Teaching Mathematics to Language Learners Accounting for Language

By

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This dissertation is dedicated to my family who sees me and loves me for who I am and to Our Lady of Guadalupe who has guided me through my higher education

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Abstract

In this study I explored the way in which mathematics was taught and discussed by bilingual (Spanish and English) teachers in order to understand what language practices teachers use to support bilingual students mathematics learning. Three elementary school teachers were observed during their mathematics lessons. The study found that it is difficult to disentangle mathematics from language but that there is a pattern of entanglement as teachers move from one participant structure to the other. As teachers moved from whole group to small group to individual participant structures, there were shifts from a broad use of language toward a more direct and specific mathematical language – what I refer to as “bilingual math teacher moves”. Seven *bilingual math teacher moves* were identified (a) tricky words, (b) context, (c) explanations, (d) redirecting questions, (e) re-voicing and clarifying ideas, (f) directing questions, and (g) directly modeling.

> *Lo que yo hice fue que para que, tome seis, y tres más tres es seis, y dos si le quitas 10 es 10 y después le quitas tres son 13 y allí esta.*
>
> *What I did so that it would, I took six, and three plus three is six, and two if you take 10 its 10 and then you take three so there's 13 and that is that.*

Nancy, a fifth grade student in a bilingual classroom, was sharing with her class her strategy for finding half of 26. Her teacher was unable to fully follow her but after a few questions she was able to understand that Nancy was subtracting half from each digit and adding at the end to find 13, half of 26. Nancy's answer though linguistically complex and difficult to follow was mathematically valid. This solution strategy was not a traditional one but explanations like hers are common in a classroom that supports teaching with understanding (Carpenter & Lehrer, 1999), where a teacher must make sense of a student's mathematical conceptions in the ways that they express them. Nancy's explanation did not follow a simple sentence structure that would allow for her explanation to be clearer. Her example demonstrated a unique use of language in order to describe a mathematical strategy. Some may say that Nancy's use of language was academically weak or that she was unable to use the mathematical language correctly. However upon listening closely and asking the right follow-up questions Nancy's solution strategy becomes clearer and her original explanation is seen as valid. Nancy's teacher was fluent in both Spanish and English and Nancy's first language was Spanish.

Nancy's example is of interest because of the ways in which the classroom community and the teacher's practice with respect to mathematics and language supported Nancy's participation in mathematics. Nancy's explanation required her teacher to have flexibility with the Spanish language and mathematics. Focusing on Nancy's use of the Spanish language or mathematical language is not enough to understand her solution strategy. In order to understand

Nancy's example it is necessary to understand the usage of language as a whole in the instruction and learning of mathematics in a bilingual classroom.

In this article, I focus on three elementary bilingual mathematics classrooms in order to identify the language practices around the teaching and learning of mathematics. Specifically my analysis focuses on identifying teachers' responses to language and math entanglement that is present during mathematics instruction. Latinos are the fastest growing group of students in the United States public schools (Kohler & Lazarín, 2007) and now one in four children has at least one immigrant parent, that is a parent who is foreign born and 60% of the parents of children under eight are from Mexico or Central America (Fortuny, Hernandez, & Chaudry, 2010; Hernandez, Denton, & Macartney, 2008). This fact makes understanding what teachers do to support bilingual students learning of mathematics that much more imperative. Nancy's explanation is not an uncommon one, however, her experience in a classroom where her ideas are fully explored is.

Literature Review

In this review of literature, I first provide a context for bilingual education in the U.S. followed by a discussion on language and mathematics. The discussion on language and mathematics begins by defining the entanglement between language and mathematics and this is extended through a discussion on: (a) the beliefs about language and mathematics and (b) mathematics teaching through an understanding of language.

Advocates for bilingual instruction believe that instruction in a student's native language will help the student meet his/her needs in the English language while maintaining high academic standards **(**August, Calderon, Carlo & Nuttal, 2006; Cummins, 2001; Thomas & Collier, 2002; Turner & Celedón-Pattichis, 2011**).** Studies have found that bilingual instruction produces higher test scores than the English as a Second Language (ESL) instruction on academic content assessments and measures of reading comprehension (August & Shanahan 2008; Genesee, Lindholm-Leary, Saunders, & Christian, 2006; Goldenberg 2008; Saunders & Goldenberg 1999; Thomas & Collier 2002, 2003). These studies have also found that there is a significant difference in English proficiency when students are instructed in their native language. English language learners (ELLs) instructed in their native language have higher English proficiency then ELLs who do not receive native language support.

Research has shown that many teachers consider mathematics to be a universal language, and that they believe the teaching and learning of mathematics does not require a vast amount of language proficiency in the language in which mathematics is being taught (Fernandes, 2012; McLeman, Fernandes, & McNulty, 2012; Rolka, 2004). Other teachers see teaching mathematics to ELLs or to students in the teacher's nonnative language as uncomplicated given the computational/procedural nature of mathematics (Rolka, 2004, Tevebaugh, 1998). Nancy's example demonstrates that regardless of the language used by the teacher or student, children use a wide range of non-mathematical language during a mathematics lesson that requires close attention.

Understanding Language and Mathematics in Teaching ELLs

When teaching mathematics to ELLs, teachers tend to take three approaches based on their beliefs about mathematics and language. The first is the belief that English needs to be taught first in order to properly teach mathematics. The second is the belief that mathematics is a universal language and thus teaching mathematics to ELLs is not complicated by the students limited English language proficiency. Lastly, there are teachers who believe that mathematics cannot be disentangled from the language it is taught in and thus they must attend to the various mathematical registers/language of the classroom.

English First. Some teachers believe that in order to properly teach mathematics to second language learners they must first focus on the second language. This belief is based on an assumption that students will not be able to engage in the classroom and teachers will not be able to instruct mathematics unless the student is English proficient (Barwell, 2005; Fernandes, 2012; McLeman et al., 2012; Walker, Ranney, & Fortune, 2005). McLeman and colleagues (2012) surveyed 292 pre-service teachers from 12 urban teacher education programs and found that those with limited exposure to ELL issues and knowledge of a foreign language prior to the education program had more deficit models of ELLs than those whose had prior exposure. Their analyses of the survey questions revealed that pre-service teachers with deficit models believed that the use of a student's native language hinders the learning of mathematics, and that learning English is more important than maintaining students' native languages. The teachers with more knowledge of language learner issues did not all align with asset based views suggesting that pre-service teachers need more than exposure to ELL issues. Mathematics instruction from teachers who have this belief primarily focuses on a child's language attainment rather than on mathematical content knowledge. These teachers attempt to separate language from mathematics.

Is Mathematics a Universal Language? Researchers have shown that many teachers consider mathematics to be a universal language (Rolka, 2004; Tevebaugh, 1998). Research also shows that teachers believe the teaching and learning of mathematics does not require a vast amount of language proficiency in the language in which mathematics is being taught (Fernandes, 2012; McLeman et al., 2012; Rolka, 2004). Other teachers see teaching mathematics to ELLs or teaching mathematics to students in the teacher's nonnative language as uncomplicated given the computational/procedural nature of mathematics (Rolka, 2004; Tevebaugh, 1998).

Some scholars see mathematics as an academic language with its own vocabulary and set of norms. Mathematics as an academic language along with the development of that language has been discussed in research as a mathematical *register*. A mathematical register is situated within the social context in which it is constructed; thereby, refuting the notion that mathematics is a universal language.

> The notion of 'developing a language' means, therefore, adding to its range of social functions. This is achieved by developing new registers. A register is a set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings. We can refer to a 'mathematics register', in the sense of the meanings that belong to the language of mathematics (the mathematical use of natural language, that is: not mathematics itself), and that a language must express if it is being used for mathematical purposes. In order to express new meanings, it may be necessary to invent new words; but there are many different ways in which a language can add new meanings, and inventing words is only one of them (Halliday, 1978, p. 195).

Mathematical Language within a Language

Pimm (1987) extends the notion of mathematical registers by proposing that mathematics is a language with its own system of meaning making, which includes specialized terms, and the use of everyday language with specialized meaning, specialized expressions, and sentence construction that might be unfamiliar to everyday speech.

Although this may appear to support the notion of mathematics as a universal language some argue that more than one register exists. Farrugia (2009) and Moschkovich (2005, 2007) suggest that mathematical registers are situated within a culture, a community, or group who have informally developed this register. As a result, classrooms have multiple mathematical registers represented by the students and the teacher. These multiple registers lead to varied way of linguistically expressing mathematical knowledge. Furthermore, researchers argue that teachers and students with different mathematical registers may have difficulty when trying to transfer from one register to the other because they may lack full understanding of both registers (Moschkovich, 2005, 2007; Ron, 1999). Teachers must then decipher whether the students are struggling with their math knowledge or with expressing what they know about the math. Differing registers make this distinction more difficult to observe given that neither the teacher nor the student fully understand the register of the other.

Mathematical registers are developed by a group of people within a cultural context (Halliday, 1978), which supports other scholars' views that mathematics is not culture-free (D'Ambrosio, 1985; Presmeg, 1998). A teacher who believes that mathematics cannot be disentangled from language must be aware of how language and mathematics are confounded.

Suggestions for Teaching Mathematics to ELLs

Lass (1988) made several suggestions for improving mathematics instruction for bilinguals, which include having teachers: develop the students' native and English language; teach mathematics to bilingual children bilingually; recognize mathematical language but not as a universal language; teach in a culturally relevant manner; be familiar with out of school practices; and have an overall appreciation for how the native language can be used to teach mathematics effectively. Moschkovich (2013) extended this list of recommendations to include:

focus on students' mathematical reasoning, not accuracy in using language; focus on mathematical practices, not language as single words or vocabulary; recognize the complexity of language in mathematics classrooms and support students in engaging in this complexity; and treat everyday and home languages as resources, not obstacles. Moschkovich (2013) also lists outs way in which mathematics instruction for ELLs should align with the Common Core State Standards along with a list of how to adapt mathematics instructional materials to meet the needs of ELLs. Andersson (1977) also claimed that effective bilingual and foreign language teachers are both biliterate and bicultural, having an in depth knowledge of both languages and cultures. These suggestions all point towards the need for teachers to be knowledgeable of the interconnectedness of language and mathematics.

Teaching Mathematics through Language

The phrase 'teaching mathematics through language' refers to teaching with an awareness of the impact of language (mathematical, everyday, and home language) present during a mathematics lesson. Bilingual teachers, are not just teachers who share their native language with their students, they are also teachers who learn their students' native language as a second language. This can lead to difficulties when the teacher and students do not share a common language in which both can express their ideas fluidly (Ballantyne, et. al., 2008). Kasule and Mapolelo (2005) state "each learner's mother tongue [native language] is the key to the world and a means of alleviating the abstract nature of classroom learning events" (p. 602). When instruction is in one's native language from a non-native speaker the ideas presented in the classroom might not be the ones the instructor intended (Saat & Othman, 2010), students may be using a different mathematical register. Kasule and Mapolelo also argue that mathematics is a different language in itself, different from the language of instruction. Thus, it is not enough for teachers to know their students' native language; the teacher must also know mathematics in the students' native language.

When teaching mathematics in their second language, teachers face difficulties that are particular to mathematics. Moschkovich (2007) found that there can be an overemphasis on vocabulary, which narrows the mathematical communication opportunities for bilingual learners. Nonnative teachers cannot rely solely on translations of mathematical terms; they must be able to use the translated terms in conversations with students. Ron (1999) supports this idea by stating that many bilingual teachers do not possess the specialized vocabulary needed for mathematical instruction. She argues that there is more to the communication of mathematical ideas in a different language than just a mere translation.

Given that mathematics teaching is highly oral, the language demands in the teaching and learning of mathematics are substantial (Janzen, 2008; Lager, 2006; Ron, 1999). A teacher of mathematics not only uses and hears mathematical language but also everyday language in their math lesson and thus must attend to issues of language (Lager 2006; Ron 1999). A bilingual teacher is exposed to and must make sense of these mathematics registers in more than one language (Ron, 1999). An effective bilingual mathematics teacher is one that connects everyday language with mathematical language in both languages of instruction (Ron, 1999). The increased knowledge of the varying mathematics registers in the classroom makes this connection effective. However not all bilingual teachers possess the specialized vocabulary needed for mathematical instruction; the effective communication of mathematical ideas in a different language is more than a translation and one single register (Ron, 1999).

Teaching mathematics to ELLs has been found to be closely linked to understanding language as a resource (Gómez, Kurz, & Jimenez-Silva, 2011; Planas & Setati-Phakeng, 2014).

Mathematics teachers must also be language teachers given that they must be aware of their students' abilities to engage with mathematics through language. Difficulties with language during a mathematics lesson can become part of the mathematical exploration and thus enhance the learning experience (Elbers & de Haan, 2005). Elbers and de Haan found that students who were also ELLs skipped through discussions of everyday meaning of words and focused on the mathematical meaning of the word. Students' response to language and mathematics has been studied to some degree, but there are fewer studies of how teachers respond to students' use of language during a mathematics lesson. Morgan, Craig, Schuette, and Wagner (2014) claim that much of the focus on language and mathematics has been on focusing on specific mathematical constructs and that more research should be conducted on the more general issues. They suggest three areas of study, one of which is the knowledge and skills that teachers might use in order to support student's linguistic competence.

In this study I set out to understand the language utilized by bilingual teachers during their mathematics instruction. The study focused on identifying what language patterns were present during different participant structures where mathematical discussions were occurring in which the teacher was present. This is a shift from the literature that focuses on the student's acquisition of mathematical knowledge through language towards an understanding of how language and mathematics is negotiated by a bilingual teacher of children with varying levels of bilingualism. The following question and sub questions guided this study: How does a bilingual (Spanish and English) teacher's usage of language impact her mathematics teaching? (a) How were issues of language present during mathematics teaching? (b) How do teachers respond to issues of language present during mathematics teaching?

In the sections that follow, I outline the theoretical perspectives that guided my research and then describe the methods used in the study. This is followed by a presentation of the findings and a discussion that relates the complexities between language and mathematics.

Theoretical Perspectives

Ethnomathematics and Socio-cultural Theory

The above demonstrates that mathematics is not language and culture free. D'Ambrosio (1985) defined ethnomathematics as "the mathematics which is practiced among identifiable cultural groups, such as national-tribal societies, labor groups, children of a certain age bracket, professional classes, and so on (pg.45)." There is common misconception that mathematics is a language of itself that can be taught and learned as you would a foreign language (Rolka, 2004). Burton (2002) writes “I understand mathematics as an artefact of the culture in which it is created. Its observed universality, therefore, is a cultural creation and not to do with any inherent and person independent features.” Indeed mathematics is an artifact of the culture in which it is created however mathematics is created throughout the life of any one particular child. There have been a significant number of studies that show that different ethnic cultures process mathematical problems differently (Mendes, 2007; Moschkovich, 2007; Musanti, Celedón-Pattichis, & Marshall, 2009).

A sociocultural approach to learning emphasizes that all learners have cultures and that their cultures define who they are as learners. Sociocultural approaches first systematized by Vygotsky and his colleagues were explained as “based on the concept that human activities take place in cultural contexts, are mediated by language and other symbol systems and, and can be best understood when investigated in their historical development” (John-Steiner & Mahn, 1996,

p. 191). This approach emphasizes the interdependence of social and individual construction of knowledge.

If human activities take place in cultural contexts that are mediated by language than a teacher of bilingual and ELLs must take into account the language and culture of her students and its effects on their learning. John-Steiner and Mahn state, “in classrooms in which there is coparticipation, cooperative learning, and joint discovery, environments are created in which students are able to build upon the culturally shaped knowledge and value systems they bring to school” (1996, p. 201). Adding this to ethnomatematics tells us that mathematics is created through students’ engagement in the classroom that is mediated by language.

Supporting Mathematics Learning in Bilingual Classrooms

Franke, Kazemi and Battey (2007) summarize three features of classroom practices that they see as most central to understanding the teaching and learning of mathematics: creating mathematical classroom discourse, developing classroom norms, and building relationships that aid in mathematical learning. Under discourse they emphasize the importance of engaging students in classroom conversations, re-voicing of student ideas and the importance of tasks. In classrooms with ELLs it is important to recognize that mathematics is not merely a set of vocabulary and norms; this belief narrows the mathematical discourse opportunities for bilinguals (Moschkovich, 2007). Classrooms norms include student autonomy where a student is able freely express their ideas including the support or opposition to another student’s idea. For bilingual mathematics learners it is important that these students are allowed to express their ideas in their native language (Cummins, 2001; Thomas & Collier, 2003) while being encouraged to express these in their second language. Students bring with them a vast amount informal and formal knowledge of mathematics that teachers need to be receptive to. Franke and colleagues (2007) write “building relationship with students allows teachers to challenge assumptions about who students are and what they bring to the mathematics classroom in a way that support creating opportunities for participation and mathematical learning” (p. 242). Moschkovich (2005, 2007) argues that ELLs can have difficulties when transferring from one mathematical language/register to the other, because they lack a full understanding of both languages/registers. It is important for teachers to be aware that ELLs struggle with issues of language during their mathematics instruction.

Methods

Methodology

Qualitative methods were used for this study as they allowed for a comprehensive understanding of how teachers enacted various language practices to support their students mathematical understanding (Patton, 1990). In order to examine the relational nature of mathematics and language in elementary bilingual classrooms, I found that a case study design was the best way to document the ways in which three bilingual (Spanish and English) teachers used language during their mathematics teaching. The three teachers were observed during their mathematics lessons and later participated in a focus group in order to better understand the language complexities present during their mathematics teaching. Case study methodology allows for an in-depth look at phenomena within real-life contexts (Creswell, 2007; Merriam, 1998; Yin, 2009) by its focus on collecting detailed information in order to understand the intricacies within phenomena (Stake, 1995; Yin, 2009). Within this case study the data collected (videos, field notes, and focus group) focused on the teachers use of language in the mathematics classroom. The connections between language and mathematics in teaching is difficult to discuss, the three teachers selected for this study provided a deeper understanding of the role that language plays during their mathematics lessons. A case study design was selected because it provided a way to describe this particular situation in greater detail, in context, and holistically (Patton, 1990).

Participants and Location

The bilingual teachers who chose to participate in this study were elementary school teachers from a Community School associated with a large public university on the west coast. The Community School was a partnership between the University, the local city school district, and the local community. The K-12 bilingual school was surrounded by two separate ethnic communities, Latino and Korean. The Community School offered a multi-age learning environment; in the elementary grades students were grouped in multi-age dens: kindergarten and first grade, second and third grade, and finally fourth and fifth grade. The community school offered dual-language instruction where elementary grade classes were taught in English and Spanish or English and Korean. The elementary school followed a 90-10 bilingual instruction model, which had children begin Kindergarten and 1st grade with instruction that was 90% in Spanish and 10% in English. The percentage of English instruction increased as students moved through elementary school and ended with 50% of the instruction in English and 50% in Spanish.

In 2009, the Community School opened with 340 students in kindergarten to fifth grade encompassing their Lower School. In the 2011-2012 school-year the school grew to its capacity approximately 1,000 K-12 students. During the 2013-2014 school-year the school was 80% Latino and 14% Asian and 81% of the students were low-income. In the 2013-2014 school-year 75% of the Lower School students were classified as Limited English Proficient/English Language Learners. The Community School faculty in the 2013-2014 included 42 teachers, 24% male and 76% female. Fifty one percent of the faculty including the principal were Latino, 29%

were Asian, 22% were Caucasian, and 3% were African American. The faculty was 86% bi- or tri-lingual and about half of them were alumni of the partner university.

The three teachers who participated in this study where from the Lower School teaching second to fifth grade split into two multi-age groups ($2^{nd}/3^{rd}$ or $4^{th}/5^{th}$ grade). The teachers varied in their language background but all three where bilingual in Spanish and English. The three teachers would be categorized as native speakers of the Spanish language but have had different experiences with Spanish.

Ms. Lorenzo. Ms. Lorenzo was a Mexican American teacher who taught a second and third grade mixed classroom. Ms. Lorenzo attended school in the United States and received instruction in English. Growing up Ms. Lorenzo spent her summers in Mexico where several of her family members were teachers. Ms. Lorenzo's classroom included 12 boys and 13 girls where 2 of her students were non Spanish speakers. Ms. Lorenzo followed a 70-30 model for bilingual teaching and taught mathematics in English on Wednesdays and in Spanish the other four days of the school week.

Ms. Álvarez. Ms. Álvarez was a Peruvian teacher who taught a fourth and fifth grade mixed classroom. Ms. Álvarez was born in Peru and attended Peruvian schools in Lima until the sixth grade when she moved to the United States. Initially, Ms. Álvarez was placed in the English as a Second Language program during her instruction in the United States. Ms. Álvarez' classroom included 19 boys and 15 girls. Two of her students were Spanish dominant, three were English dominant, and one student was a non-Spanish speaker but bilingual in English and another language. Ms. Álvarez followed a 50-50 model for bilingual teaching and taught mathematics in English on Monday and Wednesday, and in Spanish on Tuesday, Thursday, and

Friday. Her instruction on Monday and Wednesday was on average 15 minutes longer than on the other three days of the week.

Mr. Sánchez. Mr. Sánchez was a Mexican-American teacher who taught a fourth and fifth grade mixed classroom. Mr. Sánchez received instruction in English in the United States but grew up speaking Spanish to his elders and English to his cousins. Mr. Sánchez took Spanish classes in high school and two courses during his undergraduate studies. Mr. Sánchez' classroom included 17 boys and 17 girls and had two girls who were Spanish dominant and their individual and group work was primarily in Spanish. Mr. Sánchez followed a 50-50 model for bilingual teaching however he mostly taught mathematics in English. He taught one class in Spanish during my observations per my request.

Data Collection

The data for this study was collected during the 2014 spring semester. I spent the 2013 fall semester shadowing a university professor in an attempt to learn about the teachers, the students, and the school. During this time the teachers in the 2nd and 3rd grade den and the teachers in fourth and fifth grade den who were Spanish-English bilingual teachers were invited to participate in the study. The three above teachers volunteered to participate and data collection began on March 17, 2014 and ended on April 10th, 2014.

Classroom mathematics lessons were video-taped and observational field notes were written after each day's observations. The teachers were observed during the majority of their mathematics lesson that lasted anywhere between 30-75 minutes. Table 1 sets forth the details of the video observations.

Table 1

Video observations

Teacher	# of hours recorded	# of lessons in Spanish	# of lessons in English

Ms. Lorenzo	4 hrs 33 mins	7	0
Ms. Álvarez	6 hrs 38 mins	4	3
Mr. Sánchez	5 hrs 23 mins	1	6

The teachers were not asked to change their teaching practice on account of the study. However, Mr. Sánchez' offered to teach one math lesson in Spanish after a conversation on the use of language. The purpose of the video was to capture the dialogue that occurred during mathematics instruction. A video-camera was used rather than an audio recorder because the video had the added benefit of capturing student and teacher work. The camera was focused on the teacher, and followed the teacher as he or she moved across the room.

Upon completion of the in-classroom observations, teachers where asked to participate in a focus group to discuss the observations. Morgan and Kreuger (1993) write "the advantage of focus groups is that the exchanges among the participants help them to clarify for themselves just what it is that their opinion or behavior depends on" (p.18). The focus group was an ideal forum for the teachers to discuss their language and mathematics teaching practices. The focus group gave the teacher's an opportunity to view their mathematics lessons and discuss various themes. In particular the teachers were asked about their personal beliefs on the connection between language and mathematics. They also clarified those aspects of the video-taped mathematics lessons that were difficult for me to follow or understand.

Participant Structures

The three teachers in this study organized their mathematics lessons in similar ways; all three used the same participant structures in their mathematics lessons. Each developed their participant structures in similar ways but enacted them differently. The participant structures were common to many elementary mathematics classrooms and included: whole group, small group, and individual student-teacher interactions.

Whole group. During whole group instruction the primary activity was students sharing their mathematical ideas and strategies; direct instruction was rarely if ever seen. During the mathematics lessons whole group instruction provided the teachers an opportunity to discuss mathematical ideas and concepts they also utilized whole group time to set up the problem of the day. Ms. Lorenzo used whole group time for warm-ups, problem of the day instructions, and to review mathematical concepts. For the warm-up Ms. Lorenzo was always observed giving her students four numbers for which they were too find three numbers that could be grouped together under at least one mathematical concept (even numbers, greater than the fourth number, multiples, etc.). After the warm-up was complete she either started a new problem of the day or reviewed and extended the previous one. Ms. Álvarez conducted whole group with a set schedule, she began by reviewing a few selected homework problems. She then had a mathematical discussion with the students in which they discussed a series of math problems in order to arrive at an overarching mathematical concept. Lastly, she started a new problem of the day, or continued the problem of the day from the previous day. Mr. Sánchez conducted his whole group in various ways; however, whole group instruction was always used as a time to discuss a mathematical concept – what he called math talk. Problems of the day were also introduced during whole group instruction. All the students in the three classrooms were asked to sit on a carpet in front of white board during whole group instruction. All three teachers gave students the opportunity to share their ideas with the whole group and/or within smaller groups during the whole group participant structure.

Small group. Two of the three teachers used small group instruction as a time to clarify any misconceptions that students might have had on either a mathematical concept or the problem of the day. Ms. Lorenzo pulled out a few students and worked with them to address difficulties with a mathematical concept or task. This was usually done by talking through a problem of the day. Ms. Álvarez used small group as a time to work with the group of students who were confused or had questions on the problem of the day. She began on the circle rug and went through the problem again and if students still had concerns she moved on to a special table to work through the problem with a smaller group of students. The third teacher Mr. Sánchez had students share their ideas and strategies during small group. He worked around the room addressing questions the small groups might have had and guided students through the problems of the day.

