix.

**Expected shadow value of internal funds and expected marginal costs**

Another key question still left on the table is how intermediate goods producers form expectations on their marginal costs and shadow value of internal funds.

As shown in Equation 3.25, the realized shadow value is a step function. When intermediate goods producers pay dividends, the value of operating cash flows (internal funds) is just one, since they don't need to pay for financing costs. However, when they issue equity $(D_{it}) < 0$, they have to pay for financing costs, which then increases the value of their operating cash flows, that is, $\frac{1}{1-\varphi} > 1$.

Besides, in this model, hiring one more unit of labor does not only mean paying for one more unit of wage but also implies the decline in operating cash flows of intermediate goods producers, which then affects the shadow value of internal funds. Therefore, the realized marginal costs contain two parts: the first part involves the wage payments and the second part involves the shadow value of internal funds.

Expected shadow value of internal funds is given by:

$$E_t^z[\gamma_{it}] = \int_0^{z_{it}^E} 1F(z) + \int_{z_{it}^E}^{\infty} \frac{1}{1-\varphi} dF(z) = 1 + \left(\frac{\varphi}{1-\varphi}\right)[1 - \Phi(z_{it}^*)] \tag{3.39}$$

where $z_{it}^E$ denotes the cut-off idiosyncratic cost level that satisfies $D_{it} = 0$,which I refer as an external financing trigger. Since we assume idiosyncratic cost shocks $z_{it}$ follows a lognormal distribution ($\log z_{it} \sim N(-0.5\sigma_z^2, \sigma_z^2)$), I standardize $z_{it}^E$ into $z_{it}^*$, such that the expected value can be expressed in terms of standard normal CDF $\Phi(\cdot)$.

$z_{it}^E$ and $z_{it}^*$ are given by:

$$z_{it}^E = \frac{P_{it}^M s_{it} - P_t f}{W_t y_{it}^{\frac{1}{\alpha}}} \tag{3.40}$$

$$z_{it}^* = \frac{\log z_{it}^E + 0.5\sigma^2}{\sigma} \tag{3.41}$$

Expected marginal costs is given by:

$$E_t^z[mc_{it}] = E_t^z[z_{it}\gamma_{it}] \left(\frac{W_t y_{it}^{\frac{1-\alpha}{\alpha}}}{\alpha}\right) \tag{3.42}$$

It is easy to see from Equation 3.39 that when external financing trigger $z_{it}^E$ goes down, intermediate goods producers' expected shadow value of internal funds will go up, since lower $z_{it}^E$ leads to more costly external funds, which therefore increases the value of firms' operating cash flows (internal funds).

#### 3.4.2.3 Within-sector Symmetric Equilibrium

A key feature of stock-out avoidance model is that to make firms choose identical choices, firms need to face the same marginal costs of production. Here, I follow Crouzet and Oh (2016) to assume that the marginal costs of production are the same across firms. In other words, after the idiosyncratic demand shocks realize, there is a distributor in each sector who reallocates sector-level inventories across intermediate goods producers, such that each intermediate goods producer has exactly the same end-of-period inventories. Therefore, when all intermediate goods producers choose the same prices and on-shelf goods, they then have exactly the same production plan, and therefore the same expected marginal costs of production. Since the focus of the model is to compare sector-level sales, inventories, and inventory-to-sales ratio between sectors with different levels of operating leverage, this assumption is innocuous in the sense it does not generate differential behaviors across sectors. Besides, since intermediate goods producers within a sector pay the same fixed operating cost, they also have the same expected flow-of-funds, therefore, the same expected shadow value of internal funds. Taken together,in the same sector, intermediate goods producers make the same optimal price and on-shelf goods decisions, that is, $P_{it}^M = P_t^M$, $a_{it} = a_t$, and implicitly, $v_{it}^* = v_t^*$, $y_{it} = y_t$.

For intermediate goods producers who receive an idiosyncratic demand shock that is greater than $v_t^*$, they stock out, in other words, their actual sales in period t is just their on-shelf goods $a_t$. $1 - F(v_t^*)$ is thus the stock-out probability. Therefore, the sector-level final sales can be written as follows:

$$q_t = \int_0^1 q_{it} di = \int_0^{v_t^*} v dF(v) \left( \frac{P_t^M}{P_t} \right)^{-\theta} c_t + (1 - F(v_t^*)) a_t \tag{3.43}$$

Recall that final goods producers evaluate intermediate goods that stock out by their shadow values given by Equation 3.17. Therefore, the sector-level price can be expressed as follows:

$$P_t = \left( \int_0^{v_t^*} v dF(v) + (v_t^*)^{1-\frac{1}{\theta}} \int_{v_t^*}^{\infty} v^{\frac{1}{\theta}} dF(v) \right)^{\frac{1}{1-\theta}} P_t^M \tag{3.44}$$

The sector-level unsold inventories are therefore given by:

$$Invt_t = a_t - q_t \tag{3.45}$$

Thus, the sector-level inventory-to-sales ratio is :

$$IS_t = \frac{a_t - q_t}{q_t} = \frac{v_t^* F(v_t^*) - \int_0^{v_t^*} v dF(v)}{\int_0^{v_t^*} v dF(v) + v_t^* [1 - F(v_t^*)]} \tag{3.46}$$

which is increasing in $v_t^*$.

Sector-level production is total sector-level final sales plus sector-level inventory investment:

$$y_t = q_t + [invt_t - (1 - \delta) invt_{t-1}] \tag{3.47}$$

Since dividends and labor hiring decisions are made after the realization of idiosyncratic cost shocks, $D_{it}$ and $h_{it}$ are heterogeneous even within the sector and depend on their realized idiosyncratic cost shocks. However, it is easy to derive the sector-level labor demand and dividends, which are given by:

$$h_t = \int_0^1 h_{it} di = \left[ \frac{y_t}{exp[0.5\alpha(1+\alpha)\sigma_z^2]} \right]^{\frac{1}{\alpha}} \tag{3.48}$$

$$D_t = \int_0^1 D_{it} di = P_t^M q_t - W_t h_t - P_t \phi \tag{3.49}$$

### 3.4.3 Market Clearing

Households supply labors while intermediate goods producers in each sector demand labor. Therefore, the labor market clearing condition is:

$$l_t = n_t = h_t^H + h_t^L \tag{3.50}$$

where $l_t$ denotes total labor supply from households, $n_t$ denotes total labor demand from high-cost sector $h_t^H$ and low-cost sector $h_t^L$.

