

BETTING ON RISK: MODELING INVESTMENT IN PREPAREDNESS IN A COURNOT MARKET

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*“to
solve simple problems: that’s
the most
satisfying thing, it
gives you a chance and it
gives everything else a chance
too.*

*we were made to accomplish the easy
things
and made to live through the things
hard.”*

-Charles Bukowski

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Abstract

This dissertation models the interaction of firms in a game in which they must decide whether to invest in disaster preparedness (e.g., through emergency planning, stockpiling of inventories, etc.). We develop the model in a general n -player form, but focus on the case of a duopoly to illustrate the interactions between participants in the market. The firms are assumed to share a single market represented by the Cournot model, in which firms reach their decisions independently (i.e., without collusion).

We consider two alternatives for preparedness cost: one where the preparedness cost is a fixed cost that is independent of the production level; and one where the preparedness cost is a variable cost per unit of production. In each game, we find and characterize the equilibrium solution(s). In some cases, our model yields multiple equilibria.

We also perform sensitivity analysis to study the effects of parameter variations on the outcome of the competition. These variations focus on both environmental factors (e.g., the reservation price of the good, or the probability of disaster) and firm-specific factors (e.g., the variable cost of production). Moreover, we address the effects of equilibrium mixing strategy of other firms through calculation of equilibrium mixing probabilities. Finally, we compare the equilibrium with the social optimum, and discuss mechanisms for achieving the socially optimal level of investment in preparedness.

1 Introduction

Disaster preparedness is a critical issue for private businesses, since insufficient preparedness can adversely affect a firm's ability to supply key products and services. By contrast, good preparedness may enable a firm to outperform its competitors in times of disaster, and thereby achieve greater market share.

According to the Federal Emergency Management Agency (FEMA), since 1953, there has been an average of about 33 disasters per year. The number of declared major disasters nearly doubled in the 1990's compared to the previous decade, and has increased 22% more since the 1990's¹. In fact, just the first nine months of 2011 saw ten \$1 billion-plus disasters, according to FEMA.

When firms get caught up in disaster situations, they often struggle to survive. For example, the Institute for Business and Home Safety (2006) estimated that at least 25% of those businesses that close following disasters do not reopen, and many that do reopen struggle to stay in business. Many businesses are just one catastrophic event from going out of business or declaring bankruptcy. According to the World Bank (2010), the annual average direct economic losses from floods and storms in the U.S. total more than \$12 billion. Moreover, the indirect impacts (e.g., business interruptions, supply-chain disruptions) of disasters are even greater than the direct impacts (World Bank, 2010).

According to Assistant Secretary of Defense Paul Stockton, catastrophic events can cause social and political consequences far worse than those of

¹http://www.fema.gov/news/disaster_totals_annual.fema

Hurricane Katrina². For example, in 2010, FEMA simulated an earthquake of about 7.7 in magnitude, and found that casualties would be 10 times the number in Hurricane Katrina, with direct economic losses of about \$300 billion (FEMA, 2011). The Pentagon is preparing for such apocalyptic events, whether due to natural disasters, pandemics, or weapons of mass destruction.

Disasters can affect not only the financial health of a firm, but also its long-term reputation. For example, Chopra and Sodhi (2004) note that on March 17, 2000, a massive surge in the electrical grid in Albuquerque started a fire at Royal Philips Electronics, damaging a large number of microchips. Nokia Corporation and Telefon AB L.M. Ericson had both been major customers of the Philips plant. Unlike Ericson, which waited for the Philips plant to resume operation, Nokia immediately began to divert its chip orders to other suppliers. As a result, Nokia suffered relatively little, while Ericson lost \$400 million in sales. Since then, Ericsson has implemented a new system of risk management that applies not only to its own organization, but also to its key suppliers (Norrman and Jansson, 2004).

Kleindorfer and Kunreuther (1999) report that more economic assets are being built in disaster-prone areas now than in the past. Perrow (2007) suggests shrinking the vulnerability of targets, e.g., reducing the growth of population centers in areas prone to natural disasters. Oh et al. (2009) recommend avoiding investment in disaster-prone areas as a way to prevent losses from natural disasters. Still, some firms may knowingly choose to bear the risk of a possible disaster in the hope of gaining some benefit (e.g., taking advantage

²<http://www.thedailybeast.com/newsweek/2011/09/04/time-to-brace-for-the-next-9-11.html>

of more lucrative markets), while others choose not to take such risks.

Preparedness enables firms to stay operational (or reopen quickly) after a major disaster. However, disaster planning is ignored in many contemporary financial risk-management models (Oh et al., 2009). A recent crisis-preparedness survey of C-level executives, general counsel, and risk managers conducted by Pillsbury-Levick (2011) find that 38% of companies do not even have a crisis plan. Also, among those companies that have a crisis plan in place, only 29% feel highly confident that they could handle a major crisis effectively. Although 24% of respondents say that a natural disaster had affected their company in the past three years, only 19% say their plans underwent major changes after the incident. This is despite the fact that respondents identify natural disasters (e.g., hurricanes, earthquakes, or tornadoes) as the second most likely scenarios to negatively affect their companies.

Covington and Simpson (2006) review the disaster-preparedness literature, and highlight the lack of consensus on even such topics as which issues are most important to consider. In the context of natural disasters, Kousky and Zeckhauser (2006) stress the importance of considering the negative externalities associated with the actions of individual or policy makers. They refer to actions that create negative externalities as *JARing* actions (i.e., actions that Jeopardize Assets that are Remote in time, distance, or likelihood), and point out that decision makers often fail to account for the costs that their actions impose on others.

Berger et al. (2008) identify three major barriers that can prevent disaster preparedness from being effective. The first is related to unrecognized

negative externalities created by actions of individuals or firms. The second barrier to effective preparedness is the fact that disasters have low probabilities but high consequences, leading myopic decision makers to underestimate their importance. Finally, the third barrier is that firms sometimes choose to locate in disaster-prone zones due to unique characteristics of such zones, thereby exposing themselves to a high risk of disaster. Perhaps as a result of such barriers, the level of corporate commitment to emergency preparedness is typically much lower than recommended. For example, Smith et al. (2007) surveyed businesses on both pandemic planning and general disaster preparedness. They found that most businesses had started planning for a pandemic, but few had exercised their plans.

In addition to the barriers noted above, we speculate that the frequently observed lack of investment in emergency planning may be in part because of competitive pressures. After all, if one firm invests in disaster preparedness but the disaster does not materialize for a number of years, the firm that invested may lose out on market share or profitability relative to lower-cost competitors, and may even go out of business due to its higher cost structure before it has the opportunity to benefit from its investment in preparedness. Thus, one might expect the optimal level of investment in a competitive market to be lower than it would be for a monopolist that could simply pass the cost of preparedness on to its customers.

In this dissertation, we explore this hypothesis by focusing on the preparedness behavior of firms that compete in a market. We model the impact of both disasters and preparedness costs on a firm's profit. Then, we characterize the

conditions under which firms can be expected to participate in the market. Next, we build a game-theoretic model for participating firms, to study their interactions and the effect of the resulting Nash equilibrium decisions on preparedness investment. Finally, we compare the Nash equilibrium decisions to the socially preferred alternative and discuss ways in which the social optimum can be achieved.

2 Literature Review

In this section, we review literature on both externalities (i.e., uncompensated interdependencies; Cornes and Sandler, 1996) and competition.

2.1 Externalities

Economists have long been concerned about externalities (Scitovsky, 1954). Early approaches treated externalities as small deviations from equilibrium (Coase, 1960). Subsequently, however, Ayres and Kneese (1969) emphasized that externalities can be inherent to the operation of an economic system, and are not necessarily small.

Technological externalities arise when the action of one firm or individual directly affects the profit or utility of another firm or individual; by contrast, pecuniary externalities are those in which the action of one firm or individual affects other firms or individuals only through the price of a product (Greenwald and Stiglitz, 1986). Both types of externalities can be either positive (when a beneficial action by one firm also benefits other firms), or negative (when an action that is beneficial to one firm imposes costs on other firms); see Pindyck and Rubinfeld (2001). An example with positive externalities is the use of Lojack, an undetectable vehicle-retrieval system (Ayres and Levitt, 1997). If enough cars are equipped with such a system, this can produce a big enough reduction of theft risk that it may no longer be worthwhile for the remaining unequipped cars to install Lojack. An example with negative externalities, due to Berger et al. (2008), concerns landowners who build on a

floodplain, consequently decreasing the ability of the land to absorb rain and increasing the chance of flooding.

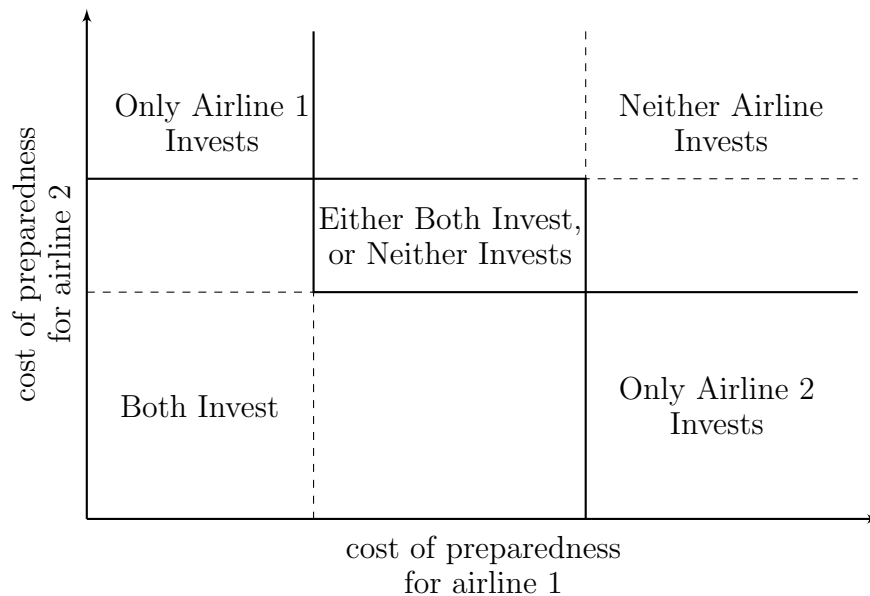
Recently, a number of researchers have become concerned about (positive or negative) externalities arising from investments in security. In particular, Keohane and Zeckhauser (2003) study externalities that arise from the protective behavior of the potential victims of security threats, which could be either individuals or firms. Individuals or firms can take action to either avoid harm (e.g., relocating from an urban area to the suburbs) or reduce harm (e.g., enforcing tighter security measures). Such actions can produce either positive or negative externalities. These externalities can be categorized under technological externalities since protective actions of individuals or firms directly affect the utility of others. For example, in the case of a firm moving its headquarter from an urban area to the suburbs, this firm's action will produce positive externalities for the urban area (by decreasing its attractiveness), but negative externalities for the suburb (by increasing its attractiveness). The existence of such externalities results in a Nash equilibrium that is not socially optimum; see Keohane and Zeckhauser (2003). Moreover, such externalities can adversely affect the benefits of even centralized investments in preparedness. For example, if government provides higher security in a metropolitan area, that will reduce the probability of an attack, but will therefore also decrease the incentive to relocate. Therefore, the total welfare loss may be the same as in the case without any government investment in security. Keohane and Zeckhauser (2003) indicate that government interventions will tend to be more effective when the individuals or firms receiving government security are

heterogeneous.

Keohane and Zeckhauser (2003) summarize the possible externalities as follows: *Discouragement* of attacks refers to the positive externalities that arise when individuals or firms decide to protect themselves. For example, if some of owners of skyscrapers in a downtown area reinforce the structures of their towers to resist possible attacks, the downtown will face a lower probability of attack overall, so other individuals and firms located in the area will benefit from the actions of a few building owners. However, such actions can also deflect the threat to less protected individuals or firms, producing negative externalities. The threat may be either *diverted* to less protected individuals or firms in the same area (e.g., if reinforcing some skyscrapers shifts the threat to unprotected skyscrapers), or *displaced* to other areas (e.g., if the threat shifts to a city that is less well protected). Finally, *containment* refers to positive externalities that do not affect the behavior of a potential attacker. For example, if one apartment owner in a building takes precautionary measures to protect his apartment against a possible fire (e.g., installing smoke detectors), other apartment owners will face a lower risk of fire. Another example is when a computer user connected to a network installs antivirus software on his computer, benefiting other computer users connected to that network.

Heal and Kunreuther (2003) and Kunreuther and Heal (2003) study *containment* (i.e., technological externalities arising from the interdependency between individuals or firms facing a common threat). In particular, Heal and Kunreuther (2003) and Kunreuther and Heal (2003) identify several classes of security externalities based on their resulting Nash equilibria.

Figure 1: Nash Equilibria as a function of preparedness costs



In class 1, preparedness investments by one individual firm eliminate the direct risk faced by that entity, but not the risk of contamination from other entities. Therefore, preparedness investment gives only partial protection in this class. However, since firms that invest in preparedness can no longer contaminate other firms, investment in preparedness results in positive externalities. Heal and Kunreuther (2004) give an example of an airline that, by incorporating more thorough baggage screening, reduces the risks to other airlines to which it transfers bags. Figure 1 indicates that in this case, if two airlines both face moderate preparedness costs, then there are multiple Nash equilibria: either both airlines invest in heightened baggage screening; or neither airline invests.

In class 2, investments in preparedness protect firms even against contam-

ination by unprotected firms (i.e., preparedness guarantees full protection for the investing entity). As a comparison with the airline example above, Heal and Kunreuther (2004) cite El Al, which checks even baggage being transferred from other airlines. Another example is vaccination (Heal and Kunreuther, 2005), since vaccination can protect against exposure to unvaccinated people. In this case, moderate investment costs result in investment by one agent or the other but not both (unlike in class 1, where moderate costs result in both agents making the same choice at equilibrium).

Finally, class 3 deals with non-defensive investments that generate positive externalities. An example is investment in research and development. In particular, investing in knowledge produces positive externalities for competing firms (if we assume that the benefits of research are at least partially transferable). The resulting Nash equilibrium in this class of problems is similar to that of class 2. The more firms invest in research and development, the greater the chance that another firm will benefit from that investment.

Zhuang et al. (2007) extend the static model of Kunreuther and Heal (2003) to allow for security threats that materialize over time, and incorporate the defenders' discount rates in the model. Their model resembles that of class 1 above. They note that the presence of firms with extremely high or low discount rates can make investment in preparedness undesirable even for other firms with more modest discount rates, who would otherwise choose to invest. For example, if some firms have extremely high discount rates, and therefore choose not to invest, it may no longer be worthwhile for other firms to invest, if they will eventually be affected by contamination from the more

myopic firms in any case.

Bier et al. (2007) examine the problem of decision makers responsible for defensive investments in the face of a strategic attacker. Their model also gives rise to negative externalities. In other words, if a defender protects one potential target, other targets become more attractive to the attacker, resulting in a race to the top. However, Bier et al. (2007) showed that a single centralized defender can internalize these externalities.

2.2 Competition

So far, we have focused on only externalities involving investments in preparedness. Now, we turn our focus to models of competition in the market. Economic models of competition can be categorized into three broad genres (Etro, 2007): (1) competition on quantity; (2) competition on price; and (3) competition for the market.

In the first category, firms compete on quantity. The underlying assumption in these models is that firms cannot easily adjust their output level on a daily basis or when faced with new market conditions (e.g., due to capacity or contractual constraints). Therefore, they find a single overall optimal level of production that maximizes their objective (e.g., profit) and the price will be determined in the market (based on supply and demand).

In particular, Cournot (1838) studied a market for a homogeneous good, where firms independently choose their optimal output levels, and the price is determined by supply and demand³. His equilibrium concept resembles the

³for an excellent review of Cournot model see <http://www.vanderbilt.edu/econ/faculty/Daughety/CournotCompetition-Daughety-webversion.pdf>

Nash equilibrium (Nash, 1950, 1951). Due to its rather simple but useful nature, the Cournot model has been used extensively to study market behavior. For example, Borenstein et al. (1997) use the Cournot-Nash equilibrium concept to study the interaction of competing firms in the electricity industry, to analyze the potential for market power (i.e., the condition in which a firm has enough power to set and charge a price higher than the competitive price). Deo and Corbett (2008) model the interaction of firms in a two-stage Cournot competition where firms face yield uncertainty in the production of influenza vaccine. In the first stage, firms decide whether to enter the market; then, in the second stage, they decide on their production levels. Deo and Corbett show that yield uncertainties reduce the total equilibrium output of influenza vaccine in this model. Novshek (1980) showed that when we focus only on the equilibrium output of a specific good, rather than the whole economy, the equilibrium outcome when all firms are assumed to be price takers can be approximated by a Cournot equilibrium if there are no barriers for firms entering or exiting the market. In a Cournot model, if entry of new firms is allowed, the resulting equilibrium is known as a Marshall equilibrium (Marshall, 1890), in which firms continue to enter the market until the profit of all participating firms is zero.

Under the same assumption of competing on quantity, if the market has leader-follower characteristics, we have a Stackelberg equilibrium (von Stackelberg, 1934). The leader firm (i.e., the firm with a competitive advantage) decides on its action first. Then the follower firms, after observing the action of the leader firm, decide on their actions (e.g., production levels). Etro (2008)

study the behavior of firms in a Stackelberg competition with endogenous entry (i.e., where firms enter the market when participating firms are profiting) and showed that when firms compete on quantity, the leader tends to set their production level more than the production level of their followers. The concept of a Stackelberg equilibrium has also been used extensively in other areas. For example, Pita et al. (2008) used this approach to allocate security resources at the Los Angeles International Airport.

In the second type of competition model, firms compete on price. This results in the so-called Bertrand equilibrium (Bertrand, 1883). In contrast to the Cournot equilibrium, firms are assumed to be able to adjust their production quantities easily. Therefore, each firm finds the optimal price that maximizes its payoff (e.g., profit), and consumers buy from the firm that offers the lowest price. Therefore, in Bertrand equilibrium any given firm can clear the market (i.e., meet all the demand) by setting a lower price than all the other firms and producing the total demanded quantity. Also, when firms cannot match a prevailing low market price, one option is to differentiate their product so that they can set a higher price in the Bertrand competition and strategically benefit from it. Changing the quality of a good or bundling two goods together are some of the techniques used for this purpose. This strategy allows firms to effectively move to another existing market where they can compete on price more profitably; see Banker et al. (1998).

Finally, the third form of competition is when firms compete to establish new markets, which may be more important in some industries than price or quantity. For example, models of research and development are often of this

form (Dixit, 1988; Quirnbach, 1996). By pursuing research and development, firms can “conquer future markets,” as Etro (2007) has pointed out.

In this work, we use the concept of Cournot-Nash equilibrium to model firms competing in the face of a possible disaster. Cournot-Nash equilibrium seems appropriate because: (1) it provides a rather simple framework for studying the behavior of firms in markets; and (2) in disaster situations, firms are constrained by their post-disaster capacity (e.g., due to shortages of input materials), making other competition forms potentially unrealistic (e.g., Bertrand competition in which firms are assumed to adjust their production quantities as needed to achieve the optimal price). Moreover, we assume that the action of firms affect each other through the market price (i.e., pecuniary externalities). Therefore, price needs to be determined in the market based on the production levels of firms. We initially allow firms to have at most two distinct production quantities: one for the post-disaster situation; and the other for the non-disaster case. We later contrast this modeling approach with a case where firms pick a single production quantity that they will produce in both disaster and non-disaster situations.

3 Assumptions and Notation

As noted above, given the likelihood of a disaster (e.g., hurricane, pandemic), and the potential consequences to a firm (e.g., bankruptcy), it would seem reasonable for firms to invest heavily in preparedness (e.g., maintain excess inventory or contractual relationships⁴ with backup suppliers). However, firms can be constrained by competitive pressures. If a firm invests in preparedness, and a disaster does not materialize, that firm may lose market share to lower-cost competitors. Therefore, we study the interaction of competing firms, and address the question of when investing in preparedness will be optimal for a firm in a competitive market.

In particular, we consider a Cournot market where the price of a given good is a decreasing function of its total output in the market. The total output is the sum of the outputs of all individual firms. We assume that each firm is profit maximizing and we determine the Cournot-Nash equilibrium in which each firm sets its own output, given the outputs of all the other firms, in order to maximize its expected profit in the face of a disaster. We believe that the assumption of profit maximizing is a reasonable one in the context of our model, although Fiat et al. (2011) showed that if competing firms in a Cournot market act to maximize revenue, they will end up recovering more profit at equilibrium than if they act to maximize profit. The question of whether firms actually do maximize their profit is beyond the scope of our model. For example, Tirole (1993) has argued that the principle-agent nature

⁴As noted by Jensen and Meckling (1976), the essence of the firm is its contractual relations with suppliers, customers, creditors, etc., as well as employees.

of most firms (i.e., the fact that firm managers are usually not the owners) may be an indication that firms do not always maximize profit⁵.

As discussed earlier, the selection of the Cournot model in our case seems reasonable, since in the other non-cooperative equilibrium, the Bertrand equilibrium, it is assumed that firms can easily adjust their output levels to any desired quantity, which may not be reasonable after a disaster. Note, however, that as the number of competing firms in the market grows, the market price in the Cournot model converges to marginal cost, giving the same result as in the Bertrand model.

We assume that decisions regarding investments in preparedness must be made before finding out whether a disaster will occur, but that firms will produce their post-disaster output levels (which may not equal pre-disaster levels) after a disaster has occurred. This assumption seems realistic, since many firms do cease to produce (or at least are unable to maintain their pre-disaster levels of production) after disasters.

Although our model does not consider some aspects of market dynamics (e.g., technological advances in production of a good), we believe that it can nonetheless provide a clear understanding of the effects of various parameters on the market equilibrium (at least in the short term, for constant technologies), and in particular the effects of different incentive policies on the welfare of consumers and producers.

Our model considers a one-time decision by firms of whether to invest in preparedness for a disaster, which is assumed to occur with a known probabil-

⁵For arguments for and against the assumption that firms maximize profit, see Tirole (1993) and Pindyke and Rubinfeld (2004).

ity. Investment in preparedness is assumed to completely protect a firm from the effects of a disaster if one occurs. We first assume that the preparedness cost is fixed (i.e., does not depend on the production level), and then model a case with variable cost of preparedness (i.e., where the preparedness cost depends on the desired post-disaster production level). If a firm chooses not to invest in preparedness and a disaster materializes, then that firm faces an increase in the variable cost of production. Thus, all else equal, if a disaster does not occur, firms that choose not to invest in preparedness will have a competitive advantage; however, if a disaster does occur, firms that did invest will have a competitive advantage. In particular, the marginal cost of production for an individual firm will be affected by both its preparedness decision and whether a disaster materializes. Therefore, firms that do not invest might end up having a larger marginal cost of production in the event of disaster. This modeling approach allows us to capture the effects of firms' preparedness decisions on each others production quantities, as well as their own production.

In Section 4, we formulate the general model of interaction between participating firms in the market. Then, in Section 5, we focus on the case where the preparedness cost is a variable cost (i.e., depends on the post-disaster production level), and in Section 6, we explore the case where the cost of preparedness is fixed. In Section 7, we present multiple numerical results for the case of fixed cost of preparedness. Section 8 offers an alternative approach to modeling the interaction of firms in the market; namely, firms are assumed to choose a single production quantity to be maintained under all conditions, rather than different production quantities for disaster and no-disaster situations. Section

9 focuses on finding the social preferred alternative and its comparison with the competitive outcome. Finally, Section 10 concludes and discusses future extensions for our model. The notation used in this dissertation is summarized in Table (1).

Table 1: Notation

n	Number of firms in the market
s	(s_1, s_2, \dots, s_n) , vector of investment strategies, where $s_i = 1$ if firm i invests in preparedness and $s_i = 0$ otherwise
$s_{\sim i}$	Vector of investment strategies of all firms except firm i
d	Indicator variable, $d = 1$ if a disaster occurs and $d = 0$ otherwise
p	Probability of disaster
$q_i^{(s_i, s_{\sim i}, d)}$	Production quantity of firm i , given the investment strategies of all other firms and whether a disaster happens
$Q^{(s, d)}$	Total production quantity in the market; i.e., $\sum_{i=1}^n q_i^{(s_i, s_{\sim i}, d)}$
f_i	Variable cost of production of firm i in the absence of a disaster (or if firm i has invested in preparedness)
f'_i	Variable cost of production of firm i in the event of a disaster, if firm i has not invested in preparedness
δ_i	Increase in post-disaster variable cost of production, $\delta_i = f'_i - f_i > 0$
C_i	Fixed cost of investing in preparedness for firm i
c_i	Variable cost of investing in preparedness for firm i

Table 1: (continued)

$\mathcal{R}_i^{(s,d)}$	Revenue of firm i , given the investment strategies of all firms and whether a disaster happens
$\mathcal{C}_i^{(s,d)}$	Total cost of firm i , given the investment strategies of all firms and whether a disaster happens
$\mathcal{P}_i^{(s,d)}$	Profit of firm i , given the investment strategies of all firms and whether a disaster happens, $\mathcal{P}_i^{(s,d)} = \mathcal{R}_i^{(s,d)} - \mathcal{C}_i^{(s,d)}$
$\pi^{(s,d)}$	Price as a function of the total production quantity $\pi^{(s,d)} = a - bQ^{(s,d)}$, where $a > 0$ is the reservation price and $-b < 0$ is the slope

4 General Model

In this section, we build a general model for our preparedness game, which can be applied to situations with either fixed or variable costs of preparedness. Assume that there are $n \geq 2$ firms in the market, all producing the same good. All firms are oligopolists and profit maximizers, and simultaneously choose whether to invest in preparedness ($s_i = 1$) or not ($s_i = 0$). We will call the collection of those investment decisions the “preparedness state,” denoted by the vector $s = (s_1, s_2, \dots, s_n)$. Similarly, let $s_{\sim i}$ be the vector of preparedness decisions of all firms except firm i .

We define the profit of firm i , $\mathcal{P}_i^{(s,d)}$ for $i = 1, 2, \dots, n$, to be the total revenue (production quantity times market price), minus the sum of the fixed and variable costs of production and preparedness. Note that profit will in general depend on both the preparedness state s and the disaster condition d :

$$\begin{aligned} \mathcal{P}_i^{(s,d)}(q^{(s,d)}) &= \overbrace{\mathcal{R}_i^{(s,d)}(q^{(s,d)})}^{\text{Revenue}} - \overbrace{\mathcal{C}_i^{(s,d)}(q_i^{(s,d)})}^{\text{Total Cost}} \\ &= \underbrace{\pi^{(s,d)}(Q^{(s,d)})}_{\text{Price}} \underbrace{q_i^{(s,d)}}_{\text{Quantity}} - \underbrace{\mathcal{C}_i^{(s,d)}(q_i^{(s,d)})}_{\text{Total Cost}} \end{aligned} \quad (1)$$

where $q^{(s,d)} = (q_1^{(s,d)}, q_2^{(s,d)}, \dots, q_n^{(s,d)})$. As above, we define $q_{\sim i}^{(s,d)}$ to be the vector of production quantities of all firms except firm i . (Note: for the sake of brevity, in what follows, we simply write $\mathcal{P}_i^{(s,d)}$ instead of $\mathcal{P}_i^{(s,d)}(q^{(s,d)})$, $\mathcal{R}_i^{(s,d)}$ instead of $\mathcal{R}_i^{(s,d)}(q^{(s,d)})$, and $\mathcal{C}_i^{(s,d)}$ instead of $\mathcal{C}_i^{(s,d)}(q_i^{(s,d)})$.)

The market price depends on the production quantities of all firms. For simplicity, we let the price be given by an affine inverse demand function

$a - bQ^{(s,d)}$ (for $a > 0$ and $b > 0$), where $Q^{(s,d)} = \sum_{j=1}^n q_j^{(s,d)}$ is the total production level under preparedness state s and disaster condition d , and $Q_{\sim i}^{(s,d)} = \sum_{\substack{j=1 \\ j \neq i}}^n q_j^{(s,d)}$ is the total production of all firms except firm i .⁶

The total cost is the sum of the production costs (i.e., fixed and variable costs of production) and the preparedness cost, either fixed or variable:

$$C_i^{(s,d)} = VC_i^{(s,d)} + FC_i^{(s,d)} \quad (2)$$

where $VC_i^{(s,d)}$ and $FC_i^{(s,d)}$, respectively, are the total variable costs (including cost of production and variable cost of preparedness, if any), and the fixed preparedness cost of firm i , if any. Although this cost function does not include a fixed cost of production, we believe that it is nonetheless a reasonable choice since we are primarily interested in modeling the preparedness decisions of firms that are already participating in the pre-disaster market, not firms that might enter the market only after a disaster. (As we will show later, such firms might exist in the real world; however, it would be difficult to identify them prior to a disaster, so the question of under what conditions such firms would enter the market is of little practical interest.) Also, we are assuming that the market is in a stable condition so any non-myopic firms with negative expected profit (due to high fixed costs of production) would have exited the market already. Moreover, identifying those firms that will produce in the market in advance will enable us to simplify the Karush–Kuhn–Tucker (*KKT*) conditions required to obtain the optimal levels of production, by

⁶For some examples in the literature that used an affine inverse demand function, see: Harrington (1995), Goyal and Netessine (2007), and Dub and Manchanda (2005).

changing the constrained profit maximization problem to an unconstrained problem. Finally, eliminating the fixed cost of production does not affect the optimal production quantities of firms. Therefore, we ignore the fixed cost of production. However, in order to have a general model that encompass all the different firms with their unique characteristics, we initially develop the model without excluding any firm and only restrict our model to participating firms when we go about computing firms' payoffs (i.e., expected profits) to use in our game-theoretic model.

Combining the variable costs of production and preparedness gives the realized variable cost, $RVC_i^{(s_i,d)}$, as the coefficient of the quantity $q_i^{(s,d)}$. In other words, $VC_i^{(s,d)} = RVC_i^{(s_i,d)} q_i^{(s,d)}$.

4.1 Finding the Optimal Production Quantities

Assume that the vector of preparedness decisions of all firms, s , is given. We can then solve the simultaneous profit-maximization problem for all firms in order to find their optimal production quantities conditional on the vector s . This step can be viewed as the second stage of our model, where firms solve for their optimal production quantities given the vector of preparedness investment decisions. Later, we will solve the first stage, where firms determine their optimal preparedness strategies.

We model firms as maximizing their total expected profit $E[\mathcal{P}_i^{(s)}]$, although the optimal level of production in disaster situations can be determined independently of the optimal level of production in non-disaster situations. In

particular, we model the optimization problem for firm i as follows:

$$\begin{aligned}
\text{maximize} \quad & E[\mathcal{P}_i^{(s)}] = p(\mathcal{P}_i^{(s,1)}) + (1-p)(\mathcal{P}_i^{(s,0)}) \\
\text{s.t.} \quad & q_i^{(s,0)} \geq 0 \\
& q_i^{(s,1)} \geq 0
\end{aligned} \tag{3}$$

Later, we let λ_i^0 be the Lagrange multiplier for the first inequality, λ_i^1 be the Lagrange multiplier for the second inequality, and λ_i be the vector of $(\lambda_i^0, \lambda_i^1)$.

Thus, firms have at most two distinct production quantities: one for the disaster situation; and one for the non-disaster case. In Section 8, we contrast this modeling approach with the case where firms pick a single quantity to produce in both disaster and non-disaster situations.

Since each firm's optimal level of production depends on the production levels of the other firms (through the market price), all n maximization problems must be solved simultaneously to find the optimal production quantity for each firm. Note that the profit in each case is a concave function of $q_i^{(s,d)}$, since the second derivative of the profit with respect to the production quantity equals $-2b$, which is less than zero:

$$\begin{aligned}
\frac{\partial^2 \mathcal{P}_i^{(s,d)}}{\partial q_i^{(s,d)^2}} &= \frac{\partial^2}{\partial q_i^{(s,d)^2}} \left((a - b \sum_{j=1}^n q_j^{(s,d)}) q_i^{(s,d)} - RVC_i^{(s,d)} q_i^{(s,d)} - FC_i^{(s,d)} \right) \\
&= \frac{\partial}{\partial q_i^{(s,d)}} \left((a - b \sum_{j=1}^n q_j^{(s,d)}) - b q_i^{(s,d)} - RVC_i^{(s,d)} \right) \\
&= -2b
\end{aligned}$$

Therefore, we can solve for the optimum production quantity by writing the *KKT* conditions for each maximization problem and solving the resulting

simultaneous equations⁷. First, we set up the Lagrangian function as follows:

$$\mathcal{L}_i(q_i^{(s,d)}, \lambda_i) = E[\mathcal{P}_i^{(s)}] + \lambda_i^0 q_i^{(s,0)} + \lambda_i^1 q_i^{(s,1)} \quad (4)$$

Now we can write down the KKT conditions for firm i as follows:

$$\begin{aligned} \frac{\partial \mathcal{L}_i}{\partial q_i^{(s,0)}}(q_i^{(s,0)}, \lambda_i) = 0 &\Rightarrow (1-p) \frac{\partial \mathcal{P}_i^{(s,0)}}{\partial q_i^{(s,0)}} + \lambda_i^0 = 0 \\ \frac{\partial \mathcal{L}_i}{\partial q_i^{(s,1)}}(q_i^{(s,1)}, \lambda_i) = 0 &\Rightarrow p \frac{\partial \mathcal{P}_i^{(s,1)}}{\partial q_i^{(s,1)}} + \lambda_i^1 = 0 \\ 0 \leq q_i^{(s,0)} \perp \lambda_i^0 &\geq 0 \\ 0 \leq q_i^{(s,1)} \perp \lambda_i^1 &\geq 0 \end{aligned} \quad (5)$$

where $0 \leq q_i^{(s,d)} \perp \lambda_i^d \geq 0$ is equivalent to the following set of relations:

$$q_i^{(s,d)} \geq 0, \lambda_i^d \geq 0, \text{ and } q_i^{(s,d)} \lambda_i^d = 0 \quad (6)$$

The above system of equalities and inequalities describes the critical points of the optimization problem. With n firms, we will have $4n$ equations in $4n$ unknowns $(q_i^{(s,0)}, q_i^{(s,1)}, \lambda_i^0, \lambda_i^1)$. We let $q^* = (q_1^*, q_2^*, \dots, q_n^*)$ and $\lambda^* = (\lambda_1^*, \lambda_2^*, \dots, \lambda_n^*)$ be the resulting optimal values.

We can characterize the optimal solution for firm i according to whether each of the Lagrange multipliers is zero or greater than zero. If $(\lambda_i^0 > 0, \lambda_i^1 > 0)$, then $q_i^{*(s,1)} = q_i^{*(s,0)} = 0$. In this case, firm i will not be in the market.

