Markets with Frictions

by

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Abstract

Chapter 2: This chapter studies the emergence of middlemen (intermediation) and its consequences for welfare and redistribution. In Rubinstein and Wolinsky's seminal model on intermediation, middlemen only create value when they have an exogenous advantage in their search speed. I show that this result depends critically on the restriction that middlemen can only carry indivisible quantities of goods. Taking into account the intensive margin of production and allowing them to carry divisible quantities, middlemen with comparatively high bargaining power can create value even when they have a disadvantage in their search cost. While the presence of middlemen can improve welfare, equilibria remain suboptimal due to search and participation externalities. I describe a multi-instrument tax-subsidy scheme that controls participation levels by producers and middlemen and restores efficiency.

Chapter 3: This chapter studies the relationships between money and middlemen both serving as critical market institutes to facilitate trades with frictions. Focusing on the middlemen who are good at enforcing repayment from buyers, I find that money and middlemen can be substitutes and complements: when credit condition is poor in retail market for trades between sellers and buyers, middlemen and money become substitutes and inflation encourages the use of repayment; when credit condition is

poor in the wholesale market for trades between sellers and middlemen, middlemen and money become complements and inflation discourages repayment. While the existence of middlemen with advantage at enforcing repayment mitigates buyers' payment friction in retail market, buyers can be better or worse off with middlemen depending on the level of inflation.

Chapter 4: This chapter studies long-run effect of inflation on employment and investment in a new monetarist model with a frictional labor market, and a frictional goods market with different market structures. Higher inflation leads to higher unemployment, less capital accumulation and fewer firm entry. These effects are robust to goods market structures. It is shown that unemployment benefits lead to more unemployment, less firm entry and less capital. However, depending on the market structure, active firms may become larger or smaller. The model is tractable and delivers many analytic results.

Chapter 1

Introduction

Intermediation (middlemen) has long been recognized both empirically and theoretically as an important instrument that mitigates market frictions, including limited commitment, asymmetric information and difficulties in coordinating trades. Middlemen emerge as they are better at addressing some of these frictions to make a transaction. In the empirical literature, evidence has been provided to show the contribution of intermediation as well as the variation of its extent across markets. A common thread in my dissertation is to study how market instruments, including money and intermediation, ameliorate market frictions, and their effects on welfare.

In Chapter 2, "Middlemen in Search Models with Intensive and Extensive Margins", I develop a model to study the emergence of intermediation and its consequences for welfare by incorporating both number of trade (extensive margin) and quantity per trade (intensive margin) which are endogenously determined. This chapter contributes to the existing empirical and theoretical literature in the following way. Given endogeneity issues in studying middlemen, some identification difficulties in empirical work can be addressed by the framework proposed in my paper as it provides a theoretical structure on how market frictions give rise to intermediation and how welfare would be affected by intermediation at different friction levels. Theoretically, Rubinstein and Wolinsky (1987) first formalize a theoretical framework with a search-bargaining model to study the emergence of intermediation. Literature spurred by their paper study middlemen given different advantages they have at mitigating market frictions. A common assumption in these papers is that goods are indivisible, i.e. quantity per trade is restricted to be only 0 or 1. This restriction neglects a key feature in markets, namely, producers and middlemen can adjust their trading quantities to accommodate market frictions. Moreover, with the restricted quantity, bargaining power can only affects the distribution of a surplus, while with an adjustable quantity it also affects the surplus to be shared. I depart from this setup and find that participation and effects on welfare of intermediation depend crucially on quantity per trade being unrestricted. I show that participation of middlemen would be underestimated (overestimated) without considering the intensive margin. In addition, in pervious versions without the intensive margin, middlemen reduce welfare when their only advantage is their rent extraction ability, while in this chapter such intermediation can be welfare-improving. This new insights can shed light on empirically and policy relevant issues, including the debate about middlemen contributing to efficiency versus acting as "bloodsuckers" that only buy low and sell high without contributing to value added.

Rising from Chapter 2 is an interesting topic of the interaction of intermediation and payment frictions. Chapter 3, studies the role of middlemen who are better than producers at enforcing repayment from consumers. While middlemen can be encouraged by providing consumers the option of using credit such that inflation cost associated with holding money is avoided, middlemen themselves can suffer from inflation if credit is not perfect in their frictional trades with producers. Depending on market structure in terms of payment frictions in up-stream and down-stream submarkets for middlemen's resale activities, money and intermediation, as two of the most important instruments that ameliorate market frictions, can be substitutes as well as complements. By incorporating both relationships and studying their interactions, results in Chapter 3 shed new light on the effect of inflation and middlemen on consumers' welfare in markets with search and payment frictions.

Besides intermediation, monetary policy and effects of inflation are also within the scope of my dissertation. Given the fact that most monetary transactions are decentralized, it is natural to study related issues in a model with frictions. Interestingly, different market structures give rise to results that can explain the coexistence of various market outcomes witnessed in an economy. Chapter 4, "Inflation, Investment and Unemployment: A New Monetarist Framework", studies long-run effect of inflation on employment and investment in a new monetarist model with a frictional labor market, and a frictional goods market with different market structures. This chapter shows that higher inflation leads to higher unemployment, less capital accumulation and fewer firm entry. These effects are robust to structures of goods market. It is also found that unemployment benefits lead to more unemployment, less firm entry and less capital. However, depending on the market structure, active firms may become larger or smaller. Tractability is maintained in the model and it delivers many analytic results.

Chapter 2

Middlemen in Search Models with Intensive and Extensive Margins

2.1 Introduction

It has been argued in empirical work that market frictions, including the problems of limited commitment, asymmetric information and difficulties in coordinating trades, are ubiquitous and explains the emergence of middlemen by their superior capacity at addressing some of these obstacles. The measurement of intermediation is a difficult problem. Given endogeneity issues in studying middlemen, empirical work in the area suffers from the lack of a theoretical structure on how market frictions give rise to intermediation. Theoretically, first formalized by Rubinstein and Wolinsky (1987), intermediation is studied in a model with explicit frictions. In their paper, intermediation is an equilibrium and is efficient if and only if intermediaries have an advantage over producers in their ability to search for consumers. A sizable literature studies intermediation using versions of Rubinstein and Wolinsky's model with middlemen having various advantages at mitigating market frictions. Similar to Rubinstein and Wolinsky, a stark restriction in these papers is that goods are indivisible, i.e. quantity per trade is either 0 or 1. This restriction neglects a key feature, namely, that producers and middlemen can mitigate search frictions by adjusting their trading quantities and consequently influence patterns of trade and social welfare. The quantity involved in direct trade, i.e. between producers and consumers, and indirect trade, i.e. between middlemen and consumers, is a critical dimension for the study of intermediation, especially when we want to have a model that is empirically and policy relevant.[1]

In this chapter, I develop a search-bargaining model which departs from the restriction of indivisible goods and allows quantity per trade to be endogenously decided by agents' strategic choices. In particular, I ask when intermediation emerges in equilibrium, what effects it exerts on welfare and redistribution, and how the extent of intermediation is determined across markets, including the intensive and extensive margins.

New insights are derived on equilibrium and efficiency by taking both margins into account. Equilibrium is inefficient on the two margins and the conventional way of restoring efficiency by setting bargaining powers correctly can only achieve efficiency on one of the two. For the effects of intermediation on welfare and redistribution,

[1] For instance, Philippon (2014), which measures financial intermediation in the U.S. over the past 130 years, finds that the share of financial intermediation in the U.S. GDP varies over time and most of these variations can be explained by corresponding changes in the quantity of intermediated assets.

I show that welfare can be improved by middlemen even if their only advantage is their bargaining power. This result is different from the existing literature with quantity per trade to be either 0 or 1 (Farboodi, Jarosch and Menzio (2017) and Masters (2007, 2008)) in which intermediation is a rent extraction activity. I find that the effect on welfare depends critically on the quantity restriction per trade. With adjustable quantity, social value created by intermediation not only comes from the number of trades but also the size of each trade. Intuitively, middlemen's bargaining power not only influences the distribution of a total surplus but also crucially affects the trading quantity and therefore the total surplus to be shared. This explains why intermediation in the literature without the intensive margin is welfare-reducing while here welfare losses can be outweighed by gains. Furthermore, I show that welfare is redistributed with intermediation in such a way that, when the search cost is small, welfare is improved for producers and middlemen while consumers are worse off, then as the search cost increases, all agents are better off. These new findings are given by the economics of divisible goods.

The chapter explains the existence and welfare effects of intermediation based on two features. One is search costs. Facing the same meeting rate with consumers, if middlemen bear a relatively higher search cost, there can be no intermediation. The other feature is the meeting-specific terms of trade: payment and quantity in a direct trade can be different from those in an indirect trade, and they depend on producers and middlemen's bargaining powers. Given the same total surplus in direct and indirect trades, if middlemen are better are extracting rents from consumers, they can earn a larger share of surplus. On top of that, the intensive margin allows the total surplus to be affected by bargaining power through quantity. Therefore, a superior bargaining power not only gives a larger share, but also a larger total surplus.

The environment is as follows. I consider a market with three types of agents, consumers, producers and middlemen, and use a search-bargaining framework related to recent work on models of monetary and asset markets, especially Lagos and Wright (2005). Tractability is maintained by alternating periods of decentralized and centralized exchange: first producers and middlemen choose whether to trade with each other in the wholesale market. It is followed by the retail market which is also frictional. In the retail market, there can be direct trade, indirect trade or both depending on strategic choices of producers and middlemen. They are eligible to participate in retail if they have inventory ready and pay a search cost. After the retail market, agents reconvene in a centralized settlement period. The retail market is a two-sided market where both producers and middlemen serve as sellers. Alternating decentralized and centralized market enables me to study endogenous money holdings by consumers, including how the extent of intermediation is affected by payment frictions and policies. Money and intermediation can be regarded as substitutes since they both ameliorate market frictions, and they also serve as complements in the sense that an increase in inflation lowers money holdings and hurts middlemen.

The first part of the chapter characterizes equilibrium. I show that there can be direct trade, indirect trade or both in the retail market. Also I study how participation, production, intermediation and quantity per trade depend on parameters. Equilibrium is such that participation of producers and middlemen are increasing in their own bargaining power with consumers and decreasing in their search costs while quantity per trade is always increasing. This is because of the quantity endogenously depends on their strategic participation decisions. Intuitively, when search costs go up, there are few sellers in the retail market, resulting in a higher buyer-seller ratio and a higher chance for each seller to meet a consumer. Therefore any seller, a producer or a middleman, who chooses to participate, will carry a larger quantity.

Given two dimensions of differences between producers and middlemen, a disadvantage in one dimension, say a higher search cost for middlemen, is not necessarily enough to keep intermediation from emerging. Intuitively, intermediation would emerge if expected net gains (i.e. surplus subtracted by sunk costs) created by bargaining advantage are high enough to cover their disadvantage in search cost. On top of that, an increase in search costs magnifies this effect, in the sense that a higher bargaining power induces a higher quantity per trade. It suggests that, when the search cost for middlemen is larger than for producers, there is still intermediation if middlemen are substantially more skilled at bargaining than producers. Especially when search costs increase, middlemen's advantage in bargaining power would support their participation even with a larger disadvantage in search cost. With quantity per trade being taken into account, participation of intermediation reacts to parameters differently from the literature with indivisible goods.

The second part of the chapter analyzes efficiency of equilibrium and fiscal interventions. This is a natural question since equilibrium in a frictional economy is typically inefficient. I solve a social planner's problem for the optimal numbers of producers and middlemen, and trading quantities. Given random search and bargaining, equilibrium is suboptimal in both margins. Inefficiency in the intensive margin is created by a holdup problem, in the sense that the quantity is too low since costs are sunk before a sale is made. Inefficiency in the extensive margin is associated with the Hosios condition, in the sense that one's participation is not efficient if its contribution to the number of meetings is not correctly reflected in bargaining power. In the literature with indivisible goods/assets, bargaining power is enough to restore efficiency as there is only one margin. This does not work for inefficiency on two margins. Fiscal interventions are designed to restore efficiency by proportional subsidies on quantity per trade, and lump-sum taxes or subsidies on participations.

The third part of the chapter explores, without interventions, whether intermediation can be socially beneficial. Although efficiency can be restored with fiscal policy, such interventions need observations of individual's bargaining powers. I want to study, without interventions, if a second best can be achieved in equilibrium with intermediation.

These findings are related to the debate about middlemen contributing to efficiency versus acting as "bloodsuckers" that only buy low and sell high without contributing to value added. I explore effects of intermediation on total welfare and redistribution when middlemen's only advantage is bargaining power. To this end, I compare the total and individual welfare between an economy without middlemen (non-intermediated economy) and an economy with middlemen (intermediated economy). Total welfare depends on the average quantity per trade and the number of sellers. Given the same number of sellers in the two economies, the average quantity per trade is higher in the intermediated economy than with nonintermediated. Now given the same search cost, when it is small, the number of sellers is the same in the two economies, then as it becomes larger, the number of sellers in the intermediated economy is higher because middlemen's participation is encouraged by their superior bargaining power. In fact, total welfare is increasing in the average quantity per trade because of the holdup problem, while changes in the number of sellers can create welfare gains by improving the number of trades but also losses by enlarging total sunk costs. I show that, under some conditions, total welfare is always improved with intermediation and it is redistributed in a way that, when search cost is low, producers and middlemen are better off while consumers are worse off, then as search cost becomes higher, all agents are better off.[2]

2.2 Literature Review

In Rubinstein and Wolinsky (1987), goods are indivisible and meeting rates are exogenous. Their main findings are that, firstly intermediation emerges if middlemen are exogenously faster than producers at meeting consumers, and secondly equilibrium is efficient. A sizable literature spurred by Rubintein and Wolinsky's model studies emergence of intermediation in product markets. Yavas (1994) studies intermediaries who trade only one goods with a superior ability at allocating traders of different values. Biglaiser (1993), Li (1998, 1999) and Biglaiser et al. (2017) study intermediaries with superior information on quality of goods. Masters (2007, 2008) analyzes intermediation among agents who are different in their production cost and bargaining power. In Johri and Leach (2002), Shevichenko (2004), and Smith (2004), intermediation caters to end users with heterogenous tastes by holding inventories of a variety of goods. Watanabe (2010, 2013) uses directed search and price posting to study intermediaries with large inventories therefore capable of serving more con-

[2]In extensions, I also study the impact of payment and credit frictions as well as directed search with price posting. With directed search and price posting in wholesale and retail markets, equilibrium is optimal on the two margins without interventions. With the impact of imperfect payment and credit, equilibrium results are qualitative robust, while quantitatively different from results in the baseline model with perfect credit. Results are available on my webpage https://gracexungong.weebly.com/

sumers at a time. Watanabe et al. (2016) study a monopoly middleman as a market maker who provides platforms for traders. The crucial difference between my setup and all these papers is that they restrict the quantity per trade to be either 0 or 1, either with restrictions on sellers' capacity or consumers' demand. As a result, effects on patterns of trade and welfare of intermediation only come from the extensive margin. In my environment, the quantity per trade is endogenous and the effect on this quantity of middlemen's comparatively high bargaining power is at the center of my analysis. Urias (2017) has some commonality in the environment to this chapter but focuses on monetary exchange. His paper takes existence of middlemen as given by assuming producers are not allowed to trade directly with consumers, while I endogenize market structure by exploring the determinants and extent of middlemen and additionally analyze their welfare effects.

A related strand of literature studies intermediation in frictional financial markets. Duffie et al. (2005) study intermediation in over-the-counter asset markets. In their paper, investors are heterogenous in their valuation and asset holding position of an indisivible asset. Dealers are good at helping investors get their valuation and asset holding aligned faster. Lagos and Rocheteau (2009) relax the restriction on asset holdings and study how results change with a dispersion of asset positions. Similarly, I allow unrestricted inventory holdings in a product market. In contrast, the motivation of trade in their paper is driven by heterogeneous valuations in assets. Another paper that explores the importance of divisibility is Golosov et al. (2004). They relax the indivisibility assumption in Wolinsky (1990) and Blouin and Serrano (2001) to study the allocation efficiency and information diffusion with asymmetric information about asset values. Although my goal here is different - to analyze the emergence and welfare effects of intermediation - I share their interest in allowing divisibility and endogenizing the size of trade. Farboodi, Jarosch and Menzio (2017) and Farboodi, Jarosch and Shimmer (2017) develop models with heterogeneous agents in their bargaining power in the former paper and search speed in the latter. My chapter like Farboodi, Jarosch and Menzio (2017) explores the welfare effect of intermediation as a rent extraction activity. Where we differ is, I depart from restricting quantity per trade to be 0 or 1 and find that welfare can be improved with intermediation.

Models with similarities to the one proposed here are Nosal et al. (2015, 2016). Nosal et al. (2015) generalize Rubinstein and Wolinsky (1987) by allowing more general bargaining power and search cost. The chapter focuses on when intermediation is active and when it is essential. Nosal et al. (2016) allow agents to choose whether to be producers or middlemen. Also they apply this model to asset markets and find existence of multiple equilibria. The main differences with my work are that their papers use indivisible goods and exogenous meeting rate for sellers to meet buyers. The setup in my chapter instead is such that goods are fully divisible and endogenously determined, and all meeting rates depend on agents' strategic decisions.[3] These assumptions lead to different implications in terms of the extent of intermediation and its social function, resulting in a higher (lower) participation of middlemen and their welfare-improving effect.

The heterogeneous role of intermediation in terms of extent across industries has been extensively studied in empirical literature. Atack and Passell (1994) take an approach different from Spulber (1996a) to measure the size of intermediation by estimating the employment distribution in the U.S. during 1840-1990 in three sec-

[3]Some of the literature, e.g. Nosal et al. (2016) or Shevichenko (2004), allows middlemen to hold multiple units, but they still trade $x \in \{0, 1\}$ in each meeting.

tors - primary (agriculture), secondary (manufacture), and tertiary (services). They show that the proportion of labor force in service sector constantly increased, that is, the ratio of labor in service sector to the total labor in agriculture plus manufactures increased from 0.08 to over 1 in the 150 years.[4] More empirical evidence is well estimated in the literature on intermediation in financial markets. Philippon (2014) measures financial intermediation in the U.S. over the past 130 years and shows variation in its share of GDP over time with a peak just below 9% in 2010. For specific markets, consider the federal fund market for instance, Demiralp et al. (2004) estimate the share of the daily volume of fed fund transactions represented by brokered fed funds is about one-third in 2003, Ashcraft and Duffie (2007) report that nonbrokered transactions represented 73% of the volume of fed fund trades in 2005, and Afonso and Lagos (2014) investigate the fed funds intermediated by commercial banks in the last 2.5 hours of the trading session and estimate the proportion by thes brokered transactions to be 40% throughout 2005-2010.

In what follows, Section 2.3 describes the environment, including the setup, decision rules and terms of trade. Section 2.4 defines and characterizes the equilibrium which can involve only direct trade, only indirect trade, or both. Section 2.5 analyzes efficiencies and proposes fiscal interventions. Section 2.6 explores effects on welfare and redistribution of middlemen when their only advantage is their bargaining power. Section 2.7 concludes.

[4] Pang and Shi (2010).

2.3 Environment

2.3.1 Setup

There is a $[0,1]$ continuum of agents in three types, C, P and M for consumers, producers and middlemen. The population of each type is fixed and denoted as N_i, $i \in \{C,\ P,\ M\}$, $N_c + N_p + N_m = 1$. Time is discrete and continues forever[5]. In each period there are three sub-periods: a wholesale market, WM, followed by a retail market, RM, both of which are decentralized markets, and finally a frictionless Arrow-Debreu market AD where agents settle debts accumulated in WM or RM. There is a divisible consumption good x traded in WM and RM, which is fully perishable across periods while storable within a period. Let the quantity of good x be denoted as q. This consumption good is only valued by C with utility $u(q)$ and produced by P with cost $c(q)$. Although M cannot produce this consumption good, they can buy it from P and sell it to C. The preference in AD is the same as that in Lagos and Wright (2005), depending on consumption of generic good, X, and labor supply l. All agents use credits as payment instrument when trading in decentralized markets. Then in AD they rebalance their credit and repay their debts, if there is any, and consume good X, which only exists in AD and is produced by their own labor l. While participation of C is passive here, it is interesting to study participation of M and P, and moreover the quantities of trades which can only be considered if decentralized good is *divisible*.

We say that P is active in WM if he is willing to trade with M upon meeting and active in RM if, after failing to meet anyone in WM, he chooses to produce and

[5]For some of the issues addressed here, a three-period model would suffice, but for my extensions on endogenous money and credit, and monetary policy, a model with infinite horizon is critical.

pay a lump-sum search cost γ_p at the end of WM in order to trade with C in RM. We assume P has a limited amount of raw materials for production so that he can only produce once within a period, therefore if he has traded with M in WM then he is out of the market, otherwise he can choose whether to continue to participate in RM to search for C. This assumption is used in the baseline model and relaxed in Appendix. For middlemen, we say that M is active in WM if he is willing to trade with P upon meeting and active in RM if he has traded with P and also pay a lump-sum search cost γ_m such that he has decentralized goods ready to trade with C.

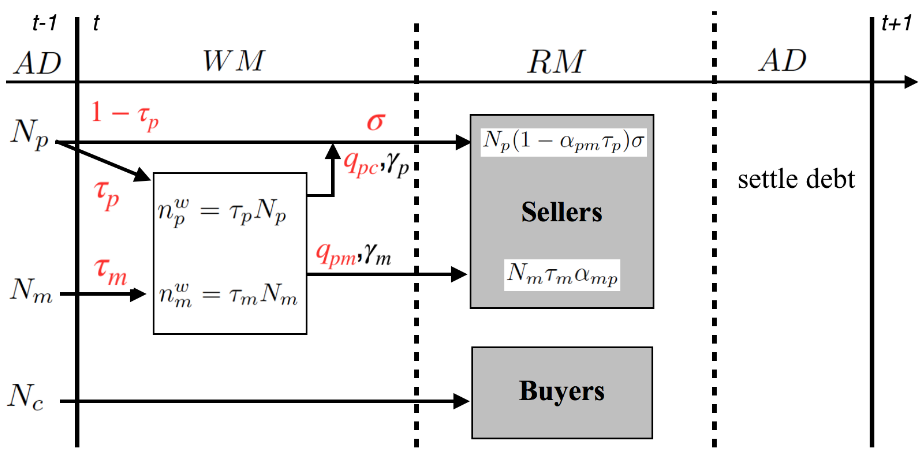


Figure 2.1: Timeline

Participation and bargaining among C, P and M take place in two consecutive two-sided markets WM and RM as shown in Figure 2.1. In the first sub-period, WM, there are only active P and M. $P(M)$ is active if he/she wants to trade in WM and production happens upon meeting. For inactive P he will produce in this period if choosing to trade with C in the following RM. For inactive M he is out of the market. All production is assumed to take place in WM and it makes P

and M who will be active in RM symmetric in the sense that they both bear sunk costs of carrying inventories for RM. When P and M consider whether they should participate in WM, they share the same decision rule, that is, whether the value of their total trading surplus is positive. As P can only produce once within a period, there are two choices to be compared: he can either sell x to M or, instead, produce for C. If he sells x to M, he saves a search cost γ_p and also avoids the search friction in RM. However, he loses the future benefit from potential trade with C in RM which might be more profitable than selling it to M right now. This assumption is relaxed in Appendix by allowing P to produce again after trading with M and it shows that the presented results are qualitatively robust. Now for M, since he can only buy x in WM, a trading decision has to be made based on the tradeoff if buying x from P. If he buys x in WM, he has a chance to sell it to C in RM and share their trading surplus. Nonetheless, the prerequisite is that, he should spend a search cost γ_m to earn such a chance of meeting. Moreover if he fails to meet C in RM, he would bear the cost of search and the cost of inventory purchase since he does not enjoy any utility from consuming x, both of which are sunk at the time of RM. Like in Nosal, Wong and Wright (2015), the search cost is different for M and P, which is an important assumption that influences the role of intermediation. More interestingly, what makes our model novel and generate new results is that, because of the assumption that good x is divisible, not only the equilibrium measure of participants but also the equilibrium quantities of x that P sells to M and C would be affected by search costs such that there are two channels for search costs γ_p and γ_m to exert their effects on equilibrium sets.

In the second subperiod, RM, there are three types of agents, all of C, active P

and active M. By active it means that only those of P and M who have goods and paid search costs before entering RM are eligible to trade with C. Specifically, given search cost γ_p is paid, P is eligible to trade with C if he chooses not to trade with M in WM, or he wants to trade with M but the M he meets does not want to buy from him, or he fails to meet M in WM. For M, he is eligible to trade if he has traded with P in WM and paid search cost before RM starts. As it is assumed that every agent searches only once in each decentralized market, M would not buy from P in RM since has he buys from P in RM there is no chance for him to sell and x is not storable across periods. In RM, C is labeled as buyers (B) while the active P and M are both labeled as sellers (S) such that RM is a two-sided market in which the market tightness is decided by the buyer-seller ratio. After trading activities in WM, the measure of active P and M as well as meeting probabilities need to be updated from WM to RM.

Meeting technology follows a constant return to scale meeting function and depend on the measure of active agents only in each decentralized market. Let n_i^W denote the measure of active type i agents in WM and n_i^R in RM, and α_{ij} the meeting rate at which type i meets type j, then in WM we have

$$\alpha_{pm} = M(1, \frac{n_m^W}{n_p^W}) \tag{2.1}$$

$$\alpha_{mp} = \frac{\alpha_{pm}}{n_m^W / n_p^W} \tag{2.2}$$

in RM we have

$$\alpha_{pc} = \alpha_{mc} = \alpha_{sb}(\tfrac{n_c^R}{n_p^R+n_m^R}) = M(1, \frac{n_c^R}{n_p^R + n_m^R}) \tag{2.3}$$

$$\alpha_{cp} = \alpha_{cm} = \alpha_{bs}(\tfrac{n_c^R}{n_p^R+n_m^R}) = \tfrac{\alpha_{sb}}{n_c^R/(n_p^R+n_m^R)} \tag{2.4}$$

2.3.2 Terms of Trade and Decision Rules

We assume that in the baseline model with perfect credit agents split the surplus consistent with generalized Nash or Kalai solutions of many strategic bargaining games. θ_{ij} is denoted as the bargaining power for type i when meeting type j. In a match the bargaining result is a pair of payment and quantity, denoted as y_{ij} and q_{ji}, meaning that a payment of y_{ij} is transferred from type i (buyer) to type j (seller) in terms of credit for goods sold by j to i. Σ_{ji} is denoted as the total surplus where i is the buyer and j is the seller. Note that since x is fully perishable across periods, M would not purchase from P more than what he would sell to C, implying $q_{mc} = q_{pm}$.

Now we consider decision rules. For C he enters and trades whenever he can so it is trivial. For P and M there are two decisions: one is for both of them on whether to trade in WM , denoted as τ, which will be proved to depend on the same condition for P and M later; the other is for P on whether to participate in RM if he has not traded in WM, denoted as σ. There is no decision for M on whether to participate in RM for the reason that, if he chooses to trade with P and successfully trades with P it is irrational not trading with C, and if he chooses not to trade with P or chooses to but fails to meet P then he has no goods to sell to C and obviously no decision to be made in this case.

Given the decision rules (τ, σ) and the populations of each type (N_c, N_p, N_m), the meeting probabilities can be rewritten as follows,

$$\alpha_{pm} = M(1, \frac{n_m^W}{n_p^W}) = M(1, \frac{N_m\tau}{N_p\tau}) = M(1, \frac{N_m}{N_p}) \quad (2.5)$$

$$\alpha_{mp} = \frac{\alpha_{pm}}{n_m^W/n_p^W} = \frac{M(1, \frac{N_m}{N_p})}{\frac{N_m}{N_p}} \quad (2.6)$$

$$\alpha_{pc} = \alpha_{mc} = \alpha_{sb}(\frac{n_c^R}{n_p^R + n_m^R}) = M(1, \frac{N_c}{N_p\sigma(1-\alpha_{pm}\tau) + N_m\alpha_{mp}\tau}) \quad (2.7)$$

$$\alpha_{cp} = \alpha_{cm} = \alpha_{bs}(\frac{n_c^R}{n_p^R + n_m^R}) = \frac{M(1, \frac{N_c}{N_p\sigma(1-\alpha_{pm}\tau)+N_m\alpha_{mp}\tau})}{\frac{N_c}{N_p\sigma(1-\alpha_{pm}\tau)+N_m\alpha_{mp}\tau}} \quad (2.8)$$

where in WM the vector of active measure of agents are given by $(n_m^W, n_p^W) = (N_m\tau, N_p\tau)$, and in RM $(n_c^R, n_p^R, n_m^R) = (N_c, N_p\sigma(1-\alpha_{pm}\tau), N_m\alpha_{mp}\tau)$. Notice that $N_p\alpha_{pm} = N_m\alpha_{mp}$. Given divisibility of good x, it is enabled to study not only when middlemen is essential but also what would be the trading quantities.

Let us start from the last subperiod AD. Let $V_i^A(y)$ be the value function for type i agents in AD with credit position of y. V_i^W and $V_i^R(q)$ the value functions in WM and RM, where q is the inventory position of x. Note that y can be either positive, or negative if it is debt.

Firstly we consider the problem for P. In AD the value function for P is

$$\begin{aligned} V_p^A(y) &= \max_{X,l}\{U_p(X) - l + \beta V_p^W\} \quad (2.9) \\ s.t. \quad & X = l - y + T \end{aligned}$$

where T is tax, $\beta \in (0,1)$ is the discount rate across periods. We set T to 0 in the baseline model but it is of use in monetary version and in versions with tax/subsidies in RM or WM. Substituting X for l, the FOC is

$$[X]: \quad U'(X) \quad = 1$$

and ENV is

$$V_p^{A'}(y) = 1 \tag{2.10}$$

implying that $V_p^S(y)$ is linear in y, and it is also linear in y for the value functions of M and C in SM.

