

**HOW COOPERATING TEACHERS' FEEDBACK PROVIDES OPPORTUNITIES FOR
PRESERVICE TEACHERS TO LEARN TO TEACH**

By

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DEDICATION

This dissertation is dedicated to my family (Yasin, Dilara, Mom, Dad, Haley, and Laurel) for their ongoing love, support, and humor through this process, and to the cooperating teachers in my study (Sara, Adam, and Ethan) who inspired me by their work with their student teachers.

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**INTRODUCTION:
WHY THIS STUDY?**

Excerpt 1

The student teacher is standing in front of the class, wrapping up the lesson warm-up. The CT gestures to the student teacher to come over to meet with him at the side of the room.

ETHAN (a cooperating teacher): Come here for a second. This is something to think about. We're 16 minutes in the class. I don't know if you save time in the warm-up. Just a thought to think about it.

SETH (a preservice teacher): Maybe only go through one of the homework problems.

ETHAN: We have 30 minutes left. The other side is I think it'll go a little quicker, if it feels like we're rushing for the rest of the lesson. I'm not sure. I'm not certain.

Excerpt 2

The CT and student teacher are sitting at a table in their classroom reflecting on their morning lessons.

JANE (a preservice teacher): I have a question about James. Does he usually do work?

SARA (a cooperating teacher): It depends whether he's on his meds or not. I have the feeling that right now he's not on his meds because he is grumpy.

JANE: He's very grumpy.

SARA: He doesn't talk, like he'll even like cover up his mouth to speak to you. He's just one of those kids that's so reflective of the kind of night he had at home, whether he's on his meds or not. He's got a crappy home life. You know so, I think he's off right now. Sometimes he's right on, sometimes he's off. It's hard to tell which James you're going to get. But he's absorbing even if he doesn't, like, it's not worth it to push him enough so that he gets mad and has to leave the room and ends up in [the Assistant Principal's] office. Last year when he was a 6th grader I would go down and visit the 6th grade math teacher's class in the mid-day because I had sort of the same schedule and James would be in the main office. Nine times of ten he was sitting in there because he got kicked out.

Excerpt 3

Prior to grading a test, the CT and PST discuss the solutions for the problems. As they look at a test question about a 30°-60°-90° triangle, the teachers have the following dialogue.

ANNA (a preservice teacher): How did we say the side of this triangle is equal to 4?

ADAM (a cooperating teacher): Because that side was 8, so that's 4, so that's 4 root 3 ($4\sqrt{3}$). Because we'd have corresponding angles, so that side would be 4, and then this side would be 8, and this is $4\sqrt{3}$.

ANNA: That diagram is just kind of weird that it's clearly not drawn to scale.

ADAM: Yeah.

ANNA: Because then they're saying that triangle is the same as that one.

ADAM: Uh huh.

During my four years as a university supervisor, I was privy to the conversations and interactions that cooperating teachers (CTs) and preservice teachers (PSTs) had throughout the school day. As I worked with CTs over multiple semesters and gained familiarity with their classrooms, schools, and instructional styles, I began to inquire about and note what aspects of teaching the CTs emphasized when working with the PSTs. For example, I knew that all of the CTs were apt to have a conversation with the PSTs at some point during the semester about the PST's pacing of lessons, conversations such as the one shared in the first excerpt provided above. I also gained a sense of which aspects of teaching particular CTs were likely to highlight with their respective PSTs. For example, one CT who taught in a middle school classroom would have extended conversations with her PSTs about the students, conversations such as the one shared in the second excerpt provided above, whereas another CT who taught in a high school classroom would have long discussions with his PSTs about the mathematics content emphasized in the lessons and tests, discussions such as the one shared in the third excerpt provided above. From observing these conversations and interactions between the CT/PST pairs, I began to wonder about what learning opportunities the CTs were providing for the PSTs and how these learning opportunities opened possibilities for or constrained the PSTs' learning. This was the genesis of my dissertation research, the experience that led me to ask the broader question of how CTs shape and influence PSTs' learning in the field placement.

Considering the breadth of ways that CTs and PSTs communicate and interact with one another in the field placement, my dissertation research focuses on how the teachers communicate and interact while the CT gives feedback to the PST. My study focuses on this particular aspect of the field placement because I feel it is one of the most important ways that CTs help PSTs learn to teach, since it is an opportunity for the CT to (1) emphasize and critique specific aspects of the PST's conception and enactment of teaching; and (2) highlight additional aspects of teaching that the PST can attend to and incorporate in her future instruction. In other words, the CTs' feedback potentially "makes" particular aspects of teaching "visible" to the PSTs, thereby providing an opportunity for the PSTs to develop their conceptions and enactments of teaching. Therefore, my research seeks to understand and describe the learning opportunities that CTs provide for PSTs as they give feedback to the PSTs.

Cooperating teachers give feedback to PSTs in a variety of ways for a variety of reasons. For example, sometimes a CT gives feedback in the middle of the PSTs' lessons to help the PST become aware of or reflect on an aspect of her instruction, as is illustrated in the first example provided above. At other times, a CT gives feedback as the teachers engage in a particular "everyday" teaching activity to reinforce or modify an aspect of the PST's understanding or enactment teaching, as is illustrated in the second example provided above. At still other times, a CT gives feedback as the teachers reflect on their instruction to help the PST become aware of a new aspect of teaching, as is illustrated in the third example provided above. My dissertation investigates the following three questions: (1) what aspects of teaching does CTs' feedback potentially help PSTs learn about; (2) how does CTs' feedback function to potentially help PSTs learn about these aspects of teaching; and (3) how can CTs give feedback in a way that helps the PSTs build on and construct their own conception and enactment of teaching. While my study

focuses on CTs who teach secondary mathematics, I believe my research and findings are relevant to CTs in all disciplines and grade levels since it addresses aspects of cooperating teachers' feedback that are not disciplinary-specific or grade level-specific.

This dissertation contains three parts. **Part I** is a literature review situating my research in the field of mathematics teacher education. This part provides the following: (1) an explanation of why it is important to understand and research what learning opportunities CTs provide for PSTs and why my research focuses on the learning opportunities provided as CTs give feedback to PSTs; (2) what it means for CTs to “give” feedback to PSTs and what other researchers have reported about CT feedback; and (3) how my research builds on and extends the past research examining CT feedback. **Parts II and III** then respond to the three questions posed above. **Part II** is a “traditional” journal article intended for mathematics teacher educators and researchers. This article shares select findings from my study regarding the aspects of teaching that the PSTs had the opportunity to learn about while the CTs gave feedback (i.e., the aspects of teaching that the feedback emphasized) and how the feedback served to potentially help the PSTs learn these aspects of teaching. I wrote this article because there have not been any articles in the field of teacher education describing the way in which the feedback serves to potentially reinforce or modify aspects of PSTs' conceptions and enactment of teaching. **Part III** is an article intended for CTs. This article provides suggestions for how CTs can give feedback in a way that provides opportunities for PSTs to “build on” and “construct” their own understandings and practices of teaching. In other words, it addresses how CTs can provide learning opportunities based on constructivist learning principles as they give feedback to the PSTs.

PART I:

**A REVIEW OF THE LITERATURE ON THE FEEDBACK THAT
COOPERATING TEACHERS GIVE TO PRESERVICE TEACHERS**

The Importance of Researching the Feedback that Cooperating Teachers Give to Preservice Teachers

The Problem

Preparing preservice teachers (PSTs) to teach reform mathematics curricula and instruction in their classrooms is a challenge in mathematics teacher education. Since the release of the National Council of Teachers of Mathematics' national curriculum standards in 1989, mathematics educators and researchers have advocated for "reform" curricula and instruction based on constructivist and socio-cultural theories of learning. However, this "problem-based," "student-centered" instruction is often different from the curricula and instruction experienced by many PSTs in their K-12 schooling (Ball 1988; Ball, 1990), and is difficult for PSTs to enact (Borko, Underhill, Brown, Jones, & Jones, 1993). As a result, mathematics teacher educators continue to strive to find ways of providing learning experiences for PSTs that support the PSTs in learning about reform instruction as well as how to enact this type of instruction in their work with students.

Preservice mathematics teachers enter teacher education programs with conceptions (Philipp, 2007)¹ of "what mathematics is, what it means to learn mathematics, what one teaches when teaching mathematics, what the roles of the teacher and the students should be, and what constitutes evidence of student knowledge and criteria for judging correctness, accuracy, or acceptability of mathematical results and conclusions" (Philipp, Flores, Sowder, & Schappelle, 1994, citing Thompson, 1991), due to their experiences as students in K-12 schooling, a phenomenon referred to as the "apprenticeship of observation" (Lortie, 1975). For example, due to her experiences receiving "traditional" mathematics instruction in K-12 schooling, the PST

¹ Philipp (2007) defines a conception as "a general notion or mental structure encompassing beliefs, meanings, concepts, proposition, rules, mental images, and preferences." Thus, it subsumes beliefs and many of the other associated constructs.

may have a conception of mathematics education in which: mathematics is a static body of knowledge to be “discovered” by people; learning entails students memorizing and practicing prescribed problem-solving strategies;² teaching entails the teacher giving students a “correct” problem-solving strategy, examples demonstrating how the prescribed method is used, and practice problems that they can individually solve using the prescribed method; the role of the teacher is “knowledge-giver” and the student is “passive knowledge-recipient;” and evidence of student knowledge entails the student giving the correct answer.

While in her teacher education program, the PST may learn about alternative types of mathematics instruction that are based on different conceptions of mathematics education. For example, the program may teach the PST about “reform” mathematics instruction. This type of mathematics instruction is based on the conception that: mathematics is a dynamic body of knowledge constantly being (re)constructed by people; learning entails students constructing their own understanding and knowledge of mathematics in collaboration with other people;³ teaching entails accessing and building on students’ prior knowledge, validating and soliciting students’ “natural” problem-solving strategies, facilitating students’ work in groups, and leading class discussion comparing and relating the multiple problem-solving strategies generated by the students; the role of the teacher is “facilitator” and the student is “active knowledge-creator;” and evidence of student knowledge entails the student sharing her process of finding the correct answer. Therefore, while in the teacher education program, the PST must change or be supported in changing her current conception of mathematics education to the conception taught in and advocated by the teacher education program, as well as must learn how to enact instruction based on this new conception of mathematics education.

² “Traditional” mathematics instruction is based on behaviorist learning theory.

³ “Reform” mathematics instruction is based on constructivist and socio-cultural learning theories.

How Teacher Educators Help Promote Preservice Teachers' Learning and Development

The teacher educators in the program can provide opportunities to support PSTs in changing their current conceptions of education, and to help them learn to enact instruction based on this new conception. Research has indicated that teacher educators do this through providing learning opportunities that do such things as: surfacing, challenging, and interrupting the PSTs existing conceptions of mathematics education; helping the PSTs learn about a new conception of mathematics education; and/or supporting the PSTs in learning to enact instruction based on the new conception of mathematics education (e.g., Ball, 1988; Ball, 1990; Ghouseini & Herbst, 2015; Lampert, Franke, Kazemi, Ghouseini, Turrou, Beasley, Cunard, & Crowe, 2013). For example, Ball (1988) describes the components of a unit that she gives the PSTs in her mathematics methods course in order to surface, challenge, and interrupt the PSTs' existing conceptions of mathematics education, and help the PSTs learn about a new conception of mathematics education. Ghouseini and Herbst (2015) and Lampert et al. (2013) describe the guided "rehearsals" of reform-based instructional activities and teaching methods that PSTs do in mathematics methods courses in preparation for leading the activities with students.

While teacher educators in both the coursework and fieldwork components of teacher education programs can potentially provide all of these types of experiences, teacher educators in the fieldwork component are particularly well-situated to provide the second and third types of experiences for PSTs (i.e., help the PSTs learn about a new conception and support the PSTs in learning to enact instruction based on this new conception). During the fieldwork component, an experienced teacher, often referred to as the cooperating teacher (CT), hosts and supervises one or more PSTs in her classroom (or alternative authentic learning context) so that the PST(s) can gain experience doing such things as interacting with and assessing students, planning and

teaching lessons, and observing instruction led by experienced teachers, among other things. Cooperating teachers are particularly well-situated to provide opportunities for PSTs to learn about a new conception of mathematics education and how to enact instruction based on the new conception since they provide opportunities for the PST to both observe the CT lead instruction and lead parts of the instruction herself. However, the ways in which the CTs help the PSTs develop a new conception of education based on their experiences observing and leading instruction depend on the additional practices that the CTs implement to support the PSTs' learning. Therefore, the next section describes how CTs promote PSTs' learning.

How Cooperating Teachers' Promote Preservice Teachers' Learning

Cohen, Raudenbush, and Ball's (2003) instructional triangle, shown in Figure 1, is useful for framing how CTs promote PSTs' learning. The instructional triangle is comprised of four primary relationships: (1) the relationship between the CT and the PST, hereafter referred to as the CT-PST relationship; (2) the relationship between the CT and the content of teacher education, hereafter referred to as the CT-C relationship; (3) the relationship between the PST and the content of teacher education, hereafter referred to as the PST-C relationship; and (4) the relationship between the CT and the PST-C relationship, hereafter referred to as the CT-PST/C relationship. These relationships are constituted and negotiated within the context of the field placement, as indicated by the "context" circle surrounding the CT, PST, content of teacher education, and connecting arrows in Figure 1.^{4 5}

⁴ Cohen, Raudenbush, and Ball's (2003) instructional triangle draws on Lampert's (2001) conceptualization of the practice of teaching, which frames teaching in terms of the relationships that exist among a teacher, students, and content. The practice of teaching is comprised of four primary relationships, which are the relationship between (1) the teacher and students; (2) the teacher and content; (3) the students and content; and (4) the teacher and the relationship between the students and content. The description of the practice of teacher education provided in this paper largely draws on Lampert's description of the practice of teaching.

⁵ Cohen, Raudenbush, and Ball's (2003) instructional triangle originally describes the relationships among a teacher educator, a PST, and the content of teacher education. This paper situates the instructional triangle in the field placement, specifically focusing on the relationships among the CT, the PST, and the content of teacher education.

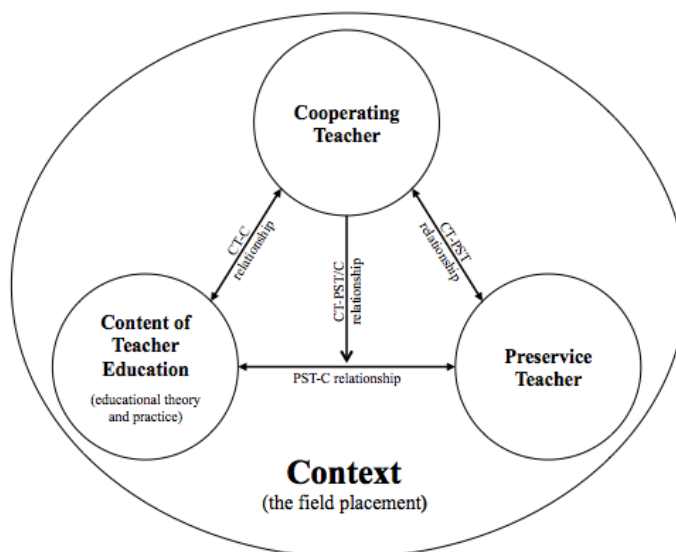


Figure 1. Cohen, Raudenbush, and Ball's (2003) instructional triangle describing the practice of teacher education in context of the field placement, as it is comprised of the relationships among a CT, a PST, and the content of teacher education

The CT-PST relationship, indicated by the bidirectional arrow between the CT and PST in Figure 1, refers to the ways in which the CT and PST establish and maintain a collaborative relationship that allows them to work together in the field placement. For example, the pair may establish shared goals and expectations so they can have a common vision for and understanding of their work together. The CT-C relationship, indicated by the bidirectional arrow between the CT and content of teacher education, refers to the CT's conception and enactment of education. For example, when explaining to the PST her reasons for using particular instructional methods for teaching fraction multiplication, the CT will justify the method based on her understanding of what the students should know about fraction multiplication and what activities promote student understanding of fraction multiplication.

The PST-C relationship, indicated by the bidirectional arrow between the PST and content of teacher education, refers to the PST's conception and enactment of education. For example, the PST will develop an understanding of how to teach fraction multiplication through

“studying” (Lampert, 2001) fraction multiplication (e.g., observing or leading a lesson on fraction multiplication, referencing a textbook or district standards to identify what students are expected to learn about fraction multiplication). The CT-PST/C relationship, indicated by the unidirectional arrow between the CT and the bidirectional arrow between the PST and content of teacher education, refers to the ways in which the CT supports the PST in developing a conception and enactment of education. In other words, it refers to how the CT promotes the PST’s learning. For example, the CT will give feedback to the PST about what representations of fraction multiplication the PST should have used during her lesson in order to help the PST understand how to teach fraction multiplication.

The “practice of teacher education” extends across all of the space containing and connecting the CT, PST, and content of teacher education since these relationships are negotiated simultaneously as the CT and PST work together in the field placement. This means that each relationship is influenced by the other three relationships. For example, the CT-PST/C relationship is influenced by CT-PST, CT-C, and PST-C relationships. In other words, every practice that the CT enacts to promote the PST’s learning is influenced in particular ways by how the CT and PST work together, the CT’s conception and enactment of education, and the PST’s conception and enactment of education (or how the CT perceives this to be). In addition, since the CT-PST/C relationship influences the PST-C relationship, each practice that the CT enacts to promote the PST’s learning provides particular opportunities for the PST-C relationship to develop. Furthermore, these relationships are not static but are constantly changing and negotiated as the CT and PST work together over time.

The CT-PST/C relationship is comprised of practices that the CT does to promote the PST's learning and development.⁶ These practices might include modeling instruction for the PST, observing the PST lead aspects of instruction, giving feedback to the PST based on the observed instruction, reflecting on the CT's or PST's instruction with the PST, explaining and justifying the CT's instruction to the PST, and having the PST explain and justify her (the PST's) instruction (Anderson & Shannon, 1988; Brooks & Sikes, 1997; Collins, Brown, and Holum, 1991; Fletcher, 2000). Since the CT-PST/C relationship is influenced by the other three relationships, each practice that the CT enacts to promote the PST's learning is influenced in particular ways by how the CT and PST work together, the CT's conception and enactment of education, and the PST's conception and enactment of education (or how the CT perceives this to be). In addition, since the CT-PST/C relationship influences the PST-C relationship, each practice that the CT enacts to promote the PST's learning provides particular opportunities for

⁶ Research has indicated that the practices that a CT does as a teacher educator to promote the PST's learning depends on the approach that the CT takes when working with the PST (Borko & Mayfield, 1995; Brooks & Sikes, 1997; Collins, Brown, & Holum, 1991; Fernandez & Erbilgin, 2009; Furlong & Maynard, 1995; Valencia, Martin, Place, & Grossman, 2009). For example, Collins, Brown, and Holum (1991) describe the practices that CTs use when using the apprenticeship approach versus cognitive apprenticeship approach. Collins, Brown, and Holum (1991) write that when using the apprenticeship approach, the CT does the activities of modeling, scaffolding, fading, and coaching in order to help the PST learn how to do the external processes entailed in teaching. When modeling, the CT demonstrates the everyday teaching activities since it is assumed that the PST will develop a conceptual model of the processes entailed in completing the task as she watches the CT teach. When scaffolding, the CT supports the PST in teaching by either having the PST do most of the everyday teaching activities or by providing hints as to how the PST can do some or all of the teaching. Related to scaffolding is fading, in which the CT removes the support (i.e., scaffolding) over time so that the PST increasingly takes on more of the teaching responsibilities. When coaching, the CT provides "hints, challenges, scaffolding, feedback, modeling, reminders, and new tasks" (Collins, Brown, and Holum, 1991, p. 2) to assist the PST as she teaches. When coaching, the CT gives feedback to the PST. In contrast, Collins, Brown, and Holum (1991) and Enkenberg (2001) write when using the cognitive apprenticeship approach, the CT does all of the activities involved in the apprenticeship approach (modeling, scaffolding, fading, and coaching), as well as the additional activities of explaining, articulating, reflecting, and exploring (Collins, Brown, & Holum, 1991) in order to help the PST learn about both the external and internal processes entailed in teaching. When explaining, the CT shares her rationale for using the instructional methods in the way she does. When articulating, the CT has the PST verbally articulate her (the PST's) knowledge, reasoning, or problem-solving processes as she teaches. When reflecting, the CT has the PST compare the problem-solving processes that she (the PST) uses while teaching with the problem-solving processes used by the CT as the CT teaches. When exploring, the CT has the PST carry out tasks on her own so that she can teach in ways that are of interest to her without guidance and coaching from the CT, in order for the PST to gain autonomy from the CT. Borko and Mayfield (1995) report that CTs' approaches to working with PSTs are based on how they view their roles as CTs and how they think PSTs learn.

the PST to develop her conception and enactment of education. For example, when giving feedback, the CT might provide an opportunity for the PST to develop her understanding of strategies that she can use to solicit student participation in her lessons or to develop her practice of soliciting student participation during a lesson.

Giving feedback is a practice that CTs commonly enact to promote PSTs' learning (Anderson & Shannon, 1988; Brooks & Sikes, 1997), albeit in varying ways (Borko & Mayfield, 1995; Valencia, Martin, Place, & Grossman, 2009). In addition, I believe that giving feedback is one of the most important practices for helping PSTs develop a conception and enactment of instruction for two reasons. First, it is an opportunity for a CT to emphasize and critique specific aspects of the PST conception and enactment of instruction at a given point in time. Second, it is an opportunity for a CT to highlight additional aspects of instruction that the PST can attend to and incorporate in her future conception and enactment of instruction. Therefore, my research seeks to understand and describe the learning opportunities that CTs provide for PSTs as they give feedback to the PSTs. Since this is the focus of my research, the remainder of this paper will first provide an overview of the literature describing and examining how CTs give feedback to PSTs, and then describe my own research with respect to this literature base.

How Cooperating Teachers' Give Feedback

When considering how CTs give feedback, it is necessary to conceptualize what feedback is and how it is "given." Therefore, this section first provides a conceptualization of what feedback is and then shares the ways in which scholars and researcher have described and characterized how CTs "give" feedback to PSTs.

Defining the Concept of Feedback

Scholars in multiple academic fields (e.g., education, medicine, psychology, rocket science) have defined the concept of feedback. Ende (1983) describes feedback as “information that a system uses to make adjustments in reaching a goal,” and cites Weiner’s (1954) description of feedback as “the control of a system by reinserting into the system the results of its performance” which the system then uses “to change the general method and pattern of the performance” (cited by Ende, 1983, p. 777). The system has a mechanism through which the performance data produced by the system is “fed back” into the system so that the system can regulate or initiate change within itself based on that performance data. Weiner terms this “learning” because it implies a circular flow of information through the system: the system produces information about its performance, which it then analyzes to regulate and modify its future performance. Scholars, particularly in the field of education, use this notion of feedback to describe the way in which people use information about their current performance (e.g., actions, thinking, behavior) to modify and shape their future performance.

In education, Hattie and Timperley (2007) build on these notions, defining feedback as “information provided by an agent (e.g. teacher, peer, book, parent, self, experience) regarding aspects of [the learner’s] performance or understanding” (p. 81) as the learner performs a task. In the context of the field placement, an “agent” (the CT) gives the “learner” (the PST) information about the PST’s teaching performance and understanding of education, which the PST then uses to shape her own future teaching performance and understanding of education.

