

Diverse evidence, independent evidence,
and Darwin's arguments from anatomy
and biogeography

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preface

Among all the philosophy seminars in which I participated, two in particular had a formative influence on my development as a philosopher, and on my thoughts about my dissertation plan. Those seminars were “Evidence and Evolution” taught by Elliott Sober, and “William Whewell” taught by Malcolm Forster. The plan that I hatched in the wake of those seminars was basically to address a set of epistemological problems in evolutionary biology by applying a Whewellian perspective on the philosophy of science. I gathered a set of inference problems and illustrative scientific arguments from evolutionary biology all of which seemed to turn on something like Whewellian consilience; they included examples from Darwin’s *Origin*, contemporary biogeography, phylogenetic inference, and contemporary arguments for the common ancestry of all life. I hoped that studying these particular inference problems would illuminate the epistemology of consilience, and vice versa.

As it happened, the first case study—Darwin’s geographical distribution argument—proved difficult enough on its own, and I never got around to writing on the other examples (as my prospectus committee predicted). In a second major development, as I proceeded I found that writing on Darwin’s reasoning in particular, and also on

consilience in general, required speaking to different target audiences. This pulled the two sides of the project in somewhat different directions, resulting in the two-part structure of this dissertation. The final product is less integrated than I had initially imagined. It is a series of deeply interrelated essays, with some biggish gaps in between, and a lot of interesting loose ends. Not a book, it is more like one half of one book, plus one half of another. Writing it has been a fabulous learning experience, and has left me with more exciting research questions than I know what to do with.

Many people helped me through this learning process and deserve a lot of the credit for whatever is good and valuable in this dissertation. The three (reading) members of my committee deserve the most thanks. Elliott, my thesis advisor, has provided constant feedback, advice, ideas, and encouragement since even before I joined the graduate program. I have also worked very closely with Malcolm ever since that Whewell seminar. What I've written in this dissertation owes very direct intellectual debts to the work of both Elliott and Malcolm. I was very fortunate that Mike Titelbaum joined our department along the way; his input and scrutiny has been extremely helpful in framing and sharpening my ideas.

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owe much of what I've learned about interdisciplinary thinking and interdisciplinary communication to my experiences in those seminars. Particularly relevant for my dissertation was the iteration of Botany 940 (co-taught with Ken Cameron and Ken Sytsma) in which we read Darwin's *Origin of Species*. I'm thankful for that opportunity to go through the book with a mixed group of biologists and philosophers.

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Help also came in the form of money. A Graduate Research Fellowship from the National Science Foundation (2007) changed my life. I wrote some of this work while

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My apologies to any individuals or organizations that I have inadvertently neglected in these acknowledgments.

Chapter 1

Introduction

In science, as in everyday life, multiple pieces of evidence that are diverse in character, or that arrive from different quarters, sometimes work together, pointing to the same conclusion. We tend to find this particularly convincing. But is our intuitive reaction correct? Does a notion of *kinds* of evidence, or of *diversity* in the character of the evidence, really have a place in rational inductive inference, or is it only, so to speak, the total quantity of evidence that matters? One label for the phenomenon of multiple sources of evidence working together for greater effect is “consilience”. In this dissertation, I investigate the epistemology of consilience from two directions: first, by reconstructing the reasoning involved in an important scientific argument from Darwin’s *Origin* that ostensibly appeals to consilience, and second, by attempting to formally model the epistemological value of consilience in probabilistic terms.

Darwin

Darwin's *Origin* is a good place to start for several reasons. The importance of a hypothesis being supported by different kinds of evidence is frequently emphasized in the philosophical literature commenting on Darwin's argument in the *Origin*, and Whewell's term "consilience" occurs frequently in that literature. Indeed, Darwin's argument in the *Origin* is among the scientific arguments—if not *the* scientific argument—most frequently discussed in connection with consilience. But the term has become (at least in the *Origin* literature) a label for poorly understood theory-observation relations rather than a tool for elucidation those relations.

For example, Thagard (1978), Recker (1987), and Waters (2003) apply Whewell's take on scientific methodology to the tasks of articulating the content of the theory there presented, spelling out exactly how Darwin's many supporting observations relate to that theory, and illuminating the rhetorical structure of his argument. I believe the conclusions of these investigations are correct *as far as they go*, but that they remain superficial due to a vague and inadequate reformulation of Whewell's key term "consilience". These analyses employ roughly the same definition, according to which a hypothesis enjoys consilience to the degree that it "explains separate classes of facts" (where the "explains" relation remains entirely unanalyzed).¹

The prevalence of this slack formulation is understandable given that Whewell

¹One might wonder whether Whewell *influenced* Darwin's scientific methodology. After all, Whewell was both a personal acquaintance of Darwin and one of Britain's highest authorities on the methodology of science (along with John Herschel and John Stuart Mill). This question is discussed in Ghiselin (1969), Ruse (1975), Thagard (1977), and Hodge (1991). Despite the historical and cultural proximity of Darwin to Whewell, I will bracket questions about influence, and about whether Darwin knowingly perceived (or presented) his own methodology in Whewellian terms.

sometimes glosses his own view in such terms.² Also, Whewell’s more rigorous presentation of consilience is quantitative in a way that may appear to preclude application to a scientific treatise such as the *Origin*, which does not contain a single mathematical formula. But defining consilience in terms of explanation leaves the former as obscure as the latter, and in fact Whewell does not do so. (He does sometimes use the term “explains” informally, as will I). The notion of “separate phenomena” or “different kinds of facts” is also in need of clarification, in Whewell’s own writing as well as in modern appropriations of his terminology. I will approach the topic of consilience from a more rigorously Whewellian perspective, informed in particular by the Whewell interpretation of Forster (1988, 2011).

The part of Darwin’s reasoning that I will examine is his biogeography argument for common ancestry. Here Darwin argues that distinct species share common ancestry (and that evolution must therefore have occurred) based on how species resemble one another anatomically *and* how they are distributed geographically around the globe. How do those two kinds of observation work together? In Chapter 3, I present a new answer to this question. But first, in Chapter 2, I critically assess an alternative account of the same observations (Sober 1999, 2008, 2011)—an account that, were it correct, would render my own superfluous.

²See Whewell (1858/1989a, 153, 159), and (1860/1989b, 331)

Diversity

Beyond Darwin's *Origin* there is the broader question of what evidential diversity is—what makes observations *different kinds?*—and why it matters to the epistemology of science. In Chapter 4, I address a special case of evidential diversity and Whewellian consilience sometimes called *the agreement of independent measurements*, that is instantiated, among many other places, in Darwin's geographical distribution argument. I analyze the epistemic import of such agreement from an abstract, formal perspective using the framework of Likelihoodist epistemology. In Chapter 5, I expand on the results of Chapter 4 to address the issue of evidential diversity more generally, taking the extra step from likelihoodist to Bayesian epistemology, the common language of much literature on the subject.

Each of the four substantive chapters has its own, more detailed introduction that motivates the concerns of that chapter more specifically and better identifies the audience and literature to which that particular part of the dissertation is addressed.

Part I

Darwin

Chapter 2

Modus Darwin Reconsidered

2.1 Introduction

The common ancestry of all extant life on Earth is a central tenant of modern evolutionary biology. In his book *On the Origin of Species* (henceforth “*Origin*”) Darwin took a giant step towards establishing this fact. Of course Darwin could address only the portion of life on Earth of which nineteenth-century naturalists were aware, and he wavered on whether there was a single primordial species, or some smallish number bigger than one (Darwin 1859/2003, 483–4). None the less, Darwin argued forcefully that vast swaths of life trace back to a common ancestors; indeed this was his main conclusion in *Origin* (Darwin 1859/2003, 6). While entirely qualitative and remarkably under-articulated by today’s standards, his arguments were enormously convincing to his contemporaries (Bowler 1989; Larson 2004).

In a series of publications, Elliott Sober (Sober 1999; Sober and Steel 2002; Sober

2008, 2011) has sought to clarify and formalize Darwin's defense of common ancestry, and to generalize Darwin's reasoning to encompass contemporary thinking about newer evidence for common ancestry. Sober's project is thus part exegesis, part epistemology: *How does Darwin argue?*, and *How does that argument justify common ancestry?* In answer to the first question, Sober attributes to Darwin the following argument form:

Similarity, ergo common ancestry. This form of argument occurs so often in Darwin's writings that it deserves to be called *modus Darwin*. The finches in the Galapagos Islands are similar; hence, they descended from a common ancestor. Human beings and monkeys are similar; hence, they descended from a common ancestor. The examples are plentiful, not just in Darwin's thought, but in evolutionary reasoning down to the present. (Sober 1999, 265)

To address the *epistemological* question, Sober sets out to formalize *modus Darwin* with mathematical rigor, ultimately deriving the force of the argument form from the *Law of Likelihood* (explained below).

In this essay I review and critique Sober's analysis of Darwin's reasoning. In the first stage of my analysis, I bracket exegesis and concentrate instead on the epistemic merits of the argument form *modus Darwin* as Sober understands it. I will argue that *modus Darwin* cannot rationally support Darwin's common ancestry hypothesis. From this conclusion it follows that *either* Darwin's reasoning was flawed (he gave bad reasons for a true conclusion) *or* he did not employ *modus Darwin* as Sober understands it.

I then move on to address Sober’s application of *modus Darwin* to Darwin’s geographical distribution observations—a variant of the argument form that could be summarized as: *Proximity, ergo common ancestry*. I find less to fault in this argument form, thought of in the abstract, though I argue that it does not illuminate Darwin’s reasoning in the *Origin*.

2.2 Modus Darwin

Sober derives the normative force of *modus Darwin* from the *Law of Likelihood* (Hacking 1965; Royall 1997; Sober 2008), according to which an observation supports one hypothesis over another whenever that observation is more to be expected supposing the one hypothesis were true, compared with supposing the other hypothesis were true. More formally, observation o favors hypothesis h_1 over hypothesis h_2 if and only if $p(o|h_1) > p(o|h_2)$. Mapping this framework onto Darwin’s reasoning requires identifying an observation o , and two hypotheses h_1 and h_2 .

Similarity between two species (or larger taxa) is the observation o . The hypothesis h_1 is *common ancestry* (CA), which says that the two species descended from a single ancestor species. The alternative hypothesis h_2 is *separate ancestry* (SA), meaning that the two species’ lineages trace back to separate origin-of-life events. These are, however, only the rough, qualitative statements of o , h_1 , and h_2 . To evaluate the inequality $p(o|h_1) > p(o|h_2)$ Sober must specify the observation more rigorously, and then formally characterize the hypotheses as stochastic (chancy) processes that can produce such outcomes with some probability.

Regarding the observation o , any two organisms are similar in some ways and dissimilar in others. It seems there are infinitely many ways to measure similarity. Which is the right yardstick? Sober defers talk of *overall* similarity to begin with a simpler and more tractable observation: that two species share the same trait on a single *dichotomous character*. A dichotomous character is one that has just two possible states, for example an insect might have wings or lack them, or the edge of a leaf might be smooth or serrated. Coding morphology in terms of dichotomous characters typically masks more continuous underlying variation, but dichotomous characters are adequate in many scientific contexts, and they provide a convenient starting point for the formalization of *modus Darwin*.

2.2.1 A single dichotomous character

Letting o be the observation that two species share the same trait on a single dichotomous character, does o favor CA over SA *sensu* the Law of Likelihood? To generate the required conditional probabilities Sober repurposes the idealizations and mathematical framework of contemporary phylogenetic inference, as follows. Represent the two species as categorical variables X and Y , each of which can take states $\{0, 1\}$, standing for the two possible states of the dichotomous character. So o is both species in the same state (either $X = 0$ & $Y = 0$, or $X = 1$ & $Y = 1$). Each hypothesis is then characterized by a schematic genealogy for the two species, *plus* a stochastic model describing how the character variable changes states as it moves along a line in the genealogy (Figure 2.1).

The model of character-state evolution (applied in the same way to all solid

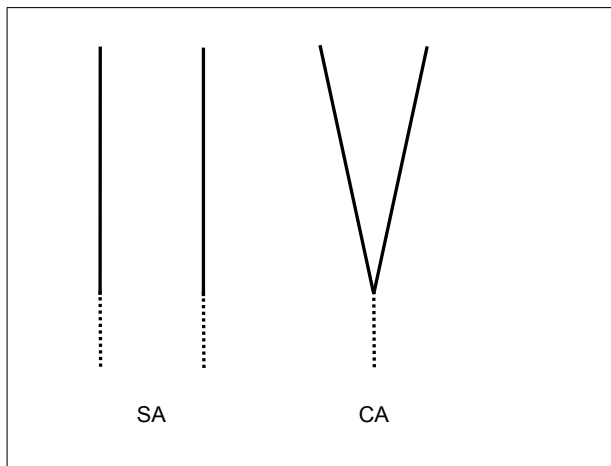


Figure 2.1: Schematic diagrams illustrating lineages postulated by the separate ancestry (SA) and common ancestry (CA) hypotheses

lines in both Figure 2.1 schematics) works as follows.¹ Each solid line has a *length* representing a number of time steps (all four lines are the same length). The variable associated with each line starts in one state or the other, and then undergoes that many time steps of evolution. At each step there is a small probability that the variable changes from its present state to the other state. (Two such state-change probabilities are required: $0 \rightarrow 1$ and $1 \rightarrow 0$, which need not be equal.) The probability of changing states depends only on the current state of the variable. Thus the longer the stretch of lineage, the greater the chance that the character variable will change states along that stretch.

¹While Darwin's primary target in *Origin* was a non-evolutionary, *creationist* version of the separate ancestry hypothesis, Sober prefers to reconstruct *modus Darwin* using a separate ancestry hypothesis that fully embraces evolutionary change. The idea is that this choice leaves the basic form of Darwin's reasoning intact, with the added benefit of illuminating the fundamental similarity between Darwin's reasoning and subsequent arguments made *within* the context of evolutionary theory.

In which state does a variable begin? The starting state of a variable is determined by a random draw from a probability distribution over the state space $\{0, 1\}$. The difference between CA and SA is only that for SA, the beginning states of the two variables are drawn independently from that distribution, whereas for CA just one random draw is required because both lineages' variables will begin in the same state (think of this as the point just before speciation).

With CA and SA so characterized, Sober proves the following result: for X and Y to end up in the same state at the end of the process is more probable on CA than on SA *regardless of lineage length, state-change probabilities, and the starting-state distribution* (Sober 2008, chap. 4).² In other words, two species found in the same state always favors CA over SA. It isn't hard to understand intuitively why this is so. If the state-change probabilities are small relative to the branch length, then the most probable outcome along any branch is that a variable won't change states at all. In this case, since CA puts the two species in the same state at the start, chances are good that they will both still be in the same state at the tips. The chances of ending in the same state are somewhat smaller on SA, since X and Y may or may not *begin* in the same state. As the probability of state change along a branch increases (due either to long branch lengths or high state-change probabilities), $p(o|CA)$ and $p(o|SA)$ converge to the same value, though $p(o|CA)$ must always be a little bit higher. The opposite is true for species found in *different* states: mismatches always favor SA over CA.

²With these very minor assumptions: the starting-state distribution gives non-zero probabilities to both states, the transition probabilities are non-extreme (*i.e.*, $\neq 0$ and $\neq 1$), and branch lengths are finite.

2.2.2 A single multistate character

Sober generalizes the preceding treatment to also cover *multistate* characters, where the variables X and Y can now take any number of states $\{1, 2, \dots, n\}$ and correspondingly more state-change probabilities are needed: one for every possible transition from one state to another ($i \rightarrow j$, for all $i, j \in \{1, 2, \dots, n\}$). Sober shows that, here too, X and Y in the same state at the end of the process is more probable on CA than on SA. Mismatches on multistate characters, however, are more complicated. Some mismatches will still favor CA, while others will favor SA (depending on the state-change probabilities).

2.2.3 Overall similarity

Next Sober aggregates many characters to arrive at something like *overall* similarity. Given a whole set of observed characters—some matches and some mismatches, which hypothesis is favored *overall*? Simply comparing the number of matches to mismatches doesn't answer the question. Some single-character observations favor more strongly than others. In other words, the *directions* of the single-character likelihood inequalities do not, on their own, provide enough information for aggregating. Supposing that the process by which each trait evolves is probabilistically independent of that governing every other trait,³ overall similarity favors CA over SA if and only if the product of the likelihood ratios (one from each observed trait)

³While this assumption is certainly not true, it is a standard idealization in, *e.g.*, phylogenetic inference from genetic data (thinking of each nucleotide site, or sometimes each codon, as a trait).

is greater than one, in mathematical notation:

$$\prod_{i=1}^m \frac{p(o_i|CA)}{p(o_i|SA)} > 1 \quad (2.1)$$

where m is the number of individual traits observed. Calculating the individual likelihood ratios $p(o_i|CA)/p(o_i|SA)$ requires additional assumptions about the details of the model of character evolution, in other words knowledge of the process by which the trait evolved (whether by drift or selection, if by selection then how strong and in what direction, and the time scale involved); see pp. 295–314, (Sober 2008) for details.

2.3 The correspondence problem

Now for my criticism. I will argue that Sober’s characterization of the observations themselves—the “similarity” in *similarity, ergo common ancestry*—illegitimately rigs his likelihood comparison in favor of the common ancestry hypothesis. My argument for this conclusion begins with an objection that Sober recognizes and discusses. But I will argue that the objection is both more serious and more general than Sober acknowledges.

Sober’s discussion of the *modus Darwin* inference form goes beyond Darwin’s own thinking to encompass modern reasoning about data that Darwin lacked. Discussing the application of *modus Darwin* to modern genetic sequence data, Sober identifies a possible stumbling block. Let’s call it the *correspondence problem*. To appreciate the worry, first think about how sequence data are used in *phylogenetic inference*. (Phy-

logenetic inference *assumes* common ancestry among a group of species and seeks to discover the particular shape of their genealogical tree.) Sequence-based phylogenetic inference uses a small stretch of DNA from each species. But phylogeneticists don't simply draw a random sequence of DNA from each species. They use "corresponding" sequences. In the final step of establishing sequence correspondence, two DNA sequences are *aligned*, by sliding the one along the other and stopping when the number of matching sites is greatest. But as Sober points out, this process seems out of place in the context of *modus Darwin*:

At first glance, alignment seems not to make sense in this problem. Since matching at a site is evidence for CA, aligning the sites so as to maximize matching seems to load the dice in favor of the common ancestry hypothesis. But the problem is deeper. If two sequences have a common ancestor, it makes sense to say that a site in one sequence "corresponds" to a site in the other; this correspondence means that the two sites derive from a site in their common ancestor. But if there was no such common ancestor, what would alignment even mean? If we want to *test* the separate-ancestry hypothesis rather than just *assume* from the outset that it is false, we need to rethink the question of how sequence data can be used. (Sober 2008, 291)

In response to this worry, Sober first points out that his *modus Darwin* likelihood comparison can be carried out regardless of whether the sequences "correspond". Choose any sequence of length n from the genome of species A , and another from

that of species B . Treat the two sequences as states of a single character with 4^n possible states (four possible nucleotides for each site). Then:

By the argument given earlier, this matching counts as evidence of CA. There is no need to align the sites to say this. The same point applies when the sequences (each n sites long) drawn from the two species do not match perfectly. They will then occupy different states of a single character that has 4^n possible states. Whether this difference between the two species favors CA or SA depends on the rules of evolution that govern how this complex character evolves. . . . The question is simply whether the observed mismatch has a higher probability of arising under the common-ancestry or the separate-ancestry hypothesis. To answer this question, all that is needed is the two *unaligned* sequences and a reasonable model for the process of sequence evolution. (Sober 2008, 291)

From this point about *unaligned* (*i.e.*, non-corresponding) sequences, Sober draws the following conclusion about *aligned* sequences:

An inference that begins with aligned sequences is valid to the extent that it mimics the verdicts of the procedure that uses unaligned sequences. When this is true, aligning sequences is not loading the dice. (Sober 2008, 291)

But this will not do. It is true that whether sequences are aligned makes no difference to the *question*; that question is always: Is the observation more probable on the

CA or SA hypothesis? The *answer*, however, does depend on whether sequences are aligned. For this reason the verdict of the procedure that uses aligned sequences will be systematically different from that of the procedure that uses unaligned sequences. Some example calculations will illustrate.

Unaligned sequences will match at about one site in four. Is this more probable on CA or SA? In the quotation above, Sober treats a DNA sequence as a single character with 4^n possible states, but alternatively we can treat each site as one character, where the resulting n likelihood ratios are aggregated as per Equation 2.1. I choose the latter method here to simplify calculations. The probabilities $p(obs.|CA)$ and $p(obs.|SA)$ depend on the details of the model of evolution, but for example, suppose that all sites evolve independently by drift, with all four nucleotides equally probable as starting states. Plot (a) in Figure 2.2 shows likelihood ratios $p(o|CA)/p(o|SA)$ with branch length increasing from left to right. That ratio is below 1 for all branch lengths, meaning that matching at one in four sites always favors SA over CA.⁴ Moreover, one in four sites matching is typical of unaligned sites regardless of what species are compared. Applying *modus Darwin* to unaligned sequences favors *separate ancestry*, not common ancestry. It doesn't matter whether you compare a bacterium with an elephant, or a human with her identical twin.

On the other hand, with the same assumptions about process, observing that two sequences match at 50% of their sites favors *common ancestry* for all but extremely

⁴Note that Sober doesn't describe overall similarity in terms of *x% matching*—he does not explicitly use any measure of overall similarity (such as % of traits matching). Rather, when considering the overall evidential weight of a set of traits, the observation is just the states of each species for each trait. In this particular case, however, the % of matching sites between the two DNA sequences is a *sufficient statistic* of that more complete description of the data, for the calculation of the likelihood ratio $p(o|CA)/p(o|SA)$.

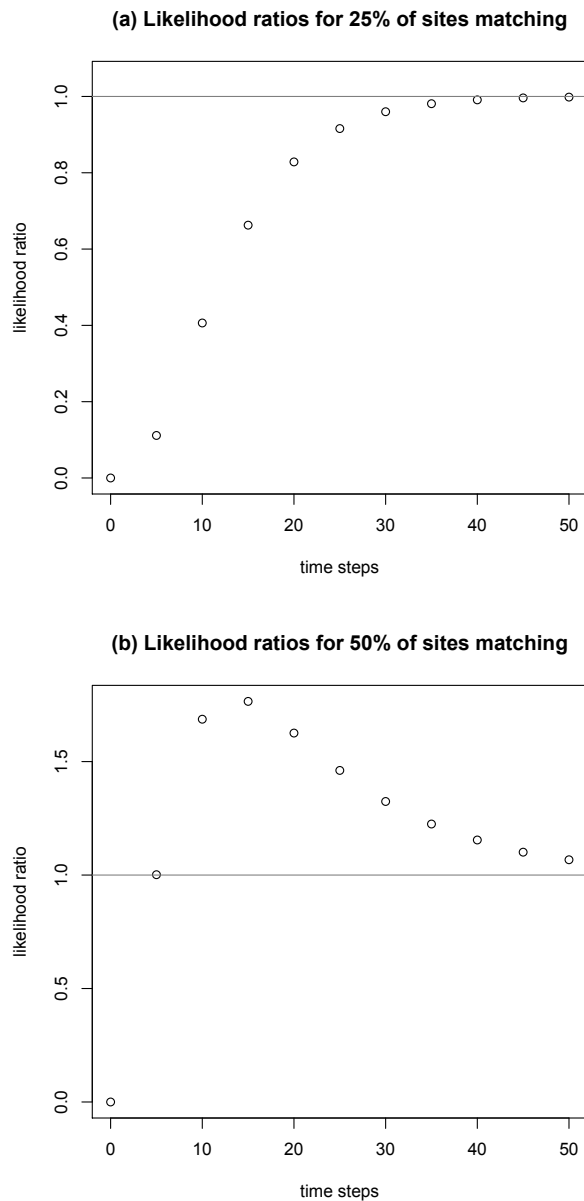


Figure 2.2: Values for the likelihood ratio $p(obs.|CA)/p(obs.|SA)$ at different numbers of time steps of discrete time evolution, assuming that each site evolves independently, starting from a uniform distribution over the four states, and with state-change probabilities $p(i \rightarrow j) = 0.01$ for $i \neq j$ (*i.e.*, drift). The two plots show likelihood ratios for the observation of DNA sequences matching at 25% of sites, and at 50% of sites.

short branch lengths (if branch lengths are short enough, then any mismatches at all become extremely improbable on CA, since on that hypothesis all sites start off matching; see plot (b) in Figure 2.2). Aligned DNA sequences match at far greater than 50% of sites, so aligned sequences favor common ancestry on the assumption—ubiquitous in the methodology of phylogenetic systematics—of sites evolving independently by drift. Thus if the *modus Darwin* inference that begins with aligned sequences is valid *only* to the extent that it mimics the verdicts of the procedure that uses *unaligned* sequences, then it is not valid at all. Aligned sequences favor CA, while unaligned sequences favor SA.

