From Brains to Games and Back Again:

Accessing the Magnitude of Fractions and Ratios Across Contexts

By

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Abstract

Many children learn about fractions and ratios as topics in the math classroom, but even before we learn to count we are constantly encountering these numerical relationships in the things we see, hear, and feel (e.g. a third of a cookie, the varying intensity and rhythm of a drum). Educational researchers have proposed that the ways we understand fractions and ratios in these two contexts may be fundamentally intertwined. However, little research has explored the specific question of how our access to the meaning of a symbolic fractions (e.g. 1/2) is related to our perception of nonsymbolic visual ratios. Using an educational neuroscience approach, this dissertation connects three separate empirical approaches to compare symbolic and nonsymbolic magnitude processing from the perspectives of cognitive psychology, neuroscience, and educational digital media research. First, analyses of magnitude comparison performance (speed and accuracy) in computer-based tasks, reveal how adults can rapidly access a sense of magnitude from symbolic and nonsymbolic stimuli within and across formats. Second, analyses of neural activity during symbolic, nonsymbolic and cross format comparisons, indicate adults show considerable overlap in the regions the brain sensitive to changes in fraction magnitude. Third, analyses of magnitude comparison performance in the context of and educational math game (Fractions War) shows how many effects of Study 1 and 2 observed in the lab do and do not extend to data observed in digital informal learning contexts. Based on this work, I argue that there remains a great potential for using interactive media experiences as interventions to test the generalizability of numerical cognition theories and increase collaboration between neuroscience, psychology, and educational researchers.

Chapter 1 – General Introduction and Background

The Importance and Difficulty of Fractions

Among all the topics, concepts, and procedures that students will learn in the math classroom, the acquisition of fraction knowledge may be one of the most important. Fractions introduce students to fundamental properties and concepts of numbers that extend beyond natural number knowledge, such as part-whole relationships and rational number magnitudes (Siegler & Lortie-Forgues, 2014). Previous studies have demonstrated that fractions knowledge can predict future mathematics achievement (Bailey et al., 2012; Siegler et al., 2012), and is implicated in the formation of early algebraic reasoning (Booth et al., 2014; Booth & Newton, 2012; DeWolf et al., 2015). Thus, fractions knowledge has been referred to as a critical competency within math education (National Math Advisory Panel, 2008).

Unfortunately, many people face great difficulties with mastering fraction knowledge (Siegler, Fazio, Bailey, & Zhou, 2013; National Math Advisory Panel, 2008). One way that difficulties with fractions manifest is when students misapply inappropriate whole number knowledge and strategies to support their reasoning with fractions (Ni & Zhou, 2005). This phenomenon, often referred to has whole-number bias (Bonato et al., 2007; Rinne et al., 2017) or the natural number bias (Gómez & Dartnell, 2019; Vamvakoussi, 2015), can be described as a preference or tendency to rely on whole-number knowledge, when rational number representations are not as strong, precise, or efficient (Alibali & Sidney, 2015).

The question of why fraction knowledge is so difficult to master is a matter of current debate among numerical cognition and math education researchers (Berch, 2017; Lewis et al., 2015; Möhring et al., 2016). Some researchers propose that whole-number biases can be explained by *innate constraints* of the cognitive capacities that humans share. Proponents of innate constraints theories argue that the foundational cognitive systems for understanding numbers have evolved to understand natural numbers but not necessarily the meaning of ratios and proportions (Feigenson et al., 2004; Geary, 1995). Innate number systems, namely the Approximate Number System (ANS), are described as perceptual systems that enable a sense of discrete quantities, and in turn provide an intuitive basis upon which people can come to understand the meaning of whole numbers (Piazza, 2010). The argument follows when we “push number representations further to embrace fractions, square roots, negative numbers and complex numbers, they move even further from the intuitive sense of number provided by the core systems” (Feigenson et al., 2004, p. 313).

Other researchers suggest that difficulties with fractions emerge as a consequence of educational conventions and the sequencing of math topics through development (Post et al., 1993). For instance, early math education places a much larger emphasis on whole number concepts prior to introduction of fractions. Given fractions are composed of whole number components, students may struggle to understand how a new set of rational number rules applies to numbers symbols that they already have a strong whole-number intuitions about (Gelman, 2015; Ni & Zhou, 2005).

Other researchers point to the role of education in a different way. Citing the emerging evidence that humans do have an innate capacity beginning early in life to perceive ratio relationships (Jacob, Vallentin, & Nieder, 2012; Duffy, Huttenlocher, & Levine, 2005; McCrink & Wynn, 2007), some researchers have argued that this intuitive sense of rational numbers has not been fully realized as a powerful educational foundation (Lewis, Matthews, & Hubbard, 2015). In other words, difficulties learning fractions may be partially addressed by emphasizing educational approaches that leverage this intuitive and perceptual sense of ratio. Thus, this *ratio processing system* (RPS), may, like the ANS for whole numbers, provide a neurocognitive startup tool upon which a grounded understanding of fractions can be built. Overreliance on educational approaches that emphasize an interpretation of fractions as discrete and countable parts may bolster the tendency of students to rely on whole number background knowledge, rather than forming new conceptions of relationally defined magnitudes (Matthews & Ellis, 2018). This leads to the critical question of whether educators and instructional designers can create learning experiences that encourage a deeper understanding of what these otherwise meaningless and complex number symbols mean.

In this dissertation, I examine and discuss a fundamental piece of ratio processing theory and how it applies to the understanding of fractions. Specifically, the theory proposes than an intuitive perceptual understanding of ratios can provide a foundation upon which education can ground a stronger understanding of fractions (even when presented as number symbols). Yet before we can address this theoretical relationship as a mechanism for fractions learning, we must evaluate more basic questions regarding how similarly or differently people understand the meaning of perceptually defined ratios and fractions represented as number symbols. Are the internal representations of magnitude that people access from these different external fraction and ratio representations compatible, the same, or completely distinct? How easily can people find shared meaning across these formats? In this dissertation I aim to address these questions using an interdisciplinary approach, which tests hypotheses of RPS theory with traditional psychological and neuroscience methods and evaluates a model of using educational games to provide critical tests of how lab-based results correspond to real-life educational contexts.

Representational Fluency with Fraction and Ratios

One way of studying the cognitive mechanisms involved in fractions knowledge, and the sources of difficulties acquiring this knowledge, is to consider broader theoretical questions about how humans encode the meaning of otherwise meaningless symbols (Harnad, 1990; Leibovich & Ansari, 2016; Reynvoet & Sasanguie, 2016). A central cognitive construct that connects the three studies in this dissertation is *representational fluency* with fractions, which I define as the ability to quickly and effortlessly translate between different (external) representations of fractions and ratios by accessing a common internal representation of their meaning. Perceptual fluency has been previously described as having the ability to rapidly identify common concepts across different external representations (e.g. numerical visualizations and graphs) (Kellman et al., 2008) and having readily accessible perceptual knowledge resulting from experience with these visual representations (Rau et al., 2015). Thus, the concept of representational fluency describes the critical capacity to connect knowledge of fractions as a symbolic representation to a sense of ratio magnitude understood via the perceptions of nonsymbolic ratios.

In this dissertation I will use the term *representation* to refer either to an external representation of fractions and ratios or an internal mental representations of the meaning these external representations refer to (G. A. Goldin & Kaput, 1996). In studying how people understand these external representations, this dissertation will generally examine the processing of two distinct *formats: symbolic fractions* (e.g. 1/2) and *nonsymbolic ratios* (e.g. a line half as long as another). In comparing how individuals understand these two external representations, the aim will be to gain insight into the relationship between the internal representations of meaning that people access from symbolic and nonsymbolic stimuli. Furthermore, this dissertation attempts to gain additional insight into the nature of internal representations by observing *neural representations* of magnitude processing from these external forms. Specifically, I will explore how neural activations represent biological manifestations of processing symbolic and nonsymbolic stimuli and activating an internal sense of the magnitudes they represent.

In regard to the internal representation of fractions meaning, it is important to note that there is no one singular *meaning* that fractions must refer to (Kieren, 1976). Fractions can represent multiple rational number concepts such as part whole relationships, ratios, division, discrete quantities and continuous quantities (Behr et al., 1983). In this dissertation I focus on how fractions represent ratio relationships and in doing so represent rational number magnitudes that are *relationally defined*, meaning a sense of magnitude that emerges from the relationship between two values (Kalra, Matthews, et al., 2020; Matthews & Ellis, 2018). Therefore, in my analysis of perceptual fluency with fractions, I will be exploring the extent to which people can quickly and accurately access a sense of relationally defined magnitudes, be they presented as a symbolic (numerical) numerator and a denominator, a ratio of two line lengths, a ratio of circle areas, or even as the relationship between the value of two playing cards. In the three empirical studies presented in the following chapters, I operationalize the construct of representational fluency in the context of magnitude comparison performance.

This dissertation evaluates the relationship between external, internal, and neural representations of fractions and ratios with the goal of describing mechanisms of numerical cognition which may generalize to adult educated populations. Some theories of math education take strong stances against the assumption that there are internal mental representations of mathematical concepts which we as humans necessarily share (Steffe & Kieren, 1994; von Glassersfeld, 1995). From this perspective external representations of fractions and ratios do not *contain* meaning that people can access, but rather these external forms can only elicit the meaning that individuals construct from their own unique existing knowledge (R Lesh & Doerr, 2003). These perspectives offer valuable lenses for appreciating the individualistic nature of math knowledge, and emphasize how attempts to describe internal representations of math knowledge in general terms will inevitably be imperfect and incomplete (G. A. Goldin & Kaput, 1996). Nevertheless, I argue that it is still valuable to study whether there are common foundations of knowledge that we can generalize across individuals despite the individualistic manifestations of that knowledge across people. Adopting the assumption that there are internal representations of knowledge at this general level, affords the study of how commonalities emerge because of shared educational experiences or even by our nature of sharing the same human bodies and nervous systems.

This dissertation aims to draw inferences about human numerical processing with fractions and ratios by first testing this assumption that there are meaningful commonalities in neural and cognitive processing, which we as humans typically share.

From this premise, we can then study how factors such as educational experiences, biological development, and specific expertise build from these common cognitive structures to create the richness of individual differences in understanding fractions and ratios.

Mathematical Cognition across Contexts

Most numerical cognition research into the topic of fractions knowledge and its development has been performed within the context of the research lab via controlled experimental tasks. This can be a serious limitation to the ecological validity of research when the goal is to discover empirical knowledge that has the potential to impact math instruction or help teachers gain greater insights into the minds of their students. The question then becomes how to connect the scientific rigor of the controlled lab study with the access to learners while they are in their true learning environments (Han et al., 2019). One key to forging this connection is reflecting on how expressions of human cognition are shaped by the context in which we as researchers observe and measure them. In the traditional controlled lab setting, features of the environment and the stimuli are carefully designed to strip away extraneous details in order to test specific hypotheses, yet as a consequence the researcher creates a very unique context for studying behavior relative to how that behavior exists in the world. Thus efforts to ask the same research questions across contexts of the lab and real-life contexts are critical for testing of numerical cognition theories aimed at explaining mechanisms of math learning (Ansari & Coch, 2006; Han et al., 2019; Rosenberg-Lee, 2018; Varma et al., 2008).

In this dissertation, I will discuss how collaboration between cognitive researchers and educational game designers can be one impactful way to achieve this goal (Chapter 4).

The notion that video games and the behaviors within these *virtual* experiences represent a contexts of learning in *real-life* may be controversial, especially when we consider the ways that these experiences can be designed so players can roleplay as a make-believe persona or use the game as an outlet to escape from reality (Molesworth, 2009). Nevertheless, these virtual designed experiences are gaining a real presence in the lives of adults and children in industrialized societies as forms of entertainment and increasingly as forms of education. As of a 2016 survey, teens and tweens (8- to12-year-olds) spend about 9 hours and 6 hours respectively in front of screens, to access social media, watch TV and play video games (Rideout, 2016). Furthermore, the presence of commercial or educational games in classrooms, homes and informal learning settings continues to grow (Brom et al., 2010; Hainey et al., 2016). In situations where education must rely largely on virtual and digital media to engage students with the learning content (such as the 2020 COVID-19 pandemic), there is a critical need for empirical knowledge about how the design of these experiences affects learning and the assessment of student knowledge. Collaborations between cognitive science researchers and educational game designers create the possibility to both observe constructs of numerical cognition in these virtual real-life experiences and in turn deepen our understanding of effective educational game design.

Educational Video Games as Research Tools

To discuss the effects of the educational video game context on fraction knowledge, it is important to define what this context entails, in reference to both the specific game used in this dissertation and to educational video games in general. Generating a definition of educational games is not straightforward. In the past, different researchers have tried to identify different key features which all games share (Garris, Ahlers, Driskell, et al., 2002;

Gee, 2007). Other researchers deny the idea that there is a definitive list of attributes which separate games from other activities and state that games fit in the same semantic category because they bear a *family-resemblance* to one another (Arjoranta, 2014; Wittgenstein, 1953).

In this dissertation, the goal in defining an educational video game context is to enable research which can begin to identify specific mechanisms unique to games, which may create a different psychological experiences than cognitive tasks designed for controlled laboratory-based research. Features common among games can be applied to the design of experimental tasks and educational activities (e.g. simulations), yet the inclusion of these features does not necessarily make them games(Crookall et al., 1987). In a complementary way, games can be implemented in ways that can make the player feel more like they are completing an obligatory task.

Therefore, when I describe the experimental manipulation of task vs game in Study 3 (Chapter 5), I outline several features common to many (but not all) educational games (Wittgenstein, 1953) and hypothesize how the manipulation of these features across contexts lead to psychological changes. For instances, there are differences in the goal of these designed experiences. While both math games and math tasks are designed to have individuals interact with some math content, the primary goal of the task is to assess and observe abilities while the primary goal of the educational game is to instruct and engage players in a fun activity. Furthermore, there are differences in the aesthetic design of these activities. The design of experimental tasks often includes the stripping away of extraneous details that may create distractions or introduce confounds that obscure the interpretation of results (Han et al., 2019); the design of educational games often includes more visual detail and artistic expression to create an experiences which is aesthetically pleasing. There are also differences in ways that performance is motivated to express the extent of an individual's abilities and knowledge. In experimental tasks, we instruct participants to give their best effort, provide simple feedback, and rely on the participants' desire to provide valid data. In educational games, motivation is encouraged by design features such as competitions, reward systems, and increasing challenges which rely on players desire to win and experience the success of overcoming challenges. Additional distinctions between tasks and games are discussed in following chapters with further elaboration on how these distinctions do not always create clear and separate theoretical contexts. Nevertheless, I argue that attention to these distinctions and the cooccurrence of more game-like or task-like attributes can help identify the family resemblance of activities which we can group into either category.

Visual Representations of Math in Games

In a math game, players are presented with visual representations of the educational content (e.g. fractions and ratios), which must also fit the game's theme, narrative, or genre. The effectiveness of visual representations in video games as tools to help players learn requires the player to have some knowledge of the educational concepts and *representational competencies* to accurately and fluently understand how those visuals correspond to the educational concepts (Gilbert, 2005; Rau & Matthews, 2017). For example, slices of pizza could be used in a game to teach now fractions represent parts of a whole, but the educational effectiveness of this approach depends on the student's ability to see and understand this correspondence between the visuals and formal math concepts.

In this dissertation, I explore a specific case of using visual representations in games when those visual representations are made from repurposing traditional playing cards. Playing cards are a cultural artifact which can be adaptively used to play a countless number of different games, and across an adult population there may be a wide variety of expertise with playing cards and their visual form. In my analysis of gameplay data, I investigate whether this prior expertise with playing cards outside of the educational game context leads to measurable differences in how fluently and accurately adults access a sense of fraction magnitude from playing cards in a game. In other words, can individual differences in playing card expertise and fluency mediate gameplay performance in *Fractions War*? Addressing this question has the potential to elucidate ways that applying general theories of magnitude processing with visual ratios to the design of real-life learning experiences requires specific attention to the nature of visual representations and learners' previous experiences with these visual representations.

Summary of Studies

From brains to games, this dissertation explores the question of how people understand rational number magnitudes when they are presented with different formats and in different contexts. In Chapters 2 and 3, I examine neurocognitive theories regarding the nature of internal representations underlying ratio processing and fraction knowledge within the controlled experimental environment of the research lab and the fMRI scanner In Chapter 4, I review arguments for why educational games can play a powerful role in educational and neuroscience research as tools for testing specific interventions and assessing players knowledge though the analysis of gameplay data. In Chapter 5, I put this argument to the test by using an educational game, *Fractions War*, to observe if visual representations of content in games and broader gameplay context shape the players abilities to fluently reason with fraction and ratio magnitudes. Through this interdisciplinary approach, I aim to identify and connect theories of learning across fields of research to observe where these perspectives converge and where they each elucidate unique can critical aspects of developing fractions knowledge.

In Chapter 2, I explore the question of whether perceiving the magnitude a visual ratio and accessing the magnitude of a fraction rely on a similar internal representations of rational number magnitude. I approach this question by examining the similarities and differences in speed and accuracy of magnitude judgements made with pairs of symbolic fractions, nonsymbolic ratios, and symbolic-nonsymbolic cross-format stimuli. Specifically, the goals of this study were to evaluate (a) whether adults show evidence of holistic magnitude processing in all formats (including cross-format judgements), (b) whether cross-format processing is more difficult than within symbolic or nonsymbolic formats, and (c) whether adults show evidence of efficient nonsymbolic ratio processing abilities. Across two experiments I evaluate these research questions and additionally explore how the complexities of these symbolic and nonsymbolic forms interact with holistic magnitude processing.

In Chapter 3, I utilize functional magnetic resonance imaging (fMRI) to further explore the question of whether similar regions of the brain support the understanding of rational number magnitudes when those magnitudes are presented as visual ratios or symbolic fractions. Using whole-brain analyses, I evaluated the localization of brain regions sensitive to magnitude processing within each of formats and identified shared and non-shared areas supporting an understanding of these two external formats. Moreover, we examined whether brain regions sensitive to holistic magnitude converged within specific regions of the parietal cortex associated with magnitude processing.

In Chapter 4, I present a review of educational games research and theory to discuss the potential of utilizing educational games as a tool to extend numerical cognition and math education research. First, I address how the dynamic and visual nature of video games may be used to study how learners develop representational competencies with visual representations in digital learning contexts. Second, I discuss how researchers thus far have assessed the generally accepted assumption that situating learning content in games should improve engagement and positive emotional associations with educational content. Lastly, I describe how games have been put forth as a means of *stealth assessment* where gameplay and associated data can serve as formal assessment to support learning in the game.

In Chapter 5, I present a series of experiments that embody these arguments for the use of games in numerical cognition research. In doing so, I evaluate the viability of this empirical approach and examine whether the behaviors observed in Studies 1 and 2 replicate in an educational game context. By contrasting adult performance in analogous magnitude comparison activities contextualized within either an assessment-based comparison task or an educational math game I evaluate the effect of context on magnitudes processing with symbolic and nonsymbolic ratios. Specifically, I evaluate the effect of repurposing known visual forms (playing cards), as visual representations of fractions and ratios, and how these effects interact with prior expertise and fluency with these visual forms.

In Chapter 6, I conclude with a summary of the findings observed across the three studies to illustrate how neurocognitive, learning science, and multimedia theories of learning can work together to bridge an understanding of how people access the meaning of fractions and ratios in applied settings. The studies in this dissertation progress from detailed analyses of seemingly simple line ratios and fractions, to more complex forms of circle ratios and double-digit fractions, and onto representations of these symbolic and nonsymbolic forms embedded in an educational video game. Through this progression, I describe how neurocognitive theories regarding the perceptual foundations of fractions knowledge work together with perceptual learning theories to articulate how people may construct understandings and fluencies of visual representations upon these foundational systems. Moreover, perceptual learning theories work together with multimedia learning theories to explain how expertise and representational competencies interact with the cognitive load of rich digital environments. Lastly, the use of educational games as research tools to test applied theories of learning can serve as a catalyst for these rich interdisciplinary projects, and in turn may build applied knowledge about how these powerful educational can be optimized in design and application.

Chapter 2 –Evidence for an Association Between Fractions and the Ratios They Represent

Introduction

A fundamental question in numerical cognition research is how people come to understand the meaning of number symbols (e.g. 2, “ten”, ¾). This question, often posed as the *symbol grounding problem* for numbers, asks how people can interpret symbolic tokens, such as Arabic numerals, as the magnitudes they refer to and understand their meaning in a way external to the arbitrary symbolic shapes of numerical notation (Harnad, 1990). One debated solution to this question is that sensory experiences with nonsymbolic quantities and magnitudes develop internal representations of number meaning, which then may ground the meaning of number symbols (for review see Leibovich & Ansari, 2016; Reynvoet & Sasanguie, 2016). Previous research has largely focused on whole number processing, but more recently this work has extended to study rational number processing with fractions and ratios (Ischebeck et al., 2009; Siegler et al., 2011).

Studying fractions as symbolic representations of magnitudes presents new perspectives on the symbol grounding problem with numbers relative to previous research with whole number processing. The symbol grounding problem with fractions involves asking how the mind represents forms of numerical meaning beyond whole number concepts, such as non-integer magnitudes, proportions, and ratios. Whereas a single whole number corresponds to a specific magnitude, infinitely many equivalent fractions can refer the same rational number magnitude using a more complex syntactical structure. Thus, studying fractions also involves identifying how the mind parses meaning from a fraction’s bi-partite structure (numerator over denominator). Specifically, how do people integrate the relationship *between* numbers to access a sense of *relatively defined magnitudes* (Matthews & Ellis, 2018)? Previous studies indicate that adults and children can perform this integrative process to access a fraction's holistic magnitude (Binzak & Hubbard, 2020; Schneider & Siegler, 2010). However, further research is necessary to understand the nature of this integration process. In the current study, we aimed to address this by directly comparing how magnitude processing with symbolic fractions relates to the processing of nonsymbolic ratios, instantiated in the relative size of two visually defined magnitudes.

In this study, we also examined how adults access a common sense of magnitude when judgements of rational number magnitude are made across symbolic and nonsymbolic formats. There are many features that distinguish symbolic fractions and nonsymbolic ratios including their visual form and the contexts in which we interact with them. *Symbolic fractions* are rational numbers represented by two numeric components vertically separated by a vinculum line. Using this bipartite structure, fractions can symbolically represent a multitude of rational number concepts such as proportions, parts of a whole, and ratios (Behr et al., 1983). On the other hand, *nonsymbolic fractions* **are** the instantiations of proportions, ratios, and parts of a whole in the world. They can be observed in multiple modalities (e.g. sound, shape, time) and in naturally occurring phenomena or human-made percepts, such as visual examples made for educational purposes.

We are specifically interested in how people understand the magnitude of rational numbers when they are represented symbolically by common fractions or instantiated visually in a nonsymbolic part-part relationship. By directly contrasting magnitude processing with these formats, we explored the ways processing these formats are similar and unique. Specifically, evidence that magnitude comparison behaviors are modulated by the magnitudes of these numbers irrespective of their external form would support the existence of abstract mental representations of rational numbers (Dehaene et al., 1998). To further test this idea, we examined whether adults could access a common magnitude code across formats. Evidence for a common magnitude code, which can enable cross-format reasoning, would support the possibility that prior experiences with nonsymbolic perceptual ratios may help ground an understanding of symbolic fractions.

Symbolic Fraction Representations of Magnitude

Research into how people learn the meaning of symbolic fractions has practical educational implications and stands to expand the empirical understanding of human numerical cognition more broadly. Knowledge about rational number concepts provides a foundation for many advanced mathematical ideas integral to occupations and everyday life. For example, rational number knowledge can be applied to understand the scale of an architect's model, the ratio of ingredients in a recipe, or the acceleration of a new car. In the U.S. educational system, fractions are introduced years after students are introduced to natural numbers (Common Core Standards Initiative, 2015). Therefore, learning about fractions often involves introducing new number properties, like how numbers are not always countable entities and infinite numerical magnitudes exist between integer values(Siegler et al., 2011; Siegler & Lortie-Forgues, 2014). Sometimes these new features of fractions may seem in conflict with their whole-number experiences, such as how the multiplication of a number by a proper fraction can make a smaller product or dividing a number by a fraction can make a larger quotient. Furthermore, students must learn about fractions' symbolic structure and develop an understanding of how two numerical components can represent one magnitude, i.e., how the magnitude of a number can be understood as the size of one value relative to another.

Multiple studies have highlighted how learning about fractions is difficult, especially relative to whole number learning (Lortie-Forgues et al., 2015; Ni & Zhou, 2005), which has spurred multiple theories regarding how fraction knowledge develops and whether rational number concepts are compatible with human's numerical intuitions. Evolutionary theories regarding the core systems for numerical cognition suggest that fractions are difficult to learn because there are no pre-existing cognitive structures to support an intuitive sense of rational number meaning in the ways that there are systems to support whole number understanding (Feigenson et al., 2004). Other theories focus on how children's predispositions to think of numbers like whole numbers, as countable entities with unique successors, may hinder the acquisition of fractions knowledge (Gelman & Williams, 1998). From these perspectives fraction knowledge may be seen as necessarily an extension of symbolic whole number knowledge. This may explain why fraction instruction is often delayed until multiple years after whole number instruction. However, others have pointed out that delaying fractions instruction in the sequence of formal math learning may make the issue of introducing new rational number concepts and syntax more difficult, because students first establish whole number-based assumptions then rearrange them to build up new fractions knowledge (Gelman, 2015). Furthermore, some reject the assumption that fraction knowledge must necessarily be taught as an extension of whole number knowledge and suggest that perceptual abilities people have about nonsymbolic ratios and proportions could provide a more intuitive foundation for fractions knowledge if utilized effectively in early education (Lewis et al., 2015).

Previous studies examining how individuals understand the magnitudes of symbolic fractions have focused on whether people are biased towards *componential* processing of a fraction's individual whole number parts or whether people can integrate the relationship between the components to access a fraction's *holistic* magnitude (Meert et al., 2009; Schneider & Siegler, 2010; Zhang et al., 2014). Mixed evidence for componential and holistic processing across studies indicates that adults tend to process fractions differently given the task demands (Fazio et al., 2016; Toomarian & Hubbard, 2018). For instance evidence for componential processing in magnitude comparison tasks emerge when numerators or denominators carry sufficient information for success on a task (e.g. judging 1/7 is larger than 1/9 because numerators are constant and 7 is smaller than 9) (Bonato et al., 2007; Toomarian & Hubbard, 2018). However, evidence that people can access a holistic sense of a fractions magnitude emerges in tasks where numerators and denominators are unique in the fractions being compared (Meert et al., 2010).

Evidence for holistic magnitude processing is often inferred from the presence of significant *numerical distance effects* (NDE) in response times and accuracy. Specifically, numerous studies using symbolic fraction magnitude comparison tasks have observed NDEs with response times and error rates decreasing as the differences between magnitudes comparison pairs increase (Binzak & Hubbard, 2020; DeWolf et al., 2014; Schneider & Siegler, 2010). Crucially, these distance effects were significantly related to holistic fraction/ratio magnitudes above and beyond the influence of the fraction's specific components (Ischebeck et al., 2009). NDEs observed with symbolic numbers have long been interpreted as a response function resembling how individuals would discriminate physical stimuli, such as the length of lines (Moyer & Landauer, 1967), yet direct and careful comparison between symbolic and nonsymbolic stimuli is necessary to test this interpretation.

Nonsymbolic Instantiations of Ratio Magnitudes

In addition to evaluating a magnitude or a quantity as *how many* and *how much* of an absolute value (e.g. items in a set, or height of figure), people can also evaluate a magnitude as how much *more* or *less* the magnitude is *relative* to a standard. These perceivable relationships between magnitudes, hereafter referred to as nonsymbolic ratios, can take multiple forms. For example, a tree half as tall as another or a sound half as loud as another are nonsymbolic ratios that people may quantify with the symbolic label 1/2, but also may perceive without any symbolic associations. Nonsymbolic ratios may also be human-made iconic representations of rational number concepts, such as visualizations in education and data visualization (e.g. pie charts) which strip away detailed surface features to help individuals *see* the rational number concepts and magnitudes.

Researchers in the past several years have proposed that a fundamental aspect of human numerical cognition is the ability to automatically perceive these nonsymbolic ratios in the world and integrate ratio magnitude information into decision making (Jacob et al., 2012). Lewis, Matthews, & Hubbard (2015) proposed that ratio processing may be supported by a set of neurocognitive architectures referred to as the *Ratio Processing System* (Lewis et al., 2015). Similarly, others have proposed that ratio sensitivity across different modalities is evidence for a fundamental ratio code, and that such a code may support different domains of numerical cognition under a common generalized magnitude code (Bonn & Cantlon, 2017).

Outside of the math classroom there are multiple examples of ratio processing guiding human and nonhuman decision making, which suggest that these forms of nonsymbolic reasoning are evolutionarily adaptive traits (Jacob et al., 2012). For example, studies on human attractiveness indicate that preferences are influenced by the proportions of human features (e.g. waist-to-hip ratios in women or waist-chest ratios in men) more so than overall size or weight (Gangestad & Scheyd, 2005; Singh, 1993). Studies of lion and chimpanzee behavior show that decisions about whether to attack or retreat from a rival group of animals are based on the proportional size of their group relative to the other(McComb et al., 1994; M. L. Wilson et al., 2002). These examples describe how nonsymbolic ratio processing may occur via an implicit integration of perceptual cues in the environment to guide actions without explicit enumeration or formal instruction.

Formal definitions of rational number concepts, such as ratio and proportion can be used to describe ratio processing, but the ability to perceive these numeric properties in the world appears to emerge independent of formal mathematical knowledge. Multiple studies have presented evidence that young children, including infants, have abilities to perceive proportionally defined magnitudes far before any formal math instruction, let alone instruction with rational numbers. Using a habituation paradigm measuring looking preference, 5- to 7-month-old infants were able to detect differences in visually presented ratios when the magnitudes of the ratios deviated by a factor of 2 (McCrink & Wynn, 2007). When asked to match lengths of a stimulus across contexts, 4- year-old's ability to encode the extent of the stimulus was reliant on the proportional relationship of the length and a given standard (Duffy et al., 2005).

Studies of proportional reasoning with children also indicate that nonsymbolic processing abilities are influenced by the form in which these proportions are presented in. In a study with 6-, 8- and 10- year-olds, children at all ages were able to successfully perform a proportional reasoning game when the proportions were presented as two continuous entities but failed when the same proportions were presented as two entities with discrete equally-sized parts (Jeong et al., 2007). Although visual ratios with discrete parts may allow observers to more precisely enumerate the values presented in each entity, the authors conclude that visualizations with discrete parts may prompts erroneous counting strategies and a fixation on the whole number parts. Ratios composed of continuous entities, on the other hand, are not countable and thus may prompt observers to focus their attention to the proportional relationship. These examples of ratio processing among young children illustrate both the capacities humans have to perceive ratio relationships without formal instruction and the ways that these abilities may be influenced by surface features of the ratios, which may support or distract from accurate performance. Furthermore, these observations open new and interesting questions regarding the similarities and differences between sensory-based ratio processing and formal ratio and fraction knowledge, as well as the educational potential to use nonsymbolic ratios during the development of fraction knowledge.

Associations between Symbolic and Nonsymbolic Fraction Knowledge

Evidence for the commonalities in symbolic fraction and nonsymbolic ratio processing have largely come from studies examining how these abilities covary in adult and child populations. For example, performance on perceptually-based ratio tasks has been shown to be a unique predictor of fraction knowledge in adults, even when controlling for multiple cognitive processes that may support fraction knowledge, such cognitive control and whole number processing (ANS acuity) (Matthews et al., 2016). Likewise, individual differences in nonsymbolic ratio magnitude processing also predict variation in fractions knowledge among elementary school aged children (Möhring et al., 2016). Furthermore, studies with children have examined the relationship between fractions knowledge and precision with number line estimation tasks (NLE), which may rely on proportional reasoning to mapping number symbols onto continuous spatial referents (Newcombe et al., 2015). These studies reveal that NLE performance measured in 3rd and 5th grade, is a significant unique predictor of fractions knowledge measured one year later (Hansen et al., 2015; Jordan et al., 2013). These studies suggest that differences in nonsymbolic magnitude acuity across individuals may relate to current and future understanding of symbolic fractions, however the precise degree and nature of these links remain largely unknown.

Few studies have directly explored relationship between symbolic and nonsymbolic fraction magnitude processing. Some previous studies have observed ways that manipulating the nonsymbolic font size of numerical components in a symbolic fraction can directly influence the precision with which individuals can accurately compare symbolic fractions' holistic magnitude value (Matthews & Lewis, 2017; c.f. Kallai & Tzelgov, 2009). These ratio congruity effects illustrate the automaticity of nonsymbolic ratio processing and how nonsymbolic ratio processing may compete with symbolic fraction processing for shared cognitive. In a series of studies, Matthews and Chesney (2015), examined magnitude comparison performance across symbolic fractions and nonsymbolic ratios, instantiated in the form of uncountable dot ratios and circle area ratios. The authors also observed that adults can make quick and accurate cross-format judgements and that these judgements produce numerical distance effects, suggesting that adults can access a sense of rational number magnitudes in ways that are not dependent on enumeration or estimation of natural number values. However, in this study the authors also observed that adults exhibit a consistent bias in how they judged the point of subjective equality between nonsymbolic ratios and symbolic fractions. Specifically, adults consistently evaluated nonsymbolic ratios as larger than their true value, relative to symbolic fractions. These studies present evidence that magnitude processing with symbolic fractions and nonsymbolic ratios can converge on compatible representations of rational number magnitude (if not the same), but the calibration of accessing specific number magnitudes from visually defined nonsymbolic ratios and symbolic fractions can be imprecise. Further experimentation is necessary to more fully understand how people access a common sense of magnitude between separate symbolic or nonsymbolic formats, and how this process differs from comparing magnitudes within format.

Current Study

In this study, we conducted two experiments to explore how adults access the magnitudes of symbolic and nonsymbolic fractions within and across formats. In both experiments participants completed the same magnitude comparison task. We asked participants to choose the larger of two fractions presented simultaneously in the form of symbolic fractions, nonsymbolic ratios, or mixed symbolic-nonsymbolic pairs. In Experiment 1, symbolic fraction pairs were composed of two single-digit irreducible fractions (e.g. 2/7), nonsymbolic ratios were presented as part-to-part ratios of two line lengths, and mixed pairs included one symbolic fraction and one nonsymbolic line ratio. In

Experiment 2, we expanded the list of possible comparisons. In addition to the three comparison conditions in Experiment 1, symbolic fraction pairs were presented with double-digit components, nonsymbolic ratios were presented as the area of two circles, and mixed pairs included one symbolic fraction (single digit) and one nonsymbolic circle ratio.

We chose to use to use part-to-part ratios of line lengths and circle areas in the current study to specifically observe ratio processing as accessing a sense of rational number magnitude by evaluating the magnitude of one entity relative to another. Specifically, we chose to present ratios as two continuously defined entities without discrete parts. As shown in studies with children, ratios that are presented as entities with discrete parts may prompt additional strategies, such as counting the whole number values of the entities, and thus distract participants from focusing on the relative magnitude of one entity compared to the other (Jeong et al., 2007). By comparing magnitude processing with symbolic fractions to this part-to-part integration of nonsymbolic ratios within the same task and using the same magnitudes, we aimed to observe the similarities and differences in this integration process across formats.

Our study of magnitude processing within and across symbolic and nonsymbolic formats was guided by the following research questions. First, do people show evidence of holistic magnitude processing, in the form of significant NDEs, during comparisons of symbolic fractions, nonsymbolic ratios and mixed pairs? If so, how do these NDEs differ when comparisons are made with symbolic fractions relative nonsymbolic ratios? Second, is the process of comparing magnitudes across formats more difficult than comparing fractions or ratios within the same format? Third, do people show biases in the point of subjective equality when making cross-format comparisons, as has been observed in previous studies (Matthews & Chesney, 2015)?

Experiment 1

Introduction

In Experiment 1, we examined magnitude processing of symbolic fractions and nonsymbolic ratios using a speeded magnitude comparison paradigm. Specifically, participants were asked to indicate the larger of two fractions in three different conditions: paired symbolic fractions (FF), paired nonsymbolic line ratios (LL) and mixed (MX) symbolic/nonsymbolic cross-format pairs (Figure 2.1). The nonsymbolic ratios used in this experiment, were composed of two line lengths. Thus, we specifically compared magnitude processing between symbolic fractions and nonsymbolic ratios, where elements of those ratios were uncountable and continuous line lengths. In our analyses of magnitude processing in these three conditions we tested four hypotheses.

- Hypothesis 1: If adults accessing the holistic magnitude of each symbolic fraction or nonsymbolic ratio, then numerical distance effects should be significant in all conditions.
- Hypothesis 2: If magnitude processing with nonsymbolic ratios occurs via sensory-based processes without explicit enumeration then line ratio performance should be faster than symbolic fraction processing.
- Hypothesis 3: If symbolic and nonsymbolic processing rely on separate magnitude codes, then cross-format comparisons should require more processing time than within=format judgements translate meaning across format-specific magnitude codes.
- Hypothesis 4: If subjective magnitudes accessed from symbolic or nonsymbolic stimuli lead individuals to the true magnitudes represent, then the point of subjective equality during cross-format judgements should equal the true point of equality

First, we tested whether adults show evidence of *holistic magnitude processing* in all three conditions. If people make magnitude decisions by accessing the holistic magnitude of each symbolic fraction or nonsymbolic ratio, then NDEs should be significant in all conditions (H1). Furthermore, effects based on holistic magnitude distances should be significant even when controlling for componential characteristics of the fraction pairs, which may facilitate magnitude comparison strategies in parallel with or in place of holistic magnitude processing (e.g. comparing only numerator values).

Second, we tested whether magnitude processing is *more efficient* with nonsymbolic ratios than symbolic fraction representations. If magnitude processing with nonsymbolic can occur via sensory-based processes, then this processing may occur without explicit enumeration or translation to a symbolic fraction form. In contrast, the symbolic orthography of fractions offers no direct perceptual cues to the magnitudes they represent. Therefore, accessing a representation of rational number meaning with fractions may require additional processing steps to encode the fractions symbolic form. In the current experiment, evidence for more efficient nonsymbolic ratio processing should be seen in lower error rates and faster RTs during within-format comparisons of nonsymbolic ratios than comparisons of symbolic fractions (H2).

Third, we tested whether additional processing stages, or *translation costs*, are necessary to compare the magnitudes of across distinct external representations. Specifically, we examined whether cross-format comparisons are more difficult than comparisons between fractions with the same format. If understanding symbolic and nonsymbolic fractions depends on accessing separate magnitude codes, then comparisons between a nonsymbolic ratio and a symbolic fraction should require additional processing stages to access unique magnitude codes for each format and then translate meaning across these magnitude codes (H3). In this study, evidence for additional translation processing in magnitude comparisons with mixed fractions should be seen in higher response times and error rates relative to within format comparisons. Alternatively, if an understanding of both formats can be supported by a common magnitude code, then comparing a symbolic fraction and a nonsymbolic ratio should require accessing this code via similar processes and without cross-format translation steps. Thus, performance (RT and ER) with mixed pairs resembling a mixture of symbolic and nonsymbolic processing, which is no worse or no better than performance within either format, would be evidence in support of the hypothesis that symbolic and nonsymbolic processing can be supported by a common magnitude code or highly compatible magnitude codes.

