Hunting for the SNARC:
An Investigation of Implicit and Explicit Associations between Numbers and Space

By

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To my grandmothers

Safta and Sue

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Abstract

From numbered city blocks to lines on a measuring cup, numbers and space are intertwined across many aspects of our daily lives. These spatial-numerical associations (SNAs) are believed to reflect an internal, spatial conceptualization of numerical magnitudes, a representation that can be accessed more implicitly (i.e. without conscious awareness) or more explicitly. Despite the fact that implicit and explicit SNAs for whole numbers have been explored in both behavioral and neuroimaging contexts, few studies have investigated SNAs for fractions and many unanswered questions still remain. In this dissertation, I tested children, typical adults, and adults with synesthesia to address several of these outstanding questions, including: do children have SNAs for fraction magnitudes? If so, at what age do implicit and explicit SNAs emerge? Are individual differences in fraction SNAs related to how well people do on fractions tests, basic math tests, or algebra? Lastly, are implicit and explicit SNAs related to each other and consistent within individuals? In a theoretical review and a series of three empirical studies, I found that SNAs for fractions differed greatly between individuals, but that this variability was not a reliable indicator of internal cognitive representations or educationally-relevant outcome measures such as math achievement. Implicit and explicit SNAs for fraction magnitudes diverged in mid-childhood and differentially accounted for variability in mathematical outcome measures, with explicit SNAs more likely to account for differences in fraction test scores in both kids and adults. These results suggest that explicit measures of SNAs may be more reliable indices of internal magnitude representations than more implicit measures, a conclusion that has implications both for basic research on numerical cognition and educational interventions.

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"We have sailed many months, we have sailed many weeks,
(Four weeks to the month you may mark),
But never as yet ('tis your Captain who speaks)
Have we caught the least glimpse of a Snark!

"We have sailed many weeks, we have sailed many days,
(Seven days to the week I allow),
But a Snark, on the which we might lovingly gaze,
We have never beheld till now!

. . .

"I engage with the Snark—every night after dark—
In a dreamy delirious fight:
I serve it with greens in those shadowy scenes,
And I use it for striking a light:

"For the Snark's a peculiar creature, that won't
Be caught in a commonplace way.
Do all that you know, and try all that you don't:
Not a chance must be wasted to-day!"

"The Hunting of the Snark" – Lewis Carroll

Chapter 1: General Introduction & Background

Imagine that you are running late to an important meeting in an unfamiliar building. As you race through the entrance, you notice that there is no map and no directory. Your meeting is in office #228. You race up the stairs to the second floor and are faced with a long corridor extending to either side of you. The office directly in front of you is labeled #217. Do you turn to the right or to the left? For most people, the decision would be to turn to the right, as many people (particularly English-speakers) conceptualize larger numbers as lying to the right of smaller numbers (Dehaene, Bossini, & Giraux, 1993). This scenario, while hypothetical, highlights one of the many ways that numbers and space are closely coupled. From simple measurement tools to complex geometric proofs, there is no question that our conceptualizations of numerical magnitude are profoundly spatial (Dehaene, 1997). Although the relationship between numbers and space has been studied for years, many questions about the origin and cognitive utility of spatial-numerical associations (SNAs) remain unanswered. In this dissertation, I will address some of these yet unanswered questions surrounding the nature of SNAs and how they may impact educationally-relevant outcomes.

Spatial-Numerical Associations

In an early psychological study of SNAs, Dehaene and colleagues (1993) found that when participants were asked to indicate whether a number they saw was even or odd, they were consistently faster to respond to the larger numbers with their right hand and smaller numbers with their left. This phenomenon, known as the "Spatial-Numerical Association of Response Codes" (SNARC) effect, refers to the culturally specified association of numerical magnitude with sides of space (Dehaene et al., 1993; Fias, Brysbaert, Geypens, & D'Ydewalle, 1996). Importantly, this effect is independent of handedness or sensory modality, and has been reliably produced in many group-level studies since its discovery (for reviews, see Hubbard, Piazza, Pinel, & Dehaene, 2005; Wood, Willmes, Nuerk, & Fischer, 2008). This effect is one of the most oft-cited and well-studied manifestations of spatial-numerical associations (SNAs) and is thought to reflect an internal linear continuum for numbers, or a mental number line (MNL). The mental number line also helps to explain the cognitive phenomenon known as the "distance effect," in which numbers that are farther apart (or more distant on a number line) are more readily distinguished than numbers that are closer together (Moyer & Landauer, 1967; Sekuler & Mierkiewicz, 1977).

Despite consistency in many group-level analyses of SNAs, recent studies have demonstrated that there is in fact much inter-individual variability (e.g. Cipora & Nuerk, 2013; Georges, Hoffmann, & Schiltz, 2017; Hoffmann, Mussolin, Martin, & Schiltz, 2014). That is, when participant data is analyzed individually and not averaged as part of the group, the direction and magnitude of the effect varies widely. While some people exhibit classic SNARC effects, others have no discernible spatial-numerical association and still others have a reverse effect. In light of these differences, there has been a proliferation of studies attempting to explain this variability and relate it to educationally-relevant cognitive domains. A reliable link between an individual's SNA and other cognitive skills/outcomes has the potential to inform the way numerically or spatially relevant topics are approached and will be a central focus of the studies in this dissertation.

The Importance of Fractions

Since its discovery, the SNARC effect has been tested with many types of numerical stimuli, including positive and negative integers, single and multi-digit numbers, and number words (e.g. Dehaene, Bossini, & Giraux, 1993; Fischer & Rottmann, 2005; Nuerk, Iversen, &

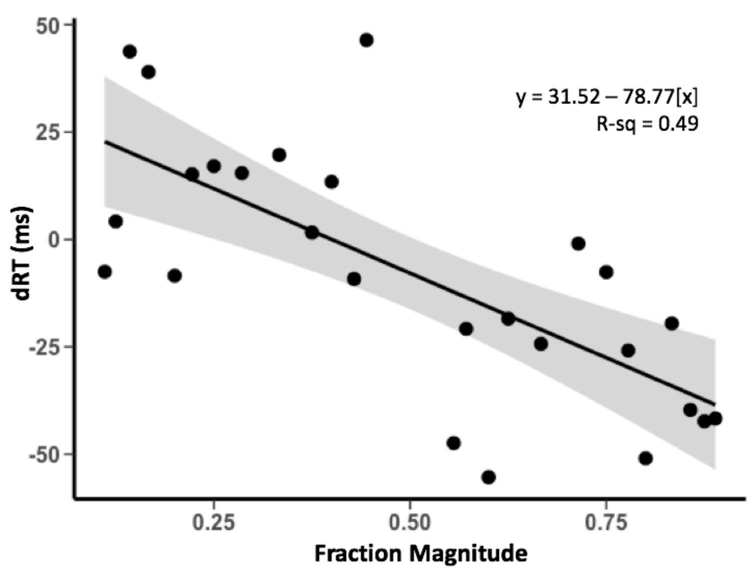
Willmes, 2004). However, relatively few have investigated the SNARC for fraction magnitudes— only two prior to this year (Bonato, Fabbri, Umiltà, & Zorzi, 2007; Liu, Xin, Lin, & Thompson, 2013). This dearth is quite glaring, particularly given the importance of fraction knowledge for a wide range of academic outcomes, including standardized math test scores (Booth & Newton, 2012). Performance on fraction tests at age 10 has been shown to uniquely predict successful understanding of higher-order concepts such as algebra in high school (Siegler et al., 2012). The role of fractions as a foundational skill even extends across cultures. In a large longitudinal study with data from the United States and United Kingdom, elementary school students' fractions knowledge uniquely predicted future math achievement in high school (Siegler et al., 2012), even after controlling for many other factors (e.g. verbal measures, family income, and education).

Having a sense for *holistic* fraction magnitudes appears to be particularly important, according to the integrated theory of numerical development (Siegler, Thompson, & Schneider, 2011). In this view, numerical development is described as a broadening understanding of magnitudes, beginning with representations of non-symbolic magnitudes, linking magnitudes to symbols, and extending to understanding of all rational magnitudes (Siegler, 2016; Siegler et al., 2011). Siegler and Pyke (2013) showed that high-achieving middle school students were more likely to rely on overall fraction magnitude when doing fraction tasks, while low achievers were more likely to focus on the numerator or denominator value, supporting the hypothesis that stronger holistic mental representations of fraction magnitudes may lead to higher levels of overall math achievement. Thus, one might expect that as we develop a better understanding of fraction magnitude, we strengthen our internal representations of their magnitudes on our MNL, and thus would exhibit an increasingly robust SNARC effect for fractions. However, this hypothesis had not—until recently—been empirically tested.

The Fractions SNARC

While two previous studies have investigated the SNARC effect for fractions (Bonato et al., 2007; Liu et al., 2013), there were critical issues that limited the interpretability of their results. Namely, the stimulus set of fractions was extremely limited, and in several of the experiments, the numerator never varied. To address these flaws and more completely consider whether fractions truly elicit a classic SNARC effect, which would indicate holistic processing of fractions magnitude, I conducted a series of three experiments that first replicated and subsequently extended the standard magnitude comparison task for fractions (Toomarian & Hubbard, 2018b).

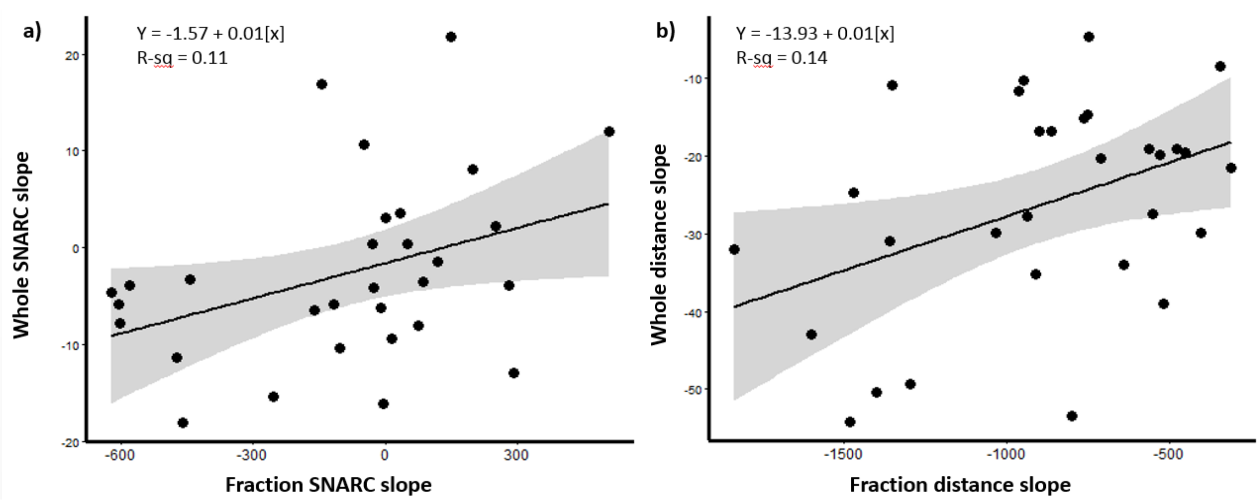
Figure 1. The SNARC Effect for Fraction Magnitudes.



Note. dRT = difference of reaction times = Right - Left hand RT

In this previous work, I demonstrated that adults do indeed exhibit a SNARC effect for fraction stimuli, particularly when the stimulus set is appropriately varied and strategic factors are minimized (Figure 1; reproduced from Toomarian & Hubbard, 2018b). When participants compared the magnitude of given fractions to the standard ½, they were overall faster to respond to larger overall fraction values on the right and smaller fractions on the left, consistent with the classic SNARC effect. This was calculated by subtracting median reaction times for left and right-hand responses (dRT = $RT_{right} - RT_{left}$) for each magnitude and comparing the resulting slope to zero. A significantly negative slope—as demonstrated in Figure 1—is taken as evidence of a classic SNARC effect. In a separate group of participants, I showed that the strength of individuals' SNARC effects for fractions was significantly correlated with their SNARC effects for whole number stimuli (Figure 2). Moreover, individual distance effects were also correlated across the two stimulus types. This would suggest that participants represent holistic fraction magnitudes on the same internal, linear continuum that they use to represent whole numbers.

Figure 3. Consistency for SNARC and Distance Effects



Note. Within-subjects comparison of fraction and whole number tasks with respect to a) SNARC effect slopes and b) distance effect slopes. Shaded regions = 95% confidence interval (reproduced from Toomarian & Hubbard, 2018b).

Dissertation Overview

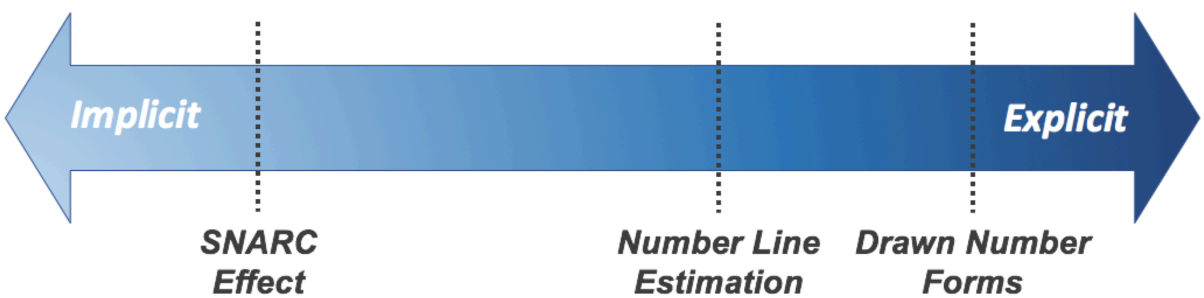
This dissertation builds on my previous work by exploring additional questions regarding SNAs more broadly, and the fractions SNARC specifically. In Chapter 2, I present a theoretical perspective on the origins of SNAs, in which I assert that both evolutionary and cultural factors play a role in co-constructing the mental number line (Toomarian & Hubbard, 2018a). In the first empirical study (Chapter 3), I take an individual differences approach to studying the fraction SNARC effect in a sample of adults. I investigate how the SNARC differs between individuals and attempt to define the relationship between the fractions SNARC and other educationally-relevant outcome measures, such as number line estimation and arithmetic. In Chapter 4, I extend this question to a sample of 3rd and 6th graders to investigate 1) do children exhibit a SNARC effect for fractions? and 2) do fraction SNAs relate to other cognitive capacities in childhood? Chapter 5 describes various hallmarks of SNAs in a sample of number-form synesthetes, to test if explicit awareness, usage, and access to internal SNAs would relate to differences in more implicit SNAs. Together, the findings from this body of work inform both basic scientific questions (e.g. what factors shape and influence spatial-numerical associations) and matters of relevance for pedagogy (e.g. are measures of SNAs useful as learning assessments or tools?).

Key Distinctions

Throughout this dissertation, several terms will recur that necessitate some initial clarification, which I will attempt to do here. One key distinction is that of implicit/explicit processing and internal/external representations. Here, I use “internal” and “external” to refer to the nature of the representations themselves. Internal representations are akin to mental representations, in that they exist strictly in the mind. External representations are physical, tangible manifestations, such as printed or drawn number lines. “Implicit” and “explicit” refer to the process or method of accessing the representation. For instance, the SNARC effect allows for implicit access to one’s (internal) mental number line, whereas asking someone to draw their conceptualization of an ordered sequence (such as numbers) is inherently more explicit.

Throughout the papers that comprise this dissertation, I will discuss many different paradigms and processes used to examine SNAs; each of these measures differs in their relative levels of "explicitness." Thus, rather than adhering to a strict dichotomy between "implicit" and "explicit" processing, I find it more appropriate to conceptualize the distinction as lying on a spectrum. As illustrated in Figure 3, various measures lie along different points on the spectrum of explicitness. The rejection of a dichotomous model of implicitness/explicitness is not a wholly unique suggestion; Karmiloff-Smith (1986) laid out a multi-phase model (primary, secondary, and tertiary) for "progressive representational explicitation" (p. 102), asserting that a dichotomous model was insufficiently rich to describe conceptual understanding. While her primary focus for this model is language, it can easily be extended to apply to more general cognitive development, or as in the case of SNAs, spatial and numerical development.

Figure 3. Spatial-Numerical Associations on a Spectrum of Explicitness



Note. Measures of spatial-numerical associations employed in this dissertation, placed along the spectrum of explicitness. SNARC= Spatial-Numerical Association of Response Codes

Such a distinction also has practical ramifications. Relative to something like a number line estimation task (in which numbers are placed on a physical number line), the SNARC effect that emerges from a magnitude comparison task is definitively more implicit; there is no external number line representation, just a putative mental number line. Now, consider the distinction between the SNARC derived from a parity judgement and that from a magnitude judgement—

are they both simply implicit relative to the number line estimation task? Because numerical magnitudes are mapped to space but parity is not, the parity task should be considered a more implicit method of accessing internal SNAs, despite the fact that both paradigms yield a SNARC effect. The conceptualization of a spectrum allows room for such distinctions, where the parity SNARC is more implicit than the magnitude SNARC, which is less implicit than number line estimation, and so on. These distinctions aid in evaluating various measures of SNAs relative to each other and allow for clearer conceptualizations of how they may affect educationally-relevant outcomes.

Chapter 2: A Theoretical Perspective on the Origins of Spatial-Numerical Associations

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Introduction

The link between numbers and space is evident in a wide range of mathematical contexts. From early finger counting and basic measurement tools, to the Cartesian coordinate system and complex geometric proofs, our conceptualization of numerical magnitude is deeply spatial (Dehaene, 1997). Despite differences in how these associations manifest, the spatial mapping of numbers is a universal cognitive strategy that has been explored in both behavioral and neuroimaging contexts. The recent increase in empirical work probing the nature of these spatial-numerical associations (SNAs) leads to several pertinent questions: Why does the human brain link numbers with space? To what extent are SNAs biologically determined vs. culturally mediated? How can broad theories of developmental cognitive neuroscience (DCN) be integrated with theories of numerical and spatial development? What factors determine individual differences in the strength of these SNAs, and what are the implications for learning about math and numbers? These questions have traditionally been addressed either from a "nature" or "nurture" approach; we argue for a more integrated view of these questions. To demonstrate the strengths of such an approach, we present a critical review of the existing

[1] This chapter appears as published (with exception of minor organizational differences) and with the permission of the coauthor.

literature on SNAs with a focus on development, to produce the first comprehensive review of the developmental cognitive neuroscience of spatial-numerical associations.

This review is critical to a principled discussion of how spatial-numerical associations may be leveraged to improve instructional approaches. To this end, we first detail the behavioral nature of these associations, with a specific focus on the extent to which SNAs are the result of a pre-specified, intrinsic mapping vs. shaped through experience. Second, we describe the cognitive and neural basis for these associations. We will then situate this body of empirical work within cognitive and neuroscientific theoretical frameworks. Finally, we explicitly address how both DCN theories and numerical and spatial development theories converge and allow for spatial-numerical associations to be influenced over the course of development. We conclude the review with a brief discussion of relevant training studies and their implications, along with identifying promising avenues for future educational neuroscience research on numbers and space.

Spatial-Numerical Associations

A wide range of behavioral paradigms have documented a mental association between numerical magnitudes and space. In a seminal study by Dehaene and colleagues (1993), participants were asked to classify a number as either odd or even, a task unrelated to the magnitude of the numbers they were processing. Participants were consistently faster to make their responses on the right for larger numbers and on the left for smaller numbers, suggesting subconscious activation of a mental representation of numerical magnitude. This Spatial Numerical Association of Response Codes (SNARC effect) has proven to be quite robust (Hubbard et al., 2005; Wood et al., 2008): the effect emerges regardless of dominant hand (Dehaene et al., 1993), when hands are crossed (Dehaene et al., 1993; but see Wood et al., 2006), with foot pedal responses (Schwarz & Müller, 2006), across modalities (Nuerk, Wood, & Willmes, 2005; Schwarz & Keus, 2004), when cued implicitly (e.g. Fischer et al., 2003), in early-blind individuals (Crollen, Dormal, Seron, Lepore, & Collignon, 2013), and even when participants are tested on the phonemic content of number words (Fias et al., 1996). The effect is also flexible and can be shaped by task constraints (Georges, Schiltz, & Hoffmann, 2014; Li et al., 2016).

While the SNARC is the most oft-cited metric of spatial-numerical associations in cognition, it is important to note that phenomena other than the classic SNARC demonstrate the widespread influence of SNAs on cognition. For example, the implicit association between numbers and sides of space can induce covert attentional shifts, with attention orienting to the left or right following small or large cues, respectively (Dodd, Van der Stigchel, Adil Leghari, Fung, & Kingstone, 2008; Fischer et al., 2003; Galfano, Rusconi, & Umiltà, 2006; Goffaux, Martin, Dormal, Goebel, & Schiltz, 2012; Nicholls, Loftus, & Gevers, 2008; Ranzini, Dehaene, Piazza, & Hubbard, 2009; Ristic & Kingstone, 2006; Salillas, El Yagoubi, & Semenza, 2008; Schuller, Hoffmann, Goffaux, & Schiltz, 2014; Schwarz & Keus, 2004). This finding has been called the attentional-SNARC (Att-SNARC). In the original experiment (Fischer et al., 2003), participants were instructed to respond to a target stimulus on either the left or right side of the screen. When these targets were preceded by an irrelevant numerical digit, participants were faster to respond to left-side targets when preceded by digits with relatively smaller numerical magnitude, and to right-side targets when preceded by larger magnitudes. While this effect has been replicated numerous times, there have also been a number of failed replications (e.g. Fattorini et al., 2015; Zanolie and Pecher, 2014). As a result of these contradictory results, the

Att-SNARC is currently the subject of a massive Registered Replication effort[2] (Colling & Holcombe, 2017).

Other paradigms that have demonstrated links between numbers and space include number-to-position/number line estimation tasks (e.g., Berteletti et al., 2010); studies of patients with hemispatial neglect (e.g. Zorzi et al., 2002; see pg. 29); number line bisection tasks (e.g. Calabria and Rossetti, 2005); attention biased in auditory space (e.g. Mitchell et al., 2012); grayscale tasks (e.g. Nicholls et al., 2008); and nonsymbolic numerosity tasks such as those used by non-human primates (e.g. Gazes et al., 2017; Rugani et al., 2015). Each paradigm offers a unique advantage or perspective from the others. For instance, the grayscale task employed by Nicholls et al. (2008) avoided the necessity for any lateralized responses, asking participants to verbally judge the relative magnitude of numbers and select the darker of two vertically-arranged grayscales. They found evidence of spatial congruency, such that smaller numbers were associated with the left and larger numbers with the right, even in the absence of lateralized responses.

These demonstrated associations between numerical magnitude and sides of space have led researchers to hypothesize the existence of a mental number line (MNL). This internal, spatially organized linear continuum extends horizontally, with larger numerical magnitudes typically located incrementally to the right on the line-- though as will be discussed in the following section, the orientation of the MNL appears to be culturally determined. Representations on the MNL are dynamic and flexible based on boundaries and task requirements (Dehaene et al., 1993). People are consistently faster and more accurate to distinguish between numbers that are more distant than those that are closer together, a

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[2] Our lab directly participated in this Registered Replication Report, contributing data from 60 participants to the multi-site and multi-cultural investigation. Results are forthcoming.

phenomenon known as the *distance effect* (Dehaene, Dupoux, & Mehler, 1990; Moyer & Landauer, 1967). This is taken as a reflection of a linear, analog mental representation of number (Restle, 1970; but see pg. 23 for a discussion of alternative accounts). Greater representational overlap between close numbers results in less accurate and slower comparisons than when comparing more distant number pairs. We note, however, that while the distance effect provides support for the mental number line hypothesis, it is not necessarily evidence of a *spatial* association with numerical magnitude. Indeed, comparison of event related potentials (ERP) during numerical comparison (Dehaene, 1996) and numerical cuing paradigms (Ranzini et al., 2009) suggest that comparison is associated with modulations of an early component (200 ms after stimulus onset) while spatial mappings are associated with later ERP components (300-500 ms after stimulus onset). Thus, numerical comparison and spatial mappings likely depend on separate stages of processing, with comparison occurring prior to spatial mappings (discussed in greater depth on pg. 31). We contend that the distance effect, despite its name, is an index of magnitude processing rather than SNAs, per se. Thus, in this review, we focus primarily on the SNARC and related effects as indices of spatial magnitude processing.

One additional issue is whether the SNARC depends on cardinal or ordinal numerical information. Cardinality refers to the total number of items in a set and is tied to concept of numerical magnitude, whereas ordinality refers to numerical sequencing and is relative in nature. This was first investigated by Gevers et al. (2003) by testing for the presence of SNARC-like effects with non-numerical ordered sequences (letters and months), in which they found evidence of spatial coding. Subsequent studies have replicated and extended these findings (Di Bono & Zorzi, 2013; Gevers, Reynvoet, & Fias, 2004) However, other studies suggest spatial coding is specific to numbers, not ordered sequences more broadly (e.g. Zorzi et al., 2006). For instance, in an investigation of the Att-SNARC paradigm with both numbers and non-numerical sequences (letters, days, months), the Att-SNARC only reliably emerged for numbers, or when order was relevant to the non-numerical stimulus (e.g. “Is the letter shown before or after ‘K’?”) (Dodd et al., 2008). Interestingly, a recent within-subjects investigation showed a dissociation between cardinal and ordinal SNARC effects, suggesting that these effects rely on separate cognitive mechanisms (Schroeder, Nuerk, & Plewnia, 2017). Thus, while the ordinality/cardinality distinction is an important one, we focus here primarily on cardinality as it relates to numerical magnitude, with additional mentions of ordinality where appropriate.

Building an SNA: Innate Prespecification or Cultural Acquisition?

Historically, SNAs have often been viewed from a “nature” or “nurture” theoretical standpoint. *Innate prespecification* theories have typically focused on findings from preverbal infants and non-human animals to demonstrate that SNAs are universal and occur even without specific cultural influences. Conversely, *cultural acquisition* theories have focused on findings that the direction of SNAs varies across cultures, and that there is a long developmental process of acquiring such SNAs, which may also involve other culturally mediated acquisitions such as finger counting. Before discussing the cognitive and neural theories that inform this debate, we first review behavioral evidence that has typically been taken as support for either a strong innate or cultural view.

Innate Prespecification

Evidence from Infants and Preliterate Children

If the SNARC effect is indeed the result of an innate mechanism that privileges the link between numbers and space, there should be evidence of it from a young age, before infants have been introduced to cultural activities, such as reading or writing. Thus, studies of humans in the period prior to formal schooling are crucial for building an understanding of the origin of SNAs (for a review, see McCrink and Opfer, 2014). Such studies reduce the influence of explicit instruction and experience with classroom number lines as possible sources or reinforcements of a defined link between numbers and space. To test for spatial-numerical preferences in infants, researchers typically employ non-symbolic representations of quantity (controlled for other psychophysical properties). One study using non-symbolic numerosity arrays and line lengths found that 8 month old American infants associate an increasing series of numerosity arrays to an increasing series of line lengths (de Hevia and Spelke, 2010; Experiment 1). The same group found that at 7 months, infants show a preference for numerical sequences of dot arrays and line lengths that increase in the rightward direction, rather than the leftward direction (de Hevia, Girelli, Addabbo, & Cassia, 2014). These studies by de Hevia and colleagues will be discussed further in the context of general magnitude processing, but provide important early evidence of SNAs in infants.

There is also recent evidence showing that infants as young as 8-9 months orient their visual attention in response to task-irrelevant numerical cues (Bulf, de Hevia, & Macchi Cassia, 2016). This study was an adapted version of the classic Att-SNARC paradigm used to test for endogenous shifts of attention in adults (M. H. Fischer et al., 2003; Posner, 1980). Similarly, Bulf et al. found that infants were faster to orient their attention to targets on the side of space congruent with the relative size of a previously presented non-symbolic numerical display (either dots or a shape). These studies with preverbal infants provide evidence for an early sensitivity to ordinality and increasing magnitudes, and support accounts of SNA development that propose an innate predisposition to map numerosities to sides of space.

However, there is a key limitation to this conclusion. Crucially, these studies were all conducted in American infants, and demonstrated a bias toward the culturally-appropriate direction. While hemispheric lateralization has been argued as one source for this preferential mapping, it is also possible that in just seven months of life, these infants picked up environmental cues leading to their preference. For example, by age 4, preliterate preschoolers already show evidence of a culturally congruent SNA (Patro and Haman, 2012; see also Tversky et al., 1991). Thus, in order to definitively state that the results point to an innately-determined preference, future studies should be conducted with preverbal infants raised in a culture with a predominantly right-to-left reading and writing direction, such as Arabic-speaking populations. If these infants also demonstrate a preference for left-to-right increasing numerosities and line-length mappings, which would be incongruent with cultural cues, this would provide more definitive evidence for an innate mapping uninfluenced by environmental cues.

Spatial-Numerical Associations in Other Species

While preverbal infants are the key population of interest for elucidating the impact of cultural experience on shaping SNAs, animal studies can address larger questions of the possible evolutionary nature of these associations. Humans are the only species with formal, culturally-transmitted reading and writing systems, which may differentially affect the nature of mental associations we form. Exploring whether other species also show evidence of a mental number line, or at least a preference for spatial orientations, is crucial for understanding how and why SNAs emerge in humans.

Evidence from animal studies of birds and monkeys has bolstered the argument that there is an evolutionary or biological basis for spatial-numeric associations that arises prior to and independent of experience. Chicks less than a week old and adult nutcrackers both demonstrate a leftward bias in an ordinal rotation task (Rugani, Kelly, Szelest, Regolin, & Vallortigara, 2010). After being trained to identify the fourth or sixth item in a vertically-oriented display, both groups were more likely to select the fourth or sixth item from the left once the display was shown in a horizontal orientation during the test condition, suggesting an inherent leftward spatial bias in birds. A later study by the same group further demonstrated that chicks associate smaller numerosities with the left and larger numerosities with the right (Rugani et al., 2015). In a series of three experiments, chicks consistently preferred the panel on the left when shown displays with fewer elements, and chose the panel on the right for more numerous displays. The authors conclude that this is the first evidence of a MNL in untrained, inexperienced animals.

Despite the tantalizing evidence that such comparative studies of SNAs/the MNL provide, it is important to consider the limitations of the claims made by the authors of these studies. Rugani et al. (2015) produce compelling evidence for a categorical mapping between numbers and sides of space. However, as Nunez and Fias (2015) argue, the results demonstrate, at best, an association but not a true mapping, since there is no evidence of a linear distance effect or a real baseline understanding of chick behavior. They argue that chicks may not even be an appropriate animal model for studying human spatial cognition due to their lack of a corpus callosum, which might magnify hemispheric differences/lateralization (for a reply to these objections, see Rugani et al., 2015; Shaki and Fischer, 2015).

In a study with rhesus macaques (Drucker & Brannon, 2014), the ordinal rotation paradigm used by Rugani et al. (2010) revealed a left-to-right oriented SNA in monkeys as well. Importantly, a number of the issues that arose with the study of chicks are not applicable in this context (e.g. there is no evidence of a side or handedness bias in monkeys, and the primate brains are less lateralized than birds' brains). Furthermore, chimps who were trained to sequence single-digit Arabic numerals when randomly spaced on a screen were later faster to respond during a test to small digits on the left and large digits on the right (Adachi, 2014). Despite the fact that ordinality was explicitly taught, the chimps received no explicit, directional training, and yet a left-to-right order was spontaneously preferred. One possible explanation for this leftward bias is the right hemisphere dominance for visuospatial tasks (Vallortigara & Rogers, 2005). These studies demonstrate that there is reason to believe that SNAs are at least partially biologically determined.

Crucially, not all studies of non-human primates have provided such strong support for an innate and consistent SNA. A recent study found spatial mappings of number in orangutans and gorillas, but also demonstrated notable inter-individual variability, as well as flexibility in spatial orientations following reversal of task instructions (Gazes et al., 2017). This pattern of results demonstrates not only that non-human primates can and do represent numerical concepts spatially, but that these representations are flexible and presumably influenced by external factors. Although these data support the view that SNAs have deep evolutionary foundations (see also Haun et al., 2011), it is clear that biological factors cannot fully account for the development of SNAs.