Describing How Cooperating Teachers Give Feedback to Preservice Teachers

Scholars in fields such as education (e.g., Borko & Mayfield, 1995; Brinko, 1993; Fernandez & Erbilgin, 2009; Fletcher, 2000; Valencia, et al., 2009), medicine (e.g., Cantillon & Sargeant, 2008; Ende, 1983; Hewson & Little, 1998; Thomas & Arnold, 2011), and psychology

(e.g., Hoffman, Hill, Holmes, & Frietas, 2005) have described and empirically researched how experienced practitioners give feedback to new practitioners in these fields. Their work has attended to a variety of aspects of how feedback is given, including logistical aspects of when and how the feedback is or can be given (e.g., Ende, 1983; Scheeler, Ruhl, & McAfee, 2004), the topics discussed during the feedback conversations (e.g., Akcan & Tatar, 2010; Borko & Mayfield, 1995; Clarke, 2001; Coulon, 2004; Valencia et al., 2009), the dynamics and interactions that occur between the two people during the feedback conversation (e.g., Bertone, Chaliés, Clarke, & Méard, 2006; Fernandez and Erbilgin, 2009), the effects of supervisory practices on PSTs' development (e.g., Byra, 1994), and how the feedback is affected by the way in which the CTs view their role as a CT and how they view PST learning (e.g., Borko & Mayfield, 1995; Valencia et al., 2009).

The research on how CTs give feedback has emphasized the influence that the CT-PST relationship has on the feedback, as well as how the feedback influences the PST-C relationship. Therefore, the remainder of this section will describe how the literature has indicated these relationships affect one another.

How the CT-PST Relationship Influences How Cooperating Teachers Give Feedback. Scholars and researchers have emphasized that the CT-PST relationship influences how CTs give feedback, particularly when the feedback is critical of the PST's instruction (e.g., Ende, 1983; Fletcher, 2000). Literature on mentoring in the field of education has stressed the importance of this relationship in promoting communication between the CT and the PST (Stanulis & Russel, 2000) and in fostering the PST's personal and professional development (Anderson & Shannon, 1998). For example, Anderson & Shannon (1988) write that the mentor (e.g., CT) and mentee (e.g., PST) should develop a caring relationship in which the mentor

“nurtures” the PST by providing an environment to foster the mentee’s personal and professional growth, “sponsors” the mentee by “protecting” the mentee from challenges in the teaching context, and “befriends” the mentee by accepting and relating to the mentee. Brooks and Sikes (1997) write that the mentor should have interpersonal skills such as honesty, openness, sensitivity, enthusiasm, a sense of humor, organization, self-awareness, and reflectiveness. Over time, the CT-PST relationship may change as the PST gains experience teaching, leading the CT and PST to develop more of a collegial relationship as they work as “critical friends,” “co-enquirers,” or “partners” who share responsibility for instruction (Furlong & Maynard, 1995).

Ende (1983) and Fletcher (2000) additionally describe the ways in which an experienced practitioner and novice practitioner should work together. In the field of medical education, Ende (1983) explains that feedback should be “undertaken with the teacher and trainee working as allies, with common goals,” and that the teacher and learner should agree on the goals, location, and time of the feedback session, and should develop a physically/spatially and emotionally “relaxed” environment that allows them to come to a shared consensus on the trainee’s overall performance, shortcomings, and future goals. In the field of teacher education, Fletcher (2000) writes “the dynamics of giving feedback are many and complex” (p. 83) because the trainee feels pressure to mimic what the CT does. Furthermore, the CT should be honest, should empathize with the PST, should anticipate the PST’s expectations, should consider the PST’s past experiences in other classrooms which may have had different expectations of the PST, should support and challenge the PST, should begin and end the feedback session on a positive note, and should help the mentee not feel isolated by using the term “we” when working together.

How the CT-C Relationship Influences How Cooperating Teachers Give Feedback.

Scholars and researchers have also emphasized how the CT-C relationship influences how CTs

give feedback. In the field placement, the CT-C and PST-C relationships are framed based on the CT's conceptualization of education since the CT typically determines what type of mathematics instruction is enacted in the classrooms as well as what aspects of the instruction the PST has the opportunity to do during her time in the field placement (Feiman-Nemser & Buchmann, 1989). For example, if the CT has a traditional conception and enactment of mathematics education, she might give feedback aimed to help the PST improve how she (the PST) transmits the content to the students or selects problems to have the students practice using the memorized problem-solving strategies provided by the teachers. Alternatively, if the CT has a reform conception and enactment of mathematics education, she might give feedback aimed to help the PST improve how she (the PST) solicits and builds on students' prior knowledge or facilitates student group work while the students work on the class activities and problems. Therefore, the CT-C relationship influences the content of the CTs' feedback to PSTs.

How Cooperating Teachers' Feedback Influences the PST-C Relationship. Past research has also described the ways in which the feedback should attend to the PST-C relationship. Many scholars and researchers have described the "feedback content" (Scheeler, Ruhl, & McAfee, 2004). This refers to the topics that the teachers discuss as the CT gives feedback to the PST (e.g., Akcan & Tatar, 2010; Borko & Mayfield, 1995; Clarke, 2001; Coulon, 2004; Valencia et al., 2009). For example, Borko and Mayfield (1995) report that the CTs and PSTs in their study discussed four domains of teacher knowledge in their pre- and post-lesson conferences. These four domains were pedagogical knowledge, knowledge of students, pedagogical content knowledge specific to mathematics instruction, and mathematical knowledge. While stating that all four domains of knowledge were mentioned in most of the

conferences to varying extents, the researchers do not explain how the CTs use their practice of giving feedback to potentially reinforce or modify the PSTs' understanding of these domains.

Van Houten (1980) differentiates among different kinds of feedback content, in particular “positive,” “negative,” and “corrective.” Positive feedback refers to feedback highlighting what the student did well, whereas negative feedback refers to feedback indicating what the student did not do well. He argues that positive feedback is “optimally effective,” regardless of whether it indicates when students “have increased the number of [things] they have done correctly” or “decreased the number of [things they have] done incorrectly” (p. 62). Corrective feedback refers to feedback indicating both the type and extent of errors made by the student, and how the student can remedy those errors. Van Houten asserts that corrective feedback is important for student learning (p. 64) because it helps the student address errors in her performance by exposing her to the correct way to perform something (p. 63). Ultimately, this assists the student in learning how to self-regulate and self-monitor her own progress (Hattie & Timperley, 2007).

Hattie and Timperley describe four levels for the focus of feedback content, and state that the effectiveness of the feedback depends on the level at which the feedback is given. The four levels are: (1) feedback about how well the learner is accomplishing or performing the task (i.e., corrective feedback); (2) feedback about the process that the learner is using to complete the task; (3) feedback about how the learner can self-regulate or self-monitor her own progress in completing the task to meet her goals (e.g., “You already know the key features of the opening of an argument. Check to see whether you have incorporated them in your first paragraph.”); and (4) feedback about the “self” or learner that is not directly related to the task (e.g., “you are a great student” or “great effort”). Hattie and Timperley state that feedback about the process that the learner is using to complete a task (#2) and how the learner can self-regulate and self-monitor

her own progress (#3) are useful in helping the learner “deeply” process and master the tasks, while feedback about how well the learner is accomplishing or performing the task (#1) is useful in helping the learner use the task information to learn how to self-regulate and self-monitor her own progress. (p. 90-91). They write that feedback about the self (#4) is the least effective level of feedback as it does not give the learner a sense of how she is progressing to meet her goals.

Van Houten (1980), Hattie and Timperley (2007), and Ende (1983) are all critical of feedback content that is about the learner but not about the extent to which the learner’s work was done correctly, as determined by the extent to which the learner met the end goals. Hattie and Timperley (2007) further argue that feedback gauging how well the learner is performing given how the teacher wants the learner to perform⁷ is a component of what makes feedback more “effective” as it is “targeted” to the learner’s “appropriate level” and seeks to reduce the distance between what the learner understands and what the teacher wants the learner to understand (p. 86). As feedback is given over time, it should continue to reduce the distance between the learner’s current level of understanding and performance and the learner’s goals for understanding and performance, until the learner achieves her goals and is thus prompted to set new goals.

Hattie and Timperley (2007) describe an alternative aspect of how the CTs’ feedback influences the PST-C relationship; they frame feedback in terms of how the feedback helps a learner “fill the gap” between what the learner understands and what the teacher wants the learner to understand. This framing of feedback suggests that the teacher and learner establish goals for the learner’s understanding and performance, and then use those goals to measure the

⁷ Hattie and Timperley (2007) refer to feedback gauging how well the learner is performing given how the teacher wants the learner to perform as “corrective feedback.” Their use of the term “corrective feedback” should not be confused with Van Houten’s (1980) description of corrective feedback that is defined as feedback indicating the type and extent of errors made by the student, and how the student can remedy those errors.

learner's developing understanding and performance. Ende (1983) emphasizes this idea as well, stating that the teacher should give information that indicates to the learner how she is currently performing in relation to the pre-established, long-term goals for the learner's performance, so that the learner can make progress toward attaining those goals.⁸

Hattie and Timperley additionally describe the "cognitive processes" that a teacher might employ when giving corrective feedback to change the learner's cognition and the resulting cognitive changes that a learner makes as a result of receiving corrective feedback. They write that teachers give corrective feedback through "cognitive processes" such as helping the learner restructure her understandings, confirming whether the learner is correct or incorrect, indicating to the learner whether "additional information is available or needed," pointing to future directions that the learner can take, and/or providing alternative strategies that the learner can use to "understand particular information" (p. 82). Butler and Winne (1995) further describe how feedback functions to reinforce or modify the information in a learner's memory. They write that feedback functions in the following five ways:

First, when students' conceptual understandings or beliefs are consistent with instructional objectives, feedback can *confirm* that condition. Second, if students lack information, feedback can help students *add* information, thereby elaborating and enriching prior knowledge. Third, where elements of prior knowledge are incorrect or prior beliefs are inappropriate, feedback can provide information to replace or *overwrite* those propositions. Fourth, if students' understandings are basically correct, they still may need to *tune* those understandings, for example, by discriminating between concepts or by specifying conditions for applying learned rules. Fifth, if students hold false theories

⁸ Although writing about how experienced doctors give feedback to doctors-in-training, Ende's ideas about how feedback is given can be extended to the field of education to describe how CTs give feedback to PSTs.

that are incompatible with new material to be learned, they may need to completely *restructure* schemata with which information in the domain is represented. (p. 265)

In conclusion, this paper conceptualizes CT feedback as the information that the CT gives to the PST about the PST's teaching performance and understanding of education that the PST then uses to shape her future teaching performance. When analyzing CT feedback, a researcher must consider both the feedback content and how the feedback helps the PST (or provides opportunities for the PST to) "fill the gap" between the PST's current understanding and performance and the end goals for the PST's understanding and performance.

Researching the Opportunities that Cooperating Teachers Provide that (Potentially) Assist the Preservice Teachers' Learning and Development of Teaching

As a researcher, my work seeks to understand what learning opportunities CTs provide for PSTs as they give feedback. I am interested in researching this aspect of feedback because there have not been any past studies indicating the ways in which feedback "functions" (Butler & Winne, 1995) to potentially help the PSTs develop a conception of teaching and enact instruction based on that conception. While researchers have detailed the content discussed by the teachers in their feedback conversations (e.g., Akcan & Tatar, 2010; Borko & Mayfield, 1995; Clarke, 2001; Coulon, 2004; Valencia et al., 2009) and described the dynamics and interactions that occur between the teachers as the CTs give feedback to the PSTs (e.g., Bertone, Chaliés, Clarke, & Méard, 2006; Fernandez and Erbilgin, 2009), none have explained how the content emphasized and the interaction between the CT and PST provide opportunities for the PST to develop a conception of teaching and learn to enact instruction based on this conception. In other words, they describe the topics talked about as the CTs give feedback and how CTs "give"

feedback without considering how the feedback acts to potentially influence or shape how a PST makes sense of, views, and enacts instruction.

My work builds on and extends past studies investigating the feedback that CTs give to PSTs in the field placement in three additional ways. First, it investigates the feedback that CTs give PSTs across the entire school day, as opposed to during specific or specified times during the day. The majority of research studies conducted to date have investigated the feedback that CTs give during pre- or post-observation meetings (e.g., Akcan & Tatar, 2010; Borko & Mayfield, 1995; Fernandez & Erbilgin, 2009; Hawkey, 1998), as opposed to during the entire school day, including while the PST leads instruction. While it is helpful to know what aspects of teaching CTs help PSTs notice and attend to before and after they lead instruction, these studies do not capture and reflect the feedback that CTs give at other times while conversing and interacting with the PSTs. As Valencia et al. (2009) reported, CTs and PSTs do not always have frequent or systematic discussions about specific lessons, but instead tend to converse at the end of the school day or during brief intervals throughout the day. Therefore, it is important to examine the feedback that CTs give to PSTs throughout the school day. Furthermore, these studies have critiqued the content of the feedback, noting its superficial or non-existent emphasis on content knowledge and pedagogical content knowledge (e.g., Akcan & Tatar, 2010; Borko & Mayfield, 1995; Valencia et al., 2009). While this critique is perhaps fair of the feedback given during pre- and post-observation meetings, it may not reflect the feedback content given at other times during the school day, such as while the teachers plan lessons together. Therefore, this study seeks to understand the feedback that CTs give to PSTs throughout the entire school day, as opposed to during pre- and post-lesson observation meetings alone.

The second way in which this study builds on and extends past research concerns how it relates to the type of instruction that is implemented in the CT's and PST's classroom. Framing the PST's learning as increasing participation in a community of mathematics teaching practice (Lave & Wenger, 1991), my research considers what aspects of teaching, as they are viewed, understood, and enacted by community members, the PSTs have the opportunity to learn about and develop. While many past studies have, again, described the feedback content, none of them have explained how the content is reflective of the kind of instruction implemented by the teachers in that context.

Finally, the third way in which this study builds on and extends past research concerns its explicit focus on understanding how PSTs learn to teach reform mathematics instruction. The past research studies in the field of mathematics education investigating how CTs give feedback to PSTs have not been selective in this way. Given the emphasis in the field of mathematics education on having teachers use reform mathematics curricula and instruction in their classrooms, it is important to understand how the learning opportunities provided by CTs help PSTs learn to lead this type of curricula and instruction in particular. Therefore, my work aims to examine the feedback practices of CTs who help PSTs learn to use reform mathematics curricula and instruction. In doing this, I hope to prompt a conversation in the broader mathematics education community as to what aspects of reform mathematics instruction CTs (and other teacher educators) help PSTs attend to (as the reform mathematics instruction is conceptualized and enacted by teachers in the field placement context), and how CTs help PSTs develop a conception and enactment of reform mathematics instruction through giving feedback.

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PART II:

**LEARNING TO IMPLEMENT REFORM MATHEMATICS INSTRUCTION: HOW
COOPERATING TEACHERS' FEEDBACK PROVIDES OPPORTUNITIES FOR
PRESERVICE TEACHERS IN LEARNING TO TEACH**

Introduction

Preparing preservice teachers (PSTs) to teach reform mathematics curricula and instruction in their classrooms is a challenge in mathematics teacher education. Since the release of the National Council of Teachers of Mathematics' national curriculum standards in 1989, mathematics educators and researchers have advocated for "reform" curricula and instruction based on constructivist and socio-cultural theories of learning. However, this "problem-based," "student-centered" instruction is often different from the curricula and instruction experienced by many PSTs in their K-12 schooling (Ball, 1988; Ball, 1990), and is difficult for PSTs to enact (Borko, Underhill, Brown, Jones, & Jones, 1993). As a result, mathematics teacher educators continue to strive to find ways of providing learning experiences for PSTs that support the PSTs in learning about reform instruction, as well as how to enact this type of instruction in their work with students.

Past research has investigated and demonstrated some of the opportunities that teacher educators in the coursework component of teacher education programs (e.g., course instructors) provide to help PSTs learn about reform instruction and how to enact this type of instruction in their work with students (e.g., Eisenhart, et al., 1993; Ghouseini & Herbst, 2015; Lampert et al, 2013; Peressini, Borko, Romagnano, Knuth, & Willis, 2004; Putnam & Borko, 2000). However, few studies have examined and described what opportunities teacher educators in the fieldwork component of teacher education programs (e.g., cooperating teachers) provide to help PSTs learn about reform instruction and how to enact it (one notable exception is Borko & Mayfield, 1995).

The fieldwork component of teacher education programs can be a particularly productive site for supporting PSTs in learning about a particular type of instruction and how to it (e.g., reform mathematics instruction). During the fieldwork component, the PST spends an extended

period of time in a classroom (or alternative authentic learning context) working with an experienced teacher who mentors her, often referred to as a cooperating teacher (CT). While working together, the CT and PST typically share responsibility for instructing the CT's class(es). The CT often determines what teaching activities entailed in the instruction (e.g., planning and leading lessons, assessing student work, and communicating with colleagues and parents) the PST has the opportunity to do and how the PST enacts these activities (Feiman-Nemser & Buchmann, 1989). Through working with the CT, the PST has the opportunity to both observe the CT model instruction and to gain first-hand experience enacting instruction (Anderson & Shannon, 1988; Brooks & Sikes, 1997),

In addition to modeling instruction for the PST and deciding which teaching activities the PST does, the CT might enact other practices that provide additional opportunities for the PST to learn about instruction and how to enact it. These practices often include observing the PST lead instruction, giving feedback to the PST based on the observed instruction, and/or reflecting on or discussing aspects of the CT's or PST's instruction with the PST (Anderson & Shannon, 1988; Brooks & Sikes, 1997). Through engaging the PST in these learning opportunities, the CT helps the PST make sense of, view, and enact teaching in the ways the CT does (Lave & Wenger, 1991), ideally providing opportunities for the PST to develop a vision, set of understandings, dispositions, practices, and tools of teaching (Hammerness, Darling-Hammond, Bransford, 2005). In other words, the PST learns about teaching and how to enact it through engaging in the learning opportunities provided by the CT.

In an effort to understand what opportunities CTs provide to help PSTs learn about teaching and how to enact it, this study examines the learning opportunities provided as CTs enact one particular practice that supports PSTs' learning; this is the particular practice of giving

feedback to PSTs. This study focuses on this specific practice since it is one that is enacted by many CTs (Anderson & Shannon, 1988; Brooks & Sikes, 1997), albeit in varying ways (Borko & Mayfield, 1995; Valencia, Martin, Place, & Grossman, 2009). In addition, I believe that giving feedback is one of the most important practices for helping PSTs learn about teaching and how to enact it for two reasons. First, it provides an opportunity for a CT to emphasize and critique specific aspects of how the PST makes sense of, views, and enacts teaching at a given point in time. Second, it provides an opportunity for a CT to highlight additional aspects of teaching that the PST can attend to and incorporate in her future instruction.

This study builds on and extends past studies investigating the feedback that CTs give to PSTs in the field placement in two important ways. First, the majority of research studies conducted to date have investigated the feedback that CTs give during pre- or post-observation meetings (e.g., Akcan & Tatar, 2010; Borko & Mayfield, 1995; Fernandez & Erbilgin, 2009; Hawkey, 1998), as opposed to during the entire school day, including while the PST leads instruction. While it is helpful to know what aspects of teaching CTs help PSTs notice and attend to before and after they lead instruction, these studies do not capture and reflect the feedback that CTs give at other times while conversing and interacting with the PSTs. As Valencia et al. (2009) reported, CTs and PSTs do not always have frequent or systematic discussions about specific lessons, but instead tend to converse at the end of the school day or during brief intervals throughout the day. Therefore, it is important to examine the feedback that CTs give to PSTs throughout the school day. Furthermore, these studies have critiqued the content of the feedback, noting its superficial or non-existent emphasis on content knowledge and pedagogical content knowledge (e.g., Akcan & Tatar, 2010; Borko & Mayfield, 1995; Valencia et al., 2009). While this critique is perhaps fair of the feedback given during post-observation meetings, it may not

reflect the feedback content given at other times during the school day, such as while the teachers plan lessons together. Therefore, this study seeks to understand the feedback that CTs give to PSTs throughout the entire school day as opposed to during pre- and post-lesson observation meetings.

The second way in which this study builds on and extends past research concerns its focus on understanding how CTs use the practice of giving feedback to assist the PSTs in developing particular aspects of teaching. Past studies have detailed the content discussed by the teachers in their feedback conversations (e.g., Akcan & Tatar, 2010; Borko & Mayfield, 1995; Clarke, 2001; Coulon, 2004; Valencia et al., 2009) and described the dynamics and interactions that occur between the teachers as the CTs give feedback to the PSTs (Bertone, Chaliés, Clarke, & Méard, 2006; Fernandez and Erbilgin, 2009). However, these studies do not explain how the content emphasized or the interaction between the CTs and PSTs provide opportunities for the PSTs to learn about a particular aspect of the instruction and how to enact it. In other words, they describe the topics talked about as the CTs give feedback and how CTs “give” feedback without considering how the feedback acts to potentially influence or shape how the PST makes sense of, views, and enacts instruction. For example, Borko and Mayfield (1995) report that the CTs and PSTs in their study discussed four domains of teacher knowledge in their pre- and post-lesson conferences. These four domains were pedagogical knowledge, knowledge of students, pedagogical content knowledge specific to mathematics instruction, and mathematical knowledge. While stating that all four domains of knowledge were mentioned in most of the conferences to varying extents, the researchers do not explain how the CTs use their practice of giving feedback to potentially reinforce or modify the PSTs’ understanding of these domains.

Therefore, this study seeks to understand how the CTs' feedback functions to potentially help the PSTs develop certain aspects of teaching.

Given all of this, this study addresses the following research questions:

1. What aspects of teaching does the feedback emphasize?
2. How do CTs use the practice of giving feedback to assist the PSTs in potentially developing these aspects of teaching?

Theoretical Framework: Learning to Teach in the Field Placement How Preservice Teachers Learn to Teach in a Community of Practice

This study uses Lave and Wenger's (1991) theory of situated learning to describe how PSTs learn to teach while working with CTs in the field placement. According to situated learning theory, learning is framed as "increasing participation in communities of practice" (Lave & Wenger, 1991, p. 49) in which "learning, thinking, and knowing are relations among people in activity in, with, and arising from the socially and culturally structured world" (p. 51). In using the term "community of practice," Lave and Wenger refer to a "community of practitioners" (Lave, 1991, p. 64) or a group of people who perform a profession or craft, such as teaching, in the same way. Through participation in a given professional teaching community in the field placement,⁹ the *newcomer*¹⁰ PST begins to make sense of, view, and enact teaching in the ways shared by others in the teaching community, particularly their *oldtimer*¹¹ CT. Therefore, through participating in a community of reform mathematics teaching practice, the PST has the opportunity to make sense of, view, and enact teaching in the ways that the CT and other community members make sense of, view, and enact reform mathematics teaching.

⁹ The teaching community may be classroom-based, school-based, district-based, etc.

¹⁰ Lave and Wenger (1991) use the term "newcomer" to refer to a person who is new to a profession.

¹¹ Lave and Wenger (1991) use the term "oldtimer" to refer to people who have experience in a profession.