Fourth, we tested for the presence of *cross-format biases* in the point of subjective equality within the magnitude comparisons, whereby the magnitude of nonsymbolic ratios appear to be perceived as slightly larger relative to the magnitude of symbolic fractions (H4). Previous findings using dot ratios and circle ratios (Matthews & Chesney, 2015), have shown a bias whereby the point at which adults judge a fraction to be of equal magnitude to a nonsymbolic ratio occurs when the symbolic fraction is actually slightly larger (or vice versa, when the nonsymbolic ratio is slightly smaller). Here we tested whether the mixed comparison format bias replicate when adults made mixed comparisons with line ratios.

Methods

Participants

24 undergraduate students (20 females, M_{age} = 20.1 years, range = 18-28) participated for course credit. All participants were right-handed native English speakers.

All participants' accuracies in each format were above our inclusion cutoff of 70%. We chose a sample size of 24 assuming effects of similar size to previous research (Binzak & Hubbard, 2020), and to fully counterbalance experimental blocks.

Procedure and design

Participants completed six blocks of 36 trials. Each block contained an equal number of trials from the three format conditions (12 trials per format), and the order of trials was randomized for each participant. We manipulated comparison difficulty via *numerical distance*, operationalized as the absolute value of distance between magnitudes in a given pair (i.e., $|Fraction_1 - Fraction_2|$). We presented 36 unique pairs within each block with numerical distances ranging from near (min = 0.048) to far (max = 0.75). Block order was counterbalanced across participants, and through all six blocks participants compared all 36 unique pairs in each format twice (n = 216 trials). We instructed that participants make their as quickly and accurately as possible.

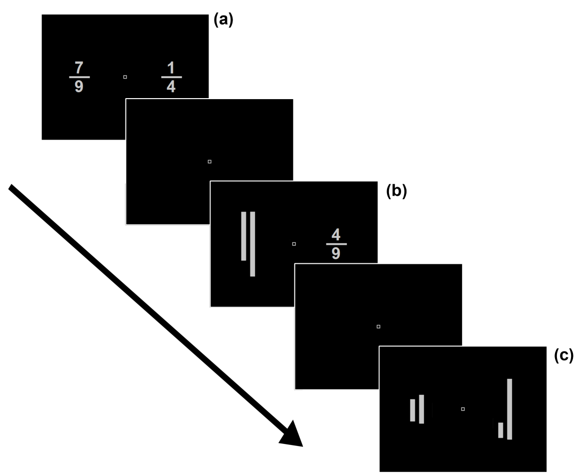

Figure 2.1:The cross-format fraction comparison paradigm for Experiment 1 presented three format conditions randomly ordered within blocks. The three conditions included (a) symbolic fractions (FF), (b) mixed comparisons (MX), and (c) nonsymbolic line ratio comparisons (LL).

Symbolic Fraction Stimuli

The symbolic fractions used in the FF and MX conditions were made up of the 27 single-digit irreducible proper fractions. To minimize participants' reliance on strategies based on symbolic fractions' component parts during FF comparisons, we selected 36 fraction pairs that contained a mixture of relationships between the fractions' holistic distances and component distances (Fazio et al., 2016; Meert et al., 2010). We included four categories of fraction pairs defined by how the numerator or denominator component values may be seen as congruent or incongruent with holistic magnitude judgements (Table 2.1), to investigate the influence component congruencies. See Appendix A for a full list of symbolic fraction pairs within each component congruence category.

Table 2.1

Categories of component congruency

Component Congruency	Description	Symbolic Example	Nonsymbolic Example (pixel length)
Same* Denominator (+ N / =D)	Pairs have the same denominator.	7/9 – 8/9	85:154 – 121:162+
Congruent Numerator & Denominator (+N/+D)	The larger fraction has a larger numerator value and a larger denominator value	1/5 – 3/4+	34:302 – 114:183
Incongruent Denominator (+N / -D)	The larger fraction has a larger numerator but a smaller denominator.	1/6 – 6/7	16:144 – 85:154
Incongruent Numerator (-N/ +D)	The larger fraction has a smaller numerator value	2/7 – 1/3	36:40 – 60:241

Note *There were no nonsymbolic ratio pairs with the exact same denominator line length, therefore coded line ratio comparisons as having similar denominator lengths if the differences between denominator lines was less than 10 pixels. + Since the lengths of line ratios were randomly generated, the same pairs when presented as symbolic fractions and nonsymbolic line ratios were not necessarily assigned to the same component congruency category within each format.

Nonsymbolic line ratio stimuli

Using the 27 single-digit irreducible fractions used to create the symbolic stimuli, line ratio images were created to instantiate the exact same rational number magnitudes. Line ratios were composed of two vertical lines side by side, with the line length on the left representing the same value as a symbolic fraction's numerator and the line on the right representing the same value as a symbolic fraction's denominator. The vertical position of the smaller (numerator) line on the left was randomized across each nonsymbolic ratio image so that it was presented within the bounds of the larger (denominator line) on the right and without aligning to either edge of the line on the right. Two sets of line ratio images were created for this experiment to account for correlations that can emerge between a line ratio's numerator or denominator line length and the ratio's holistic magnitude. For three of the six experimental blocks, one set of line ratio images was constructed to minimize the numerator-to-holistic magnitude correlation, and another set minimizing the denominator-to-holistic magnitude correlation was constructed for the other three blocks.

The numerator-controlled set of 27 line ratio images (one for each of the 27 single-digit irreducible fractions) was created by randomly generating a numerator length between 30-200 pixels for each image and then constructing the proportional denominator length to create the proper line ratio. In instances where the corresponding denominator value was larger than 350 pixels, the image was discarded, and the process was repeated with a new randomly generated numerator value. This procedure was conducted three times, creating 3 sets of 27 images, and the set of line ratios with the lowest correlation between numerator line lengths and ratio magnitude within the set was used in the experiment. The final set of numerator-controlled line ratios had a numerator length to holistic magnitude correlation of 0.340, a denominator length to holistic magnitude correlation of -0.682, and summed numerator-denominator length to holistic magnitude correlation of -0.416.

The denominator-controlled set of 27 line ratio images (one for each of the 27 single-digit irreducible fractions) was created by generating a denominator length between 130-300 pixels for each image, and then constructing proportional the numerator length to create the proper line ratio. After three iterations this process, the set of line ratios with the lowest within-set correlation between denominator line lengths and ratio magnitudes was used in the experiment. The final set of denominator-controlled line ratios had a numerator length to holistic magnitude correlation of 0.834, a denominator length to holistic magnitude correlation of -0.220, and summed numerator-denominator length to holistic magnitude correlation of 0.380.

Analysis

Effects of format and numerical distance

To evaluate if adults show evidence of holistic magnitude processing and to compare efficiency of processing across formats we modeled response times and error rates with linear mixed models using the lme4 (Bates et al., 2015) and afex (Singmann et al., 2015) packages in R (R Development Core Team, 2016). First, we tested for the presence of numeric distance effect slopes in the response time (RT) and error rate (ER) data. Specifically, we tested whether the distance between the pairs holistic magnitudes was a significant negative predictor of RT and ER in each of the three formats (H1). In our models, we began with the assumption that the magnitude of distance effect slopes may differ across formats and tested whether models accounting for interactions between distance and format accounted for significantly more variance than models assuming no differences.

Next, we tested for format effects on performance, to compare the efficiency of magnitude processing with each format (H2, H3). We chose to use the absolute value of the pairs' holistic numeric distances (hear after referred to as simply *distance*) as our continuous predictor in the model. Using the emmeans package (Lenth, 2020) in R, we estimated the marginal mean RT and ER at specific numeric distances and conducted pairwise comparisons between formats (H2). For, most analyses we tested for differences in mean RT and ER assuming the distance between the pairs was equal to 0.366 (the mean of all distances presented in the stimuli).

In our mixed models we tested for fixed effects of varying the *format* (categorical predictor: symbolic, nonsymbolic, & mixed), *numerical distance* (as a continuous predictor), and interactions between these predictors, while accounting for the within-subjects structure of our data by estimating the crossed random effects for each participant. This constituted the maximal model justified by our experimental design (Barr et al., 2013), and allowed us to properly estimate the standard errors of coefficients based on the number of participants in the sample and not on every single observation. When fitting the maximal model resulted in convergence errors or singular fits, we verified parameter estimates and the significance of effects by rerunning the models with a simplified random effects structure. These follow up results are presented only in instances where the simplified model did not support the maximal model.

Within Format Analyses

Additional follow up analyses were conducted within each format condition to check whether format specific features of symbolic fractions and nonsymbolic ratios may have impacted holistic magnitude processing. First, we tested for the presence of *component congruency* effects whereby comparisons of holistic magnitude may have been influenced by the magnitudes of numerator and denominator components. Second, we tested for evidence that participants may have relied on a strategy of judging magnitude based on the *gaps* between numerators and denominators*.* Specifically, within symbolic fractions and nonsymbolic ratio comparison participants may in some part base judgements of which stimulus is larger, on an evaluation of which stimuli has the smaller gap between numerator and denominator components, regardless of how large those components are. To test for these effects, we added predictors of component congruency and numerator-denominator gaps (separately) separately to our mixed models of absolute distance predicting RT and tested whether these features predict differences in RT and whether NDEs based on holistic magnitude distances remain significant when controlling for these features.

To test for evidence of cross-format biases in the point of subjective equality during symbolic-nonsymbolic mixed comparisons (H4), we fit a logistic model to estimate probability of adults selecting symbolic fractions as larger across the range of mixed format distances (symbolic magnitude – nonsymbolic magnitude). Using the resulting logistic function, we were able to calculate the point (the differences in magnitudes across formats) at which adults would be equally likely chose the symbolic fraction or nonsymbolic ratio as larger. Finally, using the random effects parameters specific to each participant, we determined if the group mean point of subjective equality was significantly shifted from zero.

Drift diffusion model analysis of comparisons across formats

We conducted follow-up analyses using a drift diffusion model (DDM) approach to further evaluate how format effects manifest in multiple stages of the magnitude comparison process. The DDM approach incorporates RTs and ERs into a single analysis that estimates three key decision-making parameters: the rate participants accumulate evidence (*drift rate*), the carefulness with which participants make judgments (*decision boundaries*), and the time to encode a stimulus and time to make a physical response (*non-decision time*) (Ratcliff & McKoon, 2008). We estimated these three parameters using the *Fast-DM* program (Voss & Voss, 2007).

Results

Missing Data Points and Erroneous Responses

Prior to data analysis, erroneous anticipation responses (less than 250ms) and missed trials (no response entered) were cleaned from the data. Our anticipation cutoff of 250ms was based on previous research using diffusion models to examine performance on numerical tasks (Ratcliff et al., 2015). No participants missed more than two responses within any of the format condition blocks. Across all participants and format conditions, cleaning missed responses removed 0.3% of the total data. No additional anticipation trials needed to be removed.

Response Times Across Formats

On average, participants made magnitude comparisons with all conditions in less than 1300ms. Results of the linear mixed model analysis are presented in Table 2.2.

Consistent with Hypothesis 1, responses in all formats demonstrated significant numeric distance effects (NDEs), with RTs decreasing as the distance between pairs increased Figure 2.2. NDE Slopes for FF and MX comparisons did not differ but NDE slope for nonsymbolic (LL) comparisons was slightly flatter (less negative) than symbolic fraction (FF) and mixed pair comparison slopes (MX).

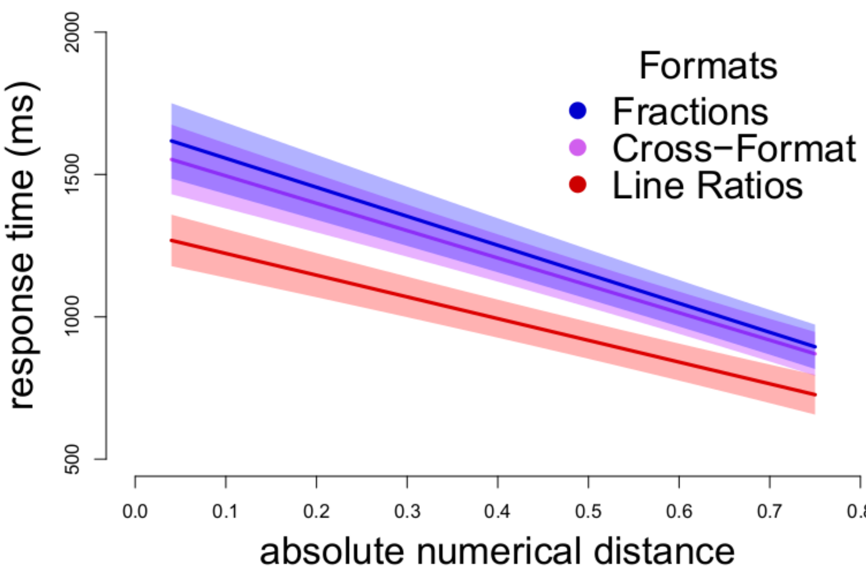

Figure 2.2 Linear predictions of conditional mean response times show that line ratio comparisons (LL in red) are fastest across all numerical distances relative to fraction-fraction (FF in blue) and mixed comparisons (MX in purple) across the . Shaded bands depict 95% confidence intervals of the linear predicted mean response time.

Consistent with Hypothesis 2, the linear mixed model analysis revealed evidence for more efficient magnitude processing with nonsymbolic line ratios than symbolic fractions and mixed pairs. LL response times (RT) were 266ms faster than FF, and 220ms faster than MX comparisons (Table 2.2). Follow up analyses necessary to address the significant interaction between format and NDE slopes confirmed that LL comparisons were associated with significantly faster responses than FF or MX comparisons at both near (dist = 0.1; LL - FF: $\beta = -334.8$, $t = -8.62$, $p < 0.001$; LL – LF: $\beta = -273.8$, $t = -7.050$, $p < 0.001$) and far distances, (dist = 0.7; LL - FF: β = -180.4, t =-5.66, p < 0.001; LL – LF: β = -153.3, t = -5.221, p < 0.001).

Table 2.2

Estimated marginal mean response times and distance effect slopes

Fixed Effect Estimates					Pairwise Comparisons	
Format	EMM RT	SE	df	95% CI	LL	MX
LL	1019	35.7	23	[945, 1093]		
MX	1240	44.9	23	[1147, 1332]	t=10.6, p<.001, d_r=.522	
FF	1286	51.0	23	[1180, 1391]	t=9.83, p<.001, d_r=.632	t=1.75, p=.208, d_r=.110
Format × Distance	NDE Slope	SE	df	95% CI	LL	MX
LL	-762	66.4	23	[-899, -624]		
MX	-963	89.3	23	[-1148, -778]	t=2.09, p=.114, d_r=.477	
FF	-1020	77.4	23	[-1180, -860]	t=3.13, p=.013, d_r=.612	t=0.63, p=.806, d_r=.135

Note: Estimated marginal means (EMM) indicate the predicted mean response time in milliseconds within each format where absolute distance = 0.3. Numerical distance effect slope estimates (NDE) were significant and negative in all instances, as the range of the 95% confidence interval (CI) does not include zero. Degrees of freedom were estimated using the Kenward-Rodger approximation

We did not observe any strong evidence to support Hypothesis 3, and the prediction that cross-format comparison would require additional processing to translate between symbolic and nonsymbolic representations of magnitude. Specifically, average RTs were no slower for MX responses than for FF responses. In fact, the mean RT difference trended in the opposite direction: FF judgments required slightly more processing time than MX judgments in this sample. The nonsignificant difference between FF and MX RTs suggests a few interesting possible interpretations. First, magnitude processing with mixed pairs may not be a straightforward combination of fast line ratio processing and slow symbolic fraction processing. Mean MX RTs were 86.9ms greater than the average of FF and LL RTs (1152.65ms), which may correspond to either a rapid cross-format translation process or an alternative processing step all together. For instance, if processing the magnitude of a line ratio within format is a rapid visual processes but processing line ratio magnitude in the mixed format involves mapping the visual stimuli onto a magnitude code shared with symbolic fraction stimuli, then some additional processing may occur in the mixed format that is not translating nonsymbolic representations to symbolic representations or vice versa.

Error rates across formats

Participants made few errors, with a total group mean ER of 5.9% (SD = 2.8%). Consistent with results in RTs, ERs showed distance effects in all three conditions (Figure 2.3), with error rates dropping off rapidly as distances increased from 0 to 0.1 and approached errorless performance once distances reached 0.6 (H1). The full logistic model indicated no significant interactions between numerical distance effects and format. We therefore dropped this interaction when evaluating format effects.

As seen in Table 2.3 analyses of error rates across formats revealed no evidence of more efficient line ratio processing (H2) or cross-format translation costs (H3). First, average error rates were not significantly lower for LL comparisons relative to MX (BF = 0.100) or FF (BF = 0.053). Second, error rates among MX comparisons were not higher than FF comparisons (BF = 0.018). Although these results do not add additional support Hypothesis 2 or 3, the lack of format effects on ER does suggests that format effects on RT were not compromised by speed-accuracy tradeoffs.

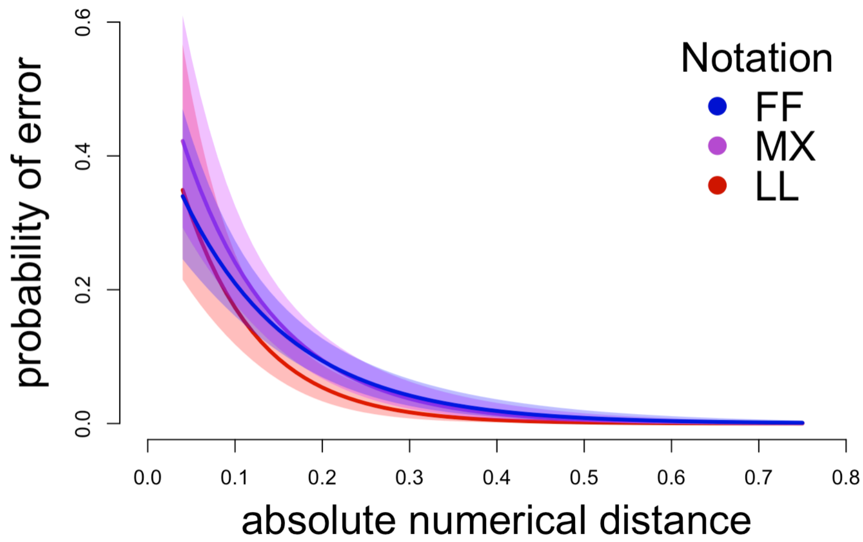

Figure 2.3: Logistic predictions of conditional error rate show that errors increased as numerical distances approach zero. Shaded bands depicting 95% confidence intervals overlap between fraction-fraction (FF), cross-format (MX), and line-line (LL) conditions illustrating that the distance x format interaction and main effects among formats were not significant.

Table 2.3

Logistic Mixed Model Regression Results for Error Rate

	Fixed Effect Estimates				Pairwise Comparisons	
	Estimate log odds	SE	95% CI (log odds)	Odds Ratio	LL	MX
Format						
LL	-5.06	0.360	[-5.49, -4.08]	0.006		
MX	-4.34	0.334	[-4.98, -3.67]	0.013	z=1.94, p=.129, OR=1.58	
FF	-4.08	0.312	[-5.02, -3.80]	0.017	z=1.52, p=.293, OR=1.45	z=0.43, p=.903, OR= 0.917
Distance	-10.7	1.05	[-12.8, -8.63]	2.3E-4		

Note: Higher likelihood of errors correspond to greater estimated marginal means of format on the log odds scale. Odds ratio transformation of these estimates indicates the probability of making an error relative to the probability of making an accurate judgement. Estimated means and pairwise comparisons are evaluated where absolute distance = 0.366, which is the mean of distances presented in the stimuli. Pairwise contrasts are performed on the log odds ratio scale, and corresponding odd ratios (OR) indicate the probability of an error in the condition along the row relative to condition in the column header.

Cross-format point of subjective equality bias

Previous studies have observed that when people compare magnitudes across mixed symbolic-nonsymbolic pairs, patterns of RT and ERs indicate that people overestimate the size of nonsymbolic fractions relative to symbolic fractions(Matthews & Chesney, 2015). In this experiment, we aimed to determine if this subjective equality bias was also present in line-ratio to single digit fraction comparisons and to assess how big the bias is. To determine the magnitude of the bias, we fit a logistic function on the data predicting the probability of picking a symbolic fraction as a function of the distance between the symbolic and nonsymbolic fraction. Unlike our analysis of error rates that examined performance across the absolute value of pair distances, cross-format distance was entered into the model as the symbolic fraction magnitude minus the nonsymbolic fraction magnitude (range = -.75 - .75). We then fit a logistic mixed model, estimating the likelihood of individuals judging the symbolic fraction as larger across this range of cross-format distances, while accounting for random variation across participants. We used this model to estimate the point of subjective equality (PSE), as the numeric distance between symbolic and nonsymbolic stimuli at which the probability of picking a symbolic fraction was .5. Finally, we tested whether group mean PSE was significantly different from zero (no PSE bias), using a one-sample t-tests on PSEs calculated individually for each participant.

Consistent with previous analysis of cross-format comparisons with fractions (Matthews & Chesney, 2015), the pattern of responses to mixed pairs indicates that the participants' point of subjective equality (PSE) was biased towards seeing line ratios as a larger magnitude relative to equivalent symbolic fractions. Model results, shown in Figure 2.4, indicated that the point at which adults are equally as likely to judge a symbolic fraction as larger or a nonsymbolic fraction as larger is when the magnitude of a symbolic fractions is 0.043 larger than a nonsymbolic fraction, and mean PSE across all subjects was greater than zero, $t(23) = 3.04$, $p = .006$.

Similar to the approach used by (Matthews & Chesney, 2015), we evaluated whether this PSE bias should be used to correct our identification of correct and incorrect responses. We identified that adjusting the distance between symbolic and nonsymbolic pairs based on the PSE bias would not change our coding of response accuracy. This is because the trial with the closest numerical distance (0.047), 1/3 vs 2/7, still has a larger distance than the PSE.

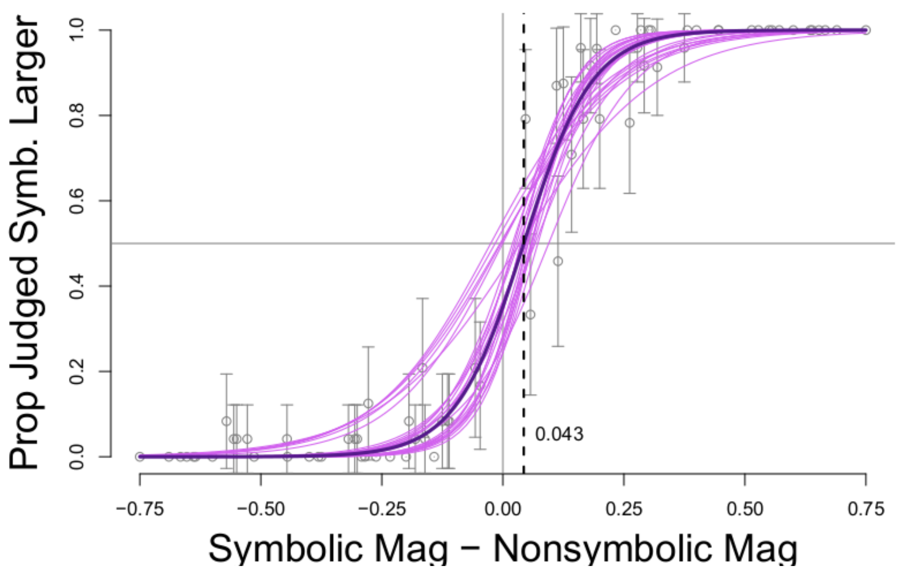

Figure 2.4. Logistic mixed model estimates of ER probability in cross-notation pairs. Light purple logistic curves depict random effect estimates of individual participants, and dark purple logistic curves depict the group level fixed effects model. Points where these lines cross the .5 probability line (grey) indicate points of subjective equality (PSE), or the distance between symbolic fractions and nonsymbolic ratios where individuals would judge the stimuli as equal. The group mean point of subjective equality is rendered in green.

Drift diffusion model parameters

Effects of Format on Drift Rates. Average drift rates, representing the rate of evidence accumulation, were significantly different across the three format conditions, F(2,46)=14.1, p < .005 (see Figure 2.5.). Specifically, adults made magnitude comparisons more efficiently in the line-line condition (M = 2.94, SD = 1.41) than the line-fraction (M = 2.10, SD = 1.04) and fraction-fraction conditions (M = 1.95, SD = 0.67). Mean differences between the line-fraction and fraction-fraction conditions were not significant, and thus no evidence was found to suggest that the rate of evidence accumulation for mixed line-fraction comparisons was any less efficient than fraction-fraction comparisons.

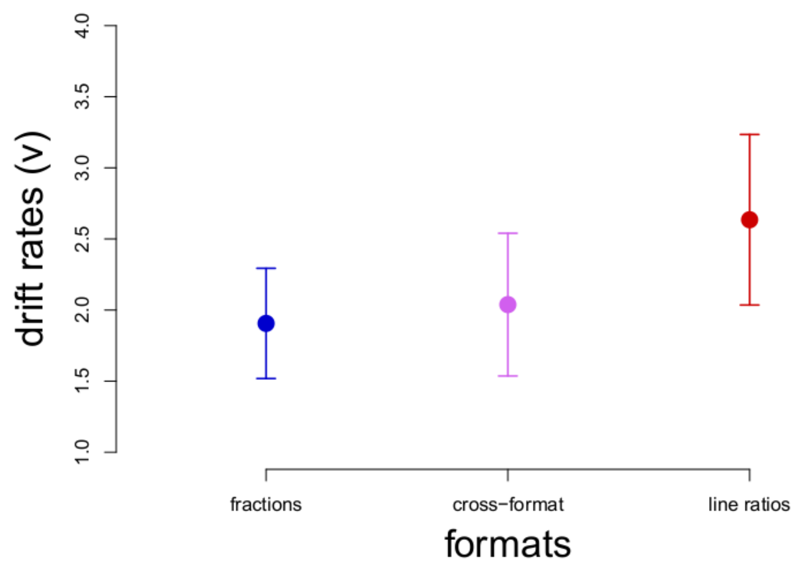

Figure 2.5. Group mean drift rates across formats. Error bars depict 95% confidence intervals around the mean. Higher values of drift rate correspond to more efficient evidence accumulation.

Effects of Format on Decision Boundaries. As seen in Figure 2.6 below, we observed no statistical differences between decision boundaries estimated for each format condition, $F(2,46) = 1.21$, $p = 0.25$. Therefore, diffusion model estimates do not indicate that differences in mean response times observed in this study were due to speed accuracy tradeoffs or approaching trials with fractions any more carefully than trials without.

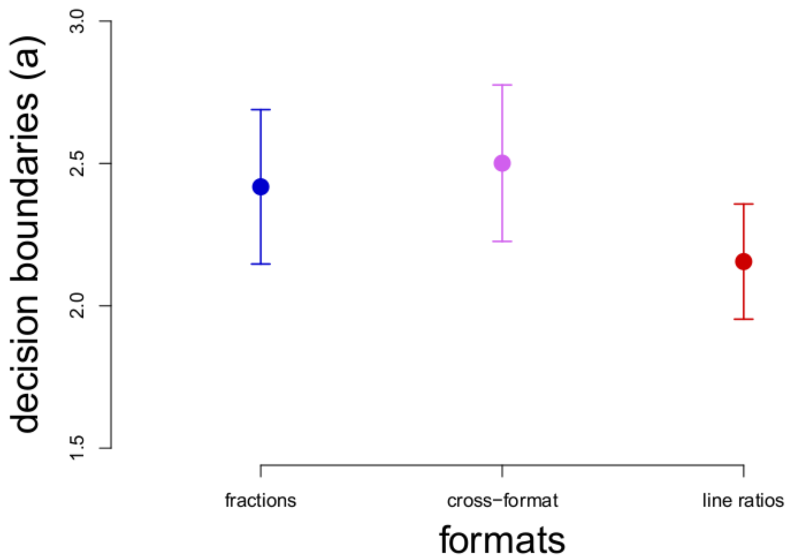

Figure 2.6.Group mean decision boundaries across formats. Error bars depict 95% confidence intervals around the mean. Higher decision boundary values correspond to more careful performance.

Effects of format on non-decision time. As seen in Figure 2.7, within-subjects comparisons of non-decision time (sum of stimulus encoding and the motor response), show significant differences between formats, $F(2,46) = 9.34$, . Specifically, longer times were estimated for fraction-fraction comparisons ($M = 0.62$, $SD = 0.13$) than line-line ($M = 0.54$, $SD = 0.10$) and mixed line-fraction comparisons ($M = 0.57$, $SD = 0.09$), however differences were not observed between line-fraction and line-line conditions.

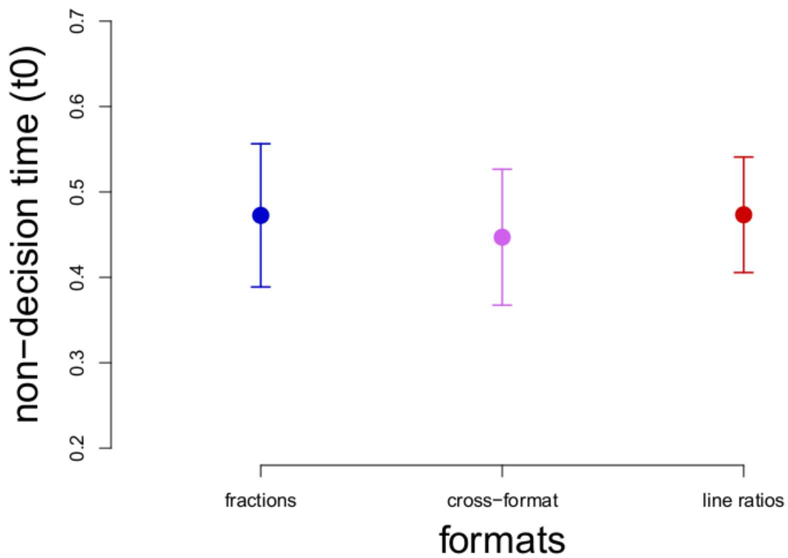

Figure 2.7. Group mean non-decision times for each format condition. Error bars depict 95% confidence intervals around the mean. Higher values correspond to more encoding processing and time to generate a physical response time.

Within-notation follow-up analyses

In the analyses of RT presented above we observed a significant negative numerical distance effect slope for the Fraction-Fraction and Line-Line comparison conditions, indicating that the holistic magnitude of fractions and ratios, and specifically the numerical distance between these magnitudes, is a significant predictor of processing time. Nevertheless, there are component-based features of symbolic fractions which may influence task performance (Pearn & Stephens, 2004). Here we evaluated the possibilities that strategies focusing on *component congruency* and numerator-denominator *gaps* may have influenced holistic magnitude processing.

Symbolic Fraction Component Congruency Effects. Previous studies have indicated that features of numerator and denominator components can influences how people make holistic fraction comparisons. Specifically, when comparing two symbolic fractions, the relative magnitude of the two fraction’s numerators or denominators may bias the evaluation of each fraction's holistic magnitude. Here we refer to these relationships between components of fraction pairs and holistic distances of fraction pairs as *component congruency* relationships (Meert et al., 2009). We identified four possible component congruency relationships (Table 2.1). We coded the component congruency of our FF pairs based on whether a comparison of the components (numerator to numerator or denominator to denominator) would lead to a response congruent with the overall magnitude. Specifically, if the larger fraction in the pair had a larger numerator, then the pair would be coded as numerator congruent. Likewise, if the larger fraction in the pair had a larger denominator, the pair would be coded as denominator congruent. Previous studies examining holistic and componential processing of fractions denominator congruence in the opposite direction conventions to define denominator congruence in magnitude comparison tasks, (Ischebeck et al., 2009) based on the fact that when numerator value is held constant the size of a denominator is inversely related to Conversely, since denominator size and holistic magnitude are indirectly related (increasing denominators decreases fraction magnitude), denominators were congruent when the larger fraction had the smaller denominator.

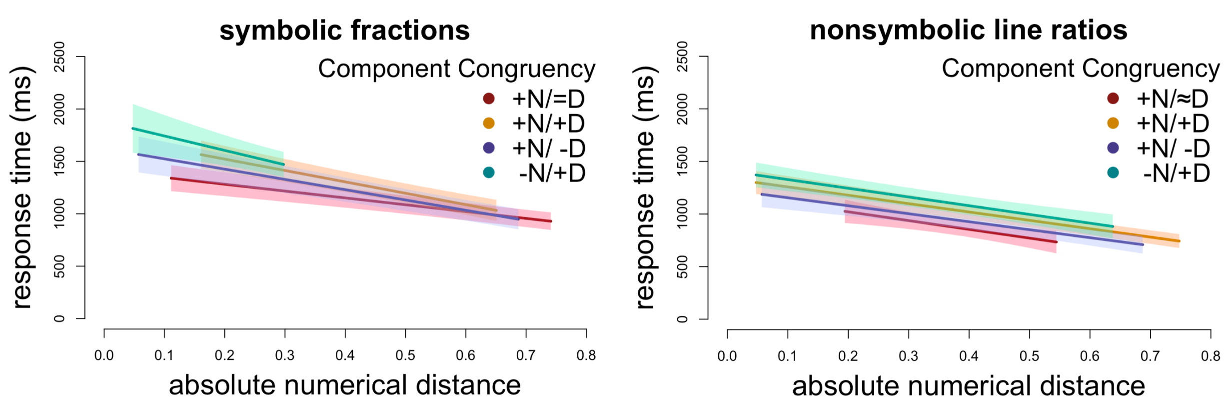

Figure 2.8. Linear predictions of distance effects on RT across FF and LL pairs with different component congruency relationships.

As seen in Table 2.4, results of the mixed effects model indicated that NDE slopes were significant and negative for FF pairs with each of the four component congruency relationships. These findings do not rule out the influence component congruency features may have on magnitude comparison performance, but they also show that our observations of holistic distance effects are not driven by these componential features. As seen in Figure 2.8, the range of numerical distances for each section of FF pairs with different component congruency relationships was not equivalent. Of note, pairs with incongruent numerators in this experiment have a maximum numerical distance of 0.31. Thus, we compared the estimated mean RTs between FF pairs with different component congruency relationships at a numeric distance of 0.3, where all types of pairs overlap.

Pairwise comparisons of mean RTs revealed that pairs with common denominators had the fastest RTs, which were 197ms faster than pairs with congruent numerators and denominators, 114ms faster than pairs with congruent numerators and incongruent denominators, and 252ms less than pairs with incongruent numerators. FF comparisons with incongruent numerators had the slowest mean RTs in the sample, but it was not significantly higher than pairs with congruent numerators and denominators, or pairs with congruent numerators and incongruent denominators. Among pairs with congruent numerators, the differences in RT between pairs with congruent denominators and pairs with incongruent denominators was not significant.

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Table 2.4

Mean response times and distance effect slopes in symbolic comparison pairs across component congruency

	Fixed Effect Estimates				Pairwise contrasts		
Component Congruency	EMM RT	SE	df	95% CI	+N /=D	+N /+D	+N / -D
+N /=D	1216	54.1	23.0	[1104, 1328]			
+N /+D	1330	67.2	23.0	[1191, 1469]	t=-2.93, p=.036, d_r=-.271		
+N / -D	1413	63.6	23.0	[1294, 1532]	t=-5.93, p<.001, d_r=-469	t=-2.11, p=.182, d_r=-.198	
- N /+D	1468	57.5	22.9	[1336, 1599]	t=-5.35, p<.001, d_r=-.599	t=-2.40, p=105, d_r=-.328	t=-1.20, p=.632, d_r=-.131
Component Congruency × Distance	NDE Slope	SE	df	95% CI	+N /=D	+N /+D	+N / -D
+N /=D	-650	100	23.0	[-858, -442]			
+N /+D	-987	115	22.5	[-1225, -748]	t=2.38, p=.109, d_r=0.80		
+N / -D	-1084	139	23.0	[-1370, -797]	t=2.80, p=.047, d_r=1.03	t=0.59, p=.933, d_r=0.230	
- N /+D	-1376	396	22.2	[-2197, -554]	t=1.86, p=.272, d_r=1.73	t=0.93, p=.787, d_r=0.926	t=0.74, p=.879, d_r=0.696

Note: Estimated marginal means (EMM) indicate the predicted mean response time in milliseconds within each category of component congruency where absolute distance = 0.3. Numerical distance effect slope estimates (NDE) were significant and negative in all instances, as the range of the 95% confidence interval (CI) does not include zero. Degrees of freedom were estimated using the Kenward-Rodger approximation

Table 2.5

Mean response times and distance effect slopes in nonsymbolic comparison pairs across component congruency

	Fixed Effect Estimates				Pairwise contrasts		
Component Congruency	EMM RT	SE	df	95% CI	+N /=D	+N /+D	+N / -D
+N /=D	937	44.7	23	[845, 1030]			
+N /+D	1004	43.0	23	[916, 1093]	t=-1.79, p=.304, d_r=.186		
+N / -D	1099	42.3	23	[1011, 1186]	t=-4.27, p=.002, d_r=.449	t=-2.39, p=.108, d_r=.262	
- N /+D	1161	47.7	23	[1062, 1260]	t= -5.12, p<.001, d_r=.623	t=-3.38, p=.013, d_r=.437	t=-2.21, p=1.52, d_r=.174
Component Congruency × Distance	NDE Slope	SE	df	95% CI	+N /=D	+N /+D	+N / -D
+N /=D	-833	257.5	23	[-1366, -300]			
+N /+D	-795	82.9	23	[-967, -624]	t=0.14, p=.999, d_r=0.105		
+N / -D	-769	133.3	23	[-1044, -493]	t=0.23, p=0.996, d_r=0.179	t=0.18, p=0.998, d_r=0.074	
- N /+D	-818	139.1	23	[-1106, -530]	t=0.05, p=.999, d_r=0.043	t=0.15, p=.999, d_r=0.063	t=0.27, p=.993, d_r=0.137

Note: Estimated marginal means (EMM) indicate the predicted mean response time within each category of component congruency where absolute distance = 0.3. Numerical distance effect slope estimates (NDE) were significant and negative in all instances, as the range of the 95% confidence interval (CI) does not include zero. Degrees of freedom were estimated using the Kenward-Rodger approximation

Nonsymbolic Component Congruency Effects. Nonsymbolic line ratios were presented as two continuous lines, and analogous to how symbolic fractions may have numerator or denominator components with congruent or incongruent relationships to the holistic magnitudes of a pair, the length of the lines in the ratio may provide cues that bias participants to compare magnitudes congruent with or incongruent to the holistic magnitudes. We assigned nonsymbolic comparison pairs to the same categories of component congruency as the symbolic fractions as seen in Table 2.1

Gap Strategy effects of symbolic fraction comparisons An additional strategy that participants may have applied while making their magnitude comparison judgements is a *gap strategy.* The gap strategy relies on the assumption that smaller (proper) fractions have a larger gap between their numerator and denominator (Denominator - Numerator = Gap) (Morales et al., 2020). If individuals use this strategy, they would be biased to respond that the larger fraction is the one with the smaller gap. This strategy will often result in an accurate response. For instance, 8/10 has a gap of 2 and 6/10 has a gap of 4. However, there are cases where this strategy is an invalid, such as how 2/5 has a smaller gap and also is a smaller fraction than 6/10.

It is important to rule out the possibility of confounding gap strategy effects when evaluating NDEs as evidence of holistic magnitude processing. Making an estimate of a fraction's size relative to another via the gap strategy and evaluating a fraction's magnitude via a proportional judgement of magnitude between numerator and denominator both involve an integration of information from a fractions bipartite structure, yet only the later may result in access to a holistic sense of magnitude. To evaluate the extent to which gap strategy use may have influenced our observation of significant NDEs, we calculated the gap distances between each FF pair (fraction 1 gap – fraction 2 gap), and using a mixed effects model evaluated whether RT NDEs based on magnitude distances remained significant while controlling for variance in RTs explained by gap distances.