If $(\lambda_i^0 = 0, \lambda_i^1 > 0)$, then it follows that $q_i^{(s,1)} = 0$ and firm i will not par-

⁷Novshek (1985) proves the existence of an n -firm Cournot equilibrium for a single homogeneous good if the marginal revenue of each firm is decreasing when the total output of other firms increases.

participate in the post-disaster market. In this case, we can simplify the KKT conditions as follows:

$$\begin{aligned}
a - bQ_{\sim i}^{(s,0)} - 2bq_i^{(s,0)} - RVC_i^{(s,0)} &= 0 \\
p(a - bQ_{\sim i}^{(s,1)} - RVC_i^{(s,1)}) + \lambda_i^1 &= 0 \\
0 \leq q_i^{(s,0)} \perp \lambda_i^0 &\geq 0 \\
\lambda_i^1 &\geq 0
\end{aligned} \tag{7}$$

Therefore, we will have $q_i^{(s,0)} = \frac{1}{2b}(a - bQ_{\sim i}^{(s,0)} - RVC_i^{(s,0)})$ and $\lambda_i^1 = -p(a - bQ_{\sim i}^{(s,1)} - RVC_i^{(s,1)}) > 0$. The resulting expected profit will be given by

$$\begin{aligned}
E[\mathcal{P}_i^{(s)}] &= p(-FC_i) + (1-p)(\mathcal{P}_i^{(s,0)}) \\
&= p(-FC_i) + (1-p)(\mathcal{R}_i^{(s,0)} - \mathcal{C}_i^{(s,0)}) \\
&= p(-FC_i) + (1-p)((a - bQ_{\sim i}^{(s,0)})q_i^{(s,0)} - RVC_i^{(s,0)}q_i^{(s,0)} - FC_i) \\
&= (1-p)((a - bQ_{\sim i}^{(s,0)} - RVC_i^{(s,0)})q_i^{(s,0)}) - FC_i
\end{aligned} \tag{8}$$

If $(\lambda_i^0 > 0, \lambda_i^1 = 0)$, then we will have $q_i^{(s,0)} = 0$, and $q_i^{(s,1)} = \frac{1}{2b}(a - bQ_{\sim i}^{(s,1)} - RVC_i^{(s,1)})$ and firm i will participate only in a disaster market. The expected profit will then be $p((a - bQ_{\sim i}^{(s,1)} - RVC_i^{(s,1)})q_i^{(s,1)}) - FC_i$. Finally, if $(\lambda_i^0 = 0, \lambda_i^1 = 0)$, then the optimal production quantities for firm i will be $q_i^{(s,0)} = \frac{1}{2b}(a - bQ_{\sim i}^{(s,0)} - RVC_i^{(s,0)})$ and $q_i^{(s,1)} = \frac{1}{2b}(a - bQ_{\sim i}^{(s,1)} - RVC_i^{(s,1)})$.

One way to solve the system of KKT equations in (93) is to use comple-

mentarity functions, as follows:

$$\begin{aligned}
(1-p)(a - bQ_{\sim i}^{(s,0)} - 2bq_i^{(s,0)} - RVC_i^{(s,0)}) + \lambda_i^0 &= 0 \\
p(a - bQ_{\sim i}^{(s,1)} - 2bq_i^{(s,1)} - RVC_i^{(s,1)}) + \lambda_i^1 &= 0 \\
\psi_{FB}(q_i^{(s,0)}, \lambda_i^0) &= 0 \\
\psi_{FB}(q_i^{(s,1)}, \lambda_i^1) &= 0
\end{aligned} \tag{9}$$

where $\psi_{FB}(\alpha, \beta) = \sqrt{\alpha^2 + \beta^2} - (\alpha + \beta)$ is the Fischer-Burmeister complementarity function (Facchinei and Pang, 2003). It is straightforward to see that when $\psi_{FB}(\alpha, \beta) = 0$, the product of α and β must be equal to zero. Moreover, if $\alpha\beta = 0$, then either $\alpha = 0$ and $\beta \geq 0$ or $\beta = 0$ and $\alpha \geq 0$.

However, Konnov (2001) and Murphy et al. (1982) have shown that the system of n maximization problems above (for either disaster or non-disaster conditions) can also be formulated as a convex quadratic problem:

$$\underset{q \in \mathbb{R}_+^n}{\text{minimize}} \quad \frac{1}{2}q^T(bM_{n \times n})q + (RVC - ae)^Tq \tag{10}$$

where $e = [1, 1, \dots, 1]_{n \times n}$, $RVC = (RVC_1, RVC_2, \dots, RVC_n)$, and

$$M_{n \times n} = \begin{bmatrix} 2 & 1 & \dots & 1 \\ 1 & 2 & \dots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \dots & 2 \end{bmatrix}.$$

Note that M is a symmetric positive-definite matrix. Therefore, the convex quadratic problem above has a unique optimal solution, which is also the

unique Nash equilibrium of the oligopolistic market. The solution of this convex quadratic problem will give the set of optimal production quantities of all firms. More importantly, this solution set will be an array of non-negative numbers. A firm that cannot compete in the market will be assigned an optimal production quantity of zero, while firms that can compete will be assigned positive optimal production quantities.

However, it is also possible to identify and remove firms that are guaranteed not to participate in the market before solving the optimization problem, using the proposition below. This will then allow us to solve an unconstrained rather than a constrained optimization problem, yielding positive optimal production quantities in closed form. (For the proof of the following proposition, see Appendix A.)

Proposition 4.1.1 *Let there be n firms in the market, and let the investment decisions and disaster or non-disaster conditions be given by (s, d) . Assume, without loss of generality, that firms are sorted with respect to their values of RVC, such that $RVC_1^{(s_1, d)} \leq RVC_2^{(s_2, d)} \leq \dots \leq RVC_n^{(s_n, d)}$. Now, let l be an integer such that $\rho(l)^{(s, d)} \geq 0$ and $\rho(l+1)^{(s, d)} < 0$, where*

$$\rho(i)^{(s, d)} = a - (i+1)RVC_i^{(s_i, d)} + \sum_{j=1}^i RVC_j^{(s_j, d)}.$$

Then firms with index values $i \leq l$ will have non-negative optimal production quantities, and firms with index values $i > l$ will have optimal production quantities of zero.

Therefore, we can eliminate firms whose values of $\rho(i)^{(s, d)}$ are negative, and

solve the following reduced-size unconstrained convex quadratic problem⁸,

$$\underset{q \in \mathbb{R}^l}{\text{minimize}} \quad \frac{1}{2} q^T (bM_{l \times l}) q + (RVC - ae)^T q \quad (11)$$

where $l^{(s,d)} = |\{i | \rho(i)^{(s,d)} \geq 0\}|$ is the number of firms with non-zero production quantities under conditions (s, d) . For example, let $a = 30$ and $\mathcal{I}_i^{(s,d)}$ be the set of indices of firms with RVC s less than or equal to $RVC_i^{(s,d)}$. Now, assume that there are five firms in the market, with values of RVC_i as given in Table 2 below. Table 2 also gives the values of \mathcal{I}_i and $\rho(i)$ for $i = 1, 2, \dots, 5$, for some arbitrary condition (s, d) . For instance, $\mathcal{I}_1 = \{1, 2, 4, 5\}$, because $RVC_i^{(s,d)} \leq 15$ for $i = 1, 2, 4, 5$. Plugging $|\mathcal{I}_1| = 4$ into the expression for $\rho(1)$, gives $\rho(1) = 30 - (4+1)(15) + (15+5+5+10) = -10 < 0$. Thus, in this example, only firms 2, 4, and 5 can participate in the market, since $\rho(i) \geq 0$ for $i = 2, 4, 5$. Therefore, we have $l = 3$.

Table 2: Example of \mathcal{I}_i and $\rho(i)$

i	$RVC_i^{(s,d)}$	$\mathcal{I}_i^{(s,d)}$	$ \mathcal{I}_i^{(s,d)} $	$\rho(i)^{(s,d)}$
1	15	$\{1, 2, 4, 5\}$	4	$-10 < 0$
2	5	$\{2, 4\}$	2	$25 \geq 0$
3	20	$\{1, 2, 3, 4, 5\}$	5	$-35 < 0$
4	5	$\{2, 4\}$	2	$25 \geq 0$
5	10	$\{2, 4, 5\}$	3	$10 \geq 0$

Without loss of generality, we can re-index the participating firms under conditions (s, d) so that their indices range from 1 to l . These l firms are then guaranteed to participate in the market based on the discussion above.

⁸Remember, from Section 4, that we assume there are no fixed costs of production.

Given the objective function in Equation (11), we have $\nabla(\frac{1}{2}q^T(bM_{l \times l})q + (RVC - ae)^T q) = bM_{l \times l}q + (RVC - ae)$. Then $\nabla(\frac{1}{2}q^T(bM_{l \times l})q + (RVC - ae)^T q) = 0$ when

$$q^* = \frac{1}{b}M_{l \times l}^{-1}(ae - RVC). \quad (12)$$

It is simple to show that if

$$M_{l \times l} = \begin{bmatrix} 2 & 1 & \cdots & 1 \\ 1 & 2 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 2 \end{bmatrix} \quad \text{then} \quad M_{l \times l}^{-1} = \frac{1}{l+1} \begin{bmatrix} l & -1 & \cdots & -1 \\ -1 & l & \cdots & -1 \\ \vdots & \vdots & \ddots & \vdots \\ -1 & -1 & \cdots & l \end{bmatrix}$$

Thus, by doing the matrix multiplication, the optimal production quantity of firm i is:

$$q_i^{*(s,d)} = \frac{1}{b} \left[\frac{l}{l+1} (a - RVC_i^{(s,d)}) - \frac{1}{l+1} \sum_{j \neq i} a - RVC_j^{(s,d)} \right] \quad (13)$$

which, after rearranging, gives the following result.

$$q_i^{*(s,d)} = \frac{1}{(l+1)b} (a - (l+1)RVC_i^{(s,d)} + \sum_{j=1}^l RVC_j^{(s_j,d)}) \quad (14)$$

Equivalently, let $\Delta_{\sim i}$ be the sum of the incurred increases in variable production cost of all firms other than firm i , due to some number of those firms not investing in preparedness. Thus, $\Delta_{\sim i}$ can be interpreted as the benefits flowing to firm i in the case of a disaster, from the increases in production costs of firms that did not invest in preparedness. Likewise, let Φ_i be the sum of the differences between the production costs of firm i and the other firms

participating in the market; i.e., $\Phi_i = \sum_{j=1}^l (f_j - f_i)$. Then we can rewrite the expression for the optimal production quantity of firm i in terms of $\Delta_{\sim i}$ and Φ_i as follows:

$$q_i^{*(s,d)} = \frac{1}{(l+1)b} (a - f_i - l\delta_i(1 - s_i)d + \Phi_i + d\Delta_{\sim i}) \quad (15)$$

Based on this result for the optimal production quantities of firms, we can calculate the profit of each firm given the preparedness states of all firms:

- If firm i is participating in the non-disaster market (i.e., $\rho(i)^{(s,0)} \geq 0$), then its profit is given by the equation below, where $TRVC^{(s,0)} = \sum_{k=1}^n RVC_k^{(s_k,0)}$ is the total realized variable cost of all the n firms:

$$\begin{aligned} \mathcal{P}_i^{(s,0)} &= \left(a - b \left(\sum_{j=1}^n \frac{1}{b(n+1)} (a - (n+1)RVC_j^{(s_j,0)} + TRVC^{(s,0)}) \right) - RVC_i^{(s_i,0)} \right) \\ &\quad \times \left(\frac{1}{b(n+1)} (a - (n+1)RVC_i^{(s_i,0)} + TRVC^{(s,0)}) \right) - FC_i^{(s_i,0)} \\ &= \left(a - \frac{n(a + TRVC^{(s,0)})}{n+1} + TRVC^{(s,0)} - RVC_i^{(s_i,0)} \right) \\ &\quad \times \left(\frac{1}{b(n+1)} (a - (n+1)RVC_i^{(s_i,0)} + TRVC^{(s,0)}) \right) - FC_i^{(s_i,0)} \\ &= \frac{1}{b(n+1)^2} (a - (n+1)RVC_i^{(s_i,0)} + TRVC^{(s,0)})^2 - FC_i^{(s_i,0)} \end{aligned} \quad (16)$$

- If firm i participates in the post-disaster market (i.e., $\rho(i)^{(s,1)} \geq 0$), then its profit is given by the equation below, where $TRVC^{(s,1)} = \sum_{k=1}^l RVC_k^{(s_k,1)}$ is the total realized variable cost of firms that participate in the market

in the case of a disaster.

$$\begin{aligned}
\mathcal{P}_i^{(s,1)} &= \left(a - b \left(\sum_{j=1}^l \frac{1}{b(l+1)} \left(a - (l+1) RVC_j^{(s_j,1)} + TRVC^{(s,1)} \right) \right) - RVC_i^{(s,1)} \right) \\
&\quad \times \left(\frac{1}{b(l+1)} \left(a - (l+1) RVC_i^{(s_i,1)} + TRVC^{(s,1)} \right) \right) - FC_i^{(s_i,1)} \\
&= \left(a - \frac{l(a + TRVC^{(s,1)})}{l+1} + TRVC^{(s,1)} - RVC_i^{(s,1)} \right) \\
&\quad \times \left(\frac{1}{b(l+1)} \left(a - (l+1) RVC_i^{(s_i,1)} + TRVC^{(s,1)} \right) \right) - FC_i^{(s_i,1)} \\
&= \frac{1}{b(l+1)^2} \left(a - (l+1) RVC_i^{(s_i,1)} + TRVC^{(s,1)} \right)^2 - FC_i^{(s_i,1)}
\end{aligned} \tag{17}$$

- If firm i does not participate in the non-disaster market (i.e., $\rho(i)^{(s,0)} < 0$), it will have an optimal production quantity of zero. Its profit in this case would be $-FC_i^{(s_i,0)}$. In other words, such a firm's profit will be either negative and equal to the cost of preparedness (in the case of a fixed cost of preparedness), or zero (in the case of a variable cost of preparedness).
- Finally, if firm i cannot participate in the post-disaster market (i.e., $\rho(i)^{(s,1)} < 0$), then firm i will not produce and its profit will be $-FC_i^{(s_i,1)}$.

Now, the expected profit for a firm can be calculated as the weighted sum of the disaster and non-disaster profits, weighted by the disaster and non-disaster probabilities, respectively. Note that our model allows for the possibility of that firms enter the market only after a disaster, due to the fact that the new post-disaster prices are sufficiently high to compensate for their high variable costs of production. However, it is difficult to identify such firms prior to a

disaster. Therefore, from now on we restrict our attention to the case where all n firms participate in the non-disaster market. In the case of a disaster, firms with indices less than or equal to l are assumed to participate in the post-disaster market and the rest do not participate. Therefore, we are not considering firms with $\rho(i)^{(s,0)} < 0$ (i.e., firms that cannot produce in the non-disaster market), even if they might enter the post-disaster market (if $\rho(i)^{(s,1)} \geq 0$ or $\rho(i)^{(s,1)} < 0$). Later, we will use this expected profit to find which preparedness strategy, $s_i = 0$ or $s_i = 1$, will yield a higher expected profit from firm i .

- If $\rho(i)^{(s,0)} \geq 0$ and $\rho(i)^{(s,1)} \geq 0$, then

$$\begin{aligned} E[\mathcal{P}_i^s] = & p \left(\frac{1}{b(l+1)^2} (a - (l+1)RVC_i^{(s_i,1)} + TRVC^{(s,1)})^2 - FC_i^{(s_i,1)} \right) \\ & + (1-p) \left(\frac{1}{b(n+1)^2} (a - (n+1)RVC_i^{(s_i,0)} + TRVC^{(s,0)})^2 \right. \\ & \left. - FC_i^{(s_i,0)} \right) \end{aligned} \quad (18)$$

- If $\rho(i)^{(s,0)} \geq 0$ and $\rho(i)^{(s,1)} < 0$, then

$$\begin{aligned} E[\mathcal{P}_i^s] = & p (-FC_i^{(s_i,1)}) \\ & + (1-p) \left(\frac{1}{b(n+1)^2} (a - (n+1)RVC_i^{(s_i,0)} + TRVC^{(s,0)})^2 \right. \\ & \left. - FC_i^{(s_i,0)} \right) \end{aligned} \quad (19)$$

Note that the above expressions for expected profit are conditional on the investment decisions of the firms, s .

4.2 Preparedness Game

Now, we can build a game-theoretic model of preparedness investment (i.e., the first stage of our general model), as follows. Assume that there are n firms in the market, $N = \{1, 2, \dots, n\}$, where each firm has only two pure strategies (i.e., invest in preparedness, $s_i = 1$, or not, $s_i = 0$). Then, if firm i invests in preparedness with probability $\sigma_i \in [0, 1]$, σ_i is a mixed strategy for firm $i \forall i = 1, 2, \dots, n$, while $\sigma = \{\sigma_1, \sigma_2, \dots, \sigma_n\} \in [0, 1]^n$ is a mixed strategy profile. Therefore, the likelihood of obtaining a specific investment profile $s = (s_1, s_2, \dots, s_n)$ is $\prod_{j \in N} \sigma_j^{s_j} (1 - \sigma_j)^{(1-s_j)}$. Now, the expected payoff for firm i under the mixed strategy profile σ is as follows:

$$u_i(\sigma) = \sum_{s \in \mathcal{S}} E[\mathcal{P}_i^{(s)}] \cdot \left(\prod_{j \in N} \sigma_j^{s_j} (1 - \sigma_j)^{(1-s_j)} \right) \quad (20)$$

Moreover, let σ_i^* be the best response of firm i to the set of strategy profiles $\sigma_{\sim i}$ of all other firms, such that $u_i(\sigma_i^*, \sigma_{\sim i}) \geq u_i(\sigma_i, \sigma_{\sim i}) \forall \sigma_i$. Then the strategy profile σ^* is a Nash equilibrium of the game $\mathcal{G} = \langle \mathcal{S}, \omega \rangle$, with action space $\mathcal{S} \triangleq [0, 1]^n$ and payoffs $\omega = (u_1(\sigma), u_2(\sigma), \dots, u_n(\sigma))$, if

$$u_i(\sigma_i^*, \sigma_{\sim i}^*) \geq u_i(\sigma_i, \sigma_{\sim i}^*) \quad \forall i \in N \quad \text{and} \quad \forall \sigma_i \in [0, 1] \quad (21)$$

In general, for an arbitrary payoff function, and when the set of strategy profiles is not closed and convex, the existence, uniqueness, and convergence of a Nash equilibrium for the game shown in Equation (21) are not guaranteed (Scutari et al., 2010). However, for each firm i , we know that (1) the set of choice probabilities $[0, 1]$ is closed and convex, and (2) the payoff function

$\omega_i = u_i(\sigma_i, \sigma_{\sim i})$ is continuously differentiable in σ and convex in σ_i for every fixed $\sigma_{\sim i}$. Therefore, our game \mathcal{G} has at least one pure Nash equilibrium.

One approach to finding this equilibrium is to reduce the problem above to an equivalent variational inequality (VI), based on the work of Facchinei and Pang (2009), as follows. In particular, since the strategy set for each firm, $[0, 1]$, is closed and convex, and the payoff function $\omega_i = u_i(\sigma_i, \sigma_{\sim i})$ is continuously differentiable in σ and convex in σ_i for any fixed $\sigma_{\sim i}$, the game \mathcal{G} is equivalent to the variational inequality problem $\text{VI}(\mathcal{S}, \Omega)$, where $\Omega(\sigma) \triangleq \{\nabla_{\sigma_i} \omega_i(\sigma)\}_{i=1}^n$ is the Jacobian of the collection of each firm's payoff and $\mathcal{S} \triangleq [0, 1]^n$ is the action space.

Following Facchinei and Pang (2009), we can use the fact that the interval $[0, 1]$ is convex and the payoff $u_i(\sigma_i, \sigma_{\sim i})$ is differentiable with respect to σ_i . Thus, we can write the KKT conditions for firm i as follows:

$$\begin{aligned} 0 &\leq \sigma_i \perp \nabla_{\sigma_i} u_i(\sigma) + \lambda_i \geq 0 \\ 0 &\leq \lambda_i \perp \sigma_i - 1 \leq 0 \end{aligned} \tag{22}$$

The nonlinear-complementarity formulation of the game, using the concatenation of the above KKT conditions for all i , is as follows:

$$0 \leq \begin{pmatrix} \sigma \\ \lambda \end{pmatrix} \triangleq \mathbf{z} \perp \mathbf{F}(\mathbf{z}) \triangleq \begin{pmatrix} (\nabla_{\sigma_i} u_i(\sigma) + \lambda_i)_{i=1}^n \\ (1 - \sigma_i)_{i=1}^n \end{pmatrix} \geq 0 \tag{23}$$

The above characterizes the equilibria of the game in Equation (21). Finding the equilibria for specific numerical values can be done easily using the Gambit software (McKelvey et al., 2010), which computes Nash equilibria in finite,

non-cooperative games.

The first step in solving our game numerically is to write down the expected profits of firms that participate in the non-disaster market in terms of their investment decisions. To begin, we define the notation ϕ_i and φ_i as follows:

$$\phi_i = \begin{cases} \delta_i & \text{for a fixed cost of preparedness} \\ \delta_i - \frac{c_i}{p} & \text{for a variable cost of preparedness} \end{cases}$$

$$\varphi_i = \begin{cases} C_i & \text{for a fixed cost of preparedness} \\ 0 & \text{for a variable cost of preparedness} \end{cases}$$

If firm i can participate in the market in the non-disaster condition only (i.e., $\rho(i)^{(s,0)} \geq 0$ and $\rho(i)^{(s,1)} < 0$), then using Equation (19), we can rewrite its expected profit as follows:

$$\begin{aligned} E[\mathcal{P}_i^s] &= p(-FC_i^{(s_i,1)}) \\ &\quad + (1-p)\left(\frac{1}{b(n+1)^2}\left(a - (n+1)RVC_i^{(s_i,0)} + TRVC^{(s,0)}\right)^2 - FC_i^{(s_i,0)}\right) \\ &= \frac{1-p}{b(n+1)^2}\left(a - (n+1)f_i + \sum_{k=1}^n f_k\right)^2 - \varphi_i s_i \end{aligned}$$

In this case, such a firm will never invest in preparedness (i.e., $s_i = 1$), since doing so will incur a preparedness cost φ_i , without generating a non-zero optimal production quantity to compensate for the preparedness cost.

If firm i can produce in both non-disaster and disaster conditions (i.e., $\rho(i)^{(s,0)} \geq 0$ and $\rho(i)^{(s,1)} \geq 0$), then its expected profit is given by Equation (18). We can rewrite its expected profit as a function of its investment decision

s_i as follows:

$$\begin{aligned}
E[\mathcal{P}_i^s] &= p \left(\frac{1}{b(l+1)^2} (a - (l+1)RVC_i^{(s_i,1)} + TRVC^{(s,1)})^2 - FC_i^{(s_i,1)} \right) \\
&\quad + (1-p) \left(\frac{1}{b(n+1)^2} (a - (n+1)RVC_i^{(s_i,0)} + TRVC^{(s,0)})^2 - FC_i^{(s_i,0)} \right) \\
&= p \left(\frac{1}{b(l+1)^2} (a - (l+1)(f_i + \delta_i - \phi_i s_i) + \sum_{k=1}^l f_k + \delta_k - \phi_k s_k)^2 \right) \\
&\quad + (1-p) \left(\frac{1}{b(n+1)^2} (a - (n+1)f_i + \sum_{k=1}^n f_k)^2 \right) - \varphi_i s_i \\
&= \frac{p(l\phi_i)^2}{b(l+1)^2} s_i^2 + \left(\frac{2pl\phi_i}{b(l+1)^2} (a - l(f_i + \delta_i) + \sum_{\substack{k=1 \\ k \neq i}}^l f_k + \delta_k - \phi_k s_k) - \varphi_i \right) s_i \\
&\quad + \frac{p}{b(l+1)^2} (a - l(f_i + \delta_i) + \sum_{\substack{k=1 \\ k \neq i}}^l f_k + \delta_k - \phi_k s_k)^2 + \frac{1-p}{b(n+1)^2} (a - (n+1)f_i \\
&\quad + \sum_{k=1}^n f_k)^2
\end{aligned}$$

Note that these results are based on the assumption that the decisions of the other firms (i.e., $s_{\sim i}$), were given. However, in reality, firms may need to decide whether to invest in preparedness in the absence of knowledge about the actions of other firms (e.g., in the case of simultaneous moves by the players). Even if we assume that firm i can estimate the values of the parameters affecting other firms' decisions within an acceptable range (e.g., based on the fact that they are all similar companies producing the same good in the same market), predicting the actual actions of those other firms (i.e., decisions regarding investing in preparedness) might be difficult or even impossible, at least in cases where the decisions are interdependent (i.e., for parameter values that yield multiple Nash equilibria). In this case, it is possible to find equilibrium mixed strategies, in which each firm will invest in preparedness

with a probability that would make other firms indifferent between investing and not investing.

In general, mixing will happen only when neither investing nor not investing is a dominant strategy for firm i (since we would expect a rational Bayesian player to play a strictly dominant strategy at all times if one exists). In such a case, a Bayesian equilibrium will occur when each firm mixes with the correct probability, and that mixing probability is also the prior probability assumed by the other firm. In order to find the mixed strategy Nash equilibria for a general n -player game, we can use the *Fundamental Theorem of Mixed Strategy Nash Equilibria*:

Theorem 1 *Let $\sigma = (\sigma_1, \sigma_2, \dots, \sigma_n)$ be a mixed strategy profile for an n -player game. For any player $i = 1, 2, \dots, n$, let $\sigma_{\sim i}$ represent the mixed strategies used by all the players other than player i , let S_i be the finite set of pure strategies available to player i , and let $u_i(s, \sigma_{\sim i})$ be the payoff to player i when playing s against $\sigma_{\sim i}$ (where $s \in S_i$). Then σ is a Nash equilibrium if and only if the following two conditions hold:*

1. *If $s, s' \in S_i$ both occur with positive probability in σ_i , then $u_i(s, \sigma_{\sim i}) = u_i(s', \sigma_{\sim i})$*
2. *If $s \in S_i$ occurs with positive probability in σ_i and $s' \in S_i$ occurs with zero probability in σ_i , then $u_i(s, \sigma_{\sim i}) > u_i(s', \sigma_{\sim i})$.*

We start by assuming that a mixed-strategy Nash equilibrium (i.e., $\sigma = (\sigma_1, \sigma_2, \dots, \sigma_n)$) exists. In other words, each firm has a mixed-strategy Nash equilibrium where $s_i = 1$ with probability σ_i , and $s_i = 0$ with probability

$(1 - \sigma_i)$. Based on this assumption, we can write down the payoff of firm i as a function of its investment decision, given the mixing strategies of all other firms (i.e., $\sigma_{\sim i}$). Since all firms are assumed to play mixed strategies, we will have n equations (each in terms of expected payoffs) in n unknowns (σ_i for $i = 1, 2, 3, \dots, n$).

A reasonable starting point for studying such interactions is to look at a two-player game (i.e., $n = 2$). In this case, we can solve the above system of equations analytically. For example, consider the case with a fixed cost of preparedness. Moreover, assume a duopoly where both firms participate in the post-disaster market. In such a case, both equations are linear, and we can directly solve for the unknown σ_i . In particular, firm 1 would invest in preparedness with probability σ_1 , chosen such that firm 2 would be indifferent between investing in preparedness and not investing. If firm 2 invests in preparedness, then its payoff would be equal to $\sigma_1 E[\mathcal{P}_2^{*(1,1)}] + (1 - \sigma_1) E[\mathcal{P}_2^{*(0,1)}]$. On the other hand, if firm 2 does not invest in preparedness, then its payoff would be equal to $\sigma_1 E[\mathcal{P}_2^{*(1,0)}] + (1 - \sigma_1) E[\mathcal{P}_2^{*(0,0)}]$. By equating these two payoffs, we can find the mixing probability of firm 1 as follows:

$$\begin{aligned} \sigma_1 E[\mathcal{P}_2^{*(1,1)}] + (1 - \sigma_1) E[\mathcal{P}_2^{*(0,1)}] &= \sigma_1 E[\mathcal{P}_2^{*(1,0)}] + (1 - \sigma_1) E[\mathcal{P}_2^{*(0,0)}] \\ \Leftrightarrow \sigma_1 &= \frac{E[\mathcal{P}_2^{*(0,1)}] - E[\mathcal{P}_2^{*(0,0)}]}{(E[\mathcal{P}_2^{*(1,0)}] - E[\mathcal{P}_2^{*(1,1)}]) + (E[\mathcal{P}_2^{*(0,1)}] - E[\mathcal{P}_2^{*(0,0)}])} \quad (24) \\ \Leftrightarrow \sigma_1 &= \frac{4p\delta_2(a + f_1 - 2f_2 + \delta_1 - \delta_2) - 9bC_2}{4p\delta_1\delta_2} \end{aligned}$$

One finding from the above result is that as the fixed cost of preparedness, f_2 , of firm 2 increases, firm 1 will invest in preparedness with smaller probability; in other words, firm 1 can make firm 2 indifferent between its pure strategies at a smaller expected preparedness cost.

Moreover, since pure strategies are simply special cases of mixed strategies, we can identify one pure strategy for firm 1 by identifying the conditions under which the mixing probability from Equation 24 would be greater than or equal to one:

$$\begin{aligned}
 \sigma_1 &= \frac{4p\delta_2(a + f_1 - 2f_2 + \delta_1 - \delta_2) - 9bC_2}{4p\delta_1\delta_2} \geq 1 \\
 &\Leftrightarrow 4p\delta_2(a + f_1 - 2f_2 + \delta_1 - \delta_2) - 9bC_2 \geq 4p\delta_1\delta_2 \\
 &\Leftrightarrow 4p\delta_2(a + f_1 - 2f_2 - \delta_2) - 9bC_2 \geq 0 \\
 &\Leftrightarrow C_2 \leq \frac{4p\delta_2}{9b}(a + f_1 - 2f_2 - \delta_2)
 \end{aligned} \tag{25}$$

This result indicates that if firm 1 always invests, firm 2 would invest in preparedness only if its fixed cost of preparedness was less than or equal to $\frac{4p\delta_2}{9b}(a + f_1 - 2f_2 - \delta_2)$. Similarly, if the mixing probability for firm 1 is less than or equal to zero, then firm 1 will never invest in preparedness, and firm 2 would choose not to invest in preparedness if its fixed cost of preparedness is greater than or equal to $\frac{4p\delta_2}{9b}(a + f_1 - 2f_2 - \delta_2) + \frac{4p\delta_1\delta_2}{9b}$.

For $n > 2$, we will have non-linearity in the equations. The payoff of firm i can be expressed in terms of the mixing probabilities for all firms recursively. Moreover, as we mentioned earlier, firms choose their mixing strategy so that other firms will be indifferent between investing in preparedness and not investing in preparedness. Therefore, for a given firm i we can write its payoff with and without preparedness investment, and equate the two; the first two steps

of the expansion of the resulting equality; i.e., $u_i(\sigma_i = 1, \sigma_{\sim i}) = u_i(\sigma_i = 0, \sigma_{\sim i})$ (using the mixing probabilities of firms j and $j + 1 \neq i$) are as follows:

$$\begin{aligned}
& \sigma_j u_i(\sigma_i = 1, \sigma_j = 1, \sigma_{\sim(i,j)}) + (1 - \sigma_j) u_i(\sigma_i = 1, \sigma_j = 0, \sigma_{\sim(i,j)}) \\
& = \sigma_j u_i(\sigma_i = 0, \sigma_j = 1, \sigma_{\sim(i,j)}) + (1 - \sigma_j) u_i(\sigma_i = 0, \sigma_j = 0, \sigma_{\sim(i,j)}) \\
\Leftrightarrow & \sigma_j \sigma_{j+1} u_i(\sigma_i = 1, \sigma_j = 1, \sigma_{j+1} = 1, \sigma_{\sim(i,j,j+1)}) \\
& + \sigma_j (1 - \sigma_{j+1}) u_i(\sigma_i = 1, \sigma_j = 1, \sigma_{j+1} = 0, \sigma_{\sim(i,j,j+1)}) \\
& + (1 - \sigma_j) \sigma_{j+1} u_i(\sigma_i = 1, \sigma_j = 0, \sigma_{j+1} = 1, \sigma_{\sim(i,j,j+1)}) \\
& + (1 - \sigma_j) (1 - \sigma_{j+1}) u_i(\sigma_i = 1, \sigma_j = 0, \sigma_{j+1} = 0, \sigma_{\sim(i,j,j+1)}) \\
& = \sigma_j \sigma_{j+1} u_i(\sigma_i = 0, \sigma_j = 1, \sigma_{j+1} = 1, \sigma_{\sim(i,j,j+1)}) \\
& + \sigma_j (1 - \sigma_{j+1}) u_i(\sigma_i = 0, \sigma_j = 1, \sigma_{j+1} = 0, \sigma_{\sim(i,j,j+1)}) \\
& + (1 - \sigma_j) \sigma_{j+1} u_i(\sigma_i = 0, \sigma_j = 0, \sigma_{j+1} = 1, \sigma_{\sim(i,j,j+1)}) \\
& + (1 - \sigma_j) (1 - \sigma_{j+1}) u_i(\sigma_i = 0, \sigma_j = 0, \sigma_{j+1} = 0, \sigma_{\sim(i,j,j+1)})
\end{aligned}$$

Once we have all n equations in terms of the mixing probabilities, we can then solve for the mixing strategy of each firm.

Datta (2010) describes a numerical solution method to find all the Nash equilibria. Datta also describes how Gambit solves for all the Nash equilibria of a finite game. In particular, Gambit uses Taylor series of each system of polynomial equations over subdivisions of the product of simplices to see if a solution can be found in that box. Gambit skips that simplex if no solution can exist in that simplex; however, if a solution can exist, Gambit finds that root using Newton's method and attempts to verify that no other solution is in that simplex. If it cannot verify that, Gambit subdivides the simplex and

looks at the resulting new simplexes. In our work here, we rely on Gambit's approach to solve for all the Nash equilibria of our game. Later, we will discuss the application of these results in finding the rational mixing probability under uncertainty.

5 Model with Variable Cost of Preparedness

In this section, we consider the case with a variable cost of preparedness, given by c_i per unit of production. This might correspond, for example, to a situation in which firms contract in advance to have raw material available during a disaster, presumably at a higher price than under normal circumstances. In this case, each firm must decide ahead of time what production quantity $q_i^{(s,1)}$ it will choose under disaster conditions, and must pay the amount $c_i q_i^{(s,1)}$ to invest in preparedness, regardless of whether the disaster materializes.

