

Professor Playfair's lectures on natural philosophy : [handwritten lecture notes, 1818-1819]. Vol. II

Brodie, William

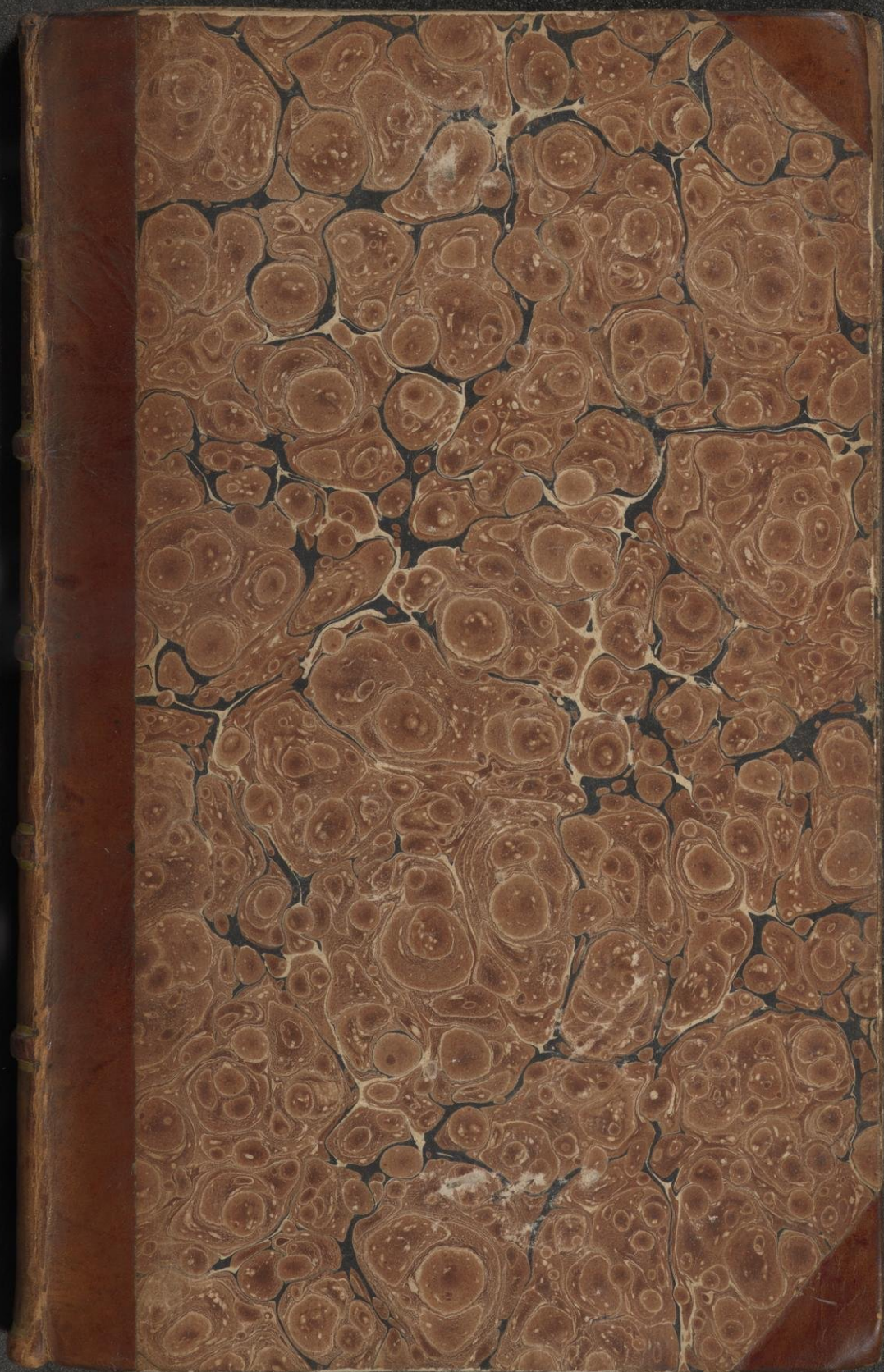
Edinburgh, Scotland: [publisher not identified], 1818

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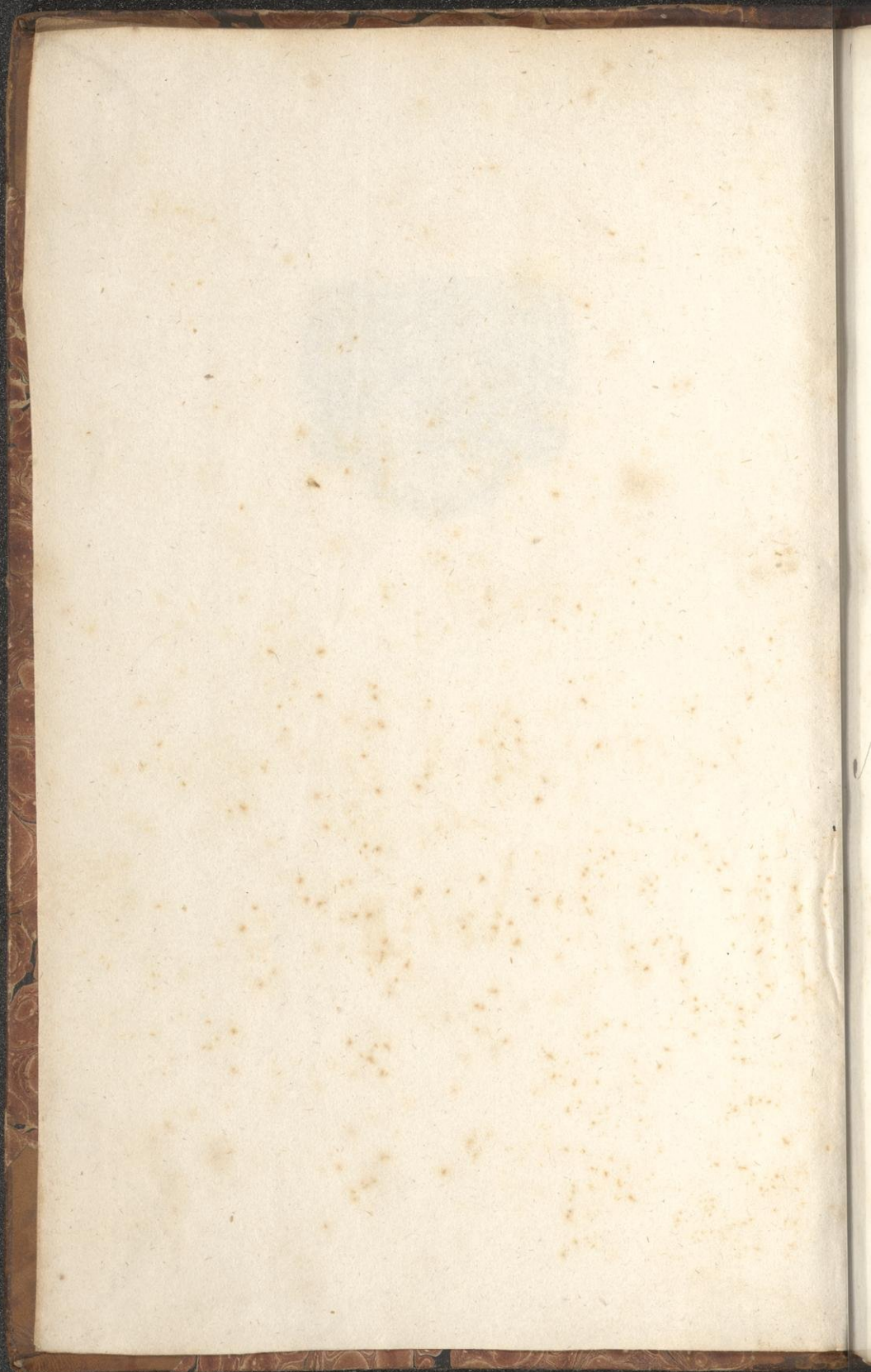
James Thompson

London

Natural Philosophy

Vol. I

W. B. Smith
London



Professor Playfair's
Lectures
on
Natural Philosophy

Vol. II

Wm Brodie

Edin^r 1818-19

Karl Books
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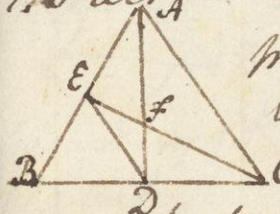
Prof. Dr. J. J. Schaeffer
Lecturer
on
Botany & Zoology

Vol. I

Wm. B. Schaeffer
1818

But the momenta would it will be equal, Now the centre of Gravity is found to be in P 2 and also in C 11 therefore it must be in the point of their intersection R, which therefore is well demonstrated to be the centre of Gravity of these different bodies —

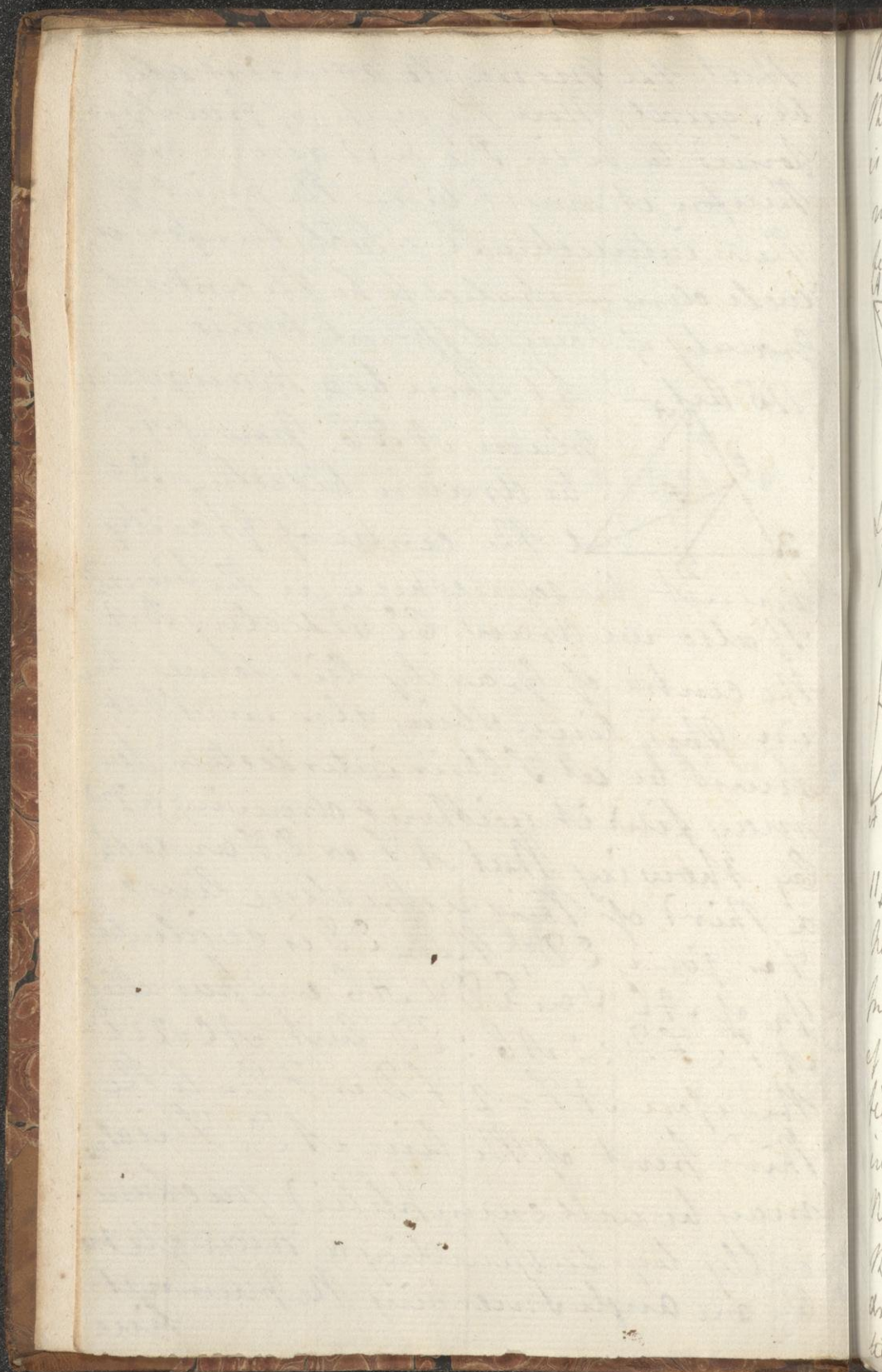
115th Lemma



Let there be a triangular plane ABC. Now if AD be drawn bisecting BC the centre of Gravity must be somewhere in the line AD

If also we draw CE bisecting BA the centre of Gravity lies somewhere in this line & being also in AD it must be at F their intersection. We may find it without drawing a 2nd by showing that AF or DF are each a third of their respective lines.

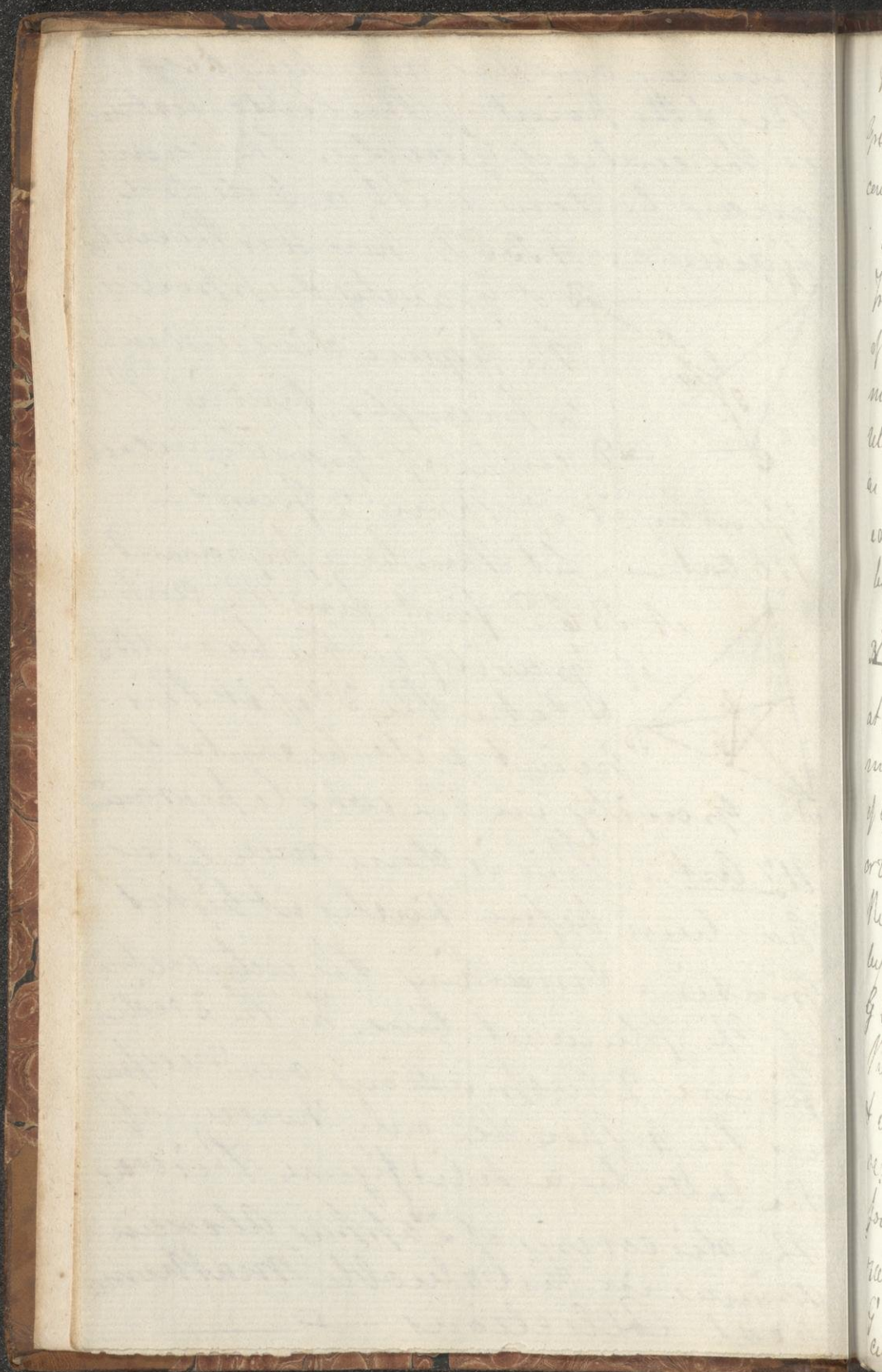
For join ED, then ED is evidently $\frac{1}{2}$ of AC & as ED & AB are parallel $AF : FD :: AB : ED$ but $AC = 2ED$ therefore $AF = 2 FD$ or $FD = \frac{1}{3}$ of the line AD. This also may be well exemplified mechanically by suspending a Triangle by one angle & marking the plummet line



Now by another ⁽¹⁴⁸⁾ marking also
 this & the point of their intersection
 is the centre of Gravity. The same
 may be done with a 4 sided
 figure as $ABCD$, here H is the centre
 of Gravity supposing
 the figure divided into
 2 Triangles finding the
 centre of Gravity in each
 & finding it of those 2 parts —

116 Art — Let there be a pyramid
 $ABCD$ first find the centre
 of Gravity in the base ABC
 & take the 3rd of it this
 point will be centre of
 Gravity in the whole pyramid.

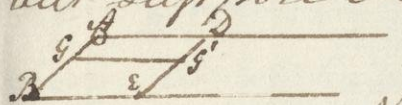
117 Art — This is done merely as
 has been before hinted at by just
 joining & marking the intersection
 of the plummet lines. In the 3 sided
 figure 2 suspensions are necessary
 in the 4 there are only two with
 the latter be a solid figure. This was
 the discovery of Pappus Alexan-
 drianus in his valuable Mathema-
 tical Collections —



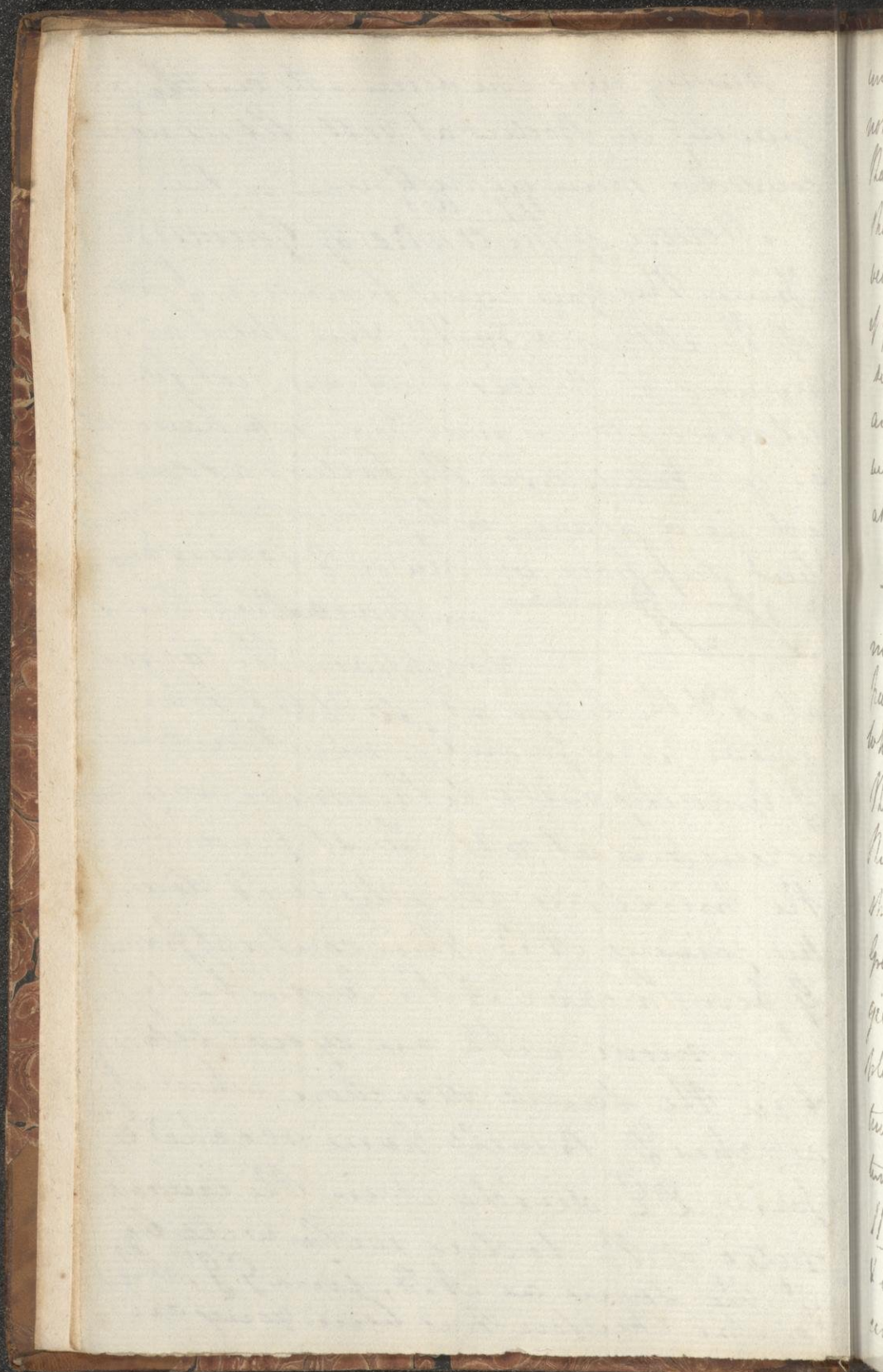
Having now considered the centre of Gravity in Bodies at rest, let us now consider them in motion — or the

Motion of the centre of Gravity —

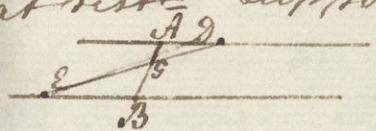
From this you may find cen. of Grav. of the Moon & Earth, & in short any number of Bodies that are not fixed relatively to one another — We have only as yet considered the bodies as connected in a plane, or by some other means, but suppose we have 2 bodies moving

 in parallel lines

I suppose the larger at A & the lesser at B & let them move uniformly, now their centre of Gravity will either move uniformly or remain at rest, First find, from the principles already laid down by joining A.B. their centre of Gravity, G being nearer A the larger body; if they move with an equal velocity & in the same direction when A reaches D, B will have reached E, join D.E divide it in the inverse ratio of the bodies which will be G' the same as A.B, join G.G', the centre therefore has been going on



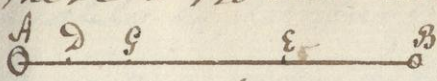
uniformly in GG' , ^(1.59) & supposing it
not uniform we could very easily prove
that it must be so, & if the Bodies
should move in any line & with any
velocity still the motion of the centre
of Gravity would be the same. If we
suppose the bodies to act upon one
another still the centre of Gravity
will either move uniform or remain
at rest. Suppose the bodies placed



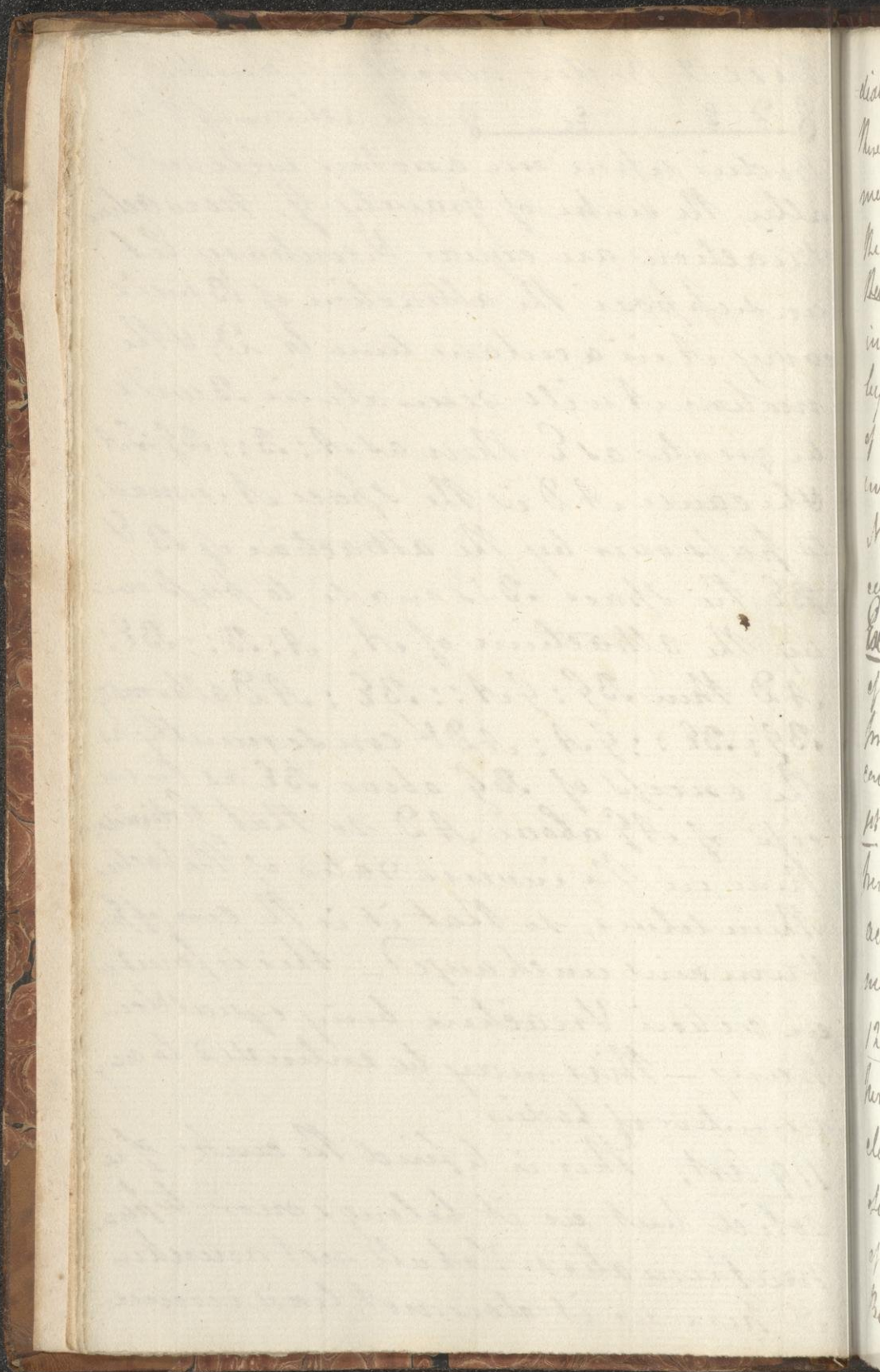
at A & B , & A the
larger, suppose they
move in opposite directions, & they
have equal quantities of motion,
when A reaches D , B arrives at E .
Then two triangles are similar &
the line AG & GB are in the same
straight line so that the centre of
Gravity has not moved. I have not
given you the demonstration com-
plete as it would take considerable
time, but shall take another oppor-
tunity.