### 3.4.4 Calibration

A period in the model equals one quarter. Parameters for inventory are from Kryvtsov and Midrigan (2012). Financing costs $\varphi$ and standard deviation of i.i.d cost shocks $\sigma_z$ follow Gilchrist et al. (2017). $f_h$ and $f_l$ are chosen such that steady-state operating leverage is 74% for high-cost sector, and 26% for low-cost sector. Standard deviation of monetary growth shocks is chosen to generate 25 basis point increase in real interest rate. Other parameters are standard in the literature.

## 3.5 Model Mechanisms and Simulations

In section 3.4, I developed a two-sector New Keynesian Model with heterogeneous fixed operating costs between sectors. In this section, I will present the simulated results, discuss the model mechanism based on the first-order conditions, and provide intuition for a better understanding of the model.

As discussed in section 3.4, the model economy faces an aggregate monetary growth shock. To study the effects of monetary shock, I therefore simulate the model with

| Parameter | Value | Description |
|---|---|---|
| $\beta$ | 0.99 | Standard discount factor |
| $\eta$ | 0.4 | Frisch elasticity of labor supply =2.5 |
| $h$ | 0.37 | Habit intensity |
| $\alpha$ | 0.67 | Labor share of income |
| $\eta_w$ | 2 | Steady-state wage markups=0.5 |
| $\delta$ | 0.0362 | Monthly inventory depreciation rate=0.011 |
| $\theta$ | 6.7 | Elasticity of substitution across varieties, steady-state markup= 1.21 |
| $\sigma_v$ | 0.604 | SD of i.i.d idiosyncratic demand shock, equilibrium stock-out probability=0.05 |
| $\varphi$ | 0.3 | External financing costs, Gilchrist et al. (2017) |
| $\sigma_z$ | 0.9 | SD of i.i.d idiosyncratic productivity shock, Gilchrist et al. (2017) |
| $f_h$ | 0.21 | Fixed operating costs of high-cost sector; Steady-state operating leverage=0.74 |
| $f_l$ | 0.071 | Fixed operating costs of low-cost sector; Steady-state operating leverage=0.26 |
| $\xi$ | 2 | Elasticity of substitution across sectors, standard |
| $\omega_H$ | 0.5 | Sector size of high-cost sector |
| $\omega_L$ | 0.5 | Sector size of low-cost sector |
| $\sigma_m$ | 0.0265 | SD of monetary growth shocks;1sd shock leads to 25bp increase in real interest rate. |
| $\psi_w$ | 0.5 | Wage adjustment costs |
| $\psi_p$ | 0.5 | Price adjustment costs |

an one-time shock to monetary growth and graph the impulse response functions of variable of interests. In all cases, I consider a negative monetary growth shock that leads to a 25 basis point increase in real interest rate at impact, as shown in plot (a) in Figure 3.3. The model is simulated with baseline calibration, as in section 3.4.4. Firms in high-cost sector pay a fixed operating costs $f_h = 0.21$ with a 74% steady-state operating leverage, while firms in low-cost sector pay a fixed operating costs $f_l = 0.071$ with a 26% steady-state operating leverage.

### 3.5.1 Key Mechanism

In this section, I explain the main empirical finding of the paper: in response to contractionary monetary shock, high fixed cost firms cut few inventories than their low fixed cost counterparts. That is, even though inventories of both sectors fall, high fixed cost firms are left with more inventories than low fixed cost firms.

The key model mechanism is as follows: a contractionary monetary shock reduces aggregate household demand for both high and low fixed costs sectors. In face of a lower household demand and thus expected sales and revenues, high fixed cost firms cut more prices than their counterparts in order to attract household demand and thus increase expected sales and avoid costly external financing. This mechanism can be seen in graphs (a), (b), and (c) of Figure 3.2. Compared with low fixed cost sector, the expected value of internal funds of high fixed cost sector increases much more, because they are more likely to raise costly external funds due to the high fixed operating costs they have to pay every period. Higher value of internal funds motivate them to cut output prices in order to attract household demand (cross-sector elasticity of substitution is greater than 1, as in other multi-sector models). Therefore, relative output price $\frac{P_t^H}{P_t^L}$ falls and relative household demand $\frac{c_t^H}{c_t^L}$ rises.

In face of a larger household demand, high fixed cost sector therefore holds more on-shelf goods in order to avoid stock-outs, thus relative on-shelf goods $\frac{a_t^H}{a_t^L}$ increases as shown in graph (d) of Figure 3.2. High fixed cost sector is therefore left with more sector-level unsold inventories, since all firms in the high fixed cost sector choose a large amount of on-shelf goods ex-ante while the only a small fraction of firms actually stock out at the end-of-period. As shown in graph (e) of Figure 3.2 relative unsold inventories $\frac{Inv_t^H}{Invt}$ increases through this mechanism. In the calibrated model, steady-state stock-out probability is chosen to be 5%, as in other papers in the literature. This is also why relative final sales $\frac{q_t^H}{q_t^L}$ moves almost the same as relative household demand.

