Essays on the Human Capital, Worker Mobility, and Labor Markets

by

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To my PhD cohort.

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ABSTRACT

This dissertation includes two essays on worker mobility in the labor market and the role of human capital. The first chapter studies how workers sort into different positions within and across firms. Firms use internal promotion and external hires to fill managerial positions. However, the role of the internal labor market is typically ignored in the workhorse theory of worker mobility. To quantify the aggregate effects of internal promotion on labor market efficiency, we develop a novel job search model, which gives rise to an equilibrium sorting pattern of heterogeneous workers, firms, and jobs. We calibrate the model using administrative data from Denmark. The estimates reveal that external labor market friction and general human capital are crucial for understanding firm hiring strategies. The counterfactual experiment shows that banning internal promotion decreases aggregate production by about 6.8%, although promotion corrects the overinvestment of firm-specific human capital. Moreover, the counterfactual results imply that the general and firm-specific human capital investment would be 50% lower when shutting down promotion, which suggests the importance of promotion margin in understanding why firms provide training to workers.

The second chapter examines the impact of minimum wage policy on firm behavior, with a particular focus on capital investment and employment decisions. While extensive research has explored how minimum wages affect employment, less attention has been paid to their effects on firm-level performance. I study the 2004 minimum wage reform in China using detailed manufacturing firm-level data and a difference-in-differences identification strategy. The empirical results indicate that higher minimum wages lead to increased capital investment and average wages, accompanied by a reduction in employment. The effects are more pronounced among low-wage firms, where both average wages and capital stock rose more rapidly, resulting in a 0.95 percentage point increase in capital per worker for every 1 percent rise in the minimum wage. To assess the welfare implications, I develop and calibrate a labor search model incorporating a holdup issue: when firms invest more in capital, they owe their workers higher bargaining wages. The model replicates key features of China's capital and labor markets prior to the reform and predicts that minimum wage hikes improve capital allocation efficiency among low-wage firms. A counterfactual simulation suggests that a 10 percent reduction in the real interest rate would yield comparable gains in capital and wage levels as the 2004 reform, but with a lower unemployment rate.

1 INTERNAL MEETS EXTERNAL: THE ROLE OF PROMOTION IN THE LABOR MARKET

1.1 Introduction

Labor reallocation is a fundamental aspect of labor market dynamics (Davis et al. (2006); Lentz and Mortensen (2010); Moscarini and Postel-Vinay (2018)). While substantial research has examined the effects of mobility across employers, often prompted by layoffs or quits, less attention has been given to worker reallocation within the same firm. The within-firm job mobility, though seldom explored in macroeconomic contexts, may contribute significantly to productivity growth and labor reallocation—effects that are potentially as impactful as those arising from inter-firm transitions.[1]

A primary reason within-firm job mobility is often overlooked in macro-labor models is the complexity involved in modeling both internal and external mobility under labor market frictions. Established models of employment dynamics (e.g., Burdett and Mortensen (1998), Postel-Vinay and Robin (2002)) emphasize external reallocation due to computational constraints and data limitations. Modeling intra-firm mobility in such environments is challenging, as it requires tracking individual workers within firms if workers are heterogeneous, leading to computational issues such as the curse of dimensionality. Consequently, a number of significant questions remain unclear: What determines firms' internal promotion decisions? What trade-offs do firms consider when choosing between external managerial hiring and promoting existing ones? Would the economy operate more efficiently with reduced reliance on internal labor markets?

In this paper, we investigate firms' managerial hiring strategy and the role of internal promotion on aggregate output and labor market efficiency. To do so, we build a rich and novel labor search model with two-sided heterogeneity and

[1]To our knowledge, Fujita, Moscarini, and Postel-Vinay (2024) is the first to acknowledge the potentially important impact of within-employer job changes for macro-labor outcomes.

endogenous human capital investment that accounts for on-the-job search and job mobility within firms to shed new light on the questions above. We specifically focus on vertical mobility within the firm, defined as promotion. We calibrate our model to Danish matched employer-employee data from 2010 to 2018. Combining the model and the administrative data allows us to quantify the effects of external friction and internal friction, modeled as general and firm-specific human capital investment, via managerial hiring strategies on labor reallocation and aggregate production.

We first turn to our main contribution. We construct a novel labor search model that extends the work of Herkenhoff, Lise, Menzio, and Phillips (2024) (henceforth HLMP) by incorporating organizational hierarchy. In the model, a firm is a collection of independent teams, each consisting of two positions: a worker and a manager. Job seekers search both off- and on-the-job. The key innovation of the model is that firms can fill managerial positions either through internal promotion or external hiring. To address external labor market frictions, firms invest in two types of training: general human capital training and firm-specific human capital training of their employees, enhancing their readiness for future promotions. Consequently, firms face the trade-off between the search friction gap between the external managers and the worker market and the quality of the available candidate pool. If a firm opts for internal promotion (i.e., promoting an employee to a managerial role), it must address the vacancy created in the low-hierarchy workforce. Conversely, they are subjected to search frictions if they recruit externally for the managerial position. One key economic mechanism of the model is that the interaction between human capital and external labor market friction provides a novel insight into internal promotion.

How does the internal promotion margin affect the labor market efficiency? We show there are three potential channels. The first and second sources of externalities are through human capital investment. Firms may overinvest in workers' firm-specific human capital to retain potential managers. The inefficiency is more salient among low-productivity firms since they are at the low rung in the external labor market, and the employees in those firms are most likely to reallocate to a better job. To put it another way, internal promotion amplifies the inefficiency resulting from firm-specific human capital. On the other side of the coin, promotion partially solves the underinvestment of general human capital among low-productivity firms. Consequently, the relative importance of firm-specific human capital and general human capital to low-productivity firms determines the size of inefficiency and output. The last channel is through the sorting pattern between workers, managers, and firms depending on the substitutability among the three parties.

To understand firms' hiring decisions and quantify the impacts of internal promotion on match firm hiring strategies, worker allocation, and aggregate productivity, we calibrate the model to the Danish labor market using several administrative datasets on firms and their employees. The detailed matched employer-employee data provides a straightforward measurement of within- and between-firm job mobility. The goal of calibration are two folds: (1) to uncover the elasticity of substitution between workers, managers, and firms, and (2) to measure the size of internal and external labor market friction in the labor markets for workers and managers, and (3) to separately identify the relative importance of general and firm-specific human capital in firm-sponsored training process. To be consistent with the model, we map the occupation codes to two levels of the job hierarchy: (production) workers and managers. Key empirical moments include the quarterly transition rates between two jobs conditional on firm change, the share of managers moving to another managerial jobs conditional on employment-to-employment (E2E) transition followed by internal promotion, wage growth upon promotion within the firm, and distribution of internal promotion share over the firm productivity distribution.

To assess firms' promotion strategies, we measure the ratio of internally promoted managers relative to all newly hired managers by a firm from 2010 to 2018. The non-degeneration distribution of the metric shows that firms conduct heterogeneous strategies even within the industry. Specifically, approximately one-third of managers are appointed through internal promotion. We further explore the connection between these variations in strategy and their firm-level characteristics. Ordinary least square estimates indicate that firm productivity plays a crucial role in the probability of promotion, even conditional on firm size. Specifically, higher productivity firms are more inclined to promote managers internally. This pattern remains true within and between industries, as well as after controlling for additional firm attributes and the characteristics of the labor market in which the firm operates.

We present two findings. First, we employ the calibrated model to analyze key drivers behind firms' managerial hiring strategies. To do so, we vary the internal and external labor frictions through three counterfactual experiments. In the first and second experiments, we reduce the specific and general training costs by 10%. This experiment leads to an increase by 3% of the average internal promotion share. In the second and third experiment, we decrease the matching efficiency of the manager market to be 10% lower than the baseline value. We find that the average internal promotion share is 7% higher. These findings underscore that labor market frictions and general human capital accumulation significantly impact firms' managerial hiring choices. Additionally, we show that although labor market friction has a larger impact on variations in hiring strategy across firms, low-productivity firms exhibit twice the sensitivity to human capital investments compared to high-productivity firms. These results suggest substantial welfare changes could be made by optimizing general and firm-specific human capital strategies, particularly within low-productivity firms.

Second, we use the calibrated model to quantitatively assess the role of promotion in aggregate output and welfare. Specifically, we consider a hypothetical economy in which firms are restricted from filling managerial positions through internal promotions but can still invest in human capital. In this scenario, external hiring becomes the sole means of filling managerial roles, as commonly assumed in the literature. Our findings indicate that aggregate production is approximately 6.8% lower than in the baseline model. This suggests that internal promotion has a significant impact on the economy. The outcome is expected given that models with internal labor mobility typically offer greater flexibility. We decompose the result into direct and indirect effects to understand the economic mechanism.

The direct effect operates through the human capital accumulation channel. Recall that the main function of firm-specific training is to retain the worker. Therefore, when the option to hire internally is excluded, the incentive for training is dampened. Consequently, aggregate production decreases due to reduced accumulation of both general and firm-specific human capital.

The indirect effect, our second channel, function through the sorting dynamics between workers and firms. As highlighted in Lentz and Roys (2015) on firm-specific training, low-productivity firms may overinvest in employees' firm-specific human capital but underinvest in their general human capital in conjunction with on-the-job search under sequential wage-setting à la Postel-Vinay and Robin (2002). Our findings align with this perspective, emphasizing that internal promotion amplifies these aggregate effects by offering firms a mechanism to retain potential managerial candidates, thereby increasing the option value of firm-specific training. In this first exercise, we demonstrate that general training instead of firm-specific human capital accounts for the main reason of internal promotion. Therefore, the negative effects of less general human capital investment dominates the positive effects of less firm-specific human capital investment, which leads to a drop in aggregate output.

1.2 Related Literature and Contributions

This paper makes several contributions to the literature. First, the paper makes a theoretical contribution to the equilibrium theory of frictional labor markets. The novel sorting pattern in the equilibrium among three parties - workers, firms, and jobs - arises from the interaction between across-firm mobility, as in the standard labor search models crucially relying on the modularity of match surplus (Shimer and Smith (2000); Bagger and Lentz (2019)), and an operative promotion margin. The within-firm mobility in our model is built on Kiyotaki and Lagos (2007), Andersen and Svarer (2007), and more recently Hong (2022), Freund (2022) and Herkenhoff et al. (2024). They provide an innovative way of slicing a firm into worker-worker matches by focusing on teamwork. Differing from the literature, a production team in our model incorporates the minimal organizational structure with two hierarchies as in the literature of Personnel economics (Lazear (1995); Lazear and Shaw (2007); Caliendo et al. (2015)) instead of symmetric roles as coworkers.

Second, the paper contributes to the understanding of internal training. Unlike the studies of promotion in Personnel economics, we focus on the results of promotion prospects on workers instead of incentive motivation and wage compensation within firms as in Tournament theory (Lazear and Rosen (1981); Green and Stokey (1983)). Two closely related literatures that follow a similar thought pipeline are Friedrich (2016) and Pastorino (2024). Both papers emphasize that internal promotion reinforces positive worker-firm sorting patterns under information friction. Friedrich (2016) provides empirical evidence that promotion amplifies positive sorting between entry-level workers and firms and thus wage inequality, while Pastorino (2024) further shows that returns to learning resulting from the dynamics of promotions account for approximately one-quarter of the cumulative wage growth and dispersion over the first 7 years at the firm.

Third, the paper contributes to the understanding and estimating of firm-specific and general human capital on aggregate welfare through a novel channel: promotion within the firm in three dimensions. Motivated by firm-sponsored general training, Acemoglu and Pischke (1999) is the first paper studying the general and firm-specific human capital in the frictional labor market. Lentz and Roys (2015) generalize the Acemoglu and Pischke (1999) argument in a partial equilibrium and find that in the general equilibrium, low-productivity firms overinvestment of firm-specific human capital and underinvestment general human capital if firms are able to extract rents from other firms. In our model, we provide a novel insight into the role of firm-specific and general human capital on aggregate welfare through internal promotion that the existence of promotion amplifies the underinvestment of general human capital but corrects the overinvestment of firm-specific human capital among low-productivity firms. Accordingly, eliminating internal promotion may be welfare-improving if firms provide more firm-specific training than general training.

Fourth, by distinguishing the employment trajectories of two types of managers, the paper also contributes to the measures of firm-specific and general human capital.[2] There has been a debate over the quantitative importance of the returns to the two types of human capital over the life cycle. Old measures (Altonji and Shakotko (1987); Topel (1991); Altonji and Williams (2005)) heavily rely on the Mincer (1958) model to estimate the returns to tenure as a proxy for firm-specific human capital by leveraging the life-cycle wage profile of workers. This method is vulnerable to measurement errors in the wage data and tenure. On the contrary, our method utilizes high-frequency employment history records to be free from the concerns above.

Lastly, the paper makes a quantitative contribution to the aggregate effects of the promotion channel on labor market outcomes.

Outline. The rest of this paper is organized as follows. Section 3 provides an overview of the data facts regarding firms' managerial hiring strategies. Section 4 presents the structural model of managerial hiring. Section 5 details the calibration. Section 6 discusses the analysis of the calibrated model and role of internal promotion on aggregate output and welfare. Section 6 concludes.

1.3 Institutional Background and Data

Data and Sample Selection

Data sources. We use Danish administrative datasets from 2008 to 2019. We combine several administrative registers constructed and maintained by Statistics Denmark. The backbone of the data is a comprehensive matched employer-employee (MEE) set on individual labor market histories, the BFL dataset (*Beskæftigelse for lønmodtagere*). BFL records all employment relationships with start and end dates

[2] There is a long-lasting discussion on the return to different types of specific human capital, i.e, firm-specific human capital, occupation/career-specific human capital, industry-specific human capital. Kambourov and Manovskii (2009) extends the framework of Parent (2000) to study the return to occupation-specific human capital. Carrington and Zaman (1994), Neal (1995) and Kim (1998) argue that that industry-specific human capital plays a crucial role in understanding the wage dynamics and the wage losses for displaced workers. Sanders and Taber (2012) provides a comprehensive survey on the role of heterogeneous human capital on life-cycle wage growth.

for each employment spell, which are recorded at a daily frequency. Within each employment spell, it provides the critical information in our setting - occupation codes, which is absent in the U.S MEE dataset, e.g., Longitudinal Employer-Household Dynamics (LEHD). This dataset records all individuals who have worked in Denmark since 2008 and contains information about earnings and hours worked, all recorded monthly.

We collect background information on workers such as age, gender, labor market experience from IDAN registry. We supplement the worker-level data with workers' highest education attainment, including graduation date from the highest education attainment from the UDDA registry.

Finally, for firm-level data, we use the FIRM dataset, an annual dataset that reports, among others information, data on sales, value-added, and firm size, gross salaries and profits.[3] The unit of observation is the firm rather than the establishment. For the rest of the paper, we interchange firm and establishment since we do not have information about establishments.

We merge all datasets mentioned above via unique worker identifiers and firm identifiers. The comprehensive data provides us with a straightforward way to measure worker mobility and the process of climbing up the job hierarchy within the firm. The details of data construction are described below.

Occupational Classification. As we mentioned above, one of the advantages of using Danish data is the availability of detailed occupational codes. Firms, not individual workers, report occupational codes. After 2010, Danish occupations were categorized using the Danish International Standard Classification of Occupations, or DISCO-08 codes.[4] DISCO-08 is a six-digit, five-level classification used to classify and aggregate information on work functions from various statistical surveys.[5] DISCO-08 makes it possible to compare people with the same job function,

[3]Firm size measures the number of full-time workers. All the firm-level balance sheet variables are reported at the end of November of each year.

[4]Before 2010, Danish occupations were categorized using DISCO-88 codes. To provide a consistent occupation classification before and after 2010, we adopt a crosswalk used in Humlum (2022). See `https://www.andershumlum.com/codes` for details.

[5]We provide a supplementary description on DISCO-08 in the *Appendix A.1*. `https://sites.`

regardless of the employee's formal title and education. It is crucial in our setting to understand that promotion is not just about wage growth. Instead, employees in different positions have distinct job responsibilities and require different skills within a team. However, since employees do not report the occupation codes, a reasonable concern is whether they are accurate and updated frequently. To reduce the measurement error, we impute missing occupation codes[6] and correct highly-misspecified occupation codes under some circumstances. The details are illustrated in the *Appendix A.1*.

Employer-to-employer Transition. The spell data (BFL) documents detailed coverage of the working history of the entire working population, which provides us a straightforward way to measure employer-to-employer (E2E)[7] mobility without concerns about the potential time-aggregation bias, compared to other data sources commonly available at the monthly frequency or quarterly frequency, such as the Current Population Survey (CPS) data or the administrative Longitudinal Employer-Household Dynamics (LEHD) matched employer-employee dataset.[8]

Job Hierarchy. Constructing an economically meaningful and consistent job hierarchy within and across firms is challenging for two potential issues. The first concern is whether firms share a common understanding of the firm hierarchy,

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`google.com/site/econrunevejlin/danish-data-1` provides a comprehensive understanding of the history of occupation codes in Denmark.

[6]There are approximately 17.6% (18.9%) occupation codes missing in terms of worker-firm-spell observations (unique workers) during the sample period. Missing occupation codes could be due to several reasons. First, reporting DISCO codes is not mandatory for companies in the Agriculture and Fishery industry and for companies with less than ten employees. Secondly, some companies only report annually. As BFL is a short-term statistics, codes of such annually reporting companies will sometimes not come early enough to make it to the BFL register. To further explore which firms do not report occupation codes, we compute the share of workers without an occupation code within each firm across distinct firm characteristics. We find that firms with employment less than 20 see a larger share of missing occupations than their counterpart. The details are illustrated in the *Appendix A.1*.

[7]We define E2E as worker mobility across firms with unemployment gap less than 14 days as in the standard search literature.

[8]The CPS data is designed to measure month-to-month transitions in workers' labor force status. Nevertheless, the potential time aggregation bias is not guaranteed to be completely eliminated. Shimer (2012) and Mukoyama (2014) propose different methods to correct for time aggregation bias in employer-to-employer transition rates. Bertheau and Vejlin (2022) quantify the time aggregation bias using Danish administrative data.

especially across the industry. For example, firms in the financial sector only hire a few manual workers and even low-skill workers. On average, the job titles in these firms are higher than those of other firms. Thus, a granular categorization of job hierarchy might cause more measurement errors. Given the structure of our model, we assume that there are two hierarchies within each firm: managerial jobs and non-managerial jobs. Promotion, both internally and externally, is defined as movements from non-managerial jobs to managerial jobs. The second issue is whether the internal promotion is more relevant to some workers than others. Since promotion decisions are associated with optimal training investments and effort input (Pergamit and Veum (1999), Melero (2010)), firms may only invest in potential managers given the costly training process. To deal with this, we follow the method proposed by Caliendo et al. (2015), which infers firm hierarchy through worker mobility upon their current one-digit occupation codes across two employment spells. First, we can see that workers with manager jobs (i.e., ISCO code 1) are more likely to stay in their job or change to another manager job.We define them as top managers (layer 1). Secondly, we find that there are dominating transitions between managers and other positions. Specifically, workers with occupation codes 2, 3, and 4 are more likely to be promoted than workers in other groups.[9] Hence, occupation codes 2, 3, and 4 are also categorized as white-collar workers. The other occupation codes (5 to 9) are categorized as blue-collar workers with few transitions to managerial jobs.

To test the validity of our measurement, we examine the mobility of workers in two aspects: (1) career trajectory by tenure within the firm in Figure A.4; (2) occupation distribution of workers' first jobs in Figure A.5. Lastly, to show that this classification of employees into layers is a meaningful economic classification, we compute the wage distribution of workers in different layers in Table 1.1.

Sample Selection. Here, we describe our sample cuts. The main sample consists of all private-sector firms. We focus on the prime working-age workers aged 25 to 60. We exclude self-employment, public sector worker-firm spells, and

[9] ISCO code 2 groups professionals. ISCO group 3 groups technicians and associate professionals. ISCO code 4 groups clerical support workers.

workers with missing occupational codes. We drop workers with multiple jobs holding at different firms at any given period in our sample period. The detailed sample selection steps are reported in *Appendix A.1*.

To have a meaningful notion of internal labor markets, we focus on white-collar workers (ISCO group 1-4) since promotion is most relevant for them.[10] Lastly, we focus on firms with at least 20 employees and at least five managerial hires over the sample period.[11] Furthermore, we exclude firms in some industries because firm-level accounting data is unavailable or most firms are not for-profit firms.[12] The final sample consists of 5,133,911 observations with 5,432 unique firms and 756,468 unique workers from 2008 to 2019.

Table 1.1 represents the summary statistics of workers. As we can see, non-managerial employees tend to be younger and have less working experience than managers, implying that human capital accumulation is an important process for promotion. Moreover, male employees dominate managerial jobs.[13] Table 1.2 represents the summary statistics of firms by industry. As demonstrated from the table, there are some differences across industries. For example, firms in the construction sector, on average, are less likely to promote workers to managerial positions. Meanwhile, they are also younger and smaller in firm size, managers get paid less, and they supervise fewer workers compared with firms in other industries.

[10]We show the transition matrix of all workers who get promoted either within firms or between firms conditional on their current hierarchy in *Appendix A.1*. Table A.2, A.3, and A.4 measure the mobility pattern across hierarchies both within- and across-firm.

[11]This threshold reduces small sample bias in the subsequent analysis of managerial hiring within firms. As noted above, reporting occupation codes for firms below 10 employees is not mandatory. This size cutoff aims at selecting firms with a meaningful role in internal labor markets as is common in managerial literature, see, e.g., Bloom et al. (2012) while capturing a large part of the Danish labor markets.

[12]Specifically, we exclude firms in the following industries; agriculture , mining , utilities , finance , education , hospitals , non-profit organizations and public administrations.

[13]This is an interesting data fact worth further exploration. However, it is beyond the focus of this paper.

Institutional Background

Several features of the Danish labor market and detailed databases make it a good setting for studying internal and external mobility. First, Denmark is known for its "flexicurity" model of labor market policies, which is characterized by high employer-to-employer and occupational mobility compared with other European countries and comparable to the US Kreiner and Svarer (2022).

Hiring an employee both internally and externally shares similar processes, and the human resource specialist takes steps to create a hiring process. Candidates look for available jobs and then apply for the positions.[14] Job candidates are interviewed before employers make decisions on whom to hire.

Empirical Evidence

In what follows, we aim to measure firms' managerial hiring strategies and how common they are among firms. We construct the following variable to measure the frequency of internal promotion of each firm by

$$\text{Internal Promotion Share} = \frac{\text{Number of managers promoted internally}}{\text{Total number of managers hired between 2010 to 2018}}. \tag{1.1}$$

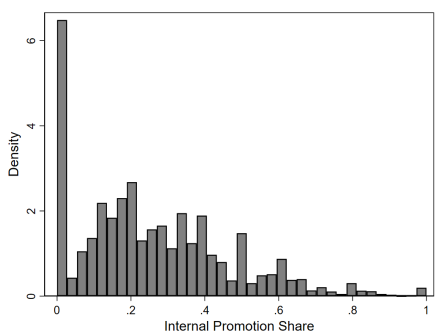
The Internal Promotion Share represents the ratio of managers promoted internally from 2010 to 2018 of each firm. Compared with the alternative measurement of the promotion rate of workers, the variable is independent of the size of low-level workers. We show two data facts as below.

Data Fact 1: Firm conduct heterogeneous strategies

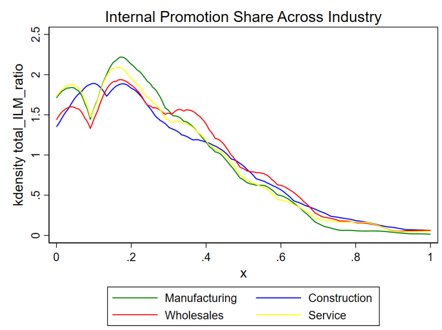
The left panel of Figure 1.1 illustrates the distribution of internal promotion share (IPS) across all firms. It is informative of firms' hiring strategy in two aspects. First, there is a substantial heterogeneity in the use of internal promotion. The mean of the share of internal promotion is 0.3, and the median is 0.25. In particular, there is

[14] According to the Labour and Employment Law of Denmark, firms do not have to post a vacancy before hiring regardless of internal hiring or external hiring and corporate types.

a mass point at 0, implying that many firms solely poach managers externally.[15] To control for the heterogeneity practice across industries, we plot the within-industry IPS in the right panel of Figure 1.1. It is evident that internal promotion is ubiquitous even within industry with a standard deviation of 2.3. Moreover, most of the variation happens within industry instead of across industry. Second, a significant share of firms conduct mixed hiring strategies, suggesting that managerial hiring strategy cannot be fully explained by the fixed characteristics of firms.



(a) Raw Data



(b) Within Industry

Figure 1.1: Internal Promotion Share Distribution

Note: Figure plots the distribution of the internal promotion share (IPS), which is defined as the percentage of managers hired from internal promotion, from 2010 of 2018 of each firm. Panel (a) shows the raw distribution of IPS across industries and Panel (b) shows the distribution of IPS within industry.

Data Fact 2: High-productivity firms use more internal promotion

To explore which firm productivity are related to the observed variation in the usage of internal promotion, we begin by estimating the following linear regression model:

$$IPS_{jt} = \beta z_{jt} + \beta_1 X_{jt-1} + \eta_j + \zeta_t + \epsilon_{jt} \tag{1.2}$$

where IPS_{jt} represents the internal promotion share of firm j, z_{jt} is measured by

[15]We delve into the firms with zero internal promotion share in *Appendix A.1*. We show that a significant share of firms that strictly prefer external hiring hire less than ten managers during the sample period.

log of value added per worker in year t, i.e.,

$$z_{jt} = \frac{\text{Value added}}{\text{Firm size}}.$$

To control for the lagged firm characteristics, we introduce X_{jt-1} to the right-hand side of the Equation 1.2 by including average manager size, worker size and sales from previous year. We find $\beta = +0.0071^{***}$, which implies that high-productivity firms are more likely to promote workers than low-productivity firms.

Data Fact 3: Limited slot constraint and vacancy chain

A crucial question regarding mangerial strategy is that how the vacancies are created in the production process. Specifically, are managerial positions created flexiblly enough following the business performing cycle or emerged due to manager leaving as in Qiu (2022)? Bianchi et al. (2023) find that there is evidence supporting slot constraint that a managerial vacancy is posted when the old manager leaves. To test the slot constraint hypothesis in the Danish labor market, we run the following event study to study the relationship between worker promotion and manager leaving.

$$y_{jt} = \sum_{s=-3, s\neq -1}^{s=4} \beta_s D_{j,t+s} + \Theta X_{jt} + \eta_t + \delta_j + \epsilon_{jt}, \tag{1.3}$$

where the main dependent variable on the left-hand side is an indicator of whether this is at least one worker get promoted within the firm j next month t, i.e $y_{jt} = 1$ if internal promotion took place. The main event here is whether the firm j experienced manager leaving in month $t-1$, that is, $D_{j,t-1} = 1$ if at least one manager left the firm j in month t otherwise 0. Equation 1.3 includes a vector of month-year fixed effects η_t to control for time-invariant unobserved differences in firm managerial hiring behavior. It also captures the seasonal variation in labor market demand (i.e., ridership typically peaks in the last two months of the year). We also include a vector of firm fixed effects $\delta_j$firm-invariant unobserved differences. To account for firm-level growth trends, we include lagged the firm-level characteristics X_{jt}, which includes the ratio of managers to production workers, sales per employee and value-added per employee.

Before the showing the results, there are several caveats worth noticing. First, since firms may have different hiring strategies if a manager leaves due to different reasons. For example, a firm tend to hire a replacement if the old manager was poached by other firms or left voluntarily, while the firm is less likely to hire if the manager was fired due to negative shocks to the firm. Therefore, $D_{j,t-1} = 1$ if at least one manager left the firm due to employment-to-employment. Second, firms may plan ahead and make movements if they are informed of the manager leaving ahead, we consider the actual leaving date to be up to 6 months before the actual leaving date of the manager.

Figure 1.2 shows our main event study results, $\beta_s, s \in [-3, 4]$, following Equation 1.3. Each coefficient estimate gives the difference in monthly probability in a particular month relative to the reference period. Under the estimator, the reference period is all months in the sample prior to any estimated pre-treatment coefficients. Two months following the manager leaving, we find that the probability of internal promotion for the firms experiencing manager leaving is 1.5 percentage point (5.12%) higher than those not.[16]

1.4 Model

This section develops and characterizes a random search model of the labor market with search friction, featuring heterogeneity on three sides - workers, firms, and jobs - to study the sorting pattern among three parties and aggregate labor market implications. The key innovation to the search literature is the existence of job ladder *within the firm* and thus a new source of job mobility, within-firm job mobility.

Environment

The model is formulated in discrete time and assumes stationarity of the labor market. In any period, the economy is populated by a continuum of risk-neutral workers with measure normalized at unity, a continuum of teams with endogenous

[16]The mean of the probability of internal promotion is 0.293.

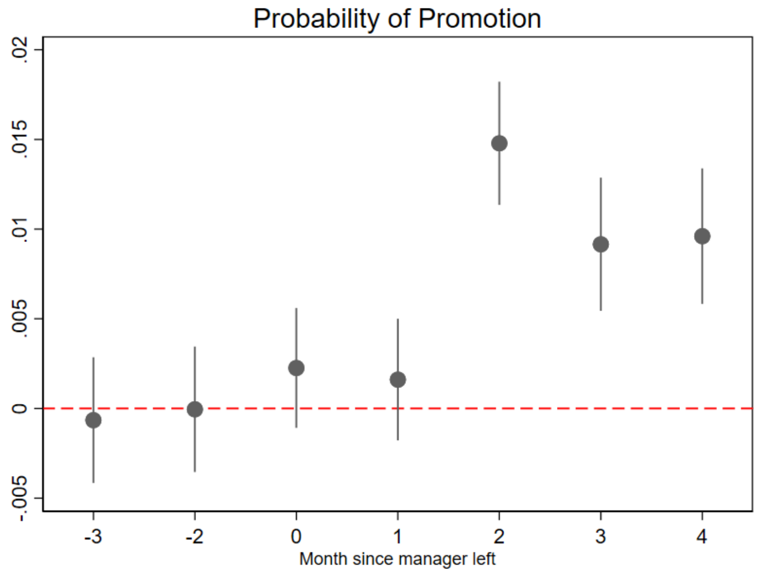


Figure 1.2: Internal Promotion Share Distribution

Note: Figure plots the estimates and 95% confidence interval of β_s using a version of Equation 1.3 where D_j is the month when a manager left the firm. The red horizontal line indicates the null hypothesis. Controls include firm fixed effects, month-year fixed effects, lagged the firm-level characteristics X_{jt} such as the ratio of managers to production workers, sales per employee, value-added per employee, and an indicator for manager leaving voluntarily. Standard errors clustered at firm level.

measure, and an exogenous measure TF of firms. Agents are forward-looking. Without loss of generality, we assume that the firms and workers share a common discount rate $\beta \in (0, 1)$. Firms all produce an identical homogeneous good. Workers inelastically supply one indivisible unit of labor and maximize the present value of net flow income.