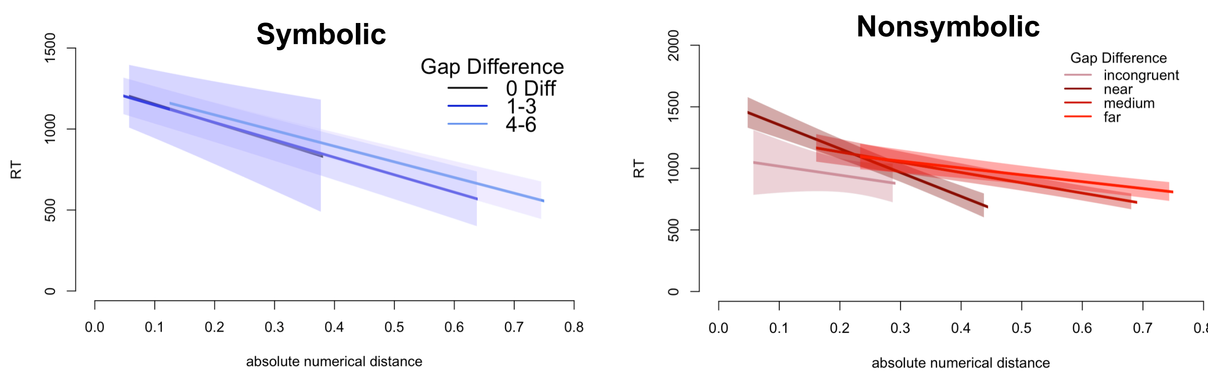

Figure 2.9: Distance effect slopes across gap distances for symbolic fractions and nonsymbolic ratios. (a) Linear estimates of distance effects when fraction pairs had no gap differences (0 Diff), a small gap difference (1-3), and a large gap difference (4-6), show significant negative NDE slopes in each subsection and how larger gap differences are associated with slightly greater mean RTs. (b) Linear estimates of distance effects when line ratio pairs had incongruent gap distances, gap distances less than 75 pixels (near), gap distances between 75 and 125 pixels (medium), and gap distances greater than 125 pixels (far)

Results of the mixed model depicted in Figure 2.9 indicated that both holistic distance, $F(1,22.7) = 86.8$, $p < .001$, and gap distances, $F(1,23.0) = 6.23$, $p = .02$, were significant predictors of variance in FF RTs, but the interaction between these factors was not significant, $F(1,22.8) = 0.22$, $p = .646$. Specifically, when controlling for effects of gap distances, we still observed that RTs decrease as the distance between fraction magnitudes increase, $b = -1095.4$, $t = -9.4$, $p < .001$. Furthermore, we observed that increasing gap distances among FF pairs gap is associated with a slight increase in RTs, $b = 27.4$, $t = 2.5$, $p = 0.019$.

Nonsymbolic Gap Strategy Effects. We further explored whether a strategy focusing on the gaps between the two lines in the nonsymbolic ratios could explain significant variance in how adults make magnitude judgements. To assess the effect of gap strategies in a way parallel to our analysis of FF comparisons, we calculated the difference in line length of each ratio in pixels (gaps) and then calculated the gap difference for each LL pair. In our stimuli this included a few incongruent gap distance pairs (n = 3), where ratio in the pair also had the larger gap, but for all other trials the larger fraction had the sampler gap (n = 69). Thus, gap distances in our stimuli ranged from negative (incongruent) gap distances of -33 to positive gap distances of 275 pixels.

Results of the mixed model indicated that both holistic distance, $F(1,23.0) = 84.1$, $p < .001$, and gap distances, $F(1,23.0) = 8.21$, $p = .009$, and the interaction between these factors, $F(1,23.0) = 5.01$, $p = .035$ were significant predictors of variance in FF RTs. As seen in Figure 2.9, distance effect slopes varied across different gap distances, $\beta = 2.15$, $t(22.8) = 2.24$, $p = 0.035$, with predicted slopes being significantly negative at a gap distance of zero, $\beta = -1256$, $t(8.6) = -9.2$, $p < .001$, and becoming flatter as gap distances increase. When we analyzed incongruent gap trials separately (Figure 2.9) we observed that adults were relatively fast to make these comparison, slope estimates were flatter and not statistically significant, $\beta = -723.9$, $t(25.7) = -1.05$, $p = .304$. When controlling for variation explained by distance, the model indicated that RTs increase as gap distances increase, $\beta = 0.923$, $t(23.3) = 2.87$, $p = .009$.

Experiment 1 Discussion

To further understand how magnitude processing with symbolic fractions relates to the processing on nonsymbolic ratios, we directly compared holistic magnitude processing when adults compared magnitudes within and across these stimuli. We tested four key hypotheses relevant to observe the similarities and differences in processing across these formats. First, we observed that response time and error rates varied as a function of the differences between symbolic and nonsymbolic fractions rational number magnitudes (NDEs). Second, faster performance in the nonsymbolic relative to symbolic condition was consistent with the hypothesis that accessing an underlying representation of rational number magnitude is more efficient with nonsymbolic line ratio stimuli than symbolic fractions (H2). Third, we observed that comparing mixed pairs, and the processing magnitude across formats, is no more difficult than the comparison of two symbolic fractions (H3). In these mixed pairs we also observed that the representation of holistic magnitude accessed via these different formats is not equivalent. Judgements were made such that the magnitudes accessed via nonsymbolic ratios were slightly larger than the magnitudes accessed via equivalent symbolic fractions.

Experiment 2

Introduction

In Experiment 1, we examined the similarities and differences between accessing magnitudes of single-digit fractions and analogous forms of magnitude processing with nonsymbolic line ratios. These two forms of stimuli represent a limited range the kinds of symbolic fractions and nonsymbolic ratios that people may interact with. In Experiment 2, we examined how the different forms of symbolic and nonsymbolic stimuli effect holistic magnitude processing using the same magnitude comparison task. Specifically, in Experiment 2, participants were asked to indicate the larger of two magnitudes in six different conditions: fraction pairs with single-digit components (e.g., 4/5 FF_S), fraction pairs with double-digit components (e.g., 44/55 FF_L), pairs of nonsymbolic line ratios (LL), pairs of nonsymbolic circle ratios (CC), mixed line ratio and fraction pairs (LF) and mixed circle ratio and fraction pairs (CF).

Consistent with Experiment 1, we tested whether adults show evidence of holistic magnitude processing in all six conditions. If accessing the magnitude of symbolic fractions and nonsymbolic ratios of all forms involves and integration of parts to access a relationally defined sense of holistic magnitude, then RT and ER patterns in all conditions should resemble classic NDEs as seen in Experiment 1 (H1). Additionally, we assessed whether componential features of fraction pairs significantly interact with these processes.

The line ratios used in Experiment 1 use variation along one dimension (length) to instantiate part-to-part ratios, and this may be seen as one of the simplest forms a nonsymbolic ratio may take. In Experiment 2, we added nonsymbolic circle ratios to the stimulus set in addition to line ratios. With circle ratios, identifying the holistic magnitude involves understanding the magnitude of a two-dimensional area of one circle relative to another. By comparing line-ratio and circle ratio performance to symbolic fraction performance (FF_S), we further tested the prediction that magnitude processing with visually defined ratios is more efficient than processing symbolic formats (H2.1). This allowed us to test if the efficient nonsymbolic line ratio processing we observed in

Experiment 1 would replicate, and if it would generalize to the nonsymbolic processing of circle ratios. Furthermore, by comparing RT and ER patterns of different nonsymbolic ratio forms we tested the prediction that processing a more visually complex two-dimensional circle area ratio would not be as efficient as processing one-dimensional line length ratios (H2.2).

Cross-format comparisons in Experiment 1 indicated that accessing a common sense of magnitude across formats was no more difficult than processing the magnitude of two fractions. Specifically, cross-format comparisons were neither slower nor more error prone than symbolic fraction comparisons. These findings were consistent with the hypothesis that symbolic and nonsymbolic processing may involve the same or highly compatible mental representations of magnitude (H3). In Experiment 2, we tested whether these findings replicate with cross-format line ratio and nonsymbolic pairs and we explored whether this pattern would generalize to the processing of cross-format pairs with circle ratios.

Adding the circle-ratio to the symbolic fraction condition also allowed us to further explore questions regarding the presence of biases in the point of subjective equality during cross-format judgements. First, we tested whether we could replicate the PSE bias findings observed with line-ratios and symbolic fractions in Experiment 1. Second, we were able to investigate whether an overestimation of nonsymbolic ratios relative to symbolic fractions would extend to cross format comparisons with circle ratios and symbolic fractions, which would replicate the findings observed by (Matthews & Chesney, 2015). Third, Matthews and Chesney (2015) observed a relatively smaller PSE bias during symbolic-nonsymbolic comparisons with circle ratios relative to when the nonsymbolic ratio was a dot ratio. One plausible factor of nonsymbolic ratios which may influence these shifts in PSE bias may be their visual complexity. To further explore the presence of these biases and why they occur, we aimed to determine if the visual complexity of a nonsymbolic ratio may directly relate to the magnitude of these biases. Specifically, we predicted that more complex circle ratios relative to line ratios may lead to larger PSE biases of judging nonsymbolic stimuli as larger magnitudes relative to symbolic fractions (H4).

Similar to how we may characterize line ratios a simple nonsymbolic representation of magnitude, the fractions in Experiment 1 were composed of only single digit irreducible fractions. These fractions may also represent a relatively simple symbolic representation of magnitude. In Experiment 2, we introduced a condition where symbolic fractions would be presented as double-digit fractions (equivalent to the single-digit fraction stimuli). By comparing single-digit (FF_S) and double-digit fractions (FF_L), we tested the effect of increasing the complexity of symbolic representations of magnitude and whether adults access to holistic magnitudes from symbolic fractions is differentially affected by componential features when those components are increased to larger values. We hypothesized that increased symbolic fraction complexity in the form of larger component values would be directly related to RT and ER (H5).

Furthermore, in Experiment 1 participants saw comparisons of symbolic, nonsymbolic, and mixed pairs randomly intermixed within each block. In Experiment 2, we were tested whether the effects observed in Experiment 1 would replicate when participants saw comparisons blocked into separate format conditions.

Methods

Participants

44 undergraduate students (36 females, M_{age} = 20.3 years, range = 19-27) participated for course credit. All participants were right-handed native English speakers and naïve to our research questions. Consistent with Experiment 1, we used the same inclusion criterion that accuracy in all conditions must be greater than 70%. Data from four participants were excluded from our analyses due to accuracy that did not satisfy this criterion. As a result, data from 40 undergraduate students (32 females, M_{age} = 20.4, range = 19-27) were included in our analyses.

Procedure and Design

Participants completed six blocks of the same magnitude comparison task used in Experiment 1. Each block contained trials from one of the six format conditions, and the six condition blocks were presented in the same order across all participants. Similar to Experiment 1, comparison difficulty was manipulated as *numerical distance*, and again each block contained 36 unique pairs, and participants viewed each pair once within each condition (n = 216 trials). A break screen appeared in the middle of a block. We instructed that participants make their responses as quickly and accurately as possible.

Symbolic Fraction Stimuli Similar to Experiment 1, stimuli for each of the six format conditions were based on 36 single-digit irreducible fraction pairs. These fraction pairs and their magnitudes were nearly identical to the pairs used in Experiment 1, except for two substitutions necessary to improve the balance of pairs with different component congruency relationships (see Appendix A). To support a deeper analysis of component congruency and gap strategies, we created fraction stimuli with double-digit components, by scaling up our single-digit fraction by a random number between 3 and 10. The resulting components of this scaling were then randomly adjusted so the magnitudes of the resulting fractions' magnitudes were close to single digit stimuli but could simply be reduced to this simplified form.

Nonsymbolic Ratio Stimuli We used the same sets of line ratio stimuli in Experiment 2 as Experiment 1. In the block of within-format line ratio comparisons, the first half of the trials used line ratio images from the set created to minimize the numerator to holistic magnitude correlation, and the second half of the trials used images from the set created to minimize the denominator to holistic magnitude correlation.

Circle ratio images were created using the same approach as the line ratios in Experiment 1 but were defined as the ratio of one circle area relative to the other circle area. Furthermore, to properly display these circle ratios on the screen, these ratios were vertically aligned with the "numerator" circle above the "denominator" circle.

Analyses

Analyses of distance effects (H2), format affects (H2, H3, H5), and cross-format PSE (H4) were conducted using mixed effects models (linear for RT and logarithmic for ER), with the same approach as Experiment 1. Tests of our primary hypotheses were conducted after applying our inclusion criteria for the data. Subject level inclusion criteria were the same as Experiment 1.

To identify features of ratios and fractions that may bias magnitude comparison performance or interact the holistic magnitude processing of the stimuli, we examined the data for specific fraction pairs that had error rates higher than chance performance. In the case of cross-format pairs, identification of items with more incorrect responses than correct responses across the experimental group were further analyzed for biases in the symbolic to nonsymbolic point of subjective equality. Pairs with high error rates in symbolic and nonsymbolic pairs where further analyzed for *component congruency* and component *gap distance* effects.

Results

Missing data points and erroneous responses

Prior to data analysis, erroneous anticipation responses (less than 250ms) and missed trials (no response entered) were cleaned from the data. Our anticipation cutoff of 250ms was based on previous research using diffusion models to examine performance on numerical tasks (Ratcliff et al., 2015). No participants missed more than two responses within any of the format condition blocks. Across all participants and format conditions, cleaning missed responses removed 0.3% of the total data. Erroneous anticipation responses were only observed in the data of two participants and removing these responses excluded 6.9% of one participant's data and 0.9% of the others. Neither participant was excluded from our analyses.

Response times

Numerical distance effects Replicating Experiment 1, responses in the LL, LF and FF_S formats demonstrated significant distance effects with RTs decreasing as the distance between pairs increased (Figure 2.10). Furthermore, these significant distance effects were also observed in CC, CF, and FF_L response times (Figure 2.10). These results support our hypothesis (H1) that magnitude processing with symbolic fractions and nonsymbolic ratios is sensitive to the holistic magnitude of the stimuli, both within the same format and across formats.

Results of the linear mixed model analysis are presented in Table 2.6. Pairwise comparisons of NDE slope magnitudes across all six formats, identified multiple instances where distance and format effects interacted, and other instances where no difference was observed. Symbolic fraction comparisons with large components (FF_L) and small components (FF_S), as well line-fraction cross-format comparisons (LF) showed the steepest distance effects, which were all significantly more negative than LL, CC, and CF comparisons, but did not significantly differ from one another. These shallower, yet significantly negative, slopes observed in LL, CC, and CF responses were not significantly different from one another.

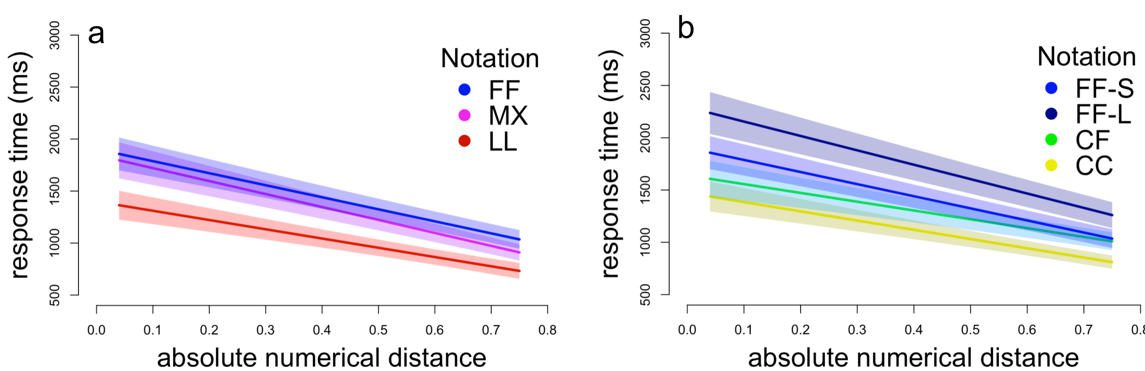

Figure 2.10: Linear predictions of conditional mean response times show that (a) line ratio comparisons (LL) are fastest across all numerical distances relative to fraction-fraction (FF) and mixed comparisons (MX). Furthermore, (b) circle ratio comparisons (cc) were faster than circle ratio-fraction (CF) and FF comparisons. Shaded bands depict 95% confidence intervals of the linear predicted mean response time.

Table 2.6

Estimated marginal means (EMM) and slopes

	Fixed Effect Estimates				Pairwise contrasts				
Format	EMM RT	SE	df	95% CI	LL	CC	LF	CF	FF_S
LL	1069	47.0	39	[974, 1164]					
CC	1145	48.5	39	[1047, 1243]	t=-2.67, p=.052, d_r=-0.17				
LF	1383	61.0	39	[1259, 1506]	t=-7.70, p<.001, d_r=-0.61	t=-5.57, p<.001, d_r=-0.52			
CF	1328	61.5	39	[1204, 1453]	t=-5.95, p<.001, d_r=-0.57	t=-4.41, p<.001, d_r=-0.36	t=-1.85, p=.265, d_r=-0.12		
FF_S	1473	59.1	39	[1354, 1593]	t=-10.57, p<.001, d_r=-0.78	t=-8.89, p<.001, d_r=-0.64	t=-2.25, p=.128, d_r=-0.18	t=-3.41, p=.008, d_r=-0.28	
FF_L	1776	78.8	39	[1617, 1935]	t=-14.28, p<.001, d_r=-1.39	t=-12.25, p<.001, d_r=-1.23	t=-9.80, p<.001, d_r=0.76	t=-8.61, p<.001, d_r=-0.87	t=7.38, p<.001, d_r=0.59
Format × Distance	NDE Slope	SE	df	95% CI	LL	CC	LF	CF	FF_S
LL	-891	100.1	38.9	[-1093, -688]					
CC	-884	96.6	39.0	[-1079, -689]	t=0.07, p=.999, d_r=-0.02				
LF	-1250	107.6	38.9	[-1467, -1032]	t=2.92, p=.029, d_r=0.70	t=3.15, p=.016, d_r=0.80			
CF	-841	101.2	38.9	[-1046, -637]	t=0.44, p=.971, d_r=0.11	t=-0.36, p=.984, d_r=0.08	t=-3.39, p=.008, d_r=-0.89		
FF_S	-1157	96.6	39.0	[-1352, -961]	t=2.31, p=.113, d_r=0.52	t=2.38, p=.098, d_r=0.53	t=-0.81, p=.849, d_r=-0.18	t=2.74, p=.044, d_r=0.61	
FF_L	-1385	102.3	38.9	[-1592, -1178]	t=4.03, p=.001, d_r=0.96	t=4.07, p=.001, d_r=0.98	t=1.12, p=.0.681, d_r=0.26	t=4.08, p=.001, d_r=1.06	t=1.91, p=.240, d_r=0.44

Note. Model estimates of mean RT were evaluated at the mean numerical distance among the stimuli (distance = 0.375).

Nonsymbolic ratio processing Consistent with Hypothesis 2.1, the linear mixed model analysis (Table 2.6) revealed evidence for more efficient nonsymbolic ratio processing relative to performances with symbolic fractions and mixed pairs. These findings replicated those observed in Experiment 1 with line ratios and extend to analogous nonsymbolic processing of circle areas. Specifically, LL response times (RT) were 313ms faster than LF comparisons, 404ms faster than FF_s comparisons, and 711ms faster than FF_L comparisons. CC RTs were 183ms faster than CF comparisons, 327ms faster than FF_s comparisons, and 635ms faster than FF_L comparisons.

The pairwise comparison of nonsymbolic ratio processing between line ratios and circle ratios revealed that adults made accurate line ratio judgements 76ms faster than circle ratios, however this difference did not reach statistical significance. Therefore, we did not observe strong support for our hypothesis (2.2) that the processing of circle ratios, defined by their relative areas, would be less efficient than the processing of line ratios defined the relative lengths of one-dimensional lines.

Cross-Format Magnitude Processing Consistent with the hypothesis that symbolic and nonsymbolic processing may involve the same or highly compatible mental representations of magnitude (H3), we observed that cross-format LF magnitude processing was neither slower or more error prone than within-format processing of FFs pairs and cross-format CF magnitude processing was in fact more efficient than symbolic processing of FFs pairs. These differences in processing cross-format and response times resemble a pattern where cross format comparisons a mixture of fast circle ratio processing and slower symbolic fraction processing. Furthermore, the pattern of these cross-format effects does not provide positive evidence to indicate that these cross-format comparisons required difficult steps to translate between highly dissimilar representations of magnitude for each format.

Effects of symbolic fraction complexity on RT Consistent with the hypothesis that encoding of symbolic fraction stimuli should require additional forms of processing when the complexity of the symbolic form increases (H5), we observed that FF_L pairs with double digit denominators and reducible terms required 307ms more time to compare than FF_S pairs with single digit irreducible fractions.

Error Rates

Cross-format point of subjective equality bias As seen in Figure 2.11, item level analysis of error rates, indicated that there were a number of cross-format pairs with group mean error rates higher that chance performance. In both LF and CF comparisons, trials with high error rates were composed of pairs where distances approached zero and the correct answer was to choose the symbolic fraction as larger. This pattern of error rates highly resembles that observed by Matthews and Chesney (2015). Analysis of participant's cross-format point of subjective equality bias was necessary to observe why group mean ER on some comparison pairs was far above this value representing chance performance. Results of the PSE analysis with cross-format pairs are presented in Figure 2.12. We observed a pattern of responses to mixed pairs indicating that the participants' PSE was biased towards seeing line ratios and circle ratios as larger magnitudes relative to equivalent symbolic fractions. These results replicate the PSE bias observed in Experiment 1, indicating that participants' PSE was reached when the magnitudes of symbolic fractions were 0.056 larger than the magnitude of the visual nonsymbolic line ratios. Individual model estimates indicated that on average PSEs with LF pairs were significantly greater than zero, $t(39) = 4.52$, $p < .001$. Cross-format judgements between circle ratios and fractions, indicated that participants' PSE occurred when the magnitude of symbolic fractions was 0.163 larger than the magnitude of circle ratios. Individual model estimates indicated that on average PSEs with CF pairs were significantly greater than zero, $t(39) = 13.55$, $p < .001$. Using a paired t-test, we further confirmed that on average the PSE bias during CF comparisons was significantly greater than the PSE bias observed during LF comparisons.

Similar to the approach used by (Matthews & Chesney, 2015), we evaluated whether the PSE biases should be used to correct our identification of correct and incorrect responses in LF pairs. Specifically, with the knowledge that on average participants PSE is biased, we can adjust the numeric distances between cross-format pairs using the magnitude of biases specific to LF and CF pairs. Furthermore, this process can potentially recode comparison pairs with very close numeric distances as having a different correct answer than defined by our experimental design. Specifically, this occurs when a smaller nonsymbolic ratio in the pair is redefined by PSE adjustment as being perceived as a magnitude larger than the symbolic stimuli.

In the line-fraction cross format condition, performing PSE adjustment to the distance between symbolic and nonsymbolic pairs did not change the correct answer coding of any trials (Figure 2.11) This is due to the fact that the trial with the closest numerical distance (0.047), 1/3 nonsymbolic vs 2/7 symbolic, has a larger nonsymbolic fraction and a smaller symbolic fraction. Adjusting for the bias in this case increases the numerical distance between the pairs rather than flipping what is subjectively correct. The second closest numerical distance in the stimuli (distance = 0.057) was slightly larger than the Line-Fraction bias in PSE (0.056). In this case the symbolic fraction 6/7 is larger than the nonsymbolic 4/5, but the distance is just far enough that the PSE adjustment does not change which of the two stimuli would be viewed as subjectively larger. Thus, the group mean error rate for the 6/7-4/5 cross-format pair is greater than .5 (M = 0.75, SD = 0.44), even after bias adjustment, and is therefore this trial was excluded from our subsequent analysis of distance effects on error rate.

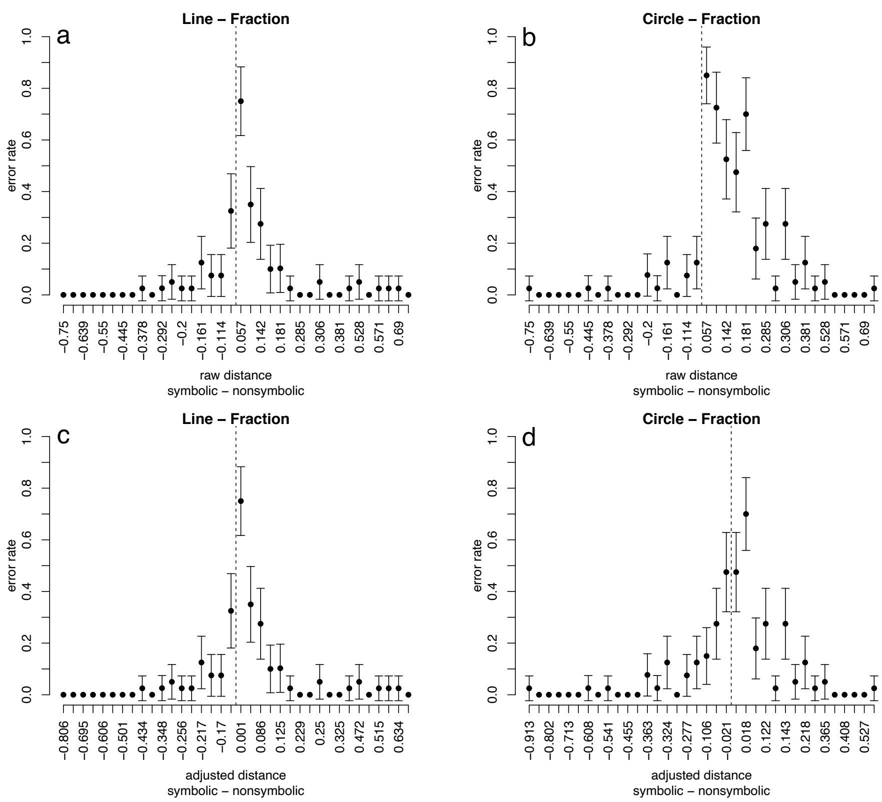

Figure 2.11. Group mean error rates for each cross-format comparison pair. Error bars indicate 95% confidence intervals of the group mean. Dotted line indicates the veridical point of subjective equality defined by the stimuli's true magnitudes. Note: Distances are presented as equally spaced across the x-axis for illustrative purposes, and do not precisely reflect the differences in cross-format distances between items.

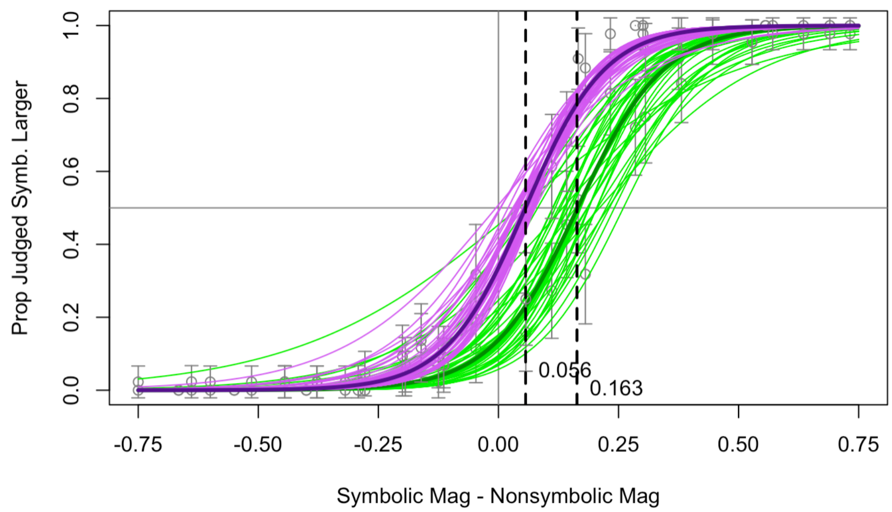

Figure 2.12. Logistic model estimates of individual (random effects) and group level (fixed effects) probability to judge a symbolic fraction as larger than the nonsymbolic ratio across varying distances between stimuli. Model estimates of LF and CF comparisons are show in purple and green, respectively, and group level estimates are shown in darker trend line. Points where these lines cross the .5 probability line (grey) indicate points of subjective equality (PSEs), or the distance between symbolic fractions and nonsymbolic ratios where individuals would judge the stimuli as equal. Vertical dashed black lines indicate group mean points of subjective for each format.

Within the circle-fraction trials the PSE bias was 0.162, which was greater than the absolute distance of 7 mixed pairs. As was seen with the line-fraction trials, the accuracy coding of some trials was not affected by the PSE adjustment because the nonsymbolic circle ratio already had the larger magnitude and thus made the bias adjusted distances larger than the distances design by our stimulus design. In 3 of these 7 pairs (e.g. 4/5 nonsymbolic vs 6/7 symbolic), adjusting the numeric distance between pairs to correct for PSE bias, inverted the correctness coding to indicate that the smaller nonsymbolic ratio in the pair could be subjectively seen as the larger stimuli Figure 2.11. However, the group mean error rate for the pair 4/9 nonsymbolic vs 5/8 symbolic was worse that chance performances (M = 0.70, SD = 0.46), even after PSE bias adjustment. Therefore, this trial was excluded from our subsequent analyses of distance effects on ER.

Format and distance effects on ER All analyses of format effects on the probability of ER across numeric distances were conducted using PSE adjusted distances, and specific comparisons pairs with ERs above chance performance were excluded prior to model fitting. As was done with mixed pairs, we calculated the group mean accuracy at the item level, to determine which pairs participants were more likely to incorrectly judge the smaller stimulus as larger. We observed only two trials with mean error rates above chance performance (.5), and both were found in the symbolic fraction condition with larger components (FF_L). These two trials, 10/16 vs 39/55 and 22/27 vs 50/58 had group mean error rates of 0.625 and 0.615, respectively. These trials were excluded from our logistic models of ER over numeric distances but are further discussed in our analyses of component congruency and gap distances below.

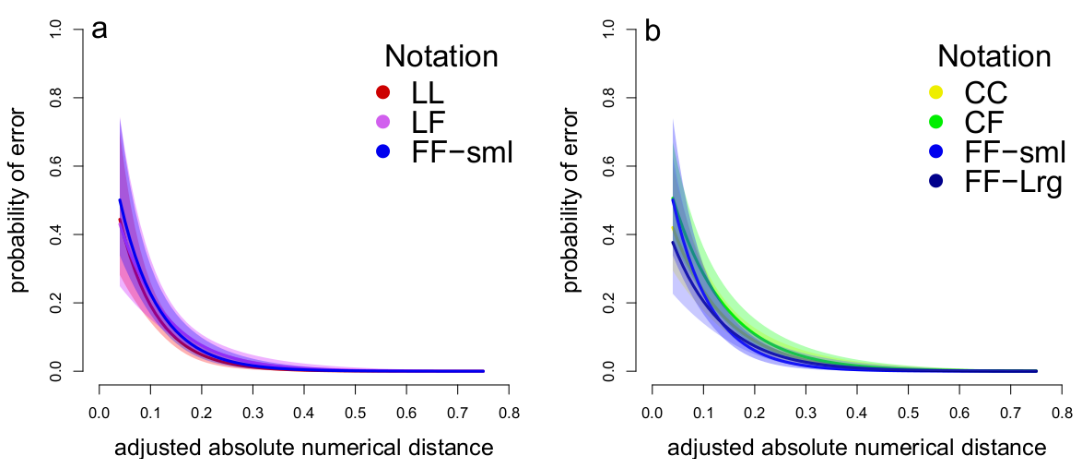

Figure 2.13. Logistic model estimates of error rate probability across absolute numerical distances adjusted for PSE biases. (a) Results replicate findings of Experiment 1, indicating that among nonsymbolic line ratio (LL), cross-format line ratio-fraction (LF), and symbolic fraction pairs with single digit components (FF_S) error rates increased as distances approached zero, but these distance effects did not interact between formats. (b) Results from nonsymbolic circle ratio (CC), cross-format circle ratio-fraction (CF) and symbolic fraction pairs with larger components (FF_L), show a similar pattern of distance effects in all conditions, with no significant format by distance interactions. Shaded bands depict 95% confidence intervals

Among all included comparison pairs across the six format conditions participants on average made a low percentage of errors (5.8%, SD = 2.7%). Consistent with RT results, the logistic mixed models of ERs showed a significant effect of distance, $F(1,90)=102.3$, $p < .001$. As seen in

Figure 2.13, distance effects were observed in all six format conditions, with error rates dropping off rapidly as distances increased from 0 to 0.2 and approached errorless performance once distances reached 0.4 (H1). The full logistic model indicated no significant interactions between numerical distance effects and format, $F(5,90)=4.28$, $p = 0.51$. We therefore dropped this interaction when evaluating format effects.

As seen in Table 2.7, analyses of error rates across formats revealed no significant differences in ER across the six format conditions. Therefore, we did not observe evidence in ER to support the hypotheses that adults should be more efficient at nonsymbolic than symbolic magnitude processing (H2.1), adults should be more efficient at line ratio processing than circle ratio processing (H2.2), that cross-format processing should be more difficult than within format processing (H3), or that single-digit FF processing should be more efficient than double-digit FF processing (H5). Although this the lack of format effects on ER does not provide positive evidence in support of our hypotheses, it does provide greater assurance that the format effects observed in RTs were not compromised by speed-accuracy tradeoffs.

Table 2.7

Logistic Mixed Model Regression Results for Error Rate and pairwise comparisons

	Fixed Effect Results				Pairwise comparisons				
Format	Estimate	SE	95% CI	Odds Ratio	LL	CC	LF	CF	FF_S
LL	-4.82	0.269	[-5.35, -4.30]	0.006					
CC	-4.45	0.256	[-4.95, -3.95]		z=-1.716, *p*=.521, OR=0.69				
LF	-4.61	0.263	[-5.13, -4.10]	0.013	z=-0.924, p=.941, OR=0.81	z=0.903, *p*=.976, OR=1.17			
CF	-4.32	0.260	[-4.83, -3.81]		z=-2.379, *p*=.164, OR=0.60	z=-0.702, *p*=.982, OR=0.88	z=-1.414, *p*=.718, OR= 0.75		
FF_S	-4.64	0.274	[-5.18, -4.10]		z=-0.780, *p*=.970, OR=0.83	z=0.903, *p*=.946, OR=1.21	z=0.123, *p*=.999, OR=1.03	z=1.630, *p*=.579, OR=1.38	
FF_L	-4.64	0.278	[-5.19, -4.10]	0.017	z=-0.794, p=.969, OR=0.83	z= 0.849, *p*= .958, OR=1.21	z=-0.119, p=.999, OR=0.97	z=1.447, *p*=.698, OR=1.38	z=0.00, *p*=.999, OR=1.00
Distance	-11.2	0.774	[-12.7, -9.7]	2.9E-6					

Note: Higher likelihood of errors corresponds to greater estimated marginal means of format on the log odds scale. Odds ratio transformation of these estimates indicates the probability of making an error relative to the probability of making an accurate judgement. Estimated means and pairwise comparisons are evaluated where absolute numerical distance = 0.371, the mean of distances presented in the stimuli. Pairwise contrasts are performed on the log odds ratio scale, and corresponding odd ratios (OR) indicate the probability of an error in the condition along the column relative to condition in the row header.

Within-notation follow-up analyses

Consistent with the analyses conducted in Experiment 1, we further evaluated whether our identification of holistic distance effects may occur in parallel or are disrupted by additional features of symbolic fractions and nonsymbolic ratios.

Symbolic Fraction Component Congruency Effects Results of the mixed effects model, presented in Table 2.8, indicated that NDE slopes were significant and negative for single-digit FF pairs within each of the four component congruency relationships. Pairwise comparisons of mean RT between FF pairs with different component congruency relationships were evaluated at an absolute distance of 0.3, where all types of pairs overlap. Pairwise comparisons of mean RTs revealed that pairs with common denominators had the fastest RTs, which were 296ms faster than pairs with congruent numerators and denominators, 414ms faster than pairs with congruent numerators and incongruent denominators, and 442ms less than pairs with incongruent numerators. FF comparisons with incongruent numerators had the slowest mean RTs in the sample, but it was not significantly higher than pairs with congruent numerators and denominators, or pairs with congruent numerators and incongruent denominators. Among pairs with congruent numerators, the differences in RT between pairs with congruent denominators and pairs with incongruent denominators did not reach statistical significance.

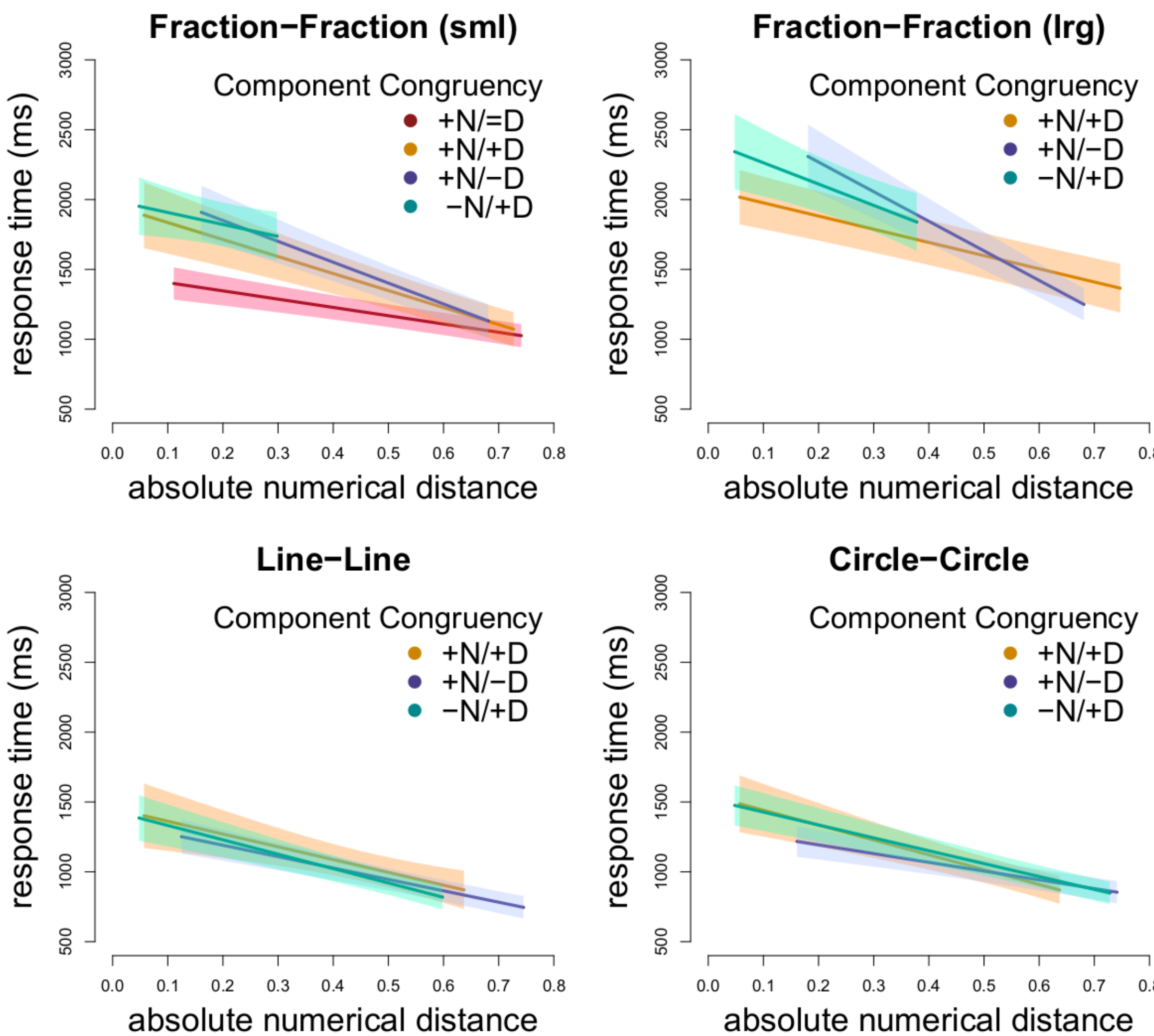

Figure 2.14: Linear predictions of distance effects on RT across symbolic fraction pairs with single digit components (sml) and double digit components (lrg) and nonsymbolic line ratio and circle ratio pairs with different component congruency relationships.

