ESSAYS ON INTERNATIONAL TRADE NEGOTIATIONS

by

Patricia Anne Mueller

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The dissertation is approved by the following members of the Final Oral Committee:
L. Kamran Bilir, Assistant Professor, Economics
Robert W. Staiger, Professor, Economics
Charles M. Engel, Professor, Economics
Thomas F. Rutherford, Professor, Agricultural & Applied Economics

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Abstract

Chapter 1: I examine the welfare benefit of committing to a trade agreement when a politically-motivated government faces monopolistically competitive firms lobbying for tariff protection against imports. In my model, lobbying is costly to producers: each firm must pay a portion of the industry's upfront-lobby-formation fee as determined by a sequential bargaining game. I show that monopolistically competitive producers under-hire capital to avoid paying a larger share of lobbying costs. As a result, more varieties are produced than is socially optimal, and each firm operates at a higher-than-optimal marginal cost. Commitment to a trade agreement leads to a consolidation of firms in the market and a reduction in the tariff level. I show that a government benefits from committing to a trade agreement (i) on goods with very elastic or very inelastic demand, (ii) when the weight the government places on receiving political contributions from lobbyists is small, and (iii) when producers are strong bargainers and are able to capture the majority of the rents from protection.

Chapter 2: In a model in which a small-country government faces domestic political-economy pressure and uncertainty over its terms of trade, a welfare-maximizing government benefits from having access to both rigid and flexible tariff policy options. I show that a welfare-maximizing government may prefer to join both a deep-integration regional trade agreement (helping it commit to free trade with key trading partners, thus correcting production distortions resulting from rent-seeking by politically organized sectors), and a large flexible agreement like the World Trade Organization (providing the flexibility in tariff levels and the institutional structure to pursue temporary tariff protection in response to shocks or unfair trade practices). My paper builds on the framework of Maggi and Rodríguez-Clare (1998), adding uncertainty over world prices and adding the option to join a trade agreement with the flexibility of a temporary escape clause.

1 Commitment and Trade Agreements with Monopolistic Competition and Costly Lobby Formation

1.1 Introduction

There is no doubt about the importance of domestic political pressure as a determinant of trade policy around the world. However, there is room to improve the way domestic lobbying is handled in trade policy models. In this paper, I present a new approach in which, firms do not directly negotiate with the government over tariff protection, instead firms must hire a lobbyist to negotiate on behalf of the industry as a whole. This feature combined with a monopolistically competitive production structure presents a more realistic picture of the role domestic political-economy pressure plays in observed tariff policies. I find that when the lobbying sector is monopolistically competitive, firms choose to be smaller (in terms of capital investment) to avoid having to pay a large share of the upfront-lobbying cost, which is convex in the size of the firm's capital investment. I also demonstrate that when a government has the opportunity to use a trade agreement to correct political-economy-induced distortions, it always benefits from correcting the distortions in industries that produce very homogeneous or very differentiated goods.

Over recent decades, liberalization of world trade has raced forward in a variety of forms with an increasingly diverse array of countries. At the founding of the General Agreement on Trade and Tariffs (GATT) in 1947, only 23 countries comprised the agreement, the majority of which were developed nations. Since then, the world's largest multilateral trade agreement has undergone changes to its name, to its most pressing purpose, and to the composition of its membership. The GATT evolved into the World Trade Organization (WTO), with 164 member countries, comprised overwhelmingly of developing and least-developed nations. This has brought the conversation on trade and development to the forefront of the WTO negotiations. Regional trade agreements (RTAs) have seen an even larger expansion in popularity and are formed among an equally diverse set of nations.

For example among WTO nations alone, the number of RTAs entering into force each year since 2001 has nearly doubled from 6.9 per year from 1995 to 2001 to 12.4 per year from 2001 to 2015.

In this paper, I present a theoretical framework that explains many of the diverse trade policy actions taken by countries in trade liberalization. In my model these decisions are mainly a product of three key parameters: (i) the degree to which varieties of an imported good are substitutable, (ii) the degree of skill with which a government negotiates with domestic special-interest lobbies (the government's "bargaining strength"), and (iii) the relative importance the government places on receiving political contributions from lobbyists (the government's "political economy weight"). I examine the impact of these variables on domestic producers and social welfare while eliminating the ability of a country to have a general equilibrium effect on the world market: I assume the home country is "small" in the sense that the home country's trade policy has no influence over the foreign tariff levels, the price of foreign varieties abroad, or the number of foreign varieties produced.

In my model, a small-country government faces domestic political pressure to grant tariff protection to monopolistically competitive lobbying sectors. I find that industries in which firms choose to lobby feature a more-than-socially-optimal number of firms producing in the industry. Commitment to a trade agreement then leads to the exit of firms from industries facing tariff cuts and the consolidation of resources into fewer, larger firms producing at lower marginal costs.

Similar to large-country models, the cost of a production distortion is not fully borne by the home country. Monopolistically competitive markets with variety-loving consumers ensure even small-country producers have market power, where market power is defined narrowly as the ability of firms to affect the world price, even if it has no influence on foreign tariff levels. Consequently, the degree to which domestic producers enjoy market power is a key determinant of whether the contributions made by politically-important industries will outweigh the cost of tariff-induced firm-entry distortions in the home economy, or if the welfare losses from the higher tariff are too great, leading the government to choose to commit to lower tariffs by way of a trade agreement.

My model predicts that the weaker a country's institutions are (in terms of their ability to resist special interest pressures), the more varied the government's trade policies should be. To demonstrate, consider a country with very strong institutions and little corruption: firms in such a nation are unable to capture rents from protection and therefore will make little or no contributions to secure a higher tariff. Additionally, even if the firms in an industry form a lobby to negotiate for a higher tariff on import-competing goods, the strong-institution government places little value on political contributions and would, therefore, prefer to commit to a trade agreement in order to correct the market-power-driven manipulation of domestic prices by distorted investment in the lobbying sector.

If a government has weaker institutions—due to highly valuing political contributions or due to a relatively weak bargaining position relative to domestic special-interest groups—then the government may be better off allowing production distortions and receiving the political payoff from lobbyists. To better understand the relationship between political economy factors and trade policy, think of the political motivation (the political economy weight) and the bargaining strength as having two possible values, high or low, giving four possible combinations of political economy weight and bargaining strength. Three of the four combinations are important in my analysis: the model predicts that the governments which choose to commit to a trade agreement tend to be (1) very politically motivated but weak bargainers, (2) strong bargainers with little political economy motivation, or (3) both minimally politically motivated and weak bargainers.[1] All else equal, the result I just delineated is true for my model and it is the same result that arises with perfectly competitive producers (Maggi and Rodríguez-Clare, 1998). What makes my result distinct from the perfectly competitive model is the way the substitutability of product varieties

[1] The fourth type of government, the strong bargainers that are highly politically motivated, never prefers to commit to a trade agreement because they value the political contributions and are skilled at negotiating to receive them.

affects the policy choice for cases (1) and (2). Specifically, in my model the politically motivated weak bargainers (case (1)) benefit from committing to a trade agreement which lowers tariffs on relatively homogeneous goods and not on relatively differentiated goods. Strong-bargaining non-politically-motivated governments (case (2)) prefer the opposite: committing to lower tariffs on relatively differentiated goods, not on more homogeneous goods.

Demonstrating this point, the diversity of trade policy preferences can be observed through the countries with which a government forms a trade agreement. A broad way to categorize trade agreement membership is whether the contracting nations are " like" countries, trading mainly in differentiated varieties of similar products, or if the contracting nations are "unalike" countries, trading predominantly in unrelated products following the pattern of Heckscher-Ohlin, comparative-advantage-based trade. My model predicts that for "small" countries (typically thought of as "developing" countries, which have weaker institutions and are more susceptible to political influence), there will likely be significant variation in the decision to join "like" or "unalike" country agreements. On the other hand, for countries with strong institutions using trade agreements mainly to correct terms-of-trade manipulation, there is no reason focus trade-negotiating efforts on forming trade agreements any specific country type: when the government has "strong institutions," any political-economy-driven distortions are small compared to distortions generated by terms-of-trade manipulation. When a country does not seek commitment, but instead seeks a solution to the terms-of-trade driven prisoner's dilemma, however, there is no reason to favor joining agreements with one type of country versus another.

Examining data from the Design of Trade Agreements (DesTA) database reveals a significant difference between the trade agreements joined by each country type (Dür et al., 2014). The Design of Trade Agreements (DesTA) database categorizes the features of around 870 regional trade agreements (Dür et al., 2014). These patterns are suggestive of what my model predicts—when joining a TA based on the desire to escape political pressure, the country's choice of joining a like-country agreement versus an unalike-country agreement depends on the product basket a country exports and its own ability to extract the rents from protection from the domestic producers receiving protection.

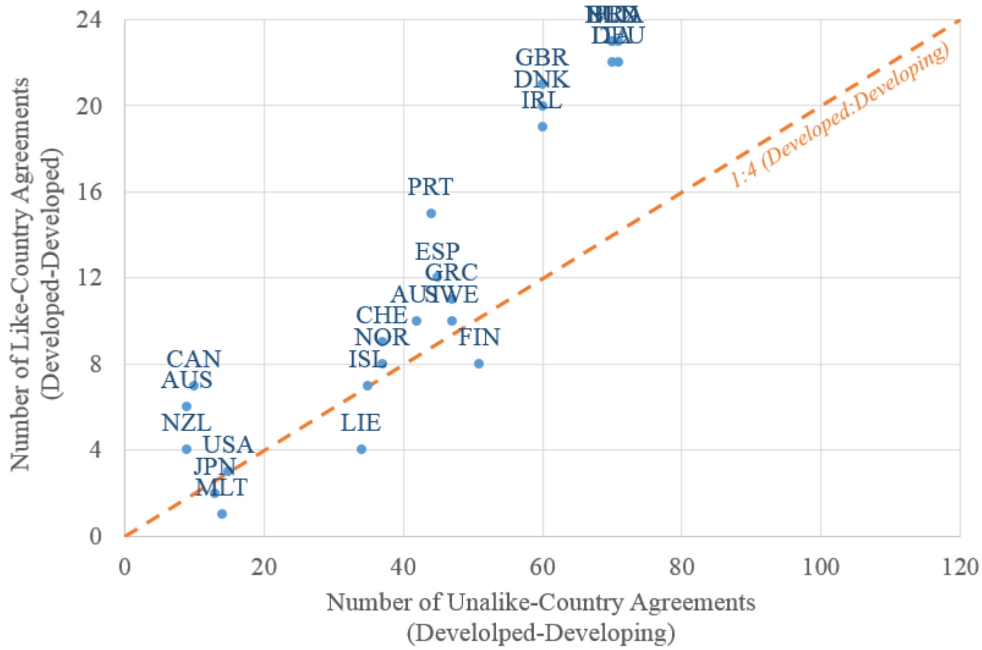
Using data only on WTO members in the DesTA dataset, consisting of 26 developed and 112 developing WTO members (32 of which are "least-developed" according to the UN), as a broad comparison of trade agreement preferences, Figure 1.1 shows the number of like-country (developed-developed or developing-developing) agreements versus the number of unalike-country agreements (developed-developing). This rough comparison shows a large difference between the trade policy choices of developed countries compared to the policy choices of developing countries. Figure 1.1a shows that developed countries exhibit a strong pattern in the ratio of like-country to unalike-country agreements joined: developed countries join approximately 22% like-country and 78% unalike-country agreements, which is very close to the sample shares of their potential partners, 18% of its potential partners are developed and 82% are developing. Figure 1.1b, on the other hand, follows no such pattern. Looking at the individual countries in Figure 1.1b shows that the ratio of unalike-country to like-country agreements varies greatly across observations, following no discernible pattern.

The basic structure of my model draws on Chang (2005), which is a monopolistically competitive version of Grossman and Helpman's "Protection for Sale" model (1994): a small-country government decides its tariff levels by maximizing a weighted welfare function, with the extra weight placed on political contributions the government receives from lobbying industries. Each country produces a perfectly competitive good which is traded freely and is defined to be the numeraire good. Any additional sectors are monopolistically competitive and may receive tariff protection. Only the monopolistically competitive sectors lobby for tariff protection. Firms offer to pay political contributions to the government in exchange for additional tariff protection.

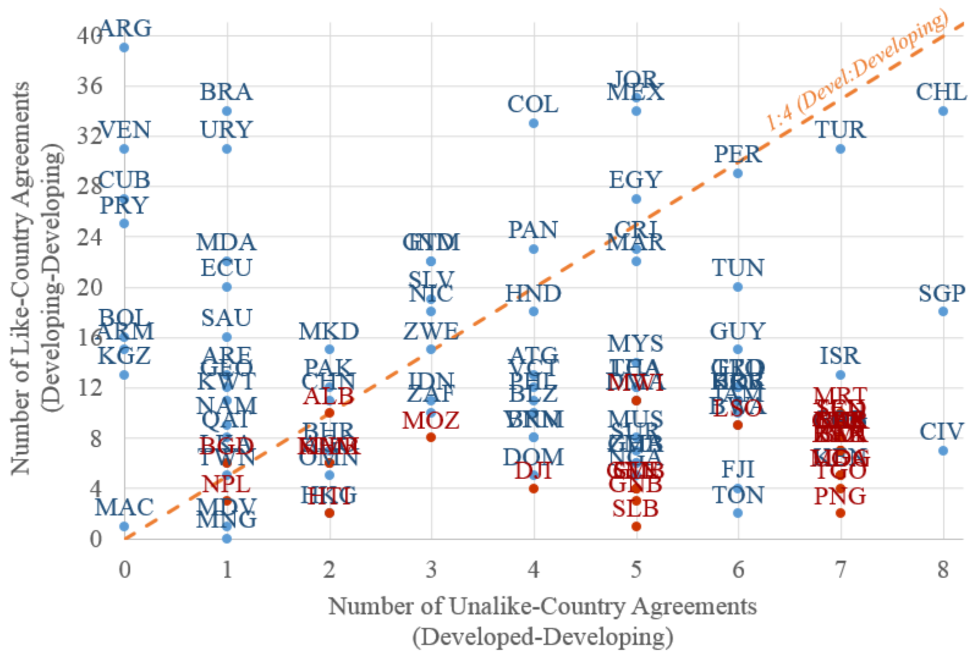
Unlike a perfectly competitive model in which the social optimum is free trade, Chang

Figure 1.1: Preferential trade agreement membership of WTO countries by partner type. The vertical axis is the number of like-country agreements of which each is a member, and the horizontal axis is the number of unalike-country agreements.

(a) Developed countries' PTA memberships. The orange-dashed line is a reference line based on the ratio of developing to developed countries in the WTO, approximately 4:1.



(b) Developing countries' PTA memberships, divided by developing countries (as self-declared to UN/WTO) in blue and least-developed countries (as determined by the UN) in red. The orange-dashed line is a reference line based on the ratio of developed to developing countries in the WTO, approximately 1:4.



shows that with monopolistic competition the endogenous import tariff is always positive, even for sectors that do not lobby the government for tariff protection. Furthermore, with monopolistic competition the level of tariff protection is strictly increasing in the import penetration of an industry regardless of whether or not the sector lobbies the government for protection. This is in contrast to Grossman and Helpman (1994), in which the level of protection is increasing in an industry's import penetration only if the sector lobbies.

To then understand how the commitment motive shapes a government's trade policy choice, I mimic the trade policy analysis of Maggi and Rodríguez-Clare (1998). The authors begin from Grossman and Helpman (1994), adding in an ex-ante capital-hiring decision. Capital is allocated between two perfectly competitive industries which produce a numeraire good using land and capital and a non-numeraire good using capital only. After capital is allocated, it is immobile. The government then sets the domestic price for the capital-intensive good using an import tariff. The tariff-protected industry exogenously forms a lobby which offers to pay the government political contributions in exchange for tariff protection.

Maggi and Rodríguez-Clare (1998) show that producers of the manufactured good hire more than the efficient amount of capital in expectation of tariff protection. Then, because capital is fixed when the government and the lobby negotiate over the division of rents from protection, the lobby's contribution only compensates the government for the losses due to the tariff's distortionary impact on prices; the lobby does not compensate the government for losses due to the ex-ante distortions in production. Therefore, if the government's share of the ex-post rents from protection is not enough to compensate it for the initial overallocation of capital to the manufacturing sector, then the government is made better off by committing to a free trade agreement before firms hire capital, inducing resources to be allocated efficiently across sectors. Whether or not the government gains from committing to a trade agreement depends on two key model parameters: the political economy weight on the lobbying contributions and the government's bargaining strength relative to the lobby. Generally, the government commits to a trade agreement if the political economy weight on contributions is small or if the government is a weak negotiator.

I study the government's trade policy choice using a strategy similar to Maggi and Rodríguez-Clare (1998). I consider the impact of lobbying on firm entry when the protected sector is monopolistically competitive, and I show how this distortion determines when a government will commit to a trade agreement. First, my findings confirm Maggi and Rodríguez-Clare's intuition extends to monopolistic competition. Then, the majority of my analysis is focused on the relationship between the government's trade policy and product-specific characteristics, namely the degree to which products are differentiated. The degree of differentiation is an obvious factor in determining trade policy, originally established in Gros (1987). Gros demonstrated that with monopolistic competition, an optimal tariff is increasing in the degree of product differentiation even if the country is "small" relative to the rest of the world.

Following the example of Maggi and Rodríguez-Clare (1998), I add an initial resource-allocation decision to the monopolistically competitive industries. Originally in the Chang (2005) model, each firm hired a fixed amount of the sector-specific capital stock to enter the market. Once firms entered the market, they then produced the good subject to an exogenous unit-labor-input requirement. To introduce ex-ante capital hiring similar to Maggi and Rodríguez-Clare (1998) to my model, I change the initial "fixed-entry cost" standard to many monopolistic competition models to a variable cost: instead of paying a fixed amount of sector-specific capital to enter the market, I make it so the amount of capital a firm pays also affects the firm's unit-labor-input requirement for production according to a decreasing and convex transformation function. As a result, each firm has an ex-ante profit-maximization decision, hiring capital from a fixed sector-specific endowment. Ex post, once the government sets the tariff level, firms hire labor to maximize short-term profits. Without a trade agreement, monopolistically competitive producers have the option to pay to organize a lobby to then negotiate with the government over the tariff level. With a trade agreement, the government unilaterally sets the tariff according to its commitment when it joined the trade agreement before firms entered the market.

The endogeneity of forming a lobby is a key addition to my model. In Maggi and Rodríguez-Clare (1998), as well as in Grossman and Helpman (1994) and Chang (2005), lobby formation is costless, thus there is no disadvantage to lobbying. In my model, however, an industry must pay an upfront lobbying-participation fee before negotiations occur. Conceptually, this fee represents the cost to firms of hiring lobbyists and organizing with other firms in the industry to conduct negotiations with the government. I derive the upfront cost as the Shapley value of a cooperative game in which firms share the cost of lobbying. My setup of costly lobby formation loosely matches the lobbying process in the United States.[2] As a result of their ability to influence the government's tariff choice, firms distort their ex-ante capital hiring in order to secure a greater amount of tariff protection ex post.

Consistent with Maggi and Rodríguez-Clare (1998), I show that the government uses commitment to a trade agreement to counteract the production distortions created by the rent-seeking behavior of politically active industries. For my model and theirs, this distortion is dependent on there being a factor of production that is only mobile ex ante. In Maggi and Rodríguez-Clare (1998), rent-seeking leads to an inter-sectoral distortion in capital hiring and an increase in production of the tariff-protected good. In my model capital is sector-specific, so lobbying distorts the intra-sectoral allocation of capital ex-ante. I show that lobbying leads to an increase in firm entry with each firm producing fewer units of its variety.[3] Additionally, because labor remains mobile ex-post firms always have the ability to distort production to improve profits. This is a standard feature of the monopolistic

[2]In the US, a lobbyist is typically paid based on the type of the project, the amount of work involved, the size of the client, and the expertise of the firm in the specific area and hired on an annual basis (lob). In the United States it is illegal for a lobbyist to be paid on commission: whether or not the outcome of bargaining is successful, the lobbyist must be paid. Additionally, firms are prohibited by law from offering or paying a bonus to a lobbyist as an incentive contingent on the success of bargaining.

[3]Assuming capital is sector specific is what eliminates intersectoral capital distortions. Alternatively, if I had an economy wide capital hiring problem in which all industries hired capital from the same pool, an intersectoral distortion in capital would also exist.

competition model. Given that I restrict the model so the only policy instruments available to the government are an import tax or import subsidy, the government is unable to correct the labor distortion to force the first-best social-welfare-maximizing production levels. Therefore, the government's decision to commit to a trade agreement is only influenced by the ex-ante production distortion.

Another related paper is Dhingra (2013). Dhingra examines how trade policy affects investment in product variety and in cost reduction. She argues that papers such as Krugman (1980) and Melitz (2003) miss an important channel of welfare gains from trade by focusing only on trade's effect on entry and exit of firms. She establishes a theoretical model in which monopolistically competitive firms invest in "branding" ex ante to differentiate their products from competitors. Once a brand is established, firms make a second-stage investment in which their resources are divided in two pools: (i) invest in new varieties, defraying the fixed cost of "branding" the firm's output and improving market share; or (ii) invest in lowering production costs and improving productivity. Dhingra demonstrates that liberalizing trade results in each firm producing fewer varieties and investing in lowering costs. This subsequently allows consumers to purchase a more diverse set of products (in terms of variety of brands) at lower prices. Her model and mine agree that without a trade agreement firms sell products at a higher price, but Dhingra's result is derived from the limited investment in varieties by the multi-product firms and my results is derived from the desire of firms to free-ride on lobbying efforts of the industry as a whole.

An additional result of my model is the prediction that the Metzler paradox arises in some situations. The Metzler paradox describes a scenario in which a reduction in tariffs results in a rise in the relative price of imports. In my model, when the government commits to a trade agreement, two things happen to the relative price of imports: (1) the price of foreign varieties in the home market falls due to the reduced tariff level, and (2) the price of domestic varieties in the domestic market falls as producers invest more in reducing their marginal cost of production in order to better compete with foreign producers. The Metzler paradox occurs when the production adjustment exceeds the tariff reduction.

The rest of the paper proceeds as follows: in Section 1.2, I define the model. In Section 1.3, I derive the basic producer equilibrium and the government's tariff choice when the government commits to a tariff level ex ante, before firms enter the market. In Section 1.4, I derive the producer equilibrium and tariff choice given the tariff is set after firms enter the market and hire capital and given that firms lobby the government for additional tariff protection. In Section 1.5, I define the government's welfare-maximizing policy choice – to join a trade agreement or not – as determined by the underlying model parameters.

1.2 Basic Theoretical Model

Generally speaking, my model is a small country, two input, two good, partial-equilibrium model. In the model, the home government faces political pressure from domestic lobbyists to grant tariff protection in exchange for a political contribution from the lobbying sector. The government has the option to allow the lobbying to occur or to commit to a trade agreement before firms enter the market.

The home country is "small" relative to the rest of the world, implying it is unable to influence production or the tariff policy decisions abroad. The home country (home, no "$*$") trades with the rest of the world (foreign, "$*$"). There are two sectors producing in both the home and foreign country. Sector 1 produces a homogeneous good using only labor, and the price of the sector 1 good is normalized to one. Sector 2 produces several varieties of a differentiated good using labor and sector-specific capital.[4] The home country is endowed with a fixed supply of labor L and capital K. Factors of production are not mobile across countries. N identical consumers comprise the home country (N^* in the

[4]My model retains the structure of production in Chang (2005) and in Grossman and Helpman (1994), in which the same simplifying assumption is used. This restriction on the model allows me to abstract away from any intersectoral misallocation of capital resources. Instead, my model generates predictions as to the degree to which lobbying generates distortions in the trade-off between firm entry and each firm's investment in lowering the cost of production.

rest of the world), and each consumer has an identical endowment of labor to supply and capital to rent to firm owners.

The sector 1 good is traded freely and costlessly across countries. In the differentiated goods sector, the government can implement either an import tariff or an import subsidy on foreign-produced varieties of a differentiated good from sector 2. I restrict the government's policy options to only include import policies. The tariff, $\tau > 0$, is defined as one plus the ad valorem import tariff rate if $\tau > 1$ (with $\tau < 1$ for an import subsidy).[5] Similarly, $\tau^* > 0$ represents the foreign import policy.[6]

The model takes place in two broad stages. The dividing line between these stages is the setting of the tariff level, referring to when the government sets the tariff and the lobbying industry makes political contributions. Ex ante, firms enter the market and hire capital; ex post, the government and lobby negotiate over the tariff level, the division of rents from tariff protection, and firms hire labor and set prices.

Consumption. Utility is quasilinear in consumption of the sector 1 good. Consumption of the sector 2 good is a CES aggregation of consumption of home-produced (h) and foreign-produced (f) varieties. A representative consumer's utility function is

$$U = q_1 + u_2(q_2), \tag{1.1}$$

with $u_2(q_2) = e_2 \ln q_2$. This structure of $u_2(q_2)$ implies total expenditure on varieties of good 2 is constant, equal to e_2. The CES consumption index for varieties of the sector 2 good is

$$q_2 \equiv \left(\sum_{m=1}^{n} q_{h2m}^{\frac{\sigma-1}{\sigma}} + \sum_{m=1}^{n^*} q_{f2m}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \tag{1.2}$$

[5]The Chang (2005) paper allows a more complete set of policy instruments. The full set of instruments is an import tariff $(\tau > 1)$, an import subsidy $(\tau < 1)$, an export subsidy $(\tau^{exp} > 1)$, and an export tax $(\tau^{exp} < 1)$.

[6]This structure implicitly assumes the government's tariff policy cannot be defined narrowly enough to assign a different tariff for each variety of the sector 2 good. Firms are symmetric in equilibrium, so this assumption does not affect the government's policy choice. The focus on import policies is both to simplify my analysis and due to the WTO's emphasis on eliminating export policies as a trade barrier.

in which q_{h2m} (q_{f2m}) is the consumption of domestic (foreign) variety m, with n (n^*) varieties of the sector 2 good produced at home (abroad), and in which $\sigma > 1$ is the elasticity of substitution between varieties. The rest of the world has identical preferences, but with e_2^* not necessarily equal to e_2.

The CES price index for the sector 2 good is

$$P \equiv \left(\sum_{m=1}^{n} p_{hm}^{1-\sigma} + \sum_{m=1}^{n^*} p_{fm}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}, \tag{1.3}$$

with p_{hm} and p_{fm} defined as the prices of home and foreign-produced varieties, respectively. Therefore, expenditure on the differentiated good can be written $e_2 = Pq_2$.

Production. In sector 2, firm m produces variety m by hiring capital k_m and labor l_m:

$$Q_m \equiv \frac{1}{\lambda(k_m)} l_m,$$

with unit-labor-input requirement $\lambda(k_m) > 0$, and firm m's total output of the differentiated good Q_m. Unique to my model, each firm owner is able to invest in the "technology" of its own production by hiring capital to lower its unit-labor-input requirement. Capital investment by firm m reduces the unit-labor-input requirement according to the function $\lambda(k_m)$. More specifically, the functional form is such that $\lambda(k_m) > 0$ and $\lambda'(k_m) < 0$ for all $0 < k_m < \infty$, and $\lambda(0) \to \infty$.[7] Production of the sector 1 good uses only labor and has a unit-labor-input requirement equal to one in both the home and foreign countries: production is $Q_1 = L_1$ in the home country and $Q_1^* = L_1^*$ in the foreign country. Sector 1, therefore, pins down the wage at one globally. Given the universal wage is equal to one, the unit-labor cost is equal to the unit-labor-input requirement, $\lambda(k_m)$, and is equal to a firm's marginal cost of production.

A result of making the marginal-labor-input requirement dependent on capital is that the model exhibits increasing returns to capital in production ex-ante and constant returns

[7] I restrict the function $\lambda(\cdot)$ to be identical for all firms the home market. A scenario in which this function varies by sector and by firm would produce an environment similar to Melitz (2003).

to labor ex-post. While this is typically avoided in order to prevent the model being driven to corner solutions, my model still yields interior solutions for firm entry and capital hiring. [8]

Because my focus is on the home country's decisions, I treat the foreign unit-labor-input requirement as exogenous. Each of the n^* foreign firms in sector 2 are identical, with unit-labor-input requirement $\lambda^* > 0$. Firm m produces $Q^*_m = (1/\lambda^*)l^*_m$ units of variety m of the sector 2 good. I also assume the total number of varieties of the sector 2 good produced, $n + n^*$, is large enough that a change in the supply of one variety does not impact the CES price index in either country.

Domestic and foreign prices. Ex post, sector 2 firms hire labor to maximize profits given a fixed number of firms in the industry and fixed amount of capital each firm employs. Given k_m, profit-maximization implies that firm m charges a constant markup over its marginal cost of production:

$$p_{hm}(k_m) = \frac{\sigma}{\sigma - 1}\lambda(k_m), \tag{1.4}$$

for $m \in \{1, \ldots, n\}$. This price is the "factory-gate price" and is also the price at which variety m is sold in the home market. In the foreign market, $p_{hm}(k_m)$ is further marked up by the tariff, τ^*, giving the foreign price $p^*_{hm} = \tau^* p_{hm}$. The price of foreign-produced varieties can be similarly defined: $p^*_{fm} = \frac{\sigma}{\sigma-1}\lambda^*$, for $m \in \{1, \ldots, n^*\}$. Given ad valorem tariff level τ in the home country, the price of the foreign good in the home country is defined $p_{fm} = \tau\, p^*_{fm}$. At the ex-post profit-maximizing prices, the market for labor clears, giving $L_1 + \sum_{m=1}^{n} l_m = L$.

[8] This use of the "increasing returns" ex-ante also has support in the work of other authors' work, a specific example of which is Dhingra (2013). In her paper, Dhingra allows for firms to invest in reducing unit cost according to $c(\omega) = c - c\omega^{1/2}$, with $\omega \in [0, 1]$, giving a cost function $c'(\omega) < 0$ and $c''(\omega) > 0$, where firms allocate resources to either improving ω or to increasing the range of products it produces. Dhingra's "$c(\cdot)$" function operates in a manner very similar to my $\lambda(\cdot)$ function. In both instances: the unit cost decreasing and convex; it is fixed after initial investment; and production ex-post is identical to standard monopolistic competition models, in which the unit-labor-input requirement is exogenous and firms hire labor to maximize profits.

Demand functions for variety m of the home-produced and of the foreign-produced sector 2 goods are

$$q_{h2m} \equiv d_{hm}(p_{hm}(k_m), q_2, P) = q_2 \times \left(\frac{p_{hm}(k_m)}{P}\right)^{-\sigma} \text{ for } m \in \{1, \ldots, n\}, \text{ and} \tag{1.5}$$

$$q_{f2m} \equiv d_{fm}(p_{fm}, q_2, P) = q_2 \times \left(\frac{p_{fm}}{P}\right)^{-\sigma} \text{ for } m \in \{1, \ldots, n^*\}, \tag{1.6}$$

respectively, with q_2 defined in equation (1.2). Demand for the home and foreign varieties of the sector 2 good in the foreign market are then defined similarly: the respective demand functions for variety m of the home-produced and the foreign-produced sector 2 goods in the foreign market are

$$q^*_{h2m} \equiv d^*_{hm}(p^*_{hm}(k_m), q^*_2, P^*) = q^*_2 \times \left(\frac{p^*_{hm}(k_m)}{P^*}\right)^{-\sigma} \text{ for } m \in \{1, \ldots, n\}, \text{ and} \tag{1.7}$$

$$q^*_{f2m} \equiv d^*_{fm}(p^*_{fm}, q^*_2, P^*) = q^*_2 \times \left(\frac{p^*_{fm}}{P^*}\right)^{-\sigma} \text{ for } m \in \{1, \ldots, n^*\}, \tag{1.8}$$

with the foreign price index, P^*.

Ex-ante firm profits. The firm-entry and capital-hiring decisions are made based on the firm's ex-ante profit function. For domestic firm m facing residual demand for its variety at home and abroad as defined in equations (1.5) and (1.7), ex-ante profits are

$$\begin{aligned} &\Pi_m(k_m, r, \mathbf{P}) = \\ &(p_{hm}(k_m) - \lambda(k_m))\Big[N d_{hm}\Big(p_{hm}(k_m), q_2, P\Big) + N^* d^*_{hm}\Big(p^*_{hm}(k_m), q^*_2, P^*\Big)\Big] - r\, k_m, \end{aligned} \tag{1.9}$$

where $\mathbf{P} = (P, P^*)$ is a vector of price indexes and r represents the rental price of capital for the capital specific to sector 2 in the domestic market.[9] Notationally, define the operating profits of the firm specifically as $\pi_m(k_m, \mathbf{P}) = (p_{hm}(k_m) - \lambda(k_m))[N d_{hm}(p_{hm}(k_m), q_2, P) + N^* d^*_{hm}(p^*_{hm}(k_m), q^*_2, P^*)]$, which are positive in equilibrium. In the ex-ante equilibrium,

[9]The above equation assumes there are no "iceberg" trade costs, or other types of deterioration of the exported goods before they arrive in the export markets for sale. The addition of an iceberg cost would give the profit function $\Pi_m(k_m, \mathbf{P}, r) = (p_{hm}(k_m) - \lambda(k_m))[N d_{hm}(p_{hm}(k_m), q_2, P) + (1+\phi) N^* d^*_{hm}(p^*_{hm}(k_m), q^*_2, P^*)] - r k_m$.

firms enter the market until profits are zero, taking as given the foreign production and tariff levels. After a firm enters the market, it hires capital to maximize ex-ante profits given the fixed total number of firms in the industry and the capital hiring decisions by the other $n-1$ firms.

Social welfare. The home country's social welfare is equal to the sum of factor income, producer welfare (which is zero ex ante, but important to include explicitly since the operating profits are positive ex post and play an important role in determining the tariff), consumer surplus, and tariff revenue:

$$\begin{aligned} W(\mathbf{k}, n, r, \tau) = L + rK + \sum_{m=1}^{n} \Big(\pi_m(k_m, \mathbf{P}) - rk_m\Big) \\ + \mathit{TR}(\tau, q_{f2}, p_f^*) + N\, \mathit{CS}(P), \end{aligned} \tag{1.10}$$

where $\mathbf{k} \equiv \{k_1, \ldots, k_n\}$ is a vector of all n firms' capital-hiring decisions, $\mathit{TR}(\tau, q_{f2}, p_f^*) \equiv Nn^*\Big(\tau - 1\Big)p_f^* q_{f2}$ is tariff revenue from imports of the n^* foreign varieties, and consumer surplus from the sector 2 good is $\mathit{CS}(P) \equiv U - q_1 - P\, q_2 = u_2(d_2(P)) - P\, d_2(P)$.

Political structure. The political environment is composed of two key actors in the monopolistically competitive sector: the government and the sector-specific lobby. In my model, if firms in sector 2 value additional tariff protection highly enough, then the firms organize to form a lobby. I assume that a lobby forms among all firms in a sector. Note that lobbying takes place in the tariff-setting stage of the model only, and not in the initial stage in which the government decides whether or not to join a trade agreement.[10]

In negotiations with the government, the lobby's objective is to maximize the total welfare of the sector 2 firm owners. Assuming that the firm owners comprise a negligible share of the population, their total welfare is equal to the sum of net revenues:

$$\sum_{m=1}^{n} \Pi_m(k_m, r, \mathbf{P}) - c\, k_m, \tag{1.11}$$

[10]Maggi and Rodríguez-Clare (2007) present a variant of their 1998 model in which firms are able to lobby the government in the trade agreement formation stage. Such an extension would be a possible expansion of this model.

with $c \geq 0$ defined as the lobbying contribution per-unit of capital employed.[11]

In negotiations with the lobbying industry, the government's objective is to maximize the sum of social welfare and political contributions from the lobbyists:

$$G \equiv W(\mathbf{k}, n, r, \tau) + a\, n\, c\, k_m, \tag{1.12}$$

in which a is the government's political economy valuation of the lobbying contributions. The government is politically motivated if $a > 0$.[12]

The government and the lobby negotiate over the division of rents from tariff protection, where the threat point of bargaining is for the government to set the domestic tariff at the ex-post welfare-maximizing level (given a fixed n and k_m) and for the lobby to pay no contribution to the government. Therefore, the lobby and the government negotiate over how to divide the social welfare cost to the government, $W^u - W^{cxp} < 0$, and the welfare benefit to firms the lobbying sector, $\pi^u - \pi^{cxp} > 0$, where u denotes the government is "uncommitted" to a trade agreement and cxp denotes the threat point ("commitment ex post"). The government has exogenous Nash bargaining strength $\gamma \in [0, 1]$ and the lobby has bargaining strength $(1 - \gamma)$. Upon the successful conclusion of bargaining, the lobby collects contribution amount $c\, k_m$ from each of the n firms in sector 2, to be paid to the government.[13]

New to my model, in order to engage in bargaining with the government, firm m in a lobbying sector must pay upfront participation cost, $\theta(k_m)$, meaning the total upfront

[11] As discussed by Maggi and Rodríguez-Clare (1998), assuming that firm ownership is concentrated in a negligible portion of the population allows the aggregate revenue of sector 2 firm owners to be reduced from $\sum_{m=1}^{n} \Pi_m(k_m, r, \mathbf{P}) - c\, k_m + \chi_m[L + TR(\tau, q_f, p_f^*) + CS(P)]$, where χ_m represents the measure of the population which owns firm m, to the firm-owner-welfare function in equation (1.11).

[12] I define a the same way as in Maggi and Rodríguez-Clare (1998). For comparison, in Grossman and Helpman (1994) and in Chang (2005), G and a are defined such that the government values the lobby contributions more than the domestic welfare for any $0 < a < 1$, where the relative value of the lobby's political contributions is decreasing in the size of a. Finally, constancy of a across sectors is consistent with all three models.

[13] Stiglitz (1986) points out that with regard to a model of monopolistic competition, it is difficult to know what a "firm" is. As a result, having anything in the model that needs a firm to self-identify presents a problem. In my model, I avoid this problem by assuming lobbying costs are collected based on the amount of capital a firm employs.

cost paid by sector 2 firms is $\sum_{m=1}^{n} \theta(k_m)$, and the total cost a given firm pays for lobbying is $\tilde{c}(k_m) \equiv ck_m + \theta(k_m)$ if lobbying succeeds and $\theta(k_m)$ if lobbying fails. The lobbying participation cost, $\theta(\cdot)$, is positive, increasing, and convex in the amount of capital employed by the firm: $\theta'(k_m) > 0$ and $\theta''(k_m) > 0$ for $0 < k_m < \infty$, and $\theta(0) = 0$. The $\theta(\cdot)$ function is the Shapley value which results from the game. In equilibrium, because all firms in a sector are symmetric: if one firm pays the upfront lobbying cost, then all firms pay it. If the upfront lobbying cost is too large relative to the gains a firm receives from a higher tariff, then the lobby does not form, regardless of whether or not the government has joined a trade agreement.[14]

The convexity of the lobbying cost can be thought of as a reduced form free-rider problem: small firms free ride on the lobbying efforts of larger firms, forcing the large firms to pay a greater share of the upfront lobbying cost. This aligns well with another paper in this vein: Bombardini (2008) models a lobbying free-rider problem using a variant of Grossman and Helpman (1994) in which firms lobbying for tariff protection pay a fixed entry fee to participate in lobbying. She finds the smallest firms (in terms of capital employment) end up free-riding on the lobbying efforts of larger firms.

To simplify the impact of the upfront lobbying cost on the rest of the model, I assume that after it is paid, the upfront cost of lobbying is redistributed to consumers in the home country through a lump-sum transfer. As a result of this redistribution and due to the quasi-linearity of preferences, the upfront cost of lobbying has no impact on the aggregate quantities consumed or on social welfare.[15] This is why lobbying income is not visible in the social welfare function in equation (1.10).

A final note before moving on to the model solution: it is simple to generalize the model to include many monopolistically competitive sectors. This is due to the assumption that capital endowments are sector-specific and the assumption that firm ownership is

[14]The derivation of $\theta(\cdot)$ is given in Appendix A.1.

[15]This is also how Bombardini (2008) handles her fixed cost of lobbying. There are other ways to treat the upfront cost within the model that could potentially lead to further distortions (for example, if the upfront cost of lobbying were treated as an expenditure of resources). I only want to examine the impact of the lobbying cost on production, therefore including a direct distortion to welfare is unnecessary.

concentrated in a negligible share of the population. The sector-specificity of capital means there would be no intersectoral competition for resources ex-ante. The concentration of firm ownership means a firm owner receives a negligible share of consumer surplus, so unlike in Grossman and Helpman (1994), firm owners have no reason to lobby for lower tariffs in other sectors. Combined, these assumptions imply the production and tariff decisions for the monopolistically competitive sectors are separable, which means generalizing to many sectors does not change the within-sector findings.

1.3 Ex-Ante Tariff Commitment, No Lobbying

Excluding the possibility of lobbying demonstrates the simplest form of the model, in which the government sets the tariff to maximize social welfare and firms maximize profits without the added benefit of lobbying for tariff protection. Without the possibility of lobbying, the government is able to choose the tariff level ex ante, before firms enter the market. I call this the ex-ante "commitment" solution, and its equilibrium variables are denoted by a "c" superscript.

Solving the model with ex-ante tariff commitment proceeds by backward induction. I solve for the ex-ante equilibrium firm entry and capital hiring given a fixed tariff level in Section 1.3.1. I derive the government's ex-ante social-welfare-maximizing tariff, τ^c, in Section 1.3.2. Finally, I discuss the impact of the elasticity of substitution across varieties on the producer and government decisions in Section 1.3.3.

1.3.1 Ex-Ante Producer Equilibrium

Each firm m chooses (i) whether or not to enter the market, determining the number of varieties of the sector 2 good produced, n, and (ii) how much capital to hire after entry, which is denoted k_m for a representative firm. In this ex-ante commitment version of the model, these decisions are made after the government has chosen the tariff level. In this section, I assume model parameters are such that there exists an interior solution for each producer equilibrium variable.[16]

Using backward induction, begin with the producer's capital-hiring decision: domestic firm m hires k_m to maximize its ex-ante profits (equation (1.9)), taking as given the rental price of capital, r, the number of firms in the sector, n, the pricing decisions of the other $n-1$ firms at home, $p_{h(\text{-}m)}$, and the prices of the other n^* firms abroad, p_f^*. The profit-maximization equation below has been simplified using the envelope condition from the firm's ex-post price-setting problem.[17] The first-order condition of firm m's profit is

$$\frac{d\Pi_m}{dk_m} = \frac{1}{\sigma - 1}\Bigg[\lambda'(k_m)\Big(Nd_{hm}\Big(p_h(k_m), q_2, P\Big) + N^* d^*_{hm}\Big(p^*_h(k_m), q^*_2, P^*\Big)\Big) + \lambda(k_m)\left(N\frac{d\,d_{hm}\Big(p_h(k_m), q_2, P\Big)}{d\,k_m} + N^*\frac{d\,d^*_{hm}\Big(p^*_h(k_m), q^*_2, P^*\Big)}{d\,k_m}\right)\Bigg] - r.$$

Because $\lambda(\cdot)$, the unit-labor-input-requirement function, is identical across firms, sector 2 firms in the home country are symmetric. A representative firm's best-response capital-hiring rule, $\tilde{k} \equiv \tilde{k}(r, n, \tau)$, is

$$r = -\lambda'\Big(\tilde{k}\Big)\Big[Nd_h\Big(p_h(\tilde{k}), q_2, P\Big) + N^* d^*_h\Big(p^*_h(\tilde{k}), q^*_2, P^*\Big)\Big]. \tag{1.13}$$

A representative firm's optimal capital-hiring equates the marginal cost of hiring additional capital with the marginal cost-reduction benefit from a fall in the unit-labor input requirement. The second-order condition for profit-maximization is

$$-\left(\lambda''\Big(\tilde{k}\Big) - \sigma\frac{\lambda'(\tilde{k})^2}{\lambda(\tilde{k})}\right)\Big[Nd_h\Big(p_h(\tilde{k}), q_2, P\Big) + N^* d^*_h\Big(p^*_h(\tilde{k}), q^*_2, P^*\Big)\Big] < 0. \tag{1.14}$$

With CES preferences, total demand for a variety of the differentiated good is always positive. Therefore, the second-order condition can be simplified to $\lambda''(\tilde{k}(r,n,\tau)) - \sigma\frac{\lambda'(\tilde{k}(r,n,\tau))^2}{\lambda(\tilde{k}(r,n,\tau))} >$

[16] The conditions for the existence of an interior solution are defined in Appendix A.2.1.

[17] The envelope condition arises from the firm setting a domestic price of their product to maximize profits, giving $d_{hm}(p_h, q_2, P) = -\big(\frac{1}{\sigma-1}\big)\lambda(k)\frac{d\,d_{hm}(p_h,q_2,P)}{d\,p_h}$ and $d^*_{hm}(\tau^* p_h, q_2, P) = -\big(\frac{1}{\sigma-1}\big)\lambda(k)\frac{d\,d^*_{hm}(\tau^* p_h,q_2,P)}{d\,p_h}$.

0. The second-order condition indicates that a profit-maximizing $\tilde{k}$ exists only if the $\lambda(k)$ is adequately convex.

Given the capital-hiring rule defined in equation (1.13), the rental price of capital adjusts so the market for capital clears. The equilibrium rental price of capital, $\hat{r}(n,\tau)$, given a fixed number of firms in the industry, n, is defined implicitly by

$$n \cdot \tilde{k}(\hat{r}(n,\tau), n, \tau) = K. \tag{1.15}$$

The profit-maximizing k given $\hat{r}(n,\tau)$ is denoted $\hat{k}(n,\tau) \equiv \tilde{k}(\hat{r}(n,\tau), n, \tau)$.

The free entry of firms then drives ex-ante profits for a firm in sector 2 down to zero. Given the profit-maximizing capital-allocation rule from equation (1.13) and the capital-market-clearing condition in equation (1.15), the equilibrium number of firms in the industry, $\bar{n}^c(\tau)$, is defined by a firm's zero-profit condition:

$$\begin{aligned}
&\bar{r}^c(\tau)\bar{k}^c(\tau) = \\
&\quad \frac{1}{\sigma-1}\lambda(\bar{k}^c(\tau))\Big[N d_h\Big(p_h(\bar{k}^c(\tau)), q_2\Big(\bar{k}^c(\tau), \bar{n}^c(\tau), \tau\Big), P\Big(\bar{k}^c(\tau), \bar{n}^c(\tau), \tau\Big)\Big) \\
&\qquad + N^* d_h^*\Big(p_h^*\Big(\bar{k}^c(\tau)\Big), q_2^*\Big(\bar{k}^c(\tau), \bar{n}^c(\tau), \tau^*\Big), P^*\Big(\bar{k}^c(\tau), \bar{n}^c(\tau), \tau^*\Big)\Big)\Big],
\end{aligned} \tag{1.16}$$

with $\bar{k}^c(\tau) \equiv \hat{k}(\bar{n}^c(\tau), \tau)$, $\bar{r}^c(\tau) \equiv \hat{r}(\bar{n}^c(\tau), \tau)$, and with the price index defined by $P(k,n,\tau) = \left(\frac{\sigma}{\sigma-1}\right)\left(n\lambda(k)^{1-\sigma} + n^*(\tau\lambda^*)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$, given the symmetry of firms, and the consumption index, $q_2(k,n,\tau) = e_2/P(k,n,\tau)$. Equation (1.16) shows that firms enter the market until the entry cost, $\bar{r}^c\bar{k}^c$, is equal to the firm's ex-post operating profits.

In Proposition 1.1 below, I simplify the three equilibrium conditions to just two equations determining $\bar{n}^c$ and $\bar{k}^c$ which are independent from τ. Therefore, moving forward I drop the dependence of $\bar{n}^c$ and $\bar{k}^c$ on τ.

Proposition 1.1 (Ex-ante producer equilibrium). *Given tariff level τ and that firms are unable to lobby for tariff protection, a competitive ex-ante producer equilibrium is defined by the best-response capital-hiring and firm-entry rules $(\bar{k}^c, \bar{n}^c)$ and price $\bar{r}^c(\tau)$, for which the following statements* *must be true:*

1. *Each firm hires capital, $\bar{k}^c$, to maximize its own ex-ante profits (equation (1.13)).*
2. *The rental price of capital, $\bar{r}^c(\tau)$, is such that the fixed capital supply, K, equals capital demand (equation (1.15)).*
3. *Firms enter sector 2 freely until there are $\bar{n}^c$ firms in the market, at which point the ex-ante profits of operating in sector 2 are equal to zero (equation (1.16)).*

The above points combined give two equilibrium conditions which define $\bar{k}^c$ and $\bar{n}^c$:

$$\lambda(\bar{k}^c) + (\sigma - 1)\lambda'(\bar{k}^c)\,\bar{k}^c = 0, \;\; \textit{and} \tag{1.17a}$$

$$\bar{n}^c\,\bar{k}^c = K, \tag{1.17b}$$

in which $\bar{k}^c$ is established by the combination of the zero-profit and profit-maximization conditions, equation (1.17a), and $\bar{n}^c$ is then determined by (1.17b), subject to the second order condition for profit-maximization, $-\left(\lambda''(\bar{k}^c) - \sigma\lambda'(\bar{k}^c)^2/\lambda(\bar{k}^c)\right) < 0$.

Equation (1.17a) is found by combining equations (1.13) and (1.16). Equation (1.17a) states that the marginal cost savings from an additional unit of capital hired (the right-hand side of equation (1.13)) must be equal to the average revenue per unit of capital hired (the right-hand side of equation (1.16) divided by k). The second-order condition restricts the functional form of $\lambda(\cdot)$: $\lambda(k)$ must be convex in k for the ex-ante equilibrium to exist.

Before moving on to solve for the tariff, further consider the impact of a change in τ on the producer equilibrium variables. As I mentioned previously, when the government chooses the tariff level ex-ante, the firm-entry and capital-hiring conditions are independent from τ.[18]. The full effect of a change in τ is absorbed by the rental price of capital, with

$$\frac{d\bar{r}^c(\tau)}{d\tau} = -\lambda'(\bar{k}^c)\left[\frac{\partial q^c_{h2}}{\partial P}\frac{\partial P(\bar{k}^c, \bar{n}^c, \tau)}{\partial \tau}\right]. \tag{1.18}$$

[18] In Appendix A.5.1, I fully prove that a change in τ does not effect $\bar{k}^c$ or $\bar{n}^c$

To maintain equilibrium following a change in τ, the rental price of capital adjusts to fully offset the effect of a tariff change on the domestic price index: the above derivative is equal to the derivative of equation (1.13), the firm's profit-maximization condition, with respect to τ, holding fixed the firm-level prices. Thus, because the rental price adjustment maintains the equality of equation (1.13) for a fixed level of k, there is no adjustment in capital and subsequently there is also no need for n to respond to the change in τ.

Proposition 1.2 (Impact of a change in τ on producer equilibrium)**.** *Assuming the existence of a producer equilibrium with allocations $(\bar{k}^c, \bar{n}^c)$ and rental price of capital $\bar{r}^c(\tau)$, a change in the tariff level has no effect on the equilibrium $\bar{k}^c$ and $\bar{n}^c$. The rental price of capital increases according to equation (1.18) to offset the effect of a tariff increase on firm profits.*

1.3.2 Ex-Ante Social-Welfare-Maximizing Tariff Choice

The government sets the tariff level to maximize domestic social welfare, given the ex-ante producer-equilibrium variables $\bar{k}^c$, $\bar{r}^c(\tau)$, and $\bar{n}^c$. The derivative of the government's welfare with respect to the tariff level is

$$\begin{aligned}\frac{dW(\bar{k}^c, \bar{n}^c, \tau)}{d\tau} &= N\bar{n}^c(p_h(\bar{k}^c) - \lambda(\bar{k}^c))\,\frac{\partial q^c_{h2}}{\partial \tau} + Nn^*\,(\tau - 1)\,p_h(\bar{k}^c)\frac{\lambda^*}{\lambda(\bar{k}^c)}\,\frac{\partial q^c_{f2}}{\partial \tau},\\ &= N\bar{n}^c(p_h(\bar{k}^c) - \lambda(\bar{k}^c))\,\frac{\partial q^c_{h2}}{\partial \tau} + N\bar{n}^c\left(\frac{\tau - 1}{\tau}\right)p_h(\bar{k}^c)\frac{z^c\sigma + 1}{z^c(\sigma - 1)}\,\frac{\partial q^c_{h2}}{\partial \tau},\end{aligned} \tag{1.19}$$

where $z \equiv (n\,p_h(k)\,q_{h2})/(n^*\,p_f\,q_{f2}) = (n/n^*)\,(\lambda(k)/(\tau\lambda^*))^{1-\sigma}$ is the inverse import penetration, the market share of domestic products relative to the market share of foreign products at the tax-included home-country price.[19] Equation (1.19) can always be satisfied with a finite tariff without imposing any restriction on the parameter values. I show later that this

[19]The foreign equivalent, the market share abroad of foreign-produced sector 2 varieties relative to home-produced products at the tax-included foreign country price, is $z^* \equiv (n^*/n)\big(\lambda^*/(\tau^*\lambda(k))\big)^{1-\sigma}$. Equation (1.19) can be verified using the fact that $\partial q_{f2}/\partial\tau = -\big(\tau\lambda^*/\lambda(k)\big)^{-1}\big[\big(\sigma/(\sigma-1)\big)(n/n^*) + 1/(\sigma - 1)\big(\tau\lambda^*/\lambda(k)\big)^{1-\sigma}\big](\partial q_{h2}/\partial\tau)$.

is not the case for the tariff set in response to lobbying.[20]

Using that $\partial q_{h2}/\partial \tau > 0$, the social-welfare-maximizing tariff rule, τ^c, is implicitly defined below by equation (1.20) in Proposition 1.3.

Proposition 1.3 (Ex-ante tariff social-welfare-maximizing tariff commitment)**.** *Given the government sets the tariff before production decisions are made, the social-welfare-maximizing tariff level, τ^c, given production in the sector is in ex-ante equilibrium, is implicitly defined by*

$$\frac{\tau^c - 1}{\tau^c} = \left(\frac{\sigma - 1}{\sigma}\right) \frac{1}{\big(\sigma + (1/z^c)\big)}, \tag{1.20}$$

subject to the second-order condition

$$\frac{\tau^c}{1 + z^c} < \frac{\sigma}{\sigma - 1}, \tag{1.21}$$

where z^c is the inverse import penetration evaluated at the commitment production and consumption levels.

From equation (1.20), it is clear that the social-welfare-maximizing trade policy is an import tariff on foreign varieties of the differentiated good (i.e. $\tau^c > 1$), consistent with Chang (2005) and other monopolistically competitive models.[21] Specifically, the tariff definition above is equivalent to Chang's tariff definition evaluated where firm owners comprise a negligible share of the population and sectors are not politically organized.

Drawing from the discussion in Chang (2005), τ^c is composed of two key effects working in opposite directions. These effects are a "monopoly-power" effect and an "import-

[20]To verify a finite solution for τ exists, look at the second term of equation (1.19). First, as τ goes to zero, $\lim_{\tau\to 0}((\tau-1)/\tau)\,(z\sigma+1)/(z(\sigma-1)) = -\infty$. As τ approaches infinity, $\lim_{\tau\to\infty}((\tau-1)/\tau)\,(z\sigma+1)/(z(\sigma-1)) = \big(\sigma/(\sigma-1)\big)$. For a tariff policy $\tau \in (0,\infty)$ such that $dW(\bar{k}^c, \bar{n}^c, \tau)/d\tau$ to exist, it must be true that $1/\sigma - \big(\sigma/(\sigma-1)\big) < 0$. This condition is trivially satisfied for all $\sigma \in \mathbb{R}$.

[21]Gros (1987) and Flam and Helpman (1987) are key papers which originally demonstrated the welfare-maximizing tariff for a monopolistically competitive industry in a small country is strictly positive. My tariff differs from the Gros (1987) optimal tariff due to the fact that Gros assumes that the marginal costs of production are equal across countries, $\lambda(k) = \lambda^*$. In terms of my model definitions, the combination of these assumptions yields an optimal tariff $\tau = \frac{\sigma}{\sigma-1} + \frac{1}{\sigma-1}\,z$. Relaxing the assumption of the marginal costs being equal, my tariff is equivalent to Gros's finding.

penetration" effect. The monopoly-power effect is negatively related to the tariff level: an increase in σ leads to a decrease in a firm's monopoly power, which then leads to a rise in the tariff level. The import-penetration effect works in the opposite direction: an increase in σ leads to an increase in $1/z^c$ (and in $1/(\sigma + (z^c)^{-1})$ as well), and therefore, a decrease in the tariff.

This tariff is equivalent to the monopolistically competitive model in Chang (2005) and the original Grossman and Helpman (1994) given there is no political economy motive, and the share of the population represented by firm owners in sector 2 (α_L in the Grossman-Helpman model) is negligible. The tariff is different from the standard "optimal tariff" for a social-welfare-maximizing government, given the standard "optimal tariff" for a perfectly competitive industry is equal to the inverse of the export-supply elasticity. In my model this is not true, because the home country has no terms-of-trade influence and because the lobbying sector is monopolistically competitive instead of perfectly competitive. To make the comparison concrete, the export-supply elasticity for the model is $\xi^c = -(\sigma z^c + 1)/(z^c + 1)$, which means the tariff can be written:

$$\frac{\tau^c - 1}{\tau^c} = \left(\frac{\sigma - 1}{\sigma}\right)\left(\frac{z^c}{z^c + 1}\right)\left(-\frac{1}{\xi^c}\right).$$

Therefore, the social-welfare-maximizing tariff in my model is decreasing in the export-supply elasticity, consistent with the perfectly competitive optimal tariff, but it is also influenced by the monopoly power of the firm and the relative size of the home country's consumer market.

As for the second-order condition in equation (1.21), $1/(z^c + 1)$ is the equilibrium market share of foreign products in the domestic market given the tariff level τ^c, and $\sigma/(\sigma - 1)$ is the degree of monopoly power enjoyed by a firm in the domestic market. In order for a tariff to be welfare-maximizing, it must be true that the tariff multiplied by the foreign market share is larger than the monopoly power of a firm.

1.3.3 Behavior of Ex-Ante Producer Equilibrium and Tariff Level

In this section, I demonstrate that an increase in the substitutability of varieties leads to an increase in the capital hiring by each firm and a decrease in the number of varieties produced. Note that although k, n, r, and τ are dependent on σ and other exogenous model parameters, I have not written them as a function of the exogenous parameters in an effort to simplify the model notation.

First, the relationship between ex-ante production and the elasticity of substitution across varieties is most easily found by differentiating equation (1.17a) with respect to σ. The derivative of capital hiring with respect to σ is

$$\frac{d\bar{k}^c}{d\sigma} = \frac{-\lambda'(\bar{k}^c)\bar{k}^c}{(\sigma-1)\lambda''(\bar{k}^c)\bar{k}^c + \sigma\lambda'(\bar{k}^c)}. \tag{1.22}$$

First because $\lambda'(k) < 0$, the numerator is positive. The denominator is positive, which can be seen using equation (1.17a) to eliminate $\bar{k}^c$, which then allows the denominator to be rewritten as $\Big(-\lambda(\bar{k}^c)/\lambda'(\bar{k}^c)\Big)\Big(\lambda''(\bar{k}^c) - \sigma\Big(\lambda'(\bar{k}^c)^2/\lambda(\bar{k}^c)\Big)$, which is positive given the second-order condition for the social-welfare-maximizing tariff. Therefore, the ex-ante equilibrium $\bar{k}^c$ is increasing in σ.[22]

Proposition 1.4 (Impact of change in σ on ex-ante producer equilibrium)**.** *Given the market is in ex-ante equilibrium and monopolistically competitive firms are maximizing profits such that* $\lambda''(\bar{k}^c) - \sigma\Big(\lambda'(\bar{k}^c)^2/\lambda(\bar{k}^c)\Big) > 0$*, an increase in the elasticity of substitution,* σ*, leads to an increase in the equilibrium capital hiring of each firm,* $d\bar{k}^c/d\sigma > 0$*. Given the fixed capital endowment, this means an increase in* σ *also leads to a decrease in the number of firms,* $d\bar{n}^c/d\sigma < 0$*.*[23]

The number of the firms in the home country is decreasing as goods become more substitutable. When varieties are relatively interchangeable, there is less benefit to a firm from producing its own variety, leading to an ex-ante equilibrium in which fewer firms enter the market. As a result of fewer firms entering, each firm hires more capital which

[22] $d\bar{k}/d\sigma$ is fully derived in Appendix A.5.2.
[23] In Appendix A.6.1 I also establish that $\lim_{\sigma\to 1} d\bar{n}^c/d\sigma = -\infty$ and $\lim_{\sigma\to\infty} d\bar{n}^c/d\sigma = 0$.

reduces unit-labor costs. This finding makes intuitive sense: as varieties become more substitutable, firms prefer to produce fewer varieties at a lower cost, taking advantage of economies of scale in the absence of greater monopoly power.

To ensure that the model is consistent with standard trade models, I assume that $\lambda^* = \lim_{k\to\infty} \lambda(k)$. This assumption ensures the government's social-welfare-maximizing trade policy is free trade for an industry in which demand is perfectly elastic.[24]

Next, the derivative of τ^c with respect to σ is

$$\frac{d\tau^c}{d\sigma} = \frac{(\tau^c)^2 z^c\left(\left(\frac{1}{\sigma}\right)^2 - \frac{\tau^c-1}{\tau^c} + \frac{\tau^c-1}{\tau^c}\ln\left(\frac{p_f^c}{p_h(\bar{k}^c)}\right)\frac{1}{z^c}\right)}{(\sigma z^c+1)-(\sigma-1)(\tau^c-1)}, \tag{1.23}$$

which is derived by taking the total derivative of equation (1.20) with respect to σ.[25] Unlike $\bar{k}^c$ and $\bar{n}^c$, the sign of the derivative of τ^c with respect to σ is non-monotonic. This non-monotonicity of the tariff in σ is a key feature of the tariff equations both with and without ex-ante tariff commitment because it is one of the key factors underlying the nonmonotonicity of the governments trade policy preference which I discussed in the introduction and will show later on. In this section, I examine the nature of the tariff's non-monotonicity in σ given firm entry and capital hiring are held constant. In Section 1.4.5, I explore how the addition of a firm-entry effect to equation (1.23) changes the overall relationship between the tariff level and the substitutability of varieties.

[24]If the optimal solution to the model is commitment to free trade (i.e. $\tau^c = 1$) and the optimal capital hiring is such that the domestic price is equal to the foreign price $p_h(\bar{k}^c) \equiv \frac{\sigma}{\sigma-1}\lambda(\bar{k}^c) = p_f^* \equiv \frac{\sigma}{\sigma-1}\lambda^*$, the tariff equation from equation (1.20) becomes

$$\frac{1-1}{1} = \left(\frac{\sigma-1}{\sigma}\right)\frac{1}{\left(\sigma + \frac{n^*}{\bar{n}^c}(1)^{1-\sigma}\right)}.$$

For the above equation to be true, given that the number of firms in the industry is decreasing in σ, when $\sigma \to \infty$

$$\lim_{\sigma\to\infty}\left(\frac{\sigma-1}{\sigma}\right)\frac{1}{\left(\sigma+\frac{n^*}{\bar{n}^c}(1)^{1-\sigma}\right)} = \lim_{\sigma\to\infty}\frac{1-\frac{1}{\sigma}}{\left(\sigma+\frac{n^*}{\bar{n}^c}\right)} = \frac{1-\frac{1}{\infty}}{\infty+\frac{n^*}{0}} = \frac{1}{\infty} = 0,$$

which means the tariff equation for $\tau^c = 1$ and $\lambda(\bar{k}^c) = \lambda^*$ is satisfied. This means $\lambda^* = \lim_{k\to\infty}\lambda(k)$. For my numerical solution, I use $\lambda(k) = \mathbf{e}^{\frac{1}{k}}$. Therefore, this means that for the numerical solution to the model, I must have $\lambda^* = \lim_{k\to\infty}\lambda(k) = \lim_{k\to\infty}\mathbf{e}^{\frac{1}{k}} = 1$.

[25]This derivation is provided in Appendix A.5.2.

As defined by equation (1.23), first notice that the denominator of the derivative is positive given the second-order condition for τ^c (equation (1.21)). Therefore, the sign of $d\tau^c/d\sigma$ is determined by the term in the numerator:

$$\frac{1}{\sigma^2} - \left(\frac{\tau^c - 1}{\tau^c}\right) + \left(\frac{\tau^c - 1}{\tau^c}\right) \ln\left(\frac{p_f^c}{p_h(\bar{k}^c)}\right) \frac{1}{z^c}. \tag{1.24}$$

If this term is positive, the equilibrium tariff is increasing in σ. Substituting in equation (1.20) allows more intuitive analysis of the first two terms. The terms can be rewritten as

$$\left[- \underbrace{\frac{z^c}{z^c + 1}}_{(Market\ Share)} + \underbrace{\left(\frac{1}{\sigma - 1}\right)^2}_{d/d\sigma(Market\ Power)} \right] \left(\frac{\sigma - 1}{\sigma}\right) \left(\frac{\tau^c - 1}{\tau^c}\right) \left(\frac{z^c + 1}{z^c}\right). \tag{1.24a}$$

The first two terms of equation (1.24) combined lead to an increase in the social-welfare-maximizing tariff if the lost market power from an increase in σ is greater than the market share of home producers. Loosely, if $z/(z+1)$ is close to one and home producers dominate the home market, equation (1.24a) is positive if $\sigma < 2$. On the other hand, if $z/(z+1)$ is close to zero and foreign producers control the domestic market, equation (1.24a) is positive for all values of $\sigma > 1$. The sign of the final term in equation (1.24) is dependent on the sign of the natural log of the home country's terms of trade for the differentiated good. This term is positive if the tariff-included price of foreign varieties, p_f^c, exceeds the price of the domestic varieties, p_h^c.