Cultural Acquisition

Cross-Cultural Differences

While SNAs appear quite robustly across various tasks and populations, there is significant variation in the direction of these associations across cultures (for a review, see Göbel et al., 2011). In the original set of studies describing the classic left-right SNARC in French speaking participants, Dehaene et al. (1993) also describe a null effect in a group of Iranian subjects, who read and write Farsi from right-to-left but have bidirectional numerical structures (for similar results in: Hebrew speakers, see Shaki et al., 2009; Arabic-English biliterates, see Zebian, 2005). Notably, the strength of the left-to-right SNARC increased in individuals who had spent increasing amounts of time in France, suggesting that increased exposure to left-to-right ordered activities reshapes SNAs. Different linguistic notations of numerical stimuli can even shape spatial associations within the same individual. Chinese readers exhibit a left-to-right SNA for Arabic numerals, but a top-to-bottom SNA for Chinese number words, congruent with the directionality for each notation (Hung, Hung, Tzeng, & Wu, 2008).

The prevailing explanation for these differences is the strong influence of directional cultural activities, such as reading, writing and counting direction (Nuerk et al., 2015). Indeed, this account receives support from studies such as Zebian's (2005) study of Arabic-speaking illiterates, who showed no evidence of a directional SNARC. Some also suggest that linguistic factors (e.g. Imbo et al., 2012) or finger counting can account for the mapping of SNAs. A recent cross-cultural study of parent-child interactions during a spatial task showed that parents modeled behaviors for their child in a culturally-consistent manner, highlighting parent interaction as a likely avenue for early spatial biases (McCrink, Caldera, & Shaki, 2017). In order to explicitly test these theories of spatial-numerical development, studies of SNAs prior to language development and in those with limited or varied cultural experience are necessary.

In addition to informing questions of SNA directionality, cross-cultural studies have also challenged the existence and form of these mappings. Dehaene et al. (2008) investigated numerical representations in the Munduruku, a remote, indigenous group in Brazil with little access to formal education and a restricted number vocabulary. The Munduruku appeared to employ a logarithmic mapping for both symbolic and nonsymbolic numbers, whereas Western individuals represent symbolic numbers linearly. The authors interpret this as evidence for 1) a universal tendency to link numbers and space, 2) an early intuition to represent numbers logarithmically, and 3) a shift to linear representations as a consequence of formal education (though this interpretation has since been challenged e.g., Cantlon et al., 2009; Núñez, 2011). Similar investigations with the Yupno in Papua New Guinea, who have received little-to-no formal schooling, have shown that they do indeed map numbers to space but in a categorical fashion (i.e., they place numbers on the respective endpoints in a number line task)(Núñez, Cooperrider, & Wassmann, 2012). The extent of this categorical mapping varied as a function of experience with formal school environments. These findings, along with more recent work on linear ordering in the same population (Cooperrider, Marghetis, & Núñez, 2017), have been taken as evidence that extensive cultural experience is required to regiment/reinforce a specific mapping type. In contrast to Western participants who consistently rely on linear ordering, the Yupno have limited exposure to this principle, and thus more readily employ categorical mappings (Cooperrider et al., 2017). Such investigations with remote populations are valuable for deepening our understanding of how cultural norms likely reinforce SNA strength, type, and directionality.

Development of Spatial-Numerical Associations

Since the initial discovery of behavioral SNAs in adults by Dehaene and colleagues (1993), the question of how and when these associations develop has been addressed by many researchers using a variety of methods, including behavioral and neuroimaging studies of infants, typically developing children, nonhuman primates and young animals of other species (e.g. baby chicks). Understanding the developmental trajectory and species specificity of spatial mappings of numerical magnitude will aid in elucidating their underlying mechanism.

Several studies have investigated spatial-numerical associations in literate schoolchildren using classic SNARC paradigms and number line estimation tasks but have yielded inconsistent results. The first developmental study of the SNARC used a parity task to successfully demonstrate the emergence of the effect as early as age 9, but it was notably absent at age 7-- the youngest age group in the study (Berch, Foley, Hill, & Ryan, 1999). Interestingly, these effects appeared to attenuate with increasing age, with a SNARC only emerging for "odd" responses in 11-13 year olds. It is possible that these effects may have been modulated by differing strategies for classifying the parity of presented digits, or a developing understanding of parity more generally.

Later studies attempted to disambiguate these findings with slight task manipulations and additions. For instance, van Galen & Reitsma (2008) found that the SNARC emerged at 7 years for a magnitude relevant task, but at 9 years for an attentional, non-magnitude-relevant task. A distance effect was present across all the tested age ranges, including adults. These findings have been bolstered by a more recent developmental study that found SNARC effects in 7- and 8-year-olds, but not at age 6, despite significant distance effects at all age groups (Gibson & Maurer, 2016). White, Szucs, & Soltesz (2012) also found that magnitude-relevant SNARC effects emerged a year earlier than a SNARC based on a parity judgment. These studies suggest that while even young children likely represent numbers on a mental number line, more implicit, automatic representations are strengthened with age.

Results from a study of kindergarteners by Hoffmann et al. (2013) have challenged these results by employing a color discrimination task, returning to the "magnitude-irrelevant" nature of the parity judgment task. A sample of 5-6 year olds completed both the classic magnitude-judgement SNARC task (i.e., indicated whether the number was larger or smaller than 5) and a color-judgement task (i.e., indicated whether the number was red or green), with the order of tasks counterbalanced. They found evidence of group-level SNARC effects in response to the color-discrimination task, but not the numerical magnitude comparison task. The authors suggest that the SNARC may have indeed been present in younger age groups of other studies but did not appear due to the explicit nature of the task. It is worth noting, however, that these results were subject to order effects, such that only the group who did the color task *after* the magnitude task exhibited a SNARC, inviting the possibility that priming effects may have driven these results. The absence of a SNARC for magnitude relevant tasks in young children has received additional support from Chan and Wong (2016), who showed that kindergarteners exhibit a left-right spatial bias in response to ordinal but not magnitude information.

The pattern of results for distance and SNARC effects from *all* of the studies discussed above may imply that access to numerical magnitude (as measured by the distance effect) may be dissociated from spatial representations (measured by the SNARC). These studies also imply a dynamic developmental trajectory for linking ordinality and cardinality, as well as for attending to other properties of number such as parity. Although there has been a recent effort to construct a taxonomy for SNA development (Patro, Nuerk, Cress, & Haman, 2014), it appears that ultimately, there is little consensus as to what exactly this developmental trajectory looks like, particularly in the school-age years (Georges et al., 2017; Gibson & Maurer, 2016; Schneider, Grabner, & Paetsch, 2009). Rather, these studies highlight that the development of SNAs in schoolchildren is variable and complex, and likely the result of interacting intrinsic and extrinsic factors. A stronger understanding of how children begin to structure their spatial-numerical mental associations is crucial both from a theoretical perspective and for informing pedagogical techniques.

The Role of Visuomotor Skills

A leading hypothesis for the development of SNAs is that learned skills, specifically reading and finger-counting, play a key role in determining the profile of SNAs across individuals and cultural populations. Both activities, addressed by the “common reading account” of SNA development, are modeled from a young age and are generally consistent over time and by geographical region (Nuerk et al., 2015). Though children do not formally learn to read for the first few years of life, subtle cues from their surroundings (e.g. watching an adult scan the page of a magazine, or playing early counting games and activities), may lay the cognitive foundation for SNAs to later emerge. Recent evidence from cross-sectional studies of SNAs across the lifespan have lent additional support for this hypothesis (Hoffmann, Pigat, & Schiltz, 2014; Ninaus et al., 2017). SNAs are not only present across all age groups but appear to strengthen with age (Wood et al., 2008), suggesting that prolonged exposure to environmental cues may reinforce these associations.

Finger-counting habits were first linked to the profile of the SNARC in a study by Fischer (2008), in which participants were classified as either “left-starters” or “right-starters” depending on the hand they chose to start with when counting to ten. Somewhat surprisingly, a majority of Scottish English speakers started counting on the left, regardless of hand dominance, which would be congruent with the direction of their SNARC and reading direction. Additionally, left-starters exhibited stronger individual SNARC effects, providing support for the “manumerical cognition” hypothesis of SNA development (see also Fischer and Brugger, 2011) . This account even appears to hold up in cross-cultural comparisons. Lindemann et al. (2011) showed that Western subjects more likely to start counting on the thumb on their left hand and Middle-Easterners more likely to start counting with the pinky finger of their right hand, consistent with direction of the MNL discussed in other cross-cultural research (e.g. Shaki et al., 2009).

These surprising results have one critical limitation- with the exception of one study (Riello & Rusconi, 2011), the data on finger counting was collected by asking participants to *imagine* counting on their fingers, and then write the numbers next to a diagram of a pair of hands (or enter values on the computer version, as in Lindemann et al., 2011). This method of finger counting may not actually be representative of an implicit association between fingers and numerical magnitudes, but rather, just a recapitulation of an SNA formed by the MNL. That is, by asking participants to first imagine how they would count on their fingers rather than simply asking them to count out loud and recording their responses, it is impossible to disentangle the contribution of their internal SNA from their true, body-based finger counting habit. In support of this possibility, Brozzoli et al. (2008) found that when spatial and finger representations were placed in competition in a numerical touch perception paradigm, spatial representations dominated. Specifically, when participants' right hands were face down, they responded faster to a tactile stimulus on the pinky/little finger after presentation of a large number relative to presentation of a small number, a pattern that was reversed when participants' palms faced upward. These results reflect dominance of an extrapersonal spatial mapping over a body-based mapping, highlighting the possible confound present in current finger-counting studies.

As it is likely that finger counting is just one of the many contributors to the formation and direction of SNAs, it is helpful to understand the degree to which finger counting influences SNAs in individuals. Fabbri & Guarini (2016) found that finger counting habits impacted performance on an implicit numerical task (digit-string bisection) but not an explicit numerical task (number-to-position). Furthermore, several studies have shown the SNARC to be malleable and dependent on task demands and characteristics (Bächtold, Baumüller, & Brugger, 1998; M. H. Fischer, Mills, & Shaki, 2010; Hung et al., 2008; Pfister, Schroeder, & Kunde, 2013; Shaki & Fischer, 2008). One study demonstrated that training for approximately fifteen minutes on a certain direction of finger counting, either congruent or incongruent with reading direction, impacted the presence and direction of the resulting SNARC effect (Pitt & Casasanto, 2014). Specifically, training American participants to count on their fingers from right-to-left reversed the SNARC in a significant number of participants, thereby extinguishing the overall group-level SNARC. This malleability supports the view that SNARC effects are not wholly intrinsic and specified early in development, but rather that experience molds SNAs into forms befitting of the local environment and task demands.

The Working Memory Account

Contrary to the “common reading” and “manumerical cognition” theories of SNA development is the working memory (WM) account, which proposes that links between numbers and space emerge as a result of temporary position coding in working memory, rather than as a result of more stable, long-term associations (Abrahamse, Van Dijck, & Fias, 2016; Fias & Dijck, 2016; van Dijck & Fias, 2011). Specifically, items in an ordered sequence are hypothesized to be indexed in WM during task execution; this positional coding then results in SNARC effects that reflect specific task constraints. There is now substantial evidence in support of this view (Aiello, Merola, & Doricchi, 2013; Fias, van Dijck, & Gevers, 2011; Herrera, Macizo, & Semenza, 2008; Rotondaro, Merola, Aiello, Pinto, & Doricchi, 2015; Santens & Gevers, 2008; van Dijck & Fias, 2011; van Dijck, Gevers, Lafosse, Doricchi, & Fias, 2011; van Dijck, Gevers, Lafosse, & Fias, 2012). For example, the SNARC effect appears to be diminished during a simultaneous visuospatial—but not phonological—WM task, while the distance effect is unaffected by dual-task WM demands (Herrera et al., 2008).

The WM account has recently been challenged by Cheung and Lourenco (2016), who employed order judgement tasks for both number and letter pairs. Pairs were shown in either "ascending" (earlier-to-later in the ordinal sequence) or "descending" (later-to-earlier in the ordinal sequence) conditions. Their finding that ascending numbers—but not letters—were associated with the right side of space and descending with the left is consistent with the MNL model and not WM model. Several other recent accounts have demonstrated that the ordinal position effect and SNARC effect are not mutually exclusive, thus supporting both models (Ginsburg & Gevers, 2015; Huber, Klein, Moeller, & Willmes, 2016). Taken together, these accounts are in line with our view that multiple influences contribute to the pattern and profile of SNARC effects.

The Verbal Coding Account

In yet another account of number-space links, some argue that SNARC effects are the result of verbal tags or coding. One such case is polarity correspondence (Proctor & Cho, 2006), in which left is associated with small/odd (negative poles) and right associated with large/even (positive poles). While this account has received some support (Landy, Jones, & Hummel, 2008; Proctor & Xiong, 2015; Santens & Gevers, 2008), the polarity correspondence principle has generally been ruled out as the underlying mechanism for the SNARC (Bonato, Zorzi, & Umilta, 2012; Di Rosa et al., 2017; Dollman & Levine, 2016; Leth-Steensen & Citta, 2016; Santiago & Lakens, 2015; Shaki, Petrusic, & Leth-Steensen, 2012), mostly due to evidence from modulators of the SNARC effect such as reading habits or cross-cultural factors.

This associative account is related to the measurable phenomenon of the MARC effect (Markedness Association of Response Codes), in which there is a *linguistic* association between numerical words and “odd” or “even,” resulting in a parity by response side interaction (Berch et al., 1999; Nuerk et al., 2004). Berch et al. (1999) found evidence of a MARC effect for children in Grades 6 and 8, but not for younger children in Grades 2-4, suggesting increasing dominance of linguistic factors with increasing age. However, the trajectory and relative contributions of verbal and spatial coding of number is still under debate. Imbo et al. (2012) tested the hypothesis that children initially associate numbers with space because they are surrounded by number lines, rulers, etc. in setting such as classrooms and subsequently develop verbal recoding of magnitudes (e.g. associating small numbers with the *word* “left” rather than the left side of space). However, their results revealed that verbal associations were in fact stronger than spatial associations in developing children (9- and 11-year olds), lending support to the claim that early verbal associations subsequently influence spatial coding (see also Gevers et al., 2010). Ultimately, verbal and visuospatial cues may both act as additional external cues to reinforce internal spatial representations of number, though the exact profile of these influences is yet unknown. Crucially, SNAs appear to emerge prior to the earliest evidence of a MARC, including in children who have yet to master even/odd or left/right distinctions (Berch et al., 1999; van Galen & Reitsma, 2008). Taken together with the evidence that verbal codes do indeed play a role in SNAs in older children, we suggest that verbal codes serve to reinforce spatial associations, rather than induce them. Indeed, the strength of the verbal code relative to spatial associations is neither surprising (given the crucial role of language in knowledge construction) nor damaging to other accounts of SNA development. Rather, in our view, the verbal coding account is parsimonious with both the manumerical cognition and common reading accounts, in that they all suggest that cultural factors influence the way we conceptualize the relationship between numbers and space and play a part in a multi-faceted, complex development of SNAs--though the existence of such associations may be supported by prespecified mechanisms.

A wide array of behavioral studies has provided compelling evidence for ontogenetic specification of SNAs, but also demonstrates flexibility and modification due to cultural environment. With these complementary mechanisms in mind, we now turn to the underlying neural mechanisms for supporting evidence of a phylogenetically ancient, ontogenetically early capacity for integrating space and number.

Neural Underpinnings of Space and Number

Distributed Processing of SNAs

Spatial and numerical processing both rely on a network of frontoparietal regions in the brain. Converging evidence from patient studies, monkey homologues, and extensive neuroimaging in humans has demonstrated a representational overlap between numbers and space specifically in parietal cortex (e.g. Hubbard et al., 2009, 2005; Knops et al., 2009), with particular attention paid to the intraparietal sulcus (IPS) in the posterior parietal cortex (PPC). However, in the quest to make sense of the complex pattern of findings regarding the nature of number-space associations, it has become increasingly clear that the neural circuitry underlying SNAs is not restricted to parietal regions. Rather, as we will demonstrate in this section, there is increasing evidence of more distributed processing of SNAs across frontoparietal networks. Here, we discuss some of these recent advances in our understanding of the neural relationship between numbers and space, with a focus on how both frontal and parietal regions—particularly IPS/PPC—contribute to number-space associations.

Evidence from Neuropsychological Studies

Neuropsychological studies of patients with parietal injuries have long suggested a link between numbers and space. For example, Gerstmann's Syndrome includes a classic tetrad of symptoms including deficits in numerical and spatial skills: acalculia, left–right confusion, finger agnosia and dysgraphia (difficulties with writing)(Benton, 1992; Gerstmann, 1940). Mayer et al. (1999) identified a case of pure Gerstmann's syndrome due to a small lesion in the white matter beneath the left angular gyrus. After substantial testing of all the elements of Gerstmann's syndrome, the authors suggested that the common deficit linking the symptoms in this patient was a deficit in visuospatial manipulations, which is consistent with our hypothesis of numerical–spatial interaction in the parietal lobe. Interpretation of symptom-association data remains complicated because it could be due to the mere anatomical proximity of functionally-distinct systems. Indeed, Rusconi et al. (2009) performed high-resolution fMRI to map the cortical regions involved in Gerstmann's syndrome and found that nearby parietal regions are involved in tasks related to the Gerstmann's tetrad. Although no single cortical region was implicated for the four tasks, DTI tractography from these functional regions indicated a small white matter pathway—consistent with the location of the white matter lesion in Mayer et al. (1999)—that may be the common locus of injury in Gerstmann's.

Although previous studies have broadly implicated parietal cortex in numerical and spatial processing, the first causal evidence for the specific role of PPC in SNAs comes from studies of patients with hemispatial neglect (e.g. Zorzi et al., 2002), as well as healthy patients using the same methodology to study pseudoneglect (Umiltà, Priftis, & Zorzi, 2009). Patients with right parietal brain lesions will neglect stimuli presented to the contralateral (left side), and thus often skew to the right when asked to mark the middle of a physical line (a standard line bisection task). Zorzi et al. (2002) demonstrated that these patients showed a similar rightward bias when asked to verbally report the middle number in a given range (e.g. erroneously reporting that 14 was the midpoint between 11 and 15), despite their ability to do calculations and compare numerical magnitudes well. Repetitive transcranial magnetic stimulation (rTMS) administered to healthy participants (i.e. those without brain damage) has corroborated these findings. Specifically, rTMS to PPC led to neglect-like results on a number-line bisection task, while inhibition of occipital areas did not (Göbel, Calabria, Farnè, & Rossetti, 2006). These early results highlighted the highly spatial nature of the putative MNL as well as the striking similarity of the MNL—a mental representation—to a physical line.

However, there have been several critiques of this early account of the link between spatial neglect and number line bisection. For instance, Doricchi et al. (2005) showed that individuals with known brain damage who demonstrated lateral deviation on a number-interval bisection task had lesions in prefrontal brain regions and impairments in spatial working memory, while neglect patients without lateral number-interval deviation had no prefrontal lesions. While impairment in spatial working memory for those with neglect on the MNL can be seen as support for the WM account of SNAs, spatial WM and spatial attention are known to overlap quite significantly at both the behavioral (e.g. Awh and Jonides, 2001) and neural (Ikkai & Curtis, 2011; Silk, Bellgrove, Wrafter, Mattingley, & Cunnington, 2010) levels. A case study by van Dijck et al. (2011) bolsters the case for WM involvement in spatial-numerical representations. Their patient with left hemisphere-damage demonstrated right-side neglect on a variety of representational tasks and in physical space, but left-side neglect on tasks recruiting the MNL (number-interval bisection). In addition to finding that the patient had impaired verbal WM span, they attributed this discrepancy to difficulty with verbal sequences represented spatially (see also Aiello et al., 2013, 2012 for accounts of neglect not attributed to representational space). These findings highlight an important dissociation between perceptual and representational neglect (see also Coslett, 1997; Guariglia et al., 2013; Priftis et al., 2006; Rotondaro et al., 2015; Wansard et al., 2016) and provide initial support for distributed neural processing of SNAs.

More research on perceptual and representational neglect has the potential to further disambiguate the role of various networks subserving the MNL. To date, most studies have focused only on associations and dissociations between number line and physical line bisection, based on the apparent task similarities. However, we argue that a closer examination of the task demands suggest that deficits in physical line bisection would be observed with perceptual neglect, whereas deficits in number line bisection should be observed only in cases of representational neglect. In the majority of cases where perceptual and representational neglect co-occur, we would predict an association between physical and mental number line bisection; in cases where they dissociate, we would we predict similar dissociations between physical line bisection and mental number line bisection. To our knowledge, no study has systematically investigated these four phenomena in the same patients to establish whether and how perceptual and representational neglect relate to mental number line bisection.

Neural Overlap in Parietal Regions

In a classic, early investigation of the neural locus of SNAs, Fias et al. (2001) hypothesized that common processing mechanisms in parietal cortex create a “neural overlap” for spatial orientation and numerical magnitude. To test this, they disambiguated the importance of various features of numerical comparison, specifically number orientation, color, and shape, by measuring differences in response time based on responses to these features, much like a parity task (see also Lammertyn et al., 2002). For example, in one color variant, participants responded to whether the number was presented in red or green; in the orientation variant, responses were based on seeing either a horizontal or vertical line superimposed on the number. Even though numerical magnitude was irrelevant to task performance, a SNARC effect only emerged for orientation judgments (processed by the dorsal visual stream, like number), but not for color or shape (processed in the ventral stream) (but see Hoffmann et al., 2013 for evidence of a color-discrimination SNARC in children). Mitchell et al. (2012) extended these results cross-modally to auditory stimuli, demonstrating a SNARC for orientation but not color judgements of an onscreen stimulus following auditory number words.

Despite this empirical support for the neural overlap account, this line of reasoning has been challenged on several fronts. For example, the observation that dorsal-stream regions are involved in color processing—specifically in anterior and middle IPS—suggests that number and color processing might overlap, contrary to the neural overlap account (Claeys et al., 2004). Additionally, Hoffmann et al. (2013) successfully elicited a SNARC effect based on color judgements of numerical stimuli in a sample of young children, which would potentially be implausible by the neural overlap account. Note, however, that one explanation for this finding might be the possibility of more widespread color-processing pathways in children compared to adults, leading to greater neural overlap between number and color processing. Lastly, if the SNARC effect is related to response selection (e.g. Keus et al., 2005) rather than stimulus properties (e.g. as proposed by Mapelli et al., 2003), the neural overlap cannot hold. We argue, however, that given the functional dissociations between anterior IPS (aIPS) and posterior IPS (pIPS), with pIPS activation modulated by spatial position of numbers (e.g. Kanayet et al., 2018), the neural overlap account is not invalidated by discoveries such as color processing in parietal regions. Rather, testing this theory simply requires an investigation with higher resolution. For example, a functional magnetic resonance imaging (fMRI) study using color and orientation SNARC tasks, focused on disentangling involvement of IPS subregions, may help to further elucidate the extent of representational overlap between various features of numerical processing. Furthermore, whether the SNARC emerges as a result of semantics/stimulus properties or response selection is still an open debate, with recent evidence suggesting that the SNARC varies as a result of both factors and cannot be considered a unitary phenomenon (Basso Moro et al., 2017; see also Koten et al., 2011). Ultimately, the neural overlap account has provided a useful starting point for investigations of neural underpinnings of SNAs, but recent investigations can and should move beyond this account.

Spatial-Numerical Processing in Parietal Cortex

There have been independent literatures on both numerical (e.g. Dehaene et al., 2003; Piazza et al., 2007) and spatial processing (e.g. Silk et al., 2010) that have each separately implicated parietal regions. However, until recently, few studies have directly investigated the neural systems that support the links between numbers and space. Studies that directly investigate numerical and spatial processing simultaneously can offer more reliable evidence of shared neural representations between numbers and space. However, there is currently a dearth of neuroimaging studies that specifically investigate number-space mappings, as it has only been in the past decade that such studies have been conducted.

In one such study, Knops et al. (2009) tested the hypothesis that cortical circuits for spatial attention are related to mental arithmetic by training a multivariate classifier on directional eye movements. They found that regions of PPC and frontal eye fields (FEF) involved in saccadic eye movements were also recruited during mental arithmetic of both symbolic and non-symbolic numerosities, suggesting that mental representations of magnitudes involves shifting attention in space. Notably, despite this functional overlap, the multivariate classifier used in the study only significantly decoded arithmetic operations in PPC, not FEF. While it's possible that the smaller size of FEF relative to the posterior superior parietal lobule (PSPL) could account for this, it also suggests that calculation specifically recruits parietal rather than frontal spatial mechanisms. Indeed, a recent fMRI study (Mathieu et al., 2017) has shown that even the mere presentation of arithmetic operators leads to increased activation within spatial regions (PSPL and FEF), consistent with the more specific role suggested by Knops et al (2009).

Cutini et al. (2014) used functional near-infrared spectroscopy (fNIRS) to further establish the functional relationship between regions in PPC underlying spatial and numerical cognition. During a numerical magnitude comparison task, they found increased hemodynamic response related to the SNARC effect in bilateral IPS that was modulated by numerical distance, in addition to left angular gyrus activation. This interaction between the SNARC and numerical distance is somewhat surprising given that other behavioral studies have suggested these effects are independent (e.g. Gibson and Maurer, 2016; Herrera et al., 2008; Toomarian and Hubbard, 2017). However, as other studies have found these effects to be correlated (e.g. Viarouge et al., 2014), more research is clearly necessary to further characterize the interplay between spatial associations of number and numerical distance. Furthermore, it is possible that fNIRS might not be a sensitive enough tool to disentangle these two responses. This debate notwithstanding, these findings do implicate both the IPS and angular gyrus as components of the frontal-parietal network underlying spatial-numerical associations.

A recent study using fMRI sought to explicitly disentangle functional regions of the IPS based on numerical magnitude or spatial position (Kanayet et al., 2018). By employing a number line estimation task with distinct windows for encoding (mentally representing the presented number) and marking (making their selection for where the number would go on the line), Kanayet and colleagues were able to distinguish between the cognitive and motor processes involved in the task. During the encoding phase, they found a functional dissociation between anterior and posterior IPS, with the former associated with numerical magnitude and the latter with spatial positioning. This finding may help account for the implication of IPS/PPC in studies of both numerical and spatial processing.

However, the role of PPC in underlying the SNARC effect has recently been challenged by Di Rosa and colleagues (2017). They found that application of transcranial direct current stimulation (tDCS) to PPC during a parity judgement task did not modulate the SNARC effect, regardless of whether the stimulation was excitatory or inhibitory. An earlier neuromodulation study (Rusconi, Turatto, & Umiltà, 2007) found that rTMS applied to posterior IPS reduced the SNARC effect, but did not eliminate it. This suggests that the posterior PPC is at least partially responsible for SNARC, but that other regions likely also play a supporting role. Taken together, these results suggest some dissociation between PPC and the SNARC effect and provide support for a more distributed network of neural regions supporting spatial-numerical associations.

In line with this conceptualization of a widespread network supporting SNAs are the findings of Koten et al. (2011), who used a unique numerical landmark task and both univariate and multivariate analytical techniques to investigate the interaction of spatial and numerical representations in the brain. In the multivariate analysis, they found evidence that the IPS, frontal eye fields and supplementary motor areas work together to integrate numerical and spatial information. Interestingly, when spatially-congruent trials were compared against incongruent trials in a classic generalized linear model (GLM) analysis, no regions showed significant activation to this contrast. This is consistent with Goffaux et al. (2012), who found bilateral parietal activation based on digit magnitude (large/small) but no evidence of SNARC in IPS/IPL during a color-discrimination cuing task with irrelevant number stimuli. Taken together, these studies underscore that there is mixed evidence for PPC as the central locus of SNAs.

Role of Frontal Regions

Beyond the IPS and angular gyrus, more anterior cortical regions also play a role in spatial-numerical associations. In an ERP study, Ranzini et al. (2009) discovered involvement of frontal components in response to endogenous shifts of attention induced by numerical stimuli, in addition to a more posterior, parietal component. Other more anterior regions of the brain, specifically right inferior frontal gyrus (rIFG) and right frontal eye fields (rFEF), have also been implicated in spatial orienting of the mental number line (Rusconi, Bueti, Walsh, & Butterworth, 2011; Rusconi, Dervinis, Verbruggen, & Chambers, 2013). These anterior attention areas may be involved in orienting attention during spatial search, particularly during tasks with magnitude-relevant components. Indeed, when rTMS was applied to these two areas to disrupt functioning, the SNARC effect during magnitude comparison disappeared, whereas it was unaffected during a parity task (Rusconi et al., 2011). Notably, overall task performance was also unaffected. These results suggest that frontal areas may also play a role in conceptual space of the MNL.

Taken together, the neural evidence reviewed in this section implicates a distributed frontoparietal network that underlies spatial-numerical associations. However, the precise nature of these networks, including how they develop, is yet unknown. Additional research that specifically and directly investigates the nature of neural representations of SNAs is still sorely needed.

Theories of Spatial and Numerical Cognition

In this section, we discuss the cognitive developmental theories that contextualize the previously discussed empirical findings. While several of these theoretical frameworks include the dimensions of space, time and number, we will limit discussion of the temporal component for the sake of brevity (for a review of time-space interactions, see Bonato et al., 2012). Instead, we will focus on how each of these theories applies specifically to spatial-numerical associations and support weak intrinsic biases that arise early in development.

A Theory of Magnitude

One of the first general conceptual frameworks that accounted for commonalities between human processing of time, space and quantity is Walsh's influential "A Theory of Magnitude" (ATOM; 2003). Inspired by Critchley's (1953) classic neuropsychological investigations of the functions of the parietal lobe, Walsh integrates more recent behavioral and neuroscientific evidence to posit that *magnitude* is the shared property and underlying basis for the associations between space, time and number. For example, in the domain of space and quantity, numerical magnitude is the determinant of distance and SNARC effects, and space, time and quantity are associated with overlapping brain regions in parietal cortex (Dehaene & Brannon, 2011; Hubbard et al., 2005).

Bueti & Walsh (2009) subsequently followed up on some of the original ATOM predictions, and delved further into the phylogenetic influences of the proposed parietal magnitude system. They posit that the parietal cortex is a logical location for the evolutionarily-advanced/late ability to count discrete objects (the first step in higher-order mathematical thinking), because the system would already be equipped to handle similar "issues" related to spatial and temporal magnitude (e.g. "nearer-farther", "faster-slower", "more than-less than"). In particular, they cite the *evolutionary efficiency* of a general magnitude representation in parietal cortex, an idea reminiscent of the more general neural theory of "neuronal recycling" (Dehaene, 2005; Dehaene & Cohen, 2007), which will be further discussed later in this chapter. Such a theory is supported by evidence of topographic maps for magnitude-relevant properties in parietal cortex (Harvey, Klein, Petridou, & Dumoulin, 2013).

Furthermore, as initially predicted by ATOM, the SNARC does indeed appear to be a specific instantiation of a much broader tendency to map quantities to space. Thus, the SNARC might be considered part of a broader "Spatial-Quantity Association of Response Codes" (SQUARC) effect (Bueti & Walsh, 2009; Kirjakovski & Utsuki, 2012; Walsh, 2003). This sensitivity to non-symbolic quantity may also be understood as a sensitivity to magnitude more broadly (Leibovich, Katzin, Harel, & Henik, 2016). Bueti & Walsh used the visual system as a model for understanding magnitude processing: initially, areas coding for color, motion and form in visual cortex were thought to be discrete, but we now know that neurons simultaneously code for several properties, such as color and orientation, within the visual system (e.g. Rentzeperis et al., 2014). Similarly, the research on numerical cognition currently seeks to identify individual neurons tuned to specific numerosities, when in reality these neural populations may serve several functions related to magnitude. One finding that is consistent with this "multiplexing" model comes from Tudusciuc and Nieder (2007). They found that cells in primate PPC responded not only to a preferred numerosity, but also that the same cells responded preferentially to specific line-lengths (see also Tudusciuc and Nieder, 2009). Interestingly, some cells in primate PPC that are tuned to number are also tuned to visual motion direction (Nieder, Diester, & Tudusciuc, 2006). More sophisticated neuroimaging techniques, such as multivariate pattern analysis, may help to elucidate the subtle nature of these possible differentiations in parietal cortex.

