ESSAYS IN MACROECONOMICS

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To my family

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Abstract

This dissertation consists of three independent essays in macroeconomics. They examine macroeconomic issues and their underlying mechanism. The first essay studies how central banks learn and respond to data uncertainty from a cross-country perspective. The second essay explores the pro-cyclical behavior of household and corporate credit in emerging economies. The third essay extends time-series forecasting methods using non-linear dynamic coupling between the US and Canadian industrial production indices.

The first chapter of the dissertation studies monetary policy from a cross-country perspective. Cross-country estimates of Taylor rules suggest that higher data uncertainty is associated with a more inertial behavior of interest rates. Data uncertainty is measured by the volatility of differences between real-time data and their revisions. Using a simple structural model with Kalman filter learning, I replicate the cross-country pattern of the inertial behavior. More inertial behavior results not because central banks gradually adjust interest rates in the face of data uncertainty, but because the central banks' inference about the true data is correlated with past interest rates. Thus, I endogenize the inertial behavior of interest rates as resulting in part from the learning process.

The second chapter explores the pro-cyclical behavior of household and corporate credit in emerging economies. Standard consumption-investment theory predicts counter-cyclical (pro-cyclical) behavior of household (corporate) credit whereby households' consumption-smoothing and firms' investment motives are aligned. Counter to the theoretical symbiosis consistent with U.S. data, it is demonstrated that the pro-cyclical behavior of both household and corporate credit in emerging economies, rationalized by households' leveraged investing in domestic assets and followed by large responses in asset values, engenders competition between them and hinders the growth of small and medium businesses. The empirical findings suggest another way of understanding the credit cycle puzzle in emerging economies,

counter-cyclical behavior of real interest rate and large consumption volatility.

The third chapter studies the coupling of industrial production indices of the United States and Canada using a non-linear autoregressive model. Estimation of the exponential smooth transition autoregressive (ESTAR) model in the literature is improved with an expanded set of specifications, and I identify the dynamic linkage between the United States and Canada and evaluate the forecast performance of each model. The results show the nonlinear autoregressive model with bilateral trade linkage to outperform other models suggested by existing studies.

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Chapter 1

Monetary Policy under Data Uncertainty: Interest-Rate Smoothing from a Cross-Country Perspective

1.1 Introduction

This essay examines differences in monetary policy across countries with respect to the extent to which their interest rates are slow to adjust. This is related to the quality of the data on aggregate output and the inflation rate that the country produces. We demonstrate that countries with more data uncertainty are slower to adjust their interest rate, and this is largely explained by the central banks' learning process. The novelty of this essay is that the central bank is considered as an active learner: the central bank observes data with noise and makes inferences about the true data before making policy decisions.

A key question in monetary policy is how responsive central banks are to current information about the level of GDP and inflation rate when they are determining their policy interest rate. The standard theories say that a central bank that is choosing its interest rate in a forward-looking manner should not place any weight on past interest rate when it is deciding what its current interest rate should be. However, we observe in all countries that there is at least some sluggishness to the interest rate. Moreover, the sluggishness varies considerably across countries. The sluggishness is measured using Taylor rule equation, which prescribes short-term interest rate based on inflation and the output gap. For each country, policy interest rate is regressed on inflation rate, output gap, and lagged interest rate, and the estimated coefficient on lagged interest rate indicates the sluggishness.

This essay explains both the sluggishness and the heterogeneity, and that has to do with the fact that data as its disseminated is imperfect. The data uncertainty is measured by the standard deviation of differences between real-time data and their revisions after a year. We find that countries with more data uncertainty tend to have more sluggishness in the interest rate. For example, Nigeria has greater data uncertainty and more sluggish interest rate than Canada. This is not only for Canada and Nigeria, but a general data pattern among 40 countries in the sample.[1]

To explain this pattern, we build a simple New Keynesian model and allow central bank to learn about the true data from noisy observations using Kalman filter. The model consists of three components: Phillips curve, IS curve, and a monetary policy rule. Central bank's objective is to minimize a loss function, which is sum of variances in inflation rate, output gap, and interest-rate shocks. Based on all available information, central bank forms belief about the true data and chooses its responds to expected inflation rate and output gap and also decides how responsive it will be by choosing the weight on lagged interest rate.

The noisy data are defined as the true data plus noise, where the noise components follow MA(1) process. Noise tends to be persistent over time in the data, and we use MA(1) to pick up the persistence. In each period, central bank observes the noisy data and decides how much weight should be placed on the new noisy information versus its past belief and past information. The weighted sum becomes the central bank's inference about the true data, and this is the standard Kalman filter learning with MA(1) noise. The assigned weight on new noisy information is called "Kalman gain," and this decreases as the data become noisier. Based on the inferences, central bank chooses the optimal responses to inflation rate

[1] In accordance with conventional belief, the cross-country Taylor rule estimates suggest that higher data uncertainty is associated with more inertial behavior of interest rates. This is often referred to as *intrinsic interest-rate smoothing* in the literature: the higher the data uncertainty, the greater the reduced-form estimate on lagged interest rate. I use the terms, the sluggishness to the interest rate, the inertial behavior of interest rates, and interest-rate smoothing, interchangeably since both terms are originated from reduced-form Taylor rule estimations in the literature. Another term, gradual adjustment, is defined differently from those two, and it is the incremental adjust of interest rates by central banks. There has not been clear distinctions among the terms in the literature. I argue that interest-rate smoothing is mostly a consequence of the *learning process*, rather than central banks' *gradual adjustment* of interest rates. Central banks' real-time beliefs about true data are indistinguishable from the gradual adjustment in ex-post data, both of which are largely picked up by the Taylor rule's reduced-form estimate on the lagged interest rate.

and output gap and decides the optimal weight on lagged interest rate within the monetary policy rule.

We find an interesting result from this model. As data become noisier, central bank's optimal weight on the lagged interest rate becomes smaller. This is because of the following reasons. First, the learning process effectively filters out the noise in the data. Second, the noise tends to be persistent, and the learning process helps getting the additional information from the predictable portion of the noise process.

This is not contradicting the empirical cross-country finding. The data pattern is not representative of the degree of gradual adjustment but rather is an artifact of what we as the econometrician observe from ex-post data. The model is simulated hundred thousand times and a series of ex-post data for each level of data uncertainty are generated. We run the same reduced-form Taylor rule regression for each level of data uncertainty and demonstrate that the coefficient on lagged interest rate increases in data uncertainty. The key explanation is that the learning component is missing in the reduced-form Taylor rule estimation on the ex-post data. The coefficient on lagged interest rate is overestimated because the central bank's belief is not included in the regressions. This is a typical omitted variable bias, and this bias increases in data uncertainty. This essay explains the sluggishness in policy interest rate with central banks' learning process when they face measurement error in the real-time data.

Contribution

This study distinguishes between the sluggishness and central banks' gradual adjustment and demonstrates that the two may not move together. Whereas the conventional view in the literature considers the sluggishness evidence of central banks' gradual adjustment (Bernanke, 2004), I show the gradual adjustment to be directly induced by central banks' desire to avoid interest-rate surprises, that is, the variance of changes in interest rates in the loss function.

This essay endogenizes the sluggishness as resulting in part from central banks' learning process. Reduced-form Taylor rule estimators in the literature often include lagged interest rate as an independent variable, which substantially increases explanatory power. Including lagged interest rate is commonly justified by the assumption that central banks partially adjust the interest rate, which implies the gradual adjustment as the key explanation for the sluggishness. I show the sluggishness to exist and increase in the level of data uncertainty even if central banks' gradual adjustment of interest rates is muted by excluding the variance of changes in interest rates from the loss function. The sluggishness can be fully endogenized in the model by the learning process to which much of it accrues and the gradual adjustment of interest rates.

Literature review

This essay is closely related to the literature that studies the effect of data uncertainty on monetary policy. For example, Rudebusch (2001) and Orphanides (2003*b*) find that noisy economic data may lead to cautious and timid responses of policymakers. However, in empirical studies, as noted by Rudebusch (2001, 2006), such inducement toward timidity appears fairly modest. These studies focus on the effect of existence versus non-existence of data uncertainty, and all of them consider only the US economy. Because that the United States has a relatively low level of data uncertainty, the scope of previous studies is quite limited.[2] In contrast, I consider the cross-country variation in *magnitudes* of data uncertainty and study policymakers' optimal responses.

There are three strands of literature rationalizing interest-rate smoothing: reducing interest-rate volatility, exploiting the expectation channel for monetary policy, and responding optimally to data and model uncertainty. The first strand of the literature emphasizes the costs and benefits of interest-rate smoothing which is arising from its effects on financial stability (Cukierman, 1991; Rudebusch and Svensson, 1999; Stein and Sunderam, 2015). The second strand of the literature analyzes the benefits of interest-rate smoothing coming from its ability allowing central banks to steer private-sector expectations by inducing history dependency in the policy rate (Levin, Wieland and Williams, 2003; Rotemberg and Woodford, 1999; Woodford, 1999, 2003). The last strand of the literature explores the benefit of interest-rate smoothing that is arising from its ability to better manage the uncertainties about data, model parameters, or the structure of the economy faced by the central

[2]Cross-country variation in data uncertainty is described in Figure 1.2 (a).

bank (Brainard, 1967; Milani, 2007; Sack, 1998, 2000; Rudebusch, 2001; Söderström, 2002; Orphanides, 2003*a*).[3]

None of these papers shows a clear distinction between interest-rate smoothing and central banks' gradual interest-rate adjustment nor decreasing in the degree of gradual adjustment in the face of data uncertainty. In contrast, I decompose interest-rate smoothing into gradual interest-rate adjustment, which is caused by central banks' motive to avoid interest-rate surprises, and component in the learning process correlated with past interest rates.

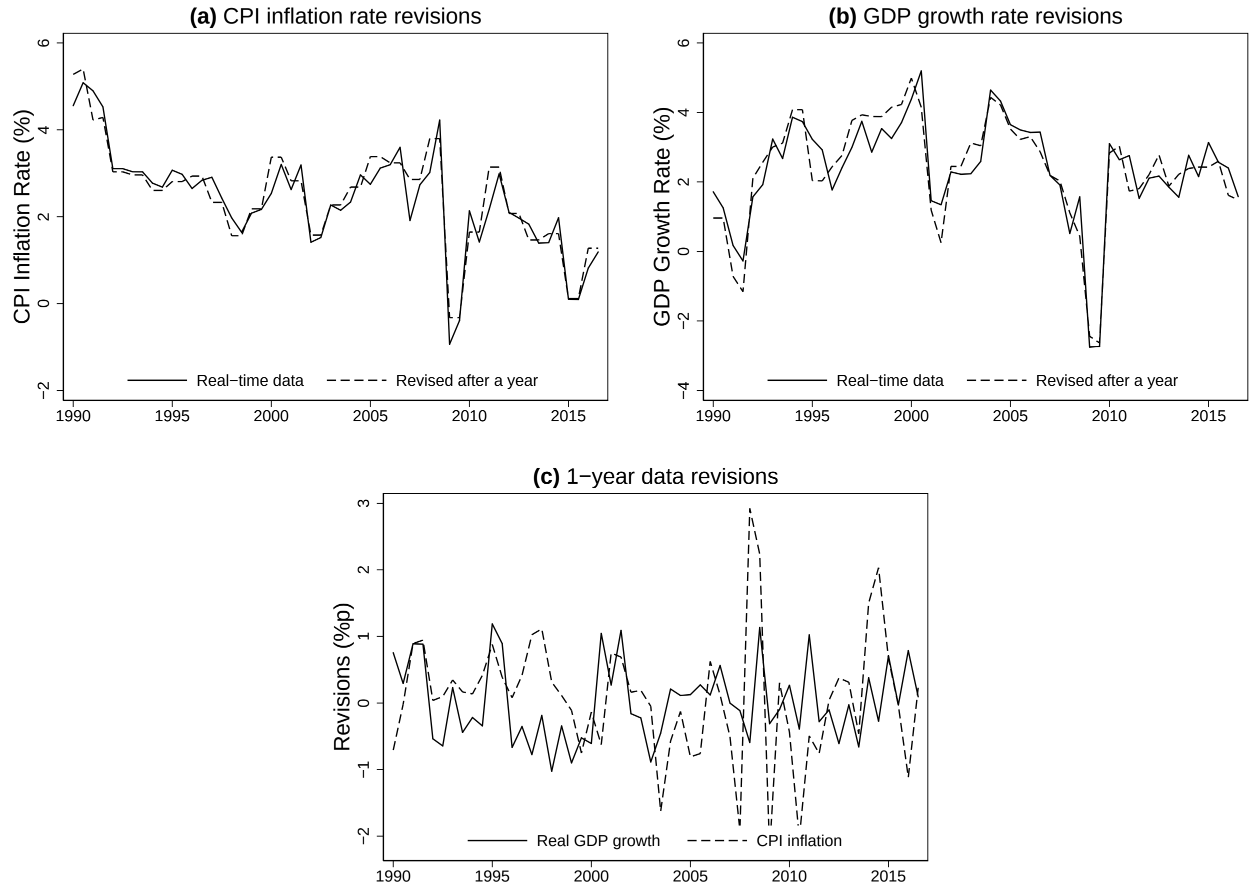
Policy implications

We can think about how central banks can improve their monetary policy under data uncertainty. There are two approaches we can think of: improving learning ability and improving data quality. This research provides a framework that helps analyzing these competing options. We consider two benchmark cases in addition to the learning policy. First, central bank always observes perfect information. Second, central bank observes noisy data and naively take the face value without making any inference. We compare the welfare loss under the learning policy and the naive policy at different levels of data uncertainty and demonstrate that learning policy is always better than the naive policy. The gain from learning is increasing and convex in data uncertainty. Given the cost information of each option, we can conduct a cost-benefit analysis and decide which option is more cost-effective.

The remainder of the chapter proceeds as follows. Section 1.2 provides cross-country comparison of monetary policies under data uncertainty. Section 1.3 introduces the Rudebusch and Svensson (1999) model with learning. Section 1.4 presents the model estimation along with reduced-form estimation on simulated data. Section 1.4.5 discusses policy implications of the study. The last section concludes.

[3]See Coibion and Gorodnichenko (2012) for an overview of recent literature.

Figure 1.1: CPI and GDP data revisions in the US



Note: Historical data on CPI inflation and GDP growth rate from IMF's World Economic Outlook (WEO) are used. Panel (a) and (b) show real-time data and revised data after a year in percentage, and panel (c) shows the simple differences between the real-time data and revised data after a year ($revisions_t$ (%p) = $revised\ data_t$ (%) − $real\text{-}time\ data_t$ (%)). The means of revisions in CPI inflation and real GDP growth are close to zero.

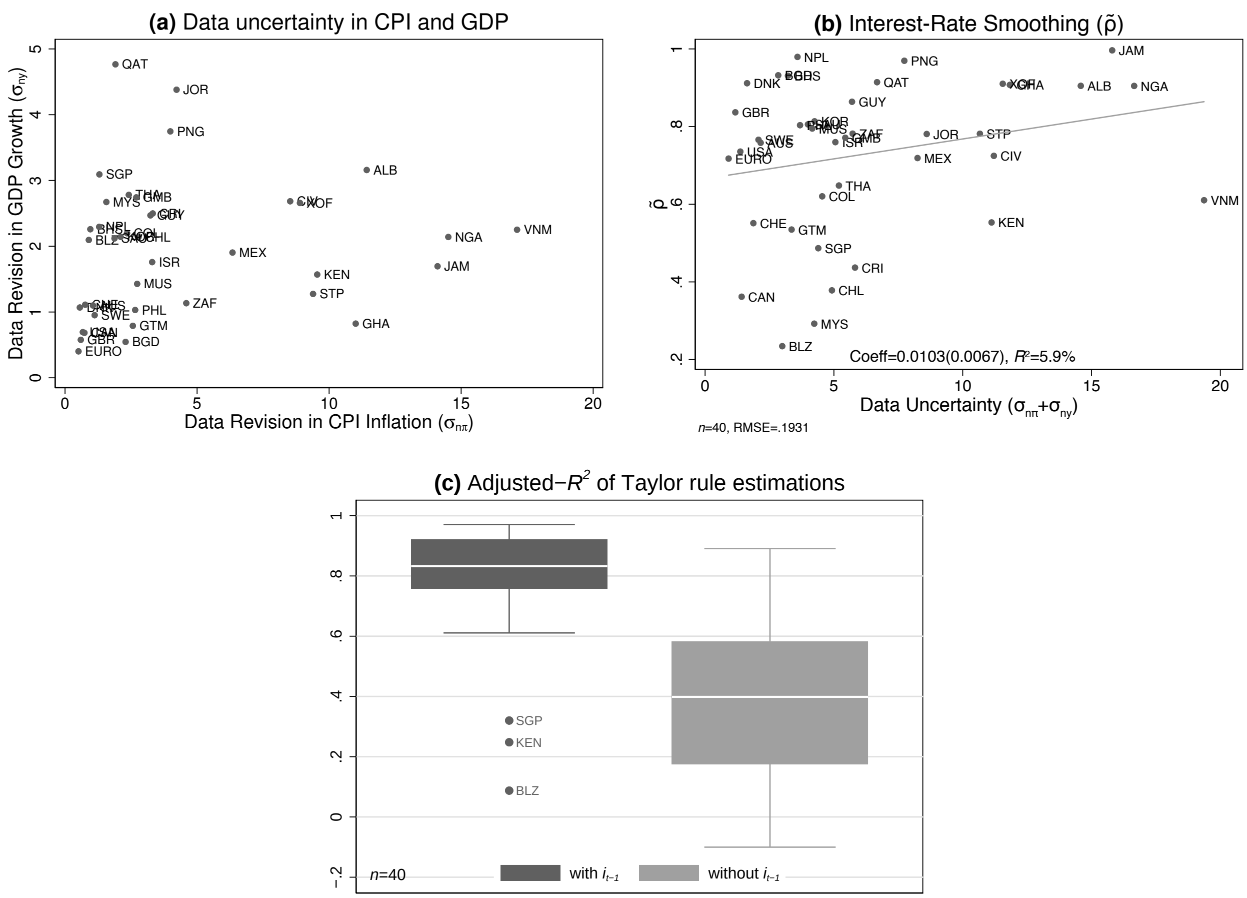
1.2 Cross-Country Observation

The conventional belief — a positive relationship between data uncertainty and interest-rate smoothing — is shown as a cross-country scatter plot in figure 1.2 (b).[4] Data uncertainty ($\sigma_{n\pi} + \sigma_{ny}$) is measured by the linear combination of volatilities in the differences between real-time data and their revisions, and interest-rate smoothing ($\widetilde{\rho}$) is measured by the esti-

[4]The forty sample countries include: Albania, Australia, Bahamas, Bangladesh, Belize, Canada, Chile, Colombia, Costa Rica, Côte d'Ivoire, Denmark, Euro area, Gambia, Ghana, Guatemala, Guyana, Israel, Jamaica, Jordan, Kenya, Korea, Malaysia, Mali, Mauritius, Mexico, Nepal, Nigeria, Papua New Guinea, Philippines, Qatar, São Tomé and Príncipe, Saudi Arabia, Singapore, South Africa, Sweden, Switzerland, Thailand, United Kingdom, United States, and Vietnam.

mated coefficients on lagged interest rate in Taylor rule. Note that countries with high data uncertainty tend to have high weight on the lagged interest rate. The measurements and estimation methods are described as the following.

Figure 1.2: Data uncertainty and cross-country comparison of interest-rate smoothing ($\widetilde{\rho}$)



Note: Panel (a) presents the cross-country variation of data uncertainty in CPI and GDP data. Partial-adjustment Taylor rule, $i_t = (1-\widetilde{\rho})[k + \widetilde{g_\pi} E_t \pi_{t+1} + \widetilde{g}_y y_t] + \widetilde{\rho} i_{t-1} + \varepsilon_t$, is estimated for each country and panel (a) reports $\widetilde{\rho}$ for the 40 sample countries. Panel (c) reports adjusted-R^2s of the 40 estimations, including and excluding lagged interest rate in the Taylor rule.

1.2.1 Interest-rate smoothing

Partial-adjustment Taylor rule

I assume that within each operating period the central bank has a monetary policy target for the nominal short-term interest rate, i_t^*, that is based on the state of the economy. In the baseline case, I assume that the target interest rate depends on both expected inflation and output.

$$i_t^* = \bar{i} + g_\pi(E[\pi_{t+n}|\Omega_t] - \pi^*) + g_y(E[x_t|\Omega_t] - x_t^*) \tag{1.1}$$

where $\bar{i}$ is the long-run equilibrium nominal rate, π_{t+n} is the rate of inflation between periods t and $t+n$, x_t is real output, π^* is target inflation, x^* is potential output, and Ω_t is the central bank's information set at time t.

I assume that the monetary policy-related interest rate partially adjusts to target, as follows:

$$i_t = (1-\rho)i_t^* + \rho i_{t-1} + \eta_t \tag{1.2}$$

where the parameter $\rho \in [0,1]$ captures the degree of interest-rate smoothing, and the exogenous random shock to the interest rate, η_t, is i.i.d. Defining $k \equiv \bar{i} - g_\pi \pi^*$ and $y_t \equiv x_t - x_t^*$, equation (1.1) becomes

$$i_t^* = k + g_\pi E[\pi_{t+n}|\Omega_t] + g_y E[y_t|\Omega_t] \tag{1.3}$$

Combining equation (1.2) and (1.3), we get

$$i_t = (1-\rho)[k + g_\pi E[\pi_{t+n}|\Omega_t] + g_y E[y_t|\Omega_t]] + \rho i_{t-1} + \eta_t \tag{1.4}$$

Rewriting the policy rule in terms of realized variables at the current period and setting $n = 1$, we have

$$i_t = (1-\rho)[k + g_\pi E_t \pi_{t+1} + g_y y_t] + \rho i_{t-1} + \varepsilon_t \tag{1.5}$$

where the error term ε_t is a linear combination of the exogenous disturbance η_t and the forecast errors of inflation and output. Parameters in equation (1.5) is estimated in each country, and the details are described below.

Estimation of the rule parameters

Parameters in the Taylor rule (equation (1.5)) are estimated for each country with least squares using quarterly data on monetary policy related interest rates, inflation rates, and output gap from 1990 to 2008. The data are drawn from the IMF's International Financial Statistics (IFS) and Thomson Reuters' Datastream. The output gap is defined by the percentage difference between real GDP and estimated potential GDP, and the potential GDP is measured by the quadratic trend of real GDP.

Rewriting equation (1.5) with country subscript c,

$$i_{t,c} = (1 - \widetilde{\rho_c})[k_c + \widetilde{g_{\pi,c}} E_t \pi_{t+1,c} + \widetilde{g_{y,c}} y_{t,c}] + \widetilde{\rho_c} i_{t-1,c} + \varepsilon_{t,c} \tag{1.6}$$

where $i_{t,c}$ is the monetary policy-related (nominal) interest rate, $\pi_{t+1,c}$ is the rate of inflation between periods t and $t+1$, and $y_{t,c}$ is output gap. Monetary policy responses to inflation rate and output gap are captured by the rule parameters $\widetilde{g_{\pi,c}}$ and $\widetilde{g_{y,c}}$, respectively, and k_c captures country-specific equilibrium real interest rate and target inflation together. The expected inflation rate, $E_t\pi_{t+1}$, is measured by the four quarter average inflation rate in percent following Rudebusch (2001) (i.e., $E_t\pi_{t+1} = \frac{1}{4}\sum_{j=0}^{3} \pi_{t-j}$).[5] Note that the expression, $[k + g_\pi E_t \pi_{t+1} + g_y y_t]$, represents the Taylor rule suggested optimal monetary policy. The parameter of interest, $\widetilde{\rho_c} \in [0, 1]$, captures the degree of interest-rate smoothing in country c. Estimated $\widetilde{\rho}$ for the sample countries are reported in figure 1.2 (b), and adjusted-R^2 for the regressions are reported as box plots in figure 1.2 (c).

1.2.2 Data uncertainty

Data uncertainty is measured by the volatility of the discrepancies between the noisy data and the true data, where the initially released real-time data and revised data after a year are taken as proxies for the noisy and the true data, respectively. It is formally defined as $(\sigma_{n\pi} + \sigma_{ny})$ where $\sigma_{n\pi}$ and σ_{ny} are standard errors of the discrepancies in inflation rate and GDP gap, respectively. I estimate $\sigma_{n\pi}$ and σ_{ny} using historical data from the IMF's World Economic Outlook (WEO), and the sample period is from 1990 to 2008.[6] The detailed description of estimating $\sigma_{n\pi}$ and σ_{ny} is the following.

[5] Alternative measures for $E_t\pi_{t+1}$, for example, linear trend forecasting, can be considered.

[6] The WEO data are released twice a year, in the spring and the fall. The WEO database consists of historical and current macroeconomic data and forecasts submitted by country teams before being vetted by the IMF's Research Department for internal and multilateral consistency. The spring WEO is released in May until the year of 2001 and in April thereafter; the fall WEO is typically released in October, and occasionally in September. Historical WEO data are publicly available for the period from 1990 to 2017. I collect real GDP growth and CPI inflation data from this database, and each report includes revised estimates for the past years, real-time estimates for the current year, and forecasts based on the current information. An example of the historical WEO dataset is presented in table A.1 in the appendix.

Measuring data uncertainty

The real-time noisy indicators on inflation rate, π_t^n, and output gap, y_t^n, are defined as:

$$\pi_t^n = \pi_t + n_t^\pi \tag{1.7}$$

$$y_t^n = y_t + n_t^y \tag{1.8}$$

where π_t and y_t are true inflation rate and output gap, and n_t^π and n_t^y are the contemporaneous measurement errors that plague the policymaker in real time, with standard errors $\sigma_{n\pi}$ and σ_{ny}, respectively, and uncorrelated with π_t and y_t.

Note that the noise in the data appears to be quite persistent over time, as shown in figure 1.1. In order to capture this persistence, the noises in inflation rate and GDP gap, n_t^π and n_t^y, are modeled as first-order moving average (MA(1)) processes:

$$n_t^\pi = \epsilon_t^\pi + \theta^\pi \epsilon_{t-1}^\pi \tag{1.9}$$

$$n_t^y = \epsilon_t^y + \theta^y \epsilon_{t-1}^y \tag{1.10}$$

where ϵ_t^π and ϵ_t^y are normally distributed with mean zero and variance $\sigma_{\epsilon\pi}^2$ and $\sigma_{\epsilon y}^2$, respectively.[7]

I measure each country's data uncertainty using linear combinations of $\sigma_{n\pi}$ and σ_{ny}, keeping the ratio between the two. One important econometric issue is that the parameters in Taylor rule (equation (1.5)) are estimated using quarterly data, while the sources of data for estimating data uncertainty (historical WEO issues) are released only twice a year, second and fourth quarter. Moreover, the WEO issues are not exactly semiannual because each issue reports real GDP growth and CPI inflation for the current entire year, not for the current half year.