Firms and Teams

Each firm consists of a collection of independent teams.[17] Firms differ with respect to two dimensions. The first dimension is their intrinsic type. Firms in the labor market are heterogeneous in their permanent productivity $z \sim \Gamma(z)$. The second difference comes from the composition of workers in each team. A team consists of two positions: production workers (henceforth workers) and managers. There is at most one employee for each position in each team. For example, in a firm with one manager and three workers, our setting implies three teams. In the first team, the manager matches with one worker. The second and the third teams have one worker for each.

We find it worthwhile to provide intuition for our modeling choice of within-firm job mobility. There are two ways to model the promotion process since promotion is a joint decision made by the firm and the worker. One can model the career progression from the perspective of firms, i.e., how workers compete with their coworkers and exert optimal effort to win the promotion prize as in the tournament models (e.g., Waldman (1984)). A disadvantage of this modeling choice is the curse of dimensionality. Indeed, we need to keep track of each of her coworkers' status if workers are heterogeneous in their human capital level.[18] An alternative way of understanding the promotion is through the lens of workers. This paper follows this assumption. We assume that each worker has the chance of being promoted within the firm, with distinct initial ability and accumulated human capital. Managers and workers are randomly assigned. This modeling choice is more tractable than

[17]This assumption differs from the constant return to scale (CRS) assumption. Under CRS, firms make strategic decisions for the worker composition of each team within the firm simultaneously. Specifically, given the type of workers, the firm optimally allocates workers to managers and decides whether the team should search for candidates. In the standard one-vacancy-one-firm models such as Mortensen and Pissarides (1994), these two assumptions are equivalent. However, under our model, the current assumption is better than the CRS assumption since our assumption makes the model tractable. Under the CRS assumption, firms must keep track of each team's state. See the discussion below for details.

[18]Theoretically, the promotion decision seems trivial if all candidates are homogeneous. Empirically, promoted workers typically are paid at the top of their cohort, which rejects the assumption of random promotion. Thus, a natural trade-off arises between the heterogeneity of workers and the measure of workers.

the first method, although this model abstracts away competition among workers.[19]

At any point in time, a team is in some state $y = (z, \vec{S}_W, \vec{S}_M)$, where $\vec{S}_W$ and $\vec{S}_M$ are the worker's and manager's skill vector respectively, which is introduced in next subsection. For example, a firm has no employee if it's at state $y = (z, 0, 0)$. At state $y = (z, \vec{S}_W, 0)$, the firm has only one worker of type $\vec{S}_W$. At state $y = (z, 0, \vec{S}_M)$, the firm has only one manager of type $\vec{S}_M$. A team's production function $f : Y \to \mathbb{R}_+$ with $Y \equiv \bar{K} \times \bar{K}$ depends on the labor composition of the team. Specifically, teams operate a constant elasticity of substitution technology that turns workers' knowledge level into f units of output.

Workers

All workers are in one of the two labor force states — the employed workers (e) and the unemployed who are not working but search for a job (u). Workers are born with heterogeneous two-dimensional skill sets: general human capital g and managerial skills $m \in \{0, 1\}$. $m = 0$ represents that the worker has no managerial skills, while $m = 1$ represents the worker knows how to supervise the team.[20] We assume that the probability of drawing $m = 1$ is $s \in (0, 1)$. On the second dimension of the skill set, a worker's type is represented by their general human capital skill, which is characterized by a human capital ladder $g \in \{g_1, g_2, ..., g_N\}$ with $g_i < g_{i+1}$. We assume that employees accumulate general human capital on-the-job modeled as learning by doing (LBD).[21] Human capital evolves stochastically as in Taber and Vejlin (2020). With probability $p(e_g)$, the employee's general skill advances to the next level, i.e., $P(g_{i+1}|g_i) = p(e_g)$. The speed of accumulation is a function of training effort e_g, which is jointly determined by the worker and the firm given a convex training cost function $C(e_g)$.

[19] We discuss potential model extensions in section 1.4.

[20] For simplicity, we assume that m is a binary variable. Although allowing more values is manageable, understanding the returns of managerial skills is not our main interest. In addition, the differences in managerial abilities among all CEOs are negligible (e.g. Terviö (2008) and Gabaix and Landier (2008) find that the wage inequality between CEOs is mainly due to variations in firm characteristics).

[21] This assumption is made, among others, in Huggett, Ventura, and Yaron (2011), Bagger, Fontaine, Postel-Vinay, and Robin (2014), and Taber and Vejlin (2020).

We assume that employed workers are also heterogeneous in their third dimension of the skills - firm-specific human capital f with $f \in \{f_1, f_2, ..., f_M\}$ with $f_i < f_{i+1}$. [22] We assume firm-specific human capital f is a perfect substitute of general type g at the current firm, and thus an employee's productivity is $h = f + g$. An employed accumulates firm-specific human capital on-the-job at an endogeneous rate $p(e_f)$. The accumulation speed is a function of training effort e_f, jointly determined by the worker and the firm given a convex training cost function $C(e_f)$. Note that firm-specific human capital is not required for promotion. What matters is the total human capital h.[23] Formally, a worker is characterized by the tuple (g, m, f, x), i.e., human capital level, managerial skill, firm-specific human capital, and employment status. At any point in time, let $x \in X = \{u, \bar{k}\}$ be workers' employment state, where u means that a worker is unemployed, while $\bar{K} = \{0, 1, 2, ..., N \times M\}$ represents coworker's type (f, g). Since not every worker (manager) has a manager (worker) in a team, $\bar{k} = 0$ refers the situation in which they work alone.

The distribution of workers implies a distribution of teams. The measure of one-worker teams with firm type z is $n^z_{10} = \sum_i e_{i0}(z, i)$, the measure of one-manager teams with firm type z is $n^z_{01} = \sum_i e_{0i}(z, i)$, and the measure of full teams with firm type z is $n^z_2 = \sum_i \sum_j e_{ij}(z, i, j)$. The rest of teams are empty teams n^z_0.

Production Technology

Since workers and managers perform distinct job responsibilities, it is natural to assume that the productivity of skills varies between workers and managers, captured by ω.[24] To capture all possible cases of and the manager's skills set $\vec{S}_M$,

[22]In our model, the firm-specific human capital combines both firm-specific human capital and task-specific human capital or occupation-specific human capital. It is interesting to disentangle the two specific human capital from each other on wage dynamics. We view it as a future plan.

[23]This assumption aligns with the fact that workers from all levels of initial education background are promoted into managerial positions in the data.

[24]This assumption differs from Herkenhoff, Lise, Menzio, and Phillips (2024). They assume that the team production function is symmetric regarding the production weights of two workers since they focus on the interaction between coworkers, who are assumed to be in the same hierarchy.

the production function is assumed to follow

$$y(z, \vec{S}_W, \vec{S}_M) = f_0\left[z^{\alpha} + [\omega h_W^{\eta} + (1-\omega) m_M \mathbb{1}(h_M > \underline{h}) h_M^{\eta}]^{\frac{\alpha}{\eta}}\right]^{\frac{1}{\alpha}}, \quad (1.4)$$

where α determines the degree of substitutability of firm productivity z and team production, and η governs the degree of substitutability of two employees within the team.

If $\eta > 1$, the team production function is submodular, i.e., $\frac{\partial y_z^2}{\partial h_W \partial h_M} < 0$, which implies a social planner prefers to allocate a high-productivity manager to match with a low-productivity worker condition on $\omega < \frac{1}{2}$. Symmetrically, if $\eta < 1$, the team production function is supermodular, i.e., $\frac{\partial y_z^2}{\partial h_W \partial h_M} > 0$, a social planner prefers to allocate a high-productivity manager to match with a high-productivity worker condition on $\omega < \frac{1}{2}$.[25]

To distinguish the different roles of workers and managers, we impose two assumptions: (1) a worker's managerial skill is not valued at the production jobs, while they are required for managerial positions in a team; (2) there is a lower bound for a manager's production skill for the manager to produce a positive marginal value to the coaltion. The second assumption nests the idea of Caliendo et al. (2011) and Caliendo et al. (2015) that the core function of a manager is to supervise workers by solving the problems beyond their abilities. We relax this assumption by allowing a worker's human capital greater than her manager's human capital because a significant share of managers are paid less than their workers. A manager has a higher marginal production in a team if $\omega < \frac{1}{2}$. The

[25]Caliendo et al. (2015) estimate a production function of the internal organization of French manufacturing firms and show that the average wages of pre-existing layers fall among firms that expand by adding layers. These results should not be understood as a signal of submodular production function since they find the number of hours and workers in pre-existing layers also increase. The number of workers and managers in our setup is not carefully modeled. Consequently, we do not have a premise of the value of ρ.

production also implies that one worker (manager) can also produce alone with

$$y(z, \vec{s}_W, \vec{\emptyset}) = \begin{cases} f_0 \left[z^\alpha + [\omega h_W^\eta]^{\frac{\alpha}{\eta}} \right]^{\frac{1}{\alpha}} & \eta > 0 \\ f_0 z & \eta \leqslant 0, \alpha > 0 \\ 0 & \eta \leqslant 0, \alpha \leqslant 0, \end{cases}$$

and

$$y(z, \vec{\emptyset}, \vec{s}_M) = \begin{cases} f_0 \left[z^\alpha + [(1-\omega)\mathbb{1}(m_M = 1) h_M^\eta]^{\frac{\alpha}{\eta}} \right]^{\frac{1}{\alpha}} & \eta > 0 \\ f_0 z & \eta \leqslant 0, \alpha > 0 \\ 0 & \eta \leqslant 0, \alpha \leqslant 0. \end{cases}$$

If the firm does not employ any workers, it produces according to $y(z, \vec{\emptyset}, \vec{\emptyset}) = f_0 z$.

Optimal General and Firm-specific Human Capital Training

One of the natural reasons that facilitate the internal labor market is to accumulate human capital. A large amount of literature shows that general and firm-specific human capital is important in workers' careers. Although the firm-specific skill increment can only be applied to the current firm, it may expedite career progression for workers and shield them from labor market friction. Becker (2009) argues that employers bear all the costs in firm-specific training when labor markets are competitive. However, in frictional labor markets, it is intuitive that the firm and the employee bargain over both wages and training investment in a contract and share the training costs as in Acemoglu and Shimer (1999). Both parties decide on the optimal training efforts, jointly balancing the cost and benefit of the investment. Since both employees and firms are risk-neutral, the optimal contract is not uniquely determined, but the optimal training investment is uniquely determined. An alternative contract is that values could be delivered through training that firms bear all the specific training costs but pay a lower wage. The wage schedule is solved through backloading.

The human capital accumulation probability $p(e_i) \in [0,1]$ depends on the training efforts $e_i, i \in \{g, f\}$. Assume that $p(e_i)$ is an increasing and concave function in e_i, i.e., $p'(e_i) > 0, p''(e_i) < 0$. Assume that the cost function $C(e_i)$ is an increasing and a convex function in e_i, i.e., $C'(e_i) > 0$, $C''(e_i) > 0$. These two assumptions imply that high-productivity firms put more efforts on training their workers. The production weight ω determines the value of firm-specific training, i.e when $\omega > \frac{1}{2}$, workers who are more likely to be promoted receive more training.

Search and Matching

Both employed and unemployed workers search off- and on-the-job. However, due to computational constraints, we assume that workers must undergo a job search to transition to another team within the same firm.[26]

External labor market frictions are characterized by an aggregate matching function with a job finding rate less than 1. Two assumptions are made to simplify computation without loss of generality. First, a full team is assumed not to hire workers and substitute the current employee with the prospective employee. Second, teams post vacancies with position information. No position switching is allowed if the match is made. Thus, we assume two segmented labor markets for job seekers: workers and managers, and search and matching happens randomly within each market. Due to perfect information, job seekers without manager skills can only search the market for workers, while candidates with managerial skills optimally search in one of the two markets.[27] The benefit of doing so instead of all job seekers in one labor market is that workers without managerial skills do not crowd out candidates eligible for managerial jobs. Assume that the match-

[26]The main difficulty is that the state variable space will have infinite dimensions if workers are allowed to move within the firm freely. Otherwise, the state variables include all workers' skill sets. This assumption implies that the probability of transition across teams within the firm faces the identical level of labor market friction as those external candidates.

[27]Two ways to understand this setting. Firstly, candidates with managerial skills spend time searching in one market and the rest of the time searching in the other. An alternative way to understand it is that at each period, some ratio of workers of one type search in one labor market, and the rest of them search in the other. This assumption can be easily relaxed that every job seeker can search in both markets. The search behavior generates unnecessary inefficiencies.

ing function between teams and workers and between teams and managers are $M_1(S, V)$ and $M_2(S, V)$ respectively, where S represents the measure of total effective searchers, and V is the measure of vacant positions in each market. S includes unemployed workers who actively search, whose search intensity is taken as the unit and hence normalized to 1, employed workers, and managers who search on the job with search intensity $\xi \in R^+$. We further assume that the search intensity of the unemployed in the manager market is ξ_u.

Teams with one manager search in the worker market, and teams with one worker search in the manager market. Empty teams search in both markets and place the job seeker to the optimal position upon hiring. Both matching functions are assumed to exhibit constant returns to scale. Thus the worker contact rate per search intensity of market $i \in \{1, 2\}$ is $p_i(\theta) = M_i/S_i$ and the job contact rate $q_i(\theta) = M_i/V_i$, where $\theta_i := V_i/S_i$ defines the effective labor market tightness. To be specific, the probability of meeting in the worker market is $\sum_i \lambda_u u_i^{m=0} + u_i^{m=1} + \lambda_e e_i^{m=0} + e_i^{m=1}$, while the probability of meeting in the manager market is $\sum_i \lambda_u u_i^{m=1} + \lambda_e e_i^{m=1}$.

Promotion: Internal Versus External

Given the independence between teams within the firm, internal promotion sorely refers to the transition to the managerial position *within the same team*. The rest of vertical movements, from workers to managers, are considered as external promotion.

Upon the realization of the human capital level, the firm and the worker jointly decide whether to promote the worker to the manager position. To be promoted, two requirements have to be met. First, the managerial position is vacant.[28] The assumption aligns with the data that demotion and firing are rare in most labor

[28]Bianchi, Bovini, Li, Paradisi, and Powell (2023) find evidence of spillover patterns consistent with older workers blocking the careers of their younger colleagues within firms with limited promotion opportunities. Using Danish administrative data, we find the probability of replacement firing, defined as employer-to-employer mobility with a wage cut, and demotion is negligible, given the firing cost is zero. This assumption can easily be relaxed by replacement hiring as in Herkenhoff et al. (2024) without loss of generality.

markets. Second, given the dynamic consideration, the worker's human capital level h is higher than the team-specific threshold $\bar{h}(z)$. The two assumptions imply that there are two ways for the worker to be promoted within the firm. To distinguish two scenarios, we name the promotion without a manager leaving as the type-I promotion because the worker is promoted due to skill upgrading and the promotion with a manager leaving as a type-II promotion.

Bargaining and Wage Determination

We adopt a standard multi-party bargaining game designed to ensure the decisions in a coalition are made jointly efficiently. Hence, wages are pure transfers between employees and firms. Therefore, the wage bargaining process is not our primary focus.

The bargaining process is similar to the one used in one-vacancy-one-firm models (e.g., Cahuc, Postel-Vinay, and Robin (2006)), in which wages are determined via a sequential auction if external negotiation occurs. The caveat is that a firm-worker match may include more than one worker. Therefore, more assumptions are needed to ensure the decisions are jointly efficient. Following Bilal, Engbom, Mongey, and Violante (2019), we state a set of extra assumptions on the contractual environment to make the model tractable: (1) workers' utility is a linear function of their total earnings, (2) internal negotiation happens whenever either the incumbent workers or the firm has a credible threat. Assumption 1 ensures the wages are sorely served as a transfer among the three parties;[29] Assumption 2 ensures the team always makes the optimal decisions. Both assumptions ensure the coalition's problem can be written as joint surplus value functions, introduced in section 1.4.

Workers come into contact with firms with empty positions. Upon meeting, the firm and the worker match if and only if the gains from trade are positive. If the firm and the worker successfully match, the worker becomes an employee of the firm, depending on the promised position. The worker then captures a fraction γ

[29]A log utility functions as in Huang and Qiu (2022) disrupts the alignment of workers and firms.

of the gains from trade, and a fraction $1-\gamma$ is captured by the firm, with $\gamma \in [0,1]$ being worker bargaining power.

Timing

Every period is divided into four stages: entry and exit, training and promotion, search-and-matching, and production.

At the entry-and-exit stage, each individual permanently exits the labor market with probability $\sigma \in (0,1)$. Hence, in aggregate, a measure σ of workers exits the labor market. The measure of labor who exit the labor market is replaced by an equal measure of workers who enter the labor market. Specifically, the fraction of entering workers with type (h_k, m) is π_k, with $\pi_k \geqslant 0$ and $\sum \pi_k = 1$. All workers who enter the labor market are unemployed. On top of that, a worker also faces the opportunity of internal promotion if their incumbent manager exits the labor market or the match is destroyed exogenously.

At the learning stage, a worker's (manager) general and firm-specific human capital evolves according to a probability distribution function that depends on the worker's (manager's) type k (l), the worker's (manager's) employment state x, and the firm type z.

At the search-and-matching stage, both hiring teams and job seekers optimally choose one labor market to search in. Workers and teams meet, match, and separate within each labor market. Several events could happen mutually exclusively, as in Bagger et al. (2014). Firms post a vacancy on the external labor market. Upon meeting, the worker and the firm match—in the sense that the worker is hired by the firm—if and only if the gains from trade are positive. An employed worker becomes unemployed for exogenous reasons with probability $\delta_w \in (0,1)$ ($\delta_m \in (0,1)$) depending on the job position.

At the production stage, teams in state y produce output according to the production function $F(y)$, pay some of the output to their employees as wages and earn the remaining output as profits. Unemployed workers of type i home produce $b_i > 0$ units of output.

Value Functions

To define an equilibrium, we need to introduce some notation. We denote U_i^P as the value of unemployment to an unemployed person of type i at the beginning of the production stage. We denote as S^S the joint value of a firm and its employees at the beginning of the search-and-matching stage. We denote as S^S the joint value of a firm and its employees at the beginning of the production stage. We also need some notation to describe the stationary distribution of workers and firms. We denote $e(i)$ as the measure of workers of type i who are in an employment state, while $u(i)$ is the measure of unemployed workers of type i. We denote $s(y)$ as the measure of firms in state y. The distributions $\{e, u, s\}$ are measured at the beginning of the search-and-matching stage.

Production Stage

The value of unemployment to a worker of type i is such that

$$U_i^P = b_i + \beta \left[U_i^P - \sigma U_i^P + (1-\sigma)\gamma \max \left\{ \underbrace{\sum_{y_W} p_W(\theta_W) s(y_W) D_W}_{\text{Search in the worker market}}, \underbrace{\sum_{y_M} p_M(\theta_M) s(y_M) D_M}_{\text{Search in the manager market}} \right\} \right], \tag{1.5}$$

where $D_l = [v(i, y_l) - U_i^P]^+, l \in \{W, M\}$. The value of unemployed workers is a combination of flow value and option value. The unemployed worker enjoys a flow utility of $b(i)$. The option value includes several events. The worker exits the labor market with rate σ. Upon survival, the unemployed worker chooses one labor market to search in. With probability $\{p_l(\theta_l)s(y_l)\}_{l=W,M}$, a function of the equilibrium labor market tightness and distribution of team within each market, the unemployed worker contacts an outside firm at status y in the labor market l. However, this is not the matching probability but the meeting probability. The new match is made if the surplus of the trade $v(i, y_l) - U_i^P$ is positive, denoted as $[\cdot]^+ = \max\{\cdot, 0\}$.

Conditional on a successful matching, the worker extracts a fraction γ of surplus of the trade. Like in a multi-worker firm setting, we follow Stole and Zwiebel (1996) and assume that each employee's outside option at a team y, $v(i, y)$, is determined by his marginal contribution to the team, also referred to as Shapley Value. The marginal contribution equals to the value of a team with the employee subtract the value of a team without the employee, i.e., the maximum value that is attributed to the employee. It is dependent on his/her human capital level h_i and firm status y as follows:

$$v(i, y) = \begin{cases} S_1^P(z, i, 0)(S_1^P(z, 0, i)\}) - S_0^P(z) & \text{if} \quad y = p(z, 0, 0) \\ S_1^P(z, i, j) - S_1^P(z, 0, j) & \text{if} \quad y = p(z, 0, j) \\ S_1^P(z, j, i)\} - S_1^P(z, j, 0) & \text{if} \quad y = p(z, j, 0) \end{cases}$$

At the beginning of the production stage, the joint value of state y to a firm and its employees is such that

Empty Team The Hamilton-Jacobi-Bellman (HJB) equation in the steady state for an empty team with no workers is

$$S_0^P(z) = \beta S_0^S(z). \tag{1.6}$$

Note that the value of an empty team is not zero in the equilibrium as we abstract away the free entry condition, which connotes an infinite job creation elasticity.[30]

One-worker Team The HJB equation for a production unit consisting of a z-type firm and a worker with human capital h_i $S_1(z, i, 0)$ is written as

$$\begin{aligned} S_1^P(z, i, 0) = y(z, i, 0) + \beta \Bigg\{ \sigma S_0^S(z) + (1 - \sigma) \max_{e_g \geqslant 0} \Big[p(e_g)\bar{S}_1(z, i', 0) \\ + (1 - p(e_g))S_1^S(z, i, 0) - C(e_g) \Big] \\ + (1 - \sigma) \max_{e_f \geqslant 0} \Big[p(e_f)\bar{S}_1(z, i', 0) + (1 - p(e_f))S_1^S(z, i, 0) - C(e_f) \Big] \Bigg\}, \end{aligned} \tag{1.7}$$

[30]Recently, Mercan and Schoefer (2020) Elsby, Gottfries, Michaels, and Ratner (2022), Qiu (2022) explore the vacating channel on the job creation, worker reallocation at establishment level, and volatility and persistence of job creation and its implications over the business cycle.

where $\bar{S}_1(z, i, 0) = \max\{S_1^S(z, i, 0), S_1^S(z, 0, i), S_0^S(z) + U_i\}$. The team produces a flow output $y(z, i, 0)$. With probability σ, the worker exits the labor market permanently at the beginning of the next period. two parties make a joint decision on the optimal effort e_g and e_f to put into general training and firm-specific training to maximize the expected value of training conditional on survival. With probability $p(e_g)(p(e_f))$, the worker's general (firm-specific) human capital appreciates to the next level $g_{i'} = g_{i+1}$ $(f_{i'} = f_{i+1})$, advancing the joint value to $\bar{S}_1(z, i', 0)$. Upon human capital accumulation, a promotion decision and a separation decision is made simultaneously by the worker and the firm. If the joint value of a firm and a manager is higher with dynamic consideration of future external hiring, i.e $S_1^S(z, 0, i) > S_1^S(z, i, 0)$, the worker is promoted to be a manager, otherwise not. Volunteer separation is possible if the $S_0^S(z) + U_i > \max\{S_1^S(z, i, 0), S_1^S(z, 0, i)\}$.

One-manager Team The HJB equation for a production unit consisting of a z-type firm and a manager with human capital h_i $S_1(z, 0, i)$ is as follows

$$
\begin{aligned}
S_1^P(z, 0, i) = y(z, 0, i) + \beta \Bigg\{ \sigma S_0^S(z) + (1-\sigma) \max_{e_g \geqslant 0} \Big[p(e_g)\bar{S}_1(z, 0, i') \\
+ (1 - p(e_g))\bar{S}_1(z, 0, i) - C(e) \Big] \\
+ (1-\sigma) \max_{e_g \geqslant 0} \Big[p(e_g)\bar{S}_1(z, 0, i') + (1 - p(e_g))\bar{S}_1(z, 0, i) - C(e_g) \Big] \Bigg\},
\end{aligned}
\tag{1.8}
$$

where $\bar{S}_1(z, 0, i) = \max\{S_1^S(z, 0, i), S_0^S(z) + U_i\}$. Similar to the one-worker team, the manager has the chance to accumulate general and firm-specific human capital. Differing from the one-worker team, promotion is not available to the manager. Yet, it does not imply no human capital investment in managers (especially when $\lambda < \frac{1}{2}$ as a higher human capital boosts output level).

Full Team The HJB equation for a production unit consisting of a z-type firm and a worker with human capital h_i and a manager with human capital h_j $S_2(z, i, j)$

is written as

$$S_2^P(z,i,j) = y(z,i,j) + \beta\Bigg\{\sigma^2 S_0^S(z) + \sigma(1-\sigma)\Big[\mathbb{E}_{i'} S_1^L(z,i_1',0) + \mathbb{E}_{i_2'} S_1^L(z,i_2',0)\Big] + \beta(1-\sigma)^2\Big[\mathbb{E}_{i_1'} S_1^L(z,i_1',i_2,0) + \mathbb{E}_{i_2'} S_1^L(z,i_1,i_2',0)\Big]\Bigg\} \tag{1.9}$$

A full team produces a flow output $y(z,i,j)$. Both the worker and the manager realize the training outcome as described above.

Search-and-matching Stage

The external labor market opens after the internal labor market. Now, we describe the value functions of firms and workers at the beginning of the search-and-matching stage.

Empty Team The joint value function of an empty team is as follows

$$S_0^S(z) = S_0^P(z) + (1-\gamma)\max\Bigg\{\underbrace{\sum_{i,x}\tilde{q}^W(i,x)[S_1^P(z,i,0) - S_0^P(z) - \nu(i,x)]^+,}_{\text{Search in the worker market}} \underbrace{\sum_{i,x}\tilde{q}^M(i,x)[S_1^P(z,0,i) - S_0^P(z) - \nu(i,x)]^+}_{\text{Search in the manager market}}\Bigg\}. \tag{1.10}$$

An empty team with productivity z enters one of the labor markets. The value of an empty team consists of a flow value and an option value of searching. The team contacts a worker of type (i,x) with meeting rate $\tilde{q}(i,x)$ in one of the two labor markets, which is defined as

$$\tilde{q}(i,x) = \begin{cases} q(\theta)u_i & \text{if} \quad x = \; u \\ q(\theta)e(z,i,0) & \text{if} \quad x = \; e(z,i,0) \\ q(\theta)e(z,0,i) & \text{if} \quad x = \; e(z,0,i) \\ q(\theta)e(z,i,j) & \text{if} \quad x = \; e(z,i,j). \end{cases}$$

The contact rate of each team in each market is regulated by the vacancy filling rate $q(\theta)$, a function of market tightness θ, and the distribution of job seekers across working states (i, x) with measure $e(i, x)$. The $\nu(i, x)$ denotes the job seeker's current outside option, defined as the marginal contribution to the current coalition. Formally,

$$\nu(i,x) = \begin{cases} U_i & \text{if} \quad x = u \\ S_1^P(z,i,0) - S_0^P(z) & \text{if} \quad x = e(z,i,0) \\ S_1^P(z,0,i) - S_0^P(z) & \text{if} \quad x = e(z,0,i) \\ S_2^P(z,i,j) - \max\{S_0^P(z) + U_j, S_1^P(z,0,j)\} & \text{if} \quad x = e(z,i,j) \\ S_2^P(z,j,i) - \max\{S_1^P(z,j,0), S_1^P(z,0,j), S_0^P(z) + U_j\} & \text{if} \quad x = e(z,j,i). \end{cases}$$

The value of a manager with type j in a full team with measure $e(z, i, j)$ is the joint vale of the team at the production stage deduct the value of the team with the worker i and the possibility of type-II promotion.

One-worker Team The joint value of a firm and a worker is defined as

$$\begin{aligned} S_1^S(z,i,0) &= S_1^P(z,i,0) + \delta_w \left[S_0^P(z) + U_i - S_1^P(z,i,0) \right] \\ &+ \sum_y s(y)\gamma \left[\nu(i,y) - [S_1^P(z,i,0) - S_0^P(z)] \right]^+ \\ &+ \sum_{j,x} \tilde{q}^M(j,x)(1-\gamma) \left[S_2^P(z,i,j) - S_1^P(z,i,0) - \nu(j,x) \right]^+ . \end{aligned} \tag{1.11}$$

With the probability δ_W, the worker exogeneously separates from employment into unemployment and the team becomes vacant. With the probability $s(y)$, the worker moves to another team through on-the-job search. With the probability $q(j, x)$, the team searches externally in the manager market.

One-manager Team The joint value of a firm and a manager is defined as

$$\begin{aligned} S_1^S(z,0,i) &= S_1^P(z,0,i) + \delta_M\Big[S_0^P(z) + U_i - S_1^P(z,0,i)\Big] \\ &+ \sum_y s(y)\gamma\Big[\nu(i,y) - (S_1^P(z,0,i) - S_0^P(z))\Big]^+ \\ &+ \Bigg[\sum_{j,x} \tilde{q}^W(j,x)(1-\gamma)\Big[S_2^P(z,j,0,i) - S_1^P(z,0,i) - \nu(j,x)\Big]^+ - c_\nu\Bigg]^+. \end{aligned} \tag{1.12}$$

The two main differences between the value of one-manager teams and one-worker teams come from the following: (1) the manager in the one-manager team faces a different probability of transition to unemployment with probability δ_M; (2) the team searches in the worker market with the meeting probability of $\tilde{q}^W$ after paying the vacancy posting cost.