Several accounts have posited that the parietal cortex processes magnitude representations independent of dimension (Bueti & Walsh, 2009; Holloway & Ansari, 2010; Piazza et al., 2007), including numerical and spatial magnitudes (Cohen Kadosh et al., 2005; Pinel, Piazza, Le Bihan, & Dehaene, 2004) and possibly temporal magnitudes as well (Gijssels, Bottini, Rueschemeyer, & Casasanto, 2013). Such accounts provide support for the central principles of ATOM. For instance, the right IPS is thought to play a particularly important role in format-independent numerical magnitude representations, with increasing specialization occurring with increasing age (Holloway & Ansari, 2010). Additionally, the IPS appears to respond to ordinality as well as cardinal numerical magnitude (Franklin and Jonides, 2009; but see Van Opstal et al., 2009). Although the same neural regions are activated for ordinal and cardinal number processing, machine-learning techniques can distinguish between cardinal and ordinal number processing within IPS (Zorzi, Di Bono, & Fias, 2011). Notably, a recent study demonstrated a dissociation between the neural correlates for time-space and number-space associations in the parietal lobe. Riemer et al. (2016) used transcranial magnetic stimulation (TMS) to inhibit the IPS immediately prior to several tests of response code association. They found that this impairment modulated tasks of time-space association but not number-space associations (measured by the SNARC). Further studies are needed to more fully explain and replicate these results, as they are the first to demonstrate such a strong dissociation between the MNL and the mental time line and thus challenge a central prediction made by ATOM.

Generalized Magnitude System

While ATOM describes magnitude processing in adults from a neural and psychological perspective, it does not offer a framework for how this magnitude system develops. To what extent do magnitude representations overlap, and what is their developmental trajectory? Do infants have an initially undifferentiated sensitivity to magnitudes, which then distinguishes among dimensions through experience in the world, or does experience with these dimensions lead to an extraction of magnitude as the underlying link? The answers to these questions have great potential to influence our understanding of SNAs, but studies with adults are insufficient for understanding whether the relationship between magnitude and dimensions such as quantity, time, size, space, etc. occurs in a top-down or bottom-up fashion. Rather, data from developmental samples afford the necessary perspective for such broad questions.

The two possible developmental trajectories noted above lead to several hypotheses that have been empirically tested. If humans are born with an initially undifferentiated "generalized magnitude system," (GMS) there should be evidence of transfer and interference effects across magnitude dimensions early in development (Newcombe, Levine, & Mix, 2015). Early work by Piaget (1952) opened a window into the intertwined development of number and spatial extent, with his use of a number conservation task. Children were likely to say that an array of objects with more space between them had "more" than an equally numerous, less spaced-out array. This effect persisted until middle childhood, suggesting that these dimensions required time and experience to differentiate from their shared base of understanding. More recently, in a set of two experiments with 9-month old (preverbal) infants, Lourenco and Longo (2010) demonstrated associative transfer effects across the dimensions of size, numerosity, and duration. Preverbal infants expect that larger objects will be more numerous, last longer, and generalize across related dimensions of magnitude. Similar results have been described by de Hevia and colleagues, with evidence that both 8-month old infants (de Hevia & Spelke, 2010) and even hours-old neonates (de Hevia, Izard, Coubart, Spelke, & Streri, 2014) transfer their understanding of ordered (increasing or decreasing) numerosities to ordered line lengths, and vice versa. These results demonstrate an extremely early association between numerosity and spatial extent, leaving little room for any experiential or linguistic factors to contribute to these effects. Because these effects emerge prior to language development, a verbal-spatial account of the SNARC cannot be the primary source of SNAs, though they may influence direction and strength of association later in the course of development.

While humans may initially be sensitive to both numerical and non-numerical magnitudes, these representations become more differentiated with development. For example, knowledge of both numerical and non-numerical magnitudes has been shown to predict math performance in 5-6-year olds (Lourenco & Bonny, 2016) and adults (Lourenco, Bonny, Fernandez, & Rao, 2012), yet each contributes unique variance to various measures of math knowledge. Additionally, Skagerlund and Träff (2016) found that space, time, and number all contribute to different components of math ability in 8- to 10-year old children, implying that magnitude processing underlies math achievement generally, and that these dimensions may already begin to differentiate by mid-childhood. These studies suggest that numerical and non-numerical magnitudes are only partially—rather than fully—integrated even in adulthood. In light of these and other findings, some have argued against the previously dominant view that humans have an innate "number sense," advocating instead that a developing, more general sensitivity to continuous magnitude may provide a more complete account (see Leibovich et al. (2016) and associated commentaries).

A key question arises as a result of these studies- do numbers and space have a *privileged* relationship early in development? de Hevia and Spelke (2010) argue that human infants are predisposed to link numbers and space specifically, while Lourenco and Longo (2010) assert that the associations are a result of an undifferentiated system tuned to magnitudes broadly. In our view, the presence of transfer effects for infants across dimensions of number, size, and duration certainly points toward an early, broad understanding of both numerical and non-numerical magnitude. Starr and Brannon (2015) also take this more middle-of-the-road stance, suggesting some magnitude dimensions are in place early in development (privileged), while others emerge later, after additional experiential input. In a test between number-pitch and number-space judgments, Marghetis et al. (2011) found that participants were faster to associate high pitches with larger numbers. Since they found a relationship between pitch and number that is similar to that observed for numbers and space, Marghetis et al. argue that the link between space and number cannot be privileged in the way that others suggest. Note, however, that their finding of a positive relationship between pitch and number is not inconsistent with the broad stance of a generalized magnitude system. Indeed, the authors acknowledge that their results could be explained by a sensitivity to magnitudes generally, or an even broader capacity for conceptual mapping. Since both numbers and pitches are mapped to space (e.g. Bruzzi et al., 2017; Rusconi et al., 2006), pitch may be another non-numerical magnitude that is part of the GMS. Ultimately, the degree of overlap between these magnitude mappings is still unclear, as the current findings do not allow for an interpretation that moves beyond "privileged/not privileged." Further research is needed to clearly understand any possible hierarchy among these dimensions of magnitude, or whether cultural convention establishes such a hierarchy.

Studies of atypical numerical development can further aid in understanding the role of magnitude in spatial-numerical development. Skagerlund and Träff (2014) demonstrated that children with developmental dyscalculia (DD) not only had difficulty with non-symbolic numerical approximation (reduced approximate numerical acuity), but also showed deficits in tasks related to time and space, supporting the existence of a shared mechanism for magnitude processing. Neuroimaging studies have provided some support for this hypothesis, with adjacent and overlapping magnitude representations for number, spatial extent, and duration in frontal-parietal circuits (e.g. Cohen Kadosh et al., 2008; Piazza et al., 2007). Dumontheil and Klingberg (2012) showed that extent of IPS activation during a visuospatial WM task predicted later math difficulties better than behavioral measures alone, providing support for the link between numerical and visuospatial abilities (but see Crollen and Noël, 2015). A recent study of adults with DD only partially corroborated the findings of Skagerland and Träff (2014), demonstrating impairment in numerosity and duration processing, but no impairment in length judgements (De Visscher, Noël, Pesenti, & Dormal, 2017). Additional studies with participants with math difficulties would help establish a higher level of clarity concerning intertwined processing of space, time and number, including whether any these dimensions have "privileged" status.

Integrated Theory of Numerical Development

Another useful theoretical framework for understanding the development of numerical magnitude knowledge, and particularly relevant to the development of spatial-numerical associations, is the integrated theory of numerical development (Siegler et al., 2011). In this view, numerical development is described as a broadening understanding of magnitudes, beginning with representations of non-symbolic magnitudes, linking magnitudes to symbols, and extending to understanding of all rational magnitudes (Siegler, 2016; Siegler et al., 2011). The key unifying theme throughout numerical development is a broadening understanding of numerical magnitude (Siegler, 2016), which is generally consistent with the generalized magnitude system described above. Indeed, in the updated theory, Siegler suggests that this unifying framework for numbers can and should be adapted to apply to other magnitude-relevant domains.

A key component of the integrated theory is the understanding that all numerical magnitudes can be represented on physical number lines, and thus should be mentally represented on a number line as well. In this construction and usage of a spatial, mental number line, the roles of associative learning and conceptual metaphor are critical (Núñez & Lakoff, 2005; Winter, Marghetis, & Matlock, 2015). Students must understand that the spatial representation of magnitude that they hold in their mind can be mapped onto magnitudes they experience out in the world. This concept is related to the notion of a "central numerical structure" (Case, Okamoto, Griffin, & McKeough, 1996), which also emphasizes spatial organization of a MNL, but incorporates aspects of verbal-spatial learning and cognitive flexibility as numerical understanding broadens.

This theoretical framework for numerical development has been bolstered by studies that demonstrate a predictive link between knowledge of numerical magnitude and measures of math achievement (Booth & Siegler, 2008; Fazio, Bailey, Thompson, & Siegler, 2014; Sasanguie, Göbel, Moll, Smets, & Reynvoet, 2013). For example, Booth and Siegler (2008) found that first graders' numerical magnitude representation (on a physical number line) was predictive of their arithmetic performance. Furthermore, students who looked at and/or produced visual images of numerical magnitudes on a number line before doing arithmetic problems subsequently learned those problems better. Such results reinforce the causal role of magnitude representations in simple mathematical problem solving, and dovetail with the central tenet of the generalized magnitude system. Importantly, the integrated theory does not, in either the original or expanded form, offer a prediction about whether magnitude is an innate human sensitivity or if it arises from experience with the world. Rather, it serves as "a useful unifying theme for understanding numerical development from infancy through adulthood" (Siegler, 2016, p. 353).

Taken together, ATOM, GMS, and the integrated theory of numerical development all propose that magnitude processing is a key concept that likely undergirds the association between space and number in the brain. Additionally, while these theories make similar predications regarding magnitude processing, they stem from very different academic traditions. For instance, ATOM derives from neuropsychological and neuroimaging data in adults, while GMS comes from a developmental psychology perspective. Thus, bringing together these theoretical frameworks is an important step towards a more unified understanding of magnitude development. Additionally, while a wide range of empirical studies appear to converge to support magnitude as the common construct underlying SNA, the privileged status of SNAs remains an open question. To date, these theories of numerical development have not been explicitly integrated with neural theories supporting spatial-numerical associations. In the following section, we will review relevant theories of neural development to provide this explicit integration between neural and developmental theories of SNAs.

Neural Theories of Development

A discussion of the relevant cognitive neuroscience theories of development may help to resolve the apparent debate between innately prespecified and culturally acquired views of SNAs. Indeed, a deeper understanding of the phylogenetic development of SNAs grounds several of the previously discussed theories of ontogenetic development. In this section, we primarily discuss two relevant and complementary accounts of the development of the cortical regions and functional pathways that underlie the ability to link spatial and numerical cognition. Both theories move beyond the obsolete nature vs. nurture dichotomy by allowing for *both* neural biasing and cultural modification. We also discuss how this development may lead to deep connections between space and number in mathematical thinking.

Neuronal Recycling

The ability to assign numerical magnitude to arbitrary symbols is uniquely human and is the basis for our ability to construct and comprehend complex mathematics. However, this ability is fairly recent when placed in perspective of the evolutionary timeline of the human brain. The *neuronal recycling hypothesis* aims to provide an account for this and other relatively recent, culturally-transmitted abilities such as reading (Dehaene, 2005; Dehaene & Cohen, 2007). In this view, these newer abilities co-opt evolutionarily older neural circuitry for more recent functionality, and thus are constrained by the existing structure of the brain. The designated circuits must have sufficiently close functionality to be able to support the new usage, and while the original circuits will adjust to accommodate the new skill or purpose, they will exact constraints that ultimately influence functionality (Dehaene & Cohen, 2007).

With regard to the origins of these cortical maps, neuronal recycling allows for a nuanced view of how they come into place. There is an undeniable continuity and consistency in human cortical mapping, with large-scale functional specificity preserved across cultures, such as vision primarily in occipital lobes. However, epigenetic changes based on postnatal sensory inputs refine these large-scale maps, allowing for subtle individual variation in functional specification of these regions. This theory helps to make sense of the wide array of functions that are subserved by parietal cortex, including manual grasping, shifts of attention, exact calculation, saccades, etc. For example, a cortical region in PPC devoted to shifts of spatial attention, which is a broad and important general ability, may be co-opted to shift attention along a spatial mental number line (Hubbard, Piazza, et al., 2009). In this way, cortical regions and even entire circuits may be repurposed to suit evolutionarily new and culturally specified cognitive demands. Eventually, differential experience with language, finger counting, and other cultural activities likely shape the unique neural and behavioral profile of individuals.

Interactive Specialization

Interactive specialization (IS) is a domain-general framework for the ontogenetic development of human brain functions. The IS framework accounts for shortcomings in other, more extreme theories of functional brain development by taking a more nuanced approach (Johnson, 2011). Specifically, this view supposes a bi-directional relationship between structure and function; activity-dependent interactions between cortical regions result in refined functional specificity and response properties. Furthermore, this refined theory clarifies that changes in response patterns in one region are determined by patterns of connectivity with other regions, resulting in network-wide changes in functionality, not simply one region (Johnson, 2011). This aspect of IS further exemplifies its fit as an account of SNAs, as they are the product of a frontal-parietal network that develops with age and experience. Additionally, it enhances the view that spatial cognition and numerical cognition can develop simultaneously and harmoniously, as part of a generalized magnitude system, and thereby give rise to SNAs, which occur as a result of interactive specialization.

An important consideration in the application of these theories to the development of SNAs is the role of plasticity and cognitive flexibility. Behavioral evidence has shown that SNAs not only change over an individual's course of development, but they are also flexible and easily influenced by task demands and task instructions (Georges et al., 2014; Li et al., 2016; Pfister et al., 2013). Such evidence suggests at least some functional flexibility of SNAs, consistent with the view that there is not a single, static MNL. Instead, we argue that SNAs arise from a multitude of cognitive processes and may be reflected in multiple representations (see also Basso Moro et al., 2017). We know that there is the ability to influence the SNARC effect in groups and individuals, but we do not know what the accompanying neural signature of this change would look like. Is there a sensitive period for developing SNAs? For instance, if a group without any trace of a classic SNARC effect (e.g. monolingual Hebrew-speakers) underwent training to encourage number-space mappings, would the functional specificity of frontal-parietal regions change accordingly? Would this intervention have to happen in childhood? The IS view would predict that functional specificity of these cortical regions would be less likely to change in adulthood. Carefully designed empirical studies are needed to test these hypotheses.

Co-Construction of the Mental Number Line

Integrating Developmental Cognitive Neuroscience Accounts

Both neuronal recycling and interactive specialization provide a useful structure onto which we can frame our understanding of SNAs. We posit that the evolutionarily ancient parietal structure provides an appropriate niche for high level, symbolic numerical processing, with non-symbolic/magnitude processing as its basis. This is the basis for spatial-numerical associations broadly. Then, as informed by the IS account, over the course of an individual’s development, their sensory experiences with magnitudes as well as through cultural experiences with activities such as language and gesture, influence the intra-regional functional specificity in posterior parietal regions. This sequence of events during development thus influences the unique profile of one’s SNA, and accounts for individual differences. This is how we can explain the data indicating cultural differences in the form and direction of SNAs, as well as evidence of SNAs in newborn chicks and preverbal infants. There is some intrinsic bias toward associated numbers and space, and then experience with the world and with others determines the extent and profile of the association. Indeed, Winter et al. (2015) have proposed a framework complementary to our bottom-up approach. They suggest that mental spatial mappings arise through a process of "converging cultural support," including constraints from physical embodiment and brain organization.

Additional empirical studies using developmental cognitive neuroscience methods are necessary to confirm this holistic account of the formation of SNAs. Ideally, the same human infants who participate in studies early in development, such as those by de Hevia and colleagues, would be tracked over the course of their childhood. Then, researchers could take an individual differences approach and study the factors known to contribute to variability in SNAs (e.g. exposure to other languages and cultures, early numeracy abilities, etc.). Regular functional and structural imaging of regions of interest (i.e. frontal-parietal networks) could supplement these behavioral measures, to provide a nuanced and complete account of development of SNAs. Ultimately, a better understanding of this process has the potential to be useful for informing instructional techniques and approaches to spatial and numerical activities.

An Integrated Vision of SNAs

One recurring theme throughout the investigation of the origins and developmental influences of SNAs has been whether they emerge as the result of predetermined, intrinsic factors or through interaction with extrinsic, sensorimotor experience in the world. This debate has been addressed in the context of neural functional and structural development. However, the framing of these two views as mutually exclusive or as universally true is outdated and inaccurate. The admission of an interaction between dynamic factors does not invalidate research that supports one view or the other; rather, it integrates and grounds the disparate research findings into a unified and comprehensive understanding of how space and number become intertwined in the mind.

The empirical research and theoretical approaches discussed throughout this review point to an evolutionarily ancient frontal-parietal circuit that is broadly tuned to process multiple dimensions of magnitude. These sensitivities are modeled well by theories of a generalized magnitude system as well as the integrated theory of numerical development. Neuronal recycling of frontoparietal regions provides the phylogenetic substrate for SNAs, and interactive specialization helps account for ontogenetic development. Enculturation and sensorimotor experience shape the specific profile of these SNAs, leading to the ample evidence of cross-cultural differences and cognitive flexibility of the MNL. Although the tension between biological pre-specification and experience dependency still persists, there is ultimately more room to effect change in applied domains such as pedagogy when one takes an interactive specialization approach. If spatial and numerical skills are indeed intertwined and influenced over the course of development, there is the opportunity to leverage this process for the purposes of intervention.

Implications for Learning

The developing association between numbers and space is one possible avenue through which cognitive psychology and neuroscience may be able to inform instruction. There are several reasons why this may be a worthwhile endeavor. First, while evidence of SNAs is apparent even in preverbal infants, more explicit markers such as the SNARC appear to emerge and consolidate during mid-childhood. Thus, instructional techniques employed in early to mid-childhood may differentially affect the profile of individual students' SNAs. Second, several behavioral studies have demonstrated that early numerical magnitude understanding is indeed correlated with arithmetic knowledge and is predictive of learning unfamiliar problems, even when controlling for factors such as memory and prior arithmetic knowledge (Booth and Siegler, 2008; see also Sasanguie et al., 2012). Under the assumption that SNAs arise as the result of the underlying property of magnitude, these findings highlight the importance of strong spatial understanding of magnitudes on the MNL. It is worth noting, however, that studies attempting to link individual differences in the SNARC effect to outcomes such as arithmetic performance have yielded mixed results. Some studies have failed to find a relationship between strength of SNAs in childhood and math performance (Gibson & Maurer, 2016; Schneider et al., 2009), while others have found positive evidence for such a link (Georges et al., 2017; Hoffmann et al., 2013). As detailed in a recent review by Cipora et al. (2015), links between SNAs and math skills are generally inconsistent and the directionality of this relationship (i.e. whether SNAs impact arithmetic ability or vice versa) has yet to be established. However, space-number mappings and metaphors are still powerful educational tools. Indeed, as we will discuss in this section, a number of training and remediation paradigms have demonstrated that simple, spatially-rooted, game-based interventions may improve internal representations of magnitudes on a mental number line.

The Relationship Between Spatial and Math Skills

The development of spatial and numeracy skills are unique predictors of later mathematical success and other academic outcomes (for a review, see Mix and Cheng, 2012). For example, in a review of two long-term longitudinal studies spanning from the 1960s to the present, spatial skills in high school students were predictive of whether they would go on to pursue a career in STEM fields 11 years later (Wai, Lubinski, & Benbow, 2009). In light of this predictive quality, the authors propose that spatial skills, in addition to numeracy skills, may be a useful metric for the placement of students in advanced academic programs (see Shea et al., 2001; Webb et al., 2007). Given that spatial skills and numerical abilities are predictive of later life success and also appear to be related to each other early on (Mix, Levine, Young, & Hambrick, 2016), we conjecture that their intertwined development should be uniquely predictive of mathematical proficiency. The ability to mentally represent numbers spatially, and to use those representations flexibly, should have a super-additive effect on cognitive outcomes, above and beyond just spatial or numerical abilities. Large scale, longitudinal studies investigating the development of SNAs are necessary to evaluate this hypothesis.

Training studies are necessary to establish causal links between spatial cognition and math ability. Despite the evidence that has linked spatial ability to broad STEM outcomes (Wai et al., 2009), there have been mixed results in studies employing explicit training studies. A meta-analysis of spatial training studies suggests that training does enhance spatial thinking, which improves STEM achievement, but does not provide strong evidence for a causal link (Uttal, Miller, & Newcombe, 2013). A recent study by Cornu et al. (2017) also showed evidence of such domain-specificity. Children who were trained for ten weeks on a tablet-based visuospatial intervention showed improvement only in visuospatial skills but not math abilities. However, there has been some success in improving math ability with spatial training. A ten-week, classroom-based spatial reasoning program designed in conjunction with schoolteachers resulted in improved performance on *both* spatial and mathematical outcome measures (Lowrie, Logan, & Ramful, 2017). Furthermore, Cheng & Mix (2014) showed that 6-to 8-year olds who trained on a mental rotation task for 40 minutes had improved calculation abilities as measured by a post-test, while a crossword puzzle control group did not. While these results are intriguing, there was no evidence that these effects endured, and no indication as to whether they might extend to other forms of spatial training. In another study with similar methods and an extended training period, there was no evidence of improved calculation ability following mental rotation training (Hawes, Moss, Caswell, & Poliszczuk, 2015). Notably, they employed a delayed post-test 3-6 days following the six week training, suggesting that calculation abilities may indeed improve immediately following mental rotation training, but these effects may not last. Taken together, these conflicting findings demonstrate the critical need for researchers to identify which mediating factors might explain the complex relationship between spatial training and mathematical outcomes.

Beyond spatial training studies, spatial-numerical associations may bolster the case for an individual differences approach to teaching and learning. For example, the absolute strength of preschoolers' individual SNAs led to stronger and more linear numerical representations, regardless of the direction of the SNA (Rinaldi, Gallucci, & Girelli, 2016). In another study with an individual approach, participants remembered a set of ordered stimuli better when they were presented in the direction congruent with the dominant directionality of their culture (McCrink & Shaki, 2016). However, the direction, strength, and reliance on SNAs may vary widely across a group, even within the same culture. Additionally, the relationship between individual SNAs and outcome measures of interest, such as arithmetic learning, is likely mediated by other factors (Cipora et al., 2015), such as performance on number line estimation task (Simms, Clayton, Cragg, Gilmore, & Johnson, 2016). Perhaps assessing an individual "baseline SNA" would be the optimal approach for structuring mathematics learning, or limiting interference effects from incongruent manipulatives or educational experiences.

Number Line Training

Both numerical magnitude understanding and spatial thinking are predictive of later STEM success, and training studies have shown promise for encouraging this trend. However, despite these promising attempts to train both spatial ability and underlying numerical representations, considerably fewer studies have considered the added benefit of training these skills concurrently. Several studies have investigated whether explicitly training people to link spatial and numerical constructs (e.g. number line estimation) yields improvement in either a domain-specific or domain-general fashion. These studies have almost exclusively focused on two training paradigms: number line estimation tasks and numerical board games.

Drawing from the integrated theory of numerical development, knowing that all real numbers have magnitudes that can be spatially oriented and placed on a number line is crucial for mathematical development (Siegler et al., 2011). Number line estimation tasks aim to strengthen this link between physical number line and a mental representation of numerical magnitudes. Many studies have demonstrated a link between number line estimation abilities and various mathematical competence measures (Booth & Siegler, 2006; Friso-van den Bos et al., 2015; Muldoon, Towse, Simms, Perra, & Menzies, 2013; Siegler & Opfer, 2003; Simms et al., 2016), a pattern that was recently validated by a meta-analysis of such studies in childhood (Schneider et al., 2018). Additionally, number line training has proven successful for both children with developmental dyscalculia and typical controls in improving spatial representation of numerical magnitudes and improving algebra (Kucian et al., 2011). The mechanism underlying these improvements (i.e. whether they truly strengthen underlying SNAs or are indicative of skills such as proportional reasoning) is still under debate (e.g. Link et al., 2014).

Similar to number line estimation, board games that rely heavily on gameplay components reminiscent of number lines, such as *Chutes and Ladders*, have proven successful in improving a wide range of mathematically-relevant outcomes. Playing linear board games, but not circular ones, for as little as one hour has been shown to improve low-income preschoolers' ability to compare numerical magnitudes, place numbers on a number line, and answer arithmetic problems (Ramani and Siegler, 2008; Siegler and Ramani, 2009; see also Whyte and Bull, 2008). Furthermore, kindergarteners who trained for six weeks on a linear board game improved their performance on a number line estimation task relative to the circular game, counting game, and no-game/control groups, and also improved on a calculation task (Elofsson, Gustafson, Samuelsson, & Träff, 2016). These board games that utilize a linear spatial mapping for number may be successful for several reasons. One reason is the "representational mapping hypothesis," which posits that a transparent connection between game play and internal representation of number is key for developing a strong linear MNL representation (Siegler & Ramani, 2009). Another consideration is that these links between numbers and linear space in gameplay have a long history, surviving transmission through generations and across cultures. Similar to other cultural practices that promote positive cognitive outcomes, such as nursery rhymes and folk tales, their enduring appeal may be due not only to their entertainment value but also to their inherent cognitive effects. This idea of the "wisdom of culture" (Siegler and Ramani, 2009, pg. 556) as a process that preserves and transmits cultural inventions that are well adapted to our cognitive and neural architectures may help to account for the widespread nature and usage of SNAs, and provide motivation for further studying their educational implications. Indeed, this idea is complementary to Dehaene's notion of cultural co-evolution in his neuronal recycling theory, which posits that those representations best adapted to the evolutionary constraints of our neural systems are those that are most likely to be acquired, and in turn passed along. To this end, future studies should explicitly test the extent to which effects as a result of number line training endure, and if not, what changes might be necessary to ensure that they do.

On the basis of these collective findings, some researchers have suggested that more sophisticated technology—beyond single-player, computerized number line paradigms— might be a particularly promising avenue for training the mental number line (for a review, see Moeller et al., 2015). For instance, researchers might consider using paradigms with multi-player functionality (K. Moeller et al., 2015) and/or adaptive trainings that calibrate based on individual performance (e.g. Käser et al., 2013). Motion-sensitive devices that monitor participant body movements and gestures, in addition to others grounded in the embodied cognition view of SNA development (e.g. Fischer et al., 2016), may also prove beneficial for MNL training. A full-body approach to number line training has already proven effective at improving number line estimation and standardized math scores (U. Fischer, Moeller, Bientzle, Cress, & Nuerk, 2011).

Conclusion

The predictive relationship between spatial-numerical cognition and later STEM proficiency highlights the role of SNA development as a relevant issue for learning. This review incorporates research from across several fields and many methodologies to provide a broad picture of spatial-numerical associations. From theories regarding their origins and influences, to understanding how SNAs shape learning outcomes, there is a great deal of room for debate and exploration of this topic. We propose that space and number are deeply intertwined in the mind as a result of both neural biasing and cultural influence, and that all characterizations of SNAs should take into account the interplay of many factors. Future studies should endeavor to further disambiguate the relationship between these two dimensions of magnitude and investigate how their associations contribute to individual differences in mathematical thinking.

Chapter 3: Individual Differences in Implicit and Explicit Spatial Representations of Fractions

Introduction

Recent efforts to understand predictors of mathematical achievement have begun to focus on the contribution of spatial skills in addition to numerical abilities. This initiative has widespread educational implications, as spatial ability in early teenage years predicts the eventual likelihood of pursuing advanced study in STEM (Science, Technology, Engineering and Mathematics) topics and careers in a STEM field (Shea et al., 2001; Wai et al., 2009). The combined development of spatial and numeracy skills are unique predictors of later mathematical success and other academic outcomes, with strong cross-domain links evident from early childhood (for a review, see Mix and Cheng, 2012). For instance, spatial skills at age 5 have been shown to predict standardized math scores at age 7 (Gilligan, Flouri, & Farran, 2017; Gunderson, Ramirez, Beilock, & Levine, 2012), and a number of spatial skills (e.g. mental rotation, visuospatial working memory) predict math performance throughout childhood. One possible account for these relationships is the close behavioral, cognitive, and neural link between numbers and space (e.g. Hubbard et al., 2005; Toomarian & Hubbard, 2018a).

These findings highlight just a few of the many factors that contribute to early mathematical understanding. Multiple numerical abilities likely serve as precursors to greater mathematical ability, though some may contribute more or less than others, with many competencies being closely related. For instance, in one specific study, preschool children's approximate number sense and cardinality knowledge of number words both predicted later math achievement, and cardinality was found to mediate the relationship between approximate number and math achievement (Chu, vanMarle, & Geary, 2015). Further investigation of these factors is certainly needed, particularly as they relate to classes of numbers such as fractions, which are believed to be a critical part of a strong foundation for numerical understanding and uniquely predictive of later algebra-readiness (Booth & Newton, 2012).

In the current study, we specifically investigated the relationship between measures that link spatial and numerical processing of fractions by using several measures of implicit and explicit spatial-numerical associations (SNAs). We then aimed to determine the unique contribution of these factors to multiple measures of formal math achievement, such as tests of fractions arithmetic and algebra.

Spatial-Numerical Associations and the Link to Mathematics

Spatial and numerical cognition have been studied in conjunction since at least the 19th century (Galton, 1880), with mounting evidence that both evolutionary and cultural factors contribute to the widely-evidenced link between the two (for a review, see Toomarian and Hubbard, 2018a). The link between numbers and space is supported from a number of theoretical perspectives. The mental number line (MNL) theory suggests that people have an internal representation of a number line, along which numerical magnitudes extend horizontally in the direction congruent with their primary spoken language (e.g. left-to-right for English speakers) (Dehaene et al., 1993). This internal conceptualization links numbers and space along a linear continuum. There is also theoretical support from a developmental perspective; one of the central claims of the integrated theory of numerical development (Siegler et al., 2011) is that solid mathematical understanding requires knowing that all numbers have magnitudes that can be spatially oriented and placed on number lines. Despite the theoretical basis for a link between spatial skills and numerical cognition, it is unclear whether SNAs directly influence complex cognitive functions such as mathematical thinking.

In order to measure the implicit link between numbers and space, researchers typically employ one of several behavioral tasks, the most common being a parity or numerical judgment task with spatially-coded responses. In the magnitude judgement task, participants indicate whether a number is larger or smaller than a standard reference number by using either a left- or right-side response key, while in the parity task participants indicate whether the given number is even or odd. Dehaene and colleagues (1993) were the first to demonstrate that people were consistently faster to respond to relatively smaller stimuli on the left and larger stimuli on the right during parity judgement, a phenomenon termed the Spatial Numerical Association of Response Codes—or SNARC—effect. This response pattern is often taken as evidence of a MNL (Dehaene et al., 1993; Fias et al., 1996; Hubbard, Piazza, et al., 2009; but see Abrahamse et al., 2016; Nuerk et al., 2015; Proctor and Xiong, 2015 for recent discussion of alternative explanations). Furthermore, the SNARC effect is generally viewed as an implicit, quantitative measure of a person's internal conception of spatially-oriented number and may prove to be useful in illuminating the building blocks of complex mathematical thinking. The distance effect, or the finding that numbers "closer" in numerical magnitude are more difficult to discriminate than those that are "farther" (Moyer & Landauer, 1967; Restle, 1970), is also often taken as evidence of a MNL, though it should be noted that this effect is not sensitive to spatial organization or direction.

The relationship between individual SNARC effects and formal mathematical abilities has become an emerging topic of interest, yet the nature of this relationship is still not well defined. Recent studies of the SNARC have highlighted notable variability in the strength and direction of people's SNARC effects. Despite group-level effects that indicate a classic SNARC effect, about 20-40% of individuals either have no SNARC effect or one that would suggest a right-to-left SNA (Cipora & Wood, 2017, supp. material; Wood, Nuerk, & Willmes, 2006). Unfortunately, attempts to link this variability in SNAs to mathematical proficiency have yielded mostly paradoxical findings, with greater math skill related to weaker or null SNARC effects for whole numbers in adults (Cipora & Nuerk, 2013; Hoffmann, Mussolin, et al., 2014) and children (Gibson & Maurer, 2016; Schneider et al., 2009).

However, there has been some evidence that spatial ability may account for these differences. Viarouge, Hubbard and McCandliss (2014) demonstrated that individual differences in the whole number SNARC were explained by measures of spatial cognition and distance effects. Furthermore, a group of professional engineers exhibited significant SNARC effects, while expert mathematicians did not (Cipora et al., 2016; see also Hoffmann, Mussolin, Martin, & Schiltz, 2014). This suggests that other factors, such as visuospatial/mental imagery skills or perhaps more domain-general skills rather than domain-specific ones, may be closely linked to the SNARC and act as a mediating factor between MNL representations and math outcomes.