Thus, applying *modus Darwin* to genetic sequence data produces a dilemma. Using aligned sequences loads the dice in favor of common ancestry—and the only discernible justification for doing so begs the question by assuming CA. n the other hand, using *unaligned* sequences favors separate ancestry, which is the wrong conclusion (wrong as in *false*, though not necessarily epistemically *irrational*).

2.3.1 Diagnosis

Applied fairly (*i.e.*, to unaligned sequences), Sober’s *Modus Darwin* likelihood comparison systematically supports the wrong conclusion. What has gone wrong? The problem is that the common-ancestry hypothesis that Sober *intends* to test via this likelihood comparison is not the same as the hypothesis that actually appears in that likelihood comparison. Sober’s qualitative statement of the the common-ancestry hypothesis says only that the two species from which the sequences are drawn have a common ancestor species, but if we’re going to take the likelihood comparison seri-

ously, we have to look more closely at the commitments of the precise, mathematical description of CA. After all, it is that mathematical description that generates the likelihood $p(obs.|CA)$. That description goes beyond Sober's informal statement of CA; it says that *the gene sequences themselves* share a common gene sequence ancestor, and that the two sequences are derived from that ancestor via a process approximated by the stochastic model of character state evolution described above (§§ 2.2.1–2.2.2). This is a very significant difference, because the sequence question need not settle the species question. If two sequences do have a common ancestor sequence, then (setting aside horizontal transfer) that means that the two species from which the sequences were taken have a common ancestor species. But if the two sequences do not have a common ancestor sequence (of the kind posited by the mathematical description of CA) this leaves it entirely unsettled whether the species have a common ancestor species.

For non-corresponding (unaligned) gene sequences, *the sequence-CA hypothesis is false*, even if the two *species* from which the sequences are drawn do share a common ancestor *species*. Sober intended for the *modus Darwin* likelihood comparison to discriminate between SA and CA, the hypotheses that say, respectively, that species A and species B do, and do not, have a common ancestor *species*. But what the *modus Darwin* likelihood comparison in fact assesses is how the observation of two sequences bears on the hypotheses that those particular sequences do or do not have a common ancestor *sequence*. Thus, while applying *modus Darwin* to unaligned sequences initially appeared to recommend the wrong answer, we can now see that in fact it gives the right answer to a different question. A negative answer to this

new question does not settle the question Sober set out to ask.

2.3.2 From gene sequences to anatomy

The objection that I have been discussing so far—what I’m calling the problem of correspondence—is one that Sober discusses while applying the *modus Darwin* argument form to modern genetic sequence data. So far I have argued that this problem is more serious than Sober acknowledges. Indeed, it results in Sober’s likelihood comparison *changing the subject*. Rather than revealing what the evidence says about CA versus SA, that likelihood comparison addresses a question about the history of the particular DNA sequences compared. But it gets worse: now I argue that exactly the same problem afflicts Sober’s reconstruction as applied to the anatomical observations that Darwin used to argue for common ancestry. In other words, there is nothing special about genetic sequence data; the correspondence problem is very general.

Recall that Sober’s individual *morphological* similarities consist of two species being in the same *state* for some single character. But each such observation implicitly treats the two *characters* as “the same” character seen in two different species. Imagine comparing a spider with an insect. And consider, for example, the character: number of appendages attached to the thorax. The insect has 6. What about the spider? Before you can answer you must decide what counts as the spider’s thorax. It’s the second of the *insect’s* three body sections, but the spider’s body has only two sections; one section has zero appendages, the other has 10 (8 legs plus 2 chelicerae—appendages by the mouth for grabbing food). So is the character *state* comparison

6 to 0?, 6 to 10?, 6 to 8? Comparative anatomy is full of far more difficult cases than this. The point, however, applies even where the “correct” correspondence is intuitively obvious. Consider a human and a giraffe, and compare them on the continuous character: *length of the femur*. Why compare what we call the “femur” of the human to what we call the “femur” of the giraffe? Why not compare the human’s femur to the giraffe’s humerus, or to its radius, its scapula, its anything?

Morphological *modus Darwin* presupposes a system of correspondences between the characters of one species and those of another, which system enables the comparisons of character *states* that in turn generate the matches and mismatches that constitute the observations on which *modus Darwin* operates. Of course there *was* such a system of correspondence, on which was built the taxonomy and comparative anatomy of Darwin’s time. The existence of such a system is not in question. What is in question is the legitimacy of relying on that system of correspondences in the context of a likelihood contest between CA and SA. It is not legitimate in that context for the same reasons that aligning DNA sequences is not. Very crudely, and leaving out many important caveats, the procedures for established the correspondence of body parts for the purpose of taxonomy amounted to selectively stretching, squishing, and reorienting the parts of one organism until they best lined up with those of the other. In the same way that aligning DNA sequences “loads the dice” in favor of CA, comparing traits with the help of the system of correspondences built into taxonomy assesses trait matching and mismatching in a way that is tailor-made to maximize matching.

2.4 First Conclusion

In translating the slogan *similarity, ergo common ancestry* into a rigorous argument form, Sober understands an instance of “similarity” as species X and Y both occupying the same character state. But it is a mistake, I have argued, to treat such observations as sufficiently theoretically naive to serve as an objective starting point for contrasting the likelihoods of the CA and SA hypotheses. Formulating those observations of matching and mismatching character states requires the use of a system of correspondences between the characters themselves, and the system presupposed by Sober’s reconstruction of Darwin’s reasoning—indeed the only system available to Darwin—is not neutral between the CA and SA hypotheses.

Somewhat more carefully, suppose that species X and Y are each unproblematically decomposed into n dichotomous characters. To formulate the observations of matching and mismatching, we must first assign to each character of species X a character of species Y , treating paired-up characters as instances of the same character in the different species. Each different mapping of the characters of X onto those of Y implicitly defines a different version of the common ancestry hypothesis—each version being a conjunction of n hypotheses about the ancestry of particular *character pairs*. The hypothesis of common ancestry *for the two species* can be thought of as the disjunction of all possible more specific hypotheses, each defined by a particular character mapping.

Because choosing an assignment of character correspondences is equivalent to singling out a particular version of CA, and because taxonomic practice included biases towards greater matching in the assignment of character correspondences,

and because the more matches the higher the likelihood of CA, reading off matches and mismatches using the standard anatomical vocabulary of comparative anatomy is akin to selecting only the highest-likelihood disjunct within the CA-for-species hypothesis and using this likelihood as the quantity $p(obs.|CA)$. In the same way, each possible correspondence assignment between the characters of species X and Y also defines a specific version of the separate ancestry hypothesis. Assignments of correspondence biased towards matching are, however, among the *lowest likelihood* variants of CA. So counting the matches and mismatches according to the standard correspondences of comparative anatomy amounts to selecting a low-likelihood variant of the CA hypothesis and treating this likelihood as the quantity $p(obs.|SA)$. Thus Sober's *modus Darwin* likelihood comparison is not really a comparison between the likelihoods of CA and SA, but between those of the best-fitting variant of CA and the worst-fitting (at least relatively poorly-fitting) variant of CA.

2.5 Geographical proximity

Among Darwin's supporting observations in the *Origin* are also many reports about the *geographical distribution* of species. Sober extends *modus Darwin* to apply to these geographical distribution observations as well. Sober does this by reinterpreting the stochastic model of multi-state character evolution (described above) as a model of *geographical dispersal*.

2.5.1 Proximity, ergo common ancestry

Consider a multistate character that has ten discrete character states, and label them 1–10. The stochastic model governing how the categorical variable changes values requires a ten-by-ten matrix of transition probabilities, one for each possible transition from one state to another. Now impose an extra constraint on these transition probabilities: allow positive transition probability only between neighboring states on the number line (and between a state and itself); make all other transition probabilities zero, for example:

$$\begin{pmatrix} .95 & .05 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ .05 & .9 & .05 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & .05 & .9 & .05 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & .05 & .9 & .05 & 0 & 0 & 0 & 0 & 0 \\ \vdots & & & & & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & .05 & .9 \end{pmatrix} \quad (2.2)$$

Now for the reinterpretation. Think of each of the ten states not as variants of an anatomical character, but *geographical locations* along a line (*e.g.*, islands in an archipelago), and think of state change not as morphological evolution but *geographical dispersal* (Sober 2008, 326). A species can disperse from the first location to the fifth only by passing through locations 2, 3, and 4, thus the constraint of zero probability for direct transition between non-neighboring states. “Neutral evolution within an ordered n -state character is formally just like random dispersal across an n -island archipelago.” (326)

The state-change probabilities in Equation 2.2 determine what is called the *equi-*

librium distribution of the character-state-cum-location variable, which gives the probabilities of finding the variable in each of states 1–10 after (loosely speaking) infinitely many time steps. Sober uses the probabilities from this equilibrium distribution for the probabilities of *starting out* in states 1–10 at the beginning of a branch (this is equivalent to treating the dotted lines in the Figure 2.1 as infinite in length).

Using this reinterpretation, Sober investigates a concrete formal example with ten locations and equal transition probabilities between neighboring locations (*e.g.*, ten equally-spaced islands). The equilibrium distribution generated by such transition probabilities is uniform over the ten locations. For CA, a single starting location is chosen, from which the two species (call them X and Y) begin their probabilistically independent random walks, whereas for the SA *each species gets its own starting point*, drawn independently from the same uniform distribution over the ten locations. The observation o is then the observed spatial separation between two species. Regarding the likelihoods $p(o|CA)$ and $p(o|SA)$, Sober reports that:

With ten locations, the expectation under the separate-ancestry hypothesis is that X and Y will be a bit more than three islands away from each other. If X and Y are more spatially proximate than this, then CA has the higher likelihood; if not, not. (Sober 2008, 326)

This “ten-islands” model shows how, in principle, the present geographical proximity between species X and Y can serve as evidence favoring CA over SA, or vice versa (where geographical proximity is taken as a proxy for accessibility via dispersal).

Sober then wields this application of *modus Darwin* to analyze Darwin's use of geographical distribution observations in the *Origin*. The best-known snippet of Darwin's long discussion of geographical distribution (chaps. 11 & 12) is his discussion of islands and mainlands, especially the example of the Galapagos Archipelago:

The most striking and important fact for us in regard to the inhabitants of islands, is their affinity to those of the nearest mainland, without being actually the same species. Numerous instances could be given of this fact. I will give only one, that of the Galapagos Archipelago, situated under the equator, between 500 and 600 miles from the shore of South America. Here almost every product of the land and water bears the unmistakable stamp of the American continent. (Darwin 1859/2003, 239–8)

Sober maps *modus Darwin* onto Darwin's Galapagos illustration as follows.

First Sober decomposes Darwin's comparison of the "inhabitants" of the Galapagos to those of the South American mainland into numerous more specific comparisons, each between a single species found on the Galapagos and one found on the South American mainland. For Galapagos species $\{X_1, X_2, \dots, X_n\}$ and mainland South American species $\{Y_1, Y_2, \dots, Y_n\}$, each more specific comparison concerns a pair (X_i, Y_i) consisting of one island species and one mainland species that are very similar morphologically. In the big picture, each X_i is geographically close to its partner Y_i —about 600 miles away, on a globe of diameter 25,000 miles. Sober reads Darwin as presenting the geographical proximity of the Galapagos and the South American mainland as evidence favoring the common ancestry of species X_1 and Y_1

over separate ancestry for those two species, and in the same way favoring (to introduce an abbreviation) $CA_{(X_2, Y_2)}$ over $SA_{(X_2, Y_2)}$, $CA_{(X_3, Y_3)}$ over $SA_{(X_3, Y_3)}$, $CA_{(X_4, Y_4)}$ over $SA_{(X_4, Y_4)}$, and so on.⁵

2.5.2 Criticism

Given the problems with applying *modus Darwin* to anatomical traits, it would be nice to find traits of organisms that can be compared without presupposing any notion of correspondence. It appears that geographical position is just such a trait. As such, the geographical distribution variant of *modus Darwin* completely avoids the criticism that I made above of the anatomical and genetic sequence variants of the inference form. Applied in this way to geographical proximity, *modus Darwin* does frame a theoretically viable way of evidentially distinguishing between SA and CA. Still, I will argue that this geographical distribution variant of *modus Darwin* is not an adequate reconstruction of Darwin's argument in the *Origin*.

I begin by pointing out an important caveat to the conclusion that Sober draws from the ten-islands model of dispersal. That conclusion is that the expectation under SA is that species X and Y will be a bit more than three islands away, and that observed separation below that threshold favors CA over SA. Sober's conclusion seems to suggest that the evidential import of geographical distribution observations can be understood simply on the basis of the distances and geographical layout. But

⁵Sober sees an additional inference in Darwin's reasoning about the Galapagos, regarding whether a group of organism pairs (*assuming CA is true of each*) all have the same geographical point of origin. But this reasoning is no longer *modus Darwin*, nor does it concern the argument for CA over SA.

this is not so, and it is worth demonstrating how the interpretation of geographical location observations requires more information about the dispersal process. It is correct that the expectation under SA is a bit more than three islands away, but it does not follow, and it is not true, that observed spatial separation less than that favors CA over SA. Whether smaller observed separations favor SA or CA depends on further details not specified in Sober’s description of the ten-islands dispersal model, specifically, it depends on the branch lengths—the product of the dispersal rate and the number of time steps per branch. For example, setting the probability of moving to a neighboring state to 0.01 per time step, Figure 2.3 illustrates branch length sensitivity by displaying the probability distribution over spatial separation generated by CA and SA at 5, 50, 200, and 1000 time steps. Notice that which hypothesis has the higher likelihood for distances 1, 2, and 3 changes depending on how much time has passed.

This sensitivity to branch length marks an important difference between *modus Darwin* applied to morphology and applied to geographical distribution. Setting aside the problem of correspondence, in the case of morphology (at least for dichotomous characters, and so long as we consider just one such character in isolation) the observation of a matching character is sufficient for knowing the direction of the likelihood inequality between $p(obs.|CA)$ and $p(obs.|SA)$. For the geographical distribution application on the other hand (and more generally for any quantitative trait), there is no geographical distance observation that itself guarantees that $p(obs.|CA)$ is greater than $p(obs.|SA)$. Further information about branch lengths is always needed. Since Sober’s reconstruction of Darwin’s geographical distribution

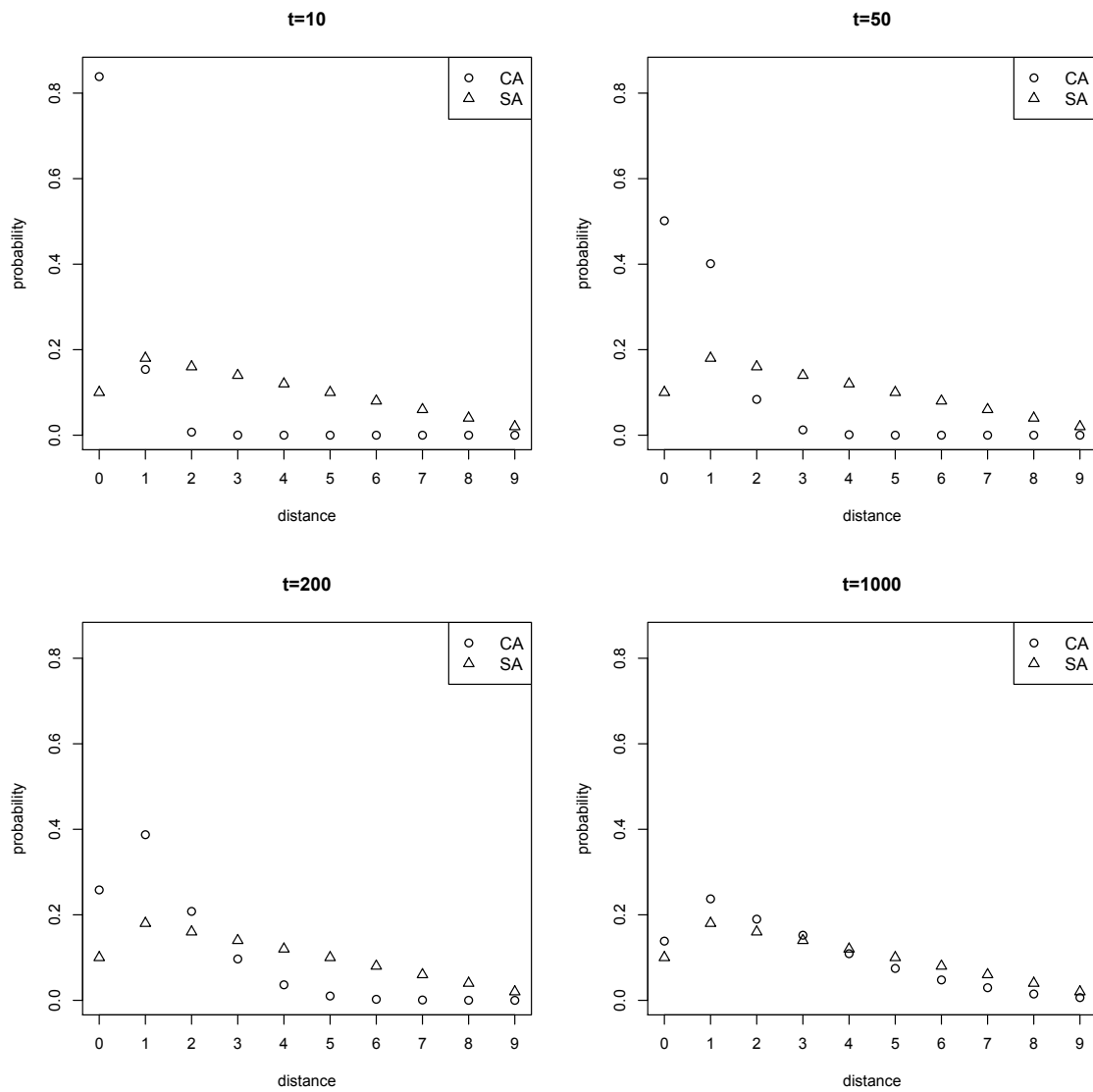


Figure 2.3: A series of probability distributions over spatial separation observed between species X and Y in Sober's "ten-islands" dispersal model. All CA distributions assume $\mu = .01$. The number of time steps (t) varies between plots. The SA distribution depends on neither μ nor t , and is the same in every plot.

argument makes no mention of branch length estimation, that reconstruction is, at best, incomplete. Perhaps *proximity, ergo common ancestry* is a part of the story, but it can't be the whole story.

There is a second, and related way in which geographical distribution *modus Darwin* is inadequate, or at least incomplete, as a reconstruction of Darwin's reasoning about biogeography and common ancestry. Recall that Sober sees in Darwin's Galapagos illustration a number of species pairs (X_i, Y_i) where the X_i are Galapagos species and the Y_i are mainland South American species. In each case, geographical proximity between the pair is the observation. The set of observations favors $CA_{(X_1, Y_1)}$ over $SA_{(X_1, Y_1)}$, $CA_{(X_2, Y_2)}$ over $SA_{(X_2, Y_2)}$, $CA_{(X_3, Y_3)}$ over $SA_{(X_3, Y_3)}$, and so on. The geographical distances between all of the species involved are represented schematically in Figure 2.4. The most striking feature of the whole geographical layout is of course not the proximity of any X to any Y , but the close proximity of all of the X_i to each other, and of the Y_i to each other. Without additional information about branch lengths, the main conclusion that *can* be drawn from the ten-islands dispersal model is that, other things being equal, the smaller the distance between species X and Y , the more plausible it is that their proximity favors CA over SA. But applying this lesson naively to the observations schematized in Figure 2.4, suggests that the primary import of these observations would be to favor CA over SA for each *within*-Galapagos species pair, as well as for each *within*-mainland pair.

Sober emphasizes that support for CA for within-Galapagos, or within-mainland species pairs is *not* the conclusion that Darwin argues for. Regarding Darwin's use of the Galapagos example, Sober says:



Figure 2.4: Schematic representation of the relative distances between species $\{X_1, \dots, X_n, Y_1, \dots, Y_n\}$ where the X_i inhabit the Galapagos archipelago and the Y_i inhabit the west coast of the South American mainland.

Darwin is not arguing that Galapagos tortoises and iguanas have a common ancestor based on the fact that they happen to live side by side. Not that he denied that they share a common ancestor, but this is not what he is here concluding. (Sober 2008, 330)

While this statement may be correct, it does not line up with the conclusion that is generated by Sober’s formal reconstruction of Darwin’s mode of weighing evidence, applied naively to the Galapagos geographical distribution observations. Sober does not apply *modus Darwin* to the Galapagos observations *naively*; rather, he does so selectively, singling out the species pairs (X_i, Y_i) . But the reasons for this selective application are not a part of Sober’s formal reconstruction of Darwin’s reasoning—this part is done, so to speak, “by hand”. If there is to be an adequate reconstruction of Darwin’s geographical distribution argument for common ancestry that includes geographical *modus Darwin*, it must also include both some estimation of branch lengths, and some rationale for the otherwise unmotivated selective application to species pairs (X_i, Y_i) .

2.6 Second Conclusion

Sober has attributed two argument forms to Darwin, informally: “*Similarity, ergo common ancestry*”, and “*Proximity, ergo common ancestry*”. Sober spells out each of these as a likelihood comparison between the common-, and separate-ancestry hypotheses. Regarding the former argument form, I have argued that the likelihood comparison that Sober makes does not pit CA against SA as it was intended to do, but rather a best-fitting variant of CA against a poorly-fitting variant of SA. This is not a rational way to assess whether the evidence favors CA or SA. If Sober’s reconstruction accurately reflects the rationale within Darwin’s evidence and reasoning, then Darwin made a bad argument. If, on the other hand, Darwin’s made a sound, rational argument for common ancestry based on morphological observations, then that argument was not *modus Darwin* as Sober reconstructs it. Regarding the latter argument form, I have acknowledged that Sober’s reconstruction frames one way that observations of geographical proximity might be used to support CA over SA, but argued that this reconstruction does not go very far in illuminating Darwin’s geographical distribution argument in the *Origin*.

In the next chapter, I develop an alternative picture of how morphological and geographical distribution observations contribute to Darwin’s argument for common ancestry.