However, for convenience in modeling, we include the preparedness cost in our model only when the disaster actually materializes, and adjust the variable cost of preparedness to account for the probability of the disaster. Thus, if a firm chooses to invest in preparedness, it is assumed to pay an amount $\frac{c_i}{p}$ per unit of production, or in other words a total of $\frac{c_i}{p} q_i^{(s,1)}$, if the disaster takes place ($d = 1$), and 0 otherwise. This modeling approach is acceptable because: (1) the variable cost of preparedness c_i affects only the post-disaster production level $q_i^{(s,1)}$, not the non-disaster production level $q_i^{(s,0)}$; and (2) we are interested only in expected profit, not in the probability distribution of a firm's profit level. In order to see the equivalence between our approach and the standard expectation, note that the expression for the expected profit of

firm i can be rewritten as follows:

$$\begin{aligned}
& p(\mathcal{P}_i^{(s,1)}) + (1-p)(\mathcal{P}_i^{(s,0)}) \\
&= p(\mathcal{R}_i^{(s,1)} - \mathcal{C}_i^{(s,1)}) + (1-p)(\mathcal{R}_i^{(s,0)} - \mathcal{C}_i^{(s,0)}) \\
&= p(\mathcal{R}_i^{(s,1)} - (f_i + \delta_i(1-s_i) + c_i s_i)q_i^{(s,1)}) + (1-p)(\mathcal{R}_i^{(s,0)} - f_i q_i^{(s,0)} - c_i s_i q_i^{(s,1)}) \\
&= p(\mathcal{R}_i^{(s,1)} - (f_i + \delta_i(1-s_i) + c_i s_i + \frac{1-p}{p} c_i s_i)q_i^{(s,1)}) + (1-p)(\mathcal{R}_i^{(s,0)} - f_i q_i^{(s,0)}) \\
&= p(\mathcal{R}_i^{(s,1)} - (f_i + \delta_i(1-s_i) + \frac{c_i}{p} s_i)q_i^{(s,1)}) + (1-p)(\mathcal{R}_i^{(s,0)} - f_i q_i^{(s,0)})
\end{aligned}$$

Thus, when a disaster happens (i.e., $d = 1$, which happens with probability p), the revenue received by firm i is $\mathcal{R}_i^{(s,1)}$, and the total cost incurred by firm i is $\mathcal{C}_i^{(s,1)} = (f_i + \delta_i(1-s_i) + \frac{c_i}{p} s_i)q_i^{(s,1)}$. Likewise, when a disaster does not happen (i.e., $d = 0$, which happens with probability $(1-p)$), the revenue is $\mathcal{R}_i^{(s,0)}$, and the total cost is $\mathcal{C}_i^{(s,0)} = f_i q_i^{(s,0)}$. Therefore, we can write the total cost incurred by firm i as a function of the state d as follows:

$$\mathcal{C}_i^{(s,d)} = d \left((f_i + \delta_i(1-s_i) + \frac{c_i}{p} s_i)q_i^{(s,d)} \right) + (1-d) \left(f_i q_i^{(s,d)} \right)$$

Rearranging the above expression using the fact that $f'_i = f_i + \delta_i$, we get:

$$\begin{aligned}
\mathcal{C}_i^{(s,d)} &= d \left((f_i + \delta_i(1-s_i) + \frac{c_i}{p} s_i)q_i^{(s,d)} \right) + (1-d) \left(f_i q_i^{(s,d)} \right) \\
&= \cancel{f_i d q_i^{(s,d)}} + \delta_i d (1-s_i) q_i^{(s,d)} + \frac{c_i}{p} s_i d q_i^{(s,d)} + f_i q_i^{(s,d)} - \cancel{f_i d q_i^{(s,d)}} \\
&= (f'_i - f_i) d (1-s_i) q_i^{(s,d)} + \frac{c_i}{p} s_i d q_i^{(s,d)} + f_i q_i^{(s,d)} \\
&= f'_i d (1-s_i) q_i^{(s,d)} - f_i d (1-s_i) q_i^{(s,d)} + \frac{c_i}{p} s_i d q_i^{(s,d)} + f_i q_i^{(s,d)} \\
&= f'_i d (1-s_i) q_i^{(s,d)} + \frac{c_i}{p} s_i d q_i^{(s,d)} + f_i (1-d(1-s_i)) q_i^{(s,d)}
\end{aligned}$$

Thus, we can interpret the total cost of firm i in the case of a variable cost of preparedness as follows:

$$\begin{aligned}
C_i^{(s,d)} = & \underbrace{f_i(1-d(1-s_i))q_i^{(s,d)}}_{\text{Normal production cost}} + \underbrace{f'_i d(1-s_i)q_i^{(s,d)}}_{\text{Disaster production cost}} \\
& + \underbrace{\frac{c_i}{p}s_i d q_i^{(s,d)}}_{\text{Preparedness cost}}
\end{aligned} \tag{26}$$

The realized variable cost of firm i in the case of a variable cost of preparedness is given by

$$RVC_i^{(s_i,d)} = f_i + \delta_i(1-s_i)d + \frac{c_i}{p}s_i d \tag{27}$$

In other words, if the disaster does not materialize ($d = 0$), the RVC_i will equal f_i ; otherwise, the RVC_i will be f_i plus either δ_i if firm i has not invested in preparedness ($s_i = 0$), or $\frac{c_i}{p}$ if firm i has invested in preparedness ($s_i = 1$). Therefore, we can rewrite the total cost of firm i in terms of the realized variable cost as follows:

$$C_i^{(s,d)} = \underbrace{RVC_i^{(s_i,d)} q_i^{(s,d)}}_{VC_i} \tag{28}$$

Here, the sum of the incurred increases in variable production cost of all firms other than firm i , due to some number of those firms not investing in preparedness, $\Delta_{\sim i}$, is equal to $\sum_{j=1, j \neq i}^n (1-s_j)\delta_j + \frac{c_j}{p}s_j$. When all firms invest in preparedness, we will have $\Delta_{\sim i} = \sum_{j=1, j \neq i}^n \frac{c_j}{p}$. Also, if all firms have the same variable cost of production, then the sum of the differences between the production

costs of firm i and the other firms, Φ_i , will be equal to zero.

We can now write down the expected profit of firms with and without investment in preparedness based on Equations (18) and (19). If $\rho(i)^{(s,0)} > 0$ and $\rho(i)^{(s,1)} > 0$ (in other words, if firm i can participate in the market in both disaster and non-disaster conditions), then we will have

$$\begin{aligned}
E[\mathcal{P}_i^s] &= p \left(\frac{1}{b(l+1)^2} (a - (l+1)RVC_i^{(s_i,1)} + TRVC^{(s,1)})^2 - FC_i^{(s_i,1)} \right) \\
&\quad + (1-p) \left(\frac{1}{b(n+1)^2} (a - (n+1)RVC_i^{(s_i,0)} + TRVC^{(s,0)})^2 \right) \\
&= \frac{p}{b(l+1)^2} \left(a - (l+1)(f_i + \delta_i - \phi_i s_i) + \sum_{\substack{k=1 \\ k \neq i}}^l f_k + \delta_k - \phi_k s_k \right)^2 \\
&\quad + \frac{1-p}{b(n+1)^2} \left(a - (n+1)f_i + \sum_{k=1}^n f_k \right)^2
\end{aligned} \tag{29}$$

In the previous section, we defined ϕ_i to be $\delta_i - \frac{c_i}{p}$ for the case with a variable cost of preparedness. Therefore, if $\phi_i > 0$ in Equation (29), then $E[\mathcal{P}_i^s]_{s_i=1} > E[\mathcal{P}_i^s]_{s_i=0}$. Similarly, if $\phi_i < 0$, then $E[\mathcal{P}_i^s]_{s_i=1} < E[\mathcal{P}_i^s]_{s_i=0}$. In other words, with a variable cost of preparedness, the investment decision of a firm i depends entirely on the relationship between its expected increase in variable cost of production, $p\delta_i$ and its variable cost of preparedness, c_i . If the expected increase in the variable cost of production due to a disaster is greater than the variable cost of preparedness, then firm i will prefer to invest in preparedness, and not otherwise. (When $p\delta_i = c_i$, firm i is indifferent between investing in preparedness and not investing, since both actions will result in the same expected profit.) Note from the above that with a variable cost of preparedness, the investment decision of any one firm is independent of the

investment decisions of other firms.

If $\rho(i)^{(s,0)} > 0$ and $\rho(i)^{(s,1)} \leq 0$ (in other words, if firm i can participate in the market in non-disaster conditions but not in disaster conditions), then its expected profit can be written as follows:

$$\begin{aligned}
E[\mathcal{P}_i^s] &= (1-p) \left(\frac{1}{b(n+1)^2} \left(a - (n+1)RVC_i^{(s_i,0)} + TRVC^{(s,0)} \right)^2 \right) \\
&= (1-p) \left(\frac{1}{b(n+1)^2} \left(a - (n+1)f_i + \sum_{k=1}^n f_k \right)^2 \right) - \varphi_i s_i \quad (30) \\
&= \frac{1-p}{b(n+1)^2} \left(a - (n+1)f_i + \sum_{k=1}^n f_k \right)^2
\end{aligned}$$

Note that here, the expected profit does not depend on the preparedness decision of firm i , since even if it chose to invest in preparedness, its post-disaster production quantity would be zero, and therefore its total cost of preparedness would also be zero. Therefore, we can assume without loss of generality that such a firm would not invest in preparedness.

We can also look at the investment behavior of a monopolist. We can show that for a monopolist with variable cost of preparedness, its optimal production quantity is equal to $\frac{1}{2b} \left(a - f - \delta(1-s)d - \frac{c}{p}sd \right)$. Moreover, comparing the expected profit for the monopolist, with and without investment in preparedness, results in $\left(\frac{c}{p} - \delta \right) \left(2a - 2f - \delta - \frac{c}{p} \right) < 0$. In other words, since $a - f - \delta - \frac{c}{p} > 0$, the ultimate decision of the monopolist depends on the comparison between $\frac{c}{p}$ and δ .

To summarize, firms that cannot cost-effectively produce in post-disaster conditions (due to their large variable cost of production) would never make a non-zero investment in preparedness, while firms that can produce in post-disaster conditions would make their investment decision based only on their

variable preparedness costs, not on the investment decisions of other firms.

To illustrate these behaviors, consider a market consisting of three firms where the components of each firm's profit function is given in Table 3, the inverse demand function is given by $500 - 0.1Q^{(s,d)}$, and the probability of the disaster is 0.05.

Table 3: Parameter Values for Three Firms Having Variable Cost of Preparedness

Parameter	Firm 1	Firm 2	Firm 3
f_i	90	90	220
δ_i	60	60	60
c_i	2	2	10

If disaster does not occur, all three firms can participate in the market. We can show this by computing $\rho(i)$ for each firm. In other words, verifying that the $\rho(i)$ for all three firms are greater than or equal to zero. Table 4 summarizes the calculations.

Table 4: Results of Participation Using $\rho(i)$ Value

i	RVC_i	\mathcal{I}_i	$ \mathcal{I}_i $	$\rho(i)$
1	90	{1, 2}	2	$410 \geq 0$
2	90	{1, 2}	2	$410 \geq 0$
3	220	{1, 2, 3}	3	$20 \geq 0$

In fact, the optimal non-disaster quantity of each firm is $q_1^{(s,0)} = 1350$; $q_2^{(s,0)} = 1350$; $q_3^{(s,0)} = 50$.

However, when disaster occurs, $\rho(3)$ is always negative regardless of the investment decisions of the other two firms. Therefore, we can conclude that firm 3 cannot cost-effectively produce in the post-disaster market, and consequently never invests in preparedness.

Now, let's consider the investment behavior of firms 1 and 2, given that firm 3 can participate only in the non-disaster market (i.e., firm 3 will produce the optimal non-disaster quantity in Equation (14), which translates into an expected profit of 237.5). The payoff of firm 1 can be written using Equation (29) as follows:

$$\begin{aligned}
E[\mathcal{P}_1^s] &= \frac{0.05}{0.1(2+1)^2} (500 - (2+1)(90 + 60 - \phi_1 s_1) \\
&\quad + (90 + 60 - \phi_1 s_1 + 90 + 60 - \phi_2 s_2) + \frac{2}{0.05} s_2)^2 \\
&\quad + \frac{1-0.05}{0.1(3+1)^2} (500 - (3+1)90 + (90 + 90 + 220))^2 \\
&= 173138 + \frac{1}{18} (350 + 2\phi_1 s_1 - \phi_2 s_2)^2
\end{aligned} \tag{31}$$

Now, we can compare the expected profit of firm 1 with and without preparedness investment and compare the results to see when investing results in a higher expected profit. The expected profit of firm 1 if investing in preparedness is given by $173138 + \frac{1}{18}(350 + 2\phi_1 - \phi_2 s_2)^2$. Similarly, its expected profit without investing in preparedness is given by $173138 + \frac{1}{18}(350 - \phi_2 s_2)^2$. Therefore, investing in preparedness results in a higher expected profit for firm 1 if $173138 + \frac{1}{18}(350 + 2\phi_1 - \phi_2 s_2)^2 > 173138 + \frac{1}{18}(350 - \phi_2 s_2)^2$. Simplifying this expression will result in $\phi_1 \left(\frac{700}{9} + \frac{2}{9}\phi_1 - \frac{2}{9}s_2\phi_2 \right) > 0$. In this example, it is straightforward to see that the expression in the parentheses is always positive, since the two firms are homogeneous. However, as we have showed before, even in cases where firms are not homogeneous, the resulting expression is still positive. Therefore, whether investment will result in a higher expected profit for firm 1 depends solely on the sign of ϕ_1 . Equivalently, we can show the same

result for firm 2.

We can represent this game as follows:

Table 5: Strategic Representation of Investment Game

		Firm 2	
		Invest	Not Invest
Firm 1	Invest	$\begin{cases} 180,743 \\ 180,743 \end{cases}$	$\begin{cases} 181,588 \\ 179,188 \end{cases}$
	Not Invest	$\begin{cases} 179,188 \\ 181,588 \end{cases}$	$\begin{cases} 179,943 \\ 179,943 \end{cases}$

The Nash equilibrium of this game is both firms investing in preparedness. Of course, for other parameter values, the equilibrium could be that neither firm invests. However, since a firm's decision does not depend on the behavior of other firms, there will never be an equilibrium in which only one of two homogeneous firms invests. Due to the simple nature of these results and the lack of interesting dependencies between the investment decisions of different firms, no further analysis of this case is required.

6 Model with Fixed Cost of Preparedness

In this chapter, we assume a fixed cost of preparedness. In particular, we assume that the preparedness cost does not depend on the actual production level of the firm. This might be the case, for example, when a firm agrees to pay an upfront cost to insure itself from future increases in the variable cost of production (say, by entering into a contract with its supplier).

With a fixed cost of preparedness, the total cost can be written as

$$\begin{aligned}
 C_i^{(s,d)} = & \underbrace{f_i [1 - d(1 - s_i)] q_i^{(s,d)}}_{\text{Normal production cost}} + \underbrace{f'_i d(1 - s_i) q_i^{(s,d)}}_{\text{Disaster production cost}} \\
 & + \underbrace{s_i C_i}_{\text{Preparedness cost}}
 \end{aligned} \tag{32}$$

Here, the realized variable cost $RVC_i^{(s_i,d)}$ is given by

$$RVC_i^{(s_i,d)} = f_i + \delta_i(1 - s_i)d \tag{33}$$

In other words, if the disaster does not materialize ($d = 0$) or firm i has invested in preparedness ($s_i = 1$), then RVC_i will equal f_i ; otherwise, RVC_i will equal $f'_i = f_i + \delta_i$. Therefore, we can rewrite the total cost of firm i in terms of the realized variable cost, as follows:

$$C_i^{(s,d)} = \underbrace{RVC_i^{(s_i,d)} q_i^{(s,d)}}_{VC_i} + \underbrace{s_i C_i}_{FC_i} \tag{34}$$

Before we proceed with our analysis, let's consider the behavior of a monopolist. We assume that the monopolist faces a possible disaster and the

preparedness cost is a fixed cost. The profit function for this monopolist $\mathcal{P}_M^{(s,d)}$ can be written as $(a - bq_M^{(s,d)})q_M^{(s,d)} - (f + \delta(1-s)d)q_M^{(s,d)} - sC$. We can compute its optimal production quantity by equating the first order condition for the problem of maximizing its profit with respect to quantity, as follows:

$$\frac{\partial}{\partial q_M^{(s,d)}} \mathcal{P}_M^{(s,d)} = 0 \Rightarrow a - 2bq_M^{(s,d)} - f - \delta(1-s)d = 0 \Rightarrow q_M^{*(s,d)} = \frac{a - f - \delta(1-s)d}{2b}$$

Now, we can compute its profit under disaster ($d = 1$) and non-disaster ($d = 0$) conditions and compute its expected profit as follows:

$$\begin{aligned} E[\mathcal{P}_M^s] &= p\mathcal{P}_M^{(s,1)} + (1-p)\mathcal{P}_M^{(s,0)} \\ &= p\left(\left(a - b\left(\frac{a-f-\delta(1-s)}{2b}\right) - f - \delta(1-s)\right)\frac{a-f-\delta(1-s)}{2b} - sC\right) \\ &\quad + (1-p)\left(\left(a - b\left(\frac{a-f}{2b}\right) - f\right)\frac{a-f}{2b} - sC\right) \\ &= \frac{p(a-f-(1-s)\delta)^2 + (1-p)(a-f)^2}{4b} - sC \end{aligned} \quad (35)$$

Comparing the expected profit of a monopolist with and without preparedness investment we can find the threshold for the preparedness decision of a monopolist that justifies investing in preparedness as follows:

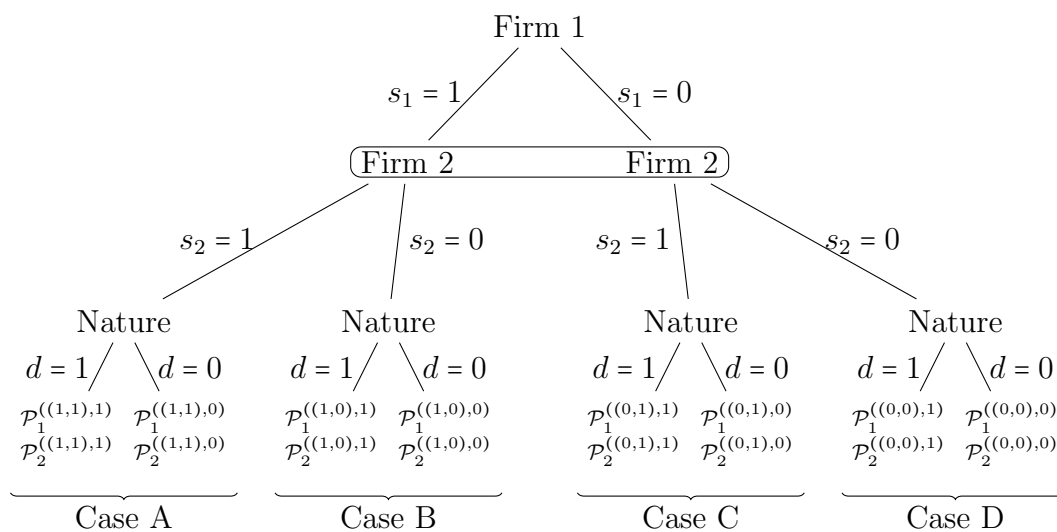
$$\begin{aligned} E[\mathcal{P}_M^1] &> E[\mathcal{P}_M^0] \\ \Rightarrow \frac{p(a-f)^2 + (1-p)(a-f)^2}{4b} - C &> \frac{p(a-f-\delta)^2 + (1-p)(a-f)^2}{4b} \\ \Rightarrow \frac{(a-f)^2}{4b} - C &> \frac{p(a-f-\delta)^2 + (1-p)(a-f)^2}{4b} \\ \Rightarrow C &< \frac{p\delta(2a-2f-\delta)}{4b} \end{aligned} \quad (36)$$

Therefore, for values of the fixed of preparedness that satisfies the inequality above, the monopolist will choose to invest in preparedness.

6.1 Duopoly

We first consider a market consisting of $n = 2$ firms. Although moves are simultaneous, we show the game in extensive form in Figure 2.

Figure 2: Two-Person Game in Extensive Form



There are four cases shown in Figure 2, corresponding to the four possible preparedness decisions of the two firms. In each of the four cases, firms 1 and 2 will find their expected profit as a weighted sum of the profits with and without disaster. Then, they find their best response and choose their preparedness decision accordingly. Given the parameters of the model, we can find Nash equilibria for this game. The decision of each firm can be modeled as how a function of the fixed cost of preparedness is, and Nash equilibrium(s) can be found for different values of the preparedness cost.

Since both firms are assumed to participate in the non-disaster market

(i.e., $q_i^{*(s,0)} > 0$ for $i = 1, 2$), their optimal non-disaster production quantities can be determined from Equation (14) as follows: $q_1^{*(s,0)} = \frac{1}{3b}(a - 2f_1 + f_2)$; $q_2^{*(s,0)} = \frac{1}{3b}(a - 2f_2 + f_1)$. Moreover, we can use Equations (16) and (17) to write down the expected for each of the four cases.

In Case A, where both firms invest in preparedness, $s = (1, 1)$, their expected profits are given by

$$\begin{aligned}
E[\mathcal{P}_1^{(1,1)}] &= p(\mathcal{P}_1^{((1,1),1)}) + (1-p)(\mathcal{P}_1^{((1,1),0)}) \\
&= p\left(\frac{1}{b(2+1)^2}(a - (2+1)f_1 + (f_1 + f_2))^2 - C_1\right) \\
&\quad + (1-p)\left(\frac{1}{b(2+1)^2}(a - (2+1)f_1 + (f_1 + f_2))^2 - C_1\right) \quad (37) \\
&= \frac{1}{9b}(a - 2f_1 + f_2)^2 - C_1 \\
&= b\overset{\circ}{E}_1^2 - C_1
\end{aligned}$$

Similarly, $E[\mathcal{P}_2^{(1,1)}] = b\overset{\circ}{E}_2^2 - C_2$, where $\overset{\circ}{E}_1 = \frac{1}{3b}(a - 2f_1 + f_2)$ and $\overset{\circ}{E}_2 = \frac{1}{3b}(a - 2f_2 + f_1)$. In a sense, $\overset{\circ}{E}_i$ can be viewed as the initial endowment of a firm.

In Case B, where only firm 1 invests in preparedness, $s = (1, 0)$, then firm 1 can participate in the post-disaster market (since its RVC would be the same as in the non-disaster case), but firm 2 can participate in the market only if $a - 2(f_2 + \delta_2) + f_1 > 0$. If firm 2 can participate in the post-disaster market,

then the expected profits for firm one is given by

$$\begin{aligned}
E[\mathcal{P}_1^{(1,0)}] &= p(\mathcal{P}_1^{((1,0),1)}) + (1-p)(\mathcal{P}_1^{((1,0),0)}) \\
&= p\left(\frac{1}{b(2+1)^2}(a - (2+1)f_1 + (f_1 + f_2 + \delta_2))^2 - C_1\right) \\
&\quad + (1-p)\left(\frac{1}{b(2+1)^2}(a - (2+1)f_1 + (f_1 + f_2))^2 - C_1\right) \\
&= p\left(\frac{1}{9b}(a - 2f_1 + f_2 + \delta_2)^2 - C_1\right) \\
&\quad + (1-p)\left(\frac{1}{9b}(a - 2f_1 + f_2)^2 - C_1\right) \\
&= \frac{1}{9b}(a - 2f_1 + f_2)^2 + \frac{2p\delta_2}{9b}(a - 2f_1 + f_2) + \frac{p\delta_2^2}{9b} - C_1 \\
&= b\overset{\circ}{E}_1^2 + \frac{2p\delta_2}{3}\overset{\circ}{E}_1 + \frac{p\delta_2^2}{9b} - C_1
\end{aligned} \tag{38}$$

Also, the expected profit for firm two is given by

$$\begin{aligned}
E[\mathcal{P}_2^{(1,0)}] &= p(\mathcal{P}_2^{((1,0),1)}) + (1-p)(\mathcal{P}_2^{((1,0),0)}) \\
&= p\left(\frac{1}{b(2+1)^2}(a - (2+1)(f_2 + \delta_2) + (f_1 + f_2 + \delta_2))^2\right) \\
&\quad + (1-p)\left(\frac{1}{b(2+1)^2}(a - (2+1)f_2 + (f_1 + f_2))^2\right) \\
&= p\left(\frac{1}{9b}(a - 2f_2 + f_1 - 2\delta_2)^2\right) \\
&\quad + (1-p)\left(\frac{1}{9b}(a - 2f_2 + f_1)^2\right) \\
&= \frac{1}{9b}(a - 2f_2 + f_1)^2 - \frac{4p\delta_2}{9b}(a - 2f_2 + f_1) + \frac{4p\delta_2^2}{9b} \\
&= b\overset{\circ}{E}_2^2 - \frac{4p\delta_2}{3}\overset{\circ}{E}_2 + \frac{4p\delta_2^2}{9b}
\end{aligned} \tag{39}$$

If firm 2 cannot participate in the post-disaster market, because $a - 2(f_2 + \delta_2) +$

$f_1 \leq 0$, then the expected profit for firm 1 is given by

$$\begin{aligned}
E[\mathcal{P}_1^{(1,0)}] &= p(\mathcal{P}_1^{((1,0),1)}) + (1-p)(\mathcal{P}_1^{((1,0),0)}) \\
&= p\left(\frac{1}{b(1+1)^2}(a - (1+1)f_1 + (f_1))^2 - C_1\right) \\
&\quad + (1-p)\left(\frac{1}{b(2+1)^2}(a - (2+1)f_1 + (f_1 + f_2))^2 - C_1\right) \\
&= p\left(\frac{1}{4b}(a - f_1)^2 - C_1\right) \\
&\quad + (1-p)\left(\frac{1}{9b}(a - 2f_1 + f_2)^2 - C_1\right) \\
&= pb\left(\frac{a - f_1}{2b}\right)^2 + (1-p)b\mathring{E}_1^2 - C_1
\end{aligned} \tag{40}$$

Also, the expected profit for firm 2 is given by

$$\begin{aligned}
E[\mathcal{P}_2^{(1,0)}] &= p(\mathcal{P}_2^{((1,0),1)}) + (1-p)(\mathcal{P}_2^{((1,0),0)}) \\
&= p(0) + (1-p)\left(\frac{1}{b(2+1)^2}(a - (2+1)f_2 + (f_1 + f_2))^2\right) \\
&= (1-p)\left(\frac{1}{9b}(a - 2f_2 + f_1)^2\right) \\
&= (1-p)b\mathring{E}_2^2
\end{aligned} \tag{41}$$

Similarly, in Case C, where only firm 2 invests in preparedness, but firm 1 still produces in the post-disaster market because $a - 2(f_1 + \delta_1) + f_2 > 0$, then similar to Equations (38) and (39) we have

$$\begin{aligned}
E[\mathcal{P}_1^{(0,1)}] &= b\mathring{E}_1^2 - \frac{4p\delta_1}{3}\mathring{E}_1 + \frac{4p\delta_1^2}{9b} \\
E[\mathcal{P}_2^{(0,1)}] &= b\mathring{E}_2^2 + \frac{2p\delta_1}{3}\mathring{E}_2 + \frac{p\delta_1^2}{9b} - C_2
\end{aligned} \tag{42}$$

If firm 1 does not participate in the post-disaster market, then similar to

Equations (40) and (41) we have

$$\begin{aligned} E[\mathcal{P}_1^{(0,1)}] &= (1-p)b\mathring{E}_1^2 \\ E[\mathcal{P}_2^{(0,1)}] &= pb\left(\frac{a-f_2}{2b}\right)^2 + (1-p)b\mathring{E}_2^2 - C_2 \end{aligned} \quad (43)$$

Finally, in Case D, where neither firm invests in preparedness, $s = (0, 0)$, then there are four possible cases. If both firms participate in the post-disaster market, then the expected profit for firm 1 would be

$$\begin{aligned} E[\mathcal{P}_1^{(0,0)}] &= p(\mathcal{P}_1^{((0,0),1)}) + (1-p)(\mathcal{P}_1^{((0,0),0)}) \\ &= p\left(\frac{1}{b(2+1)^2}(a - (2+1)(f_1 + \delta_1) + (f_1 + \delta_1 + f_2 + \delta_2))^2\right) \\ &\quad + (1-p)\left(\frac{1}{b(2+1)^2}(a - (2+1)f_1 + (f_1 + f_2))^2\right) \\ &= p\left(\frac{1}{9b}(a - 2(f_1 + \delta_1) + (f_2 + \delta_2))^2\right) \\ &\quad + (1-p)\left(\frac{1}{9b}(a - 2f_1 + f_2)^2\right) \\ &= b\mathring{E}_1^2 + \frac{p(-2\delta_1 + \delta_2)}{3}\left(2\mathring{E}_1 + \frac{-2\delta_1 + \delta_2}{3b}\right) \end{aligned} \quad (44)$$

Similarly, for firm 2 would be

$$E[\mathcal{P}_2^{(0,0)}] = b\mathring{E}_2^2 + \frac{p(-2\delta_2 + \delta_1)}{3}\left(2\mathring{E}_2 + \frac{-2\delta_2 + \delta_1}{3b}\right) \quad (45)$$

If only firm 1 participates in the post-disaster market, then for firm 1 we have

$$\begin{aligned}
E[\mathcal{P}_1^{(0,0)}] &= p(\mathcal{P}_1^{((0,0),1)}) + (1-p)(\mathcal{P}_1^{((0,0),0)}) \\
&= p\left(\frac{1}{b(1+1)^2}(a - (1+1)(f_1 + \delta_1) + (f_1 + \delta_1))^2\right) \\
&\quad + (1-p)\left(\frac{1}{b(2+1)^2}(a - (2+1)f_1 + (f_1 + f_2))^2\right) \\
&= p\left(\frac{1}{4b}(a - f_1 - \delta_1)^2\right) \\
&\quad + (1-p)\left(\frac{1}{9b}(a - 2f_1 + f_2)^2\right) \\
&= pb\left(\frac{a - f_1 - \delta_1}{2b}\right)^2 + (1-p)b\mathring{E}_1^2
\end{aligned} \tag{46}$$

Similarly, for firm 2 we have

$$\begin{aligned}
E[\mathcal{P}_2^{(0,0)}] &= p(\mathcal{P}_2^{((0,0),1)}) + (1-p)(\mathcal{P}_2^{((0,0),0)}) \\
&= p(0) + (1-p)\left(\frac{1}{b(2+1)^2}(a - (2+1)f_2 + (f_1 + f_2))^2\right) \\
&= (1-p)\left(\frac{1}{9b}(a - 2f_2 + f_1)^2\right) \\
&= (1-p)b\mathring{E}_2^2
\end{aligned} \tag{47}$$

Conversely, if only firm 2 participates in the market, then we have

$$\begin{aligned}
E[\mathcal{P}_1^{(0,0)}] &= (1-p)b\mathring{E}_1^2 \\
E[\mathcal{P}_2^{(0,0)}] &= pb\left(\frac{a - f_2 - \delta_2}{2b}\right)^2 + (1-p)b\mathring{E}_2^2
\end{aligned} \tag{48}$$

Finally, if neither firm participates in the post-disaster market, their expected profits are as follows (since no firm will realize a profit in the case of a disaster):

$$\begin{aligned}
E[\mathcal{P}_1^{(0,0)}] &= (1-p)b\mathring{E}_1^2 \\
E[\mathcal{P}_2^{(0,0)}] &= (1-p)b\mathring{E}_2^2
\end{aligned} \tag{49}$$

Let's first consider the case where $a - 2(f_1 + \delta_1) + f_2 > 0$ and $a - 2(f_2 + \delta_2) + f_1 > 0$. We show the resulting game in Table 6.

Table 6: Strategic Representation of Investment Game For Two Participating Firms

		Firm 2	
		Invest	Not Invest
Firm 1	Invest	$\begin{cases} b\dot{E}_1^2 - C_1 \\ b\dot{E}_2^2 - C_2 \end{cases}$	$\begin{cases} b\dot{E}_1^2 + \frac{2p\delta_2}{3}\dot{E}_1 + \frac{p\delta_2^2}{9b} - C_1 \\ b\dot{E}_2^2 - \frac{4p\delta_2}{3}\dot{E}_2 + \frac{4p\delta_2^2}{9b} \end{cases}$
	Not Invest	$\begin{cases} b\dot{E}_1^2 - \frac{4p\delta_1}{3}\dot{E}_1 + \frac{4p\delta_1^2}{9b} \\ b\dot{E}_2^2 + \frac{2p\delta_1}{3}\dot{E}_2 + \frac{p\delta_1^2}{9b} - C_2 \end{cases}$	$\begin{cases} b\dot{E}_1^2 + \frac{p(-2\delta_1 + \delta_2)}{3}(2\dot{E}_1 + \frac{-2\delta_1 + \delta_2}{3b}) \\ b\dot{E}_2^2 + \frac{p(-2\delta_2 + \delta_1)}{3}(2\dot{E}_2 + \frac{-2\delta_2 + \delta_1}{3b}) \end{cases}$

We now explore the conditions that result in a pure Nash equilibrium. If $C_1 < \frac{4p\delta_1}{3}\dot{E}_1 - \frac{4p\delta_1^2}{9b}$ and $C_2 < \frac{4p\delta_2}{3}\dot{E}_2 - \frac{4p\delta_2^2}{9b}$, then both firms investing in preparedness is the unique Nash equilibrium. We refer to these upper bounds on C_i as \underline{C}_i . In other words, \underline{C}_i is the largest value for C_i where firm i does not invest in preparedness regardless of the investment decision of the other firm.

If both preparedness costs are high enough (in other words, if $C_1 > \frac{4p\delta_1}{3}\dot{E}_1 - \frac{4p\delta_1^2}{9b} + \frac{4p\delta_1\delta_2}{9b}$ and $C_2 > \frac{4p\delta_2}{3}\dot{E}_2 - \frac{4p\delta_2^2}{9b} + \frac{4p\delta_1\delta_2}{9b}$), then both firms not investing in preparedness is the unique Nash equilibrium. We refer to these lower bounds on C_i as \overline{C}_i , indicating the lowest value for C_i where firm i invests in preparedness regardless of the investment decision of the other firm.

Only firm 1 investing would be a Nash equilibrium if $C_1 < \frac{4p\delta_1}{3}\dot{E}_1 - \frac{4p\delta_1^2}{9b} + \frac{4p\delta_1\delta_2}{9b}$ and $C_2 > \frac{4p\delta_2}{3}\dot{E}_2 - \frac{4p\delta_2^2}{9b}$. Similarly, only firm 2 investing would be a Nash

equilibrium if $C_1 > \frac{4p\delta_1}{3} \overset{\circ}{E}_1 - \frac{4p\delta_1^2}{9b}$ and $C_2 < \frac{4p\delta_2}{3} \overset{\circ}{E}_2 - \frac{4p\delta_2^2}{9b} + \frac{4p\delta_1\delta_2}{9b}$. Therefore, for $\frac{4p\delta_1}{3} \overset{\circ}{E}_1 - \frac{4p\delta_1^2}{9b} + \frac{4p\delta_1\delta_2}{9b} < C_1 < \frac{4p\delta_1}{3} \overset{\circ}{E}_1 - \frac{4p\delta_1^2}{9b}$ and $\frac{4p\delta_2}{3} \overset{\circ}{E}_2 - \frac{4p\delta_2^2}{9b} + \frac{4p\delta_1\delta_2}{9b} < C_2 < \frac{4p\delta_2}{3} \overset{\circ}{E}_2 - \frac{4p\delta_2^2}{9b}$, either firm investing in preparedness would be a pure Nash equilibrium. Moreover, since both pure strategies of firms are best responses, there also exist mixed-strategy Nash equilibria. We can compute the mixing probability of each firm based on the fact that in such a case, firm i invests in preparedness with probability σ_i in order to make the other firm indifferent between investing and not investing: So, for firm 1, we have

$$\begin{aligned}
& \sigma_1(b\overset{\circ}{E}_2^2 - C_2) + (1 - \sigma_1)(b\overset{\circ}{E}_2^2 + \frac{2p\delta_1}{3} \overset{\circ}{E}_2 + \frac{p\delta_1^2}{9b} - C_2) \\
&= \sigma_1(b\overset{\circ}{E}_2^2 - \frac{4p\delta_2}{3} \overset{\circ}{E}_2 + \frac{4p\delta_2^2}{9b}) + (1 - \sigma_1)(b\overset{\circ}{E}_2^2 + \frac{p(-2\delta_2 + \delta_1)}{3}(2\overset{\circ}{E}_2 + \frac{-2\delta_2 + \delta_1}{3b})) \\
\Rightarrow \sigma_1 &= \frac{4p\delta_2(3b\overset{\circ}{E}_2 + \delta_1 - \delta_2) - 9bC_2}{4p\delta_1\delta_2}
\end{aligned} \tag{50}$$

Similarly, the mixing probability σ_2 of firm 2 can be calculated as

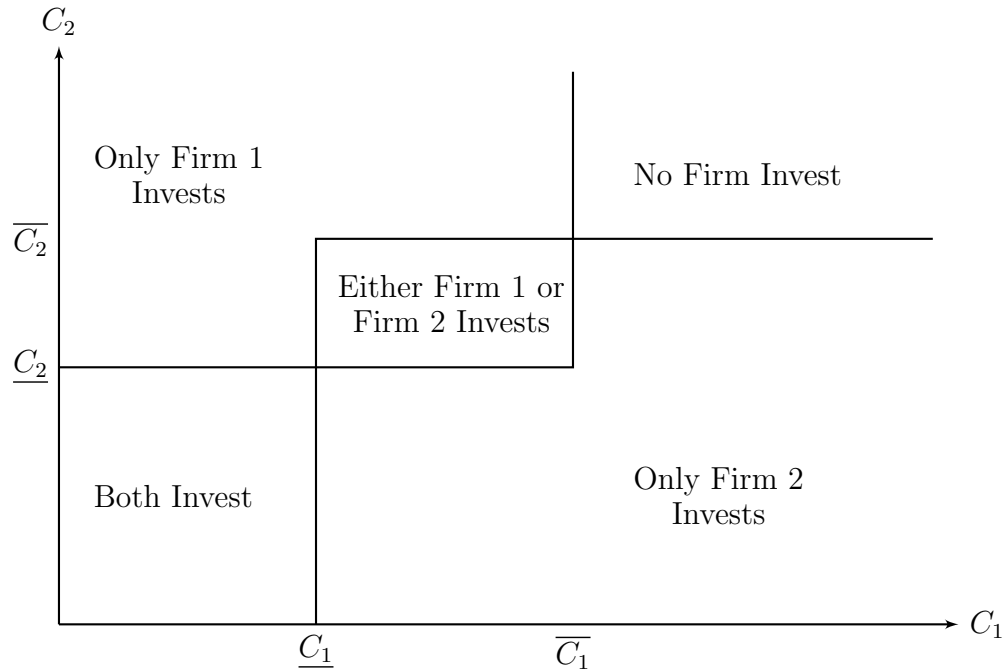
$$\sigma_2 = \frac{4p\delta_1(3b\overset{\circ}{E}_1 + \delta_2 - \delta_1) - 9bC_1}{4p\delta_1\delta_2} \tag{51}$$

Both of these mixing probabilities satisfy the condition that $0 < \sigma_i < 1$, since we have assumed that $\frac{4p\delta_i}{3} \overset{\circ}{E}_i - \frac{4p\delta_i^2}{9b} < C_i < \frac{4p\delta_i}{3} \overset{\circ}{E}_i - \frac{4p\delta_i^2}{9b} + \frac{4p\delta_i\delta_{-i}}{9b}$.