To determine the behavior of τ^c for changes in σ, look at the value and derivative of τ^c for extreme values of σ.[26] First, looking at equation (1.20), as σ approaches one, $\lim_{\sigma\to 1}\left(\frac{\tau^c - 1}{\tau^c}\right) = (0)\frac{1}{1+(1/\infty)} = 0$ given that $\lim_{\sigma\to 1} z^c = \bar{n}^c/n^* = \infty$, which then means $\lim_{\sigma\to 1}\tau^c = 1$. Additionally, this means the home firms dominate the home market, resulting in equation (1.24a) being positive. The sign of the natural log term, however, takes more work to determine.[27]

[26]Production for $\sigma \to 1$ is derived in Appendix A.4. Given production becomes Cobb-Douglas as $\sigma \to 1$, the appendix section shows that with Cobb-Douglas production, $z = n/n^*$.

[27]I include the full derivation of $\lim_{\sigma\to 1} d\tau^c/d\sigma > 0$ and $\lim_{\sigma\to\infty} d\tau^c/d\sigma = 0$ in Appendix A.6.1.

Next, as σ approaches infinity, the global market for the differentiated good becomes perfectly competitive, and I have already established $\lim_{\sigma\to\infty} p_h(\bar{k}^c) = p_f^c$. In turn, this means the limit of the first-order condition is zero, given that $\lim_{\sigma\to\infty} \frac{1}{\sigma^2} - \left(\frac{\tau^c-1}{\tau^c}\right) + \left(\frac{\tau^c-1}{\tau^c}\right) \ln\left(\frac{p_f^c}{p_h(\bar{k}^c)}\right)\frac{1}{z^c} = 0$. Taking the second derivative of τ^c and simplifying using the limits of τ^c and $d\tau^c/d\sigma = 0$ and given equation (1.24) is zero, I find that the second-order condition is negative. Therefore, as $\sigma \to \infty$, the tariff level approaches a minimum at $\tau^c = 1$. Therefore, τ^c is nonmonotonic in σ.

Proposition 1.5 (Impact of change in σ on equilibrium tariff, τ^c). *The tariff is non-monotonically related to the elasticity of substitution of varieties, σ. The derivative of the tariff with respect to σ is defined in equation (1.23). For σ close to one, $\tau^c = 1$ and is increasing in σ. As σ approaches infinity, τ^c is again approaching one and is at a local minimum. For intermediate values of σ, the tariff is increasing in σ where*

$$\frac{1}{\sigma^2} - \left(\frac{\tau^c - 1}{\tau^c}\right) + \left(\frac{\tau^c - 1}{\tau^c}\right) \ln\left(\frac{p_f}{p_h(\bar{k}^c)}\right)\frac{1}{z^c} > 0,$$

and decreasing in sigma where the above is negative.

Recall from the introduction that one of the results of my model is that the government always prefers to commit to a trade agreement covering products with $\sigma \to 1$ and when σ is very large. This result stems from the fact that the welfare benefit to the government of joining a TA is nonmonotonic in σ, with the nonmonotonicity of the welfare benefit being in part driven the nonmonotonicity of the tariff level in σ.

I compare this tariff equation's behavior with the behavior of the politically optimal tariff level in Section 1.4.5. The numerical estimate of τ^c is depicted in Figure 1.2.[28]

Returning again to discussion of the monopoly-power and import-penetration effects, the change in the slope of τ^c from positive to negative as σ increases reflects a change in which effect is dominant: for small enough σ, the firm's diminished monopoly power

[28]Parameter values and function specifications used in the numerical solution are given in Table A.1 in Appendix A.8.

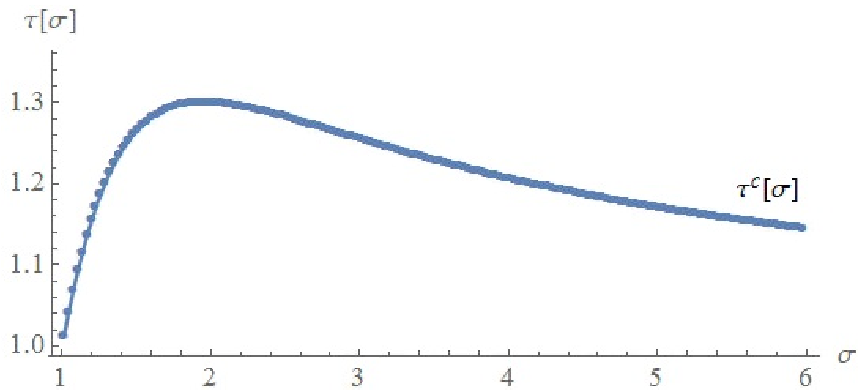


Figure 1.2: Equilibrium tariff level with ex-ante tariff commitment, given the best response capital hiring and firm entry, $\bar{k}^c$ and $\bar{n}^c$, and the capital-market clearing rental price of capital, $\bar{r}^c(\tau)$.

from an increase in σ leads the government to increase tariff protection for domestic firms. Once σ reaches a certain level, subsequent increases in σ lead to a large enough increase in import penetration that the tariff falls, allowing consumers in the home country to benefit from the increase competition and lower prices for the differentiated good.

1.4 Ex-Post Tariff Selection and Domestic Lobbying

In this section, the government is no longer able to commit to a tariff level before firms enter and hire capital, and producers are free to lobby the government for tariff protection. I use the "u" superscript to indicate that variables are "unconstrained" by any ex-ante commitments.

Again, I proceed by backward induction. In Section 1.4.1, I solve the Nash bargaining problem for the government-welfare-maximizing tariff, τ^u, and contribution, c^u. In Section 1.4.2, I solve for firm entry and capital hiring, given the contribution and tariff rules. In Section 1.4.3 I discuss the effect of the political economy weight on the tariff level and the ex-ante producer equilibrium. Section 1.4.4 provides a brief discussion of the potential for the Metzler paradox to arise for certain values of a, and I introduce an assumption to ensure the Metlzer paradox does not occur. Finally, I discuss the impact of changes in the elasticity of substitution on the producer equilibrium variables and the tariff in Section

1.4.5.

1.4.1 Lobbying Problem: Tariff and Lobby Contribution

In the Nash bargaining problem, the tariff schedule is found by maximizing the sum of the government welfare and the politically-weighted welfare of lobbying firms with respect to the domestic tariff level, τ. To ensure a finite solution for the tariff policy exists, assume the values of a and σ are such that $(1+a) < \left(\frac{\sigma}{\sigma-1}\right)\sigma$.[29] The total politically weighted welfare of the government and the lobby, $TW(k,n,r,\tau)$, is

$$\begin{aligned} TW(k,n,r,\tau) &= [W(k,n,\tau) + a\,n\,c\,k] + a\,n[\Pi(k,r,\mathbf{P}) - c\,k] \\ &= L + (1+a)n\,\pi(k,\mathbf{P}) - a\,r\,K + TR(k,n,\tau) + N\Big(u(d(P)) - P\,d(P)\Big). \end{aligned}$$

Taking the derivative of the above equation with respect to τ gives

$$\begin{aligned} \frac{dTW(k,n,r,\tau)}{d\tau} &= (1+a)Nn\,(p_h(k) - \lambda(k))\,\frac{\partial q_{h2}}{\partial \tau} + Nn^*\,(\tau-1)\,p_f^*\frac{\partial q_{f2}}{\partial \tau}, \\ &= (1+a)Nn\,(p_h(k) - \lambda(k))\,\frac{\partial q_{h2}}{\partial \tau} - Nn\left(\frac{\tau-1}{\tau}\right)\frac{z\sigma+1}{z(\sigma-1)}\,p_h\,\frac{\partial q_{h2}}{\partial \tau}. \end{aligned} \tag{1.25}$$

Notice in equation (1.25) that $\lim_{\tau\to 0}\left(\frac{\tau-1}{\tau}\right)\left(\frac{z\sigma+1}{z(\sigma-1)}\right) = -\infty$ and that $\lim_{\tau\to\infty}\left(\frac{\tau-1}{\tau}\right)\left(\frac{z\sigma+1}{z(\sigma-1)}\right) = \frac{\sigma}{\sigma-1}$. In order for $\frac{dTW(k,n,r,\tau)}{d\tau}$ to be equal to zero for a value of $\tau \in (0,\infty)$, given that $\lim_{\tau\to 0}\frac{dTW(k,n,r,\tau)}{d\tau} = \infty$, it must be true that $\lim_{\tau\to\infty}\frac{dTW(k,n,r,\tau)}{d\tau} < 0$. Looking at equation (1.25), this condition is satisfied if $(1+a)\frac{1}{\sigma} - \left(\frac{\sigma}{\sigma-1}\right) < 0$. Therefore, finite solutions for the endogenous tariff level, τ, exist if $(1+a) < \left(\frac{\sigma}{\sigma-1}\right)\sigma$.

Using the fact that $\partial q_{h2}/\partial\tau > 0$, equation (1.25) can be solved to find the government's optimal tariff policy given the government is unconstrained by a trade agreement.

Proposition 1.6 (Tariff without a trade agreement)**.** *Given a producer equilibrium, (k,n), and given the values of a and σ are such that $(1+a) < \left(\frac{\sigma}{\sigma-1}\right)\sigma$, the government's ex-post politically*

[29] This restriction on a and σ is further explained following equation (1.25).

optimal tariff, $\tau^u(k,n)$, is characterized by the equation

$$\frac{\tau^u(k,n)-1}{\tau^u(k,n)} = \left(\frac{\sigma-1}{\sigma}\right)\frac{(1+a)}{\left(\sigma + z^u(k,n)^{-1}\right)}, \tag{1.26}$$

subject to the second-order condition

$$\frac{\tau^u(k,n)}{1+z^u(k,n)} < \frac{\sigma}{\sigma-1}, \tag{1.27}$$

where $z^u(k,n)$ is the inverse of import penetration given the tariff level $\tau^u(k,n)$.

As was true of τ^c in Proposition 1.3, the unconstrained optimal tariff policy is an import tariff, $\tau^u(k,n) > 1$. Further analysis of the behavior of τ^u and a comparison of this tariff to τ^c is given in Proposition 1.15 (in Section 1.4.5) after I derive the ex-ante producer equilibrium variables (in Section 1.4.2).

First, the second-order condition is unchanged from Section 1.3.2: in order for the tariff to be ex-post politically optimal, the monopoly power must be larger than the home producer's market share times the tariff level. Next, consistent with τ^c and with Chang (2005), it remains true that for a fixed k and n the tariff is increasing in the degree of import penetration and decreasing in the monopoly power of firms in the industry. When domestic firms lobby, the key difference in the tariff definition is the presence of the political economy parameter in the numerator. As a result, for fixed k and n, an increase in the political influence of firms increases so does the politically optimal tariff level.[30]

The second half of the bargaining solution is the contribution the lobbying industry pays to the government. The contribution is determined by Nash bargaining between the government and the lobby, where the government has bargaining strength $\gamma \in [0,1]$ and the lobby has bargaining strength $1-\gamma$. The government and the lobby negotiate over the division of the rents from tariff protection, given a fixed n, k, and τ. If bargaining fails, the

[30] I examine the effect of σ and a allowing k and n to adjust in Section 1.4.5, once I have solved the ex-ante producer equilibrium in Section 1.4.5.

government's threat point is the "ex-post commitment" tariff, $\tau^{cxp}(k,n) < \tau^{u}(k,n)$, which is the tariff that maximizes ex-post social welfare equal to equation (1.26) evaluated at $a = 0$. The resulting contribution per unit of capital hired is

$$c^{u}(k,n) = \left(\frac{1-\gamma}{a}\right)\frac{W(k,n,\tau^{cxp}) - W(k,n,\tau^{u})}{n\,k} + \gamma\frac{\pi(k,\mathbf{P}^{u}) - \pi(k,\mathbf{P}^{cxp})}{k}, \tag{1.28}$$

where $\tau^{cxp} \equiv \tau^{cxp}(k,n)$ and $\tau^{u} \equiv \tau^{u}(k,n)$. In total each firm in the lobbying sector pays $\widetilde{c}^{u}(k,n) \equiv c^{u}(k,n)\,k + \theta(k)$, the contribution to the government plus the upfront cost of lobbying.

The contribution the lobby pays to the government is a weighted average of the lost social welfare from the increase in tariff protection over the threat point, τ^{cxp}, and the welfare gains by the lobbying industry resulting from the additional tariff protection. This structure of contributions satisfies the requirement of "truthful contributions" from Grossman and Helpman (1994), meaning the lobby's willingness to pay for a marginal increase in the tariff level is equal to its marginal contribution to the government.

Given the definition of the contribution, the government will always choose to grant the higher level of tariff protection if it has not committed ex-ante to a trade agreement. To demonstrate this, note first that if the government sets the tariff to maximize ex-post social welfare, then it receives total welfare $G^{cxp} = W(k,n,\tau^{cxp})$. If it grants the politically optimal tariff protection, however, it receives $G^{u} = W(k,n,\tau^{u}) + a\,n\,k\,c^{u}(k,n)$. Assuming that firm owners lobby for tariff protection and using the definition of the contribution rule to simplify, the government chooses to grant the politically-optimal tariff protection if

$$\begin{aligned}\gamma W(k,n,\tau^{u}(k,n)) + (1-\gamma)(W(k,n,\tau^{cxp}(k,n)))\\ + \gamma a n\left(\pi(k,\mathbf{P}^{u}) - \pi(k,\mathbf{P}^{cxp})\right) > W(k,n,\tau^{cxp}(k,n)),\end{aligned} \tag{1.29}$$

with $W(k,n,\tau^{u}(k,n)) < W(k,n,\tau^{cxp}(k,n))$ and $\pi(k,\mathbf{P}^{u}) > \pi(k,\mathbf{P}^{cxp})$. In the perfectly competitive model, Maggi and Rodríguez-Clare (1998) show that this inequality is always satisfied, meaning if there is no trade agreement, the government always imposes the politically-motivated τ^u instead of τ^{cxp}. In my model, this is not obviously true without taking into account the lobbying participation constraint.

Because there is an upfront lobbying cost, firms only organize to form a lobby if the net payoff from successfully lobbying is greater than the payoff of not lobbying, if $\pi(k, \mathbf{P}^u) - c^u\, k - \theta(k) > \pi(k, \mathbf{P}^{cxp})$.[31] Again using equation (1.28) to simplify, firms form a lobby if

$$\begin{aligned} \theta(k) < -\left(\frac{1-\gamma}{a\,n}\right)\Big(W(k,n,\tau^{cxp}(k,n)) - W(k,n,\tau^{u}(k,n))\Big) \\ + (1-\gamma)\Big(\pi(k,\mathbf{P}^u) - \pi(k,\mathbf{P}^{cxp})\Big). \end{aligned} \tag{1.30}$$

A firm is willing to participate in negotiations if the upfront lobbying cost is smaller than the firm's share of the total rents from tariff protection. If equation (1.30) is not true, then firms do not participate in lobbying and the government sets the tariff $\tau^{cxp}(k, n)$.[32] Comparing the inequalities in equations (1.29) and (1.30), it is clear that if equation (1.30) is true, then equation (1.29) must also be true. Therefore, if model parameters are such that the lobby forms, then the government is always better off by granting more tariff protection in exchange for the contribution from the lobby.

Assumption 1.7 (Lobbying occurs)**.** *Given a producer equilibrium, (k, n), and given the values of a and σ are such that $(1 + a) < \left(\frac{\sigma}{\sigma-1}\right)\sigma$, assume the upfront lobbying cost is such that*

$$\begin{aligned} \theta(k) < -\left(\frac{1-\gamma}{a\,n}\right)\Big(W(k,n,\tau^{cxp}(k,n)) - W(k,n,\tau^{u}(k,n))\Big) \\ + (1-\gamma)\Big(\pi(k,\mathbf{P}^u) - \pi(k,\mathbf{P}^{cxp})\Big), \end{aligned} \qquad \text{(eq. 1.30)}$$

which means producers in the monopolistically competitive sector choose to form a lobby.

Once the lobby decides to engage in bargaining, $\theta(k)$ is a sunk cost. As a result, the

[31] I use the strict inequality assuming that if firms are indifferent between forming a lobby and not, they choose not to organize.

[32] Going a step further, if the firms infer by backward induction that they will not to participate in lobbying, then ex-ante production decisions will reflect this expectation. This means $\bar{n}^c$ firms will enter and hire $\bar{k}^c$ units of capital.

lobby's "truthful" contribution, $c^u(k,n)$, is unchanged by adding in the upfront lobbying cost. For the remainder of the paper, I assume that the participation constraint is always satisfied.

1.4.2 Ex-Ante Producer Equilibrium

Assume model parameters are consistent with the existence of an interior solution.[33] The solution for the ex-ante producer equilibrium is found by using the same methods as Section 1.3.1, with the addition of the contribution rule, $\tilde{c}(k_m) \equiv c^u\, k_m + \theta(k_m)$, to the firm's profit function.

In the home country, firm m hires capital amount k_m to maximize its profits. With lobbying, profit-maximization must take into account the total cost of lobbying, $\tilde{c}(k_m)$. In equilibrium, all firms in the home industry are identical. A representative firm's capital-hiring rule, $\tilde{k} \equiv \tilde{k}(r,n)$, is implicitly defined by the equation

$$\begin{aligned} r + \tilde{c}'(\tilde{k}) = -\lambda'\Big(\tilde{k}\Big)\Big[& N d_h\Big(p_h(\tilde{k}), q_2(\tilde{k},n,\tau), P(\tilde{k},n,\tau)\Big) \\ & + N^* d_h^*\Big(p_h^*(\tilde{k}), q_2^*(\tilde{k},n,\tau), P^*(\tilde{k},n,\tau)\Big)\Big], \end{aligned} \tag{1.31}$$

given a fixed rental price of capital and number of firms in the industry, and where the derivative of the total contribution with respect to k is $\tilde{c}'(\tilde{k}) = c + \theta'(\tilde{k})$. At the profit-maximizing $\tilde{k}(r,n)$, the cost of hiring an additional unit of capital inclusive of the marginal increase in lobbying costs is equal to the firm's marginal benefit from decreased labor costs. The second-order condition for profit maximization is

$$\frac{d^2\Pi}{dk^2} = -\left(\lambda''\Big(\tilde{k}\Big) - \sigma\frac{\lambda'(\tilde{k})^2}{\lambda(\tilde{k})}\right)\Big[N d_h\Big(p_h(\tilde{k}), q_2, P\Big) + N^* d_h^*\Big(p_h^*(\tilde{k}), q_2^*, P^*\Big)\Big] - \theta''(\tilde{k}), \tag{1.32}$$

where the dependence of the price and quantity indexes on $(\tilde{k}, n, \tau)$ has been dropped to conserve space. In Section 1.3.1, the second-order condition was negative as long as

[33] The conditions for an interior solution are fully expanded in Appendix A.2.2.

$\lambda''\left(\tilde{k}\right) - \sigma\lambda'(\tilde{k})^2/\lambda(\tilde{k}) > 0$. Here, because $\theta''(k) > 0$, the assumption from Section 1.3.1 that $\lambda''\left(\tilde{k}\right) - \sigma\lambda'(\tilde{k})^2/\lambda(\tilde{k}) > 0$ for all $k \in (0, \infty)$ guarantees that equation (1.32) is also true for all $k \in (0, \infty)$.

Given the capital-hiring rule, $\tilde{k}(r, n)$, and fixed number of firms in the industry, n, the rental price of capital adjusts to clear the market for capital. The capital-market-clearing rental price, $\hat{r}(n)$, is

$$n \cdot \tilde{k}(\hat{r}(n), n) = K. \tag{1.33}$$

Denote the profit-maximizing capital-hiring rule when the rental price of capital is $\hat{r}(n)$ as $\hat{k}(n) \equiv \tilde{k}(\hat{r}(n), n)$.

Finally, assuming capital is $\hat{k}(n)$ and the rental price of capital is $\hat{r}(n)$, the free entry of firms drives the profits for each firm down to zero. The equilibrium number of firms, $\bar{n}^u$, is

$$\begin{aligned} \bar{r}^u\bar{k}^u + \tilde{c}(\hat{k}) = \frac{1}{\sigma - 1}\lambda(\bar{k}^u)\Big[& N d_h\Big(p_h(\bar{k}^u), q_2\Big(\bar{k}^u, \bar{n}^u, \tau\Big), P\Big(\bar{k}^u, \bar{n}^u, \tau\Big)\Big) \\ & + N^* d_h^*\Big(p_h^*\Big(\bar{k}^u\Big), q_2^*\Big(\bar{k}^u, \bar{n}^u, \tau\Big), P^*\Big(\bar{k}^u, \bar{n}^u, \tau\Big)\Big)\Big], \end{aligned} \tag{1.34}$$

where $\bar{k}^u \equiv \hat{k}(n)$ and $\bar{r}^u \equiv \hat{r}(n)$, and $\tilde{c}(\bar{k}^u) = c\,\bar{k}^u + \theta(\bar{k}^u)$.

As is true without lobbying, the profit maximization and zero-profit equations, (1.31) and (1.34), combined define the firm's ex-ante equilibrium capital hiring. Comparing these equations to the same equations from Section 1.3.1, equations (1.13) and (1.16), it is immediately evident that lobbying only leads to an ex-ante production distortion if the total lobbying cost, $\tilde{c}(\bar{k}^u)$ is non-linear. In theory, either the upfront cost, $\theta(k)$, or the contribution to the government, $c\,k$, could satisfy this role. I build my model such that the non-linearity derives from the upfront lobbying cost. This allows me to keep my model as similar as possible to prior analysis done by Grossman and Helpman (1994) and Maggi and Rodríguez-Clare (1998) assumptions on c. Namely, that each individual firm takes c as given in making ex-ante production choices, which makes $\theta(k)$ the only option for generating the non-linearity.

Proposition 1.8 (Ex-ante producer equilibrium)**.** *Competitive ex-ante producer equilibrium in the home country's monopolistically competitive sector is defined by best-response rules* $(\bar{k}^u, \bar{n}^u)$ *and price* $\bar{r}^u$ *for which the following statements must be true:*

1. *Each firm hires capital amount,* $\bar{k}^u$*, to maximize its profits (equation (1.31)).*
2. *The rental price of capital,* $\bar{r}^u$*, is such that the fixed capital supply,* K*, equals capital demand (equation (1.33)).*
3. *Firms freely enter until there are* $\bar{n}^u$ *firms in the industry, at which point the ex-ante profits of operating in the monopolistically competitive sector are equal to zero (equation (1.34)).*

The three above points combine to give two equilibrium conditions which define $\bar{k}^u$ *and* $\bar{n}^u$*:*

$$\begin{aligned}(\sigma-1)\Big(\theta(\bar{k}^u) - \theta'(\bar{k}^u)\bar{k}^u\Big) &= \\ &\Big(\lambda(\bar{k}^u) + (\sigma-1)\lambda'(\bar{k}^u)\bar{k}^u\Big)[Nq_{h2}(\bar{k}^u,\bar{n}^u,\tau) + N^* q^*_{h2}(\bar{k}^u,\bar{n}^u,\tau)], \ \ \textit{and}\end{aligned} \tag{1.35a}$$

$$\bar{n}^u\,\bar{k}^u = K, \tag{1.35b}$$

with $\bar{k}^u$ *is determined by equation (1.35a) and* $\bar{n}^u$ *determined by equation (1.35b), subject to the second-order condition for profit maximization,*

$$-\left(\lambda''(\bar{k}^u) - \sigma\frac{\lambda'(\bar{k}^u)^2}{\lambda(\bar{k}^u)}\right)[Nq_{h2}(\bar{k}^u,\bar{n}^u,\tau) + N^* q^*_{h2}(\bar{k}^u,\bar{n}^u,\tau)] - \theta''(\bar{k}^u) < 0, \tag{1.36}$$

with the demand for a firm's output at home and abroad defined by $q_{h2}(\bar{k}^u,\bar{n}^u,\tau) \equiv d_h(p_h(\bar{k}^u), q_2(\bar{k}^u,\bar{n}^u,\tau), P(\bar{k}^u,\bar{n}^u,\tau))$ *and* $q^*_{h2}(\bar{k}^u,\bar{n}^u,\tau) \equiv d^*_h(p^*_h(\bar{k}^u), q^*_2(\bar{k}^u,\bar{n}^u,\tau), P^*(\bar{k}^u,\bar{n}^u,\tau))$*, respectively.*

The convexity of the upfront lobbying cost reveals that $\bar{k}^u < \bar{k}^c$. This is evident because the upfront lobbying cost is convex, $\theta(k) - \theta'(k)\,k < 0$ must be true for any value of k, which in turn means that the equilibrium capital hiring as defined by equation (1.35a) must be such that $\lambda(\bar{k}^u) + (\sigma-1)\lambda'(\bar{k}^u)\bar{k}^u < 0$. Drawing on the assumption for a unique interior ex-ante production solution, I know that $\lambda(k) + (\sigma - 1)\lambda'(k)k < 0$ for any $k < \bar{k}^c$. Therefore, it must be true that $\bar{n}^u > \bar{n}^c$: when lobbying occurs, there is over-entry of firms into the market ex ante. Intuitively, in expectation of the tariff negotiations with the government, a given firm owner hires less capital so it pays a smaller share of the upfront lobbying cost while still reaping the benefit of the tariff protection. Given each individual firm prefers to be smaller, free entry of firms leads to a larger overall number of firms in the industry.

The existence of an ex-ante production distortion, and the direction of that distortion, is dependent on the non-linearity of the upfront lobbying cost.[34] This is clear from the fact that if $\theta(\cdot)$ were linear, then the ex-ante producer equilibrium condition would be unchanged from Proposition 1.1. Therefore, the value of $\theta(\bar{k}^u) - \theta'(\bar{k}^u)\bar{k}^u$ is a good proxy for the degree of distortion of the ex-ante producer equilibrium which a trade agreement has potential to correct.

Proposition 1.9 (Effect of lobbying on firm entry and capital hiring)**.** *Given the existence of an interior ex-ante producer equilibrium both with and without ex-ante tariff commitment, firm entry is greater when the government allows lobbying by domestic producers,* $\bar{n}^u > \bar{n}^c$*.*

Moving forward, I refer to the value of $\lambda(k) + (\sigma - 1)\lambda'(k)k$ as the "wedge" in ex-ante production between $\bar{k}^c$ and $\bar{k}^u$. To further illustrate this point, rearrange equation (1.35a):

$$\left(\underbrace{\frac{\theta(\bar{k}^u)}{\bar{k}^u}}_{\substack{\text{avg. lobby}\\ \text{cost}}} - \underbrace{\theta'(\bar{k}^u)}_{\substack{\text{marg. lobby}\\ \text{cost}}} \right) = \left(\underbrace{\frac{\left(\frac{1}{\sigma-1}\right)\lambda(\bar{k}^u)}{\bar{k}^u}}_{\substack{\text{avg. operating}\\ \text{profit per } k}} - \underbrace{\left(-\lambda'(\bar{k}^u)\right)}_{\substack{\text{marginal labor}\\ \text{cost reduction}}} \right) \underbrace{\left[N q^u_{h2} + N^* q^{*u}_{h2}\right]}_{\substack{\text{Firm } m\text{'s total}\\ \text{output}}} .$$

The left-hand side is the difference between the average lobbying cost per unit of capital and the marginal lobbying cost of additional capital. It is only non-zero when lobbying occurs and lobbying costs are nonlinear. The right-hand side is equal to the average operating profit per unit of capital (equal to the price, p_h, minus the unit-labor-cost, $\lambda(k)$, times total output) minus the marginal reduction in labor costs from hiring additional capital ($-\lambda'(k)$

[34] This is also discussed in Appendix A.1.

times total output). This ex-ante production wedge is zero for $\bar{k}^c$ and negative for the lobbying equilibrium $\bar{k}^u$. Because this wedge appears in many places in the model, I will simply denote it as $w_\lambda(k) \equiv \lambda(k) + (\sigma - 1)\lambda'(k)k$.

1.4.3 Effect of Political Economy on Tariff and Ex-Ante Producer Equilibrium

In this section, I derive the effect of a change in the political economy weight on the producer equilibrium variables and on the tariff level. With the help of a simplifying assumption, I show that firm entry is decreasing in the political economy weight $(d\bar{n}^u/da < 0)$ and the ex-post politically optimal tariff is increasing $(d\tau^u/da > 0)$.

First, I solve for the total derivative of the tariff with respect to a, given $\bar{k}^u$ and $\bar{n}^u$ are also dependent on a. The total derivative of τ^u with respect to the political economy parameter by taking the total derivative of equation (1.26), giving

$$\frac{d\tau^u}{da} = \frac{\partial \tau^u}{\partial a} + \left(\frac{w_\lambda(\bar{k}^u)}{\bar{n}^u \lambda(\bar{k}^u)}\right) \frac{(\tau^u - 1)\tau^u}{(\sigma z^u + 1) - (\sigma - 1)(\tau^u - 1)} \left(\frac{d\bar{n}^u}{da}\right), \tag{1.37}$$

where the partial derivative of the tariff with respect to a is

$$\frac{\partial \tau^u}{\partial a} = \frac{(\frac{\sigma-1}{\sigma})(\tau^u)^2 \, z^u}{(\sigma z^u + 1) - (\sigma - 1)(\tau^u - 1)}. \tag{1.38}$$

This partial derivative is always positive, given the denominator is the second-order condition for the government's welfare-maximizing tariff problem.[35] The indirect effect of a on τ^u is unclear without first solving for $d\bar{n}^u/da$, given that the indirect effect of a change in a is inversely related to $d\bar{n}^u/da$. Differentiating equation (1.35a) with respect to a given that τ^u is dependent on a, the first-order impact of a change in a on the number of firms in a

[35]The complete derivation of equations (1.37) and (1.38) is provided in Appendix A.5.3.

sector is

$$\frac{d\bar{n}^u}{da} = \frac{w_\lambda(\bar{k}^u)\left[N\frac{q^u_{h2}}{P^u}\frac{\partial P^u}{\partial \tau}\frac{\partial \tau^u}{\partial a}\right]}{\frac{-(\bar{k}^u)^2}{\bar{n}^u}\left(-\left(\lambda''(\bar{k}^u) - \sigma\frac{(\lambda'(\bar{k}^u))^2}{\lambda(k^u)}\right)[TD^u] - \theta''\right) - w_\lambda(\bar{k}^u)\left[\mathbf{N}\frac{\mathbf{q}^u_{h2}}{\mathbf{P}^u}\frac{d\mathbf{P}^u}{dn}\right]}, \tag{1.39}$$

where the tariff level is $\tau^u \equiv \tau^u(\bar{k}^u, \bar{n}^u)$; the total demand for home-produced varieties is $TD^u \equiv Nq^u_{h2} + N^*q^{*u}_{h2}$; the effect of a change in $\bar{n}^u$ on demand for home varieties by way of the price index is $\mathbf{N}\frac{\mathbf{q}_{h2}}{\mathbf{P}}\frac{d\mathbf{P}}{dn} \equiv N\frac{q_{h2}}{P}\left(\frac{dP}{dn}\right) + N^*\frac{q^*_{h2}}{P^*}\left(\frac{dP^*}{dn}\right)$; and finally the total derivative of the price index with respect to n is $\frac{dP}{dn} \equiv \frac{\partial P}{\partial k}\frac{d\hat{k}}{dn} + \frac{\partial P}{\partial n} + \frac{\partial P}{\partial \tau}\left(\frac{\partial \tau}{\partial n} + \frac{\partial \tau}{\partial k}\frac{d\hat{k}}{dn}\right)$.[36]

Beginning the examination of $d\bar{n}^u/da$, the numerator of equation (1.39) is a product the ex-ante production wedge and the direct impact of change in a on demand for home-produced varieties holding constant $\bar{k}^u$ and $\bar{n}^u$. The ex-ante production wedge is negative, and the price index for the home country is increasing in a. As a result, the numerator of equation (1.39) must be negative.

The denominator of equation (1.39) is less straightforward to sign. The first main term in the denominator, $-(\bar{k}^u)^2/\bar{n}^u\left(-\left(\lambda''(\bar{k}^u) - \sigma\lambda'(\bar{k}^u)^2/\lambda(\bar{k}^u)\right)[TD^u] - \theta''\right)$, is equal to the firm's second-order condition for profit maximization (equation (1.32)) times $(d\bar{k}^u/d\bar{n}^u * \bar{k}^u) \equiv -(\bar{k}^u)^2/\bar{n}^u$. As a result, this term must always be positive. The second half of the denominator, $w_\lambda(\bar{k}^u)\left[\mathbf{N}\frac{\mathbf{q}^u_{h2}}{\mathbf{P}^u}\frac{d\mathbf{P}^u}{dn}\right]$, is the first-order effect on demand resulting from the home and foreign price indexes' response to a change in $\bar{n}^u$ times the ex-ante production wedge. Given the ex-ante production wedge is negative, if $\mathbf{N}\frac{\mathbf{q}^u_{h2}}{\mathbf{P}^u}\frac{d\mathbf{P}^u}{dn}$ is positive, then the entire denominator must also be positive. The total effect of n on the price indexes, $\mathbf{N}\frac{\mathbf{q}^u_{h2}}{\mathbf{P}^u}\frac{d\mathbf{P}^u}{dn}$, is composed of two parts: (i) the effect of a change in $\bar{n}^u$ on home and foreign demand by way of the price indexes while holding τ^u fixed, which is negative, and (ii) the effect of a change in $\bar{n}^u$ on demand by way of the effect on the price index through τ^u, which is

[36] The fully expanded definition of $\mathbf{N}\frac{\mathbf{q}^u_{h2}}{\mathbf{P}^u}\frac{d\mathbf{P}^u}{dn}$ is

$$\mathbf{N}\frac{\mathbf{q}_{h2}}{\mathbf{P}}\frac{d\mathbf{P}}{dn} = \frac{-w_\lambda(\hat{k})}{(\sigma-1)n\lambda(\hat{k})}\left[Nq_{h2}\frac{\left(\frac{z}{z+1}\right)(\sigma z+1) - (\sigma-1)(\tau-1)}{(\sigma z+1) - (\sigma-1)(\tau-1)} + N^*q^*_{h2}\left(\frac{1}{z^*+1}\right)\right]. \tag{1.40}$$

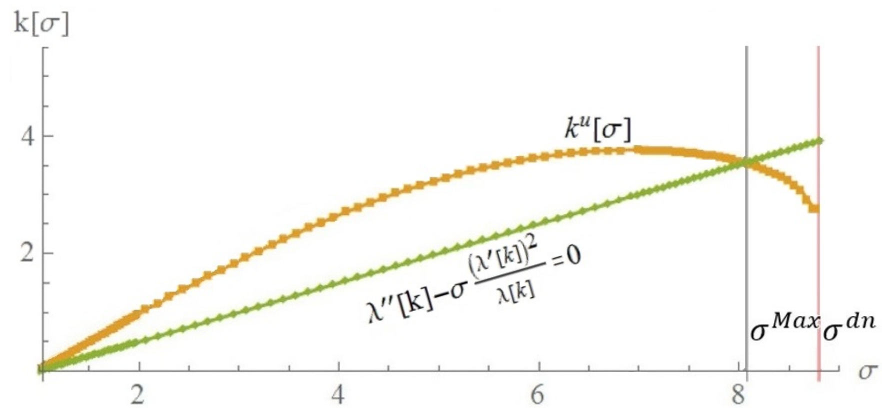


Figure 1.3: The relevant range of elasticities of substitution, $(1, \sigma^{Max})$, is restricted such that $\bar{n}^u$ and $\bar{k}^u$ are continuous over the range of possible σ values. Requiring that the second-order condition from equation (1.32) holds when evaluated at $\bar{k}^u$ satisfies this requirement. The green line shows the (σ, k) combinations for which $\lambda''(k) - \sigma(\lambda'(k))^2/\lambda(k) = 0$. the second-order condition with ex-ante tariff commitment is satisfied at $\bar{k}^u$, which is true for any σ below σ^{Max}.

positive. Overall, $\mathbf{N}\frac{\mathbf{q}^u_{h2}}{\mathbf{P}^u}\frac{d\mathbf{P}^u}{dn}$ is positive if the direct effect of $\bar{n}^u$ on demand is larger than the indirect effect coming through τ^u. Define σ^{dn} as the elasticity of substitution for which the denominator of equation (1.39) is equal to zero.

To eliminate σ^{dn} from my analysis, I restrict the values of σ to a "relevant range," $\sigma \in (1, \sigma^{Max})$, where $\sigma^{Max} < \sigma^{dn}$ is the value of σ for which the second-order condition with tariff commitment, equation (1.14), evaluated at $\bar{k}^u$ is positive. This second-order condition is more restrictive than equation (1.32). Figure 1.3 shows that by assuming equation (1.32) holds when evaluated at $\bar{k}^u$, firm entry $\bar{n}^u$ and capital hiring $\bar{k}^u$ are continuous over the full range of sigma: the range $(1, \sigma^{Max})$ does not include the discontinuity which occurs at σ^{dn}.[37]

Assumption 1.10 (Elasticity of substitution $\sigma \in (1, \sigma^{Max})$). *Moving forward, assume σ falls within the range $(1, \sigma^{Max})$, where for any $\sigma < \sigma^{Max}$, $\lambda''\big(\bar{k}^u(\sigma)\big) - \sigma(\lambda'\big(\bar{k}^u(\sigma)\big))^2/\lambda\big(\bar{k}^u(\sigma)\big) > 0$.*

Return now to the derivative of τ^u from equation (1.37). The effect of a change in a on τ^u is the sum of a direct effect, $\partial\tau^u/\partial a > 0$, plus the indirect effect from an adjustment

[37]More discussion on restricting the range of σ to $\sigma \in (1, \sigma^{Max})$ is given preceding the definition of Assumption A.4 in Appendix A.2.2. Using the numerical solution, I am also able to demonstrate that σ^{Max} and σ^{dn} are increasing in a.

in the number of firms in the industry, $(\partial\tau^u/\partial\bar{n}^u)(d\bar{n}^u/da)$, which is positive (negative) if $d\bar{n}^u/da < (>)0$. With some additional work, I also find that if $d\bar{n}^u/da < (>)0$, then $d\tau^u/da > (<)0$ for all a. Using Assumption 1.10, I can go one step further and say that $d\bar{n}^u/da < 0$ and $d\tau^u/da > 0$ for all a.[38] The behavior of τ^u and $\bar{n}^u$ for a change in a is given in Proposition 1.11.

Proposition 1.11 (Impact of political weight a on τ, $\bar{n}^u$)**.** *Given the existence of an interior producer equilibrium, $(\bar{k}^u, \bar{n}^u)$, and that Assumptions 1.7 and 1.10 hold, total firm entry is decreasing in the political economy weight $(d\bar{n}^u/da < 0)$ and the ex-post tariff resulting from lobbying is increasing in the political economy weight $(d\tau^u/da > 0)$. Additionally, although τ^{cxp} is not directly dependent on a, $d\tau^{cxp}/da$ is also negative, due to τ^{cxp}'s dependence on $\bar{n}^u$, with $d\tau^u/da > d\tau^{cxp}/da$ for all $a \in (0, \infty)$ given $\partial\tau^u/\partial a > 0$.*

The relationship between the tariff level and a as compared to the ex-ante commitment tariff level is given in Figure 1.4.

I can now take a closer look at the relationship between τ^u and τ^c. First, suppose that $\bar{n}^u = \bar{n}^c$. If this is the case, then for any $a > 0$, the fact that $d\tau^u/da > 0$ means $\tau^u > \tau^c$ must be true. Notice also that $d\bar{n}^u/da < 0$, means that as a continues to rise, $\bar{n}^u$ is moving toward $\bar{n}^c$ (but with $\bar{n}^u > \bar{n}^c$ remaining true given the convexity of $\theta(k)$ is what determines whether firm entry increases or decreases upon joining a trade agreement). Therefore, when a is very large such that $\bar{n}^u \approx \bar{n}^c$, it is true that $\tau^u > \tau^c$. Given that the tariff is continuous in a, this means that there must be some political economy weight, call it $a^\tau > 0$, such that for a fixed σ, $\tau^u > \tau^c$ only if $a > a^\tau$. This is shown in Figure 1.5, which depicts the relative price of foreign varieties in the home country over a range of values of a for a fixed σ. Figure 1.5 also includes a^{MP}, where for any $a < a^{MP}$ commitment to a trade agreement results in a rise in the relative price of foreign varieties, a Metzler paradox.

[38]The full derivation of both of these points is in Appendix A.5.3. In my numerical solution, I have been unable to discover a set of parameters for which equation (1.39) is positive, which reinforces the decision to assume the denominator of $d\bar{n}^u/da$ is positive made by Assumption 1.10.

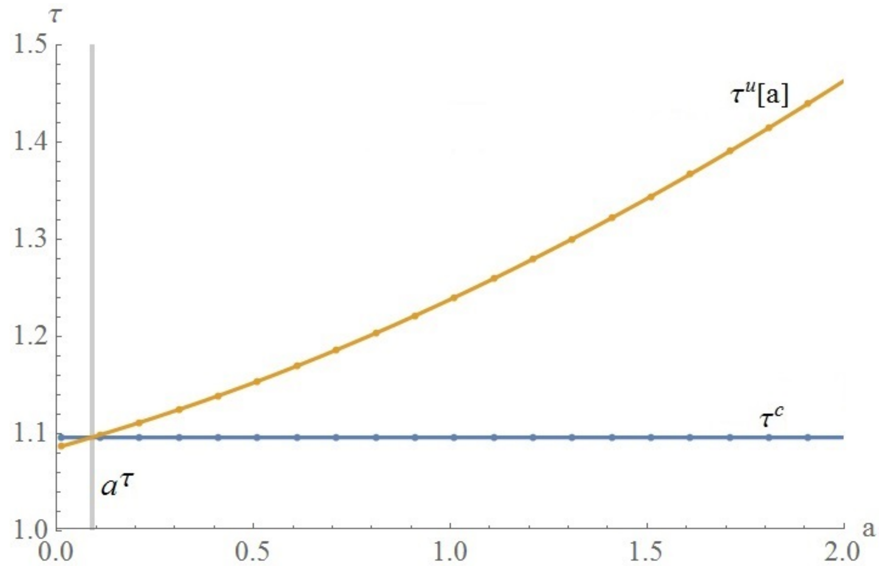


Figure 1.4: Equilibrium tariff levels, given the best response capital hiring and firm entry and the capital-market clearing rental price of capital for $\sigma = 509/64$. The tariff level without commitment to a trade agreement, τ^u, is greater than with a trade agreement, τ^c, for any $a > a^\tau$, with τ^u increasing and convex in a.

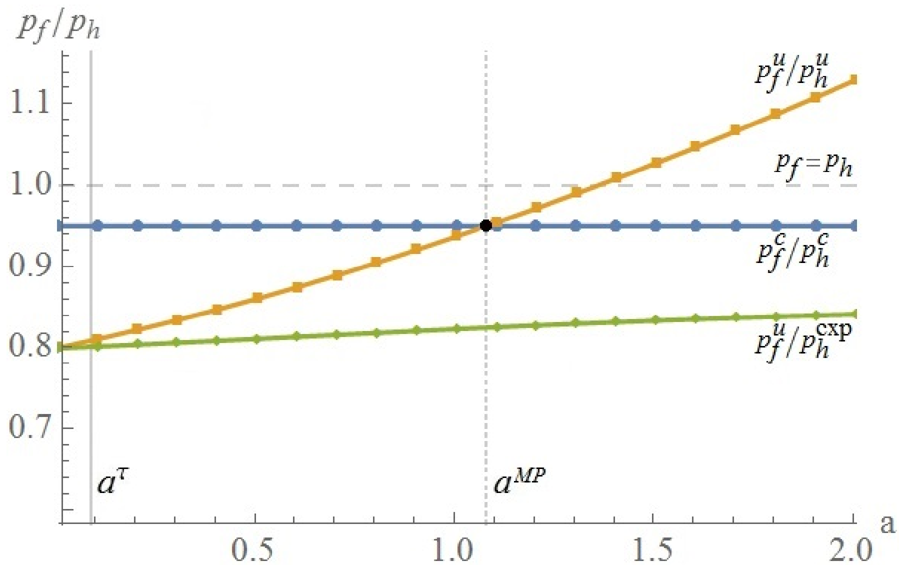


Figure 1.5: The relative price of foreign varieties compared to home varieties when the government commits to a trade agreement ("c"), does not commit to a trade agreement and bargaining succeeds ("u"), or does not commit to a trade agreement and bargaining fails ("cxp"), for $\sigma = 509/64$. The Metzler paradox always arises if joining a trade agreement causes the domestic price to go from p_f^u/p_h^{cxp} to p_f^c/p_h^c. For $a < a^{MP}$, joining a trade agreement results in an increase in the relative price, $p_f^u/p_h^u < p_f^c/p_h^c$.

Proposition 1.12 (τ^u versus τ^c when a is near zero)**.** *Given the existence of interior ex-ante producer equilibrium both with and without a trade agreement and given that* $(1+a) < \left(\frac{\sigma}{\sigma-1}\right)\sigma$*, for any fixed* σ *when* $a = 0$ *the tariff level is lower without a trade agreement,* $\tau^u|_{a=0} < \tau^c$*. There exists a political economy weight,* $a^\tau > 0$*, such that for any* $a < a^\tau$ *the government's ex-post politically optimal tariff is lower than the tariff level with ex-ante tariff commitment,* $\tau^u < \tau^c$*.*

Notice that because $\bar{n}^u > \bar{n}^c$, the domestic price of home varieties, p_h, actually rises when the government commits to a trade agreement. This is due to the fact that when the government commits to a trade agreement, fewer firms enter the market, which drives up the unit-labor cost of production, giving $\lambda(\bar{k}^c) > \lambda(\bar{k}^u)$. As a result, there is potential for the Metzler paradox to arise, wherein ex-ante commitment to a trade agreement causes the relative price of foreign varieties to increase rather than decrease. I further explore the role of the Metzler paradox in Section 1.4.4.

1.4.4 Potential for Metzler Paradox

Before I continue with the model, it is worthwhile to discuss the fact that the Metzler paradox arises in my model for some parameter values. By definition, the Metzler paradox is a scenario in which commitment to lowering tariffs paradoxically leads to an increase in the relative price of imports. Beyond the discussion in this section, I do not focus my analysis on the cases in which there is or is not a Metzler paradox.

In my model, the Metzler result stems from the dual effects of a trade agreement on relative prices: the trade agreement not only affects the foreign price in the home country, p_f, through the tariff level, but it also causes production adjustments which lower the price of home varieties, p_h. In terms of my model, the Metzler paradox arises where $p_f^u/p_h^u < p_f^c/p_h^c$, rearranged and simplified to

$$\frac{\lambda(\bar{k}^u)}{\lambda(\bar{k}^c)} > \frac{\tau^u(\bar{k}^u, \bar{n}^u)}{\tau^c}.$$

Keeping in mind that $\lambda'(k) < 0$ and $\bar{k}^c > \bar{k}^u$, the above condition indicates that joining a trade agreement creates a Metzler paradox when the percentage change in the tariff level from joining a trade agreement is less than the percentage improvement in labor productivity that results from committing to a trade agreement and correcting the ex-ante production distortion.[39]

I already showed in Section 1.4.3 that the Metzler paradox arises when $a = 0$. Establishing this is straightforward, noting that the tariff level when a is equal to zero is $\tau^u|_{a=0} = \tau^{cxp}$, and given the derivative of τ with respect to n,

$$\frac{\partial \tau}{\partial n} = \left(\frac{w_\lambda(k)}{n\lambda(k)}\right) \frac{(\tau - 1)\tau}{(\sigma z + 1) - (\sigma - 1)(\tau - 1)}, \tag{1.41}$$

which is clearly negative for any $n > \bar{n}^c$, given $w_\lambda(k) < 0$ and $(\sigma z + 1) - (\sigma - 1)(\tau - 1) > 0$ when $n > \bar{n}^c$. Because $\bar{n}^u > \bar{n}^c$ when a is close to zero, τ^u must be lower than τ^c. Because $\lambda(\bar{k}^u) > \lambda(\bar{k}^c)$ and $\tau^u|_{a=0} < \tau^c$, committing ex-ante to a trade agreement results in an increase in the relative price of imports. This also means that any scenario in which the the lobbying sector's participation constraint (equation (1.30)) is not satisfied or if lobbying fails ($\tau = \tau^{cxp}$), commitment to a trade agreement results in a rise in the relative price of imported varieties.

Proposition 1.13 (Potential for Metzler paradox)**.** *In the model, joining a trade agreement has the potential to result in an increase in the relative price of imported varieties, the Metzler paradox, such that for certain parameter values* $\tau^u(\bar{k}^u, \bar{n}^u)\lambda^*/\lambda(\bar{k}^u) < \tau^c\lambda^*/\lambda(\bar{k}^c)$*. The Metzler paradox is most easily observed when the tariff without a trade agreement is* τ^{cxp}*, which results when lobbying does not occur or lobbying fails. In terms of the political economy weight,*

1. *There is a cutoff political economy weight,* a^τ*, such that for* $a < a^\tau$ *the tariff level increases following commitment to a trade agreement, implying* $\tau^c > \tau^u$ *(Figures 1.4 and 1.5);*

[39]Note that the Metzler paradox never arises for the foreign country since $p_h^{*c}/p_f^* < p_h^{*u}/p_f^*$ as long as $\bar{k}^c > \bar{k}^u$.

2. *There is a cutoff political economy weight, a^{MP}, for which $a < a^{MP}$ implies the Metzler paradox occurs, and with $a^{MP} > a^{\tau}$ (Figure 1.5);*

1.4.5 Behavior of Ex-Ante Producer Equilibrium and Tariff Level

Now, consider the relationship of $\bar{n}^u$ and τ^u with σ. As was the case for τ^c, the government's tariff without commitment to a trade agreement is nonmonotonic in σ. Firm entry without ex-ante tariff commitment, $\bar{n}^u$, is also nonmonotonic in σ, standing in contrast with $\bar{n}^c$ which is monotonically falling in σ.

Beginning with the behavior of τ^u, notice that the limiting behavior of τ^u is the same as was true for τ^c. Using the definition of τ^u from equation (1.26), when σ approaches one $\lim_{\sigma \to 1} \left(\frac{\sigma-1}{\sigma}\right)\frac{(1+a)}{(\sigma+1/z^u)} = \left(\frac{0}{1}\right)\frac{(1+a)}{1+1/z^u} = 0$, which means $\lim_{\sigma \to 1} \tau^u = 1$. Alternatively, when σ goes to infinity $\lim_{\sigma \to \infty} \left(1 - \frac{1}{\sigma}\right)\frac{(1+a)}{(\sigma+1/z^u)} = \left(1 - \frac{1}{\infty}\right)\frac{(1+a)}{\infty+1/z^u} = 0$, which also means $\lim_{\sigma \to \infty} \tau^u = 1$.[40]

The derivative of the tariff with respect to the elasticity of substitution is

$$\frac{d\tau^u}{d\sigma} = \frac{\partial \tau^u}{\partial \sigma} + \frac{\tau^u(\tau^u - 1)}{(\sigma z^u + 1) - (\sigma - 1)(\tau^u - 1)} \left(\frac{w_\lambda(\bar{k}^u)}{\bar{n}^u \lambda(\bar{k}^u)}\right) \frac{d\bar{n}^u}{d\sigma}, \tag{1.42}$$

with the partial derivative defined

$$\frac{\partial \tau^u(\bar{k}^u, \bar{n}^u)}{\partial \sigma} = \left(\tau^u\right)^2 \frac{\left(\frac{1}{\sigma}\right)^2 (1+a) z^u - \left(\frac{\tau^u - 1}{\tau^u}\right) z^u + \left(\frac{\tau^u - 1}{\tau^u}\right) \ln\left(\frac{p_f^u}{p_h(\bar{k}^u)}\right)}{(\sigma z^u + 1) - (\sigma - 1)(\tau^u - 1)}. \tag{1.43}$$

The direct effect of a change in σ on the tariff level (equation (1.43)) matches the functional form of $d\tau^c/d\sigma$ from equation (1.23), so I forgo re-visiting the analysis here. The indirect effect of σ on τ is the new addition to the tariff derivative, which results from the effect of the ex-ante production decisions on τ^u. Equation (1.42) is distinguished from $d\tau^c/d\sigma$ due to the influence of firm entry on the tariff level without commitment. The coefficient on the firm-entry effect is negative, meaning τ^u is inversely related to firm entry $\bar{n}^u$. To finish

[40] Both of these are true regardless of what $\lim_{\sigma \to 1} z^u$ and $\lim_{\sigma \to \infty} z^u$ are.

characterizing the behavior of τ^u. I must first establish the relationship between firm entry andσ.

To solve for the effect of a change in σ on firm entry, I implicitly differentiate equation (1.26) with respect to σ, using $d\bar{k}^u/d\sigma = -(\bar{k}^u/\bar{n}^u)(d\bar{n}^u/d\sigma)$, yielding the total derivative of $\bar{n}^u$ with respect to σ:

$$\frac{d\bar{n}^u}{d\sigma} = \frac{-\left(\frac{1}{\sigma-1}\right)^2 \lambda(\bar{k}^u)[TD^u] + \left(\frac{1}{\sigma-1}\right) w_\lambda(\bar{k}^u) \left[N\left(\frac{\partial q^u_{h2}}{\partial \sigma} + (\sigma-1)\frac{q^u_{h2}}{P^u}\frac{\partial P^u}{\partial \tau}\frac{\partial \tau^u}{\partial \sigma}\right) + N^* \frac{\partial q^{*u}_{h2}}{\partial \sigma}\right]}{\frac{-(\bar{k}^u)^2}{\bar{n}^u}\left(-\left(\lambda''(\bar{k}^u) - \sigma\frac{(\lambda'(\bar{k}^u))^2}{\lambda(\bar{k}^u)}\right)[TD^u] - \theta''(\bar{k}^u)\right) - w_\lambda(\bar{k}^u)\left[\mathbf{N}\frac{\mathbf{q}^u_{h2}}{\mathbf{P}^u}\frac{d\mathbf{P}^u}{dn}\right]}, \tag{1.44}$$

where the denominator is unchanged from equation (1.39) and is therefore positive due to Assumption 1.10. The numerator of $d\bar{n}^u/d\sigma$ is composed of two parts: (i) the impact of σ on the per-unit net revenue holding demand fixed, and (ii) the impact of σ on demand holding the per-unit net revenue fixed.[41]

The effect of an increase in σ on the per-unit net revenue is always negative. An increase in σ decreases the market power of a producer, leading to a fall in its per-unit net revenue. The direct effect of an increase in σ on total demand for the differentiated good is nonmonotonic given $\partial\tau^u/\partial\sigma$ and $\partial q^u_{h2}/\partial\sigma$ are nonmonotonic in σ.[42] The numerator of $d\bar{n}^u/d\sigma$ is negative where $d\tau^u/d\sigma$ is positive. This results from the fact that if $\partial\tau^u/\partial\sigma > 0$ then $\partial q^u_{h2}/\partial\sigma > 0$, and adding in the positive first term means $d\bar{n}^u/d\sigma < 0$. Then given $\partial\tau^u/\partial\sigma > 0$ and $\partial\tau^u/\partial n < 0$, $d\tau^u/d\sigma$ must be positive overall. Alternatively when $\partial\tau^u/\partial\sigma < 0$, I am not able to determine the sign of $d\bar{n}^u/d\sigma$.[43]

Proposition 1.14 (Impact of change in σ on ex-ante producer equilibrium)**.** *Given the existence of an interior ex-ante producer equilibrium,* $(\bar{k}^u, \bar{n}^u)$*, that* $(1+a) < \left(\frac{\sigma}{\sigma-1}\right)\sigma$*, and that*

[41]The full algebraic expansion of the numerator can be found in Appendix A.5.3, equation (A.22).

[42]The partial derivative of $\partial\tau^u/\partial\sigma$ is given in equation (1.43) and

$$\frac{\partial q_{h2}}{\partial \sigma} = (q_{h2})\Big[\frac{1}{\sigma(\sigma-1)} + \frac{1}{z+1}\ln\Big(\frac{p_f}{p_h(k)}\Big)\Big] \text{ and } \frac{\partial q_{f2}}{\partial \sigma} = (q_{f2})\Big[\frac{1}{\sigma(\sigma-1)} - \frac{z}{z+1}\ln\Big(\frac{p_f}{p_h(k)}\Big)\Big].$$

$\partial q^u_{h2}/\partial\sigma$ is nonmonotonic due to the presence of the relative price term. The relative prices p^u_f/p^u_h and p^{cxp}_f/p^u_h are non-monotonic in σ and are less than one when σ is very low or very high.

[43]I am able to show analytically that if $\partial q^u_{h2}/\partial\sigma < 0$, then $\partial\tau^u/\partial\sigma < 0$. However, because $\partial q^{*u}_{h2}/\partial\sigma$ is still positive, I cannot make a conclusion regarding the sign of the numerator of $d\bar{n}^u/d\sigma$.

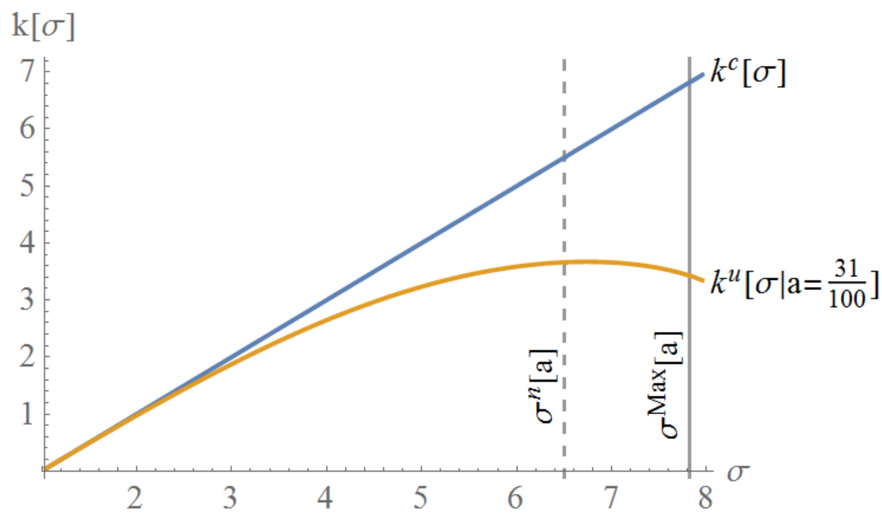


Figure 1.6: Equilibrium capital hiring, $\bar{k}^c$ and $\bar{k}^u$, for different elasticities of substitution (σ). The equilibrium capital hiring with commitment to a trade agreement, $\bar{k}^c$, is always greater than the capital hiring without commitment to a trade agreement, $\bar{k}^u$, and the difference between the two increasing as goods within the sector become more homogeneous and with the maximum elasticity, σ^{Max}, defined in Assumption 1.10. The value of σ where $\bar{k}^u$ achieves its maximum, σ^n, is labeled as well.

Assumptions 1.7 and 1.10 hold, the equilibrium number of firms in the industry, $\bar{n}^u$, may either be increasing or decreasing in σ. For low values of σ where $\partial\tau^u/\partial\sigma > 0$, firm entry is decreasing in σ $(d\bar{n}^u/d\sigma < 0)$. For larger σ, the sign is uncertain.[44]

The numerical solution for $\bar{k}^u \equiv K/\bar{n}^u$ is depicted in Figure 1.6, where I graph $\bar{k}^u$ instead of $\bar{n}^u$ given the scale of $\bar{k}^u$ is more conducive to visual comparisons than is $\bar{n}^u$ (which approaches infinity as $\sigma \to 1$). The value of σ where $\bar{n}^u$ achieves its minimum is σ^n.

The behavior of τ^u for changes in σ when σ is low can be solved using the behavior of $\bar{n}^u$ from Proposition 1.14.[45]

Proposition 1.15 (Impact of change in σ on tariff level, τ^u)**.** *Given the existence of an interior producer equilibrium, $(\bar{k}^u, \bar{n}^u)$, that $(1+a) < \left(\frac{\sigma}{\sigma-1}\right)\sigma$, and that Assumptions 1.7 and 1.10 hold, the government's politically optimal tariff as $\sigma \to 1$ and $\sigma \to \infty$ is $\lim_{\sigma\to 1}\tau^u = 1$ and $\lim_{\sigma\to\infty}\tau^u = 1$.*

[44]This uncertainty is further discussed in Appendix A.5.3 following equation (A.22).

[45]An additional feature established by the numerical solution to the models is that the size of the production distortion is increasing monotonically in σ. I briefly explore this in Appendix A.5.4, deriving the analytical solution for $\sigma \in (\sigma^n, \sigma^{Max})$ and for all σ when a is large.

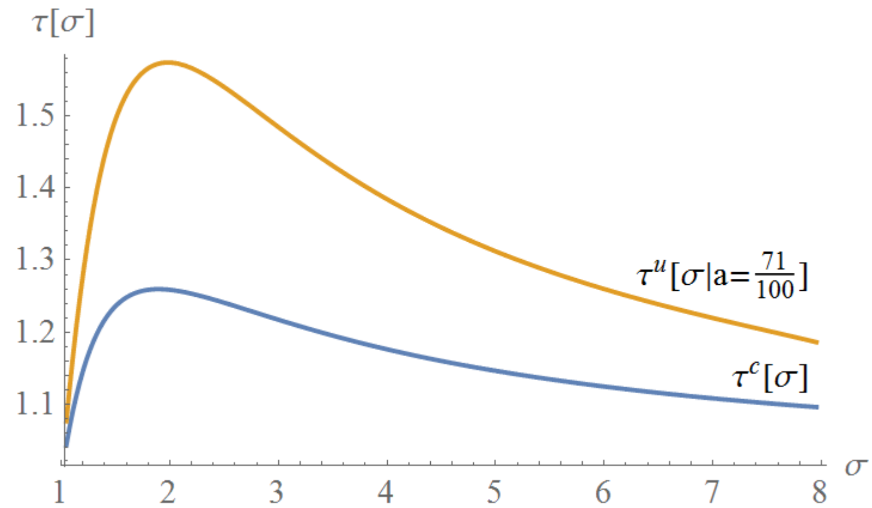


Figure 1.7: Equilibrium tariff levels given an interior ex-ante producer equilibrium exists and $a > a^\tau$. The tariff level without commitment to a trade agreement is always greater than with a trade agreement, and both are increasing when the elasticity of substitution, σ, is low and both are decreasing when σ is high.

For the full range of σ, τ^u may either be increasing or decreasing in σ. For low values of σ where $\partial\tau^u/\partial\sigma > 0$, firm entry is decreasing in σ $(d\bar{n}^u/d\sigma < 0)$ which in turn means that $d\tau^u/d\sigma > 0$. For larger σ, the sign is unclear analytically.

The behavior of τ^u over the full range of σ is demonstrated using the numerical solution in Figure 1.7, with the value of τ^c also plotted for comparison.

1.5 Government Decision: Trade Agreement vs. Political Gains

Before firms enter the market and before lobbying occurs, the government sets its trade policy to maximize its total welfare. The government has access to two policies: (i) setting the tariff level ex ante via a trade agreement and disallowing lobbying in the later stages of the model, or (ii) setting the tariff level ex post, allowing lobbying by domestic producers for tariff protection. There are four parameters of interest on which I base my trade policy analysis: the weight the government places on political contributions, a; the government's bargaining strength, γ, relative to the lobbying sector's bargaining strength,

$1-\gamma$; the substitutability of varieties of the differentiated good, σ; and finally I briefly discuss the role of the foreign consumers' share of total expenditure on the differentiated good, $(N^*e_2^*)/(Ne_2+N^*e_2^*)$.

For the algebraic analysis, the government's welfare with commitment to a trade agreement is defined $G^c \equiv W(\bar{k}^c, \bar{n}^c, \tau^c)$, and welfare without a trade agreement is $G^u \equiv G(\bar{k}^u, \bar{n}^u, \tau^u) = W(\bar{k}^u, \bar{n}^u, \tau^u) + a\,\bar{n}^u c^u(\bar{k}^u, \bar{n}^u)\,\bar{k}^u$. The equation of interest in this section is the difference between the two, the government's welfare benefit from commitment to a trade agreement:

$$\begin{aligned} G^c - G^u = {} & W(\bar{k}^c, \bar{n}^c, \tau^c) - W(\bar{k}^u, \bar{n}^u, \tau^{cxp}) \\ & - \gamma\left[\Big(W(\bar{k}^u, \bar{n}^u, \tau^u) - W(\bar{k}^u, \bar{n}^u, \tau^{cxp})\Big) + a\bar{n}^u\Big(\pi(\bar{k}^u, \mathbf{P}^u) - \pi(\bar{k}^u, \mathbf{P}^{cxp})\Big)\right], \end{aligned} \tag{1.45}$$

where the above equation has been simplified using the definition of the contribution rule (equation (1.28)). If $G^c - G^u > 0$, then the government prefers to join a trade agreement and commit ex-ante to the level of tariff protection. I arranged the terms in equation (1.45) to demonstrate that $G^c - G^u > 0$ if the social-welfare gains of correcting the ex-ante production distortion are larger than the government's shares of ex-post rents from tariff protection, which is what the government gives up by committing ex ante to the trade agreement.

I begin in Section 1.5.1 by examining how the effect of a and γ on trade policy compares to Maggi and Rodríguez-Clare's perfectly competitive model by deriving versions of their paper's two main propositions regarding trade policy preference. I show that while the details of the behavior of $G^c - G^u$ may be different from the perfectly competitive model, the general behavior of the government in my model matches Maggi and Rodríguez-Clare's findings. I show that there exists a "policy-indifference curve" for combinations of (a, γ) that is decreasing and convex, such that for any (a, γ) combination below the indifference curve, the government prefers to commit to a trade agreement. The consistency of my results with their findings is in itself interesting, because while both models use the trade agreement to correct ex-ante production distortions Maggi and Rodríguez-Clare's model shows the use of a trade agreement to correct intersectoral distortions in the allocation of capital and I focus on trade agreements used to correct intrasectoral distortions in production (discussed in Section 1.2).