In order to resolve these issues, I introduce the following econometric technique. Let $\pi_{t,0}$ be the first released estimate of CPI inflation for the year of t and $\pi_{t,1}$, $\pi_{t,2}$ and $\pi_{t,3}$ be the following semi-annual revisions of it. $\pi_{t,0}$ releases in a second quarter, and $\pi_{t,1}$ releases in the

[7]The assumption of the persistent noise in the real-time data accords with previous studies. See Orphanides (2001) for a detailed argument. Rudebusch (2001) uses AR(1) for modeling the noise in GDP gap and MA(3) for inflation rate. Onatski and Williams (2003) use AR(1) for both. I use MA(1) for both, and this assumption is useful under the structure of data releases as described in the next page.

fourth quarter in the same year. $\pi_{t,2}$ and $\pi_{t,3}$ release in the second and the fourth quarter of the following year. Assume that it takes four quarters to finalize the data since their first releases, then $\pi_{t,0} \sim \pi_{t,3}$ can be expressed as the following:[8]

$$\begin{aligned}
\text{Real-time data:} \quad \pi_{t,0} &= \tfrac{1}{4}\left[\pi^n_{t,q1} + \pi^n_{t,q2} + E_{q2}[\pi_{t,q3} + \pi_{t,q4}]\right] \\
\text{First revision:} \quad \pi_{t,1} &= \tfrac{1}{4}\left[\pi^n_{t,q1} + \pi^n_{t,q2} + \pi^n_{t,q3} + \pi^n_{t,q4}\right] \\
\text{Second revision:} \quad \pi_{t,2} &= \tfrac{1}{4}\left[\pi_{t,q1} + \pi_{t,q2} + \pi^n_{t,q3} + \pi^n_{t,q4}\right] \\
\text{Third revision:} \quad \pi_{t,3} &= \tfrac{1}{4}\left[\pi_{t,q1} + \pi_{t,q2} + \pi_{t,q3} + \pi_{t,q4}\right]
\end{aligned} \tag{1.11}$$

where $\pi_{t,q1} \sim \pi_{t,q4}$ are true CPI inflation in each quarter, and $\pi^n_{t,q1} \sim \pi^n_{t,q4}$ are noisy indicators of them. All variables above are annualized. $\pi_{t,0}$ consists of two noisy indicators for the first and second quarters and two forecasts for the third and fourth quarters, and $\pi_{t,1}$ consists of noisy indicators for all four quarters. $\pi_{t,2}$ consists of two true inflation rates for the first and the second quarters, because it has been four quarters since $\pi_{t,0}$ was initially released, and two noisy indicators for the third and the fourth quarters. $\pi_{t,3}$ consists of true inflation rates for all four quarters.[9] From equation (1.7) and (1.11), note that

$$\begin{aligned}
\pi_{t,1} - \pi_{t,2} &= \tfrac{1}{4}\left[n^\pi_{t,q1} + n^\pi_{t,q2}\right] \\
\pi_{t,2} - \pi_{t,3} &= \tfrac{1}{4}\left[n^\pi_{t,q3} + n^\pi_{t,q4}\right]
\end{aligned} \tag{1.12}$$

Since n^π_t is assumed to follow MA(1), $\{\theta^\pi, \sigma^2_{\epsilon\pi}\}$ in equation (1.9) can be estimated with observations $(n^\pi_{t,q1} + n^\pi_{t,q2})$ and $(n^\pi_{t,q3} + n^\pi_{t,q4})$ by matching the moments:

$$Var(\pi_{t,1} - \pi_{t,2}) = \left(\tfrac{1}{4}\right)^2 [1 + (1+\theta^\pi)^2 + (\theta^\pi)^2]\sigma^2_{\epsilon\pi} \tag{1.13a}$$

$$\begin{aligned}
E(\pi_{t,1} - \pi_{t,2})(\pi_{t,2} - \pi_{t,3}) &= \left(\tfrac{1}{4}\right)^2 E n^\pi_{q3} n^\pi_{q2} \\
&= \left(\tfrac{1}{4}\right)^2 E(\epsilon^\pi_t + \theta^\pi \epsilon^\pi_{t-1})(\epsilon^\pi_{t-1} + \theta^\pi \epsilon^\pi_{t-2}) \\
&= \left(\tfrac{1}{4}\right)^2 \theta^\pi \sigma^2_{\epsilon\pi}
\end{aligned} \tag{1.13b}$$

and I use the same technique for estimating $\{\theta^y, \sigma^2_{\epsilon y}\}$ in equation (1.10) using WEO's semi-

[8] On average 79% of inflation and GDP data revisions happen within four quarters from the initial release in the United States and 73% in all sample countries.

[9] One may argue that the second quarter inflation rate ($\pi^n_{t,q2}$) in the real-time data ($\pi_{t,0}$) should also be a forecast rather than a noisy observation since the WEO data are released at the beginning of the second and fourth quarters. This issue can be resolved by adding one more revision ($\pi_{t,4}$) to the list (equation (1.11)) and modifying the assumption as the following: it takes about a quarter for the IMF research department to receive and organize new data before releasing them on WEO database.

annual releases on real GDP growth.

Once $\{\theta^\pi, \sigma^2_{\epsilon\pi}\}$ and $\{\theta^y, \sigma^2_{\epsilon y}\}$ are estimated, $\sigma_{n\pi}$ and σ_{ny} can be calculated from:

$$\begin{aligned} \sigma^2_{n\pi} &= \sigma^2_{\epsilon\pi}(1 + (\theta^\pi)^2) \\ \sigma^2_{ny} &= \sigma^2_{\epsilon y}(1 + (\theta^y)^2) \end{aligned} \tag{1.14}$$

and figure 1.2 (a) describes the cross-country variation in $\sigma_{n\pi}$ and σ_{ny}.

1.2.3 Robust pattern

Figure 1.2 (b) reports estimated $\widetilde{\rho}$ for the sample countries with regard to the level of data uncertainty in each country, where $\widetilde{\rho}$ is the coefficient on lagged interest rate in the Taylor rule equation (1.6). Note that $\widetilde{\rho}$ is positively related with the level of data uncertainty. The inclusion of the lagged interest rate in the Taylor rule estimation can be rationalized by the substantial increase in adjusted-R^2 in figure 1.2 (c).

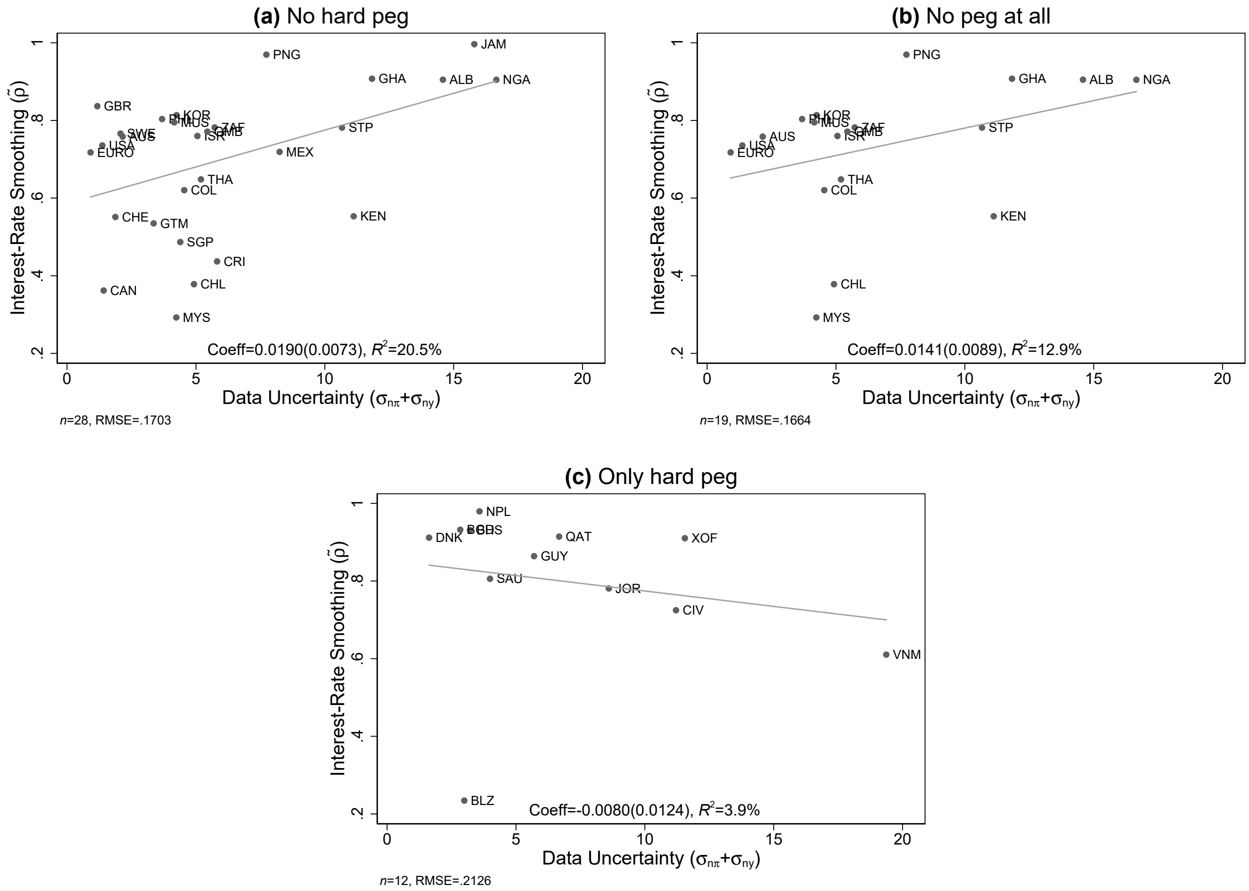
This cross-country pattern is robust to controlling currency pegs, country income levels, exchange rates, and federal funds rates, as reported in figure 1.3, 1.4 and 1.5.

Figure 1.3 shows that the cross-country pattern is robust in the subset of sample countries without hard currency peg. Panel (a) reports the scatter plot of the subset of the countries without a hard currency peg. Panel (b) reports the scatter plot of the subset of the countries without any currency peg (hard peg and soft peg). Panel (c) reports the scatter plot of the subset of the countries only with hard currency peg, and the scatter plot implies that the relationship does not hold among those countries with a hard currency peg. This is not surprising because domestic data uncertainty would not affect a central bank's monetary policy if its currency is perfectly pegged to another currency.

Figure 1.4 shows that the cross-country pattern is robust when country income levels are considered. Panel (a), (b) and (c) report the scatter plot within each subset of the sample countries with high, middle, and low income, respectively. The cross-country pattern among low-income countries is also consistent with the others if countries with a hard currency peg are eliminated.

The cross-country pattern is robust when real effective exchange rates and federal funds rates are controlled. Real effective exchange rate and federal funds rate are added in addition

Figure 1.3: Cross-country comparison of interest-rate smoothing ($\widetilde{\rho}$) regarding currency pegs



Note: These figures present subsets of the scatterplot in Figure 1.2 (b) with regard to exchange rate regimes. The classifications of exchange rate regimes are based on Shambaugh (2004) and Klein and Shambaugh (2008).

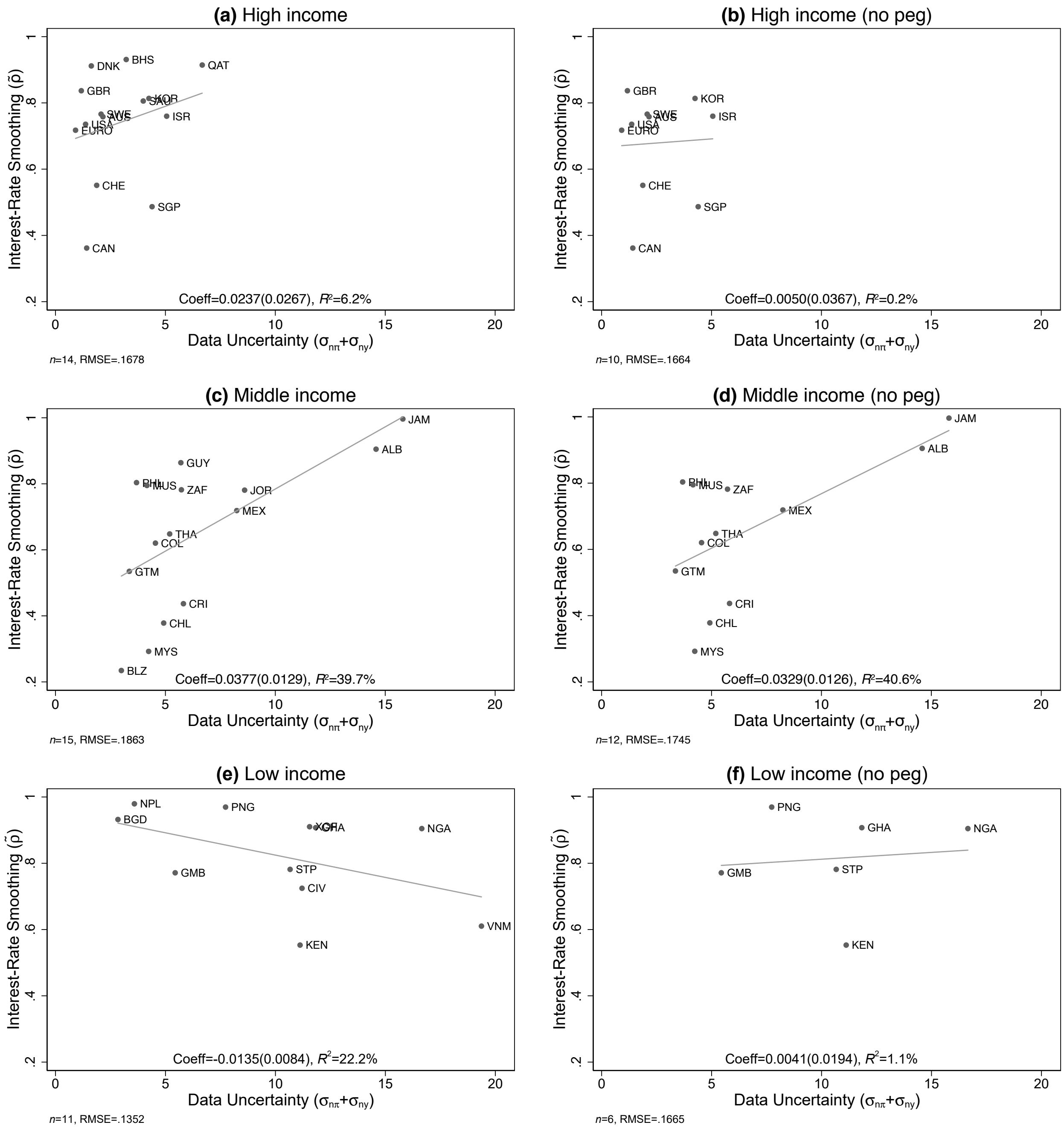
to the baseline equation (equation (1.6)). I estimate the following specifications of the partial-adjustment Taylor rule for each country c using the least squares:

$$i_{t,c} = (1 - \widetilde{\rho_c})[k_c + \widetilde{g_{\pi,c}}\pi_{t+1,c} + \widetilde{g_{y,c}}y_{t,c} + \widetilde{g_{RER,c}}RER_t] + \widetilde{\rho_c}i_{t-1,c} + \varepsilon_{t,c} \tag{1.15}$$

$$i_{t,c} = (1 - \widetilde{\rho_c})[k_c + \widetilde{g_{\pi,c}}\pi_{t+1,c} + \widetilde{g_{y,c}}y_{t,c} + \widetilde{g_{FFR,c}}FFR_t] + \widetilde{\rho_c}i_{t-1,c} + \varepsilon_{t,c} \tag{1.16}$$

where $i_{t,c}$ is annualized monetary policy-related interest rate, $\pi_{t+n,c|t}$ is n-period ahead annualized CPI inflation rate forecast, $y_{t,c|t}$ is output gap measured by the percentage difference between real GDP and estimated potential GDP, $RER_{t,c}$ is real effective exchange rate (CPI base), and FFR_t is the annualized federal funds rate. The horizon of the inflation forecast is chosen to be one year. The scatter plots are reported in figure 1.5, and the cross-country

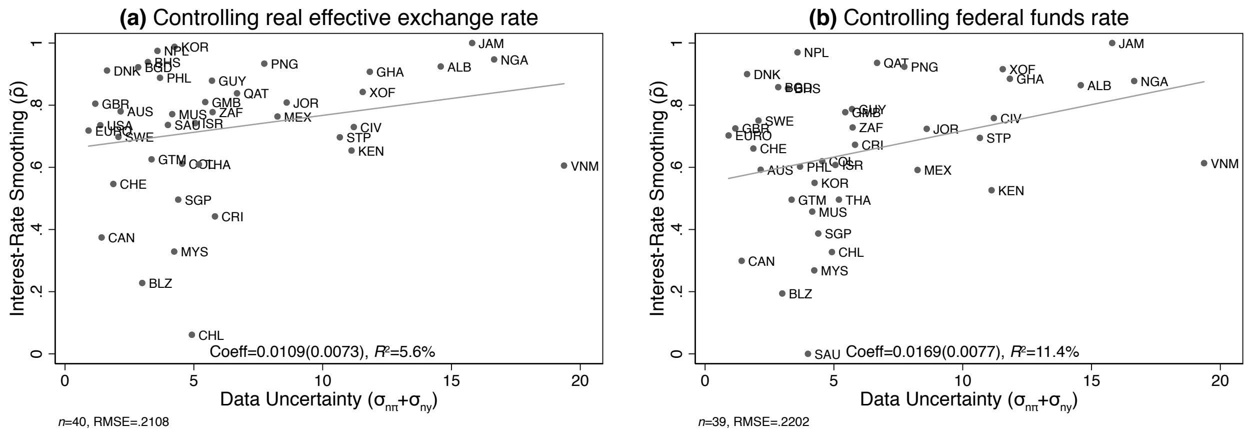
Figure 1.4: Cross-country comparison of interest-rate smoothing ($\widetilde{\rho}$) by income level



Note: These figures present subsets of the scatterplot in Figure 1.2 (b) by country income level. The classifications of country income levels are based on the World Development Indicators database of the World Bank. In this database, income is measured using gross national income (GNI) per capita, in U.S. dollars, which is converted from local currency using the World Bank Atlas method. The classifications of exchange rate regimes are based on Shambaugh (2004) and Klein and Shambaugh (2008).

pattern is robust in both panels.

Figure 1.5: Cross-country comparison of interest-rate smoothing ($\widetilde{\rho}$) controlling real effective exchange rate and federal funds rate



Note: Panel (a) and (b) report estimated $\widetilde{\rho}$ for 40 sample countries, based on the equation (1.15) and (1.16), respectively. Real effective exchange rate (RER_t) data are CPI-based and drawn from IMF's IFS dataset, and the annualized federal funds rates (FFR_t) are drawn from FRED dataset by the Federal Reserve Bank of St. Louis. Panel (b) does not include the United States.

1.3 Theoretical Framework

The robust cross-country pattern–the higher data uncertainty, the higher weight on lagged interest rate in the estimated Taylor rule–is further explored within a simple New Keynesian framework. I introduce a model that describes the economy in which a central bank minimizes its loss by choosing the monetary policy rule parameters. Three types of monetary policies are considered: monetary policy under perfect information, naive monetary policy, and monetary policy with learning.

1.3.1 Model setup

The model is taken from Rudebusch and Svensson (1999) with some modifications. The optimal policy rules are derived in a simple model of output and inflation:

$$\pi_t = \alpha_0 + \alpha_{\pi 1}\pi_{t-1} + \alpha_{\pi 2}\pi_{t-2} + \alpha_{\pi 3}\pi_{t-3} + \alpha_{\pi 4}\pi_{t-4} + \alpha_y y_{t-1} + \varepsilon_t \tag{1.17}$$

$$y_t = \beta_0 + \beta_{y1}y_{t-1} + \beta_{y2}y_{t-2} + \beta_r(\bar{\imath}_{t-1} - \overline{\pi}_{t-1}) + \eta_t \tag{1.18}$$

where π_t is inflation rate, y_t is output gap, i_t is interest rate, $\bar{\imath}_t = \frac{1}{4}\Sigma_{j=0}^{3} i_{t-j}$, and $\overline{\pi}_t = \frac{1}{4}\Sigma_{j=0}^{3}\pi_{t-j}$.[10] It is assumed that the central bank minimizes the variation in inflation around its target π^*, in the output gap (y_t), and in changes in the interest rate (Δi_t). Expected loss equals the weighted sum of unconditional variances,

$$E[L_t] = Var[\overline{\pi}_t - \pi^*] + \lambda_y Var[y_t] + \lambda_i Var[\Delta i_t] \tag{1.19}$$

where $\Delta i_t = i_t - i_{t-1}$, and the parameters $\lambda_y \geq 0$ and $\lambda_i \geq 0$ are defined as the relative weights on output stabilization and interest-rate smoothing, respectively, with respect to inflation stabilization. The central bank has the forward-looking policy rule in the form of:

$$i_t = (1-\rho)(k + g_\pi \pi_{t+1|t} + g_y y_{t|t}) + \rho i_{t-1}, \tag{1.20}$$

where $\pi_{t+1|t}$ and $y_{t|t}$ are the central bank's forecast and inference on the future inflation, π_{t+1}, and the true output gap, y_t, and chooses the optimal values for g_π, g_y, and ρ. The Taylor rule sets the interest rate in quarter t on the basis of real-time noisy indicators on inflation, π_t^n, and output gap, y_t^n, which are defined as:

$$\pi_t^n = \pi_t + n_t^\pi \tag{1.21}$$

$$y_t^n = y_t + n_t^y \tag{1.22}$$

n_t^π and n_t^y are the contemporaneous measurement errors that plague the policymaker in real time, with standard errors $\sigma_{n\pi}$ and σ_{ny}, respectively, and uncorrelated with π_t and y_t. The measurement errors, n_t^π and n_t^y, are modeled as first-order moving average (MA(1)) processes:

$$n_t^\pi = \epsilon_t^\pi + \theta^\pi \epsilon_{t-1}^\pi \tag{1.23}$$

$$n_t^y = \epsilon_t^y + \theta^y \epsilon_{t-1}^y \tag{1.24}$$

where ϵ_t^π and ϵ_t^y are normally distributed with mean zero and variance $\sigma_{\epsilon\pi}^2$ and $\sigma_{\epsilon y}^2$, respectively.

Depending on the central bank's information and policy type, I introduce three monetary policy cases: perfect information and naive monetary policy as benchmark cases and

[10]The model is in quarterly frequency.

monetary policy with learning for the main analysis. For each monetary policy case, the expectation on future inflation rate ($\pi_{t+1|t}$) and the expectation on current GDP gap ($y_{t|t}$) differ, and these lead to different reactions of the central bank.

1.3.2 Perfect information

Under the assumption that the central bank always has perfect information (i.e., $\pi_{t|t} = \pi_t$ and $y_{t|t} = y_t$ in equation (1.20)), optimal policy is described by:

$$i_t = (1 - \rho^P)(k + g_\pi^P E_t[\pi_{t+1}] + g_y^P y_t) + \rho^P i_{t-1} \tag{1.25}$$

where $E_t[\pi_{t+1}]$ is the rational expectation on π_{t+1} using all current and past perfect information at time t. The central bank optimally chooses its response parameters, g_π^P, g_y^P and ρ^P, to minimize the loss function (equation (1.19)). These response parameters are invariant to the magnitude of noise in the data because the central bank always observes and responds to the true data.

1.3.3 Naive monetary policy

The policymaker may take the noisy indicators at face value without making any inference about the true inflation rate and GDP gap (i.e., $\pi_{t|t} = \pi_t^n$ and $y_{t|t} = y_t^n$ in equation (1.20)). The optimal policy is described by:

$$i_t = (1 - \rho^N)(k + g_\pi^N E_t[\pi_{t+1}] + g_y^N y_t^n) + \rho^N i_{t-1} \tag{1.26}$$

where $E_t[\pi_{t+1}]$ is the rational expectation on π_{t+1} using all available information at time t, which can be derived from the Phillips curve (equation (1.17)) using current and past noisy data.[11] The central bank optimally chooses rule parameters, g_π^N, g_y^N and ρ^N, such that minimize the loss function (equation (1.19)). Since the true data are not observable, the central bank chooses different rule parameters as data become noisier.

[11] At the time of decision, the central bank's information set includes recent four quarters of noisy indicators $\{\pi_t^n, \pi_{t-1}^n, \pi_{t-2}^n, \pi_{t-3}^n; y_t^n, y_{t-1}^n, y_{t-2}^n, y_{t-3}^n\}$ and historical true data $\{\pi_{t-4}, \pi_{t-5}, \ldots, \pi_{t-\infty}; y_{t-4}, y_{t-5}, \ldots, y_{t-\infty}\}$ assuming that the noisy indicators are revised and become true values after four quarters.

1.3.4 Monetary policy with learning

Naive monetary control is efficient in the absence of noise but is inefficient when noise is present in the data since the policymaker can reduce her loss using forecastable components in the noise process. By implementing the Kalman filter, the central bank makes optimal inferences on the inflation rate and GDP gap given available information.