The Bellman equations for P in WM and RM are given by

$$\begin{aligned} V_p^W &= \alpha_{pm}[y_{mp} - c(q_{pm}) + V_p^R(0)] + (1-\alpha_{pm})[-c(q_{pc}) + V_p^R(q_{pc})] && (2.11) \\ V_p^R(q) &= \alpha_{sb}[y_{cp} + V_p^A(0)] + (1-\alpha_{sb})V_p^A(0) - \gamma_p && (2.12) \\ &= \alpha_{sb}y_{cp} + V_p^A(0) - \gamma_p && (2.13) \end{aligned}$$

The Bellman equations for M in AD, WM and RM are

$$
\begin{aligned}
V_m^A(y) &= \max_{X,l}\{U_m(X) - l + \beta V_m^W & (2.14)\\
s.t. \quad & X = l - y + T \\
V_m^W &= \alpha_{mp}[V_m^R(q_{pm}) - y_{mp}] + (1-\alpha_{mp})V_m^A(0) & (2.15)\\
V_m^R(q) &= \alpha_{sb}[V_m^C(q_-q_{mc}) + y_{cm}] + (1-\alpha_{sb})V_m^A(0) - \gamma_m \\
&= V_m^A(0) + \alpha_{sb}y_{cm} - \gamma_m & (2.16)
\end{aligned}
$$

where $q_{pm} = q_{mc}$.

The Bellman equations for C in AD, WM and RM are

$$
\begin{aligned}
V_c^A(y) &= \max_{X,l}\{U_c(X) - l + \beta V_c^W\} & (2.17)\\
s.t. \quad & X = l - y + T \\
V_c^W &= V_c^R & (2.18)\\
V_c^R &= \alpha_{bs}[u(q_{pc}) + V_c^A(-y_{cp})] + \alpha_{bs}[u(q_{mc}) + V_c^A(-y_{cm})] \\
&\quad +(1-2\alpha_{bs})V_c^A(0) \\
&= V_c^A(0) + \alpha_{bs}[u(q_{pc}) - y_{cp})] + \alpha_{bs}[u(q_{mc}) - y_{cm}] & (2.19)
\end{aligned}
$$

Now we analyze the bargaining outcomes in WM and RM. In the baseline model with perfect credit agent i gets a fraction θ_{ij} of total surplus in a trade with j. In WM, there is only bargaining between active M and P given by

$$\max_{q_{pm}} \quad V_m^R(q_{pm}) - y_{mp}$$
$$s.t \quad V_m^R(q_{pm}) - y_{mp} = \theta_{mp}\Sigma_{mp}$$

where $\Sigma_{pm} = y_{mp} - c(q_{pm}) + V_p^R(0) + V_m^R(q_{pm}) - y_{mp}$. By linearity of $V_i^S(y)$ and $q_{pm} = q_{mc}$, the total surplus of trade between M and P can be rewritten as

$$\Sigma_{pm} = [\alpha_{sb}\theta_{mc}u(q_{pm}) - c(q_{pm}) - \gamma_m] - [\alpha_{sb}\theta_{pc}u(q_{pc}) - c(q_{pc}) - \gamma_p] \tag{2.20}$$

Using the constraint, the objective function is

$$\max_{q_{pm}} \theta_{mp}\{[\alpha_{sb}\theta_{mc}u(q_{pm}) - c(q_{pm}) - \gamma_m] - [\alpha_{sb}\theta_{pc}u(q_{pc}) - c(q_{pc}) - \gamma_p]\}$$

FOC for q_{pm} is given by

$$FOC[q_{pm}]: \ \alpha_{sb}\theta_{mc}u'(q_{pm}) = c'(q_{pm})$$

Therefore the bargaining solution when M meets P in WM is given by (q_{pm}, y_{mp}) satisfying

$$\begin{cases} c'(q_{pm}) & = \alpha_{sb}\theta_{mc}u'(q_{pm}) \\ y_{mp} & = \theta_{mp}[\alpha_{sb}\theta_{pc}u(q_{pc}) - c(q_{pc}) + c(q_{pm}) - \gamma_p] \\ & +\theta_{pm}[\alpha_{sb}\theta_{mc}u(q_{pm}) - \gamma_m] \end{cases} \tag{2.21}$$

In RM, there are two types of meetings, one is for C and P, the other is for C and M. The bargaining problem when C meets P is already solved in the last subperiod WM as all the production takes place in WM. The optimal q_{pc} is given by $\alpha_{sb}\theta_{pc}u'(q_{pc}) = c'(q_{pc})$. Therefore when C meets P in RM the terms of trade are given by (q_{pc}, y_{cp}) satisfying

$$\begin{cases} c'(q_{pc}) & = \alpha_{sb}\theta_{pc}u'(q_{pc}) \\ y_{cp} & = \theta_{cp}\Sigma_{pc} \end{cases} \tag{2.22}$$

where $\Sigma_{pc} = u(q_{pc})$. By $q_{mc} = q_{pm}$, the terms of trade between C and M are given by (q_{pm}, y_{cm}) satisfying

$$\begin{cases} c'(q_{pm}) & = \alpha_{sb}\theta_{mc}u'(q_{pm}) \\ y_{cm} & = \theta_{cm}\Sigma_{mc} \end{cases} \tag{2.23}$$

where $\Sigma_{mc} = u(q_{mc}) = u(q_{pm})$.

Now we consider the decision rules (τ, σ).

Lemma 2.1. M wants to trade with P if and only P wants to trade with M.

The trading decision τ, same for M and P in WM follow the rule given by:

$$\tau = \begin{cases} 1 \\ [0,1] \\ 0 \end{cases} \quad if \ \Sigma_{pm} \begin{cases} > 0 \\ = 0 \\ < 0 \end{cases} \tag{2.24}$$

The decision σ for P depends on the expected payoff for P in RM given by

$$\sigma = \begin{cases} 1 \\ [0,1] & if \ \alpha_{sb}\theta_{pc}\Sigma_{pc} - c(q_{pc}) - \gamma_p \begin{cases} > 0 \\ = 0 \\ < 0 \end{cases} \\ 0 \end{cases} \tag{2.25}$$

2.4 Equilibrium

We are now ready to define an equilibrium. Denote $\mathbf{V}$ as a vector of value functions for $C,\ P\ and\ M,\ \mathbf{y} = (y_{mp}, y_{cp}, y_{cm}),\ \mathbf{q} = (q_{pm}, q_{pc}),$ we have

Definition 2.1. An equilibrium is a list $< \mathbf{V}, \mathbf{q}, \mathbf{y}, \tau, \sigma >$ such that: $\mathbf{V}$ satisfy value functions given $\mathbf{q}$, $\mathbf{y}$, $\tau,\ \sigma$; ($\mathbf{q},\ \mathbf{y}$) satisfy the bargaining solutions given $\mathbf{V}$, $\tau,\ \sigma$, and ($\tau,\ \sigma$) satisfy the best response conditions given $\mathbf{V},\ \mathbf{q},\ \mathbf{y}$.

Given γ_p and γ_m positive, there are four classes of equilibria as shown in Figure 2.2, 2.3 and 2.4 with details described below. The first class of equilibrium is with $\tau = 0$ and $\sigma = 0$, which means both wholesale and retail market shut down, thus neither direct trade from producers to consumers, nor indirect trade from middlemen to consumers. The second class of equilibria is with $\tau = 0$ and $\sigma > 0$, meaning wholesale market shuts down while retail market is open with only producers and no intermediation, and in this case there is only direct trade but no indirect trade. The third class of equilibria is with $\tau > 0$ and $\sigma = 0$, which means both wholesale and retail market are open, and the retail market is operated by intermediation only with no producer, that is, there is only indirect trade but no direct trade. The last class of equilibria is with $\tau > 0$ and $\sigma > 0$, which means both wholesale and retail market are open and the retail market is operated by both producers and intermediation, thus there are both direct and indirect trade.

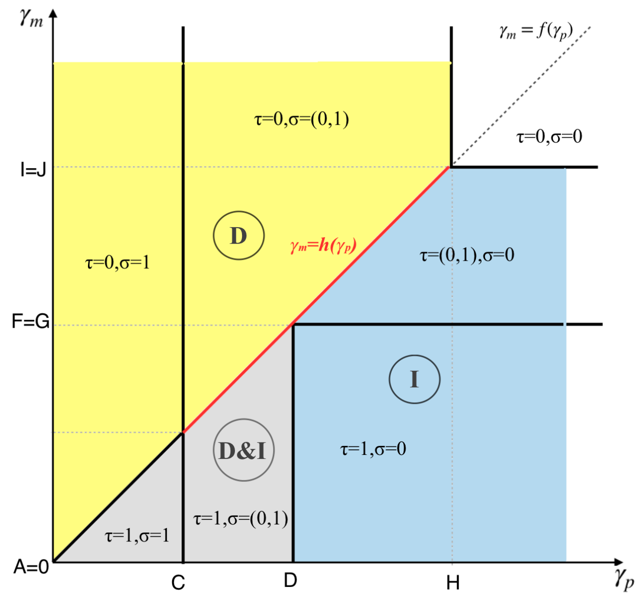


Figure 2.2: Equilibrium Set with $\theta_{mc} = \theta_{pc}$

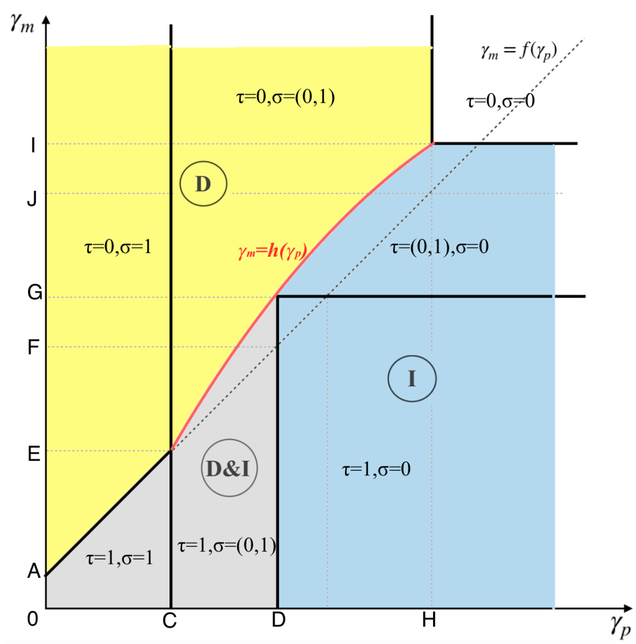


Figure 2.3: Equilibrium Set with $\theta_{mc} > \theta_{pc}$

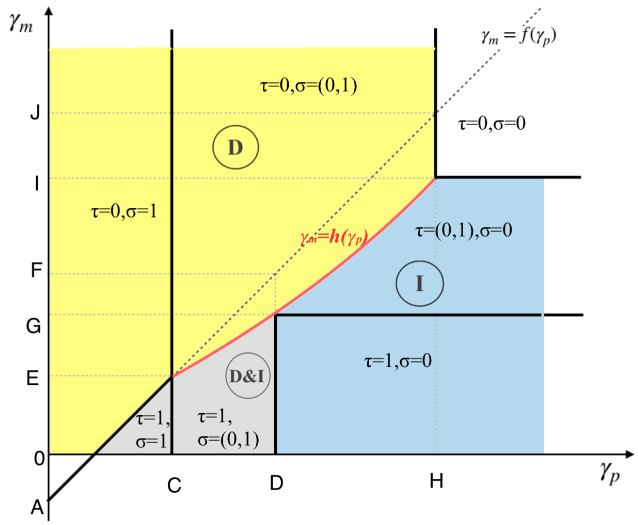


Figure 2.4: Equilibrium Set with $\theta_{mc} < \theta_{pc}$

Before the analysis of equilibrium outcomes, recall that the trading quantity $q_{pc}(\alpha_{sb})$ is solved by $c'(q_{pc}) = \alpha_{sb}\theta_{pc}u'(q_{pc})$ and $q_{mc}(\alpha_{sb})$ by $c'(q_{pm}) = \alpha_{sb}\theta_{mc}u'(q_{pm})$, given α_{sb} being different from case to case depending on (γ_p, γ_m).

To begin the analysis, consider the equilibrium with $\tau = 0$ and $\sigma = 0$. This is an equilibrium when $\Sigma_{pm} \leq 0$ and $\alpha_{pc}\theta_{pc}\Sigma_{pc} - c(q_{pc}) - \gamma_p \leq 0$. Let $\bar{\alpha_{sb}}$ be the value of α_{sb} when $\tau = 0$ and $\sigma = 0$, and $\bar{\alpha_{sb}} = 1$. Then $\tau = 0$ iff $\gamma_m \geq I$ and $\gamma_p \geq H$, where

$$I \equiv \bar{\alpha_{sb}}\theta_{mc}u([q_{mc}(\bar{\alpha_{sb}})] - c[q_{mc}(\bar{\alpha_{sb}})] \tag{2.26}$$

$$H \equiv \bar{\alpha_{sb}}\theta_{pc}u[q_{pc}(\bar{\alpha_{sb}})] - c]q_{pc}(\bar{\alpha_{sb}})] \tag{2.27}$$

where $q_{pc}(\bar{\alpha_{sb}})$ and $q_{mc}(\bar{\alpha_{sb}})$ are solved by $c'(q_{pc}) = \bar{\alpha_{sb}}\theta_{pc}u'(q_{pc})$ and $c'(q_{pm}) = \bar{\alpha_{sb}}\theta_{mc}u'(q_{pm})$.

Lemma 2.2. An equilibrium with $\tau = 0$ and $\sigma = 0$ exists if $\gamma_p \geq H$ and $\gamma_m \geq I$, where H and I are defined 2.26 and 2.27.

Next consider the second class of equilibrium with $\tau = 0$ and $\sigma > 0$, when the wholesale market shuts down thus in the retail market there is no intermediation and only direct trades from producers to consumers. There are two equilibria in this class: one with $\tau = 0$ and $\sigma = 1$, and the other with $\tau = 0$ and $\sigma \in [0, 1]$.

To start with this class consider the pure strategy equilibrium $\tau = 0$ and $\sigma = 1$. This is the case when $\alpha_{sb} = \alpha_{sb}(\frac{N_c}{N_p})$, and $\tau = 0$ iff $\Sigma_{pm} \leq 0$ and $\sigma = 1$ iff $\gamma_p \leq \alpha_{pc}\theta_{pc}\Sigma_{pc} - c(q_{pc})$, so that retail market opens with all producers participating while no intermediation since neither producers nor middlemen want to trade in WM. It is easy to check that $\Sigma_{pm} \leq 0$ iff $\gamma_m \geq f(\gamma_p)$ and $\gamma_p \leq \alpha_{pc}\theta_{pc}\Sigma_{pc} - c(q_{pc})$ iff $\gamma_p \leq C$ where

$$
\begin{aligned}
f(\gamma_p) \quad &\equiv \quad \gamma_p + \{\alpha_{sb}(\frac{N_c}{N_p})\theta_{mc}u[q_{mc}(\frac{N_c}{N_p})] - c[q_{mc}(\frac{N_c}{N_p})]\} \\
&\quad -\{\alpha_{sb}(\frac{N_c}{N_p})\theta_{pc}u[q_{pc}(\frac{N_c}{N_p})] - c[q_{pc}(\frac{N_c}{N_p})]\} \qquad (2.28) \\
C \quad &\equiv \quad \alpha_{sb}(\frac{N_c}{N_p})\theta_{pc}u[q_{pc}(\frac{N_c}{N_p})] - c]q_{pc}(\frac{N_c}{N_p})] \qquad (2.29)
\end{aligned}
$$

When $\gamma_p = 0$, $\gamma_m = f(\gamma_p) = A$, and when $\gamma_m = 0$, $\gamma_p = f^{-1}(\gamma_m) = B$, where

$$
\begin{aligned}
A \quad &\equiv \quad \alpha_{sb}(\frac{N_c}{N_p})\theta_{mc}u[q_{mc}(\frac{N_c}{N_p})] - c[q_{mc}(\frac{N_c}{N_p})] \\
&\quad -\{\alpha_{sb}(\frac{N_c}{N_p})\theta_{pc}u[q_{pc}(\frac{N_c}{N_p})] - c[q_{pc}(\frac{N_c}{N_p})]\} \qquad (2.30)
\end{aligned}
$$

$$B \equiv \alpha_{sb}(\frac{N_c}{N_p})\theta_{pc}u[q_{pc}(\frac{N_c}{N_p})] - c[q_{pc}(\frac{N_c}{N_p})]$$
$$\{\alpha_{sb}(\frac{N_c}{N_p})\theta_{mc}u[q_{mc}(\frac{N_c}{N_p})] - c[q_{mc}(\frac{N_c}{N_p})]\} \quad (2.31)$$

Lemma 2.3. An equilibrium with $\tau = 0$ and $\sigma = 1$ exists if $\gamma_p \leq C$ and $\gamma_m \geq f(\gamma_p)$, where $f(\gamma_p)$ and C are defined in 2.28 and 2.29.

Next consider the equilibrium with $\tau = 0$ and $\sigma \in [0, 1]$ when some but not all of the producers participating in RM. In this case $\alpha_{sb} = \alpha_{sb}(\frac{N_c}{N_p\sigma})$, and $\tau = 0$ iff $\Sigma_{pm} \leq 0$, that is, $\gamma_m \geq \alpha_{sb}\theta_{pc}u[q_{pc}(\alpha_{sb})] - c[q_{pm}(\alpha_{sb})]$, and $\sigma = [0, 1]$ iff $\gamma_p = \alpha_{sb}\theta_{pc}u[q_{pc}(\alpha_{sb})] - c[q_{pc}(\alpha_{sb})]$. By $\gamma_p = \alpha_{sb}\theta_{pc}u[q_{pc}(\alpha_{sb})] - c[q_{pc}(\alpha_{sb})]$ and $\alpha_{sb} = \alpha_{sb}(\frac{N_c}{N_p\sigma})$, we have $\sigma = \sigma(\gamma_p)$ and thus $\alpha_{sb} = \alpha_{sb}(\gamma_p)$. Substituting $\alpha_{sb}(\gamma_p)$ for α_{sb} in $\gamma_m \geq \alpha_{sb}\theta_{pc}u[q_{pc}(\alpha_{sb})] - c[q_{pm}(\alpha_{sb})]$, then $\tau = 0$ iff $\gamma_m \geq h(\gamma_p)$ where

$$h(\gamma_p) \equiv \gamma_p + \alpha_{sb}(\gamma_p)\theta_{mc}u[q_{pm}(\gamma_p)] - c[q_{pm}(\gamma_p)]$$
$$- \alpha_{sb}(\gamma_p)\theta_{pc}u[q_{pc}(\gamma_p)] - c[q_{pc}(\gamma_p)] \quad (2.32)$$
$$= \gamma_p + \alpha_{sb}(\gamma_p)\theta_{mc}u[q_{pm}(\gamma_p)] - c[q_{pm}(\gamma_p)] \quad (2.33)$$

when $\gamma_p = C$, and $\alpha_{sb}(\gamma_p)\theta_{pc}u[q_{pc}(\gamma_p)] - c[q_{pc}(\gamma_p)] = 0$ by $\sigma = [0, 1]$. Then $\sigma = 1$, $\alpha_{sb} = \alpha_{sb}(\frac{N_c}{N_p})$, and $\gamma_m \geq h(\gamma_p = C) = E$ where

$$E \equiv \alpha_{sb}(\frac{N_c}{N_p})\theta_{mc}u[q_{pm}(\frac{N_c}{N_p})] - c[q_{pm}(\frac{N_c}{N_p})] \quad (2.34)$$

and when $\gamma_p = H$, where H is defined in (18), then $\sigma = 0$, $\alpha_{sb} = \bar{\alpha_{sb}}$, and $\gamma_m \geq$

$h(\gamma_p = H) = I$ where

$$I \equiv \bar{\alpha_{sb}}\theta_{mc}u[q_{pm}(\bar{\alpha_{sb}})] - c[q_{pm}(\bar{\alpha_{sb}})] \tag{2.35}$$

Lemma 2.4. An equilibrium with $\tau = 0$ and $\sigma \in [0, 1]$ exists iff $\gamma_m \geq h(\gamma_p)$ and $\gamma_p = \alpha_{sb}\theta_{pc}u[q_{pc}(\alpha_{sb})] - c[q_{pc}(\alpha_{sb})]$, where $h(\gamma_p)$ is defined in 2.32 and $\alpha_{sb} = \alpha_{sb}(\frac{N_c}{N_p\sigma})$.

Next consider the third class of equilibria with $\tau > 0$ and $\sigma = 0$. In this case wholesale market is open and in retail market there is only intermediation and no direct trade. There are two candidates to be considered, one with $\tau = 1$ and $\sigma = 0$, and the other with $\tau \in [0, 1]$ and $\sigma = 0$. For $\tau = 1$ and $\sigma = 0$ to be an equilibrium, we need $\Sigma_{pm} \geq 0$ and $\gamma_p \geq \alpha_{sb}\theta_{pc}u[q_{pc}(\alpha_{sb})] - c[q_{pc}(\alpha_{sb})]$, where $\alpha_{sb} = \alpha_{sb}(\frac{N_c}{N_m\alpha_{mp}})$. $\Sigma_{pm} \geq 0$ iff $\gamma_m \leq G$ and $\gamma_p \geq \alpha_{sb}\theta_{pc}u[q_{pc}(\alpha_{sb})] - c[q_{pc}(\alpha_{sb})]$ iff $\gamma_p \geq D$, where

$$G \equiv \alpha_{sb}(\frac{N_c}{N_m\alpha_{mp}})\theta_{pc}u[q_{pc}(\frac{N_c}{N_m\alpha_{mp}})] - c[q_{pm}(\frac{N_c}{N_m\alpha_{mp}})] \tag{2.36}$$

$$D \equiv \alpha_{sb}(\frac{N_c}{N_m\alpha_{mp}})\theta_{pc}u[q_{pc}(\frac{N_c}{N_m\alpha_{mp}})] - c[q_{pc}(\frac{N_c}{N_m\alpha_{mp}})] \tag{2.37}$$

Lemma 2.5. An equilibrium with $\tau = 1$ and $\sigma = 0$ exists iff $\gamma_m \leq G$ and $\gamma_p \geq D$, where G and D are defined in 2.36 and 2.37.

For $\tau \in [0, 1]$ and $\sigma = 0$ to be an equilibrium we need $\Sigma_{pm} = 0$, which holds iff $\gamma_m = \alpha_{sb}\theta_{mc}u[q_{pm}(\alpha_{sb})] - c[q_{pm}(\alpha_{sb})]$, and $\gamma_p \geq \alpha_{sb}\theta_{pc}u[q_{pc}(\alpha_{sb})] - c[q_{pc}(\alpha_{sb})]$, where $\alpha_{sb} = \alpha_{sb}(\frac{N_c}{N_m\alpha_{mp}\tau})$. By $\gamma_m = \alpha_{sb}\theta_{mc}u[q_{pm}(\alpha_{sb})] - c[q_{pm}(\alpha_{sb})]$, we have $\tau = \tau(\gamma_m)$, then $\alpha_{sb} = \alpha_{sb}(\gamma_m)$. Substitute $\alpha_{sb}(\gamma_m)$ for α_{sb} in $\gamma_p \geq \alpha_{sb}\theta_{pc}u[q_{pc}(\alpha_{sb})] - c[q_{pc}(\alpha_{sb})]$, then $\sigma = 0$ iff $\gamma_p \geq F(\gamma_m)$ where

$$F(\gamma_m) \equiv \gamma_m + \alpha_{sb}(\gamma_m)\theta_{pc}u[q_{pc}(\gamma_m)] - c[q_{pc}(\gamma_m)] \tag{2.38}$$

When $\gamma_m = G$, then $\tau = 1$, $\alpha_{sb} = \alpha_{sb}(\frac{N_c}{N_m \alpha_{mp}})$, and $\gamma_p \geq F(\gamma_m = G) = D$. When $\gamma_m = I$, then $\tau = 0$, $\alpha_{sb} = \bar{\alpha_{sb}}$, and $\gamma_p \geq F(\gamma_m = I) = H$.

Lemma 2.6. An equilibrium with $\tau \in [0, 1]$ and $\sigma = 0$ exists iff $\gamma_m = \alpha_{sb}\theta_{mc}u[q_{pm}(\alpha_{sb})] - c[q_{pm}(\alpha_{sb})]$ and $\gamma_p \geq F(\gamma_m)$, where $F(\gamma_m)$ is defined in 2.38 and $\alpha_{sb} = \alpha_{sb}(\frac{N_c}{N_m \alpha_{mp} \tau})$.

Next consider the last class of equilibria with $\tau > 0$ and $\sigma > 0$. There are two candidates in this class: one is with $\tau = 1$ and $\sigma = 1$, and the other $\tau = 1$ and $\sigma \in [0, 1]$. First consider the equilibrium with $\tau = 1$ and $\sigma = 1$. This is pure strategy equilibrium in which both WM and RM are open with all of middlemen and producers participating, and $\alpha_{sb} = \alpha_{sb}(\frac{N_c}{N_p})$. $\tau = 1$ iff $\Sigma_{pm} \geq 0$, and $\sigma = 1$ iff $\gamma_p \leq \alpha_{sb}\theta_{pc}u[q_{pc}(\alpha_{sb})] - c[q_{pc}(\alpha_{sb})]$. To have these conditions be satisfied, we need $\gamma_p \leq C$ and $\gamma_m \leq f(\gamma_p)$.

Lemma 2.7. An equilibrium with $\tau = 1$ and $\sigma = 1$ exists iff $\gamma_m \leq f(\gamma_p)$ and $\gamma_p \leq C$, where C is defined in (20) and $f(\gamma_p)$ in (19).

For the other equilibrium in this class $\tau = 1$ and $\sigma \in [0, 1]$, $\tau = 1$ iff $\Sigma_{pm} \geq 0$, that is, $\gamma_m \leq \alpha_{sb}\theta_{mc}u[q_{pm}(\alpha_{sb})] - c[q_{pm}(\alpha_{sb})]$ and $\sigma \in [0, 1]$ iff $\gamma_p = \alpha_{sb}\theta_{pc}u[q_{pc}(\alpha_{sb})] - c[q_{pc}(\alpha_{sb})]$, where $\alpha_{sb} = \alpha_{sb}(\frac{N_c}{N_p \sigma(1-\alpha_{pm}) + n_m \alpha_{mp}})$. This is an equilibrium when WM is open with all middlemen and producers participating while some but not all of those producers who fail to meet a middleman choose to participate in RM. By $\gamma_p = \alpha_{sb}\theta_{pc}u[q_{pc}(\alpha_{sb})] - c[q_{pc}(\alpha_{sb})]$, we have $\sigma = \sigma(\gamma_p)$, then $\alpha_{sb} = \alpha_{sb}(\gamma_p)$. Substituting

$\alpha_{sb}(\gamma_p)$ for α_{sb} in $\gamma_m \leq \alpha_{sb}\theta_{mc}u[q_{pm}(\alpha_{sb})] - c[q_{pm}(\alpha_{sb})]$, then $\tau = 1$ iff $\gamma_m \leq g(\gamma_p)$ where

$$g(\gamma_p) \equiv \gamma_p + \alpha_{sb}(\gamma_p)\theta_{mc}u[q_{pm}(\gamma_p)] - c[q_{pm}(\gamma_p)] \tag{2.39}$$

When $\gamma_p = C$, then $\sigma = 1$, $\alpha_{sb} = \alpha_{sb}(\frac{N_c}{N_p})$, and $\gamma_m \leq g(\gamma_p) = E$. And when $\gamma_p = D$, then $\sigma = 0$, $\alpha_{sb} = \alpha_{sb}(\frac{N_c}{N_m\alpha_{mp}})$, and $\gamma_m \leq g(\gamma_p) = G$.

Lemma 2.8. An equilibrium with $\tau = 1$ and $\sigma \in [0,1]$ exists iff $\gamma_m \leq g(\gamma_p)$ and $\gamma_p = \alpha_{sb}\theta_{pc}u[q_{pc}(\alpha_{sb})] - c[q_{pc}(\alpha_{sb})]$, where $g(\gamma_p)$ is defined in 2.39 and $\alpha_{sb} = \alpha_{sb}(\frac{N_c}{N_p\sigma(1-\alpha_{pm})+n_m\alpha_{mp}})$.

For completeness we have two other equilibria, $\tau \in [0,1]$ and $\sigma = 1$, and the other $\tau \in [0,1]$ and $\sigma \in [0,1]$, but both of them are possible only for a measure 0 set of parameters.

Proposition 2.1. *With γ_p and γ_m both positive, equilibrium exists and is generically unique. The equilibrium set is as shown in Figure 2.2, 2.3 and 2.4 for the cases when $\theta_{pc} = \theta_{mc}$, $\theta_{pc} < \theta_{mc}$ and $\theta_{pc} > \theta_{mc}$. For some parameters intermediation is essential.*

Lemma 2.9. *$\gamma_m = g(\gamma_p)$, $\gamma_m = F^{-1}(\gamma_p)$ and $\gamma_m = h(\gamma_p)$ are the same curve while $g(\gamma_p)$ is defined with $\gamma_p \in [C,D]$, $F^{-1}(\gamma_p)$ with $\gamma_p \in [D,H]$ and $h(\gamma_p)$ with $\gamma_p \in [C,H]$, and $h'(\gamma_p) > 0$.*

As shown in the graphs, intermediation is essential - i.e. some allocation can not be achieved without intermediation when γ_p is too large even if producers have an advantage over middlemen in terms of bargaining power. What is more important and novel in this model is that, because of divisibility of goods and endogenous α_{ij},

Figure 2.2, 2.3 and 2.4 display how the equilibrium regimes are different because of advantage(or disadvantage) in bargaining power for middlemen over producers when trading with consumers. These results would not be demonstrated without neither divisibility of goods or endogenous meeting technology.