Full Team The joint value of a firm and a manager and a worker is defined as

$$\begin{aligned} S_2^S(z,i,j) &= S_2^P(z,i,j) + \Big[\delta_M(\max\{\hat{S}_1(z,i,0), \hat{S}_1(z,0,i)\} + U_j) \\ &+ \delta_W(\hat{S}_1(z,0,j) + U_i) - 2S_2^P(z,i,j)\Big] \\ &+ \lambda_e \sum_y s(y)\gamma \max\Big\{\nu(i,y) - [\hat{S}_2(z,i,j) - \hat{S}_1(z,j,0)], 0\Big\} \\ &+ \lambda_e \sum_y s(y)\gamma \max\Big\{\nu(j,y) - [\hat{S}_2(z,i,j) - \hat{S}_1(z,i,0)], 0\Big\}. \end{aligned} \tag{1.13}$$

A full team does not actively search for a replacement, which differs from Herkenhoff et al. (2018). One caveat is that the worker could be promoted if the manager leaves due to exogenous separation δ_M or one-the-job poaching.

Final Stage

We describe the value functions $\hat{S}$ to finalize the equilibrium. After the search-and-matching process is complete but before the firm has had the option of firing, the joint value functions of state y to a firm and its employees is such that

$$\hat{S}_0(z) = S_0^P(z) \tag{1.14}$$

$$\hat{S}_1(z, i, 0) = \max\{S_1^P(z, i, 0), S_0^P(z) + U_i\} \tag{1.15}$$

$$\hat{S}_1(z, 0, i) = \max\{S_1^P(z, 0, i), S_0^P(z) + U_i\} \tag{1.16}$$

$$\hat{S}_2(z, i, j) = \max\{S_2^P(z, i, j), S_1^P(z, i, 0) + U_j, S_1^P(z, j, 0) + U_i\} \tag{1.17}$$

Law of Motion

To formally express how this distribution evolves over time, we need to introduce some notation to describe the equilibrium policy functions. Let e, u denote the distribution of workers across employment states at the beginning of the search-and-matching stage in the current period. Let e_1, u_1 denote the distribution of workers after the search-and-matching process has taken place but before the firms have had the chance to fire their employees. We start with the law of motion of teams with one manager, which is the novel element of the model. Three channels affect the inflows and outflows of one-manager teams: (1) hiring, (2) vacating or destruction, and (3) internal promotion. First, an empty team posts a vacancy in the manager market and matches with an external candidate. Second, positions are vacated by workers in full teams quitting their jobs at rate δ_W, endogenously determined by job-to-job transitions and employment-to-unemployment transitions. The vacated positions, therefore, add to the pool of job openings available for job seekers. Lastly, workers in the one-worker teams accumulate enough human capital and are promoted to managers. All three channels of vacancy flows are endogenous. The law of motion of one-manager teams $e^1(z, 0, i)$ of manager with human capital

h_i in firm type z can be written as

$$
\begin{aligned}
e^1(z,0,i) = {} & u_i \lambda_u p_0(z) h_0(u_i)(1-W) \\
& + \sum_{z'} e(z',i,0)\lambda_e p_0(z) h_0(i)(1-W) \\
& + e(z,0,i)\left\{1-\delta-\sum_{i,x} q_i(x) h_{z,0,i}(x)\right\} + \sum_{z'} e(z',0,i)\lambda_e p_0(z) h_0(i)(1-W) \\
& + \sum_{z'}\sum_{j} e(z',i,j,0)\left\{\lambda_e p_0(z) h_0(i)(1-W)\right\} \\
& + \sum_{z'}\sum_{j} e(z',i,0,j)\left\{\lambda_e p_0(z) h_0(i)(1-W)\right\} \\
& + \sum_{z'}\sum_{j} e(z',j,0,i)\left\{\lambda_e p_0(z) h_0(i)(1-W)\right\} \\
& + \sum_{j} e(z,j,0,i)\left\{\delta+\sum_{y} \lambda_e p_y h_y(j)\right\} \\
& + \sum_{z'}\sum_{j}\sum_{i_2} e(z',i,i_2,j)\left\{\lambda_e p_0(z) h_0(i)(1-W)\right\} \\
& + e(z,i_2,j,i)\left\{\lambda_e p_0(z) h_0(i)(1-W)\right\},
\end{aligned}
\tag{1.18}
$$

The laws of motion of other types of teams are more standard. Readers interested in the complete formation of stationary distribution can directly skip to Appendix A.2.

Stationary Equilibrium

The steady-state equilibrium is defined as a set of value functions U, S^s, S^L, S^P for each state and type of the teams, a set of hiring policies for each origin and destination pair, a set of promotion policies for one-worker teams, a distribution of workers, teams, and jobs across labor market statuses and the resulting labor market tightnesses $\theta_i, i \in \{W, M\}$, such that the HJB equations in Section 1.4 hold and the laws of motion in Section 1.4 balance inflows and outflows (i.e., give zero net flows).

Model Properties

This section discusses key implications of the model regarding the interplay between human capital accumulation, job search and promotion policies. These insights help comprehend the mechanisms through which the internal labor market influences labor market allocation. To understand properties of the dynamic model, we show the properties of a one-period static model. The timing in the period is as follows. First, firms and their employees make optimal training and promotion decisions. Then, job seekers and firms meet in the external labor markets searching for potential jobs. Lastly, production starts after the matching ends.

Proposition 1 (Worker type and internal promotion). In equilibrium, suppose there exist a promotion threshold of worker human capital $\bar{h}(z)$ of each type of firm z that the firm is indifferent between internal promotion and external hiring. When $\omega < \frac{1}{2}, \rho < 1$, and $\alpha < 1$, (1) any worker with $h > \bar{h}(z)$ is promoted internally; (2) If $z' > z$, the threshold satisfies $\bar{h}(z') > \bar{h}(z)$. When $\omega > \frac{1}{2}, \rho < 1$, and $\alpha < 1$, there exists a upper bound $\tilde{h}(z)$ such that for $\forall h > \tilde{h}(z)$, the worker will not be internally promoted.

Proof. Recall that the value of the internal promotion in a dynamic world can be summarized as

$$S^S(z, 0, h) = \underbrace{\sum_{j,x} q_w(h, j, x)(1-\gamma)\left[S^P(z, j, h) - S^P(z, 0, h) - \nu(j, x)\right]}_{\text{Promotion Internally (ILM)}}^{+}. \quad (1.19)$$

The value of external hiring is

$$S^S(z,h,0) = \underbrace{\sum_{j,x} q_m(h,j,x)(1-\gamma)\Big[S^P(z,h,j) - S^P(z,h,0) - \nu(j,x)\Big]}_{\text{Hiring Externally (ELM)}}^{+}. \quad (1.20)$$

Under the two assumptions, we can simplify the equation (1) and (2) to be

$$S^S(z,0,h) = \underbrace{\sum_j q_w(h,j)}_{\text{contact rate in the worker market}} (1-\gamma)\Big[\underbrace{y(z,j,h) - y(z,0,h) - U(j)}_{\text{Surplus of the trade}}\Big]^{+}, \quad (1.21)$$

and

$$S^S(z,h,0) = \underbrace{\sum_j q_m(h,j)}_{\text{contact rate in the manager market}} (1-\gamma)\Big[\underbrace{y(z,h,j) - y(z,h,0) - U(j)}_{\text{Surplus of the trade}}\Big]^{+}. \quad (1.22)$$

The value of internal promotion (external hiring) consists of two parts: (1) the vacancy filling rate in the manager (worker) market, which is a function of the current employee's productivity, and (2) the surplus of the trade upon meeting.

Now let's look at the terms in the bracket more carefully. Define $S_E = y(z,j,h) - y(z,0,h) - \nu(j,x)$ and $S_I = y(z,h,j) - y(z,h,0) - \nu(j,x)$. Then,

$$S_E - S_I = y(z,j,h) - y(z,0,h) - (y(z,h,j) - y(z,h,0)). \quad (1.23)$$

Given the worker's type h, and the production function

$$y(z,h,j) = f_0\Big[z^\alpha + [\omega h^\rho + (1-\omega)j^\rho]^{\frac{\alpha}{\rho}}\Big]^{\frac{1}{\alpha}},$$

It is straightforward to verify that if $\omega < \frac{1}{2}$ and $\rho < 1$,

$$S_E - S_I < 0, \quad \frac{\partial(S_E - S_I)}{\partial h} > 0, \tag{1.24}$$

and if $\alpha < 1$,

$$\frac{\partial(S_E - S_I)}{\partial z} < 0, \tag{1.25}$$

Then for $\forall h > \bar{h}$, the value of external hiring is

$$\begin{aligned} S^S(z,0,h) &= \sum_j q_w(h,j)(1-\gamma)\Big[y(z,j,h) - y(z,0,h) - U(j)\Big]^+ \\ &> \sum_j q_w(h,j)(1-\gamma)\Big[y(z,h,j) - y(z,h,0) - U(j)\Big]^+ \\ &> \sum_j q_m(h,j)(1-\gamma)\Big[y(z,h,j) - y(z,h,0) - U(j)\Big]^+ \\ &= S^S(z,h,0). \end{aligned} \tag{1.26}$$

The second row follows directly from the equation 1.24, and the third row is true if the elasticity of vacancy filling rate respect to surplus of the trade in the worker market is larger than the elasticity of vacancy filling rate respect to surplus of the trade in the manager market.

Next, to compare the threshold across firm type z, we assume the promotion threshold of human capital of worker in a type-z firm is $\bar{h}(z)$. Suppose there is another firm with type $z' > z$, the value of external hiring is

$$\begin{aligned} S^S(z',0,\bar{h}(z)) &= \sum_j q_w(\bar{h}(z),j)(1-\gamma)\Big[y(z',j,\bar{h}(z)) - y(z',0,\bar{h}(z)) - U(j)\Big]^+ \\ &> \sum_j q_m(\bar{h}(z),j)(1-\gamma)\Big[y(z',j,\bar{h}(z)) - y(z',0,\bar{h}(z)) - U(j)\Big]^+ \\ &> \sum_j q_m(\bar{h}(z),j)(1-\gamma)\Big[y(z',\bar{h}(z),j) - y(z',\bar{h}(z),0) - U(j)\Big]^+ \\ &= S^S(z',\bar{h}(z),0). \end{aligned} \tag{1.27}$$

Therefore, to make both sides of the equation equal, we need a $\bar{h}(z') > \bar{h}(z)$.

To summarize, proposition 1 states that if both $\alpha < 1$, $\rho < 1$, given any value of ω, there exists an promotion set for each type of firms. Only the workers with labor productivity in the set will be promoted internally conditional on $m = 1$. One caveat is that since g and f are perfect substitutes in the current firm, high-g workers may not necessarily get promoted.

Both general or firm-specific human capital training costs determine the internal labor friction and speed of human capital accumulation. We first show that high-type firms invest more on worker human capital in the following proposition.

Proposition 2 (Training Effort and firm type z). In equilibrium, high-z firms invest more on general human capital if $\alpha < 1$.

Proof: Recall that the value function of a firm $(z, h, 0)$ at the beginning of learning stage is represented as

$$S_1^P(z,h,0) = y(z,h,0) + \beta\Bigg\{\sigma S_0^S(z) + (1-\sigma)\max_{e_g \geqslant 0}\Big[p(e_g)\bar{S}_1(z,h',0) + (1-p(e_g))S_1^S(z,h,0) - C(e_g)\Big] + (1-\sigma)\max_{e_f \geqslant 0}\Big[p(e_f)\bar{S}_1(z,h',0) + (1-p(e_f))S_1^S(z,h,0) - C(e_f)\Big]\Bigg\}, \tag{1.28}$$

where $\bar{S}_1(z,h',0) = \max\{S_1^S(z,h',0), S_1^S(z,0,h')\}$ represents the chance of promotion. Take the first order condition with respect to e_g and e_f, we have

$$p'(e_g)\Big[\bar{S}_1(z,h',0) - S_1^S(z,h,0)\Big] - C'(e_g) = 0, \tag{1.29}$$

and

$$p'(e_f)\Big[\bar{S}_1(z,h',0) - S_1^S(z,h,0)\Big] - C'(e_f) = 0. \tag{1.30}$$

Denote $V(z,i) = \bar{S}_1(z,h',0) - S_1^S(z,h,0)$. Equation (11) can be rewritten as

$$p'(e_g)V(z,i) = C'(e_g). \tag{1.31}$$

Since $p'(e_g) > 0, p''(e_g) < 0$ and $C'(e_g) > 0, C''(e_g) > 0$, the optimal general training effort e_g is uniquely pinned down. Moreover, since $\frac{\partial V(z,i)}{\partial z} > 0$, the cross-derivative of e_g with respect to z is derived from taking the partial derivative of equation (13) with respect to z stated as follows

$$p''(e_g)\frac{\partial e_g}{\partial z}V(z,i) + p'(e_g)\frac{\partial V(z,i)}{\partial z} = C''(e_g)\frac{\partial e_g}{\partial z}. \tag{1.32}$$

By rearranging the equation (9), we can derive the cross-derivative of e_g with respect to z as follows

$$\frac{\partial e_g}{\partial z} = p'(e_g)\frac{\partial V(z,i)}{\partial z}\frac{1}{\left[C''(e_g) - p'(e_g)V(z,i)\right]} > 0. \tag{1.33}$$

Therefore, more productive firms invest more on worker's general and firm-specific human capital. Additionally, the option for internal promotion has a distributional effect of training across firm types. To see why, one of the role of the internal promotion channel is to retain the worker. However, since high-type firms are more competitive in the external labor market than low-type firms, it dampens the incentive for firm-specific training.

As is stated in the first and second proposition, high-type firms invest more on worker training and have a higher criteria of promotion. However, the relationship between internal promotion share and firm type z is ambiguous.

Model Discussion

The baseline model abstracts away multiple features to keep the model simple without distraction from the main focus of our model. This subsection discusses the intuition and implications for possible extensions to the baseline model.

Multi-worker teams The first feature we abstract away is the possibility of a many-to-one team structure in which a manager supervises more than one production worker in a team as in the standard Garicano (2000) style models. Theoretically, we could extend the number of workers to be any positive integer. The more realistic model has several predictions. First, the promotion decision would depend on workers' absolute *and* comparative advantages, as in the canonical Roy model (Roy (1951)). To see why, consider the scenario of two workers competing for the internal promotion opportunity. The worker with managerial skills will be promoted even though the other worker may have a higher general skill. The more productive worker wins the competition if both candidates have managerial skills. Moreover, if the training efforts are endogenously determined, promotion is an incentive to exert more effort, as in the tournament models (e.g., Lazear and Rosen (1981)).

Second, the relationship between firm type and the span of control, defined as the number of workers per manager, reconciles the data pattern that high-type firms have a larger span of control. To see why, if the production function is supermodular in terms of firm type z and manager human capital h, in tandem with a manager's marginal contribution to the coalition is larger than a worker's marginal contribution, high-productivity firms hire more workers out of precautionary consideration as in DeVaro and Morita (2013).

However, to cope with the curse of dimensionality, a model with one manager and two workers could be the optimal modeling choice under the computational constraint.

Information Friction and Symmetric Learning Information friction arises naturally when studying the economic mechanisms behind internal versus external hiring. Indeed, the literature emphasizes the role of information friction in understanding firms' promotion decisions (e.g., Friedrich (2016), Pastorino (2019)). Information friction provides an additional incentive to invest to fill managerial positions internally and procure firm-specific human capital. Our model does not separate information friction from external labor market friction. There are several methods to model information frictions.

First, information friction can be modeled as the screening cost upon meeting in the external labor market. Data on vacancy postings is required to identify the screening cost and the matching elasticity of the manager matching function. An alternative way is to assume firms and workers have limited information on the worker's managerial skills at the beginning of employment. All firms, including the incumbent one, gradually learn the workers' type by observing their production level (DeVaro and Morita (2013), Pastorino (2024)).

Asymmetric Learning and Moral Hazard Asymmetric information in a dynamic model with an infinite number of periods also generates complicated interaction among firms as promotion sends signals about the potential quality of the manager. Therefore, firms may hold off promotion by underinvesting their training efforts as in Waldman (1984). Yet, the external labor market friction and inflexible organizational structure prevent firms from choosing inefficiently low promotion rates.

1.5 Calibration

Guided by theoretical and empirical results shown above, we discuss the calibration strategies in this section. We aim to (1) estimate the friction in the internal labor market and the external labor market (2) separately identify the importance of general human capital training investment from firm-specific human capital training investment through matching both micro- and macro-moments characterizing the Danish Labor market from 2010-2018. We calibrate the model at a quarterly frequency.

Parameterization

Training Investment We assume the cost of training conditional on efforts e_i with $i \in \{g, f\}$ is a quadratic function $\lambda_i e_i^2$, and the human capital accumulation probability is $p(e_i) = p_0 \frac{e_i}{p_1 e_i + 1}$. Without data on training effort or training costs, we are not able to identify the coefficients λ_i and p_0 separately.[31] Thus, we normalize p_0 to be 1. We further assume $p_1 = 2$ to make sure the probability of a two-employee team is positive if both of them acquire the human capital advancement.

Firm Size To map the model to the data in a nontrival way,[32] we assume there is a fixed set of firms with measure TF. Upon entering the market, they draw a constant and permanent productivity z from a given distribution. Then they decide the optimal number of teams $n(z)$ with the understanding the equilibrium value of

[31]For simplicity, assume the surplus of training is $s(z, h)$. The first order of condition of optimal training effort can be written as

$$\frac{1}{(p_1 e_i + 1)^2} s(z, h) = 2 \frac{\lambda_i}{p_0} e_i.$$

It is clear that we can identify $\frac{\lambda_i}{p_0}$ but not separate λ_i from p_0.

[32]Due to the data limitation, we do not observe individual team composition in the data.

an empty team, $S_0(z)$. Assume there is no fixed costs of creating a team, but the marginal cost of creating a team is ζ. So the cost function is $C(n) = \zeta n^2$. Therefore, firms' problem is

$$V(z) = \max_{n \geqslant 0} \{nS_0(z) - C(n)\} \tag{1.34}$$

where $C(n)$ is an increasing and convex function. Take the first order condition of Equation 1.34, we have,

$$C'(n) = S_0(z) \tag{1.35}$$

Since $S_0(z)$ is increasing in z, the optimal number of teams is increasing in terms of productivity, i.e $\frac{\partial n(z)}{\partial z} > 0$. Thus, high-productivity firms have potential larger size.

External Calibration

We set the quarterly discount rate to the unconventional value of $\beta = 0.987$ that corresponds to an annual interest rate of 5%. We calibrate the worker bargaining power to 0.2.[33]

Distribution We assume that the distribution of the permanent type of firm follows from a Beta distribution $Z \sim \mathtt{Beta}(\phi_0^f, \phi_1^f)$, and the initial distribution of the general human capital of the entire working population follows from a Beta distribution $g \sim \mathtt{Beta}(\phi_0^w, \phi_1^w)$. We further assume the share s of workers with managerial skill $m = 1$ is uniformly distributed across worker's initial general human capital. We discretize the general and firm-specific types of the worker into four equally spaced grid points of each.

To identify the distribution of workers, we set the parameters of the general skill distribution, ϕ_0^w, ϕ_1^w to match the mean and standard deviation of the highest educational achievement of those who enter the labor market for the first time. The

[33] In the standard job search model such as Mortensen and Pissarides (1994), the worker bargaining power is assumed to be 0.5. However, in the papers with careful estimation, the worker's bargaining power ranges from 0.188 (Lise et al. (2016)) to 0.72 (Shimer (2005)). We argue that the value of γ has little effect on our results since wages serve as pure transfers between firms and employees, the choice of worker's bargaining power does not affect mobility decisions.

identification strategy for uncovering the distribution of managerial skills needs more attention since it is harder to measure a worker's managerial skill. We provide more discussion in the internal calibration section.

The entry-and-exit process of workers is described by the probability σ with which a worker permanently exits the labor market and replaced by the probability distribution π from which a worker who enters the labor market draws his initial human capital. The average duration of working life in Denmark is around 35 years for the duration of working life. Therefore, the arrival rate of out of labor force shock is set to $\sigma = 0.0084$ to match the expected life-time working time.

Unemployment production function The unemployed worker's production function is described by the the flow value of unemployment b_i. We follow the specification from Herkenhoff et al. (2018) in which b_i is assumed to be equal to a fraction b_0 of the output produced alone and set $b_0 = 0.6$[34].

Matching function As standard in the labor search literature, we assume the two matching functions have Cobb-Douglas functional forms. For each market $i \in \{worker, manager\}$, the matching function is assumed to be $M_i = A_i S_i^{\eta_i} V_i^{1-\eta_i}$, where A_i represents the matching efficiency, and η_i represents the matching elasticity. The matching elasticities of two matching functions η_w and η_m are assumed to be 0.5 as in Petrongolo and Pissarides (2001).[35]

Internal Calibration and Identification

Ideally, we would identify the model parameters taking advantage of a natural experiment. However, instead of looking for such experiments, we internally estimate the remaining parameters using Simulated Method of Moments (Gourieroux et al. (1993)), as collected in Θ. To implement the approach we calculate a vector of moments from the data $m(\Theta_0)$. Counterparts to these moments, defined

[34]Freund (2022) target a ratio of 0.63 in a similar setting using the German administrative data.

[35]Theoretically speaking, we should use the vacancy-level data to back out the elasticity parameters in the matching functions. To the best of our knowledge, the STAR from the ministry of labor in Denmark collecting nationwide vacancy posting information is the fittest data for estimating the two parameters η_w and η_m.

as $m(\Theta)$, are then constructed based on the simulated moments from the model, given a parameter vector Θ. In particular, to derive precise simulated moments without compromising computation speed, we simulate two versions of the model. In the first version, we simulate the model at individual worker level for 100,000 workers. In the second version, we exogenously assign each team to each firm for $m \times 100,000 = 5,440$ unique firms at the beginning of the simulation and then simulate the model at individual team level. The benefit of simulating two versions of the model is to avoid replacement sampling at each period of time.[36] For the first version of the simulation, we simulate the model for 140 quarters (35 years). For the first version of the simulation, we simulate the model for 2000 periods to derive a stationary wage distribution. The procedure then is to iterate on the parameters so as to bring the moments constructed from the simulated data as close as possible to the ones estimated from the data. Parameters are jointly disciplined by matching the simulated moments to data counterparts. Specifically we find a value for Θ to maximize

$$\Theta = \left\{ s, \delta_w, \delta_m, A_w, A_m, \xi, \xi_u, \lambda_f, \lambda_g, \phi_0^f, \phi_1^f, f_0, \alpha, \eta, \omega, \underline{h} \right\}$$

The corresponding estimator is shown below,

$$\hat{\Theta} = \arg\min_{\Theta} [m(\Theta_0) - m(\Theta)]' D [m(\Theta_0) - m(\Theta)]$$

where D is a diagonal weighting matrix whose diagonal is the inverse of the squared value of the data moments. We explain the identification strategies in detail.

External Labor Friction The seven parameters in Θ : $\delta_w, \delta_m, A_w, A_m, \xi, \xi_u, \xi_w$

[36] Suppose we only simulate at worker level, we have to keep track of their team number to compute firm-level statistics. However, including team number as a state variable is nontrival since a team does not vanish even if it is empty. A team may have more than 2 people if we randomly assign a worker to a team. Similarly, it is unnecessary to keep track of worker ID in each team for the sake of computation time. Another advantage of team-level simulation is to measure within-firm promotion accurately. The theoretical model avoids within-firm promotion in subtle way under the law of large numbers assumption. In the simulation with finite number of teams, within-firm-across-team promotion will be counted as internal promotion.

discipline the external labor market friction. We target the relevant steady state level of the corresponding worker flow rate across jobs conditional on employer-to-employer transition. Specifically, the employment-to-unemployment EU rate of all workers and managers identifies the job destruction rate δ_w and δ_m. The unemployment-to-employment (UE) rate identify the relative searching intensity ξ. We identify the two matching efficiencies of the matching functions of each labor market and information friction parameter ξ_w using the time series data on four job-to-job ($J2J$) rate between two positions across firms, i.e. manager-to-worker ($M2W$), manager-to-manager ($M2M$), worker-to-worker ($W2W$), and worker-to-manager ($W2M$) that are three-month moving average of mobility.

Identify Managerial Skills and Threshold of Managerial Human Capital To uncover the latent managerial skill of workers, we leverage the life-cycle simulation of each worker to identify the size of s and the threshold of required human capital $\underline{h}$ by targeting the share of unemployed workers whose first job is a manager conditional upon flow out of the unemployment pool within a year. However, given the quarterly unemployed-to-manager rate, we are only able to identify s and $\underline{h}$ jointly. Therefore, to separately identify the two parameters, we also target the mean of IPS. The higher s, the higher the IPS is.

Internal Labor Friction The two training cost parameters, λ_g and λ_f, governing the marginal training costs determine the internal labor market friction. First of all, the overall size of λ_g and λ_f are pinned down by the share of internal promotion given the external labor market friction calibrated above. The internal promotion share points to the difference in the size of external labor market friction and marginal costs of internal promotion. In addition, the ratio of managers over the life cycle also identifies the speed of human capital accumulation.

In the second step, to disentangle general training and firm-specific training without observing the firm-level training data, we compare the employment trajectories of two types of managers. Type-1 managers are those who are promoted to managerial positions within the current firm, while type-2 managers are those who enter the current firm as managers. The discrepancy between the probabilities of the next job being managers conditional on employment-to-employment identifies the relative importance of general and firm-specific human capital on internal promotion, and thus disciplines the size of λ_g and λ_f. The logic behind this identification strategy is that if internal promotion is mostly due to general human capital accumulation, the gap between two data moments is expected to be close. However, suppose firm-specific human capital is crucial for internal promotion. In that case, managers who are promoted internally are significantly less likely to move to another managerial job through employment-to-employment transition.

Production function The six key parameters in $\Theta : \lambda, f_0, \alpha, \eta, \omega$ models the firm production function. The challenges of identifying the parameters are two-fold: (1) we do not directly observe team structure and output in the data; (2) we do not observe latent type of firms and employees. In this subsection, we describe the relevant moments for the identification the parameters. Parameters are jointly disciplined by matching the simulated moments to data counterparts. We use the following moments to estimate the parameters.

Specifically, we follow two steps in identifying the firm production function. In the first step, we estimate firm fixed effect as a proxy for firm type z following Arcidiacono et al. (2012). Two arguments illustrates our choice of methodology. First, the firm type z is broader than the definition of the total factor productivity (TFP) of a firm because the literature assumes a Hicks-neutral production function that TFP is factor-augmenting.[37] Second, the standard AKM-style (Abowd et al. (1999)) wage regression is not appropriate for our setting since they abstract away the effects of coworkers or hierarchies estimating individual wages, even after correcting the bias in Kline et al. (2020). In the second step, we describe firm classification concept based on a revealed preference argument - poaching rank as in Bagger and Lentz (2019)[38] to identify the sorting across hierarchy within the firm. We describe the details below.

First, following Arcidiacono et al. (2012), we estimate a peer effect model by incorporating a within-firm hierarchy component into the canonical AKM econo-

[37]We estimate TFP using previous method such as Olley and Pakes (1992), Levinsohn and Petrin (2003) and Ackerberg et al. (2015) in the appendix to validate our fixed effect estimates.

[38]Other papers that use job-to-job transitions as revealed preference are Sorkin (2018) and Taber and Vejlin (2020).

metric specification as expressed below.

$$y_{ijlt} = \alpha_i + \beta\bar{\alpha}_{-i,j,-l,t} + \chi_j + X_{it} + \epsilon_{it}. \tag{1.36}$$

where y_{ijlt} is the log hourly wage of individual i at hierarchy level l in firm j in month t. We assume it is a function of individual fixed effects α_i, which captures an employee's unobserved permanent characteristics such as ability, peer effect $\bar{\alpha}_{-i,j,-l,t}$, and firm fixed effects χ_j, which measures the firm unobserved permanent characteristics such as average productivity. The covariates X_{it} are the time-varying observables,including age squared, tenure, tenures squared,log firm size, occupation fixed effects, industry fixed effects, and year fixed effects. ϵ_{it} is the error term.

To valid our estimates of firm fixed effects, we show the correlation with several standard measures of firm productivity, including: (1) labor productivity, which is measured as value added per worker (2) worker and manager wage rank, measured as average worker wage and average manager wage respectively. We can see that they are highly correlated.[39]

Across-firm wage inequality The firm production complementarity α governs the elasticity between firm type z and team production. We target the across-firm wage inequality. The cutoff value of α is 1. When $\alpha > 1$, firm type z and team production are submodular. Therefore, high-type z matches low-skill workers and managers. By contrast, when $\alpha > 1$, firm type z and team production are supermodular. Therefore, high-type z value high-skill workers and managers.

Covariance between worker and manager wage The team production complementarity η governs the elasticity between the manager and the worker. We target the covariance between worker and manager wage to pin down the size of η after controlling the covariance between the firm type z and the team.

Wage ratio between worker and manager The production weight ω governs the relative importance of a manager relative to a worker in the production process,

[39]The covariance between our estimates and labor productivity is 0.29, the covariance between our estimates and worker wage rank is 0.49 he covariance between our estimates and manager wage rank is 0.39.

which pins down the wage ratio of managers and workers.

Wage level and wage ratio Lastly, as a normalization parameter, f_0 is pinned down by the average wage.

Calibration Results and Model Fit

Table 1.3 summarizes the parameter description and parameter values, including the internally calibrated in the model and externally calibrated. There are several calibration results that worth discussion.

First, the estimates of two matching functions imply that the quarterly vacancy meeting rate of a managerial job is 0.0152. Compared with the vacancy meeting rate of a production job, which is 0.4272, it only takes about two and half quarters to meet a production worker. The caveat is that these are the meeting rate instead of the filling rate since to make a match, the offer value has to be larger than the outside option.

Second, the team production complementarity parameter η is smaller than 1, meaning that the production function is supermodular within a firm, pointing to positive sorting between hierarchies. This value is close to the team production elasticities estimated in previous literature studying how production teams are formed among coworkers (e.g, Herkenhoff et al. (2018), Freund (2022), Hong (2022)). However, in our setting, a manager and a worker are in a superior-subordinate relationship instead of in the same hierarchy. Moreover, both the fact that η is close to 1, which suffices a linear production function and production weight being close to 0.5 signals the strong complementarity between a worker and a manager. This is not surprising given the relatively high education achievement and small wage gap between managers and workers compared to other economies, such as the US.