Kalman filtering with noisy information

The model described in section (1.3.1) has a state-space representation,

$$X_{t+1} = AX_t + Bi_t + \boldsymbol{\nu}_{t+1} \tag{1.27}$$

The 10×1 vector X_t of state variables, the 10×10 matrix A, the 10×1 column vector B, and the 10×1 column disturbance vector $\boldsymbol{\nu}_t$ are given by

$$X_t = \begin{bmatrix} 1 \\ \pi_t \\ \pi_{t-1} \\ \pi_{t-2} \\ \pi_{t-3} \\ y_t \\ y_{t-1} \\ i_{t-1} \\ i_{t-2} \\ i_{t-3} \end{bmatrix}, A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha_0 & \alpha_{\pi 1} & \alpha_{\pi 2} & \alpha_{\pi 3} & \alpha_{\pi 4} & \alpha_y & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_0 & -\beta_r/4 & -\beta_r/4 & -\beta_r/4 & -\beta_r/4 & \beta_{y1} & \beta_{y2} & \beta_r/4 & \beta_r/4 & \beta_r/4 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \beta_r/4 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \boldsymbol{\nu}_t = \begin{bmatrix} 0 \\ \varepsilon_t \\ 0 \\ 0 \\ 0 \\ \eta_t \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The central bank's observation equation is given as

$$Z_t = CX_t + \boldsymbol{w}_t \tag{1.28}$$

where

$$Z_t = \begin{bmatrix} \pi_t^n \\ y_t^n \end{bmatrix}, C = \begin{bmatrix} e_2 \\ e_6 \end{bmatrix}, \boldsymbol{w}_t = \begin{bmatrix} n_t^\pi \\ n_t^y \end{bmatrix},$$

and e_j denotes a 1×10 row vector with element j equal to unity and all other elements equal to zero. Z_t is the observation vector which consists of noisy indicators, C_t is the observation model that maps the true state space into the observed space, and $\boldsymbol{w}_t$ is the vector of observation noises.[12]

Optimal Kalman gain K (10×2 matrix) and predicted error covariance $P_{t|t-1}$ (10×10 matrix) are specified as

$$\begin{aligned} K &= P_{t|t-1}C^T(CP_{t|t-1}C^T + V_w)^{-1} \\ P_{t|t-1} &= A(P_{t|t-1} - KCP_{t|t-1})A^T + V_\nu \end{aligned} \tag{1.29}$$

where V_ν (10×10 matrix) and V_w (2×2 matrix) are variance-covariance matrices of ν_t and w_t, respectively.

The central bank's optimal inference $X_{t|t}$ and forecast $X_{t+1|t}$ are in recursive form as

$$\begin{aligned} X_{t|t} &= X_{t|t-1} + K(Z_t - Z_{t|t-1}) \\ &= (I - KC)AX_{t-1|t-1} + (I - KC)Bi_{t-1} + KZ_t \end{aligned} \tag{1.30}$$

$$X_{t+1|t} = AX_{t|t} + Bi_t \tag{1.31}$$

where $X_{t|t-1}$ denotes predicted (a priori) state estimate and $Z_t - Z_{t|t-1}$ denotes innovation (measurement pre-fit residual).[13] In each period, by observing noisy data, the central bank learn about the noise process and decide how much weight (K) will be placed on upcoming noisy observation while the rest of the weight ($I - K$) will be placed on the prior inference.

Optimal monetary policy with learning

The central bank's policy rule in equation (1.20) can be written as

$$i_t = (1 - \rho)(k + GX_{t|t}) + \rho i_{t-1} \tag{1.32}$$

where

$$G = \begin{bmatrix} g_\pi & g_y \end{bmatrix} \begin{bmatrix} e_2 A \\ e_6 \end{bmatrix} \tag{1.33}$$

and e_j denotes a 1×10 row vector with element j equal to unity and all other elements equal to zero.[14]

[12]Standard Kalman filter assumes zero-mean Gaussian white noise for $\boldsymbol{w}_t$. In this essay, I introduce an auxiliary random process so that $\boldsymbol{w}_t$ follows MA(1) within the Kalman filter. Please refer to appendix (A.1) for more details.

[13]Note that $Z_{t|t-1} = CX_{t|t-1}$ and $X_{t|t-1} = AX_{t-1|t-1} + Bi_{t-1}$.

[14]Note that $\pi_{t+1|t} = e_2 X_{t+1|t} = e_2(AX_{t|t} + Bi_t) = e_2 AX_{t|t}$ and $y_{t|t} = e_6 X_{t|t}$.

The central bank optimally chooses response coefficients, g_π, g_y and ρ, minimizing the loss function (equation (1.19)) based on the central bank's inferences ($X_{t|t}$).

1.4 Results

Using the model described above, I estimate the optimal monetary policy rule parameters, g_π, g_y and ρ, under the three information and policy cases. Then, I run reduced-form Taylor rule regressions on simulated data, which are generated by the same model, using the specification in section (1.2.1). I show that the optimal level of gradual adjustment (ρ) derived from the model and the reduced-form estimate on the lagged interest rate ($\widetilde{\rho}$) move toward the opposite directions when the central bank faces more data uncertainty. I describe the results and explain the underlying intuition in this section.

The model in the previous section is calibrated using the following parameters. The parameters in Phillips and IS curves (equations (1.23) and (1.24)) are estimated in the data, and the parameters in the loss function are assumed to be $\lambda_y = 1$ and $\lambda_i = 0.5$, as apprear in Rudebusch and Svensson (1999).[15] Later in this section, I consider the loss function with $\lambda_i = 0$ in a purpose of muting the central bank's cautious motive. The parameters in MA(1) noise processes (equations (1.23) and (1.24)) are estimated for 40 sample countries in the historical data, and, for example, they are $\theta^\pi = 0.51$ and $\theta^y = 0.58$ in the United States.

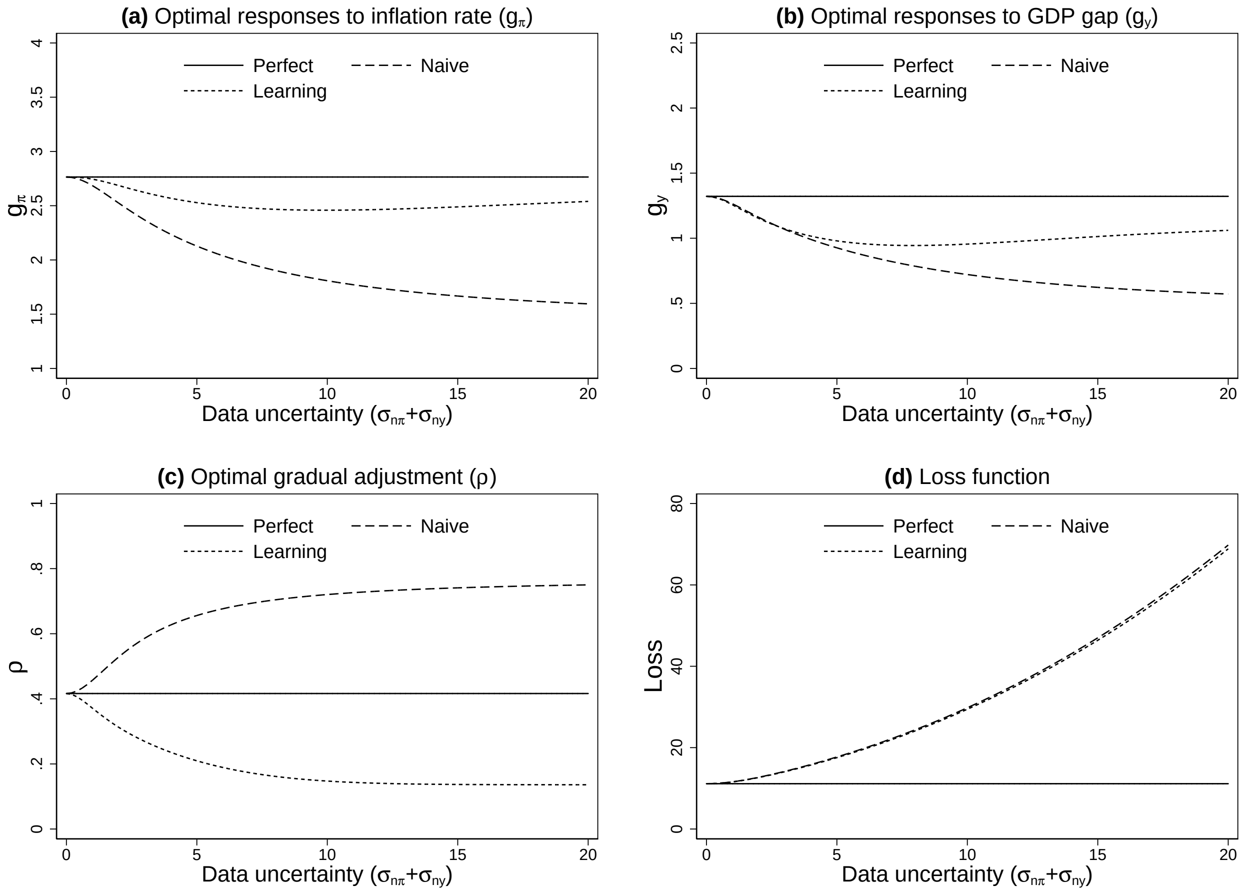
1.4.1 Optimal monetary policy under data uncertainty

Depending on information and policy type, the central bank optimally responses to noise in the data. The optimal rule parameters, g_π, g_y and ρ, are estimated using the grid search and reported in figure 1.6. The optimal responses and the associated loss under perfect information, naive policy, and learning policy are described as solid lines, long-dashed lines, and short-dashed lines, respectively.

Under the perfect information case, the central bank observes data without error and

[15] It is called a "strict" inflation targeting if only inflation enters the loss function ($\lambda_y = \lambda_i = 0$). A "flexible" inflation targeting allows other goal variables in the loss function. See Rudebusch and Svensson (1999) for detailes.

Figure 1.6: Optimal rule parameters and loss function subject to data uncertainty in US



Note: Figure 1.6 reports the responses of optimal monetary policy rule parameters, g_π, g_y and ρ, under perfect information, naive monetary policy, and monetary policy with learning, to noise in the data. The parameters are estimated with grid search.
The estimated parameters in Phillips and IS curves (equation (1.17) and (1.18)) are described as:

$$\pi_t = \underset{(0.09)}{0.08} + \underset{(0.08)}{0.67}\pi_{t-1} - \underset{(0.10)}{0.08}\pi_{t-2} + \underset{(0.10)}{0.29}\pi_{t-3} + \underset{(0.08)}{0.12}\pi_{t-4} + \underset{(0.03)}{0.15}y_{t-1} + \varepsilon_t, \sigma_{\epsilon\pi} = 1.007$$

$$y_t = \underset{(0.10)}{0.19} + \underset{(0.08)}{1.17}y_{t-1} - \underset{(0.08)}{0.27}y_{t-2} - \underset{(0.03)}{0.09}(\bar{\imath}_{t-1} - \overline{\pi}_{t-1}) + \eta_t, \sigma_{\epsilon y} = 0.822$$

and the estimated parameters in MA(1) noise processes (equations (1.23) and (1.24)) are $\theta^\pi = 0.51$ and $\theta^y = 0.58$.

responds to the true inflation rate and GDP gap. The optimal responses and loss are independent of data uncertainty. On the other hand, the central bank's responses, g_π and g_y, under naive and learning policies generally decrease in data uncertainty. Panel (d) reports that loss strictly increases in data uncertainty under both policy cases.

The optimal degree of gradual adjustment (ρ), which is the parameter of interest, increases in data uncertainty under the naive policy and decreases in data uncertainty under the learning policy, as described in panel (c). The optimal degree of gradual adjustment

(ρ) is determined by the tradeoff in the loss function, which consists of three components: $Var[\overline{\pi}_t - \pi^*]$, $Var[y_t]$, and $Var[\Delta i_t]$. Each component varies over ρ, and the central bank orchestrates the contribution of each component by choosing the optimal ρ that minimize the loss function. Note that $Var[\overline{\pi}_t - \pi^*]$ and $Var[y_t]$ increase in ρ, while $Var[\Delta i_t]$ decreases in ρ.[16] If $\lambda_i = 0$, which implies that the loss is independent of interest-rate volatility, then the optimal ρ is zero regardless of data uncertainty in all three cases, because both $Var[\overline{\pi}_t - \pi^*]$ and $Var[y_t]$ uniformly increase in ρ. If $\lambda_i > 0$, then the optimal ρ is greater than or equal to zero, and it increases in data uncertainty under the naive policy and decreases in data uncertainty under the learning policy.

The main reason why ρ moves differently under different policy case can be explained with the relative contribution of $Var[\Delta i_t]$ to the loss: the relative contribution of $Var[\Delta i_t]$ in the loss function increases in data uncertainty under the naive policy and decreases in data uncertainty under the learning policy, as described in figure 1.7. Using the Taylor rule in equation (1.32), $Var[\Delta i_t]$ can be written as

$$\begin{aligned} Var[\Delta i_t] &= Var\left[[(1-\rho)(k + GX_{t|t}) + \rho i_{t-1}] - [(1-\rho)(k + GX_{t-1|t-1}) + \rho i_{t-2}]\right] \\ &= Var\left[(1-\rho)(G\Delta X_{t|t}) + \rho \Delta i_{t-1}\right] \end{aligned} \tag{1.34}$$

where $\Delta X_{t|t} = X_{t|t} - X_{t-1|t-1}$ and $\Delta i_{t-1} = i_{t-1} - i_{t-2}$. Since $Var[\Delta i_t] = Var[\Delta i_{t-1}]$ in steady state,

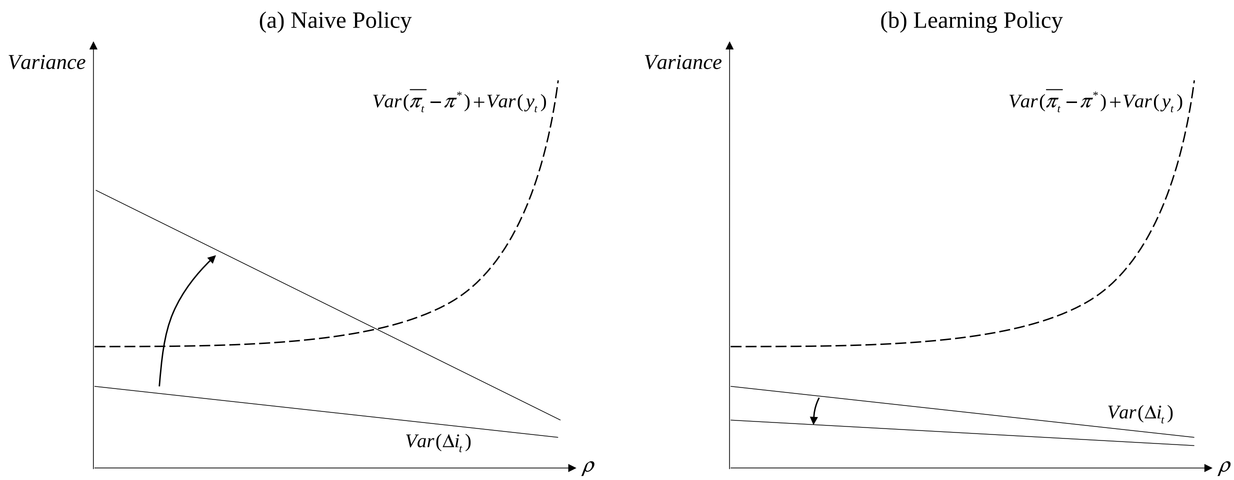
$$Var[\Delta i_t] = \frac{1-\rho}{1+\rho} Var(G\Delta X_{t|t}) + \frac{2\rho}{1+\rho} Cov(G\Delta X_{t|t}, \Delta i_{t-1}) \tag{1.35}$$

where $\frac{1-\rho}{1+\rho}$ is a decreasing function and $\frac{2\rho}{1+\rho}$ is an increasing function in ρ. Fixing everything else constant, $Var[\Delta i_t]$ decreases in ρ because $Var(G\Delta X_{t|t})$ is positive and $Cov(G\Delta X_{t|t}, \Delta i_{t-1})$ is negative in the absence of noise.[17] When the data become noisier, $Var(G\Delta X_{t|t})$ and $Cov(G\Delta X_{t|t}, \Delta i_{t-1})$ change differently under different policy.

[16] If ρ approaches to one, then the interest rate process becomes close to a unit root process, whose variance diverges to infinity. Noting that π_t and y_t are functions of current and lagged interest rates in equation (1.17) and (1.18), $Var[\overline{\pi}_t - \pi^*]$ and $Var[y_t]$ increase in ρ. On the other hand, $Var[\Delta i_t]$ decreases in ρ since high ρ reduces volatility in interest rates.

[17] For $0 \leq \rho \leq 1$, $\frac{1-\rho}{1+\rho}$ is convex and monotone decreasing in ρ from 1 to 0, and $\frac{2\rho}{1+\rho}$ is concave and monotone increasing in ρ from 0 to 1.

Figure 1.7: Decomposition of the loss function



Note: These diagrams describe the shifts of $Var(\Delta i_t)$ subject to data uncertainty under naive policy and learning policy, respectively, adjusting the scales. Higher data uncertainty increases the slope of $Var(\Delta i_t)$ curve under the naive policy and decreases the slope of $Var(\Delta i_t)$ under the learning policy. The shape of $[Var(\overline{\pi}_t - \pi^*) + Var(y_t)]$ curves relatively does not change much under both policies.

Naive monetary policy

The optimal choice of ρ increases in data uncertainty because $Var[\Delta i_t]$ curve in the figure 1.7 becomes steeper due to the following reason. When there is data uncertainty, the central bank's responses, g_π and g_y, become less useful since they are the responses not only to the true inflation and GDP gap but also to the noise. If the central bank keeps ρ fixed, it causes high fluctuations in interest rates resulting in greater $Var[\Delta i_t]$ in the loss function and a steeper $Var[\Delta i_t]$ curve in the figure 1.7 (a). Therefore, the central bank raises ρ to stabilize the interest rates.

An algebraic explanation is the following. Note that $X_{t|t} = Z_t$ under the naive policy, then equation (1.35) becomes

$$Var[\Delta i_t] = \frac{1-\rho}{1+\rho} Var(G\Delta Z_t) + \frac{2\rho}{1+\rho} Cov(G\Delta Z_t, \Delta i_{t-1}) \tag{1.36}$$

$Var[\Delta i_t]$ curve in the figure 1.7 (a) becomes steeper as data uncertainty increases because: (i) $\frac{1-\rho}{1+\rho}$ decreases in ρ and $Var(G\Delta Z_t)$ increases very much in noise; (ii) the second term, $\frac{2\rho}{1+\rho}$ increases in ρ and $Cov(G\Delta Z_t, \Delta i_{t-1})$ decreases in noise (from negative to further negative), when evaluated at empirically relevant parameter values. The marginal benefit of increasing

ρ becomes large.

Monetary policy with learning

The optimal responses to the inflation rate and GDP gap are moderate under learning policy, compared to those under the naive policy, because the central bank's inferences on the true data are more informative than noisy data. The loss is also smaller under the learning policy. One interesting finding is that the optimal interest-rate smoothing parameter, ρ, decreases in data uncertainty.

The central bank's optimal choice of ρ decreases in data uncertainty because the learning process (i) can serve to effectively filter the noise, and (ii) tease out from the persistence of the noise additional information about the true data. The learning process helps the central bank to make decisions based on the much less noisy information. The second point needs to be more discussed. The noise in the data is assumed to follow MA(1) process, and the estimated θ^{π} and θ^{y} are both positive in almost all countries in the sample, implying a positive autocorrelation in the noise. A simple thought experiment helps in understanding how the central bank learns the additional information from the persistent noise.

Let's say there was a positive noise shock in the past period, then that means the optimal policy in that period resulted in a higher interest rate than it would be under perfect information. Due to a positive autocorrelation in the noise process, the chance of getting another positive noise shock in the current period is more likely. If the central bank increases or keeps ρ constant, then the interest rate in the current period will likely be higher than the central bank wants because the interest rate in the past period is already high. Therefore, the central bank reduces ρ.

The same logic applies to the other way. If there was a negative noise shock in the past period, then that means the optimal policy in that period resulted in a lower interest rate than it would be under perfect information. Due to a positive autocorrelation in the noise process, the chance of getting another negative noise shock in the current period is more likely. If the central bank increases or keeps ρ constant, then the interest rate in the current period will likely be lower than the central bank wants because the interest rate in the past period is already low. Therefore, the central bank reduces ρ. In both cases, the central bank can reduce interest rate volatility by lowering ρ.

This thought experiment can be supported by figure 1.8 (a). If noise is persistent, or the noise process has a positive autocorrelation ($\theta > 0$), then optimal ρ decreases in data uncertainty. On the other hand, if noise is not persistent, or the noise process has a negative autocorrelation ($\theta < 0$), then optimal ρ increases in data uncertainty.[18]

As mentioned above, the optimal degree of gradual adjustment (ρ) is determined by the tradeoff in the loss function. The central bank's optimal choice of ρ decreases in data uncertainty because $Var[\Delta i_t]$ curve in the figure 1.7 (b) becomes flatter as data uncertainty increases. An algebraic explanation is the following.

Note that $X_{t|t}$ under the learning policy is given in equation (1.30), then equation (1.35) is

$$Var[\Delta i_t] = \frac{1-\rho}{1+\rho} Var(G\Delta X_{t|t}) + \frac{2\rho}{1+\rho} Cov(G\Delta X_{t|t}, \Delta i_{t-1}) \tag{1.37}$$

where

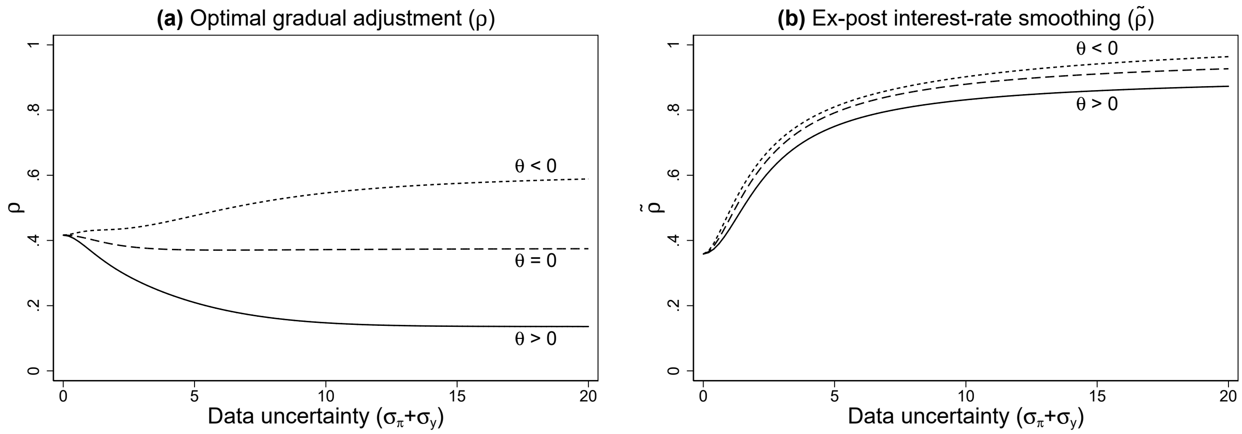
$$\Delta X_{t|t} = (I - KC)A\Delta X_{t-1|t-1} + (I - KC)B\Delta i_{t-1} + K\Delta Z_t \tag{1.38}$$

$Var[\Delta i_t]$ curve in the figure 1.7 (b) becomes flatter as noise increases because (i) $\frac{1-\rho}{1+\rho}$ decreases in ρ and $Var(G\Delta X_{t|t})$ increases little in noise; (ii) the second term, $\frac{2\rho}{1+\rho}$ increases in ρ and $Cov(G\Delta X_{t|t}, \Delta i_{t-1})$ increases very much in noise (from negative to positive), when evaluated at empirically relevant parameter values. The marginal benefit of increasing ρ becomes small.

Under the learning policy, $Var(G\Delta X_{t|t})$ increases little in noise because the learning process effectively filters the noise. The variance is not zero since, even if the learning process is effective, it cannot completely eliminate the noise in the data. $Cov(G\Delta X_{t|t}, \Delta i_{t-1})$ increases very much in noise because the learning process helps the central bank to gain some information from the persistence in the noise. If noise is not persistent, or the noise process has a negative autocorrelation ($\theta < 0$), then this covariance decreases in noise, same as in the naive policy case.

[18] When the noise is white noise ($\theta = 0$), the optimal ρ decreases when data uncertainty is small because the central bank is not sure if the shock is from the noise or from Phillips and IS curves. Note that Phillips and IS curves have shock terms, ε_t and η_t, and their standard errors are 1.007 and 0.822, respectively, in the United States. If data uncertainty is big enough, then the central bank believes that most of the shock is coming from data uncertainty, rather than Phillips and IS curves, and the optimal ρ does not change.

Figure 1.8: Optimal gradual adjustment (ρ) and ex-post interest-rate smoothing ($\widetilde{\rho}$) with varying signs of θ under learning policy



Note: Panel (a) reports the estimated optimal gradual adjustment (ρ) under learning policy when θ is positive, zero, and negative. Panel (b) reports the reduced-form estimate of interest-rate smoothing ($\widetilde{\rho}$) for each θ.

1.4.2 Ex-post estimates using simulated data

I run reduced-form Taylor rule regressions on simulated data from the model and show that, when the central bank faces more data uncertainty, the reduced-form estimate on the lagged interest rate ($\widetilde{\rho}$) may move differently from the optimal level of gradual adjustment (ρ) derived from the model.

Using the optimal responses described in the previous section, I simulate the model and generate 100,000 observations (and the first 20,000 observations are dropped) given each level of data uncertainty from 0 to 20. Using the simulated data, I replicate the empirical finding that higher data uncertainty leads to more interest-rate smoothing (i.e., the greater $\widetilde{\rho}$) by estimating the reduced-form specification in section (1.2.1), that is

$$i_t = (1 - \widetilde{\rho})[k + \widetilde{g_\pi} E_t \pi_{t+1} + \widetilde{g_y} y_t] + \widetilde{\rho} i_{t-1} + \varepsilon_t \tag{1.39}$$

where the expected inflation rate, $E_t\pi_{t+1}$, is measured by the four quarter average inflation rate in percent. The reduced-form estimates, $\widetilde{g_\pi}$, $\widetilde{g_y}$ and $\widetilde{\rho}$, are based on *ex post* observation, and they are reported in figure (1.9).

Note that the central bank's optimal inference $X_{t|t}$ is correlated with i_{t-1}. Combining equation (1.30) and (1.32), the policy rule under the learning policy can be expressed as

$$i_t = (1-\rho)(k + G[(I-KC)AX_{t-1|t-1} + (I-KC)Bi_{t-1} + KZ_t]) + \rho i_{t-1} \tag{1.40}$$

and the reduced-form estimate on i_{t-1} consists of not only ρ but also coefficients on the correlating components in the central bank's belief (let's call it δ, then $\widetilde{\rho} = \rho + \delta$). Figure (1.9) reports responses of the reduced-form estimates, $\widetilde{g_\pi}$, $\widetilde{g_y}$ and $\widetilde{\rho}$, with regard to data uncertainty.