Consider the equilibrium 1 regime when at least one decentralized market is open. Firstly, position of $\gamma_m = f(\gamma_p)$ relative to $\gamma_m = \gamma_p$ is different according to the relative magnitute of θ_{pc} and θ_{mc}. This can also be reflected from the intercept value A of $\gamma_m = f(\gamma_p)$ being different. When $\theta_{pc} = \theta_{mc}$, then $\gamma_m = f(\gamma_p) = \gamma_p$, imply that with the same bargaing power for P and M, trading decision $\tau$is always 0 as long as middlemen has advantage in search cost over producers. When $\theta_{pc} < \theta_{mc}$, then $\gamma_m = f(\gamma_p)$ always is above $\gamma_m = \gamma_p$, and the area in between implies that, with an advantage in bargaining power, middlemen want to trade with producers even if they bear higher search cost than producers. When $\theta_{pc} > \theta_{mc}$, then $\gamma_m = f(\gamma_p)$ is below $\gamma_m = \gamma_p$, and the area in between implies that the advantage in search cost does not necessarily make middlemen be willing to trade since it is eroded by the disadvantage in lower bargaining power compared with producers. Secondly, while we can say that $\tau = 0$ as long as $\gamma_m > f(\gamma_p)$ for equilibria with pure strategy, we cannot conclude the same for equilibria with mixed strategy. The curve that seperating mixed equilibria with $\tau = 0$ from those with $\tau > 0$ is $\gamma_m = h(\gamma_p)$, and it is also different in Figure 2.2, 2.3 and 2.4. In Figure 2.2 with $\theta_{pc} = \theta_{mc}$, $\gamma_m = h(\gamma_p)$ overlap with $\gamma_m = f(\gamma_p)$ and $\gamma_m = \gamma_p$, implying as long as middlemen and producers share the same bargaining power in a meeting with a consumers, they always want to trade in WM as long as middlemen has advantage in search cost. In Figure 2.3 with $\theta_{pc} < \theta_{mc}$, $\gamma_m = h(\gamma_p)$ is above $\gamma_m = f(\gamma_p)$, extending the area of mixed stratege with $\tau > 0$ from what is below $\gamma_m = f(\gamma_p)$. In Figure 2.4 with $\theta_{pc} < \theta_{mc}$, $\gamma_m = h(\gamma_p)$ is below $\gamma_m = f(\gamma_p)$, extending the area of mixed stratege with $\tau = 0$ from what is above $\gamma_m = f(\gamma_p)$.

In summary, under the symmetric assumptions for middlemen and producers in bearing sunk cost of carrying inventory and meeting consumers at the same speed in RM, it is shown in Figure 2.2, 2.3 and 2.4 that bargaining powers and search costs can still give rise to essentiality of intermediation. More importantly, bargaining power and search cost can now exert effect via two channels: intensive margin and extensive margin. Therefore there are two curves, $\gamma_m = f(\gamma_p)$ and $\gamma_m = h(\gamma_p)$, served as thresholds on the existence of intermediation, and these two curves are different unless the bargaining power is the same for producers and middlemen. All these differences are contributed by economics of divisibility of goods and endogenous meeting technology.

Lemma 2.10. *When $\theta_{mc} = \theta_{pc}$, then $A = 0$, $F = G < I = J$. When $\theta_{mc} > \theta_{pc}$, then $A > 0$, $F < G < J < I$. When $\theta_{mc} < \theta_{pc}$, then $A < 0$, $G < F < I < J$. Also given $\gamma_p \in [C, H]$, $h(\gamma_p)$ is part of an upward-sloping curve, $h'(\gamma_p) > 1$ if $\theta_{mc} > \theta_{pc}$, $h'(\gamma_p) < 1$ if $\theta_{mc} < \theta_{pc}$, and $h'(\gamma_p) = 1$ if $\theta_{mc} = \theta_{pc}$.*

Although the results given by the divisibility of goods is involved in the above graphical analysis, the attention is focused on the outcome of equilibrium participation decision, which is the extensive margin. What would be interesting to consider in detail with divisible goods is the intensive margin, that is, quantity of goods (q_{pm}, q_{pc}) traded in WM and RM. Also notice that compared with the equilibrium outcome with indivisible goods, the number of candidate equilibria with mixed strategies are extended with divisible goods since now the equilibrating force is allowed to work through two channels: both the number of active agents as well as the trading quantities are adjustable.

To start with the analysis on trading quantities (q_{pm}, q_{pc}), consider (q_{pm}, q_{pc}) in pure strategy equilibria $(\tau, \sigma) \in \{(1,1), (0,1), (1,0), (0,0)\}$. For $(\tau, \sigma) = (1,1)$ and $(\tau, \sigma) = (0,1)$, the meeting probabilities are the same, $\alpha_{sb} = \alpha_{sb1} \equiv M(1, \frac{N_c}{N_p})$, and $(q_{pm}, q_{pc}) = (q_{pm1}, q_{pc1})$ where q_{pm1} is solved by $\alpha_{sb1}\theta_{mc}u'(q_{pm1}) = c'(q_{pm1})$, q_{pc1} by $\alpha_{sb1}\theta_{mc}u'(q_{pm1}) = c'(q_{pm1})$. For $(\tau, \sigma) = (1,0)$, $\alpha_{sb} = \alpha_{sb2} \equiv M(1, \frac{N_c}{N_m \alpha_{mp}})$, where $\alpha_{mp} = M(\frac{N_p}{N_m}, 1)$, and, similarly, $(q_{pm}, q_{pc}) = (q_{pm2}, q_{pc2})$ where q_{pm2} is solved by $\alpha_{sb2}\theta_{mc}u'(q_{pm2}) = c'(q_{pm2})$, q_{pc2} by $\alpha_{sb2}\theta_{mc}u'(q_{pm2}) = c'(q_{pm2})$. For $(\tau, \sigma) = (0,0)$, $\alpha_{sb} = \bar{\alpha_{sb}}$, and $(q_{pm}, q_{pc}) = (\bar{q_{pm}}, \bar{q_{pc}})$ where$\bar{q_{pm}}$ is solved by $\bar{\alpha_{sb}}\theta_{mc}u'(\bar{q_{pm}}) = c'(\bar{q_{pm}})$, $\bar{q_{pc}}$ by $\bar{\alpha_{sb}}\theta_{mc}u'(\bar{q_{pc}}) = c'(\bar{q_{pc}})$.

In pure strategy equilibria, obviously $\alpha_{sb1} \leq \alpha_{sb2} \leq \bar{\alpha_{sb}}$, and it is proved in the Appendix that:

Proposition 2.2. $\frac{\partial q_{pm}}{\partial \theta_{mc}} \geq 0$, $\frac{\partial q_{pc}}{\partial \theta_{pc}} \geq 0$, $\frac{\partial q_{ij}}{\partial \alpha_{sb}} \geq 0$, $\frac{\partial q_{ij}}{\partial (\alpha_{sb}\theta_{ij})} \geq 0$, $ij \in \{pc, pm\}$

Therefore $q_{pm1} \leq q_{pm2} \leq \bar{q_{pm}}$, and $q_{pc1} \leq q_{pc2} \leq \bar{q_{pc}}$.

Next consider (q_{pm}, q_{pc}) in mixed strategy equilibria. For $(\tau, \sigma) = (1, [0,1])$, $\alpha_{sb} = \alpha_{sb3} \equiv M(1, \frac{N_c}{N_p \sigma(1-\alpha_{pm}) + N_m \alpha_{mp}})$. Let q_{pm3} and q_{pc3} denote the quantities in this equilibrium. It is easy to check that when $\gamma_p = C$, then $\sigma = 1$, $\alpha_{sb3} = \alpha_{sb1}$, $q_{pm3} = q_{pm1}$ and $q_{pc3} = q_{pc1}$. Generally q_{pm3}, q_{pc3} and σcan be solved in terms of γ_p by

$$\begin{cases} \gamma_p = \alpha_{sb3}\theta_{pc}u(q_{pc3}) - c(q_{pc3}) \\ \alpha_{sb3}\theta_{pc}u'(q_{pc3}) = c'(q_{pc3}) \\ \alpha_{sb3}\theta_{mc}u'(q_{pm3}) = c'(q_{pm3}) \end{cases} \tag{2.40}$$

For $(\tau, \sigma) = (0, [0,1])$, $\alpha_{sb} = \alpha_{sb4} \equiv M(1, \frac{N_c}{N_p \sigma})$. Let q_{pm4} and q_{pc4} denote the quantities in this case. When $\gamma_p = C$, then $\sigma = 1$, $\alpha_{sb} = \alpha_{sb1}$, $q_{pm4} = q_{pm1}$, $q_{pc4} = q_{pc1}$. When $\gamma_p = H$, $\sigma = 0$, $\alpha_{sb} = \bar{\alpha_{sb}}$, $q_{pm4} = \bar{q_{pm}}$, and $q_{pc4} = \bar{q_{pc}}$. Generally q_{pm4}, q_{pc4} and σcan be solved in terms of γ_p by

$$\begin{cases} \gamma_p = \alpha_{sb4}\theta_{pc}u(q_{pc4}) - c(q_{pc4}) \\ \alpha_{sb4}\theta_{pc}u'(q_{pc4}) = c'(q_{pc4}) \\ \alpha_{sb4}\theta_{mc}u'(q_{pm4}) = c'(q_{pm4}) \end{cases} \tag{2.41}$$

For $(\tau, \sigma) = ([0,1], 0)$, $\alpha_{sb} = \alpha_{sb5} \equiv M(1, \frac{N_c}{N_m \alpha_{mp} \tau})$. Let q_{pm5} and q_{pc5} denote the quantities in this equilibrium. When $\gamma_m = G$, then $\tau = 1$, $\alpha_{sb} = \alpha_{sb2}$, $q_{pm5} = q_{pm2}$, $q_{pc5} = q_{pc2}$. When $\gamma_m = I$, $\tau = 0$, $\alpha_{sb} = \bar{\alpha_{sb}}$, $q_{pm5} = \bar{q_{pm}}$, and $q_{pc5} = \bar{q_{pc}}$. Generally q_{pm5}, q_{pc5} and τ can be solved in terms of γ_m by

$$\begin{cases} \gamma_m = \alpha_{sb5}\theta_{mc}u(q_{pm5}) - c(q_{pm5}) \\ \alpha_{sb5}\theta_{pc}u'(q_{pc5}) = c'(q_{pc5}) \\ \alpha_{sb5}\theta_{mc}u'(q_{pm5}) = c'(q_{pm5}) \end{cases} \tag{2.42}$$

In the class of mixed strategy equilibria, when $\sigma \in [0,1]$, it is proved that:

Proposition 2.3. *When $\sigma \in [0,1]$, $\frac{\partial \sigma}{\partial \gamma_p} < 0$, $\frac{\partial \alpha_{sb}}{\partial \gamma_p} > 0$, $\frac{\partial q_{pm}}{\partial \gamma_p} > 0$, and $\frac{\partial q_{pc}}{\partial \gamma_p} > 0$.*

Proposition 2.4. *When $\tau \in [0,1]$,$\frac{\partial \tau}{\partial \gamma_m} < 0$, $\frac{\partial \alpha_{sb}}{\partial \gamma_m} > 0$, $\frac{\partial q_{pm}}{\partial \gamma_m} > 0$, and $\frac{\partial q_{pc}}{\partial \gamma_m} > 0$.*

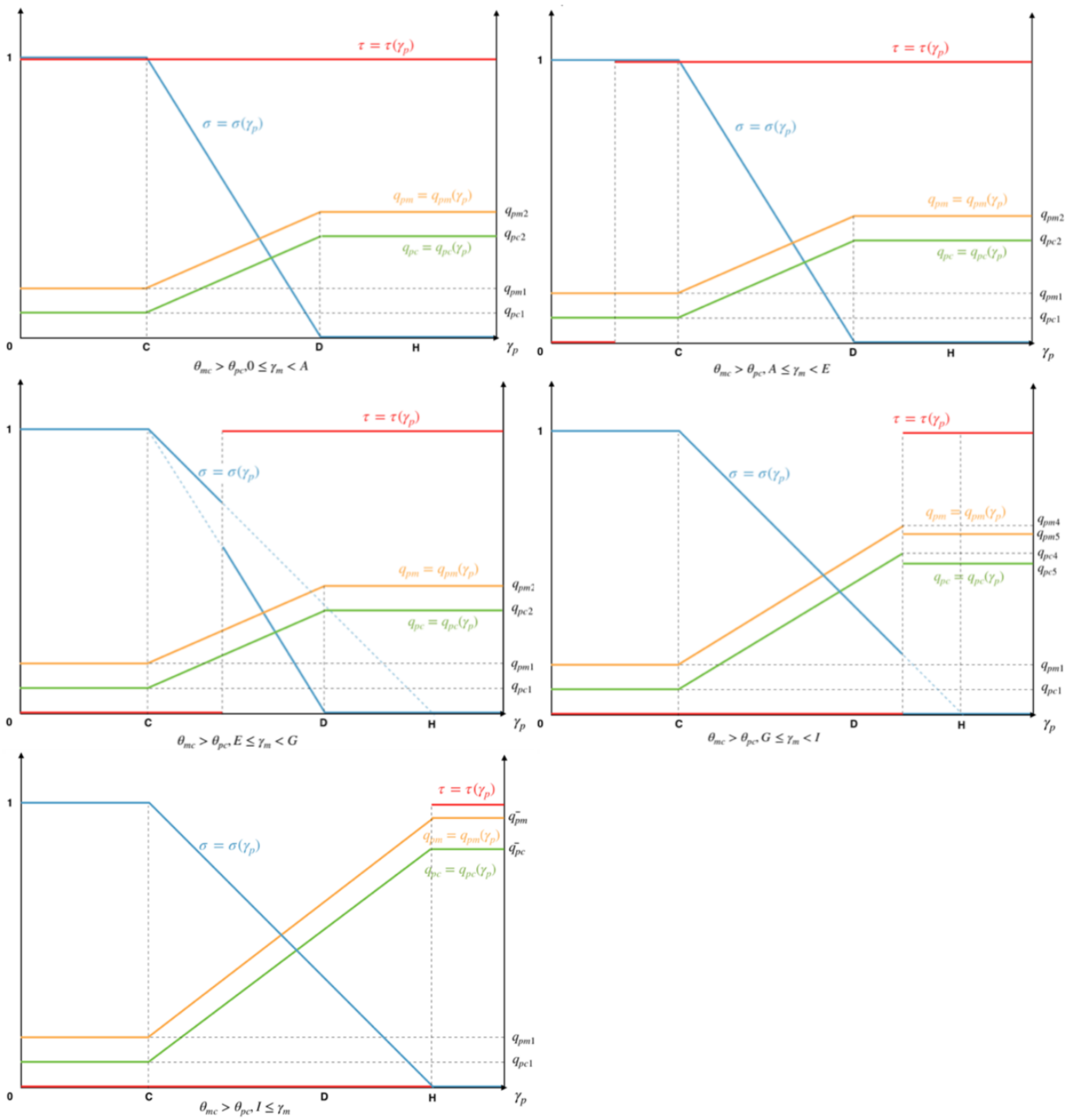


Figure 2.5: Changes in σ,τ and q_{ij} *w.r.t.* γ_p, given γ_m and $\theta_{mc} > \theta_{pc}$

Based on Figure 2.3 with $\theta_{mc} > \theta_{pc}$ as an example, Figure 2.5 illustrates how q_{pm}, q_{pc}, $\tau$and σ change in response to γ_p for a given γ_m in equilibrium. Based on the range from which γ_m is chosen, there are five possible outcomes describing the relationships. Generally, intensive margin and extensive margin move in the opposite directions when in mixed strategy equilibria. Also in some cases there are discrete changes in intensive and/or extensive margin.

Specifically the left one at the top illustrates the outcome when $0 \leq \gamma_m < A$. The right one at the top illustrates the outcome when $A \leq \gamma_m < E$, which is the same as when $0 \leq \gamma_m < A$ except $\tau = 0$ for small γ_p , and at the point of $\gamma_p = f^{-1}(\gamma_m)$ there is a discrete change of τ, jumping upward from 0 to 1.

The left one in the middle is for the outcome when $E \leq \gamma_m < G$. In this case, there are discontinuities in both extensive margin at the same point of γ_p, while not in intensive margin. Given γ_m, the value of equilibrium σ drops down from σ_4 to σ_3 at the point when $\gamma_p = h^{-1}(\gamma_m)$, where σ_4 is the value of σ in equilibrium with $\tau = 0$ and $\sigma \in [0, 1]$, and σ_3 is the value of σin equilibrium with $\tau = 1$ and $\sigma \in [0, 1]$. When $\gamma_m = E$, $\sigma_4 = \sigma_3 = 1$, and when $\gamma_m = G$, $\sigma_4 = \alpha_{pm}$ and $\sigma_3 = 0$. As γ_m increases from E to G, the drop in σ increases from 0 to α_{pm}. Also given γ_m, the value of equilibrium τ jumps up from 0 to 1 at the same point of γ_p when σ drops down.

The right one in the middle shows the outcome when $G \leq \gamma_m < I$. In this case, there are discontinuities in both intensive margin and extensive margin. σ and τ jump at $\gamma_p = h^{-1}(\gamma_m)$ for a given $\gamma_m \in [G, I)$. The value of equilibrium τ jumps up from 0 to 1 at $\gamma_p = h^{-1}(\gamma_m)$ and σ drops to 0. And the discrete drop in σ gets smaller as γ_p increases. Specifically the value of σ before dropping down to 0 is solved by

$$\begin{cases} h^{-1}(\gamma_m) = \alpha_{sb4}\theta_{pc}u(q_{pc4}) - c(q_{pc4}) \\ \alpha_{sb4}\theta_{pc}u'(q_{pc4}) = c'(q_{pc4}) \\ \alpha_{sb4} = \frac{N_c}{N_c + N_p\sigma} \end{cases}$$

In this case, there are also discrete changes in the equilibrium quantity (q_{pc}, q_{pm}). However it is ambiguous whether the quantity increases or decreases at the point of discontinuity and this would depend on the specific form of utility and cost function.

The last graph at the bottom in Figure 2.5 shows the change in intensive and extensive margin with respect to γ_p when $I \leq \gamma_m$.

In summary, the equilibrium σ is 1 when γ_p is small and goes down to 0 as γ_p increases for any given γ_m. For equilibrium τ, middlemen's advantage over producers in search cost is reduced as γ_m increases and thus the set of γ_p supporting $\tau = 1$ shrinks. This result is obvious as shown in Figure 2.5. Also notice that, as γ_p changes, whenever there is jump in σ there is also a jump in τ in the opposite direction. Moreover, the equilibrium q_{pc} stays the same whenever $\sigma$is constant while q_{pc} moves in the opposite direction with σ when σ changes. As for q_{pm}, although it seems not responsive to changes in τ, it changes whenever q_{pc} changes since both are affected by the market tightness in RM. With divisible good, the search cost not only give rise to endogenously adjustment in the composition of active sellers, but also the adjustment of equilibrium quantities as well such that we can study the change in extensive margin and intensive margin in response to different levels of search costs.

2.5 Efficiency and Fiscal Intervention

It is natural to study the efficient outcome since the above equilibrium analysis is based on a market with frictions. For a social planner, the problem is given by

$$\max_{\tau^o,\sigma^o,q^o_{pm},q^o_{pc}} \quad N_p\tau^o\alpha^o_{pm}[-c(q^o_{pm})] + N_m\tau^o\alpha_{mp}[\alpha^o_{sb}u(q^o_{pm}) - \gamma_m]$$
$$+N_p\sigma^o(1-\tau^o\alpha_{pm})[\alpha^o_{sb}u(q^o_{pc}) - c(q^o_{pc}) - \gamma_p] \tag{2.43}$$

where $\alpha^o_{sb} = M(1, \frac{N_c}{N_p\sigma^o(1-\alpha_{pm}\tau^o)+N_m\alpha_{mp}\tau^o})$. By using $N_p\alpha_{pm}$ to substitute for $N_m\alpha_{mp}$ in the optimization problem and dividing the function by N_p, then it is the same as solving

$$\max_{\tau^o,\sigma^o,q^o_{pm},q^o_{pc}} \quad Z \equiv \tau^o\alpha_{pm}[\alpha^o_{sb}u(q^o_{pm}) - c(q^o_{pm}) - \gamma_m] + \sigma^o(1-\tau^o\alpha_{pm})[\alpha^o_{sb}u(q^o_{pc}) - c(q^o_{pc}) - \gamma_p] \tag{2.44}$$

Given α^o_{sb}, $q^o_{pm} = q^o_{pc} = q^o$ is solved by

$$\alpha^o_{sb}u'(q^o) = c'(q^o) \tag{2.45}$$

Qualitatively the graph of efficient set is similar to the graph of equilibrium set when $\theta_{pc} = \theta_{mc}$.

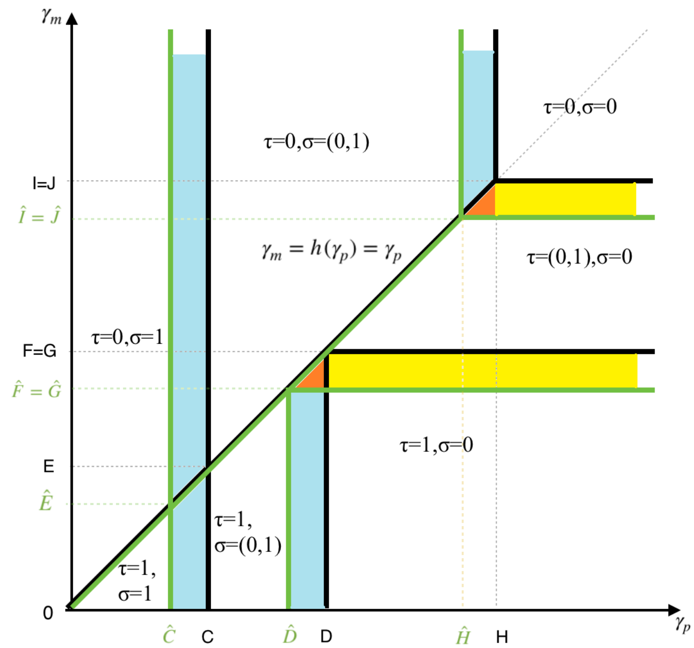


Figure 2.6: Inefficiency of the Extensive Margin When $\theta_{mc} = \theta_{pc}$

Proposition 2.5. *The efficient outcome exists and is generically unique, as shown in Figure 2.6 with green boundaries.*

To compare equilibrium with efficient outcome, there are two margins to be considered, extensive margin and intensive margin. Alternatively, we can also compare the range of γ_p and γ_m that support the same extensive margin in equilibrium and social planner's problem, and check the difference in intensive margin.

To get some intuition we can set intensive margin in equilibrium to be efficient by controling some parameters and focus on the extensive margin. Recall that q_{pm} is solved by $\alpha_{sb}\theta_{mc}u'(q_{pm}) = c'(q_{pm})$, q_{pc} by $\alpha_{sb}\theta_{pc}u'(q_{pc}) = c'(q_{pc})$, and q^o by $\alpha_{sb}u'(q^o) = c'(q^o)$. By setting $\theta_{pc} = \theta_{mc} = 1$ in equilibrium then $q_{pm} = q_{pc} \equiv q^e = q^o$. The comparison is as shown in Figure 2.6.

To begin with the analysis of extensive margin, consider the conditions on γ_p and γ_m such that $(\tau, \sigma) = (1, 1)$, and $\alpha_{sb1} = \alpha^0_{sb1}$. In equilibrium, γ_p and γ_m should satisfy

$$\begin{cases} \gamma_p & \leq \alpha^o_{sb1} u(q^o_1) - c(q^o_1) \\ \gamma_m & \leq \gamma_p \end{cases}$$

In social planner's problem, γ_p and γ_m should satisfy

$$\begin{cases} \gamma_p & \leq (\alpha^o_{sb1})^2 u(q^o_1) - c(q^o_1) \\ \gamma_m & \leq \gamma_p \end{cases}$$

Obviously, to support $(\tau, \sigma) = (1, 1)$, the ranges of γ_p and γ_m in equilibrium are no less than those in efficiency. Therefore, in this simple case when producers and middlemen have full bargaining power, although the intensive margin (q_{pm}, q_{pc}) and extensive margin τ equilibrium are efficient, the extensive margin σ is not. This is an example of too many producers participating in equilibrium since the efficient outcome is $\sigma \in [0, 1]$ while in equilibrium $\sigma = 1$ for $\gamma_p \in [(\alpha^o_{sb1})^2 u(q^o) - c(q^o), \alpha^o_{sb} u(q^o) - c(q^o)]$.

Next consider a case of inefficiency when there is too many middlemen participating in equilibrium. Consider the outcome of $(\tau, \sigma) = ([0, 1], 0)$, by which $\alpha_{sb5} = \alpha^o_{sb5}$, then in equilibrium, γ_p and γ_m should satisfy

$$\begin{cases} \gamma_p & \geq \gamma_m \\ \gamma_m & \in [G, I] \end{cases}$$

In social planner's problem, γ_p and γ_m should satisfy

$$\begin{cases} \gamma_p & \geq \gamma_m \\ \gamma_m & \in [G^o, I^o] \end{cases}$$

where $G^e = \alpha_{sb2} u(q_{pm2}) - c(q_{pm2})$, $I^e = \bar{\alpha_{sb}} u(\bar{q_{pm}}) - c(\bar{q_{pm}})$, $G^o = (\alpha^o_{sb2})^2 \theta_{mc} u(q^o_{pm2}) - c(q^o_{pm2})$, $I^o = (\bar{\alpha^o_{sb}})^2 \theta_{mc} u(\bar{q^o_{pm}}) - c(\bar{q^o_{pm}})$, and q_{pm}, q_{pc} and σ in equilibrium are the same as the efficient outcomes since $\theta_{pc} = \theta_{mc} = 1$. Nonetheless it is not always the case for τ. To prove this, it is easy to check that $G^o < G^e$, $I^o < I^e$. When $\gamma_m \in \{[G^o, G], [I^o, I]\}$, there would be too many middlemen participating: when $\gamma_m \in [G^o, G]$, $\tau^e = 1$ in equilibrium while $\tau^o \in [0, 1]$ in optimality; when $\gamma_m \in [I^o, G]$, $\tau^e \in [0, 1]$ in equilibrium while $\tau^o = 0$ in optimality.

Next consider a case when there are too many middlemen as well as too many producers participating in equilibrium. Consider $(\tau, \sigma) = (1, 0)$ in equilibrium, then γ_p and γ_m should satisfy

$$\begin{cases} \gamma_p & \geq D^e \\ \gamma_m & \leq G^e \end{cases}$$

In social planner's problem, γ_p and γ_m should satisfy

$$\begin{cases} \gamma_p & \geq D^o \\ \gamma_m & \leq G^o \end{cases}$$

where $D^e = \alpha_{sb2} u(q_{pc2}) - c(q_{pc2})$, $D^o = (\alpha^o_{sb2})^2 u(q^o_{pc2}) - c(q^o_{pc2})$. Still intensive margin are efficient given full bargaining power for producers and middlemen in retail market but the extensive margin in not efficient in equilibrium. It is easy to check that $D^o < D^e$, and there are both too many active producers and middlemen: when

$\gamma_p \in [D^o, D^e]$, there are too many active producers since $\sigma^e \in [0,1]$ while $\sigma^o = 0$; when $\gamma_m \in [G^o, G^e]$, there are too many active middlemen since $\tau^e = 1$ while $\tau^o \in [0,1]$.

Proposition 2.6. *Given $\theta_{pc} = \theta_{mc} \equiv \theta_{sb} = 1$, the intensive margin is equilibrium is efficient while extensive margin is not. The inefficiency is as shown in Figure 2.6 in which there are too many producers in the blue shaded areas and too many middlemen in the yellow shaded areas.*

Now we relax the assumption of $\theta_{pc} = \theta_{mc} = 1$ such that the intensive margin is not necessarily efficient. Consider $\theta_{pc} = \theta_{mc} \equiv \theta_{sb} < 1$, then it is easy to check $q_{pm} = q_{pc} \equiv q^e < q^o$, the intensive margin in equilibrium is less than the efficient outcome. For extensive margin, it is proved in the Appendix that efficiency is achieved for a measure zero set of bargaining power therefore the extensive margin is neither efficient.

Generally, with $\theta_{pc} = \theta_{mc} \equiv \theta_{sb} < 1$, for a given regime of equilibrium, intensive margin (q_{pm}, q_{pc}) are solved by $\alpha_{sb}\theta_{mc}u'(q_{pm}) = c'(q_{pm})$ and $\alpha_{sb}\theta_{pc}u'(q_{pc}) = c'(q_{pc})$, and the extensive margin σ is determined by whether $B^e_{\gamma_p} - \gamma_p$, that is, $\alpha_{sb}\theta_{pc}u(q_{pc}) - c(q_{pc}) - \gamma_p$ is positive, negative or zero for a given γ_m. In efficiency, to support the same (τ, σ) as in equilibrium, the extensive margin is determined by whether $B^o_{\gamma_p} - \gamma_p$, that is, $(\alpha_{sb})^2 u(q^o) - c(q^o) - \gamma_p$ is positive, negative or zero, and q^o is solved by $\alpha_{sb}u'(q^o) = c'(q^o)$. Graphically, $B^e_{\gamma_p}$ and $B^o_{\gamma_p}$ are the curves dividing $\gamma_p - \gamma_m$ space into regimes. For a given equilibrium outcome of (τ, σ), when $\theta_{pc} = \theta_{mc} = \theta_{sb} < 1$, $q_{pm} = q_{pc} \equiv q^e < q^s$, intensive margin in equilibrium is less than efficiency. For extensive margin, there are two cases to consider. Suppose we want to compare the regimes of a given (τ, σ) in $\gamma_p - \gamma_m$ plane between equilibrium and efficiency. If $\theta_{pc} = \theta_{mc} \geq \alpha_{sb}$, then $\alpha_{sb}\theta_{sb}u(q^o) - c(q^o) > (\alpha_{sb})^2 u(q^o) - c(q^o)$. As $\alpha_{sb}\theta_{sb}u(q) - c(q)$ is maximized at $q = q^e$, then $\alpha_{sb}\theta_{sb}u(q^e) - c(q^e) > \alpha_{sb}\theta_{sb}u(q^o) - c(q^o)$. Therefore we have $\alpha_{sb}\theta_{sb}u(q^e) - c(q^e) > (\alpha_{sb})^2 u(q^o) - c(q^o)$, $B^e_{\gamma_p} > B^o_{\gamma_p}$, implying that there are too many active middlemen and producers in equilibrium. If $\theta_{pc} = \theta_{mc} < \alpha_{sb}$, there can be too many or too few active middlemen and producers in equilibrium than in efficiency depending on bargaining powers which affect both intensive margin and extensive margin. For a given (τ, σ), if θ_{sb} is such that $\alpha_{sb}\theta_{sb}u(q^e) - c(q^e) > (\alpha_{sb})^2 u(q^o) - c(q^o)$, then $B^e_{\gamma_p} > B^o_{\gamma_p}$ implying that there are too many active middlemen and producers; if θ_{sb} is such that $\alpha_{sb}\theta_{sb}u(q^e) - c(q^e) < (\alpha_{sb})^2 u(q^o) - c(q^o)$, then $B^e_{\gamma_p} < B^o_{\gamma_p}$ implying that there are too few active middlemen and producers. The equilibrium outcome of extensive margin is efficient only if θ_{sb} satisfies $\alpha_{sb}\theta_{sb}u(q^e) - c(q^e) = (\alpha_{sb})^2 u(q^o) - c(q^o)$, however this is possible only for a measure 0 set of θ_{sb}, therefore we conclude that when $\theta_{pc} = \theta_{mc} \equiv \theta_{sb} < 1$, in equilibrium both intensive and extensive margins are not efficient.