Lave and Wenger write that, in order for the PST to gain membership in the community of practice, the newcomers must have the opportunity to participate in the community in two ways. First, they must have the opportunity to “legitimately participate” in the community of practice by performing some of the authentic activities done by community members. Second, they must have the opportunity to “peripherally participate” in the community of practice by observing additional activities done by community members that they do not have the opportunity to try firsthand. Through both legitimately and peripherally participating in the community of practice, the newcomer gains a broad sense of the range of activities entailed in the profession. In the context of the field placement, this means that the PST should have the opportunity to both legitimately participate by doing some of the authentic everyday classroom activities (e.g., planning and leading lessons, interacting with students, assessing student work, collaborating with other teachers) and “peripherally participate” by observing the CT and other teachers do additional everyday teaching activities (e.g., planning and leading a parent-teacher conference, being observed by an administrator, making placement recommendations for students). Through participating in these ways, the PST gains a broad sense of the range of activities entailed in teaching, thereby helping her to make sense of, view, and enact teaching in the ways done by the community members.

This study seeks to understand how the CTs use the practice of giving feedback to assist PSTs in making sense of, viewing, and enacting teaching as they participate together in a community of teaching practice. Furthermore, while PSTs develop multiple aspects of teaching when spending time with CTs, there are particular aspects of teaching that teacher educators and researchers argue are particularly important for PSTs to develop. Therefore, this study is also

interested in focusing on how the CTs' feedback helps PSTs potentially develop these aspects of teaching.

To frame these aspects of teaching, this study uses Hammerness, Darling-Hammond, and Bransford's (2005) framework for teacher learning in a community of practice. Hammerness, Darling-Hammond, and Bransford (2005) write, "new teachers learn to teach in a community that enables them to develop a *vision* for their practice; a set of *understandings* about teaching, learning, and children; *dispositions* about how to use this knowledge; *practices* that allow them to act on their intentions and beliefs; and *tools* that support their efforts" (p. 385). A vision for practice includes "images of the possible" and "a sense of where they are going and how they are going to get students there" (p. 385). Understandings of teaching signify "deep knowledge of the content, pedagogy, students, and social context," while dispositions of teaching entail "habits of thinking and action regarding teaching and children" (p. 385). New teachers additionally develop specific practices that allow them to "develop, practice and enact a beginning repertoire" of instructional techniques, as well as to use "conceptual and practical resources" and tools to assist in their practice (p. 385). Therefore, this study uses the five dimensions identified by Hammerness et al. as analytic categories to determine which aspects of teaching the CTs' feedback emphasized, thereby providing the opportunity for the PSTs to potentially develop the desired aspects of teaching.

How Cooperating Teacher Feedback Helps Preservice Teachers Learn to Teach

To define what it means for CTs to give feedback to PSTs, this study uses Hattie and Timperley's (2007) description of feedback, which they describe as "information provided by an agent (e.g. teacher, peer, book, parent, self, experience) regarding aspects of [a person's] performance or understanding" (p. 81) as the person performs a task. Appropriating this

description of feedback to the field placement, this study defines feedback as information provided by the CT regarding the PST's vision, understandings, dispositions, practice, and tools of teaching. In identifying the aspects of teaching emphasized in the feedback, this study describes the "feedback content" (Scheeler, Ruhl, & McAfee, 2004, p. 397), which refers to the information about teaching discussed by the teachers.

In order to understand how the CTs use the practice of giving feedback to potentially help the PSTs develop the specified aspects of teaching, this study draws on Butler and Winne's (1995) description of how feedback functions to reinforce or modify the information in a learner's memory. Butler and Winne describe the four functions used in this study as follows:

First, when students' conceptual understandings or beliefs are consistent with instructional objectives, feedback can *confirm* that condition. Second, if students lack information, feedback can help students *add* information, thereby elaborating and enriching prior knowledge. Third, where elements of prior knowledge are incorrect or prior beliefs are inappropriate, feedback can provide information to replace or *overwrite* those propositions. Fourth, if students' understandings are basically correct, they still may need to *tune* those understandings, for example, by discriminating between concepts or by specifying conditions for applying learned rules. (p. 265)

Appropriating this definition to how CTs give feedback to PSTs in the field placement, this study seeks to understand how the CTs' feedback serves to potentially help the PSTs confirm, add to, overwrite, or tune their vision, understandings, disposition, practices, or tools of teaching.

Therefore, this study uses these functions as analytic categories to determine how the CTs use the practice of giving feedback to potentially help the PSTs develop a vision, understandings, dispositions, practices, and/or tools of teaching.

Research Methods

This qualitative research study is an exploratory collective case study of the feedback that CTs give to PSTs in classrooms implementing reform mathematics curricula and instruction. It focuses on the “bounded system” (Creswell, 2007) of one secondary mathematics teacher education program to look in-depth at three CT/PST pairings affiliated with the program located at different field placement sites. This teacher education program emphasized and advocated for reform mathematics instruction, and placed many of its PSTs in classrooms/field placement sites that use reform mathematics curricula and instruction.

Participants

This case study focuses on three pairs of CTs and PSTs working together in suburban secondary schools over the course of one semester. The cooperating teachers were selected because they had multiple years of experience working with PSTs, regularly engaged in everyday teaching activities with their PSTs (e.g., planning lessons, assessing student work, reflecting on instruction, communicate with colleagues and parents), and implemented reform mathematics curricula and instruction. In the first classroom, Sara (the CT) and Jane (the PST) taught 7th grade mathematics in a mid-sized school district. Sara was a white female who had taught 7th grade mathematics at her school for 25 years and had 10 years of experience working with PSTs. In the second classroom, located in the same school district, Adam (the CT) and Anna (the PST) taught high school geometry. Adam was a white male who had taught high school mathematics at his school for 13 years and had five years of experience working with PSTs. At the time of the study, Adam was one of the two mathematics department heads at his school and was one of Anna’s two CTs. In addition, a first year Special Education teacher co-planned and co-taught the geometry course with Adam and Anna. In the third classroom, in a

different mid-sized school district, Ethan (the CT) and Seth (the PST) taught 8th grade mathematics. Ethan was a white male who had taught for 16 years, nine of which were as an 8th grade teacher. The PSTs (Jane, Anna, and Seth) were white undergraduate students in their last semester of a two-year Secondary Mathematics teacher education program at a large mid-western university. During this semester, the PSTs spent the entire school day with their respective CTs as fulltime student teachers. The semester began at the end of January and ended in early June.

Data Collection

Data collection included classroom observations, and copies of classroom artifacts and instructional materials. The classroom observations captured the teachers' conversations and interactions throughout the school day since the CTs had potential opportunities to give feedback to the PSTs throughout the entire school day. The observation data was collected throughout the semester. The data included the following: (1) audio recordings of the teachers' conversations during non-instructional times (i.e., planning-time periods, time between class periods, and time before or after school); (2) observational notes of the teachers' conversations during class periods taught by the PST; (3) observational notes describing the teachers' interactions during non-instructional times and class periods taught by the PSTs (e.g., materials used and referenced by the teachers, gestures used by the teachers, and the teachers' physical location and proximity to one another); (4) copies of artifacts created and used by the teachers during the observations (e.g., lesson plans, assessments, and curricular materials); and (5) copies of the curricular materials for the courses taught by the teachers that are available online.

In the first classroom, nine planning-time periods, 15 class periods taught by Jane (the PST), and four of Sara and Jane's interactions/conversations outside of class and planning time

were observed. In the second classroom, eight planning-time periods, 10 class periods taught by Anna (the PST), and 14 of Adam and Anna's interactions/conversations outside of class and planning time were observed. In the third classroom, nine planning-time periods, 20 class periods taught by Seth (the PST), and 13 of Ethan and Seth's interactions/conversations outside of class and planning time were observed. The number of class periods taught by each PST includes multiple periods taught by the given PST on the same day. For example, Anna taught two class periods in a row during the last few weeks of her student teaching experience, so four of the 10 total class periods taught by Anna observed by the researcher occurred on two days (with two lessons occurring on each day). The audio recordings of the teachers' conversations during non-instructional time were transcribed for data analysis.

Data Analysis

In order to determine what aspects of teaching the feedback emphasizes and how the CTs used the practice of giving feedback to assist the PSTs in potentially developing these aspects of teaching, the analytic approach first identified the *feedback exchanges* that occurred between the teachers, then used Hammerness, Darling-Hammond, and Bransford's (2005) framework for teacher learning in a community of practice to code the *feedback content* of the exchanges and Butler and Winne's (1995) description of how feedback functions to code the *feedback function*.

Unit of analysis: CT/PST feedback exchanges. This work draws on Ong, deHaes, Hoos, and Lammes (1995)'s description of Bensing's (1991) definition of the exchange of information. Ong et al. write, "The exchange of information consists of information-giving and information-seeking" (p. 902). In this study, a feedback exchange occurred when the CT and PST engaged in acts of information-giving and information-seeking that were related based on a

“problem of practice” (Lampert, 1985). More specifically, the problem of practice regarded an aspect of how the PST made sense of, viewed, or enacted mathematics teaching.

In general, the feedback exchanges began in one of two ways. The first way occurred when the CT gave information to the PST about something the PST had said or done. For example, one CT made a comment to the PST that she had projected her voice well, or while another indicated that the PST should call on a particular student by pointing to the student during the PST’s lesson. The second way in which the feedback exchanges began occurred when the PST asked the CT a question about an aspect of teaching (i.e., was seeking information) or made a statement about an aspect of teaching (i.e., gave information) which the CT then gave information in response to. For example, one PST asked the CT what the students had previously learned about a concept and the CT responded by describing how the concept was presented in previous lessons taught earlier in the year. The exchanges ended when the CT finished giving information about a particular problem of practice.

The following two examples illustrate the ways in which the exchanges began and ended. In the first example, the PST begins the exchange by asking the CT what the students had previously learned about the concept of congruence. In doing so, she signals a problem of practice in relation to situating this concept and lesson in the broader curriculum. The CT replies, “Early in the year, so the way we did transformations. So we did translations, rotations, reflections and we talked about things are congruent when you, if you can do those things and basically get them to lie perfectly on top of one another. So that’s where they learned congruence. But it’s, that was chapter 1 or 2. So it’s been a long time and we’ve had them translating and rotating doing those things but I think congruence has really been lost a bit from all that.” The CT’s response first addresses the problem of practice shared by the PST since he

provides information about what the students had learned about congruence in previous chapters. In the end of the comment, the CT shifts from talking about this problem of practice when commenting that the concept “has really been lost,” meaning that the students have forgotten some of what they had previously learned. This shift signaled the end of the exchange since it concerns a different topic unrelated to the problem of practice.

In the second example, the PST begins the exchange by telling the CT that the students did not make the content connections that she had wanted them to make in her lesson. In doing so, she signals a problem of practice in relation to anticipating the difficulties that students will have in learning the content. The CT replies, “Yeah, anticipating what they'll run up against is a big one. Trying to kind of figure out what...their issues are going to be so. And never be afraid to pause and wait. Like when you said, "Let's open up to page 300 whatever" and then you started within seconds talking about it. Give them 15, 20 seconds to open their book...” The CT’s response first addresses the problem of practice shared by the PST since she provides information about a practice that the PST can use to troubleshoot the issue she faced in her lesson. In the end of the comment, the CT shifts from talking about this problem of practice when addressing a new problem of practice regarding how to pace instruction.

At times, the CT gave the information over multiple turns of talk if the PST made intermittent comments indicating her understanding of or agreement with the information given by the CT. Therefore, some of the exchanges contained multiple turns of talk between the teachers. For example, in one exchange, the PST asked the CT how to help students solve a particular problem. The CT started describing an instructional method that the PST could use. The PST then interrupted the CT to make a comment, after which the CT finished her description of the instructional method. In other words, the exchange included all of the comments made by

the PST and CT but it took two full turns of talk for the CT to give all of the information to the PST about how she could help students solve the problem.

Coding CT/PST feedback exchanges. The analysis sought to identify two aspects of the feedback in each exchange. First, it identified the content in order to understand what aspects of the teaching the feedback emphasized. Second, it identified the feedback function in order to understand how the CTs used the practice of giving feedback to assist the PSTs in potentially developing the aspects identified. Several analytic questions guided the analysis: *What aspects of teaching did the teachers emphasize in the feedback exchanges? What aspects were not emphasized? How were multiple aspects addressed in a feedback exchange, if at all? What similarities and differences occurred in the ways in which the CTs used the feedback to assist the PSTs in potentially developing these aspects of teaching?*

To address these analytic questions, each exchange was first coded to indicate which of the five aspects of the teaching were emphasized in the *content* of the exchange. Next, each exchange was coded to indicate which of the four *functions* the feedback served to help the PST potentially develop the aspect(s) of teaching identified.

Feedback content codes. To analyze the feedback content, Hammerness, Darling-Hammond, and Bransford's (2005) framework for teacher learning in a community of practice was used to code the aspects of teaching emphasized by the teachers in each exchange. Each exchange was coded for all of the categories of content that occurred in the exchange. Table 1 lists and describes the five codes used to indicate the feedback content.

The first code indicated content regarding a vision of reform mathematics instruction. Drawing on Feiman-Nemser's (2001) description of vision and examples of teachers' vision provided by Cochran-Smith and Lytle (1999, 2006), this study used this code to identify content

Table 1: Analytical framework used to code feedback content, based on Hammerness, Darling-Hammond, and Bransford's (2005) framework for teacher learning in a community of practice

u	Description	Example(s) in the Data
Vision	<p>Images of the possible and desirable instruction both in the PST's classroom and in broader education. This includes images of what the students and teachers do inside and outside of the classroom, what is entailed in the work of teaching, the role of schools in society, and the politics of schooling.</p>	<p><u>HOW TEACHER PACES INSTRUCTION:</u> The CT says, "And never be afraid to pause and wait...a lot of times I'll put the lesson up here and when they're doing teamwork-homework check and then I'll put the timer up here, so I'll say, 'Okay, finish your corrections. We're on page 333.' They know to get ready for teamwork time and then I put usually four minutes on the timer when they're...doing the teamwork-homework check."</p>
Understanding	<p>The PST's "deep knowledge of the content, pedagogy, students, and social context" (Hammerness, Darling-Hammond, & Bransford, 2005) that guides or should guide her teaching. This includes content knowledge, general pedagogical knowledge, curriculum knowledge, pedagogical content knowledge, knowledge of learners and their characteristics, and knowledge of educational contexts (Shuman, 1987).</p>	<p><u>CURRICULUM:</u> <i>The PST asks "Where have [the students] seen congruent before?"</i> The CT replies, "Early in the year, so the way we did transformations. So we did translations, rotations, reflections and we talked about things are congruent when you, if you can do those things and basically get them to lie perfectly on top of one another. So that's where they learned congruence...that was chapter 1 or 2. So it's been a long time and we've had them translating and rotating doing those things."</p>
Disposition	<p>An attitude, value, or belief that guides or should guide the PST's behavior as a teacher. This includes persistence in working with students, "the belief that all students can learn" (NCATE, 2008), "thinking dispositions" (Tishman, Jay & Perkins, 1993), self-awareness and self-reflection (Garmon, 2004), and a commitment to social justice (Garmon, 2004).</p>	<p><u>ALL STUDENTS HAVE VALUABLE INSIGHTS:</u> The CT says, "If some non-honors who had good insights, try and solicit from them because those honors kids are going to, they're the ones who'd be sitting in class with their hands up all day long."</p>

Practice	An activity that the PST did, should have done, or could do to promote and support student learning. This includes how to select and lead an instructional activity, grade student assessments, solicit student participation in class, transition between activities during the lesson, pace instruction, and communicate with parents.	<p><u>PROJECTING STUDENT WORK:</u> <i>The PST puts a piece of paper on the document camera. The CT says, “Can you slide it over? We can’t see it”</i></p> <p><u>SPEAKING:</u> The CT says, “Your tone and your voice projection was great. I was at different points around the room and I could hear you very clearly where I was so that was really good.”</p>
Tool	The conceptual and practical tools of teaching (Grossman, Smagorinsky, & Valencia, 1999) that the PST uses, should use, or could use to promote and support student learning. This includes “principles, framework, and ideas about teaching, learning and [disciplinary] acquisition that teachers use as heuristics to guide decisions about teaching and learning” and “classroom practices, strategies, and resources that ...[have] local and immediate utility” (Grossman, Smagorinsky, & Valencia, 1999, pp. 13-15).	<p><u>TIMER:</u> The CT says, “So, you know, pause and give them, I’m a big timer person. Give them a minute.”</p> <p><u>STRATEGY TO FACILITATE STUDENT GROUPWORK:</u> <i>The PST says, “So maybe even that [problem] they could do, ‘You have to explain this to your elbow partner.’”</i> The CT says, “Well, they could. What about where they, you write the problem on the top of a sheet of paper, everyone’s got a different color pen, and as they work through solving for x they spin it around the table because there’s a lot of steps.”</p>

regarding images of the possible and desirable instruction both in the PST’s classroom and in broad education (images of what the students and teachers do inside and outside of the classroom, what is entailed in the work of teaching, the role of schools in society, and the politics of schooling).

The second code indicated content regarding understandings of reform mathematics instruction. Drawing on Hammerness, Darling-Hammond, and Bransford’s (2005) description of understanding, this study used this code to identify content regarding the PST’s “deep knowledge of the content, pedagogy, students, and social context” (Hammerness, Darling-Hammond, & Bransford, 2005) that guides or should guide her teaching. This code was used for exchanges in which the teachers communicated about content knowledge, general pedagogical knowledge,

curriculum knowledge, pedagogical content knowledge, knowledge of learners and their characteristics, and knowledge of educational contexts (Shuman, 1987).

The third code indicated content regarding dispositions of reform mathematics instruction. Drawing on the National Council for Accreditation of Teacher Education's (2008) definition of dispositions, this study used this code to identify content regarding an attitude, value, or belief that guides or should guide the PST's behavior as a teacher (e.g. persistence in working with students, believing that all students can learn, commitment to social justice).

The fourth code indicated content regarding practices of reform mathematics instruction. Drawing on Lampert's (2009) definition of practice and Ball and Forzani's (2009) description of this study used this code to identify content regarding an activity that the PST did, should have done, or could do to promote and support student learning. This code was used for exchanges in which the teachers communicated about how to select and lead an instructional activity, grade student assessments, solicit student participation in class, transition between activities during the lesson, pace instruction, and communicate with parents.

The fifth code indicated content regarding tools of reform mathematics instruction. Using Grossman, Smagorinsky, and Valencia's (1999) description of tools, this study used this code to identify content regarding the conceptual and practical tools of teaching (Grossman, Smagorinsky, & Valencia, 1999) that the PST uses, should use, or could use to promote and support student learning. This code was used for exchanges in which the teachers communicated about "principles, framework, and ideas about teaching, learning and [disciplinary] acquisition that teachers use as heuristics to guide decisions about teaching and learning" and "classroom practices, strategies, and resources that...[have] local and immediate utility" (Grossman, Smagorinsky, & Valencia, 1999, pp. 13-15).

Feedback function codes. To analyze how the CTs used the practice of giving feedback to assist the PSTs in potentially developing the aspects of teaching identified, Butler and Winne's (1995) description of how feedback functions was used to code for how the feedback served to potentially confirm, add to, overwrite, or tune aspects of the PSTs' vision, understandings, dispositions, practices, and tools of teaching. Each exchange was coded for all of the functions that the feedback served in the exchange. Table 2 lists and describes the four codes used to indicate the feedback function.

The first code indicated feedback that functioned to potentially confirm the PST's vision, understandings, dispositions, practices, or tools of teaching. This feedback indicated that an aspect of the PST's vision, understandings, dispositions, practices, or tools of teaching is correct. The second code indicated feedback that functioned to potentially add to the PST's vision, understandings, dispositions, practices, or tools of teaching. This feedback provided information

Table 2: Analytical framework used to code feedback function, based on Butler and Winne's (1995) categories of the functions of feedback for student understanding and beliefs

n	Description	Example in the Data
Confirm	The feedback indicates that an aspect of the PST's vision, understandings, dispositions, practices, or tools of teaching is correct.	<p><u>Conceptualization:</u> The PST says, "What I did notice from the flex grouping though is that they knew how to write the equations better when they were given a scenario." The CT replies, "Right. Oh yeah. For sure. They're rock stars with that."</p> <p><u>Enactment:</u> The CT says, "Your tone and your voice projection was great. Like I was at different points around the room and I could hear you very clearly where I was, so that was really good."</p>

Add To	The feedback that gives information to enrich an aspect of the PST's vision, understandings, dispositions, practices, or tools of teaching.	<p><u>Conceptualization:</u> The PST asks, "Where have they seen congruent before?" The CT replies, "Early in the year, so the way we did transformations. So we did translations, rotations, reflections, and we talked about things are congruent when you, if you can do those things and basically get them to lie perfectly on top of one another. So that's where they learned congruence. But it's, that was chapter 1 or 2. So it's been a long time and we've had them translating and rotating doing those things."</p> <p><u>Enactment:</u> The CT says, "Maybe next time you do...when they get it, see what they can do by themselves without so much of our help. Try and start stepping back."</p>
Overwrite	The feedback that gives information to replace an incorrect aspect of the PST's vision, understandings, dispositions, practices, or tools of teaching.	N/A
Tune	The feedback that gives information to make a small change to an aspect of the PST's mostly correct vision, understandings, dispositions, practices, or tools of teaching.	<p><u>Conceptualization:</u> The teachers are planning a lesson on finding the best fit line for a data set. The PST says, "Do you have, we've done like least square regression things. I'm sure the students are not doing that right now. But do you have specific ways that you do best fit line? Or just pick two points and average it?" The CT says, "You know, I always tell them that they should have as many points above the line as they do below and that it should try to kind of go through some of them. So you'll see like for this one, you've got three points above..." The PST says, "Three points below." The CT says, "...and three points below and it kind of goes through those three points, so that's always how we've talked about it."</p> <p><u>Enactment:</u> The PST projects an image on the document camera. The CT says, "Can you slide it over [PST name]? We can't see it."</p>

to enrich an aspect of the PST's existing vision, understandings, dispositions, practices, or tools of teaching.

The third code indicated feedback that functioned to potentially write over an aspect of the PST's vision, understandings, dispositions, practices, or tools of teaching. This feedback provided information to replace an incorrect aspect of the PST's existing vision, understandings, dispositions, practices, or tools of reform mathematics instruction. The fourth code indicated feedback that functioned to potentially tune the PST's vision, understandings, dispositions, practices, or tools of teaching. This feedback provided information to make a small change to an aspect of the PST's existing mostly correct vision, understandings, dispositions, practices, or tools of teaching that was mostly correct.

Once all of the feedback exchanges in the transcripts and field notes were coded, the exchanges for each CT/PST pair as well as the collective CT/PST pairs were compiled for each content and function code. The next section shares the results of the analysis.

Findings

The analysis of the feedback exchanges indicates that the feedback emphasized all five types of feedback content (i.e., a vision, understandings, dispositions, practices, and tools of teaching). The types of feedback content emphasized most often were understandings and practices of teaching, as well as the practical tools of teaching referenced or used in those understandings and practices. Tables 3 and 4 summarize the feedback content regarding aspects of the PSTs' understanding and practices of teaching across all three classrooms since these were the aspects of teaching primarily emphasized in the feedback.