Under the learning policy, the optimal level of gradual adjustment (ρ), calibrated in the model, decreases in data uncertainty, as described in figure 1.6 (c). On the other hand, the reduced-form estimate on the lagged interest rate ($\widetilde{\rho}$) increases in data uncertainty. This explains the *intrinsic interest-rate smoothing* in the literature: the higher the data uncertainty, the greater the reduced-form estimate on the lagged interest rate. Interest-rate smoothing is mostly a consequence of the *learning process*, rather than central banks' *cautious motive*. Central banks' real-time belief about true data is indistinguishable from the cautious motive in ex-post data, both of which are largely picked up by the Taylor rule's reduced-form estimate on the lagged interest rate.

1.4.3 No-cautious-motive constraint

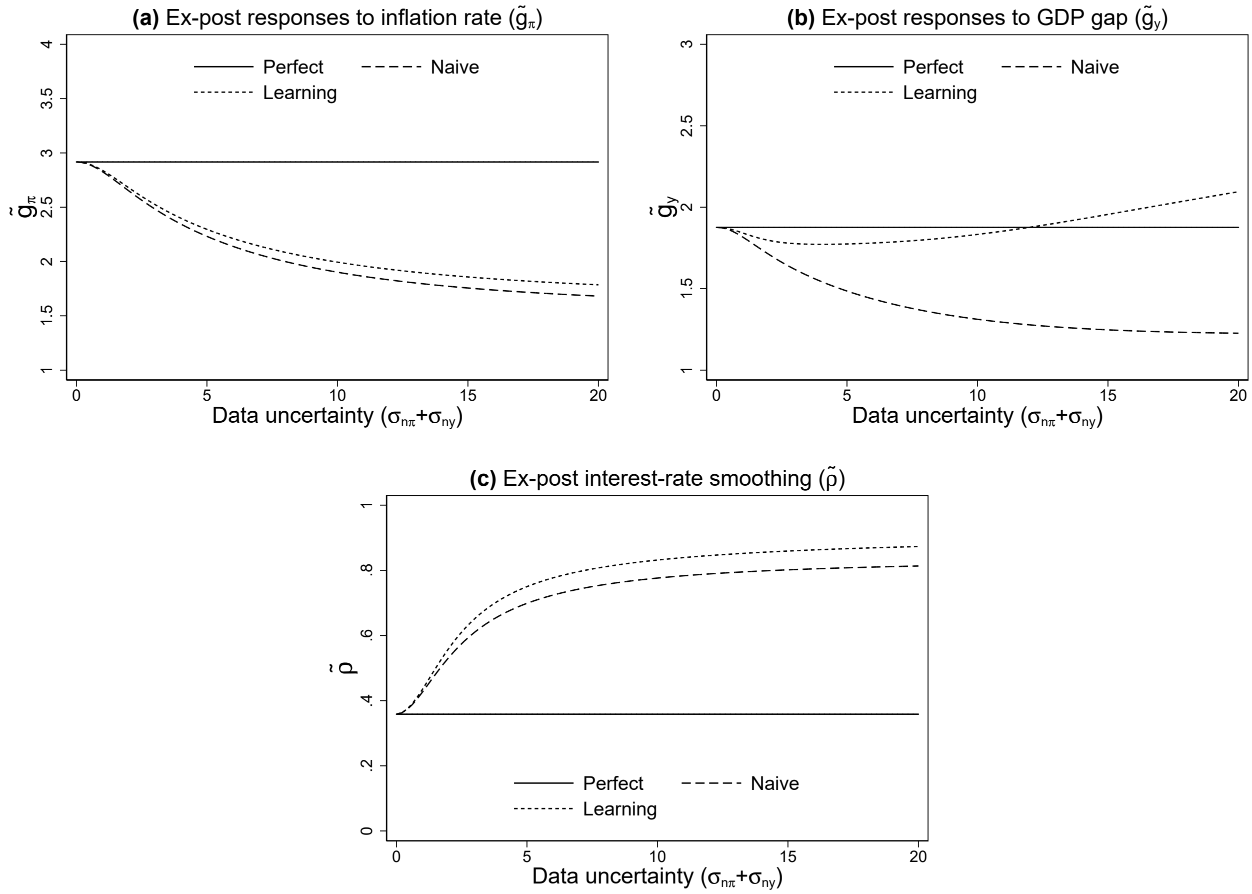
If the central bank has $\lambda_i = 0$, this implies no cautious motive ($\rho = 0$), and the monetary policy rule with learning in equation (1.32) becomes:

$$i_t = k + GX_{t|t} \tag{1.41}$$

Under the additional restriction, the optimal rule parameters, g_π and g_y, can be estimated from the model, and the reduced-form estimates, $\widetilde{g_\pi}$, $\widetilde{g_y}$ and $\widetilde{\rho}$ in equation (1.39), can be obtained from the simulated data. The estimation results are reported in figure 1.10. Panel (a) reports the optimal responses of the rule parameters under the constraint, and panel (b) reports the ex-post estimates of the rule parameters. Panel (c) reports the optimal level of caution (ρ) and interest-rate smoothing parameter ($\widetilde{\rho}$). Panel (d) reports the loss under each monetary policy given the additional constraint.

Since there is no cautious motive ($\lambda_i = 0$), ρ becomes zero, and interest-rate smoothing

Figure 1.9: Reduced-form estimates on ex-post rule parameters using simulated data

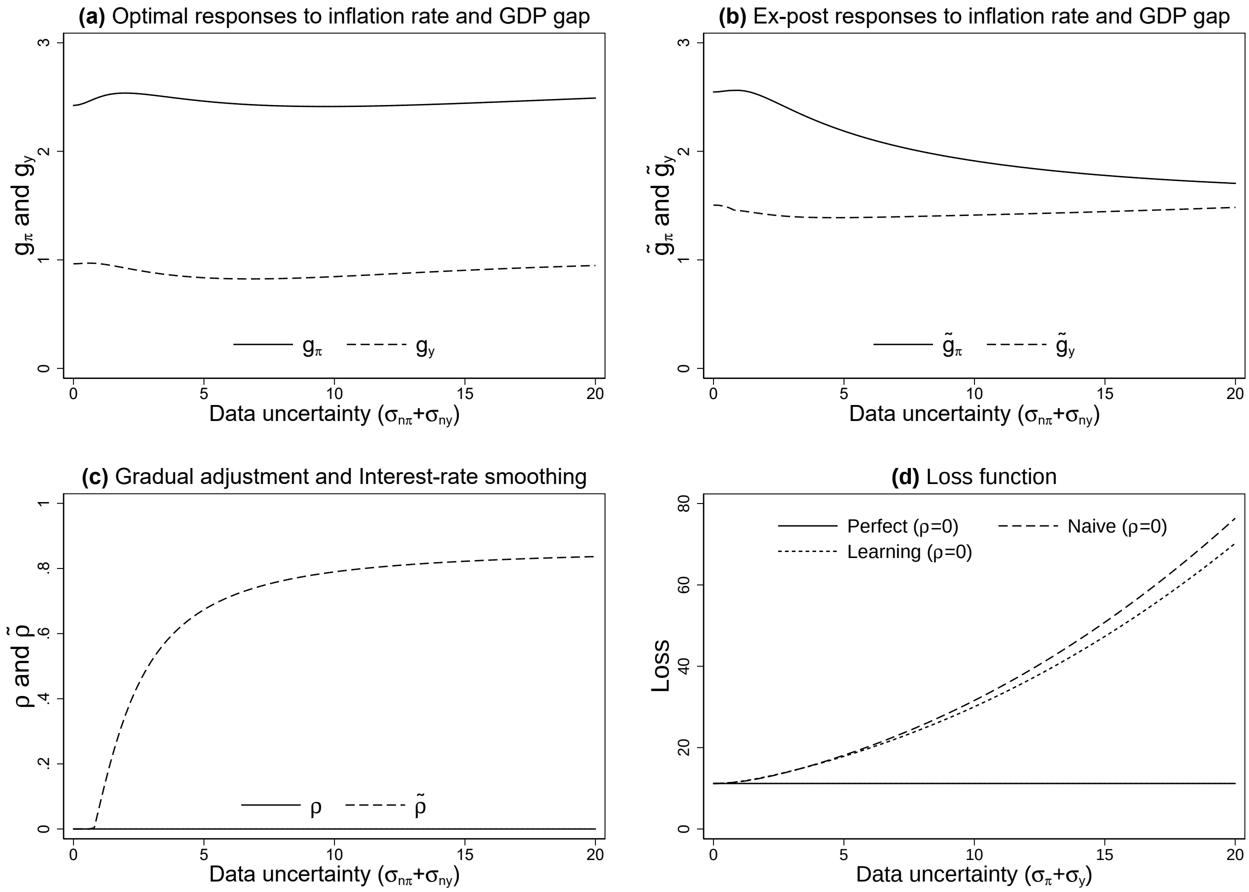


Note: Figure 1.9 reports reduced-form estimates of the Taylor rule (equation (1.39)) based on simulated data from the model. Estimated $\widetilde{g_\pi}$, $\widetilde{g_y}$ and $\widetilde{\rho}$ are reported under perfect information, naive monetary policy, and monetary policy with learning.

($\widetilde{\rho}$) is completely induced by the learning process ($\widetilde{\rho} = \delta$). The reduced-form estimate on the lagged interest rate ($\widetilde{\rho}$) increases in data uncertainty even though the central bank does not conduct gradual adjustment (i.e., $\rho = 0$). A positive estimate on lagged interest rate does not necessarily mean that the central bank gradually adjusts the interest rates.

I show interest-rate smoothing ($\widetilde{\rho}$) to exist and increase in the level of data uncertainty even if central banks' cautious motive is muted by excluding the variance of changes in interest rates from the loss function. Interest-rate smoothing ($\widetilde{\rho}$) can be fully endogenized in the model by the learning process (δ) when there is no cautious motive. If the central bank has some caution, then interest-rate smoothing ($\widetilde{\rho}$) can be decomposed into the learning process (δ) and gradual adjustment (ρ).

Figure 1.10: Optimal responses and ex-post estimates under learning muting cautious motive



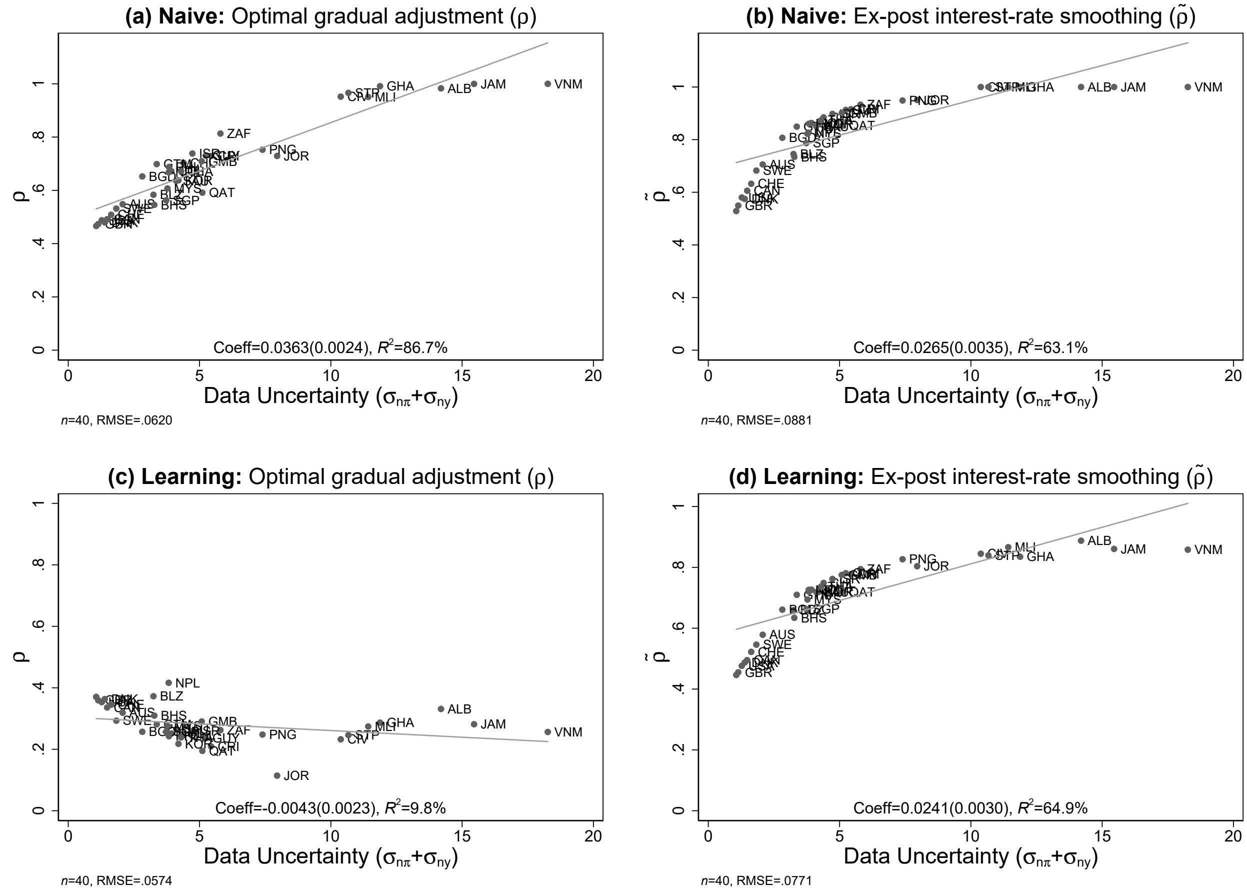
Note: Figure 1.10 reports the optimal responses and ex-post estimates of the Taylor rule when central bank's cautious motive is muted ($\lambda_i = 0$). The loss function does not include the variance of the changes in interest rates, and this leads ρ become zero. Panel (a) reports optimal monetary policy rule parameters, g_π and g_y, under monetary policy with learning. The parameters are estimated with grid search. Panel (b) reports reduced-form estimates of the Taylor rule (equation (1.39)) based on simulated data from the model. Estimated $\widetilde{g_\pi}$ and $\widetilde{g_y}$ are reported under monetary policy with learning. Panel (c) reports ρ and $\widetilde{\rho}$ under monetary policy with learning.

1.4.4 Cross-country anlysis

Country-specific data uncertainty parameters $\{\sigma_{n\pi}, \sigma_{ny}, \theta^\pi, \theta^y\}$ are estimated from the historical WEO data. Each country has different level and relative composition of $\sigma_{n\pi}$ and σ_{ny}, and the levels of noise persistence, θ^π and θ^y, also differ across countries. Given data uncertainty in each country, the model is calibrated to estimate the optimal level of gradual adjustment parameter (ρ) and its *ex-post* reduced-form estimate ($\widetilde{\rho}$) under the naive and learning policy, and these are reported in figure 1.11.

Panel (a) and (b) show that, under the naive policy, both ρ and $\widetilde{\rho}$ increase in data

Figure 1.11: Cross-country estimates and optimal parameters under naive and learning policy



Note: Country-specific data uncertainty parameters $\{\sigma_{n\pi}, \sigma_{ny}, \theta^{\pi}, \theta^{y}\}$ are estimated from the historical WEO data, and the same model is calibrated for each country. Country-specific level of persistence in the noise and ratio in data uncertainty are allowed.

uncertainty. On the other hand, panel (c) and (d) show that, under the learning policy, ρ and $\widetilde{\rho}$ move differently — ρ decreases in data uncertainty and $\widetilde{\rho}$ increases in data uncertainty. Given that the learning model described in this essay is relatively simple compared to the efforts and investment in central banks, panel (d) is most likely the cross-country pattern that we observe in figure 1.2 (b).[19]

[19] The limitation of this practice is that I use the same model to estimate ρ and $\widetilde{\rho}$ for all sample countries. Estimating the Phillips and IS curves for each country requires further discussions, and it is left for future study.

1.4.5 Policy implications

We can think about how central banks can improve their monetary policy under data uncertainty. There are two approaches we can think of: improving learning ability and improving data quality. This research provides a framework that helps analyzing these competing options. We consider two benchmark cases in addition to the learning policy. First, central bank always observes perfect information. Second, central bank observes noisy data and naively take the face value without making any inference. We compare the welfare loss under the learning policy and the naive policy at different levels of data uncertainty and demonstrate that learning policy is always better than the naive policy.[20] The gain from learning is increasing and convex in data uncertainty. Given the cost information of each option, we can conduct a cost-benefit analysis and decide which option is more cost-effective.

1.5 Conclusion

The conventional belief about the relationship between the level of data uncertainty and interest-rate smoothing has been re-examined. The cross-country comparison of coefficients on the lagged interest rate supports the conventional belief, and I could replicate this with an appropriate calibration of the Rudebusch and Svensson (1999) model. However, the interest-rate smoothing is present in the reduced-form estimation of the Taylor rule, not because central banks gradually adjust the interest rates nor central banks are more cautious in the face of data uncertainty, but because the reduced-form estimates are obtained with *ex-post* data, in which the central banks' beliefs are not distinguishable, and because the central banks' inference about the true data is correlated with past interest rates.

This essay distinguishes between interest-rate smoothing and central banks' gradual interest-rate adjustment and demonstrates that the two may not move together. Whereas the conventional view in the literature considers interest-rate smoothing as evidence of central banks' cautious and gradual adjustment, I show that central banks' gradual interest-rate adjustment is directly induced by central banks' preference to avoid interest-rate surprises,

[20] See loss functions in figure 1.6 (d) and figure 1.10 (d).

that is, the variance of the changes in interest rates in the loss function.

This essay endogenizes interest-rate smoothing as a result of the central banks' learning process. I show that interest-rate smoothing exists and increases with data uncertainty, even if central banks' cautious motive is completely muted, by taking out the variance of the changes in interest rates from the loss function. Much of interest-rate smoothing comes from the learning process, and it can be fully endogenized by the cautious motive and the learning process in the model.

Chapter 2

Household and Corporate Credit in Emerging Economies: Symbiosis or Competition?[1]

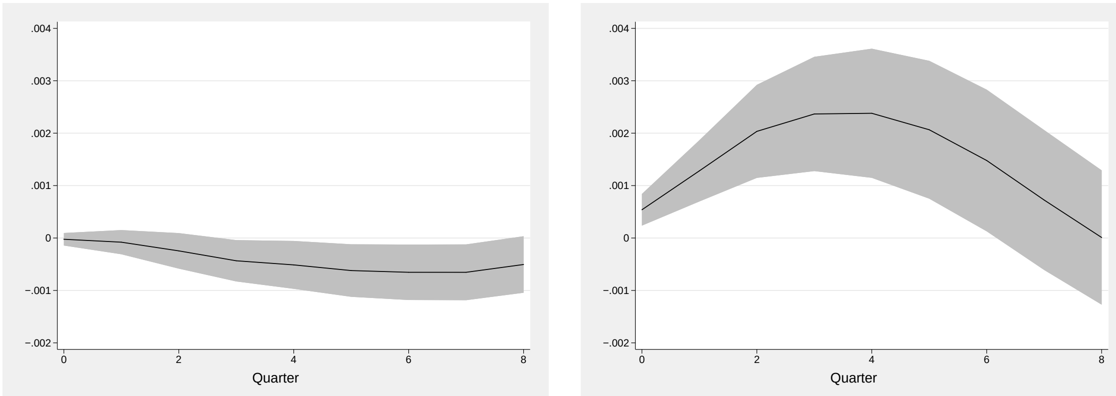
2.1 Introduction

Standard consumption-investment theory (see, for example, Aiyagari (1994), Krusell and Smith (1998), Chang and Kim (2007), and Krusell, Mukoyama and Sahin (2010)) implies that households' tendency to save (borrow) during boom (recession) periods encourages (discourages) investment by firms. The theory, premised on the insight that households are the subject of consumption smoothing, firms of productive investment, predicts household credit to be counter-cyclical and corporate credit pro-cyclical. This essay, using vector autoregressive (VAR) models to investigate the cyclical patterns of corporate and household credit in both developed and emerging economies, sheds light on how households' leveraged investment motives engender pro-cyclical household credit in the latter countries.

Panels (a) and (b) in Figure 2.1, depict, for the United States and South Korea, respectively, the impulse responses of the household credit ratio (=the ratio of household credit to total private sector credit) to a positive productivity shock. Whereas the impulse response of the U.S. economy is consistent with the prediction of standard consumption-investment theory, that of the Korean economy reveals a much volatile and inverted U-shaped pattern. Indeed, the volatile and pro-cyclical response of the household credit ratio in South Korea

[1] I thank Seung-Gyu Sim at Aoyama Gakuin University and Daeyup Lee at the Bank of Korea for collaborating this second essay. Financial support from the Bank of Korea is gratefully acknowledged. We assume responsibility for any errors, and for the views expressed in this essay, which do not necessarily reflect the official opinion of the Bank of Korea.

Figure 2.1: The impulse responses of the household credit ratio



(a) United States (b) South Korea

[Figure 2.1] shows the impulse response functions of household credit ratios (=the ratio of household credit to total private sector credit) subject to a positive productivity shock from 1980:Q1 to 2016:Q4. Panels A and B show the impulse response in the United States and South Korea, respectively. Time-series data used in [Figure 2.1] are publicly available on the web site of the Bank for International Settlements (BIS).

indicates that household credit outgrows corporate credit in response to productivity shocks. Moreover, this pro-cyclical response of the household credit ratio is not unique to the Korean economy, being observed as well in the emerging economies of Brazil, Chile, South Africa, and Thailand.[2] Figure 2.1 requires a new approach in understanding the credit market of emerging economies, pointing out that the previous literature relying on standard consumption smoothing theory and treating household credit as a mirror image of corporate credit may miss dominant players in the credit market.

Our analysis of the South Korean data reveals households' leveraged investment in domestic assets, funded by collateral credit based on their asset holdings, to generate and consolidate a strong linkage between household credit and asset values. Furthermore, data provided by the Bank of Korea (BOK) reveals almost no impulse response in the corporate credit of small- and medium-size South Korean businesses after the foreign currency crisis of 1997, whereas the response of domestic household credit has been substantial and significant during the same period. Apparently, the growth of small- and medium-size businesses has been hindered by households' leveraged investment which is rationalized by its interaction with asset values. In addition, such positive interaction has been sustained in response to

[2]Evidence pertaining to other developing countries is presented in Section 2.

not only domestic productivity, but also U.S. interest rate, shocks, notwithstanding that an increase in the U.S. interest rate is expected to discourage asset investment and reduce asset values. This result implies that the leveraged investment by household in emerging economies can dominate and/or substitute not only corporate investment but also international capital flow, which amplifies aggregate risk of those economies.

Neumeyer and Perri (2005) document business cycles to be more volatile in a sample of emerging than in developed economies, real interest rates to be counter-cyclical, and consumption to be more volatile than output. Fernandez and Gulan (2015), by incorporating financial friction a la Bernanke, Gertler and Gilchrist (1999) into an otherwise standard real business cycle model of a small open economy, show a positive productivity shock to increase not only output, but also the value of collateral assets, thereby reducing the country premium and increasing the credit supply. However, previous studies have paid little attention to households' leverage investment motives and the implied relationship between corporate and household credit, whether symbiosis or competition, and if competition, whether household or corporate dominates. Our empirical findings provide another way to understand the puzzle posed by Neumeyer and Perri (2005): that the pro-cyclical, dominant growth of (collateral) household credit, rationalized by the accompanying growth of asset values, stimulates the pro-cyclical credit supply, generates counter-cyclical interest rates, and amplifies consumption volatility in emerging economies.[3]

The essay proceeds as follows. Section 2.2 briefly describes the data and explains the econometric model we employ. Sections 2.3 focuses on the relationship between corporate and household credit, whether symbiosis or competition, and Section 2.4 interprets the credit market outcome driven by the competitive relationship between them through the lens of Neumeyer and Perri (2005). Section 2.5 concludes.

[3] Han (2017) shows that introducing into a small open economy model collateral constraints for households and limited enforcement constraints for the banking sector á la Gertler, Kiyotaki et al. (2010) and Gertler and Karadi (2011) reconciles the stylized fact in emerging economies.

2.2 The Model

2.2.1 Data description

Real GDP growth (g_t), based on data drawn from the International Financial Statistics (IFS) which is managed by the International Monetary Fund (IMF), is defined as the percentage change in real GDP relative to the corresponding quarter of previous year. We use stock index as a proxy for aggregate domestic asset value. House price index was considered, but not used, as a proxy for aggregate domestic asset value because housing price data are not sensitive to changes in real GDP growth and sensitivity substantially varies with location.[4] Asset value growth (a_t) is defined by the annual percentage change from the corresponding quarter of the previous year, in the KOSPI for Korea and S&P500 for the United States, respectively. Small firm credit (s_t) data drawn from Korea's Financial Supervisory Service are provided by the Bank of Korea. Korean and U.S. 3-year government bond yields (i_t) are from, respectively, the Bank of Korea's Economic Statistics System (ECOS) and Federal Reserve Bank of St. Louis's Federal Reserve Economic Data (FRED).[5]

Time-series data for total credit to non-financial corporations and households are publicly available on the web site of the Bank for International Settlements (BIS). Total credit to households and non-financial corporations data are core debt in USD billions. We generate three credit variables from the BIS data: household credit ratio (hr_t), corporate credit growth (f_t), and household credit growth (h_t).[6] Household credit ratio (hr_t) is defined as the proportion of total credit to households to the sum of total credit to non-financial

[4] In an ideal situation where the credit market clears without any friction, the portfolio choice theory expects that stock price index and house price index move together with high correlation and that the impulse responses of those indices have similar patterns. However, there are two reasons that stock index is a better proxy for aggregate domestic asset value. First, considering households' heterogeneous amenity value, liquidity constraint, search friction and so on, we hardly argue that the observed price and quantity are the outcome of the housing market clearing process. In this case, impulse responses of house price index do not properly reflect households' investment motive. Second, impulse responses of house prices to the productivity shock differ across areas, locations, and houses. The house price index inevitably suffers from substantial measurement and aggregation errors.