Table 2.8

Mean RTs and distance effect slopes in FF (small) pairs across component congruency

	Fixed Effect Estimates				Pairwise contrasts		
Component Congruency	EMM RT	SE	df	95% CI	+N /=D	+N /+D	+N / -D
+N /=D	1287	49.2	39	[1187, 1386]			
+N /+D	1583	84.9	39	[1412, 1755]	t=-4.33, p<.001, d_r=-0.64		
+N / -D	1701	81.6	39	[1536, 1866]	t=--6.20, p<.001, d_r=-0.89	t=-2.64, p=.055, d_r=-0.25	
- N /+D	1729	86.9	39	[1553, 1905]	t=--5.61, p<.001, d_r=0.95	t=-1.89, p=.250, d_r=0.31	t=0.41, p=.976, d_r=0.06
Component Congruency × Distance	NDE β	SE	df	95% CI	+N /=D	+N /+D	+N / -D
+N /=D	-592	115	39	[-825, -359]			
+N /+D	-1186	181	39	[-1553, -819]	t=2.72, p=.047, d_r=1.27		
+N / -D	-1496	179	39	[-1859, -1133]	t=4.23, p<.001, d_r=1.94	t=1.62, p=0.382, d_r=0.67	
- N /+D	-848	396	39	[-1652, -43.3]	t=0.63, p=.923, d_r=0.55	t=-0.82, p=.844, d_r=-0.73	t=1.54, p=.428, d_r=-1.39

Note: Estimated marginal means (EMM) indicate the predicted mean response time within each category of component congruency where absolute distance = 0.3. Numerical distance effect slope estimates (NDE) were significant and negative in all instances, as the range of the 95% confidence interval (CI) does not include zero. Degrees of freedom were estimated using the Kenward-Rodger approximation.

Results of the mixed effects model with fraction-fraction comparisons and large components, shown in Table 2.9, indicated that NDE slopes were significant and negative within each of the four component congruency relationships. Pairwise comparisons of mean RTs revealed that pairs with congruent numerators and denominators had the fastest RTs, which were on average (at a distance of 0.3) 265ms faster than pairs with congruent numerators and incongruent denominators and168ms faster than pairs with congruent numerators and incongruent denominators. FF_L comparisons with congruent numerator and incongruent denominators had the slowest mean RTs in the sample, but they were not significantly slower than pairs with incongruent numerators and congruent denominators.

Table 2.9

Mean RTs and distance effect slopes in FF (large) pairs across component congruency

	Fixed Effect Estimates				Pairwise contrasts	
Component Congruency	EMM RT	SE	df	95% CI	+N /+D	+N / -D
+N /+D	1786	86	39	[1612, 1960]		
+N / -D	2051	100	39	[1848, 2254]	t=-411, p<.001, d_r=-0.45	
- N /+D	1954	93.6	39	[1765, 2143]	t=-2.56, p=.038, d_r=-0.28	t=1.37, p=.367, d_r=0.16
Component Congruency × Distance	NDE β	SE	df	95% CI	+N /+D	+N / -D
+N /+D	-936	128	38.3	[-1194, -678]		
+N / -D	-2098	220	38.8	[-2543, -1653]	t=4.69, p<.001, d_r=2.49	
- N /+D	-1490	411	37.2	[-2323, -657]	t=1.30, p=.403, d_r=1.19	t=-1.44, p=.331, d_r=-1.31

Note: Estimated marginal means (EMM) indicate the predicted mean response time within each category of component congruency where absolute distance = 0.3. Numerical distance effect slope estimates (NDE) were significant and negative in all instances, as the range of the 95% confidence interval (CI) does not include zero. Degrees of freedom were estimated using the Kenward-Rodger approximation

Nonsymbolic Component Congruency Effects Nonsymbolic line ratios in Experiment 2 were presented as the ratio two continuous lines and the ratio of two circle areas. We conducted analogous analyses of component congruency effects as symbolic fractions based on the visual extent of the ratio components. There were not enough pairs within the LL and CC conditions, where the lengths of the denominator line lengths and circle areas were equivalent or close to equivalent to include the *same denominator* category in our analysis of nonsymbolic congruency effects.

Table 2.10

Mean RTs and distance effect slopes in LL pairs across component congruency

	Fixed Effect Estimates				Pairwise contrasts	
Component Congruency	EMM RT	SE	df	95% CI	+N /+D	+N / -D
+N /+D	1184	70.4	39	[1039, 1324]		
+N / -D	1109	50.8	39	[1006, 1212]	t=1.67, p=.230, d_r=.170	
- N /+D	1125	56.1	39	[1012, 1239]	t=0.96, p=.603, d_r=.132	t=-0.411, p=0.911, d_r=-0.04
Component Congruency × Distance	NDE (b)	SE	df	95% CI	+N /+D	+N / -D
+N /+D	-922	255.6	38.7	[-1439, -405]		
+N / -D	-815	99.9	38.9	[-1017, -613]	t=-0.43, p=.904, d_r=0.25	
- N /+D	-1025	160.3	38.6	[-1349, -701]	t=0.37, p=.928, d_r=0.24	t=1.26, p=.428, d_r=0.50

Note: Estimated marginal means (EMM) indicate the predicted mean response time within each category of component congruency where absolute distance = 0.3. Numerical distance effect slope estimates (NDE) were significant and negative in all instances, as the range of the 95% confidence interval (CI) does not include zero. Degrees of freedom were estimated using the Kenward-Rodger approximation

Results of the mixed effects model with LL comparisons, shown in Table 2.10, revealed that NDE slopes were significant and negative within each of the three component congruency subsets of the LL pairs. We did not observe a significant interaction between distance and component congruency, $F(2,37.7) = 0.80$, $p = .458$, which can be seen in Figure 2.14 where distance effect slopes regardless of component congruency are parallel. Consistent with our analysis of mean RTs in the Symbolic Fraction condition, we tested for differences at the absolute distance of 0.3. Results of the mixed model revealed no significant main effect of component congruency on mean RTs, $F(2,37.7) = 0.89$, $p = .458$.

Table 2.11

Mean RTs and distance effect slopes in CC pairs across component congruency

	Fixed Effect Estimates				Pairwise contrasts	
Component Congruency	EMM RT	SE	df	95% CI	+N /+D	+N / -D
+N /+D	1223	67.8	39	[1086, 1360]		
+N / -D	1130	49.4	39	[1030, 1230]	t=2.32, p=.065, d_r=.237	
- N /+D	1242	54.7	39	[1132, 1353]	t=-0.48, p=.882, d_r=0.05	t=-3.46, p=1.52, d_r=0.29
Component Congruency × Distance	NDE (b)	SE	df	95% CI	+N /+D	+N / -D
+N /+D	-1050	180	38.6	[-1414, -687]		
+N / -D	-627	113	39.0	[-856, -397]	t=2.25, p=.075, d_r=-1.08	
- N /+D	-919	102	38.8	[-1125, -713]	t=-0.74, p=.741, d_r=-0.33	t=2.23, p=.078, d_r=0.74

Note: Estimated marginal means (EMM) indicate the predicted mean response time within each category of component congruency where absolute distance = 0.3. Numerical distance effect slope estimates (NDE) were significant and negative in all instances, as the range of the 95% confidence interval (CI) does not include zero. Degrees of freedom were estimated using the Kenward-Rodger approximation

Results of the mixed effects model with CC comparisons are presented in Table 2.11. Similar to the line ratio comparison results, we saw a significant main effect of holistic distance, $F(2,37.7) = 81.6$, $p < .001$, with significant negative NDE slopes in each of the component congruency categories of CC comparisons. Results also revealed a main effect of component congruency, $F(37.7) = 5.85$, $p = .006$, and a significant interaction between distances and component congruency effects, $F(2, 37.78) = 3.43$, $p = .043$. Pairwise comparisons of NDE slopes between component congruency categories indicated these differences did not reach statistical significance, and likewise pairwise comparisons of estimated mean RT did not reveal significant differences.

Symbolic Fraction Gap Strategy Effects We conducted the same gap strategy analysis of FF_S pairs and adapted this analysis to examine if gap strategy effects with FF_L pairs. As we observed in Experiment 1, the FF_S comparison stimuli contained pairs with zero gap distance, and gap distances as large as 6. The FF_L comparison stimuli included 4 pairs where gap distances were incongruent (the larger fraction had a larger gap between numerator and denominator values), only one pair with no gap difference, and congruent gap differences as larger as 59.

Results of the mixed models with single digit symbolic fractions indicated that both holistic distance, $F(1,38.7) = 43.9$, $p < .001$, and gap distances, $F(1,38.8) = 7.63$, $p = .009$, were significant predictors of variance in FF RTs, but the interaction between these factors was not significant, $F(1,38.7) = 0.05$, $p = .832$. As seen in Figure 2.15, these results were consistent with symbolic comparison in Experiment 1 indicating that we still see significant distance effects , $b = -1284.6$, $t = -12.4$, $p < .001$, when controlling for effects of gap distances. Furthermore, we observed that when controlling for distance effects, increases in gap distances was associated with a slight increase in RTs, $b = 10.0$, $t = 2.8$, $p = 0.007$.

Results of the mixed model among symbolic fractions with large components revealed a significant interaction between distance and gap effects, $F(1,38.6) = 13.2$, $p < .001$. We explored the nature of this interaction by evaluating the direction and significance of distance effect slopes within subsections of the data with varying gap distances. As seen in Figure 2.15, this follow up analysis revealed that the significant main effect of distance, $F(1,38.8) = 184.1$, $p < .001$ can be observed across all gap distances when the gaps within the larger fraction were smaller than gaps of the smaller fraction. However, estimated distance effects among responses to pairs with incongruent gap distances were neither significant nor negative, among these pairs (n = 4 trials), $b = 460$, $t = 0.82$, $p = 0.419$. Therefore, we see that among fractions with large components, gap distances have minimal to no effect on distance effect slopes when these gap distances are consistent with a gap strategy, however holistic magnitude processing may be interrupted or effected when larger fractions also have larger gaps.

Nonsymbolic Ratio Gap Strategy effects Analyses of gaps and gap distances among nonsymbolic ratio stimuli used the same approach as Experiment 1, with the addition of calculating gap distances among circle ratios (CC). We defined circle ratio gaps as the difference in visual area between the circles in pixels, and gap distance as the difference between the two circle ratios' gaps. Given the extremely large difference in the scale of line ratio gap distances (LL_{range}= [-33, 266]) in pixel length and circle ratio gap distances CC_{range}=[-8176, 42276] in pixel area, all predictors were scaled and centered when evaluating the significance and interaction of distance and gap effects

Results of the mixed model with LL comparisons indicated that when controlling for effects of gap distances, holistic distance predicted significant variance in RTs, $F(1,38.9) = 7.79$, $p = .005$. However, when controlling for holistic distances, the effect of gap distances did not reach significance, $F(1,38.9) = 3.05$, $p = .088$. Furthermore, the interaction between these factors, $F(1,38.7) = 8.17$, $p = .007$ was a significant predictor of variance in RTs. The nature of this interaction, presented in Figure 2.15, shows that distance effect slopes varied across different gap distances, β = 50.3, t = 2.86, p = 0.006, with negative distance effect slopes at small gap distances becoming flatter as gap distances increase. When analyzed separately (Figure 2.15), slope estimates for incongruent gap trials were not significantly negative, β = -1033.8, t(38.2) = -1.08, p = .284, however this result is based on a small number of trials in the LL condition (n = 3). When controlling for variation explained by distance, the model indicated that RTs decrease as gap distances increase, β = -41.3, t = -1.75, p = .089.

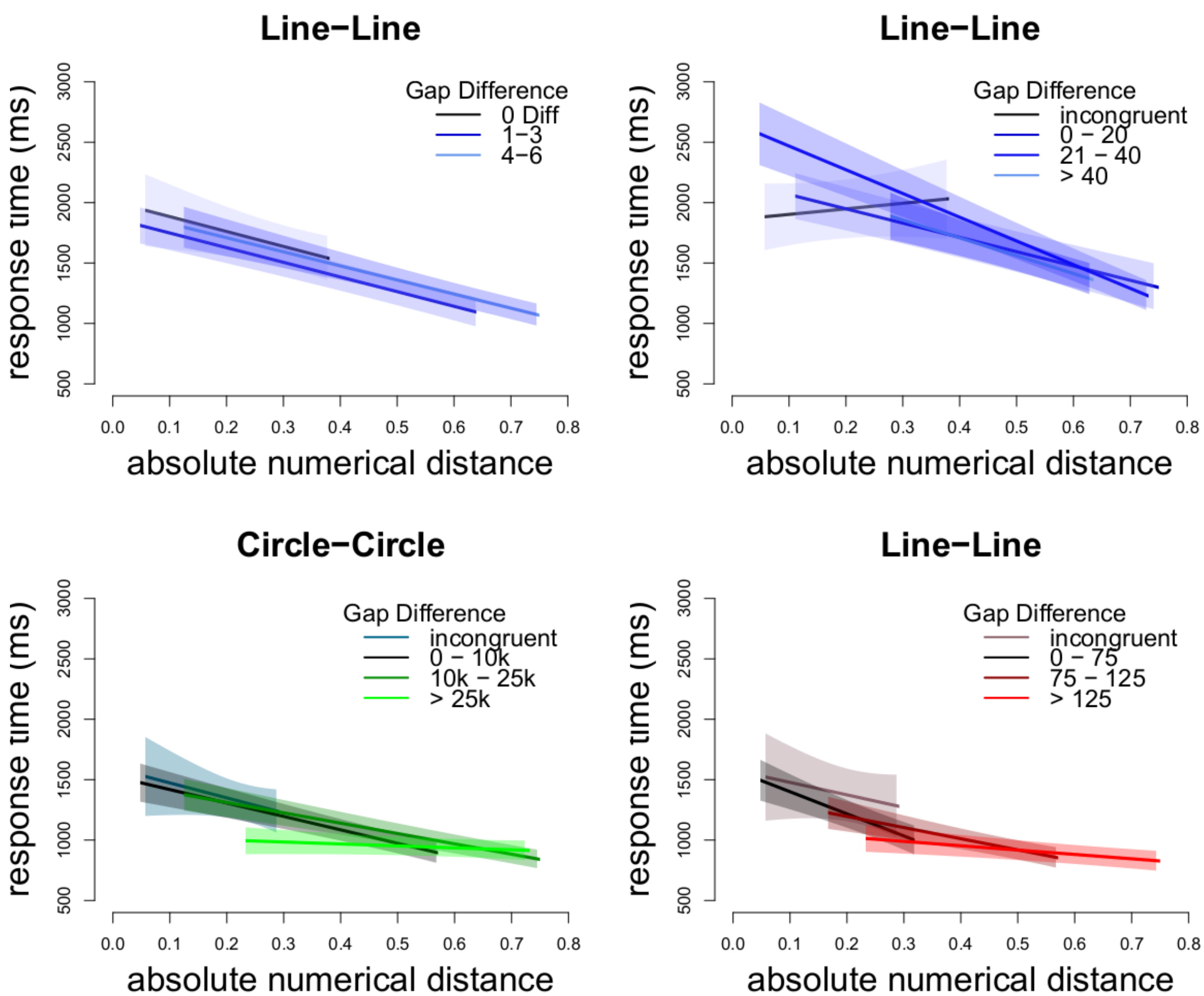

Figure 2.15. Distance effect slopes across gap distances for symbolic fractions and nonsymbolic ratios. Binned gap differences in symbolic fractions (a/b) indicate the difference in the symbolic magnitude value between gaps in each fraction. Gap differences with nonsymbolic ratios indicate the differences in pixel length (c) or pixel area (d).

Results of the mixed model with CC comparisons indicated that holistic distance, $F(1,39.0) = -81.3$, $p < .001$, gap distances, $F(1,38.9) = 5.69$, $p = .022$, and the interaction between these factors, $F(1,39.0) = 9.02$, $p = .005$, were significant predictors of variance in RTs. This interaction, where distances effect slopes were modulated by varying gap distances is shown in Figure 2.15. Specifically, predicted slopes were negative at a gap distance of zero and became flatter as gap distances increased. When we analyzed incongruent gap trials separately we observed that slope estimates were not statistically significant, $\beta = -1239$, $t = -1.47$, $p = .15$ (Figure 2.15), however these results are based on a small number of incongruent gap trials (n = 4) that participants completed in the CC condition. When controlling for variation explained by distance, the model indicated that RTs decrease as gap distances increase, $\beta = -34.3$, $t = -2.39$, $p = .019$.

Discussion

Across two experiments, we tested five hypotheses to explore the similarities and differences between magnitude processing with symbolic fractions and nonsymbolic visually defined ratios. First, we consistently observed numerical distance effects (NDEs) in all symbolic and nonsymbolic conditions, indicating that response times and error rates varied as a function of the stimuli's holistic rational number magnitudes (H1). Second, we observed more efficient nonsymbolic processing relative to symbolic conditions and identified how nonsymbolic ratio processing may become slightly less efficient when presented in a more complex visual form. (H2). Third, we observed that adults can compare magnitudes of cross-format pairs as efficiently as comparing pairs within symbolic notation (H3), however biases exist in point of subjective equality adults see when comparing cross-format pairs (H4). Lastly, we observed that the complexity symbolic fraction stimuli can slow the speed of comparison performance.

Evidence for holistic magnitude processing

In the current experiments we aimed to evaluate the presence and nature of holistic magnitude processing across symbolic and nonsymbolic formats via an analysis NDEs. In both experiments we observed significant and negative NDE slopes in all symbolic, nonsymbolic and cross format conditions. Previous studies with adults and children have similarly observed holistic magnitude processing in the form of NDEs with symbolic fractions (Binzak & Hubbard, 2020; DeWolf et al., 2014; Meert et al., 2010). However, only a few studies have directly compared response patterns with symbolic fractions to those for nonsymbolic ratio processing within the same subjects and using the same comparison magnitudes (Kalra, Binzak, et al., 2020; Mock et al., 2018). The present study presents the first comparison of NDEs across formats with adults using a continuous and large range of numerical distances to provide more detailed insights into the nature of these response functions.

Results of the present experiments indicate that NDEs in RT were not equivalent across stimuli, pointing to differences in how efficiently and precisely people can access a holistic sense of magnitude from symbolic fractions and nonsymbolic ratios. Specifically, we observed that nonsymbolic magnitude processing can occur more efficiently than symbolic or cross-format processing and that cross-format processing is no less efficient that within-format symbolic processing (discussed in greater details in Sections 4.2 and 4.3 below). We further explored these differences, by examining ways that magnitude comparison with fractions may involve adaptive strategy use when there are multiple cues that people can use to make decisions (Fazio et al., 2016; Huber et al., 2014). For instance, the magnitude of fractions' and ratios' components as separate entities may provide information to bias magnitude comparison judgements in ways that reinforce or contradict information based solely on holistic magnitude processing (Bonato et al., 2007; Meert et al., 2009; Obersteiner & Tumpek, 2016; Toomarian & Hubbard, 2018).

Critically, follow up analyses of componential effects on within-format comparisons revealed that in almost all cases NDEs remained significant and negative even when controlling for component-based effects. Relative to nonsymbolic ratio processing, we observed that magnitude processing speed and NDE slopes were modulated by symbolic fraction component features to a greater degree. The strongest example of this was observed in Experiment 2, when multi-digit symbolic fraction comparisons did not show a significant NDE among pairs with incongruent gap distances. However, this observation was based on only four pairs in the current sample, which may explain the large standard errors observed around this slope estimate. Given the exploratory nature of these follow up analyses of componential effects, it is a matter of future research to disentangle how the difference in componential effects between symbolic and nonsymbolic processing stem from phenomena such as less confidence with fractions, less precise representation of holistic magnitude with fractions, or other entrenched whole-number biases. The presence of componential effects in the current data present further support for the arguments that multiple forms of magnitude processing with fractions and ratios may occur in concert with one another (Faulkenberry & Pierce, 2011; Fazio et al., 2016; Huber et al., 2014). Nevertheless, even when we account for the alternative effects of component congruency and gap distances, we observe that patterns processing time and accuracy are still associated with holistic magnitude processing.

Evidence for Efficient Ratio Processing

Proponents of the ratio processing system theory argue that human cognition is supported by an innate and biologically ancient perceptual system sensitive to the magnitudes of nonsymbolic ratios (Jacob et al., 2012; Lewis et al., 2015). Ratio processing can be described as a largely perceptual process by which estimates of relative magnitude can be made quickly and accurately, and without conversion to alternative (e.g. symbolic) representations (Matthews & Chesney, 2015). The results of the current experiments support the hypothesis that nonsymbolic ratio processing with visual stimuli is more efficient than analogous magnitude processing with symbolic fraction stimuli. Specifically, adults made accurate comparisons of line ratios and circle ratios faster than comparisons of symbolic fractions, and models of these effects based on the current data indicate that this efficiency holds whether the numeric value of stimuli are close or very distant.

One alternative explanation of efficient magnitude processing with nonsymbolic stimuli relative to symbolic fractions, is that continuously defined part-to-part ratios are visually more simplistic than symbolic fractions. Regarding how visual complexity may influence ratio processing, we observed in Experiment 2 that line ratio comparisons (LL) were made more quickly than circle ratio comparisons (CC). If we characterize circle ratios, defined by the relative size of their two-dimensional areas, as more complex than line ratios, defined by their one-dimensional line lengths, then these results support the hypothesis that nonsymbolic ratio processing is directly related to visual complexity. Future research is necessary specify the how features of visual complexity may affect the efficiency of nonsymbolic processing and how these effects manifest with different nonsymbolic stimuli (e.g. dot ratios).

Differences between nonsymbolic ratios and symbolic fractions may also be explained by the varying familiarity and experiences participants had with the stimuli. For instance, symbolic fractions may be more visually complex, but they are also representations of rational number magnitude which participants (college-aged adults) in this study have likely seen before. On the other hand, the nonsymbolic ratios used in these experiments were novel visual representations. Furthermore, these stimuli were carefully designed so that judgments needed to be based on intensive holistic ratio magnitudes rather than the extensive lengths of either component. Thus, it is interesting to see that adults still exhibited more efficient holistic magnitude processing with nonsymbolic stimuli. When we also consider the presence of component congruity and gap distance effects specific to symbolic fraction processing, it is possible that familiarity with symbolic fractions may bring addition prior knowledge and biases which may make holistic magnitude processing less efficient.

Processing of magnitude across representations

Beyond a comparison of symbolic fraction and nonsymbolic ratio processing within the same format, the current study observed ways that holistic magnitudes can be compared across these formats. Based on research observing comparisons of whole numbers in symbolic and nonsymbolic forms, some researchers have proposed that the underlying magnitude representations accessed from these separate external formats may become estranged (Lyons et al., 2012). Evidence supporting this conclusion came from the observation of translation costs, where additional cognitive processing to map between these separate formats may have made the processing time significantly greater than the time to compare magnitudes within the same format (400ms). Interestingly, the current experiments did not observe analogous translation costs when magnitude comparisons were made between symbolic fractions and nonsymbolic ratios. In fact, performance during to cross-format comparisons relative to within-format symbolic fraction comparison in Experiment 1 and Experiment 2 trended towards the opposite effect, where mixed comparisons were made faster on average than within-format symbolic comparisons. Consistent with previous arguments, these results indicate that the internal representations of magnitude accessed from these two distinct symbolic and nonsymbolic representations are not estranged but rather highly compatible or even convergent on a common magnitude code (Bonn & Cantlon, 2017; Matthews & Chesney, 2015).

The analyses of cross-format comparison in the present experiments also extends previous findings showing that the point of subjective equality (PSE) during these judgements is not symmetrical (Matthews & Chesney, 2015). Specifically, we observed biases in response patterns showing that participants on average made comparison judgements as if the magnitude of nonsymbolic stimuli was larger than the true visually defined magnitude. Furthermore, we observed that these PSE biases are stronger among cross-format comparisons with circle ratios than with line ratios. Further research focused specifically on these effects is necessary to disentangle how the magnitude of PSE biases can be explained by the visual complexity of nonsymbolic stimuli (e.g. 2D areas vs 1D lengths), the overall size of stimuli (e.g. narrow lines vs larger surfaces). Based on our findings, we argue that either visual attribute may introduce additional cues during visual processing to create biases of over estimation within nonsymbolic ratios.

Conclusion & Future Directions

The findings of this study illustrate multiple ways that symbolic fraction and nonsymbolic ratio processing are both similar and distinct. The observations of significant NDEs in symbolic, nonsymbolic, and cross-format comparisons suggests that adults can engage in analogous forms of holistic magnitude processing based on the relationship between fractions' and ratios' component parts. Furthermore, observations of successful magnitude processing across symbolic and nonsymbolic formats provides further evidence that format specific processes can converge on highly compatible magnitude representations or even a common magnitude code. The findings of this study are consistent with arguments that the ability to represent one magnitude relative to others creates a generalizable ratio code that supports magnitude processing across different dimensions and sensory modalities (Bonn & Cantlon, 2017).

Beyond questions of *if* symbolic and nonsymbolic processing are associated, further research is necessary to address *how* these abilities develop over time and experience. Previous findings indicate that nonsymbolic ratio processing abilities emerge prior to formal fractions instruction (Duffy et al., 2005; McCrink & Wynn, 2007) and can predict individual differences in future fraction knowledge (Hansen et al., 2015; Jordan et al., 2013). Thus, we have argued that perceptual experience with nonsymbolic ratios may develop foundational systems upon which educational approaches may ground students' sense of symbolic fraction magnitudes (Kalra, Binzak, et al., 2020; Lewis et al., 2015). The current findings are compatible with this developmental theory, but further research is necessary to fully understand the developmental and educational implications our results. Specifically, training studies and longitudinal approaches are necessary to observe how forms of symbolic and nonsymbolic magnitude processing develop over time and in response to response to educational experiences. If active experiences with perceptual representations of ratio relationships can ground a conceptual understanding of fraction magnitudes (Lewis et al., 2015; Matthews et al., 2016; Sidney et al., 2017), then continued research may identify novel instructional approaches to advance this knowledge. Working towards this goal stands to advance the theories of numerical cognition with rational numbers and human abilities to understand complex numerical symbols.

Acknowledgements

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Chapter 3 – Neural Associations of Magnitude Processing with Fractions and Ratios

Introduction

Fractions are an important topic for developing higher order mathematical understanding (Booth et al., 2014; Siegler et al., 2013), yet much remains unknown about how the mind represents the meaning of fractions. Efforts to explain how people encode the meaning of symbolic fractions have led to several formally articulated theories regarding how this meaning is accessed and represented in the brain. In this study, we aimed to explore predictions of Ratio Processing System (RPS) theory (Jacob et al., 2012; Lewis et al., 2015). This theory proposes that humans develop intuitive abilities to perceive and distinguish ratios in the world (e.g. the relative height of two trees, the proportion of green marbles in a jar, etc.), and the neurocognitive systems supporting ratio processing support this sense for ratios. Furthermore, this theory proposes that perceptual and embodied experiences with ratios over time (and the corresponding neural systems) are foundational for understanding the symbolic ratio magnitudes that are commonly encountered in formal school environments and daily adult life (e.g. common fractions ½, ¼). One major prediction of this theory is that an understanding of fractions as holistic (relationally defined) can be grounded in a perceptually based sense of nonsymbolic ratio processing

In support of this theorized relationship between symbolic fraction and nonsymbolic ratio processing, previous research as observed that individual differences in ability to compare nonsymbolic ratios can be a significant predicter of performance on a (predominantly symbolic) fractions tests and even algebra achievement (Matthews et al., 2016). In the previous chapter of the dissertation, we observed that adults can compare holistic magnitudes between symbolic and nonsymbolic cross-format stimuli and can do so as efficiently as comparing fractions within the same format. However, measures of magnitude comparison behavior, such as response time (RT) and error rate (ER), are not sufficient to make strong claims about the internal representations of magnitude meaning that people access from symbolic and nonsymbolic stimuli.

Relatively few studies have examined the neural basis of rational number processing in humans (DeWolf et al., 2016; Ischebeck et al., 2009; Jacob & Nieder, 2009b, 2009a; Mock et al., 2018). Ischebeck et al. (2009) used a symbolic fraction comparison task to demonstrate that holistic distance between fractions inversely modulated activation in the right intraparietal sulcus (IPS), a region often implicated in numerical processing (Arsalidou & Taylor, 2011). Using an fMRI adaptation paradigm, Jacob and Nieder (2009a) observed that regions of the IPS were sensitive to magnitude changes in fractions regardless of whether deviant magnitudes were presented as symbolic fractions or fraction words (German). Using a similar paradigm, the same research group also observed evidence for neural adaptation and recovery in response to the magnitude of nonsymbolic visual part-to-part ratios in bilateral parietal and frontal cortex (Jacob & Nieder, 2009b). These results suggest that regions of the IPS are important for an understanding of fraction magnitudes and may encode this meaning in an amodal magnitude code underlying forms of magnitude understanding with various external representations including nonsymbolic visual ratios

The hypothesized format-independent ratio magnitude representation suggested by Jacob and Nieder (2009a) was further tested by DeWolf et al. (2016), in which they used fMRI during a magnitude comparison task with three different stimulus types (integers, symbolic fractions, and decimals). The inclusion of decimals in addition to fractions and integers allowed for a distinction to be made between number representation (base-10 numbers vs. fractions) and number type (natural vs. rational). By using both univariate and multivariate analysis approaches, the authors found that numerical distances in all formats modulated activity in the IPS. These findings implicate the IPS as a common neural substrate for both rational and natural numbers, though there may be some representational differences between base-10 numbers and symbolic fractions.

Notably, none of the studies described thus far have simultaneously investigated nonsymbolic and symbolic ratios. To address this limitation, a recent investigation by Mock, Huber et al. (2018) looked neural representations of rational number processing with symbolic (fractions and decimals) and nonsymbolic representations (pie charts and dot ratios) in the same group of participants. Among the widespread activation patterns observed for each format, they found overlap in the IPS for both symbolic and nonsymbolic ratio magnitudes. In characterizing this more widespread activation, the authors attribute activation in frontal areas to greater executive function/strategy use, and occipital regions as visual attention/encoding, implying an extended ratio processing network. However, the nonsymbolic stimuli used by Mock et al. (2018) included only two forms of nonsymbolic ratios: pie charts that show part whole relationships and integrated dot arrays that show discrete part-part relationships. Testing whether brain encodes format-independent ratio magnitude representations requires further investigation with different nonsymbolic stimuli.

Current Study

Previous studies have examined symbolic (DeWolf et al., 2016; Ischebeck et al., 2009; Jacob & Nieder, 2009a) and nonsymbolic (Jacob & Nieder, 2009b) separately. Notably, the only study to examine neural representations of symbolic and nonsymbolic ratios in the same participants have examined employed block designs to study the processing of magnitude comparisons within the same format (Mock et al., 2018). However, comparing symbolic and nonsymbolic processing as separate processes does not directly explore hypotheses regarding whether a common magnitude code is accessed or can be accessed across both external representations. In the current investigation, we specifically aimed to address this limitation by examining both magnitude comparison of symbolic and nonsymbolic fractions within format and the comparison of symbolic and nonsymbolic mixed pairs. Specifically, we employed a novel cross-format magnitude comparison task and functional neuroimaging to specifically test the RPS theory that the perception of nonsymbolic ratios and an understanding of a symbolic fraction's magnitude converges on common or compatible representations of magnitude meaning psychologically and physically in the brain.

Furthermore, we sought to explore the neural representations of nonsymbolic ratio processing with part-to-part ratios that are continuously defined (see Figure 3.1). Processing of this ratio form has been studied using an adaptation paradigm (Jacob & Nieder, 2009b) but not in studies using a magnitude comparison task. To examine how adults connect the meaning of nonsymbolic ratios to symbolic fractions, we included trials with cross-format magnitude comparisons. Whereas Mock et al. employed a blocked fMRI design, our event-related design allowed for direct comparison of cross-format ratio representations (i.e. a symbolic fraction directly compared to a line ratio). Using this approach, we conducted the first investigation of the neural representations of symbolic fraction and continuous nonsymbolic ratio magnitude processing within and across formats.

Methods

Participants

Twenty-six young adults from the University of Wisconsin-Madison community participated in the experiment. All participants were right-handed, native English-speaking adults with normal or corrected vision and no history of psychiatric, neurological, or developmental disorders. Participants gave written consent to complete the experimental procedure approved by the biomedical research ethics committee of the University of Wisconsin – Madison (IRB # 2013-1607) and received compensation for their participation ($50). One participant was excluded from analysis due to excessive scanner movement (movements between functional images greater than 1.75mm in more than half of the functional scans) and another due to a technical difficulty with the scanner. The average age of the remaining 24 participants (16 female) was 22 years old (SD = 3 years and 2 months).

Design and Procedure

The study consisted of one fMRI session. After providing consent and completing an MRI safety screening, participants received instructions on the experimental task and completed 12 practice trials (different from the experimental trials) outside of the scanner. Participants were given the instructions again and a second attempt at the practice trials if they made 5 or more errors on their first attempt. No participants failed to meet this criterion after the second attempt. Participants then proceeded to the MRI scan room to begin the experiment. MRI scanning began with the acquisition of a high-resolution anatomical image of the whole brain, followed by the acquisition of fMRI data during the experimental task.

Experimental Task

To investigate the neural activity associated with how adults process magnitude meaning within and across symbolic fractions (e.g. ½) and nonsymbolic ratios (e.g. the relative length of two lines), participants completed a cross-format magnitude comparison task in MRI scanner. This cross-format magnitude comparison task was directly adapted from the paradigm used in Experiment 1 of Chapter 2. Specifically, we measured response times, accuracy, and neural data while participants made magnitude comparison judgements in three different comparison *formats* (shown in Figure 3.1): symbolic fraction (FF), nonsymbolic line ratio (LL), and cross-format fraction-line (FL).

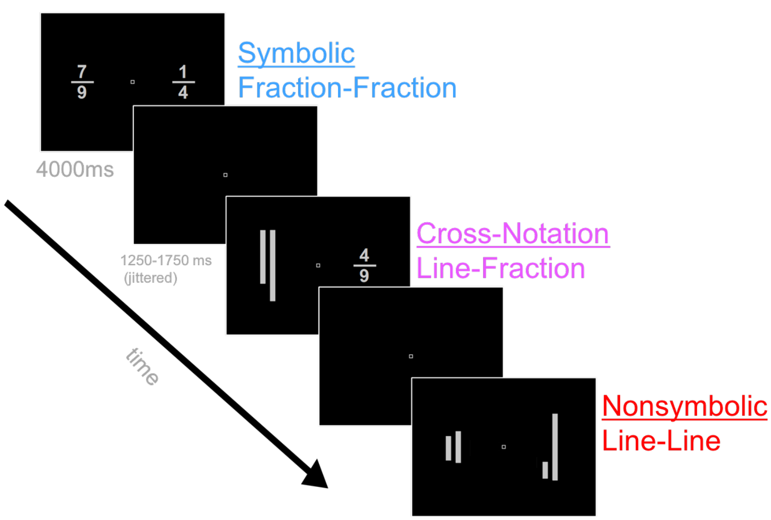

Figure 3.1. Cross-format magnitude comparison task stimuli. Three format conditions of symbolic fractions, nonsymbolic line ratios, and cross-format comparisons were intermixed and presented sequentially.

In order to observe the effect of magnitude processing on performance and associated brain activity, the 36 trials presented in each run also varied in terms of their *numerical distance*, or the absolute value of the difference between magnitudes of the comparison pair stimuli. The magnitudes of numerical distances across the 36 pairs ranged from near (min = 2/7 vs. 1/3 = 0.048) to far comparisons (max = 1/8 vs 7/8 = 0.75). To facilitate our analysis of neural activity associated magnitude processing, the 36 numerical distances presented in our fraction pairs were evenly split into three bins: near, medium, and far comparisons. (See *Table 3.1*).

Table 3.1

Distribution of absolute numerical distances among comparison pairs

	Trials/Block	Min	Max	Mean
Near	12	0.048	0.233	0.144
Medium	12	0.262	0.446	0.341
Far	12	0.514	0.750	0.613

Functional data associated with the magnitude comparison task in each format and distance bin was acquired using a 3 x 3 event-related design. Specifically, six experimental runs presented trials with the same 36 numerical distances presented in one of the three comparison formats (symbolic, nonsymbolic, or mixed). Across the whole experiment (six blocks) each participant completed each of the 36 comparisons twice in each format, once with the larger fraction on the right and the other on the left. Since cross-format trials can be presented in four possible combinations of larger-smaller and symbolic-nonsymbolic orientations, half of the participants completed blocks with two of the possible combinations (e.g. $1/5_{\text{Nonsymbolic}}$ □ $3/4_{\text{Symbolic}}$ & $3/4_{\text{Symbolic}}$ □ $1/5_{\text{Nonsymbolic}}$) and the other half completed blocks of the other two possible combinations (e.g. $1/5_{\text{Symbolic}}$ □ $3/4_{\text{Nonsymbolic}}$ & $3/4_{\text{Nonsymbolic}}$ □ $1/5_{\text{Symbolic}}$). The order of the blocks, which varied the format and orientation of each comparison pair, was counterbalanced across participants.

Fraction Stimuli

Symbolic fraction stimuli in the study included all 27 unique single-digit irreducible fractions. From all possible paired comparisons of these fractions, a sample of 36 pairs was selected to maximize the diversity of unique fractions sampled in each distance bin and to minimize the participant's reliance on any one componential comparison strategy. Specifically, comparison pairs included a balanced selection of fraction pairs that shared the same denominator (common denominator), pairs where the larger fraction had a larger numerator and a smaller denominator than the smaller fraction (congruent numerator and incongruent denominator), the larger fraction had a larger numerator and denominator than the smaller fraction (congruent numerator and denominator), and the larger fraction had a smaller numerator and denominator than the smaller fraction (incongruent numerator). Comparison pairs at far distances, however, did not include any numerator incongruent trials because no such pairs exist at distances greater than .306.

Line Ratio Stimuli

Line ratios were composed of two vertical lines representing a part-part ratio of the line length on the left relative to the line length on the right (See Figure 3.1). Using the list of 36 unique comparison pairs chosen for the symbolic stimuli, two sets of line ratio stimuli were generated for all single digit irreducible fraction values using the same procedure as described Chapter 1. Two sets of stimuli were created to minimize the correlations between numerator line lengths and holistic magnitude in one set and denominator-holistic magnitude correlations in the other set. For each participant, blocks used numerator-controlled and denominator controlled nonsymbolic stimuli in alternating order.

Task timing parameters

Each participant's full set of functional data was acquired over six sequential scans (blocks) lasting 3 minutes and 40 seconds. The start of each functional block was synched to first TR of the fMRI sequence via a TTL pulse. Each block began with 10 seconds (5 TRs) of a waiting screen, to allow the magnetic field to stabilize prior to data collection

Trials were presented for 4000ms and participants could enter their responses during that time. In between each trial a light grey fixation box was presented in the center of the screen. Intertrial-intervals (ITI) were jittered around a mean duration of 1500ms (range = 1250 – 1750ms).