Chapter 3

Pattern as observation: Darwin's geographical distribution argument

3.1 Introduction

In his book *On the Origin of Species* Darwin presented a theory and a large collection of supporting observations. How does the theory relate to those observations? This question is at the center of any attempt at normative evaluation of Darwin's argument. The simplest and most fundamental approaches to evaluating hypotheses in light of observation require the hypothesis to tell us what we should expect to observe in nature were that hypothesis correct (or more generally, how *probable* various observations would be, were the theory correct). According to most philosophical analyses, however, the theory Darwin put forward falls short of this standard.¹ Were

¹Sober's work on *modus Darwin*, discussed in the previous chapter, is a notable exception.

it true, as Darwin claimed, that all species trace back to one or a few common ancestors, and that natural selection is the primary means of modification, it wouldn't follow that tigers should have sharp teeth, that grasses should have a wide geographical distribution, or that beetles should be so prolific. Nor does Darwin's theory tell us how *probable* any of these outcomes are—not even qualitative, ballpark probabilities. This apparent shortcoming introduces an interpretive problem that all philosophical commentators on the *Origin* must face. How can Darwin have made a *good* scientific argument for his theory without having compared what actually obtains in nature with what *should* be observed were his theory correct?

The philosophical literature on the *Origin* offers a variety of solutions to this problem. In lieu of saying how probable the observations are, Darwin's theory is said to *explain* the observations, where the 'explains' relation is left unanalyzed (Thagard 1978; Recker 1987; Hodge 1991; Waters 2003), or to *fit* the observations *after post hoc adjustment*, where the assumptions used in the fitting are testable in principle (Lloyd 1983; Recker 1987), or, similarly, to *provide a framework* within which a speculative, yet ultimately testable, historical narrative (leading up to the observed event) can be formulated (Kitcher 1993, 2003). The empirical observations compiled in the *Origin* are then said to *support* Darwin's theory over the alternatives in virtue of one or more of the following: the *number* of observations that the theory can explain (or that it can be made consistent with, or about which a story can be told), the number of *different kinds* of observations explained, the *novelty* of these kinds, the prior plausibility or familiarity of the *causes* cited in the explanations (*vera causa*), and the *economy* with which the theory does so much explaining.

In this essay I argue that *some* of Darwin's observations are just what we should expect to observe were his theory correct. Thus I reject a presupposition of the interpretive problem posed above, a presupposition that motivates and shapes all of the analyses referenced above. If my conclusion is correct, those analyses misrepresent how Darwin's theory relates to (at least some of) his supporting observations, and consequently, how those observations support the theory evidentially. Of course Darwin's hypothesis doesn't say *exactly* how probable any observation would be supposing the hypothesis were correct (his theorizing was entirely qualitative), but I will argue that *when the relevant observations are properly understood*, his hypothesis does generate a degree of expectation that is concrete enough to compare favorably with the alternatives. This conclusion does not constitute a full normative analysis of the rational epistemic import of these observations for Darwin's argument, but philosophers with a variety of ideas about confirmation should find this conclusion relevant to evaluating that epistemic import.

The particular observations that I will discuss come from Darwin's survey of *geographical distribution* in chapters eleven and twelve (I motivate this decision below), and my analysis of those observations turns on a very general, and under-appreciated philosophical point about theory-observation relations. Indeed this essay is as much a case study illustrating this general philosophical point as it is a targeted analysis of Darwin's geographical distribution argument. A simple example (adapted from Forster (1988)) will introduce this general philosophical point.

Suppose you are to receive two data sets, *A* and *B*, each reporting the outcomes of fifty coin tosses. And suppose I tell you my hypothesis about the process that

generated those data. My hypothesis says that a single coin was tossed one hundred times, and that data sets A and B record tosses 1–50, and 51–100, respectively. My hypothesis is silent on whether the coin is fair or biased. The probability this coin lands heads on a given toss could be anywhere between zero and one, but whatever this unknown probability, it is constant across flips of the coin (and the outcome of each flip is probabilistically independent of past and future flips). What should you expect to observe in the data were my hypothesis correct? A natural place to start is the outcome of the first coin toss. What does my hypothesis predict about the first entry in data set A ? Which is more probable ‘heads’ or ‘tails’, and by how much? My hypothesis cannot answer this question. Loosely speaking, it *doesn’t say enough* to generate any expectations about the outcome of that coin toss. And the same goes for any other specific entry in either data set. You could look over the data one entry at a time: ‘heads’, ‘heads’, ‘tails’, ‘heads’, and so on, of each data point asking ‘*To what degree is this expected were my hypothesis correct?*’, and you would never get an answer. (Of course things would be different if my hypothesis also specified the coin’s probability of landing heads, but it *does not* specify that value.)

Now step back from the individual coin toss outcomes and look at some more abstract features of the data. If my hypothesis were correct, what should we expect to observe regarding the *frequency* of heads in data set A ? Again, this all depends on the unknown probability of the coin landing heads on any given toss. The same goes for the frequency of heads in data set B , and also for the overall frequency of heads in the total data set A plus B . But consider the following—now *very* abstract—feature of that total data set: the *difference* between the frequency of heads in A and that

in B . My hypothesis *does* generate expectations about this feature of the total data set. My hypothesis says that both data sets were generated by tossing *the same coin*, so the frequency of heads in the two data sets (whatever frequency that is) should be roughly the same—in other words the difference between the frequencies should be close to zero. This prediction holds regardless of the coin’s (unknown) probability of landing heads.² This simple coin-tossing example illustrates how a hypothesis can bear a loose, non-expectations-generating relation to every individual datum within a set of observations, while at the same time sticking its neck out when it comes to certain abstract, ‘high level’ features of the same set of observations.³

The moral of the example is familiar within the field of statistics, but is less well appreciated among philosophers of science and has been largely overlooked in philosophical analyses of Darwin’s argument in the *Origin*. Darwin’s most compelling evidence consists of large-scale *patterns* observable in nature. Like the difference-between-frequencies feature of the coin-tossing data, the observed patterns of geographical distribution that Darwin presents in support of his hypothesis are abstract, ‘high level’ features of a large set of smaller-scale observations (that this species is found here, and that species is found there, *etc.*). And just like my hypothesis in the coin-tossing example, Darwin’s hypothesis bears only a loose relation to each of

²The probability distribution assigned to the statistic $|frequency_A - frequency_B|$ by the binomial model for the total data set (that’s the technical name for my hypothesis) *is* a function of the coin’s probability (θ) of landing heads, *i.e.*, that statistic is not (what is called) *ancillary* for the model. But for all values of θ , the most probable frequency difference is zero, and differences greater than 0.2 are extremely improbable.

³The relationship between my single-coin hypothesis and a small observed value for the difference-between-frequencies statistic can also be described (as per Forster (2007)) in terms of a good cross-validation score across a specific partition of the full data set that divides part A from part B.

the individual, local observations that make up the observation set, but nonetheless enjoys a tighter, expectations-generating relation to an abstract feature of that observation set. It is correct that Darwin's theory doesn't tell us whether or to what degree *any of the particular, local observations* are to be expected were the theory correct. But it is a mistake to think that because the hypothesis generates no expectations about 'lower-level' observations that it generates none about the 'higher-level' observations. My main task in what follows is to rigorously characterize the 'high-level' patterns from Darwin's discussion of geographical distribution in a way that both faithfully captures the observations Darwin presents, and at the same time makes apparent the (expectations-generating) relation between his hypothesis and those observations.

What I have above called 'abstract', or 'high-level' features of a set of observations can also be describe as *logically weakened descriptions* of that set of observations. The frequency of heads is logically weaker than the actual sequence of heads and tails, and the *difference* between frequencies is weaker still. Looking at the coin example from this perspective, it is unsurprising that successive weakenings of an observation's description will eventually produce a description that is probable on, even entailed by, the hypothesis. For example the observation that each coin came up either heads or tails. So it is important to point out that the difference-between-frequencies observation is not trivial in the sense that the same would be predicted by every non-trivial hypothesis. Consider as an alternative to my same-coin hypothesis one that says each data set was generated by a different coin: a two-coin hypothesis. The two coin hypothesis lets each coin have its own unknown bias, allowing the

frequencies of heads to diverge in the two data sets, and therefore does not predict that the difference between frequencies should be small.

The notion of logical strength applies to hypotheses as well, and can be used to encapsulate the difference between the accounts of Darwin's evidence referenced above and what I will present here. Where a hypothesis appears to lack any commitments about what should be observed in the data were that hypothesis correct, there are, broadly speaking, two ways to bridge the gap. One way is to logically *strengthen* the hypothesis: if the hypothesis doesn't say enough to make any predictions, then let it say more. This is the approach taken by the accounts referenced above. On those accounts, Darwin's hypothesis makes contact with real-world observations with the help of *added* assumptions used in fitting the hypothesis to data, or in spelling out a narrative within the framework of the hypothesis. The other way to bridge the gap is to logically *weakening* the description of the observations. In the analysis to follow, I take the second route.

I have spoken above of 'Darwin's theory' or 'Darwin's hypothesis'. More specifically, I will be working with his hypothesis of *common ancestry*. While Darwin is nowadays revered first and foremost as the author of the theory of natural selection, he says plainly in the introduction to the *Origin* that his main conclusion is that diverse species *share ancestry* (Darwin 1859/2003, 6). He opens with a scaled-back version of this conclusion: that the species within any one *genus* have descended from a single ancestor species (1859/2003, 6), but as the book proceeds he expands the scope of the claim until, in the final paragraph, he extols the 'grandeur' of his view of life, 'having been originally breathed into a few forms or into one' (1859/2003,

490). And if morphologically disparate species trace back to the same ancestor, then those species must have been transformed over time (*i.e.*, shared ancestry entails *that* evolution has occurred). It is a separate matter *how evolution works*. This is where natural selection comes in. To his main conclusion, Darwin adds ‘Furthermore, I am convinced that Natural Selection has been the main but not exclusive means of modification’ (1859/2003, 6).

Common ancestry is, moreover, the part of Darwin’s proposal that his audience actually bought. After the *Origin*, the scientific community quickly accepted common ancestry as fact, while the competence of natural selection to produce evolutionary change was widely debated well into the twentieth century (Larson 2004). In this paper I follow Waters (2003) and Sober (2011) in reading Darwin’s defense of *common ancestry* as separable and largely independent of whether natural selection is in fact the primary means of modification. I set aside natural selection and address only Darwin’s main conclusion, the hypothesis of common ancestry (and therefore evolution by some means or other).

Darwin’s case for common ancestry includes evidence from the geological succession of fossil remains (chap. 10), the geographical distribution of extant life (chaps. 11 & 12), embryology, morphology, and the nature and methods of biological taxonomy (chap. 13). I take Darwin’s discussion of *geographical distribution* to provide his strongest argument for shared ancestry, so I restrict my attention to that topic. (Within geographical distribution, I bracket Darwin’s chapter-ten discussion of the distribution of *fossil remains* to focus on the distribution of *living forms* examined in chapters eleven and twelve—though I do this only to keep the subject matter

manageable.)

3.2 Overview of chapters eleven and twelve

To orient readers, I begin by briefly outlining the whole of chapters eleven and twelve. Darwin begins chapter eleven with a review of the major trends in the observed geographical distribution of species, recognizing three major trends (the following reports are pitched at a very abstract level, so don't be alarmed if their meaning is not immediately clear):

The first great fact which strikes us is, that neither the similarity nor the dissimilarity of the inhabitants of various regions can be accounted for by their climatal and other physical conditions. (346)

... A second great fact which strikes us in our general review is, that barriers of any kind, or obstacles to free migration, are related in a close and important manner to the differences between the productions of various regions. (347)

... A third great fact, partially included in the foregoing statements, is the affinity of the productions of the same continent or sea, though the species themselves are distinct at different points and stations. (349)

Each of these pithy statements is followed by a page or two of explanation and examples (discussion of which I put off till the next section). This brief review of observed trends is clearly intended to support Darwin's theory of evolution, and to

undermine his primary target, ‘the theory of creation’. But *how* it is supposed to do this is not spelled out.⁴ The text leaves the evidential import of the first great fact entirely implicit at this point. The outlines of an argument are more visible regarding facts two and three. For those facts, Darwin’s theory appears to provide (loosely speaking) an *explanation*. Discussing the ‘similarity’, or ‘bond’ connecting species within the same continent or sea (fact two), or otherwise inhabiting regions easily accessible by migration (fact three), he says ‘This bond, on my theory, is simply inheritance . . .’ (350). Roughly, the idea seems to be that morphological similarity and geographical proximity go together because they are both consequences of recent shared ancestry.

Darwin’s survey takes up about six pages. Most of the remaining sixty pages devoted to geographical distribution are taken up by what are essentially replies to objections. The objections arise because Darwin’s approach to explaining the second and third great facts commits him to a position in a debate already ongoing within the (broadly creationist) mainstream: Was there at most one geographical creation site per species, or were some species created more than once, at separate geographical locales (352)? Darwin posits one localized speciation event per species, which puts him on the ‘single geographical origin’ side of the debate. This position, however, faced a large collection of problem cases, each concerning a species with a discontinuous range and ostensibly remote chances of migration or dispersal between the isolated parts. (The advocate of multiple origins finds it more reasonable

⁴Far from novel observations, all three facts were (at least at this level of description) well-known among naturalists who studied geographical distribution. See Nelson (1978) for discussion of these facts—especially the first—in the work of Darwin’s predecessors and contemporaries, including Linnaeus, Buffon, Candolle, Prichard, Humboldt, and Lyell.

to posit separate creation events at each of the distant locales.) To address these anomalies Darwin takes a long digression into the various means by which organisms are dispersed, and the climatic, geological, and ecological factors that shape distribution. Citing ice ages, land bridges, ocean currents, floods, icebergs, hitchhiking, and many other mechanisms, as well as deploying a few qualitative theorems derived from his theory of natural selection, Darwin does his best to make the problem cases compatible with the single origin thesis. This exercise takes up the rest of chapter eleven and most of twelve.

Darwin saves for last the anomalous cases of the same species found on islands and mainlands separated by hundreds of miles of open sea. While discussing these he shifts away from deflecting problem cases and back to big-picture trends, relating a few that are specific to islands, among them ‘the most striking and important’ being the ‘affinity [of the inhabitants of islands] to those of the nearest mainland, without being actually the same species’ (397). The Galapagos and Cape Verde archipelagos provide illustrations (397–9). (This islands-and-mainlands trend is really just a special case of the second and third ‘great facts’ from Darwin’s survey at the beginning of chapter eleven, while Darwin’s *comparison* of Galapagos to Cape Verde provides another illustration of the first ‘great fact’. In what follows, I stick to the examples that appear at the beginning of chapter eleven, as they are more fully described than the islands-and-mainlands illustrations.)

3.3 The great facts: what, exactly, are the relevant observations?

From here on I restrict attention to the ‘great facts’ from Darwin’s review of geographical distribution. In this section I rigorously characterize those facts themselves. In the next section I describe how (two of) those facts relate to Darwin’s hypothesis of common ancestry, and how they relate to some relevant competing hypotheses. Darwin’s discussion of the ‘great facts’ is multi-layered, including talk of specific, local observations such as that species x inhabits region y , as well as observations at several levels of abstraction above that. In this respect, Darwin’s geographical distribution observations resemble those discussed above in the coin-tossing example. Again, my goal is to clearly articulate the ‘high level’ observations such that they can be seen as expected, were Darwin’s common ancestry hypothesis correct, even though the same can *not* be said for the ‘lower level’ observations.

The first ‘great fact’ says that similarity between the ‘inhabitants’ of different regions does not track similarity between the ‘climatal and other physical conditions’ of the regions themselves. For example:

... we may compare the productions of South America south of lat. 35° with those of north of 25° , which consequently inhabit a considerably different climate, and they will be found incomparably more closely related to each other, than they are to the productions of Australia or Africa under nearly the same climate. (347)

To paraphrase, the Patagonian plains of South America below 35° share ‘nearly the

same climate' with parts of Southern Africa and Australia, yet the inhabitants of Patagonia are more similar to those found in a *different climate* (South America north of 25°) than to the inhabitants of either Southern Africa or Australia.

The same regions illustrate the second and third 'great facts' as well. The second fact reports that barriers to migration are 'related in a close and important manner' to differences between regional biotas, which Darwin clarifies by noting the 'great difference in nearly all the terrestrial productions of the New and Old worlds ...' and 'between the inhabitants of Australia, Africa, and South America ... for these countries are almost as much isolated from each other as is possible.' (347). The same goes for marine fauna separated by 'impassable barriers, either of land or of open sea' (348), and to a lesser degree, for regional biota separated by 'lofty and continuous mountain ranges, ... great deserts, and sometimes even large rivers.' Fact three is just the other side of the same coin: that within a given continent or a sea, the inhabitants of neighboring regions are more similar (though not the same). To paraphrase facts two and three: relative similarity of the inhabitants of different regions is generally observed to go together with relative *accessibility* (via migration/dispersal) between those regions. (The first fact, recall, said that relative similarity of inhabitants does *not* reliably go together with relative similarity of the *environments* of those regions.)

All three great facts turn on the 'similarity' or 'dissimilarity' of the living things ('inhabitants', 'productions') of one region compared, *en masse*, to those of another region. But how can such miscellaneous collections be judged more or less 'similar'? What is the metric? One simple approach might be to judge similarity by the

number of species the two biota have in common. But Darwin goes beyond this simple approach to recognize similarity where ‘the species themselves are distinct’ (349). As I understand him, Darwin reads two biotas as similar to the degree that species found in each biota tend to be matched by a *similar species* in the other. Conversely, he reads them as dissimilar to the degree that the *most* similar species found in the other biota tends to be relatively dissimilar. This first-pass analysis reduces similarity between *biotas* to similarity between *species*. But what does it mean for two species to be similar?

Darwin’s use of the terms ‘affinity’ (349) and ‘related’ (349), as well as the phrase ‘species of the same genus’ (349) indicate that the relevant metric is *taxonomic relatedness*. Taxonomic relations issue from the groups-within-groups structure of biological classification. That structure partitions any set of taxa into mutually exclusive groups, with each group itself being similarly partitioned, and so on for the subgroups.⁵ For a given number of taxa, there are a finite number of distinct groups-within-groups arrangements. Each arrangement can be notated using parentheses to represent groupings, for example the arrangement for the taxa {cardinal, penguin, robin} is: (robin, cardinal) penguin. Such groupings express only *relative* taxonomic relations: the cardinal and the robin are each closer to each other than either is to the

⁵O’Hara (1991) divides taxonomic representations in the forty years prior to publication of the *Origin* into the Quinarian (1819–1840), and mapmaking (1840–1859) periods. While there is considerable diversity within each period, the dominant classificatory forms strongly emphasize nested groupings as explained in the text. Quinarian classification requires exactly five subgroups within every group and includes additional relationships that cross-cut the nested groupings. Some of what O’Hara calls ‘maps’ also include supplementary cross-cutting relationships. Darwin calls attention to the groups-within-groups nature of classification while discussing his principle of divergence: ‘It is a truly wonderful fact – the wonder of which we are apt to overlook from familiarity – that all animals and all plants throughout all time and space should be related to each other in group subordinate to group, in the manner which we everywhere behold . . .’ (128).

penguin; but the arrangement says nothing about *how close* any bird is to any other. (Taxonomic *ranks* such as ‘family’, ‘genus’, *etc.*, offer a nominal yardstick of *absolute* taxonomic relatedness. Darwin, however, dismisses these ranks as arbitrary.)

In light of the relations provided by the taxonomy of Darwin’s day, I propose the following formalization of his notion of *biota* similarity. Consider biotas A , B , and C . Choose an arbitrary species from one biota, say from A , and then identify that species’ closest taxonomic relations that are native to biotas B and C . Label the species a , b , and c . Species b is by design taxonomically closer to species a than anything else in biota B is. Likewise for species c and biota C . But this leaves open the relative taxonomic relations among the three species. Say biotas A and B are more similar to each other than either is to C if and only if the taxonomic relations among species chosen in the manner just described are typically $(ab)c$. In other words, biotas A and B are more similar iff the taxonomically most closely related species in B tends to be even closer than that in C . (See Figure 3.1 for a visual aid.) On this reading, each of Darwin’s statements about the relative similarity of the “inhabitants” of different regions can be understood as a summary of many smaller-scale facts about the *taxonomic relations* among representative species drawn (as described above) from the biotas of those regions.⁶

After describing the ‘great facts’ in terms of whole biota and the biota comparison language that I have just interpreted, Darwin then relates two additional illustrations

⁶Species a may have several equally close taxonomic relations in biotas C and B , in which case admit all of them (b_1, b_2, \dots , and c_1, c_2, \dots) and expand the definition of relative biota similarity to: ‘Say biotas A and B are more similar to each other than either is to C if and only if the taxonomic relations among species chosen in the manner just described are typically $((ab)c)$, or in the case of multiple closest taxonomic relations $((a, b_1, b_2, \dots) c_1, c_2, \dots)$.’

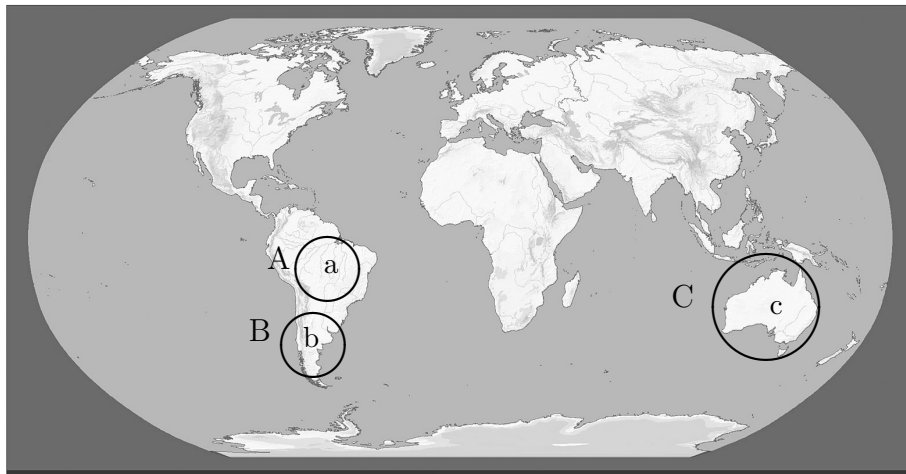


Figure 3.1: As I interpret Darwin’s biota comparison language, biotas A and B are more “similar” to each other than either is to biota C because the taxonomic arrangement for species a, b, and c, drawn from their respective biotas using the rule described in the text, is typically $(ab)c$. Note that order from left to right carries no meaning; $(ab)c$ is synonymous with $(ba)c$, $c(ba)$, and $c(ab)$.

at the more detailed level of representative taxa. (It is partly on the basis of these that I have interpreted the meaning of his biota comparison language.) First example:

The plains near the Straits of Magellan are inhabited by one species of Rhea (American ostrich), and northward the plains of La Plata by another species of the same genus; and not by a true ostrich or emeu, like those found in Africa and Australia under the same latitude. (349)

The (arid) southern-most plains of South America house (what is now called) Darwin's rhea. The greater rhea is a close taxonomic relative and inhabits the plains somewhat to the north, in a very accessible region with a different climate. The closest taxonomic relatives found in the arid regions of Africa and Australia are the ostrich and emu, respectively. (See Figure 3.2 for a visual aid.) The two rheas are taxonomically more closely related to each other than either is to the ostrich or the emu. And the two regions of South America are far more accessible to each other than either is to Australia or southern Africa. The relations of accessibility between these four *regions* thus mirror the taxonomic relations between these four *species* that inhabit them. (Lumping the locales by environmental similarity, on the other hand, puts Darwin's rhea together with the ostrich and emu, creating a grouping that conflicts with the taxonomic relations.) The top half of Table 3.1 summarizes these grouping statements.

