

SIGNS OF IN/EQUALITY:
A HISTORY OF REPRESENTATION AND REFORM IN ELEMENTARY SCHOOL
MATHEMATICS FROM THE 1950s TO THE PRESENT

submitted to the Graduate School of the
University of Wisconsin-Madison
in partial fulfillment of the requirements for the
degree of Doctor of Philosophy

By

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Date of final oral examination: March 31, 2014

Month and year degree to be awarded: May 2014

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To Jesse and Ada, for whom there are not enough words.

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Acknowledgements

In addition to my loving and supportive family I would also like to thank the members of the committee. Tom, without your guidance and patient teaching, this project would not have been the intellectual endeavor it proved to be. I thank you for the countless time you spent reading and commenting on words that never even made it to these pages. Although sometimes difficult to hear, I appreciated your careful questioning that pushed my thinking in directions I did not think were possible.

Daniel, thank you for your continued support throughout this research project. From the very beginning, your pointed questions and critiques were coming from a place of intellectual curiosity and for that I thank you. Most importantly, I truly appreciate your willingness to make the journey to Madison to raise provocative questions and challenge my sense of history in person!

Anita, I appreciate the numerous ways in which you contributed to my graduate studies. In presenting me with the opportunities to work with undergraduates in the program, you unknowingly provided me with the intellectual playground for my research. More than that, I thank you for understanding what I was trying to provoke with my research and see in more nuanced ways than I could how this provocation both related to and diverged from studies of math education.

Dawnene, thank you for walking with me this past year. From volunteering to be a part of my committee, to raising important questions, to offering praise, I knew that you were there for me as a continual source of encouragement.

Gay, since meeting with you last year I have not forgotten your excitement and interest in my project – even then in its fledgling stages. Your words of wisdom and questions about how to clarify and create boundaries where there were none were influential to say the least.

In addition to the members of the committee, I would like to thank Professor Bernadette Baker for providing me with some of the most delicious choices in the intellectual candy store that is the Department of C & I at UW-Madison. Your always stimulating and captivating courses incited new lines of thought and questioning that will remain influential for me in ways that cannot yet be imagined.

Professor Paola Valero, during your time at UW as a visiting scholar, you had a profound impact on my study of curriculum, particularly with math education as my site of inquiry. Even from afar, your insights and pointed critiques served as reminders throughout my research and writing that intellectual work is not a form of magic.

Not least, I would like to thank some of my fellow graduate students. In particular, Ezequiel Gomez Caride and Weili Zhao, you have been integral partners in my learning. I appreciate your willingness to read and discuss at length drafts of each of these chapters. Manali Sheth, Jason Salisbury, Irene Resenly, and Bryn Jaastad, thank you for keeping me accountable and creating space for me to think hard and write more. To members of the Wednesday group, thank you for engaging me in a way of reading and discussing that was unlike any other I had encountered in my schooling. Together, you create something special that allowed me to learn with, not merely from, others.

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Prologue

Writing a dissertation is a pedagogical practice. As such, it brings with it a particular way of thinking about what it is. It also carries rules for how one might do it. These rules for thinking about and the practices of doing a dissertation – as a thing – construct certain kinds of people.

It constructs the kinds of people who can sit for a long time. By a computer. Looking at something, thinking about it, writing about it, studying it. Standing still so long that what we thought we were looking at changes. It is, in effect, a moving target. Never sitting still for too long. Like us. It is this motion that makes things difficult to track. Difficult to trace.

In the beginning of this project, I admittedly had many points in which I questioned what I was doing. Believe me, so did other people. It took some time to articulate what I thought to be the problem and then figure out how to track it. The equal sign, as the object of my research, has been a bit of a moving target. Where is it? Where isn't it? Which equal signs to focus on? Why? What about this? Why not? Oh, look over here. It was easy to feel lost. At times I was. Among those who helped me find my way, it was my husband that most patiently expressed that when I felt most unsure, maybe I wasn't asking the questions that needed to be asked.

Writing a dissertation also constructs people as a focused kind. What do focused people do? What do they not do? When do they sleep? How do they finish a research project? Before it eats them alive? If you are reading this, then I have survived and in so doing, have maybe become what seems to the observer as 'more focused'.

To do so, I also needed to become a writer of an academic kind. That is a person who can articulate coherent, clear, and rational thoughts in some sort of logical order. I am not always so logical. I am not always so clear. Yet, I do think there is a method to what may seem to be my madness. This madness comes in many forms and is provoked by the raising of questions – Why? Why not? Could it be like this? How come? How does that make sense? Has it always been like that?

These kinds of questions – like those my daughter likes to ask me – are what motivated the research that follows. The madness piercing this project was first recognized as intrigue. It was interesting to me, that the imperative to learn the equal sign (=) as a fact of the modern curriculum intersected with cultural, political, and social notions of in/equality. This relationship became even clearer in the projects of curriculum reform aimed at *math for all*. Yet, beyond that, it was a little murky.

I realized I had to be the kind of person that is willing to stick it out. Have doubts. Be unsure. Let it – as in the equal sign project – marinate. Keep finding focus. Keep diving in. Keep tackling it as a real problem. Don't quit. You might be onto something. I think. I hope.

I also had to be someone who was prepared to go with it. Let it take me somewhere and show me what to notice. How to think. How to look. Not always so clearly about what not to look at. But there will always be more. There is always something I didn't see. Didn't write. Didn't say.

But what I think I did say is something like this: While the equal sign might seem like a neutral fact of the school curriculum, it is a pedagogical practice. Much like a

dissertation. It makes people into certain kinds. Similarly, it tells us who we should not be. In that way, it is also a cultural practice. As such, the knowledge of the equal sign organizes who children are and who they should be as the foundation of their identity as a particular kind of person – the math learner.

This making of certain kinds of people is possible because what comes to count as school mathematics is filtered into the spaces of the school through psychologies aimed at producing knowledge about who children are and should be as 'learners'. As a scientific object, the child as a math learner is placed under scrutiny to be studied, measured, and understood. So as to make it better. Faster. More effective. More efficient.

In this process of making math into math education, each child is seen as possible to shape and remake into a mathematical kind of person. This individual, distinct from the others is both imagined and real. She is imagined as one who does not like math. He is seen as the child who is not motivated to learn it. His friend wants an extra challenge. She is the girl who needs first to build self-esteem then can worry about the mathematics. He is the child who just gets it. Together, with other kids of their kind, they form groups. Identified as certain kinds of learners – giving further distinction to what it means to learn mathematics in the classroom. Sorted and classified into seeming simple categories of can/cannot and will/will not learn math.

This seems somewhat counter intuitive to the relationship of equality that learning the equal sign seems to imply. One of equality, similarity, sameness. In that relationship, however, a paradox emerges and the equal sign can no longer be just that.

It becomes a subject of the curriculum – given attention, life, value, breath, and the possibility of producing (and changing) both identity and difference.

Thinking with the equal sign, then, might provoke an encounter with some of the hopes/fears we have about change – how does it happen? How do you plan it? How can it be better than what we have now? So much uncertainty in projects of reform. Yet, we find comfort in trying to know something about ourselves. About our situations. The circumstances of our lives. The values and beliefs that entangle with our thoughts and actions. The assumptions that ground the very living and learning we might be able to do. What is possible to see and say about people, about the rules that order their lives and learning, is the stuff of educational research.

The people I have been focused on since the beginning of this project, while not the immediate ‘actors’ in the narrative, are children. The madness that continued to motivate my project was the way in which it is possible to see, talk about, and think about children as mathematical kinds of learners. The kind of people who could be assigned identity, represent the world with symbols, calculate unknowns, isolate variables, establish equality, and solve for inequality. As math learners, children have become the kinds of people who could be scrutinized and studied as a scientific object so as to make mathematics equally available and accessible to them all. In the management of what is equal and what is not, the child also became a subject – governed by certain rules for how/how not to see, think, and act.

As the uneasy feeling that came with the inclusion/exclusion of particular kinds of children into the all, I began to develop the equal sign project as a way to continue

exploring the seemingly natural and automatic things that are possible to think about identity, representation, difference, inclusion, and equivalence that manage the making of in/equality. First turned in as a two and a half page paper, a certain professor thought it was interesting and provoked me think about it more. In that intellectual challenge, he has been the greatest of teachers. On the following pages, you will find the paper that started the equal sign project. What comes after is the development of *more*.

Paper written in association with C&I 714 “Research and Evaluation Paradigms in Curriculum and Instruction”, Thomas S. Popkewitz
Submitted February 28, 2011

How are difference and sameness inscribed with value through the acceptance of the equal sign as a fact? How do notions of sameness as equality and difference as inequality become a ‘fact’ of the school curriculum and function in social contexts? How are scientific styles of reasoning engaged in a governing relationship with the concept of equality as sameness as a fact? How do the conceptual tools of governmentality, potentiality, and subjectivity relate to my thinking about equality as factuality?

In *A History of the Modern Fact*, Mary Poovey (1998) suggests that the modern fact is an epistemological unit that organizes knowledge projects. By examining how facts come to be constituted, she attempts to understand the limits of factual knowledge, epitomized by its peculiar construction as universal, common, public, objective, transparent, and disinterested. Following Poovey, I will consider the equal sign to be an accepted modern fact, and suggest that as such, it should be subjected to scrutiny as an object of study.

The equal sign appearing to represent equality as sameness as part of the mathematics curriculum is a relatively modern event that can be considered historically. Turning to the historical sets necessary temporal and spatial limits. In doing this, I am not trying to locate an origin of the concept of equality or the invention of the equal sign. Rather, I am trying to think about how the discourse around the equal sign circulates and produces the equal sign as an historical fact that privileges and represents equality as a relationship of sameness within a system of public knowledge.

With space being carved out for it in the school curriculum, the equal sign is considered to be objective and public knowledge, as a truth to be disseminated; taught and learned in a formalized, standardized, rule governed and systematized way (Porter, 1995). Ways of knowing, or epistemologies, are organized through an acceptance of equality as sameness within this system. Consequently, political, social, and educational projects that appeal to notions of equality, organized around the privileging of sameness over difference, can be related to the production of and tension around the equal sign. As a question of epistemology, it is also a question of social order that “encourage[s] the historian to consider how domains of knowledge production overlap,” (Poovey, 1998, p. 19). Domains of knowledge production that are of interest here are discourses that seek to produce and reproduce systematic knowledge of the equal sign as a thing.

By suggesting that the equal sign is not a given object that exists, but rather that it is a historically produced thing that operates through discourse allows me to engage equality as an analyzable object and an active subject that functions conceptually and historically to produce social effects. The making of the equal sign as a thing of science introduces the idea that this “thing itself is no longer an entity in its obscurity, an object presupposed by language and the cognitive process, but it is instead that *through which* it is knowable, *its own knowability and truth* (Agamben, 1987, p. 22-23). We are able to know equality through the equal sign because the equal sign is being acted upon and acting through itself within scientific styles of reasoning. It is both a knowable thing of science and a scientific subject with the potential to act.

Important here is what negotiates the relation between knowability and potentiality; the thing itself. Agamben (1999) applied the concept of potentiality as a matter of human potential, capacity, or faculty. It is the potential not to pass into actuality that is often ascribed to non human beings while the potential to act is assigned to humans. Yet, if equality is knowable and things are known through the equal sign as fact, then the equal sign in his sense is able to act and engage in relations with humans as a non-human actor. In exchanges between humans and the equal sign, as representative of equality, both have the potential to act, to do this or that, and also the potential not to. The equal sign, created and made a thing of science, negotiates the manner in which relations with it can occur. It both acts and is acted upon.

Therefore, understanding the equal sign as a fact cannot be reduced to a description of social construction, as social assumes and is restricted to a fixed understanding of social as human (Mitchell, 2002). Forgotten, erased, made invisible, is the idea that often it is through interactions with non-human actors (things) that “so-called human agency draws its force,” (ibid, p. 29). This agency compels humans to act, as having potential to make decisions. The potential of equality as a fact and act of sameness gets translated into human projects and has social effects. How the equal sign comes to count as a universal fact of the science of math negotiates the relations that can occur with and through equality. What seems to count as equality is a matter of how equality is valued and evaluated in reference to sameness.

This point is elaborated by Hacking’s (2002) discussion of the self-authenticating characteristic of scientific styles of reasoning where he suggests that new ways of

finding out the truth create the conditions and criteria of what counts as truth. Equality as sameness is a factual truth because it is truth based upon relationships between numbers or variables, which authorizes its existence as a thing of the science of mathematics. Numbers, as Poovey (1998) suggests, are pre- or non-interpretive. Therefore, any fact linked to a relationship with numbers is generally unquestioned. At the same time that equality is made into a fact, it also determines what counts as equality through interactions with people, institutions, kinds of knowledge, experts, and classifications.

My questioning and interest in this topic is guided by the idea that the equal sign has also become a subject with the potential of engaging in relationships. If how something is created negotiates the manner in which relations with it can occur, then there are certain possibilities for acting on, through and with equality. In this sense, equality as a fact of sameness governs and is governed by relations with classifications, people, institutions, knowledge, and experts, all elements in a framework that interact dynamically to “make up people,” (Hacking, 2002). The dynamic interactions of these elements and the relations they produce are part of an attempt to make up, or fabricate, the sameness that operates through and gives meaning to the concept of equality as a fact of similarity.

Projects of equality, then, generally become efforts to reproduce sameness. Neither equality nor the sameness that defines its ontological status are put into question. Yet, lived experience would suggest over and over again that equality does not exist. Putting the equal sign, as both an historical object and a social subject, to

question is bound to be contentious. Acknowledging that this is a political project, it is not my intention to achieve consensus through consciousness. Change as an effect of agreement and awareness are elements of a certain theory of change. I operate under the assumption that change will unfold if we are open to interrogating ideas that are taken for granted as factual and have therefore been left unquestioned. I hope to allow the political to emerge and open the concept of equality to discussion by questioning the production of equal sign as fact. Therefore, the ideal of consensus does not seem fully realizable nor desirable here. Consensus assumes and values objectivity as a way to produce truth (Porter, 1995). The possibility and the goal of consensus is a foundation of scientific thought; a style of reasoning that is self-authenticated through the concept of the fact. As self-authenticating, scientific consensus is always contingent upon discourses and relations through which ideas are produced as facts. Exploring the equal sign as a unit of systematic knowledge and a way that knowledge is produced allows us to consider the limitations of and possibilities for equality as a governing mentality.

CHAPTER 1

The Equal Sign and Naming the Problem of In/Equality

“Mathematics is the great equalizer”

Edward James Omos as Jaime Escalante in *Stand and Deliver*
(Musca & Menendez, 1988)

Introduction

The equal sign is taken for granted in teaching elementary school mathematics. Yet, it provides children ways to think about equivalence that are not only mathematical but also organize their learning to understand and negotiate their daily life. From learning about money to modeling experiences as algebraic functions, as the equal sign appears in the school math curriculum it orders children’s thinking about equality in mathematics by assigning identities to things of the world - as expressions of equivalence and non-equivalence. However, when looking at the curriculum, the equal sign is never merely about mathematical thinking. Instead, its use in the school entangles with discourses about how to think about equality and identity that have cultural dimensions beyond pure mathematics. Both the sign and the comparative logic it embodies organize both the curriculum and the broader discourses of In/equality in math education. The ordering of thought about equality is central to calculating how children are to participate in school and society as ‘equals’.

The dissertation research is to explore how the equal sign is related to historical, political, and social understandings of equality. Generally, I focus on the entanglement between seemingly distinct notions of mathematical and cultural equality embedded in contemporary math education reforms and research. More specifically, my study

investigates the equal sign (=) not merely as a mathematical symbol, but as a cultural practice related to questions about how to establish equivalence between children and their “equalized” opportunities to math education. Central to the analysis is how the practices of comparing, ordering, measuring, and sorting that work to establish equivalence also carry rules for how to organize difference. For this reason, I consider the equal sign as part of the problematic of how to see, think about, and manage in/equality as a relationship between identity, equivalence, and difference.

In this chapter, I introduce the problem of the research as well as my approach to studying mathematics education as a field of cultural politics. This entails a discussion of school mathematics on many levels. As such, the initial point of entry into the study of the equal sign is the contemporary math reform projects aimed at *math for all*. In this discussion, I specifically attend to how the elementary math education has become intertwined with concerns for social inclusion, equity, and equality. Highlighting the *logic of equality* that organizes the *math for all* reforms, I name the problem of in/equality in the curriculum as an issue of representation and difference. Then, in relating the curriculum to cultural practices and particular ways reasoning about equality, I discuss how the use of the equal sign in the curriculum works to organize styles of thinking and acting that fabricate children as certain kinds of people. This leads to a consideration of how the mathematical content of the curriculum is not simply about math, but assembles with discourses of educational sciences to order truths about who children are and should be. Drawing upon contemporary research and reform efforts, I then explore how the mathematical content of the equal sign intersects with cultural distinctions to

organize who is – and is not – part of the *all*. To conclude this chapter, I lay out the questions that will guide my inquiry into how the principles embodied in the equal sign historically produces a particular knowledge of equality, organizes notions of equivalence and difference, and generates practices of inclusion and exclusion.

Arguing that the equal sign is related to both a theory of *social* equality and a seemingly distinct *mathematical* equality taught in schools does not lead to an evaluation of children, best practices, or better curriculum. Rather, it is to consider how cultural, political, and pedagogical ways of reasoning intersect to order the subject(s) of school math and express equality for *all* as an objective of its reform.

The Equal Sign, Equality and Ordering Mathematics for All

The equal sign figures importantly in the ordering of elementary school mathematics. Its placement as a primary focus of math reform and research in the United States makes that clear. For example, Falkner, Levi, and Carpenter (1999) are not alone in concluding the importance of children learning equality as a relationship of sameness (e.g. MacGregor & Price, 1999; Carpenter, Franke & Levi, 2003). Importantly, learning the equal sign is connected to larger concerns in math education. That is, it is embedded in a *mathematics for all* movement (see Choike, 1996; Fennema & Meyer, 1989; Frankenstein, 1995; Oakes, 1985; Secada, 1995). As part of this effort to reform school math, focus is placed on granting equal access and opportunity to all students. The reason underpinning this research and reforms is that with knowledge of the equal sign, children will have a better understanding of equality, greater access to higher

levels of mathematics, more academic opportunities, and an overall improvement in economic and social standing.

As related to the goals of *math for all*, it seems that learning mathematics is to produce social equality – expressed as *all* students having the same opportunity, access, and possibilities in school and in life as *equals*. With that, reform projects assume that social, economic, and educational inequalities can be minimized if *all* children have the opportunity to learn mathematics. Circulating within this logic, the equal sign (=) and the child's understanding of equality are represented as neutral and ahistorical tools to achieve that aim.

In general, then, mathematics is considered a “great equalizer” (Musca & Menendez, 1988). Simultaneously, however, higher levels of mathematics (i.e., algebra, geometry, calculus) are taken as “gatekeepers” to success and opportunity (Witzel & Ricommini, 2011). Students on one side of the gate seem to achieve to record measures. The others appear to lag behind. How can the gap in achievement be closed? How can more children learn more mathematics more efficiently? What are the best practices being used by teachers who effectively teach higher level reasoning? How might the differences in children be understood so as to guide instruction toward more equitable teaching? How can U.S. schools compete mathematically on a global scale? How can teaching *mathematics for all* work to produce equality in society?

The above questions seem to frame various goals and approaches to math education reforms and research in the United States. Included in these approaches are curriculum reform efforts that are situated within traditions of teaching a *critical*

mathematics or *mathematics for social justice* (Frankenstein, 1987; Gutstein, 2003, 2006; Gutstein & Peterson, 2005; Skovsmose, 1994). Focused on the solutions that mathematics education can provide to the problem of social inequalities and exclusions, much of the scholarship aims to offer methods for progress – construed as better teaching, more efficient learning, a more inclusive classroom, or an improved curriculum for all children. With an impulse for inclusion to move society toward equality and prosperity for *all*, the above studies and questions may be necessary to school math reform, but they are not sufficient. Let me explain.

Related to particular theories of critical pedagogy, critical mathematics and mathematics for social justice can broadly be seen as situated within an emancipatory pedagogy – that is, a pedagogy presumed to free students from an otherwise oppressive and oftentimes hidden curriculum. Drawing predominantly upon the work of Paulo Freire, many of the explorations into this critical formulation of a “problem posing” approach to mathematics education are described in opposition to an oppressive “banking method”. The assumption that the critical approaches aim to upend is that a banking – or filling the empty vessel – method uses various psychological means to oppress the student and cut him/her off from knowledge production.

As a way to situate my research in relation to what are given as critical mathematics, it is important to introduce a question asked in *Jacques Rancière: Education, Truth and Emancipation* (Bingham & Biesta, 2010). That is, “Just what sort of human being is being assumed by the pedagogies that would otherwise claim to emancipate the human being?” (p. 63). The presumed emancipatory counterpart to

oppressive banking method is to apply a problem posing technique as a means by which to bring the child from one psychological state to another – from astounded by the teacher to astounding, from empty to full, from being for someone else or for being for oneself (ibid, p. 69). In other words, the critical method is to apply psychological knowledge about who children are and should be to (re)define what math education should do for them in a way that is presumed to lead either to oppression or to freedom.

From this psychological perspective, the child is granted as either free or oppressed. As such, it follows that the work of both a critical mathematics and mathematics for social justice is to create the conditions for those who are not free to become free. Yet, in this formulation the task is to restore a psychological order to the child in ways that are already decided upon in advance. This image of the child as psychological is one that progresses, moving developmentally toward emancipation, benefiting from a pedagogical method in order to attain what is given as freedom within a notion of social justice/injustice. Therefore, attention to questions of who is/is not free, who does/does not read, write, and speak the world with mathematics, who does/does not learn within a critical mathematics are inadequate since the very mode of reasoning embodied in the 'counter' curriculum assembles with psychological classifications of the child to inscribe divisions and distinctions that exclude.

My approach to mathematics as a site of curriculum studies is to call into question how the image of the child as part of *math for all* has been historically seen through psychological lenses in ways that predetermine who children are and who they should be. This consistent application of psychological rules to the 'improvement' of

education is seen within an “order and harmony that the sciences of the individual and collective soul are said to offer a society,” (ibid, p. 71). Yet, the very modes of reasoning about the equivalences (as a form of harmony) that describe and bind the individual to the collective are unquestioned even when they continuously produce ‘other’ souls as different and excluded from the social order.

As an introduction to the problematic that frames my research, this chapter gives attention to how the rules of inclusion and exclusion are embodied in the very translations of math into the curriculum to make school mathematics. Situating the following discussion within concerns of *mathematics for all*, I argue how the equal sign (=) carries a particular logic of equality that organizes notions of equivalence and difference through which to compare children as the cultural embodiment of notions of in/equality and justice. My concern is with how the logic of equivalences works in and through the ordering of the school curriculum to reinsert notions of inequality as differences between children. This is what I refer to as the problem, or paradox, of in/equality. With this in mind, the research “is not a matter of taking pleasure in paradox, but of fundamental inquiry into the meanings of knowledge, teaching and learning: not an amusing journey into the history of pedagogy, but a matter of timely philosophical reflection on the way in which pedagogical logic and social logic bear on each other,” in the making of school mathematics for all (Rancière in Bingham & Biesta, 2011, p.1).

Governing Equality: The Cultural Logic of Reforming Curriculum

During the last two decades of the twentieth century, clear reference was made to notions of equality, equity, and inclusion in math educational reforms in the U.S. With others, I have considered this the *math for all* movement within which an emphasis was placed on raising the expectations for all students to learn and excel in mathematics. This can be seen in *An Agenda for Action*, where the National Council for Teachers of Mathematics (NCTM, 1980) call for a higher level of mathematical skill and competence for *all*. In another influential document, the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989), “opportunity for all” was formulated as one of four societal goals for mathematics education. The document refers to equal opportunity and outcomes, apparently related to a notion of equity. Equity, in this relation to discourses about social equality, generally includes three aspects: equal opportunity to learn mathematics, equitable treatment (based on the perception and description of being equal) and equal educational outcomes (Fennema & Meyer, 1989). Equality and equity seem to be implied in one another and have both been assumed to fit into the discussion and (re)direction of school mathematics.

My aim in the research is not to distinguish between notions of equity and equality in this contemporary (re)formulation of math education. Instead, it is to consider the particular ways of reasoning about how children and the content of the curriculum are organized in the conduct of schooling. Within what Dorothea Olkowski (1999) refers to as a “logic of representation”, I consider the ways of thinking and acting about equality (and equity) to assume the identity of an individual, represented as distinct from

and comparable to others. This comparison is made possible given this logic by which similarities can be established between things (and people) and differences categorized. As related to the commitments to equality and equity mentioned above, both seem to take for granted the representation of people and their worlds as fixed and, therefore, possible to compare. Central to this research is to consider how this logic operates in the ordering of school subjects to produce norms and practices of equivalence and difference.

Significantly, the cultural norms of sameness and difference circulating within the reforms provide a comparative reference by which to see children in relation to others. As an entry point into the discussion of identity and norms of representation, the notion of fabrication is used to consider how certain kinds of people are given identities as a way to understand the problem of difference and inequality through practices of classification and categorization. This issue of difference and in/equality will be approached through the notion of fabricating, or “making up people” (Hacking, 2002). Within the reasoning of equality bound to the discourses of inclusion, children are given traits as certain kinds of people and ordered on a continuum of value – from high achieving and successful to struggling or low achieving. The presumed differences between children are then reduced to gaps, or inequalities, and used as distinctions that rationalize further categorizations so as to establish some sort of equivalence. Therefore, the production of this logic of equality that works to establish equivalence and non-equivalence between certain *kinds of people* is a central focus in this study of the equal sign.

Considering how the use of the equal sign in elementary school math is related to cultural inscriptions of equivalence and difference is a way to explore the effects of power on the formation of subjects and subjectivities in the math curriculum. To borrow from Judith Butler (1992), I consider the equivalences and differences that form subject identities to be “contingent foundations”. They are contingent in the sense that they are effects of particular cultural conditions and historical forces – always open to contestation. This is neither to exclude the question of agency nor to discount the experiences of individual students and teachers in the school. However, it is to displace *experience* as the subject of this research.

Narrated through identity and representational categories, appeals to individual experiences of learning and teaching can authorize the claims that the curriculum itself provides access, opportunity, and success as experiences of equality or inequality. Never raising “questions about the constructed nature of experiences, about how subjects are constituted as different in the first place, about how one’s vision is structured – about language (or discourse) and history” risks maintaining the very distinctions one wishes to unsettle (Scott, 1992, p. 25). With attention given to the historical discourses that fabricate subjects and intern possibilities for seeing, acting, and thinking, my study seeks to disrupt the neutrality of mathematics education by attending to the power/knowledge relationship articulated with the politics of identity, norms of representation, and rules for in/exclusion.

Importantly, this power is not an overt or oppressive ‘hidden curriculum’. Operating as a series of effects in the curriculum, the productive powers aligned with

knowledge generate particular ways of knowing and acting in the world (Foucault, 1980). This knowledge of people and their world is inserted into the social and cultural spaces of the school and works to organize the curriculum as well as the conduct of the learner, as a kind of person it assumes. Visible at the level of discourse, the rules and standards organizing the knowledge of equality and equivalence are not simply ideas but are given materiality in the curriculum. That is, they are embodied in the subjects of school mathematics as cultural practices of how to see, think, and act.

With this in mind, I find the notion of governmentality (Foucault, 1991) helpful in considering how the equal sign is bound to discourses of equality, issues of power and politics, and circulates within systems of knowledge that govern the cultural norms and standards as the rationality by which individuals are to live (Foucault, 1980).¹ From this perspective, I do not take governmentality as a theory to be applied to my study. Rather, in seeing the equal sign to embody rules organizing a particular knowledge about equality, I ask questions about how the pedagogical practices embedded in school mathematics historically form and (re)form the subjects of school and society. Specifically, this view is to place the pedagogical uses of the equal sign and the understanding of equality as linked to cultural and social values, practices, and assumptions that define ways of thinking and acting.

To explore the relation between the equal sign and the organization of equality in the contemporary *math for all* reforms, I consider the sign as more than a symbol of

¹ This is not an attempt to explicate, reproduce or glorify Michel Foucault as the single author on mentalities that govern modes of living and schooling. In reading Foucault alongside others (e.g. Bloch, Kennedy, Lightfoot, & Weyenberg (Eds), 2006; Cruikshank, 1999; Popkewitz, 2000) this project explores the math curriculum as related to governing the self and others.

mathematical logic. That is, I look at the ways in which the equal sign is given meaning in the curriculum as a cultural practice that orders the thoughts and actions of the child as a “technology of the self” (Foucault, 1988). Specifically, with reference to pedagogical strategies that operate through the equal sign, I examine the principles of equality embodied in the problems children are to solve. These are not statements of pure mathematics but are translated into the curriculum to organize how the individual is to see equivalence and think about its production within a *system of reason* (Foucault, 1977). This particular way of thinking assembles with the cultural, political, and historical norms to organize notions of equivalence and difference by which to determine equality. To return to the earlier discussion, this system carries with it the principles and practices that “keep the gate” by marking boundaries of inclusion and exclusion, so to speak. The emphasis in the research is on how the rules of thought that order knowledge of the equal sign assemble with knowledge systems of the child to identify (and differentiate) what children should know and do as particular kinds of people.