Figure 3 below shows the Nash equilibria as a function of whether the preparedness costs C_i are small ($C_i < \underline{C}_i$), medium ($\underline{C}_i < C_i < \overline{C}_i$), or large ($C_i > \overline{C}_i$) where $\underline{C}_i = \frac{4p\delta_i}{3} \overset{\circ}{E}_i - \frac{4p\delta_i^2}{9b}$ and $\overline{C}_i = \underline{C}_i + \frac{4p\delta_i}{9b} (\sum_{j=1, j \neq i}^2 \delta_j)$.

Here, the sum of the incurred increases in variable production cost of all firms other than firm i , due to other firm not investing in preparedness, $\Delta_{\sim i}$, is equal to $\delta_{\sim i}$. Also, if all firms have the same variable cost of production, then the

Figure 3: Nash Equilibria for Fixed Cost of Preparedness in a Duopoly



sum of the differences between the production costs of firm i and the other firms, Φ_i , would be zero. These boundaries can be rewritten in terms of $\Delta_{\sim i}$ and Φ_i as follows:

$$\underline{C}_i = \frac{4p\delta_i}{9b}(a - f_i + \Phi_i - \delta_i) \quad \overline{C}_i = \underline{C}_i + \frac{4p\delta_i}{9b}(\Delta_{\sim i}|_{s_{\sim i} = 0}) \quad (52)$$

As above, we see that if the fixed cost of preparedness for both firms is between \underline{C}_i and \overline{C}_i , then each firm's decision will depend on the other firm's preparedness decision.

We can see that if one firm's fixed cost of preparedness is small, then no matter what the other firm does, it will invest in preparedness. However, as

its fixed cost of preparedness gets larger, investing in preparedness becomes unprofitable unless the other firm chooses not to invest. Finally, when the fixed cost of preparedness for a firm is sufficiently large, then that firm cannot afford to invest in preparedness under any market condition.

Now, let's consider the case where $a - 2(f_1 + \delta_1) + f_2 < 0$ and $a - 2(f_2 + \delta_2) + f_1 < 0$, so that non-investing firms cannot participate in the post-disaster market. The strategic representation of this game is shown in Table 7.

Table 7: Strategic Representation of Investment Game when Non-Investing Firms Cannot Participate in the Case of a Disaster

		Firm 2	
		Invest	Not Invest
Firm 1	Invest	$\begin{cases} b\dot{E}_1^2 - C_1 \\ b\dot{E}_2^2 - C_2 \end{cases}$	$\begin{cases} pb\left(\frac{a-f_1}{2b}\right)^2 + (1-p)b\dot{E}_1^2 - C_1 \\ (1-p)b\dot{E}_2^2 \end{cases}$
	Not Invest	$\begin{cases} (1-p)b\dot{E}_1^2 \\ pb\left(\frac{a-f_2}{2b}\right)^2 + (1-p)b\dot{E}_2^2 - C_2 \end{cases}$	$\begin{cases} (1-p)b\dot{E}_1^2 \\ (1-p)b\dot{E}_2^2 \end{cases}$

As before, for a small enough fixed cost of preparedness, both firms will invest in preparedness, and for a large enough fixed cost of preparedness, neither of them will invest in preparedness. For intermediate values, either firm investing can be a Nash equilibrium. The thresholds for investing in preparedness in this case are as follows:

$$\begin{aligned} \underline{C}_i &= pb\dot{E}_i^2 \\ \overline{C}_i &= pb\left(\frac{a-f_i}{2b}\right)^2 \end{aligned} \tag{53}$$

Similarly, the mixing probability of firm i in the case of multiple Nash equilibria (i.e., when either firm investing is a Nash equilibrium) is as follows:

$$\sigma_i = \frac{pb\left(\frac{a-f_{\sim i}}{2b}\right)^2 - C_{\sim i}}{pb\left(\frac{a-f_{\sim i}}{2b}\right)^2 - pb\mathring{E}_{\sim i}^2} \quad (54)$$

As before, it is straightforward to show that $0 < \sigma_i < 1$.

Our next case is when $a - 2(f_1 + \delta_1) + f_2 > 0$ and $a - 2(f_2 + \delta_2) + f_1 < 0$, so that the only firm that can participate in a post-disaster market without investing in preparedness is firm 1. The strategic representation of this game is shown in Table 8.

Table 8: Strategic Representation of Investment Game when Firm 1 Can Participate in the Case of a Disaster Even without Investing

		Firm 2	
		Invest	Not Invest
Firm 1	Invest	$\begin{cases} b\mathring{E}_1^2 - C_1 \\ b\mathring{E}_2^2 - C_2 \end{cases}$	$\begin{cases} pb\left(\frac{a-f_1}{2b}\right)^2 + (1-p)b\mathring{E}_1^2 - C_1 \\ (1-p)b\mathring{E}_2^2 \end{cases}$
	Not Invest	$\begin{cases} b\mathring{E}_1^2 - \frac{4p\delta_1}{3}\mathring{E}_1 + \frac{4p\delta_1^2}{9b} \\ b\mathring{E}_2^2 + \frac{2p\delta_1}{3}\mathring{E}_2 + \frac{p\delta_1^2}{9b} - C_2 \end{cases}$	$\begin{cases} pb\left(\frac{a-f_1-\delta_1}{2b}\right)^2 + (1-p)b\mathring{E}_1^2 \\ (1-p)b\mathring{E}_2^2 \end{cases}$

The thresholds for investing in preparedness are as follows:

$$\begin{cases} \underline{C}_1 = \frac{4p\delta_1}{3}(\dot{E}_1 - \frac{\delta_1}{3b}) \\ \overline{C}_1 = \frac{p\delta_1}{4b}(2a - 2f_1 - \delta_1) \end{cases} \quad \begin{cases} \underline{C}_2 = pb\dot{E}_2^2 \\ \overline{C}_2 = pb\dot{E}_2^2 + \frac{2p\delta_1}{3}\dot{E}_2 + \frac{p\delta_1^2}{9b} \end{cases} \quad (55)$$

The corresponding mixing strategy for the case of multiple Nash equilibria is given by

$$\sigma_1 = \frac{p(3b\dot{E}_2 + \delta_2)^2 - 9bC_2}{p\delta_1(6b\dot{E}_2 + \delta_1)} \quad \sigma_2 = \frac{9p\delta_1(2a - 2f_1 + \delta_1) - 36bC_1}{p\delta_1(18a - 18f_1 - 48b\dot{E}_1 + 7\delta_1)} \quad (56)$$

The case where $a - 2(f_1 + \delta_1) + f_2 < 0$ and $a - 2(f_2 + \delta_2) + f_1 > 0$ is similar.

So far, we have shown that in a duopoly, when the preparedness cost is a fixed cost, each firm's preparedness decision is independent of the other firm's decision when the preparedness cost is either extremely small or extremely high; in the former case, a firm will invest in preparedness regardless of the investment decision of the other firm, and in the latter case, a firm will not invest in preparedness regardless of the investment decision of the other firm. However, for moderate preparedness costs, each firm's decision will depend on the other firm's preparedness decision.

Next, in Section 6.2 below we analyze the preparedness investment decisions of multiple homogeneous firms competing in a single market.

6.2 Multiple Homogeneous Firms

We now consider a market consisting of n homogeneous firms. Note that, as before, these n firms are assumed to be able to participate in the non-disaster

market. Therefore, based on Equation (14), we have $a - f_i > 0$ for each firm; moreover, since we are dealing with homogeneous firms, we can suppress the subscripts and simply write $a - f > 0$. Also, note that when a firm invests in preparedness, it will definitely be able to produce in the post-disaster market, since in that case $RVC_i^{(1,1)}$ would be equal to $RVC_i^{(1,0)}$.

Let m be the total number firms that have invested in preparedness. If $l = m$, where l refers to the total number of firms participating in the post-disaster market (with or without preparedness investment), then the expected profit of such firms, using Equation (18), will be as follows:

$$E[\mathcal{P}_i^s] = \frac{p}{b(m+1)^2}(a-f)^2 + \frac{1-p}{b(n+1)^2}(a-f)^2 - C \quad (57)$$

Some firms that have not invested in preparedness may also be able to produce in the post-disaster market (bringing the total number of firms that participate in the post-disaster market to $l > m$), but we need to determine how many non-investing firm can participate, if any. To answer this question, we will first compute $\rho(i)$ of each firm described in Section 4 to check whether the market is saturated by those firms that have invested in preparedness. In order to see whether a non-investing firm i can produce in the post-disaster market, we compute $\rho(i)^{(s,1)}$ and see if it is positive (indicating that such a firm can participate in the post-disaster market). Note that, since we are dealing with homogeneous firms in this section, the same criterion will apply for all non-investing firms since $\rho(i)^{(s,1)} = \rho(j)^{(s,1)} \forall i, j$. We can compute $\rho(i)^{(s,1)}$ using the fact that the total number of firms participating in the post-disaster market if a non-investing firm can participate is equal to the total number of

firms⁹ (i.e., $|\mathcal{I}_i^{(s,1)}| = n$).

$$\begin{aligned}
\rho(i)^{(s,1)} &= a - (|\mathcal{I}_i^{(s,1)}| + 1)RVC_i^{(s_i,1)} + \sum_{j \in \mathcal{I}_i^{(s,1)}} RVC_j^{(s_j,1)} \\
&= a - (n + 1)(f + \delta) + \sum_{j=1}^n RVC_j^{(s_j,1)} \\
&= a - (n + 1)(f + \delta) + \sum_{j=1}^m f + \sum_{j=m+1}^n (f + \delta) \\
&= a - (n + 1)(f + \delta) + mf + (n - m)(f + \delta) \\
&= a - f - (m + 1)\delta \\
&= a - f - (l + 1)\delta
\end{aligned} \tag{58}$$

Therefore, if $a - f - (l + 1)\delta > 0$, or equivalently, if $l \leq \frac{a - f - \delta}{\delta}$, then a firm that has not invested in preparedness can enter the market (at least if no other non-investing firms have entered the market). More specifically, an unprepared firm can participate in the post-disaster market if at most $\frac{a - f - \delta}{\delta}$ firms are participating. This is basically the decision faced by the first non-investing firm that seeks to enter the post-disaster market. However, once one such firm enters the market, the next non-investing firm faces a comparison between $l + 1$ and $\frac{a - f - \delta}{\delta}$, since now the number of participating firms has increased by one (with l investing firms and one non-investing firm participating in the post-disaster market). This process can be repeated until no other non-investing firm can participate in the post-disaster market, and the market is stable. Therefore, we have the number of participating firms in a pure strategy

⁹Note that firms that have invested in preparedness can participate, moreover, all the non-investing firms have the same *RVC*.

equilibrium as follows:

$$l^* = \begin{cases} n & \text{if } \frac{a-f-\delta}{\delta} \geq n \\ \lfloor \frac{a-f-\delta}{\delta} \rfloor & \text{otherwise} \end{cases} \quad (59)$$

Now, we can write down the expected profit for a non-investing firm based on its participation in the post-disaster market. If a non-investing firm does not participate in the market, then its expected profit would be simply:

$$E[\mathcal{P}_i^s] = \frac{1-p}{b(n+1)^2} (a-f)^2 \quad (60)$$

However, if a non-investing firm does participate in the post-disaster market, its expected profit, using Equation (18), would be given by:

$$\begin{aligned} E[\mathcal{P}_i^s] &= \frac{p}{b(l+1)^2} (a - (l+1)(f+\delta) + mf + (l-m)(f+\delta))^2 + \frac{1-p}{b(n+1)^2} (a-f)^2 \\ &= \frac{p}{b(l+1)^2} (a-f - (m+1)\delta)^2 + \frac{1-p}{b(n+1)^2} (a-f)^2 \end{aligned} \quad (61)$$

where l is the total number of participating firms in the post-disaster market, m is the total number of investing firms in the post-disaster market, and n is the total number of firms participating in the non-disaster market.

We can also write down the expression for the expected profit of firms that have invested in preparedness, as follows:

$$E[\mathcal{P}_i^s] = \frac{p}{b(l+1)^2} (a-f + (l-m)\delta)^2 + \frac{1-p}{b(n+1)^2} (a-f)^2 - C \quad (62)$$

Note that, when $l = m$, the above expression would be equal to Equation (57).

So far we calculated the expected profit of a given firm under a given market condition (i.e., how many firms can participate in the post disaster market). However, the ultimate decision of firms depends on which action would result in a higher expected profit. In order to tie the results from the above equations to individual firms' investment decisions based on the magnitude of the preparedness cost, C , we can compare the expected profits of an individual firm with and without investment. Let's first consider a firm that is currently investing in preparedness, but is considering whether it is beneficial to switch to not investing. Assuming that there are currently m firms investing in preparedness, the resulting number of investing firms after this change would be $m - 1$. However, depending on whether the new total number of firms being able to participate in the post-disaster market, $l - 1$, is greater or less than $\frac{a-f-\delta}{\delta}$, we have two outcomes.

If $l - 1 > \frac{a-f-\delta}{\delta}$, the consequence of firm i changing its decision from investing to not investing will force firm i to stop producing in the post-disaster market. Therefore, using Equations (60) and (62), we can compare its expected profits with and without preparedness, as follows:

$$\begin{aligned}
& E[\mathcal{P}_i^{(s_i=1, s_{-i})}] > E[\mathcal{P}_i^{(s_i=0, s_{-i})}] \\
& \frac{p}{b(l+1)^2} (a-f+(l-m)\delta)^2 + \frac{1-p}{b(n+1)^2} (a-f)^2 - C > \frac{1-p}{b(n+1)^2} (a-f)^2 \quad (63) \\
& C < \frac{p}{b(l+1)^2} (a-f+(l-m)\delta)^2
\end{aligned}$$

Thus, firm i will invest in preparedness and participate in the market if and only if preparedness cost is sufficiently small.

Conversely, if $l - 1 < \frac{a-f-\delta}{\delta}$, then the consequence of changing from investing

to not investing in preparedness will not affect the production behavior of firm i (since if that inequality is satisfied, the firm could produce with or without investment). However, the number of investing firms will decrease by one when such a firm changes its investment decision (so m^{old} should be replaced by $m - 1$ in calculating $E[\mathcal{P}_i^{(s_i=0, s_{\sim i})}]$). We can again compare the expected profit of such a firm with and without preparedness, using Equations (61) and (62), as follows:

$$\begin{aligned}
& E[\mathcal{P}_i^{(s_i=1, s_{\sim i})}] > E[\mathcal{P}_i^{(s_i=0, s_{\sim i})}] \\
\Rightarrow & \frac{p}{b(l+1)^2} (a-f+(l-m)\delta)^2 + \frac{1-p}{b(n+1)^2} (a-f)^2 - C \\
& > \frac{p}{b(l+1)^2} (a-f-(m^{old}+1)\delta)^2 + \frac{1-p}{b(n+1)^2} (a-f)^2 \quad (64) \\
\Rightarrow & C < \frac{p}{b(l+1)^2} (a-f+(l-m)\delta)^2 - \frac{p}{b(l+1)^2} (a-f-m\delta)^2 \\
\Rightarrow & C < \frac{pl\delta}{b(l+1)^2} (2(a-f-m\delta) + l\delta)
\end{aligned}$$

Again, firm i invests in preparedness if its preparedness cost is sufficiently small, but the threshold for what count as small is different, because now deciding not to invest does not prevent firm i from producing in disaster conditions.

To summarize, when there are multiple homogeneous firms competing in the market, those firms that have invested in preparedness can certainly participate in the post-disaster market. However, among those firms that have not invested in preparedness, only some of them may be able to participate in the post-disaster market; the remainder of the unprepared firms will not have a positive post-disaster optimal production quantity.

7 Numerical Results for Fixed Cost of Preparedness

In order to demonstrate the interaction between competing firms and also to show how we can use $\rho(i)$ to initially find the participating firms and then compute the Nash equilibria of a given game, we first construct a base-case model with two homogeneous firms (Section 7.1), where the parameters of the profit functions are chosen to reflect a realistic market environment. The base-case values and ranges for sensitivity analysis used throughout the remainder of this section are shown in Table 9. These values were chosen so that (1) the variable cost of production, f_i , is small relative to the reservation price a (so that firms are likely to produce at equilibrium), and (2) the increase in the variable cost of production, δ_i , does not exceed the variable cost of production under non-disaster conditions, f_i .

We then conduct sensitivity analysis to study the effects of market factors (the probability of disaster p , the reservation price a , and the slope of the price function b), other firms' firm-specific factors (variable costs of production f_j , and the increases in their variable costs of production δ_j), and the effects of equilibrium mixing strategy of other firms s_j . The results of our sensitivity analyses highlight the importance of those parameters to firms' preparedness decisions.

Then in Section 7.2 we increase the number of homogeneous firms in the market from $n = 2$ to $n = 5$ (in which the market is dominated by a small number of large firms; Neghenebor and Willis, 2000). This enables us to see

Table 9: Parameter Values for Homogeneous Firms in the Base Case and Sensitivity Runs

Parameter	Base Case	Lower Bound	Upper Bound
a	500	10	1000
b	0.1	0.1	10
p	0.05	0.01	1
f_i	90	10	1000
δ_i	60	10	1000
C_i	4000	10	10000

the effects of more firms competing in the market.

We start by assuming that these five firms are homogeneous. Of course, the assumption of only homogeneous firms is also not realistic. Therefore, to explore the effects of heterogeneity, we study a market of 5 firms, in which 4 are homogeneous (with parameter values equal to the base case in Table 9), but the 5th firm displays some different characteristic of interest; either a different variable cost of production f_5 , or a different post-disaster increase in the variable cost of production.

7.1 Duopoly Example: Homogeneous Firms

Here, we consider a market consisting of two homogeneous firms, and construct the corresponding optimization problem. The profit function of each firm is the total revenue minus the total cost. Using the price function $a - b(\sum_{i=1}^2 q_i^{(s,d)})$, we can write the following:

$$\begin{cases} \mathcal{P}_1^{(s,d)} = (a - b(q_1^{(s,d)} + q_2^{(s,d)}))q_1^{(s,d)} + (f + \delta(1 - s_1)d)q_1^{(s,d)} - s_1C \\ \mathcal{P}_2^{(s,d)} = (a - b(q_1^{(s,d)} + q_2^{(s,d)}))q_2^{(s,d)} + (f + \delta(1 - s_2)d)q_2^{(s,d)} - s_2C \end{cases} \quad (65)$$

Each firm will choose its production quantity to maximize its profit given a market condition (s, d) , subject to a non-negativity constraint for the production quantity. Therefore, we need to solve this joint maximization problem for all possible market conditions (i.e., all 8 combinations of s_1 , s_2 , and d for the case of a duopoly). However, the optimal production quantity of a given firm will be the same when $d = 0$ as when both firms invest in preparedness and a disaster occurs. Also, using the symmetry between these two firms, we need to solve only three joint optimization problems, as discussed below.

- No firm invests in preparedness (i.e., $s = 0, 0$) and a disaster happens (i.e., $d = 1$). Since the firms are homogeneous and neither invests in preparedness, we have $RVC_1 = RVC_2 = 90 + 60 = 150$. Then we have $\rho(1) = \rho(2) = 500 - (2 + 1)150 + (150 + 150) = 350 \geq 0$ so both firms can produce. Moreover, their optimal production quantities are as follows:

$$\begin{aligned} q_1^{*((0,0),1)} = q_2^{*((0,0),1)} &= \frac{1}{(2+1)0.1} (500 - (2+1)150 + (150+150)) \\ &= \frac{3500}{3} \approx 1,166.667 \end{aligned} \tag{66}$$

with a total market production quantity of 2,333.333.

- Only firm 1 invests in preparedness (i.e., $s = (1, 0)$ and $d = 1$). We now have, $RVC_1 = 90$ and $RVC_2 = 150$. For firm 1, we have $\rho(1) = 500 - (1 + 1)90 + (90) = 410 \geq 0$, and for firm 2, we have $\rho(2) = 500 - (2 + 1)150 + (90 + 150) = 290 \geq 0$. Their corresponding optimal levels of

production are as follows:

$$\begin{cases} q_1^{*((1,0),1)} = \frac{1}{(2+1)0.1} (500 - (2+1)90 + (90 + 150)) = \frac{4700}{3} \approx 1,566.667 \\ q_2^{*((1,0),1)} = \frac{1}{(2+1)0.1} (500 - (2+1)150 + (90 + 150)) = \frac{2900}{3} \approx 966.667 \end{cases} \quad (67)$$

with a total market production quantity of 2,533.333. (The case when only firm 2 invests in preparedness is symmetrical.)

- Finally, when both firms invest in preparedness (i.e., $s = (1, 1)$), or when no disaster occurs ($d = 0$), we have $RVC_1 = RVC_2 = 90$. Moreover, both firms can participate in this market since $\rho(1) = \rho(2) = 500 - (2+1)90 + (90 + 90) = 410 \geq 0$. The optimal production quantity of each firm can be calculated as follows:

$$\begin{aligned} q_1^{*((1,1),1)} = q_2^{*((1,1),1)} &= \frac{1}{(2+1)0.1} (500 - (2+1)90 + (90 + 90)) \\ &= \frac{4100}{3} \approx 1,366.667 \end{aligned} \quad (68)$$

with a total market production quantity of 2,733.333.

Tables 10 summarizes the above post-disaster optimal quantities. (Note that the optimal non-disaster production quantities of both firms are equal to the case where both firms invest.)

Based on the above optimal production quantities, we can compute the payoff of each firm (i.e., its expected profit $E[\mathcal{P}_i^s] = p\mathcal{P}_i^{(s,1)} + (1-p)\mathcal{P}_i^{(s,0)}$) under the various investment scenarios (i.e., both invest, either one invest, or none invests), as follows:

Table 10: Optimal Production Quantity

		Disaster	
Both firms invest	$\begin{cases} q_1^{*((1,1),1)} = \frac{4100}{3} \\ q_2^{*((1,1),1)} = \frac{4100}{3} \end{cases}$	$Q^{*((1,1),1)} = 2,733.3$	
Only firm 1 invests	$\begin{cases} q_1^{*((1,0),1)} = \frac{4700}{3} \\ q_2^{*((1,0),1)} = \frac{2900}{3} \end{cases}$	$Q^{*((1,0),1)} = 2,533.3$	
Only firm 2 invests	$\begin{cases} q_1^{*((0,1),1)} = \frac{2900}{3} \\ q_2^{*((0,1),1)} = \frac{4700}{3} \end{cases}$	$Q^{*((0,1),1)} = 2,533.3$	
No firm invests	$\begin{cases} q_1^{*((0,0),1)} = \frac{3500}{3} \\ q_2^{*((0,0),1)} = \frac{3500}{3} \end{cases}$	$Q^{*((0,0),1)} = 2,333.3$	

- If both firms invest in preparedness, their payoffs would be as follows:

$$\begin{aligned}
E[\mathcal{P}_1^{(1,1)}] = E[\mathcal{P}_2^{(1,1)}] &= (500 - 0.1(\frac{4100}{3} + \frac{4100}{3}))\frac{4100}{3} - 90\frac{4100}{3} - 4000 \\
&= \frac{1645000}{9} \approx 182,777.778
\end{aligned} \tag{69}$$

with a total expected profit of 365,555.556.

- If only firm 1 invests in preparedness, then the corresponding expected

profits are as follows:

$$\left\{ \begin{array}{l} E[\mathcal{P}_1^{(1,0)}] = 0.05\left((500 - 0.1\left(\frac{4700}{3} + \frac{2900}{3}\right))\frac{4700}{3} - 90\frac{4700}{3} - 4000\right) \\ \quad + 0.95\left(\frac{1645000}{9}\right) = \frac{1671400}{9} \approx 185,711.111 \\ E[\mathcal{P}_2^{(1,0)}] = 0.05\left((500 - 0.1\left(\frac{2900}{3} + \frac{4700}{3}\right))\frac{2900}{3} - 150\frac{2900}{3}\right) \\ \quad + 0.95\left(\frac{1645000}{9} + 4000\right) = \frac{1639000}{9} \approx 182,111.111 \end{array} \right. \quad (70)$$

(or conversely when only firm 2 invests in preparedness) with a total expected profit of 367,822.222.

- Finally, if none of the firms invest in preparedness, then the expected profit of each firm is as follows:

$$\begin{aligned} E[\mathcal{P}_1^{(0,0)}] = E[\mathcal{P}_2^{(0,0)}] &= 0.05\left((500 - 0.1\left(2\frac{3500}{3}\right))\frac{3500}{3} - 150\frac{3500}{3}\right) \\ &\quad + 0.95\left(\frac{1645000}{9} + 4000\right) \\ &= \frac{1658200}{9} \approx 184,244.444 \end{aligned} \quad (71)$$

with a total expected profit of 368,488.889.

Now, we can construct our game-theoretic model based on the above calculated payoffs for each firm. The normal-form game is as shown in Table 11. Examining this game reveals that both firms investing in preparedness is the unique Nash equilibrium of this game, since for both firms, investing is the dominant strategy. However, this would of course not hold true for other parameter values.

Table 11: Strategic Representation of Investment Game For Two Participating Firms

		Firm 2	
		Invest	Not Invest
Firm 1	Invest	$\begin{cases} 182,777.778 \\ 182,777.778 \end{cases}$	$\begin{cases} 185,711.111 \\ 182,111.111 \end{cases}$
	Not Invest	$\begin{cases} 182,111.111 \\ 185,711.111 \end{cases}$	$\begin{cases} 184,244.444 \\ 184,244.444 \end{cases}$

7.1.1 Sensitivity Analysis: Effects of Environment

Let's consider the duopoly example above, based on the numerical values in Table 9. Here, we begin by performing one-way sensitivity analysis for the reservation price, a . The results show that as a gets larger, more firms will invest in preparedness. Moreover, if the reservation price is large enough to compensate for the cost of preparedness, all firms will choose to invest in preparedness (Figure 4(a)). (Note that in order to show a wide range of values for the preparedness cost C , we used a logarithmic scale on the y-axis.)

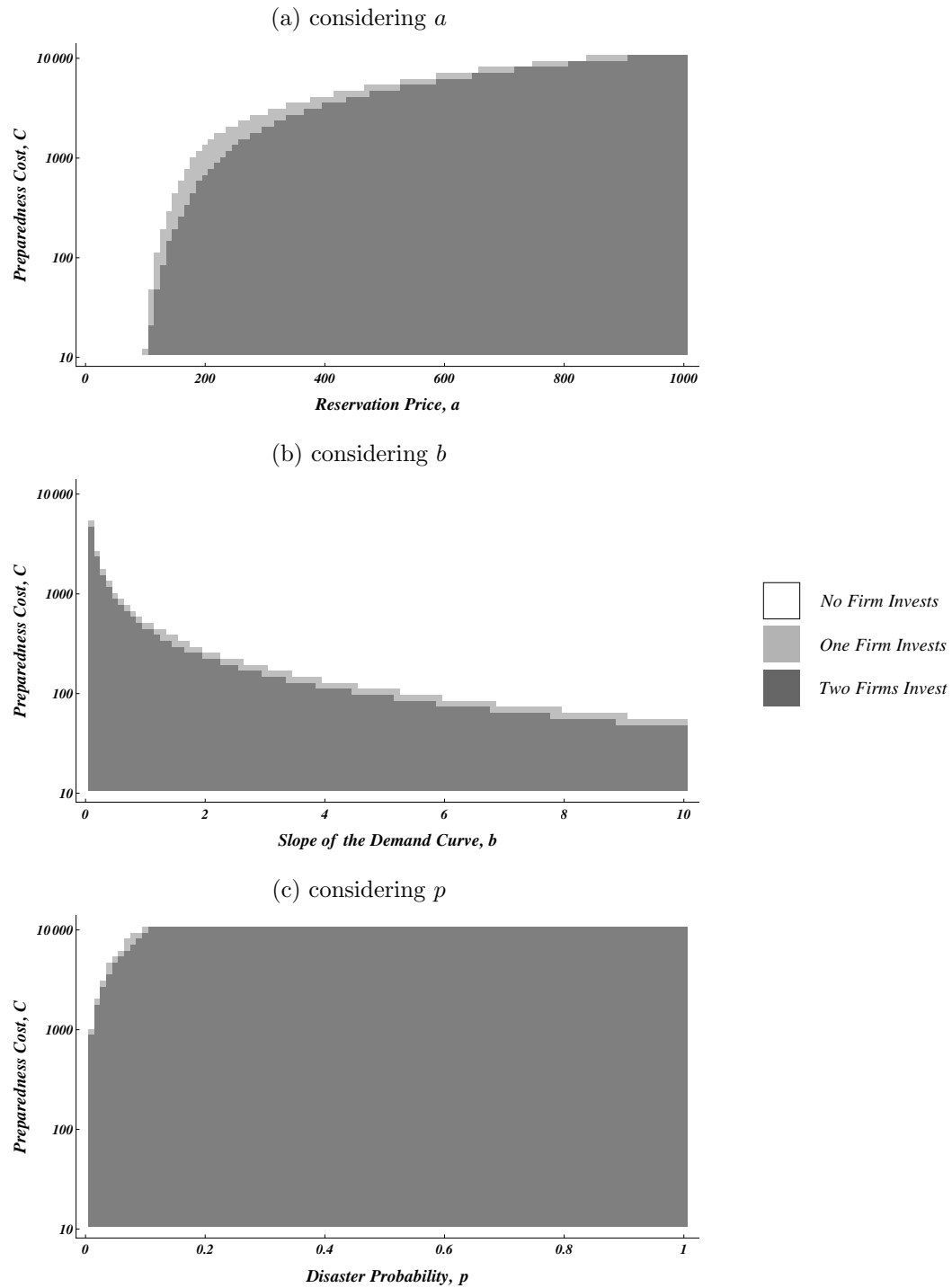
For example, consider the case where the fixed cost of preparedness, C , is approximately 1,000. Reading off the Nash equilibrium as the reservation price, a , varies from 0 to 1000, we see that for small values of the reservation price (i.e., less than or equal to about 170), the Nash equilibrium is for no firms to invest in preparedness (as indicated by the number 0 in the graph). When the reservation price is larger than 230, then the Nash equilibrium is for both firms to invest in preparedness (as indicated by the number 2 in the

graph). For reservation prices in between these values, the Nash equilibrium is for only one firm to invest in preparedness (as indicated by the number 1 in this graph).

Next, we perform a one-way sensitivity analysis for the slope of the demand curve, b . This result shows that as b gets larger, all firms will eventually choose not to invest in preparedness (Figure 4(b)). Choosing a fixed cost of preparedness of $C = 1000$ shows that when the slope of the demand curve is less than about 0.4, then both firms investing in preparedness, and when the slope of the demand curve is bigger than 0.6, then no firm invests in preparedness. Finally, for intermediate values, either firm investing in preparedness can be a Nash equilibrium.

Finally, Figure 4(c) shows the result of a one-way sensitivity analysis for the probability of disaster, p . In particular, as a disaster becomes more probable, firms begin to invest in preparedness to avoid the consequences of the disaster. Here for instance, we see that for a preparedness cost of 1000, if the probability of a disaster is greater than 1%, then both firms will want to invest in preparedness.

Figure 4: One-way Sensitivity Analysis for market factors in a Duopoly, Based on the Pure Nash Equilibrium



7.1.2 Sensitivity Analysis: Effects of Realized Variable Cost of Production

Now, we examine the effects of firm-specific factors. Equation (14) gives the optimal production quantity of a participating firm, as a function of the realized variable costs of production of other firms (conditional on knowledge of their investment decisions, s_j). According to Equation (33), the realized variable cost of production of a firm j , $RVC_j^{(s_j, d)}$, is given by $f_j + \delta_j(1 - s_j)d$. Therefore, we conduct sensitivity analysis on the variable cost of production f_j , and the post-disaster increase in the variable cost of production (δ_j). However, since we are dealing with homogeneous firms, we assume that the same numerical values of these quantities apply to all firms. (Later, in Section 7.2, we consider heterogeneous firms.)

Figure 5(a) shows the result of a one-way sensitivity analysis for the variable cost of production, f . In particular, it shows that for a given fixed cost of preparedness, as f becomes larger, fewer firms will invest in preparedness.