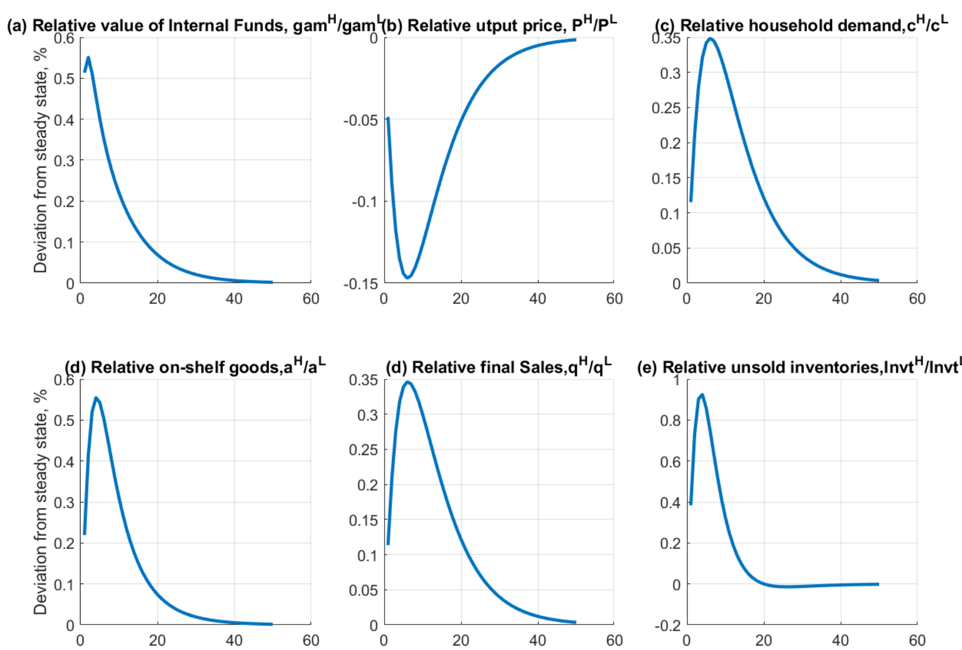



**Figure 3.2:** IRFs to a Contractionary Monetary Shock

### 3.5.2 Explaining Impulse Responses Functions

Figure 3.3 shows the responses of aggregate consumption, labor, and nominal wage to a contractionary monetary shock. In response to an interest rate hike, households decrease their aggregate consumption demand. The decline in aggregate demand further leads to decrease in aggregate output and labor demand, which I will discuss in details later. Both the decline in aggregate consumption and labor hired lead to decrease in nominal wage through households intra-temporal condition Equation 3.9, as shown in plot (d) in Figure 3.3. The direct implication of declining nominal wages is the decrease in expected marginal costs of production for intermediate goods producers in plot (a) of Figure 3.4 .

However, the convex adjustment costs of changing nominal wages prevent households from changing their wages too much at once, which induces a slow decline in nominal wage, and therefore, slow decline in expected marginal costs.

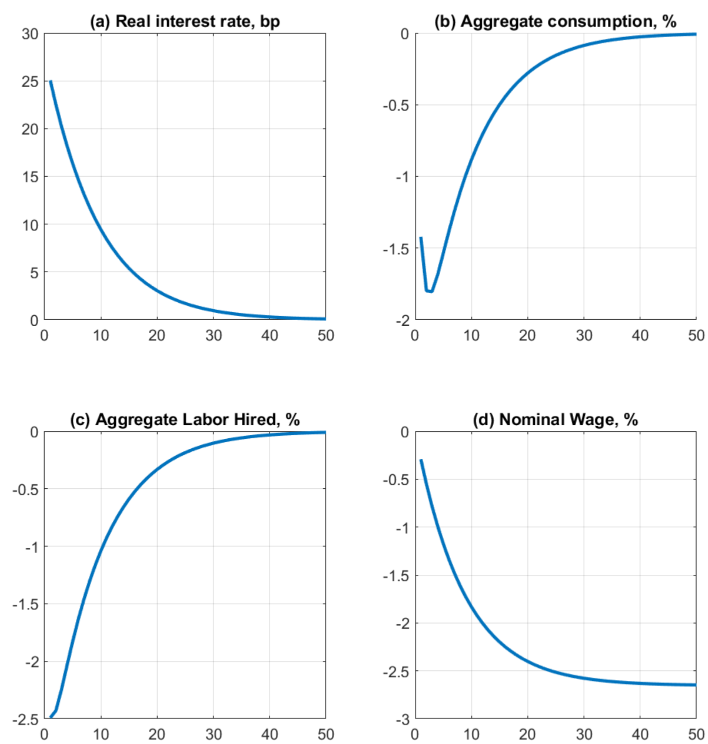



**Figure 3.3:** IRFs to a Contractionary Monetary Shock

The huge drop in aggregate consumption demand $c_t$ leads to drop in intermediate goods producers' expected sales $s_t$. This drop in expected sales $s_t$, combined with small initial drop in nominal wage $W_t$ due to nominal rigidity, translates to a huge decline in expected operating cash flows of intermediate goods producers, which implies that intermediate goods producers are more likely to raise costly external funds to cover realized production costs after productivity uncertainty resolves. This effect can be seen from the external financing trigger below,

$$z_t^E = \frac{P_t^M s_t - P_t f}{W_t y_t^{\frac{1}{\alpha}}} \tag{3.51}$$

The drop in right-hand side leads to a decline in external financing trigger $z_t^E$, which then translates to increase in expected shadow value of internal funds $E^z[\gamma_t]$. Intuitivle, when $z_t^E$ goes down, it is more likely for intermediate goods producers to pay financing cost $\varphi$ in order to get external funds for compensating the decline in operating cash flows.

$$E^z[\gamma_t] = \int_0^{z_t^E} 1F(z) + \int_{z_t^E}^{\infty} \frac{1}{1-\varphi} dF(z) \tag{3.52}$$

Plot (b) of Figure 3.4 shows the rise in expected shadow value of internal funds in response to a contractionary shock for both sectors. More importantly, a drastic difference in expected shadow value of internal funds $E^z[\gamma_t]$ between high-cost sector and low-cost sector can be easily seen in plot (b) of Figure 3.4 as well. The differential fixed operating costs between sectors drive this difference. Facing the same degree of decline in aggregate consumption demand, and thus drop in expected revenue $P_{it}^M s_{it}$, intermediate goods producers in high-cost sector are more likely to raise external funds due to the high fixed operating cost they pay. In other words, the external financing trigger declines much more for intermediate goods producers in high-cost sector, which then leads to much larger increase in expected shadow value of internal funds based on Equation 3.52.