The following discussion is to highlight how children see, think and act is largely determined by what sense they have made – not only of the mathematics but also of the world around them. Therefore, the logic of the curriculum aimed at *understanding* and *producing* (knowledge of) equality is explored by examining how the equal sign functions as a *modern fact*.

The Modern Fact, Modeling Mathematics, and the Mind

Before pedagogy can be organized so as to improve learning for *all* children, the equal sign must be taken as a stable and objective object of knowledge. If everyone is

to learn and understand it, then the equal sign can be thought of as what Mary Poovey (1998) characterizes as a “modern fact”. Suggesting that modern facts organize knowledge based upon their apparent truth, I follow her to consider how the fact of equality is historically organized by fields of knowledge that intersect to give meaning to the equal sign.

As a ‘fact’ of the elementary school curriculum, the equal sign is rarely seen as anything other than a symbol of mathematical thinking. Yet, it acts within historical and cultural ways of knowing that intersect in the forming of pedagogy. That is, the sign is to be learned in relation to psychologies of learning that translate mathematics into something that children can use to think about their ‘real’ life. The process that translates mathematics into math education is important as it generates cultural ways of thinking and acting that are never merely about mathematics. Popkewitz (2004) refers to this process of translation as “an alchemy” – or a transformation of the content of mathematics that gets linked to pedagogical practices through overlapping fields of knowledge that organize both mathematics and learning. Such alchemies are necessary as children are not mathematicians. Yet, to make the equal sign intelligible as a fact of the curriculum, the knowledge of mathematical equality interacts with and resembles the tools of psychology that order notions of identity and difference in the practices of how children are to learn.

Integral to this research is how learning about equivalence crosses into territories of equality and inequality through the cultural principles generated about sameness and difference. Importantly, these principles, as rules ordering who is/is not part of the all,

change over time through the changing psychologies that continuously produce new distinctions and classifications about who is/is not the child who learns math. More specifically, these changes can be seen in how the psychological truths shift. During the New Math reforms after WWII, the psychologies that assembled to reform mathematics emphasized that all children could learn to master and think creatively with mathematics due to its universal structure. In the next Back to Basics period of reform, the tools of psychology ordered learning as a process of discovery by which the child was to be motivated to come to what was seen as his or her own understanding. As will be evident in the examination of the contemporary Standards Based movement, the principles of psychology assumed learning as a social construction that required the tools of language as the representation of confident mathematical seeing, thinking, and speaking.

Briefly, I want to talk about the equal sign itself as related to particular historical conditions that relate to the reason and sense of equality as a fact of school math. We do not have to go far into the past to recognize an historical moment where the equal sign could not be considered a fact as such. Math historian Florian Cajori (1928) suggests that, “the beginning of the eighteenth century may be roughly designated as the time when all competition with other symbols practically ceased,” (p. 304). This shift from a seemingly haphazard selection of various notations to a more common adoption of the equal sign was, for Cajori, related to the work of Newton and Leibniz.

While situated in a historical discussion of math as a disciplinary knowledge, this moment is also historically related to the use of mathematics and practices of

quantification in the formation of the modern nation state. Numbers, particularly statistics, were deemed useful in providing the state with knowledge of the people, unified and rendered calculable as a population (Scott, 1998). This new order of the government brought with it the production of a system of standard measurement to calculate life expectancy, death rates, rent, wages, prices, taxes, as well as length, weight, and volume in the markets of resources (Poovey, 1998). The regulated measures and symbols in accounting procedures can be linked to Enlightenment notions that the state can be known and planned to promote equality and freedom (Popkewitz, 2011). Within this reason, equal measures seemed to play a part in thinking about and seeing the abstractions of equality and justice.

It also became possible within this historical and cultural moment to accept the equal sign as the sign of equality when the social value of equality was linked to man and his mathematical knowledge of a distinct natural world. This possibility is situated in a broader shift in thought that revisioned the centrality of mathematical knowledge as associated with the presumed natural equality of man. Expressed as *all men being equal in their capacity to reason*, this principle of equality can be attributed to the (re)location of reason within the mind of man, rather than within a sovereign, all-knowing God (Cassirer, 1951). With God no longer as the driver of human action and thought, man and his rationality were to uncover knowledge of a nature that seemed unbounded and unpredictable. Mathematics was to contribute to this knowledge and provide an objective view of an underlying structure representing an otherwise uncertain world (Daston & Galison, 2007). Therefore, as the equal sign appeared in the encyclopedias

and scholarly work in the eighteenth century, it was given a common value, as a *tool of thought* that seemed to provide objective ways of knowing and ordering the world.

This historical detour importantly relates to how math has become part of the rationality organizing how a child is to see and think about the equal sign. As part of the “alchemy” described before, the equal sign enters the social and cultural spaces of the school and carries principles of thinking about equality that are more than mathematical. In other words, mathematics pedagogy interacts with rules of reasoning about equivalences and equality that operate in the curriculum to order two notions. One is the rationality of mathematics given as a field of thought that models the world. Another is the logic and learning of mathematics through educational psychology.

This rationality is built upon abstractions of the child’s mind - that mathematical ‘facts’ and numerical statements of equivalence often work to make visible! Ordered by principles of equality, the child’s (abstract) thinking is taken as the mode by which to develop all children’s mathematical reasoning. Simultaneously, there is an identity and equivalence given to children’s minds, modeled through mathematics (e.g. Piaget, 1950). Given as the space where the capacity to reason is presumed to rest – the abstraction of mind has been presumed as an object of knowledge through which to know the child (Danziger, 1997). In the curriculum, this knowledge is linked to certain practices of behavior and linked to psychological notions of motivation, desire, and interest.

Given by the principles of psychology and modeled by the rationality ascribed to mathematics, the abstraction of the child’s mind works to signify the kind of child that

can internalize the cultural ways of reasoning about equality with the use of the equal sign. This child embodies a hope for future progression toward equality and prosperity for all. In this hope is also a “double gesture” (Popkewitz, 2008) articulated as a fear of the child who will not meet the norms set by psychology to express what constitutes learning.

Next, I attend to the ways in which knowledge about mathematics and learning assemble to organize the use and value of the equal sign in school math practices. This is explored in how the use of numbers, symbols, and objects tie together with knowledge of children’s thinking to operate in the school as techniques to develop the reason of equality in schools.

Mathematical Objects and Styles of Reasoning

In school mathematics, the equal sign rarely appears alone. It makes sense in relation to other symbols and objects. Together, the equal sign and numbers organize problems for children to solve about identity and difference from which to understand the fact of equality. When children are asked to determine this equality, they are to see and think about the numerical statements of equivalence as an objective representation of the world.

Are these math sentences true (T) or false (F)? Why?

$$4 + 5 = 9$$

$$7 = 3 + 4$$

$$8 = 8$$

$$10 - 5 = 2 + 3$$

$$8 + 2 = 10 + 4 \text{ (Beatty \& Moss, 2007, p. 31).}$$

As an apparently useful way of representing, measuring, and comparing people and things, the numerical inscriptions work alongside the equal sign to give it meaning as either true or false. However, numbers are not sense makers in and of themselves (Poovey, 1998). Rather, they are given validity as social facts, and therefore involved in constructing the social world, not just representating it (Porter, 1995). In this way, school math problems work as a strategy for constructing a truth about equivalence that is seen as a mathematical form of reasoning. Yet it is more than that - it is learning how to see difference, sameness, equalities and inequalities as a form of knowing.

This problem solving can be considered to inscribe in the child what Ian Hacking (2002) refers to as a “style of scientific reasoning.” Embodying a way of thinking, the statements with numbers related by the equal sign carry rules about how to systematically evaluate equality as a relationship of sameness. This importantly highlights how the use of “numbers in the representation of the world is predicated upon procedures of classification and separation of the ‘identical’ and ‘different’ which results in the building of a rigid perception of reality,” (Patriarca, 1996, p. 9). That rigidity is inscribed in the expressions of sums yielding equal (or not) results. These rules then are to be applied in a logical and precise way to other instantiations of equality represented with the equal sign.

As the truth-value of the math problems make up how children are to see the world, they also provide rules for how to act and reason. In the relationship between the numerical statements of equivalence and the sense that children are to make of them is a particular notion of ordering and classifying given as the rationality of math to be

internalized by the child. This rationality is formed within the process of alchemy through which math as a school subject becomes a tool for reasonable thought. Children, however, are not born to see and think (or to be seen and thought about) in this way. Instead, the problems of mathematics order certain rules, not only about equality but also about the child's modes of thinking and participating in school.

Walkerdine (1988) discusses how a notion of rationality in children is produced through learning school mathematics. In its relation to principles of psychology, mathematics education is explored as a cultural and political inscription ordering who children are. Within the categories that differentiate between certain kinds of children, psychological classifications provide a link relating the child's internal thought with particular behaviors. In this association, children's "number sense" is presumably based upon abilities, intuitions, awareness, feelings, and innate capacities (Berch, 2005). While children who have not yet developed this sense are distinguished from those that have. Similarly, the child identified as "mathematically disabled" (Witzel & Riccomini, 2011) does not seem to reason or problem-solve in the same way as 'other' children. Rather than being an inherent trait, these psychological distinctions of difference are related to the ways in which mathematics in schools is tied to discourses of knowledge about who children are and how they think.

Establishing Equivalence: Developing the “Reason” of Equality through the Discourses of the Learning Sciences

As a *tool* of thought, the rationality ascribed to mathematics organizes what comes to count as ‘rational’. This is in how it has become possible to model children’s thinking through ordering principles of educational sciences. One need only to look to psychometrics to see how the measures of intelligence, personalities, and abilities assemble with notions of equivalence to organize a ‘scientific’ knowledge of learning. Here, the abstraction of the mind and how it works is identified with certain behaviors, given as equivalent to ‘learning’ and mathematical reasoning. Embedded in the problems to solve are rules of how this form of thought should be developed.

Where mathematical content interacts with scientific discourses of educational psychology, equality is given meaning in the curriculum as some thing to learn. That is, together with the language and logic of mathematics, the learning sciences organize and classify how children can ‘best’ embody the rules of thought instantiated in the equal sign. In school math, the knowledge of equality intersects with learning theories of Thorndike, Piaget, and Vygotsky, among others, (Lamdin & Walcott, 2007) to order mathematics pedagogy by knowing the student. With the rules of equality and equivalence to be internalized and demonstrated as learning by children, the style of reasoning embedded in the sign provides a way to govern how a child should think and act. That is, the practices of pedagogy give children a particular sense of the world and their relation to it. This sense becomes what is rational and reasonable – as a standard for judging the child’s achievements and failures. Within this comparative cultural

framework, the child can also judge his or her own experience and competence in relation to others.

An article entitled, “Teaching the Meaning of the Equal Sign to Children with Disabilities: Moving from Concrete to Abstractions,” provides a way of seeing how this rationality is inscribed in the use of the equal sign (Beatty & Moss, 2007). The research is to substitute children’s “intuitive protoquantitative understanding of equality,” (p. 30) so as to “support the development of a more sophisticated and mathematically useful understanding of the equal sign,” (p. 28). Translating children’s understanding into an objective knowledge with numerical symbols and objects, the project works to represent what children intuit but generally “misunderstood” (p. 29) (See figure 1.1). The aim, then, is to “find efficient technologies that replace children’s ‘intuitive’ reasoning in which misconceptions are contained with new sets of rules for acting and seeing,” (Popkewitz, 2008, p. 140). Embedded in the notion of misconception is a certain truth-value of equality in the world, arrived at through the learning of mathematics.

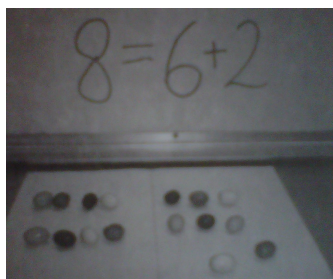


Figure 1.1 Embodying practices of identifying and comparing, the equal sign inscribes rules for organizing what is equal, equivalent and different. (Beatty & Moss, 2007, p. 33).

The new rules for acting and seeing require children to consider numbers, symbols, and other objects as a relationship ordered to representing sameness. To

establish equality, the child must learn that the concrete objects on the table stand in to represent abstract numbers. What is important is that in order to progress or develop, the child is to demonstrate a particular style of thinking through which they translate cultural objects (M&M's, numbers, symbols) to mediate equality's meaning. What children should see and how they should see it are techniques that operate through the curriculum to order a style of learning that the child should internalize and practice as problem solving.

What number goes in the box?

$$6 - 2 = \square$$

$$5 + 7 = \square + 8 \text{ (Beatty and Moss, 2007, p. 32)}$$

To answer "What number goes in the box?" the child should see the mathematical statements as a strategy for establishing equivalence. Specifically, the child is to compare what lies on one side of the equal sign to another. This embodies both a mode of learning and a style of reasoning about how to represent and compare things as equivalent. In this way, the expression $6 - 2 = \square$ is much more than a mathematical question.

Importantly, this problem solving also organizes thought and action as a logic of equivalence to make people objects of comparison. That is to say, within a comparative style of reasoning it becomes possible to see children, as objects of the sciences of psychology, on a continuum of achievement, understanding, or ability. These categories presumably identify how children should think and act to solve the school math problems. At the same time, these characteristics are to represent the inner workings of

the child's mind and are made visible as the materialization of learning. In solving the math problem above, children were to manipulate and quantify objects, order a linear thought process, and articulate a rationale by responding (correctly) to the question "How do you know?"

The rationality of psychology is to know the child's mind and how thinking processes function. In rendering the subjects of the school as intelligible and manageable, the principles of psychology work to organize the standards of participation as practices the child is to internalize through learning the content of the curriculum. The interaction of educational psychology with math curriculum appears as a seemingly objective strategy for organizing knowledge of children and their thoughts. However, pedagogy organized through the educational science of psychology embodies a way of comparing to *make people equivalent*. This equivalence is established by what the child should see, think, and do as a math learner to solve problems involving the equal sign. Ironically, the ordering of the child's thinking as 'rational' also produces divisions about who is not this child. This paradox is the focus of the next section.

Inequality as Equality: Calculations of Difference

Given meaning through mathematics and its use in the curriculum by learning psychologies, the equal sign works to organize identities and differences of people and the objects in their world. In the previous section, I worked to make explicit the relation between the discourses of mathematical styles of thought and learning that classify knowledge of equality and children as particular kinds of people. In this assemblage, the equal sign inscribes a comparative logic about how to produce the individual who

reasons about equality as sameness. The intersection of mathematical and educational discourses gives the equal sign its value and intelligibility in the curriculum as a fact embodying rules according which to develop this reason.

Drawing from contemporary research in teaching the equal sign, I have attended to how the equal sign operates in the curriculum through models that give a fixed quality to knowledge and people. This was explored in the true/false example whereby what counts as math is validated through the use of numbers as telling the truth and inscribing a style of reasoning about equality. Embodied in that style of reasoning are specific sets of practices by which a child is to demonstrate learning. Importantly, the same principles that organize equality as a fact in the curriculum also manage the identification of children, ordered as the same or not.

To further examine this production of difference in teaching and learning equality, I look at how the *reason* of the equal sign and equality are used to represent and compare children's abilities to "think mathematically". In a chapter entitled "Equality", Carpenter, Franke, & Levi (2003) begin with a visual representation of children's responses to the question "What number would you put in the box to make this a true number sentence? $8 + 4 = \square + 5$." Numerical inscriptions represent and organize children's answers, their grade level, and the frequency of response:

Grade	<i>Response/Percent Responding</i>			
	7	12	17	12 and 17
1 and 2	5	58	13	8
3 and 4	9	49	25	10
5 and 6	2	76	21	2

Figure 1.2. This translation and categorization of children's numerical response into a statistic entails the use of numbers to render things (and people) in the world as fixed, objective, and factual. (Falkner, Levi, & Carpenter, 1999 quoted in Carpenter, Franke, & Levi, 2003,)

As a strategy for objectifying things in the world, numbers (and the notion of equivalence they embody) stand in to signify children's thinking. Not only that, they establish a fixed norm as the standard by which to compare children. The percentages are used as measurements to sort students' responses, as compared to "most", "typical", or "common" (p. 1). The psychologies of learning and the rules of mathematical thinking contribute to this organization, granting equality and equivalence as a technical practice of learning mathematics as well as a cultural in providing ways of establishing equivalence (and non-equivalence) between children.

This is clear in how "children with learning disabilities" are represented (Beatty & Moss, 2007). The individual's qualities ("severe disabilities with sequencing, decoding text, long term working memory, central auditory processing disorders, attentional deficits with and without hyperactivity" p. 30) stand in as an evaluative difference. This places the child associated with certain characteristics on a continuum of normative sameness whereby the logic of equality reinserts difference as inequality. "These" children are compared to "typically developing children" (p. 35) and both seem to

identify distinct kinds of people, one being desirable and the other being the difference that stands in the way of *all* children knowing mathematics.

As mentioned earlier, embedded in the notion of *all* is a representational logic that entails identification, or making identical. Resolving and reducing the world into identifiable and fixed categories that appear manageable seems particularly convenient in times of need:

“Given the proportion of students with learning disabilities (5% to 8% according to Shalev et al. 2000) and the current move to *integrate* students with learning disabilities into general classroom environments, there is a *need* to examine remedial mathematics education in general and specifically how *these* students can be helped to be part of “algebra for all” (Beatty & Moss, 2007, p. 28, my italics).

Notice how “these” children are taken as a fixed category, proportions of a segmented whole, numerical objects, and different kinds of people. The remediation and management of this group in the school is organized by a similar logic that sets the task in school mathematics to fix inequality by identifying difference and making same. While seeming to operate in distinct registers, discourses of social and mathematical equality require a representation of stabilized identities and distinctions to determine and produce equivalence. Yet, the normalizing process of reducing *all* to a common measure represents difference as deviance and inequality, justified through the reason of equality.

Rethinking the Equal Sign, (Re)framing the Questions

By drawing attention to how the equal sign in elementary school math is linked to cultural rules of equality, equivalence, and difference, the dissertation research will

discursively analyze the principles that order the use of the equal sign in the contemporary program aimed at *math for all*. This entails a close textual reading of the program within a method that historicizes the system of reason by which a particular knowledge is generated about equivalence. To understand how those equivalences act to historically define sameness and difference, I ask:

- *What are the cultural principles generated in the elementary algebra curriculum reforms from the 1950s to the present that order and relate the equal sign with cultural statements of in/equality?*
- *What cultural practices are embodied in the principles as rules that fabricate children as having particular identities and comparative differences? How do the practices change over time?*
- *What historical principles about children, teaching, and difference make this way of thinking and organizing the curriculum reasonable?*

These questions introduce an important historical element of the research that requires further elaboration – a discussion to which Chapter 2 is devoted.

CHAPTER 2

Giving Pause to a Swinging Pendulum: A Discussion of Method

“In our attempts to improve the learning of mathematics, we will always remain torn between two concerns: *our concern about the learner and our concern about the quality of the mathematics being learned*. Because of this ever-present tension, we are repeatedly thrown from one extreme solution to another. It appears impossible to bring the swinging pendulum to a stop.”

(Sfard, 2003, p. 386, my italics).

Theories of change embodying ideals of progress, equality, and inclusion frame the proverbial pendulum of math education reform. That is, it is not mathematics per se that is the focus of the curriculum. In the broad field of education, mathematics has been given particular focus in relation to the notion of progress – on individual, societal, national and international levels. Within this scope, it has become clear that questions of inclusion and equality have intersected with the mathematics curriculum. However, in efforts to name what works, prove best practices, and solve the problem of inequality, representations of both the student and content in the curriculum have been assumed. Even more, these assumptions are ironically related to notions of difference and identity in ways that take for granted the cultural politics of math in schools.

To this point, the discussion has focused on how the equal sign in the math curriculum works to organize particular ways of knowing equality and establishing equivalence. In reading the math curriculum as a technology that organizes the conduct of the self and schooling, I have situated the equal sign within a system of reason about equality. Constituted by discourses that assemble cultural, historical, and political

principles and practices to give meaning to equality as a fact of sameness, this system provides the rules by which to organize equality as a way of seeing, thinking, and knowing in the world. Therefore, the meaning of the equal sign (=) in the curriculum is never simply mathematical. It also frames notions of difference and identity that categorize children as kinds of people who think and act according to certain 'truths'. In highlighting the ways in which math in the curriculum is not neutral or natural, my aim has been to provoke thought about how the equal sign works to govern a logic of equality that produces and maintains representations of children as objects that can be measured, compared, and calculated so as to be included in the *all*.

In relation to the present efforts to achieve equality and equity, learning mathematics has emerged as the force to mobilize individuals, society, and the nation toward a future that seems more equal and just. The constant motion back and forth in curriculum reforms, considering *content* on one end and the *learner* on the other has assumed both and left the relationship between them unquestioned. To give pause to the pendulum, I take the equal sign (=) and its inscriptions in the contemporary math curriculum in the United States as the object of my research. This is to consider how the school subjects (both the content and the student) are deeply entangled with the fabrication of identity and difference that give meaning to in/equality and its logic. The contemporary *math for all* reform discourse has served as the entry point into this discussion as a way to introduce the problematic of my study. More than that, it also frames the approach to my historical analysis of the equal sign and notions of in/equality organizing the curriculum.

In this chapter, I elaborate the important methodological implications for analyzing the discourses that intersect to give meaning to the equal sign (=) in school math problems and historically organize the knowledge of equality as the production of equivalence and difference. First, I articulate the approach to this study of math curriculum as a history of the present. In this, I consider *math for all* as an historical event that can be examined within a grid of cultural practices and rules. Then, with the event and historical grid as analytical tools, I explore two historical conditions that make *math for all* possible as an educational reform project in the present. These conditions importantly relate to drawing temporal boundaries in a discussion that emphasizes a post World War II shift in the rationality given to mathematics and math education in the planning of society. Finally, taking the post war period as an historical marker, I outline the narrative that will archive the relationship between the equal sign, school mathematics, and a shifting cultural logic of equality.

A Cultural History of the Present

In exploring the logic of equality embedded in contemporary reform discourses, it becomes apparent that the premise *all children can and should learn mathematics* is extended by conditions of *when, if, and because*. In the statement of *all* children learning mathematics are rules that embody the limits of the reason of equality and the boundaries of in/exclusion. These rules, again, are not merely about mathematics. That is, the use of the equal sign in the school curriculum inscribes ways of thinking about equality and establishing equivalence that intersect with cultural and historical standards about who children are and who they should be.

Methodologically, I approach this study as a *cultural history of the present* (Popkewitz, Franklin, & Pereyra, 2001). This is a strategy to study how cultural norms about equality, equivalence, and difference are historically constituted and brought into the present as a way of reasoning about the subjects of school reform (see also Lesko, 2012; do Ó, Martins, & Paz, 2013). In this approach, I consider *math for all* as an event. This importantly highlights how it has not always been possible to think of elementary mathematics education as part of producing and maintaining a just and equal society.

Taking *math for all* as an event involves an examination the historical conditions that intersect to give meaning to the logic of equality embedded in the present reforms. To do this, I place the use of the equal sign in *math for all* reforms within a “grid of intelligibility” (Foucault, 1971). That is, both the sign and the logic organizing reforms are explored in how they are given meaning within a larger system of knowledge about equality. The ways of knowing and acting that (re)organize math education circulate as discourses about how citizens of the nation should live their lives as well as how children should learn mathematics. Therefore, this cultural history of present math reforms provides an approach to understanding the modes of thinking, seeing, and acting that make this project of reform and change ‘reasonable’ – even when its implications can be seen as ‘unreasonable’.

The historical project, then, is to understand both the conditions of possibility as well as the effects of the ways in which discourses related to inclusion, equality, and equivalence embedded in contemporary math for all reforms are mobilized through the equal sign as a ‘fact’ of the curriculum. In the sections that follow, I will discuss the

methodological moves that are involved in approaching contemporary elementary math education reforms as a cultural history of the present. This historical reading of the curriculum refers to the ways in which the logic of equality inscribed in the equal sign is rendered historical – conditioned by particular cultural, social, and political phenomena. Considered as an alternative to a traditional history of math education, this ‘method’ is not to provide a linear and causal narrative of how math reforms changed as the effect of historical agents. Rather, the equal sign itself is taken as an historical ‘actor’ involved in producing the knowledge of equality and the subject that is to learn it.

A Present, Conditioned by the Past

My interest is in how the present efforts to improve math education engender an *assumption of social progress towards equality and prosperity for all that has been left unquestioned*. This assumption, and importantly its placement in this cultural history of school mathematics, operates on two conditions. One, the field of mathematics has historically been linked to progress through the belief in its ‘practical use’ in ordering and planning the future. As a field related to discourses of scientific objectivity and precision, *mathematics in general is seen to engender notions of reason and rationality* (Daston & Galison, 2007). To see this, one can refer back to the eighteenth century narrative exploring the intersecting shifts in the location of reason, the use of mathematics in modeling the world, and the social value given to that standard of equivalence and measurement involved in the enlightenment rationality of planning society. Historically, the notions of progress and precision as the ‘reason’ embedded mathematics are visible

in 18th century France. Mathematics and math education for all people, including women, was linked to the “perfectability of society” (de Condorcet, 1796).

The place of mathematics in the social and cultural progress of society can be seen in contemporary reasoning of the importance of learning algebra:

“...algebraic reasoning in its many forms, and the use of algebraic representations such as graphs, tables, spreadsheets, and traditional formulas are among the most powerful intellectual tools that our civilization has developed. Without some form of symbolic algebra there could be no higher mathematics and no quantitative science; hence no technology and modern life as we know them. Our challenge then is to find ways to find the power of algebra (indeed, all mathematics) available to all students,” (Kaput, 1999, p. 134).

The entanglement of mathematical reasoning with notions of science and technology inscribes images of “our civilization” as embodying powerful and modern ways of living. Specific to the United states, this image of math as useful for planning and securing the future of society is expressed in national reports that hold mathematics education as central in the claim that “the eminence, safety, and well-being of nations have been entwined for centuries with the ability of *their people* to deal with sophisticated quantitative ideas,” (National Mathematics Advisory Panel, 2008, p. xi, my italics added). This centuries-old idea can be seen in how math as a technology for planning a modern and progressive civilization entangled with 19th century narratives identifying the nation with notions of progress and rationality. Historian David Nye (1994) explores how the relationship between science, technology, and the nation’s “ideas about itself” are embodied in the “technological sublime”. Taking skyscrapers, railroads, and satellites, as objects invested with sublimity, Nye considers these, and

other things, in how they have historically come to stand as the representation of reason and rationality and assemble with notions of America's national identity.

As a condition of how mathematics education in the United States has been linked to the progress of the nation, the logic ascribed to the *technological* was (re)visioned both during and after World War II. With that, mathematics was (re)imagined as useful and practical for making the modern nation and saving it from the "crisis" caused by the "breath-taking movement into the a new technological era," (College Entrance Examination Board (CEEB), 1959). The historical link between technology, national identity, and modern life embedded in the technological sublime gave mathematics education a part in the narrative of how the nation and its citizens would survive the 'crisis' and, more than that, be exceptional. Even more explicitly, math was expressed as a "tool of modern life" (ibid).

This relationship between math and planning society intersects with a second historical phenomenon that conditions the possibility for thinking of mathematics as a tool for making modern society. That is the (re)emergence of the social and pedagogical sciences as a way of ordering modes of living as 'rational' people. Connected to broader social goals of understanding society by understanding the individual, this rationality is embedded in the bond between mathematics, technology, and progress through planning the future of the nation. In this, *math for all* in the present is assumed to develop the 'right' kinds of reasoning, and people, required for securing the future promises of a liberal and egalitarian society.

The recognition of this present, made possible in relation to cultural practices, directs attention to three main curricular reforms in math education in the U.S. during the second half of the 20th century. Although elementary mathematics was in the curriculum before this historical moment, the aims of *math for all* in the present are related to particular social, cultural, historical, and political phenomena that can be located in the post World War II period.

Drawing Historical and Discursive Boundaries

Within this historical frame of post World War II to the present, children's achievement in elementary mathematics has become linked to notions of progress, prosperity, and equality for the nation and its citizens. For this reason, I consider the emergence of "New Math" (late 1950s-early 1970s), "Back to Basics" (1970s- early 1980s), and "Standards-Based" (1980-early 1990s) reforms² as important historical shifts in reasoning about how math education is to function in the progress of society by making the child into a math learner³. I turn to these historical markers to consider how the equal sign embodies not only mathematical knowledge but is inscribed in social,

² The time frames are borrowed from Klein (2003) and are useful in providing boundaries for primary and secondary document selection. However, they are not used to suggest that as one period 'ended' another 'began'. Rather, they can be seen as overlapping historical trajectories that shape and are shaped by cultural practices and principles – giving meaning to the content of the curriculum and the subjectivities of the 'learner'.