Individual. Individual interactions between students and the teacher were short, given that the teacher had multiple students that needed individual assistance. All three teachers used the time where students were working on their own to rotate around the classroom and observe and assist student's work. Teachers stopped to work with individual students if the student asked for help or if the teacher observed something they wanted to correct or to further clarify. These interactions were about assessing where the students were on a given task and moving them forward. The teachers did not use individual interactions to explore students' conceptions on the task rather they focused this time to directly address students' mathematical understanding.

Data Analysis

The data analysis began with the transcription of the videotaped observations followed by a partitioning of the transcription and corresponding field notes into the three different participant structures. Considering the theoretical perspectives from above the data analysis sought to investigate the ways in which bilingual teachers engaged students with various language backgrounds in mathematics through their use of language. Each teacher's field notes and transcripts were analyzed using an open coding system (Corbin & Strauss, 1990) and principles of grounded theory (Glaser & Strauss, 1967) to compare the types of language/registers they employed during the three different participant structures. Open coding was selected so that categories and subcategories of the language and mathematics interactions could emerge freely and on their own without the influence of any predicted categories. This was followed by an analysis of categories across teachers for similarities and differences. The categories or codes were divided into three overarching themes: mathematics, classroom environment, and language used in mathematics. The first theme had the majority of coded data which was expected given that this was a study of mathematics lessons. The second theme though interesting did not offer answers to the research questions. These two themes were included in the data analysis given that language is affected by both the type of the mathematics and classroom practices. The third theme and its codes presented answers to the research questions. The focus group was also coded with the same codes as the observation transcripts and field notes. The focus group data helped to better understand the mathematics lessons and teacher decisions.

Study Teachers' Description of Language and Mathematics

The findings from this study show that a teacher's awareness of the language used in their classroom can have a positive impact on the mathematical learning of the student. The three teachers in this study had their own individual understanding of language as a resource in their mathematics teaching (Gómez et al., 2011; Planas & Setati-Phakeng, 2014). During the focus group session Ms. Lorenzo says the following about her belief of language and mathematics:

> Ms. Lorenzo: I really think that language is important overall of course, vocabulary and things like that. But just I feel like we really want the kids to be able to explain their thinking. And I think a lot of that like first we need to make sure that when we pose a

> problem that they really understand the words. So that we can anticipate that is really important sitting down and me anticipating all these things that are going to happen. And that goes into me knowing my students their vocabulary and that depends on their reading ability. So it's just like all these things that come into effect when you are preplanning is vocabulary and preplanning these many sentence frames, you are pre-planning small groups that are going to take place.

Ms. Lorenzo focused on the understanding of her student's language abilities and their ability to understand the problem of the day she posed. She believed that it was important to use the knowledge she had of her students to anticipate her students reactions to a problem and thus be better able to assist them. Ms. Lorenzo's emphasis was not just on mathematics vocabulary but rather on how the words use effect students' ability to learn (Moschkovich, 2005, 2007; Ron, 1999). Ms. Álvarez, on the other hand, focused on language within the context of the student she said the following:

> Ms. Álvarez: Something like that I always came into my mind about the language of mathematics. For me it goes back to a lot of things from my own experience. And I think with our children with the community that we work with this is something that must be done; I think language when it comes to written language. When we say learning a new language or a third language, sometimes frustrations can come into place. But I think language and mathematics even though its complex there is a simplicity when it comes to numbers. And I think trying to provide that along with language is something that I try to do all the time. Whether it is in Spanish or it is in English. So that they can feel successful. I think especially a lot of our kids whose native language is Spanish being

> able to throw those academic words on continuous bases and to question their thinking continuously is something that I try to do.

Ms. Álvarez thought of language through her own experience learning a second language and believed that it was important for students to learn academic words as they were learning mathematics. Ms. Álvarez said mathematics gave her the opportunity to challenge a student's language while allowing the student to feel successful. Ms. Álvarez used difficulties with language as an opportunity to build mathematical understanding (Elbers & de Haan, 2005). She also believed that it was important to teach mathematics to bilingual children bilingually and to build both languages academically (Lass, 1988). Mr. Sánchez, however, is cautious about academic language and said the following:

> Mr. Sánchez: I think with me I am a bit more cautious by that I mean cautious with the academic language. With that, like once we start doing multiples or whatever I want to make sure that there's enough background knowledge or prior knowledge to kind of be able to attach that academic language. Um also with this idea of when to picking up on Ms. Álvarez' video what I think I do a little bit more now is not the academic language but just the explanation like speaking in complete sentences. There saying it this way maybe I should be a little bit more literal. So that if I am writing something down or if I am re-voicing it. So that there is conflict so that there is a bit more conflict. I think that in my class there is a bit more conflict on the concepts on the mathematical understanding as opposed to…. it kind of goes hand and hand; the language in order to talk about the concepts.

Mr. Sánchez began his explanation of how he saw mathematics and language as two separate entities, the language around mathematical concepts and the academic mathematical language.

He began by focusing on academic and mathematical language and his attention to a student's understanding and background knowledge of mathematics before they attached academic language to a concept. However, Mr. Sánchez realized that there are high language demands in the discussion of concepts that occurred in his classroom. He could not isolate language from mathematics. Through this focus group discussion Mr. Sánchez was able to recognize that mathematics teaching is highly oral and requiring special attention to language (Janzen, 2008; Lager, 2006).

Though it is difficult to discuss the interplay between language and mathematics through these teachers' explanations we are able to see the different perspectives that come in to play. Ms. Lorenzo focused on language and reading ability of her students in thinking of the word problems she would pose. Ms. Álvarez focused on the academic language opportunities that teaching mathematics offered her students. While Mr. Sánchez saw the importance of his student's explanations and thinking through a mathematical concept. These three teachers did not attempt to separate out language from mathematics nor did they see language as either Spanish or English, rather they saw language as the language used in teaching and understanding mathematics. Andersson (1977) claimed that an effective bilingual teacher was both biliterate and bicultural and these teachers demonstrated their effectiveness by being mindful of the ways language played a role in their mathematics classroom. The findings suggests that despite the differences in their views of language and mathematics the three teachers followed similar patterns of distinguishing between issues of language, issues of mathematics, and issues of both language and mathematics.

Findings – Classroom Lessons

The data included 20 mathematics lessons; seven in Ms. Álvarez and in Ms. Lorenzo's classroom and six in Mr. Sánchez' classroom. I have chosen to highlight three mathematics lessons, one from each of the three classrooms. Each of these mathematics lessons are one full class period of observation and are representative of a typical mathematics lesson in each of the classrooms. These three lessons stood out from the onset because of their themes and different approaches to language and these lessons were also discussed during the focus group. These lessons were also chosen because they clearly highlight the seven *bilingual math teacher moves* that will be discussed in the second section of findings. The lessons also demonstrated a movement from discussions in which mathematics and language were more difficult to disentangle towards a more direct mathematical language usage in which deciphering problems between mathematics or language was easier. The lessons contained three general participant structures, whole group, small group, and individual interactions, Ms. Álvarez' lesson contained all three while Mr. Sánchez and Ms. Lorenzo contained two of the three participant structures. I will begin by giving a short account of each of the lessons. (Please refer to the Appendix for the shorten transcript and translation of the lessons). The findings section continues towards a discussion of the lessons along with examples from other days of observation to enhance the understanding of the teacher moves. In this discussion I include small snapshots of the transcript. Instances where the language used was not about the mathematics lesson were omitted along with quotes in regards to classroom management. All three lessons were in Spanish and thus I present the Spanish transcript and English translation. The presentation and discussion of these lessons was deliberately written with a language and mathematics perspective, the snapshots were chosen such that they highlight language usage during a mathematics lesson.

Doce Veces Cinco (Twelve Times Five) Lesson

The doce veces cinco lesson occurred on March 18, 2014 during the second observation in Ms. Lorenzo's classroom. During the first observation the day before the students were given a problem of the day in which they were dividing out a number of cupcakes. However, on this day the majority of the time was spent with a warm-up followed by a discussion of a student's solution strategy for the word problem from the day prior. The students were given four numbers during their warm-up for which they were to select three that belonged in a group: 13, 5, 75, and 60. Several students presented their groupings; however, Brandon's grouping was used by the teacher to discuss in greater detail. Brandon proposed that 5, 75, and 60 belong in a group because they were multiples of five. He explained that the number 60 contained 12 fives because there were 2 fives in ten thus there must be 12 fives in 60. This lead to a lengthy discussion on how the number 60 was made up of 12 fives. Ms. Lorenzo did not capture Brandon's strategy and substituted it for 60 being composed of two sets of 6 times 5. She then proceeded to ask students to work in small groups and find the number of fives in 75.

Cubrecama (Bedspread) Lesson

The cubrecama lesson occurred on March 24, 2014 during the fourth observation in Ms. Álvarez classroom. The previous lesson involved students taking percentages of numbers and this lesson was a continuation of the work on percentages that they had been doing. The whole group discussion was divided into three sections the first a review of homework, the second a discussion on percentages, and third the unpacking of the problem of the day. The review of the homework involved students sharing their solution strategies for 50% of $75 and 25% of $40. The discussion on percentages had students finding 75% of $100; during this discussion students made connections to quarters and elaborated on the connection between $100 and 100%. The problem of the day was:

La señorita Álvarez necesita comprar una sobrecama para su mama. En la tienda Target el precio regular de un cubrecama es ___ sin embargo el cubrecama esta en oferta por el ___ del precio regular. ¿Cuál es el valor del descuento? ¿Cuánto pagaría? En la tienda Sears el mismo cubrecama para su mama cuesta ___ en esta tienda el cubrecama esta en descuento por ___ del precio regular. ¿Cuál es el valor del descuento? ¿Cuánto pagaría? ¿Para ahorrar dinero cual cubrecama debería comprar la señorita Álvarez?

Ms. Álvarez needs to buy a bedspread for her mother. At the Target store the regular price of one bedspread is ____[1] however, the bedspread is on sale for ____ of the regular price. What is the value of the discount? How much would she pay? At the store, Sears, the same bedspread for her mother costs ____ at this store the bedspread is on sale for ___ of the regular price. What is the value of the discount? How much would she pay? To save money which bedspread should Ms. Álvarez buy?

In reading the word problem out loud Ms. Álvarez recognizes that cubrecama might be a difficult word for her students and offers a definition and description immediately than continues to unpack the problem. The small group gave students a second opportunity to unpack the problem of the day and to discuss solution strategies. The students then worked individually on their problem while Ms. Álvarez worked individually with students.

Bombear Agua (Pumping Water) Lesson

The bombear agua lesson occurred on April 3, 2014 during the fourth observation in Mr. Sánchez' classroom. In this observation the students were given a problem of the day that had them explore place value by dividing large numbers by powers of ten. Mr. Sánchez had the

[1] The _____ or blank space is used in the three classrooms and is read out loud by saying the word blank. These blanks are read out loud as blanks given that the students will select a number set to fill in the blanks. The students understand that the blank is a stand-in for the number they will utilize to solve the problem.

students join him on the circle carpet and unpack the problem of the day. He quickly realized that the students were struggling to understand the problem. The students had particular trouble understanding what the word bombear (to pump) meant and thus had a difficult time deciphering the mathematical operation they were asked to perform. The students in Mr. Sánchez' classroom attempted to model the action of the problem, however, the action was not that of pumping but rather placing. The students were unsatisfied with modeling and proceeded to have a lengthy discussion about the action of the problem including a description of the object used to pump. Mr. Sánchez and the students deciphered the meaning of bombear that ends with him drawing a picture of a pump and then the students proceeded to work individually on the problem of the day. During the individual interactions Mr. Sánchez pushes student thinking by questioning students work and answers questions from several students about their own solution strategies.

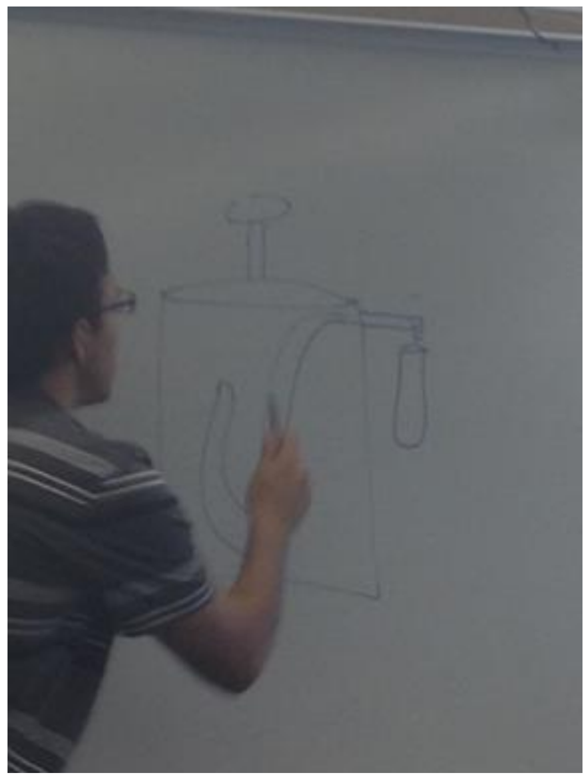


Figure 1. Mr. Sánchez' Pump

Findings – Analysis of Classroom Accounts

The findings are presented based on analysis of data from all the classroom mathematics lessons and the conversations in the focus group. A total of 22 categories came to the forefront and were coded for. Of these 22 codes four codes were language related such as code switching, 3 codes were mathematics related such as instances of math connections, 4 codes were related to classroom practices such as verbal rewards. The remaining 11 codes were reanalyzed and rethought of as seven *bilingual math teacher moves* present during a mathematics lesson. Some codes in the eleven overlapped with each other for example teacher intervention included instances of directly modeling. The seven *bilingual math teacher moves* that emerged from the data were: tricky words, context, explanations, redirecting questions, re-voicing and clarifying ideas, directing questions, and directly modeling. These teacher moves were places in which the teachers and students were negotiating between issues of language and issues of mathematics and often it was difficult to distinguish one over the other and thus became issues of both language and mathematics.

Table 2

Bilingual Math Teacher Moves

Bilingual Math Teacher Moves	# of Instances	Definition	Example
Tricky Words	15	Words that caused student confusion or words teacher's identified as potentially difficult	Repartir, meaning to divide out, heard and understood as partir, to cut, in Ms. Lozano's classroom.
Context	25	The context of a word problem or of a student's explanation of a the word problem or mathematical concept	A student sharing how he played with toy cars in order to understand a word problem about speeds in Mr. Sánchez' classroom.
Explanations	25	A student's explanation of a language complexity, word problem, or math concept	A student explaining how she arrives at a half of a two digit number by subtracting half the ones and half the tens, then adding the ones and tens in Ms. Álvarez' classroom.
Redirecting Questions	60	Teacher's use of questions to direct students away from a mathematical strategy or idea.	Mr. Sánchez asking his students what the whole represents when you are taking

			100% off.
Re-voicing and Clarifying Ideas	79	Teacher’s re-voicing of student ideas in order to clarify or highlight the student’s idea	Ms. Álvarez clarifying a student’s thought that 50% is half.
Directing Questions	61	Teacher’s questions to move students towards a particular mathematical strategy or idea.	Ms. Lozano asking one of her students to identify the difference between digits and units.
Directly modeling	17*	Teacher’s direct modeling of a mathematical concept or problem.	Ms. Lozano pointing to the two as a digit and showing two fingers to represent two digits.

*There were also 54 instances that were coded for teacher intervention that included forms of directly modeling.

These teacher moves represented and described some of the confounding factors between language and mathematics that were present during the mathematics lessons. These teacher moves did not necessarily occur in only one participant structure but rather they are primarily found in one participant structure over the others. Tricky words, context, and explanations tended to occur during whole group. Redirecting questions and re-voicing and clarifying ideas were primarily found in the small group, while directing questions and directly modeling were primarily found in the individual participant structure.

Tricky Words

Tricky words were defined as words that caused student confusion or words that were identified by the teacher as potentially difficult; these words included math vocabulary along with everyday words and phrases. Throughout the 20 observations there were 15 instances that were coded for tricky words – those words became or were seen as potentially difficult by the teacher. Given the structure of the classroom in which students were presented with a problem of the day during whole group and were asked to unpack these word problems tricky words were primarily found in the whole group participant structure. In the first teacher lesson Ms. Álvarez was confronted with cubrecama and sobrecama, which she used interchangeably, as a tricky word. She anticipated that this word might be difficult so Ms. Álvarez gave a definition of cubrecama as soon as she read the word problem.

> Mtra. Álvarez: No se olviden que tenemos que desempacar el problema para hacer exactamente lo que nos pide. ¿La primer, la primera pregunta es, de qué se trata la historia que información tienes que te puede ayudar a resolver el problema? ¿Qué información necesitas? Voy a leer el problema y por favor escuchen. La Señorita Álvarez necesita comprar una sobrecama. Algunas personas no utilizan esa palabra para describir lo que va encima de una cama. Aparte de las almohadas hay una cubrecama por eso se llama cubre cama, o se llama sobre, o se llama cobija.
>
> *Ms. Álvarez: Do not forget that we have to unpack the problem so that we can do exactly what it is asking us. The first, the first question is what is the story about? What information do you have in order to solve the problem? What information do you need? I am going to read the problem, please listen. Ms. Álvarez needs to buy a bedspread. Some people do not utilize this word to describe what goes on top of a bed. Aside from the pillows there is a bedspread, that is what it is called a bedspread, or it is called (Sobrecama, cobija)*
>
> Sam: Colcha
>
> *Sam: Colcha (another name for bedspread often referring to quilt)*
>
> Mtra. Álvarez: O colcha, colcha también se dice. Entonces a visualizar.
>
> *Ms. Álvarez: Or colcha, colcha is also used. Okay so let's visualize.*

During the focus group Ms. Álvarez said the following about this tricky word: "I can't spend a whole time on it because I need to get to the problem so I just went into synonyms because cubrecama comes right away because it's the way I call things that's the way I grew up" (Focus

Group Transcript, 4/9/2014). Ms. Álvarez anticipated that the students may not understand the word cubrecama because of their cultural background being different from hers; most students were of Mexican descent while she was Peruvian. Though the word problem first used the word sobrecama Ms. Álvarez, in her focus group transcript, focused on the word cubrecama acknowledging that this was a word that she grew up with, sobrecama was more often used by her students. In her definition of the cubrecama and sobrecama Ms. Álvarez also acknowledged and accepted colcha as a valid synonym for cubrecama. Having cleared up any confusion Ms. Álvarez then asked the students to visualize the word problem, given that the students could now visualize a bedspread.

Mr. Sánchez on the other hand did not anticipate that the word bombear would become problematic in his classroom. At the start of the lesson Mr. Sánchez asked the students to unpack the word problem as they normally would through a discussion. The students wanted to be able to describe the action of the word problem accurately. One student believed that Trader Joes wanted to sell water bottles while another student mentioned that they wanted to fill water bottles. A student modeled the placement of water from one source to the next. All of these actions would have produced a valid solution given that they all corresponded to the division of two numbers; however the students were unsatisfied, because they wanted to correctly model the pumping action.

Mattie: Mister some people might be getting confused because of bombear.

Mtro. Sánchez: Okay tres dos uno. Okay so esa palabra bombear que quiere decir la palabra bombear. Yeah Alfonso

Mr. Sánchez: Okay three two one. Okay so that word to pump what does that word to pump mean. Yeah Alfonso.

Alfonso: Rellenar

Alfonso: To fill-up.

Mtro. Sánchez: Rellenar,

Mr. Sánchez: To fill-up.

Estudiantes: Llenar

Student: To fill.

Mtro. Sánchez: Llenar, rellenar, llenar.

Mr. Sánchez: To fill, to fill-up, to fill.

During the focus group discussion Mr. Sánchez acknowledged that he did not anticipate this confusion occurring but also gave an explanation as to why he dedicated a vast amount of time deciphering the word bombear instead of quickly defining the word like Ms. Álvarez had with cubrecama.

> Mr. Sánchez : Well, they talked about it, is it going to help you is it going to, knowing the definition knowing what this word means is it going to help you understand the problem. In this case it did. So that's why I dedicated more time (Focus Group Transcript, 4/9/2014).

Mr. Sánchez offered the distinction between these two types of tricky words. The word cubrecama may be a tricky word for some students but fully understanding what this word is, will not inform the students about the problem as a whole. This is a sentiment Ms. Álvarez also acknowledged when she mentioned wanting to get into the actual problem. Through the discussion of bombear the students began to understand more fully that this was a division problem rather than a multiplication, in which the first amount given was the amount being divided by the second number. It is difficult to distinguish whether students struggled with the wording of the bombear problem or if their struggle was mostly due to being unable to visualize a pumping water action. The discussion about cubrecama was not particularly for mathematical understanding it however allowed students to visualize the object being purchased. Cubrecama could be replaced by another object without altering the problem, bombear could not. The two teachers distinguished tricky words by its importance within the context of the problem of the day.

On a different day of observation in Ms. Álvarez' classroom the phrase "sales price" and its translation to "el precio de la oferta" offered a third type of difficulty with words and phrases. These are words that mathematically offer confusion, in a problem of the day Ms. Álvarez asked the students for the amount paid and the sales price (the discounted amount). This is difficult given that sales price is often the amount paid not the amount saved.

> Ms. Álvarez: Cuesta $49. Pero esta de oferta y el precio es la mitad. Es una mitad y cuesta $49. ¿Cuál es el precio de la oferta cuánto pago Mary por los jeans? ¿Qué puedes hacer, para encontrar o averiguar eso?
>
> *Ms. Álvarez: It costs $49. But it is on sale the price is half. It is half and it costs $49. What is the sales price and how much did Mary pay for the jeans. What can you do to find or search for that?*
>
> Eduardo: Es que tengo que buscar el precio de los jeans.
>
> *Eduardo: It's that I have to find the price of the jeans.*
>
> Ms. Álvarez: ¿Bueno cuál es el precio de la oferta, primero, y después cuánto pago Mary por los jeans? ¿Que puedas tratar de hacer para comenzar? Perdón.
>
> *Ms. Álvarez: Okay what is the sales price, first, and then how much did Mary pay for the jeans? What can you try to do to get started? Sorry*

Eduardo: Hacer otro

Eduardo: Make another.

Ms. Álvarez: Un par es uno nada más. Un par son los que tiene que ver con las piernas. ¿Qué puedes hacer para empezar?

Ms. Álvarez: One pair is one only. A pair is the ones that have to do with the legs. What can you do to start?

Eduardo: Los jeans

Eduardo: The jeans

Ms Álvarez: Aha, cuánto cuesta, el precio regular.

Ms. Álvarez: Aha, how much does it cost, the regular price?

Eduardo: [points to $49]

Ms. Álvarez: Cuarenta y nueve. Y la oferta es que está a mitad de precio. Cuál es la oferta y cuánto va a pagar.

Ms. Álvarez: Forty nine. And the sale is that it is half the price. What is the sale and how much is she going to pay.

Eduardo: La oferta es el 50%

Eduardo: The sale is the 50%.

Ms. Álvarez: La oferta es de 50% de 49. ¿Qué puedes hacer? Quiero que pienses una estrategia, okay.

Ms. Álvarez: The sale is 50% of 49. What can you do? I want you to think of a strategy, okay.

In this example Ms. Álvarez is able to distinguish between problems of language versus problems of mathematics. Eduardo was struggling with understanding what sales price meant and the wording of the problem did not assist him in making this distinction in the rewording of the phrase the mathematics embedded comes to the forefront.

Context

For the purposes of this paper context referred to the setting and the actions of the problem of the day. Context was further extended to include the ways in which the student understood and related to the setting and actions of a mathematical problem. Throughout the 20 observations there were 25 instances that were coded for context – times where context was discussed by either the teacher or the students. During whole group instruction students discussed, unpacked, and reviewed problems of the day. It is in the whole group participant structure that students shared out their understanding of the word problem which included their familiarity and personal experience with the context of the problem of the day. A teacher conducting whole group discussion must be aware of how their understanding of the problem and context is the same and different from that of their students. This included the familiarity and knowledge the student may have had with the setting and actions of the problem of the day.

The cubrecama lesson offered a context that students were familiar with, the purchasing of an item at a reduced rate. Ms. Álvarez in a previous math lesson informed the students that they would be doing percentages something that they need in order to get the lowest price possible when shopping, to which the students responded that this was a discount. Understanding what the problem was about was not difficult for the students given that there warm-up and homework included taking percentages of a whole number.

The bombear agua lesson offered a context that the students were uncertain about. Given that bombear was a tricky word and it was also the action for the problem of the day. The students were unable to fully visualize the word problem. At the start of the math lesson one student believed that Trader Joes was selling water bottles rather than filling them, perhaps because grocery stores more commonly sell water bottles than package them. Students were also unsatisfied with modeling an action that though mathematically valid would not produce a valid interpretation of the word problem. It would be very difficult to place water into a water bottle using your hands. Vincent knew that he disagreed with action however was unable to fully verbalize his position.

Mtro. Sánchez: ¿Mas o menos umm Vincent que quieres decir tú?

Mr. Sánchez: More or less umm Vincent what did you want to say?

Vincent: Que es que umm lo que, que tienes que hacer es como es que casi no le entiendo porque no tiene los números umm pero umm creo que lo está tratando de decir es que tienes que ver cuántos galones vas a llenar.

Vincent: That it is that umm what I that you have to. It's because I don't really understand because it doesn't have the number umm but umm I think that it is trying to say is that you have to see how many gallons you are going to fill.

The discussion of the problem of the day mostly revolved around this tricky word given that the context was not useful for understanding the problem.

In Ms. Lorenzo's classroom on a separate occasion the students were asked a problem of the day in which a geologist went down to the river to collect small rocks and place an equal amount in each bag. A geologist collecting rocks might not be familiar to the students but the idea of collecting rocks is.