Then in Sections 1.5.2 and 1.5.3, I turn my focus to the effect on trade policy preference of changes in the substitutability of varieties, σ, and the foreign share of total expenditures, $m^* \equiv (N^* e_2^*)/(Ne_2 + N^* e_2^*)$. I first focus on how the foreign share of total expenditures influences the government's policy choice in Section 1.5.2. Using the numerical solution I demonstrate that an increase in m^* leads to a rise in the government's welfare benefit from committing to a trade agreement. Regarding the influence of the elasticity of substitution, in Section 1.5.3 I show that $G^c - G^u$ retains the nonmonotonicity in σ observed throughout the production and price-setting stages. Using the numerical solution, I show that $G^c - G^u > 0$ for $\sigma \to 1$ for all $a \in \Big(0, \big(\frac{\sigma}{\sigma-1}\big)\sigma - 1\Big)$ and $\gamma \in [0,1]$, possibly becoming negative as σ rises if a or γ are large enough, but then eventually again becoming positive as σ becomes large.

Before I continue, I need to introduce some additional terminology. Throughout Section 1.5 (especially in Section 1.5.3), I discuss the government's welfare benefit of joining a trade agreement in the context of a trade-off between two factors: (i) the social-welfare gains from correcting the ex-ante production distortion (XAD); and (ii) the ex-post compensation (XPC) the government receives from granting the political-economy-induced level of tariff protection given that production is at the distorted $(\bar{k}^u, \bar{n}^u)$ equilibrium. Using equation (1.45), the XAD and XPC comprise the welfare benefit from joining a trade agreement as follows:

$$\begin{aligned} G^c - G^u = & \underbrace{\Big[W(\bar{k}^c, \bar{n}^c, \tau^c) - W(\bar{k}^u, \bar{n}^u, \tau^{cxp})\Big]}_{\text{Ex-Ante Distortion Correction, } XAD(\sigma,a)} \\ & - \underbrace{\gamma\Big[W(\bar{k}^u, \bar{n}^u, \tau^u) - W(\bar{k}^u, \bar{n}^u, \tau^{cxp}) + a\bar{n}^u\Big(\pi(\bar{k}^u, \mathbf{P}^u) - \pi(\bar{k}^u, \mathbf{P}^{cxp})\Big)\Big]}_{\text{Ex-Post Compensation, } XPC(\sigma,a,\gamma)}. \end{aligned} \tag{1.46}$$

If $XAD(\sigma, a) > XPC(\sigma, a, \gamma)$, then the government chooses to commit to a trade agreement.

If $XAD(\sigma, a) \leq XPC(\sigma, a, \gamma)$, then the government does not join a trade agreement. The ex-ante distortion is positive for any $\bar{n}^u \neq \bar{n}^c$ and $\tau^{cxp} \neq \tau^c$, given social welfare is maximized when the tariff is τ^c and firm entry is $\bar{n}^c$. The ex-post compensation is also positive given $XPC > 0$ is equivalent to assuming the lobby-participation constraint holds (Assumption 1.30).

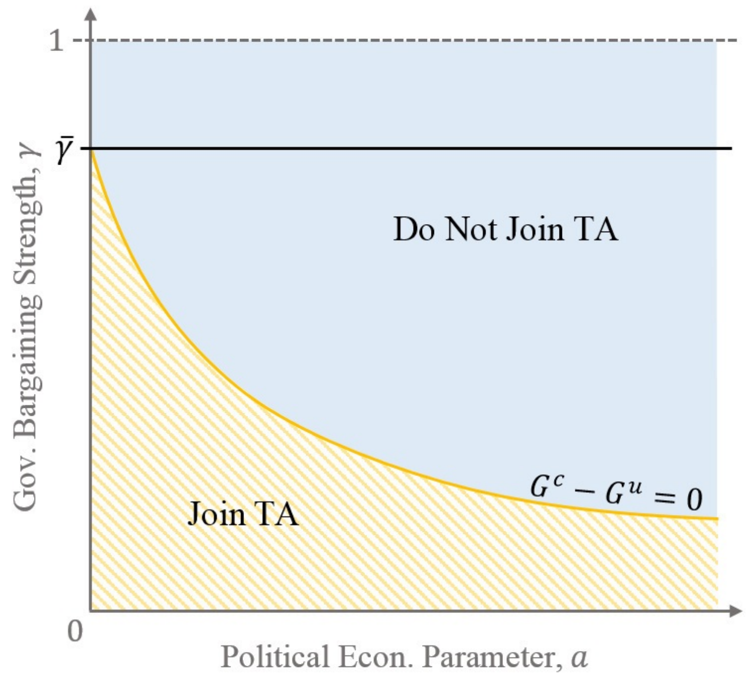
1.5.1 Comparison to Maggi and Rodríguez-Clare (1998) Policy Preferences

First, Maggi and Rodríguez-Clare's Proposition 2 shows that with perfectly competitive producers there is a maximum relevant government bargaining strength, $\bar{\gamma}$, such that for any $a > 0$ the government prefers to join a trade agreement only if $\gamma < \bar{\gamma}$. This result does not hold in my model due to the behavior of $G^c - G^u$ when a is near zero. As I discuss further below (equation (1.51)), the government prefers to join a trade agreement for all values of γ when a is equal to zero. Therefore as an alternative to $\bar{\gamma}$, I present my model's "minimum relevant" bargaining strength, $\bar{\bar{\gamma}}$, such that for $\gamma < \bar{\bar{\gamma}}$ the government prefers to join a trade agreement for all values of $a > 0$. Figure 1.9 depicts $\bar{\gamma}$ and $\bar{\bar{\gamma}}$ in Subfigures 1.8a and 1.8b, respectively. Note that the difference in the behavior of $G^c - G^u$ for changing γ in my model is in part due to the fact that in the monopolistically competitive model the bargaining strength only affects the division of rents from tariff protection, whereas in Maggi and Rodríguez-Clare's model it effects the Nash bargaining tariff level and the ex-ante production distortion as well.

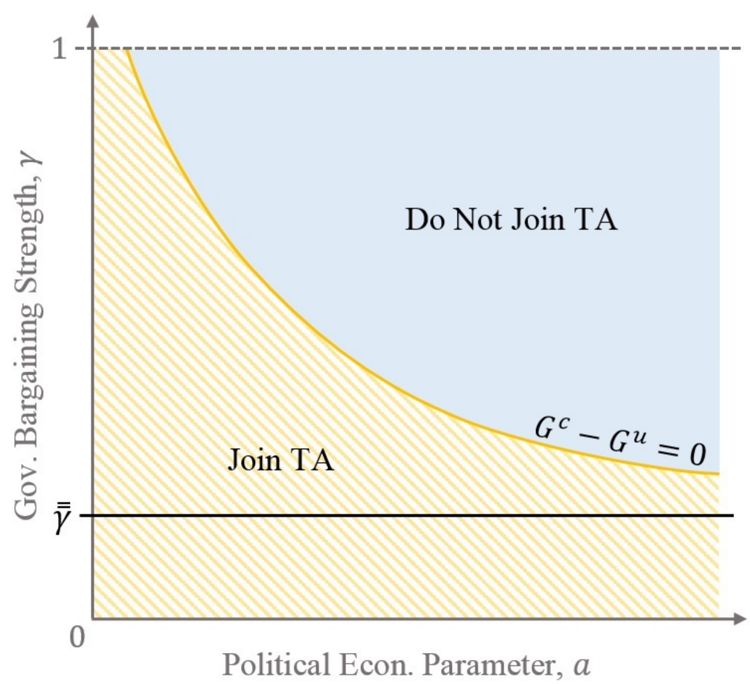
In their paper, Maggi and Rodríguez-Clare establish the existence of $\bar{\gamma}$ in four steps. First, they show that when $\gamma = 0$, the government's optimal tariff policy is free trade. Then, they show that when $\gamma = 1$, the government does not join a trade agreement for any values of $a > 0$. Then in steps three and four, they show that $G^c - G^u$ is decreasing and continuous in γ.

With monopolistically competitive producers, when the government's bargaining power

Figure 1.8: The relationship between γ, a, and the Government's trade policy preference for a fixed level of σ. $G^c - G^u = 0$ is the government's policy-indifference curve, showing the (a, γ) combinations for which the government's welfare benefit of committing to a trade agreement is equal to zero.



(a) Perfect Competition: As predicted in Maggi and Rodríguez-Clare's 1998 Proposition 2



(b) Monopolistic Competition: As predicted in Proposition 1.16

is $\gamma = 0$ it remains true that the government is better off by committing to a trade agreement for all values of $a > 0$. To see this, when $\gamma = 0$ equation (1.45) is

$$G^c - G^u|_{\gamma=0} = W(\bar{k}^c, \bar{n}^c, \tau^c) - W(\bar{k}^u, \bar{n}^u, \tau^{cxp}), \tag{1.47}$$

which is greater than zero given that (i) social welfare is maximized at $\bar{n}^c$ and τ^c, and (ii) as long as $\theta(k) > 0$, $\bar{n}^u \neq \bar{n}^c$. That $W^c > W^{cxp}$ can also be demonstrated using the derivative of social welfare with respect to firm entry. For reference, the derivative of politically weighted total welfare with respect to n is $(\partial TW(k,n,\tau)/\partial k)(dk/dn) + (\partial TW(k,n,\tau)/\partial n)$, which can be expanded:

$$\frac{\partial TW(k(n), n, \tau)}{\partial n} = \Big(\frac{1}{\sigma - 1}\Big) w_\lambda(k) \\ * \left[N q_{h2}(\tau) \Big(\frac{\sigma}{\sigma - 1}\Big) \left(\left(\frac{\tau - 1}{\tau} \right) \frac{1}{z} + 1 \right) + (1 + aI) N^* q^*_{h2} \left(\frac{z^*}{z^* + 1} \right) \right], \tag{1.48}$$

where I is an indicator equal to one if producers successfully lobby the government and zero otherwise, with $TW(k(n), n, \tau) = W(k(n), n, \tau) + aIn\pi(k, P)$, and output is $q_{h2}(\tau) \equiv q_{h2}(k, n, \tau)$ and $q^*_{h2} \equiv q^*_{h2}(k, n, \tau^*)$.[46] Evaluating equation (1.48) for when the government joins a trade agreement, $\partial W(\bar{k}^c, \bar{n}^c, \tau^c)/\partial n = 0$ given that $w_\lambda(\bar{k}^c) = 0$. Additionally, for any $k < \bar{k}^c$, given that $w_\lambda(k) < 0$, the derivative of social welfare with respect to n is negative. Therefore, it must be true that $W^{cxp} < W^c$, which means equation (1.47) is greater than zero and the government prefers a trade agreement. Just as in Maggi and Rodríguez-Clare (1998), when $\gamma = 0$ the government is better off committing to a trade agreement.

Turning to the other extreme, when $\gamma = 1$, Maggi and Rodríguez-Clare found that the government preferred not to join a trade agreement for all values of $a > 0$. In my model, this is no longer true. To see this, consider the benefit to the government of joining a trade

[46] Which means that $TW(\bar{k}^c, \bar{n}^c, \tau^c) = W(\bar{k}^c, \bar{n}^c, \tau^c)$ and $TW(\bar{k}^u, \bar{n}^u, \tau^{cxp}) = W(\bar{k}^u, \bar{n}^u, \tau^{cxp})$.

agreement when $\gamma = 1$:

$$G^c - G^u|_{\gamma=1} = W(\bar{k}^c, \bar{n}^c, \tau^c) - W(\bar{k}^u, \bar{n}^u, \tau^u) - a\bar{n}^u\Big(\pi(\bar{k}^u, \mathbf{P}^u) - \pi(\bar{k}^u, \mathbf{P}^{cxp})\Big). \qquad (1.49)$$

The government's benefit from joining a trade agreement is positive if a is small enough that the politically weighted ex-post producer gains from securing tariff level τ^u, $a\,\bar{n}^u\Big(\pi(\bar{k}^u, \mathbf{P}^u) - \pi(\bar{k}^u, \mathbf{P}^{cxp})\Big) > 0$, do not outweigh the social-welfare gains from ex-ante commitment to a tariff level, $W(\bar{k}^c, \bar{n}^c, \tau^c) - W(\bar{k}^u, \bar{n}^u, \tau^u) > 0$. Given that social welfare is maximized at $W(\bar{k}^c, \bar{n}^c, \tau^c)$, the social welfare gains from ex-ante commitment are always positive. When $a = 0$, the ex-post producer gains are equal to zero, which means equation (1.49) is positive. When a is large, the social welfare gains of a trade agreement are swamped by the politically weighted producer surplus gains meaning equation (1.49) is negative.

To show that $G^c - G^u|_{\gamma=1}$ is negative when a is large, suppose the value of σ is such that a can become very large without violating the condition for the finite tariff solution, $(1 + a) < \Big(\frac{\sigma}{\sigma-1}\Big)\sigma$. Consider what happens to equation (1.49) as $a \to \infty$. Because $\bar{n}^u \to \bar{n}^c$ as a grows, the social welfare is $W^{cxp} \to W^c$. As a result, the government's lobbying participation constraint (equation (1.29)) as $a \to \infty$ is $\gamma(W^u - W^c) + \gamma a\bar{n}^u(\pi^u - \pi^{cxp}) > 0$. Therefore, given equations (1.29) and (1.30) are satisfied, $G^c - G^u|_{\gamma=1,a\to\infty} < 0$.[47]

Stepping aside from analysis of equation (1.49), consider the role the participation constraint plays. My discussion thus far does not take into account the firm's lobbying participation constraint (equation (1.30)). For example, if $\gamma = 1$, then the participation constraint dictates that firms only participate in lobbying if the upfront lobbying cost, $\theta(\bar{k}^u)$, is equal to zero. If $\theta(\bar{k}^u) = 0$, then I know from the ex-ante producer equilibrium equation (equation (1.35a)) that $\bar{n}^u = \bar{n}^c$ and $\bar{k}^u = \bar{k}^c$. Therefore, taking into account the lobbying

[47] Using this analytical strategy is least helpful for intermediate values of σ where the size of a is most constrained by $a < (\frac{\sigma}{\sigma-1})\sigma - 1$. When the maximum a is relatively low, relying on $W^{cxp} \to W^c$ no longer makes sense, leaving the model open to the possibility that there is no $a < (\frac{\sigma}{\sigma-1})\sigma - 1$ for which $G^c - G^u|_{\gamma=1} < 0$. Without using $W^{cxp} \to W^c$, there is no analytical way to establish the sign of $G^c - G^u|_{\gamma=1,a\to(\frac{\sigma}{\sigma-1})\sigma-1}$. Using numerical solution, however, I can establish that $G^c - G^u|_{\gamma=1,a\to(\frac{\sigma}{\sigma-1})\sigma-1} < 0$ is always true. An example of this is given in Appendix A.8, Figure A.15.

participation constraint implies that $G^c - G^u|_{\gamma=1} = 0$ for all $a > 0$ and the government is indifferent between joining a trade agreement or not.

If $\gamma = 0$, on the other hand, then firms participate in negotiations if $\theta(\bar{k}^u) < -(1/(a\bar{n}^u))$ $(W^{cxp} - W^u) + (\pi^u - \pi^{cxp})$. Whether or not this inequality is satisfied is not obvious. The participation constraint is ultimately unimportant though, because when $\gamma = 0$ the government always chooses to commit to a trade agreement given that $G^c - G^u|_{\gamma=0} = W^c - W^{cxp} \geq 0$, with the equality holding if $\bar{n}^u = \bar{n}^c$.

For the purposes of understanding the behavior of $G^c - G^u$, whether or not the lobby forms is irrelevant. Therefore, for the remainder of the section I assume the participation constraint is satisfied for all $a > 0$ and $\gamma \in [0, 1]$, which also means that $\bar{n}^u \neq \bar{n}^c$.[48]

Finally, consider the derivative of $G^c - G^u$ with respect to γ,

$$\frac{d(G^c - G^u)}{d\gamma} = W^{cxp} - W^u - a\,\bar{n}^u\,(\pi^u - \pi^{cxp})\,. \tag{1.50}$$

This derivative is monotonically negative for all $a > 0$ given the government's lobbying participation constraint holds (equation (1.29)). When $a = 0$, $d(G^c - G^u)/d\gamma = 0$ since $W^{cxp} = W^u|_{a=0}$. Combining this with the other observations regarding the effect of γ on $G^c - G^u$ establishes my model's version of Maggi and Rodríguez-Clare's Proposition 2:

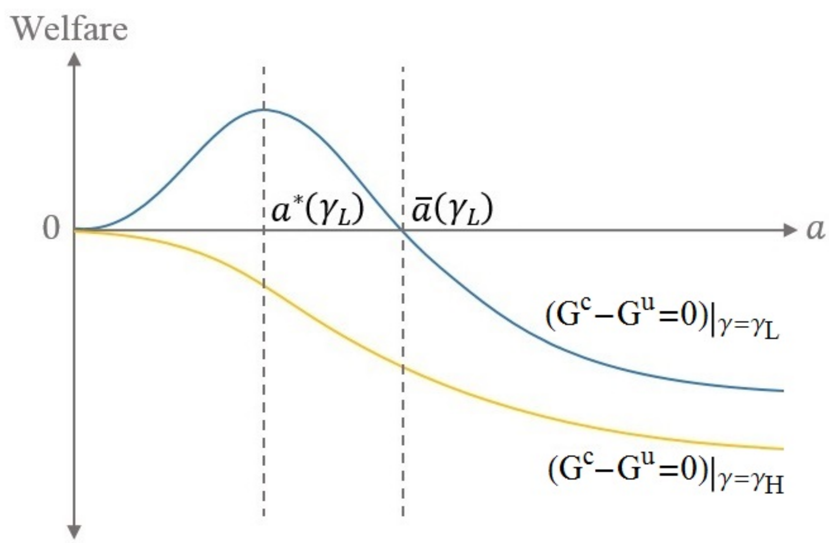
Proposition 1.16 (Relationship between γ and trade policy)**.** *Given the existence of interior ex-ante producer equilibria $(\bar{k}^c, \bar{n}^c)$ and $(\bar{k}^u, \bar{n}^u)$, that $(1 + a) < \left(\frac{\sigma}{\sigma-1}\right)\sigma$, and that Assumptions 1.7 and 1.10 hold, there exists a government bargaining strength, $\bar{\bar{\gamma}} \in (0, 1)$, such that $G^c - G^u < 0$ if and only if $\gamma > \bar{\bar{\gamma}}$.*

Figure 1.9 shows how my model's prediction here compares to the prediction based on the perfectly competitive production structure in Maggi and Rodríguez-Clare (1998).[49]

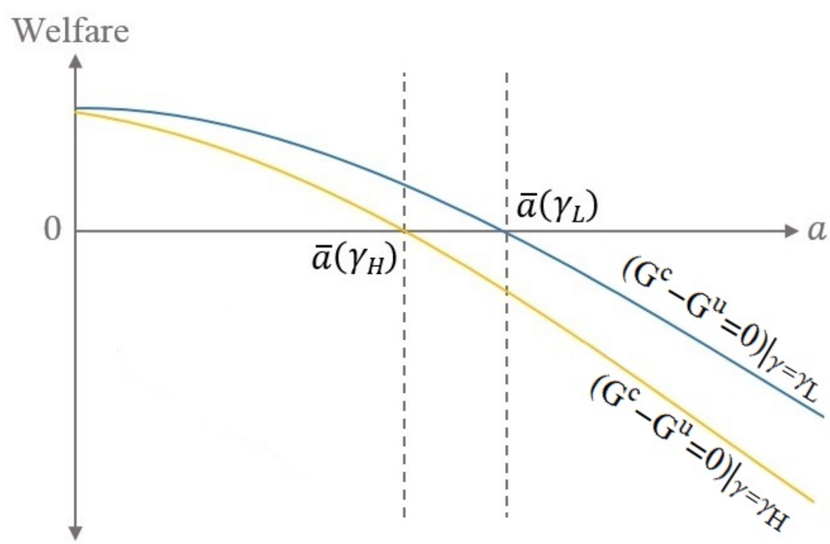
Next, Maggi and Rodríguez-Clare's Proposition 3 examines the behavior of $G^c - G^u$ for changes in a. The authors show that when γ is low, $G^c - G^u$ is at a minimum when $a = 0$,

[48] Figure A.17 in Appendix A.8 shows how the participation constraint fits into Figure 1.8b.
[49] In Maggi and Rodríguez-Clare's model, $\bar{\bar{\gamma}} = 0$.

Figure 1.9: The relationship between the political economy weight, a, and the welfare benefit of commitment to a trade agreement, $G^c - G^u$, for a fixed level of σ and two values of γ.



(a) Perfect Competition: As predicted in Maggi and Rodríguez-Clare's 1998 Proposition 3



(b) Monopolistic Competition: As predicted in Proposition 1.17

reaches a maximum at $a = a^*$, and eventually becomes negative at $a = \bar{a}$. Then they show that when γ is high, $G^c - G^u$ is equal to zero and at a maximum when $a = 0$, meaning $G^c - G^u$ is negative for all $a > 0$ and the government never prefers to commit to a trade agreement. Figure 1.9b shows the comparison of Maggi and Rodríguez-Clare's findings and my model.

With monopolistically competitive production when $\gamma > \bar{\bar{\gamma}}$, I find generally that when a is low the government prefers a trade agreement, and when a is high the government prefers no trade agreement. Overall my result is consistent with Maggi and Rodríguez-Clare's low-γ finding.

To establish these points, first look at the government's trade policy preference when $a = 0$. Simplifying equation (1.45) for $a = 0$, given that the tariff resulting from political negotiations is the same as the threat point tariff (i.e. $\tau^u|_{a=0} = \tau^{cxp}$), the government's welfare benefit from commitment to a trade agreement is

$$G^c - G^u|_{a=0} = W(\bar{k}^c, \bar{n}^c, \tau^c) - W(\bar{k}^u, \bar{n}^u, \tau^{cxp}), \tag{1.51}$$

which is equal to equation (1.47), and the discussion regarding equation (1.47) establishes that if $\theta(\bar{k}^u) > 0$, then the equation is positive. When the government places no value on political contributions, the lobbying sector is unable to compensate it for social welfare losses from the ex-ante production distortion, thus the government is better off committing to a trade agreement.

My finding is in contrast with Maggi and Rodríguez-Clare, who show that when $a = 0$, $G^c - G^u = 0$. Intuitively, this difference between the perfectly competitive model and the monopolistically competitive model derives from the difference in socially optimal tariff policies. With perfectly competitive producers, the social-welfare-maximizing tariff level is equal to zero. With monopolistically competitive producers, the social-welfare-maximizing tariff is positive. As a result, producers in monopolistically competitive industries can use their production decisions to manipulate the government's tariff choice even without lobbying, which is why $\tau^c \neq \tau^{cxp}$ for $a = 0$. On the other hand, producers in perfectly competitive sectors face free trade if there is no lobbying, which means $\tau^c = \tau^{cxp}$, eliminating any potential benefit of distorting production decisions. Thus, while with perfect competition $G^c - G^u|_{a=0} = 0$, in my model $G^c - G^u|_{a=0} > 0$ even without a political-economic motivation for the small country to commit to a trade agreement, the government benefits from preventing the ex-ante production distortion and chooses to join a trade agreement.[50]

To establish the behavior of $G^c - G^u$ when a is high, Maggi and Rodríguez-Clare use the limited size of the capital stock to show that with perfectly competitive producers there exists a political economy weight, a_K, such that for any $a > a_K$ the entire stock of capital is allocated to the lobbying sector and $d\bar{k}^u/da = 0$. Thus for any a larger than a_K, the welfare gain from correcting the ex-ante production distortion remains constant but the political economy loss from preventing lobby contributions continues to grow $(d(G^c - G^u)/da < 0$ for any $a > a_K)$. Therefore $G^c - G^u$ must eventually become negative as $a > a_K$ continues to rise.

My findings regarding the behavior of $G^c - G^u$ when a is large mirror those of Maggi and Rodríguez-Clare. Establishing the result for the full range of $\sigma \in (1, \sigma^{Max})$ is made difficult for two reasons. First, because capital distortions are intrasectoral in my model $\bar{k}^u > K$ is feasible. Second, as I mentioned following equation (1.49) in footnote 47, it is only possible to consider $a \to \infty$ if σ is either very low or very large since $(1 + a) < \left(\frac{\sigma}{\sigma-1}\right)\sigma$ must hold.

When $a \to \infty$, recalling that $\bar{n}^u \to \bar{n}^c$, the social welfare is $W^{cxp} \to W^c$. Therefore, the welfare benefit of commitment to a trade agreement can be written

$$G^c - G^u|_{a\to\infty} = -\gamma \left[W^u - W^{cxp} + a\bar{n}^u\Big(\pi^u - \pi^{cxp}\Big)\right],$$

which is negative given the government's lobbying participation constraint (equation (1.29)) is satisfied.[51] Because $G^c - G^u|_{a=0} > 0$, there must also exist an $\bar{a}(\gamma) \geq 0$ where $G^c - G^u|_{a=\bar{a}(\gamma)} = 0$.

[50] Note again that this scenario would actually violate the firm's lobby participation constraint, given that when $a = 0$, the firms only choose to lobby if $\theta(\bar{k}^u) = 0$, which means there would be no production distortion and, therefore, $G^c - G^u|_{a=0}$ would be equal to zero.

[51] The same issue as footnote 47 arises again here: this analytical strategy does not work for intermediate values of σ where the size of a is most constrained by $(1 + a) < \left(\frac{\sigma}{\sigma-1}\right)\sigma$: when the maximum a is relatively low, relying on $W^{cxp} \to W^c$ no longer makes sense. Using numerical solution I establish that $G^c - G^u|_{a\to(\frac{\sigma}{\sigma-1})\sigma-1} < 0$ is always true, shown for two combinations of σ and γ in Appendix A.8, Figure A.15.

Next, consider the slope of $G^c - G^u$ for changes in a. Given that when a is low $G^c - G^u$ is positive and when a is high the opposite is true, there must be a range of a for which $d(G^c - G^u)/da < 0$. Whether $G^c - G^u$ is monotonically decreasing in a, however, is unclear without additional work. First, the definition of $d(G^c - G^u)/da$ is

$$\begin{aligned}\frac{d(G^c - G^u)}{da} &= -\gamma\left(\frac{\partial TW^u}{\partial a} + \frac{\partial TW^u}{\partial n}\frac{d\bar{n}^u}{da}\right) - (1-\gamma)\left(\frac{\partial W^{cxp}}{\partial n}\frac{d\bar{n}^u}{da}\right) \\ &\quad + \gamma a \frac{d(\bar{n}^u \pi^{cxp})}{dn}\frac{d\bar{n}^u}{da} + \gamma \bar{n}^u \pi^{cxp},\end{aligned} \tag{1.52}$$

where the derivative of the industry's total ex-post operating profits for changes in a, $d(\bar{n}\pi)/da$ is

$$\frac{d(\bar{n}\pi)}{dn} = \frac{w_\lambda(\bar{k})}{\sigma\bar{n}\lambda(\bar{k})}\left(\frac{d\bar{n}}{da}\right)\left[Ne_2\frac{z}{(z+1)^2}\frac{(\sigma z+1)}{(\sigma z+1)-(\sigma-1)(\tau-1)} + N^*\frac{e_2^*}{\tau^*}\frac{z^*}{(z^*+1)^2}\right] > 0, \tag{1.53}$$

and equation (1.52) is simplified using the envelope conditions for τ^{cxp} and τ^u, and with the derivative of welfare with respect to firm entry, $\partial TW/\partial n = (\partial TW(k,n,\tau)/\partial k)(dk/dn) + (\partial TW(k,n,\tau)/\partial n)$, defined in equation (1.48). The first line of equation (1.52) is negative given that $\partial TW^u/\partial a = \bar{n}^u\pi^u$ is positive, $\partial TW^u/\partial n$ and $\partial W^{cxp}/\partial n$ are negative (shown in equation (1.48)), and $d\bar{n}^u/da$ is negative. In the second line is the effect of a change in a on the ex-post operating profits, which is positive given the second-order condition for the tariff and given the derivative of operating profits with respect to n is defined in equation (1.53). Overall the sign of equation (1.52) is unclear. Therefore, I proceed by examining the sign of $d(G^c - G^u)/da$ using the extreme values of a.

When $a = 0$, equation (1.52) can be simplified given that $\tau^u|_{a=0} = \tau^{cxp}$ and also $W^u = W^{cxp}$ and $\pi^u = \pi^{cxp}$. This yields the simplified derivative

$$\left.\frac{d(G^c - G^u)}{da}\right|_{a=0} = -\frac{\partial W^{cxp}}{\partial n}\frac{d\bar{n}^u}{da}, \tag{1.54}$$

which is easy to see is negative. Therefore when $a = 0$, $G^c - G^u$ is positive and decreasing in a for all values of γ. As a point of reference, if $\bar{n}^u|_{a=0} = \bar{n}^c$ (as is the case in the perfectly competitive model), then equation (1.54) would be equal to zero, which would be consistent with Maggi and Rodríguez-Clare's low-γ result. Supposing that $\bar{n}^u|_{a=0} = \bar{n}^c$, the benefit from commitment to a trade agreement is at a local minimum when $a = 0$ regardless of the value of γ. For Maggi and Rodríguez-Clare's model, this is true if the government in their model is a relatively weak bargainer.[52]

When a becomes very large, firm entry $\bar{n}^u \to \bar{n}^c$ implies that $w_\lambda(\bar{k}^u) \to 0$. Using the definition of $\partial TW/\partial n$ from equation (1.48) and the definition of $d(\bar{n}\pi)/dn$ from equation (1.53) to simplify the derivative of $d(G^c - G^u))/da|_{a\to\infty}$ gives

$$\left.\frac{d(G^c - G^u)}{da}\right|_{a\to\infty} = -\gamma\bar{n}^u\left(\pi^u - \pi^{cxp}\right),$$

which is negative for all $\gamma \in (0, 1]$. This derivative again only applies when σ is such that the maximum possible a can go to infinity. For intermediate values of σ, this strategy does not work.

To determine the sign of $d(G^c - G^u)/da$ for intermediate values of σ, simplify the problem by looking at $d(G^c - G^u)/da$ for $\gamma = 0$ or $\gamma = 1$. First when $\gamma = 0$, equation (1.52) becomes

$$\left.\frac{d(G^c - G^u)}{da}\right|_{\gamma=0} = -\frac{\partial W^{cxp}}{\partial n}\frac{d\bar{n}^u}{da},$$

which is negative for all values of a given that $\partial W^{cxp}/\partial n$ and $d\bar{n}^u/da$ are both negative. Therefore, when γ is very low, $d(G^c - G^u)/da < 0$ for all values of a.

Next when $\gamma = 1$, $d(G^c - G^u)/da$ is only margainally simplified compared to equation (1.52):

$$\left.\frac{d(G^c - G^u)}{da}\right|_{\gamma=1} = -\frac{\partial TW^u}{\partial n}\frac{d\bar{n}^u}{da} - \bar{n}^u\left(\pi^u - \pi^{cxp}\right) + a\frac{d(\bar{n}^u\pi^{cxp})}{da}.$$

Therefore, $G^c - G^u|_{\gamma=1}$ is decreasing in a if the total of the direct effect of a on $G^c - G^u$ (which is equal to $-\bar{n}^u(\pi^u - \pi^{cxp})$) and the indirect effect on TW^u through n $((\partial TW^u/\partial n)(d\bar{n}^u/da))$

[52]The full derivation is in Appendix A.7.1.

is low enough so the politically weighted effect of a on the threat point operating profits $(d(\bar{n}^u\pi^{cxp})/da)$ does not overpower it. Then using the fact that $d(G^c - G^u)/da$ is linear in γ, if $d(G^c - G^u)/da|_{\gamma=1} < 0$ then it must be true that $d(G^c - G^u)/da < 0$ for all $\gamma \in [0, 1]$.[53] Because $G^c - G^u$ is positive when a is low and is negative when a is high, there is a value of a, $\bar{a}(\gamma) > 0$, such that for $a < \bar{a}(\gamma)$ the government prefers to join a trade agreement. When the political economy weight is large, $a > \bar{a}(\gamma)$, the government prefers to not join a trade agreement and to receive the lobby's contribution.[54]

Proposition 1.17 (Effect of a on preferred trade policy)**.** *Assume the existence of interior ex-ante producer equilibria $(\bar{k}^c, \bar{n}^c)$ and $(\bar{k}^u, \bar{n}^u)$, that $(1+a) < \left(\frac{\sigma}{\sigma-1}\right)\sigma$, and that Assumptions 1.7 and 1.10 hold. Given $\gamma > \bar{\bar{\gamma}}$ and holding σ constant, the following points regarding the relationship between the government's trade policy preference and the political economy weight are proven true:*

1. *For $a = 0$, $G^c - G^u$ is positive and at a local maximum.*
2. *If a is high enough, $G^c - G^u$ is negative and decreasing in a.*
3. *There exists a political economy weight, $\bar{a}(\gamma)$, such that for any $a > \bar{a}(\gamma)$, $G^c - G^u < 0$.*[55]

1.5.2 Government Policy Preference and Foreign Expenditure Share

Now shift focus to the role in determining trade policy played by variables unique to the monopolistically competitive model: (i) the foreign consumers' share of total expenditures, $m^* \equiv (N^*e_2^*)/(Ne_2 + N^*e_2^*)$, and (ii) the elasticity of substitution across varieties, σ.

Begin with the effect of a change in foreign expenditure share, m^*, on the government's preferred trade policy. In terms of model parameters, this can be most easily demonstrated by allowing a change in $N^*e_2^*$ while holding Ne_2 constant. For this analysis, it is helpful to

[53]The cross-partial derivative is

$$\frac{d^2(G^c - G^u)}{dad\gamma} = -\left(\frac{\partial TW^u}{\partial n} - \frac{\partial W^{cxp}}{\partial n}\right)\frac{d\bar{n}^u}{da} - \bar{n}^u\left(\pi^u - \pi^{cxp}\right) + a\frac{d(\bar{n}^u\pi^{cxp})}{da}.$$

[54]Figure A.15 in Appendix A.8 demonstrates that $G^c - G^u$ is decreasing in a for all values of γ.

[55]$\bar{a}(\gamma)$ is not written as a function of σ, as σ is held constant for this section's analysis.

first establish how changes in foreign expenditure, $N^*e_2^*$, effect ex-ante producer variables and the tariff policy. First, because the ex-ante producer equilibrium condition (equation (1.17a)) depends only on the unit-labor-input requirement and the elasticity of substitution, $d\bar{n}^c/d(N^*e_2^*) = 0$. Additionally, the foreign expenditure does not factor into τ^c directly or indirectly, leaving $d\tau^c/d(N^*e_2^*)$ also equal to zero. Without ex-ante tariff commitment, firm entry is influenced by foreign expenditure, which is clear because $N^*e_2^*$ does appear in equation (1.35a) through $N^*q_{h2}^*$. Solving for $d\bar{n}^u/d(N^*e_2^*)$ shows that firm entry is decreasing in foreign expenditure, given that $d\bar{n}^u/d(N^*e_2^*)$ has the same as the denominator as $d\bar{n}^u/da$ (which is positive given Assumption 1.10) and with the numerator equal to

$$\frac{w_\lambda(\bar{k}^u)}{\bar{n}^u\lambda(\bar{k}^u)}\,\frac{1}{\tau^*}\left(\frac{1}{\sigma}\right)\left(\frac{1}{z^{*u}+1}\right), \tag{1.55}$$

which is negative. Equation (1.55) indicates growth in foreign expenditure drives up the relative benefit to firms of consolidating resources into fewer, larger firms which are more competitive abroad. Thus as foreign expenditure grows, ex-ante production shifts toward $(\bar{k}^c, \bar{n}^c)$. Because $\bar{n}^u$ is dependent on foreign expenditure, this implies that both τ^u and τ^{cxp} are also dependent on foreign expenditure indirectly through firm entry even though they are not directly dependent on $N^*e_2^*$. Given that τ^u and τ^{cxp} are both decreasing in $\bar{n}^u$, this implies that the tariff levels are both increasing in foreign expenditure.

Returning to the analysis of the welfare benefit of commitment to a trade agreement, the derivative of $G^c - G^u$ with respect to $N^*e_2^*$ is

$$\begin{aligned}\frac{d(G^c - G^u)}{d(N^*e_2^*)} &= \left(-\gamma\frac{\partial TW^u}{\partial n} - (1-\gamma)\frac{\partial W^{cxp}}{\partial n}\right)\frac{d\bar{n}^u}{d(N^*e_2^*)} + \gamma a\frac{d(\bar{n}^u\pi^{cxp})}{d(N^*e_2^*)} \\ &\quad + \left(\frac{1}{\sigma}\right)\frac{1}{\tau^*}\left(\frac{1}{z^{*c}+1} - \frac{1}{z^{*u}+1}\right),\end{aligned} \tag{1.56}$$

Beginning with the first line, the sign of the indirect effect, $d(G^c - G^u)/d\bar{n}^u = \Big[\partial(G^c - G^u)/\partial\bar{n}^u) + (\partial(G^c - G^u)/\partial\tau^{cxp})(d\tau^{cxp}/d\bar{n}^u)\Big]\Big(d\bar{n}^u/d(N^*e_2^*)\Big)$, is unclear. The first two parts of the indirect effect, $-\gamma(\partial TW^u/\partial n)$ and $-(1-\gamma)(\partial W^{cxp}/\partial n)$, are positive times the firm-entry derivative which is negative. The third part of the indirect effect, $d(\bar{n}^u \pi^{cxp})/d(N^* e_2^*)$, is

$$\begin{aligned}\frac{d(\bar{n}^u \pi^{cxp})}{d(N^* e_2^*)} &= \left(\frac{1}{\sigma}\right) \frac{w_\lambda(\bar{k}^u)}{\bar{n}^u \lambda(\bar{k}^u)} \left(\frac{d\bar{n}^u}{d(N^* e_2^*)}\right) \\ &* \left[N e_2 \frac{z^{cxp}}{(z^{cxp}+1)^2} \frac{(\sigma z^{cxp}+1)}{(\sigma z^{cxp}+1)-(\sigma-1)(\tau^{cxp}-1)} + N^* \frac{e_2^*}{\tau^*} \frac{z^{*u}}{(z^{*u}+1)^2} \right],\end{aligned} \tag{1.57}$$

which is easily seen to be positive given $w_\lambda(\bar{k}^u) < 0$ and $d\bar{n}^u/d(N^* e_2^*) < 0$.[56]

The direct effect of a change in $N^* e_2^*$ comes only through the effect on a firm's ex-post foreign operating profits, equal to $n(p_h(k) - \lambda(k))(N^* q_{h2}^*)$. The effect of an increase in foreign expenditure is positive given that $1/(z^{*c}+1) - 1/(z^{*u}+1)$ (the change in foreign market share of home producers) is positive. This indicates that the benefit to the government of commitment to a trade agreement grows as the share of total expenditures by domestic consumers falls.[57]

Therefore, the overall sign of $d(G^c - G^u)/d(N^* e_2^*)$ is unclear analytically.[58] Using the numerical solution demonstrates that the positive direct effect is larger than the negative indirect effect, indicating that $G^c - G^u$ is increasing in foreign expenditure share.[59]

The effect of a change in foreign expenditure share is shown in Figure 1.10, where I plot the government's policy preference for (a, σ) combinations given three different values of the foreign expenditure share m^* holding constant $\gamma = 1/2$. The figure shows that the area of (a, σ) combinations for which $G^c - G^u > 0$ is decreasing as the foreign expenditure share increases. The figure shows that as the size of the home country as a share of the world market is at its largest ($m^* = 25/26$ in the figure), the set of (a, σ) combinations for which the government benefits from commitment to a trade agreement is at its largest. When the

[56] The indirect effect is simplified using the envelope conditions for the government's tariff-setting problem, which is why it only depends on the τ^{cxp} derivative, specifically the tariff derivative for τ^{cxp} is $(\partial(G^c - G^u)/\partial\tau^{cxp}) = \gamma a(\partial(\bar{n}^u \pi^{cxp})/\partial\tau^{cxp})$, and the τ^c and τ^u derivatives are $\partial(G^c - G^u)/\partial\tau^c = 0$ and $\partial(G^c - G^u)/\partial\tau^u = 0$, respectively.

[57] The algebraic comparison of z^{*c} and z^{*u} is discussed in Appendix A.3.

[58] In Appendix A.7.2 I do some further algebraic expansion of the derivative.

[59] Figure A.16 in Appendix A.8 demonstrates the numerical solution for the direct versus indirect effects of a change in $N^* e_2^*$ as defined in equation (1.56).

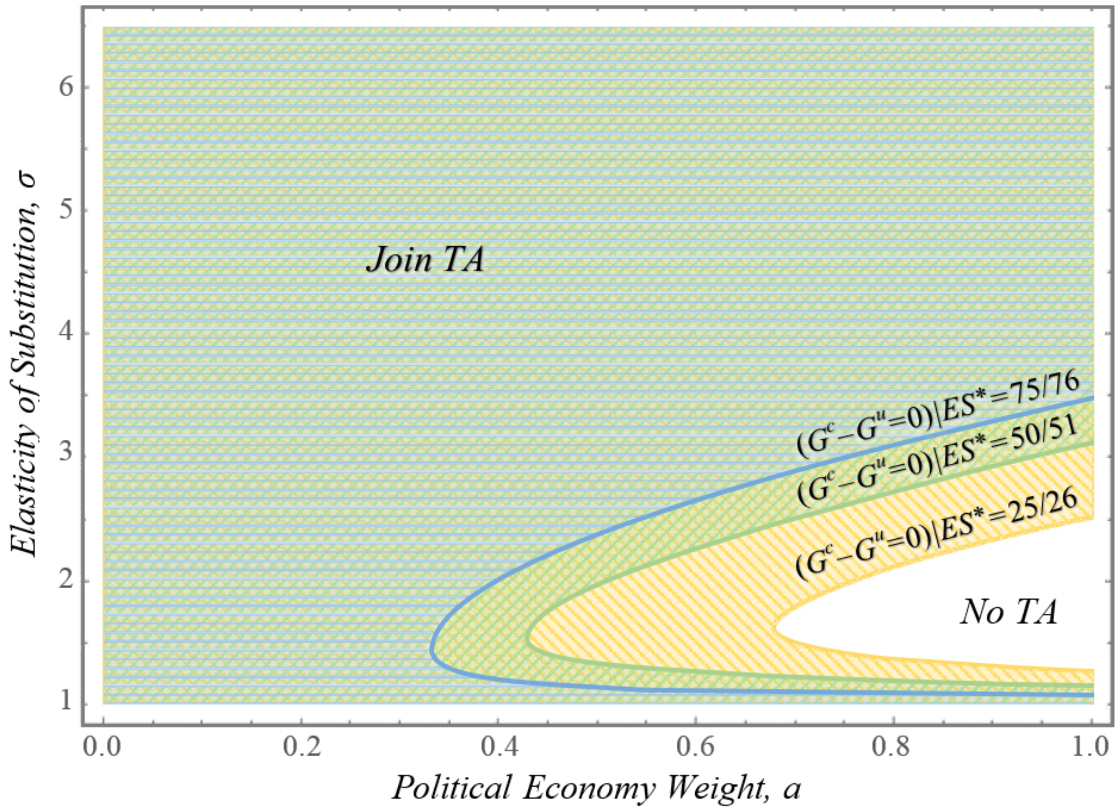


Figure 1.10: Government trade policy preference for different foreign expenditure shares, $m^* \equiv (N^* e_2^*)/(N e_2 + N^* e_2^*)$, given a fixed $\gamma = 1/2$. Each region shows combinations of σ and a for which the government is better off by committing to a trade agreement ex ante. Where for any (a, σ) pair, if $G^c - G^u|_{m^*=m_0^*} > 0$ then $G^c - G^u|_{m^*<m_0^*} > 0$.

home country becomes smaller on the world market ($m^* = 50/51$ and $m^* = 75/76$), there is a smaller set of parameters for which a trade agreement is beneficial. Additionally, for any given (a, σ) combination where $G^c - G^u|_{m^*=m_0^*} > 0$, it is also true that $G^c - G^u|_{m^*<m_0^*} > 0$.

Conjecture 1.18 (Effect of foreign expenditure share, m^*, on preferred trade policy)**.** *Assume the existence of interior ex-ante producer equilibria $(\bar{k}^c, \bar{n}^c)$ and $(\bar{k}^u, \bar{n}^u)$, that $(1 + a) < \left(\frac{\sigma}{\sigma-1}\right)\sigma$, and that Assumptions 1.7 and 1.10 hold. Given $\gamma > \bar{\bar{\gamma}}$, the effect of an increase in total foreign expenditure is defined by equation (1.56),*

$$\frac{d(G^c - G^u)}{d(N^* e_2^*)} = \left(-\gamma \frac{\partial TW^u}{\partial n} - (1-\gamma)\frac{\partial W^{cxp}}{\partial n}\right)\frac{d\bar{n}^u}{d(N^* e_2^*)} + \gamma a \frac{d(\bar{n}^u \pi^{cxp})}{d(N^* e_2^*)}$$
$$+ \left(\frac{1}{\sigma}\right)\frac{1}{\tau^*}\left(\frac{1}{z^{*c}+1} - \frac{1}{z^{*u}+1}\right) < 0, \tag{1.56}$$

where second line of the equation is positive, equal to the direct effect of $N^*e_2^*$ *on* $G^c - G^u$*. The first line is the indirect effect of* $N^*e_2^*$ *on* $G^c - G^u$*, the sign of which is unclear analytically. Using the numerical solution, I show that the indirect effect is negative and large, which results in* $d(G^c - G^u)/d(N^*e_2^*) < 0$ *overall.*

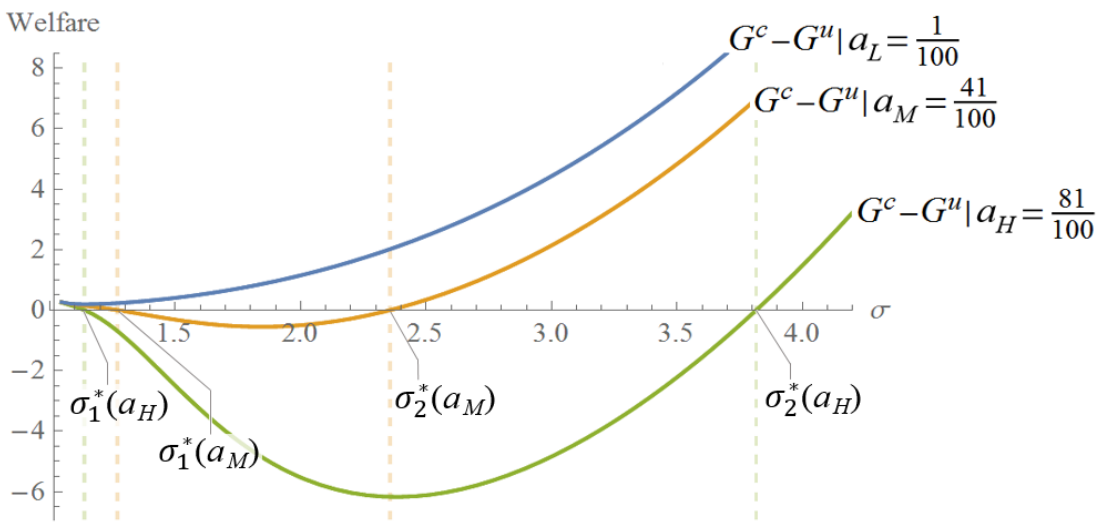
Therefore, the government's welfare benefit from committing to a trade agreement falls when the size of foreign consumer expenditures rises relative to home consumer expenditures on the differentiated good. Intuitively, when the foreign market is very large the importance of sales abroad to producer surplus increases, which in turn means that a firm benefits more by investing in lowering its unit-labor cost through additional capital investment than it does from distorting its production and receiving the politically-motivated ex-post tariff protection. The result overall is that when the foreign market is very large, the potential loss to firms from being less competitive internationally prevents them from greatly distorting ex-ante production decisions with or without a trade agreement (reflected in the fact that firm entry is decreasing in $N^*e_2^*$, equation (1.55)). Subsequently, when the government makes the decision to commit to a trade agreement, when foreign markets are very large, there is little ex-ante distortion to correct, making the gains to the government of correcting the ex-ante production distortion relatively small.

1.5.3 Government Policy Preference and Elasticity of Substitution

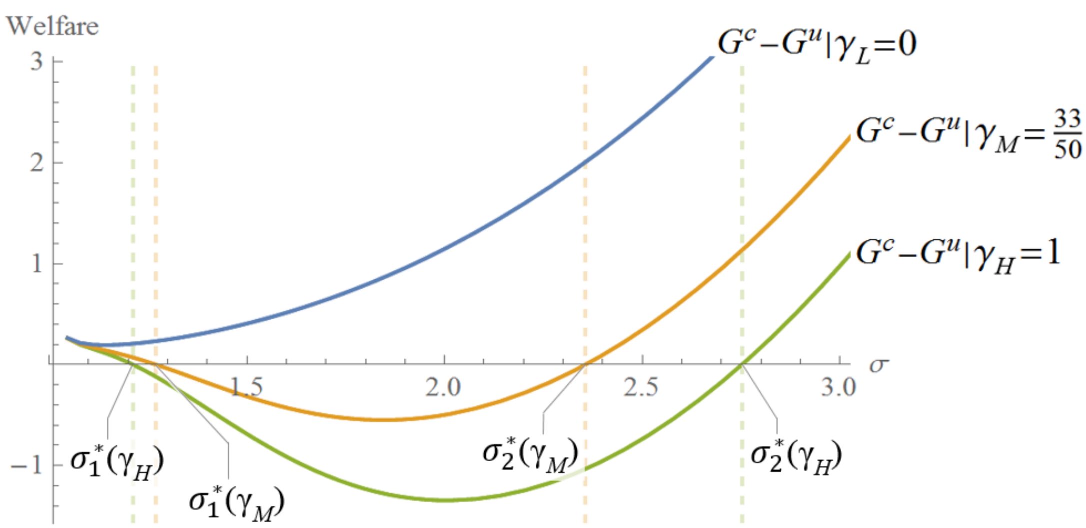
Next, consider the relationship between the government's benefit from commitment to a trade agreement, $G^c - G^u$, and the elasticity of substitution across varieties, σ. Characterizing the effect of σ on $G^c - G^u$ is far more difficult than the analysis of the effects of a and γ on $G^c - G^u$. This is due to the fact that firm entry $\bar{n}^u$, tariffs τ^c, τ^u, and τ^{cxp}, and the direct effect of σ on $G^c - G^u$ are all nonmonotonic in σ. Therefore, it is not surprising that $G^c - G^u$ is also nonmonotonic in σ.

Figure 1.11 demonstrates the nonmonotonicity of $G^c - G^u$ in σ using the numerical solution, with Figure 1.11a showing $G^c - G^u$ for different values of a holding γ constant

Figure 1.11: Welfare benefit of joining a trade agreement, $G^c - G^u$, for changes in σ. $G^c - G^u$ is decreasing in σ when σ is low and is increasing in σ when σ is high. $G^c - G^u$ is positive for all a when γ or a are near zero. The range of values for which the government does not join a TA is $[\sigma_1^*, \sigma_2^*]$, with the size of the range increasing in a or γ.



(a) Welfare benefit of joining a trade agreement for $\gamma = 33/50$ and $a \in \{1/100, 41/100, 81/100\}$.



(b) Welfare benefit of joining a trade agreement for $a = 41/100$ and $\gamma \in \{0, 33/50, 1\}$

and Figure 1.11b showing $G^c - G^u$ for different values of γ holding a constant. In each panel, the blue line corresponds to the findings in Section 1.5.1 that when a or γ are very low, $G^c - G^u$ is always positive. The figure also demonstrates a key finding of my model: for higher values of a and γ, $G^c - G^u$ remains positive when σ is close to one or when σ is high, and is only negative for an intermediate range of values of σ.

The remainder of this section proceeds in three parts: (i) demonstrating algebraically that the government always prefers a trade agreement when σ is very low; (ii) relating the effect of σ on $G^c - G^u$ to the region plot from Section 1.5.1 (Figure 1.8b); and (iii) showing how for a fixed value of a, the (σ, γ) combination for which $G^c - G^u = 0$ and $d(G^c - G^u)/d\sigma = 0$ can be used to characterize the range of σ for which the government does not join a trade agreement, with the range of σ where the government prefers not to join a TA labeled as $\sigma \in [\sigma_1^*, \sigma_2^*]$ in Figure 1.11. Point (ii) is depicted in Figure 1.13; point (iii) is depicted later in Figures 1.12 and 1.14, summarized in Conjecture 1.21. Beyond point (i), the majority of the section relies on the numerical solution.

The limit of $G^c - G^u$ as $\sigma \to 1$ is shown below using a re-written version of equation (1.45):

$$\begin{aligned}
\lim_{\sigma \to 1}\Big(G^c - G^u\Big) = Ne_2 \Bigg[& \frac{1}{\sigma}\underbrace{\left(\frac{z^c}{z^c+1} - \frac{z^{cxp}}{z^{cxp}+1}\right)}_{\to 0 \text{ for } \sigma \to 1} + \underbrace{\left(\frac{\tau^c - 1}{\tau^c}\right)\left(\frac{1}{z^c+1}\right) - \left(\frac{\tau^{cxp}-1}{\tau^{cxp}}\right)\left(\frac{1}{z^{cxp}+1}\right)}_{\to 0 \text{ for } \sigma \to 1}\Bigg] \\
& + N^*\frac{e_2^*}{\tau^*}\Bigg[\underbrace{\frac{1}{z^{*c}+1} - \frac{1}{z^{*u}+1}}_{\to 0 \text{ for } \sigma \to 1}\Bigg] + Ne_2\Bigg[\underbrace{\ln\left(\frac{\tau^{cxp}}{\tau^c}\right)}_{\to 0 \text{ for } \sigma \to 1} + \underbrace{\left(\frac{1}{\sigma-1}\right)\ln\left(\frac{1}{z^{cxp}+1}\Big/\frac{1}{z^c+1}\right)}_{>0 \text{ as } \sigma \to 1}\Bigg] \\
& - \gamma Ne_2\Bigg[(1+a)\frac{1}{\sigma}\underbrace{\left(\frac{z^u}{z^u+1} - \frac{z^{cxp}}{z^{cxp}+1}\right)}_{\to 0 \text{ for } \sigma \to 1} + \underbrace{\left(\frac{\tau^u-1}{\tau^u}\right)\left(\frac{1}{z^u+1}\right) - \left(\frac{\tau^{cxp}-1}{\tau^{cxp}}\right)\left(\frac{1}{z^{cxp}+1}\right)}_{\to 0 \text{ for } \sigma \to 1}\Bigg] \\
& - \gamma Ne_2\Bigg[\underbrace{\ln\left(\frac{\tau^{cxp}}{\tau^u}\right)}_{\to 0 \text{ for } \sigma \to 1} + \underbrace{\left(\frac{1}{\sigma-1}\right)\ln\left(\frac{1}{z^{cxp}+1}\Big/\frac{1}{z^u+1}\right)}_{\to 0 \text{ for } \sigma \to 1}\Bigg] = \infty,
\end{aligned} \tag{1.58}$$

where the sign of the natural log terms is derived in Appendix A.7.3 using l'Hopital's rule.[60]

Proposition 1.19 (Government policy preference as elasticity of substitution, σ, approaches one). *Assume the existence of interior ex-ante producer equilibria* $(\bar{k}^c, \bar{n}^c)$ *and* $(\bar{k}^u, \bar{n}^u)$*, that* $(1+a) < \left(\frac{\sigma}{\sigma-1}\right)\sigma$*, and that Assumptions 1.7 and 1.10 hold. Regardless of the values of* a *and* γ*, when* $\sigma \to 1$ *the government always benefits from committing to a trade agreement, as shown in equation (1.58).*

Establishing the sign of $G^c - G^u$ for other values of σ analytically quickly becomes prohibitively complex. I have a brief discussion of $\lim_{\sigma \to \sigma^{Max}} G^c - G^u$ in Appendix A.7.3, but am unable to sign this equation. Therefore, I rely on the numerical solution as shown in Figure 1.11 regarding the behavior of $G^c - G^u$ over the full range of σ.

Conjecture 1.20 (Government policy preference as elasticity of substitution, σ, approaches σ^{Max}). *Assume the existence of interior ex-ante producer equilibria* $(\bar{k}^c, \bar{n}^c)$ *and* $(\bar{k}^u, \bar{n}^u)$*, that* $(1+a) < \left(\frac{\sigma}{\sigma-1}\right)\sigma$*, and that Assumptions 1.7 and 1.10 hold. Regardless of the values of* a *and* γ*, Figure 1.11 demonstrates that when goods are very homogeneous (as* $\sigma \to \sigma^{Max}$*, the government always benefits from committing to a trade agreement.*

Breaking the $G^c - G^u$ equation into the welfare gains from correcting the ex-ante-production distortion (XAD) and the government welfare losses from foregoing the ex-post political contributions (XPC) provides additional insight into what happens when σ is very high or very low. Using the definitions of XAD and XPC given in equation (1.46), Figure 1.12 shows that as $\sigma \to 1$, the governments benefit XAD is increasing and the ex-post compensation goes to zero. Over the full range of σ values, the XAD curve is initially decreasing in σ and then increasing. The XPC curve is initially increasing in σ and then decreasing for high values of σ.[61] Recall from the discussion regarding τ^c and τ^u that

[60] Although at first glance it appears that $G^c - G^u$ reaches a local maximum as $\sigma \to 1$, examining Figure 1.11 more closely demonstrates that when σ is very near one $G^c - G^u$ is decreasing in σ, consistent with the finding that $\lim_{\sigma \to 1} G^c - G^u = \infty$.

[61] Figure 1.12 is cut off at $\sigma = 3$ to better show how changes in a affect the range of σ for which the government joins a trade agreement, which made it necessary to cut off the figure before the range of σ where $d(XPC)/d\sigma < 0$.

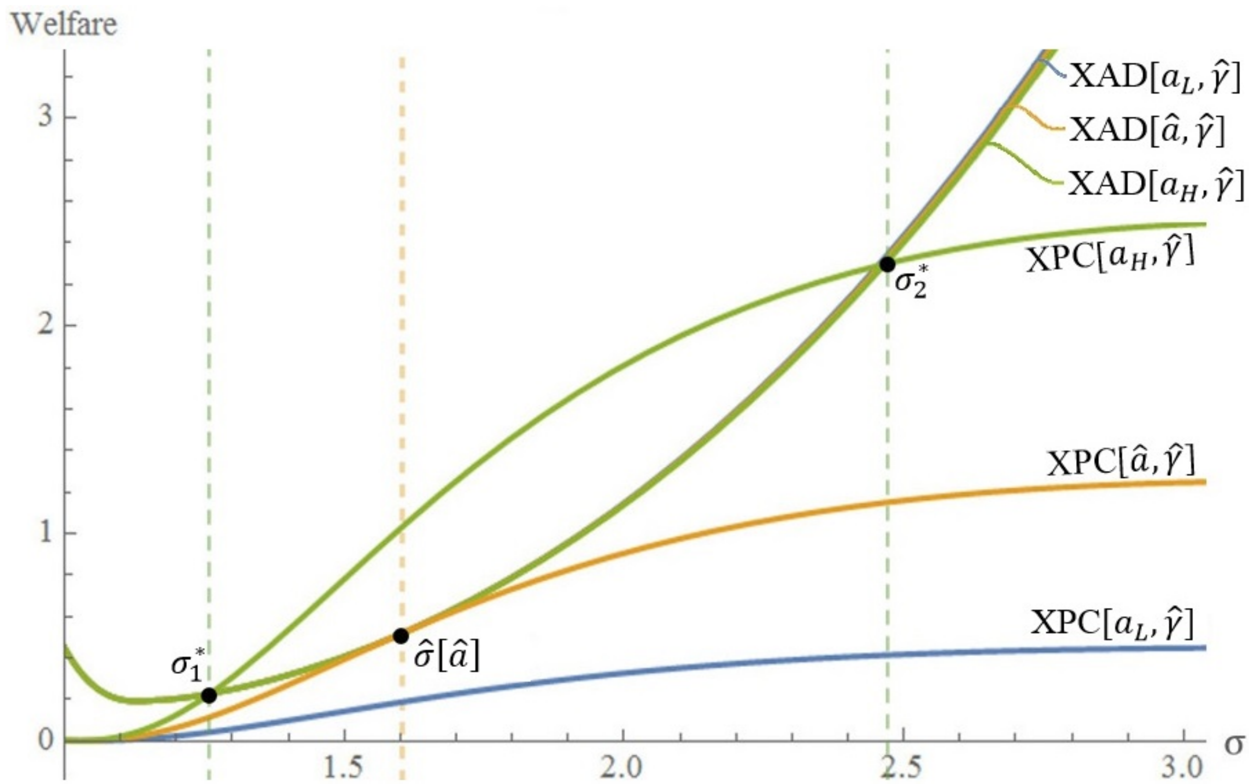


Figure 1.12: The ex-ante distortion (XAD) caused by not committing to a trade agreement compared to the ex-post compensation (XPC) the government receives for the tariff protection for goods with varying elasticities of substitution, σ. The figure shows that for a fixed $\hat{\gamma}$, there is an equilibrium $\hat{\sigma}(\hat{a})$ and $\hat{a}$, such that for any $a < \hat{a}$ the government will choose to commit to a trade agreement for every product, regardless of elasticity of substitution. For $a > \hat{a}$, the government chooses not to join a trade agreement for products with elasticity of substitution $\sigma \in [\sigma_1^*(a, \gamma), \sigma_2^*(a, \gamma)]$.

this pattern matches the fact that when σ is very low or very high, the ad valorem tariff rate approaches zero ($\tau \rightarrow 1$) both with and without a trade agreement. As a result, the government is not able to extract enough of a political contribution from firms to make up for the social welfare losses from the ex-ante production distortion.

Continuing with Figure 1.12, the figure also shows that for any increase in a, there is a large upward shift of the XPC curve and a small upward shift of the XAD curve. Regardless of the size of a or γ, the benefit to the government of correcting the ex-ante production distortion is relatively stable. Ex-post compensation, however, shifts greatly from changes in bargaining strength or political pressure.

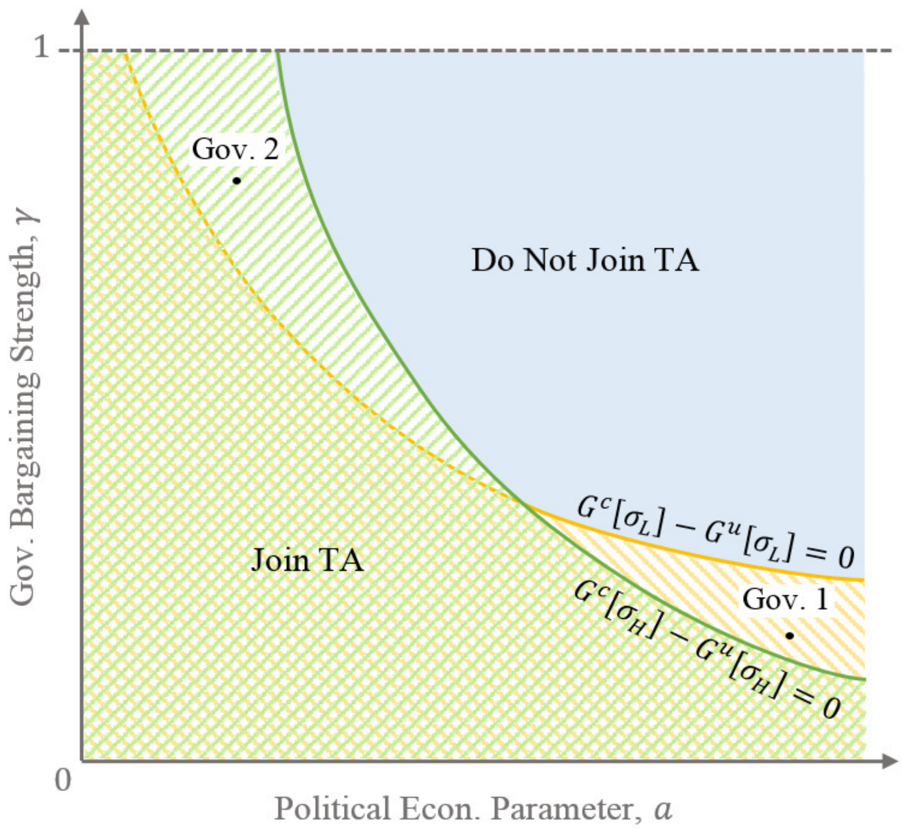


Figure 1.13: Government preference to join a trade agreement or not, comparing G^c and G^u for differing elasticities of substitution (σ), given convex upfront lobbying cost $\theta(k)$. The two lines on the figure represent combinations of political-economy weight, a, and government bargaining strength, γ, for which the government is indifferent between joining a trade agreement or not (where $G^c - G^u = 0$). For Government 1, which has a high a and a low γ, it will only choose to join a trade agreement and commit ex-ante to a tariff if it trades in the relatively more homogeneous good, good L, and it will not commit to a tariff ex-ante on good H.

Given that Figures 1.11 and 1.12 show that for a given (a, γ) combination there are two values of σ for which $G^c - G^u = 0$, consider how this impacts the region plot of the government's policy preferences for different (a, γ) combinations (Figure 1.8b). Figure 1.13 shows that an increase in the substitutability of varieties (moving from σ_L to σ_H) leads to a downward shift and outward (or clockwise) rotation of the policy-indifference curve.[62] There are two (a, γ) combinations labeled on Figure 1.13 which represent two "types" of governments. The figure depicts two potential products, good L and good H, each with its

[62]Figure 1.11 demonstrates using the numerical solution the fact that for any given (γ, a) combination there are two values of σ for which $G^c - G^u = 0$.

own demand elasticity, σ_L and σ_H, respectively. The point labeled "Government 1" shows that a weak bargainer that is highly politically motivated uses a trade agreement to tie its hands on the tariff it sets on good L, the more differentiated good, while not placing an ex-ante restriction on the tariff it sets on imports of good H, the more homogeneous good. Government 2 prefers the opposite: it prefers a trade agreement restricting its tariff on good H and not good L. Government 2's policy is comparable to a trade agreement being formed between "dissimilar" countries which mainly trade in distinct product categories, reflecting a Ricardian pattern of trade according to comparative advantage. On the other hand, Government 1's policy is comparable to a trade agreement between "similar" countries which trade in varieties of similar product categories.