³ I consider the learner as a historical signification. I use it throughout to designate a theory/practice about the identification of subjects of the school as particular kinds of people. Learner is historically distinguished from pupil or student, for example, to express a subject of the school that is subjected to the scrutiny of the teacher. For an elaborated discussion of this see Popkewitz's (2001) *Production of reason and power: Curriculum history and intellectual traditions* in Popkewitz, Franklin, & Pereyra (Eds.)

cultural, and political norms that generate and maintain notions of difference and identity operating in current reforms.

Beginning in the 1950s with the mobilization of the New Math curriculum, I conduct a discourse analysis by engaging in a close examination of the archival relationships between the equal sign and discussions of in/equality in these three periods. Analysis at the level of discourse is to examine the cultural norms and standards that shape the terms of inclusion and give meaning to what counts as 'equal' (Foucault, 1980; Rabinow, 1984). This is carried out at an inter-textual level, meaning that I will investigate the relation of the 'mathematical' use of the equal sign to cultural narratives that shape its understanding in math textbooks, teaching resources, educational reform literature, policy documents, and educational sciences research published between 1950 and the present. Looking across these documents, I join linguistic traditions of discourses analysis with inter-textual analysis to examine the principles and practices that order the 'common sense' of equality, organize notions of identity and difference, and frame how children are to participate and be included in school mathematics.

Placing the equal sign (=) and its relation to discourses about equality at the center of analysis is to reveal how threads of thought weave together to historically produce and maintain identities of certain kinds of people. These identities emerge at particular historical moments, not as given but as effects of power that operate through discourse and become visible as the cultural expression of language and practices (Foucault, 1972). In this way, the equal sign can be seen as an object of discourse,

framing and framed by particular ways of knowing and being in the world. The analysis draws upon the idea that the knowledge, or logic, of equality that circulates in the school curriculum is not only to do with mathematics. Rather, it melds together with and organizes words, gestures, interactions, actions, values, and 'truths' to gain meaning, enact specific identities and frame practices as ways of living in the world. In other words, the equal sign in the school curriculum has meaning only in and through the cultural practices that constitute the discourses that frame its pedagogical 'understanding'.

How the equal sign is used to construct knowledge, frame pedagogical practices, and order children's identities is discursively related to notions of equality and difference. This was explored in the discussion in Chapter 1 about how children were to answer "What number would you put in the box to make this a true number sentence? $8 + 4 = \square + 5$." (Falkner, Franke & Levi, 2003). Translating the numerical response to identify children's 'mathematical thinking' requires a relationship between the equal sign, cultural practices, and discourses that have implications for how identity is constructed. The 'typical' categorization is not neutral or ahistorical; its meaning is entangled with the use of and practices implicit in teaching and learning the sign.

The equal sign and its discursive meaning in the curriculum is not of little consequence. The political implications are visible in how the sign operates in the curriculum to build and maintain a 'reality' about what constitutes difference and how equivalence is established as a comparison between children. Since identities and differences are enacted as modes of establishing equivalence through the equal sign

and its logic, the study of how it operates in the curriculum is central to issues of what constitutes 'equity' and 'justice' in math education for all.

Implied in centering the analysis on the historical production of knowledge and subjectivities is a displacement of the *a priori* subject as the 'cause' and 'agent' of historical change. This is evident in two ways throughout the dissertation. First, and as discussed earlier, the child's identifying traits are taken as a fabrication, not as natural and neutral categories of distinction. Rather than simply ideas, the psychological traits are given materiality and made visible as characteristics of how children are expected to think and act. At the intersection of mathematics and educational sciences it becomes clear how the discourses of these overlapping fields of knowledge give shape to the boundaries around what it means to be a *math learner* and how that matters in relation to issues of in/equality.

Exploring the fabrication of identity and difference is related to the second effect of displacing the *a priori* subject as the center of historical analysis. That is, the sources of historical change are not attributed to agents, authors, and actors. The history is not given as an account of how people have acted to change the course and shape of school mathematics outside of political, cultural, and social influences. Rather, historical actors are thought of as "conceptual personae", to borrow from Deleuze and Guattari (1991). In other words, bringing particular works and authors into this narrative is not to focus on the people themselves, but rather how they stand as 'figures of thought' to consider how particular modes of reasoning are given materiality through them.

In Chapter 2, for example, the work of Jean Piaget and Jerome Bruner is entangled in the analysis of how the child's reasoning about equivalence was related to notions of social progress and planning during the period of reform that followed WWII. The close reading of 'their' texts is not to understand their intentions or how they 'caused' the New Math educational reforms. Instead, considering the authors as conceptual personae, the discourse analysis operates to understand how principles from educational psychology assembled with the mathematics curriculum to inscribe cultural norms and standards for relating the child's mathematics to traits of citizenship.

While the analysis of discourses that relate the equal sign and equality to knowledge and effects of power is not to foreclose the possibilities for human action and agency, autonomous actors are not positioned as the cause of change or the makers of history. This project, therefore, engenders a particular notion of change. The actions to be 'made' and the changes that count as reform are considered within a larger system of reasoning and "it is through locating changes in the rules of reason that we can think about change," (Popkewitz, 2001, p. 168). That is, in exploring the rules that organize equality as a cultural practice, new possibilities for seeing, knowing, and acting may emerge. Framed by the concern that equality is granted as neutral and logical while embodying ways of knowing in the curriculum that 'unreasonably' mobilize action, my research works to unsettle the reason of equality – not simply as a goal of math curriculum reform, but as a challenge to it.

Archiving as Traces of the Past

Locating historical changes in how the equal sign relates to cultural theses of equality and works to fabricate differences between children entails the notion of *archiving*. In the sense that I am using it, *the* archive is not some 'thing' neatly filed on a shelf, already compiled, waiting to be discovered (Stoler, 2009). Rather, archiving is a process, constructed by tracing historical relations between categories, fields of knowledge, contexts, objects, and practices as discourses that assemble to give meaning to the equal sign in school mathematics and the notions of difference and identity it embodies. This archiving involves the collection, selection, and interpretation of historical materials that organize the cultural reason of equality in the elementary math curriculum.

While initial documents, curriculum materials, and secondary sources have been chosen, structuring the 'method' to my research as the work of archiving a history of the present by no means lays out a neat and orderly path that has been determined up front. Nevertheless, the focus in exploring the use of the equal sign is to understand the logic of equality that relates the practices of pedagogy to the premise that *all children can and should learn mathematics*. More specifically, I work to document how this statement is extended and defined by conditions of *when*, *if*, *by* and *for*. These conditions importantly embody cultural rules that define limits of the reason of equality and the rules of in/exclusion embedded in the *all*. Archiving the historical conditions that give meaning to the contemporary program of *math for all* is not to suggest that it is unimportant. On the contrary, it is to ask questions about how it has come to matter and

it what ways it is historically related the production of identity and difference that define in/equality.

With this in mind, Chapter 3: *New Math and Making the Child a Mathematical Citizen* examines the reorganization of curriculum from post WWII to the early 1970s that emphasized children's learning of equality and equivalence as foundational to learning mathematics. Interestingly, this placement of the structures of equivalence and equality is given as central to *all students at all levels* understanding mathematics as a "tool for modern life". Learning equivalence and equality, however, was not simply about math. It is to be understood within a broader cultural frame whereby the curriculum was (re)organized according to principles of national progress in post war America, notions of social planning, and the aims for educational equality.

The use of the equal sign (=) in New Math texts produced by the School Mathematics Study Group are textually related to cultural narratives about making rational and responsible citizens of the nation to secure the technological future of the United States. Drawing upon cultural history, political theory, policy documents, and an educational psychology that presumed children could master the content of mathematics given its 'universal' structures, this chapter examines how the New Curriculum produced new ways of thinking about the relationship between the child's development in mathematics and the progress of the nation as an effect of a "mathematical" form of creativity, innovation, and independence. This link would serve as an important condition for establishing the identities and differences that

distinguished the psychological traits of the child as a *mathematical citizen* from the child who was not that.

In Chapter 4: *Back to the Basics: Inventing the Mathematical Self*, I focus on how the teaching of equality in the math curriculum from the early 1970s to the 1980s seemed to emphasize practical application and skills as the core of instruction. As compared to the previous reform emphasizing *understanding* structures and mastering the underlying logic of mathematics, the Back to Basics movement was apparently distinguished by attending to the discovery of basic arithmetic operations as fundamental skills and competences. In highlighting these changes in math education journals and texts, I seek to understand the changes as an effect of how emphasis was placed on learning basic arithmetic operations as a skill – not only for making sense of and doing mathematics, but also for understanding the self. Analyzing school math problems as translated through a psychology that assumed children as capable of being equally motivated, I examine how the use of the equal sign in the curriculum produced cultural narratives and images of the child as a self-responsible, -reliant, -directed and -motivated individual.

As related to social questions of in/equality, the analysis relates the curricular shift to notions of individualism in the 1970s and 1980s whereby unequal outcomes or opportunities were no longer deemed the responsibility of society – but of the motivated individual. More than about basic skills in mathematics, then, the analysis highlights how the practices of math pedagogy were organized through new tools of psychology that placed particular (not universal) skills, abilities and interests as ‘basic’ and ‘natural’

traits of each individually motivated child, inscribed in the curriculum as a *mathematical self*.

Chapter 5: *Standardizing Mathematical People: The Power in the Child as Problem Solver* attends to the “Standards-Based” reform movement that emerged in the 1980s. In this chapter, I look at the use of the equal sign in the *Curriculum and Evaluation Standards* (NCTM, 1989; NCTM, 2000) in relation to other policy documents that aimed to develop norms for what children should know and do in learning math in the elementary school. The examination of these texts is to understand how expressions of the equal sign inscribed a notion of mathematical power as a way to organize who the child is and should be as a confident kind of *mathematical person and problem solver*.

Tracing textual relationships that give meaning to mathematical power and how it is to be produced, I analyze how research in mathematics education and changing psychologies assembled to give characteristics and value to the child’s development of this power as a solution to the problem of “mathematical illiteracy”. Equivalences between children and their traits as mathematical kinds of people were to serve as the standard by which to determine who did/did not have mathematical power – expressed by certain ways of seeing, thinking, and talking about the world.

In Chapter 6, *Crossing the Equal Sign: Making Math Education and the Cultural (Re)Production of the Child*, I discuss the ways in which the equal sign in the mathematics curriculum has been historically related to cultural rules and practices that shift and (re)form the identities and differences of children. Broadly retracing the

historical and cultural trajectories that relate the equal sign to narratives and images of the child as a *mathematical citizen*, *self*, and *person with mathematical power*, I highlight the contingencies upon which differences are produced to give meaning to equality in the making of mathematics for all. This chapter concludes with a discussion of how this research contributes to the study of schooling and curriculum as well as issues of difference and equality. As related to that, particular emphasis is given to a consideration of how studying the math curriculum as a cultural history of the present might provide alternative ways of considering the limits of reform and the possibilities of change.

Summary

While my study takes elementary math curriculum reforms and research in the United States as the site of inquiry, it aims to speak more broadly to how equality is instantiated as a cultural practice in the curriculum. In arguing that the equal sign is related to a theory of equality that is not merely mathematics itself, I relate the cultural norms of equality to the organization of school math practices to consider the historical, cultural, and political sensibilities through which equality is expressed as an object(ive) of the curriculum.

Historically situating my discussion of math education within equality discourses embodied in contemporary math for all projects is to highlight the relations between equality, identity, and difference that give meaning to present reforms and instructional practices. Unraveling the historical relations is aimed at understanding the political implications of reforms that produce difference as inequality, identifying some children

as equals while others are not. With in/equality as both a problematic of the curriculum and a question open for consideration, alternative ways of thinking about educational reform and children may emerge. In this project then, is the hope that with an understanding of the past as conditions that shape the present, change in the form of rethinking what is 'known' might be possible. This hope embodies a double gesture of fear – a fear that even within the well intentioned efforts to (re)organize mathematics education as the “great equalizer”, the ways of thinking that shape the dividing practices embedded in the curriculum will continue to go unquestioned.

The double gesture of hope and fear (Popkewitz, 2004) will emerge as a theme throughout the dissertation and draws attention to curriculum reforms projects as related to cultural concerns of risk and crisis coupled with expectations of progress toward a more equal and just society. In the articulation of the hopes and fears of the curriculum is the inscription of the identity of a kind of child who embodies the 'right' characteristics and traits to improve the nation. This relationship between the development of the child's mathematical reasoning and securing the identity of the United States is discussed in the next chapter as it became visible in the curricular reforms that claimed to bring something 'new' to mathematics education.

CHAPTER 3

New Math and Making the Child a Mathematical Citizen

As a convenient event in the history of math education, the launching of the Sputnik satellite by the Soviet Union seems to provide the reason for the math curriculum reforms that took place in the 1950s and 1960s in the U.S. The historical explanation that Sputnik prompted a reorganization of both the science and math curriculum illuminates how, on the surface, the changes in math education were linked to discourses of anti-intellectualism, incompetence, and national security (Dow, 1999). However, the changing role of mathematics in the nation was not simply caused by the satellite or the threat of Cold War. In this chapter, I argue how, beneath the surface, the New Math curriculum assembled with political and social sensibilities to construct a narrative about a new kind of citizen in the post war period.

It has not always been the case that mathematics education seems integral to the making of future citizens. This relationship between children's mathematics, national identity, and questions of social progress is a historical one that emerged during the New Math reforms of the 1950s and 60s. In this chapter, I examine the emergence of this relationship and explore how it provided the conditions to see children as *mathematical citizens* – a collective group whose responsibility and participation in society is organized through the logic given to mathematics. In this historical moment, math educational reform was importantly expanded to include all students, at all levels. More than that, with math as a “tool of modern life”, *math education for all* was intricately bound to modes of living as rational, intelligent, and innovative citizens.

A central argument throughout the chapter is that the mathematics children were to do in schools was not simply about mathematics – but organized cultural rules for how they were thought to belong as part of the collective *all*. That is, the New Curriculum assembled with cultural inscriptions of creativity to produce new rules for how children should think and act as future citizens of the nation. This cultural element of equivalence, equality and inclusion will be explored in the use of the equal (=) sign in the New Math curriculum as it embodies principles about how to live the ‘modern life’ as a creative, innovative, and free thinker. These traits, given through psychological rules that assumed all people could become masters of mathematically creative modes of thought, worked to differentiate who is and is not the citizen. As such, the equal sign is not simply an example but is analyzed to understand how similar ways of reasoning about ‘mathematical’ equality carry rules for how to identify what is equivalent, and, in that, what constitutes difference.

Tools of Modern Life and Planning a New Curriculum for All Students

During and immediately after World War II, mathematics and science were ascribed value as a “tool of modern life” (College Entrance Examination Board (CEEB), 1959). One need only look to the introduction to a teacher textbook created by the School Mathematics Study Group (1966a) to see the importance placed on mathematics and cultural modes of living,

“The increasing contribution of mathematics to the culture of the modern world, as well as its importance as a vital part of the scientific and humanistic education, has made it essential that the mathematics in our schools be both well selected

and well taught – at all levels, from the kindergarten through the graduate school,” (n.p.).

An important shift to call attention to here is how math at all levels, including elementary math, was presumed as part of its vital and cultural importance. In this, mathematics taught “at all levels” signified a new focus in planning the curriculum during this period of reform. This was in contrast to earlier decades where forms of abstract math considered to be abstract were thought to be valueless, dangerous, and impractical, particularly for children (see Langer, 1931). Yet, in the years following the war, a rigorous math education was to start at the earliest levels as part of living as an “intelligent citizen,” (National Council of Teachers of Mathematics (NCTM), 1945; Wagner, 1960).

Including and investing in elementary mathematics in the reform of the New Curriculum brought with it new ways of reasoning about how children might be considered part of an intelligent citizenship – capable of living a modern life defined by norms of participation organized around notions of creativity, independent, flexible, and methodical thought. These norms, as standards of living this life, were inscribed in early elementary curriculum texts as particular modes of thinking, seeing, and acting and become the focus of the chapter.

Before the analysis of elementary curriculum textbooks, I first consider the historical intersection between science, math, education, and psychologies emphasizing learning as a process of mastery. This ‘meeting’ conditioned the possibility of a link between the use of the equal sign in New Math textbooks and the cultural norms of representation whereby children were all given equivalence as mathematical kinds of

learners, capable of mastering its mode of thought. Then, in three sections, I examine the use of the equal sign ($=$) in school math texts. I focus on math textbooks created by the School Mathematics Study Group (MSG) because this group played a significant role in creating and implementing the New Math curriculum for the elementary schools. Importantly, the specific textbooks and problems selected for analysis are not meant to be representative of the entire New Math movement. Instead, they are examined as a way to understand how certain ways of thinking about and establishing equivalences as a foundation of the math curriculum assembled with psychological classifications of mathematical creativity to ascribe identity to *all* children as innovative, flexible, independent and methodical.

School Math Study Group and Psychology as the Truth

Headed by Edward Begle and funded by the National Science Foundation, the MSG represented a collaborative effort of mathematicians, math educators, elementary teachers, and educational psychologist to “lead the way toward inspiring a more meaningful teaching of Mathematics, the Queen and Servant of the Sciences,” (School Mathematics Study Group, 1966a, n.p.). Together, they were seeking ways to expand the teaching of quality mathematics to more people so as to influence the future by way of science. The rationality given to mathematics as a “Queen” and “Servant” of the sciences entangled with a seemingly rational planning of the curriculum and produced ways of seeing children as part of mathematics and its service to the nation.

Importantly, the task of reforming the elementary curriculum was not merely a copy and paste of what modern mathematicians were doing. That is, mathematics was

to be translated into math education via the psychological rule of mastery so that all children could *understand* and internalize predetermined patterns of thinking and acting. In the task of translation, children were grouped together as belonging to a cohesive group with common psychological traits that presumed them all as capable of learning the structure of mathematics. These traits, given in the language of a psychology that presumes to know who children are and how they learn, are read as part of the process of “alchemy” (Popkewitz, 2004), or translation, by which mathematics became a tool for making the modern life.

The analysis of the math problems in children’s textbooks intersects with such psychological ‘truths’ about how children learned. Important texts, such as *A Study of Thinking* (Bruner, Goodnow, and Austin, 1956), *The Process of Education*⁴ (Bruner, 1960) and *The Psychology of Intelligence* (Piaget, 1950), are explored in terms of how the logic of psychology operated to give equivalence to all children so as to rationalize how they *all* were to learn and understand the content of the New Curriculum that was more than just mathematical thinking. In this rationalization, the cultural principles of creativity operated as psychological tool to identify the rules by which to make modern mathematics intelligible to children.

By making visible how the logic of equivalence and equality embedded in the equal sign (=) in the New Curriculum overlapped with psychological modes of identifying children and characteristics of their thinking, I examine how the foundations of New

⁴ This text was written as the result of a 1959 conference at Woods’ Hollow between ‘experts’ in the fields of education, learning sciences, science and mathematics. The National Academy of Sciences, the RAND Corporation, and the U.S. Air Force funded the collaboration aimed at creating curriculum and improving the teaching of math and science.

Mathematics were organized so as to include *all students* in modern math and life. In this, I draw attention to what were given as “mathematical competences” that embodied a particular way of reasoning about equivalence and inclusion. On one hand, these foundations of the elementary curriculum were to help “all students, with their varying interests and abilities” understand the underlying structures of mathematics (Begle, 1958, p. 616). On another, they embodied particular civic virtues “required for effective and intelligent living in our culture,” (CEEB, 1959, p. 7).

Set Theories of the New Curriculum

The introduction to a 1st grade math text published by the School Mathematics Study Group stated that, “one of the prerequisites for the improvement of teaching mathematics in our schools is an improved curriculum...which reflects recent advances in mathematics itself,” (MSG, 1966a, n.p.). Set theory represented one of these advancements. As a branch of modern algebra, set theory was to provide children with an understanding of the structures and patterns that organized numerical relation and operations. Explicitly, its placement in the curriculum was “an effort to help children in learning to appreciate the precision in making statements to describe things,” (MSG, 1961, p. 1). This appreciation was linked to helping children understand that “the language of sets is an effective way to express ideas, not only in mathematics, but also in many other fields of knowledge,” (ibid). Implicit here is a style of thinking that related mathematics to an importance given to science as an accurate and effective way to see and communicate about the world.

In the first year texts, this precise use of vocabulary was emphasized throughout both the student and teacher texts – particularly as a way to understand the mathematical notion of equivalence. Equivalence and the exact terminology that would give it meaning were underscored as a way to prepare children for the notion of number and the inequality of number (SMSG, 1963, p. 4). In the first years of mathematics education, then, children were to make sense of the comparative logic of equivalence expressed in the language of more than and less than.

Before interacting with numbers and symbols, like the equal sign ($=$), children were expected to use specific language so as to compare and establish equivalence and non-equivalence between sets of objects. In the student text, these relationships were to be determined through a description and comparison of visual representations (see figure 3.1).

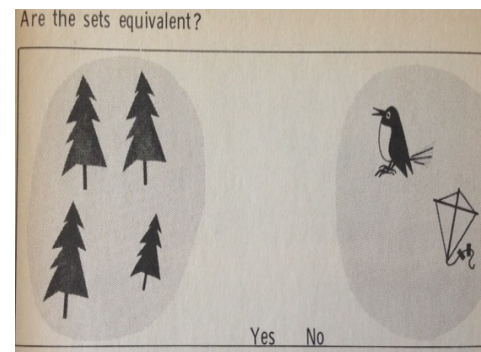


Figure 3.1 SMSG, 1966b, p. 17

According to the comparative rules embodied in the language, the sets are not equivalent because there are more members in the set containing trees. This verbal expression can be understood as more than descriptive. That is, embedded in the use of language to understand equivalence is a particular way of thinking that can be characterized as grouping objects into classes (sets) and responding to them (describing them) in terms of what they hold in common. On one hand, making things equivalent, seemed to express a comparative reality. On another, it entailed a mode of

thought that psychologically involved “an act of invention,” (Bruner, Goodnow, & Austin, 1956, p. 2).

This act of invention involved in grouping things together to determine equivalence can be related to the “imagination” required for developing the concept of number (SMSG, 1966d, p.1). By identifying and making the 4 trees into members of a set, children should also “imagine” many other sets of objects signifying that number. Significantly, this *act of invention* is not merely a free association of things into patterns but inventions with certain rules of ordering and classifying. The rules for grouping objects together were bound to the rules organizing how to see and talk about the equivalence and non-equivalence between objects in the world.

Inventing equivalent groups as sets of objects, identified by imaginations of number so as to develop a mathematical understanding about equivalence was not only to do with mathematics. That is, this pedagogical practice related the mathematical competences to particular cultural standards required for living as an intelligent citizen, capable of seeing and creating solutions. This is clear in how the link between language use and imagination was bound to “evidence for the belief that a student who *understands* the solutions to systems...can *create* his own methods,” (UICSM, 1956 quoted in Bidwell & Clason, p. 658). Evidently, if students could systematically see and describe relationships between objects, they would be able to think creatively and independently.

While teachers were supposed to “encourage the child to use his own method of thinking,” this thinking was not independent from the modes of thought embodied in the

manipulation of language, images, and physical objects for understanding equivalence (MSG, 1966e, p. 400). That is, the creation of “his own method” was organized through the ordering principles of equivalence for ‘seeing’ objects in the world as included in the same set.

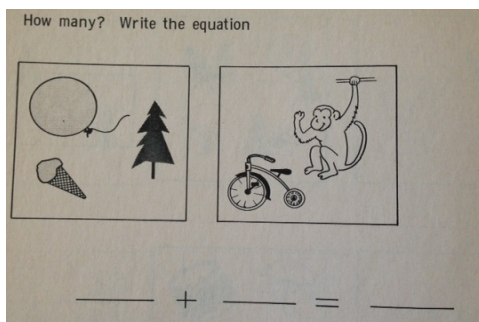


Figure 3.2 (MSG, 1966b, p. 118)

Imagination, creativity, and independence were not simply mathematical skills. They were psychological traits that children were to internalize so as to have methods for creating solutions to problems they had not yet

encountered, like those in figure 3.2. As a new mode of representing relationships of

equivalence, this expression of the equal sign (=) embodied supposedly imaginative and creative method for answering “How Many?” on one’s own.

Prompted by the directive, “Write the equation,” children were to, in the language of the text, “imagine” the set of objects as 3 and 2 and join them together. Creating a solution to the problem of equality expressed by ____ + ____ = ____ required a way of reasoning about distinct objects as members of a set and imagining an identity that would create an equivalence between them. This way of thinking about ‘mathematical’ equality was not only about building mathematical competences. It also worked to establish equivalences between children in that they were given traits as creative, inventive, and precise in their use of language and methods for problem solving in school math.

To return to the earlier discussion, the expansion of and investment in the elementary New Math curriculum was linked to notions of living the modern life. This modern life, outside of the spaces of the school, was given meaning with cultural narratives of creativity that inscribed characteristics of inventiveness and imagination in all citizens. In post war America, creativity was deemed as a valuable personality trait of the kind of citizen who would develop autonomously while part of a collective citizenship (Cohen-Cole, 2009). As opposed to conformist, rigid, and closed-minded thinking, creativity was presumed to organize society according to democratic values in line with “unity in diversity” way of thinking.⁵ That is, society could be united even in its diversity. The organization of unity presumed a consensus as to how it would be defined. In this value given to creativity, there was also universality – as something that could be developed in, and unite, everyone.

This cultural ideal of technically producing creativity for the prosperity of the democratic nation entangled with the reorganization of the math curriculum. In other words, given the ‘right’ conditions, all children could be creative. The right conditions were given in the curriculum as the pedagogy of equivalence and equality explored above. All children could presumably use precise language, think independently, and develop their own methods to as a way to understand equivalence and the equal sign as foundations of the New Curriculum. Yet, more than about mathematical thinking, cultural norms of creativity were translated through the principles of psychology to

⁵ *Unity in diversity* emerged as a particular way of thinking about the organization of both school and society in the post war period. This call can be seen, for example, in James B. Conant (1948), *Education in a Divided World: The Function of the Public Schools in Our Unique Society*.

inscribe the 'right' conditions for living and learning as inventive and imaginative children.

This was not only an educational matter. Sociology, political science, and social psychology also studied creativity as a trait of citizenship in relation to notions of a modern and progressive society. A number of so-called experts identified creativity, innovativeness, and autonomy as characteristics of the *kind of citizen* a modern and progressive society required (Maslow, 1943; Doob, 1960; Gardner, 1963; Almond & Verba, 1963). More than a cultural ideal, this psychological trait was ascribed to people through the measurement of civic behaviors and democratic attitudes (Herman, 1995). Specifically, these measures were to organize and compare cultures of democracy to those that were not. In some cases, psychological classifications were used to equate the model citizen with a score measuring high and low values of the traits people were to embody as signifying features of the *American*.

Organized according to a similar way of reasoning about identities (of sets of objects and people), the problems of equality and equivalence in the math curriculum related to the logic of producing similarities between children. The logic of equivalence embodied in the practice of grouping distinct things together to give common identities and establish a space of collective belonging was not only a mathematical calculation or measurement. It entailed a cultural way of inscribing characteristics to creative kinds of people and providing the rules of rationality by which to develop these traits in children through math as a tool of modern life.

Set Materials and the Materiality of Abstract Thought: Solving for the Unknown

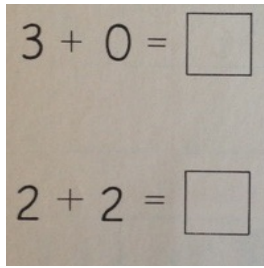


Figure 3.3 SMSG,
1966b. p. 120.

As the first year progressed, children would encounter open addition and subtraction expressions as the core of their problem solving. Different in their form, these problems were presumed to be more abstract, given the symbolic logic (see figure 3.3). While it was acknowledged that children might not be able to think abstractly, physical materials were to aid in their problem solving. It was thought

that, “working with concrete materials is essential as it provides every child with a method for finding the sum,” (SMSG, 1966a, p. 214). In other words, these objects, like blocks, pennies, or toys, embodied a strategy for finding the unknown if used correctly (ibid, p. 236). “Regardless of intellectual level,” children could join together sets of objects to systematically solve the problems (SMSG, 1966d, p. 2).

This method was not merely a way of working with the curriculum materials to produce understanding. It also embodied a cultural expression of the kind of child who was capable of thinking abstractly and the mode of thought that made that capability. In reorganizing the curriculum, it was taken that “what is most important in teaching basic concepts is that the child be helped to pass progressively from concrete thinking to the utilization of more conceptually adequate modes of thought,” (Inhelder quoted in Bruner, 1960, p. 38). With “concrete thinking” as one of the limits point of the trajectory of a “natural thought process”, a more conceptual form of thought was presumably the other. Though not expressed in those terms, abstract thinking was given as a higher and more adequate form of thinking and observable as a kind of “mental mobility,” (ibid, p. 42).

This mental mobility required a child to see beyond the immediacy of the concrete materials while thinking with them.