Ms. Lorenzo: ¿Eso es mi pregunta para ellos como sabían que ella estaba coleccionando en grupos? Voy a leer el problema porque quiero que ella use el texto para decirme que porque infirió, escuchen, porque infirió que ella estaba, que ella estaba coleccionando esas piedras en grupos. Voy a leerlo. La geóloga Alice estaba buscando piedras chiquitas y triangulares por un rio. Llevaba mmm bolsas para poner sus piedras. Puso mmm piedras en cada bolsa. ¿Cuantos piedras encontró Alice? ¿Qué leí que le dio una idea que estaba coleccionando esas rocas en grupos?

Ms. Lorenzo: That is my question how did you know that she was collecting in groups? I am going to read the problem because I want her to use the text to tell me why she inferred, listen, why did she infer that she was, that she was collecting those rocks in groups. I am going to read it. The geologist Alice was looking for rocks that were small and triangular by a river. She had mmm (blank) bags to put her rocks in. She put mmm rocks in each bag. How many rocks did Alice find? What did I read that gave you the idea that I was collecting rocks in groups?

Julie: Que ella las metió en bolsas.

Julie: That she put them in bags.

Ms. Lorenzo: Las metió en bolsas.

Ms. Lorenzo: That she put them in bags.

Julie: Cada bolsa.

Julie: Each bag.

Ms. Lorenzo: Cada bolsa.

Ms. Lorenzo: Each bag

Kevin: En cada bolsa

Kevin: In each bag

Ms. Lorenzo: Dice puso mmm piedras en cada bolsa. Entonces para usted que se estaba imaginando cuando decía que estaba poniendo en grupos en las bolsas.

Ms. Lorenzo: It says mmm rocks in each bag. So then for you what where you imagining when it said that she was putting them in groups in the bags.

Julie: Que ella los recogió y luego los metía en grupos.

Julie: That she picked them up and that she then put them in, in groups.

Ms. Lorenzo: Nos podría dar un ejemplo.

Ms. Lorenzo: Could you give us an example.

Julie: Que ella los metió en grupos como de diez en diez.

Julie: That she put them in, in groups of ten and ten. (Mr. Lorenzo Classroom Transcript, 2014.03.28)

Ms. Lorenzo made it clear that the problem of day had language cues for the students to use in order to better understand the mathematics. Ms. Lorenzo does not separate language from mathematics but rather focuses on its interplay. In part the context in this example is the geologist collecting rocks but also the idea that there is a placement of objects in groups. This idea is grasped and discussed through an understanding that “cada uno” or “each one” implies a very deliberate mathematical action.

Explanations

‘Explanations’ refers to a bilingual math teacher move that occurred when students were asked to share out their ideas and strategies that may be have been hard for the teacher to decipher. Throughout the 20 observations there were 25 instances that were coded for explanations – times when students gave a hard to follow mathematical explanation. The teachers in this study paid close attention to student's mathematical explanation and language; their responses varied. In the previous two discussions on tricky words and context teachers had to be mindful of the students' knowledge outside of the mathematics in the problems of the day; with explanations the teachers also had to be mindful of a student's mathematical understanding.

In the homework review to the cubrecama lesson several students offered explanations of how to take a percentage from a number. One of the homework problems asked students to find 25% of $40.

Mtra. Álvarez: El número que da, que es $40. ¿Entonces si no es 4 por 20, seria 4 por qué? ¿Eduardo, sabes?

Ms. Álvarez: The number that gives, that gives $40. So then if it is not 4 times 20, it would be 4 times what? Eduardo do you know?

Eduardo: Diez.

Eduardo: Ten.

Mtra. Álvarez: ¿Cuatro por qué?

Ms. Álvarez: Four times what?

Eduardo: Cuatro por 10.

Eduardo: Four times 10.

Mtra. Álvarez: ¿Cuatro por 10 cuánto te da 4 por 10?

Ms. Álvarez: Four times 10, how much does 4 times 10 give you?

Estudiantes: Cuarenta.

Students: Forty.

Mtra. Álvarez: Entonces eso significa si es un cuarto. Y esto lo roto, lo quiebro, lo divido, en cuatro partes iguales. Primero pongo el 40.

Ms. Álvarez: So then this means if it is one fourth. And this I break, I break apart, I divide out in four equal parts. First I put the 40.

Eduardo: Y después eso es cero. Y después lo que hice es puse 10 en cada uno.

Eduardo: And then that is zero. And then what I did is I put 10 in each one.

Mtra. Álvarez: Diez, 20, 30. Y eso significa que 25% sería igual a lo que dijo Dalia.

Ms. Álvarez: Ten, 20, 30. And this means 25% this would be the same as what Dalia said.

In finding 25% of $40, Eduardo mentioned that in the division of the number line into 4 parts first there was a zero then he put 10 into each one. Ms. Álvarez interpreted this as Eduardo counting by tens, though in his explanation he never says that he did this nor does he use the numbers 20 or 30. Seperating out what is understood from what is known mathematically is difficult in particular if what is being understood and what is known are two separate valid ideas.

In the warm-up to the cubrecama lesson students were to find 75% of $100 which led to a discussion about 100%.

Mtra. Álvarez: ¿Quién está de acuerdo con lo que acaba de decir 100% es igual a 100? ¿Eso es verdad en todos los casos?

Ms. Álvarez: Who agrees with what he just said that 100% is the same as 100? Is that true in all cases?

Bobby: No, porque 100% es igual a 100 dólares.

Bobby: No, because 100% is the same as 100 dollars.

Mtra. Álvarez: En este caso, pero siempre es siempre la verdad.

Ms. Álvarez: In this case, but is this is this always true.

Bobby: Si

Bobby: Yes

Mtra. Álvarez: ¿Tienes un ejemplo que no sea verdad?

Ms. Álvarez: Do you have an example where this is not true?

Bobby: Como 100% puede ser de otra

Bobby: Like 100% can be of another

Mtra. Álvarez: Excelente.

Ms. Álvarez: Excellent.

In the discussion of 100% Bobby said that 100% could be something else, but never finished his thought. However, Ms. Álvarez interpreted this as Bobby saying that 100% of any number is that number and 100% is not always $100. Ms. Álvarez then asks Johnny for his views on 75% of $100.

Mtra. Álvarez: Estas diciendo que en vez de utilizar 100 dólares utilizaste un dólar. ¿Hacer esa conexión como te ayudo?

Ms. Álvarez: You are saying that instead of utilizing 100 dollars you utilized one dollar. How did making that connection help you?

Johnny: Yo hice coras (Spanglish word for quarters).

Johnny: I did quarters.

Mtra. Álvarez: ¿Perdón?

Ms. Álvarez: Excuse me?

Johnny: Yo hice coras para

Johnny: I made quarters in order to

Mtra. Álvarez: Coras, okay, eso significa que 25 centavos. ¿Y después que hiciste?

Ms. Álvarez: Quarters, okay, that means 25 cents. And then what did you do?

Johnny: Umm yo sabía que 25 por 3 era 75.

Johnny: Umm I knew that 25 by 3 was 75.

Mtra. Álvarez: Tú dices que 25 por 3 es igual a 75.

Ms. Álvarez: You are saying that 25 by 3 is the same as 75.

Johnny: Y 75 es 3 sobre 4.

Johnny: And 75 is 3 over 4.

Mtra. Álvarez: Y 75 tú crees que es 3 sobre 4. ¿Pero esto es 75 dólares o 75 centavos?

Ms. Álvarez: And 75 you believe is 3 over four. But this is 75 dollars or 75 cents?

Johnny: Centavos, no dólares, dólares.

Johnny: Cents, no dollars, dollars.

Johnny's explanation of taking 75% of $100 by using quarters and one dollar was not difficult to follow given that this is a commonly used connection. This connection offered Ms. Álvarez the opportunity to confirm that Johnny and the rest of the class fully understood this strategy. The examples in Ms. Álvarez' cubrecama lesson point to the importance of listening to what the student is explaining and connecting it to the mathematics, not simply listening to one or the other.

Doce veces cinco was a lesson in which explanations as a bilingual math teacher move was prevalent. The math lesson revolved around Brandon's explanation of the number 60 and what Ms. Lorenzo and his classmates' understood of Brandon's explanation. Brandon began his explanation by stating that 5, 60, and 75 belong together because they all counted by fives. He further explained that there were 12 fives in 60 because there were two fives in 10. Ms. Lorenzo did not capture this idea fully and concluded that Brandon knew that six times five gave 30 and

30 was half of 60 thus 12 times 5 gave 60. She further demonstrated this to the rest of the class by circling the two sets of six fives.

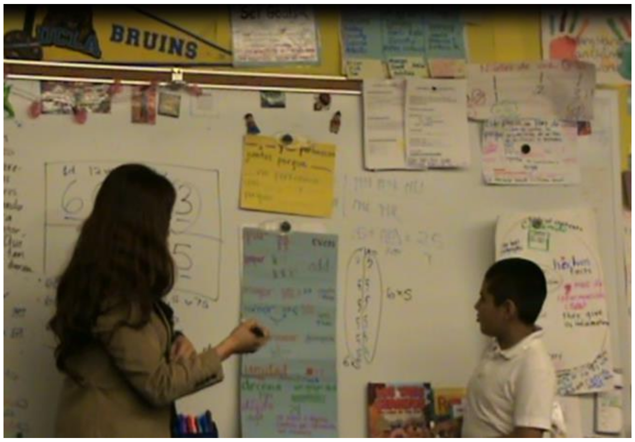


Figure 2. Ms. Lorenzo's Circling of Fives

Ms. Lorenzo continued to ask Brandon about the fives he had written down in order to verify her conclusion.

> Brandon: Era, porque si, debemos de multiplicar con el seis para que sean cinco debo de separar, hacer dos veces seis y acá está el seis, uno, dos, tres, cuatro, cinco, seis.
>
> *Brandon: It was, because, we have to multiply with the six so that it could be five I have to separate, make two times six and here is the six, one, two, three, four, five, six.*
>
> Mtra. Lorenzo: Oh no entonces no es seis entonces es dos piensa en eso. ¿Okay, Brandon aquí, aquí hay un grupo de qué?
>
> *Ms. Lorenzo: Oh no so then it's not six it is two think about that. Okay, Brandon here, here is a group of what?*
>
> Brandon: Aquí hay seis veces cinco.
>
> *Brandon: Here is six times five.*
>
> Mtra. Lorenzo: Seis veces cinco.
>
> *Ms. Lorenzo: Six times five.*

Brandon: Y aquí a un lado a 6 veces 5. Yo solo le tuve que, para que fuera 6 tiene que ser 6 más 6 doce, 12 veces 5.

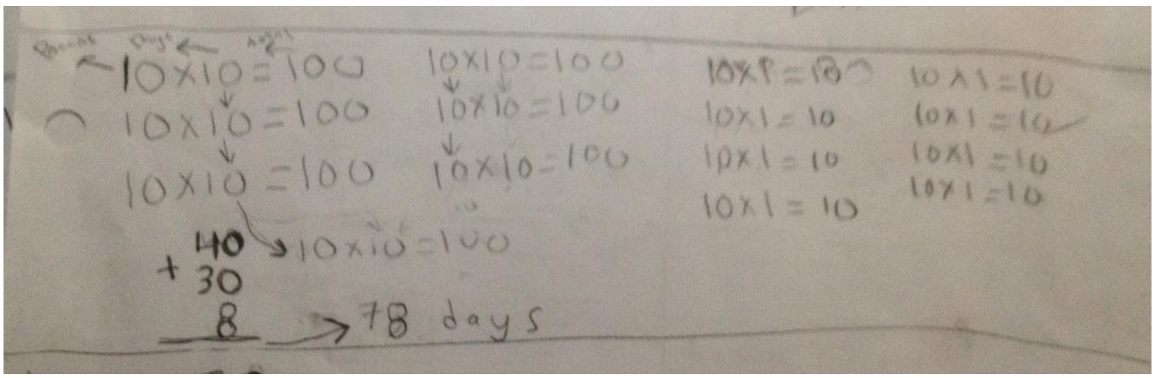
Brandon: And here to the side there is 6 times 5. I only had to so, so that it was 60 it had to be 6 plus 6, 12, 12 times 5.

Mtra. Lorenzo: Entonces usted sabía que 6 veces 5 era 30 y uso esa información. Para encontrar cuántas veces 6 por 5 hacia 60. Okay ahorra si entendí eso es lo que está diciendo la mitad.

Ms. Lorenzo: So then you knew that 6 times 5 is thirty and you used that information. So that you could find how many times 6 by 5 make 60. Okay, so no I understand that it is what you are saying half.

This explanation was difficult for Ms. Lorenzo to follow, during the focus group Ms. Lorenzo mentioned that she continued to be confused by Brandon's explanation. She believed that Brandon knew that 6 times 5 was 30 and there were 2 thirties in 60. Ms. Lorenzo said the following: "Porque cuando estaba hablando el de las dos veces cinco. Apuntaba y está viendo ten, ten, ten, ten I saw it like it was this way like it was thirty. Going down (Because when he was talking about the twelve times five he pointed and saw ten, ten, ten, ten…)." Brandon however, was aware that Ms. Lorenzo was confused and continued to explain in different ways his knowledge of fives and tens. In the transcript above he shifted his explanation to better fit Ms. Lorenzo's in that he now says that there are two groups of six and five. Given that Brandon's explanation and Ms. Lorenzo's understanding were valid mathematical arguments it made the understanding of what was being said more difficult to distinguish from what was being understood.

In the following example from a different observation Mr. Sánchez has the students reviewing their solution strategies for the following problem of the day: *An elephant eats 10 pounds of food a day. How many days would it take the elephant to eat 780 pounds of food?* Mr. Sánchez projects a student's strategy and asks another student to describe what he was noticing. During the explanation of his observation, Mr. Sánchez asks him to give more language by labeling his explanation.



Mr. Sánchez: Alright we will start with Gilberto and then Nydia. Alright Gilberto in your own words so far there are numbers and labels but let's see if you can describe what is happening.

Gilberto: What I think he's doing is he's doing 10 times 10 is a 100 because. He did 10 times 10 seven times and then yeah seven times. And then he got to 700 when he added it and then he did 10 times one 8 times and that was 80. So since the 100 that he did 10 times 10 is 10 days and the other 100 is 20 and the other 100 is 300 until 700 is 70 days. And then he added the 80 days the 10 times one and then he got 78 days.

Mr. Sánchez: Now let's add a bit more language to it. So you were using like the numbers, and the numbers being multiplied, we are talking about days but there are these key words that we are supposed to use

Students: The pounds

Mr. Sánchez: The pounds so Nydia let's see if we can add the pounds to your explanation.

Nydia: Umm what I think it is 10 pounds times 10 pounds times 10 pounds and did this for 70 days is 700. And then they did 10 times one, 10 pounds times one 8 times. And then they added 40 plus 30 plus 8 and they got 78 days.

Mr. Sánchez: Seventy eight days. So Nydia is saying that they are multiplying 10 pounds times 10 pounds.

Students: No

Mr. Sánchez: Ten pounds times

Students: Ten days

Mr. Sánchez: Ten days. So this first 10 pounds represents 10 pounds.

Students: Ten pounds each day.

Mr. Sánchez: Each day. Ten pounds times and then you say

Students: Ten days.

Mr. Sánchez: Will give us 100.

Students: Pounds (Mr. Sánchez Classroom Transcript, 2014.04.08)

Mr. Sánchez follows the student's explanation however he reminds the students that there are key words that they are supposed to use. Thus he asks another student to use the label pounds in the explanation. By asking students to use labels in their explanations Mr. Sánchez is able to better identify students' whether their misconception is language based or mathematical. He then was able to discuss with the students how labeling the two quantities allowed them to produce a third label that accounted for the mathematical concept. In this case if the elephant eats 10 pounds in one day for 10 days thus, he eats 100 pounds in 10 days.

Redirecting Questions

Redirecting questions were questions that teachers used to guide students away from a mathematical strategy or idea. Throughout the 20 observations there were 60 instances that were coded for redirecting questions – times when the teacher redirected a student away from a mathematical strategy or idea. Small group time was a time for students to voice their ideas and strategies, or lack of, about a given task. Redirecting questions occurred most often during small group time when students were voicing their intended solution strategies for the problem of the day or the task at hand.

In the cubrecama lesson Ms. Álvarez asked her students what they believed the problem was about during the small group participant structure. The students responded that this was about a bedspread being on sale; though this was true Ms. Álvarez reminded the students that there was an original price that the students had to be aware of.

Mtra. Álvarez: ¿Muy bien, que más sabemos sobre esa primera parte del problema? ¿Aton?

Ms. Álvarez: Very good, what else do we know about that first part of the problem? Aton?

Aton: De que el cubrecama esta en oferta en la tienda.

Aton: That the bedspread is on sale at the store.

Mtra. Álvarez: Sabemos que esta cubrecama esta en oferta pero hay una cantidad del precio regular. Excelente, esa es el cubrecama en la tienda Target. ¿Puede leer la segunda parte por favor?

Ms. Álvarez: We know that the bedspread is on sale but there is a quantity for the regular Price. Excellent that is the bedspread at the Target store. Can you read the second part, please?

Calling attention to the original price allows the students to think not only of the context of the problem but also the mathematics that must be performed in order to solve the problem. Upon dismissing the students Ms. Álvarez asked her students to inform her of their intended solution strategies.

Mtra. Álvarez: ¿Puedes utilizar unas de estas estrategias de acá, que estrategia te llama la atención? Y qué estrategia puedes adoptar, la línea numérica, quebrar los números, sumar cual, multiplicar, tienes que escoger una para comenzar.

Ms. Álvarez: Can you utilize one of the strategies over here, which strategy catches your attention? And what strategy can you adopt, the numeric line, decomposing numbers, adding what, multiplications, you have to pick one to start.

Paco: Yo ya escogí el 79 con 50%.

Paco: I already picked the 79 with 50%.

Mtra. Álvarez: ¿Excelente que estrategia vas a utilizar?

Ms. Álvarez: Excellent and what strategy will you use.

Paco: Línea numérica.

Paco: Number line.

This was not done by simply asking, but by listing to different strategies that they had been working on earlier. This question was redirecting because Ms. Álvarez was offering the students potential solution strategies that were presented earlier during the warm-up and not just one solution strategy.

During the small group interaction for the doce veces cinco lesson Ms. Lorenzo asked Alexa and Mauricio to explain how they arrived at 15 fives in order to make 75.

Mtra. Lorenzo: ¿Siete veces cinco cuánto es? ¿Aquí tenía cuánto es 7 veces 5? ¿Treinta y cinco entonces otra vez siete veces cinco cuántos son aquí?

Figure 3. Students' Strategy for Number of Fives in 70

Ms. Lorenzo: Seven times 5 is how much? Here I had, how much is 7 times? Thirty-five so then again 7 times five is how many here?

Alexa: Cinco, 10, 15, 20, 25, 30, 35.

Alexa: Five, 10, 15, 20, 25, 30, 35.

Mtra. Lorenzo: ¿Treinta y cinco entonces cuánto es 35 más 35?

Ms. Lorenzo: Thirty-five so then how much is 35 plus 35?

Alexa: Setenta

Alexa: Seventy.

Mtra. Lorenzo: Setenta. Buena estrategia ahorra pensaron en lo que dijo Brandon.

Ms. Lorenzo: Seventy. Good strategy so now did you think about what Brandon said.

Alexa: Si dos veces 7 de 5.

Alexa: Yes, two times 7 fives.

Mtra. Lorenzo: Oh entonces pensaron más bien en 7 veces 5 y esto los puede ayudar a pensar a llegar a ver cuántas veces cinco hay en 75.

Ms. Lorenzo: Oh so you guys thought of 7 times 5 and this could help you think to see how many times five there are in 75.

When Ms. Lorenzo sees the two columns of seven fives she asked the students whether they knew that this was 35. Alexa did not know that seven times five is 35 given that she began to count by fives to arrive at 35. Alexa responded a bit confused and Ms. Lorenzo asked again; it was this line of questioning that moved the students away from Brandon's idea of two fives making up 10 to knowing that 7 times five was 35 half of 70. We know that this line of questioning was redirecting the student when Alexa responded that she did use Brandon's idea she used "two times seven of fives."

In the following example from a different observation Ms. Lorenzo has joined a small group that was thinking and sharing their ideas about the warm-up. The students had been given four numbers of which they are to group three under a mathematical concept; the numbers given were 22, 55, 14, and 11. The students in the small group had noticed that the number 55 was made up of a 5 and 5 but also that this was a multiple of 5. Ms. Lorenzo used guiding questions in order to get the students to think and discuss 22, 55, and 11 as numbers that have the same numeral in the ones and tens place.

Kevin: Cincuenta y cinco tiene cinco y cinco.

Kevin: Fifty five has five and five.

Ms. Lorenzo: Cinco y cinco.

Ms. Lorenzo: Five and five.

Kevin: Y también 22 y 11.

Kevin: And also 22 and 11.

Daniel: Porque son pares si cuentas cinco y cinco no van a llegar porque si cuentas a 20 te faltan dos.

Daniel: Because they are pairs if you count five and five it is not going to reach because if you count to 20 you are missing two.

Ms. Lorenzo: ¿Entonces cual es otra manera de decir si hubiera contado de cinco y cinco? ¿Qué otra palabra usamos para describir que es un número que pertenece a la familia de cinco, como se?

Ms. Lorenzo: So then what is another way to say if you would have counted in five and five? What other word do we use to describe a number that belong in the family of five, what is it?

Karla: Múltiple

Karla: Multiple

Ms. Lorenzo: Múltiple, ya descubrimos que 22 no es un múltiple de cinco. ¿Qué más observas Karla?

Ms. Lorenzo: Multiple, we had discovered that 22 is not a multiple of five. What else are you observing Karla?

Daniel: Ya se

Daniel: I know

Ms. Lorenzo: Vean específicamente en las decenas y las unidades. Okay espere.

Ms. Lorenzo: Look specifically at the tens and the units. Okay wait.

Karla: Cinco y cinco

Karla: Five and five

Ms. Lorenzo: Vea las decenas y unidades de cada uno 22. ¿Qué tienen las decenas y las unidades?

Ms. Lorenzo: Look at the tens and the units of each one 22. What does the tens and the units have?

As the transcription continues Ms. Lorenzo guides the students through redirecting questions from thinking of pairs of numbers to multiples of eleven. Ms. Lorenzo builds off of the students' articulation of counting by fives into multiples of a different number. The students were not lost in the questions about multiples of eleven. This is seen when the students share their ideas to the rest of the class further into the transcription and they are able to use unifix cubes in groups of eleven to demonstrate the multiples Ms. Lorenzo had previously guided them through.

Re-voicing and Clarifying Ideas

The teachers in the study often re-voiced and clarified ideas a student had about a given mathematical concept or problem of the day. Throughout the 20 observations there were 79 instances that were coded for re-voicing and clarifying ideas, notably re-voicing and clarifying occurred during all three participant structures. For the purpose of this study I focus on instances of re-voicing and clarifying ideas during small group. The voicing and clarifying ideas during small group differed from the other two participant structures in that these ideas were more specific and directly related to the mathematics of a task, rather than about the task itself. The language during this time was specific to mathematics.

The small group in Ms. Álvarez' classroom was set up to clarify the problem of the day. She began by slowly rereading the word problem stopping after reading each one of the sections of the problem to ensure that the students understood the entire problem of the day. In the process of asking a redirecting question Ms. Álvarez also re-voiced and clarified a student's idea. The student believed that the cubrecama problem was about a bedspread being on sale, Ms.

Álvarez agreed but also extended to include the existence of a regular price. In the process of asking students for their solution strategies Ms. Álvarez was also clarifying which solution strategies would offer a valid mathematical solution.

The small group in Ms. Lorenzo's classroom also offered the teacher the opportunity to learn about her students ideas and to re-voice them or clarify them. Ms. Lorenzo observed the work her students had written on their white boards and believed her students approach to finding the number of fives was to find the number of fives in half of 70. Though she had redirecting questions for the students, her questions are also re-voicing what she believed her students ideas were. Ms. Lorenzo asked Alexa and Mauricio what seven times five was in three different ways in one exchange. To further clarify what she believed her students work was she asked the students what 35 plus 35 was. Voicing and clarifying ideas were not necessarily what the student was saying but rather what the teacher perceived the student was saying. When these two ideas did not match re-voicing became a redirection of the student's idea.

The next example occurred in Ms. Álvarez's classroom, she has dismissed the students to work on the problem of the day. The problem of the day is similar to the cubrecama lesson where students were too find the price of a pair of jeans that has been discounted. She has asked the students who do not understand the problem of the day to stay behind for them to review it together.

Ms. Álvarez: Okay who can read the problem? Okay go for it.

Victoria: Mr. Sánchez needs a new pair of jeans. His wife, Mary, went shopping at Macy's because they were having a sale on jeans.

Ms. Álvarez: Okay let's stop it there. What does Daniel need?

Students: A new pair of jeans.

Ms. Álvarez: Okay let's stop, next question. Who is going to help him get a new pair of jeans?

Students: His wife Mary.

Ms. Álvarez: Let me ask you another question. Where is his wife going to?

Students: Macys

Ms. Álvarez: Okay so far we know that Mr. Sánchez needs a pair of jeans, okay. And then his wife Mary is going to go get them for him. So it's on the board, yes. Okay so we have that so far. Let's continue reading. Go for it.

Patricia: A pair of jeans was on sale for blank off the regular price.

Ms. Álvarez: Okay let's stop. Let's visualize that. So we have some jeans and they are on sale. It says sale, right. So that means that the price of the jeans is going to be less or lower than the regular price. We don't know, okay, next. Go ahead and read the question.