As indicated in table 3, all three of the CTs gave feedback to their respective PST about their understanding of: mathematics (e.g., order of operations, triangle similarity, vertical

angles), general pedagogy (e.g., strategies for communicating with parent emails, facilitating student group work, or soliciting student participants), mathematics-specific pedagogy (e.g., an instructional method for teaching the Pythagorean Theorem, the sequence in which students should work on the problems when taking a test, characteristics of problems that make it easier for students to solve), the curriculum (e.g., the scope and sequence of the curriculum, how a concept is presented across the chapters, the essential question for a lesson), and the teachers' educational context (e.g., administrators' visits to classrooms, the mathematics teachers' implementation of standards-based grading at the school, the classroom policies regarding corrections and make-up work). The CTs in classrooms 1 and 2 gave additional feedback about the PSTs' understanding of students (e.g., whether a student is medicated or has a poor home life, a student's level of confidence or attitude in mathematics class). Appendix 1 provides examples of the CTs' feedback regarding each of these understandings.

As indicated in table 4, all three of the CTs gave feedback to their respective PST about the PST's practice of: assisting students in working on a problem or activity; attending to classroom management; beginning an instructional activity; engaging students in a lesson or an instructional activity; managing classroom resources and instructional materials; modifying the instructional activities, materials, or assignments; selecting the task(s) or problem(s) for an instructional activity or assessment; talking about class expectations with students; pacing a lesson; and using a concept-specific instructional strategy. The CT in classroom 1 gave additional feedback about the PST's practice of anticipating student difficulties, ending an instructional activity, grading student work, grouping students, involving colleagues in conversations with students, pacing speech, promoting student justification, soliciting multiple student answers, transitioning between parts of a lesson or activity, and using her voice. The CT

Table 3: Summary of the Feedback Content Regarding Aspects of the Preservice Teachers' Understanding of Teaching Across Classrooms

<u>All Classrooms</u>		
<ul style="list-style-type: none"> • Content Knowledge (e.g., order of operations, triangle similarity, vertical angles) • General Pedagogical Knowledge (e.g., writing parent emails, strategies for facilitating student group work, strategies for soliciting student participants) • Curriculum Knowledge (e.g., the scope and sequence of the curriculum, how a concept is presented across the chapters, the essential question for a lesson) • Pedagogical Content Knowledge (e.g., an instructional method for teaching the Pythagorean Theorem, the sequence in which students should work on the problems when taking a test, characteristics of problems that make it easier for students to solve) • Knowledge Of Educational Contexts (e.g., administrators' visits to classrooms, the mathematics teachers' implementation of standards-based grading at the school, the classroom policies regarding corrections and make-up work) 		
<u>Classroom 1</u>	<u>Classroom 2</u>	<u>Classroom 3</u>
<ul style="list-style-type: none"> • Knowledge of Learners and their Characteristics (e.g., whether a student is medicated or has a poor home life) 	<ul style="list-style-type: none"> • Knowledge of Learners and their Characteristics (e.g., a student's level of confidence or attitude in mathematics class) 	<ul style="list-style-type: none"> • N/A

in classroom 2 gave additional feedback about the PST's practice of grading student work, leading a class discussion, selecting student participants, sharing the upcoming schedule with students, summarizing an instructional activity, and transitioning between parts of a lesson or instructional activity. The CT in classroom 3 gave additional feedback about the PST's practice of calling student by the correct name, communicating with parents, identifying when a student has a question, sharing the upcoming schedule with students, and taking attendance. Appendix 2 provides examples of the CTs' feedback regarding each of these practices.

The analysis also indicates that the CTs used the practice of giving feedback to potentially confirm, add to, or tune the PSTs' understandings and practices of teaching, as well as the practical tools of teaching referenced or used in those understandings and practices. None of the CTs used the practice of giving feedback to potentially overwrite an aspect of the PSTs' vision, understandings, dispositions, practices, or tools of teaching. While the CTs' feedback did

Table 4: Summary of the Feedback Content Regarding Aspects of the Preservice Teachers' Practice of Teaching Across Classrooms

<u>All Classrooms</u>		
<ul style="list-style-type: none"> • Assist students in working on a problem or activity • Attend to classroom management • Begin an instructional activity • Engage students in a lesson or an instructional activity • Manage classroom resources and instructional materials • Modify the instructional activities, materials, or assignments • Select the task(s) or problem(s) for an instructional activity or assessment • Talk about class expectations with students • Pace a lesson • Use a concept-specific instructional strategy 		
<u>Classroom 1</u>	<u>Classroom 2</u>	<u>Classroom 3</u>
<ul style="list-style-type: none"> • Anticipate student difficulties • End an instructional activity • Grade student work • Group students • Involve colleagues in conversations with students • Pace speech • Promote student justification • Solicit multiple student answers • Transition between parts of a lesson or activity • Voice tone and projection 	<ul style="list-style-type: none"> • Grade student work • Lead a class discussion • Select student participants • Share upcoming schedule with students • Summarize an instructional activity • Transition between parts of a lesson or activity 	<ul style="list-style-type: none"> • Call student by correct name • Communicate with parents • Identify when a student has a question • Share upcoming schedule with students • Take attendance

not serve to potentially confirm, add to, or tune aspects of the PSTs' vision or dispositions of teaching, the CTs sometimes referenced these aspects of teaching while giving feedback that served to potentially add to or tune one of the PSTs' understandings or practices of teaching. Similarly, at times, the CTs referenced a practice when giving feedback that functioned to potentially add to or tune one of the PSTs' understandings or referenced an understanding when giving feedback that served to potentially add to or tune one of the PSTs' practices.

Throughout the school day, the CT/PST pairs in all three classrooms engaged in similar kinds of day-to-day teaching activities together. This included planning lessons, leading instruction, grading student work, reflecting on instruction, and interacting with parents, colleagues, and administrators. In taking the time to do these activities with the PSTs, the CTs had the opportunity to give feedback to the PSTs about multiple kinds of understandings and practices. For example, through planning lessons and grading student work together, the CTs had the opportunity to emphasize aspects of their PST's understanding of the mathematics content, methods for teaching specific concepts, the students' misconceptions of the concepts, and how the PST could support the students' learning by communicating with parents and colleagues. In leading instruction and reflecting on the PST's lessons together, the CTs had the opportunity to give their PST feedback about aspects of instruction such as how to begin or modify an instructional activity, engage students in the lesson, manage the classroom resources or instructional materials, pace the lesson, or use a specific instructional strategy to teach a concept. Therefore, this case illustrates the breadth of content emphasized in the feedback as well as how the feedback functioned to help the PSTs potentially develop their understanding and practice of teaching as the teachers engaged in these everyday teaching activities.

This section describes and provides examples of the feedback from one of the three classrooms since this case is representative of the feedback content and how the feedback functioned across all three classrooms. In order to contextualize this case, this section first provides background information about the CT and what instruction typically looked like in the classroom. Next, it shares an overview and provides examples of the feedback that the CT gave the PST over the course of the semester. Finally, it provides a summary explaining how these

examples illustrate commonalities in the feedback content and function across all three classrooms.

Sara

Background. Sara (the CT) taught 7th grade mathematics at a high-performing middle school in a mid-sized school district. The school had white students predominantly (approximately 75%), although there were students of other races and ethnicities as well (Asian or Pacific Islander, Hispanic, Black, and American Indian/Alaska Native). In addition, approximately 13% were “students with disabilities,” 24% were “economically disadvantaged,” and 4% were “limited English proficient.” All of the classes in the school were integrated.

Sara had taught mathematics at the school for 25 years, and had been a CT for PSTs for 10 years of that time. At the school, she was in one of the two seventh grade “blocks.” Each block had one teacher from each discipline (English, Mathematics, Science, and Social Studies), in addition to support staff for the special education students and English Language Learners (ELLs). The teachers in each block all worked with the same students, although the students were grouped differently for each academic class. The school day had eight 50-minute periods and a 30-minute lunch period. Sara taught one “honors” class during period 1 and four “regular” classes during periods 2, 3, 7, and 8. Sara had her “planning time” during periods 4 and 5. The 7th grade lunch period fell between these two periods. Period 6 was an “academic resource” period when the teachers met with students struggling in their respective disciplines.

Sara interacted with the other teachers in her block daily. The teachers in the block formally met during period 4 at least twice per week to share information about the students’ in- and out-of-school lives/experiences, to discuss and strategize how to address student issues, and to touch base about their upcoming individual and grade-level activities (so that the teachers

were all on the “same page”). The teachers in the block often visited each other’s classrooms throughout the school day to share anecdotes about events that had happened in their classes or to inquire about how a particular student was doing in another class, among other things. The teachers in the block often ate lunch together, depending on their individual workloads, and during this time they talked about both professional and personal topics.

While Sara regularly interacted with the other teachers in her block, she rarely worked with the other mathematics teachers at the school, including the mathematics teacher in the other 7th grade block. In addition, once per month, she attended “vertical” meetings with some of the mathematics teachers from her school, the other middle school in the district, and the high school to discuss topics relevant across all of the schools and grade levels. The district had adopted a highly-regarded and widely-used reform mathematics curriculum named College Preparatory Mathematics (CPM) for their middle school and high school courses a few years earlier. When first learning how to implement this curriculum, Sara had worked with the other mathematics teachers in the school; however, she did not continue to do so once she and the others had become comfortable using the materials and resources in their respective classrooms. She was viewed as a “model teacher” in the district based on how she implemented the curriculum. Occasionally, other mathematics teachers in the district visited her classroom to observe how she enacted the curriculum.

Sara and Jane’s instruction. In general, the curricular materials significantly shaped Sara’s instruction since all of the middle school and high school teachers were expected to use the lessons, resources, and pacing guidelines as designed and intended. Sara used the *Core Connections Course 2* curriculum in her regular classes (the curriculum intended for 7th grade mathematics classes), and the *Core Connections Course 3* curriculum in her honors class (the

curriculum intended for 8th grade mathematics classes). Appendix 3 provides a copy of some of the materials provided for one chapter in the Core Connections Course 2, which was the curriculum taught in Sara's 7th grade mathematics class (visit for <http://cmp.org/textbooks/> additional resources and course materials).

The curriculum for each course contains multiple “chapters” divided into “sections” organized around “core” mathematics topics. Each section contains multiple lessons with problems that help students develop a “deep” understanding of the content. Each lesson has specified “core” problems, which the students must do during the lesson in order to “meet” the objective(s), as well as “non-core” problems, which are extra problems that the teacher can assign. Each lesson also includes a “closure” activity that the class can do to summarize the main points of the lesson after working on the core problems.

The teacher-version of every chapter provides detailed information about the sections and lessons in a given chapter. This includes: a “teacher guide” listing the number of “50-minute” class periods, objective(s), materials needed, and homework problems for each lesson in the chapter; how the chapter relates to other chapters in the course; recommendations for the individual and team tests used to assess the students’ learning at the end of the chapter; and a copy of the student pages with the answers for each problem. The chapter also contains a plan for each lesson. The lesson plan describes the lesson objective(s), the “core” problems, the lesson overview, strategies the teacher could use to facilitate the lesson, the closure activity, how to make the lesson “universally accessible” to all of the students (including English Language Learners and students with special needs), and “study team strategies” that the teacher can use to facilitate the students’ work in teams.

Sara regularly used the curricular materials and resources when planning and leading instruction. At the beginning of every chapter, she mapped out (in her lesson-planning book) all of the lessons in the chapter based on the information and pacing guidelines provided in the teacher materials. Generally, during periods 4 or 5 on the day before she taught a given lesson, Sara read through the lesson and prepared the necessary instructional materials (e.g., made copies of worksheets or assessments, tracked down manipulatives). She additionally used the “web-based” resources needed for particular lessons.

Sara’s lessons had the same structure most days, closely mirroring the structure of the CPM lessons. At the start of class, Sara first led a warm-up. Next, she introduced the problems that the students would work on during the lesson that day, making sure that the students understood the definitions of the terms emphasized in the lesson, as well as what they were expected to do while working on the problems. Next, Sara gave the students time to work in “teams” on the problems while she circulated to help the students work in their team, to find out what the teams were talking about, and to ask questions about the problem-solving strategies the teams used to solve the problems. At times, Sara interrupted the teamwork time to troubleshoot an issue faced by multiple students or teams. After having the students work on the problems, Sara then had the class convene to do the closure activity. She ended class by assigning the homework. Throughout the lesson, Sara encouraged the students to “employ problem-solving strategies,” “question,” “investigate,” “analyze,” “gather and construct evidence,” and “communicate” their thinking (College Preparatory Mathematics, 2004a).

While Sara facilitated the overall lesson, the students were expected to facilitate and manage their own work as a team, as was required by the curriculum. Most of the teams had four students with each student assigned to one of the following four “roles:” facilitator,

recorder/reporter, resource manager, and problem manager. The facilitator was expected to “[get] the conversation started among the team members and [make] sure the team understands the entire problem before they begin.” The recorder/reporter was expected to “[give] an update on the team’s progress, and [organize] and [introduce] the team’s report to the class.” The resource manager was expected to “[collect] the materials and resources that the team needs, [call] the teacher if there is a team question, and [organize] clean up.” The problem manager was expected to “[enforce] the use of norms and [encourage] participation, [find] compromises, and [substitute] for absent jobs” (College Preparatory Mathematics, 2004b). The students generally worked with the same team members for an extended period of time (e.g., a month), although the roles rotated among the students during that time.

The physical layout of Sara’s classroom also assisted her instruction. The classroom had eight tables spaced around the room with four chairs at each table (numbered one through four). The tables and chairs were situated so that all of the students could see the front of the classroom. In the left front corner of the classroom was the teachers’ area. This area contained Sara’s desk, a desk for her PST, a computer (on Sara’s desk), and teachers’ instructional materials distributed across the tops of both desks (e.g., the lesson-planning book, the curricular materials, work that needed to be graded, and a cord to connect the document camera with the computer).

In the center of the classroom were a document camera on a rolling cart and a stool, both facing the front of the classroom, as well as a timer on the rolling cart. Sara used these throughout her daily lessons to project images of the student-version of the textbook, student work, and class notes (written by her), and to pace her lessons. A whiteboard, projector screen, and two bulletin boards hung in the front of the classroom. The images placed on the document

camera were projected onto the projector screen throughout the class period. On the bulletin boards were posted the “word wall” and “vocabulary.” The word wall listed the name and definition of the key vocabulary words that the students had learned in recent lessons. The vocabulary board listed words (without their definition) that the students had learned in earlier chapters. Along the right side of the classroom was another whiteboard on which Sara wrote information for the students (the “big question” for the daily lesson, the team role corresponding to the chair number, and a calendar of the nightly homework problems for each class for the entire week). The classroom also contained two additional computers, various shelves and storage units containing additional manipulatives and materials used during instruction (e.g., algebra tiles, index cards, graph paper, rulers, string, wind up toys), and copies of the student- and teacher-versions of the curricular materials for all of the chapters.

Sara’s feedback. Jane (the PST) began working with Sara in late January. During her first few days in the classroom, Jane observed Sara teaching all of the classes (periods 1, 2, 3, 7, and 8). During the first week in February, Jane began teaching the honors class during period 1, which included planning and leading lessons, and assessing student work. In mid-March, she began teaching the afternoon sections of the regular class during periods 7 and 8. In general, the curricular materials determined what and how Jane led instruction since she was expected to implement the lessons as described in the lesson plans. For the regular class, Jane taught the same lessons that Sara taught in the same ways that Sara led instruction.

Given their schedule, Sara and Jane primarily spent time during periods 4 or 5 reflecting on the lessons they had taught that morning, planning their upcoming lessons, assessing student work, talking with colleagues about a pressing issue, or communicating with parents. While reflecting on the lessons they had taught that morning, Sara and Jane would discuss how Jane’s

lesson went in the honors class, and what Jane should keep in mind when leading the lesson in the regular class during periods 7 and 8, given how the lesson had gone when Sara had taught it during periods 2 and 3. Jane generally used the same lesson and instructional methods in periods 7 and 8 that Sara had used to teach periods 2 and 3. Many days, the teachers also talked briefly at the end of the day to quickly debrief something that had happened in one of the afternoon classes. They would then sometimes continue their conversation the following day. Sara rarely gave Jane feedback during her lessons or between periods 7 and 8.

An overview of Sara's feedback. Over the course of the semester, Sara's feedback provided opportunities to confirm, add to, or tune multiple aspects of Jane's understanding and practices of teaching. The feedback about Jane's understanding addressed Jane's knowledge of mathematics (e.g., content emphasized in specific lessons), pedagogy (e.g., methods for communicating with parents, grouping students, soliciting student participation, and facilitating and monitoring student engagement), mathematics-specific pedagogy (e.g., instructional methods for teaching specific concepts, strategies students use to solve a problem, and student difficulties or misconceptions with particular concepts), students (e.g., the personal characteristics of or information about a student, how a student learns or behaves in class, and a student's experiences in past mathematics classes), the curriculum (e.g., lesson-specific content, the scope and sequence of the curriculum), and the teachers' educational context (e.g., how the teachers, administrators, students, and/or parents interact with one another, and how the school schedule affects the students' work time). The feedback about Jane's practice addressed Jane's enactment of planning lessons (e.g., anticipating student difficulties with a lesson, creating student groups, and writing the lesson objective on the side whiteboard), leading lessons (e.g., giving directions when introducing the problems, using a specific instructional strategy to teach a particular

concept, modifying the lesson during instruction, soliciting multiple student answers during the closure, and pacing lessons), assessing students (e.g., grading homework and tests, helping students during a test, and talking with students about assessment content), and communicating with colleagues and parents (e.g., sharing information with colleagues about students' in-school experiences, strategizing how to address a student issue, emailing parents).

In order to illustrate how the feedback served to potentially confirm, add to, and overwrite these aspects of Jane's understanding and practice of teaching, this section shares two excerpts from a conversation that Sara and Jane had during period 4 on a day in mid-February. During their conversation, the teachers planned their upcoming lessons, reflected on the lesson Jane had taught the day before, discussed how to grade the test they had given to their regular classes that day, and talked about the weekly homework quiz. The first excerpt occurred while the teachers reflected on the lesson Jane had taught the day before. This excerpt provides examples of feedback that functioned to potentially confirm, add to, or tune aspects of Jane's practice of teaching. The second excerpt occurred while the teachers discussed how to grade the test they had given to the regular classes that day. The second excerpt also provides examples of feedback that functioned to potentially confirm, add to, or tune aspects of Jane's understanding of teaching.

Examples of Sara's feedback about Jane's practices. The first excerpt occurred while the teachers were reflecting on the lesson that Jane had taught the day before. This excerpt contains five feedback exchanges that occurred back-to-back in the teachers' conversation. The exchanges occurred in the sequence provided. In these exchanges, Sara uses her feedback to confirm Jane's practices of transitioning and speaking, to add anticipating student difficulties and

using wait time to Jane's practices, and to tune Jane's practice of asking questions. Appendix 4 contains a transcript of this entire excerpt.

Exchange 1: Feedback about Jane's practice of asking questions. The first exchange begins with Jane telling Sara that she thought her lesson was "rough" because she had not remembered whether she was supposed to ask the students a particular question during the class activity, based on the questions she had recalled reading in the lesson summary prior to teaching the lesson. In other words, Jane felt that she had not enacted the practice of asking questions during the activity well. Sara says the following in reply:

Yep. And when you're up there, if you've gone through the lesson plan and you've read what the flow should be and everything is about the quadrilaterals, that doesn't mean that you can't turn and say, "Well triangles. If there are $\frac{6}{9}$ here quadrilaterals, then how many ninths here should be the triangles?" Because they kind of did feel, I sense they felt, neglected.

In saying this, Sara provides feedback that gives information about a small change that Jane can make to how she implements her practice of asking questions. The information that Sara provides is that Jane can ask questions not provided in the CPM lesson summary so long as the new questions are aligned with the lesson's objectives. Therefore, Sara uses this feedback to tune Jane's practice of asking questions. In this example, Sara does not use the feedback to add to or overwrite Jane's practice because she does not indicate that Jane needs to enact a new practice or that Jane has an entirely incorrect practice of asking questions that needs to be replaced.

Exchange 2: Feedback about Jane's practice of transitioning. The second exchange begins with Jane telling Sara that she felt her "flow" in and out of the activity (i.e., transitions)

was “rough or rocky.” In other words, Jane felt that she had not enacted the practice of transitioning well. Sara says the following in reply:

I think it was fine. You'll get the feel for how to transition as you get more experience with it.

In saying this, Sara provides feedback that explicitly says that Jane transitioned into and out of the activity correctly. Therefore, Sara uses this feedback to confirm Jane’s practice of transitioning.

Exchange 3: Feedback about Jane’s practice of anticipating student difficulties. The third exchange begins with Jane telling Sara that, during the lesson, she had noticed that the students had difficulty making the mathematical connection that Jane wanted them to make as they worked on a particular problem. Jane mentions that she had not anticipated that the students would have this difficulty prior to teaching the lesson, and tells Sara what she had told a student in order to help the student make the desired mathematics connection. Sara says the following in reply:

Yeah, anticipating what they'll run up against is a big one. Trying to kind of figure out what their issues are going to be.

In saying this, Sara provides feedback that gives information about a practice that Jane can implement in order to remedy the problem she encountered during her lesson. The new practice Sara describes is anticipating the issues that students will have during the lesson (i.e., anticipating student difficulties). Therefore, Sara uses this feedback to add anticipating student difficulties to Jane’s practice.

Exchange 4: Feedback about Jane's practice of using wait time. The fourth exchange immediately follows the third exchange. This exchange entails Sara saying the following:

And never be afraid to pause and wait. Like when you said, "Let's open up to page 300 whatever" and then you started within seconds talking about it. Give them 15, 20 seconds to open their book because there was a lot of rustling around going on. I'm a big timer person. Give them a minute or, a lot of times I'll put the lesson up here and when they're doing teamwork-homework check and then I'll put the timer up here, so I'll say, "Okay, finish your corrections. We're on page 333." They know to get ready for teamwork time and then I put usually four minutes on the timer when they're doing the teamwork-homework check. It just seems to be a good time and that gives them time to put their folders back and stuff. But you'll get a good feel for the pacing of it and stuff as you move forward.

In saying this, Sara provides feedback that gives information about a practice that she wants Jane to do, which is "to pause and wait" (i.e., use wait time). This information includes why it is problematic if Jane does not enact this practice during her instruction ("there was a lot of rustling around going on"), what this practice entails ("give them 15, 20 seconds"), and how she enacts it in her own instruction ("I put usually four minutes on the timer when they're...doing the teamwork-homework check. It just seems to be a good time and that gives them time to put their folders back and stuff"). In this exchange, Sara uses this feedback to add using wait time to Jane's practice.

Exchange 5: Feedback about Jane's practice of speaking. The fifth exchange immediately follows the fourth exchange. This exchange entails Sara saying the following:

Your tone and your voice projection was great. I was at different points around the room and I could hear you very clearly where I was.

In saying this, Sara provides feedback that indicates that Jane spoke well (i.e., correctly) during her lesson. Sara indicates this by telling Jane that her “tone” and “voice projection” were “great.” In her feedback, Sara gives information about how she determined whether Jane was enacting this practice well, which was to see if she could hear Jane “clearly” when standing “at different points” in the room. In this exchange, Sara uses this feedback to confirm Jane’s practice of speaking.