The “considerable and extensive manipulation” of physical objects was to serve as a bridge to a presumably more adequate mode of thought that no longer relied upon an interaction with the physical environment. In other words, the identification of objects with images and numbers was to move children from a concrete world tied to the specificity of things to a more general and abstract mode of reasoning. Solving problems like $2 + 2 = \square$ (shown above), entailed the use of materials for children to model what is known to find out what is not. This mode of thought was characterized as “a way of arranging knowns and unknowns in equations so that the unknowns are made knowable,” (Bruner, Goodnow, & Austin, 1956, p.7). The unknown could now be seen so as to *understand* (i.e. master) the known.

This way of thinking about knowns and unknowns related the manipulation of set objects in the curriculum to a cultural way of reasoning about the systematic planning of the future. In *Excellence: Can we be Equal and Excellent too?* (Gardner, 1961) a concern was expressed that “we don’t even know what skills may be needed in the years ahead. That is why we must train our ablest young men and women in the fundamental fields of knowledge, and equip them to understand and cope with change. That is why we must give them the critical qualities of mind and the durable qualities of character that will serve them in circumstances we cannot even predict,” (Gardner, 1961, p. 53). In this statement about planning for an unknown future, it was known that science and math were “fundamental fields of knowledge”. More than that, the qualities

of mind and character were part of the curriculum, as traits given to children as open to the unknown and methodical in their problem solving.

Inscribed in the use of materials as a method of problem solving, these traits that children were to internalize were assumed to be useful in solving future and unforeseeable problems. As variations on the same set of rules, the materials were to aid children in methodically modeling what could not be known or seen with things that could. In this way, the goal was to take thought beyond the immediacy of objects where there is “more than reality involved, since the world of the possible becomes available for construction and since thought become free from the world. Mathematical creativity is an illustration of this new power,” (Piaget, 1950, p. 151). This form of “mathematical creativity” was to incite thinking about equality and equivalence that was not tied to the immediate and concrete world. This apparently allowed children freedom to think independently. While not immediately present in every child, this independent and abstract thought was presumed possible to develop, just as creativity was.

In thinking independently, finding their ‘own’ methods, and freeing their thought, children could not use the materials however they wanted. Like the use of language and acts of invention involved in understanding equivalence, the materials involved in solving expressions with the equal sign = and numeric symbols entailed some rules. These rules were mathematical in terms of organizing modes of thinking for solving problems but they were also cultural with regard to how they ascribed identities to children. Using the materials seemed to require a “thoughtful” and “meaningful” manipulation on the part of the child (MSG, 1966d, p. 2). Meaningful and thoughtful

manipulation was not using the materials however children pleased. Their use could be understood in relation to the rule that, “with the meaningful manipulation of concrete objects, children become active participants and not merely passive watchers or listeners,” (ibid).

The use of materials in solving problems with the equal sign $=$ was to serve as a method for moving from concrete to more mobile forms of thinking and finding unknowns from knowns. In this reorganization of the curriculum around notions of an abstract and mathematically creative mode of thought, the equal sign embodied notions of equivalence and equality that the child was to understand (through the use of materials). Their use, interestingly, was organized according to a logic that operated through psychology to classify certain kinds of thinking as higher, more adequate, and aligned with the creativity that assumed cultural importance in the context of learning as mastery. More than styles of mathematical thinking, these classifications were embedded in expressions of the equal sign gave visibility to children’s unknown and internal thought processes as knowable, flexible, abstract, and independent.

Reversibility of Thought and Modeling the Unknown

At the end of the first year of instruction in the School Mathematics Study Group texts, children were held to a new standard of thinking implied in problems of a different kind. The child was to undo, or mentally reverse addition and subtraction (see figure 3.4).

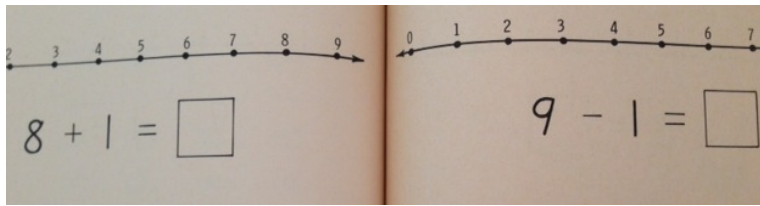


Figure 3.4. SMSG, 1966c, p. 206-207

Reversing the addition with subtraction entailed an application of the underlying rules of the logic of equality. That is, the child would need to recognize a distinction between 8, 1, and 9 as members of sets that could be joined together to form relationships. No longer relying on objects and images, the reversal, expressed here as subtraction, required what were given as “more conceptually adequate modes of thought,” (Inhelder quoted in Bruner, 1960, p. 38) as the expression of higher form of reasoning. More than a math problem, this reversal of thinking represented the trajectory toward the outer limit points of human thought. That is, it modeled and was modeled by what were given as the highest levels of the thinking process – mathematical creativity and abstraction.

Granted as a psychological truth, the abstraction of abstract thinking was given a ‘scientific’ meaning in reference to the “reversibility of thought,” (Piaget, 1950, p. 42). The undoing of mental processes, or reversing “intellectual operations” as expressed in the above math problem, was taken to represent the highest form of thinking as an observable practice. To give another general illustration, dividing a collection of objects into four equal piles then combining them together again to find the original quantity stood for a child’s reversal of thought. However, still tied to the immediate and concrete world, this way of thinking was presumed difficult as “the young child perceives changes as operating in one direction without being able to grasp the idea that certain

fundamental features of things remain constant over change, or that if they change the change is reversible,” (Inhelder quoted by Bruner, 1960, p. 41). In other words, children do not *naturally* reverse their thinking or see the world beyond an immediate ‘reality’.

Yet, as a psychological characteristic of human thinking, reversible and abstract forms of reasoning could apparently be developed even if they weren’t ‘natural’ for children.

In this development of abstract thought, it was not simply the tools and language of psychology that were of service to mathematics in forming math education in the New Curriculum. This relationship seemed to work both ways. More specifically, a symbolic logic gave visibility to what was presumed as the highest level of thinking, a reversible thought process:

$$y - x = x' \text{ or } y - x' = x \text{ (Piaget, 1950, p. 42).}$$

Representing reversibility, the expression embodied a generalized pattern of the development of abstract and logical reasoning that seemed to repeat over time and in all people. In making apparent an otherwise unseen thought process, the equation provided rules for how to organize the curriculum so as to bring the underlying structures of mathematics “within the intellectual reach of children of all ages,” (Bruner, 1960, p. 32). The problem solving like that in figure 3.4, then, followed the model of an internal thought process and a general pattern of reversible styles of reasoning.

As a model of a common and systematic thinking process, the logic embodied in the equal sign organized a rule of reasoning whereby children’s thinking seemed to naturally move toward a higher form of thought that was to be seen as abstract and mathematically creative in its reversibility. As a fundamental part of the internal thinking

“mechanism”, this process was presumed to not yet to function at this highest, most perfect form in young children (Piaget, 1950). Regardless, in mapping this pattern of thought, the meeting of mathematics and psychological principles of creativity that presumed access to a universally understandable structure given to mathematics would organize the curriculum and children’s thinking on a continuum from lower to higher forms.

While the equation giving expression to the reversibility of thought can be seen as a familiar pattern of problem solving in the curriculum, it was not just about helping children learn to think at higher levels. This model of abstract reasoning also worked as a psychological tool deemed useful in ‘scientifically’ establishing equivalences between children, grouped together by common models of thought. With children apparently having the same internal and innate structure underlying the development of their process of thinking, it seemed reasonable that all children could master the foundations of modern mathematics and function according to the same operations and systems of reasoning. Following the same linear sequence of mathematical practices and rules in the curriculum, understanding would presumably develop toward reversibility.

Interestingly, as Piaget’s work entered the United States it not only raised a discussion about *when* children could learn certain things and perform higher mental operations. It also evoked questions about *how* that learning could happen more effectively and efficiently (Resnick & Ford, 1981; Bruner, 1960). This question of efficiency as tied to the transportation and translation of Piaget’s studies in the United States can be related to the notions of “intelligent and effective” citizens entangling with

math reforms. Understood within a broader cultural frame, this concern for a more systematic and effective progression through the levels of thought could be historically related to the emergence of a shared language with computer programming and cybernetics. Exploring systems and their organizational logics, engineers and mathematicians translated symbolic models into technical tools for calculating unknowns. This computing machine, and others like it, were programmed to solve problems effectively and efficiently.

These qualities were given in a mode of thinking that organized equivalence between mathematical logic and human problem solving (represented in the expression of reversibility). The linear process of computing unknowns, expressed within a symbolic logic, was taken as a model resembling how humans solved problems (Newell, Shaw, & Simon, 1958; Simon & Newell, 1970). While linear equations represented by a symbolic logic translated the work of the computer's "mind" into a language intelligible to humans, they also seemed to identify how the human mind might function as a systematic, orderly, and methodical computing machine.

To this point, I have examined how patterns of reasoning about children and the organization of curriculum are embodied in the use of the equal sign. These patterns did not only inscribe rules for understanding mathematical notions of *equivalence* and *equality*. They also carried a cultural way of reasoning about the identity and equivalences of children, as a group of creative thinkers that could be included into *all students*. Given their interaction with research and development in the fields of mathematics and a psychology that presumed a trajectory of thought toward mastery as

understanding, the problems children were to solve also worked to order their thinking as reversible, flexible, innovative and efficient. More than traits that organized creative mathematical thinking, these were qualities that would identify children as mathematical citizens of the nation.

The cultural logic of New Math, however, was not isolated to the curriculum and educational research. Therefore, to understand how the imperative to reform elementary mathematics was historically tied to the making of citizenship, the New Math curriculum must be placed within a broader “grid of intelligibility,” (Foucault, 1971). Next, I consider how the logic of equivalence and equality that organized the New Curriculum assembled with patterns of thought about how to technically reorganize society.

Planning the Future by Planning Citizens

Looking beyond Sputnik or the Cold War as the ‘reason’ for the reforms, I attend to the intersecting historical trajectories that came together to formulate new ways of thinking about the relationship between children, their process of learning mathematics, and the progress of society. Generally, I look at how the promises of science and mathematics overlapped with the importance granted to the social sciences as a mode of planning the modern life. Tied with the rising notion of ‘expertise’, the consensus given to the social sciences seemed to provide a way of ordering the kinds of people who would contribute to the idea of a united and democratic nation. Together, these conditions made it possible for the cultural narratives about citizenship to entangle with the remaking of math education in the post war period.

Embedded in the relation between math and the production of the modern society is a revisioning of what historian David Nye (1994) calls the “technological sublime”. The “objects invested with sublimity”, such as railroads and skyscrapers, linked the nation’s “ideas about itself” and its future to the production of what was taken as progress (p. xiv). In the United States, both during and after the war, the reason and rationality ascribed to the *technological* was reimagined with mathematics as a useful tool in planning the modern life. In this historical link between technology, national identity, and modern society, both science and mathematics (as the language of science) assembled with the narrative of how the nation would progress.

This hope invested in the rationality of scientific and mathematical thinking is visible in how calculations were supposed to produce solutions to technical problems. The Electronic Numerical Integrator and Computer (ENIAC) appeared during the war embodied a way of thinking about the practicality of mathematics for effectively solving problems. Its main function was to decode algebraic expressions to solve for unknown variables of wind resistance, distance, and speed thereby calculating artillery-firing tables for the U.S. Army. In their use, mathematical models were to plan future trajectories by finding and solving for the unknown. ‘Thinking’ logically and flexibly to compute an answer was seen as a way to produce a secure nation and a better life for citizens.

The logic given to math was deemed useful in solving not only physical problems – like the firing of missiles – but social ones as well. In a seemingly practical application of math for modeling social phenomenon, it was assumed that “measurement in

psychology and physics are in no sense different. Physicists can measure when they can find the operations by which they may meet the necessary criteria; psychologists can do the same,” (Reese, 1943, p. 49). This was visible in the use of the expression $y - x = x'$ or $y - x' = x$ to psychologically model the thought process in its reversible, flexible, and highest form of expression.

The uses of science and math in modeling society was linked to the belief that the ‘social’ operated as a system of variables that could be measured, predicted, and controlled. As mentioned earlier, this was given expression in the use of psychological scales to quantify democratic, open minded, and independent types of people. Beyond psychological factors, scientific research and mathematical models were deemed to be useful in organizing the economic development of society. Related to the issue of economic development, theory about investment assembled with post war ideals about building the Great Society and fighting a War on Poverty. These were not simply ideas about how to improve the standard of living but were linked to practices of using scientific research and mathematical modeling to plan society.

In this formulation, the social sciences were positioned as the “policy sciences” (Lasswell & Lerner, 1951), a characteristic of its 19th century formation but given a more public expression in the post war years. Sociologist Daniel Bell (1962) discussed the public beliefs and consensus about the role of social science in developing policies that would technically produce a democratic society. By arguing an “end of ideology”, Bell explored notions of societal change from the assumption that society’s complexity required scientific studies at a systematic level. New ways of organizing “the framework

of freedom, to increase living standards of the majority of people and at the same time maintain or raise cultural levels,” according to Bell, was tied to understanding society, as a cohesive organization of social groups (p. 38). This commonsense can be related to practices of developing social and educational policies based on a distinction given to certain kinds of people that did not seem to meet the “living standards” or “cultural levels” of expected societal participation.

Within the goals of economic development and raising the standard of living, the policy sciences undoubtedly contributed to planning the reduction of poverty.

Represented as a social issue that could be addressed through the welfare state as a planned and strategic intervention, poverty required a group with which the social condition could be identified. Once identified, the management of funding was to organize equal opportunities and access for those not living up to the cultural standard. In order to move society forward, toward equality and prosperity for all, it seemed that particular kinds of people required an additional investment, particularly in their education. Policies, organizations, and programs, such as the Department of Health, Education, and Welfare (1953), the Elementary and Secondary Education Act (1965) and Head Start (196) focused on the development of certain groups that were seen as not contributing to the progress of society.

Under the umbrella of making investments in *human capital* these reforms and policies could be rationalized as part of the process by which society would build abilities, competencies, and skills in areas considered to be useful knowledge (Becker, 1964). Human capital, or potential for earning, was taken as a factor of health, wealth,

and education – all measures that could be managed. As the calculation of a standard of living, these skills and abilities were linked to the economic development of society and were not to be left to chance – particularly in the face of poverty. While skills and abilities could be measured on an individual level, an emphasis on collective belonging and social cohesion would make way for the psychologies to expand and provide models, not of individual behavior, but of common thought processes that could be used to understand patterns of behavior in society.

More specifically, the scientific study of character was taken up by psychology with the goal of understanding the kinds of people who would contribute to and participate in the social collective of the nation (Cohen-Cole, 2009). From this perspective, where social sciences intersected with policy, psychological research was “perceived as a vehicle that will assist in bringing about the American Creed of equality, fair play, and minimal group conflict,” (Darley, 1952, p. 719). Embodied in this statement is the assumption that social concerns, such as coherence, fairness, and justice should be managed within the technical model of understanding the social. Here, psychology was to provide ways to understand problems, identify their causes, and manage a solution by identifying the kinds of citizens, as exemplary Americans, who would “bring coherence to America’s increasingly complex and diverse culture,” (Cohen-Cole, 2009, p. 220). Identifying traits of citizenship entailed giving meaning to the forms of character and modes of living that would move the nation toward democratic ideals of equality and prosperity for all. In relation to a national identity defined in terms of modern,

technological, and progressive, citizenship was given equivalence with notions of freedom of thought, logical reasoning, and creativity.

To return to the rationality given to math in planning the future of the nation, it seemed that the practices of grouping distinct things together and giving common identities to establish collective belonging embodied a logic of equality and equivalence – made visible as traits of the mathematical citizen in the New Curriculum. As a member of the set of all people belonging to the nation, the citizen could plan and be planned. With daily modes of living ordered by practices of calculating the unknown and applying modes of abstract reasoning to solve future problems, the child was identified in the curriculum as the image of rationality and progress, internalizing not only mathematical competencies but also civic virtues for living as the mathematical citizen.

However, while the elementary math curriculum ordered the competences and traits identifying children as an equal part of the *all*. Organized by a logic of equivalence and equality, it also made visible the differences by which to divide them. Next, I explore how cultural theses of in/equality were enacted in the emergence of new ways of classifying the children who did not belong to the all.

New Math and the Cultural Construction of In/Equality

“If all students are helped to the full utilization of their intellectual powers, we will have a better chance of surviving as a democracy in an age of enormous technological and social complexity,” (Bruner, 1960, p. 10).

The application of scientific and mathematical reasoning discussed above embodied a way of thinking that not only determined what constituted a social problem

but also how to organize a solution. Within this, math education reform at the elementary level was aimed at expanding modern mathematics to students at “all levels, from the kindergarten through the graduate school,” (MSG, 1966a, n.p.). More than that, New Math was situated within social and cultural politics concerned with planning a more egalitarian society in which all citizens had the opportunity to learn and live the modern life.

In post war United States, educational opportunity was increasingly considered integral to assuring all Americans the chance to better their life. *Brown vs. The Board of Education* invoked the public conception of the importance of equal educational opportunity as well as education as the right of a free citizenry. This consensus on the role of education together with the belief in public welfare paved the way for a federal policy aimed at reorganizing education with opportunity and equality as a central concern. Situated within the War on Poverty and the Great Society social reforms in the United States, the Elementary and Secondary Education Act (ESEA) was formulated as a public law in 1965. With a primary focus of granting educational rights, access, and opportunity, the concern was in “meeting the special educational needs of educationally deprived children,” (ESEA, 1965, p. 27). Ordered by a particular way of reasoning about the right to an equal and quality education for all students, the law embodied cultural theses about who was not equal. The “educationally deprived” seemed to express a new kind of social problem of school success and human capital – whereby an economic investment could solve the ‘problem’ of poverty and the educational inequalities associated with its public expression.

On the surface, the policy solution to deprivation rested in creating the ‘right’ learning environments by reallocating funding, resources, and materials so as to reduce inequality in terms of economic access and opportunity. However, as a cultural distinction, not just an economic one, the inequalities associated with “culturally” and “educationally deprived children” did not simply rest in the environment but were placed in the child. Educational success for ‘these’ children seemed to require an arrest and reversal of “the course of intellectual retardation in the culturally deprived” (Ausubel, 1963, p. 464). The curriculum was one of many techniques necessary to “providing him with an optimal learning environment as early as possible in the course of his educational career,” (ibid).

This was made explicit in the introduction to a revised edition of an SMSG 1st year text wherein “culturally deprived children” emerged as a distinct group with whom the curriculum was concerned (SMSG, 1966a, preface). The preface of this revision articulated the hope that the new edition would better meet the needs of the “disadvantaged” and “deprived”. Embedded in this hope was a fear of these children – as a group who seemed to lack the ‘mathematical’ competences to participate in school and society. In the curriculum, this distinction operated to give the “disadvantaged” psychological traits, expressed as different forms of rationality and participation.

To see this, however, one needs to look outside of the math textbooks to cultural texts concerned with understanding “deprived children”. An article entitled, “The Overlooked Positives of Disadvantaged Groups,” articulates the “disadvantaged” as a group understood according to the “mental style or way of thinking characteristics of

these people,” (Riessman, 1964, p. 225). Given psychological distinctions within a logic operating to give them equal educational access and opportunities, this group could presumably be understood in reference to several defining qualities. One distinction granted that, “slowness is an important feature of their mental style,” (ibid). The slowness with which deprived children apparently performed intellectual tasks could be understood in contrast to the child whose “mental style” and thought process was efficient and methodical.

Slowness was attributed to this child representing a “physical style of learning” and a thought process that “persists in one line and is not flexible or broad” (ibid, p. 226). Both of these characteristics of deprived children are given in reference to children who think beyond the physical realm, in the abstract, as well as those who are able to reverse their thinking. While the physical style was given attention in the curriculum through the use of physical materials in solving for equality and understanding equivalence, the presumption was that this was a method to progress a ‘natural’ trajectory of thought toward abstraction – a mode of reasoning the “deprived” seemed to lack. Distinguished from abstract, flexible, and efficient styles of thought inscribed as the standard in the curriculum, thinking tied to the physical and set on one track would seem to classify this group of children as different and excluded.

Interestingly, however, the “single-minded” quality was also articulated in reference to notions of creativity. Like flexibility of thought, open-mindedness, and abstract thinking, one track thinking seemed to offer a considerable potential for creativity (ibid). This creativity, however, was of a different kind. In fact, “the mental style

of the socially and economically disadvantaged learners resembles the mental style of *one type* of highly creative persons. Our schools should provide for the development of these unique, untapped national sources of creativity,” (ibid, p. 231). This different form of creativity was articulated in relation to other features identifying the thought processes of those considered “deprived”. In *The Culturally Deprived Child* (Riessman, 1962) these modes of thinking were characterized as: problem-centered rather than abstract-centered approach; the freedom from being word-bound; the equalitarianism in informality; the avoidance of competitiveness; emphasis on collaboration over individualism; a spatial rather than temporal perspective (Riessman, 1962).

Among others, these traits and the creativity they presumed were still framed as a deficit compared to the other, mathematical modes of creative thinking, even in their “positive” orientation. That is to say, the child who was not correctly and independently using the physical materials, following the rules bound to language use, formally seeing equivalence and equality in the images and set objects, developing an abstract and higher order of thinking, and using efficient problem solving methods would not develop the ‘kind’ of creativity ascribed to the mathematical citizen in the New Math curriculum. Creativity served as to psychologically order ‘these’ children as belonging to, yet distinct from, the group of all students within the logic of equalizing their access to and participation in school math.

Here, equality and equal educational opportunities were to be granted on the condition of all children internalizing the cultural norms of the ‘right’ kind of creative, flexible, abstract, methodical, and efficient thought processes. These children, as

citizens who participated in school and society according to these rules, would always already have the right and opportunity to educational e/quality outlined in the policies and curriculum. On one hand, the logic of the policies and interventions aimed at the culturally and educationally deprived embodied a goal of improving the unequal conditions of schooling for certain groups. On the other, the deprived and disadvantaged child was identified in relation to the child who embodied the traits of the intelligent and mathematical citizen. The standard of comparison for *all* served to categorize the children who were included as part of the all while simultaneously distinguishing the non-equivalent group for which the right to an equal and quality education were named.

The cultural norms of difference that distinguish children as certain kinds relates to the reformation of the math curriculum. On the surface, this is clear in how the notion of difference operated to categorize children that did not seem to have equal access to math education. Below the surface, however, the relationship between the representation of difference and the New Math curriculum is not as obvious.

Organized by and organizing particular standards for thinking about what is equivalent, what is equal, and, in that, what is different, the logic of equality embodied in the equal sign ($=$) resembles the cultural rules of identity and difference ascribed to children. That is, the rules for thinking about how to group children together, as members of an inclusive set, were similar to the standards for thinking about how to establish equivalence between set objects in mathematics. Identified by common factors, both the children and the classroom materials could be considered equal on the

basis of similarities. For the set objects, these similarities were represented as a numerical identity. Children, in addition to being seen as numbers, were identified by a presumed common capacity to think creatively, abstractly, and flexibly. This way of reasoning about the equivalences between children provided a way to think about how they were to be considered part of *all students, at all levels*, or not.

Progress, Prosperity, and Equality = Children's Mathematics

Presently, it seems so obvious that children's mathematical reasoning is linked to their participation in society. What may be less apparent, however, is how this relationship historically assembled to produce new ways of seeing a connection between children's math, notions of citizenship, and progress of the nation. The New Math reforms that mobilized in the post war period in the United States have provided a compelling site to explore the emergence of this relationship. Not least because of how cultural narratives about intelligent and effective citizens entangled with the goals of the New Curriculum in producing mathematical competencies for the modern life.

Drawing attention to how the elementary math curriculum was about much more than competencies in New Mathematics, I have explored how its reorganization worked to make children carriers of the modern life – inscribed in the rules of creativity as a cultural standard by which to live this life. Intersecting with norms of representation, given as tools of psychology, the equal sign carried cultural rules organizing how children were to participate as mathematically creative kinds of citizens, involved in developing the future of the nation. As creative, independent, abstract, and flexible yet

methodical in their thinking, children and their process for learning mathematics were taken as integral to the progress of a post war, democratic nation that embraced a sense of unity in its diversity and planning for the unknown. Clearly, this was not just about learning mathematics.

Emerging as the image of rationality and progress, children were to internalize the psychological traits as norms of citizenship and rules for belonging as part of *all*. Yet, in determining the standards of inclusion into the *all*, the very rules of equality and equivalence that organized who was the mathematical citizen also classified who was not that – distinguishing characteristics of “deprived” and “disadvantaged” children who seemed to embody different forms of creativity as the expression of inequality.

As the investment in children’s math education continued to assemble with social commitments to provide equal opportunities for *all*, descriptions and calculations of student (under)achievement, inequality, and incompetence emerged. With these concerns, the New Math Curriculum became the target of pointed criticism in the early 1970s. In the Back to Basics period of reform that followed, failure in mathematics was not historically new. What was new were the ways in which it would be articulated as a problem to solve at the individual level. In this, elementary math education seemed to be disarticulated from notions of citizenship. Though, without the historical entanglement of mathematics education and citizenship, the emergence of the *mathematical self* as the basic factor of progress and prosperity would not have been possible. This emergence is the focus of the next chapter.

CHAPTER 4

Back to the Basics: Inventing the Mathematical Self

In a May 1969 issue of the *Arithmetic Teacher* a new footer appeared:

“Excellence in Mathematics Education – For All” (p. 421). This call for excellent math in all schools for all children remained on each page thereafter. As if reminding the reader of the ways in which mathematics education had once again become an issue of educational reform, *excellence for all* seemed to demand something new.

The appearance of this message coincided with rising concerns about the previous New Mathematics curriculum that began to emerge around the 1970s. In some ways, this reorganization of mathematics was thought of as a transition away from New Math toward basic math skills (Kline, 1973). In other ways, this particular period is thought to blend into New Math, as part of what historically came before the now so familiar Standards-based reforms (Hiebert, 1997). So, generally speaking the Back to Basics reform does not get much individual attention in the history of math education. Yet, it served as an important event in the history of working to organize math for *all* children.

Here, I continue to examine how the *logic of equality* that is embedded in the notion of *mathematics for all* organized new ways to see and think about children. Looking at how the mode of establishing a cultural norm of *all* changed after the New Math curriculum, this chapter focuses on the Back to Basics reform of math education that occurred primarily during the 1970s in the United States. As in the previous chapter,

the inscriptions of identity and difference are explored as a cultural set of principles through the use of the equal sign (=) in the curriculum.

On one level, I see the equal sign as a fundamental element of the skills-based curriculum that emerged in this period. As a symbol of mathematical logic, the equal sign ordered how children were to think of fundamental arithmetic relationships of equivalence and non-equivalence. On another level, I examine the use of the equal sign in the curriculum as embodying a cultural way of reasoning about equivalences and differences between children.

In relation to Chapter 3, this chapter highlights how the inscriptions of identity and difference re-signify the use of the equal sign in the reorganization of math curriculum by representing the child as a *mathematical self* – a child whose traits of individuality and sense of self (in relation to the all) are inscribed in and through the Basics curriculum. Here, the distinctions to which the equal sign relates to represent cultural identities and differences shift from notions of citizenship to traits of individuality. In this, the practices of math pedagogy assembled with psychological norms as tools with which to identify the child's sense of mathematics within the language of self-motivation and discovery.

Attending to the Basics in Math: The Individual, Intuition, and Interest

"We do need to get back to the basics, but it is essential that we first identify the basics we want to get back to," (W. Ross Winterowd quoted in Brodinsky, 1977, p. 523).

While the Back to Basics movement may be considered a move away from what came before in the New Math curriculum, it also expressed important continuities in

terms of a fundamental link between mathematics education and the (re)organization of social and cultural life. What characterized the Basics movement, then, were new modes of organizing this life – not only as a citizen, unified by a collective notion of belonging, but also as a unique and distinct individual. In this section, I will briefly lay out some of the distinguishing features of the Back to Basics movement in mathematics education, particularly as they embodied cultural notions of individual interests and intuition as elements of a “natural motivation” to learn math.

In reorganizing mathematics for all, this shift to make math about fundamental skills required a determination of the *basics* to get back to. No longer simply about making intelligent citizens of the nation, mathematics education was related to “the development of life (or survival) skills – that is, competencies needed for personal growth and for successful existence as a citizen, consumer, jobholder, taxpayer, and member of a family,” (Brodinsky, 1977, p. 524). Implicit in this statement of math skills as integral to life skills is a double layer of *competencies* whereby knowledge learned in school was to be related to one’s participation in society as an individual – given multiple forms of expression and responsibility. This individual, responsible for personal survival and growth, would stand as the standard of excellence, achievement, and opportunity.

So, basic skills in mathematics were related to essential skills of *personal* survival and growth. Thus, the rules that identified and inscribed the “basics to get back to” also gave meaning to how to survive and grow in this life and what abilities that required.