Lastly, the probability of the next job being a manager conditional on employment-to-employment of two types of managers is close, which suggests the dominance of general human capital in the determination of promotion decisions.

Table 1.4 reports the model fit in the labor market worker flow rates as well as firm structure and wages. As shown in the table, in general, the model is able to match the level of labor market transition and wage payment across jobs well. To assess the model's ability in reproducing the life-cycle career, we plot the probability of being a manager of unemployed who find a job within one year since layoff for ten years in Figure 1.3.

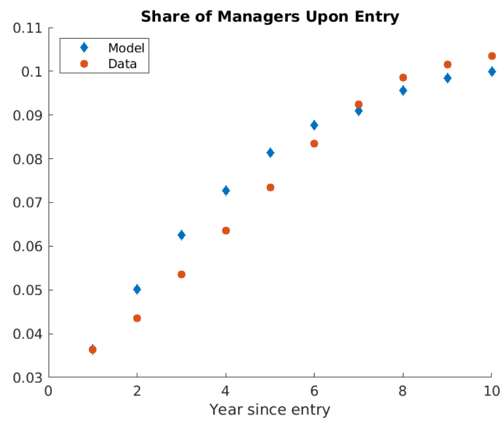


Figure 1.3: Model Fit: Manager Share Over Time
Notes: This figure plots the probability of a managerial job over the first ten years of career of unemployed.

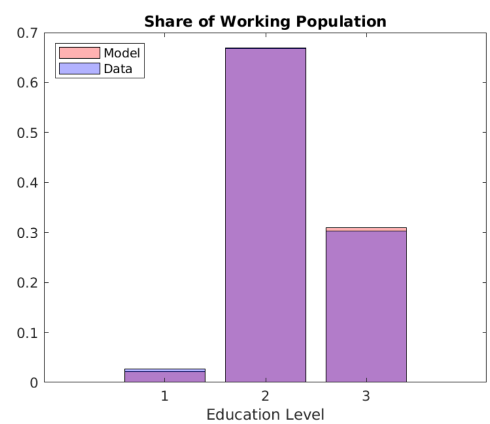


Figure 1.4: Model Fit: Initial Distribution of Labor by Education Category
Notes: This figure plots the initial distribution of the working population in the data and the model.

Next, we examine how well the model can fit the poaching rank over firm type

z. First of all, as we can see that the model is able to match the average poaching rank level of both workers and managers.

1.6 Counterfactual Experiments

In this section, we quantify the role of internal labor markets on the aggregate economy through the lens of the calibrated model. In subsection 1.6, we show how firms' managerial hiring strategies react to . In subsection 1.6, we examine the role of internal promotion on aggregate productivity. Lastly, we ask how to increase productivity by changing two frictions.

Why firms conduct internal promotion?

To understand firms' managerial hiring decisions, we quantify the effects of two channels on firms' internal promotion share and aggregate moments. Specifically, we conduct two experiments. In the first experiment, we set the λ_f to be 10% lower than the baseline. In the second experiment, we set the A_m to be 10% lower than the baseline. The results are shown in Table 1.5.

With the decline of firm-specific training costs or matching efficiency of the manager market, the internal labor market friction is smaller, and thus internal promotion share is larger. Since team do not search for managers externally but search for workers, it is more difficult for managers to reallocate and less workers search in the manager market. In the former case, the worker market is not affected by the shock to the manager market that both the job seekers and vacancies increase proportionally and thus we see a small negative effect on the E2E rate of workers, which signals the existence of a vacancy chain. However, in the second experiment, firms substitute managers for workers, and thus more vacancies posted than job seekers, boosting up the E2E rate of workers.

We draw two main findings in the table 1.5. First of all, both firm-specific human capital investment and labor market friction contribute significantly to firms' promotion decisions. Second, the internal promotion share is sensitive to the change of external labor market friction that the change of internal promotion share due to decrease in external labor market friction is three To examine firms' hiring strategy across firm productivity, we illustrate the firms' investment change across firm type z in figure 1.5. The left panel plots the percentage change of IPS due to changes in training costs and search friction, and right panel plots the ratio of two percentage changes. It is evident that firms among all types use more internal labor market when hiring a manager. However, there is a heterogeneous treatment effects across firm types that low-type firms are the most responsive to the change of both internal and external labor market frictions. Moreover, low-type firms' investment decisions are more sensitive to the change of firm-specific training costs.

Figure 1.5: Managerial Hiring Strategy Heterogeneity

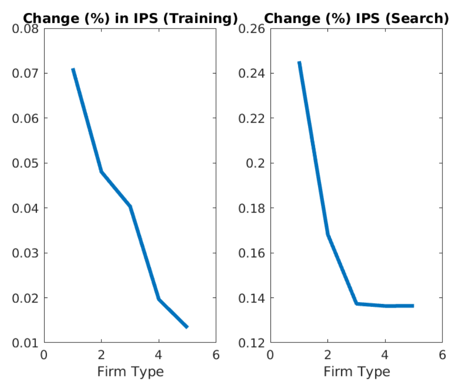


Figure 3.a

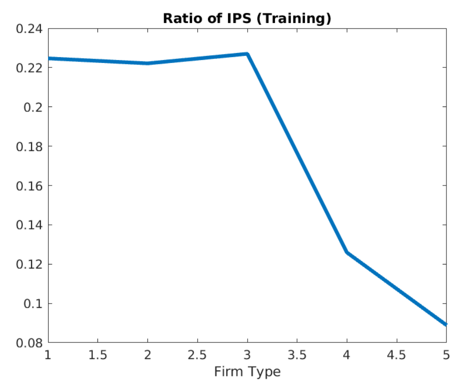


Figure 3.b

Notes: This figure plots the response of managerial hiring strategy across firm type z. The left panel plots the percentage change of internal promotion share due to firm-specific training costs and search efficiency in the manager market. The right panel plots the ratio of two changes.

Internal Promotion and Aggregate Output

In this subsection, we use model estimates to understand the role of the internal labor market in determining aggregate production and aggregate welfare, measured by total production less of the training costs. Specifically, we conduct an extreme exercise in which internal promotion is banned but keep every aspect of the model the same to see how internal labor markets affect the economy on the aggregate level. The results are shown in Table 1.6. The column (1) records the statistics of the average internal promotion share, the unemployment rate, the aggregate output, and the welfare of the baseline model with internal promotion, while the column (2) records that of the counterfactual economy in which internal promotion is shut down.

By comparing column (1) and (2) of Table 1.6, we see that the aggregate output and welfare are 6.76% and 6.57% lower when promotion is forbidden. Moreover, shutting down internal promotion raises the unemployment rate significantly by 62.59%, and decreases the transition rate of workers by 55.5% and that of managers by 18.69%. The results are not surprising since the counterfactual economy is more restrictive than the baseline model, and thus the option value of employment is lower and makes unemployment more appealing to labor. Although the sign of the changes in output is reasonable, multiple channels intertwine with each other. We explore the possible mechanisms in the next subsection.

Mechanisms

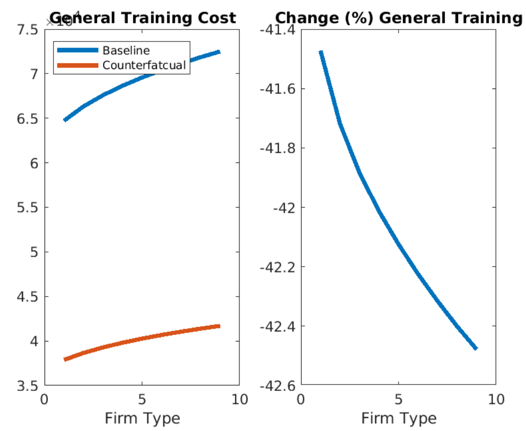
To understand how internal promotion affects the aggregate output and welfare, we plot firms' heterogeneous training strategies across firm types and the changes in stationary distribution among firms, workers, and jobs when we shut down promotion.

First, internal promotion raises the outside option of a worker if a worker is swayed to another firm because the worker could be promoted if qualified in the future. Recall that both general and firm-specific training provide an incentive to promote workers internally. Therefore, shutting down internal promotion decreases workers' skill levels. We plot the human capital investment in Figure 1.6 of workers and in Figure 1.7 of managers under both scenarios. First of all, as predicted by proposition 2, high-type firms provide more training to their workers. Second, when internal promotion is prohibited, firms across all productivity put less effort on firm-specific training and general training. Moreover, high-productivity firms see larger drops in both types of training investment since they benefit most from promotion. The explanation is that banning internal promotion decreases the option value of training regardless of the type of training. And thus, firms put less effort into training. However, shutting down internal promotion also dampens the positive sorting between firms and low-level jobs in the equilibrium. Good workers move from high-type firms to low-type firms, incentivizing low-type firms to invest more in training.

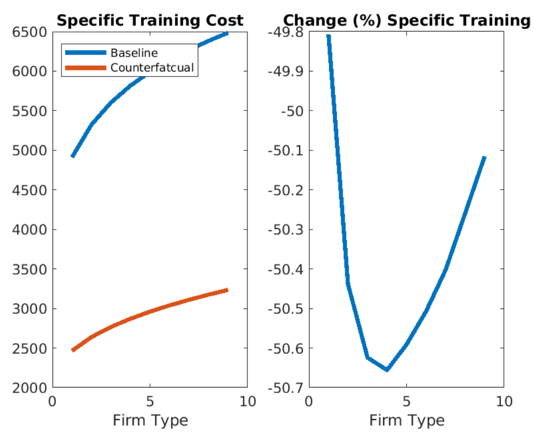
Yet, contrary to the prediction of training investment of workers, firms increase firm-specific and general training of managers. The effects are more salient among high-productivity firms. The intuition behind it is that the poaching rate of a manager is lower when switching off internal promotion, and thus the value of a manager is higher due to external labor market friction. Therefore, it is more valuable to train managers.

As in Figure 1.8, which plots the average human capital of managers and workers in full matches conditional on firm type. The solid(dotted) blue curve represents the average human capital, i.e., $h = g + f$, of workers in a full team under the baseline(counterfactual) model with(without) promotion, while the solid(dotted) blue curve represents the average human capital of managers in a full team in the baseline(counterfactual) model with(without) promotion. The figure delivers two pieces of information. First, worker's human capital is an increasing function with respect to firm type z, which is mainly due to the supermodularity between firm type and team production in the outside nest of the firm production function. However, a manager's human capital is a flat function with respect to firm type z, suggesting there is a small dispersion in managers' productivity across firm productivity. This result echoes the findings in Gabaix and Landier (2008) in understanding the CEO payment across firms. The second message is that the human capital of workers is lower and the human capital of managers is higher when shutting down internal promotion as shown in Figure 1.7.

To see how general equilibrium changes, we show the sorting pattern between



(a) General Training

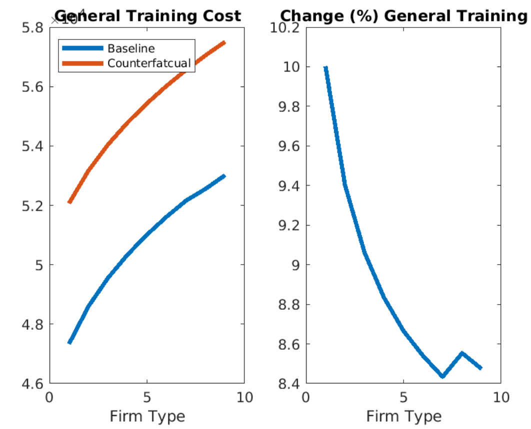


(b) Specific Training

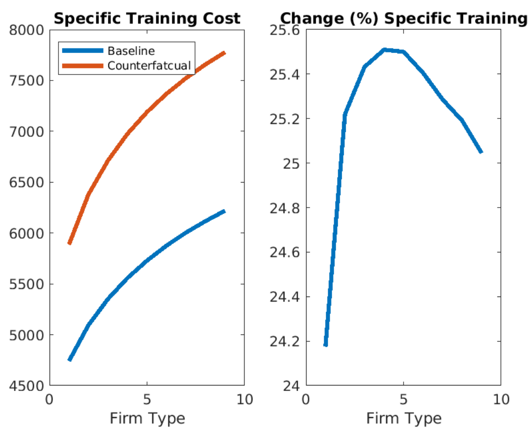
Figure 1.6: Change of General and Specific Training on Workers and Firm Type

Notes: This figure plots the optimal training investment by firms and workers and changes between the baseline model and counterfactual exercise. Panel (a) plots general training costs and panel (b) plots firm-specific training costs.

firms, workers, and jobs in Figure 1.8, which plots the average human capital of managers and workers in full matches conditional on firm type. Banning internal promotion has two effects on organizational structure. First, the measure of full teams decreases and the measure of one-worker teams increases due to the lower vacancy-filling rate of the managerial positions shown in Figure 1.9. The second effect is a general equilibrium effect regarding the changes of sorting patterns be-



(a) General Training



(b) Specific Training

Figure 1.7: Change of General and Specific Training on Managers and Firm Type

Notes: This figure plots the optimal training investment by firms and managers and changes between the baseline model and counterfactual exercise. Panel (a) plots general training costs and panel (b) plots firm-specific training costs.

tween firms, labor, and jobs. Promotion affects the stationary distribution in the equilibrium through two channels. First, since workers can no longer be promoted internally, high-productivity firms lose the privilege competing for potential managerial talents in the entry-level market by promising a high option value to job seekers. As a result, workers with high general skills and managerial skills worker hired at high-productivity firms working as production workers are now poached

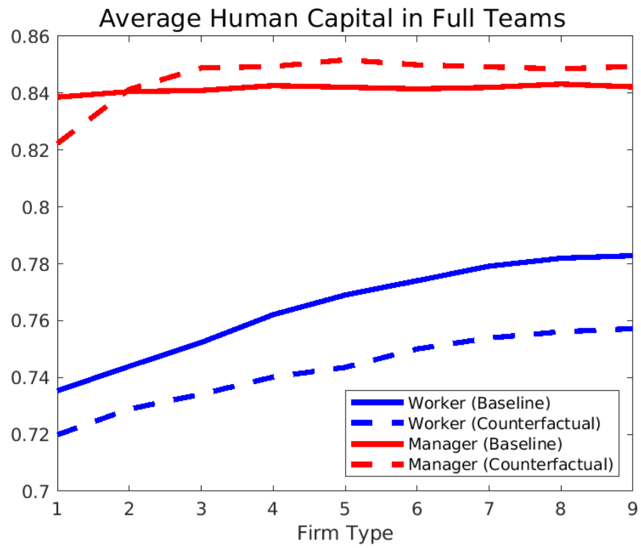


Figure 1.8: Average Labor Productivity

Notes: This figure plots the average human capital, i.e., $h = g + f$, of workers (blue solid line) and managers (red solid line) across firm type z under the baseline model. The blue dotted line and the red dotted line represent the average human capital of workers and managers respectively in counterfactual exercise (no promotion).

by medium-productivity firms and sway to managerial positions. The hypothesis is supported by Figure 1.8 that the slope of worker's human capital across firm type is flatter without promotion than with promotion.

To observe how the match distribution changes in a more straightforward way, we show the joint distribution between workers and managers with heterogeneous human capital in full teams in Figure 1.10. By comparison, matches are concentrated in pairs with workers having managerial skills. This is because more labor with $m = 1$ are hired into low-hierarchy jobs in the counterfactual economy as shown on the top of Figure 1.10. We plot the change of the distribution of one-manager teams in B.1.

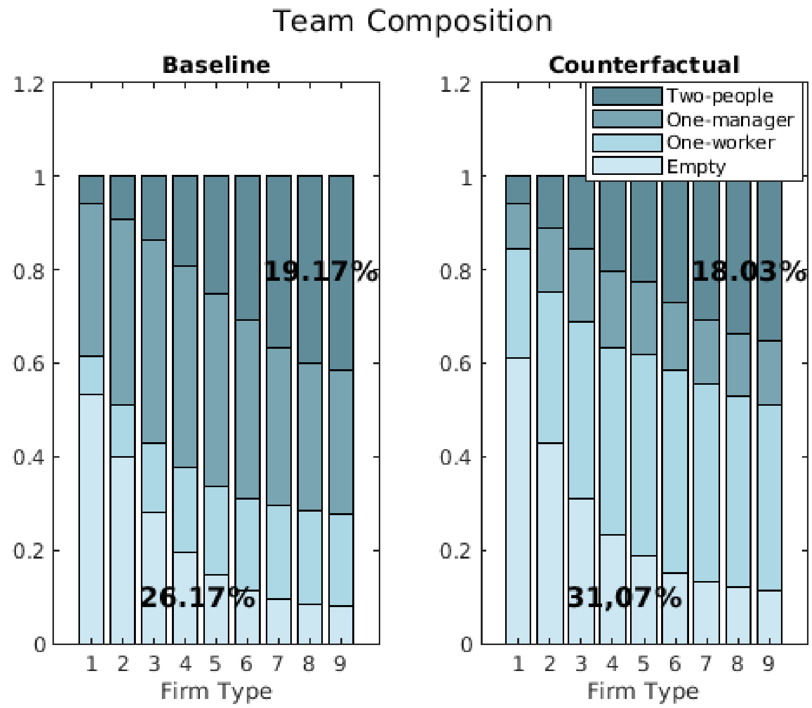


Figure 1.9: Composition of Teams by Firm Type

Notes: This figure plots the composition of four types of teams across firm type z. The areas from bottom to top represent the measure of empty teams, one-worker teams, one-manager teams, and full teams (a worker with a manager) respectively. The left panel shows the distribution of teams under the baseline model and the right panel shows the distribution of teams under counterfactual exercise (no promotion).

1.7 Concluding Remarks

Not all employment movements are subject to labor market friction. An employee can switch jobs within the firm if an opportunity arises. Conceptually, the internal margin introduces a channel of employment transition, whereas the conventional theory conceptualizes all transitions are external and frictional.

In this paper, we focus on a specific type of internal mobility - promotion. Grossmann (2007) and Minni (2023) show that good managers play a vital role in the effective functioning and boost the productivity of firms. We first document empirical facts that support and highlight the prevalence of the internal labor market

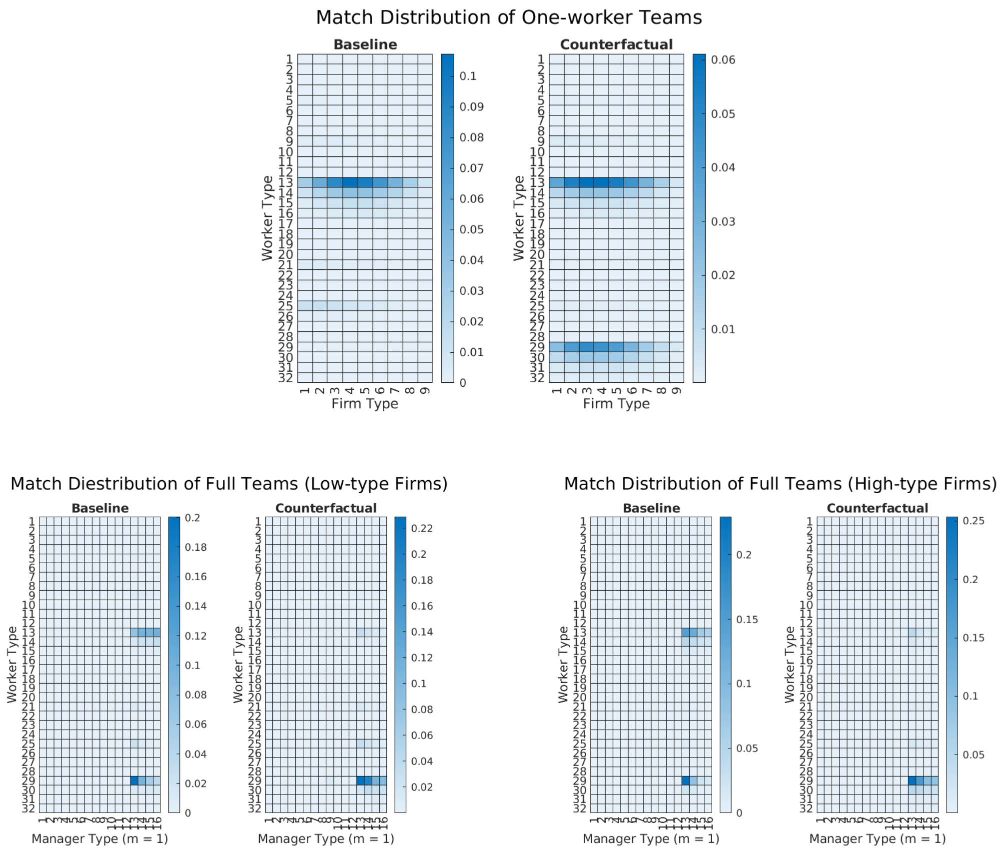


Figure 1.10: Match Distributions

Notes: The figures plot the distribution of matches across firm type z under the baseline model and counterfactual exercise (no promotion). The figure on the top illustrates the distribution of one-worker teams and the bottom two figures illustrate the distribution of full teams.

in managerial hiring. First, the paper provides aggregate-level evidence that internal promotion is widely adopted both within and across industries, emphasizing the empirical relevance of the internal mobility channel. Second, a micro-level analysis examines the relationship between internal promotion share and firm-level characteristics, especially productivity. A positive estimate indicates that high-productivity firms prefer promotion over external hire than low-productivity firms.

We then develop a rich and novel labor search model under frictional external labor market friction, featuring job mobility within and between firms to shed light on firms' managerial hiring strategy and the role of internal labor markets on aggregate output and welfare. In the model, a firm consists of two types of jobs: production jobs and managerial jobs, which form a within-firm job ladder. Promotion takes place whenever the managerial position is vacant. Firms strategically set hiring rules, giving consideration to external labor market friction and the qualifications of the internal candidate. A calibrated model reproduces firm-level heterogeneous managerial hiring strategy observed in the data.

Recognizing the promotion margin under frictional labor markets is crucial to understanding how promotion affects aggregate welfare. In the first channel, promotion affects the sorting pattern between firms and entry-level jobs since the internal promotion channel makes the sorting incentives between firms and workers pass down from high-level jobs to entry-level jobs. The second channel is through the general and firm-specific human capital accumulation channel. Internal promotion provides an incentive to retain the worker. As a result, the incentive of training is dampened when the possibility of internal promotion is eliminated. Therefore, the aggregate production is lower due to less human capital accumulation.

The analysis so far points to several possible avenues for future research. First, although the current paper extends the standard one-worker-one-firm model to the two-worker-one-firm model to incorporate the promotion margin in a minimal setup as a first step, it is a natural next step to understand richer implications of promotion on firm dynamics with an explicit organizational structure. For example, allowing for worker competition at non-managerial positions nests the Perseonnel literature focusing on incentives.

Second, this paper points to the importance of firm-specific and general human capital accumulation in promotion decisions, providing a novel insight into the heated debate on non-compete contracts. If the knowledge required for managers is firm-specific rather than general, banning non-compete contracts might be welfare-improving through the lens of our model.

Third, this paper abstracts away other possible mechanisms that may distort firms' managerial hiring strategies. For example, firms may prefer external hiring because the new blood stimulates innovation and growth. Therefore, it is an exciting direction to study the interaction between internal promotion and innovation dispersion across firms.

Lastly, the model strives for simplicity and ignores the life cycle of firms. Do firms at different ages conduct heterogenous managerial hiring strategies? How does internal promotion affect firm growth? These are interesting questions to delve into to understand the life cycle of firms.

Table 1.1: Summary Statistics: All Employee

	Non-manager	Manager
Age	41.51	44.75
	(9.67)	(8.22)
Tenure	4.04	4.32
	(2.40)	(2.38)
Gender	0.46	0.25
	(0.50)	(0.43)
Education	2.36	2.41
	(0.50)	(0.51)
Experience	8.37	9.24
	(5.09)	(5.42)
log(hourly wage)	5.35	5.80
	(0.42)	(0.56)
Observations	306,375	42,873

Note: Table sourced from the Integrated Database for Labour Market Research (IDA) and BFL in Denmark.Tenure and experience are measured in years. Gender $= 0 (= 1)$ represents male (female). Education $= 1$ if a worker's highest education level is primary education; Education $= 2$ if a worker's highest education level is secondary education; Education $= 3$ if a worker's highest education level is college or above. Hourly wages are real wages. Standard deviations are in the parenthesis.

Table 1.2: Summary Statistics: All Firms

	Total sample	Manufacturing	Construction	Wholesales	Service
Firm age	29.87	36.09	28.97	34.18	22.54
	(22.56)	(25.45)	(21.54)	(23.49)	(17.32)
Employment	3727.40	2393.21	490.98	7508.63	2147.57
	(7789.14)	(4063.13)	(498.91)	(12341.27)	(3344.88)
Wage ratio	1.95	2.01	1.68	1.91	1.98
	(0.73)	(0.62)	(0.49)	(0.91)	(0.65)
Span of control	16.91	12.28	9.29	14.38	23.40
	(29.77)	(14.44)	(13.45)	(35.61)	(32.90)
Promotion share	0.36	0.37	0.27	0.40	0.35
	(0.21)	(0.21)	(0.18)	(0.20)	(0.22)
Value added per employee	0.58	0.84	0.54	0.42	0.54
	(0.55)	(0.60)	(0.33)	(0.40)	(0.57)
Sales	6133.76	9172.15	906.52	10003.92	1895.77
	(12275.94)	(16269.57)	(1079.29)	(14514.86)	(3367.00)
Unique number of firms	4036	1051	273	1321	1356

Note: Table sourced from the firm-level data (FIRM) in Denmark. Firm age are measured in years. Employment represents the firm size in terms of full-time workers. Wage ratio is the ratio of average managers' wage to average workers' wage. Span of control is defined as number of workers per manager. Promotion share measures the percentage of managers hired from internal promotion from 2010 of 2018 of each firm. Standard deviations are in the parenthesis.

Table 1.3: Parameter Estimates

Parameters	Description	Value
Externally calibrated		
β	Quarterly discount rate	0.987
σ	Death rate	0.0084
η_w	Matching elasticity of worker market	0.5
η_m	Matching elasticity of manager market	0.5
Internally calibrated		
Distribution		
Labor Market Transition		
δ_w	Exogenous destruction rate of workers	0.0300
δ_m	Exogenous destruction rate of managers	0.0160
A_w	Matching efficiency	0.4445
A_m	Matching efficiency	0.0141
λ_f	Specific Human Capital investment cost function	4535.4991
λ_g	General Human Capital investment cost function	203.6139
Production Function		
f_0	Normalization	1015.1009
η	Team production elasticity	0.9491
α	Firm production elasticity	0.8597
ω	Production weight	0.4680
$\underline{h}$	Human capital threshold of managers	0.7079

Notes: This table reports parameters and calibrated values.

Table 1.4: Auxiliary Model and Data

Moments	Data	Model
Quarterly W2W rate	0.0384	0.0360
Quarterly W2M rate	0.0009	0.0011
Quarterly M2W rate	0.0123	0.0116
Quarterly M2M rate	0.0246	0.0221
Quarterly EU rate of workers	0.0300	0.0301
Quarterly EU rate of managers	0.0160	0.0167
Quarterly U2W rate	0.3814	0.3696
Quarterly U2M rate	0.0110	0.0199
Share of M2M conditional on promotion	0.6255	0.5867
IPS	0.2811	0.2645
log(Average workers' wage)	5.3759	5.3562
log(Average managers' wage)	5.9567	6.0039
SD of workers' wage	5.3931	5.3075
SD of managers' wage	5.6720	5.4826
Cov(Worker wage, Manager wage)	0.4400	0.4314
Across-firm wage inequality	56.4495	55.2198
Share of First Managerial Job	0.0363	0.0362

Notes: This table lists the target moment informative to identifying the parameters in data and the counterpart of simulation results in the axillary model, including average internal promotion (IPS), quarterly unemployment rate, worker-to-worker (W2W), worker-to-manager (W2M), manager-to-worker (M2W), manager-to-manager (M2M), unemployment-to-worker (U2W), unemployment-to-manager (U2M), and statistics of wages of workers and managers. All the transition rates are seasonally adjusted in the data, quarterly averaged, logged, and HP-detrended.

Table 1.5: Internal Promotion and Changes of Friction

	E2E rate (W)	E2E rate (M)	IPS	Total output
(1) Baseline	0.0371	0.0337	0.2645	854.5433
(2) $\lambda_f \downarrow$	0.0371	0.0331	0.2655	856.7463
$\Delta\% = (2) - (1)$	0.48% $\downarrow$	11.22% $\downarrow$	8.60% $\uparrow$	2.56% $\uparrow$
(3) $A_m \downarrow$	0.0339	0.0325	0.2696	848.4887
$\Delta\% = (3) - (1)$	2.16% $\uparrow$	11.73% $\downarrow$	25.03 % $\uparrow$	1.17% $\uparrow$

Table 1.6: Counterfactual: No Promotion

	(1) Baseline	(2) No Promotion	(3) $\Delta = ((2) - (1))/(1)$
IPS	0.2811	0	-
Unemployment	0.0671	0.1091	62.59% ↑
Job finding rate (worker market)	0.4624	0.4061	12.18% ↓
Job finding rate (manager market)	0.0131	0.0241	83.97% ↑
Vacancy filling rate (worker market)	0.4274	0.4866	13.86% ↑
Vacancy filling rate (manager market)	0.0152	0.0083	45.39% ↓
Output	854.5433	796.8138	6.76% ↓
Welfare	843.4255	788.0449	6.57% ↓

Notes: This table summarizes the (unweighted) average internal promotion (IPS), quarterly unemployment rate, worker-to-worker (W2W), worker-to-manager (W2M), manager-to-worker (M2W), manager-to-manager (M2M), aggregate output, and total welfare, which is equivalent to subtracting training costs from output under the baseline model (column 1) and counterfactual (column 2) exercises without promotion. The final column is computed as the percentage change of column (2) compared to column (1).

A.1 Appendix: Data and Theoretical

Key Variable Explanation

Occupation Code DISCO-08 is a 6 digit variable which contains information about the type of job held by an individual. The first digit places the employee within one of the following broad categories:

The subsequent digits contain further information on the nature of the job. Using the first two digits the labor market is segmented into 36 groups and using the third digit as well the number of groups is increased to 147. For four, five, and six digits the number of groups is 492, 774, and 791, respectively.