Examples of Sara's feedback about Jane's understandings. The second excerpt occurred while Sara and Jane discussed the test they had given in class that day. This excerpt contains four feedback exchanges that did not occur back-to-back in the teachers’ conversation; however, they did occur in the sequence provided. In these exchanges, Sara uses her feedback to confirm Jane’s understanding of student difficulties, to add students’ problem-solving strategies for finding the y-intercept and an instructional strategy that teachers can use to help students’ learn how to write the equation representing a pattern to Jane’s understanding, and to tune Jane’s understanding of writing the equation representing a pattern. Appendix 5 contains a transcript of this entire excerpt.

Exchange 1: Feedback about Jane's understanding of student difficulties. The first exchange begins with Jane telling Sara that one of the students had struggled as he worked on

one of the test questions. In other words, Jane shares her understanding of the student's difficulty with mathematics. Sara says the following in reply:

He was, right, he was really struggling with, "Well why can't I do figure 0?" because they have been for all their patterns, tables, graphs.

In saying this, Sara provides feedback that indicates that Jane is correct in thinking that that the student struggled with the mathematics problem. Sara indicates this by reiterating Jane's comment that the student struggled and by describing what the student struggled with ("why can't I do figure 0?"). Therefore, Sara uses this feedback to confirm Jane's understanding of student difficulties.

Exchange 2: Feedback about Jane's understanding of students' problem-solving strategies for finding the y-intercept. The second exchange begins with Jane asking Sara how the students can find the y-intercept of the equation. In asking this, Jane indicates that she does not have an understanding of the strategies that the students can use to find the y-intercept. Sara says the following in reply:

Well, then they draw the graph and they can see that the y-intercept is -2. And then they can make the connection.

In saying this, Sara provides feedback that gives information about a strategy that the students can use to find the y-intercept. Therefore, Sara uses this feedback to add students' problem-solving strategies for finding the y-intercept to Jane's understanding.

Exchange 3: Feedback about Jane's understanding of an instructional strategy. The third exchange begins with Jane asking Sara how she helped the students learn how to write the equation representing a pattern. In asking this, Jane indicates that she does not have an

understanding of an instructional strategy that teachers can use to help students' learn about the equation. Sara says the following in reply:

That's how from the very beginning of the year when we started doing pattern-table-rule-graph in chapter 3. That's how it is. We look for the pattern, then we do the growth first, then you find the constant, which seems to be this 3 [in figure 1]. However, it's really, the intercept is -2 at the 0 figure. So that's just how we've kind of pounded it through. Once you can get a couple of these you can make the graph and scale it at 1s if possible. Then they can do a slope triangle, which they have a lot of experience from last year, and then they can figure out the y-intercept based on that and then is it really moving over 5 for every 1 that we go up on that rise.

In saying this, Sara provides feedback that gives information about how the curriculum had taught the students to find the equation. This information includes what chapters the class learned about this concept and the strategy that the students had learned to use in order to find the equation representing a pattern. Therefore, Sara uses this feedback to add an instructional strategy that teachers can use to help students' learn how to write the equation to Jane's understanding.

Jane continues the exchange by asking Sara a question clarifying that the students use the strategy for finding the equation that Sara just described instead of an alternative strategy. Sarah says the following in reply:

We're going to get more into detail in how to do those conversions later in chapter 7, in the middle of chapter 7. It's going to be a big one for the kids.

In saying this, Sara provides feedback that gives information about when the students will learn about the alternative strategy that Jane just mentioned. Therefore, Sara uses this feedback to

continue adding to Jane's understanding of an instructional strategy that teachers can use to help students learn how to write the equation representing a pattern.

Exchange 4: Feedback about Jane's understanding of students' problem-solving strategies for writing the equation representing a pattern. The fourth exchange begins with Jane asking Sara whether the students would use the equation to make the table or vice versa when finding the equation representing the pattern. Since this exchange occurred right after the previous exchange in which Sara describes how the curriculum had taught the students to find the equation, Jane presumably has some knowledge of the strategy that the students would use to find the equation. However, in asking this question, Jane indicates that she is still unclear about some aspects of the strategy. Sara says the following in reply:

Usually the table's the first thing. So the y is the total number of tiles, so like here's 8 tiles and 13. They can do that first and then if they can go and find the growth. "Well that's the five times then. Okay. So then what does this have to be?" So then they can actually say, "Well okay. Five, ten, eleven, twelve, thirteen. This is five times three but two less. Oh well then every one of those is, well okay I've got five, ten, fifteen, sixteen, seventeen, eighteen, so that's, you know, five times four but then two less. Okay, two less."

In saying this, Sara provides feedback that gives information about how the students will use the table to find the equation. Therefore, Sara uses this feedback to tune Jane's understanding of the students' problem-solving strategies for writing the equation representing a pattern.

An example of Sara's feedback that referenced multiple aspects of teaching. As mentioned at the beginning of this section, although Sara's feedback did not serve to potentially confirm, add to, overwrite, or tune aspects of Jane's vision or dispositions of teaching, Sara

sometimes referenced these aspects of teaching while giving feedback that served to potentially add to or tune one of Jane's understandings or practices of teaching. In addition, at times, Sara referenced a practice when giving feedback that functioned to potentially add to or tune one of Jane's understandings or referenced an understanding when giving feedback that served to potentially add to or tune one of Jane's practices. This section revisits one of the exchanges shared above to illustrate how Sara's feedback referenced a vision, disposition, and understanding of practice while potentially adding using wait time to Jane's practices of teaching.

Sara says the following:

And never be afraid to pause and wait. Like when you said, "Let's open up to page 300 whatever" and then you started within seconds talking about it. Give them 15, 20 seconds to open their book because there was a lot of rustling around going on...I'm a big timer person. Give them a minute or, a lot of times I'll put the lesson up here and when they're doing teamwork-homework check and then I'll put the timer up here, so I'll say, "Okay, finish your corrections. We're on page 333." They know to get ready for teamwork time and then I put usually four minutes on the timer when they're... doing the teamwork-homework check. It just seems to be a good time and that gives them time to put their folders back and stuff. But you'll get a good feel for the pacing of it and stuff as you move forward.

In saying this, Sara provides feedback that references a vision of what it looks like for the teacher to use wait time during a lesson. For example, through describing how she herself implements this practice, Sara shares a vision of when and how a teacher can use a timer to pace her instruction (e.g., she puts the textbook and timer on the document camera and then sets the timer

for four minutes) and how the use of a timer benefits the students (e.g., they have time to “do” the teamwork-homework check and put their folders back). In the feedback, Sara also references dispositions of teaching. For example, when describing how the lack of time affected the students (i.e., they were “rustling around” while Jane was talking about the lesson), Sara indicates a value she holds that the students should be attentive during the lesson. In the feedback, Sara additionally references an understanding of why Jane should use this practice and how Jane can implement this practice. For example, Sara shares understandings of how the students engage in the lesson when not given “enough” transition time (i.e., they “rustle around”) and of strategies that Jane can use to pause and wait during her lesson (e.g., waiting 15 to 20 seconds after opening the textbook at the start of the lesson introduction and putting four minutes on the timer during the homework-teamwork check). Therefore, while the overall feedback served to add using wait time to Jane’s practices of teaching, the feedback also referenced multiple other aspects of teaching.

Discussion

The findings of this study indicate that, when giving feedback, the CTs provided opportunities to help the PSTs develop their understandings and practices of teaching, as well as the practical tools of teaching referenced or used in these understandings and practices. This finding is consistent with research reporting that CTs and PSTs discuss content knowledge, pedagogical knowledge, pedagogical content knowledge, and knowledge of students in their feedback conferences (e.g., Borko & Mayfield, 1995; Valencia et al., 2009). The kinds of understandings and practices emphasized in the feedback reported in this study are also consistent with Akcan & Tatar’s (2010) finding that CTs give feedback about issues and topics

that are specific to the instruction, activities, and students in their own classroom, as opposed to broader educational contexts.

This study also found that the CTs did not provide feedback about the PSTs' visions and dispositions of teaching, although the CTs did reference visions and dispositions of teaching when giving feedback that functioned to add to or tune aspects of the PSTs' understandings and practices of teaching. To clarify, the findings indicate that the feedback did not serve to confirm, add to, or tune aspects of the PSTs' visions and dispositions of teaching. These findings suggest that, if and when the PSTs shared their visions and dispositions of teaching with the CTs, the CTs did not give them feedback about it. In referencing these aspects of teaching, the CTs shared their own visions or dispositions of teaching with the PSTs, but did not address the PSTs' visions or dispositions of teaching. Researchers have noted the importance and necessity of addressing PSTs' visions and dispositions of teaching. Feiman-Nemser (2001) writes that it is through developing a vision of teaching, PSTs' visions "inspire and guide their professional learning," "connect important values and goals to concrete classroom practices," and "construct a normative basis for developing and assessing their teaching and their students' learning" (p. 1017). The National Council for Accreditation of Teacher Education (2002) writes that dispositions assist PSTs in implementing the knowledge and skills learned in teacher education programs in their own classrooms. It is possible that the CTs provided an opportunity for the PSTs to develop visions and dispositions of teaching when they referenced these aspects of teaching; however, it was beyond the scope of this study to describe how, by referencing an aspect of instruction, the CTs provide opportunities for the PSTs to develop those aspects of teaching.

Another notable aspect of the feedback content was its emphasis on understandings and practices specific to mathematics education. Past studies have criticized CTs feedback for its superficial or lack of attention to content knowledge and pedagogical content knowledge (e.g., Borko & Mayfield, 1995; Valencia et al., 2009). Counter to the findings of these studies, which have primarily collected data during the teachers' pre- and post-observation feedback sessions, this study found that all of the CTs provided opportunities for the PSTs to develop aspects of their understandings and practices of mathematics teaching. While some of the exchanges addressing these topics were brief, others were not, such as the example used to illustrate how Sara helped Jane develop her broader understanding of how to teach students about writing the equation representing a pattern. Furthermore, considering that researchers in the field of Mathematics education have identified particular understandings and practices that PSTs "need" to learn about and explore how to enact while in teacher education programs (e.g., the TeachingWorks project at the University of Michigan), the findings of this study give rise to potential future studies investigating the opportunities that CTs provide while giving feedback (and enacting other practices promoting PST learning) to help the PSTs develop these specific understandings and practices of Mathematics teaching.

The findings of this study also indicate that the CTs' practices of giving feedback provided opportunities for the PSTs to develop their understandings and practices in three ways. These three ways were to potentially confirm, add to, or tune aspects of the PSTs' understandings and practices. This finding is one contribution that this study makes to the literature since no other researchers have indicated how CTs' practices of giving feedback serve to help PSTs develop particular aspects of teaching. Since it was beyond to scope of this study to determine the ways in which the feedback actually affected the PSTs' learning, future studies

need to determine the ways in which the PSTs' understandings and practices change after they receive feedback from the CTs, in particular whether it changed in the ways intended by the CTs.

One additional factor worth mentioning when considering the feedback is the potential role that the curriculum played in shaping the aspects of teaching discussed by the teachers. All of the classrooms used mandated, highly structured curricula that detailed what the teachers should do throughout all parts of their lessons. In using these materials, the teachers had few opportunities to engage in conversations about what content should be taught in lessons and units, and what learning activities would help the students best learn and understand the content identified. Thus, in their discussions about the lessons, the teachers made many declarative statements about the lesson content and activities, as opposed to negotiating and debating what should be taught and how it should be taught, which may have allowed more opportunities for the CTs to give feedback to the PSTs (assuming the CTs would have solicited the PSTs' suggestions for the lesson content and activities). It follows that, since the curriculum was implemented by the teachers in the ways written and intended, the finding that the CTs' feedback did not function to potentially overwrite the PSTs' practices and understandings may have been due to the fact that the PSTs did not have the agency or opportunity to teach in any way but the specified way. Therefore, they did not have the opportunity to share an understanding or practice that needed to be "overwritten." Their conversations with the CTs revolved around making sense of the curricular materials, as opposed to the strengths and weaknesses of the curricular materials.

Implications

This study has implications for mathematics teacher education and research. First, this study has implications for providing professional development opportunities for CTs in order to

support them in giving feedback to PSTs. Considering that CTs often receive little preparation in how to mentor PSTs (Anderson, 2007), it is important that they engage in professional development opportunities that help them develop the knowledge and skills needed to help PSTs learn about multiple aspects of teaching, included those emphasized in this study. The findings from this study suggest that CTs may need additional support in developing their repertoire of practice in order to engage PSTs in conversations about a broader range of topics in order to give them feedback about aspects of teaching in addition to understandings and practices. Therefore, teacher education programs can provide support and professional development opportunities for CTs to assist them in developing the skills elicit other aspects of PSTs' conceptualization of teaching, in particular their vision and dispositions, in order to give the PSTs feedback about these additional aspects of teaching.

The findings of this study also demonstrate the potential ways in which the use of a prescribed curriculum influences the feedback content. Given that all three classrooms used highly-structure, mandated curricula, many aspects of the lesson planning and enactment were outside of the teachers' control. Furthermore, since the teachers used the curricular materials in the ways that they were written, the teachers' conversations generally centered on how to implement the instructional activities as provided, as opposed to critically analyzing the curricular materials and debating aspects of the lessons (e.g., what mathematics content should be taught, how that content should be taught). Therefore, when placing PSTs in classroom using prescribed curricula, teacher education programs must mindful to provide alternative opportunities for PSTs to engage in lesson planning activities that allow them to receive feedback about their understandings and practices related to these additional teaching activities. The program should also provide opportunities for PSTs to critically analyze and discuss the

ways in which the curriculum provides affordances and constraints on their instruction and students' learning. In addition, future research should examine the feedback content of CT/PST pairing who engage in creating their own lessons and/or critically analyze the curricular materials.

Since this study highlights one of the ways that CTs promote PST learning in the field placement, future research should also continue to describe the learning opportunities that CTs provide for PSTs as they engage in other practices promoting the PSTs' learning (e.g., modeling instruction for PSTs, planning lessons with PSTs) and investigate how the feedback is actually received and/or taken up by PSTs. In particular, researchers should identify if and how PSTs have the opportunity to develop their vision and dispositions of teaching since these are two essential aspects of teaching that PSTs must have the opportunity to develop but did not in this study. Since this paper describes the breadth of feedback content emphasized in the feedback as opposed to the frequency with which each aspect was emphasize, future studies and analysis can also consider the temporal dimensions of when feedback is given to establish potential trends in content over the course of their time working with PSTs. In addition, considering that this study frames the field placement as a community of teaching practice, future studies should investigate the ways in which and extent to which the CTs' feedback actually serves to help the PSTs make sense of, view, and enact these aspects of teaching in ways the ways done by the CT and other members in the community of mathematics teaching practice.

Appendix 1: Examples of the Cooperating Teachers' Feedback Regarding the Preservice Teachers' Understanding of Teaching

Content Knowledge

- **Classroom 1:** The class is playing jeopardy. The question "84 x 10 to the fifth power" is projected on the board and the class discusses the answer. The PST says to the CT, "Where would you say the parenthesis are?" The CT says, "84 x (10⁵). How did you interpret it?" The PST says, "I did the 84 and 10 have the parenthesis." The CT says, "No, I have 10 to the fifth." The PST says, "It's own power."
- **Classroom 2:** The teachers are finding the solutions for a test question about a 30°-60°-90° triangle. The PST says, "How did we say [the side of this triangle] is equal to 4?" The CT says, "Because that [side of the triangle] was 8, so that's 4, so that's 4 root 3 (4√3)... Because we'd have corresponding angles, so [that side] would be 4, and then this [side] would be 8 and this is 4√3." The PST says, "That's just kind of weird that [the diagram is] clearly not drawn to scale" The CT says, "Yeah."
- **Classroom 3:** During a lesson, the students offer suggestions for what is true about the measure of "vertical angles." In response to one of the student's suggestions, the PST says to the class, "They're congruent. That would actually be a good thing to add to the definition." The CT affirms this.

General Pedagogical Knowledge

- **Classroom 1:** The PST and CT are talking about writing parent emails. The PST says, "It seems like you also, from what you've seen, especially like with [student name], 'This is happening' not 'this is how I...' " The CT says, "This isn't my interpretation of [what happened in class]." The PST says, "Like [the email is] more factual based instead of like..." The CT says, "Yeah, what I'm thinking." The PST says, "It's not 'I'm mad at your kid' or anything like that." The CT says, "Right."
- **Classroom 2:** The teachers are talking about which study team strategies to use during an activity. The CT suggests a strategy called the swap meet that they could do (in which "one person would get up and then go to another table and swap their information."). The PST says, "Or like I've done it where they work on it at their tables and then they get in new groups that has like each person from a different group so then they all share out." The CT says, "Uh huh." The PST says, "1, 2, 3, 4 [groups]." The CT says, "Uh huh. Alright, well alright." The PST says, "We have more than 4 groups but..." The CT says, "Sixth block I've got swap meet in my brain so I think that's what I'm going to do."
- **Classroom 3:** When planning a lesson with an activity in which the students are silent, the PST asks, "Do they look at [the table] and then volunteer to go up?" The CT says, "Yeah. The theory is they don't even raise their hand."

Curriculum Knowledge

- **Classroom 1:** The PST says, "Something that I personally just want to focus on and I didn't really get a big chance to last semester is kind of looking at the whole year and what [the students] have gotten and what they're going to... What the book is really getting them, because it seems like the... particular to the abstract is the big one." The CT says, "Concrete to the abstract. Yeah, with the algebra tiles."

- **Classroom 2:** The teachers planning a lesson. The PST asks, “Where have they seen congruent before?” The CT says, “Early in the year, so the way we did transformations. So we did translations, rotations, reflections and we talked about things are congruent when you, if you can do those things and basically get them to lie perfectly on top of one another. So that’s where they learned congruence. But it’s, that was chapter 1 or 2. So it’s been a long time and we’ve had them translating and rotating doing those things but I think congruence has really been lost a bit from all that.”
- **Classroom 3:** The teachers are modifying the slides for an upcoming lesson. As they look at the slide stating the lesson’s essential question, the PST says, “The essential question is good but I feel like almost like ‘What are the benefits of combining it into one equation.’” The CT says, “Is a better question?” The PST says, “Yeah.” The CT says, “Okay. I have no problem with that. Do you like ‘What are the benefits of combining,’ what was the question?” The PST says, “This was ‘What are the advantages and disadvantages of using one equation rather than two or more equations?’” The CT says, “All right”

Pedagogical Content Knowledge

- **Classroom 1:** The teachers talk about doing a lesson on the Pythagorean Theorem. The PST says, “Have you seen the starburst, like when you build the squares with starbursts?” The CT says, “No... We build the squares and we actually count the squares and add them up and realize that these two [squares] do equal this third [square].” The PST says, “I’ve seen it where they put starburst and because they’re squares so it’s like a unit square on each one of [the “legs” of the triangle]. And then you take the starbursts from [the legs] and you line them all up... onto the hypotenuse and then you can see that it’s the same volume and then they get to eat starbursts or their square candy.” The CT says, “That would be so fun... Yes, let’s do it.”
- **Classroom 2:** The teachers are discussing the questions they are going to give on a test. The PST says, “I mean maybe we should say [to the students] ‘Work on this [problem] first.’ Because I mean this one like, although I don’t feel like it’s hard it’s just a lot of steps. And I feel like it’s going to be hard for all of them to be on the same page if they’re working like with teams. But it might be a good team [problem].” The CT says, “So maybe... we’d like them to do three but if they don’t get there, if they run out of time then so be it. So start with [problems 4 and 5]. I mean and... if they don’t recall the 30-60-90 relationships [for problem 3], which we haven’t made a huge priority, then things go slower. I mean I think... someone amongst them will come up with it but...” The PST says, “Yeah. I think this [problem] is better than that [problem].” The CT says, “Yeah. So do this last.”
- **Classroom 3:** The teachers are reflecting on a lesson they taught earlier in the day. The PST says, “What I did notice from the flex grouping though is that they knew how to write the equations better when they were given a scenario.” The CT says, “Right. Oh yeah. For sure. They’re rock stars with that.”

Knowledge Of Learners And Their Characteristics

- **Classroom 1:** The PST says, “I have a question about [student name] ... does he usually do work or is it just...” The CT says, “It depends whether he’s on his meds or not. I have the feeling that right now he’s not on his meds because he is grumpy. He doesn’t talk, like he’ll cover up, he’ll even like cover up his mouth to speak to you and I’ll say ‘[Student name] you need to speak clearly. I don’t understand you.’ So he’s just one of those kids that’s so reflective of the kind of night he had at home... he’s got a crappy home life.” The PST later says, “Okay.

So I was just trying to get him to do something with the tiles and it seemed like more of a battle.” The CT says, “He’s a tough one. He’s really stubborn. He shuts down.”

- **Classroom 2:** The teachers are creating a list of things they need to remember to do. The PST says, “Try and get [student name 1] to come to the...[All School Resource block], because she says she’s been going to chem a lot but she’s just struggling on the little things and then losing her confidence.” The CT says, “She and [student name 2].” The Special Education Teacher says, “Yes. Well [student name 2] feeds her the attitude.” The PST says, “ [Student name 2] doesn’t even look at the math.” The CT says, “Yeah.”

Knowledge Of Educational Contexts

- **Classroom 1:** The PST says, “I know [the Dean of Students] came in a while ago and then I think [the Associate Principal] came in the other day. If they’re popping into the class, especially like 7th and 8th hour when I’m teaching it, I should assume it’s for kids and not that they’re watching me.” The CT says, “[The Associate Principal], right. [The Associate Principal] comes in a lot. He used to come in more frequently. They’ve been really busy but he’ll, they used to come in weekly anytime of the day he would stop and ask you ‘What’s the big question?’ ...components of the essential question. And it’s just to get out of the main office and ... normalcy. Some time for them so it doesn’t mean that they’re, but they certainly could.”
- **Classroom 2:** The PST says, “One other thing I really like about like how they do math curriculum here is like mastery and developing ideas like in algebra we are literally our tests are mastery is on the front two sides, developing...” The CT says, “And then, but then, what’s happening is we got, we’ve done all this and put the structures in place for standards-based grading but then most of the teachers are in this middle zone and it’s a bad place to be, it’s almost worse than doing it all one way or the other.”
- **Classroom 3:** The PST says, “Do you allow corrections on the reflection?” The CT says, “No... and I would even say not back to that first quiz...the other one at least started it earlier.” The PST asks, “When do they have to make up the homework by? I know it’s by the end of the unit.” The CT says, “Put in next Wednesday. With him as many missing as he has, it’ll help his grade

Appendix 2: Examples of the Cooperating Teachers' Feedback Regarding the Preservice Teachers' Practice of Teaching

Anticipate student difficulties

- Classroom 1: The PST says, "Something that I didn't necessarily notice would be a problem...was when I put all the labels up there and a lot of students didn't make the connection that they would have to do parallelograms and other quadrilaterals and subtract the two or make one big one and cut it in half..." The CT says, "Yeah, anticipating what they'll run up against is a big one. Trying to kind of figure out what that's going to be, what their issues are going to be so."