The so-called failures of New Math – given as math’s abstractness, disinterest, and inward turn – brought with them ideals of what would constitute the basic education for life in a progressive and democratic society. In grappling with *Why Johnny Can’t Add: The Failure of New Math*, Kline (1973) suggested that the emphasis on structure and abstraction was the reason, not only for Johnny’s⁶ failure but also for math’s inadequate contribution to the development of science and society. More specifically, while the modes of abstract reasoning emphasized in the previous New Math curriculum seemed to “destroy the spirit and life”, a more “natural motivation” could be found in the study of problems that would “revivify math by the air of reality,” (ibid, p. 149). In terms of curriculum redesign, math in its most basic form was to be motivating and interesting – not abstract or removed from what constituted the problems of real and everyday life.

In terms of curriculum redesign, the appeal to notions of application and interest as a form of *natural motivation* was to be understood in relation to the idea that “the basic approach to all new subject matter at all levels should be intuitive,” (ibid, p. 157). Together, the notions of interest and intuition seemed to provide the means by which individuals could be motivated to excel in math. If school math was more interesting and connected to who children are and how they think, then they would presumably want to learn it.

This articulation of basic math as inherent modes of reasoning, instinct, and awareness shaped the terms by which all children would be seen to have equal

⁶ Johnny is the name used by Klein (1973). I take the use of his name to represent the emerging impulse to situate mathematics education within each child, as a distinct individual. Throughout, I use the names taken from the original source documents. Although seeming to represent a particular child, any of the names could be substituted in the narrative without changing the way in which notions of ‘mathematical’ individuality and motivation were determined as cultural norms of identity with which to classify difference.

opportunities to learn math. Supported by research concluding that even “infants have ample opportunity to learn about number, repetition, regularity, differences in magnitude, equivalence, causality, and correlation,” (Ginsburg, 1977, p. 30) notions of mathematical intuition would serve to create equivalences between all children, whose learning was motivated by what were taken as natural mathematical insights and skills.

However, the assumptions of a natural intuition and motivation that were to organize the math curriculum and provide equal opportunities for success were not biological distinctions. This distinction between nature and culture is expressed as unproblematic in the following discussion of the math curriculum,

“it is hard to see how any child, rich or poor, Western or non-Western can grow up in an environment which does not offer him or her the natural opportunity to learn about basic aspects of quantity. In this sense, environments are all similar and quantity is universally available. We shall see later that there is a sense in which *all children take advantage of this natural opportunity*,” (ibid, p. 31, my italics).

Although expressed as “natural opportunities”, the opportunities for children to “take advantage of” are not simply natural. It is a cultural standard that establishes a sort of equivalence between all children – represented as having the same opportunities.

What those opportunities were and how to take advantage of them would have implications in how the idea of a basic, intuitive, and motivating mathematics was to be made available to *all children*.

While it was possible to think that all children could take advantage of the opportunity to learn basic math, it would become clear how some children did not adequately do so. When Kenneth, a child interviewed to better understand his

“challenges” with math, “interpreted the + and = in terms of actions to be performed,” his interpretation was seen as one that “can lead to trouble:

I: How would you read this [$\square = 3 + 4$]

K: ...Blank equals 3 plus 4.

I: O.K. What can you say about that, anything?

K: It's backwards! [He changed it to $4 + 3 = \square$] You can't go, 7 equals 3 plus 4.”
(ibid, p. 84).

Kenneth's opportunity to “see” the relation of equality was missed by an apparently limited, or backwards, logic. That is, he should have known that $\square = 3 + 4$ is the same as $4 + 3 = \square$. It is Kenneth's interpretation, then, that is taken as backwards. Expressing a truth of mathematical equality, the equal sign and its terms create a relationship between 4, 3, and $\square$ that can be ordered only in particular ways. In not seeing equality as a relationship of like terms, Kenneth's interpretation was deemed different, outside of the norm. More than an expression of a basic math skill, the inscription of equality above also provided a way of organizing a personal and individualized form of understanding – different from everyone else's. As a way to understand how Kenneth sees, thinks about, and communicates “his” mathematical understanding, the logic of equivalence and difference embedded in the equal sign distinguishes Kenneth's interpretations and opportunity to learn from ‘others’.

Johnny, who has failed and been failed by New Math, was not the same as Kenneth. Each was presumed individuality in reference to cultural norms of interest, motivation, and intuition that established equivalence as well as difference between them. Similar to how the terms of equality could be reordered and seen as identical

($\square = 3 + 4$ to $4 + 3 = \square$), Kenneth and Johnny could be seen as distinct individuals – yet equal in their differences.

This signification of individuality importantly relates the organizing logic of math in schools to the notion of equality underlying excellence in math for all. This occurs through the psychology that defines the child's unique traits and differences as necessary to understand as a way to think about producing equal opportunities for every one to learn math. But, this individuality could not be understood without reference to some norm that represents an imagined *all*.

The above analysis of the equal sign is no mere example. It provides a way to examine how cultural principles of identity and difference in what appears as mathematics in the curriculum is more than mathematics – ordering the thoughts and actions of the child as a 'naturally' but differently motivated individual, capable of learning Basic mathematics. This ordering is visible at the intersection of research in the psychology of individual math learning and basic math skills that serve as the foundation of the elementary math curriculum. In the next section, I explore this intersection as a way to understand how a particular style of reasoning about equality, equivalence, and difference represents children as individuals – distinct in their abilities, skills, and interests.

Motivated to Learn Mathematics: Equalizing Opportunity and Access

With a “natural motivation” to learn basic math skills linked to children's interests, intuitions, and individuality, it seemed necessary to determine how to produce, maintain, and differentiate this motivation. Particularly in the context of *excellence in mathematics*

education – for all, the focus on motivation to learn math assembled with a concern for equality of opportunity. So, while equal opportunities for all children to learn math could not technically be guaranteed, this equality was thought to be measureable as the effect of an “equality of optimum motivation,” (Nicholls, 1979). In fact, it seemed fairly straightforward that, “we can say that someone is not developing his or her potential in, for example, mathematics, if that person is not optimally motivated to learn mathematics,” (ibid, p. 1072). With motivation as a factor of success in school mathematics, it would make sense for the curriculum to be organized to enhance the motivation of each individual student.

As a historical classification, motivation was deemed useful in the language of psychology to distinguish children who were/were not achieving (Danziger, 1997). The invention of this distinction was to give finer ‘scientific’ classifications to understand and talk about differences between children. At the same time, motivation has become a category by which to establish equivalence between *all* children, as equally capable of being ‘motivated’. During the Back to Basics movement, it was this cultural notion of identity that was given as a way to establish a standard of equivalence by which to make sense of how all children could excel in mathematics.

Though it would seem that *all* children are or can be motivated to learn mathematics, it would become clear that some children could not excel. This was emphasized in *The Psychology of Mathematical Abilities in Schoolchildren* (Kilpatrick & Wirszup, 1976). Stating, “even with perfect teaching, *individual differences in mathematical abilities* will always occur – some will be more able, others less. Equality

will never be achieved in this respect,” (ibid, p. 6-7). Yet, this was not the end of hope for equal access and opportunity for all to excel in mathematics. Together with the investigation of individual differences in interest and abilities in math education, it was thought that equal opportunities could be produced as an effect of equalizing children’s motivations.

Self Discovery: Starting with the Child

A common sense idea organizing the Basics curriculum was that meaningful and motivating mathematics started with *each child*. By starting with the individual, rather than the mathematics, pedagogy was to “encourage young children to respond to situations using their own ideas, language patterns,” (Price Troutman, 1973, p. 427). This way of thinking about the child as an individual, defined by his/her “own” ideas and language, required a way of thinking about this individuality and what its expression entailed.

This can be seen in a discussion of “Child-created Mathematics” (Cochran, Barson, & Davis, 1970). In solving the open sentence $\square - 3 = \Delta$, a child named Leslie chose numbers so as to create equivalence between $\square - 3$ and Δ on either side of the equal sign. This required her to substitute the $\square$ and Δ with chosen numbers so as to identify the unknowns with known quantities. Then, to represent this equivalence as a comparison between the two quantities, Leslie produced a graph by translating the numbers and shapes to points on a grid. Through her ‘creation’ (selecting numbers and plotting points), the shapes were given identity – representable and fixed as both a quantity and a point in space.

The openness of the expression provided more than an opportunity for this child to create a mathematical understanding of equality as a relationship of like/unlike terms. In producing this graph, the act of identifying the mathematical objects that gave meaning to the equal sign – open symbols, numbers, point on a graph – represented a way for Leslie to create an understanding of her self and the choices she was to make as an individual. Apparently not satisfied with the way her graph of $\square - 3 = \Delta$ looked, she expressed that she wanted her graph to be more “straight up”. Rather than correct her language use (by introducing the term “vertical”) the teacher was to provide Leslie with the opportunity to follow her hunch that the $\square$ and Δ were related and were not the same. In this, the child appeared to make choices about the language and task direction while also discovering that changing the $\square$ would change the Δ . This discovery was to leave her feeling more satisfied with and motivated by mathematics.

On one level, developing basic math skills was associated with the ability to identify the shapes as distinct numbers and create a relationship of mathematical equality. On another level, the mathematics intersected with cultural expressions of individual identity. Given in psychological terms, children’s interests, abilities, and feelings were categories deemed useful in motivating each individual child given their differences. This logic of cultural distinction assembled with the math. Related by the equal sign ($=$), to distinguish $\square$ and Δ while at the same time making them equivalent it is necessary to identify the terms that hold their differences constant. In the case of the math equation, the 3 served to differentiate as well as identify the $\square$ and Δ . In a similar

way, a term of comparison must identify Leslie (or any other child) as different from, yet related to, another.

Defining terms of equivalence between children, psychological research emerged examining how individuals thought about and learned mathematics differently. The focus on differential abilities invited new ways to distinguish between “the gifted or less able child,” (Inskeep, 1970, p. 195). Within this range, “mathematical precociousness” provided a psychological category to identify the child presumed to be mathematically talented. The classification and the traits attributed to this child were articulated in the language of interests, values, and personality. Further, the “mathematically precocious” or “gifted” child was characterized as a person who, “liked finding out things, discovering things, and learning things,” (Stanley, Keating, & Fox, 1974). S/he also appeared to have a positive attitude toward and a general “likeness of math”. Together these traits served to explain student success in school math. More than that, they provided a way for children to “see themselves” in their math classes. This seeing, however, was more than a reflection of who the child ‘naturally’ was. It was an invention of a particular kind of self – motivated by self-discovery and general feelings of personal satisfaction.

Creating her own mathematics was to be seen as an opportunity for Leslie to discover and enact what were given as “her” mathematical interests and intuitions. Not only that, it was to provide her with a way to feel personally fulfilled and express an unbound sense of self as a motivated individual, capable of excelling in math. Not every child would achieve like Leslie, however. Though, since “children’s self-concept is usually bound up with their intellectual achievements,” it became increasingly part of the

common sense to identify individual strengths and weaknesses in learning math as factor impacting learning (Ginsburg, 1977, p. 130). Both were to be understood so as to provide all children with a better self-concept and an equal opportunity to be motivated to learn.

Informally interviewing children with “severe difficulties in the learning of arithmetic,” was seen as a way to “discover” strengths and weaknesses, as though they were already there. In solving $21 - 5 = 24$, a child named Bob seemed to use an incorrect written strategy. The research emphasized Bob’s self-realization that, “Oh, I think I added” (ibid, p. 134). The mathematical understandings seemed unimportant although the logic of difference and equivalence was embodied in the curriculum. The fact Bob came to his own awareness of was to be understood as Bob having an intuitive sense that he had done something wrong “formally”. Presumably, if Bob could identify when he made a mistake in translating his “informal” mathematical logic to “formal” strategies, then he could eventually find a way to establish equality between $21 - 5$ and its equivalent term. Until then, his self-awareness would motivate future success.

Organizing particular ways of thinking about identity and equivalence, the use of the equal sign is not isolated as pure mathematical reasoning. Translated through the language of psychology and connected to notions of self-expression and -discovery, the equal sign embodied a cultural way of identifying the child as motivated to learn mathematics.

This relation of mathematical logic to cultural principles of equality also functioned to produce a notion of difference. That is, certain kinds of kids were

presumed capable of creating, discovering, and expressing their 'own' mathematical ideas while others were not. This distinction was to be understood in terms of the psychological promise of "learning by discovery" (Gardner Thompson, 1973, p. 344). In solving the following series of problems:

$$\begin{aligned} 1+1 &= \\ 1+1+1 &= \\ 1+1+1+1 &= \end{aligned}$$

"Caesar didn't seem to grasp that we were going in sequence and just adding one more each time," (ibid, p. 344). In fact, this inability to intuitively "grasp" the pattern seemed to reinforce his use of what was taken as an inefficient counting strategy. In addition, "Caesar didn't seem to realize that problems like $8+1 = \square$ and $1+8 = \square$ were the same," (ibid, p. 346). Given as obvious realizations, that Caesar did not immediately discover, the problems served as a way to identify his lack of self-awareness and rationalize the need for "continued practice" of predetermined concepts. The practice of establishing equivalence between like problems provided a way to produce a distinction between Caesar and the children that did have the ability to 'realize' and internalize the mathematical rules. Again, as more than mathematical rules, the principles of identity and difference embodied in the use of the equal sign framed cultural standards by which to compare Caesar to the child who learned by discovery and was motivated by one's own sense of self-realization.

Enthusiasm, Choice, and Control: Games of Winners and Losers

“Fostering enthusiasm through child-created games” provided another mode of motivating learning, for children of all abilities (Price Troutman, 1973, p. 428). Following the assumption that “children are highly motivated to explain games clearly to others,” (Golden, 1970, p. 114) games were to provide children with the opportunity to express mathematical ideas while encouraging cooperation and collaboration. Organized according to the rule that games provided a pleasurable medium through which to develop basic mathematical skills, they also embodied rules that ordered how the child was to enact that pleasure and play.

As part of the Basic Curriculum, games were not to be enjoyed at the expense of building math skills. Rather, they were supposed to play an integral part in developing understanding. This included approaches to “Mastering the basic math facts with dice,” (Gosman, 1973). In this game, the dice and the mathematical expressions they represent are “loaded with activities to help children master *basic number facts* and *enjoy the experience*,” (ibid, p. 330). In one turn of “Multiplication-addition double trouble” children might multiply the number on the dice by 5, following the formula $5 \times \#$ on the dice = total score. The math seemed fairly straightforward and basic. Mastering basic facts required children to think about each turn as establishing a single value (score) from the dice and the given number.

Aiming to reach a combined score of 100 without going over, the child would be expected to add each value to the total that came before it giving a numerical identity to the total scores that were to be compared so as to determine the winner of the game.

On one level, solving equations like $5 \times \# \text{ on the dice} = \text{total score}$ ordered children's understanding of how to establish basic equivalencies between numbers given in multiple forms. On another level, the equal sign carried rules for how to reason about and construct basic equivalences between objects that would otherwise have no relation or basis for comparison.

Thinking about and seeing equivalence in a given number, a dice, a series of single values, and a total score arranged the game and children's thinking about how to play. But this mode of thinking was not just about mathematics – it was also about the game of making choices within a cultural system of a “game” in which self-realization was to occur. Like mastering the math skills by following a formulaic method, enjoying the experience of playing the game not to be left to chance. Instead, kids were to enjoy that the game itself was “completely under the control of the children. They must roll the dice, keep track of their scores, and decide to pass or play after each throw of the dice,” (ibid, p. 331). This required children to make choices at every turn. How they made their choices was thought to impact not only the outcome of the game, but also the extent to which they enjoyed it as a mode of living.

While all kids could presumably learn math from and enjoy the game, only some of them would win. Nevertheless, playing the game was to “guarantee participants an experience that is like the experience they would have in the real world,” (Cruickshank & Tefler, 1980, p. 77). The experiences of taking turns, calculating outcomes, keeping score, and deciding what to do next were to provide all children with opportunities to “make decisions and live with the consequences,” (ibid). In this way, solving $5 \times \# \text{ on}$

the dice = total score was not only about following rules of mathematical logic. It was linked to a cultural standard of motivation by which children were to embody norms for personal decision-making, even if in the end they (represented by their total score) were deemed unequal to their competitor. The game was organized to offer equal opportunities and there was a value ascribed to children by which to identify, compare, and differentiate them as winners or not.

Living with the consequences of the game presumed some natural outcome of winning or losing – as the effect of one's own calculated interest, enthusiasm, and effort. Articulated as the opportunity “for participants to solve difficult problems themselves rather than to observe the way someone else solves them,” (ibid, p. 76) the game was to model some notion of what children would encounter in the ‘real’ world. That is, no one was going to do the work for them. They would need to decide to do it for themselves and take advantage of the situation. Together, “these exercises may give pupils a greater sense of control over their future,” (ibid, p. 79). This sense of control was determined by how one played the game and what choices were made to impact the future outcome. With the game ultimately determined by how children were identified and given equivalence with a numerical score, the total score became more than that. Continuously multiplying 5 by the # *on the dice* = a measure of personal interest, decision-making ability, and sense of control over one's life.

The equal sign (=) used in the formula for play embodied a method of establishing equivalence between unrelated terms and making them equal to a value. Given as a way to identify both the result of the game and the child capable of winning,

calculating the score was more than a mathematical skill. In relating basic math facts to cultural principles for how children were to make their own choices and control their own future, it embodied a skill for living as a self-regulating individual. Inscribing this as the standard, the mode of reasoning about identity carried in the equal sign also organized principles of difference. In other words, more than a series of basic facts, the equivalence calculated by throws of the dice became a mode of comparing things that otherwise seemed incomparable – children’s abilities to control situations and make decisions.

The comparison was most easily made by using scores to determine the winner and loser. Whereas losing the game could be attributed to an unequal score, it could also be seen as child’s lack of control over the game and inability to make wise decisions. Importantly, this could also represent a child’s incompetence with even the most basic facts. As mentioned earlier, the game was also about math skills. If a child was not learning them, then motivating him/her to learn them was to psychologically order self-directing thoughts and actions. That is, if a child does not want to play the game, cannot control it, or continuously loses, that can be rearranged.

Seen as a “psychological block” to learning even the most basic skills, a child’s self-concept as a loser or failure intersected with a notion of “self-fulfilling prophecy,” (Dreyfuss, 1973, p. 489). A child’s idea of him or her self in the math classroom was thought to either advance or inhibit future success. This student “had already decided somewhere back in the grades that the mention of the words like *inches*, *feet*, or *yards* spelled FAILURE,” (ibid, p. 488). Comparing things that otherwise seemed

incomparable did not seem to make sense. In this case, a sort of “mental paralysis” engendered the belief that something was too difficult or could not be done. Yet, this could be undone.

As a way to provide equal opportunities for children to be motivated to learn math, certain practices of math pedagogy were linked to a psychologically (re)ordering of self-directing thoughts and actions. So that any child could have the skills to play the math game, the equal sign reinscribed expressions of difference as a mode of mathematical thought that was thought to *undo the mental paralysis*. By “start[ing] out with a simple equality: If 2 zobos = 5 dingbats, then how many dingbats do 4 zobos equal? ...the students responded admirably,” (ibid, p. 488). That is, now believing they could solve problems that they previously thought were impossible, the students were “surprised”, “happy”, and “relieved” and presumably motivated to continue learning.

This equality between zobos and dingbats seemed to provide a way for the child to think about the difference between the unlike terms by resignifying their identity and making it at once strange and familiar. Now, no longer thinking of units of measure, children were supposed to compare the zobos to the dingbats – presumed as non-equivalent and unnecessary to identify. One did not need to know what a zobo was to compare it to a dingbat. S/he just needed to follow the rule of making equivalences based upon what were already given as different. A dingbat was always already not a zobo.

At the same time, and following a similar way of reasoning about difference, the expression of equality in $2 \text{ zobos} = 5 \text{ dingbats}$ inscribed a psychological fix for the child

who was thought of as non-equivalent to the one who was seen as self-directed, enthusiastic, and in control. This difference distinguished the kind of child who seemed to need a more direct form of instruction, did not appear to motivate him or her self, and continuously fulfilled his 'own' predetermined sense of failure.

The use of the equal sign in mastering basic math skills was about developing mathematical modes of reasoning about equality and equivalence that embodied rules for thinking about identity and difference. Yet, it also entangled with the principles of psychology to identify individuality and ascribe norms of self-direction, -concept, -awareness, and -control as traits of the child motivated to learn math. In its various inscriptions, then, the equal sign was much more than a symbol of mathematical equality. It carried cultural standards for living as a self-motivated individual, as the basic element of all children having an equal opportunity to excel in school math.

To this point, I have looked at how the equal sign in the Basics curriculum organized a logic of equivalence and difference that worked to inscribe notions of identity as psychological traits of the motivated and self-directed individual. In the next section, I discuss how making the child motivated to learn mathematics historically assembled within broader social and cultural shifts that would give intelligibility to the skills that would seem basic to living and surviving in 1970s United States.

Basic Principles of Self-Expression, Individuality, and Personal Choice

To understand the shifts in curriculum that connected success in mathematics education to motivating the mathematical self, I place the Basics curriculum within a grid of historical, cultural, and political trajectories. This is in order to see how the child's

identity as a particular kind of self made sense within this context of reform. With that, it is to explore how mathematical and cultural ways of thinking about equality and establishing equivalence intersect outside of what is taken as the mathematics curriculum.

In other educational projects of reform the individual was given as a basic and fundamental component of social progress within a web of economic, political, and cultural challenges about rights, equality, and freedom. In *The 1970s: A New Global History from Civil Rights to Economic Inequality*, Borstelmann (2011) asserted that these challenges of reform were undergirded by a “spirit of egalitarianism and inclusiveness that rejected traditional hierarchies and lines of authority, asserting instead the equality of all people,” (p. 3) This notion of equality and inclusion is expressed as a logic of mathematics embodied in the equal sign, in how identity, equivalence and difference are taken as representable in order to make this “spirit” visible.

The sociological, psychological, and cultural histories provide a way to see the connection between the particular discursive distinctions and classifications embodied in the use of the equal sign in the curriculum. More than a textual relationship, norms of identity and difference connect the logic of equivalence expressed in the equal sign that bear a logical resemblance to cultural standards in the curriculum as elements of the child’s freedom, choice, motivation, and self-expression by which to live one’s life.

Defining individuality and marking difference could be conceived, not merely as self-serving, but within a broader responsibility for others, acceptance of difference, and

an appreciation for pluralism (Yankelovitch, 1998). At the same time, this way of thinking about equality and inclusion entailed a consensus as to what constituted this difference, who was identified as the other, and what forms of plurality were to be appreciated. Thinking about equality within a diverse society required a norm by which to determine a basis on which equality was granted.

In a society that valued the individual in relation to others, differences and inequalities in the social, political, and economic challenges that characterized one's daily life were simultaneously troubled and presumed to be mutable. In this, unequal results and differential opportunities were taken to be the result of, and therefore altered by, individual actions. In the context of social sciences, the value given to reason and rationality as the basis for deciding was accompanied by a cultural pattern of thought that gave significance to notions of individual choice as the basis of good decision making (Heyck, 2012).

Importantly, this interest in the individual also encoded new norms for discovering, identifying, and planning the self (Bellah, et al., 1985; Thomson, 1992). Working on the 'self' was to be in the name of gaining a greater sense of freedom, finding one's place in society, and inspiring one's own commitments to personal fulfillment. In an issue of New York Magazine devoted to the "Me Decade", journalist Tom Wolfe (1976) noted that "the new alchemical dream is: changing one's personality – remaking, remodeling, elevating, and polishing one's very *self*...and observing, studying, and doting on it." Describing what seemed to be a new American preoccupation with self-awareness, Wolfe pointed to an individual and inward turn. This

was not his view alone. Social psychologist Louis Zurcher (1977) conceived what he considered to be the mutable self as a response to rapid social change and an increasing fear of uncertainty of the future. In a similar sociological vein, Ralph Turner (1976) examined the impulse of individuals to identify what seemed to be inner, more personal thoughts and actions. Granted as more 'real' than actions that aligned with what one 'should' do, personal decisions seemed to be an expression of an internalized 'want'.

This relates to an earlier discussion of Leslie, enacting a sense of personal freedom to formulate and, in a sense discover, her own mathematics, rather than following the predetermined rules given by the teacher. This form of creating understanding for one self and directing one's own satisfaction served as a cultural standard ascribed to the child motivated to learn. Given as a rule to identify math learning, this notion of motivation aligned with more nuanced ways to identify the very traits associated with motivation to learn. One such measurement tool, the California Psychological Inventory (Gough, 1969) assigned equivalencies to traits of sociability, self-acceptance, well-being, responsibility, and self-control as factors of children's mathematical success. As individuals identified capable of discovering these psychological characteristics of self, all children could be seen to have equal opportunities to choose to be motivated and control their own excellence in math.

The Promise and Problem of Motivation: Excellence for All?

If the math in schools started with the natural abilities, skills, and interests of children, rather than with abstract mathematical ideas, then each and every child could

be motivated to master basic math skills. In this reorganization of the curriculum, motivation and its psychological traits served as the norm by which to identify equal opportunities for success. While this psychology seems to have little to do with mathematics, the very notion of in/equality could not make sense without a way of thinking about how to establish equivalence by assigning identity to things so as to compare them.

Concerns for unequal opportunities to learn math were not new to this period of reform. In fact, *The Equality of Educational Opportunity Study* (1966), pointed to the very problem of educational inequality among particular groups in the previous decade. The problem is how this inequality begins to get (re)assigned distinctions and classifications as traits of certain kinds of people. Here, I explore how representations of identity and equivalence produced new ways to think about equal opportunities as well as how to make sense of inequality.

The relationship that bound basic math skills in the curriculum to modes of living as a self-regulating, focused, and motivated individual also distinguished the kinds of children whose sense of self was in need of reshaping. In contrast to studies discussed earlier that explored mathematical precocity, “everywhere teachers are searching for ways to reexamine their understanding of mathematics teaching and learning as they work to resolve the problems of the inner city child who is mathematically disabled,” (Keiffer & Greenholz, 1970, p. 588). Emerging to give expression to notions of difference and inequality, the *inner* city represented more than a geographical location.

Presumed to have “problems of motivation,” the notion of difference ascribed to this child was given as a factor of what s/he was not in reference to the cultural standard of motivation. The common sense in teaching the child seen to have problems motivating was that, “what was taught is less important than how it was taught. It matters greatly that each student experience sufficient success to strengthen his confidence and pride in himself, to improve his self-esteem, and to encourage him to exert effort,” (ibid, p. 590). This pedagogical redirection from *what* to *how* seemed to subvert basic skills in math to reorder the “adverse effects of self-efficacy” (ibid, p. 589). With this focus, it was thought that “the fear of failure will be diminished, and the student will realize that his achievement is directly related to his own effort,” (ibid, p. 595). Effort and the achievement linked to that effort were given as a choice.

It would seem that remaking this child’s self-image had little to do with basic mathematical reasoning. However, the very principles of choice, effort, and self-efficacy embedded in the problem of motivation could not be understood without reference to notions of non-equivalence and difference linked as cultural logic in the rules of mathematics. In “An example of a mathematics instructional program for disadvantaged children,” (Kaplan, 1970) the distinction of disadvantage intersected with the psychological language of choice and freedom to direct one’s own learning toward basic mathematical understandings as well as a new sense of self.

To learn addition and subtraction operations with zero, “disadvantaged children” were supposed to select correct responses from multiple choices. Through this, the child was to develop a certain set of “desired behaviors”, of which choosing the correct

answer to $X + 0 = \underline{\quad}$ was one. “Immediately after responding, the student can confirm his choice by turning a page and comparing his response with the correct answer,” (ibid, p. 333). Making this choice and developing the behavior of choosing correctly from limited options had to do with understanding that the X would always be equivalent to itself, unchanged by the addition of 0. Given as the neutral term by which to determine the equality, 0 provides X with its identity as X .

Related to cultural principles of motivation and the discovery of self inscribed in the equal sign, the disadvantaged child could only be represented in relation to a term that maintained its distinction while representing a neutral base from which to make comparisons, like 0. The ability to decide what to mark in the $= \underline{\quad}$ as the “correct answer” required the “disadvantaged child” to see the X as what it was not. In similar ways, naming the unmotivated or disadvantaged child enacted cultural rules of ascribing traits of difference, in comparison to the norm of identity. In other words, the distinctions that gave meaning to the “inner city” and “disadvantaged” child’s mathematics were given in reference to child identified as the motivated other.

Conclusion

In this chapter, I have explored how the logic of equality embedded in what seems to be a purely mathematical symbol is transmogrified into a cultural way of representing identity and difference. By examining the use of the equal sign as related to the production of basic math skills, I have aimed to highlight how a cultural reason of equality is inscribed in and through the use of the equal sign in the math curriculum to produce new ways of classifying difference.

As it intersected with psychological notions of self-control, -esteem, and -direction, the equal sign itself embodied cultural principles ordering who the child is and should be as a motivated kind of self. I have referred to this as the *mathematical self* to highlight the ways in which children's sense of individuality was organized through the relation of the equal sign and cultural modes of reasoning about equality. In this way, the equal sign was much more than a symbol encompassing basic math skills. It inscribed standards by which to identify the child as a kind of person who could be motivated to learn and excel in math – while learning basic skills to survive in life!