118 Art. — Suppose 2 Bodies A & B
& let A be the greater, find the
centre of Gravity G . Suppose

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These 2 Bodies attract one another
 The action of the
Bodies upon one another will not
alter the centre of Gravity G. Now action
& reaction are equal & contrary let
us suppose the attraction of B will
carry A in a certain time to D, & the
motion A will generate in B will
be greater as E than as A: B :: BG: EA
The cause AD is the space A is made
to pass over by the attraction of B &
BE the space B is made to pass over
by the attraction of A, A: B :: BE:
AD then BG: GA :: BE: AD alternately
BG: BE :: GA: AD & consequently as
the excess of BG above BE as the ex-
cess of AG above AD so that G divide,
them in the inverse ratio of the bodies
themselves, so that it is the cen. of grav.
& remains unchanged - this is founded
on action & reaction being equal & con-
trary - this may be extended to any
number of bodies

119 Art. This is to find the centre of a
solid but as it belongs more to pure
mathematics I shall not mention
it here as it does not lead imme-

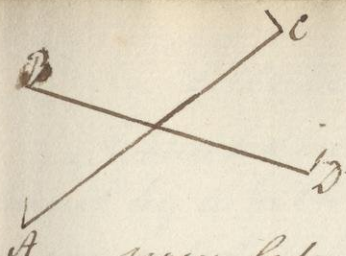


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diately to the subject before us & as
 there are now better methods - this
 may be considered as the latest of
 the General Principles of Mechanics,
~~that can be ascribed to the French~~
 in the year 40 of the last century
 by D'Alembert it is a full example
 of action & reaction, & I have not
 understanding this principle
 Newton in one of his most diffi-
 cult propositions went wrong
Example 4th To give an account
 of the 2 demonstrations of the
 principles of the centre of Gravity
 contained in Article 105 mainly
1st An Indirect demonstration
 from the equality of action & re-
 action 2nd The Geometrical De-
 monstrations of Archimedes -
120 Art - Now what is laid down
 here is not easy to communicate
 clearly, I trust you will be made
 sensible of it however by Recd
 of a diagram suppose these
 Bodies A B C D of any magnitude

[Faint, illegible handwriting, likely bleed-through from the reverse side of the page.]

(153)



and also fixed, by
a kind of framework
or in any other manner

A now let us suppose each of them
receives an impetus we know it must
communicate a part of it to the other
three let represent the impulse
now in what manner will this motion
be distributed among them—

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Mechanical Powers

A mechanical power is an Instrument by which the effect of a given force is increased, while the force itself remains the same. Thus you may have the hand of man alone, but there are various contrivances that the effect of that force may be varied to an enormous extent. They are six in number. 1. The Lever, 2. The Wheel & Axle, 3. The Pulley, 4. The Wedge, 5. The Screw, 6. The Gunpowder Machine.

This division is very ancient, with the exception of the latter which is not always added. I have in the sequel to show you another Instrument, which may be considered as another of the Mech. Powers.

The Lever

123. A Lever is an inflexible rod, moving about a centre or fulcrum, having force applied to 2 or more points in it. It is considered as an inflexible rod, but strictly speaking we have nothing inflexible, we are to consider them as a Mathematical Line abstracting from all weight. This power was known long ago particularly by

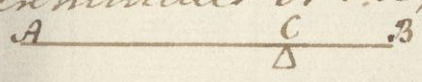
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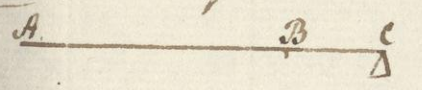
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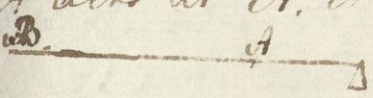
Pappus Alexandrinus who gives a dissertation on all the Mechanical powers. We have reason however to believe that they were known before his time certainly by Archimedes from the wonders he performed at the siege of Syracuse. There are 2 kinds of Levers, 1st such as in the Crow bar. It has 2 points at its extremities A & B, to the longer end the



power is applied to the shorter the resistance, & the point C the centre of motion is called the Fulcrum. Now the 2nd kind of Lever is where the Fulcrum



C is at the end, & the points to which the forces are applied are on the same side of the fulcrum. There is also a 3rd but it ought not to be separated from the 2nd it is where the power, it acts at A, it is this kind that



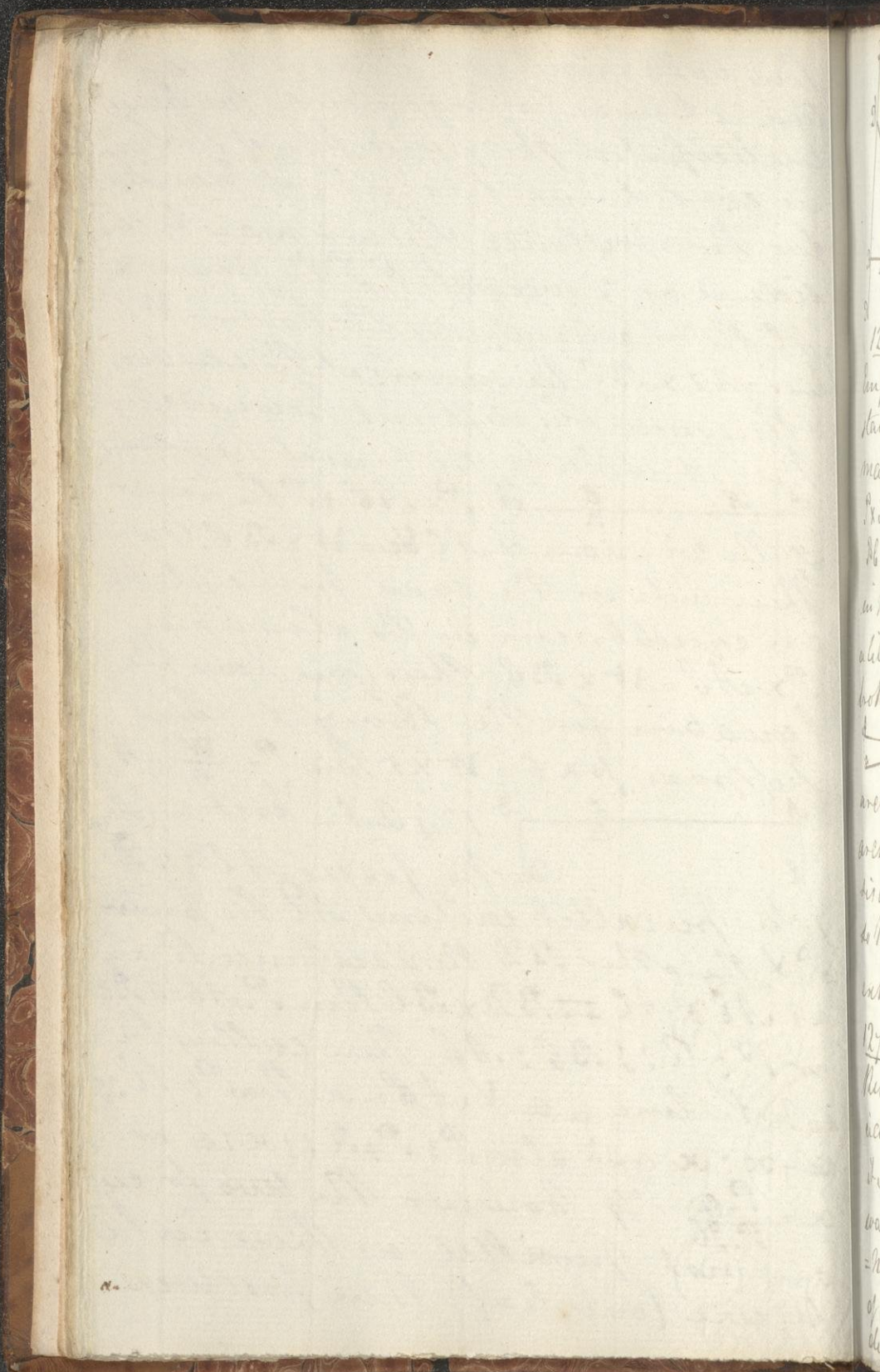
Nature uses in the animal Kingdom.

There should as I have already said be only 2. I shall distinguish them therefore by the 1st & 2nd kind. In the first if the power move downwards the resistance moves upwards this is called heterogomous, the 2nd when

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They both move in the same direc-
 tion is termed homogeneous. We have
 anticipated the principle of the Lever
 in considering the centre of Gravity
 for the resistance & power may be con-
 sidered as 2 weights & P the power $\times$
 AC the momentum of the power = W the
 weight $\times$ BC the mom: of the resistance
 & there may be different powers acting
 But it will be the sum of the powers
 $\frac{A}{A'} \frac{C}{\Delta} B$ $P \times AC + P$ the power
 of the 2^d man $\times D'C'E = W \times BC$. Now
 then will on the same principle be
 an equilibrium in the other when
 $P \times AC = W \times BC$ then you have the
 Equation for the straight Lever.
 Suppose $p \times 6 = W \times 1$ then $P = \frac{W}{6}$

$\frac{A}{e} \frac{C}{\Delta} B$ 124 Art. First suppose
 the forces AC & BD
 to be parallel calling AC the power
 P & the other BD the resistance R and
 as $AC \times AC = BD \times BC$ then $P \cdot AC = R \cdot BC$
 or $P : R :: BC : AC$ then calling the
 whole line = a & $AC = x$ then $P : R ::$
 $a - x : x$ and also $P : P + R :: x : a$ or
 $x = \frac{P \cdot a}{P + R}$ If however the two forces
 are not parallel as then in the
 figure (over leaf) then produce

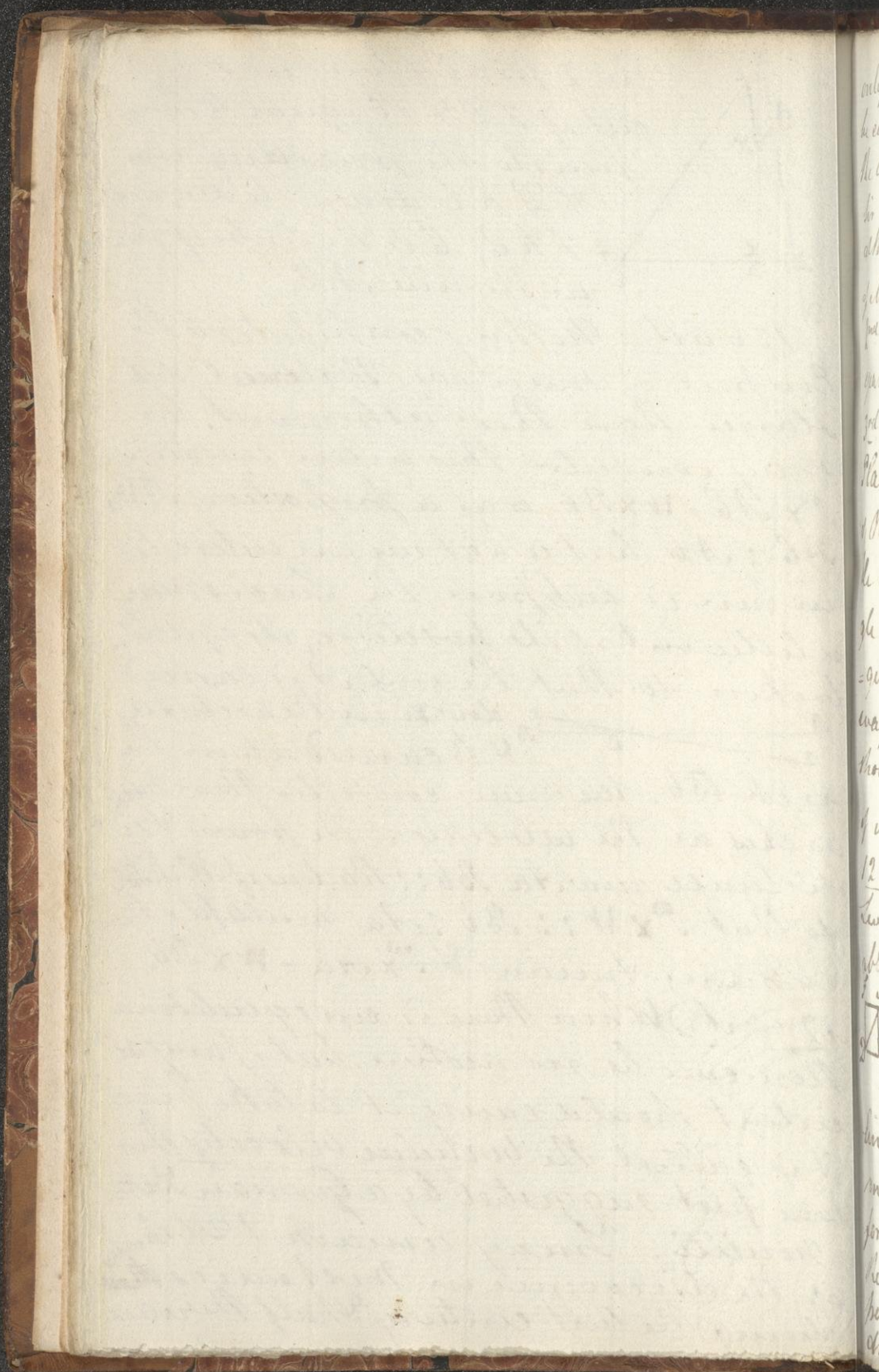




The 2 fore ^(15°) meet in F
cut of $F D$ & $F E$ equal to them
finish the parallelogram
 $F D' K E'$ draw the diagonal
A $F K B$, C is the centre of $gr.$
E in the line $A B$.

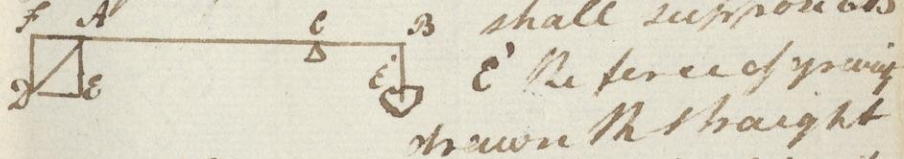
126 art. - Nothing contributes to the Empire of man over Material substance than this Instrument. We may consider this as an Equation $P \times AB = W \times Bb$ or as a proportion $P:W::Bb:AB$, but what we are interested in here is suppose the Lever is moved a little out of its position, its equilibrium broken so that the end A is carried down in the arch a & B carried up in the arch Bb . We may consider these small arches as the velocities of the power & resistance now $Aa:Bb::\text{Radius of } AC:R:BC$ so that $P \times W::Bb:Aa$ multiply the extremes & means & $P \times Aa = W \times Bb$.

127 Art. - When there is an equilibrium there can be no motion, but if any accident should cause it to take place it is called the virtual velocity. This was first suggested by a German Ber-nouilli. I may remark that nearly of the discoveries in mechanics ^{the} during the last century & half there are

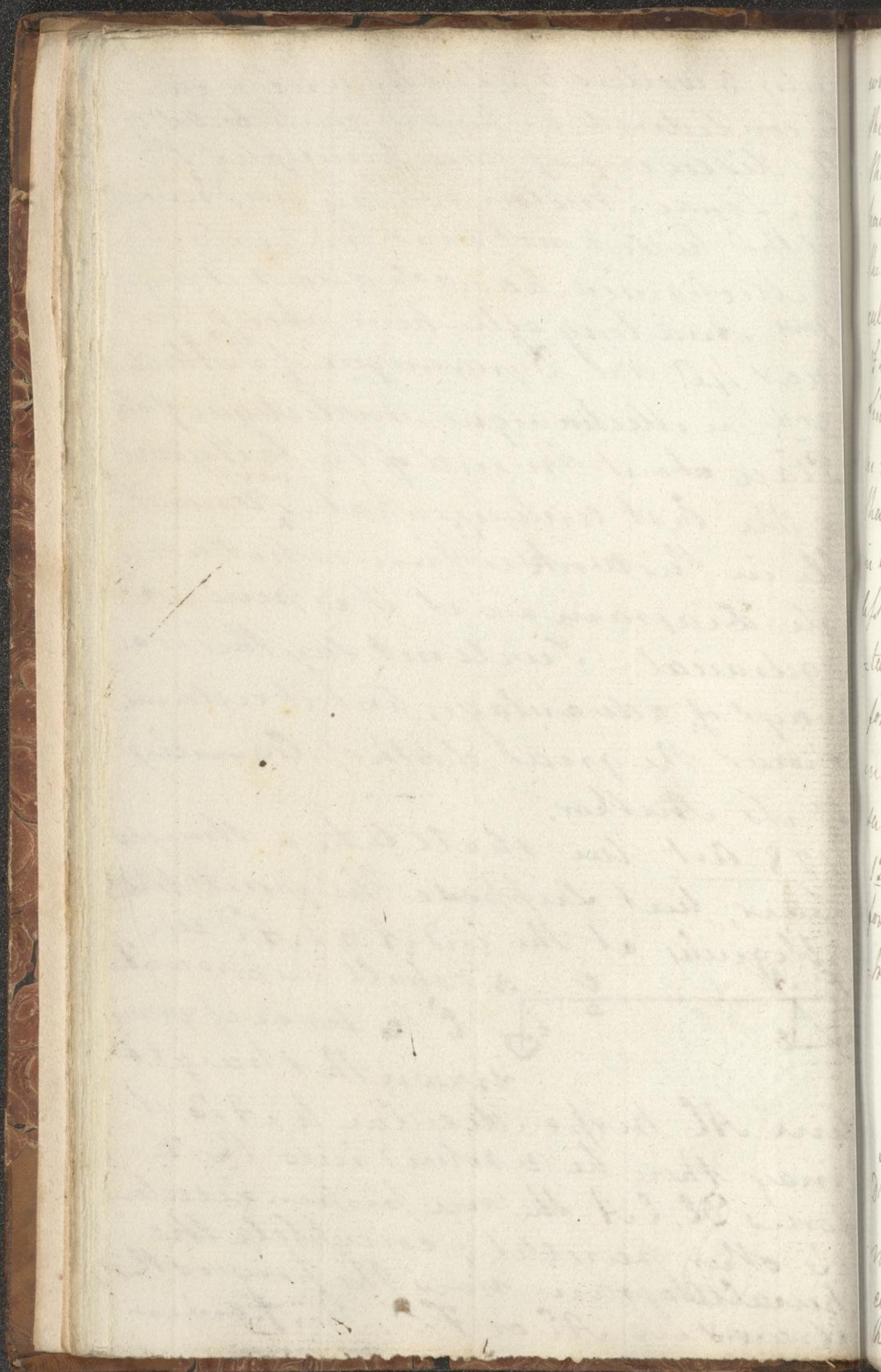


only 3 writers & 3 Books which can be considered as having contributed to the discovery of new principles 1st Sir Isaac Newton who in his *Principia* altho' he did not treat particularly of Mechanics, has advanced it much 2nd And long after him, about the year 40 the *Dynamique* of D'Alembert 3rd The *Mechanique Analytique* of La Place about the end of the last years of the last century, what is remarkable in this work is there is not a single Diagram in it, it is purely algebraical - I will not say this is always of advantage, but it certainly shows the great depth of reasoning of its Author.

128 Art we shall take a straight line, but suppose the force applied obliquely at the end A as A D we shall suppose B

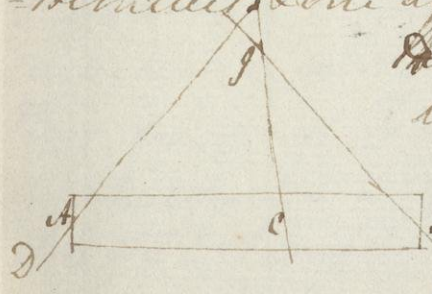


drawn the straight line AC perpendicular to AB, it may then be resolved into the 2 forces DE, EA the one perpendicular the other parallel; complete the parallelogram, now the power that draws in AE or FD, this power



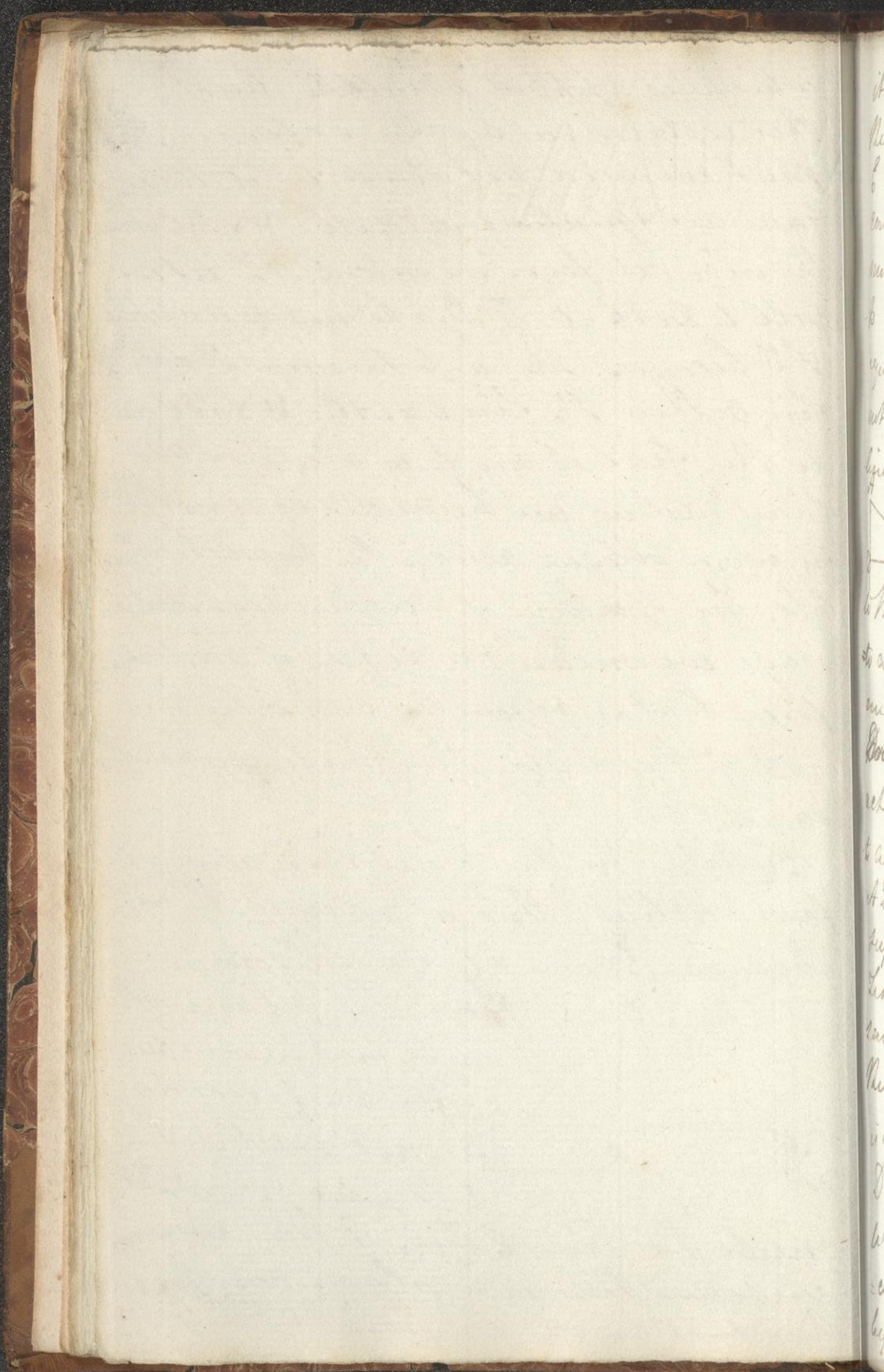
which is applied parallel but
this is overcome by the Fulcrum, so
that we need not consider it. We shall
have an Equation $AE \times AB = W \times BC$ now
the original force we called AD . Let us
call $\angle DAB = \theta$ If DA be made inclined
 ED becomes the hypotenuse $ED = R$
 $\sin \theta$ then $P \times \sin \theta \times AB = W \times BC$ and
as the sine of an θ is always less
than radius we have discovered that
in every oblique action the power becomes
less. In speaking of Mechanical Advan-
tage we mean the degree of additional
force that is given by using the long
end of the lever. Purchase means the
same.