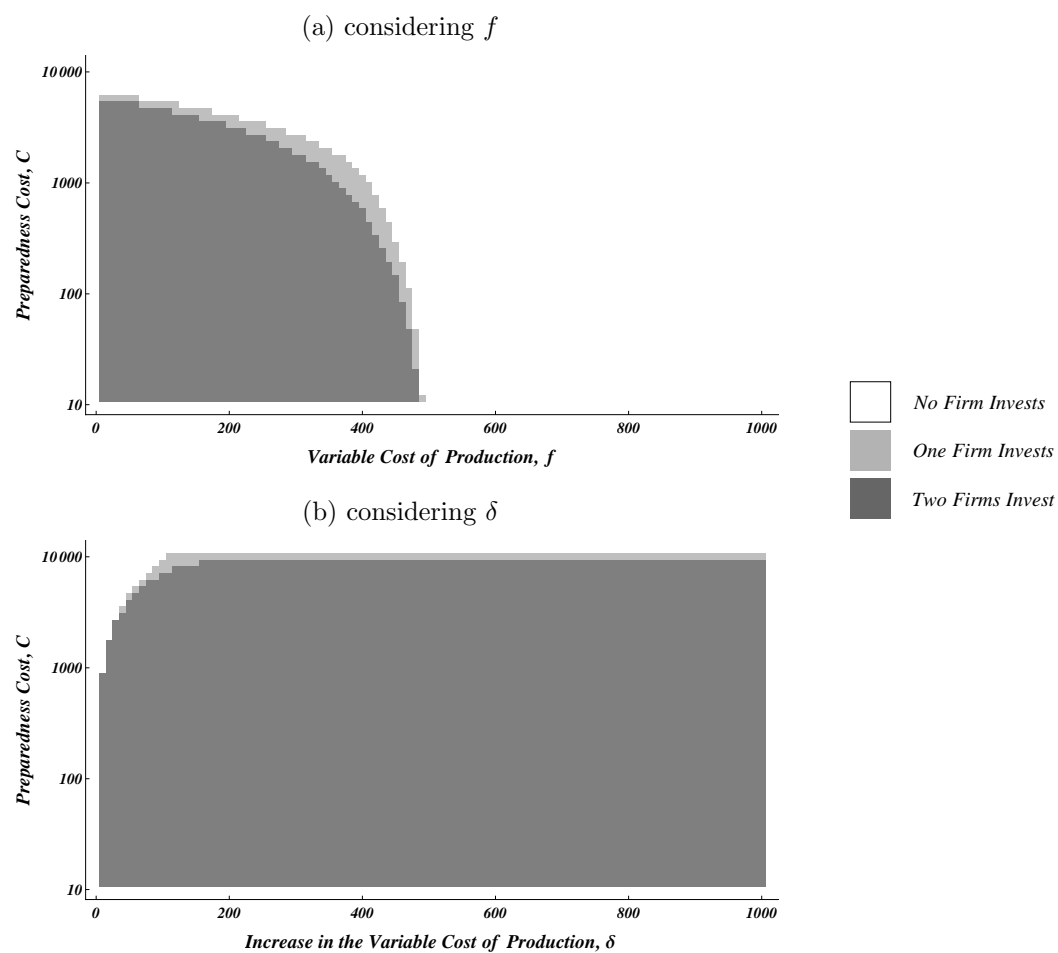
The result of a one-way sensitivity analysis for the increase in the variable cost of preparedness, δ , is shown in Figure 5(b). Here, for a given level of preparedness cost (say, 1000), if δ is small, then the Nash equilibrium is for no firms to invest in preparedness. However, when δ becomes greater than 20, then the Nash equilibrium is for both firms to invest in preparedness. We should note that if the fixed cost of preparedness becomes extremely large, then we would expect to see no firms investing in preparedness regardless of the magnitude of the increase in the variable cost of preparedness. This is due to the fact that although investing in preparedness will remove any

future increases in the variable cost of production, but since the fixed cost of preparedness is extremely large, investing will result in a negative profit and consequently negative expected profit for such a firm. We can illustrate this fact for a monopolist. The expected profit of a monopolist when investing in preparedness and when not investing in preparedness is as follows:

$$\begin{aligned} E[\mathcal{P}_m^1] &= \frac{(a-f)^2}{4b} - C \\ E[\mathcal{P}_m^0] &= \frac{(a-f)^2}{4b} - (1-p) \frac{2a-2f-\delta}{4b} \delta \end{aligned} \tag{72}$$

Therefore, if $C > (1-p) \frac{2a-2f-\delta}{4b} \delta$, then not investing in preparedness (and consequently not being able to produce in the case of a disaster) would be the decision of that firm. Hence, the results of our sensitivity should be viewed with consideration of such facts. Moreover, the results are applied to the ranges of sensitivity analysis we chose at the beginning. Similar conclusion will apply for a duopoly and an oligopolistic market.

Figure 5: One-way Sensitivity Analysis for firm-specific factors in a Duopoly, Based on the Pure Nash Equilibrium



7.1.3 Sensitivity Analysis: Effects of Equilibrium Mixing Strategies

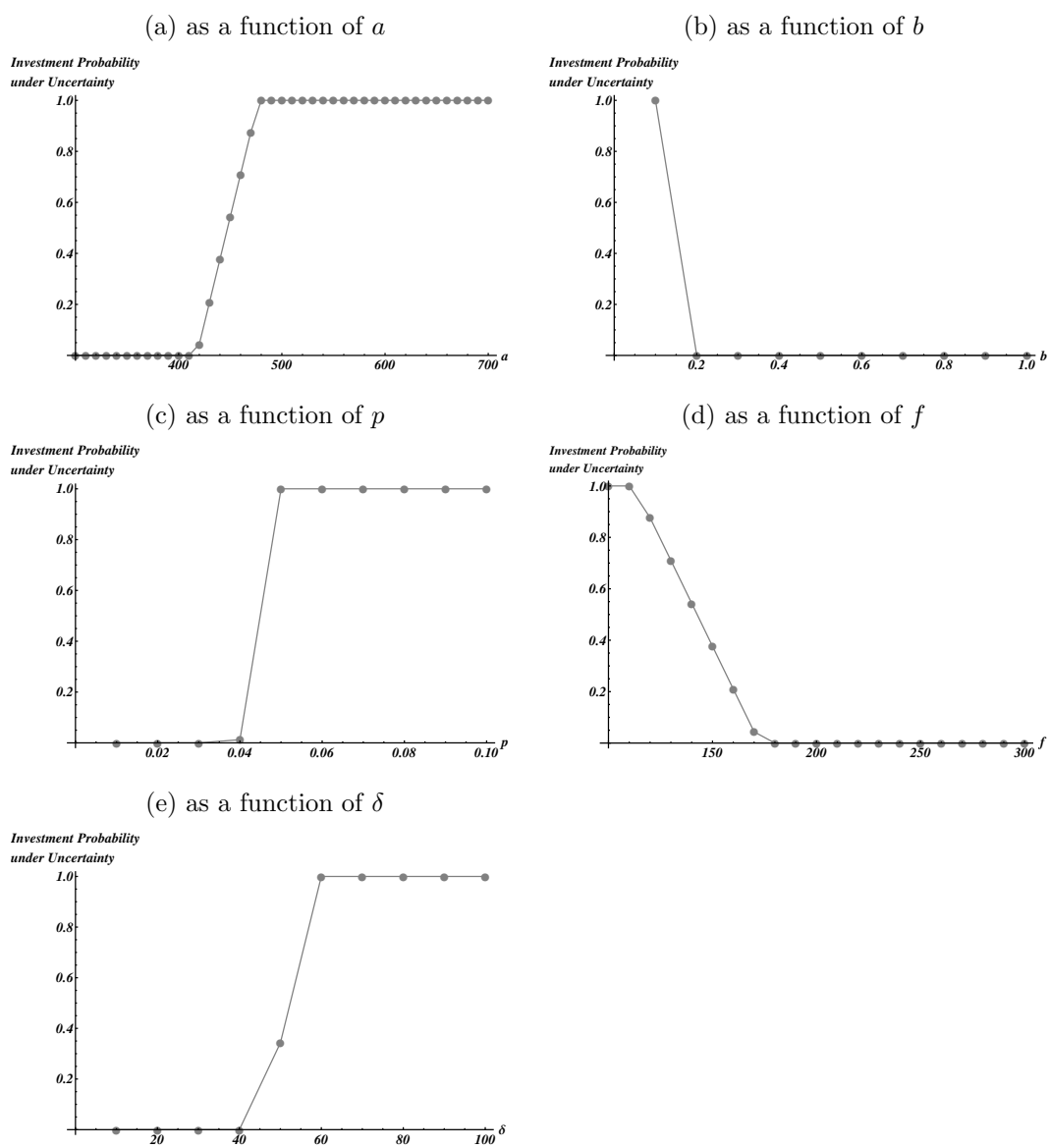
Our final analysis focuses on the effects of changes in equilibrium mixing strategy of firms' investment decisions. When neither investing nor not investing is a dominant strategy, we have shown that firms can choose a mixed strategy, with a mixing probabilities such that other firms are indifferent between investing in preparedness and not investing.

We have shown, in Section 6, how to compute the mixing probability of each firm. For example, if the reservation price is 400, then using the rest of the values from our base-case, we can compute the mixing probability of both firms as follows (using Equations (50) and (51)):

$$\sigma_1 = \sigma_2 = \frac{4 * 0.05 * 60(400 - 90) - 9 * 0.1 * 4000}{4 * 0.05 * 60 * 60} = 0.167 \quad (73)$$

Furthermore, the resulting mixing probabilities can be computed by Gambit. Mixing probabilities for two homogeneous firms corresponding to region where one firm investing is the pure Nash equilibrium, as a function of market factors and firm-specific factors, are shown in Figure 6.

Figure 6: Investment probabilities for two homogeneous firms



7.2 Oligopoly Example

We start by assuming that there are five homogeneous firms in the market, with the same base-case values as in Table 9. The optimal production quantities of these firms under different market conditions can be calculated using the same procedure as for the duopoly, and are summarized in Table 12. In this table, we use the fact that homogeneous firms with the same investment decision will have the same optimal quantity. Therefore, it is adequate to display only optimal quantities for those that have invested in preparedness, q_I^* , and those that have not invested in preparedness, q_{NI}^* . Moreover, the optimal non-disaster production quantities of firms are equal to the case where all firms invest in preparedness. As in the previous section, the payoff of each firm under a given set of investment strategies can also be calculated.

We now analyze the sensitivity of our results to variations in the input parameters. In particular, we perform three types of analysis for the model with a fixed cost of preparedness. Initially, we study the effects of the environment in which firms are producing (i.e., the probability of disaster, the reservation price, and the slope of the price function). Then, we examine the effects of firms' variable cost of production and post-disaster increase in the variable cost of production. Finally, we investigate the effects of equilibrium mixing strategy of other firms.

Throughout our sensitivity analysis, we also introduce heterogeneity into the market. First, we consider a market of five firms, in which four are homogeneous (with parameter values equal to the base case in Table 9), but the fifth has a larger variable cost of production, $f_5 = 135$ (compared to $f = 90$

Table 12: Optimal Production Quantities for 5 Homogeneous Firms (Base Case)

Number of Firms Investing in Preparedness		Disaster
5	$q_I^* = \frac{4100}{6}$	$Q^{*(s,1)} = 3,416.7$
4	$\begin{cases} q_I^* = \frac{4700}{6} \\ q_{NI}^* = \frac{1100}{6} \end{cases}$	$Q^{*(s,1)} = 3,316.7$
3	$\begin{cases} q_I^* = \frac{5300}{6} \\ q_{NI}^* = \frac{1700}{6} \end{cases}$	$Q^{*(s,1)} = 3,216.7$
2	$\begin{cases} q_I^* = \frac{5900}{6} \\ q_{NI}^* = \frac{2300}{6} \end{cases}$	$Q^{*(s,1)} = 3,116.7$
1	$\begin{cases} q_I^* = \frac{6500}{6} \\ q_{NI}^* = \frac{2900}{6} \end{cases}$	$Q^{*(s,1)} = 3,016.7$
0	$q_{NI}^* = \frac{3500}{6}$	$Q^{*(s,1)} = 2,916.7$

for the homogeneous firms). Next, we look at a market in which the fifth firm has a larger post-disaster increase in its variable cost of production, $\delta_5 = 135$ (compared to $\delta = 60$ for the homogeneous firms). In general, we would expect a firm with a larger variable cost of production to invest in preparedness for larger values of the reservation price a and the disaster probability p than in the base case, and stop investing in preparedness for smaller values of the slope of the demand curve b . Conversely, we would expect a firm with a larger post-

disaster increase in its variable cost of production to invest in preparedness for smaller values of the reservation price a and the disaster probability p , and keep investing in preparedness even for larger values of the demand curve b .

7.2.1 Sensitivity Analysis: Effects of Environment

Consider an oligopoly of five homogeneous firms, as described above. We first perform one-way sensitivity analysis for the reservation price, a . We are interested in studying how the pure Nash equilibrium changes as we vary a . The result of a one-way sensitivity analysis for the reservation price shows that as a gets larger, more firms will invest in preparedness. In particular, if the reservation price is large enough to compensate for the cost of preparedness, all firm will choose to invest, as shown in Figure 7(a). (Note that, in order to show a wide range for the preparedness cost C , we use a logarithmic scale for the fixed cost of preparedness.) Figure 7(b) shows a one-way sensitivity analysis for the reservation price in a market where one firm has a larger variable cost of production, $f_5 = 135$, compared to the base case of 90. We can see that such a firm does not invest in preparedness unless the reservation price is quite high, as shown by the arrow. By contrast, if the fifth firm has a larger post-disaster increase in its variable cost of production, $\delta_5 = 135$, compared to the base case of 60, then for a given fixed cost of preparedness, such a firm will invest in preparedness even with a relatively small reservation price, as shown by the arrow in Figure 7(c). By investing in preparedness, such a firm can prevent incurring a large increase in its variable cost of production.

Next, we perform one-way sensitivity analysis for the slope of the demand

curve, b . Figure 8(a) shows that as b gets larger, fewer firms will choose to invest in preparedness. Performing one-way sensitivity analysis for the slope of the demand curve in a market where the fifth firm has a larger variable cost of production ($f_5 = 135$ instead of 90) shows that such a firm stops investing in preparedness if the slope of the demand curve is even moderately large, as shown by the arrow in Figure 8(b). By contrast, if the fifth firm has a larger post-disaster increase in its variable cost of production ($\delta = 135$ instead of 60), then Figure 8(c) illustrates that it will invest in preparedness even for somewhat larger values of b . For example, the arrow in Figure 8(c) shows that the firm with a larger post-disaster increase in its variable cost of production keeps investing in preparedness for larger values of b than in the homogeneous.

Finally, Figure 9(a) shows the result of one-way sensitivity analysis for the probability of disaster, p . In particular, this figure shows that as a disaster becomes more likely, more firms choose to invest in preparedness to avoid the consequences of the disaster. In a market where the fifth firm has a larger variable cost of production, Figure 9(b) shows that such a firm will not invest in preparedness unless the probability of a disaster is quite large, as shown by the arrow in Figure 9(b). In fact, when the preparedness cost is sufficiently large (e.g., $C = 10,000$), we can see that such a firm will not invest in preparedness even if a disaster is guaranteed. However, if the fifth firm has a larger post-disaster increase in its variable cost of production, then it will want to invest in preparedness even for relatively small values of the disaster probability, as shown by the arrow in Figure 9(c).

Figure 7: One-way Sensitivity Analysis for a in an Oligopoly, Based on the Pure Nash Equilibrium

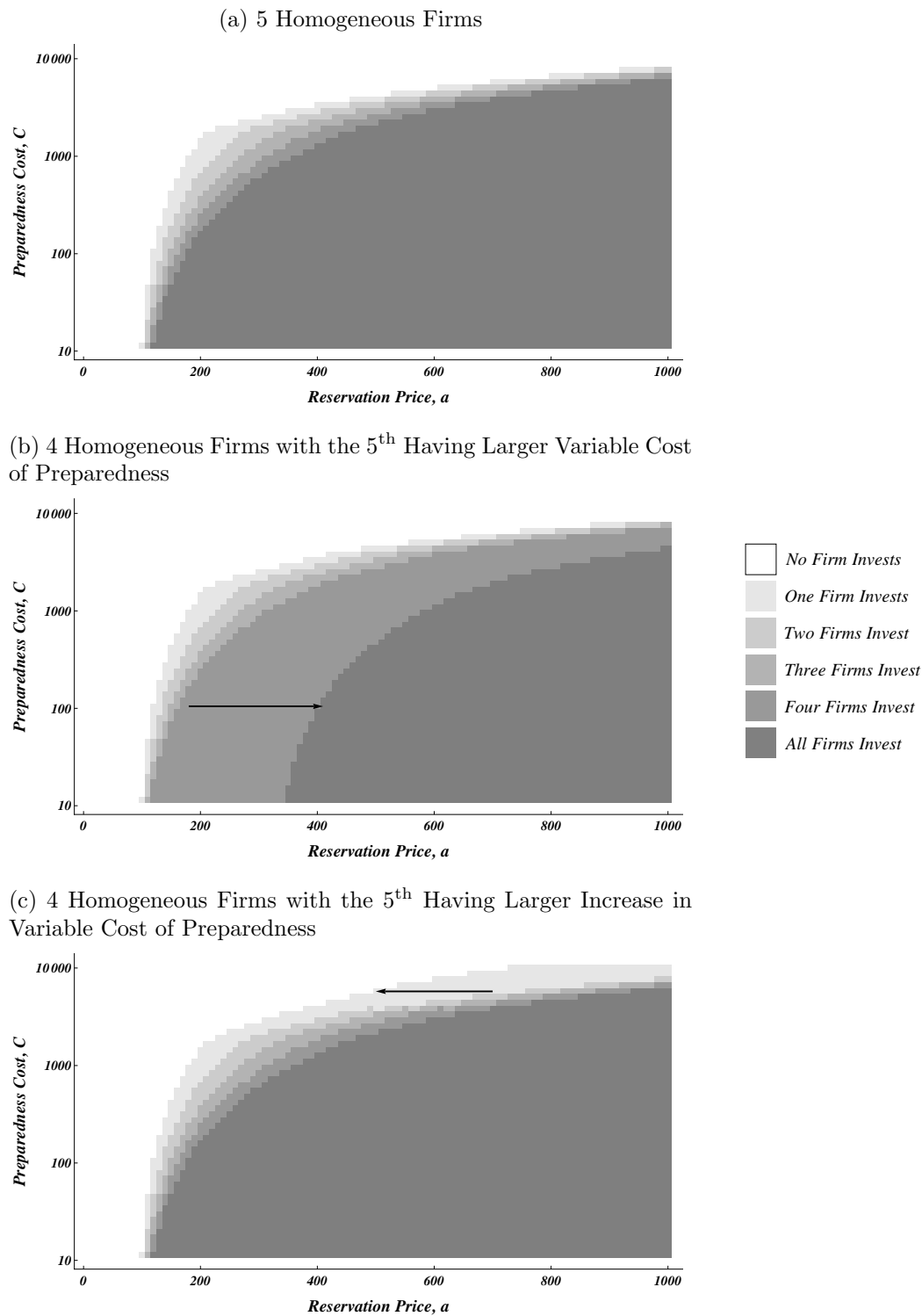
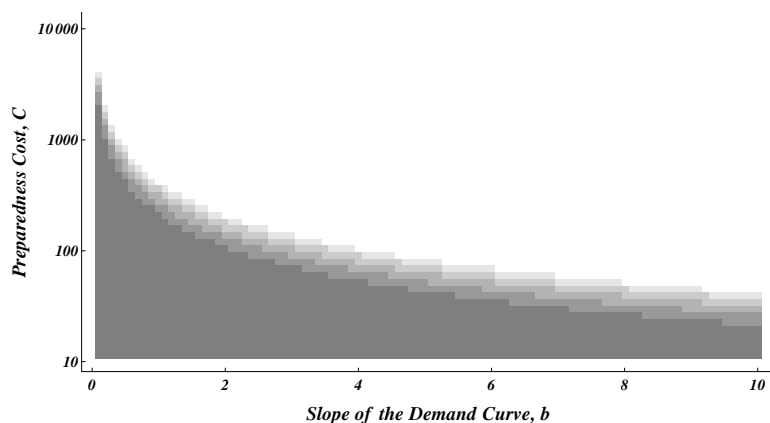
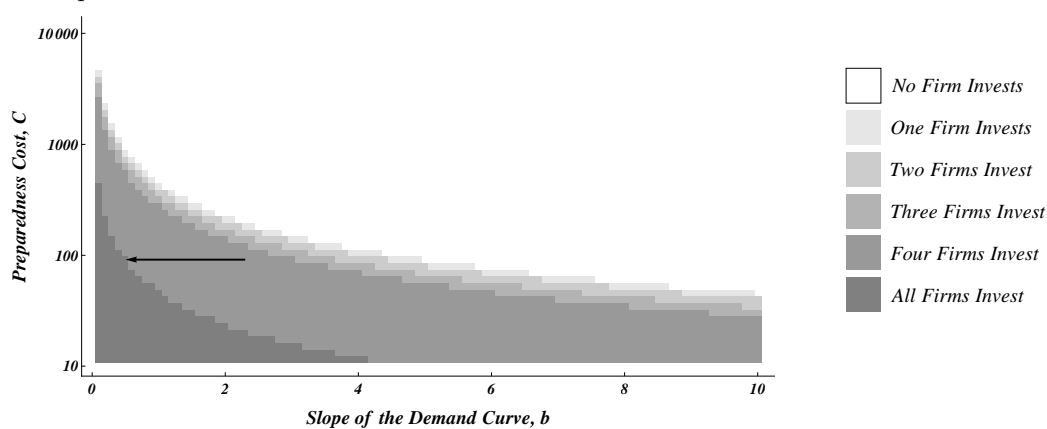


Figure 8: One-way Sensitivity Analysis for b in an Oligopoly, Based on the Pure Nash Equilibrium

(a) 5 Homogeneous Firms



(b) 4 Homogeneous Firms with the 5th Having Larger Variable Cost of Preparedness



(c) 4 Homogeneous Firms with the 5th Having Larger Increase in Variable Cost of Preparedness

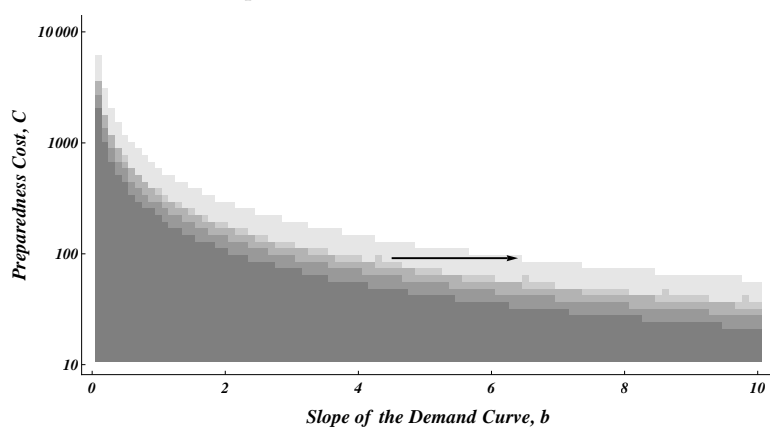
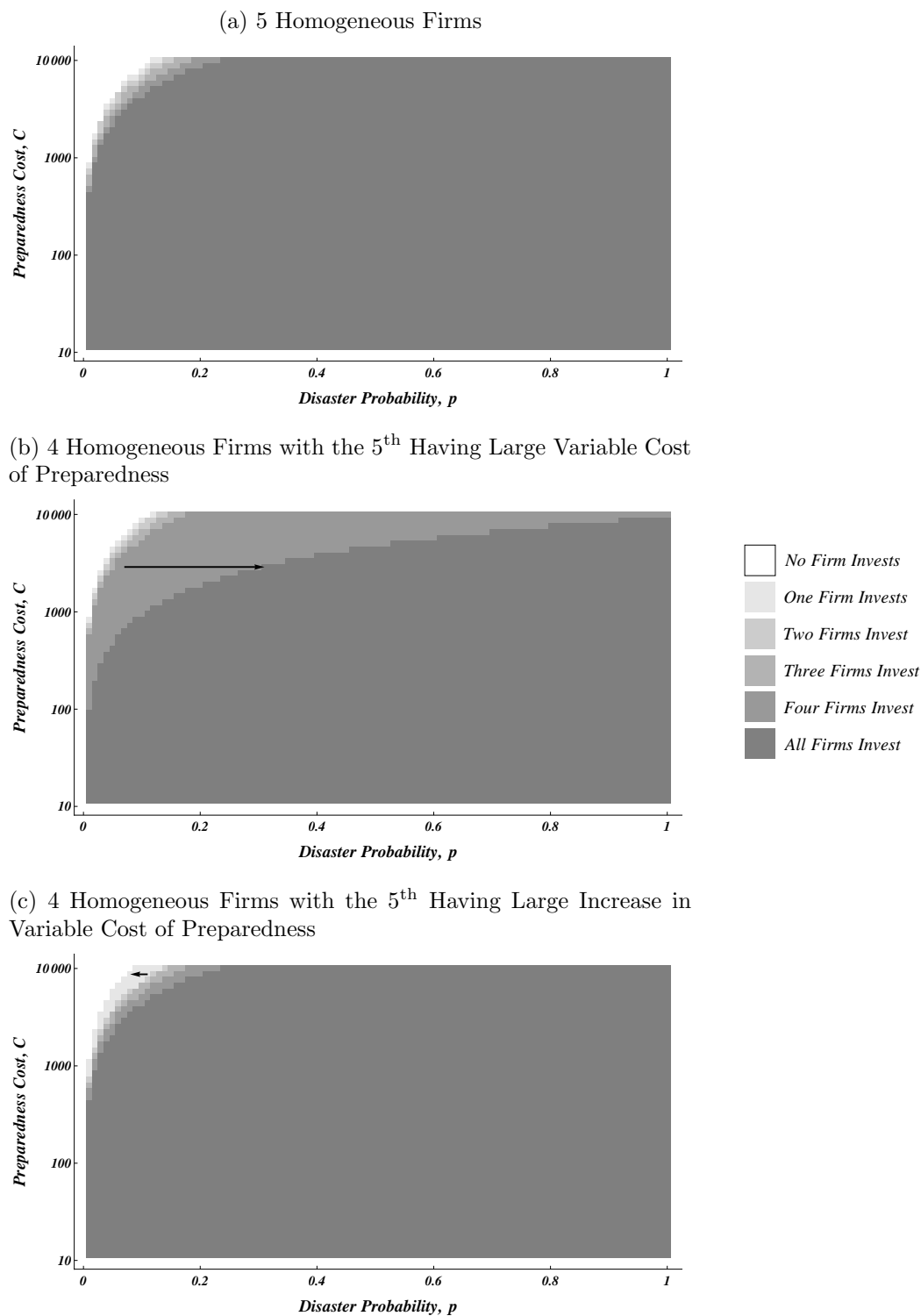


Figure 9: One-way Sensitivity Analysis for p in an Oligopoly, Based on the Pure Nash Equilibrium



7.2.2 Sensitivity Analysis: Effects of Realized Variable Cost of Production

We now examine the sensitivity of the equilibrium to firm-specific factors. In particular, we focus on firms' variable costs of production (f), and the increases in their variable costs of production (δ), since these factors make up the firms' realized variable costs of production.

Given an oligopoly market with five homogeneous firms, Figure 10(a) shows the result of a one-way sensitivity analysis for the variable cost of production, f . For a given fixed cost of preparedness, the number of firms that invest in preparedness in a pure Nash equilibrium decreases as the variable cost of production f increases. This is due to the fact that when firms are able to increase their production (due to investing in preparedness), this drives down the market price, until the price is so low that investing in preparedness (in order to be able to produce in the post-disaster market) is no longer economically justified. We now allow the fifth firm to have a different variable cost of production ($f_5 = 135$ instead of 90). We can see that when the variable cost of production of the other firms is much smaller than f_5 , firm 5 will not invest in preparedness, since it does not possess a competitive advantage at that point, as shown by the left arrow in Figure 10(b). However, when the variable cost of production of the other firms is much larger than f_5 , they stop investing in preparedness for smaller values of f than in the base case, as shown by the right arrow in Figure 10(b), while the fifth firm continues investing in preparedness and producing. When the fifth firm has a larger post-disaster increase in its variable cost of production ($\delta_5 = 135$ instead of 60), Figure 10(c)

shows that such a firm keep investing in preparedness for higher variable costs of production than other firms. For example, at $C = 2,500$, the arrow in Figure 10(c) shows that the firm with the larger post-disaster increase in its variable cost of production keeps investing in preparedness for larger values of f than in the base case.

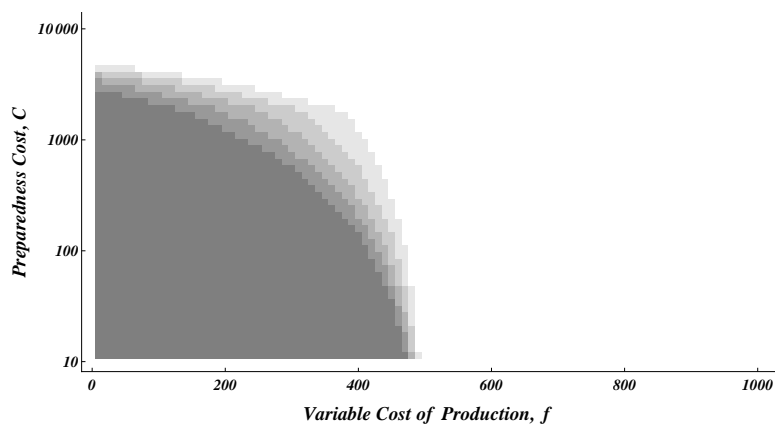
The results of a one-way sensitivity analysis for the increase in the variable cost of preparedness, δ , is shown in Figure 11(a). In this case, for a given fixed cost of preparedness, the number of firms that invest in preparedness in a pure Nash equilibrium is increasing in the post-disaster increase in the variable cost of preparedness. In other words, as the consequence of not investing in preparedness becomes more severe, then more firms will choose to invest in preparedness at equilibrium. However, for a given fixed cost of preparedness, there is a limit on the number of firms that will invest in preparedness in the pure Nash equilibrium, no matter how large the post-disaster increase in the variable cost of production gets. For example, at $C = 10,000$, at most one firm will invest in preparedness in the pure Nash equilibrium, regardless of the value of δ . This is because, although other firms might be able to produce after a disaster if they invested in preparedness, the large fixed cost of preparedness would eliminate their profit. Therefore, such firms will choose not to invest in preparedness, even at the cost of not being able to produce in the event of a disaster.

Allowing the fifth firm to have a larger variable cost of production significantly affects its preparedness decision. First, it dramatically reduces the preparedness cost such a firm would be willing to pay, as shown by the vertical

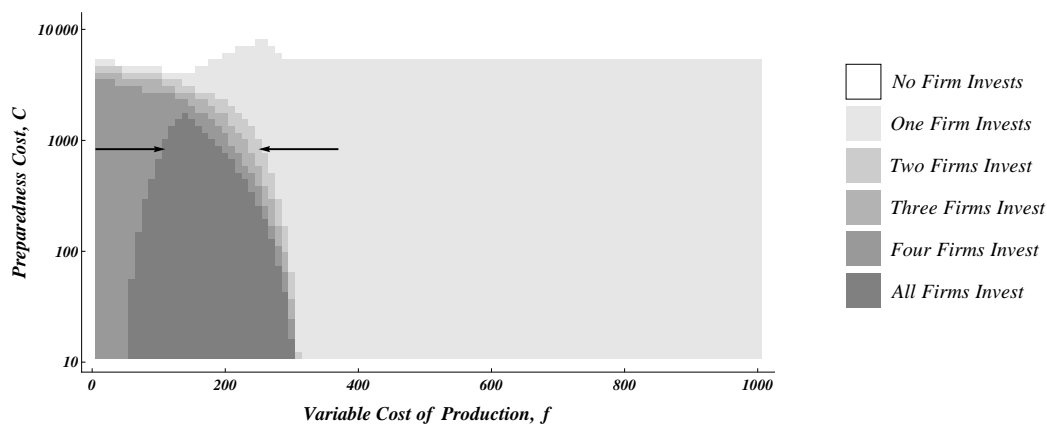
arrow in Figure 11(b). Second, a firm with a larger variable cost of production will not invest in preparedness unless the post-disaster increase δ is slightly larger than in the base case. For example, if $C = 360$, the fifth firm will not invest in preparedness until δ is at least 10 (compared to investing for all values of δ in the case of homogeneous firms), as shown by the horizontal arrow in Figure 11(b). However, when the fifth firm has a larger increase in its variable cost of production, Figure 11(c) shows that it will choose to invest in preparedness even when other firms have smaller values of δ than in the base case. The arrow in Figure 11(c) shows this change for $C = 2,500$.

Figure 10: One-way Sensitivity Analysis for f in an Oligopoly, Based on the Pure Nash Equilibrium

(a) 5 Homogeneous Firms



(b) 4 Homogeneous Firms and the 5th Having Large Variable Cost of Preparedness ($f_5 = 135$)



(c) 4 Homogeneous Firms and the 5th Having Large Increase in Variable Cost of Preparedness ($\delta_5 = 135$)

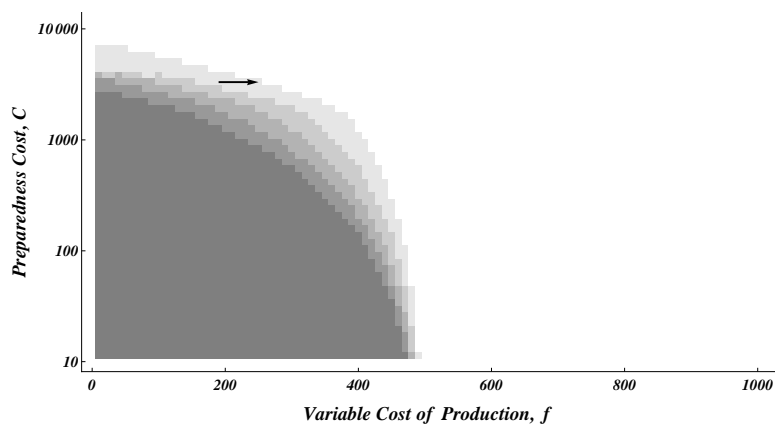
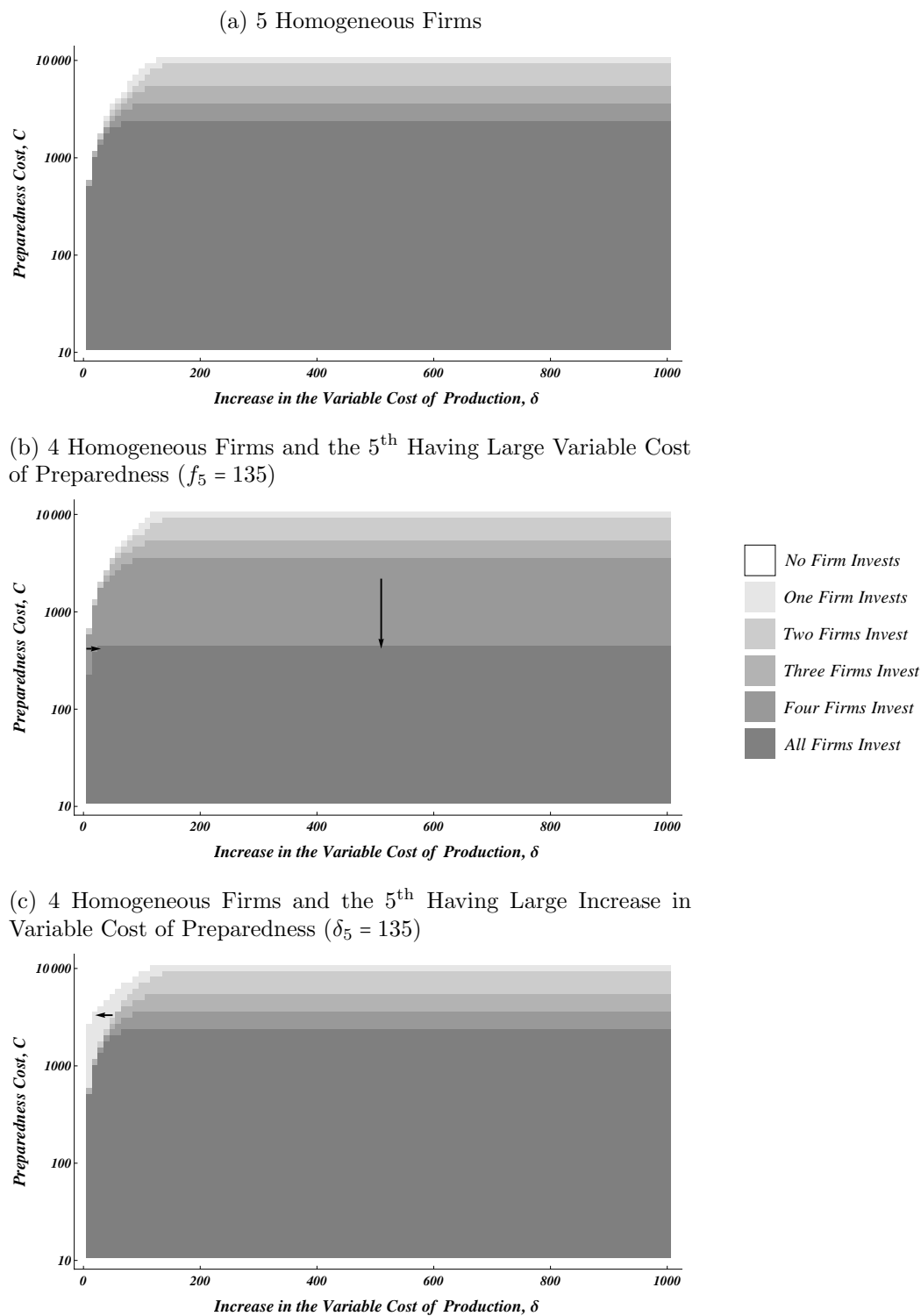


Figure 11: One-way Sensitivity Analysis for δ in an Oligopoly, Based on the Pure Nash Equilibrium



7.2.3 Sensitivity Analysis: Effects of Other Firms' Decisions

We now consider the effects of other firms' investment decisions on the preparedness investment of a given firm. When neither investing nor not investing is a dominant strategy, firms may choose a mixed strategy that makes other firms indifferent between investing and not investing in preparedness. The resulting mixing probabilities can be computed by Gambit. Assuming the base case values, the unique Nash equilibrium is for no firm to invest in preparedness. Therefore, in order to illustrate our results we deviate from the base case values in computing the investment probabilities of firms.