Importantly, I emphasized how the shift that distinguished the New Math from the Back to Basics curriculum was not only about making the mathematics more “basic”. In other words, the reform also signified a shifting focus in the psychology as it intersected with mathematics that presumed the child as an individual, defined by one's “own” interests and feelings that could be managed so as to make motivation. In that, psychological truths identified the child, not as capable of mastering the abstract math content that was the same for everyone, but as able to discover the most basic principles for him or her self.

During the Back to Basics reforms, the inscriptions of equivalence by which to represent children and their mathematics shifted from expressing norms of citizenship to individuality. This mode of establishing a cultural norm of *all* continued to change as math education reformed once again. This can be seen in how the distinctions to which the equal sign relate to represent cultural differences changed to classify notions of *mathematical power* and *problem solving* during the Standards-Based movement that

mobilized in the 1980s. *All children* were thought capable of learning mathematics because math was about problem solving – something all people had to power do. Following similar ways of reasoning about identity, equivalence, and difference, the mathematical logic embedded in the Standard math problems overlapped with cultural distinctions to classify children's problem solving based upon abilities to think, speak, and act as a mathematical kind of person. The emergence of this new way of seeing children is the focus of the next chapter.

CHAPTER 5

Standardizing Mathematical People: The Power in the Child as a Problem Solver

“Students construct meaning as they learn mathematics. They use what they are taught to ***modify their prior beliefs and behavior***, not simply to record and store what they are told. It is students’ acts of construction and invention that build their ***mathematical power*** and enable them to ***solve problems*** they have never seen before,”
(National Research Council (NRC), 1989, p. 59)

The above statement was released to the United States public in a document entitled *Everybody Counts: Report to the Nation on the Future of Mathematics Education* (NRC, 1989). The National Academy of Sciences, the National Academy of Engineering, and the Institute of Medicine, acting through the National Research Council, put out this urgent call to ‘revitalize’ and ‘rebuild’ mathematics education. Looking ahead to the year 2000, it seemed evident that, “to participate fully in the world of the future, America must tap the power of mathematics,” (ibid, p. 1). This *power of mathematics* emerged as a key focus in the reorganization of math education in the U.S. during the 1980s. In this emphasis, it was presumed that *mathematical power* was a characteristic of all people that could be brought out through a reformed math curriculum.

As a common bond that all people shared, this power of mathematics intersected with cultural notions of identity and equality. In this chapter, I continue to explore how a particular logic of equality, embedded in the notion of *mathematics for all*, ordered how to see and think about children as part of the *all*. Like the previous chapters focused on the New Math reform and the Back to Basics movement, I continue to examine the

cultural ways of reasoning about what is equal, equivalent, and different through the use of the equal sign (=) in the math curriculum.

In comparison to the previous chapters, I highlight how the inscriptions of identity and difference are (re)signified during the Standards-Based reforms. This is made visible in how the ways of reasoning about equality embodied in the use of the equal sign (=) resemble and relate to shifting cultural standards of identity and difference. So, even though the rules of reasoning about equivalence, identity, and difference that link the math equal sign to the cultural logic appear the same, they are not. More directly, the norms of identity that are to include all children into the *math for all* shifted from ideals of citizenship in the New Math reforms, to distinctions of individuality during the Back to Basics movement, to standards of personhood during the reforms of the 1980s. In this way, the equal sign continues to order notions of equality, enacted as cultural practices and rules for how children are to think, speak, and see as mathematical kinds of people.

Focusing on the principles and practices of equality that organized the *Curriculum and Evaluation Standards for Mathematics* (National Council of Teachers of Mathematics (NCTM) 1989), I argue that the Standards math curriculum was about much more than normalizing expectations for math pedagogy. In this, the reorganization of curriculum intersected with principles of identity and difference to organize cultural standards by which to order children as people with *mathematical powers* who could *solve problems* of everyday life. Expressed in the language of psychology, this ordering of identity, equivalence, and difference made it reasonable to think that standards would

“ensure that *all students benefit equally* from the opportunities provided by mathematics,” (NRC, p. 89, my italics).

(Re)Setting the Standards: The Power of Problem Solving for All People

The common sense that *everybody counts* implied a cultural way of reasoning about the *equal benefit of math for all*. Marking a shift from a focus on individual differences in math learning (that characterized the Back to Basics movement), the Standards period of reform was to be organized according to this general notion of everybody. Importantly, *Everybody Counts* (NRC, 1989) was not the only document publicly expressing the need for math to change so as to bring to the fore the *mathematical powers* of all people. In fact, later that same year, the *Curriculum and Evaluation Standards* (NCTM, 1989) suggested that, “if students are exposed to the kinds of experiences outlined in the *Standards*, they will gain *mathematical power*,” (ibid, p.5). In line with this belief the *Standards* sought to provide the expectations for what that power was and how it was to be utilized.

Given as a “capacity of mind of increasing value,” mathematical power was presumably situated within the mind and would appear as “mathematical modes of thought,” (*Everybody Counts*, 1989, p. 32). Assumed as something common to all people, mathematical thinking was taken as something that everybody did. This can be seen in discussions of *logical-mathematical* kinds of intelligence that were thought to frame one of many, yet universal, ways of thinking about and seeing the world (Gardner, 1983). By placing a natural capability to think *logically* and *mathematically* always

already in the mind, all people could be seen as having in them the power to see and reason in particularly mathematical ways.

Interestingly, the mathematical modes of thought were not simply to do with mathematics. The emphasis given to the so-called ‘mathematical’ capacities of mind were related to problem solving in general. Problem solving was given as the focus of school mathematics (NCTM, 1980, p.2) and linked to considerations about how students lived their life outside of school. Articulated quite clearly in *The World Book of Math Power* (Johnson, 1983) mathematics was thought of as a “how-to-science, a friend and ally, a way of solving problems of daily life,” (ibid, p. 492). To use math as a way to solve these everyday problems, it was reasoned that, “students need to view themselves as capable of growing mathematical power to make sense of new problem situations in the world around them,” (NCTM, 1989, p. 6). Growing this mathematical power, through the image of seeds already planted in everyone’s mind, seemed imperative to understanding, interpreting, and solving problems in the world as a moral obligation to others.

From the basis that all humans inherently have mathematical power and this mathematical power helps them solve real problems in the world, it was thought that everybody is a mathematician and does mathematics somehow consciously (NRC, 1989; NCTM, 1989). School mathematics, then, was to “endow all students with a realization that doing mathematics is a common human activity,” (NCTM, 1989, p. 6). As a *capacity of mind* framing day-to-day activities, the naturalness and rationality given to

mathematical thinking served as a standard way to establish similarity among all people.

Taken as a normal way of living in the world, mathematical modes of thought and problem solving were presumed to be common cultural practices binding people together, across both time and space. That is, making sense of the world through ‘mathematical’ sensibilities seemed to be universal for people no matter where or when they lived. For the ancient Greeks, math and its powers served as a logical way to see, think about, and order the complex and chaotic world (Johnson, 1983, p. 19). Expressed with this example of the ancient logic, the abstraction of mathematical power and problem solving was represented as a universal and symbolic way of ordering the world, according to predictable patterns.

Consider “a pattern such as the following:

$$\begin{aligned} 2 + 3 &= 3 + 2 \\ 9 + 5 &= 5 + 9 \\ 27 + 58 &= 58 + 27 \\ 132 + 6 &= 6 + 132 \end{aligned} \text{ (ibid, p. 20)}$$

This pattern, represented as a series of whole number sums, seemed to be generalizable as a logical rule. “For counting numbers, called n and m here, it is always true that $n + m = m + n$,” (ibid, pg. 21). So, no matter the ordering, two discrete things can be added together to establish an equal identity. This symbolic expression of equivalence assumes an “If...then...” rule: If $2 + 3 = 5$ and $5 = 2 + 3$, then it follows that $2 + 3 = 3 + 2$. Accepted as a truth of mathematical logic, this expression of equivalence and identity inscribed a universal law of equality.

In the discussion of mathematical power and its modes of thought, this use of the equal sign (=) reinscribed a truth by which to establish equality, as a reduced and deduced form of universal logic. At the same time, it intersected with a cultural standard by which to identify what was given as the power to think logically and mathematically. This power relied upon what was considered logical thought, ordered by the ability to see patterns and generalize relationships between objects in the world.

More than a representation of how people sort out and reduce the complexity of the world around them, mathematical thought also served as a cultural norm by which to judge animal rationality and ability to solve problems. It would seem that an animal's problem solving process could be made visible by identifying certain behaviors as 'mathematical'. Taking the example of counting crows, mathematical thinking was considered a means of survival for the crow. In having the ability to see people as discriminate objects from a bird's eye view, the crow appeared to count and solve problems based on the mathematical information taken from the environment (Johnson, 1983). If birds could think and reason mathematically to problem solve, certainly all humans could.

However, there appeared to be differences in the crow's mathematical powers as compared to people. This distinction was grounded on the assumption that "since crows cannot use language, they have developed the ability to judge larger quantities by sight," (ibid, p. 14). So, although the crows could see the world from a mathematical perspective and solve problems based upon that information, they were not able to communicate their mathematical thinking. The crow cannot name or speak about the

thought process that keeps it out of the field while the number of people increases. This key distinction signified language as an important factor identifying the power of mathematics and how it was to be used.

This assumption that language represented a distinct human mode of mathematical thought was not new to the Standards period of reform in terms of organizing how children were to learn mathematics. Vocabulary and its precise use were granted as integral to understanding foundational concepts of equivalence and equality during the New Math curriculum reforms. During the Back to Basics movement, *The Psychology of Learning Math*, (Skemp, 1971) asserted that, “speech (which has to be learnt) is essential for the formation and use of the higher order concepts which, collectively, form our scientific and cultural heritage,” (p. 28). However, during the reorganization of math education in the 1980s, the association between language and the representation of mathematical thinking framed the norms by which to *speak and think mathematically*. These cultural norms not only organized the Standards by which to evaluate what was given as mathematical power, they also gave new meaning to how the child was to be seen as a specific kind of person, called the problem solver.

Math as a Language and Tool for Becoming a Mathematical Problem Solver

“It takes careful planning to create a curriculum that capitalizes on children’s intuitive insights and language,” (NCTM, 1989, p. 16).

If all children could learn and achieve in mathematics because they had the “intuitive insights and language” to do so, then it made sense that the reorganization of

the curriculum would seek to “capitalize” on these capacities. On one level, the value of math was justified in how it served as a universal communicative tool, useful for expressing, interpreting, and translating the world. On another level, the representation of math as a language rationalized the equivalence between all children and their natural capacity to learn mathematics.

In *Speaking Mathematically* (Pimm, 1987) the “intellectual powers of children evidenced in the mastery of their first language” served as the standard by which to deem all children capable of expressing an inherent mathematical power (p. 197-198). That is, math was described as a natural power, like the ability to learn language, that the Standard-Based math curriculum was expected to foster. If all children had intuitive abilities to learn language *and* inherent mathematical-logical frames of thought, then it would follow that they could all be taught to “grow in their ability to communicate mathematically and use higher-level thinking processes,” (NCTM, 1989, p. 23). Communication in general, and the use of language in particular, appeared to nurture this growth. In the use of psychology as a tool to translate the abstraction of mathematics and its presumed power into math education, language became a lens through which to see both the growth and distinctions of a mathematical kind of person.

In this way, “everybody counts” was not merely a catch phrase. It invoked a commonsense way of thinking about who *everybody* is, how they think, and what they do. This reasoning about everybody, as an equal part of all people, set the Standards expectations for “what children should do when they learn mathematics,” (ibid). Clearly, all children would be expected to use mathematics as a language and mode of

reasoning to interpret, translate, and communicate about everyday and unfamiliar problems. This is important in how mathematical forms of language and reasoning served to produce a common way of making children into what were considered mathematical kinds of people that fit in with the “everybody”.

This way of organizing the equivalence given to children is explored in and through the use of the equal sign ($=$) in the National Council of Teachers of Mathematics (NCTM) *Curriculum and Evaluation Standards* (1989) math problems. In this, the Standard use of the equal sign ($=$) organized what words, thoughts, and behaviors were expected and accepted of children as using what would be seen as math powers to express what they should know and be able to do as mathematical problem solvers.

As the practices of problem solving intersected with the rules of a psychology that situated learning within a social relationship, a new way of seeing children as mathematical kinds of people emerged. In this, the mathematical power and problem solving was not only to do with mathematics but rather provided an opportunity to “get inside students’ heads” (Schoenfeld, 1987) and give meaning to what it meant to be a person who could see, think, and speak mathematically. Identifying children as social beings who learn in various ways, yet always through interactions with others and the world, the tools of psychology assembled with the logic of mathematics to create a commonsense about who the mathematical problem solver was and should be. Importantly, the abstraction of mathematical power (and the person who had it) was not a given, but was created in the remaking of mathematics.

The analysis that follows works to make visible the process of alchemy whereby psychological research about how people see, think, and speak about the world in presumably mathematical ways was translated into the Standards curriculum as problems for children to solve. One on hand, the equal sign was to encourage children to use what were given as mathematical powers. One another, the equal sign embodied a cultural thesis that ordered standard rules for thinking and communicating in everyday, mathematical ways and produced a way of reasoning about how to make children mathematically powerful problem solvers.

Modeling with Math: Representing Problems and Metacognitive Awareness

Standard #2 of the K-4 curriculum, “Mathematics as Communication” includes an activity in which groups of children were to use language and counters to represent the problem $14 - 5 = \square$. Here, the emphasis was not so much for children to find the correct answer. Rather, the modeling was so that they could “see how problems that appear to be different in fact share the same underlying structure,” (NCTM, 1989, p. 27). In other words, children were supposed to recognize that $14 - 5 = \square$ is a common way of representing a problem about Maria and some pencils, Eddie and several balloons, and Nina and Pedro’s seashells.

Yet, this similarity was not necessarily so obvious, or easy to “see”. The shared structure to the problems was to become visible through a mathematical discussion of how they were different – as a way to see how they were alike. In many ways, this entailed coming to a consensus about what the problem was and how to represent it. Maria’s “problem” was to determine how many pencils she started with after she added

5 more and then had 14. Eddie's "problem" was to figure out how many of his 14 balloons he lost to be left with 5. Nina and Pedro's "problem" was to compare their number of shells.

Modeling with both the counters and the mathematical expression was related to an importance in the curriculum placed on "translating a problem or idea in a new form," (ibid, p. 26). This translation involved decoding the text, communicating to establish a consensus about what the problem is, modeling the problem with counters, and reorganizing the model into a symbolic representation. Therefore, the focus of using mathematics as communication was to establish a common way for children to see and communicate about problems in their world – given their differences. So, in spite of the various contexts in which the problems were situated, children were to talk through the distinctions, reduce them, and agree to represent their equivalence as a general problem of difference. In other words, the problems were all alike in that the relationship between 14, 5, and $\square$ in each situation was to be organized by the rules of subtraction and given as $14 - 5 = \square$.

The mathematical problems of equivalence and difference used as the Standard were related to the psychological problem of identifying who the problem solver is and what s/he does. In fact, as the Standards were in development, *Mathematical Problem Solving* (Schoenfeld, 1985) was a topic of research emerging to examine the "spectrum of behaviors that comprise people's mathematical thinking" and "to explain what goes on in their heads as they engage in mathematical tasks," (p. 5). This spectrum was bound by the limits of language and symbolic expressions to model a form of thinking

that could be recognized as drawing upon and having control over relevant resources, knowledge, and strategies.

As a mode of establishing equivalence between people as “good problems solvers”, the psychological explanation of the social behaviors that were presumed to represent internal thoughts provided a cultural norm by which to identify the expression of math power in people. In addition to the behaviors given as this spectrum, the person that exhibited mathematical power was expected to self-monitor as a way to assess and control problem solving as a process. This evaluation was based upon an assumption that good problem solvers were equivalent in their ability to know “what to do with the facts at hand” (ibid, p. 33). In other words, “the issue is not what the students know, it is how they use that knowledge” that made them successful in solving non-routine problems in mathematics (ibid, p. 32).

Within the concerns about standardizing problem solving methods in mathematics, the consciousness of one’s thoughts and organization of one’s actions seemed to establish a cultural norm about the kinds of people who make good problem solvers – as the demonstration of mathematical power. This model of thought resembled how children were expected to see similarity in the problems of difference and represent their common structure as $14 - 5 = \square$. Embedded within the rules for establishing equality between things that did not seem to be alike but shared a similar organization was a particular way of ordering equivalence as well as a way to regulate difference. This was not just in the mathematics but transmogrified into cultural modes

of reasoning about how equivalences could be organized as standards for children and these normalized behaviors.

Organized by the rules of psychology that presumed learning as embedded in a process of social construction, the use of mathematical knowledge would become identifiable as particular behaviors. Specifically, practices of self-monitoring, self-regulating, and evaluating cognitive activity were all associated with a general problem-solving process (Silver, in Schoenfeld 1987, p. 49). More than components of mathematical thinking, these were psychological traits that ascribed a specific identity to people who were aware of, able to see, and could think about their own thinking. Entangled in a standard “process model” of problem solving, *metacognitive awareness* emerged as a psychological characteristic to identify this person. It was presumed that “awareness of one’s intellectual behavior is a prerequisite for learning to change it,” (Schoenfeld, 1987, p. 199). Operating below the surface, at a “metalevel”, were processes of planning, monitoring, and evaluating that it seemed one must be aware of in order to change intellectual behaviors, engage in problem solving, and learn.

Given as traits of the good problem solver, meta-awareness and self-regulation ordered the thoughts and actions of children in their ability to communicate mathematically. Evaluating this Standard, then, introduced the possibility of seeing children as *unable* to change their thoughts into mathematical language. In this assessment, however, it was acknowledged that it might not be the mathematical understanding that is wrong, but rather an “inability” to translate and communicate the thought process that represented it (NCTM, 1989, p. 27). The difference between

children, while similar in their innate capacity for language and mathematics, was expressed as an inability to interpret what was going on in their head in an ordered and cognizant way. So, the neutrality given to language that appeared to give equivalence to all children also inserted a notion of difference in solving Standard problems.

Related to the psychological norm of metacognition – as the guide to changing and transforming intellectual behaviors – even when kids seem to be wrong, what was “of primary importance is *the value that children derive from reflecting on their responses*,” (NCTM, 1989, p. 27, my italics). Embedded in this statement about the “value that children derive” is a norm of equivalence by which to determine difference. More specifically, within the value given to the ability to see, assess, communicate, and plan one’s mathematical thinking was a distinction of the kinds of people who did not yet meet the standard, but could. That is, the children who were seen as unable to communicate in the universal language of mathematics would still be able to grow in this ability if they were aware of and could reinterpret their thinking as a way to take stock of and persevere in their problem solving.

So, while finding the “right” answer did not appear to be the focus of the activity, there was a “right” way to think about $14 - 5 = \square$ as the representation of equivalence in the language of mathematics. The presumed universality given to math as a language was to provide children with a way to express ‘mathematical power’ – inscribed as cultural traits of a new kind of person who could think about, model, communicate, and change mathematical modes of thought.

Mathematics as Reasoning: Strategy for Making Sense and Building Confidence

“Mathematics does, however, provide one of the few disciplines in which the growing student can, by exercising only the power inherent in his or her own mind, reach conclusions with full assurance,”
(NRC, 1989, p. 4)

As a rule, learning mathematics was not purely a matter of cognition. While the previous discussion highlighted the idea that all children would benefit from math education because they could communicate and reflect upon their mathematical thinking, math learning was also contingent upon and entangled with affective dimensions. Historically speaking, both the “pre-sixty” and “post-sixty” conceptions of mathematics learning were conceived as inadequate given that, “neither view considered the psychological ways children develop cognitively *and* affectively,” (Price Troutman, p. 427, 1973). In other words, before the previous Back to Basics movement during the 1970s, the child’s mathematical learning process was articulated primarily in cognitive terms as the mind was seen as a masterful computing device. Emerging during the Basics reform, then, was a new psychological language that intersected with the reorganization of math education to express the child’s personal interests and motivations to learn math.

What was given as the affective dimension of math learning gave sensibility to the idea that “a major goal of mathematics instruction is to help children *develop the belief* that they have the power to do mathematics and that they have control over their own success or failure,” (NCTM, 1989, p. 29). Articulated in the discussion of Standard #3 *Mathematics as Reasoning*, children’s math power was to emerge not only through

learning math as a language, but also through an internalization of the belief that mathematics makes sense and that they can make sense of it. Therefore, the power to benefit from mathematics seemed to rely upon an ability to build knowledge of one's own reliability, certainty, and assured sense of direction.

Particular problems were thought to “[help] children see that mathematics makes sense,” (ibid). In solving $35 - 19 = \square$, the emphasis was, once again, not on children computing the correct answer. Instead, this problem was posed in relation to the idea that “children need to know that being able to explain and justify their thinking is important and that how a problem is solved is just as important as its answer,” (ibid). In that, while children might be able to identify 16 as the correct response, the goal was for them to have a strategic and analytical way to think about the problem that was more than simply filling in the box.

A series of questions posed by the teacher, then, were thought to illuminate the inherent logic organizing how children should see and think about mathematical equivalence as reasonable, but not necessarily self-evident:

“Do you think it would help to know that $35 - 20 = 15$?”

“How would it help to think of 19 as $15 + 4$?”

“Would it help to count on from 19 to 35?”

(ibid, p. 30)

The above questions were meant to provoke discussion. At the same time they also implied strategies for seeing the sense built into the related expressions of equality. To the first question, children were to respond, “Yes”. Knowing that $35 - 20 = 15$ is a true and valid expression was supposed to help them think strategically about the problem

$35 - 19 = \square$. By comparing the 19 to 20 as 1 less, children should reason that the 15 compared to the $\square$ would be 1 more. Reframed as a different, yet related problem, $35 - 20 = 15$ was thought to provide a sound way of seeing and rationalizing a mathematical relationship of equivalence between $35 - 19$ and $\square$.

This way of making sense of the expression appealed to an authority of the identity given to whole numbers that ordered the relationships between them. In other words, the meaning ascribed to numbers meant that they could be seen as identical (such as 35 and 35) and different (like 20 and 19). As a series of ordered and predictable patterns, the identity given to numbers formed the relationship by which to determine such equivalence and difference. In the Standards, the ability to distinguish between numbers, understand how they were related, and predict their patterns was referred to as “number sense” (ibid, p. 38). This *sense of number* entangled with the equal sign (=) was the basis on which children should be able to recognize “valid arguments” about mathematical equivalence and non-equivalence.

However, this way of assigning identity of discrete objects and the relationship between them (mediated by the equal sign =) was not simply mathematical. Rather, the logic that ordered the validity of numbers and their relationships resembled cultural principles of identity and equality that organized how children were to build valid arguments as a way to build confidence and a belief in their abilities to learn mathematics. Said another way, “number sense” was not only a way of thinking about numbers but was related to psychological notions of how that sense was built.

As mentioned earlier, the discussion about $35 - 19 = \square$ in connection to other expressions was to help children see that “mathematical reasoning cannot develop in isolation...the ability to reason is a process that grows out of many experiences that *convince* children that mathematics makes sense” (NCTM, 1989, p. 31). This convincing was about showing children that math made sense so that they would believe in, be aware of, and appreciate it. The experiences that were not isolated and were based upon interactions with others that were thought to convince children that math just made sense. Not just for the sake of liking math, it was thought that if children believed in and saw the power of mathematics in many situations, they could become more sure and certain that they had the power in them to know, do, and show it.

Ordering the beliefs of children had to do with what were given as the affective dimensions of learning math mentioned earlier. Not necessarily distinct from matters of cognition, it was thought that the feelings, perceptions, and beliefs of children were intricately related to how they conducted themselves as problem solvers. This idea was discussed in *Mathematical Problem Solving* (Schoenfeld, 1985), as an important text at the intersection of math education and psychology focused on what good problem solvers know and know how to do. The study maintained that, “the performance of most intellectual tasks takes place within the context established by one’s perspective regarding the nature of those tasks. Belief systems shape cognition,” (ibid, p. 35). This statement embodies a cultural norm, expressed in psychological language, that identifies a link between how one’s views of the world shape how one thinks and acts.

While this was historically not a new association, the affective and cognitive considerations in learning mathematics were re-signified during the Standards reform movement. Marking a shift in emphasis from all children as *self-motivated individuals*, the Standards reform organized and maintained that all children could learn math as *strategic and confident problem solvers*. Motivation was still an important psychological factor in learning math but it became reassembled as interest fueled by the certainty that math could be done. Yet, whether or not children wanted to learn math assembled with psychological considerations of confidence – ordering how children should see and believe in the value given to math as a way to build certainty in their problem solving as mathematically powerful kinds of people.

This confidence framed a standard approach to all students learning math since it was thought that the confidence could be built from the validity granted to math itself. In fact, the Standards maintained that the assessment of mathematical power, “should examine students’ disposition toward mathematics, in particular their confidence in doing mathematics and the extent to which they value mathematics,” (NCTM, 1989, p. 205). Taken as psychological dispositions of people who engaged in effective problem solving, confidence and an intentional appreciation of math were associated with particular learning behaviors and therefore, possible to evaluate. The “good problem solver” could use a variety of strategies and approaches to test the validity of arguments, check in with his/her thought process, and persevere to logical and confident conclusions (Schoenfeld, 1987). Even when unsure, children could work through what they knew and plan what to do next. The shaping and reshaping of

mathematical thought, then, was tied to a reorganization of the child's value system as well as their value as a person.

Organized by psychological principles identifying effective problem solving strategies, children were seen as people who could make sense of, plan, communicate, rethink, and represent a well-organized, logical, and justifiable mathematical thought process. This assumption could be seen in how children were expected to discuss the problem $35 - 19 = \square$. As a Standard by which to evaluate mathematical power, confidence in mathematical reasoning was to be represented by students' participation in math discussions aimed to uncover an inherent logic of what they are doing, how they are thinking, and why (Schoenfeld, 1985, 1987; Johnson, 1983). As such, the mathematical notions of identity and equivalence embodied in expression like $35 - 19 = \square$ interacted with cultural modes of representing the child as capable of using math as a language and mode of reasoning to communicate his/her believing in and being in control of the logic of one's "own" conclusions.

This mode of characterizing a cultural standard of problem solving gave equivalence to all children – as having equal opportunities to think and speak mathematically so as to benefit from math education. In this, children's participation was to be organized around the assumption that they can contribute to knowledge production and decision-making in the classroom. Yet, the benefit of this liberal form of participation was based upon the ability to contribute to mathematical problem solving in ways that were already determined by the Standards. In that, the certainty with which one could reason mathematically characterized children and their participation around

the abilities to plan, strategize, assess, rethink, and, therefore, feel confident about their powers to problem solve.

In this, the rationality and value given to mathematical identity as a valid truth overlapped with the cultural rules of how to identify children as able to solve problems – or not. Any sense of insecurity, uncertainty, incoherence, disbelief, or unwillingness to share would be recognized as an inability to express mathematical power. In modeling the problem $26 - 17 = \underline{\quad}$ a teacher (T) and pupil (P) “discuss” how this should be done:

T: ...you start it off and tell me what you are doing.
 P: *Put one there.*
 T: No. Let's start from the very beginning. Six take away seven. Can you do it?
 P: *No.*
 T: No. Why can't you do it?
 P: *Cos it's...bigger number on the bottom.*
 T: All right. Because six is smaller than seven. All right? So what do we do then?
 We go to the...
 P: *The...the units*
 T: No. What column's that? The tens column. Right. And what do we do there?
 P: *We cross that one out...and then we put one there.*
 T: We take a...
 P: *Er...er...er...*
 T: ...What do we take from the tens column? We take a ten don't we. One ten.
 P: *Put one there*
 T: Yes you've got one left there. And where do you put the ten you've taken?
 P: *There*
 T: You put them in the units column, right. How many units have you got?
 P: *Twenty-six*
 T: No. Put your ten in the units column. No. No. Come on, you go to the units column and you take a ten. Where do you put the ten? We put it in the...units column, don't we. Like we did there, and there, and there, and there.
 (Pimm, 1987, p. 65-66)

In this discussion, a particular standard of participation and consensus is embedded in the practices of this problem solving. The repetitive instantiation of “we” in

statements such as “We take a...” and “What do we do now?” embodies a way of reasoning about the normalized conventions of how this problem should be thought about, what should be done to solve it, and what this problem solving should look and sound like. As the student “responds” to the teacher’s prompts and the feedback she or he receives is “No”, it is clear that the rules organizing the conversation and how this problem is to be solved also carries with it an expression of what isn’t being done. This student does not seem able use mathematics as a language, as seen in how general statements and imagined physical gestures “*There*” are rephrased into a different, mathematical language as “You put them in the ones column, right.”

More than that, the student does not seem to have a clear vision or plan for how to strategically and methodically solve the problem. Given the opportunity to start where s/he wants, the teacher immediately stops and offers a different plan to start at “the beginning”. Aside from appearing as unable to start and develop a plan, this student also seems unable to follow how “we” would normally solve by modeling it. In asking, “So, what do we do then? We go to the...” the teacher seems to limit the student respond to a place based action. Offering a somewhat stuttered response of “*The...the units*” this student is once again placed outside of the expectations with an otherwise certain “No.” His or her uncertainty and seeming inability to express a clear and logical thought process appears again as “*Er...er...er...*” Finally, at the end of this exchange it seems apparent that this student has difficulty seeing the patterns in how to model and solve this type of problem, and so needs reminding that putting the ten in the units column is “Like we did there, and there, and there, and there.”