129 Art. - In the lever there always 3
forces acting, one at each of the ex-
-terminities & one at the Fulcrum.



Here the plane AB
will not be in Equi:
for the forces do
not meet in the
same point, BE

make the line CG in G then we may
consider this as a plane compound
hinging in it the points F & G .

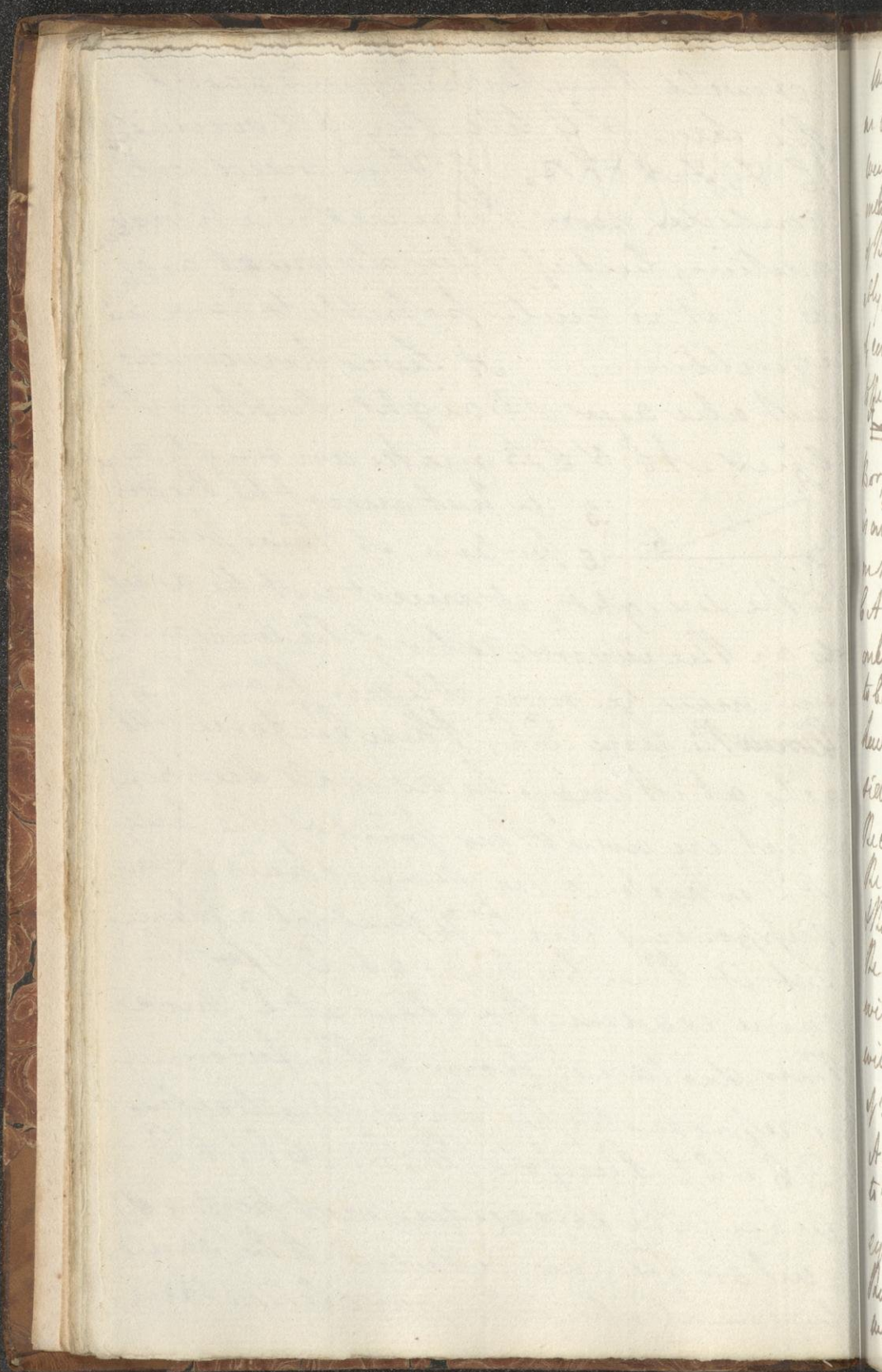


it will then turn round about
the Lever & G till they all coincide.
C F, F A, & F B, & F we need not
consider, now then inspire to produce
motion, but if they do meet at H
to it is quite possible to have an
equilibrium.

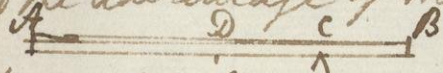
A Lever however is
not always straight suppose the
lines A C & C B make an angle, now



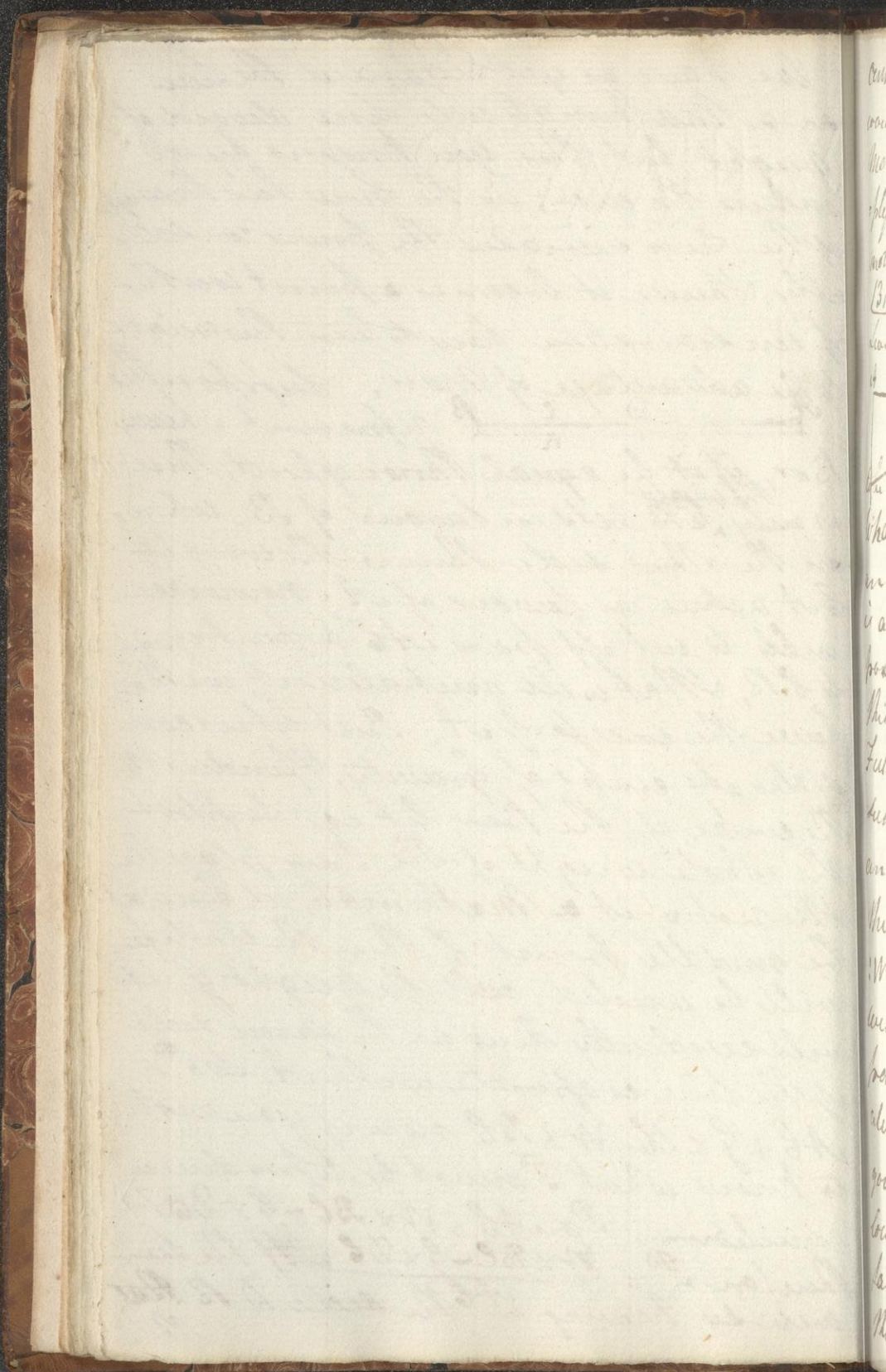
what must be the pro-
portion of these powers
to the weight, it must not be as before
to as the inverse ratio of the lines as the
one may be more oblique than the other.
Draw the line DE, then the force that
acts at A may be as well supposed
to act in any other part of the line
AD which is the same straight line
supposing this to represent a plane.
Let it then be fixed at D, & for the
same reason the other at E, hence
their distance from C the fulcrum
is represented by the perpendiculars
DB & EC therefore $D : E :: CE : CD$.
When the forces are not perpendi-
-cular they are reduced to the same
by completing the parallelogram,



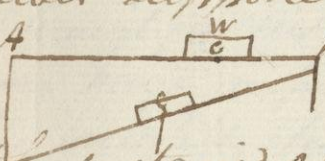
We have as yet regarded the Lever
as a Mathematical line devoid of
weight but this you know is by no
means the case; in the Crow bar the weight
of the Lever increases the power consider-
ably, thence it becomes a point worthy
of consideration how to turn this weight
to the advantage of Man. Suppose this



represent a heavy
Bar, if it be equal throughout, there
is only ^{the weight} CB acts in favour of B , whereas
on the other side there is the weight
 CA acting in favour of A . now we have
only to cut off from AC a part equal
to CB , that will balance it we then
have the excess of A . But let us con-
sider its centre of Gravity, & finding it
the centre of the Bar, let us suppose
the whole weight of the Bar placed there
the rest of it a Mathematical line; at
the middle point D then the whole force
will be united, call the weight G , it
will evidently bring on the same side
of the line cooperate with A . So $P \times$
 $AB + G \times DC = W \times CB$ now if you wish
to know what P must be to produce an
equilibrium $P \times AC = W \times CB - G \times DC$
therefore $P = \frac{W \times CB - G \times DC}{AC}$ If the Lever
were so heavy on the side D B that
the



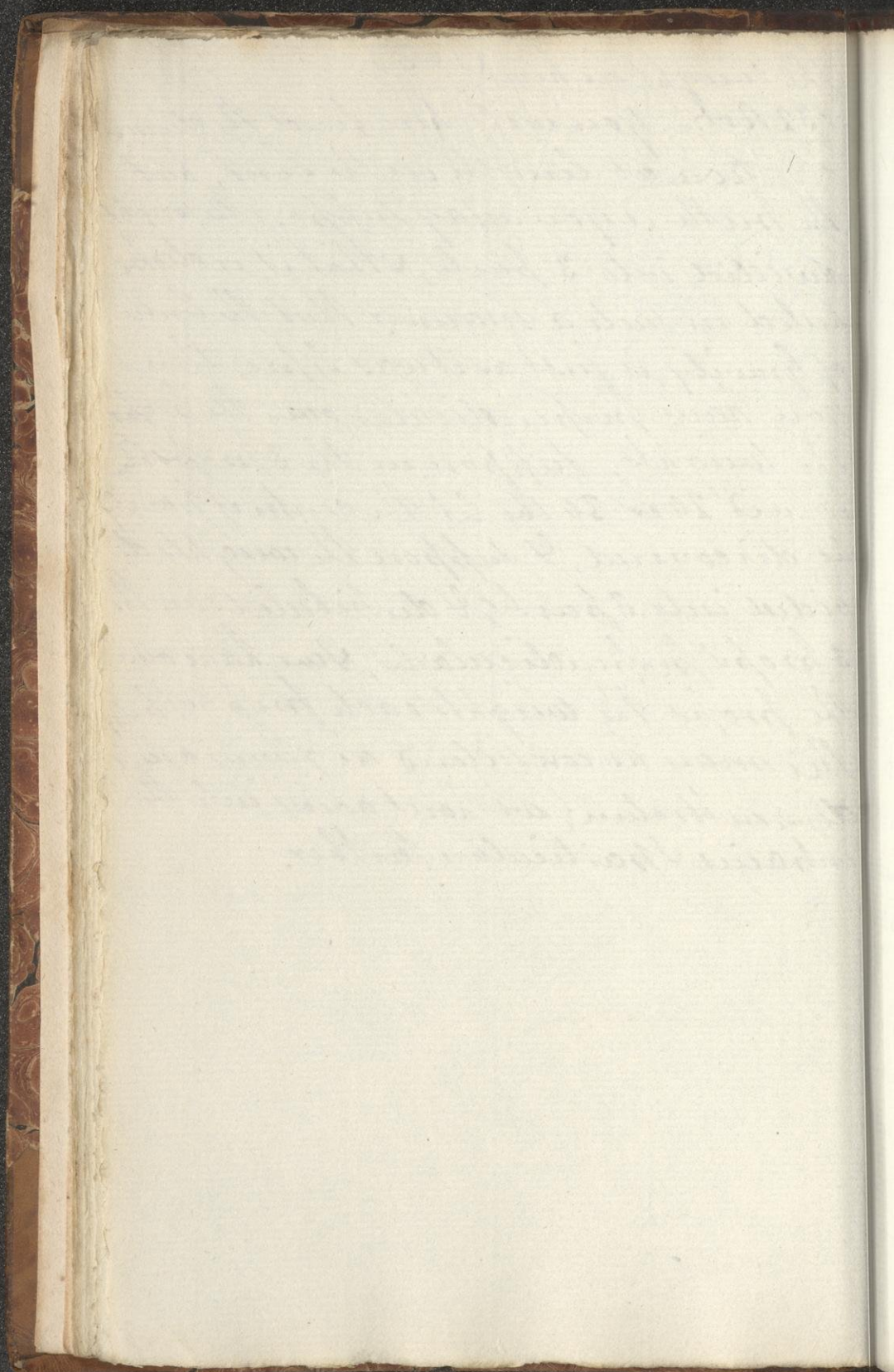
Centre of Gravity was ⁽⁶²⁾ placed there, it
would then be in disadvantage. The
Momentum is always shown by Multi-
-plying the force from the distance of
motion.

[31] Art. Suppose there is here a straight
Lever supported by props at A & B
 Then is a Lead weight.
Now the whole weight
is sustained at C the centre of Gravity.
What will be the weight at A to give
an Equilibrium $A \times AB = W \times CB$, This
is a Lever of the 2^d kind for we sup-
pose B the fulcrum. Now let us invert
this & suppose A the real immovable
Fulcrum, then in order that B may
sustain a weight equal to produce
an Equi: $B \times AB = W \times CA$ therefore
this quantity $A \times AB :: B \times AB :: W \times CB$
 $: W \times AC$ or $A : B :: CB : AC$; Thus the
weight rests in B the worst. And
from this principle the weight will
always press upon that man, who is
going down any inclination is the
lowest, for the centre of Gravity will
fall nearer him, & the more inclined
the plane is the more secure will be

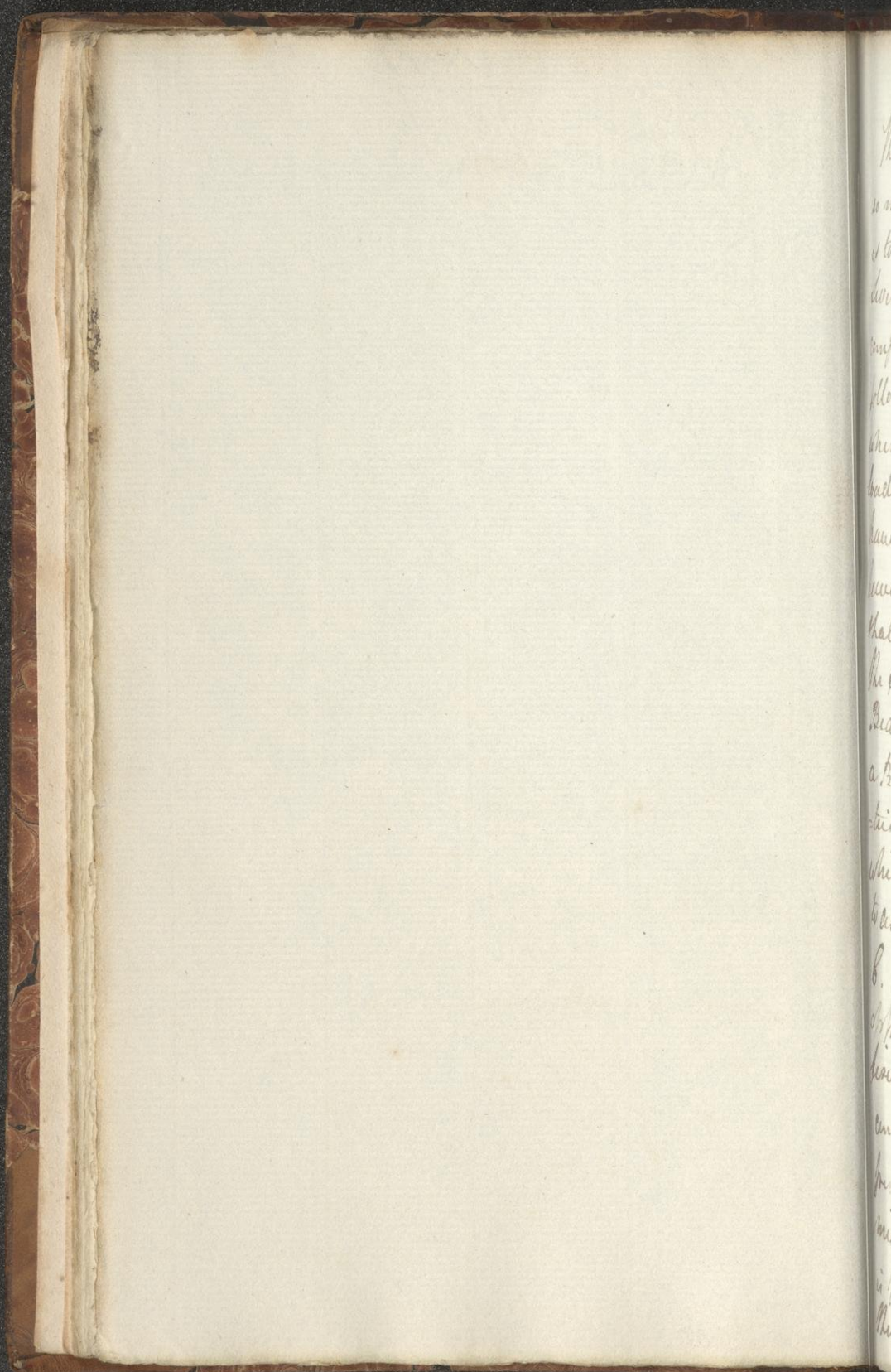
[illegible]

The weight on him. (163)

132 Art. - You will here find the demonstration at length in the Text, but the truth is you may suppose the weight divided into 3 parts, & that it is distributed in such a manner that the Centre of Gravity, is just as it was before; then you have perpendicular over the props the weights, suppose on the 3 props there is laid 20 or 30 lbs, let the centre of Gravity be discovered, & suppose the weights divided into 3 parts, & distributed over the 3 props perpendicularly, & we have over the props the weights each prop supports; this may be considered as giving a full demonstration, without going into the intricacies & particulars further.



(164)

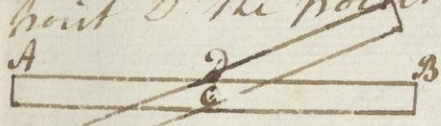


(165)
of the Balance

The use of this Instrument is not so much to increase the power of man as to extend his knowledge. It is a lever with equal Arms, used for comparing weights, it should have the following properties: 1 It should rest when in an horizontal position when loaded with equal weights. 2 It should have great sensibility. 3 It should have great stability. Then terms we shall explain immediately. To specify the line or knife edges on which the Beam turns, suppose we have here a beam of any kind, from the 2 extremities of which we are to have weights, which we are supposed to have brought to accurate equiv. Bisect it equally at C. Now I let us suppose the 2 weights opposite in the beam as if they were fixed at A & B (figure over leaf) & C this centre of Gravity, if the point of suspension be accurately placed in the middle in whatever position the beam is placed it will remain at rest, now this would be an imperfect construction for

x wants explanation

a balance, you could not have a distinct mode of telling whether there was an Equilibrium. Now let us suppose the Beam is supported at the point D. The point of suspension



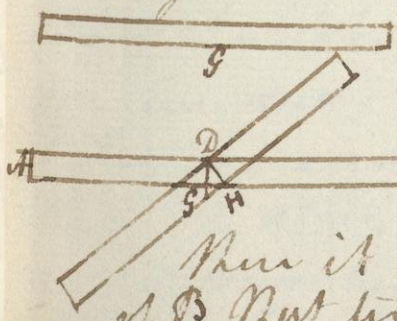
Let us suppose it be at A a little

more than at B, it will then incline towards A, now it has the greater gravity on the higher side, which will counteract the additional weight at A so that it will come to rest in the oblique position, by its throwing up the centre of gravity.