Besides, this heterogeneous rise in expected shadow value of internal funds also leads to the difference in expected marginal costs between sectors in plot (a) of Figure 3.4. The larger increase in expected shadow value of internal funds of high-cost sector leads to smaller decrease in their expected marginal costs, since the expected marginal costs of intermediate goods producers are also affected by the shadow value of internal funds, which can be seen from Equation 3.42.

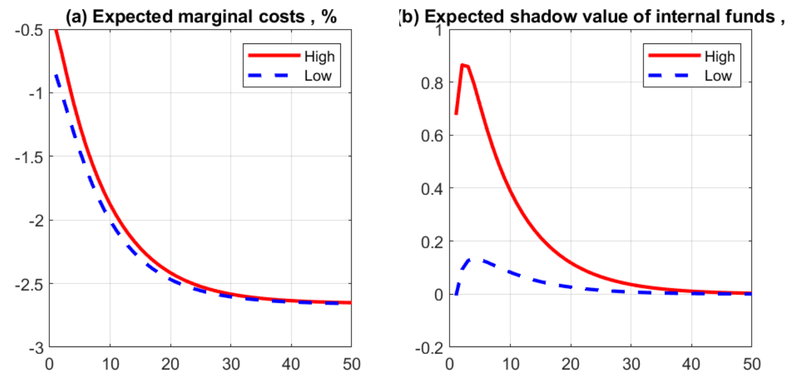



**Figure 3.4:** IRFs to a Contractionary Monetary Shock

As derived in section 3.4, sector-level inventory-to-sales ratio is given by:

$$IS_t = \frac{v_t^* F(v_t^*) - \int_0^{v_t^*} v dF(v)}{\int_0^{v_t^*} v dF(v) + v_t^*[1 - F(v_t^*)]} \tag{3.53}$$

which is increasing in $v^*$, the cut-off demand level. As discussed in section 3.4 and shown in Equation 3.54, $v^*$ is determined by three elements: households demand $c_t$, intermediate goods producers' markup $\mu_t$, and return to inter-temporal production substitution $r_t^I$. The right-hand side of Equation 3.54 is decreasing in $\mu_t$ and $r_t^I$. When markup $r_t^I$ is high, stock-outs are especially costly, since the profits lost by failing to make a sale is greater. Higher markups thus lead firms to lower stock-out probability $1 - F(v_t^*)$ by choosing more on-shelf goods relative to demand. Besides, when return to inter-temporal production substitution is high, it is profitable for firms to produce intermediate goods today and store them as inventories to save the high costs of production next period, which also implies that firms should lower the probability of stock-outs, and therefore more unsold inventories can be left for next period.

$$1 - F(v_t^*) = \frac{1 - r_t^I}{\mu_t - r_t^I} \tag{3.54}$$

$$\mu_t = \frac{P_t^M}{\frac{E^z[mc_{it}]}{E^z[\gamma_{it}]}} \tag{3.55}$$

$$r_t^I = \frac{(1-\delta)E_t[Q_{t,t+1}E^z[mc_{i,t+1}]]}{E^z[mc_{it}]} \tag{3.56}$$

In addition, Equation 3.57 is the optimal flexible price for intermediate goods. A key

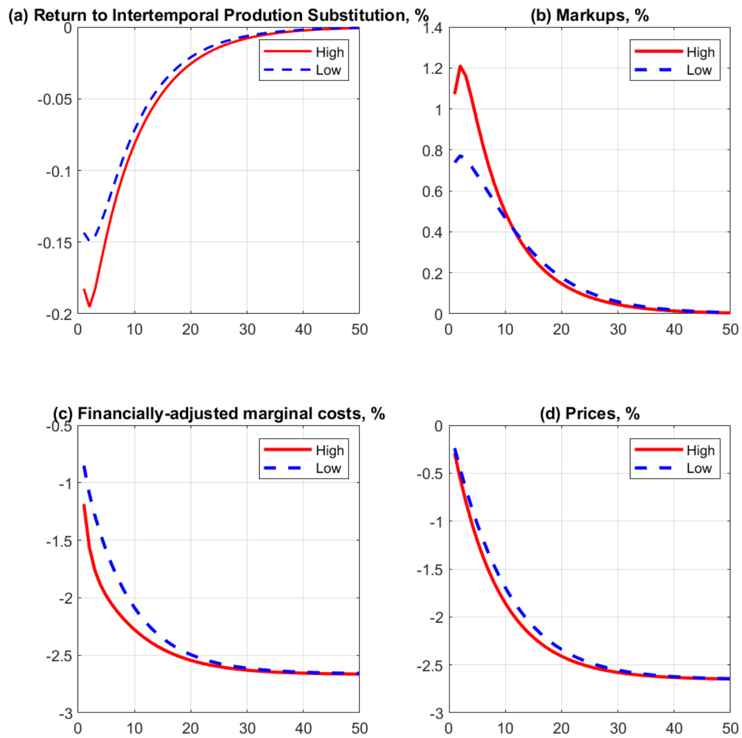



**Figure 3.5:** IRFs to a Contractionary Monetary Shock

feature of the stock-out avoidance model is that the optimal flexible price features an endogenous markup, which is affected by firms' stock-out probability. When stock-out probability is high, sales are less sensitive to price changes (more constrained by on-shelf goods), leading to a lower markup. In addition, optimal flexible prices in this model are also affected firms' expected value of internal funds. When firms' expected value of internal funds goes up, it is optimal for firms to lower prices to facilitate sales, thereby increasing operating cash flow and avoiding costly external financing. Following Gilchrist et al. (2017), I refer $\frac{E^z[mc_{it}]}{E^z[\gamma_{it}]}$ as *Financially-adjusted marginal costs.*

$$P_t^M = \frac{\theta}{\theta - 1 - \frac{v_t^*[1-F(v_t^*)]}{\int_0^{v_t^*} vdF(v)}} \frac{(1-\delta)E_t[Q_{t,t+1}E^z[mc_{i,t+1}]]}{E^z[\gamma_{it}]} \tag{3.57}$$

Figure 3.5 shows the changes in return to inter-temporal production substitution, markups, financially-adjusted marginal costs, and intermediate goods prices in response to a contractionary shock.