Number Line Estimation and the Link to Mathematics

While the SNARC effect reveals an implicit link between numerical magnitudes and space, experimental paradigms using physical number lines attempt to more *explicitly* probe participants' underlying spatial conceptions of number (see Chapter 1, Figure 3). Perhaps the most common such paradigm is the Number Line Estimation (NLE) task, in which participants place a given number on a physical, horizontally-oriented line that typically includes labeled endpoints (e.g. Siegler and Opfer, 2003). Performance on the task is classically measured in terms of acuity and/or the linear fit of participant responses. This paradigm is widely used in the numerical cognition literature, as it provides a concrete link between physical and mental spatial representations of numerical magnitudes.

Several studies have now demonstrated a link between number line estimation ability and math achievement (Booth & Siegler, 2006; Friso-van den Bos et al., 2015; Muldoon et al., 2013; Siegler & Opfer, 2003; Simms et al., 2016), with greater acuity on NLE tasks associated with higher math ability. These findings have been validated by a recent developmental meta-analysis of such studies (Schneider et al., 2018), which found a strong correlation between number line estimation ability and measures of mathematical competence, including counting, arithmetic, school grades, and standardized test scores. The link between number line estimation and stronger internal magnitude representations has been extended to training studies using linear gameplay elements. Studies of board games that rely heavily on gameplay components reminiscent of number lines, such as *Chutes and Ladders*, have demonstrated a positive effect on a range of mathematically-relevant outcomes (Ramani & Siegler, 2008; Siegler & Ramani, 2009; Whyte & Bull, 2008), including numerical magnitude comparison, counting ability, and more formal number line estimation tasks.

Some scholars contend that the relationship between NLE performance and math proficiency can be attributed to other, related cognitive factors, many of which are spatial in nature. For instance, Simms et al. (2016) found that visuospatial abilities mediated the relationship between linearity of NLE responses and math achievement in children aged 8-10 years. Interestingly, Gunderson et al. (2012) found that number line performance mediated the relationship between spatial skills and early calculation abilities. Taken together, these studies point to intertwined development of spatial ability and numerical estimation abilities underlying later math achievement.

The Importance of Fractions

Notably, the entirety of this new research has focused solely on SNAs (and specifically the SNARC effects) for whole numbers. This is surprising, as recent behavioral studies have repeatedly demonstrated links between basic numerical abilities and individual differences in fraction knowledge. In middle school, fraction magnitude knowledge and whole number division have been shown to predict individual differences in both fraction arithmetic and standardized math test scores (Siegler & Pyke, 2013). Furthermore, high-achieving students are more likely to rely on overall (holistic) fraction magnitude when doing fraction tasks, while low achievers are more likely to focus on the components, supporting the hypothesis that stronger holistic mental representations of fraction magnitudes leads to higher levels of overall math achievement (for similar evidence related to math learning disabilities, see Mazzocco, Myers, Lewis, Hanich, & Murphy, 2013). DeWolf, Bassok & Holyoak (2015) demonstrated that measures of *relational* fraction knowledge and placing decimals onto number lines were the best predictors of algebra performance. The predictiveness of relational fraction concepts may be supported by an underlying ratio-processing system (RPS), which is sensitive to nonsymbolic ratios such as line length comparisons (Lewis, Matthews, & Hubbard, 2015). Acuity of the RPS is also related to formal math achievement, including performance on symbolic fraction tasks and algebra achievement scores (Matthews, Lewis, & Hubbard, 2016), bolstering the claim that holistic fraction magnitude processing is key for later math learning.

As evidence emerges that fractions provide a foundation for later achievement in mathematics, researchers have also begun to investigate the developmental predictors of elementary school children's fraction knowledge. A longitudinal study by Ye et al. (2016) demonstrated the importance of number line estimation, division and multiplication with whole numbers, as well as nonsymbolic proportional reasoning, on later fraction knowledge. Additionally, Schneider et al. (2018) found that the relationship between NLE and math achievement became stronger with age, a pattern that could be attributed to fraction knowledge. Jordan and colleagues (2013) found that performance on a number line estimation task was the *largest* independent contributor to both conceptual and procedural fraction knowledge, highlighting the importance of spatial-numerical associations for fraction understanding. As a number line estimation task is essentially an explicit measure of internal representations of the number line, this finding indicates that an implicit measure of spatial-numerical associations (e.g. the fraction SNARC) might be similarly sensitive.

In line with this prediction and previous work on the SNARC effect for whole numbers, fractions have indeed elicited a group-level classic SNARC effect (Toomarian and Hubbard, 2018b). Inasmuch as whole number SNAs may be related to spatial or math-related outcomes, this inter-individual variability in the fractions SNARC may be an important signature of differences in holistic fraction processing and mathematics ability more broadly. However, the link between the fraction SNARC and individual differences in math achievement has not yet been explored. Furthermore, no studies have investigated the possibility that a more explicit number line estimation task may mediate the relationship between the implicit fractions SNARC effect and spatial/mathematical measures. While Schneider et al. (2009) found that a parity based SNARC effect for whole numbers did not predict conceptual knowledge of decimal fractions and that a decimal NLE task did, it is unclear whether these findings would hold if fractions were used to elicit a SNARC instead. An independent effect of the fractions SNARC on mathematical outcome measures would further support the critical role of spatial processing in fraction processing and proportional reasoning (Möhring, Newcombe & Frick, 2015).

The Present Study

This study aimed to investigate the link between implicit spatial representations of fractions in adults and explicit measures of numerical/mathematical knowledge by focusing on three central questions: 1) which factors predict individual differences in spatial representations of fractions? 2) to what extent is the SNARC effect distinct from other indices of numerical processing (e.g. the distance effect and number line estimation) and 3) do spatial representations of fractions, as measured by the fractions SNARC and NLE task, uniquely account for differences in math achievement in university undergraduates?

With respect to the first two research questions, our predictions were largely influenced by theoretical considerations. If people consistently rely on the mental number line when comparing numerical magnitudes, that would imply 1) that SNARC effects are distinct from other basic factors, such as IQ, and 2) associations between the distance effect, SNARC effect, and performance on a number line estimation task. As for whether the fractions SNARC and NLE performance would predict math achievement in our sample, we did not have strong *a priori* predictions due to the conflicting nature of relevant theory and past research. Theoretically, a stronger internal spatial-numerical representation (i.e. mental number line) should be associated with higher mathematical achievement. Additionally, nonsymbolic ratio comparison has been shown to predict university algebra scores (Matthews et al., 2016), and NLE performance has been associated with greater mathematical competence (Schneider et al., 2018). However, the SNARC effect with whole numbers has not been positively associated with math proficiency (e.g. Cipora et al., 2016; Hoffmann et al., 2014). In light of these inconsistent findings, we hypothesized that the slope of participants' fraction SNARC effects and NLE performance might uniquely account for variability in more domain-specific outcome measures, such as a formal test of fraction knowledge and a standardized measure of basic math skills, but would not predict algebra scores.

Methods & Measures

Participants and Procedure

One hundred and six undergraduate students were recruited for this study. However, no data was collected for one participant, as the session was disrupted shortly after the start. Thus, the final sample consisted of 105 adults, aged 18-43 (mean= 20.39 years, SD= 2.83), who participated in this study for course credit. All components of the study were approved by the Institutional Review Board (IRB#2013-1346). Computerized experiments were programmed with E-prime 2.0.8.90a (Psychology Software Tools, Sharpsburg, PA) on a Dell Optiplex 390 Desktop PC (3.1 GHz, 4 GB RAM) running Windows 7.0 64-bit operating system. Visual stimuli were presented on a Dell UltraSharp U2212H 21.5″ flat-screen monitor at a resolution of 1024 × 768 and a refresh rate of 60 Hz.

Measures

The study session lasted approximately 1.5 hours, during which time participants completed several measures, in following order:

Fraction Comparison. Participants compared all 26 single-digit, irreducible fractions to the standard fraction ½, indicating with a keyboard response if the fraction was larger or smaller than the standard. In an exact replication of Experiment 2 from Toomarian & Hubbard (2018b), each fraction appeared eight times, with response side counterbalanced across two blocks and two different run orders. A total of 10 practice trials preceded each block, which included visual feedback. A central fixation cross appeared for 600 ms, followed by a blank screen for 1000 ms and the target fraction for 3000 ms or until a response was detected. Fraction stimuli were approximately 1.8 cm wide and 2.7 cm tall (1.5° × 2.8° visual angle). Left button presses corresponded to the ‘d’ key, and right button presses corresponded to the ‘k’ key on the keyboard (distance = 8.5 cm).

Median reaction times were calculated for each fraction magnitude for each participant, resulting in either a positive or negative sloping regression line (Fias et al., 1996; Lorch & Myers, 1990). Negative slopes indicate a classic SNARC effect (small magnitudes associated with the left, large with right), and positive slopes indicate the reverse. Data from this task yielded several outcome measures: an individual SNARC effect, individual distance effect, overall reaction time (RT), and overall accuracy. It is important to note that this task is based on a direct magnitude comparison rather than the classic parity judgement primarily because fractions cannot be classified as even or odd.

Number Line Estimation (NLE). This computerized number-to-position task included both proper fractions on a 0-1 number line and improper fractions on a 0-5 number line (adapted from Torbeyns, Schneider, Xin, & Siegler, 2014). Specifically, participants estimated the position on a number line that corresponded with the fraction displayed at the top of the screen. On the basis of these estimates, we calculated the percent absolute error (PAE) score for each participant (PAE = [|answer – correct answer|/numerical range]). Thus, smaller PAE values indicate higher acuity for fractions.

Fraction Knowledge Assessment (FKA). This written assessment of fraction knowledge is comprised of items largely drawn from the TIMSS and NAEP (Matthews et al., 2016) . Items were intended to assess both procedural (e.g. “1/10 + 3/5 = __”) and conceptual (e.g. “How many fractions are possible fractions are between ¼ and ½?”) fraction knowledge. The assessment had a total possible score of 38 points; percentage correct was used as a quantitative measure of general fraction knowledge for each participant.

Wechsler Abbreviated Scale of Intelligence, Second Edition (WASI-II). This standardized assessment was used to quickly generate an estimate of IQ. Administration of two subtests—Vocabulary and Matrix Reasoning (MR)—yielded the Full Scale IQ 2 (FSIQ-2). Scores for Matrix Reasoning were also used as a measure of abstract problem solving, inductive reasoning and spatial reasoning.

Placement Exams. Participants provided consent for the study team to obtain placement test scores from university administration. All students entering the University of Wisconsin system take a required series of math and English placement tests, comprised of Basic Mathematics, Algebra, Trigonometry, English, and Reading scores. Of particular theoretical interest are the Basic Math and Algebra scores, which have strong internal consistency (Cronbach's α = .90) and have been linked to nonsymbolic ratio processing ability (Matthews et al., 2016). Scores are standardized on a scale ranging from 150-850 points.

Analyses & Results

The accuracy threshold for inclusion was 80%, but all participants who completed the session exceeded this threshold. Missing data due to various technical issues (e.g. computer error, fire alarms) resulted in several participants without data for all of the measures conducted in a session. Additionally, placement test scores were unavailable for 19 participants. Thus, the following analyses describe results from slightly different samples, dependent on which measures were available for each participant. Sample sizes for each analysis are listed in Table 1, along with descriptive statistics. Diagnostic analyses revealed two influential points (as measured by Cook's d). These outlier points reflected extreme but not implausible values, and removal of these two points did not meaningfully change the regression results. Thus, all possible data points were retained in the following models. Due to incongruous scaling of the measures, all reported beta values reflect standardized regression coefficients. Outcome measures were not standardized. There was no evidence of multicollinearity among the factors included in the model, as evidenced by variance inflation factors less than 10.

Table 1. Descriptive Statistics

Measure	n	Mean (SD)
Fraction comparison		
Reaction time (RT)	99	749.44 (137.24)
Accuracy (ACC)	99	0.96 (0.02)
SNARC slope (SNARC)	99	-75.57 (276.32)
Distance Effect slope (DIST)	99	-912.85 (373.67)
Fraction Knowledge Assessment % (FKA)	100	84.11 (10.28)
Number Line Estimation (PAE)	94	6.89 (2.75)
Algebra Exam (ALG)	86	585.00 (101.80)
Basic Math Exam (MBSC)	86	629.19 (104.87)
WASI- Full-Scale IQ (FSIQ)	102	104.33 (10.50)
Matrix Reasoning (MR)	102	49.81 (8.26)
Vocabulary (VOCAB)	102	55.36 (6.57)

Note. Descriptions of each measure include the abbreviation used in subsequent analyses. Reaction time measured in milliseconds. SNARC = spatial-numerical association of response codes

Distance and SNARC Effects

As predicted, there was a significant group-level distance effect, both when average RTs were regressed on magnitude (β= -840.11, F[1,11]=105.8, p<.001) and when individual distance effects were tested against zero in a one-sample t-test (β= -912.85, t[1,98]=-24.31, p=.007). Consistent with Toomarian and Hubbard (2018b), individual SNARC slopes were overall significantly less than zero (β= -75.57, t[1,98]=-2.72, p<.001), indicating a group-level classic SNARC effect for fractions.

Correlational Analyses

Simple bivariate correlations for all measures in the study are listed in Table 2. There was no correlation between the distance effect and SNARC effect ($r = 0.05$, $p = 0.622$). When accounting for the possible mediating role of reaction time, the correlation was still non-significant ($p=0.54$). The fractions SNARC was correlated with both acuity on the NLE task (PAE; $r=0.23$, $p=0.029$) and basic math ability (MBSC, $r=-0.26$, $p=0.018$), meaning that increasingly negative SNARC slopes were associated with lower PAE scores (greater acuity) on the fractions NLE task and better basic math scores. Lower PAE was also associated with higher scores on the fractions task (FKA; $r=-0.42$, $p<0.001$), higher accuracy on the fraction comparison task (ACC; $r=-0.33$, $p=0.001$), basic math scores ($r=-0.26$, $p=0.024$), and algebra scores (ALG; $r=-0.26$, $p=0.023$).

Table 2. Bivariate Correlations

	FKA	SNARC	FSIQ	RT	ACC	DIST	PAE	MBSC	ALG	MR
SNARC	-0.15	1								
FSIQ	0.26**	-0.09	1							
RT	-0.14	-0.01	0.06	1						
ACC	0.26**	0.01	0.11	0.20*	1					
DIST	0.09	0.05	0.03	-0.69***	-0.27**	1				
PAE	-0.42***	0.23*	0.02	0.19	-0.33**	-0.06	1			
MBSC	0.43***	-0.26*	0.36***	-0.02	0.09	-0.06	-0.26*	1		
ALG	0.33**	-0.17	0.33**	-0.18	0.15	0.09	-0.26*	0.70***	1	
MR	0.26**	-0.13	0.86***	-0.01	0.10	0.12	-0.07	0.29**	0.34**	1
Vocab	0.18	-0.01	0.76***	0.11	0.07	-0.04	0.10	0.33**	0.18	0.36***

Note. ***p < .001, **p < .01, *p < .05

Predicting the SNARC Effect

To investigate our first research question of which factors predict the SNARC effect, we used linear regression to model the following equation: $SNARC_i = \alpha + \beta_1 MR + \beta_2 Vocab + \beta_3 PAE + \beta_4 RT + \beta_5 ACC + \varepsilon$ (see Table 3). The only significant factor in the specified model was performance on the number line estimation task. When holding all other factors constant, for every standard deviation increase in PAE (i.e. decreasing acuity), the SNARC slope is expected to increase by 82.88 ($t=2.76$, $p=0.007$), resulting in an increasingly positive slope. In other words, acuity for a physical number line task—as measured by PAE—uniquely predicts the degree to which participants activate holistic fraction magnitudes on their (implicit) mental number. Indices of general intelligence, reaction times, and accuracy did not meaningfully influence the fraction SNARC. This provides some validation that the fraction SNARC effect is a valuable measurement of internal spatial-numerical associations and is distinct from other measures of task performance. However, this model predicted relatively little variance in SNARC slopes, suggesting that other factors (not measured in this investigation) have greater influence on the variability in individuals' SNARC effects.

Table 3: Regression Analysis for Variables Predicting SNARC Effect Slope

Variable	*β*	*SE*
Intercept	-74.51	28.31
WASI- MR	-15.71	32.42
WASI - Vocab	-8.85	30.43
Number Line Est. (PAE)	81.12*	31.10
RT	-10.95	29.95
ACC	52.00	30.77
R-squared	0.086	
Adjusted R-Squared	0.032	

Note. $*p < .05$. β represents standardized regression coefficients. $n = 90$

Contributions to Fraction Knowledge

Next, we aimed to test the unique contributions of SNARC slopes and PAE to procedural and conceptual fraction knowledge, as measured by the FKA. To do this, we conducted a three-step hierarchical regression analysis that introduced SNARC and then PAE to the reduced model containing other basic cognitive factors that could influence FKA scores (see Table 4). Because participants with any missing values for SNARC, PAE or FKA were excluded from analysis, 88 participants were retained for this analysis. Step 1 included only mean RT, mean accuracy, and full scale IQ, which together accounted for 14% of the variance in FKA scores (F[3,84]=5.45, p=.002). All of these factors on their own predicted FKA scores. When SNARC slopes were added in Step 2, only an additional 1% of variance in FKA scores was accounted for, and it was not significantly improved from the reduced model (F[1,83]=3.003, p=0.09). In the third step, PAE from the NLE task was added to the model, which increased the amount of explained variance in FKA scores to 23%, a significant improvement in model specification (F[1,82]=9.35, p=0.003) compared to the model in Step 2.

Notably, there was no evidence of multicollinearity among the factors included in the model, as evidenced by relatively small variance inflation factors (SNARC slope = 1.16; PAE= 1.29; RT=1.17, ACC=1.35, IQ=1.03). When all other basic cognitive factors and the SNARC are controlled for, FKA scores decrease by 0.03 points for each standard deviation increase in percent absolute error for the fractions number line task. To summarize, scores on a fraction test were significantly predicted by an explicit number line estimation task but not by an implicit measure of spatial-numerical associations for fractions, contrary to our initial hypothesis.

Table 4: Hierarchical Regression Analysis for Variables Predicting FKA Score

	Regression 1		*Regression 2*		*Regression 3*	
Predictor Variable	*β*	*SE*	*β*	*SE*	*β*	*SE*
RT	-0.02	0.01	-0.02*	0.01	-0.01	0.01
ACC	0.03**	0.01	0.03**	0.01	0.02	0.01
FSIQ	0.02*	0.01	0.02*	0.01	0.03*	0.01
SNARC			-0.02	0.01	-0.01	0.01
Number Line Est. (PAE)					-0.03**	0.01
R^2		0.13		0.15		0.23
ΔR^2				0.02		0.07**

Note. *p < .05, **p < .01, All reported R^2 are adjusted. n = 88

Contributions to Basic Math Skills

To investigate the relative contributions of implicit and explicit processing of SNAs to basic math skills, we conducted another three-step hierarchical regression analysis, with progressive introduction of the SNARC effect and then PAE score as predictors. The first model contained the same initial predictors as the previous model for FKA scores, namely RT, ACC, and FSIQ (see Table 5). Because participants with any missing values for SNARC, PAE or MBSC were excluded from analysis, 73 participants were retained for this analysis.

This first regression model explained 7% of the variance in scores for basic math skills ($F[3,69]=2.78$, $p=.05$). In this reduced sample, only FSIQ predicted scores on MBSC, meaning that when holding all other factors constant, each standard deviation increase in FSIQ is associated with a 38.19 point increase in MBSC score. The addition of SNARC slopes explained 1% more variance, though according to a partial F-test, this model was not a significant improvement ($F[1,68]=1.58$, $p=.21$). The last step—adding in PAE—resulted in a slightly better model and explained an additional 3% of variance in MBSC scores ($F[1,67]=4.13$, $p=0.05$). For each standard deviation increase in PAE (indicating reduced acuity), MBSC scores decrease by 26.69 points, controlling for changes in ACC, RT, FSIQ, and SNARC.

Table 5: Hierarchical Regression Analysis for Variables Predicting Basic Math Score

	Regression 1		*Regression 2*		*Regression 3*	
Predictor Variable	*β*	*SE*	*β*	*SE*	*β*	*SE*
RT	-5.10	11.87	-7.29	11.95	-1.98	11.97
ACC	2.01	11.74	4.96	11.92	-4.79	12.61
FSIQ	38.19**	13.62	36.35**	13.64	38.31**	13.37
SNARC			-15.52	12.35	-8.72	12.53
Number Line Est. (PAE)					-26.69*	13.14
R^2		0.07		0.08		0.11
ΔR^2				0.01		0.03*

Note. *p < .05, **p < .01, All reported R^2 are adjusted. n = 73

Contributions to Algebraic Knowledge

The last outcome measure we tested was score on a standardized algebra exam. This outcome measure was motivated by findings that college students' nonsymbolic ratio judgements significantly predicted algebra placement exam scores (Matthews et al., 2016). To test whether either the SNARC or PAE predicted algebra scores, we conducted another three-step hierarchical regression analysis to investigate the relative contributions of implicit and explicit measures of SNAs to ALG. These models followed the same structure as the previous two hierarchical regression models, with basic cognitive factors in the initial model, followed by progressive introduction SNARC and PAE score (Table 6). Due to incomplete cases, 73 participants were retained for analysis.

In the initial model, only RT was a significant predictor of algebra test scores (p=0.008), and 12% of the variance in ALG was explained by the model. When SNARC was introduced, the model actually explained less variance, when the number of factors was considered (adj-R^2=0.11). Adding PAE to the model explained an additional 1% of variance from the first model, though neither of the subsequent models were any better than the first (*1 v. 2*: F[1,68]=0.29, p=0.59; *2 v. 3*: F[1,67]=2.35, p=0.13), indicating that neither implicit not explicit measures of SNAs have predictive power over algebra test scores. In the final model, only RT and FSIQ significantly predicted ALG. Thus, while holding all other variables in the final regression constant, ALG scores decrease by 25.57 points for every standard deviation increase in RT; they increase by 24.22 points for every standard deviation increase in FSIQ.

Table 6: Hierarchical Regression Analysis for Variables Predicting Algebra Scores

	Regression 1		*Regression 2*		*Regression 3*	
Predictor Variable	*β*	*SE*	*β*	*SE*	*β*	*SE*
RT	-28.35	10.49**	-29.19	10.66**	-25.57	10.82*
ACC	17.02	10.38	18.15	10.64	11.50	11.39
FSIQ	23.59	12.04	22.88	12.17	24.22	12.08*
SNARC			-5.95	11.02	-1.32	11.32
Number Line Est. (PAE)					-18.19	11.87
R^2		0.12		0.11		0.13
ΔR^2				-0.01		0.02

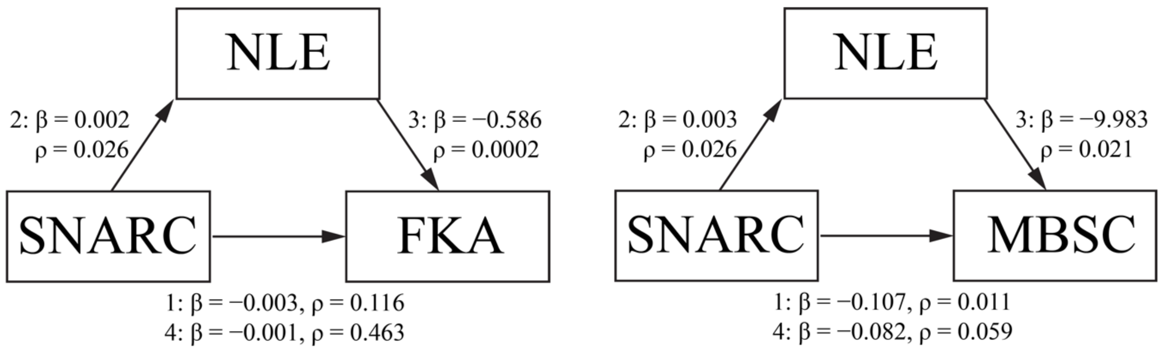
Note. *p < .05, **p < .01, All reported R^2 are adjusted. n = 73

Mediation Analyses

Despite the extensive planned analyses, it is unclear whether SNARC slopes and PAE scores contribute uniquely to our outcomes of interest, specifically FKA and MBSC scores. We employed mediated path analyses to determine whether acuity on the NLE task—as measured by PAE—mediated the relationship between the SNARC and our two outcome measures of interest. We did not have reason to believe that there was any mediation in the case of ALG scores, since neither measure was predictive of ALG scores in prior analyses. Additionally, while the independent variable predicting the dependent variable is often regarded as a necessary condition for conducting mediation analyses (Baron & Kenny, 1986), recent guidelines have supported mediation analysis without such a relationship in certain cases (Shrout & Bolger, 2002). For instance, in cases when theory would predict such a relationship and sample sizes are relatively small, mediation analysis may be conducted with bootstrapped confidence intervals. Thus, although SNARC did not predict FKA scores, we proceeded with mediated path analysis nonetheless. To test whether PAE mediates the relationship between SNARC and our two dependent measures (FKA and MBSC), we conducted path analysis with mediation using the 'lavaan' package in R (Rosseel, 2012). Variables are unstandardized. We used the full information maximum-likelihood imputation approach for missing values.

In Model A (Figure1), the only direct effect was between NLE and FKA scores; adjusting for SNARC slopes, every 1-unit increase in PAE is associated with a decrease of $b=0.568$ ($SE=0.16$, $p<0.001$) in FKA score. There was no indirect effect, and thus no evidence of full mediation $ab=-0.001$ ($SE=0.0008$, $p=0.204$). A bias-corrected bootstrapped 95% confidence interval based on 10,000 samples included zero [-0.003, 0.0001], confirming that there is no evidence of mediation in this model.

Figure 1. Mediated Path Analysis



Note. SNARC=Spatial-Numerical Association of Response Codes; FKA=Fraction Knowledge Assessment; NLE=Number Line Estimation, representing percent absolute error (PAE) values; MBSC= Basic Math

In Model B, we tested for mediation between SNARC and MBSC score. Independent of PAE, a one-unit increase in SNARC slope is associated with 0.107 decrease in MBSC score (SE=0.044, p=0.014). Every unit increase in SNARC slope is associated with an a= 0.003 (SE=0.001, p=0.028) increase in percent absolute error (PAE) on the NLE task. Adjusting for SNARC slopes, every 1-unit increase in PAE is associated with a decrease of b=9.983 (SE=4.400, p=0.023) in MBSC score. There was no indirect effect, and thus no evidence that PAE score mediated this association ab=-0.026 (SE=0.019, p=0.184). A bias-corrected bootstrapped 95% confidence interval based on 10,000 samples included zero [-0.077, 0.0002], confirming that there is no evidence of full mediation in this model. However, there was a significant total effect for the model (SE=0.044, p=0.015), indicating that the model fit the data well and is evidence that PAE may at least partially mediate the relationship between SNARC and MBSC.

Discussion

In this study, we investigated the relationship between implicit and explicit measures of SNAs, including the link to formal math abilities. First, we successfully replicated our previous work demonstrating that a classic SNARC for fraction magnitudes emerges at the group-level

(Toomarian & Hubbard, 2018b) and for the majority of adult individuals. This replication in a separate, larger sample of adults supports the assertion that people can and do represent fractions holistically under appropriate task constraints.

We then moved past group level effects to investigate a second question: which factors influence individual differences in participants' SNARC effects. Performance on a number line estimation task, which included whole numbers and fractions, was uniquely predictive of individual SNARC slopes. Importantly, this relationship emerged even while controlling for factors such as response time, overall accuracy, and two IQ subtests. That accuracy and reaction time in the comparison task were not associated with SNARC slopes indicates that the SNARC is measuring a unique, spatial ability that cannot be accounted for by basic processing speed or ability to do the task. These results are theoretically supported by the mental number line hypothesis; if the SNARC is a measure of reliance on a right-to-left spatially oriented mental number line, greater reliance on this internal number line (evidenced by more negative SNARC slopes) should be related to acuity on a similarly oriented, external number line task. However, Schneider et al. (2009) found no relationship between NLE performance and the parity SNARC in kids, thereby challenging this interpretation of the results. Instead, they argue that the internal and external number line cannot be equated, at least early in development.

Our results indicate that NLE has greater predictive power than the SNARC for multiple outcome measures, which suggests some degree of dissociation between these two measures. One explanation for this dissociation may be that the fractions SNARC, by nature of being more implicit than the NLE task, has a weaker effect and may not have much influence to exert on explicit outcome measures. This is in contrast to the NLE task, which has both theoretical (e.g. Siegler et al., 2011) and empirical (e.g. Gunderson et al., 2012; Resnick et al., 2016; Thompson and Siegler, 2010; Ye et al., 2016) support for its role in fractions learning and math proficiency. A recent study demonstrated that number line training but not area model training improved performance on an untrained fraction magnitude comparison task, highlighting the utility of an external spatial-numerical representation (Hamdan & Gunderson, 2017).

In this study, there was no evidence of a correlation between the distance effect and SNARC effect. Previous studies with whole numbers have yielded mixed evidence on the relationship between the distance and SNARC effects; Viarouge, Hubbard and McCandliss (2014) found a correlation between these measures, while Gibson and Maurer (2016) did not. Interestingly, Schneider et al. (2009) found a significant correlation in one experiment, but not in a subsequent experiment.[3] While both effects are often taken as evidence supporting the mental number line hypothesis, there is a key difference between the two effects: only the SNARC effect reflects a directional/spatialized association. With this difference in mind, it is not difficult to imagine that these effects might dissociate within subjects, particularly for stimuli such as common fractions, for which the cognitive processing mechanisms are still not well understood.

Lastly, neither the fractions SNARC nor PAE predicted algebra placement exam scores, despite PAE being a significant predictor of fraction knowledge and basic math skills. This suggests that more implicit processing of spatial-numerical representation may not be as readily recruited during higher-order mathematical concepts, but rather may serve as a foundation for thinking about simpler problems involving rational magnitudes. This would cohere well with the recent finding that the ability to place decimals, but not fractions, on number lines was one of the best predictors of algebra performance (DeWolf, Bassok & Holyoak, 2015).

[3] Beyond just significance testing, these studies also found markedly different correlation coefficients for the relationship between SNARC and distance effect: Viarouge et al. (2014): r=0.52; Schneider et al. (2009): r=0.25 (Exp. 1) & r=-0.03 (Exp. 2); Gibson & Maurer (2016): r=-0.06; the current study: r=0.05

Limitations

Here we would like to note several aspects of the current research that may limit the interpretability of the results. First, as previously mentioned, the sample size was moderately reduced for each analysis due to missing data points across various measures. This issue was perhaps most significant for the hierarchical regressions with MBSC and ALG as the dependent variables, since the placement tests were the variables for which there were the most missing data points. While this reduction affected the degrees of freedom, decreased the adjusted R-squared, and increased the possible influence of outliers, it is important to note that the total *n* never dipped below the number required for a medium effect size and there were no marginal effects.

Additionally, recent simulation work on detecting reliable SNARC effects with various sample sizes, stimulus repetitions, and effects has provided guidelines for obtaining results of moderate effect (Cipora & Wood, 2017). Specifically, studies are recommended to test a minimum of twenty participants and with twenty repetitions per stimulus. While our sample size exceeds this minimum requirement, there are only eight repetitions per stimulus in the task from which we draw our individual SNARC slopes. That said, our stimulus set contains four times the number of individual numerical stimuli as classic SNARC paradigms (24 vs. 8), thus offsetting the reduction in the number of trials per stimulus. Thus, the overall experiment time would be unreasonably long if we were to collect twenty observations per stimulus per condition and would thus compromise the integrity of the data. Furthermore, because this recommendation stems from the desire to control for intra-individual variability, we argue that our wide range of fraction magnitudes in fact serves a similar purpose; by increasing the number of points on the mental number line to which participants are asked to respond, we are effectively controlling for this variability in an analogous fashion.

Conclusion

In this study, we investigated how individual spatial representations of fractions relate to explicit fraction knowledge and two other formal measures of math achievement. We observed significant group-level SNARC and distance effects based on overall fraction magnitude, with notable individual variability. Performance for the number line estimation task was correlated with SNARC slopes and predicted significant variance in SNARC slopes even when accounting for factors such as overall accuracy and matrix reasoning ability. Multi-step regressions revealed that NLE performance was a significant predictor of fraction test scores and basic math skills but the SNARC was not, indicating that working with an explicit number line may be a stronger predictor of domain-specific and domain-general math abilities than more implicit number line processing of fractions. Neither individual SNARC effects nor NLE performance were significant predictors of algebra scores. This suggests that the mental number line may not be as readily recruited during higher-order mathematical concepts, but rather may be a foundation for thinking about simpler problems involving rational magnitudes.