Let us now consider the different properties of the Balance 133 Art:

to explain merely from the Nature of the thing the Sensibility on the one hand & the Stability on the other

suppose first the centre of Gravity is at G then when the Balance comes into the oblique position the centre of gravity is thrown up to the point H



position the centre of gravity is thrown up to the point H

then it is the superior weight of B that tends to restore it or the weight at H the stability depends on

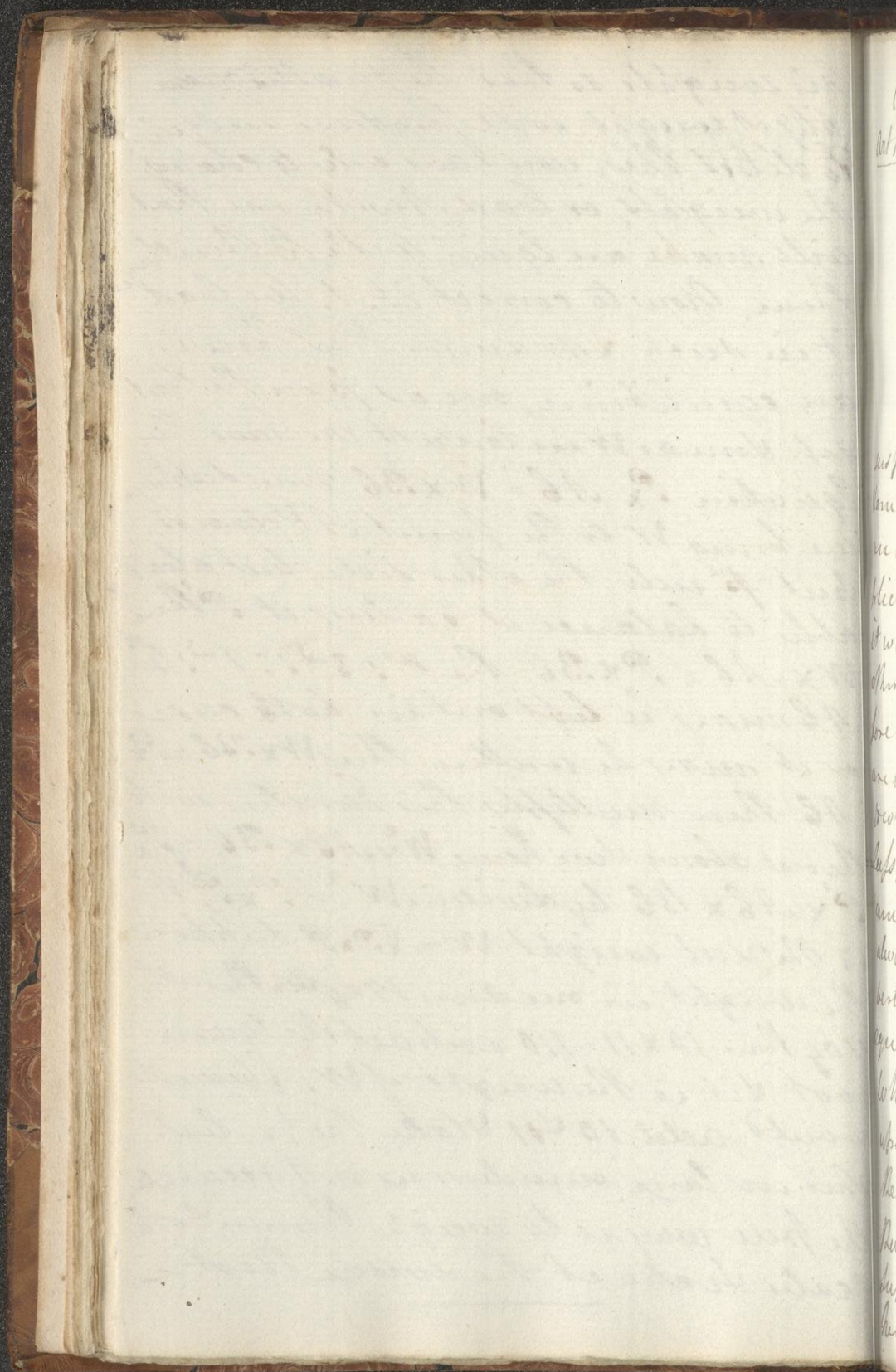
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(167)
It is greater, represented by b the
expression for the sensibility, then
will $\frac{a}{b}$ & to make it more sensible
must be increased - if b is made
too small there will be no stability
you must multiply b into the weight
of the beam the stability therefore into
 $\frac{a}{b}$. To have one accurately made
^{Wb} determine what is the weight of a
& b the centre of which is most difficult
construct a new balance, whatever may
be the length of the new beam form it
upon the same proportions as the re-
create one you have already before
you. Formerly the beam was always
made solid now however they are
made of 2 hollow tubes connected at the
base, in the construction of a new ba-
lance we must have a stop board at
the bottom to stop the oscillations

134 Art - When a Balance is false
how we may detect & correct it
1. By one arm being made longer
than the other, you of necessity give
more momentum to the long arm.
A man therefore with great diligent
intention would be careful to put

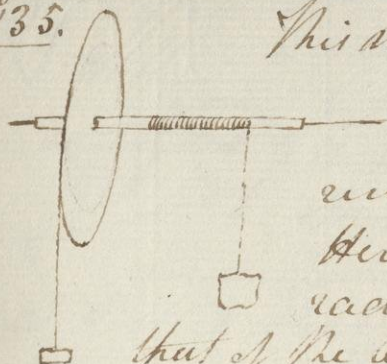
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his weights so that the smallest quan-
tity of weight will produce an equi:
To detect this, you have only to change
the weights, or load it with one that
will make an Equi: with B then
them. Now to correct it. 1. We load
it in such a manner that there is
an equilibrium, one as p in the short
est arm as W in longest we have the
Equation $P \times AB = W \times BC$ Now suppose
we bring W to the point A & do not
put p into the other side, but adjust
it to balance it calling it P then
 $W \times AB = P \times BC$ the pt. 3^d :: 2^d :: 4th
AB may be left out in both cases
or it may be written thus $W \times BC = P \times$
AB then multiply this Equation with
the one above & we have $W^2 \times AB \times BC = P \times$
 $P \times AB \times BC$ by division $W^2 = P \times P$ W
is the real weight $W = \sqrt{P \times P}$ Suppose
the weight in one arm 10 oz & the other
11 oz then $10 \times 11 = 110$ extract the square
root & it is the weight of W. Chemists
would add 10 & 11 & take the 1/2, but
this in large numbers is not accurate,
the true way is to weigh them in both
scales & extract the square Root &



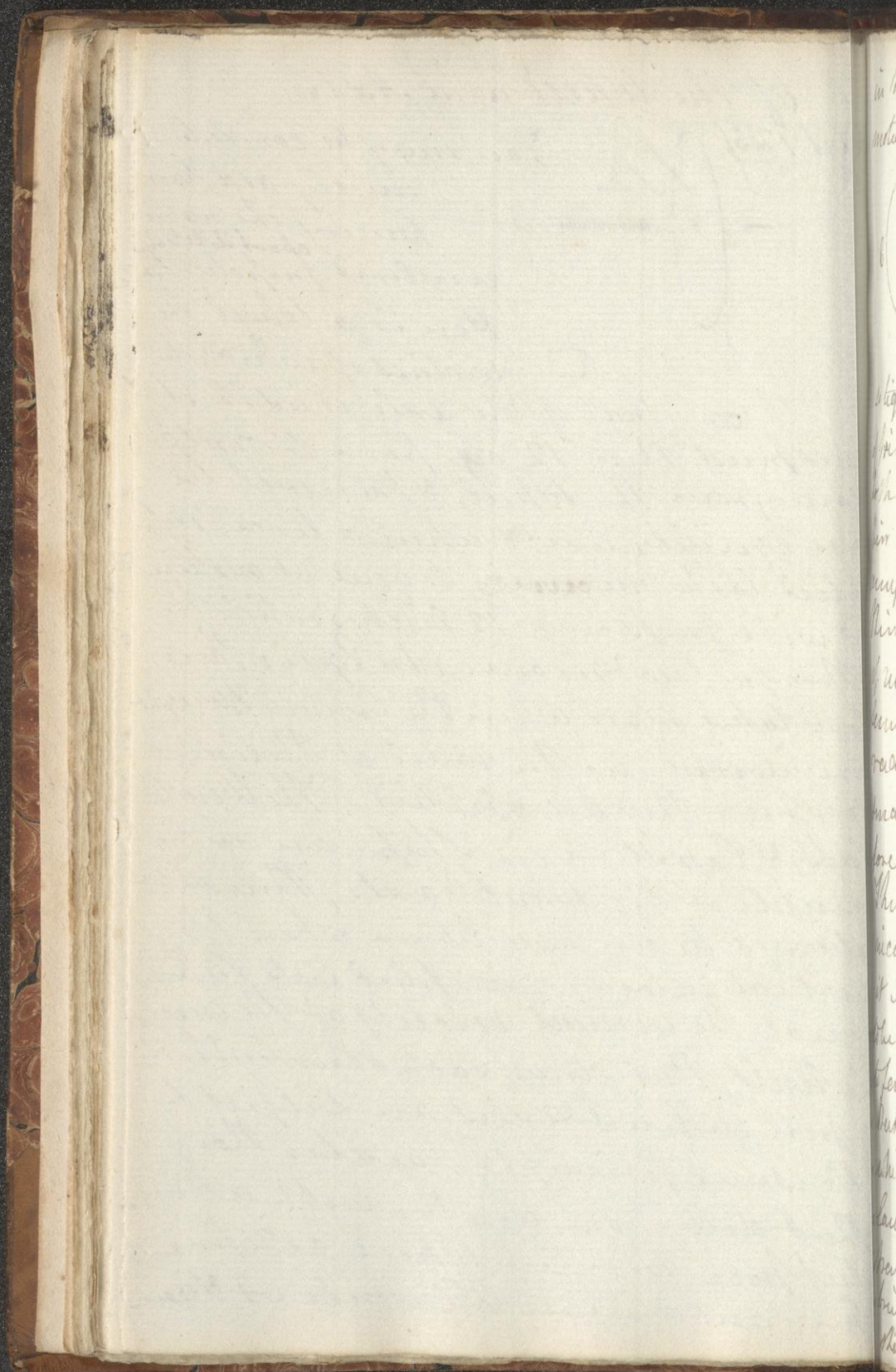
(169)
Of the Wheel and Axle

Art 135.

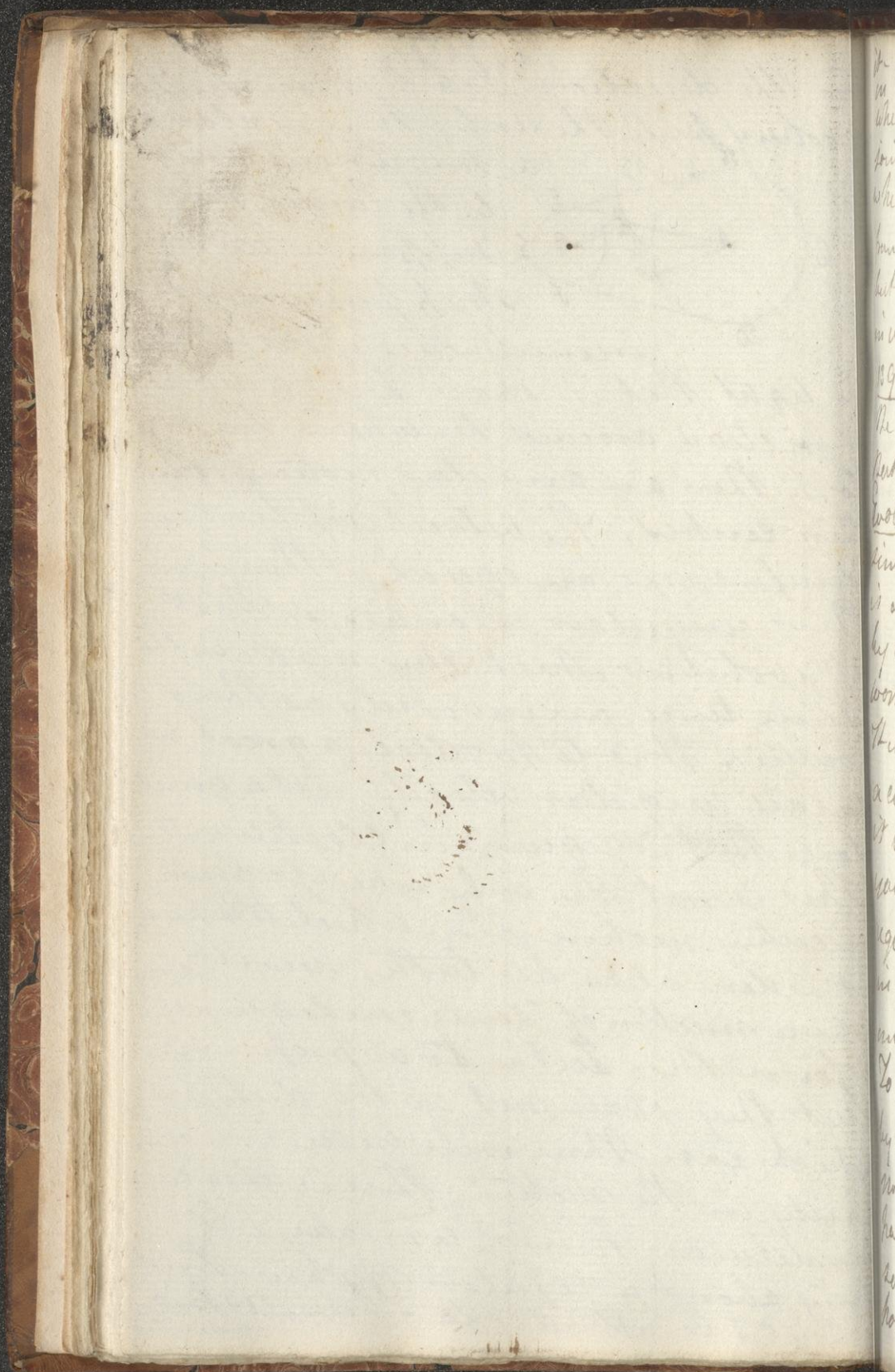


This may be considered
merely as a lever
having its motion
inclined ^{about its fulcrum} perpendicular.

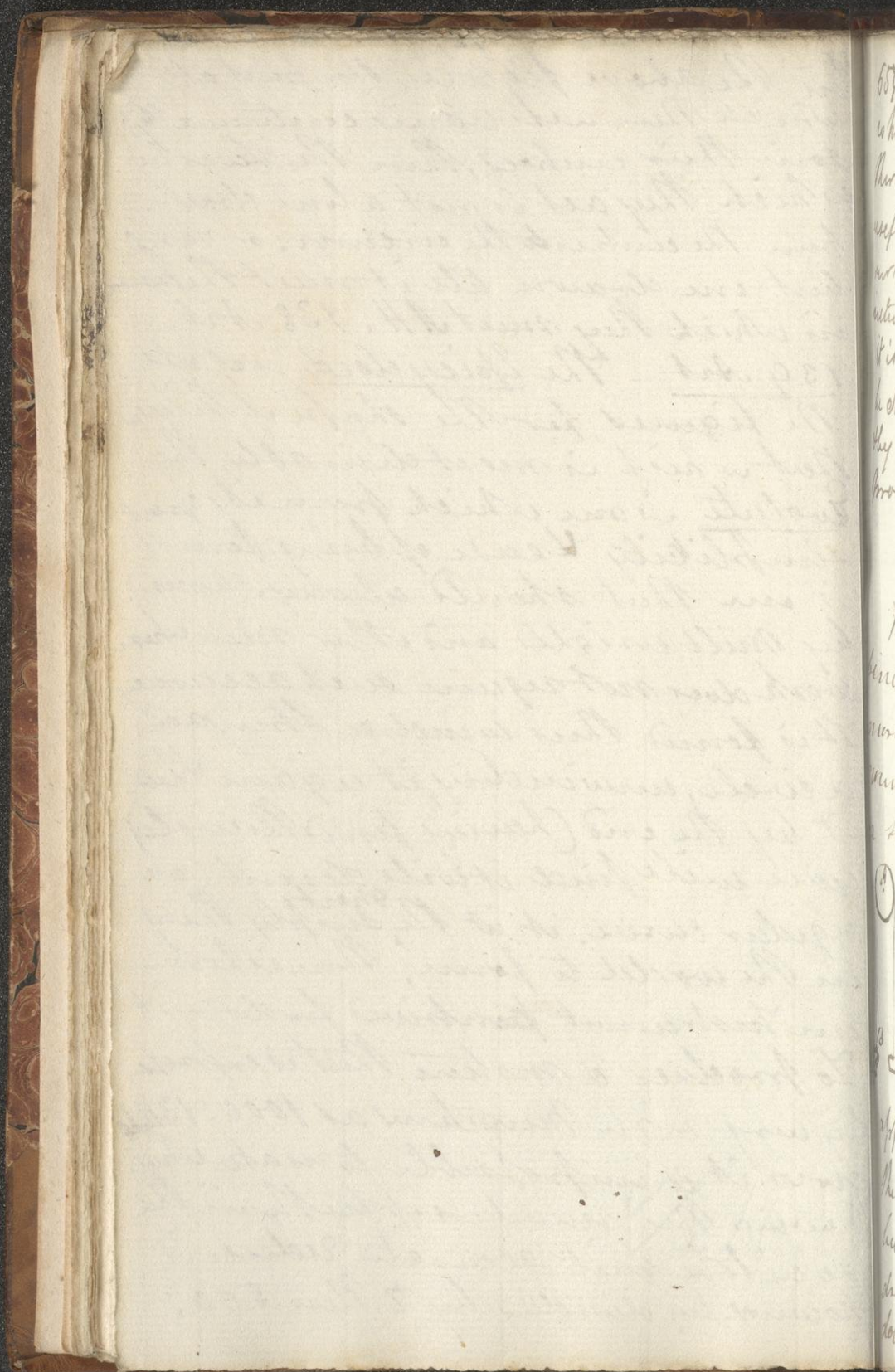
Here is a wheel the
radius of which to
that of the axle is as 12:1,
suspend then 12 oz. from the axle
down from the wheel, & there will be
an equilibrium; & when a force is ap-
plied to the one ~~corner~~, to give it motion
it will pass over 12 inches while the
other passes over one. An Equi: there-
fore takes place when the power & weight
are inversely as the radii of the circles
to which they are applied. The wind-
lass, & Capstan in Ships are an ex-
ample of the wheel & axle, there will
always be an equilibrium when the
vertical velocity multiplied into the power
equal the vertical velocity of the weight.
Wheels that drive each other by teeth
upon different axes are subject to
the same principle, as also those
that drive one another with a string,
suppose there is a wheel as represents
here moving on the axle A & going



in the direction B C D it receives its
motion from the axle; There is also a
smaller one times



it - ⁽¹⁷¹⁾ The above figure, the part at
which these act varies continually;
join their centres; then the Lower by
which they act is not a line drawn
from the centre to the circumference; or radius
but one drawn till it meet the point
in which they meet A.H. 138 Art-
139 Art - The Epicycloid is of all
the figures per the shape of the Lute
but which is most desirable, The
Evolute is one which from its great
simplicity & ease of being formed
is one that should always be used
by mill wrights and other men whose
work does not require such accuracy
It is formed thus wind a string round
a circle, unwinding it again holding
it by the end (having fixed the circle)
you will find it will describe a very
regular curve, it is the ^{most} simply thing
in the world to form, there is however
an instrument contrived for drawing it
To produce a motion that is expressed
by very high numbers as 1000: 13446
now it is impossible to make wheels
having that number of teeth in them
so as to be quite accurate reduce it
however by dividing by 2 then 500:



65708 & it may be reduced to 14:54
which is low enough; There is ano-
ther number which is remarkably
useful 113: 355. Sometimes in finding
our motion accurate we must place
intermediate wheels, per a train as
it is called by Clock makers. I shall
be obliged to direct your attention by
they again to them let us now
proceed to the

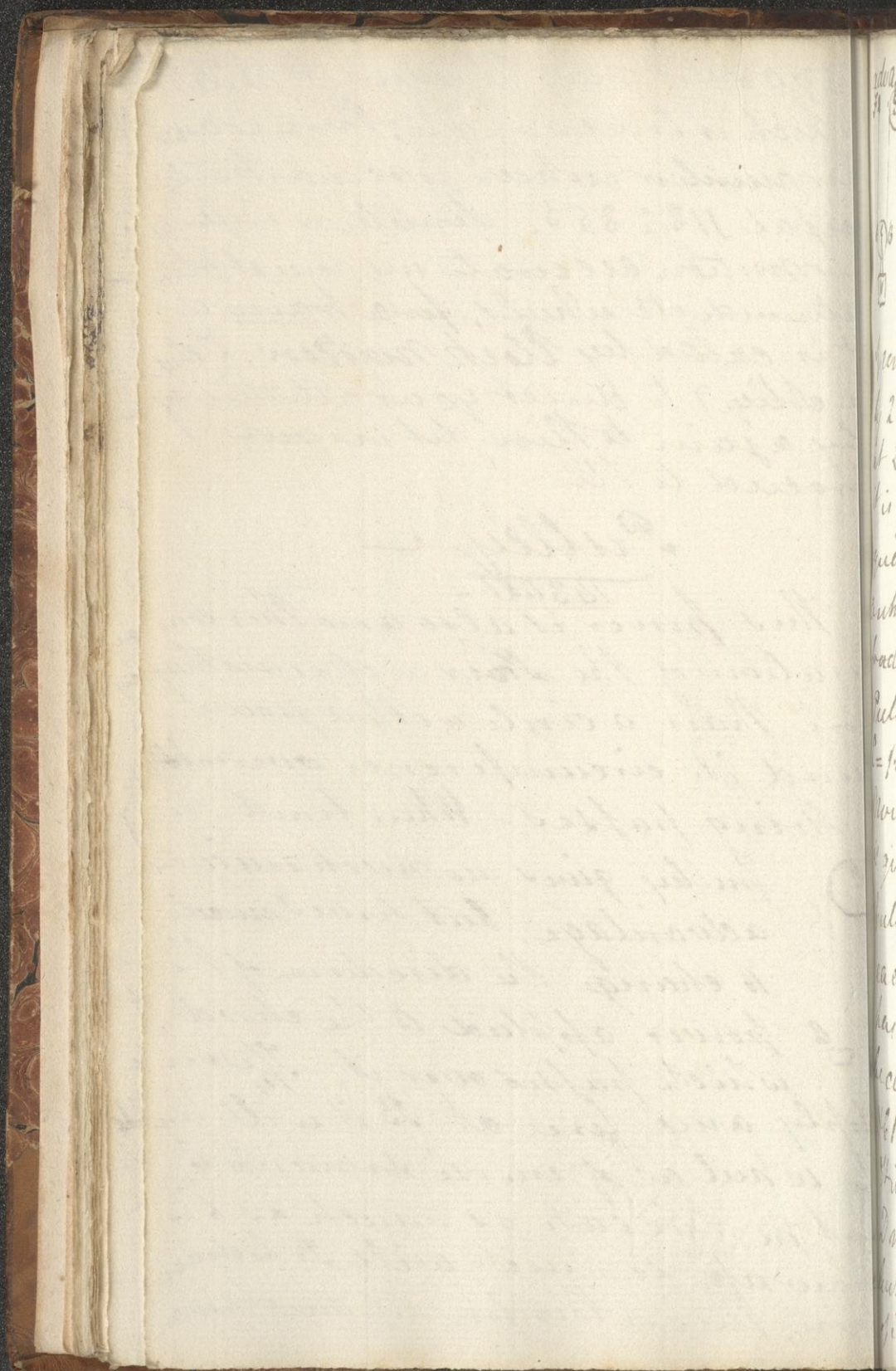
Pulley

143 alt.

This power is also another com-
bination of the Lever - it is nothing
more than a circle with a groove
round its circumference over which
a string passes - When fixed the



pulley gives no mechanical
advantage, but serves merely
to change the direction of the
power applied to the chord
which passes over it. If you
apply any force at B it will turn
the wheel as of course draw up C,
but in this case as much as C is
draw up, so much will B be drawn
down, there is therefore no mechanical

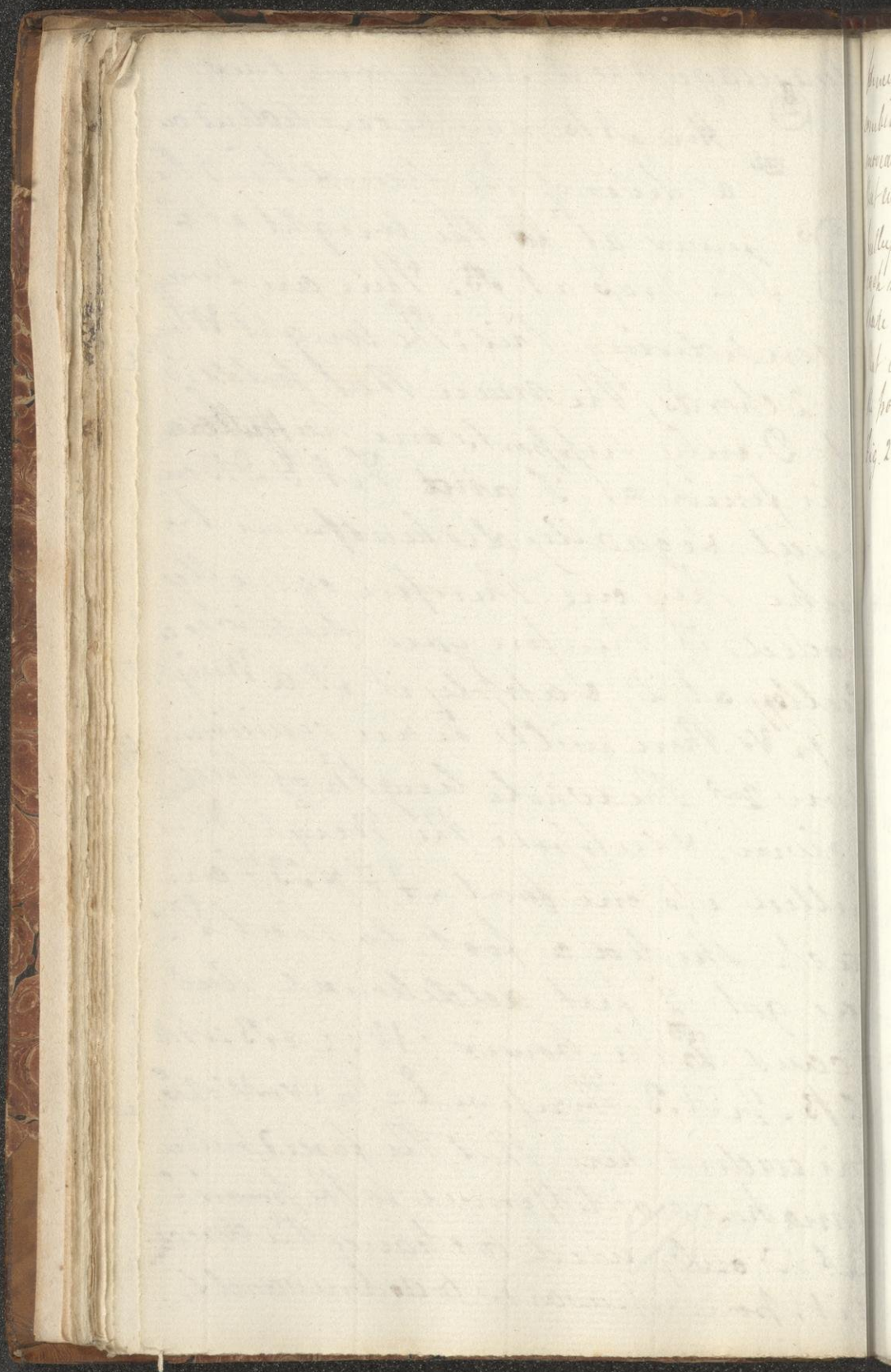


(173)
advantage to be derived from this.