Darwin's second more detailed example addresses a group of South American rodents and closely related taxa spread over North America and Eurasia:

On these same plains of La Plata, we see the agouti and bizcacha, animals having nearly the same habits as our hares and rabbits and belonging to

<i>taxonomy:</i>	((greater rhea , Darwins rhea) emu) ostrich
<i>accessibility:</i>	(greater rhea , Darwins rhea) emu, ostrich
<i>environment:</i>	greater rhea (Darwins rhea, emu, ostrich)
<i>taxonomy:</i>	((agouti , vizcacha , coypu , capybara) (beaver, muskrat)) (hares, rabbits)
<i>accessibility:</i>	(agouti , vizcacha , coypu , capybara) beaver, muskrat, hares, rabbits
<i>environment:</i>	(agouti, vizcacha, hares, rabbits) (beaver, muskrat, coypu, capybara)

Table 3.1: A representation of Darwin’s flightless birds and rodents examples. In each case, taxonomic relations are juxtaposed with groupings according to migratory accessibility, and environmental similarity of the corresponding geographical regions. Bold text highlights the groups on which taxonomy and accessibility agree. Incomplete statements such as (abc)d should be read as disjunctions, consistent with any arrangement that respects the stated grouping(s), in this case ((ab)c)d, *or* (a(bc))d, *or* ((ac)b)d.

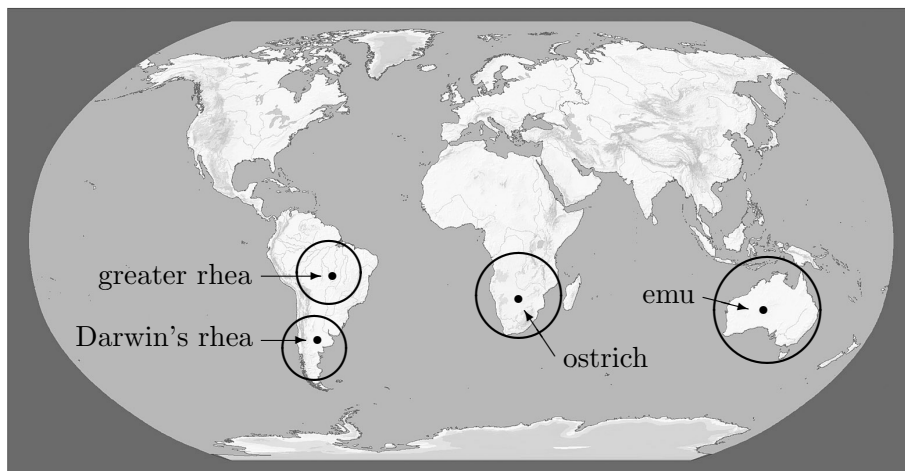


Figure 3.2: A map showing the regions and species featured in Darwin’s flightless birds example. The two rheas are closest taxonomic relations and inhabit the two regions with the greatest pair-wise mutual accessibility. The ostrich and emu are the rheas’ closest taxonomic relations found in Africa and Australia respectively, but each is taxonomically more distantly related to the rheas than the rheas are to each other. The South America-Africa and South America-Australia environment pairs are less accessible than the two South American regions are to one another.

the same order of Rodents,⁷ but they plainly display an American type of structure. We ascend the lofty peaks of the Cordillera and we find an alpine species of bizcacha; we look to the waters, and we do not find the beaver or muskrat, but the coypu and capybara, rodents of the American type. (349)

The phrase ‘American type’ refers to the morphological similarities that mark the South American rodents as closest taxonomic relations. Regarding environmental similarity, the agouti and vizcacha have the same general external appearance (‘habit’) as the hares and rabbits of Eurasia and North America, making them fit for the same environments (Darwin appears to reason), while the semi-aquatic coypu and capybara occupy roughly the same environmental niche as the beaver and muskrat in North America. Thus accessibility relations again mirror the taxonomic relations, while grouping by similarity of environment conflicts with taxonomy (see the bottom half of Table 3.1).

I can summarize my reading of the ‘great facts’ by starting from the ground floor observations and working up to the ‘high-level’ ones. (For brevity, I do this only for the second and third facts.) The simplest observations each report the geographical range of a single species (or somewhat larger taxon). Darwin’s flightless birds illustration, for example, includes four such observations, reporting the geographical ranges of Darwin’s rhea, the greater rhea, the ostrich, and the emu.

⁷Rabbits and hares were moved out of *Rodentia* and into the sister order *Lagomorpha* in the early 20th century. This development doesn’t undermine Darwin’s argument. (There are closer taxonomic relations in Europe and America, but these are still taxonomically more distant than the South American rodents are to each other.)

Combined with some knowledge of the organisms' migratory capabilities, the simple location observations determine which species are, from their present locales, more accessible to which other species. In the case of the flightless birds, the two rheas (inhabiting neighboring regions of South America) make the most accessible pair, while all other pairings (greater rhea with ostrich, ostrich with emu, *etc.*) are considerably less accessible as the paired species inhabit different continents. These accessibility relations can be summarized as per the 'accessibility' row of Table 3.1, where placement inside parentheses indicates greater migratory accessibility with anything else inside the parentheses than with anything outside. Notice that this summary statement says nothing about where any of the species are found. The summary statement *leaves behind information* that was present in the simple observations, and expresses an abstract feature of the whole set of simple observations.

Next, the abstract summary of accessibility relations is compared with preexisting, off-the-shelf *taxonomic classifications* for the same taxa. (Far from raw, naive observations, these taxonomic classifications are themselves the *products* of the theory and practice of biological taxonomy applied to observations of comparative morphology—but I bracket discussion of how taxonomists produced these classifications, since here Darwin simply takes these products for granted.) Continuing with the flightless birds example, the abstract summary of accessibility relations is seen to be *congruent* with the taxonomic relations (*i.e.*, the accessibility and taxonomic arrangements contain some of the same groupings, and do not cross-cut one another) as displayed in Table 3.1. The statement of congruence between taxonomic and accessibility groupings now expresses a *very* abstract feature of an even larger

set of simple observations. The total data set now includes both the simple location observations *and* the comparative morphology observations that underly taxonomic relations.

Finally, through the language of biota comparisons the second and third ‘great facts’ state that congruence between taxonomic relations and accessibility groupings is the norm, or is at least a *trend* among many such comparisons, for appropriately chosen groups of taxa. Surely there are instances of *inconsistency*—perhaps they are common. But Darwin assures us that agreement between taxonomic and accessibility groupings (as expressed in the second and third ‘great facts’) ‘...is a law of the widest generality, and every continent offers innumerable instances’ (349) and again, ‘Innumerable other instances could be given’ (349).

3.4 How the observations relate to theory

Recall that the first ‘great fact’ says (loosely speaking) that relative similarity of the inhabitants of different regions is disassociated from relative similarity of the *environments* of those regions. The second and third facts together say that relative similarity of inhabitants is positively associated with relative *accessibility* between regions. Darwin takes the first ‘great fact’ to contradict a particular flavor of theistic creationism (*cf.* natural theology, The Bridgewater Treatises). At this point I set aside the first fact, and with it Darwin’s pointed attack on this particular, and somewhat feeble target. I focus instead on the observed positive association of taxonomic proximity and migratory accessibility—expressed in facts two and three—which Dar-

win takes to support his own hypothesis. In the previous section I described those observations rigorously; now I discuss their relation to Darwin’s hypothesis of common ancestry, and to some alternative hypotheses.

3.4.1 Darwin’s hypothesis

What is Darwin’s common ancestry hypothesis? It says that all species, extant and extinct, trace back to one or a few common ancestor species. But it says more than this. I think Darwin’s common ancestry hypothesis is best understood as an *interpretation* of the taxonomic arrangements produced by the biological classification of his time. Groups-within-groups taxonomic arrangements express relative taxonomic ‘relatedness’. But what did ‘relatedness’ mean? What is classification really *about*? Minimally, a given classification was both a summary of observed morphological similarities, and a predictive hypothesis regarding similarities in traits not yet observed, or not taken into account in making the classification. Beyond this superficial agreement, different naturalists had different ideas about the true nature of taxonomic relations. A taxonomic classification might represent a blueprint in the mind of the creator, or a map of the physiologically possible adult forms, or it might indicate the constraints inherent in the embryological development of living organisms (Winsor 2009). Or—at least for varieties within a species, and perhaps even species within a genus—taxonomic relations might express *genealogical relations* among the taxa. As (Winsor 2009, 44) explains:

To us it may seem paradoxical that naturalists should use the word ‘related’ without agreeing on its meaning, but actually this tolerance en-

abled them to make progress as a scientific community.

The situation of naturalists in this period can be compared to that of contemporary philosophers vis-a-vis ethics. Two philosophers can agree that ‘murder is wrong’, even while entertaining different theories about what it means for an act to be wrong. They agree on an ethical claim while disagreeing about meta-ethics. Naturalists before Darwin could and often did agree on classification claims even as they disagreed about (so to speak) meta-classification.

Every groups-within-groups classification can also be represented as a branching tree structure (see Figure 3.3). Many naturalists already believed that subgroups within the same species, or even the same genus, were related genealogically. Viewing classifications as trees, some of the twigs at the very tips of the tree were already taken to indicate genealogical relations. Darwin pushed this interpretation to the extreme, claiming that classifications were genealogical *all the way down*. All (or almost all) living things are related by genealogy, he said, and the closer the taxonomic relation, the more recent the shared ancestor. In summary, Darwin’s common ancestry hypothesis says: that all living things trace back to one or a few common ancestor species, that the ancestry of life has the form of a branching tree, and that the shape of this tree is revealed (at least approximately) in the taxonomic classifications of Darwin’s time.

3.4.2 Were Darwin’s hypothesis correct

Now supposing this hypothesis were true, why should we expect to see the association of taxonomic relatedness and migratory accessibility expressed in facts two and three?

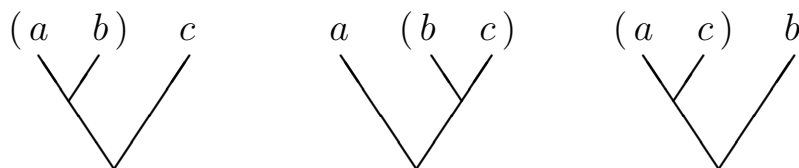


Figure 3.3: An illustration of the one-to-one correspondence between groups-within-groups classificatory arrangements and (rooted) tree structures.

To begin, let's step back from the highest-level observation—the *trend* of agreement between taxonomic and accessibility-based groupings for appropriately chosen sets of taxa—and consider just a *single instance* of such agreement, for example the flightless birds. Regarding the taxonomic grouping, Darwin's interpretation of taxonomic relations as genealogical relations goes along with treating the *practice* of taxonomy as a (disguised) method of *phylogenetic inference* (inferring the genealogical relations among a set of taxa). Indeed it *was* largely accurate as a method of phylogenetic inference—assuming the results of today's molecular-based methods are not too far off the mark. What about the accessibility grouping? Consider how the accessibility groupings are generated: look at the all-things-considered migratory accessibility between each pair of species under consideration, then group together the ones that are the most mutually accessible, then pull in the next most accessible, and so on. It is formally the same as applying a primitive method of tree construction (*e.g.*, neighbor-joining, or UPGMA) to a matrix of pair-wise genetic or character 'distances' (Table 3.2). I view this procedure for constructing accessibility groupings as *another method of phylogenetic inference*. The reason it should *work* is that species that split more recently have had more time to disperse further apart, and conversely, species that

split more recently can't have gotten too far apart since their speciation.⁸ So long as grouping taxa on the basis of pair-wise migratory accessibility has some value as a method of phylogenetic inference, then its results should *tend* to agree with taxonomic relations simply because they are two ways of inferring the same thing.

	emu	ostrich	g.r.rhea	D.rhea
Darwin's rhea	2	2	1	.
greater rhea	2	2	.	
ostrich	2	.		
emu	.			

Table 3.2: Pairwise migratory accessibility (using cartoon numbers) arranged to display the formal identity with a matrix of pair-wise character ‘distances’ as used by some methods of tree construction. The two rheas are the most accessible, while all other pairs are about equally (and much less) accessible. (For simplicity, I assume that migratory accessibility is symmetric.)

This discussion connects directly back to the coin-tossing example. In that example the simple, lowest-level observations were the individual coin toss outcomes (heads, heads, tails, *etc.*). And there were two sets of these simple observations: *A* and *B*. In Darwin's biogeography observations, the two data set are (1) the set of simple geographical location observations that underly the accessibility groupings, and (2) the set of morphological observations that underly taxonomic classifications. In the coin-tossing example, my *hypothesis* was that both data sets were generated by tossing the same coin (with unknown bias). Just as that hypothesis is silent about whether heads or tails is more probable on any individual coin toss, Darwin's common ancestry hypothesis has nothing to say about where any particular organism

⁸For a more formal argument, see Sober's probabilistic reconstruction of Darwin's ‘space-time principle’ (Sober 2008, 326)

should be found on the globe, or what morphological characteristics an organism should display.

Now let's step up one level of abstraction. Just as the *frequency* of heads in one data set is a summary of the data that abstracts away from the individual coin toss outcomes, an *accessibility grouping* such as ((D.rhea, g.rhea) ostrich, emu) is a summary of, and a function of, a set of simple location observations. My 'single-coin' hypothesis in the coin-tossing example is silent about what frequency of heads should be observed in either data set, *when one or the other data set is viewed in isolation*. In the same way, when geographical distribution is viewed in isolation, Darwin's common ancestry hypothesis has no commitment to taxa being distributed in a way that corresponds to any particular grouping. Likewise, if morphology is looked at in isolation, the common ancestry hypothesis is (before the results of taxonomists are known) equally compatible with any taxonomic arrangement.

When it comes to the *difference* between the frequency of heads in *A* and *B*, my 'single-coin' hypothesis does make a commitment, it predicts that this value should be near zero. The frequency of heads in *A* is not just any abstract feature of data set *A*, it is the best (maximum likelihood) estimate of the coin's probability of landing heads on any given toss. Similarly, the frequency of heads in *B* is the best estimate of the probability of landing heads for the coin that generated data set *B*. My 'single-coin' hypothesis says that these two numbers are estimates of the *same quantity*, so the difference between them should be small (see Note 2 for mathematical details). In the same way, Darwin's common ancestry hypothesis says that the taxonomic classification and the accessibility grouping for the four flightless birds

are two estimates for which tree is the true genealogical tree that underlies *both* the simple morphological observations and the simple geographical location observations. Thus the two groupings should be the same, or at least similar. Congruence between the taxonomic classification and the partial grouping based on accessibility is the analogue of a small value for the difference-between-frequencies statistic in the coin-tossing example.⁹

Finally, we have the highest-level observation: the *trend* of agreement between taxonomic and accessibility groupings, as expressed in facts two and three through the language of biota comparisons. Darwin doesn't say exactly how often consistency is observed as opposed to inconsistency—the rodents and birds are two examples of consistency, and 'Innumerable other instances could be given', but there are surely also many examples of inconsistency. I will characterize this highest-level observation conservatively, as agreement between taxonomic and accessibility groupings, for appropriately chosen taxa, at a frequency greater than chance (where we can understand 'chance' as the frequency that would result from drawing a grouping uniformly at random from all possible groupings, and having that agree with the taxonomic classification).¹⁰ To summarize very broadly, my claim is that this highest-level ob-

⁹A tighter analogy can be established by introducing a quantitative measure of the degree of similarity between two trees. See, *e.g.*, Robinson and Foulds (1981); Penny et al. (1982); Penny and Hendy (1985) for such measures.

¹⁰More precisely, treat the accessibility grouping as drawn uniformly at random from among all possible groupings *that are comparably complete*. For example, Darwin's flightless birds illustration features 4 taxa and a single 2-member accessibility grouping. The number of distinct 2-member subsets is $\binom{4}{2} = \frac{4!}{2!(2!)} = 6$, but only one is consistent with the taxonomic classification. Such agreement should occur 'by chance' only 1 in 6 times. The rodents illustration features 8 taxa and a single 4-member accessibility grouping. The number of distinct 4-member subsets is $\binom{8}{4} = \frac{8!}{4!(4!)} = 70$. Such agreement should occur 'by chance' only 1 in 70 times. (Adding a fifth South American rodent—an alpine species of viscacha appearing in Darwin's example but left out of the main text

ervation should be observed, were Darwin’s common ancestry hypothesis correct. Were congruence observed at or below chance, this would be surprising, and difficult to reconcile with Darwin’s hypothesis. For those who use ‘predicts’ in the formal sense that does not imply anything about the sequence of events in time, Darwin’s common ancestry hypothesis *predicts* this observation.

3.4.3 Objections

Before moving on to consider some competing hypotheses and how those relate to the same geographical distribution observations, I should address two objections to the preceding analysis.

First objection: I have glossed both the taxonomic and the accessibility groupings for a set of taxa (*e.g.*, the flightless birds) as *phylogenetic trees*, each inferred from a different data set. This gloss is an essential part of my argument that agreement between the groupings is to be expected were Darwin’s common ancestry hypothesis correct. But this talk of phylogenetic trees takes a very theory-laden perspective on the observations. Have I not begged the question against hypotheses that deny evolution and common ancestry by describing the observations themselves in terms that only an advocate of common ancestry could accept?

Response: Notice that the same objection could be raised in the coin-tossing example. An intuitive way of understanding why the difference-between-frequencies statistic should be near zero, were my ‘single-coin’ hypothesis correct, is to note that in this case the two frequencies would be two estimates of the single coin’s bias, so

for simplicity—brings the ‘chance’ frequency to 1 in 126.)

they should be roughly the same. This perspective on the observation is theory-laden, and would be unacceptable to anyone who rejects my hypothesis. But this doesn't change the fact that the difference-between-frequencies statistic itself is a perfectly objective function of the data. The statistic itself is the observation; the added gloss about agreement between two estimates of the same quantity is a theory-laden explanation of what that statistic means and why certain values should be observed. Similarly, an observation of agreement between taxonomic and accessibility groupings is a function of morphology, geographical location, and migratory capacity. Darwin and his opponents can agree about all of these things, and so they can agree about the observations. My talk of two methods of phylogenetic inference is an added, theory-laden explanation of what those groupings mean and why agreement is to be expected (at least with frequency greater than chance) were Darwin's common ancestry hypothesis correct.

Second objection: My argument depends on the idea that a very crude method of grouping taxa by migratory accessibility can function as a workable method of phylogenetic inference. But in fact accessibility is typically a dreadful proxy for genealogical relatedness. For example, North American bison are much, much more closely related to the water buffalo of Asia than to the prairie dogs underfoot, not to mention the prairie grass, the ants, the microbes in the soil. For almost any choice of taxa, judging genealogical relatedness based on relative migratory accessibility would do no better than putting all possible trees into a hat and reaching in blindfolded.

Response: Geographic distribution contains some information about genealogical relations, but the world is small and migration is fast. The noise of dispersal quickly

obscures the signal of genealogy.¹¹ Grouping by accessibility is indeed a very limited method of phylogenetic inference. There is, however, a set of circumstances under which it will perform best: pair-wise migratory accessibility will carry the most information about genealogy where (1) at least some speciations within the tree are recent (so that dispersal has not erased all trace of genealogy), and (2) observed differences in migratory accessibility between species pairs is large (and thus less plausibly attributed to accidents of dispersal). The observations that I have described above include *only* sets of taxa that satisfy these circumstances. On my reading of Darwin’s biota comparisons, representative taxa should be among each other’s closest taxonomic relations that are found in each locale—this arranges for (1). And Darwin’s emphasis on contrasting within-continent biota pairs versus between-continent biota pairs (see fact three) accords with (2). Thus the way in which the *regions* are chosen, and then the way in which *taxa* are chosen from those regions, works to limit the sets of taxa that enter into the observations to those for which geographical distribution is informative about genealogical relations.

3.4.4 Alternative hypotheses

I’ve argued above that the trend in agreement between taxonomic and accessibility groupings, expressed jointly by Darwin’s ‘great facts’ two and three, is to be expected were his theory of common ancestry correct. Can the same be said of the alternatives to Darwin’s hypothesis?

¹¹Somewhat more formally: supposing that daughter species disperse via probabilistically independent random walks, the likelihood function over all possible trees flattens out very quickly, so that relative accessibility data no longer contains any information about phylogeny.

In connection with the first great fact, Darwin discusses an alternative theory of geographical distribution, according to which each species came into existence as-is, and in just the right environment for that species to thrive. Supposing this hypothesis were true, would it follow that taxonomic and accessibility groupings should be systematically congruent? It would, *if* some additional assumptions were true: (1) that species more closely related taxonomically were fit for more similar environments, and (2) accessibility between regions were correlated with similarity of environment. But neither (1) nor (2) is true. Given that accessibility between regions is (at least at a large geographical scale) uncorrelated with similarity between the environments of regions, this hypothesis arguably predicts that taxonomy-accessibility congruence should occur at chance frequency, which fits very poorly the observations.

A more relaxed variant of the preceding biogeographical theory is implicit in Darwin's discussion of single versus multiple creation sites for a single species (what I've called, in my overview of chapters eleven and twelve, Darwin's "replies to objections"). This theory allows for different locations of origin for each species, and even for different populations within a species, but it does not put any constraints on where those locations should be. This theory is perfectly *consistent* with the observed trend of accessibility-taxonomy congruence, but it would be equally consistent with any possible geographical distribution observations.

A third alternative can do better, but is also more similar to Darwin's own theory. In Darwin's time, some amount of evolution and common ancestry was generally accepted. Few doubted, for example, that subspecies within a species, or varieties within a species could trace back to a common ancestor population in the past.

Some went so far as to allow that species within a genus all evolved from a common ancestor. Such *limited* common ancestry theories will generate the same expectations about geographical distribution as Darwin's theory, so long as the species included within a single congruence observation all fall within the same subspecies, species, or genus, according to how much evolution and common ancestry the particular alternative theory allows.

In Darwin's description of the rodent example (p. 51 above) he notes that the lowest taxonomic rank that includes all of the taxa taking part in the congruence is the *order*. This is two full Linnaean ranks above genus. So the trend of congruence between taxonomic and geographical groupings does hold for at least some groups of taxa broad enough to discriminate between Darwin's very general common ancestry hypothesis and the more limited ones accepted by many of his contemporaries. But for all that the geographical distribution observations have to say, it would remain open for an opponent of Darwin's general common ancestry theory to accept only as much common ancestry and evolution as is indicated by the smallest inclusive taxonomic rank of the taxa that participate in the trend. Ultimately, the limit of the ability of the biogeographical evidence to distinguish between Darwin's general common ancestry hypothesis and more conservative variants of it is determined by the limit of the method of inferring a genealogical tree from pairwise migratory accessibility observations, as discussed above in the second objection of § 3.4.3. (Darwin does not discuss this limit, nor these limited common ancestry hypotheses.)

3.5 Phylogenetic congruence

Before concluding, I wish to briefly point out how my analysis of Darwin's geographical distribution observations places the form of those observations into the context of modern evolutionary biology. The agreement between genealogical trees inferred for the same taxa from different data sets, as described above, will be recognized by contemporary evolutionary biologists as a form of (what is now called) 'phylogenetic congruence'. Phylogenetic congruence simply means the congruence of two genealogical trees, each inferred from a different data set. The innumerable instances of observed taxonomy-geography congruence summarized by Darwin's second and third great facts is each a qualitative and perhaps somewhat crude analogue of the measures of tree similarity (Robinson and Foulds 1981) used in tests of phylogenetic congruence in contemporary phylogenetic systematics (Huelsenbeck et al. 1996), historical biogeography (Wiley 1988), symbiotic evolution (Funk et al. 2000), and several other subfields of evolutionary biology. Darwin's congruences are different, however, in that each of these modern types of congruence (including those of modern biogeography) uses morphological or genetic data for *both* trees, while Darwin's congruences use morphological data for one tree and geographical data for the other.