(Ms. Álvarez Classroom Transcript, 2014.03.20)

The transcription continues with Ms. Álvarez asking similar questions after every phrase in the word problem. What we see in this example is that Ms. Álvarez's language is directly about the word problem. She asks the children specific and direct questions about what they understood of the problem; however, she is not looking for a lengthy description but short and straightforward answers that allow her to know if the students are struggling with the language of the word problem or the mathematics of the problem.

Directing Questions

Directing questions was a strategy used by the teachers in order to guide students towards a particular mathematical strategy. Throughout the 20 observations there were 61 instances that were coded for directing questions – times in which the teacher asked questions that directed students towards a particular mathematical strategy. These questions primarily occurred during individual interactions when students were working one-on-one with the teacher on a mathematical problem. Directing questions differed from redirecting questions given that they no longer attempted to guide a student away from a strategy but rather were questions that guided students towards a given mathematical strategy or idea.

In the cubrecama lesson Ms. Álvarez began her interaction with Benjy by first assessing his work. Benjy responded to her questions by confirming that he understood what the numbers in the problem represented.

Mtra. Álvarez: ¿Restas cómo vas a utilizar como vas a comenzar? ¿Qué vas a restar?

Ms. Álvarez: Subtractions how are you going to use how are you going to start? What are you going to subtract?

Benjy: Cien.

Benjy: One hundred.

Mtra. Álvarez: ¿Cien, y de donde sacas el 100?

Ms. Álvarez: One hundred, and where are you getting the 100?

Benjy: Porque acá dice que cuesta.

Benjy: Because over here it says that it costs.

Mtra. Álvarez: Cien dólares

Ms. Álvarez: One hundred dollars

Benjy: Y acá dice que tiene descuento de un décimo

Benjy: And over here it says the discount is one tenth.

Mtra. Álvarez: ¿Décimo, y como vas averiguar eso? ¿Qué estrategia te puede ayudar? ¿Vas a utilizar una qué?

Ms. Álvarez: One tenth, and how are you going to find that out? What strategy can help you? You are going to use a what?

Benjy: Línea numérica.

Benjy: Number line.

In discussing the solution strategy Ms. Álvarez used direct questions to ensure that Benjy knew what strategy to use. She asked Benjy what strategy he would be using; when he answered that he would be using subtraction Ms. Álvarez asked specific questions about the intended strategy. When Benjy was unable to answer these questions Ms. Álvarez asked questions about using a different strategy and ended the question with the pronoun "una" meaning a feminine one, indicating a number line. This can be deduced by noticing that this was the only strategy of the ones she offered towards the end of the small group that had a feminine pronoun. Ms. Álvarez proceeded to ask another series of directing questions in this interaction when she asked Benjy what the numbers he had written represented and into how many parts was he going to divide his number line.

Mtra. Álvarez: Okay, excelente. ¿Ahorra cómo la vas hacer? Décimo así es como se divide. [Classroom management] ¿Y cuál es el valor de acá? ¿Muy bien, y en cuántas partes lo estas dividiendo, cuántas partes?

Ms. Álvarez: Okay, excellent. Now how are you going to do it? One tenth like that is how you divide it. [Classroom management] And what is the value over here? Very good, and in how many parts are you dividing it, how many parts?

Benjy: Un décimo.

Benjy: A tenth.

Mr. Sánchez and Gloria had an interaction similar to the one Ms. Álvarez had with Benjy. Mr. Sánchez began his interaction assessing what Gloria had written down as her solution strategy. He listened to her explanation and asked about a piece of her solution this caused her to reread the word problem and focus on having 57,000. He then asked her to explained what the numbers in her solution represented.

Mtro. Sánchez: Cincuenta y siete mil. Yo allí sé que son 105 botellas, que pueden llenar. ¿Dime que representan estos números de acá?

Mr. Sánchez: Fifty seven thousand. I know that right there are 105 bottles that they can fill. Tell me what these numbers over here represent.

Gloria: Esos números representan los cinco galones. Porque sé que estos son dos galones, dos galones.

Gloria: Those numbers represent the five gallons because I know that these are two gallons, two gallons.

Mtro. Sánchez: ¿So, son dos galones aquí, el 100 o el cinco?

Mr. Sánchez: So there are two gallons here the 100 or the five?

Gloria: Esos dos son galones.

Gloria: Those two are gallons.

Mtro. Sánchez: ¿Los números de afuera representan galones o botellas?

Mr. Sánchez: The numbers on the outside represent gallons or bottles.

Gloria: Estaba contando por dos para tener 10 galones de agua.

Gloria: I was counting by twos to have 10 gallons of water.

Mtro. Sánchez: Okay

Mr. Sánchez: Okay

Gloria: Y hice algo mal

Gloria: And I did something wrong

Mtro. Sánchez: Y dijiste que cometiste un error.

Mr. Sánchez: And you said that you made a mistake

Gloria decided to separate the 10 gallon water bottles into two 5s, because she wanted to count by twos. Ms. Sánchez identified that this strategy could be the source of the confusion and asked Gloria to label the five with bottles or gallons. Gloria did not answer the question but rather recognized that she had made a mistake. The direct questions in the interaction allowed Gloria to see that she had made a mistake. Mr. Sánchez ended this interaction by asking her to work with a different number set for which she would be better able to use the strategy of counting by tens. The individual participant structure allowed Mr. Sánchez and Ms. Álvarez to directly focus on mathematics; issues with understanding the problem have presumably been dealt with in the previous two participant structures.

The next two examples occur one after the other in Ms. Álvarez's classroom around the discounted jeans problem of the day; she is going around the classroom asking the students to talk about their solution strategies. Ms. Álvarez is very direct with the questions she is asking her students. She wants to know that they are able to do the problem, when she finds that they understood the she then asks verifying questions.

Ms. Álvarez: What do you think?

Jackie: Times two.

Ms. Álvarez: What is the price, what is the sales price of the jeans?

Jackie: Three times

Ms. Álvarez: So if the whole price is 24 dollars. Okay what is the sales price, what chunk is what allows it to be on sale? So what is that percentage of that sale price? So what do you have here? Okay so if Mary is only paying this quantity and she is getting a, the discount from that quantity. You are saying that quantity is eight dollars. Okay, so that would be the sales price. Okay, so how much would she pay with that sales price. That's what you should, that's what you need to respond. (Ms. Álvarez Classroom Transcipt, 2014.03.20)

Ms. Álvarez: How is it going? Okay which number, which number did you chose?

Dalia: The second one

Ms. Álvarez: You are still working on the second set. What strategy did are you using.

Dalia: The one I did for homework

Ms. Álvarez: Which is what?

Dalia: The 3 times what is equal to 24?

Ms. Álvarez: So three times what is equal to 24? And what did you get?

Dalia: eight.

Ms. Álvarez: So you are saying that, so what is what is one third. That is the sales price.

Dalia: it is

Ms. Álvarez: How much is she paying? (Sixteen dollars). Because why, what is your discount?

Dalia: (Sixteen dollars), eight dollars

Ms. Álvarez: Eight dollars, okay wonderful, did you think of a second way of doing this. Do you have a second strategy? (Ms. Álvarez Classroom Transcipt, 2014.03.20)

In the two examples above Ms. Álvarez asked questions about the student's solution strategies. She wanted to know what they understood of the problem, how they did the problem, and if they could distinguish between the two numbers the problem asks them to find. This direct line of questioning gets at disentangling the mathematics from the language. This is easier to do at the individual participant structure because of the previous discussions during whole group and small group participant structures.

Directly Modeling

During individual interactions teachers guided the student through the mathematical tasks that were assigned. Throughout the 20 observations there were 17 instances that were coded for directly modeling. There was also a second code for teacher intervention that included instances of directly modeling; teacher intervention was coded 54 times. It was during interventions that the teacher often directly modeled a mathematical concept or problem. Teachers modeled a given mathematical concept or word problem throughout the three participant structures; however, for the purpose of this study directly modeling was a bilingual math teacher move that occurred during individual interactions. The mathematical lessons presented do not show a clear instance of directly modeling thus an example that is representative of directly modeling is presented here.

In the following example Ms. Lorenzo had put up four numbers, 4, 8, 12, and 21 that the students were to group together three numbers under a mathematical concept. One of Ms. Lorenzo's students, Mauricio, believed that 21, 12, and 8 all had two digits and thus belong in a group. Ms. Lorenzo explained to Mauricio that there were digits and that there were numbers and explained the difference between them. When Mauricio seemed to understand this difference Ms. Lorenzo informed Mauricio that she disagreed with his grouping of three numbers and asked the child how many digits were in the number eight, and he responded eight. At this point Ms. Lorenzo attempted a different strategy she began to directly model the two digits by pointing at the digits in 12 and 21. She pointed at the digit two and said one digit than at the number one and said two digits in the number 21, and the same with 12. This direct modeling still proved to be difficult for Mauricio to understand as he still believed that there were 8 digits in the number 8. Ms. Lorenzo then continued her modeling by drawing out 8 lines and showing that number eight represented eight things but only had one digit. When he understood this he also realized that 8 contained the number four and that adding another four would give him 12, thus finding a completely different grouping. Mauricio found the concept of digit difficult to understand, he could point to the two digits in 12 and 21, but instead of realizing these were two digits he observed the number one and the number two. The direct modeling of the eight lines for Mauricio helped him make the distinction between the number eight and the number of digits.

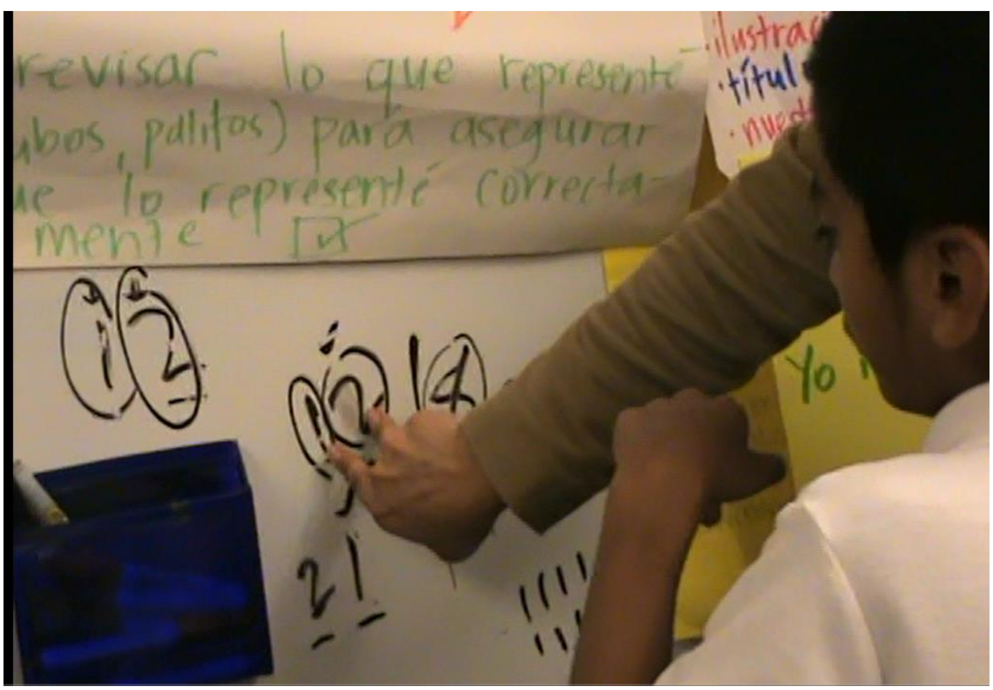


Figure 4. Digits in 12 and 21

During this same day of observation Ms. Lorenzo gathered a group of students she felt were having a hard time modeling a word problem. The word problem has two cousins sharing out 12 cupcakes. She devised a system for the students to follow when they use manipulatives and modeled their word problems. Prior to this part of the transcription Ms. Lorenzo has placed on the white board in front of the students five steps to follow when doing a sharing out problem. The transcription picks up after they have set out 12 cupcakes and Ms. Lorenzo has asked them to always look back and verify if they have set out the right amount of manipulatives.

Ms. Lorenzo: Okay muy bien, en el problema nosotros tenemos 12 pastelitos y los estábamos compartiendo justamente entre dos primos verdad. ¿Cómo puedo representar lo que yo sé? ¿Qué se, sé que hay 12 pastelitos, qué use para representar los pastelitos?

Ms. Lorenzo: Okay very good, in the problem we have 12 cupcakes and we were sharing them fairly between two cousins right. How can I represent what I know? What do I know, I know that there are 12 cupcakes and what did I use to represent the cupcakes?

Alice: Usaste los cuadritos.

Alice: You used the little squares.

Ms. Lorenzo: Entonces cuando sabemos algo me voy a preguntar sé que hay 12 pastelitos. ¿Qué sabemos?

Ms. Lorenzo: So then when we know something I am going to ask myself, I know that we have 12 cupcakes. What do we know?

Alice: Sabemos que hay 12 pastelitos.

Alice: We know that there are 12 cupcakes.

Ms. Lorenzo: Okay vamos a ver como ella los representa. Y usted está observando. Okay puede decirnos en voz alta como los está organizando. Queremos saber lo que usted está pensando.

Ms. Lorenzo: Okay we are going to see how she represents it. And you are observing. Okay can you tell us in a loud voice how you are organizing it. We want to know what you are thinking.

Alice: Yo los estoy organizando de seis en seis. Porque hace 12.

Alice: I am organizing them six in six. Because it makes 12.

Ms. Lorenzo: Hmm 6 y 6 hacen 12. Muy bien entonces nos puede enseñar. Yo escuche que James hizo dos, cuatro, seis.

Ms. Lorenzo: Hmm 6 and 6 makes 12. Very good so then can you show us. I heard James do two, four, six. (Ms. Lorenzo Classroom Transcript, 2014.03.27)

The transcription continues in a similar fashion in which Ms. Lorenzo does most of the talking and models the ways in which she would like the students to solve a similar problem. Ms. Lorenzo in this example is directly targeting students' mathematical performance and actions as they think through solving a problem using manipulatives. Ms. Lorenzo uses direct modeling with manipulatives to connect to what Alice is expressing.

Discussion and Conclusion

The research question and its follow up questions for this study were the following: How does a bilingual (Spanish and English) teacher's usage of language impact her mathematics teaching? (a) How were issues of language present during mathematics teaching? (b) How do teachers respond to issues of language present during mathematics teaching? The answer to these questions is found in the seven *bilingual math teacher moves* presented including the ways that language and mathematics was confounded. The teachers' responses during the focus group give insight into their connections between language and mathematics; in particular their insights on the difference between, language, mathematical language, and the use of language in mathematics.

Types of Mathematics Language

Research has shown that mathematics teaching is highly oral and thus teachers need to attend to issues of language (Janzen, 2008; Lager, 2006; Ron 1999). The ways in which the *bilingual math teacher moves* appeared through the three participant structures demonstrated what language needs to be attended to and when. The language used in these bilingual classrooms during the mathematics lesson flowed from common language used for general understanding of a concept to direct language used for application on a mathematical task.

Kasule and Mapolelo (2010) argue that mathematics language differs from that of the language of instruction; this also extends to the use of everyday language in the teaching and learning of mathematics (Lager, 2006; Ron 199) The first three *bilingual math teacher moves* presented were commonly used during the whole group participant structure. The whole group structure was used by the three teachers to present and discuss mathematical concepts and problems of the day; the language used was both mathematical and nonmathematical. Franke and colleagues (2007) call for classroom norms that encourage students to engage in mathematical discourse. Given, the structure of these three bilingual classrooms the students freely discussed their understanding and asked questions openly. The time spent unpacking Brandon's explanation of 12 times five was not uncommon in the classrooms. Ms. Lorenzo allowed Brandon to give his explanation of 12 times five, but she also asked for clarification given that his explanation was hard to follow. In Mr. Sánchez' classroom children advocated for themselves and other students, Mattie during the bombear agua lesson informed Mr. Sánchez that students may be finding the word bombear difficult. The whole group participant structure had teachers attending to the ways in which everyday language affects mathematical understanding and teaching (Lager, 2006; Ron, 1999).

The language attended to during the small group participant structure was mainly mathematical and specifically addressed student's ideas about the mathematical task. The small group participant structure was primarily used to clarify student's thoughts and thus the language was no longer general but rather specific. Farrugia (2009) and Moschkovich (2005, 2007) argue that more than one mathematical register is present in a classroom and that this may cause difficulty for the student. During small group interactions the teacher attempted to address and assists students with the different mathematical registers and difficulties with language they might have encountered during the whole group interaction. Ms. Álvarez' small group interaction revolved around the cubrecama problem of the day and was specific to helping the students understand the problem and offer potential solution strategies. Ms. Álvarez had the students reread the problem of the day and she asked questions that redirected students towards different solution strategies.

The language attended to during the individual participant structure was direct and purely about understanding what the mathematical task was. Moschkovich (2005, 2007) and Ron (1999) discuss teaching mathematics as being more than an understanding of the technical and academic language present in a mathematics lesson. However, during the individual interactions the language used by the teacher was mostly technical and directly related to mathematical concepts. The teacher's asked direct questions about the students work and thinking process and asked questions that moved the students towards a particular mathematical strategy or idea. Ms.

Álvarez began her individual interaction with Benjy by first inquiring about his work and then guiding him towards the use of a number line. Benjy responded in a similar fashion, by answering her questions directly and demonstrating the knowledge he had about the problem. Ms. Álvarez's questions were not about what he understands of the number line or the overall word problem; her questions were about his intended solution strategy. In her direct modeling of two digits versus the digit number two Ms. Lorenzo used direct mathematical language. She did not ask Mauricio to explain what digits were but rather she asked questions to guide Mauricio to the right conclusion. Mauricio's responses were similar to Benjy in that they answered Ms. Lorenzo's question to the best of his knowledge. The language during the individual participant structure revolved around the students intended solution strategies and was directly mathematical.

The language used in the three participant structures showed that it was difficult for the study teachers to disentangle mathematics fully from language. However, as they move from one participant structure to the other we are able to observe that the majority of the entanglement was found in the whole group session versus the other two participant structures where it was easier to decipher if this was an issue of language versus an issue of mathematics. This is because the majority of the mathematical concept building happens in that whole group session where the students are negotiating what they understand whether it's the word problem as a whole or the mathematical concept present. When students discuss what and how they understand a concept the language that they use is very much tied into their understanding of mathematics. When students were first learning a concept it was more entangled given that they were trying to use whatever it is that they have and know and express it to understand something new. Thus, it is difficult to distinguish if a student is struggling to express their ideas versus struggling with the mathematical concept. When the teacher and student are interacting in the individual participant structure they have presumably already attacked the issues of building the concept thus the teacher moves towards making sure that the concept was built correctly.

Researchers argue that mathematics teachers of ELLs require more than just mathematical language (Lager, 2006; Moschkovich, 2005, 2007; Ron, 1999), this study demonstrated that this is particularly true during the whole group participant structure. During small group and individual participant structures teachers use more mathematical and academic language because this is the issue at hand.

Teachers' Response to Language

Teachers' responses to the *bilingual math teacher moves* during their mathematics lessons demonstrated their ability to be aware of the effects of language present in their classroom. During the cubrecama lesson Johnny used the word coras during his explanation of 75% of $100. Though the word Coras in Spanish is a name for American Indian people in western Mexico, the meaning Johnny was using was the Spanglish word for quarters. Ms. Álvarez used Pimm's (1987) extension of mathematical register to include a word with a special meaning in everyday language that assisted with a mathematical concept. Ms. Álvarez was aware of Johnny's use of the word and accepted as a valid term, allowing the adaptation of this word commonly used by her students. Though, Ms. Lorenzo did not fully understand Brandon's explanation of the number of fives in 60, she validated his response by building off of it. Elbers and de Haan (2005) found that when students discussed difficult words in small group they focused on the meaning behind mathematical language and discussions of everyday language were overlooked. Unlike the students in Elbers' study, in this study teachers responded to tricky words. The tricky words included mathematical vocabulary and everyday language. In particular in Mr. Sánchez' response to bombear (pumping) demonstrated his ability to be flexible and aware of the language difficulties this word had for his students. Likewise Ms. Álvarez anticipated the difficulty that the word cubrecama could have for her students. Overall, teacher responded to issues of language by being aware and accepting of the language their students used.

Implications and Future Research

The three study teachers responded to issues of language in slightly different ways that were representative of their beliefs of language and mathematics. What must not be forgotten is that the three teachers were expert math teachers and that they were fluent in both English and Spanish and that they were able to move fluidly from one language to the other in order to meet their students' needs. If these issues in regards to the interplay between mathematics and language were present in these classrooms what can we expect from teachers who are not expert math teachers or are not fluently bilingual being able to move fluidly between one language and the other. Understanding where and how some issues of language and mathematics are present with outstanding teachers will help inform places were teacher education can inform future teachers of this interplay. This calls for the education community to rethink how bilingual teachers use language throughout their teaching. Professional development and the teaching of pre-service teachers should reflect the different language demands that are presented in a bilingual classroom.

This study demonstrated the need to further investigate the connection between all aspects of language and their effect on the teaching and learning of mathematics. The teachers in this study demonstrated that their personal awareness of language had an impact on their mathematics teaching. Future research should include the study of how an increase awareness of language impacts mathematics teaching and learning. That is, would further discussion with teachers about the impact of language on mathematics have an effect on mathematics teaching and subsequently on student learning?

Appendices

Appendix A

Cubrecama (Bedspread)

	Mtra. Álvarez 2014.03.24 Transcription	Ms. Álvarez 2014.03.24 Translation
Problem of the Day	La señorita Álvarez necesita comprar una sobrecama para su mamá. En la tienda Target el precio regular de un cubrecama es ___ sin embargo el cubrecama esta en oferta por el ___ del precio regular. ¿Cuál es el valor del descuento? ¿Cuánto pagaría? En la tienda Sears el mismo cubrecama para su mamá cuesta ___ en esta tienda el cubrecama esta en descuento por ___ del precio regular. ¿Cuál es el valor del descuento? ¿Cuánto pagaría? ¿Para ahorrar dinero cual cubrecama debería comprar la señorita Álvarez?	Ms. Álvarez needs to buy a bedspread for her mother. At the Target store the regular price of one bedspread is ____ however, the bedspread is on sale for ____ of the regular price. What is the value of the discount? How much would she pay? At the store, Sears, the same bedspread for her mother costs ____ at this store the bedspread is on sale for ___ of the regular price. What is the value of the discount? How much would she pay? To save money which bedspread should Ms. Álvarez buy?
Whole Group: Homework Problem 1	Mtra. Álvarez: ¿Quién está de acuerdo con Magdalena? Entonces lo que hizo Kamilah estuvo correcto esto también esta correcto pero lo que se le olvido a Kamilah es que cuando se divide en partes iguales tienes que utilizar toda la cantidad por completo. ¿Viste lo que hizo Magdalena? Okay, muy bien entonces es treinta y siete punto cinco. Ahora 25% de $40. Esta es una estrategia que acaba de, uno de sus compañeros acaba de utilizar más la estrategia de en donde el número se quebró, correcto. Que fue lo que hizo Alexis y ustedes lo continuaron. Hable con la persona (a lado) de como averiguamos 25% de $40. De su punto de vista.	Ms. Álvarez: Who agrees with Magdalena? So then what Kamilah did was correct, this is also correct but what Kamilah forgot is that when you divide into equal parts you have to utilize that entire amount. Did you see what Magdalena did? Okay, great so then thirty-seven point five. Now 25% of $40. This is one strategy that one, that one of your classmates has just used where the number was broken, right. That was what Alexis did and you guys continued it. Talk to the person (next to you) about how we find out 25% of $40. Give your point of view.