[5] U.S. 3-year treasury bond data span the period 1980:Q1 to 2016:Q4; Korean 3-year government bond yield data are available only from 1995:Q2 to 2016:Q4.

[6] One may concern that household credit data may include sales credit, increases in which may induce a significant growth of household credit. But such concern may be allayed by the observation that response in household credit growth to the real GDP shock is substantially weaker in the United States, which is expected to have a higher credit card usage rate, than in Korea.

corporations and total credit to households, in each quarter. Corporate credit growth (f_t) is defined as the percentage change in total credit to non-financial corporations, household credit growth (h_t) as the percentage change in total credit to households, relative to the corresponding quarter of the previous year. Observations are quarterly and, unless stated otherwise, span the period 1980:Q1 to 2016:Q4. All variables are seasonally adjusted and detrended using the Hodrick-Prescott filter with smoothing parameter of 1,600.

We employ credit data on the United States, South Korea, and eight other emerging economies from the BIS dataset, which are Brazil, Chile, Mexico, New Zealand, Singapore, South Africa, Thailand, and Turkey. Figure 2.2 reports the responses of household (h_t) and corporate (f_t) credit growth to a productivity shock (g_t) in the United States, South Korea, Brazil, Chile, South Africa, and Thailand. Note that patterns of relative responses of credit growth are similar among South Korea, Brazil, Chile, South Africa, and Thailand. In the United States, responses of corporate credit growth are more positive and stronger than those of household credit growth, whereas the response patterns are the opposite in the other countries.[7] Mexico, New Zealand, Singapore, and Turkey are not included in the figure because their credit response patterns lie between those of the United States and South Korea.

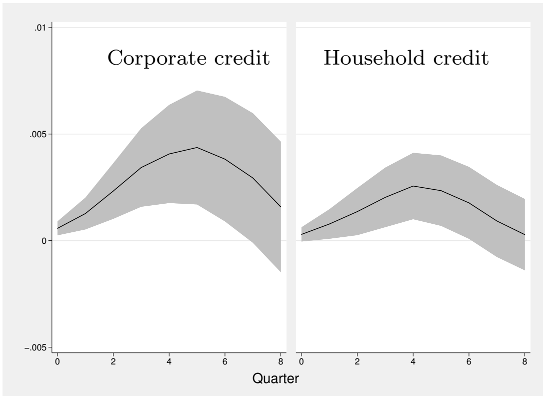
Summary statistics for all variables are reported in Table 2.1. Standard deviations reveal all variables to be more volatile in Korea than in the United States, as reported by other studies in the literature. Correlations between real GDP growth (g_t) and household credit ratio (hr_t) are positive and similar in magnitude in both countries. Correlations between real GDP growth (g_t) and household credit growth (h_t), and between real GDP growth (g_t) and corporate credit growth (f_t), are higher for Korea than for the United States, the magnitude of the differences being much greater for the latter. Correlation between asset value growth (a_t) and household credit ratio (hr_t) is positive in the United States and negative in Korea, but these do not necessarily define the relationship between those variables because the persistency of responses is not taken into account.[8] This simple correlation analysis having

[7]Slightly positive responses of U.S. household credit growth to a productivity shock are attributed to sales credit, consumption in durable goods, and households' investment motive. In contrast, considering relatively small proportion of sales credit in consumption, the responses of Korean and emerging economies' household credit growth to a productivity shock are remarkably strong and significant.

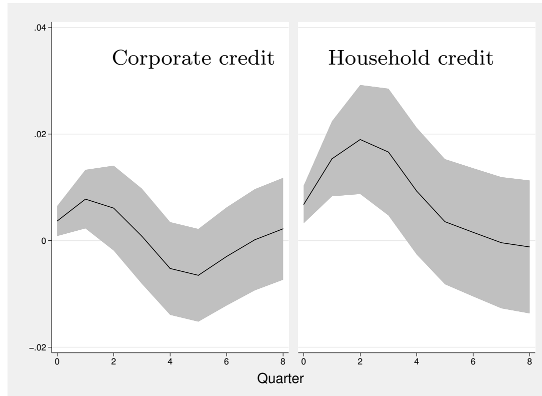
[8]Stock prices are high frequent while household credits are relatively low frequent, and the relative

Figure 2.2: Impulse responses of corporate and household credit growth

Panel A: United States and South Korea

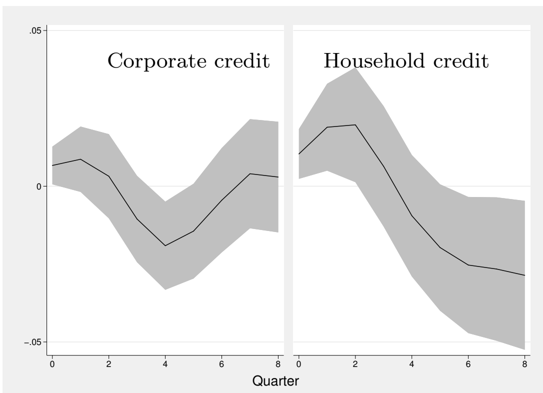


(a) United States (1980:Q1-2016:Q4)

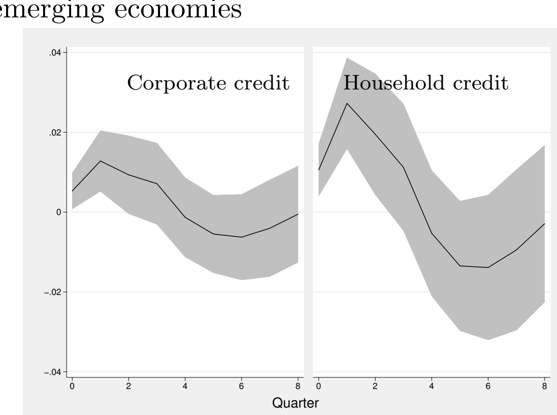


(b) South Korea (1980:Q1-2016:Q4)

Panel B: Other emerging economies

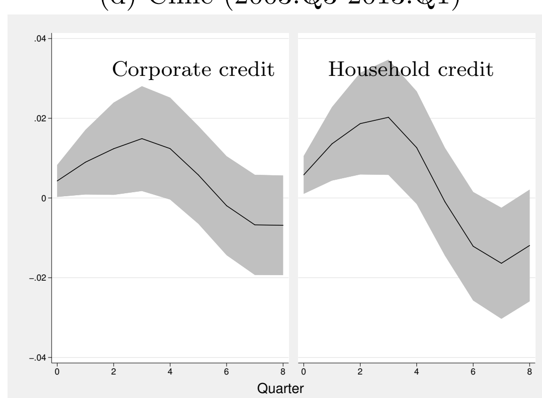


(c) Brazil (1995:Q4-2016:Q4)



(d) Chile (2003:Q3-2013:Q1)

(e) South Africa (2008:Q4-2016:Q4)



(f) Thailand (1994:Q1-2016:Q4)

[Figure 2.2] shows the impulse response functions of corporate (left) and corporate (right) credit subject to a positive productivity shock. IRFs for Chile and South Africa use 4 lags due to relatively shorter time series, IRFs for other countries, 8 lags. Baseline specification ($\mathbf{x}_t = [g_t \quad \mathbf{z}_t']'$ and $\mathbf{z}_t = [a_t \quad h_t \quad f_t]'$,) is estimated. Results are qualitatively robust to lag selection. Solid lines depict point estimates of impulse responses; shaded area represents the 95% confidence interval.

persistency of the variables may differ in each country.

Table 2.1: Summary statistics, quarterly data

United States		g_t	a_t	hr_t	h_t	f_t	b_t	$i_t^{\dagger}$
Standard deviation		1.630	0.118	0.006	0.017	0.024	0.197	0.667
Quarterly autocorrelation		0.821	0.853	0.956	0.936	0.944	0.614	0.933
	g_t	1	0.409	0.306	0.506	0.192	0.288	0.276
	a_t		1	0.471	0.272	-0.073	0.130	0.288
	hr_t			1	0.368	-0.126	0.220	0.113
Correlation matrix	h_t				1	0.486	0.098	0.460
	f_t					1	0.045	0.549
	b_t						1	0.147
	i_t	-	-	-	-	-	-	1

South Korea		g_t	a_t	hr_t	h_t	f_t	$i_t^{\dagger}$	$s_t^{\ddagger}$
Standard deviation		2.897	0.255	0.010	0.102	0.072	0.863	0.055
Quarterly autocorrelation		0.852	0.852	0.939	0.865	0.839	0.794	0.927
	g_t	1	0.556	0.284	0.712	0.592	-0.400	0.051
	a_t		1	-0.103	0.565	0.464	-0.266	-0.034
	hr_t			1	0.470	0.271	-0.047	0.137
Correlation matrix	h_t				1	0.836	-0.400	0.097
	f_t					1	-0.507	0.236
	s_t	-	-	-	-	-	1	0.515
	i_t	-	-	-	-	-	-	1

[Table 2.1] reports the standard deviations, quarterly autocorrelations and correlation matrices of real GDP growth (g_t), asset value growth (a_t), household credit ratio (hr_t), household credit growth (h_t), corporate credit growth (f_t), U.S. Treasury bill rates (b_t), Korean and U.S. 3-year government bond yields (i_t), and small firm credit growth (s_t). All variables are seasonally adjusted and detrended using the Hodrick-Prescott filter with smoothing parameter of 1600. Columns tagged with † span 1995:Q2-2016:Q4 (87 obs) and ‡ span 2000:Q1-2016:Q4 (65 obs). All other columns span 1980:Q1-2016:Q4 (145 obs).

limited ability to uncover relationships among the variables, we employ in the following sections more rigorous methods to analyze the cyclicality in corporate and household credit behavior.

2.2.2 The empirical model specification

A positive productivity shock constitutes an incentive for corporations to borrow and invest for future profit, but for households engenders concerns related to consumption smoothing in order to maximize lifetime utility and financial investment aimed at growing wealth. Faced with a positive productivity shock, households must decide whether to save (or pay back previous debt) to smooth consumption or borrow and invest in equities and real estate in hopes of growing their wealth. To disentangle the consumption smoothing motive and leverage investment motive, we employ the following *pth* order vector autoregressive (VAR) model for macroeconomic data in Korea, abstracting from deterministic terms

$$\mathbf{x}_t = \Phi_1 \mathbf{x}_{t-1} + \Phi_2 \mathbf{x}_{t-2} + \cdots + \Phi_p \mathbf{x}_{t-p} + C u_t \tag{2.1}$$

where $\mathbf{x}_t = [g_t \quad \mathbf{z}_t']'$ and C is a lower-triangular matrix and $\mathbf{u}_t$ a vector of mutually orthonormal structural shocks, that is, $E\mathbf{u}_t\mathbf{u}_t' = \mathbf{I}_t$.[9] Variable g_t, which denotes real GDP growth, is used as a proxy for a productivity shock. The identification scheme implies that productivity shocks affect financial markets contemporaneously, a common assumption in the literature (Uribe and Yue, 2006). We believe our identification strategy to be natural inasmuch as decisions to invest and issue credit take time to plan and implement. It is also reasonable to assume that financial markets can react pretty quickly to news about the state of the current business cycle in the economy. Consider the following specifications for $\mathbf{z}_t$:

$$\begin{aligned} \mathbf{z}_t &= [a_t \quad h_t \quad f_t]', \\ \mathbf{z}_t &= [a_t \quad h_t \quad f_t \quad s_t]', \quad \text{and} \\ \mathbf{z}_t &= [a_t \quad h_t \quad f_t \quad i_t]', \end{aligned} \tag{2.2}$$

where a_t is stock price index (e.g., KOSPI, and S&P500), hr_t household credit ratio (out of total credit to households and firms), h_t household credit growth, f_t corporate credit growth, s_t small firm credit growth, and i_t 3-year government bond yields or the U.S.-Korea bond yield spread.

In order to check whether the interaction between asset value growth (a_t) and household credit growth (h_t) is sustained or not, another specification for $\mathbf{x}_t$ is considered:

$$\mathbf{x}_t = [b_t \quad \mathbf{z}_t']', \tag{2.3}$$

where b_t denotes U.S. Treasury bill rates.[10] Combined with specifications (2.2), the direct and indirect effects of U.S. interest rate shocks on household credit growth (h_t) and asset

[9] We employed an intercept, linear time trend, and quadratic term for our VAR estimations. Estimates are qualitatively similar when we remove the quadratic trend term. See Kim and Lee (2018) for a similar example. We use 8 lags for our VAR estimation, unless otherwise stated. Our results are robust to using other lag-length selections suggested by information criteria, such as AIC, BIC, and HQIC.

[10] U.S. Treasury bill rates are considered as short-term interest rates, wheras 3-year government bond yields are considered as longer-term interest rates.

value growth (a_t) can be analyzed. The results are reported in Section 2.3.2.

As a robustness check to allay the concern that asset value data are more frequent than credit variables, we perform the analysis with different model specifications including changes in lag number. We are interested in the j-period ahead orthogonalized impulse response functions (OIRF) of $\mathbf{z}_t$ to a productivity shock, defined as follows:

$$IRF(j) = E(z_{t+j}|u_{g,t} = 1, \Omega_{t-1}) - E(z_{t+j}|\Omega_{t-1}) \tag{2.4}$$

where $u_{g,t}$ is the structural shock to g_t in the first equation and Ω_{t-1} the adaptive information set at time $t-1$.[11] Note that g_t is ordered first in $\mathbf{x}_t$, meaning that real GDP growth is not influenced by other variables in $\mathbf{z}_t$ within one quarter.

2.3 The Macroeconomic Ecology

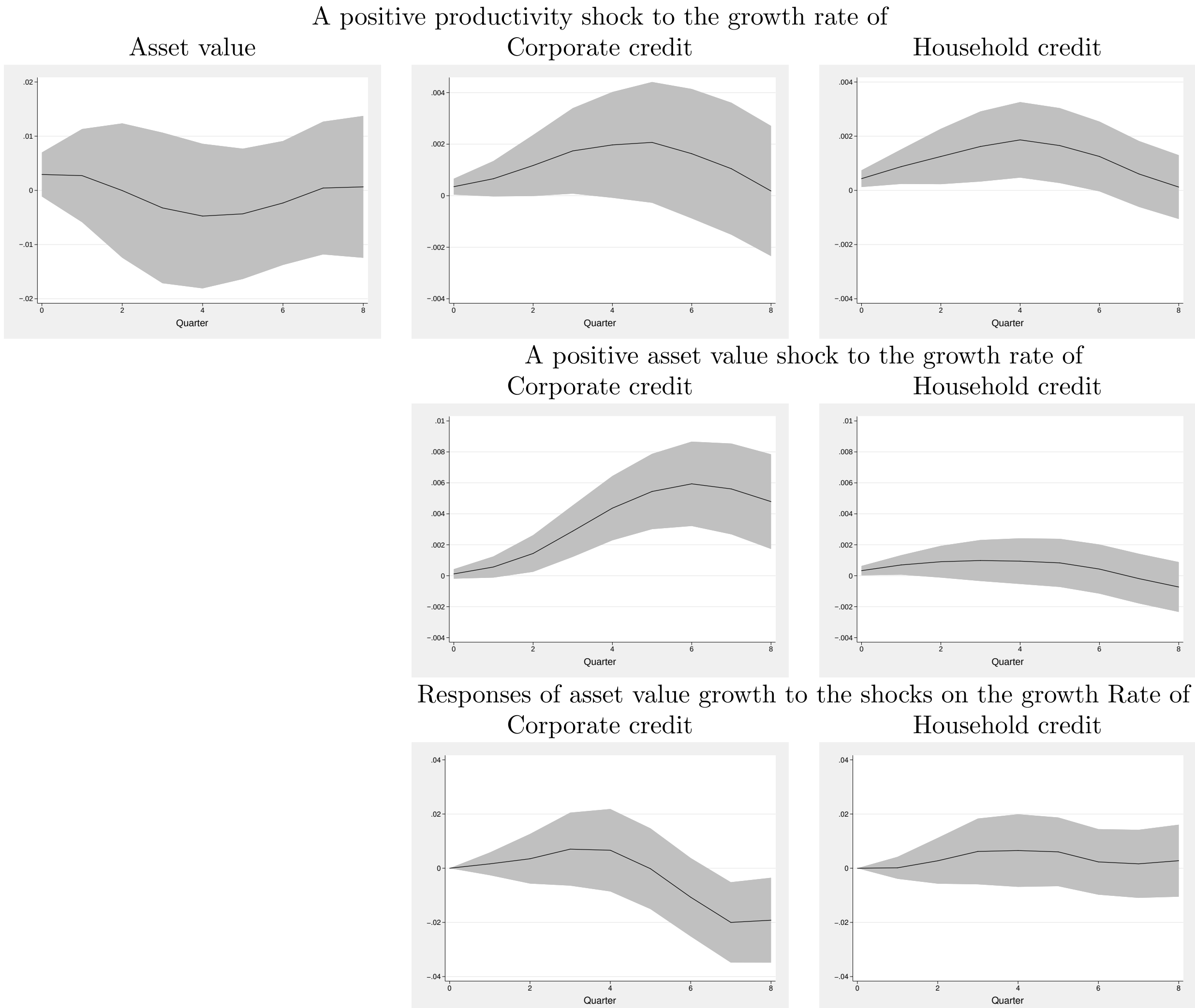
2.3.1 The symbiotic credit behavior in the U.S.

The productivity shock may affect corporate and household credits through domestic asset value growth, and it may affect as well the asset value through corporate and household credit. To identify both channels, we estimate the VAR model with $\mathbf{z}_t = [a_t \quad h_t \quad f_t]'$ using U.S. data. The implied impulse response functions are reported in Figure 2.3. The first graph reports responses of asset value (a_t) to a productivity shock ($g_t \rightarrow a_t$), which is not significant. The subsequent graphs report responses of corporate (f_t) and household (h_t) credit growth to a productivity shock (g_t), ($g_t \rightarrow f_t$ and $g_t \rightarrow h_t$). Both corporate and household credit growth respond positively, then gradually die out, but the responses are moderate. Responses to a positive shock in asset value growth (a_t), as can be seen in the graphs in the second row, are strongly positive for corporate (f_t) but not for household (h_t) credit growth ($a_t \rightarrow f_t$ and $a_t \rightarrow h_t$). The two graphs in the third row reveal that the shocks on the corporate (f_t) and household (h_t) growth have little effect on asset value growth (a_t), ($f_t \rightarrow a_t$ and $h_t \rightarrow a_t$).

Households' choices of financial activities during a boom period include consumption smoothing (i.e., paying back debt carried from the past or saving for the future) and financial

[11] That is, the information set has the property: $\Omega_{t-1} \supseteq \Omega_{t-2} \supseteq \Omega_{t-3} \supseteq \cdots$.

Figure 2.3: Impulse responses of asset value & credit growth in US (1980:Q1-2016:Q4)



[Figure 2.3] shows the impulse response functions of asset value and corporate and household credit in the United States subject to a positive productivity shock. Solid lines depict point estimates of impulse responses; the shaded area depicts the 95% confidence interval.

investment (i.e., investing current wealth or borrowing in equities and real estate in order to grow their wealth). Although the impulse responses of household credit growth to the productivity shock is significantly pro-cyclical, these are not necessarily interpreted as the evidence of households' dominant leverage investment motive, because these are affected by sales credit, durable goods consumption, and so on. Indeed, noting that the responses of household credit growth (asset value growth) to the asset value growth (household credit growth) are not so significant, we conclude that households' investment motive in the U.S. economy does not significantly dominate households' consumption smoothing motive.

2.3.2 The competitive credit behavior in Korea

Responses to productivity shock

We estimate the VAR model with $\mathbf{z}_t = [a_t \quad h_t \quad f_t]'$ using Korean data. Impulse response functions from all period (1980:Q1-2016:Q4) and post-currency crisis period (2000:Q1-2016:Q4) are reported in Figures 2.4 and 2.5, repectively.[12]

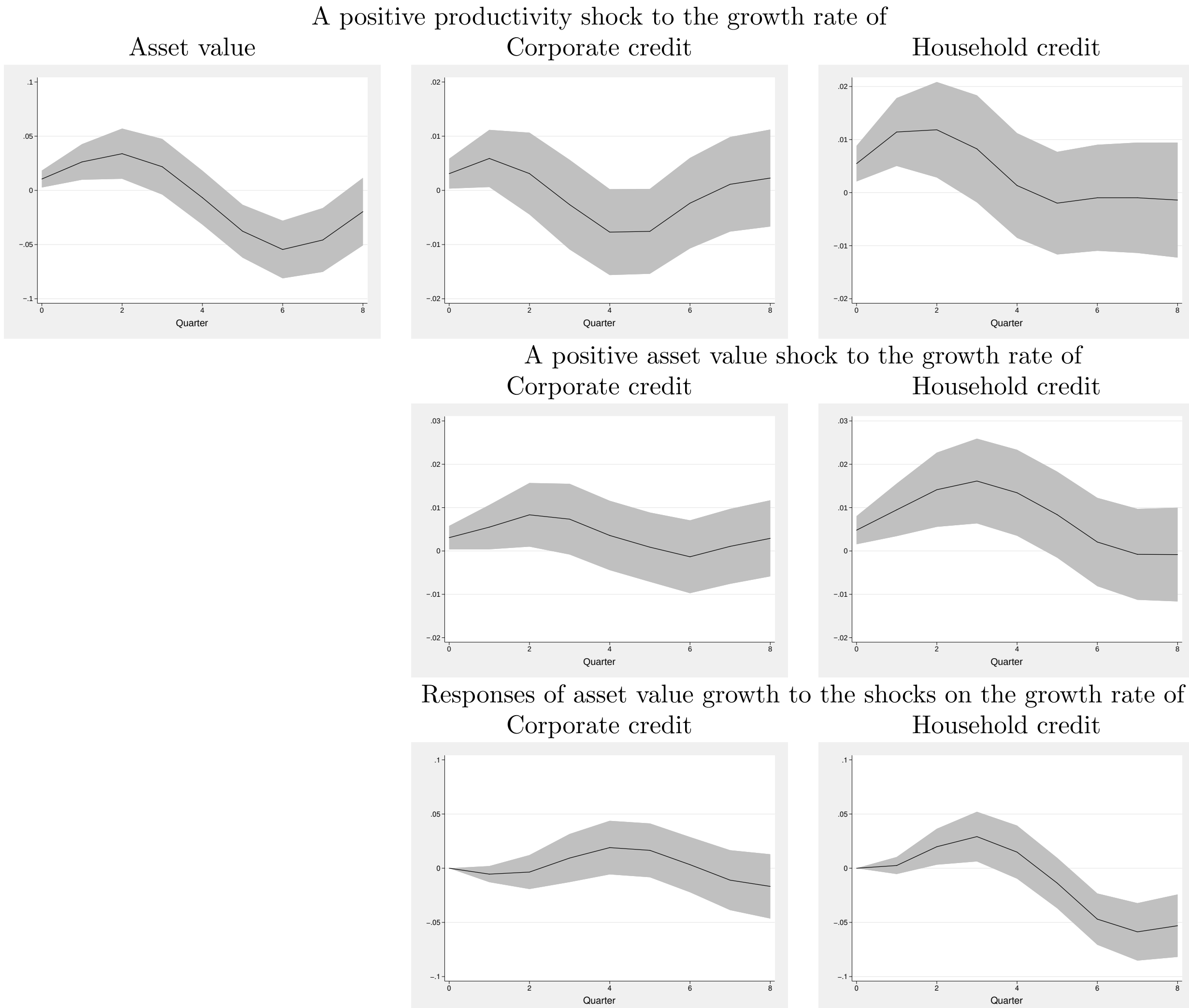
The first graphs in Figures 2.4 and 2.5 report the responses of asset value growth (a_t) to a productivity shock (g_t), ($g_t \rightarrow a_t$). In contrast to the United States, aggregate asset value in Korea rises significantly in response to a positive productivity shock, which result is robust across specifications. The second and third graphs Figures 2.4 and 2.5 report the responses of corporate (f_t) and household (h_t) credit growth to a productivity shock (g_t), ($g_t \rightarrow f_t$ and $g_t \rightarrow h_t$). The responses of household credit growth are stronger than the responses of corporate credit growth across specifications and periods. The initial responses of corporate and household credit growth to a productivity shock are positive in both Figures 2.4 and 2.5.

The two graphs in the second row in Figures 2.4 and 2.5, report the responses of corporate (f_t) and household (h_t) credit growth to a positive shock in asset values (a_t), ($a_t \rightarrow f_t$ and $a_t \rightarrow h_t$). The all-period responses of corporate and household credit growth are both significantly positive, but in contrast to the U.S. economy, the responses are larger for household than for corporate credit growth. The post-currency crisis responses of corporate and household credit growth are consistent with the all-period results.

The two graphs in the third row in Figures 2.4 and 2.5 report the responses of asset value growth (a_t) to the shocks to corporate (f_t) and household (h_t) credit growth ($f_t \rightarrow a_t$ and $h_t \rightarrow a_t$). That the responses to household credit growth are significant and stronger than the responses to corporate credit growth in Figures 2.4 supports our conjecture that Korean households having strong leverage investment borrow from financial institutions and buy domestic assets like equity and real estate, to grow their wealth. The post-currency crisis responses of asset value growth (a_t) are significantly negative to the corporate credit

[12] Cho and West (2003) report significant changes in economic structure during the *Korean Currency Crisis of 1997*. Following their viewpoint we additionally analyze the impulse responses after the currency crisis.