For example, if we let $a = 530$ instead of 500, Figure 12(a) shows the probability of investing for those firms choosing a mixed strategy (when other firms are guaranteed not to invest in preparedness). In this case, one firm investing is the pure Nash equilibrium. Figure 12(a) shows that as the number of firms choosing a mixed strategy increases, each such firm is less likely to invest in preparedness. Under complete uncertainty (i.e., when no firms choose the pure strategy of not investing in preparedness), all five firms invest with probability 0.026. This can thus be considered a rational mixing probability under uncertainty. Figure 12(b) shows how the rational mixing probability varies as a function of the reservation price a for three different markets: (1) a market with five homogeneous firms; (2) a market with four homogeneous firms and one firm with a larger variable cost of production (i.e., $f_5 = 135$ vs. $f = 90$); and (3) a market with four homogeneous firms and one firm with a larger post-disaster increase in its variable cost of production (i.e., $\delta_5 = 135$ vs. $\delta = 60$). As the reservation price increases, the probability of investing in

preparedness increases in all three cases, but the investment probability for the four homogeneous firms is highest when the fifth firm has a larger variable cost of production, and lowest when the fifth firm has a larger post-disaster increase in its variable cost of production (since in that case, the fifth firm will itself invest with a larger probability).

Figure 12(c) shows the probability of investing for those firms choosing a mixed strategy when their parameter values are equal to those in base case, but letting $p = 0.06$ instead of 0.05. The pure Nash equilibrium in this case is for two firms to invest in preparedness. Thus, when two firms are already investing, the probability of any other firm investing at equilibrium will be zero. However, when either no or only one firm is investing in preparedness, then other firms can choose a mixed strategy. Therefore, we show two sets of mixing probabilities. One is for the case where no firm is guaranteed to invest in preparedness (and three, four, or five firms choose a mixed strategy). (Note that if only one or two firms choose a mixed strategy, they would invest with probability one. Therefore, that case is not shown here.). The other set of mixing probabilities is for the case where one firm invests in preparedness as a pure strategy (and two, three, or all four of the other firms choose the mixed strategy). We can see that the probability of any one firm investing in preparedness decreases as more firms choose a mixed strategy. Moreover, knowing that one firm has definitely invested in preparedness dramatically decreases the probability of investing in preparedness for those firms choosing a mixed strategy. Under complete uncertainty, all firms invest with probability 0.265.

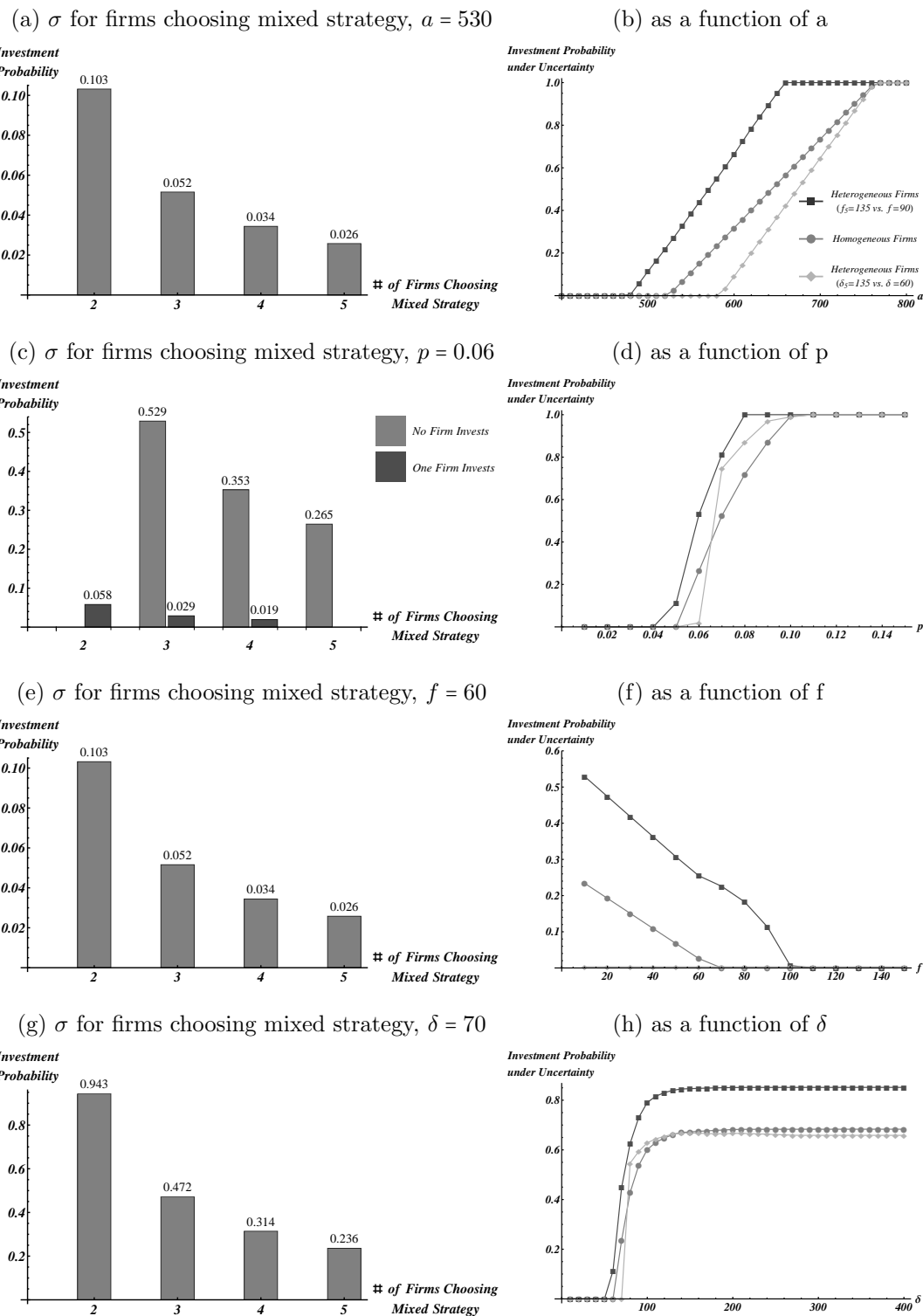
Figure 12(d) shows the rational mixing probability under uncertainty as a function of the probability of disaster, p , again for all three markets (one with five homogeneous firms, another one with four homogeneous firms and one that has a larger variable cost of production, and the last with four homogeneous firms and one that has a larger post-disaster increase in its variable cost of production). As the probability of disaster increases, the probability of investing in preparedness increases in all three cases. However, the probability of investing in preparedness for the homogeneous firms is highest in the case where one firm has a larger variable cost of production.

Figure 12(e) shows the probability of investing in preparedness for firms choosing a mixed strategy for parameter values equal to those in the base case, but letting $f = 60$ instead of 90. In this case, the only pure Nash equilibrium is for one firm to invest in preparedness. Therefore, Figure 12(e) shows the investment probability when two, three, four, or five firms choose a mixed strategy. As before, the rational mixing probability in this case is for all firms to invest in preparedness with probability 0.026. Figure 12(f) shows the rational mixing probability under uncertainty for the homogeneous firms, as a function of the variable cost of production. If one firm has a sufficiently large value of δ , then the other firms will not invest in preparedness at all, because the fifth invests with such high probability. For the homogeneous case and the case where the fifth firm has a different variable cost of production, then as the variable cost of production increases, the probability of investing in preparedness decreases. In addition, the investment probability when all firms choose a mixed strategy is lower in the homogeneous case than in the case

where one firm has a different variable cost of production. As illustrated in Figure 12(f), firms stop investing in preparedness altogether when the variable cost of production is sufficiently large.

Finally, Figure 12(g) shows the probability of investing in preparedness for firms choosing a mixed strategy for the base case values, but letting $\delta = 70$ instead of 60 (since with base case parameter values, the unique Nash equilibrium is for no firm to invest in preparedness). The rational mixing probability under complete uncertainty in this case equals 0.236. Figure 12(h) shows the rational mixing probability under uncertainty for the homogeneous firms as a function of the post-disaster increase in the variable cost of production. As δ increases, the probability of investing in preparedness increases in all three cases, but the investment probability of the homogeneous firms is highest when one firm has a larger variable cost of production, since the fifth firm will not invest with high probability in that case. Moreover, Figure 12(h) shows that when the post-disaster increase in the variable cost of production for the homogeneous firms is sufficiently large, all four homogeneous firms will choose a mixed strategy.

Figure 12: Investment Probabilities of Homogeneous Firms in an Oligopoly



8 An Alternative Model

The approach presented in Section 4 is of course not the only way of modeling the interaction between firms in a market. Until now, we have assumed that firms can adjust their production quantities in the event of a disaster. However, an alternative modeling approach might assume that each firm is committed to maintaining the same production quantity regardless of whether a disaster occurs (e.g., because of contractual requirements, or because the firms provide an essential service).

In this case, rather than determining separate disaster and non-disaster production quantities, firm i must choose a single production quantity $\tilde{q}_i^{(s)}$ to maximize its expected profit. In other words, under a given preparedness state, firm i chooses a single quantity to produce regardless of whether a disaster occurs. Under this alternative model, we show that the optimal production quantity of any given firm is less than or equal to the weighted average of the post-disaster and non-disaster optimal production quantities in the original model (weighted by the probability of a disaster). (Note, however, that this result depends critically on the assumption of risk-neutrality of the decision maker.)

We limit our analysis to firms that have a fixed cost of preparedness, since this was the more interesting case in our original model. Using equation (14), we can calculate the optimal production quantities for firm i under both disaster and non-disaster conditions as follows. Note that we limit ourselves only to firms that participate in the market in non-disaster conditions, so that the cardinality of the subset of firms with non-zero production quantities, $|\mathcal{I}_i^{(s,0)}|$,

is equal to n .

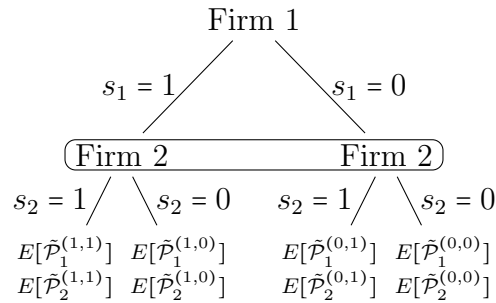
$$q_i^{*(s,1)} = \frac{1}{b(l+1)} \left(a - (l+1)RVC_i^{(s_i,1)} + \sum_{j=1}^l RVC_j^{(s_j,1)} \right) \quad (74)$$

$$q_i^{*(s,0)} = \frac{1}{b(n+1)} \left(a - (n+1)f_i + \sum_{j=1}^n f_j \right) \quad (75)$$

(However, the number of firms that would participate in the market in disaster conditions if they had the opportunity to adjust their production quantities after a disaster, $l = |\mathcal{I}_i^{(s,1)}|$, is less than or equal to n)

The weighted average of these quantities would be $pq_i^{*(s,1)} + (1-p)q_i^{*(s,0)}$. We now show that the optimal overall production quantity $\tilde{q}_i^{*(s)}$ is always less than or equal to the above weighted average. To do so, we determine the expected profit of each firm, and compute the optimal production quantities under a given preparedness state s . Let $E[\tilde{\mathcal{P}}_i^{(s)}]$ be the expected profit of firm i in this alternative model. Figure 13 below shows the extensive form of this alternative game for a duopoly.

Figure 13: Alternative Two-Person Game in Extensive Form



We can write down the expression for $E[\tilde{\mathcal{P}}_i^{(s)}]$ as follows

$$E[\tilde{\mathcal{P}}_i^{(s)}] = \left(a - b \sum_{j=1}^n \tilde{q}_j^{(s)} - f_i - p(1 - s_i)\delta_i \right) \tilde{q}_i^{(s)} - C_i s_i \quad (76)$$

The optimal quantities that maximize this expected profit can now be calculated by differentiating the above expression with respect to $\tilde{q}_i^{(s)}$ and setting the result equal to zero. Since there are n firms in the market, we will have n equations to solve simultaneously. These equations are similar to the expression in (12), but with the difference that $R\tilde{V}C_i^{(s_i)}$ will replace $RVC_i^{(s_i, d)} = f_i + \delta_i(1 - s_i)d$, where $R\tilde{V}C_i^{(s_i)} = f_i + \delta_i(1 - s_i)p$. Using this approach, we find the optimal overall production quantity of each *participating* firm as follows:

$$\tilde{q}_i^{x(s)} = \frac{1}{(1+n)b} \left(a - f_i - np(1 - s_i)\delta_i + p\Delta_{\sim i} + \Phi_i \right) \quad (77)$$

If $\left| \mathcal{I}_i^{(s,1)} \right| = n$, then the above equation for the optimal production quantity in

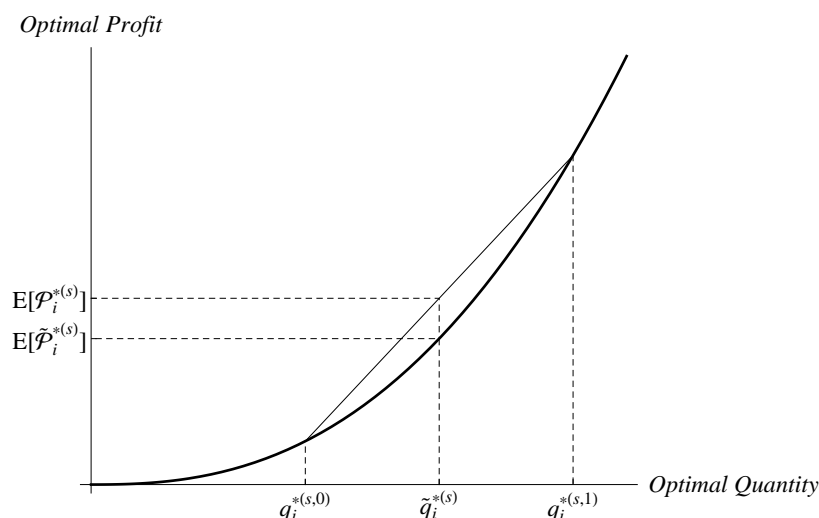
the alternative model is equal to $pq_i^{*(s,1)} + (1-p)q_i^{*(s,0)}$:

$$\begin{aligned}
& pq_i^{*(s,1)} + (1-p)q_i^{*(s,0)} \\
&= p\left(\frac{1}{b(n+1)}\left(a - (n+1)RVC_i^{(s_i,1)} + \sum_{j=1}^n RVC_j^{(s_j,1)}\right)\right) \\
&\quad + (1-p)\left(\frac{1}{b(n+1)}\left(a - (n+1)f_i + \sum_{j=1}^n f_j\right)\right) \\
&= \frac{p}{b(n+1)}\left(a - (n+1)(f_i + \delta_i(1-s_i)) + \sum_{j=1}^n f_j + \delta_j(1-s_j)\right) \\
&\quad + \frac{1-p}{b(n+1)}\left(a - (n+1)f_i + \sum_{j=1}^n f_j\right) \\
&= \frac{1}{b(n+1)}\left(a - (n+1)(f_i + p\delta_i(1-s_i)) + \sum_{j=1}^n f_j + p\delta_j(1-s_j)\right) \\
&= \frac{1}{b(n+1)}\left(a - f_i - np\delta_i(1-s_i) + p\Delta_{\sim i} + \Phi_i\right)
\end{aligned} \tag{78}$$

This quantity is equal to that in Equation (77). Similarly, we can show that when $|\mathcal{I}_i^{(s,1)}| < n$ (i.e., when not all firms would choose to produce after a disaster if they could adjust their production quantities), then $\tilde{q}_i^{*(s)} < pq_i^{*(s,1)} + (1-p)q_i^{*(s,0)}$. This results holds since for firms that cannot participate in a post-disaster market, their optimal production quantity is zero. This shows that the quantity firms produce in the alternative case is never greater than the weighted average of the quantities in the original model under the two possible cases of disaster and no disaster.

Moreover, it is straightforward to show that the expected profit under this alternative model will always be less than in the original model (and in fact strictly less, except when the disaster probability p is zero). This is because firms have less flexibility in this alternative model, and in particular cannot wait to find out whether a disaster happens before setting their production

Figure 14: Alternative Expected Profit vs. Original Expected Profit



quantities. Mathematically, if we assume that there are n participating firms in the market with profit functions of the form $\mathcal{P}_i = (a - b(\sum_{j=1}^n q_j) - f_i)q_i$ and optimal production quantities q_i^* , then the optimal level of profit for each firm as a result of the optimal level of production is $\mathcal{P}_i^* = \frac{1}{b}q_i^{*2}$. Hence, we get the graph shown in Figure 14.

While this alternative model may be realistic in some circumstances, we believe that firms will usually have the option of adjusting their production quantities after a disaster. For example, firms may continue to serve preferred clients with whom they already have contracts in place, but reduce the amount of product they sell on the “spot market,” or may default on contracts even at the expense of some penalty charges. Likewise, even providers of essential services such as electricity may put higher priority on some customers (e.g., hospitals), and lower priority on others (e.g., private residential customers).

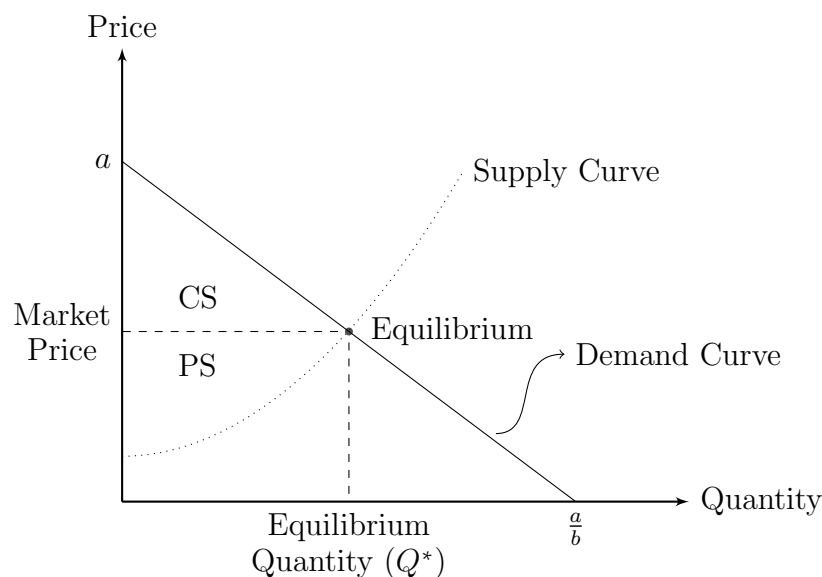
9 Social Optimum

In this section, we extend our model to address another important question – namely, how the equilibrium from our competitive model compares to the social optimum. The motivation for this question is that, if competitors are producing a product that is essential during an emergency (such as antibiotics), then if the consequences of their competitive decisions are too adverse (e.g., insufficient investment in preparedness, leading to inadequate availability of antibiotics), the government may wish to intervene (e.g., by using coordinating mechanisms to push firms toward a more desirable equilibrium). By contrast, if no disaster happens, we assume that firms would be left alone to compete and produce their equilibrium amounts.

The first step in addressing this question is to construct a measure of how good a given set of investment strategies is in terms of social benefit. Here, we are interested in ensuring the socially optimum investment decisions of firms, not the socially optimum total quantity produced by these firms, in part because it seems more credible for government to encourage or subsidize investment in preparedness than to dictate production quantities. In this regard, one could propose different measures of social benefit. For example, the total economic surplus to both producers and consumers might seem to be a good candidate for measuring benefit. This quantity is represented by the area above the supply curve and below the demand curve in Figure 15. In particular, the consumer surplus, CS , is the area under the demand curve and above a horizontal line representing the market price; the producer surplus, PS , is the area above the supply curve and below the horizontal line representing

the market price. (Note that since we do not know the supply curve explicitly, we instead compute producer surplus as the sum of all individual firms' profits.)

Figure 15: Consumer Surplus (CS) and Producer Surplus (PS)



However, since producer surplus is measuring the monetary benefit to producers from producing (rather than consuming) a good, one could theoretically omit producer surplus from the analysis and concentrate only on the consumer surplus (i.e., the benefit to consumers of enabling them to consume at a particular price). Intuitively, this might seem to be a good measure for the kind of benefit that an essential good could provide during a disaster. Mathematically, we can compute the consumer surplus given the total optimal production quantity, $Q^{*(s,d)}$, as follows:

$$CS^{(s,d)} = \int_0^{Q^{*(s,d)}} \pi^{(s,d)}(Q^{(s,d)})dQ - \int_0^{Q^{*(s,d)}} \pi^{(s,d)}(Q^{*(s,d)})dQ \quad (79)$$

Assuming that the price function is affine (as before), the above quantity equals $\frac{b}{2}(Q^{*(s,d)})^2$, a quadratic function. Therefore, more quantity means greater consumer surplus, and ultimately a greater social benefit. Let's consider a duopoly, where both firms can participate in the post-disaster market, with or without investing in preparedness, but the firms need not be homogeneous. The total optimal production quantities under different post-disaster market conditions are as follows:

$$\begin{aligned}
\text{Both firms invest: } Q^{*((1,1),1)} &= \frac{1}{3b}(2a - f_1 - f_2) \\
\text{Only firm 1 invests: } Q^{*((1,0),1)} &= \frac{1}{3b}(2a - f_1 - f_2 - \delta_2) \\
\text{Only firm 2 invests: } Q^{*((0,1),1)} &= \frac{1}{3b}(2a - f_1 - f_2 - \delta_1) \\
\text{No firm invests: } Q^{*((0,0),1)} &= \frac{1}{3b}(2a - f_1 - f_2 - \delta_1 - \delta_2)
\end{aligned} \tag{80}$$

(Note that the non-disaster total optimal production under any market condition is also equal to $Q^{*((1,1),1)}$.)

Comparing these quantities reveals that in a post-disaster market, the total optimum production quantity will always be maximized when both firms invest in preparedness. Therefore, if consumer surplus were the measure of social optimality, then the socially preferred decision would always be for all firms to invest in preparedness. Not only does this result ignore the cost of production, but it will sometimes result in over-investment in preparedness. Therefore, we will use the total economic surplus as a metric for the socially preferred alternative. In particular, we compare the economic surplus under each possible set of investment decisions to find the decision that generates the highest economic surplus.

In order to compute the economic surplus for the more general case of n firms for $n \geq 2$, $ES^{(s,d)}$, we need to compute both the producer surplus and the consumer surplus. As noted above, we are most interested in achieving the social optimum when a disaster happens. Therefore, we compute the consumer and producer surplus only for the case when a disaster happens ($d = 1$). Using the optimal production quantities from Section 4, the economic surplus can be written as follows:

$$\begin{aligned}
ES^{(s,1)} &= CS^{(s,1)} + PS^{(s,1)} \\
&= \frac{b}{2}(Q^{*(s,1)})^2 + \sum_{i=1}^l \mathcal{P}_i^{*(s,1)} \\
&= \frac{b}{2}\left(\sum_{i=1}^l q_i^{*(s,1)}\right)^2 + \sum_{i=1}^l \mathcal{P}_i^{*(s,1)} \\
&= \frac{b}{2}\left(\sum_{i=1}^l \frac{1}{b(l+1)}(a - (l+1)RVC_i^{(s_i,1)} + TRVC^{(s,1)})\right)^2 \\
&\quad + \sum_{i=1}^l \left(\frac{1}{b(l+1)^2}(a - (l+1)RVC_i^{(s_i,1)} + TRVC^{(s,1)})^2 - FC_i^{(s_i)}\right) \\
&= \frac{1}{2b(l+1)^2}\left(\sum_{i=1}^l (a - (l+1)RVC_i^{(s_i,1)} + TRVC^{(s,1)})\right)^2 \\
&\quad + \frac{1}{b(l+1)^2}\sum_{i=1}^l (a - (l+1)RVC_i^{(s_i,1)} + TRVC^{(s,1)})^2 - \sum_{i=1}^l FC_i^{(s_i)}
\end{aligned} \tag{81}$$

where as before, $TRVC^{(s,1)} = \sum_{k=1}^l RVC_k^{(s_k,1)}$ is the total of the realized variable costs of production of all participating firms, and $l^{(s,d)} = |\{i | \rho(i)^{(s,d)} \geq 0\}|$ is the total number of firms with non-zero production quantities.

This expression for the economic surplus can be rewritten in terms of the preparedness decisions s_i of the individual firms as follows:

$$\begin{aligned}
ES^{(s,1)} &= \frac{z}{2} \left(\sum_{i=1}^l (X_i + Y_i) \right)^2 + z \sum_{i=1}^l (X_i + Y_i)^2 - \sum_{i=1}^l (s_i C_i) \\
&= \frac{z}{2} \left(\sum_{i=1}^l X_i \right)^2 + \frac{z}{2} \left(\sum_{i=1}^l Y_i \right)^2 + z \left(\sum_{i=1}^l X_i \right) \left(\sum_{i=1}^l Y_i \right) \\
&\quad + z \left(\sum_{i=1}^l X_i^2 \right) + z \left(\sum_{i=1}^l Y_i^2 \right) + 2z \left(\sum_{i=1}^l (X_i Y_i) \right) - \sum_{i=1}^l (s_i C_i)
\end{aligned} \tag{82}$$

where

$$\begin{aligned}
X_i &= a - (l+1)(f_i + \delta_i) + \sum_{j=1}^l (f_j + \delta_j) \\
Y_i &= (1+l)\delta_i s_i - \sum_{j=1}^l (\delta_j s_j) \\
z &= \frac{1}{b(l+1)^2}
\end{aligned} \tag{83}$$

(Note that the quantity X_i does not depend on the investment decision of any firm.)

We can simplify some of the terms in the above expression as follows:

$$\begin{aligned}
\left(\sum_{i=1}^l Y_i \right)^2 &= \left(\sum_{i=1}^l (\delta_i s_i) \right)^2 \\
\sum_{i=1}^l Y_i^2 &= ((1+l)^2 - l - 2) \sum_{i=1}^l (\delta_i s_i)^2 - 2(l+2) \sum_{i=1}^l \left(\sum_{j=i+1}^l (\delta_i s_i \delta_j s_j) \right) \\
\sum_{i=1}^l (X_i Y_i) &= \left(a + (2+l) \sum_{i=1}^l (f_i + \delta_i) \right) \sum_{i=1}^l (\delta_i s_i) - (l+1)^2 \sum_{i=1}^l ((f_i + \delta_i) \delta_i s_i) \\
\sum_{i=1}^l X_i \sum_{i=1}^l Y_i &= \left(la - \sum_{i=1}^l (f_i + \delta_i) \right) \sum_{i=1}^l (\delta_i s_i)
\end{aligned} \tag{84}$$

Substituting these back into the original expression for economic surplus, we

have:

$$\begin{aligned}
ES^{(s,1)} &= \frac{z}{2} \left(\sum_{i=1}^l X_i \right)^2 + z \left(\sum_{i=1}^l X_i^2 \right) + z \left(l^2 + l - \frac{1}{2} \right) \left(\sum_{i=1}^l (\delta_i s_i)^2 \right) \\
&\quad + z \left(a(l+2) + (4+2l-1) \sum_{i=1}^l (f_i + \delta_i) \right) \left(\sum_{i=1}^l (\delta_i s_i) \right) \\
&\quad - 2z(l+1)^2 \left(\sum_{i=1}^l ((f_i + \delta_i) \delta_i s_i) \right) - \sum_{i=1}^l (s_i C_i) \\
&\quad + (z - 2z(l+2)) \sum_{i=1}^l \left(\sum_{j=i+1}^l (\delta_i s_i \delta_j s_j) \right)
\end{aligned} \tag{85}$$

Letting $s_i^2 = s_i$, because $s_i \in \{0, 1\}$, we can further simplify the expression for the economic surplus as follows:

$$\begin{aligned}
ES^{(s,1)} &= \frac{z}{2} \left(\sum_{i=1}^l X_i \right)^2 + z \left(\sum_{i=1}^l X_i^2 \right) \\
&+ \sum_{i=1}^l \left[\left(z \delta_i \left((2+l)a - 2(1+l)^2 f_i - \left(\frac{5}{2} + l(3+l) \right) \delta_i + (3+2l) \sum_{i=1}^l (f_i + \delta_i) \right) - C_i \right) s_i \right] \\
&\quad + (z - 2z(l+2)) \sum_{i=1}^l \left(\sum_{j=i+1}^l (\delta_i \delta_j s_i s_j) \right)
\end{aligned} \tag{86}$$

The formulation above shows that with no investment in preparedness, the level of economic surplus would simply equal

$$ES^{(0,1)} = \frac{z}{2} \left(\sum_{i=1}^l X_i \right)^2 + z \left(\sum_{i=1}^l X_i^2 \right) \tag{87}$$

If firm i invests in preparedness, then the economic surplus would increase by the coefficient of s_i in Equation (86), which is

$$z \delta_i \left((2+l)a - 2(1+l)^2 f_i - \left(\frac{5}{2} + l(3+l) \right) \delta_i + (3+2l) \sum_{i=1}^l (f_i + \delta_i) \right) - C_i \tag{88}$$

The last term in Equation (86) shows the effect of any two firms investing in

preparedness on the economic surplus. The coefficient for $s_i s_j$ is $(z - 2z(l + 2))\delta_i \delta_j$, which is always less than zero, since z and l are both positive. Therefore, if both firms i and j invest in preparedness, the economic surplus will be smaller than what would be predicted based on the effects of the two firms individually investing in preparedness.

To formulate an optimization problem that finds the optimal set of firms that should invest in preparedness to maximize the economic surplus, we extend our current notation for the number of participating firms l to reflect the fact that under different investment strategies s , we could have different number of firms participating in the post-disaster market, $l^{(s)}$. Also, we update the notation for κ_i and $\kappa_{i,j}$ in the same manner. Thus, we can write our new optimization problem as follows:

$$\begin{aligned}
Max_s \quad & \sum_{i \in \mathcal{I}_i^{(s,1)}} (\kappa_i^{(s)} - C_i) s_i + \frac{1}{2} \sum_{(i,j) \in \mathcal{I}_{i,j}^{(s,1)}} \kappa_{i,j}^{(s)} s_i s_j \\
& + \frac{1}{2b(l^{(s)} + 1)^2} \left(\sum_{i \in \mathcal{I}_i^{(s,1)}} (a - (l^{(s)} + 1)(f_i + \delta_i) + \sum_{j \in \mathcal{I}_i^{(s,1)}} f_j + \delta_j) \right)^2 \\
& + \frac{1}{b(l^{(s)} + 1)^2} \left(\sum_{i \in \mathcal{I}_i^{(s,1)}} (a - (l^{(s)} + 1)(f_i + \delta_i) + \sum_{j \in \mathcal{I}_i^{(s,1)}} f_j + \delta_j) \right)^2 \\
s.t. \quad & s_i \in \{0, 1\} \quad \forall i \in \{1, 2, \dots, n\} \\
& \mathcal{I}_i^{(s,1)} = \{j \mid RVC_j^{(s_j,1)} \leq RVC_i^{(s_i,1)}\} \quad \forall i \in \{1, 2, \dots, n\} \\
& \rho(i)^{(s,1)} = a - (|\mathcal{I}_i^{(s,1)}| + 1)RVC_i^{(s_i,1)} + \sum_{j \in \mathcal{I}_i^{(s,1)}} RVC_j^{(s_j,1)} \quad \forall i \in \{1, 2, \dots, n\} \\
& l^{(s)} = Max_i \{ |\mathcal{I}_i^{(s,1)}| \mid \rho(i)^{(s,1)} \geq 0 \} \\
& \kappa_i^{(s)} = \frac{\delta_i}{b(l^{(s)} + 1)} \left((2 + l^{(s)})a - 2(1 + l^{(s)})^2 f_i \right. \\
& \quad \left. - \left(\frac{5}{2} + l^{(s)}(3 + l^{(s)}) \right) \delta_i + (3 + 2l^{(s)}) \sum_{i \in \mathcal{I}_i^{(s,1)}} (f_i + \delta_i) \right) \\
& \kappa_{i,j}^{(s)} = \frac{1 - 2(l^{(s)} + 2)}{b(l^{(s)} + 1)} \delta_i \delta_j
\end{aligned} \tag{89}$$

This formulation allows us to find the subset of firms that should invest

in preparedness in order to maximize the economic surplus. For the purpose of our numerical studies, we find the optimal set of investment strategies that produces the maximum level of economic surplus by comparing all possible combinations of investment strategies.

Now, our task is to compare the social optimum to the Nash equilibrium. We are interested in whether the social optimum is the Nash equilibrium (or one of several Nash equilibria), and if not, how we can achieve the socially preferred outcome.

In Figure 16(a), consider the point representing the base case (i.e., $a = 500$ and $C = 4000$), as shown by the circle. We can see that the pure Nash equilibrium at this point is for no firm to invest in preparedness. We can also read off the corresponding social optimum at this point from Figure 16(b), indicating the economic surplus will be maximized when all firms invest in preparedness. Therefore, in this case, the social optimum is not the Nash equilibrium. In order to steer the preparedness decisions of firms toward the socially preferred alternative, we need to decrease the preparedness cost, as shown by the arrow in Figure 16(a). Figure 16(c) indicates the amount by which investment would need to be subsidized to move firms from the pure Nash equilibrium to the corresponding social optimum. Therefore, we need to spend approximately $4,000 - 1,900 = 2,100$ per firm, or approximately 10,500 in total, to ensure that the pure Nash equilibrium contains the socially preferred outcome. In Figure 16(c), the darker the region, the more subsidy is required to achieve the social optimum. Similarly, Figures 17 through 20 show this comparison as a function of the slope of the demand curve, the variable

cost of production, the disaster probability, and the increase in the variable cost of production, respectively. (Note that when the social optimum involves some firms not investing, the subsidy would be provided only to those firms that invest, reducing the cost of achieving the social optimum.)