To deal with the missing and highly misspecified occupation codes, we impute these in three possible scenarios. In the first scenario, the missing occupation codes are in the middle of a worker's worker-firm working spell, defined as a uninterrupted employment history in a specific firm. We impute it to the previous occupation code if the previous and after occupation codes are identical. In the second scenario, the missing occupation code is the first observation in a worker-firm. We assume it is due to probation. We impute the occupation code if the following two occupations are not missing and identical. In the third scenario, we cope with the misspecified occupation codes lie in the first observation for each worker-firm spell. We correct them to the second occupation codes if the first occupation belong to DISCO group 1 but the second and third occupation codes are different.

Missing Occupation Code In our sample used for measuring worker mobility, we see approximately 17.8% observations with missing occupation codes. To explore whether missing occupation codes is firm-specific or industry-specific, we construct a variable

$$M_j = \frac{\text{Number of workers without an occupation code}}{\text{Total number of workers}}$$

of each firm j. On average, 17% of workers in the same firm do not have an occupation code. We plot the distribution of this variable across firms and industries

Table A.1: Statistics Denmark's Subject Classification (DISCO-08)

First digit	Job responsibility	Included industry
0	Military work	
1	Management work	Management of main activities of all industries
2	Work that requires knowledge at the highest level	Engineering, health-care, researching, law and finance
3	Work that requires knowledge at a medium level	Technical positions (shipping and aviation, process management, information and communication work)
4	General office and customer service work	Secretary and customer service
5	Service and sales work	Kitchen work, hairdressing, child care, police and firefighter
6	Agricultural, forestry, and fishing	
7	Craftsmanship	Carpentry, welding, textile and clothing, installation and reparation of electronic devices
8	Operator and assembly work as well as transport work	
9	Other manual work	

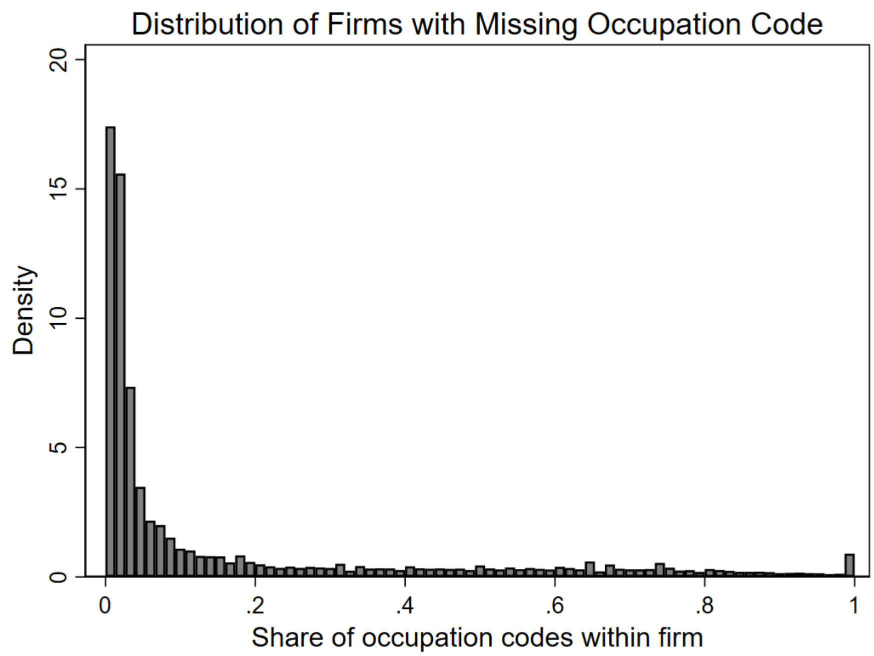


Figure A.1: Distribution of Firms with Missing Occupation Codes

in figure A.1. Generally, the missing occupation codes could due to both sides of the labor market. We first examine whether the missing occupation codes is more salient among some type of employees. For example, firms may opt to not report low-skill workers in order to save operation costs. By observation, it is unlikely that the missing occupation code is worker-specific. We find that workers with a missing occupation code this year may have an occupation code next year. Next, to explore whether this feature is firm-specific or industry-specific, we examine two characteristics: (1) firm size (2) industry.

Figure A.2 illustrates the distribution of M_j conditional on average firm size. The left panel plots the distribution of the missing ratio among firms with less than 20 workers, and right panel plots the distribution of the missing ratio among firms with larger than 20 workers. It is clear that small firms have both a higher ratio of workers with missing occupation codes and a larger dispersion across firms. Specifically, 49% of workers without occupation codes among small firms, while

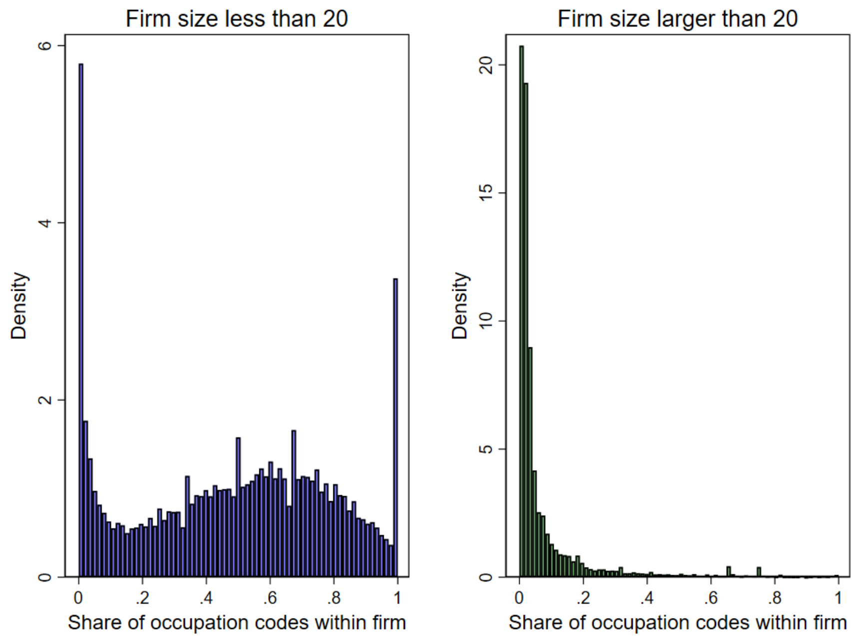
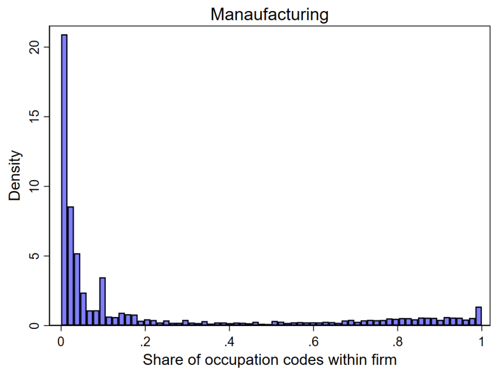


Figure A.2: Distribution of Firms with Missing Occupation Codes by Firm Size

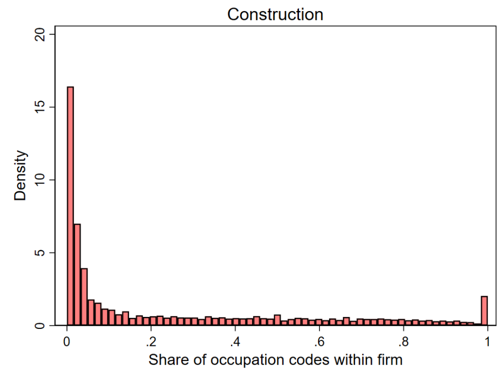
this figure is 8% among large firms. Meanwhile, the standard deviation is 0.3 for small firms while is 0.15 for large firms.

Figure A.3 the distribution of M_j of four main industries: manufacturing, construction, wholesales, and service. By first examination, we find the share of missing occupation codes in the manufacturing and construction industries is larger than the rest of two industries. The means (standard deviations) of the share of missing occupations are 0.22 (0.32), 0.25 (0.31), 0.18 (0.26), and 0.15 (0.24) respectively.

In sum, we find firms with missing occupation codes reported are concentrated in firms with employment sizes less than 20, and in the manufacturing and construction industries.

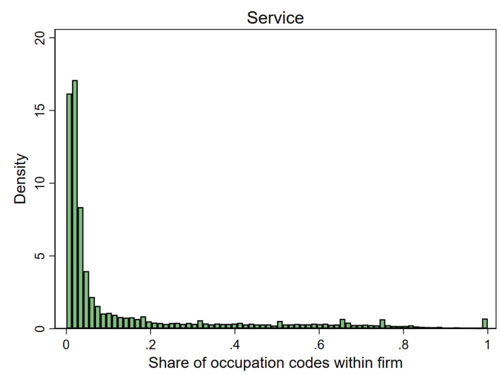


Manufacturing



Construction

Wholesales



Service

Figure A.3: Distribution of Firms with Missing Occupation Codes by Industry

Sample Selection

Main job We correct the starting and end dates of overlapped observations in the following three scenarios. In the first scenario, two observations completely overlap in the sense that the starting date of observation A is later than the starting date of the other observation B but the end date of observation A is before the end date of B. Then we drop the observation A. In the second scenario, two observations partially overlap. If two observations have the same firms. We combine two observations if two observations have identical occupation codes and combine two observations, otherwise change the starting date of the latter observation be the end date of the first observation.

Starting from the raw matched employer-employee data, we apply the following sample selection criteria.

First, 985,220 observations (about 1.60% of observations) in the raw data have overlaps in spell start time and end date for the same person, meaning that the worker works for multiple firms at the same time or work for different positions. In this case, it is necessary to assign the main job. Specifically, it can be one of the following five scenarios:

- Two spells are completely overlapped in the sense that spell B has later start date and earlier end date than firm A ABA. We drop the spell B regardless whether firm A and B are identical, and label firm A as the main job firm. (21,311,665 observation deleted).

- First, we discard observations on part-time workers with weekly hours less than 28 hours (21,311,665 observation deleted).

Worker Transition

Table A.2: Hierarchy Transition Matrix Upon Internal Promotion

		Next Job		
		1	2	3
Current Job	1	91.55	7.40	1.04
	2	1.21	96.93	1.86
	3	0.27	3.52	96.21

Notes: Table sourced from the spell-level data (BFL) in Denmark using the hierarchy definition defined in section 1.3. Layer 1 includes all managerial jobs (ISCO 1), layer 2 includes all jobs in ISCO 2-4, and layer 3 includes all jobs in ISCO 5-9. Each row represents the conditional probability between two consecutive jobs of each employed worker.

Table A.3: Hierarchy Transition Matrix Upon Internal Promotion (Internal)

		Next Job		
		1	2	3
Current Job	1	91.55	7.40	1.04
	2	1.21	96.93	1.86
	3	0.27	3.52	96.21

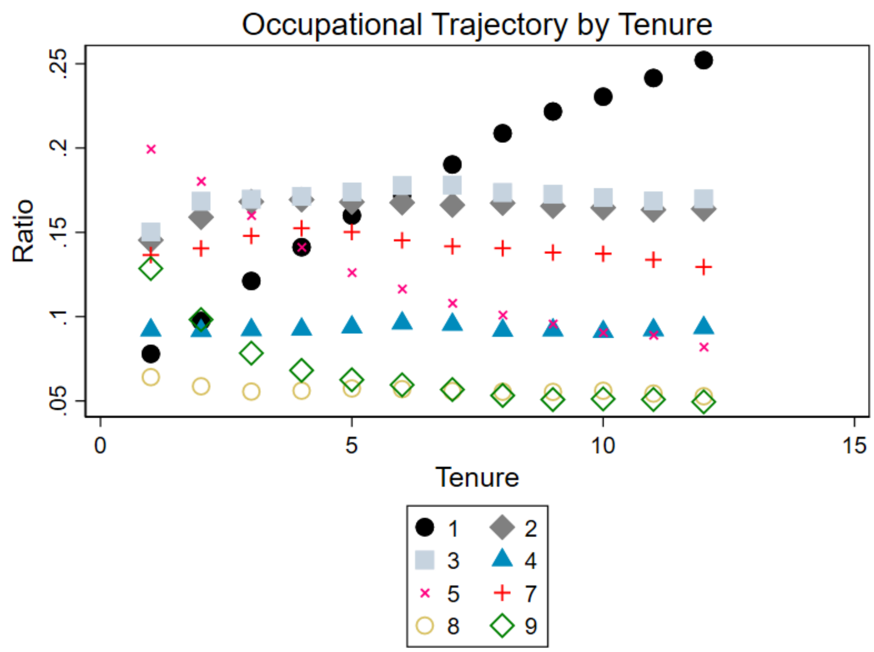
Notes: Table sourced from the spell-level data (BFL) in Denmark using the hierarchy definition defined in section 1.3. Layer 1 includes all managerial jobs (ISCO 1), layer 2 includes all jobs in ISCO 2-4, and layer 3 includes all jobs in ISCO 5-9. Each row represents the conditional probability between two consecutive jobs of each employed worker conditional within-firm mobility.

Table A.4: Hierarchy Transition Matrix Upon Internal Promotion (External)

		Next Job		
		1	2	3
Current Job	1	91.55	7.40	1.04
	2	1.21	96.93	1.86
	3	0.27	3.52	96.21

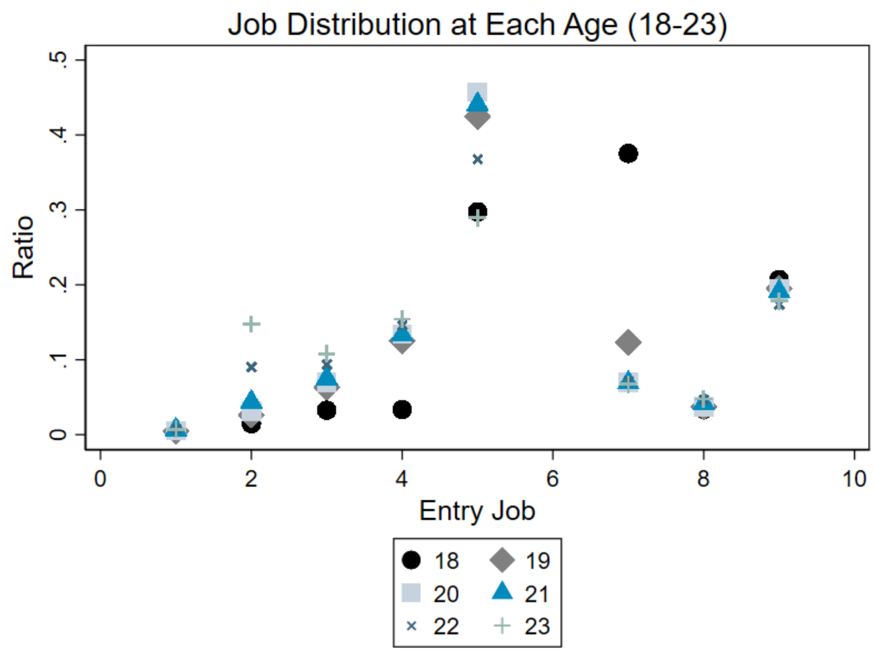
Notes: Table sourced from the spell-level data (BFL) in Denmark using the hierarchy definition defined in section 1.3. Layer 1 includes all managerial jobs (ISCO 1), layer 2 includes all jobs in ISCO 2-4, and layer 3 includes all jobs in ISCO 5-9. Each row represents the conditional probability between two consecutive jobs of each employed worker conditional across-firm mobility.

Figure A.4: Career Trajectory by Tenure



Notes: Figure plots the all occupational trajectory by tenure conditional on workers' first one-digit occupation using the Danish matched employer-employee data cover the period from 2018 to 2019. Workers aged 25-60 with full-time jobs are included. Occupations in agriculture, forestry and fishing related (DISCO CODE = 6) are dropped.

Figure A.5: Occupation Distribution of First Job



Notes: Figure plots the distribution of first-digit occupations by age conditional on workers' first observation of labor market entry using the Danish matched employer-employee data cover the period from 2018 to 2019. Workers aged 25-60 with full-time jobs are included. Occupations in agriculture, forestry and fishing related (DISCO CODE = 6) are dropped.

A.2 Appendix: Theoretical Analysis

Stationary Distribution

The measure u_i^1 of unemployed workers of type i is given by

$$
\begin{aligned}
u_i^1 = u_i \left[1 - \lambda_u \sum_y p_y P_y\right] &+ \delta \sum_z [e(z,i,0) + e(z,0,i)] \\
&+ \delta \sum_j [e(z,i,j,0) + e(z,i,0,j) + e(z,j,0,i)] \\
&+ \delta \sum_j \sum_{i_2} [e(z,i,i_2,j,z) + e(z,i_2,j,i,z)]
\end{aligned} \tag{37}
$$

where $h_y(u_i) = 1$ if the unemployed worker is hired, $h_y(u_i) = 0$ otherwise.

The measure $e^1(z, i, 0)$ of employed workers of type i without a coworker in firm type z is given by

$$
\begin{aligned}
e^1(z, i, 0) = {} & u_i \lambda_u p_0(z) h_0(u_i) W \\
& + e(z, i, 0) \left\{ 1 - \delta - \sum_{i,x} q_i(x) h_{z,i,0}(x) \right\} + \sum_{z'} e(z', i, 0) \lambda_e p_0(z) h_0(i) W \\
& + \sum_{z'} e(z', 0, i) \lambda_e p_0(z) h_0(i) W \\
& + \sum_{j} e(z, i, j, 0) \left\{ \delta + \sum_{y} \lambda_e p_y P_y(j) \right\} \\
& + \sum_{z'} \sum_{j} e(z', i, j, 0) \lambda_e p_0(z) h_0(i) W \\
& + \sum_{j} e(z, i, 0, j) \left\{ \delta + \sum_{y} \lambda_e p_y h_y(j) \right\} \\
& + \sum_{z'} \sum_{j} e(z', i, 0, j) \lambda_e p_0(z) h_0(i) W \\
& + \sum_{z'} \sum_{j} e(z', j, 0, i) \lambda_e p_0(z) h_0(i) W \\
& + \sum_{z'} \sum_{j} \sum_{i_2} e(z', i, i_2, j) \lambda_e p_0(z) h_0(i) W + e(z', i_2, j, i) \lambda_e p_0(z) h_0(i) W
\end{aligned}
\tag{38}
$$

where $I(z, i, 0) \in \{0, 1\}$ refers to the internal promotion, and $W \in \{0, 1\}$ refers to the hiring decision of an empty team whether the worker is hired as a worker

The measure $e^1(z, i, j, 0)$ of employed workers of type i with a coworker j in firm type z is given by

$$
\begin{aligned}
e^1(z,i,j,0) = {} & u_i \lambda_u p_{z,j,0} h_{z,j,0}(u_i) W \\
& + e(z,i,0)\left\{ \sum_{j,x} q_j(x) h_{z,i,0}(x) W \right\} + \sum_{z'} e(z',i,0)\lambda_e p_{z,j,0} P_{z,j,0} W \\
& + \sum_{z'} e(z',0,i)\lambda_e \sum_{y} p_{z,j,0} P_{z,j,0} W \\
& + e(z,i,j,0)\left\{ 1 - 2\delta - \sum_{i_2,x} q_{i_2}(x) h_{z,i,j,0}(x) \right\} \\
& + \sum_{z'} \sum_{i_2} e(z',i,i_2,0)\lambda_e p_{z,j,0} P_{z,j,0} W \\
& + \sum_{z'} \sum_{i_2} e(z',i_2,0,i)\lambda_e p_{z,j,0} P_{z,j,0} W \\
& + \sum_{z'} \sum_{j'} e(z',i,0,j')\left\{ \lambda_e \sum_{y} p_{z,j,0} P_{z,j,0}(i) W \right\} \\
& + \sum_{i_2} e(z,i,j,i_2,z)\left\{ \delta + \lambda_e \sum_{y} p_y P_y(i_2) \right\} \\
& + \sum_{z'} \sum_{j'} \sum_{i_2} e(z',i,i_2,j')\lambda_e p_{z,j,0} P_{z,j,0}(i) W \\
& + \sum_{z'} \sum_{j'} \sum_{i_2} e(z',i_2,j',i)\lambda_e p_{z,j,0} P_{z,j,0}(i) W
\end{aligned}
\tag{39}
$$

where $p \in \{0,1\}$ refers to the poaching decision. Specifically, p_i in a full team refers to whether i is poached by some other firms.

Now we move the next stage. Let e^2 denote the distribution of workers at the beginning of the production stage (i.e., after the firm has had the option of firing some of its employees and instant promotion). The measures u^2, $e^2(z,i,0)$, $e^2(z,0,i)$, $e^2(z,i,j)$

are given by

$$
\begin{aligned}
u_i^2 &= u_i^1 + [e^1(z,i,0) + e^1(z,0,i)]d + \sum_j [e^1(z,i,j) + e^1(z,j,i)]d_i \\
e^2(z,i,0) &= e^1(z,i,0)(1-d)(1-I) + \sum_j e^1(z,i,j)d_j(1-d_i)[1-I] \\
e^2(z,0,i) &= e^1(z,i,0)I + e^1(z,0,i)(1-d) + \sum_j e^1(z,i,j)d_j(1-d_i)I \\
&\quad + \sum_j e^1(z,j,i)d_j(1-d_i) \\
e^2(z,i,j) &= e^1(z,i,j)(1-d_i)(1-d_j)
\end{aligned} \tag{40}
$$

where $d \in \{0,1\}$ refers to the (voluntary) separation decision.

Let e^3 denote the distribution of workers at the beginning of the entry-and-exit stage of next period (i.e. after the workers' human capital accumulation process is completed). The measures $u^3, e^3(z,i,0), e^3(z,0,i), e^3(z,i,j)$ are given by

$$
\begin{aligned}
u_i^3 &= u_i^2 + \lambda \sum_z \Big[e^2(z,i-1,0) + e^2(z,0,i-1) \\
&\quad + \lambda \sum_j [e^2(z,i-1,j) + e^2(z,j,i-1) + e^2(z,i,j-1) + e^2(z,j-1,i)] \Big] d_i \\
e^3(z,i,0) &= (1-\lambda)e^2(z,i,0) + \lambda e^2(z,i-1,0)(1-d) \\
&\quad + \lambda \Big[\sum_j e^2(z,i-1,j) + e^2(z,i,j) \Big] d_j(1-d_i) \\
e^3(z,0,i) &= (1-\lambda)e^2(z,0,i) + \lambda e^2(z,0,i-1) + \lambda \Big[\sum_j e^2(z,i-1,j) + e^2(z,i,j-1) \Big] \\
&\quad + \lambda \Big[\sum_j e^2(z,j,i-1) + e^2(z,j-1,i) \Big] \\
e^3(z,i,j) &= (1-\lambda)^2 e^2(z,i,j) \\
&\quad + \Big[\lambda(1-\lambda)[e^2(z,i-1,j) + e^2(z,i,j-1)] + \lambda^2 e^2(z,i-1,j-1) \Big]
\end{aligned} \tag{41}
$$

Finally, we close this subsection by describing the distribution of workers at different states. Let e^+ denote the distribution of workers at the beginning of the search-and-matching stage of next period (i.e., after the entry-and-exit process is completed). The measures $u^+, e^+(z,i,0), e^+(z,0,i), e^+(z,i,j)$ are given by

$$\begin{aligned} u_i^+ &= (1-\sigma)u_i^3 + \sigma\pi_i \\ e^+(z,i,0) &= (1-\sigma)e^3(z,i,0) + \sigma(1-\sigma)(1-d_i)(1-I)\sum_j e^3(z,i,j) \\ e^+(z,0,i) &= (1-\sigma)e^3(z,0,i) + \sigma(1-\sigma)(1-d_i)I\sum_j e^3(z,i,j) \\ &\quad + \sigma(1-\sigma)(1-d_i)\sum_j e^3(z,j,i) \\ e^+(z,i,j) &= (1-\sigma)^2 e^3(z,i,j) \end{aligned} \tag{42}$$

Wage Determination

A worker's(manager's) wage w is the difference between the net present value and continuation value. As mentioned in Section 1.4, wages are subjective to outside option, depending on firm type z and human capital of all employees h_w and h_m. Therefore, any changes in the current team composition affects wages. We first describe the outside option updating rule and then describe wage equations. Assume that a worker's current outside option is U_i. At any period of time, her updated outside option O_i' follows

$$O_i' = \min\{\max\{O_i, U_i\}, S(i,-i) - S(0,-i)\}.$$

A worker's updated outside option is larger than the unemployment value and may be larger than the current outside option O_i, but it cannot exceed the maximum value offered by the production unit. Thus, O_i' must satisfies that $W(U_i) = U_i$ if $O_i' = U_i$, where $W(U_i)$ is the next present value.

I now lay out the detailed expression of the wage equations below. The net present value of an employed worker with a skill set $h = (g, f, m)$ who works alone with an outside option O follows

$$
\begin{aligned}
W_h^{10}(O) = w + \beta \mathbb{E}_{g',f'} \Bigg\{ & W_{h'}^{10}(O') + \sigma \Big[0 - W_{h'}^{10}(O') \Big] + \delta \Big[U_{h'} - W_{h'}^{10}(O') \Big] \\
& + \sum_{i,x} q(i,x) \mathbb{1}(\text{hire i}) \Big[W_{h'}^{21}(O') - W_{h'}^{10}(O') \Big] \\
& + \sum_{y} q(y) \Big[\max \Big\{ W_{h'}^{10}(O'), \min\{(1-\gamma)v(h',y) + \gamma v_{h'}, \gamma v(h',y) \\
& + (1-\gamma)v_h'\} \Big\} - W_{h'}^{10}(O') \Big] \Bigg\}.
\end{aligned} \tag{43}
$$

A worker's next present value, $W_h^{10}(O)$, is equivalent to a flow wage, w, and the expected option value, which depends on the realization of two-dimension human capital g' and f' after learning. Given any value of new human capital levels, the continuation value becomes $W_{h'}^{10}(O')$. Several events could happen exclusively. With probability σ, the worker exits the labor market with zero continuation value. With probability δ, the worker becomes unemployment with continuation value to be $U_{h'}$. With probability $q(i,x)$, the team meets a manager with skill set i and employment status x, the worker's continuation value is $W_{h'}^{21}(O')$ conditional on hiring the manager. With probability $q(y)$, the worker is contacted by a hiring team at status y. Two possible events could happen. If the maximum value offer by current team, $v_{h'}$ is larger than the poaching offer, $v(h',y)$, the worker stays at the current team with continuation value to be $max\{\gamma v(h',y) + (1-\gamma)v_{h'}', W_{h'}^{10}(O')\}$ and the outside option is updated to $v(h',y)$. Conversely, if $v_{h'}$ is smaller than $v(h',y)$, the worker sways to the new match with continuation value updates to $max\{(1-\gamma)v(h',y) + \gamma v_{h'}, W_{h'}^{10}(O')\}$ and outside option updates to $v_{h'}$.

Similarly, the net present value of an employed manager with a skill set $h =$

(g, f, m) who works alone with an outside option O follows

$$
\begin{aligned}
W_h^{01}(O) = w + \beta \mathbb{E}_{g',f'} \Bigg\{ & W_{h'}^{01}(O') + \sigma \Big[0 - W_{h'}^{01}(O') \Big] + \delta \Big[U_{h'} - W_{h'}^{01}(O') \Big] \\
& + \sum_{i,x} q(i,x) \mathbb{1}(\text{hire i}) \Big[W_{h'}^{22}(O') - W_{h'}^{01}(O') \Big] \\
& + \sum_{y} q(y) \Big[\max \Big\{ W_{h'}^{01}(O'), \min\{(1-\gamma)v(h',y) + \gamma v_{h'}, \\
& \gamma v(h',y) + (1-\gamma)v_h'\} \Big\} - W_{h'}^{01}(O') \Big] \Bigg\}.
\end{aligned} \tag{44}
$$

The net present value of an employed worker with a skill set $h = (g, f, m)$ who works in a full production unit with an outside option O follows

$$
\begin{aligned}
W_h^{21}(O) = w + \beta \mathbb{E}_{g',f'} \Bigg\{ & W_{h'}^{21}(O') + \sigma \Big[0 - W_{h'}^{21}(O') \Big] + \delta \Big[U_{h'} - W_{h'}^{21}(O') \Big] \\
& + [\sigma + \delta + \sum_{y} q(y) \mathbb{1}(\text{manager poached})] \\
& \Big[\max\{W_h^{10}(O'), W_h^{01}(O')\} - W_{h'}^{21}(O') \Big] \\
& + \sum_{i,x} q(i,x) \mathbb{1}(\text{hire i}) \Big[W_{h'}^{22}(O') - W_{h'}^{21}(O') \Big] \\
& + \sum_{y} q(y) \Big[\max \Big\{ W_{h'}^{21}(O'), \min\{(1-\gamma)v(h',y) + \gamma v_{h'}, \\
& \gamma v(h',y) + (1-\gamma)v_h'\} \Big\} - W_{h'}^{21}(O') \Big] \Bigg\}.
\end{aligned} \tag{45}
$$

For brevity, I only describe the terms that are substantially different from equation 43. The second line describes the event that the manager leaves with probability, $[\sigma + \delta + \sum_y q(y) \mathbb{1}(\text{manager poached})]$, the worker may continue as a worker with continuation value $W_h^{10}(O')$ or promoted to be a manager with continuation value

$W_h^{01}(O')$.