Figure 2.4: Impulse responses of asset value & credit growth in Korea (1980:Q1-2016:Q4)

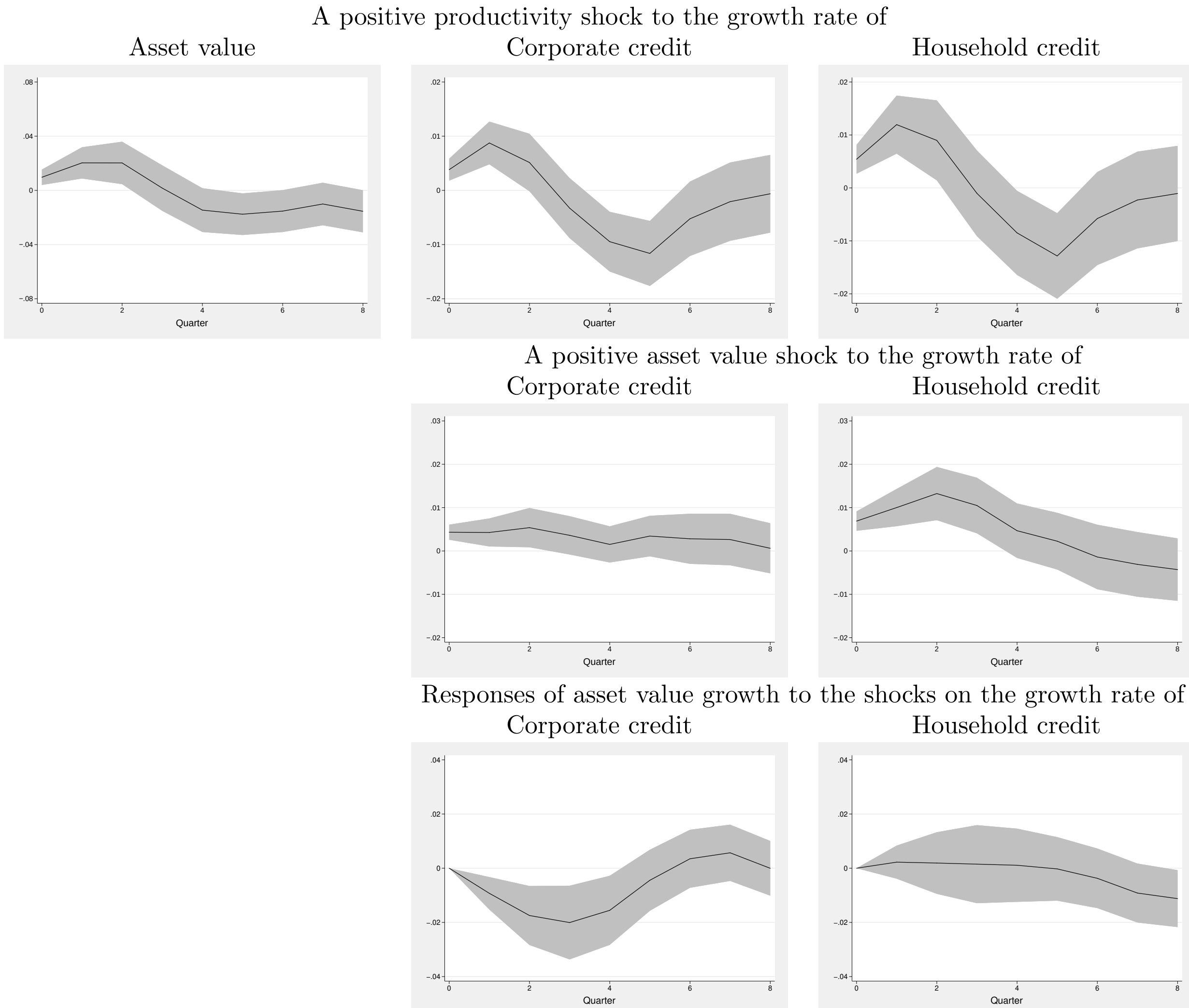


[Figure 2.4] shows the impulse response functions of asset value and corporate and household credit in Korea subject to a positive productivity shock. Solid lines depict point estimates of impulse responses; the shaded area depicts the 95% confidence interval.

growth (f_t) shocks and insignificant to the household credit growth (h_t) shocks, and these do not reject our conjecture. Relative to corporate credit growth (f_t), household credit growth (h_t) is more positively sensitive to asset value growth (a_t), and the latter is more positively sensitive to the former as well.

So far, we have not distinguished small firms from large firms in Korea. To examine their distinct responses, we examine small firm credit growth by estimating the VAR model with $\mathbf{z}_t = [a_t \quad h_t \quad f_t \quad s_t]'$ using Korean data, and the results are reported in Figure 2.6. The responses of small firm credit growth (s_t) to a productivity shock (g_t) are almost nonexistent,

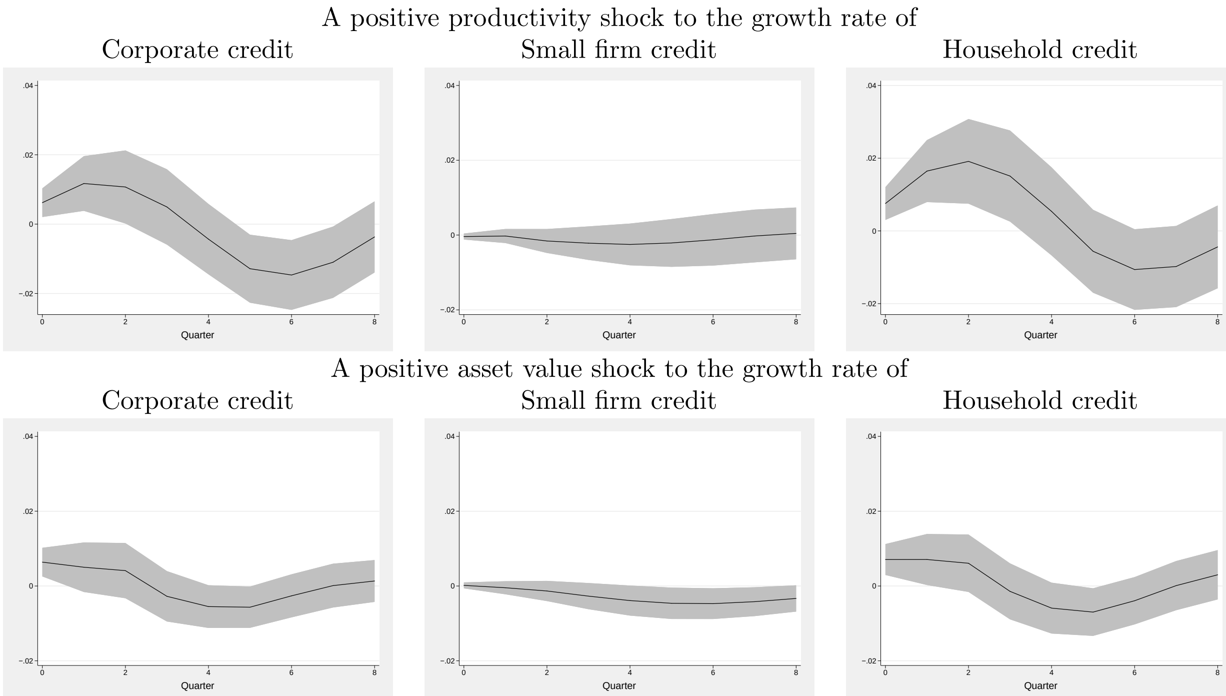
Figure 2.5: Impulse responses of asset value & credit growth in Korea (2000:Q1-2016:Q4)



[Figure 2.5] shows the impulse response functions of asset value and corporate and household credit in Korea subject to a positive productivity shock. Solid lines depict point estimates of impulse responses; the shaded area depicts the 95% confidence interval.

and to a asset value growth shock (a_t) slightly negative, ($g_t \to s_t$ and $a_t \to s_t$). We may infer from these results that collateral-equipped households that borrow to invest in equities and real estate crowd out corporations, especially small firms, in the domestic credit market. Although some large firms still borrow from financial institutions and fund equity issues, small firms lacking collateral are largely excluded from the credit market. The view that lending to small firms is much riskier than lending to households that possess collateral may be optimal for financial institutions. However, it is not optimal for the economy as a whole because of the attendant potential to increase structural risk. High occupancy of collateral

Figure 2.6: Impulse responses of asset value & credit growth after financial crisis in Korea (2000:Q1-2016:Q4)



[Figure 2.6] shows the impulse response functions of asset value and corporate and household credit in Korea subject to a positive productivity shock. Solid lines depict point estimates of impulse responses; the shaded area depicts the 95% confidence interval.

household credit induces household idiosyncratic risk to be synchronized with domestic asset value risk. In contrast, idiosyncratic shocks by small firms can be diversified through financial institutions' portfolio.[13]

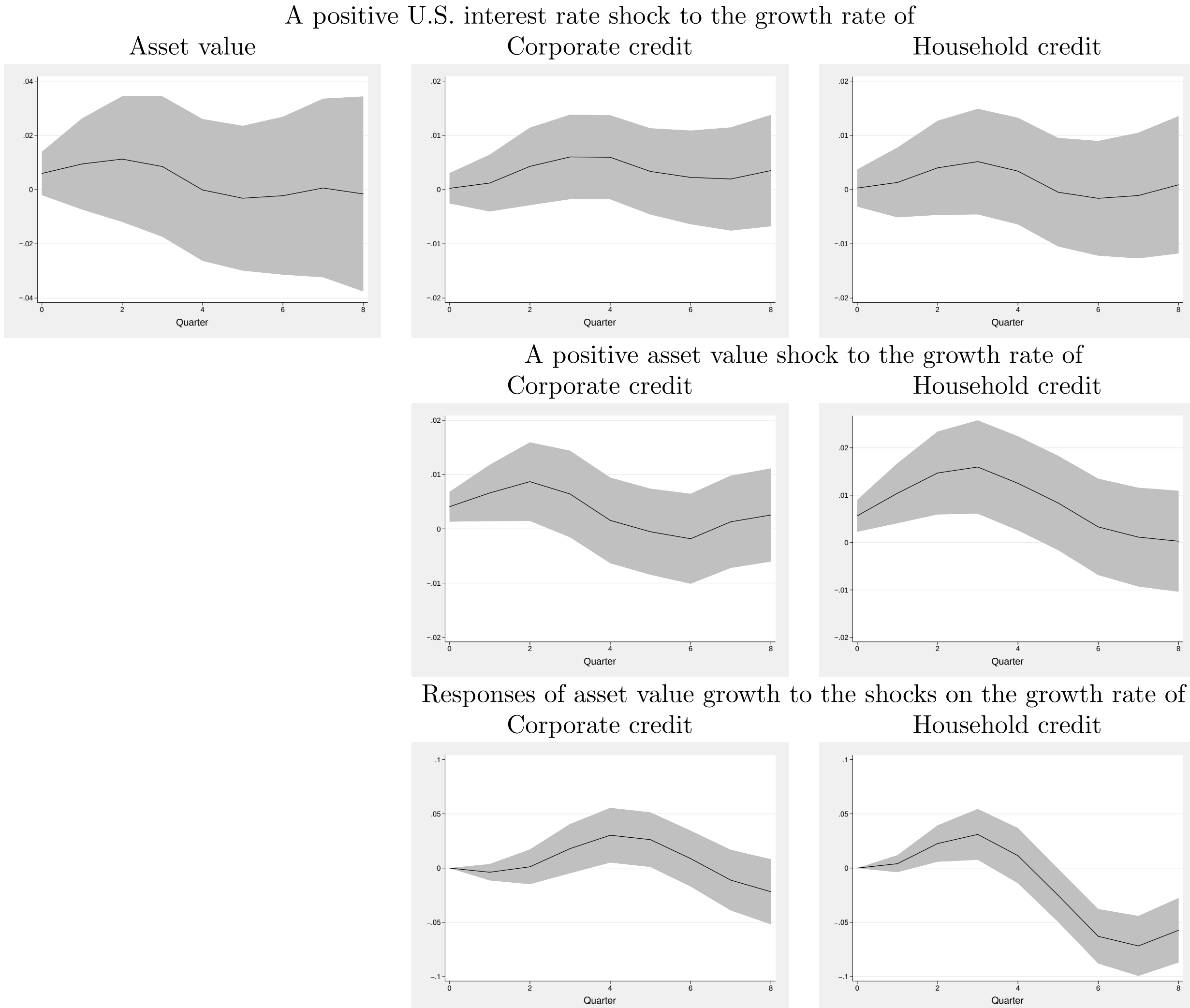
Responses to the U.S. interest rate shock

Uribe and Yue (2006) document U.S. interest rate shocks to explain approximately 20% of movement in aggregate activity in emerging economies, and the effects of U.S. interest rate shocks on aggregate output and gross domestic investment to be significantly negative.

In order to analyze the effect of U.S. interest rate shocks on domestic credit market in Korea, we replace the productivity shock (g_t) variable in Figures 2.4 and 2.5 with the

[13] The results in Figure 2.6, although they explain correlations among the variables in the data, do not explain causality. That financial institutions in emerging economies increase household and decrease small firm credit during booms, and decrease household and increase small firm credit during recessions, reflects financial institutions' portfolio choice problem with scarce resources.

Figure 2.7: Impulse responses to US interest rate in Korea (1980:Q1-2016:Q4)

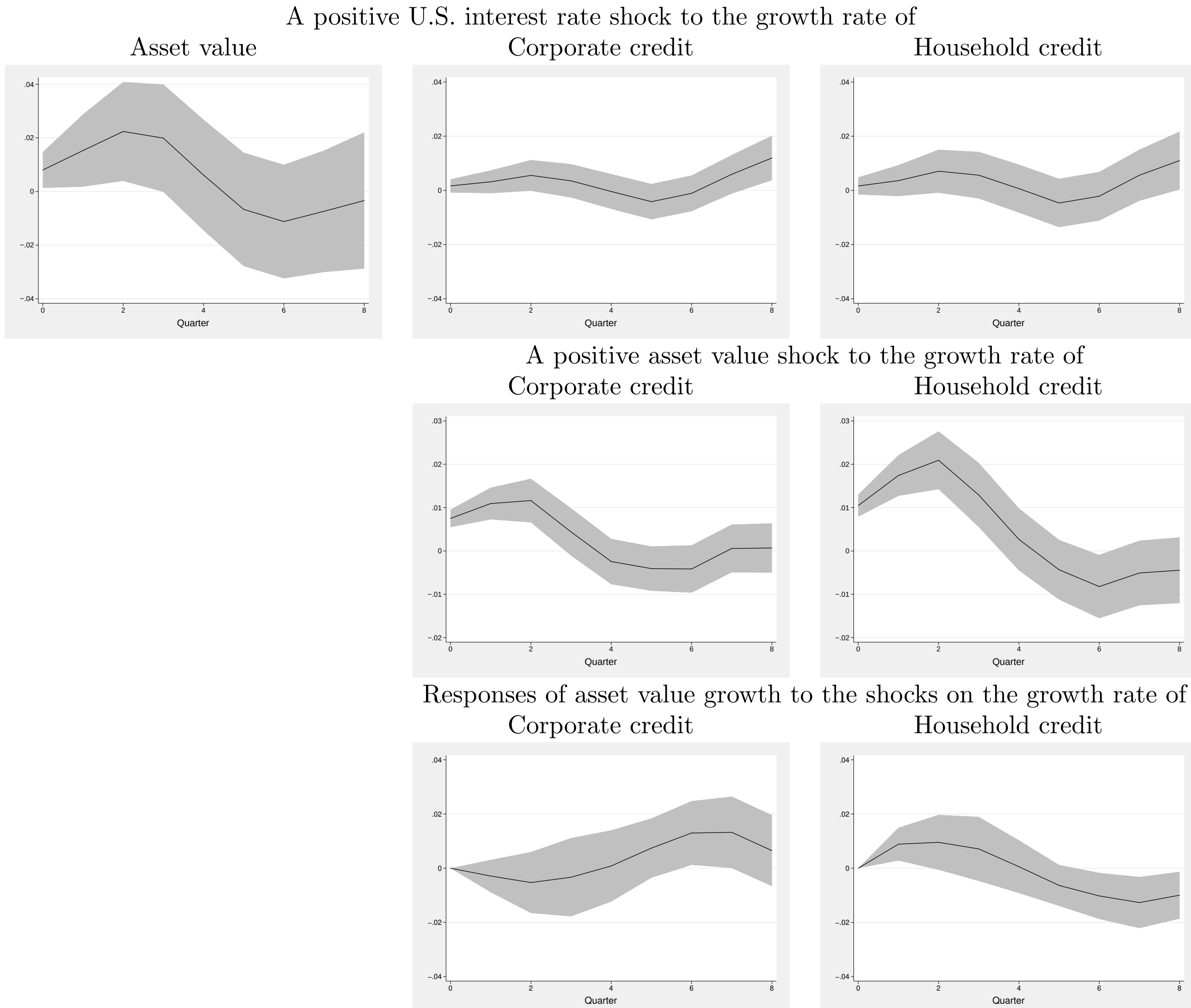


[Figure 2.7] shows the impulse response functions of asset value and corporate and household credit in Korea subject to a U.S. interest rate shock. Solid lines depict point estimates of impulse responses; the shaded area depicts the 95% confidence interval.

U.S. interest rate shock (b_t) variable, defined as the percentage change from the previous quarter in the secondary market rate of the 3-month treasury bill, seasonally adjusted and detrended using the Hodrick-Prescott filter with smoothing parameter of 1,600. The baseline VAR model is estimated with $\mathbf{z}_t = [a_t \quad h_t \quad f_t]'$, and the corresponding impulse response functions are reported in Figure 2.7 for all period (1980:Q1-2016:Q4) and Figure 2.8 for post-currency crisis (2000:Q1-2016:Q4).

Responses of asset value growth (a_t) to a U.S. interest rate shock (b_t) are gentle and not significant across specifications in Figure 2.7. The responses of corporate (f_t) and household

Figure 2.8: Impulse responses to US interest rate in Korea (2000:Q1-2016:Q4)



[Figure 2.8] shows the impulse response functions of asset value and corporate and household credit in Korea subject to a U.S. interest rate shock. Solid lines depict point estimates of impulse responses; the shaded area depicts the 95% confidence interval.

(h_t) credit growth to a U.S. interest rate shock (b_t) are slightly positive but insignificant. These findings are consistent with the literature, and do not reject the conventional belief that a positive U.S. interest rate shock reduces aggregate domestic asset value and, consequently, the borrowing activities of corporations and households. Now, the following results are new and worth noted. The second row reports that the responses of corporate (f_t) and household (h_t) credit growth to an asset value growth (a_t) are both positive and significant, and the responses of household credit growth are stronger than those of corporate credit growth. The third row reports that the responses of asset value growth to the corporate credit growth are slightly negative and insignificant, and the responses of asset value growth to the household credit growth are significantly positive. These results reconfirm the strong interaction between domestic asset value and household credit growth.

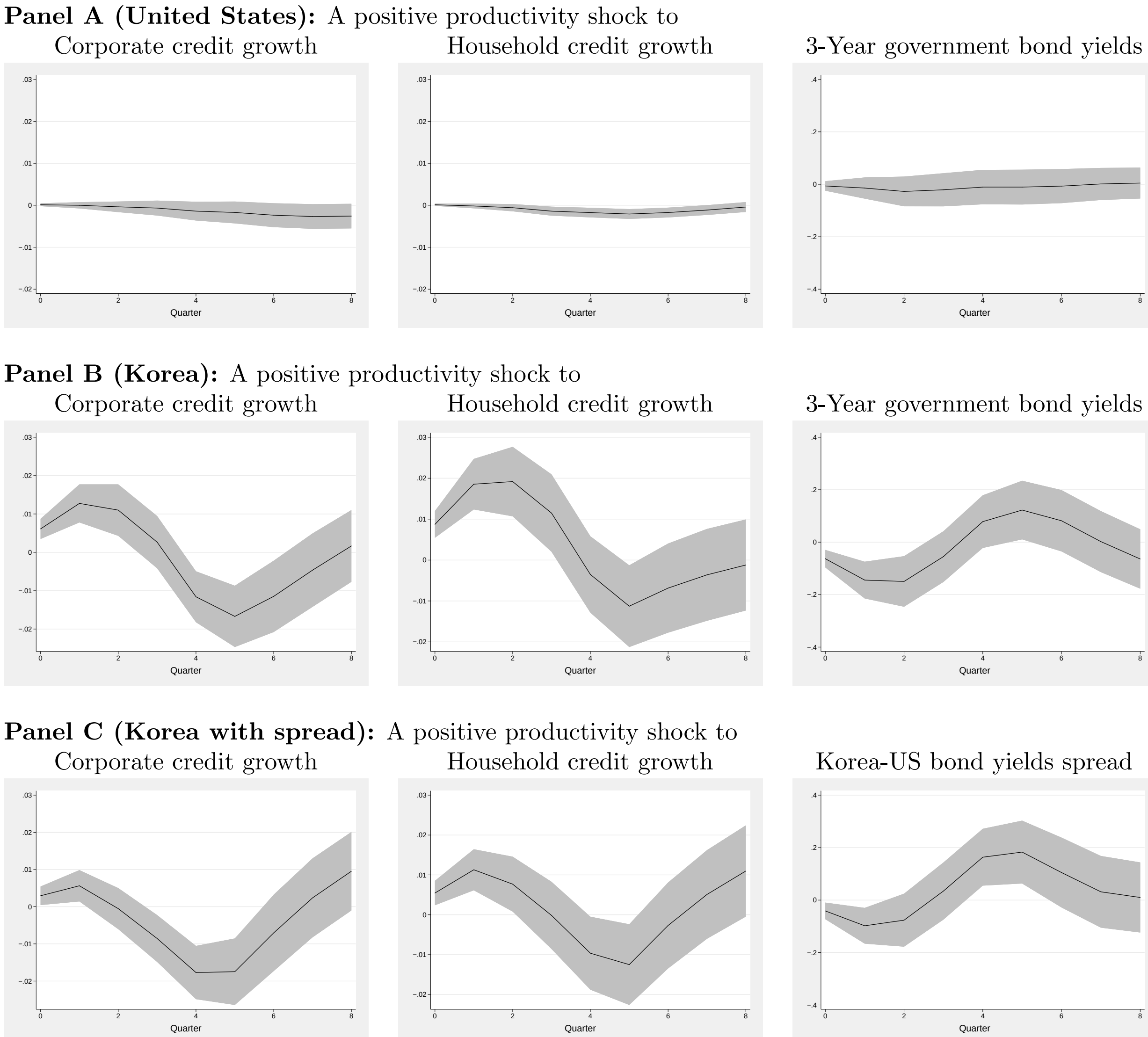
Post-currency crisis responses strengthen this inference, as observed in Figure 2.8. The responses of corporate (f_t) and household (h_t) credit growth to a U.S. interest rate shock (b_t) are slightly positive and insignificant, same as in the previous figure. Note that the positive response of asset value growth (a_t) to an increase in the U.S. interest rate (b_t) is somewhat counter-intuitive, as an increase in the U.S. interest rate should be negatively reflected in the country spread and drive a reduction in domestic asset value. This unusual behavior is explained in the subsequent graphs. The second row reports that the responses of corporate (f_t) and household (h_t) credit growth to an asset value growth (a_t) are both positive and significant, and the responses of household credit growth are stronger than those of corporate credit growth. The third row reports that the responses of asset value growth to the corporate credit growth are slightly negative and insignificant, and the responses of asset value growth to the household credit growth are significantly positive. These findings support our argument that the unusual growth of asset value to an increase in the U.S. interest rate is driven by households' leveraged investments. Korea has experienced a rapid recovery and growth in its asset value right after the currency crisis, and households have further developed their leveraged investment motive.

2.4 Counter-Cyclical Interest Rate

2.4.1 The interest rate puzzle

So far, we have analyzed the linkage between household credit growth and asset value growth with an emphasis on households' leverage investment motive. We now turn to analysis on the behavior of private sector credit (quantity) and interest rates (price) in emerging economies. In particular, we focus on how households' investment motive affects credit market clearing, which was previously characterized by counter-cyclical interest rates in Fernandez and Gulan (2015); Han (2017); Neumeyer and Perri (2005).

Figure 2.9: Impulse responses of credit growth & gov't bond yields (1995:Q2-2016:Q4)



[Figure 2.9] shows the impulse response functions of corporate and household credit, 3-year government bond yields (i_t), and U.S.-Korea 3-year government bond yield spreads ($i_t^{KR} - i_t^{US}$) subject to a positive productivity shock. Solid lines depict point estimates of impulse responses; the shaded area depicts the 95% confidence interval.

The VAR model with $\mathbf{z}_t = [a_t \quad h_t \quad f_t \quad i_t]'$, and corresponding impulse responses using U.S. and Korean data are reported in Figure 2.9. The first two columns report responses of corporate and household credit growth, and the last column reports 3-year government bond yields in the United States in Panel A and Korea in Panel B, respectively. Note that because time series for 3-year Korean government bond yields are available only since 1995, we use the quarterly time series data from 1995:Q1 to 2016:Q4.[14]

Panels A and B jointly indicate that all responses are much stronger in Korea than in the United States. For direct comparison, we maintain the same scale on the vertical axes across panels. Whereas the responses of private sector credit growth and 3-year government bond yields are all negligible and insignificant in the United States, the initial responses of private sector credit growth are significantly positive, and the 3-year government bond yields are significantly negative in Korea. Also those responses are quite persistent in the Korean economy. This empirical finding implies that private sector credit volume (quantity) in the Korean credit market is pro-cyclical, while the interest rate (price) is counter-cyclical. This finding is also confirmed in Panel C as well, which replaces the domestic interest rate with the Korea-U.S. bond yield spread. Since the Korea-U.S. bond yield spread, calculated as the Korean 3-year government bond yield minus the U.S. 3-year treasury bond yield. U.S. government bonds is considered risk-free assets, the Korea-U.S. bond yield spread is interpreted as a normalized interest rate in Korea.

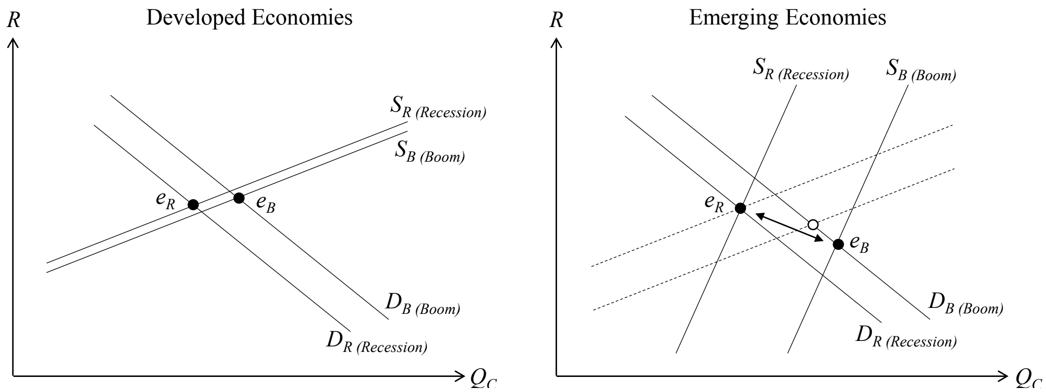
Figure 2.9 suggests an interesting approach to the counter-cyclical interest rate puzzle in emerging economies suggested by Fernandez and Gulan (2015). That is, the counter-cyclical behavior of the interest rate is accompanied by the pro-cyclical corporate and household credit, the former of which should play a key role in credit market clearing in emerging economies.