Finally, using the numerical solution I can characterize the range of σ for which the government joins a trade agreement based off of the value of σ for which $G^c - G^u = 0$ and $d(G^c - G^u)/d\sigma = 0$. Returning to Figure 1.12, $G^c - G^u = 0$ and $d(G^c - G^u)/d\sigma = 0$ are true where XAD and XPC are tangent, labeled on the figure as $\hat{\sigma}(a)$. One benefit of focusing on the tangency between XAD and XPC is that it narrows down the number of parameters to consider: the point of tangency is determined by the value of a where $XAD(\hat{\sigma}(a), a) = XPC(\hat{\sigma}(a), a, \hat{\gamma}(a))$ and $\partial XAD(\hat{\sigma}(a), a)/\partial\sigma = \partial XPC(\hat{\sigma}(a), a, \hat{\gamma}(a))/\partial\sigma$. In other terms, the point of tangency represents the (a, γ) combination for which the government will join a TA for all but on value of σ.

Algebraically, $\hat{\gamma}(a)$ and $\hat{\sigma}(a)$, are characterized by the following two equations:

$$\begin{aligned} W^c\Big(\hat{\sigma}(a)\Big) &= \hat{\gamma}(a)\, W^u\Big(\hat{\sigma}(a), a\Big) + (1-\hat{\gamma}(a))\, W^{cxp}\Big(\hat{\sigma}(a), a\Big) \\ &\quad + a\,\hat{\gamma}(a)\,\bar{n}^u\Big(\hat{\sigma}(a), a\Big)\Big(\pi^u\Big(\hat{\sigma}(a), a\Big) - \pi^{cxp}\Big(\hat{\sigma}(a), a\Big)\Big), \quad \text{and} \end{aligned} \tag{1.59}$$

$$\begin{aligned} \left[\frac{\partial W^c}{\partial\sigma} + \frac{\partial W^c}{\partial n}\frac{d\bar{n}^c}{d\sigma} + \frac{\partial W^c}{\partial\tau}\frac{d\tau^c}{d\sigma}\right] &= (1-\hat{\gamma}(a))\left[\frac{\partial W^{cxp}}{\partial\sigma} + \frac{\partial W^{cxp}}{\partial n}\frac{d\bar{n}^u}{d\sigma} + \frac{\partial W^{cxp}}{\partial\tau}\frac{d\tau^{cxp}}{d\sigma}\right] \\ + \hat{\gamma}(a)\left[\frac{\partial TW^u}{\partial\sigma} + \frac{\partial TW^u}{\partial n}\frac{d\bar{n}^u}{d\sigma} + \frac{\partial TW^u}{\partial\tau}\frac{d\tau^u}{d\sigma}\right] &- \hat{\gamma}(a)\left[a\,\bar{n}^u\frac{d\pi^{cxp}}{d\sigma} + a\,\pi^{cxp}\frac{d\bar{n}^u}{d\sigma}\right], \end{aligned} \tag{1.60}$$

where the dependence of the firm entry, welfare, and profit terms in equation (1.60) on σ and a is suppressed for the sake of simplicity.

$\hat{\gamma}(a)$ is the "political-indifference frontier," denoting the combinations of (a, γ) for which (i) the government weakly prefers a trade agreement for all values of σ; and where (ii) joining a trade agreement is strictly preferable for all $\sigma > 1$ if $\gamma < \hat{\gamma}(a)$. In other words, for $\gamma < \hat{\gamma}(a)$ domestic lobbyists have no influence on trade policy. Algebraically, establishing the behavior of $\hat{\gamma}(a)$ for changes in a is relatively straightforward. Beginning with the sign of $\hat{\gamma}(a)$, my work in Section 1.5.1 on the behavior of $G^c - G^u$ for changes in a and γ is sufficient. That is because using the $G^c - G^u = 0$ notation instead of $XAD = XPC$, the behavior of $\hat{\gamma}(a)$ can be found by taking the total derivative of $G^c - G^u = 0$ with respect to γ:

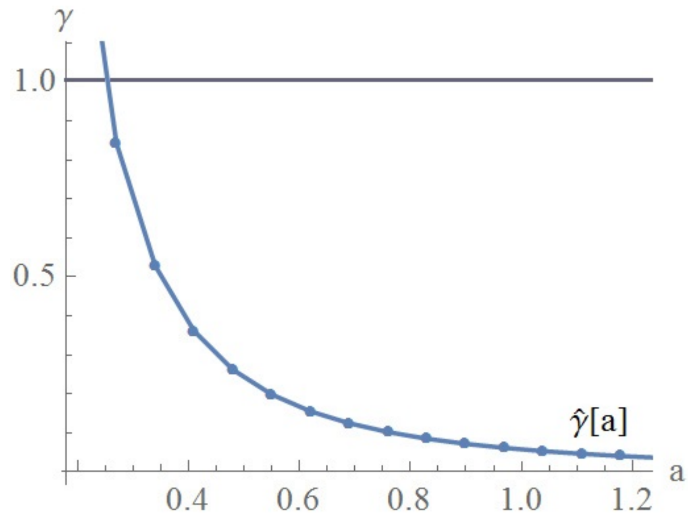
$$\frac{d\hat{\gamma}(a)}{da} = -\left(\frac{d(G^c - G^u)}{da}\right) \Big/ \left(\frac{d(G^c - G^u)}{d\gamma}\right),$$

where the derivatives with respect to σ cancel out at the point of tangency. Because $d(G^c - G^u)/da$ and $d(G^c - G^u)/d\gamma$ are negative, $d\hat{\gamma}(a)/da$ is also negative, which is shown in Figure 1.14a. Using the numerical estimate of the model, Figure 1.14a shows that $\hat{\gamma}(a)$ is decreasing in a, which is consistent with the behavior of the policy-indifference curves as shown in the region plots of $G^c - G^u = 0$. Figure 1.14a also shows that when the political economy weight is very low, $\hat{\gamma}(a)$ goes to one. When $\hat{\gamma}(a) \geq 1$, it means that a is low enough such that for any value of γ or σ, the government always prefers to join a trade agreement. The value of a for which $\hat{\gamma}(a) = 0$ would be equivalent to the minimum of $\bar{a}(\gamma, \sigma)$ over all values of $\gamma \in [0, 1]$ and $\sigma \in (1, \sigma^{Max})$.

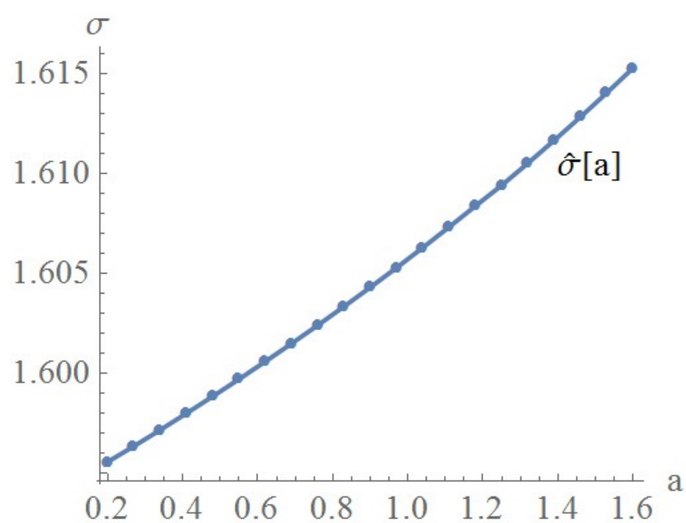
$\hat{\sigma}(a)$ is mainly useful in understanding how the range of σ for which the government does not join a trade agreement shifts for changes in a. The usefulness of this parameter in understanding the $[\sigma_1^*(a, \gamma), \sigma_2^*(a, \gamma)]$ range is most easily established by an example.[63] Comparing two parameter sets $(\hat{\sigma}(\hat{a}), \hat{\gamma}(\hat{a}), \hat{a})$ and $(\hat{\sigma}(a'), \hat{\gamma}(a'), a')$ such that $\hat{a} < a'$ demonstrates that (i) the lower bound falls slightly, $\sigma_1^*(\hat{a}, \gamma) > \sigma_1^*(a', \gamma)$, and (ii) the upper bound

[63] Figure A.14 in Appendix A.7.4 uses the numerical solution to illustrate how $\sigma_1^*(a, \gamma)$ and $\sigma_2^*(a, \gamma)$ shift in a and γ.

Figure 1.14: The effect of the political economy weight, a, on the values of $\hat{\gamma}(a)$ and $\hat{\sigma}(a)$.



(a) The $\hat{\gamma}(a)$ for which the XAD and XPC curves are tangent is decreasing and convex in a.



(b) The $\hat{\sigma}(a)$ for which the XAD and XPC curves are tangent is increasing and convex in a.

rises significantly, $\sigma_2^*(\hat{a},\gamma) < \sigma_2^*(a',\gamma)$. These effects combined result in an overall rise in $(\sigma_1^* + \sigma_2^*)/2$, the average σ for which the government does not join a trade agreement. Using the numerical solution, Figure 1.14b shows that $\hat{\sigma}(a)$ is also increasing in a. Therefore, the upward movement of $(\sigma_1^* + \sigma_2^*)/2$ is reflected in the upward movement of $\hat{\sigma}(a)$, which allows me to use the movement of $\hat{\sigma}(a)$ to serve as a proxy for the range of σ for which the government does not join a trade agreement, $[\sigma_1^*(a,\gamma), \sigma_2^*(a,\gamma)]$.

Algebraically, $d\hat{\sigma}(a)/da$ can be found using the total derivative of equation (1.60) with respect to a. Written in terms of $G^c - G^u$, the resulting equation for $d\hat{\sigma}(a)/da$ is

$$\frac{d\hat{\sigma}(a)}{da} = -\left(\frac{d^2(G^c - G^u)}{d\sigma da} + \frac{d\hat{\gamma}(a)}{da}\frac{d^2(G^c - G^u)}{d\sigma d\gamma}\right) \Bigg/ \left(\frac{d^2(G^c - G^u)}{d\sigma^2}\right). \tag{1.61}$$

Because the point of tangency is where $G^c - G^u$ is both equal to zero and at its maximum, it must be true that for $\sigma = \hat{\sigma}(a)$ and $\gamma = \hat{\gamma}(a)$, $d^2(G^c - G^u)/d\sigma^2 < 0$. Therefore, the sign of $d\hat{\sigma}(a)/da$ is determined by the sign of the numerator, $(d^2(G^c - G^u)/d\sigma da) + (d\hat{\gamma}(a)/da)(d^2(G^c - G^u)/d\sigma d\gamma)$, which is the sum of the cross-partial derivatives.[64]

Conjecture 1.21 (Behavior of $\hat{\sigma}(a)$ and $\hat{\gamma}(a)$ as depicted in the numerical solution)**.** *Assume the existence of interior ex-ante producer equilibria $(\bar{k}^c, \bar{n}^c)$ and $(\bar{k}^u, \bar{n}^u)$, that $(1 + a) < \left(\frac{\sigma}{\sigma-1}\right)\sigma$, and that Assumptions 1.7 and 1.10 hold. For a given value of a, XAD and XPC are tangent when $\sigma = \hat{\sigma}(a)$ and $\gamma = \hat{\gamma}(a)$, with $d\hat{\sigma}(a)/da > 0$ and $d\hat{\gamma}(a)/da < 0$ according to the numerical solution.*

1.6 Conclusion

In this paper, I demonstrated that a government may prefer to join a trade agreement in order to correct ex-ante production distortions, even though by joining a trade agreement the government must forego the political contributions which lobbying firms pay in exchange for tariff protection.

[64] The cross-partial derivatives are defined in Appendix A.7.4.

My model showed that lobbying by firms in a monopolistically competitive sector leads to intrasectoral distortions in production, specifically firms over-enter the lobbying industry and each firm under-invests in cost reduction. To prevent the production distortion, the government has the option of an ex-ante commitment to a trade agreement which eliminates the domestic political pressure the government otherwise faced from lobbyists. Joining a trade agreement results in fewer firms entering the industry and each firm producing at a lower marginal cost.

I showed that after firm entry and capital are fixed, the tariff-setting rules are unchanged from the original "Protection for Sale" model in Grossman and Helpman (1994) and the monopolistically competitive version from Chang (2005). With or without a trade agreement, the welfare-maximizing trade policy is an import tariff, which stands in contrast with the perfectly competitive model from Maggi and Rodríguez-Clare (1998) in which the welfare-maximizing policy without a trade agreement is free trade.

I demonstrated that the government makes its decision to commit to a trade agreement based on the trade-off it faces between correcting the ex-ante production distortion by joining the trade agreement or receiving the ex-post compensation from lobbyists by not joining the agreement. I showed that as the political economy weight increases or as the government's bargaining strength increases, the government is less likely to prefer to commit to a trade agreement, a result which is unchanged from the perfectly competitive model introduced in Maggi and Rodríguez-Clare (1998). I also demonstrated that the elasticity of substitution across varieties has a nonlinear effect on the government's trade policy: when goods are very homogeneous or very differentiated, the government prefers to join a trade agreement. For an intermediate range of elasticities, the government will prefer not to join a trade agreement if it values political contributions highly if the government is skilled at bargaining with the lobby. This stands in contrast with Levy (1997), in which he uses a median-voter model to show that the median voter supports a trade agreement between countries trading in differentiated goods (high σ) without consideration for the government's political economy weight or bargaining strength.

Finally, my model shows that as the size of the home country relative to the world market falls, the government will prefer to join a trade agreement for a smaller range of products. This results from the fact that when the home country is very small on the world market, the ex-ante production distortion with lobbying is very small. Firms realize that competitiveness on the world market will generate more profits than petitioning the government to impose higher tariffs on their foreign competitors to improve sales at home.

Because my model generates a trade policy prediction dependent on four key factors: two political factors, the degree of product differentiation, and the share of world expenditure on a product, I am able to reflect the variety observed in preferential trade agreements joined by small countries. My next steps with this model are then to test the predictions empirically, to see if the products on which small countries seek to commit to lowering tariffs generally include the most differentiated and most homogeneous goods. Additionally, I can use different proxies for the political economy parameters to examine if the countries with whom a government joins a preferential agreement reflect the model's predictions.

2 Afraid of Commitment: Why Small Countries Join Trade Agreements with Escape Clauses

2.1 Introduction

Three months prior to the unsuccessful conclusion of the World Trade Organization's (WTO) Doha round of negotiations, the WTO hosted a seminar – "Cross-Cutting Issues in Regional Trade Agreements" – intended to address the problem of WTO member countries using regional trade agreements (RTAs) as a substitute for securing tariff reductions through the WTO's multilateral negotiations. Then-Director-General Roberto Azevêdo delivered a closing address in which he spoke urgently of the need to focus on multilateral negotiations: "For the sake of the multilateral system, and all those who stand to benefit from it, I think we have to find a solution to our current problems and put our work here at the WTO back on track. And we have to do it quickly. Time is not on our side" (WTO). While focusing efforts on forming regional trade agreements (RTAs) does undercut the WTO bargaining rounds, in this paper I will argue that it is unreasonable to ask smaller WTO members to limit their use of such agreements.[1]

In this paper I demonstrate that for many WTO members, regional trade agreements may not be functioning as a substitute for multilateral negotiations but are instead a necessary policy tool to accomplish trade policy goals. For small countries, the use of regional agreements in conjunction with WTO membership potentially reflects a political-economy motive for joining a trade agreement: a politically-motivated government may need a stronger tariff commitment than the WTO can provide in order to prevent distortionary rent-seeking behavior by import-competing industries. Additionally, my theory shows that

[1]Note that I use "regional trade agreement" to refer to any trade agreement between WTO member countries entered into under GATT Article XXIV or the Enabling Clause. My usage of the term matches the WTO's use of "regional trade agreement." Distinct from regional trade agreements, the GATT Articles also use "regional trade agreement" to refer to unilateral tariff concessions granted to a sub-group of WTO members. PTAs in this sense would still fit the predictions of my model as long as any tariff concessions are enforced by trading partners.

the inclusion of an escape clause – a temporary measure which allows a government to increase tariff levels in response to a harmful shock – in a regional trade agreement does not unravel the commitment created through the regional agreement. Finally, my model suggests that a WTO member country using a free trade agreement with escape clauses to maximize government welfare would actually be better off joining an RTA in which it can negotiate non-zero tariff bindings, but such agreements are technically disallowed by WTO rules (discussed further below).

If the government is joining a trade agreement specifically to avoid political pressure, then the one truly essential feature of an agreement is enforcement. As I discuss in greater depth below, the WTO's structure provides almost no enforcement for small countries seeking to maintain low tariff levels.

The WTO provides flexibility to a small-market member in two key ways: (i) explicitly in the WTO Articles and Decisions and (ii) implicitly due to the nature of how the WTO is enforced. First, the WTO has explicit flexibility built into its rules: countries are allowed flexibility in their tariffs through (i) non-zero tariff bindings, which for smaller countries are frequently it above the pre-agreement tariff levels, (ii) escape clauses, which allow a government to increase a tariff temporarily if import-competing industries are facing injury, and (iii) special allowances for developing countries which require the concession of minimal tariff cuts to be a member of the WTO.

The first measure of flexibility is the use of non-zero tariff bindings. In the WTO, countries are actually negotiating over tariff bindings, not negotiating over the applied tariff levels, and these bindings are not necessarily lower than the pre-agreement applied tariff levels. If the binding is higher than the applied tariff for a product, then there is "overhang," which allows a government to increase its tariff unilaterally without any notification to the WTO.[2]

[2]The benefits of tariff-binding overhang for both large and small countries has been extensively studied. Bagwell and Staiger (2005) show that the flexibility in the form of binding overhang improves government welfare. Beshkar and Bond (2017) examine the flexibility preferences of a government facing political-economic uncertainty in a "cap-and-escape" agreement – where a government sets a maximum tariff level and can access an EC only through a costly investigation. Their model concludes that larger countries will

The WTO also has multiple escape clauses built in that allow a country to temporarily protect an industry from surges in competition due to trade liberalization, unfair trading practices, or other economic shocks.

To add to the flexibility, commitments made by governments within the WTO become even weaker if a country is classified as "developing" or "least-developed." The "Differential and More Favourable Treatment, Reciprocity and Fuller Participation of Developing Countries" Decision, negotiated during the Tokyo Round, states "The developed countries do not expect reciprocity for commitments made by them in trade negotiations to reduce or remove tariffs and other barriers to the trade of developing countries." The Decision then goes on to state that "Having regard to the special economic difficulties and the particular development, financial and trade needs of the least-developed countries, the developed countries shall exercise the utmost restraint in seeking any concessions or contributions for commitments made by them to reduce or remove tariffs and other barriers to the trade of such countries" (WTO, 2012).

Second, the WTO has implicit flexibility from the small country's perspective. In the WTO, cooperation is a result of the principal that tariff concessions must be both reciprocal and mutually advantageous. In other words, the WTO requires that tariff concessions benefit each party of the agreement equally and that members apply the same tariff levels for imports from all WTO-member trading partners (referred to as the most-favored nation (MFN) tariffs). Enforcement takes place through the maintenance of the "balance of concessions," and any changes in tariffs that throw off this balance may be subject to retaliatory actions from other WTO members.

Retaliation as an enforcement mechanism, however, does not work for a small country. Bagwell and Staiger (1999) show that in a static model, for a politically motivated welfare-maximizing government, the principal of reciprocity translates into requiring that

tend to have no tariff overhang and will therefore use the EC for flexibility, and small countries will tend to have tariff overhang, therefore allowing them to use temporary protection without undergoing any costly investigations. I will not be considering the role of tariff overhang in this model, however, and will instead assume that the government will set a tariff binding that is lower than the political Nash tariff level through the use of a regional free trade agreement.

concessions do not impact the terms of trade for any of the member countries. Therefore, any violation of the agreement would be evident from the resultant shift in terms of trade in favor of the violator. It is straightforward to then conclude that retaliatory actions are not likely against small countries, since by definition a small country cannot impact its terms of trade.[3]

The WTO can facilitate enforcement through another mechanism: GATT Article XXIV on Regional Trade Agreements requires that "the duties and other restrictive regulations of commerce [...] are eliminated on substantially all the trade between the constituent territories in products originating in such territories" (GATT Article XXIV, paragraph 4(b)). Therefore, WTO membership coupled with a rigid regional trade agreement would seemingly successfully provide commitment against protectionist pressure from domestic lobbyists, with the additional benefit of the market access which comes with most-favored-nation (MFN) status as a WTO member.[4]

I argue that small countries are not using the WTO as a commitment device, but instead use the WTO for market access and seek commitment against domestic political pressure through regional trade agreements. Consider a small country that joins the WTO and a more rigid regional trade agreement. By joining an RTA, a small country can secure a deeper level of tariff commitment than it can with the WTO. The country will be held accountable for its tariff commitments by way of the regional agreement, because agreement members have an incentive to retaliate. It will also enjoy the benefits of the WTO by securing access to the MFN tariff levels for all member countries.

Before moving on, I also want to note that there is significant evidence that regional trade agreement members frequently use the WTO dispute settlement body for within-RTA

[3]There is a large body of literature on the use of disputes and retaliation by large countries against small countries for a variety of purposes. Bagwell and Staiger (1990), for example, discuss the important role of repeat interactions in determining cooperation within the agreement and in determining use of dispute measures. Another paper in this literature, Busch and Reinhardt (2006), discusses of the role of third parties in dispute-settlement proceedings. A third example is Subramanian and Watal (2000), in which the authors suggest the use of intellectual-property rights as an alternative means for enforcement for developing countries.

[4]The welfare benefit of receiving MFN status is not a focus of my model. For analysis on the benefits of gaining MFN status as a small country see Bagwell and Staiger (1999).

disputes. The adjudication of a trade dispute or escape request through WTO institutions does not contradict my model if the dispute is filed between members of the same regional agreement. Access to the WTO's institutions, such as the dispute settlement body, is an example of another key benefit of WTO membership, but I do not explicitly include this in my theoretical model. The World Trade Report 2011, published by the WTO, presents evidence that the WTO dispute settlement is used to adjudicate within-RTA disputes. Analysis of all disputes filed between 1995 and 2005 shows that an average of 30 percent of WTO disputes were between members of the same regional trade agreement, with the number of intra-RTA disputes increasing over time (WTO, 2011). Furthermore, the share of disputes that are over subsidies and safeguards is disproportionately greater when disputes are between members of the same RTA compared to non-intra-RTA disputes. Because the use of safeguards is constrained by the trade environment, not the trade practices of other countries, safeguards are much less likely to be manipulated as a protectionist tool. Additionally, subsidy disputes reflect an effort by an RTA to reduce or eliminate subsidies, which implies the RTA is seeking deeper integration among RTA members (WTO, 2011). This observation lends additional credibility to the idea that regional agreements are used as a stronger form of commitment than the WTO.

Within the literature on trade agreement structure and small-country preferences, the work of Staiger and Tabellini (1987), Grossman and Helpman (1994), and Maggi and Rodríguez-Clare (1998) are most relevant to this paper. Staiger and Tabellini (1987) focus on the role of time-inconsistent preferences in trade policy determination. They show that with uncertainty over terms-of-trade, a welfare-maximizing goverment may prefer to commit to a low tariff ex ante, thus limiting reducing the overinvestment of capital in the import-competing industry ex ante and of labor ex post, but this policy is not credible without some form of ex-ante commitment to a low-tariff policy.

In the same vein, Maggi and Rodríguez-Clare (1998) examine the role production distortions and political-economy pressure play in determining trade policy. They add an initial resource-allocation decision to the Grossman and Helpman (1994) "Protection for Sale" model. Maggi and Rodríguez-Clare show that without the ability to commit to free trade before producers hire resources, domestic lobbying always leads to positive tariff protection and an overallocation of resources to the tariff-protected industry. Because capital is allocated before the government sets the tariff, the the compensation the government receives in exchange for the tariff protection does not take into account the ex-ante welfare losses due to the production distortion: the lobby's contribution only takes into account the effect of the tariff on profits and government welfare. Therefore, the government benefits from committing to a free trade agreement ex ante if its share of the ex-post rents from tariff protection (at the distorted production levels) are smaller than the fall in social welfare due to the ex-ante production distortion.

Beshkar and Bond (2017) examine whether a government prefers to join a cap-and-escape trade agreement – in which countries negotiate tariff bindings and have access to escape clauses – or a trade agreement with tariff bindings and no escape clauses. Countries in the model vary in size and face a political-economy parameter that is uncertain, and only observable by trading partners by way of a costly investigation. Therefore in order to use the escape clause, a government must pay to investigate and reveal the political-economy weight of its trading partners. Beshkar and Bond demonstrate that a government of a large country is able to achieve efficient tariff levels through a cap-and-escape agreement, thereby making the cap-and-escape agreement the government's preferred policy. Beshkar and Bond also show that a small country prefers to join an agreement in which the government negotiates over tariff bindings and there is no escape clause. Then in the event which the realized political-economy pressure is high, raising the tariff to the bound rate is a more flexible and less costly means of escape. Their finding regarding the small-country policy preference, however, does not allow for ex-ante distortions in production.

This paper proceeds in two main sections: first, I add a "flexible trade agreement" policy option to the simple two-country, two-sector, perfectly competitive model from Maggi and

Rodríguez-Clare (1998). In my version of the model, before the world price is revealed the government chooses between three policies: (i) to join a free trade agreement; (ii) to join a free trade agreement with flexibility to raise the tariff if the observed world price is low; or (iii) to not join a trade agreement and set the tariff to maximize politically-weighted welfare after the world price is revealed. After trade policy is chosen and the world price is revealed, capital is allocated between a politically-organized import-competing industry and an industry producing a freely-traded numeraire good. Following the production decisions, the government and lobby engage in bargaining over the tariff level and size of the political contribution to the government.

When the government is able to choose between three policy types – a rigid free trade agreement, no trade agreement, or a free trade agreement with an escape clause – the policy choice boils down to a trade-off between two key parameters: the weight the government places on the contributions from the lobbyists (the political-economy motivation) and the amount of rents it expects to be paid in exchange for tariff protection (based on the government's bargaining strength relative to the lobbyists). When the government has a relatively small amount of bargaining strength in relation to the lobby, it is eager to make some form of commitment, because the government can recover relatively little rent from the protection it grants. The motive to commit due to a weak bargaining position, however, may be counterbalanced by the distorted weight they place on the contributions they receive from the lobby. When the value the government places on the lobby's contributions is high enough (relative to the welfare of the rest of the economy), the government will want to be able to grant protection to the lobby. The model predicts the following:

(i) the most politically motivated governments (i.e. those that place the greatest value on lobby contributions) and the governments with the most bargaining strength relative to their domestic lobby will never join a trade agreement,

(ii) the governments that place the least value on political contributions or those that have the weakest bargaining positions will prefer to join the rigid free trade agreement, and

(iii) for a middle range political-economy values and of bargaining strength, the government will want to have the flexibility to use the escape clause.

The logic behind this is that government type (i) will either be gaining so much from the political contribution (since the government values the welfare of the lobby so highly) or it will capture so large a portion of the rents of granting tariff protection that the government will never want to use a policy of free trade. The government that fits type (ii) is either so unmotivated politically or so weak relative to their political contributors that it will be unable to recover enough rent from protection to justify the ex ante overinvestment in the import-competing industry caused by capitalists knowing that the government's best response to such an overinvestment will be to grant the tariff protection the lobby seeks. Therefore it will join a strict free trade agreement in which its hands are completely tied from protecting domestic industries. The government that is type (iii) is the one that benefits from the lesser degree of commitment it finds in the FTA with an escape clause. These governments are those either with relatively little bargaining strength that value the contributions from the lobby high enough that they still want to grant some protection to the lobby or they are governments that are not as weak when it comes to bargaining but may not value the lobby contribution enough to justify the larger capital-distortion-generated welfare losses from not joining a trade agreement in any form.[5]

Then I examine how the addition of a tariff-binding trade agreement – in which countries must bind tariffs at or below the Nash political levels – effects a small country's trade policy preference. I show that if given the choice, a government will always prefer a tariff-binding agreement to an FTA with an escape clause. This policy option is precluded by Article XXIV, however, which makes a FTA with an escape clause a second-best alternative.

The rest of the paper proceeds as follows: in Section 2.2, I define the model. The subsequent sections define the model equilibria for three trade policy options: Section 2.3,

[5] Figure 2.3 in Section 2.6 depicts this graphically.

free trade agreement; Section 2.4, no trade agreement; and Section 2.5, free trade agreement with escape clause. In Section 2.6, I discuss the the government's trade policy preference when it can choose between the first three trade policies. In Section 2.7, I solve model when the government commits to a tariff-binding trade agreement ex ante, and in Section 2.8, I show that the government would always prefer being able to commit to a tariff binding ex ante to joining a free trade agreement with an escape clause.

2.2 Basic Model Setup

Consider a small-country two-input, two-good model as in Maggi and Rodríguez-Clare (1998), in which the small-country government faces political-economy pressure to grant tariff protection to a lobbying industry. The country produces two products, each in a perfectly competitive sector. Sector 1 (the numeraire sector) produces using capital and labor, and sector 2 produces using only capital. There is a continuum of workers, normalized to 1, and each worker endowed with the same amount of labor to provide. The country is endowed with K units of capital. Ownership of capital is concentrated in a negligible share of the population.[6]

Because production in both sectors is perfectly competitive, the price of the sector 2 good is determined by the world price and the tariff level. Therefore, the government can set the price in the domestic market given the world price, p^*. There is uncertainty over the world price, with the world price being revealed in the stage after capital is allocated between the two sectors.[7] The world price, $p^* = p_2^*/p_1^*$, is a random variable $p^* \in \{p_L^*, p_H^*\}$ where $p_L^* < p_H^*$, $Pr(p^* = p_H^*) \equiv \pi$, and $Pr(p^* = p_L^*) \equiv 1 - \pi$.

[6] As discussed by Maggi and Rodríguez-Clare (1998), assuming that capital is concentrated in a negligible portion of the population allows the revenue of capital owners in the protected industry to be reduced to $(p - c)k_2$. If capital is not concentrated in a negligible share of the population, then the revenue of capital owners would also include their share of consumer surplus, which complicates the trade policy preferences of capital owners.

[7] Bagwell and Staiger (2005), for example, point out that many trade agreements are negotiated years in advance of when the actual tariffs are put into place, making uncertainty in political pressures or prices very relevant.

Consumption. Utility is quasilinear in consumption of the sector 1 good. Utility from consumption of the sector 2 good is quadratic. A representative consumer's utility function is

$$U = u(q_2) + q_1, \tag{2.1}$$

with $u(q_2) = v\, q_2 - (q_2)^2/2$. The quadratic structure of utility means that demand for the sector 2 good is linear in the price, $d_2(p) = v - p$.

Production. Production of the sector 2 good is linear in the amount of capital the sector employs $Q_2 = k_2$. The marginal product of capital in sector 2 is then just p. Production of the sector 1 good is constant returns to scale (and therefore diminishing returns to capital): $Q_1 = F(k_1, L)$.

Both sectors are perfectly competitive. The home country is endowed with K units of capital and L units of labor.[8]

Social welfare. The social welfare function is the sum of factor income, tariff revenue, and consumer surplus. Producer surplus is not in the welfare equation because production is competitive. The full social welfare function is

$$\begin{aligned} W(p, k_1, k_2) = {} & \underbrace{F_2(k_1, L) * T}_{\text{Labor Income}} + \underbrace{F_1(k_1, L) * (k_1) + p * k_2}_{\text{Capital Income}} \\ & + \underbrace{r(p, k_2)}_{\text{Tariff Revenue}} + \underbrace{\{u(d(p)) - p * d(p)\}}_{\text{Consumer Surplus}}. \end{aligned} \tag{2.2}$$

Tariff revenue is defined by the equation $r(p, k_2) \equiv (p - p^*)\Big(q_2 - Q_2\Big) = (p - p^*)\Big(d(p) - k_2\Big)$, where $d(p) - k_2$ (domestic demand minus domestic production) is the excess demand for the sector 2 good, imported from abroad.

Political structure. Suppose the capital owners in sector 2 exogenously organize to form a lobby. In sector 1, neither workers nor capital owners are organized. The lobby collects contributions proportionate to the amount of capital in the manufacturing sector

[8] In studying the effect of lobbying on capital allocations, k_2 is the capital allocation in the politically-organized sector and as a result is the variable of interest.

(all capital owners in the sector contribute). Denote the contribution capital owners in sector 2 pay per unit of capital as c. The total contribution the government receives in exchange for granting tariff protection to sector 2 producers is $c\,k_2$.

The lobby's objective is to maximize the net revenue of the capital owners, which given that capital owners are a negligible fraction of the population, is equal to $(p - c)\,k_2$. The government's objective is to maximize social welfare and political contributions from the lobbyists: $G = W(p, k_2) + a\,c\,k_2$, where a is the government's politically-weighted valuation of lobby contributions. The government is politically motivated to protect sector 2 if $a > 0$.

Trade policy options. The government chooses it trade policy from four options:

(i) *No Trade Agreement* (*"NoTA"*): The government chooses to not join any trade agreement, the resulting tariffs represent the Nash political outcome.

(ii) *Rigid Free Trade Agreement* (*"FTA"*): The government chooses to join a trade agreement in which it agrees to set the tariff at zero, regardless of the world prices.

(iii) *Free Trade Agreement with Escape Clause* (*"EC"*): The government chooses to join a trade agreement in which it sets the tariff at zero (as was the case in the rigid trade agreement), unless the world prices are such that the country's terms of trade are poor, in which case it may use a temporary "escape" in which the government sets a higher domestic price.

(iv) *Tariff-Binding Trade Agreement* (*"Binding"*): The government joins a trade agreement in which the tariff level is bound prior to the realization of the world prices. In this scenario the government does not have access to an escape clause. This will allow the government to commit to the maximum level of protection ex ante and alleviate some of the pressure to protect domestic industries.

Timing of the model. The following list characterizes the timing of the model for each trade policy the government can choose:

S0: The government chooses which of the four policy options to implement based on a problem maximizing domestic welfare. If the government chooses to join a binding TA, then it will also set a tariff binding, t^{Max}.

S1: Capital owners allocate capital to each sector. Because the investors are small, they are assumed to behave nonstrategically. The decisions are summarized by k_2, given $K = k_1 + k_2$.

S2: The world relative price, p^*, is revealed.[9]

S2(b): (*EC Only.*) Given the government chose to join an FTA with an escape clause, if the world price was revealed to be low in stage 2, then the government decides whether or not to evoke the escape clause.

S3: The government determines the domestic prices based on any trade agreement rules and the lobby pressures they face from their domestic industry. This stage takes three general forms:

FTA/EC: If the government joined the rigid FTA or if it joined the FTA with an escape and the world price was high, then it will set $p = p^*$ and the lobby will not make any contribution to the government.

NoTA/EC: If the government did not join a trade agreement or if it joined an FTA with an escape clause and evoked the EC in stage 2(b), then organized industries lobby the government for protection. The negotiations between the lobby and the government are assumed to take the form of Nash bargaining, in which the government has bargaining strength γ and the lobby's bargaining strength is $1 - \gamma$. The threat point for bargaining is no contribution from the lobby and domestic prices equal to world prices.

[9]Assume that the low realization of p^* is such that $a \geq \big(2(F_1(0, L) - p_L^*)\big)/\big(K(1-\gamma)\big)$. This ensures that the political equilibrium allocations for either realization of the price will be interior.

Binding: If the government joined a tariff-binding trade agreement, then Nash bargaining will occur the same as in the previous point, but with the additional constraint that the the protection level cannot exceed the agreed to binding level, t^{Max}. The government will then set $p = p_j^* + t$.

The rest of the paper proceeds through the solution to each trade agreement type, solving for the optimal resource-allocation and tariff-setting problems by backward induction. I solve the simplest version of the model first – the tariff and production equilibria when the government commits to a strict free trade agreement – in Section 2.3. Then in Section 2.4, I solve the second of the two original trade policy options from the Maggi and Rodríguez-Clare (1998) model: the model with no commitment to a trade agreement. I also demonstrate that in expectation, the government's trade policy preference without uncertainty is the same as with uncertainty as long as the distribution of the world price is mean-preserving. The discussion of No TA versus FTA is located in Appendix B.1.

Then, I proceed to one of the new trade policy options: in Section 2.5, I solve for the optimal tariff and resource allocation when the government joins an FTA with an escape clause. In Section 2.6, I discuss the the government's trade policy preference when it can choose between the first three trade policies, FTA, No TA, and FTA with EC. I show that the flexibility of the escape clause benefits governments with political-economy parameters or bargaining strengths that fall near the border of the FTA and No TA regions, drawing some countries that were in the FTA region and some from the No TA region. Finally in Section 2.7, I solve for the optimal tariff and resource allocations when the government joins a tariff-binding trade agreement, and in Section 2.8, I show that if the small country could ensure its tariff bindings were enforced by WTO partners, then it would prefer to use a tariff binding over using an escape clause to achieve flexibility in its tariff levels.

Walking through the model, one trade policy at a time, I will build my argument for the co-existence of commitment theory and flexibility in the context of trade agreements. I will start with a brief comparison between the free trade agreement and not joining a trade agreement. This solution will be key in showing how the government's preferences change with the addition of escape clauses, and it will allow me to establish that the results from Maggi and Rodríguez-Clare (1998) are unchanged by the addition of uncertainty over world prices to the model. An expanded version of this section is in Appendix B.1.

2.3 Ex-Ante Commitment to Free Trade Agreement

If the government joins a rigid FTA, then it sets the domestic price exactly equal to the realization of the world price. Given the government's commitment and the uncertainty over the price, capitalists will allocate capital to equalize the expected returns in each industry at the world price. This simplifies to them setting k_1 such that $F_1(K-k_2, L) = E[p^*]$. Call the allocation that satisfies this equation k_2^*.[10]

2.4 Politically Optimal Tariffs and Domestic Lobbying

Suppose now that the government chooses not to commit to free trade.

Stage 3. Suppose the capital allocations are fixed and all uncertainty has been resolved, with $p^* = p_j^*$, $j \in \{H, L\}$. The outcome of the Nash bargaining, $\widetilde{p}(k_2; p_j^*)$ and $\widetilde{c}(k_2; p_j^*)$, must satisfy two equations. First, $\widetilde{p}(k_2; p_j^*)$ must maximize the joint surplus of the lobby and the government, $W(p, k_2) + a(c\,k_2 + (p - c)k_2)$ for all k_2, where the surplus for the lobby and the contributions from the lobby are weighted by a. The resulting equation is the government's best response price-setting strategy for any k_2, given p_j^*:

$$\widetilde{p}(k_2; p_j^*) = p_j^* + a\,k_2. \tag{2.3}$$

[10]Assume that the free trade capital allocation is interior – the equilibrium amounts of capital allocated to each sector are nonzero regardless of the world price realization. To ensure this, assume that $F_1(K, L) < p_L^* < p_H^* < F_1(0, L)$.

This is the ex-post price-setting schedule for the government, taking the capital allocation k_2 as given.[11] The second half of the political equilibrium is the contribution schedule. The lobby's contribution, $\widetilde{c}(k_2; p_j^*)$, is the solution to the Nash bargaining problem, where the threat point is $p = p_j^*$ and $c = 0$. It is straightforward to show that the solution is

$$\widetilde{c}(k_2; p_j^*) = \frac{(1-\gamma)(W(p^*, k_2) - W(p, k_2))}{a\,k_2} + \gamma\,(p - p_j^*) = \left(\frac{1+\gamma}{2}\right) a\,k_2. \tag{2.4}$$

The optimal contribution of the lobby for any given level of investment k_2 is a weighted average of the lost welfare due to tariff protection and the gain in welfare the capitalists receive from protection (the change in the prices).

Stage 1. Now, given the best response rules for any given level of investment, the capitalists allocate capital to equalize the expected returns to each industry, given the uncertainty over the world price. If there is no value of k_2 for which the returns are equalized, however, then the capitalists will allocate all the capital in the sector earning the higher return. If there is an allocation of capital such that the expected returns are equalized across sectors, the capitalists will allocate to equalize the returns to each industry.

Define $p^0(k_2)$ as the price at which expected returns, net of contributions, are equalized across sectors given k_2. Continuing with their notation, the equal returns price is given by the equation

$$p^0(k_2) = \left(\frac{1+\gamma}{2}\right) a\,k_2 + F_1(K - k_2, L). \tag{2.5}$$

The intersection of the equal returns line and the expected value of the government's best response price strategy will dictate the ex-ante equilibrium allocation, where the expected price set by the government is $E[\widetilde{p}(k_2)] = \pi(p_H^* + a\,k_2) + (1-\pi)\,(p_L^* + a\,k_2)$. The resulting

[11] It is necessary here to add in a condition on the price and on the demand intercept, v. As Maggi and Rodríguez-Clare pointed out in their paper, due to the linearity of the demand function, a problem arises if the price in sector 2 rises higher than v: the demand for the sector 2 good becomes negative when p surpasses v. Furthermore, they note that this would also imply that, since p is chosen given a fixed k_2, any increase in the price above v would essentially lead to a zero-cost transfer from consumers to the owners of capital in sector 2. The government and lobby then have the incentive to increase the price as much as possible. To prevent these two issues from arising, Maggi and Rodríguez-Clare assume that the feasible values of p are restricted be less than v and that $v > p_H^* + a\,K$, which guarantees the upper bound of the price will never be reached.

ex-ante equilibrium allocation is exactly equivalent to that in Maggi and Rodríguez-Clare (1998), but with the allocation being a function of the expected world price taking the place of the actual.

So, if there exists some k_2 where the expected price is equal to $p^0(k_2)$, then capital will be allocated to each sector. If $E[\widetilde{p}(k_2; p_j^*)] < p^0(k_2)$ for all k_2, then all capital will be located in sector 1. If $E[\widetilde{p}(k_2; p_j^*)] > p^0(k_2)$ for all k_2, then all capital will be located in sector 2. The point at which they are equal, denoted $\widehat{p}$, is the unique interior solution and the ex-ante equilibrium expected domestic price. The resulting allocations can be summarized by

$$k_2(a) = \begin{cases} 0 & \text{if } E[\widetilde{p}(\widehat{k}_2(a); p_j^*)] < p^0(\widehat{k}_2(a)), \\ \widehat{k}_2(a) & \text{if } E[\widetilde{p}(\widehat{k}_2(a); p_j^*)] = p^0(\widehat{k}_2(a)), \\ K & \text{if } E[\widetilde{p}(\widehat{k}_2(a); p_j^*)] > p^0(\widehat{k}_2(a)), \end{cases} \tag{2.6}$$

with $\widehat{k}_2(a)$ being the allocation of capital that satisfies $E[\widetilde{p}(k_2; p_j^*)] = p^0(k_2)$. These results are depicted in Figure 2.1.

Figure 2.1 shows the ex-post relationship between the capital allocation (k_2) and the price (p). Under free trade, the equal-returns allocation is determined by the intersection of $F_1(K - k_2, L)$ and p^*. If the government does not join a trade agreement, then the intersection of the government's best-response price-setting rule $(E[\widetilde{p}(k_2; p_j^*)])$ and the equal returns line $(p^0(k_2))$ determines the Nash/political equilibrium allocation. The figure shows that the presence of lobbying leads to an overallocation of resources to the protected industry, given that the productively efficient allocation is k_2^*. The assumption on production required for Figure 2.1 to be true is that $-F_{11}(K - k_2, L) > \big(p_H^* - F_1(K, L)\big)/K + \big((1-\gamma)/2\big)a$. This ensures that (1) the equal returns outcome falls between zero and K for either realization of the world price, and (2) that there is a unique capital allocation that equalizes returns over the same range. In other words, this assumption implies there will be a unique, interior solution for the Nash allocation, $\widehat{k}_2$.[12]

[12]It is immediately evident that the first finding of Maggi and Rodríguez-Clare (1998) is unchanged:

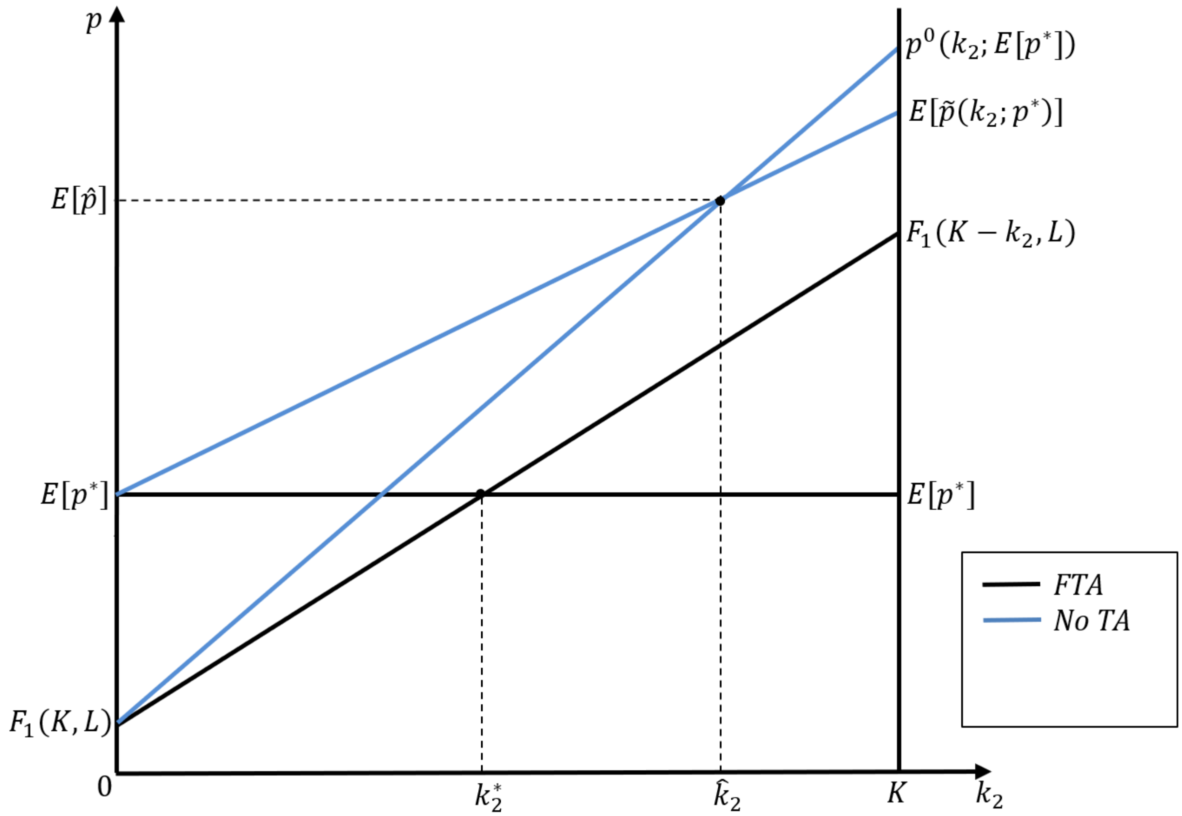


Figure 2.1: Equilibrium capital hiring under free trade or without a trade agreement.

Proposition 2.1. *Given that there is a unique, interior solution such that* $-F_{11}(K - k_2, L) > \Big(p_H^* - F_1(K, L)\Big)/K + \Big((1-\gamma)/2\Big)a$*, the "political" or Nash equilibrium allocation,* $\widehat{k}_2(a) > k_2^*$*, is decreasing in* γ*. Furthermore,* $\widehat{k}_2(a) > k_2^*$ *for all* $\gamma < 1$ *(and* $\widehat{k}_2 = k_2^*$ *for* $\gamma = 1$*). The expected price without the free trade agreement,* $E[\widehat{p}(k_2; p^*)]$*, is strictly greater than* $E[p^*]$*, and is decreasing in* γ*, which is easy to see given the equal returns equation (equation (2.5)). These allocations, given that the distribution of the world price is mean-preserving, are identical to the certainty equivalent in Maggi and Rodríguez-Clare (1998).*

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the level of overinvestment in sector 2, even under uncertainty over prices, is decreasing in the bargaining strength of the government, γ. Graphically, this is evident because for higher levels of γ, the slope of the equal returns line will increase, leading to a drop in the equilibrium level of investment in sector 2. Analytically, this can be shown by implicitly differentiating the equal returns outcome with respect to sigma, which gives $\partial k_2/\partial\gamma = (a\,k_2)/(2F_{11}(K - k_2, L) - (1-\gamma))$, where denominator is greater than zero given the assumption of the unique, interior solution. Notice also that, because the uncertainty over the world price is a mean-preserving spread from the certainty equivalent, the allocations determined by Figure 2.1 are the same as in Maggi and Rodríguez-Clare (1998).

2.5 Ex-Ante Commitment to a Free Trade Agreement with Escape Clause

Suppose now that the government has chosen to join a trade agreement with an escape clause. The escape clause is subject to a use constraint: the government can only evoke the escape clause if the country's terms of trade deteriorate enough below a certain level. In my model there are only two potential realizations of the world price, so "if terms of trade deteriorate enough" is equivalent to the escape clause being available if the realization of the world price is $p^* = p^*_L$.

Solving the model with an escape clause requires one main addition to the FTA and No TA solution methods from Sections 2.3 and 2.4: the government is facing one of two potential paths on the outset: if world prices are high $(p^* = p^*_H)$, then the government does have access to the escape clause; if world prices are low $(p^* = p^*_L)$, then the government can choose if it wants to use the escape clause or not. When p^*_H is realized, the solution follows the FTA solution from Section 2.3, and ultimate outcome is free trade.

The model only substantially changes if the price realization is p^*_L. Therefore, I solve stages 2 and 3 below for $p^* = p^*_L$. Then in stage 1, I solve for the equilibrium capital allocation, $\widehat{k}_2^{EC}$, and show how it compares to capital allocations k^*_2 and $\hat{k}_2$.

2.5.1 Model Outcome when the World Price is Low

Suppose that the world price realized is p^*_L. Thus, the government will be able to use the escape clause if it chooses.

Stage 3. If the government evokes the escape clause, then $\widetilde{p}(k_2; p^*_L)$ will be the solution a maximization problem very similar to the one period model:

$$\widetilde{p}(k_2; p^*_L) = \arg\max_p W(p, k_2^{EC}; p^*_L) + apk_2^{EC},$$

with the EC superscript used to denote that the government is in a trade agreement with an escape clause. The result of this maximization problem is unchanged from Section 2.4. The ex-post price is $\widetilde{p}(k_2^{EC}; p_L^*) = p_L^* + a\, k_2^{EC}$.

It is then straightforward to show that the second half of the Nash bargaining solution is unchanged as well. Therefore, the Nash bargaining solution is $[\widetilde{p}(k_2^{EC}; p_L^*), \widetilde{c}(k_2^{EC})]$, with $\widetilde{p}(k_2^{EC}; p_L^*) = p_L^* + a\, k_2^{EC}$ and $\widetilde{c}(k_2^{EC}) = \Big((1+\gamma)/2\Big) a\, k_2^{EC}$.

If the government does not evoke the escape clause in stage 2(b), then it implements $p = p^*$ in accordance with the trade agreement and the lobby contributes nothing.

Stage 2(b). The government's choice will be determined based on the price realization and the allocation of capital to each sector. The government will evoke the escape clause if $G(\widetilde{p}(k_2; p_L^*), k_2)$ is larger than $G(p_L^*, k_2)$:

$$G(\widetilde{p}(p_L^*, k_2), k_2) - G(p_L^*, k_2) > 0,$$
$$\frac{\gamma}{2}\, a^2 (k_2)^2 > 0.$$

When the government does not evoke the EC, it is clear that the same problem from the original model arises: they are implementing free trade without an ex ante commitment, leading the government to always prefer to evoke the escape clause when it has access to it. From here forward, I will take it as given that if the government joins a trade agreement with an escape clause, that it will evoke the escape clause if the world price is p_L^*. This results from all factors of production being held constant ex-post, leaving the government to choose whether to evoke the escape clause or not without having any ability to affect the overall production in each industry. Therefore, all they can effect in the returns equation is the size of the political rents of granting protection (as shown in equation (2.7)), which will be greater than zero if they have any bargaining strength and if they value political contributions at all.[13]

[13]If they have no bargaining strength or if they do not value political contributions, however, they are indifferent between evoking and not evoking the escape clause. To simplify the solution, however, I will assume that when they are indifferent between the two, they will evoke the EC.

Stage 1. The expected value allocating one unit of capital to sector 2, equal to $F_1(K - k_2, L)$, must equal the expected value of allocating it to sector 1, equal to $p - (1-\pi)\,\widetilde{c}(k_2)$. The equal returns line for the capitalists when they are in a trade agreement with an escape clause is given by the equation

$$p^{0EC}(k_2) = F_1(K - k_2, L) + (1-\pi)\left(\frac{1+\gamma}{2}\right) a\, k_2, \tag{2.7}$$

which is not directly dependent on the realization of the world price. The capitalists will be comparing this to the price they expect the government to set, $E[p] = \pi(p_H^*) + (1-\pi)(\widetilde{p}(k_2; p_L^*)) = E[p^*] + (1-\pi)\, a\, k_2$.

It is easy to show that the allocation when the escape clause is evoked is strictly decreasing in γ, given the existence of a unique, interior solution. The behavior of the government's price-setting rule is unchanged from Section 2.2.

Proposition 2.2. *Given that there is a unique, interior solution such that* $-F_{11}(K - k_2, L) > (p^* - F_1(K, L))/K + \big((1-\gamma)/2\big)a$*, the equilibrium allocation,* $\widehat{k}_2^{EC}$*, and the price when the escape clause is evoked,* $\widehat{p}^{EC}$*, are decreasing in* γ*. And in relation to the other capital allocations:*

(*a*) $\widehat{k}_2^{EC} > k_2^*$ *for all* $\gamma < 1$ (*and* $\widehat{k}_2^{EC} = k_2^*$ *for* $\gamma = 1$)*. The price when the escape clause is evoked,* $\widehat{p}^{EC}$ *is strictly greater than* p^**, and is decreasing in* γ*, which is easy to see given the equal returns equation* (*equation* (2.7))*.*

(*b*) $\widehat{k}_2^{EC} < \widehat{k}_2$ *for all* $\gamma < 1$ (*and* $\widehat{k}_2^{EC} = \widehat{k}_2$ *for* $\gamma = 1$)*. The price when the escape clause is evoked,* $\widehat{p}^{EC}$ *will be smaller than* $\widehat{p}$*. As it is depicted in Figure 2.2, due to the fact that the allocation,* k_2^{EC} *is smaller than under no trade agreement.*

As when the government does not join a trade agreement, the government's welfare when it joins a trade agreement with an EC, $G(\widehat{p}(\widehat{k}_2^{EC}; p^*), \widehat{k}_2^{EC})$, is increasing in γ (i) directly through the term $(1-\pi)\frac{\gamma}{2}(a\widehat{k}_2^{EC})^2$, and (ii) indirectly through $\widehat{k}_2^{EC}$. More analysis will be done on the behavior with respect to γ in Section 2.6. For now, it is enough to point out that

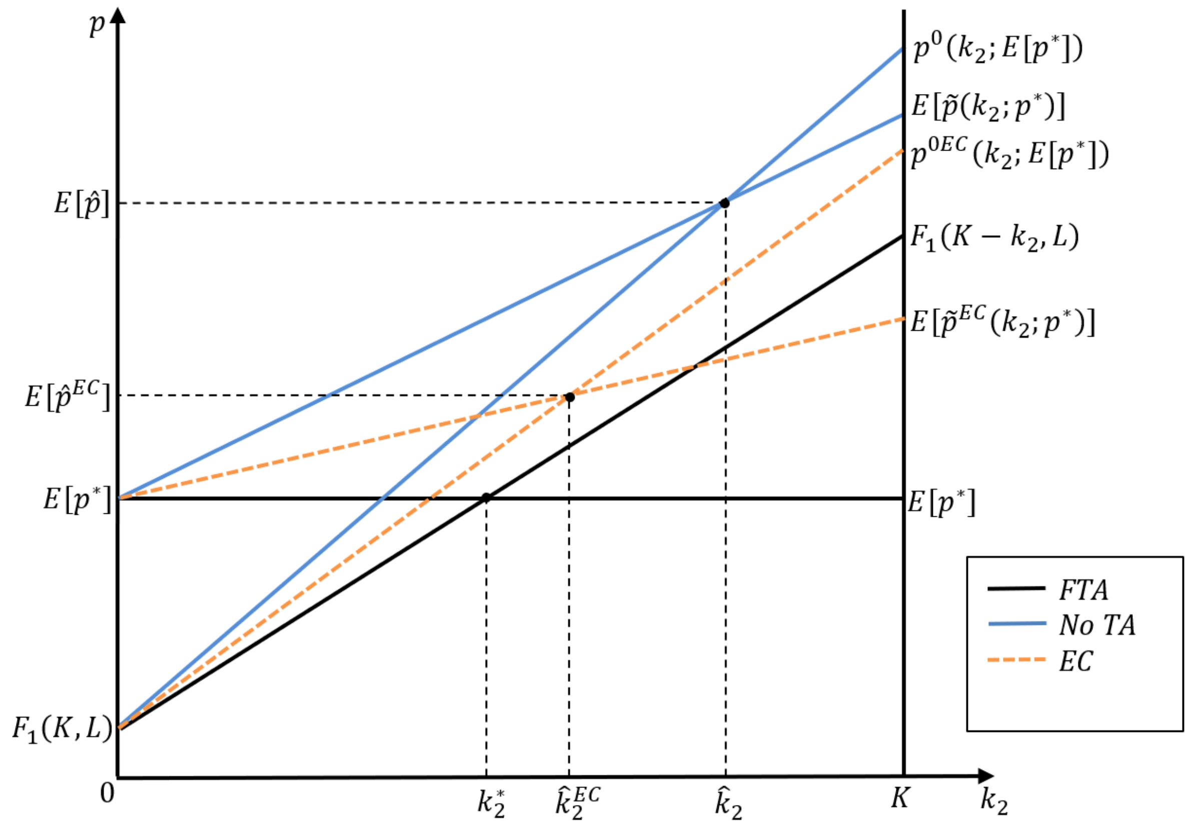


Figure 2.2: Equilibrium capital hiring under free trade, with an EC agreement, and without a trade agreement.

the behavior is similar to that of when the government does not join a trade agreement, leading to a alternative Proposition B.1:

Proposition 2.3. *There exists some cutoff level of bargaining strength,* $\overline{\overline{\gamma}}$*, such that if and only if* $\gamma < \overline{\overline{\gamma}}$, $E[G^c(p^*, k_2^*)] > E[G(\widehat{p}(\widehat{k}_2^{EC}; p^*), \widehat{k}_2^{EC})]$ *for all* a *will be true.*

Similarly to the case with no trade agreement, when $\gamma > \overline{\overline{\gamma}}$, the size of a becomes an important determinant of the government's decision. Without going through all of the derivations again, the behavior of $E[G(\widehat{p}(\widehat{k}_2^{EC}; p^*), \widehat{k}_2^{EC})]$ can be summarized by the following version of Proposition B.2:

Proposition 2.4. *For small* γ, $E[G(\widehat{p}(\widehat{k}_2^{EC}; p^*), \widehat{k}_2^{EC})]$ *is nonmonotonic: it reaches a minimum at some value of* a*, call it* a^{**}*, and then becomes greater than* $E[G^c(p^*, k_2^*)]$ *at some value of* a*, call it* $\widehat{\widehat{a}}$*,* $a^{**} < \widehat{\widehat{a}}$*.*

2.6 Comparing Government Welfare for No TA, FTA, and FTA with EC.

Now I can turn to a closer examination of when the government will choose each policy from the first three choices considered: not joining a trade agreement, joining a rigid free trade agreement, or joining a free trade agreement with an escape clause. How the government values political contributions, a, and the government's bargaining strength, γ, each play an important role in determining the government's choice of trade policy. The relationship between the government's decisions and these parameters will build off of those already derived in Appendix B.1 regarding the choice between committing to free trade and no trade agreement, and in Section 2.5.1 regarding the choice between using an escape clause and committing to free trade.

To decide on which policy to implement, the government will compare the expected welfare functions of all three. The expected welfare equations are

$$E[G^c(p^*, k_2^*)] = W(E[p^*], k_2^*) + \frac{1}{2}\text{Var}(p^*) \equiv G^{FTA},$$
$$E[G(\widetilde{p}(\widehat{k}_2; p^*), \widehat{k}_2)] = W(E[p^*], \widehat{k}_2) + \frac{1}{2}\text{Var}(p^*) + \frac{\gamma}{2}a^2(\widehat{k}_2)^2 \equiv G^{NoTA}(a), \text{ and}$$
$$E[G(\widetilde{p}(\widehat{k}_2^{EC}; p^*), \widehat{k}_2^{EC})] = W(E[p^*], \widehat{k}_2^{EC}) + \frac{1}{2}\text{Var}(p^*) + (1-\pi)\frac{\gamma}{2}a^2(\widehat{k}_2^{EC})^2 \equiv G^{EC}(a).$$

To simplify notation, I dropped the dependence of $\widehat{k}_2$ and $\widehat{k}_2^{EC}$ on a. I have also assigned each equation a new name to simplify the notation. New notation in this section includes a new cutoff of a: the value of a for which $\widehat{k}_2^{EC}(a) = K$ will be referred to as $a_{\widehat{k}_2^{EC}}$. Given that $\widehat{k}_2(a) > \widehat{k}_2^{EC}(a)$ for all a, it must be true that $a_{\widehat{k}_2} < a_{\widehat{k}_2^{EC}}$.

First, consider the effect of γ on the welfare functions. For $\gamma = 0$, the government will strictly prefer a free trade agreement, with $G^{FTA} > G^{EC} > G^{NoTA}$. For $\gamma = 1$, the government will strictly prefer to not join a trade agreement, with $G^{NoTA} > G^{EC} > G^{FTA}$. This will be true both ex-ante and ex-post.

Now, consider the role of the political-economy parameter, and its relation to the bargaining strength, in the government's decision of trade agreement type. The following results are relatively straightforward to derive and are (for the most part) derived explicitly in Appendix B.2:

1. As the size of γ increases, the difference between $\widehat{k}_2(a)$ and $\widehat{k}_2^{EC}(a)$ decreases for all a (but with $\widehat{k}_2(a) > \widehat{k}_2^{EC}(a)$ remaining true). This is evident given that the equal returns lines, p^0 and p^{0EC}, get closer together for all a as γ increases.

2. For small enough γ, G^{NoTA} and G^{EC} are decreasing and less than G^{FTA} in the right neighborhood of $a = 0$. Furthermore, $G^{EC} > G^{NoTA}$ in the right neighborhood of $a = 0$. This indicates that for any γ small enough such that $G^{EC''}(0) < 0$, the government welfare for each type of trade agreement is ordered $G^{FTA} > G^{EC} > G^{NoTA}$ in the right neighborhood of $a = 0$.

3. For large enough γ, $G^{NoTA}(a)$ and $G^{EC}(a)$ are increasing in the right neighborhood of $a = 0$, with $G^{NoTA''}(0) > G^{EC''}(0)$. Furthermore, it is also evident that for large enough γ, $G^{NoTA'}(a) > G^{EC'}(a) > 0$ for all $a > 0$. Thus, the government welfare for each type of trade agreement is ordered $G^{NoTA} > G^{EC} > G^{FTA}$ for all $a > 0$ when γ is large.

4. If $\gamma > 0$, for large enough a, $G^{NoTA'}(a) > G^{EC'}(a) > 0$.

5. When $\gamma > 0$ is small enough, starting with the order $G^{FTA}(a) > G^{EC}(a) > G^{NoTA}$:

   (i) $G^{NoTA}(a)$ and $G^{EC}(a)$ must each reach a minimum at some point between $a = 0$ and $a = a_{\widehat{k}_2^{EC}}$;

   (ii) $G^{NoTA}(a)$ and $G^{EC}(a)$ must then start increasing in a and become greater than G^{FTA} by the point which $a = a_{\widehat{k}_2^{EC}}$;

   (iii) once $a \geq a_{\widehat{k}_2^{EC}}$, the order must be $G^{NoTA}(a) > G^{EC}(a) > G^{FTA}$.

6. Suppose $\gamma > 0$ is small enough such that $G^{NoTA''}(0) < 0$ but large enough such that $G^{NoTA''}(0) > 0$. In the right neighborhood of $a = 0$, $G^{EC}(a) > G^{FTA} > G^{NoTA}(a)$. For

any value of a, the government will never prefer to join a free trade agreement and $G^{EC}(a) > G^{FTA}$. $G^{NoTA}(a)$ will reach a minimum at some point between $a = 0$ and $a = a_{\widehat{k}_2}$. For any $a \geq a_{\widehat{k}_2^{EC}}$, $G^{NoTA}(a) > G^{EC}(a) > G^{FTA}$.

Proposition 2.5. *For small γ, the government will join an FTA for any a below $\min\{\widehat{\widehat{a}}, \widehat{a}\}$. Then, if $\widehat{\widehat{a}} > \widehat{a}$, it will not join a trade agreement for any $a > \widehat{a}$. If $\widehat{\widehat{a}} < \widehat{a}$, it will join a trade agreement with an EC for any $a^c > a > \widehat{\widehat{a}}$ and it will not join a trade agreement if $a > a^c$. For moderate $\gamma > 0$, the government will never prefer to join an FTA. It will join with an EC for any value of a lower than some level, call it a^c and it will not join a trade agreement for $a > a^c$. For γ large enough, the government will prefer not to join a trade agreement for all a.*

The relationship between the government's welfare and the values of the parameters a and γ is depicted in Figure 2.3. The graph is of a specified model, in which production of the sector 1 good is Cobb-Douglas, with the output elasticity of capital equal to ζ.[14]

It is important to note that due to the structure of the model, any benefit to the government from having access to an escape clause is not contingent on when the government is permitted to use it: Figures 2.4a and 2.4b demonstrate that the (a, γ) combinations for which the government chooses each type of trade agreement are unchanged if the escape clause is used when $p^* = p_L^*$ or $p^* = p_H^*$, respectively.[15]

The benefit the government receives from the addition of the escape clause is a result of the restricted use of the escape clause. This result indicates that the model is not providing insight as to why the government would join a trade agreement with an escape clause, specifically. Instead, the model is showing that the government would like to have some intermediate tool, where it is able to commit to protection that is higher than free trade and lower than the Nash outcome. The benefit of such a tool being that the allocation distortion decreases, meaning that the amount of the rents that the capital owners are able to capture

[14] Figure 2.4 has more detailed information about the specification used. The graph here simply depicts the general result. It also includes the cutoff value on each axis as a point of reference for the reader.