The current study informs our understanding of the relative contributions of more implicit (SNARC) and explicit (NLE) processing of fractions, but it is still unknown whether these relations are consistent from childhood to adulthood. Developmental studies—particularly with continuous age data—are necessary to better understand how spatial and numerical conceptions influence mathematical thinking. Future studies should investigate this relationship with 1) a larger, more educationally-diverse sample, and 2) additional spatial tasks as covariates.

Chapter 4: The Development of the Fractions SNARC

Introduction

Since the initial discovery of behavioral SNAs in adults by Dehaene et al. (1993), researchers have been interested in understanding how and when various types of spatial-numerical associations develop. Understanding how this cognitive association matures can not only inform our basic understanding of numerical and spatial cognition, but may also have important educational implications. For instance, number line training—which builds on spatial-numerical links—may be a useful tool for improving mental magnitude representations and algebraic reasoning in both typically developing children and those with math difficulties (Kucian et al., 2011; K. Moeller et al., 2015). Despite the interest and potential for translational insights, there is little clarity regarding the developmental trajectory of SNAs in humans. Investigations on the topic suggest that more explicit, magnitude-relevant tasks evoke stronger SNAs than more implicit paradigms (such as parity judgement), but the age at which these effects emerge are still unclear. Additionally, despite the key theoretical and empirical importance of fractions (Siegler et al., 2012, 2011), a developmental study of implicit SNAs (such as the SNARC effect) for fractions has been conspicuously absent from the literature. In this study, we address this gap by investigating the development of implicit and explicit SNAs for fractions in a sample of 3rd and 6th grade students.

The Development of SNAs

Several studies have investigated the nature of SNAs for whole numbers in schoolchildren using classic SNARC paradigms and number line estimation tasks, but findings from these studies have been largely inconsistent. The first developmental study of the SNARC successfully demonstrated emergence of the SNARC effect in a parity task for children as young as age 9, but found that it was absent at age 7 (Berch et al., 1999). Yang et al. (2014) used the parity task in a cross-sectional study of Chinese children from kindergarten to 6th grade and found evidence of a SNARC effect in nearly all age groups. To better understand which factors would influence the SNARC, other studies have employed slight task manipulations and additions. For instance, van Galen & Reitsma (2008) found that the SNARC emerged at 7 years for a magnitude relevant task, but not until 9 years for a magnitude-irrelevant task. Distance effects were present across all the tested age ranges, including adults. These findings have been bolstered by more recent developmental study. Gibson and Maurer (2016) found evidence of SNARC effects in 7- and 8-year-olds, but not at age 6, despite distance effects being present at all age groups. White, Szucs, & Soltesz (2012) also found that magnitude-relevant SNARC effects emerged a year earlier (~age 6) than a SNARC based on a parity judgment (~age 7). Taken together, these studies indicate that tasks in which magnitude is directly relevant may elicit a SNARC effect earlier than tasks in which magnitude is not central, but the exact age at which this occurs is yet unknown.

This interpretation of the development of the SNARC has been challenged by studies showing the opposite pattern of results. A study of kindergarteners by Hoffmann et al. (2013) used a color discrimination (magnitude-irrelevant) task in a sample of 5 and 6-year-old participants. They found evidence of group-level SNARC effects for the color-discrimination task, where magnitude was not central to the task, but not for the classic numerical magnitude comparison task. They argue that it may be possible that the SNARC was indeed present in the younger age groups of other studies, but they did not appear due to the explicit nature of the task. This sees counterintuitive given the multiple studies that have shown that a magnitude-relevant SNARC appears prior to more implicit tests. Critically, only the group who did the color task *after* the magnitude task exhibited a SNARC, inviting the possibility that priming effects may have influenced this pattern of results. In any case, the absence of a SNARC for magnitude relevant tasks in young children has received additional, recent support from Chan and Wong (2016), who showed that kindergarteners exhibited a left-right spatial bias in response to ordinal but not magnitude information.

Relating Individual Differences to Math Achievement

Beyond simply testing for the presence of a SNARC effect in school-aged children, several recent studies have investigated how the extent of spatial-numerical associations in individuals may relate to mathematical performance. Unfortunately, this approach has not yielded much clarity as to the role of SNAs in cognitive and numerical development. In the first such study, Schneider et al. (2009) found no relationship between the parity SNARC in $5^{th}/6^{th}$ grade students and scores on a math test. Math scores were, however, significantly predicted by performance on a number line estimation task. Similarly, Gibson and Maurer (2016) found no relationship between the SNARC and Test of Early Mathematics Ability (TEMA) in 6-8 year old children. Contrary to these results, Hoffmann et al. (2013) found that basic number knowledge in kindergartners was related to the strength of SNARC in a magnitude-based but not color-judgement task (which perhaps accounts for the lack of a group-level SNARC on the magnitude-based task). The same group found that stronger SNARC effects in 3^{rd} and 4^{th} graders were related to stronger arithmetic abilities, but not visuospatial skills (Georges et al., 2017). Taken together, these conflicting results highlight the need for additional research into the interplay between developing mental number-space mappings and various mathematical outcome measures. Additionally, studies are needed that employ multiple tasks designed to measure SNAs, as convergence of those measures would strengthen any resulting developmental account.

Developing SNAs for Fractions

These studies of the SNARC in children have highlighted that the development of this association is variable and complex, and likely the result of interacting intrinsic and extrinsic factors. While the developmental trajectories of implicit and explicit SNAs for whole numbers are still somewhat opaque, it is also unclear whether the findings from developmental studies of whole numbers generalize to other classes of numbers, such as fractions. Fractions are of particular interest given their predictive power for higher level math, such as algebra (e.g. Booth and Newton, 2012), and other math achievement. Furthermore, whereas it was previously thought that only a reverse SNARC emerged for fraction comparisons in adults (Bonato et al., 2007), our recent work has demonstrated that a classic SNARC does indeed emerge for fractions when the stimuli and paradigm are designed appropriately and include a wide range of fraction magnitudes (Toomarian and Hubbard, 2018b; Chapter 3). Given the importance of fractions for later mathematical success, it is striking that, to date, there have been no developmental studies of the fractions SNARC.

The Current Study

In this study, we aimed to shed new light on the development of SNAs by investigating whether children implicitly represent fraction magnitudes on an MNL, including the extent to which that is related to other cognitive processes and academic outcomes. The specific research questions of this study were: 1) do children in grades 3 and 6 (~9 and 12 years old, respectively) have a group-level SNARC when comparing fraction magnitudes? and 2) do individual differences in the SNARC/NLE relate to differences in outcome measures, specifically scores on a pencil-and-paper fraction knowledge assessment (FKA)?

Due to previous research that has demonstrated emergence of the SNARC later in childhood, we predicted that 6th graders would be more likely to have a fractions SNARC effect than 3rd graders, particularly given their more extensive experience with symbolic fraction magnitudes. This familiarity with symbolic fractions implies more automatic activation of holistic fraction magnitudes, thereby resulting in an advantage for 6^{th} graders over 3^{rd} graders in demonstrating a SNARC. Based on the data presented in Chapter 3, I also predicted that SNARC, NLE and FKA scores would all be correlated, with NLE possibly mediating the relationship between SNARC and FKA. As the first investigation of the SNARC effect for fractions in a developmental sample, this study will add to the literature on both the development of SNAs and the development of fraction magnitude understanding.

Methods

Participants

This study was conducted as part of a large, ongoing longitudinal study on the development of fractions knowledge. Participants in this study were 3^{rd} and 6^{th} grade students (in the 2017-2018 school year), the majority of whom had also taken part in this research the prior school year (2016-2017), as 2^{nd} and 5^{th} graders, respectively. One participant who skipped 3^{rd} grade (i.e. went from 2^{nd} to 4^{th} grade) was still coded as a 3^{rd} grader for the purposes of this study. An *a priori* power analysis for group-level effects determined that a sample of 50 children for each grade level would be required for a medium effect size. Thus, in order to account for potential exclusions, the recruitment goal for the study was either 80 participants in each age group or as many as possible prior to August 1, 2018. As of that cut-off date, 135 children participated in the study (71 3^{rd} graders, 64 6^{th} graders). However, data from nineteen children were not included in the final data set, either due to technical issues (n=5), experimenter error (n=12), participant noncompliance (e.g. repeatedly removing hands from response keys) (n=7), or a diagnosed learning difficulty (n=1). Thus, the final sample contained 110 total participants (58 3rd graders and 52 6th graders). All participants in this sample had normal or corrected vision, had no diagnosis of any learning difficulties, and were fluent English speakers. Participants were compensated for their time with cash ($15) and a small toy. All procedures were approved by the UW-Madison Health Sciences Institutional Review Board (#2016-0665).

Procedure

Data was collected over the course of a 90-minute testing session; participants were given the option to take a short break in the middle of the session, with breaks between tasks as needed. All sessions occurred in a quiet testing environment. Sessions were comprised of multiple written and computer-based tasks, including fraction magnitude comparison, Woodcock-Johnson-III Spatial Relations and Math Fluency subtests, number line estimation tasks, and paper-and-pencil fractions test. These measures—described in greater detail below—were collected along with additional measures that were relevant for the overall longitudinal study, but not relevant to the current investigation and thus will not be detailed here.

Fraction Knowledge Assessment. This measure is similar to the fractions assessment administered to adults in Chapter 3 (see also Matthews et al., 2016), but the formatting and some items were adapted to be more developmentally appropriate. Thus, 3rd and 6th graders received slightly different forms of this assessment, though the assessments were similar in format and content. The total possible score was 39 points for 3rd graders and 37 points for 6th graders; all FKA scores have been converted to percentages for ease of comparison across grade levels.

Number Line Estimation. This computerized number-to-position task is a variant of the number line estimation task described in Chapter 3 (Toomarian, Meng & Hubbard, in prep), modeled from Hansen et al. (2015). Participants placed fractions on number lines ranging from 0-1, 0-2, and 0-5. Percent absolute error scores were calculated for each participant (PAE = [|answer – correct answer|/numerical range]). Smaller PAE values indicate higher acuity for fractions.

Fraction Magnitude Comparison. Participants compared 26 single-digit, irreducible fractions to the standard fraction ½, indicating with a keyboard response if the fraction was larger or smaller. All stimulus characteristics and timings exactly replicated Toomarian & Hubbard (2018b). To adapt task administration for use with children, the instructions were edited to include more child-friendly language, and two additional practice trials with distances closer to ½ were added to the beginning of each block (12 trials rather than 10).

The difference between median reaction times for left and right-hand responses (dRT = $RT_{right} - RT_{left}$) were calculated for each fraction magnitude for each participant, and were fit with a linear regression, resulting in either a positive or negative sloping regression line. The slope of this line was used as our measure of the fractions SNARC effect. In addition to the fractions SNARC, we collected overall reaction time (RT), overall accuracy (ACC) and individual distance effect (slope of linear regression of RT against numerical distance from ½).

Woodcock-Johnson (WJ) III standardized assessment. While participants completed multiple subtests of the WJ, the subtests of primary interest were Math Fluency and Spatial Relations. All reported values are raw scores (J. Moeller, 2015).

1) *Math Fluency*. A timed (three minute) test of simple addition, subtraction and multiplication problems; goal is to complete as many problems correctly as time allows. Score determined by number of items answered correctly (total possible: 160)

2) *Spatial Relations*. Participants select the component parts of a target shape; parts may be flipped or rotated as difficulty increases. Total possible score is 81 points.

Analyses and Results

The following analyses describe results from slightly different samples, depending on which measures were available for each individual participant. Sample sizes for each measure are listed in Table 1, along with descriptive statistics. As the main measures of interest in this study, particularly with respect to development, the SNARC and distance effects were analyzed separately for each grade level. On account of additional sample size reduction (described below), further correlational and regression analyses were collapsed across grades.

Table 2. Descriptive Statistics

	3rd Grade		6th Grade	
Measure	**n**	**Mean (SD)**	**n**	**Mean (SD)**
Age (years)	42	8.81(0.84)	46	11.95(0.67)
Fraction comparison				
Reaction time (RT)	42	1259(529.6)	46	1070.3(461.6)
Accuracy (ACC)	42	0.802(0.40)	46	0.874 (0.33)
SNARC slope (SNARC)	42	-235.2(474.3)	46	25.36(356.6)
Distance Effect slope (DIST)	42	-925.9(568)	46	-993.8(472)
Fraction Knowledge Assessment % (FKA)	37	0.80(0.16)	36	0.73 (0.15)
Number Line Estimation (PAE)	34	0.20(0.073)	22	0.094(0.05)
WJ-III Spatial Relations (SPATIAL)	42	67.86(6.98)	46	70.93(5.24)
WJ-III Math Fluency (MATH)	42	63.33(18.23)	46	80.46(22.26)

Note. Descriptions of each measure include the abbreviation used in subsequent analyses. Reaction time measured in milliseconds. WJ = Woodcock-Johnson; spatial relations and math fluency given as raw scores.

Fraction Comparison Task

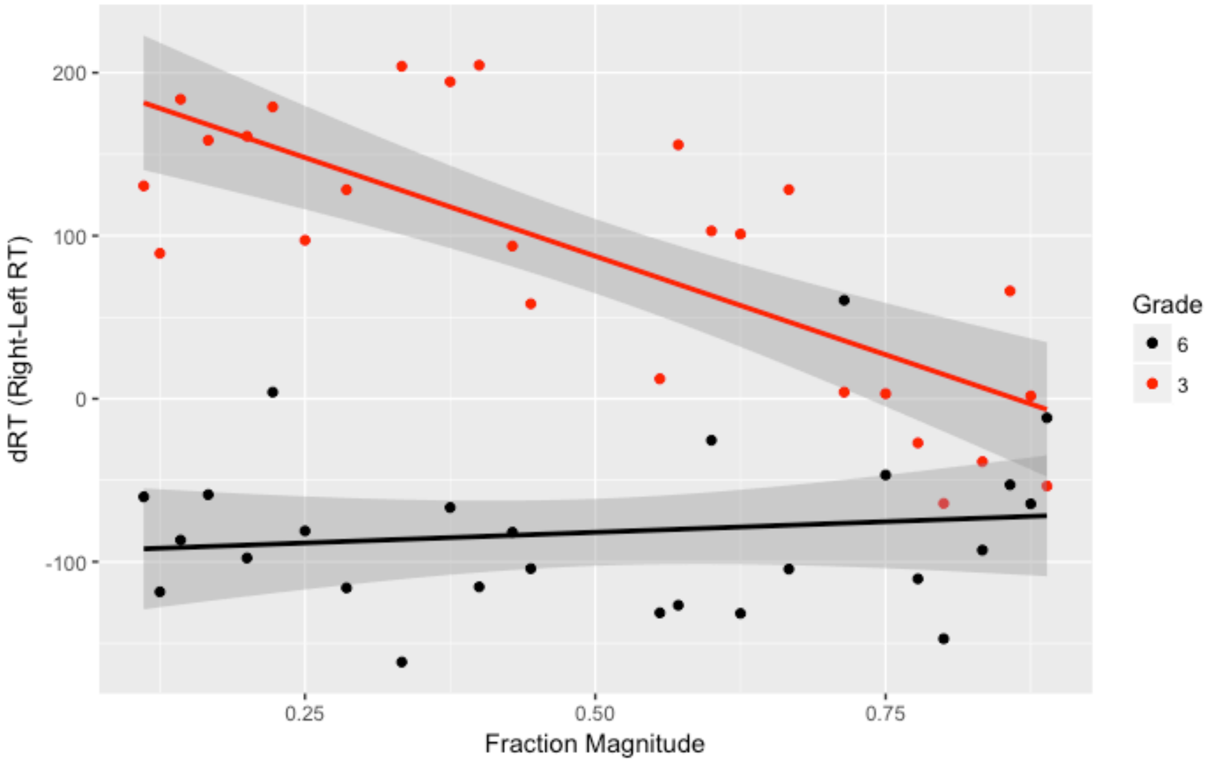
All twelve practice trials were excluded prior to analysis. Trials with RT less than 300 ms were also excluded (5.7% of trials). Previous studies of the SNARC in children have all used whole number stimuli, and thus were comparatively less difficult magnitude comparisons than the fraction comparison task used in this study. Exclusion criteria based on accuracy have ranged across experiments, from 50-75% correct responses for the parity SNARC (e.g. Georges et al., 2017; Hoffmann et al., 2014; Schneider et al., 2009) to 75-80% for the magnitude SNARC (e.g. Gibson and Maurer, 2016; Hoffmann et al., 2013; van Galen and Reitsma, 2008). In the current study, participants with less than 60% accuracy in the fraction comparison task were excluded from analysis, a threshold reflecting increased tolerance for errors due to the increased difficulty of the task, yet still above chance (50%). This threshold resulted in the removal of 16 3rd graders and 6 6th graders.

Comparisons between grade levels were conducted using Welch's two-sample t-test for unequal variances. In line with our expectations, 6th graders were significantly faster ($t[75.9]=4.055$, $p<0.001$) and more accurate ($t[81.5]=-3.49$, $p<0.001$) at comparing fractions than the 3rd graders.

Distance and SNARC Effects.

Individual distance effects for 3rd graders were tested against zero in a one-sample t-test, revealing a significant group-level distance effect ($\beta=-993.78$, $t[45]=-14.26$, $p<0.001$). Sixth graders also showed a significant group-level distance effect ($\beta=-925.89$, $t[41]=-10.56$, $p<0.001$), indicating that both grade levels responded faster to fractions more distant from ½ and slower for fractions closer to ½. Individual SNARC slopes for 3rd graders were significantly less than zero ($\beta=-235.16$, $t[41]=-3.21$, $p=0.003$), indicating a group-level classic SNARC effect for fractions in 3rd graders. Interestingly, SNARC slopes for 6th graders did not differ significantly from zero ($\beta=35.36$, $t[45]=0.48$, $p=0.632$), meaning that there was no group-level SNARC effect for older children. Additionally, the SNARC effects for each grade differed significantly from each other ($t(81.5)=-3.49$, $p<0.001$). Group level SNARC effects are displayed in Figure 1.

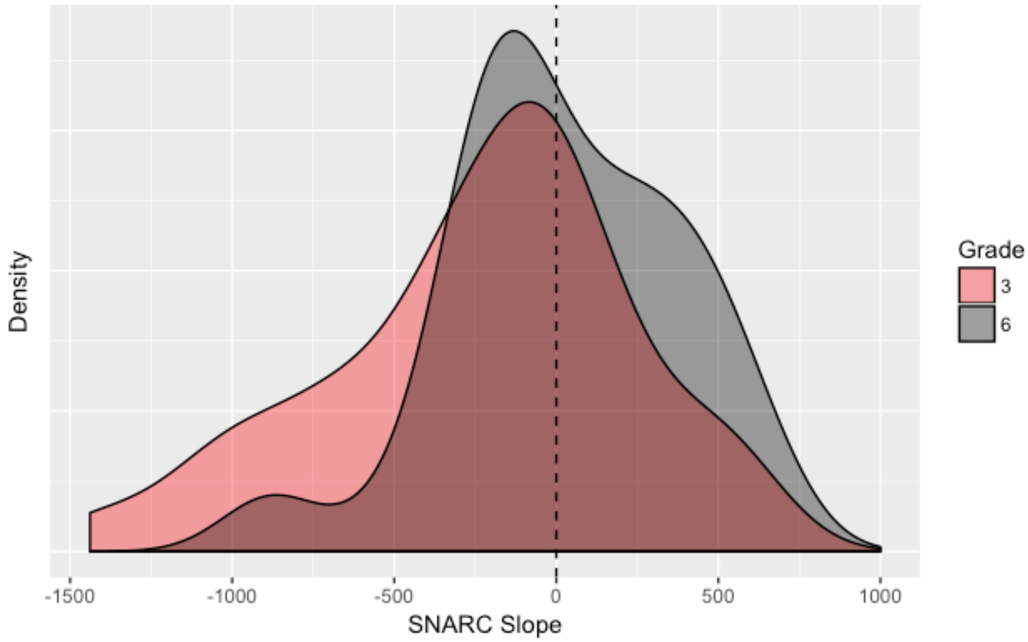
Figure 1. SNARC Effects by Grade



Note. 3rd grade SNARC effects vs. 6th grade SNARC effects. dRT=right-left hand median reaction times.

Within the two age groups, 28(14) of the 42 3rd graders had a negative(positive) SNARC slope, while 24(22) of the 46 6th graders had negative(positive) slopes. This skew is evident in the density plot of SNARC slopes by grade level in Figure 2. Both distributions center generally slightly to the left of (less than) zero, but there are more 3rd graders with negative slopes and fewer of them with positive ones, likely accounting for the group-level SNARC effect for that grade level.

Figure 2. Distribution of Individual SNARC Slopes by Grade



Note. Distribution density plot for 3rd and 6th grade SNARC slopes.

Correlational Analyses

Bivariate correlations for all measures in the study are listed in Table 2 (additional correlational data can be found in Appendix A). Consistent with our previous findings in adults (Chapter 3; Toomarian, Meng & Hubbard, in prep), there was no correlation between the distance effect and SNARC effect (r = 0.02, p=0.82). When accounting for the possible mediating role of reaction time, the correlation was still non-significant (p=0.54). Secondly, contrary to our previous findings with adults, there was no relationship between the strength of the SNARC and NLE acuity as measured by PAE (r=-0.10, p=0.45). There was a significant correlation between SNARC and overall RT (r=-0.33, p=0.002), indicating that people who responded more slowly were more likely to have a stronger negative SNARC slope. Additionally, the SNARC was not correlated with FKA, math fluency, or spatial reasoning ability, whereas PAE was significantly correlated with all of these measures.

Table 3. Bivariate Correlations

	FKA	SNARC	RT	ACC	DIST	PAE	SPATIAL	MATH
FKA	1.00							
SNARC	-0.22	1.00						
RT	-.11	-0.33**	1.00					
ACC	0.37**	-0.04	-0.15	1.00				
DIST	-0.24*	0.02	-0.41***	-0.48***	1.00			
PAE	-0.46***	-0.10	0.35**	-0.49***	0.08	1.00		
SPATIAL	0.34**	0.02	-0.12	-.33**	-0.28**	-0.39**	1.00	
MATH	0.36**	-0.07	-0.34**	0.54***	-0.11	-0.56***	0.10	1.00

Note. ***p < .001, **p < .01, *p < .05

Predicting the SNARC Effect

To investigate which factors predict individual SNARC effects, we used linear regression to model the following equation: $SNARC_i = \alpha + \beta_1 RT + \beta_2 ACC + \beta_3 MATH + \beta_4 SPATIAL + \beta_5 AGE + \varepsilon$. All measures except the outcome measure were standardized. While the model explained a significant amount of variance in SNARC slopes (adj-R^2= 0.12, $F[5,82]=3.37$, $p=0.008$), the only significant factor in the specified model was overall RT on the fraction comparison task. When holding all other factors constant (including age), for every standard deviation increase in mean RT, the SNARC slope is expected to decrease by 149.69 ($t=-3.05$, $p=0.003$), resulting in an increasingly negative slope. In other words, reaction times uniquely predict the degree to which participants respond to fraction magnitudes in a manner congruent with a left-to-right spatially oriented number line, with longer RTs leading to more negative SNARC effects.

Contributions to Fractions Knowledge

Lastly, we tested whether more implicit (SNARC) or explicit (NLE) measures account for variance in fraction test scores (FKA). Due to the smaller-than-anticipated sample size and thus limited statistical power, additional factors were not included in this reduced model, and no hierarchical regressions were conducted. Linear regression was used to model the following equation: $FKA_i = \alpha + \beta_1 SNARC + \beta_2 PAE + \varepsilon$. All measures except the outcome measure were standardized. Together, SNARC and PAE explain 19% of the variance in FKA scores (adj-R^2= 0.19, $F[2,48]=6.83$, $p=0.002$), with PAE as a significant predictor ($t=-3.59$, $p<0.001$). When holding SNARC slope constant, for every standard deviation increase in PAE, scores on the FKA will decrease by 3.33 percent. Put another way, decreases in acuity on the number line estimation task uniquely predict worse performance on a paper-and-pencil fractions knowledge test.

Discussion

In this study, we investigated whether 3rd and 6th grade students exhibited a spatial-numerical association for fraction magnitudes, as well as the relationship between individual children's internal representations and formal math assessments. For the first time, we have demonstrated that 3rd graders demonstrate a classic SNARC effect during a fraction magnitude comparison task, while 6th graders do not. Furthermore, we found that fraction distance effects, SNARC effects, and PAE on the fraction NLE task were not correlated within individuals. PAE was related to fraction test scores, spatial reasoning, and math fluency assessments, whereas the SNARC effect was not. These findings suggest that the SNARC effect and number line estimation ability have separable developmental trajectories and cognitive contributions, which informs our emerging understanding of how implicit and explicit SNAs develop.

Previous work on the SNARC effect in children has suggested that explicit SNAs emerge prior to more implicit ones, but the exact age at which these associations surface is still an open question. Some of the differences that appear between studies may be due to differences in sample characteristics and/or paradigm choice. For instance, it is unclear if SNAs for parity and magnitude judgements can be reasonably compared or if the relative developmental trajectories of these two tasks differ meaningfully. This consideration, in addition to the fact that no studies have investigated the development of the SNARC for fractions, means that the only predictions we had for this investigation were theoretical. Specifically, we predicted that in accordance with several accounts of the SNARC (e.g. the common reading account (Nuerk et al., 2015)), representations on the MNL might be strengthened over time, with increased exposure to physical number lines and directional/cultural cues. Thus, 6th graders would be more likely to demonstrate a SNARC effect. However, our results demonstrate quite the opposite: 3rd graders were more likely to have a classic SNARC effect for fractions than 6th graders.

This result is consistent with past studies that have demonstrated evidence of a magnitude-based SNARC for whole numbers at approximately 7-8 years of age (e.g. Gibson and Maurer, 2016; van Galen and Reitsma, 2008). Additionally, our finding that longer reaction times were predictive of more negative SNARC slopes—even when controlling for age—is consistent with Gevers et al.'s (2006) model that greater reaction time leads to a larger SNARC effect, particularly for magnitude judgements relative to parity judgements.

With respect to the relationship between the SNARC and math skills, our results largely echo findings from previous studies that have demonstrated little correspondence between the two. For instance, Schneider et al. (2009) found that NLE performance—but not the SNARC—was a good predictor of math achievement in children. In their review on the topic, Cipora, Patro & Nuerk (2015) make the case that all SNAs are not created equal in terms of their relationship to arithmetic skills. Specifically, the SNARC has weaker predictive power than NLE and is particularly susceptible to mediating factors. Our correlational results are largely consistent with this view. PAE was much more strongly related to math fluency and fraction knowledge than SNARC slopes.

One key difference from previous studies of the development of the SNARC is the increased difficulty level of our task relative to others. Firstly, comparing fraction magnitudes is generally more difficult than whole number comparison (e.g. Toomarian & Hubbard, 2018b). Secondly, in accordance with recent research that has demonstrated the number of stimulus repetitions necessary to produce a reliable estimate of the SNARC (Cipora & Wood, 2017), our paradigm had many more trials (418) than many early studies of the SNARC (e.g. 88 trials in Schneider et al. (2009)). This impacts not only the strength and reliability of the SNARC effects resulting from these tasks, but also impacts performance due to diminished attention. One might imagine diminished motivation and/or attention for a comparatively long and tedious experimental task. This is a consideration that should not be taken lightly, both in comparing results across studies and in the design of future studies.

Lastly, many more participants than anticipated were excluded from analysis due to low accuracy, particularly 3rd graders. While we can only speculate as to the true reason for the low accuracy, it is likely that the aforementioned characteristics of the paradigm led to poorer performance. For these reasons, and in spite of the compelling theoretical motivation, the SNARC may not be an effective implicit measure of SNAs for use with young children. Rather, the NLE task for fractions may be a more appropriate alternative to the SNARC for driving our understanding of internal magnitude representations for fractions. Future studies aiming to understand more about the relationship between implicit and explicit magnitude processing should consider whether the SNARC offers affordances that cannot be derived from the NLE task, particularly considering its greater predictive power. Longitudinal studies and investigations with slightly older age groups would better inform our understanding of how SNAs for fractions crystalize in development.

Chapter 5: The SNARC Effect in Number Form Synesthetes

Paper currently under review as: Toomarian, E.Y., Gosavi, R.S., & Hubbard, E.M.[4] "Implicit and Explicit Spatial-Numerical Representations Diverge in Number-Form Synesthetes"

Introduction

From numbered city blocks to measuring cups for cooking, numbers and space are intertwined in many aspects of our daily lives. In the numerical cognition literature, these associations are generally referred to as spatial-numerical associations (SNAs). While the majority of empirical research on the integration of numerical and spatial thinking has been conducted in the last several decades, observational studies of SNAs in fact date back to the late 19th century. Galton (1880, 1881) was the first to describe cases of people with "the power of visualising numerals" (p. 252, 1880), or the ability to use internal spatial layouts to represent and manipulate numbers. He describes the ability as consisting of "the sudden and automatic appearance of a vivid and invariable 'Form' in the mental field of view, whenever a numeral is thought of, and in which each numeral has its own definite place" (p.88, 1881). While these descriptions refer to somewhat atypical spatial-numerical representations, this is the first known documentation of spatial associations for numbers and shaped much of the later research on SNAs.

The concept of an internal, spatial conception of number was further articulated nearly a century later by Restle (1970), who formally postulated the mental number line (MNL). Building on findings from Moyer and Landauer (1967), Restle found that when comparing two quantities—a single number and the sum of two numbers—participants' error rates and response

[4] This chapter appears as submitted (with exception of minor organizational differences) and with the permission of the coauthors.

times varied as a function of the distance between the two quantities and the magnitude of the numbers being compared. Specifically, greater distances and numerical magnitudes yielded faster and more accurate responses, while the opposite was true for smaller differences and numerical magnitudes. Since this pattern of responses is similar to what one might expect when using a physical number line to make numerical judgements, Restle hypothesized that people were using a mental analog of the physical number line when making numerical judgements and operations (see also Dehaene, Dupoux, & Mehler, 1990).

The first demonstration of a clearly spatial link between numbers and sides of space came another two decades later, when Dehaene, Bossini, & Giraux (1993) simply asked people to determine whether a given number was even or odd. If participants were in fact using a mental analog of a number line to make these judgements, this should theoretically be reflected in their response times. Indeed, across a range of experimental manipulations, participants were consistently faster to respond to small numbers on the left side of space and large numbers on the right, even though the parity judgement was irrelevant to numerical magnitude. Referred to as the Spatial Numerical Association of Response Codes (SNARC) effect, this finding has been taken as evidence of an internal, linear representation of number that has smaller magnitudes extending to the left and larger magnitudes extending to the right (for reviews: Hubbard, Piazza, Pinel, & Dehaene, 2005; Wood, Willmes, Nuerk, & Fischer, 2008). While the directionality of this effect is widely believed to be culturally mediated, the link between numerical magnitudes and sides of space is evident across cultures and contexts (for reviews: Göbel, Shaki, & Fischer, 2011; Toomarian & Hubbard, 2018a).

As SNAs have become a topic of increasing intrigue and debate, several paradigms have been employed to elucidate the nature of the putative MNL. The classic SNARC effect is one commonly used measure, elicited by either a parity (even/odd) judgement or magnitude judgement. Evidence of a SNARC is derived from the slope of the regression line when regressing the difference in right-left reactions times (dRT) on numerical magnitude (Fias et al., 1996; Lorch & Myers, 1990). Negative regression slopes indicate an internal representation of numbers that is spatially oriented from left-to-right. Another common paradigm is the number line estimation (NLE) task, in which participants are typically asked to place a given number on a number line with labeled endpoints (i.e. a bounded number line). Performance on this task is typically measured by the acuity of responses for each number. If the MNL is indeed a common cognitive representation that influences and aids in numerical judgements, then there should be correspondence between internal representations of quantity (the MNL) and tasks such as NLE, which utilizes a structurally-similar external representation. Thus, this task can theoretically be used in conjunction with the SNARC to test for consistency between internal and external spatial-numerical representations.