Proposition 2.7. *Given $\theta_{pc} = \theta_{mc} \equiv \theta_{sb} < 1$, in any outcome of (τ, σ), the intensive margin in equilibrium is always less than the efficient outcome , $q_{pm} = q_{pc} < q^o$. Also the extensive margin is inefficient and there can be too many or too few producers and/or middlemen participating in equilibrium.*

Additionally for the case when $\theta_{pc} = \theta_{mc} < \alpha_{sb}$, we can derive a cutoff curve $\alpha_{sb} = \alpha(\theta_{sb})$ from

$$\begin{cases} \alpha_{sb}\theta_{sb}u(q^e) - c(q^e) & = (\alpha_{sb})^2 u(q^o) - c(q^o) \\ \alpha_{sb}\theta_{sb}u'(q^e) & = c'(q^e) \\ (\alpha_{sb})^2 u'(q^o) & = c'(q^o) \end{cases}$$

For a given α_{sb}, there is a unique value of θ_{sb} such that $\alpha_{sb}\theta_{sb}u(q^e) - c(q^e) = (\alpha_{sb})^2 u(q^o) - c(q^o)$ since the left hand side is increasing in θ_{sb} and right hand side is a constant. Since θ_{sb} is exogenously given while α_{sb} is endogenously determined in response to γ_p and γ_m, there can coexist too many or too few middlemen and producers in $\gamma_p - \gamma_m$ space. For a regime in $\gamma_p - \gamma_m$ space such that $\alpha_{sb} > \alpha(\theta_{sb})$ then are too many middlemen and producers; for a regime of $\gamma_p - \gamma_m$ space such that $\alpha_{sb} < \alpha(\theta_{sb})$ then there are too few. For when there would be too few participation, consider a case when $\theta_{pc} = \theta_{mc} \equiv \theta_{sb} < 1$, then for the regime with $(\tau, \sigma) = (0, 0)$ in Figure 2.3, we have too few active middlemen or producers. In this regime, $\alpha_{sb} = \bar{\alpha_{sb}} = 1 > \theta_{sb}$. To support $(\tau, \sigma) = (0, 0)$ in equilibrium, γ_p should be more than $\theta_{sb}u(q^e) - c(q^e)$, and in efficiency γ_p should be more than $u(q^o) - c(q^o)$. If we think of q^o to be the value of q^e in equilibrium when $\theta_{sb} = 1$, then because $\frac{\partial\{\theta u[q(\theta)] - c[q(\theta)]\}}{\partial\theta} = \theta u(q) > 0$ and $\theta_{sb} < 1$, then $\theta_{sb}u(q^e) - c(q^e) < u(q^o) - c(q^o)$. We can conclude that when $\gamma_p \in [\theta_{sb}u(q^e) - c(q^e), u(q^o) - c(q^o)]$ and $\gamma_m \leq \theta_{sb}u(q^e) - c(q^e)$, $\tau^e = \tau^o = 0$, while $\sigma^e = 0$ while $\sigma^o = [0, 1]$, implying there are too few active producers in equilibrium. When $\gamma_m \in [\theta_{sb}u(q^e) - c(q^e), u(q^o) - c(q^o)]$ and $\gamma_p < \theta_{sb}u(q^e) - c(q^e)$, $\sigma^e = \sigma^o = 0$, while $\tau^e = [0, 1]$ and $\tau^0 = 0$, implying there are too few active middlemen in equilibrium. When $\gamma_p \in [\theta_{sb}u(q^e) - c(q^e), u(q^o) - c(q^o)]$, $\gamma_m \in [\theta_{sb}u(q^e) - c(q^e), u(q^o) - c(q^o)]$ and $\gamma_m < \gamma_p$, $(\tau^e, \sigma^e) = ([0, 1], 0)$ while $(\tau^o, \sigma^o) = (1, [0, 1])$, implying there are both too few middlemen and producers in equilibrium. Alternative, if $\theta_{pc} = \theta_{mc} \equiv \theta_{sb} = 0$, there would also be too few participation since there is no bargaining power for middlemen and producers who bear sunk costs of having inventory before making a sale in retail market.

The comparison between equilibrium and efficiency on intensive margin and extensive margin sheds light on how bargaining power should be set such that equilibrium is efficient. For the intensive margin, it suggests $\theta_{pc} = \theta_{mc} = 1$ like Lagos and Wright, that is, producers and middlemen who are held up by consumers should have full bargaining power. For extensive margin, it suggests $\theta_{pc} = \theta_{mc} = \theta_{sb} = \frac{\partial m(n_s,n_b)}{\partial_s}\frac{n_s}{n_b} = \alpha_{sb}$ like Hosios, that is, the bargaining power of producers(middlemen) should reflect the elasticity of matching function contributed by their participation.

Obviously it is impossible to have efficiency in extensive and intensive margin by only using bargaining power since it can be set to correct only one of the two inefficiencies at a time. How can we get efficiency for general $(\theta_{mp}, \theta_{mc}, \theta_{pc})$? We propose that proportional subsidies on trading quantities (q_{pm}, q_{pc}) and lump-sum taxes or subsidies on middlemen and producers' participation can be one approach.

Suppose we are choosing proportional taxes (t_p, t_m) and lump-sum taxes (T_p, T_m) such that $(\tau^e, \sigma^e) = (\tau^o, \sigma^o)$ and $(q^e_{pm}, q^e_{pc}) = (q^o_{pm}, q^o_{pc})$. To begin with the analysis, consider proportional subsidies (t_p, t_m) on intensive margin given extensive margin is efficient. Suppose we want to support (τ^o, σ^o). In order to give producers and middlemen incentive such that they will carry (q^o_{pm}, q^o_{pc}) to RM, which is more than (q^e_{pm}, q^e_{pc}), (t_p, t_m) should satisfy

$$\begin{aligned} t_p &= (1-\theta_{pc})\alpha^o_{sb}u'(q^o) \\ t_m &= (1-\theta_{mc})\alpha^o_{sb}u'(q^o) \end{aligned}$$

where q^o is solved by $\alpha^o_{sb}u'(q^o) = c'(q^o)$, and $t_p > 0$, $t_m > 0$.

Next consider lump-sum taxes(or subsidy) (T_p, T_m) given intensive margin is efficient with proportional taxes. To support (τ^o, σ^o) as an equilibrium outcome, (T_p, T_m) are given by

$$\begin{aligned} T_p &= (\alpha^o_{sb} - \theta_{pc})\alpha^o_{sb}u(q^o) \\ T_m &= (\alpha^o_{sb} - \theta_{mc})\alpha^o_{sb}u(q^o) \end{aligned}$$

Moreover T_p is a subsidy if $\alpha^o_{sb} > \theta_{pc}$, and a tax if $\alpha^o_{sb} < \theta_{pc}$. Similarly T_m is a subsidy if $\alpha^o_{sb} > \theta_{mc}$, and a tax if $\alpha^o_{sb} < \theta_{mc}$.

2.6 Effects of Intermediation on Welfare and Redistribution

In this section, I consider effects of intermediation on welfare and redistribution. Specifically, I consider intermediation as a rent extraction activity, in the sense that middlemen have a higher bargaining power than producers when trading with consumers. To this end, I abstract away from the difference in search costs, i.e.$\gamma_m = \gamma_p$, such that I focus on the bargaining power effect to compare welfare and its distribution in two economies: one is an economy with producers who have a higher bargaining power than middlemen when trading with consumers in the retail market, i.e $\theta_{mc} < \theta_{pc}$; the other is an economy with middlemen having a higher bargaining power than producers when trading with consumers in the retail market, i.e $\theta_{mc} > \theta_{pc}$. By the equilibrium results shown in Figure 2.3 in Section 2.4, middlemen would emerge endogenously with a bargaining power advantage for the reason that they are better than producers at extracting rents from consumers. To simplify notations, denote

$\gamma \equiv \gamma_p = \gamma_m$ since search cost is the same for producers and middlemen.

In Subsection 2.6.1 and 2.6.2 , I study the welfare and distribution of a non-intermediated economy and an intermediated economy. I find that, total welfare and each consumer's welfare are improving in quantity per trade per meeting. Increase in the number of sellers can create welfare gains by promoting the number of trades but also welfare losses by incurring higher total costs paid on production and search. I show the effect of entry decisions on the number and composition of sellers with increasing search cost: when search cost is small, the number of sellers are the same in both economies, and as search cost becomes larger, there are more sellers in the intermediated economy than the non-intermediated. In Subsection 2.6.3, I study the effects of intermediation on welfare and redistribution by comparing the total welfare and distribution of the two economies. I find that, in the view of intermediation as a rent extraction activity, economy can be better with intermediation, in which all types of agents are better off as well. These results can shed new light on this debate and related issues – about middlemen contributing to efficiency versus acting as "vampires" that only buy low and sell high without contributing value added.

2.6.1 Non-intermediated Economy

We first look for the equilibrium outcome in a non-intermediated economy, i.e. when the economy have producers with a bargaining advantage over middlemen, i.e. $\theta_{mc} < \theta_{pc}$. As a result, in equilibrium middlemen do not participate such that the wholesale market is removed and retail market is the only decentralized market. Also since producers are the only sellers if the retail market is open, buyer-seller ratio and meeting rates would only change with producers' participation. Let $\hat{\sigma}$ denote producers' participation decision, $\hat{\alpha_{sb}}$ the probability a seller (i.e. producer) meets a buyer (i.e. consumer), $\hat{n_s}$ the number of producers participating in the retail market, and $(\hat{q_{pc}},\hat{y_{pc}})$ terms of trade. Then $\hat{n_s} = \hat{\sigma} N_p$, $\hat{\alpha_{sb}} = \alpha_{sb}(\frac{N_c}{\hat{n_s}}) = (\frac{N_c}{\hat{n_s}})^{\alpha}$. Similar to the decision rule of σ in Section 2.3, $\hat{\sigma}$ is given by

$$\hat{\sigma} = \begin{cases} 1 \\ [0,1] & if \;\; \hat{\alpha_{sb}}\theta_{pc}u(\hat{q_{pc}}) - c(\hat{q_{pc}}) - \gamma \begin{cases} > 0 \\ = 0 \\ < 0 \end{cases} \\ 0 \end{cases} \tag{2.46}$$

where $\hat{q_{pc}}$ is given by $\hat{\alpha_{sb}}\theta_{pc}u'(\hat{q_{pc}}) = c'(\hat{q_{pc}})$. Note that similar to the analysis in Section 2.3, $\hat{q_{pc}}$ is decided by the product of $\hat{\alpha_{sb}}$ and θ_{pc}, so $\hat{q_{pc}} = q_{pc}(\hat{\alpha_{sb}}\theta_{pc})$, and $\hat{y_{pc}} = \theta_{pc}u(\hat{q_{pc}})$.

We find that

$$\hat{\sigma} = \begin{cases} 1 \\ [0,1] & if \;\; \gamma \in \begin{cases} [0,C] \\ [C,H] \\ [H,+\infty) \end{cases} \\ 0 \end{cases} \tag{2.47}$$

in which C and H are defined in 2.29 and 2.27 in Section 2.4. Then for a given γ, an equilibrium for this economy without intermediation is given by $(\hat{\sigma},\hat{q_{pc}},\hat{y_{pc}})$. The equilibrium exists and is unique.

Now consider the total welfare and distribution in the economy. Let $\hat{W}(\theta_{pc},\gamma)$ denote the total welfare, $\hat{W}_i(\theta_{pc},\gamma)$ the welfare for an agent of type $i \in \{C,P\}$, then

$$\hat{W}(\theta_{pc},\gamma) = (N_c)^{\alpha}(\hat{n_s})^{1-\alpha}u(\hat{q_{pc}}) - \hat{n_s}[c(\hat{q_{pc}}) + \gamma] \tag{2.48}$$

in which the first term represents welfare gains given by the number of trade $(n_b)^\alpha(\hat{n_s})^{1-\alpha}$ and utility created per trade $u(\hat{q_{pc}})$, the second terms is for the welfare cost given by the number of producers $\hat{n_s}$, and production and search costs, $c(\hat{q_{pc}})+\gamma$, paid by each producer before trading. Welfare for each consumer and producer are given by,

$$\hat{W}_c(\theta_{pc},\gamma) = (\frac{N_c}{\hat{n_s}})^{\alpha-1}(1-\theta_{pc})u(\hat{q_{pc}}) \quad (2.49)$$

$$\hat{W}_p(\theta_{pc},\gamma) = (\frac{N_c}{\hat{n_s}})^{1-\alpha}\theta_{pc}u(\hat{q_{pc}}) - c(\hat{q_{pc}}) - \gamma \quad (2.50)$$

in which again $\hat{q_{pc}}$ is endogenously determined by $(\frac{N_c}{\hat{n_s}})^{1-\alpha}\theta_{pc}u'(\hat{q_{pc}}) = c'(\hat{q_{pc}})$.

Lemma 2.11. *$\hat{W}(\theta_{pc},\gamma)$, $\hat{W}_c(\theta_{pc},\gamma)$ are increasing in $\hat{q_{pc}}$.*

Depending on γ, equilibrium outcome of $\hat{\sigma}$ is different so we calculate welfare accordingly. When $\gamma \in [0, C]$, then $\hat{\sigma} = 1$, $\hat{n_s} = N_p$, $\hat{\alpha_{sb}} = (\frac{N_c}{N_p})^\alpha$, $\hat{q_{pc}} = q_{pc}[(\frac{N_c}{N_p})^\alpha\theta_{pc}]$, then welfare and distribution are given by

$$\hat{W}(\theta_{pc},\gamma \in [0,C]) = (N_c)^\alpha(N_p)^{1-\alpha}u(\hat{q_{pc}}) - \hat{n_s}[c(\hat{q_{pc}})+\gamma] \quad (2.51)$$

$$\hat{W}_c(\theta_{pc},\gamma \in [0,C]) = (\frac{N_c}{N_p})^{\alpha-1}(1-\theta_{pc})u(\hat{q_{pc}}) \quad (2.52)$$

$$\hat{W}_p(\theta_{pc},\gamma \in [0,C]) = (\frac{N_c}{N_p})^{1-\alpha}\theta_{pc}u(\hat{q_{pc}}) - c(\hat{q_{pc}}) - \gamma \quad (2.53)$$

When $\gamma \in [C, H]$, then $\hat{\sigma} = [0, 1]$, $\hat{n_s} = \hat{\sigma}N_p$, $\hat{\alpha_{sb}} = (\frac{N_c}{\hat{\sigma}N_p})^\alpha$, $\hat{q_{pc}} = q_{pc}[(\frac{N_c}{\hat{\sigma}N_p})^\alpha\theta_{pc}]$, then welfare and distribution are given by

$$\hat{W}(\theta_{pc},\gamma \in [C,H]) = (N_c)^{\alpha}(\hat{\sigma}N_p)^{1-\alpha}u(\hat{q_{pc}}) - \hat{n_s}[c(\hat{q_{pc}})+\gamma] \tag{2.54}$$

$$\hat{W}_c(\theta_{pc},\gamma \in [C,H]) = (\frac{N_c}{\hat{\sigma}N_p})^{\alpha-1}(1-\theta_{pc})u(\hat{q_{pc}}) \tag{2.55}$$

$$\hat{W}_p(\theta_{pc},\gamma \in [C,H]) = (\frac{N_c}{\hat{\sigma}N_p})^{1-\alpha}\theta_{pc}u(\hat{q_{pc}}) - -c(\hat{q_{pc}}) - \gamma \tag{2.56}$$

When $\gamma \in [H,+\infty)$, then $\hat{\sigma}=0$, $\hat{n_s}=0$, no producer participates therefore the retail market shuts down and welfare is zero.

2.6.2 Intermediated Economy

Now by introducing middlemen into the non-intermediated economy, I consider an intermediated economy . As we shut down any difference in search cost and focus on the difference in bargaining powers between producers and middlemen, now intermediation can be regarded as a rent extraction activity. I want to check effects on welfare and redistribution of intermediation.

The advantage in bargaining power endogenously generates intermediation as shown in the Figure 2.3 in $\gamma_p - \gamma_m$ space with $\theta_{mc} > \theta_{pc}$ in Section 2.4. As we assume $\gamma_p = \gamma_m = \gamma$, the set of potential equilibria is given by a 45^0 line through the origin in Figure 2.3. There are four candidate equilibrium outcomes, but depending the bargaining ratio $\frac{\theta_{mc}}{\theta_{pc}}$, there can be two cases as shown in Figure 2.7.

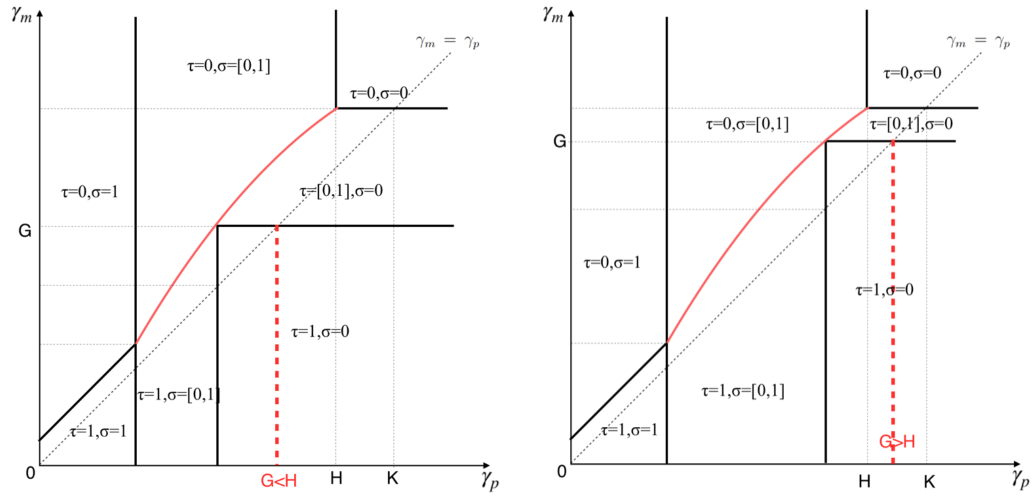


Figure 2.7: Two Cases of Equilibrium Outcomes w.r.t. $\frac{\theta_{mc}}{\theta_{pc}}$

The left graph shows the equilibrium outcome when $1 < \frac{\theta_{mc}}{\theta_{pc}} \leq \frac{\bar{\alpha_{sb}}}{\alpha_{sb}(\frac{N_c}{N_m \alpha_{mp}})}$, and the right graph shows when $\frac{\bar{\alpha_{sb}}}{\alpha_{sb}(\frac{N_c}{N_m \alpha_{mp}})} < \frac{\theta_{mc}}{\theta_{pc}}$, in which again $\bar{\alpha_{sb}}$ is the highest possible value of α_{sb}. The difference is given by different relationships between G and H, defined in 2.36 and 2.27. The left graph is for $H \geq G \equiv G_1$ and in the right graph for $H < G \equiv G_2$. Consider an example that shows why this difference matters. Given $\gamma = H$, the equilibrium in the left panel is $(\tau, \sigma) = ([0,1], 0)$ while in the right is $(\tau, \sigma) = (1, 0)$, clearly τ is higher in the latter for the same γ. This is because in the latter the bargaining power ratio is higher in favor of middlemen and encourages middlemen's participation. This generates different patterns of equilibrium with respect to γ. Now we characterize equilibrium for the two cases. For $\gamma \in [0, D]$, equilibria are the same for both cases: $(\tau, \sigma) = (1, 1)$ for $\gamma \in [0, C]$, and $(\tau, \sigma) = (1, [0, 1])$ for $\gamma \in [C, D]$. Then they become different for $\gamma > D$: when $1 < \frac{\theta_{mc}}{\theta_{pc}} \leq \frac{\bar{\alpha_{sb}}}{\alpha_{sb}(\frac{N_c}{N_m \alpha_{mp}})}$, $(\tau, \sigma) = (1, 0)$ for $\gamma \in [D, G_1]$, $(\tau, \sigma) = ([0, 1], 0)$ for $\gamma \in [G_1, K]$; when $\frac{\bar{\alpha_{sb}}}{\alpha_{sb}(\frac{N_c}{N_m \alpha_{mp}})} < \frac{\theta_{mc}}{\theta_{pc}}$, $(\tau, \sigma) = (1, 0)$ for $\gamma \in [D, G_2]$, $(\tau, \sigma) = ([0, 1], 0)$ for $\gamma \in [G_2, K]$. Note $K = I$ by $\gamma_m = \gamma_p$, and I is defined in 2.26.

Now we consider the welfare and its distribution in this economy. Let $W(\theta_{mc}, \theta_{pc}, \gamma)$ denote the total welfare and $W_i(\theta_{mc}, \theta_{pc}, \gamma)$ denote the welfare for an agent of type $i \in \{C, P, M\}$. Again the number of sellers in the retail market is $n_s = n_p^R + n_m^R = N_p\sigma(1 - \alpha_{pm}\tau) + N_m\alpha_{mp}\tau$. Then total welfare and individual welfare are given by outcomes in the retail market. For total welfare,

$$\begin{aligned} W(\theta_{mc}, \theta_{pc}, \gamma) &= (N_c)^{1-\alpha}(n_s)^{\alpha}\{\frac{n_p^R}{n_s}u(q_{pc}) + \frac{n_m^R}{n_s}u(q_{mc}) \\ &\quad -n_s[\frac{n_p^R}{n_s}c(q_{pc}) + \frac{n_m^R}{n_s}c(q_{mc}) + \gamma] \end{aligned} \tag{2.57}$$

in which the first term is for welfare gains and the second for welfare losses. For individual welfare,

$$W_c(\theta_{mc}, \theta_{pc}, \gamma) = (\frac{N_c}{n_s})^{\alpha-1}[\frac{n_p^R}{n_s}(1 - \theta_{pc})u(q_{pc}) + \frac{n_m^R}{n_s}(1 - \theta_{mc})u(q_{pm})] \tag{2.58}$$

$$W_p(\theta_{mc}, \theta_{pc}, \gamma) = (\frac{N_c}{n_s})^{1-\alpha}\theta_{pc}u(q_{pc}) - c(q_{pc}) - \gamma + \alpha_{pm}\theta_{pm}\Sigma_{pm} \tag{2.59}$$

$$W_m(\theta_{mc}, \theta_{pc}, \gamma) = \alpha_{mp}(1 - \theta_{pm})\Sigma_{pm} \tag{2.60}$$

in which again α_{pm} and α_{mp} are meeting rates in the wholesale market define in 2.5 and 2.6, q_{pc} and q_{pm} in 2.22 and 2.21, and Σ_{pm} in 2.20. Clearly total welfare and its distribution depend on numbers of sellers, proportions of middlemen among sellers and quantity per trade.

Lemma 2.12. *$W(\theta_{mc}, \theta_{pc}, \gamma)$, $W_c(\theta_{mc}, \theta_{pc}, \gamma)$ are increasing in q_{pc} and q_{pm}.*

No matter $1 < \frac{\theta_{mc}}{\theta_{pc}} \leq \frac{\bar{\alpha_{sb}}}{\alpha_{sb}(\frac{N_c}{N_m\alpha_{mp}})}$ or $\frac{\bar{\alpha_{sb}}}{\alpha_{sb}(\frac{N_c}{N_m\alpha_{mp}})} < \frac{\theta_{mc}}{\theta_{pc}}$, the same four candidates of equilibrium exist as γ increases, so we can just derive welfare and its distribution of the four candidates and then match them with different cases of $\frac{\theta_{mc}}{\theta_{pc}}$. First consider $(\sigma, \tau) = (1, 1)$. Then $n_p^R = N_p(1 - \alpha_{pm})$, $n_m^R = N_m\alpha_{mp}$, $n_s = N_p$, $\alpha_{sb} = (\frac{N_c}{N_p})^\alpha$, $q_{pc} = q_{pc}(\alpha_{sb}\theta_{pc}) = q_{pc}[(\frac{N_c}{N_p})^\alpha\theta_{pc}]$, $q_{pm} = q_{pm}(\alpha_{sb}\theta_{mc}) = q_{pm}[(\frac{N_c}{N_p})^\alpha\theta_{mc}]$, $\Sigma_{pm} = \alpha_{sb}\theta_{mc}u(q_{pm}) - c(q_{pm}) - [\alpha_{sb}\theta_{pc}u(q_{pc}) - c(q_{pc})]$. Note $N_m\alpha_{mp} = N_p\alpha_{pm}$ is given by the fact that, the number middlemen meet producer, $\tau N_m\alpha_{mp}$, must equal to the number producers meet middlemen $\tau N_p\alpha_{pm}$, so $N_m\alpha_{mp}$ is substituted with $N_p\alpha_{pm}$ in calculation. Now total and individual welfare are given by

$$
\begin{aligned}
W(\theta_{mc}, \theta_{pc}, \gamma &\in [0, C]) = (N_c)^{1-\alpha}(N_p)^\alpha\{(1 - \alpha_{pm})u(q_{pc}) + \alpha_{pm}u(q_{mc})\} - N_p[(1 - \alpha_{pm})c(q_{pc}) + \alpha_{pm}c \\
W_c(\theta_{mc}, \theta_{pc}, \gamma &\in [0, C]) = (\frac{N_c}{N_p})^{\alpha-1}[(1 - \alpha_{pm})(1 - \theta_{pc})u(q_{pc}) + \alpha_{pm}(1 - \theta_{mc})u(q_{pm})] \\
W_p(\theta_{mc}, \theta_{pc}, \gamma &\in [0, C]) = (\frac{N_c}{N_p})^{1-\alpha}\theta_{pc}u(q_{pc}) - c(q_{pc}) - \gamma + \alpha_{pm}\theta_{pm}\Sigma_{pm} \\
W_m(\theta_{mc}, \theta_{pc}, \gamma &\in [0, C]) = \alpha_{pm}(1 - \theta_{pm})\Sigma_{pm}
\end{aligned}
$$

Now consider $(\sigma, \tau) = (1, [0, 1])$. We have $n_p^R = \sigma N_p(1 - \alpha_{pm})$, $n_m^R = N_m\alpha_{mp}$, $n_s = \sigma N_p N_p(1 - \alpha_{pm}) + N_m\alpha_{mp}$, $\alpha_{sb} = (\frac{N_c}{n_s})^\alpha$, $q_{pc} = q_{pc}(\alpha_{sb}\theta_{pc}) = q_{pc}[(\frac{N_c}{n_s})^\alpha\theta_{pc}]$, $q_{pm} = q_{pm}(\alpha_{sb}\theta_{mc}) = q_{pm}[(\frac{N_c}{n_s})^\alpha\theta_{mc}]$, $\Sigma_{pm} = \alpha_{sb}\theta_{mc}u(q_{pm}) - c(q_{pm}) - [\alpha_{sb}\theta_{pc}u(q_{pc}) - c(q_{pc})]$. Total and individual welfare are derived the same way by 2.57-2.60. In this case, the composition of sellers is tilted towards middlemen further than the last case, so the probability is higher for a consumer to meet a seller who is a middlemen. Also, q_{pc} and q_{pm} are higher than in the last case since they are increasing in α_{sb} , which increases in a reduction in the number of sellers. Now consider $(\tau, \sigma) = (1, 0)$. $n_p^R = 0$, $n_s = n_m^R = N_m\alpha_{mp} = N_p\alpha_{pm}$, $q_{pc} = 0$, $q_{pm} = q_{pm}[(\frac{N_c}{N_p\alpha_{pm}})^\alpha\theta_{mc}]$, $\Sigma_{pm} = \alpha_{sb}\theta_{mc}u(q_{pm}) - c(q_{pm})$. In this case all producers participate only in the wholesale market and stop entering the retail market so that middlemen are the only sellers.

Also q_{pm} is higher than in the last two cases. Lastly, consider $(\tau, \sigma) = [(0,1), 0]$. We have $n_p^R = 0$, $n_s = n_m^R = \tau N_m \alpha_{mp} = \tau N_p \alpha_{pm}$, $q_{pc} = 0$, $q_{pm} = q_{pm}[(\frac{N_c}{\tau N_p \alpha_{pm}})^\alpha \theta_{mc}]$, $\Sigma_{pm} = \alpha_{sb}\theta_{mc}u(q_{pm}) - c(q_{pm})$. In this case n_s is further reduced than in the last case, therefore q_{pm} is even higher.

2.6.3 Welfare and Redistribution

We now in a position to compare the welfare of these two economies and study the effect of intermediation on welfare redistribution. Again welfare analysis is based on different ranges of γ. Let $\hat{W}(\theta_{pc}, \hat{\sigma}) \equiv W(\theta_{pc}, \gamma)$ relabel welfare in the non-intermediated economy, where $\hat{\sigma}$ is the participation decision at a given γ, and $W(\theta_{mc}, \theta_{pc}, \tau, \sigma)$ in the intermediated economy, where $\tau$and σ are the participation decisions for a given γ. Before we start, recall $\frac{\partial q_{pm}}{\partial \alpha_{sb}} \geq 0$, $\frac{\partial q_{pc}}{\partial \alpha_{sb}} \geq 0$, $\frac{\partial q_{pm}}{\partial \theta_{pm}} \geq 0$, $\frac{\partial q_{pc}}{\partial \theta_{pc}} \geq 0$.