[15] For the numerical solution, the endowments of capital and labor $K = 1$ and $L = 1$. The maximum feasible political-economy weight is $amax = 100$. The prices are $p_L^* = 1/2$ or $p_H^* = 7/6$. Production is Cobb-Douglas with $F(k, l) = (k)^{1/2}(l)^{1/2}$.

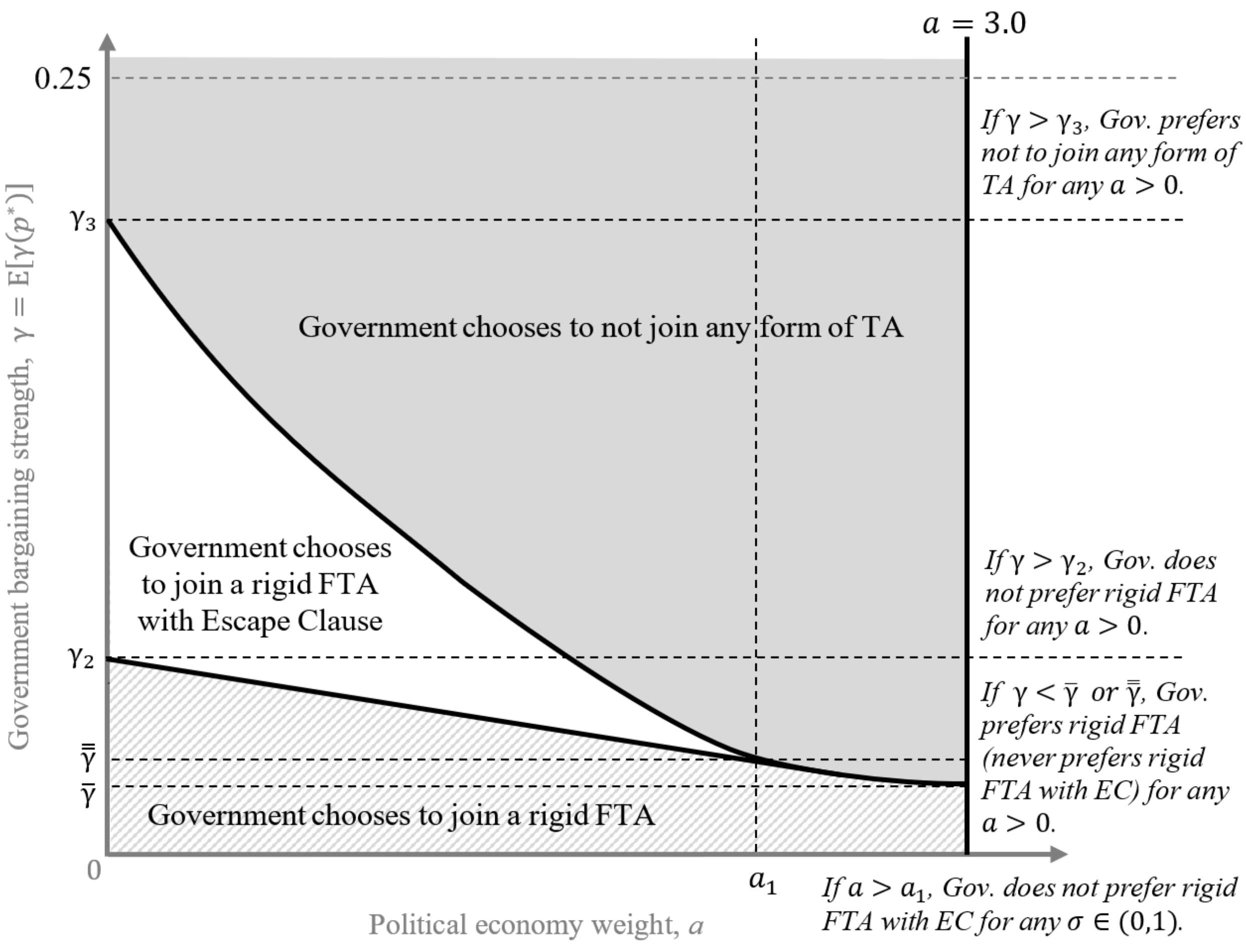


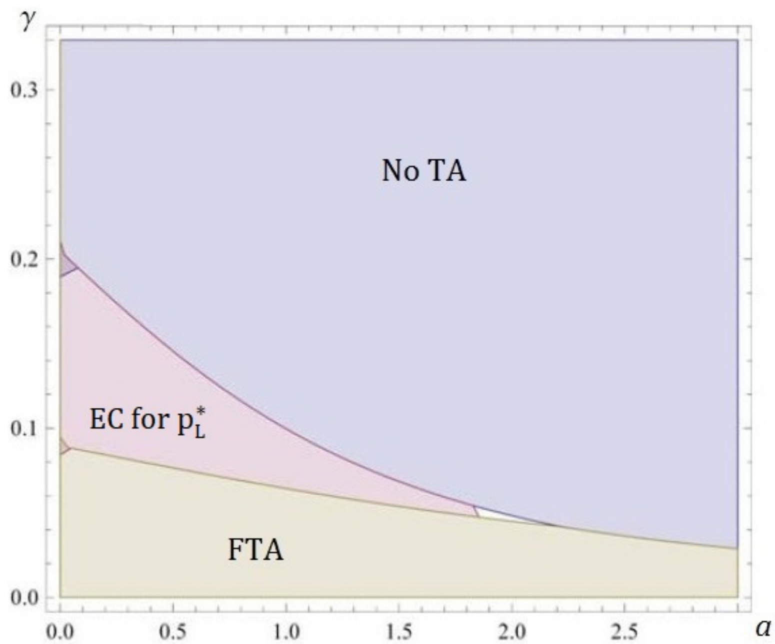
Figure 2.3: Government preferences between trade agreements for values of a and γ.

through the ex ante distortion is smaller. In the next section, to further illustrate this point, I will give the government the option of joining a trade agreement with a nonzero tariff, and then show that they do not value having access to the escape clause option when they are in such an agreement.

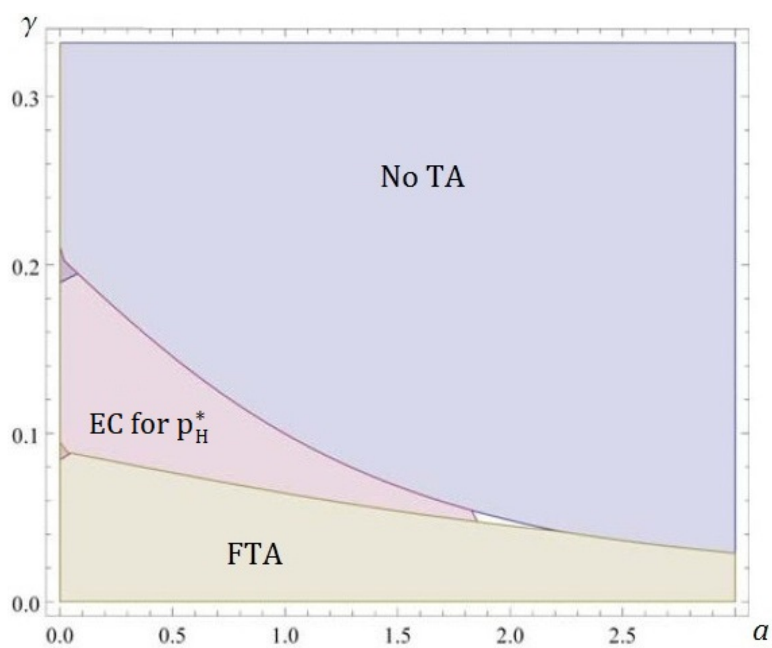
2.7 Ex-Ante Commitment to Tariff Level

Stage 3. The government and the lobby will bargain over the prices and contributions, given the that the protection level has been bound as some level, t^{Max}, during the initial stage. Therefore, the government's price-setting rule is to set $p_j = p_j^* + t$ for $j \in \{H, L\}$, where $t \leq t^{Max}$. The tariff rule will be the value of t that maximizes the joint surplus for

Figure 2.4: Government choice between FTA, FTA with an EC, or No TA. The political-economy weight, a, and bargaining strength, γ, combinations for which a government chooses to join a free trade agreement (FTA), join a flexible trade agreement (EC), or not join a trade agreement (No TA). The (a, γ) combinations for which the government prefers EC are identical whether the EC is used when world prices are low (which is the standard structure of escape clause use) or if the EC is used when world prices are high.



(a) Escape clause can only be evoked if the world price is low.



(b) Escape clause can only be evoked if the world price is high.

the lobby and the government. In the same vein as the escape clause use in Section 2.5.1, the government at the point in which the tariff level, t, is decided will always choose to implement $t = t^{Max}$. Therefore, it will be assumed that there is never any excess space between the tariff binding and the applied tariff in this model.

The lobby will then pay the government according to the contribution rule derived from Nash bargaining,

$$\widetilde{c}^t(k_2, t) = \left(\frac{1+\gamma}{2}\right)\left(\frac{t^2}{\gamma t + (1-\gamma) a\, k_2}\right), \tag{2.8}$$

where the t superscript is used to denote that the trade policy in place is a tariff-binding trade agreement.

It is worthwhile to note that the division of the rents from tariff protection resulting from Nash bargaining has a very similar structure to divison of rents from previous sections. As an illustration, suppose that the model parameters are such that politically optimal tariff is $\widehat{t} = a\, k_2$. If this is the case, then the Nash bargaining solution, $(\widetilde{p}^t, \widetilde{c}^t)$ is identical to $(\widetilde{p}, \widetilde{c})$ (the outcome when the government does not join a trade agreement).

Stage 2. The world relative price, p_j^*, $j \in \{H, L\}$, is revealed.

Stage 1. Capital is allocated such that the expected domestic price of the good, $E[p^t] = \pi\, \widetilde{p}^t(t; p_H^*) + (1-\pi)(\widetilde{p}^t(t; p_L^*)) = E[p^*] + t$, is equal to the ex-ante equilibrium equal returns price,

$$p^{0t}(k_2, t) = F_1(K - k_2, L) + \left(\frac{1+\gamma}{2}\right)\left(\frac{t^2}{\gamma\, t + (1-\gamma) a\, k_2}\right), \tag{2.9}$$

giving some equal returns allocation of capital which depends on the binding tariff level set by the government. For now, rather than solving for this allocation explicitly, I will just refer to this capital level as $k_2^t(t)$ and where $\widehat{k}_2^t = k_2^t(\widehat{t})$ is the optimal capital allocation at the optimal tariff level.

Stage 0. The government chooses the tariff binding t^{Max} to maximize its expected welfare, given that in Stage 3 it will implement tariff level $t = t^{Max}$. The government's decision problem is,

$$\max_t E[G(p^t, k_2^t(t))] = \max_t W(E[p^*], k_2^t(t)) + \frac{1}{2}\,\text{Var}(p^*) + \frac{\gamma}{2}\,t^2\left(\frac{2a\,k_2^t(t) - t}{\gamma t + (1-\gamma)a\,k_2^t(t)}\right) \equiv G^t.$$

The solution to this problem gives some optimal tariff binding level, $t^{Max} = \widehat{t}$. As with the capital allocation, I will not solve explicitly for $\widehat{t}$.

2.8 Comparison of Government Welfare with a Tariff Binding Agreement or an FTA with an Escape Clause

Earlier, I left the allocation of capital and the binding tariff level unspecified. Due to the structure of the model, the explicit solutions for each variable is too complex to be intuitive and intractable, respectively. Due to this limitation of the model, I will compare the government's welfare in the tariff-binding trade agreement to the free trade agreement with an escape clause by assuming a value for $\widehat{k}_2^t$ and $\widehat{t}$, then comparing government welfare at the assumed solutions.

Assume for now that the government chooses a tariff binding such that the capital allocation is the same as if the government had chosen to join the FTA with an EC, where I define t^{EC} as the tariff level that ensures $k_2^t(t) = \widehat{k}_2^{EC}$. If the government welfare at this capital allocation for any value of the model parameters, then it must be true that the government is weakly better off in the binding trade agreement than in the FTA with an EC. In other terms given $k_2^t(t) = \widehat{k}_2^{EC}$, if $t = t^{EC} \neq \widehat{t}$, then the government is strictly better off than in the EC, and if $t = t^{EC} = \widehat{t}$, then the government is indifferent between the two trade policies.

The tariff level necessary to ensure this outcome can be solved for as a function of the capital allocation and model parameters:

$$t^{EC} = \frac{1}{2}\left((1-\pi)\gamma - \left((\gamma(1-\pi)-2)^2 - 4\pi\right)^{1/2}\right) a\,\widehat{k}_2^{EC}. \tag{2.10}$$

To determine whether the government is better off with the flexible trade agreement or the rigid one, examine the difference in the value of the government welfare function, assuming that in the trade agreement with the nonzero binding and the agreement with the escape clause the capital allocation to the protected industry is $\widehat{k}_2^{EC}$:

$$\begin{aligned} G^{EC} &= W(E[p^*], \widehat{k}_2^{EC}) + \frac{1}{2}\text{Var}(p^*) + (1-\pi)\frac{\gamma}{2}\, a^2(\widehat{k}_2^{EC})^2, \text{ and} \\ G^{t} &= W(E[p^*], \widehat{k}_2^{EC}) + \frac{1}{2}\text{Var}(p^*) + \frac{\gamma}{2}\, t^2 \left(\frac{2a\widehat{k}_2^{EC} - t}{\gamma\, t + (1-\gamma)a\widehat{k}_2^{EC}}\right), \end{aligned}$$

where similarly to the definition of G^{EC} established in Section 2.6, $G^t \equiv E[G(p^t, k_2^t(t))]$.

Given the capital allocations, the only difference between the two government welfare equations is now in the final term. The government welfare for each trade policy is compared in a straightforward manner by making use of the definition for t in equation 2.10:

$$\begin{aligned} G^t - G^{EC} = &\left[\frac{\gamma}{2}(1-\pi)\left(a\widehat{k}_2^{EC}\right)^2\right] \\ &- \left[\frac{\gamma}{2}(1-\pi)\left(a\widehat{k}_2^{EC}\right)^2\left(1 + \frac{1}{2}(2-(1-\pi)\gamma) + \left((2-\gamma(1-\pi))^2 - 4\pi\right)^{1/2}\right)\right], \end{aligned}$$

which can easily be shown to be greater than zero for any values of the model parameters $a > 0$, $\pi \in (0,1)$, and $\gamma \in (0,1)$.

Thus, without solving explicitly for the optimal tariff level, $\widehat{t}$, it is clear that the option to impose a nonzero tariff binding is strictly preferable to the government to the ability to use the escape clause.

Proposition 2.6. *Assume there is a unique, interior solution such that* $-F_{11}(K - k_2, L) >$

$\Big(p_H^* - F_1(K,L)\Big)/K + \Big((1-\gamma)/2\Big)a$. *Any* (a,γ) *combination for which the government prefers a free trade agreement with an escape clause over a strict FTA and over no trade agreement* (*i.e.* $G^{EC}(a,\gamma) > G^{FTA}$ *and* $G^{EC}(a,\gamma) > G^{NoTA}(a,\gamma)$), *the government would be made strictly better off by begin able to commit to a tariff binding ex ante* (*i.e.* $G^{EC}(a,\gamma) < G^t(a,\gamma)$).

2.9 Conclusion

In this paper, I address the use of regional trade agreements by small countries in conjunction with WTO membership. My theory focuses on the trade policy preferences of a small country facing uncertainty over world prices and domestic political-economy pressure to protect a lobbying sector. Building on prior work by Maggi and Rodríguez-Clare (1998), I demonstrate that the presence of uncertainty over world prices does not substantially change their results: a government will benefit from committing to a trade agreement when it is either relatively weak at bargaining with lobbyists over rents from protection or when it places little weight on receiving political-economy contributions from lobbyists.

After establishing that the original findings of Maggi and Rodríguez-Clare (1998) remain true, I demonstrate that a slightly more flexible trade policy option is beneficial to governments whose political-economy and bargaining parameters place is near the border of the "join a free trade agreement" region of political-economy parameters initially. The main type of flexibility I consider is a free trade agreement with an escape clause that can only be used if there is a harmful shock to world prices. I show that the and FTA with an escape clause option results in a larger range of governments types will choose to join some form of trade agreement. I also showed that the benefit to small countries of joining a trade agreement with the escape clause is not due specifically to the ability to increase tariff levels when conditions are bad. The benefit of the escape is that it allows the government to commit to a non-zero tariff level in expectation.

This finding is significant because of its application to WTO members and their use of regional trade agreements. Small countries within the WTO cannot commit to lower tariff levels the WTO structure itself: because the country is small, cheating and raising tariffs does not affect the welfare of any other WTO members, and as a result there is no incentive for trading partners to retaliate. Therefore the small country must seek outside means of commitment to lowering tariffs. Commitment by way of a regional trade agreement is a good option, especially given that the WTO places rules on RTAs that are enforceable for all countries: the rules require that tariffs "are eliminated on substantially all the trade between the constituent territories in products originating in such territories" (GATT Article XXIV, paragraph 4(b)).

In the final section of my paper, I showed that in the presence of uncertainty over world prices, a government that is moderately-skilled at negotiating with the domestic lobbyists or that places a moderate-to-low value on receiving political-economy contributions is best off if it can commit ex-ante to a tariff binding, $t > 0$. In other words, if the tariff-binding agreement is an option, the government will never choose an FTA with an escape clause. The use of FTAs with escape clauses, however, demonstrate a second-best "approximation" of the tariff-binding agreement. The only way for a politically-motivated government in the WTO to commit to a positive tariff level is to join a regional trade agreement with an escape clause, which gives it a positive tariff in expectation.

A Commitment and Trade Agreements with Monopolistic Competition and Costly Lobby Formation

A.1 Cooperative Lobbying: Deriving Lobbying Cost, $\theta(\cdot)$, using Shapley Values

Given the key mechanism behind the ex-ante production distortion from lobbying is the firm's upfront lobbying cost, $\theta(k_m)$, in this section I micro-found the behavior of the function. Consider a cooperative bargaining model in which all firms in a lobbying industry must contribute to pay the upfront lobbying cost. Using the concept of the Shapley value, I show how a change in the capital hired by a firm impacts the share of lobbying costs the firm is responsible for paying. If an increase in the capital employed by a given firm leads to a more-than-linear increase in the share of upfront lobbying firms it must pay, then this would be evidence that $\theta(\cdot)$ should be convex.

To find the Shapley value, suppose that the number of firms in a representative monopolistically competitive sector, n, represents a discrete number of firms in the industry. Denote by $\mathcal{N}$ the set of all firms in the sector. Firms in the sector must coordinate their lobbying efforts to communicate to the government what they are willing to contribute as a group to secure a higher level of tariff protection. Anecdotally, suppose that the firms in the industry form a lobbying coalition, with one representative for the industry conveying offers to the government for the purposes of the negotiation. In order to allow each firm to convey their offer to the industry lobbyist, the industry must construct a building in which the meetings can occur. The upfront cost of lobbying, therefore, represents the cost to build such a building. Assuming each firm takes turns to meet with the industry lobbyist (as opposed to all lobbyists meeting at once), the size of the building need only be large enough to accommodate the largest lobbying contingent. Therefore, the share of the cost paid by firm m, θ_m, reflects the size of the lobby it employs. This construct is similar to the cost-sharing airport-building model in Littlechild and Owen (1973), in which firms must pay an airport to help build a runway, and the cost of building a runway is shared across the flights which will use the airport, given the type of airplane and the number of flights which will land on the runway.

Building on Littlechild and Owen (1973), define the following discrete example of cost sharing. Suppose that each firm m which intends to lobby the government will hire some number of lobbyists to lobby on the firm's behalf, which costs $t(k_m)$. Suppose also the cost of hiring lobbyists is monotonic and increasing in the number of lobbyists employed, $t'(k_m) > 0$, for all $k_m \in [0, \infty]$, with $t(0) = 0$. Without loss of generality, assume the index representing the types of firms $i = 1, \ldots n^{k-type}$ are ordered so that $0 = t(k_0) < t(k_1) < \ldots < t(k_{n^{k-type}})$. There are subgroups of firm "types", divided by the amount of capital which each firm employs, k. There are n^{k-type} total types of firms. The set of firms that are type i is $\mathcal{N}_i$, which is composed of n_i total firms. The cost of hiring lobbyists for firm m is given by the function $T(k_m) = \max_i\{t_i\}$, given $\mathcal{N}_i \cap \mathcal{S} \neq \emptyset$, and with $T(\emptyset) = 0$.

The Shapley value is the vector of the n real-valued numbers which defines the amount of the total $T(\mathcal{N})$ paid by each of the n firms, if all n firms join the lobbying coalition. Given the Shapley value is $\phi(T) = (\phi_1(T), \ldots, \phi_n(T))$, the entry for firm $m \in \{1, \ldots, n\}$ is given by the following formula:

$$\phi_m(T) = \sum_{\substack{\mathcal{M} \subseteq \mathcal{N} \\ m \in \mathcal{M}}} \frac{(|\mathcal{M}| - 1)!(n - |\mathcal{M}|)!}{n!} [T(\mathcal{M}) - T(\mathcal{M} - \{m\})], \text{ for } m \in \mathcal{N}, \tag{A.1}$$

with $|X|$ defined as the number of elements in set X, and with $\sum_{m \in \mathcal{N}} \phi_m(T) = T(\mathcal{N})$. This can then be simplified using the definition of $T(\mathcal{M})$,

$$\phi_m(T) = \sum_{k=1}^{i} \frac{t(k_k) - t(k_{k-1})}{r_k}, \text{ for } m \in \mathcal{N}_i,\ i = 1, \ldots, n. \tag{A.2}$$

To find the behavior of the Shapley value, I look at what happens when firm m moves from one size group, i, to the adjacent firm size group, $i + 1$. I assume that any feasible firm size is represented by a subset of firms. In other words, any possible k is some k_j with $j \in \mathcal{N}_i$, with n_i firms total of size k_m. I then see what happens to ϕ_m if firm m increases its size from k_i to k_{i+1}. If firm m increases its size from k_i to k_{i+1}, with m now in $\mathcal{N}_{i+1}$, therefore decreasing n_i by one and increasing n_{i+1} by one, the Shapley value for firm m is now $\phi'_m(T) = \sum_{k=1}^{i+1} \Big(t(k_k) - t(k_{k-1}) \Big) / r'_k$. Therefore, the difference between the Shapley value for firm m after increasing in size from k_i to k_{i+1},

$$\phi'_m(T) - \phi_m(T) = \frac{t(k_i) - t(k_{i-1})}{\Big(\sum_{h=i+2}^{j} n_h \Big) + n'_{i+1}}. \tag{A.3}$$

Given the ordering of the elements of k, the above equation is positive. Therefore, this discrete approximation of the model predicts that the cost to firm m of lobbying is increasing in the capital hired by a firm. To approximate the second order condition, compare the above to firm m shifting from i to $i+1$: $\phi'_{m2}(T) - \phi_{m2}(T) = (t(k_{i+1}) - t(k_i)) / \Big(\Big(\sum_{h=i+3}^{j} n_h \Big) + n'_{i+2} \Big)$. For the Shapley value to be "convex," it needs to be true that $\phi'_m(T) - \phi_m(T) < \phi'_{m2}(T) - \phi_{m2}(T)$, or in other words, it must be true that the same change in capital employed by firm m must induce a larger increase in the Shapley value if the beginning capital level is greater. Using equation (A.3) for $i-1$ to i and the definition for i to $i+1$, the Shapley value is convex if

$$\begin{aligned} (t(k_i) - t(k_{i-1})) &\left(\sum_{h=i+3}^{j} n_h + n'_{i+2(m2)} \right) \\ &< (t(k_{i+1}) - t(k_i)) \left(\sum_{h=i+3}^{j} n_h + n'_{i+1} + n_{i+2} \right). \end{aligned} \tag{A.4}$$

To distinguish between the change in n_i for the shift from $i-1$ to i and from i to $i+1$, I add "$(m2)$" to the subscript for the new n's for the shift from i to $i+1$. In the above equation, given that only firm m changes its capital hiring, $n'_{i+2(m2)} = n_{i+2} + 1$. Whether or not the inequality in (A.4) is true will depend on the assumptions made regarding $t(\cdot)$. Given the assumption that the size of t is strictly increasing in the index i, we know that

$t(k_i) - t(k_{i-1}) = t(k_{i+1}) - t(k_i)$ if $t(\cdot)$ is linear, and $t(k_i) - t(k_{i-1}) < t(k_{i+1}) - t(k_i)$ if $t(\cdot)$ is convex. Assuming that $t(k_i) - t(k_{i-1}) \leq t(k_{i+1}) - t(k_i)$ for the n terms, $\sum_{h=i+3}^{j} n_h + n'_{i+2(m2)} < \sum_{h=i+3}^{j} n_h + n'_{i+1} + n_{i+2}$ is sufficient to ensure that the Shapley value is increasing in the size of the firm (in terms of capital employment) at an increasing rate. Because the inequality can be simplified to

$$n'_{i+1} > n'_{i+2(m2)} - n_{i+2},$$

and because when there is a discrete set of firms, and with $n'_{i+2(m2)}$ being the new number of firms in $\mathcal{N}_{i+2}$ after one firm moves up in type, going from $\mathcal{N}_{i+1}$ to $\mathcal{N}_{i+2}$. Therefore, the inequality becomes $n'_{i+1} > (1 + n_{i+2}) - n_{i+2} = 1$. Because n'_{i+1} is the number of firms of type $i+1$ once firm m moves from type i to type $i+1$, as long as $\mathcal{N}_{i+1} \neq \emptyset$, then $n_{i+1} \geq 1$ and $n'_{i+1} > 1$. Therefore, as long as $t(\cdot)$ is strictly increasing and either linear or convex, the inequality in equation (A.4) holds and the Shapley value, firm m's share of the upfront lobbying cost, is convex in the amount of capital employed by the firm.

A.2 Interior Assumptions for Equilibrium

I derive the requirements for an interior solution to exist for the solution with commitment to a trade agreement first (in Appendix A.2.1). Then I derive the conditions for existence of an interior solution when the government does not join a trade agreement in Appendix A.2.2.

A.2.1 Interior Assumptions: Ex-Ante Commitment (Section 1.3)

In order for an interior solution for the capital-hiring rule to exist, it must be true that there is a value of capital, $k \in (0, \infty)$, for which the marginal profit of hiring an additional unit of capital is zero.[1] To better understand the necessary assumptions to guarantee such a solution exists, consider the marginal revenue product of capital ($MRPK$) and the marginal

[1] I allow $k > K$ so as to not arbitrarily rule out $n \in (0, 1)$ as an equilibrium solution to the model.

cost of capital (MCK) functions for firm m:

$$MRPK_m = -\sigma\lambda'(k_m)\Big[Nd_{hm}\Big(p_h(k_m), q_2, P\Big) + N^* d^*_{hm}\Big(p^*_h(k_m), q^*_2, P^*\Big)\Big], \qquad \text{(eq. 1.13a)}$$

$$\begin{aligned} MCK_m = &-(\sigma-1)\lambda'(k_m) \\ &* \Big[Nd_{hm}\Big(p_h(k_m), q_2, P\Big) + N^* d^*_{hm}\Big(p^*_h(k_m), q^*_2, P^*\Big)\Big] + r. \end{aligned} \qquad \text{(eq. 1.13b)}$$

Given that the total demand for the output of variety m is strictly increasing in the capital allocation k, it can be shown that the unit-labor-input-requirement function is the only relevant factor in determining the slope of equations (eq. 1.13a) and (eq. 1.13b).[2] The derivative of each of these equations with respect to the capital-hiring for a representative firm (given all firms are symmetric in equilibrium) is

$$\frac{dMRPK}{dk} = -\sigma\left(\lambda''(k) - \sigma\frac{\lambda'(k)^2}{\lambda(k)}\right)\Big[Nd_h\Big(p_h(k), q_2, P\Big) + N^* d^*_h\Big(p^*_h(k), q^*_2, P^*\Big)\Big],$$
$$\frac{dMCK}{dk} = -(\sigma-1)\left(\lambda''(k) - \sigma\frac{\lambda'(k)^2}{\lambda(k)}\right)\Big[Nd_h\Big(p_h(k), q_2, P\Big) + N^* d^*_h\Big(p^*_h(k), q^*_2, P^*\Big)\Big].$$

Recall that the profit maximizing capital allocation, $\tilde{k}(r,n)$, $MRPK = MCK$ and $\lambda''(\tilde{k}(r,n)) - \sigma\lambda'(\tilde{k}(r,n))^2/\lambda(\tilde{k}(r,n)) > 0$. Therefore, in the neighborhood of the profit-maximizing capital allocation, $|dMRPK/dk| > |dMCK/dk|$.

Placing restrictions on the corners of $MRPK$ and MCK ensures the lines cross at least once. In order for an intersection of $MRPK$ and MCK to fall in the range $(0,\infty)$, it is sufficient to assume that (i) for $k \to 0$, $MRPK > MCK$ and (ii) for $k \to \infty$, $MRPK < MCK$.

Assumption A.1 (Interior $\tilde{k}(r,n)$)**.** *In order to ensure there is an interior solution for $\tilde{k}(r,n,\tau)$ for any given values of r and n, assume that:*

1. *As $k \to 0^+$, $r < -\lambda'(k)\Big[Nd_h(p_h(k), q_2, P) + N^* d^*_h(p^*_h(k), q^*_2, P^*)\Big]$;*

2. *As $k \to \infty$, $r > -\lambda'(k)\Big[Nd_h(p_h(k), q_2, P) + N^* d^*_h(p^*_h(k), q^*_2, P^*)\Big]$;*

[2] The total demand is $Nd_{hm}(p_h(k_m), q_2, P) + N^* d^*_{hm}(p^*_h(k_m), q^*_2, P^*)$, referred to here as TD. It is straightforward to show the total demand for variety m is increasing in k_m, with $[\partial TD/\partial p_h] dp_h(k)/dk = [TD]\ \lambda(k_m)^{-1}\lambda'(k_m)(-\sigma)$.

Establishing conditions for an interior solution for the rental price of capital is less straightforward: the value of r can only be restricted to be greater than zero, as there is no immediately clear upper bound for the rental price. To achieve market clearing in the capital market, the total demand for capital, $K^d(r)$, must equal the fixed capital supply, K, as shown earlier in equation (1.15). To better understand the necessary conditions to ensure an interior solution, first take a closer look at the first-order behavior of the capital demand function, $K^d(r)$, given a fixed number of firms in the industry, n, and the best-response capital-hiring rule, $\tilde{k}(r,n)$:

$$\frac{dK^d(r)}{dr} = n\left(\frac{\partial \tilde{k}(r,n)}{\partial r}\right). \qquad \text{(eq. 1.15a)}$$

Looking back at the definition of $\partial \tilde{k}(r,n)/\partial r$, the above equation is equal to

$$\frac{dK^d(r)}{dr} = \frac{-n}{\left(\lambda''(\tilde{k}) - \sigma\frac{\lambda'(\tilde{k})^2}{\lambda(\tilde{k})}\right)[TD] + (\sigma-1)\lambda'(\tilde{k})\left[N\frac{q_{h2}}{P}\frac{\partial P}{\partial k} + N^*\frac{q^*_{h2}}{P^*}\frac{\partial P^*}{\partial k}\right]},$$

which is decreasing in r for all values of r given that $K^d(r) = n\,\tilde{k}(r,n)$.

In order for the ex-ante equilibrium r to be interior, it must be true that for $r = 0$, $K^d(0) > K$ and as $r \to \infty$, $K^d(r) < K$. Because the capital supply is fixed, and therefore $dK^s(r)/dr = 0$, the market-clearing rental price is also unique as long as the prior two conditions hold.

Assumption A.2 (Interior $\hat{r}(n,\tau)$)**.** *Given that total demand for capital is $K^d(r) = n\,\tilde{k}(r,n)$ for all $r \in (0,\infty)$, and that $\lambda''(\tilde{k}(r,n)) - \sigma\lambda'(\tilde{k}(r,n))^2/\lambda(\tilde{k}(r,n)) > 0$, an interior capital-market-clearing $\hat{r}(n)$ exists if:*

1. *As $r \to 0^+$, $n\,\tilde{k}(r,n) > K$;*

2. *As $r \to \infty$, $n\,\tilde{k}(r,n) < K$.*

Looking at the conditions for the existence of an interior $\tilde{k}(r,n)$ in Assumption A.1, it is evidence that for $r = 0$, the marginal revenue of hiring an additional unit of capital is always greater than the marginal cost. The demand for capital will head to infinity, ensuring Assumption A.2.1 will hold given $k = \tilde{k}(r,n)$. As $r \to \infty$, the marginal cost of capital is driven to infinity, leading the demand for capital to fall to zero, ensuring Assumption A.2.2 will always hold given $k = \tilde{k}(r,n)$.

Finally, in order for an interior solution to exist for the ex-ante equilibrium number of firms in the industry, there must exist some $n \in (0,\infty)$ for which profits are equal to zero. A firm making the decision of whether or not to enter the market faces total revenue (R) and total cost (TC) functions as follow:

$$R = \frac{\sigma}{\sigma-1}\lambda(\hat{k}(n))\Big[N d_h\Big(p_h(\hat{k}(n)), q_2, P\Big) + N^* d_h^*\Big(p_h^*(\hat{k}(n)), q_2^*, P^*\Big)\Big], \qquad \text{(eq. 1.16a)}$$

$$TC = \lambda\Big(\hat{k}(n)\Big)\Big[N d_h\Big(p_h(\hat{k}(n)), q_2, P\Big) + N^* d_h^*\Big(p_h^*(\hat{k}(n)), q_2^*, P^*\Big)\Big] + \hat{r}(n)\,\hat{k}(n). \qquad \text{(eq. 1.16b)}$$

where $\hat{k}(n) \equiv \tilde{k}(\hat{r}(n), n)$ is the capital-hiring rule and $\hat{r}(n)$ is the capital-market-clearing rental price of capital.

To establish the existence of an intersection between the two curves, consider how each changes in response to a change in the number of firms, given the ex-ante-equilibrium capital-hiring rule, $\tilde{k}(r,n)$, and rental price of capital, $\hat{r}(n)$. The first-order behavior of the firm's total revenues and total costs are given by the following equations:

$$\frac{dR}{dn} = -\sigma\left(\lambda'(\hat{k})\frac{d\hat{k}(n)}{dn}[Nq_{h2} + N^*q_{h2}^*] - \lambda(\hat{k})\left[N\frac{q_{h2}}{P}\frac{dP}{dn} + N^*\frac{q_{h2}^*}{P^*}\frac{dP^*}{dn}\right]\right),$$

$$\frac{dTC}{dn} = -(\sigma-1)\left(\lambda'(\hat{k})\frac{d\hat{k}(n)}{dn}[Nq_{h2} + N^*q_{h2}^*] - \lambda(\hat{k})\left[N\frac{q_{h2}}{P}\frac{dP}{dn} + N^*\frac{q_{h2}^*}{P^*}\frac{dP^*}{dn}\right]\right) + \frac{d\hat{r}(n)}{dn}\hat{k}(n).$$

where $\hat{k} \equiv \hat{k}(n)$, $q_{h2} \equiv d_h\Big(p_h(\hat{k}(n)), q_2, P\Big)$, and $q_{h2}^* \equiv d_h^*\Big(p_h^*(\hat{k}(n)), q_2^*, P^*\Big)$ for notational simplicity, and with the first-order behavior of the rental price of capital, $d\hat{r}(n)/dn$,

$$\frac{d\hat{r}(n)}{dn} = \frac{\hat{k}}{n}\left(\lambda''(\hat{k}) - \sigma\frac{\lambda'(\hat{k})^2}{\lambda(\hat{k})}\right)[Nq_{h2} + N^* q^*_{h2}] - \lambda(\hat{k})\left[\mathbf{N}\frac{\mathbf{q}_{h2}}{\mathbf{P}}\frac{d\mathbf{P}}{dn}\right], \tag{A.5}$$

with $\hat{k} \equiv \hat{k}(n)$ and $\mathbf{N}\frac{\mathbf{q}_{h2}}{\mathbf{P}}\frac{d\mathbf{P}}{dn} \equiv N\frac{q_{h2}}{P}\frac{dP}{dn} + N^*\frac{q^*_{h2}}{P^*}\frac{dP^*}{dn}$. Therefore, the total revenue function is clearly decreasing in the number of firms for all $n > 0$, given that $d\hat{k}(n)/dn = -\hat{k}(n)/n$. The slope of the total cost function can then be defined in terms of the total revenue function's slope: $dTC/dn = ((\sigma - 1)/\sigma)(dR/dn) + (d\hat{r}(n)/dn)\hat{k}(n)$. Given equation (A.5) and that for profit maximization at $\hat{k}(n)$ it must be true that $\lambda''(\hat{k}) - \sigma\lambda'(\hat{k})^2/\lambda(\hat{k}) > 0$, a sufficient condition to ensure an interior solution exists for the ex-ante equilibrium number of firms, $\bar{n}^c$ is to require that as $n \to 0$, that $R > TC$, and for $n \to \infty$, that $R < TC$. Assuming that an interior ex-ante equilibrium exists, consider what the slope of R and TC are when $n = \bar{n}^c$. When $n = \bar{n}^c$, it can be shown that

$$\begin{aligned}
\frac{dR(\bar{n}^c)}{dn} &= -\sigma\lambda'(\bar{k}^c)\frac{d\hat{k}(\bar{n}^c)}{dn}[Nq^c_{h2} + N^* q^{*c}_{h2}] < 0,\\
\frac{dTC(\bar{n}^c)}{dn} &= -\sigma\lambda'(\bar{k}^c)\frac{d\hat{k}(\bar{n}^c)}{dn}[Nq^c_{h2} + N^* q^{*c}_{h2}]\\
&\quad + \frac{(\bar{k}^c)^2}{\bar{n}^c}\left(\lambda''(\bar{k}^c) - \frac{\sigma\lambda'(\bar{k}^c)^2}{\lambda(\bar{k}^c)}\right)[Nq^c_{h2} + N^* q^{*c}_{h2}].
\end{aligned}$$

At an interior ex-ante equilibrium, it must be true that $dR(\bar{n}^c)/dn < dTC(\bar{n}^c)/dn$, or in other terms the revenue an additional firm would earn by entering the market is less than the total costs the firm incurs. Therefore, to ensure a solution exists for which $R = TC$, the natural assumption is to require that $R > TC$ as $n \to 0$ and that as $n \to \infty$, $R < TC$.

Assumption A.3 (Interior $\bar{n}^c$)**.** *Given the definitions of the total revenue and total cost functions in equations (eq. 1.13a) and (eq. 1.13b), respectively, given $\lambda''(\hat{k}(n)) - \sigma\lambda'(\hat{k}(n))^2/\lambda(\hat{k}(n)) > 0$ for all $r, n \in (0, \infty)$, and given that $K^d(\hat{r}(n)) = n\,\tilde{k}(\hat{r}(n), n) = K$, an interior ex-ante equilibrium firm-entry condition $\bar{n}^c$ exists if the following assumptions hold:*

1. *As $n \to 0^+$, $\frac{1}{\sigma-1}\lambda\big(\hat{k}(n)\big)[Nq_{h2} + N^* q^*_{h2}] > \hat{r}(n)\hat{k}(n)$;*

2. *As* $n \to \infty$, $\frac{1}{\sigma-1}\lambda\Big(\hat{k}(n)\Big)[Nq_{h2} + N^*q^*_{h2}] < \hat{r}(n)\hat{k}(n)$.

A.2.2 Interior Assumptions: Ex-Post Tariff Selection with Lobbying (Section 1.4)

In order for an interior solution for the capital-hiring rule to exist, it must be true that there is a value of capital, $k \in (0, \infty)$, for which the marginal profit of hiring an additional unit of capital is zero (where equation (1.31) is true). To derive the conditions for an interior solution to exist, first look at the marginal revenue product of capital ($MRPK$) and the marginal cost of capital (MCK) functions for firm m:

$$MRPK_m = -\sigma\lambda'(k_m)\Big[Nd_{hm}\Big(p_h(k_m), q_2, P\Big) + N^*d^*_{hm}\Big(p^*_h(k_m), q^*_2, P^*\Big)\Big], \tag{A.6a}$$

$$\begin{aligned} MCK_m = &-(\sigma-1)\lambda'(k_m)\Big[Nd_{hm}\Big(p_h(k_m), q_2, P\Big) + N^*d^*_{hm}\Big(p^*_h(k_m), q^*_2, P^*\Big)\Big] \\ &+ r + c + \theta'(k_m). \end{aligned} \tag{A.6b}$$

Given that the total demand for the output of variety m is strictly increasing in the capital allocation k, it can be shown that the unit-labor-cost function is the only relevant factor in determining the slope of equations (A.6a) and (A.6b). The derivative of each of these equations with respect to the capital-hiring for a representative firm is

$$\begin{aligned} \frac{dMRPK}{dk} &= -\sigma\left(\lambda''(k) - \sigma\frac{\lambda'(k)^2}{\lambda(k)}\right)\Big[Nd_h\Big(p_h(k), q_2, P\Big) + N^*d^*_h\Big(p^*_h(k), q^*_2, P^*\Big)\Big], \\ \frac{dMCK}{dk} &= -(\sigma-1)\left(\lambda''(k) - \sigma\frac{\lambda'(k)^2}{\lambda(k)}\right)\Big[Nd_h\Big(p_h(k), q_2, P\Big) + N^*d^*_h\Big(p^*_h(k), q^*_2, P^*\Big)\Big] \\ &\quad + \theta''(k_m). \end{aligned}$$

Recall that for a given allocation of capital to be a maximum, $MRPK = MCK$ and $\Big(\lambda''(k) - \sigma\lambda'(k)^2/\lambda(k)\Big)\Big[Nd_h(p_h(k), q_2, P) + N^*d^*_h(p^*_h(k), q^*_2, P^*)\Big] + \theta''(k) > 0$. Comparing these two equations reveals that as long as $\lambda''(k) - \sigma\frac{\lambda'(k)^2}{\lambda(k)} > 0$, the convexity of $\theta(k)$ implies that $dMRPK/dk < dMCK/dk$.

Notice next that for large σ, the ex-ante commitment second-order condition, $\lambda''(k) - \sigma \frac{\lambda'(k)^2}{\lambda(k)} > 0$, does not necessarily hold for $\bar{k}^u$, this is because the second-order condition with ex-post tariff selection has additional "wiggle room" from the $\theta''(k)$ term. Therefore, there are values of σ for which $\lambda''(\bar{k}^u) - \sigma \frac{\lambda'(\bar{k}^u)^2}{\lambda(\bar{k}^u)} < 0$ but $\Big(\lambda''(\bar{k}^u) - \sigma\lambda'(\bar{k}^u)^2/\lambda(\bar{k}^u)\Big)[TD] + \theta''(\bar{k}^u) > 0$ still holds. To slightly simplify analysis moving forward, I restrict my analysis to focus on a "relevant" range of σ, which I define as the values of σ for which $\lambda''(\bar{k}^u) - \sigma\frac{\lambda'(\bar{k}^u)^2}{\lambda(\bar{k}^u)} > 0$. Define σ^{Max} as the elasticity for which $\lambda''(\bar{k}^u) - \sigma\frac{\lambda'(\bar{k}^u)^2}{\lambda(\bar{k}^u)} = 0$.

Given that for the relevant range of σ the slope of the marginal revenue product of capital is more negative than the marginal cost of capital, placing restrictions on the corners of $MRPK$ and MCK ensures the lines cross at least once. In order for an intersection of $MRPK$ and MCK to fall in the range $(0, \infty)$, it is sufficient to assume that (i) for $k \to 0$, $MRPK > MCK$ and (ii) for $k \to \infty$, $MRPK < MCK$.

Assumption A.4 (Interior $\tilde{k}(r,n)$)**.** *In order to ensure there is an interior solution for $\tilde{k}(r,n)$ for any given values of r and n, assume that:*

1. *Elasticity of substitution is $\sigma \in (1, \sigma^{Max})$, so that*

$$\lambda''(\tilde{k}(r,n)) - \sigma\frac{\lambda'(\tilde{k}(r,n))^2}{\lambda(\tilde{k}(r,n)} > 0;$$

2. *As $k \to 0^+$, $r + c^u(k,n) + \theta'(k) < -\lambda'(k)\Big[Nd_h(p_h(k), q_2, P) + N^*d_h^*(p_h^*(k), q_2^*, P^*)\Big]$;*

3. *As $k \to \infty$, $r + c^u(k,n) + \theta'(k) > -\lambda'(k)\Big[Nd_h(p_h(k), q_2, P) + N^*d_h^*(p_h^*(k), q_2^*, P^*)\Big]$;*

Next, consider the requirements for capital-markan interior solution for the rental price of capital. The capital market clears if the total demand for capital, $K^d(r)$, is equal the fixed capital supply, K, as shown earlier in equation (1.33). Because the capital stock is constant, a unique interior solution for the rental-price of capital is guaranteed if the capital demand is (i) monotonic, and (ii) greater than (less than) K as $r \to 0$ and less than (greater than) K as $r \to \infty$. First, look at the slope of the capital demand equation. The first-order behavior of the capital demand function, $K^d(r)$, given a fixed number of firms in the industry, n, and the best-response capital-hiring rule, $\tilde{k}(r,n)$:

$$\frac{dK^d(r)}{dr} = n\left(\frac{d\tilde{k}(r,n)}{dr}\right). \tag{1.33a}$$

Looking back at the definition of $\partial\tilde{k}(r,n)/\partial r$, the above equation is equal to

$$\frac{dK^d(r)}{dr} = \frac{-n}{\frac{\partial c^u}{\partial k} + \theta''(\tilde{k}) + \left(\lambda''(\tilde{k}) - \sigma\frac{\lambda'(\tilde{k})^2}{\lambda(\tilde{k})}\right)\left[Nq_{h2} + N^* q^*_{h2}\right] + (\sigma-1)\lambda'(\tilde{k})\left[\frac{\mathbf{q}_{h2}}{\mathbf{P}}\frac{d\mathbf{P}}{dk}\right]},$$

with $\tilde{k} \equiv \tilde{k}(r,n)$, $c^u \equiv c^u(k,n)$, and $\mathbf{N}\frac{\mathbf{q}_{h2}}{\mathbf{P}}\frac{\partial\mathbf{P}}{\partial k} \equiv N\frac{q_{h2}}{P}\left(\frac{\partial P}{\partial k} + \frac{\partial P}{\partial \tau}\frac{\partial \tau}{\partial k}\right) + N^*\frac{q^*_{h2}}{P^*}\frac{\partial P^*}{\partial k}$. It can be shown that $dK^d(r)/dr$ is decreasing in r for all values of r given that $K^d(r) = n\,\tilde{k}(r,n)$. Therefore, the ex-ante equilibrium r is unique and interior, it must be true that for $r = 0$, $K^d(0) > K$ and as $r \to \infty$, $K^d(r) < K$. Because the capital supply is fixed, and therefore $dK^s(r)/dr = 0$, the market-clearing rental price is also unique as long as the prior two conditions hold.

Assumption A.5 (Interior, unique $\hat{r}(n)$)**.** *Given that total demand for capital is $K^d(r) = n\,\tilde{k}(r,n)$ for all $r \in (0,\infty)$, and that $\lambda''(\tilde{k}(r,n)) - \sigma\lambda'(\tilde{k}(r,n))^2/\lambda(\tilde{k}(r,n)) > 0$, a unique interior capital-market-clearing $\hat{r}(n)$ given a fixed n exists if the following assumptions are satisfied:*

1. *As $r \to 0^+$, $n\,\tilde{k}(r,n) > K$;*
2. *As $r \to \infty$, $n\,\tilde{k}(r,n) < K$.*

Assumption A.4 shows when r is close to zero, the marginal revenue of hiring an additional unit of capital is always greater than the marginal cost. Therefore, the demand for capital goes to infinity, ensuring Assumption A.5.1 holds. Alternatively, as r approaches infinity, the marginal cost of capital also goes to infinity, leading to the demand for capital to fall to zero, ensuring Assumption A.5.2 will always hold given $k = \tilde{k}(r,n)$.

Finally, in order for an interior solution to exist for the ex-ante equilibrium number of firms in the industry, there must exist some $n \in (0,\infty)$ for which profits are equal to zero.

A firm making the decision of whether or not to enter the market faces total revenue (R) and total cost (TC) functions as follow:

$$R = \frac{\sigma}{\sigma-1}\lambda(\hat{k}(n))\Big[Nd_h\Big(p_h(\hat{k}(n)), q_2, P\Big) + N^* d_h^*\Big(p_h^*(\hat{k}(n)), q_2^*, P^*\Big)\Big], \tag{A.7a}$$

$$\begin{aligned} TC = \lambda\Big(\hat{k}(n)\Big)\Big[Nd_h\Big(p_h(\hat{k}(n)), q_2, P\Big) + N^* d_h^*\Big(p_h^*(\hat{k}(n)), q_2^*, P^*\Big)\Big] \\ + \hat{r}(n)\,\hat{k}(n) + c(\hat{k}(n), n)\,\hat{k}(n) + \theta(\hat{k}(n)), \end{aligned} \tag{A.7b}$$

where $\hat{k}(n) \equiv \tilde{k}(\hat{r}(n), n)$ is the capital-hiring rule and $\hat{r}(n)$ is the capital-market-clearing rental price of capital. An intersection between the two lines can be established using their derivatives:

$$\begin{aligned} \frac{dR}{dn} &= -\sigma\left(\lambda'(\hat{k})\frac{d\hat{k}(n)}{dn}[Nq_{h2} + N^* q_{h2}^*] - \lambda(\hat{k})\left[\mathbf{N}\frac{\mathbf{q}_{h2}^u}{\mathbf{P}^u}\frac{d\mathbf{P}^u}{dn}\right]\right), \\ \frac{dTC}{dn} &= -(\sigma-1)\left(\lambda'(\hat{k})\frac{d\hat{k}(n)}{dn}[Nq_{h2} + N^* q_{h2}^*] - \lambda(\hat{k})\left[\mathbf{N}\frac{\mathbf{q}_{h2}}{\mathbf{P}}\frac{d\mathbf{P}}{dn}\right]\right) \\ &\quad + \frac{d\hat{r}(n)}{dn}\,\hat{k} + \frac{dc(\hat{k}(n), n)}{dn}\,\hat{k}, \end{aligned}$$

where $\hat{k} \equiv \hat{k}(n)$, $q_{h2} \equiv d_h\Big(p_h(\hat{k}(n)), q_2, P\Big)$, $q_{h2}^* \equiv d_h^*\Big(p_h^*(\hat{k}(n)), q_2^*, P^*\Big)$, and $\mathbf{N}\frac{\mathbf{q}_{h2}}{\mathbf{P}}\frac{d\mathbf{P}}{dn} \equiv N\frac{q_{h2}}{P}\frac{dP}{dn} + N^*\frac{q_{h2}^*}{P^*}\frac{dP^*}{dn}$. Expanding and simplifying the price index term gives

$$\begin{aligned} \mathbf{N}\frac{\mathbf{q}_{h2}}{\mathbf{P}}\frac{d\mathbf{P}}{dn} &= \frac{-w_\lambda(\hat{k})}{(\sigma-1)n\lambda(\hat{k})} \\ &\quad * \left[Nq_{h2}\frac{\left(\frac{z}{z+1}\right)(\sigma z+1) - (\sigma-1)(\tau-1)}{(\sigma z+1) - (\sigma-1)(\tau-1)} + N^* q_{h2}^*\left(\frac{1}{z^*+1}\right)\right]. \end{aligned} \tag{1.40}$$

Equation (1.40) is composed of two main parts: (i) the effect of a change in n on home and foreign demand by way of the price indexes while holding τ fixed and given $\hat{k}(n) = K/n$, which is positive, and (ii) the effect of a change in n on demand by way of the effect on the price index through τ, which is negative. The derivative of the rental price of capital is

$$\begin{aligned}\frac{d\hat{r}(n)}{dn} =& \frac{\hat{k}}{n}\left(\theta''(\hat{k}) + \left(\lambda''(\hat{k}) - \sigma\frac{\lambda'(\hat{k})^2}{\lambda(\hat{k})}\right)[TD]\right) - \frac{dc(\hat{k}(n),n)}{dn} \\ &+ \frac{\hat{k}}{n}(\sigma-1)\lambda'(\hat{k})\left[\mathbf{N}\frac{\mathbf{q}_{h2}}{\mathbf{P}}\frac{\partial\mathbf{P}}{\partial k}\right] - (\sigma-1)\lambda'(\hat{k})\left[\mathbf{N}\frac{\mathbf{q}_{h2}}{\mathbf{P}}\frac{d\mathbf{P}}{dn}\right],\end{aligned} \tag{A.8}$$

with $\hat{k} \equiv \hat{k}(n)$, $\mathbf{N}\frac{\mathbf{q}_{h2}}{\mathbf{P}}\frac{d\mathbf{P}}{dn} \equiv N\frac{q_{h2}}{P}\frac{dP}{dn} + N^*\frac{q^*_{h2}}{P^*}\frac{dP^*}{dn}$, and $TD \equiv Nq_{h2} + N^*q^*_{h2}$. The signs of the slope of the total revenue and total cost equations are unclear due to the non-monotonicity of the $\mathbf{N}\frac{\mathbf{q}_{h2}}{\mathbf{P}}\frac{d\mathbf{P}}{dn}$ terms.

The slope of the total cost function can be defined in terms of the total revenue function: $dTC/dn = ((\sigma-1)/\sigma)(dR/dn) + (d\hat{r}(n)/dn)\,\hat{k}(n) + (dc(\hat{k}(n),n)/dn)\,\hat{k}(n)$. Given the definition of $d\hat{r}(n)/dn$ (defined in equation (A.8)), the definition of $dc(\hat{k}(n),n)/dn$, and the second-order condition for profit maximization, an interior solution exists for $\bar{n}^u$ if as $n \to 0$, $R > TC$, and for $n \to \infty$, that $R < TC$. Assuming that an interior equilibrium exists for the model with ex-ante tariff commitment, consider what the slope of R and TC are when $n = \bar{n}^c$, given that the solution without commitment shows $\bar{n}^u > \bar{n}^c$. When $n = \bar{n}^c$, it can be shown that

$$\begin{aligned}\frac{dR(\bar{n}^c)}{dn} &= -\sigma\lambda'(\bar{k}^c)\frac{d\hat{k}(\bar{n}^c)}{dn}[Nq^c_{h2} + N^*q^{*c}_{h2}] < 0, \text{ and} \\ \frac{dTC(\bar{n}^c)}{dn} &= -\sigma\lambda'(\bar{k}^c)\frac{d\hat{k}(\bar{n}^c)}{dn}[Nq^c_{h2} + N^*q^{*c}_{h2}] + \frac{(\bar{k}^c)^2}{\bar{n}^c}\left(\lambda''(\bar{k}^c) - \frac{\sigma\lambda'(\bar{k}^c)^2}{\lambda(\bar{k}^c)}\right)[Nq^c_{h2} + N^*q^{*c}_{h2}], \\ &\implies \frac{dR(\bar{n}^c)}{dn} < \frac{dTC(\bar{n}^c)}{dn},\end{aligned}$$

showing that for $n < \bar{n}^u$, the total revenue is less than the total cost. To ensure that the lines cross only onces, I assume a slightly more restrictive condition:

Assumption A.6 (Interior $\bar{n}^u$). *Given that the definitions of the total revenue and total cost functions in equations (A.6a) and (A.6b), respectively, given* $\left(\lambda''(\tilde{k}(r,n)) - \sigma\lambda'(\tilde{k}(r,n))^2/\lambda(\tilde{k}(r,n))\right)$ $[TD] + \theta''(\tilde{k}(r,n)) > 0$ *for all* $n \in (0,\infty)$, *and given the rental price of capital* $\hat{r}(n)$ *is such that* $K^d(\hat{r}(n)) = n\,\tilde{k}(\hat{r}(n),n) = K$, *an interior ex-ante equilibrium firm-entry condition* $\bar{n}^c$ *exists if the* *following assumptions hold:*

1. *As* $n \to 0^+$, $\frac{1}{\sigma-1}\lambda\Big(\hat{k}(n)\Big)[Nq_{h2} + N^* q^*_{h2}] > \hat{r}(n)\,\hat{k}(n) + c\,\hat{k}(n) + \theta(\hat{k}(n))$;
2. *As* $n \to \infty$, $\frac{1}{\sigma-1}\lambda\Big(\hat{k}(n)\Big)[Nq_{h2} + N^* q^*_{h2}] < \hat{r}(n)\,\hat{k}(n) + c\,\hat{k}(n) + \theta(\hat{k}(n))$.

The assumption implies that if $n < \bar{n}^u$, then the firm's total revenue is greater than its costs, implying firms in the industry are earning positive profits, which in turn leads to more firms entering the market. On the other hand, if $n > \bar{n}^u$, then firms earn negative profits and will exit the industry.

A.3 Behavior of Market Share for Changes in σ

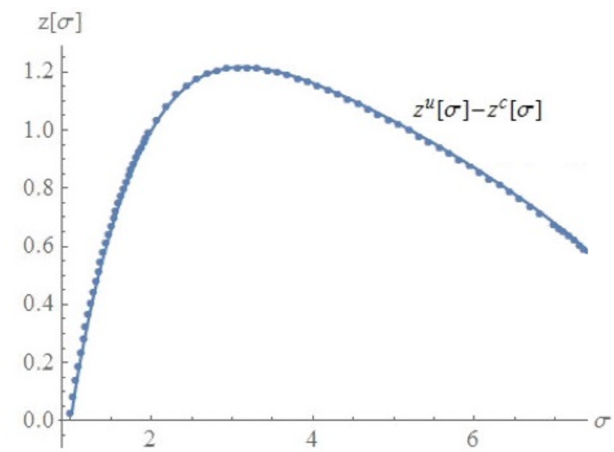
A key factor in making comparisons within the model is how the trade policy impacts the home country's market share in sales both at home and abroad, where $z/(z+1)$ is the market share of home-produced varieties at home and $1/(z^*+1)$ is the market share of home-produced varieties abroad. The comparative values of z and z^* across varying policy types is difficult to determine. In all, I need to understand how z^c, z^{cxp}, z^u, z^{*c}, and z^{*u} compare. For this discussion, recall

$$\begin{aligned} z &\equiv \frac{n\,p_h\,q_{h2}}{n^* p_f\,q_{f2}} = \frac{n}{n^*}\left(\frac{\lambda}{\tau\lambda^*}\right)^{1-\sigma} = \frac{n}{n^*}\left(\frac{p_h}{p_f}\right)^{1-\sigma}, \text{ and} \\ z^* &\equiv \frac{n^*\,p^*_f\,q^*_{f2}}{np^*_h\,q^*_{h2}} = \frac{n^*}{n}\left(\frac{\lambda^*}{\tau^*\lambda}\right)^{1-\sigma} = \frac{n^*}{n}\left(\frac{p^*_f}{p^*_h}\right)^{1-\sigma}. \end{aligned}$$

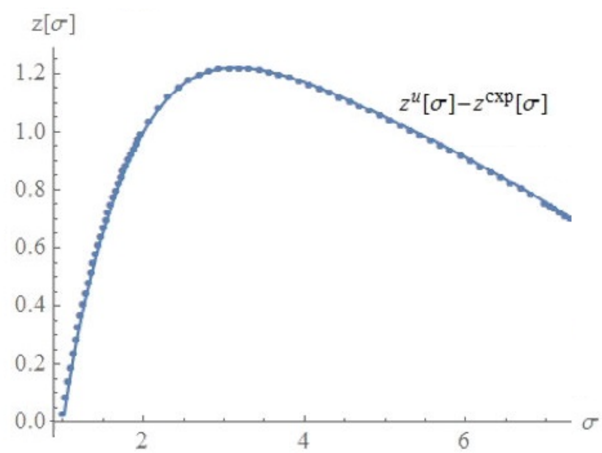
The following points characterize the relationship between the four variables, which is also depicted using the numerical solution in Figure A.1:

1. z^c vs. z^u: If I assume that the Metzler paradox does not occur, then joining a trade agreement leads to a fall in expenditure on home varieties. In other terms, because $\bar{n}^u > \bar{n}^c$, as long as $p^c_f/p^c_h < p^u_f/p^u_h$ then it will necessarily also be true that $z^c < z^u$.

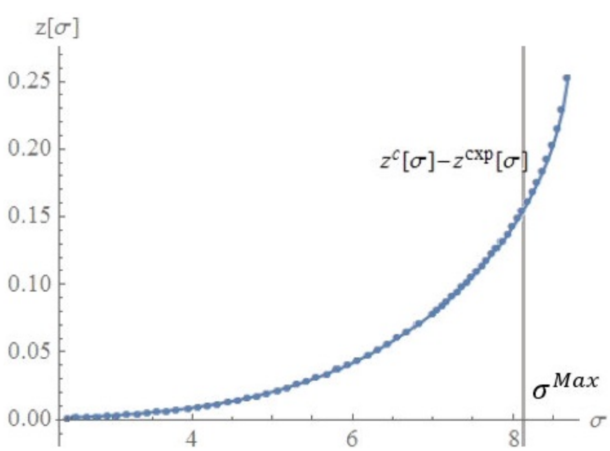
Figure A.1: Comparative sizes of z^c, z^{cxp}, z^u, z^{*c}, and z^{*u}. The top-left graph shows the total effect of the ex-ante production distortion and political economy tariff protection on z. The top-right graph shows the effect of lobbying on z, given ex-ante production is $(\bar{k}^u, \bar{n}^u)$. The bottom-left graph shows how z is impacted by the change in capital allocation, given social-welfare maximizing tariffs. Finally, the bottom-right graph shows the effect of ex-ante production distortion of z^*.



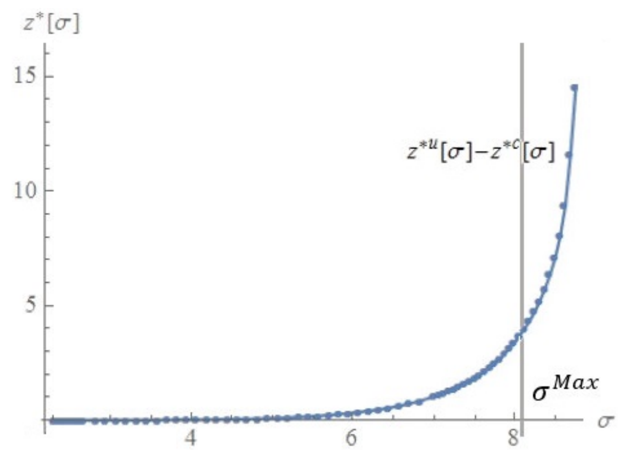
(a) Comparison of z^u and z^c.



(b) Comparison of z^u and z^{cxp}.



(c) Comparison of z^c and z^{cxp}.



(d) Comparison of z^{*u} and z^{*c}.

Algebraically,

$$\frac{p_f^c}{p_h^c} < \frac{p_f^u}{p_h^u} \implies \left(\frac{p_h^c}{p_f^c}\right)^{1-\sigma} < \frac{\bar{n}^u}{\bar{n}^c}\left(\frac{p_h^u}{p_f^u}\right)^{1-\sigma} \implies z^c < z^u.$$

If the Metzler paradox does occur, then it is unclear analytically if z^c or z^u is larger. Using the numerical solution, I find $z^c < z^u$ for all model parameters. The relationship between z^c and z^u for one set of model parameters is depicted in Figure A.1a.

2. $z^u > z^{cxp}$: Lobbying decreases the import penetration of foreign varieties, $z^u > z^{cxp}$. This can be proven to be true very easily, given the ex-ante production decisions are the same and that $\tau^{cxp} < \tau^u$. The relationship between z^u and z^{cxp} for one set of model parameters is depicted in Figure A.1b.

3. $z^c > z^{cxp}$: The ex-ante production distortion increases the import penetration of foreign varieties at the social-welfare maximizing tariff levels. This is evident by simplifying the definitions of z^c and z^{cxp}: $z^c > z^{cxp}$ if $\bar{n}^c\Big(\lambda(\bar{k}^c)\Big)^{1-\sigma}\Big(\tau^c\Big)^{\sigma-1} > \bar{n}^u\Big(\lambda(\bar{k}^u)\Big)^{1-\sigma}\Big(\tau^{cxp}\Big)^{\sigma-1}$. This is established by two facts. First, $\tau^{cxp} < \tau^c$ for all values of σ and a (shown in Proposition 1.12). Second, $\bar{n}\Big(\lambda(\bar{k})\Big)^{1-\sigma}$ is decreasing in $\bar{k}$, with $\frac{d}{d\bar{k}}\Big((\bar{k}/K)(\lambda(\bar{k}))^{\sigma-1}\Big) = w_\lambda(\bar{k})\Big(\lambda(\bar{k})\Big)^{\sigma-2}(1/K)$, which means $\bar{n}^c\Big(\lambda(\bar{k}^c)\Big)^{1-\sigma} > \bar{n}^u\Big(\lambda(\bar{k}^u)\Big)^{1-\sigma}$. Therefore, $z^c > z^{cxp}$. The relationship between z^c and z^{cxp} for one set of model parameters is depicted in Figure A.1c.

4. $z^{*c} < z^{*u}$: The distortion in ex-ante production decisions leads to a rise in import penetration of domestic varieties abroad, $z^{*c} < z^{*u}$. Simplifying the problem, $z^{*c} < z^{*u}$ if $\bar{n}^c\Big(\lambda(\bar{k}^c)\Big)^{1-\sigma} > \bar{n}^u\Big(\lambda(\bar{k}^u)\Big)^{1-\sigma}$. In point 3 above, I showed that this inequality holds using the derivative of $\bar{n}\Big(\lambda(\bar{k})\Big)^{1-\sigma}$ with respect to $\bar{k}$. The relationship between z^{*c} and z^{*u} for one set of model parameters is depicted in Figure A.1d.