Similarly, the net present value of an employed manager with a skill set $h = (g, f, m)$ who works in a full production unit with an outside option O follows

$$
\begin{aligned}
W_h^{22}(O) = w + \beta \mathbb{E}_{g',f'} \Bigg\{ & W_{h'}^{22}(O') + \sigma\Big[0 - W_{h'}^{22}(O')\Big] + \delta\Big[U_{h'} - W_{h'}^{22}(O')\Big] \\
& + [\sigma + \delta + \sum_y q(y)\mathbb{1}(\text{worker poached})]\Big[W_h^{01}(O') - W_{h'}^{22}(O')\Big] \\
& + \sum_{i,x} q(i,x)\mathbb{1}(\text{hire i})\Big[W_{h'}^{22}(O') - W_{h'}^{22}(O')\Big] \\
& + \sum_y q(y)\Big[\max\Big\{W_{h'}^{22}(O'), \min\{(1-\gamma)\nu(h',y) + \gamma\nu_{h'}, \\
& \gamma\nu(h',y) + (1-\gamma)\nu_h'\}\Big\} - W_{h'}^{22}(O')\Big]\Bigg\}.
\end{aligned}
\tag{46}
$$

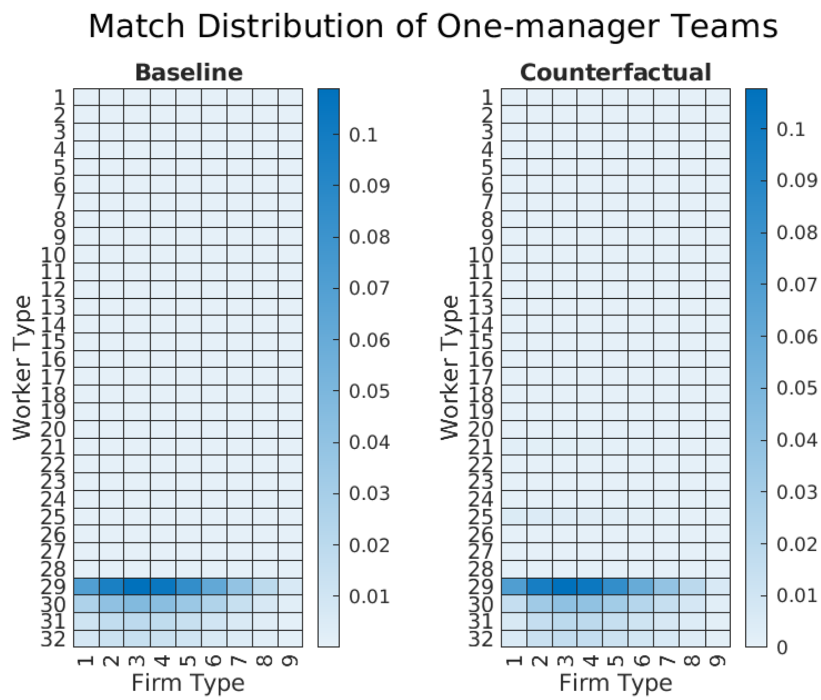


Figure B.1: Composition of Teams by Firm Type

Notes: This figure plots the composition of four types of teams across firm type z. The areas from bottom to top represent the measure of empty teams, one-worker teams, one-manager teams, and full teams (a worker with a manager) respectively. The left panel shows the distribution of teams under the baseline model and the right panel shows the distribution of teams under counterfactual exercise (no promotion).

2 MINIMUM WAGE EFFECT ON FIRM CAPITAL AND LABOR CHOICE

2.1 Introduction

Researchers have extensively studied the effects of minimum wages on workers' wages and employment (Stigler (1946); Card and Krueger (1995); Card and Krueger (2000); Dube et al. (2010)). However, less attention has been paid to the effects of minimum wages on firm-level strategies, particularly capital investment decisions. It remains unclear whether firms respond to rising labor costs from minimum wage reforms by substituting labor with capital.

Theoretical predictions about the effects of an exogenous increase in the price of labor vary between non-frictional and frictional labor market models. In a competitive framework, the overall impact of a minimum wage hike on capital investment depends on the magnitude and direction of substitution and scale effects. This paper proposes an alternative mechanism: firms may be incentivized to increase capital investment not due to substitution effects, but as a response to holdup problems in wage bargaining. I aim to quantify how much of the observed capital response can be attributed to this holdup mechanism.

We begin by presenting new empirical evidence on the effects of a large, nationwide minimum wage reform in China on firm-level capital investment, employment, and wage incidence. Between 2002 and 2006, 286 cities across 30 provinces implemented over 1,990 adjustments to local minimum wage levels.[1] As shown in Figure 2.1, both nominal and real minimum wages increased sharply—by more than 10% in 2004 alone—and continued rising thereafter. The perceived scale and enduring nature of this distinctive policy change enable us to address the concern that many of the minimum wage increases examined in recent labor economics literature are, in fact, relatively small and transient (Sorkin (2013), Harasztosi and Lindner (2019)). Notably, many of the most affected firms operated in the manufacturing

[1] Provinces are the equivalent of US states.

sector, which produces tradable goods (Hau et al. (2016)).

This substantial, discrete increase in the minimum wage facilitates the implementation and evaluation of a range of relatively credible difference-in-differences estimators. In the first part of the paper, we focus on the capital investment, employment, and wage effects of the minimum wage reform. Specifically, using annual firm-level data in the manufacturing sector, I measure the impact of the minimum wage reform in 2004 on the capital-to-labor ratio for all types of firms in China by leveraging the variation of the binding extent of minimum wage across firms and across-time variations within the firm. The results suggest that low-wage firms prior to the minimum wage reform raised wages, decreased employment, and increased capital. Specifically, a 1% increase in the minimum wage in 2004 led to a 0.95% increase in capital of low-wage firms relative to high-wage firms.

To further explore the mechanisms at play, I extend a simple Diamond-Mortensen-Pissarides (DMP) style labor search model by incorporating endogenous capital decisions. Due to the labor market friction, wages are determined by Nash Bargaining and positively depend on firms' total profit, which is an increasing function of their capital stock. However, since firms bear all the investment costs, negotiating means that workers will take part of the pie. This reduces the incentives for investment if firms cannot pass capital costs through to workers, which is called the holdup problem. The minimum wage as a wage guarantee can mitigate the firm's holdup concern. As a result, the model predicts that some low-wage firms will increase their capital under the minimum wage.

Then, I calibrate the model to the Chinese economy by fitting firm-level behavior before the 2004 minimum wage reform. Specifically, I use the China Industry Business Performance Data (CIBPD) and aggregate economic moments from the National Bureau of Statistics of China (NBS) and the Ministry of Labour and Social Security (MLSS) to discipline the magnitude of labor market friction and holdup problem. The calibrated model is able to match the key features of China's capital and labor markets prior to the reform.

The main findings are three-fold. First, effective minimum wage policies increase capital investment among low-wage firms that remain in operation. Moreover, consistent with the empirical evidence, the model generates a decreasing pattern of capital growth conditional on firms' pre-reform wage levels.

Second, an increase in the minimum wage results in compression of the wage distribution and positive spillovers on higher wages, as documented in Lee (1999). This arises because the minimum wage increases the value of unemployment, which raises the bargaining position of all workers. Third, the model shows that the 2004 minimum wage reform enhanced overall social surplus, as the gains from improved capital investment outweighed the losses from reduced vacancy creation.

Finally, I conduct a counterfactual experiment to compare the reform's effects with those of alternative policies. The results indicate that a 10 percent reduction in the real interest rate could achieve similar gains in capital and wage levels as the 2004 minimum wage reform, but with a smaller increase in unemployment.

The paper is organized as follows. Section II provides a brief discussion of some of the related literature. Section III introduces the institutional background and the data used in this paper. Section IV describes a simple holdup model. Section V describes the main calibration strategy and validation test. Section VI performs a counterfactual experiment. Section VII concludes, and the Appendix contains proofs and technical details.

2.2 Related Literature and Contributions

This paper makes several contributions to the literature. First, the paper is related to the literature studying the effect of the minimum wage on firm behavior. Previous studies found mixed results on the employment effects of minimum wages in China across regions and demographic groups. For example, using provincial data of rural migrants data, Wang and Gunderson (2011) finds no negative employment effects in the prosperous and growing Eastern region, while Fang and Lin (2020) finds significant adverse effects on employment in the Eastern and Central regions of China, leveraging granular household survey data. Hau et al. (2016) also confirms a negative employment growth across firm types from 2002 to 2008. Despite several decades of evidence, the minimum wage remains a highly controversial policy across countries, depending on the employment skill type and the relative size of minimum wage growth. Given the large and persistent increase in the minimum wage in our setting, it is not surprising that we find a large negative impact among low-wage firms. In addition to the effects of minimum wage on employment margin, Hau et al. (2016), Mayneris et al. (2018), and Harasztosi and Lindner (2019) examine how firm profitability and productivity changed in response to the 2004 China minimum wage reform using the same firm-level data as I do and argue that minimum wage hikes accelerate total factor productivity growth. This paper also complements studies on changes of other margins, such as output prices (Aaronson (2001), Harasztosi and Lindner (2019)), firm exit (Aaronson et al. (2018); Luca and Luca (2019)), firm profitability (Draca et al. (2011)), firm size (Smith (1999)), the full scope of establishments' behavior (Chen et al. (2019)) in US, and profitability of private firms in Vietnam (Cuong (2017)) and in Hungary (Harasztosi and Lindner (2019)).

My work also contributes to the literature on the effects of the minimum wage under the frictional labor market and match models. Flinn (2006) is the first paper analyzing the effect of changes in minimum wages on labor market outcomes and welfare using a continuous-time search model. He shows that minimum wage increases can be welfare-improving to labor market participants on both the supply and demand sides of the labor market. However, in his model, labor is the only production input. In my model, incorporating the capital market with the labor market like most of the neoclassical growth models generates interesting interactions and provide a new explanation for the increases in capital investment other than production factor substitution . Zhang and Todd (2025) develops a spatial general equilibrium job search model focusing on the effects of local and federal minimum wage policies on workers with different education levels and labor market transitions. Blömer et al. (2018) explore unemployment in Germany, while Engbom and Moser (2018) study earning inequality in Brazil. However, all of these papers focus on the effects of the minimum wage on workers, not firms, while my paper examines the effect of the minimum wage through the lens of firms' input decisions.

My model is inspired by Acemoglu and Shimer (1999), which is the first paper incorporating capital into a labor search model. The difference between my model and the Acemoglu and Shimer (1999) is the main cause of the holdup issue. In my model, holdup problem arises due to the precommittment of capital in a wage bargaining protocol, while Acemoglu and Shimer (1999) relies on the risk aversion of workers to make firms inefficiently underinvest capital. My work most closely resembles Bauducco and Janiak (2018) who study the holdup problem of minimum wage in the US. Another difference between this paper and Bauducco and Janiak (2018) is that I assume the heterogeneity of firms due to the availability of firm-level data, which helps me to examine the heterogeneity of firm behavior. However, in their paper, firm exit and entry are exogenous. Unlike developed countries, high-frequency entry and exit is an important feature of developing countries like China. I endogenize exit rate by constructing a Mortensen and Pissarides (1994) style model. hlA minimum wage in a oligopsonistic model as in Kaas and Madden (2008) predicts no effects on employment, which contradicts our empirical results on employment.

Another contribution of my model is that I provide a micro foundation for Nash Bargaining with minimum wage constraints. Previous literature such as Flinn (2006) and Cahuc et al. (2006) assume Nash Bargaining. I show that under specific conditions, Rubinstein alternating offer game generates the same result as in Nash Bargaining.

2.3 Institutional Background and Data

Minimum Wage Reform in China

China introduced its first minimum wage regulations in 1993. By July 1994, these regulations were formally integrated into the national Labor Law. By 1995, 24 provinces and municipalities had established their own minimum wage standards. However, enforcement was initially weak, and compliance rates remained low (Mayneris et al. (2018)). Enforcement of minimum wage policies was improved substantially after then (see Figure 2.3).

In December 2003, the Chinese government announced a new minimum wage law. To balance regional inequality in development, the Ministry of Labor and Social Security initiated a nationwide reform in March 2004. The new minimum wage law improved and expanded the previous one in many aspects. The reform measures emphasized: (1) an extension of coverage to village enterprises, self-employed businesses, and part-time workers; employers were not to include items such as overtime pay or traveling supplements as part of the wage when calculating minimum wages (Wang and Gunderson (2012)). (2) A new hourly minimum wage rule was introduced into the new labor law. (3) More inspection and enforcement work on the part of local governments strengthened the compliance of the law. The penalty for non-compliant enterprises increased from 20% – 100% of the wage owed to 100% – 500% (Hau et al. (2016)). (4) Minimum wages were adjusted more frequently. As different Chinese provinces have very different living standards, the local government (here, this could be a province or a city) has the right to set its own minimum wage according to local GDP, unemployment, product prices, etc. Typically, following the national requirements, provincial governments set out multiple minimum-wage classes for the region as a whole, and each city and county within the region chooses the appropriate minimum wage level (Mayneris et al. (2018)). For example, the minimum wage level is the same across districts in Shanghai, while the Beijing government sets different minimum wage levels for urban districts and rural areas. The law required that the local authority adjust the minimum wage level every one to three years based on economic conditions.

Minimum Wage Data and Firm Data

I employ two main sources of data. I mainly use the China Industry Business Performance Data (CIBPD), which contains firm-level information. The CIBPD is an annual survey data starting from 1995. Data was collected by the National Bureau of Statistics (NBS) in China. It includes a very detailed balance-sheet data for all industrial state-owned (SOE) and non-state-owned enterprises (NSOE) with sales above 5 million Yuan.[2] For much of the analysis, I only use data from 2002 to 2006 because China joined the World Trade Organization (WTO) in December 2001. In addition, the Labor Contract Law and the Union Law were tightened in December 2006, which improved workers' bargaining power (Yao and Zhong (2013)), I restrict the data to end before 2007 since the sales numbers are believed unreliable after 2007 (Brandt et al. (2014)) .[3]

The data on minimum wages at the city-level was collected from various official websites such as China Labour Net and the Ministry of Human Resources and Social Security (MOHRSS). The MOHRSS includes both city-level monthly minimum wages and hourly minimum wages from 1998-2007. I am not able to observe working hours in the firm data and thus I only use monthly minimum wage. To match minimum wage data to the annual reporting frequency of the firm data, I define annual for each city as the average level if there were several minimum wage adjustments occur during the calendar year. The nationwide average real minimum wage is depicted in Figure 2.1.[4]

Sample Selection and Summary Statistics

The raw data comprise 2,615,016 firm-year observations, corresponding to 666,554 distinct firms. One limitation of this data is that I do not have information at the worker level. This means that I am not able to use the distribution of wages within a firm to measure firm-level exposure to minimum wage hikes. Instead, I follow the literature and use the firm-level average wage, defined as the ratio of the total wage bill to the number of employees(Draca et al. (2011)), to measure firm exposure to the minimum wage reform.

Another complexity of the Chinese data is the unusual way the capital stock is measured.[5] Following Brandt and Zhu (2010), I assume a 9% yearly depreciation

[2]At the exchange rate of 8.27 RMB per USD (in force between January 1997 and July 2005), this amounts to 605,000 USD.

[3]Approximately 30% of the above-scale firms' data is missing.

[4]All wages are in real terms, deflated by the Chinese CPI with base year 2000.

[5]In each year fixed assets are reported three ways: (i) "original fixed assets" is the sum of

Figure 2.1: Real Minimum Wage

Note: Figure plots the average real minimum wage from 1998 - 2007 in China. Minimum wages at the city-level were collected from various official websites such as China Labour Net and the Ministry of Human Resources and Social Security (MOHRSS). All wages are in real terms, deflated by the Chinese CPI with base year 2000.

rate and deflate nominal capital with the investment deflator to impute real capital every year.[6] To overcome difficult measurement issues, I clean the data and create a panel dataset as follows: (1) I drop firm-year observations with missing or nonpositive values for firm-level total wage bill, employment, real (value-added) output, real capital. (2) I drop firms with fewer than 8 employees and cities with less than 20 firms. Reported average wages, defined as the ratio of total wage

past investments at historical prices, (ii) "net fixed assets" is original fixed assets less accumulated depreciation, and (iii) "total fixed assets" is net fixed assets with construction materials and ongoing construction added (Brandt et al. (2014)).

[6]See Brandt et al. (2014) for more details on real capital imputation.

bill to total number of employees, may not be reliable for these firms (Mayneris et al. (2018)).[7] (3) To exclude outliers, I drop the top and bottom 1% of firm-year observations with unusually high or low value of real wages per worker, real capital per worker. My final sample contains 301,379 unique firms and 820,003 firm-year observations during the period 2002-2006.

Table 2.1 presents summary statistics on firm numbers and aggregate employment, capital and capital to labor ratio. It also provides information about compliance rates, defined as the ratio of firms whose average wage is less than local minimum wage, and national average minimum wage growth rates. It is clear that the compliance rate surged to about 97% after 2004. Moreover, both the mean wage and capital saw a rapid growth after 2004.

Table 2.1: Summary Statistics: All Manufacturing Firms

Year	Observations	Minwage Growth	Mean Wage (Yuan)	Mean Capital (Thousand Yuan)	Compliance
2001	90,949	0.0946	874.8208	67.8978	0.9126
2002	104,438	0.0946	914.2543	69.5842	0.9110
2003	124,131	0.0074	983.3533	69.6285	0.9266
2004	188,540	0.1716	1161.066	70.0935	0.9738
2005	189,230	0.0983	1314.616	74.3835	0.9742
2006	213,665	0.0524	1548.395	81.1460	0.9729

Note: All mean variables are weighted by firm size. All variables are in real terms. Minwage growth is defined as the average minimum wage growth nationwide. Mean wage is the total wage payment divided by the total number of workers, and Mean capital is the real capital value divided by the total number of workers. Both are measured in Chinese currency Yuan. The compliance rate is computed by the portion of total number of firms that pay above local minimum wage. More details on variable construction are in the *Appendix*.

Previous literature raised concerns about weak enforcement in China. To show the strict enforcement of the 2004 minimum wage policy, Figure 2.2 depicts the distribution of firm average wages before and after the 2004 minimum wage reform, and Figure 2.3 depicts the distribution of firm average wage to local minimum wage ratio before and after minimum wage reform. Both figures show that the 2004 reform is well implemented.

[7] See *Appendix A.1* for the definition and computation of these variables.

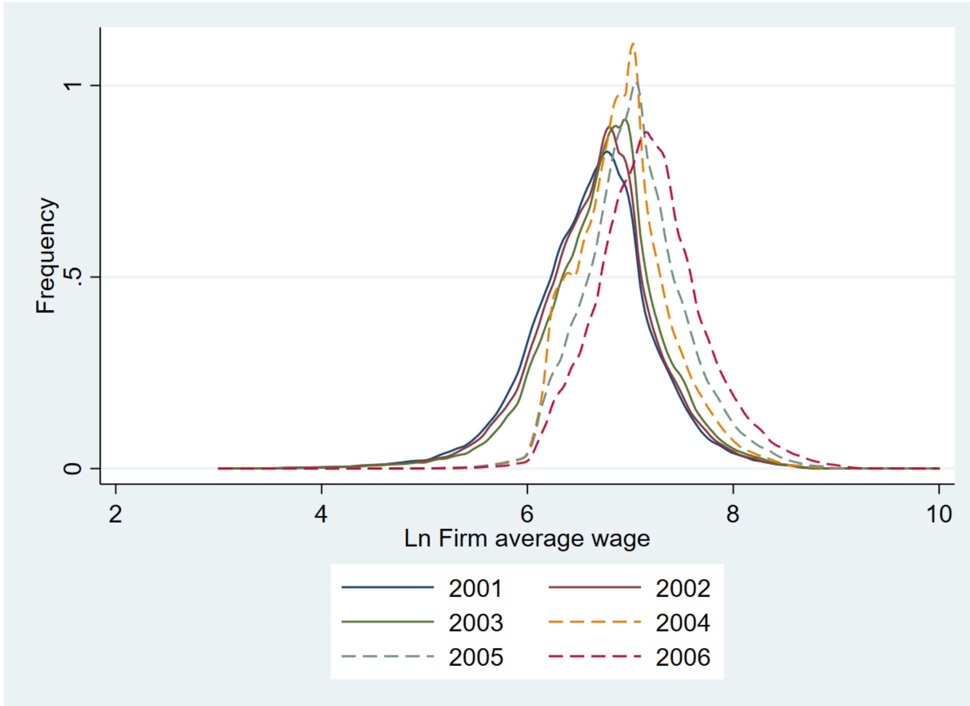


Figure 2.2: Firm-level Average Wage Distribution

Note: Figure plots the average wage across firms from 2001 - 2006 in China. Firm-level average wage is measured by the total wage payment per worker. All wages are in real terms, deflated by the Chinese CPI with base year 2000.

Empirical Strategy

The primary goal is to estimate how the 2004 minimum wage reform affected firm capital, employment, and wage decisions. As is standard in the few papers considering the effects of minimum wages with firm-level data (Draca et al. (2011)), I define the firms whose average wage is less than the local minimum wage level next year as "exposed" or treated, as they are certainly the most affected by the minimum-wage rise, while other firms may also be affected in equilibrium. Furthermore, to measure the degree of treatment and to consider heterogeneity in treatment intensity, I also define an alternative the treatment variable as the percentage deviation of a firm's average wage to the next period's local minimum wage. Firms in the treatment group at period t are those with positive exposure

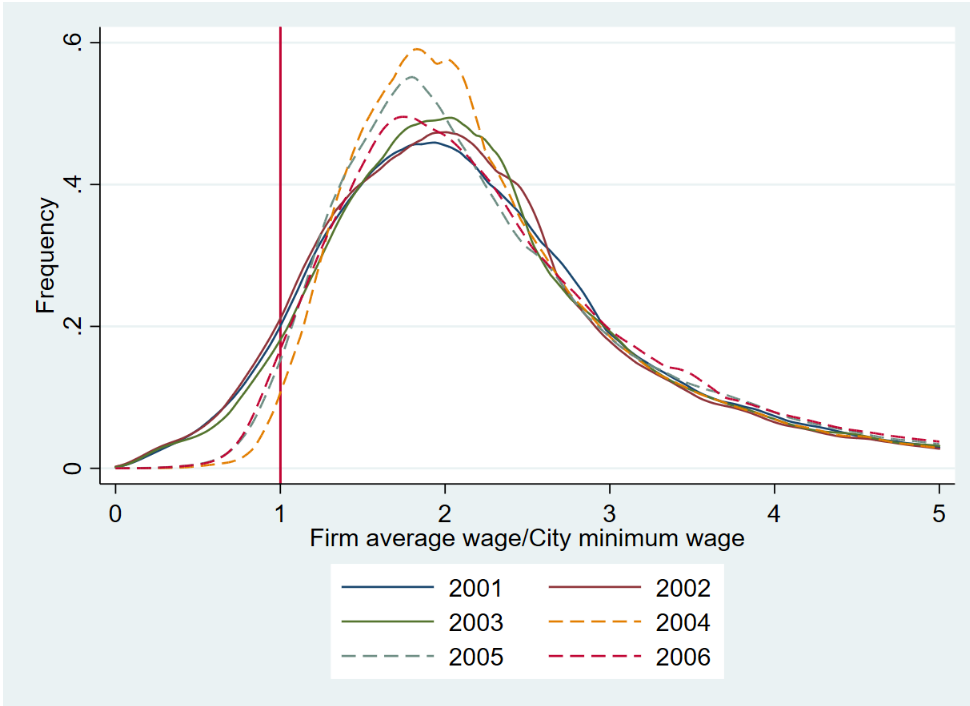


Figure 2.3: Firm-level Average Wage and Minimum Wage

Note: Figure plots the ratio of firm average wage to local minimum wage from 2001 - 2006 in China. Firm-level average wage is measured by the total wage payment per worker. Minimum wages at the city-level were collected from China Labour Net and the Ministry of Human Resources and Social Security (MOHRSS). All wages are in real terms, deflated by the Chinese CPI with base year 2000.

(see definition below), while firms with an average wage higher at t is higher than the local minimum wage at $t+1$ are in the control group. Pre-reform refers to the period before 2004. The identification strategy exploits variation in the degree of exposure to the local minimum wage to compare firm-level outcomes of firms that were bound by the minimum wage to those without a labor payment constraint. Our identifying assumption is that the level of minimum wage was uncorrelated with firm-specific trends in wage.

I use a difference-in-differences to quantify the minimum Wage effect on capital to labor ratio using data from the 2002 - 2006 observations. The baseline econometric model is the following:

$$y_{fsct} = \alpha * Exp_{ft} + \beta * Exp_{ft} * \mathbb{1}(t \geqslant 2004) + Z_{f,t-1} + \mu_f + \theta_{st} + \eta_{ct} + \epsilon_{fsct}, \quad (2.1)$$

where y_{fsct} are firm-level outcome variables including capital to labor ratio, firm size, and average wage for firm f in sector s in city c in year t. The parameter of interest β measures the average treatment effect of the minimum wage reform on low-wage firms after 2004. To capture the heterogeneous degree of constraint, Exp_{ft} is a continuous variable that is defined as the percentage of deviation of the average wage of firm at year $t-1$ and the local minimum wage at year t, $Exp_{ft} = max\{-\frac{wage_{t-1} - minwage_t}{minwage_t}, 0\}$. The larger Exp_{ft}, the more exposed to the minimum wage. To make sure our empirical analysis is consistent with the model, I also include lagged firm-level controls, $Z_{f,t-1}$, including dummies for foreign and exporting firms, as well as $\ln(\text{wage})$, $\ln(\text{capital})$ and $\ln(\text{labor productivity})$, which is defined as log of value added per worker. Equation 2.1 includes a vector of agency-by-mode fixed effects μ_f to control for time-invariant unobserved differences in outcomes across firms. We also include a vector of city-by-year fixed effects η_{ct} and sector-by-year fixed effects θ_{st} to account for city-specific growth trends and sector-specific differences in firm growth. For example, firms in coastal China grow faster due to trade.

We weight the regression in Equation 2.1 by firm size to capture the impact of the minimum wage reform on the average employment and to account for potential heteroskedasticity from firms of different sizes. The specification also echoes the one-firm-one-vacancy model introduced in the next section. To account for arbitrary serial correlation in travel behavior within a given metro area, standard errors ϵ_{fsct} in Equation 2.1 are clustered by city.

Table 2.2 shows our main results following Equation 2.1. Each coefficient estimate gives the difference in outcome variables in the post-policy period relative to the pre-policy period. As we can see that minimum wage increases wage, decreases employment and increase capital to employment ratio. Moreover, following the implementation of the new minimum wage policy, I find the average wage in low-wage firms rose faster and has a larger growth rate of capital that capital per worker increased 0.95 percentage point per 1 percent below minimum wage after the implementation of the 2004 Reform.

However, we are not able to separately identify the channel how it works. In the next section, I build a holdup model without labor-capital substitution effect. The goal is to testify whether a pure holdup model is able to explain the increases of capital per worker.

Table 2.2: Difference-in-Differences Regression Results

variables	(1) Wage	(2) Employment	(3) Capital/Employment
Exposed Firm	-27.536***	7.872***	-8.679***
	(0.450)	(0.274)	(0.323)
Exposed Firm	1.580***	-1.261***	0.949***
x Post-reform	(0.312)	(0.204)	(0.234)
Firm FE	Yes	Yes	Yes
Sector-Year FE	Yes	Yes	Yes
City-Year FE	Yes	Yes	Yes
R-squared	0.94	0.94	0.89
Observations	295,213	295,289	295,289

Note: Observations in the table are aggregated at the city-year level from 2002-2006. All regressions include firm fixed effects, region-by-year fixed effects, and sector-by-year fixed effects. Standard errors are reported are clustered at the city level. Regressions weighted by firm size. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. All wages are in real terms, deflated by the Chinese CPI with base year 2000.

2.4 A Simple Holdup Model

In what follows, I consider a Mortensen and Pissarides (1994) style model in which a firm consists of a finite number of independent vacancies with free entry. The key feature of the model is that firms commit to the investment in capital before bargaining with workers.

Environment

The model is formulated in discrete time and assumes stationarity of the labor market. Agents are forward-looking. Without loss of generality, we assume that the firms and workers share a common discount rate $\beta \in (0, 1)$.

Labor Market Agents. In any period, the economy is populated by a continuum of risk-neutral and homogeneous workers with measure normalized at unity. Workers inelastically supply one indivisible unit of labor and maximize the present value of net flow income. All workers are in one of the two labor force states — the employed workers (e) and the unemployed who are not working but search for a job (u). On the firm side, firms are ex-ante homogeneous in their permanent productivity $z \sim \Gamma(z)$. Firms all produce an identical homogeneous good. There are two types of jobs among active jobs: filled jobs that are producing (J) and vacant jobs that are not producing but recruiting (V). Jobs can be destroyed and exit the labor market.

Production Technology and Capital Investment. To produce, all firms rent capital k each period at cost r to produce following the production function per job $F(z, k) = zk^{\alpha}$ with $\alpha \in [0, 1]$, which is assumed to be increasing and concave in k. I assume that capital fully depreciates every period, and the scrap value to be zero. I assume capital investment is complete prior to bargaining, and thus investment costs are not part of the bargaining surplus.

Search and Matching. Each period, vacant jobs enter the labor market with the average productivity shock z_a and search for unemployed workers after paying a periodical vacancy posting cost $c > 0$. We assume that only unemployed workers actively search for jobs. Workers and vacancies randomly meet in the labor market. The labor market frictions are characterized by an aggregate matching function, $M(U, V)$, where U is the measure of the unemployed and V is the measure of vacant positions. The matching function is assumed to exhibit constant returns to scale, and satisfies the property $M(0,V) = M(U,0) = 0$. Vacancies posted by firms are filled with probability $q(\theta) = \frac{M}{V} = M(\frac{U}{V}, 1)$ each period. Likewise, unemployed workers find jobs with probability $\theta q(\theta) = \frac{M}{U} = M(1, \frac{V}{U})$. Thus, the ratio of aggregate vacancies to aggregate unemployment, $\frac{V}{U} = \theta$, is a sufficient statistic for the job filling and job finding probabilities in the model.

Wage Determination and Minimum Wage. Given the labor market friction, it is natural to assume that wages are determined through bargaining between both parties. Therefore, we adopt a standard bargaining game designed to ensure the decisions in a coalition are made jointly efficiently. Hence, wages are pure transfers between employees and firms. Workers come into contact with firms with vacant jobs. Upon meeting, the firm and the worker match if and only if the total surplus is positive. If the firm and the worker successfully match, the worker becomes an employee of the firm, depending on the promised position. The worker then captures a fraction ψ of the gains from trade, and a fraction $1 - \psi$ is captured by the firm, with $\psi \in [0, 1]$ being worker bargaining power.