Numbers and Space in Synesthetes

An additional approach to better understand spatial-numerical associations is to study those who acutely experience these spatial-numerical associations, such as those initially described by Galton. For the majority of people, spatial-numerical associations are a largely *implicit* phenomenon, meaning they are not consciously experienced. The SNARC effect highlights a relationship that emerges independent of task instructions, and without conscious effort. Indeed, this implicit association can be discretely modulated and primed (e.g. Bächtold, Baumüller, & Brugger, 1998; Hung, Hung, Tzeng, & Wu, 2008). While tasks such as number-line estimation essentially require integration of spatial and numerical abilities, participants may not be consciously aware of this integration when completing the task. However, for a unique subpopulation, associations between numbers and space are never far from their minds. When Galton (1880, 1881) reported descriptions of people's vivid and distinct mental forms for numbers and other ordered sequences, he was not simply describing random representational idiosyncrasies; rather, he was likely describing a variant of what is now recognized as a form of synesthesia (Cytowic, 1989; Simner & Hubbard, 2013).

Sequence-space synesthesia is a condition in which people have *explicit* associations between ordinal sequences (e.g. days, months, hours, letters, numbers) and a specific location in three-dimensional space (Eagleman, 2009; Jonas & Jarick, 2013). In our particular variant of interest—number form (NF) synesthesia—people report vivid, automatic and consistent mental layouts for numerical sequences (Galton, 1880, 1881; Sagiv, Simner, Collins, Butterworth, & Ward, 2006; Seron, Pesenti, Noël, Deloche, & Cornet, 1992). These synesthetic associations are not restricted to horizontal, linear forms but rather, can take any number of shapes and characteristics (Galton, 1880; Jonas & Jarick, 2013; Seron et al., 1992). Additionally, these mental representations are unique and differ from synesthete to synesthete, variability that may be attributable to differences in developmental visuospatial experiences (Price & Pearson, 2013). While the exact nature of the form differs between synesthetes, number forms have been found to be consistent within individuals over time (Piazza, Pinel, & Dehaene, 2006; Seron et al., 1992).

NF synesthesia offers a unique lens through which we can investigate SNAs, and to better understand the relation between implicit and explicit spatial-numerical associations. NF synesthetes are typically consciously aware of their number-space associations, as demonstrated by their ability to describe and draw their forms in detail. This provides an ideal set-up for comparison between implicit and explicit number representations. Some researchers have hypothesized that explicit NFs and the implicit SNARC may rely on shared neural mechanisms of co-activation, specifically in regions of parietal cortex underlying spatial and ordinal/numerical processing (Hubbard, Piazza, Pinel, & Dehaene, 2005; Tang, Ward, & Butterworth, 2008, but see Eagleman, 2009). A unitary mechanism for implicit and explicit spatial-numerical processing would predict correspondence between behavioral measurements of internal and external representations.

Correspondence Between Representations

Previous research has primarily focused on assessing the correspondence between explicit synesthetic reports and implicit SNAs. Observing SNARC effects that go in non-canonical directions (e.g., right-to-left or top-to-bottom) for synesthetes whose NFs go in non-canonical directions has been taken as evidence corroborating synesthetes' subjective reports. In one such study, Jarick et al. (2009) tested two synesthetes with vertical (bottom-to-top) forms for the numbers 1-10, and horizontal forms for larger numbers (10-20 left to right; 21-40, 41-60, etc. right to left). They employed two different tasks both aimed at measuring implicit spatial-numerical associations- the classic parity-judgement SNARC task (Dehaene et al., 1993) and an implicit cuing task (Fischer et al., 2003). Across both tasks, Jarick et al. found that synesthetes with non-canonical number forms exhibited SNARC effects only when responses were oriented congruently with their forms, but not when responses were oriented incongruently. The authors argue that this congruency is effectively an empirical verification of the synesthetes' number forms and that explicit synesthetic forms likely underlie implicit SNARC effects.

These findings of compatibility have also been found using the Size Congruity Effect (SiCE) in synesthetes (Gertner, Henik, Reznik, & Cohen Kadosh, 2013), an effect which tests automatic processing of number (Henik & Tzelgov, 1982). When presented with pairs of numbers that differ in physical and numerical size, people are faster when these two properties are congruent (e.g., the larger value is also physically larger). Gertner et al. (2013) found that synesthetes exhibited a SiCE only when their internal number forms were congruent with the task orientation. The same has been shown for distance effects in NF synesthetes (Gertner, Henik, & Cohen Kadosh, 2009).

Prior to these studies demonstrating general consistency between implicit and explicit spatial-numerical representations in NF synesthetes, Piazza, Pinel and Dehaene (2006) reported an investigation of the SNARC in a synesthete (“SW”) with a curvilinear, right-to-left number form. While SW performed better on number comparison tasks that were congruent with his highly irregular, non-canonical number form than on those that were incongruent, he lacked a SNARC effect in parity judgement tasks, both when stimuli were presented centrally on the screen and when presented peripherally. If synesthetic number forms can be considered explicit mental number lines, one might have predicted that SW’s right-to-left association would have yielded a “reverse” SNARC effect, which would indicate faster responses to smaller numbers on the right and larger numbers on the left. However, SW effectively had no evidence of an association one way or the other, as his SNARC slope did not differ significantly from zero.

In another case study of an NF synesthete, Hubbard, Piazza, Pinel & Dehaene (2009) tested “DG,” who had spatial associations for 58 different ordinal sequences, including integers, on a variety of numerical tasks. His non-canonical number form—curved in a C-shape from bottom-to-top for 1-10, then horizontal—led to several task-specific predictions about his performance. The overall prediction was consistency between his form (bottom-to-top/left-to-right) and task performance. While his explicit reports of non-canonical SNAs were largely congruent with the data from implicit tests, such as a numerical cued-detection task, there were some inconsistencies. For instance, DG did not have a significant horizontal or vertical SNARC effect, despite the prediction that he would have a vertical SNARC. This was the case for both parity and magnitude judgements.

It is difficult to generalize the findings of this and other single-case studies of NF synesthetes to all NF synesthetes, particularly given the inconsistent results. Indeed, in the intervening decade or so since these studies were conducted, our understanding of the SNARC effect and SNAs in general has become much more nuanced. We now know that the SNARC effect is known to vary widely between individuals. Recent estimates place the percentage of people who have significantly negative slopes at around 70% (Cipora & Wood, 2017, supplementary material), highlighting the need for larger investigations of NF synesthesia that move beyond single case designs. To address this need, Jonas et al. (2014) tested the SNARC in a group of NF synesthetes whose forms all extended in the canonical left-to-right direction. In a series of two experiments, they found that the SNARC for synesthetes did not differ from non-synesthetic controls, which suggests some dissociation between the cognitive representations used in the parity SNARC task and the cognitive phenomena that give rise to NF synesthesia.

The Present Study

In the present study, we aimed to resolve the discrepancies evident in earlier studies testing the correspondence between implicit and explicit spatial-numerical representations in NF synesthetes. While previous research has paved the way for investigating SNAs in NF synesthetes, there are important limitations and opportunities for deeper understanding which we address in the current study. First, previous studies have primarily highlighted instances of congruence between implicit and explicit measures of SNAs in synesthetes, effectively ignoring occasions when that correspondence is imperfect. Additionally, the majority of previous investigations of the SNARC in NF synesthetes have been single case studies, in which one synesthete has been studied thoroughly but has not been directly compared to other synesthetes undergoing the same protocol. Multiple-case study designs have the advantage of allowing in-depth investigations, but also permit developing a richer understanding of the similarities and differences across NF synesthetes, and informing the degree to which we may generalize our findings to SNAs more broadly (see Hubbard & Ramachandran, 2005). In this study, we employed a multiple-case study design in a group of synesthetes with both canonical and noncanonical NFs. This approach simultaneously offers the opportunity for the in-depth investigations afforded by single-case design while also allowing generalizations to be made by comparing across synesthetes. Additionally, we used two magnitude-relevant tasks rather than just one to allow us to more directly and thoroughly explore the extent of correspondence between implicit and explicit SNAs. Lastly, little is known about how NF synesthetes represent numbers beyond integers, specifically rational numbers. To investigate this, we used both fraction and whole number stimuli in our comparison tasks. Therefore, the current study builds on previous research investigating SNAs in NF synesthetes by addressing the aforementioned limitations and extending the reach of the investigation.

General Methods and Procedures

All sessions took place in a quiet testing room. For computer tasks, participants were seated in a comfortable chair at a distance of approximately 68 cm from the computer monitor. Experiments were programmed with E-prime 2.0.10 (Psychology Software Tools, Sharpsburg, PA) on a Dell Optiplex 390 Desktop PC (3.1 GHz, 4 GB RAM) running Windows 7.0 64-bit operating system. Stimuli were presented on a Dell UltraSharp U2212H 21.5″ flat-screen monitor at a resolution of 1024 × 768 and a refresh rate of 60 Hz. Left button presses corresponded to the ‘D’ key, and right button presses to the ‘K’ key on a standard keyboard (distance = 8.5 cm).

Recruitment of all NF synesthetes was conducted as part of a separate, ongoing recruitment process for synesthetes of all subtypes. As part of that process, participants were recruited via announcements made in large undergraduate classes, mass emails, and paper flyers posted around the local community. All communications briefly described various synesthetic experiences and individuals who believe they may qualify are invited to the lab for an initial semi-structured interview session. All participants were screened to confirm their synesthetic experiences using a broad synesthesia questionnaire designed to validate their synesthetic experience(s) and detail the nature of their associations (Ramachandran & Hubbard, 2001). This questionnaire asks about the phenomenology of any and all synesthetic experiences, the perceptual reality of these experiences, and any connections between these experiences and the physical environment. During this initial screening, all potential subtypes were noted, including sequence-space/number-form synesthesia. All verified synesthetes in this database who indicated that they experienced synesthetic number-forms were re-contacted for the current study, yielding a sample of eight sequence-space synesthetes. All eight participants were female; the age of each participant is listed in Table 1. Participants were compensated $12/hour in cash for participation. The study protocol was approved by the university institutional review board (#2014-0691).

Session 1 Methods

In the first session, participants took part in an additional semi-structured interview that specifically focused on the nature of their sequence-space associations. As a part of the “Sequence-Space/Number-Form Questionnaire” participants were asked to provide details about basic phenomenology, the nature of their visualizations, the development of these associations (e.g. age of onset), distinct representations for whole numbers and fractions, and basic demographic and background information (Appendix). All participants provided hand-drawn depictions of their representations for the number ranges 1-10, 1-100, and fractions between 0-1, in addition to their verbal descriptions. These self-reported descriptions aided in our characterization of each participant's number forms, as drawing a number-form often required transforming multidimensional representations into two-dimensional space. For all tasks, participants were instructed to respond to the best of their ability.

Participants also completed an unconstrained number placement task, in which participants were shown a number at the top of an otherwise blank, black screen, and instructed to click on the position in space that best corresponded to the location of that number on their number-form. Participants first responded to a subset of 20 positive integers (range: 1-40, modeled after Hubbard et al., 2009), followed by a set of 27 irreducible, single-digit fractions between 0 and 1 (modeled after Toomarian & Hubbard, 2018b). For both stimulus sets, the largest and smallest numbers were presented first and second, respectively, to provide cognitive landmarks/reference points. All other numbers presented in a random order. Since the stimuli were not presented in order, this task served as a basic validation and quantification of each synesthete's implicit number form.

Session 2 Methods

All eight participants who took part in the first session were invited to return for an additional session. Of those eight, four participants returned. The participants from the first session who did not return either did not respond to our inquiries or had moved away from the study location. The second session lasted approximately one hour and was comprised of two types of types of tasks: magnitude comparison and number-line estimation.

Magnitude comparison

Participants completed two magnitude comparison tasks, from which we extracted individual SNARC effects. In both tasks, participants were instructed to compare the magnitude of a centrally presented number (the target) to a reference number provided at the start of the task. In each trial, participants saw a central fixation cross (600 ms), followed by a blank screen (1000 ms) and the target number (3000 ms or until a response was detected). In the first task, participants compared whole numbers (1-9) to the reference 5. There was a total of 416 trials, with the eight unique target numbers randomly presented 26 times in each of two blocks. In one block, smaller numbers corresponded to a left button press and larger number to a right button press, while the other block had this correspondence reversed (i.e. smaller/right, larger/left). Ten practice trials preceded each block, which included visual feedback.

After completing the task with whole numbers, participants compared single-digit, irreducible fractions to the reference ½, in a task exactly replicating the fractions comparison task used in Toomarian and Hubbard (Toomarian & Hubbard, 2018b). Each of the 26 fraction targets appeared eight times, yielding 416 total trials, with response side counterbalanced across two blocks. A total of 10 practice trials with visual feedback preceded each block. Participants did not receive feedback on test trials. All participants completed the whole number comparison task first, followed by the fraction comparison task. For both tasks, two synesthetes (SY1 and SY4) first responded to small numbers on the left and large on the right, whereas the other two synesthetes (SY2 and SY8) first responded to small numbers on the right and large on the left. All participants switched directions for the second block of the task. Magnitude relevant tasks were used rather than parity judgement to be able to draw conclusions across both the whole number and fractions SNARC effects, as there is no equivalent of parity for fractions.

We predicted that synesthetes with left-to-right (canonical) number forms would exhibit a classic SNARC effect (faster for small numbers on the left and large numbers on the right) for whole numbers. The classic SNARC effect reflects the directional organization (left-to-right) that would be congruent with their reported forms. We predicted that synesthetes who reported a non-canonical directionality (right-to-left) of their number forms would exhibit a reverse SNARC effect. Additionally, because participants self-reported very robust explicit number forms for whole numbers, particularly for positive integers 1-10, but weaker associations for fractions, we predicted a correspondingly weaker trend for implicit fractions representations. However, we hypothesized that the directionality of the SNAs for fractions would still correspond to the reported, explicit form.

Number line estimation

In the number line estimation task, participants were presented with a number at the top of the screen and asked to use a computer mouse to click on the appropriate position for that number on the bounded number line below. This number-to-position task included two separate blocks, one consisting of proper fraction stimuli to be placed on a 0-1 number line and one consisting of improper fractions to be placed on a 0-5 number line. Critically, in order to investigate whether performance differed when the bounded number lines were congruent with participants' reported forms, each number line task was administered in both the canonical direction (0 on left end, 5 on right end) and the non-canonical (5 on left end, 0 on right end) direction. Participants completed the NLE task in the order corresponding to their canonicity, such that they all first completed the task in the orientation that was congruent with their reported form, followed by the orientation that was incongruent with their form. This resulted in SY1, SY2, and SY4 completing the left-to-right orientation first, while SY8 did the right-to-left orientation first. Responses yield a measure of percent absolute error (PAE) for each participant (Booth & Siegler, 2006, 2008). Smaller PAE values indicate higher acuity.

We predicted that PAE would be smallest, indicating greater acuity, on the version of the task congruent with the direction of each synesthete's number form. PAE for the 0-1 task may be slightly greater, since most of the participants reported weaker associations for numbers in that range. Additionally, we were able to compare performance of synesthetes to a group of non-synesthetic controls who completed the same task in the left-to-right orientation as part of a separate study. We predicted that synesthetes whose reported forms were congruent with this orientation should show greater acuity than the controls.

Results

Session 1 Results

Key insights from the Sequence-Space/Number-Form Questionnaire are detailed in Table 1. All eight synesthetes reported having a spatial form for calendars in some way, though the units of the calendar were not consistent (e.g. not all had forms for weeks). Seven synesthetes reported the ability to zoom in/out of their spatial representations, either in their mind's eye or projected in space. Five synesthetes reported being aware of at least one immediate family member who also had synesthesia of some kind, with four of those identifying their mother as a synesthete. Conjuring mental images was generally very easy for our sample, though reports of navigation and memory ability varied widely (e.g. for places or dates). All synesthetes recall having these associations since early childhood, with nearly all citing that these associations have existed for as long as they can remember. All synesthetes reported that their forms were elicited whenever presented with or thinking about numbers (as well as other stimuli for which they have forms), with the forms often fading shortly after removal of the stimulus or shifting attention.

Descriptions of each synesthete's number form for fractions and whole numbers are listed in Table 2, based on the synesthetes' drawings and self-reported descriptions. Of the eight synesthetes, five reported ascending left-to-right NFs, one reported a right-to-left NF, and two others reported vertical bottom-to-top NFs. These forms have been classified in relation to the canonical, left-to-right oriented number line. For the purposes of this study, in which we are focusing on horizontal orientation, synesthetes who reported vertical or right-to-left number orientations are considered to have a non-canonical organization of their number form.

Responses from the unconstrained number placement task were plotted to obtain a two-dimensional visualization of each participant's number form. The output from each participants' number placement task was compared with the drawings provided in the interview portion of the session. Figure 1 shows this comparison for two participants (see Supplementary Material for all plots). As predicted, these computer mouse-placement tasks largely validated the written/drawn reports of synesthetes' forms, though some multidimensional aspects of the reported forms were diminished. This form effectively served as a qualitative, rapid test-retest reliability of each synesthetes' number form.

Table 1. *Descriptions of select questionnaire responses for each participant. "Age": age at test; "Zoom": reported ability to zoom in or out on reported number form; "Family": immediate family members who are known to experience synesthesia of any type; "Mental Images": self-reported ease (scale:1-9) of constructing mental images.*

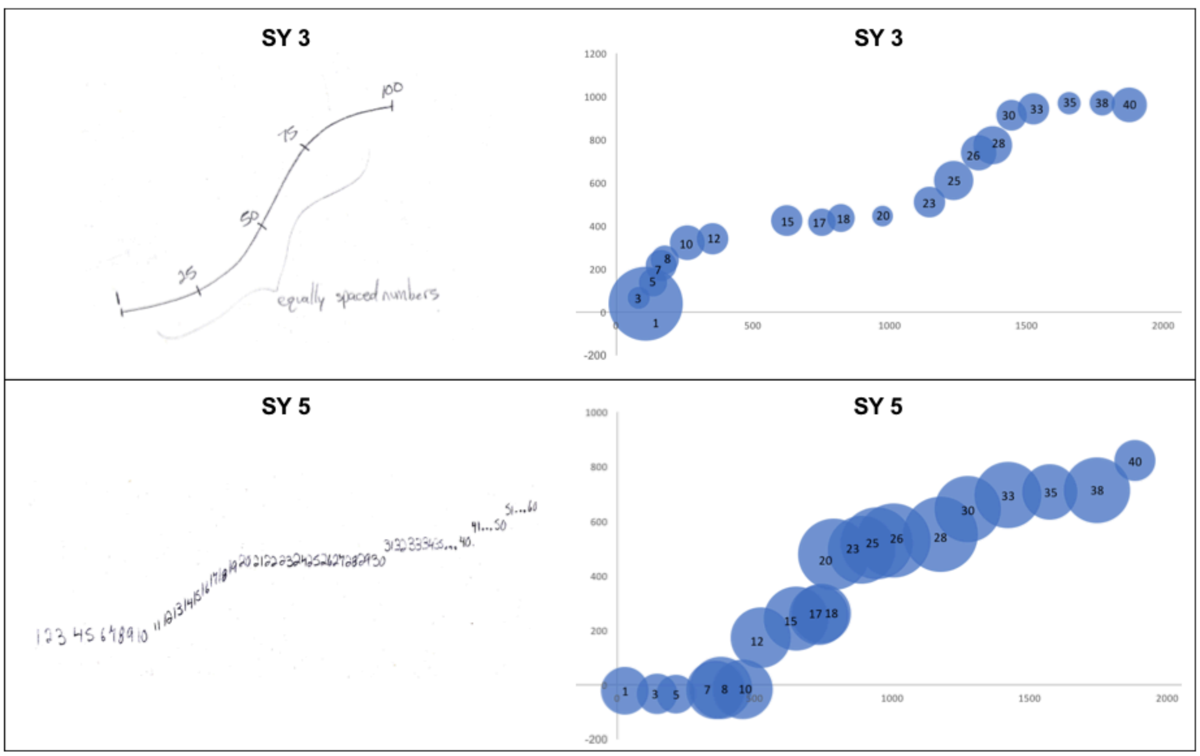
Synesthete	Age	Onset age	Other Forms	Zoom	Family	Mental Images	Other Notes
SY1	20	Elementary school/unknown	Calendar (months, days, years); Temperature	Yes	No	Easy (2)	Finds math "relaxing" and enjoyable; very good memory for places/dates/time; spatial form for years begins at 2000; no fraction or decimal form, but has form for negative numbers
SY2	19	Unknown/always	Grapheme-color; Calendar; Temperature Alphabet;	Yes	Yes- father	Easy (1)	Some numbers "weighted"/bolded/"special"; fractions "nicer" than decimals; prime numbers "spiky"
SY3	22	Unknown/always	Calendar (months, weeks, days, years); Time; Temperature; Alphabet	Yes	Yes-mother	Easy (2)	Finds math easy; fractions more linear than whole numbers; good memory for dates; recalls detailed dreams
SY4	18	Unknown/always	Calendar (months, days, years)	No	Yes- mother, brother	Easy (1)	Finds math difficult, including arithmetic; thinks NF hinders math ability; Year 2000 has same spatial position as 0 on NF
SY5	19	Unknown/always	Calendar; Alphabet; Songs	Yes	Yes-mother	Moderate (4)	Negative numbers separate from positive; finds math difficult but enjoys algebra; form for decimals but not common fractions; poor memory and navigation skills
SY6	31	Unknown/always	Calendar (months, days, years); Roman numerals; Alphabet; Temperature	Yes	No	Easy (3)	Finds math easy; feels NF helps with arithmetic; negative numbers represented until -10; no fraction form; good memory for dates/times/locations
SY7	18	Unknown/always	Calendar (months, years); Temperature; Alphabet	Yes	Yes- mother, sister	Easy (3)	Numbers are gendered; NF fades when focusing too hard on it; form consistent except certain numbers' colors/genders; has NF for fractions/decimals; uses NF for basic math; poor memory for dates; poor navigation
SY8	57	4-5 yrs	Calendar (years, weeks, days); Time; Alphabet	Yes	No	Difficult (7.5)	Extensive mathematical training; reliant on finger counting in early years; calendar is a ribbon; bad at facial recognition

Table 2. Descriptions of synesthetic number forms.

Synesthete	Age (yrs)	Fraction Form	Whole Number Form	Horizontal Canonicity
SY1	20	Horizontal left to right	Horizontal, ascending left to right, stacked by decade	Canonical
SY2	19	Horizontal left to right	Horizontal, ascending left to right, stacked by decade	Canonical
SY3	22	Horizontal left to right	Sigmoidal, ascending left to right	Canonical
SY4	18	Horizontal left to right	Horizontal, slightly descending left to right	Canonical
SY5	19	Horizontal left to right	Horizontal, ascending left to right, stacked by decade	Canonical
SY6	31	No form	Vertical, ascending linearly, stacked by tens	N/A (vertical)
SY7	18	Vertical, linear, ascending	Vertical, ascending linearly, logarithmic	N/A (vertical)
SY8	57	Horizontal right to left	Horizontal, ascending right to left, stacked by decade	Non-canonical

Note. Descriptions of synesthetic number forms for each participant. Canonicity refers to left-to-right horizontal orientation.

Figure 1. Correspondence of Representations Across Tasks



Note. Samples from two synesthetes (top: SY 3, bottom: SY 5) of the correspondence between drawn number forms (left) and corresponding output from the unconstrained number placement task (right). Data plots for all other synesthetes available as supplementary material.

Session 2 Results

Magnitude Comparison Results

Practice trials were excluded prior to analysis, as well as trials for responses that occurred less than 300 ms or greater than 2000 ms after stimulus onset, consistent with previous analyses of this task. Across all participants, this resulted in 0.5% of trials being excluded for the fractions task and 0.4% of trials being excluded for the whole number task. Overall accuracy was high for comparisons of both fractions (mean=0.947, SD=0.223) and whole numbers (mean=0.976, SD=0.152). Overall reaction times were calculated only on correct trials (fractions: mean=664.19, SD=224.84; whole numbers: mean=503.14, SD=137.57). Accuracies and reaction times for each synesthete are listed in Table 3.

Table 3. Results of individual SNARC effect analyses.

	Accuracy		RT							
Synesthete	***Mean***	***SD***	***Mean***	***SD***	***B***	***SE B***	***Int.***	***t***	***p***	R^2
Whole SNARC										
SY1	1.00	--	459.45	77.52	15.66	3.00	-80.23	5.23	0.002*	0.81
SY2	0.96	0.20	508.26	124.51	7.77	4.37	-59.65	1.78	0.13	0.34
SY4	0.97	0.16	505.00	150.29	-14.23	5.74	91.04	-2.48	0.047*	0.51
SY8	0.98	0.15	538.91	166.29	-25.18	6.15	90.73	-4.10	0.006*	0.74
Fraction SNARC										
SY1	0.99	0.10	560.01	111.01	19.73	34.60	-20.00	0.57	0.57	0.01
SY2	0.93	0.26	659.51	207.16	-182.77	105.32	1.83	-1.74	0.095	0.11
SY4	0.92	0.26	696.92	275.07	121.63	86.01	2.19	1.41	0.17	0.08
SY8	0.94	0.23	741.05	236.51	-277.28	113.60	165.64	-2.44	0.022*	0.20

Note. *p<0.05; whole number t-statistics based on 6 degrees of freedom; fraction t-statistics based on 24 degrees of freedom

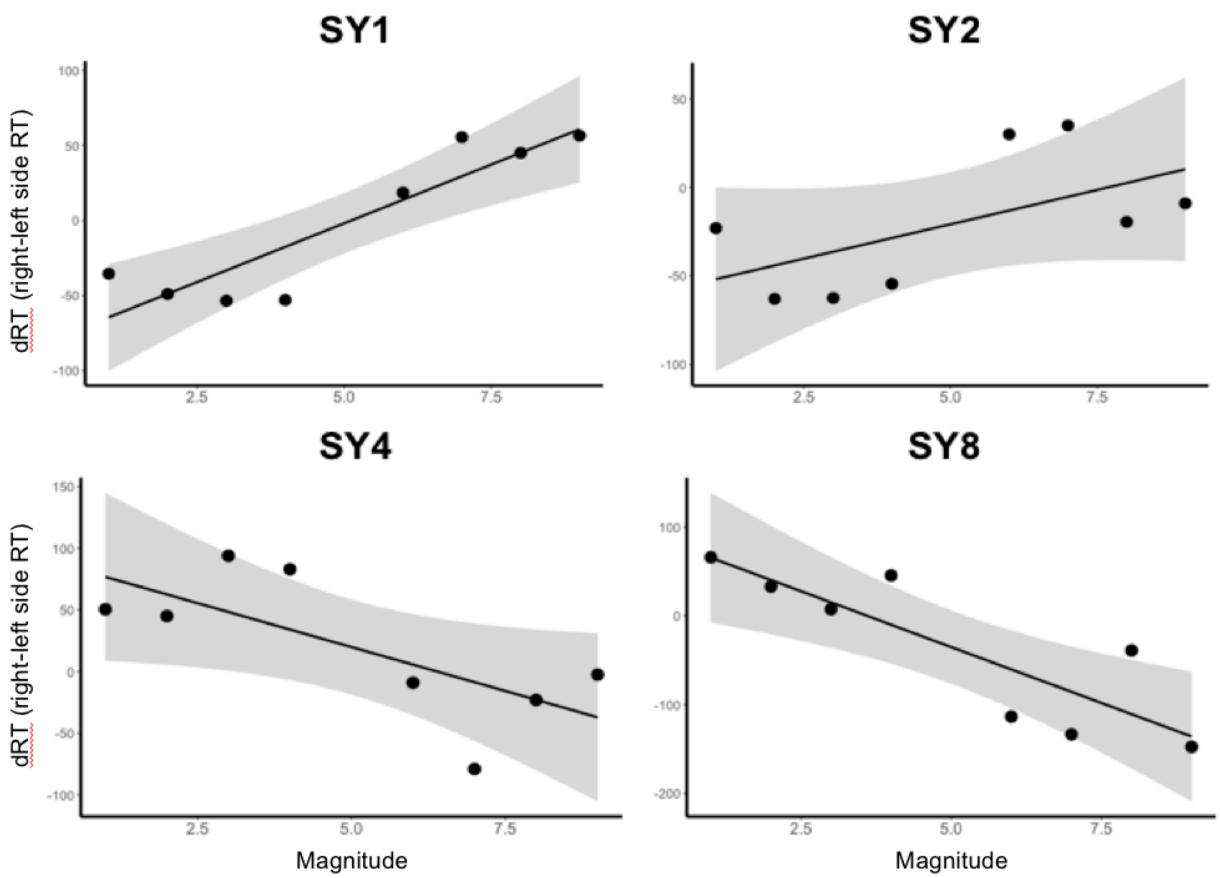
For each of the magnitude comparison tasks, the difference between the left and right hand median reaction times (dRTs) were regressed onto numerical magnitude for individual participants, yielding a regression slope. These slopes—one for fractions and one for whole numbers—were used as quantitative measurements of the strength and direction of each synesthetes' implicit SNARC and are listed in Table 2. We did not test for a group level SNARC effect due to the small size of the group (n=4), but rather tested each participant individually.

Three of the four synesthetes has a significant SNARC effect for whole number comparisons. See Figure 2 for plots of the whole number SNARC effect. SY4 demonstrated a significantly negative SNARC slope for whole numbers, indicating a canonical implicit representation of number, which is congruent with her explicit reports. Notably, however, the other three synesthetes demonstrated a response-coding for whole numbers that was incongruent with the direction of their reported spatial number forms. SY1 explicitly reported an internal number form extending in the canonical direction, but had a significantly positive SNARC slope, indicating faster responses to large numbers on the left and small numbers on the right. Similarly, SY2 had a positive SNARC slope despite reporting a canonically-orientated number form, though because the effect was not significant, she effectively had no evidence of a SNARC. Conversely, SY8 described her form as being in the non-canonical right-to-left orientation but exhibited a classic SNARC effect (negative slope) for both whole numbers and fractions. None of the other three synesthetes had a significant SNARC effect for fractions.

Number line estimation results

The PAE was calculated for each of the stimulus ranges in each of the number line orientations (PAE = [|answer – correct answer|/numerical range] x 100). Smaller PAE values indicate higher acuity; mean PAE scores are presented in Table 3. Data from the number-to-position tasks largely support the contradictory findings from the SNARC analyses detailed above. We expected that synesthetes would show larger PAE scores (indicating lower acuity) for the number line orientation incongruent with their explicit forms. This was the case for SY2 and SY4, but not for SY1 or SY8, who had greater acuity (smaller PAE) for the conditions incongruent with their reported forms. These were the same synesthetes who showed incongruencies in the SNARC task.

Figure 2. Individual SNARC Slopes



Note. Individual SNARC slopes for each of the synesthetes from session 2. Shaded regions indicate 95% confidence intervals.

We also compared each of the synesthetes to a group of non-synesthetic controls recruited as a part of a separate study (n=92). The controls completed the canonically oriented NLE task and had a group mean PAE of 6.87 (SD=2.75). We used established methods for comparing a single case to a control sample using a t-test (Crawford & Garthwaite, 2012), the results of which can be found in Table 4. None of the synesthetes had canonical PAE scores that significantly differed from the controls.

Table 4. Number Line Estimation for Session 2 Synesthetes

	Canonical number line							*Non-canonical number line*		
Syn	**NLE 0-1**	**NLE 0-5**	**Avg. PAE**	**t**	**p**	z_{cc}	**Syn**	**NLE 1-0**	**NLE 5-0**	**Avg. PAE**
1	5.04	5.99	5.37 (5.92)	-0.551	0.58	-0.55	***1***	2.90	4.54	3.46 (2.94)
2	2.31	7.86	4.20 (5.18)	-0.977	0.33	-0.98	***2***	4.10	11.81	6.73 (8.48
4	5.17	9.92	6.79 (7.81)	-0.038	0.97	-0.03	***4***	7.36	10.21	8.34 (12.45)
8	3.47	11.07	6.06 (10.33)	-0.301	0.76	-0.30	***8***	13.60	7.87	11.65 (17.94)

Note. NLE= number line estimation, PAE = percent absolute error; Avg. PAE is the average PAE across both NLE tasks t-tests are based on 91 degrees of freedom; standard deviations are listed in parentheses

Discussion

In this study, we investigated implicit and explicit spatial-numerical associations in a group of number-form synesthetes. Synesthetes exhibited a wide range of distinct number forms, which differed in shape, extent, and direction. In the subgroup that underwent additional testing, we found marked incongruencies between their reported number forms and the results of two cognitive behavioral paradigms, namely a magnitude-based SNARC task and a number line estimation task. Only one of the synesthetes demonstrated a SNARC effect for fractions. We discuss these findings below, in light of previous research on NF synesthesia and studies of SNAs in the general population.