Figure A.1 demonstrates these comparisons using the numerical model. The top-left graph (item 1 in the list above) shows that $z^u > z^c$, which means domestic producers' home-market share falls if the government commits to a trade agreement, with the loss in market share being at its largest near the σ where τ^u and τ^c reach their maximum points.

The top-right graph (item 2 in the list above) shows that $z^u > z^{cxp}$, meaning domestic producers' home-market share is greater when lobbying is successful given ex-ante production is $(\bar{k}^u, \bar{n}^u)$, and the size of the benefit to market share from the success of lobbying efforts is greatest near the σ where τ^u and τ^{cxp} are maximized.

The bottom-left graph (item 3 in the list above) shows that $z^c > z^{cxp}$, meaning domestic producers' home-market share is improved by correcting the ex-ante production distortion conditional on lobbying not occurring ex-post. As goods become more homogeneous, the benefit of correcting the production distortion grows at an increasing rate, which is consistent with the fact that the size of the ex-ante production distortion is also increasing in σ.

The bottom-right graph (item 4 in the list above) shows that the home producers' foreign-market share, $1/(z^* + 1)$, improves from correcting the ex-ante production distortion, with the benefit to home producers' foreign-market share increasing as σ increases.

A.4 Production and Tariff Levels for Minimum σ

Now, suppose the elasticity of substitution approaches $\sigma \to 1$ or $\sigma \to \sigma^{Max}$. In the CES model, this lower bound is the special case of Cobb-Douglas production. The upper bound is the value of σ for which $\lambda''(k) - \sigma(\lambda'(k))^2/\lambda(k) = 0$. I do not explore the limits for $\sigma \to \sigma^{Max}$ here.

First, for the lower bound of σ specifically, the demand index approaches a Cobb-Douglas function. To examine production when $\sigma \to 1$, first simplify the consumption index given the symmetry of production across firms at home and symmetry across firms abroad:

$$\lim_{\sigma\to 1} q_2 = \lim_{\sigma\to 1} \left(\sum_{m=1}^{n} q_{h2m}^{\frac{\sigma-1}{\sigma}} + \sum_{m=1}^{n^*} q_{f2m}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} = (n+n^*)^{\frac{\sigma}{\sigma-1}} \left(\eta q_{h2}^{\frac{\sigma-1}{\sigma}} + (1-\eta) q_{f2}^{\frac{\sigma-1}{\sigma}} \right),$$

where $\eta \equiv n/(n+n^*)$ is the relative share of varieties produced in the home country compared to the total number of varieties produced in the world. Taking natural logs of each side and finding the limit as $\sigma \to 1$ then gives

$$\begin{aligned}\lim_{\sigma \to 1} q_2 &= (n+n^*)^{\infty} \left(\prod_{m=1}^{n} q_{h2m}^{\frac{1}{n+n^*}} \prod_{m=1}^{n^*} q_{f2m}^{\frac{1}{n+n^*}} \right), \\ &= (n+n^*)^{\infty} \left(q_{h2}^{\frac{n}{n+n^*}} q_{f2m}^{\frac{n^*}{n+n^*}} \right) = \infty \text{ if } n+n^* > 1.\end{aligned}$$

Additionally, the price index becomes

$$\begin{aligned}\lim_{\sigma \to 1} P = \lim_{\sigma \to 1} &\left(\sum_{m=1}^{n} p_{hm}^{1-\sigma} + \sum_{m=1}^{n^*} p_{fm}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}, \\ &= \left(n p_h^{0^-} + n^* p_f^{0^-} \right)^{\frac{1}{0^-}} = (n+n^*)^{-\infty} = 0 \text{ if } n+n^* > 1.\end{aligned} \tag{A.9}$$

The consumer's demand for a home variety can be found by maximizing the CES consumption index, subject to the consumer's budget constraint $e_2 = np_h q_{h2} + n^* p_f q_{f2}$, giving demand functions:

$$q_{h2} = \left(\frac{e_2}{n+n^*} \right) \frac{1}{p_h}, \quad \text{and} \quad q_{f2} = \left(\frac{e_2}{n+n^*} \right) \frac{1}{p_f}.$$

Because demand is Cobb-Douglas, the price of each good can only be discussed in relative terms, where

$$\frac{q_{h2}}{q_{f2}} = \frac{p_f}{p_h}.$$

Any combination of (p_h, p_f) satisfies this condition, because demand for each variety adjusts to maintain this equality. Moving forward, I assume that (p_h, p_f) are finite. Using the CES price-setting rule, $p_h = \frac{\sigma}{\sigma-1}\lambda(k)$, the limit is $\lim_{\sigma \to 1} p_h = \frac{1}{0}\lambda(k)$. To ensure that the price is finite, the limit of $\lambda(k)$ would need to offset the limit of $\frac{\sigma}{\sigma-1}$:

$$\text{For finite price, would need:} \quad \lim_{\sigma \to 1} p_h = \lim_{\sigma \to 1} \frac{\frac{\sigma}{\sigma-1}}{(1/\lambda(k))} = \frac{\infty}{\infty},$$

which implies that $\lambda(k)$ would need to go to zero. Given that by definition $\lambda'(k) < 0$, $\lambda(0) \to \infty$, and $\lim_{\sigma\to\infty} \lambda(k) = \lambda^*$, this is not possible. Therefore, the price of the home variety will not remain finite. Instead the markup goes to infinity, driving the relative demand for the home good to zero.

Production with Ex-Ante Tariff Commitment (Section 1.3). Look next at how σ influences the firms' production decisions. First, I examine the effect on capital hiring. Marginal revenue and marginal cost as $\sigma \to 1$ are:

$$
\begin{aligned}
MRPK_m &= -\sigma\lambda'(k_m)\Big[Nd_{hm}\Big(p_h(k_m), q_2, P\Big) + N^* d^*_{hm}\Big(p^*_h(k_m), q^*_2, P^*\Big)\Big], \\
\lim_{\sigma\to1} MRPK_m &= \lim_{\sigma\to1}\left(-\sigma\lambda'(k_m)\left[N\left(\frac{e_2}{n+n^*}\right)\frac{1}{p_{hm}(k_m)} + N^*\left(\frac{e^*_2/\tau^*}{n+n^*}\right)\frac{1}{p^*_{hm}(k_m)}\right]\right), \\
&= \lim_{\sigma\to1}\left(-(\sigma-1)\frac{\lambda'(k_m)}{\lambda(k_m)}\left[N\left(\frac{e_2}{n+n^*}\right) + N^*\left(\frac{e^*_2/\tau^*}{n+n^*}\right)\right]\right),
\end{aligned}
$$

$$
\begin{aligned}
MCK_m &= -(\sigma-1)\lambda'(k_m)\Big[Nd_{hm}\Big(p_h(k_m), q_2, P\Big) + N^* d^*_{hm}\Big(p^*_h(k_m), q^*_2, P^*\Big)\Big] + \bar{r}, \\
\lim_{\sigma\to1} MCK_m &= \lim_{\sigma\to1}\left(-(\sigma-1)\lambda'(k_m)\left[N\left(\frac{e_2}{n+n^*}\right)\frac{1}{p_{hm}(k_m)} + N^*\left(\frac{e^*_2/\tau^*}{n+n^*}\right)\frac{1}{p^*_{hm}(k_m)}\right] + r\right), \\
&= \lim_{\sigma\to1}\left(-(\sigma-1)\frac{\lambda'(k_m)}{\lambda(k_m)}\left(\frac{\sigma-1}{\sigma}\right)\left[N\left(\frac{e_2}{n+n^*}\right) + N^*\left(\frac{e^*_2/\tau^*}{n+n^*}\right)\right] + r\right),
\end{aligned}
$$

where the price is $p_{hm}(k_m) = \frac{\sigma}{\sigma-1}\lambda(k_m)$. Therefore, if $\lim_{\sigma\to1} \lambda'(k_m)/\lambda(k_m) > -\infty$, the marginal revenue is always less than the marginal cost, driving capital hiring k to zero, implying that $\bar{n} \to \infty$ would need to be true.

The revenue and total cost functions when $\sigma \to 1$ are:

$$
\begin{aligned}
R &= \frac{\sigma}{\sigma-1}\lambda(\hat{k}(n))\Big[Nd_h\Big(p_h(\hat{k}(n)), q_2, P\Big) + N^* d^*_h\Big(p^*_h(\hat{k}(n)), q^*_2, P^*\Big)\Big], \\
\lim_{\sigma\to1} R &= \lim_{\sigma\to1}\frac{\sigma}{\sigma-1}\lambda(\hat{k}(n))\left[N\left(\frac{e_2}{n+n^*}\right)\frac{1}{p_h(\hat{k}(n))} + N^*\left(\frac{e^*_2/\tau^*}{n+n^*}\right)\frac{1}{p^*_h(\hat{k}(n))}\right], \\
&= \left[N\left(\frac{e_2}{n+n^*}\right) + N^*\left(\frac{e^*_2/\tau^*}{n+n^*}\right)\right] > 0,
\end{aligned}
$$

where the final line is obtained by substituting in $p_h(k) = \frac{\sigma}{\sigma-1}\lambda(k)$.

$$TC = \lambda\Big(\hat{k}(n)\Big)\Big[N d_h\Big(p_h(\hat{k}(n)), q_2, P\Big) + N^* d_h^*\Big(p_h^*(\hat{k}(n)), q_2^*, P^*\Big)\Big] + \hat{r}(n)\,\hat{k}(n),$$

$$\begin{aligned}\lim_{\sigma\to 1} TC &= \lim_{\sigma\to 1} \lambda\Big(\hat{k}(n)\Big)\left[N\left(\frac{e_2}{n+n^*}\right)\frac{1}{p_h(\hat{k}(n))} + N^*\left(\frac{e_2^*/\tau^*}{n+n^*}\right)\frac{1}{p_h^*(\hat{k}(n))}\right] + \hat{r}(n)\,\hat{k}(n),\\ &= \left(\frac{0}{1}\right)\left[N\left(\frac{e_2}{n+n^*}\right) + N^*\left(\frac{e_2^*/\tau^*}{n+n^*}\right)\right] + \hat{r}(n)\,(0) = 0,\end{aligned}$$

again where the final line is obtained by substituting in $p_h(k) = \frac{\sigma}{\sigma-1}\lambda(k)$. In this case, the firm's revenue always exceeds the total costs for $n < \infty$, which means that firms will enter so that

$$\lim_{\sigma\to 1} \bar{n}^c = \infty.$$

Now, use this finding to re-check the limiting behavior of the marginal cost and product functions. First, given that $\bar{n}^c \to \infty$ and $\bar{n}^c\bar{k}^c = K$, it must hold that $\bar{k}^c \to 0$. This is consistent with profit maximization if the MRPK is less than the MCK for all $k > 0$. Simplifying the MRPK and MCK equations for a representative variety given $\bar{n}^c \to \infty$ and supposing $\bar{k}^c \in (0,\infty)$ gives

$$\begin{aligned}\lim_{\sigma\to 1} MRPK - MCK &= \lim_{\sigma\to 1}\left(-\left(\frac{\sigma-1}{\sigma}\right)\frac{\lambda'(\bar{k}^c)}{\lambda(\bar{k}^c)}\left[N\left(\frac{e_2}{\bar{n}^c+n^*}\right) + N^*\left(\frac{e_2^*/\tau^*}{\bar{n}^c+n^*}\right)\right] - \bar{r}^c\right),\\ &= -(0)\frac{\lambda'(\bar{k}^c)}{\lambda(\bar{k}^c)}\left[N e_2(0) + N^*\frac{e_2^*}{\tau^*}(0)\right] - \bar{r}^c = -\bar{r}^c.\end{aligned}$$

Therefore, the marginal revenue product of capital is lower than the marginal cost of hiring capital, driving down the per-firm demand. In other terms,

$$\lim_{\sigma\to 1} \bar{k}^c = 0.$$

Production without Tariff Commitment (Section 1.4). When $\sigma \to 1$, the marginal revenue product of capital and marginal cost of capital equations are

$$\begin{aligned} MRPK_m &= -\sigma\lambda'(k_m)\Big[Nd_{hm}\Big(p_h(k_m), q_2, P\Big) + N^* d^*_{hm}\Big(p^*_h(k_m), q^*_2, P^*\Big)\Big], \\ \lim_{\sigma\to 1} MRPK_m &= \lim_{\sigma\to 1}\left(-\sigma\lambda'(k_m)\left[N\left(\frac{e_2}{n+n^*}\right)\frac{1}{p_{hm}(k_m)} + N^*\left(\frac{e^*_2/\tau^*}{n+n^*}\right)\frac{1}{p^*_{hm}(k_m)}\right]\right), \\ &= \lim_{\sigma\to 1}\left(-(\sigma-1)\frac{\lambda'(k_m)}{\lambda(k_m)}\left[N\left(\frac{e_2}{n+n^*}\right) + N^*\left(\frac{e^*_2/\tau^*}{n+n^*}\right)\right]\right), \end{aligned}$$

$$\begin{aligned} MCK_m &= -(\sigma-1)\lambda'(k_m)\Big[Nd_{hm}\Big(p_h(k_m), q_2, P\Big) + N^* d^*_{hm}\Big(p^*_h(k_m), q^*_2, P^*\Big)\Big] \\ &\qquad + r + c + \theta'(k_m), \\ \lim_{\sigma\to 1} MCK_m &= \lim_{\sigma\to 1}\Bigg(-(\sigma-1)\lambda'(k_m)\left[N\left(\frac{e_2}{n+n^*}\right)\frac{1}{p_{hm}(k_m)} + N^*\left(\frac{e^*_2/\tau^*}{n+n^*}\right)\frac{1}{p^*_{hm}(k_m)}\right] \\ &\qquad + r + c + \theta'(k_m)\Bigg), \\ &= \lim_{\sigma\to 1}\Bigg(-(\sigma-1)\frac{\lambda'(k_m)}{\lambda(k_m)}\left(\frac{\sigma-1}{\sigma}\right)\left[N\left(\frac{e_2}{n+n^*}\right) + N^*\left(\frac{e^*_2/\tau^*}{n+n^*}\right)\right] \\ &\qquad + r + c + \theta'(k_m)\Bigg), \end{aligned}$$

where the original definition of the price, $p_{hm}(k_m) = \frac{\sigma}{\sigma-1}\lambda(k_m)$.

As for firm entry, the total revenue function when $\sigma \to 1$ is

$$\begin{aligned} R &= \frac{\sigma}{\sigma-1}\lambda(\hat{k}(n))\Big[Nd_h\Big(p_h(\hat{k}(n)), q_2, P\Big) + N^* d^*_h\Big(p^*_h(\hat{k}(n)), q^*_2, P^*\Big)\Big], \\ \lim_{\sigma\to 1} R &= \lim_{\sigma\to 1}\frac{\sigma}{\sigma-1}\lambda(\hat{k}(n))\left[N\left(\frac{e_2}{n+n^*}\right)\frac{1}{p_h(\hat{k}(n))} + N^*\left(\frac{e^*_2/\tau^*}{n+n^*}\right)\frac{1}{p^*_h(\hat{k}(n))}\right], \\ &= \left[N\left(\frac{e_2}{n+n^*}\right) + N^*\left(\frac{e^*_2/\tau^*}{n+n^*}\right)\right] > 0, \end{aligned}$$

where the final line is obtained by substituting in $p_h(k) = \frac{\sigma}{\sigma-1}\lambda(k)$. The total cost function when $\sigma \to 1$ is

$$TC = \lambda\Big(\hat{k}(n)\Big)\Big[N d_h\Big(p_h(\hat{k}(n)), q_2, P\Big) + N^* d_h^*\Big(p_h^*(\hat{k}(n)), q_2^*, P^*\Big)\Big] + \hat{r}(n)\,\hat{k}(n),$$

$$\lim_{\sigma\to 1} TC = \lim_{\sigma\to 1} \lambda\Big(\hat{k}(n)\Big)\left[N\left(\frac{e_2}{n+n^*}\right)\left(\frac{\sigma-1}{\sigma}\right)\frac{1}{\lambda(\hat{k}(n))} + N^*\left(\frac{e_2^*/\tau^*}{n+n^*}\right)\left(\frac{\sigma-1}{\sigma}\right)\frac{1}{\lambda(\hat{k}(n))}\right]$$
$$+ \hat{r}(n)\,\hat{k}(n) + c^u(\hat{k}(n), n)\,\hat{k}(n) + \theta(\hat{k}(n)),$$
$$= (0)\left[N\left(\frac{e_2}{n+n^*}\right) + N^*\left(\frac{e_2^*/\tau^*}{n+n^*}\right)\right] + \hat{r}(n)\,(0) + c^u(0,n)\,(0) + \theta(0) = 0,$$

again where the final line is obtained by substituting in $p_h(k) = \frac{\sigma}{\sigma-1}\lambda(k)$. In this case, the firm's revenue always exceeds the total costs for $n > 0$, which means that firms will enter so that

$$\lim_{\sigma\to 1} \bar{n}^u = \infty.$$

Now, use this finding to re-check the limiting behavior of the marginal cost and product functions. First, given that $\bar{n}^u \to \infty$ and $\bar{n}^u\bar{k}^u = K$, it must hold that $\bar{k}^u \to 0$. This is consistent with profit maximization if the MRPK is less than the MCK for all $k > 0$. Simplifying the MRPK and MCK equations for a representative variety given $\bar{n}^u \to \infty$ and supposing $k \in (0, \infty)$ gives

$$\lim_{\sigma\to 1} MRPK - MCK =$$
$$\lim_{\sigma\to 1}\left(-\left(\frac{\sigma-1}{\sigma}\right)\frac{\lambda'(k)}{\lambda(k)}\left[N\left(\frac{e_2}{\bar{n}^u+n^*}\right) + N^*\left(\frac{e_2^*/\tau^*}{\bar{n}^u+n^*}\right)\right] - \bar{r}^u - c - \theta'(k_m)\right),$$
$$= -(0)\frac{\lambda'(k)}{\lambda(k)}\left[N e_2(0) + N^*\frac{e_2^*}{\tau^*}(0)\right] - \bar{r}^u - c - \theta'(k_m) = -\bar{r}^u - c - \theta'(k_m).$$

Therefore, for any $k \in (0, \infty)$, marginal revenue product of capital is lower than the marginal cost of hiring capital, driving down the per-firm demand. In other terms,

$$\lim_{\sigma\to 1} \bar{k}^u = 0.$$

A.5 Derivations of First-Order Conditions of k, n, and τ

A.5.1 Ex-Ante Tariff Commitment: Behavior for change in τ

The total derivative of the capital-hiring rule with respect to the rental price of capital is[3]

$$1 = -\frac{\partial\tilde{k}(r,n,\tau)}{\partial r}\lambda''(\tilde{k})[q_{h2}(\tilde{k},n,\tau) + N^* q^*_{h2}(\tilde{k},n,\tau)] \\ - \lambda'(\tilde{k})\left[N\frac{dq_{h2}(k,n,\tau)}{dk} + N^*\frac{dq^*_{h2}(k,n,\tau)}{dk}\right]\frac{\partial\tilde{k}(r,n,\tau)}{\partial r}.$$

where the demand functions are written as $q_{h2}(k,n,\tau) \equiv d_h\Big(p_h(k),\, q_2(k,n,\tau)\ ,P(k,n,\tau)\Big)$ and $q^*_{h2}(k,n,\tau) \equiv d^*_h\Big(p^*_h(k), q^*_2(k,n,\tau), P^*(k,n,\tau)\Big)$. Simplifying the above equation shows the derivative of the capital-hiring best-response function, $\tilde{k}(r,n,\tau) \equiv \tilde{k}$ with respect to the rental price of capital is equal to

$$\frac{\partial\tilde{k}(r,n,\tau)}{\partial r} = \frac{-1}{\left(\lambda''(\tilde{k}) - \sigma\frac{\lambda'(\tilde{k})^2}{\lambda(\tilde{k})}\right)[Nq_{h2} + N^*q^*_{h2}] + (\sigma-1)\lambda'(\tilde{k})\Big[N\frac{q_{h2}}{P}\frac{\partial P}{\partial k} + N^*\frac{q^*_{h2}}{P^*}\frac{\partial P^*}{\partial k}\Big]}. \tag{A.10}$$

The denominator is positive, given the profit-maximizing conditions in part 1 of Proposition 1.1, and that total demand for a variety of good 2 is decreasing in the price index. This means equation (A.10) is negative: holding fixed the number of firms in the industry, an increase in the rental price of capital leads to a decrease in the amount of capital each firm hires.

The total derivative of capital with respect to the number of firms in the home country's sector 2 is also found by implicitly differentiating the firm's profit-maximization condition. Given some arbitrary function, $r(n,\tau)$, the total derivative of k with respect to n is

$$\frac{d\tilde{k}}{dn} = \frac{-\frac{dr(n,\tau)}{dn} - (\sigma-1)\lambda'(\tilde{k})\left[N\frac{q_{h2}}{P}\frac{\partial P}{\partial n} + N^*\frac{q^*_{h2}}{P^*}\frac{\partial P^*}{\partial n}\right]}{\left(\lambda''(\tilde{k}) - \sigma\frac{\lambda'(\tilde{k})^2}{\lambda(\tilde{k})}\right)[Nq_{h2} + N^*q^*_{h2}] + (\sigma-1)\lambda'(\tilde{k})\left[N\frac{q_{h2}}{P}\frac{\partial P}{\partial k} + N^*\frac{q^*_{h2}}{P^*}\frac{\partial P^*}{\partial k}\right]}, \tag{A.11}$$

[3]The envelope condition from the firm setting a domestic price of their product to maximize profits, given a fixed k_m and n is used to simplify, where by the envelope condition $d_{hm}(p_h, q_2, P) = -\big(\frac{1}{\sigma-1}\big)\lambda(k)\frac{d\, d_{hm}(p_h,q_2,P)}{d\, p_h}$ and $d^*_{hm}(\tau^* p_h, q_2, P) = -\big(\frac{1}{\sigma-1}\big)\lambda(k)\frac{d\, d^*_{hm}(\tau^* p_h,q_2,P)}{d\, p_h}$.

with $\tilde{k} \equiv \tilde{k}(r(n,\tau),n,\tau)$ for notational simplicity.

The third and final first-order condition for the capital-hiring rule, the derivative of $\tilde{k}(r,n,\tau)$ with respect to the tariff level, holding r and n fixed, is

$$\frac{\partial \tilde{k}(r,n,\tau)}{\partial \tau} = \frac{-(\sigma-1)\lambda'(\tilde{k})\Big[N\frac{q_{h2}}{P}\frac{\partial P}{\partial \tau}\Big]}{\Big(\lambda''(\tilde{k})-\sigma\frac{\lambda'(\tilde{k})^2}{\lambda(\tilde{k})}\Big)\left[Nq_{h2}+N^*q^*_{h2}\right]+(\sigma-1)\lambda'(\tilde{k})\Big[N\frac{q_{h2}}{P}\frac{\partial P}{\partial k}+N^*\frac{q^*_{h2}}{P^*}\frac{\partial P^*}{\partial k}\Big]}.$$

Given arbitrary functions $r \equiv r(n(\tau),\tau)$ and $n \equiv n(\tau)$, the total derivative of capital with respect to the tariff level is

$$\frac{d\tilde{k}(r,n,\tau)}{d\tau} = \frac{-\frac{dr(n(\tau),\tau)}{d\tau}-(\sigma-1)\lambda'(\tilde{k})\Big[N\frac{q_{h2}}{P}\frac{\partial P}{\partial \tau}\Big]-(\sigma-1)\lambda'(\tilde{k})\left[\mathbf{N}\frac{\mathbf{q}_{h2}}{\mathbf{P}}\frac{\partial \mathbf{P}}{\partial n}\right]\frac{dn(\tau)}{d\tau}}{\Big(\lambda''(\tilde{k})-\sigma\frac{\lambda'(\tilde{k})^2}{\lambda(\tilde{k})}\Big)\left[Nq_{h2}+N^*q^*_{h2}\right]+(\sigma-1)\lambda'(\tilde{k})\Big[N\frac{q_{h2}}{P}\frac{\partial P}{\partial k}+N^*\frac{q^*_{h2}}{P^*}\frac{\partial P^*}{\partial k}\Big]}, \tag{A.12}$$

with $\tilde{k} \equiv \tilde{k}(r(n(\tau),\tau),n(\tau),\tau)$. Again, the denominator of this equation is clearly positive and the numerator is unclear without knowing the first-order behavior of the rental price of capital and the number of firms in the industry given a change in τ.

Next, consider the first-order behavior of the rental price of capital, beginning with the behavior of r with respect to a change in n. Implicitly differentiating the capital-market-clearing condition from equation (1.15) with respect to the number of firms in the industry, given the firm's best-response hiring rule gives

$$n\frac{\partial \tilde{k}(r,n,\tau)}{\partial r}\frac{\partial \hat{r}(n,\tau)}{\partial n}+n\frac{\partial \tilde{k}(r,n,\tau)}{\partial n}+\hat{k}=0.$$

Using the definition of the derivatives $\partial\tilde{k}(r,n,\tau)/\partial r$ and $\partial\tilde{k}(r,n,\tau)/\partial n$ (from equations (A.10) and (A.11), respectively) to simplify, the derivative of the rental price of capital with respect to the number of firms in the industry is

$$\begin{aligned}\frac{d\hat{r}(n,\tau)}{dn} = \frac{1}{n}\left(\lambda(\hat{k})+(\sigma-1)\lambda'(\hat{k})\hat{k}\right)&\left[\mathbf{N}\frac{\mathbf{q}_{h2}}{\mathbf{P}}\frac{\partial \mathbf{P}}{\partial k}\right]\\ +\frac{\hat{k}}{n}&\left(\lambda''(\hat{k})-\frac{\sigma\lambda'(\hat{k})^2}{\lambda(\hat{k})}\right)[TD],\end{aligned} \tag{A.13}$$

with $\hat{k} \equiv \hat{k}(n,\tau) \equiv \tilde{k}(\hat{r}(\tau),n,\tau)$, $\mathbf{N}\frac{\mathbf{q}_{h2}}{\mathbf{P}}\frac{\partial \mathbf{P}}{\partial k} \equiv N\frac{q_{h2}}{P}\frac{\partial P}{\partial k} + N^*\frac{q^*_{h2}}{P^*}\frac{\partial P^*}{\partial k}$, and $TD \equiv Nq_{h2} + N^*q^*_{h2}$ for notational simplicity. Recalling equation (1.17a), if $n = \bar{n}^c(\tau)$, then the first term in $d\hat{r}(n,\tau)/dn$ is equal to zero, leaving only the second term. Holding fixed τ, the rental price of capital is increasing in the number of firms in sector 2.

By combining equation (A.13) with equation (A.11), I can now finish solving for the derivative of $\hat{k}(n,\tau)$ with respect to n, given that the profit-maximization condition in equation (1.13) and the capital-market-clearing condition in equation (1.15) both hold:

$$\frac{\partial \hat{k}(n,\tau)}{\partial n} = \frac{-\hat{k}(n,\tau)}{n}, \tag{A.11$'$}$$

which is exactly what this derivative must be for the response in k_2 to a change in n to maintain capital-market-clearing.

Given an arbitrary firm-entry condition, $n(\tau)$, and given firms in sector 2 hire capital according to $\tilde{k}(r,n,\tau)$, the total derivative of $\hat{r}(n(\tau),\tau)$ with respect to τ is

$$\begin{aligned}\frac{d\hat{r}(n(\tau),\tau)}{d\tau} = \frac{\partial \hat{r}(n,\tau)}{\partial \tau} + \frac{dn(\tau)}{d\tau}\frac{\lambda(\hat{k})\hat{k}}{n(\tau)}\left(\lambda''(\hat{k}) - \sigma\frac{\lambda'(\hat{k})^2}{\lambda(\hat{k})}\right)[Nq^c_{h2} + N^*q^{*c}_{h2}] \\ + \frac{dn(\tau)}{d\tau}\frac{1}{n(\tau)}\left(\lambda(\hat{k}) + (\sigma-1)\lambda'(\hat{k})\hat{k}\right)\left[N\frac{q^c_{h2}}{P}\frac{\partial P}{\partial k} + N^*\frac{q^{*c}_{h2}}{P^*}\frac{\partial P^*}{\partial k}\right],\end{aligned} \tag{A.14}$$

where the partial derivative of $\hat{r}(n,\tau)$ with respect to τ is

$$\frac{\partial \hat{r}(n,\tau)}{\partial \tau} = -(\sigma-1)\lambda'(\hat{k})\left[N\frac{q^c_{h2}}{P}\frac{\partial P}{\partial \tau}\right],$$

and with $\hat{k} \equiv \hat{k}(n(\tau),\tau)$.

Now, turn to the first-order behavior of firm entry, beginning with the derivative of n with respect to τ. Based on the firm's zero-profit condition from equation (1.16), given the TA-equilibrium capital-hiring rule $\hat{k}(n,\tau)$ and the capital-market-clearing rule $\hat{r}(n,\tau)$, the implicit derivative of firm-entry, $\bar{n}^c(\tau)$, with respect to the tariff level is

$$\begin{aligned}
&\bar{r}^c(\tau)\frac{d\bar{k}^c(\tau)}{d\tau} + \bar{k}^c(\tau)\frac{d\bar{r}^c(\tau)}{d\tau} \\
&= \lambda(\bar{k}^c(\tau))\left[N\frac{q^c_{h2}}{P^c}\frac{\partial P^c}{\partial \tau}\right] + \lambda(\bar{k}^c(\tau))\left[N\frac{q^c_{h2}}{P^c}\frac{\partial P}{\partial n} + N^*\frac{q^{*c}_{h2}}{P^{*c}}\frac{\partial P^{*c}}{\partial n}\right]\frac{d\bar{n}^c(\tau)}{d\tau} \\
&+ \left(\lambda(\bar{k}^c(\tau))\left[N\frac{q^c_{h2}}{P^c}\frac{\partial P^c}{\partial k} + N^*\frac{q^{*c}_{h2}}{P^{*c}}\frac{\partial P^{*c}}{\partial k}\right] - \lambda'(\bar{k}^c(\tau))\left[Nq^c_{h2} + N^*q^{*c}_{h2}\right]\right)\left(\frac{d\bar{k}^c(\tau)}{d\tau}\right),
\end{aligned}$$

with firm entry $\bar{n}^c \equiv \bar{n}^c(\tau)$, rental price of capital $\bar{r}^c(\tau) \equiv \hat{r}(\bar{n}^c(\tau), \tau)$, $\bar{k}^c \equiv \hat{k}(\bar{n}^c(\tau), \tau)$, and capital hiring $d\bar{k}^c(\tau)/d\tau \equiv d\tilde{k}(\bar{r}^c(\tau), \bar{n}^c(\tau), \tau)/d\tau$. Plugging in the definitions of the derivatives of $\bar{r}^c(\tau)$ and $\bar{k}^c(\tau)$ and using the equilibrium condition from equation (1.17a) to simplify, the total derivative of the firm-entry condition with respect to τ is

$$\frac{d\bar{n}^c(\tau)}{d\tau} = \frac{-\frac{\bar{n}^c}{\bar{k}^c}\left[N\frac{q_{h2}}{P}\frac{\partial P}{\partial \tau}\right]\left(\lambda(\bar{k}^c) + (\sigma-1)\lambda'(\bar{k}^c)\bar{k}^c\right)}{(w_\lambda(\bar{k}^c))^2\left[\mathbf{N}\frac{\mathbf{q}_{h2}}{\mathbf{P}}\frac{\partial \mathbf{P}}{\partial k}\right] - \bar{k}^c\left(\lambda''(\bar{k}^c) - \frac{\sigma\lambda'(\bar{k}^c)^2}{\lambda(\bar{k}^c)}\right)\left[Nq^c_{h2} + N^*q^{*c}_{h2}\right]}, \tag{A.15}$$

with the dependence of producer equilibrium variables on τ dropped for notational simplicity. Again, recalling the equilibrium condition in equation (1.17a), if the model is in equilibrium, then $d\bar{n}^c(\tau)/d\tau = 0$.

Now that I have derived the derivative $dn(\tau)/d\tau$, I can further solve for the impact of a change in τ on the rental price of capital. Using equation (A.15) to simplify equation (A.14), the total derivative of $\bar{r}^c(\tau)$ with respect to τ becomes

$$\frac{d\bar{r}^c(\tau)}{d\tau} = -\lambda'(\bar{k}^c(\tau))\left[\frac{\partial q_{h2}}{\partial P}\frac{\partial P}{\partial \tau}\right]. \tag{A.14'}$$

In words, any change in the firm's revenues due to a change in the tariff level chosen by the government is completely offset by a change in the rental price of capital.

The definition of $d\bar{r}^c(\tau)/d\tau$ then allows me to simplify the equilibrium first-order conditions for capital-hiring and firm-entry:

$$\frac{d\bar{k}^c(\tau)}{d\tau} = 0, \quad \text{and} \quad \frac{d\bar{n}^c(\tau)}{d\tau} = 0. \qquad \text{(A.12}'\text{), (A.15}'\text{)}$$

Because the impact of changes in the tariff level on a firm's profits is offset by the change in $\bar{r}^c(\tau)$, neither $\bar{k}^c$ nor $\bar{n}^c$ are impacted by a change in τ. The findings of this section are summarized in Proposition 1.2.

A.5.2 Ex-Ante Tariff Commitment: Behavior of Producer Eq'm, Tariff Level

The most straightforward way to find the impact of a change in σ on the equilibrium amount of capital hired by each firm and on the number of varieties of the sector 2 good produced is to examine the equilibrium conditions which characterize the values of k and n. Recall from Proposition 1.1 (equation (1.17a) specifically) that in equilibrium, $\lambda(\bar{k}^c) + (\sigma - 1)\lambda'(\bar{k}^c)\bar{k}^c = 0$, given that the capital-market-clearing condition, $\bar{n}^c\,\bar{k}^c = K$, also holds. Implicitly differentiating the the producers' equilibrium condition with respect to σ and solving for $d\bar{k}^c/d\sigma$ gives

$$\lambda'(\bar{k}^c)\frac{d\bar{k}^c}{d\sigma} + \lambda'(\bar{k}^c)\bar{k}^c + (\sigma-1)\lambda''(\bar{k}^c)\bar{k}^c\frac{d\bar{k}^c}{d\sigma} + (\sigma-1)\lambda'(\bar{k}^c)\frac{d\bar{k}^c}{d\sigma} = 0,$$

$$\frac{d\bar{k}^c}{d\sigma} = \frac{-\lambda'(\bar{k}^c)\bar{k}^c}{(\sigma-1)\lambda''(\bar{k}^c)\bar{k}^c + \sigma\lambda'(\bar{k}^c)}. \qquad \text{(eq. 1.22)}$$

The capital hired by each firm is increasing in σ if $(\sigma-1)\lambda''(\bar{k}^c)\bar{k}^c + \sigma\lambda'(\bar{k}^c) > 0$. Given that $\bar{k}^c = -\lambda(\bar{k}^c)/\big((\sigma-1)\lambda'(\bar{k}^c)\big)$, this simplifies to $-\big(\lambda(\bar{k}^c)/\lambda'(\bar{k}^c)\big)\big(\lambda''(\bar{k}^c) - \sigma\lambda'(\bar{k}^c)^2/\lambda(\bar{k}^c)\big) > 0$. Given that $\lambda''(\bar{k}^c) - \sigma\lambda'(\bar{k}^c)^2/\lambda(\bar{k}^c) > 0$ must be true for the second-order condition of profit maximization, it must be true that the value of $\bar{k}^c$ is increasing in σ. Using the market clearing condition, the derivative with respect to $\bar{n}^c$ is

$$\frac{d\bar{n}^c}{d\sigma} = \frac{-\lambda'(\bar{k}^c)\bar{k}^c}{(\sigma-1)\lambda''(\bar{k}^c)\bar{k}^c + \sigma\lambda'(\bar{k}^c)} \left(\frac{-\bar{n}^c}{\bar{k}^c}\right),$$

which can be shown to be negative. These results are summarized in Proposition 1.4 in Section 1.3.3.

Finally, consider the first-order behavior of the tariff with respect to a change in the elasticity of substitution across varieties of the differentiated good. The partial derivative of the tariff level, τ^c, with respect to σ is

$$\frac{d\tau^c}{d\sigma} = \frac{(\tau^c)^2 z^c \left(\left(\frac{1}{\sigma}\right)^2 - \frac{\tau^c-1}{\tau^c} + \frac{\tau^c-1}{\tau^c}\ln\left(\frac{p_f}{p_h(\bar{k}^c)}\right)\frac{1}{z^c}\right)}{(\sigma z^c+1) - (\sigma-1)(\tau^c-1)}, \qquad \text{(eq. 1.23)}$$

The sign of equation (1.23) is difficult to discern due to the presence of the term $\ln\left(p_f/p_h(\bar{k}^c)\right)$, the natural log of the home country's terms of trade.[4] The home country's terms of trade are $p_f/p_h(\bar{k}^c) = (\tau^c\lambda^*)/\lambda(\bar{k}^c)$. Further discussion of equation (1.23) is included in Section 1.3.3.

[4] To solve for this derivative, the partial derivative of the price indexes with respect to σ is

$$\begin{aligned} \frac{\partial P}{\partial \sigma} &= \frac{1}{\sigma-1} P\Big[-\frac{1}{\sigma} - \frac{z}{z+1}\ln\Big(\frac{P}{p_h(k)}\Big) - \frac{1}{z+1}\ln\Big(\frac{P}{p_f}\Big)\Big], \text{ and} \\ \frac{\partial P^*}{\partial \sigma} &= \frac{1}{\sigma-1} P^*\Big[-\frac{1}{\sigma} - \frac{1}{z^*+1}\ln\Big(\frac{P^*}{p_h^*(k)}\Big) - \frac{z^*}{z^*+1}\ln\Big(\frac{P^*}{p_f^*}\Big)\Big], \end{aligned} \qquad \text{(A.16)}$$

and the partial derivatives of the demand for the home- and foreign-produced goods are

$$\frac{\partial q_{h2}}{\partial \sigma} = (q_{h2})\Big[\frac{1}{\sigma(\sigma-1)} + \frac{1}{z+1}\ln\Big(\frac{p_f}{p_h(k)}\Big)\Big] \text{ and } \frac{\partial q_{f2}}{\partial \sigma} = (q_{f2})\Big[\frac{1}{\sigma(\sigma-1)} - \frac{z}{z+1}\ln\Big(\frac{p_f}{p_h(k)}\Big)\Big], \qquad \text{(A.17)}$$

where the foreign demand for a home variety is $\partial q_{h2}^*/\partial\sigma = q_{h2}^*\left[1/(\sigma(\sigma-1)) + z^*/(z^*+1)\ln(p_f^*/p_h^*(k))\right]$.

A.5.3 Ex-Post Tariff Selection with Lobbying: Behavior of Producer Equilibrium and Bargaining Solution

To begin, examine the impact on the tariff, $\tau^u \equiv \tau^u(\bar{k}^u, \bar{n}^u)$, of a change in the political economy parameter, a. The partial derivative of τ^u with respect to a is

$$\frac{\partial \tau^u}{\partial a} = \frac{\left(\frac{\sigma-1}{\sigma}\right)(\tau^u)^2\ z^u}{(\sigma z^u + 1) - (\sigma - 1)(\tau^u - 1)}, \tag{A.18}$$

which is positive given the tariff's SOC for welfare-maximization.[5] Using capital-market clearing, the definition of the derivative of $\tau^u \equiv \tau^u(\bar{k}^u, \bar{n})$ with respect to a is:

$$\frac{d\tau^u}{da} = \frac{\tau^u(\tau^u - 1)}{(\sigma z^u + 1) - (\sigma - 1)(\tau^u - 1)} \left[\left(\frac{1 + a}{\sigma z^u + 1} \right) + \left(\frac{w_\lambda(\bar{k}^u)}{\bar{n}^u \lambda(\bar{k}^u)} \right) \frac{d\bar{n}^u}{da} \right]. \tag{eq. 1.37}$$

Therefore, it is only true that τ^u is increasing in a when the derivative of $\bar{n}^u$ with respect to a is such that

$$\frac{d\bar{n}^u}{da} < \left(\frac{\sigma - 1}{\sigma} \right) \left(\frac{\tau^u}{\tau^u - 1} \right) \frac{-\bar{n}^u \lambda(\bar{k}^u) z^u}{w_\lambda(\bar{k}^u)}. \tag{A.19}$$

If $d\bar{n}^u/da$ is negative, this inequality is trivially satisfied since the right hand side is positive. If $d\bar{n}^u/da > 0$, on the other hand, I need to know the definition of $d\bar{n}^u/da$ to draw any conclusions regarding the sign of $d\tau^u/da$ for if $d\bar{n}^u/da < 0$.

The first-order response of $\bar{n}^u$ to a change in the political pressure, a, is

$$\frac{d\bar{n}^u}{da} = \frac{w_\lambda(\bar{k}^u) \left[N \frac{q^u_{h2}}{P^u} \frac{\partial P^u}{\partial \tau} \frac{\partial \tau^u}{\partial a} \right]}{\frac{-(\bar{k}^u)^2}{\bar{n}^u} \left(- \left(\lambda''(\bar{k}^u) - \sigma \frac{(\lambda'(\bar{k}^u))^2}{\lambda(\bar{k}^u)} \right) TD^u - \theta''(\bar{k}^u) \right) - w_\lambda(\bar{k}^u) \left[\mathbf{N} \frac{\mathbf{q}^u_{h2}}{\mathbf{P}^u} \frac{d\mathbf{P}^u}{dn} \right]}, \tag{eq. 1.39}$$

where $TD^u \equiv \left[N q^u_{h2} + N^* q^{*u}_{h2} \right]$ is total demand for a firm's output; $\mathbf{N} \frac{\mathbf{q}_{h2}}{\mathbf{P}} \frac{d\mathbf{P}}{dn} \equiv N \frac{q_{h2}}{P} \frac{dP}{dn} + N^* \frac{q^*_{h2}}{P^*} \frac{dP^*}{dn}$ is the total change demand following a change in the price index due to a change in n, holding fixed the price p_h, and with $\frac{dP}{dn} \equiv \frac{\partial P}{\partial k} \frac{dk}{dn} + \frac{\partial P}{\partial n} + \frac{\partial P}{\partial \tau} \left(\frac{\partial \tau}{\partial n} + \frac{\partial \tau}{\partial k} \frac{dk}{dn} \right)$.

To determine the sign of equation (1.39), first examine the numerator. In order for the

[5]The envelope condition from the firm setting a domestic price of their product to maximize profits, given a fixed k_m and n is used to simplify, where by the envelope condition $d_{hm}(p_h, q_2, P) = -(\frac{1}{\sigma-1})\lambda(k)(d\, d_{hm}(p_h, q_2, P)/d\, p_h)$ and $d^*_{hm}(\tau^* p_h, q_2, P) = -(\frac{1}{\sigma-1})\lambda(k)(d\, d^*_{hm}(\tau^* p_h, q_2, P)/d\, p_h)$.

ex-ante equilibrium condition from equation (1.35a) to hold, it must be true that $w_\lambda(\bar{k}^u) < 0$. Therefore, given that $\partial P^u/\partial\tau > 0$ and $\partial\tau^u/\partial a > 0$, the numerator of equation (1.39) is negative.

Next, examine the denominator of equation (1.39). The first main term in the denominator, $-(\bar{k}^u)^2/\bar{n}^u\left(-\left(\lambda''(\bar{k}^u) - \sigma\lambda'(\bar{k}^u)^2/\lambda(\bar{k}^u)\right)[TD^u] - \theta''\right)$, is equal to the firm's second-order condition for profit maximization (equation (1.32)) times $(d\bar{k}^u/d\bar{n}^u * \bar{k}^u) \equiv -(\bar{k}^u)^2/\bar{n}^u$. As a result, this term must always be positive. The second half of the denominator, $w_\lambda(\bar{k}^u)\left[\mathbf{N}\frac{\mathbf{q}^u_{h2}}{\mathbf{P}^u}\frac{d\mathbf{P}^u}{dn}\right]$, is the first-order effect on demand resulting from the home and foreign price indexes' response to a change in $\bar{n}^u$, then multiplied by the ex-ante production wedge. Given the ex-ante production wedge is negative, if $\mathbf{N}\frac{\mathbf{q}^u_{h2}}{\mathbf{P}^u}\frac{d\mathbf{P}^u}{dn}$ is positive, then the entire denominator must also be positive. In equation (1.40) I expand and simplify the price index term, showing that it is composed of two main parts: (i) the effect of a change in $\bar{n}^u$ on home and foreign demand by way of the price indexes while holding τ^u fixed and given $\bar{k}^u = K/\bar{n}^u$, which is positive, and (ii) the effect of a change in $\bar{n}^u$ on demand by way of the effect on the price index through τ^u, which is negative.

Substituting in equation (1.40), the denominator of equation (1.39) is

$$\frac{(\bar{k}^u)^2}{\bar{n}^u}\left(\theta''(\bar{k}^u) + \left(\lambda''(\bar{k}^u) - \sigma\frac{\lambda'(\bar{k}^u)^2}{\lambda(\bar{k}^u)}\right)[Nq^u_{h2} + N^* q^{*u}_{h2}]\right) + \frac{(w_\lambda(\bar{k}^u))^2}{(\sigma-1)\bar{n}^u\lambda(\bar{k}^u)}\left[Nq^u_{h2}\frac{\left(\frac{z^u}{z^u+1}\right)\frac{\sigma z^u+1}{\sigma-1} - (\tau^u - 1)}{\frac{\sigma z^u+1}{\sigma-1} - (\tau^u-1)} + N^* q^{*u}_{h2}\left(\frac{1}{z^{*u}+1}\right)\right]. \tag{A.20}$$

Overall, a sufficient assumption for the denominator of equation (1.39) to be positive is to assume that the direct effect of $\bar{n}^u$ on demand is larger than the indirect effect coming through τ^u (i.e. that equation (1.40) is positive). Rather than the restrictive sufficient condition, I make use of a less-restrictive assumption – that equation (A.20) is greater than zero. If the denominator of $d\bar{n}^u/da$ is positive, given that the numerator is negative, then the number of firms in the lobbying sector is decreasing in a. Figure A.2 shows that the denominator is greater than zero, for changes in σ for a fixed a. These findings regarding

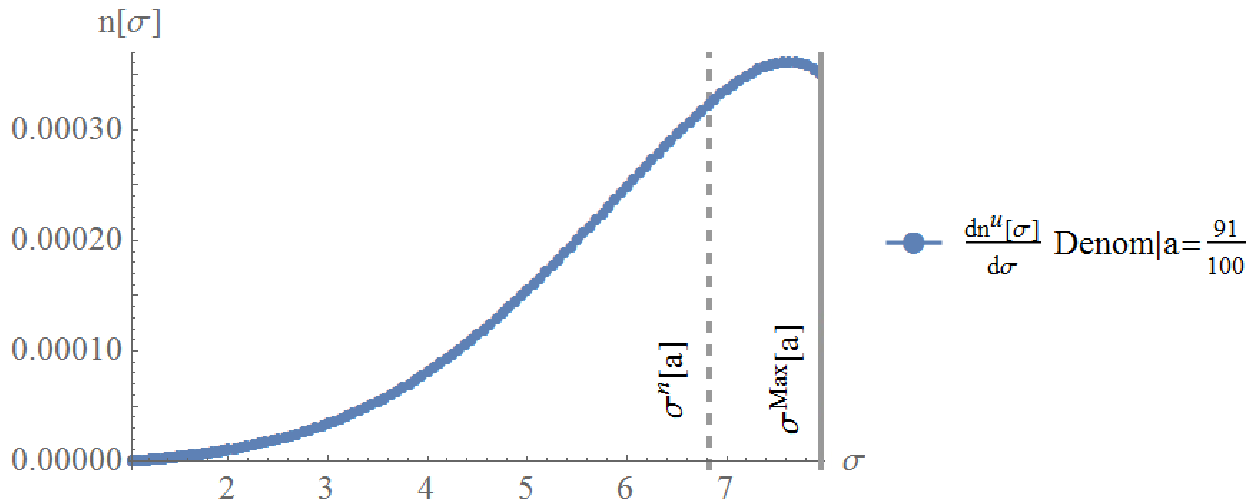


Figure A.2: Derivative of ex-post firm entry when tariffs are set ex post, given the best response tariff, τ^u, where σ^{Max} is the maximum relevant value of σ, defined by the point where the firm's second-order condition for profit maximization is equal to zero. The value of σ for which the denominator of $d\bar{n}^u/d\sigma$ is equal to zero, σ^{dn}, is not pictured and occurs around $\sigma = 9$. I restrict the relevant range of σ to $\sigma < \sigma^{Max}$.

the first-order behavior of $\bar{n}^u$ in response to a change in a are summarized in Section 1.4.5 in Proposition 1.11.

Returning to analysis of the behavior of $d\tau^u/da$, I can now use the definition of $d\bar{n}^u/da$ from equation (1.39) to simplify and solve the inequality from equation (A.19). First suppose that $d\bar{n}^u/da > 0$, which means the denominator of $d\bar{n}^u/da$ must be negative. Simplifying the inequality from equation (A.19), it can be shown that $d\tau^u/da > 0$ is not possible. Instead, if $d\bar{n}^u/da > 0$, then it must be true that $d\tau^u/da < 0$ given that

$$\frac{(\bar{k}^u)^2}{\bar{n}^u}(-kSOC^u) + \frac{(w_\lambda(\bar{k}^u))^2}{(\sigma-1)\bar{n}^u\lambda(\bar{k}^u)}\left[Nq^u_{h2}\left(\frac{z^u}{z^u+1}\right) + N^* q^{*u}_{h2}\left(\frac{1}{z^{*u}+1}\right)\right] > 0,$$

where $(kSOC^u) \equiv -\Big(\lambda''(\bar{k}^u) - \sigma\lambda'(\bar{k}^u)^2/\lambda(\bar{k}^u)\Big)[TD^u] - \theta''(\bar{k}^u) < 0$. The above inequality is true for the long-run equilibrium firm entry and capital hiring given the second-order condition for profit maximization is satisfied.

In summary, if $d\tau^u/da > (<)0$, then $d\bar{n}^u/da < (>)0$ must be true. Additionally, assuming that the direct effect of a change in the ex-ante producer equilibrium on demand

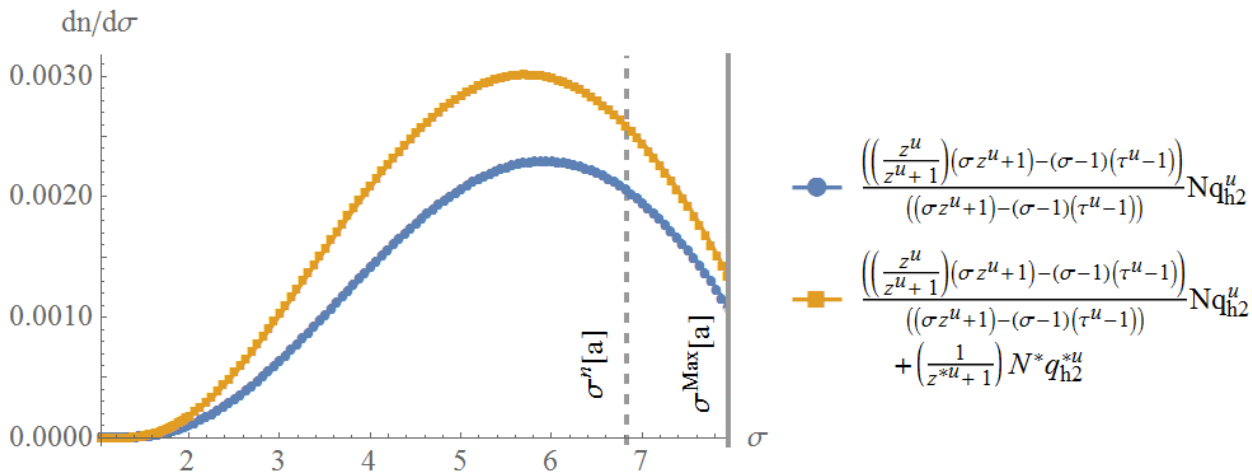


Figure A.3: The impact of a change in $\bar{n}^u$ on the domestic demand by way of the price index, $N\frac{q^u_{h2}}{P}\left(\frac{\partial P^u}{\partial k}\frac{d\bar{k}^u}{dn}+\frac{\partial P}{\partial n}\right)$ and the total effect on demand for home varieties by way of home and foriegn price indexes, $\mathbf{N}\frac{\mathbf{q}^u_{h2}}{\mathbf{P}^u}\frac{\partial \mathbf{P}^u}{\partial n}$, given $a = 91/100$. On the graph are two critical sigmas: σ^n is the value of sigma where $d\bar{n}^u/d\sigma = 0$; σ^{Max} is the cutoff of the elasticities I consider defined in Assumption 1.10.

for home varieties, $\mathbf{N}\frac{\mathbf{q}^u_{h2}}{\mathbf{P}^u}\frac{\partial \mathbf{P}^u}{\partial n} \equiv N\frac{q^u_{h2}}{P}\left(\frac{\partial P^u}{\partial k}\frac{d\bar{k}^u}{dn}+\frac{\partial P}{\partial n}\right) + N^*\frac{q^{*u}_{h2}}{P^{*u}}\left(\frac{\partial P^{*u}}{\partial k}\frac{d\bar{k}^u}{dn}+\frac{\partial P^{*u}}{\partial n}\right)$, is larger than the indirect effect of the same change by way of the tariff, $N\frac{q^u_{h2}}{P^u}\left(\frac{\partial P^u}{\partial \tau}\frac{d\tau^u}{dn}\right)$, implies that $d\bar{n}^u/da < 0$. Figure A.3 demonstrates these price index derivatives for the numerical solution. These findings are summarized in Proposition 1.11 in Section 1.4.5.

Next, examine the impact of a change in σ on the model. As was true for τ^c, the direct impact of σ on τ^u is difficult to sign analytically. The total derivative of the tariff with respect to the elasticity of substitution is

$$\frac{d\tau^u}{d\sigma} = \frac{\partial \tau^u}{\partial \sigma} + \frac{\tau^u(\tau^u-1)}{(\sigma z^u+1)-(\sigma-1)(\tau^u-1)}\left(\frac{w_\lambda(\bar{k}^u)}{\bar{n}^u\lambda(\bar{k}^u)}\right)\frac{d\bar{n}^u}{d\sigma}, \qquad \text{(eq. 1.42)}$$

with the partial derivative of the tariff with respect to σ, holding fixed $\bar{k}^u$ and $\bar{n}^u$, defined

$$\frac{\partial \tau^u(\bar{k}^u,\bar{n}^u)}{\partial \sigma} = \left(\tau^u\right)^2 \frac{\left(\frac{1}{\sigma}\right)^2(1+a)z^u - \left(\frac{\tau^u-1}{\tau^u}\right)z^u + \left(\frac{\tau^u-1}{\tau^u}\right)\ln\left(\frac{p^u_f}{p_h(\bar{k}^u)}\right)}{(\sigma z^u+1)-(\sigma-1)(\tau^u-1)}. \qquad \text{(eq. 1.43)}$$

Using equations (1.42) and (1.43) to find how $d\bar{n}^u/d\sigma$ and $d\tau^u/d\sigma$ relate, it can be shown that τ^u is increasing in σ if

$$\frac{d\bar{n}^u}{d\sigma} < \left(\frac{\bar{n}^u \lambda(\bar{k}^u)}{w_\lambda(\bar{k}^u)}\right)\left[\left(\frac{\sigma-1}{\sigma}\right)(z^u+1)\left(\frac{z^u}{z^u+1}-\left(\frac{1}{\sigma-1}\right)^2\right)-\ln\left(\frac{p_f^u}{p_h(\bar{k}^u)}\right)\right]. \tag{A.21}$$

Unlike with the inequality in equation (A.19), the sign of the right-hand side is no longer strictly positive. Now, the right hand side is either positive or negative, depending on the sign of the partial derivative of the tariff, $\partial\tau^u/\partial\sigma$.

The derivative of firm entry with respect to the elasticity of substitution is given in equation (1.44). It can be expanded to the following:

$$\frac{d\bar{n}^u}{d\sigma} = \frac{-\left(\frac{1}{\sigma-1}\right)^2 \lambda(\bar{k}^u)[TD^u] + \left(\frac{1}{\sigma-1}\right) w_\lambda(\bar{k}^u)\left[N\left(\frac{\partial q_{h2}^u}{\partial\sigma} + (\sigma-1)\frac{q_{h2}^u}{P^u}\frac{\partial P^u}{\partial\tau}\frac{\partial\tau^u}{\partial\sigma}\right) + N^*\frac{\partial q_{h2}^{*u}}{\partial\sigma}\right]}{-\frac{(\bar{k}^u)^2}{\bar{n}^u}\left(-\left(\lambda''(\bar{k}^u) - \sigma\frac{(\lambda'(\bar{k}^u))^2}{\lambda(\bar{k}^u)}\right)[TD^u] - \theta''(\bar{k}^u)\right) - w_\lambda(\bar{k}^u)\left[\mathbf{N}\frac{\mathbf{q}_{h2}^u}{\mathbf{P}^u}\frac{d\mathbf{P}^u}{dn}\right]}. \tag{eq. 1.44}$$

Due to Assumption 1.10, I already know the denominator of equation (1.44) is positive. The numerator is composed of two main parts: (i) the impact on the per-unit net revenue holding demand fixed, and (ii) the impact on demand holding the per-unit net revenue fixed. The first term of the numerator is negative: an increase in σ decreases the market power of a producer, leading to a fall in the per-unit net revenue. The second term is non-monotonic. From the definitions of $\partial q_{h2}^u/\partial\sigma$ in equation (A.17) and $\partial\tau^u/\partial\sigma$ in equation (1.43), it can be shown that if $\partial\tau^u/\partial\sigma > 0$, then $\partial q_{h2}^u/\partial\sigma > 0$ as well, which means the second term of the numerator is negative, making the whole numerator negative. This indicates that if $\partial\tau^u/\partial\sigma > 0$, then $d\bar{n}^u/d\sigma$ must be positive. Additionally, it can be shown that if $\partial q_{h2}^u/\partial\sigma < 0$, then $\partial\tau^u/\partial\sigma < 0$, but the relationship between these variables and $\partial q_{h2}^{*u}/\partial\sigma$ is unknown making the sign of the numerator still unclear.

To further examine the behavior of firm entry, a more simplified version of the numerator of the firm-entry derivative is

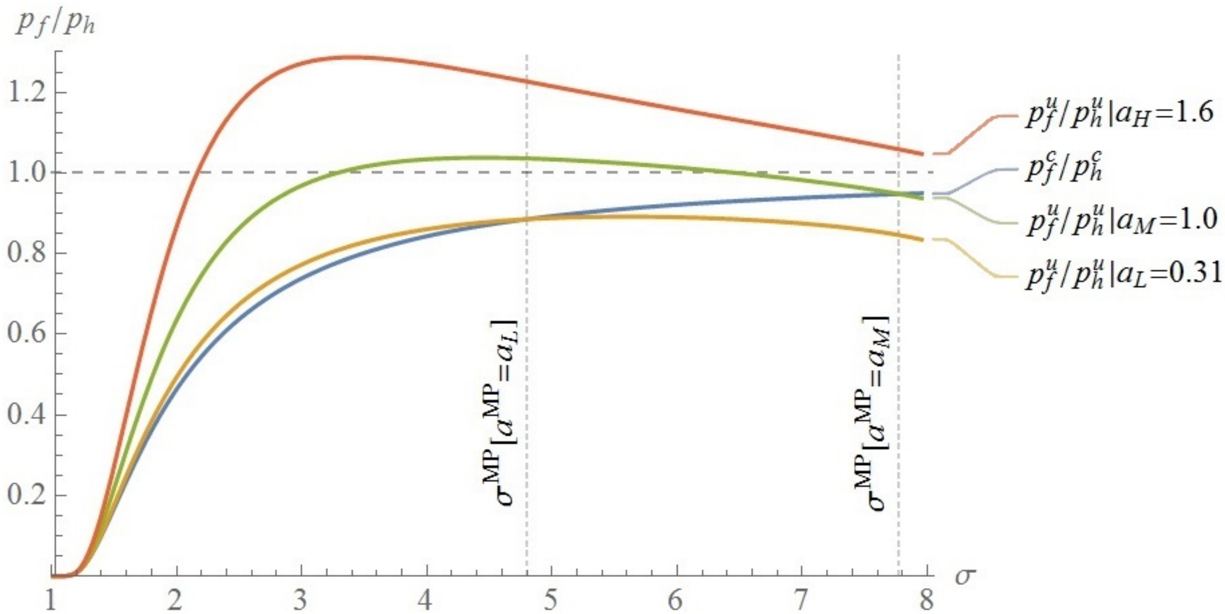
$$
\begin{aligned}
&-\frac{(\lambda^u - \lambda^{u\prime}\bar{k}^u)}{\sigma(\sigma-1)}[TD^u] - w_\lambda(\bar{k}^u)\left[Nq^u_{h2}\frac{\frac{1}{\sigma}(\tau^u-1)(\sigma-1)^2\left(\left(\frac{1}{\sigma-1}\right)^2 - \frac{z^u}{z^u+1}\right)}{(\sigma z^u+1)-(\sigma-1)(\tau^u-1)}\right] \\
&- w_\lambda(\bar{k}^u)\left[Nq^u_{h2}\frac{\left(\frac{1}{\sigma-1}\right)^2\frac{\sigma z^u+1}{z^u+1}}{\frac{\sigma z^u+1}{\sigma-1}-(\tau^u-1)}\ln\left(\frac{p^u_f}{p^u_h}\right) + N^* q^{*u}_{h2}\left(\frac{z^{*u}}{z^{*u}+1}\right)\ln\left(\frac{p^*_f}{p^{*u}_h}\right)\right].
\end{aligned}
\tag{A.22}
$$

Begin with the first line of equation (A.22). The first term, $\frac{1}{\sigma}\left(\frac{1}{\sigma-1}\right)(\lambda^u - \lambda^{u\prime}\bar{k}^u)(TD^u)$, is clearly negative given $\lambda'(k) < 0$. The second term may be either positive or negative. I discussed the sign of $(1/(\sigma-1))^2 - (z^u/(z^u+1))$ when I derived $d\tau^c/d\sigma$ and $d\tau^u/d\sigma$. This term is positive when σ is low and is negative for high values of σ. Overall, this means that when σ is high the first line of equation (A.22) is definitely negative. When σ is low, the first line may be either positive or negative.

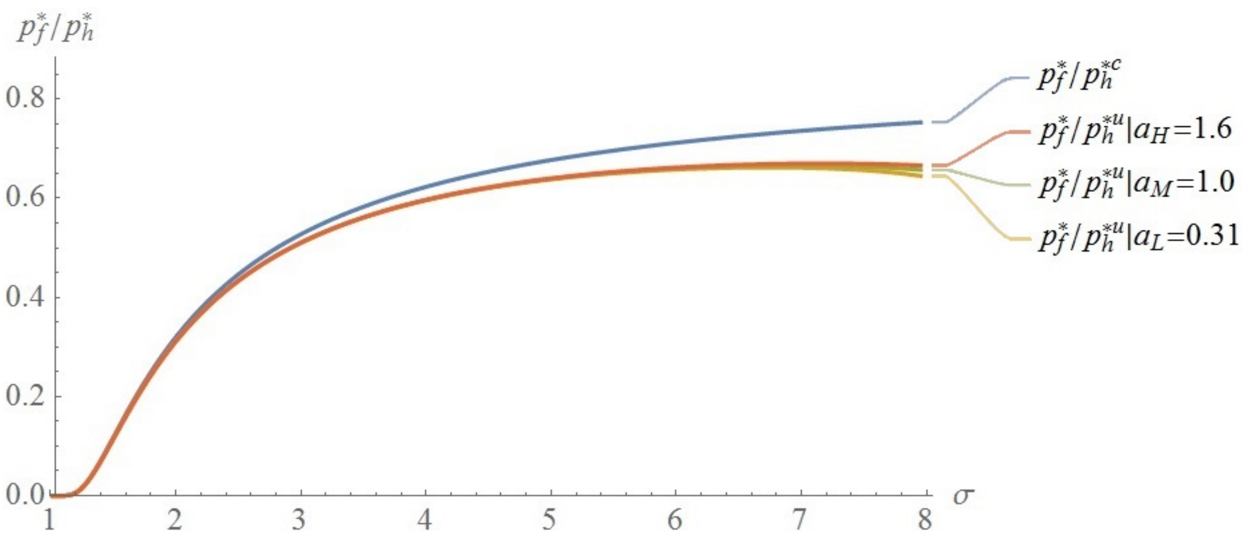
Given the coefficient on the second line of equation (A.22) is positive ($-w_\lambda(\bar{k}^u) > 0$) the sign of the term overall is determined by the log of the domestic relative price, $\ln(p^u_f/p^u_h)$, and the foreign relative price, $\ln(p^*_f/p^{*u}_h)$. Before I begin analytical work on these terms, note that Figure A.4 shows the relative prices at home and abroad for different values of a over a range of σs.

First, $\ln(p^*_f/p^{*u}_h)$ is always negative. This must be true given that $\lambda(\bar{k}^c) > \lambda^*$ for $\sigma \in (1,\infty)$ and that $\lambda(\bar{k}^u) > \lambda(\bar{k}^c)$. The domestic relative price $\ln(p^u_f/p^u_h)$ is nonmonotonic in σ. To demonstrate this, I can establish that when σ is either very low or very high $\ln(p^u_f/p^u_h) < 0$. As $\sigma \to 1$, capital hiring $\bar{k}^u \to 0$, which means that $p^u_h \to \infty$ pushing p^u_f/p^u_h toward zero. Also driving down p^u_f/p^u_h is the foreign price, since $\lim_{\sigma\to 1}\tau^u = 1$ which means $\lim_{\sigma\to 1} p^u_f/p^u_h = (\lambda^*/\infty)$, resulting in $\ln(p^u_f/p^u_h) < 0$. As σ gets very large, p^u_f again approaches p^*_f since $\lim_{\sigma\to\sigma^{Max}}\tau^u$ is finite. When $\sigma \to \sigma^{Max}$, $\bar{k}^u$ is decreasing toward zero, which again drives up $\lambda(\bar{k}^u)$ since $\lim_{k\to 0}\lambda(k) = \infty$. Therefore, it must be true that for σ very high or very low, the $\ln(p^u_f/p^u_h)$ term is negative. As a result, the second line of equation (A.22) is negative when σ is very low or very high. The high range of σ for which $p^u_f/p^u_h < 1$ does not

Figure A.4: The relative price of foreign varieties compared to home varieties for changes in σ at home, p_f^u/p_h^u, and abroad, p_f^*/p_h^{*u}, for three values of a. In both panels, the blue line demonstrates the relative price at the social-welfare maximizing production and tariff levels. The relative price for the politically optimal solution is given for three values of a: the yellow line is $a_L = 0.31$; the green line is $a_M = 1.0$; and the orange line is $a_H = 1.6$. The panels show that p_f^u/p_h^u is non-monotonic in σ and that an increase in a shifts p_f^u/p_h^u upward and p_f^*/p_h^{*u} downward.