3.6 Conclusion

I have offered a new analysis of Darwin's geographical distribution observations, and of how those observations relate to Darwin's theory of common ancestry. I take

these observations to support Darwin's theory of shared ancestry over the alternatives discussed above in virtue of Darwin's theory predicting those observations (no implication regarding the sequence of events in time intended) while the alternatives either predict much less taxonomy-geography congruence or make no predictions and are equally compatible with any amount of congruence. My analysis stands in contrast to those cited in § 3.1, according to which Darwin's theory does not tell us what we should expect to observe in nature were the theory correct. The novelty of my analysis has two main sources. First, I've taken to heart the lesson illustrated by the coin-tossing example from section 3.1, namely that a hypothesis can sometimes stick its neck out regarding certain abstract, or 'high-level' features of a *set of observations* even while generating no expectations about any individual datum within the set. And second, I have characterized the major trends from Darwin's survey of geographical distribution in a novel way that identifies just such a 'high-level' observations (or 'patterns'), namely the congruences of accessibility-based groupings with taxonomic relations for appropriately chosen groups of taxa.

Part II

Diversity

Chapter 4

The Confirmational Significance of Agreeing Measurements

4.1 Introduction

The *agreement of independent measurements* occurs when a theoretically posited quantity is measured via multiple, and (in some sense) ‘independent’ methods, and those measurements agree (*cf.* Forster 1988). The phenomenon is also called ‘the method of overdetermination of constants’ (Norton 2000), and ‘the consilience of inductions’ (Whewell 1858/1989a). Judging by the scientific episodes most studied and celebrated by philosophers, the phenomenon is of central importance to confirmation in science. The agreement of independent measurements played a key role in confirming, *e.g.*, Newton’s theory of gravity (Forster 1988; Harper 2007), the wave theory of light (Whewell 1858/1989a), Darwin’s theory of common ancestry (Chap-

ter 3 of this dissertation), the atomic theory of matter (Salmon 1984; Norton 2000), the charged particle (electron) theory of cathode rays (Norton 2000), and the theory of plate tectonics (Koolage 2008). In the present essay I propose a new, formal account of the phenomenon's epistemic significance, and contrast my proposal with a more established approach to the same problem.

The agreement of independent measurements is often treated under the 'diversity of evidence' heading (where the 'independence' of individual measurements is taken to enhance the 'diversity' of a total *set* of observations that includes those measurements). But that approach (in its current form) does not adequately acknowledge the hierarchical structure that is characteristic of hypothesis spaces in science. Specific scientific hypotheses are nested within more general hypotheses, and those are nested within hypotheses more general still. Within such a structured hypothesis space, the diversity of evidence approach locates the evidential significance of agreeing measurements at the nitty-gritty level of parameter estimates—agreement warrants extra confidence that the measured value is accurate. While not incorrect, this result is incomplete, and does little to explain the perceived significance of agreeing measurements in the history of science. My proposal complements existing accounts by identifying, *in addition*, warrant for the 'higher level' theory that *posits* the measured quantity. It is the confirmation of this higher level hypothesis—more so than the very specific hypothesis that a parameter takes a certain value—that explains the historical significance of the real scientific examples.

Regarding formal methodology, I will judge the evidential import of an observation via the *Law of Likelihood* (Hacking 1965; Edwards 1984; Royall 1997). That is, I

treat an observation as supporting one hypothesis over another if that observation is more probable supposing the one hypothesis were correct, compared with supposing the other hypothesis were correct. More formally, observation o favors hypothesis h_1 over hypothesis h_2 if: $p(o|h_1) > p(o|h_2)$. Likelihoodists and Bayesians of all stripes can agree that such comparisons are the basis of confirmation. I wish to bracket the finer, quantitative issues about how strongly a hypothesis is favored or confirmed by an observation (Fitelson 2011). My proposal concerns the more basic issue of exactly which hypotheses and observations to label o , h_1 , and h_2 , such that the import of agreeing measurements can be better appreciated within the framework of Bayesian epistemology broadly understood.

4.2 Examples of the Phenomenon to be Analyzed

A couple of examples will indicate the flavor of the phenomenon to be analyzed. The following quotations from (Whewell 1858/1989a), and (Norton 2000), frame scientific work by Thomas Young and Jean Baptiste Perrin respectively:

And what was no less striking a confirmation of the truth of the [wave] theory [of light], *Measures* of the same element deduced from various classes of facts were found to coincide. Thus the Length of a luminiferous undulation, calculated by Young from the measurement of *Fringes* of shadows, was found to agree very nearly with the previous calculation from the colours of *Thin plates*. (Whewell 1858/1989a, 154)

...Perrin was able to report roughly a dozen different methods for es-

timating N [Avogadro's number] and they all gave values of N in close agreement. In the conclusion to *Les Atoms*, Perrin tabulated the resulting estimates of N from methods based on: viscosity of gases (kinetic theory), vertical distribution in dilute emulsions, vertical distribution in concentrated emulsions, Brownian movement . . . , density fluctuations in concentrated emulsions, critical opalescence, blueness of the sky, diffusion of light in argon, black body spectrum, charge of microscopic particles, radioactivity . . . (Norton 2000, 73)

As Norton paraphrases Perrin's argument, 'The case for the reality of atoms and molecules lay in this agreement . . . ' (Norton 2000, 73). Although each individual measurement of N concerns *the size of the atom*, the agreement of these measurements is said to confirm the general theory *that matter consists of atoms* (also see Salmon 1984). Similarly, in the first quotation above, Whewell says that agreement between two measurements of the wavelength of light confirms the theory that light is made of waves (not the more specific hypothesis that lightwaves have such and such length).

It is this type of inference—confirmation for the higher level theory, based on agreeing measurements of a quantity *posited within the theory*—that I will reconstruct formally in what follows. Again, this is not to deny that such agreement can *also* confirm the measured value for the quantity posited within the theory, *if that theory is already taken to be true*. But Perrin's main conclusion was that matter is made of atoms. Young's main conclusion was that light is a wave. (Analogous statements hold for the other examples from §1.)

4.3 Measurement Formally Characterized

To begin my analysis of the agreement of measurements, I first abstractly characterize the phenomenon itself. I formally characterize *measurement* as the statistical procedure of *parameter estimation*. Parameter estimation requires a *statistical model*—a family of probability distributions, each associated with a particular value for the model’s adjustable parameter (or with a vector of values, if the model has multiple parameters). Given a set of data, the highest likelihood distribution (or distributions) within the family can be identified, and the associated parameter value (or interval) is the parameter estimate. On this characterization of measurement, the statistical model’s adjustable parameter is the quantity to be measured, and estimation of that parameter’s value, as just described, is a measurement. For example, suppose we want to measure the mass of an object using a spring scale. Like any measuring device, our scale is imperfect. Suppose that its readings are normally distributed around the true mass of the object that is hung from it. This supposition is the statistical model. We produce a set of data by hanging the object, observing the reading, removing the object, then repeating the procedure a number of times. These data are then used to estimate the *mean* of the normal distribution from which the individual readings were treated as random draws. This estimate is a *measurement* of the object’s mass.

What then, is the *agreement of measurements*? Suppose we have two disjoint datasets, and a statistical model for each. The two models needn’t be the same, and each may include adjustable parameters that the other does not, but they must both contain an adjustable parameter representing the quantity to be measured.

Each model is fitted to its respective data set, generating two vectors of parameter estimates, and two estimates of the *shared parameter*, *i.e.*, two measurements of the quantity to be measured. (I understand ‘agreement’ between measurements as a matter of degree, and I will quantify this precisely in the worked example below.) Continuing with the spring scale example, suppose we measure the mass of the same object again, this time (as astronauts are ‘weighed’ in space) by applying a known force to the object, observing its resulting motion, calculating acceleration, and finally working back to the object’s mass by applying $f = m \times a$. In this case the data are a set of (*position, time*) points, and the statistical model is a Newtonian equation of motion with a stochastic element representing observation error. The object’s acceleration is estimated from these data via the model, and mass is calculated from acceleration and force. This estimate is a second *measurement* of the object’s mass. (While the two example measurements just described—the spring scale and the applied force—are *intuitively* ‘different’, or ‘independent’ methods, note that I have not yet characterized the ‘independence’ of measurements. I address this in what follows.)

4.4 Agreement as Observation

With the phenomenon formally characterized, I turn to its epistemic significance. I first illustrate my approach using the simplest possible case of the agreement of measurements. Say we will make two measurements of the mass of an object, using two separate spring scales. Let there be two data sets with 20 points each

$x_a = \{x_1, x_2, \dots, x_{20}\}$ and $x_b = \{x_{21}, x_{22}, \dots, x_{40}\}$, corresponding to forty scale readings, twenty from each scale, all using the same object. For each data set employ the location-normal model with known variance $\sigma^2 = 1$, and unknown mean μ . I.e., model a says that the twenty points x_a are drawn from twenty independent and identically distributed random variables $\{X_1, X_2, \dots, X_{20}\}$, each normal with variance $\sigma^2 = 1$, and mean μ_a . Model b says the same about points $x_{21} - x_{40}$, with mean μ_b . Under the location-normal model, the maximum-likelihood estimator for the value of μ is the *mean* of the data set. So the two maximum-likelihood estimates for the true values of μ_a and μ_b are \bar{x}_a and \bar{x}_b respectively.

Now I introduce a super-model that expresses the assumption, required for the agreement of measurements, that \bar{x}_a and \bar{x}_b are two estimates (measurements) of the *same quantity*. This super-model is the location-normal model treating all forty random variables $\{X_1, X_2, \dots, X_{40}\}$ as independent, with identical normal distributions, with variance $\sigma^2 = 1$ and mean μ ($\mu = \mu_a = \mu_b$). And to quantify the degree of agreement between the two measurements of the parameter μ , I define the following statistic of the *total data set* $\{x_1, x_2, \dots, x_{40}\}$: $\bar{x}_a - \bar{x}_b$. The closer this statistic is to zero, the greater the agreement between measurements. Call this the *agreement statistic*.¹ What does the super-model predict about the value of the agreement statistic? With respect to the super-model, this particular statistic is what is called *ancillary*, meaning that the probability assigned to observed values of the statistic does not depend on the value of the adjustable parameter. By definition, such

¹I don't mean to privilege the formula $\bar{x}_a - \bar{x}_b$ over other ways of quantifying agreement, e.g., $|\bar{x}_a - \bar{x}_b|$, or $(\bar{x}_a - \bar{x}_b)^2$. Either of these alternatives can be substituted for the simple difference statistic used in the text without affecting my conclusions.

statistics contain no information about the parameter value, and are thus completely useless for parameter estimation. The model itself assigns probabilities to observed values of an ancillary statistic, regardless of the value of the model's adjustable parameter. In the case of the statistic $\bar{x}_a - \bar{x}_b$ and the super-model introduced above, it is easy to understand why the super-model alone generates these probabilities. If all 40 random variables $\{X_1, X_2, \dots, X_{40}\}$ have the same distribution, then regardless of what μ is, we should expect $\{x_1, x_2, \dots, x_{20}\}$ and $\{x_{21}, x_{22}, \dots, x_{40}\}$ to be clustered around roughly the same value. Qualitatively, the super-model should assign higher probability to values of the agreement statistic near zero, and assign lower probability to large positive or negative values. The actual distribution is shown as the solid line in figure 4.1.

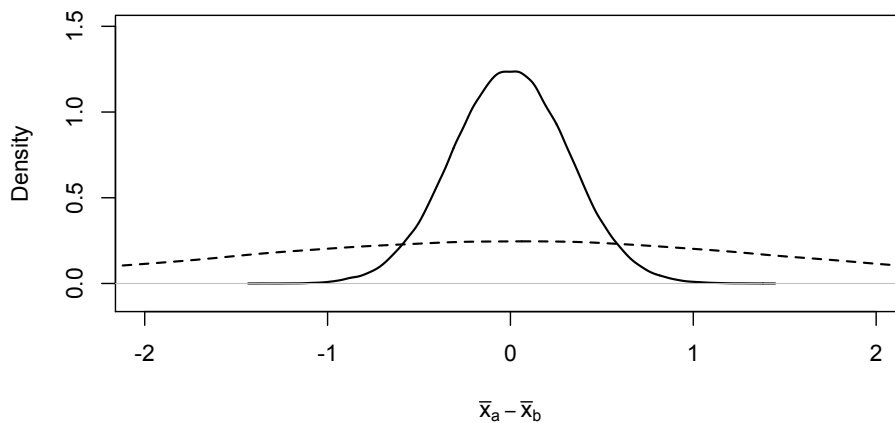


Figure 4.1: Probability density distributions assigned to the agreement statistic by the one-parameter (solid line) and two-parameter (dashed line) super-models. (Distributions approximated via simulation.)

To summarize what I've done so far, I first treat measurement as parameter estimation, and the *agreement of measurements* as agreement between two estimates, based on disjoint data sets, of a single parameter shared by two statistical models. To formally encode the idea that the parameter appearing in both statistical models is the *same quantity*, I introduce a super-model that comprises the two models plus the assumption that $\mu_a = \mu_b$. Then I characterize the agreement of measurements as a statistic of the total data (in this case $\bar{x}_a - \bar{x}_b$), and *I treat the degree of agreement itself as an observation*. I must emphasize this last part because it is the key to my approach. It may initially seem unintuitive (or worse) to treat the degree of agreement between two *estimates* of a posited quantity as an *observation*. Admittedly, it is a very abstract, 'high-level' observation. Yet the agreement statistic is a straight-forward function of the total data set, and is thus entirely determined by the data.² And so long as we can calculate a probability function for the statistic, nothing prevents us from treating it as an observation within our statistical framework. For my simplest-case example, I have displayed the probability distribution assigned to the agreement statistic by the super-model. My next step is to introduce a competing super-model, and calculate the distribution that *it* assigns to the same agreement statistic. I will then locate the epistemic significance of the agreement of independent measurements in the likelihood favoring of the one super-model over the other, given observed values of the agreement statistic near zero.

²Compare the 'higher-level regularities in the data' in Forster's (1988) discussion of Whewellian methodology, Sober's (1999) observation of 'matching' character states between two species, and the treatment of *differences* between AIC scores in Forster and Sober (2011).

4.5 A Competing Super-model

Let's temporarily set aside the super-model discussed above, and go back to the two data sets $x_a = \{x_1, x_2, \dots, x_{20}\}$ and $x_b = \{x_{21}, x_{22}, \dots, x_{40}\}$, and the two separate location-normal statistical models with known variance $\sigma^2 = 1$, and unknown means μ_a and μ_b respectively. (Recall the example of a single object weighed twenty times on each of two spring scales.) Now I introduce an alternative super-model. I want this alternative to lack the commitment to a common mechanism underlying the two data sets, so where previously I assumed that $\mu_a = \mu_b$, now I remove that constraint. This alternative super-model is simply a composite of the two separate location-normal models, which retains both adjustable parameters μ_a and μ_b . Call the original super-model the *one-parameter* super-model, and call this alternative the *two-parameter* super model.

What does the two-parameter super-model say about the agreement statistic $\bar{x}_a - \bar{x}_b$? The agreement statistic is not ancillary for the two-parameter model, meaning that the distribution assigned to that statistic depends on the values of the adjustable parameters μ_a and μ_b . All on its own, the two-parameter super-model doesn't say enough—it is too logically weak—to predict anything about that statistic. We can, however, use a standard Bayesian technique to generate a distribution over the agreement statistic by logically strengthening the two-parameter super-model hypothesis. We can assume *prior probability* distributions for the parameters μ_a and μ_b , and then 'integrate out' those priors. Think of this logically strengthened hypothesis as describing two layers of stochastic processes. The data $\{x_1, x_2, \dots, x_{40}\}$ are generated by first drawing values for μ_a and μ_b from their respective prior distributions,

and then drawing data points $\{x_1, x_2, \dots, x_{20}\}$ and $\{x_{21}, x_{22}, \dots, x_{40}\}$ from normal distributions with means μ_a and μ_b respectively. The first level of stochastic processes is intended to represent uncertainty about the true values of the parameters, and the two-level process incorporates that uncertainty into the super-model's predictions about the data. Adding priors in this way logically strengthens the two-parameter super-model enough to generate predictions about the agreement statistic, and I employ this procedure in order to contrast the two super-models vis-à-vis observed values for the agreement statistic.

Exactly what the augmented two-parameter super-model predicts about the agreement statistic of course depends on what priors are built into that hypothesis. But qualitatively, the likelihood comparison between the two super-models is not very sensitive to the choice of priors. Here is one example calculation. For convenience, I start by assuming that the priors for μ_a and μ_b are normal, and independent. Let the variance of each distribution be $\sigma^2 = 25$ to represent a moderate degree of uncertainty about those parameter values. The resulting distribution assigned to the agreement statistic further depends only on the *difference* between the means of the two prior distributions, not on their individual values. The smaller the difference between the two means, the more the two-parameter super-model predicts agreement between measurements, so it is most charitable to make the difference zero. Intuitively, the resulting distribution over the agreement statistic should be fairly flat, since the high variance in the first level of the stochastic process means there is a large range of reasonably probable divergences between μ_a and μ_b , and so the probability mass is spread more widely over possible values for $\bar{x}_a - \bar{x}_b$. The actual

distribution is shown as the dashed line in figure 4.1. (The distribution is centered on zero only because I have made the means of the two priors equal.)

Comparing the two distributions pictured in figure 4.1, the one-parameter super-model has much higher likelihood for observed values of the agreement statistic near zero. It is this likelihood comparison between the two super-models which, on my account, expresses the epistemic significance of the agreement of independent measurements.³

4.6 Application

So far I have provided a concrete illustration, framed in abstract mathematical terms. It remains to be explained how the competing hypotheses that are salient within the real scientific episodes characterized as ‘the agreement of independent measurements’ are relevantly similar to the two super-models from my illustration.

The real world analogues of the one-parameter super-model are hypotheses that posit a quantity that is not (colloquially speaking) directly observable, but can (ac-

³Regarding sensitivity to the choice of priors, any difference between the means of the two priors will shift the mean of the resulting distribution for the agreement statistic away from zero, making the likelihood comparison with the single-parameter super-model even more dramatic. The effect of increasing or decreasing the variance of the priors depends on how close the two means are, but the likelihood of the two-parameter super-model, given an observed value of the agreement statistic near zero, can approach that of the one-parameter super-model only if the variance of the priors is very low, *and* their means are very close to one another. In subjective terms this means that the agent is very confident that $\mu_a = \mu_b$ in which case the two-parameter super-model collapses to the one-parameter super-model; in this case it is no concern that comparing likelihoods no longer distinguishes the two hypotheses.

ording to its hypothesized nature) be measured in multiple ways. For example, in Newton's physics the *mass of an object* can be measured by observing how much the object stretches a spring, or by observing how much it accelerates when a force is applied (§3). Likewise, the wave theory of light posits a *wavelength*, and the atomic theory of matter posits a *number of particles* in a standard unit of a substance. Each posited quantity was (eventually) measurable in a variety of ways. These are the one-parameter hypotheses.

The real-world analogues of the *two-parameter super-model* are harder to characterize as a group since these hypotheses vary a great deal in how fully and explicitly they are articulated. They lie on a scale from full-fledged alternative scientific theory to vague skeptical worry. Despite the variation exhibited in that dimension, I will endeavor to explain how they all share the relevant similarity to the two-parameter super-model from my illustration. To do this, I must go back and discuss an aspect of my formal illustration that I glossed over in the first pass.

Returning to the formal illustration, consider the dual nature of the quantity \bar{x}_a , the mean value of the data set $x_a = \{x_1, x_2, \dots, x_{20}\}$. On the one hand, \bar{x}_a is the maximum-likelihood estimate of the value of the parameter μ_a . Call this the *theoretical perspective* on \bar{x}_a . But at the same time, \bar{x}_a is merely the result of a mathematical operation applied mechanically to the data set x_a . Call this the *observational perspective* on \bar{x}_a . Notice that while the two super-models share the same observational perspective on \bar{x}_a , they take contradictory theoretical perspectives on \bar{x}_a . We might say that they offer different *interpretations* of \bar{x}_a . The one-parameter super-model interprets \bar{x}_a as the best estimate of the single parameter μ which also underlies data

set $x_b = \{x_{21}, x_{22}, \dots, x_{40}\}$. The two-parameter super-model interprets \bar{x}_a as the best estimate of the parameter μ_a (which parameter exerts no influence on data set x_b —the stochastic process underlying data set x_b being governed by a separate parameter μ_b). The common thread among real world analogues of my two-parameter super-model is that they offer a more limited, local *interpretation* of a single measurement.

On the full-fledged scientific theory end of the spectrum, take for example the Ptolemaic theory of the solar system as an alternative to the Copernican theory. Ptolemy put the Earth at the center of the solar system, and decomposed the *apparent* motion of each planet (as viewed from Earth) into an orbit around Earth (the *deferent*), plus a second, smaller orbit (the *epicycle*) that circles a point moving along the deferent. It turns out that the Ptolemaic epicycle captures the component of apparent planetary motion that is in fact contributed by the motion of the Earth around the Sun. In effect, Ptolemy (unknowingly) took the motion of the Earth around the Sun and displaced it to another location within his picture of the solar system—but another location from which it could make the same contribution to the overall motion of a planet relative to the Earth. Thus the relative motion of the Sun and Earth is replicated within the Ptolemaic model *for each planet*. A Ptolemaic ‘super-model’ addressing two planets plus the Earth will then include one parameter for the period of the first planet’s epicycle, and another parameter for the period of the second planet’s epicycle. The corresponding Copernican super-model, however, will treat the estimates of those two parameter values as two estimates of the same quantity, *viz.*, the period of the Earth’s orbit around the Sun.

At the other end of the spectrum we have less fully articulated ideas about a measurement being an ‘artifact’ of the measuring procedure, or the measuring device, or of the particular experimental setup generating the data (*cf.* Hacking 1985). The single-parameter hypothesis interprets the measurement as an estimate of a property of the entity under study, which property will naturally be constant across repeated measurements or measurements using different techniques. The alternative ‘hypothesis’ interprets the measurement as a property of the dust on the microscope lens, or of a glitch in the computer software, or of a one-off spike in emissions from the factory down the road, *i.e.*, as an estimate of some quantity that is of less general significance and that would not be expected to influence attempted measurements of the target property on other occasions or through other media. Fully articulating such alternative hypotheses would involve positing separate parameters underlying the results of separate measurement attempts on other occasions or through other media, as per the two-parameter super-model in my illustration.

4.7 Evidential Diversity?

Now that my proposal is on the table, I contrast it with an alternative approach to the same problem, and say where I think that approach falls short. For purposes of formal analysis, the agreement of independent measurements is usually lumped under the rubric of ‘evidential diversity’. The literature on evidential diversity is a response to the widely experienced intuition that collections of confirming observations that are more ‘diverse’, ‘varied’, or ‘heterogeneous’ (or, equivalently, the *parts* of which

are ‘independent’ or ‘of different kinds’) confirm a hypothesis more strongly than otherwise comparable collections that are ‘narrow’, ‘homogeneous’, or ‘of the same kind’. Hempel’s discussion of the ‘criteria of confirmation and acceptability’ provides a standard expression of the idea. Hempel first discusses how, as the number of supporting observations already cited grows larger, the confirmation effected by a new supporting observation grows smaller. He then adds the following caveat:

This remark must be qualified, however. If the earlier cases have all been obtained by tests of the same kind, but the new finding is the result of a different kind of test, the confirmation of the hypothesis may be significantly enhanced. For the confirmation of a hypothesis depends not only on the quantity of the favorable evidence available, but also on its variety: the greater the variety, the stronger the resulting support.
(Hempel 1966, 33–34)

Notice how Hempel sets up the problem. He addresses a set of observations *each of which* individually confirms the hypothesis in question, and then gestures at a notion of ‘variety’ within such sets, and a relationship between this ‘variety’ and the sum total of confirmation provided by the set. The challenge is then to rigorously define the ‘variety’ (or related property) of a collection of individually confirming observations in such a way that one’s normative epistemic theory (typically Bayesian epistemology) pronounces a hypothesis better confirmed when the supporting observations have more of that property (*e.g.*, Sober 1989; Earman 1992; Howson and Urbach 1993; Wayne 1995; Fitelson 2001; Bovens and Hartmann 2003).