Assist students in working on a problem or activity

- Classroom 1: When discussing the difficulty that a student had while solving a problem, the PST says, "Because what I was trying to get with him [when working with him] was, 'What was your rule when you're going up? So it's the same rule but the opposite when you're going down. So when you get to figure 1 that's 3. Then when you get to figure 0 you still use the same rule.'" The CT says, "Right, but 'What is your intercept on y?'" The PST says, "Right."
- Classroom 2: The CT says, "And one thing [the Special Education teacher] and I were talking is maybe next time you do, well I, when they get it see what they can do by themselves." The PST says, "Completely individual." The CT says, "Without so much of our help." The PST says, "Sure." The CT says, "Try and start stepping back." The PST says, "Yeah. Gradual release?" The CT says, "Uh huh." The PST says, "Okay."
- Classroom 3: During a lesson, the CT points to the PST to indicate that the student needs help.

Attend to classroom management

- Classroom 1: The teachers are talking about what to do during the next class with a student who was difficult in class that day. The CT says, "My suggestion on Monday is to just remove him. Call [the Dean of Students] or have [the Special Ed teacher] take him down to the office if he is going behave [that way]."
- Classroom 2: The CT says, "And then 7th block if they're not listening and just, walk in the middle of the room..." The PST says, "Stop." The CT says, "...and just say, you know, 'Can I have you attention?' and then just wait. Make it, like a really awkward silence or just try and get that. Don't try and talk over them." The PST says, "Uh huh." The CT says, "All right."
- Classroom 3: A student leaves class and the CT says to the PST, "Did you just tell [student name] he could leave?" The PST says he did not. The CT walks out of the room after the student.

Begin an instructional activity

- Classroom 1: The PST shows the jeopardy board on the doc cam and stands by the projector screen. The CT walks over to the PST. The CT says, "Wait until everyone sits down."
- Classroom 2: The PST puts 6-2 on doc cam and has all students do all problems. The PST says some tables will do some problems. The CT does an action or says something to

indicate that the students should do all of the problems. The PST tells the class to do all of the problems.

- Classroom 3: The CT points to the note that he had written during the PST's lesson which says, "Don't tell them the page number." The CT says, "I just don't want to forget. We did the tile thing and moved them over, you said, 'Look on page 7 of your book.' I don't know if it's a big deal if they'll look on page 7...I don't know if it matters."

Call student by correct name

- Classroom 3: The PST calls on a student. The CT says to the PST, "That's not [student name]. That's [a different student name]."

Communicate with parents

- Classroom 3: The PST is writing the draft of an email to parents. The PST says, "Do you allow corrections on the reflection?" The CT says, "No...and I would even say not back to that first quiz...the other one at least started it earlier." The PST asks, "When do they have to make up the homework by? I know it's by the end of the unit." The CT says, "Put in next Wednesday. With him as many missing as he has, it'll help his grade...Did she ask if he could retake [the quizzes]?" The PST says, "Yeah." The CT says, "Fine...let's put it this way then. 'Although he can...'" The CT continues telling the PST what he should write in the email, and then takes over typing the email. The PST later asks, "Can you conclude it better than that?" indicating the conclusion he had written for the email. The CT says, "No" and types more in the email. The CT says, "Alright, 'Please see attached spreadsheet.' Did you attach a spreadsheet? ... [The mother] jumped over us at one point too to the principals or something. [The student] didn't do his packet either. I don't think she even mentions the first quiz... Oh it's, the quiz is called investigations." The PST says, "So we should just..." The CT says, "I guess we could put it in."

End an instructional activity

- Classroom 1: The class is playing jeopardy. The PST tells the class that they need to behave better since the other classes have "been able to handle it." The CT says, "Well let's finish this round."

Engage students in a lesson or an instructional activity

- Classroom 1: The teachers are talking about what to do about a student who the PST was frustrated with in class. The CT says, "So it's what we can do to keep him, as much as we can do to keep him in class and listening to what we're doing."
- Classroom 2: The CT says, "Just say 'Hey I saw you do this. Tell me what these last steps were' and then we can move through it just so they have the model so most things would be a minute and it's easier for them to pay attention."
- Classroom 3: In the middle of a lesson, the CT says, "Hey [ST]...have someone come up and do [something with the diagram on the board]."

Grade student work

- Classroom 1: The teachers are grading homework. The PST says, "Seven out of ten?" The CT says, "Twelve. Oh we're doing ten aren't we. Six out of Ten. Got about two-thirds of it done. The ones he did do are right."

- Classroom 2: The PST says, “So [student name] says up here correctly and then she just, so you think 3.5?” The CT says, “Yeah, 3 or 3.5. Depends how, yeah.” The PST says, “I mean her.” The CT says, “So I think it’s still a 3.” The PST says, “Three. Okay.”

Group students

- Classroom 1: Some of the students leave to read at an elementary school leaving six or so students in the class. The CT says, “You can make two teams.” The PST says, “I know, I thought of that but decided not to for team homework check.”

Identify when a student has a question

- Classroom 3: The CT says to the PST, “Hey you got a question there,” indicating that a student has a question.

Involve colleagues in conversations with students

- Classroom 1: The teachers are talking about what to do about a student who the PST was frustrated with in class. The CT says, “Let’s talk to today during [academic resource] about that and we can even involve [English teacher]. She needs to know that this is the deal.”

Lead a class discussion

- Classroom 2: The CT and ST stand next to the doc cam. Both teachers look at a sheet on the doc cam. The PST says they are talking about “leading the class.” The CT looks at the clock and says to ST, “We get this out here...got that, write that on the board and ask them what it represents...what that and that represent, and tell the honors kids to do that” as he points to parts of the problems on the sheet on the doc cam that they are looking at.

Manage classroom resources and instructional materials

- Classroom 1: The teachers are talking about the “big question” that the PST wrote on the board earlier that day. The CT says, “Well it’s readable so that’s all that matters.”
- Classroom 2: The PST stands in the middle of the class putting the three triangular prisms together to make a cube. The CT says “[ST] they can’t see”
- Classroom 3: During a lesson, the PST tries to write on the SMARTBoard. The CT says, “You got the marker in your hands” indicating that the PST cannot go on to the next slide until he puts all the pens in the bay.

Modify the instructional activities, materials, or assignments

- Classroom 1: The teachers are talking about the lesson led by the PST the day before in which each student was assigned a shape. The PST says that while teaching she struggled to remember all of the parts of the activity detailed in her lesson plan. The CT says, “Yep. And when you’re up there, if you’ve gone through the lesson plan and you’ve read what the flow should be and everything is about the quadrilaterals that doesn’t mean that you can’t turn and say, ‘Well triangles, if there are six-ninths here quadrilaterals, then how many ninths here should be the triangles?’”
- Classroom 2: In between class periods, the CT asks, “Do you have any questions?” The PST says, “I think a lot of people are having a hard time...I think they have a hard time reflecting ... finding two triangles...Yeah, I think they’re not really seeing the point of this one. Two

triangles are formed so they are just triangles to do that but...” The CT says, “So maybe stealing someone’s picture and saying, ‘These are the two triangles that are similar.’”

- Classroom 3: In between class periods the CT says to the PST, “Okay, I’m just going to get rid of some stuff first.” The CT alters a few of the slides. The PST points to the SMARTBoard screen which has the lesson projects and says, “I think I might do the graph during that.” The CT says, “The one thing I’m curious about though, our question is systems of equations...if the questions is what systems and we want them to describe this at the end, we probably should get to question c. Actually seeing the equations and putting them together.” The PST asks, “More than I just did?” The CT says, “Shorten that up a bit and spend a little more time saying, ‘Now we have these equations’... You don’t want to give it away with the equations ... The equations share a common variable. I almost felt the estimating part went a little long...so maybe here you just, maybe just pick one, really stress 1 and 4, check these and people have the answer.”

Pace speech

- Classroom 1: The CT says, “You didn’t speak too quickly. A lot of times people when they first start teaching ‘Budadada. Slow down, it’s okay.’ You did great. So that was really good.”

Promote student justification

- Classroom 1: The PST says, “I think it’s most of the time getting familiar with the lesson, because...I went from a really old book to you know examples like that...So kind of getting adjusted to that and what they want us to do versus you know like how can I, I like to make the kids think more, I think like during the minute math too, not necessarily that this doesn’t, I’m sorry I’m not trying to go there, um, but like ‘Okay so your answer is 2. What is your, like you have to give more explanation that just 2.’” The CT says, “So justify. It’s all about justify, justify, justify. Like today, was it 3rd period, we were talking about our follow-up from our number line activity and it was like crickets. ‘Table 8. Come on [student name]’ ... called on table 8 because I know he knew, like ‘Come on give me information about what did the dot on [problem] four mean.’”

Select the task(s) or problem(s) for an instructional activity or assessment

- Classroom 1: The teachers are selecting a jeopardy board for the class to do. The PST says, “I found another one.” The CT looks at the board the PST suggested. The CT says, “Let’s try it.”
- Classroom 2: The CT says, “You want to get like a problem that’s accessible to those kids and how much better they do with this one than we had assessing.”
- Classroom 3: The teachers are discussing which problems to give the students. The PST indicates a problem and says, “They shouldn’t need to be decimals...we can do that one too.” The CT says, “Yeah. Do that one too.”

Select student participants

- Classroom 2: The PST says, “You’ve got anything else that you’ve noticed?” The CT says, “No. It was good. It just, you know, especially with, the honors kids are going to have the answers. Try it, especially if some non-honors who had good insights try and solicit from them because those honors kids are going to, they’re the one’s who’d be sitting in class with

their hands up all day long.” The PST says, “Yeah. I think that’s another thing is they’re like shouting out sometimes before giving everyone else a chance to think so maybe be a little bit more aware of that. Okay. Alright.” The CT says, “And then, yeah this is just a suggestion.” The PST says, “Yeah.”

Share upcoming schedule with students:

- Classroom 2: The CT says “do you want to tell them what’s up with the rest of the semester?” The PST says “yes” and then tells the students what they’re up to.
- Classroom 3: While planning their lesson for the day, the CT says, “Let’s also, apparently these guys have a science and social studies quiz, and they also have a math quiz on Friday, if you just let them know to get ready.”

Solicit multiple student answers

- Classroom 1: The PST says, “We talked about it in our methods class...like somebody offers an answer and you go ‘Okay that’s one answer. Are there any other answers or?’ You know not necessarily being ‘yes’ or ‘no’ but let’s talk about that...And then not necessarily even giving them the right answer...But just being like ‘Well this is an option and this is an option and what do you guys think?...Okay, move on.’ And not saying, ‘Yes.’...” The CT says, “Well you should try it in 7th and 8th period and 1st period.”

Summarize an instructional activity

- Classroom 2: The CT says, “In some cases a quick summary from you like “okay we did this this and this” boom. I think even now some of your discussions, you’re getting too much student input. Just put it down because they’ve done, some of them have done the whole problem, most of them, well all of them should have probably made it more than halfway so if it’s this or...” The PST says, “Wrap it up.” The CT says, “You just saw someone who did it and did a good job on it.”

Talk about class expectations with students

- Classroom 1: The PST says, “[Student name] and [second student name], I think, were like ‘Is the closure going to be on this week’s homework check? Can it be due Monday? There’s a lot of problems.’ And I was like...” The CT says, “No.” The PST says, “... ‘I think you can handle.’” The CT says, “You’re in accelerated math.”
- Classroom 2: The CT says, “And with 7th block I think we should give them a talk at the start of class and maybe just say “you know what” especially if the air conditioning is broken again.” The PST says, “Oh my god.” The CT says, “And it’s hot just say ‘you know what, we all know it’s hot and people complaining about it.’” The PST says, “And adults don’t feel the hot less.” The CT says, “It does not help so can we please not complain about the temperature. We’re all feeling it but complaining isn’t going to change the temperature of the classroom.’ And just maybe something positive after that. ‘Okay we’ve got 90 minutes, we’re near the end of the year. There’s a handful of things we expect you to master and we need your concentration and we didn’t feel that we had it last class mostly.’” Just some sort of speech like that just to...” The PST says, “Yeah, pump it up.” The CT says, “That you can’t shut down yet.” The PST says, “Yeah no kidding. So then, we’re less than a month.”

- Classroom 3: The PST explains how he had graded a test and tells students when test corrections are due. A student then asks when corrections are due. The CT says to the PST, “Didn’t you just say that? I wouldn’t answer it”

Take attendance

- Classroom 3: The CT says to ST, “Did you take attendance 4th hour?” The PST shakes head “no.” The CT says jokingly, “You’re a terrible person.”

Pace a lesson

- Classroom 1: The CT says, “And never be afraid to pause and wait. Like when you said, ‘Let’s open up to page six hundred whatever, three hundred whatever’ and then you started within seconds talking about it. Give them 15, 20 seconds to open their book because there was a lot of rustling around going on. So you know pause, and give them, I’m a big timer person, give them a minute.”
- Classroom 2: The CT says, “And then just one thing to keep track of is try and set time constraints. Um, I mean, like I noticed, I think it was the first problem that we did in 6th hour last time, um, you didn’t set time and it started to spill over. Because I think there was like a moment where you were ready to have the class discussion and then another team called you over and”
- Classroom 3: The CT says, “This is something to think about. We’re 16 minutes in the class. I don’t know if you save time in the warm-up... Just a thought to think about it now that we’re...” The PST says, “Maybe only go through one of the homework...” The CT says, “We have 30 minutes left. The other side is I think it’ll go a little quicker if it feels like we’re rushing for the rest of the lesson. I’m not sure. I’m not certain.”

Transition between parts of a lesson or instructional activity

- Classroom 1: The PST says, “I think the flow of getting into the activity and out of the activity is, was a little rough or rocky. I wasn’t entirely sure.” The CT says, “I think it was fine. You’ll get the feel for how to transition as you get more experience with it.”
- Classroom 2: The CT says, “For the record, we’re just talking about making the transitions more smooth.” The PST says, “So like comparing, we’ll say, ‘Okay, a pyramid first. And then how the cone compares. And then tying those two together and then do the sphere.’ Saying, ‘We already know how to find the volume of this cylinder.’” The CT says, “I feel like it might be worth even coming back to it at the end doing this is as hard as it is... So three cones made a prism.” The PST says, “Yeah, say, ‘How many pyramids?’” The CT says, “‘But is that, so now let’s look at the cone.’”

Use a concept-specific instructional strategy

- Classroom 1: The teachers are discussing a question on linear growth. The PST says, “What I was trying to get with [a student] was what was your rule when you’re going up. So it’s the same rule but the opposite when you’re going down. So when you get to figure 1 that’s 3, then when you get to figure 0 you still use the same rule.” The CT says, “Right, but what is your intercept on y.”
- Classroom 2: The CT says, “For the record, we’re just talking about making the transitions more smooth.” The PST says, “So like comparing, we’ll say, ‘Okay, a pyramid first. And then how the cone compares. And then tying those two together and then do the sphere.’”

Saying, ‘We already know how to find the volume of this cylinder.’” The CT says, “I feel like it might be worth even coming back to it at the end doing this is as hard as it is...So three cones made a prism.” The PST says, “Yeah, say, ‘How many pyramids?’” The CT says, “ ‘But is that, so now let’s look at the cone.’”

- Classroom 3: In the middle of a lesson, the CT says to the PST, “You wrote equation...get questions just keep that in mind that equal sign with expression. I don’t know if when you took all three of them if you wan to do that just because those are the three we had, so just something to think about.”

Use voice

- Classroom 1: The CT says, “Your tone and your voice projection was great. Like I was at different points around the room and I could hear you very clearly where I was so that was really good.”

Chapter 6 Teacher Guide

Section	Lesson	Days	Lesson Objectives	Materials	Homework
6.1	6.1.1	1	Comparing Expressions	<ul style="list-style-type: none"> • Algebra tiles • Lesson 6.1.1A-B Res. Pgs. 	6-6 to 6-11
	6.1.2	1	Comparing Quantities with Variables	<ul style="list-style-type: none"> • Algebra tiles • Lesson 6.1.1A Res. Pg. • Lesson 6.1.2 Res. Pg. 	6-17 to 6-22
	6.1.3	1	One Variable Inequalities	<ul style="list-style-type: none"> • Algebra tiles • Lesson 6.1.1A Res. Pg. • Poster graph paper • Marker • Sticky dots 	6-29 to 6-34
	6.1.4	1	Solving One Variable Inequalities	<ul style="list-style-type: none"> • Algebra tiles • Lesson 6.1.1A Res. Pg. 	6-42 to 6-47
6.2	6.2.1	1	Solving Equations	<ul style="list-style-type: none"> • Algebra tiles • Lesson 6.2.1 Res. Pg. 	6-54 to 6-59
	6.2.2	1	Checking Solutions and the Distributive Property	<ul style="list-style-type: none"> • Algebra tiles • Lesson 6.2.1 Res. Pg. 	6-65 to 6-70
	6.2.3	2	Solving Equations and Recording Work	<ul style="list-style-type: none"> • Lesson 6.2.3 Res. Pg. • Colored pencils 	6-81 to 6-86 and 6-87 to 6-92
	6.2.4	1	Using a Table to Write Equations from Word Problems	None	6-97 to 6-102
	6.2.5	2	Writing and Solving Equations	<ul style="list-style-type: none"> • Lesson 6.2.5 Res. Pgs. • Poster paper • Markers • Tape 	6-104 to 6-109 and 6-110 to 6-115
	6.2.6	1	Cases With Infinite or No Solutions	<ul style="list-style-type: none"> • Algebra tiles • Lesson 6.2.1 Res. Pg. 	6-122 to 6-127
	6.2.7	1	Choosing a Solving Strategy	<ul style="list-style-type: none"> • Algebra tiles • Lesson 6.2.1 Res. Pg. 	6-134 to 6-139
Chapter Closure		Various Options			

Total: 13 days plus optional time for Closure and Assessment

Overview of Chapter

In Section 6.1, students compare expressions to determine whether one is greater than the other by simplifying. To do this, they use algebra tiles on an “Expression Comparison Mat.” They investigate the legal moves of removing zero pairs from one or both sides of the mat and removing or adding balanced (matching) sets of tiles from each side of the mat as they try to simplify expressions without changing the relationship between them. In Lesson 6.1.3, students will begin to record the expressions they have compared symbolically as inequalities. They learn how to represent solutions to an inequality on a number line and to interpret the meaning of a number line graph in a specific situation.

In Section 6.2, students begin working with Equation Mats. Building from their work comparing expressions in Section 6.1, they transition to looking for values that make two expressions equal. Students learn how to write an equation, solve for a variable, and record their solving steps using algebraic notation. Students will also learn to verify their solutions by evaluating equations for a specific value. They will work with equations that have no solutions as well as those that have infinite solutions.

While students practice writing and solving equations, they return to their work with the 5-D Process and begin to write and solve equations that summarize relationships found in word problems. Students construct equations after completing the Describe/Draw and Define steps of the process and possibly one or two trials. They see that solving the equation is another strategy for solving the word problem once the variables have been defined.

Common Core State Standards for Mathematical Practices

In Chapter 6, **making sense of problems** is your main goal for Mathematical Practices. Students will be **using appropriate tools**, expression mats with algebra tiles, to focus on comparing algebraic expressions. Throughout the chapter, it is recommended that you focus very clearly on the skills of **abstract and quantitative reasoning**, **construction of viable arguments and critiquing of others’ reasoning**, and **attention to precision**, both in student communication with others and their use of tools.

Where Is This Going?

Work with solving equations in this chapter lays groundwork for topics that extend into algebra. Students will revisit techniques of solving equations in Chapter 7, when they encounter equations with fractional coefficients. Students will also apply these skills and their understanding of inverse operations to solve proportional equations in Chapter 7.

Suggested Assessment Plan for Chapter 6

For complete discussion and recommendations about assessment strategies and grading, refer to the Assessment section of this Teacher Edition.

Participation Quiz	Student Presentations	Portfolios
<p>Lesson 6.1.2 Problems 6-12 and 6-13 A Participation Quiz will encourage students to talk about how they are manipulating tiles on the expression comparison mat and to justify their work to their teammates.</p>	<p>Lesson 6.2.5 Study teams can prepare poster presentations that can stand-alone or be shared verbally. Presentations should demonstrate how students interpret word problems to define relationships between quantities and to write equations.</p>	<p>Lesson 6.2.7 Students can choose one of the problems from this lesson to include in their portfolio. Their explanation of the problem could include how they chose a strategy to use, how that strategy helped them to arrive at an answer, and why their answer is reasonable.</p>

Ideas for Team Test

In a team test, it is appropriate to assess some of the same ideas as those on the individual test, as well as some newer material, but in a team worthy form. For example, teams could be asked to represent and compare two expressions presented in a word problem. Teams could also be asked to write and solve an equation from a word problem, using the 5-D Process to help them organize information. See the online Assessment Bank for more ideas.

Ideas for Individual Test

Students compare expressions in Section 6.1. They use Expression Comparison Mats and algebra tiles to represent expressions, simplify them, and determine whether one expression represents a greater value than the other (if possible). At this point in the course, expect students to comfortably rewrite single expressions by combining like terms, making zeros, and doing simple distribution.

Section 6.1 also introduces students to representing inequalities on a number line. It is reasonable to expect students to represent expressions such as $x \leq 5$ or the solution to the inequality $12 > 2x + 1$ on a number line.

Section 6.2 focuses on solving equations. Working with algebra tiles and Equation Mats, students apply simplification strategies to solving an equation and use algebraic notation to show simplification steps. Students also learn to verify their solutions by evaluating equations for a specific value. Throughout the section, students are expected to have access to algebra tiles to help them represent and solve equations. At this point, it is appropriate to individually assess students' solving skills using equations that can be reasonably represented using the tiles and to expect students to record their work algebraically. It is recommended that you wait to assess finding solutions to equations with values that students must visualize (rather than build) until students have had additional practice in homework.

Section 6.2 also extends work with word problems to writing equations that represent those problems. Using the 5-D Process to organize information and define variables, students learn to write equations that they can then solve, either before or after completing a series of trials to find

a solution. Students will continue to practice writing equations for word problems in homework in later chapters; it is appropriate that assessment focus on defining variables and summarizing the work and solution represented in a 5-D Process table with variable expressions at this time.

It is strongly recommended that **more than half** of each test be made up of material from previous chapters. Along with any previous material, it is now appropriate to test students' ability to:

- Compare two expressions, as in problems 6-17, 6-102, and CL 6-141.
- Represent a simple inequality on a number line, as in problems 6-29, 6-42, 6-92, and CL 6-145.
- Solve a simple linear equation and record steps in symbols, as in problems 6-81, 6-98, 6-104, 6-110, 6-122, and CL 6-140.
- Evaluate expressions, including checking solutions to equations, as in problems 6-44, 6-65, 6-81, 6-90, 6-112, 6-123, and 6-139 (e).
- Solve problems using the 5-D Process, including defining variables and summarizing relationships using variable expressions, as in problems 6-70, 6-83, 6-97, 6-106, 6-111, and CL 6-144.
- Mastery of Checkpoint 6: Writing and evaluating algebraic expressions such as in problems 6-10, 6-20, 6-31, 6-44, 6-66, 6-67, 6-90, 6-112, and 6-139.

Lesson 6.1.1 How do these compare?