Lemma 2.13. $\frac{\partial[\alpha_{sb}u(q) - c(q)]}{q} \geq 0$*, for* $q \in \{q_{pc}, q_{pm}\}$*.*

For $\gamma \in [0, C]$, in the non-intermediated economy $\hat{\sigma} = 1$ and in the intermediated economy $(\tau, \sigma) = (1, 1)$. So we have $\hat{n_s} = n_s = N_p$ and $\hat{\alpha_{sb}} = \alpha_{sb}$. Also by $\frac{\partial q_{ij}}{\partial(\alpha_{sb}\theta_{ij})} > 0$, $ij \in \{pc, pm\}$ and $\theta_{mc} > \theta_{pc}$, $\hat{q_{pc}} = q_{pc} < q_{pm}$. This implies the number sellers, therefore buyer-seller ratio and the extensive margin, i.e. number of trades are the same. But the expected welfare in the non-intermediated economy is less than that in the intermediated economy. This is because in the former, the quantity per trade is always $\hat{q_{pc}}$, while in the latter there are some intermediated trade with a larger volume of q_{pm}, so the expected welfare is higher in the latter. Formally, the difference in welfare between the two economies is

$$
\begin{aligned}
& W(\theta_{mc}, \theta_{pc}, \tau = 1, \sigma = 1) - \hat{W}(\theta_{pc}, \hat{\sigma} = 1) \\
= \quad & (N_c)^{1-\alpha}(n_s)^{\alpha}\{\alpha_{sb}u(q_{pm}) - c(q_{pm}) - [\alpha_{sb}u(q_{pc}) - c(q_{pc})]\} \quad > 0
\end{aligned}
$$

It is positive by Lemma 2.13, with $q_{pm} > q_{pc}$ by $\theta_{mc} > \theta_{pc}$. Therefore total welfare in the intermediated economy is higher than in the non-intermediated. For welfare redistribution, we have

$$W_p(\theta_{mc}, \theta_{pc}, \tau = 1, \sigma = 1) - \hat{W}_p(\theta_{pc}, \hat{\sigma} = 1) = \alpha_{pm}\theta_{pm}\Sigma_{pm} > 0$$

$$W_m(\theta_{mc}, \theta_{pc}, \tau = 1, \sigma = 1) = \alpha_{mp}\theta_{mp}\Sigma_{pm} > 0$$

$$
\begin{aligned}
& W_c(\theta_{mc}, \theta_{pc}, \tau = 1, \sigma = 1) - \hat{W}_c(\theta_{pc}, \hat{\sigma} = 1) \\
& = (\tfrac{N_c}{N_p})^{\alpha-1}\alpha_{pm}[(1-\theta_{mc})u(q_{pm}) - (1-\theta_{pc})u(q_{pc})]
\end{aligned}
$$

where $\Sigma_{pm} = \alpha_{sb}\theta_{mc}u(q_{pm}) - c(q_{pm}) - [\alpha_{sb}\theta_{pc}u(q_{pc}) - c(q_{pc})]$. As shown above, producers and middlemen are better off in the intermediated economy. However for consumers, it is not determined. Intuitively, in the intermediated economy, consumers' welfare gains are created by a larger expected total surplus since upon meeting a middleman they can enjoy q_{pm}, however there are also welfare losses associated with a lower share $1 - \theta_{mc}$ of total surplus. Depending on parameter values, the welfare gains can outweigh the losses.

For $\gamma \in [C, D]$, in the non-intermediated economy $\hat{\sigma} = [0, 1]$ and in the intermediated $(\tau, \sigma) = (1, [0, 1])$. $n_s = N_p\sigma(1 - \alpha_{pm}) + N_m\alpha_{mp}$, and $\hat{n_s} = \hat{\sigma}N_p$. Given the number of sellers in each market and $\theta_{mc} > \theta_{pc}$, we have $\hat{\alpha_{sb}} = \alpha_{sb}$, $\hat{q_{pc}} = q_{pc} < q_{pm}$, $n_s = \hat{n_s}$ and $\sigma < \hat{\sigma}$, proved in the Appendix. Intuitively, as search cost increases, less producers participate in both economies while all middlemen participate because of their higher bargaining power. Moreover, in the intermediated economy, the reduction of producers is larger than in the non-intermediated one for two reasons: one is that producers in the former have an option to sell goods to middlemen when search cost climbs up, the other is that the retail market would be too crowded to be profitable on the sellers' side if $\sigma < \hat{\sigma}$, given middlemen take part of the positions. $\sigma < \hat{\sigma}$ implies that, while $n_s = \hat{n_s}$, the composition of sellers is tilted towards middlemen compared the case of $\gamma \in [0, C]$, in the sense that among all sellers, a larger proportion of them become middlemen as search cost increases. Then there is a higher chance for consumers to meet a seller who is a middleman and enjoy a larger volume of goods than in a meeting with producers. Similar to the last case, formally welfare change and redistribution are given by

$$W(\theta_{mc}, \theta_{pc}, \tau = 1, \sigma = [0, 1]) - \hat{W}(\theta_{pc}, \hat{\sigma} = [0, 1])$$
$$= (N_c)^{1-\alpha}(n_s)^{\alpha}\{\alpha_{sb}u(q_{pm}) - c(q_{pm}) - [\alpha_{sb}u(q_{pc}) - c(q_{pc})]\} > 0$$

$$W_p(\theta_{mc}, \theta_{pc}, \tau = 1, \sigma = [0, 1]) - \hat{W}_p(\theta_{pc}, \hat{\sigma} = [0, 1]) = \alpha_{pm}\theta_{pm}\Sigma_{pm} > 0$$

$$W_m(\theta_{mc}, \theta_{pc}, \tau = 1, \sigma = [0, 1]) = \alpha_{mp}\theta_{mp}\Sigma_{pm} > 0$$

$$W_c(\theta_{mc}, \theta_{pc}, \tau = 1, \sigma = [0,1]) - \hat{W}_c(\theta_{pc}, \hat{\sigma} = [0,1])$$
$$= (\tfrac{N_c}{n_s})^{\alpha-1}\{\alpha_{pm}(1-\theta_{mc})u(q_{pm}) - [1-\sigma(1-\alpha_{pm})](1-\theta_{pc})u(q_{pc})\}$$

where $\Sigma_{pm} = \alpha_{sb}\theta_{mc}u(q_{pm}) - c(q_{pm}) - [\alpha_{sb}\theta_{pc}u(q_{pc}) - c(q_{pc})]$. Again, welfare is improved with intermediation, producers and middlemen are better off, consumers can be better off, worse off depending on parameter values.

In the above two cases, both economies has the same number of sellers in the retail market and each producer brings the same quantity of goods. Welfare is higher in the intermediated economy because among all sellers some are middlemen who bring a larger volume of goods, and welfare is increasing in the quantity per trade in equilibrium. This justifies the important role of intensive margin introduced in models of intermediation. Also by $\frac{\partial q_{ij}}{\partial(\alpha_{sb}\theta_{ij})} > 0$, $ij \in \{pc, pm\}$, in these two cases the difference between q_{pm} and $\hat{q_{pc}}$ is only generated by the difference between θ_{mc} and θ_{pc}.

Now for $\gamma > D$, we need to consider the discussion of $\frac{\theta_{mc}}{\theta_{pc}}$ in Subsection 2.6.2 because it would generate different pairs of equilibrium to be compared.

We first consider the case when $1 < \frac{\theta_{mc}}{\theta_{pc}} \leq \frac{\bar{\alpha_{sb}}}{\alpha_{sb}(\frac{N_c}{N_m\alpha_{mp}})}$ and $H > G \equiv G_1$. For $\gamma \in [D, G_1]$, in the non-intermediated economy $\hat{\sigma} = [0,1]$ and in the intermediated $(\tau, \sigma) = (1, 0)$. Now in the latter economy, producers stop entering and middlemen are the only sellers in the retail market. $n_s = N_m\alpha_{mp}$, $\hat{n_s} = \hat{\sigma}N_p$. Also $n_s > \hat{n_s}$, $\alpha_{sb} \leq \hat{\alpha_{sb}}$, $\hat{q_{pc}} \leq q_{pm}$, proved in Appendix. Intuitively, $\sigma = 0$ in the intermediated economy while $\hat{\sigma} = [0,1]$ in the other indicates that the retail market in the former economy is more crowded for producers to profit from. In fact, the two economies are the same for producers except in the intermediated economy middlemen's participation is promoted by their bargaining power advantage and can make the retail market too crowded for producers to make profits. For the quantity per trade q_{pm} and $\hat{q_{pc}}$, as implied by $\frac{\partial q_{ij}}{\partial(\alpha_{sb}\theta_{ij})} > 0$, $ij \in \{pc, pm\}$, here $\theta_{mc} > \theta_{pc}$ while $\alpha_{sb} \leq \hat{\alpha_{sb}}$. It is proved that $\alpha_{sb}\theta_{mc} > \hat{\alpha_{sb}}\theta_{pc}$, indicating that the negative effect of lower meeting rate is outweighed by the positive effect of higher bargaining power in the intermediated economy. Intuitively, $\tau = 1$ implies that middlemen make a positive profit in the intermediated economy, and $\hat{\sigma} = [0, 1]$ indicates producers make a zero free profit in the non-intermediated. Given the same search cost, the reason for a positive profit must be the effect of $\theta_{mc} > \theta_{pc}$ outweighs $\alpha_{sb} \leq \hat{\alpha_{sb}}$. Therefore $\hat{q_{pc}} \leq q_{pm}$. Now we consider total welfare and redistribution. Formally, total welfare in the two economies are given by

$$\begin{aligned}\hat{W}(\theta_{pc}, \hat{\sigma} = [0, 1]) &= (N_c)^{\alpha}(\hat{n_s})^{1-\alpha}u(\hat{q_{pc}}) - \hat{n_s}[c(\hat{q_{pc}}) + \gamma] \\ W(\theta_{mc}, \theta_{pc}, \tau = 1, \sigma = 0) &= (N_c)^{\alpha}n_s^{1-\alpha}u(q_{mc}) - n_s[c(q_{mc}) + \gamma]\end{aligned}$$

Different from the previous two cases when there is only change in the intensive margin, now both intensive margin, i.e. size of trade, increases from $\hat{q_{pc}}$ to q_{mc}, and extensive margin, i.e. number of producers, increases from $\hat{n_s}$ to n_s. Increase in the intensive margin always creates welfare gains since the holdup problem keeps the quantity per trade inefficiently low in equilibrium. Increase in the extensive margin, however, creates welfare gains as well as losses. As shown above, with a higher number of sellers n_s, welfare is improved by a higher number of meetings but at the cost of a higher sunk payment on search and production. Without specifying parameter values, welfare can be improved, reduced. For welfare redistribution, we found that

$$W_p(\theta_{mc}, \theta_{pc}, \tau = 1, \sigma = 0) - \hat{W}_p(\theta_{pc}, \hat{\sigma} = [0, 1]) = \alpha_{pm}\theta_{pm}\Sigma_{pm} > 0$$

$$W_m(\theta_{mc}, \theta_{pc}, \tau = 1, \sigma = 0) = \alpha_{mp}\theta_{mp}\Sigma_{pm} > 0$$

$$\begin{aligned} & W_c(\theta_{mc}, \theta_{pc}, \tau = 1, \sigma = 0) - \hat{W}_c(\theta_{pc}, \hat{\sigma} = [0, 1]) \\ = & (\tfrac{N_c}{n_s})^{\alpha-1}(1 - \theta_{mc})u(q_{pm}) - (\tfrac{N_c}{\hat{n_s}})^{\alpha-1}(1 - \theta_{pc})u(\hat{q_{pc}}) \end{aligned}$$

So producers and middlemen are better off in the intermediated economy, while consumers can be better off, worse off because while they benefit from a larger volume of goods per trade and a higher meeting rate, they also suffer from a smaller share from total surplus.

Now consider $\gamma \in [G_1, K]$, in the non-intermediated economy $\hat{\sigma} = [0, 1]$ and in the intermediated $(\tau, \sigma) = ([0, 1], 0)$. In fact, compared with the last case, as search cost increases, the total number of sellers reduces further in both economies, and in the intermediated economy some middlemen stop entering now. $n_s = \tau N_m \alpha_{mp}$ and $\hat{n_s} = \hat{\sigma} N_p$. Now $n_s > \hat{n_s}$, $\alpha_{sb} \leq \hat{\alpha_{sb}}$, and $\hat{q_{pc}} = q_{pm}$, proved in Appendix. Intuitively, when search cost becomes larger, middlemen become indifferent between participating or not as they earn a zero profit, same as producers who participate in the non-intermediated economy. For a given search cost, $n_s > \hat{n_s}$ is generated by middlemen's advantage in bargaining power, and a zero profit is made even though their entry results in a lower buyer-seller ratio than that in the non-intermediated economy. Similar to the last case, total welfare in the two economies are given by

$$\begin{aligned}
\hat{W}(\theta_{pc}, \sigma = \hat{[0,1]}) &= (N_c)^{\alpha}(\hat{n_s})^{1-\alpha}u(\hat{q_{pc}}) - \hat{n_s}[c(\hat{q_{pc}}) + \gamma] \\
W(\theta_{mc}, \theta_{pc}, \tau = 1, \sigma = 0) &= (N_c)^{\alpha} n_s^{1-\alpha} u(q_{mc}) - n_s[c(q_{mc}) + \gamma]
\end{aligned}$$

Again, total welfare can be higher or lower with intermediation.

In summary, when $1 < \frac{\theta_{mc}}{\theta_{pc}} \leq \frac{\bar{\alpha_{sb}}}{\alpha_{sb}(\frac{N_c}{N_m \alpha_{mp}})}$, for $\gamma \in [0, D]$, economy is better off with intermediation, and for $\gamma \in [D, K]$, economy can be better or worse off. For any γ, producers and middlemen are better off, while consumers can be better or worse off with intermediation.

Now consider the case when $\frac{\theta_{mc}}{\theta_{pc}} \geq \frac{\bar{\alpha_{sb}}}{\alpha_{sb}(\frac{N_c}{N_m \alpha_{mp}})}$ and $H < G \equiv G_2$. For $\gamma \in [D, G_2]$, the results are the same as in the case of $1 < \frac{\theta_{mc}}{\theta_{pc}} \leq \frac{\bar{\alpha_{sb}}}{\alpha_{sb}(\frac{N_c}{N_m \alpha_{mp}})}$ with $\gamma \in [D, G_1]$. For $\gamma \in [G_2, K]$, in the non-intermediated economy $\hat{\sigma} = 0$ and in the intermediated economy $(\tau, \sigma) = ([0, 1], 0)$. This indicates that the retail market is closed in the non-intermediated economy, but still open in the intermediated economy. Clearly, welfare improves with intermediation as it achieves allocations which would not exist in the non-intermediated economy.

In summary, when $\frac{\theta_{mc}}{\theta_{pc}} \geq \frac{\bar{\alpha_{sb}}}{\alpha_{sb}(\frac{N_c}{N_m \alpha_{mp}})}$, for $\gamma \in [0, D]$ and $[G, K]$ the economy is better off with intermediation, for $\gamma \in [G, K]$ the economy can be better or worse off. For any γ, producers and middlemen are better off, while consumers can be better or worse off with intermediation.

While change in total and consumers' welfare can have uncertainty with general parameter, I find that with small α in the meeting function $M = (n_b)^{\alpha}(n_s)^{1-\alpha}$, that is sellers' entry dominately decides the number of meetings, then for any γ, economy is better off with intermediation, producers and middlemen are always better off, while consumers are worse when $\gamma \in [0, D]$, and better off when $\gamma > D$. This speaks to the issue of disintermediation and it implies that, when search frictions (i.e. search costs) are small, economy is better off with intermediation while welfare is redistributed such that consumers are worse off, and when search frictions increase, all agents are better off with intermediation.

Proposition 2.8. *Given $\gamma_m = \gamma_p = \gamma$, and a small α in $M = n_b^{\alpha} n_s^{1-\alpha}$, (1) when γ is small, intermediation improves total welfare, producers and middlemen are better off while consumers are worse off; (2) as γ increases, all agents are better off with intermediation.*

Proposition 2.9. *Given $\gamma_m = \gamma_p = \gamma$, $\theta_{mc} > \theta_{pc}$, (1) if $\theta_{mc} - \theta_{pc}$ is small, when γ is low, welfare improves with M; as γ increases, welfare can increases, decreases, and for any γ, P and M are weakly better off, C can be better or worse off; (2) if $\theta_{mc} - \theta_{pc}$ is big, when γ is not too big, same as in 1; when γ is large, markets are closed iff M are allowed, agents are better off with intermediation.*

2.7 Conclusions

This chapter has studied the intermediation theory in a search-matching based environment in the spirit of Rubinstein and Wolinsky (1987). The model incorporates divisible goods and an endogenous meeting technology to allow questions related with intensive margin as well as extensive margin to be studied. Compared with models of intermediation with indivisible good, it allows new results to be demonstrated as equilibrium forces can now work in more ways than one. I have proved existence, generic uniqueness of equilibrium and compare it with the efficient outcome. Compared with previous work on middlemen with indivisible goods, I can support a larger set of equilibrium with participation of middlemen. Also in response to changes in parameters, there is co-movement of entensive and intensive margins, and the symmetry of producers and middlemen in their meeting probability in retail market allows advantage (disadvantage) in bargaining power and search cost play an important role in the equilibrium outcome. In the efficiency analysis I find that bargaining power, which has been used in labor and monetary search literature to restore efficiency, is no longer effective here. Since there are inefficiencies on both intensive and extensive margins, if we set bargaining power correctly to have efficiency in one margin we would lose efficiency in the other, thus the chapter gives some policy suggestions: for a general set of bargaining power, we should assign proportional subsidy on the intensive margin and lump-sum taxes or subsidy on the extensive margin. Both proportional taxes and lump-sum taxes (subsidies) should be taken into account if there is any difference between producers and middlemen in their bargaining powers. I also explore, without policy intenventions, the effect of intermediation on welfare and redistribution. Taking the view of intermediation as a rent extraction activity, I show that welfare can be improved with intermediation and all agents are better off. This result can speak to the social function of intermediation and issues on disintermediation. All these new findings are given by the economics of divisible good and endogenous meeting technology.

There are many interesting applications to be considered. It would be interesting to allow goods not fully depreciate so that the distribution of inventory over time can affect agents' decisions and terms of trade. For example, we can allow multiple subperiods of wholesale and retail markets before the settlemen period. It is also desirable to endogenize agents' choice to be producers or middlemen and check changes in the composition of sellers, and the corresponding equilbrium and efficiency results. Furthermore introducing endogenous money or limited credit would be of course within my scope to study issues related with monetary policy and inflation, including cycles and volatility, and in this point of view, middlemen and money can be substitutes but also complements.

2.8 Appendix

Proof of Lemma 2.1

Proof. This can be proved by the property of generalized Nash bargaining with perfect credit. It is obvious that M and P would want to trade if each of them gets non-negative surplus from trading. M gets a proportion of θ_{mp} from the total surplus when trading with P, therefore as long as total surplus Σ_{pm} is non-negative, M would be better off trading with P. Similarly, P would want to trade as well if Σ_{pm} is non-negative. Thus the trading decisions for M and P in WM follow the same rule. This can also be proved by considering if there is any profitable deviation for P. Suppose one P chooses not to participate in WM when there are M participating. Then his expected payoff is $V_p = \max\{\alpha_{sb}\theta_{pc}u(q_{pc}) - c(q_{pc}) - \gamma_p, 0\} + V_p^A$. If he deviates and trade with M in WM, his expected payoff is $V_p = \alpha_{pm}\theta_{mp}\Sigma_{pm} + \max\{\alpha_{sb}\theta_{pc}u(q_{pc}) - c(q_{pc}) - \gamma_p, 0\} + V_p^A$, where α_{sb} are the same if deviating or not since P and M are one-for-one, in the sense that if a marginal P trades with M, the market tightness in retail market is the same as the case if this marginal P goes to RM by himself. Therefore, given

M's participation, i.e. $\Sigma_{pm} \geq 0$, if one P deviates to trade with M, all P would trade with M. So as long as M participates in WM, P has an additional chance of getting $\alpha_{pm}\theta_{mp}\Sigma_{pm}$ from participating in WM besides $\max\{\alpha_{sb}\theta_{pc}u(q_{pc}) - c(q_{pc}) - \gamma_p, 0\}$ that he gets from participating in RM. ■

Proof of Lemma 2.10 and Proposition 2.1

Proof. First, we prove $B < C < D < H$ and $G < J$. Recall that

$$\begin{aligned} B &= [\alpha_{sb1}\theta_{pc}u(q_{pc1}) - c(q_{pc1})] - [\alpha_{sb1}\theta_{pm}u(q_{pm1}) - c(q_{pm1})] \\ C &= \alpha_{sb1}\theta_{pc}u(q_{pc1}) - c(q_{pc1}) \\ D &= \alpha_{sb2}\theta_{pc}u(q_{pc2}) - c(q_{pc2}) \\ H &= \bar{\alpha_{sb}}\theta_{pc}u(\bar{q_{pc}}) - c(\bar{q_{pc}}) \end{aligned}$$

where $\alpha_{sb1} = M(1, \frac{N_c}{N_c+N_p})$, $\alpha_{sb2} = M(1, \frac{N_c}{N_c+N_m\alpha_{mp}})$, and $\bar{\alpha_{sb}} = 1$. Obviously $B < C$ since $\alpha_{sb1}\theta_{pm}u(q_{pm1}) - c(q_{pm1}) > 0$.

It is easy to compare C , D and H by checking that in general $\alpha_{sb}\theta_{pc}u(q_{pc}) - c(q_{pc})$ in monotonically increasing in α_{sb},

$$\begin{aligned} \frac{\partial[\alpha_{sb}\theta_{pc}u(q_{pc}) - c(q_{pc})]}{\partial\alpha_{sb}} &= \theta_{pc}[u(q_{pc}) + \alpha_{sb}u'(q_{pc})\frac{\partial q_{pc}}{\partial\alpha_{sb}}] - c'(q_{pc})\frac{\partial q_{pc}}{\partial\alpha_{sb}} \\ &= \theta_{pc}u(q_{pc}) > 0 \end{aligned}$$

Since $\alpha_{sb1} < \alpha_{sb2} < \bar{\alpha_{sb}}$ in C, D and H, then $C < D < H$.

Second, we prove that when $\theta_{mc} = \theta_{pc}$ then $A = 0$, $F = G$, $J = I$ and $h'(\gamma_p) = 1$; when $\theta_{mc} > \theta_{pc}$ then $A > 0$, $F < G$, $J < I$ and $h'(\gamma_p) > 1$; when $\theta_{mc} < \theta_{pc}$ then

$A < 0$, $F > G$, $J > I$ and $h'(\gamma_p) < 1$. Recall that

$$\begin{aligned} A &= \alpha_{sb1}\theta_{mc}u[q_{pm}(\alpha_{sb1})] - c[q_{pm}(\alpha_{sb1})] \\ &\quad -\{\alpha_{sb1}\theta_{pc}u[q_{pc}(\alpha_{sb1})] - c[q_{pc}(\alpha_{sb1})]\} \end{aligned}$$

For a given α_{sb}, $\frac{\partial\{\alpha_{sb}\theta u[q(\theta)]-c[q(\theta)]\}}{\partial\theta} = \alpha_{sb}u(q) + [\alpha_{sb}\theta u'(q) - c'(q)]\frac{\partial q}{\partial\theta} = \alpha_{sb}u(q) > 0$. When $\theta_{mc} > \theta_{pc}$, $\alpha_{sb}\theta_{mc}u[q_{pm}(\theta_{mc})] - c[q_{pm}(\theta_{mc})] > \alpha_{sb}\theta_{pc}u[q_{pc}(\theta_{pc})] - c[q_{pc}(\theta_{pc})]$ therefore given $\alpha_{sb} = \alpha_{sb1}$, $A > 0$. Similarly, if $\theta_{mc} < \theta_{pc}$, $A < 0$, and if $\theta_{mc} = \theta_{pc}$, $A = 0$,

Next consider part $\gamma_m = h(\gamma_p) = \alpha_{sb}\theta_{mc}u(q_{pm}) - c(q_{pm})$ in which α_{sb}, q_{pm} can be expressed in terms of γ_p by solving

$$\begin{cases} \gamma_p & = \alpha_{sb}\theta_{pc}u(q_{pc}) - c(q_{pc}) \\ c'(q_{pm}) & = \alpha_{sb}\theta_{mc}u'(q_{pm}) \\ c'(q_{pc}) & = \alpha_{sb}\theta_{pc}u'(q_{pc}) \end{cases}$$

Taking derivative w.r.t. γ_p on both side of $\gamma_m = h(\gamma_p)$,

$$h'(\gamma_p) = \frac{\partial\alpha_{sb}}{\partial\gamma_p}\{\theta_{mc}u(q_{pm}) + [\alpha_{sb}\theta_{mc}u'(q_{pm}) - c'(q_{pm})]\frac{\partial q_{pm}}{\partial\alpha_{sb}}\} = \frac{\partial\alpha_{sb}}{\partial\gamma_p}\theta_{mc}u(q_{pm}) = \frac{\theta_{mc}u(q_{pm})}{\theta_{pc}u(q_{pc})}$$

Using Proposition 2.2, when $\theta_{mc} > \theta_{pc}$, then $\theta_{mc}u(q_{pm}) > \theta_{pc}u(q_{pc})$, $h'(\gamma_p) > 1$, implying $h(\gamma_p)$ is above $f(\gamma_p)$ for $\gamma_p \geq C$, and vice versa. Moreover when $\theta_{mc} > \theta_{pc}$, as $h(\gamma_p) > f(\gamma_p)$ for $\gamma_p \geq C$, then $G = h(\gamma_p = D) > f(\gamma_p = D) = F$, $I = h(\gamma_p = H) > f(\gamma_p = H) = J$. Similarly, when $\theta_{mc} < \theta_{pc}$, as $h(\gamma_p) < f(\gamma_p)$ for $\gamma_p \geq C$, then $G < F$, $I < J$.

Now check the slope and curvature of $\gamma_m = h(\gamma_p) = \gamma_p + \alpha_{sb}(\gamma_p)\theta_{mc}u[q_{pm}(\gamma_p)] - c[q_{pm}(\gamma_p)] - \alpha_{sb}(\gamma_p)\theta_{pc}u[q_{pc}(\gamma_p)] - c[q_{pc}(\gamma_p)]$ for $\gamma_p \in [D, H]$. By $\frac{\partial h(\gamma_p)}{\partial \gamma_p} = \frac{\theta_{mc}u(q_{pm})}{\theta_{pc}u(q_{pc})}$, we have $\frac{\partial h(\gamma_p)}{\partial \gamma_p} > 1$ if $\theta_{mc} > \theta_{pc}$, and $0 < \frac{\partial h(\gamma_p)}{\partial \gamma_p} < 1$ if $\theta_{mc} < \theta_{pc}$. To decide curvature we check the second order derivative of $h(\gamma_p)$ with respect to γ_p.

$$
\begin{aligned}
\frac{\partial^2 h(\gamma_p)}{\partial \gamma_p^2} &= \frac{\partial \frac{\theta_{mc}u(q_{pm})}{\theta_{pc}u(q_{pc})}}{\partial \alpha_{sb}} \frac{\partial \alpha_{sb}}{\partial \gamma_p} \\
&= [\theta_{mc}u'(q_{pm})u(q_{pc}) - \theta_{pc}u'(q_{pc})u(q_{pm})] \frac{\theta_{mc}\theta_{pc}}{[\theta_{pc}u(q_{pc})]^3} \\
&\cong \frac{\partial q_{pm}}{\partial \alpha_{sb}} u'(q_{pm})u(q_{pc}) - \frac{\partial q_{pc}}{\partial \alpha_{sb}} u'(q_{pc})u(q_{pm}) \\
&\cong \frac{u'(q_{pm})}{u(q_{pm})} \frac{\partial q_{pm}}{\partial \alpha_{sb}} - \frac{\theta_{pc}u'(q_{pc})}{u(q_{pc})} \frac{\partial q_{pc}}{\partial \alpha_{sb}}
\end{aligned}
$$

Since $\frac{u'(q_{ij})}{u(q_{ij})} \frac{\partial q_{ij}}{\partial \alpha_{sb}}$ is decided by θ_{ij} given other parameters, we just need to check how $\frac{u'(q_{ij})}{u(q_{ij})} \frac{\partial q_{pc}}{\partial \alpha_{sb}}$ response to θ_{ij} to find the sign for $\frac{u'(q_{pm})}{u(q_{pm})} \frac{\partial q_{pm}}{\partial \alpha_{sb}} - \frac{\theta_{pc}u'(q_{pc})}{u(q_{pc})} \frac{\partial q_{pc}}{\partial \alpha_{sb}}$. For simplicity, we drop the subscription for calculation in this step.

$$
\begin{aligned}
\frac{\partial [\frac{u'(q)}{u(q)} \frac{\partial q_{pm}}{\partial \alpha_{sb}}]}{\partial \theta} &= \frac{\partial [\frac{u'}{u} \frac{\theta u'}{c'' - \alpha\theta u''}]}{\partial \theta} \\
&= \frac{numerator}{[u(c'' - \alpha\theta u'')]^2}
\end{aligned}
$$

where numerator is

$$
\begin{aligned}
&\alpha\theta(u')^2[3u''u - (u')^2] - \theta(u')^2u(c''' - \alpha\theta u''')\tfrac{\partial q}{\partial \theta} \\
&= \alpha\theta(u')^2[3u''u - (u')^2] - \theta(u')^2u(c''' - \alpha\theta u''')\tfrac{\alpha u'}{c'' - \alpha\theta u''}
\end{aligned}
$$

Depending the form of cost and utility functions, $\gamma_m = h(\gamma_p)$ can be concave or convex. But as shown above, if $h(\gamma_p)$ is concave (convex) when $\theta_{mc} > \theta_{pc}$, then it would be convex (concave) when $\theta_{mc} > \theta_{pc}$. We illustrate an example of a concave $h(\gamma_p)$ when $\theta_{mc} > \theta_{pc}$ in this chapter. ■

Proof of Proposition 2.2

Proof. Generally $q_{ij} \in \{q_{pc}, q_{mc}\}$ is solved by $\alpha_{sb}\theta_{ij}u'(q_{ij}) = c'(q_{ij})$. Taking derivative w.r.t. α_{sb} on both side, then $\frac{\partial q_{ij}}{\partial \alpha_{sb}} = \theta_{ij}\frac{(u')^2}{c''u'-c'u''} > 0$. Similarly, $\frac{\partial q_{ij}}{\partial \theta_{ij}} = \alpha_{sb}\frac{(u')^2}{c''u'-c'u''} > 0$. Then let η_{ij} denote $\alpha_{sb}\theta_{ij}$, then $\frac{\partial q_{ij}}{\partial \eta_{ij}} = \frac{u'(q_{ij})}{c''(q_{ij})-\eta_{ij}u''(q_{ij})} \geq 0$. ■