The observations and hypotheses in my illustration above do not satisfy (or at least need not satisfy) this setup. Recall that the competing hypotheses in my example were the one- and two-parameter super-models for the total data set $x = \{x_1, x_2, \dots, x_{40}\}$. What the diversity of evidence approach calls the observations would be those associated with the two ‘independent’ measurements, *viz.*, \bar{x}_a and \bar{x}_b (or x_a and x_b , the difference will not matter). The problem is that neither of these observations individually confirms the one-parameter super-model. If you look at only one half of the total data set, *e.g.*, $x_a = \{x_1, x_2, \dots, x_{20}\}$, the two super-models say exactly the same thing about those data (and about any statistic of those data). Both supermodels say that $x_a = \{x_1, x_2, \dots, x_{20}\}$ came from 20 independent and identically distributed random variables, each normal with known variance $\sigma^2 = 1$, and unknown mean. The only difference between the two super-models is whether that mean is given by the parameter μ_a or the parameter μ , but this difference is immaterial if we restrict our attention to x_a . (The same can be said, *mutatis mutandis*, of x_b .)

In terms of the spring scale example from §3, the one-parameter hypothesis says that two different scales measure the *same property*, and the two-parameter hypothesis says that the two scales measure *different properties*, or at least *may* measure different properties. Naturally, weighing an object on only the first scale does not discriminate between the two hypotheses. (And neither does weighing an object on only the second scale.) More formally, neither super-model (all on its own) assigns any probability to \bar{x}_a . A likelihood comparison vis-à-vis \bar{x}_a can be made by adding *prior distributions* over both μ_a and μ and calculating likelihoods as per the proce-

dure described in §5. But if those two priors are the same, as would seem to facilitate a fair comparison, then the two super-models will assign exactly the same probability to \bar{x}_a (and to any other statistic of x_a). Thus, neither measurement favors one super-model over the other. The way that I have identified the relevant observations and hypotheses puts the inference problem that I am addressing outside of the set up to which the diversity of evidence literature is addressed.

The diversity of evidence approach may address some of the same scientific scenarios, but focusses on a different aspect of the problem by directing attention to different hypotheses and observations. In contrast to my treatment of the agreement between measurements as *an observation* (in the form of the agreement statistic), the diversity of evidence approach treats each individual measurement as an observation. The question is then: What do \bar{x}_a and \bar{x}_b each individually confirm? If we set aside the two-parameter super-model, and assume that the one-parameter super-model is correct, then similar observed values for \bar{x}_a and \bar{x}_b each confirm a similar range of values for the parameter μ (recall that \bar{x} is the maximum-likelihood estimate of μ for the normal model). It is at this point that the issues raised in the quotation from Hempel become relevant. How ‘diverse’ is the observation set $\{\bar{x}_a, \bar{x}_b\}$, (or, equivalently, how ‘independent’ are \bar{x}_a and \bar{x}_b)? And how ought this degree of diversity impact our beliefs about the true value of the parameter μ ? The key point here is that treating the two measurements as each confirming the same value (or range of values) for the parameter μ *presupposes* the one-parameter hypothesis, whereas what I have tried to show is how the agreement of measurements *confirms* the one-parameter hypothesis.

Mapping this distinction back onto the motivating scientific examples, the diversity of evidence approach *presupposes* the wave theory of light, and then addresses how ‘diversity’ among measurements helps confirm a value for the wavelength. The diversity of evidence approach *presupposes* the atomic theory of matter, and addresses confirmation for hypotheses about the size of the atom. The diversity of evidence approach *presupposes* the Copernican model of the solar system, and addresses confirmation for hypotheses about the period of the Earth’s orbit. In contrast, I have tried to show how agreement between measurements of the wavelength of light can confirm the wave theory of light, how agreement between measurements of the size of the atom can confirm the atomic theory of matter, and how agreement between measurements of the period of the Earth’s orbit can confirm the Copernican model of the solar system.

4.8 Conclusion

Seeing that multiple, ‘independent’ measurements of a quantity agree, one intuitive conclusion is that the value about which the measurements agree is *correct* (and moreover, the greater the ‘independence’, the more confidence is warranted). But there is another, more basic (yet less obvious) conclusion, which is equally intuitive once made explicit: that the several procedures used for measurement in fact measure the same property. The first conclusion, which is the subject of the diversity of evidence literature, presupposes the second. I have pointed to historically detailed philosophical work suggesting that the second conclusion is at least as important

as the first within the scientific episodes that are described as the agreement of independent measurements and which partially motivate the diversity of evidence literature. I have provided a template for formal reconstruction and rationalization of this second and more basic element within the motivating scientific episodes. The key innovation is to treat the degree of agreement between measurements *as a single observation* (a statistic of a total data set). Hypotheses that posit a single property underlying multiple measurement attempts will tend to assign a higher probability to close agreement between measurements, as compared to hypotheses that posit different parameters underlying different measurement attempts.

Chapter 5

Evidential Diversity in Hierarchically Structured Hypothesis Spaces

5.1 Introduction

In the previous chapter, I introduced an approach to modeling the epistemology of agreeing measurements, and very briefly contrasted this approach with existing philosophical work on the related topic of evidential diversity. In the present chapter I generalize this approach to agreeing measurements to directly address the epistemic intuition motivating the diversity of evidence literature, namely that a set of supporting observations that is diverse in character provides better evidence than an otherwise comparable set of supporting observations that lacks diversity or is “all

the same kind”. This will involve defining “diversity” and explaining in what sense diverse evidence is “better” evidence. Existing formal work on the value of diverse evidence uses the Bayesian framework predominantly, and I will adopt that framework here, and discuss in detail how the account that I present compares to existing Bayesian work on the diversity of evidence.

The real scientific examples that I take to motivate interest in the epistemological problem that I am addressing include some of those mentioned in the introduction to Chapter 4. But they also include countless more mundane examples from both science and everyday reasoning. These are the kind of examples that I discussed at the end of § 4.6, in which the alternative hypotheses are usually expressed informally as worries about the reliability of a method of measurement, rather than as alternative scientific theories. As an example of such a more mundane instance, consider the famous episode of Florin Périer’s measurement of the height of a column of mercury in an early barometer, at the top of the *Puy de dôme*.

The *Puy de dôme* is a high lava dome in south-central France, and Périer was the brother-in-law of Blaise Pascal. Pascal wanted to know how mercury-column readings at high altitude compared to those made near sea level, and Périer lived near the *Puy de dôme*, so Pascal asked Périer to make the trek, and the reading, for him. Périer and an entourage scaled the dome and made the measurement. But not just one measurement:

To make sure they took measurements in five places at the top, on one side and the other of the mountain top, inside a shelter and outside, but the column heights were all the same. (Boring 1954)

The result of the measurement (*i.e.*, the height of mercury observed) was very significant to Pascal’s broader theorizing about hydrostatics, but setting aside this broader scientific importance of the result, notice that *to get that result*, P erier et al. made a number of separate measurements under *diverse* conditions at the top of the dome.

5.2 Formal Example

5.2.1 Setup

I will continue to use parameter estimation as a general model of measurement. Because it is intuitive and accessible, I begin with an example using coin tossing (*i.e.*, the geometric statistical model). Consider some observations of coin toss outcomes $\{t, h, h, \dots\}$. Suppose we have four methods of collecting such data, methods A , B , C , and D . And suppose that 20 outcomes are collected by each method. (Ultimately, I will call the four data sets different “kinds” of observation, and say that the *total* data set is “diverse” in virtue of including these several kinds.) Now consider four hypotheses about the process(es) that generated these data, hypotheses 1–4. All four hypotheses agree that the data represent the outcomes of coin tosses. But they disagree in what are called their *homogeneity assumptions*.

Two data sets are homogeneous with respect to a given statistical property if that property is constant across those data sets. For example, (labeling the data set gathered by collection method A as a , that gathered by collection method B as b , and so on) the frequency of heads might be the same in data sets a and b , in which case those data sets are homogeneous with respect to that statistical property. We

must, however, distinguish between the *sample frequency* and the coin's *probability of landing heads*. The former is the observed frequency of heads in the actual data; the latter is the corresponding property of the process that generated the data. (In the long run, the two have the same value, but in small data sets they may diverge due to sampling error.) Homogeneity typically refers to processes rather than outcomes, in which case data sets a and b may be homogeneous despite small differences in their sample frequencies—in other words, it is the *probability* of heads that is constant across the two data sets.

Continuing to spell out the hypotheses, about data collected by any single collection method all four hypotheses say the same thing: that each coin toss is an independent draw from the distribution $p(\textit{heads}) = \theta$, $p(\textit{tails}) = 1 - \theta$, where θ is unknown but constant. The four hypotheses *disagree* about whether θ is constant *across* data collection methods. Using θ_i to indicate $p(\textit{heads})$ for data collected by method i , hypothesis 1 says that $\theta_a = \theta_b = \theta_c = \theta_d$. The other hypotheses make different homogeneity assumptions, as depicted in Table 5.1; each partitions the total data into at least two parts, and allows for different values for $p(\textit{heads})$ within different parts of the total data. To complete the probabilistic description of the four hypotheses, let each say that the θ_i are drawn from a uniform probability density distribution over the interval $[0,1]$. Where θ_i and θ_j are not constrained to be equal, they are treated as independent draws from the uniform distribution.

	A	B	C	D
hypothesis 1	$\theta_a = \theta_b = \theta_c = \theta_d$			
hypothesis 2	$\theta_a = \theta_b = \theta_c$			θ_d
hypothesis 3	$\theta_a = \theta_b$		θ_c	θ_d
hypothesis 4	θ_a	θ_b	θ_c	θ_d

Table 5.1: Hypotheses 1–4 make different *homogeneity assumptions*. Hypothesis 1 says that $p(\text{heads})$ is constant across all four data collection methods, whereas hypothesis 4 says that $p(\text{heads})$ may be (and probably is) different for each method. Hypotheses 2 and 3 are intermediate in how much they partition the total data into parts inhomogeneous with respect to $p(\text{heads})$.

5.2.2 Analysis

Now I wish to show, given the above setup, how a kind of evidential diversity can be important to the relative confirmation of hypotheses 1–4. Recall that 20 coin toss outcomes are collected by each of four data collection methods. Suppose that the observed frequencies of heads in those four data sets are: $\bar{a} = 0.4$, $\bar{b} = 0.5$, $\bar{c} = 0.35$, and $\bar{d} = 0.4$. The observations that I will consider are *comparisons* between these four observed frequencies. Representing heads by 1 and tails by 0, the frequency of heads in data set x is the average of the values in the set, written as \bar{x} . The observations I will consider are the observed values of the following statistics of the total data: $|\bar{a} - \bar{b}|$, $|\bar{a} - \bar{c}|$, and $|\bar{a} - \bar{d}|$. Given the frequencies supposed above, those values are: 0.1, 0.05, and 0, respectively. (What is important for my purposes in these numbers is only that the four frequencies fall within a small range, so that the difference statistics are nearish to zero.)

I'll begin with a simple, qualitative analysis of what these observations say about hypotheses 1–4, and then move on to a rigorous Bayesian treatment. Consider the

observation that $|\bar{a} - \bar{b}| = 0.1$. Roughly, such a small difference in frequency between data sets a and b is what we should expect to see supposing any of the hypotheses that constrain θ_a and θ_b to be equal. Hypotheses 1–3 all include that constraint. Hypothesis 4, on the other hand, says that θ_a and θ_b are independent draws from the uniform distribution over $[0,1]$, on which supposition such a small difference between \bar{a} and \bar{b} is at least somewhat less probable. So the first observation reflects equally well on hypotheses 1–3, and somewhat less well on hypothesis 4. In the same way, the observed small value for $|\bar{a} - \bar{c}|$ is just what we should expect supposing any hypothesis that constrains θ_a and θ_c to be equal (those being hypotheses 1 and 2), but a bit of a coincidence on hypotheses that treat θ_a and θ_c as independent draws from the uniform distribution (hypotheses 3 and 4). Finally, the difference between \bar{a} and \bar{d} is also small, and this observation is more favorable to hypothesis 1 than to any of the other hypotheses since only by hypothesis 1 requires θ_a and θ_d to be equal.

The formal Bayesian analysis of the example follows the outlines of the informal one just given. Figure 5.1 displays the probability distributions that single-theta and independent-theta hypotheses assign to all possible values of the statistics $|\bar{a} - \bar{b}|$. By “single-theta” I mean hypotheses 1–3, all of which assume that data collected by methods A and B are homogeneous with respect to $p(\textit{heads})$. For the statistic $|\bar{a} - \bar{b}|$, hypothesis 4 is the only “independent-thetas” hypothesis, meaning that hypothesis 4 treats data collected by methods A and B as generated by two values of $p(\textit{heads})$, drawn independently from the uniform distribution over $[1,0]$. The corresponding probability distributions for statistics $|\bar{a} - \bar{c}|$ and $|\bar{a} - \bar{d}|$ are exactly the same; what

changes, in proceeding from one statistic to the next, is only which of hypotheses A , B , C , and D count as single-theta and which count as independent-theta hypotheses. Drawing all likelihoods from Figure 5.1, and assuming a uniform prior over the four hypotheses to begin with, Figure 5.2 displays the posterior probabilities for those hypotheses after sequential conditionalization on the observations that $|\bar{a} - \bar{b}| = 0.1$, $|\bar{a} - \bar{c}| = 0.05$, and $|\bar{a} - \bar{d}| = 0$.

To compare this example to that from the previous chapter, the important conceptual difference so far is that in last chapter's example, there were two hypotheses with different homogeneity assumptions (though I didn't use that term there), as opposed to the four hypotheses in the current example. The "one-parameter super-model" treated the whole data set as homogeneous with respect to the parameter μ , while the "two-parameter super-model" partitioned the total data in two. The latter hypothesis treated each half of the data as internally homogeneous with respect to μ , while treating the whole data set as (potentially) inhomogeneous. In the current example each of four hypotheses posits different homogeneity assumptions. Now I introduce a second important difference.

The mathematical example in the previous chapter concerned the contrast between how two hypotheses (the one- and two- parameter super-models) related to a single data set. The difference in how those two hypotheses related to the total data set could be described as a difference in how much each hypothesis unified those data. And more generally, this is the kind of contrast that is relevant to characterizing unification, namely two hypotheses compared via their relations to the *same data set*. Implicit in Figure 4.1 (page 75) is a second kind of contrast, a contrast between

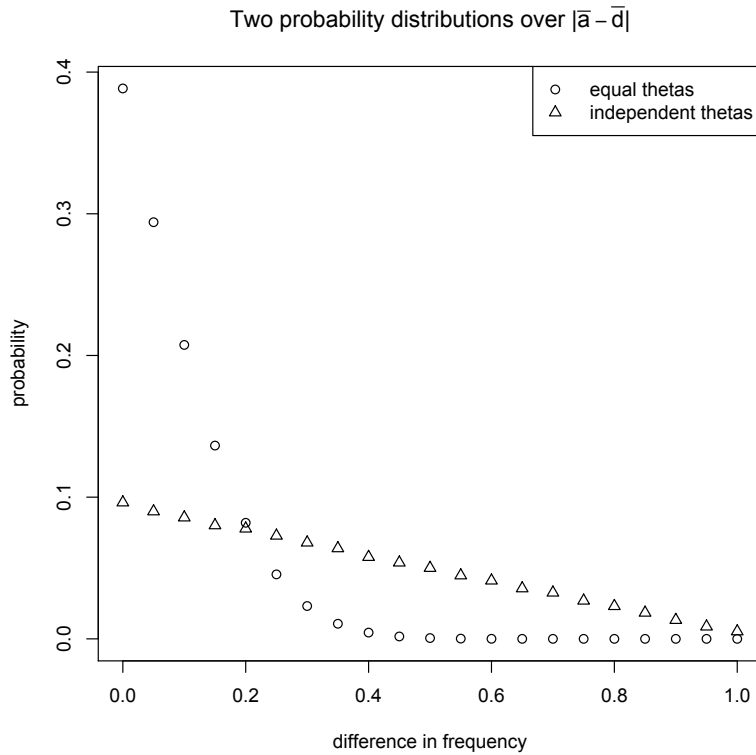


Figure 5.1: Two probability functions over possible values for the statistic $|\bar{a} - \bar{b}|$. The picture is exactly the same for the statistics $|\bar{a} - \bar{c}|$, and $|\bar{a} - \bar{d}|$. What changes from observation to observation is how many of the four hypotheses are “equal thetas” versus “independent thetas” for the relevant data collection methods. Likelihoods for each step of Bayesian conditionalization come from this plot (specifically, from the values at “difference in frequency” = 0.0, 0.05, and 0.1).

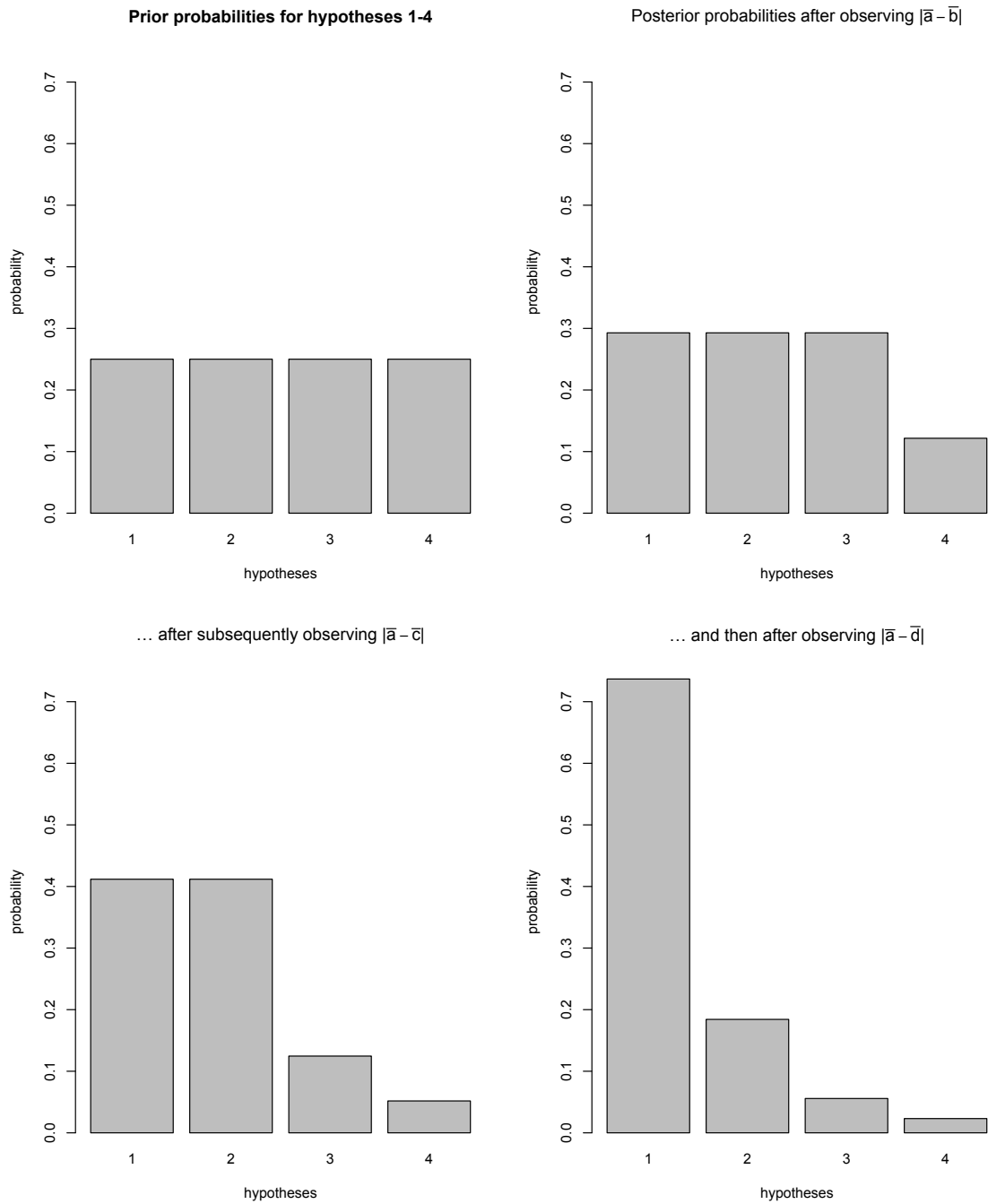


Figure 5.2: Four probability distributions over hypotheses 1–4. Clockwise from the upper left, (1) a uniform prior distribution, (2) the posterior distribution after conditionalizing on the observation that $|\bar{a} - \bar{b}| = 0.1$, (3) the posterior after subsequently conditionalizing on the observation that $|\bar{a} - \bar{c}| = 0.05$, and (4) the final distribution after conditionalizing on the observation that $|\bar{a} - \bar{d}| = 0$.

alternative possible *observations*, *viz.*, between total data sets that entail small values of that statistic and other total data sets that entail larger values of that statistic. The former kind of data set favored the one-parameter super-model, the later kind favored the two-parameter super-model. These two kinds of data sets, however, do not differ in *diversity*. Characterizing the evidential importance of *diversity* can be done only by holding fixed the hypothesis (or set of hypotheses) while considering alternative possible data sets that differ in how diverse they are. The new element that I now introduce to the coin tossing example above is an alternative possible data set that is less diverse, but otherwise comparable to the data set already described.

The data set already described consisted of 80 outcomes total, 20 collected by each of four data collection methods, labeled *A*, *B*, *C*, and *D*. Now consider as an alternative data set 80 outcomes collected by just methods *A* and *B*, 40 outcomes from each method. Call the two parts of this alternative total data set a_{40} and b_{40} (the subscripts are needed to distinguish these data sets from the *a* and *b* from above). To make this alternative data set otherwise comparable to the first, let $\overline{a_{40}} = \bar{a} = 0.4$ and $\overline{b_{40}} = \bar{b} = 0.5$. This new data set is less diverse in the sense that it contains outcomes collected by only two methods rather than by four. But it is otherwise similar to the original data set in that the total number of outcomes is the same (80), and the frequencies of heads among outcomes gathered by methods *A* and *B* are also the same.

Suppose that this second, less diverse data set were observed instead of the first. How would that change the support for hypotheses 1–4? Previously, I considered three statistics of the total data set: $|\bar{a} - \bar{b}|$, $|\bar{a} - \bar{c}|$, and $|\bar{a} - \bar{d}|$. But only one

of those statistics can be formulated in this second scenario, namely $|\bar{a} - \bar{b}|$. By stipulation, that statistic has the same value that it did in the original scenario. Just as before, the small observed value for $|\bar{a} - \bar{b}|$ is more probable on those hypotheses that make $\theta_a = \theta_b$ than on those that treat θ_a and θ_b as independent draws from the uniform distribution $[0,1]$. The former include hypotheses 1–3, the latter only hypothesis 4. So hypotheses 1–3 are favored over hypothesis 4. But the import of the evidence ends there. Without outcomes gathered by methods C and D , there are no observations that could possibly discriminate between hypotheses 1–3; all three hypotheses say exactly the same thing about all possible data gathered by methods A and B . So the evidential difference between the original and less diverse data sets is seen in the *range of alternative hypotheses over which the observations favor hypothesis 1*. The more diverse set of observations favors hypothesis 1 over each of the other three, while the less diverse set favors hypothesis 1 over hypothesis 4, while leaving hypotheses 2 and 3 on par with hypothesis 1.