First, the gradual decline in expected nominal marginal costs in plot (a) of Figure 3.4 suggests that it is always cheaper to produce next period. Thus, return to inter-temporal production substitution is always negative, as shown in plot (a) of Figure 3.5. This would motivate firms to choose a lower inventory-to-sales ratio by increasing stock-out probability.

Secondly, the decline in expected marginal costs, combined with a sharp increase in expected shadow value of internal funds, lead to large decrease in firms' *Financially-adjusted marginal costs* in plot (c) of Figure 3.5. In other words, it is optimal for firms to sharply decrease their output prices to increase household demand and thus avoid liquidity shortage and costly external financing. However, due to convex adjustment costs of changing nominal price, it is especially costly to decrease output prices at once, thus the actual output prices chosen by firms are higher than their optimal levels suggested by Equation 3.57, which leads to an increase in firms' markups for both sectors, as shown in plot (b) in Figure 3.5. More importantly, high future expected marginal costs in high-cost sector compensate for their strong motive to decrease prices (caused by high expected shadow value of internal funds), thus the actual output prices chosen by firms in high-cost sector are just slightly lower than those of low-cost counterparts in plot(d) of Figure 3.5. As a result, firms in high-cost sector experience a much larger increase in markup, defined as the actual prices over *Financially-adjusted marginal costs*, than their low-cost counterparts, as shown in plot (b) in Figure 3.5.

In addition, plot (a) and Plot (b) in Figure 3.5 suggest that the increased markup, in response to a contractionary shock, is much larger than the negative return to inter-temporal production substitution for both sectors. That is, overall, it is optimal for firms to lower their stock-out probability. This force is much stronger for high-cost sector, since they experience a much larger increase in markup than their low-cost counterparts.

Plot (b) in Figure 3.6 shows the response of sales. Both the decrease in households' demand and temporal high prices caused by price adjustment costs contribute to the huge decrease in firms' final sales. The decline in sales is slightly less severe for firms in high-cost sector, since their output prices are slightly lower than those of firms in low-cost sector.

Plot (c) in Figure 3.6 shows the response of inventory. First, the decrease in household demand caused by negative money supply shock increases $v^*$, thereby lowering the stock-out probability for all intermediate goods producers. This effect reduces the value of having on-shelf goods, and thus end-of-period inventories, which leads firms to cut their inventories in response to a contractionary shock. However, the huge rise in markups increase firms' profit loss when failing to make a sale, thus preventing firms from cutting inventories too much. This effect is especially prominent for firms in the high-cost sector that experience a larger rise in markups. As a result, high-cost sector cut much fewer inventories than their low-cost counterparts. Again, since two sector go through similar decline in sales, high-cost sector thus have a higher inventory-to-sales ratio than low-cost sector, as shown in plot (a) of Figure 3.6.

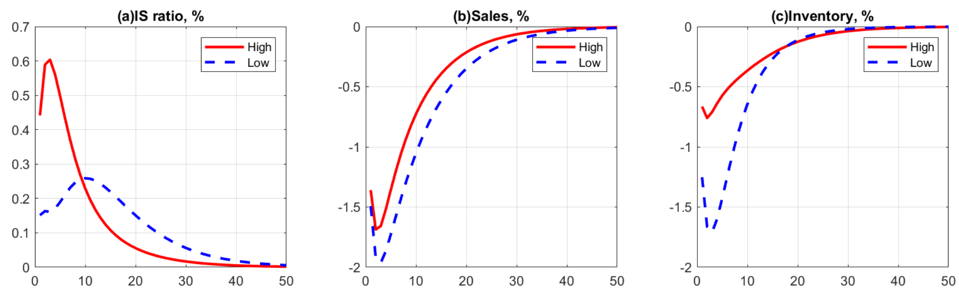



**Figure 3.6:** IRFs to a Contractionary Monetary Shock

In summary, I discussed the main mechanism through which operating leverage differentiate firms' inventory-to-sales ratio, sales, and inventories response to monetary policy shocks. The model qualitatively matches the empirical patterns documented in section 3.3: in response to a contractionary monetary shock, high operating leverage firms adjust to a higher inventory-to-sales ratio than their low operating leverage by cutting fewer inventories.

### 3.5.3 Understanding Model Ingredients

To facilitate a better understanding of the model mechanism, I then discuss and present IRFs with different model specifications.

#### 3.5.3.1 Role of External Financing Costs

The key mechanism driving the heterogeneous responses between high-cost sector and low-cost sector is the different expected shadow value of internal funds that firms in different sectors have. In response to a decrease in expected operating cash flows caused by declining household demand, firms in high-cost firms are more likely to raise costly external funds due to the high fixed operating costs they pay, and thus internal funds are becoming more valuable for them. That is, as long as there is no wedge between internal funds and external funds, there should be no difference in behaviors between sectors, even though these two sectors pay different fixed operating costs.

Figure 3.7 shows the IRFs with baseline parameters but $\varphi = 0$. It is easy to see when $\varphi = 0$, $\gamma_{it} = 1$ and $E^z[\gamma_{it}] = 1$ all the time. As shown in plot (d) of Figure 3.7, expected shadow value of internal fund do not respond to a contractionary shock anymore. The model then reduces to a standard stock-out avoidance inventory model. Even though firms in different sectors still pay different operating costs in this case, there is no difference in behaviors between them anymore.

#### 3.5.3.2 Role of Fixed Cost and Price Rigidity

In this subsection, I evaluate the role of fixed cost and price rigidity in driving the model results. In doing so, I set the size of one sector to be zero, therefore, the model reduces to an one-sector model, and I also set habit intensity h to be zero, therefore, there are no humped shaped responses in IS ratio and sales.