While the above discussion can be read as a standard interaction between a teacher and student, it can also be considered in relation to the process of alchemy whereby mathematics becomes math education in the social spaces of the school. It is in this translation that certain normative values about a *mathematically powerful problem solver* work to order and make visible a cultural thesis about who is/is not this child. In ways that seem almost natural, the cultural theories about what children should know and be able to do interact with the subjects of school math to make the abstraction of mathematical power ‘real’, as a measurable goal of the Standards.

To this point, I have examined the use of the equal sign (=) in the organization of the curriculum around particular Standards. This has been to provoke a way of thinking about how the rules for establishing identity and equivalence are visible in the Standard expectations for knowing, seeing, and representing equality as a truth of mathematical logic. In this way, the principles organizing the problems in the Standards have been enacted as norms for how kids should speak and think mathematically in schools.

Within this standard approach to developing and evaluating what was expressed as mathematical power, the child above could be considered as unsure and unable to represent – either in mathematical language or models – what was seen as a mathematical view of the world. Yet, as a view of the world, mathematics also carried a normative and cultural set of values that are about more than mathematics. In more ways than one, the mathematical was linked to the cultural inscription of everyday people living an everyday life. The activities that framed what kids should know and be able to do aimed to emphasize “connections between mathematics and the real world

and [encourage] children to recognize and use a variety of situations and problem structures,” (NCTM, 1989, p. 12). This instantiation of the “real world” brings with it an assumption of what was considered real in the world as well as what constitutes a problem in it.

Problems in the “Real World”: Mathematical Illiteracy

The aim of the Standards-Based curriculum was to provide experiences for all students to: value math, be confident in one’s abilities, become a mathematical problem solver, and communicate and reason mathematically (ibid, p. 5-6). These goals valued the kind of person that appreciated math, respected its certainty, understood and spoke its language, and saw through its perspective to solve the problems of everyday life. As discussed in the analysis of the equal sign ($=$), these aims were instantiated in how children were to see, think, speak, represent, and communicate mathematically – as the expression of mathematical power. Given that “math power is a term important for everyone” (Johnson, 1983, p. 8), it served to provide a general expectation for *everyone* and frame the common sense about how *all* children should know and do mathematics for living and surviving in the world.

Yet, as the expression of equivalence between all people based upon an inherent capacity to learn math, the instantiation of *math power* also embodied a notion of difference. Everybody counts, but in a particular way. Directly, “math power can mean the *difference* between confidence and insecurity; vitality and boredom; goals and regrets,” (ibid). The expressions of confidence, vitality, and goals can be related to the psychological inscriptions of identity presumed to represent the person who could think

and speak mathematically. Again, this person was not ‘everyone’ but was classified and expected to confidently plan, assess, reflect upon, regulate, represent, and communicate a well-ordered, logical, and valid (mathematical) thought process. Embedded in the equivalence given to children as equally capable of possessing and demonstrating mathematical power was a comparative frame with which to see children who could not draw upon their mathematical power to benefit from all that mathematics was thought to provide.

This notion of difference became visible in relation to the distinctions given above – insecurity, boredom, and regret – in discussions of the problem named “innumeracy”. Taken as “an inability to cope with common quantitative tasks,” innumeracy and its consequences were presumed to be “magnified by the very insecurity that it creates,” (Steen, 1990, p. 216). In naming this problem, “innumeracy” was associated with psychological notions of apprehension, fear, and uncertainty (ibid). Even more, it assembled with concerns over a culturally and historically specific form of “math anxiety” that was visible as expressions of helplessness, guilt, unwillingness, and boredom (Chavez and Widmer, 1982; Cockcroft, 1982; Stodolsky, 1985). As a problem in the pedagogy of mathematics, the differences that defined the innumerate as insecure, disengaged, ignorant, and unacceptable were conceived within and emerged alongside the language of the alchemy that produced a literacy – of a mathematical kind.

Taking as granted that, “numeracy is to mathematics as literacy is to language,” (Steen, 1990, p. 211) then it could be reasoned that, “numeracy and literacy are the entwined complements of human communication,” (ibid, p. 212). In its various forms,

“functional literacy,” “cultural literacy,” “scientific literacy,” “environmental literacy,” and “quantitative literacy” set the historical and cultural backdrop for the demand of a specific form of “mathematical literacy” as the equivalent expression of numeracy (ibid). The rising cultural expectations of numeracy and literacy emerged in the Standards to further define what this mathematical form of literacy would look like. In this, it became clear that “the intent of these goals is that students will become mathematically literate,” (NCTM, 1989, p. 6). As an expression of mathematical power, mathematical literacy was taken as an integral part of learning how to draw upon what had been granted as a power to see, think, and communicate ‘mathematically’ in and about the world.

Becoming mathematically literate and realizing the ideal of mathematical power, however, had little to do with learning mathematics. This was clear in how it seemed self-evident, that “mathematical literacy is essential as a foundation for democracy in a technological age,” (NRC, 1989, p. 8). This link between democracy and the role the mathematically literate were to have in maintaining its foundation was made reasonable in relation to numeracy, as the mode of thinking and acting about the world in ways that seemed ‘mathematical’. The sense of democracy entangled with the production of the mathematically literate was clear in cultural statements regarding how, “with numeracy comes increased confidence for individuals to gain control over their lives and their jobs. Numeracy provides the ability to plan, to challenge, and to predict,” from a series of choices and alternatives (Steen, 1990). Gaining a sense of control and confidence would seem to provide individuals with the tools of thought and action by which to plan their future through making rational decisions in the present. Improving mathematical

literacy, or numeracy, inscribed a mathematical model or interpretation of the world that was thought to make a better kind of person – and, in turn, somehow produce a better kind of worker.

The production of “mathematically literate workers” within the Standards based curriculum was organized in line with the goal of “economic survival and growth” (NCTM, 1980, p. 3). Emerging in a web of technological, democratic, and economic considerations, the mathematical kind of person was presumed capable of sustaining his/her own livelihood while contributing to the “world of work in the twenty-first century,” (NRC, 1989, p. 11). This world was presumed to be, “less manual but more mental; less mechanical but more electronic; less routine but more verbal; and less static but more varied,” (ibid). As such, certain kinds of people were thought to be best “prepared to cope confidently with quantitative, scientific, and technological issues,” both now and in the future (ibid, p. 12). Those people were seen as ‘mathematical’ – able to see, think, represent, and speak about the world in logical, ordered, planned, and methodical yet flexible ways in the face of uncertainty and unpredictability.

To return to the equal sign ($=$), while its use in the curriculum ordered standard notions of identity and equivalence, its organizing principles gave a particular nuance to how children were to be seen as mathematically literate. In other words, the math kind of person was not born as such, but was to be made through the cultural inscription of traits in the math curriculum that assembled to create a commonsense of who that person is and should be. Carrying a cultural thesis about mathematics as the “right” view of the world, the math that made it into the spaces of the Standards also operated

to give civic meaning and normative value to those that were seen as mathematically literate. With mathematical literacy as the common way of framing the democratic identity of the nation, its citizens, its workforce, and its schoolchildren, a notion of mathematical illiteracy emerged as the distinction from this norm.

This difference was expressed in relation to the production of cultural distinctions that seemed to threaten a democratic sense of equality and opportunity. That is, “apart from economics, the *social and political consequences of mathematical illiteracy* provide alarming signals for the survival of democracy in America,” (ibid, pg. 13, my italics). In *Innumeracy: Mathematical Illiteracy and Its Consequences* (Paulos, 1988), the social and political consequences were given as the embodiment of various inadequacies, insufficiencies, and an overall lack of ability or awareness of how to function as a productive member of this democratic society.

The mathematical illiterate was characterized as the kind of person that had “a lack of numerical perspective, an exaggerated appreciation for meaningless coincidence, a credulous acceptance of pseudosciences, and an inability to recognize social trade-offs,” (ibid, p. 5-6). Apparently, these people could not appreciate the sense of certainty a mathematical perspective would lend to one’s approach to living and problem solving in the social world. To use another language, they were an insecure and passive group who left life to chance, could not access benefits or assess risk, and were otherwise unplanned, unpredictable, and unreliable. These kinds of people – based upon their presumed inability to think and speak mathematically – were seen as

less productive and overall lacking the power to act in intentional, rational, and organized ways.

Mathematical literacy and illiteracy served to divide and differentiate kinds of people – who would either support the nation in its growth or risk its economic, political, and cultural demise. Quite directly, “we are at risk of becoming a divided nation in which knowledge of mathematics supports a productive, technologically powerful elite while a dependent, semiliterate majority, disproportionately Hispanic and Black, find economic and political power beyond reach,” (NRC, 1989, p. 14). This divide, would be drawn along economic, linguistic, and racial lines and serve as another cultural expression marking the difference between the mathematically il- and semi- literate and the norm of the mathematically literate person who could represent the world and their place in it in well-ordered and justifiable ways.

In a similar style of comparative thought that validated standards of identity and difference in the mathematical formula for equality, the mode of representing the mathematically literate as equivalent with progress and growth classified the mathematically illiterate as the cultural expression of difference. Generally accepted as unacceptable, mathematical illiteracy emerged as a social problem and provided new ways to see and think about the concern of organizing mathematics for all. The cultural organization of *all* seemed to be entangled with a concern that “changing demographics have raised the stakes for all Americans. Never before have we been forced to provide true equality in opportunity to learn,” (ibid, p. 19). This common sense was the impulse behind the Algebra Project that emerged during the 1980s to provide Black children in

the U.S. with both the opportunities and identities as mathematical kinds of people entangled with mathematical literacy, as discussed in *Radical Equations: From Mississippi to the Algebra Project* (Moses & Cobb, 2001).

Reasoning about and *providing true equality* embodied a particular way of assigning identity and difference to establish equality and formulate how that problem would be solved. In this way, the cultural statement about the expectation of equality bears resemblance to the Standard way of approaching equality as a problem, expressed by the equal sign ($=$). That is, the particular system of reason in the curriculum that orders the solving of equality inscribes difference as the problem of inequality. In this way, the Standard modes of reasoning about equality, equivalence, and difference embodied in the equal sign ($=$) were not simply mathematical. Rather, they worked into the cultural imperative to force, organize, and manage a “true equality” wherein the mathematically illiterate were differentiated as the risk – the real world problem in solving for equality.

Standards Revision and The Common Core of Mathematical Proficiency

By the year 2000, the goals of tapping into the power of mathematics were translated into a new iteration of the *Principles and Standards of School Mathematics* (NCTM, 2000). In this revision, *equity* served as a founding principle by which “all students, regardless of their personal characteristics, backgrounds, or physical challenges, can have access to high-quality mathematics instruction,” (ibid, p.12). This “high-quality mathematics” would be envisioned as the condition for equity in access and attainment for *all* students. Carrying the same logic as the 1989 Standards, while

seeming to have no regard for personal and individual characteristics, the 2000 Standards would continue to inscribe traits of the child who appeared as the mathematical person.

This is clear in how the updated version (re)envisioned who could learn math and how, as articulated in the “Process Standards”. Providing a particular way of seeing and thinking about “people who reason and think analytically” (ibid, p.56) the process standards framed the rules by which these kinds of mathematical people *acquire* and *apply* the content knowledge of school math. Saliently, this process of gaining and using “mathematical ideas” drew upon a mode of “mathematical communication,” that was given as “a way of sharing ideas and clarifying understanding,” (ibid, p.60). Assumed to be able to see and think about the world in mathematical ways, all students were thought capable of accessing a high-quality math education through the translation of “mathematical ideas” in to a “mathematical language”.

It was this mathematical mode of communication that was to help students solve mathematical problems and “acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations that serve them well outside the mathematics classroom,” (ibid, p.62). In the 1989 Standards, confidence – as a trait of mathematical kinds of people – was to be built through doing mathematics and using its language to participate in what was psychologically ordered as a social process of learning. Yet, in the 2000 (re)formulation of the Standards, the language and lens of mathematics was already given as a perspective from which mathematical kinds of people saw and spoke in presumably confident ways about the world. As such, all

students were expected to participate (and thereby gain access to an equitable and high-quality education) by engaging in “conversations in which mathematical ideas are explored from multiple perspectives,” (ibid, p. 63).

This multiplicity of perspectives, however, was formed within what was already presumed as a confident and “mathematical” mode of communication. In other words, to even be recognized as an ‘other’ perspective, one would need to follow the rules and use the language that ordered the mathematical conversations as clear, convincing, and precise – in both their content and form. Mathematical communication was to be organized according to the reasoning that all children already had the language and ideas that were deemed “mathematical”. In this way, all children were considered to be the same and so it could be reasoned that all could learn math “regardless of personal characteristics”.

Importantly, “personal characteristics” were not completely disregarded. Instead, new qualities of the individual student emerged in relation to the logic of equivalence and difference that organized the notion of equity and equality embedded in both the 1989 and 2000 NCTM Standards. This can be seen in the most recent revision of standards, the Common Core Standards for Mathematical Practice (NGA & CCSSO, 2012). In the updated version, the psychological principles of confidence, clarity, and certainty that described the mathematically powerful and literate were rearticulated as traits of the “mathematically proficient” (ibid, p. 6). In this way, they were not exactly the same qualities as those that were given to establish equivalence between children in the

previous iterations of the Standards. Rather, they assembled with new classifications and distinctions, ascribed as personal traits of a mathematically proficient kind.

Nevertheless, in the organization of the Common Core, the standard that established children as equals in their ability to communicate mathematically would continue to be the norm by which to further categorize and differentiate who is/is not the *mathematically proficient* child. This is evident in how the Standards for Mathematical Practice expect that “mathematically proficient students try to communicate precisely to others,” and “they state the meaning of the symbols they choose, including the equal sign consistently and appropriately,” (ibid, p. 7). The use of the equal sign and its organizing logic of equivalence and difference to manage the expectations of the child in Common Core Standards resembles the rules ordering how children were to develop a certain form of mathematical creativity in the New Math curriculum. However, now translated through the lens of a psychology that takes language as the tool by which children engage in learning as a social process, the standards set the expectation that elementary students – by virtue of an already assumed mathematical language – will “give carefully formulated explanations to each other,” (ibid). Precision, accuracy, careful formulation, and efficiency were no longer linked to the cultural classification of mathematical creativity but were historically reassigned as traits of the person seen to be proficient in the ability to *communicate mathematically with others*.

Just as the expression of mathematical creativity was not just about mathematical thinking, neither was mathematical proficiency. Rather, in the curriculum it assembled with the psychological principles of learning to order learning as a standard

process and social practice of communicating with others. From this psychological lens that ordered the principles of learning mathematics, the common expectations for children gave an identity to mathematically proficient kinds of people. Given in the expectation of how children should talk about the equal sign (as not just an example!), are qualities of this kind of proficient person in relation (and communication) with others.

Organized as personal characteristics of consistency, accuracy, appropriateness, and precision, the mathematically proficient child emerged as yet another Standard mathematical kind of person. Importantly, as the identity of children that are to be seen as mathematical kinds shifts historically from expressions of mathematical creativity, motivation, confidence and proficiency, so too did the cultural signification of difference. That is, the emergence over time of the child imagined as disadvantaged, unmotivated, and illiterate resignified the categories by which difference was to be understood. With the subtle shift from mathematical power to mathematical proficiency in the revision of the Standards, the category of difference that was produced within the aims of creating equal access to high quality mathematics was given further nuance and finer visibility. It could now be seen as a characteristic of certain kinds of children.

Marked as an imagined 'other' with whom the mathematically proficient student communicates, the English language learner was given distinction as a category of difference in the introduction to the Common Core State Standards for Mathematics. The section, "Understanding Mathematics" explains how the "Standards define what students should understand and be able to do" (ibid, p. 4). In this, it is also made explicit what the Standards do not do, as "it is also beyond the scope of the Standards to define

the full range of supports appropriate for English language learners and for students with special needs,” (ibid). As certain kinds of people, the English language learner and student with special needs are classified as not already part of the *all*. They are to be included in the “opportunity to learn and meet the same high standards” through the use of “appropriate accommodation to ensure maximum participation” (ibid). This accommodation of difference, aimed at inclusion and participation in high quality mathematics, however, maintains the categories of equivalence and difference by which this child as the imagined other is excluded in the first place – always in comparison to a common identity given to the mathematically and linguistically proficient child who does not require modification.

Operating within an entangled social and pedagogical logic of equivalence and difference, the shifting cultural categories that standardized mathematical kinds of people became visible in the emergence and revision of formalized Standards for school mathematics. The discussion into the present articulation of Standards has been to broaden the scope whereby it becomes visible that the classifications that define who is/is not learning mathematics have historically entangled with shifting economic, gendered, racial, and linguistic categories to further narrow the boundaries of in/exclusion into math for all.

Reforming People and the Limits of Standardizing Equality

“**All**, regardless of race or class or economic status, are entitled to a fair chance and to the tools for **developing their individual powers of mind and spirit** to the utmost. This promise means that **all children...** can hope **to attain the mature and informed judgment** needed to **secure gainful employment**, and **to manage their own lives**, thereby serving not only their own interests but also the progress of society itself,”

(National Commission on Excellence in Education (NCEE), 1983, p. 9)

Largely in response to *A Nation at Risk* (NCEE, 1983) and *An Agenda for Action: Recommendations for School Mathematics in the 1980s* (NCTM, 1980), the NCTM Standards emerged as an attempt to overhaul math education so as to improve the economic, technological, and social lives of the nation and its people. This chapter has explored how the people that the nation called upon to secure its future were of a particular ‘mathematical’ kind who were seen as capable of making logical decisions, rationalizing their actions according to this logic, and managing their future by appreciating math as a tool for problem solving in the present. More directly, I have examined how the imperative to reform math for all worked to organize the standards and expectations for how children should think and act as these mathematical kinds of people.

To briefly summarize, the equivalence given to children as mathematical problem solvers was explored in this chapter through an analysis of the principles of reason that organized the use of the equal sign ($=$) in Standard math problems. Again, the equal sign was not examined merely as a symbol of mathematical logic. Rather, it was analyzed as a cultural practice that ordered notions of equivalence, identity, and difference. This relation between the mathematical and cultural inscriptions of equality was given emphasis in how the ordering of truth about equality and its logic in the math curriculum became entangled with tools of psychology that sought to understand and evaluate the child as a kind of person who interacts with the world and interprets it through a mathematical lens and language.

It was this relation, visible as a process of translation of math into math education, that worked to order a cultural truth about who the child should be as a mathematical kind of person. As ‘mathematical’, *everyone* was presumed to have inherent powers to use the language and reason of mathematics to solve the problems of everyday life. Through an appeal to a democratic notion of math as a universal model of this life, apparently *natural* modes of communicating and thinking mathematically established equivalences between children as common in their ability to gain mathematical literacy and have a greater sense of control over their present and future life choices, chances, and opportunities. As a distinction from the previous Back to Basics reform that emphasized individual differences, the use of the equal sign was reorganized from the perspective that “school mathematics should...transcend the cultural diversity of our nation,” (NRC, 1989, p. 19). This shift in emphasis provided new ways of ordering how children were to be seen in relation to others, defined by the same standards. Importantly, this way of seeing and evaluating children was not only to do with mathematics but emerged with the abstraction of math power that was given visibility as a cultural norm.

With mathematics education and its cultural value already tied to notions of citizenship during the New Mathematics movement, it became sensible to think during the Standards-Based movement that, “it is vitally important for society that *all* citizens benefit equally from high-quality mathematics education,” (NRC, 1989, p. 7). Yet, the Standards by which to evaluate children and how they did/did not benefit from mathematics education were not articulated solely in terms of participating as the right

kind of citizen. Rather, the Standards were rationalized as a way to develop the nation and improve one's chances for full participation in this nation, as equal kinds of people. In this way, notions of citizenship connected to math education reassembled with cultural principles of identity and equivalence to reinscribe who constituted "everybody".

As related to the concern about making mathematics accessible and equally beneficial for *all*, the logic of equality embedded in the common sense that "everybody counts" was explored here as cultural standards about who was everybody. This could be seen in how the use of the equal sign ($=$) was linked through the language of psychology to standards framing what all children should know and do to 'confidently' express math powers in the social space of the school. As methodical, organized, planned, and reflective, mathematically literate kinds of people represented the normalized ways of interpreting the world through the power of math as a language and mode of reasoning.

This mode of organizing equality and equivalence signified a shift in the Standard math reform from what historically came before. Recall how during the New Math reforms, all children were thought capable of learning math because they were all presumed to follow a similar and naturally progressing trajectory of thought that was based upon and leading to the formation of mathematical creativity. Giving identity to children as math learners who were to be innovative, independent, flexible, and imaginative of an abstract world removed from physical reality, the rules of equivalence embodied in the equal sign also organized difference as disadvantaged children whose thoughts were tied to a physical realm.

Granted as unable to express a mathematical creativity, this child would be given further distinction in the Back to Basics movement as having problems with motivation. As the cultural standard of mathematical creativity was resignified through the lens of a psychology that presumed learning as a process of discovery, rather than mastery, some children would be characterized as unmotivated to discover both the mathematics and the sense of self-esteem and control that it seemed to engender. With self-motivation as the factor driving success and inclusion, the inscription of the *all* during the Back to Basics movement entangled with the rules organizing the use of the equal sign (=) in the Standard math problems and framed the expectations for the person that would be motivated to benefit from the power of mathematics into the 21st century.

In a similar style of comparative thought, distinct ways in which certain kinds of people were not counted as part of everybody emerged as new ways of seeing people who lack confidence in their powers to formulate, communicate, and rethink the mathematical modes of communication that organized the problems of everyday life. The child who was imagined as a risk to the nation's progress and democratic survival was marked by a difference in ability to read, write, and speak about the world mathematically. Seen as mathematically illiterate, this person was presumed as unreliable, uncertain, and otherwise unproductive in terms of the economic, technological, and social production that was valued in the calculated effort to manage equality.

Generally speaking, discussions of math reform in the United States during the 1980s make some reference to what were considered the "Math Wars". In the narrative

of this historical marker, the battle lines are drawn between the content and skills, theory and method, text and context. Yet, in this chapter, I have in some ways stepped to the side of what has been commonly viewed as the battle ground about teaching all children good mathematics. That is to say, in an attempt to understand the systems of reasoning operating below the surface to reorganize the curriculum, I have drawn attention to the terms, rules of consensus, and conditions that have made this war possible. Without the importance and value given to math education in making a democratic and prosperous nation, citizen, individual, and people, a war about teaching children math would not be thinkable.

Emerging as the focus of this chapter, the image of the child as the mathematical problem solver was and still is integral to the narrative about how math education should be organized for all. As the practices of math pedagogy intersected with the principles of identity and difference expressed in the language of psychology, the standard problems of school math entangle with cultural norms categorizing and distinguishing what it meant to be a person who could *see, think, and speak mathematically* about the everyday world. Yet, this kind of person also embodies the limits of the reason of equality and how its very modes of representation reinscribed cultural notions of difference, as the inequality that reform is to solve.

CHAPTER 6

Crossing the Equal Sign:

Making Math Education and The Cultural (Re)Production of the Child

*After awhile making a proof is like making a calculation.
There are certain things you automatically do.
You move with closed eyes, clenched eyes
unseeing eyes, no eyes.
You move, sometimes with no brain.
After awhile crossing the implication sign is like crossing the equal sign.
After awhile a proof is collapsed to a point.*

-Marion Deutsche Cohen (2007)

This poem has served as a guide for me in historically *crossing the equal sign* to consider how its use in the elementary math curriculum carries cultural, political and social modes of organizing notions of in/equality. Following traces of the sign in the curriculum from the 1950s to the present, I have examined how the rules that order the pedagogy of equality instantiated in the equal sign are not situated outside of culture or history as ‘pure’ mathematics. Rather, they are embedded in the very systems of reason through which the knowledge of equality comes to matter in the teaching and learning of school math.

Inspired by the poem’s opening “after while”, I have historically analyzed *math for all* as an event that shapes and is shaped by the cultural ways of reasoning in which equality and the equal sign both seem automatic – as taken for granted truths in the (re)organization of the math curriculum. In my crossing of the equal sign, I have attempted to open up space to reconsider how instruction, curriculum, policies, and research have become entangled with principles of identity and difference that (re)organize who children are and should be. This opening has been to bring attention

to the commonsense and apparently ‘natural’ ways of seeing children as ‘mathematical’ kinds of citizens, individuals, and people. With this in mind, this chapter will not aim to collapse the historical data, as proof, to a point so as to provide calculations for “what to do” in the management of in/equality. Instead, it will continue to question the very sensibility by which this seems possible.

The chapters, both separately and together, have aimed to explore how the use of the equal sign (=) in the reorganization of the math curriculum intersects with cultural principles of identity and difference. More than that, the analysis has been a way to examine how those principles give intelligibility to both the mathematical notion of equality and the child that is to learn it, as part of the cultural expression of *all*. At one level, the equal sign (=) has served as a foundational fact organizing how children are to *know and do* equality in school mathematics. On another, in order to make sense in the cultural and social spaces of the school the sign has been translated through the changing language of psychology to identify who children are and who they should be as certain kinds of math learners. The analysis, then, has provided a way to see how the principles of identity and difference intersect as both cultural and mathematical ways of knowing and acting upon equality, as a logic carried in the equal sign.

As such, in this chapter, I will continue discussing the ways in which the equal sign (=) in the school math curriculum has historically related to cultural rules and practices, as ‘truths’, that give materiality to the identities and differences by which to classify and compare children as part of/not part of the all. In this, I emphasize how what has appeared as a universal notion of the “fact” of equality is, in fact, historical. By

that, I mean that the use of the equal sign in the curriculum has been open to change and has resignified its relation to cultural inscriptions of equality that continuously disrupt its commonsense as a neutral fact of school mathematics.

To broadly retrace the historical and cultural trajectories that relate the equal sign to narratives and images of the child as *mathematical kinds*, I highlight the contingencies upon which these classifications have been fabricated to give further distinction to what is equal, equivalent, and different in the making of math for all. This entails a consideration of how the psychologies that translated mathematics into the cultural spaces of the school organized a process of “alchemy” (Popkewitz, 2004) whereby the equality that seemed to be merely math transformed into much more than that. Most directly, as an interpretive lens by which to filter math learning into the school, psychology provided the cultural rules around which to organize this ‘learning’ as developing norms of creativity, motivation, and confidence in all children. In reiterating how this mathematics has come into the schools as a way of shaping who children are and should be, I provoke how this modeling is not simply representing what is there but is involved in making something in the process. In this, the child as a mathematically confident, motivated, and creative kind has been questioned as an empirical reality and treated instead as an historical construction that makes it appear as real.

This chapter concludes with a discussion of the possible implications of this study and speaks to alternative trajectories of exploration that extend beyond the scope of what I have done here. In this way, the conclusion is not necessarily a conclusion but an

opening that considers how this study of math curriculum relates to international and comparative educational research across multiple fields and sites of inquiry.

After awhile...

As I developed a way of seeing how the equal sign was related to and entangled with a narrative of how to create a math curriculum that seemed universal for all, I started to *unsee* the equal sign as merely a mathematical truth. In this, the universality of the knowledge of equality was disrupted. This disruption is visible as change that “deprives the self of the reassuring stability of life and nature...It will uproot its traditional foundations and relentlessly disrupt is pretended continuity. This is because knowledge is not made for understanding; it is made for cutting,” (Foucault, 1984, p. 88). After awhile, I began to see the change that characterized the problem of reform as (re)making children into certain kinds of people.

In a field of education that seems unchanging, disrupting the fact of the equal sign and its logic that organizes how to see, think about, and act upon equality allowed me to see cracks in the foundation of *math for all*. It was in these cracks that I could see how differences were produced as distinctions from the creative, motivated, and confident learner of math that gave sensibility to the organization of math for all.

This mathematical learner, I argued, has been identified as a particular “kind of person” (Hacking, 2002) who can use math as a tool for living a rational, logical, and certain kind of life – in an uncertain world. This vision of using math “as a tool for modern life,” (CEEb, 1959) emerged as part of organizing the elementary curriculum

“for all students, at all levels” (SMSG, 1966a) during the New Math period, but was not specific to it. That is, across all three periods of reform – New Math, Back to Basics, and Standards-Based – mathematics was granted as a reasonable mode of thought by which to (re)organize cultural and social life and manage what were taken as democratic notions of progress, prosperity, and equality.

In the following three sections, I will give attention to how the ways of thinking about equality and equivalence have shifted and reassigned the cultural inscriptions of identity to (re)organize children and their learning of mathematics. This will serve, not necessarily as a summary, but as a way to emphasize how shifting psychological norms intersected to give meaning to the math learning for all children and worked to classify difference and give finer ‘scientific’ distinction to who is/is not the *all*.

Each section will revisit the particular periods of reform and will also be couched in a discussion of the present. This is to provoke how, while the reforms can be given distinct temporal demarcations, the features that assembled to characterize the math learner circulate and change over time and are also visible in the present as taken for granted ways of seeing the child as mathematical.