2.4.2 Illustrative interpretation

Fernandez and Gulan (2015) argue that an increase (decrease) in the value of collateral assets provided by corporate firms and held by domestic financial intermediaries in boom (recession) encourages substantial capital inflow (outflow) in emerging market. Then, they attribute the counter-cyclical behavior of interest rate to the immature financial market practice in emerging economies. As an alternative argument, we have shown that the key players in the collateral credit market are not corporate firms but household. Especially, an expected rapid growth in asset values in emerging economies stimulates households' investment motive and

[14]Relative to Panel A pertaining to the U.S. economy, we obtain similar impulse responses using longer time series data.

Figure 2.10: Domestic credit market



[Figure 2.10] describes the pro-cyclical and counter-cyclical movements of interest rates in developed and emerging economies, respectively. R denotes interest rate, Q_C quantity of credit, S credit supply, D credit demand, and e credit equilibrium in the domestic credit market.

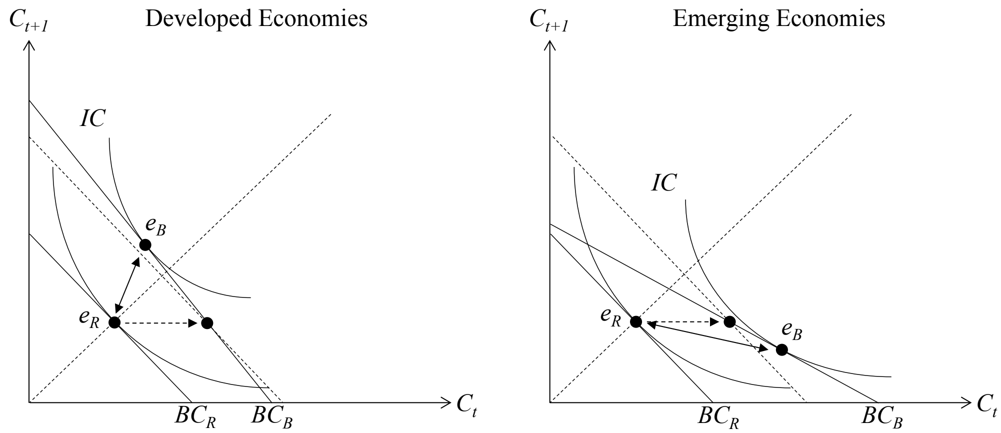
incentivizes them to actively participate in the credit market.

Counter-cyclical interest rate

When we normalize recession, credit supply and demand curves shift out in a boom in both developed and emerging economies ($S_R \rightarrow S_B$ and $D_R \rightarrow D_B$), as illustrated in Figure 2.10. The left panel shows developed economies' interest rate changes to be pro-cyclical movements ($e_R \rightarrow e_B$), the right panel emerging economies' interest rate changes counter-cyclical fluctuations ($e_R \rightarrow e_B$). Much of emerging economies' credit being issued on the basis of domestic collateral, a positive productivity shock increases domestic asset value and thereby domestic credit supply.[15] As shown before, the responses of asset value growth to the productivity shock are moderate and insignificant in the U.S. economy (Figure 2.3), but substantial and significant in the Korean economy (Figures 2.4 and 2.5). The credit supply curve thus shifts out more in emerging than in developed economies in Figure 2.10. In addition to the foregoing shifts, credit supply curves should be steeper in emerging than in developed economies owing to the credit supply in the former being relatively inelastic. In

[15] Garcia-Cicco, Pancrazi and Uribe (2010) embed an exogenous shock of country-specific risk premium to reconcile the observed data in emerging economies. A formula for credit supply as a function of aggregate net worth of all domestic financial institutions, recently derived by Fernandez and Gulan (2015), implies that credit supply depends on the value of aggregate collateral debt.

Figure 2.11: Two-period intertemporal consumptions



[Figure 2.11] describes two-period intertemporal consumption fluctuations in developed and emerging economies. C_t and C_{t+1} denote consumption level in period t and $t+1$, respectively. BC denotes the intertemporal budget constraint, IC the intertemporal indifference curve, y the income bundle, and e the intertemporal consumption equilibrium. R and B denote recession and boom, respectively. Income changes are indicated with dotted, equilibrium consumption changes with solid, arrows.

the right panel of Figure 2.10, the magnitudes of imaginary (dotted lines) and actual (solid lines) supply curve shifts are the same in Q_C, but the latter lines have steeper slopes.[16] It makes interest rate more counter-cyclical with less elastic credit supply. At last, we would like to emphasize that these are supply-driven results, not demand-driven results. If the demand curve shifted rightward further, the interest rate should be pro-cyclical in emerging economies, which is not consistent with our findings.

Consumption volatility

Consider a two-period intertemporal consumption decision problem assuming the income and consumption bundles in recession, e_R, to be on a 45 degree line in Figure 2.11. Given a positive productivity shock in the first (boom) period, the budget line shifts out in parallel fashion. (This is an imaginary budget line, shown as dotted in the figure.) Standard consumption-investment theory predicts consumption smoothing via saving in the first pe-

[16]The dotted lines indicate that imaginary supply curves have the same slope as those for developed economies. Open dots denote the imaginary credit equilibrium, which accounts for the shifts in credit supply and demand in emerging economies.

riod and borrowing in the second period, implying an outward shift and slight clockwise rotation of the intertemporal budget line ($BC_R \rightarrow BC_B$), as described in the left panel of the figure. This new budget line is slightly steeper than the imaginary budget line owing to the pro-cyclical interest rates in developed economies. The new equilibrium consumption bundle in a boom, e_B, shows increases in consumption in both periods.

In emerging economies, due to counter-cyclical interest rate changes, the intertemporal budget line shifts outward more in the first than in the second period. This new budget line is flatter than the budget line in developed economies, as described in the right panel of the figure. The new equilibrium consumption bundle in a boom (e_B) shows the consumption level to be much higher in the first than in the second period. Notice that it also makes households to consume further than their first period income as well. This provides an illustrative example of high consumption volatility in emerging economies.

2.5 Conclusion

Our impulse response function analysis indicates that rapid recovery from the currency crisis in South Korea encouraged households as well as firms to make leveraged investments through collateral debt. Impulse responses of corporate and household credit to a positive productivity shock were thus positive in South Korea. In the emerging economies of a number of countries including Brazil, Chile, South Africa, South Korea, and Thailand, given a positive productivity shock, household credit outgrows corporate credit such that households' leveraged investments on assets dominate firms' productive investments.

Our finding is consistent with the stylized facts documented by Neumeyer and Perri (2005). The pro-cyclical behavior of collateral-based credit secured by households, which dominates that of corporate credit, amplifies consumption volatility and generates counter-cyclical behavior in real interest rates in emerging economies, attributed by many previous studies to financial friction, policy uncertainty, and, hence, secured loans for non-financial sectors.

Chapter 3

Dynamic Coupling of the U.S. and Canadian Industrial Production Indices

3.1 Introduction

This chapter studies the coupling of industrial production indices of the United States and Canada using a non-linear autoregressive model. Estimation of the exponential smooth transition autoregressive (ESTAR) model in the literature is improved with an expanded set of specifications, and I identify the dynamic linkage between the United States and Canada and evaluate the forecast performance of each model. The results show the non-linear autoregressive model with bilateral trade linkage to outperform other models suggested by existing studies.

The dynamics of coupled systems may have very different behavior from the dynamics of their constituent sub-systems. The properties of a coupled system, such as synchronization and chaos, depending on the nature of the coupling mechanism, and there is an extensive literature on this topic in mathematics, physics, chemistry, and biology. Anderson and Ramsey (2002) introduce non-linear economic relationships between the U.S. and Canadian industrial production indices using the variable amplitude coefficient model, introduced by Ozaki (1981).[1] They estimate the impact of dynamic linkages between indices by assuming that the linkages are represented by functions of current and past indices. However, their study is limited in terms of modeling the dynamic coupling system and identification of the

[1] Ozaki (1981)'s model is a variation of ESTAR model.

dynamic linkages.

In this chapter, Anderson and Ramsey (2002)'s work is improved with expanded data and different model specifications. The bilateral trade between the two countries is used as the linkage variable, and the augmented model provides a better forecast performance than other model specifications.

3.2 Data

The data used in Anderson and Ramsey (2002) consist of 458 seasonally adjusted monthly observations on the industrial production indices for the United States and Canada. The data source is the IMF financial series database, and the available sample covers the period dating from January 1957 to February 1995. They are transformed into a monthly series of annual

Figure 3.1: Industrial production of Canada and the Unites States.

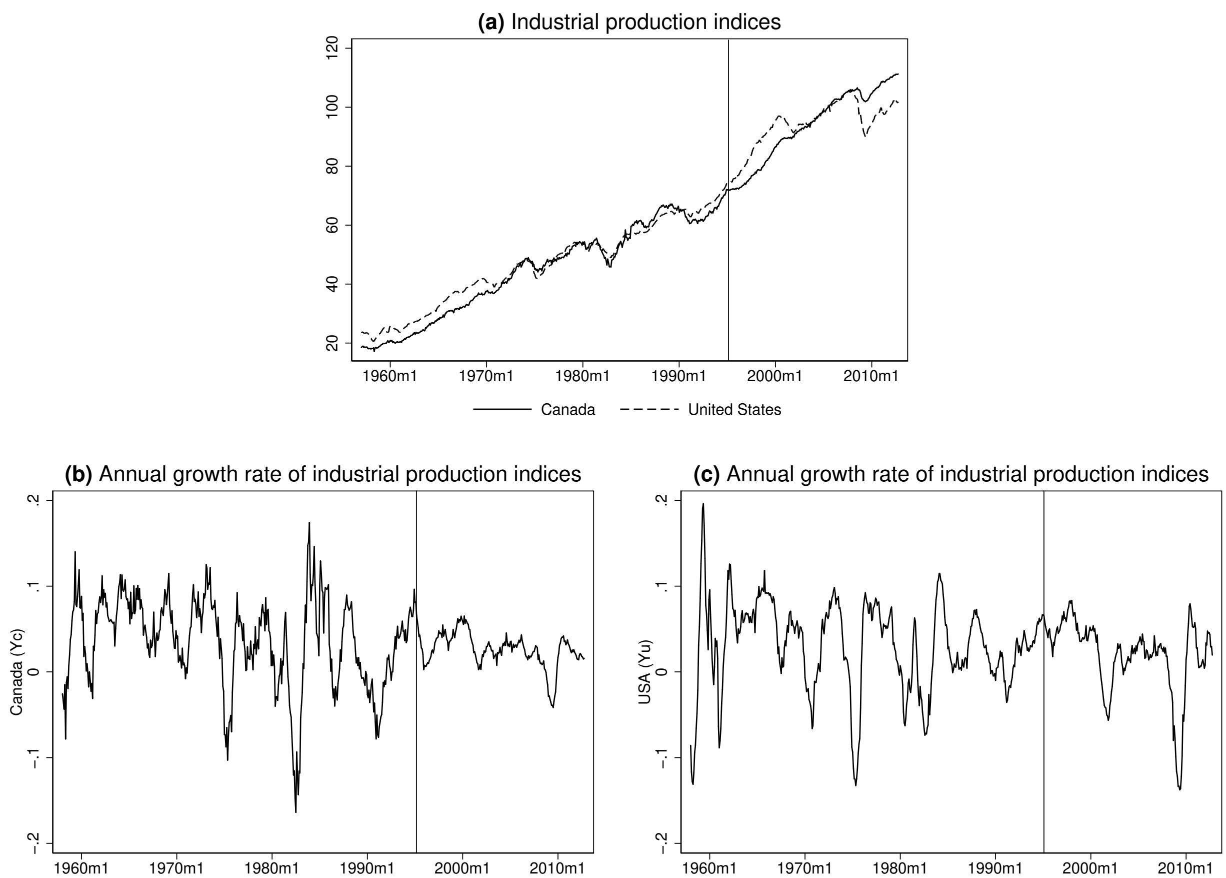


Table 3.1: Annual growth rates of the U.S. and Canadian industrial production indices

	1958:1-2012:10		2004:1-2012:10	
	Canada (y_{ct})	USA (y_{ut})	Canada (y_{ct})	USA (y_{ut})
Mean	3.26	2.69	1.85	0.78
Median	3.36	3.19	2.30	2.20
Maximum	17.41	19.60	4.53	7.94
Minimum	-16.39	-13.76	-4.16	-13.76
Standard deviation	4.53	5.00	2.00	4.86
Second raw moment	31.14	32.22	7.42	23.96
Skewness	-0.69	-0.78	-1.57	-1.69
Kurtosis	4.67	4.41	4.86	5.24
P-value of unit root test	$<$0.01	$<$0.01	$<$0.21	$<$0.16
P-value of linearity test	0.0020	0.0150	0.0013	0.4238

(1) An augmented Dickey-Fuller test is used for the unit root test. The null hypothesis is that the growth rate (y_t) is a unit root process against the alternative hypothesis that it is stationary. It uses the t-statistic associated with y_{t-1} in the regression of Δy_t on a constant, y_{t-1} and 12 lags of Δy_t.
(2) The linearity test is associated with the null hypothesis that the growth rate of industrial production indices (y_t) is a linear AR(p) process. The alternative hypothesis is that it is a STAR(p) process in which the AR coefficients vary with y_{it-d}. See Teräsvirta (1994) for more details. The reported p-values above support an alternative hypothesis where coefficients of an AR specification vary with y_{it-1}.
(3) Units are in percentage change.

growth rates by taking the twelfth differences of natural logarithms, and this leaves 446 monthly observations for the empirical analysis. In this essay, I add 212 more observations from the same database, and the total data cover the period dating from January 1957 to October 2012. Figure 3.1 describes annual growth rates (y_t) of the industrial production indices in the United States and Canada, where the right-hand side of the black vertical line in each graph represents the extended data. Summary statistics for annual growth rates of the U.S. and Canadian industrial production indices are provided in Table 3.1.

I use monthly bilateral trade data between the United States and Canada as linkage variables, and they cover the period dating from January 1968 to March 2004 for exports from Canada and from January 1980 to March 2004 for exports from the United States. The trade data are drawn from Statistics Canada's key socioeconomic database (CANSIM), and they are seasonally adjusted using moving average filter. Figure 3.2 describes the levels (x_t) and the annual growth rates (z_t) of bilateral trade between the United States and Canada, and the summary statistics are provided in Table 3.3.

Table 3.2: Annual growth rates of the U.S. and Canadian bilateral exports (1969:1-2004:3 for Canada, 1981:1-2004:3 for the United States)

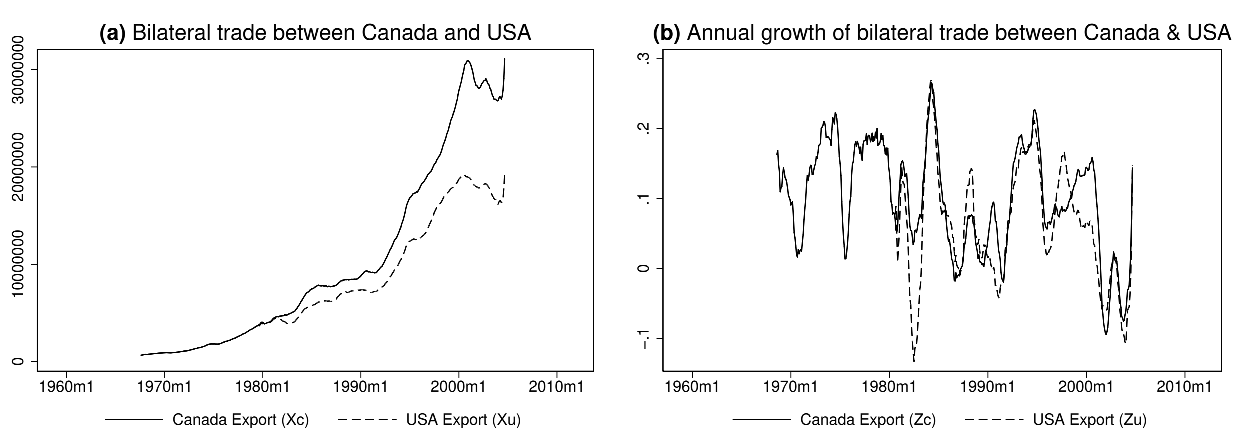
	Canada (z_{ct})	USA (z_{ut})
Mean	10.05	6.03
Median	9.93	6.17
Maximum	26.55	26.92
Minimum	-9.43	-13.30
Standard deviation	7.64	8.34
Skewness	-0.32	0.07
Kurtosis	2.44	2.56

3.3 Empirical Models

Anderson and Ramsey (2002) propose a dynamic coupling between the U.S. and Canadian industrial production indices. They use the nonlinear exponential autoregressive model, proposed by Ozaki (1981), to estimate the impact of dynamical linkages between indices, and they assume that linkages affecting y_{jt} can be represented by simple functions which incorporate y_{it} with $(i \neq j)$. Ozaki (1981)'s model has a couple of advantages. First, the model is also easy to estimate. Second, it provides an effective approximation to a wide variety of underlying continuous-time differential equations in a sense that its coefficients are functions of the state space. Denoting y_{jt} as the business cycle indicator for country j, the Ozaki (1981) model is given by

$$y_{jt} = \Phi_{j0} + \sum_{k=1}^{k=p} \Phi_{jk}\left(y_{jt-d_j}\right) y_{jt-k} + \varepsilon_{jt} \tag{3.1}$$

Figure 3.2: Bilateral trade between Canada and the United States.



without allowing for linkage effects and

$$y_{jt} = \Phi_{j0} + \sum_{k=1}^{k=p} \Phi_{jk} \left(y_{jt-d_j} \right) y_{jt-k} + \sum_{r} \varsigma_r L_{jr(t-dr)} + \varepsilon_{jt} \tag{3.2}$$

with allowing for linkage effects, where

$$\Phi_{jk} \left(y_{jt-d_j} \right) = \Phi_{jk0} + \pi_{jk} \left(y_{jt-d_j} \right) \exp \left[-\gamma_j \left(y_{jt-d_j} - c_j \right)^2 \right], \tag{3.3}$$

$$\pi_{jk} \left(y_{jt-d_j} \right) = \pi_{jk0} + \pi_{jk1} y_{jt-d_j} + \pi_{jk2} y_{jt-d_j}^2 + \pi_{jk3} y_{jt-d_j}^3 + \cdots + \pi_{jkm} y_{jt-d_j}^m. \tag{3.4}$$

Equation (3.1) and (3.1) indicate that they are dealing with an AR process, but with coefficients that are functions of the state space associated with y_{jt-d_j}. The lag of d_j in $\left(y_{jt-d_j} \right)$ indicates that they allow for a variety of alternative lag lengths in defining the relevant state space. d is the transmission lag of a linkage variable, and r is an indicator for the linkage variable. Equation (3.3) shows the variation in the state-dependent coefficients. The exponential term is the key component that generates the variation. It allows the weight of the polynomial terms in equation (3.4) to vary from zero to one. It is a zero weight when the exponent approaches infinity and a full weight when the exponent is zero. The use of a polynomial in equation (3.4) allows for different types of nonlinear behavior. The model can capture nonlinearities by setting $m > 1$ in the differential equations.

For a non-linear linked model (linked model 1), they consider the possible linkage variables:

$$\begin{aligned} y_{ut} \\ \Delta y_{ut} \\ \Delta y_{ct} - \Delta y_{ut} \\ \Delta y_{ct} \cdot \Delta y_{ut} \\ \frac{y_{ct} - y_{ut}}{y_{ct} + y_{ut}} \end{aligned} \tag{3.5}$$

and various lags of each. These represent five alternative ideas for linking the U.S. and Canadian output profiles. However, they are not really linkage variables. They justify that they are not focusing on identification issues. In addition to their linkage variables, I also consider the following linkage variables as alternatives (linked model 2):

$$\begin{aligned} z_{ut} \\ \Delta z_{ut} \\ \Delta z_{ct} \cdot \Delta z_{ut} \\ zi_{ut} \\ \Delta zi_{ut} \\ \Delta zi_{ct} \cdot \Delta zi_{ut} \end{aligned} \tag{3.6}$$

where z_{jt} is annual growth of bilateral export from country j and zi_{jt} is annual growth of bilateral industrial export from country j.

I set the scaling parameter $\gamma = 10$ to ensure sufficient variation in the exponential function in order to avoid ill-conditioning in the likelihood. The lag length $p = 12$ is chosen by AIC with maximum lag of 12, in contrast to $p = 72$ in Anderson and Ramsey (2002). The parameters $m = 1$ and $d = 1$ are fixed due to computational complexity. The parameters $\mu_{y^2_{t-1}}$ and $\mu_{y_{t-1}}$ are the mean of y^2_{t-1} and y_{t-1}, respectively, and these ensure that the exponential functions fluctuate around one.

Anderson and Ramsey use $\exp\left[-10\left(y^2_{jt-1} - \mu_{y^2_{jt-1}}\right)\right]$ to approximate the original transition function, $\exp\left[-\gamma_j\left(y_{jt-d_j} - c_j\right)^2\right]$, to simplify the estimation. The regression models (1)-(4) become:

$$y_{jt} = \Phi_{j0} + \sum_{k=1}^{k=12}\left[\Phi_{jk0} + (\pi_{jk0} + \pi_{jk1}y_{jt-1})\exp\left[-10\left(y^2_{jt-1} - \mu_{y^2_{jt-1}}\right)\right]\right]y_{jt-k} + \varepsilon_{jt}, \tag{3.7}$$

$$\begin{aligned} y_{jt} = \Phi_{j0} &+ \sum_{k=1}^{k=12}\left[\Phi_{jk0} + (\pi_{jk0} + \pi_{jk1}y_{jt-1})\exp\left[-10\left(y^2_{jt-1} - \mu_{y^2_{jt-1}}\right)\right]\right]y_{jt-k} \\ &+ \sum_r \varsigma_r L_{jr(t-dr)} + \varepsilon_{jt}. \end{aligned} \tag{3.8}$$

However, I use the original transition function, $\exp\left[-10\left(y_{jt-1} - \mu_{y_{jt-1}}\right)^2\right]$, rather than their approximation because I do not find any benefit from it, so the regression models I use

Table 3.3: Transition variables for modeling the growth rates of the U.S. and Canadian industrial production indices (1958:1-2012:10)

	Canada (T_{ct})	USA (T_{ut})
Mean	0.98	0.98
Median	0.99	0.99
Maximum	1.00	1.00
Minimum	0.68	0.75
Standard deviation	0.04	0.04
Skewness	-4.03	-3.18
Kurtosis	24.39	13.89

Note: The transition variables are defined by $T_{jt} = \exp\left[-10\left(y_{jt-1} - \mu_{y_{jt-1}}\right)^2\right]$ for $j \in \{c, u\}$.

are:

$$y_{jt} = \Phi_{j0} + \sum_{k=1}^{k=12}\left[\Phi_{jk0} + (\pi_{jk0} + \pi_{jk1}y_{jt-1})\exp\left[-10\left(y_{jt-1} - \mu_{y_{jt-1}}\right)^2\right]\right]y_{jt-k} + \varepsilon_{jt}, \tag{3.9}$$

$$\begin{aligned} y_{jt} = \Phi_{j0} + \sum_{k=1}^{k=12}\left[\Phi_{jk0} + (\pi_{jk0} + \pi_{jk1}y_{jt-1})\exp\left[-10\left(y_{jt-1} - \mu_{y_{jt-1}}\right)^2\right]\right]y_{jt-k} \\ + \sum_{r}\varsigma_r L_{jr(t-dr)} + \varepsilon_{jt}. \end{aligned} \tag{3.10}$$

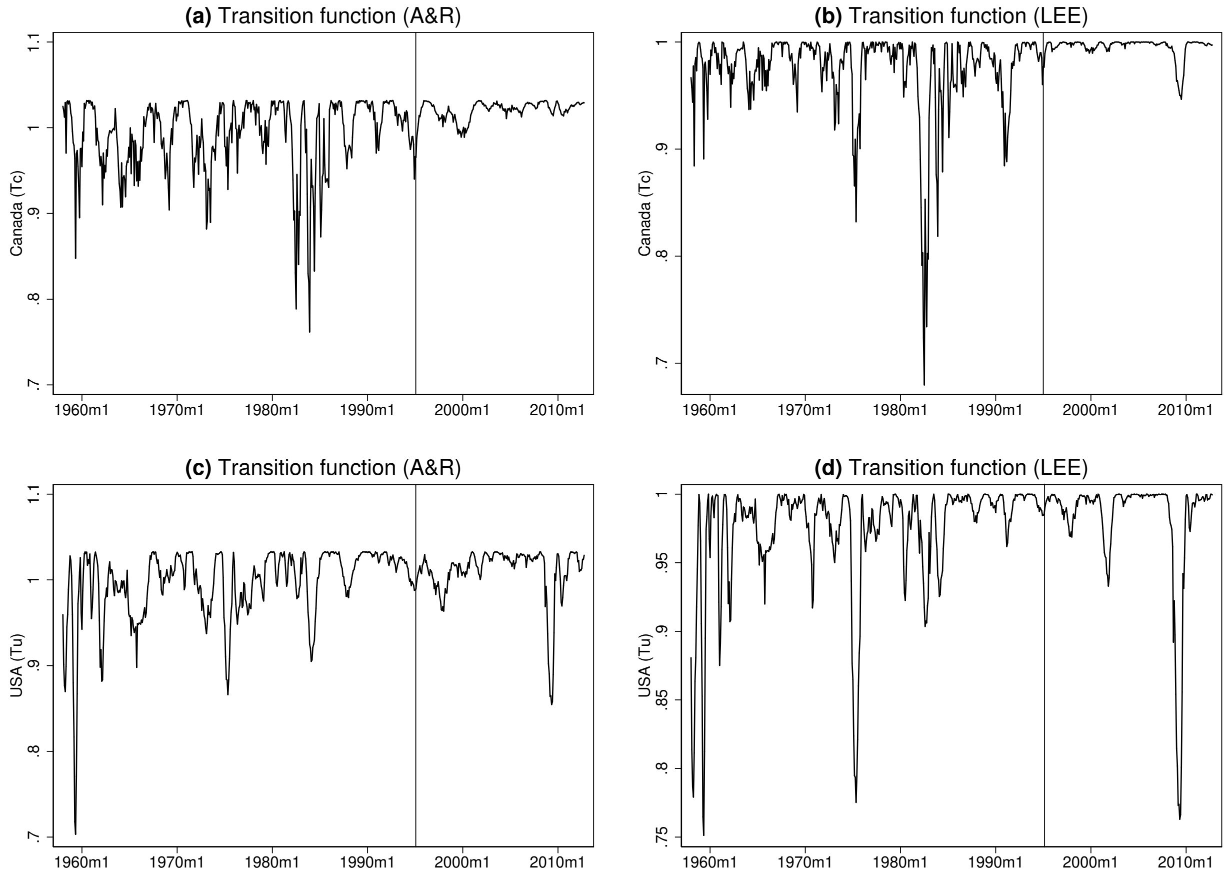
Plots of the exponential transition functions are provided in Figure 3.3. Left panel describes the approximated transition functions by Anderson and Ramsey, and the right panel describes the original version of the exponential transition functions that I use. The right-hand side of the black vertical line in each graph is calculated with the extended data. Summary statistics for the original version of the exponential transition functions are provided in Table 3.2.