Whole Group: Homework Problem 2	Mtra. Álvarez: El número que da, que es $40. ¿Entonces si no es 4 por 20, seria 4 por qué? ¿Eduardo, sabes? Eduardo: Diez. Mtra. Álvarez: ¿Cuatro por qué? Eduardo: Cuatro por 10. Mtra. Álvarez: ¿Cuatro por 10, cuánto te da 4 por 10? Estudiantes: Cuarenta. Mtra. Álvarez: Entonces eso significa si es un cuarto. Y esto lo roto, lo quiebro, lo divido en cuatro partes iguales. Primero pongo el 40. Eduardo: Y después eso es cero. Y después lo que hice es puse 10 en cada uno. Mtra. Álvarez: Diez, 20, 30. Y eso significa que 25% sería igual a lo que dijo Dalia. ¿Okay, entonces clase cual es el 25%? Estudiantes: Diez. Mtra. Álvarez: Diez dólares.	Ms. Álvarez: The number that gives, that gives $40. So then if it is not 4 times 20, it would be 4 times what? Eduardo do you know? Eduardo: Ten. Ms. Álvarez: Four times what? Eduardo: Four times 10. Ms. Álvarez: Four times 10, how much does 4 times ten give you? Students: Forty. Ms. Álvarez: So then this means if it is one fourth. And this I break, I break apart, I divide out in four equal parts. First I put the 40. Eduardo: And then that is zero. And then what I did is I put 10 in each one. Ms. Álvarez: Ten, 20, 30. And this means 25% this would be the same as what Dalia said. Okay, so class what is 25%? Students: Ten Ms. Álvarez: Ten dollars.
Whole Group: Warm-Up	Mtra. Álvarez: ¿Setenta y cinco dólares por qué? Bobby: Porque si 100, si 100% es igual a cien, porque. Mtra. Álvarez: ¿Quién está de acuerdo con lo que acaba de decir 100% es igual a 100? ¿Eso es verdad en todos los casos? Bobby: No, porque 100% es igual a 100 dólares. Mtra. Álvarez: En este caso, pero siempre es siempre la verdad. Bobby: Si Mtra. Álvarez: ¿Tienes un ejemplo que no sea verdad? Bobby: Como 100% puede ser de otra Mtra. Álvarez: Excelente. Entonces	Ms. Álvarez: Seventy five dollars why? Bobby: Because if 100, if 100% is the same as 100, because. Ms. Álvarez: Who agrees with what he just said that 100% is the same as 100? Is that true in all cases? Bobby: No, because 100% is the same as 100 dollars. Ms. Álvarez: In this case, but is this is this always true. Bobby: Yes Ms. Álvarez: Do you have an example where this is not true? Bobby: Like 100% can be of another Ms. Álvarez: Excellent. So it depends on the quantity that is how much it is.

	depende de la cantidad eso es cuánto es. Correcto, voy a esperar. ¿Usted que piensa cuánto es? Bobby: Setenta y cinco Mtra. Álvarez: ¿Johnny usted está de acuerdo, porque? Johnny: Porque yo umm, umm es que yo umm si lo cuentas porque umm compare a un dólar. Mtra. Álvarez: Estas diciendo que en vez de utilizar 100 dólares utilizaste un dólar. ¿Hacer esa conexión como te ayudo? Johnny: Yo hice coras. Mtra. Álvarez: ¿Perdón? Johnny: Yo hice coras para Mtra. Álvarez: Coras, okay, eso significa que 25 centavos. ¿Y después que hiciste? Johnny: Umm yo sabía que 25 por 3 era 75. Mtra. Álvarez: Tú dices que 25 cinco por 3 es igual a 75. Johnny: Y setenta y cinco es 3 sobre 4. Mtra. Álvarez: Y setenta y cinco tú crees que es 3 sobre 4. ¿Pero esto es 75 dólares o 75 centavos? Johnny: centavos, no dólares, dólares	Correct, I am going to wait. What do you think it is? Bobby: Seventy five. Ms. Álvarez: Johnny are you in agreement, why? Johnny: Because I umm, umm, it's because I umm, if you count them because umm I compare it to one dollar. Ms. Álvarez: You are saying that instead of utilizing 100 dollars you utilized one dollar. How did making that connection help you? Johnny: I did quarters. Ms. Álvarez: Excuse me? Johnny: I made quarters in order to. Ms. Álvarez: Quarters, okay, that means 25 cents. And then what did you do? Johnny: Umm I knew that 25 by 3 was 75. Ms. Álvarez: You are saying that 25 times 3 is the same as 75. Johnny: And 75 is 3 over 4. Ms. Álvarez: And 75 you believe is 3 over 4. But this is 75 dollars or 75 cents? Johnny: Cents, no dollars, dollars.
Whole Group: Unpacking problem	Mtra. Álvarez: No se olviden que tenemos que desempacar el problema para hacer exactamente lo que nos pide. ¿La primer, la primera pregunta es de qué se trata la historia? ¿Qué información tienes que te puede ayudar a resolver el problema? ¿Qué información necesitas? Voy a leer el problema y por favor escuchen. La señorita Álvarez necesita comprar una sobrecama. Algunas personas no	Ms. Álvarez: Do not forget that we have to unpack the problem so that we can do exactly what it is asking us. The first, the first question is what is the story about? What information do you have in order to solve the problem? What information do you need? I am going to read the problem, please listen. Ms. Álvarez needs to buy a bedspread. Some people do not utilize this word to describe what goes

	utilizan esa palabra para describir lo que va encima de una cama. Aparte de las almohadas hay una cubrecama por eso se llama cubre cama, o se llama sobre, o se llama cobija. Sam: Colcha Mtra. Álvarez: O colcha, colcha también se dice. Entonces a visualizar.	on top of a bed. Aside from the pillows there is a bedspread, that is what it is called a bedspread, or it is called (Sobrecama, cobija) Sam: Colcha (another name for bedspread) Ms. Álvarez: Or colcha, colcha is also used. Okay so let's visualize.
Small Group: Start	Mtra. Álvarez: Momentito, acérquese más, okay, ojitos y orejitas para acá. Paco, por favor me lee la primera parte del problema. Paco: La señorita Álvarez necesita comprar una sobrecama para su mamá. En la tienda Target el precio regular de un cubrecama es ___ sin embargo el cubrecama esta en oferta por el ___ del precio regular. ¿Cuál es el valor del descuento? ¿Cuánto pagaría? Mtra. Álvarez: ¿Una pregunta, Alexis de que se trata esa primera parte? Alexis: De que la Señorita A. necesita comprar un cubrecama para su mamá. Mtra. Álvarez: ¿Muy bien, que más sabemos sobre esa primera parte del problema? ¿Aton? Aton: De que el cubrecama esta en oferta en la tienda. Mtra. Álvarez: Sabemos que esta cubrecama esta en oferta pero hay una cantidad del precio regular. Excelente, esa es el cubrecama en la tienda Target. ¿Puede leer la segunda parte por favor?	Ms. Álvarez: Wait a minute, come closer, okay, little eyes and little ears over here. Paco, please can you read the first part of the problem. Paco: Ms. Álvarez needs to buy a bedspread for her mother. At the Target store the regular price of one bedspread is ____ however, the bedspread is on sale for ____ of the regular price. What is the value of the discount? How much would she pay? Ms. Álvarez: A question, Alexis what is the first part about? Alexis: About Ms. Álvarez needing to buy a bedspread for her mother. Ms. Álvarez: Very good, what else do we know about that first part of the problem? Aton? Aton: That the bedspread is on sale at the store. Ms. Álvarez: We know that the bedspread is on sale but there is a quantity for the regular price. Excellent that is the bedspread at the Target store. Can you read the second part, please?
Small Group: End	Mtra. Álvarez: ¿Excelente, quien está listo para el primer pasó? ¿Quién está listo, quien puede comenzar? Okay,	Ms. Álvarez: Excellent, who is ready for the first step? Who is ready, who can start? Okay, Angie, can you read it

	Angie can you read it in English and then I'll talk to you in a minute, okay. ¿Si cual estrategia? ¿Puedes utilizar unas de estas estrategias de acá, que estrategia te llama la atención? Y qué estrategia puedes adoptar, la línea numérica, quebrar los números, sumar cual, multiplicar, tienes que escoger una para comenzar. Paco: Yo ya escogí el 79 con 50%. Mtra. Álvarez: ¿Excelente que estrategia vas a utilizar? Paco: Línea numérica	in English and then I'll talk to you in a minute, okay. Yes, what is your strategy? Can you utilize one of the strategies over here, which strategy catches your attention? And what strategy can you adopt, the numeric line, decomposing numbers, adding what, multiplications, you have to pick one to start. Paco: I already picked the 79 with 50%. Ms. Álvarez: Excellent and what strategy will you use. Paco: Number line.
Individual	Mtra. Álvarez: ¿Excelente, eso tenemos que saber cuánto es el precio regular y cuánto tiene de descuento? Entonces qué es lo que tengo que es lo que yo tengo que averiguar. Benjy: Para ahorrar dinero cual cubrecama debería comprar la Señorita Álvarez. Mtra. Álvarez: ¿Cómo puedes comenzar para averiguar y darme y decirme como cual cubrecama debo de comprar? ¿Cómo vas a comenzar? ¿Qué estrategia vas a utilizar? Benjy: Como restas. Mtra. Álvarez: ¿Restas cómo vas a utilizar como vas a comenzar? ¿Qué vas a restar? Benjy: Cien. Mtra. Álvarez: Cien, y de donde sacas el 100. Benjy: Porque acá dice que cuesta. Mtra. Álvarez: Cien dólares Benjy: Y acá dice que tiene descuento de un décimo. Mtra. Álvarez: ¿Décimo, y como vas	Ms. Álvarez: Excellent, that is something that we need to know what the regular price is and what is the discount? So then what it that I have to what is do I have to find out. Benjy: To save money which bedspread should Ms. Álvarez buy. Ms. Álvarez: How can you start to find out and to give me to tell me which bedspread should I buy? How are you going to start? What strategy are you going to utilize? Benjy: Like subtractions. Ms. Álvarez: Subtractions how are you going to use how are you going to start? What are you going to subtract? Benjy: One hundred. Ms. Álvarez: One hundred, and where are you getting the 100. Benjy: Because over here it says that it costs. Ms. Álvarez: One hundred dollars. Benjy: And over here it says the discount is one tenth. Ms. Álvarez: One tenth, and how are

	averiguar eso? ¿Qué estrategia te puede ayudar? ¿Vas a utilizar una qué? Benjy: Línea numérica. Mtra. Álvarez: Okay, excelente. ¿Ahorra cómo la vas hacer? Décimo así es como se divide. Aarón ahorita voy a preguntarte lo que estás haciendo para que me digas que estrategia estas utilizando, okay. ¿Y cuál es el valor de acá? ¿Muy bien, y en cuántas partes lo estas dividiendo, cuántas partes? Benjy: Un décimo.	you going to find that out? What strategy can help you? You are going to use a what? Benjy: Number line. Ms. Álvarez: Okay, excellent. Now how are you going to do it? One tenth like that is how you divide it. Aaron I am going to go ask you questions about what you are doing, so you can tell me what strategy you are using, okay. And what is the value over here? Very good, and in how many parts are you dividing it, how many parts? Benjy: A tenth.

Appendix B

Doce Veces Cinco (Twelve times Fives)

<table>
<tr><th></th><th>Mtra. Lorenzo 2014.03.18 Transcription</th><th>Ms. Lorenzo 2014.03.18 Translation</th></tr>
<tr><td>Warm-Up</td><td><table><tr><td>60</td><td>13</td></tr><tr><td>5</td><td>75</td></tr></table></td><td><table><tr><td>60</td><td>13</td></tr><tr><td>5</td><td>75</td></tr></table></td></tr>
<tr><td>Whole Group: Warm-Up 1st Grouping</td><td>Mtra. Lorenzo: Okay listos, cinco, cuatro, tres, dos, uno. Okay todos tienen ideas así que. Todos tienen muchas ideas así que voy a pedir que Max comparta. ¿Cuáles tres pertenecen juntos y porque?
Max: Sesenta no pertenece a 13, 75, y 5.
Estudiantes: ¿Porque?
Max: Porque 60 es par.
Mtra. Lorenzo: ¿Cómo sabes que 60 es par? Él tiene una manera muy distinta de explicar cómo sabe que 60 es par no lo había escuchado antes.
Max: Porque ya sabemos que seis tiene pareja entonces también cuatro</td><td>Ms. Lorenzo: Okay, ready five, four, three, two, one. Okay everyone has ideas so then. Everyone has ideas so I am going to ask Max to share. Which three belong together and why?
Max: Sixty does not belong with 13, 75, and 5.
Students: Why?
Max: Because 60 is even.
Ms. Lorenzo: How do you know 60 is even? He has a very different way to explain how he knows 60 is even, I had not heard before.
Max: Because we already know that six has a partner, so then four also has a partner.</td></tr>
</table>

	tiene pareja. Mtra. Lorenzo: Okay so seis tiene pareja o seis es par otra manera de decirlo y cuatro es par. Lo voy a poner aquí. Okay seis y cuatro son par. Max: Entonces si los juntamos junto va a hacer 10. Mtra. Lorenzo: Okay, right. Max: Y si seguimos poniendo seis y cuatro, seis y cuatro van a llegar hasta 60. Mtra. Lorenzo: Cuántas veces seis y cuatro necesito para llegar a 60?	Ms. Lorenzo: Okay so six has a partner or six is even is another way to say it and four is even. I am going to put it here. Okay six and four are even. Max: So then if we joined them together it's going to be 10. Ms. Lorenzo: Okay, right. Max: And if we continue six and four, six and four, it's going to reach 60. Ms. Lorenzo: How many times six and four do I need to reach 60?
Whole Group: Warm-Up 3rd Grouping	Brandon: Porque solo le debo de contar en múltiples porque acá es sesenta. Dos, cinco, 10, 15, 20, 25, 30, 35, 40, 45. Daniel: Tienes 50. Brandon: Oh, estaba multiplicando, 10, 20, 30, 40, 50, 60. Mtra. Lorenzo: Y usted dijo algo de seis. Brandon: Era, porque si, debemos de multiplicar con el seis para que sean cinco debo de separar, hacer dos veces seis y acá está el seis, uno, dos, tres, cuatro, cinco, seis. Mtra. Lorenzo: Oh no entonces no es seis entonces es dos piensa en eso. ¿Okay, Brandon aquí, aquí hay un grupo de qué? Brandon: Aquí hay seis veces cinco. Mtra. Lorenzo: Seis veces cinco. Brandon: Y aquí a un lado a 6 veces 5. Yo solo le tuve que, para que fuera 6 tiene que ser 6 más 6 doce, 12 veces 5. Mtra. Lorenzo: Entonces usted sabía que 6 veces 5 era 30 y uso esa información. Para encontrar cuántas	Brandon: Because I only have to count by multiples because over here is 60. Two, five, 10, 15, 20, 25, 30, 35, 40, 45. Daniel: You have 50. Brandon: Oh, I was multiplying, 10, 20, 30, 40, 50, 60. Ms. Lorenzo: And you said something about six. Brandon: It was, because, we have to multiply with the six so that it could be five I have to separate, make two times six and here is the six, one, two, three, four, five, six. Ms. Lorenzo: Oh no so then it's not six it is two think about that. Okay, Brandon here, here is a group of what? Brandon: Here is six times five. Ms. Lorenzo: Six times five. Brandon: And here to the side there is 6 times 5. I only had to so, so that it was 60 it had to be 6 plus 6, 12, 12 times 5. Ms. Lorenzo: So then you knew that 6 times 5 is 30 and you used that information. So that you could find

	veces 6 por 5 hacia 60. Okay ahorra si entendí eso es lo que está diciendo la mitad. Okay ven hay manera de hacer. Okay entonces quiero que hablen con su pareja como podemos saber cuántas veces 5 hay en 75. Usando lo que hizo Brandon. ¿Miren que 6 veces 5 es 30 como podemos usar esa información?	how many times 6 by 5 make 60. Okay, so no I understand that it is what you are saying half. Okay so then I want you to talk to your partner about how we can know how many times 5 is in 75. Using the same strategy as Brandon. Look 6 times 5 is 30 how can we use that information?
Small Group: Warm-Up	Mauricio: Veces cinco. Mtra. Lorenzo: ¿Siete veces cinco cuánto es? Aquí tenía cuánto es siete veces cinco. ¿Treinta y cinco entonces otra vez siete veces cinco cuántos son aquí? Alexa: Cinco, 10, 15, 20, 25, 30, 35. Mtra. Lorenzo: ¿Treinta y cinco entonces cuánto es 35 más 35? Alexa: Setenta Mtra. Lorenzo: Setenta. Buena estrategia ahorra pensaron en lo que dijo Brandon. Alexa: Si dos veces 7 de 5. Mtra. Lorenzo: Oh entonces pensaron más bien en 7 veces 5 y esto los puede ayudar a pensar a llegar a ver cuántas veces cinco hay en 75. Mauricio: Siete más 30 va hacer 100 más 30 más 30. Mtra. Lorenzo: Que es lo que escribió, 7 veces 5 es 35 y dos veces, otra vez 7 veces 5 es 35. Okay listos vamos a regresar deje todos sus materiales allí.	Mauricio: Times five. Ms. Lorenzo: Seven times 5 is how much? Here I had, how much is 7 times 5. Thirty-five so then again 7 times five is how many here? Alexa: Five, 10, 15, 20, 25, 30, 35. Ms. Lorenzo: Thirty-five so then how much is 35 plus 35? Alexa: Seventy. Ms. Lorenzo: Seventy. Good strategy so now did you think about what Brandon said. Alexa: Yes, two times 7 fives. Ms. Lorenzo: Oh so you guys thought of 7 times 5 and this could help you think to see how many times five there are in 75. Mauricio: Seven plus 30 is going to be 100 plus 30 plus 30. Ms. Lorenzo: What did you write, 7 times 5 is 35 and two times one more time 7 times 5 is 35. Okay ready we are going to return please leave your materials there.
Whole Group: Sharing out ideas	Karen: Lo que nosotros estábamos tratando. Lo que yo estaba pensando es que si pusiéramos 6 veces 5 iguala a 30. Mtra. Lorenzo: Okay, iguala a, oh okay. ¿Hicieron eso y cuánto es 30 y	Karen: What we were trying. What I was thinking was if we put 6 times 5 is the same as 30. Ms. Lorenzo: Okay, it equals, oh okay. You did that and how much is 30 and 30?

30?
Ingrid: Sesenta
Mtra. Lorenzo: ¿Sesenta y después que hicieron?
Ingrid: Le agregamos 3 veces 5.
Mtra. Lorenzo: ¿Sesenta y otros cinco más es?
Estudiantes: Sesenta y cinco.
Mtra. Lorenzo: ¿Sesenta y cinco y otros cinco?
Estudiantes: Setenta.
Mtra. Lorenzo: ¿Setenta y otros cinco?
Estudiantes: Setenta y cinco.
Mtra. Lorenzo: Setenta y cinco entonces vamos a circular las veces que tenemos cinco. Seis veces 5, circular 6 veces 5 y acá tenemos 6 veces 5 y aquí le agregamos.
Ingrid: Tres veces 5.
Mtra. Lorenzo: Tres veces 5, cuántas veces 15 va ver, 6 a ver cuántas veces 5 seis y 6
Todos: Doce
Mtra. Lorenzo: Doce y 3 más.
Ingrid: Quince.
Mtra. Lorenzo: Quince entonces 15 veces 5 es 75. Listos okay, Alexa nos puede ensenar lo que ustedes estaban pensando.
Alexa: Nosotros estábamos pensando 7 veces 5.
Mtra. Lorenzo: ¿Y porque 7 veces 5?
Alexa: Porque 7 veces 5 y otros 7 veces 5 son 70.
Brandon: Uso la misma estrategia pero solamente le agrego 2 veces 5 más a 60 porque 60 más 10 son igual a 70.

Ingrid: Sixty.
Ms. Lorenzo: Sixty and then what did you do?
Ingrid: We added 3 times 5.
Ms. Lorenzo: Sixty and another five is?
Students: Sixty-five
Ms. Lorenzo: Sixty-five and another five?
Students: Seventy.
Ms. Lorenzo: Seventy and another five?
Students: Seventy-five.
Ms. Lorenzo: Seventy-five so then we are going to circle the number of fives we have. Six times 5, I circle 6 times five and here we have 6 times 5 and here we add.
Ingrid: Three times 5.
Ms. Lorenzo: Three times 5, how many times 15 is there, 6 let's see how many times 5, six and 6.
All: Twelve.
Ms. Lorenzo: Twelve and 3 more.
Ingrid: Fifteen.
Ms. Lorenzo: Fifteen so then 15 times 5 is 75. Ready okay, Alexa can you show us what you guys were thinking.
Alexa: What we were thinking was 7 times 5.
Ms. Lorenzo: And why 7 times five?
Alexa: Because 7 times 5 and another 7 times 5 are 70.
Brandon: She used the same strategy but only she added 2 times 5 more to 60 because 60 plus 10 is the same as 70.

Appendix C

Bombear Agua (Pumping Water)

	Mtro. Sánchez 2014.04.03 Transcription	Mr. Sánchez 2014.04.03 Translation
Problem of the Day	*Trader Joes vende agua en botellas de ___ galones. Si bombean ____ galones de agua. ¿Cuántas botellas pueden rellenar?*	*Trader Joes sells water in ___ gallon bottles. If they pump ____ gallons of water, how many bottles can they fill?*
Whole Group: Introduction of POD	Mtro. Sánchez: Discutimos eso un poco más. ¿Qué quiere decir llenar, rellenar o bombear? ¿Alguien puede describir eso? Andy. Andy: Es como el M&Ms, M&Ms POD. Como lo que necesitábamos poner los M&Ms en cada bolso para saber cuántos M&Ms. Mtro. Sánchez: Alguien nos puede demos… Andy nos puedes demostrar esta acción. Andy: ¿Con que? Mtro. Sánchez: Dijiste como la acción con tus manos con tu cuerpo. Haber parece, parece, parece. Estudiante: Okay I am the bag. Mtro. Sánchez: ¿Ese es el del M&Ms o del agua? Andy: M&Ms. Mtro. Sánchez: Okay alright estamos de acuerdo con la acción de Andy? que estamos poniendo algo. Estudiante: I disagree. Mtro. Sánchez: ¿Mas o menos umm Vincent que quieres decir tú? Vincent: Que es que umm lo que, que tienes que hacer es como es que casi no le entiendo porque no tiene los números umm pero umm creo que lo está tratando de decir es que tienes que ver cuántos galones vas a llenar.	Mr. Sánchez: Let us discuss a bit more. What does to fill, to fill-up, to pump mean? Can someone describe this? Andy. Andy: It is like the M&Ms, M&Ms POD. Like what we were doing was putting M&Ms in every bag to know how many M&Ms. Mr. Sánchez: Can someone demon…Andy can you demonstrate this action. Andy: With what? Mr. Sánchez: You said that the action like with your hands your body. Let's see stand-up, stand-up, stand-up. Student: Okay I am the bag. Mr. Sánchez: This is the M&Ms or the water? Andy: M&Ms. Mr. Sánchez: Okay alright do we agree with Andy's action? That we are putting something. Student: I disagree. Mr. Sánchez: More or less umm Vincent what did you want to say? Vincent: That it is that umm what I that you have to. It's because I don't really understand because it doesn't have the number umm but umm I think that it is trying to say is that you have to see how many gallons you are going to fill.

Whole Group: Bombear as Problematic	Mattie: Mister some people might be getting confused because of bombear. Mtro. Sánchez: Okay tres dos uno. Okay so esa palabra bombear que quiere decir la palabra bombear. Yeah Alfonso Alfonso: Rellenar Mtro. Sánchez: Rellenar, Estudiante: Llenar Mtro. Sánchez: llenar, rellenar, llenar. Alguien más la palabra bombear, es un verbo. A ver qué estás haciendo, Gilberto. Tres dos uno. Gilberto tu hizo esto. Gilberto: Pumping Mtro. Sánchez: Mattie Mattie: A veces cuando las pelotas no tienen aire como le llaman una bomba para.	Mattie: Mister some people might be getting confused because of pumping. Mr. Sánchez: Okay three two one. Okay so that word to pump what does that word to pump mean. Yeah Alfonso. Alfonso: To fill-up. Mr. Sánchez: To fill-up. Student: To fill. Mr. Sánchez: To fill, to fill-up, to fill. Someone else the word to pump, it's a verb. Let see what you are doing. Gilberto. Three two one. Gilberto you did this. Gilberto: Pumping. Mr. Sánchez: Mattie. Mattie: Sometimes when the balls don't have air they call it a pump in order to.
Whole Group: Bombear Explanation	Mtro. Sánchez: So muchos están diciendo que hay como, como Alfonso dijo que hay una manguera podemos ver que tenemos como un tanque de agua verdad. Y para sacar, para sacar el agua cual es el verbo que van usar, van a bombear. So vamos a inflar una pelota con aire que vamos hacer. Vamos a empujar. Podemos imprimir arriba. Push, what's push. Empujar, estamos empujando, estamos empujando Mtra. V: Like presionando Mtro. Sánchez: Estamos empujando, y el aire como muchos, la presión verdad, la presión va a mandar el agua así para que salga por la manguera que tenemos. Roger: Porque esta toda el agua llena	Mr. Sánchez: So many are saying that there is, like Alfonso said that there is a hose we can see that we have like a tank right. And to take out, to take out the water what is the verb you are going to use, you are going to pump. So we are going to inflate a ball with air what are we going to do. We are going to push. We can push on top. Push, what is push. Push, we are pushing, we are pushing. Ms. V: Like pressing. Mr. Sánchez: We are pushing, and the air like many, the pressure right, the pressure is going to send the water like this so that it comes out of the hose that we have. Roger: Because it is full of water and if.

	y si ya.	
	Mtro. Sánchez: Entonces regresamos a los números que nos dio Janet. Si, si venden botellas de dos galones y el tanque de agua es de 20 galones cuántas botellas podemos rellenar. Hable con su pareja.	Mr. Sánchez: So then let's return to the numbers that Janet gave us. If, if they sell bottles of two gallons and the tank of water is 20 gallons how many bottles can we fill. Talk to your partner.
Individual	Mtro. Sánchez: ¿Cómo vas a comenzar la pregunta que piensas?	Mr. Sánchez: How are you going to start the question what do you think?
	Carlos: Trader Joes vende agua en botellas de dos galones. ¿Si bombean 20 galones de agua, cuántas botellas pueden rellenar? Veinte.	Carlos: Trader Joes sell water in bottles of two gallons. If they pump 20 gallons of water, how many bottles can they fill? Twenty.
	Mtro. Sánchez: So 20 botellas que pueden llenar de agua.	Mr. Sánchez: So 20 bottles that they can fill with water?
	Carlos: No 10.	Carlos: No 10.
	Mtro. Sánchez: ¿Son 10, porque? ¿Porque es 10?	Mr. Sánchez: So they are 10, why? Why is it 10?
	Carlos: Porque es lo que hicimos. Porque yo sé que puedo contar dos. Dos, cuatro, seis, ocho, diez. Y si cuento hasta 10 pero me quedan 10.	Carlos: Because it is what we did. Because I know I can count by two. Two, four, six, eight, ten. And if I count to 10 but then I'm left with 10.
	Mtro. Sánchez: Okay:	Mr. Sánchez: Okay
	Carlos: So 10 y 10 son 20.	Carlos: So 10 and 10 is 20.
Individual	Mtro. Sánchez: Haber dime lo que estás haciendo.	Mr. Sánchez: Okay, tell me what you are doing.
	Gloria: Lo que hice para, es que lo partí en la mitad y lo compartí en cinco. Entonces sé que cinco por 10 es 10 veces 57 galones de agua. Y las dividí pero me sobraban siete galones.	Gloria: What I did to, is that I cut in half and I shared it in five. So then I know that five times 10 is 10 times 57 gallons of water. And I divided it but I was left with seven gallons.
	Mtro. Sánchez: Pero yo sé que hiciste algo más, Que hiciste con los otros. ¿Dónde está tu unidad?	Mr. Sánchez: But I know that you did something else. What did you do with the others? Where is your unit?
	Gloria: Trader Joes vende agua en botellas de diez galones. Si bombea 57 galones de agua cuántas botellas puede rellenar.	Gloria: Trader Joes sells water in bottles of 10 gallons. If they pump 57 gallons of water how many bottles can they fill.
	Mtro. Sánchez: Cincuenta y siete	Mr. Sánchez: Fifty seven.