Apparatus

Experimental stimuli were presented in light gray text on a black screen mounted at the end of the MRI machine bore. Participants were able to see the screen using a mirror mounted on the scanner's head coil above the participant. The presentation of stimuli was controlled from outside of the scanner room using on a Windows 8 PC to run E-prime 2.0.8.90a (Psychology Software Tools, Shapsburg, PA). Participants laid in the supine position in the MRI scanner bore and were given a button box to respond to the stimuli using index and middle finger button presses. Participants were instructed to indicate as quickly and as accurately as possible whether the larger of two fractions presented side-by-side appeared on the left (index finger press) or the right (middle finger press) side of the screen.

MRI Data Acquisition

Brain imaging was conducted using a General Electric 3-Tesla MRI scanner (GE Medical Systems, Waukesha, WI) equipped with a 32-channel array head coil (Nova Medical). First, the brain anatomy of each participant was collected with high-resolution T1-weighted anatomical images (3-D T1-weighted inversion recovery fast gradient-echo; 256 x 256 in-plane resolution; 256mm FOV; 176 axial slices, 1mm thickness). Next, whole-brain functional images of the BOLD signal were acquired using a T2-weighted echo planar imaging sequence (38 sagittal slices, 3mm thickness; 128x128 matrix; 224x224 mm field of view (FOV); repetition time (TR)/echo time (TE)/Flip, 2000ms/22ms/75°; voxel size of 1.75x1.75x3mm). Images were collected in ascending-interleaved order. Acquisition resulted in 120 volumes (full brain images) for each functional run.

Imaging analysis

MRI Preprocessing. Structural and functional brain imaging analysis was conducted using Brain Voyager QX 2.8 (Brain Innovation, Mastrich, The Netherlands). Each individuals imaging data underwent the following preprocessing procedure. The first 5 volumes of each functional run, during which participants waited for the task to begin, were excluded to account for the stabilization of magnetic saturation. Functional images were then corrected for differences in slice time acquisition using a sinc interpolation algorithm (ascending-interleaved), adjusted for head motion using trilinear sinc interpolation, and images were cleaned of low-frequency noise using a high-pass temporal filter (GLM-Fourier) with a cut off of 2 sines/cosines cycles. Functional images were then coregistered with the T1-anatomical anatomical image, smoothed with an 8mm full width at half maximum Gaussian smoothing kernel, and transformed into Talairach Space (Talairach & Tournoux, 1988). fMRI runs were excluded from a participant's dataset if the run included a head movement greater than 1.75mm between functional volumes, and participants were excluded if this occurred in 3 or more of the 6 functional runs.

Analysis Contrasts

Neural distance effects across formats. An initial whole-brain analysis was conducted to explore the regions of the brain sensitive to changes in the holistic magnitudes of the comparison pairs. We conducted a random effects analysis on the whole-brain BOLD signal to identify regions of the brain showing greater activity during comparisons of near distances relative to far distances. This pattern of activation is referred to as the *neural distance effect*. Next, we conducted the same contrast within each format separately to identify regions of the brain where neural distance effects are observed specific to each format and where significant regions overlap across formats. We then conducted two conjunction analyses to statistically confirm which regions were recruited similarly across formats.

Format-specific neural distance effects. We conducted a series of contrasts between formats to characterize the difference in magnitude processing between conditions. First, we examined regions where distance effects during symbolic fraction comparison were greater than nonsymbolic line ratio comparison, and regions where distance effects during nonsymbolic comparison was greater than symbolic. Second, we examined regions where cross-format comparisons evoked stronger neural distance effects than within-format comparisons (Fraction-Fraction and Line-Line).

Cluster threshold corrections for multiple comparisons. All contrast analyses were initially run with a statistical threshold of α = 0.001. All resulting statistical maps were further corrected for multiple comparisons using Brain Voyager's cluster level statistical threshold estimator (Goebel et al., 2006) to avoid false positive results and invalid cluster inferences (Eklund et al., 2016). Based on the assumption that areas of true neural activity will be observed in signal changes within contiguous voxels (Forman et al., 1995), we ran Monte Carlo simulations of whole-brain neural activity to estimate cluster-level false positive rates. After 1000 iterations, we identified the minimum cluster size with a false positive rate (α) below 0.05 and applied this cluster threshold to our contrast and conjunction analyses. Specifically, significant clusters of neural activation composed of fewer voxels than the cluster level threshold were excluded from our final results.

ROI Analysis

Two regions of the brain commonly implicated in the processing of number magnitudes are the right and left the intraparietal sulci (IPS) (Ashkenazi et al., 2008; Dehaene et al., 2003; Piazza et al., 2004). In a meta-analysis of neural imaging studies Arsalidou and Taylor (2011) identified regions of the brain which were the most commonly associated with numerical processing and mental calculation. Results of this meta-analysis showed that the left and right inferior parietal lobe, specifically within the IPS, were among the regions with the highest activation likelihood estimation. The studies included in this meta-analysis were largely focused on whole number processing, and in our analyses we tested whether the IPS regions identified by Arsalidou and Taylor (2011) were also sensitive to numerical processing with fractions and ratios.

To measure the neural activity from these two a priori regions of interest, we centered two 10mm x 10mm x 10mm cubes around the two IPS coordinates identified by Arsalidou and Taylor (2011). From these regions we extracted the average BOLD signal for each participant at each cross section of the experiment's 3 format (Fraction-Fraction, Line-Line, Cross-format) x 3 distance bin (Near, Medium, Far) design. Using these numeric values, we conducted a 3 x 3 within-subjects ANOVA and planned pairwise comparisons to evaluate if the neural signal within these ROI regions increased as comparison distances became smaller for all three formats.

Results

Behavioral Results

Effect of Format and Distance on Reaction Times

We analyzed the effect of format and distance on response times using a 3 format x 3 distance bin within-subjects ANOVA (Type 3)[1]. Results revealed a significant main effect of format, $F(2,46) = 72.4$, $p < .001$, $\eta_p^2 = .86$. a significant main effect of distance, $F(2,46) = 147.0$, $p < .001$, $\eta_p^2 = .76$, and a significant interaction between these effects, $F(4,92) = 8.30$, $p < .001$, $\eta_p^2 = .27$. Group mean RTs of each format and distance are presented in Figure 3.2. Pairwise comparisons within each format revealed that RTs showed a significant numerical distance effect, in which RTs became significantly longer at medium distances than far distances, and significantly longer at near distances than medium distances. Pairwise comparisons within each distance bin revealed that participants were always fastest to

[1] Distance effects were analyzed using an ANOVA approach with binned data to be consistent with the planned fMRI analyses.

respond to nonsymbolic line ratios. No differences were observed between RTs during symbolic and cross-format comparisons and near and far distances, but at medium distances cross-format comparisons were slightly faster on average than symbolic comparisons.

Effect of Format and Distance on Error Rates.

We analyzed the effect of format and distance on average error rates using a 3 format x 3 distance bin within-subjects ANOVA (Type 3). Results revealed error rates varied across distance bins, $F(2,46) = 76.7$, $p < .001$, $\eta p2 = .77$, but no differences were observed formats, $F(4,92) = 1.38$, $p = .262$, $\eta p2 = .06$. Group mean RTs of each format and distance are presented in Figure 3.2.. Pairwise comparisons within each format revealed significant numerical distance effect in mean ERs, in which errors were significantly lower at medium distances than near distances. ERs and medium and far distances were very low, and no significant differences were observed between these distance bins in any of the comparison formats.

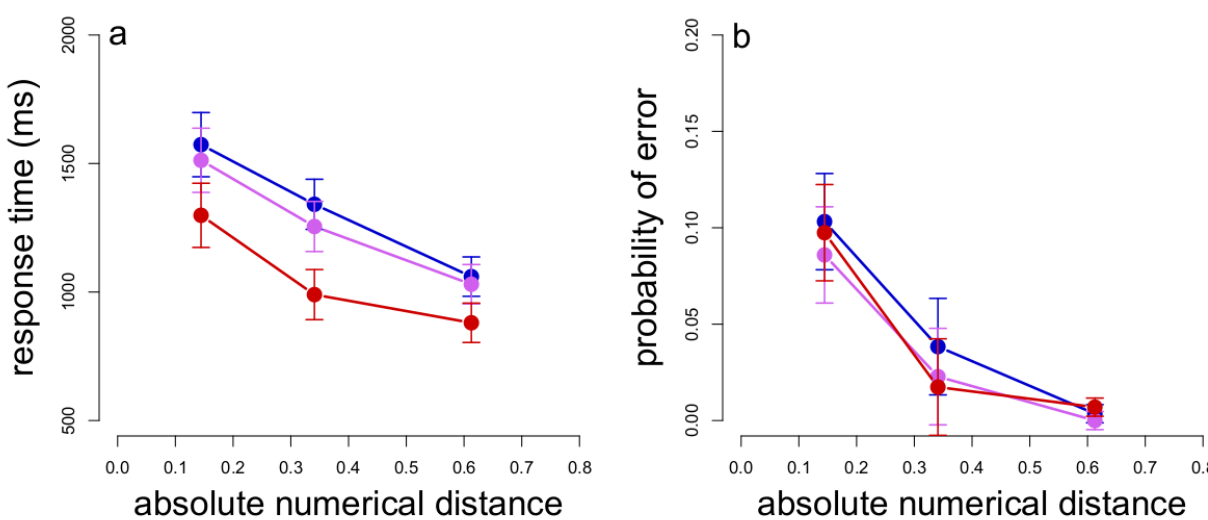

Figure 3.2. Group mean response times (a) and error rates (b) for symbolic fraction (blue), nonsymbolic ratio (red) and mixed comparisons (magenta). Means are binned at near, medium, and far numerical distances and centered over the mean distances within each bin.

Imaging Results

Whole brain near vs far contrasts.

In our first analysis we analyzed the regions of the brain which were modulated by holistic magnitude distances of the comparison pairs without differentiating regions specific to each of the three formats. Specifically, we conducted a whole-brain voxel-wise t-test to examine which areas of the brain were significantly modulated by the manipulation of numerical distances, as seen in the contrast of activity associated with near and far magnitude comparisons.

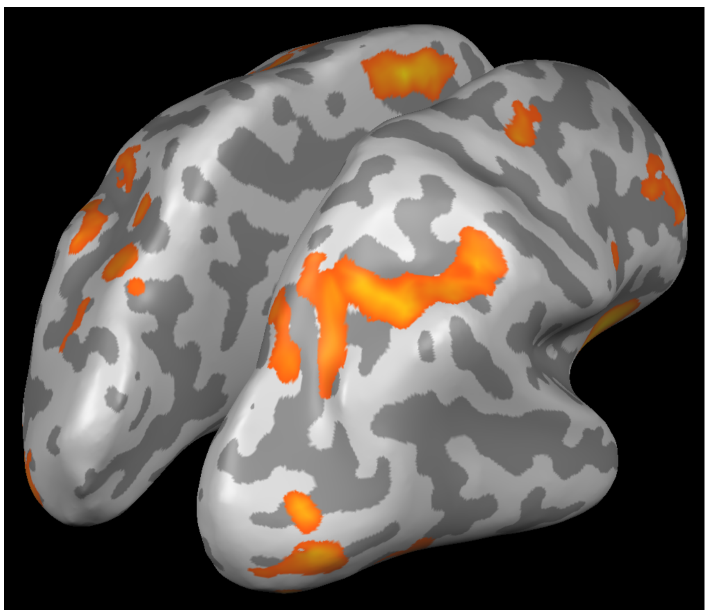

Figure 3.3. Regions showing significant neural distance effects (near activation > far activation) using all formats on an inflated brain.

Results of this analysis revealed several large regions across the parietal, frontal and occipital lobes which showed significantly greater activity during near comparisons relative to far comparisons. All regions where significant clusters of activation were observed and the location of peak near-far differences within these regions are presented in the Appendix C. These significant regions included large regions of bilateral activation along the intraparietal sulci, bilateral frontal lobe activation of the anterior insula, lateral and inferior regions of the occipital lobe (bilateral) including the left fusiform gyrus, and the premotor area.

Conjunction of neural distance effects across formats.

Analysis of neural distance effects observed within each comparison format revealed multiple regions specific to each format showing greater activity during near comparisons relative to far comparisons. As seen in Figure 3.4, patterns of significant neural distance effects within each format have unique regions sensitive to changes in comparison magnitudes and regions where effects overlap across formats.

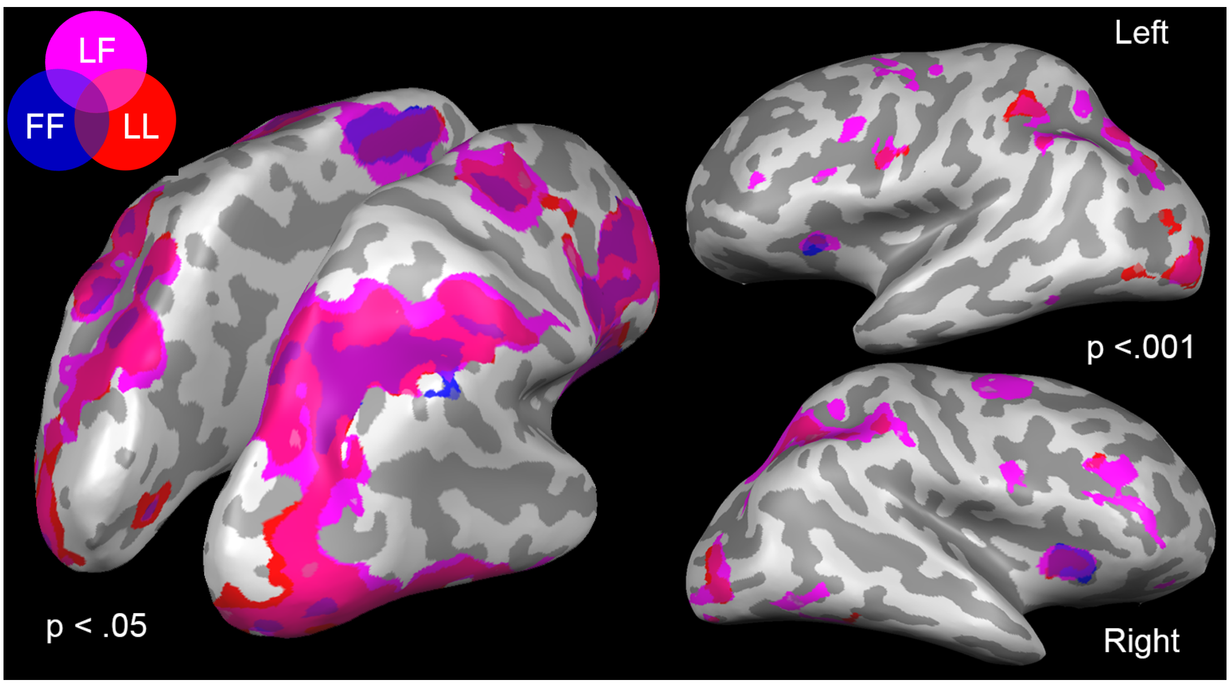

Figure 3.4. Regions showing significant neural distance effects (near activation > far activation) during symbolic (blue), nonsymbolic (red) and cross-format (magenta) comparisons visualized on an inflated brain. (a) Canted view from the posterior right side of the brain shows regions of significant distance effects at $\alpha = 0.05$, to illustrate the broad extent of regions sensitive to changes in magnitude. (b/c) Lateral views of the left and right hemisphere show regions of significant distance effects at $\alpha = 0.001$, highlight regions showing where effects survived strict statistical cutoffs to correct for multiple comparisons.

Conjunction analyses of neural distances effects in within-format symbolic and nonsymbolic conditions revealed seven neural regions where activity was significantly modulated by the magnitude of comparison pairs in both conditions. These regions included the left and right superior parietal lobule, right anterior insula, right inferior frontal gyrus, left ventrolateral PFC, and bilateral supplementary motor area. When cross-format comparisons were added to the conjunction analysis, four neural regions showed shared significant distance effects in all three conditions. Specifically, these regions included the right superior parietal lobule, right anterior insula, and bilateral supplementary motor area.

Table 3.2

Regions modulated by neural distances across formats

			FF + LL				FF + LL + FL			
Hem.	Region	BA	x	y	z	t	x	y	z	t
R	(IPS) superior parietal lobule	7	29	-56	36	4.57	29	-56	36	4.57
L	(IPS) superior parietal lobule	7	-28	-62	39	4.49				
R	inferior frontal gyrus	44	50	19	33	4.69				
R	Anterior Insula	13	32	16	6	5.30	32	16	6	5.30
L	ventrolateral PFC	46	-40	40	6	5.24				
R	supplementary motor area	8	5	16	48	5.92	5	16	48	5.92

Neural Localization of Contrasts across Format

We conducted whole-brain voxel-wise t-tests to examine which areas of the brain activated to a greater degree during symbolic fraction comparison than nonsymbolic fraction comparison, and which regions showed the opposite effect (Figure 3.5a). Results of this contrast analysis revealed several regions where neural activity was greater during symbolic magnitude comparisons than nonsymbolic magnitude comparison. Specifically, greater activation was observed during symbolic fraction comparisons in the right anterior insula, left inferior frontal gyrus, left fusiform gyrus (fusiform face area), left intraparietal sulcus, and the supplementary motor area. Results of this contrast also revealed several regions where neural activity was greater during nonsymbolic magnitude comparisons than symbolic magnitude comparisons. Specifically, greater activation was observed during nonsymbolic comparisons in the right inferior parietal lobule, right visual association area, parahippocampus, and superior occipital gyrus.

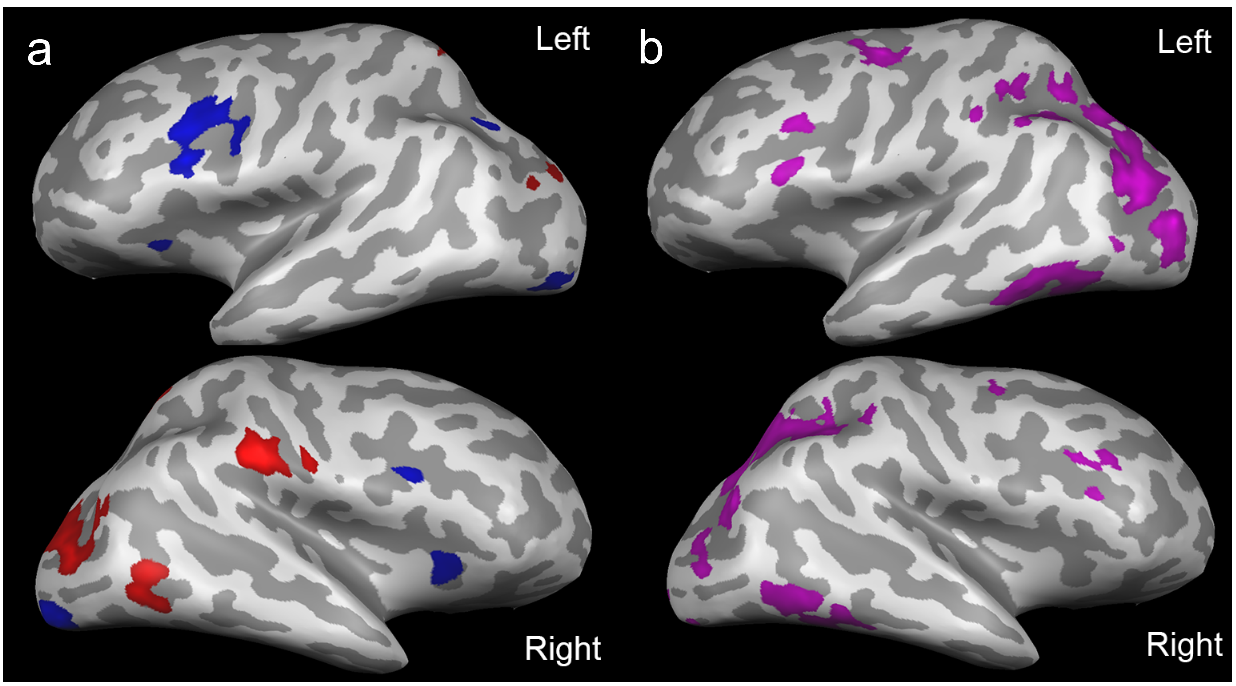

Figure 3.5 Regions showing different levels activity across format conditions. (a) Clusters of activation shown in blue depict regions of the brain with significantly greater activity during the comparison of symbolic fractions than activity during nonsymbolic ratios. Clusters shown in red depict regions showing significantly greater activation in the nonsymbolic comparison condition. (b) Clusters of activation shown in magenta depict regions of the brain with significantly greater activity during cross-format comparisons than within format comparisons. All regions are significant at $\alpha = 0.001$.

Neural Localization of Cross-Format Processing

We also conducted a whole-brain voxel wise t-test to examine which areas of the brain showed greater levels of activity during cross-format magnitude comparisons than when magnitudes were presented in the same format. This contrast revealed clusters across the brain which remain significant after corrections for multiple comparisons. Specifically, bilateral regions of the fusiform gyrus, left intraparietal sulcus, right inferior frontal gyrus, and bilateral primary motor cortex showed significantly greater activity during cross-format comparisons than within-format comparisons.

ROI Analysis of Neural Distance Effects in Bilateral IPS

To test whether a priori regions of the IPS showed sensitivity to changes in comparison pair magnitudes, we tested for the presence of distance effects across bins of near, medium, and far magnitude comparisons using a within-subjects ANOVA (Type 3). We conducted this analysis of distances bins within each format condition separately and included the *hemisphere* of the ROI as a within-subjects factor (left and right) in the ANOVA to examine whether distance effects differed in the left or right IPS. Locations of our a priori ROIs and mean beta weights (activation) extracted from these regions during magnitude comparisons across format and distance are presented in Figure 3.6.

ROI Distance Effects with Nonsymbolic Ratios.

Average IPS activation during nonsymbolic line ratio comparisons showed significant differences across distance bins, $F(2,46) = 15.7$, $p < .001$, $\eta_p^2 = .41$, but activation across hemispheres was not significantly different, $F(1,23) = 0.36$, $p = .556$, $\eta_p^2 = .02$. The interaction between distance and hemisphere was also not significant, $F(2,46) = 1.40$, $p = .256$, $\eta_p^2 = .06$.

Pairwise comparisons of bins revealed that in the left IPS ROI, activity was significantly greater during near comparisons (M = 0.29, SD = 0.21) than during medium comparisons (M = 0.17, SD = 0.15), $t(46) = 3.84$, $p = .001$, and far comparisons (M = 0.12, SD = 0.18), $t(46) = 5.20$, $p < .001$. Activity in medium and far comparisons was not significantly different, $t(46) = 1.36$, $p = .369$.

The same pattern of distance effects was observed in the right IPS ROI. Activity was significantly greater during near comparisons (M = 0.29, SD = 0.23) than during medium comparisons (M = 0.14, SD = 0.18), $t(54.5) = 4.41$, $p < .001$, and far comparisons (M = 0.10, SD = 0.21), $t(54.5) = 5.48$, $p < .001$. Activity in medium and far comparisons was not significantly different, $t(54.5) = 1.07$, $p = .892$.

ROI Distance Effects with Symbolic Fractions.

Average IPS activation during symbolic fraction comparisons also showed significant differences across distance bins, $F(2,46) = 4.06$, $p = .024$, $\eta_p^2 = .15$, and a significant difference across hemispheres, $F(1,23) = 4.73$, $p = .040$, $\eta_p^2 = .17$. The interaction between distance and hemisphere was not significant, $F(2,46) = 1.07$, $p = .353$, $\eta_p^2 = .04$.

Pairwise comparisons of bins revealed that in the left IPS ROI, activity was not significantly greater during near comparisons (M = 0.21, SD = 0.17) than during medium comparisons (M = 0.17, SD = 0.16), $t(46) = 1.48$, $p = .311$ or during far comparisons (M = 0.14, SD = 0.18), $t(46) = 2.32$, $p = .063$. Activity in medium and far comparisons was not significantly different, $t(46) = 0.84$, $p = .680$. The lack of distance effects in the left IPS cannot be attributed to a lack of neural activation in this regions, as activation during symbolic comparisons (collapsed across bins) was in fact greater in the left IPS ROI than in the right ROI, $t(23) = 2.17$, $p = 0.043$, but not modulated by the magnitudes of the comparison pairs.

Pairwise comparisons of bins revealed a different pattern of significant distance effects in the right IPS ROI. Activity was significantly greater during near comparisons (M = 0.17, SD = 0.19) than during medium comparisons (M = 0.10, SD = 0.15), $t(46) = 2.4$, p = .049, and far comparisons (M = 0.09, SD = 0.20), $t(46) = 2.74$, $p = .023$. Activity in medium and far comparisons was not significantly different, $t(46) = 0.30$, $p = .951$.

ROI Distance Effects with Cross-Format Comparisons.

Average IPS activation during cross-format comparisons showed a pattern of results similar to nonsymbolic ratio comparisons, with significant differences across distance bins, $F(2,46) = 30.9$, $p < .001$, $\eta_p^2 = .57$, and no significant difference across hemispheres, $F(1,23) = 1.61$, $p = .217$, $\eta_p^2 = .07$. The interaction between distance and hemisphere was also not significant, $F(2,46) = 2.94$, $p = .063$, $\eta_p^2 = .11$. Pairwise comparisons of bins revealed that in the left IPS ROI, activity was significantly greater during near comparisons (M = 0.32, SD = 0.18) than during medium comparisons (M = 0.23, SD = 0.15), $t(46) = 3.63$, $p = .002$, and far comparisons (M = 0.15, SD = 0.17), $t(46) = 6.29$, $p < .001$. Activity curing medium comparisons was also greater than during far comparisons, $t(46) = 2.66$, $p = .029$. In the right IPS ROI, activity also varied across all three distance bins in the form of typical distance effects. Activity was significantly greater during near comparisons (M = 0.31, SD = 0.22) than during medium comparisons (M = 0.19, SD = 0.18), $t(67.7) = 4.50$, $p < .001$, and far comparisons (M = 0.10, SD = 0.18), $t(67.7) = 7.65$, $p < .001$. Activity during medium comparisons was significantly greater than far comparisons, $t(67.7) = 3.15$, $p = .008$.

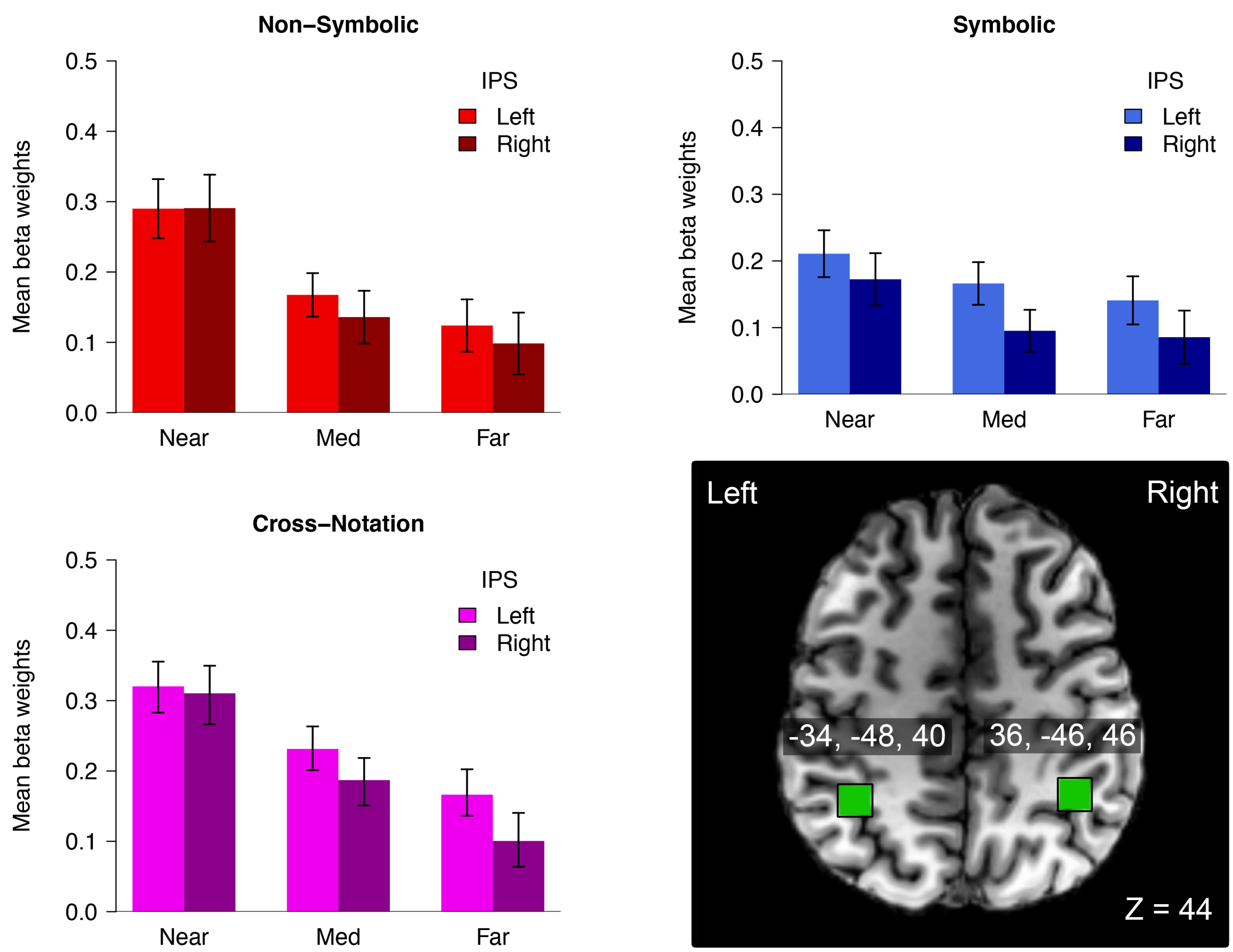

Figure 3.6. Mean beta weights observed within a priori regions of interest centered on IPS coordinates identified by (Arsalidou & Taylor, 2011). Different patterns of activation are shown for nonsymbolic (red), symbolic (blue), and cross-format comparisons (magenta).

Discussion

In this study we examined the neural representation of magnitude processing with symbolic fractions, nonsymbolic ratios, and cross-format pairs, to further explore theories that specific regions of the brain support magnitude processing independent of the external format. Few studies have previously explored the neural representations symbolic fraction magnitudes (DeWolf et al., 2016; Ischebeck et al., 2009; Jacob & Nieder, 2009a) and nonsymbolic ratio magnitudes (Jacob & Nieder, 2009b). These investigations using varying paradigms across separate populations consistently implicated a frontoparietal network of brain regions sensitive to the magnitudes of symbolic and nonsymbolic stimuli. Only one previous study has directly tested the argument that symbolic and nonsymbolic forms of rational numbers share common neural representations of magnitude within the same participants (Mock et al., 2018). The present experiment presents a novel extension of this research. Specifically, this is the first study to compare the neural representations of symbolic fraction to nonsymbolic ratio processing, when those ratios are presented as continuously defined part-to-part line ratios. Furthermore, this is the first study to examine the nature of neural representations supporting magnitude process across symbolic and nonsymbolic formats. Our findings are largely consistent with these previous investigations, and further implicate frontoparietal regions of the brain as locations where format-independent representations rational number meaning may reside.

Behavioral results showed significant response time and error rate distance effects in all three formats, replicating previous behavioral findings presented in Study 1 of this dissertation. These findings suggest that responses to magnitude comparisons made by participants in this study involved holistic magnitude processing of symbolic, nonsymbolic and cross-format stimuli. These findings provide additional evidence that our observed neural effects correspond to the underlying subprocesses of involved in accurately mapping external symbolic and nonsymbolic stimuli to corresponding representations of holistic magnitude.

In comparing the neural localization of distance effects specific to processing symbolic and nonsymbolic (excluding cross-format comparisons) we identified a set of regions similarly modulated by magnitudes in both formats. Specifically, this included bilateral regions of the IPS in the superior parietal lobule and frontal regions of the right inferior frontal gyrus, right anterior insula, and the left ventrolateral PFC. All of these regions, with the exception of the left ventrolateral PFC, are included in the regions Ischebeck et al. (2009) describes as the *fraction comparison network*. Furthermore, DeWolf et al. (2016) observed that these regions show greater activation to symbolic fractions relative to symbolic whole numbers. The current finding that these regions are also activated during magnitude processing of nonsymbolic ratios provides new evidence that aspects of magnitude processing localized in these regions may be format independent.

Interestingly, we identified regions showing common distance effects in our conjunction analysis in the frontal cortex, which previous studies of symbolic and nonsymbolic fractions did not observe. Conversely, we did not observe common distance effects among occipital regions which had been previously observed (Mock et al., 2018). What remained consistent across these studies was the identification of the right intraparietal sulcus as a region sensitive to changes in magnitude independent of the format presented. The IPS has often been implicated magnitude processing with symbolic whole numbers and nonsymbolic quantities (Ashkenazi et al., 2008; Dehaene et al., 2003; Fias et al., 2003; Holloway & Ansari, 2009; Piazza et al., 2004). Furthermore, the co-localization of regions supporting numerical and spatial processing in the IPS have spurred to arguments that the two processes share a common amodal (generalized) magnitude code (Hubbard et al., 2005; Walsh, 2003). Findings of the current study present new evidence that these theories specific to whole number magnitude processing should expand to the explain an understanding of rational number magnitude represented in symbolic or nonsymbolic forms (Bonn & Cantlon, 2017; Jacob et al., 2012; Lewis et al., 2015).

Our a priori regions of interest, based on IPS coordinates commonly associated with numerical processing with whole numbers (Arsalidou & Taylor, 2011), were located at slightly more anterior regions of the IPS than the regions identified by our whole-brain conjunction analysis. Our analyses of distance effects within these a priori ROIs in the IPS revealed a pattern of activation consistent with our whole-brain contrasts. Specifically, when we examined differences in activity across bins within each format, we observed distance effects in left and right IPS ROIs during nonsymbolic line ratio comparison and cross-format comparisons. However, during symbolic fraction comparisons, distance effects were only observed in the right IPS ROI. The nonsignificant neural modulation of activity in left IPS during symbolic fraction comparison at the whole-brain level (significant only at liberal thresholds, see Figure 3.4) and within the a priori ROI, does not necessarily mean that this region was not activated during these comparisons. In fact, relative to activity during nonsymbolic line ratio comparisons, our whole brain contrast revealed a cluster of parietal activation near the ROI where the left IPS showed greater activation during symbolic comparisons.

Results of our conjunction and ROI analyses implicate regions of shared magnitude processing and support the theory that representations of magnitude in these regions may not be format specific. However, conducting these group-level contrasts included spatial smoothing to account for individual differences in neural anatomy. This approach, however, cannot rule out the possibility that distinct format specific populations of neurons may coexist within similar regions of the cortex (Dehaene et al., 1998). Future investigations exploring hypotheses of format-independent representations of magnitude and corresponding mechanisms supporting magnitude processing will benefit from methods which assess the similarity of neural signals at the level of neural patterns, rather than the coarse localization of voxel-wise contrasts. Additional analysis at the subject level with unsmoothed fMRI day may also allow us to examine these representations and their convergence across individuals at a higher resolution.

Beyond theories that magnitude processing with fractions may share common or compatible magnitude representations as nonsymbolic ratio processing, proponents of the Ratio Processing System argue that such compatibility may support fractions learning. Future studies using the same magnitude comparison paradigm may be able to test the developmental and educational validity of this theory through the use of longitudinal and intervention experiments. By tracking the development of neural regions sensitive to magnitude processing in symbolic and nonsymbolic formats across years of primary education, these studies may test the assumption that magnitude knowledge with symbolic fractions builds upon regions specialized for the processing of nonsymbolic ratios. Furthermore, intervention studies will be necessary to determine how neural specialization may be driven by specific educational experiences.

Chapter 4 –The potential role of video games as tools to study math cognition

Introduction

Video games have permeated our culture and drawn the attention of many children and adults around the world. The ways that video games engage players' interest, attention, and resilience in the face of extraordinary challenges has inspired educators and researchers to see the potential for video games to be valuable learning tools. In studying whether and how educational game experiences achieve learning goals, there also lies a potential for video games to be used as the tools with which educational and cognitive psychology researchers can test hypotheses in settings beyond the research lab. In the following, I present a review of educational research with video games, with a specific focus on games designed for math learning. The goal of this review is to explore the idea that these designed experiences have specific value as a research tools to better understand the mechanisms of numerical cognition and math learning.

A large contingent of research on video games has focused on negative effects these experiences, such as whether violent games encouraged violent behavior (Anderson, 2004; Gentile et al., 2004), how sedentary behavior was related to incidence of obesity (Gordon-Larsen et al., 2000; Marshall et al., 2004), or how excessive gameplay habits can be detrimental to academic progress (Rideout et al., 2005) or become addictive behavior (Andreassen et al., 2016). However, some have argued that this focus on negative consequences of video games has shifted empirical discourse too far towards overgeneralizing the negative consequences playing video games (Ferguson et al., 2011). Critical reviews of the literature on violent video games have revealed inconsistent evidence for the commonly assumed negative consequences, but have more consistently revealed positive cognitive effects, such as improved spatial attention and visual acuity (Bejjanki et al., 2014; Ferguson, 2007; Green & Bavelier, 2003). In efforts to get more players out of their gaming chairs, more video games are utilizing artificial reality (AR) and motion capture mechanics to make video game play an active full body experiences (e.g. PokemonGo, Ring Fit Adventure, Beat Saber). Others researchers have identified many ways that video games are in fact excellent learning contexts, and effective theories of learning in video games closely demonstrate the theories of learning upheld by researchers in the field of cognitive science (Gee, 2003a).

Video game designers and educators alike have also taken notice of how video game players become engaged by the gaming experience and asked if it is possible for students to be as engaged with formal educational content presented in the form of an educational game (Gee, 2007). This has led to the use of commercial games in formal education contexts (Squire, 2005) and the design of novel *educational games* with the intention for players to encounter specific educational content in play-based, entertaining, and interactive ways. With the proliferation of video games developed and utilized for formal and informal educational settings, has come an abundance of research studying the cognitive and affective effects of these experiences (Connolly et al., 2012; Hainey et al., 2016). From this work, some researchers have argued that educational games are not only phenomena to be studied, but also powerful tools for studying the psychological phenomena which occur when they are played. By advancing methods to interpret game data as markers for learning, games can be utilized as both interventions and assessments (Halverson & Owen, 2014). The potential to utilize educational games in learning science and cognitive science research extends across academic content areas, age groups and learning contexts (Shute & Rahimi, 2017). The current review explores how educational games have been utilized mathematical cognition research thus far and identifies valuable directions for future investigations.

In evaluating how video games can be used as tools for studying math cognition, this review explores three arguments in favor of this methodological approach. First, video game technology introduces possibilities for *stealth assessment* where gameplay data can unobtrusively be collected without breaking players engagement with the game or introducing pressures associated with being in a traditional context of cognitive assessment (Shute, 2011; Wideman et al., 2007). Second, the flexibility of math games as an audiovisual medium can help researchers to study the role of using concrete visual representations in supporting a deeper understanding of mathematics (S. A. Barab et al., 2010; Fey, 1989). Third, engaging math games create a learning context to study the role of informal play-based experiences in the formation of affective dispositions towards mathematics (Hoffman & Nadelson, 2010; Ke, 2008). In addressing these three arguments, this review will describe how “research-based recommendations” for the design of educational media may be better reframed as causal hypotheses which can be critically tested by observing their efficacy function in real educational environments (A. P. Goldin et al., 2014; Han et al., 2019; Rosenberg-Lee, 2018)

Math games as a means of stealth assessment

In support of empirical goals to understanding the cognitive and emotional mechanisms of learning math, the flexibility of the digital video game medium presents a wide diversity of opportunities to observe learning in contexts outside of the research lab.