Figure 16: Social Optimum and Comparison with Nash Equilibrium (using base case values), as a function of a

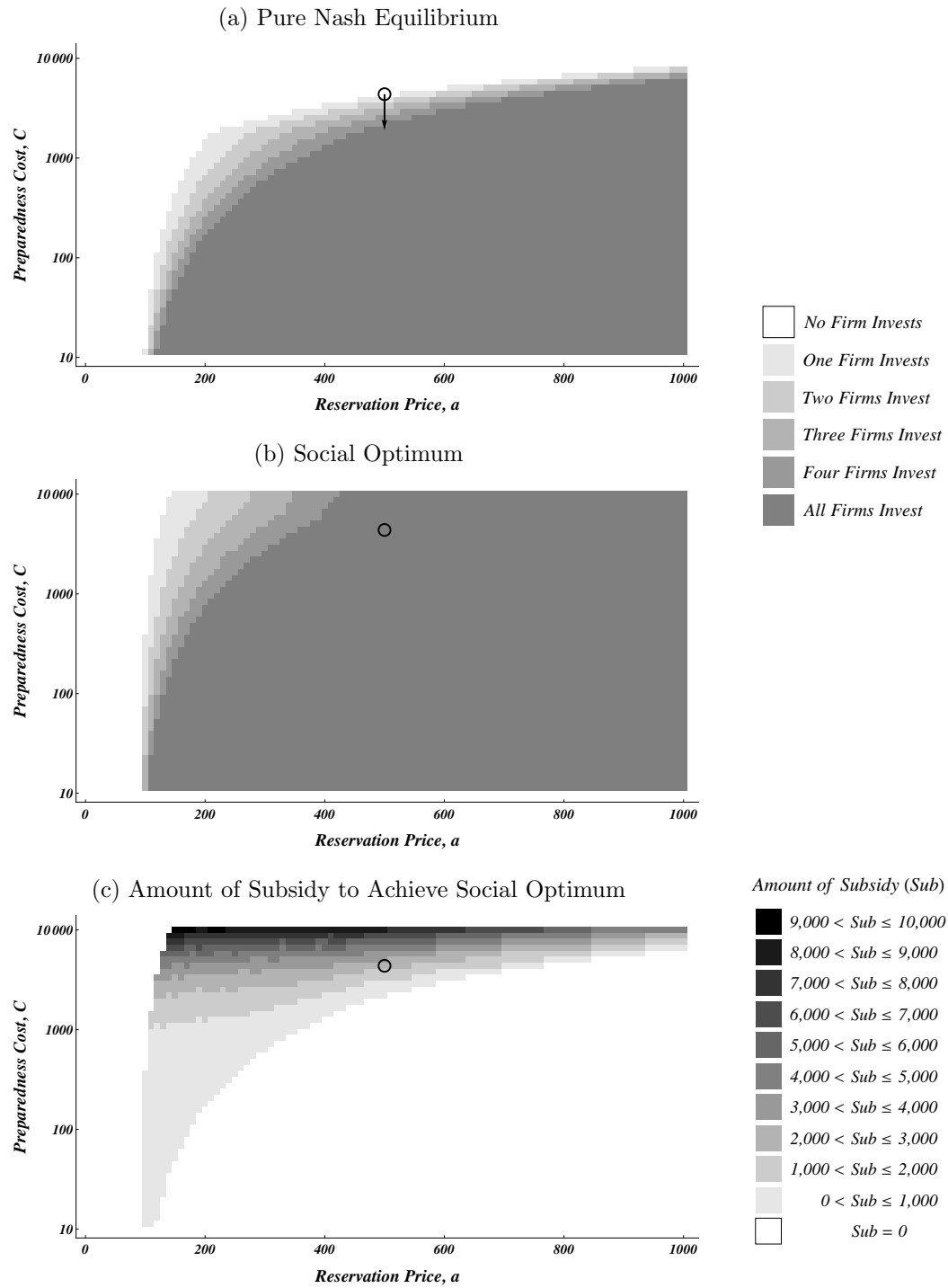


Figure 17: Social Optimum and Comparison with Nash Equilibrium (using base case values), as a function of b

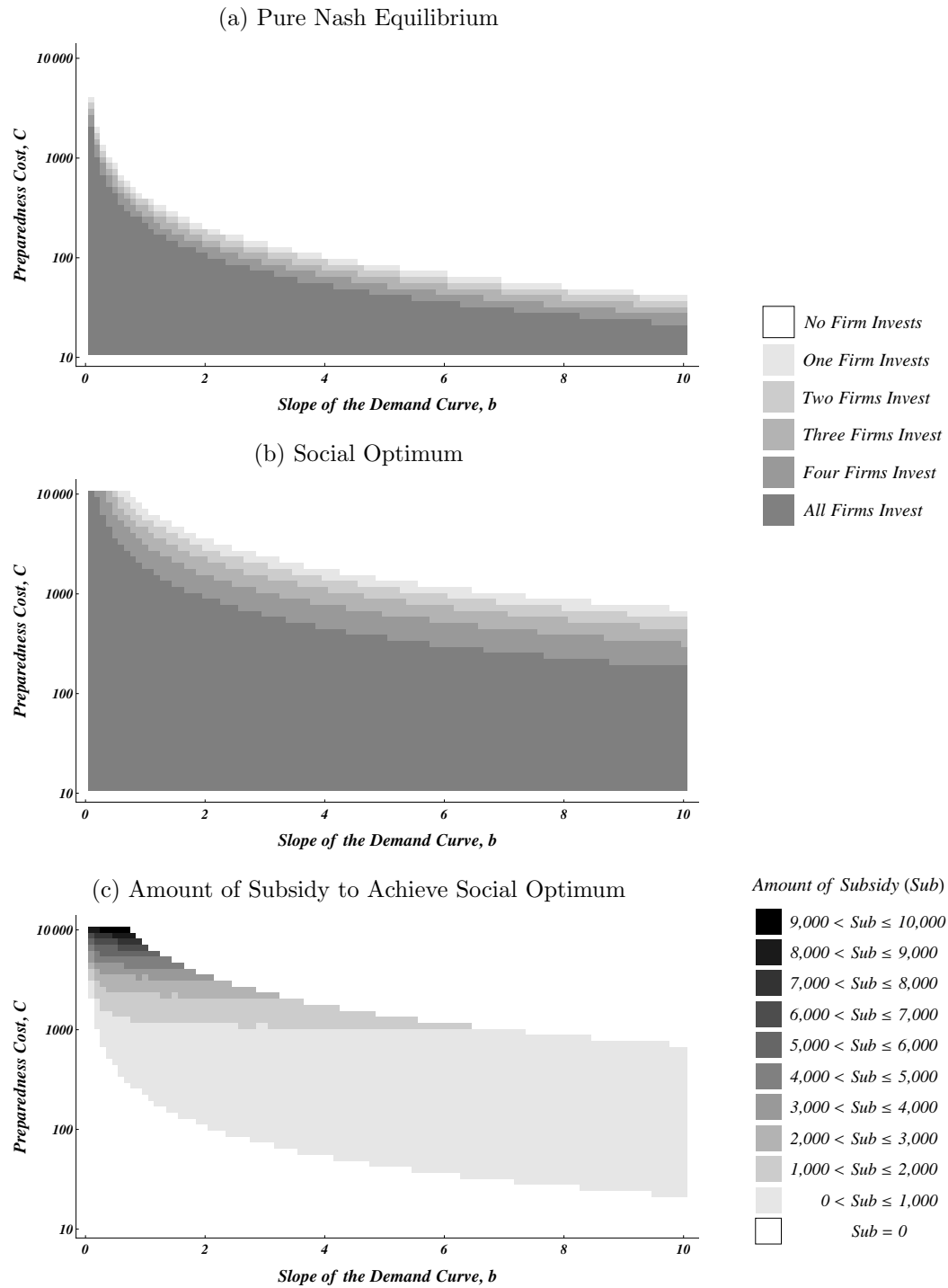


Figure 18: Social Optimum and Comparison with Nash Equilibrium (using base case values), as a function of p

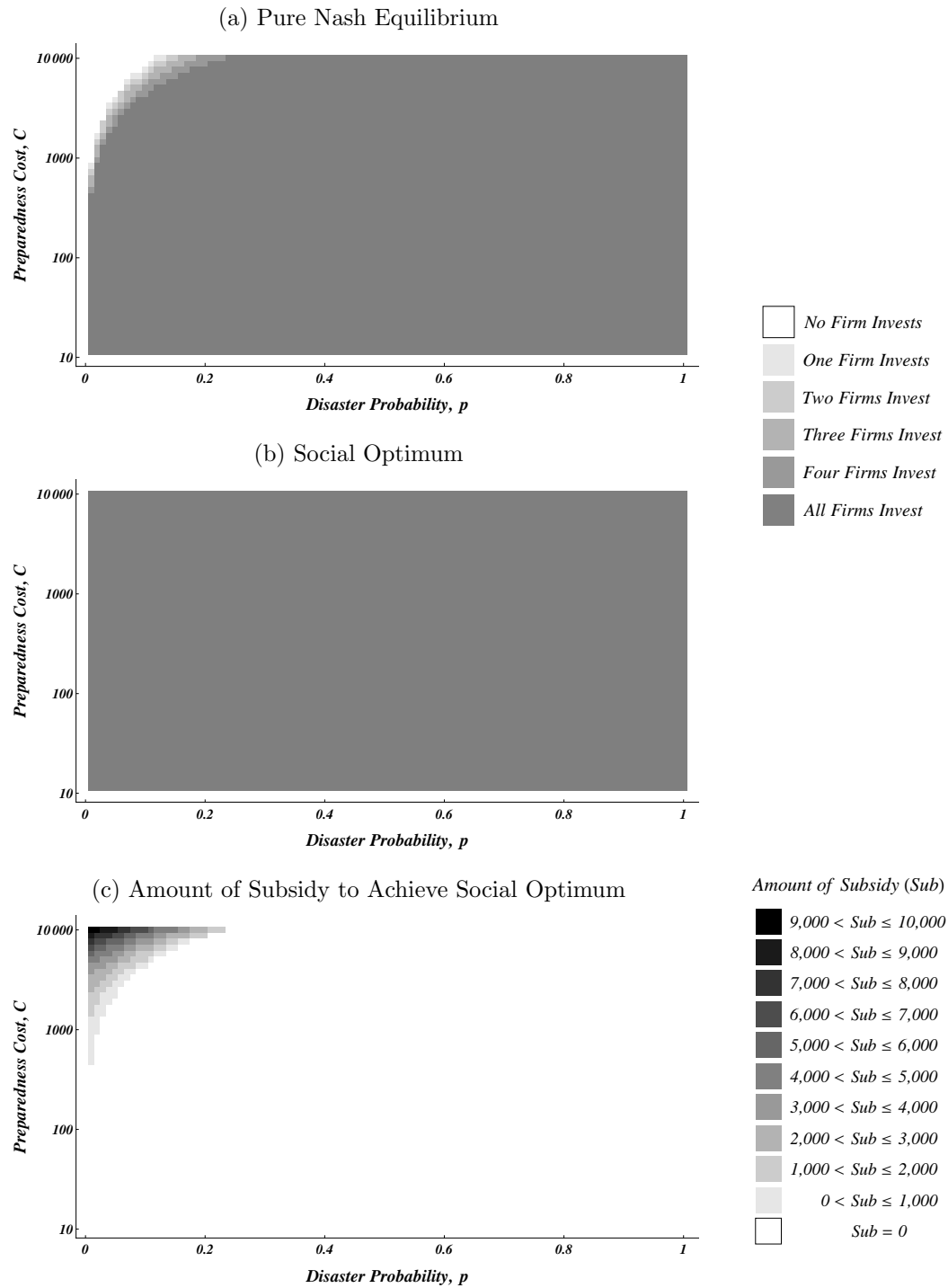


Figure 19: Social Optimum and Comparison with Nash Equilibrium (using base case values), as a function of f

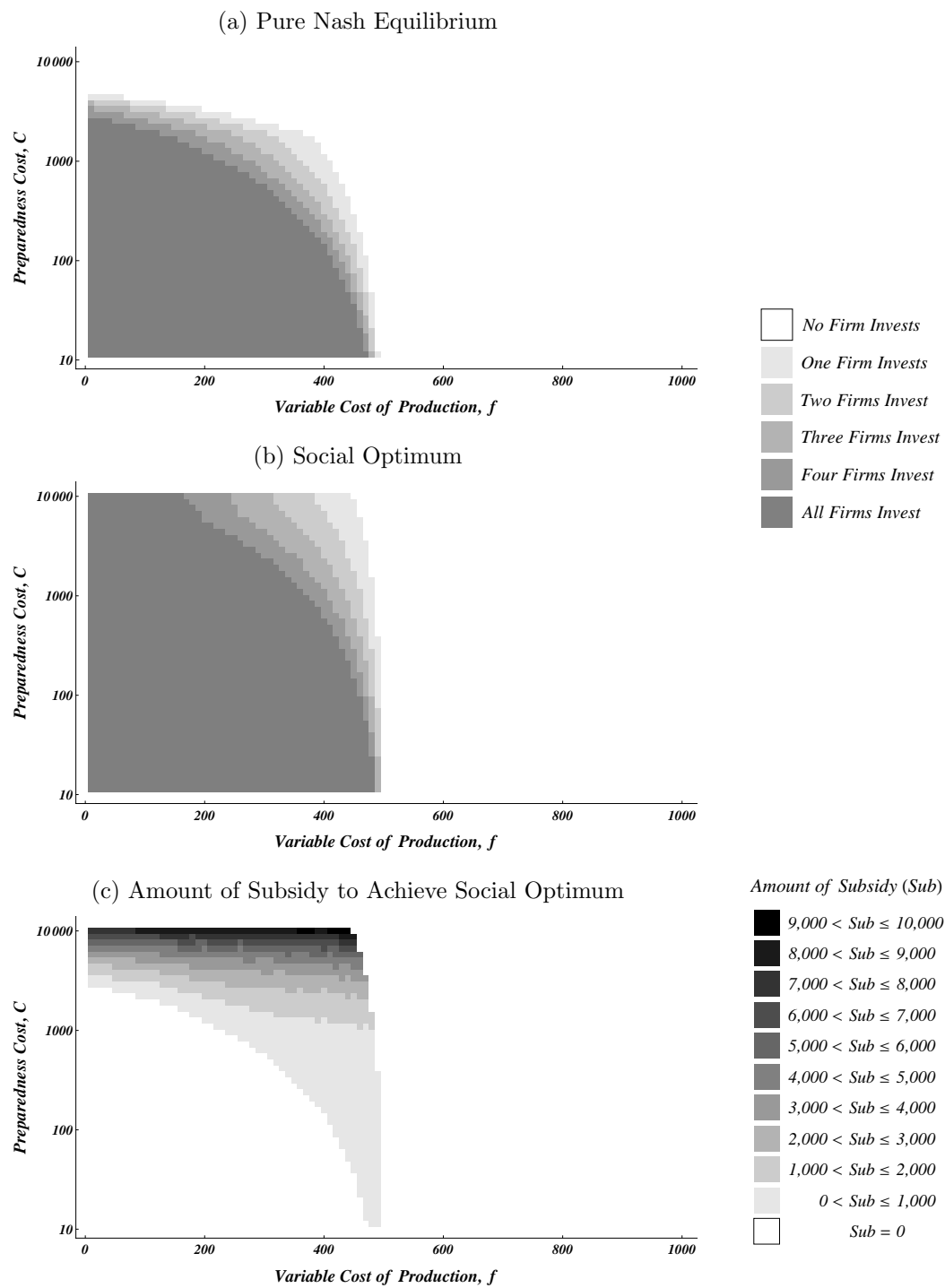
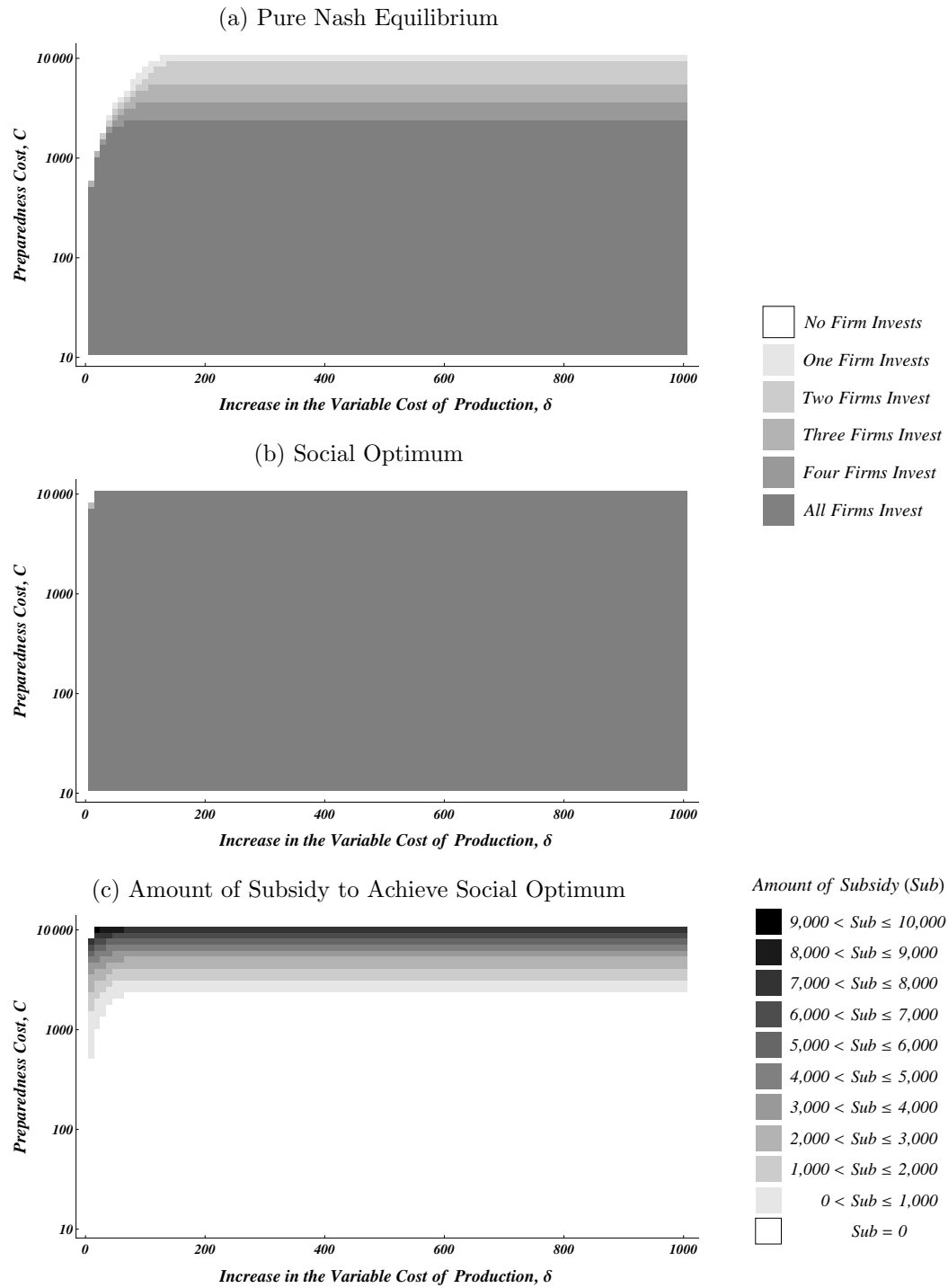


Figure 20: Social Optimum and Comparison with Nash Equilibrium (using base case values), as a function of δ



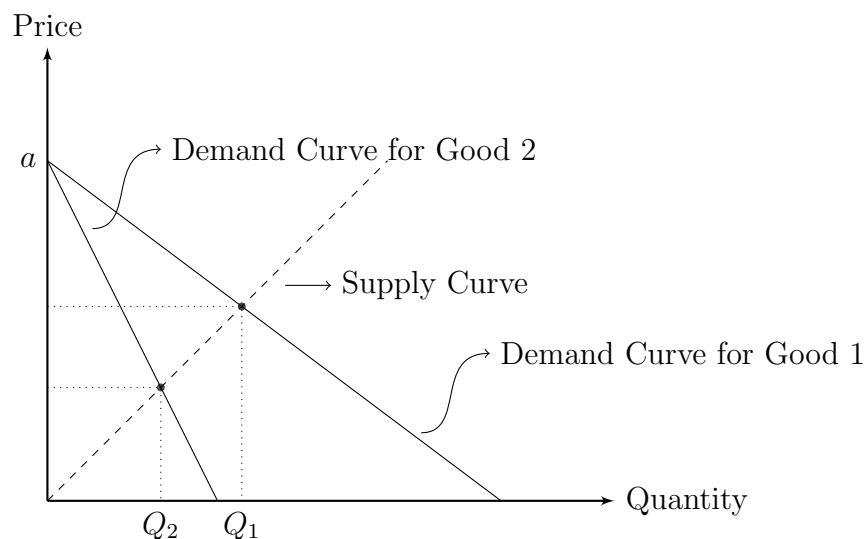
In order to see when achieving the social optimum will be important, we can look at how critical goods are in the post-disaster market. For example, under our assumption of a linear demand curve, we can characterize goods according to their slope of the demand function ($-b$). The values of this parameter may help to indicate which goods are likely to be important during a disaster (e.g., antibiotics) and which are not as important (e.g., office chairs). In particular, we would anticipate that important goods (e.g., goods that are essential during a disaster) would tend to have relatively inelastic demand curves, since inelastic demand is associated with goods that have low availability of substitutes (Frank, 2008), are highly necessary (Parkin et al., 2002), and cannot easily be produced in large quantities on short notice (Goodwin et al., 2009). For example, Figure 21 below depicts the demand curves for two goods, where demand for good 1 is relatively inelastic compared to that for good 2. In this example, both goods have the same reservation price, a , but differ in the slope of their demand curves ($b_1 > b_2$).

To put the elasticity of demand into perspective, note that it measures the responsiveness of the demanded quantity to changes in price. For a linear demand curve, the price elasticity of demand, PED , is as follows:

$$PED = \frac{\partial Q}{\partial P} \frac{P}{Q} = \left(\frac{-1}{b}\right) \left(\frac{a - bQ}{Q}\right) = 1 - \frac{a}{bQ} \quad (90)$$

Here, PED is always less than or equal to zero, since quantities greater than

Figure 21: Illustration of the Price Elasticity of the Demand Curve



$\frac{a}{b}$ would result in a negative price¹⁰. Therefore, in Figure 17(c), the larger the value of the slope b (*ceteris paribus*), the more important a particular good will be, and the larger the subsidy that would be required to induce firms to undertake the socially optimum level of investment in preparedness.

We now consider duopolies, where the two participating firms are not homogeneous. Assume that the price function is given by $500 - 0.1Q^{(s,d)}$, and the disaster probability is 0.05. Clearly, when the social optimum is the Nash equilibrium, then no intervention is necessary. For example, consider the pa-

¹⁰Most estimates for the *PED* of different goods are based either on historical data, or on current surveys of consumer preferences. For example, Samuelson and Nordhaus (2001) report a *PED* of -0.31 for medical insurance in the U.S., assuming constant elasticity of demand for medical insurance. Since this is greater than -1, the demand for medical insurance is inelastic, reflecting the fact that people do not easily choose to forgo medical insurance. In another example from the literature, Pindyck (1979) estimated the short-run (i.e., one-year) *PED* of gasoline to be -0.11, compared to the long-run (20-year) *PED* of -1.17. Similarly, Anderson et al. (1997) report that the short-run *PED* for airline travel is -0.1, compared to a long-run *PED* of -2.4.

parameter values shown in Table 13. The unique Nash equilibrium of that game is for both firms to invest in preparedness, and the economic surplus will be maximized when both firms invest in preparedness. Therefore, this situation does not require any interventions.

Table 13: Parameter Values where the Nash Equilibrium is the Social Optimum

Parameter	Firm 1	Firm 2
f_i	100	50
δ_i	20	20
C_i	1000	1000

By contrast, if the social optimum is one of multiple Nash equilibria, then cost-free coordinating mechanisms will exist that achieve the social optimum; in such cases, just provision of information on “best practices” may be sufficient to ensure that the social optimum becomes a “focal equilibrium” (Schelling, 1960). For example, for the parameter values in Table 14, either firm investing in preparedness is a Nash equilibrium. However, the socially preferred investment strategy is for firm 2 to invest. In this situation, sharing information regarding the socially preferred alternative (e.g., encouraging firms to invest in preparedness if they have a small variable cost of production but a large post-disaster increase in the variable cost of production) would in theory be enough to achieve the social optimum.

Finally, in cases where the social optimum is not an equilibrium (or no Nash equilibrium exists), then subsidies would be required to achieve the social optimum. These subsidies may not in general be equal for the participating firms. For example, if the firms in the market have parameter values as given

Table 14: Parameter Values where One of Multiple Nash Equilibria is the Social Optimum

Parameter	Firm 1	Firm 2
f_i	270	90
δ_i	20	60
C_i	300	7200

in Table 15, the social optimum is for both firms to invest in preparedness. However, either firm investing in preparedness is a Nash equilibrium.

Table 15: Parameter Values where the Nash Equilibrium is not the Social Optimum

Parameter	Firm 1	Firm 2
f_i	90	95
δ_i	55	60
C_i	5000	5000

Here, if we target only one firm for subsidies, then we will not achieve the socially preferred alternative. Both firms must receive subsidies, but the amount of subsidy required by each firm will not be equal. In fact, firm 1 requires more subsidy in order to invest than firm 2, since firm 1 has a lower variable cost of production for disaster and non-disaster conditions. The minimum amount of subsidy firm i requires to invest in preparedness, Sub_i , can be calculated by comparing its current fixed cost of preparedness to the

lower bound in Equation (52), as follows:

$$\begin{aligned}
 Sub_1 = C_1 - \underline{C}_1 &= \frac{4p\delta_1}{9b}(a - f_1 + \Phi_1 - \delta_1) = \\
 5,000 - \frac{11}{0.9}(500 - 90 + 5 - 55) &= 5,000 - 4,400 = 600 \\
 Sub_2 = C_2 - \underline{C}_2 &= \frac{4p\delta_2}{9b}(a - f_2 + \Phi_2 - \delta_2) = \\
 5,000 - \frac{12}{0.9}(500 - 95 - 5 - 60) &= 5,000 - 4,533.333 = 466.667
 \end{aligned} \tag{91}$$

If we assume that we are constrained to give equal subsidies to both firms, however, then we would need to offer a subsidy of 600 to both firms.

To summarize, we have considered the case of multiple homogeneous firms (where we were able to find the magnitude of subsidy required to achieve the social optimum), and also the case of two heterogeneous firms (where the socially preferred option may be achieved either with subsidies, or by simply providing information to firms, when multiple equilibria exist). It may be argued that subsidies are not likely to be politically acceptable in practice; however, governments have numerous other ways to encourage investment in preparedness: 1) legislation or regulation (Kunreuther and Heal, 2003); 2) direct state provision of goods and services needed for producer preparedness; 3) fiscal policy interventions such as taxes (Schelling, 1978; Gladwell, 2002; Zhuang et al., 2007); or 4) requiring investment in preparedness for firms that wish to bid on government contracts. In some cases, even just provision of information on the nature of recommended investment could serve as a subsidy, by reducing the amount of staff time firms need to devote to researching and choosing good investment strategies.

10 Conclusions and Future Directions

In this work, we modeled the preparedness investment decisions of firms participating in a Cournot market in which firms independently choose their optimal output levels, and the price is determined by supply and demand. We assumed that profit-maximizing firms determine whether to invest in preparedness by comparing their expected profits with and without investing in preparedness, assuming that for either preparedness decision (i.e., invest or not invest), each firm would produce the corresponding optimal production quantity. We computed the optimal production quantities of each firm, and built a game-theoretic model to represent their interaction. We considered two cases of preparedness: one where the preparedness cost depends on the level of production; and one where the preparedness cost is fixed. We showed that firms facing a variable preparedness cost choose their optimal actions independently, with no dependence on the investment decisions of other firms. However, when preparedness involves a fixed cost, we showed that for intermediate values of the preparedness cost, some firms will choose to invest in preparedness (in order to obtain an advantage over their competitors in the event of a disaster), while other firms will forgo preparedness investment in order to become low-cost producers in the non-disaster market. This conclusion supports our hypothesis that competition can lead firms not to invest in preparedness.

We studied the sensitivity of our results to variations in the input parameters. The results showed that as the reservation price, the probability of a disaster, or the post-disaster increase in the variable cost of preparedness

increases, more firms will choose to invest in preparedness. However, as the slope of the demand curve or the variable cost of production increases, fewer firms will invest in preparedness. Of course, if the fixed cost of preparedness is extremely large, firms cannot economically invest in preparedness at all, since doing so will result in a negative profit even when the optimal production quantity is positive. Finally, we compared the Nash equilibrium to the socially optimum investment strategy, and determined the magnitude of subsidies or incentives required to achieve the social optimum.

We believe that our results to date can already be useful in understanding the interaction of firms in the face of a possible disaster, and the mechanisms for possible interventions in the market. However, our model could also usefully be extended.

First, it would be interesting to consider the investment behavior of firms in a multi-period framework (e.g., to study the effects of negative profits or carryovers). For example, firms that have invested in preparedness in a previous period, may be able to invest less in the next period, especially if a disaster did not materialize yet (Zhuang, 2010).

Another possible extension to our model would be to allow new firms that were not participating in the non-disaster market to enter the post-disaster market. We have showed that such firms can exist. The fact that high post-disaster prices could justify their entrance into the market is an encouraging feature, providing more ways to meet consumer needs. It may also be worthwhile to consider heterogeneous goods, in which case the goods produced by different firms are not perfect substitutes.

Yet another extension to our model would be to consider the effects of disaster preparedness in more complex supply chains. First, investing in preparedness is typically not a binary decision; firms may need to choose among a number of preparedness strategies, such as maintaining higher levels of inventory versus choosing a more reliable (but higher cost) supplier. Second, if we consider a multi-level supply chain, we see that preparedness decisions must be made by firms at all levels of the supply chain. Thus, for example, when a manufacturer is choosing between more and less reliable suppliers, the reliability of those suppliers is generally not exogenous, but rather chosen by the suppliers' own decisions about whether to maintain higher levels of inventory, work with more reliable lower-level suppliers, etc. Thus, firms at different levels of a supply chain can be modeled as determining their preparedness investments simultaneously and endogenously, as is done in other types of multi-echelon supply-chain management.

One problem with “blackboard economics”¹¹ such as our analysis is that we did not consider the real-world behavior of firms. In our case, we assumed that firms are maximizing a known profit function, something that is known not to always be the case in practice (Simon, 1979). In particular, we assumed that rational decision makers can observe all relevant input variables, perform the needed computations, and make an unbiased decision. However, research in the field of behavioral decision making has shown that people are far from perfect decision makers, and prone to numerous biases (Aldag, 2012). For example, some firms might follow the investment strategies used by their com-

¹¹This term was coined by Ronald Coase.

petitors, even if not optimal for their situations. Likewise, Kunreuther and Michel-Kerjan (2011) has shown that many decision makers tend to invest in preparedness immediately after a disaster, but cease investing too soon; encouraging long-term commitments to investment strategies (e.g., through contracts) could mitigate this problem. Observing the actual preparedness behavior of firms through surveys of managers (e.g., Smith et al., 2007 and Pillsbury-Levick, 2011) could also help to develop more realistic models of firm behavior.

Finally, using realistic values for our model parameters could increase the accuracy and applicability of our results in practice. Thus, case studies focusing on estimating these parameters in particular markets could be useful.

11 Bibliography

- Aldag, R. (2013). Bounded Rationality and Satisficing (Behavioral Decision Making Model). Encyclopedia of Management Theory (ed: Eric Kessler).
- Aldag, R. (2012). Distinguished Scholar Invited Essay Behavioral Decision Making: Implications for Leadership and Organizations. *Journal of Leadership & Organizational Studies* 19,2:133-141.
- Anderson, P. L., R. McLellan, J. P. Overton, and G. Wolfram. (1997). The Universal Tuition Tax Credit: A Proposal to Advance Parental Choice in Education. <http://www.mackinac.org/archives/1997/s1997-04.pdf>
- Ayres, I. and S. D. Levitt. (1998). Measuring Positive Externalities from Unobservable Victim Precaution: An Empirical Analysis of Lojack. *The Quarterly Journal of Economics* 113,1:43-77.
- Ayres, R. U. and A. V. Kneese. (1969). Production, Consumption, and Externalities. *The American Economic Review* 59,3:282-297.
- Banker, R. D., I. Khosla, and K. K. Sinha. (1998). Quality and Competition. *Management Science*, 44,9:1179-1192.
- Berger, A., C. Kousky, and R. Zeckhauser. (2008). Obstacles to Clear Thinking About Natural Disasters: Five Lessons for Policy. *Risking House and Home: Disasters, Cities, Public Policy* (eds.: J. M. Quigley and L. A. Rosenthal). Berkeley Public Policy Press. Berkeley, CA.
- Bertrand, J. (1883). Book Review of *Theorie Mathematique de la Richesse Sociale* and of *Recherches sur les Principes Mathematiques de la Theorie des Richesses*. *Journal de Savants* 67: 499-508.
- Bier, V. M., S. Oliveros, and L. Samuelson. (2007). Choosing What to Protect: Strategic Defense Allocation Against an Unknown Attacker. *Journal of Public Economic Theory* 9,4:563-587.

- Borenstein S., J. Bushnell, and C. Knittel. (1997). A Cournot-Nash Equilibrium Analysis of the New Jersey Electricity Market. http://www.ucei.berkeley.edu/PDF/sb_jersey.pdf
- Brown, M. A. (2001). Market failures and barriers as a basis for clean energy policies. *Energy Policy* 29:1197-1207.
- Chopra, S. and M. S. Sodhi. (2004). Managing Risk to Avoid Supply-Chain Breakdown. *MIT Sloan Management Review* 46,1.
- Clarke, L. (1999). *Mission Improbable*. The University of Chicago Press.
- Coase, R. H. (1937). The Nature of the Firm. *Economica* 4:386-405.
- Coase, R. H. (1960). The Problem of Social Cost. *Journal of Law and Economics* 3:1-44
- Comfort, L. (1999). *Shared Risk: Complex Systems in Seismic Response*. New York: Pergamon.
- Cornes, R. and T. Sandler. (1996). *The theory of Externalities, Public Goods and Club Goods* (2nd ed.). Cambridge University Press.
- Cournot, A. (1838). *Researches into the Mathematical Principles of the Theory of Wealth*. New York: The Macmillan Company.
- Covington, J. and D. M. Simpson. (2006). An Overview of Disaster Preparedness Literature: Building Blocks for an Applied Bay Area Template. <http://hazardcenter.louisville.edu/pdfs/wp0602.pdf>
- Davidson, C. and R. Deneckere. (1986). Long-Run Competition in Capacity, Short-Run Competition in Price, and the Cournot Model. *The RAND Journal of Economics* 17,3:404-415.
- Datta, R. S. (2010). Finding All Nash Equilibria of a Finite Game Using Polynomial Algebra. *Journal of Economic Theory*. 42:55-96.
- Deo, S. and C. Corbett. (2008). Cournot Competition under Yield Uncertainty: The Case of the U.S. Flu Vaccine Market. *Manufacturing & Service Operations Management*. 11,4:563-576.