(a) The relative price of foreign varieties compared to home varieties at home, p_f^u/p_h^u, for changes in σ.



(b) The relative price of foreign varieties compared to home varieties abroad, p_f^*/p_h^{*u}, for changes in σ

necessarily occur before σ^{Max}. Therefore, the observed pattern may be one of three things: (i) a is low results in $p_f^u/p_h^u < 1$ for all $\sigma \in (1, \sigma^{Max})$; (ii) when a is high $p_f^u/p_h^u > 1$ for a range $\sigma \in (\sigma_1, \sigma_2) \subset (1, \sigma^{Max})$; (iii) when a is very large $p_f^u/p_h^u > 1$ for any $\sigma \in (\sigma_1, \sigma^{Max})$. When a is very large, however, the value of σ above which $p_f^u/p_h^u < 1$ shifts upward, potentially resulting in this point occurring for a value of sigma greater than σ^{Max}.

For intermediate values of σ, the effect of a on the sign of $\ln(p_f^u/p_h^u)$ is evident using the numerical solution. Figure A.5 demonstrates how a effects τ^u and $\bar{k}^u$ for two "mid-range" values of σ. The figure shows that τ^u is convex and increasing in a while $\bar{k}^u$ is convex in a when a is low and concave when a is high, with the inflection point decreasing as σ rises.[6] The figure also shows how the behavior of $\bar{k}^u$ translates into the relative price level specifically, since $p_f^u/p_h^u = \tau^u\lambda^*/\lambda(\bar{k}^u)$. Still examining the sign of $\ln(p_f^u/p_h^u)$ for the mid-range σs, as $a \to \left(\frac{\sigma}{\sigma-1}\right)\sigma - 1$ the tariff level is increasing at a rate faster than $\lambda(\bar{k}^u)$ is falling, which leads to $p_f^u > p_h^u$. When a is very low, τ^u is at its lowest and $\lambda(\bar{k}^u)$ is at its highest, potentially leading to $p_f^u < p_h^u$.

Return to Figure A.4a. For the low value of a depicted in Figure 1.5, the relative price never goes above one. For a high value of a, p_f^u/p_h^u does rise above one. While signing $\ln(p_f^u/p_h^u)$ would be useful for determining the sign of $d\bar{n}^u/d\sigma$, returning to Figure A.4a it is clear that the sign of $\ln(p_f^u/p_h^u)$ does not determine whether $d\bar{n}^u/d\sigma$ is positive or negative: $\bar{n}^u$ is nonmonotonic in σ for all values of a.

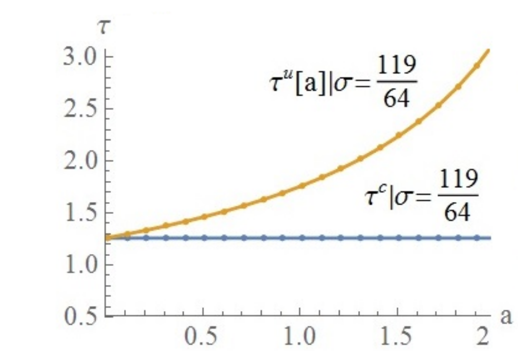
Combining these observations regarding equation (A.22) to look at the numerator as a whole, I am unable to draw conclusions regarding the sign of $d\bar{n}^u/d\sigma$ without relying on the numerical solution to the model.

A.5.4 Comparison of $\bar{k}^c$ and $\bar{k}^u$ for changes in σ

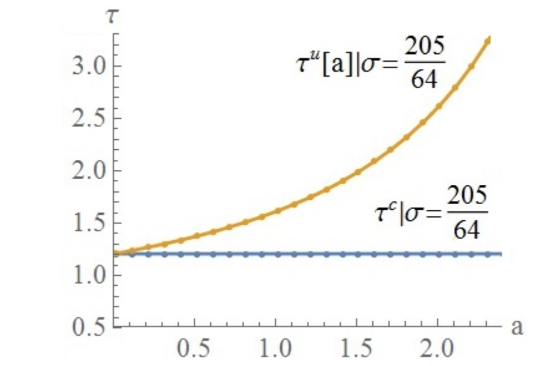
In addition to showing the behavior of $\bar{n}^u$ in σ, Figure 1.6 also demonstrates an interesting feature of the size of the production distortion: the size of the wedge between $\bar{k}^c$ and $\bar{k}^u$ is

[6] A key reason I am unable to establish these second derivatives analytically is because $d^2\tau^u/da^2$ and $d^2\bar{n}^u/da^2$ depend on the third-derivatives $\lambda'''(k)$ and $\theta'''(k)$, which cannot be signed intuitively.

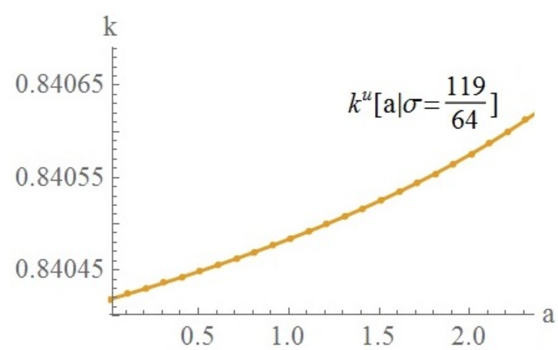
Figure A.5: The behavior of the tariff, τ^u, capital hiring, $\bar{k}^u$, and the unit-labor-input requirement, $\lambda(\bar{k}^u)$, for changes in a. Both the tariff level and capital are increasing in a, but the tariff is convex while capital hiring is initially convex and then concave in a, with the inflection point falling as σ rises.



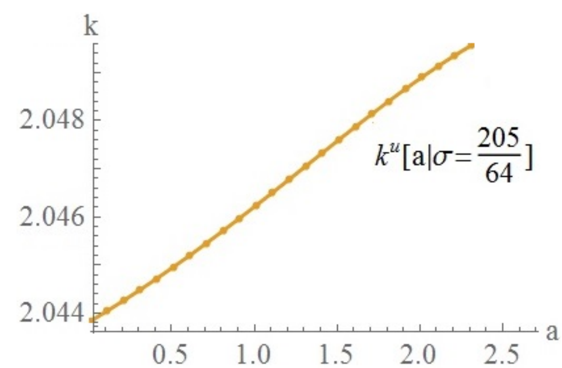
(a) Tariff levels when σ is low.



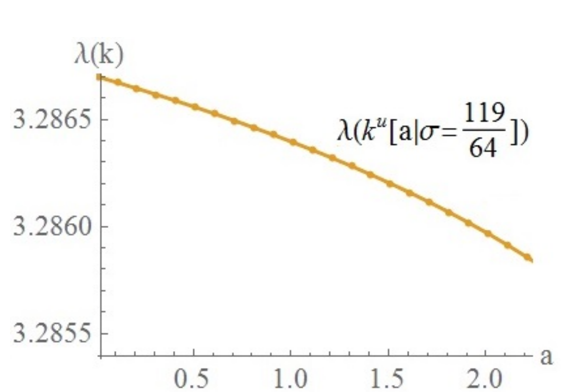
(b) Tariff levels when σ is high.



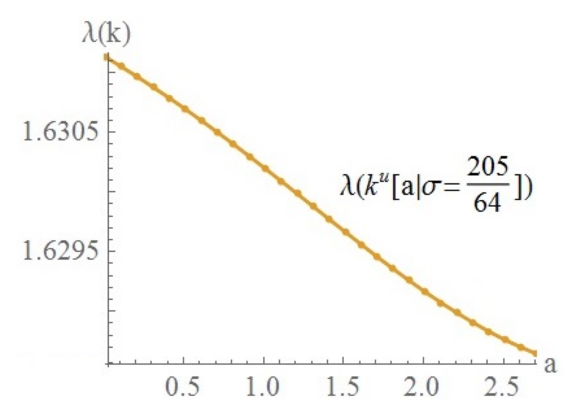
(c) Capital hiring when σ is low.



(d) Capital hiring when σ is high.



(e) Labor-input requirement when σ is low.



(f) Labor-input requirement when σ is high.

monotonically increasing in σ. Taking as given that $\bar{k}^u$ is nonmonotonic in σ, it is simple to establish that the gap between $\bar{k}^u$ and $\bar{k}^c$ is increasing over the range of σ for which $d\bar{k}^u/d\sigma < 0$ since $d\bar{k}^c/d\sigma > 0$ for all $\sigma > 0$. Proving $\bar{k}^c - \bar{k}^u$ is increasing for low σ values where $d\bar{k}^u/d\sigma > 0$ is much more complicated.

Using equation (1.17a) to slightly rearrange the original definition of $d\bar{k}^c/d\sigma$ (equation (1.22)) the numerators and denominators of $d\bar{k}^c/d\sigma$ and $d\bar{k}^u/d\sigma$ can be compared

$$\begin{aligned}
\frac{d\bar{k}^c}{d\sigma}\text{ Num.} &= \left(\frac{1}{\sigma-1}\right)^2 \lambda(\bar{k}^c),\\
\frac{d\bar{k}^u}{d\sigma}\text{ Num.} &= \left(\frac{1}{\sigma-1}\right)^2 \lambda(\bar{k}^u)\\
&\qquad - \left(\frac{1}{\sigma-1}\right)\frac{w_\lambda(\bar{k}^u)}{TD^u}\left[N\left(\frac{\partial q_{h2}^u}{\partial\sigma} + (\sigma-1)\frac{q_{h2}^u}{P^u}\frac{\partial P^u}{\partial\tau}\frac{\partial\tau^u}{\partial\sigma}\right) + N^*\frac{\partial q_{h2}^{*u}}{\partial\sigma}\right],\\
\frac{d\bar{k}^c}{d\sigma}\text{ Denom.} &= \bar{k}^c\left(\lambda''(\bar{k}^c) - \sigma\frac{(\lambda'(\bar{k}^c))^2}{\lambda(\bar{k}^c)}\right),\\
\frac{d\bar{k}^u}{d\sigma}\text{ Denom.} &= \bar{k}^u\left(\left(\lambda''(\bar{k}^u) - \sigma\frac{(\lambda'(\bar{k}^u))^2}{\lambda(\bar{k}^u)}\right) + \frac{\theta''(\bar{k}^u)}{TD^u}\right) - \frac{\bar{n}^u}{\bar{k}^u}\frac{w_\lambda(\bar{k}^u)}{TD^u}\left[\mathbf{N}\frac{\mathbf{q}_{h2}^u}{\mathbf{P}^u}\frac{d\mathbf{P}^u}{dn}\right].
\end{aligned}$$

Beginning with the parts of the numerator and denominator which the equations have in common, compare $d\bar{k}^c/d\sigma$ and $d\bar{k}^u/d\sigma$ when $a \to \infty$. When $a \to \infty$, $\bar{k}^u \to \bar{k}^c$ and therefore $w_\lambda(\bar{k}^u) \to 0$, which means $d\bar{k}^u/d\sigma$ is defined very similarly to $d\bar{k}^c/d\sigma$ but with the addition of the $\theta''(k)$ term in the denominator. Therefore, because $\theta(\cdot)$ is convex, the denominator of $\bar{k}^u$ is greater than the denominator of $\bar{k}^c$, which means that $d\bar{k}^u/d\sigma < d\bar{k}^c/d\sigma$, which is consistent with the observation that $\bar{k}^u - \bar{k}^c$ is increasing for all values of σ. This observation is limited to the values of σ for which the maximum a for which a finite tariff solution exists, $a < \left(\frac{\sigma}{\sigma-1}\right)\sigma - 1$, is very large. For the mid-range values of σ where the maximum a is most limited, this observation cannot be applied.

Still examining the terms which both derivatives have in common, using the fact that $\bar{k}^c > \bar{k}^u$, the numerator of $d\bar{k}^c/d\sigma$ is larger when it is evaluated at $\bar{k}^u$ since $\lambda'(k) < 0$. The denominator of $\bar{k}^c$ is less straightforward, but using the numerical solution I can establish that $\bar{k}^c\Big(\lambda''(\bar{k}^c) - \sigma\lambda'(\bar{k}^c)^2/\lambda(\bar{k}^c)\Big) < \bar{k}^u\Big(\lambda''(\bar{k}^u) - \sigma\lambda'(\bar{k}^u)^2/\lambda(\bar{k}^u)\Big)$ for all values of σ when a

is low. When a is high, then for σ high enough the inequality is reversed.

The price-index term from the $\bar{k}^u$ numerator can be expanded:

$$
\begin{aligned}
&-\left(\frac{1}{\sigma-1}\right)\frac{w_\lambda(\bar{k}^u)}{TD^u}\left[N\left(\frac{\partial q_{h2}^u}{\partial \sigma}+(\sigma-1)\frac{q_{h2}^u}{P^u}\frac{\partial P^u}{\partial \tau}\frac{\partial \tau^u}{\partial \sigma}\right)+N^*\frac{\partial q_{h2}^{*u}}{\partial \sigma}\right]= \\
&-\frac{w_\lambda(\bar{k}^u)}{(\sigma-1)}\left(\frac{1}{\sigma}\left(\frac{1}{\sigma-1}\right)+\frac{1}{\sigma}\frac{Nq_{h2}^u}{TD^u}\frac{(\tau^u-1)(\sigma-1)^2}{(\sigma z^u+1)-(\sigma-1)(\tau^u-1)}\left(\left(\frac{1}{\sigma-1}\right)^2-\frac{z^u}{z^u+1}\right)\right. \\
&\qquad\left.+\frac{Nq_{h2}^u}{TD^u}\ln\left(\frac{p_f^u}{p_h^u}\right)\frac{\left(\frac{1}{z^u+1}\right)(\sigma z^u+1)}{(\sigma z^u+1)-(\sigma-1)(\tau^u-1)}+\frac{N^*q_{h2}^{*u}}{TD^u}\left(\frac{z^{*u}}{z^{*u}+1}\right)\ln\left(\frac{p_f^*}{p_h^{*u}}\right)\right),
\end{aligned}
$$

First, note that the coefficient on the term overall is positive. Next, the second line is a positive, $\frac{1}{\sigma}\left(\frac{1}{\sigma-1}\right)$, plus a term containing the non-monotonic portion of the tariff derivative, $(1/(\sigma-1))^2 - z^u/(z^u+1)$, which is initially positive in σ and eventually becomes negative for σ large enough, as was shown in the discussion of the τ^c derivative. Next, the final part is also difficult to sign. This is demonstrated by looking at the relative price terms. First, $\ln(p_f^u/p_h^u)$ is only ever positive if a is high enough. Furthermore, if a is high enough such that there exists a range of σ for which $\ln(p_f^u/p_h^u) > 0$, when σ is very high or very low, $\ln(p_f^u/p_h^u)$ will still be negative. This was also shown in Figure A.4a. The foreign-price term on the other hand, $\ln(p_f^*/p_h^{*u})$, is always negative given that $\lambda(k) > \lambda^*$ for all $k < \infty$. Therefore, the overall sign of the term is unclear.

Next, from the denominator of $d\bar{k}^u/\sigma$,

$$
\begin{aligned}
&-\frac{\bar{n}^u}{\bar{k}^u}\frac{w_\lambda(\bar{k}^u)}{TD^u}\left[\mathbf{N}\frac{\mathbf{q}_{h2}^u}{\mathbf{P}^u}\frac{d\mathbf{P}^u}{dn}\right]= \\
&\quad-\frac{(w_\lambda(\bar{k}^u))^2}{(\sigma-1)\lambda(\bar{k}^u)\bar{k}^u}\left[\frac{Nq_{h2}^u}{TD^u}\frac{\left(\frac{z^u}{z^u+1}\right)(\sigma z^u+1)-(\sigma-1)(\tau^u-1)}{(\sigma z^u+1)-(\sigma-1)(\tau^u-1)}+\frac{N^*q_{h2}^{*u}}{TD^u}\left(\frac{1}{z^{*u}+1}\right)\right].
\end{aligned}
$$

Using the numerical solution, I establish that $\left(z^u/(z^u+1)\right)(\sigma z^u+1)-(\sigma-1)(\tau^u-1)$ is positive for all parameter values I test, but I am unable to establish this fact analytically. This was also the reason I needed Assumption 1.10 to establish the sign of the full denominator of $d\bar{n}^u/d\sigma$. The fact that this term is positive reflects that the direct effect on the price index of a change in n is larger than the indirect effect which occurs by way of the effect of n on τ.

Combining all of these observations, I am only able to pin down the relationship between $\bar{k}^c$ and $\bar{k}^u$ for low values of σ when $a \to \infty$, in which case I show that $d\bar{k}^u/d\sigma < d\bar{k}^c/d\sigma$ for all values of $\sigma > 1$.

A.6 First-Order Conditions of k, n, τ for Maximum, Minimum σ

This section examines the derivatives of production and the tariff levels both with and without ex-ante commitment. It builds on the discussion of production from Appendix A.4. I discuss the behavior of the variables with ex-ante tariff commitment first in Appendix A.6.1, looking at firm entry and the tariff as $\sigma \to 1$ and $\sigma \to \infty$. Then in Appendix A.6.2 I discuss the behavior of variables without ex-ante tariff commitment. In this second section I am restricted to look at the behavior for $\sigma \to \sigma^{Max}$, since the relevant range of σ is restricted in the no-trade-agreement solution.

A.6.1 Limiting Behavior of k, n, and τ with Tariff Commitment

The behavior of $\bar{k}^c$ can be found by using the ex-ante equilibrium conditions, $\lambda(\bar{k}^c) + (\sigma - 1)\lambda'(\bar{k}^c)\bar{k}^c = 0$ and $\lambda''(\bar{k}^c) - \sigma\lambda'(\bar{k}^c)^2/\lambda(\bar{k}^c) > 0$, and simplifying the derivative equation from Proposition 1.4. The behavior as σ reaches its extreme values is:

$$\lim_{\sigma\to 1}\frac{d\bar{n}^c}{d\sigma} = \lim_{\sigma\to 1}\frac{\lambda'(\bar{k}^c)\bar{n}^c}{-\left(\lambda(\bar{k}^c)/\lambda'(\bar{k}^c)\right)\left(\lambda''(\bar{k}^c) - \sigma\lambda'(\bar{k}^c)^2/\lambda(\bar{k}^c)\right)} = \frac{(\infty)(-\infty)}{(0)\,(>0)} = -\infty.$$

where the fact that $-\lambda(\bar{k}^c)/\lambda'(\bar{k}^c) = \left(\frac{1}{\sigma-1}\right)(1/\bar{k}^c)$ establishes that $\lim_{\sigma\to\infty} -\lambda(\bar{k}^c)/\lambda'(\bar{k}^c) = \infty$.

Alternatively, when $\sigma \to \infty$, the firm entry derivative is

$$\lim_{\sigma\to\infty} \frac{d\bar{n}^c}{d\sigma} = \lim_{\sigma\to\infty} \frac{\lambda'(\bar{k}^c)\bar{n}^c}{-\Big(\lambda(\bar{k}^c)/\lambda'(\bar{k}^c)\Big)\Big(\lambda''(\bar{k}^c) - \sigma\lambda'(\bar{k}^c)^2/\lambda(\bar{k}^c)\Big)} = \frac{(0)(0)}{-(\infty)(>0)} = 0.$$

given that I established earlier $\lim_{\sigma\to\infty}\lambda(\bar{k}^c) = \lambda^*$.

The behavior of $\bar{n}^c$ for changes in σ is summarized in Proposition 1.4.

Recall that production for $\sigma \to 1$ is derived in Appendix A.4. Given production becomes Cobb-Douglas as $\sigma \to 1$, $z^c \equiv \Big((\bar{n}^c N p^c_h q^c_{h2})/(n^* N p^c_f q^{*c}_{h2})\Big)$ is $\lim_{\sigma\to 1} z^c = \bar{n}^c/n^*$ and $\lim_{\sigma\to 1}\tau^c = 1$ Rearranging equation (1.23), the limit of the derivative of τ^c with respect to σ is

$$\lim_{\sigma\to 1}\frac{\Big(\tau^c\Big)^2\left(\Big(\frac{1}{\sigma}\Big)^2 - \frac{\tau^c-1}{\tau^c} + \frac{\tau^c-1}{\tau^c}\ln\Big(p^c_f/p^c_h\Big)\frac{1}{z^c}\right)}{\Big(\sigma + \frac{1}{z^c}\Big) - (\sigma-1)(\tau^c-1)\frac{1}{z^c}},$$
$$= \frac{(1)\left((1) - (0) + (0)\ln\Big((1)\lambda^*\Big)\frac{1}{\infty} - \ln\Big(\lambda(\bar{k}^c)\Big)\frac{1}{\infty}\right)}{((1)+\frac{1}{\infty}) - (0)(0)\frac{1}{\infty}}.$$

The only part of the limit which requires more work to solve is the final term in the numerator. Because $\lim_{\sigma\to 1}\lambda(\bar{k}^c) = \infty$, the final term is ∞/∞ and needs to be solved using l'Hopital's rule. Before applying l'Hopital's rule, I rearrange the terms in the numerator and denominator giving

$$\lim_{\sigma\to 1}\frac{-\tau^c(\tau^c-1)}{(\sigma z^c+1) - (\sigma-1)(\tau^c-1)}\ln\Big(\lambda(\bar{k}^c)\Big) = \lim_{\sigma\to 1}\frac{\frac{d}{d\sigma}\Big(-\ln\Big(\lambda(\bar{k}^c)\Big)\Big)}{\frac{d}{d\sigma}\left(\frac{1}{\tau^c}\left(\frac{(\sigma z^c+1)}{(\tau^c-1)} - (\sigma-1)\right)\right)}$$
$$= \lim_{\sigma\to 1}\frac{-(\tau^c)^2(\tau^c-1)^2(\lambda'(\bar{k}^c)/\lambda(\bar{k}^c))\frac{d\bar{k}^c}{d\sigma}}{-\Big(\frac{(\sigma z^c+1)}{(\tau^c-1)} - (\sigma-1)\Big) + \frac{\tau^c}{\tau^c-1}\Big(-(\sigma z^c+1)\frac{d\tau^c}{d\sigma} + \Big(z^c + \sigma\frac{dz^c}{d\sigma}\Big)\Big) - \tau^c(\tau^c-1)},$$

where $dz^c/d\sigma = z^c\ln\Big(p^c_f/p^c_h\Big) + (\sigma-1)(z^c/\tau^c)(d\tau^c/d\sigma)$. Using this definition of $dz^c/d\sigma$, the denominator of the limit can be rewritten

$$\lim_{\sigma\to 1} z^c \Bigg[-\left(\sigma + \frac{1}{z^c}\right) + (\sigma-1)(\tau^c - 1)\frac{1}{z^c} - \tau^c(\tau^c-1)^2\frac{1}{z^c} + (\tau^c-1)\tau^c$$
$$+ (\tau^c - 1)\sigma \ln\left(\frac{p_f^c}{p_h^c}\right) + \left(-(\sigma-1) - \left(1+\frac{1}{z^c}\right)\tau^c\right)\frac{d\tau^c}{d\sigma}\Bigg]$$
$$= (\infty)\Bigg[-(1) + (0) - (0) + (0) + (0)\ln(\lambda^*) - (0)(1)\ln(\infty) + ((0)-(1))\frac{d\tau^c}{d\sigma}\Bigg],$$

which means that regardless of what the limits of $d\tau^c/d\sigma$ and $(\tau^c - 1)\sigma \ln\left(\lambda(\bar{k}^c)\right)$, the denominator of $\lim_{\sigma\to 1} d\tau^c/d\sigma$ goes to either positive or negative infinity. The limit of the numerator is

$$\lim_{\sigma\to 1} -(\tau^c)^2(\tau^c-1)^2(\lambda'(\bar{k}^c)/\lambda(\bar{k}^c))\frac{d\bar{k}^c}{d\sigma} = -(1)^2(0)^2\left(\frac{1}{\infty}\right)\left(\frac{1}{-\infty}\right) = 0,$$

therefore the limit of the $\ln(p_h^c)$ term in $d\tau^c/d\sigma$ is equal to zero. As a result, $\lim_{\sigma\to 1} d\tau^c/d\sigma > 0$.

Next, as σ approaches infinity, the global market for the differentiated good becomes perfectly competitive, and I have already established $\lim_{\sigma\to\infty} p_h(\bar{k}^c) = p_f^c$. In turn, this means the limit of the first-order condition is zero, given that $\lim_{\sigma\to\infty} \frac{1}{\sigma^2} - \left(\frac{\tau^c-1}{\tau^c}\right) + \left(\frac{\tau^c-1}{\tau^c}\right)\ln\left(\frac{p_f^c}{p_h(\bar{k}^c)}\right)\frac{1}{z^c} = 0$. Taking the second derivative of τ^c and simplifying given that $\lim_{\sigma\to\infty} \tau^c = 1$, given equation (1.24) is zero, and given that $\lim_{\sigma\to\infty} d\tau^c/d\sigma = 0$, I find that the second-order condition is negative. Therefore, as $\sigma \to \infty$, the tariff level approaches a minimum at $\tau^c = 1$. Given that $\lim_{\sigma\to 1} \tau^c = 1$, there must be a range of σ for which the tariff level is increasing.

The behavior of the τ^c when $\sigma \to 1$ or $\sigma \to \infty$ is summarized in Proposition 1.5.

A.6.2 Limiting Behavior of k, n, and τ without Tariff Commitment

The derivative of the tariff, τ^u, with respect to the elasticity of substitution across varieties is defined in equation (1.42), with the derivative τ^{cxp} equal to equation (1.42) if $a = 0$. As was true for the model with tariff commitment, production becomes Cobb-Douglas as

$\sigma \to 1$, which in turn means that $z^u \equiv \left((\bar{n}^u N p_h^u q_{h2}^u)/(n^* N p_f^u q_{h2}^{*u})\right)$ is $\lim_{\sigma\to 1} z^u = \bar{n}^u/n^*$ and $\lim_{\sigma\to 1} \tau^u = 1$. Therefore, the tariff derivative is

$$\lim_{\sigma\to 1}\frac{d\tau^u}{d\sigma} = \lim_{\sigma\to 1}\frac{\left(\tau^u\right)^2\left(\left(\frac{1}{\sigma}\right)^2 - \left(\frac{\tau^u-1}{\tau^u}\right) + \left(\frac{\tau^u-1}{\tau^u}\right)\ln\left(p_f^u/p_h^u\right)\frac{1}{z^u} + \left(\frac{\tau^u-1}{\tau^u}\right)\left(\frac{w_\lambda(\bar{k}^u)}{\bar{n}^u\lambda(\bar{k}^u)}\right)\frac{d\bar{n}^u}{d\sigma}\right)}{\left(\sigma + \frac{1}{z^u}\right) - (\sigma-1)(\tau^u-1)\frac{1}{z^u}}$$
$$= \frac{(1)\left(1 - 0 + (0)\ln\left(\lambda^*/\lambda(\bar{k}^u)\right)\right)(0) + (0)\left(\frac{0}{(\infty)(\infty)}\right)\frac{d\bar{n}^u}{d\sigma}\Big)}{(1+0)-(0)}.$$

First, the $\ln(\bar{k}^u)$ term has already been simplified in the context of $\bar{k}^c$, where I showed that the limit of the term was equal to zero. Therefore, the only relevant term left to further examine is the term which contains the firm entry derivative. I revisit the limit of $d\tau^u/d\sigma$ following the discussion of the limit of $d\bar{n}^u/d\sigma$.

The derivative of firm entry with respect to the elasticity of substitution is given in equation (1.44). Here, I will examine the limit of the $d\bar{k}^u/d\sigma$ as it was expanded in Appendix A.5.4, noting that the limit of $z^{*u} \equiv ((n^* N^* p_f^* q_{f2}^{*u})/(n^* N^* p_h^* q_{h2}^{*u}))$ is $\lim_{\sigma\to 1} z^{*u} = \lim_{\sigma\to 1} n^*/\bar{n}^u = 0$ given production approaches Cobb-Douglas as $\sigma \to 1$. To simplify the problem consider the limit of the numerator and denominator separately, where I have divided both through by $w_\lambda(\bar{k}^u)$ and multiplied through by $(\sigma - 1)$. First, the limit of the numerator is

$$\lim_{\sigma\to 1}\left\{\left(\frac{1}{\sigma-1}\right)\frac{\lambda(\bar{k}^u)}{w_\lambda(\bar{k}^u)} - \frac{1}{TD^u}\left[N\left(\frac{\partial q_{h2}^u}{\partial\sigma} + (\sigma-1)\frac{q_{h2}^u}{P^u}\frac{\partial P^u}{\partial\tau}\frac{\partial\tau^u}{\partial\sigma}\right) + N^*\frac{\partial q_{h2}^{*u}}{\partial\sigma}\right]\right\}$$
$$= \lim_{\sigma\to 1}\left\{\frac{1}{\sigma}\left(\frac{\lambda(\bar{k}^u) - \lambda'(\bar{k}^u)\bar{k}^u}{w_\lambda(\bar{k}^u)}\right) - (s^u)\frac{\frac{1}{\sigma}(\tau^u-1)\left(1-(\sigma-1)^2\frac{1}{1+(1/z^u)}\right)}{\left(\sigma+\frac{1}{z^u}\right) - (\sigma-1)(\tau^u-1)}\right.$$
$$\left. - \left((s^u)\ln\left(\frac{p_f^u}{p_h^u}\right)\frac{\frac{1}{1+(1/z^u)}\left(\sigma+\frac{1}{z^u}\right)}{\left(\sigma+\frac{1}{z^u}\right) - (\sigma-1)(\tau^u-1)\frac{1}{z^u}} + (1-s^u)\frac{z^{*u}}{z^{*u}+1}\ln\left(\frac{p_f^*}{p_h^{*u}}\right)\right)\right\},$$
$$= (1)\,(>1) - (s^u)(0) + (s^u)(1)\ln\left(\lambda(\bar{k}^u)\right) + (1-s^u)(0)\ln\left(\lambda(\bar{k}^u)\right).$$

where $s \equiv (Nq_{h2})/(Nq_{h2} + Nq_{h2}^*)$ is the share of total sales by domestic firms at home, with

$\lim_{\sigma\to 1} s = (Ne_2)/(Ne_2 + N^* e_2^*/\tau^*)$. Next, the limit of the denominator of $d\bar{k}^u/d\sigma$ is

$$\lim_{\sigma\to 1}\left\{\frac{(\sigma-1)\bar{k}^u}{w_\lambda(\bar{k}^u)}\left(\left(\lambda''(\bar{k}^u)-\sigma\frac{\left(\lambda'(\bar{k}^u)\right)^2}{\lambda(\bar{k}^u)}\right)+\frac{\theta''(\bar{k}^u)}{TD^u}\right)-\frac{(\sigma-1)\bar{n}^u}{(TD^u)\bar{k}^u}\left[\mathbf{N}\frac{\mathbf{q}_{h2}^u}{\mathbf{P}^u}\frac{d\mathbf{P}^u}{dn}\right]\right\},$$
$$=\lim_{\sigma\to 1}\left\{\frac{1}{\lambda'(\bar{k}^u)}\left(\frac{(\sigma-1)\lambda'(\bar{k}^u)\bar{k}^u}{w_\lambda(\bar{k}^u)}\right)\left(\left(\lambda''(\bar{k}^u)-\sigma\frac{\left(\lambda'(\bar{k}^u)\right)^2}{\lambda(\bar{k}^u)}\right)+\frac{\theta''(\bar{k}^u)}{TD^u}\right)\right.$$
$$\left.-\frac{1}{\bar{k}^u}\frac{w_\lambda(\bar{k}^u)}{\lambda(\bar{k}^u)}\left((s^u)\frac{\frac{1}{1+(1/z^u)}\left(\sigma+\frac{1}{z^u}\right)-(\sigma-1)(\tau^u-1)\frac{1}{z^u}}{\left(\sigma+\frac{1}{z^u}\right)-(\sigma-1)(\tau^u-1)\frac{1}{z^u}}+(1-s^u)\left(\frac{1}{z^{*u}+1}\right)\right)\right\},$$
$$=\frac{1}{-\infty}(>1)(>0\text{ or }+\infty)-\frac{1}{0}(<1)\,(1)=-\infty.$$

The limit of the denominator must be $-\infty$, because the limit of the second term is $-\infty$. This can be seen without solving for the first term, which is unclear and may be either $-\infty$ or 0^-.[7]

The remaining terms in the limit which create a problem in determining the limit of $d\bar{k}^u/d\sigma$ are the two numerator terms with $\ln(\lambda(\bar{k}^u))$. However, the complexity of the dependency of these terms on σ prevents me from finding the limit of these terms. Therefore, I rely on the numerical solution to finish solving the limit and I find that $\lim_{\sigma\to 1} d\bar{k}^u/d\sigma = 0$ and $\lim_{\sigma\to 1} d\bar{n}^u/d\sigma = -\infty$. This result is consistent with the finding in Appendix A.4 that $\lim_{\sigma\to 1}\bar{n}^u = \infty$ and given that when $\sigma \in (1, \sigma^{Max})$, $\bar{k}^u > 0$ which in turn means $\bar{n}^u$ is finite for $\sigma \in (1, \sigma^{Max})$.

Return now to the tariff derivative. Because $\lim_{\sigma\to 1} d\bar{n}^u/d\sigma = -\infty$, the final term of the tariff derivative needs further work, since it is equal to $0 * \infty$. Expanding this term, with

[7] Note that this does not violate Assumption 1.10, which states the denominator is positive, because to solve the problem I divided the numerator and the denominator through by $w_\lambda(\bar{k}^u) < 0$.

$dz^u/d\sigma = z^u \ln\left(p_f^u/p_h^u\right) + (\sigma-1)(z^u/\tau^u)(d\tau^u/d\sigma)$,

$$\begin{aligned}
\text{Num.: } & \lim_{\sigma\to 1} -(2\tau^u-1)\frac{d\tau^u}{d\sigma}\left(\frac{w_\lambda(\bar{k}^u)}{\bar{k}^u\lambda(\bar{k}^u)}\right)\frac{d\bar{k}^u}{d\sigma} + \tau^u(\tau^u-1)\left((\sigma-1)\lambda''(\bar{k}^u)+\sigma\lambda'(\bar{k}^u)\right)\frac{d\bar{k}^u}{d\sigma} \\
& \quad + \lambda'(\bar{k}^u)\bar{k}^u - \tau^u(\tau^u-1)\frac{w_\lambda(\bar{k}^u)}{(\bar{n}^u\lambda(\bar{k}^u))^2}\left(-\frac{\bar{n}^u}{\bar{k}^u}\frac{d\bar{k}^u}{d\sigma}\lambda(\bar{k}^u)+\bar{n}^u\lambda'(\bar{k}^u)\frac{d\bar{k}^u}{d\sigma}\right), \\
\text{Denom.: } & \lim_{\sigma\to 1} 1-(\tau^u-1)\frac{1}{z^u} - (1-(\sigma-1)(\tau^u-1))\frac{1}{z^u}\ln\left(\frac{\tau^u\lambda^*}{\lambda(\bar{k}^u)}\right) \\
& \quad - (\sigma-1)\frac{1}{\tau^u}\left((1-(\sigma-1)(\tau^u-1))+\tau^u\right)\frac{1}{z^u}\frac{d\tau^u}{d\sigma} \\
& = 1-(0)(0)-(1-(0)(0))(0)\ln(\lambda^*)+(1-(0)(0))(0)\ln\left(\lambda(\bar{k}^u)\right) \\
& \quad - (0)(1)\left((1-(0)(0))+1\right)(0)\frac{d\tau^u}{d\sigma},
\end{aligned}$$

but I cannot make use of l'Hopital's rule, because I cannot solve the second derivative $d^2\bar{n}^u/d\sigma^2$. Making use of the numerical solution, I can show that this term goes to zero, which in turn implies the overall limit is $\lim_{\sigma\to 1} d\tau^u/d\sigma > 0$.

A.7 Deriving Government Trade Policy Preference

A.7.1 Behavior of Welfare for changes in a if $a = 0$ and $\bar{n}^u|_{a=0} = \bar{n}^c$

To determine whether the benefit to the government is convex or concave in a, next look at the second-derivative $d^2(G^c - G^u)/da^2$:

$$\begin{aligned}
\left.\frac{d^2(G^c-G^u)}{da^2}\right|_{a=0} = & -\frac{\partial W^u}{\partial n}\frac{d^2\bar{n}^u}{da^2} - \frac{\partial^2 W^u}{\partial \tau^2}\left(\frac{d\tau^u}{da}\right)^2 \\
& - \frac{\partial^2 W^u}{\partial n^2}\left(\frac{d\bar{n}^u}{da}\right)^2 - 2\frac{\partial^2 W^u}{\partial\tau\partial n}\left(\frac{d\bar{n}^u}{da}\frac{d\tau^u}{da}\right).
\end{aligned}$$

Note that γ is missing from the equation due to the fact that when $a = 0$, the u and u outcomes are equivalent. Using the definition of the derivative of the tariff with respect to

a to simplify, the above equation can then be written:

$$
\begin{aligned}
\left.\frac{d^2(G^c - G^u)}{da^2}\right|_{a=0} &= -\frac{\partial W^u}{\partial n}\frac{d^2\bar{n}^u}{da^2} \\
&\quad - \frac{\partial^2 W^u}{\partial \tau^2}\left(\frac{w_\lambda(\bar{k}^u)}{\bar{n}^u\lambda(\bar{k}^u)}\frac{\tau^u(\tau^u - 1)}{(\sigma z^u + 1) - (\sigma - 1)(\tau^u - 1)}\right)^2\left(\frac{d\bar{n}^u}{da}\right)^2 \\
&\quad - \left[\frac{\partial^2 W^u}{\partial n^2} + 2\frac{\partial^2 W^u}{\partial\tau\partial n}\frac{w_\lambda(\bar{k}^u)}{\bar{n}^u\lambda(\bar{k}^u)}\frac{\tau^u(\tau^u - 1)}{(\sigma z^u + 1) - (\sigma - 1)(\tau^u - 1)}\right]\left(\frac{d\bar{n}^u}{da}\right)^2.
\end{aligned}
\tag{A.23}
$$

The sign of the second derivative is dependent on the second derivative of welfare with respect to firm entry and with respect to the tariff level. These derivatives are given by the following two equations:

$$
\left.\frac{\partial^2 W^u}{\partial \tau^2}\right|_{a=0} = Ne_2\frac{1}{(z^u + 1)^2}\left(\frac{1}{\tau^u}\right)^3\Big[(\tau^u - 1)(\sigma - 1) - (\sigma z^u + 1)\Big],
\tag{A.24}
$$

which is negative given that the second-order condition for the government's welfare-maximization dictates that $(\tau^u - 1)(\sigma - 1) - (\sigma z^u + 1) < 0$. Therefore, $(\partial^2 W^u/\partial\tau^2)|_{a=0} < 0$. The second derivative of welfare with respect to firm entry is

$$
\begin{aligned}
\left.\frac{\partial^2 W^u}{\partial n^2}\right|_{a=0} &= -Ne_2\frac{(\bar{k}^u)^2}{(\bar{n}^u)^2\lambda(\bar{k}^u)}\left(\frac{1}{z^u + 1}\right)\left[\left(\left(\frac{\tau^u - 1}{\tau^u}\right) + z^u\right)\left(\lambda''(\bar{k}^u) - \sigma\frac{\left(\lambda'(\bar{k}^u)\right)^2}{\lambda(\bar{k}^u)}\right)\right. \\
&\qquad \left. + \left(\left(\frac{\tau^u - 1}{\tau^u}\right) + \frac{z^u}{z^u + 1}\left(\left(\frac{\tau^u - 1}{\tau^u}\right) + z^u\right)\right)\frac{(w_\lambda(\bar{k}^u))^2}{(\sigma - 1)\left(\bar{k}^u\right)^2\lambda(\bar{k}^u)}\right] \\
&\quad - \left(\frac{1}{\sigma}\right)N^*\frac{e_2^*}{\tau^*}\frac{(\bar{k}^u)^2}{(\bar{n}^u)^2\lambda(\bar{k}^u)}\frac{z^{*u}}{(z^{*u} + 1)^2}\left[2\frac{(w_\lambda(\bar{k}^u))^2}{(\bar{k}^u)^2\lambda(\bar{k}^u)} + (z^{*u} + 1)\left(\lambda''(\bar{k}^u) - \sigma\frac{\left(\lambda'(\bar{k}^u)\right)^2}{\lambda(\bar{k}^u)}\right)\right].
\end{aligned}
\tag{A.25}
$$

where assuming that σ is such that $\lambda''(\bar{k}^u) - \sigma\lambda'(\bar{k}^u)^2/\lambda(\bar{k}^u)\big) > 0$ (i.e. that $\sigma < \sigma_{Max}$), the second derivative $(\partial^2 W^u/\partial n^2)|_{a=0} < 0$. The final derivative necessary to complete equation (A.23) is the cross-partial derivative, $(\partial^2 W^u/\partial n\partial\tau)|_{a=0}$. The cross-partial derivative of welfare with respect to firm entry and the tariff level is

$$
\left.\frac{\partial^2 W^u}{\partial n\partial\tau}\right|_{a=0} = Ne_2\left(\frac{w_\lambda(\bar{k}^u)}{\bar{n}^u\lambda(\bar{k}^u)}\right)\frac{1}{(z^u + 1)^2}\left(\frac{1}{\tau^u}\right)\left(\frac{z^u}{\sigma z^u + 1}\right)\left(\frac{\sigma - 1}{\sigma}\right),
\tag{A.26}
$$

which is also negative given that $w_\lambda(\bar{k}^u) < 0$.

Combined, equations (A.24) through (A.26) can be shown to be positive by using the definition of the tariff from equation (1.26) to simplify. The only remaining term from equation (A.23) is the second derivative of firm entry times the derivative dW^u/dn. Because I am examining $d^2(G^c - G^u)/da^2|_{a=0}$ with a focus on where $\bar{n}^u = \bar{n}^c$, I know that $w_\lambda(\bar{k}^u) = 0$. Therefore, the term featuring the second derivative of firm entry is equal to zero. Using this to further simplify $d^2(G^c - G^u)/da^2|_{a=0}$, the derivative is equal to

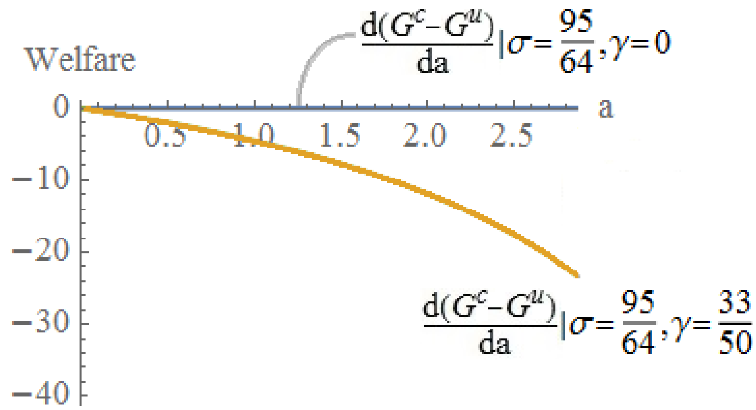
$$\left.\frac{d^2(G^c - G^u)}{da^2}\right|_{a=0} = (\sigma - 1)\left(\frac{\bar{k}^u}{\bar{n}^u}\right)^2 \frac{1}{\lambda(\bar{k}^u)}\left(\lambda''(\bar{k}^u) - \sigma\frac{\left(\lambda'(\bar{k}^u)\right)^2}{\lambda(\bar{k}^u)}\right)$$
$$* \left[Ne_2\left(\left(\frac{\tau^u - 1}{\tau^u}\right) + z^u\right) + \left(\frac{1}{\sigma}\right)N^*\frac{e_2^*}{\tau^*}\left(\frac{z^{*u}}{z^{*u}+1}\right)\right]\left(\frac{d\bar{n}^u}{da}\right)^2,$$

which must be positive given the second-order condition for profit maximization. Therefore, if $\bar{n}^u|_{a=0} = \bar{n}^c$ (as was true in Maggi and Rodríguez-Clare (1998)), then my model maintains their Proposition 3 finding for γ low – that the benefit to the government of committing to a trade agreement is at a local minimum. For reference, the first-order behavior of $G^c - G^u$ with respect to a is depicted in Figure A.6a and Figure A.6b for low σ and high σ, respectively.

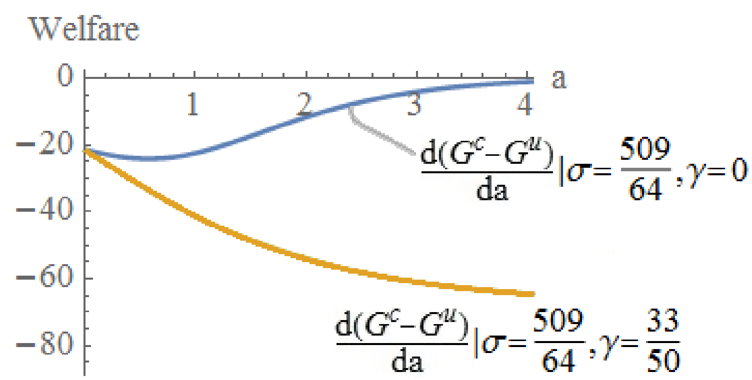
A.7.2 Government Policy Preference Foreign Expenditure Share

In this subsection, I expend upon the derivatives of the foreign consumers' share of total expenditures, $m^* \equiv (N^*e_2^*)/(Ne_2 + N^*e_2^*)$. In terms of model parameters, this can be most easily demonstrated by allowing a change in $N^*e_2^*$ while holding Ne_2 constant. The derivative of $G^c - G^u$ with respect to $N^*e_2^*$ is given in equation (1.56),

$$\frac{d(G^c - G^u)}{d(N^*e_2^*)} = \left(-\gamma\frac{\partial TW^u}{\partial n} - (1-\gamma)\frac{\partial W^{cxp}}{\partial n}\right)\frac{d\bar{n}^u}{d(N^*e_2^*)} + \gamma a\frac{d(\bar{n}^u\pi^{cxp})}{d(N^*e_2^*)}$$
$$+ \left(\frac{1}{\sigma}\right)\frac{1}{\tau^*}\left(\frac{1}{z^{*c}+1} - \frac{1}{z^{*u}+1}\right),$$



(a) Derivative of $G^c - G^u$ with respect to a, low σ.



(b) Derivative of $G^c - G^u$ with respect to a, high σ.

Figure A.6: Derivative of $G^c - G^u$ with respect to a, $d(G^c - G^u)/da$. $d(G^c - G^u)/da$ is always negative and is concave in a when σ is low and becomes convex in a when σ is high and γ is low.

where the social welfare derivatives are defined in equation (1.48) and the derivative of the total operating profits; $d(\bar{n}^u \pi^u)/d(N^* e_2^*)$ is defined in equation (1.57) and is greater than zero; and finally where the derivative of firm entry, $d\bar{n}^u/d(N^* e_2^*)$, is negative and is defined in equation (1.55).

To determine the sign of $d(G^c - G^u)/d(N^* e_2^*)$, consider equation (1.55) for the extreme values of γ and the extreme values of a. Suppose that $a \to \infty$, as I have discussed multiple times previously this means that $\bar{n}^u \to \bar{n}^c$. Simplifying equation (1.56) using the fact that $\lim_{a\to\infty} w_\lambda(\bar{k}^u) = 0$ and $z^{*u} \to z^{*c}$, the derivative is $\lim_{a\to\infty} d(G^c - G^u)/d(N^* e_2^*) = 0$. To determine if this is a minimum or a maximum, look also at the second derivative,

$$\begin{aligned}\frac{d^2(G^c-G^u)}{d(N^*e_2^*)^2} &= \left(-\gamma\frac{\partial TW^u}{\partial n}-(1-\gamma)\frac{\partial W^{cxp}}{\partial n}+\gamma a\frac{d(\bar{n}^u\pi^{cxp})}{dn}\right)\frac{d^2\bar{n}^u}{d(N^*e_2^*)^2}\\ &\quad+\frac{d}{d(N^*e_2^*)}\left[\left(-\gamma\frac{\partial TW^u}{\partial n}-(1-\gamma)\frac{\partial W^{cxp}}{\partial n}+\gamma a\frac{d(\bar{n}^u\pi^{cxp})}{dn}\right)\right]\frac{d\bar{n}^u}{d(N^*e_2^*)}\\ &\quad-\left(\frac{1}{\sigma}\right)\frac{1}{\tau^*}\left(\frac{z^{*c}}{(z^{*c}+1)^2}-\frac{z^{*u}}{(z^{*u}+1)^2}\right)\left(\frac{w_\lambda(\bar{k}^u)}{\bar{n}^u\lambda(\bar{k}^u)}\right)\frac{d\bar{n}^u}{d(N^*e_2^*)},\end{aligned}$$

which is also equal to zero when $\bar{n}^u \to \bar{n}^c$, meaning when $a \to \infty$, $G^c - G^u$ is at an inflection point.

When $a = 0$ on the other hand, the derivative becomes

$$\left.\frac{d(G^c-G^u)}{d(N^*e_2^*)}\right|_{a=0} = -\frac{\partial W^{cxp}}{\partial n}\frac{d\bar{n}^u}{d(N^*e_2^*)}+\left(\frac{1}{\sigma}\right)\frac{1}{\tau^*}\left(\frac{1}{z^{*c}+1}-\frac{1}{z^{*u}+1}\right).$$

In this case, the sign of the derivative again remains unclear. The first term is negative since $\partial W^{cxp}/\partial n < 0$ and $d\bar{n}^u/d(N^*e_2^*) < 0$. The second term is positive given that $z^{*c} < z^{*u}$, as was discussed in Appendix A.3. Notice as well that the derivative when $a = 0$ is equal to $d(G^c - G^u)/d(N^*e_2^*)|_{\gamma=0}$, which implies the sign of $d(G^c - G^u)/d(N^*e_2^*)|_{\gamma=0}$ is indeterminate as well.

Next, suppose that $\gamma = 1$:

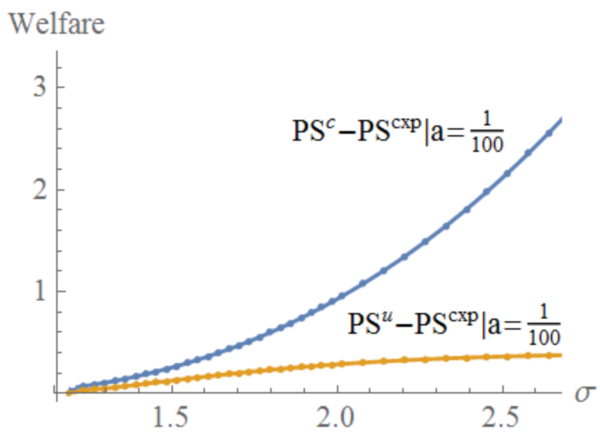
$$\left.\frac{d(G^c-G^u)}{d(N^*e_2^*)}\right|_{\gamma=1} = \left(-\frac{\partial TW^u}{\partial n}+a\frac{d(\bar{n}^u\pi^{cxp})}{dn}\right)\frac{d\bar{n}^u}{d(N^*e_2^*)}+\left(\frac{1}{\sigma}\right)\frac{1}{\tau^*}\left(\frac{1}{z^{*c}+1}-\frac{1}{z^{*u}+1}\right),$$

the sign of which is unclear due to the fact that $\partial TW^u/\partial n$ and $d(\bar{n}^u\pi^{cxp})/dn$ are negative, making the overall sign of the $d\bar{n}^u/d(N^*e_2^*)$ term unclear.

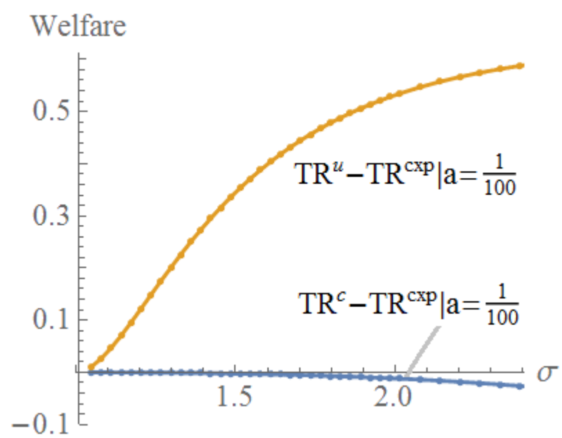
A.7.3 Behavior of $G^c - G^u$ for Changes in σ

In this section, I show that $\lim_{\sigma\to 1}(G^c - G^u) > 0$. For context, Figure A.7 shows how the welfare-benefit of commitment to a trade agreement breaks down into producer surplus,

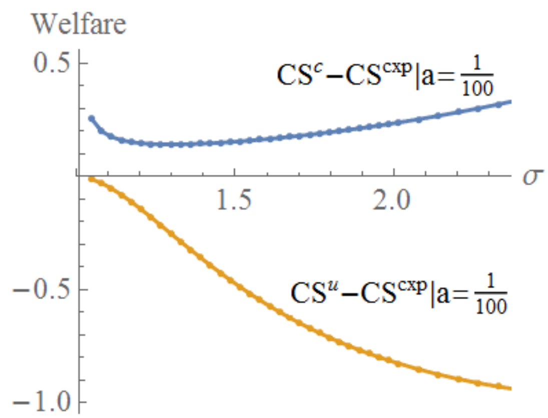
Figure A.7: The value of $G^c - G^u$ for changes in σ, separated into producer surplus, consumer surplus, and tariff revenue. As σ approaches one, the tariff revenue and producer surplus terms go to zero. Only $CS^c - CS^{cxp}$ does not approach zero as $\sigma \to 1$.



(a) Producer surplus for changes in σ.



(b) Consumer surplus for changes in σ.



(c) Consumer surplus for changes in σ.

consumer surplus, and tariff revenue.

Algebraically, the limit of $G^c - G^u$ as $\sigma \to 1$ is

$$
\begin{aligned}
&\lim_{\sigma\to 1}\Big(G^c - G^u\Big) \\
&= Ne_2\Bigg[\frac{1}{\sigma}\Bigg(\underbrace{\frac{z^c}{z^c+1} - \frac{z^{cxp}}{z^{cxp}+1}}_{\to 0 \text{ for } \sigma\to 1}\Bigg) + \underbrace{\Bigg(\frac{\tau^c-1}{\tau^c}\Bigg)\Bigg(\frac{1}{z^c+1}\Bigg) - \Bigg(\frac{\tau^{cxp}-1}{\tau^{cxp}}\Bigg)\Bigg(\frac{1}{z^{cxp}+1}\Bigg)}_{\to 0 \text{ for } \sigma\to 1}\Bigg] \\
&- \gamma Ne_2\Bigg[\frac{1+a}{\sigma}\Bigg(\underbrace{\frac{z^u}{z^u+1} - \frac{z^{cxp}}{z^{cxp}+1}}_{\to 0 \text{ for } \sigma\to 1}\Bigg) + \underbrace{\Bigg(\frac{\tau^u-1}{\tau^u}\Bigg)\Bigg(\frac{1}{z^u+1}\Bigg) - \Bigg(\frac{\tau^{cxp}-1}{\tau^{cxp}}\Bigg)\Bigg(\frac{1}{z^{cxp}+1}\Bigg)}_{\to 0 \text{ for } \sigma\to 1}\Bigg] \\
&+ N^*\frac{e_2^*}{\tau^*}\Bigg[\underbrace{\Bigg(\frac{1}{z^{*c}+1}\Bigg) - \Bigg(\frac{1}{z^{*u}+1}\Bigg)}_{\to 0 \text{ for } \sigma\to 1}\Bigg] + Ne_2 \underbrace{\ln\Bigg(\frac{P^{cxp}}{P^c}\Bigg)}_{\to \infty \text{ for } \sigma\to 1} - \gamma Ne_2 \underbrace{\ln\Bigg(\frac{P^{cxp}}{P^u}\Bigg)}_{\to 0 \text{ for } \sigma\to 1}.
\end{aligned}
$$

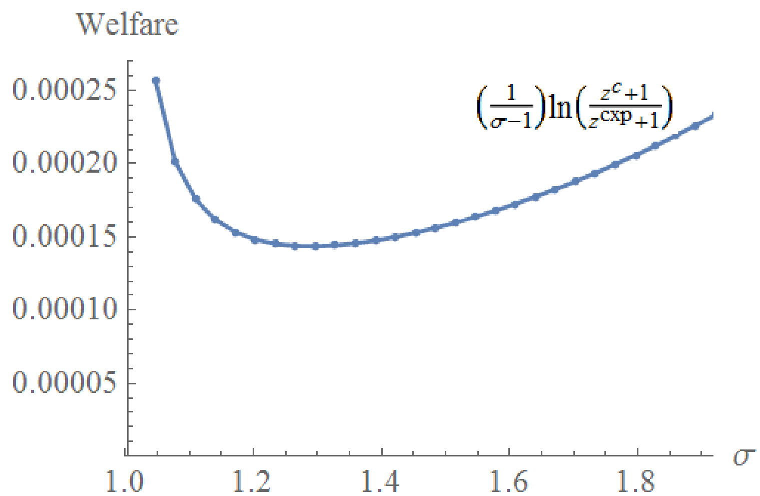
The two terms requiring further discussion are the two price index terms, $\ln(P^{cxp}/P^u)$ and $\ln(P^{cxp}/P^c)$. Expanding these terms using the definition of the price indexes, their limits are

$$
\begin{aligned}
\lim_{\sigma\to 1}\ln\Bigg(\frac{P^{cxp}}{P^c}\Bigg) &= \lim_{\sigma\to 1}\frac{\ln(1/(z^{cxp}+1))}{\sigma-1} - \lim_{\sigma\to 1}\frac{\ln(1/(z^c+1))}{\sigma-1}, \\
&= \lim_{\sigma\to 1}\frac{(1/(z^{cxp}+1))\frac{dz^{cxp}}{d\sigma}}{1} - \lim_{\sigma\to 1}\frac{\ln(1/(z^c+1))\frac{dz^c}{d\sigma}}{1}, \\
&= \lim_{\sigma\to 1}\underbrace{\Bigg(\frac{z^{cxp}}{z^{cxp}+1}\Bigg)}_{\to 1 \text{ as } \sigma\to 1}\Bigg(\underbrace{\ln\Bigg(\frac{\tau^{cxp}\lambda^*}{\lambda(\bar{k}^u)}\Bigg)}_{\to 0 \text{ as } \sigma\to 1} + \underbrace{(\sigma-1)\frac{1}{\tau^{cxp}}\frac{d\tau^{cxp}}{d\sigma}}_{\substack{\to (0)(1)(>0)=0 \\ \text{as } \sigma\to 1}} + \underbrace{\frac{w_\lambda(\bar{k}^u)}{\bar{n}^u\lambda(\bar{k}^u)}\frac{d\bar{n}^u}{d\sigma}}_{\substack{\to (-\infty)(-\infty)=\infty \\ \text{as } \sigma\to 1}}\Bigg) \qquad \text{(A.27)} \\
&- \lim_{\sigma\to 1}\underbrace{\Bigg(\frac{z^c}{z^c+1}\Bigg)}_{\to 1 \text{ as } \sigma\to 1}\Bigg(\underbrace{\ln\Bigg(\frac{\tau^c\lambda^*}{\lambda(\bar{k}^c)}\Bigg)}_{\substack{\to (1)(-\infty)=0 \\ \text{as } \sigma\to 1}} + \underbrace{(\sigma-1)\frac{1}{\tau^c}\frac{d\tau^c}{d\sigma}}_{\substack{\to (0)(1)(>0)=0 \\ \text{as } \sigma\to 1}}\Bigg) = \infty,
\end{aligned}
$$

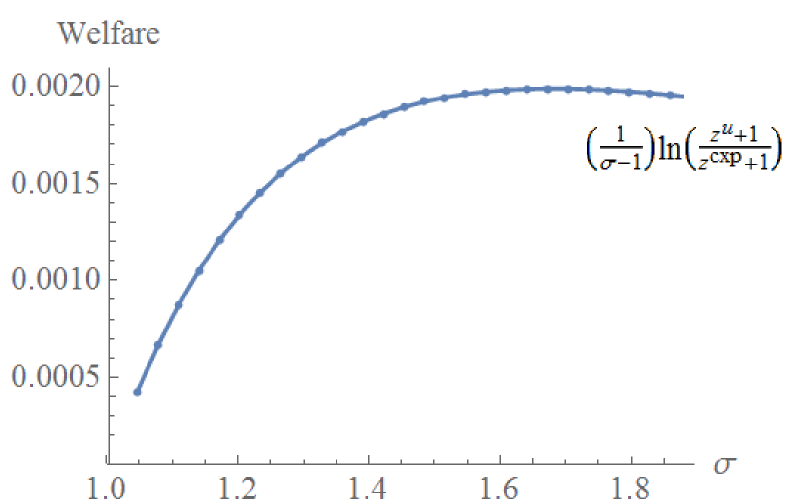
where the limits of $d\tau^{cxp}/d\sigma$ and $d\bar{n}^u/d\sigma$ are derived in Appendix A.6.2. Calculating the limit for $\ln\Big(P^{cxp}/P^u\Big)$ shows that $\lim_{\sigma\to 1}\ln\Big(P^{cxp}/P^u\Big) = 0$ given that the only nonzero term in $\ln\Big(P^{cxp}/P^c\Big)$ is the firm-entry derivative term, $w_\lambda(\bar{k}^u)/(\bar{n}^u\lambda(\bar{k}^u))\,(d\bar{n}^u/d\sigma)$ which is equal for u and cxp. The limit of the price index terms is also demonstrated using the numerical solution in Figure A.8.

Overall, this implies that $\lim_{\sigma\to 1} G^c - G^u = \infty$ and the government will always prefer

Figure A.8: Behavior of consumer surplus terms, $CS^c - CS^{cxp}$ and $CS^u - CS^{cxp}$. The left-hand panel depicts the part of $CS^c - CS^{cxp}$ which does not go to zero as $\sigma \to 1$, as shown in equation (A.27). The right-hand panel shows that for $CS^u - CS^{cxp}$ the same term does approach zero as $\sigma \to 1$.



(a) Consumer surplus, $CS^c - CS^{cxp}$, for changes in σ.



(b) Consumer surplus, $CS^c - CS^{cxp}$, for changes in σ.

to join a trade agreement when $\sigma \to 1$.

When $\sigma \to \sigma^{Max}$, I am unable to determine whether $G^c - G^u$ is positive or negative analytically because I cannot make use of any limiting behavior. Numerically, however, the above limit is shown to be positive in Figure 1.11.

Next, for completeness of the discussion of the behavior of $G^c - G^u$ for changes in σ, consider the derivative of the government's welfare benefit from commitment to a trade agreement for changes in σ. The derivative of $G^c - G^u$ with respect to σ is

$$\begin{aligned}\frac{d(G^c - G^u)}{d\sigma} = \frac{\partial W^c}{\partial \sigma} &- \gamma\left(\frac{\partial TW^u}{\partial \sigma} + \frac{\partial TW^u}{\partial n}\frac{d\bar{n}^u}{d\sigma}\right) \\ &- (1-\gamma)\left(\frac{\partial W^{cxp}}{\partial \sigma} + \frac{\partial W^{cxp}}{\partial n}\frac{d\bar{n}^u}{d\sigma}\right) + \gamma a \frac{d(\bar{n}^u \pi^{cxp})}{d\sigma},\end{aligned}$$

which is simplified using the envelope condition for the government's tariff-setting problem and using the fact that $\partial W^c / \partial n = 0$.

For reference, $G^c - G^u$ is shown in Figure 1.11 for varying a and γ levels.[8] Figure 1.11 indicates that when a is close to zero, the government prefers to join a trade agreement for all values of σ and γ (blue line in bottom panel). When a is large enough, then for any $\gamma > \bar{\bar{\gamma}}$ there exists a $\sigma \in (1, \sigma^{Max})$ for which the government prefers not to join a trade agreement. These ranges of σ are delineated by the color-coded shaded regions with the range of σ where the government weakly prefers no trade agreement equal to $[\sigma_1^*(a,\gamma), \sigma_2^*(a,\gamma)]$. The top panel shows that as γ increases holding a constant, σ_1^* decreases and σ_2^* increases. The bottom panel shows the same is true when a increases holding γ constant.

To algebraically examine $d(G^c - G^u)/d\sigma$, first look more closely at the derivatives of W^c, TW^u, and W^{cxp} with respect to σ. To begin, first remember the total politically weighted welfare equation is $TW = W + aIn\pi$ where I is an indicator equal to one if producers successfully lobby the government and zero otherwise. The partial derivative of total

[8]Note that this figure then translates into the region plot, Figure 1.13, in Section 1.5.3.

welfare with respect to σ is

$$\begin{aligned}\frac{\partial TW}{\partial \sigma} = Ne_2 \Bigg\{ & - \frac{1}{z+1}\left(\frac{1}{\tau}\right)\frac{1}{\sigma-1}\ln\left(\frac{\tau p_f^*}{p_h}\right) \\ & - \frac{N^* e_2^*/\tau^*}{N\, e_2}\frac{(1+aI)}{\sigma}\frac{z^*}{(z^*+1)^2}\ln\left(\frac{\tau^* p_h}{p_f^*}\right) + \frac{1}{(\sigma-1)^2}\ln\left(\frac{1}{\bar{n}}\left(\frac{z}{z+1}\right)\right) \\ & + \frac{1}{\sigma^2}\left[\frac{\sigma}{\sigma-1} - (1+aI)\frac{z}{z+1} - (1+aI)\frac{N^* e_2^*/\tau^*}{N\, e_2}\frac{1}{z^*+1}\right]\Bigg\}.\end{aligned} \tag{A.28}$$

The most problematic terms in the above equation are the natural log terms. First, it is not clear whether $\bar{n}$ is greater than or less than one for either $\bar{n}^c$ or $\bar{n}^u$. In addition, whether $z/(z+1)$ is greater than or less than one is also not evident.