I compare two extreme cases, fixed operating cost f=0.3 and f=0, the corresponding steady-state operating leverage are 97% and 0%. The key difference is the response of expected shadow value of internal funds, as in graph (d) of Figure 3.8. When fixed cost is zero, the value of internal fund goes down, since contractionary monetary shock not only brings down expected values but also brings down wages. In this case, firms do not have incentive to cut their prices since they also benefit from decreasing wage payment. When fixed cost is really high, firms therefore have strong incentive to cut prices to increase expected revenues and avoid costly external financing. This difference further results in the differences in other variables.

Besides, price rigidity is important in explaining the rise in IS ratio and markups after a contractionary shock, as shown in Figure 3.8. Without price rigidity, IS ratio goes down, which is inconsistent with the empirical results. As discussed in 3.5 and Kryvtsov and Midrigan (2012), counter-cyclical markup due to price rigidity is key to generate increase in IS ratio in response to a contractionary monetary shock for a stock-out avoidance inventory model. As shown in graph (h) of Figure 3.8, without price rigidity, markup barely responds to a contractionary shock and IS ratio declines. The calibration in the paper follows the literature to ensure a counter-cyclical markup and thus rise in IS ratio in response to a contractionary monetary shock.

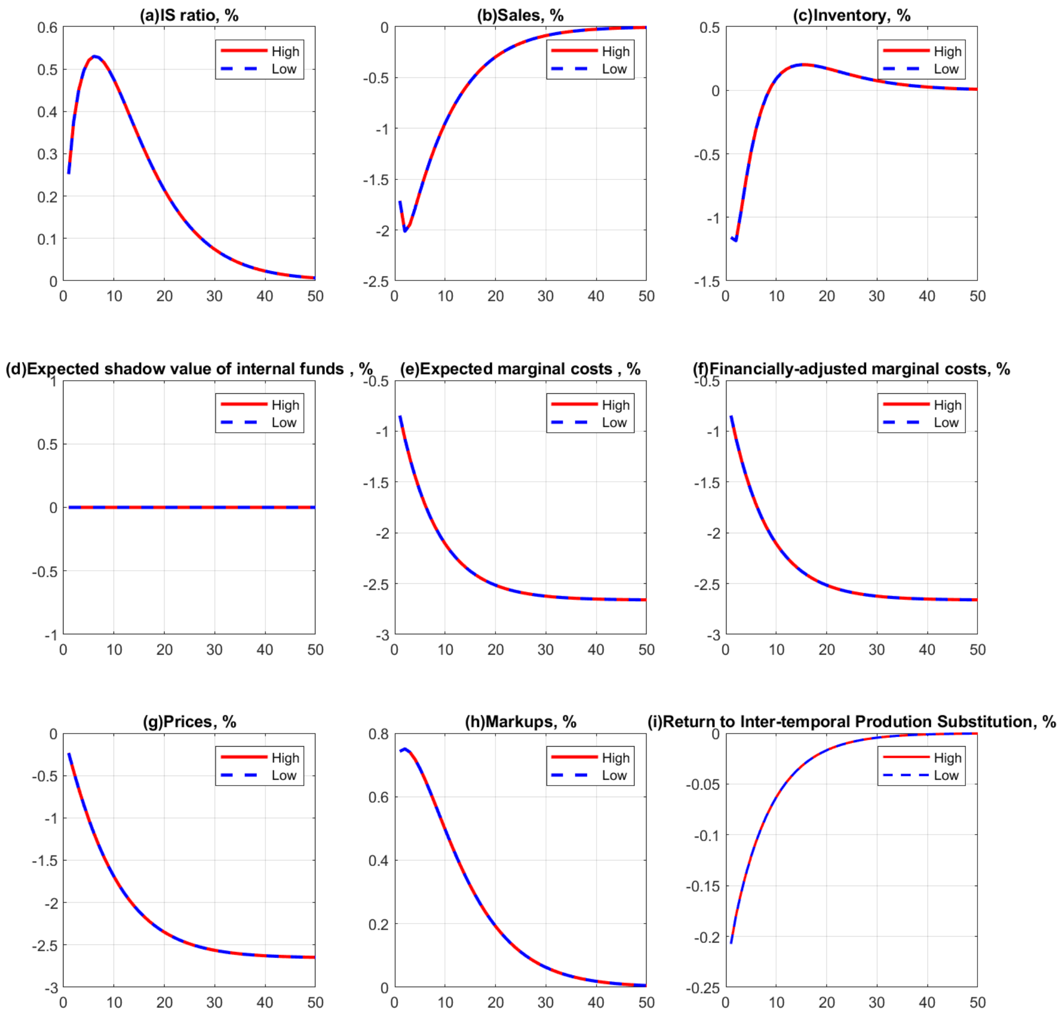



**Figure 3.7:** IRFs to a Contractionary Monetary Shock without Financial Frictions

The focus of this paper is how fixed operating cost affects markups and thus IS ratio in response to monetary shocks. As shown in Figure 3.8, the rise of markup in an one-sector economy with high fixed cost is much larger than that in an one-sector economy with low fixed cost. That is because high fixed cost serves as a mechanism that induces firms to slash their output prices to preserve internal funds and avoid costly external funds. With price rigidity, the prices chosen deviate from their optimal prices, leading to large markups.