More formally, starting again with a uniform prior over hypotheses 1–4, the posterior distribution after conditionalization on the observation that $|\overline{a_{40}} - \overline{b_{40}}| = 0.1$ is exactly the same as that after observing $|\bar{a} - \bar{b}|$ from the original, more diverse data set, *i.e.*, the upper right plot of Figure 5.2.¹

¹The two posteriors are the same because, for all i, j , and x :

$$\frac{p(|\overline{a_{40}} - \overline{b_{40}}| = x | \text{hypothesis } i)}{p(|\overline{a_{40}} - \overline{b_{40}}| = x | \text{hypothesis } j)} = \frac{p(|\bar{a} - \bar{b}| = x | \text{hypothesis } i)}{p(|\bar{a} - \bar{b}| = x | \text{hypothesis } j)}$$

despite that for all x and i :

$$p(|\overline{a_{40}} - \overline{b_{40}}| = x | \text{hypothesis } i) \neq p(|\bar{a} - \bar{b}| = x | \text{hypothesis } i)$$

5.3 “Diversity” and “Kinds”

On the approach to evidential diversity just illustrated, what counts as “diverse”? What does it mean for observations to be “different kinds”? On this approach, neither term applies to observations on their own. Rather, “diversity” and “kinds” are defined only relative to the set of hypotheses under consideration. Two data sets are different kinds of data if any of the hypotheses under consideration *say that they are* by partitioning the total data set into parts and treating those parts as inhomogeneous with respect to the measured quantity (or potentially so), where the two data sets in question come from different parts of the partition. Roughly, an observation set is more diverse the more such partitions of the total observation set are stipulated by salient competing hypotheses.

5.4 Application

The formal example above can be mapped schematically onto the case of Périer’s mercury tube measurements mentioned in §5.1. Recall that Périer made a series of measurements under varying circumstances at the top of the peak: in a sheltered area and out in the open, and on different sides of the peak. These differing circumstances of measurement correspond to the different data collection methods A , B , C , and D in my formal example. From the perspective of hypothesis 1 in the formal example, each observed frequency (\bar{a} , \bar{b} , \bar{c} , and \bar{d}) is a measurement of the single quantity θ . Each of hypotheses 2–4, on the other hand, treats one or more of \bar{b} , \bar{c} , and \bar{d} as something other than an additional measurement of the quantity measured by \bar{a} .

What about the hypotheses 1–4? The Périer analogue of hypothesis 1 is the hypothesis that all of the measurements made at the top of the *Puy de dôme* are measurements of the same theoretically posited quantity: the air pressure at that altitude. In other words, that hypothesis treats observations collected at the different locations as homogeneous with respect to the measured quantity. The other hypotheses are ones that posit some *heterogeneity* within that total observation set. For example, one possible alternative hypothesis says that mercury tube readings are influenced significantly by whether they take place on the windward or the leeward side of the peak. Another says that sunlight influences the readings. Every such hypothesis partitions the total data set into parts that are generated by somewhat different underlying processes, like hypotheses 2–4 in the formal example.

Perhaps a more accurate formalization of such alternative hypotheses as the one that has sunlight influencing the mercury height would include a parameter for air pressure that *is* constrained to be constant across measurement contexts, but where each context introduces *additional* parameters that also influence the result of the reading. I have avoided this in my formal example only for simplicity of presentation. There are many ways to instantiate statistical heterogeneity. For present purposes, in terms of the likelihoods displayed in Figure 5.1, the effect should not be very different.

5.5 The Correlation Approach

In the rest of this chapter I give a brief overview of the main strand of existing Bayesian literature on evidential diversity, and explain how what I've said so far fits into that literature. This is the approach to modeling the epistemology of evidential diversity that Wayne (1995), Steel (1996), and Bovens and Hartmann (2003) call the “correlation approach”.

Speaking in terms of *similarity* between observations (where an observation *set* is less diverse the more similar its constituent observations are to one another) Howson and Urbach (1993) write:

This idea of similarity between items of evidence is expressed naturally in probabilistic terms by saying that e_1 and e_2 are similar provided $P(e_2|e_1)$ is higher than $P(e_2)$; and one might add that the more the first probability exceeds the second, the greater the similarity. This means that e_2 would provide less support if e_1 had already been cited as evidence than if it was cited by itself. (159–60)

Granted the stipulated relations between e_1 and e_2 , their claim that e_2 would provide less support for h had e_1 already been cited as evidence follows directly from the Bayesian conditionalization rule, according to which the posterior probability of a hypothesis h after making observation e_2 is:

$$P(h|e_2) = P(h) \times \frac{P(e_2|h)}{P(e_2)} \tag{5.1}$$

As $P(e_2)$ is the denominator of the factor $P(e_2|h)/P(e_2)$ by which the prior probability of the hypothesis h is multiplied to arrive at the posterior probability of h , increasing $P(e_2)$ decreases that posterior probability. In this way, h is better confirmed by observations e_1 and e_2 if those two observations are such that learning e_1 does not raise the probability of e_2 , as opposed to being such that learning e_1 *does* raise the probability of e_2 . In short, the less similar e_1 and e_2 , the more they jointly confirm h . Earman (1992) points to the same mechanism for more “varied” observation sets to confirm more strongly than less varied ones.

Wayne (1995) labeled the approach taken by Howson and Urbach (1993) and Earman (1992) the “correlation approach”,² and repackaged it by defining a *measure* of similarity, S :

$$S(e_1, e_2) = \frac{p(e_2|e_1)}{p(e_2)} = \frac{p(e_1 \& e_2)}{p(e_1)p(e_2)}, \quad (5.2)$$

and more generally, for $(e_1, e_2, e_3, \dots, e_n)$:

$$S(e_1, e_2, e_3, \dots, e_n) = \frac{p(e_1 \& e_2 \& \dots \& e_n)}{p(e_1)p(e_2) \dots p(e_n)}. \quad (5.3)$$

Myrvold (1996) adopts S and further explores its role in Bayesian confirmation by defining a “conditional form” of S :

$$S(e_1, e_2, \dots, e_n|h) = \frac{p(e_1 \& e_2 \& \dots \& e_n|h)}{p(e_1|h)p(e_2|h) \dots p(e_n|h)}. \quad (5.4)$$

²Strictly speaking, the term “correlation” is misused here. Correlation describes a relationship either between random variables, or between event types, while Wayne uses it to describe a relationship between individual events.

Myrvold assumes the ratio measure of the degree of confirmation of h by o (*i.e.*, he expresses the degree of confirmation with the ratio $p(h|o)/p(h)$) and he uses S to rearrange the mathematical expression for the degree of confirmation of hypothesis h by observations $\{e_1, e_2, e_3, \dots, e_n\}$ into the following form:

$$\frac{p(h|e_1 \& \dots \& e_n)}{p(h)} = \frac{p(h|e_1)}{p(h)} \times \dots \times \frac{p(h|e_n)}{p(h)} \times \frac{S(e_1, \dots, e_n|h)}{S(e_1, \dots, e_n)}, \quad (5.5)$$

calling the final term on the right hand side of the equation the “interaction term”. The equation above expresses the confirmation of h by $\{e_1, e_2, e_3, \dots, e_n\}$ as a product of how much the e_i each individually confirm h and the interaction term.

Informally, the numerator of the interaction term is the degree of “similarity” (in the sense described by Howson and Urbach) among the observations $\{e_1, e_2, e_3, \dots, e_n\}$ as judged by the hypothesis h , while the denominator is the unconditional similarity among those observations. The interaction term is thus a measure of how much the assessed similarity among the observations is increased by supposing that h were true. So Myrvold’s decomposition of the expression $P(h|e_1 \& \dots \& e_n)/P(h)$ shows that, other things being equal (*viz.* the individual degrees of confirmation of h by the e_i being held constant), the greater the interaction term, the greater the confirmation of h by $\{e_1, \dots, e_n\}$ jointly.

Wheeler (2009) relabels Myrvold’s interaction term as the “focussed correlation” of $\{e_1, \dots, e_n\}$ relative to hypothesis h , introducing the notation $For_h(e_1, \dots, e_n)$:

$$For_h(e_1, \dots, e_n) = \frac{S(e_1, \dots, e_n|h)}{S(e_1, \dots, e_n)} = \frac{\frac{p(e_1 \& e_2 \& \dots \& e_n|h)}{p(e_1|h)P(e_2|h) \dots p(e_n|h)}}{\frac{p(e_1 \& e_2 \& \dots \& e_n)}{p(e_1)p(e_2) \dots p(e_n)}}. \quad (5.6)$$

Wheeler (2009); Wheeler and Scheines (2011); Schlosshauer and Wheeler (2011) investigate the relationship between focussed correlation and confirmation. Wheeler and Scheines (2011) show that given assumptions (a) and (b) below, for hypothesis h and evidence sets $e = \{e_1, e_2\}$ and $e^* = \{e_1, e_3\}$, if the focussed correlation of e by h is greater than that of e^* by h , then the confirmation of h by e is greater than that by e^* , *on any one of several of common measures of confirmation*. The conditions are: (a) that the e_i each individually confirm h , and (b) that $P(h|e_i) = P(h|e_j)$ for all i & j . (On the ratio measure, this result follows trivially from Mryvold's equation; the result of Wheeler and Scheines (2011) is more general.) Schlosshauer and Wheeler (2011) relax assumption (b), and demonstrate that if $For_h(e_1, e_2)$ is greater than $For_h(e_1, e_3)$ but e_3 individually confirms h *a lot* more than e_2 individually confirms h , the higher individual confirmation within e^* can outweigh the higher focussed correlation of e , and e^* will confirm h more strongly than e does. They give inequalities that have to hold for better focussed correlation to entail better confirmation, and vice versa.

5.5.1 My proposal and the correlation approach

The way that I have spelled out diversity among observations, and the value of that diversity for confirmation, *can* also be modeled within the so-called correlation approach. I will demonstrate this by recasting my coin example in terms of (using Wheeler's terminology) focussed correlation. Then I will discuss how my original presentation differs in emphasis from the focussed-correlation reformulation, and what I take my approach to add to the existing work on the correlation approach.

Viewing my coin example from the focussed correlation perspective, the relevant observations are not the frequency *differences* $|\bar{a} - \bar{b}|$, $|\bar{a} - \bar{c}|$, and $|\bar{a} - \bar{d}|$ as in my earlier presentation of the example, but rather the full data sets: a , b , c , and d , or instead (the results will be the same) the frequencies \bar{a} , \bar{b} , \bar{c} , and \bar{d} . The upper right plot of Figure 5.3 (light colored bars) displays the results of starting with a uniform prior over hypotheses 1–4 and conditionalizing not on the frequency differences as I did previously, but on the conjunction $\bar{a} \& \bar{b} \& \bar{c} \& \bar{d}$. The lower right plot shows the results of instead conditionalizing on the observation: $\bar{a}_{40} \& \bar{b}_{40}$ (again the lighter colored bars). The dark colored bars in the upper and lower right plots repeat results from the corresponding earlier calculations for comparison, displaying the posterior probabilities after conditionalizing on $\{|\bar{a} - \bar{b}|, |\bar{a} - \bar{c}|, |\bar{a} - \bar{d}|\}$, and $\{|\bar{a} - \bar{b}|, |\bar{a} - \bar{c}|\}$ respectively. As the comparisons show, there is little difference in the posteriors between conditionalizing on the frequency *differences* (dark bars), and conditionalizing on the frequencies themselves (light bars). The difference that diversity makes to the posterior distribution (four kinds of observation versus two kinds) is apparent using either approach to describing the observations.

Moreover, the focussed correlation values of the two different observation sets, relative to each of hypotheses 1–4, do manage to express the features of the example that lead to the posterior probabilities displayed in Figure 5.3. Returning to Myrvold’s (1996) decomposition of the expression $p(h|o)/p(h)$ (motivated by the ratio measure of confirmation), plugging in the observation set $\{\bar{a}, \bar{b}, \bar{c}, \bar{d}\}$, and using Wheeler’s (2009) notation for focussed correlation yields the following expression for the degree of confirmation of h_i by $\bar{a} \& \bar{b} \& \bar{c} \& \bar{d}$:

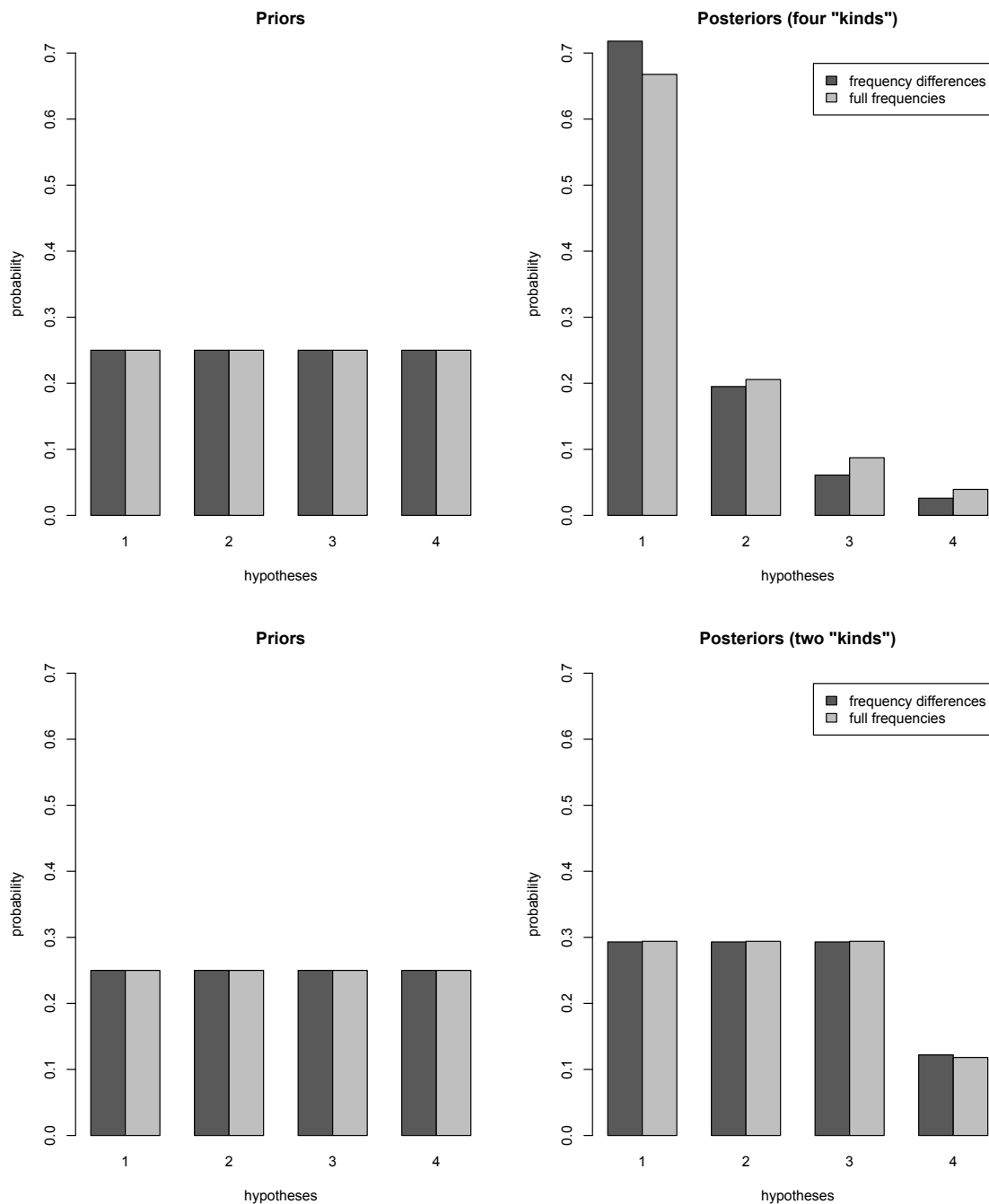


Figure 5.3: Probability distributions over hypotheses 1–4. The top row shows uniform prior distributions (left) and posterior distributions after conditionalizing on four kinds of data (right). The bottom row shows uniform prior distributions again (left) and posterior distributions after conditionalizing on two kinds of data (right). The frequency difference statistics are used as the observations for the darker bars, whereas the frequencies themselves are the observations for the lighter colored bars.

$$\frac{p(h_i|\bar{a} \& \bar{b} \& \bar{c} \& \bar{d})}{p(h_i)} = \frac{p(h_i|\bar{a})}{p(h_i)} \times \frac{p(h_i|\bar{b})}{p(h_i)} \times \frac{p(h_i|\bar{c})}{p(h_i)} \times \frac{p(h_i|\bar{d})}{p(h_i)} \times For_{h_i}(\bar{a}, \bar{b}, \bar{c}, \bar{d}) \quad (5.7)$$

The right hand side of Equation 5.7 can, in this particular case, be simplified, since for all i

$$\frac{p(h_i|\bar{a})}{p(h_i)} = \frac{p(h_i|\bar{b})}{p(h_i)} = \frac{p(h_i|\bar{c})}{p(h_i)} = \frac{p(h_i|\bar{d})}{p(h_i)} = 1, \quad (5.8)$$

so the first four terms of the right hand side of Equation 5.7 can be ignored, and the ratio of posterior to prior probabilities for any h_i is simply equal to the value of the focussed correlation expression $For_{h_i}(\bar{a}, \bar{b}, \bar{c}, \bar{d})$.

Using the alternative, less diverse observation set $\{\bar{a}_{40}, \bar{b}_{40}\}$, Myrvold's decomposition of $p(h|o)/p(h)$ becomes:

$$\frac{p(h_i|\bar{a}_{40} \& \bar{b}_{40})}{p(h_i)} = \frac{p(h_i|\bar{a}_{40})}{p(h_i)} \times \frac{p(h_i|\bar{b}_{40})}{p(h_i)} \times For_{h_i}(\bar{a}_{40}, \bar{b}_{40}). \quad (5.9)$$

Again, no individual observation confirms any h_i on its own, *i.e.*, for all i :

$$\frac{p(h_i|\bar{a}_{40})}{p(h_i)} = \frac{p(h_i|\bar{b}_{40})}{p(h_i)} = 1. \quad (5.10)$$

So the ratio of posterior to prior probabilities for any h_i is again equal to the value of the focussed correlation expression, $For_{h_i}(\bar{a}_{40}, \bar{b}_{40})$. Table 5.2 displays the focussed correlation values for both the four-kinds and the two-kinds observation sets, relative to each of hypotheses 1–4. If you refer back to Figure 5.3, you can confirm that the

“full frequencies” posterior probabilities are, for each hypothesis, equal to the prior times the focussed correlation from Table 5.2.

	$\bar{a}, \& \bar{b} \& \bar{c} \& \bar{d}$	$\bar{a}_{40} \& \bar{b}_{40}$
h_1	2.671	1.176
h_2	0.823	1.176
h_3	0.349	1.176
h_4	0.157	0.473

Table 5.2: The table displays eight values for the expression that Wheeler (2009) calls the “focussed correlation” of the observations relative to a hypothesis: that of observation sets $\{\bar{a}, \bar{b}, \bar{c}, \bar{d}\}$ and $\{\bar{a}_{40}, \bar{b}_{40}\}$ relative to hypotheses 1–4.

In short, what the preceding formulas and figures show is that the notion of focussed correlation, already widely discussed in the literature, appears to adequately capture the epistemically relevant differences between the four-kinds and two-kinds data sets—what I have called a difference in diversity. Nonetheless, my analysis of the four-hypothesis coin example goes beyond the existing literature in several ways.

While focussed correlation is a very abstract mathematical feature of theory-observation relations, I have constructed a concrete example that actually generates, from the ground up, the probabilistic relations required for two data sets to differ in focussed correlation relative to a set of hypotheses. My concrete example can also be schematically mapped onto numerous scientific and everyday examples that share the relevant features of the Périer episode discussed in Section 5.1. I have built a bridge between formalism and case study.

Moreover, my example expands the scope of the diversity of evidence literature by demonstrating the application and relevance of both the concept of diversity and

the mathematics of the so-called correlation approach within a type of scenario that falls outside of the topic's usual domain. The standard idea is that the evidential importance of diversity among observations comes into play in scenarios in which the individual observations each already support or confirm the hypothesis in question. Indeed this is an explicit assumption of the formal results derived in Wheeler and Scheines (2011) and Schlosshauer and Wheeler (2011). But this assumption is violated in my example; there, none of the individual observations confirms *any* hypothesis over any other.³ The diversity within a set of observations can be epistemically relevant even where no individual observation is.

Finally, I have applied the formalism of Bayesian epistemology in a non-standard way by identifying the key statistics of the full data set (the difference-between-frequencies statistics), and treating those as the observations—feeding *them* into the Bayesian updating mechanics rather than a more complete description of the data. One attractive feature of this way of doing things is that the statistic of the data that the formalism treats as the observation actually corresponds to what is intuitively meaningful in those observations. In the coin example, it really doesn't matter what the frequencies themselves are (\bar{a} , \bar{b} , and so on); the hypotheses only differ on what they predict about the frequency *differences*. Neglecting the identification of the key data statistics and treating the frequencies themselves as the observations is to throw a lot of irrelevant (or nearly irrelevant) information into the formalism—this does less to illuminate the epistemology of the inference problem, even if the posterior

³This follows directly from the fact that $p(obs.|h_i) = p(obs.|h_j)$ for all $obs. \in \{\bar{a}, \bar{b}, \bar{c}, \bar{a}_{40}, \bar{b}_{40}\}$ and all $i \in \{1, 2, 3, 4\}$. In Bayesian updating, there can be no change in posterior probabilities without likelihood differences between hypotheses.

distribution that comes out the back end of the Bayesian updating mechanics is nearly the same.

An additional consequence of treating the key data statistics as the observations rather than using a more complete description of the data is that this shift can reduce the influence of prior probabilities on the likelihoods $p(obs.|h_i)$ that govern updating. Recall that in the example above, each of hypotheses 1–4 included one or more parameters θ , and that in addition, each hypothesis stipulated a uniform distribution over $[0, 1]$ as the *source* of each parameter θ . I built in those uniform distributions for simplicity, but real scientific hypotheses typically do not come with probability distributions from which their parameters are drawn. In the absence of such a distribution as a proper part of the hypothesis, Bayesian epistemology has the agent's subjective prior distribution over the parameter space do the job. The larger the role of such priors in the updating process, the less objective the inference.

In the example above, when the observation is a difference-between-frequencies statistic (*e.g.*, $|\bar{a} - \bar{b}|$) and the hypothesis is a single-theta hypothesis for that statistic (*e.g.*, h_1), the probability $p(obs.|h)$ is hardly affected by the prior over θ . That is because h_1 predicts a small value for $|\bar{a} - \bar{b}|$ regardless of what value θ takes. But where the observation includes the frequencies themselves (*e.g.*, \bar{a} & \bar{b}) the probability $p(obs.|h)$ now depends very heavily on the prior distribution over θ because $p(obs.|h)$ takes very different values for different values of θ , and that prior distribution determines how those widely differing values are weighted to establish the average value of $p(obs.|h)$ that will appear in the Bayesian updating factor. (But I have not yet systematically investigated the influence of different priors on confirmation in such

scenarios, and I must leave that for another occasion.)

Wrap Up

Most of the concluding has already been done in the closing discussions of the preceding chapters. So here I will give a brief big-picture synopsis, with some reminders about how the various parts connect to one another.