Relative to traditional psychology experiments where participants come into the lab, video game data can be collected portably and at the participant's convenience. Furthermore, the development of data aggregators built within the programing of games can support the collection of data during the course of gameplay (Owen et al., 2014). Shute (2011) defines the term *stealth assessment* as a process of gathering data during the course of learning to not only measure abilities but also provide formative knowledge for the digital environment to maintain the learners state of being full engaged and absorbed in an ongoing activity, referred to as *flow*. Shute, Hanson, & Almond (2008) argue this approach increases assessment reliability and validity by minimizing potential effects of test anxiety the learner would normally experience if they knew they were being assessed. Owen and Halverson (2014) argue that these approaches can provide key indices of basic achievements learners accomplish in the game, and also generate rich data to assess how players responded to failure, use learning supports within the game, and improve performance over time. From a design perspective this data can allows researchers and game developers measure the effectiveness of game play features, across players and then use that knowledge to design better versions of the game design. Furthermore, by continuously updating models of the learner's knowledge, these forms of computer-based assessment can provide real time feedback and adapt content to varying states of the learner's knowledge (Shute & Rahimi, 2017).

In-game data that tracks player actions and progress through the game takes a much more massive form when data is collected across large populations of players playing the same game on online platforms. For example, *Math Garden* is an online platform for practicing math skills that allows players to track their progress and receive appropriately difficult activities based on adaptive game mechanics maintain a level of difficulty following the player's skill development (Maas & Nyamsuren, 2017). From this immense database of over 20 million player responses, the authors performed detailed analyses on separate items to draw strongly powered inferences about the human capacities to reason with number. Such datasets represent an attractive option for advancing the ways in which theories of numerical cognition can be tested across wide populations.

The strengths of these data collection approaches to measure behaviors and virtual actions provide exciting avenues to test theories of learning based on the implementation of adaptive learning algorithms (Torrente et al., 2009), manipulating gameplay variables, and collecting in-time data from players actively engaged in the learning context (Shute et al., 2017). Through programmatic changes in gameplay features such as the presence of a narrative context (Swart et al., 2017), the ordering of gameplay levels (Kim & Shute, 2015) or the form of in-game feedback (Tsai et al., 2015) researchers can test specific causal predictions of learning while developing formative knowledge to inform future game design.

Video games as audiovisual representations of math concepts in informal contexts

Beneath the artful design that makes video games enjoyable to play, video games are programs executing the presentation of visuals and sounds, detecting player's actions, and responding to those actions with specific feedback. By this very nature, digital math games present unique methodological advantages for studying how people interpret visual and audio representations designed to convey and teach mathematical ideas. The notion of creating educational math games to research learning in these contexts has existed for many years (Bright et al., 2017; Ernest, 1986), yet modern advancements in technology continue to change the landscape of what kinds of games are possible and how people interact with these experiences. What has remained consistent over the years is the argument that *math games* can support a deeper understanding of mathematics through audio-visual representations of mathematical concepts. Learning theories in math education have long emphasized the use of tangible and perceptual manipulatives to help students grasp (metaphorically and literally) novel concepts (Bruer, 2001; Dienes, 1960) and math games have been put forth as a way to instantiate this concept digitally in video games and physically in board games. Testing the viability of math games as a means to help students gain a deeper understanding of math concepts typically presented in the math classroom as formal definitions and symbolic notations, can also be a test of these broader learning theories of perceptual learning, multimedia learning and grounded cognition.

Using Math Games to Reify Mathematics

Goals to inspire interest in mathematics using computer games are intertwined with goals of math games to provide experiences and simulations that put mathematical abilities into practice. Towards this end, video game researchers have argued that that a specific strength of interactive digital media is in the ability to design experiences that reify abstract concepts and simulate phenomena in the world that are otherwise difficult to perceive (Corredor et al., 2014). While some educational games, such as scientific simulations, present specific facts and phenomena for learners to observe and appreciate, topics of mathematics and number knowledge are more abstract. Mathematical operations and concepts can be exemplified by audiovisual instantiations but are not defined by any one example. Math educators must help their students develop both *procedural knowledge*, in the form of abilities to carry out specific action sequences (e.g. counting or arithmetic operations), and *conceptual knowledge*, meaning the implicit and explicit understanding of mathematical ideas and why they hold true across multiple contexts (Rittle-Johnson et al., 2001). Learning goals of math games may focus on either procedural or conceptual knowledge, yet the audio-visual experience afforded by math games may create greater opportunities for deepening conceptual knowledge.

Whereas, game features can motivate learners to drill math facts and improve fluency with mathematical procedures, these experiences do not directly help learners understand how to apply those operations in external contexts (Goldstone et al., 2008). Conversely, it has been argued that a key opportunity for math games to promote learning is in the ability for learners to experience and interact with multimedia representations of mathematical concepts through play and in doing so to gain a grounded or intuitive understanding (Fey, 1989; Hanna, 2000). In the following section, I present cognitive theories of numerical and mathematical development, and review how educational games have been used in the empirical study of these theories. Furthermore, I put forth arguments for how advances in video game technology can be leveraged to address these theories in the future.

Using games to study theories of grounded cognition

According to grounded theories of cognition, mathematical knowledge, such as the meaning of number symbols and the understanding of mathematical operations, is based on human perceptual and active experience situated in the physical world (Barsalou, 2008) Some researchers have proposed that an abstract understanding in language and mathematics depends on drawing metaphorical connections to our bodily states and their situated knowledge (Lakoff & Johnson, 1999). For example, we can describe the number 2 as *smaller* than 7, and the distance between these magnitudes as *further* apart than 3 and 5 to convey the abstract meaning of relative numerical magnitude. Grounded theories argue against the notion that math and number knowledge is encoded in memory independent of the perceptual and affective context in which the knowledge is learned and used. Thus, conceptual understanding of mathematics and constructing strong representations of meaning relies on partial simulations of perceptual components (Barsalou, 1999) and embodied referents (Hauk et al., 2004) of past experience relevant to the mathematical concepts and procedures at hand.

Grounded theories of math learning closely align with longstanding theories in mathematics education about the use of concrete experiences to introduce abstract mathematical concepts. Dienes (1960) argued that by having students interact with multiple concrete examples of quantity and magnitude, skillfully crafted to demonstrate the structures of mathematics, they can then extract the information necessary to support conceptual knowledge and abilities (Dienes, 1960). Drawing from the writings of Bruner (1966), Resnik & Ford (1981) present the argument that instruction should follow the learner's natural progression of enactive experiences to graphical representations to symbolic representations of knowledge, known formally as *concreteness fading*. Thus each step in this process is seen as prerequisite for successful understanding in the subsequent stage, and recent research in perceptual learning have shown that there may be unique benefits to sequencing learning experiences in this order (Fyfe et al., 2014; McNeil & Fyfe, 2012). In other words, developing the proper understanding that $(3 + 2)^2 \neq 3^2 + 2^2$ needs to begin before students ever see the equations written out.

One way in which the predictions of grounded cognition can be explored in the video game context is in studies that examine the effectiveness of games as anchoring activities that illustrate numerical properties and mathematical concepts in graphical representations. Beyond math games that automate drill and kill practice with mental calculations, some strategy games present mathematical concepts in visuo-spatial puzzles. For example, in the game *Refraction* (2013) players must place devices to divide a power source into the correct fraction of power to properly fuel rocket ships, calling upon players to conceive of fraction magnitudes and the operation of division. In the game *Treefrog Treasure* (2011) players are introduced to a series of barriers in the form of number lines and must direct a frog character to break through at specific numerical values. These games exemplify educational experiences that allow players to enact up on visual representations of mathematics and offer multiple opportunities to test theories of grounded cognition. For example, randomized control studies with games like these prior to formal instruction compared to the opposite order could test theories that learning new mathematical topics should begin with interactive experiences. Furthermore, through a manipulation of graphics in these games, artistic game features such as narrative, animation, or naturalistic renderings (Swart et al., 2017), intervention studies could further test theories of concreteness fading that argue for the transition of instruction from concrete to abstract.

Using games to test fundamental theories of numerical development

Grounded theories have been widely proposed and supported by cognitive science and neuroscience researchers studying the mechanisms underlying how humans come to understand the meaning of number symbols and form abstract conceptions of whole-numbers (Butterworth, 1999; Dehaene, 2011; Nieder & Dehaene, 2009) and fraction values

(Jacob et al., 2012; Lewis et al., 2015). For example, neuroscientists have observed that neural activation in a common brain region (intraparietal sulcus, IPS) is sensitive to changes in perceived magnitude, irrespective of the magnitude's symbolic or nonsymbolic form (Nieder, 2013; Piazza et al., 2007). These findings support the hypothesis that IPS may represent an abstract and common magnitude code for meaning of distinct entities and number symbols alike (Dehaene et al., 1998; Fias et al., 2003; Piazza et al., 2004). These findings have laid the foundation for causal claims that early childhood experiences and intuitions within nonsymbolic numerical instantiations are fundamental to the development of symbolic whole number understanding (Dehaene, 2011) and higher-order mathematical reasoning (Izard et al., 2011). While correlational and longitudinal studies in this field have largely supported this claim, Bugden et al. (2017) argue that training studies that directly test the use of nonsymbolic number experiences using pre-post assessments and neuroimaging techniques are necessary to understand how these theories can be applied in educational techniques. Towards this goal, research has shown that training approximate calculation with non-symbolic representation of quantities can have a causal benefit for arithmetic performance with symbolic numeral (Park et al., 2016). Other studies suggest that manipulating nonverbal spatial representations of number are the key mechanism by which stronger understandings of number are forged (Park & Brannon, 2014).

In a limited number of studies, math games have been used to situate nonsymbolic to symbolic mapping experiences in a naturalistic learning environment. In a study by (Whyte & Bull, 2008), experience playing board games that represent numbers along a linear depiction of magnitude improved preschoolers' numerical abilities including symbolic to nonsymbolic number estimation. Likewise, Wilson and collegues (2009) observed that playing *The Number Race*, a digital game that requires the comparison of symbolic and non-symbolic quantities improved numerical abilities, especially among those who struggled with math and number. Kucian and colleagues (2011) observed using a pre-post fMRI design that experience playing, *Rescue Calcularis*, a number line estimation-based game, led to improvements in numerical estimation for students with and without specific math disabilities. Participants with math disability in this study also showed positive changes in the recruitment of neural systems associated with the processing of numerical tasks. These findings further support the argument that educational games that ground numerical concepts (such as magnitude and ordinality) may provide important grounding to support an understanding of these concepts when present in symbolic form. Moreover, these studies provide examples of how video games can bridge cognitive theories of psychology and neuroscience with learning experiences designed for actual educational applications.

Game Contexts to Study Perceptual Learning and Transfer

An additional interpretation of how the integration of concrete experiences in math education supports the transfer of mathematical understanding across contexts, is put forth by theories of *perceptual learning.* Specifically, Goldstone and colleagues (2010) claim that the development of expertise involves updating perceptual interpretations of math experiences to fluently recognize relevant details and patterns across varied learning situations. Privileging the education of symbolic formalisms prior to or in absence of concrete referents ingrains a narrow understanding of the content and limits opportunities for learners to understand how math equations apply to problems outside of the math textbook (Braithwaite et al., 2016; Goldstone et al., 2010).

Situated cognition theories argue that knowledge is strongly linked to the context and goals of the learning experience (Lave & Wenger, 2012; Strauss & Lave, 1990), and from this perspective there is little to support the notion that knowledge transfers to new learning contexts. The issue of whether learning in educational games can transfer knowledge to new contexts, has been put forth as a major limitation in the use of educational games (Schroeder & Kirkorian, 2016). Gladstone, Landy, and Son (2005) offer the possibility that in "grounding" knowledge, the mechanisms underlying the ways we transform concrete knowledge into abstract representations work to allow people to recognize how the concrete forms math concepts can be observed.

Using Math Games to Foster Math Interest

Helping students thrive in the fields of science, technology, engineering, and mathematics requires providing educational opportunities that foster the development of knowledge as well as inspire motivation to persist in the field. Likewise, performance in mathematics and the pursuit of opportunities to learn math are ultimately influenced by the beliefs, attitudes, emotions, and values students hold towards the content (Hidi et al., 2004). The study of learning in math games may provide powerful insights into how emotional dispositions towards mathematics form, how they are related to math achievement, and the role of learning experiences in forming these relationships. In this section, I present three perspectives on the hypothesized roles math games may play in development of personal interest in mathematics and the affective effects of gameplay on the learning experience. First, I review theories of interest development, and discuss how math games have been used to test their hypothesized role in initiating interest through entertaining experiences. Second, I discuss evidence regarding the experiences of flow and engagement during gameplay, and how game features such as narrative framing, adaptive challenges, and social interaction relate to this experience. Third, I discuss the nature of video games as external motivators of effort and theories regarding how players respond to and persist through challenges in the game context

Affective Experience in the Development of Math Knowledge

In a description of the traditional state of mathematics education, Lockhart (2009) laments the unfortunate disservice our schools systems do to students by teaching mathematics only as strict facts to memorize and precise procedures to follow for the vast majority of primary education while withholding applied, creative, and theoretical conceptions of mathematics for STEM majors in college. Such conventions, Lockhart argues, are analogous to teaching music to students without letting them perform or teaching art principles for years before letting children create a painting. The argument follows that conventional structures of math education fail to give students insights into why mathematics is interesting and useful. In line with this argument, cognitive researchers have also argued that instruction of facts, algorithms, and operations without providing operable contexts to apply this learning may in fact be contributing to the prevalence of math anxiety and a lack of overall engagement (Brown et al., 1989).

A large amount of research pertaining to the relationship between affective dispositions towards mathematics and mathematical abilities has actually focused on the ways that math anxiety constricts math abilities and encourages the avoidance of math activities (Ashcraft, 2002). For example, neuroscience research into the neural mechanisms of affective experience during math performance has explored how individuals experience math anxiety. Specifically, high levels of anxiety in anticipation of doing math is associated with higher activation of brain regions related the experience of physical pain (Lyons & Beilock, 2012a, 2012b). While neuroscience research has examined the mechanisms of positive reinforcement in decision-making contexts, much is still unknown about the role of engagement in the development of math abilities and knowledge.

Models of how math interests develop stem from general theories of interest development, which describe the role of emotional reactions during early math experiences (Hidi & Renninger, 2006) as well as the relationship between affective dispositions and mathematical abilities (Fisher et al., 2012). Here I adopt a dual definition of *interest* outlined by Hidi and colleagues (2004), as both *situational interest*, meaning an emotional state induced by external factors of an experience (Knogler et al., 2015) and *individual interest*, meaning an individual's personal relationship with topic or domain-specific activity and their propensity to reengage with those activities. Thus, the development of math interests can be described broadly as the establishment of sustained individual interests as driven by the accumulation of math experiences inciting situated interest. Other theories of academic motivation focus more so on metacognitive factors, such as self-efficacy beliefs (Bandura, 1977), personal goals to achieve mastery of mathematics (Ames, 1992), and perceived value of acquiring skills or content knowledge (Eccles, 1983). These primarily cognitive-based theories remain useful for conceptualizing the relationship between motivation and math abilities. However, they do not account for subconscious aspects of motivation that are initiated by emotional experiences in learning contexts (Hidi, 1990; Krapp, 2002).

Empirical investigations into the early development of math interest and how it relates to the foundations of math and number knowledge have revealed some indication that affective dispositions form early and in nonacademic contexts. In a study with low-income preschoolers, Fisher and colleagues (2012) identified that early math abilities significantly related to engagement in a free play numerical task. Furthermore, they observed that over the course of 6 months, early levels of engagement in situated contents predict future levels of math achievement and signs of competency predict higher amounts of goal-directed play. Similarly, Yang and colleauges (2014) observed that efforts to provide remedial support for 1st grade students with low-SES backgrounds not only improved abilities but also led to increases in measures of interest in mathematics and confidence. Fisher and colleagues (2012) argue that such findings support Ma and Kishor's (1997) model of math development as a reciprocal relationship between the interest and abilities, whereby early interests motivate greater engagement with math content and stronger understanding from this engagement promotes more enjoyable learning.

Studies with early elementary school students have observed that students exposed to higher occurrences of informal number and math related activities at home, such as board games, card games, cooking, and shopping, showed significantly higher levels of mathematical skills in kindergarten and early elementary years (Lefevre et al., 2009; Purpura et al., 2017). Similarly, in an intervention study utilizing a digital aid to situate early number and math concepts in entertaining stories, the amount of use at home was associated with greater math learning across first grade, especially among students whose parents experience significant anxiety with math (Berkowitz et al., 2015).

Video Games as Triggers for Situated Interest

Video games are receiving attention in education for their potential to engage players in fun interactive experiences with rich educational content (Mayo, 2009). Furthermore, the success of an educational game, in the form of players mastering the educational content, relies on creating engaging challenges and experiences that players want spend time completing. Garris, Ahlers, and Driskell (2002) describe an input - output model of learning in math games that emphasizes a cyclical process of user actions, game feedback, and user reflections on the experience, to argue that this cycle is the source of engagement and the user's judgments are critical to whether this cycle persists. Specifically, video games are designed to elicit emotional reactions from the player, through the use of game elements such as rewards, obstacles, and game narrative (Squire, 2003). Games can also provide immediate feedback, unlimited chances to try again, and options for the student to scale difficulty based on his or her needs. Through these features, math games are intended to reframe homework and drill exercises as fun competitions or goal directed challenges.

Many game features included to create entertaining experiences overlap with psychological theories of reward-based learning. Specifically, learning can be stimulus-driven, when rewards are paired with the delivery of the information, feedback-driven, when rewards follow the stimulus, or motivationally driven by the anticipation of the reward to come (Adcock et al., 2006). Furthermore, the use of games for learning builds on the theory that the pursuit of winning and the excitement of competition invite learners to develop emotional associations with the content that extend to different learning domains (Immordino-Yang & Damasio, 2007).

Studies comparing the differences between math learning in game-based and paper and pencil contexts with 5th grade children have shown that games have positive effects in task engagement (Ke, 2008). Studies with middle school students have shown that having the ability to choose math examples and personalize the learning context significantly increased the experience of situated interest and task effort, especially among students with lower individual interests (Høgheim & Reber, 2015). Similarly, with high school students, Squire recognized that students who normally struggle in academic contexts showed great enthusiasms and engagement learning through a complex strategy game, yet high achieving students were more vocal about labeling the games as a waste of time (Squire, 2005). Likewise, studies with college-aged adults demonstrated that the main reasons for players to seek out and play video games is to feel entertained and relaxed, even when challenging forms of gameplay lead to failure, however self-reported reasons to play games adults never mention an intention to learn (Hoffman & Nadelson, 2010). One observation regarding the potential for educational games to initiate situated interest in formal educational settings is that, across studies and ages, games appear to have a more positive effect on young students relative to older students, whose level of interest may be more established. For those who do find greater excitement from game-based learning relative to traditional learning contexts (Squire, 2005), these positive effects further emphasize the critical role that informal learning experiences have in enabling learners to pursue their personal learning goals on their own and with peers (Barron, 2006). Furthermore, these studies provide additional evidence for the conclusion that informal math experiences prior to formal math education are directly related to the dispositions that learners bring to the math classroom.

Using Games to Promote Engagement

Flow and Situated Interest in Math

One goal of educational video games is to deeply engage with the learning content (Mayo, 2009). Research on the effect of playing video games has also advanced the conceptualization of what the construct of engagement means. Congruent with the notion that engagement is a psychological experience, video game researchers refer to the positive mental state where the player perceives an ideal fit between his or her skills and the task demands as *flow*. Through an appreciation of flow states in games, neuroscience researchers have begun to understand the neural mechanisms underlying engagement. By giving adults appropriately challenging math problems and comparing neural activity during this task to conditions of induced boredom and overload, Ulrich and colleagues (2014) induced flow states. Results showed that flow states were associated with increases in activation in regions of the brain associated with cognitive control (left anterior IFG) and decreased activation in regions associated with processing negative arousal (amygdala). Additional studies are necessary to replicate the manifestation of flow states with video games in the MRI scanner, yet these findings do present exciting evidence regarding the flow states. One possible mechanism by which games induce a productive learning context is by situating players in appropriately challenging experiences that are enjoyable and engage capacities to fixate on a task and suppresses negative affect in the pursuit of accomplishing a goal. Furthermore, the study of flow in games has provided evidence for the importance of peers and the social context in engagement. Direct comparisons of human versus computer-controlled opponents while playing games has shown that playing against humans can be associated with greater self-reported feelings of flow, enjoyment, and presence in the game experience (Weibel et al., 2008). These findings have been replicated in neural investigations of video game play, with results showing that playing games against human opponents is associated with greater activation in reward processing areas of the brain compared to when the opponent was a computer (Kätsyri et al., 2013).

Using Games to Encourage Persistence through Failure

One hope in designing educational games is to frame learning in a context that encourages players to persist through increasing challenges. An activity central to most video game designs is the notion of leveling up. When players level up in video games, they are introduced to new abilities, more complex scenarios, and even harder challenges to overcome. Despite these challenges, leveling up is not designed to discourage players but rather it becomes an achievement that players get to enjoy. In mathematics education, course sequences and the units within also present a structure of increasing challenges, but failure in these contexts is greatly discouraged and comes with real consequences to career and academic aspirations. Theories of motivation from a cognitive perspective have focused on how motivational goals are related to academic achievement (Ames, 1992), and how beliefs about self-efficacy guide one's motivations to excel (Bandura, 1977). Findings showing that video game players enjoy the challenge of games and persist even in the face of failure (Malone, 1981; Przybylski et al., 2010), have motivated the hypothesis that educational games could inspire the same joy, resilience, and intrinsic motivation with educational content (Gee, 2003a).

Math video games that fall under the category of drill and practice games are generally welcomed into the classroom as reward activities or homework since they align closely to the exercises normally assigned on a worksheet. Despite the close resemblance to more traditional math homework, the game interface has been shown to encourage students to complete more examples and practice for longer stretches of time than paper-based activities (J. Lee et al., 2004). Additionally, controlled comparisons between arithmetic fact practice in a game setting and on paper show some evidence that gameplay was more enjoyable and associated with improvements in working memory capacity (Núñez Castellar et al., 2014).

An Argument for the Use of Video Games as Research Tools

Math games are increasingly incorporated into the math classroom following two primary hypotheses. First, math games provide rich audio-visual experiences and goal-based scenarios for learners to interact with mathematical concepts and form richer understanding of mathematical structures. Second, math games create enjoyable situations to initiate interest in mathematics, inspire deeper engagement, and motivate players to persist through difficult mathematical concepts. This review explored how games research has tested these hypotheses thus far and presented a vision of how this work can advance in the future.

Advancing the empirical understanding of how learning experiences develop both cognitive capacities and affective associations is specifically important as digital multimedia become continually more present in schools and the workplace. The field of educational research on video games continues to grow (Connolly et al., 2012; Hainey et al., 2016), yet there is still much to be learned as digital technology continues to advance and enable new and exciting educational games. Moreover, it is important that the growing presence of educational games in the math classroom be met by research that elucidates the efficacy of these learning experiences. Studying these effects not only provides insight into the formative evaluation of educational games, but also offers an opportunity to test theories of numerical cognition, memory, interest, and motivation that are proposed to support learning and mathematical knowledge.

Furthermore, educational games offer particularly powerful methods for measuring players' interaction with mathematical content through authentic learning situations. Specifically, these games offer opportunities to observe learning through play using methods that do not interrupt the experience of flow in gameplay at home or in labs. Here, I agree with the descriptions of stealth assessment put forth by Shute and colleges (Dicerbo et al., 2017; Shute, 2011), that measuring learning in unobtrusive ways can critically increase the validity and reliability of performance measures by taking pressure off of the learning experiences and engaging learners in an immersive experience. Furthermore, I argue through creative manipulations of the video game environment, future researchers may full take advantage of the video game medium to directly test predictions about the roles that different game features play in the learning process. Such manipulations can be made to test specific causal claims of mathematical development.

In research aimed to uncover the fundamental mechanisms by which symbolic knowledge of mathematics is tied to grounded experiences, we see clear examples of how video games can be utilized in testing the causal predictions of long-standing theory. Specifically, theories about the fundamental role of nonsymbolic number abilities in supporting symbolic number knowledge have been translated into actual learning interventions, and the effects of these interventions have been tested through behavioral and neuroimaging measures (A. J. Wilson & Dehaene, 2009). Furthermore, new educational technologies inspired by embodied theories of learning offer new opportunities to test causal assumptions of those theories (V. R. Lee, 2014). Nathan & Walkington (2017) argue that this research also must extend to articulating how commercial games conceived upon or marketed as incorporating movement into mathematics instruction, do or do not represent valid instantiations of embodied theory. Advancing these fields of research stands to provide critical real-life tests of numerical and mathematical cognition theories while generating usable knowledge about how these theories can inform the development of effective number games.

In addition to their value as a research tool, teachers can use math video games to support or supplement their current instruction and assess knowledge. Among the genre of math games, titles vary across whether the design experience aims to reify math concepts in graphical and interactive representations or encourage the practice of math facts and operations. A discussion of how learning occurs in these game-based contexts raises interesting questions regarding how players perceive mathematical meaning from graphical representations embedded in games and the extent to which practice in games transfers to the formation of mathematical knowledge outside of the game. The theories and empirical studies presented in this review generally support the theory that there is a great potential for math games to provide formative learning experiences for understanding math in deeper ways. This stance, of course, comes with the recognition that video games do not have a single specific form, and the abilities to improve the design of math games are only limited by the current state of technology and the imagination of game designers. Yet even among simple forms of math games, their educational utility is perhaps best defined by their relative strengths compared to traditional forms of instruction. Relative to math textbooks with static images, textual descriptions, and practice problems, educational games may provide a more dynamic, visual driven, and engaging teaching tool to supplement formal math instruction.

As described in this paper, studying learning in the context of games can provide unique insights into the development of mathematical abilities and interests. In turn, these insights will hopefully lead to theories and frameworks for the design of effective educational video games. The translation from research to development is difficult and requires an iterative process some researchers have suggested mirrors engineering more so than science (Nathan & Sawyer, 2014). These efforts will be strengthened by existing guidance established by fields of design-based research to apply empirical results to the formative design of new and better technologies and learning environments (S. Barab & Squire, 2004).

The second hypothesis explored in this review states that educational games should be used in math education to promote the development of positive emotional dispositions towards mathematics. Research examining the role of early math experiences on abilities and interest in math shows that engaging in more informal numerical experiences before attending school, such as card games and linear board games, is positively associated with higher levels of math interest and abilities (Siegler & Ramani, 2008). According to the studies reviewed in this chapter, it appears that as children grow older their new experiences with math are less likely to change established dispositions towards the utility and form of mathematics. For example, when high school students were introduced to a complex strategy game, Squire (2005) observed that students who do well in in traditional education contexts, where knowledge is composed of concrete facts and ideas to memorize, tended to shy away from strategy games that require active learning approaches including exploration, failure, and hypothetical scenarios. Such a stance suggests that an important goal of early education is not merely to excite learners but to help learners become comfortable with learning through goal-oriented scenarios and applying critical thinking skills. In this review, the role of math games in creating goal-oriented scenarios was primarily discussed as a way to enhance learners' knowledge of mathematics and how to apply this knowledge. Moreover, the idea of giving young learners the chance to see how useful and interesting mathematics is when applied to relevant problems may be a primary mechanisms by which learners develop sustained individual interest in mathematics and pursue opportunities to use math academically and in future careers.

In addition to cognitive and affective hypotheses regarding the benefits of math games on the development of math knowledge discussed in this review, there are additional perspectives for research in this field to pursue going forward. For instance, attention to the role of the social context in math learning has led to insights about the multiple ways that parents actions and dispositions towards math and science can influence student's self-perceptions and values about these academic topics (Bleeker & Jacobs, 2004). For instance, introducing technology supports that motivate caregivers to spend more time with children talking about mathematics can have positive benefits in math achievement (Berkowitz et al., 2015; Levine et al., 2011), and especially for parents who experience math anxiety. The study of learning in the context of math games may provide additional valuable avenues to explore these social aspects of learning through games. For example, games can frame learning in scenarios with interesting social dynamics such as cooperative and competitive play. Questions about whether math games can bolster engagement via competition, or spark productive collaborative meaning making with peers have yet to be fully explored. Furthermore, questions about the social nature of games extend to the study of how learners engage differently with real or virtual agents in the game play environment. These questions present important avenues to better understand how math games can and should be incorporated into the classroom while also elucidating the role of social interaction in the development of mathematical knowledge.

Among all of the positive results and perceived potential for educational games to have a great impact on the future of education, it remains important to balance enthusiasm with limitations of this research and recommendations for appropriate amounts of screen time for young children (Radesky et al., 2015). Conducting research on the role of math games in the development of knowledge and interest can also rely on assumptions that video games are clear in their presentation of the learning materials and effective in presenting a fun game that children want to play. If players are bored or dislike the experience, poor engagement with the educational intervention will obscure any cognitive or affective effects. Furthermore, more research is necessary to appreciate the role of math teachers in helping students draw valuable connections across formal math instruction and the representations of mathematics in video game contexts.

As audio-visual, cellular, and computer technologies continue to develop so too will opportunities to express the structures of mathematics in creative and engaging ways. Educational math games are a relevant, valuable, and interesting context for understanding the mechanisms of numerical and mathematical cognition. Continued incorporation of math games into programs of cognitive, social, and neuroscience research has the potential to generate valuable insights to push these fields of research and the digital design of math instruction forward (Howard-Jones et al., 2011).

Chapter 5 – Accessing the Magnitudes of Fractions and Ratios though Playing Cards.

Introduction

Video games are receiving attention in education for their potential to engage players in fun interactive experiences with rich educational content (Mayo, 2009). Video games are designed to elicit emotional reactions from the player using game elements such as rewards, obstacles, and game narrative (Squire, 2003). Through this design educational video games can portray educational content in unique audio-visual representations. One question around the implementation of educational video games is whether the creative and artistic depictions of learning content fosters deeper engagement with the content or makes the content less accessible. In the current study we investigated this question as it pertains to accessing the meaning of fractions and ratios in an educational math game called *Fractions War.*

Visual Complexity in Educational Materials

One aspect of designing good educational games, is crafting visual representations that depict the educational content in ways that the learner can understand and develop new knowledge. (Plass et al., 2009). This requires attention to the ways visuals may test the limits of visual attention and perception (Desimone & Duncan, 1995) In multimedia research, Cognitive Load Theory outlines three different ways that learning experiences draw on the limited resources of working memory (Sweller, 1999). Specifically, the theory describes three different types of cognitive load. *Intrinsic load* is inherent complexity of the learning content. *Extraneous load* pertains to the processing of additional, unnecessary, or nonessential information. And *germane load* corresponds to the mental effort the learner invests into the activity. In regard to the current study, efficient representations of fractions and ratios follow the coherence principal (Mayer, 2002) and eliminate extraneous details so individuals to focus solely on the relation between values. Likewise, controlled experimental stimuli in traditional cognitive tasks follow this coherence principal to purely observe performance with the specific construct of interest. However, video games and educational materials often use naturalistic or complex visual representations that carry additional extraneous load for people to grapple with or inhibit. It is through repeated experiences with these complex visual representations that individuals can fluently extract meaningful information amidst their extraneous details (Green & Bavelier, 2003; Kellman et al., 2008; Kellman & Massey, 2013; Rau et al., 2017).

Previous studies observing the ways that people understand and misunderstand fractions reveal that the whole number parts of fraction components can distract from the holistic magnitude that emerges from the parts relation to one another (Bonato et al., 2007; Obersteiner & Tumpek, 2016; Toomarian & Hubbard, 2018; Zhang et al., 2014). However, in Chapter 2 we observed that influences of these componential features do not appear to have an analogous effect when these components are visual parts of nonsymbolic ratios. Specifically, the use of visual ratios broken into discrete parts can actually be a less efficient way convey a proportion than if the parts of the ratio were continuous extends, because the discrete parts invite extraneous and misleading whole number counting strategies. However, these simplified controlled stimuli of cognitive research tasks, does not necessarily reflect how symbolic and nonsymbolic processing occurs in authentic learning environments. Educational math games offer a new context in which we can observe magnitude processing with these external formats.

Cognitive and Affective Learning Goals

According to the cognitive affective theory of multimedia learning (CATLM) proposed by (Mayer & Moreno, 2010) learning is mediated by motivational factors directly effecting engagement, and these metacognitive factors effect learning by directly influencing the levels of cognitive processing and affect. Theoretical arguments for the relationship between emotion and cognition lay out a spectrum from these two phenomena being separate and interactive (Ames, 1992; Bandura, 1977) to inseparable and codependent (Immordino-Yang & Damasio, 2007). In a review of educational video games across multiple educational topics, Connolly and colleagues observed mixed evidence for the argument that these games lead to positive cognitive effects (e.g. academic achievement), but more consistent evidence that these games created a more enjoyable learning context. To this, Connolly and colleagues (2012) argue that with so many people playing and enjoying video games, the actual state of enjoyment is forgotten as a benefit of game play. Similarly Mayer (2014) observed that as long as additional decorative elements of a learning environments do not overload learners with high amounts of extraneous load, these approaches can lead to higher levels of personal interest and motivation. Furthermore, these affective effects may in turn have positive cognitive effects in the future.

One goal in the audio-visual design of games is to create an aesthetically pleasing form. What qualifies as aesthetically pleasing is a matter of subjective preferences and conventions of a game's genre, but unlike researchers designing an experimental task game designers are encouraged to use their artistic expression in service of creating the appropriate visual experiences. Video games can be immensely complex 3D worlds (e.g. *BioShock Infinite*, 2013) or a minimalist 2D scrollers (e.g. *Limbo*, 2010), yet the goal is to create a visual aesthetic which invites the player to feel a part of a digital context. In educational games, visual design goals are not necessarily in direct service of learning goals, especially when educational content is embedded into fictional stories (e.g. role-playing games) or designed into adaptations of established game genres (e.g. card games). Nevertheless, these decisions can be made to evoke positive emotions during the learning experiences (Plass et al., 2014), which may in turn mediate the effectiveness of the learning experiences.

Fractions War

Fractions War is a digital educational game (for iOS tablet devices) adapted from the classic card game, *War*. In the game *Fractions War* depicts fractions and ratios as the relationship between two card values. To play the game, a deck of playing cards is split evenly between two players and one hand at a time the players flip over their top two cards and arrange them vertically to form fractions. For example, the 2 of hearts and the 5 of clubs would be arranged into the fraction 2/5 and have a holistic magnitude of 0.4. The player whose fraction has the higher magnitude wins the hand and it is each player's goal to be the first to identify who the winner is and swipe all four cards on the table into the winner's deck. When the two fractions have the same magnitude, players must try to be the first to "Declare War" prompting a high stakes comparison of another pair of fractions with the potential to win more cards. The game ends when one of the two players has collected all of the cards into their deck, but the winner of the game is the player who earned the most points from making the fastest and most accurate judgements of magnitudes.

The core game mechanic of fractions war is the comparison of fraction and ratio magnitudes. This magnitude comparison activity is commonly employed in numerical cognition research to observe how quickly and accurately participants process the magnitudes of two values and determine which value is larger, which value is smaller, or if they are the same (e.g. Binzak & Hubbard, 2020; DeWolf et al., 2014; Holloway & Ansari, 2009; Matthews & Chesney, 2015). In addition to the specific learning goals of *Fractions War* (See Appendix D), the game was designed to be an assessment tool for teachers or researchers to observe how players understand these quantities in an informal context. By embedding data collection features into the game, *Fractions War* can be used as a form a stealth assessment, where records of player performances can be gathered without interrupting the educational game experiences (Shute, 2011). In the current study, we tested the viability of using *Fractions War* as a tool of studying magnitude processing by comparing performance in the game to more traditional magnitude comparison tasks.

Games vs Tasks

In defining *Fractions War* as a *digital educational game* which can be used as a research tool, it is necessary to distinguish how games like *Fractions War* are a qualitatively different experiences to the more common cognitive lab task. Some researchers argue that the immense variety of video games makes it so that they cannot be defined by rigid boundaries. Rather, there are ways in which games resemble one another by the features they share even when any two games are rarely composed of entirely the same features (Arjoranta, 2014; Wittgenstein, 1953). Therefore, our goals in defining games and a game-based context are to highlight general features of tablet-based digital games and make key distinctions of how these features differ from general features of controlled research tasks.

First, the goals of educational video games are to be experiences in which learners can interact with and learn from educational content, and to be an entertaining and fun environment to play with. The goals of tasks are to observe valid and reliable human performance, to test specific cognitive theories and hypotheses. Second, educational video games are typically very creative and aesthetically pleasing digital environments to learn in. Cognitive tasks purposely exclude the use extraneous features that may bias or cofound the results. Third, educational video games are activities that people get to *play*, whereas tasks are things that people are obligated to do.

Experiment 1

Current Experiment

A primary goal of this experiment is to first validate the use of *Fractions War* as an effective method of stealth assessment, which can engage a player in a learning experiences and measure magnitude comparison performances. Before we can draw comparisons of symbolic and nonsymbolic magnitude processing across game and task contexts, it is important to first determine how reasonable this comparison is. The numerical distance effect is a robust effect that is consistently observed with numeric and nonsymbolic comparison tasks, and thus we are interested in whether this effect can be detected within *Fractions War*'s game data. Specifically, we predicted that the time for players to make magnitude judgements in the game would be inversely related to the numerical distance between the fractions and ratios presented in the cards.

Next, we explored the extent to which the game context impacted magnitude comparison performances and format effects. In previous comparisons of symbolic and nonsymbolic magnitude comparison discussed in Chapter 2, we observed that adults can more efficiently compare the magnitudes of nonsymbolic ratios than symbolic fractions. Here we explored whether this format effect generalizes to the game context, where symbolic fractions (numerals) and nonsymbolic ratios (relative quantity of pips) are presented on playing cards.

Furthermore, we aim to evaluate the effect of the game context within symbolic and nonsymbolic comparison. If the game environment, through its mechanics of competition, aesthetic appeal, and play-based learning, lead to greater engagement or focus during magnitude comparisons, then performance in the game group may be greater than the task group. On the other hand, if playing cards are an inefficient visual representation, aesthetic design becomes distracting extraneous detail, or the play-based nature of the game encourage people to more freely make mistakes, then performance in the game group may be worse than the task group. In our comparison of context effects on performance within symbolic and nonsymbolic formats, our primary metric of performance is error rates.

Third we aimed to explore the potential for a game like Fractions War to be a helpful predictive measure for evaluating fraction knowledge more broadly. Knowledge of how symbolic fractions and nonsymbolic ratios represent magnitudes has been identified as a significant predictor of external measures of fractions knowledge (Matthews et al., 2016), therefore we aimed to determine if these abilities measured in Fractions War game are also associated with performance on a paper and pencil- based fraction knowledge assessment.

Lastly, we explored the causal hypothesis that interacting with math content within a fun educational game may prompt players to form positive affective associations with that content. Specifically, we examined whether playing *Fraction War* lead to measurable differences in self-reported attitudes towards math in general or fractions specifically.

Methods

Participants

Game group. 35 undergraduates (28 females, Mage = 19.9, range = 18-24) completed the experiment in the game group. After applying the inclusion criterion that accuracy in all conditions must be greater than 70%, data from three participants were excluded from our analyses. As a result, data from 32 undergraduate students (25 females, Mage = 19.8, range = 18-24) were included in our analyses.

Task group. 44 undergraduate students (36 females, Mage = 20.3 years, range = 19-27) completed the experiment in the task group. After applying the inclusion criterion that accuracy in all conditions must be greater than 70%, data from four participants were excluded from our analyses. As a result, data from 4 undergraduate students (32 females, Mage = 20.4, range = 19-27) were included in our analyses.