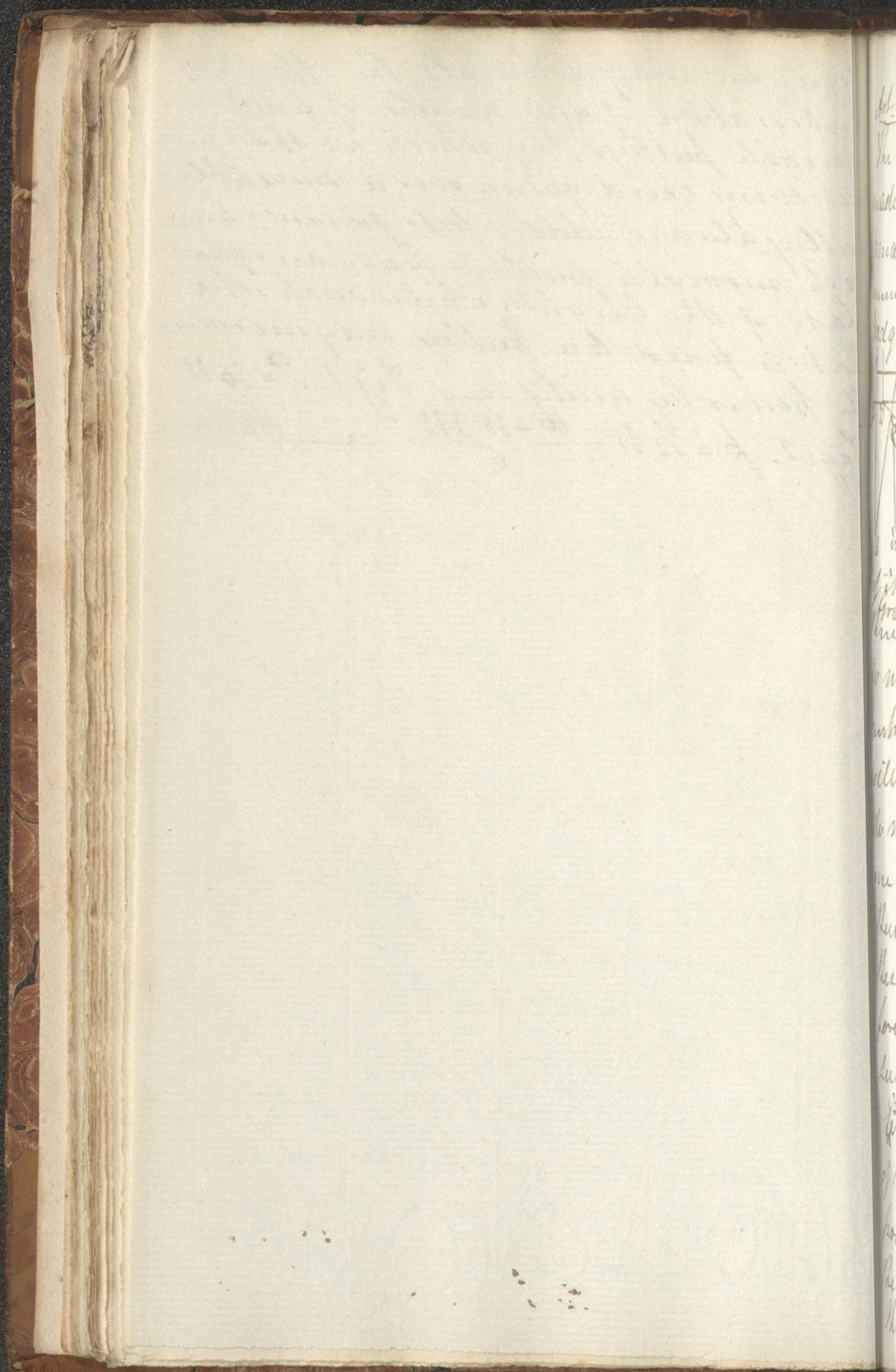
The diagram shows a fixed pulley at the top. A rope passes over it, with one end attached to a point A and the other end attached to a weight W. A second rope is attached to point A, passes over a movable pulley B, and its other end is attached to a weight W. The movable pulley B is also attached to a weight W. The text describes this as a lever of the second kind with the fulcrum at A, the power at B, and the weight at C.

Then AB may be considered as
a lever of the second kind the
power at B the weight at C
the prop at A . There are 2 ways

of considering this: The weight W hangs
by 2 chords, the man that holds it
at D only supports one half, because
it is fixed at F and F & D are
equal, & equally distant from the
center, they are therefore equally
loaded, if therefore you suppose a
Pulley at D , & apply it it a weight
 $E = \frac{1}{2} W$ there will be in equilibrium.
Now 2nd the whole length of the chain
is given, & suppose the weight W is
pulled up one foot A & B are
each shortened a foot, so that DE
has got 2 feet additional, and
because E the power: $W :: CB : AB$
 $CB = \frac{1}{2} AB$ therefore $E = \frac{1}{2} W$ or $W = 2E$.
It is evident here that the fixed pulley
 D makes no difference to the power E ,
but is only used to change the direction
of it, from upwards to downwards.

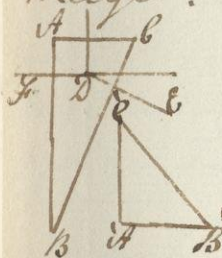


Since we may estimate the effect of a combination of any number of fixed or moveable pulleys; by which we shall find that every chord going over a moveable pulley always adds 2 bits, powers; hence each moveable pulley bears an equal share of the weight, while each rope that is fixed to a pulley, only increases the power by unity e. Fig 1. $P = \frac{1}{6}W$
Fig. 2. $p = \frac{1}{2}W = \frac{W+W+W}{6}$ e. Hutton



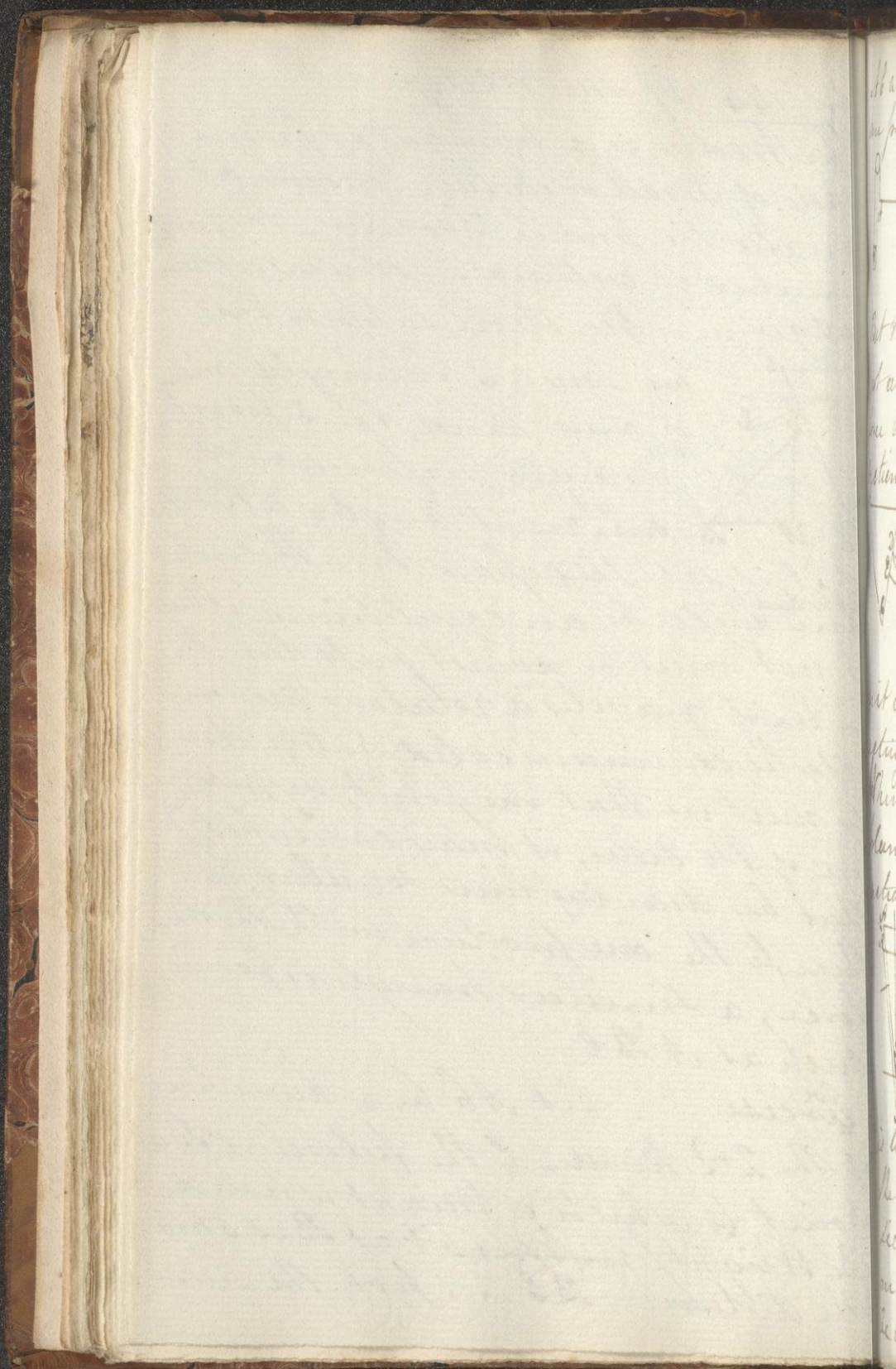
Art. 144 Of the Wedge

The Wedge is a triangular prism made of Wood or Metal. It is usual to make the prism Isosceles. The great number of cutting Instruments are Wedges. We shall suppose that

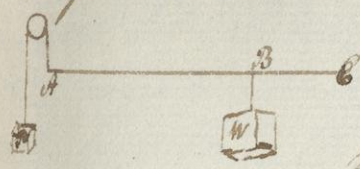


we have a Triangular prism of any kind, the 3 forces drawn pressing on it ^{at right angles to the sides} of it if all these pass thro' the centre of gravity. There will be an equilibrium, if they do not meet or do not pass thro' the centre of gravity a rotatory Motion will be communicated, untill they do meet in that one point, by producing one of the sides, it may easily be shown that by drawing lines similar to these, to the one produce with the other force, a similar triangle is formed. Such as $\triangle B'CE$.

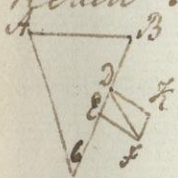
Exercise. Let AB be a heavy lever of the 2nd kind - C the fulcrum Ba point to which a Weight is fixed, the Weight, the Weight at B is 56lb the distance BC is 1 foot, the lever



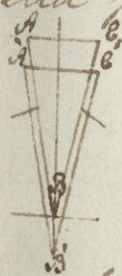
AB a rod of Iron. Every foot weighs
 one pound. P is a weight of 11 lb; what
 must be the weight
 of AC that the whole
 may be in equilibrium.



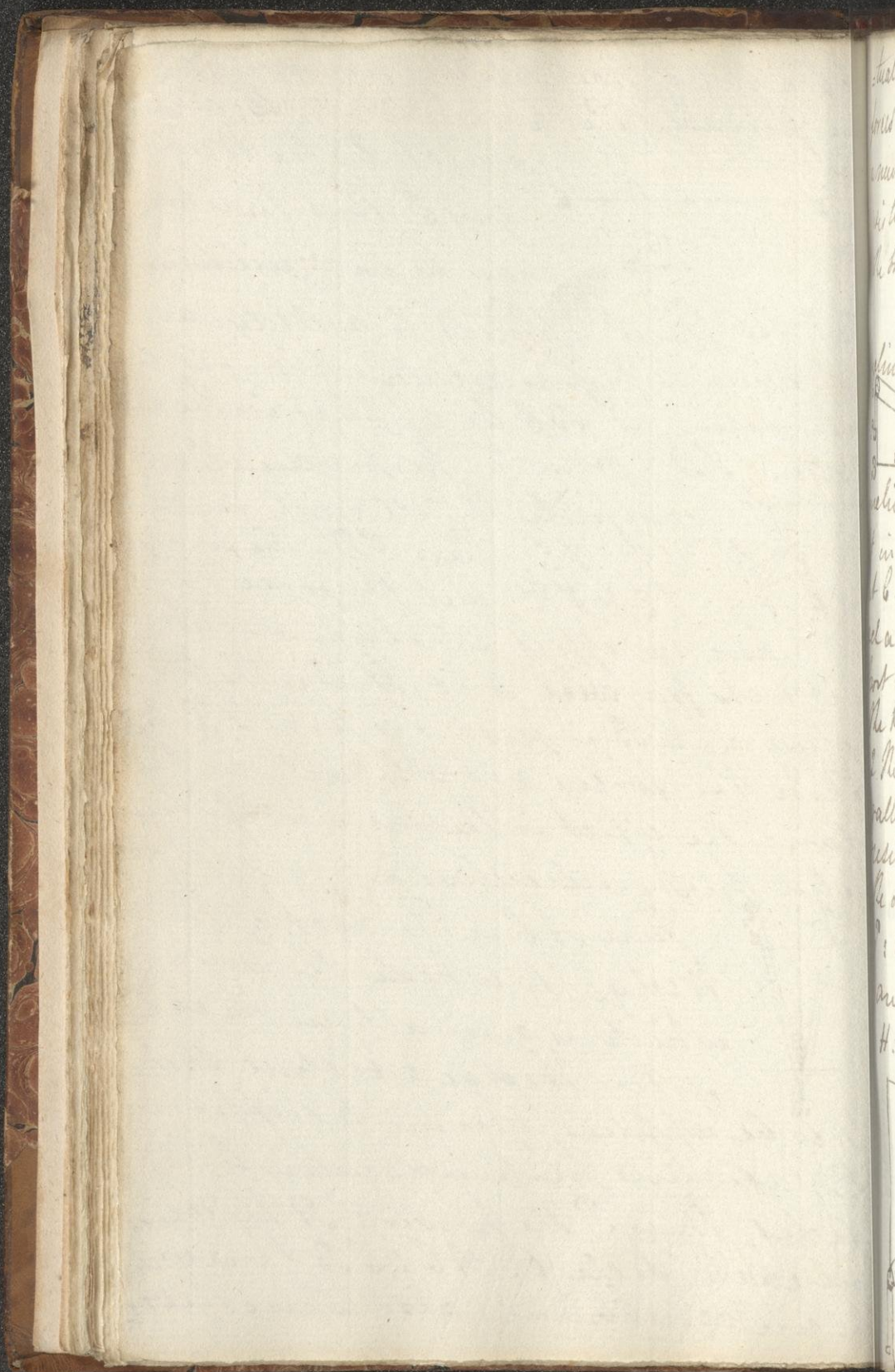
But the forces applied to a Wedge are
 not always perpendicular to it. Let the
 force acting at BB be applied in the di-
 rection FD. Draw FE perpendicular to
 complete the rectangle under
 FK, FE. Then FD = the force
 FE, & FK, but the force FK



has no effect whatever on the Wedge
 as it acts parallel to it. Hence the force
 acting on CB is just the force FE = FD
 When the forces act obliquely on the
 plane the effect is the same as if they
 acted perpendicular to it

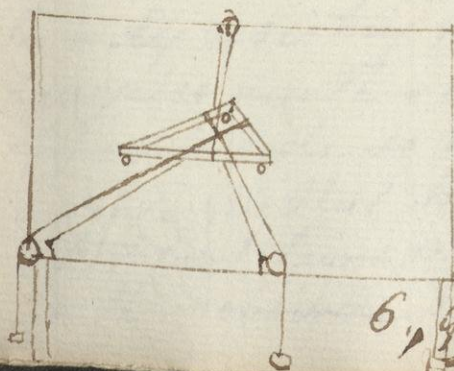


Here AC is the back of the
 Wedge, to which the power
 which is generally in the case
 of the Wedge a violent stroke
 is to be applied, you are to suppose
 the resistance perpendicular to the
 sides, then P the power: R the resist:
 on each side & AC:CB then will
 be an equilibrium, also when the force

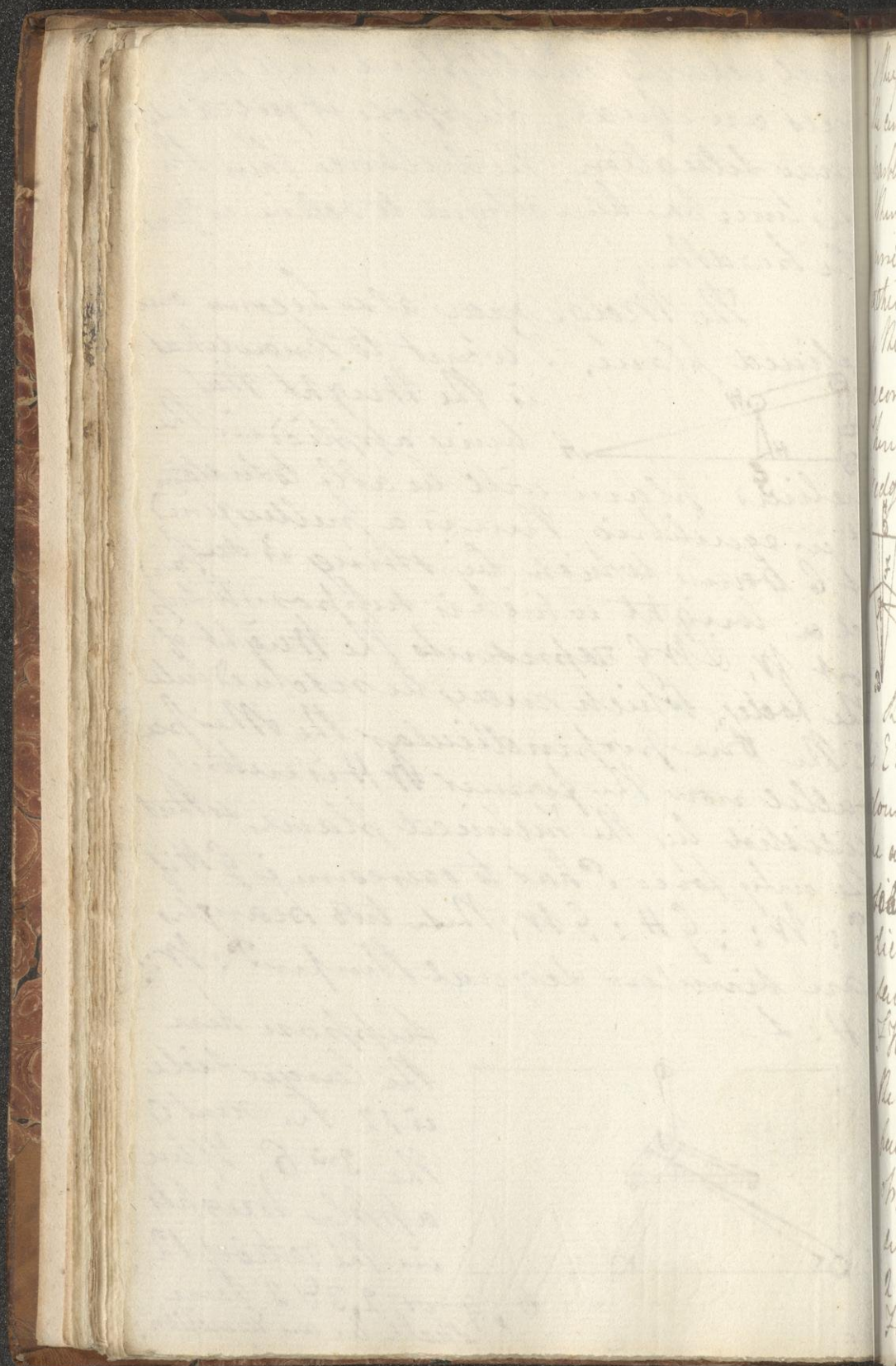


= total velocity multiplied into the forces are equal - Suppose it forced into a new situation. The distance then the resistance has been obliged to retire is equal to the breadth.

The Wedge may also become an inclined plane. I want to know what is the Weight that being applied in the inclined plane will be able to sustain it in equilibrium. Now is a pulley fixed at C bound to which by string is suspended a weight which is supposed to support W. W represents the weight of the body, which may be resolved into 2 the one perpendicular the other parallel now the former WH is entirely resisted by the inclined plane, so that the only force P has to overcome is GH if $P : W :: GH : GW$, these two triangles are similar & equal therefore $P : W :: H : L$.

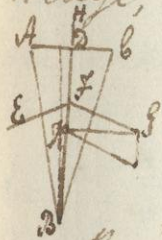


Suppose here the longer side is 12 the next 9 the 3rd 6. If we apply weights in the ratios 12 6, 9 or 2, 3 & 2 there shall be an Equilibrium.

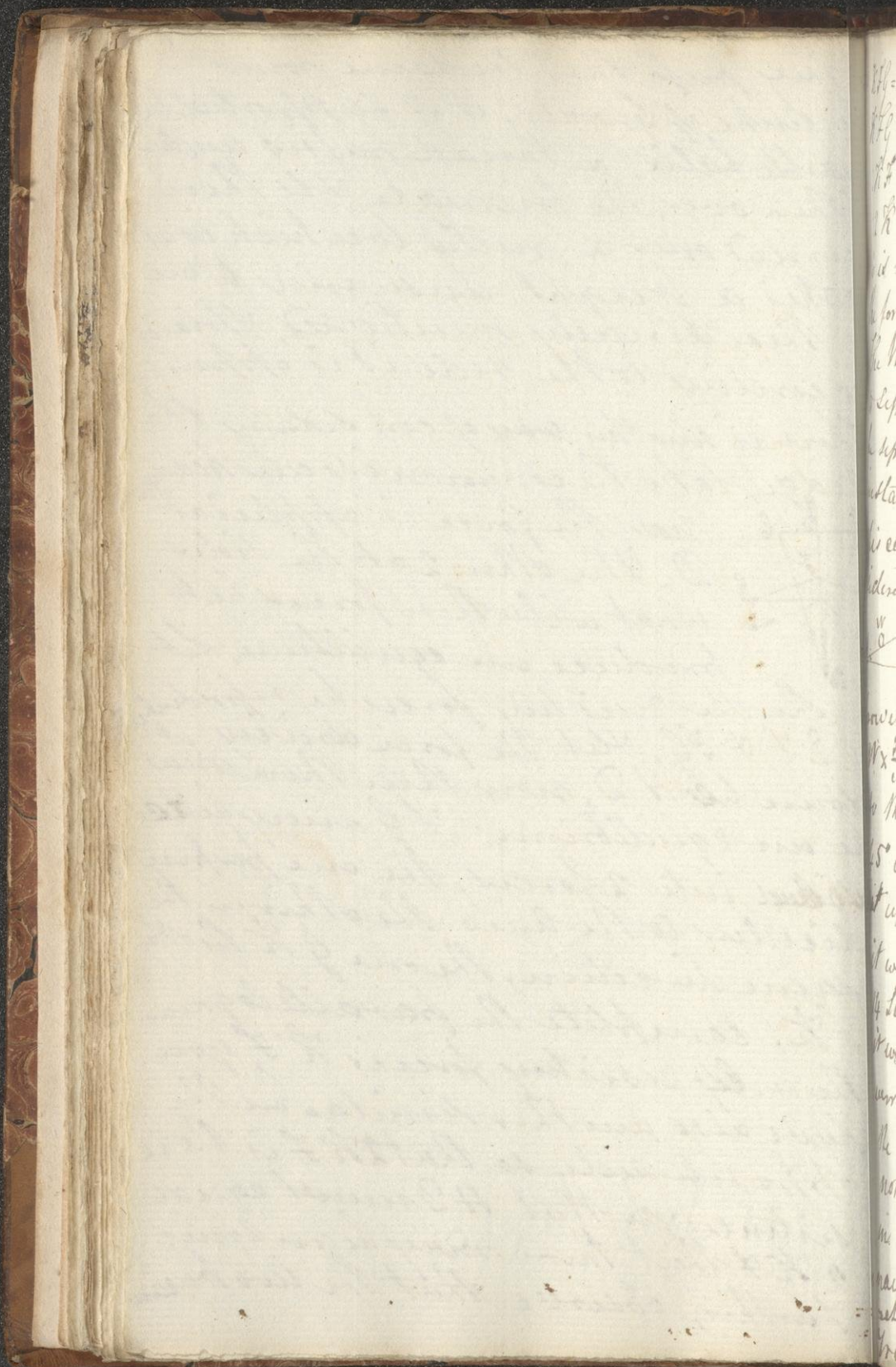


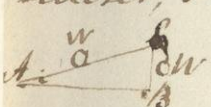
if they pass thro' the same point
 the centre of gravity, it is supported by
 marble balls, a thread passes under
 them over the triangle & is then
 carried over a pulley to which is at-
 tached a weight answering to one
 of these already mentioned, & placed
 according to the side it is opposite