Following Flinn (2006), the minimum wage $\bar{w}$ acts solely as a side constraint on the Nash-bargaining problem. Specifically, the set of feasible wages of the negotiation problem is now restricted to $[\bar{w}, +\infty]$.

Idiosyncratic Shocks and Exit. At the end of each period, production jobs face idiosyncratic production shocks, arriving with probability $1 - \lambda$ and jobs draw an independent and identically distributed shock from the distribution $F(z)$ for their new productivity. Existing firms decide whether to fire their worker and exit the market or keep producing.

Value Functions

In this section, I focus on the steady state equilibrium of the economy. Hence, all variables are free of subscript t. I start by presenting the Hamilton-Jacobi-Bellman (HJB) equations in the steady state.

Assuming that the price of the product is normalized to be 1. the HJB equation for a producing job is characterized as:

$$J(z) = \max_k \{F(z,k) - w(z,k) - rk\} + \beta[(1-\lambda)\int \max\{J(z'), V\}dF(z') + \lambda J(z)]. \quad (2.2)$$

The value of a filled job consists of two parts. The first part in the maximum of the residual profit of output $F(z,k)$ net wage $w(z,k)$ and capital investment cost rk borne by the firm. The second part represents the expected value depending on the state of the next period. With probability λ, the job draws a new productivity shock z' from the distribution and makes the exit decision based on the realization of the shock, based on a cutoff rule that $z > z_R^f$, where z_R^f is defined as the reservation shock level that a firm is indifferent between staying and exiting, i.e., $J(z_R^f) = V$. Conditional on surviving, the job continues to produce with a value $J(z')$.

The HJB equation for a vacant job is characterized as:

$$V = -c + \beta[q(\theta)J(z_a) + (1 - q(\theta))V]. \quad (2.3)$$

The firm of the vacant job pays a flow cost of c. With probability $q(\theta)$, the vacant job is filled by a worker and turns into a producing job with entry-level productivity z_a.

Under the free entry assumption, $V = 0$, we have

$$c = \beta q(\theta)J(z_a). \quad (2.4)$$

The posting cost is set to be the expected value of a producing job with initial productivity draw, z_a.

Wage Setting

The existence of frictions in the labor market implies that finding an alternative employment relationship to a firm or to a worker is costly. As a result, the firm and its workers must bargain over quasi-rents. When a worker and a firm meet after firm capital decision is made, the wage is decided by Nash Bargaining over joint surplus of the match. An offer is made if both parties agree on the contract. Different from the standard problem, if the firm does not make an employment offer, its "disagreement" outcome is the capital investment costs and a vacant job, $-rk$, a sunk cost independent of whether an agreement is achieved.[8] The job applicant's disagreement value is the value of unemployment, which I denote by U.

For any producing jobs, the Nash-bargained wage under the minimum wage constraint is given by

$$\begin{aligned} w(z,k) &= \arg\max_{w}(J(z)+rk)^{1-\psi}(W(z,k)-U)^{\psi} \\ \text{s.t} \quad & w(z,k) \geqslant \bar{w} \end{aligned} \tag{2.5}$$

The HJB equation for for an employed worker (e) in a job with productivity z and capital level k is characterized as:

$$W(z,k) = w(z,k) + \beta[(1-\lambda)\int \max\{W(z',k(z')),U\}dF(z') + \lambda W(z,k(z))]. \tag{2.6}$$

The employed worker gets a flow wage of $w(z,k)$ and option value of being employed — with probability $1-\lambda$, the match will receive a new shock. The worker then optimally decides to exit the labor force depending on the realization of the productivity shock, based on a cutoff rule that $z > z_R^w$, where z_R^w is defined as the reservation shock level that a worker is indifferent between staying and joining the unemployment pool voluntarily, i.e., $W(z_R^w, k(z_R^w)) = U$. One caveat is that z_R^w may not be aligned with z_R^f. In particular, if $z_R^w \geqslant z_R^f$, the worker will always stay employed if the match survives.

The HJB equation for unemployment is characterized as:

$$U = b + \beta\theta q(\theta)W(z_a, k(z_a)) + \beta[1-\theta q(\theta)]U, \tag{2.7}$$

An unemployed worker receives an unemployment benefit b and finds a job with probability $\theta q(\theta)$. With probability $1-\theta q(\theta)$, an unemployed worker stays unem-

[8] I assume the owners of firms can work on side to pay for the debt instead of default.

ployed next period.

The bargaining problem from Equation (5)implies that

$$W(z,k) - U = \frac{\psi}{1-\psi}[J(z) + rk]. \tag{2.8}$$

The surplus from an employment relationship for a worker is the additional utility a worker obtains from working in her current firm over and above the utility she obtains from unemployment. Another message delivered by the surplus sharing condition (2.8) is that $z_R^w < z_R^f$ given that the value of a producing job is increasing in terms of productivity z.[9]

Capital Choice Without Minimum Wage Constraint

I start with solving the steady state equilibrium assuming minimum wage is not binding for all firms. There exists a steady state for optimal decisions, i.e $\theta, z_R^f, k^*(z)$ that are constant. The solution is a textbook one. Details are in the *Appendix*.

Define $S(z)$ to be the joint bargaining surplus of a match of a firm with productivity z and capital k and a worker

$$S(z,k) = J(z) + rk + W(z,k) - U, \tag{2.9}$$

Adding up the value functions from (2), (3), (6), (7), and making use of the sharing rule (8), the joint bargaining surplus $S(z,k)$ can be written as

$$\begin{aligned} S(z,k) = F(z,k) - b + \beta \Bigg[&\lambda S(z,k(z)) + (1-\lambda)\int S(z',k(z'))dF(z') \\ &- \theta q(\theta)\psi S(z_a,k(z_a)) - [(1-\lambda)\int rk(z')dF(z') + \lambda rk(z)] \Bigg]. \end{aligned} \tag{2.10}$$

[9]It is straightforward to see that when $z = z_R^w$, $W(z_R^w, k^\star(z_R^w)) = U$ given the definition of z_R^w. Under the surplus sharing condition (2.8), it implies that $J(z_R^w) + rk(z_R^w) = 0 < J(z_R^f) + rk(z_R^f) = rk(z_R^f)$. It is easy to prove that $J(z) + rk$ is increasing in z. Therefore, $z_R^w < z_R^f$. It is intuitive since a firm needs higher productivity to reimburse for the sunk capital investment cost. As a result, a worker will stay employment as long as the match is active, i.e., $z_R^w = z_R^f$ in the equilibrium.

Endogenous market tightness θ and reservation shock z_R in equilibrium are decided jointly and uniquely by job destruction (JD) curve (11) and job creation (JC) curve (12)

$$S(z_R, k(z_R)) = z_R k(z_R)^{\alpha} - b + \beta \bigg[\lambda S(z_R, k(z_R)) + (1-\lambda) \int S(z', k(z')) dF(z') \\ - \theta q(\theta) \psi S(z_a) - [(1-\lambda) \int_{z_R}^{z_u} rk(z') dF(z') + \lambda rk(z_R) \bigg]. \tag{2.11}$$

$$c = \beta q(\theta) \bigg[(1-\psi) S(z_a, k(z_a)) - rk(z_a) \bigg]. \tag{2.12}$$

With knowledge of θ, JD curve is monotonically increasing in z under the assumption that parameters satisfy $\frac{1}{1-\alpha}[\frac{r(1-\psi\beta\lambda)}{(1-\psi)\alpha z_l}]^{\frac{\alpha}{\alpha-1}} > \beta\lambda r$. JD slopes up because it is easier to find a job when market tightness θ is higher, so the opportunity cost of employment is higher and thus more job destruction. JC slopes down because a higher reservation z_R makes firms more likely to exit the market, and thus more job destruction.

The steady-state condition for the unemployment rate of the model is derived from the Beveridge curve. The flow out of unemployment equals the flow into unemployment at any productivity level. The endogenous job separation rate is $(1-\lambda)F(z_R)$ and the job matching rate per unemployed worker is $\theta q(\theta)$, so the equation for the Beveridge curve is

$$u = \frac{(1-\lambda)F(z_R)}{(1-\lambda)F(z_R) + \theta q(\theta).} \tag{2.13}$$

Given θ, z_R and the sharing rule (2.8), we can derive Nash Bargaining wage

$w(z,k)$ as

$$w(z,k) = (1-\psi)\left[b + \beta\theta q(\theta)\psi S(z_a)\right] + \psi\left\{zk^{\alpha} - \beta\left[\lambda rk + (1-\lambda)\int_{z_R}^{z_u} rk(z')dF(z')\right]\right\} \tag{2.14}$$

One important feature is that bargained wage this period negatively depends on next period's holdup problem. This is because a firm chooses its capital level every period, and firm owners take this into consideration and deduct the option value.

Proposition 1. The optimal capital choice $k^*(z)$ when minimum wage is not binding is given by

$$k^*(z) = \left[\frac{r(1-\psi\beta\lambda)}{(1-\psi)\alpha z}\right]^{\frac{1}{\alpha-1}} \tag{2.15}$$

Proof. After plugging the bargained wage (2.14) into the flow utility of the match and taking the first order condition (FOC) with respect to k, we can derive the optimal capital investment.

Capital Choice Under Minimum Wage Constraint

Next, we solve for the stationary equilibrium when the minimum wage binds for some firm. In that scenario, a worker chooses to accept minimum wage or a bargained wage, that is, a firm's wage is $\tilde{w}(z,k;\bar{w}) = \max\{w(z,k),\bar{w}\}$.[10] To simplify the model, I assume that a worker prefer taking the minimum wage to bargaining without loss of generality.[11] Then the firm's per period profits are

$$\pi = \max_k\{zk^{\alpha} - \tilde{w}(z,k,\bar{w}) - rk\} = \max_k\{zk^{\alpha} - \max\{w(z,k),\bar{w}\} - rk\} \tag{2.16}$$

[10]I provide micro foundations for this game by solving a Rubinstein alternating offer game with a minimum wage constraint. See proof and more details in the *Appendix A.1*.

[11]An alternative assumption is to assume that workers randomize according to a mixed strategy as in Bauducco and Janiak (2018). It makes a difference in their environment, in which workers are heterogeneous. Under some parameters, no such a pure strategy exists in the equilibrium when the minimum wage binds. However, since workers are assumed to be homogeneous in our setup, a pure strategy always exists.

$$k^*(z) = \arg\max_k \{zk^\alpha - \max\{w(z,k), \bar{w}\} - rk\} \tag{2.17}$$

where $w(z,k)$ is defined by (2.15).

I will prove optimal capital rule takes the form

$$k^*(z) = \begin{cases} k(z) = \left[\frac{r(1-\psi\beta\lambda)}{(1-\psi)\alpha z}\right]^{\frac{1}{\alpha-1}} & \text{if } z \geqslant z_w \\ \tilde{k}(z) & \text{if } z \in (z_e, z_w) \\ \hat{k}(z) = (\frac{r}{\alpha z})^{\frac{1}{\alpha-1}} & \text{if } z \leqslant z_e, \end{cases}$$

where $\tilde{k}(z)$ is defined such that $w(z, \tilde{k}(z)) = \bar{w}$, that is

$$\begin{aligned} \bar{w} &= (1-\psi)[b + \beta\theta q(\theta)[W(z_a) - U]] \\ &\quad + \psi\{z\tilde{k}(z)^\alpha - \beta[\lambda r\tilde{k}(z) + (1-\lambda)\int_{z_R}^{z_u} rk(z')dF(z')]\}. \end{aligned} \tag{2.18}$$

This $\tilde{k}(z)$ is the capital level such that any firm is indifferent between paying the minimum wage $\bar{w}$ and the Nash Bargaining wage $w(z,k)$. Because $\frac{\partial w(z,k)}{\partial k} > 0$. $w(z,k) \geqslant \bar{w}$ if and only if $k \geqslant \tilde{k}(z)$.

Taking first-order condition (FOC) of flow utility (2.16) with respect to capital k, we have

$$\alpha z(k^*(z))^{\alpha-1} - \frac{\partial w}{\partial k} - r = 0 \tag{2.19}$$

$$\frac{\partial w}{\partial k} = \begin{cases} 0 & \text{if } k^*(z) < \tilde{k}(z) \\ \frac{\partial w(z,k)}{k} & \text{if } k^*(z) \geqslant \tilde{k}(z). \end{cases}$$

Proposition 2: Define z_w as the reservation shock such that $w(z_w, k(z_w)) = \bar{w}$. There exists a unique z_w.

Proof: Taking the first-order condition (FOC) of $w(z,k)$ in (2.14), we have

$$\frac{\partial w(z,k)}{\partial k} = \psi[z\alpha k^{\alpha-1} - \beta\lambda r]. \tag{2.20}$$

As a result, $\frac{\partial k}{\partial z} > 0$ and $\frac{\partial w(z,k)}{\partial k} > 0$ if and only if $\psi[z_R \alpha k(z_R)^{\alpha-1} - \beta\lambda r] > 0$. Plugging the reservation productivity z_R into the firm's problem in (17), we can prove that $\frac{\partial w(z,k)}{\partial k} > 0$ if $1 > \beta\lambda$.

Therefore, if $k^*(z) \geqslant \tilde{k}(z)$, $w(z, k^*(z)) \geqslant \bar{w}$. Equation (16) implies that

$$\alpha z(k^*(z))^{\alpha-1} - \frac{\partial w(z,k)}{\partial k} - r = 0. \tag{2.21}$$

Now the firm's problem is the same as that without the minimum wage constraint, and thus $k^*(z) = k(z)$. To complete the proof, we still need to verify that $k(z) \geqslant \tilde{k}(z)$ if $k^*(z) \geqslant \tilde{k}(z)$.

According to the definition of z_w, it is the lowest shock such that the minimum wage constraint is not binding and a firm does not change its optimal capital level. By rearranging the equation (2.14), we have

$$\begin{aligned} \psi\{z\tilde{k}(z)^\alpha - \beta\lambda r\tilde{k}(z)\} &= \bar{w} - (1-\psi)[b + \beta\theta q(\theta)[W(z_a) - U]] \\ &\quad + \psi\beta(1-\lambda)\int_{z_R}^{z_u} rk(z')dF(z') \end{aligned} \tag{2.22}$$

$$\implies \frac{\partial}{\partial z}(\text{RHS}) = \psi\{(\tilde{k}(z))^\alpha + [z\alpha(\tilde{k}(z))^{\alpha-1} - \beta\lambda r]\frac{\partial \tilde{k}(z)}{\partial z}\} = 0.$$

We have proved that $z\alpha(\tilde{k}(z))^{\alpha-1} > \beta\lambda r$, implying that $\frac{\partial \tilde{k}(z)}{\partial z} < 0$.

However, $\frac{\partial}{\partial z}(k(z)) = (\frac{1}{1-\alpha})(z)^{\frac{-\alpha}{\alpha-1}}(\frac{r}{(1-\psi)\alpha})^{\frac{1}{\alpha-1}} > 0$. Therefore, if $\tilde{k}(z_R) \geqslant k(z_R)$ and $\tilde{k}(z_u) \leqslant k(z_u)$, there exists a $z_w \in [z_R, z_u]$ such that

$$\begin{aligned} \bar{w} &= (1-\psi)[b + \beta\theta q(\theta)[W(z_a) - U]] \\ &\quad + \psi\{z_w k(z_w)^\alpha - \beta[\lambda r k(z_w) + (1-\lambda)\int_{z_R}^{z_u} rk(z')dF(z')]\}. \end{aligned} \tag{2.23}$$

Now we have proved that if $z \geqslant z_w$,

$$k^*(z) = k(z) = \left[\frac{r(1-\psi\beta\lambda)}{(1-\psi)\alpha z}\right]^{\frac{1}{\alpha-1}}.$$

Proposition 3: Define z_e as the reservation shock such that $w(z_e, \hat{k}(z_e)) = \bar{w}$. This is the productivity level such that the firm invested in the efficient amount of capital and bargained with the worker having they pay the minimum wage. There exists a unique z_e and $z_e < z_w$.

Proof: According to Proposition 1, we know if the optimal capital investment level $k^*(z) < \tilde{k}(z)$, $w(z, k^*(z)) < \bar{w}$, then firm's flow profit is as follows

$$\pi = \max_{k^*}\{z(k^*)^\alpha - \bar{w} - rk^*\}.$$

Since the wage is independent of the capital choice, it is straightforward to see that

$$k^*(z) = \hat{k}(z) = (\frac{r}{\alpha z})^{\frac{1}{\alpha-1}}. \tag{2.24}$$

Now we need to make sure that $\hat{k}(z) \geqslant \tilde{k}(z)$. It's easy to see that $\frac{\partial \hat{k}(z)}{\partial z} > 0$, while $\frac{\partial \tilde{k}(z)}{\partial z} < 0$, thus, if $\tilde{k}(z_R) \geqslant \hat{k}(z_R)$ and $\tilde{k}(z_u) \leqslant \hat{k}(z_u)$, there exists a $z_e \in [z_R, z_u]$ such that

$$\begin{aligned}\bar{w} &= (1-\psi)[b + \beta\theta q(\theta)[W(z_a) - U]] \\ &+ \psi\{z_e\hat{k}(z_e)^\alpha - \beta[\lambda r\hat{k}(z_e) + (1-\lambda)\int_{z_R}^{z_u} rk(z')dF(z')]\}.\end{aligned} \tag{2.25}$$

As such z_e is the lowest shock such that a firm invests efficient capital level and minimum wage constraint is exactly binding.

Next, I show that $z_e < z_w$. Since $\hat{k}(z) > k(z)\ \forall z$, $\hat{k}(z_w) > k(z_w) = \tilde{k}(z_w)$. Therefore, $z_e < z_w$ by monotonicity of $\hat{k}(z)$ and $\tilde{k}(z)$.

So far, we have shown that $\forall z < z_e$, the optimal capital investment of firms is efficient, i.e., $k^*(z) = (\frac{r}{\alpha z})^{\frac{1}{\alpha-1}}$.

Proposition 4: For all $z \in [z_e, z_w]$, the optimal capital investment of firms $\tilde{k}(z)$ satisfies

$$\begin{aligned}\bar{w} &= (1-\psi)[b + \beta\theta q(\theta)[W(z_a) - U]] \\ &+ \psi\{z\tilde{k}(z)^\alpha - \beta[\lambda r\tilde{k}(z) + (1-\lambda)\int_{z_R}^{z_u} r\tilde{k}(z')dF(z')]\}.\end{aligned} \tag{2.26}$$

Proof: We prove the Proposition 4 by contradiction. Suppose there exists another optimal capital choice $k^{**}(z)$ and $k^{**}(z) > k^*(z) = \tilde{k}(z)$. According to Proposition 2 and 3, we can prove that if $z_e < z < z_w$, $w(z, k^{**}(z)) \geqslant \bar{w}$. Thus, $k^{**}(z)$ should be equal to $k(z)$. However, $k(z) < \tilde{k}(z)$. Contradiction.

Symmetrically, suppose that there exists another optimal capital choice $k^{**}(z)$ and $k^{**}(z) < k^*(z) = \tilde{k}(z)$, then $w_{NB}(z, k^{**}(z)) < \bar{w}$. Thus, $k^{**}(z)$ should be equal to $\hat{k}(z)$. However, $\hat{k}(z) > \tilde{k}(z)$. Contradiction.

Now, I define z_{R_m} to be the productivity level such that $J(z_{R_m}) = 0$. z_{R_m} is the threshold shock level that firms are indifferent between exiting and staying in the new equilibrium under the minimum wage constraint. It implies that $S(z_{R_m}) = \frac{rk^*(z_{R_m})}{1-\psi}$. To compare the relative level of z_{R_m} and z_e and z_w, three possible scenarios could happen.

Case 1: If $z_{R_m} < z_e$, then $w(z_{R_m}) = \bar{w}$, $k^*(z_{R_m}) = (\frac{r}{\alpha z_{R_m}})^{\frac{1}{\alpha-1}}$.

Case 2: If $z_e \leqslant z_{R_m} < z_w$, then $w(z_{R_m}) = \bar{w}$, $k^*(z_{R_m}) = \tilde{k}(z_{R_m})$

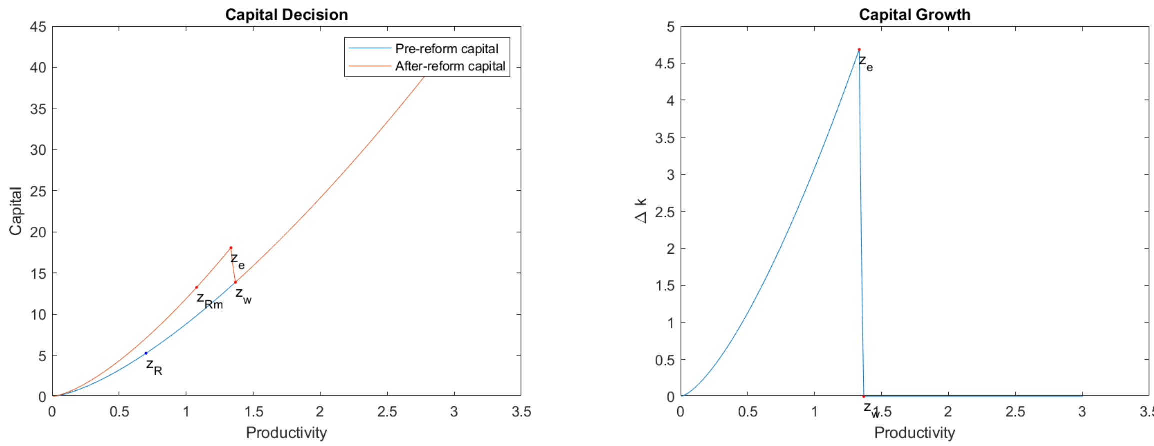
Case 3: If $z_{R_m} \geqslant z_w$, then $w(\frac{w}{R}) = w_{NB}(z_{R_m}, k(z_{R_m}))$, $k^*(z_{R_m}) = [\frac{r(1-\psi\beta\lambda)}{(1-\psi)\alpha z_{R_m}}]^{\frac{1}{\alpha-1}}$

To visualize the result, I use the calibrated parameter values to show the effects of minimum wage. Assume that a firm receives the same productivity z before and after the minimum wage reform in Figure 2.4, the firm's capital choices are shown in the left panel and its corresponding capital difference in the right panel, where the difference in capital $\Delta k(z)$ is defined as $\Delta k(z) = k_{post} - k_{pre}$. The wage payments across firms are shown in Figure 2.5.

Discussion

I devote the rest of subsection to discuss the impact of the minimum wage on capital and employment, as well as on the wage distribution. I consider the case of a production function with constant returns to scale in labor h and thus matches within a firm are independent. As a result, it is equivalent to set $h = 1$.[12] Since labor and capital are complements, without a commitment of wage contract, firms

[12] Cahuc et al. (2008) have shown that with a minimum wage constraint, it's profitable for a firm to overemploy in order to reduce wages if the production function is concave in h and wages are determined via Stole and Zwiebel (1996)



(a) Optimal Capital Level (b) Difference in Capital

Figure 2.4: Optimal Capital With and Without Minimum Wage Constraint

Note: Figure plots the optimal capital investment $k(z)$ across firm productivity z under the calibrated model. The orange line represents the optimal capital investment after the minimum wage reform given the same level of true minimum wage, while the blue line represents the optimal capital investment before the minimum wage reform. The z_R and z_{R_m} represent the productivity cutoffs for exiting with and without minimum wage binding, respectively. z_w is the productivity cutoff for firms whose wages are is equal to the minimum wage. z_e is the productivity cutoff for firms whose bargained wage is equal to the minimum wage.

tend to underinvestment in capital in the presence of a holdup problem: the firm can decrease wages by underinvesting in capital.

In this context, the existence of a minimum wage may impact employment and capital choices in different ways. To see how it changes firms' capital decisions. First, it may alleviate the holdup problem. Indeed, when the minimum wage is effective, (i.e minimum wage is binding for some firms) these firms can no longer reduce the wage of workers by underinvesting, as the minimum wage is a floor on wages. As a result, capital demand increases, and some firms, (i.e those shock less than z_e but higher than z_{R_m}) invest efficient level of capital, while firms with productivity higher than z_e but lower than z_w also increase capital to some extent.

Employment decreases because of the increases of wages and therefore lower surplus of the matches. Thus, the reservation shock z_R may increase and more

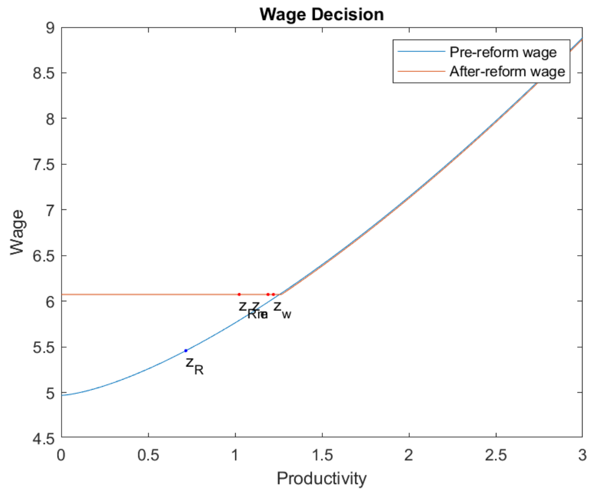


Figure 2.5: Wage levels Pre- and After-reform

Note: Figure plots the wage payments $w(z, k(z))$ across firm productivity z under the calibrated model. The orange line represents the wage payments after the minimum wage reform given the same level of true minimum wage, while the blue line represents the wage payments before the minimum wage reform. The z_R and z_{R_m} represent the productivity cutoffs for exiting with and without minimum wage binding, respectively. z_w is the productivity cutoff for firms whose wages are is equal to the minimum wage. z_e is the productivity cutoff for firms whose bargained wage is equal to the minimum wage.

firms exit the market. Since the labor choice in this one-vacancy-one firm model is binary for each firm, unemployment rate increases as more jobs are destroyed.

The minimum wage may also have effects on the wage distribution. Obviously, it directly affects the wage of low-wage workers, but it also have spillover effects on the wage of other workers if the minimum wage level is nontrival, i.e $\bar{w}$ is higher than the lowest wage payment before 2004. This is due to the capital demand effect: the incidence of minimum wage increases capital demand increases the negotiated wage of some workers, whose current firms' productivity is between z_e and z_w, because the increase of the outside option of being unemployed U may be larger than the decline of firm's surplus, especially when the minimum wage level is not too high. If the level of a minimum wage is too high, a large number of firms exiting the market will decrease the probability of finding a job, and thus the value of being unemployed could decrease.

2.5 Calibration

Guided by theoretical and empirical results shown above, we discuss the calibration strategies in this section. We aim to (1) estimate the friction in the labor market (2) separately identify the importance of general human capital training investment from firm-specific human capital training investment through matching both micro- and macro-moments characterizing the Chinese Labor market from 2002-2006. In order to fit the frequency of my firm-level data, we calibrate the model at a quarter frequency instead of a monthly frequency. I argue this is a reasonable setting because the average capital adjustment time is at least 2 years (Getzmann et al. (2014)).

Parameterization and External Calibration

We set the annual discount rate to the unconventional value of $\beta = 0.9463$ that corresponds to an annual interest rate of $r = 5.68\%$.[13]

Firm Distribution We assume that the distribution of firm productivity follows the lognormal distribution, i.e., $z \sim \mathtt{lognormal}(\mu, \sigma^2)$, with the two parameters governing the shape of the distribution.

Production Function Since we assume that the production function of a firm is constant return to scale in terms of firm size, it is natural to proceed with a Cobb-Douglas production function $F(z, K, L) = zK^{\alpha}L^{1-\alpha} = z(\frac{K}{L})^{\alpha}L$, where K is the total capital stock and L is the number of workers. Therefore, the production function of each independent producing team is $f(z, k) = zk^{\alpha}$, where k is the capital per worker and α represents the factor share of capital or the elasticity of value added

[13] In the model, r is defined as the real corporate bank loan rate. Hau et al. (2016) use a corporate bank loan rate of $r = 7.7\%$ for medium size firms and a corporate bank loan rate of $r = 8.4\%$ for small firms. Small firms refer to firms with workers less than 100, and medium size refers to firms with workers less than 1000. However, since there is no firm size in my model, and I abstract away from default possibility, I argue setting r to be the real interest rate is reasonable.

with respect to capital. Following the literature on the estimation of the production function, I run the following regression at the firm-year level

$$\ln(y_{it}) = \alpha K_{it} + (1-\alpha)L_{it} + \epsilon.$$

The elasticity of capital is estimated to be $\alpha = 0.3121$ using Levinsohn and Petrin (2003). I argue this value is in line with the literature. This value falls in the range of typically estimated for value-added production functions.[14]

Matching function As standard in the labor search literature, we assume the matching function has the Cobb-Douglas functional form. Specifically, the matching function is assumed to be $M = AU^{\eta}V^{1-\eta}$, where A represents the matching efficiency, U represents the measure of unemployed and V represents the measure of vacancies. Due to the lack of data on the vacancies and the number of job applicants, we are unable to separately identify A from η.[15] Therefore, the matching elasticity η is taken from Petrongolo and Pissarides (2001) and set to 0.3.

Nash Bargaining The size of holdup problem crucially depends on the the worker's bargaining power ψ. For the current version of the paper, I set worker's bargaining power to be 0.5.

Internal Calibration

Ideally, we would identify the model parameters taking advantage of a natural experiment. However, instead of looking for such experiments, we internally estimate the remaining parameters using Simulated Method of Moments (Gourieroux et al. (1993)), as collected in Θ. To implement the approach, we calculate a vector of moments from the data $m(\Theta_0)$. Counterparts to these moments, defined as $m(\Theta)$,

[14] Estimation of the factor share of capital vary widely. For example, Brandt and Zhu (2010) argue that α in China ranges from 0.4 to 0.42 using the same data as I do. After correcting the firm-level data matching and merging method and real capital imputation, Rudai (2015) estimate the Total Factor Productivity (TFP) of Chinese manufacturing enterprises by both Levinsohn and Petrin (2003) and Olley and Pakes (1992). He shows that the capital share is between 0.28-0.41 varying with sectors.