Regarding the relative price terms, as I discussed in Appendix A.3, $\lim_{\sigma\to 1}\ln(\tau p_f^*/p_h) = 0$ and $\lim_{\sigma\to\infty}\ln(\tau p_f^*/p_h) = 0$, with $\ln(\tau p_f^*/p_h)$ being either positive or negative for intermediate values of σ depending on model parameters. While I cannot establish the sign of these log terms, I can establish that $\ln(\tau^c p_f^*/p_h^c) > \ln(\tau^{cxp} p_f^*/p_h^u)$ and $\ln(\tau^u p_f^*/p_h^u) > \ln(\tau^{cxp} p_f^*/p_h^u)$ for all parameter values. The log of the relative price abroad is $\ln(\tau^* p_h/p_f^*) > 0$ for all parameter values.

In the threat-point welfare equation the derivative of the ex-post operating profits $\bar{n}^u \pi^{cxp}$ enters into the equation as well. The threat point terms are

$$\begin{aligned}(1-\gamma)\frac{dW^{cxp}}{d\sigma} - \gamma a \frac{d(\bar{n}^u \pi^{cxp})}{d\sigma} = {} & (1-\gamma)\left(\frac{\partial W^{cxp}}{\partial \sigma} + \frac{\partial W^{cxp}}{\partial n}\frac{d\bar{n}^u}{d\sigma}\right) \\ & - \gamma a \left(\frac{\partial(\bar{n}^u \pi^{cxp})}{\partial \sigma} + \frac{\partial(\bar{n}^u \pi^{cxp})}{\partial \tau}\frac{d\tau^{cxp}}{d\sigma} + \frac{\partial(\bar{n}^u \pi^{cxp})}{\partial n}\frac{d\bar{n}^u}{d\sigma}\right).\end{aligned}$$

Fully expanding the cxp terms demonstrates the difficulties in determining the effect of σ on $G^c - G^u$. The equation below is the sum of the cxp terms, with some notes regarding

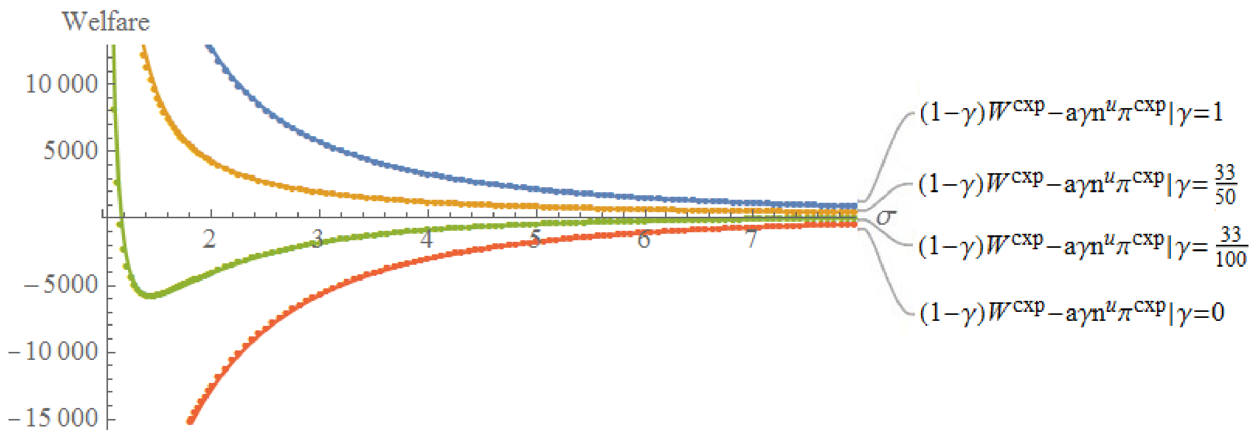


Figure A.9: Welfare $(1-\gamma)W^{cxp} + a\gamma\bar{n}^u\pi^{cxp}$ for changes in σ given $a = 181/100$.

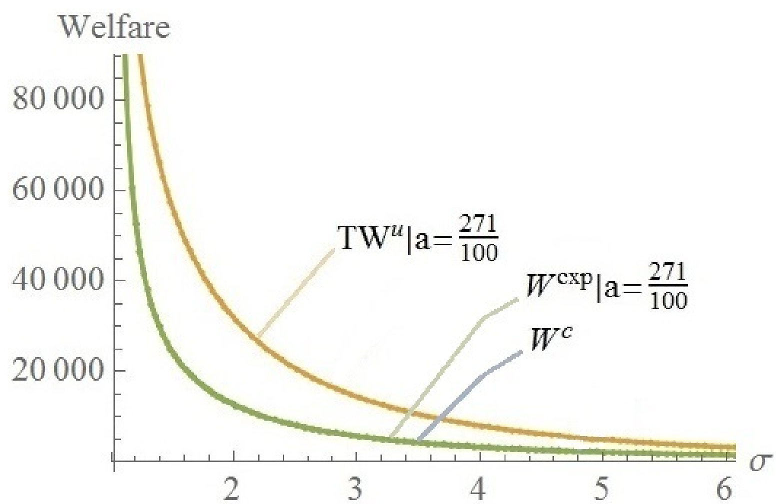
the sign of each term included:

$$
\begin{aligned}
Ne_2 \Bigg\{ & \underbrace{\frac{1}{\sigma^2}\left((1-\gamma)\frac{\sigma}{\sigma-1} - (1-\gamma(1+a))\frac{z^{cxp}}{z^{cxp}+1}\right)}_{>0 \text{ for } a \text{ low}} \underbrace{-(1-\gamma(1+a))\frac{N^* e_2^*/\tau^*}{N\, e_2}\frac{1}{\sigma^2}\frac{1}{z^{*u}+1}}_{<0} \\
& \underbrace{-\gamma\, a\frac{z^{cxp}}{(z^{cxp}+1)^2}\frac{(\sigma z^{cxp}+1)}{(\sigma z^{cxp}+1)-(\tau^{cxp}-1)(\sigma-1)}}_{<0} \underbrace{\left(\frac{1}{\sigma}\left(\frac{1}{\sigma-1}\right)(\sigma z^{cxp}+1) - z^{cxp}\right)}_{>0 \text{ when } \sigma \text{ is low}} \\
& \underbrace{-\frac{1}{z^{cxp}+1}\left(\frac{1}{\sigma-1}\right)\left(\frac{(1-\gamma)}{\tau^{cxp}} + a\gamma\frac{z^{cxp}}{(z^{cxp}+1)^2}\frac{(\tau^{cxp}-1)}{\frac{\sigma z^{cxp}+1}{\sigma-1}-(\tau^{cxp}-1)}\right)\ln\left(\frac{\tau^{cxp}p_f^*}{p_h^u}\right)}_{>0 \text{ given } \ln(\tau^{cxp}p_f^*/p_h^u)<0 \text{ in Num Soln}} \\
& \underbrace{-(1-\gamma(1+a))\frac{N^* e_2^*/\tau^*}{N\, e_2}\frac{1}{\sigma}\frac{z^{*u}}{(z^{*u}+1)^2}\ln\left(\frac{\tau^* p_h^u}{p_f^*}\right)}_{>0 \text{ if } (1-\gamma)/\gamma > a \ (\gamma \text{ low or } a \text{ low})} \underbrace{+\frac{(1-\gamma)}{(\sigma-1)^2}\ln\left(\frac{1}{\bar{n}^u}\left(\frac{z^{cxp}}{z^{cxp}+1}\right)\right)}_{<0 \text{ for } \bar{n}^u > 1}\Bigg\} \\
& + \underbrace{\frac{w_\lambda(\bar{k}^u)}{\bar{n}^u\lambda(\bar{k}^u)}\left(\frac{1}{\sigma-1}\right)\frac{d\bar{n}^u}{d\sigma}Ne_2}_{>0 \text{ when } \sigma<\sigma^n} \Bigg[\underbrace{(1-\gamma(1+a))\frac{N^* e_2^*/\tau^*}{Ne_2}\left(\frac{\sigma-1}{\sigma}\right)\frac{z^{*u}}{(z^{*u}+1)^2}}_{>0 \text{ if } (1-\gamma)/\gamma > a \ (\gamma \text{ low or } a \text{ low})} \\
& \qquad + \underbrace{\frac{z^{cxp}}{z^{cxp}+1}\left((1-\gamma)\left(\left(\frac{\tau^{cxp}-1}{\tau^{cxp}}\right)\frac{1}{z^{cxp}}+1\right) - \gamma a\frac{\left(\frac{1}{\sigma}\right)\frac{1}{z^{cxp}+1}(\sigma z^{cxp}+1)}{\frac{\sigma z^{cxp}+1}{\sigma-1}-(\tau^{cxp}-1)}\right)}_{>0 \text{ if } \gamma \text{ or } a \text{ low}}\Bigg].
\end{aligned}
$$

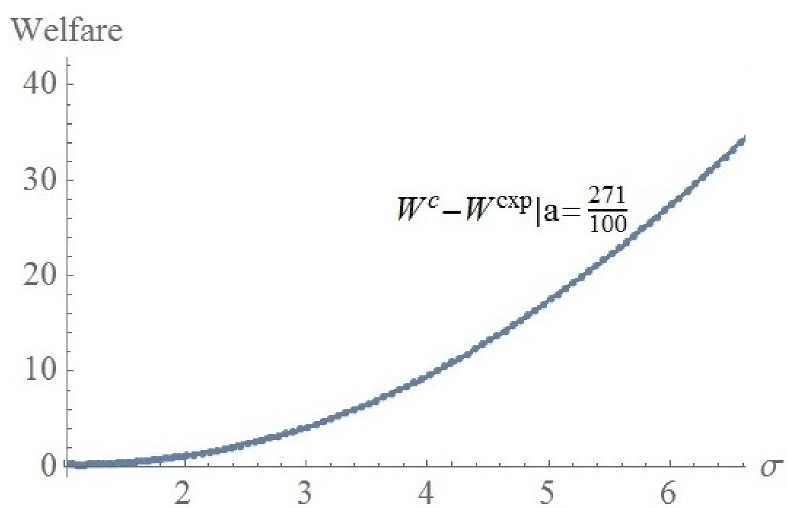
First, it is immediately clear that when a is close to zero, Figure A.9 shows how $(1-\gamma)W^{cxp} + a\gamma\bar{n}^u\pi^{cxp}$ is changing in σ.

Using the numerical solution, Figure A.10 depicts that the total welfare functions W^c,

Figure A.10: Total welfare, TW^u, W^{cxp}, and W^c, for changes in σ given $a = 271/100$. The total welfare is falling in σ for all values of σ for all a. On the left-hand panel, scale makes W^c and W^{cxp} indistinguishable. The right-hand panel depicts the difference between W^c and W^{cxp}, which is increasing in σ.



(a) Social welfare, TW^u, W^{cxp}, and W^c, for changes in σ.



(b) Social welfare, $W^c - W^{cxp}$, for changes in σ.

W^{cxp}, and TW^c are all increasing in σ, and the result holds for all values of a.

Analytically solving for the effect of a change in σ on the government's welfare benefit from commitment to a trade agreement can be understood to an extent by examining the extreme values of a and γ. Recall that Figure 1.11 depicts $G^c - G^u$ for various a and γ values for reference during the algebraic analysis in this section.

To begin work on the analytical solution, it is clear that a change in γ has a linear effect on $G^c - G^u$ while a has a much more complex relationship with welfare due to its influence on production and the tariff level. Looking at the extreme values of a and γ, first note that $d(G^c - G^u)/d\sigma$ is the same when the government has no bargaining strength, $\gamma = 0$, as when the political economy weight is equal to zero, $a = 0$:

$$\left.\frac{d(G^c - G^u)}{d\sigma}\right|_{\gamma=0} = \left.\frac{d(G^c - G^u)}{d\sigma}\right|_{a=0} = \frac{\partial W^c}{\partial \sigma} - \left(\frac{\partial W^{cxp}}{\partial \sigma} + \frac{\partial W^{cxp}}{\partial n}\frac{d\bar{n}^u}{d\sigma}\right),$$

with the equality of $d(G^c - G^u)/d\sigma|_{\gamma=0}$ and $d(G^c - G^u)/d\sigma|_{a=0}$ depending on the fact that when $a = 0$, $TW^u = W^{cxp}$. The sign of this equation is unclear due to the difficulties in comparing $\partial W^c/\partial\sigma$ and $\partial W^{cxp}/\partial\sigma$.

Next, suppose that $\gamma = 1$, in this case the derivative of the welfare benefit of committing to a trade agreement is

$$\left.\frac{d(G^c - G^u)}{d\sigma}\right|_{\gamma=1} = \frac{\partial W^c}{\partial \sigma} - \left(\frac{\partial TW^u}{\partial \sigma} + \frac{\partial TW^u}{\partial n}\frac{d\bar{n}^u}{d\sigma}\right) + a\frac{d(\bar{n}^u\pi^{cxp})}{d\sigma}.$$

Again, this provides no real insight into the behavior of $G^c - G^u$ for changes in σ, given $\bar{n}^u$ is nonmonotonic in σ and given the additional difficulties in signing the welfare partial derivatives with respect to σ.

Next, consider $a \to \infty$. When this is the case, $\bar{n}^u \to \bar{n}^c$ meaning that $\lim_{a\to\infty} W^{cxp} = W^c$. Simplifying the derivative of $G^c - G^u$,

$$\left.\frac{d(G^c - G^u)}{d\sigma}\right|_{a\to\infty} = -\gamma\frac{\partial W^u}{\partial \sigma} - \gamma a\left(\frac{d(\bar{n}^u\pi^u)}{d\sigma} - \frac{d(\bar{n}^u\pi^{cxp})}{d\sigma}\right).$$

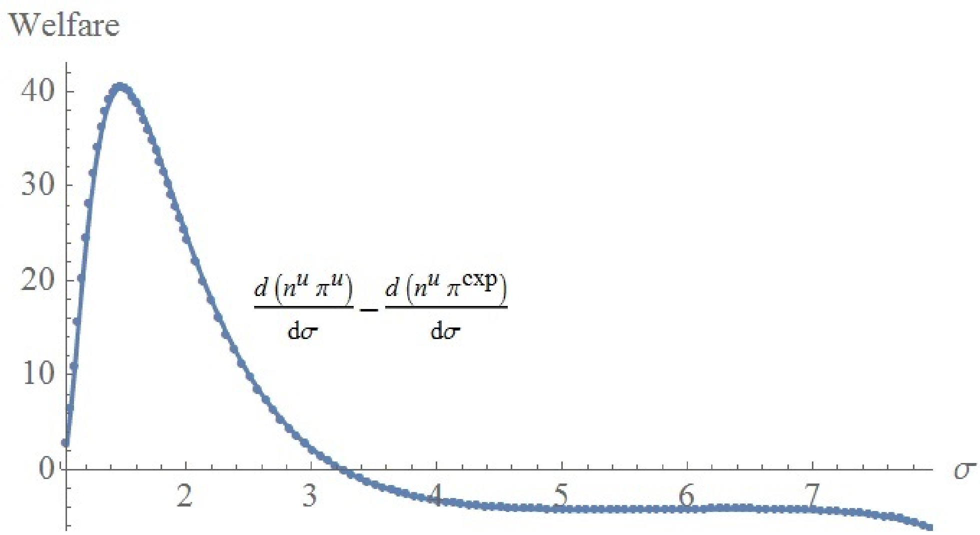


Figure A.11: $d(\bar{n}^u\pi^u)/d\sigma$ and $d(\bar{n}^{cxp}\pi^u)/d\sigma$ for changes in σ for $a = 181/100$. This basic pattern is consistent for all values of a.

Where the derivative of ex-post operating profits with respect to σ is

$$\frac{d(\bar{n}\pi)}{d\sigma} = Ne_2\Bigg\{ \underbrace{\frac{\frac{z}{(z+1)^2}(\sigma-1)(\tau-1)}{(\sigma z+1)-(\sigma-1)(\tau-1)}\left(\frac{1}{\sigma}\left(\frac{1}{\sigma-1}\right)(\sigma z+1)-z\right)}_{>0 \text{ when } \sigma \text{ low}}$$

$$\underbrace{+\frac{1}{\sigma^2}\left(\frac{z}{z+1}+\frac{N^*e_2^*/\tau^*}{Ne_2}\frac{1}{z^*+1}\right)}_{>0}$$

$$+\frac{1}{\sigma}\Bigg[\underbrace{\frac{\frac{z}{(z+1)^2}(\sigma z+1)}{(\sigma z+1)-(\sigma-1)(\tau-1)}\ln\left(\frac{\tau p_f^*}{p_h}\right)}_{<0 \text{ when } \sigma \text{ high or low}}+\underbrace{\frac{N^*e_2^*/\tau^*}{Ne_2}\frac{z^*}{(z^*+1)^2}\ln\left(\frac{p_f^*}{\tau^* p_h}\right)}_{<0 \text{ for all } \sigma}\Bigg]$$

$$\underbrace{+\frac{1}{\sigma}\left[\frac{\frac{z}{(z+1)^2}(\sigma z+1)}{(\sigma z+1)-(\sigma-1)(\tau-1)}+\frac{N^*e_2^*/\tau^*}{Ne_2}\frac{z^*}{(z^*+1)^2}\right]\frac{w_\lambda(\bar{k})}{\bar{n}\lambda(\bar{k})}\frac{d\bar{n}}{d\sigma}}_{>0 \text{ when } \sigma \text{ low}}\Bigg\}.$$

Using the numerical solution, I find that overall $d(\bar{n}^u\pi^u)/d\sigma$ and $d(\bar{n}^{cxp}\pi^u)/d\sigma$ are negative for all values of a and σ. Additionally, the numerical solution demonstrates that $d(\bar{n}^u\pi^u)/d\sigma - d(\bar{n}^u\pi^{cxp})/d\sigma$ is positive when σ is low and negative when σ is high, depicted in Figure A.11. Therefore, when σ is high, $d(G^c - G^u)/d\sigma|_{a\to\infty}$ is positive. When σ is low, it may be either positive or negative depending on whether the direct effect of σ on W^c

outweighs the effect of σ on the politically-weighted producer rents from tariff protection.

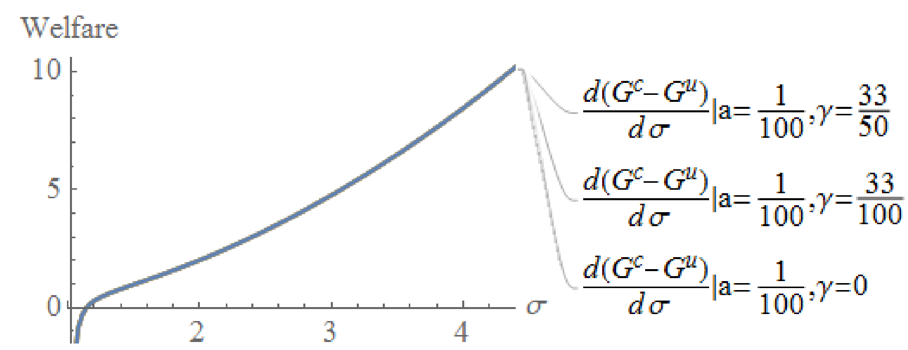
Figures A.12 and A.13 depict the behavior of $d(G^c - G^u)/d\sigma$ for changes in σ and for changes in a to help demonstrate how much the a and γ parameters affect the relationship between $G^c - Gu$ and σ. Figure A.12a shows that when a is very low, γ has little effect on the behavior of $d(G^c - G^u)/d\sigma$, with $G^c - G^u$ decreasing in σ when σ is close to one and then increasing and convex in σ for higher σ. Figure A.12b illustrates a finding that is true for higher values of a: $d(G^c - G^u)/d\sigma$ is negative when σ is close to one and is positive for higher values of σ, with the cutoff value for which $d(G^c - G^u)/d\sigma = 0$, call it $\sigma^*(a, \gamma)$, is (i) increasing in γ and (ii) nonmonotonic in a. To clarify where $d(G^c - G^u)/d\sigma = 0$ for changes in a, consider Figure A.13 which graphs $d(G^c - G^u)/d\sigma$ for changes in a.

Figure A.13 shows that for any given σ, when $\gamma > \bar{\gamma}$, there are two values of a for which $d(G^c - G^u)/d\sigma = 0$. The top-right panel shows clearly that the lower value of a for which $d(G^c - G^u)/d\sigma = 0$ is falling in γ and the upper value of σ where $d(G^c - G^u)/d\sigma = 0$ is increasing in γ. Therefore, as γ falls, the range of a for which the government prefers not to join a trade agreement is growing in γ. As σ rises, however, there is eventually a point where $d(G^c - G^u)/d\sigma > 0$ for all values of σ. Therefore, when σ is very high or very low, the government prefers to commit to a trade agreement for all values of $a \geq 0$ and $\gamma \in [0, 1]$.

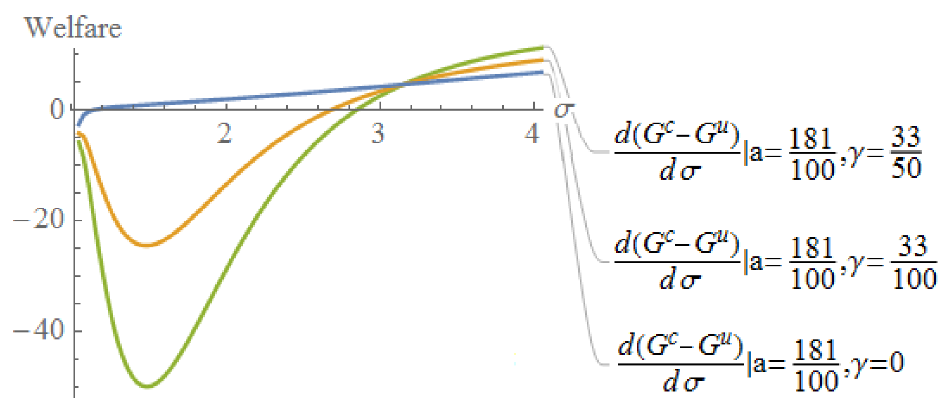
A.7.4 Derivative of $\hat{\sigma}(a)$ with respect to a

The derivative of $\hat{\sigma}(a)$ with respect to a as defined by equation (1.61) can be signed by determining if $d^2(G^c - G^u)/d\sigma da + (d\hat{\gamma}(a)/da)(d^2(G^c - G^u)/d\sigma d\gamma)$ is positive or negative. Here, I expand the two cross-partial derivatives.

Figure A.12: $d(G^c - G^u)/d\sigma$ for changes in σ. For any $\gamma \geq 0$, $d(G^c - G^u)/d\sigma$ is negative when σ is low and positive when σ is high. When a is close to zero (top), γ has little effect on $d(G^c - G^u)/d\sigma$. When a is larger (bottom), as γ increases, the variability of $d(G^c - G^u)/d\sigma$ also increases.



(a) $d(G^c - G^u)/d\sigma$ for changes in σ when a is close to zero.



(b) $d(G^c - G^u)/d\sigma$ for changes in σ when a is large.

The cross-partial derivative of $G^c - G^u$ with respect to σ and a is

$$\begin{aligned}\frac{d^2(G^c - G^u)}{d\sigma da} = & -\frac{\partial^2 G^u}{\partial n^2}\frac{d\bar{n}^u}{da}\frac{d\bar{n}^u}{d\sigma} - \frac{\partial^2 G^u}{\partial n \partial \tau}\left(\frac{d\bar{n}^u}{d\sigma}\frac{d\boldsymbol{\tau}}{da} + \frac{d\bar{n}^u}{da}\frac{d\boldsymbol{\tau}}{d\sigma}\right) - \frac{\partial^2 G^u}{\partial n \partial \sigma}\frac{d\bar{n}^u}{da} \\ & - \frac{\partial G^u}{\partial n}\frac{d^2\bar{n}}{dad\sigma} - \frac{\partial^2 G^u}{\partial \boldsymbol{\tau}^2}\frac{d\boldsymbol{\tau}}{d\sigma}\frac{d\boldsymbol{\tau}}{da} - \frac{\partial^2 G^u}{\partial \boldsymbol{\tau}\partial \sigma}\frac{d\boldsymbol{\tau}}{da} - \frac{\partial G^u}{\partial \boldsymbol{\tau}}\frac{d^2\boldsymbol{\tau}}{dad\sigma} \\ & - \frac{\partial^2 G^u}{\partial a \partial n}\frac{d\bar{n}^u}{d\sigma} - \frac{\partial^2 G^u}{\partial a \partial \boldsymbol{\tau}}\frac{d\boldsymbol{\tau}}{d\sigma} - \frac{\partial^2 G^u}{\partial a \partial \sigma}.\end{aligned} \tag{A.29}$$

Rather than writing out that the derivatives are for both τ^u and τ^{exp}, I've simplified the

Parameter	Description	Value
K	Home capital stock	1,000
L	Home labor supply	1,000
Ne_2	Home expenditure on sector 2 good	1,000
$\lambda(k)$	Home unit-labor-cost	$\mathbf{e}^{1/k}$
$\theta(k)$	Upfront cost of lobbying	$k^g,\ g = \frac{19}{16}$
n^*	Foreign number of firms in sector 2	200
τ^*	Foreign tariff on imports sector 2 good	1.15
λ^*	Foreign unit labor cost	1
σ	Elasticity of substitution of varieties	$\{\frac{65}{64}, \frac{66}{64}, \ldots, 8\}$
a	Political economy weight	$\{\frac{1}{10}, \frac{2}{10}, \ldots, 5\}$
γ	Government bargaining strength	$\{0, \frac{1}{100}, \ldots, 1\}$
$\frac{N^* e_2^*}{Ne_2 + N^* e_2^*}$	Foreign expenditure share	$\{\frac{25}{26}, \frac{50}{51}, \frac{75}{76}\}$

Table A.1: Parameter Values in Numerical Solution.

notation, with $\boldsymbol{\tau} \equiv \{\tau^u, \tau^{cxp}\}$. Next, the cross-partial derivative with respect to σ and γ is

$$\frac{d^2(G^c - G^u)}{d\sigma d\gamma} = \frac{\partial^2(G^c - G^u)}{\partial\gamma\partial n}\frac{d\bar{n}^u}{d\sigma} + \frac{\partial^2(G^c - G^u)}{\partial\gamma\partial\boldsymbol{\tau}}\frac{d\boldsymbol{\tau}}{d\sigma} + \frac{\partial^2(G^c - G^u)}{\partial\gamma\partial\sigma}. \tag{A.30}$$

The effect of changing a on the range of σs for which the government chooses to join a trade agreement according to the numerical solution is shown in Figure A.14. I find that the range of σs for which the government does not join a trade agreement is increasing as either the government's political economy weight rises or when the government's bargaining strength improves.

A.8 Numerical Solution

The values of the model parameters used in the numerical estimates are given in Table A.1. In the numerical solution, $\theta(k)$ is assumed to take the form of $\theta(k) = k^g$. Given that

$\theta''(k) > 0$ according to the Shapley value solution in Appendix A.1, g is restricted so that $g > 1$.

The specification of the unit-labor-input requirement function, $\lambda(k)$, used for the numerical solution is $\lambda(k) \equiv \mathrm{e}^{\frac{1}{k}}$. This specification is used to ensure the behavior of $\lambda(k)$ is such that there exists a unique interior equilibrium solution to the model. The key features of this function necessary for an interior solution to exist come from a couple of sources in the paper. First, in the basic assumptions on the unit-labor cost, I assume $\lambda(k) > 0$, $\lambda'(k) < 0$, and $\lambda(0) \to \infty$. Then, in order for an interior solution to exist, it needs to be true that there exists a $k \in (0, \infty)$ for which $w_\lambda(k) \equiv \lambda(k) + (\sigma - 1)\lambda'(k)k = 0$, subject also to the second-order conditions for $\bar{k}^c$ and $\bar{k}^u$.[9]

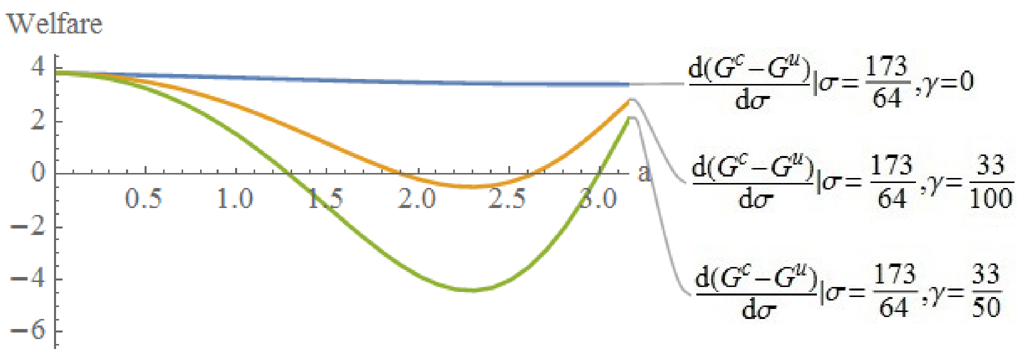
Figure A.15 demonstrates the behavior of $G^c - G^u$ for changes in a. This figure is first referenced in footnote 47, regarding the fact that the welfare benefit of committing to a trade agreement is positive when a is near zero and is negative when a is large.In this figure, I demonstrate this for a value of σ where the constraint $(1 + a) < \left(\frac{\sigma}{\sigma-1}\right)\sigma$ is nearly at its most restrictive, as I mention in footnote 51.

As mentioned in footnote 59, Figure A.16 demonstrates the numerical solution for the direct versus indirect effects of a change in $N^* e_2^*$ as defined in equation (1.56). It shows that the direct effect of a change in $N^* e_2^*$ is positive and smaller than the negative indirect effect in magnitude, resulting in a negative overall effect for all possible σ values.

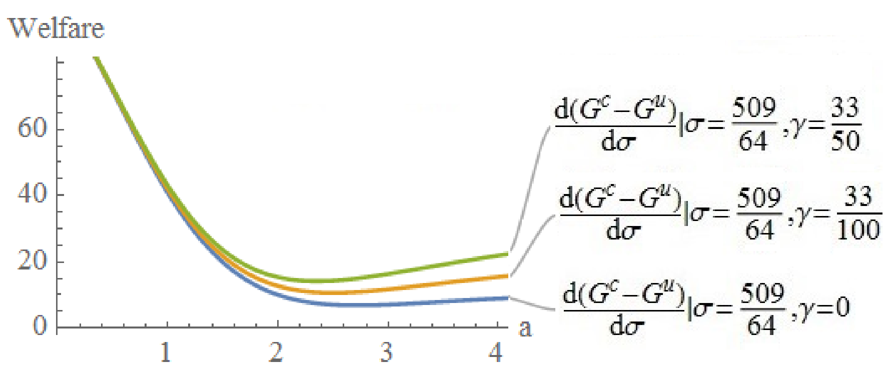
As referenced in footnote 48, Figure A.17 shows the relationship between trade policy preference and when lobbying occurs by adding the lobbying participation constraint to the region plot from Figure 1.9. The figure is labeled with the line $\gamma^{PC}(a)$, which gives the combinations of (a, γ) for which the lobbying participation constraint is equal to zero. If there is an increase in σ, then the $\gamma^{PC}(a)$ curve shifts downward. For σ large enough, there is no (a, γ) for which the lobby forms.

[9] Some examples of functional forms that do not satisfy these requirements are $\lambda(k) = k^x$ for any $x \in \mathbb{R}$ and $\lambda(k) = x \ln(k)$ for any $x < 0$.

Figure A.13: $d(G^c - G^u)/d\sigma$ for changes in a. $d(G^c - G^u)/d\sigma$ is positive for all a when $\gamma = 0$. When $\gamma > 0$, when a is low $G^c - G^u$ is concave in σ when σ is very low. When σ is high $G^c - G^u$ is increasing and convex in a when γ is close to one and a is high.

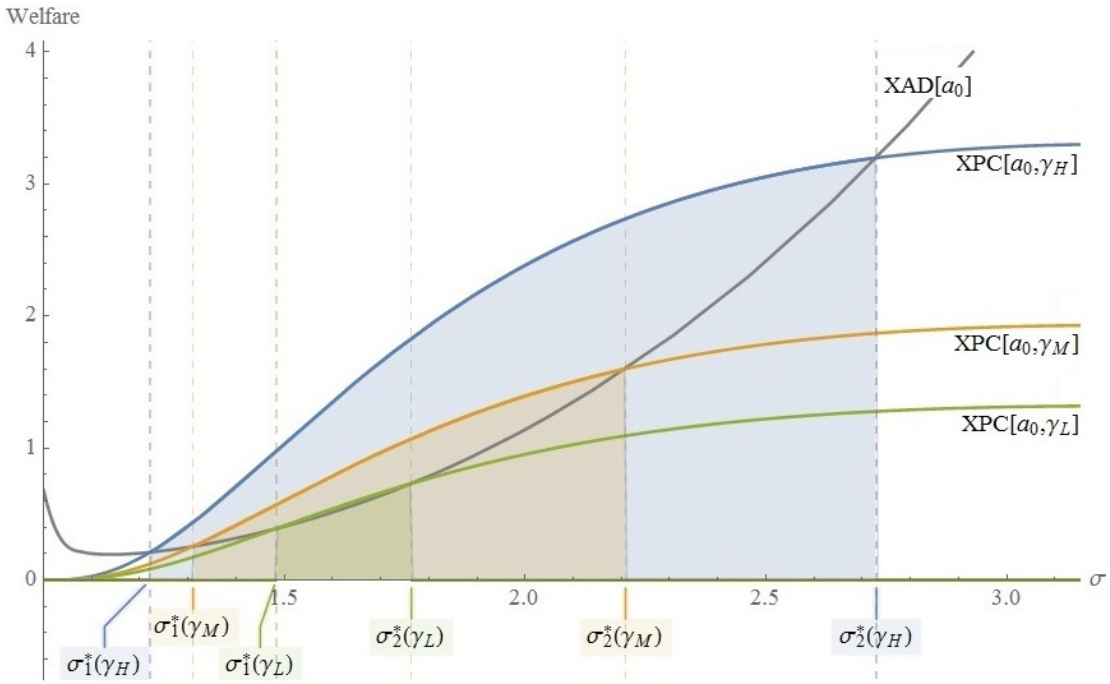


(a) $d(G^c - G^u)/d\sigma$ for changes in a when σ is low.

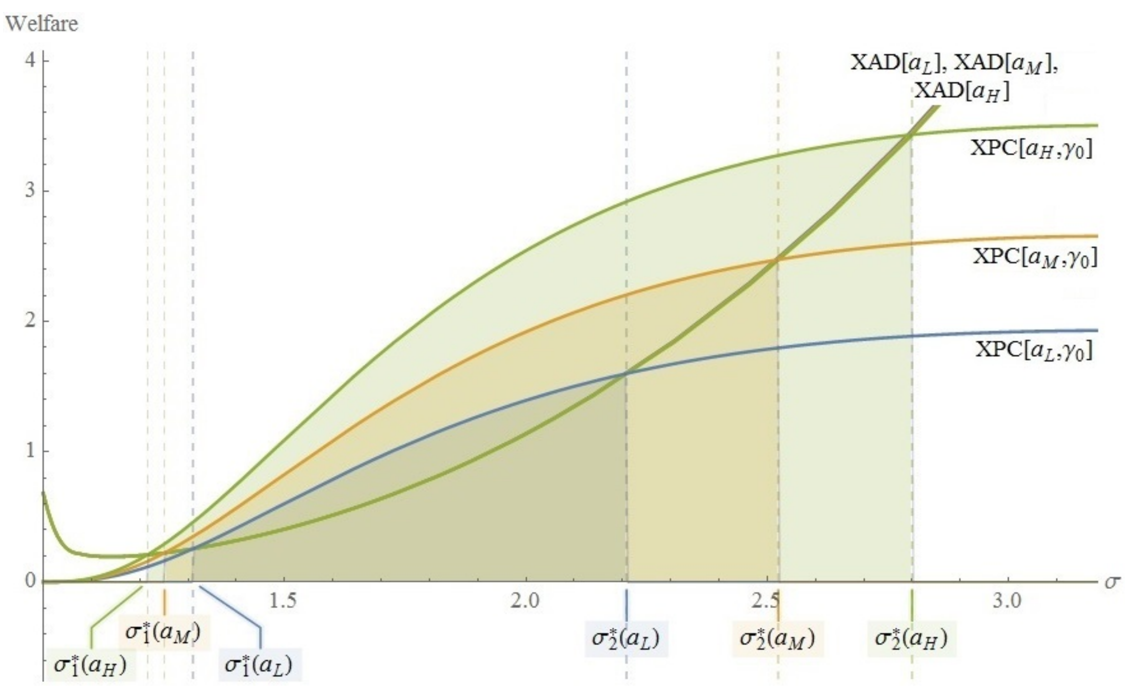


(b) $d(G^c - G^u)/d\sigma$ for changes in a when σ is large.

Figure A.14: The effect of changing a or γ on the region for which the government does not join a trade agreement, (σ_1^*, σ_2^*). The shaded regions represent the areas where XPC is above XAD, which means the government prefers not to join a trade agreement.

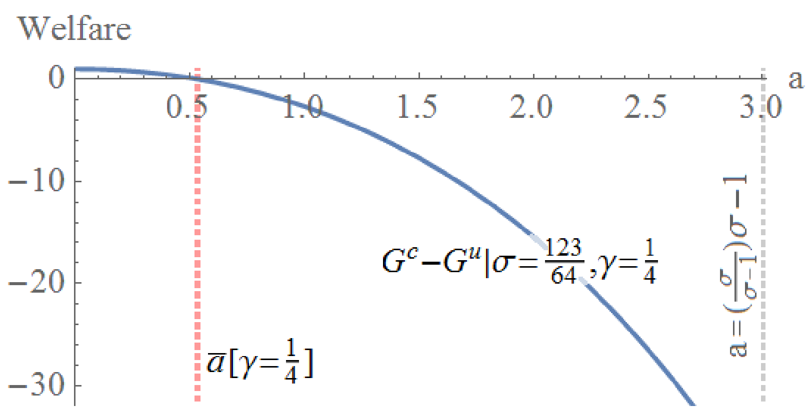


(a) Holding $a = a_0 = 43/40$ constant: as γ increases, σ_1^* falls and σ_2^* rises, making the range of σs where the government does not join a TA larger.

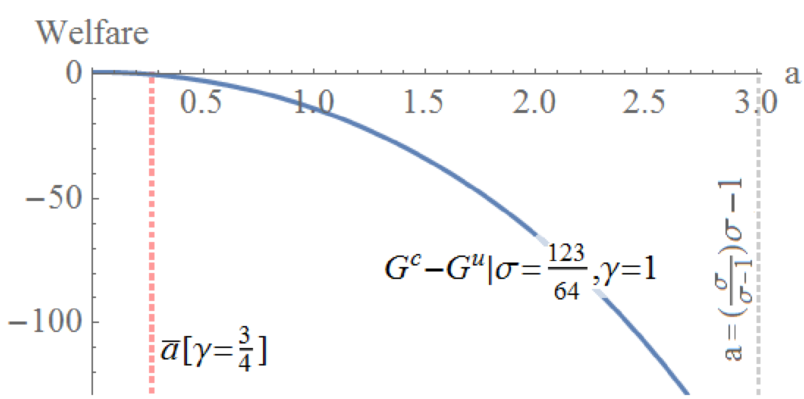


(b) Holding $\gamma = \gamma_0 = \hat{\gamma}(a = 7/8)$ constant: as a increases, σ_1^* falls and σ_2^* rises, increasing the range of σs for which the government does not join a TA.

Figure A.15: The welfare benefit of joining a trade agreement given $\sigma = 123/64$ for $\gamma = \{1/4, 1\}$. The figure shows that $G^c - G^u|_{a\to 0} > 0$ and $G^c - G^u$ is decreasing in a.

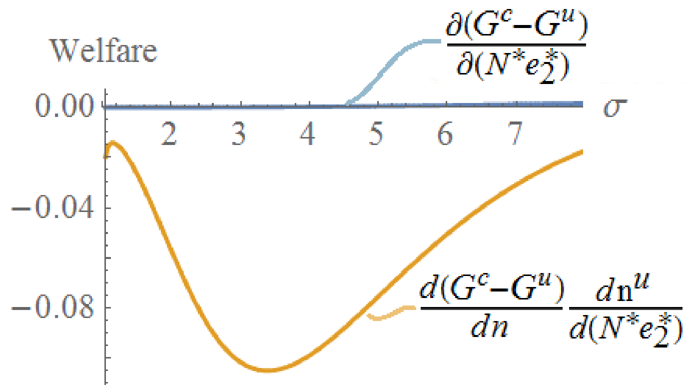


(a) $G^c - G^u$ for changes in a when γ is low.

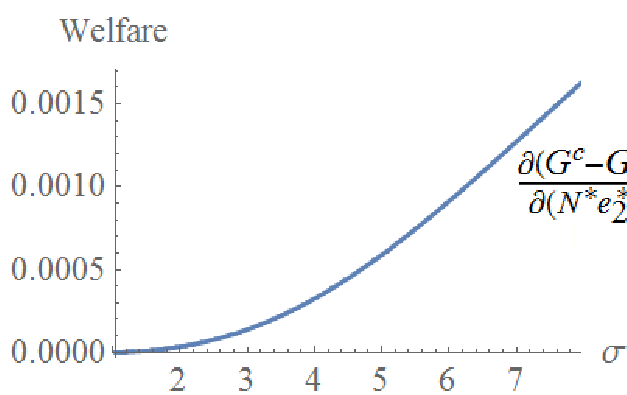


(b) $d(G^c - G^u)/d\sigma$ for changes in a when γ is high.

Figure A.16: The derivative of $G^c - G^u$ with respect to $N^* e_2^*$ for varying σs, given $a = 271/100$ and $\gamma = 1/2$. The left-hand panel shows the direct effect in blue and the indirect effect in orange. The right-hand panel shows only the direct effect.



(a) $G^c - G^u$ for changes in a when γ is low.



(b) $d(G^c - G^u)/d\sigma$ for changes in a when γ is high.

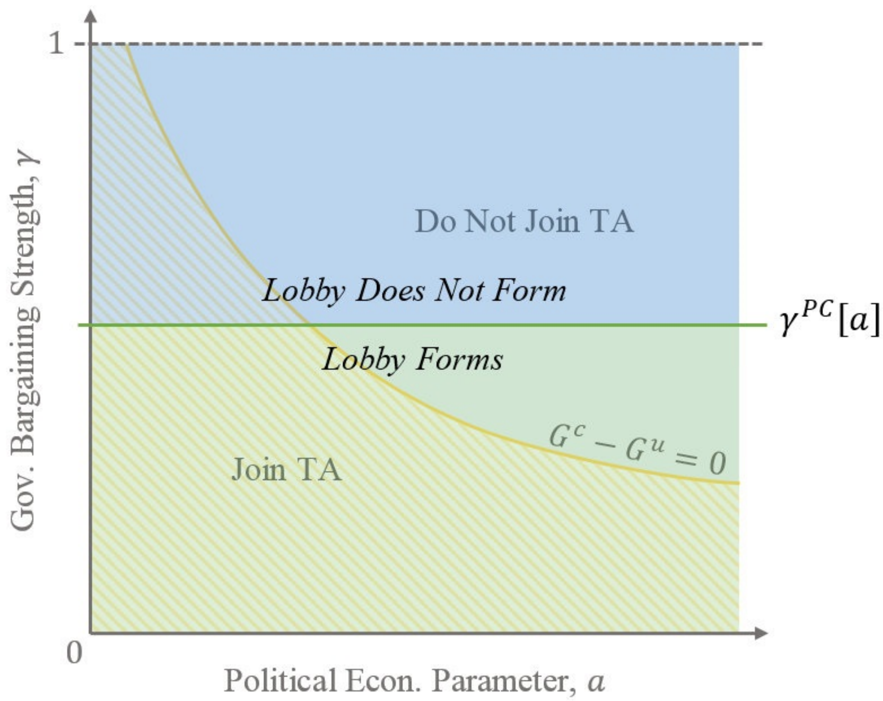


Figure A.17: Relationship between lobbying participation constraint and trade-policy preference for changes in a and γ, adding the lobbying participation constraint to the trade-policy preference figure, Figure 1.9. The line $\gamma^{PC}(a)$ indicates the combinations of (a, γ) for which the lobbying participation constraint is equal to zero.

B Afraid of Commitment: Why Small Countries Join Trade Agreements with Escape Clauses

B.1 Complete discussion of the decision between FTA and no trade agreement, from Section 2.2.

The government compares its payoff under free trade and its payoff in the political equilibrium. The key result of Maggi and Rodríguez-Clare (1998) is that the ex-ante commitment to free trade is what generates the benefit of joining the trade agreement. To illustrate this point, suppose the government has no bargaining strength. If the government does not commit to free trade initially, but instead sets prices once capital is allocated to each sector, they will only be compensated for the protection given the distorted allocation, which is less than if they were compensated also for the capital distortion. Whenever the government's bargaining strength is less than one, they will be compensated only partially for the distortion of capital. Thus, the government benefits in certain circumstances from being able to pre-commit to free trade. This is typically the case when its bargaining strength is small or it places a relatively low value on the political contributions. In this section, I will show that the Maggi and Rodríguez-Clare (1998) result holds under price uncertainty.

The government, in choosing whether to join an FTA or not, will be comparing the expected welfare function of joining versus not joining. Keeping in mind that the social welfare at the world price ($W(p^*, k_2)$) is maximized at the allocation k_2^*, the expected welfare functions simplify to

$$E[G^c(p^*, k_2^*)] = W(E[p^*], k_2^*) + \frac{1}{2}\text{Var}(p^*), \text{ and}$$
$$E[G(\widehat{p}, \widehat{k}_2)] = W(E[p^*], \widehat{k}_2) + \frac{1}{2}\text{Var}(p^*) + \frac{\gamma}{2}a^2(\widehat{k}_2)^2,$$

where G^c represents the welfare when the government is able to commit ex ante to free trade.[1]

One thing to note is that the price uncertainty has no effect on the relative sizes of the government welfare functions. This is a result of the linearity of production of the sector 2 good.[2] Before moving to the setup of the model in which the government may join a trade agreement with access to an escape clause, however, I will establish the behavior of government welfare in relation to the two key model parameters, a and γ, for the case in which the government does not join a free trade agreement.

First, consider how the government's bargaining strength, γ, enters the government's decision. Ex post, the value of γ, is only relevant in determining the division of the rents from tariff protection: because the resource allocations are fixed, γ does not affect the domestic price. Ex ante, γ is inversely related to the political allocation, $\widehat{k}_2$, which is most clear by looking at Figure 2.1: an increase in γ increases the slope of the equal returns line, thus decreasing $\widehat{k}_2$.

The government's welfare, $G(\widehat{p}(\widehat{k}_2; p^*), \widehat{k}_2)$, is increasing in γ (i) directly through the term $\frac{\gamma}{2}(a\widehat{k}_2)^2$, and (ii) indirectly through $\widehat{k}_2$. For $\gamma = 0$ (meaning the government will receive no rents from protection), the government would be strictly better off from committing to free trade. In other words, for $\gamma = 0$, $E[G^c(p^*, k_2^*)] > E[G(\widehat{p}(\widehat{k}_2; p^*), \widehat{k}_2)]$ for all a. This result holds regardless of the realization of p^*. For $\gamma = 1$, the government will receive all of the surplus from protection (equal to $(\widetilde{p}(k_2) - p^*)k_2$). So while the outcome will still be $\widehat{k}_2 = k_2^*$, the government receives contributions from the lobby equal to $a\, k_2^*$, leading to $E[G^c(p^*, k_2^*)] < E[G(\widehat{p}(\widehat{k}_2; p^*), \widehat{k}_2)]$ for all $a > 0$, which again holds regardless of the realization of p^*.

Proposition B.1. *There exists some cutoff level of bargaining strength, $\overline{\gamma}$, such that if and only if $\gamma < \overline{\gamma}$, $E[G^c(p^*, k_2^*)] > E[G(\widehat{p}(\widehat{k}_2; p^*), \widehat{k}_2)]$ for all a will be true. Therefore, for any bargaining*

[1] The dependence of the allocation on the price and a has been dropped for notational simplicity (i.e. $\widehat{k}_2 \equiv \widehat{k}_2(a)$).

[2] Relaxing the restrictiveness of this production function would generate more interesting results (in the context of comparing certainty to uncertainty). For now, however, the purpose of adding in uncertainty is to allow for the addition of an escape clause whose use is dependent on the terms of trade.

strength below the cutoff, the government will always commit to free trade. This result is identical to that of the model with certainty, as shown in Maggi and Rodríguez-Clare (1998).

When $\gamma > \overline{\gamma}$, the behavior of the government welfare function without commitment changes: the size of a is now an important determinant of the government's decision.[3] To better illustrate, consider the first-order behavior of the government's value of not joining a trade agreement:

$$\frac{\partial E[G(\widehat{p}(\widehat{k}_2;p^*),\widehat{k}_2)]}{\partial a} = W_K(E[p^*],\widehat{k}_2)\widehat{k}_2'(a;E[p^*]) + \gamma a(\widehat{k}_2)^2 + \gamma a^2(\widehat{k}_2)\widehat{k}_2'(a;E[p^*]).$$

Before moving forward with interpretation of the behavior of $E[G(\widehat{p}(\widehat{k}_2;p^*),\widehat{k}_2)]$, it is necessary to understand the behavior of $\widehat{k}_2'(a)$. The relationship between $\widehat{k}_2(a)$ and a can be derived by implicitly differentiating the equal returns outcome. This gives a first derivative $\widehat{k}_2'(a) = \Big((1-\gamma)\widehat{k}_2(a)\Big)/\Big(-2F_{11}(K-\widehat{k}_2(a),L)-(1-\gamma)a\Big)$. Based on earlier assumptions for Figure 2.1 (i.e. that there is a unique, interior solution), this derivative is greater than or equal to zero for all values of a.

Now, the fuller characterization of the government welfare function, $E[G(\widehat{p}(\widehat{k}_2;p^*),\widehat{k}_2)]$, is possible. The following results are relatively straightforward to derive:

1. For small enough γ, $E[G(\widehat{p}(\widehat{k}_2;p^*),\widehat{k}_2)]$ is decreasing in the right neighborhood of $a = 0$.

2. For large enough γ, $E[G(\widehat{p}(\widehat{k}_2;p^*),\widehat{k}_2)]$ is decreasing for all values of a and is therefore negative for all $a > 0$.

3. If $\gamma > 0$, for large enough a, $\partial E[G(\widehat{p}(\widehat{k}_2;p^*),\widehat{k}_2)]/\partial a > 0$.

4. When $\gamma > 0$ is small enough, $\partial E[G(\widehat{p}(\widehat{k}_2;p^*),\widehat{k}_2)]/\partial a$ is decreasing in a near $a = 0$ and increasing in a when $\widehat{k}_2(a) = K$, which means that $E[G(\widehat{p}(\widehat{k}_2;p^*),\widehat{k}_2)]$ must reach

[3] The behavior of the government welfare when it has committed to free trade ex ante does not depend on a due to the fact that $(\partial E[G^c(p^*,k_2^*)]/\partial a) = 0$. Therefore, only the behavior of $E[G(\widehat{p}(\widehat{k}_2;p^*),\widehat{k}_2)]$ will be explicitly discussed moving forward.

a minimum somewhere in between and then start increasing and become greater than $E[G^c(p^*, k_2^*)]$ by the point which $\widehat{k}_2(a) = K$ (call this level $a_{\widehat{k}_2}$).

Proposition B.2. *For small γ, $E[G(\widehat{p}(\widehat{k}_2; p^*), \widehat{k}_2)]$ is nonmonotonic: it reaches a minimum at some value of a, call it a^*, and then becomes greater than $E[G^c(p^*, k_2^*)]$ at some value of a, call it $\widehat{a}$, $a^* < \widehat{a}$. For large γ, $E[G^c(p^*, k_2^*)] < E[G(\widehat{p}(\widehat{k}_2; p^*), \widehat{k}_2)]$ for all a. Again, this result is identical to the one derived under certainty in Maggi and Rodríguez-Clare (1998).*

Therefore, when the government has most of the bargaining strength it will not want to commit to a free trade agreement, because it recovers a large share of the rents from protection by way of the lobby contributions. If the government has somewhat weaker bargaining strength, then they will want to commit to free trade only if the expected size of protection ($E[\widetilde{p}(k_2) - p^*]$) would be relatively large. This is the case when the political-economy parameter is large. When the premium placed on the lobby's welfare is relatively small (i.e. when a is close to zero), the price distortion in the political equilibrium is small. Just as in Maggi and Rodríguez-Clare (1998), there is incentive for the government to commit to a free trade agreement. Additionally, for a mean-preserving spread, the range of parameters for which the government will commit to free trade is unchanged. Thus, the addition of the uncertainty over world price to the original Maggi and Rodríguez-Clare (1998) model had no meaningful effect on the model under certainty.

B.2 Deriving Relationship between G^{FTA}, G^{NoTA}, and G^{EC}

To characterize the behavior of the $G(a)$ functions, I will go through steps similar to those taken by Maggi and Rodríguez-Clare in their Appendix A. For the first step of characterizing the G functions, recall the following definitions of their first derivatives with respect to a

(ignoring G^{FTA}, which is not dependent on a):

$$G^{NoTA\prime}(a) = W_K(p^*, \widehat{k}_2)\ \widehat{k}_2'(a) + \gamma a(\widehat{k}_2)^2 + \gamma a^2 \widehat{k}_2(\widehat{k}_2'(a)),$$
$$G^{EC\prime}(a) = W_K(p^*, \widehat{k}_2^{EC})\ \frac{\partial \widehat{k}_2^{EC}(a)}{\partial a} + (1-\pi)\gamma a(\widehat{k}_2^{EC})^2 + (1-\pi)\gamma a^2 \widehat{k}_2^{EC}\left(\frac{\partial \widehat{k}_2^{EC}(a)}{\partial a}\right).$$

Suppose that $a = 0$. This means both $\widehat{k}_2$ and $\widehat{k}_2^{EC}$ will be equal to k_2^*. So, at $a = 0$, $G^{FTA} = G^{EC} = G^{NoTA}$ and $G^{EC\prime}(0) = G^{NoTA\prime}(0) = 0$. The second order behavior of each is

$$G^{NoTA\prime\prime}(0) = W_{KK}(p^*, k_2^*)\ (\widehat{k}_2'(0))^2 + \gamma(k_2^*)^2, \tag{B.1}$$
$$G^{EC\prime\prime}(0) = W_{KK}(p^*, k_2^*)\left(\frac{\partial \widehat{k}_2^{EC}(0)}{\partial a}\right)^2 + (1-\pi)\gamma(k_2^*)^2, \tag{B.2}$$

where $W_{KK}(p^*, k_2^*) < 0$, given that the welfare function is maximized at k_2^*. So, if γ is small enough, then $G^{NoTA\prime\prime}(0) < 0$ and $G^{EC\prime\prime}(0) < 0$ must both be true. This suggests that when $a = 0$, for small enough γ, G^{NoTA} and G^{EC} are at a local maximum and are decreasing in the right neighborhood of $a = 0$.

Given also that the first derivatives of each of the allocations are

$$\widehat{k}_2'(0) = (1-\gamma)k_2^* / \Big(2F_{11}(K - k_2^*, L)\Big)$$
$$\frac{\partial \widehat{k}_2^{EC}(0)}{\partial a} = \Big((1-\pi)(1-\gamma)k_2^*\Big) / \Big(2F_{11}(K - k_2^*, L)\Big) = (1-\pi)\widehat{k}_2'(0),$$

I can also say that for small enough γ, $G^{EC\prime\prime}(0) > G^{NoTA\prime\prime}(0)$. This in turn implies that $G^{EC}(a) > G^{NoTA}(a)$ in the right neighborhood of $a = 0$. This will always be the case given that γ is small enough such that both second derivatives are still negative. This result is easily derived from the equations for the second derivatives at $a = 0$ combined with the restriction that both second derivatives are negative.

2. Summary: For small enough $\gamma > 0$, $G^{NoTA}(a)$ and $G^{EC}(a)$ are decreasing in $a > 0$ and less than G^{FTA} in the right neighborhood of $a = 0$.[4] Furthermore, $G^{EC}(a) > G^{NoTA}(a)$

[4] The number on this summary statement corresponds to its number in Section 2.6. This is true for the

in the right neighborhood of $a = 0$.

The last part of the above point is a result of the fact that $G^{EC''}(0) > G^{Dyn''}(0)$, which is true if γ is small enough such that the following inequality holds:

$$-(1-\pi)W_{KK}(p^*, k_2^*)(\widehat{k}_2'(0))^2 > \gamma(k_2^*)^2.$$

This inequality is equivalent to requiring that $G^{EC''}(0) < 0$, which was assumed. This indicates that for any γ small enough such that $G^{EC''}(0) < 0$, the government welfare for each type of trade agreement is ordered $G^{FTA} > G^{EC} > G^{NoTA}$ in the right neighborhood of $a = 0$.

Next, for large enough γ, it is clear that both $G^{EC''}(0)$ and $G^{NoTA''}(0)$ will be greater than zero. Furthermore, for large enough γ, $G^{EC''}(0) < G^{NoTA''}(0)$ is true because at $a = 0$, the slope of $\widehat{k}_2^{EC}(0)$ is always less than the slope of $\widehat{k}_2(0)$, where the slopes are $\Big((1-\pi)(1-\gamma)k_2^*\Big)/\Big(2F_1(K-k_2^*, L)\Big)$ and $\Big((1-\gamma)k_2^*\Big)/\Big(2F_1(K-k_2^*, L)\Big)$, respectively. Because $G^{NoTA}(0)$ and $G^{EC}(0)$ are equal to each other, I can say that the value of joining any form of trade agreement is maximized at $a = 0$ and is decreasing in the right neighborhood of $a = 0$. To show that when γ is large enough $G^{NoTA'}(a) > G^{EC'}(a) > 0$ for all $a > 0$, consider γ close to one. Since $\widehat{k}_2$ and $\widehat{k}_2^{EC}$ are very close to k_2^* when γ is close to one, the $W_K(p^*, K)$ term in each derivative will be small. Meanwhile, the political rents will be large since the government has most of the bargaining strength. Next, given that when the government does not join a trade agreement they are able to receive rents for both price realizations versus only receiving rents if the world price is low if they joined a trade agreement with an escape clause, it is easy to show that $G^{NoTA'}(a) > G^{EC'}(a) > 0$ for all $a > 0$ when γ is large.

3. Summary: For large enough γ, $G^{NoTA}(a)$ and $G^{EC}(a)$ are increasing in the right neighborhood of $a = 0$, with $G^{NoTA''}(0) > G^{EC''}(0)$. Furthermore, since $\widehat{k}_2$ and $\widehat{k}_2^{EC}$

rest of the summary statements in the section.

are very close to k_2^* when γ is close to one, it is also evident that for large enough γ, $G^{NoTA\prime}(a) > G^{EC\prime}(a) > 0$ for all $a > 0$. Thus, for large enough γ, the government welfare for each type of trade agreement is ordered $G^{NoTA}(a) > G^{EC}(a) > G^{FTA}$ for all $a > 0$.

Now, I will show that for some fixed value of $\gamma > 0$, there is some value of a high enough such that $G^{NoTA}(a)$ is larger than G^{FTA}. This is illustrated by assuming that a is large enough such that $\widehat{k}_2(a) = \widehat{k}_2^{EC}(a) = K$, and therefore $\partial\widehat{k}_2^{EC}(a)/\partial a = 0$. At such a value of $a > a_{\widehat{k}_2^{EC}}$, $G^{EC\prime}(a)$ and $G^{NoTA\prime}(a)$ are clearly positive. Additionally, $G^{NoTA}(a) > G^{EC}(a)$ by the amount $(1/2)\pi\gamma a^2 K^2$.

4. Summary: If $\gamma > 0$, for large enough a, $G^{NoTA\prime}(a) > G^{EC\prime}(a) > 0$.

Combining the information found above, it is clear that when γ is relatively small, the relationship between the G's is nonmonotonic.

5. Summary: When $\gamma > 0$ is small enough, starting with the order $G^{FTA} > G^{EC}(a) > G^{NoTA}(a)$ in the right neighborhood of $a = 0$:

   (i) $G^{NoTA}(a)$ and $G^{EC}(a)$ must each reach a minimum at some point between $a = 0$ and $a = a_{\widehat{k}_2^{EC}}$;

   (ii) $G^{NoTA}(a)$ and $G^{EC}(a)$ must then start increasing in a and become greater than G^{FTA} by the point which $a = a_{\widehat{k}_2^{EC}}$;

   (iii) once $a \geq a_{\widehat{k}_2^{EC}}$, the order must be $G^{NoTA}(a) > G^{EC}(a) > G^{FTA}$.

   Each of these offers a different potential way the functions could be interacting outside of the information derived above.

Figure B.1 shows the information that is explicitly known for the behavior of the welfare functions when $\gamma > 0$ is small.

The last case I will consider here is when the value of γ is in some moderate range for which $G^{NoTA\prime\prime}(0) < 0$ and $G^{EC\prime\prime}(0) > 0$. In this case, G^{EC} will be increasing and greater

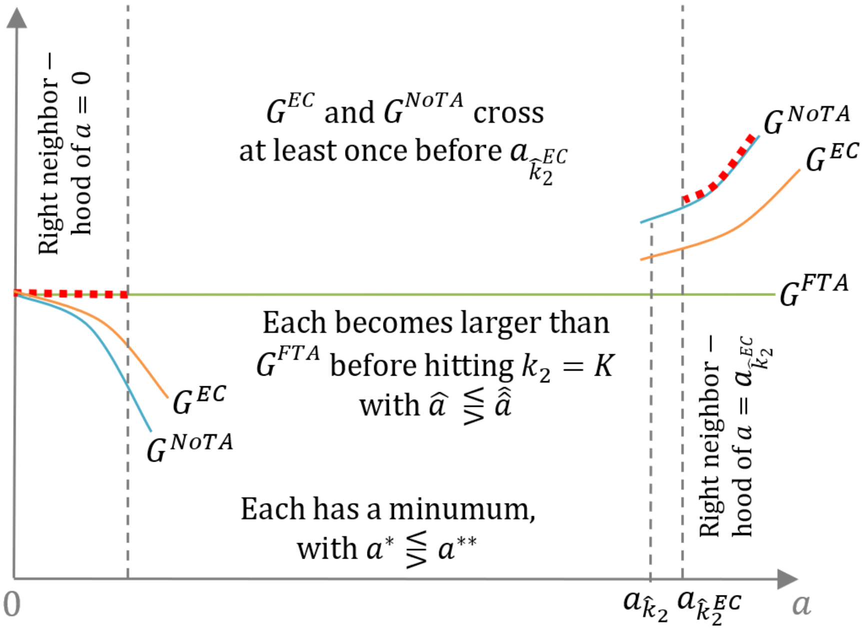


Figure B.1: Summary of known behavior of government welfare function when γ is small

than G^{FTA} in the right neighborhood of $a = 0$ and G^{NoTA} will be decreasing in that same neighborhood. As a result, in the right neighborhood of $a = 0$, the government will choose to join a trade agreement with an escape clause. Because it still must be true that $G^{NoTA}(a) > G^{EC}(a)$ for $a > a_{\widehat{k}_2^E}$, it is clear that $G^{EC}(a)$ and $G^{NoTA}(a)$ must cross at some intermediate value of a, with $G^{NoTA}(a)$ reaching a minimum at some point before they cross.

6. Summary: Suppose $\gamma > 0$ is small enough such that $G^{NoTA''}(0) < 0$ but large enough such that $G^{NoTA''}(0) > 0$. In the right neighborhood of $a = 0$, $G^{EC}(a) > G^{FTA} > G^{NoTA}(a)$. For any value of a, the government will never prefer to join a free trade agreement and $G^{EC}(a) > G^{FTA}$. $G^{NoTA}(a)$ will reach a minimum at some point between $a = 0$ and $a = a_{\widehat{k}_2}$. For any $a \geq a_{\widehat{k}_2^{EC}}$, $G^{NoTA}(a) > G^{EC}(a) > G^{FTA}$.

There are a few points to keep in mind when trying to understand which trade agreement type will be preferred for a low γ:

1. Given $\gamma > 0$ is small enough (such that there are values of $a > 0$ for which the government would prefer a free trade agreement over the other two alternatives),

and that

(i) $\widehat{\widehat{a}} < \widehat{a}$, then each of the three outcomes are possible for some value of a; or that

(ii) $\widehat{\widehat{a}} > \widehat{a}$, then the government will not choose to join a trade agreement with an escape clause for any value of a.

2. The variance of $G^{NoTA}(a)$ will be greater than the variance of $G^{EC}(a)$ for any $\gamma > 0$.

3. As the value of γ increases, the swings in the G^{NoTA} and G^{EC} functions will flatten out, the G^{EC} function flattening out more quickly. This is apparent given $G^{EC''}(0)$ becomes positive for a lower value of γ than does $G^{NoTA''}(0)$, but that no matter what the value of γ is, $G^{NoTA}(a_{\widehat{k}_2^{EC}}) > G^{EC}(a_{\widehat{k}_2^{EC}})$.

4. Given that the distance between $\widehat{k}_2$ and $\widehat{k}_2^{EC}$ is decreasing in the value of γ, the value of a at which the G^{EC} and G^{NoTA} lines cross is decreasing in γ as well. This makes sense intuitively, because if the allocations are relatively similar for each, then the main difference will be coming from the political-economy rents the government receives from granting protection, and as γ increases, so does the share of the rents the government receives, causing the benefit of the extra gains from the political rents to dominate the losses from the misallocation of resources at a lower level of a.

These observations and the other information combine to form Proposition 2.5.

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