Procedure

Before arriving at the study, participants were assigned to either the game or the task group. Participants in both groups first completed a number line estimation task. Next, they completed one of two magnitude comparison activities: a computer-based magnitude comparison task or a magnitude comparison game on a tablet device. Immediately following the game, all participants completed a fraction knowledge assessment, and a self-report survey assessing their video game habits, and affective dispositions towards math and fractions. Participants received partial course credit for completing the study.

Between-Group Magnitude Comparison Activities

Magnitude Comparison Game. The game group played the game *Fractions War* (described above) under three different game modes. Specifically participants played the game with (a) traditional playing cards containing both Arabic numerals in the corners and a nonsymbolic array of pips (e.g. diamonds, hearts, spades, clubs), (b) symbolic cards containing only 1 large Arabic numeral in the center of the card, and (c) nonsymbolic cards containing only the nonsymbolic array of pips. Participant's played the game with these three card types in 8-minute blocks with the same order: nonsymbolic, symbolic, and traditional cards.

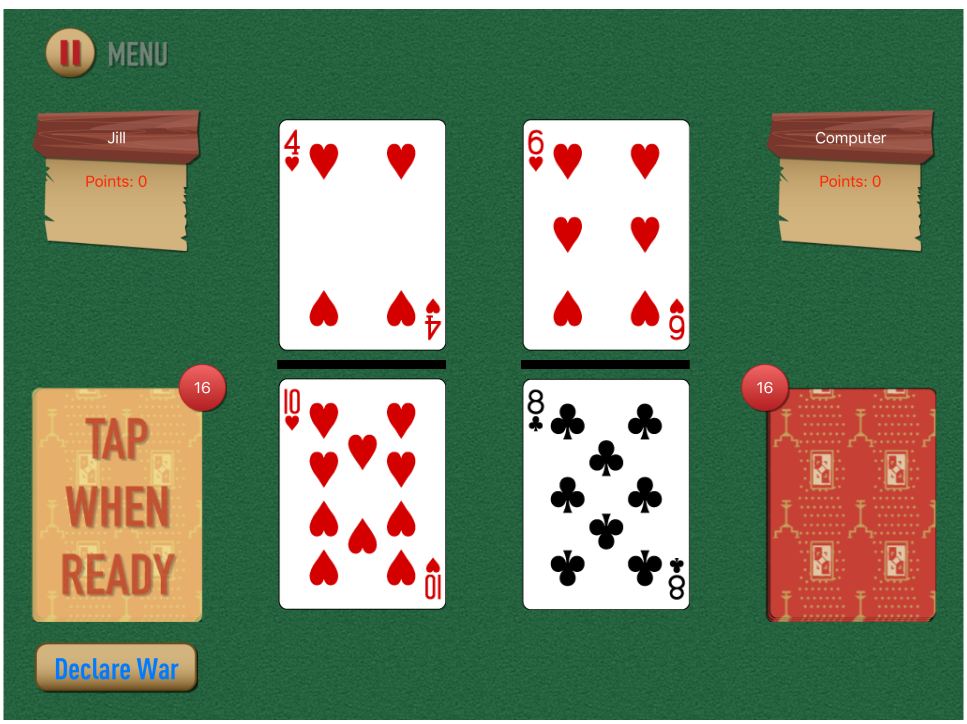

Figure 5.1. Gameplay screen of *Fractions War* showing the two fractions revealed form the player's deck (left) and the computer's deck (right). Players have a set amount of time to identify that the computer's fraction is larger and swipe the cards to the right towards the computer's deck before the computer makes that correct decision.

Comparison pairs were generated randomly by taking two cards from the player's and computer's decks and arranging them vertically with the smaller fraction on top. Card values included in the game ranged from 1-10, which excluded Jacks, Queens, and Kings normally within a 52 card deck. The game allowed for fractions to have a magnitude equal to 1, when the two cards flipped over had the same magnitude (e.g. 2 of hearts over the 2 of diamonds).

The magnitude comparison game, *Fractions War*, was administered on an Apple iPad. The computer difficulty was set to medium, which made it so participants had 5 seconds to make a response before the computer would make the correct response. Unlike the task group, participants were not directly instructed to provide their response as quickly and as accurately as possible. Instead, participants were told that the goal is to earn as many points as possible, and the way to do so is to identify the larger fraction before the computer. Consistent with the task, if participants did not respond within the 5 seconds (before the computer response), the trial was marked as a miss; if participants responded within the allotted time, their response time latency from stimulus onset was recorded. Participants initiated each trial by tapping on their deck of cards.

Magnitude comparison Task The task group completed magnitude comparisons between symbolic fractions and nonsymbolic ratios in 6 conditions (Described in Chapter 2: Experiment 2). Only three of these conditions were included in our analyses of the current experiment. The three critical conditions included in our analyses were two nonsymbolic ratio comparison conditions containing either line ratios (LL) or circle ratios (CC) and single-digit symbolic fraction comparison condition (FF). These three conditions were blocked and presented in the same order: LL, CC, FF. Each block included 36 unique pairs, which varied the numerical distance between the stimuli (range = 0.023 – 0.75). Symbolic fraction stimuli had a maximum denominator size of 9, and all fractions were proper fractions with values between 0 – 1.

The magnitude comparison task was administered on a PC using E-prime 2.0. In the task stimuli were rendered in white and presented on a black background. Comparison pairs were presented on the screen for a maximum of 5 seconds, and participants were instructed to provide their response as quickly and as accurately as possible. If participants did not respond within the 5 seconds that the stimuli were present on the screen, the trial was marked as an incorrect miss; if participants responded within the allotted time, their response time latency from stimulus onset was recorded and task proceeded immediately to the next trial. In between all trials a small fixation square was presented for a variable duration jittered around a mean duration of 1500ms (range = 1250 – 1750ms).

Measures of Math knowledge and attitudes

Number line Estimation. Prior to participants completing either the game-based or task-based comparison activity, the number line estimation task was administered on a PC using E-prime 2.0. In the task stimuli were rendered in white and presented on a black background. 27 different single-digit irreducible fractions and 27 fractions representing the same magnitudes with double-digit components were presented one at a time, centered on the top of the screen. Along the bottom of a screen a number line was presented with the values 0 and 1 labeling each end of line. Participants were instructed to use the computer mouse and click the location on the number line which corresponded to the magnitude of the fraction presented on the top of the screen.

Performances on the number line task was measured via a calculation of percent absolute error (PAE) Specifically, the absolute magnitude of difference between the target magnitude corresponding to the presented fraction and the estimated magnitude corresponding to the location participant's response was divided by the magnitude of the number line's scale (1). Average PAE_{small} and PAE_{large} was calculated for each participant by averaging the PAE of all items within the single-digit and double-digit fractions, respectively. Smaller PAE values represent a more precise estimate of the fractions true value on the number line.

Fraction Knowledge Assessment. Immediately following participants completion of the game-based or task-based comparison activity, participants completed a paper and pencil-based fraction knowledge assessment (FKA). The FKA is a 38-item assessment measuring conceptual and procedural aspects of fractions knowledge. The first half of the assessment measured participants' conceptual knowledge of fractions, operationalized as accuracy completing multiple choice and open response items evaluating abilities to order fraction magnitudes, solve word problems with fractions, and explain their reasoning. The second half of the assessment, measured participants' procedural knowledge of fractions, operationalized as accuracy solving arithmetic (addition, subtraction, multiplication & division) problems with fractions. This assessment was constructed using items taken from national and international assessments such as the National Assessment of Educational Progress and the Trends in International Mathematics and Science study (Carpenter, 1981; Hallett et al., 2012).

Self-Report Survey. Immediately following the completion of the FKA, participants completed a self-report survey with four components. The first component of the survey was designed to measure an individual's *video game habits*. This included questions inquiring how much time individuals spend playing video games, what kinds of games and platforms individuals play, and what reasons people report for play video games or not.

The second component of the survey assessed attitudes towards mathematics. These attitudes were further broken down into separate constructs of *interest in the topics of mathematics*, *interest in math behaviors*, *confidence* in math, seeing the *utility value* of math, and seeing the attainment of math knowledge as part of one's *identity*. Each construct was measured using 7-point Likert items, where individuals indicated how much statements about mathematics were very true (7) or not true at all (1). All items in the math attitudes measure were adapted from items previously developed and validated to assess attitudes towards academic topics (Harackiewicz et al., 2016)

The third component of the survey assessed attitudes towards fractions specifically. These attitudes were further broken down to three separate constructs of interest in the topic of fractions, confidence in fractions and seeing the utility value of fractions. Each construct was measured using 7-point Likert items, via the same method as the math general survey.

The fourth component of the survey was a shorted version of the revised Math Anxiety Rating Scale (Hopko, 2003). Items in this scale are designed to two aspects of math anxiety: perceived level of anxiety while learning mathematics and anxiety when being evaluated on math knowledge.

Data preparation

Data preparation steps were completed to minimize variation between groups evaluate the differences between contexts without. First, we removed remove all hands (trials) in the game data where the accurate response involved identifying equivalent fractions, since these judgements of equivalence were not present in the task group. Second, in the game data we removed all hands where the denominator of the symbolic fraction was *10* since the value of denominators in the comparison task did not exceed 9. Third, we removed all hands in the game data where the value of either fraction presented was equal to 1 (e.g. 9/9), as all fractions in the task group had magnitudes less than 1.

Results

Distance and format Effects

Response times. In support of our use of *Fractions War* as a research tool to measure magnitude processing performance, we observed the presence of distance effects in gameplay data with all card types Figure 5.2. Specifically, we evaluated we evaluated the significance of these effects by fitting a mixed effects linear model on response times using the format of the comparison and the absolute numerical distance of the pairs as fixed effect predictors. We also accounted for the within-group structure of the game data by modeling the random effects slopes of these distance effects and format differences for each participant. In order to compare the similarity of format effects across contexts, we conducted the same analysis separately with the task data.

Results of mixed model of gameplay data confirmed that response time patterns showed a significant negative relationship between RTs and numeric distances when comparisons were made with traditional playing cards $\beta = -1319$, $t(29.2) = -10.9$, $p < .001$, and that the magnitude of this slope did not differ from those observed in comparison of symbolic cards, $\beta = -24.8$, $t(29.8) = -0.15$, $p = 0.988$, or nonsymbolic cards, $\beta = -240$, $t(30.1) = -1.10$, $p = 0.523$.

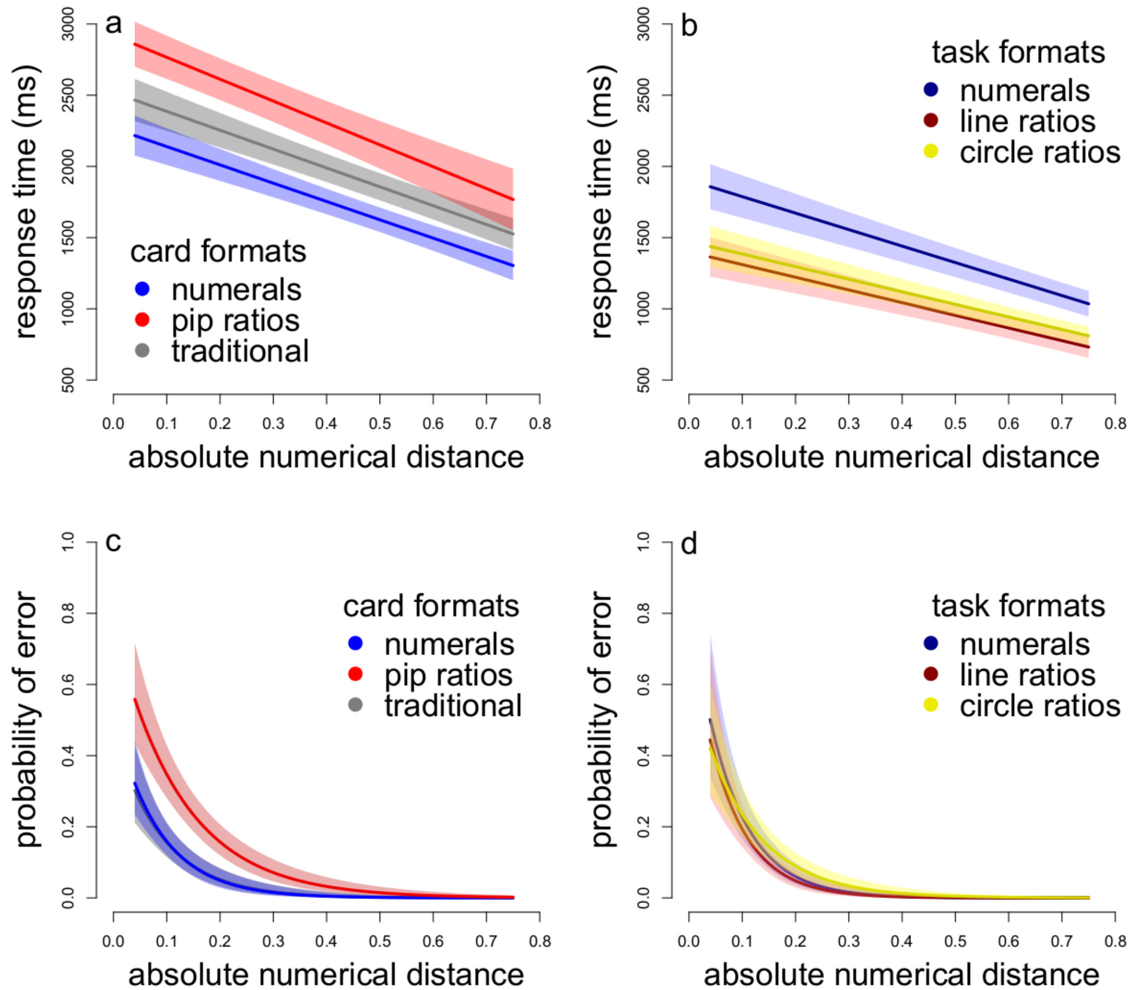

Figure 5.2. (Top) Linear mixed model estimations show group mean response times across varying numerical distances and demonstrate the presents of negative distance effect slopes in all format conditions of the (a) game and (d) task contexts. (Bottom) Logistic mixed model estimates show the mean probability of error across varying distances for the (c) game and (d) task contexts. Shaded regions indicated 95% confidence intervals around the predicted means.

In our comparison of format effects on response time across contexts, we observed a unique pattern in the game context relative to the comparison made within the task, as seen in Figure 5.2. Specifically, across all comparison distances, accurate comparisons of nonsymbolic cards in the game took about a half of a second longer than symbolic card comparisons, $\beta = 574$, $t(30.9) = 8.39$, $p < .001$. This effect is directly the opposite of what was observed in the task group, where the time to make accurate comparison of symbolic fractions took significantly longer than the comparison of nonsymbolic line ratios, β = 404.8, $t(39) = 10.6$, $p < .001$., and circle ratios, β = 328.1, $t(39) = 8.91$, $p < .001$.

Symbolic Fraction Context Effects on Error Rates

Overall mean error rates in the game group, *M* = 0.077 *SD* = 0.057, and task group, *M* = 0.060, *SD* = 0.053, were low, and the difference in ER across contexts did not reach statistical significance, $t(55.8) = 1.87$, $p = 0.067$. Mixed model fits of FF comparisons in game and task data, shown in Figure 5.2 illustrate that the low incidence of errors was more likely at when the numerical distance was very small, characteristic of distance effects. Mixed model results[2] confirmed this significant distance effect, β = -12.7, *SE* = 1.48, $p < .001$. These results show how distance effects on error rates emerge in both contexts and we do not observe any evidence to claim that the game context influenced magnitude comparison accuracy with fractions.

Magnitude comparison and fractions knowledge

We observed no group differences in FKA performances between the game, *M* = 32.2, *SD* = 3.98, and task group, *M* = 31.9, *SD* = 4.37. This exploratory analysis did not reveal any evidence to suggest that immediate performance on the FKA was in anyway influenced by the different comparison activities.

Previous studies have observed that individual differences in magnitude comparison performance are associated with broader assessments of fractions knowledge (Matthews et al., 2016). To determine if performance in *Fractions War* or our magnitude

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[2] The full mixed models indicated that the interaction term between distance and group effects was not significant, β = -1.20, *SE* = 1.89, $p = .525$, therefore this term was excluded in our tests of format and distance effects.

comparison tasks also predicts fraction knowledge, we evaluated the significance of correlations between mean error rates in each format condition and overall FKA scores.

FKA scores were not significantly correlated with Mean ERs in FF comparison made in either the game, $r(30) = 0.01$, $p = .952$ or the task, $r(38) = -1.22$, $p = .230$. Similarly, FKA scores were not significantly correlated with line ratio, $r(38) = -0.032$, $p = .847$ or circle ratio comparisons, $r(38) = 0.021$, $p = .897$, in the task. There was a modest correlation between FKA and pip ratio comparison in the game, $r(30) = -0.314$, $p = .079$, but with this sample size it was not significant.

Between group effects of math and fraction attitudes

Our analysis of self-reported attitudes towards math between groups revealed no significant differences in interest, $t(56.5) = 0.17$, $p = 0.864$, confidence, $t(59.8) = 0.16$, $p = 0.875$, perceived utility value, $t(59.0) = 0.946$, $p = 0.348$, math identity, $t(64.1) = 0.867$, $p = 0.390$, or interested in doing math activities, $t(62.4) = 0.11$, $p = 0.912$.

Experiment 1 Discussion

In this experiment we explored how magnitude comparison performance with symbolic fractions and nonsymbolic ratios is impacted when it is situated within an educational card game. First, we confirmed that *Fractions War* is a viable research tool to collect response time and error rate data of magnitude comparisons. Furthermore, we observed that response times and error rates in the game show significant numerical distance effects similar to the comparison task. These NDEs observed in gameplay were present regardless of whether participants played with the symbolic, nonsymbolic or traditional playing cards. Error rates within the symbolic fraction condition were also consistent across the game and task contexts, indicating that the game context had no observable positive or negative consequence on magnitude processing performance with fractions.

Interestingly, we observed that in the game context, participants required more processing time to make magnitude judgments with the nonsymbolic playing cards than when the cards presented fractions and in symbolic form. Whereas in the task group, this effect was reversed. Thus, we observed that the relative efficiency of nonsymbolic processing observed in the previous studies (Chapters 2 and 3), does not generalize to the context of *Fractions War* where nonsymbolic ratios are presented as the relative magnitude of two pip arrays. Due to the multiple ways in which these two contexts differed in their use of nonsymbolic visual ratios and the additional of gameplay features, it is unclear what drove the differences that we observed between groups. One possible explanation is the that pip arrays presented on the cards are a fundamentally different nonsymbolic ratio form than the continuously defined part-to-part ratios of lines or circles. The pips of playing cards present ratio components as discrete quantities, which may cue participants to count the values on each card. Thus, it may be the form of discrete nonsymbolic arrays within a small number range that slows nonsymbolic ratio processing. Alternatively, this could be driven by features of playing cards and the consequences of remove the symbolic numeral cues from cards that people may come to rely on. While traditional cards show both the numeral and the nonsymbolic array of pips, people may prefer the symbolic numeral for quickly identifying the cards value. By removing this numeral from the cards, we may have taken away a critical feature of cards, which people come to expect and develop fluency with. Furthermore, differences between groups may be explained by differences in extraneous load between contexts. *Fractions War* has an artful design with elements of competition and other unnecessary features that players are exposed to while interacting with fractions and ratios in the game, which are absent in the task. However, the results of study 1 did not present any clear evidence that these features had a negative effect overall.

As described in the methods section above, a number of fraction comparisons present in the game needed to be removed before a fair comparison between groups could be conducted. While manipulating the data after collection allowed us to compare performance between groups according to similar magnitude comparison trials, this means of data manipulation cannot remove the potential psychological impacts that some trials may have had on task performance as a whole. Therefore, these features were more carefully controlled in Experiment 2

Lastly, we did not observe evidence that playing *Fractions War* had a unique or immediate effects conceptual or procedural forms of fraction knowledge or on attitudes towards math. It is hard to draw any strong inferences from regarding why we observed no effect given the learning activity was very short and participants were adults who may have come into the study with established math knowledge and dispositions. Thus, fraction knowledge and math attitudes may be better characterized as measures of individual differences rather than a direct outcome of gameplay.

Experiment 2

Introduction

Experiment 1 tested questions regarding whether two different magnitude comparison activities lead to measurable differences symbolic fraction and nonsymbolic ratio processing. However, there were so many features of the game context which differed from the traditional experimental task, that it was difficult to draw any strong inferences about which features lead to differences across contexts and why. By recognizing these key limitations, multiple steps were taken to match the two activities, and thus more directly examine the effect of specific game features. A summary of these changes is presented in in Appendix D.

Current study

First, we aimed to better understand why nonsymbolic processing with pip ratios in the game was so much slower that symbolic fraction processing. As described above, varying forms of nonsymbolic ratios across the game and task contexts in Experient 1 made it difficult to know why ratio processing with pip ratios processing was so much slower. By contrasting gameplay with nonsymbolic cards to a well-matched dot-ratio comparison task, we aimed to rule out nonsymbolic form as a confounding variable, and more carefully test the effect using playing cards as informal representations of magnitudes. If higher errors and response time are due to features specific to the pips on a playing card, then we should replicate this finding in the game group, but not the task group. If, however, it is the case that nonsymbolic ratios with discrete and countable items hinder performance in general, then we should see higher errors and response time in both nonsymbolic conditions regardless of context.

Next, we considered how playing cards are physical (and digital) artifacts which some people have extensive experience with, and others have little exposure. Given that the visual design of typical playing cards has been very consistent for over the past 100 years (Hargrave, 2000), it is possible that some people develop specific representational competencies with playing cards (Rau, 2017; Gilbert 2005; NRC, 2006), such as the ability to see a pattern of pips and know the value it represents without counting or additional effort. This fluency may allow card players to focus less on deciphering a cards value and more on the goals and strategies of a game. Here we tested this hypothesis, that if individuals with higher levels of card playing expertise may exhibit more efficient magnitude comparison performances in *Fractions War*, if their expertise allows them to mitigate the extraneous features of the game context. Conversely, players with little to no card playing experiences may take longer to make magnitude judgements and make more errors, if their abilities to see the value of the cards are obscured by playing cards nontraditional form. Lastly, if card playing expertise is specifically beneficial to card games, then differences in expertise should have no effect on magnitude comparison performance outside of the game.

Next, we examined whether there are differences in how people judge their experience playing a comparison game relative to completing a comparison task? Educational games are suggested to be a context wherein interactions with content is associated with fun, play, and opportunities for low stakes failure. Whereas in the previous study we explored the possibilities that gameplay may lead to differences in self-reported attitudes towards math, here we investigated participants immediate reactions specific to the context of comparison activities. Therefore, we predicted that participants in the game group would report higher ratings of enjoyment completing the comparisons and would report a higher interest in reengaging with the experiences, than the task group.

Lastly, we explored the idea of engagement in games, and the idea that game features of competition, rewards, and aesthetically pleasing visuals can draw the learner's attention in ways that typical assessments and tasks do not. Specifically, after participants rated their experiences with the game and the task, they completed a symbolic fraction magnitude comparison task on the computer, with a mixture of novel fraction pairs and pairs participants saw during the comparison activity. If the experiences of comparing fractions in a game context, encourage more attention to the content or makes errors in the game more salient than the task, this may lead to better performances on the follow-up task.

Methods

Participants

Game Group. 45 undergraduates[3] completed the study in the game group. Two participants were excluded because they did not complete the game play portions of the study in full. After applying the inclusion criterion that accuracy in all game conditions must be greater than 60%, data from four participants were excluded from our analyses. As a result, data from 39 undergraduate students (33 females) were included in our analyses.

Task Group. 51 undergraduates completed the study in the task group. After applying the inclusion criterion that accuracy in all game conditions must be greater than 60%, data from three participants were excluded from our analyses. As a result, data from 48 undergraduate students (38 females) were included in our analyses.

Procedure

Prior to arrival, participants were assigned to either the game or the task group. All participants first completed two tasks to assess their expertise and fluency with playing

[3] Need to access paper documents on campus to obtain age related data.

cards. Next, all participants completed a self-report questionnaire, very similar to the one used in Experiment 1. Then, all participants completed the Fraction Knowledge Assessment. Participants then completed either three comparison tasks on the computer or three *Fractions War* games on an iPad depending on their group assignment. Immediately following the comparison activities all participants completed an activity rating to express their experience completing the games or the tasks. Next, participants completed a symbolic fraction comparison task containing trials that participants had completed in the comparison games and tasks, and novel comparison pairs. Lastly, participants completed a demographic survey and received partial course credit for completing the study.

Between-Group Magnitude Comparison Activities

Magnitude comparison game. Consistent with Experiment 1, participants in the game group played three games of *Fractions War*, using the three different cards types: traditional cards with numerals and pips, nonsymbolic cards with only pips, and symbolic cards with large numerals.] For this experiment, we picked 80 fractions culled from all fraction pairs randomly presented to the game group in Experiment 1, with a mixture of component congruencies. We then imported our custom decks into *Fractions War,* so that the game would present these fraction pairs in the same order for all participants. We randomly rearranged the order to create the lists for each of the three conditions

Magnitude Comparison task. We developed a new magnitude comparison task to complement the experience of playing *Fractions Far*, without game-base features. We evaluated how the game and task in Experiment 1, differed in unnecessary or confounding ways, and built features into the tasks to address these gaps. this task to address those differences. Frist, this included matching the conditions of the magnitude comparisons in the game, with a symbolic dot ratio comparison, a symbolic comparison, and an integrated comparison form where dot arrays and the corresponding numeral were presented together.

Card Playing Expertise

Sorting Task. The sorting task was created to evaluate participants' expertise with the visual form of playing cards. Participants were presented with a list of ten playing cards. Five of the 10 cards were presented with the traditional arrangement of pips (e.g. diamonds), and the other five were presented with pip arrangements that had been modified to nontraditional arrangements. Cards were presented on a computer screen via an online program, Qualtrics. Participants were instructed to sort the 10 cards with a drag and drop function to either the traditional or nontraditional category. Participants could get a total of 10 points in the sorting task for correctly soring all cards into the proper category.

Matching Task. The matching task was created to measure participant's fluency with recognizing the value of playing cards. In the task two numbers were presented on the screen side by side. On the left side of the screen, participants saw 1-10 diamonds presented on playing card. On the right they saw an Arabic numeral, 1-10. In half of the trials, the number of diamonds on the card matched the numeral, and in the other half the two values did not match. Participants were told to indicate if the values matched as quickly and as accurately as possible Card fluency scores were calculated by taking the average response time to trials where the magnitude of the match was greater than 3.

Self-Report Survey. We used same self-report survey as Experiment 1, except for one section of the video game habit section, in which we added items specifically asking about prior experience with playing cards.

Fraction Knowledge Assessment. We used the same FKA as experiment 1, with no changes to its form or administration.

Activity Rating. A brief rating survey was created to capture participants immediate reactions to the magnitude comparison activities. Using Likert-scale items, participants indicated how much they enjoyed the comparison activity, how challenging they found it to be and how confident they were in their performance. In reflecting on all three conditions, participants were asked to rate their overall enjoyment and their desire to complete the task again.

Results

Distance and format effects

Response time. We fit linear mixed models predicting response times across varying numeric distances and across formats (with the same specifications as Experiment 1). The mixed model of gameplay data confirmed that response time patterns showed a significant negative relationship between RTs and numeric distances when comparisons were made with traditional playing cards $\beta = -1027$, $t(37.9) = -11.4$, $p < .001$, and that the magnitude of this slope did not differ from those observed in comparison of symbolic cards, $\beta = -135.2$, $t(61.2) = -0.96$, $p = 0.338$, or nonsymbolic cards, $\beta = -255$, $t(61.3) = -1.76$, $p = 0.083$.

Results of this analysis replicated the finding Experiment 1 findings of game data showing that participants took longer to make nonsymbolic magnitude judgements between pip ratios in the game than judgements between two symbolic fractions *β = 317*, $t(38) = 2.46$, $p = .047$. The difference between nonsymbolic and symbolic comparisons in the in the task group did not reach statistical significance but trended in the same direction as the game group *β = 125*, $t(47) = 2.23$, $p = .077$, $d_r = 0.215$. The significant difference observed between formats in the game group and not in the task group may indicate that these nonsymbolic representations in the game were less efficient, however it is hard to draw strong inferences from these modest effects in both groups. Key comparison of nonsymbolic performance across task will be assessed in error rates below.

Error Rates. We fit logistic mixed models on error rates across numerical distances and formats using the same specifications as the models used in Experiment 1. Our initial model revealed a peculiar fit of error rates to the nonsymbolic comparisons in both conditions, where estimates of error exceeded beyond a chance performance (probability > 0.5). Follow-up analyses of group error rates for each nonsymbolic comparison pair revealed 8 pairs from the game and 9 pairs from the task (5 pairs in common) with group mean error rates above .5, meaning that participants at the group level were more likely to decide the fraction with the smaller magnitude is actually the larger. Examining these up these high error trials, revealed that they were all comparisons of small numerical distances (range = 0.014-0.194), and composed of ratio pairs where either the smaller pair was composed of less total dots (e.g. 2/7–1/3, 3/4–5/9) or the larger pair had larger gap distances (e.g. 2/9-1/5). These problematic trials were specific to the nonsymbolic condition.

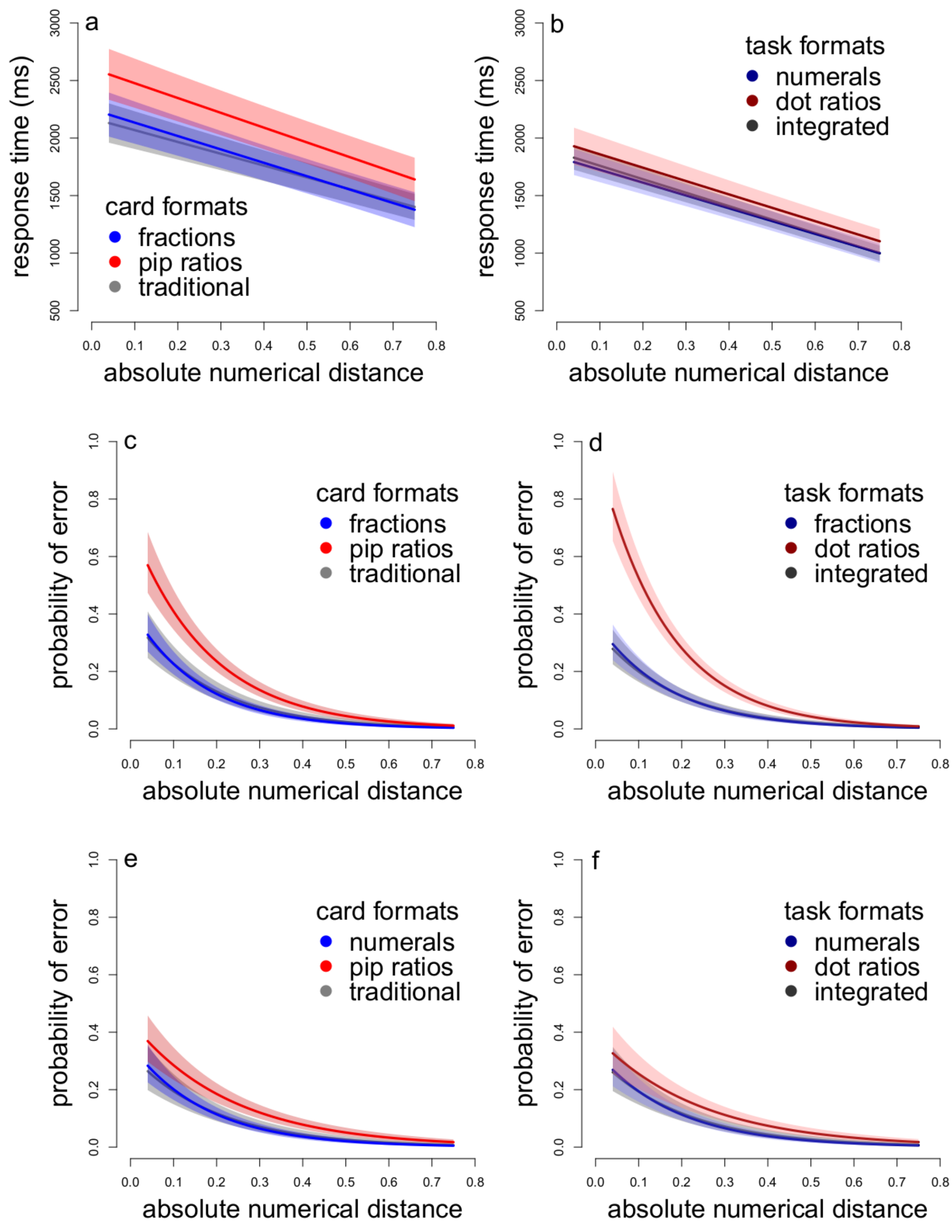

Figure 5.3. Linear mixed model estimates of response time in *Fractions War* gameplay (a) and in the comparison task (b) Logistic mixed model estimates of error rate probability in the *Fractions War* gameplay (c/e) and in the comparison task. (d/f). Shared regions indicated 95% confidence intervals around the predicted means.

After fitting the models again with these problematic trials removed from all conditions, we still observed significant format effects in both the game and task groups. Specifically, error rates in the game group were higher during nonsymbolic pip ratio comparisons than symbolic fraction, $\beta = 0.603$, *SE* = 0.09, $p < .001$, *OR* = 1.83, and integrated comparisons, $\beta = 0.586$, *SE* = 0.10, $p < .001$, *OR* = 1.80. Likewise, error rates in the task group were higher during nonsymbolic dot ratio comparisons than symbolic fraction, $\beta = 0.950$, *SE* = 0.09, $p < .001$, *OR* = 2.59, and integrated comparisons, $\beta = 0.991$, *SE* = 0.09, $p < .001$, *OR* = 2.70. These results show how magnitude processing with nonsymbolic ratios that use only a small number of dots (1-10) to represent their component magnitudes is less efficient that symbolic fraction processing regardless of the specific context or form.

Between group comparisons of error rates.

Across groups we compared the mean error rates predicted by these models at near distances (distance = 0.2) for each format. These comparisons revealed that there was no difference error rates during symbolic fraction comparison between the game, *M* = 0.18, *SE* = 0.01, and task, *M* = 0.17, *SE* = 0.02 groups, $p = 0.582$, *OR* = 1.07. Likewise, we observed no difference in error rates with traditional cards in the game, *M* = 0.05, *SE* = 0.01, and integrated forms of the task, *M* = 0.05, *SE* = 0.01 groups, $p = 0.274$, *OR* = 1.16. However, we did observe that in mean error rates during nonsymbolic fraction comparison in the game, *M* = 0.08, *SE* = 0.01, were significantly lower than in the task, *M* = 0.11, *SE* = 0.01 group, $p = 0.004$, *OR* = 0.78. Therefore, despite the additional complexity of the game context and the playing cards relative to the dot ratio stimuli, (variable suits of red and black vs black dots), participants were able to make more accurate judgments in the game among these more difficult near comparisons.

Effect of Card Playing Expertise

Performance in the playing card sorting task provides insights into how well individuals can recognize the traditional arrangement of pips representing cards values on a playing card and when these arrangements are altered. On average participants in the game group, *M* = -0.80, *SD* = 0.12, and the task group, *M* = 0.78, *SD* = 0.17, sorted a high proportion of the cards into the proper categories. We identified seven game group participants and eight task group participants who sorted all cards perfectly, but to make comparisons of high expertise and nonexperts with similar numbers between these subgroups, we defined high expertise as having a one or fewer errors on the sorting task (n_{game} = 13, n_{task} = 18) and nonexperts as having two or more error on the sorting task, (n_{game} = 24, n_{task} = 30).

First, we examined if there were differences in fraction knowledge (FKA score) across groups to confirm that differences in card expertise were not confounded by differences in math abilities. Indeed, we found no mean differences in FKA performances across groups, $F(1,81) = 0.316$, $p = .576$, expertise, $F(1,81) = 0.053$, $p = .818$, or the group X expertise interaction, $F(1,81) = 0.108$, $p = .744$.

Next, we tested whether differences in card playing expertise were associated with performance in the varying formats of the magnitude comparison activity. Specifically, we focused on effects of error rate given the differences in the RT scale between touch screen responses in the game and keyboard responses in the task. Consistent with our predictions, participants with playing card expertise had a specific advantage when making nonsymbolic judgements in the game, but not in the task. Specifically, we observed a that significant group × expertise interaction, $F(1,81) = 7.37$, $p = .008$, and significant different between groups, $F(1,81) = 13.9$, $p < .001$, but the main effect of expertise was not significant, $F(1,81) = 1.80$, $p = 0.184$. Overall participants completing the comparison task made more errors with dot ratio comparisons than participants comparing pip ratios in the game, $t(74.6) = 3.54$, $p < .001$. In the comparison task, playing card expertise had no advantage for nonsymbolic dot ratio comparison, $t(26.4) = 0.68$, $p = 0.500$. but within the game participants with playing card expertise significantly outperformed their nonexpert peers, $t(23.7) = -2.94$, $p = 0.001$. Interestingly, we did not observe any main effects of expertise when comparisons were made with symbolic fractions, $F(1,81) = 0.062$, $p = .805$, or intermixed representations, $F(1,81) = 0.074$, $p = .785$.

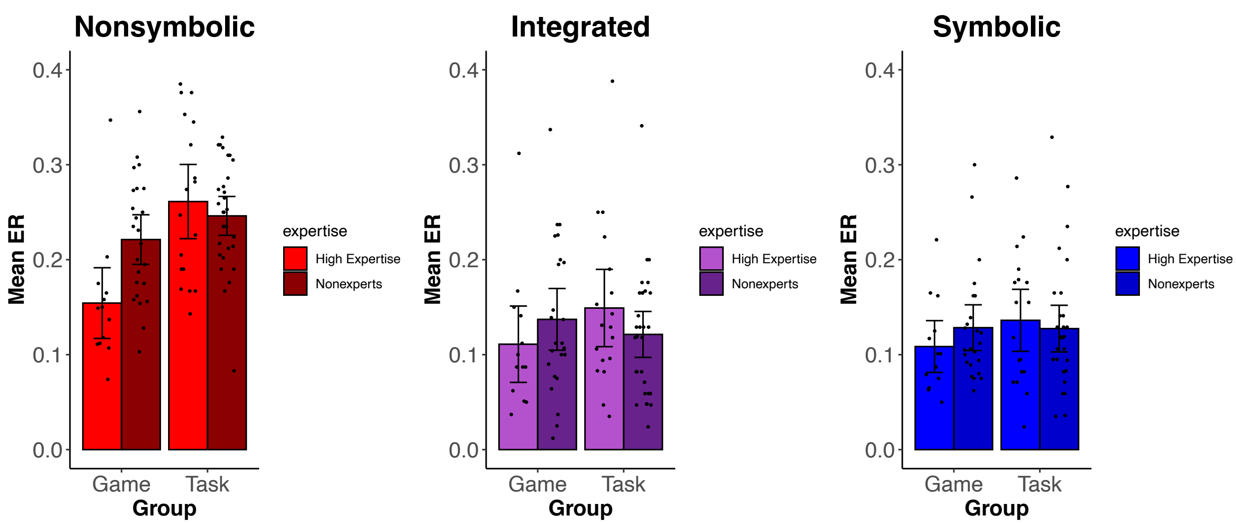

Figure 5.4. Mean error rates for participants with high levels of playing card expertise relative to nonexperts during comparisons of nonsymbolic (red), integrated (purple) and symbolic (blue) representations of fractions and ratios in the game and in the task. Error bars show 95% confidence intervals, therefore significant differences can be seen with error bars do not include the mean of another condition.