General Observations Across Synesthetes

On the whole, the findings detailed above illustrate the complex interplay between implicit and explicit numerical representations. While we expected that synesthetes' reported forms and their subsequent implicit spatial-numerical associations would be congruent, evidence from multiple tasks suggest that the picture is more nuanced than a direct correspondence between internal conceptualizations and external representations. Whereas previous research on NF synesthetes has generally demonstrated such congruency, whether in canonical or non-canonical orientations, this study is the first to demonstrate clear dissociations between implicit and explicit spatial-numerical associations in a group of NF synesthetes.

Such a dissociation may be in line with the dissociation between implicit and explicit processing described in studies of people with spatial neglect. Priftis et al. (2006) showed that patients with left neglect had impaired performance on a number line bisection task but were seemingly unaffected on tests of the SNARC. The authors suggested that this difference was due to the difference in task demands, with number line bisection requiring direct access to the MNL while the SNARC is implicit. Similarly, Ninaus et al. (2017) found that performance on a number line bisection task was not correlated with individual SNARC scores, further supporting the hypothesis that these two tasks may be supported by disparate underlying cognitive mechanisms. While in this study we saw some degree of continuity across the NLE task and the SNARC task (in that the pattern of congruency was generally preserved across tasks), the possibility that these tasks do not directly measure internal numerical conceptualizations might account for the differences we see between synesthetes' internal number forms and their explicit performance on these tasks.

Another possible explanation for the heterogeneity in our findings is the notable individual variability of implicit SNAs in the adult population more broadly. Recent studies of SNAs in nonsynesthetes have shown inconsistencies in SNAs (for both whole numbers and fractions), with much of this variability being previously masked by group-level reporting of data. For instance, Cipora et al. (2017) highlighted that for many studies reporting a group-level negative SNARC slope, between 60-80% of individual SNARC slopes were negative. Other studies with nonsynesthetes have begun exploring the factors that may impact both the strength and direction of individual SNAs, such as level of mathematics ability, visuospatial reasoning, fluency in other languages, etc. (Cipora & Nuerk, 2013; Cipora et al., 2015; Georges et al., 2017; Hoffmann, Mussolin, et al., 2014; Viarouge et al., 2014). These factors may be influencing outcomes of these tasks measuring implicit SNAs, and thus it is possible that these same factors would be influencing the synesthetes in this study as well. This possibility should be explored in future studies of NF synesthesia.

This study is also the first investigation of the fractions SNARC effect in number form synesthetes. We have previously used this same task to elicit a fractions SNARC in non-synesthetic adults (Toomarian & Hubbard, 2018b), while also showing that the fractions SNARC correlated with the SNARC for whole numbers. In this study, two synesthetes did not report their NF extending to fractions, and only one synesthete (SY8) had a SNARC effect for fractions; notably, it was in the direction contrary to the orientation of her reported form. Our results demonstrate that while synesthetes may report weak explicit representations of fractions on their NFs, they generally do not demonstrate an implicit spatial representation for fraction magnitudes. This may be due to the fact that fractions are less commonly encountered in daily life (relative to whole numbers), and thus are less reliably/robustly represented on synesthetic NFs.

Discussion of SY8

Our results highlight the heterogeneity of synesthetic number forms and implicit spatial-numerical associations, suggesting that additional factors are likely at play in determining individual spatial-numerical associations. Here, we would like to more thoroughly discuss the unique case of SY8, who described a right-to-left oriented number line but whose performance on the magnitude judgement and number line tasks indicated the opposite. There are several considerations to take into account in interpreting these results. First, it should be noted that the current study only tested for implicit SNAs in the horizontal dimension. Thus, for someone like

SY8, whose form went right-to-left but also included vertical stacking, we cannot get the complete picture of their internal conceptualizations from the these horizontally-focused tasks. Ideally, follow-up studies would incorporate paradigms similar to those used by Hubbard et al. (2009), which included vertically-oriented tasks.

Secondly, the dissociation between implicit and explicit representations in the case of SY8 may be partially explained by mathematical expertise, as she is a professor of mathematics. While it is unclear what may have influenced or encouraged the non-canonical directionality of her number form (she is monolingual and has no extensive experience with cultures that orient numbers differently), she reported that she consistently represents numbers in the opposite orientation of her internal conceptualization as a requirement of her profession. As a mathematician, she regularly switches to represent numbers going from left-to-right, such as when using Cartesian graphing coordinates and working with functions. Interestingly, she reported being able to make this translation with ease, potentially indicating a high level of cognitive flexibility. Galton's early case studies also included a report from a presumed synesthete with extraordinary calculation abilities (e.g. rapid multiplication of two fifteen-digit numbers) (1880). He reported that "when I am multiplying together two large numbers, my mind is engrossed in the operation and the idea of locality in the series for the moment sinks out of prominence" (p. 253, 1880). This would suggest that more advanced mathematical thinking may be separable from more basic numerical and ordinal thought processes.

This possibility has been empirically examined by Cipora et al. (2016), who investigated the SNARC in a group of professional mathematicians, as well as a group of engineers and non-engineer controls. While the engineers and controls both demonstrated a group-level SNARC effect, the mathematician group did not. This difference was not related to greater overall response variance or intelligence. Based on this finding, we might have expected SY8 to exhibit a flat SNARC slope in the current study, if not a positive one. However, Cipora et al. noted that while a significant effect failed to emerge at the group level, 9 of the 14 mathematicians had a negative SNARC slope. Thus, SY8's implicit SNA may indeed be consistent with those of other professional mathematicians. The authors propose that greater abstract reasoning and/or cognitive flexibility may account for the difference in SNAs between mathematicians and the other two groups. Future studies should directly investigate the link between cognitive flexibility and congruency of implicit and explicit spatial-numerical representations in NF synesthetes.

One final consideration relevant to SY8 is her age; at 57 years old, she was 37 years older than the average age of the other synesthetes from session 2. Given that most studies of the SNARC have been conducted on young adult populations, she is likely much older than the majority of participants in other studies of the SNARC. In a recent investigation of the SNARC effect across the lifespan, Ninaus et al. (2017) found that SNARC effect was stronger with increasing age, which supports the theory that directionality of the SNARC is primarily influenced by culturally congruent sensorimotor experience. SY8 had nearly 40 additional years of exposure to left-to-right numerical organizations, particularly through her work, which might be influencing her canonical SNARC effect. Of course, such an explanation would imply a marked differentiation between her explicit number form and the SNARC effect, since it implies that her lived experience with external influences overrides her own spatial number form. An additional consideration is that evidence from grapheme-color synesthetes has suggested that synesthetic associations decline in consistency with age (Meier, Rothen, & Walter, 2014). While this has not been systematically tested in NF synesthetes, it is possible that overlearned (canonical) cultural representations supersede internal, idiosyncratic representations in aging NF synesthetes.

Taken together, it is clear that several factors converge and likely interact to build an individual SNA profile. The case of SY8 highlights the confluence of factors such as occupation and cultural experience in shaping this profile. Longitudinal studies of NF synesthetes that extend beyond childhood would help to further elucidate the role of these factors.

Additional Considerations

The considerations noted above highlight the power and necessity of in-depth, mixed-methods, single-case approaches to investigating cognitive phenomena. There is much to be gained from examining complex questions from an individual perspective, while also looking across individuals for commonalities and differences, a strength of the multi-case study approach. As described by Hubbard and Ramachandran (2005), the multiple-case study approach in studies of synesthesia affords fine-grained analyses of individuals with heterogenous synesthetic profiles while also allowing for conclusions to be drawn about the group. In the case of this paper, this methodological approach led to the group-level insights that 1) NF synesthetes have generally inconsistent implicit and explicit representations of numbers, 2) the heterogeneity in number forms and implicit/explicit congruency may be more strongly related to other factors besides having NF synesthesia, and 3) NF synesthetes do not have robust implicit associations for fractions. In addition to these group-level findings, we were able to dive deeper into the phenomenology by investigating possible influences at the single-case level, such as our analysis of SY8.

These results challenge the simple prediction made by Hubbard et al. (2005) that explicit number-forms and implicit SNAs depend on the same mechanisms. In the decade since this explanation was first proposed, it has become clear that individual differences, strategy choice and other factors play critical roles in both explicit number forms and in non-synesthetic SNAs. Given the complexity of these number space mappings, it is difficult to fully account for the wide variability in these implicit and explicit measures across both synesthetes and non-synesthetic controls. While researchers have attempted to account for this variability by measuring myriad possible influences, we still do not have a strong understanding of why this intra- and inter-individual variability exists. Thus, it is important to consider alternative explanations—beyond individual cognitive differences—that may account for this pattern of results.

One possibility is that the tasks used in this and other investigations of SNAs may not actually be accessing a unitary spatial representation of number. For instance, some authors have argued that the SNARC effect is not an index of a spatial MNL, but rather that it indexes processes like order in working memory (e.g. Abrahamse, Van Dijck, & Fias, 2016), or processes associated with linguistic “markedness” of number words (e.g. Nuerk, Iversen, & Willmes, 2004). Most recently, Shaki and Fischer (2018) have contended that—contrary to the majority of their own prior work (see Fischer & Shaki, 2014, for a review)—SNAs emerge as artifacts of either explicit magnitude processing or explicit spatial- directional processing; they do not reveal spatial-conceptual links. This was supported by two novel experiments combining implicit or explicit magnitude judgements (i.e. either a parity or magnitude-based go/no-go task combined with arrows) with implicit spatial-directional cues (i.e. using a central response key for various response rules) (see also Fischer & Shaki, 2016). They revealed a SNARC only in the conditions with explicit magnitude processing, suggesting that previously reported evidence of a SNARC may have been the result of task-related bias (see also Karolis, Iuculano, & Butterworth, 2011). While this proposal is certainly compelling, more research is needed to replicate and extend these findings to other paradigms.

In the case of the NLE task, the assumption that performance is indicative of implicit number magnitude representations has been challenged on several fronts (e.g. Huber, Moeller, & Nuerk, 2014; Slusser & Barth, 2017). One consideration is that the imposed linear representation makes it difficult to determine whether a small PAE is indicative of an internal linear representation (as generally assumed) or rather, indicative of the participants simply learning how to do the task correctly. Huber, Moeller & Nuerk (2014) present two pieces of evidence to support this claim: 1) adults can be easily trained to do both linear and logarithmic versions of NLE well, and 2) young children who represented numbers logarithmically and those who represented them linearly on a paper-and-pencil number line task did not differ on a separate computerized, logarithmic training version of NLE. As this violates the expectation that those with presumed logarithmic internal representations would perform better on the logarithmic training task, the leading alternative explanation is that the task is not appropriately capturing implicit SNAs. It is also possible that NLE performance is influenced by task-specific strategies, such as proportional reasoning (e.g. Slusser, Santiago, & Barth, 2013; Slusser & Barth, 2017) or using reference points or “benchmarks” (e.g. Barth & Paladino, 2011; Siegler & Opfer, 2003). These strategies may have been differentially relied upon by our synesthetes, leading to incongruencies between our measures and their reported forms.

Another possibility—and in our view, the most likely—is that there is no *one* factor that influences implicit and explicit representations of number, but rather that multiple factors are responsible for response profiles in a person at any given point in time. Alibali and Sidney (2015) suggested that people choose from a range of possible strategies as a function of task constraints and context. The dynamic strategy choice account is one way to account for intraindividual differences evidenced in the current study. For instance, we may have seen general consistency between the unconstrained number placement task and their reported/drawn number forms because there were minimal task constraints, and the context in which numbers were presented was simple. However, the SNARC task requires a speeded choice, and the NLE tasks impose a visual structure that likely interfered with our synesthetes' number forms. It is possible that, in line with the dynamic strategy choice account, these additional task features constrained the extent to which NF synesthetes accessed their own number forms, choosing instead to rely on a different processing strategy. This theory aligns with the view that the SNARC emerges as the result of both spatial-numerical mental representations and response-selection (Basso Moro et al., 2018).

Conclusion

Taken together, the results presented in this study build upon what was previously known about NF synesthetes while also highlighting more general issues that must be reckoned with in the field. Similar to past reports of NF synesthesia, the participants in this study demonstrated a wide array of NFs. While there was notable inter-individual variability in NFs, the synesthetes' drawn forms bore great similarity to the forms derived via an unconstrained number placement task. We found that it was very uncommon for NF synesthetes to have robust implicit representations for fractions, which unsurprisingly resulted in weak implicit SNAs for fraction stimuli. Contrary to our hypotheses, performance on classic indices of implicit SNAs—the SNARC and NLE task—generally did not match up with explicit reports of synesthetic number forms. While this may indicate separate underlying cognitive mechanisms for implicit and explicit spatial-numerical representations, task-specific constraints and strategic variability could also account for these differences, and thus must be seriously considered in future investigations. To more thoroughly explore these issues, future studies should consider testing synesthetic associations using paradigms that are less susceptible to strategic effects, and which are cognitively more direct, such as spatial cuing paradigms or implicit measures of spatial processing like spontaneous eye movements. We also recommend that studies of SNAs in which participants are presumed to be non-synesthetic systematically inquire about potential synesthetic experiences to ensure homogeneity of the group. These steps will help to further refine our understanding of explicit SNAs in synesthetes and non-synesthetes alike.

Chapter 6: Discussion

The studies described here represent a multi-faceted effort to better understand the nature of implicit and explicit spatial-numerical associations. This body of work addresses a number of questions, including: how can we account for the significant inter-individual variability for fraction SNAs in adults, and does this variability correspond to any educationally-relevant outcomes? What is the profile of SNAs for fractions in childhood? Are implicit and explicit measures of SNAs (i.e. SNARC and number line estimation) consistent within individuals? If so, what are their relative contributions to various measures of mathematical cognition? As a result of the investigations detailed in this dissertation, we now know that implicit and explicit SNAs for fraction magnitudes diverge in mid-childhood and differentially account for variability in mathematical outcome measures. Performance on a number line estimation task for fractions—a more explicit measure of SNAs—appears to be a more reliable index of internal magnitude representations than the SNARC effect—a more implicit measure of SNAs. Here I will discuss the overall support for this conclusion, including relevant theoretical perspectives.

To get a broad view of the nature of implicit and explicit spatial-numerical associations for fractions, I employed multiple complementary empirical approaches. First, in Chapter 3, I replicated my own previous work by showing that a large group of neurotypical adults exhibited a classic SNARC effect for fraction magnitude comparison. This approach was extended to children in Chapter 4, in which I found that 3rd graders were more likely than 6th graders to show a SNARC effect for fractions. There was no evidence of a correlation between the SNARC effect and the distance effect for fractions in either of these studies, a relationship that for whole numbers has received mixed support despite the fact that both effects are believed to reflect processing of magnitudes on the mental number line. Additionally, in the adult sample, there was a moderate correlation between SNARC and PAE; however, no such relationship was observed in children. These two measures are both considered indices of SNAs that vary in their level of explicitness. That they were not correlated in childhood—in addition to the fact that PAE more consistently predicted variability in mathematical outcomes for both children and adult—provides strong support for the view that more implicit and more explicit SNAs have dissociable developmental trajectories.

The relationship between implicit/explicit SNAs and internal/external representations was tested in Chapter 5, in which I employed similar methods in a sample of number-form synesthetes. Despite the synesthetes' reported conscious access to their internal conceptualizations of number, there were still significant dissociations between multiple measures intended to measure internal spatial-numerical representations. Participants' SNARC effects largely contradicted their own explicitly reported/drawn number forms, which also differed in unpredictable ways from their performance on NLE tasks. Taken together, these studies highlight the extreme variability in human SNAs, and suggest that intra-individual consistency for SNAs is low even in a population with conscious access to their internal representations.

There are a number of ways to account for and make sense of the variability evident across the studies presented here (see Chapter 5 for additional detail on many of these considerations). The first emergent theme from this work is that there is no simple or direct correspondence between internal spatial-numerical conceptualizations and external representations. Rather, the relationship between internal and external representations is quite nuanced and subject to factors such as 1) the nature of the measure(s) used, 2) stimulus and environmental characteristics, and 3) general strategic variability. These factors likely converge to result in the widespread inter- and intra-individual variability evident in both this dissertation and other investigations of SNAs.

While the high variability for these implicitly-accessed SNAs might lead one to conclude that they are simply unreliable and idiosyncratic phenomena, I instead propose that these patterns of variability are in line with well-articulated theoretical perspectives on cognitive development. The first such model is overlapping waves theory (Siegler, 1996), which suggests that people choose from multiple problem-solving strategies available to them at any given time. These available strategies might even vary on a trial-by-trial basis for repetitions of the same item, but are believed to change over time due to experience and characteristics of the problem at hand. New and old preferred strategies fade in and out, respectively, over the course of development as the learner acquires additional experience. Overlapping waves theory helps to account for both intra-individual variability (learners have multiple strategies to choose from at any given time) and inter-individual differences (strategy use and choice adapt over time due to individual experience), and can help account for the inconsistent developmental timeline that has emerged for implicit SNAs, including those for fractions described here. In fact, this model has recently been successfully applied to individual strategy choice in fraction magnitude comparison (Fazio, Dewolf, & Siegler, 2015). A complementary account has been proposed by Alibali & Sidney (2015), who further assert that variability arises as a result of dynamic strategy choice, and that this strategy choice is shaped by the strength and accessibility of internal representations. Together, these theories provide a compelling account for the individual differences in SNAs evidenced in these studies and others.

Lastly, these accounts are also complementary to a predominant theory of neural development described in Chapter 2—interactive specialization (Johnson, 2011). This theory supposes a bi-directional relationship between brain structure and function, with multiple dynamic processes developing simultaneously and exerting influence on each other. As detailed in Chapter 2 (Toomarian and Hubbard (2018a)), interactive specialization can help to explain the variability, flexibility, and cultural differences in SNAs evident in a range of studies over the last three decades. Specifically, over the course of development, sensory experiences with magnitudes as well as cultural experience (e.g. language and gesture) influence the functional specificity of relevant brain regions, which in turn exert influence over cognition and spatial-numerical representations. These dynamic, emerging brain structures and functions may help to account for—and provide a mechanism for—the variability in implicit and explicit SNAs that we see over the course of development. Behavioral research supports such bidirectional influences; spatial skills and math abilities show strong cross-domain relationships throughout development, but the relationships between specific tasks change with age (Mix et al., 2016). Additionally, longitudinal data on NLE performance and math achievement shows mutual influence (NLE impacts math ability and vice versa) in early childhood years (Friso-van den Bos et al., 2015).

The studies presented here highlight the level of nuance necessary to appropriately characterize spatial-numerical associations. There is no one clear-cut influence, mechanism, or profile that has emerged from this collection of studies; rather, the primary revelation appears to be that SNAs for fractions are incredibly dynamic and subject to a range of influences. These influences may be intrinsic (e.g. competing strategies, linguistic influences) or extrinsic (e.g. stimulus properties, task demands) or more likely a combination of both. Ultimately, the SNARC effect for fractions seems to be less reliable and less predictive of higher-level cognition than more explicit assessments of internal SNAs, such as the number line estimation task.

Moving forward, this conclusion carries implications for both basic cognition research and more practical application such as math remediation. From a basic research perspective, it's apparent that the SNARC is a relatively weak and unreliable effect. While a fascinating phenomenon, particularly given the wide range of cultures and paradigms in which it has been observed, it is too easily influenced by fleeting factors to be a useful tool for inquiry. However, number line estimation tasks also make use of the close link between numbers and space, and are not only more robust than SNARC effect, but have been repeatedly shown to predict other cognitive factors (for a review, see Schneider et al., 2018). From an applied and educational perspective, these results suggest that training on number line estimation may be a more expedient and reliable way to train mental representations of number (and fractions in particular) than relying on more implicit association tasks. Number line estimation tasks specifically strengthen the link between external number lines and spatially-oriented mental representations of numerical magnitudes. Number line training has proven successful for both children with developmental dyscalculia and typical controls in improving spatial representation of numerical magnitudes and improving algebra (Kucian et al., 2011). A full-body approach to number line training using motion-sensitive devices to monitor body movements and gestures—which is grounded in the embodied cognition view of SNA development (e.g. Fischer et al., 2016)—has already proven effective at improving standardized math scores (U. Fischer et al., 2011). Ultimately, researchers should seriously consider the relative affordances of various measures of SNAs when deciding which may be most powerful for assessment and training internal conceptions of number.

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List of Appendices

Appendix A. Fraction Knowledge Assessment (FKA) for Adults

First Fraction Questionnaire

Do not start until you press the Space Bar on the computer.

Participant Code:
Date:
Time:
Computer:

1.

Circle the larger fraction in each pair. The first one is done for you.

Ⓐ $\frac{1}{4}$ $\frac{3}{4}$ Ⓑ $\frac{3}{7}$ $\frac{5}{7}$ Ⓒ $\frac{3}{5}$ $\frac{3}{4}$ Ⓓ $\frac{2}{5}$ $\frac{3}{10}$ Ⓔ $\frac{4}{5}$ $\frac{5}{6}$

2.

In which of the following are the three fractions arranged from least to greatest? Circle your answer.

Ⓐ $\frac{2}{7}$, $\frac{1}{2}$, $\frac{5}{9}$

Ⓑ $\frac{1}{2}$, $\frac{2}{7}$, $\frac{5}{9}$

Ⓒ $\frac{1}{2}$, $\frac{5}{9}$, $\frac{2}{7}$

Ⓓ $\frac{5}{9}$, $\frac{1}{2}$, $\frac{2}{7}$

Ⓔ $\frac{5}{9}$, $\frac{2}{7}$, $\frac{1}{2}$

3.

Circle the larger fraction in each pair. The first one is done for you.

Ⓐ $\frac{1}{4}$ $\frac{3}{4}$ Ⓑ $\frac{4}{9}$ $\frac{2}{3}$ Ⓒ $\frac{6}{5}$ $\frac{5}{6}$ Ⓓ $\frac{7}{6}$ $\frac{83}{90}$ Ⓔ $\frac{14}{13}$ $\frac{13}{12}$

4.

In which list of fractions are all of the fractions equivalent? Circle your answer.

Ⓐ $\frac{1}{2}$, $\frac{2}{4}$, $\frac{4}{6}$

Ⓑ $\frac{2}{3}$, $\frac{4}{6}$, $\frac{8}{12}$

Ⓒ $\frac{2}{5}$, $\frac{4}{10}$, $\frac{8}{50}$

Ⓓ $\frac{3}{4}$, $\frac{4}{6}$, $\frac{6}{8}$

5.

Which of the following numbers is SMALLEST? Circle your answer

Ⓐ $\frac{1}{2}$

Ⓑ $\frac{5}{8}$

Ⓒ $\frac{5}{6}$

Ⓓ $\frac{5}{12}$

6.

Mary and John both have pocket money. Mary spends $\frac{1}{4}$ of hers, while John spends $\frac{1}{2}$ of his.

Is it possible for Mary to have spent more than John? ______________________________

7.

Estimate the sum of $\frac{12}{13}+\frac{7}{8}$. Which of the following numbers is closest to this sum? Circle your answer.

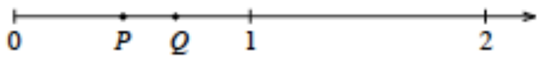
Ⓐ 1

Ⓑ 2

Ⓒ 19

Ⓓ 21

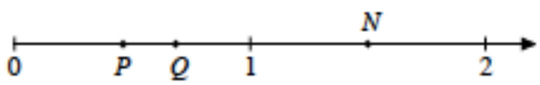
8.

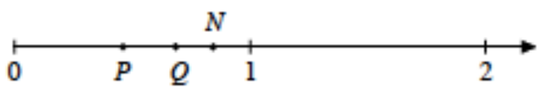


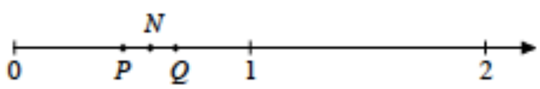
P and *Q* represent two fractions on the number line above.

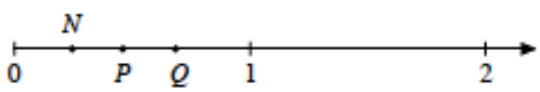
$P \times Q = N$.

Which of these shows the location of *N* on the number line?

Ⓐ 

Ⓑ 

Ⓒ 

Ⓓ 

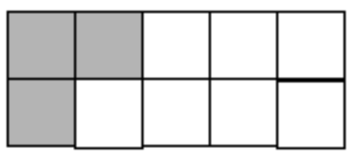
9.

How many possible fractions are between $\frac{1}{4}$ and $\frac{1}{2}$?

____

10.

In the figure, how may MORE small squares need to be shaded so that $\frac{4}{5}$ of the small squares are shaded?



Ⓐ 5

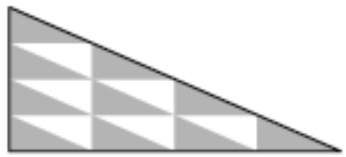
Ⓑ 4

Ⓒ 3

Ⓓ 2

Ⓔ 1

11.



In the figure above, each of the smaller triangles has the same area. What is the ratio of the shaded area to the unshaded area?

Ⓐ 5 : 3

Ⓑ 8 : 5

Ⓒ 5 : 8

Ⓓ 3 : 5

STOP

End of First Section.

If there is time remaining, feel free to complete any unfinished problems.

If there is time left and you are done, press the Space Bar on the computer.

Don't start the second section until you have pressed the Space Bar again.

12.

The fractions $\frac{4}{14}$ and $\frac{\square}{21}$ are equivalent.

What is the value of $\square$?

Ⓐ 6

Ⓑ 7

Ⓒ 11

Ⓓ 14

13.

$$\frac{3}{5}+\left(\frac{3}{10}\times\frac{4}{15}\right)=$$

Ⓐ $\frac{3}{51}$

Ⓑ $\frac{1}{6}$

Ⓒ $\frac{6}{25}$

Ⓓ $\frac{11}{25}$

Ⓔ $\frac{17}{25}$

14.

What is the value of $\frac{4}{5} - \frac{1}{3} - \frac{1}{15}$?

Ⓐ $\frac{1}{5}$

Ⓑ $\frac{2}{5}$

Ⓒ $\frac{7}{15}$

Ⓓ $\frac{3}{4}$

Ⓔ $\frac{4}{5}$

15.

$$\frac{2}{5} + \frac{5}{4} + \frac{9}{8} =$$

Ⓐ $\frac{16}{17}$

Ⓑ $\frac{41}{40}$

Ⓒ $\frac{81}{40}$

Ⓓ $\frac{111}{40}$

Answer the following questions.

16.	$\frac{2}{3}$ of 12 =
17.	$1 - \frac{5}{12} =$
18.	$\frac{1}{3}$ of 6 = $\frac{1}{4}$ of $\square$, what should $\square$ be?
19.	$3 \times 10\frac{1}{2} =$
20.	$40 \div 10\frac{1}{2} =$
21.	$\frac{3}{8} + \frac{2}{8} =$
22.	$\frac{1}{10} + \frac{3}{5} =$
23.	$\frac{1}{6}$ of $\frac{3}{4} =$

24.	$1\frac{1}{4} \times 7 =$
25.	$\frac{3}{4} \div \frac{3}{8} =$
26.	$2 \times \frac{1}{8} =$
27.	$\frac{1}{3} + \frac{1}{4} =$
28.	$1\frac{1}{5} - \frac{3}{5} =$
29.	$2\frac{1}{2} - \frac{3}{4} =$
30.	$32\frac{2}{3} + 5\frac{1}{4} =$
31.	$\frac{2}{7} + \frac{3}{4} =$
32.	$\frac{6}{55} \div \frac{3}{25} =$

STOP

End of Second Section

If there is time remaining, feel free to complete any unfinished problems (on the second test).

If there is time left and you are done, press the Space Bar on the computer.

Almost done! Before you leave, please answer the brief questions on the next two pages.

Age:

Sex: **M** **F**

Do you have normal or corrected to normal vision?: **YES** **NO**

Major (if declared):

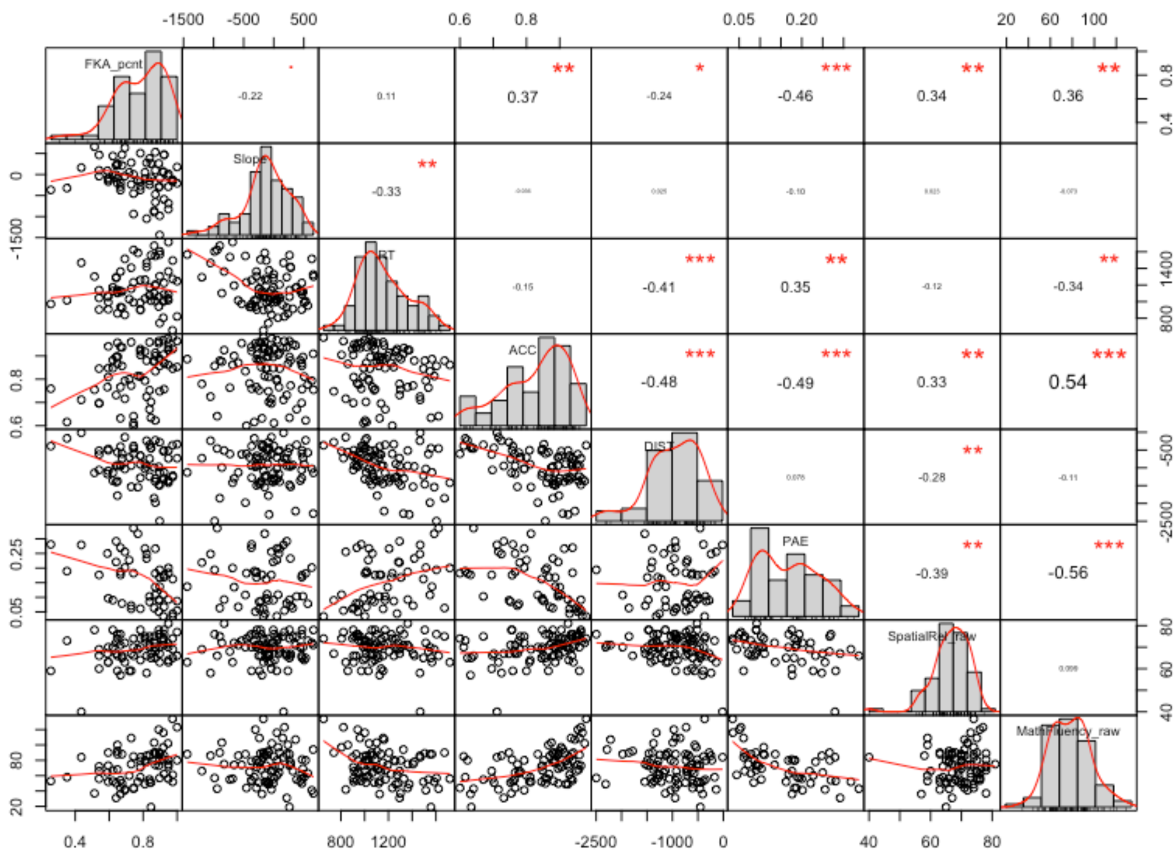
Likely major (if not declared):

What country (or countries) did you attend elementary school in?

What is your native language (or languages)?

When you chose the larger fraction on the computer, how did you make your decisions?

Appendix B. Correlation Data for Chapter 4



Appendix C. Number-Space Synesthesia Questionnaire

The following questions were presented as a 13-page questionnaire, with several inches of blank space following each question for adequate elaboration. If additional space was allotted for a question, it is noted below.

1. Number-form Synesthesia: Basic Questions

Number-form synesthesia is a phenomenon in which people *automatically* and *consistently* associate numbers with specific locations in space. Do you experience number-form synesthesia? Yes / No

Some people with number-form synesthesia also visualize other ordered sequences (e.g., letters of the alphabet, weekdays, months, temperature) spatially. Please list all sequences that you visualize spatially.

Are your synesthetic experiences very apparent or do you have to make a special effort to attend to them?

Can you ignore your number-form synesthesia? Yes / No

If ***yes***, please describe how and when you can do this.

Can you completely stop your number-form synesthesia? Yes / No

If ***yes***, please describe how and when you can do this.

How, if at all, do your number-form synesthetic associations change when you are using other languages you may know?

Do you have a form for historical year numbers (e.g., 1996)? Yes / No (Please circle one)

If ***yes***, is it identical to the form you have for numbers (e.g., does the year 1996 occupy the same location as the number 1,996)? Please describe.