3.4 Estimation of the Models

Summary statistics for the data are presented in Table 3.1 and plots are reported in Figure 3.1. The plots show that the transformed data series are approximately stationary. We assume that the transformation renders the data stationary, and this assumption is supported by the Dickey and Fuller (1979) test statistics.[2] The plots present differences between the

[2]The null hypothesis of a unit root in either y_{ut} or y_{ct} is strongly rejected.

Figure 3.3: Transition functions



Note: Left panels report Anderson and Ramsey (2002)'s approximations, and right panels report original definitions of transition function that I use.

dynamics of industrial production indices in the two series, but the sample moments are very similar in Table 3.1.

The last line of Table 3.1 contains p-values for Terasvirta-type LM tests. The null hypothesis is that the data generating processes for y_{ut} and y_{ct} are linear, while the alternative hypothesis suggests STAR(p).[3] A rejection of the null hypothesis of linearity is interpreted as a support for a variable amplitude coefficient model.

We provide forecasts produced by the nonlinear model (without a link), two linked models and a linear AR benchmark model using the same lag structure. This allows the forecasting abilities of a variety of models to be compared. Some summary information on the forecast

[3] These tests are based on Teräsvirta (1994)'s observation that a Taylor series approximation of the alternative hypothesis, STAR(p), is a linear combination of the $3 \times p$ variables defined by $y_{t-k}y_{t-d}$, $y_{t-k}y_{t-d}^2$ and $y_{t-k}y_{t-d}^3$, for $k = 1, \ldots, p$.

Table 3.4: One-step-ahead forecasts of industrial production indices

(a) Comparison of the one-step-ahead forecasts derived from different models of the growth of Canadian industrial production

	Linear AR	Nonlinear AR	Linked model 1	Linked model 2 (trade)
RMSE (A&R)	0.01818	0.01701	0.01689	N/A
RMSE (LEE)	0.01515	0.01441	0.01430	0.01278

(b) Comparison of the one-step-ahead forecasts derived from different models of the growth of the U.S. industrial production

	Linear AR	Nonlinear AR	Linked model 1	Linked model 2 (trade)
RMSE (A&R)	0.01161	0.01140	0.01140	N/A
RMSE (LEE)	0.01091	0.01037	0.01037	0.00747

(1) For comparison with Anderson and Ramsey (2002), lagged Δy_{ut} and $\Delta y_{ct} \cdot \Delta y_{ut}$ are used as linkages for the linked model 1 for Canada (and vice versa for the United States).
(2) The sample periods for linked model 2 are 1969:1-2004:3 for Canada and 1981:1-2004:3 for the US, respectively, due to the availability of bilateral trade data. The sample period of 1958:1-2012:10 is used for all other forecasts.
(3) $\Delta z_{ct} \cdot \Delta z_{ut}$ is reported for the linked model 2 (with bilateral trade). Other linkage variables are also tested, and they produce consistent results.

errors is provided in Table 3.4. First raw with the label "RMSE (A&R)" indicates root mean squared error (RMSE) calculated using Anderson and Ramsey's dataset, and the second raw with the label "RMSE (LEE)" indicates RMSE calculated using extended dataset. When we compare RMSEs derived from each model, nonlinear AR model generates smaller RMSEs than those of linear AR model, and the linked model generates smaller RMSEs than those of nonlinear AR model. These are consistent across countries and different lengths of datasets. In addition, West (1996) forecast test says that nonlinear AR model performs better than linear AR model with a p-value of 0.0144 and 0.0458, for Canada and the United States respectively. We also notice that linked model with bilateral trade (linked model 2) generates smaller RMSEs than those of the other linked model (linked model 1) that Anderson and Ramsey suggest, and this is also consistent across countries.

The root mean squared errors (RMSE) for h-step ahead forecasts are provided in Table 3.5. Sample periods of 1969:1-2004:3 for Canada and 1981:1-2004:3 for the U.S. are used with the rolling method. Comparing root mean squared errors (RMSE) across models, we can say that nonlinear AR model performs much better than linear AR model across countries and different steps. However, linked model 1 (linked model with lagged Δy_{ut} and $\Delta y_{ct} \cdot \Delta y_{ut}$) does not show smaller RMSE's than those of nonlinear AR model. Interestingly, we find that

Table 3.5: RMSE for h-step ahead forecasts

(a) Root mean squared errors (RMSE) for h-step ahead forecasts of the annual growth of Canadian industrial production

Forecast horizon (h)	Number of forecasts	Linear AR	Nonlinear AR	Linked model 1	Linked model 2 (bilateral trade)
1 step ahead	646	0.0151	0.0144	0.0143	0.0128
6 steps ahead	641	0.0303	0.0293	0.0292	0.0287
12 steps ahead	635	0.0442	0.0422	0.0422	0.0419
18 steps ahead	629	0.0452	0.0437	0.0438	0.0439
24 steps ahead	623	0.0449	0.0441	0.0441	0.0375
30 steps ahead	617	0.0449	0.0439	0.0435	0.0327

(b) Root mean squared errors (RMSE) for h-step ahead forecasts of the annual growth of the U.S. industrial production

Forecast horizon (h)	Number of forecasts	Linear AR	Nonlinear AR	Linked model 1	Linked model 2 (bilateral trade)
1 step ahead	646	0.0109	0.0104	0.0104	0.0075
6 steps ahead	641	0.0336	0.0300	0.0298	0.0198
12 steps ahead	635	0.0453	0.0422	0.0421	0.0314
18 steps ahead	629	0.0460	0.0457	0.0457	0.0329
24 steps ahead	623	0.0462	0.0463	0.0460	0.0311
30 steps ahead	617	0.0462	0.0454	0.0454	0.0297

(1) All forecasts use the rolling method.
(2) The direct method is used for the multistep forecasting. See Marcellino, Stock and Watson (2006) for performance comparison of iterated method and direct method.
(3) For comparison with Anderson and Ramsey (2002), lagged Δy_{ut} and $\Delta y_{ct} \cdot \Delta y_{ut}$ are used as linkages for the linked model 1 for Canada (and vice versa for the United States).
(4) $\Delta z_{ct} \cdot \Delta z_{ut}$ is reported for the linked model 2 (with bilateral trade). Other linkage variables are also tested, and they produce consistent results.
(5) The sample periods for linked model 2 are 1969:1-2004:3 for Canada and 1981:1-2004:3 for the US, respectively, due to the availability of bilateral trade data. The sample period of 1958:1-2012:10 is used for all other forecasts.

linked model with bilateral trade (linked model 2) generates generally smaller RMSEs than those of other models, and this is supporting evidence that bilateral trade is a good linkage variable for the non-linear economic relationships between the U.S. and Canadian industrial production indices.

3.5 Conclusion

This essay studies the coupling of the U.S. and Canadian industrial production indices. It first replicates Anderson and Ramsey (2002)'s main results with an extended dataset then it improves the forecasting performance by considering dynamic linkage variables – the U.S.

and Canadian bilateral trade data. Being consistent with Anderson and Ramsey (2002), this essay uses Ozaki (1981)'s nonlinear autoregressive model as a baseline, and a wide range of linkage variables are considered.

The essay compares the forecast accuracies of the linked, unlinked, and benchmark autoregressive models and shows that the linked models improve forecasts on average. Especially, the linked model with bilateral trade that this essay proposes outperforms the linked model suggested by Anderson and Ramsey (2002) and other linear and nonlinear models. We conclude that bilateral trade is a good linkage variable for the non-linear economic relationships between the U.S. and Canadian industrial production indices.

Appendix A

Appendix: Chapter 1

A.1 Kalman Filter with MA(1) Noises

Standard Kalman filter assumes zero mean Gaussian white noise for $\boldsymbol{w}_t$. In this essay, I introduce an auxiliary random process so that $\boldsymbol{w}_t$ follows MA(1) within the Kalman filter.[1] The MA(1) noise defined in equations (1.23) and (1.24) is equivalent to the following:

$$\begin{bmatrix} \boldsymbol{w}_t \\ \boldsymbol{\phi}_t \end{bmatrix} = \begin{bmatrix} \mathbf{0}_{2,2} & \boldsymbol{\theta} \\ \mathbf{0}_{2,2} & \mathbf{0}_{2,2} \end{bmatrix} \begin{bmatrix} \boldsymbol{w}_{t-1} \\ \boldsymbol{\phi}_{t-1} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\xi}_t \\ \boldsymbol{\xi}_t \end{bmatrix} \tag{A.1}$$

where

$$\boldsymbol{\phi}_t = \begin{bmatrix} \phi_t^\pi \\ \phi_t^y \end{bmatrix}, \boldsymbol{\theta} = \begin{bmatrix} \theta^\pi & 0 \\ 0 & \theta^y \end{bmatrix}, \boldsymbol{\xi}_t = \begin{bmatrix} \epsilon_t^\pi \\ \epsilon_t^y \end{bmatrix}.$$

$\boldsymbol{\phi}_t$ is the auxiliary vector to transfer past white noise to current period, $\boldsymbol{\theta}$ is the matrix of MA(1) parameters, and $\boldsymbol{\xi}_t$ is the vector of white noises. The upper block ($\boldsymbol{w}_t = \boldsymbol{\theta}\boldsymbol{\phi}_{t-1} + \boldsymbol{\xi}_t$) and the lower block ($\boldsymbol{\phi}_t = \boldsymbol{\xi}_t$) together produce $\boldsymbol{w}_t = \boldsymbol{\xi}_t + \boldsymbol{\theta}\boldsymbol{\xi}_{t-1}$.

Given the noise formulation (A.1), an equivalent to state-space (1.27) and (1.28) with MA(1) observation noise can be proposed:

$$\begin{bmatrix} X_{t+1} \\ \boldsymbol{w}_{t+1} \\ \boldsymbol{\phi}_{t+1} \end{bmatrix} = \begin{bmatrix} A & \mathbf{0}_{10,2} & \mathbf{0}_{10,2} \\ \mathbf{0}_{2,10} & \mathbf{0}_{2,2} & \boldsymbol{\theta} \\ \mathbf{0}_{2,10} & \mathbf{0}_{2,2} & \mathbf{0}_{2,2} \end{bmatrix} \begin{bmatrix} X_t \\ \boldsymbol{w}_t \\ \boldsymbol{\phi}_t \end{bmatrix} + \begin{bmatrix} B \\ \mathbf{0}_{2,1} \\ \mathbf{0}_{2,1} \end{bmatrix} i_t + \begin{bmatrix} \boldsymbol{\nu}_{t+1} \\ \boldsymbol{\xi}_{t+1} \\ \boldsymbol{\xi}_{t+1} \end{bmatrix} \tag{A.2}$$

[1] Please refer to Geist and Pietquin (2011) for more detailed explanations on autoregressive and moving-average noise in Kalman filter.

$$Z_t = \begin{bmatrix} C & \boldsymbol{I}_{2,2} & \mathbf{0}_{2,2} \end{bmatrix} \begin{bmatrix} X_t \\ \boldsymbol{w}_t \\ \boldsymbol{\phi}_t \end{bmatrix} \tag{A.3}$$

Therefore, we can rewrite the equation (1.27) and (1.28) as

$$X'_{t+1} = A'X'_t + B'i_t + \boldsymbol{\nu}'_{t+1} \tag{A.4}$$

$$Z'_t = C'X'_t \tag{A.5}$$

where

$$X'_t = \begin{bmatrix} X_t \\ \boldsymbol{w}_t \\ \boldsymbol{\phi}_t \end{bmatrix}, A' = \begin{bmatrix} A & \mathbf{0}_{10,2} & \mathbf{0}_{10,2} \\ \mathbf{0}_{2,10} & \mathbf{0}_{2,2} & \boldsymbol{\theta} \\ \mathbf{0}_{2,10} & \mathbf{0}_{2,2} & \mathbf{0}_{2,2} \end{bmatrix} B' = \begin{bmatrix} B \\ \mathbf{0}_{2,1} \\ \mathbf{0}_{2,1} \end{bmatrix}, \boldsymbol{\nu}'_t = \begin{bmatrix} \boldsymbol{\nu}_t \\ \boldsymbol{\xi}_t \\ \boldsymbol{\xi}_t \end{bmatrix}, C' = \begin{bmatrix} C \\ \boldsymbol{I}_{2,2} \\ \mathbf{0}_{2,2} \end{bmatrix}^T$$

Optimal Kalman gain K' (14×2 matrix) and predicted estimate covariance $P'_{t|t-1}$ (14×14 matrix) are specified as

$$\begin{aligned} K' &= P'_{t|t-1}C'^T(C'P'_{t|t-1}C'^T)^{-1} \\ P'_{t|t-1} &= A'(P'_{t|t-1} - K'C'P'_{t|t-1})A'^T + V'_\nu \end{aligned} \tag{A.6}$$

where V'_ν (14×14 matrix) is variance-covariance matrices of ν'_t.

The central bank's optimal inference $X'_{t|t}$ and forecast $X'_{t+1|t}$ are in recursive form as

$$\begin{aligned} X'_{t|t} &= A'X'_{t-1|t-1} + B'i_{t-1} + K'(Z'_t - Z'_{t|t-1}) \\ &= (I - K'C')A'X'_{t-1|t-1} + (I - K'C')B'i_{t-1} + K'Z'_t \end{aligned} \tag{A.7}$$

$$X'_{t+1|t} = A'X'_{t|t} + B'i_t \tag{A.8}$$

and the estimated values of $X'_{t|t}$ and $X'_{t+1|t}$ are entered as $X_{t|t}$ and $X_{t+1|t}$ in equation (1.30) and (1.31) after eliminating the auxiliary variables.

A.2 Additional Tables and Figures

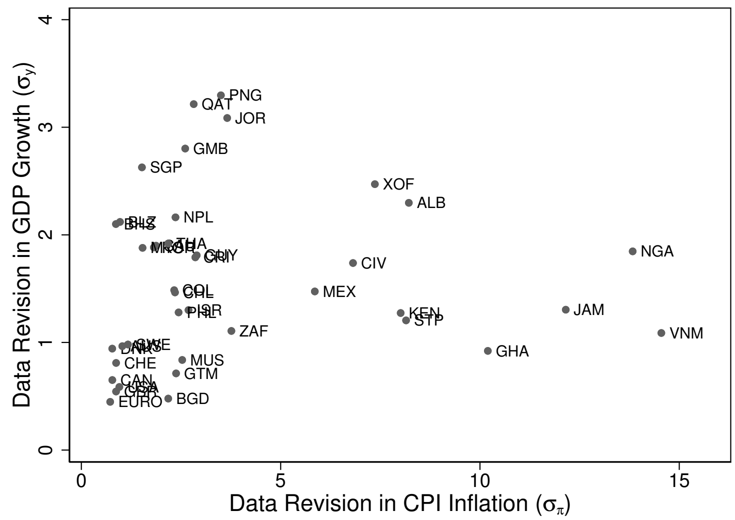
Table A.1: Example of historical WEO data (United States)

	WEO releases (real GDP growth)					
	S1990	F1990	S1991	F1991	S1992	F1992
1988	4.06	4.08	.	.	.	.
1989	2.99	2.96	3.26	3.31	.	.
1990	2.27	1.98	2.08	2.20	2.19	2.26
1991	3.09	2.44	1.19	0.86	-0.33	0.15
1992	3.30	3.42	2.93	2.79	1.42	1.10
1993	3.33	3.45	3.42	3.36	3.57	3.14
1994	3.32	3.42	3.46	3.41	3.75	3.55
1995	3.34	3.40	3.39	3.29	3.77	3.73
1996	.	.	3.39	3.29	3.62	3.74
1997	.	.	.	.	3.57	3.67

	WEO releases (CPI inflation)					
	S1990	F1990	S1991	F1991	S1992	F1992
1988	4.09	4.07	.	.	.	.
1989	4.80	4.77	4.82	4.82	.	.
1990	4.56	5.08	5.28	5.40	5.41	5.40
1991	4.19	4.51	4.90	4.53	4.23	4.28
1992	4.20	4.20	4.00	4.05	3.11	3.11
1993	4.20	4.20	3.90	3.80	3.07	3.13
1994	4.20	4.20	3.70	3.60	3.00	3.00
1995	4.20	4.20	3.60	3.40	2.90	2.90
1996	.	.	3.50	3.30	2.75	2.80
1997	.	.	.	.	2.60	2.80

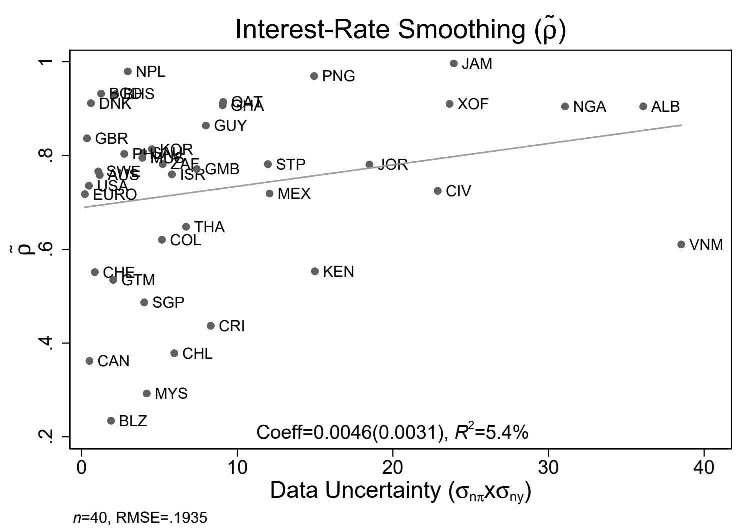
Note: This example includes the WEO issues from Spring 1990 to Fall 1992. Values are in percentage change from previous years. Sample data span from Spring 1990 to Fall 2008 for each country.

Figure A.1: Data uncertainty in CPI and GDP countries (raw standard deviation)



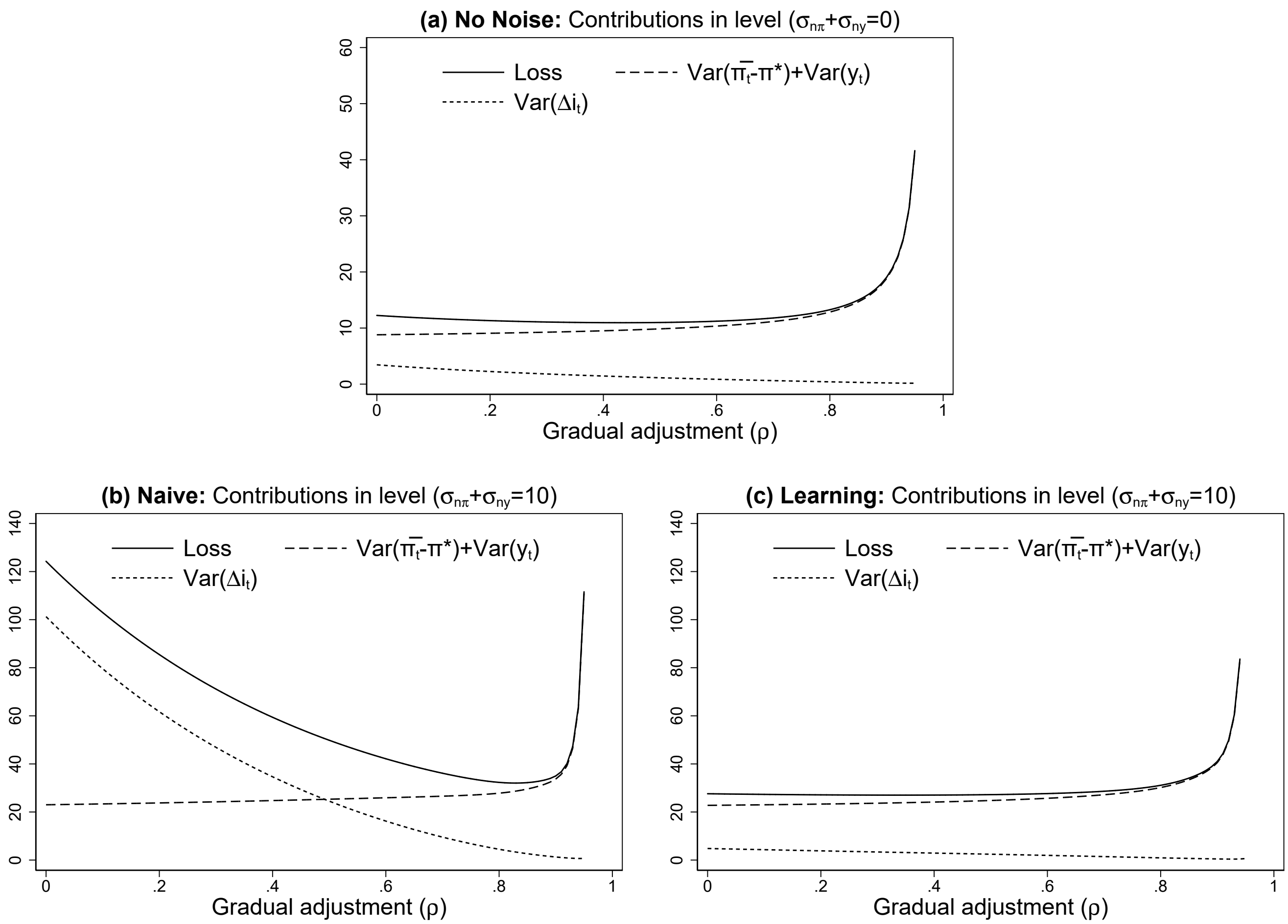
Note: Raw standard deviations are reported for data revisions in CPI inflation and GDP growth. On the contrary, figure 1.2 (a) reports standard errors implied by MA(1) noise process. Both provide the comparatively similar amount of data revisions with minor differences in magnitude.

Figure A.2: Cross-country comparison of interest-rate smoothing (product measure)



Note: An alternative measure of data uncertainty is considered: a product of standard errors ($\sigma_{n\pi} \times \sigma_{ny}$) instead of the sum of standard errors ($\sigma_{n\pi} + \sigma_{ny}$). The scale of data uncertainty increases from 20 to 40, and the overall pattern is similar to that in figure 1.2 (b).

Figure A.3: Loss decomposition (absolute contributions)



Note: Figure A.3 reports the loss decomposition under different monetary policy and level of data uncertainty. It shows the contribution of $Var[\overline{\pi}_t - \pi^*] + Var[y_t]$ and $Var[\Delta i_t]$ in levels and their associated loss with regard to the degree of gradual adjustment (ρ). Note that $Var[\overline{\pi}_t - \pi^*]$ and $Var[y_t]$ increase in ρ and $Var[\Delta i_t]$ decreases in ρ. $Var[\overline{\pi}_t - \pi^*]$ and $Var[y_t]$ diverge to infinity if ρ is close to 1. Panel (a) reports those when there is no data uncertainty, and they are same under both monetary polices. Panel (b) reports those in face of data uncertainty $((\sigma_{n\pi} + \sigma_{ny}) = 10)$ under naive policy, and panel (c) reports those in face of data uncertainty $((\sigma_{n\pi} + \sigma_{ny}) = 10)$ under learning policy. $Var[\Delta i_t]$ curve becomes much steeper in the face of data uncertainty under naive policy. $Var[\Delta i_t]$ curve becomes relatively flatter in the face of data uncertainty under learning policy. Figure A.4 reports the relative contributions of each loss components for a better obseration of the change in $Var[\Delta i_t]$ from panel (a) to (c). Figure 1.7 provides diagrams that summurize these changes.

Figure A.4: Loss decomposition (relative contributions)

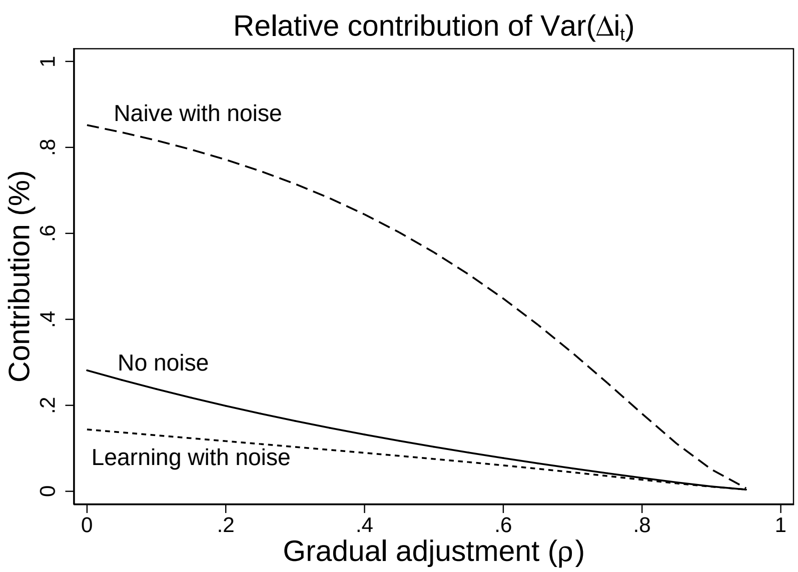


Figure A.4 reports the relative contributions of each loss components for a better observation of the change in $Var[\Delta i_t]$ under naive and learning policies. The relative contribution curve of $Var[\Delta i_t]$ becomes much steeper in the face of data uncertainty under the naive policy and flatter in the face of data uncertainty under the learning policy.

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