Discussion

In these experiments, we demonstrated ways that an educational math game, *Fractions War*, can successfully support empirical investigations of magnitude processing with fractions and ratios. First, the game provided an engaging interface where participants could play the game without data collection intruding their gameplay experiences. Second, we observed that gameplay responses followed traditional numerical distance effect patters seen also in the task group data. Third, we were able to examine magnitude comparison performances across three different card types and reveal different aspects of symbolic and nonsymbolic magnitude processing. Though our comparisons of magnitude processing data across groups we observed that using playing cards in games to represent these magnitudes does not lead to worse performances. Furthermore, by comparing the performances of playing cards experts to nonexpert peers we observed that prior knowledge with informal play-based representations of numbers, playing cards, can actually drive significantly more accurate magnitude processing of nonsymbolic ratio arrays.

On primary question of Study 2, was whether inefficient magnitude processing with pip-only playing cards was due to features of playing cards specifically or due to the processing their nonsymbolic form. Specifically, nonsymbolic ratios made with two playing cards contain two juxtaposed pips patterns (or arrays), and these arrays are presented as countable (1-10) and discrete quantities. By evaluating the performance of magnitude comparison with these pip arrays to analogous forms dot ratio comparison, we did not observe evidence that playing cards specifically made magnitude processing any harder. On the contrary, participants in the game condition were more accurate when making these ratio comparisons than participants reasoning with the controlled stimuli. Therefore, it appears that inefficient processing with discrete arrays of small quantities is not specific to playing cards but may be due to the ways these nonsymbolic forms invite inefficient strategies to enumerate these nonsymbolic quantities. This finding is consistent with previous studies showing that children have more difficulties discerning the magnitudes of nonsymbolic ratios when presented in discretized forms than with continuous forms (Abreu-Mendoza et al., 2020; Boyer & Levine, 2015).

The results of these studies lead to the conclusion that representing rational number magnitudes as the relationship between the pips of two cards is neither intuitive nor efficient, but it is a representation that people can become fluent with. Indeed, we observed that participants took the longest time to judge these ratios and still made the most errors. Specifically, item analyses in Study 2 identified instances where adults may have conflated the total number of dots in a ratio with its holistic magnitude and as a group made errors more than half of the time. However, this does not rule out the potential for these representations to be effective learning tools in *Fractions War*. Knowledge included in the learning goals of educational games are often the means by which players are able win the game, and failure states are designed into the gameplay experience in order to highlight misconceptions the player needs to realize. The single study design of these experiments allowed us to explore how adults reason with these game-based representations of magnitude. However, future research observing performance across multiple games is necessary to observe whether performance in these difficult nonsymbolic trials improve, and if improvements coincide with insights into why their initial biases where incorrect.

Critically, not all participants in our study had specific difficulties with these nonsymbolic ratios in playing cards. Participants with visual expertise in the pip patterns that represent the cards numeric values appeared to be better at nonsymbolic magnitude comparisons than their nonexpert peers. It is interesting to consider the multiple mechanisms which may have driven this effect. Base on previous research on the use of symbolic, iconic, and naturalistic visual representations some digital design researchers explain that the cognitive load of iconic representations is largely affected by prior knowledge of the learner (Lee et al, 2006; Plass et al., 2009). These improvements may come from developing "top-down" conscious methods to suppress extraneous details a selectively focus on key features (Desimone & Duncan, 1995). Alternatively, this may come from repeated experience with these representations that reasoning with them becomes automatic or in ways unconscious. And additional feature of playing cards relevant to the characterization of their pip patterns as nonsymbolic representations, is the fact that traditional playing card patterns have remained constant for hundreds of years (Hargrave, 2000). Playing cards are artifacts of culture, and individual who play games with cards may even generate preferences for particular card values or suites (Olson et al., 2012). The patterns of card's pips may for some become so familiar that they perceive these arrays as more of a symbolic representation that directly cues the number it refers to rather than a collection of entities which add up to that magnitude. Support for this notion may be seen in the results of Study 2, which showed that the inclusion of symbolic numerals on cards lead to faster and more accurate magnitude comparison performance, any of the purely nonsymbolic conditions. Thus, if experts can parse the patterns of pips on a cards as in ways similar to symbolic mappings between whole numbers and their value, then this may explain why they exhibited these specific advantages in the game when others specifically struggled.

In this study we did not observe the predicted increases in enjoyment with the game relative to task condition. To understand why this game did not elicit a stronger or more positive response in this population, it may be important to consider the age of the participants and aspects of the experimental environment. The participants in these experiments were college aged students, and the target age range for this game is children in elementary and middle school. Furthermore, aspects of the experimental conditions may not be optimal for supporting a true gameplay experience. In attempting to define what games are, scholars have emphasized that these experiences are to be contexts separate from the real world (Garris, Ahlers, & Driskell, 2002), that they are fundamentally voluntary in nature (Granic et al., 2014). The context of playing a game in a research study, although significantly different from traditional lab tasks, requires a strict administration of gameplay to ensure the data reflects a specific research design. Thus, playing games in these contexts may break a fundamental feature of fun games, which is the freedom to choose to play and explore the possibilities of the experiences through play. Future, studies with *Fractions War* and other educational math games hold the potential to further elucidate the relationship between affect and cognition in math education, especially in studying the effects of fostering positive associations. These investigations stand to broaden our understanding of mathematical cognition theories, while also developing applied knowledge about how to leverage intuitive and peculiar representations of learning in digital learning environments.

Chapter 6 – Conclusion

Summary of Findings

The work in this dissertation applies an educational neuroscience approach to the question of how people access the meaning of symbolic fractions and nonsymbolic ratios. Ratio Processing System (RPS) theory proposes that knowledge of rational number concepts, which are typically associated with formal math education, actually have fundamental roots in our perception of proportions and ratios in the world (Jacob et al., 2012; Lewis et al., 2015). Indeed research with young children indicates that the ability to discriminate visual ratios appears to emerge prior to any formal schooling (Duffy et al., 2005; Jeong et al., 2007; Newcombe et al., 2015). These findings indicate how systems in the brain may develop to support a sense of ratios that underlies spatial and numerical decision making in implicit ways. It has been proposed that these very systems may provide a foundation upon which fraction knowledge can be built (Jacob et al., 2012; Lewis et al., 2015), yet there is much to learn about nature of this theorized relationship between symbolic and nonsymbolic ratio processing. Using three separate empirical approaches, the studies presented in this dissertation addressed unique facets of symbolic and nonsymbolic magnitude processing and revealed three complementary forms of evidence that could not be revealed through any one approach.

One outstanding question about the relationship between symbolic fraction and nonsymbolic ratio processing is whether an internal representation of rational number magnitudes can be described as abstract. Just as a fractions can represent multiple forms of meaning (Behr et al., 1983; Kieren, 1979), our semantic representation of rational number magnitudes may be similarly independent to the modality and representational form with which that meaning is conveyed (Dehaene et al., 1998). In line with this theory, Bonn and Cantlon (2017) proposed that ratios are a ubiquitous and naturally dimensionless phenomena that can be seen across modalities and forms, and our abilities to represent these relationships fluidly may constitute a generalized magnitude system in the mind. A similar perspective on the connection between abstract symbol knowledge and perceptual experiences comes from proponents of grounded and embodied cognition theory (Barsalou, 2008; Goldstone & Barsalou, 1998). Such theories argue that understanding the meaning of abstract concepts, such as numbers and fractions, is reliant on perceptual and motor systems to ground their meaning, and it is through that simulation and activation of these systems that we represent the meaning of these concepts (Barsalou, 2008). In the first two studies of this dissertation, we examined the nature of understanding magnitudes with symbolic fractions and nonsymbolic ratios through these perspectives.

One criterion for identifying abstract representations of number proposed by Dehaene and colleagues (1998), states that behaviors and neural activity should be modulated by the magnitudes of these numbers independent of the format with which numbers are depicted. In the three studies of this dissertation we tested for evidence that meets these criteria. Study 1 tested for the presence of similar numerical distance effects in response times and error rates during rational number comparison with symbolic fractions and nonsymbolic ratios. Results of this study indicated that behavior in these cases was indeed significantly modulated by the relative magnitudes of the stimuli regardless of the presented format. Study 2 introduced a neuroscience approach to test for the presence of analogous modulation of neural activity based on changes in the magnitudes of symbolic fractions and nonsymbolic ratios. Results of whole-brain and region of interest analyses revealed regions of the brain that showed specific responses to relative magnitude differences for both symbolic and nonsymbolic processing. The results of both studies support the RPS view that the meaning of rational number magnitudes can be represented in abstract ways which generalize across formats. Nevertheless, nuances in the shape of these effects persist across formats, which may reflect how the external features of these two formats cue other strategies and biases during holistic magnitude processing

In Study 3, we continued to test for shared representations across formats. Furthermore, we explored how features of fractions' and ratios' visual form influenced magnitude processing. We used three different playing card formats in the game to present learners with symbolic, nonsymbolic, and intermixed representations of fractions and ratios. Gameplay data from these decisions provided further evidence that numerical distance effects emerge in performance similarly regardless of the cards' format. Interestingly, we observed the slowest performance with the most errors in games with the nonsymbolic pip-only cards. In Studies 1 and 2, efficient nonsymbolic ratio processing with continuously defined line ratios and circle ratios was taken as evidence that magnitude processing of visual ratios is an efficient perceptual process that can occur without enumeration or complex strategies. However, explaining the inefficient nonsymbolic processing of pip ratios (e.g. 5 diamonds over 6 clubs), required additional theoretical perspectives beyond Ratio Processing Theory and theories of grounded cognition.

Here perceptual learning theories (Kellman et al., 2008; Rau et al., 2017) and cognitive load theory (Mayer, 2002; Moreno & Mayer, 2009) provided perspectives to test why participants were not able to fluently connect visual representations of pip ratios in the game to rational number meaning. We observed that it was not the specific nature of playing cards or their use in a gameplay context that made the discrimination of pip arrays less efficient. Analogous and simplified dot ratios presented outside of the game context were just as slow and error prone. Thus, these results are consistent with arguments that visual ratios with discretized parts, are less efficient than ratios with continuous parts, because they cue participants to fixate on the discrete countable parts rather than the overall proportional relationships (Jeong et al., 2007). Study 3, therefore, provided further evidence that response times and errors were modulated by the magnitudes of these stimuli regardless of format, but a visual features specific to these formats may compete for cognitive resources during magnitude processing (Desimone & Duncan, 1995).

Visual Complexity and Magnitude Processing.

One common finding that emerged across the controlled experimental studies (Study 1 and 2) and game-based studies (Study 3) of this dissertation is that as the complexity of external representations made it harder for adults to attend solely to the holistic magnitude of fractions and ratios. In Study 1, we observed that adults can process two continuously defined nonsymbolic ratios—line ratios and circle ratios—faster than symbolic fractions. Furthermore, between these two continuously defined nonsymbolic ratio forms, circle ratios took significantly longer to discriminate. Multiple features of circles, such as their circumference and diameter may compete for perceptual and attentional resources even when participants are instructed to attend to the circle's area. On the other hand, these additional dimensions may not enter the minds of people observing the simple lengths of a line ratio. Likewise, we observed that among symbolic fractions, increasing the values of the components lead not only to slower responses, but also stronger effects of component congruity and gap distances. Thus, the complexity of symbolic fraction's orthographic form may also distract adults from focusing on the fractions' holistic magnitude. In the first experiment of Study 3, comparisons between nonsymbolic processing of pip ratios in *Fractions War* and processing of continuously defined line and circle ratios indicated that fluency with pip ratios from informal playing card representations was hindered by extraneous features of their complex visual forms. Results of the second experiment of Study 3 indicated that representations of ratios with discretized parts, especially with nearly countable quantities, may not facilitate an efficient perceptually based discrimination of magnitudes. Therefore, across these three studies, we observed that holistic magnitude processing can occur for line ratios, circle ratios, dot ratios, pip ratios and symbolic fractions, but also that increasing visual complexity of these external representations may obscure an efficient and more purely perceptual form of relational reasoning that was observed with simple line ratios. Further research aimed at identifying how visual features of ratios and fractions can support or distract from learning is necessary to further understand these affects and their implications for instruction. Learning about fractions and ratios from creative visual representations is part of our current math education system and understanding the ways that students misinterpret these external representations may be important for supporting the learning goals of educators and instructional designers.

Comparisons of magnitude processing with fractions and ratios between an educational game and a traditional lab task revealed ways that prior knowledge can influence magnitude processing with symbolic fractions and nonsymbolic ratios. Specifically, analyses of playing card expertise and the representational fluency which comes from this this expertise (Kellman et al., 2010), revealed how influential background experiences can be in the digital learning environment. Beyond *Fractions War* these results highlight how the choice of novel representations used to convey educational material in the games may be more or less successful across individuals depending on their prior content knowledge and fluency with those visual forms. The potential for players' gameplay experiences to vary with background knowledge is an important consideration both for the design of new educational games and in applications stealth assessment. From a design perspective, efficient understanding of educational content in video games will require the formation new representational fluencies with novel and complex visual representations (Rau & Matthews, 2017), and it is the job of instructional designers to guide learners to develop these fluencies. Furthermore, if knowledge is being assessed through gameplay, it becomes important to consider whether improved performance reflects true mastery of content knowledge or specific expertise in the representations used in the game to convey the content. Attempting to dissociate mastery of content knowledge from in gameplay from expertise specific to game design is tricky but may be addressed in multiple ways. For instance, identifying expertise with specific gameplay representations via external measurement is an initial step that may explain significant variance in gameplay performance (consistent with the approach applied in Study 3). Furthermore, games for stealth assessment may be designed to integrate multiple gameplay scenarios aimed to assess different facets of content knowledge in unique ways that are not reliant any one representation of educational content.

Training the Ratio Processing System

A key argument of RPS theory is that nonsymbolic ratio processing abilities can provide a grounding for the development of richer symbolic fraction knowledge (Lewis et al., 2015). This argument should be tested with learning interventions which directly apply forms of cross-format sense making aimed at building up representational fluencies across various visual forms. It may be that traditional conventions of teaching fractions, whether that be introducing the topic years after whole-number instruction or relying largely on the drilling of symbolic fraction operations, are contributing to the prevalence of whole number biases and overgeneralization of component-based strategies when reasoning with fractions. Thus, it may be that learned "rules" of fractions or "tricks" to reason with fractions may be getting in the way of a deeper understanding of how these symbols represent real magnitudes (Lewis et al., 2015; Siegler et al., 2011). Therefore, there may be approaches to learning about fractions that educators and students may think are unobtainable, and therefore, never attempt to attain. This elusive form of knowledge can be described as having an intuitive feel for numbers and magnitudes which comes from appreciating the relative magnitude of one value to another. Building this intuitive and grounded fluency with symbolic fractions may not be a critical skill for interacting with numbers in daily activities, but it may be a skill that when acquired enables fluency between a fractions symbolic form and a more automatic sense of the meaning these symbols represent.

Furthermore, investigations into fluency-based interventions of fraction and ratio processing may be strengthened by perspectives that consider the role of sense-making activities in conjunction in improving fluency and transfer (Rau et al., 2017). On one hand, repetitive and rapid practice in conditions which push individuals to practice and refine their intuitions with symbolic fractions and nonsymbolic ratios may help people break from rehearsed tricks and strategies with fractions. On the other hand, explicit instruction and deliberate reflection on the commonalities and differences between symbolic fractions and nonsymbolic ratios may be necessary for building deeper conceptual understandings which may support the transfer of this knowledge across contexts. Critically, these applied theories of fractions instruction should be validated by testing them within authentic learning interventions such as cognitive tutors and educational games both inside and outside the research lab.

Future applications of math games in educational neuroscience research

Games leave traces in the mind and in data.

One way to advance the field of educational neuroscience, is to embrace educational video games as an important context for testing cognitive neuroscience theories of learning (Bugden et al., 2017; Rosenberg-Lee, 2018). At the heart of this argument is the idea that these digital experiences leave traces in the mind and in data. Theories about the benefits of playing video games and specifically educational games (Granic et al., 2014; Green & Bavelier, 2003), indicate how these digital interactions leave traces in the mind. For instance, games allow players to improve their hand eye coordination, practice strategic decision making, and encounter multiple instances of failure and success. Moreover, educational video games have the potential to help players acquire new content knowledge or skills. In a complementary way, methodological arguments for the use of educational games in education and in research emphasize how cognitive effort put forth to engage with these digital experiences leaves traces in forms of gameplay data (Halverson & Owen, 2014; Shute, 2011). For game developers, this game data is typically used to track bugs in software code and determine if their original designs are resulting in an enjoyable game experience. However, from an educational or psychological research perspective, these data traces provide a digital reflection of knowledge and abilities. With this knowledge, the careful crafting of digital educational games can create powerful tools to observe and assess cognitive abilities and knowledge for educational and research purposes.

The game-based research in this dissertation exemplifies how a video games can be utilized to test specific hypotheses of cognitive processing. Video games as programed audio-visual experiences provide a medium that can be designed and manipulated to assess the educational efficacy of contrasting cases (Wideman et al., 2007). In this dissertation, several design features of *Fractions War* were utilized to examine how the visual complexity of educational representations influences players' fluency connecting these visuals to the magnitudes they represent. For instance, the ability to switch gameplay between different cards types enables a direct comparison of magnitude processing across formats while keeping the rest of the gameplay experience constant. Furthermore, the ability to design and load custom decks allows control over which magnitudes and formats players interact with in the game. Beyond *Fractions War*, similar approaches may be applied to the visual and aesthetic design of new video games with cognitive and affective goals. By programmatically studying the specific influence of gameplay features, such as the design of visual representations for learning, this approach stands to identify not only effective design principals but also insights into why these design choices matter. Furthermore, future research focused on the design of effective, intuitive, and engaging educational games stands to unravel the critical relationship between affective goals to increase enjoyment and engagement and cognitive goals to support the development of new knowledge and skills.

Studying affective and cognitive effects of game-based learning.

In real world learning contexts, factors impacting individuals' abilities to learn math and express their knowledge may be tied to their affective dispositions towards mathematics. While situational interest in mathematics may be triggered by specific experiences, developing a deep personal interests in math topics may be built up over time and across multiple learning experiences (Hidi & Renninger, 2006). In educational game design and research, a common question regards what the role of educational games could be in the developing positive emotional associations with math content (Plass et al., 2013, 2014)? Within a specific learning experience, the context that educational content is situated within may have a significant impact on the emotional response that individuals experience with that content. Educational games are suggested to be a context wherein interactions with content is associated with fun, play, and opportunities for low stakes failure (Gee, 2003b, 2007). One crucial factor in studying this phenomenon is that the assumption that educational games are a fun and engaging context to play with educational content is only as true as the player's subjective perspective. Therefore, future studies aimed at exploring the affective effects of games require an appropriate fit between the subject matter of the game and the interests of the learners engaging in these activities.

In this dissertation's exploration of whether playing a fractions game may affect math attitudes, we did not observe any large positive effects. However, these findings are not necessarily surprising given the short duration of gameplay in our studies (24 minutes) and the age of our adult participants. It is possible that for undergraduate-aged adults, small bouts of gameplay have minimal to no effect on math attitudes when these attitudes are the product of many years of formal and informal education. Thus, critical tests of whether games like *Fractions War* may influence players' affective associations with fractions, ratios, and math more generally should be done in younger populations and with longer intervention programs. Furthermore, one of the most powerful attributes of using games to conduct this research is the potential for these games to be implemented within authentic learning contexts of schools and home learning environments. Further research is necessary to better understand how gameplay situated within traditional research lab contexts relates to more self-directed forms of gameplay in school and casual gaming contexts, especially in terms of the player's enjoyment and motivation to play.

In defense of an educational neuroscience approach

Advancing educational neuroscience research involves not only conducting interdisciplinary research but also advocating for taking an interdisciplinary approach. The educational neuroscientist sees an exciting potential to integrate empirical knowledge of the human central nervous system with educational researcher's and practitioner's knowledge of learning and teaching. This potential can be simplified to the fact that our nervous systems are the biological material with which we learn. Learning creates mental changes in what we can know or do, and that necessarily corresponds with biological changes in our brains and our brain's connection to our body. Thus, neuroscience can offer a unique window into the mind to observe mechanisms of human learning that complement methods of observing changes in behaviors and abilities. If the pursuit of studying the developing mind can help us enhance teacher training, support students with learning disabilities, or make better educational resources, then there is an exciting potential for researchers studying the brain to advance these goals in new ways. Moreover, it is in our attempts to translate theories of learning from cognitive neuroscience research to real life applications that we are able to test the ecological validity of these theories. Thus, while academic silos may separate neuroscience and educational researchers for epistemological or methodological reasons, there remains a great potential that once separate fields can symbiotically advance through interdisciplinary collaboration with each other.

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Appendix A: Symbolic Fraction Pair Stimuli Lists of Study 1

Table A1

Symbolic fraction pairs used in Study 1 Experiment 1 and Study 2

Component Congruence	Pair	Distance
Common Denominator	7/9_8/9	0.111
	3/7_4/7	0.143
	2/5_3/5	0.2
	3/7_5/7	0.286
	2/5_4/5	0.4
	1/9_5/9	0.444
	2/7_6/7	0.571
	1/5_4/5	0.6
	1/6_5/6	0.667
	1/8_7/8	0.75
Congruent Numerator & Denominator	4/5_6/7	0.057
	3/5_5/7	0.114
	2/3_5/6	0.167
	1/7_4/9	0.302
	1/4_5/8	0.375
	1/3_5/7	0.381
	1/4_7/9	0.528
	1/3_8/9	0.556
	1/7_7/9	0.635
	1/4_8/9	0.639
	1/6_6/7*	0.690
Congruent numerator & incongruent denominator	1/8_2/7	0.161
	4/9_5/8	0.181
	1/6_2/5	0.233
	4/7_5/6*	0.262
	5/9_7/8	0.319
	1/8_4/7	0.446
	1/9_5/8	0.514
	1/5_3/4	0.55
	2/9_7/8*	0.652778
Incongruent Numerator & congruent denominator	2/7_1/3	0.047619
	3/8_1/2	0.125
	5/9_3/4	0.194444
	2/9_1/2	0.277778
	3/8_2/3	0.291667
	4/9_3/4	0.305556

Note: Distance is given as the absolute numerical distance between the magnitudes of the two symbolic fractions in a pair rounded to the thousandths place. Some pairs included in Experiment 1* were replaced in Experiment 2.

Table A2

Symbolic fraction pairs used in Study 1 Experiment 2 and Study 3 Experiment 1

Component Congruence	Distance	Single Digit
Common Denominator	7/9_8/9	0.111
	3/7_4/7	0.143
	2/5_3/5	0.2
	3/7_5/7	0.286
	2/5_4/5	0.4
	1/9_5/9	0.444
	2/7_6/7	0.571
	1/5_4/5	0.6
	1/6_5/6	0.667
	1/8_7/8	0.75
Congruent Numerator & Denominator	4/5_6/7	0.057
	3/5_5/7	0.114
	2/3_5/6	0.167
	1/7_4/9	0.302
	1/4_5/8	0.375
	1/3_5/7	0.381
	1/4_7/9	0.528
	1/3_8/9	0.556
	1/4_8/9	0.639
	1/7_7/8	0.732
Congruent numerator & incongruent denominator	1/8_2/7	0.161
	4/9_5/8	0.181
	1/6_2/5	0.233
	5/9_7/8	0.319
	2/9_3/5*	0.378
	1/8_4/7	0.446
	1/9_5/8	0.514
	1/5_3/4	0.55
	2/9_6/7*	0.635
	1/7_5/6*	0.69
Incongruent Numerator & congruent denominator	2/7_1/3	0.048
	3/8_1/2	0.125
	5/9_3/4	0.194
	2/9_1/2	0.278
	3/8_2/3	0.292
	4/9_3/4	0.306

Note: Distance is given as the absolute numerical distance between the magnitudes of the two symbolic fractions in a pair rounded to the thousandths place. Some pairs included in Experiment 2 replaced pairs from Experiment 1.

Appendix B: Drift Diffusion Model Analysis of Study 1 Experiment 1

Effects of Numerical Distance

Drift Rate Analysis

As seen Figure B1 below, the numerical distance between fractions being compared had a main effect on the efficiency adults can make accurate magnitude judgements, $F(2, 46) = 60.8$, $p < .001$. Specifically, drift rates estimated for comparisons at far distances (M = 4.3, SD = 1.62) were greater than medium distances and medium distances (M = 2.45, SD = 1.04) were greater than near distances (M = 1.22, SD = 0.37). Consistent with numerical distance effects it becomes easier to make accurate judgments as the numerical distance increases between the magnitudes in the comparison pair.

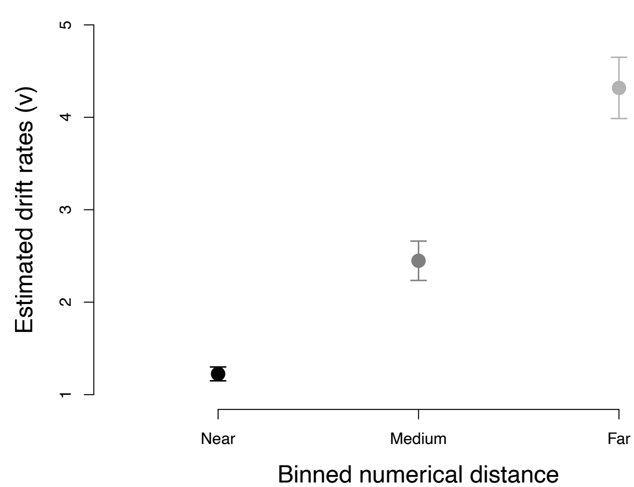


Figure B1: Group mean drift rates at near, medium, and far distances. Error bars depict one standard error of the mean. Higher values of drift rate correspond to more efficient evidence accumulation.

Decision Boundary Analysis

Within-subjects comparisons of estimated decision boundaries indicate that the manipulation of numerical distance had an effect on how carefully adults made accurate responses $F(2,46) = 5.86$, $p = 0.005$. As seen in Figure B2 below, pairwise comparisons of mean decision boundaries reveal that this main effect is driven by significantly wider decision boundaries (more careful responses) in far comparisons ($M = 2.89$, $SD = 0.94$) than near comparisons ($M = 2.23$, $SD = 0.42$). Mean estimated decisions boundaries for comparisons with a medium distance ($M = 2.63$, $SD = 1.04$) were numerically between near and far comparisons, but not significantly different than either.

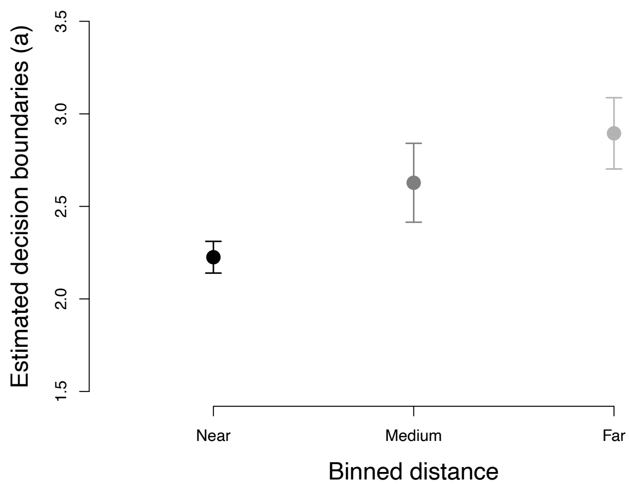


Figure B2: Group mean decision boundaries at near, medium, and far distances. Error bars depict one standard error of the mean. Higher decision boundary values correspond to more careful performance.

Non-Decision Time Analysis

Within-subjects comparison of mean estimated non-decision times show that numerical distance has a main effect on time to encoding the stimulus (and generate motor response), $F(2,46) = 11.1$, $p < .001$. Specifically, higher non-decision times were estimated for near comparisons (M = 0.65, SD = 0.13) than for medium (M = 0.59, SD = 0.12) and far comparisons (M = 0.58, SD = 0.12), but results do not indicate that there are differences between non-decision times in medium and far comparisons.

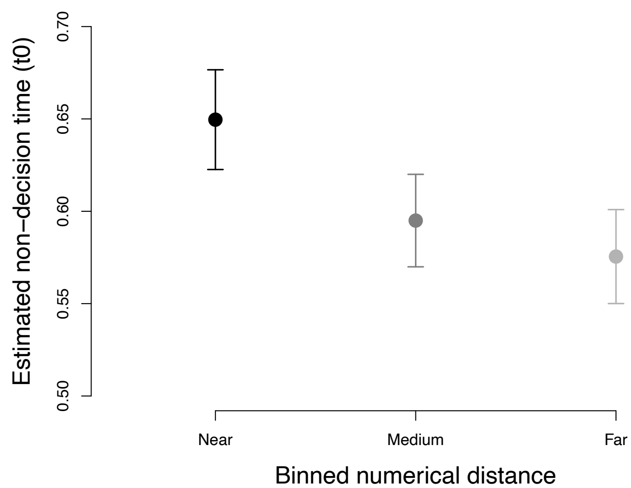


Figure B3: Group mean non-decision times at near, medium, and far distances. Error bars depict one standard error of the mean. Higher values correspond to more encoding processing and time to generate a physical response time.

Appendix C: Study 2 Supplementary Tables

Table C1

Group mean response times across formats and distance bins

	Near		Medium		Far	
Format	*M*	SD	*M*	*SD*	*M*	*SD*
LL	1305	562	989	328	880	304
LF	1516	559	1253	441	1030	302
FF	1579	607	1347	476	1059	357

Note: Means (M) and standard deviations (*SD*) of response times are rounded to the nearest millisecond. Values aggregated from subject mean RT during comparisons with nonsymbolic line ratios (LL), symbolic fractions (FF), and cross-format pairs (LF) across increasing numerical distance (near, medium, and far).

Table C2

Group mean error rates across formats and distance bins

	Near		Medium		Far	
Format	*M*	SD	*M*	*SD*	*M*	*SD*
LL	0.0971	0.059	0.017	0.027	0.007	0.020
LF	0.086	0.081	0.023	0.025	0.000	0.000
FF	0.103	0.064	0.038	0.064	0.004	0.012

Note: Means (M) and standard deviations (*SD*) of error rates were aggregated from subject mean error rates recorded during comparisons with nonsymbolic line ratios (LL), symbolic fractions (FF), and cross-format pairs (LF) across increasing numerical distance (near, medium and far).

Table C3

Regions showing significant distance effects (all conditions)

Hem.	Location	x	y	z	t
R	IPS	29	-56	39	7.47
R	lateral occipital	29	-86	-9	6.05
R	anterior insula	32	16	6	7.71
R	supplementary motor area	5	16	45	9.65
R	caudate	8	4	12	5.64
L	IPS	-28	-65	39	6.18
L	lateral occipital	-43	-86	-6	5.40
L	anterior insula	-31	19	9	7.62
L	premotor area	-46	4	33	5.41
L	fusiform gyrus	-49	-56	-15	5.35
L	middle frontal area	-43	40	6	6.40
L	caudate	-16	7	12	4.99

Note: Coordinates indicate the peak voxel within a cluster of significant activation showing the greatest difference between near and far distances.

Appendix D: Fractions War Specifications

Learning Goals of Fractions War

The primary learning goal of *Fractions War* is for players to develop their understanding of how fractions represent magnitudes. Specifically, this game aims to help players recognize ways that fraction magnitudes are defined by the relative size of their component parts, and not the overall size of their component parts. Multiple studies, including those presented in Chapter 2, have exemplified how common it is for people to fixate on the magnitudes of a fraction's components, overgeneralizing gap strategies, or apply forms of calculation when the goal is to fluently understand a fractions magnitude (Bonato et al., 2007; DeWolf et al., 2014; Fazio et al., 2016; Morales et al., 2020). Through repetitive play, *Fractions War* creates multiple instances to receive immediate feedback when responses based on incorrect assumptions lead to incorrect responses giving points to the opponent (e.g. assuming 2 of clubs / 2 of diamonds is smaller than the 9 of spades/ 10 of clubs).

Lastly, *Fractions War* takes advantage of a design common to playing cards. On a traditional playing card, you will find both a symbolic numeral indicating the value of the card in addition to a number of *pips*, or icons representing the cards suit, that visually instantiate the card's number value. In the game we have created options to separate these two numerical cues in order to allow gameplay with cards that only contain the numeral (symbolic), only show the pip arrays (nonsymbolic), or are presented in their traditional form. By allowing players to engage in the same game structure and see how both symbolic and nonsymbolic cards can represent the same relationally defined magnitudes, we hypothesize that players will come to be fluent in accessing a sense of rational number magnitudes in both stimuli.

By asking participants to engage in this extremely fast magnitude judgement, we aim to have players rely on, refine, and become confident in their intuitions towards ratios and symbolic fractions. If a major problem that students have with fractions is the misapplication of whole number concepts or an inaccurate execution of symbol-based calculation strategies, then perhaps what is lacking in traditional educational approaches are conditions where we explicitly force students to rule out these costly and inaccurate strategies.

Feature	Game	Task	Consequence	Fix
Formats in comparison tasks	• There are 3 different card type conditions: Symbolic, Nonsymbolic & Traditional Cards	• There are 3 tasks formats: LL, CC, FF_s,	• Only FF allows for direct comparison between task & Game.	• Use dot ratios instead of lines & circles in the task. • Dots w/ random pos. but same size, • Do Only FF_s during task phase
In congruent kinds of mixed comparisons	• Traditional cards have pips and numerals.	• Mixed CF & LF comparisons are cross-notation	• War with traditional cards will give experience with redundant cues while cross-notation tasks give experience mapping between representations.	• Add a condition where symbolic numerals and dot ratio are presented together.
Feedback for responses	• Visual feedback is given when individuals move the cards in the correct or incorrect direction. • ***Points awarded to player for correct responses and to comp. for incorrect***	• No Feedback is given for any response.	• Any cognitive effects that may be different between the game and completing the task may be due effects of receiving feedback on accuracy. • ***The nature of the feed back in Fractions War is purposefully rewarding and informative***	• Add simple correct/incorrect/too slow feedback to the comparison tasks that is informative but not purposefully rewarding.
Catch Trials – Trials with same mag.	• "War" hands occur in the game when two fractions are equivalent. Players identify these occasions with a button • ***War hands create an exciting moment where players can earn extra points***	• Only comparisons of inequivalent fractions were used in all tasks	• ***The "war" event is an intentional game-based feature.*** • However, individuals in the task group would not receive experience with catch trials.	• Add trials to the comparison tasks where participants need to identify equal magnitudes
Goal of the Task	• ***Goal of the game is to accumulate the most points by making the correct judgements (Win)*** • ***To Play*** • ***Hope you earn the most cards***	• Respond as quickly and as accurately as possible.	• ***We intend for the game to be more fun, more enjoyable, & more engaging.*** • Individuals are not instructed to optimize speed in the game, but they do have to answer before the computer. >> Attention is not directed specifically/only on the comparisons in the game.	• Matched instructions across contexts to include the phrase, "respond as quickly and as accurately as possible" But in the game this came with the reason based on the competition.
Instructions	• How to play screens with descriptions of how to use the touch screen to play the game	• How to use the keyboard to make judgements	• Game instructions are more complex in that they have an overall instruction of how to play the game and then specific instructions to prepare the player for each card type.	• Matched the length of the instructions at the sentence level, while keeping the number of words as close as possible.
Interface & Stimuli Presentation	• ***iPad Touch screen*** • Cards flip over and then individuals can swipe on the screen.	• E-Prime Computer task using keyboard responses • Presentation time is fast, time for computer to generate an image	• RT comparisons can not be compared in terms of absolute time from stimuli onset.	• Relative RTs (and DE slopes) between conditions could be compared, and the condition x group interaction can too • Comparing overall RT between groups requires estimating a constant (mean) difference err term that describes the difference between conditions in the response generation.
Fraction Pairs	• A random distribution of fractions composed of numbers 1-10	• A controlled/pre-selected but randomly presented number of fractions (w/ 1-9)	• Players in the game experiences a wider array of fraction pairs numeric distances.	• We can use the functionality to stack the deck in the game to match the groups. • This may involve making a decision about how we want the games to end (who gets the most cards).

Figure D1: Differences between task and game group of Study 3 Experiment 1 and how these differences were resolved in Experiment 2, to control for confounding group differences and emphasize critical group differences.

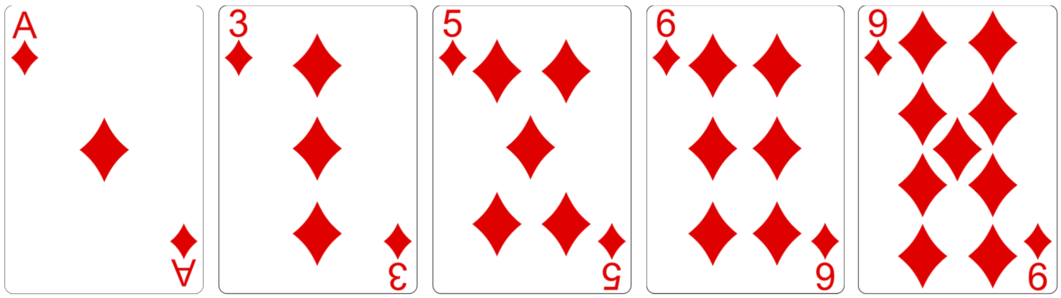
Appendix E: Playing Card Expertise Tasks

Matching Task: Speeded Number to Pip Pattern Recognition

In the matching task participants were presented with two numbers on the screen. On the left side of the screen, participants saw a number of diamonds presented on playing card. On the left they saw and Arabic numeral. In half of the trials, the number of diamonds on the card matched the numeral, and in the other half the two values did not match. Participants were told to indicate if the values matched as quickly and as accurately as possible

Nonmatching pairs in the matching task were created so that the numerical distance between the number of diamonds on the card and the mismatched numeral was 2. A distance of two was used so that mismatch values were similarly even or odd relative to the card value. For card values, 3, 4, 7, 8, the mismatch numeral was two integer values higher. For Card values 5, 6, 9, 10, the mismatch numeral was two integer values lower. For card values 1 and 2, the mismatched numeral was 2 and 1, respectively. Values 1-3 were not included in a participants' mean RTs or ERs for the task, in order to avoid responses where decisions were made with numbers that fall within a range of numbers, termed the *subitizing range,* which very easily identified visually.

Traditional



Non -Traditional

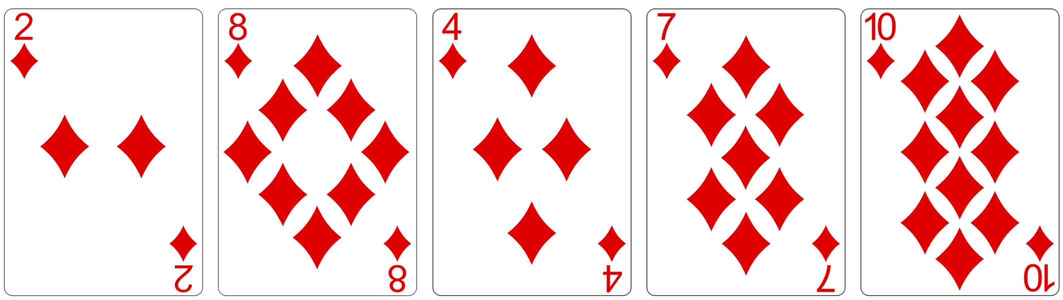


Figure E1. Images of playing cards with traditional and nontraditional pip arrangements used in the matching task and the sorting task to measure playing card expertise.