Similarly, if you have any other forms that involve numbers (such as temperature or height), do those values have the same location as the numbers in your number form (e.g., is 67 degrees Fahrenheit in the same location as the number 67)? Please describe.

Please illustrate, to the best of your ability, the locations of the numbers 0 to 1 on your number form. [Full page allocated]

Please illustrate, to the best of your ability, the locations of the numbers 1 through 10 on your number form. [Full page allocated]

Please illustrate, to the best of your ability, the locations of the numbers 1 through 100 on your number form. [Full page allocated]

2. Visualization and Experience of Number-form Synesthesia

Would thinking about one stimulus (e.g., the number 7) still bring to mind its association (i.e., its spatial location)? Would it be the same when you dream?

Is your synesthesia present when you close your eyes? Yes / No

Do you visualize your associations in your mind, or do you see them superimposed on your visual field?

Can you pinpoint the synesthetic perception in space?

Do you experience something (ex. colors, objects, or patterns) projected somewhere in front of you?

Can you point to an exact point where your visualizations occur or is your experience relatively scattered?

Can you manipulate your number form in your mind's eye, (such as being able to "zoom in" or "zoom out" on particular sections of numbers, or rotate it), or is your number form static? Please describe.

Does the location change in different environments (e.g., so that it will occupy "free" space left in a specific setting and not interfere with the perception of other objects)?

How long does your number-form synesthesia typically last when you experience it?

Does your spatial form go away immediately after the stimulus is removed or the thought disappears, or does it fade out more slowly? Please describe.

Are there any conditions under which your number-form synesthetic associations are not as strong or reliable (e.g., if you are tired, on medication, etc.)?

Do certain emotions or how hard you focus affect your number-form synesthesia? Do the associations get stronger or weaker?

Does your number-form synesthesia interfere with other things or your day-to-day activities?

Does your number-form synesthesia ever distract you? Yes / No

If ***yes***, please describe how and when this tends to happen.

3. Development of the Number-Form Synesthesia

Are you aware of any family members that experience this form of synesthesia? Yes / No. If ***yes***, which family member(s)?

At what age did you begin to experience number-form synesthesia?

Have your number-form synesthetic experiences been consistent since they developed? (For example, is the number 1 always visualized in the same location?) Yes / No

If ***no***, please describe how your associations have changed.

Can you think of any experiences that could have shaped your number-form associations?

Under what conditions do you experience number-form synesthesia?

Are there any particular triggers?

4. Whole Numbers and Fractions

Do Roman numerals or other nonsymbolic number representations have specific locations in space for you? Yes / No

Can you list different classes of numbers (e.g. whole numbers, fractions, negative numbers) for which you have a spatial association? Do you visualize them in separate forms or are they integrated?

If yes, can you describe your spatial locations for:

fractions (e.g., 3/5)? Yes / No

decimal numbers (e.g., 0.3)? Yes / No

negative numbers (e.g., -4)? Yes / No

irrational numbers (e.g. pi)?

imaginary numbers (e.g., *i*)? Yes / No

Do you find math difficult or easy? Please describe your general experience with math.

Do you feel that your number-form synesthesia has helped or hindered your math abilities? Please describe how.

Can you describe the process you would use to solve a simple equation? For example, how would you approach 7 + 5=?

Does this approach to solving math problems change for fractions? For example, ¼ + 3/5?

Do you believe you could perform mathematics or simple number operations if your synesthesia went away? Yes / No If ***no***, please explain why you feel this way.

Does your number-form synesthesia aid you when doing daily tasks involving fractions (for example, measurements when baking, dividing a cake, etc.)?

How similar or different are your associations for fractions and decimals from other whole numbers?

5. Background Information

What languages other than English, if any, can you speak, read, and/or write? (Please indicate age of acquisition and proficiency for each)

Have you ever lived or spent a significant amount of time in a country other than the United States? Yes / No

If ***yes***, which one(s)?

Are you artistic? Yes / No

If ***yes***, what kind of art do you engage in (e.g., painting, sculpture, photography, etc.)?

Would you describe yourself as having a good memory? Do you have a better for certain types of information over others (e.g., names, locations, etc.)? Please explain.

Do you have a good memory for dates and/or times? Yes / No

Do you have a good sense of direction? Do you have difficulty navigating new areas or using maps? Please describe.

Do you consider yourself to have high mental visual imagery? On a scale from 1 to 9, where 1 is very easy and 9 is very difficult, how easy or difficult is it for you to create detailed visual images in your head?

1	2	3	4	5	6	7	8	9

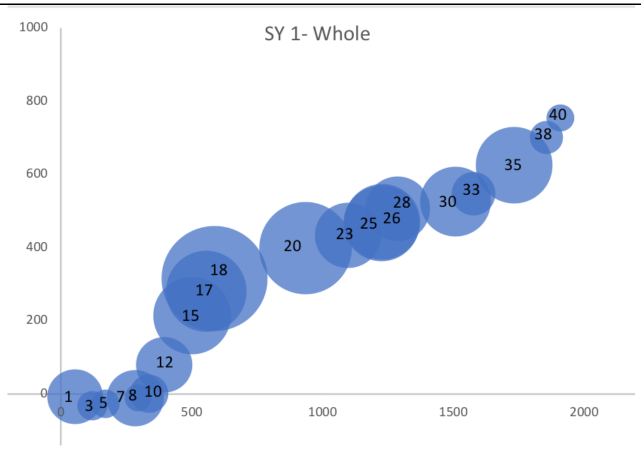
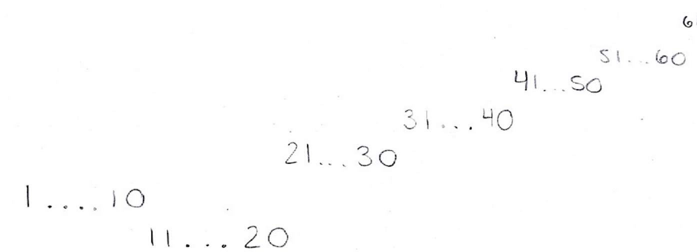
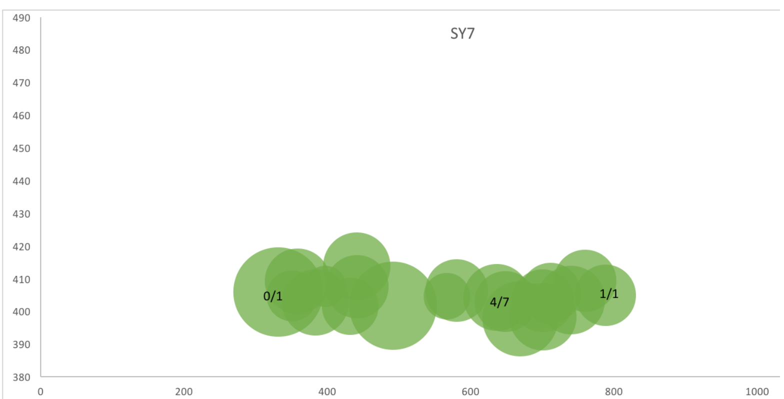
Very easy Very difficult

Please explain your answer:

Appendix D. Unconstrained Number Placement Data

The following graphs represent participant responses on the unconstrained number placement task for participants not featured in the main manuscript. For each synesthete, responses to the fractions task and whole number task are displayed next to the corresponding drawn number form. Cells containing “n/a” indicate that the participant did not report and/or did not draw a number form for fractions.

<u>Synesthete 1</u>

	
	n/a

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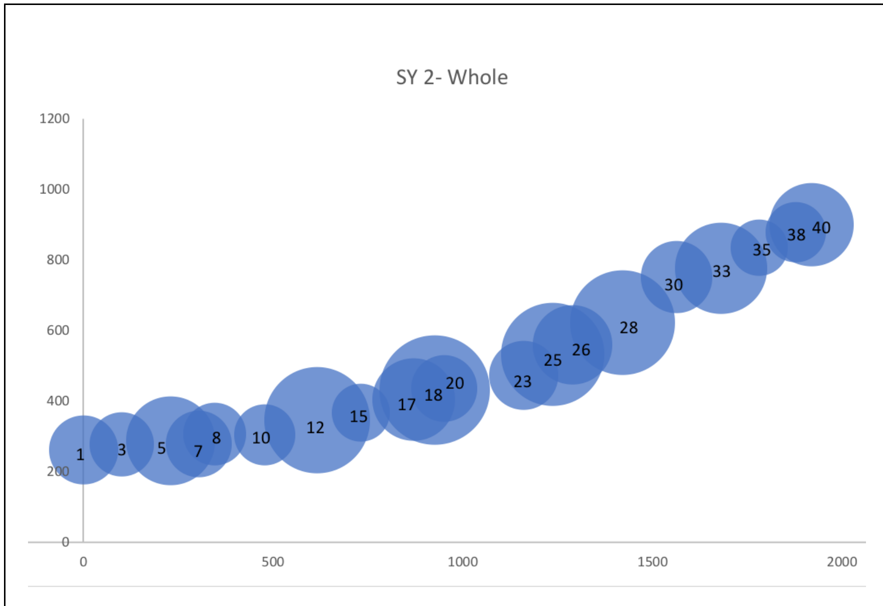

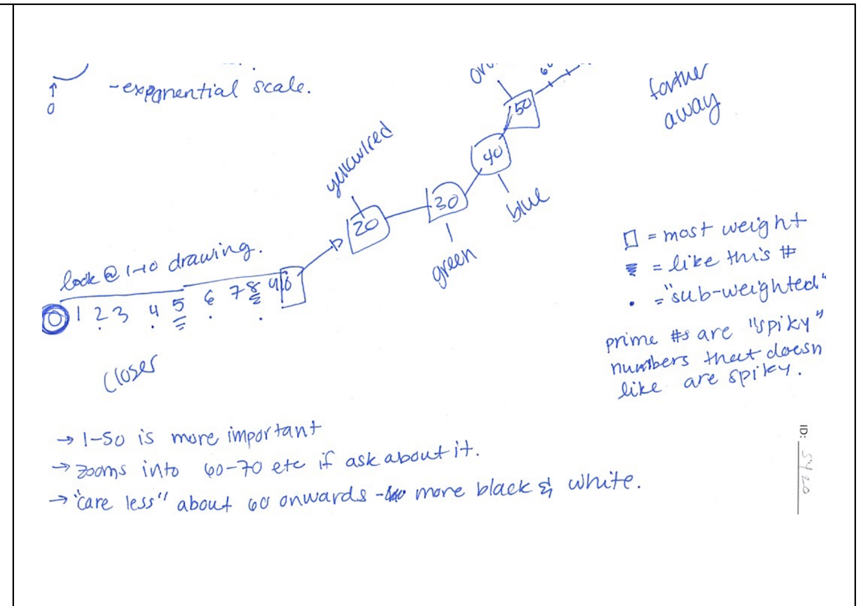

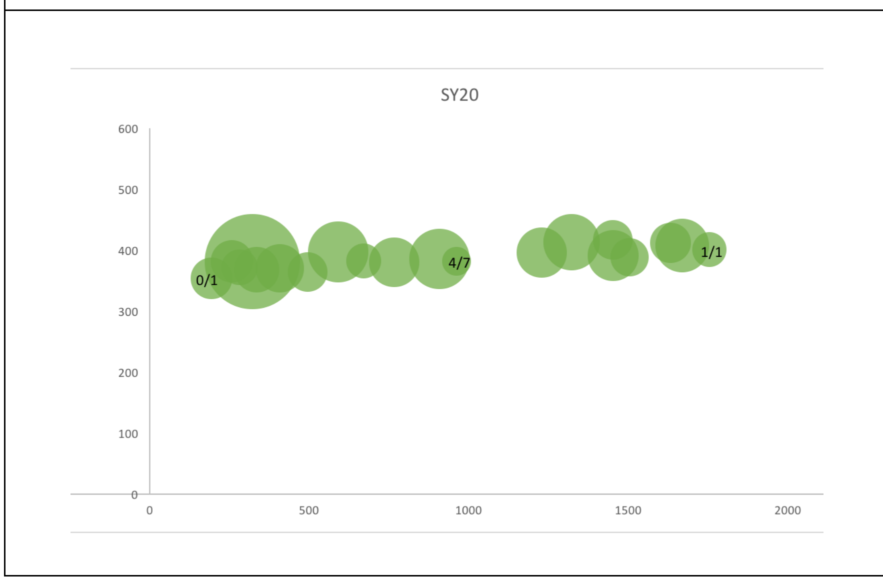

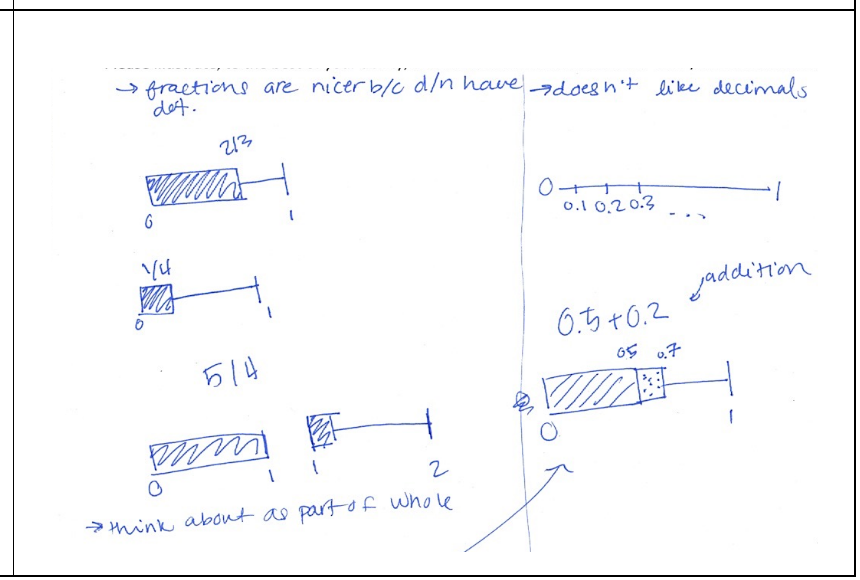

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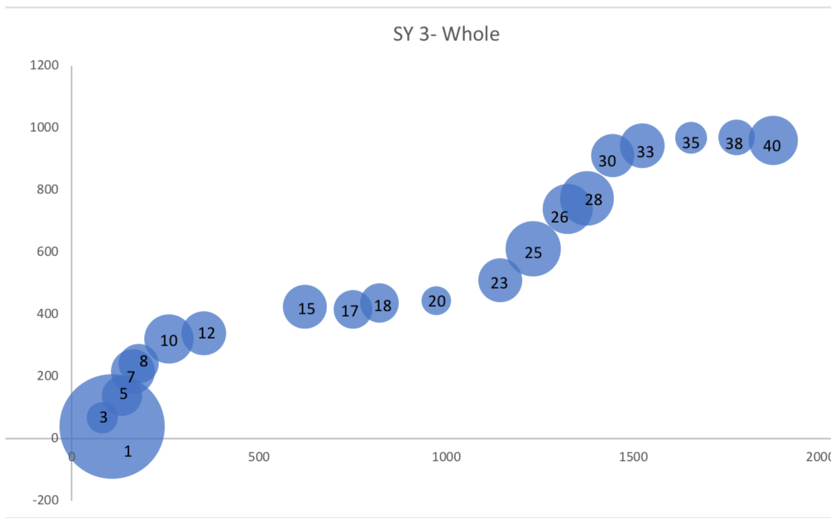

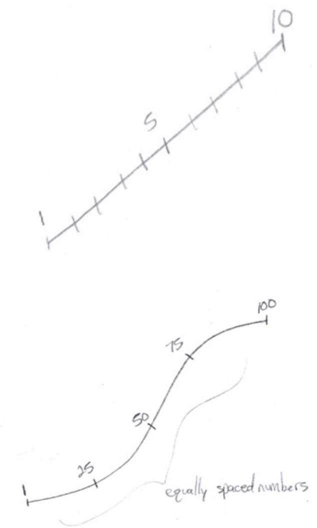

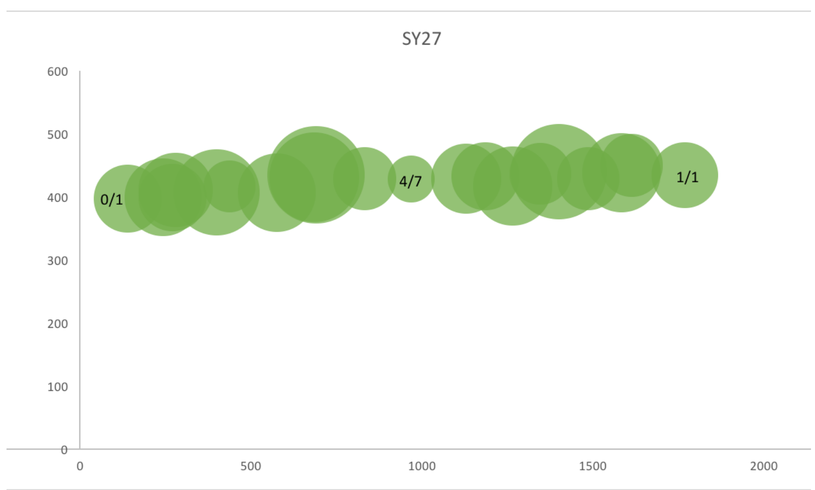

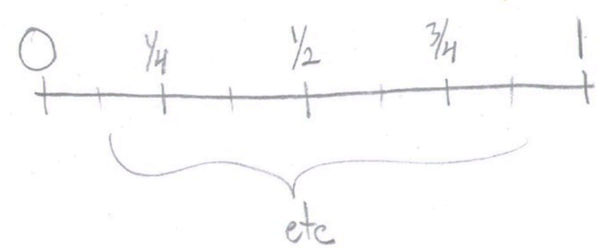

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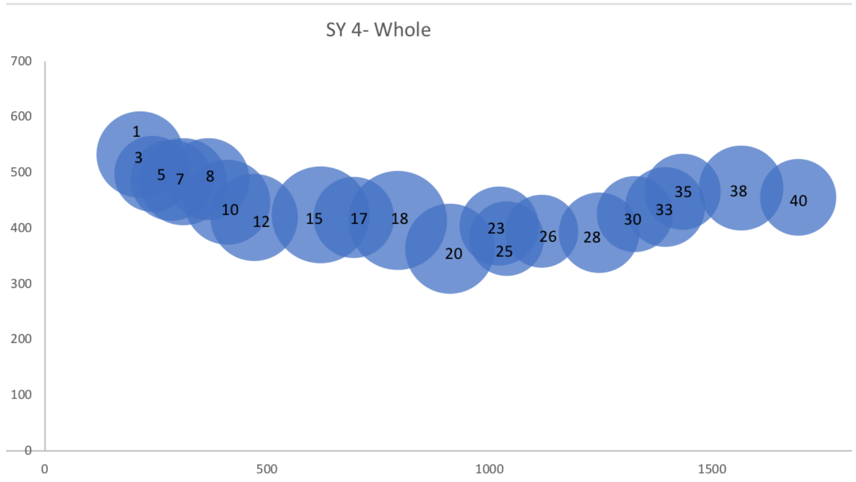
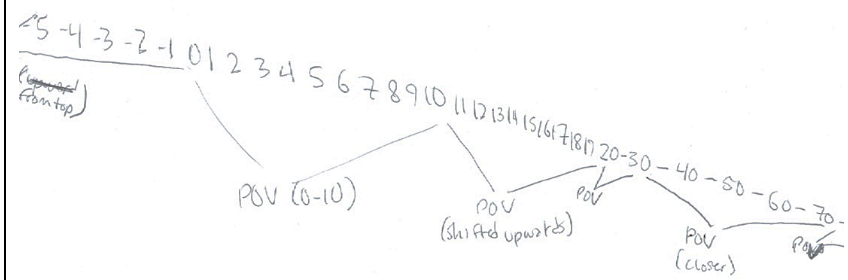
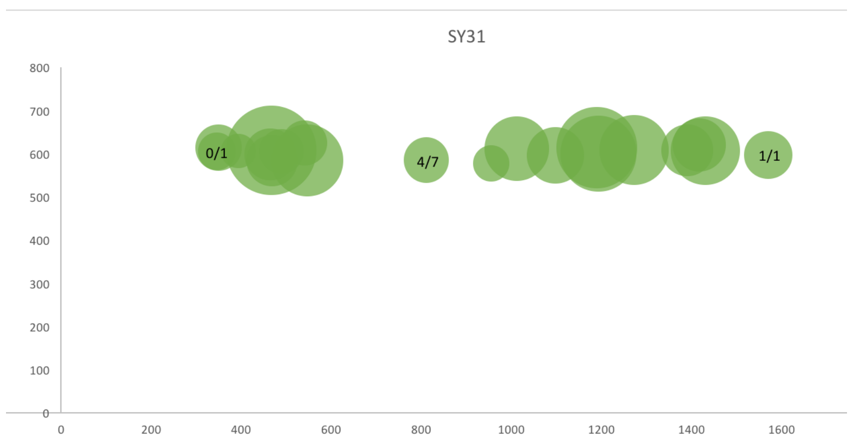
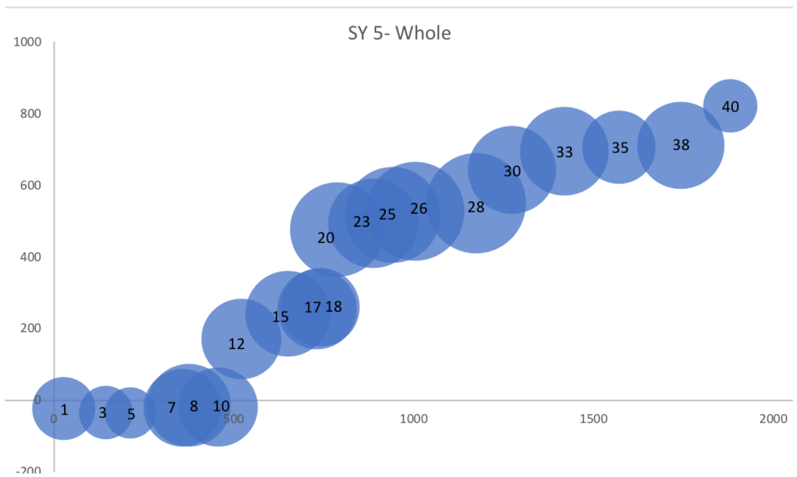


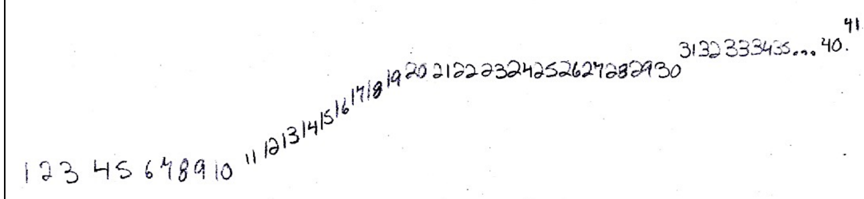


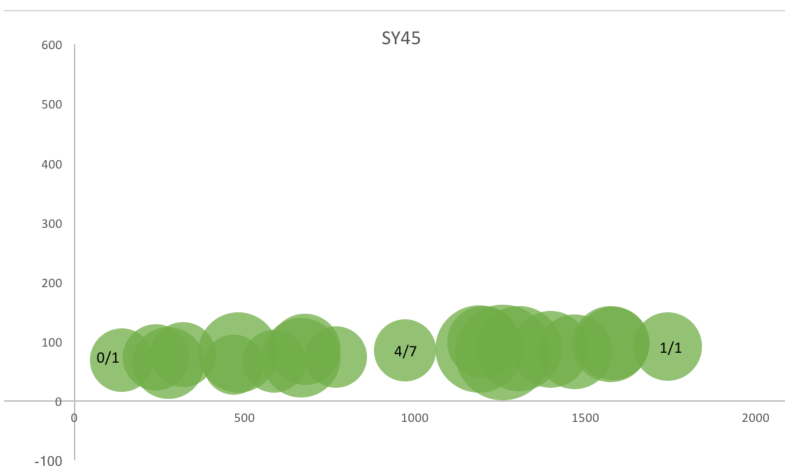


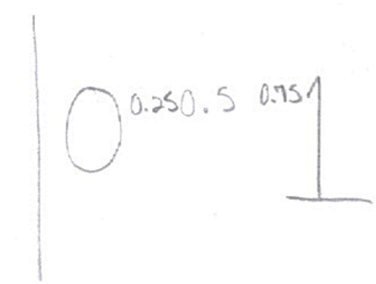
n/a

Synesthete 5

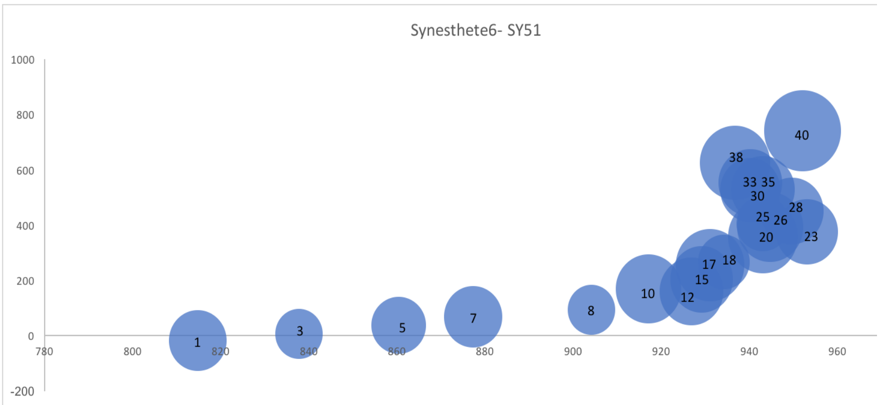


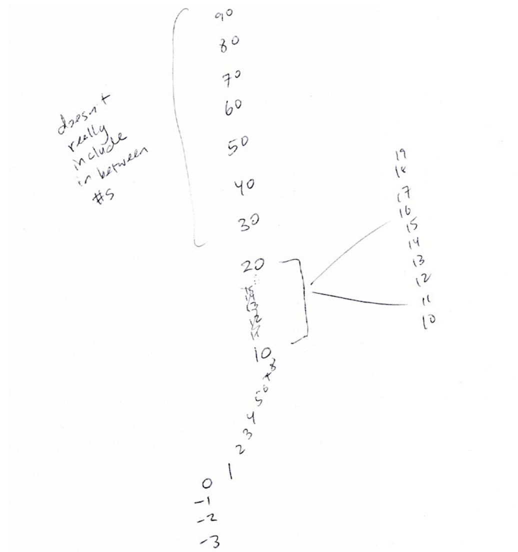


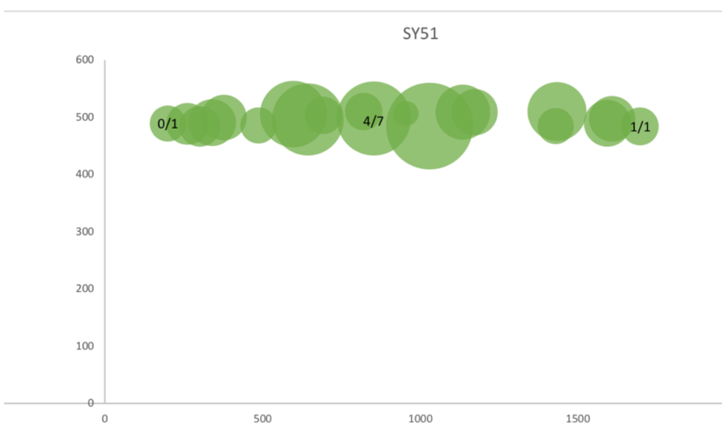




Synesthete 6







n/a

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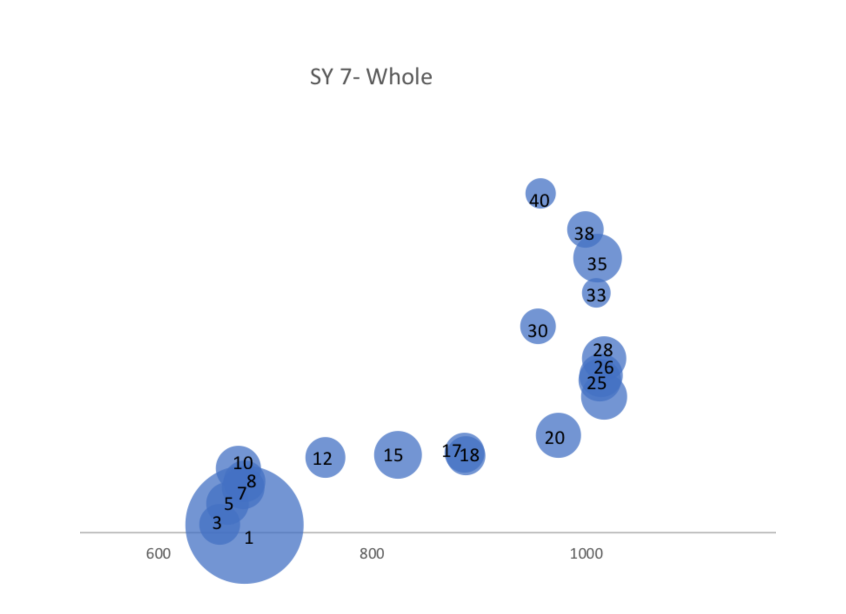

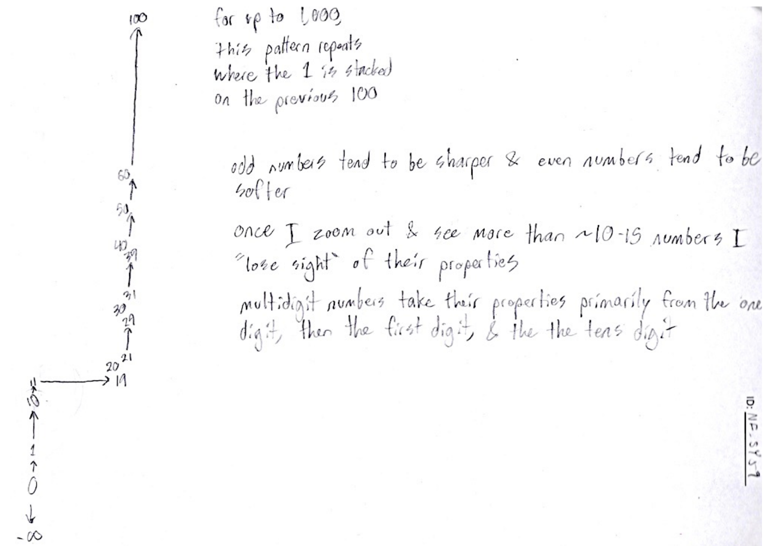

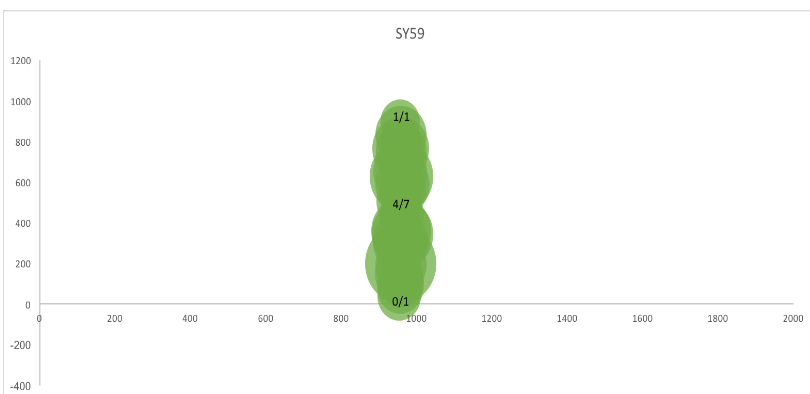

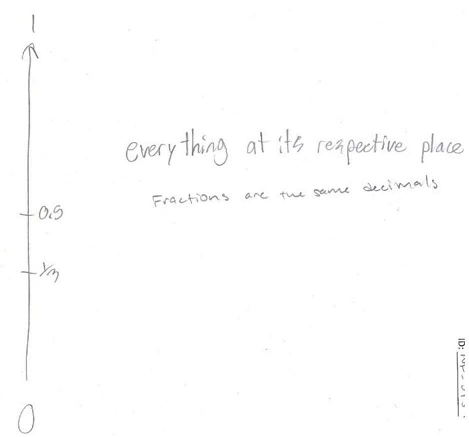

Synesthete 8