Comparing Expressions

Lesson Objective:	Students will build and simplify expressions on an expression mat to determine which of them is greater. Students will also build an understanding of “legal moves” with the algebra tiles on the Expression Comparison Mats.
Mathematical Practices:	Today’s goal is to reason abstractly and quantitatively , using algebra tiles to compare expressions. As teams discuss legal moves, students should critique the reasoning of others within their team to clarify this process in their own minds.
Length of Activity:	One day (approximately 45 minutes)
Core Problems:	Problems 6-1 through 6-4
Materials:	Algebra tiles, class set Algebra tiles for the overhead or document camera Lesson 6.1.1A Resource Page (“Expression Comparison Mat”), one per student copied onto cardstock (This resource page will be needed for all lessons in this section.) Lesson 6.1.1B Resource Page (“Comparing Expressions”), one per student
Lesson Overview:	So far in this course students have represented expressions using positive and negative algebra tiles and with symbols. They have simplified expressions by combining like terms, identifying and removing zero pairs, and applying the Distributive Property. In this lesson, students transition to comparing two expressions using an Expression Comparison Mat (Lesson 6.1.1A Resource Page). The Expression Comparison Mat allows two expressions to be compared side by side in an effort to determine which side has the greater value (or if the values are equal). Students add to their repertoire of simplifying strategies by identifying ways to simplify expressions that are being compared without altering the relationship between the two expressions.
Suggested Lesson Activity:	Begin this lesson with problem 6-1 in which students investigate how to simplify on a comparison mat – both by making zeros and/or by balancing (removing tiles from both sides). An alternate way to do this problem is with a Think-Ink-Pair-Share. Students can simplify by making zeros on one side (with three positive tiles and three negative tiles) OR they could remove three positive tiles from both sides and then remove three negative tiles from both sides. They are challenged to find two different methods to begin exploring the “simplifying by removing balanced sets” method. Students may need you to ask some additional

questions to help them see zeros or matching tiles. Regardless, when the mat is completely simplified, the expression on the left is greater because it simplifies to $+2$ while the one on the right simplifies to -3 . Students should recognize that a positive is always greater than a negative. At this point, do not offer strategies or assign certain moves as "valid" for creating equivalent expressions; rather, encourage students to justify their thinking as they make the moves by asking, "How do we know this is equivalent?" (in the case of removing a zero pair from one side) or "How do we know we have not changed which one is greater?" (in the case of removing the same number of tiles from both sides). After students have worked through problem 6-1, conduct a whole class discussion in which students share their strategies and justifications for why the moves do not change the value of the expressions. At this point, when students say that Mat A is greater than Mat B, introduce the notation $2 > -3$.

Note that answers in the text will provide only one way the relationship can be stated. Since simplification results vary depending on whether tiles are removed using zeros or by removing equal sets from both sides, the answers to comparison problems in this text will need to be interpreted as one possible result. Note that this means that the form or process of the simplification may be different, but not the answer itself. Equivalent relationships can be found by adding or subtracting equal amounts on both sides of the relationship.

Distribute the Lesson 6.1.1B Resource Page and explain that the students will be comparing several pairs of expressions. This resource page will help students show their recorded work for problems 6-2, 6-3, and 6-5 in an organized manner. Students will be using the pictures of expression mats to record their legal moves by circling and using arrows to show how they move the tiles. This will keep them from having to draw the expression mats, which is sometimes a laborious process. Symbolic representations will be introduced in later lessons. At this point, the focus is centered on the concrete and visual representations of the simplification steps and how the relationships between expressions are maintained.

Problem 6-2 is structured to allow students to see what is the same between the mats and what is different. Students should see that the number of x^2 , x , and $-x$ tiles on both sides is the same, so they are comparing -4 to -5 . As students share their strategies, ask questions such as, "Why did you ignore the x^2 , x , and $-x$?" This question prompts students to justify, and sets students up to describe another legal move, namely, that when comparing two quantities one can remove the same thing ("balanced sets") from both expressions and the relationship between the two expressions is preserved.

Note that comparing -4 and -5 is probably not an obvious task for many students. Some students will start by comparing the digits 4 and 5 and will conclude that -5 is larger. Instead of correcting them, you may want to ask, "We were able to remove x -tiles from both sides and maintain the

relationship. Can we still remove some tiles?" Encourage students to remove 4 negative unit tiles from both sides to have the values 0 and -1 . Students may find it easier to recognize that 0 is greater than a negative value. However, be sure to revisit the values of -5 and -4 and re-interpret the comparison. Another strategy is to remind students of their work in Chapter 2 with the number line. Ask, "Is -5 to the right or left of -4 ?"

Problems 6-3 and 6-4 formalize the concept of legal moves, or moves that maintain the existing relationship between the expressions being compared. It is critical today that you stop to discuss the first two legal moves and why they are mathematically valid. The first two legal moves are:

- Removing equivalent sets of tiles from both sides, and
- Removing a zero pair to form an equivalent expression on that side.

Help students recognize that since the goal is to determine which mat is greater, removing zero pairs or balanced sets of tiles from both mats preserves the relationship that was already present. If one side was greater before such a move, it will continue to be greater after the move.

Problem 6-5 gives teams a chance to practice the legal moves they have learned in order to simplify expressions. There are several parts of problem 6-5 since teams work at different rates. It is not necessary for students to complete all of the parts.

Remind students to show their moves on the Lesson 6.1.1B Resource Page.

Closure:
(8 minutes)

To close today's lesson, lead an activity with legal moves. Display an Expression Comparison Mat at the front of the room (drawing on the board, document camera with mats and tiles, magnetic tiles, etc.) with handfuls of tiles on each side. Have one student go to the board and ask for a class volunteer to tell that student a first step toward being able to compare the expressions. Then ask another student to come to the board and have another volunteer offer a legal move for the problem. Require students to justify each suggested move.

In this activity, stress that there are often several legal moves to choose from and that as students work in their teams in the upcoming lessons it is important that they clearly communicate how they are working with the tiles and expressions.

Universal Access:

Academic Literacy and Language Support: A Pictorial Word Wall can serve as a helpful support to students as they build vocabulary. Students are going to use the language of *greater than* and *less than* in this lesson, in addition to the symbols $>$ and $<$. Even though they should have seen these symbols in previous courses, taking the time to ensure that students understand both the meaning of the language and the convention of the symbols will be important in this lesson, as well as those that follow.

Have your students discuss what it means to compare numbers. During

the lesson, as the terms *greater than* and *less than* emerge and you introduce the symbols, have teams of students create a graphic for the wall that pairs the word with the corresponding symbol. If these are displayed in the classroom it will make it easy for students to reference in this lesson as well as those that follow.

Scaffolding: Some students who are building their understanding of variables may benefit from using algebra tiles in the “Review & Preview” section of this lesson. See the “Homework” section below for more detailed ideas for supporting students with these problems.

Team Strategies:

The role of the Resource Manager is very important when using manipulatives. Make sure they understand how to collect and return the materials. Make sure the Resource Managers understand all the expectation when using algebra tiles (no throwing, do not leave any on the floor, return the same amount in the container that was used...) and that they let their teammates know these expectations. You could do this with a Huddle of all Resource Managers before handing the tiles out.

Mathematical Background:

In this section you will deal with what is often one of the most troubling distinctions for students for whom the use of variables is new: the difference between an expression and an equation or inequality. Basically, an expression is any combination of numbers, variables, and mathematical operations such as $2x + 4y + 21$. It is a combination that can be evaluated if you know what x and y are, but nothing more. Most beginning students, however, want very much to make this expression equal to something, anything, and you will have to keep reminding them it is an object in itself.

If you have two expressions and set them equal to each other, then you have an equation. Often one of the expressions is simply a number, such as 25 in the equation $3x - 7 = 25$.

A somewhat more difficult idea for some students is the idea of equivalent expressions. These are two expressions that always have the same numerical value no matter what values you choose for any variable that occurs in either expression as long as you use the same value in both expressions. The following “equations” are actually equivalent expressions, some more obvious than others. If you perform “legal” algebraic operations on these equations, that is, steps that conform to the properties of real numbers, they will all always reduce to $0 = 0$.

$$2 + 2 = 4 \qquad 2(x + 3) = 2x + 6 \qquad 6x^2 - 10x = 2x(3x - 5)$$

Homework:

Problems 6-6 through 6-11

Note: Beginning with this chapter the number of “Review and Preview” problems has increased from 5 problems to 6 for each lesson. This is done to provide adequate practice with past topics and content while still maintaining 1-2 problems that are related to the current lesson each day.

If you have concerns about your students being able to complete this additional problem each day, then it is suggested that you preview the problems and only assign parts of those with multiple parts rather than skip entire problems.

Some students may benefit from using algebra tiles to represent and simplify expressions in problem 6-6. If so, give them the Algebra Tile Resource Page from Chapter 4 so that they can make a set of paper algebra tiles.

Notes to Self:

6.1.1 How do these compare?

Comparing Expressions



In Chapter 4, you worked with writing and simplifying expressions. As you wrote expressions, you learned that it was helpful to simplify them by combining like terms and removing zeros. In this lesson, you and your teammates will use a tool for comparing expressions. The tool will allow you to determine whether one expression is greater than the other or if they are equivalent ways of writing the same thing (that is, if they are equal).

Remember that to represent expressions with algebra tiles, you will need to be very careful about how positives and negatives are distinguished. To help you understand the diagrams in the text, the legend at right will be placed on every page containing a mat. It shows the shading for $+1$ and -1 . This model also represents a zero pair.



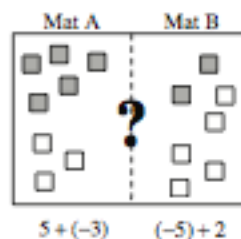
6-1. COMPARING EXPRESSIONS

Ignacio and Oliver were playing a game. Each of them grabbed a handful of algebra tiles. They wanted to see whose expression had the greater value.

Two expressions can be compared by dividing the expression mat in half to change it into an **Expression Comparison Mat**. Then the two expressions can be built side by side and compared to see which one is greater.

- Oliver put his tiles on Mat A in the picture above and described it as $5 + (-3)$.
- Ignacio put his tiles on Mat B and said it was $(-5) + 2$.

With your team, find two different methods to simplify the two expressions so you can compare them. Which side of the mat is larger? [Mat A; Simplification results vary (see the "Suggested Lesson Activity"); $2 > -3$]



- 6-2. Using your Expression Comparison Mat, build the two expressions at right. Find a way to determine which side is greater, if possible. Show your work by sketching it on the Lesson 6.1.1B Resource Page. Be ready to share your conclusion and your justification. [Mat B is greater; the x^2 , x , $-x$, and -4 can be removed from both sides since they are the same; simplification steps may vary.]



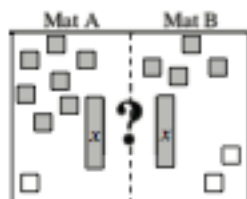
6-3. MORE COMPARING EXPRESSIONS – Is one expression greater?

Consider how you were able to compare the expressions in the previous problems. When is it possible to remove tiles to compare the expressions on the mats? In this problem, you will work with your team to identify two different “legal moves” for simplifying expressions.



Build the mat below using tiles and simplify the expressions. Record your work by drawing circles around the zeros or the balanced sets of tiles that you remove in each step on the Lesson 6.1.1B Resource Page. Which expression is greater?

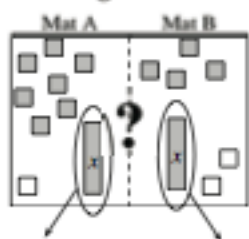
[Mat A is greater.]



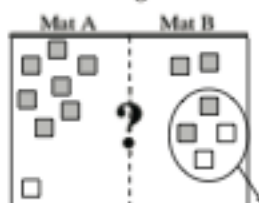
- 6-4. There are two kinds of moves you could use in problem 6-3 to simplify expressions with algebra tiles. First, you could remove zeros. Second, you could remove matching (or balanced) sets of tiles from both sides of the mat. Both moves are shown in the figures below. Justify why each of these moves can be used to simplify expressions. [Removing zero from a side does not change the expression's value; balanced sets of tiles on both sides are equal, so they can be removed without changing the relationship between the two expressions.]



Removing Balanced Sets



Removing Zeros



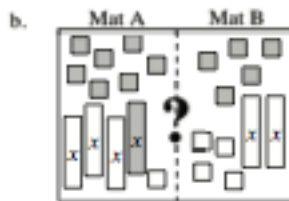
6-5. WHICH SIDE IS GREATER?

For each of the problems below, use the Lesson 6.1.1 Resource Page and:

- Build the two expressions on your mat.
- Write an expression for each side below the mats for parts (a) through (d) OR draw the tiles in the space given on the resource page for parts (e) and (f).
- Use legal moves to determine which mat is greater, if possible. Record your work by drawing circles around the zeros or the balanced (matching) sets of tiles that you remove in each problem.



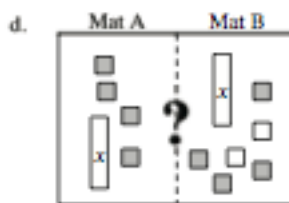
[Mat B is greater.]



[Mat A is greater.]




[They are equal.]



[Mat A is greater.]

- e. Mat A: $3x - 4 - 2$
 Mat B: $3(x - 1)$
 [Mat B is greater.]

- f. Mat A: $5 + (-3x) + 5x$
 Mat B: $x^2 + 2x + 1 - x^2$
 [Mat A is greater.]



MATH NOTES

METHODS AND MEANINGS

Inequality Symbols

- $<$ less than
- \leq less than or equal to
- $>$ greater than
- \geq greater than or equal to

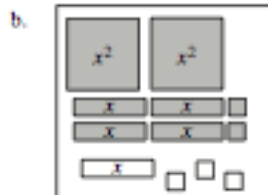
Just as the symbol “=” is used in mathematics to represent that two quantities are equal, the **inequality symbols** at right are used to describe the relationships between quantities that are **not necessarily equal**. Examples: $3 < 7$, $14 \leq 14$, $-7 < -3$, $19 \geq 14$.



- 6-6. Write the expression shown on each of the Expression Mats below. Then simplify them by making zeros and combining like terms.



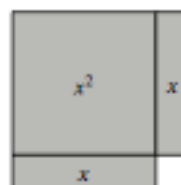
$$[x - x + 1 - 3 = -2]$$



$$[2x^2 + 4x - x + 2 - 3 = 2x^2 + 3x - 1]$$

- 6-7. Which expressions are equivalent to the perimeter of the shape? How do you know? [Parts a, c, and d match the perimeter.]

- a. $x + 3 + 3x + 1$ b. $2x + 4 + x$
 c. $4x + 4$ d. $2x + 2 + 2x + 2$



6-8. Simplify the following expressions.

- a. $-\frac{3}{4} - \frac{2}{5}$ [$-\frac{29}{20}$ or $-1\frac{9}{20}$] b. $\frac{7}{8} - \frac{2}{3}$ [$\frac{5}{24}$] c. $\frac{1}{3} - \frac{5}{6}$ [$-\frac{1}{2}$ or $-\frac{1}{2}$]
- d. $1\frac{2}{3} + (-\frac{2}{5})$ [$\frac{18}{15}$ or $1\frac{2}{3}$] e. $\frac{4}{7} - (-\frac{3}{8})$ [$\frac{31}{56}$] f. $-4\frac{1}{2} + 3\frac{1}{9}$ [$-\frac{35}{18}$ or $-1\frac{17}{18}$]

6-9. Desmond is rolling a standard six-sided number cube. He plans to roll it 72 times.



- a. About how many times would you expect Desmond to roll a 4? Why? [**About 12. There are six possible outcomes that are equally likely, and $\frac{72}{6} = 12$.**]
- b. About how many times would you expect him to roll an even number? Why? [**About 36. Half of the possible outcomes are even.**]
- c. Desmond kept track of his results for all 72 rolls. The table at right shows some of his results.

Result	Number of Outcomes
1	9
2	11
3	8
4	8

Based on his partial results, how many times did he roll a 5 or a 6? [**30 times**]

6-10. In parts (a) through (c) below, you will see pairs of quantities. For each pair of quantities, use words to write a sentence that describes the relationship. For example, "\$5, \$8" could be, "\$8 is three more than \$5."

- a. \$13, \$39 [**One possible answer: \$13 is one third of \$39.**]
- b. 25 feet, 17 feet [**One possible answer: 17 feet is 8 less than 25 feet.**]
- c. 38 lbs., 19 lbs. [**One possible answer: 38 lbs. is twice as much as 19 lbs.**]

6-11. Copy each part below on your paper. Then use the number line to help you fill in < (less than) or > (greater than) on the blank line.



- a. -5 ___ -2 b. 8 ___ -1 c. -5 ___ 0 d. -15 ___ -14
 [$-5 < -2$] [$8 > -1$] [$-5 < 0$] [$-15 < -14$]

Chapter 6 Closure What have I learned?

Reflection and Synthesis

Closure Objective: Chapter closure provides an opportunity for students to reflect about what they have learned. See the Closure section of this Teacher Edition for more information about chapter closure.

Length of Activity: Varies

Materials: For teachers using *Summarizing My Understanding*:

- Chapter 6 Closure Resource Page, one for each student
- Colored pencils, multiple colors per team

Summarizing My Understanding **Overview:** This summary activity requires students to draw explicit connections between their understanding of the rules for simplifying expressions (“legal moves”) and solving an equation on an Equation Mat and symbolically. Students will need to connect the tiles on the Equation Mat to symbols and give reasons for each step in the solution process.

Activity: Have students begin by examining Jamie’s work as presented in the text and answering the related questions. This will ensure that students make sense of the equation using the tools with which they are familiar (symbols and Equation Mat). Circulate and ask questions of teams to be sure that students note the mistake that Jamee made in her solution.

Distribute the Chapter 6 Closure Resource Page to each student. Give students time to represent the equation in three ways: on the Equation Mat, symbolically and in words. Students may approach this by solving the equation completely in one representation before moving to the next, or may do each step in each representation before moving to the next step.

Once students have finished representing the solution process in each of the three ways, Part 3 directs them to identify and color code the related steps in each representation. It is possible that some steps do not have parallel steps in each representation (such as representing the distribution on the Equation Mat). For this reason, it is important that students read the steps carefully before they decide which steps correspond, rather than assuming that each step in one representation corresponds with the next step in the sequence in the next representation. Encourage teams to talk about which steps are analogous and why as they color-code.

What Have I Learned? This section gives students the opportunity to see if they can work with the current topics at the expected level.

One way of doing the CL problems is to use a study team strategy called

the Hot Seat. Students work in teams making sure that everyone on the team understands how to do problems CL 6-140 to 6-145. Give them 15 minutes to do this. Then bring one person (say the Facilitator) from each team to the front of the classroom while the rest of the teams stay at their desks/tables. The teacher randomly selects one of the CL problems for everyone to work on at the same time. The individuals at the front each work the problem silently while their teammates are working together on the same problem at their desks/tables. After a specified amount of time the teacher tells everyone to put his/her pencil down and answers are checked. If the individual gets the problem correct, then two points are earned for their team. If the team gets the problem correct, then an additional point is earned for the team. Repeat the process with a different teammate (such as the Task Manager) coming to the front. Repeat this for as long as class time remains. The rest of the problems can be done for homework.

● **What Tools Can I Use?**

Academic Literacy and Language Support: If a deeper examination of the vocabulary and strategies to interact with vocabulary is something that your students would benefit from, then this activity might be particularly useful for chapter closure.

Begin this activity with a discussion about different strategies that students use to remember words. If students do not mention the strategy of picturing in their minds what a word means, add this idea to the list. Post the terms *positive correlation*, *negative correlation*, and *no correlation* on the board. Have students discuss with their teams how they might describe these words. Circulate and ask representatives of each team about the images these words conjure in the students' minds.

Next, task the teams to draw a sketch of what each of the terms means and to find a way to incorporate the term in the actual picture. Once they have drafts of each picture, give each student three note cards and have them put their drawing on the card.

On the back of the card, have students write a definition of each term and give three unique examples of how things are correlated.

If students find this strategy useful, try this process in the future while reading a new Math Notes box, lesson introduction, or problem in the student text.

Now might also be an appropriate time to continue building on reading comprehension strategies for times when students come across words that are unfamiliar to them.

If you are using other vocabulary strategies, such as class word walls, personal dictionaries, or vocabulary tables, you can revisit and update them, as needed, at this time.

Chapter 6 Closure What have I learned?

Reflection and Synthesis

The activities below offer you a chance to reflect about what you have learned during this chapter. As you work, look for concepts that you feel very comfortable with, ideas that you would like to learn more about, and topics with which you need more help.



① SUMMARIZING MY UNDERSTANDING

In this chapter, you have used algebra tiles and an Equation Mat as tools for solving equations. You have also represented your solution steps on an Equation Mat and with algebraic symbols. Today you will use what you have learned about equations in this chapter to show connections between all of these methods. To start, consider the following problem.

Jamee is working to solve an equation. She did the work shown below. With your team, answer the questions that follow:

Jamee's work:	Original problem:	$3(2x - 4) = 2(2x + 5)$
	Step 1:	$6x - 12 = 4x + 10$
	Step 2:	$2x = -2$



- Explain what Jamee did at each step. [She used the Distributive Property to get the second row. Then she subtracted $4x$ and 12 from both sides.]
- What is her solution? [$x = -1$]
- Is her solution correct? Justify your answer. If it is not, find her error and the correct answer. [Her solution is incorrect because she subtracted 12 from both sides instead of adding 12 to both sides. $x = 11$]

Activity continues on next page. →

① *Activity continued from previous page.*

Obtain a Chapter 6 Closure Resource Page (shown at right) from your teacher. Follow the directions below to demonstrate your understanding of solving equations with an Equation Mat, algebraically (with numbers and symbols), and in words.

Part 1: Sketch the equation on the mat on the resource page. You may also want to build it with algebra tiles.

Part 2: Complete each step to solve the equation. Represent each step on the mat, with symbols, and in words. As you work, ask questions to clarify your thinking and understanding. Make sure you can give reasons for each step.

Part 3: Color-code the matching steps in each representation. For example, if your second step is to combine like terms, label this step with green in the symbols, on the mat, and in words. Use a new color to code each step.

Answers and Support for Closure Problems

What Have I Learned?

Note: MN = Math Note, LL = Learning Log

Problem	Solution	Need Help?	More Practice
CL 6-140.	a. $3x+2 = x-2$ b. $x = -2$ c. $3(-2)+2 = -2-2$ $-6+2 = -4$ $-4 = -4$	Lessons 6.2.1, 6.2.2, and 6.2.3 MN: 6.2.1, 6.2.2, 6.2.3, and 6.2.6 LL: 6.1.2 and 6.2.3	Problems 6-54, 6-81, 6-98, 6-104, 6-110, 6-122, and 6-129
CL 6-141.	a. $x = x$. The mats are equal in value. b. No, the mats will be equal for any value of x .	Lessons 6.1.1 and 6.1.2 MN: 6.1.1 LL: 6.1.2	Problems 6-17 and 6-102
CL 6-142.	a. $P(\text{yellow or blue}) = \frac{2}{3} + \frac{1}{6} = \frac{17}{30}$. b. $P(\text{not red}) = 1 - \frac{1}{3} = \frac{2}{3}$. c. $P(\text{green}) = 1 - \frac{1}{3} - \frac{2}{3} - \frac{1}{6} = \frac{1}{10}$.	Lessons 1.2.2 and 5.2.3 MN: 1.2.3 and 5.2.5 LL: 5.2.3	Problem CL 5-151 and 6-107
CL 6-143.	a. Multiply by $\frac{1}{4}$ (or you can divide by 4 moving from shape A to shape B or multiply by 4 moving from shape B to shape A). b. On Shape A, the missing sides are 10 and 8 units; on shape B, the missing sides are 2 and 1.5 units.	Lesson 4.1.1 MN: 4.1.2	Problems CL 4-125 5-22, 5-142, and 6-99
CL 6-144.	The length is 11 in. and the width is 4 in. If $x =$ width, one possible equation would be $x + (3x-1) + x + (3x-1) = 30$.	Lessons 5.3.2, 5.3.3, and 6.2.4 MN: 5.3.3, 6.2.4 LL: 5.3.4, 6.2.4	Problems CL 5-150, 6-70, 6-83, 6-97, 6-106, and 6-111

Materials

This section contains compiled lists of all the necessary materials that do not come with the Teacher edition, materials that will need to be purchased if they are not already available as existing classroom supplies. Note: Basic teacher supplies (such as chalk or whiteboard pens, and paper for copying resource pages) are not included in these lists, nor are basic student supplies (such as scientific calculators, paper, pencils, and erasers).