“There are certain things you automatically do”

During the New Math movement in the post war United States, “mathematical creativity” (Piaget, 1950, p. 151) emerged as a condition of possibility for all children to learn modern mathematics. In other words, since children were not seen as master mathematicians, mathematical creativity – as a common characteristic of thought in all

people – would make it reasonable to think that children could learn set theory as the foundation of the New Curriculum. In the first years of this New Math curriculum, the use of the equal sign ($=$) intersected with psychologies that assumed children could master the content, given its common structure, to translate the mathematical notions of equality and equivalence into math for children. Through the lens of developing modes of creative thought, all children were assumed to follow a common trajectory of thinking from the concrete to the abstract as a way to develop psychological traits of mathematical forms of creativity.

Given as a new way to see children, mathematical creativity signified the space of who all children are and should be as math learners. Recall how the discussion of solving expressions such as $_____ + _____ = _____$ required children to invent and create a numerical representation of equality from images of penguins, monkeys, or trees. In this, the modes of reasoning about and assigning identity to equivalent objects resembled the cultural rules that ordered the creation of an identity for children as members of an inclusive set.

This was visible in how all kids were expected to use materials to solve expressions such as $2 + 2 = \square$. Embodying modes of thought that were to be internalized as “a way of arranging knowns and unknowns in equations so that the unknowns are made knowable,” (Bruner, Goodnow, & Austin, 1956, p. 7) this method of using concrete things to represent abstraction was more than a way of working with materials to produce understanding. It also carried cultural norms inscribing an identity

of the child who was able to think abstractly, precisely, and flexibly beyond the immediacy of physical objects – as the expression of mathematical creativity.

With creativity as a seemingly natural and agreed upon “aim of all science and all mathematics at all levels and in all classrooms,” then it seemed reasonable to think that all children were to be given the opportunity to participate in the mastery of mathematics (Mallinson, 1960, p. 26). As creativity became historically linked with math and science learning, it seemed almost automatic to assume its importance in reforming *math for all* – since it was given that “every person possesses creativity,” (ibid, p. 25). Therefore, it seemed reasonable to assume it as a *natural* condition of learning math.

As was discussed in Chapter 3, this form of mathematical creativity embodies limits that work to distinguish the child who is not learning. In the period of New Math, this child was imagined as socially, educationally, economically, and culturally deprived (ESEA, 1965; SMSG, 1966a). Emerging as a new kind of social problem, the deprived or disadvantaged child also became a focus of the curriculum and the policies that were concerned with its reform. Notably, the differences that seemed to represent the child who would have trouble learning math were most recognizable along economic lines. That is, “the mental style of the socially and economically disadvantaged learners resembles the mental style of *one type* of highly creative persons. Our schools should provide for the development of these unique, untapped national sources of creativity,” (Riessman, 1964, p. 231). In this statement about the kinds of people and styles of thought that could, but did not, represent nationally valued forms of creativity, social

disadvantage was equated with a calculable and precise measure of economic disparity.

Mathematics and the creativity it was thought to produce seemed to serve as both a guard against *and* a sense of security for the child who was poor, disparaged, and troubled by bleak economic circumstances. The equivalence given to mathematical creativity as the cultural norm for math learning delimits the economically disadvantaged child as the different 'other'. Yet, the fear of these kinds of children, seen as single minded, slow, spatially oriented, and physical rather than mental, was also coupled with a hope that they, too, embodied a certain style of creative thought that could be of value to the nation and its development. It was just not of the 'mathematical' kind.

In some ways, it presently seems so natural to think that mathematics and national progress go hand in hand. This is evident in the *Trends in International Mathematics and Science Study* (TIMSS) assessments of math and science knowledge amongst 4th and 8th grade students. In this survey and report, individual student, teacher, district, and state level scores (where they exist as such) are compiled to create and identify an aggregate measure of a nation's achievement as compared to others. However, in the TIMSS, one's responsibility to learn math and science and participate in building a nation's identity as successful is not reasoned in terms of creativity. In fact, it seems that little attention is given to children's creativity as the basis of math teaching, learning, and assessment. Nevertheless, the characteristics of mathematical modes of creativity are still embedded in contemporary discussions. They have merely shifted and now appear as a necessary requirement, not a condition of

possibility, for math learning. In this, the expressions of mathematical creativity and its presumed “disadvantaged other” are not the same as those that emerged in the New Math reforms.

This shift is important and visible in how the psychological traits that historically defined what emerged as an elusive, yet indisputable, form of mathematical creativity are given as a prerequisite for participation in contemporary curriculum reforms. In this, the notions of mathematical modes of creativity have merged and reassembled with psychologies that organize learning through the assumptions of discovery and social interactions. Signifying a discontinuity with the psychological traits assigned to mathematical creativity, it seems to be reassigned as something else.

Given new distinctions and classifications as part of the commonsense of “constructivism”, for example, the notion of creativity embodied in the contemporary *Everyday Math* program is based upon research maintaining that the curriculum “should encourage the children’s own construction of knowledge,” (Isaacs, Carroll, & Bell, 2001, p. 5). In this, children are expected to *create* their own mathematical knowledge, integrating it with what they bring into the classroom. Constructing a sense of the relationships between inverse operations, like addition and subtraction, children are to see a set of four facts as a “family” (McGraw-Hill, 2011, p. 169). Taking numerical identities from the image of a domino, children are presented with the problem of creating the family as a series of two addition _____ + _____ = _____ and two subtraction _____ - _____ = _____ expressions. In doing so, the child is to demonstrate a mode of mathematical creativity that is related to but distinct from traits of

the independent, innovative, flexible, methodical and abstract math learner who can and should understand inverse operations and the relations that signify them. More than that, however, this child is also expected to generate the family so as to build his/her “own”, yet common, understanding.

As the assumption of creativity in all people assembled with psychologies of mastery learning in the New Math period, mathematical creativity emerged as a condition of possibility for all children to learn math. However, as the notions of creativity merged and reassembled with psychologies based upon ideals of discovery learning, the notion of mathematical creativity traveled with the equal sign (=) and were resignified and reassigned as different psychological traits. To explore this change, we can look once again and more closely to the problem and the promise of motivation to learn mathematics that emerged during the Back to Basics movement.

“You move with closed eyes, clenched eyes”

The psychological traits of creativity embedded in the use of the equal sign (=) in the New Math curriculum provided the historical conditions whereby it would be possible to think that children could create their own mathematics in the Back to Basics movement (and in the example from the contemporary *Everyday Math* curriculum given above). Recall the discussion from Chapter 4 of Leslie and her “child-created mathematics” (Cochran, Barson, & Davis, 1970) in which she was to create a graph to explore mathematical equality with the open number sentence $\square - 3 = \Delta$? Yet, producing this graph and giving meaning to the equal sign as the basic representation of equivalence and equality was not necessarily to express a mode of what had been

constructed as mathematical creativity. So while the emphasis on “child-created” implied notions of flexible, independent, and abstract thinking, this problem solving was to provide Leslie with the opportunity to discover what was thought of as her “own” sense of mathematics and self through her individual interests, intuitions, and motivations.

Seen from the perspective of *The Psychology of Learning Mathematics* (Skemp, 1971) Leslie’s awareness and discovery of self was key to learning even the most basic mathematical concepts. This was based upon the psychological truth that each person seemed to have “his own conceptual system,” and “the actual formation of a conceptual system is something each individual has to do for himself,” (ibid, p. 28-29) as a mode of self-discovery. This way of reasoning about math learning as largely a matter of differences in individual personality, interests, and intuitions assembled with the practices of problem solving in the curriculum and reassigned the identity ascribed to the child who was/was not learning math. In this shift, the mathematics in its most basic form was to be organized around a “natural motivation” so as to bring the study of math closer to the each individual child and allow for the possibility of discovering math as an interesting and useful system of thought (Kline, 1973, p. 149).

Enhancing the so-called natural motivation of each individual student provided the psychological lens through which to envision an effort aimed to create an “equality of optimum motivation” (Nicholls, 1979). Equally motivating *all students* to the best of their own abilities, desires, and needs required a view of mathematics that moved from the outside, as a static and structured external source of knowledge to be mastered, to one

that was to be discovered and internally negotiated with one's own motivated sense of self-direction, control, and interest.

In this shift, the traits given to describe Leslie and her learning would serve as the image of the child who could express her/his own motivations to learn math.

Interestingly, these characteristics ascribed to children who could be seen as successful were determined primarily through a psychology that sought to understand the individual differences between children that were doing well in math and those that were not. In the larger scope of such studies, particular attention was given to understanding the "unexpected and disconcerting" distinctions in precocity drawn along gendered lines (Astin, 1974). With mathematically precocious girls characterized as "naturally" more sympathetic, tender, conscientious, and shy, it seemed quite clear that they were not like their male counterparts.

In this classification, it seemed that, "the only distressing trend was the finding that the parents of girls have rather average or low expectations of their daughters considering the fact that these girls are exceptional with respect to mathematical aptitudes," (ibid, p. 84). This exceptionality was based upon a comparison with the normal attributes of the precocious boy whose achievement in mathematics was apparently influenced by factors such as sociability, self-acceptance, well-being, responsibility, and self-control (Weiss, Haier, & Keating, 1974). Qualities organizing the girl's mathematically precocious sense of self were not given to understand how or why they succeeded but were taken to explain why they were not motivated to achieve to the

level of the boys. In some ways, then, Leslie's problem solving with the equal sign could be seen as the exception to the rule.

Importantly, these gendered norms and distinctions that would further categorize the child who could/could not learn math as part of the all also intersected with racial classifications. In this, the child who seemed to have problems with math was not simply presumed to lack certain forms of creativity, as was the economically and educationally "disadvantaged child" that emerged as a problem of the New Math curriculum. Still identified as the "disadvantaged" (Kaplan, 1970), this disadvantage was given finer distinction in comparison to the norms of self-control, -regulation, -discovery, and – esteem. As the cultural signification of the child equal to the motivated *all* shifted and produced a pathology represented as the "mathematically disabled", "inner city", and "problems of motivation", the child who did not seem to (want to) learn math became visible as a problem of the curriculum (Keiffer & Greenholz, 1970).

Over time, this "inner city" problem would assign trouble with math learning to students who "are mainly black, Spanish, and other minorities. They come from inner-city areas, and, in fact, thereby have developed negative attitudes toward mathematics" (Williams, 1974, p. 650). As the distinction of "inner city" moved out of the city limits, the disadvantaged and unmotivated math learner became visible as a non-white male, "minority" student. Without even seeing the child, it would become possible to assume trouble with math learning based upon, not only economic distinctions, but also gendered and racial classifications. This fabrication of the "inner city child" is tied to mathematics through a cultural thesis about who is/is not achieving. That is, it is a

cultural category that emerged to give intelligibility to certain kinds of people that appear not as abstractions, but as 'real' problems that frame how to deal with and think about these people and their place in the world, as different.

As though with eyes closed, certain children could be seen to have the choice, opportunity, and potential to do well in mathematics yet lack the motivation to do so. This link between a seemingly natural motivation to learn math and individual achievement is part of the alchemy that historically produced an image of the child who is not interested in and cannot control his/her own success in mathematics. This way of reasoning about producing *math for all* is not isolated in the period of reform in which it emerged and so moves into present discussions as the commonsense of math pedagogy.

In one such discussion, a child named Cameron was thought to have potential to learn math, but was apparently not self-motivated enough to do so. He could evaluate the expressions $5 \times 3 = \underline{\quad}$; $7 \times 2 = \underline{\quad}$; and $4 \times 6 = \underline{\quad}$ but did not want to. Seeing Cameron in this way through the problems he solved allowed for the conclusion that *if only he was more motivated, he would be interested in and able to learn math at a higher level*. Another teacher seemed frustrated with this conclusion. Speaking up later, he mentioned that as a child he was considered to be unmotivated; the one that could not seem to control his own efforts to learn; could not manage his own discovery of mathematics; could not calculate his interest and enthusiasm in the subject. *The problem was assumed to be in him and the task of the teacher was to manage a sense of motivation, control, and self-discipline he did not seem to have.*

There was no decision reached in terms of “what to do” in this conversation. Yet, the question was raised, “How else might it be possible to think and talk about this student and his mathematics if the problem of learning is not placed in him?” This vignette is not to displace the problem from the student to the teachers, thereby suggesting that *if only teachers thought harder*. Rather, it is to provoke how the taken for granted and automatic ways of thinking that relate a child’s achievement in math to matters of motivation historically (re)present what constitutes the problem and carry implications for seeing/unseeing who could learn math as well as how they are to do it.

“You move, sometimes with no brain”

While helping my daughter, Ada, with a math worksheet she brought home from preschool, I was immediately struck by how similar the problems were to those of equivalence and equality that emerged with the introduction of *set theory* during the New Math reforms. Given images of sets of objects, she was to fill in the numbers to formulate expressions _____ + _____ = _____ and solve the problems. In talking with her about the work on the sheet, she seemed to know what to do and had some language with which to talk about “plus” and “equals”.

Almost without thinking, I prompted her with questions, perhaps to “get inside her head” (Schoenfeld, 1985) to see how she was thinking about what she was doing. I posed questions like, “What do you think?” or “If you know that $2 + 2 = 4$, then what does $2 + 3 = ?$ ” to which she sometimes replied, “I don’t know”. When I asked, “Does that make sense?” about the number she wrote to answer a subtraction problem about mittens she said, “Yes it makes sense because we gave some of the mittens to the

animals outside because they are so cold.” Her answers seemed both logical and illogical, playful and precise at the same time. For the final problem on the worksheet, she wrote a number and I asked her, “How do you know?” To this, she responded with a look at me then at the images on the paper seeming to imply, *you just watched me count them, that’s how I know.*

At one point as I worked with her to translate the model of the pictures into written numerals that were to represent her thinking, she appeared to be frustrated. “I know how to do it!” she insisted. Even though it appeared as though she was not sure about *how to do it*, she looked at me and said, “I can do it like Milli!” Turning our conversation, she explained to me that Milli is a girl on Team Umizoomi, a cartoon show that uses the Common Core Standards as a guide and is “designed to help kids develop their skills with and build their confidence in problem solving” (Team Umizoomi Curriculum, 2014). The premise of each episode is to embark on a mathematical adventure so as to help children understand and appreciate “Mighty Math Powers” and “gain the confidence to solve any and every math problem that comes their way, in the show, and in their lives!” (ibid). Seeming to model what problems may arise and the necessary tools with which to think and act upon them, the Team invites children to interact with characters Milli, Geo, and Bot to use “math language as they analyze problems, compute equations, and measure and compare items,” (ibid).

The mathematics that forms the content of this show and Ada’s worksheet is about so much more than identifying math concepts. More specifically, it intersects with cultural and historical modes of reasoning about identity that order her math learning

according to psychological traits of the *confident*, *motivated*, and *creative* math learner. As such, the mighty math powers and the problems that organize these powers seem to be naturally expressed by the categories and classifications that have emerged over time in the history of children's math reform from the 1950s to the present.

While appearing as similar inscriptions of equality and the equal sign (=) from the New Math curriculum over fifty years ago, the related cultural logic of equivalence and identity that organized how Ada appeared to think, see, and act in solving the problems on her worksheet were not necessarily the same. In relation to that historical period, the psychological lens through which to see her as a math learner and the tools of translation that organized learning appeared to embody a different cultural principle of mathematical creativity. Put another way, in the present, Standards-based inscription, mathematical creativity was no longer the goal but continued to organize the reasonable ways of understanding early math learning. In this shift, mathematical creativity became a prerequisite, a necessary component of math learning that was thought to organize the kind of "play", fun, and enthusiasm that was imagined to motivate the child. This could be seen in what could be described as Ada's playful and imaginative (and perhaps illogical) story about the mittens.

Given as a fruitful ground for developing a rich mathematics understanding, the notion of creativity and playfulness embedded in Ada's problem solving could be seen as a condition of possibility for all children to be motivated to learn every day concepts of mathematics (Ginsburg, 2006). More than that, from the perspective of the Standards-Based Curriculum, that motivating playfulness could be interpreted as a

“habit of *creating* a coherent representation” so as to contextualize the problem and demonstrate the ability to “reason abstractly and quantitatively” (CCLS, 2013, p. 6, my italics). Either way, since its emergence during the Back to Basics movement, the notion of motivation has entered into the organization of math and has been part of the alchemy that operates to produce the child as mathematically creative. For all children to be seen as capable of motivating to discover their own mathematics, they also are to be thought of as able to create this math as their own.

However, as the psychological principles of learning math shifted from mastery learning, to an assumption that children would discover their own mathematics, to an organization of math learning around social and emotional relationships, the ordering of motivation shifted and became reassembled as a mode of organizing confidence. Emerging during the Standards-Based movement as a new and normalized way to see the possibility for all children to learn math, confidence became the lens through which to evaluate math power and its expression as mathematical literacy. Assembling with notions of motivation, this confidence was to be built through the use of a seemingly universal math language. As models that would motivate the child to make confident and logical sense, the representation of mathematical thought would also frame the social interaction that learning math seemed to require.

Since its emergence as a condition for *all* to learn math in particularly ‘confident’ ways, the universality of mathematical language has been questioned (Whiteford, 2009/2010). The response to “Is Mathematics a Universal Language?” introduced Melita as “a sixth grade student and recent immigrant to the United States,” (ibid, p. 277).

Based upon the claim that “for students such as Melita, the challenges of learning mathematics can be as great as those encountered when learning to read, speak, and write in English,” it seems as though “teachers must also acknowledge English Language Learners’ (ELLs) “first mathematics,” the math they bring with them (ibid). However, in this effort to include Melita is a necessary classification of what seems to be the difference between her and the other, unnamed non-ELL math learner.

With the standard expectation to use math language continuing to frame the commonsense of learning in the Common Core Standards, a new classification emerges to give finer distinction to divide the children that are included in part of the all and those that need to be made into the all. In this, Melita’s math learning is to be differentiated through new linguistic classifications that provide a way to see and think about how to “explore the differences in ELLs’ procedural and conceptual knowledge,” (ibid, p. 278). The recognition of what constitutes these differences, although framed in terms of honor, respect, and validation, are placed in the child as problems of math pedagogy. Nevertheless, seeing Melita’s math learning as an issue is rationalized in how it seems that “once we have considered these issues, we can then identify and implement ways of modifying, adapting, or differentiating classroom experiences,” (ibid, p. 282). That modification aimed at goals of inclusion, however, reinserts the category of English Language Learners as the representation of difference, in comparison to the identity given to the normal math learner who does not need modifications.

Historically entangling with economic, gendered, and racial distinctions, linguistic categories have become visible and worked to further narrow and define who is/who is

not learning mathematics. The new Common Core Standard of “mathematical proficiency,” assumes and requires a sort of linguistic proficiency, in English. Here, the comparison drawn between Melita’s efforts to read, write, and speak as a form of literacy, seems to historically make sense. Mathematical language – as the expression of a mathematical kind of literacy – has embedded in it the power to see, communicate, (re)think, and make sense of the problems that constitute the everyday. Yet, it also carries with it the limits and historical conditions whereby the possibility to include all children in the math curriculum does not make much sense and, in fact, seems impossible.

“After awhile crossing the implication sign is like crossing the equal sign”

The ‘commonsense’ of math and the social commitments of schooling are today expressed as: All students can learn math because they can learn to think creatively. Each child can learn math because s/he can be motivated to learn it. Everyone can learn math because math is a language and its use will make children confident in their abilities to solve the problems of everyday life. Historically, these statements have served to organize the reason of equality, equivalence, and inclusion embedded in *math for all* reforms. Over time, this reason has shifted – inscribing different cultural principles and standards of who is to be included in the *all*. Throughout the dissertation, I have worked to make these shifts visible by historically tracing the equal sign (=) as it has crossed over three periods of elementary math reform in the United States from the 1950s to the present.

This exploration has incited, for me, new ways of thinking the ‘reason’ of math pedagogy and its implied cultural assumptions about who children are and how they are to learn math. Embedded in this commonsense are political implications for who can learn math as part of the *all* as related to expressions of equality, equity, and inclusion. By analyzing the discourses that link the cultural logic of equality to the knowledge of the equal sign in school mathematics, I have examined how these implications for *who can learn and how* are visible as truths that generate rules governing how children are to think and act as *mathematical kinds of citizens, individuals, and people*.

As effects of a productive form of power, these associations between mathematics education and modes of living as certain kinds of children operate to organize the limits and possibilities for thinking about and acting upon the knowledge of equality, equivalence, and identity in making math for all. Discussed throughout the dissertation, the possibilities and limits emerge together as the norms that define the boundaries of inclusion and rationalize the rules of exclusion. As such, the implications of ordering how to see, think about, and establish equality are neither bad nor good, but dangerous (Foucault, 1988). This danger is evident in the paradox that becomes recognizable as the limits of equality, as a system of knowledge ordering the rules for how to include as well as how to differentiate and divide.

Yet, “if everything is dangerous, then we always have something to do,” (Foucault, 1984, p. 343). For me, this *something to do* has been to research unexplored histories and unthought of ways to study the reforms of mathematics education. This has been to disrupt the status quo and raise serious questions about the systems of

reason that relate cultural and mathematical notions of in/equality – not as stable facts but as historically particular ways of seeing and organizing the world. As historical and not necessarily natural, the taken for granted rules and practices that order pedagogical thought and action in mathematics education can be seen as open to change as the effect of power. Importantly, this power has not been studied as one that can be grabbed, taken, or given but rather as an effect that circulates and impacts the cultural, political, and social relations that frame what can be seen, said, and done in remaking math education.

By placing the equal sign (=) at the center of this historical analysis, I have been able to make visible the power that produces the reason of equality as the rules of identity and difference through which the subjects of the math curriculum are constituted as included/excluded in the all. Here, and throughout the analysis, the inscription of *subjects* implies a double meaning, as both the mathematical content that makes its way into the curriculum and its relation to the subjectivities created to identify the kinds of children that can and will succeed in mathematics. By assuming that children are made, and not born, as mathematical kinds of *citizens*, *individuals*, and *people*, I have worked to examine the cultural logic that gives sensibility to this construction.

This displacement of the *a priori* subject has had implications for how I have approached the historical exploration of what I have called the problem of in/equality that (re)organizes math curriculum, instruction, research, and reform. In this approach, I have worked to place the problem in the system of reasoning about equality – not in children, teachers, psychologists, or mathematicians. No autonomous historical actor is

the problem, the reason for change, or the resistance to it. Rather, the problem is how the historical conditions that make it possible to (re)organize mathematics education embody the very limits of the logic of equality and continuously reinscribe cultural norms of identity and representations of difference as commonsense ways of seeing, organizing, evaluating, and instructing math for all children. As such, the implications of a study such as this can not lead to a conclusion about methods for changing the way kids, mathematicians, psychologists, teachers, or policy makers think – so as to make them better, more effective, efficient, more accepting, more open to others. So, what could this mean for *what to do now*?

“After awhile a proof is collapsed to a point”

In what ways does this study offer possibilities for rethinking the links between math education and citizenship that seem so natural? Where could there be space to reconsider the ways in which mathematics education has been historically entangled with making a sense of self? Where might one look for an opening in which alternative ways of seeing ‘mathematical’ kinds of people might emerge? In rethinking the making of math and its learners, what might a challenge to the assumed and sensible notions of creativity, motivation, and confidence produce? More broadly, how does this study provoke questions about how research in the field of math education and its history might be done? Even more, how might curriculum (not only in mathematics) be studied as an historical event, open to the process of alchemy and change?

These are possible questions to ask now. There are many more and various directions in which a study like this could extend. One trajectory would be to consider

how this research relates to concerns about improving and extending education in Science, Technology, Engineering, and Mathematics (STEM) as a field of curriculum that appears to be critical to the formation of the nation and its future, in comparison to ‘others’. This unforeseen future and the promises of planning it through STEM education seems to offer possibilities for prosperity while also defining the “capacity and diversity of the STEM workforce pipeline” (STEM Education Coalition, 2014). Taking the contribution of this study as a starting point for further studies of curriculum, the particular kinds of people who are seen to be prepared for “success in college, career, and life,” (ibid) could be studied, not as objects given in the world, but as subjects fabricated in the making of schooling.

The point that has been reiterated over time is that the equal sign is more than a mathematical fact that embodies cultural practices and rules that inscribe particular ways of thinking about and organizing the knowledge of equality. To begin the research and locate the problem, I drew upon the notion of the “modern fact” (Poovey, 1998) to think about how the equal sign ‘acts’ to order notions of identity, equivalence, and difference. This has been to offer the possibility of rethinking how the very reorganization of curriculum, through its interactions with psychology as ‘truth’, produces theories about children that operate to make them appear as certain kinds of people. The *math learner*, as a general abstraction, has become a ‘real’ category that assembles with cultural distinctions to order statements and their truth-value about the child who learns math, as well as identify the child who is not that.

This image of the child as psychological was introduced in Chapter 1 as a way to speak to an inadequacy with both ‘mainstream’ and presumably alternative ‘critical’ curriculum reforms that start from the assumption that a child’s development is the means by which to achieve social equality and justice. While both approaches seem to be in opposition to one another, they are related in the sense that they are organized within a logic of equivalence and difference to represent who children are now and who they might become through a ‘emancipatory’ pedagogy of mathematics.

In this way, my study has aimed to provoke how a seemingly liberal form of equality is paradoxically embodied in the math curriculum and its reform for *all*. As a ‘kind’ of equality, liberalism presumes and requires a way of representing what is equal, equivalent, and different to give meaning to what would otherwise be an abstract hope of including all children. As the founding term of political and social organization in the U.S. is an abstraction of equality, its knowledge also serves as the foundation of children’s mathematics in schools. I do not consider this to be a historical coincidence but instead a cultural commonsense that frames a particular way of reasoning about the ordering of equality in schools. Within the concern that this relationship between the pedagogical and social logic of equality has remained unproblematic and unquestioned, I have placed the very representation of its knowledge in the equal sign and the cultural processes of making of people equivalent as central to this study.

As such, this examination of the logic of equivalence and how it acts within a system of reasoning to organize the rules by which to order difference makes a significant contribution to studies of curriculum reform. By highlighting how the

comparative style of reasoning about equality embodied in the use of the equal sign is related to notions of liberal egalitarianism, I have shown that the ways of signifying individuals as equal shift over time while continuing to (re)insert new categories of distinction that make up certain children as the representation of difference and inequality.

In connection to a discussion of *Liberal Equality* (Gutmann, 1980), this concept of equality is related to notions of social justice in ways that both *describe people* as equal beings and *justify* a more equal distribution of goods, services, and opportunities among those people (ibid, p. 2). In this, the presumed potential of liberal egalitarianism and its organization rests on the capacity, ability, and responsibility for individuals to choose to make the ‘right’ choices, so as to distribute things equally for themselves and others (ibid, p. 3). As theories of social justice and equality assemble with and enter into discussions about mathematics education, the notions of inclusion, diversity, and equity assume a particular kind of person who can make the ‘right’ choice. Yet, who that person is and what that capability requires have been left unquestioned and are assumed as the normative description of the ‘right’ kinds of people.

The choices and the choice maker (as the individual who appears to define and distribute things equally) are both bound by the historical chains that signify and represent what constitutes ‘equal’. In this, it becomes reasonable to think that some people can and should receive the opportunities that seem available to them through math education, while others cannot and should not. Translated through the norms of creativity, motivation, and confidence, the ‘reason’ of *math for all* inscribes a cultural

thesis that works to define who are the equals as well as the others. Studying the alchemy of the equal sign in school math and its cultural sensibility of equality as tied to liberalism has been to problematize how the efforts to include work to exclude.

Since its initial inscription in *The Whetstone of Witte* (Recorde, 1557), in which the Welsh mathematician apparently sought to avoid a repetition of the words ‘is equal to’, the equal sign was marked on the page as two parallel lines that would appear again and again, across time and space, to signify a particular way of thinking about and seeing one thing as equal to an other, given difference. The mathematical use of the equal sign that I have examined is one that gets into the school some 400 years later. As the content of mathematics that children are to learn as citizens, individuals, and people involved in constructing a particular kind of future for the United States, the equal sign (=) has appeared to play an integral role in organizing the principles of equality it presumes.

With both notions of identity and difference involved in the ordering of equality in curriculum reforms aimed at *math for all*, it can be said that in the recurring calls for diversity and inclusion the “greatest danger is lapsing into the representations of a beautiful soul which says: we are different but we are not opposed,” (Deleuze, 1994). It is this danger of representation that points to the conditions by which difference has been historically constructed as the exception to that which is seen as ‘naturally’ beautiful, valuable, and appreciated.

In closing, I want to distinguish the psychological image of the child from an image of the child as political. In this, the task is not to develop the child toward an

already determined goal of beauty, value, appreciation, progress, equality, or justice. Nor is the task to bring the child into the world of the political because she already “is political even before she goes to school to become autonomous and emancipated,” (Bingham & Biesta, p. 57). Perhaps, then, the task of “what to do” is to continuously provoke the categories and distinctions that historically (re)make the child in the image of psychology and (re)semblance of equality.

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