	Gloria: Mil. Mtro. Sánchez: Cincuenta y siete mil. Yo allí sé que son 105 botellas que pueden llenar. Dime que representan estos números de acá. Gloria: Esos números representan los cinco galones. Porque sé que estos son dos galones, dos galones. Mtro. Sánchez: ¿So, son dos galones aquí, el 100 o el cinco? Gloria: Esos dos son galones. Mtro. Sánchez: Los números de afuera representan galones o botellas. Gloria: Estaba contando por dos para tener 10 galones de agua. Mtro. Sánchez: Okay. Gloria: Y hice algo mal. Mtro. Sánchez: Y dijiste que cometiste un error.	Gloria: Thousand. Mr. Sánchez: Fifty seven thousand. I know that right there are 105 bottles that they can fill. Tell me what these numbers over here represent. Gloria: Those numbers represent the five gallons because I know that these are two gallons, two gallons. Mr. Sánchez: So there are two gallons here the 100 or the five? Gloria: Those two are gallons. Mr. Sánchez: The numbers on the outside represent gallons or bottles. Gloria: I was counting by twos to have 10 gallons of water. Mr. Sánchez: Okay Gloria: And I did something wrong. Mr. Sánchez: And you said that you made a mistake.
Individual	Mtro. Sánchez: ¿Y qué representan esos números? Derek: Representan el cinco y 10 que los galones o los galones que estos son los galones de agua. Y luego que estos son los que iba a dar los bottles y son 10, 10, 10, 10, y 10 y son 50. Y luego 10 a estos 10 le quito cinco y van a quedar cinco galones y los hice. Y eran siete galones so eran siete. Y luego eran 10 y cinco. Mtro. Sánchez: ¿So los siete galones que sobran podemos llenar una botella de agua? Derek: Si Mtro. Sánchez: ¿Son suficiente galones de agua? Derek: No. Mtro. Sánchez: ¿Cuántas botellas podemos llenar? Ve aquí cinco	Mr. Sánchez: And what do those numbers represent? Derek: They represent the five and 10 that the gallons or the gallons that these are the gallons of water. And that these are the ones that I was going to give the bottles and they are 10, 10, 10, 10, 10, and they are 50. And then 10 to these 10 I took away five and there are going to be five gallons and I did. And there were seven gallons so there were seven, and then there were 10 and five. Mr. Sánchez: So the seven gallons that are left can we fill one bottle of water? Derek: Yes Mr. Sánchez: Are they enough gallons of water? Derek: No. Mr. Sánchez: How many bottles can

botellas y nos sobran siete. ¿Podemos utilizar esta misma estrategia para los otros sets? Estos de aquí o los que están acá. ¿Tú crees que podamos usar la misma estrategia? Descomponiendo los números en decenas y centenas. Derek: Si. Mtro. Sánchez: Okay.	we fill? See here five bottles and we have seven left. Can we use this same strategy for the other sets? These here the ones that are over here. Do you think you can use the same strategy? Decomposing numbers into tens and hundreds. Derek: Yes. Mr. Sánchez: Okay.

Appendix D

Dos Dígitos (Two Digits)

Mtra. Lorenzo 2014.03.28 Transcription	Ms. Lorenzo 2014.03.28 Translation
Mauricio: Este es un digito.	Mauricio: This is a digit.
Mtra. Lorenzo: ¿Y qué número es?	Ms. Lorenzo: And what number is it?
Mauricio: (Dos)	Mauricio: (Two)
Mtra. Lorenzo: (Dos)	Ms. Lorenzo: (Two)
Mauricio: Y otro digito.	Mauricio: And another digit.
Mtra. Lorenzo: ¿Es uno, cuántos dígitos tiene 21?	Ms. Lorenzo: It is one, how many digits does 21 have?
Mauricio: Dos.	Mauricio: Two
Mtra. Lorenzo: Dos dígitos. ¿Cuántos dígitos, dígitos tiene 12?	Ms. Lorenzo: Two digits. How many digits does 12 have?
Mauricio: Dos	Mauricio: Two
Mtra. Lorenzo: Yo no estoy de acuerdo con usted porque usted dijo que ocho contiene dos dígitos enséñeme los dígitos. Uno, dos.	Ms. Lorenzo: I am not in agreement with you because you said that eight contained two digits show me the digits. One, two.
Mauricio: Aquí.	Mauricio: Here
Mtra. Lorenzo: ¿Okay cuántos dígitos tiene ocho?	Ms. Lorenzo: Okay how many digits does eight have?
Mauricio: Hay ocho.	Mauricio: There are eight.
Mtra. Lorenzo: Hay ocho unidades estoy de acuerdo. Verdad pero unidades es diferente que digititos. Dígitos son los números en un número. Okay sé que es difícil de ver la diferencia pero mira un digito y un digito. Entonces son dos, este es un digito, un digito. Okay un digito otro digito. ¿Okay cuántos dígitos hay aquí?	Ms. Lorenzo: There are eight units I agree. True but units are different from digits. Digits are the numbers in the number. Okay I know it is hard to see the difference but look one digit and one digit. So then there are two, this is one digit, one digit. Okay one digit another digit. Okay how digits are here?

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PART II:

A FRAMEWORK FOR TEACHING MATHEMATICS TO ELLS

Abstract

This article is a review of the literature on teaching mathematics to English language learners. Mathematics is examined through a language and mathematical register lens. The article uses three existing frameworks for the general teaching of English language learners in order to develop the framework: Knowledge Areas for Teaching Mathematics to English language learners. This framework offers four knowledge areas for the teaching of mathematics to language learners: (a) knowledge of second language development and mathematical knowledge; (b) knowledge of children's mathematics and language backgrounds; (c) knowledge of how to use language and culture to teach mathematics to English language learners; (d) knowledge of how to promote language and culture in mathematics instruction.

In this article, I review the literature and develop a framework for teaching mathematics to English language learners (ELLs). In reconceptualizing three existing frameworks specific to the general teaching of ELLs, I offer a new framework that focuses on a holistic view of mathematics teaching. I examine mathematics teaching through a language and mathematical register lens; that is that mathematics teaching has an oral component. In this literature review I suggest that there exists a limited discussion on the ways in which Bilingual and ESL teachers consider language and the multiple mathematical registers present in their classrooms.

In recent years, the US has seen an increase in immigration rates, largely due to an influx in the number of Latina/o people.[2] This increase in the immigration rate has changed the demographics in classrooms across the country. One in four children has at least one immigrant parent, that is a parent who is foreign born and 60% of the parents of children under eight are from Mexico or Central America (Fortuny, Hernandez, & Chaudry, 2010; Hernandez, Denton, & Macartney, 2008). Educating these students, whose first language is other than English, has become a topic for national debate (Gebhard, 2010; Olsen, 2009) and thus has taken on different approaches to teaching non-English speakers.

The teacher population equipped to work with these students has not increased at the same rate. Ballantyne, Sanderman, Levy and the National Clearinghouse for English Language Acquisition & Language Instruction Educational Programs (2008) found that while the majority of mainstream teachers have at least one ELL student in their classroom, and only 29.5% of the teachers with ELLs are prepared to instruct their ELLs effectively. Menken, Antunez, National Clearinghouse for Bilingual Education, and ERIC Clearinghouse on Teaching and Teacher Education (2001) found that of the 1,075 colleges and universities they looked at, less than one

[2] The 2010 US Census reported that there were 50.3 million Hispanics in the US at that time.

sixth offered pre-service teacher training focusing on instruction of Ells, addressing it either as a separate curriculum or within their teacher training programs. Furthermore, the National Center of Education Statistics (NCES) (2001) survey also revealed 26% of in service teachers had received some form of professional development regarding the instruction of ELLs, although 57% of the teachers believed that they needed additional training or instruction. These results raise the question of whether ELLs, generally, have access to adequately trained and supported teachers (Rumberger & Gándara, 2004).

This becomes more complicated when considering how mathematics is taught to ELLs. Many teachers view mathematics as a universal language (Fernandes, 2012; McLeman, Fernandes, & McNulty, 2012; Moschkovich, 2013; Rolka, 2004) which complicates their understanding about ELLs' experience with an English mathematics classroom. Perhaps even more difficult is that the vast majority of teachers teaching ELLs are either unfamiliar with the students' native language or are non-native speakers of the second language (Gebhard, 2010; Lucas, 2010). If teachers see mathematics as a universal language and are themselves unfamiliar with the students' native language they fail to see students' complications in learning mathematics. A complicating factor is that there exist a mathematical language (symbols, equations, etc.), yet this math language cannot be taught without another language (English, Spanish), thus increasing the language demands during the teaching and learning of mathematics.

This literature review will begin by setting up the political background of bilingual education and the ways this has influenced the types of programs in schools. Next, these programs are summarized, providing a glimpse into the type of instruction that teachers should be prepared to provide for this population of students. The paper then moves to discuss the way that teachers are being prepared to teach ELLs in broad terms. Then, building off of three existing frameworks, I provide a framework for examining teaching mathematics to ELLs. The concluding remarks tie the framework together as a way to answer the following two questions:

What factors contribute to the way teachers instruct mathematics to ELLs?

How can we understand the impact of the language used by the teacher and students during the teaching and learning of mathematics?

Politics of Bilingual Education

Bilingual education has had a long history of varying degrees of support in the United States. Bilingual education dates back to the 1700s where European immigrants settled in enclaves and ran their own schools. However, the federal government has since adopted different laws in regards to language and bilingual education[3]. The Bilingual Act of 1968 recognized the needs of students with limited English communication skills and encouraged school districts to use federal money to start bilingual programs in their schools. The Emergency Immigration Education Act of 1984 was passed in order to help school districts offset the cost required to educate immigrant non-English speaking students. Most of these funds were used to increase school personnel equipped with the tools (ESL training and/or bilingual certification) to help the growing immigrant population. Olsen writes "Once home of the strongest state bilingual education program in the nation, in the past two decades, California has weathered contentious and far-ranging battles over how to educate immigrants (2009, p. 818)." In 1994 California voters passed Proposition 187 that denied public benefits including education to immigrants. Proposition 187 was later overturned, but the discourse has left a lasting negative impression of immigrants and immigration to many Californians. In 1998 California voted in favor of Proposition 227 that forced all public education to be instructed in English-only; thus, many

[3] For a timeline and history on bilingual education laws and its effects see Baker, 2011.

bilingual programs were dismantled[4]. The legislation allowed for some special programs but the overall result was that of an English-only policy. There have been similar state policies throughout the United States; California is used as an example because of its history with bilingual education and its demographics. Nationally, the 2002 No Child Left Behind (US Department of Education) legislation required states and schools to make quantitative improvements in testing. This legislation also replaced the Bilingual Act of 1968 and reauthorized the Elementary and Secondary Act (ESEA) of 1965 (Sass, 2010). Title III of the legislation states that its' first purpose is to "ensure that children who are limited English proficient, including immigrant children and youth, attain English proficiency, develop high levels of academic attainment in English, and meet the same challenging state academic content and student academic achievement standards as all children are expected to meet" (*ESEA* Section 3102(1)). These changes in bilingual education over time have led educators to take multiple approaches when educating the growing population of ELLs.

Bilingual Education versus ESL Instruction

There are two main approaches to the instruction of students who are non-native English speakers: the English as a Second Language (ESL) approach and the bilingual language approach. The ESL approach emphasizes the use of the English language to help students meet their language needs with an end goal of meeting high academic standards. ELLs in ESL programs are primarily, if not exclusively, taught in English, and their native language is only used to help support their use of the English language (Faulkner-Bond et al., 2012). Content is instructed in English and presented in a way that supports students' attainment of the English language.

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[4] For a more in depth look at California's Proposition 227 see Olsen, 2009.

Advocates for the bilingual approach claim that instruction in a student's native language will aid the student meeting their needs in the English language while maintaining high academic standards (August, Calderon, Carlo, & Nuttal, 2006; Cummins, 2001; Thomas & Collier, 2002; Turner & Celedón-Pattichis, 2011). There are many programs that use the bilingual approach, but each program differs in their methods and/or end goal. There are some programs whose main goal is the improvement in students' English fluency; and there are other programs that aim to develop both the students' native language and English language skills (Faulkner-Bond et al., 2012) (See appendix A for an in depth description of language programs within each approach).

The bilingual approach has been found to produce higher test scores than those of the ESL approach on academic content assessments and measures of reading comprehension (August & Shanahan, 2008; Genesee, Lindholm-Leary, Saunders, & Christian, 2006; Goldenberg, 1999; Saunders & Goldenberg, 2010; Thomas & Collier, 2002, 2003). These studies have also found that there is a significant difference in English proficiency when students are instructed in their native language. Two-way immersion programs, programs in which the instruction is both in Spanish and English and the classroom demographics are 50% non-English speaking and 50% English speaking, have been shown to be successful for both Spanish and English speakers in their native and second languages (Howard, Genesee, & Christian, 2004). ELLs in two way immersion programs outperform ELLs in ESL and transitional bilingual programs in both content assessments and English language proficiency.

Despite studies that show the importance of two-way immersion programs, as of March 2015 the Center of Applied Linguistics has reported 454 dual immersion schools in 33 different states which are primarily K-6 schools approximately 0.3% of all schools in the US. Bitterman, Gray, Goldring and NCES (2013) reported 70,780 schools (private and public) had at least one

ELL student in their school approximately 61% of all K-12 schools for the 2011-2012 school year. Thus most students are receiving ESL or transitional bilingual instruction (whose goal is English fluency) and hence most teachers are instructing in these types of programs.

Mathematical Registers and Mathematical Language

When teaching mathematics to ELLs, teachers tend to take three approaches based on their beliefs about mathematics and language. The first is the belief that English needs to be taught first in order to properly teach mathematics. The second is the belief that mathematics is a universal language and thus teaching mathematics to ELLs is not complicated by the students limited English language proficiency. Lastly, there are teachers who believe that mathematics cannot be divorced from the language it is taught in and thus they must attend to the various mathematical registers/language of the classroom. These categories were formulated after reading through literature on teachers beliefs of language and mathematics.

English First

Some teachers believe that in order to properly teach mathematics to second language learners they must first focus on the second language. This belief is based on an assumption that students will not be able to engage in the classroom and teachers will not be able to instruct mathematics unless the student is English proficient (Barwell, 2005; Fernandes, 2012; McLeman et al., 2012; Walker, Ranney, & Fortune, 2005). Mathematics instruction from teachers who have this belief primarily focuses on a child’s language attainment rather than on mathematical content knowledge.

Is Mathematics a Universal Language?

Researchers have shown that many teachers consider mathematics to be a universal language (Rolka, 2004; Tevebaugh, 1998). Research also shows that teachers believe the teaching and learning of mathematics does not require a vast amount of language proficiency in the language in which mathematics is being taught (Fernandes, 2012; McLeman et al., 2012; Rolka, 2004). Other teachers see teaching mathematics to ELLs or teaching mathematics to students in the teacher's nonnative language as uncomplicated given the computational/procedural nature of mathematics (Rolka, 2004; Tevebaugh, 1998).

Some scholars see mathematics as an academic language with its own vocabulary and set of norms. Mathematics as an academic language along with the development of that language has been discussed in research as a mathematical *register*. A mathematical register is situated within the social context in which it is constructed; thereby, refuting the notion that mathematics is a universal language.

> The notion of 'developing a language' means, therefore, adding to its range of social functions. This is achieved by developing new registers. A register is a set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings. We can refer to a 'mathematics register', in the sense of the meanings that belong to the language of mathematics (the mathematical use of natural language, that is: not mathematics itself), and that a language must express if it is being used for mathematical purposes. In order to express new meanings, it may be necessary to invent new words; but there are many different ways in which a language can add new meanings, and inventing words is only one of them (Halliday, 1978, p. 195).

Halliday further discuses seven techniques to develop a register of mathematics: reinterpreting existing words (set, odd, and column); creating new words out of a native word stock (output, feedback, and clockwise); borrowing words from another language (infinite, series, and degree); creating new words in imitation of another language (for Spanish speakers rangulo for rectangle); inventing totally new words; creating ‘locutions’ (square on the hypotenuse), and creating new words out of non-native word stock (binomial, parabola, and figurate are not borrowed from Greek or Latin).

Mathematical Language within a Language

Pimm (1987) extends the notion of mathematical registers by proposing that mathematics is a language with its own system of meaning making, which includes specialized terms, and the use of everyday language with specialized meaning, specialized expressions, and sentence construction that might be unfamiliar to everyday speech.

Although this may appear to support the notion of mathematics as a universal language some argue that more than one register exists. Farrugia (2009) and Moschkovich (2005, 2007) suggest that mathematical registers are situated within a culture, a community, or group who have informally developed this register. As a result, classrooms have multiple mathematical registers represented by the students and the teacher. Furthermore, researchers argue that teachers and students with different mathematical registers may have difficulty when trying to transfer from one register to the other because they may lack full understanding of both registers (Moschkovich, 2005, 2007; Ron, 1999). That is a mainstream (non-certified ESL or Bilingual) teacher may be working within a particular English mathematical register and an ELL student may be working within a particular mathematical register in their native language and transferring it over to the less familiar English mathematical register of the teacher and classroom. Students may also be building both mathematical registers simultaneously or using one to inform the other.

Mathematical registers are developed by a group of people within a cultural context (Halliday, 1978), which supports other scholars’ views that mathematics is not culture-free

(D'Ambrosio, 1985; Presmeg, 1998). A teacher who believes that mathematics cannot be taught without also being mindful of language outside of mathematical language, must address the multiple mathematical registers in her classroom.

Preparing Teachers to Teach Mathematics to ELLs

Despite research on mathematical registers and the multiple register that may be operating in a given classroom, many teachers do not have the experience or expertise to teach mathematics to ELLs. Researchers have documented the need to provide low-income students who are also ELLs highly qualified teachers who can meet their needs as ELLs (Ingersoll, 2004; Rumberger & Gándara, 2004). Saunders and Goldenberg (1999) argue that all teachers who have an ELL student in their classroom, particularly mainstream secondary and elementary teachers, need to be prepared to have specific specialized instructional practices to best serve the needs of these students. Teacher preparation programs must explicitly train future bilingual and ESL teachers to be well versed in the principles of second language acquisition (de Jong & Harper, 2005; Lucas & Grinberg, 2008; Menken, et. al, 2001; Saunders & Goldenberg, 2008). Cummins (1981) states that teachers need to know strategies that make sense for students in direct ESL instruction as well as having a full understanding of the students native language use for the development of literacy skills and academic content.

Teachers of ELLs are primarily trained in four different approaches; ESL professional development, foreign language certification, ESL certification, and bilingual certification. In the first approach in-service teachers receive a wide variety of professional development that addresses how to best work with ELLs. Ballantyne and colleagues (2012) report from the NCES (2001) study that 26% of teachers had received professional development to address non-English speaking students. The second approach has teachers trained as foreign language instructors but who teach in ESL or bilingual education programs. These teachers do not receive adequate training to teach K-12 content in the foreign language in which they are certified (Shick & Nelson, 2001). They have learned the structure of the language but not how to teach content using the language. The third approach has teachers certified as ESL instructors. These teachers have had training in the ways that students develop a second language while being immersed in it. The fourth approach has teachers certified as bilingual teachers; these teachers have received a degree in bilingual education and are prepared to teach elementary or secondary content in two languages. As of 2012, only 1.5% of the public elementary and .6% of the public secondary teachers in the United States where bilingual and/or ESL certified (NCES, 2013).

Bilingual and ESL teachers need more preparation than mainstream teachers; they have to know the content, the language, and how to teach it to ELLs. Yet, ESL and bilingual teachers are given up to one extra year of coursework according to SB 2042 to be prepared to teach ELLs (Olivos & Saramiento, 2006). Olivos and Saramiento argue that a fifth year is not a viable option for all students and thus teacher certification programs are imbedding coursework in ESL and Bilingual education in other coursework. Legislation (SB 2042) does not explicate the type of standards for bilingual certification, thus bilingual teachers are not being prepared according to best practices for ELLs but rather by the general legislation standards. Part of the lack of adequate preparation of ELLs is the emphasis in the teaching standards and techniques that treat all ELLs as the same or in need of the same type of instruction (Balderrama, 2001). Watson, Miller, Driver, Rutledge, and McAllister (2005) investigated the preparation of mainstream teachers for ELLs by looking at teacher education textbooks. They looked at 25 textbooks that were most widely used based on purchase rates, and found that they did not directly discuss the concept of teaching ELLs. The instruction of ELLs was mentioned in less than 1% of each textbook in sections describing the problems that ELLs face in the classroom and the textbooks did not put forth solutions or appropriate teaching methods to address the stated problems.

Frameworks for Teaching ELLs

The view of mathematics as a universal language from inadequately prepared teachers of ELLs is problematic; given that the teachers focus is not on advancing the mathematical knowledge of their students using the students' native language as a resource. Teachers' main focus is advancing the students English language first then their mathematical knowledge. As opposed to a teacher who believes that mathematics must be taught using another language and is aware that mathematical registers are developed within a cultural context, the third type of teacher from above. The question then becomes how to adequately prepare teachers to teach mathematics to ELLs. Several researchers have put forth frameworks for preparing all teachers to teach ELLs that are both in-service and pre-service. This literature review focuses on three frameworks, Menken and colleagues (2001), Lucas and Grinberg (2008), and de Jong and Harper (2005). The first was developed for policy purposes, the second for teacher education, and the third to attend to cultural aspects. These three frameworks are used to think through and set up a fourth framework for teaching mathematics to ELLs. Menken and colleagues' (2001) framework was selected because it was prepared for the United States Department of Education Office of Bilingual Education and Minority Languages Affairs and thus has been highly cited. Lucas and Grinberg's (2008) framework was selected because it appeared in the Handbook for Teacher Education and has been highly cited. Finally de Jong and Harper's (2005) framework was selected given its emphasis on culture, which reflects the current research in mathematics education around culturally relevant teaching. Menken and colleagues' framework focuses on pre-service teachers while the other two frameworks focus on classroom teachers.

Menken and colleagues (2001) framework includes knowledge of pedagogy, linguistics, and cultural and linguistic diversity. They emphasize the importance of having teachers be exposed to a variety of instructional methods for teaching literacy and content. With respect to knowledge of pedagogy, they argue teachers need to be able to differentiate instructional methods for bilingual education programs and those used in monolingual (English) education programs. With respect to knowledge of linguistics, they state that teachers need to be able to apply research on the development of a second language. This aids in the recognition of stages and characteristic of language acquisition in order to help students through this process. With respect to knowledge of cultural and linguistic diversity, they claim teachers need to attend to student's native languages and cultures since researchers have shown that this leads to higher student achievement. This framework provides teachers with resources that help them view the commonalities of ELLs as a group and as individual ELLs.

Lucas and Grinberg (2008) emphasize four knowledge areas of language. The first knowledge area is the language backgrounds, experiences, and proficiencies of their students. Lucas and Grinberg say that teachers need to understand this knowledge area for two main reasons. The first reason is that relationships are central to healthy human development and students' academic engagement. Thus, in order for a teacher to develop a relationship with the student they must understand the students' backgrounds. The second reason they state that students' linguistic and cultural resources should be used to support students' learning. The second knowledge area that teachers need to understand is how second language development occurs. The third knowledge area that teachers need to understand is the connection between language, culture, and identity. They argue that if teachers understand the connection between language, culture and identity then they will be more likely to develop respectful, affirming attitudes toward linguistically diverse students. Furthermore, if teachers understand that the way students express themselves reflects different cultural values and expectations, they can learn not to make assumptions about students' intentions based solely on their own cultural framework. The fourth and final knowledge area that teachers need to understand is the way that language is formed, the mechanics of it, and its uses. They claim that teachers should have a full understanding of the language grammar and how students use this language to navigate their worlds. The authors write that these knowledge areas give teachers a full understanding of their students' backgrounds and approaches to learning a second language.

De Jong and Harper (2005) claim that national content standards only describe good teaching practices but do not explain the highly linguistic foundation that make these effective classroom practices. Within these good teaching practices, students are still expected to read texts, participate actively in discussion, and use language to represent their learning, either through oral reports or formal presentations of their work. Thus, they suggest that teachers need specialized knowledge about teaching ELLs in addition to the preparation that they already receive for teaching mainstream students. This specialized knowledge includes knowledge of both language and culture of their students. De Jong and Harper therefore argue for three dimensions in the good teaching of ELLs. First, teachers need to understand the process of second language acquisition and acculturation. This is an understanding of how both the first and the second language develop in similar ways but also how they differ. Teachers must also develop an understanding of how this process is manifested in the student's oral and literacy development. Second, teachers need to be aware of how language and culture play the role of media (modes) in both teaching and learning. This is an understanding of both the structure of oral/written language and colloquial versus academic English as well as how the teacher's expectations and opportunities for learning in the classroom are based on cultural assumptions that might not be familiar to the student. Third, teachers need to have linguistic and cultural diversity as a goal of curriculum and instruction. This is where teachers develop an awareness of the role language plays with ELLs and of the culture that surrounds the students both outside and inside the classroom.

Putting the three frameworks together, teachers must fully understand the backgrounds of their students both as a group and as individuals and know the different approaches to learning a second language all while having cultural diversity be a goal of instruction. Next this literature review examines across the three frameworks to provide a fourth framework that addresses teaching mathematics to ELLs (see Figure 5).