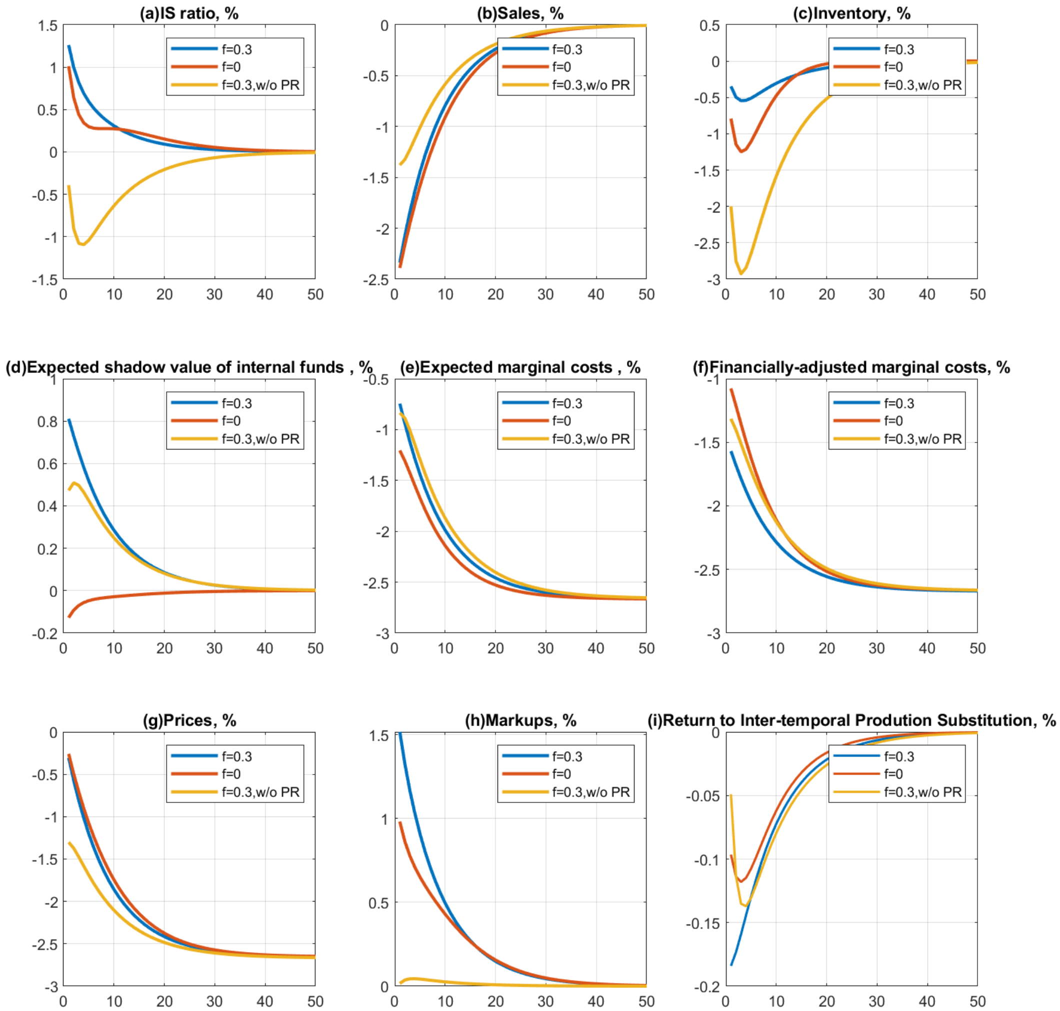



**Figure 3.8:** IRFs to a Contractionary Monetary Shock without Fixed Costs or w/o Price Rigidity

### 3.5.3.3 Role of Habit Formation

In the baseline calibration, I include habit formation in household problem in order to capture the hump-shaped decline in sales. Figure 3.9 shows the IRFs with baseline parameters but $h = 0$. In this case, there is no hump-shaped decline in sales anymore. In addition, the hump-shaped behavior in inventory-to-sales ratio disappears as well.