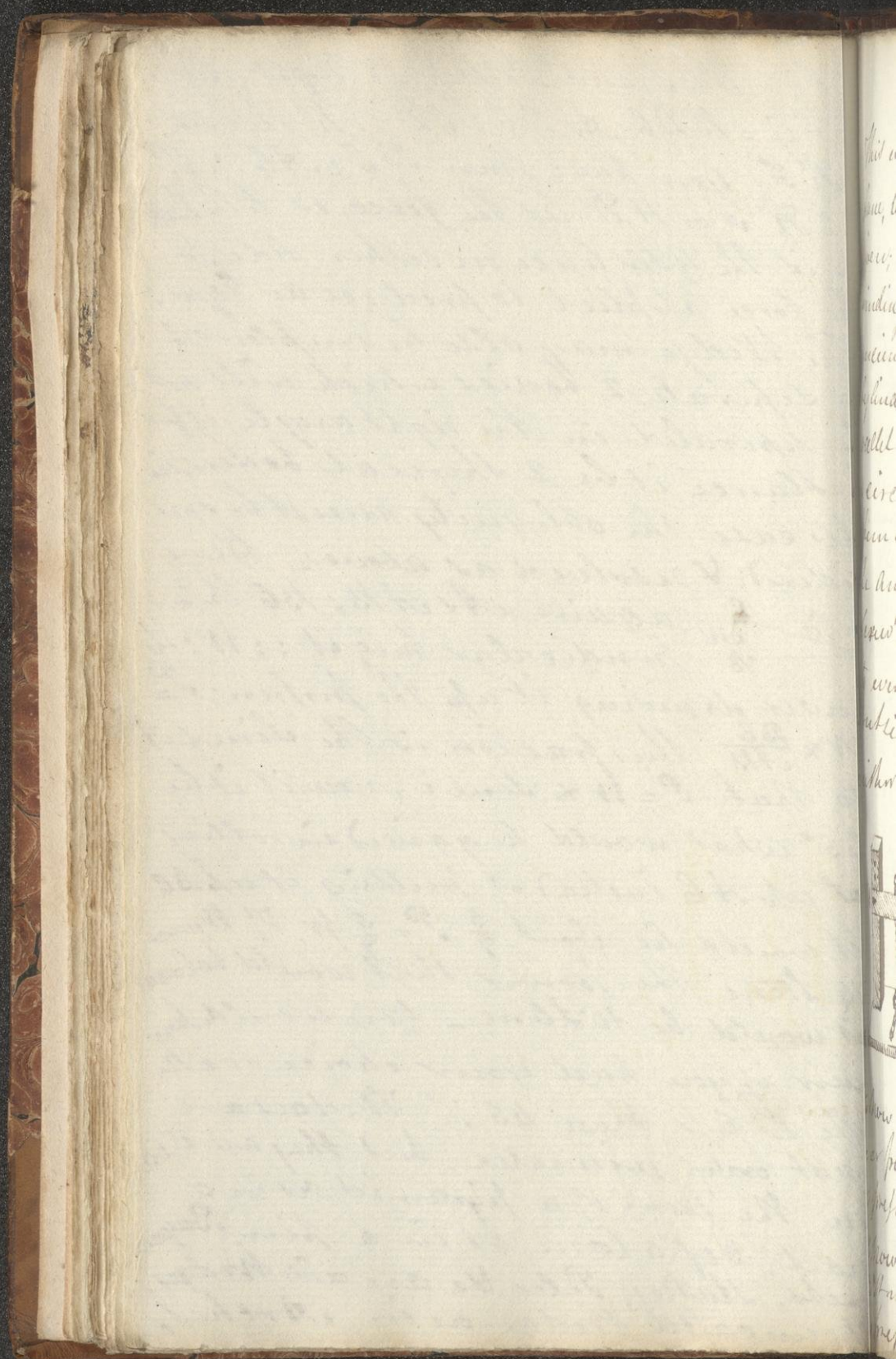
There is another way of considering the
 Wedge, take the common isosceles triangle
 now the force is applied at
 D. & the other 2 at the sides
 what will these forces be to
 produce an equilibrium. Let



the resisting forces be represented
 by EF & FG, & the force driving it
 down be HD, now then there may
 be an equilibrium; FG may be re-
 solved into 2 forces, the one perpen-
 dicular to the axis the other in the
 same direction, the one G to the other
 FK, complete the parallelogram
 the only resisting force is KF, you
 have also another similar on the
 opposite side so that 2KF is the re-
 sistance; so that HD must equal
 2KF that there may be an equi:
 Further, observe that the two basins

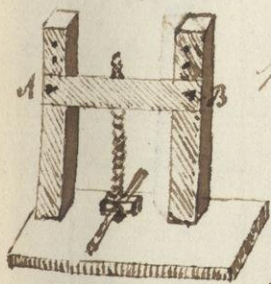


$K F G = D B C$ and as they are right angled
 $K F G = B D C$ As $B C : C D :: R$ the resist.
 $: K F$ you have then $B C : A C :: R$
 $: 2 R$ For $H D$ as the force is to the back
 so is the resistance on either side, to
 the force applied to produce an Equi.
 The Wedge may also be employed
 to separate 2 bodies which will not
 be separated in the right angle, if for
 instance it be 2 spherical bodies, in
 this case the obliquity must be con-
 sidered, & resolved as above. Here

 again As $A B : B C$ the per-
 pendicular height $:: W$ the
 power drawing it up the perpendicular $P =$
 $W \times \frac{B C}{A B}$ this fraction is the sine of A
 so that $P = W \times \text{line } i$, now if it be
 45° what would be gained in rolling
 it up $A B$ instead of pulling it up $B C$
 it would be about $\frac{5}{7}$ $P = \frac{5}{7} W$. If W were
 14 Stone the power that would balance
 it would be 10 Stone - You would have
 more if you had your choice make
 the $\angle$ less than 45° . Wedges are
 not only prismatic, but they are also
 in the form of a pyramid, as in a
 nail, & of a cone as in a pin, Bayo-
 nets, Stakes, Piles &c are all Wedges.
 Truncated Wedges as in Arches,



(180)
of the Screw

This is a Species of Wedge or Inclined plane, to which a circular motion is given; it is a spiral groove or Thread, winding round a cylinder; it is a very ancient Instrument; Imagine a cylinder & there are drawn lines parallel to the axis, & if from the base a circular line be drawn cutting them all at one angle, & the smaller the angle is the better will be the screw. The use of it is familiar to every one; it may be cut on the outside or inside, & is then termed either exterior or interior screw.

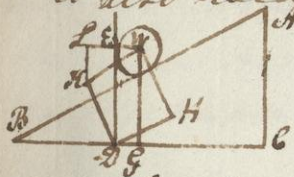


You observe this screw passes thro' this board A B which is made fast by pegs, & by turning the Lever one way it is used for compression, now if I remove the hand spike or power, which has caused the impression it would return, not so however with respect to the screw. It may not be the purpose to compress the body, not for a minute

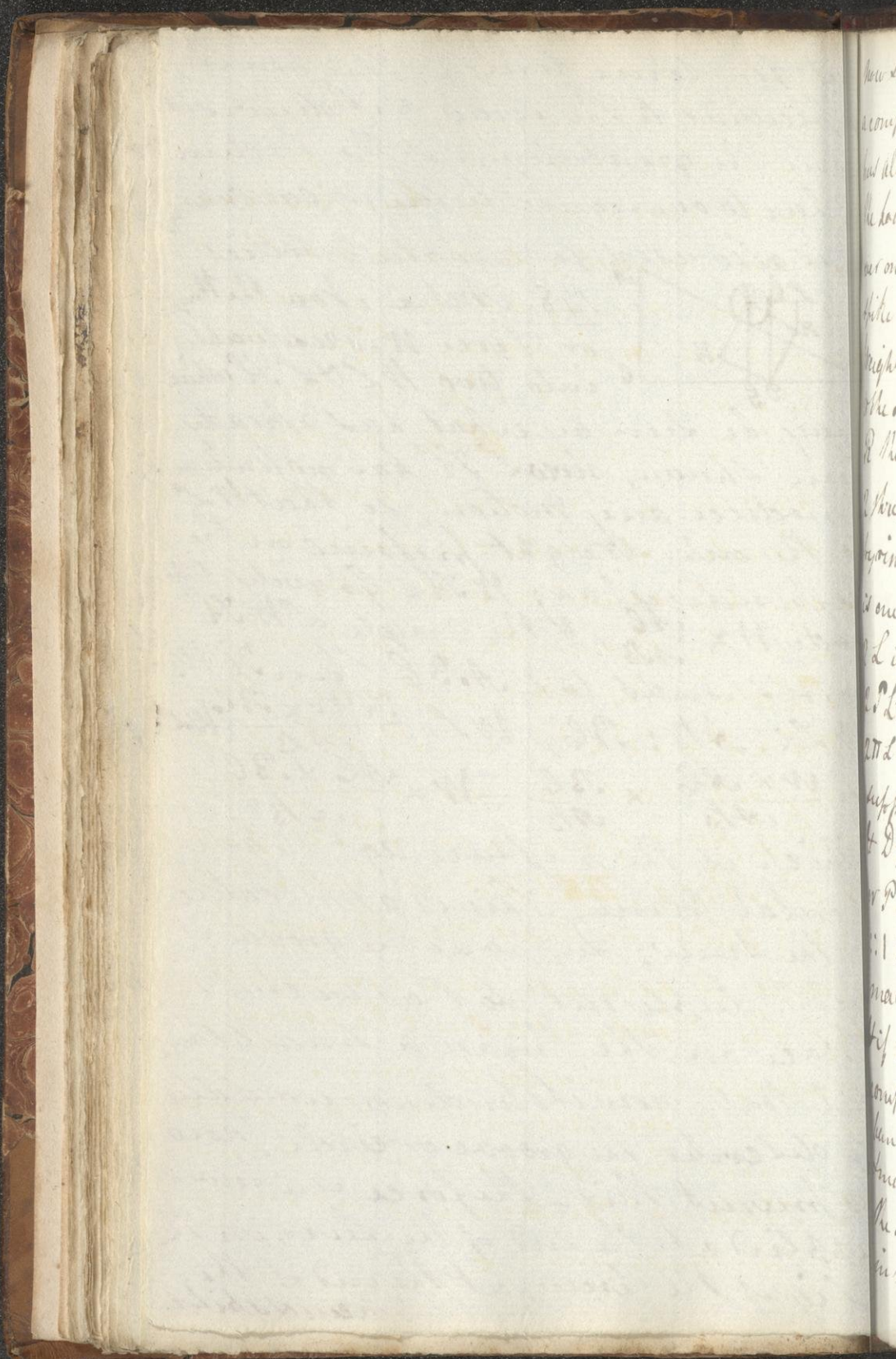
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but for some time - This power
is peculiar to the screw, & it derives it
from the great degree of the friction
it has to overcome, before it can return.
It is also used for elevating bodies.



148. Art. 2. Now the Weight or Force W is resolvable into two WL & WT which may be seen by what has already been shown, now LT has no tendency to produce any motion, so that WL is the only weight pressing on the perpendicular; $WT =$ Gravity of the body, $W \times \frac{AB}{AB}$, & the angle LWT corresponds to LAC . Then $LTW :: WL :: AB :: BC$, $WL = \frac{LTW \times BC}{AB}$
 $= \frac{W \times AB}{AB} \times \frac{BC}{AB} = W \times \frac{AB \times BC}{AB^2}$
 which is the pressure ag^t the horizontal plane. This is applicable to the screw; suppose the groove is very deeply cut so that we could place on the thread a small spherical ball, now its tendency would be to describe the groove or circle. Now to prevent this - the force is a power applied at the end of the Lever, in the Lig of the screw, at the end of the handle, & the

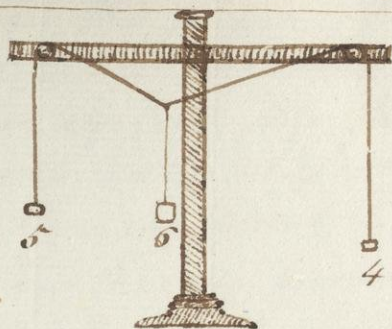


now suppose the hand has made
 a complete revolution, the spiral screw
 has also described a complete circle, &
 the body consequently has just moved
 over one thread; so that when the hand
 spike has made a complete circle the
 weight has only advanced ^{from} one thread
 to the other. Let P be the power &
 R the resistance ~~B~~ the distance of the
 2 threads L the Lever, π means the
 perimetre of the circle whose radius
 is one & is equal to 3.141659. Then
 $2L$ is the diameter of the circle & $D =$
 $2P L \pi L =$ the circumference then
 $2\pi L \times P = R \times D$ then $P : R :: D : 2\pi L$
 suppose that the Lever is a foot long
 & $D \frac{1}{10}$ part of an inch $P : R :: \frac{1}{10} : \pi \times 120$
 or $P : R :: 1 : 3600$ so that $P : R$
 $:: 1 : 3600$ so that one lb may be
 made to balance this immense weight
 & if the force was withdrawn the
 compression would remain. It has
 been used for raising buildings in Italy
 moving them from one place to another
 the Pillar of Antoninus was moved
 in this manner e

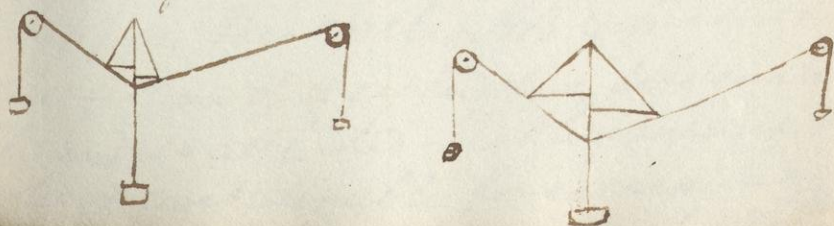
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4 P.
3 pr


(183) The Lunicular Machine



152 Art. If a body fixed to 2 or more ropes, is sustained by powers which act by means of those ropes, this Apparatus is called the Lunicular or Rope Machine. It is not reducible to any of the former except the Pulley, & scarcely that, this instrument should have been put up before to explain a different object. This is a kind of Lunicular Machine, to prevent as much as possible friction there are fixed in the cross beam 2 pulleys, over which passes a thread to which is attached weight. If the 3 weights were equal the 3 triangles would be equal.



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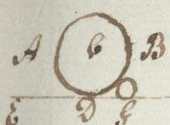
The 2 sides are 4 & 5 the diagonal 6, now the the figure the 2 Diagonals, have two perpendiculars on them which are equal, till therefore there is as much lateral force produced as 4 & 5 there will not be an equilibrium. This there is more obliquely at the 4 which produces the Equilibrium. If one of them is made as heavy as the other 2 then it will never remain at rest - and also a small weight placed in the centre will prevent the thing coming to a mathematical line however heavy the weights at these points are - it is from this that it becomes useful Mechanical Instrument.

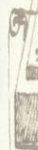
Roller S

The force of Rollers may also be considered as another mechanical power for compressing Metals - The truth is that the principle of it is merely the Wedge, with the addition of the lever. Suppose this circle AB represents the section of a cylinder, let its centre be C and so adjusted that by means of a lever considerable force may be applied

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we shall suppose there is a plane
 lying under it, & suppose
 it moves from A to B, so
 as to be made to press on
 the Bar of Copper at G, it will then
 compress it, & in doing this will
 communicate considerable heat. Nothing
 in Metals, but Percussion is able to
 produce extension, there pressure
 is seen to have the same effect, & to a
 much greater degree; There can
 be made no rectilinear angle so
 small, as the angle of contact, so
 that if you take an L of a Wedge
 an equal $\frac{1}{4}$ a degree, & another
 $\frac{1}{4}$ you may say it is double of the
 force, but here is an angle smaller
 than the angle of a Wedge, so that
 this brings pressure into a state of
 comparison with percussion. The
 angle of contact becomes always less
 the greater the radius of the circle
 so that the effect of the effect of the Rod
 here ought to be greater. The larger the
 cylinder; but you act upon it by a Lever
 so that it is the square of the Radius, on
 account of the longer Lever the Radius being
 multiplied into the Lever



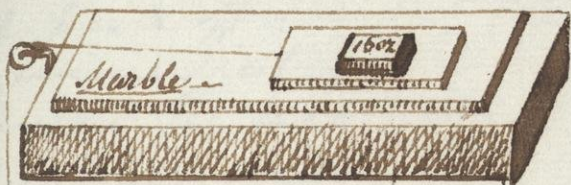


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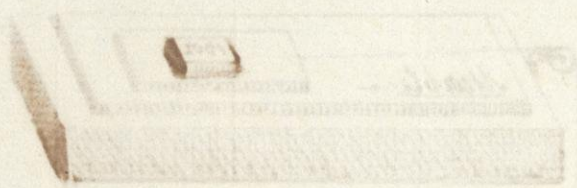
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(186)

Friction



1102 It is inseparable from all force of Mechanical action, it is sometimes of use to it, but more commonly an impediment & disadvantage. Here on this polished Marble is a piece of Mahogany loaded with 16oz, & the weight of 11oz attached to it over the pulley is only just sufficient to overcome its Friction. Friction never generates motion but always destroys it; when ever however on the contrary as we have already seen, Gravity here destroys the motion upwards, it generates a motion downwards, equal to what it had in ascending; it is quite otherwise with Friction, when the motion ceases there will be none generated in the opposite direction. See the force of Friction in Art 159. If instead of the polish on the Marble



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(187)
slab, it were made rough, the friction
would be increased, further if de-
pended on the weight, if I take the
lead from off this piece of wood it
will immediately slide down, it
is therefore the asperities of the one
entering the asperities of the other
the concavities of the one entering the
convexities of the other. It is not much
affected by the surface pressing, if I
turn this piece of Mahogany on
its side there will be no alteration
or at least so very slight as scarcely
to be perceptible. This I would wish
you to remark as it is a notion gen-
erally entertained among men that
the greater the surface the greater the
friction, but this you see from this
experiment is not the case, the rea-
son is when you have a smaller
surface, there is a greater force applied
so that the convexities of the one are
sunk more deeply into the concavities
of the other, so that it rather increases
the friction - it is the same with
respect to the broad wheel waggons,
the friction here is not increased by
the broadness of the wheel, if there should

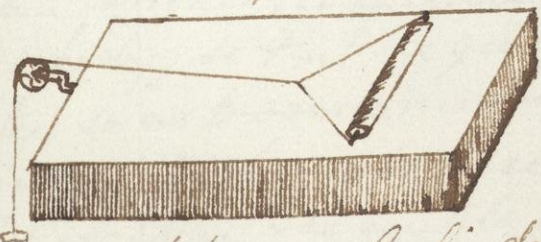
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(188)

Suppose to be an additional friction
which is seldom the case it proceeds
from the weight - Art 159 (a) & (b)

160 Art - Friction may be divided
into 2 great divisions. 1. Friction
arising from Sliding Bodies. 2.
from Rolling bodies. If instead of
this parallelepiped we had a roller
or cylinder we should ^{not} require so much
weight, thence the advantage of
wheel carriages - The first advances
towards this was the Sledge, where they
banded partly on the horse & partly on
the surface, The asperities of the one
in the case of wheel carriages are
not obliged to overcome the asperities
of the other, but on account of the lever
that is applied to it, tends to draw it out
the convexities of the concavities

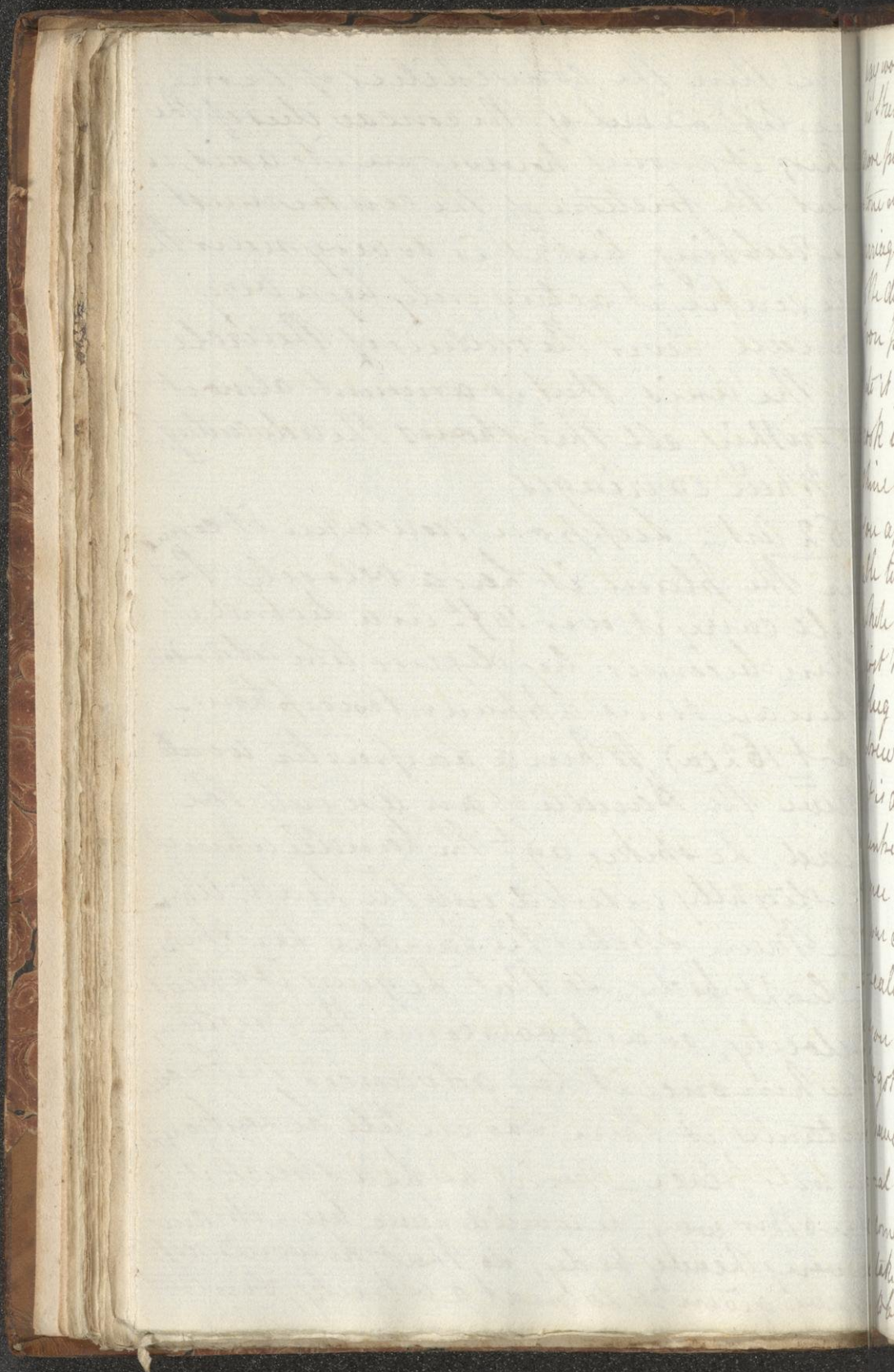


We shall suppose a cylindrical body
to be drawn along a plane, it is movable
on its centre parallel to the plane, now
suppose the 2 surfaces are serrated

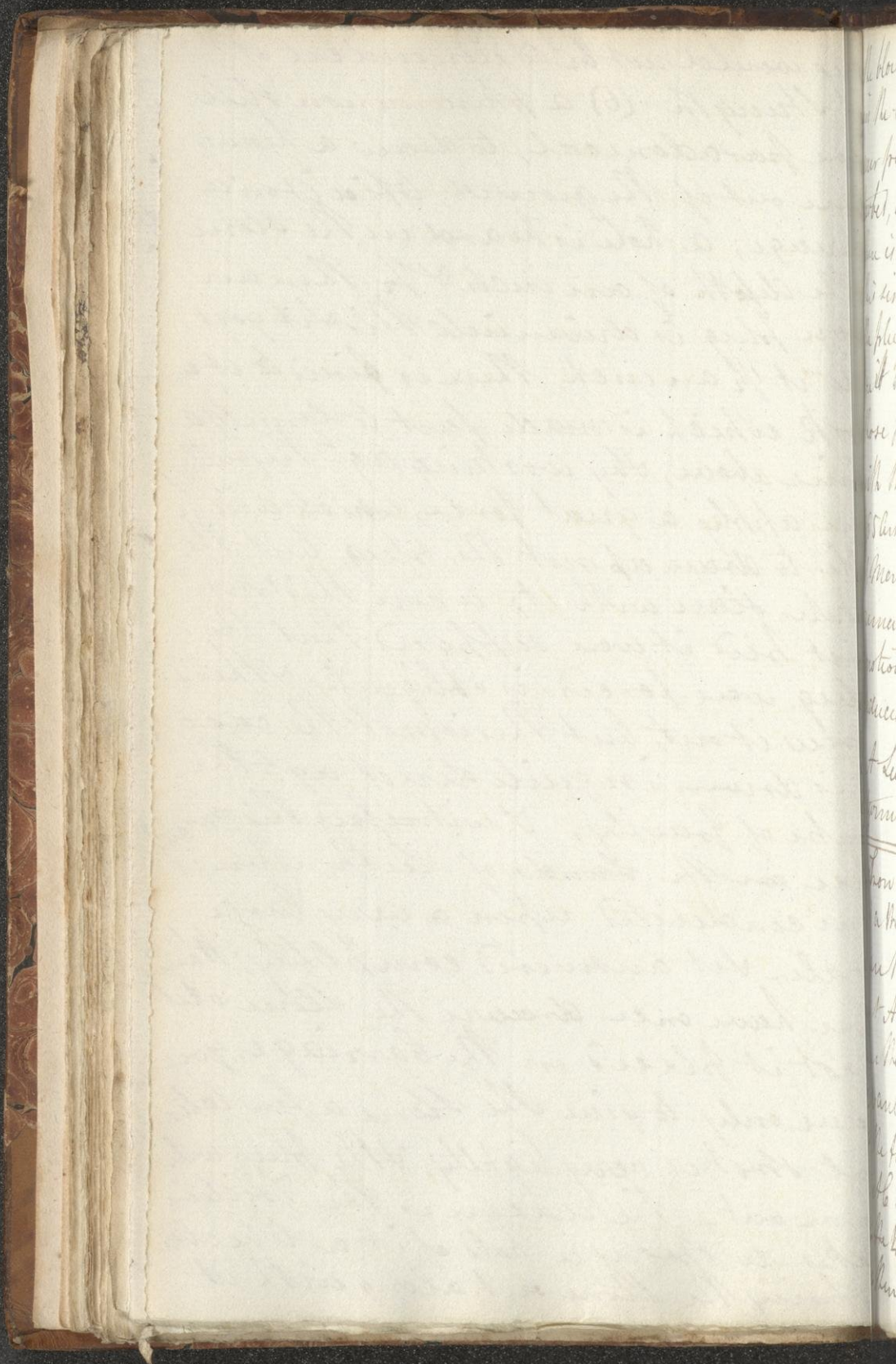


189)
here then the convexities of the one
are lifted out of the concavities of the
other; it moves however on its axis, so
that the friction of the centre must
be rubbing, but it is so very near the
the centre, it acts only by a very
small lever the radius of the circle
of the axis, that it amounts almost
to nothing all this shows the advantage
of wheel carriages