[15] Theoretically speaking, we should use the vacancy-level data to back out the elasticity parameters in the matching functions.

are then constructed based on the simulated moments from the model, given a parameter vector Θ. In particular, I simulate the model by assuming 100,000 firms for 2000 periods to derive a stationary wage distribution at the initial steady state to fit pre-reform Chinese data. The procedure then is to iterate on the parameters so as to bring the moments constructed from the simulated data as close as possible to the ones estimated from the data. Parameters are jointly disciplined by matching the simulated moments to data counterparts. Specifically, we find a value for Θ to maximize the remaining 7 parameters

$$\Theta = \left\{ A, \mu, \sigma, b, c, \lambda, z_a \right\},$$

where b be the value of non-market production, c is the flow vacancy cost each period, z_a is the initial productivity upon entry.

The corresponding estimator is shown below,

$$\hat{\Theta} = \arg\min_{\Theta} [m(\Theta_0) - m(\Theta)]' D [m(\Theta_0) - m(\Theta)]$$

where D is a diagonal weighting matrix whose diagonal is the inverse of the squared value of the data moments. I explain the identification strategies in detail.

The vector $m(\Theta_0)$ consists of two sets of moments: one is for aggregate economy and one is based on firm-level panel data from China. The first set of moments consists of the average unemployment rate and job finding rate of the Chinese labor market before 2004. The unemployment rate is informative about the flow payoff from leisure b. The larger b is, the higher the unemployment rate is. The average unemployment rate is computed by taking average of 2-year unemployment rates gathered by Ministry of Labour and Social Security.[16] Since there is no official job finding rate and vacancy filling rate, and very few studies investigate Chinese job market situations, I compute both average job finding rate and average vacancy filling rate using the China Statistical Year Book from NBS as untargeted moments.

[16]Several studies argue that the official unemployment rate was underestimated. See Giles et al. (2005), Feng et al. (2017) for more discussion.

A is then chosen to fit the appropriate job finding rate and vacancy filling rate. The cost of creating a vacancy c must match the equilibrium value of entry.

The second group of moments in $\bar{m}$ includes firm-level production inputs, specifically, wages and capital-labor ratio. Since μ and σ govern the shape of the distribution of the productivity, means for log capital and log wages and their variances across all firms. New entries are defined as those whose birth year is survey year. The entry rate of each year is computed as the ratio of total number of new entries to total number of firms in that survey year. Since this data only include firms with a lower bound of gross revenue, the true entry rate is underestimated. Given the distribution of productivity, the mean log wage and log capital across new entrants pin down the entry-level productivity z_a. The correlation between log wages of two consecutive years of each individual firm pin down the persistence of productivity λ.

Since μ and σ govern the shape of the distribution of the productivity, we use means for log capital and log wages and their variances across all firms. Note that it is straightforward to see that capital is monotonically increasing in productivity z. And same argument holds for wage and productivity after taking derivative of wage respect to z.[17] This is the main idea behind identification of productivity distribution parameters μ and σ because I can express $\ln z$ as a function of the log wage and log capital data and several parameters discussed below.

Though we externally calibrate the value of the bargaining power ψ, we illustrate the main idea behind identification strategy. The model predicts that for a low-wage firm pre-reform, it should increase its capital stock, from $[\frac{r(1-\psi\beta\lambda)}{(1-\psi)\alpha z}]^{\frac{1}{\alpha-1}}$ to $[\frac{r}{\alpha z}]^{\frac{1}{\alpha-1}}$, if it survived and received a low shock. Therefore, the size of capital growth rate identifies workers' bargaining power. As a result, I compute the average capital growth rate between 2003 and 2005 among survivors based on their 2003 wage deciles. Since the the low-wage firms are the most exposed to the reform, I target

[17] Rewrite wage equation (16), and take derivative respect to productivity as

$$\frac{\partial w(z,k^*)}{\partial z} = \psi\{\frac{r(1-\psi\beta\lambda)}{(1-\psi)\alpha}\}^{\frac{1}{\alpha-1}}[\frac{r(1-\psi\beta\lambda)}{(1-\psi)\alpha} - \beta\lambda r]z^{\frac{1}{1-\alpha}} > 0$$

where k^* is the optimal capital decision defined in (12) at firms in the lowest three deciles. I will use the rest of seven moments in the validation test.

Calibration Results and Model Fit

Table 2.3 summarizes the parameter description and parameter values, including the internally calibrated in the model and externally calibrated. Several other features of my estimation results merit comment.

Estimates of b and c are measured in terms of the numeraire — the price of products. The sunk cost of creating a vacancy is estimated to be ¥7812.7.[18] Note that this value is relatively small compared with standard search literature because c is interpreted as the cost of posting a vacancy. Yet in my model, the total cost of creating a new vacancy or hiring a worker includes two parts: vacancy posting cost and capital investment, which is ¥9916.5753.[19]

The value of leisure or home production is $b = 5.9389$. We can see that b is about 50 percent of the average wage, which is higher than the value of leisure 60 percent of labor income in Shimer (2005) because b includes value of non-market activities and leisure not just unemployment insurances. The entry-level productivity $z_a = 2.3439$ is higher than the average productivity $\mu = 0.6546$. The results are not surprising given the limitations of our data sample. A new entrant is defined as those enterprises started business and joined database with first-year revenue more than 5 million Yuan. The probability of not receiving a new shock $\lambda = 0.85$ shows that productivity is highly persistent.

Table 2.4 reports the data moments and their model counterparts. Overall, the model fits the data well in statistics. However, this model is not able to generate a large enough entry-level capital, while it overestimates entry rate, unemployment and wage correlation. The model that generates higher entry rate and unemployment is expected, because I compute the entry rate only using data on new firms whose annual sales is above 500 million Chinese Yuan, and some researchers think

[18] $7.8127 \times ¥1,000 = ¥7812.7$.

[19] $c + r * k^*(a_a) = ¥7812.7 + 0.0568 \times e^{3.6120} \times ¥1000 = ¥9916.5753$.

the official Chinese unemployment rate is too low. The true unemployment rate from 2002 to 2008 ranges up to 11% (Giles et al. (2005), Feng et al. (2017)).

Finally, to assess the model's ability in reproducing the untargeted moments, we compute the job finding rate, vacancy filling rate and variance of log capital in Table 2.5. The model predicts a much higher value of vacancy filling rate than the data counterpart I computed using Statistics Year Book, to the best of my knowledge, neither previous literature nor public data reveals true job finding rate. Hence, it is possible that there exists bias in the data moment.

Table 2.3: Parameter Estimates

Parameters	Description	Value
Externally calibrated		
β	Quarterly discount rate	0.987
r	Real Interest Rate	0.0568
α	share of capital in total output	0.3121
η	Matching elasticity of labor market	0.3
ψ	Worker Bargaining Power	0.5
Internally calibrated		
μ	Productivity distribution mean	0.6546
σ	Productivity distribution std	1.1122
z_a	Productivity distribution	2.3439
A	Matching function efficiency	1.4212
b	Unemployment benefits	5.9389
c	Vacancy costs	7.8127
λ	Shock arrival rate	0.8112

Notes: This table reports parameters and calibrated values.

Validation

In this subsection, I validate the model by examining the effect of the 2004 Minimum wage reform on firms and the aggregate economy using my calibrated model. Specifically, I set the minimum wage level in the model at the same value of data to determine the extent to which these simulated effects can capture the changes in firms' employment and capital decisions. In this experiment, I set the real minimum

Table 2.4: Auxiliary Model and Data

Targeted Moments	Data	Model Prediction
Entry rate	0.0310	0.0352
Entry lnwage	1.9824	2.0429
Entry lncapital	3.5290	3.4108
Unemployment rate	0.0441	0.0649
Average lncapital	3.5929	3.6120
Average lnwage	2.2233	2.2353
Average lnwage variance	0.3139	0.3032
Wage correlation	0.6340	0.7115

Notes: This table lists the targeted moments informative to identifying the parameters in data and the counterpart of simulation results in the axillary model, including the distribution of firm-level wages and the status of aggregate labor market. All the transition rates are seasonally adjusted in the data, quarterly averaged, logged, and HP-detrended. All wages are in real terms, deflated by the Chinese CPI with base year 2000.

Table 2.5: Untargeted Moments

Untargeted Moments	Data	Model Prediction
Job finding rate	0.5145	0.5123
Vacancy filling rate	0.4649	1
Average lncapital variance	1.3202	1.0651

Notes: This table lists the untargeted moments in data and the counterpart of simulation results in the axillary model. All the transition rates are seasonally adjusted in the data, quarterly averaged, logged, and HP-detrended.

wage level to its actual level in 2005, which is $\bar{w} = 6.0735$. Whenever possible, I focus on year 2005 because the average capital adjustment time is at least 2 year (Getzmann et al. (2014)). Each firm drew a random productivity from stationary distribution $G(z)$. Starting from 2003, each firm drawa a new shock with probability γ. I trace every survival firm in 2003 from 2004 to 2005, and compare the capital growth rate of these firms using 2005 and 2003 average capital data based on the percentile of 2003 wage.

The results of the experiment on capital growth are reported in the first ten rows of Figure 2.6 and Table 2.6. There are three aspects worth attention. First, this model is able to capture the data pattern that low-wage firms have a larger capital growth than high-wage firms. Moreover, the lower wage was in 2003, the larger capital growth was in 2005. However, this model predicts too much capital increases compared with data for low-wage firms, while too little capital growth for high-wage firms under a standard bargaining power. It was partially due to the poor persistency of productivity. Recall, our calibrated model predicts a higher correlation of productivity across periods than the data in Table 2.4. Specifically, holdup is able to explain all of the capital increases of lowest 10 percent firms, and half of the that of firms below 50 percent, while contributes nothing to the other half the firms. One thing worth noticing is that high-wage firms are predicted to have an average negative capital growth because they are more likely to receive a negative shock conditional on receiving a new productivity than low-productivity firms. This implies that there are other mechanisms that drive the capital increases for high-wage firms. Second, this model is also able to reproduce a decreasing pattern in capital growth, although it is not as smooth in the data. Third, firms in the 30th, 50th, 80th percentiles show different growth pattern. One explanation is that there are large measurement errors in these firms. Another more possible explanation is that in the real world, there is no monotonic relationship between wage and capital even without measurement error. This further rejects the assumption of constant return to scale production function.

In the Table 2.6, I focus on the effect of minimum wage reform on aggregate economy. We can see that the model predicts a lower firm exit rate and job finding rate

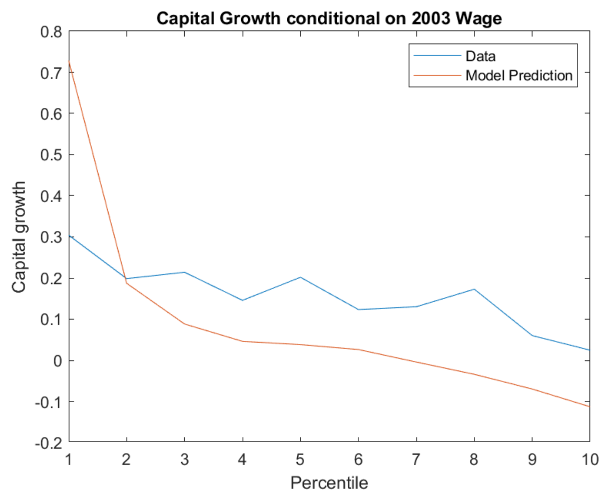


Figure 2.6: Model Fit for Capital Growth Conditional on Pre-reform Wage Percentile
Notes: This table illustrates the average capital growth rate of firms in 2005 compared to 2003 based on the percentile of 2003 wage. The capital growth is defined as $\frac{\text{Capital per worker}_{2005} - \text{Capital per worker}_{2003}}{\text{Capital per worker}_{2003}}$. The orange line represents the changes in the simulated model and the blue line represents the changes in the data.

Table 2.6: Capital Growth Across Firm

Moments	Data	Model Prediction
Average lnWage	2.7786	2.3349
Average lnCapital	4.3203	3.8391
Exit Rate	0.0879	0.0536
Unemployment Rate	0.0449	0.0965
Job finding rate	0.5244	0.5021

Notes: This table lists the moments in data and the counterpart of simulation results in the axillary model. All wages are in real terms, deflated by the Chinese CPI with base year 2000.

because a binding minimum wage decreases the surplus of some low-productivity matches. As a result, the reservation shock z_R increases that low-productivity firms exit the market, and market tightness θ decreases. However, this model overpredicts the new unemployment rate after the minimum wage reform.

To examine how the 2004 minimum wage reform on social surplus, I compute the worker surplus, producer surplus and total surplus before and after the reform shown in Table 2.7. The worker surplus consists of the utility of unemployed workers and employer workers, which is $uU + (1-u)\int_{z_R}^{z_u} W(z)dG(z)$. $G(z)$ is the Probability Density Function (PDF) of firms with productivity z in the equilibrium. The producer surplus represents the total value of incumbents, that is, $(1-u)\int_{z_R}^{z_u} J(z)dG(z)$. The total surplus is the aggregate surplus of the worker surplus and the producer surplus. There are two features worth attention. First, the worker surplus increases greatly while the producer surplus decreases a little. As a result, the total surplus of the entire economy is larger after the minimum reform. Second, the size of the producer surplus is much smaller than that of the worker surplus, which is the result of the free entry condition. Thus, the role of firms is underestimated.

Table 2.7: Social Surplus

	Before 2004	After 2004
Consumer Surplus	171.5810	223.8907
Producer Surplus	26.0309	24.9022
Total Surplus	197.6119	248.7929

Notes: This table lists the social surplus of simulation results in the axillary model. The worker surplus consists of the utility of unemployed workers and employer workers. The producer surplus represents the total value of incumbents. The total surplus is the aggregate surplus of the worker surplus and the producer surplus.

Counterfactual Experiment

In this experiment, I attempt to find the firm loan rate (interest rate) reduction that the Chinese government would need to achieve the same average increase in capital and wage. The goal is to find out the "equivalent" effect of minimum wage by targeting at the average wage and average capital under minimum wage, with the calibrated parameters. The results are shown in Table 2.8. We can see that the minimum wage reform is equivalent to about 10% reduction in the real interest rate to generate same level of wage growth and capital growth as minimum wage. However, a lower interest rate r predicts a lower unemployment and a higher job finding rate. This is due to the fact that a cheaper capital rental price increases all firms capital and wages, which in turn increases average capital and wages across the economy. As more entrants enter the market, the value of being unemployed is higher due to a higher wage and a larger chance to find a job. Therefore, at new steady state, the reservation productivity z_R increases mildly compared with the minimum wage increase, which mostly affects low-productivity firms.

Table 2.8: Counterfactual Analysis: Tax Cut

	Under Minimum Wage	9.97% Tax Deduction
Average Wage	2.3349	2.3323
Average Capital	3.8391	3.8396
Unemployment Rate	0.0944	0.0654
Job finding Rate	0.5021	0.6055

Notes: This table lists the moments in data and the counterpart of simulation results in the axillary model. All wages are in real terms, deflated by the Chinese CPI with base year 2000.

2.6 Conclusion

This paper explores the endogenous capital response of Chinese firms exposed to the minimum wage reform took place in 2004. Using firm-level data in China, I show that firms substitute labor with capital especially in the firms bind to minimum wage. Therefore, we propose a random search model with a new mechanism: wages are determined by Nash Bargaining after capital investment.

Fit to micro data from China, the model delivers several basic messages. First, this model proposes that labor market frictions cause firms to holdup capital investment, and that minimum wage increases can help lessen holdup. Moreover, this model shows that low-wage firms are more likely to increase capital than high-wage firms, and suffer more from holdup issues. Second, this model is able to generate the decreasing pattern in capital growth as in the data, which validates the model. Third, this model suggests that a mild increase in the minimum wage may increase social surplus.

The model could be extended in several directions. First, allowing for a multi-worker random search model can give us both a labor-capital substitution effect and holdup effect. We can identify these effects by comparing different industries with different capital requirements and worker bargaining power. Second, we could make the model dynamic and introduce uncertainty in future minimum wage levels. The uncertainty makes the model more realistic on how people understand the minimum wage policy. I see these extensions as interesting directions for future work.

A.1 Appendix: Data and Empirical Results

First, I follow Brandt et al. (2014) to construct real variables:

- **Output deflators**: To construct an output deflator at the most detailed level possible, I use information from the 1998-2003 surveys, for which firms report the value of their output in both nominal and real prices. Output in real terms is constructed by firms using a set of "reference" prices provided by NBS. In any given year, the ratio of nominal to real output provides an index of prices in that year relative to the base. The change in this index between year t and $t-1$ then is a measure of the firm-specific price change between the two years, which I average to the four-digit industry level. I then calculate the weighted average for each sector, using as weights each firm's current output. These changes are linked over time to provide an output deflator for each of the 423 sectors.

- **Value Added**: Real value added is constructing by separately deflating output, net of goods purchased in for resale and net of indirect taxes, and material inputs.

  Input deflators are calculated using the output deflators and information from the 2002 National Input-Output table. The sectors defined in the Input-Output table are less disaggregated than my deflator computed before. I then construct a concordance table linking IO sectors and 4-digit industries.

- **Wages**: Firms report total annual employment and several components of employee compensation include wages, employee supplementary benefits, and unemployment insurance. Firms began to report retirement and health insurance in 2003 and housing benefits in 2004. I utilize information on the first three categories to generate consistent measures of firms' total wage payment.

- **Capital and investment**: Firms do not report their fixed investment. Each firm reports information on the value of their fixed capital stock at original purchase prices, and their capital stock at original purchase prices less accumulated depreciation. As these book values are the sum of nominal values for different years, I have to make some assumptions to convert them into real values that are comparable across time and across firms. I use information from the 1993 annual enterprise survey to construct estimates at the two-digit industry level by province of the average rate of growth of the nominal capital stock between 1993 and 1998, and information on the age of each firm to calculate the firm's nominal capital stock in the year in which it was established. I assume a annual depreciation rate 9% to compute the real capital for all years up to 2007.

A.2 Appendix: Analytical Solution and Simulation

Derive Job Destruction Curve and Job Creation Curve

First, we can solve the value of being unemployed U (11). Rewrite the value of being unemployed in terms of joint surplus defined in (9) as:

$$\begin{aligned} U &= b + \beta\theta q(\theta)[W(z_a) - U] + \beta U \\ &= b + \beta\theta q(\theta)\psi S(z_a) + \beta U \end{aligned} \tag{27}$$

To compute the value of being unemployed, we need to know the value $S(z_a)$.

Similarly, we can rewrite the value of being employed $W(z)$ (10) as:

$$W(z) = w(z,k) + \beta\psi[(1-\lambda)\int_{z_R}^{z_u} S(z')dF(z') + \beta\lambda S(z)] + \beta U \tag{28}$$

where the middle two terms come from the wage bargaining decision: $W(z) - U = \psi S(z)$, $J(z) + rk(z) = (1-\psi)S(z)$ and $k(z')$ is the optimal capital choice depending on tomorrow's shock z'.

Given (25) and (26), we have:

$$W(z) - U = w(z,k) - b + \beta\psi[\lambda S(z) + (1-\lambda)\int_{z_R}^{z_u} S(z')dF(z')] - \beta\theta q(\theta)\psi S(z_a) \tag{29}$$

To a filled job with productivity z, its value is represented in terms of joint surplus as:

$$\begin{aligned} J(z) = zk^{\alpha}(z) - w(z,k(z)) - rk(z) + \beta(1-\psi)[(1-\lambda)\int_{z_R}^{z_u} S(z')dF(z') + \lambda S(z)] \\ - \beta[(1-\lambda)\int_{z_R}^{z_u} rk(z')dF(z') + \lambda rk(z)] \end{aligned} \tag{30}$$

The bargaining wage is then given by (12):

$$\begin{aligned} w(z,k) &= (1-\psi)[b+\beta\theta q(\theta)\psi S(z_a)] \\ &\quad + \psi\{zk^{\alpha} - \beta[\lambda rk + (1-\lambda)\int_{z_R}^{z_u} rk(z')dF(z')]\} \end{aligned} \tag{31}$$

$\Longrightarrow$

$$\frac{\partial w(z,k)}{\partial k} = \psi[z\alpha k^{\alpha-1} - \beta\lambda r] \tag{32}$$

$\Longrightarrow$

$$k^* = [\frac{r(1-\psi\beta\lambda)}{(1-\psi)\alpha z}]^{\frac{1}{\alpha-1}} \tag{33}$$

Adding up (25), (26) and (28), the joint bargaining surplus $S(z)$ is represented as:

$$\begin{aligned} S(z) &= zk(z)^{\alpha} - b + \beta[\lambda S(z) + (1-\lambda)\int S(z')dF(z')] \\ &- \beta\theta q(\theta)\psi S(z_a) \\ &- \beta[(1-\lambda)\int_{z_R}^{z_u} rk(z')dF(z') + \lambda rk(z)] \end{aligned} \tag{34}$$

By free entry condition,

$$\begin{aligned} (1-\psi)S(z_a) &= J(z_a) + rk(z_a) \\ &= \frac{c}{\beta q(\theta)} + rk(z_a) \end{aligned} \tag{35}$$

θ and z_R are jointly determined by

$$\begin{cases} (1-\psi)S(z_a) &= \frac{c}{\beta q(\theta)} + rk(z_a) \\ (1-\psi)S(z_R) &= rk(z_R) \end{cases}$$

$\Longrightarrow$

$$S(z_a) - S(z_R) = \frac{\frac{c}{\beta q(\theta)} + r[k(z_a) - k(z_R)]}{1-\psi} = \frac{(z_a k(z_a)^\alpha - z_R k(z_R)^\alpha) - \beta\lambda r[k(z_a) - k(z_R)]}{1-\beta\lambda} \tag{36}$$

This is the job destruction curve.

$$(1-\psi)S(z_a) = \frac{c}{\beta q(\theta)} + rk(z_a) \tag{37}$$

This is the job creation curve.

Minimum Wage under Alternating Offer Bargaining Game

Now consider another scenario: A firm and a worker bargain over wages with minimum wage constraint under alternating offers assumption. That is, either firm or worker proposes a wage offer w first, and then the other party must decide whether to accept or reject the proposal. If the other party accepts, the proposal is implemented; if not, the other party proposes another offer until the game breaks down or achieve an agreement. I show *the Subgame Perfect Equilibrium* of this problem generates same result as Nash bargaining solution.

Proof:

Assume firm and worker bargain over a finite round T within each period. Suppose the discount rate of worker within one period is $\beta_w = 1$ and the discount rate of firm within one period is $\beta_F = 1$. Bargaining breaks down with an exogenous probability s. I also assume that $W(\bar{w}) > U$. if $W(\bar{w}) \leqslant U$, minimum wage constraint is not binding for all firms, then the solution is identical to Nash Bargaining.

Assume that worker proposes an offer w_T at round T. According to Osborne and Rubinstein (1990), *the Subgame Perfect Equilibrium* is that worker's offer w_T makes the firm indifferent between accepting and rejecting. Therefore, the value of a worker can receive is the full value of the match, denoted as S, which is production revenue zk^{α}, and firm gets nothing, that is,

$$W_T(z,k,w_T) = zk^{\alpha} = S$$

$$F_T(z,k,w_T) = 0$$

Where $W_T(z,k,w_T) = w_T$ represents the value of the worker if he/she accepts the wage offer w_T, and $F_T(z,k,w_T) = zk^{\alpha} - w_T$ represents the value of the firm proposing the wage offer w_T.

At round $T-1$, firm proposes an offer w_{T-1} to make the worker indifferent between accepting immediately and rejecting now but accepting next round taking exogenous separation rate into consideration, that is,

$$W_{T-1}(z,k,w_{T-1}) = \max\{(1-s)W_T(z,k,w_T) + sU, W(z,k,\bar{w})\}$$

$$F_{T-1}(z,k,w_{T-1}) = S - W_{T-1}(z,k,w_{T-1})$$

where $W(z,k,\bar{w}) = zk^{\alpha} - \bar{w}$ is the value of being an employed worker payed at minimum wage $\bar{w}$.

At period $T-2$, worker proposes an offer w_{T-2} to make the firm indifferent between accepting immediately and rejecting now but accepting next round taking exogenous separation rate into consideration, that is,

$$W_{T-2}(z,k,w_{T-2}) = S - F_{T-2}(z,k,w_{T-1})$$

$$F_{T-2}(z,k,w_{T-2}) = (1-s)F_{T-1}(z,k,w_{T-1})$$

We can solve the game backwards until first round. There are two cases could happen: (1) $(1-s)]W_t(z,k,w_t)+sU > W(\bar{w})$ for all rounds t; (2) $(1-s)]W_t(z,k,w_t)+sU < W(\bar{w})$ at some period $\tilde{T}-1$. For the first case, minimum wage constraint is not binding, then the solution is identical to the one without minimum wage constraint. For the second case, minimum wage binds at period $\tilde{T}-1$. This is possible because the value of a worker can get is monotonically increasing when time goes to T. Now we can rewrite the values staring at $\tilde{T}-1$ and solve backwards:

$$(1-s)]W_{\tilde{T}}(z,k,w_{\tilde{T}})+sU<W(\bar{w})$$

$$W_{\tilde{T}-1}(z,k,w_{\tilde{T}-1})=\max\{(1-s)]W_{\tilde{T}}(z,k,w_{\tilde{T}})+sU,W(\bar{w})\}=W(\bar{w})$$

$$F_{\tilde{T}-1}(z,k,w_{\tilde{T}-1})=S-W_{\tilde{T}-1}(z,k,w_{\tilde{T}-1})=S-W(\bar{w})$$

At $t=\tilde{T}-2$, worker proposes an offer $w_{\tilde{T}-2}$

$$W_{\tilde{T}-2}(z,k,w_{\tilde{T}-2})=S-F_{\tilde{T}-2}(z,k,w_{\tilde{T}-2})=S-(1-s)[S-W_{\tilde{T}-1}(z,k,w_{\tilde{T}-1})]$$

$$W_{\tilde{T}-2}(z,k,w_{\tilde{T}-2})=S-(1-s)[S-W(\bar{w})]$$

$$F_{\tilde{T}-2}(z,k,w_{\tilde{T}-2})=(1-s)F_{\tilde{T}-1}(z,k,w_{\tilde{T}-1})=(1-s)[S-W_{\tilde{T}-1}(z,k,w_{\tilde{T}-1})]$$

$$F_{\tilde{T}-2}(z,k,w_{\tilde{T}-2})=(1-s)[S-W(\bar{w})]$$

At $t=\tilde{T}-3$, firm proposes an offer $w_{\tilde{T}-3}$

$$W_{\tilde{T}-3}(z,k,w_{\tilde{T}-3})=\max\{\beta_w[(1-s)]W_{\tilde{T}-2}(z,k,w_{\tilde{T}-2})+sU],W(\bar{w})\}$$

$$F_{\tilde{T}-3}(z,k,w_{\tilde{T}-3})=S-W_{\tilde{T}-3}(z,k,w_{\tilde{T}-3})$$

Note that $W_{\tilde{T}}(z,k,w_{\tilde{T}})=S-(1-s)F_{\tilde{T}+1}(z,k,w_{\tilde{T}+1})=S-(1-s)[S-W_{\tilde{T}+1}(z,k,w_{\tilde{T}+1})]$. Since $W_{\tilde{T}+1}(z,k,w_{\tilde{T}+1})\geqslant W(\bar{w})$, $W_{\tilde{T}}(z,k,w_{\tilde{T}})\geqslant W_{\tilde{T}-2}(z,k,w_{\tilde{T}-2})$. Thus, $(1-s)W_{\tilde{T}-2}(z,k,w_{\tilde{T}-2})+sU\leqslant(1-s)W_{\tilde{T}}(z,k,w_{\tilde{T}})+sU<W(\bar{w})$. We can rewrite $W_{\tilde{T}-3}(z,k,w_{\tilde{T}-3})$:

$$W_{\tilde{T}-3}(z,k,w_{\tilde{T}-3})=W(\bar{w})$$

Therefore we have proved that once minimum wage constraint binds, the only *Subgame Perfect Equilibrium* as $T\to\infty$ is that firm pays minimum wage and worker accepts assuming that a worker gets higher value than being unemployed.

Now we consider another scenario: firm proposes w_T an offer at round T, which makes the worker indifferent between accepting and rejecting, which implies

$$W_T(z, k, w_T) = \max\{W(\bar{w}), U\} = W(\bar{w})$$

$$F_T(z, k, w_T) = S - W(\bar{w})$$

At period $T - 1$, worker proposes an offer w_{T-1} to make the firm indifferent between accepting immediately and rejecting now but proposing next round taking exogenous separation rate into consideration, that is,

$$W_{T-1}(z, k, w_{T-1}) = S - F_{T-1}(z, k, w_{T-1})$$

$$F_{T-1}(z, k, w_{T-1}) = (1 - s)F_T(z, k, w_T) = (1 - s)[S - W(\bar{w})]$$

At period $T - 2$, firm proposes an offer w_{T-2} to make the worker indifferent between accepting immediately and rejecting now but proposing next round taking exogenous separation rate into consideration, that is,

$$W_{T-2}(z, k, w_{T-2}) = \max\{(1 - s)W_{T-1}(z, k, w_{T-1}) + sU, W(\bar{w})\}$$

$$F_{T-2}(z, k, w_{T-2}) = S - W_{T-2}(z, k, w_{T-2})$$

$$(1 - s)W_{T-1}(w_{T-1}) + sU = (1 - s)[sS + \bar{w}(1 - s) + sU$$

$\Longrightarrow$

$$(1 - s)W_{T-1}(w_{T-1}) + sU > W(\bar{w}) \Longleftrightarrow (1 - s)sS + (1 - s)^2 W(\bar{w}) + sU > W(\bar{w})$$

As we can see that if $(1-s)sS+(1-s)^2W(\bar{w})+sU > W(\bar{w})$ is true at any round t, minimum wage is not binding. If $(1-s)sS+(1-s)^2W(\bar{w})+sU < W(\bar{w})$, minimum wage is binding. Like I have showed above that once minimum wage is binding, it will bind every other rounds. Then the unique *Subgame Perfect Equilibrium* is that firm pays minimum wage and worker accepts or worker rejects and proposes a slightly higher wage than minimum wage, assuming that a worker gets higher value than being unemployed. If $(1-s)sS + (1-s)^2 W(\bar{w}) + sU = W(\bar{w})$, there exists mixed strategy in *Subgame Perfect Equilibrium*.

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