- Dixit, A. (1988). A General Model of R&D Competition and Policy. *The RAND Journal of Economics* 19,3:317-326.
- Dub J.-P. and P. Manchanda. (2005). Differences in Dynamic Brand Competition Across Markets: An Empirical Analysis. *Marketing Science* 24,1:81-95.
- Espey, M. (1996). Explaining the variation in elasticity estimates of gasoline demand in the United States: a meta-analysis. *Energy Journal* 17,3:49-60.
- Etro, F. (2007). *Competition, Innovation, and Antitrust: A Theory of Market Leaders and Its Policy Implications*. Springer-Verlag.
- Etro, F. (2008). Stackelberg Competition with Endogenous Entry. *The Economic Journal*, 118:16701697.
- Facchinei, F. and J.S. Pang. (2003). *Finite-Dimensional Variational Inequalities and Complementarity Problems (Vols. 1-2)*. Springer.
- Facchinei, F. and J.S. Pang. (2009). Nash Equilibria: The Variational Approach. *Convex Optimization in Signal Processing and Communication*. D.P. Palomar and Y.C. Eldar, Eds. London: Cambridge University Press, 2009, Ch. 12:443-493.
- FEMA. (2011). National Level Exercise 2011. http://www.fema.gov/pdf/media/factsheets/2011/nle11_quick_look_report.pdf
- Fiat, A., E. Koutsoupias, K. Ligett, Y. Mansour, and S. Olonetsky. (2011). Beyond Myopic Best Response (in Cournot Competition). Preceding ACM-SIAM Symposium on Discrete Algorithms 2012. Kyoto, Japan.
- Frank, R. (2008). *Microeconomics and Behavior* (7th ed.). McGraw-Hill.
- Gladwell, M. (2002). *The Tipping Point: How Little Things Can Make a Big Difference*. Boston, MA: Back Bay Books.
- Goodwin, N., J. A. Nelson, F. Ackerman, and T. Weisskopf. (2009). *Microeconomics in Context* (2nd ed.). Sharpe.

- Goyal M. and S. Netessine. (2007). Strategic Technology Choice and Capacity Investment under Demand Uncertainty. *Management Science* 53,2:192-207.
- Greenwald, B. C. and J. E. Stiglitz. (1986). Externalities in Economies with Imperfect Information and Incomplete Markets. *The Quarterly Journal of Economics* 101,2:229-264.
- Hammond, J. S., R. L. Keeney, and H. Raiffa. (1999). *Smart Choices: A Practical Guide to Making Better Decision*. Harvard Business School Press. Boston, Massachusetts.
- Harrington J. E. (1995). Experimentation and Learning in a Differentiated-Products Duopoly. *Journal of Economic Theory* 66:275-288.
- Heal, G. and H. Kunreuther. (2003). You Only Die Once: Managing Interdependent Risks. <http://opim.wharton.upenn.edu/risk/downloads/03-11-HK.pdf>
- Heal, G. and H. Kunreuther. (2004). Interdependent Security: A General Model. <http://opim.wharton.upenn.edu/risk/downloads/04-11-HK.pdf>
- Heal, G. and H. Kunreuther. (2005). The Vaccination Game. <http://opim.wharton.upenn.edu/risk/downloads/05-10-HK.pdf>
- Hirschey, M. (2000). *Managerial Economics*. Harcourt.
- Institute for Business & Home Safety. (2006). *Open for Business: A Disaster Planning Toolkit for the Small Business Owner*. <http://www.crew.org/sites/default/files/openforbusiness.pdf>
- Jensen, M. C. and W. H. Meckling. (1976). Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure. *Journal of Financial Economics* 3:305-360.
- Keohane, N. O. and R. J. Zeckhauser. (2003). The Ecology of Terror Defense. *The Journal of Risk and Uncertainty* 26,2/3:201-229.

- Kleindorfer, P. R. and H. C. Kunreuther. (1999). Challenges facing the insurance industry in managing catastrophic risks. The financing of Catastrophe Risk. Editor: K. A. Froot. The University of Chicago Press.
- Konnov, I. (2001). Combined Relaxation Methods for Variational Inequalities. Springer.
- Kousky, C. and R. Zeckhauser. (2006). JARring Actions that Fuel the Floods. On Risk and Disaster: Lessons from Hurricane Katrina (eds.: R. J. Daniels, D. F. Kettl, and H. Kunreuther). University of Pennsylvania Press, Philadelphia.
- Kousky, C., E. F. P. Luttmer, and R. J. Zeckhauser. (2006). Private Investment and Government Protection. *Journal of Risk and Uncertainty* 33:73-100.
- Kreps, D. M. and J. A. Scheinkman. (1983). Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes. *The Bell Journal of Economics* 14,2:326-337.
- Kunreuther, H. and G. Heal. (2003). Interdependent security. *Journal of Risk and Uncertainty*, 26:231-249.
- Kunreuther, H. and E. O. Michel-Kerjan. (2011). At War with the Weather: Managing Large-Scale Risks in a New Era of Catastrophes. The MIT Press.
- Kunreuther, H. and A. Rose (Eds.). (2004). The Economics of Natural Hazards. Cheltenham, UK: An Elgar Reference Collection.
- Kunreuther, H., R. Meyer, and C. Van den Bulte. (2004). Risk Analysis for Extreme Events: Economic Incentives for Reducing Future Losses. <http://fire.nist.gov/bfrlpubs/build04/PDF/b04026.pdf>
- Marshall, A. (1890). Principles of Economics. London: Macmillan and Company Ltd.
- McKelvey, Richard D., McLennan, Andrew M., and Turocy, Theodore L. (2010). Gambit: Software Tools for Game Theory, Version 0.2010.09.01. <http://www.gambit-project.org>.

- Morgan, M. G. and M. Henrion. (1992). *Uncertainty: A Guide to Dealing with Uncertainty in Quantitative Risk and Policy Analysis*. Cambridge University Press.
- Murphy, F. H., H. D. Sherali, and A. L. Soyster. (1982). A Mathematical Programming Approach For Determining Oligopolistic Market Equilibrium. *Mathematical Programming* 24:92-106.
- Nash, J. F. (1950). Equilibrium Points in n-person Games. *Proceedings of the National Academy of Sciences* 36,1:48-49.
- Nash, J. F. (1951). Non-cooperative Game. *Annals of Mathematics* 54,2:286-295.
- Neghenebor, A. I. and J. F. Willis. (2001). *Microeconomics, The Freedom to Choose*. CAT Publishing Co.
- Norrman, A. and U. Jansson. (2004). Ericsson's Proactive Supply Chain Risk Management Approach After a Serious Sub-supplier accident. *International Journal of Physical Distribution and Logistics Management* 34,5:434-456.
- Novshek, W. (1980). Cournot Equilibrium with Free Entry. *Review of Economic Studies* 47:473-486.
- Novshek, W. (1985). On the Existence of Cournot Equilibrium. *Review of Economic Studies* 52:85-98.
- Oh, K. B., C-T. B. Ho, and D. D. Wu. (2009). Natural Disaster and Financial Risk Management. *International Journal of Emergency Management* 6,2:215-226.
- Parkin, M., M. Powell, and K. Matthews. (2002). *Economics*. Harlow: Addison-Wesley.
- Perrow, C. (2007). *The Next Catastrophe: Reducing Our Vulnerabilities to Natural, Industrial, and Terrorist Disasters*. New Jersey: Princeton University Press.

- Pillsbury Winthrop Shaw Pittman's Crisis Management Team and Levick Strategic Communications. (2011). 2011 Crisis Preparedness: Survey Results. <http://www.pillsburylaw.com/siteFiles/Publications/2011CrisisResultsfinal.pdf>
- Pindyck, R. S. (1979). *The Structure of World Energy Demand*. Cambridge, MA: MIT Press.
- Pindyck R. S. and D. L. Rubinfeld. (2001). *Microeconomics* (5th ed.). Prentice-Hall.
- Pita, J., M. Jain, C. Western, C. Portway, M. Tambe, F. Ordonez, S. Kraus, and P. Parachuri. (2008). Deployed ARMOR protection: The application of a game-theoretic model for security at the Los Angeles International Airport. In *Proceedings of the 7th International Conference on Autonomous Agents and Multiagent Systems: Industry and Applications Track*, Estoril, Portugal.
- Porter, M. E. and M. P. Kramer. (2006). *Strategy & Society: The Link Between Competitive Advantage and Corporate Social Responsibility*. Harvard Business Review.
- Quirnbach, H. C. (1993). R&D: Competition, Risk, and Performance. *The RAND Journal of Economics* 24,2:157-197.
- Riley, G. (2006). *Market Failure: Government Intervention in the Market*. <http://www.tutor2u.net/economics/revision-notes/as-marketfailure-government-intervention-2.html>
- Samuelson, P. A. and W. D. Nordhaus. (2001). *Microeconomics* (17th ed.). McGraw-Hill.
- Schelling, T. C. (1960). *The Strategy of Conflict*. Harvard University Press (Reprint, illustrated and revised ed., 1980).
- Schelling, T. C. (1978). *Micromotives and Macrobehavior*. New York: Norton
- Scitovsky, T. (1954). Two Concepts of External Economies. *Journal of Political Economy* 62,2:143-151.

- Scutari, G., D. P. Palomar, F. Facchinei, and J.-S. Pang. (2010). Convex Optimization, Game Theory, and Variational Inequality Theory in Multiuser Communication Systems. *IEEE Signal Processing Magazine* 27,3:35-49.
- Simon, H. (1979). Rational Decision Making in Business Organizations. *The American Economic Review* 69,4:493-513.
- Smith, J. E. and R. F. Nau. (1995). Valuing Risky Projects: Option Pricing Theory and Decision Analysis. *Management Science* 41,5:795-816.
- Smith, P. W., K. Hansen, L. Spanbauer, and D. F. Shell. (2007). Pandemic influenza preparedness: A survey of businesses. *American Journal of Infection Control* 35,7:484-485.
- Tirole, J. (1993). *The Theory of Industrial Organization*. The MIT Press.
- von Stackelberg, H. (1934). *Market Structure and Equilibrium*. Springer-Verlag Wien New York.
- World Bank. (2000). *World Development Report 2000/2001: Attacking Poverty*. Oxford University Press, New York.
- World Bank. (2010). *World Development Report 2010: Development and Climate Change*. The World Bank, Washington D.C.
- Zhuang, J. (2010). Impacts of Subsidized Security on Stability and Total Social Costs of Equilibrium Solutions in an N-Player Game with Errors. *The Engineering Economist* 55,2:131-149.
- Zhuang, J., V.M. Bier, and A. Gupta. (2007). Subsidies in Interdependent Security with Heterogeneous Discount Rates. *The Engineering Economist* 52,1:1-19.

A Proof of Proposition 4.1.1

In Section 4.1, we showed that in order to find the set of optimal production quantities of all firms under condition (s, d) , the convex quadratic problem defined in (11) needs to be solved. *Proposition 4.1.1* explained an approach to identify firms that are guaranteed to produce a zero quantity (i.e. not participating in the market) in order to solve the optimization problem much faster and more efficiently. In this appendix, we prove that proposition.

Let's assume that there are n firms in the market under market condition (s, d) , then the solution of the convex quadratic problem below will give the set of optimal production quantities of all firms.

$$\begin{aligned} \text{minimize} \quad & \frac{1}{2}q^T(bM_{n \times n})q + (RVC - ae)^Tq \\ \text{s.t. } & q \in \mathbb{R}_+^n \end{aligned} \tag{92}$$

where $e = [1, 1, \dots, 1]_{n \times 1}$, $RVC = [RVC_1, RVC_2, \dots, RVC_n]$, and

$$M_{n \times n} = \begin{bmatrix} 2 & 1 & \dots & 1 \\ 1 & 2 & \dots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \dots & 2 \end{bmatrix}.$$

We let $q^* = (q_1^*, q_2^*, \dots, q_n^*)$ and $\lambda^* = (\lambda_1^*, \lambda_2^*, \dots, \lambda_n^*)$ be the resulting optimal values. Now we can write down the KKT conditions for the problem above as

follows:

$$\begin{aligned} bM_{n \times n}q + (RVC - ae) &= \lambda & (*) \\ 0 \leq q^{(s,d)} \perp \lambda &\geq 0 \end{aligned} \quad (93)$$

where $0 \leq q^{(s,d)} \perp \lambda \geq 0$ is equivalent to the following set of relations:

$$q^{(s,d)} \geq 0, \lambda \geq 0, \text{ and } \lambda^T q^{(s,d)} = 0 \quad (94)$$

The above system of equalities and inequalities describes the critical points of the optimization problem.

We initially claim that $q_i^{*(s,d)} = 0 \Rightarrow q_j^{*(s,d)} = 0 \forall j > i$. In order to show this relation, let's assume that $q_i^{*(s,d)} = 0$ and $q_j^{*(s,d)} > 0$ for some $j > i$. We show that this will result in a contradiction. (Note that for simplicity we are going to suppress writing (s, d)).

The i^{th} row of the the equation $(*)$ in (93) is as follows:

$$b \left(\sum_{k=1}^n (q_k^*) + q_i^* \right) + RVC_i - a = \lambda_i^* \quad (95)$$

Similarly, the j^{th} row of the the equation $(*)$ in (93) is as follows:

$$b \left(\sum_{k=1}^n (q_k^*) + q_j^* \right) + RVC_j - a = 0 \quad (96)$$

Considering (95) and the assumption that $q_i^* = 0$ we will have

$$b \left(\sum_{k=1}^n (q_k^*) \right) = a - RVC_i + \lambda_i^* \quad (97)$$

Now, by substituting the above expression into (96) we will have:

$$a - RVC_i + \lambda_i^* + bq_j^* + RVC_j - a = 0 \Rightarrow \lambda_i^* + bq_j^* = RVC_i - RVC_j \quad (98)$$

The right-hand side of the above expression is positive but the left-hand side is less than or equal to zero. This is a contradiction. Therefore, q_j^* must be equal to zero. Then by the claim above we have:

$$\begin{aligned} q_i^* &\geq 0 & \forall i \leq p \\ q_i^* &= 0 & \forall i > p \end{aligned} \quad (99)$$

Therefore, by complementary slackness, we can rewrite the (*) in (93) as follows:

$$\begin{aligned} b \left(\sum_{j=1}^p (q_j^*) + q_i^* \right) + RVC_i - a &= 0 & \forall i \leq p & \quad (\dagger) \\ b \sum_{j=1}^p (q_j) + RVC_i - a &= \lambda_i^* & \forall i > p & \quad (\ddagger) \end{aligned} \quad (100)$$

The p equations in p unknowns in (†) have the closed form solution of

$$q_i^* = \frac{1}{b(p+1)} \left[a - (p+1)RVC_i + \sum_{j=1}^p RVC_j \right] \quad \forall i \leq p \quad (101)$$

Moreover, by substituting the above closed form and the fact the $q_i^* = 0 \forall i > p$,

we can rewrite (‡) as follows:

$$\begin{aligned}
\lambda_i^* &= b \sum_{j=1}^p \left(\frac{1}{b(p+1)} [a - (p+1)RVC_j + \sum_{k=1}^p RVC_k] \right) + RVC_i - a \\
&= \frac{1}{(p+1)} [ap - (p+1) \sum_{j=1}^p (RVC_j) + p \sum_{k=1}^p (RVC_k)] + RVC_i - a \\
&= \frac{1}{(p+1)} [ap - \sum_{j=1}^p (RVC_j)] + RVC_i - a \tag{102} \\
&= \frac{1}{(p+1)} [ap - \sum_{j=1}^p (RVC_j) + (p+1)RVC_i - (p+1)a] \\
&= \frac{1}{(p+1)} [-a + (p+1)RVC_i - \sum_{j=1}^p (RVC_j)] \quad \forall i > p
\end{aligned}$$

Letting $\kappa_i(p) = a - (p+1)RVC_i + \sum_{j=1}^p RVC_j$, the solution to the KKT system with exactly p components of q being positive is

$$\begin{aligned}
q_i^* &= \frac{1}{b(p+1)} \kappa_i(p) & \forall i \leq p \\
\lambda_i^* &= \frac{-1}{(p+1)} \kappa_i(p) & \forall i > p
\end{aligned} \tag{103}$$

Therefore, for the solution to be both feasible and optimal, we need to choose p^* such that

$$\begin{aligned}
\kappa_i(p^*) &\geq 0 & \forall i \leq p^* \\
\kappa_i(p^*) &\leq 0 & \forall i > p^*
\end{aligned} \tag{104}$$

Moreover, we can show that $\kappa_i(p) \geq \kappa_{i+1}(p) \forall i$ (i.e. the monotonicity of

$\kappa_i(\cdot)$), since:

$$\begin{aligned}
\kappa_i(p) - \kappa_{i+1}(p) &= a - (p+1)RVC_i + \sum_{j=1}^p RVC_j - [a - (p+1)RVC_{i+1} + \sum_{j=1}^p RVC_j] \\
&= -(p+1)RVC_i + (p+1)RVC_{i+1} \\
&= RVC_{i+1} - RVC_i \geq 0
\end{aligned}$$

Finally, assume that there is an l such that $\kappa_l(l) \geq 0$ and $\kappa_{l+1}(l+1) < 0$ (Note that $\kappa_l(l) = a - (l+1)RVC_l + \sum_{j=1}^l RVC_j = \rho(l)$ and $\kappa_{l+1}(l+1) = a - (l+1+1)RVC_{l+1} + \sum_{j=1}^{l+1} RVC_j = \rho(l+1)$). Therefore, By the monotonicity of $\kappa_i(\cdot)$, we can show that the former condition (i.e., $\kappa_l(l) \geq 0$) implies $\kappa_i(l) \geq 0 \forall i \leq l$ and the latter condition (i.e., $\kappa_{l+1}(l+1) < 0$) implies $\kappa_i(l+1) \leq 0 \forall i \geq l+1$. Therefore, $l = p^*$ and by (103) we have shown that firms with index values less than or equal to l will have a non-negative optimal production quantity and firms with index values greater than l will have the optimal production quantity of zero.

B *Mathematica* Code

B.1 Initialization

```

SetDirectory["\\Business Preparedness\\Model"]; RVC[i_]:=fi + (1 - si) δid;
Π[i_]:=((a - b Σj=1n qj) qi - RVC[i]qi - siCi) / .
Flatten[{fs, δs, ss, sol, AB, Cs}];
EΠ[i_]:=pΠi + (1 - p)Πnd;
n = 5;
prob = 0.05;
fs = Table[fi → 90, {i, 1, n, 1}];
δs = Table[δi → 60, {i, 1, n, 1}];
S = Reverse[Tuples[{0, 1}, n]];
For[i = 1, i ≤ n, i++, Payoffi = ConstantArray[0, 2n]];
order = "";
For[m = 0, m ≤ 1, m++,
For[l = 0, l ≤ 1, l++,
For[k = 0, k ≤ 1, k++,
For[j = 0, j ≤ 1, j++,
For[i = 0, i ≤ 1, i++, o = i * 24 + j * 23 + k * 22 + l * 21 + m * 20 + 1;
order = order <> ToString[o] <> " "]]]]];

```

B.2 Iterations

MINvc = 2;

MAXvc = 3;

MINiter = 100;

MAXiter = 200;

XScale = 50;

YScale = 1;

AbsoluteTiming[Index = 1001; RUN = 1;

For[vc = MINvc, vc ≤ MAXvc, vc = vc + YScale,

Cs = Table[C_i → Round[10^{vc}, 0.001], {i, 1, n, 1}];

For[iter = MINiter, iter ≤ MAXiter, iter = iter + XScale,

GameFileName = "math" <> ToString[Index] <> ".nfg";

OutputFileName = "output" <> ToString[Index] <> ".txt";

AB = {a → iter, b → 0.1};

Put[n, AB, prob, fs, δs, Cs, "input" <> ToString[Index] <> ".txt"];

(*Optimization Problem Using Procedure*)

For[z = 1, z ≤ 2ⁿ, z++,

ss = Table[s_i → S[[z, i]], {i, 1, n, 1}];

For[d = 0, d ≤ 1, d++,

RVCs = Table[RVC_i → RVC[i]/.Flatten[{fs, δs, ss}], {i, 1, n, 1}];

qs = Table[{q_i}, {i, 1, n, 1}];

Q = SparseArray[{Band[{1, 1}] → 2b, {i-, j-}/; i ≠ j → b},

{n, n}];

```

RHS = Table[{RVCi - a}, {i, 1, n, 1}];
OBJ =  $\frac{1}{2}$ Transpose[qs].Q.qs + Transpose[RHS].qs/.
Flatten[{AB, RVCs}];
srvcs = Sort[RVCs[[All, 2]]]; arvcs = Accumulate[srvcs];
prvcs = Table[(a/.AB) - (j + 1)srvcs[[j]] + arvcs[[j]], {j, 1, n}];
level3 = If[Length[Select[prvcs, # ≥ 0&]] == 0, 0,
srvcs[[Length[Select[prvcs, # ≥ 0&]]]];
CONS = {}; For[i = 1, i ≤ n, i++,
If[RVCs[[i, 2]] > level3, CONS = Append[CONS, qi == 0]];
sol = Last[Minimize[Flatten[{OBJ, CONS}], Table[qi, {i, 1, n}]]];
If[d == 1, For[i = 1, i ≤ n, i++, Πdi = Π[i]],
For[i = 1, i ≤ n, i++, ΠIndi = Π[i]];
sol >>> "optimalQ" <> ToString[Index] <> ".txt"
For[y = 1, y ≤ n, y++,
Payoffy = ReplacePart[Payoffy, z → EΠ[y]/.p → prob]];

(*Writing the payoff matrix*)
payoffMatrix = "{\n";
For[i = 0, i ≤ 1, i++,
For[j = 0, j ≤ 1, j++,
For[k = 0, k ≤ 1, k++,
For[l = 0, l ≤ 1, l++,
For[m = 0, m ≤ 1, m++,
z = i * 24 + j * 23 + k * 22 + l * 21 + m * 20 + 1;

```



```

OutputFileName;
Run[Exec];

(*Writing the final results*)
str = OpenRead[OutputFileName];
tmp = ReadList[str, Record, RecordLists → True,
RecordSeparators → {"NE", ",", "\n"}];
Close[str];
tmp = ToExpression[
Partition[
Flatten[Drop[Flatten[tmp], {2, Dimensions[tmp][[1]], 2}], n]];
tmpp = tmp;
For[i = 1, i ≤ Dimensions[tmp][[1]], i++,
For[j = 1, j ≤ n, j++, If[0 < tmpp[[i, j]] < 1, tmpp[[i, j]] = 0.5]];
tmpp = DeleteDuplicates[tmpp];
tmp >> "ne" <> ToString[Index] <> ".txt";
tmpp >>> "neall" <> ToString[RUN] <> ".txt";
Index++; ];
RUN++; ]
]

```

B.3 Graphing Sensitivity Results

```

AbsoluteTiming[
eqN = {};
RUN = 1;
For[k = MINvc, k ≤ MAXvc, k = k + YScale,
list = ReadList["neall" <> ToString[RUN] <> ".txt"];
For[i = 1, i ≤ Length[list], i++,
TMP = {};
For[j = 1, j ≤ Length[list[[i]]], j++,
AppendTo[TMP, ToExpression[ToString[Count[list[[i, j], 1]]]]];
AppendTo[eqN, TMP]]; RUN++];
eqN = Partition[eqN,  $\frac{MAXiter - MINiter}{XScale} + 1$ ];
list = {};
RUN = 1;
Index = 1;
For[i = MINvc, i ≤ MAXvc, i = i + YScale,
For[j = MINiter, j ≤ MAXiter, j = j + XScale,
AppendTo[list, eqN[[RUN, Index, 1]]; Index++; Index = 1; RUN++];
list = Partition[list,  $\frac{MAXiter - MINiter}{XScale} + 1$ ];
font[x_, y_] := Text[Style[x, Bold, Italic, y]];
MatrixPlot[n - list, DataReversed → {True, False},
FrameTicks →
{Table[{i, font[10MINvc+(i-1)*YScale, 16]}, {i, 1,  $\frac{MAXvc - MINvc}{YScale} + 1$ }},
Table[{i, font[(i - 1) * XScale + MINiter, 16]},

```

$\left\{i, 1, \frac{\text{MAXiter}-\text{MINiter}}{\text{XScale}} + 1\right\}$,
Frame \rightarrow $\{\{\text{True}, \text{False}\}, \{\text{True}, \text{False}\}\}$, ColorFunction \rightarrow GrayLevel,
FrameLabel \rightarrow $\{\text{font}[\text{"Preparedness Cost, C"}, 20],$
 $\text{font}[\text{"Reservation Price, a"}, 20]\}$, Mesh \rightarrow All,
PlotLabel \rightarrow $\text{font}[\text{"Pure Nash Equilibrium"}, 20]$

B.4 Mixing Probabilites

```

Clear[d];
For[i = 1, i ≤ n, i++,
  ToExpression[F <> ToString[i] <> “={}”]];
Index = 1;
For[i = MINiter, i ≤ MAXiter, i = i + XScale,
  OutputFileName = “output” <> ToString[1000 + Index] <> “.txt”;
  str = OpenRead[OutputFileName];
  tmp = ReadList[str, Record, RecordLists → True,
  RecordSeparators → {“NE”, “,”, “\n”}];
  Close[str];
  tmp = ToExpression[
  Partition[
  Flatten[Drop[Flatten[tmp], {2, Dimensions[tmp][[1], 2}], n]];
  For[j = 1, j ≤ n, j++,
  ToExpression[“AppendTo[F” <> ToString[j] <> “,tmp[[All,” <>
  ToString[j] <> “]]”]]; Index++];
  RUN = 1;
  For[k = MINvc, k ≤ MAXvc, k = k + YScale,
  dRUN = ReadList[“neall” <> ToString[RUN] <> “.txt”];
  dRUN = Map [IntegerPart, Table [FromDigits [dRUN[[i, j]]/.{0.5 → 5}],
  {i, 1, Length [dRUN]} , {j, 1, Length [dRUN[[i]]}]]; RUN++
  kth = Round [0.5 * ( $\frac{MAXvc-MINvc}{YScale} + 1$ )];
  ith = Round [0.5 * ( $\frac{MAXiter-MINiter}{XScale} + 1$ )];

```

```

OP = {};
Quiet[For[i = 0, i ≤ n, i++,
AppendTo[OP,
Check[
F1[[ith,
Position[dkth[[ith]],
IntegerPart[ $\frac{\text{FromDigits}[\text{ConstantArray}[5, n]]}{10^i}$ ][[1, 1]]],
err]]]]
OP = Select[OP, # > 0 &];
BarChart[Sort[OP, Greater],
Ticks → {Table[{i, i + 1}, {i, 1, 6}], Automatic},
AxesLabel →
{Text[Style["# of Firms Choosing\nMixed Strategy", Bold,
Italic, 20, TextAlignment → Center]],
Text[Style["Investment\nProbability", Bold, Italic, 20,
TextAlignment → Center]]},
AxesStyle → Directive[Thick, Bold, Italic, 20], BarSpacing → 1.5,
ChartStyle → {Gray},
LabelingFunction → (Placed[Style[Round[#1, 10-3]/N, 18], Above] &)]
MP = {};
Index = 1;
For[i = MINiter, i ≤ MAXiter, i = i + XScale,
If[dkth[[Index]] == {0}, AppendTo[MP, 0],
If[dkth[[Index]] == {FromDigits[ConstantArray[1, n]]},

```

```

AppendTo[MP, 1],
AppendTo[MP,
F1[[Index,
Position [ $d_{kth}$ [[Index]], FromDigits[ConstantArray[5,  $n$ ]]] [[
1, 1]]]]]; Index++; If[Index > 3, Index = 1];
ListPlot[MP, Joined → True, PlotMarkers → {Automatic, 18},
AxesLabel → {Text[Style[a, Italic, 20, TextAlignment → Center]],
Text[Style["Investment Probability\under Uncertainty",
Italic, 20]]}, AxesStyle → Directive[Thick, 20],
PlotStyle → {Gray, Thick}, DataRange → {MINiter, MAXiter},
PlotRange → {{MINiter, MAXiter * 1.01}, {0, 1.01}}]

```

B.5 Social Optimum

```

AbsoluteTiming[
SumQ = {};
TotalProfits = {};
d = 1;
Index = 1;
For[iter = MINiter, iter ≤ MAXiter, iter = iter + XScale,
AB = {a → iter, b → 0.1};
OptimalQuantity =
Drop[ReadList["optimalQ" <> ToString[1000 + Index] <> ".txt"],
{1, 2n+1, 2}];
For[vc = MINvc, vc ≤ MAXvc, vc = vc + YScale,
Cs = Table[Ci → 10vc, {i, 1, n, 1}];
TMP1 = {};
TMP2 = {};
For[z = 1, z ≤ 2n, z++,
ss = Table[si → S[[z, i]], {i, 1, n, 1}];
sol = OptimalQuantity[[z]];
AppendTo[TMP1, Sum[Π[i], {i, 1, n}]];
AppendTo[TMP2,  $\frac{b}{2}$  (Sum[qk, {k, 1, n}] /. sol)2 /. AB];
AppendTo[TotalProfits, TMP1];
AppendTo[SumQ, TMP2];
];

```

```

Index++]
];
SumQ >> "SumQ-5Ha.txt";
TotalProfits >> "TotalProfits-5Ha.txt";
(*Getting Data*)
loc1 = "5Ha";
SumQ = ReadList["SumQ-" <> loc1 <> ".txt"][[1]];
TotalProfits = ReadList["TotalProfits-" <> loc1 <> ".txt"][[1]];
(*FindingBestPS, ES, andNash*)
pRES = {};
For[i = 1, i ≤ Length[TotalProfits], i++,
TempList =
Reverse[Sort[Map[IntegerDigits[#, 2, n]&,
2n - Position[TotalProfits[[i]], Max[TotalProfits[[i]]]]][
All, 1]]]];
TMP = {};
For[j = 1, j ≤ Length[TempList], j++,
AppendTo[TMP, ToExpression[ToString[Count[TempList[[j]], 1]]]]
];
AppendTo[pRES, DeleteDuplicates[TMP]]
];
pRES = Partition [pRES,  $\frac{\text{MAXvc}-\text{MINvc}}{\text{YScale}} + 1$ ];
ES = SumQ + TotalProfits;
esRES = {};

```

```

For[i = 1, i ≤ Length[ES], i++,
TempList =
Reverse[Sort[Map[IntegerDigits[#, 2, n]&,
2n - Position[ES[[i]], Max[ES[[i]]][[All, 1]]]]];
TMP = {};
For[j = 1, j ≤ Length[TempList], j++,
AppendTo[TMP, ToExpression[ToString[Count[TempList[[j]], 1]]]]
];
AppendTo[esRES, TMP]
];
esRES = Partition[esRES,  $\frac{\text{MAXvc}-\text{MINvc}}{\text{YScale}} + 1$ ];
(*Building Lists*)
plist = {};
For[i = 1, i ≤  $\frac{\text{MAXiter}-\text{MINiter}}{\text{XScale}} + 1$ , i++,
For[j = 1, j ≤  $\frac{\text{MAXvc}-\text{MINvc}}{\text{YScale}} + 1$ , j++,
AppendTo[plist, pRES[[i, j, 1]]]];
plist = Partition[plist,  $\frac{\text{MAXvc}-\text{MINvc}}{\text{YScale}} + 1$ ];
elist = {};
For[i = 1, i ≤  $\frac{\text{MAXiter}-\text{MINiter}}{\text{XScale}} + 1$ , i++,
For[j = 1, j ≤  $\frac{\text{MAXvc}-\text{MINvc}}{\text{YScale}} + 1$ , j++,
AppendTo[elist, esRES[[i, j, 1]]]];
elist = Partition[elist,  $\frac{\text{MAXvc}-\text{MINvc}}{\text{YScale}} + 1$ ];
nlist = {};
For[i = 1, i ≤  $\frac{\text{MAXvc}-\text{MINvc}}{\text{YScale}} + 1$ , i++,

```

```

For [j = 1, j ≤  $\frac{\text{MAXiter}-\text{MINiter}}{\text{XScale}} + 1, j++$ ,
AppendTo[nlist, eqN[[i, j, 1]]]];
nlist = Partition [nlist,  $\frac{\text{MAXiter}-\text{MINiter}}{\text{XScale}} + 1$ ];
TesRES = Transpose[esRES];
comlist = {};
For [i = 1, i ≤  $\frac{\text{MAXvc}-\text{MINvc}}{\text{YScale}} + 1, i++$ ,
For [j = 1, j ≤  $\frac{\text{MAXiter}-\text{MINiter}}{\text{XScale}} + 1, j++$ ,
If[MemberQ[eqN[[i, j]], TesRES[[i, j, 1]]], AppendTo[comlist, 1],
AppendTo[comlist, 0]]]];
comlist = Partition [comlist,  $\frac{\text{MAXiter}-\text{MINiter}}{\text{XScale}} + 1$ ];
mlist = {};
test = 0;
For [i = 1, i ≤  $\frac{\text{MAXvc}-\text{MINvc}}{\text{YScale}} + 1, i++$ ,
pi = i;
For [j = 1, j ≤  $\frac{\text{MAXiter}-\text{MINiter}}{\text{XScale}} + 1, j++$ ,
If[MemberQ[eqN[[i, j]], TesRES[[i, j, 1]]], AppendTo[mlist, 0],
While[pi ≥ 1&&test == 0,
If[MemberQ[eqN[[pi, j]], TesRES[[i, j, 1]]], test = 1, pi--];
If[test == 0, AppendTo[mlist, i], AppendTo[mlist, i - pi]]];
pi = i; test = 0]];
mlist = Partition [mlist,  $\frac{\text{MAXiter}-\text{MINiter}}{\text{XScale}} + 1$ ];
(*Saving Data*)
plist >> "plist-" <> loc1 <> ".txt";
elist >> "elist-" <> loc1 <> ".txt";

```

```
nlist >> "nlist-" <> loc1 <> ".txt";
comlist >> "comlist-" <> loc1 <> ".txt";
mlist >> "mlist-" <> loc1 <> ".txt";
```

(*Graphing Results*)

```
loc1 = "5Ha";
font[x_, y_] := Text[Style[x, Bold, Italic, y]];
plist = ReadList["plist-" <> loc1 <> ".txt"][[1]];
elist = ReadList["elist-" <> loc1 <> ".txt"][[1]];
nlist = ReadList["nlist-" <> loc1 <> ".txt"][[1]];
comlist = ReadList["comlist-" <> loc1 <> ".txt"][[1]];
mlist = ReadList["mlist-" <> loc1 <> ".txt"][[1]];
FORM = {DataReversed -> {True, False},
FrameTicks ->
{Table[{i, font[10MINvc+(i-1)*YScale, 16]}, {i, 1,  $\frac{MAXvc-MINvc}{YScale} + 1$ }},
Table[{i, font[(i - 1) * XScale + MINiter, 16]},
{i, 1,  $\frac{MAXiter-MINiter}{XScale} + 1$ }}},
Frame -> {{True, False}, {True, False}}, ColorFunction -> GrayLevel,
FrameLabel -> {font["Preparedness Cost, C", 20],
font["Reservation Price, a", 20]},
ColorRules->Table[i -> GrayLevel[1 - i *  $\frac{1}{n}$ ], {i, 0, n}], Mesh -> All};
MatrixPlot[Transpose[plist], FORM,
PlotLabel->font["Producer Surplus", 20]]
MatrixPlot[Transpose[elist], FORM,
```

PlotLabel->font["Economic Surplus", 20]]

MatrixPlot[nlist, FORM, PlotLabel->font["Pure Nash Equilibrium", 20]]

MatrixPlot[comlist, FORM,

PlotLabel->font["Comparison between Nash and ES", 20]]

ArrayPlot[mlist, FORM, PlotLabel->font["Amount of Subsidy", 20]]