Proof of Proposition 2.3

Proof. In equilibrium with $(\tau, \sigma) = (1, [0, 1])$, we have

$$\begin{cases} \gamma_p & = \alpha_{sb}\theta_{pc}u(q_{pc}) - c(q_{pc}) \\ \gamma_m & \leq \alpha_{sb}\theta_{mc}u(q_{pm}) - c(q_{pm}) \end{cases}$$

where $\alpha_{sb} = M(1, \frac{N_c}{N_p\sigma(1-\alpha_{pm})+N_m\alpha_{mp}})$. Also q_{pm}, q_{pc} and σ can be solved in terms of γ_p by 2.40, and

$$\gamma_p = \alpha_{sb}(\sigma)\theta_{pc}u[q_{pc}(\sigma)] - c[q_{pc}(\sigma)] \tag{2.65}$$

Taking derivative with respect to γ_p on both of 2.65,

$$\begin{aligned} 1 &= \frac{\partial \alpha_{sb}}{\partial \sigma}\frac{\partial \sigma}{\partial \gamma_p}\{\theta_{pc}u(q_{pc}) + [\alpha_{sb}\theta_{pc}u'(q_{pc}) - c'(q_{pc})]\frac{\partial q_{pc}}{\partial \alpha_{sb}}\} \\ \frac{\partial \sigma}{\partial \gamma_p} &= [\frac{\partial \alpha_{sb}}{\partial \sigma}\theta_{pc}u(q_{pc})]^{-1} < 0 \end{aligned}$$

Moreover $\frac{\partial \alpha_{sb}}{\partial \gamma_p} = \frac{\partial \alpha_{sb}}{\partial \sigma}\frac{\partial \sigma}{\partial \gamma_p} = [\theta_{pc}u(q_{pc})]^{-1} > 0$, $\frac{\partial q_{pc}}{\partial \gamma_p} = \theta_{pc}\frac{(u')^2}{c''u'-c'u''}[\theta_{pc}u(q_{pc})]^{-1} > 0$, $\frac{\partial q_{pm}}{\partial \gamma_p} = \theta_{mc}\frac{(u')^2}{c''u'-c'u''}[\theta_{mc}u(q_{pm})]^{-1} > 0$. Similarly the same results can be proved in equilibrium with $(\tau, \sigma) = (0, [0, 1])$. ■

Proof of Proposition 2.5

Proof. For a social planner, the problem is given by

$$\max_{\tau^o,\sigma^o,q^o_{pm},q^o_{pc}} N_p\tau^o\alpha^o_{pm}[-c(q^o_{pm})] + N_m\tau^o\alpha_{mp}[\alpha^o_{sb}u(q^o_{pm}) - \gamma_m] + N_p\sigma^o(1-\tau^o\alpha_{pm})[\alpha^o_{sb}u(q^o_{pc}) - c(q^o_{pc}) - \gamma_p] \tag{2.66}$$

where $\alpha^o_{sb} = M(1, \frac{N_c}{N_p\sigma^o(1-\alpha_{pm}\tau^o)+N_m\alpha_{mp}\tau^o})$. By using $N_p\alpha_{pm}$ to substitute for $N_m\alpha_{mp}$ in the optimization problem and dividing the function by N_p, then it is the same as solving

$$\max_{\tau^o,\sigma^o,q^o_{pm},q^o_{pc}} Z \equiv \tau^o\alpha_{pm}[\alpha^o_{sb}u(q^o_{pm}) - c(q^o_{pm}) - \gamma_m] + \sigma^o(1-\tau^o\alpha_{pm})[\alpha^o_{sb}u(q^o_{pc}) - c(q^o_{pc}) - \gamma_p] \tag{2.67}$$

given α^o_{sb}, $q^o_{pm} = q^o_{pc} = q^o$ is solved by

$$\alpha^o_{sb}u'(q^o) = c'(q^o) \tag{2.68}$$

$$\begin{aligned}\frac{\partial Z}{\partial \tau^o} &= \alpha_{pm}\{\alpha^o_{sb}u(q^o) - c(q^o) - \gamma_m - \sigma^o[\alpha^o_{sb}u(q^o) - c(q^o) - \gamma_p]\} \\ &\quad + \frac{\partial \alpha^o_{sb}}{\partial \tau^o}u(q^o)[\alpha_{pm}\tau + \sigma^o(1-\tau\alpha_{pm})] \\ &= 0 \end{aligned} \tag{2.69}$$

$$\frac{\partial Z}{\partial \sigma^o} = (1-\tau^o\alpha_{pm})[\alpha^o_{sb}u(q^o) - c(q^o) - \gamma_p] + \frac{\partial \alpha^o_{sb}}{\partial \sigma^o}u(q^o)[\alpha_{pm}\tau + \sigma^o(1-\tau\alpha_{pm})] = 0 \tag{2.70}$$

where

$$\frac{\partial \alpha_{sb}^{o}}{\partial \tau^{o}} = -\frac{N_c N_p \alpha_{pm}(1-\sigma^o)}{[N_c + N_p\sigma^o(1-\alpha_{pm}\tau^o) + N_m\alpha_{mp}\tau^o]^2} \tag{2.71}$$

$$\partial \alpha_{sb}^{o} = -\frac{N_c N_p (1-\alpha_{pm}\tau^o)}{[N_c + N_p\sigma^o(1-\alpha_{pm}\tau^o) + N_m\alpha_{mp}\tau^o]^2} \tag{2.72}$$

■

Extension of giving producers another chance to trade with a consumer after trading with a middleman

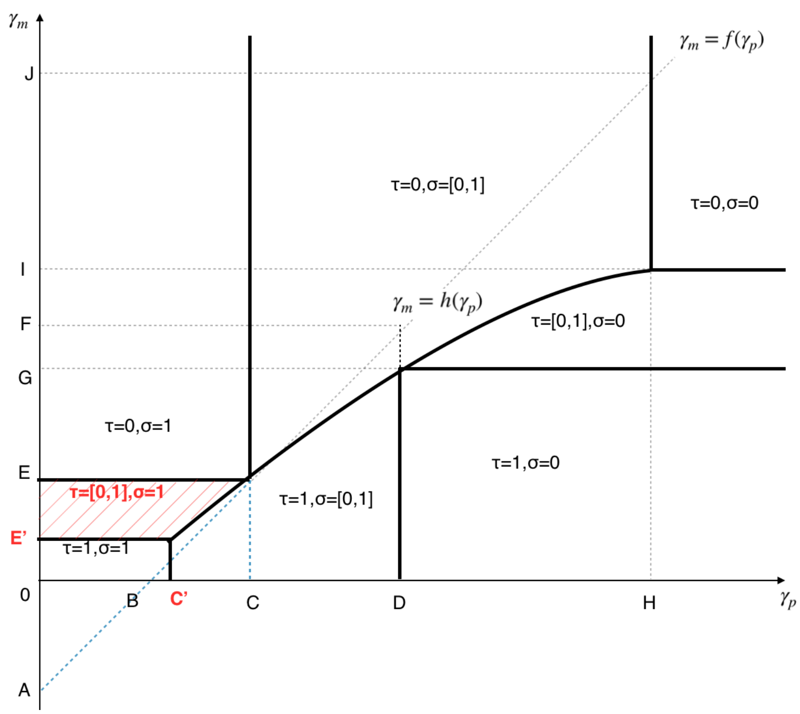


Figure 2.8: Extension: Equilibrium When $\theta_{pc} > \theta_{mc}$

Suppose we relax the assumption that producers are not eligible to produce again for retail market if they have traded with a middleman and allow they to produce again for consumers. The equilibrium set is as shown in Figure 2.8 for the cases when $\theta_{pc} > \theta_{mc}$.

Here are some interesting observations. The equilibrium set is quite similar to that in the baseline model qualitatively. However there are several differences to notice: first, there emerges a new regime with $(\tau, \sigma) = ([0,1], 1)$, as shown in the red shaded area, while in baseline model this is a regime with a zero set of parameters. Second, compared with Figure 2.4 in the baseline model, near origin $\gamma_p = \gamma_m = 0$ now the equilibrium is $(\tau, \sigma) = (1, 1)$ while in the baseline model it is $(0, 1)$. Third, the equilibrium regime with $(\tau, \sigma) = (1, [0, 1])$ is extended such that when $\gamma_p \in [C', C]$, σ is now a mixed strategy instead of 1 implying some but not all producers participate. Last, since producers can produce again, to support an equilibrium in which producers choose to produce again after trading with middlemen, the decision rule for $\tau$is now only determined by the range of γ_m without any need to be compared with γ_p. In the baseline model, this is not the case because once produce choose to trade with middlemen he can avoid the search cost γ_p in retail market. However if a producer choose to produce for retail market when he is given the chance, γ_p will be paid no matter he trades or not with a middlemen therefore γ_p is not considered when evaluating the total surplus of a producer-middleman meeting. All these differences are related to economics of giving producers the chance to producer again.

Proof of equilibrium in an economy with only producers and no middlemen in Section 2.6.1

Proof. To start with the equilibrium in this economy, consider a pure strategy equilibrium that $\hat{\sigma} = 1$, which implies $\hat{n_s} = N_p$. This equilibrium is supported by $\gamma \leq \hat{\alpha_{sb}}\theta_{pc}u(\hat{q_{pc}}) - c(\hat{q_{pc}})$, in which $\hat{\alpha_{sb}} = (\frac{N_c}{N_p})^{\alpha}$, and $\hat{q_{pc}} = q_{pc}(\hat{\alpha_{sb}})$, $y_{cp} = \theta_{pc}u[q_{pc}(\hat{\alpha_{sb}})]$. It is easy to show that when $\hat{\alpha_{sb}} = (\frac{N_c}{N_p})^{\alpha}$, $\hat{\alpha_{sb}}\theta_{pc}u(\hat{q_{pc}}) - c(\hat{q_{pc}})$ equals to the value of C in 2.29. Now consider the other pure strategy equilibrium with $\hat{\sigma} = 0$. This equilibrium is supported if $\gamma \geq \hat{\alpha_{sb}}\theta_{pc}u(\hat{q_{pc}}) - c(\hat{q_{pc}})$, in which $\hat{\alpha_{sb}} = \bar{\alpha_{sb}}$ and $\hat{q_{pc}} = q_{pc}(\hat{\alpha_{sb}})$, $y_{cp} = \theta_{pc}u[q_{pc}(\hat{\alpha_{sb}})]$. When $\hat{\alpha_{sb}} = \bar{\alpha_{sb}}$, $\hat{\alpha_{sb}}\theta_{pc}u(\hat{q_{pc}}) - c(\hat{q_{pc}})$ equals to the value of H in 2.27. Lastly, consider the mixed strategy equilbrium with $\hat{\sigma} = [0, 1]$. This equilibrium is supported if $\gamma = \hat{\alpha_{sb}}\theta_{pc}u(\hat{q_{pc}}) - c(\hat{q_{pc}})$, in which $\hat{\alpha_{sb}} = (\frac{N_c}{\hat{\sigma}N_p})^{\alpha}$ and $\hat{q_{pc}} = q_{pc}(\hat{\alpha_{sb}})$, $y_{cp} = \theta_{pc}u[q_{pc}(\hat{\alpha_{sb}})]$. Therefore

$$\hat{\sigma} = \begin{cases} 1 \\ [0,1] \\ 0 \end{cases} \quad if \ \gamma \in \begin{cases} [0, C] \\ [C, H] \\ [H, +\infty) \end{cases} \tag{2.73}$$

in which C and H. ■

Proof of Lemma 2.11

Proof. Total welfare in a non-intermediated economy is given by $\hat{W}(\theta_{pc}, \gamma)$, then

$$\begin{aligned} \frac{\partial \hat{W}(\theta_{pc}, \gamma)}{\partial q_{pc}} &= \frac{\partial\{(N_c)^{\alpha}(\hat{n_s})^{1-\alpha}u(\hat{q_{pc}}) - \hat{n_s}[c(\hat{q_{pc}}) + \gamma]\}}{\partial q_{pc}} \\ &= (N_c)^{\alpha}(\hat{n_s})^{1-\alpha}u'(\hat{q_{pc}}) - \hat{n_s}c'(\hat{q_{pc}}) \end{aligned}$$

we know that $\hat{\alpha_{sb}}\theta_{pc}u'(\hat{q_{pc}}) = c'(\hat{q_{pc}})$ and $\hat{\alpha_{sb}} = (\frac{N_c}{\hat{n_s}})^{1-\alpha}$, so

$$\begin{aligned} \frac{\partial \hat{W}(\theta_{pc}, \gamma)}{\partial q_{pc}} &= (N_c)^{\alpha}(\hat{n_s})^{1-\alpha}u'(\hat{q_{pc}}) - \hat{n_s}c'(\hat{q_{pc}}) \\ &= (N_c)^{\alpha}(\hat{n_s})^{1-\alpha}(1 - \theta_{pc})u'(\hat{q_{pc}}) \geq 0 \end{aligned}$$

Also

$$\frac{\partial \hat{W}_c(\theta_{pc},\gamma)}{\partial q_{pc}} \quad = \quad (\frac{N_c}{\hat{n_s}})^{\alpha-1}(1-\theta_{pc})u'(\hat{q_{pc}}) \geq 0$$

■

Proof of Lemma 2.12

Proof. Total welfare in an intermediated economy is given by $W(\theta_{mc},\theta_{pc},\gamma)$, similar as the proof for Lemma 2.11

$$\frac{\partial W(\theta_{mc},\theta_{pc},\gamma)}{\partial q_{pc}} = (N_c)^{1-\alpha}(n_s)^{\alpha}\frac{n_p^R}{n_s}(1-\theta_{pc})u'(q_{pc}) \geq 0$$

$$\frac{\partial W(\theta_{mc},\theta_{pc},\gamma)}{\partial q_{pm}} = (N_c)^{1-\alpha}(n_s)^{\alpha}\frac{n_m^R}{n_s}(1-\theta_{mc})u'(q_{pm}) \geq 0$$

$$\frac{\partial W_c(\theta_{mc},\theta_{pc},\gamma}{\partial q_{pc}}) = (\frac{N_c}{n_s})^{\alpha-1}\frac{n_p^R}{n_s}(1-\theta_{pc})u'(q_{pc}) \geq 0$$

$$\frac{\partial W_c(\theta_{mc},\theta_{pc},\gamma)}{\partial q_{pm}} = (\frac{N_c}{n_s})^{\alpha-1}\frac{n_m^R}{n_s}(1-\theta_{mc})u'(q_{pm}) \geq 0$$

■

Proof of Lemma 2.13

Proof. For net welfare created per trade, $\alpha_{sb}u(q_{ij})-c(q_{ij})-\gamma$, for $ij \in \{pc, pm\}$

$$\begin{aligned}\frac{\partial[\alpha_{sb}u(q_{ij})-c(q_{ij})-\gamma]}{\partial q_{ij}} &= \frac{\partial[\alpha_{sb}u(q_{ij})-c(q_{ij})]}{\partial q_{ij}} \\ &= \alpha_{sb}(1-\theta_{ij})u'(q_{ij}) \geq 0\end{aligned}$$

■

Proof of results in Subsection 2.6.3

Proof. Consider a comparison of $\hat{\sigma} = 1$ in the non-intermediated economy and $(\tau, \sigma) = (1, 1)$ in the intermediated. For extensive margin, since the number of sellers are the same in both economy $\hat{n_s} = n_s = N_p$, then $\hat{\alpha_{sb}} = \alpha_{sb}$. For the intensive margin, by $\hat{q_{pc}}$, q_{pc}, q_{pm} are given by

$$c'(\hat{q_{pc}}) = \hat{\alpha_{sb}}\theta_{pc}u'(\hat{q_{pc}}) \tag{2.74}$$

$$c'(q_{pc}) = \alpha_{sb}\theta_{pc}u'(q_{pc}) \tag{2.75}$$

$$c'(q_{pm}) = \alpha_{sb}\theta_{mc}u'(q_{pm}) \tag{2.76}$$

Given $\hat{\alpha_{sb}} = \alpha_{sb}$, and $\theta_{pc} < \theta_{mc}$, by Proposition 2.2, we have $\hat{q_{pc}} = q_{pc} < q_{pm}$.

Now consider a comparison between $\hat{\sigma} = [0, 1]$ in the non-intermediated economy and $(\tau, \sigma) = (1, [0, 1])$ in the intermediated. For extensive margin, $\hat{n_s} = \hat{\sigma}N_p$, and $n_s = N_p\sigma(1 - \alpha_{pm}) + N_m\alpha_{mp}$ in which the first term is the number of producers and the second the number of middlemen. For the intensive margin, in order to compare $\hat{q_{pc}}, q_{pc}$ and q_{pm}, we need to compare $\hat{\alpha_{sb}}$ and α_{sb}. Recall the equilibrium conditions on γ to support $\hat{\sigma} = [0, 1]$, and $(\tau, \sigma) = (1, [0, 1])$,

$$\hat{\alpha_{sb}}\theta_{pc}u(\hat{q_{pc}}) - c(\hat{q_{pc}}) = \gamma \tag{2.77}$$

$$\alpha_{sb}\theta_{pc}u(q_{pc}) - c(q_{pc}) = \gamma \tag{2.78}$$

$$\alpha_{sb}\theta_{mc}u(q_{pm}) - c(q_{pm}) \geq \gamma \tag{2.79}$$

Using 2.74 though 2.76, by 2.77 through 2.79, we know $\hat{\alpha_{sb}} = \alpha_{sb}$, and $\hat{q_{pc}} = q_{pc} < q_{pm}$. Also by $\hat{\alpha_{sb}} = \alpha_{sb}$, we have $\hat{n_s} = n_s$, $\hat{\sigma} > \sigma$.

Now consider a comparison between $\hat{\sigma} = [0,1]$ in the non-intermediated economy and $(\tau, \sigma) = (1, 0)$ in the intermediated. For the extensive margin, $\hat{n_s} = \hat{\sigma} N_p$, and $n_s = N_m \alpha_{mp}$, now the intermediated economy only have middlemen as sellers in the retail market. For the intensive margin, recall the equilibrium conditions on γ to support $\hat{\sigma} = [0,1]$, and $(\tau, \sigma) = (1, 0)$,

$$\hat{\alpha_{sb}} \theta_{pc} u(\hat{q_{pc}}) - c(\hat{q_{pc}}) = \gamma \tag{2.80}$$

$$\alpha_{sb} \theta_{pc} u(q_{pc}) - c(q_{pc}) \leq \gamma \tag{2.81}$$

$$\alpha_{sb} \theta_{mc} u(q_{pm}) - c(q_{pm}) \geq \gamma \tag{2.82}$$

Again using 2.74 through 2.76, by 2.80 and 2.81 we know $\hat{\alpha_{sb}} \geq \alpha_{sb}$ and by 2.80 and 2.82 we have $\hat{\alpha_{sb}} \theta_{pc} \leq \alpha_{sb} \theta_{mc}$. Since $\frac{\partial q_{ij}}{\partial(\alpha_{sb}\theta_{ij})}$ by Proposition 2.2, we have $q_{pc} \leq \hat{q_{pc}} \leq q_{pm}$.

Now consider a comparison between $\hat{\sigma} = [0,1]$ in the non-intermediated economy and $(\tau, \sigma) = ([0,1], 0)$ in the intermediated. For the extensive margin, $\hat{n_s} = \hat{\sigma} N_p$, and $n_s = \sigma N_m \alpha_{mp}$. For the intensive margin, by

$$\hat{\alpha_{sb}} \theta_{pc} u(\hat{q_{pc}}) - c(\hat{q_{pc}}) = \gamma \tag{2.83}$$

$$\alpha_{sb} \theta_{pc} u(q_{pc}) - c(q_{pc}) \leq \gamma \tag{2.84}$$

$$\alpha_{sb} \theta_{mc} u(q_{pm}) - c(q_{pm}) = \gamma \tag{2.85}$$

using 2.74 through 2.76, $\hat{\alpha_{sb}} \geq \alpha_{sb}$ from 2.83 and 2.84, $\hat{\alpha_{sb}} \theta_{pc} = \alpha_{sb} \theta_{mc}$ from 2.83 and 2.85, So $q_{pc} \leq \hat{q_{pc}} = q_{pm}$. Also by $\hat{\alpha_{sb}} \geq \alpha_{sb}$, we know $\hat{n_s} \leq n_s$. ■

Chapter 3

Money versus Middlemen

3.1 Introduction

This chapter studies the relationship between money and middlemen, two of market institutions that facilitate trades in the presence of different market frictions: money facilitates trades if commitment is not perfect such that repayment cannot be enforced and middlemen facilitate trades when they have advantage in searching speed, inventory holdings, information, etc.. In the seminar work of Rubinstein and Wolinsky (1987), middlemen are faster than producers at finding a consumer such that the presence of middlemen not only provide a high meeting rates but also improves the efficiency of equilibrium. There are a large size of literature as shown in Chapter 2 that study the role of intermediation but none has studies the role of middlemen with an advantage over producers at enforcing repayment from consumers. To this end, a framework with intermediation like Rubinstein and Wolinsky is a suitable environment to embed the idea. Also, to study intermediation and monetary exchanges, a framework with money is obviously needed. Since the study requires explicit description of market frictions and monetary exchange, to maintain results to be tractable, Lagos and Wright (2005) with alternating decentralized and centralized markets provides a promising framework. In what follows, Section 3.2 describes the environment, Section 3.3 characterizes equilibrium results in baseline model of a pure monetary economy , Section 3.4 introduces middlemen with the advantage of enforcing repayment to the baseline model, Section 3.5 analyzes the impact of inflation on middlemen's participation, Section 3.6 studies the effect of middlemen on consumers' welfare and Section 3.7 concludes.

3.2 Environment

3.2.1 Setup

Time is discrete and continues forever. In each period, three sub-markets open sequentially: the first two are frictional decentralized markets named wholesale market (WM) and retail market(RM) and the last one a frictionless centralized market named Arrow Debreu market(AD). There is a $[0,\ 1]$ continuum of agents in three types, C, P and M for consumer, producers and middlemen. The population of each type is fixed and denoted as N_i, $i \in \{C,\ P,\ M\}$, and $N_C + N_P + N_M = 1$. An indivisible good $q \in \{0, 1\}$ is traded in WM and RM. It can only be produced by P at cost c and enjoyed by C with utility u. Production of the good occurs upon meeting and producers can only produce once within a period. While M cannot produce they can purchase from P in WM and trade with C in RM. This good is depreciated at rate ρ in AD if it is not sold by the end of RM. The discount rate across periods is denoted as $\beta \in (0,1)$. It is assumed that $u > c > \rho$ such that P would produce upon meeting C and not if meet no one.

In WM, M and P choose whether to participate to trade. If they do they randomly meet in a bilateral way. After trading, the middleman is eligible to trade with C in RM and the producer is out of the market given he/she can only produce once within a period. The medium of exchange for trade between M and P depends on the credit condition in WM: if credit condition is poor in that sense that P cannot enforce repayment from M, money serves as the only payment instrument; if credit condition is perfect in the sense that P can enforce repayment from M, credit is chosen over money as payment method as holding money bears inflation cost.

In RM, M with inventories and P who have not traded with M in WM can potentially trade with C bilaterally following a random meeting process. In WM, P are poor at enforcing repayment from C therefore money is the only medium of exchange for C to trade with P. M, on the other hand, has an advantage over P at enforcing repayment from C thus credit is allowed for trade between C and M.

The preference in AD is the same as in Lagos and Wright (2005) which depends on the consumption of generic good X and labor supply l and all agents can work to rebalance their credit and money holding positions.

In terms of random bilateral meeting in WM and RM, it follows a constant-return-to-scale meeting technology and the number of meetings is given by $M(n_i, n_j) = n_i^{\alpha} n_j^{1-\alpha}$ where n_i and n_j are the populations of the two sides participating in exchanges. Let α_{ij} denote the rate at which a type i meets a type j, $\alpha_{ji} = \alpha(\frac{n_i}{n_j}) = \frac{n_i^{\alpha} n_j^{1-\alpha}}{n_j} = (\frac{n_i}{n_j})^{\alpha}$. In WM, the measure of active M and P are denoted as n_m^w and n_p^w, and meeting rates are $\alpha_{pm} = \alpha(\frac{n_m^w}{n_p^w})$ and $\alpha_{mp} = \frac{\alpha(\frac{n_m^w}{n_p^w})}{\frac{n_m^w}{n_p^w}}$. In RM, the measure of sellers is $n_s = n_m^R + n_p^R$ where n_m^R and n_p^R are the numbers M and P participating in RM. Therefore the meeting rates are $\alpha_{mc} = \alpha_{pc} = \alpha(\frac{n_c}{n_s})$, $\alpha_{cm} = \alpha_{cp} = \frac{\alpha(\frac{n_c}{n_s})}{\frac{n_c}{n_s}}$.

The terms of trade in both WM and RM are determined by Kalai bargaining. Let θ_{ij} denote the bargaining power of type i when trading with type j, Σ_{ij} the total trading surplus when a type i agent sells goods to type j, m_{ij} the nominal monetary transfer from type i to type j and d_{ij} the credit commitment made by type i with type j.

3.2.2 Decisions

There are decisions for all three types of agents: C's decision of whether to bring money to the market, P's decision of whether to trade with M and M's decision of whether to trades with P and whether to participate in RM. Obviously, it is irrational of M if choosing to buy from P in WM but not participating in RM, therefore there is only one decision for M, that is, whether to participate in WM. Let σ be C's money holding decision, τ_p be P's WM participation decision and τ_m be M's participation decision. All decisions are made the the beginning of each period and agents are committed to their decisions. The decision rules will be discussed in the following sections of equilibium characterization.

3.3 Baseline Model

The baseline model studies C's money holding decision when P cannot enforce repayment from C such that the credit condition is poor in RM for PC trades and M do not exist in the market. In this case P are the only potential sellers and money is the only medium of exchange as shown in Figure 3.1. There are only two submarkets RM and AD to be considered. Let σ_0 and τ_{p0} be C and P's decision in the baseline model. Let V_i^W, V_i^R, V_i^A be the value functions for type $i \in \{C,\ P\}$ in the three submarkets. Let m_c be the amount of nominal money chosen by C if they choose to trade with P. The unit value of money is ϕ in terms of numeria.

Figure 3.1: Baseline Model

Considering the problem for C, their value function is AD submarket at time t is given by

$$\begin{aligned} V_c^A(m_c) &= \max_{X,l}\{U(X) - l + \beta \max\{V_c^R(\hat{m}_c), V_c^A(0)\}\} \\ s.t. \quad & X = l + \phi[m_c - \hat{m}_c \cdot 1(\hat{m}_c > 0)] + T \\ FOC[X]: \quad & U'(X) = 1 \\ ENV[m_c]: \quad & \frac{\partial V_c^A(m_c)}{\partial m_c} = \phi \end{aligned}$$

where m_c is C's money holding at the beginning of AD in period t, $\hat{m}_c$ is the money holding chosen by C for period $t+1$ and T is the government transfer. As shown in above, $V_c^A(m_c)$ is linear in m_c. The value function for C in RM at period t is

$$\begin{aligned} V_c^R(m_c) &= \alpha_{cp}[u + V_c^A(m_c - m_{cp}) - V_c^A(m_c)] + V_c^A(m_c) \\ s.t. \quad & m_{cp} \le m_c \end{aligned}$$

where the constraint is binding in equilibrium. Also the meeting rate $\alpha_{cp} = \alpha(\frac{N_c}{N_p}) = \alpha(n_{cp})$ where $n_{cp} \equiv \frac{N_c}{N_p}$. Rearranging $V_c^R(m_c)$ we have

$$V_c^R(m_c) = \frac{\alpha(n_{cp})}{n_{cp}}(u - \phi m_c) + V_c^A(m_c)$$

The value functions for P are given by

$$\begin{aligned}
V_p^A(m) &= \max_{X,l}\{U(X) - l + \beta V_p^R(0)\} \\
s.t. \quad & X = l + \phi m + T \\
FOC[X]: \quad & U'(X) = 1 \\
V_p^R(0) &= \alpha_{pc}\sigma_0[-c + V_p^A(m_{cp}) - V_p^A(0)] + V_p^A(0) \\
&= \alpha(n_{cp})\sigma_0(-c + \phi m_c) + V_p^A(0)
\end{aligned}$$

In terms of bargaining solution, notice that given that money is the only medium of exchange and goods are indivisible, P cannot ask for more money than the amount that C have brought to RM neither can they reduce the quantity of goods sold to C as goods are indivisible. As a result, in equilibrium, C would bring just enough money to make P indifferent from trading with C or not, that is, we are at a corner solution such that C get all the surplus from trading in a monetary exchange with P. It is equivalent to the case when $\theta_{cp} = 1$ therefore $\phi m_c = c$.

In terms of decisions, P would always produce upon meeting as long as $u \geq c$.

The decision for C depends on the rule that

$$\sigma_0 = \begin{cases} 1 \\ [0,1] & if V_c^R(m_c) - V_c^A(0) \\ 0 \end{cases} \begin{cases} > (1+r)\phi_{-1}m_c \\ = (1+r)\phi_{-1}m_c \\ < (1+r)\phi_{-1}m_c \end{cases}$$

where $(1+r)\phi_{-1}m_c$ is the cost of holding money and $V_c^R(m_c) - V_c^A(0)$ is the benefit. Rearranging the terms, we have

$$V_c^R(m_c) - V_c^A(0) - (1+r)\phi_{-1}m_c = \alpha_{cp}(u-c) + \phi m_c - (1+r)\phi_{-1}m_c$$

In stationary equilibrium, using Fisher equation and bargaining solution, we have

$$\begin{aligned} & V_c^R(m_c) - V_c^A(0) - (1+r)\phi_{-1}m_c \\ = \; & \alpha_{cp}(u-c) - \phi m_c - (1+r)\phi_{-1}m_c \\ = \; & \alpha_{cp}(u-c) - ic \end{aligned}$$

Therefore

$$\sigma_0 = \begin{cases} 1 \\ [0,1] & if \;\; i - \dfrac{\alpha_{cp}(u-c)}{c} \\ 0 \end{cases} \begin{cases} > 0 \\ = 0 \\ < 0 \end{cases}$$

Let i_0^* denote the cutoff value of i above which $\sigma_0 = 1$, $i_0^* = \frac{\alpha_{cp}(u-c)}{c}$.