I began by criticizing Sober's account of Darwin's argument for common ancestry. I argued that Sober's reconstruction of how anatomical similarities provide evidence favoring Darwin's common ancestry (CA) hypothesis over a separate ancestry (SA) alternative does not effect the theory comparison that it was designed to make. Instead it compares two logically stronger hypotheses: a specific version of CA against a specific version of SA, where the more specific CA hypothesis is a high-likelihood variant (relative to the other variants of CA) and the more specific SA hypothesis is a low-likelihood variant (relative to the other variants of SA). If this characterization of Sober's likelihood comparison is accurate, then that comparison is not a rational way to assess the evidence for the more general hypotheses CA and SA.

Still in Chapter 2, I went on to also criticize Sober's application of *modus Darwin* to geographical distribution observations. There I argued that while geographical *modus Darwin* does frame a theoretically possible way to use geographical distribu-

tion observations to distinguish between the *general* CA and SA hypotheses, that argument form does not suffice to model and illuminate Darwin’s more complex geographical distribution argument.

From there I proceeded to give my own account of how morphological and geographical distribution observations relate to, and support Darwin’s universal common ancestry hypothesis over some alternatives that posit a lesser degree of common ancestry (Chapter 3). On my account, they do so not independently (as per Sober) but jointly, through “high-level” statistics of mixed (morphological + geographical) observation sets. The relevant “high-level” feature is whether taxonomic and accessibility groupings (trees) for the same set of taxa are congruent.

The congruence between trees inferred separately from different data sets can be viewed as a special case of agreement between estimates of a parameter within a statistical model, where the estimates are based on disjoint data sets. In Chapter 4, I proposed this second description as a formal characterization of the theory-observation relation variously known as the agreement of independent measurements, the overdetermination of constants, and the consilience of inductions. I located the epistemic significance of such agreement in the difference in likelihood, given a small observed value of an agreement statistic, between those hypotheses that posit a single parameter involved in generating both data sets and those that posit separate parameters underlying the two data sets (*i.e.*, between hypotheses like the one-parameter super-model and the two-parameter super-model).

In Chapter 5, I introduced the idea of the *homogeneity assumptions* of a hypothesis in order to describe the important difference between the one-, and two-parameter

supermodels from the previous chapter, and I generalized the importance of that idea through the analysis of an example in which each of *four* competing hypotheses make different homogeneity assumptions. The homogeneity assumptions of those hypotheses determine what observations will count as “different kinds” of observations in that context, and are also responsible for the greater amount of confirmation that accrues to hypotheses that treat more observation kinds as homogeneous with respect to the measured parameter (so long as the various estimates of that parameter do in fact agree—at least approximately).

Appendices

R code for probability calculations

I have presented a lot of probabilities in the preceding chapters, mainly in the charts and graphs. In every case I've stated (sometimes informally) all the relevant details about the probability models that generate those probabilities—in other words I have provided the premises from which the probabilities in the charts and graphs follow. But I have omitted the calculations. I have left those calculations behind the scenes not only because they would be opaque to many readers in my target audience, but also because they are simply not interesting or innovative in any way. I haven't proven any new mathematical results or proposed any novel formalisms. What I claim is innovative in the preceding work is located in how I have *framed* certain inference problems, and how I have *applied* standard formalisms to those problems. Once the inference problem is set up in the right way, getting the actual probabilities, likelihoods, posteriors, etc., is just standard number-crunching. Moreover, little depends on the precise numbers presented in the text. I present concrete mathematical examples in the text in order to illustrate the ideas that I hope I have conveyed more informally and intuitively, and to show that there is some rigor behind what I'm saying.

All of that said, I should still show the work so that it can be checked over if needed. I did most of my probability calculations using the (free) statistical computing environment *R*. (See <http://www.r-project.org/> for more information, or to download.) I display the code for those calculations below.

I am sure that many of the probabilities that I needed can be calculated analytically with little trouble by a competent statistician, but being out of practice with the relevant mathematics, it was often easier for me to approximate the probabilities via simulation. Consequently, much of the code below is devoted to implicitly defining complicated probability distributions in terms of sequential draws from simpler distributions, and then sampling from those over and over to get the long-run frequency of a certain event, which I then treat as the probability of that event.

Figure 2.2

```
#####
## CALCULATE P(MATCH|CA) FOR SINGLE SITE
## TRANSITION PROBABILITIES: P(i to j)=.01, i not= j
#####

n<-100000 ## n = number of runs
t<-seq(5,50,by=5) ## t = time steps vector

state<-vector(length=n)
state.2<-vector(length=n)

compare.temp<-vector(length=n)
compare<-vector(length=length(t))

for(k in t){ ## loop through time steps values
## k = current time steps value
```

```

for(j in 1:n){ ## loop through n trials; FIRST branch
temp<-1
for(i in 1:k){ ## loop through the time steps up to k
draw<-runif(1,0,1)
if(0<draw & draw<.96) temp<-temp else
{if(.96<draw & draw<.97) temp<-1 else
{if(.97<draw & draw<.98) temp<-2 else
{if(.98<draw & draw<.99) temp<-3 else temp<-4}
}
}
}
state[j]<-temp
} ## close n-trials loop; FIRST branch

for(j in 1:n){ ## loop thorough n trials; SECOND branch
temp.2<-1
for(i in 1:k){ ## loop through the time steps up to k
draw<-runif(1,0,1)
if(0<draw & draw<.96) temp.2<-temp.2 else
{if(.96<draw & draw<.97) temp.2<-1 else
{if(.97<draw & draw<.98) temp.2<-2 else
{if(.98<draw & draw<.99) temp.2<-3 else temp.2<-4}
}
}
}
state.2[j]<-temp.2
} ## close n-trials loop; SECOND branch

compare.temp<-(state==state.2) ## which trials ended in a match?
compare[k]<-mean(compare.temp) ## frequency of matching

} ## close loop through k (time steps)

#####
## CALCULATE P(MATCH|CA) AND P(MATCH|SA) FOR 25% AND 50% MATCHING
#####

compare.complete<-c(1,compare[seq(5,50,by=5)]) ## add p=1 for k=0

match<-1
mismatch<-3

```



```

match.2<-2
mismatch.2<-2

CA.likelihood<-((compare.complete)^match*
((1-compare.complete)^mismatch))

SA<-(.25^match*(.75^mismatch))
SA.likelihood<-rep(SA,11) ## p(obs.|SA) is the same for all k

ratio<-CA.likelihood/SA.likelihood ## vector of likelihood ratios

CA.likelihood.2<-((compare.complete)^match.2*
((1-compare.complete)^mismatch.2))

SA.2<-(.25^match.2*(.75^mismatch.2))
SA.likelihood.2<-rep(SA.2,11)

ratio.2<-CA.likelihood.2/SA.likelihood.2

#####
## PLOTS RESULTS
#####
quartz(width=6,height=12)
par(mfrow=c(2,1))

plot(seq(0,50,by=5),ratio, ylim=c(0,1.05), xlab="time steps", ylab="likelihood ratio",
main="(a) Likelihood ratios for 25% of sites matching")
abline(h=1,col="gray50")

plot(seq(0,50,by=5),ratio.2, xlab="time steps", ylab="likelihood ratio",
main="(b) Likelihood ratios for 50% of sites matching")
abline(h=1,col="gray50")

#####

```

Figure 2.3

```
#####
### CALCULATE SA DISTRIBUTION OVER OBSERVED SEPARATION
#####

SA.dist<-vector(length=10)

SA.dist[1]<-.01*10
SA.dist[10]<-.01*2
for(s in 1:8){ ## s is separation
SA.dist[s+1]<-2*(10-s)*.01
}

## plot(seq(0:9),SA.dist)

#####
### CALCULATE CA DISTRIBUTIONS OVER OBSERVED SEPARATION
#####

length=10000
results=vector(length=length)

mu<-.01 ## dispersal factor
t<-10 ## time steps

move=vector(length=t)
direction=vector(length=t)

for (i in 1:length){
r1<-sample(1:10,1) ## choose starting point
r2<-r1 ## same start for two species

move<-sample(0:1,t,prob=c(1-mu,mu),replace=T)
direction<-sample(c(-1,1),t,replace=T)
for (j in 1:t){
r1<-r1+move[j]*direction[j]
if(r1==11)r1<-10 else if(r1==0)r1<-1
}

move<-sample(0:1,t,prob=c(1-mu,mu),replace=T)
```

```

direction<-sample(c(-1,1),20,replace=T)
for (k in 1:t){
move<-sample(0:1,prob=c(1-mu,mu))
direction<-sample(c(-1,1),1)
r2<-r2+move*direction
if(r2==11)r2<-10 else if(r2==0) r2<-1
}

results[i]<-abs(r1-r2)
}

count.a=vector(length=10)
for(k in 1:10){
count.a[k]<-(1/10000)*sum(as.integer(results==(k-1)))
}

#####
### REPEAT THE ABOVE CODE WITH t=50, t=100, AND
### t=300, PUTTING THE RESULTS INTO VECTORS
### count.b, count.c, and count.d RESPECTIVELY
#####

#####
### PLOT RESULTS
#####

quartz(width=10, height=10)
par(mfrow=c(2,2))
proximity<-seq(0,9)

plot(proximity,count.a, pch=1, cex=1.1,lwd=1, ylab="probability",
ylim=c(0,.85),xaxp=c(0,9,9), main="t=10")
points(proximity,SA.dist,cex=1.1, lwd=1, pch=2)
legend(x="topright",c("CA","SA"), cex=1.1,lwd=1, pch=c(1,2),lty=0)

plot(proximity,count.b, pch=1, cex=1.1,lwd=1,ylim=c(0,.85),xaxp=c(0,9,9),
ylab="probability", main="t=50")
points(proximity,SA.dist,cex=1.1, lwd=1, pch=2)
legend(x="topright",c("CA","SA"),cex=1.1, lwd=1, pch=c(1,2),lty=0)

plot(proximity,count.c, pch=1, cex=1.1,lwd=1,ylim=c(0,.85),

```

```

ylab="probability",xaxp=c(0,9,9), main="t=100")
points(proximity,SA.dist,cex=1.1, lwd=1, pch=2)
legend(x="topright",c("CA","SA"),cex=1.1, lwd=1, pch=c(1,2),lty=0)

plot(proximity,count.d, pch=1, cex=1.1,lwd=1,ylim=c(0,.85),
ylab="probability",xaxp=c(0,9,9), main="t=300")
points(proximity,SA.dist,cex=1.1, lwd=1, pch=2)
legend(x="topright",c("CA","SA"),cex=1.1, lwd=1, pch=c(1,2),lty=0)

#####

```

Figure 5.1

```

#####
### CALCULATE DISTRIBUTION OVER FREQ. DIFF. FOR SAME-THETA HYPOTHESES
#####

length=10000 ## number of runs for each k
outcomespace<-c(0:20) ##outcome space for 20 tosses
temp.summands<-vector(length=21)
runs=vector(length=length)
probability.same=vector(length=21)

for(k in 0:20){ ## k is delta heads

for (i in 1:length) {

theta<-runif(1,0,1) ## draw theta from uniform distribution
dist.20.theta<-dbinom(outcomespace,20,theta)

temp.summands<-rep(0,21)
for (j in 1:(21-k)) {
temp.summands[j]<-(dist.20.theta[j]*dist.20.theta[j+k])
}
runs[i]<-2*(sum(temp.summands))

}
probability.same[k+1]<-mean(runs)

```

```

}

#####
### DISTRIBUTION OVER FREQ. DIFF. FOR INDEPENDENT-THETAS
#####

##length=10000
##outcomespace<-c(0:20)
temp.summands.forward<-vector(length=21)
temp.summands.backward<-vector(length=21)
runs=vector(length=length)
probability.independent<-vector(length=21)

for(k in 0:20){ ## k is delta heads

for (i in 1:length) {

theta.a<-runif(1,0,1) ## draw theta.a from uniform distribution
theta.b<-runif(1,0,1) ## draw theta.b from uniform distribution

dist.20.theta.a<-dbinom(outcomespace,20,theta.a)
dist.20.theta.b<-dbinom(outcomespace,20,theta.b)

temp.summands.forward<-rep(0,21)
temp.summands.backward<-rep(0,21)

for (j in 1:(21-k)) {
temp.summands.forward[j]<-(dist.20.theta.a[j]*dist.20.theta.b[j+k])
temp.summands.backward[j]<-(dist.20.theta.b[j]*dist.20.theta.a[j+k])
}
runs[i]<-(sum(temp.summands.forward)+sum(temp.summands.backward))
}
probability.independent[k+1]<-mean(runs)
}

plot(outcomespace, probability.independent)

#####
### PLOT RESULTS
#####

```

```

frequency<-seq(0,1,by=0.05)
plot(frequency, probability.same,
xlab="difference in frequency",ylab="probability")
points(frequency, probability.independent,pch=2)

```

```
#####
```

Figure 5.2

```
#####
### READ IN PROBABILITY DISTRIBUTIONS FROM PREVIOUS CALCULATION
#####
```

```

probability.same<-c(3.893528e-01, 2.951248e-01, 2.078873e-01,
1.354883e-01, 8.254216e-02, 4.565298e-02, 2.320472e-02, 1.075267e-02,
4.415064e-03, 1.689093e-03, 5.748683e-04, 1.656775e-04, 4.348670e-05,
1.000188e-05, 1.867842e-06, 3.078669e-07, 3.997927e-08, 4.153709e-09,
3.065017e-10, 1.484317e-11, 3.491893e-13)

```

```

probability.independent<-c(0.094405052, 0.089884009, 0.086016252,
0.081002561, 0.077254491, 0.072512082, 0.067904410, 0.062711094,
0.058921481, 0.054494088, 0.049658591, 0.045598551, 0.041163191,
0.035927210, 0.031135110, 0.026550621, 0.022160802, 0.017969909,
0.013634217, 0.009337743, 0.004242093)

```

```
#####
### USE THOSE DISTRIBUTIONS TO CALCULATE SERIES OF POSTERIORs
#####
```

```

prior<-c(.25,.25,.25,.25)
likelihoods.1<-c(probability.same[3],probability.same[3],
probability.same[3],probability.independent[3])
p.observation.1<-sum(prior*likelihoods.1)
posterior.1<-prior*likelihoods.1/p.observation.1

```

```

likelihoods.2<-c(probability.same[2],probability.same[2],
probability.independent[2],probability.independent[2])

```

```

p.observation.2<-sum(posterior.1*likelihoods.2)
posterior.2<-posterior.1*likelihoods.2/p.observation.2

likelihoods.3<-c(probability.same[1],probability.independent[1],
probability.independent[1],probability.independent[1])
p.observation.3<-sum(posterior.2*likelihoods.3)
posterior.3<-posterior.2*likelihoods.3/p.observation.3

#####
### PLOT RESULTS
#####

quartz(width=10,height=12)
par(mfrow=c(2,2))

barplot(prior,ylim=c(0,.75), xlab="hypotheses", ylab="probability",
names.arg=c(1,2,3,4), main="Prior probabilities for hypotheses 1-4")

barplot(posterior.1,ylim=c(0,.75), xlab="hypotheses",
ylab="probability", names.arg=c(1,2,3,4),
main = expression(paste("Posterior probabilities after observing ",
"|",bar(a)-bar(b),"|"))) )

barplot(posterior.2,ylim=c(0,.75), xlab="hypotheses",
ylab="probability", names.arg=c(1,2,3,4),
main = expression(paste( , " after subsequently observing ",
"|",bar(a)-bar(c),"|"))) )

barplot(posterior.3,ylim=c(0,.75), xlab="hypotheses",
ylab="probability", names.arg=c(1,2,3,4),
main = expression(paste( , " and then after observing ",
"|",bar(a)-bar(d),"|"))) )

#####

Figure 5.3

#####

```

```

### READ IN POSTERIOR FROM PREVIOUS CALCULATION
#####

prior<-c(.25,.25,.25,.25)
posterior.1<-c(0.2929318, 0.2929318, 0.2929318, 0.1212046)
posterior.2<-c(0.4114245, 0.4114245, 0.1253046, 0.0518465)
posterior.3<-c(0.74246336, 0.18002257, 0.05482816, 0.02268591)

#####
### CALCULATE PROBABILITY OF OBSERVING EXACTLY a, b, c, and d
### HEADS IN DATA SETS A, B, C, AND D RESPECTIVELY, GIVEN H1-H4
#####

a<-8 ## i.e. frequency 0.4
b<-10 ## 0.5
c<-7 ## 0.35
d<-8 ## 0.4

length=100000 ## number of runs in simulation
runs=vector(length=length)

for (i in 1:length) { ##### HYPOTHESIS 1

theta.ABCD<-runif(1,0,1) ## draw theta from uniform distribution

heads.A<-dbinom(a,20,theta.ABCD) ## prob. of a heads in twenty tosses
heads.B<-dbinom(b,20,theta.ABCD) ## prob. of b heads in twenty tosses
heads.C<-dbinom(c,20,theta.ABCD) ## prob. of c heads in twenty tosses
heads.D<-dbinom(d,20,theta.ABCD) ## prob. of d heads in twenty tosses

runs[i]<-heads.A*heads.B*heads.C*heads.D
}

likelihood.hyp.1<-mean(runs)

for (i in 1:length) { ##### HYPOTHESIS 2

theta.ABC<-runif(1,0,1) ## draw theta from uniform distribution
theta.D<-runif(1,0,1) ## draw theta from uniform distribution

heads.A<-dbinom(a,20,theta.ABC)

```



```
heads.B<-dbinom(b,20,theta.ABC)
heads.C<-dbinom(c,20,theta.ABC)
heads.D<-dbinom(d,20,theta.D)

runs[i]<-heads.A*heads.B*heads.C*heads.D
}

likelihood.hyp.2<-mean(runs)

for (i in 1:length) { ##### HYPOTHESIS 3

theta.AB<-runif(1,0,1)
theta.C<-runif(1,0,1)
theta.D<-runif(1,0,1)

heads.A<-dbinom(a,20,theta.AB)
heads.B<-dbinom(b,20,theta.AB)
heads.C<-dbinom(c,20,theta.C)
heads.D<-dbinom(d,20,theta.D)

runs[i]<-heads.A*heads.B*heads.C*heads.D
}

likelihood.hyp.3<-mean(runs)

for (i in 1:length) { ##### HYPOTHESIS 4

theta.A<-runif(1,0,1)
theta.B<-runif(1,0,1)
theta.C<-runif(1,0,1)
theta.D<-runif(1,0,1)

heads.A<-dbinom(a,20,theta.A)
heads.B<-dbinom(b,20,theta.B)
heads.C<-dbinom(c,20,theta.C)
heads.D<-dbinom(d,20,theta.D)

runs[i]<-heads.A*heads.B*heads.C*heads.D
}

likelihood.hyp.4<-mean(runs)
```

```
#####
### USE THOSE LIKELIHOODS TO CALCULATE POSTERIORs
### AFTER OBSERVING NUMBERS OF HEADS: a, b, c, and d
#####

prior<-c(.25,.25,.25,.25)
likelihoods.fc<-c(likelihood.hyp.1,likelihood.hyp.2,
likelihood.hyp.3,likelihood.hyp.4)
p.observation.fc<-sum(.25*likelihoods)
posterior.fc<-prior*likelihoods.fc/p.observation.fc

#####
### CALCULATE LIKELIHOODS FOR OBSERVING |a40-b40|
#####

k<-4 ## k is delta heads
length=100000 ## number of runs in simulation

outcomespace<-c(0:40) ##outcome space for 40 tosses
temp.summands<-vector(length=41)
temp.summands.backward<-vector(length=41)
runs=vector(length=length)

for (i in 1:length) { ## same-theta hypotheses

theta.AB<-runif(1,0,1)
dist.40.theta.AB<-dbinom(outcomespace,40,theta.AB)
temp.summands<-rep(0,41)
for (j in 1:(41-k)) {
temp.summands[j]<-(dist.40.theta.AB[j]*dist.40.theta.AB[j+k])
runs[i]<-2*(sum(temp.summands))
}
}
likelihood40.same.theta<-mean(runs)

for (i in 1:length) { ## independent-theta hypotheses

theta.A<-runif(1,0,1)
```

```

theta.B<-runif(1,0,1)
dist.40.theta.A<-dbinom(outcomespace,40,theta.A)
dist.40.theta.B<-dbinom(outcomespace,40,theta.B)

temp.summands<-rep(0,41)
for (j in 1:(41-k)) {
temp.summands[j]<-(dist.40.theta.A[j]*dist.40.theta.B[j+k])
temp.summands.backward[j]<-(dist.40.theta.B[j]*dist.40.theta.A[j+k])

runs[i]<-sum(temp.summands)+sum(temp.summands.backward)
}
}
likelihood40.ind.theta<-mean(runs)

likelihoods40<-c(likelihood40.same.theta, likelihood40.same.theta,
likelihood40.same.theta, likelihood40.ind.theta)

#####
### USE THOSE LIKELIHOODS TO CALCULATE POSTERIORS
#####

p.observation40<-sum(prior*likelihoods40)
posterior40<-prior*likelihoods40/p.observation40

#####
### CALCULATE PROBABILITY OF OBSERVING a40 and b40, GIVEN H1-H4
#####

a<-16 ## i.e., frequency=0.4
b<-20 ## i.e., frequency=0.5

length=100000 ## number of runs in simulation
runs=vector(length=length)

for (i in 1:length) { ## for same theta hypotheses

theta.A<-runif(1,0,1)
heads.A<-dbinom(a,40,theta.A)
heads.B<-dbinom(b,40,theta.A)

runs[i]<-heads.A*heads.B

```

```

}
likelihood40.hyp.1<-mean(runs)
likelihood40.hyp.2<-mean(runs)
likelihood40.hyp.3<-mean(runs)

for (i in 1:length) { ## for independent theta hypotheses

theta.A<-runif(1,0,1)
theta.B<-runif(1,0,1)
heads.A<-dbinom(a,40,theta.A)
heads.B<-dbinom(b,40,theta.B)

runs[i]<-heads.A*heads.B
}

likelihood40.hyp.4<-mean(runs)

likelihoods40.fc<-c(likelihood40.hyp.1,likelihood40.hyp.2,
likelihood40.hyp.3, likelihood40.hyp.4)

#####
### USE THOSE LIKELIHOODS TO CALCULATE POSTERIORS
### AFTER OBSERVING a40 AND b40
#####

p.observation40.fc<-sum(prior*likelihoods40.fc)
posterior40.fc<-prior*likelihoods40.fc/p.observation40.fc

#####
### DISPLAY RESULTS
#####

prior.matrix<-matrix(data=c(prior,prior),nrow=2, byrow=TRUE)
post.matrix<-matrix(data=c(posterior.3,posterior.fc),nrow=2, byrow=TRUE)
post.matrix.40<-matrix(data=c(posterior40,posterior40.fc),nrow=2, byrow=TRUE)

quartz(width=10,height=12)
par(mfrow=c(2,2))

barplot(prior.matrix,ylim=c(0,.7), col=c("gray35","gray75"), beside=TRUE,
xlab="hypotheses", ylab="probability", names.arg=c(1,2,3,4), main="Priors")

```

```
barplot(post.matrix,ylim=c(0,.7), col=c("gray35","gray75"),
beside=TRUE, xlab="hypotheses", ylab="probability",
names.arg=c(1,2,3,4), main = "Posteriors (four \"kinds\)\"",
legend=c("frequency differences","full frequencies") )
```

```
barplot(prior.matrix,ylim=c(0,.7), col=c("gray35","gray75"), beside=TRUE,
xlab="hypotheses", ylab="probability", names.arg=c(1,2,3,4), main="Priors")
```

```
barplot(post.matrix.40,ylim=c(0,.7), col=c("gray35","gray75"),
beside=TRUE, xlab="hypotheses", ylab="probability",
names.arg=c(1,2,3,4), main = "Posteriors (two \"kinds\)\"",
legend=c("frequency differences","full frequencies") )
```

```
#####
```

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