The Teacher Guide at the front of each chapter includes a list of the necessary and optional materials for that chapter. Specifics and details about those materials can be found in the Materials section at the beginning of each lesson. In addition, some lessons have an additional section regarding Materials Preparation. In addition, for long-range planning, the second table below summarizes those chapter specific materials.

These are the general supplies you will frequently use in the classroom:

- Algebra tiles*
- Clear tape
- Colored pencils
- Computer / projector
- Graph paper
- Markers
- Masking tape
- Meter sticks
- Poster (graph) paper
- Presentation materials**
- Rulers
- Scissors
- Sticky dots

* Algebra Models-brand algebra tiles by Classroom Products are available through CPM. An algebra tiles applet is available at www.cpm.org.

**You will need a means to display student work for presentations such as document camera or transparencies and pens.

<u>CC1 - Materials</u>	<u>CC2 – Materials</u>	<u>CC3 – Materials</u>
4x6 index cards	bubble solution and wands	cardboard
base-ten blocks	butcher paper	index cards
dry beans	can of tennis balls	measuring tape
glue sticks	clay	models of solids
licorice	coins	paper plates
multilink cubes	colored paper	pattern blocks
paper cups	construction paper	protractors
pennies	envelopes	ribbon
plastic sheet protectors	glue sticks	rice or small beans
playing cards		rope
sticky notes		straws or skewers
straws		string
zip-top bags		tracing paper

***Integer tiles are available through CPM.

Appendix 4: A Transcript of the Excerpt of Sara's Feedback About Jane's Practice of Teaching

SARA (the CT): How'd you feel about yesterday?

JANE (the PST): It was kind of rough. Generally okay. I kind of, like making sure that I hold on all the things when we're in the circle was kind of something that I didn't think I did because then there was like the triangles and I guess I didn't remember reading that in the...

SARA: Yep. And when you're up there, if you've gone through the lesson plan and you've read what the flow should be and everything is about the quadrilaterals, that doesn't mean that you can't turn and say, "Well triangles. If there are $\frac{6}{9}$ here quadrilaterals, then how many ninths here should be the triangles?" Because they kind of did feel, I sense they felt, neglected.

JANE: Yeah. And I think the flow of getting into the activity and out of the activity is, was a little rough or rocky. I wasn't entirely sure.

SARA: I think it was fine. You'll get the feel for how to transition as you get more experience with it.

JANE: Yeah, that's something that even my CT last semester was like, "Well let's work on that a little bit more and transition." So that something that I need to work on. Something that I didn't necessarily notice would be a problem until I started going around was when I put all the labels up there and a lot of students didn't make the connection that they would have to do parallelograms and other quadrilaterals and subtract the two or make one big one and cut it in half. Because there was one student, where she had done all the separate parts and she had done nine quadrilaterals and six parallelograms and I was like, "Okay, let's think about this for a second. Add all those up, it had to equal one." So kind of reinforcing that this is one whole and then we're making it into portions and the parts of a whole.

SARA: Yeah, anticipating what they'll run up against is a big one. Trying to kind of figure out what their issues are going to be. And never be afraid to pause and wait. Like when you said, "Let's open up to page 300 whatever" and then you started within seconds talking about it. Give them 15, 20 seconds to open their book because there was a lot of rustling around going on. I'm a big timer person. Give them a minute or, a lot of times I'll put the lesson up here and when they're doing teamwork-homework check and then I'll put the timer up here, so I'll say, "Okay, finish your corrections. We're on page 333." They know to get ready for teamwork time and then I put usually four minutes on the timer when they're doing the teamwork-homework check. It just seems to be a good time and that gives them time to put their folders back and stuff. But you'll get a good feel for the pacing of it and stuff as you move forward. Your tone and your voice projection was great. I was at different points around the room and I could hear you very clearly where I was. So that was really good.

Appendix 5: A Transcript of the Excerpt of Sara's Feedback About Jane's Understanding of Teaching

JANE (the PST): Well [student name] was really struggling with that.

SARA (the CT): He was, right, he was really struggling with, "Well why can't I do figure 0?" because they have been for all their patterns, tables, graphs. Well once they get [figure] one, two, three, and four, they'll get the rule "five times" because you can clearly see that it's five for every single time is the growth.

JANE: But they like plug in and find b? Or how would they...

SARA: Well, then they draw the graph and they can see that the y-intercept is -2. And then they can make the connection.

JANE: Okay, yeah. Because what I was trying to get with him was, "What was your rule when you're going up? So it's the same rule but the opposite when you're going down. So when you get to figure 1 that's 3. Then when you get to figure 0 you still use the same rule."

SARA: Right, but "What is your intercept on y?"

JANE: Right. I get it because when you plug in figure 3 that's, you're adding five three times.

SARA: Right, so you can clearly see that this is the growth.

JANE: Uh huh. I guess. How did you explain it so that they know that you're not adding, but it's actually multiplying or repeated addition? Is that just how you went about it?

SARA: That's how from the very beginning of the year when we started doing pattern-table-rule-graph in chapter 3. That's how it is. We look for the pattern, then we do the growth first, then you find the constant which seems to be this 3 [in figure 1]. However, it's really, the intercept is -2 at the 0 figure. So that's just how we've kind of pounded it through. Once you can get a couple of these you can make the graph and scale it at 1s if possible. Then they can do a slope triangle, which they have a lot of experience from last year, and then they can figure out the y-intercept based on that and then is it really moving over 5 for every 1 that we go up on that rise.

JANE: Okay. So it's not necessarily figuring out it equation-wise but putting it on the graph and then figuring out what the slope is and y-intercept.

SARA: We're going to get more into detail in how to do those conversions later in chapter 7, in the middle of chapter 7. It's going to be a big one for the kids.

JANE: Okay. Would you go from this to this usually, or you would normally go from this to this?

SARA: Usually the table's the first thing. So the y is the total number of tiles, so like here's 8 tiles and 13. They can do that first and then if they can go and find the growth. "Well that's the five times then. Okay. So then what does this have to be?" So then they can actually say, "Well okay. Five, ten, eleven, twelve, thirteen. This is five times three but two less. Oh well then every one of those is, well okay I've got five, ten, fifteen, sixteen, seventeen, eighteen, so that's, you know, five times four but then two less. Okay, two less."

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PART III:

**INVOLVING PRESERVICE TEACHERS IN THE CONVERSATION: HELPING
PRESERVICE TEACHERS BUILD ON AND CONSTRUCT THEIR UNDERSTANDING
AND PRACTICE OF TEACHING THROUGH GIVING FEEDBACK**

Introduction

Giving feedback is a practice commonly done by cooperating teachers as they work with preservice teachers. In my work as a mathematics teacher educator, I have found it to be one of the most important practices that cooperating teachers implement for two reasons. First, feedback is an opportunity for cooperating teachers to emphasize and critique specific aspects of the preservice teachers' understandings and practices of teaching. Second, it is an opportunity for cooperating teachers to highlight new understandings and practices that the preservice teachers can attend to and incorporate in future teaching. In other words, it helps the preservice teachers to think about their own understandings and practices of teaching as well as to learn about new understandings and practices that they might not otherwise have encountered.

While giving feedback is a practice widely used by cooperating teachers, research has found that cooperating teachers give feedback in a variety of ways. For example, Valencia, Martin, Place, and Grossman (2009) reported that while most of the cooperating teachers in their study gave feedback about classroom management (e.g., praised how the preservice teacher managed the classroom or told the preservice teacher to use certain management techniques), one cooperating teacher also engaged her preservice teacher in "collaborative discussions" about how the preservice teacher led instruction related to particular content. Borko and Mayfield (1995) had also noted this variation and attributed it to cooperating teachers' beliefs about how preservice teachers learn to teach, their role as a cooperating teacher, and the extent to which they can influence the preservice teachers' learning.

As a teacher educator committed to supporting teacher learning, I advocate for feedback that helps preservice teachers build on their prior knowledge and skills, and helps them construct their own understandings and practices. These types of learning opportunities are important

because, as is the case with K-12 students, preservice teachers come to teacher education programs with ideas about teaching that provide the basis of their practice as a teacher (Cross 2009, Lortie, 1975). Since these ideas of teaching serve as the foundation for their learning experiences in the teacher education program, teacher educators need to provide learning opportunities that build on and add to the preservice teachers' prior ideas of teaching. However, teacher educators have little guidance in how to make such opportunities possible. In comparison to K-12 teachers who can find assistance in implementing instructional methods based on constructivist theories of learning, teacher educators have little support in how to develop and provide similar types of learning opportunities for their preservice teachers. In my own work as a mathematics methods course instructor and university supervisor, I have experienced the challenge of providing these kinds of learning opportunities for preservice teachers, and have observed other teacher educators endure similar challenges. Therefore, in this article I share three strategies that I have found to be helpful for providing feedback that helps preservice teachers build on their prior knowledge and skills, and helps them construct their own understandings and practices. I start by articulating what I mean by cooperating teacher feedback, and then use my observations of cooperating teacher/preservice teacher pairings to elaborate on three strategies that cooperating teachers can use when giving feedback to help preservice teachers build on and construct their understandings of and practices of teaching.

What is Feedback and What Does it Mean for Cooperating Teachers to Give Feedback to Preservice Teachers?

Using Hattie and Timperley's (2007) definition of feedback, I consider CT feedback to be any kind of information provided to PSTs regarding aspects of their understanding or performance. In my observations of how cooperating teachers giving feedback to preservice

teachers, I have found that cooperating teachers give feedback to preservice teachers throughout the school day while engaging in a variety of “everyday” teaching activities together. These teaching activities include planning and leading lessons, reflecting on instruction, assessing student work, and communication with colleagues and parents. In addition, my observations suggest that there are two phases entailed in the practice of giving feedback: (1) the preservice teacher shares an understanding she has about teaching with the cooperating teacher, asks a question about an understanding or practice to the cooperating teacher, or enacts a practice that the cooperating teacher observes; and (2) the cooperating teacher provides information to the preservice teacher about the understanding shared, question asked about an understanding or practice, or practice observed (which may entail the teachers having a discussion or conversation back and forth). For example, while planning a lesson together, the preservice teacher may ask the cooperating teacher a question about the lesson, such as how the students had learned the concept targeted in the lesson earlier in the year. In response to the question, the cooperating teacher will give feedback that provides information answering the preservice teacher’s question. Alternatively, during a lesson taught by the preservice teacher, the cooperating teacher may notice that the preservice teacher did not do something correctly, such as position a piece of paper on the document camera so that all of the students can see what is written on the paper. The cooperating will then give feedback that provides information about how the preservice teacher can correctly do the action.

**Three Strategies that Cooperating Teachers Can Use When
Giving Feedback to Help Preservice Teachers Build on and Construct
Their Understandings and Practices of Teaching**

Strategy 1: Set Goals Together for the Preservice Teacher’s Learning and Track Her Efforts Toward Meeting Those Goals

Ende (1983) and Hattie and Timperley (2007) write that an important part of giving feedback is to have the teacher and learner set goals for the learner's understanding and practice which they can later refer back to and discuss in their feedback sessions. In the context of the field placement, this means that, when first working together, the cooperating teacher and preservice teacher should work together to set two or three goals for the preservice teacher's learning in the context of the field placement. The goals should help the preservice teacher over the duration of the practicum develop understandings and practices to support students' learning. For example, the following two goals are ones that I have often set and have heard cooperating teachers set with student teachers: (1) understand the "flow" of the lesson (i.e., how the lesson introduction, activity, and summary work together to support students' exploration and learning of concepts); and (2) solicit participation from a variety of students in the class (in particular those who are not the first to volunteer). When establishing the goals, the cooperating teacher should talk with the preservice teacher about how the preservice teacher will better support the students' learning in meeting these goals, and what evidence the teachers will use to determine whether or not and how the preservice teacher has "met" each goal.

Throughout the remainder of their time together, the teachers should periodically revisit the goals in their feedback conversations to determine the ways in which and the extent to which the preservice teacher is meeting them. During the feedback sessions, the cooperating teacher should use the goals to gauge changes in the preservice teacher's understanding and practice (ideally their improvement), as well as to strategize what the preservice teacher can do in the immediate future to continue working toward meeting the goals. For example, at a given point in the semester, the preservice teacher may understand the sequence of class activities (e.g., warm-up, lesson introduction, student work time, lesson summary, homework time) but not yet

understand how to use the questions posed during the lesson introduction to facilitate the students' work on the problems during the student work time or to guide the discussion about the content during the lesson summary. Therefore, the cooperating teacher can determine ways that the preservice teacher can reference and revisit the questions posed during the lesson introduction while facilitating the student work time and lesson summary.

Once the preservice teacher has "met" one of the goals, the teachers should work together to set a new goal for the preservice teacher's learning. Thus, the preservice teacher should always have two or three larger goals that they are working to meet over an extended period of time. This is not to say that the cooperating teacher cannot set short-term or lesson-specific goals with the preservice teacher for the preservice teacher's understanding and practice, but that the preservice teacher should have a few goals related to promoting student learning and understanding that will be challenging for her to meet (yet ultimately attainable).

Strategy 2: Create Opportunities for the Preservice Teacher to Contribute Her Ideas to the Conversation

While working with cooperating teachers and researching how cooperating teachers give feedback to preservice teachers, I have noticed that cooperating teachers give preservice teachers feedback throughout the school day as the teachers engage in a variety of teaching activities. For example, while planning a lesson, the preservice teacher might ask the cooperating teacher about whether she has the correct understanding of the lesson's objective, and the cooperating teacher will tell the teacher whether or not she has the correct understanding. Alternatively, while assessing student work together, the cooperating teacher might confirm that the preservice teacher should give a certain number of points for one student's answer to a particular problem.

I have observed that, as the teachers engage in their conversations during these teaching activities, the cooperating teacher is more apt to share her ideas about teaching than the

preservice teacher is. Having the cooperating teacher dominate the conversation is problematic for two reasons. First, it limits the cooperating teacher's ability to evaluate and build on the preservice teacher's prior understanding and practice since she does not have the opportunity to evaluate what aspects of instruction the preservice teacher is focused on and attuned to. Second, it limits the preservice teacher's opportunity to construct her own understanding of her instruction since she does not have the opportunity to articulate her ideas or to describe and analyze aspects of the cooperating teacher's and preservice teacher's instruction.

One way that I have seen cooperating teachers provide opportunities for preservice teachers to contribute their ideas to the teachers' conversations is by encouraging the preservice teacher to begin the conversation. The cooperating teacher can encourage the preservice teacher to begin the conversation by asking questions that prompt the preservice teacher to share her ideas first. One question that I commonly use to start my conversation with preservice teachers is, "What are you thinking about?". Ende (1983) also suggests asking the questions, "How did you think the lesson went?", "What went well?", and "What needs improvement?" Another way that I have seen cooperating teachers provide opportunities for preservice teachers to contribute their ideas to the teachers' conversations is by inviting the preservice teacher to share her ideas about teaching before the CT indicates what she thinks. For example, when discussing what questions the preservice teacher should ask in the lesson summary, the cooperating teacher can ask the preservice teacher, "What questions do you think are important to highlight in the lesson summary?", in addition to "How will these questions help the students understand the content objective of this lesson?" and "How will these questions help the students build on and synthesize the knowledge and skills they gained while working on the problems?"

Creating opportunities for preservice teachers to contribute their ideas to conversations about teaching is particularly important in our age of mandated curricula in which teachers have limited opportunities to generate their own lesson plans and instructional materials. While using prescribed curricula is helpful for teachers in many ways, particularly given the increasing demands on educators, it means that preservice teachers have few opportunities during their practicum and/or student teaching experiences to construct their own understanding of what content should be taught in lessons and units, and create lesson plans and activities that help students learn and understand the content identified. Therefore, cooperating teachers can invite preservice teachers to lead the lesson planning sessions and explain how the activities described in the lesson plans provided will help students learn the specified content.

Strategy 3: Ask the Preservice Teacher to Identify Aspects of Her Teaching That She Can Improve Upon and Strategies for Improving Those Aspects in the Desired Ways

In my analysis of the feedback that cooperating teachers give to preservice teachers throughout the school day, I have found the cooperating teachers “tell” preservice teachers information about the content, the students, the lessons, the school, and other such things. At times, the cooperating teachers share this information in response to a question asked by the preservice teachers, while at other times they simply feel inclined to provide the information to the preservice teacher. This tendency for the cooperating teachers to “tell” is particularly prevalent when they give feedback about “problematic” aspects of the preservice teachers’ instruction. In my observations, I have found that cooperating teachers often identify an aspect of the preservice teacher’s instruction that they think is “problematic,” explain what is problematic about that aspect, and make suggestions for what the preservice teacher can do to remedy the problem identified. While the cooperating teachers often share information that helps the preservice teachers address problematic aspects of their practices, the ways in which they share

the information do not allow the preservice teachers to construct their own understandings and practices.

One way that I have observed cooperating teachers “refrain” from telling is by providing opportunities for the preservice teachers to identify and strategize how to address problematic aspects of their instruction. The cooperating teacher can do this by asking the preservice teacher questions such as, “What do you want to know (or do)?”, “What would be helpful for you to understand (or do in practice)?”, “What is confusing for you (or hard for you to do) or do you not understand (or not know how to do) now?”, and “How can you do find out about (or learn how to do) the things you identified?” The cooperating teacher can also ask the preservice teacher to identify resources that she (the preservice teacher) can use to evaluate and address the problematic aspects of her instruction, and to describe how her students’ learning will benefit from having her “fix” these aspects of her instruction. In having the preservice teacher identify and strategize how to address problematic aspects of her instruction, the cooperating teacher provides opportunities for the preservice teacher to construct her own understanding and practice since it requires the preservice teacher to reflect on her own instruction and engage in problem-identification and answer-seeking processes.

Conclusion

Having cooperating teachers use the strategies described above while giving feedback to the preservice teachers may require the cooperating teachers to shift their approaches to mentoring the preservice teachers. While research has indicated that many cooperating teachers use mentoring approaches, such as the apprenticeship model, in which the cooperating teacher’s primary role is to demonstrate correct ways of thinking about and enacting instruction (Borko & Mayfield, 1995; Valencia, et al., 2009), using the mentoring practices described above requires

the cooperating teachers to also act as a “reflective coach,” “critical friend,” and “co-enquirer” (Brooks & Sikes, 1997). If we, as teacher educators, aim to provide learning opportunities for preservice teachers that help them build on their existing knowledge and skills, and help them construct their own new understandings and practices of teaching, then we need to modify the ways in which we engage with them and engage them in learning. In doing so, we can help them become educators capable of critiquing their own instruction, identifying aspects of their teaching that they can improve upon, strategizing and implementing plans to improve in the desired ways, and talking with others about their growth and learning.

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CONCLUSION:

WHERE TO FROM HERE?

Coming out of this experience and process, I have found that my research interest continues to be in understanding and describing the learning opportunities that CTs provide for PSTs. I hope to extend this work in three main ways. The first way is by doing additional analysis of my dissertation data. Over the past few months, I have become interested in “temporal” aspects of CT feedback, specifically the content discussed during specific times of the school day (e.g., instructional time versus planning-time periods versus time between class periods versus time before or after school) or at particular points in the semester. When doing this analysis, I am interested in noting the times during the school day when CTs tend to give feedback emphasizing content knowledge and pedagogical content knowledge, in order to determine whether the reason past studies have not found that CTs emphasize these in pre- or post-lesson conferences is because the CTs emphasize these aspects of teaching during other conversations with the PSTs. I am also interested in understanding the order or sequence in which the CTs discussed the various aspects of teaching with the PSTs and which aspects were discussed more frequently or regularly than others. This analysis would help me confirm or disconfirm the trends that I informally observed in the teachers’ conversations and interactions as a university supervisor (as described in the introduction to this dissertation).

The second way I intend to extend this work is by conducting research that investigates the learning opportunities that CTs provide as they enact other practices when working with PSTs. These practices include modeling instruction for PSTs and planning lessons with PSTs. In doing this work, I aim to understand the breadth of learning opportunities that CTs provide for PSTs, in particular the aspects of the community of practice that they emphasize in only one or a few of these learning opportunities as opposed to in multiple learning opportunities, and how these opportunities potentially influence the PSTs’ learning. In addition, while it is important to

make explicit the learning opportunities that PSTs have while in the field placement, it is also necessary to examine what the PSTs actually learn while engaging in these learning opportunities. Therefore, I additionally plan to examine what aspects of teaching the PSTs “actually learn” from these learning opportunities.

My focus on understanding the learning opportunities provided by CTs extends to understanding the learning opportunities provided by the other types of teacher educators who work with PSTs. I am particularly intrigued by the multiple, and potentially conflicting, communities of practice that preservice teachers are simultaneously inducted into as they work with different teacher educators in the coursework and fieldwork components of preservice teacher education programs. The questions, “How do PSTs synthesize commonalities or reconcile differences across these multiple learning opportunities?” and “How can teacher educators work individually and collectively to provide learning opportunities for PSTs that help the PSTs develop a coherent understanding of teaching?” guide my future inquiry.

Finally, the third way I intend to extend this work is by working with CTs to support them in their work with PSTs. My experiences working with CTs and PSTs as a university supervisor and mathematics teacher education researcher have confirmed my love of and desire to work in classrooms with teachers. I foresee working with teachers as an integral and central part of my future work. In the near future, I can envision organizing professional development sessions for CTs regarding how they can give feedback to PSTs. My general approach to working with teachers is as a knowledgeable colleague as opposed to an “authority” or “expert.” Therefore, while I anticipate sharing the findings of my dissertation research with them, I think they would find it equally valuable to engage in conversations about (1) the aspects of teaching they emphasize (and do not emphasize) when giving feedback and why they emphasize (or do not

emphasize) those aspects; (2) how they think their feedback shapes their PSTs' learning and how that aligns with how they ideally want it to shape their PSTs' learning; and (3) the approach they take when giving feedback (and working with the PSTs in general) and how this approach allows them to provide learning opportunities for the PSTs that reflect the way in which they think the PSTs learn to teach. Ideally, I would like to develop a model that allows me to work closely with CTs, which includes researching and providing professional development opportunities with and for them. I continue to admire the wonderful work that CTs do with PSTs, and hope to acknowledge, honor, and share their efforts on a wider scale in whatever way possible.