3.4 Model 1: Money and Middlemen as Substitutes

In this section, M who have an advantage at enforcing payment from C is introduced so that WM can potential be open as shown in Figure 3.2. This section studies the equilibrium when credit condition is poor for PC trades in RM as in Section 3.3 and credit condition is good for PM trades in WM. In this case, for C they cannot use credit but money to trade with P but can use credit to trade with M if M participate and for M they can use credit to trade with P. Let σ be C's money holding decision, τ_p be P's WM participation decision and τ_m be M's participation decision. Suppose only active M and P participate in WM.

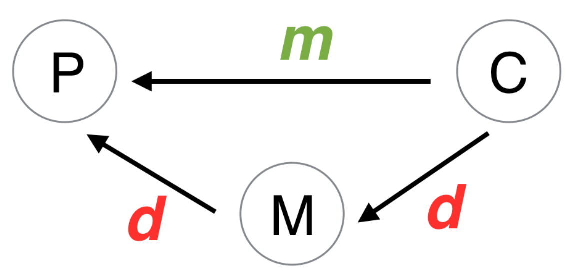


Figure 3.2: Model 1

First consider the value functions for C.

$$\begin{aligned}
V_c^A(d,m) &= \max_{X,l}\{U(X) - l + \beta\max\{V_c^R(0,\hat{m}_c), V_c^A(0,0)\}\} \\
s.t. &\quad X = l + d + \phi[m - \hat{m}_c \cdot 1(\hat{m}_c > 0)] + T \\
FOC[X]: &\quad U'(X) = 1 \\
ENV[m_c]: &\quad \frac{\partial V_c^A(0,m_c)}{\partial m_c} = \phi \\
V_c^R(0,0) &= \alpha_{cm}[u + V_c^A(-d_{cm},0) - V_c^A(0,0)] + V_c^A(0,0) \\
V_c^R(0,m_c) &= \alpha_{cp1}[u + V_c^A(0,m_c - m_{cp}) - V_c^A(0,m_c)] \\
&\quad +\alpha_{cm}[u + V_c^A(-d_{cm},m_c) - V_c^A(0,m_c)] + V_c^A(0,0)
\end{aligned}$$

The value functions for M are given by

$$\begin{aligned}
V_m^A(d,q) &= \max_{X,l}\{U(X) - l + \beta\max\{V_m^w(0,0), V_m^A(0,0)\}\} \\
s.t. &\quad X = l + d + \rho q + T \text{ and } q \in \{0,1\} \\
FOC[X]: &\quad U'(X) = 1 \\
V_m^w(0,0) &= \alpha_{mp}[V_m^R(-d_{mp},1) - V_m^A(0,0)] + V_m^A(0,0) \\
V_m^R(-d_{mp},1) &= \alpha_{mc}[V_m^A(-d_{mp} + d_{cm},0) - V_m^A(-d_{mp},1)] + V_m^A(-d_{mp},1)
\end{aligned}$$

The value functions for P are given by

$$
\begin{aligned}
V_p^A(d,m) &= \max_{X,l}\{U(X) - l + \beta \max\{V_p^w(0,0), V_p^R(0,0)\}\} \\
s.t. &\quad X = l + d + \phi m + T \\
FOC[X]: &\quad U'(X) = 1 \\
V_p^w(0,0) &= \alpha_{pm}[-c + V_p^A(d_{mp},0) - V_p^R(0,0)] + V_m^R(0,0) \\
V_p^R(0,0) &= \alpha_{pc1}\sigma[-c + V_p^A(0,m_{cp}) - V_p^A(0,0)] + V_m^A(0,0)
\end{aligned}
$$

The meeting probability in WM is given by $\alpha_{pm} = \alpha(\frac{\tau_m N_m}{\tau_p N_p})$. It is later proved that $\tau_p = \tau_m \equiv \tau$ therefore $\alpha_{pm} = \alpha(\frac{N_m}{N_p}) = \alpha(n_{mp})$ where n_{mp} is denoted as the value of $\frac{N_m}{Np}$. The number of sellers in RM is denoted as n_s which equals to $N_p(1-\tau\alpha_{pm}) + N_m\tau\alpha_{mp}$. By the identity that $N_p\tau\alpha_{pm} = N_m\tau\alpha_{mp}$, we have $n_s = N_p$. As a result, the meeting rates in RM are given by $\alpha_{pc1} = \alpha_{mc} = \alpha(n_{cp})$, where $n_{cp} = \frac{N_c}{N_p}$, the same as α_{pc} in Section 3.3. The rates at which C meet P is $\alpha_{cp1} = \frac{\alpha(n_{cp})}{n_{cp}}[1-\tau\alpha(n_{mp})] = \alpha_{cp}[1-\tau\alpha(n_{mp})]$ and the rate at which C meets M is $\alpha_{cm} = \frac{\alpha(n_{cp})}{n_{cp}}\tau\alpha(n_{mp}) = \alpha_{cp}\tau\alpha(n_{mp})$.

The total surpluses from trade are $\Sigma_{pc} = u-c$, $\Sigma_{mc} = u-\rho$, $\Sigma_{pm} = \alpha(n_{cp})\theta_{mc}(u-\rho) + \rho - c$. Similar to the argument in Section 3.3, for monetary exchange between PC trade, we have $\phi m_c = c$. For credit exchange between PM and MC, the terms of trade are given by $d_{cm} = \theta_{cm}\rho + \theta_{mc}u$ and $d_{mp} = \theta_{mp}[\alpha(n_{cp})\sigma\theta_{pc}(u-c) + c] + \theta_{pm}[\alpha(n_{cp})\theta_{mc}(u-\rho) + \rho]$.

The decision rule of σ for C is given by

$$\sigma = \begin{cases} 1 \\ [0,1] & if V_c^R(m_c) - V_c^A(0) \\ 0 \end{cases} \begin{cases} > (1+r)\phi_{-1}m_c \\ = (1+r)\phi_{-1}m_c \\ < (1+r)\phi_{-1}m_c \end{cases}$$

Rearranging the terms, in stationary equilibrium, we have

$$V_c^R(m_c) - V_c^A(0) - (1+r)\phi_{-1}m_c = \alpha_{cp}[1 - \tau\alpha_{pm}](u-c) - ic$$

Therefore

$$\sigma = \begin{cases} 1 \\ [0,1] & if \alpha_{cp}[1 - \tau\alpha_{pm}](u-c) - ic \\ 0 \end{cases} \begin{cases} > 0 \\ = 0 \\ < 0 \end{cases}$$

Lemma 3.1. *$\tau_p = \tau_m$ and $\tau_p = \tau_m = 1$ iff $\Sigma_{pm} > 0$.*

Let $\tau \equiv \tau_p = \tau_m$ and the decision rule for τ is given by

$$\tau = \begin{cases} 1 \\ [0,1] & if \alpha_{pc}\theta_{mc}(u-\rho) + \rho - c \\ 0 \end{cases} \begin{cases} > 0 \\ = 0 \\ < 0 \end{cases}$$

Definition 3.1. *The equilibrium is given by $\{V_i^w,\ V_i^R,\ V_i^A,\ \sigma,\ \tau,\ m_c,\ d_{cm}\}$ where $i \in \{C,\ P,\ M\}$ such that: (1) V_i^w, V_i^R, V_i^A satisfy the value functions, $i \in \{C,\ P,\ M\}$, (2) σ, τ satisfy the decision rules and (3) m_c, d_{cm} are given by Kalai bargaining* *solutions.*

There are four classes of equilibria in terms of decisions τ and σ described graphically in Figure 3.3.

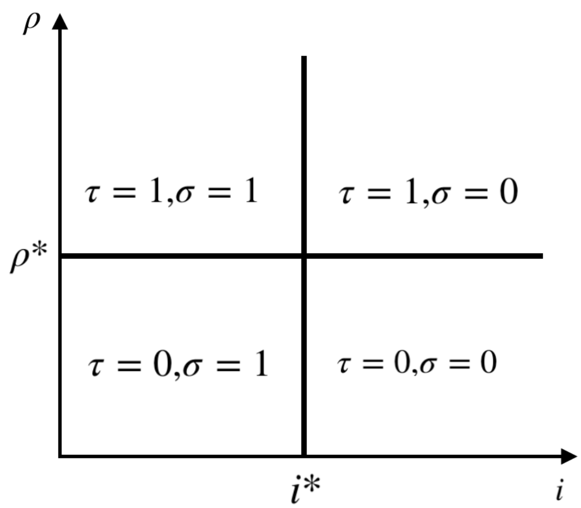


Figure 3.3: Equilibrium of Model 1

In Figure 3.3, $i^* = \frac{\alpha_{cp}[1-\alpha_{pm}](u-c)}{c}$ is the cutoff value of i above which C do not bring money to the market therefore only trade with M upon meeting. Intuitively, inflation cost of holding money is too high such that the benefit of holding money to trade with P cannot compensate the cost. Given i, $\rho^* = \frac{c-\alpha_{mc}\theta_{mc}u}{1-\alpha_{mc}\theta_{mc}}$ is the cutoff of ρ above which M would participate, that is, the return of the good must be high enough such that the cost of failing to meet C can be compensated for M.

Now consider the case when $\rho > \rho^*$ such that M always participate. Compared with the baseline model, the result in this section shows that money and middlemen are substitutes given credit condition is poor in retail market for PC trade and middlemen have advantage at enforcing repayment from C. As shown in Figure 3.4, $i^* < i_0^*$ implying that the cutoff value for i below which C choose to hold money is lower with M in the market than without. When $i \in [i^*, i_0^*]$, $\sigma_0 = 1$ while $\sigma = 0$ indicates that money and middlemen are substitutes in the sense that C stop to hold money given the participation and existence of M with repayment enforcing power.

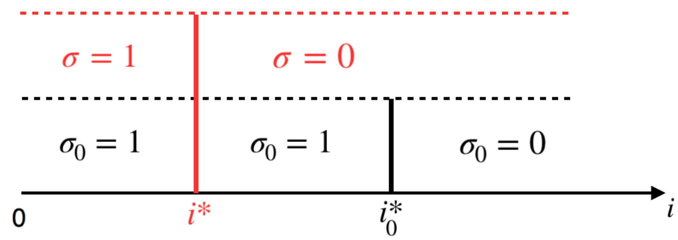


Figure 3.4: Comparison of C's Money Holding Decision w.r.t. i

Proposition 3.1. *Money and middlemen are substitutes when credit is perfect in PM and MC trade but not in PC trade.*

Money and middlemen as two market instruments that facilitate trades serve as substitutes in an environment with search and payment frictions since M who are good at enforcing repayment provide C another channel to trade without holding money ahead of meetings so that inflation cost associated with failing to meet is avoided. Moreover, M enlarge the equilibrium set over what obtained when inflation rate is high since it prevents C from carry money to trade in RM however the existence of M makes it possible for C to trade in RM without money.

3.5 Model 2: Middlemen and Money as Complements

In this section, consider the case when the setup is the same as in Section 3.4 except credit condition is poor in WM such that M have to use money instead of credit to buy goods from P as shown in Figure 3.5. Intuitively, different from Section 3.4, the participation decision of M now takes into account the cost of holding money and inflation discourages M's entry if it is too high.

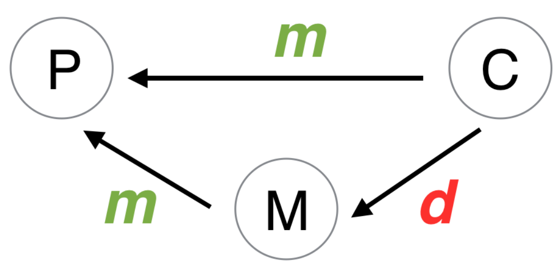


Figure 3.5: Model 2

C's problem is the same as in Section 3.4. It is proved in Appendix that the money holding decision $\hat{\sigma}$ is the same as in Section 3.4.

Consider M's problem, the value functions are given by

$$\begin{aligned} V_m^A(d, m_m, q) &= \max_{X,l}\{U(X) - l + \beta \max\{V_m^W(0, \hat{m_m}, 0), V_m^A(0,0,0)\}\} \\ s.t. \quad & X = l + \phi[m_m - \hat{m_m} \cdot 1(\hat{m_m} > 0)] + \rho q + T,\ q \in \{0,1\} \\ V_m^w(0, m_m, 0) &= \hat{\alpha_{mp}}[V_m^R(0, m_m - m_{mp}, 1) - V_m^A(0, m_m, 0)] + V_m^A(0, m_m, 0) \\ V_m^R(0,0,1) &= \hat{\alpha_{mc}}[V_m^A(d_{cm}, 0, 0) - V_m^A(0,0,1)] + V_m^A(0,0,1) \end{aligned}$$

where m_m is M's money holding and the constraint $m_m \geq m_{mp}$ is binding.

Since M cannot trade in WM if they do not hold money, for M the participation decision is the same as money holding decision. Let this decision be denoted as $\hat{\tau}$ and the decision rule is given by

$$\hat{\tau} = \begin{cases} 1 \\ [0,1] & if V_m^R(0, m_m 0) - V_m^A(0,0,0) \\ 0 \end{cases} \begin{cases} > (1+r)\phi_{-1}m_m \\ = (1+r)\phi_{-1}m_m \\ < (1+r)\phi_{-1}m_m \end{cases}$$

Rearranging the terms we have

$$V_m^R(0, m_m 0) - V_m^A(0,0,0) - (1+r)\phi_{-1}m_m = \hat{\alpha_{mp}}\Sigma_{pm} - i\phi m_m$$

Now consider P's value functions, they are given by

$$\begin{aligned} V_p^A(m) &= \max_{X,l}\{U(X) - l + \beta \max\{V_p^W(0), V_p^R(0)\}\} \\ s.t. &\quad X = l + \phi m + T \\ V_p^W(0) &= \hat{\alpha_{pm}}[-c + V_p^W(m_m) - V_p^R(0)] + V_p^R(0) \\ V_p^R(0) &= \hat{\alpha_{pc}}\hat{\sigma}[-c + V_p^A(m_c) - V_p^A(0)] + V_p^A(0) \end{aligned}$$

P always would trade as long as M or C are willing to trade with P.

Total surplus from trade are given by $\Sigma_{pc} = u - c$, $\Sigma_{mc} = u - \rho$ and $\Sigma_{pm} = \alpha_{mc}\theta_{mc}(u-\rho) + \rho - c$. Similarly, M just bring enough money to WM such that P are indifferent between trading with M and not. Therefore $\phi m_m = \phi m_c = c$ and $d_{cm} = \theta_{cm}\rho + \theta_{mc}u$.

Similar to Section 3.4, there are four classes of equilibrium in terms of M and C's decisions in $i-\rho$ space as shown in Figure 3.6. But different from Section 3.4, the cutoff value of ρ is higher. For example, to support $\hat{\tau}=1$, $\hat{\sigma}=0$, by the decision rules above, we need

$$\begin{cases} \rho \geq \frac{c-(1-\frac{i}{\alpha_{mp}\theta_{mp}+i\theta_{mp}})\alpha_{mc}\theta_{mc}u}{(1-\frac{i}{\alpha_{mp}\theta_{mp}+i\theta_{mp}})(1-\alpha_{mc}\theta_{mc})} \\ i \geq \frac{\alpha_{cp}(1-\alpha_{pm})\theta_{cp}(u-c)}{\theta_{cp}c+\theta_{pc}u} \end{cases}$$

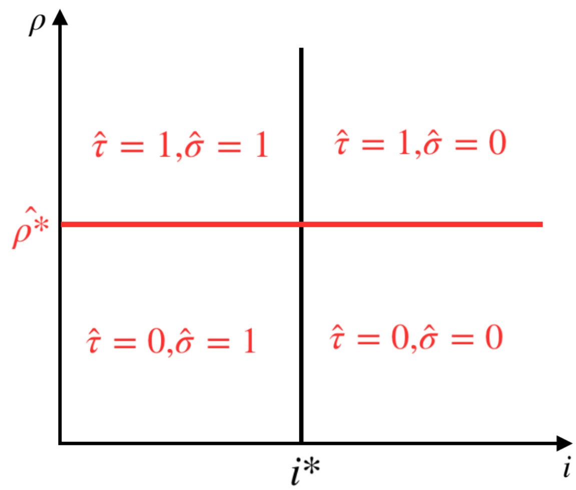


Figure 3.6: Equilibrium of Model 2

Let $\hat{\rho^*} \equiv \frac{c-\alpha_{mc}\theta_{mc}u+\frac{ic}{\alpha_{mp}}}{1-\alpha_{mc}\theta_{mc}}$. Compared with the cutoff value for ρ in Section 3.4, $\rho^* \leq \hat{\rho^*}$ for $i \geq 0$. It implies that when middlemen cannot use credit to trade with P, the good depreciation rate must be higher than in the case when credit is viable for PM trade such that M are willing to participate. Moreover, it is proved in Appendix that $\frac{\partial \hat{\rho^*}}{\partial i} > 0$, this result shows that money and middlemen are complements in the sense that inflation discourages M's participation if credit condition is poor in WM such that money is the only medium of exchange and M have to bear inflation cost to participate.

Proposition 3.2. *Money and middlemen are complements if middlemen have to use money as the only medium of exchange in trades with P in WM.*

3.6 Welfare Analysis

In this section, the effect on consumers' welfare of the participation of middlemen is studied. In order to do so, we compare the welfare of C in the baseline model and Model 1, denoted as W_c^0 and W_c, with respect to inflation rate as shown in Figure 3.4. In Model 1, we focus on the case when ρ is not too small such that M are always willing to participate, i.e. $\tau = 1$. In the following discussion, for a given inflation rate, C's welfare is compared to see whether C are better off with M than without in equilibrium.

For $i > i_0^*$, $\sigma_0 = 0$ and $\sigma = 0$. That is, C do not bring money regardless of the existence of middlemen with repayment enforcing power. In the baseline model, as C do not bring money then there is no trade, $W_c^0 = 0$. In Model 1, $W_c = \alpha_{cp}\tau\alpha_{pm}\theta_{cm}(u-\rho) > 0$. Therefore when inflation rate is highe, C are better of with the participation of M.

For $i < i^*$, $\sigma_0 = 1$ and $\sigma = 1$. That is, C bring money no matter M participate or not. In the baseline model, $W_c^0 = \alpha_{cp}(u-c) - ic$. In Model 1, $W_c = \alpha_{cm}\theta_{cm}(u-\rho) + \alpha_{cp1}(u-c) - ic$. Note that $\alpha_{cm} = \alpha_{cp}\alpha_{pm}$ and $\alpha_{cp1} = \alpha_{cp}(1-\alpha_{pm})$. We have

$$\begin{aligned} W_c - W_c^0 &= \alpha_{cp}\alpha_{pm}\theta_{cm}(u-\rho) + \alpha_{cp}[1-\alpha_{pm}](u-c) - \alpha_{cp}(u-c) \\ &= \alpha_{cp}\alpha_{pm}[\theta_{cm}(u-\rho) - (u-c)] \end{aligned}$$

By the decision rule discussed in Section 3.4, to support $\tau = 1$ and $\sigma = 1$, we have

$\alpha_{mc}\theta_{mc}(u-\rho)+\rho-c \geq 0$ by $\Sigma_{pm} \geq 0$. That is,

$$\begin{aligned} (1-\theta_{cm})(u-\rho) &\geq \frac{c-\rho}{\alpha_{mc}} \\ -u+\rho+\theta_{cm}(u-\rho) &\leq -\frac{c-\rho}{\alpha_{pc}} \\ \theta_{cm}(u-\rho)-(u-c) &\leq (1-\frac{1}{\alpha_{mc}})(c-\rho) \leq 0 \end{aligned}$$

where $1-\frac{1}{\alpha_{mc}} \leq 0$ and $c-\rho > 0$. Therefore $W_c \leq W_c^0$. When inflation is low, C are worse off with the participation of M.

For $i \in [i^*, i_0^*]$, $\sigma_0 = 1$ and $\sigma = 0$. That is, C only bring money if M do not participate. In the baseline model, $W_c^0 = \alpha_{cp}\theta_{cp}(u-c) - i[\theta_{cp}c + \theta_{pc}u]$. In Model 1, $W_c = \alpha_{cm}\theta_{cm}(u-\rho) = \alpha_{cp}\alpha_{pm}\theta_{cm}(u-\rho)$. Comparing the welfare levels,

$$\begin{aligned} W_c - W_c^0 &= \alpha_{cp}\alpha_{pm}\theta_{cm}(u-\rho) - [\alpha_{cp}(u-c) - ic] \\ &= \alpha_{cp}\alpha_{pm}\theta_{cm}(u-\rho) - \{\alpha_{cp}[\alpha_{pm} + (1-\alpha_{pm})](u-c) - ic\} \\ &= \alpha_{cp}\alpha_{pm}[\theta_{cm}(u-\rho) - (u-c)] - [\alpha_{cp}(1-\alpha_{pm})\theta_{cp}(u-c) - ic] \end{aligned}$$

where $\theta_{cm}(u-\rho)-(u-c) \leq 0$ and $\alpha_{cp}(1-\alpha_{pm})(u-c) - ic \leq 0$ by the decision rules for $\tau = 1$, $\sigma = 0$. The welfare effect on C of M's participation is not determined: if $i \leq \frac{\alpha_{cp}(u-c)-\alpha_{cp}\alpha_{pm}\theta_{cm}(u-\rho)}{c}$, then $W_c \geq W_c^0$ and C are better off with M participating in the market. Also $W_c - W_c^0$ is increasing in i, implying that the higher inflation is, the more benefit is rendered by not holding money if M are actively provide repayment option in RM. So when inflation rate is neither too high nor too low, C can be better off or worse off with M's participation. Besides, the higher inflation can potential make the choice of not holding money if M participate better.

To explain the welfare effect on C discussed above, we should notice that, the indivisibility of goods results in full bargaining power for C in monetary exchange with P while C do not necessarily have full bargaining power in trades with M. Therefore while C suffer from inflation cost in monetary trade they also benefit from having full bargaining power in such trades. As a result, depending on the level of inflation, the loss can be overwhelmed by the benefit of monetary trade such that the introduction of M can make C worse off.

3.7 Conclusion

This chapter studies the relationship between money and middlemen by building a model in the spirit of Rubinstein and Wolinsky(1987) with the framework of Lagos and Wright(2005). I focus on the type of middlemen who have advantage at enforcing repayment such that middlemen can provides credit to consumers while producers without such advantage only accepts money from consumers. When credit is perfect for trades between middlemen and producers, money and middlemen are substitutes in the sense that consumers can avoid holding money and inflation cost by trading with participating middlemen. This substitution relationship encourages repayment and the participation of middlemen when inflation rate is high. However, when credit is not perfect for trades between middlemen and producers in the sense that middlemen themselves must carry money to participate, money and middlemen become complements since inflation cost can discourage intermediation and the use of repayment. As a result, depending on the payment frictions in upstream wholesale market and downstream retail market, the two relationships between money and middlemen interact to determine the level of intermediation when middlemen have advantage at enforcing repayment from consumers. Moreover, the introduction of middlemen is not necessarily beneficial from consumers' perspective because indivisibility of goods results in full bargaining power for consumers in monetary trades with producers but partial bargaining power in trades with middlemen. The tradeoff between getting away with inflation cost and suffering from a lower bargaining power determines the effect of middlemen on consumers' welfare: consumers are better off with middlemen when inflation is high and worse off when inflation is low.

3.8 Appendix

C's problem in Section 3.5

Proof. The decision rule of $\hat{\sigma}$ for C is given by

$$\hat{\sigma} = \begin{cases} 1 \\ [0,1] \quad if V_c^R(m_c) - V_c^A(0) \\ 0 \end{cases} \begin{cases} > (1+r)\phi_{-1}m_c \\ = (1+r)\phi_{-1}m_c \\ < (1+r)\phi_{-1}m_c \end{cases}$$

where

$$V_c^R(m_c) - V_c^A(0) - (1+r)\phi_{-1}m_c = \alpha_{cp}[1 - \tau\alpha_{pm}](u-c) - ic$$

Therefore the decision rule for $\hat{\sigma}$ is the same as for σ in Section 3.4. ■

Proof of $\frac{\partial \hat{\rho}^*}{\partial i} > 0$ in Section 3.5

Proof. $\frac{\partial \hat{\rho}^*}{\partial i} = \frac{c}{\alpha_{mp}(1-\alpha_{mc}\theta_{mc})} > 0$ ■

Chapter 4

Inflation, Investment and Unemployment: A New Monetarist Framework

4.1 Introduction

Capital accumulation and market participation of firms have been studied by a sizable literature both empirically and theoretically. According to the law of one price, the view held by the majority is that firms, as the supply side in the product market, supply more when price is high and thus accumulate more on capital investment to support production. This point of view is admittedly convincing in some markets however cannot explain why there exists shortage in supply in markets with a relatively higher price which is supposed to encourage more production and capital accumulation. While there are many factors that can affect capital accumulation and firm participation, in this chapter we are interested in studying the effect of inflation and unemployment benefits on firm size and capital accumulation since labor supply and capital accumulation are closely related and monetary policy affect firm entry indirectly by influencing demand of workers whose money holding is closely related with inflation level. Previous literature only focuses on the relationship between inflation and unemployment while abstract from the consideration of capital demand and firm participation . We incorporate the firm entry and capital accumulation such that labor and monetary search models are embedded in a more general macro setting in a tractable way. This enable us to speak to issues like how inflation and unemployment policies affect firm entry and capital accumulation. We find that the implementation of monetary and unemployment policies is sensitive to market structures in the good market, that is active firms may become larger or smaller depending on whether the market is random-search with bargaining or directed search with posting.

4.2 The Model

Time is discrete and continues forever. Following Berentsen et. al. (2011), as illustrated in Figure 4.1, in each period t three markets convene sequentially: a decentralized good market as in Kiyotaki and Wright (1989), characterized by frictions detailed below; a frictional labor market as in Mortensen and Pissarides (1994); and a frictionless Arrow-Debreu market. We refer to these markets as KW, MP and AD respectively. There are a continuum of households with a unit measure and a continuum of firms with measure $N \geq 1$. In the AD, households trade a numeraire consumption good x, which is endowed to them. They save in the form of capital k

and choose money holding m. They also receive wages ω if they were employed in the last period and receive transfers T. Firms choose whether to enter by paying a fix cost c of posting a job. Upon the entry decision, they decide how much capital to rent from the households. Firms are owned by the households and transfer any profit back to the households in the form of dividend Δ. In the KW, households want to consume a KW good. Firms which have workers and capital can produce this good. Households randomly match with firms and the good is produced on spot. Due to anonymity and limited commitment, no credit is available. Money can be used to facilitate transactions. In MP, firms with vacancies randomly matched with unemployed workers. Upon a match, they negotiate over wage using Nash bargaining and any match separate with probability σ.

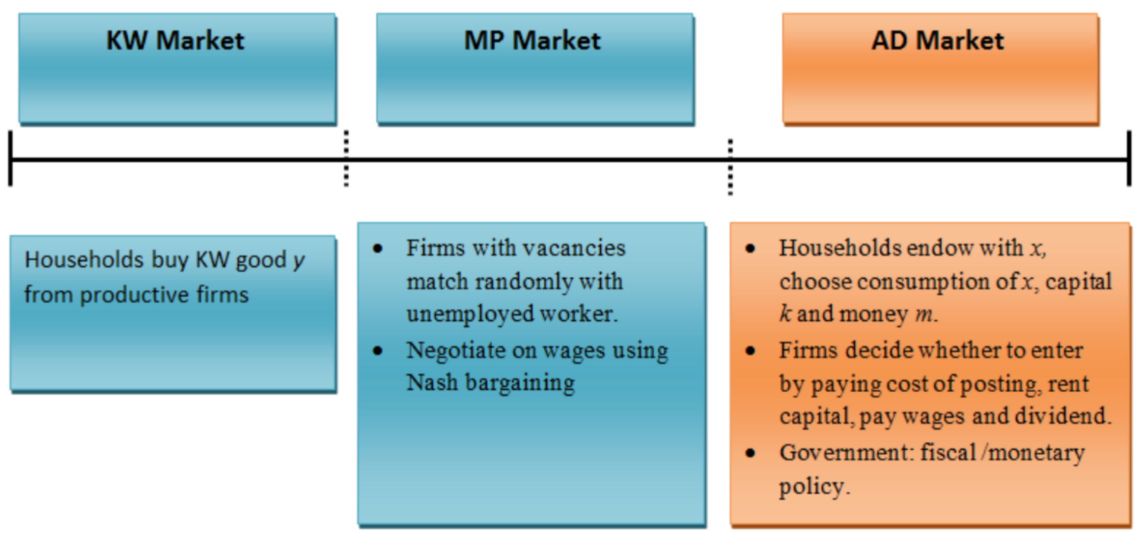


Figure 4.1: Timeline

4.2.1 AD Market

Households trade a numeraire consumption good x, adjust their money holding and invest in capital $\hat{k}$. They receive transfer from the firms Δ and pay tax T. We assume that firms have unlimited liability. Therefore, Δ can be negative if a firm does not make a sale last period. This happens if it is a new entrant, or does not make a new hire, or fails to find a customer in last KW market. Similarly, T can be positive or negative. Employed households receive wages ω from their employers. Households have linear utility in the numeraire good where the consumption can go negative. One interpretation of that is households need to work extra hours to produce x at linear disutility to pay back his obligations. If a household is employed, he receives wage ω and otherwise, he receives unemployment benefits b. His employment status is denoted by $e \in \{0, 1\}$ where $e = 1$ meaning he is employed. Hence his AD market problem is

$$\begin{aligned} W_e^h(m,k) &= \max_{x,\hat{m},\hat{k}} \left\{ x + \beta V_e^h(\hat{m},\hat{k}) \right\} \\ s.t.\ x + \phi\hat{m} + \hat{k} &= (\rho + 1 - \delta)k + \phi m + e\omega + (1-e)b - T + \Delta \end{aligned}$$

where ϕ is the value of money in terms of x, ρ is the interest rate for renting capital and δ is the depreciation rate. Substituting for the constraint:

$$W_e^h(m,k) = (\rho + 1 - \delta)k + \phi m + I_e + \max_{\hat{m},\hat{k}} \left\{ -\phi\hat{m} - \hat{k} + \beta V_e^h(\hat{m},\hat{k}) \right\} \tag{4.1}$$

where $I_e = e\omega + (1-e)b - T + \Delta$. First order conditions yield

$$\hat{m} \quad : \quad \beta \frac{\partial V_e^h(\hat{m}, \hat{k})}{\partial \hat{m}} = \phi \tag{4.2}$$

$$\hat{k} \quad : \quad \beta \frac{\partial V_e^h(\hat{m}, \hat{k})}{\partial \hat{k}} = 1. \tag{4.3}$$

Choices of money and capital are independent of last period as in LW. Envelop conditions yield

$$\frac{\partial W_e^h(m,k)}{\partial m} = \phi \quad \text{and} \quad \frac{\partial W_e^h(m,k)}{\partial k} = \rho + 1 - \delta.$$

Firms with no workers choose whether to post vacancy. Firms with workers pay wages, rent capital and choose money holding. Notice that regardless of whether it has a worker or not, a firm always choose $\hat{m} = 0$. In addition, a firm without a worker cannot produce and it is optimal to choose $\hat{k} = 0$. Therefore, a firm with no worker has value function

$$W_0^f(m,k) = \phi m - \rho k + \max\left\{0, -c + \beta V_0^f(0,0)\right\},$$

where the subscript denote its hiring status. Firms choose to post vacancies if and only if $-c + \beta V_0^f(0,0) > 0$. Throughout this chapter, we focus on the case that not all firms are active which means

$$W_0^f(0,0) = \max\left\{0, -c + \beta V_0^f(0,0)\right\} = 0. \tag{4.4}$$

A firm with a worker decide how much capital to rent. The value function is

$$W_1^f(m,k) = \phi m - \rho k - \omega + \max_{\hat{k}} \beta V_1^f(0,\hat{k}) \tag{4.5}$$

First order condition yields

$$\hat{k} : \beta \frac{\partial V_1^f(0,\hat{k})}{\partial \hat{k}} = 0. \tag{4.6}$$

And envelop conditions are

$$\frac{\partial W_1^f(m,k)}{\partial k} = -\rho \text{ and } \frac{\partial W_0^f(m,k)}{\partial m} = \frac{\partial W_1^f(m,k)}{\partial m} = \phi. \tag{4.7}$$

Lastly, total money supply grows at a constant rate μ.