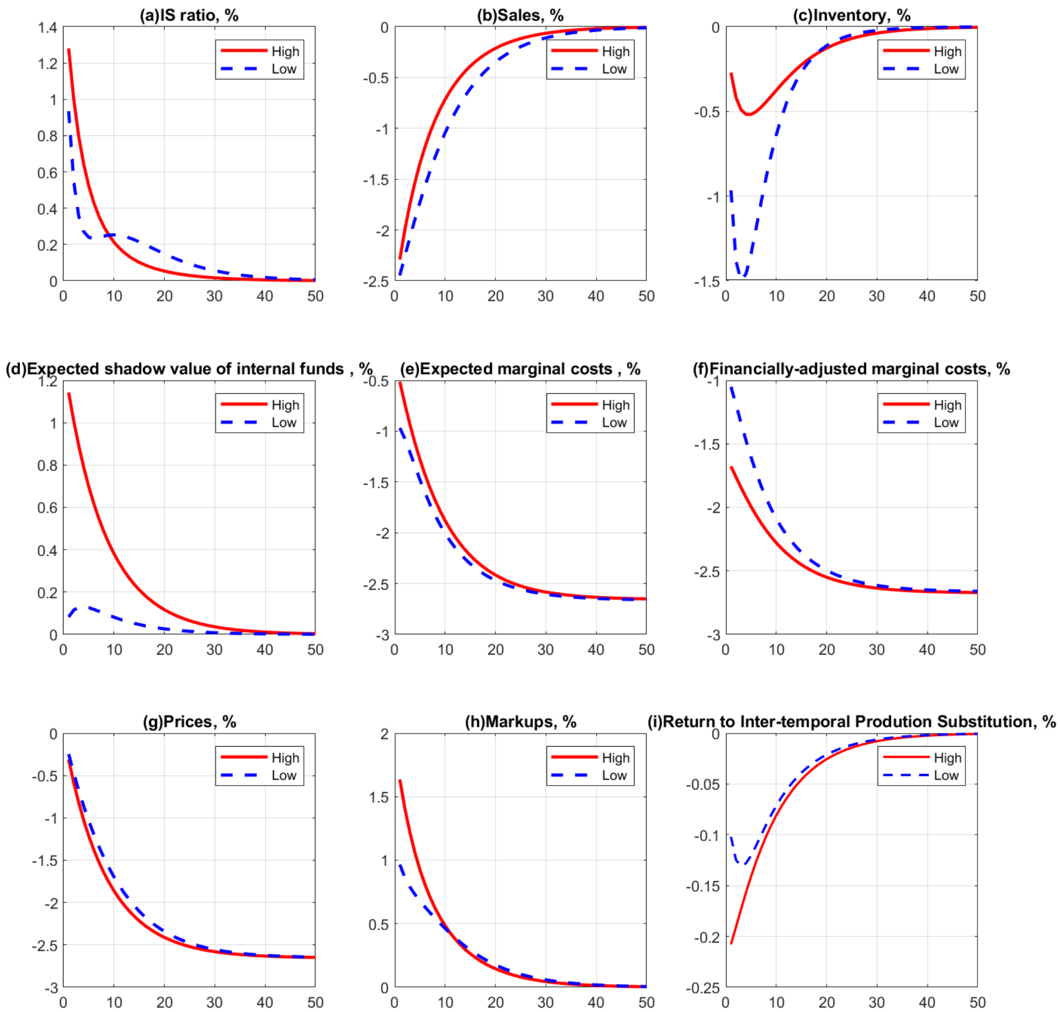



**Figure 3.9:** IRFs to a Contractionary Monetary Shock without Habit Formation

## 3.6 Monetary Policy Analysis

In this section, I use the model to examine the effects of a contractionary monetary policy on aggregate sales and aggregate inventory. To be specific, I compare the effects of a contractionary shock between two different economies. The baseline economy has two sectors with steady-state operating leverage of 74% and 26%, respectively. The other economy has two sectors with the same steady-state operating leverage of 74%, which I refer to as a high-cost economy. Other parameters are the same as those in subsection 3.4.4.

Figure 3.10 shows the response of aggregate sales and aggregate inventories to a contractionary monetary shock that leads to a 25bp increase in real interest rate. As shown in plot (b) of Figure 3.10, the effect of a contractionary monetary shock on aggregate inventories is much smaller in the high-cost economy. In the model, different fixed operating costs differentiate firms' markup changes in response to interest rate changes while not inducing much difference in the output price changes. As a result, actual sales changes are similar across economies with different levels of fixed operating costs. In contrast, economies with high operating costs tend to have more end-of-period inventories because high markups increase their demand for on-shelf goods at the beginning of the period. Taken together, high operating leverage might reduce the effects of a contractionary monetary policy shock.

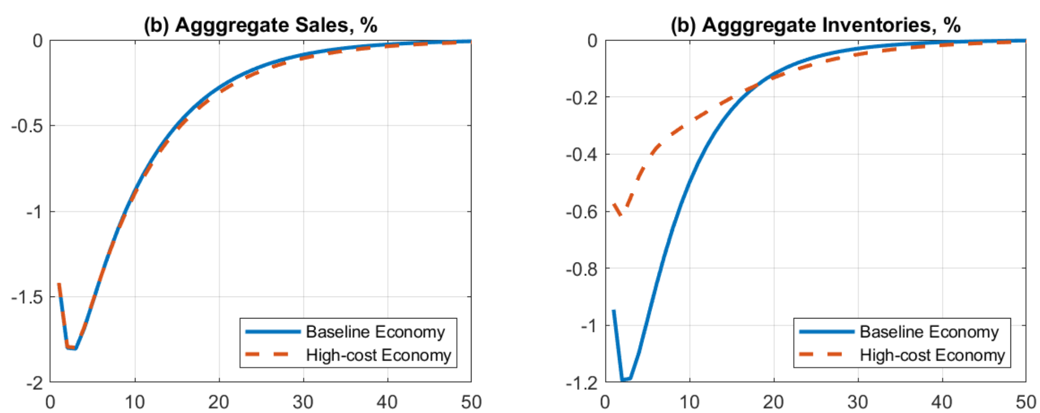



**Figure 3.10:** IRFs to a contractionary monetary shock

## 3.7 Conclusion

How do financial frictions influence the transmission of monetary policy? Contrary to the financial accelerator effects on fixed capital investment in the literature, this chapter shows both empirically and theoretically that financial frictions dampen the effects of monetary policy shocks on inventory investment. Using firm-level data combined with externally identified monetary policy shocks, I first show that following contractionary monetary policy shocks, more financially constrained firms cut much fewer inventories than their less financially constrained counterparts despite similar effects of monetary policy shocks on their sales. To explain the empirical patterns, I build a dynamic New Keynesian general equilibrium model in which firms face demand uncertainty and financial frictions and thus manage inventory to avoid stock-outs and cash flow shortfalls. When contractionary monetary policy shocks lower households' demand for goods and thus firms' expected sales and revenues, more financially constrained firms slash their goods' prices and put more inventories on the shelves to increase operating cash flows, thereby reducing costly external financing. My calibrated model successfully replicates a wide set of data features: pro-cyclical inventories and sales, counter-cyclical inventory-to-sales ratio and markups, as well as heterogeneous responses across differently financially constrained firms. Counterfactual exercises show that the aggregate effect of monetary policy is smaller in a more financially constrained economy through the inventory channel.

## 3.8 Appendix

### 3.8.1 Firm-level Data

I draw firm-quarter data for publicly listed U.S. nonfinancial firms in Compustat/CRSP Dataset. I followed the standard practices in the corporate finance literature for variable construction and sample selection, which are detailed in Appendix A.3. The sample period is from 1990Q1 to 2007Q4, when high-frequency identified monetary policy shocks discussed above are well-defined.

### 3.8.2 Variable Construction

1. *Firm size*: defined as the log of total assets (atq).

2. *Profitability*: defined as operating income after depreciation oiadpq over lagged total assets(atq).

3. *Cash flow*: defined as the sum of (ib) and (dpr) over total assets(atq).

4. *Liquid asset holding*: defined as the sum of cash and short-term investments (cheq) over total assets(atq).

5. *Leverage ratio*: defined as the ratio of total debt (sum of dlcq and dlttq) to total assets (atq).

### 3.8.3 Sample Selection

Point 1 and 2 follow the standard practices in corporate finance literature. Point 3 and 4 follow Ottonello and Winberry(2018) and Jeenas(2018).

1. I exclude firms in finance,utility, and government-related sectors

2. I exclude firms not incorporated in the United States

3. I only include firm-quarter observations from firms which are observed for at least 40 quarters during the sample period (a reasonably long time-dimension is required for firm-level fixed effects and within estimator)

4. I winsorize observations of asset size, leverage ratio, liquid asset ratio, profitability, cash flows at the top and bottom 0.5% of the distribution.

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