Framework of Knowledge Areas for Teaching Mathematics to ELLs

Preparing teachers to teach ELLs either as bilingual or ESL teachers becomes more complicated when we discuss this through the instruction of mathematics. Mathematics requires not only a specialized technical language but also everyday discourse used around mathematics, thus increasing the demand (mathematics and language) on bilingual and ESL teachers. Elementary school teachers have often been shown to have difficulties teaching mathematics either from lack of mathematical knowledge for teaching or negative affects towards mathematics (Brown, Westenskow, & Moyer-Packenham, 2012). This framework is designed to analyze teachers' practice and to inform teacher preparation educators of knowledge areas for teaching mathematics to ELLs. Mapping the recommendations of the three frameworks above to the knowledge teachers of ELLs need in order to teach mathematics, teachers would need to know: how a second language is developed and have mathematical knowledge for teaching; have knowledge of children's mathematics and language backgrounds; knowledge of how to use language and culture to teach mathematics to ELLs; and knowledge of how to promote language and culture in mathematics. In the following sections I expand on each of these knowledge areas through current and past research.

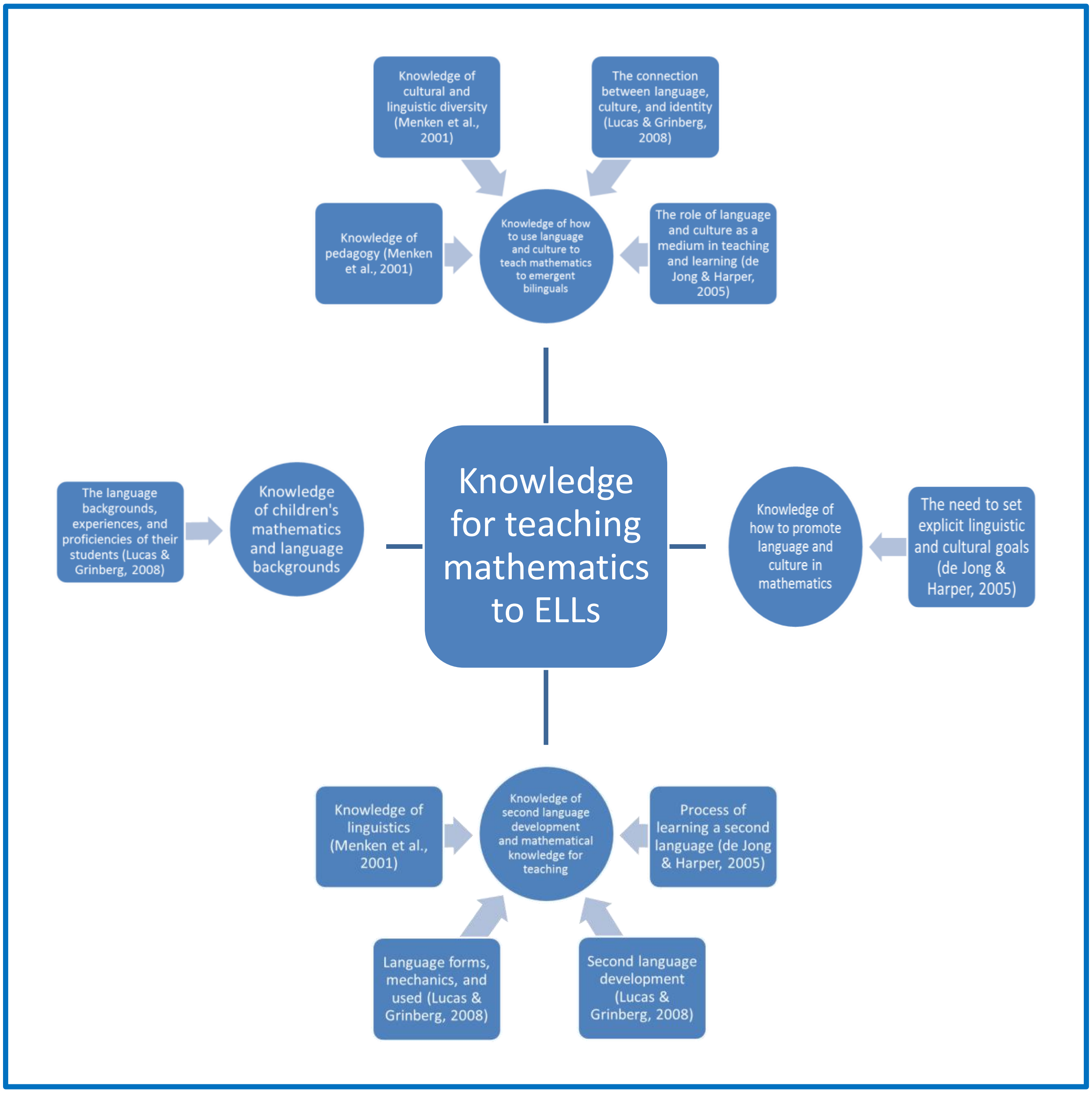

Figure 5. Framework for Teaching Mathematics to ELLs

Knowledge of second language development and mathematical knowledge.

The framework discussed here is mapped from: Menken and colleagues (2001) knowledge of linguistics; Lucas and Grinberg's (2008) second language development and language forms, mechanics, and uses knowledge areas; de Jong and Harper's (2005) process of learning a second language; and teaching mathematical knowledge for teaching[5]. (See Figure 6)

Teachers who have knowledge of linguistics understand psycholinguistics; the mental process involved in the production, cognition, and comprehension of language. Teachers also understand sociolinguistics; "the study of the interaction between linguistic, cultural, and social elements in communication as they impact learning two languages" (Menken et al., 2001, p. 11). In order to best assist students' language development teachers must have a full understanding of the language forms, mechanics, and its uses in the languages that they are teaching (Gándara, Maxwell-Jolly, & Driscoll, 2005; Valdés, Bunch, Snow, & Lee, 2005). Developing a second language requires teachers to understand the advantages of having a strong native language; see native language as a resource; understand everyday language versus academic language; constantly expose student to proficient users of the second language; provide students with language that is attainable; and understand anxiety as a hindrance to second language development (Lucas & Grinberg, 2008). The process of learning a second language has teachers understanding how students' development of oral and literacy skills in their first and second language are similar and different (de Jong & Harper, 2005).

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[5] The literature on the knowledge of second language development and mathematical content knowledge is beyond the scope of this paper (for review on the second language acquisition research see Ionin, 2013 and for mathematical content knowledge see Ball, Thames, and Phelps, 2008).

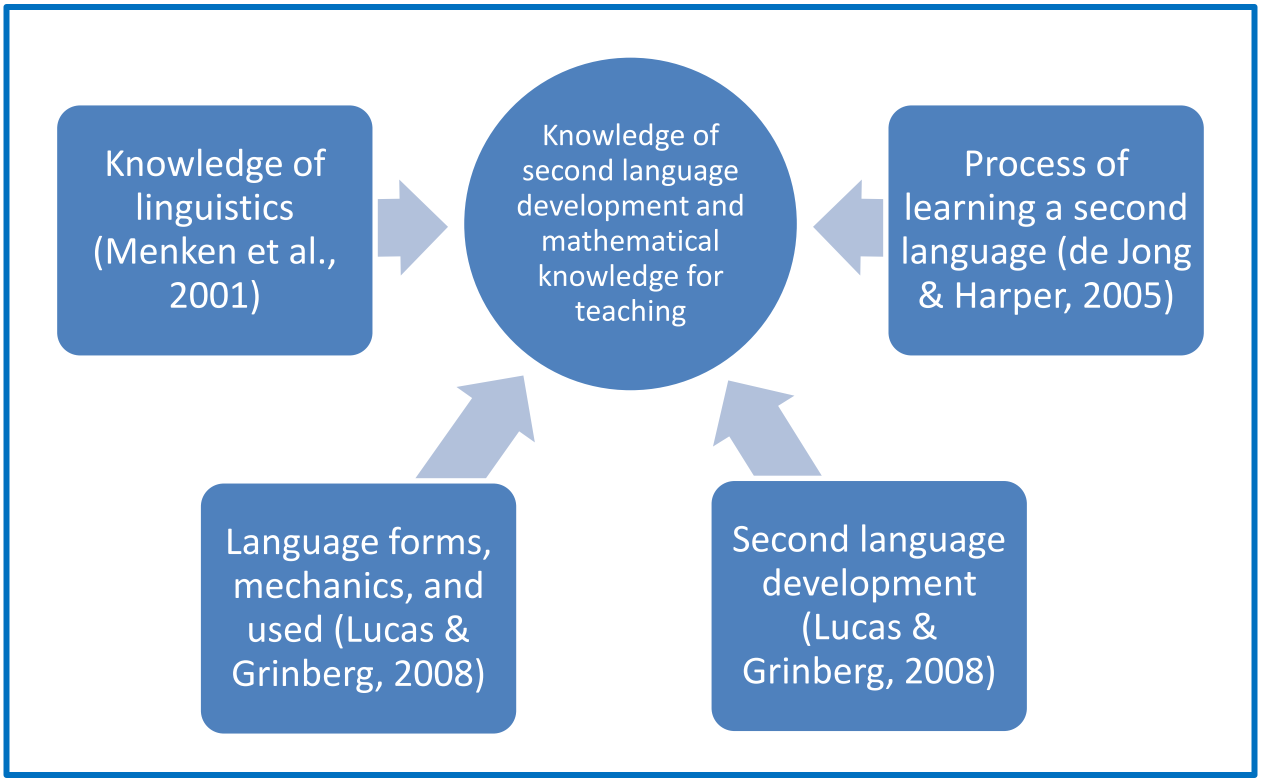

Figure 6. First Knowledge Area for Teaching Mathematics to ELLs

Knowledge of children's mathematics and language backgrounds.

Knowledge of children's mathematics and language backgrounds is a direct rethinking of Lucas and Grinberg's (2008) knowledge of language backgrounds, experiences, and proficiencies of their students to include the knowledge of students' mathematics backgrounds. (see Figure 7)

In mathematics, teachers must also understand the mathematical backgrounds of their students and consider both in-and-out of school practices and language. Research in in-school practices primarily look into the students' use of their native language in the classroom (August et al., 2006; Cummins, 2001; Thomas & Collier, 2002; Turner & Celedón-Pattichis, 2011**).** The work in funds of knowledge (Civil, 2007; González, Andrade, Civil, & Moll, 2001; Moll, Amanti, Neff, González, 1992) has looked at students out of school mathematical practices and how these practices can be brought into the classroom.

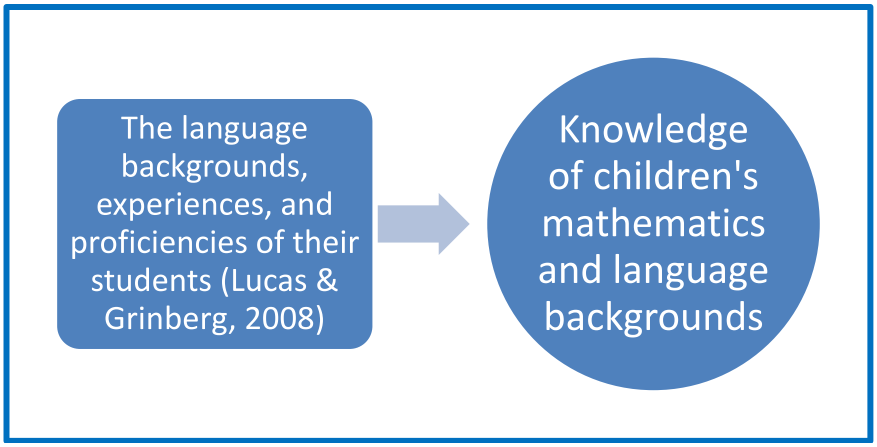

Figure 7. Second Knowledge Area for Teaching Mathematics to ELLs

Students' use of native language in their in-school mathematics practice. Research has shown that students who are able to access their native language have an increased opportunity to be successful in school (August et al., 2006; Cummins, 1981, 2001) and are better able to learn mathematics (Thomas & Collier, 2002; Turner & Celedón-Pattichis, 2011). Cummins (1981, 2001) sets forth a theoretical framework for analyzing the school failure of minority students and the lack of success with bilingual education programs. His framework analyses three types of relationships: majority/ minority societal group relations, school/minority community relationships, and educator/minority student relationships. He states, "students' school success appears to reflect both the more solid cognitive/academic foundation developed through intensive L1 [native language] instruction and the reinforcement of their cultural identity" (Cummins, 2001, p. 662).

August and colleagues (2006) found that when teachers use Spanish during their classroom instruction, they increase the transfer of academic skills that consequently increased the achievement of young bilingual and ELLs. Students who were instructed in their native language were successful in content assessments in the native language; however they were unsuccessful on the same content assessment if they were instructed in English-only. Students who had sound instruction in both Spanish and English were successful in both languages and were able to rely on their native language to navigate the English language.

Thomas and Collier (2002) conducted longitudinal studies with ELLs and language programs in order to find out how the language programs addressed ELLs' use of their native language. They researched five school districts for a total of 210,054 ELLs in various language programs. They found that, in general, native Spanish speaking students assessed on standardized mathematics tests in Spanish outperformed native English speaking students assessed on standardized mathematics tests in English. In particular, students in the two-way immersion language model (a two-way immersion model has both ELLs and non-ELLs receive instruction equally in both English and a non-English Language) out performed students tested in Spanish who were part of the 90-10 transitional bilingual model (90% native language instruction and 10% English language instruction at the start of the bilingual program by the fifth grade.

Turner and Celedón-Pattichis (2011) examined the learning opportunities of Latina/o kindergarten students in three classrooms based on the language of instruction, native language of the teacher, and native language of the students. In the first classroom the teacher was a native Spanish speaker who led his/her math lessons in Spanish to a class of native Spanish speaking students. In the second classroom, the native English speaking teacher was trained in ESL education and led her math lessons in English to a class of both English and Spanish speaking students. In the third classroom, the native Spanish speaking teacher led his/her math lessons in both Spanish and English to a class with half native Spanish speakers and half native English speakers. Turner and Celedón-Pattichis found that students in the classroom with the native Spanish speaking teacher who led her lessons in Spanish were better problem solvers then their peers in the other two classrooms. They also found that these students had more opportunities to learn and engage with mathematics. The first teacher posed questions in an informal/storytelling manner twice as often as the other two teachers. They concluded that the first teacher had more mathematical conversations and was the only teacher in which all her students learned all mathematics in their dominant language.

All four of studies varied in their methods and approach in addressing the use of a child's native language in content instruction. They all concluded that students who have high quality native language support in their classroom have academic success and that the use of native language in mathematics instruction resulted in greater mathematical success (Thomas & Collier, 2002; Turner & Celedón-Pattichis, 2011).

Some ELLs have a strong foundation in their native language and mathematics while others have had little to no formal mathematics instruction in their native language. Whiteford (2009) states that teachers need to value the culture of ELLs. In particular he emphasizes the need for teachers to validate, honor, and support the mathematics that students bring with them to U.S. classrooms. He writes:

> To do so, we [teachers] must become aware of procedures, types of math instruction, and student's current performance levels. We should be sensitive to cultural math differences that students may be experiencing. We need to make careful decisions as to whether students can continue to use procedures that differ from those taught in U.S classrooms. We must take care not to assume that a student encountering difficulties in mathematics does so because of limited language proficiency (p. 282).

For students who are U.S born researchers say that it is equally important for teachers to know the instruction that children have received before entering their classroom (Valenzuela, 1999;

Whiteford, 2009). Foote (2009) asks teachers to step outside of the classroom and observe children in multiple settings; the knowledge attain from these observations can shed light on the students' practices in the classroom.

Students' out-of-school math practices. The work in funds of knowledge (Civil, 2007; González et al., 2001; Moll et al., 1992) advocates that teachers learn the mathematical practices that students' use in their homes and communities in order to incorporate these practices in the classroom. Moll and colleagues (1992) define funds of knowledge as "historically accumulated and culturally developed bodies of knowledge and skills essential for household or individual function and well-being" (pg. 133). In their work teachers learn about the skills needed for the running of the students' household and communities and find ways to incorporate this knowledge of students' households into the classroom instruction. Civil (2007) emphasizes that funds of knowledge is about the teacher as a researcher with the goal of "learning about the community and about the resources and knowledge in their students' households" (p. 116). This gives the teacher a firsthand experience about the household and the lives of their students. Teachers who learn about the backgrounds of their students have a place to draw from and formulate academic concepts.

In looking explicitly for mathematical funds of knowledge González and colleagues (2001) found that families use mathematics throughout their daily lives; however, the household members were unfamiliar with how mathematical concepts were present in their home. This was particularly difficult because the mathematical concepts that the researchers were hoping to find were school-based academic concepts while what they found were everyday mathematics that could not be described outside of their context. For example some mothers knew how to sew articles of clothing without having to use a sewing pattern, but it was difficult to describe the mathematics they used in measurements and their decisions to cut fabric in certain ways outside of the context. The researchers state that everyday mathematical concepts such as, sewing without clothing patterns, can be used as building blocks for school concepts and likewise that school mathematical concepts can be used to understand everyday concepts.

Research in in-and-out of school practices can be reframed as work that has teachers learn about the mathematical registers that students are part of. Students are taught in school to use a mathematical register that most closely resembles what researchers call school mathematics. The work in mathematical funds of knowledge has researchers (teachers) examine mathematical practices of the home so that they are able to incorporate this in there classroom practices bringing in what some call real world mathematics.

Knowledge of How to Use Language and Culture to Teach Mathematics to ELLs

This section of the framework is derived from Menken and colleagues' (2001) knowledge of pedagogy and knowledge of cultural and linguistic diversity; Lucas and Grinberg's (2008) connection between language, culture, and identity; and de Jong and Harper's (2005) role of language and culture as a medium in teaching and learning. Incorporating these teaching into mathematics teaching requires teachers to understand not only mathematical language but their [the teacher's] own language practices around teaching mathematics. (See Figure 8)

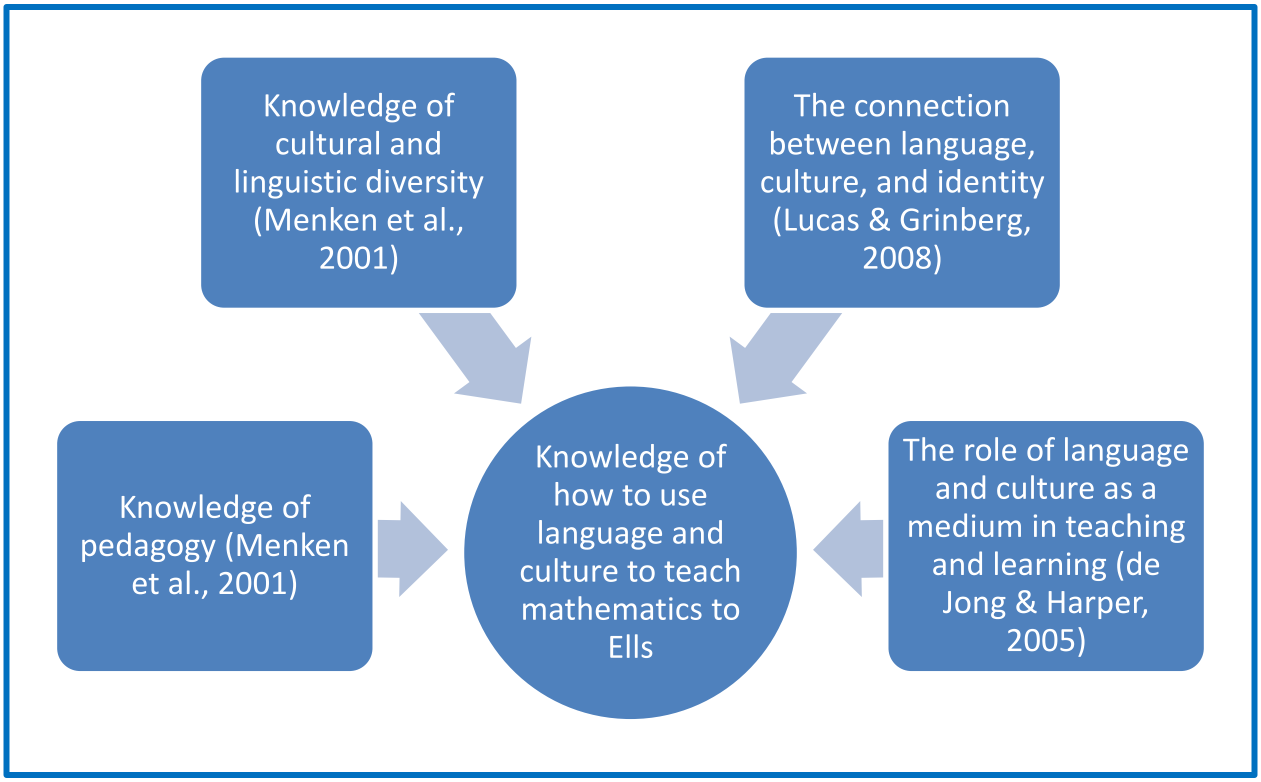

Figure 8. Third Knowledge Area for Teaching Mathematics to ELLs

Beliefs about language and mathematics. One of the challenges in preparing teachers to teach mathematics to ELLs is interrupting teacher beliefs about ELLs, mathematics, and the use of language in teaching mathematics. McLeman and colleagues (2012) surveyed 292 pre-service teachers from 12 urban teacher education programs and found that those with limited exposure to ELL issues and knowledge of a foreign language prior to the education program had more deficit models of ELLs than those whose had prior exposure. Their analyses of the survey questions revealed that pre-service teachers with deficit models believed that the use of a student's native language hinders the learning of mathematics, and that learning English is more important than maintaining students' native languages. Approximately, one third of the respondents believed that state assessments should not be offered in other languages. The 187 pre-service teachers were less likely to believe that native language hinders the learning of mathematics and that English language acquisition is more important in learning mathematics.

The teachers with more knowledge of ELL issues did not all align with asset based views suggesting that pre-service teachers need more than exposure to ELL issues.

Likewise Fernandes' (2012) extension on the above research found that even after being allowed to see the type of problems ELLs have when doing mathematics, some pre-service teachers still had conceptions of mathematics as being universal. In this study the pre-service teachers were able to see and acknowledge that language assistance made an impact on the success of students with mathematics. However, pre-service teachers still believed that teaching mathematics to ELLs was uncomplicated because mathematics is universal. Pre-service teachers maintained their original views that mathematics could be taught effectively to ELLs without the need to rely on the native language of the students. This occurred after the pre-service teachers saw ELLs successfully solve math problems. However, the researcher notes that pre-service teachers were able to acknowledge the increase language demands for ELLs and ways they could mitigate this knowledge in their classroom. Some pre-service teachers maintained their deficit models when discussing how they would teach mathematics to ELLs. There have been multiple studies that have had similar results; in which pre-service teachers indicate a belief that mathematics is universal and as such it should not be too difficult for ELLs possessing minimal language problems to learn mathematics (Barwell, 2005; Fernandes, 2012; McLeman et al., 2012; Walker et al., 2005).

De Oliveira (2011) discusses the need for teachers to recognize the struggles that ELLs have with language in a subject like mathematics that is seen as needing little outside language. In her study, De Oliveira had pre-service teachers engage in a mathematical task that was presented to them in Brazilian-Portuguese. De Oliveira first presented the mathematical task using little or no ESL teaching strategies and asked the pre-service teachers to reflect on their experiences. Then, she presented a mathematical task using ESL strategies and asked the pre-service teachers to reflect on this experience in comparison to the first and to list out the ESL strategies that they saw and thought were useful. Overall, de Oliveira found that the pre-service teachers had more empathy for ELLs and had changed their beliefs about the experiences of ELLs in mainstream classrooms after the study was completed.

Hoeffert (2009) describes her own struggle teaching ELLs after her high school opened up a center for ELLs. She was the mathematics department chair and had to adapt her instruction and aid her colleagues on how to approach teaching this new student population. She writes that 95% of her ELLs passed the state mandated algebra1 exam. She lists several strategies that she employed that she credits helped her achieve this. Hoeffert claims that it is important to: assess student's prior knowledge; use precise written and oral language even highlighting and bolding key words; using multiple modes of instruction; engaging students in mathematics daily by having them communicate orally or through written work using language as a bridge for all students to learn mathematics; students should be given feedback daily; and students should work cooperatively in groups.

Similarly, Lass (1988) made several suggestions for improving mathematics instruction for bilinguals, which include having teachers: develop the students' native and English language; teach mathematics to bilingual children bilingually; recognize mathematical language but not as a universal language teach in a culturally relevant manner be familiar with out of school practices; and have an overall appreciation for how the native language can be used to teach mathematics effectively. Andersson (1977) also claimed that effective bilingual and foreign language teachers are both biliterate and bicultural, having an in depth knowledge of both languages and cultures.

All of the research reviewed here, examined practices and beliefs for teaching mathematics to ELLs. With increased exposure teachers began to view teaching mathematics to ELLs not as uncomplicated but rather as having a greater amount of language demands. These studies suggest that as teachers became more familiar with ELLs' struggles in learning mathematics they had a more favorable disposition towards the use of native language.

Teaching in a non-native language. Most bilingual teachers in the US are native-English speakers, whereas most students taught by these teachers are non-native English speakers. This can lead to difficulties when the teacher and students do not share a common language in which both can express their ideas fluidly (Ballantyne, et. al., 2008). Kasule and Mapolelo (2005) state "each learner's mother tongue [native language] is the key to the world and a means of alleviating the abstract nature of classroom learning events" (p. 602). When instruction is in one's native language from a non-native speaker the ideas presented in the classroom might not be the ones the instructor intended (Saat & Othman, 2010), students may be using a different mathematical register. Kasule and Mapolelo also argue that mathematics is a different language in itself, different from the language of instruction. Thus, it is not enough for teachers to know their students' native language; the teacher must also know mathematics in the students' native language.