4.2.2 KW Market

For now, we assume that the terms of trade in KW market is determined by competitive pricing. Let p be the nominal price of good q. As standard in this class of models, households spend all the money they have and then $pq = m$. The KW value function for the households is

$$V_e^h(m,k) = \alpha_h[u(q) + U_e^h(0,k)] + (1-\alpha_h)U_e^h(m,k)$$

or

$$V_e^h(m,k) = \alpha_h[u(q) + U_e^h(0,k) - U_e^h(m,k)] + U_e^h(m,k), \tag{4.8}$$

where α_h is the probability that a household finds the market. For now, we assume it is exogenous. But later on, we are going to endogenize it using a matching function.

If a firm does not have a worker in the KW, it cannot produce. Therefore, it skips the KW market and move to the MP market,

$$V_0^f(m,k) = U_0^f(m,k). \tag{4.9}$$

If it has a worker and rent capital k, it can produce $f(k)$ amount of goods once it finds the market. The function $f(\cdot)$ satisfies $f' > 0$, $f'' < 0$, $f(0) = 0$ and $f'(0) = \infty$. A seller finds the market with probability α_f exogenous for now. In this event, it sells the good at market price p and get $pf(k)$. Otherwise, it continues to the MP market with its capital and worker.

$$V_1^f(0,k) = \alpha_f U_1^f[pf(k),k] + (1-\alpha_f)U_1^f(0,k). \tag{4.10}$$

4.2.3 MP Market

In the MP market, unemployed workers randomly matched with vacancies. While at the same time, matched workers and firms separate at an exogenous rate σ. Let u and v be the unemployment and vacancy rates and define $n = 1-u$ to be the employment rate. The number of matches given u and v is $\mathcal{N}(u,v)$ where $\mathcal{N}$ is a matching function with constant return to scales. Let $\gamma = v/u$ and then the matching probability for the unemployed worker is $\lambda_h = \mathcal{N}(1,\gamma)$ while the matching rate of vacancies is $\lambda_f = \mathcal{N}(1,\gamma)/\gamma$.

An unemployed worker finds a job with probability λ_h and stays unemployed with probability λ_h. Therefore, his/her MP market value function is

$$U_0^h(m,k) = (1-\lambda_h)W_0^h(m,k) + \lambda_h W_1^h(m,k). \tag{4.11}$$

Similarly, an employed worker keeps the job with probability $1-\sigma$ and becomes unemployed with probability σ. His/her value function is

$$U_1^h(m,k) = \sigma W_0^h(m,k) + (1-\sigma)W_1^h(m,k). \tag{4.12}$$

We can combine (4.11)-(4.12) with (4.1) to write

$$U_0^h(m,k) = \phi m + (1+\rho-\delta)\,k + \lambda_h W_1^h(0,0) + (1-\lambda_h)W_0^h(0,0) \tag{4.13}$$

$$U_1^h(m,k) = \phi m + (1+\rho-\delta)\,k + \sigma W_0^h(0,0) + (1-\sigma)W_1^h(0,0). \tag{4.14}$$

Therefore, the MP market value functions of workers are separable in their wealth and their employment status.

Firms with vacancies get matched with probability λ_f and the value function is

$$U_0^f(0,0) = \lambda_f W_1^f(0,0) + (1-\lambda_f)W_0^f(0,0), \tag{4.15}$$

The value function of a matched firm is

$$U_1^f(m,k) = \sigma W_0^f(m,k) + (1-\sigma)W_1^f(m,k). \tag{4.16}$$

By (4.4) and (4.5),

$$U_1^f(m,k) = \phi m - \rho k + (1-\sigma) W_1^f(0,0). \tag{4.17}$$

Wage is determined through Nash bargaining. The bargaining power for a worker is η and wage is paid in the subsequent AD market. Because the AD value functions are linear in m and k, equilibrium wage maximizes the Nash product

$$\left[W_1^h(0,0) - W_0^h(0,0)\right]^\eta \left[-\omega + \beta V_1^f\left(0,\hat{k}^f\right)\right]^{1-\eta} \tag{4.18}$$

where $\hat{k}^f$ is firms' optimal choice of capital next period. Notice that

$$W_1^h(0,0) - W_0^h(0,0) = \omega - b + \beta\left[V_1^h\left(\hat{m},\hat{k}^h\right) - V_0^h\left(\hat{m},\hat{k}^h\right)\right]$$

where $\hat{m}$ and $\hat{k}^h$ are households' optimal choice of money and capital. Then the solution is

$$\omega = (1-\eta)\left[b - \beta V_1^h\left(\hat{m},\hat{k}^h\right) + \beta V_0^h\left(\hat{m},\hat{k}^h\right)\right] + \eta\beta V_1^f\left(0,\hat{k}^f\right). \tag{4.19}$$

4.3 Steady State Equilibrium

First, denote the real price of goods in KW market as $\bar{p} \equiv \phi p$. Notice that (4.8), (4.14) and (4.13) together imply that

$$\frac{\partial}{\partial m} V_e^h(m,k) = \alpha_h \left[\frac{u'(q)}{\bar{p}} - 1 \right] \phi + \phi, \tag{4.20}$$

$$\frac{\partial}{\partial k} V_e^h(m,k) = \rho + 1 - \delta. \tag{4.21}$$

Combine (4.20) with (4.2) and evaluate at the steady state to obtain

$$i = \alpha_h \left[\frac{u'(q)}{\bar{p}} - 1 \right] \tag{4.22}$$

where $1 + i = (1 + \mu) / \beta$. In addition, (4.3) and (4.21) imply $\rho = r + \delta$. Combine this with (4.6), (4.10) and (4.17) to obtain

$$r + \delta = \alpha_f \bar{p} f' \left(k^f \right), \tag{4.23}$$

where k^f is firms' demand for capital.

To determine the equilibrium wage, notice that

$$W_1^h(m,k) - V_0^h(m,k) = \frac{w - b}{1 - (1 - \sigma - \lambda_h)\beta}, \tag{4.24}$$

$$W_1^f(0,k) = \frac{\alpha_f \bar{p} f(k) - \rho k - (1 - \sigma) w}{1 - (1 - \sigma)\beta}. \tag{4.25}$$

Substitute them into (4.19) and obtain

$$w = \frac{(1-\eta)[1-\beta(1-\sigma)]b + \eta[1-\beta(1-\sigma-\lambda_h)]\beta\left[\alpha_f \bar{p} f\left(k\right) - \rho k\right]}{(1-\eta)\left[1-\beta(1-\sigma)\right] + \eta\left[1-\beta(1-\sigma-\lambda_h)\right]}. \quad (4.26)$$

Wage is a weighted sum of the worker's cost, which is giving up the unemployment benefits b, and the firm's benefit.

Next, the MP market tightness γ is determined by firms' free entry condition,

$$V_0^f\left(0,0\right) = \lambda_f\left[-w + \beta V_1^f\left(0,k^f\right)\right] = \frac{c}{\beta}.$$

Eliminate w and $V_1^f\left(0,k^f\right)$ using (4.25) and (4.26) and recall $\rho = r + \delta$:

$$\frac{c}{\beta} = \frac{\lambda_f(1-\eta)\left[\beta\alpha_f \bar{p} f\left(k^f\right) - \beta\left(r+\delta\right)k^f - b\right]}{(1-\eta)\left[1-\beta(1-\sigma)\right] + \eta\left[1-\beta(1-\sigma-\lambda_h)\right]}. \quad (4.27)$$

where λ_f and λ_h are functions of γ. Equations (4.22), (4.23) and (4.27) together with the MP market steady state condition (4.28), KW market clearing condition (4.29) and the capital market clearing condition (4.30) determines the equilibrium. This system of equations determines six equilibrium objects.

$$\lambda_h = n\sigma/\left(1-n\right), \quad (4.28)$$

$$\alpha_h q = n\alpha_f f\left(k^f\right), \quad (4.29)$$

$$k = nk^f. \quad (4.30)$$

To simplify this system, first use (4.29) and (4.23) to eliminate q in (4.22). Then we obtain the equation (4.31). It defines $\bar{p}$ as a function of employment n. Next, notice that (4.28) defines γ as an increasing function of n, i.e., $\gamma = v(n)$, where $v(0) = 0$ and $v\left(\frac{1}{1+\sigma}\right) = \bar{\gamma}$ such that $N(\bar{\gamma}) = 1$. Substitiute this into (4.27) to eliminate λ_f. Lastly, eliminate λ_h and k^f using (4.28) and (4.23). We arrived at (4.32), which defines n as a function of $\bar{p}$. Any $(n, \bar{p})$ is an equilibrium iff it solves (4.31)-(4.32). Once we obtain $(n, \bar{p})$, other equilibrium objects can be computed accordingly.

$$i = \alpha_h \left\{ \frac{u'\left[n\frac{\alpha_f}{\alpha_h} f \circ f'^{-1}\left(\frac{r+\delta}{\alpha_f \bar{p}}\right)\right]}{\bar{p}} - 1 \right\} \tag{4.31}$$

$$(1+r)\,c = \frac{\frac{N[1,v(n)]}{v(n)}(1-\eta)\left[\beta\alpha_f \bar{p} f \circ f'^{-1}\left(\frac{r+\delta}{\alpha_f \bar{p}}\right) - \beta(r+\delta) f'^{-1}\left(\frac{r+\delta}{\alpha_f \bar{p}}\right) - b\right]}{(1-\eta)\left[1-\beta(1-\sigma)\right] + \eta\left[1-\beta(1-\sigma-\frac{n\sigma}{1-n})\right]} \tag{4.32}$$

We can draw (4.31) and (4.32) in n-$\bar{p}$ plane as in Figure 4.2. We only focus on the region that $n \in [0, 1/(1+\sigma)]$ as n cannot exceed $1/(1+\sigma)$. Equation (4.31) defines a downward sloping curve as shown by RB. Intuitively, if n is higher, the supply of KW goods is higher, which lowers $\bar{p}$. As $n \to 0$, $\bar{p}$ converges to $\bar{p}_0$ where $\bar{p}_0 = \alpha_h u'(0) / (i + \alpha_h)$. If $u'(0) = \infty$, then the RB curve asymptotes the vertical axis. Equation (4.32) defines a version of Beveridge curve as shown by BC in the graphs. It is upward sloping because higher $\bar{p}$ leads to higher profit of firms, which leads to more firm entry. As a result, employment increases. As $\bar{p}$ converges to $\bar{p}_1$ where

$$\frac{(1-\eta)\left[\beta\alpha_f \bar{p}_1 f \circ f'^{-1}\left(\frac{r+\delta}{\alpha_f \bar{p}_1}\right) - \beta(r+\delta) f'^{-1}\left(\frac{r+\delta}{\alpha_f \bar{p}_1}\right) - b\right]}{(1-\eta)\left[1-\beta(1-\sigma)\right] + \eta\left[1-\beta(1-\sigma)\right]} = (1+r)\,c,$$

n converges to 0. If $\bar{p} \to \infty$, then $n \to 1/(1+\sigma)$. These two curve intersects as long as $\bar{p}_0 > \bar{p}_1$, which can be guaranteed by i sufficiently small. If $u'(0) = \infty$, a monetary equilibrium exists regardless of the value of i. They can only intersect once as shown by point A in as shown in Figure 4.2. Consequently we have the following result.

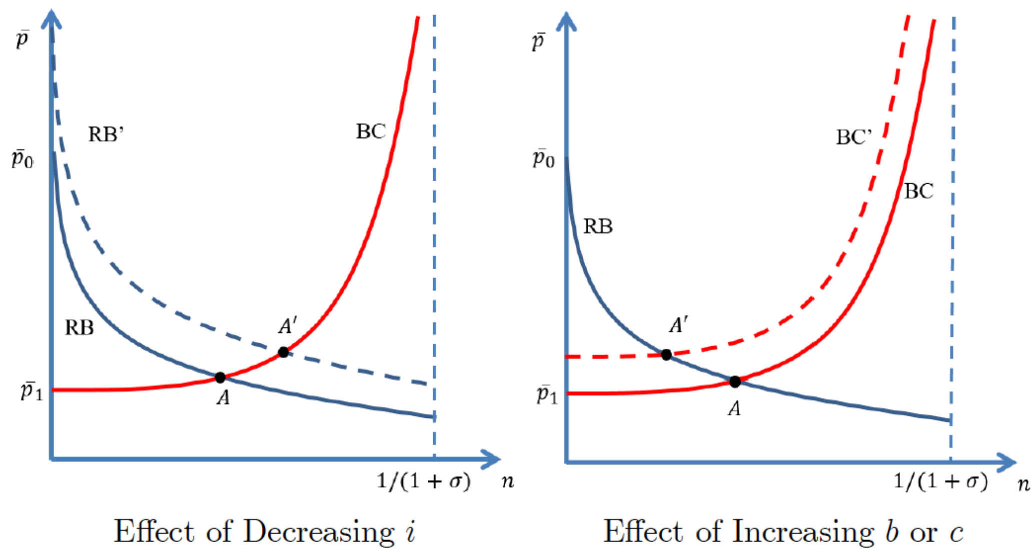


Figure 4.2: Steady State Equilibrium with Exogenous α_h and α_f

Proposition 4.1. *If α_h and α_f are exogenous, monetary steady state equilibrium exists and is unique iff* $i < \bar{\imath} \equiv \alpha_h \left[u'(0)/\bar{p}_1 - 1\right].$

We can use RB and BC curves to analysis comparative statics. For example, if the central bank lowers i, the RB curve shifts up to RB' on the left panel of Figure 4.2 while the BC curve is unaffected. The equilibrium changes from point A to point A', resulting in higher n and higher $\bar{p}$. Intuitively, lower i increases the real balances and hence increase the demand of KW good. As a result, $\bar{p}$ increases. Consequently, firms become more profitable which leads to more entry and higher employment. Also by (4.23), firms rent more capital because $\bar{p}$ increases. Therefore, there are more firm-entry and each firm becomes larger. The total stock of capital, which equals the number of productive firms multiplied by capital per firm, also increases. In addition, both consumption and output increase. On the contrary, if i increases, the RB curve shifts down and the opposite happens. If i is sufficiently large, $\bar{p}_0$ may drop below $\bar{p}_1$ and money is not valued. Consequently, no firm enters and KW market and MP market collapse.

If the government raises the unemployment benefits b or firm entry cost c increases, the RB curve is uneffected but the BC curve shifts left to BC' as shown in the right panel of Figure 4.2. The equilibrium changes from point A to point A'. Employment n decreases and $\bar{p}$ increases. Intuitively, less firm enters, which lowers supply and raises real price. Consequently, each productive firm rents more capital and becomes larger. However, households KW consumption drops and the total output drops. More results on effects of changing parameters are shown in the following proposition.

Proposition 4.2. *If α_h and α_f are exogenous, the effects of parameter changes are show in Table 4.1.*

	n	$\bar{p}$	q	k^f	k	ω
i	$-$	$-$	$-$	$-$	$-$	$-$
c	$-$	$+$	$-$	$+$	$-$	$-$
b	$-$	$+$	$-$	$+$	$-$	
α_h	$+$	$+$				
α_f						
σ	$-$	$+$	$-$	$+$	$-$	

Table 4.1: Comparative Statics

So far, we treat α_h and α_f as parameters. We can also endogenize them using matching functions. Then the equilibrium $(n, \bar{p})$ still solves (4.31) and (4.32) with $\alpha_h = M(n,1)$ and $\alpha_f = M(n,1)/n$, where M is a constant return-to-scale matching function. To simplify notations, write $\alpha_h = M(n)$ and $\alpha_f = M(n)/n$ from now on. One can show that as before the BC curve is similar as before. It is increasing and intersects the vertical axis at $\bar{p}_1 > 0$ which solves

$$\frac{(1-\eta)\left[\beta\bar{p}_1 f \circ f'^{-1}\left(\frac{r+\delta}{\bar{p}_1}\right) - \beta(r+\delta) f'^{-1}\left(\frac{r+\delta}{\bar{p}_1}\right) - b\right]}{(1-\eta)\left[1-\beta(1-\sigma)\right] + \eta\left[1-\beta(1-\sigma)\right]} = (1+r)\,c.$$

As $\bar{p} \to \infty$, $n \to 1/(1+\sigma)$. Similarly, one can show that the RB curve passes through and origin and is increasing. This is very different from the previous version. There are two factors that contribute to this change. First, now higher n increases buyer's chance to get matched. Therefore, they are willing to bring in more real balances which, in turn, enables sellers to charge higher price. On the other hand, if $n \to 0$, the real balances buyers bring in goes to 0 and hence $\bar{p} \to 0$. Second, the matching is one-to-one. As a result, higher n does not induce more supply in the market and hence does not have any negative effect on the price.

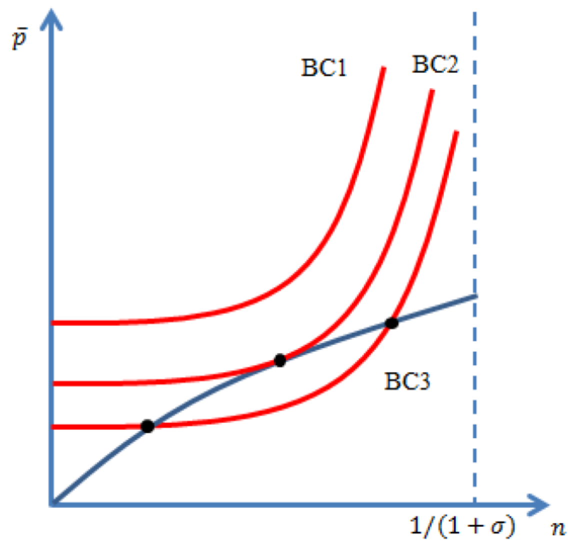
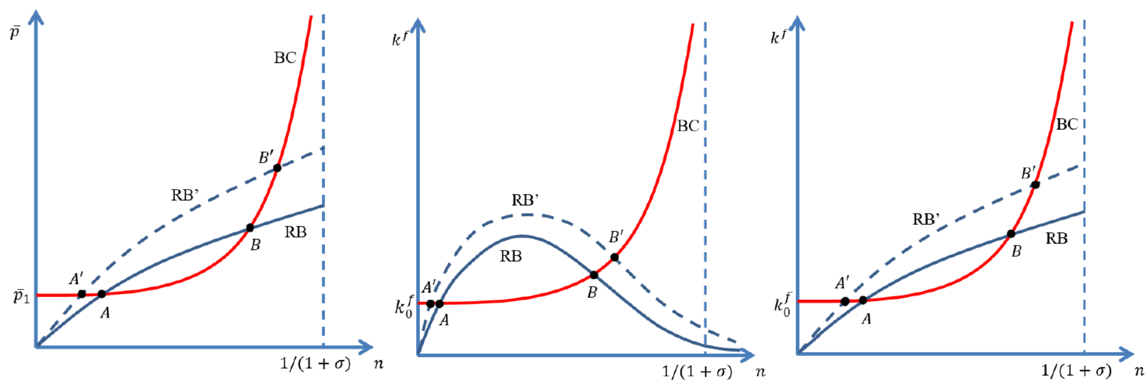


Figure 4.3: Equilibrium with Endogenous α_h and α_f

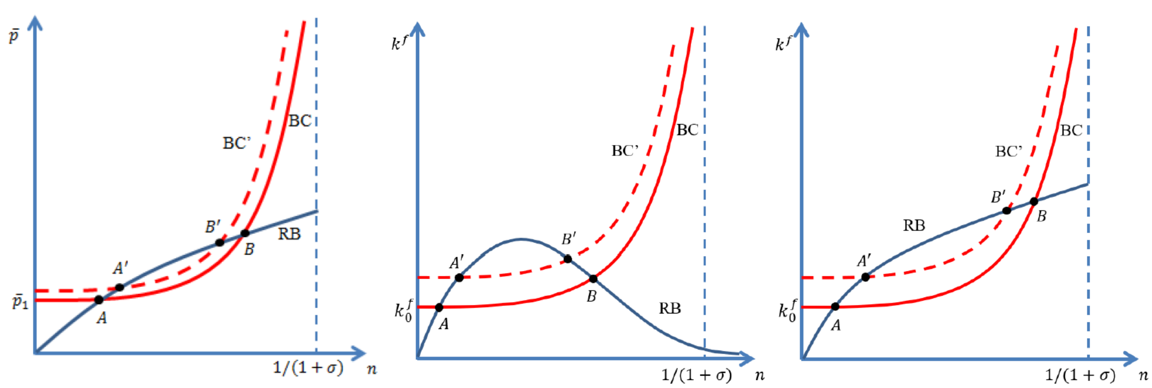
The equilibrium can be studies using Figure 4.3. Depending on the parameters, (4.32) can lead to BC1, BC2 or BC3. In all these cases, there exists a non-monetary equilibrium at $n = 0$ and $\bar{p} = 0$. If we have BC1, there does not exist any other equilibria. If we have BC2, which is tangent to the RB curve, there exists a unique monetary equilibrium at the tangent point. If we have BC3, there exist at least two monetary equilibria. There can exist more than two if the CB curve is not convex and the RB curve is not concave. And in theory, we cannot rule out this possibility. Therefore, we have the following proposition:

Proposition 4.3. *If α_h and α_f are endogenous, at least one equilibrium exists. There can exit more than* 1 *monetary equilibria.*



Effects on n and $\bar{p}$ Effects on k^f: Case 1 Effects on k^f: Case 2

Figure 4.4: Effects of Increasing i: Endogenous α_h and α_f



Effects on n and $\bar{p}$ Effects on k^f: Case 1 Effects on k^f: Case 2

Figure 4.5: Effects of Increasing b or c: Endogenous α_h and α_f

Figures 4.4 and 4.5 show how to use graphs to analyze the effect of parameter changes on the extreme monetary equilibria. Figure 4.4 illustrates what happens if i decreases. The first graph shows the effect on n and $\bar{p}$. If i decreases, the RB curve rotates counter clock-wise from RB to RB', while the BC curve is unaffected. Intuitively, lower i means higher real balances which enables firms to charge more. Therefore, the monetary equilibrium with smallest n changes from A to A', resulting in lower n and $\bar{p}$. While the equilibrium with the largest n changes from B to B'. Both n and $\bar{p}$ increases. The change of the equilibrium with the highest n coincides with the case with exogenous α_f and α_h. To see the effects on the capital holding of firms k^f and the total stock of capital in the economy k , it is useful to plot the equilibrium in the n-k^f space. We can eliminate $\bar{p}$ in (4.31)-(4.32) using (4.23) and obtain a system of equations on k^f and n. One can show that in the n-k^f space, the RB curve starts from the origin and is non-monotone in n. As $n \to \infty$, the RB curve asymptotes the horizontal axis. The BC curve takes a similar shape as in the n-$\bar{p}$. Now the equilibrium with highest n has two cases. One is that lies on the decreasing part of the RB curve as shown in the middle panel of Figure 4.4. The other case is that it lies on the increasing part of the RB curve. If i decreases, RB curve rotates counter-clockwise which results in higher k^f in the equilibrium of highest n and lower k^f in the smallest monetary equilibrium in both cases. As $k = nk^f$, the total amount of capital decreases in the monetary equilibrium with smallest n and increases in the one with largest n. The effects on the equilibrium with largest n is in line with that in the model with exogenous α_f and α_h.

Figure 4.5 shows what happens if b or c increases. The first graph shows the effect on n and $\bar{p}$. Again,the BC curve shifts up to BC' after the change. The monetary equilibrium with lowest n shifts from A to A'. This leads to higher n and higher price $\bar{p}$. While the equilibrium with highest n shifts from B to B'. Both $\bar{p}$ and n decreases. This effect is very different from what we obtain in Proposition 4.2. Here, if entry is more costly, firms enters less. This reduces the matching probability of the buyers and hence reduces their real balances. As a result, equilibrium price falls. This is in sharp contrast with the case where α_f and α_h are exogenous. There, firms are competing on quantities. Fewer firms means lower supply and therefore real price rises. The second and third panel shows the effect on k^f. The effect on the monetary equilibrium with smallest n are the same across these two cases. Both k^f and n increase, resulting in a high overall k. But the effect on highest equilibrium depends on whether the equilibrium lies on the increasing or decreasing region of RB. If it lies in the decreasing region of RB as shown in the middle panel, increase in b or c increases k^f and decrease n and the effect on k is not clear. While if it lies in the increasing region as shown in the right panel, both n and k^f drops which leads to a decrease in k.

4.4 Conclusion

This chapter studies the effect of inflation and unemployment benefits on firm entry and capital accumulation. We incorporate firms' decision in a monetary and labor search model such that the environment is embedded in a more general macro setting while maintain the tractability of results. This enables us to speak to issues like how monetary policy and unemployment policies affect capital accumulation and production. We find that unemployment benefits lead to more unemployment, less firm entry and less capital. However, depending on the market structure, active firms may become larger or smaller. In a competitive market, with less entry on the firm side, meeting probability would not be affected since there is no trading friction so that price would not drop much and terms of trade for buyers would not get too bad to reduce their holdings of real balance. As a result, each firm in the market has an incentive to accumulate enough capital and produce to match the demand.

Differently, with random search, the less entry of firms would lower the meeting probability for consumers to find a seller therefore preventing them from holding enough real balance to trade. On top of that, since there is no price competition among active firms, the terms of trade get worse, discouraging consumers from holding the same real balance as they do in a competitive market. All these two factors result in a lower level of capital accumulation of each active firms that enter the market.

4.5 Appendix

Proof of Proposition 4.1

Proof. Notice that (4.31) defines implicitly a function Φ_1 such that $\bar{p} = \Phi_1(n)$. After some algebra, one can show $\Phi_1(0) = \alpha_h u'(0) / (\alpha_h + i) = \bar{p}_0$, $\Phi_1\left(\frac{1}{1+\sigma}\right) < \infty$, and

$$\Phi_1'(n) = \frac{\frac{u'(q)}{\bar{p}} f'' \circ f'^{-1}\left(\frac{r+\delta}{\alpha_f \bar{p}}\right) + n \frac{u''(q)}{\alpha_f \bar{p}^3}(r+\delta)^2}{u''(q)\, \alpha_f f \circ f'^{-1}\left(\frac{r+\delta}{\alpha_f \bar{p}}\right) f'' \circ f'^{-1}\left(\frac{r+\delta}{\alpha_f \bar{p}}\right)} < 0,$$

where

$$q = n \frac{\alpha_f}{\alpha_h} f \circ f'^{-1}\left(\frac{r+\delta}{\alpha_f \bar{p}}\right).$$

Next, notice that (4.31) defines n as a function of $\bar{p}$, $n = \Phi_2(\bar{p})$. One can show that $\Phi_2(\bar{p}_1) = 0$ and

$$\Phi_2'(\bar{p}) = \frac{\frac{N[1, v(n)]}{v(n)} \eta \beta \alpha_f f \circ f'^{-1}\left(\frac{r+\delta}{\alpha_f \bar{p}}\right)}{(1+r)\, c\left[\left(1 - \frac{N'[v(n)] v(n)}{N[v(n)]}\right) \frac{v'(n)}{n} + \frac{1}{(1-n)^2}(1-\eta)\beta\sigma\right]} > 0.$$

An n is a monetary equilibrium iff $\Phi_2 \circ \Phi_1(n) = n > 0$. Notice $\Phi_2 \circ \Phi_1(0) = \Phi_2(\bar{p}_0) > \Phi_2(\bar{p}_1) = 0$ if $i < \bar{p}_0$. In addition, $\Phi_2 \circ \Phi_1\left(\frac{1}{1+\sigma}\right) < \Phi_2(\infty) = \frac{1}{1+\sigma}$.

Consequently, $\Phi_2 \circ \Phi_1(n) - n > 0$ at $n = 0$ and < 0 at $n = 1/(1+\sigma)$. In addition, $[\Phi_2 \circ \Phi_1(n)]' = \Phi_1'(n)\,\Phi_2' \circ \Phi_1(n) < 0$. There exists a unique n that solves $\Phi_2 \circ \Phi_1(n) = n$ by continuity. Hence, there exists a unique monetary equilibrium. ■

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