162 Art- Suppose now when it comes
on the plane it has a velocity that
will carry it over 10 ft in a second, it
then becomes by degrees to be retarded,
then some apparent exceptions -
Art 162 (ca) When a carpenter would
drive the handle of an axe into the
head, he strikes ag^t the handle when it
is slightly inserted into the head; when
the man strikes the handle he is striking
a light body, so that he gives it a great
velocity, so as to overcome the friction,
when once it has advanced the friction
retains it there, so on till he has brought
it to its place - now if he had struck it in
the other way, he would have been striking
a very heavy body, so that he would not
have given it so great a velocity, for that

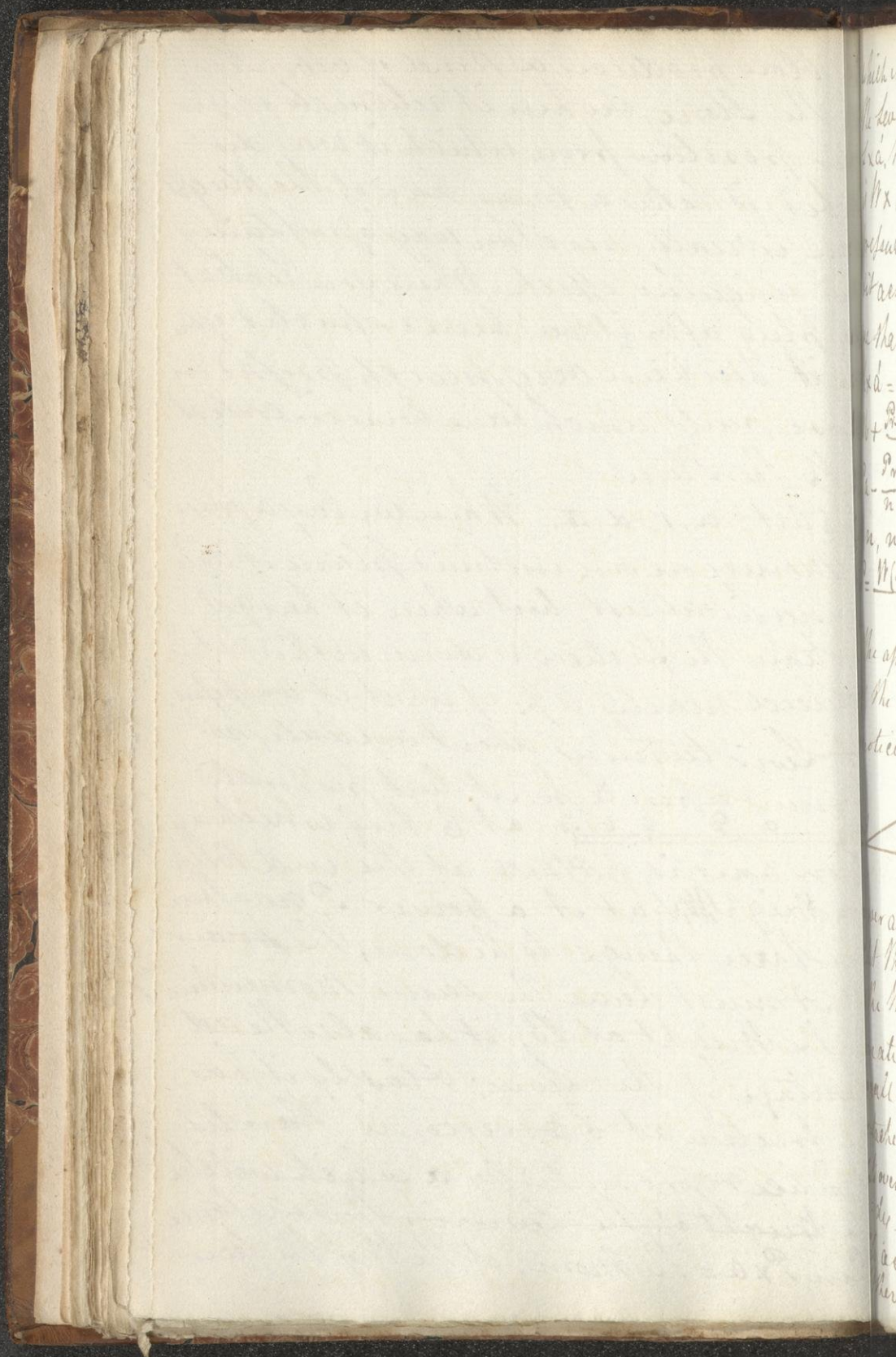


way would not be so Economical of
his Strength - (6) a Phenomenon still
more paradoxical, to draw a heavy
stone out of the ground, to rise it on a
carriage; a hole is bored in the stone
to the depth of one inch & $\frac{1}{2}$. Then an
Iron Plug is driven into this. It goes
into it $\frac{1}{2}$ an inch, then is fixed to it a
hook which is made fast to some Ma-
chine above, the working ag^t the pulley,
you apply a great force, which will be
able to draw up not the plug, but the
whole stone with it; when this way
first tried it were supposed that the
plug was forced in obliquely, & thus
draw it out, but this is not the case,
it is driven in quite direct ag^t the
centre of Gravity; I witnessed this operation
once on the Lands of Leith, when it
was conducted upon a very large
scale, & it answered completely; when
you have once drawn the stone out
& got it placed on the carriage, you
have only to give the stone a few later-
al strokes, very lightly, & the plug will
come out - The reason is the Friction
takes so strong a hold of it as to be able
to bring the stone out along with it



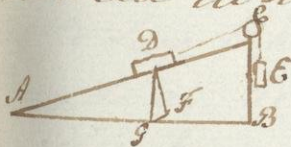
19th
The blow produced a kind of vibration
in the Stone, when it returns to its for-
mer position from which it was dis-
turbed, it takes a firm hold of the plug &
there is really no other way of explaining
this singular effect. When you look at
the plug after it has been extracted you
will find it has been very much pressed in
those parts which have been in contact
with the Stone.

165 Art. a. 1. d. IV. When we lay a piece
of money on an inclined plane it will
remain at rest, but when it has got
motion the friction is diminished & it is
reduced nearly to $\frac{1}{4}$ of what it was before.
A lever turning about an axis, not
turning upon a point but interfered
a D C B at C thro' which an
Iron axis is pressed at the end B there
is a weight W at A a power P now then
are three things to be done, The power
at A must have the same Momentum
as the weight at B; it has also the ad-
vantage of the lever; & lastly it has
the friction at C to overcome. Now let
AC be represented by a which will be
the weight of the lever in that distance.
then $P \times a =$ The Mom: of the W of the lever

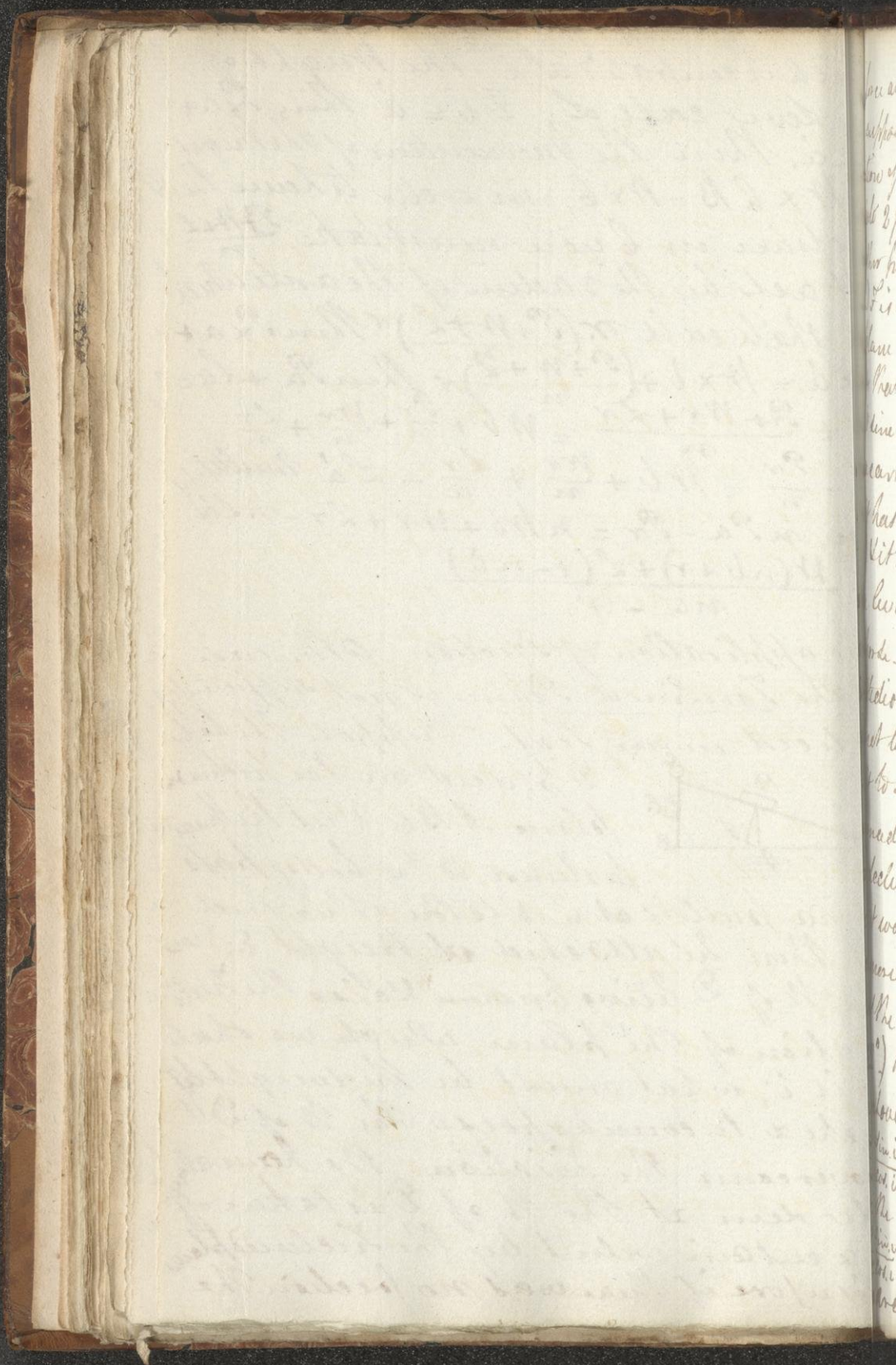


which is centralized at D the weight of
 the lever call L , $DB = a$ then $P \times a +$
 $L \times a$. Then the momentum of the weight
 is $W \times b = W \times b$, in order to have the
 pressure in b you must take $\frac{P+W+L}{n}$
 it acts by the radius of the axle which
 we shall call r . $(\frac{P+W+L}{n}) \times r$ then $P \times a +$
 $L \times a = W \times b + (\frac{P+W+L}{n}) \times r$ then $P \times a + L \times a =$
 $Wb + \frac{Pr + Wr + Lr}{n} = Wb + \frac{Pr}{n} + \frac{Wr}{n} + \frac{Lr}{n}$
 $P \times a - \frac{Pr}{n} = Wb + \frac{Wr}{n} + \frac{Lr}{n} - L \times a$ multiply
 by n , $nPa - Pr = nWb + Wr + Lr - nLa$
 $P = \frac{W(nb + r) + L(r - na)}{na - r}$

The application of Friction to the case
 of the Inclined Plane is not sufficiently
 noticed in our Text. Suppose the body

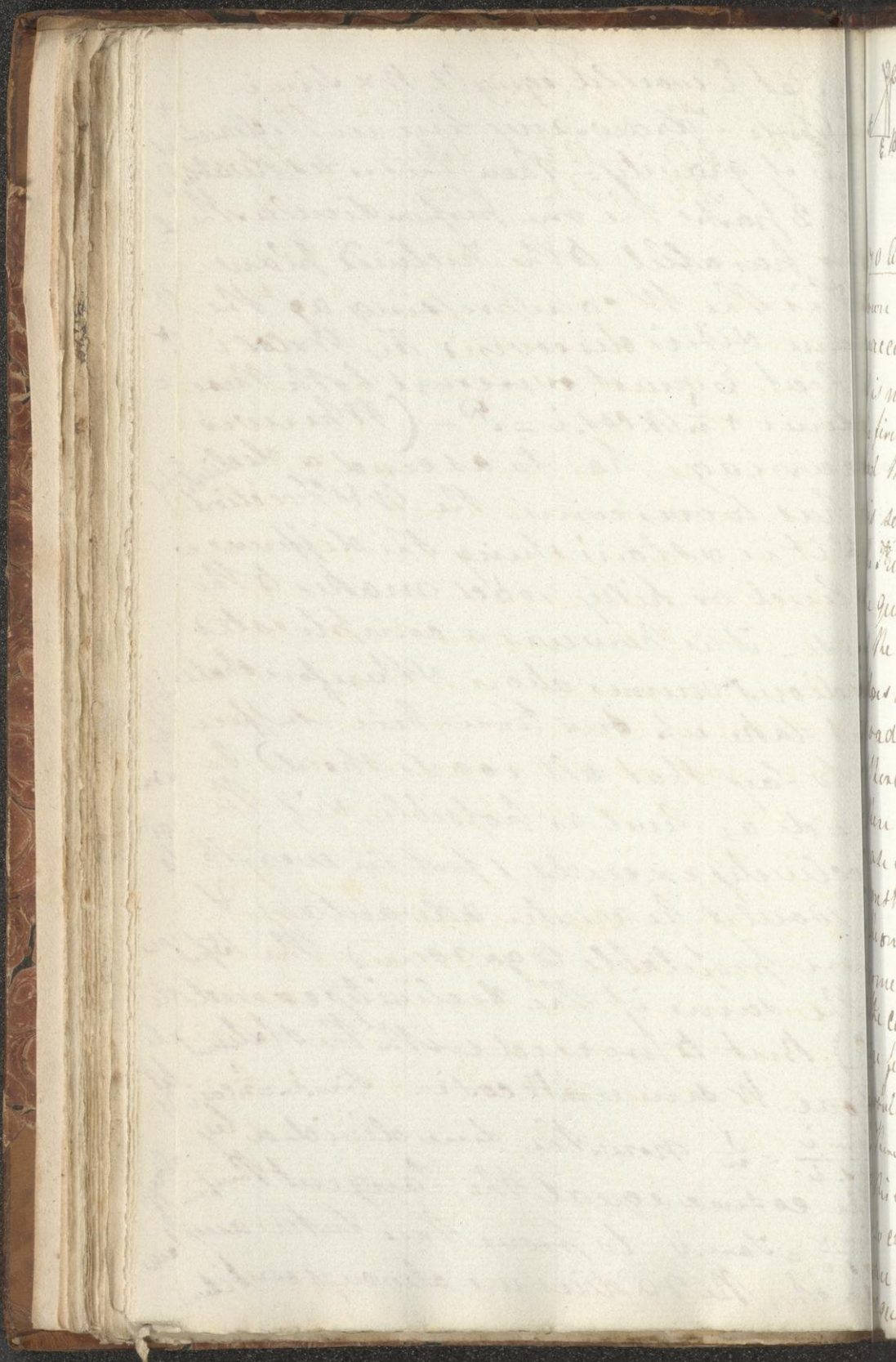


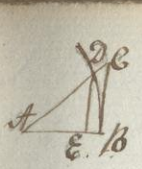
I placed on the inclined
 plane ABC . Let the string
 fastened to the body pass
 over a pulley at C . Let the other end
 let there be attached a weight E . Now
 the W of D being known & also the incli-
 nation of the plane, which we shall
 call i , what must be the weight of
 taken to counterpoise the W of D
 to overcome the Friction. We have already
 seen that the W of D is taken off
 to a certain extent by the inclined plane.
 Therefore if there was no friction the



(193)

force at E would equal $W \times \sin i$ -
suppose. "draw any line in the direction
of gravity. Then this is resolvable
into 2 parts the one perpendicular to the
plane parallel to the inclined plane.
D F is the W compressing ag^t the
plane & it is discovered by $W \cos i$.
so that E must overcome both these
 $W \sin i + \frac{1}{n} W \cos i = P$ - (Wherever
a carriage has to ascend a declivity
it has to overcome the W & friction
- It is astonishing the difference
a level or hilly road makes to the
horse - it is however a complicated
 tedious summation & therefore shall
not take up our time here, suffice
it to say that all roads should be
made as level as possible, if the
declivity exceeds 1 foot in every 15
it would be greater advantage &
more profitable to go round the 15 ft
& the same if the declivity exceeds
2") But to proceed with the statement
above $W \sin i = \frac{1}{n} W \cos i$. $\sin i = \frac{1}{n} \cos i$
 $\frac{\sin i}{\cos i} = \frac{1}{n}$ now the sine divided by
the cosine equal the Tangent therefore
 $\frac{\sin i}{\cos i} = \tan i$ to prove this take any
arch, the radius we always consider =



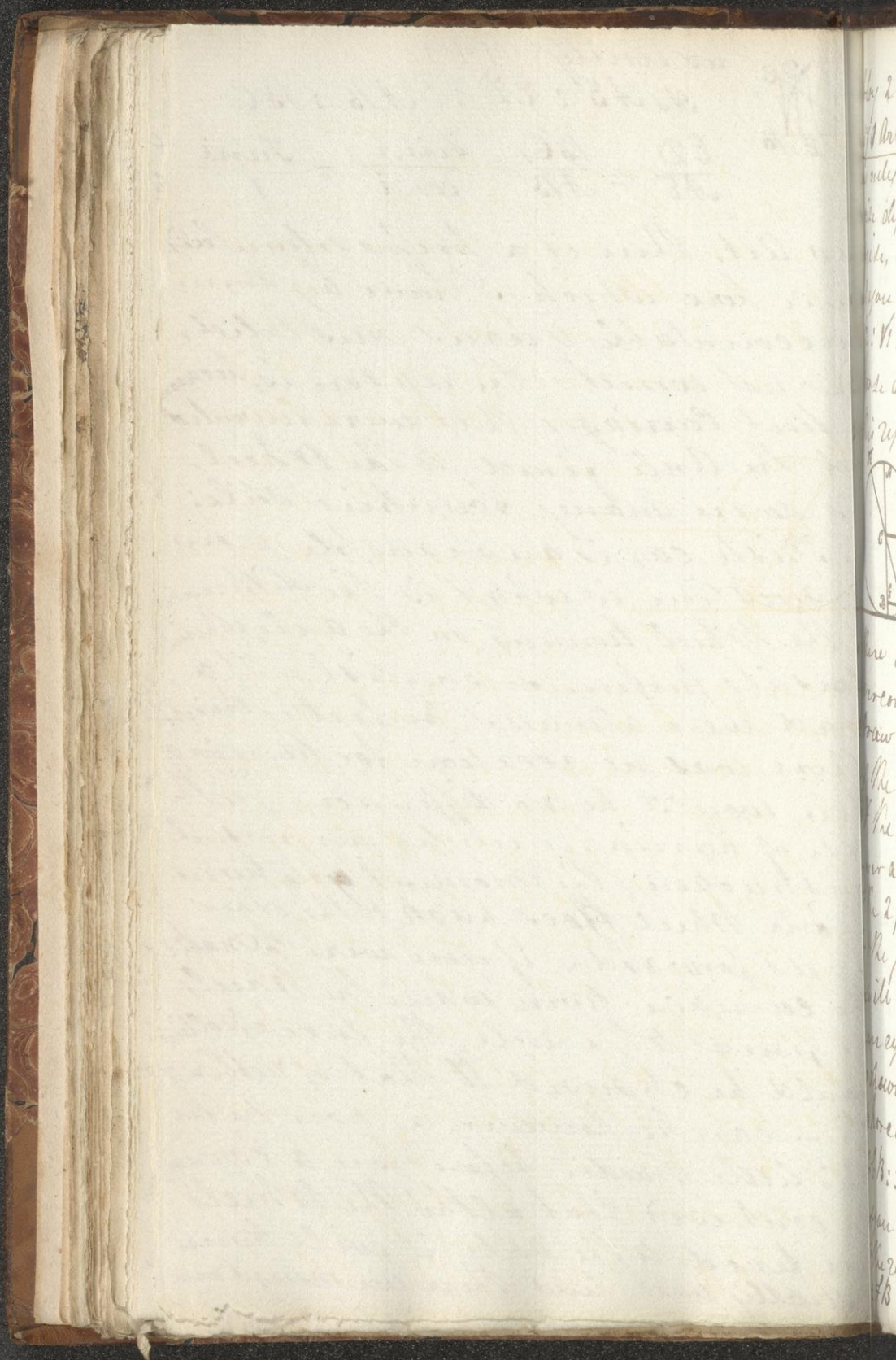


as unity (194)

$$As AE : ED :: AB : BE$$

$$\frac{ED}{AE} = \frac{BE}{AB} = \frac{\sin. i}{\cos. i} = \frac{\tan i}{1}$$

170 Art. There is a proposition laid down here which I have by some unaccountable means mislaid; it is not correct. The reason is, because the first Carriages that were invented had the axle fixed to the wheel, as so in many countries still; the Irish car is an example; now the question is what is the difference of the wheel turning on the axle, & when does its preference consist - If a road were always perfectly straight & there was no occasion for turning then would be no difference - In the case of Carriages under our present construction the moment you turn the one wheel goes back & the other comes forward - if you were to make the carriage turn when the wheels are fixed to the axle, the force of rolling would be changed to that of rubbing. You can be however as you see in this little model before you a Carriage so contrived that altho' the wheels are fixed to the axle it will turn equally well, but then we must en-





(196)

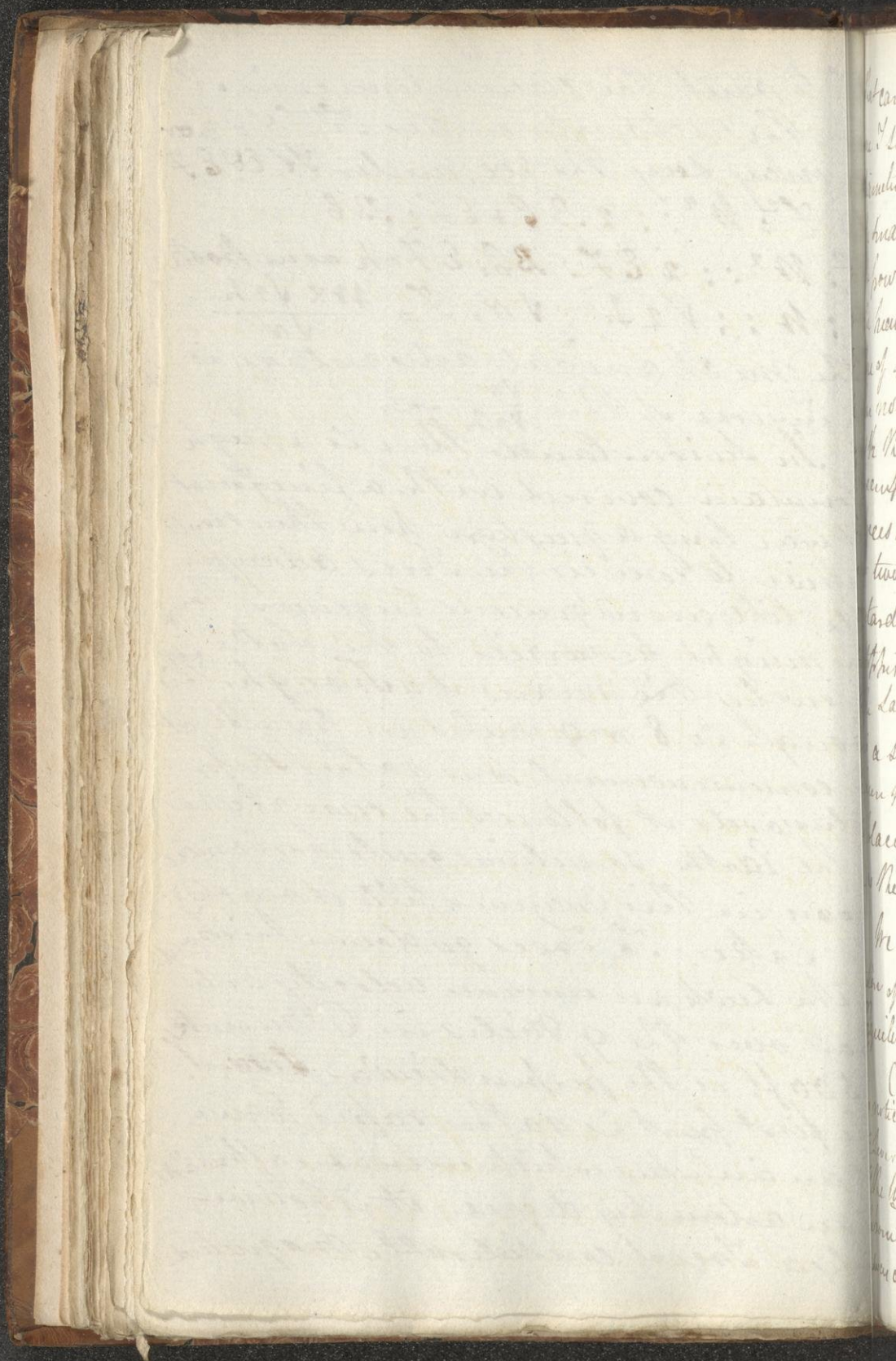
EF to meet the circumference in R
then the rectangle under RF & FE or
we may say the rec. under H & EF
the $F^2:W^2::2.BC+EF:BC$

$F^2:W^2::2.EF:BC$, $EF=h$ as in Book

$F:W::\sqrt{2h}:\sqrt{r}$, $F=\frac{W \times \sqrt{2h}}{\sqrt{r}}$

The mechanical advantage is
the inverse of this $\frac{\sqrt{r}}{\sqrt{2h}}$

In Switzerland there is a high
mountain covered with a pine forest
it was long a question how they should
contrive to convey the trees down for
sale, till one ingenious Engineer thought
they might be carried to the Lake
below by the means of a trough. This
trough is 8 or 9 miles in length, at
the commencement it is rather steep,
afterwards it follows the curvature
of the Earth, sometimes quite horizontal
& soon in this manner till it reaches
the Lake. The trees go down this trough
with such an immense velocity as to
pass over the 9 miles in 6 minutes,
2500 ft is the perpendicular descent,
the first part is rather rapid, to give
it an impetus which increases afterwards
to an astonishing degree; it is however
only a tree of considerable magnitude.