Malaysia has a new language policy in which students must now be instructed in English instead of Mandarin, thus this offers us an opportunity to see mathematical instruction in a second language when the student and instructor share the same native language. Lim and Presmeg's (2011) study revealed that both Malaysian teachers and students are struggling in mathematics. Teachers claim that they are spending more class time than usual explaining mathematical language translations (from Mandarin to English) and thus are reverting back to

Mandarin in order to teach mathematics. The Malaysian teachers believe that the problem with their teaching has to do with not having a culture in which the English language was promoted both in and out of school. The teachers also deem that it is easier to express and teach mathematical ideas in their native language.

When teaching mathematics in their second language, teachers face difficulties that are particular to mathematics. Moschkovich (2007) found that there can be an overemphasis on vocabulary, which narrows the mathematical communication opportunities for bilingual learners. Thus nonnative teachers cannot rely solely on translations of mathematical terms they must be able to use the translated terms in conversations with students. Ron (1999) supports this idea by stating that many bilingual teachers do not possess the specialized vocabulary needed for mathematical instruction. She argues that there is more to the communication of mathematical ideas in a different language than just a mere translation. In a study of Brazilian indigenous teachers, Mendes (2007) found that teachers who wrote school-like mathematical problems relied heavily on indigenous language structures rather than western traditional word problems. Some word problems that were developed did not have a deliberate question or mathematical task. It was assumed that the students would understand what the tasks were based on the context of the word problem. Mendes revealed that the Brazilian indigenous population had a cultural structure in which questions could be derived from the context of a problem without having to make the question explicit.

Taking this a step further, there have been studies that show that adult bilinguals perform arithmetic operations more quickly and efficiently in their preferred language (Marsh & Maki, 1976) or the language in which they were instructed (Gutiérrez, 2002; Moschkovich, 2007). This means that a native Spanish speaker who was instructed in English is quicker at arithmetic operations in English than in Spanish. However, Dehaene, Spelke, Pinel, Stanescu, and Tsivkin (1999) found that when given training in repetitive arithmetic tasks, one can become proficient (accuracy) in arithmetic in their second language. With enough familiarity with Spanish, arithmetic teachers can become quicker at performing arithmetic tasks in Spanish (Dehaene, et al., 1999); this does not extend to being able to discuss arithmetic. Accurately performing arithmetic tasks does not support teaching for understanding nor does it provide students the opportunity to engage in discussion about their strategies and ideas.

The research above reveals the importance of language and culture when attempting to teach any content including mathematics in a second language. Teachers must understand that the students' language and culture will affect the students' mathematical registers and the way in which they voice it. Teachers cannot rely solely on mathematical translations nor can they rely on English language structures. Moving from one mathematical register to the next can cause a misunderstanding if both or one register is not fully understood. Thus, in order for a teacher of ELLs to become an effective mathematics teacher she must have a thorough understanding of mathematical registers in the native language of the student.

Knowledge of How to Promote Language and Culture in Mathematics Instruction

Knowledge of how to promote language and culture in mathematics instruction to ELLs comes directly from de Jong and Harper's (2005) idea of having teachers set explicit linguistic and cultural goals. In the fourth knowledge area, teachers actively enact the knowledge that they have gained from the students language and mathematics background and their own understanding of how language and culture affect their teaching; the two previous knowledge areas. It uses the knowledge that teachers have gained of the students' language and mathematics and applies it to the teachers' understanding of language and mathematics. This area has not been as widely researched as the previous two and much of its work lies on how teachers bring culture into the mathematics classroom. (See Figure 9)

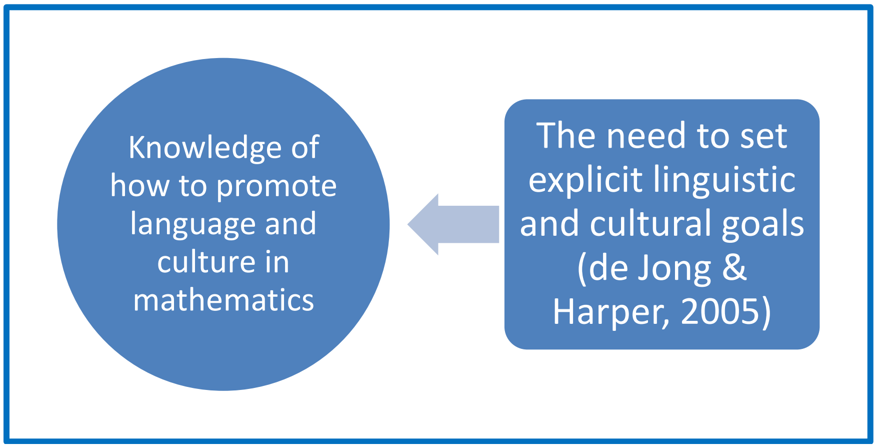

Figure 9. Fourth Knowledge Area for Teaching Mathematics to ELLs

Culturally relevant mathematics. The work on culturally relevant mathematics has teachers teach mathematics while taking into account the students' cultural context. Ladson-Billings (1995) defines culturally relevant teaching as:

> pedagogy of opposition not unlike critical pedagogy but specifically committed to collective, not merely individual, empowerment. Culturally relevant pedagogy rests on three criteria or propositions: (a) students must experience academic success; (b) students must develop and/or maintain cultural competence; and (c) students must develop a critical consciousness through which they challenge the status quo of the current social order. (p. 160)

In this article Ladson-Billings describes eight successful teachers of African American children. These teachers viewed themselves as part of the low-income African American community in which they were teaching, and saw their teaching as a way to give back to the community. These teachers all believed that all of their students could and should succeed and adapted their teaching to make this the case. In a related article Ladson-Billings (1997) goes into detail about one of these teachers and her mathematics teaching. This teacher was an active learner in the classroom; allowing students to be problem posers, thinkers, and solvers. Teaching in a culturally relevant manner asks teachers to take into account and use the knowledge that students bring with them to the classroom.

Matthews (2003) writes about four teachers implementing culturally relevant teaching in Bermuda to black Bermudan children. He puts forth four complexities that the teachers teaching in a culturally relevant manner ran into. The first is building empowering relationships, in which teachers who have robust culturally relevant teaching build intimate relationships involving students, themselves, and community. The second complexity is building on culture and fostering critical thinking formally, in which teachers use both informal and cultural knowledge to build mathematical thinking and critical consciousness of their students. The third complexity is building on cultural knowledge and fostering critical thinking informally where “teachers build on both informal and cultural knowledge; teachers foster critical mathematical thinking and personal notions of critical consciousness” (p. 72). Complexities two and three differ in that the first builds on general knowledge while the second builds on personal notions of critical consciousness. These complexities are difficult for teachers because they have to be aware on how this critical view can be related back to mathematics. Mathews writes about a classroom discussion on the number of boys and girls in class that started off as a family of addition and subtraction equations but ended as a discussion about the lives of Afghani women. The teacher in the example discussed having a difficult time relating the conversation he believed to be culturally relevant back to mathematics. The difficulty arrives from allowing the students to express their views but using classroom time efficiently to relate back to the mathematics of the original lesson. These discussions often occur organically and thus making connections to mathematics may be difficult to generate. The fourth complexity is building on cultural knowledge in which teachers build towards an informal cultural knowledge through teacher and text transmitted cultural and critical-infusions. The teachers in the study had difficulty in this area because they were trying to incorporate culture into traditional activities.

Culturally relevant mathematics teaching also incorporates a critical lens. Students and teachers use mathematics to understand their own environment and community. Gutstein, Lipman, Hernandez, and de los Reyes (1997) define critical mathematical thinking as “students making and exploring conjectures, questioning their peers and teachers, and using mathematical evidence to validate knowledge” (p. 712). In their study of teachers of primarily Mexican American students the authors developed a model for culturally relevant pedagogy in a mathematics classroom. This model includes three components: connections between becoming critical mathematical thinkers and viewing knowledge critically; connections between building on students’ informal mathematical knowledge and building on students’ cultural knowledge; and orientations to students’ culture and experience. Gutstein and his colleagues emphasize the connection between students’ culture and experiences in the classroom. Teachers need to build on students’ culture and experiences in order to have students become critical mathematical thinkers.

Tate (1995) has a similar view of culturally relevant mathematics pedagogy, in which the teacher incorporates awareness of the problems that students face in education and society. His study focuses on one teacher of African American students who has this type of pedagogy. This teacher believes that the mathematical problems in her classroom should come directly from the students; students should then research the problem within its greater context and communicate and present a resolution to the problem to the community. This is pedagogy of social change in which the teacher in this study seeks to develop democratic citizenship through mathematics.

Culturally relevant teaching and pedagogy as presented in the above studies all take the culture, community, and experiences of the students into account in teaching mathematics. Mathematics is learned through the knowledge and experiences that the student brings with them into the classroom. The above studies however do not discuss the use of language around mathematics both by the student and the teacher. However, we can envision that in these classrooms students are allowed to talk freely through their mathematical experiences connecting one mathematical register to another. Students who are able to pose their own problems and discuss their solutions are given the opportunity to use their own mathematical register and understand the mathematical register of another student or teacher.

Conclusion

This framework and its knowledge areas give insight into mathematics teaching that encourages the use of language and culture. It demonstrates the importance of understanding both the student and the teacher holistically. In the fourth knowledge area teachers are asked to promote culture and language diversity in their mathematics lessons which can be difficult to discuss and do when teachers are unfamiliar with the culture and language of their students. The studies in the previous section demonstrate the difficulty in modifying pre-service and in-service teachers' beliefs about ELLs' mathematics learning experiences.

The literature points to the lack of completeness in thinking about the affordances and limitations that language has on the instruction of ELLs in mathematics. Teachers of language minority students are presented with obstacles that they were not prepared for and have difficulty communicating with their students. Studies suggest that teachers have deficit views about teaching mathematics to ELLs (Fernandes, 2012; McLeman et al., 2012). These teachers can often think that the best teaching practices for mainstream students can be transferred to language minority students because they are “best practices.” Other studies suggest that teachers view teaching mathematics for ELLs as the same as teaching mathematics for mainstream students, with the same expectations and with few problems (Rolka, 1994; Tevebaugh, 1998).

The learning of mathematics for language minority students has been shown to be successful when teachers use the student’s native language teaching in the instruction (August et al., 2006; Cummins, 1981, 2001; Thomas & Collier, 2002; Turner & Celedón-Pattichis, 2011). Students who are allowed to use their native language during instruction perform better in mathematical assessment in comparison to students who are not allowed to use their native language. This suggests that students are able to transfer their Spanish mathematical register to English mathematical tasks.

When it comes to mathematics, many teachers believe that mathematics is universal and that teaching mathematics requires minimal language. Thus, these teachers believe that teaching mathematics to ELLs should not be difficult since mathematics is in theory the same just expressed in a different language. The problem with this mind set is that mathematics is not language-free and culture-free. Students come in to the classroom with knowledge that they have gained formally and informally, and the teacher has to use this knowledge in their instruction of the student. Halliday (1978) discusses this as mathematical registers in which vocabulary from everyday life and other subjects takes on special meanings in the mathematics classroom. For an ELL, the student not only has to navigate his/her own mathematical register in his/her native language, but has to transfer it to the English mathematical register. Therefore, it is important for a teacher to not only to know that there are potential translation issues, but also that they and the student have to decipher the language of matheamtics of the classroom as a whole but also be aware of the influence of everyday language.

Teachers whose native language is the same as their students' native language have an additional set of skills available to them than teachers who do not share the same native language as their students. These teachers are better equipped to understand the mathematical registers of their students in their native language. The studies that examine K-12 mathematics instruction by a native teacher look at the use of native language, but do not look at the type of language that native speaking teachers use in their mathematics lesson. That is, the researchers have studied whether native language is used but not how or what type of native language is used and by whom. Turner and Celedón-Pattichis (2011) addressed the type of language that was used by kindergarten teachers of Latina/o children and reported on the teacher's language background, yet they do not go into detail as to the type of instruction around language that was used by the teacher. Therefore, researchers should study the use of language and the use of mathematical registers by native teachers around mathematics in both the students' native and non-native language. This research should be extended to language and mathematical registers used by non-native teachers as a comparison in order to understand how to enhance the mathematical learning experience of all ELLs.

APPENDIX

Programs in ESL and Bilingual Approaches

	Approach	**Length**	**School Instruction**	**Goal**
Newcomer	Bilingual and ESL	1 semester to 4 years	may range from a half-day, in-school programs to a full-time, self-contained school	Acclimate students to the American school setting and prepare them for mainstream classrooms
English as a Second Language (ESL)	ESL	As needed	ESL-certified teacher provides explicit language instruction to students. Students may have a dedicated ESL class in their school day, or may receive pull-out ESL instruction	Proficiency in the English language, including grammar, vocabulary and communicational skills
Content-based ESL	ESL	As needed	ESL-certified teacher provides explicit language instruction to students. Students may have a dedicated ESL class in their school day, or may receive pull-out ESL instruction.	Although using content as a means, instruction is still focused primarily on learning English
Sheltered Instruction	ESL	As needed	Teacher provides instruction that simultaneously introduces both language and content using specialized techniques to accommodate English learners' linguistic needs. Instruction may be used for English learners'-only classrooms or for a mixed classroom with English learners and non-English learners.	Instruction focuses on teaching academic content rather than the English language itself
Transitional Bilingual	Bilingual	Starts in K or 1st grade until Grade 3 but as	Instruction begins mainly in native language and incrementally	English proficiency, native language is used as a medium but

Education		early as Grade 2 or as late as Grade 5	transitions to English	native language proficiency is not a goal
Developmental Bilingual Education	Bilingual	Starts in K or 1st grade until typically Grade 5	Instruction begins mainly in native language and incrementally transitions to English. Programs typically follow a 50-50 or 90-10 model	Typically at the end of a program there is a 50-50 language balance
Two-way immersion	Bilingual		Both Language learners and non-Language learners receive instruction in English and a non-English language. Programs follow either a 50-50 or 90-10 model	Proficiency in both English and non-English language for both Language learners and non-Language learners

Faulkner-Bond, et al. (2012) Programs recognized by the Department of Education

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PART III:

WHAT TO DO WHEN TRANSLATING MATHEMATICS IS NOT ENOUGH?

"I probably used descuento I didn't use the word rebate. Because I try to include as many synonyms as I can because the way that I speak Spanish sometimes is not something that a lot of my kids understand. I think I did it really quickly to make a connection; to my kids who are new comers."

Ms. Álvarez is a Native Peruvian bilingual teacher describing how she presented a math problem to the fourth and fifth graders in her classroom. The problem asked students to find the sales price of a pair of jeans that was on sale, and in the quote above, she described the way she shifts from one language to the other. Ms. Álvarez is aware that though she speaks Spanish her use of the Spanish language differs from that of her students. Ms. Álvarez discussed the difficulties with translating but also the need to be mindful of her students' language versus her own.

The act of translating from one language to another requires more than the knowledge of words and phrases in both languages. In order to do this effectively the translator must be aware of their own language abilities but also the language abilities of their intended audience (Moschkovich, 2005; 2007). Bilingual teachers are often confronted with this difficulty and the teaching of mathematics is no exception (Ron, 1999). In this article I will share Ms. Álvarez flexibility with language as she translates and modifies the phrase "sales price." Ms. Álvarez uses her awareness of language and its difficulties to: restate and offer clues, focus attention, rephrase by explanation, and reword future math problems.

Mathematics and Language Teaching

The teaching of mathematics in a second language can be seen as unproblematic. Some may believe that being bilingual implies knowing mathematical language in both languages (Ron, 1999). However the teaching of mathematics requires more than just mathematical language. The teaching of mathematics is often oral thus it is important to understand the language demands that bilingual teachers encounter during their mathematics lessons (Gómez, Kurz, & Jimenez-Silva, 2011; Janzen, 2008; Lager, 2006). During the 2014-2015 school year Ms. Álvarez along with two other elementary school English-Spanish bilingual teachers were observed during their mathematics lessons. I set out to find the ways in which language was confounded with mathematics.

Ms. Álvarez followed a 50-50 model where half of her lessons were taught in Spanish and the other half were taught in English. Her classroom had students with different linguistic abilities, not all students were bilingual nor did all her students speak Spanish. Though Ms. Álvarez had lessons in one language or the other she often resorted to the students' dominant language during their individual interactions to best assist the students.

Translating Mathematics

Extending on the ideas Ms. Álvarez originally expressed at the beginning of the article, she found that words and phrases were difficult to translate from one language to the other. During a mathematics lesson taught in English the students were asked to solve the following word problem written out for them in both Spanish and English:

> *Daniel necesita un par de jeans. Su esposa Mary fue de compras a Macy's porque había una oferta de jeans. Un par de jeans estaba en oferta por ½ de su precio regular. El precio regular es $40. ¿Cuál fue el precio de oferta? ¿Cuánto pago Mary por los jeans? Daniel needs a new pair of jeans. His wife, Mary, went shopping at Macy's because they were having a sale on jeans. A pair of jeans was on sale for ½ off the regular price. The regular price was $49.What was the sales price of the jeans? How much did Mary pay for the jeans?*

Ms. Álvarez unpacked the word problem during whole group discussion and further assisted students in small groups and individually. Eduardo a Spanish dominant student was having difficulty getting started with the word problem. Eduardo seemed to be having difficulties understanding what he was supposed to do with the jeans and thus could not answer the question he was posed. The following is a Spanish transcription and its translation of this individual interaction.

Table 3

Skinny Jeans Transcript

Skinny Jeans Spanish Transcript	**Skinny Jeans Translation**
Ms. Álvarez: Cuesta $49. Pero esta de oferta y el precio es la mitad. Es una mitad y cuesta $49. ¿Cuál es el precio de la oferta cuánto pago Mary por los jeans? ¿Qué puedes hacer, para encontrar o averiguar eso?	Ms. Álvarez: It costs $49. But it is on sale the price is half. It is half and it costs $49. What is the sales price and how much did Mary pay for the jeans. What can you do to find or search for that?
Eduardo: Es que tengo que buscar el precio de los jeans.	Eduardo: It’s that I have to find the price of the jeans.
Ms. Álvarez: ¿Bueno cuál es el precio de la oferta, primero, y después cuánto pago Mary por los jeans? ¿Que puedas tratar de hacer para comenzar? Perdón.	Ms. Álvarez: Okay what is the sales price, first, and then how much did Mary pay for the jeans? What can you try to do to get started? Sorry
Eduardo: Hacer otro	Eduardo: Make another.
Ms. Álvarez: Un par es uno nada más. Un par son los que tiene que ver con las piernas. ¿Qué	Ms. Álvarez: One pair is one only. A pair is the ones that have to do with the legs. What can

puedes hacer para empezar?	you do to start?
Eduardo: Los jeans	Eduardo: The jeans
Ms Álvarez: Aha, cuánto cuesta, el precio regular.	Ms. Álvarez: Aha, how much does it cost, the regular price?
Eduardo: [apunta a $49]	Eduardo: [points to $49]
Ms. Álvarez: Cuarenta y nueve. Y la oferta es que está a mitad de precio. Cuál es la oferta y cuánto va a pagar.	Ms. Álvarez: Forty nine. And the sale is that it is half the price. What is the sale and how much is she going to pay.
Eduardo: La oferta es el 50%	Eduardo: The sale is the 50%.
Ms. Álvarez: La oferta es de 50% de 49. ¿Qué puedes hacer? Quiero que pienses una estrategia, okay.	Ms. Álvarez: The sale is 50% of 49. What can you do? I want you to think of a strategy, okay.

Restating and Offering Clues

When reading the word problem in either language it is difficult to understand what Ms. Álvarez intended the students to do with the question regarding sales price. Thus, it is not surprising to see Eduardo having difficulties. Ms. Álvarez translated the phrase "sales price" into "el precio de la oferta." This translation was not something that Eduardo grasped and it may not have been the problem itself that caused confusion, rather the way it was phrased. He knew that he had to find the price of the jeans however he did not know how to do this. Ms. Álvarez restates the questions in the word problem and offers clues in order to help Eduardo understand what he was to do with the jeans. She says "¿Bueno cuál es el precio de la oferta, primero, y después cuánto pago Mary por los jeans? (Okay what is the sales price, first, and then how much did Mary pay for the jeans?)" Informing Eduardo that there are two steps in the process lets him know that there exists a difference between the sales price and the amount paid for the jeans.

Though Ms. Álvarez restated and offered Eduardo clues as to how to proceed; he talked about making another pair of jeans in order to get started. This was Eduardo's way of incorporating the jeans into his solution strategy. Elbers (2005) found that students who were also language learners skipped through discussions of everyday meaning of words and focused on the mathematical meaning of the word. Thus, Eduardo focus on making another pair of jeans may have something to do with his knowledge of jeans or his knowledge of a sale. Regardless of which of the two this is, Eduardo had difficulty in understanding Ms. Álvarez phrasing of the word problem.

Focus Attention

Ms. Álvarez then proceeded to reiterate and focused Eduardo's attention to the original price that he needed to use when she asked "Aha, cuánto cuesta, el precio regular (Aha, how much does it cost, the regular price?)." Ms. Álvarez refocused her clues by asking Eduardo for the regular price informing him that this was a place for him to start. By focusing attention on one question Ms. Álvarez was attempting to recognize where Eduardo could be confused. He points to $49, which clarified that he understood that there was an original price of the jeans, this also identifies that his confusion could be the sales price of jeans.

Rephrasing by Explanation

When Eduardo correctly points to $49 as the regular price Ms. Álvarez proceeded to rephrase the potential source of the problem when she said "la oferta es que está a mitad de precio (the sale is that it is half of the price)" This new translation of the word problem is not a direct translation rather it is an explanation of the original phrase. Eduardo picks up on this rephrasing and is able to see that the jeans were 50% off. A student who is unable to get started with a word problem is not necessarily a student struggling mathematically, but rather struggling with understanding the word problem and phrasing as a whole. In this case Ms. Álvarez was able to recognize Eduardo's struggle with the wording of the math problem. As the class continued Eduardo was able to find how much Mary paid for the jeans.

Future Rewording

After this observation Ms. Álvarez and I discussed the use of the phrase "sales price" and consequently the Spanish phrase "el precio de oferta." The discussion was on the ambiguity of the phrase that may have students believing that the sales price is the same amount as what Mary had to pay. The next observation was two school days later and the problem of the day was similar to the previous skinny jeans problem but worded differently. It read:

> *La señorita Álvarez necesita comprar una sobrecama para su mama. En la tienda Target el precio regular de un cubrecama es $79. Sin embargo el cubrecama esta en oferta por el 50% del precio regular. ¿Cual es el valor del descuento? ¿Cuánto pagaria?*
>
> *Ms. Álvarez needs to buy a bedspread for her mother. At the Target store the regular price of one bedspread is $79 however, the bedspread is on sale for 50% of the regular price. What is the value of the discount? How much would she pay?*

The phrase "sales price" was replaced with the phrase "value of the discount," this rewording generated fewer questions from the students in regards to word problem as a whole. The rewording made it clearer that Ms. Álvarez was searching for two different numbers the amount of the discount and the amount paid for the bedspread.

Ms. Álvarez went through a five step process in order to decipher the language complications that her student was having. The first step, being the most important, was understanding and being aware that language plays a role in the teaching of mathematics. This allowed her to use the following three steps to identify Eduardo's complication. She restated the questions in the word problem and offered a clue on how to get started in order to observe whether the problem was mathematical or language based. She then focused attention to one of the questions and detected that this was not the source of the problem. Finally she rephrased "sales price" by offering an explanation. The fifth step was to rethink and reword the problem as a whole to avoid future confusions.

Language as a Resource for Mathematics Teaching

Ms. Álvarez primarily possessed an awareness that language played a major role in the teaching and learning of mathematics. This awareness allowed her to move Eduardo forward and in the process modified her future lesson to better assist all of her students. The teaching of mathematics is not without language complications but with awareness of the effects of language on mathematics teachers are better able to decipher students' difficulties with language versus difficulties with mathematics. This awareness allows teachers to move their students forward linguistically and as mathematicians.

In a subsequent focus group Ms. Álvarez said the following about the importance of language and mathematics:

> *But I think language and mathematics even though its complex there is a simplicity when it comes to numbers. And I think trying to provide that along with language is something that I try to do all the time. Whether it is in Spanish or it is in English; so that they can feel successful. I think especially a lot of our kids whose native language is Spanish, being able to throw those academic words on continuous bases and to question their thinking continuously is something that I try to do.*

Ms. Álvarez was aware that the interplay between language and mathematics is complex and nuanced. Mathematics for Ms. Álvarez gave her students a platform to be challenged with their language while becoming successful in math. Ms. Álvarez has identified that the understanding of language as a resource in the classroom is closely linked to teaching of mathematics (Gómez, Kurz, & Jimenez-Silva, 2011; Planas & Setati-Phaken, 2014). Mathematics teachers both bilingual and not must also be language teachers in which they are aware of their students' abilities to engage with mathematics through language.

Ms. Álvarez used Eduardo's difficulty with the language of the problem and was able to use this to enhance the learning experience of Eduardo and his classmates. Complications with language during a mathematics lesson can be an opportunity for mathematical exploration and thus can enhance a student's opportunity to learn math from this complication (Elbers, 2005). Distinguishing between struggles with math and struggles with language particularly in a bilingual classroom is not without difficulty. However, acknowledging that both complications exist allows a teacher to be more alert about the language they use and the language their students use. Ms. Álvarez used three different steps she restated and offered clues, focused the students attention, and rephrased by using an explanation, this allowed her to better understand her student's struggle.

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