(197)

That can go at this astonishing rate, the one I saw was 100 ft long, 4 feet the diameter of its greater extremity & 2 at the smaller. This then goes to prove that the power friction acts much less on the heavy than on the light body, & a Tree of 452 cubical feet as the above has not have so much friction as one of the same number. I saw this well exemplified in the Trough. One of Trees in the desert happened to break in two, the velocity of both was evidently retarded, the one first soon after stopped & the other could scarcely reach the Lake - The Launching of a Ship is a similar circumstance - it has been presumed that an effect takes place viz. that the friction diminishes as the velocity increases -

We are now to consider the application of the Mech: powers to produce an Equilibrium -

There several Instruments were noticed, particularly applications of the Lever as the Machine for weighing Hay, the Universal Joint & the Hook's Joint from the Inventor Gen. the Fly Wheel was also exemplified by Models (etc) & used

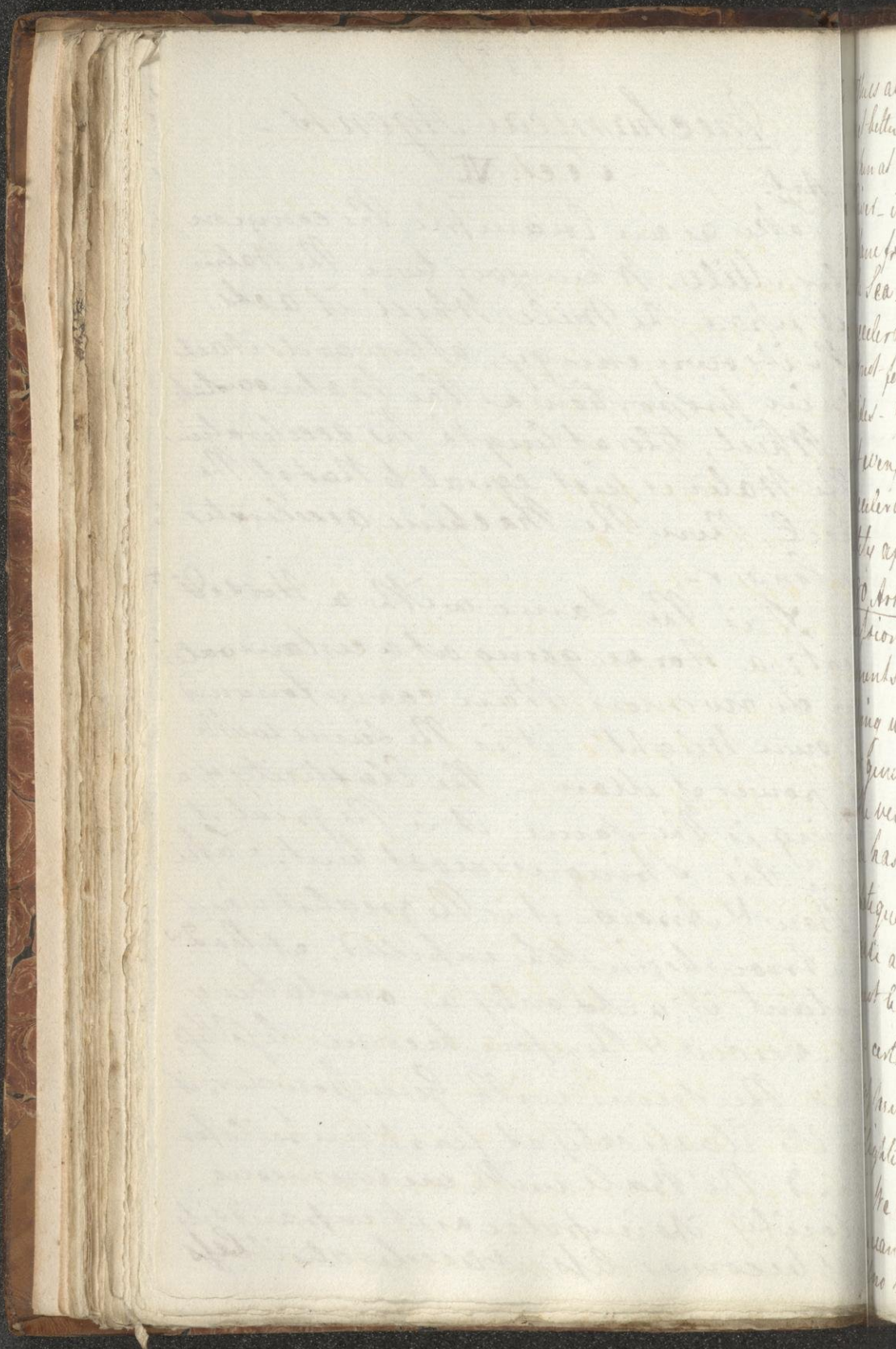
* The resistance of the air and the Sun
Seem —

Mechanical Agents -Sect. VI

177. Ant.

Take as an Example the common Water Mill. When you turn the Water first upon the Mill Wheel it acts with its own energy, afterwards it acts only in proportion as the Water overtakes the Wheel, till at length the acceleration of the Water is just equal to that of the Wheel, then the Machine accelerates no longer.

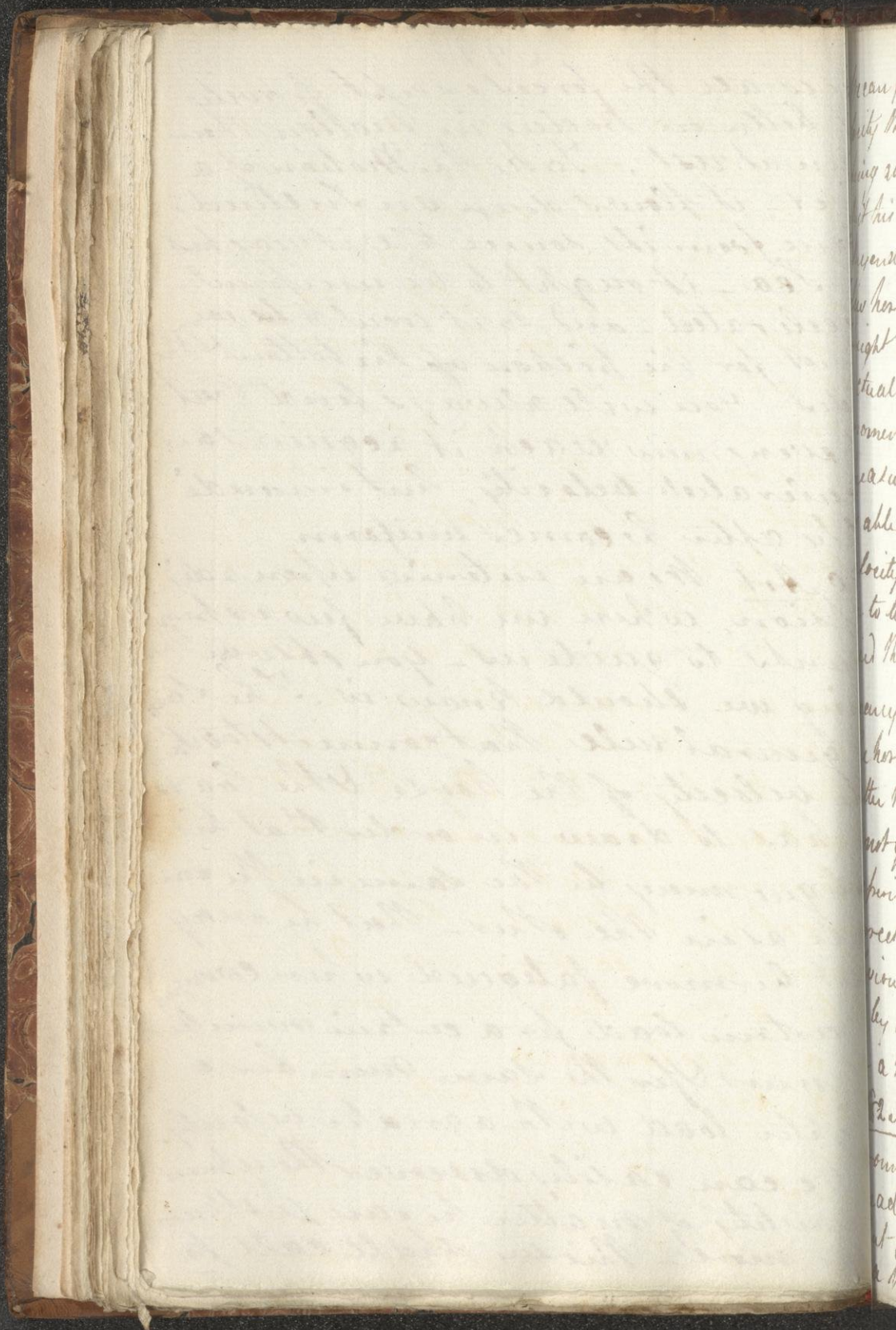
It is the same with a Horse & Cart; a Horse going at a certain rate can do no more than carry forward his own weight - it is the same with the power of Man - the Elasticity of a Spring is the same; it is the greatest when the Spring is most bent. Take a Bow & Arrow - it is the greatest when the Arrow begins to be expelled, at the 2^d instant it acts only as overtaking the Arrow & therefore becomes less. It is the same with Gun powder; it by its Elasticity at first impels forward the Ball with an enormous velocity, its impulse as it expands itself becomes less, it accelerates itself



Thus all the forces except Gravity act better on Bodies in Motion than when at rest. Take the Motion of a River - it flows down an Inclined Plane from its source till it reaches the Sea - it ought to be uniformly accelerated - and so it would be were it not for the friction of the bottom & the sides - You will always find that at every new reach it acquires an accelerated velocity, but immediately after becomes uniform.

180. Art. We are entering upon a discussion, where we have few experiments to guide us - You observe the thing we should know is - The Law or General rule that connects together the velocity of the horse & the load he has to draw - in order that his fatigue may be the same in the one case as in the other - that he may not be more fatigued when carrying a certain load for a certain number of hours than when carrying a lighter load with a greater velocity.

We can easily discover the utmost quantity of matter he can just move & no more - This we shall call p.



We can find out also I know the ve-
 locity that a horse can go without
 being able to carry forward any weight
 but his own. This we shall call C
 Beyond ten miles an hour very
 few horses could be able to carry any
 weight $C = 10$. v is the variable or
 actual velocity he has at a certain
 moment of time. The resistance is
 measured by W it is the weight he
 is able to put in motion with the
 velocity v . We may conceive the Formu-
 la to be this $W = p(1 - \frac{v}{C})$. When Euler
 tried this he found it did not answer
 many of the conditions of the power of
 the horse he substituted what answered
 better $W = p(1 - \frac{v}{C})^2$. By this Formula
 if not quite correct, we can still by
 experiments with great precision
 correct it & I should think the most
 obvious way to try it would be to try
 it by a number of men carrying each
 in a wheelbarrow

182. Art. I shall take the Formula
 grounded on experiments made by
 made by Desaguliers. He found
 that a horse drawing a weight out
 of a well by a pulley can raise

[Faint, illegible handwriting in cursive script, likely bleed-through from the reverse side of the page.]

[Marginalia on the right edge of the page, including numbers and fragments of text.]

1	450
2	338
3	242
4	200
5	162
6	98
7	50
8	6
9	2
10	0

200 lb above: for 8 hours together going at the rate of $2\frac{1}{2}$ miles an hour. We must now estimate what he can perform in 1 or 2 hours but must always take the labour of the day. Then $W = 450 \times (1 - \frac{v}{7\frac{1}{2}})^2$ we may make it more simple thus let us multiply the Numerator & denominator by 2, $1 - \frac{2v}{15}$ then $\frac{15-2v}{15}$
 $W = 450 (\frac{15-2v}{15})^2$ $W = \frac{450}{(15)^2} (15-2v)^2$ then
 $W = 2(15-2v)^2$ which gives us W expressed in lb. Let us give 6 different values

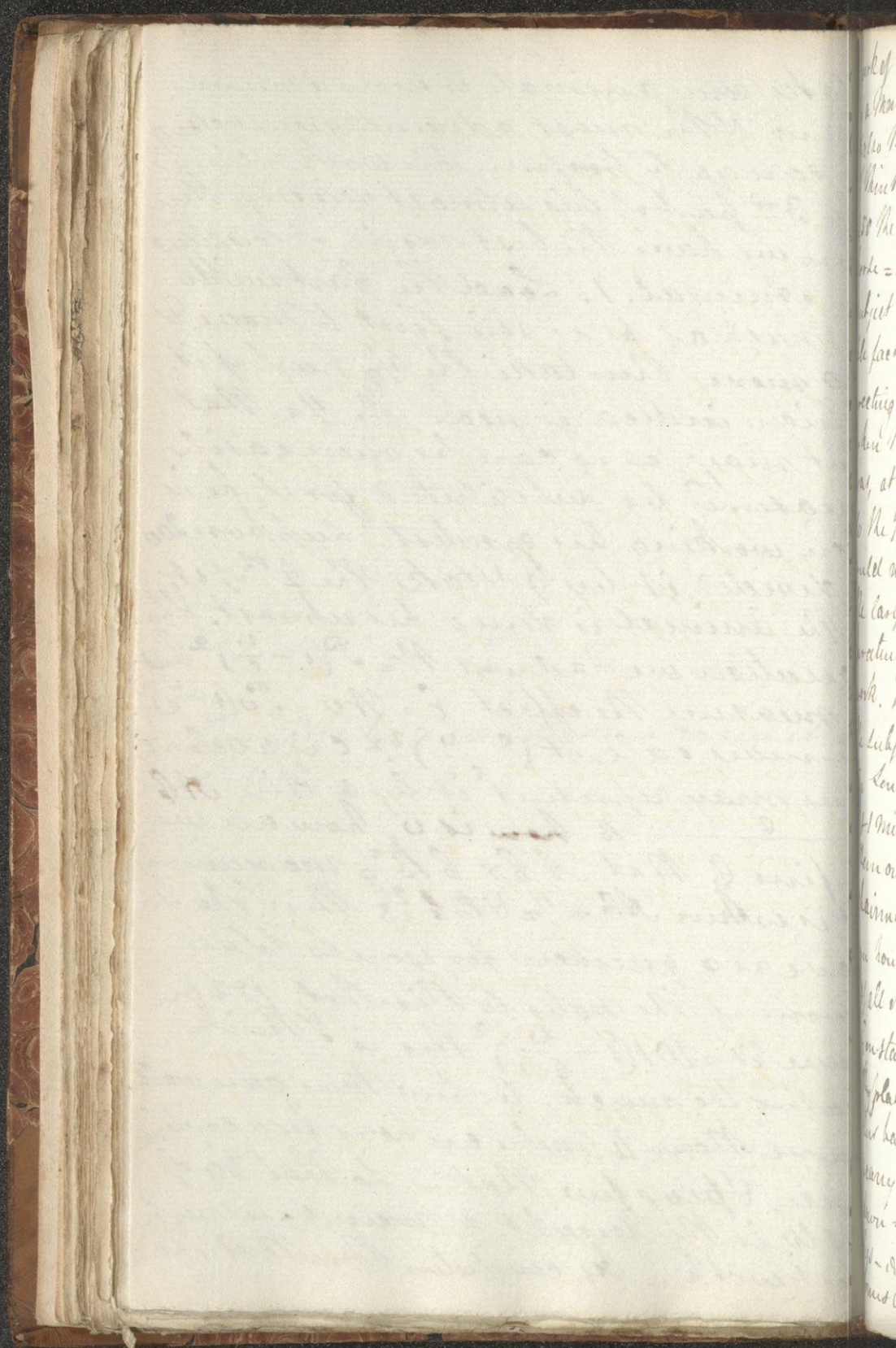
$\frac{v}{15}$	W	$\mathcal{E}$
0	450	0
1	338	338
2	242	484
$2\frac{1}{2}$	200	500
3	162	486
4	98	397
5	50	250
6	6	36
7	2	14
$7\frac{1}{2}$	0	0

see the diagram. We suppose he has no velocity then of course he will have the whole weight 450 W & there can be no effect $\mathcal{E}$. Then let us suppose he goes with a velocity equal to 1 mile an hour, what will be the weight he will be able

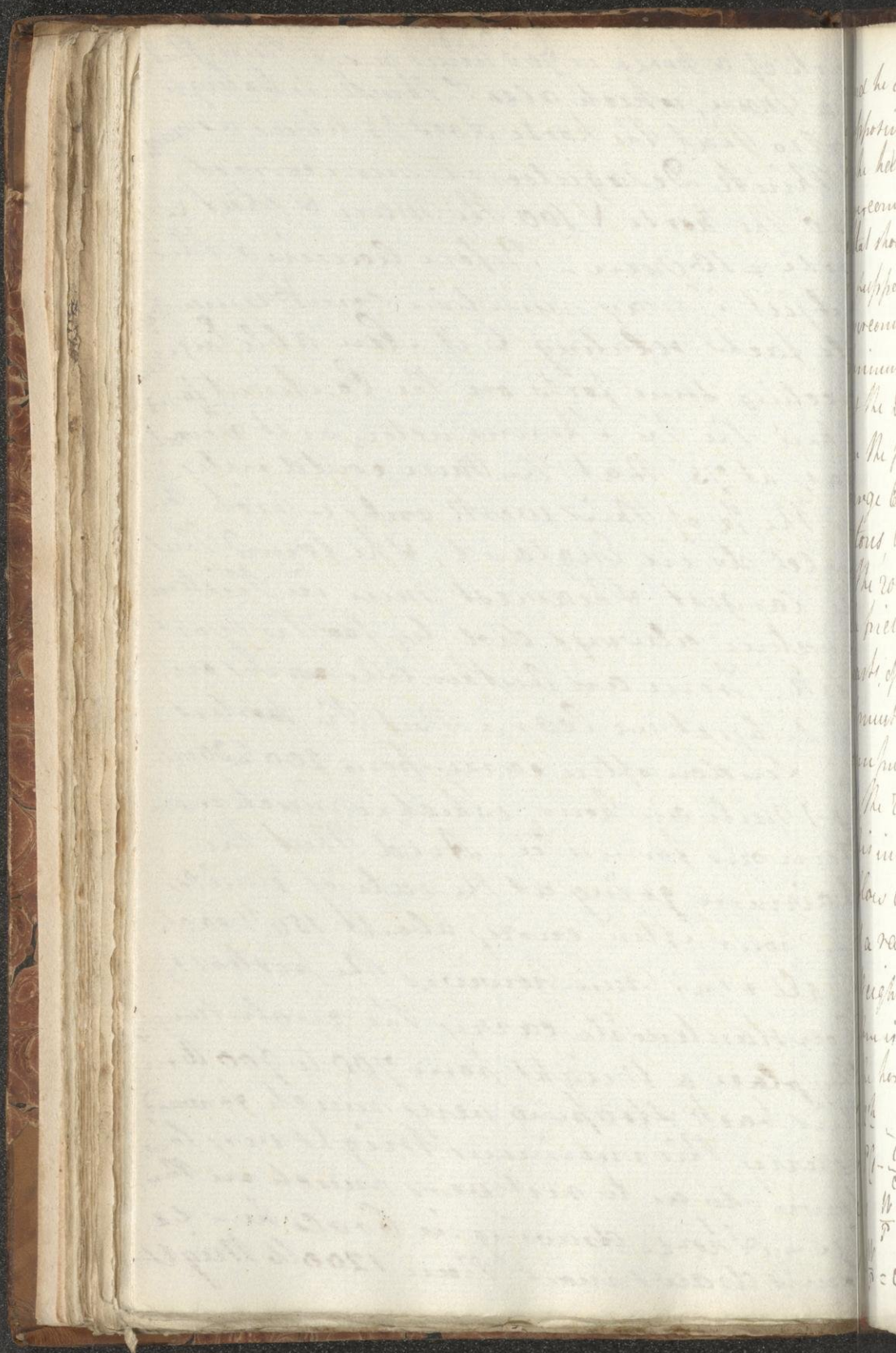
to carry, & the effect or work done. The 1×2 the velocity = $2 - 15 = 13$ raised to the square = $169 \times 2 = 338$ the weight. & the effect is found by $\times$ the weight into the velocity $1 \times 338 = 338$ the effect. In the 3rd place let us suppose the velocity = $2\frac{1}{2}$
 $2\frac{1}{2} \times 2 = 5 - 15 = 10$, $10^2 = 100 \times 2 = 200$ as above
 $200 \times 2\frac{1}{2} = 500 =$ the effect. & so on as the velocity increases the effect is diminished.

x Then will be a maximum when it =
 $v(c-v)^2$

Take any animal or even a human
 being & the most advantageous velocity
 he can go to perform his work well is
 the 3rd part of his utmost velocity. Now
 then we have the best mode of loading
 an animal. 1. Load the Cart with
 as much as he is able first to move &
 no more, then take the $\frac{4}{9}$ part of it,
 which indeed is near the $\frac{1}{2}$ that
 that may, as it can be more easily
 measured be substituted for it, he is
 then working his greatest. Suppose 200
 lb weight, it by $\frac{4}{9}$ take the $\frac{4}{9}$ of it,
 & the animal is doing his utmost. The
 Equation we had was $W = P(1 - \frac{v}{c})^2$ but
 to measure the effect $E = Wv = P(1 - \frac{v}{c})^2 v$
 we may call it $(\frac{c-v}{c})^2 \times c$ is a velocity
 we may represent it by a line AB
 $A \quad c \quad B$ how is v , how are we
 to find c that $AB \times c B^2 = \text{maximum}$,
 it is when $AB = \frac{1}{2}$ & $B = \frac{2}{3}$ This I shall
 leave as a question for you to solve
 In one of the notes to this Art 182 we
 have $W = 101(1 - \frac{v}{5.5})^2$ this is I think
 rather too much, for few men can walk
 more than 4 miles an hour for a continu-
 ance, & few men - so that $\frac{4}{9}$ of
 101 lb is the weight a man can weigh
 but with - be consider, that the



work of a horse is ⁽²⁰³³⁾ 700 times more than that
of a Man, which also I think is too high,
also that the horse does 14 times as much.
I think Desaguliers is more correct
450 the horse & 100 the man & that a
horse = 10 men. Before leaving this
subject I may mention some remarkable
facts relating to it. An able Engineer
erecting some forts on the Continent found
when he the the the character, as it generally
was, at 75, that the men could only
do the 1/2 of this work only, which they
could do in England, & he found that
the largest & heaviest men in this
nation always did by far the most
work. From an Author who writes on
the subject we learn that the porters
in London often carry from 200 to 300 lb
at 1 mile an hour, which is much more
than our formulae. And that the
chairmen going at the rate of 4 miles
an hour often carry about 150 lb each
of all their men however. The porters of
Constantinople carry the greatest weight.
They place a weight from 700 to 900 lb on
their back stooping very much forward
carry this enormous weight very low
down so as to rest very much on the
legs. A horse drawing in Coals here will
draw more than 1200 lb weight.



and he is only acting against 100 lb
supposing it pulled out of a Hole that
is he has only a $W = 100$ lb of a dead pull
overcoming the friction; This is a point
that should be observed. The great thing
is supported by the ground, he is only
overcoming the friction; in some ex-
periments which I assisted to perform
at the Duke of Bedford's in Bedfordshire
on the friction of Carriages - some
large Carts were loaded with 4 or
5 tons, & these were yoked on 4 horses,
& the road being very good & level,
the friction was found to amount to 24
parts of the whole weight; This however
I must remark is much too small a
computation or estimate, on account
of the roads seldom being equal to
this in this park, you must generally
allow a 12th part. The great advantage
of a rail way is the almost all the
weight is supported on the ground &
there is very little friction to overcome
the horse has only the 10th part of the
12th - Take the formula again $W =$
 $P(1 - \frac{c}{c})^2$ now W being given to find
 c $\frac{W}{P} = (1 - \frac{c}{c})^2 \sqrt{\frac{W}{P}} = 1 - \frac{c}{c} \times \log c$ and
 $c \sqrt{\frac{W}{P}} = c - c$ then $c = c - c \sqrt{\frac{W}{P}}$ or more
simply

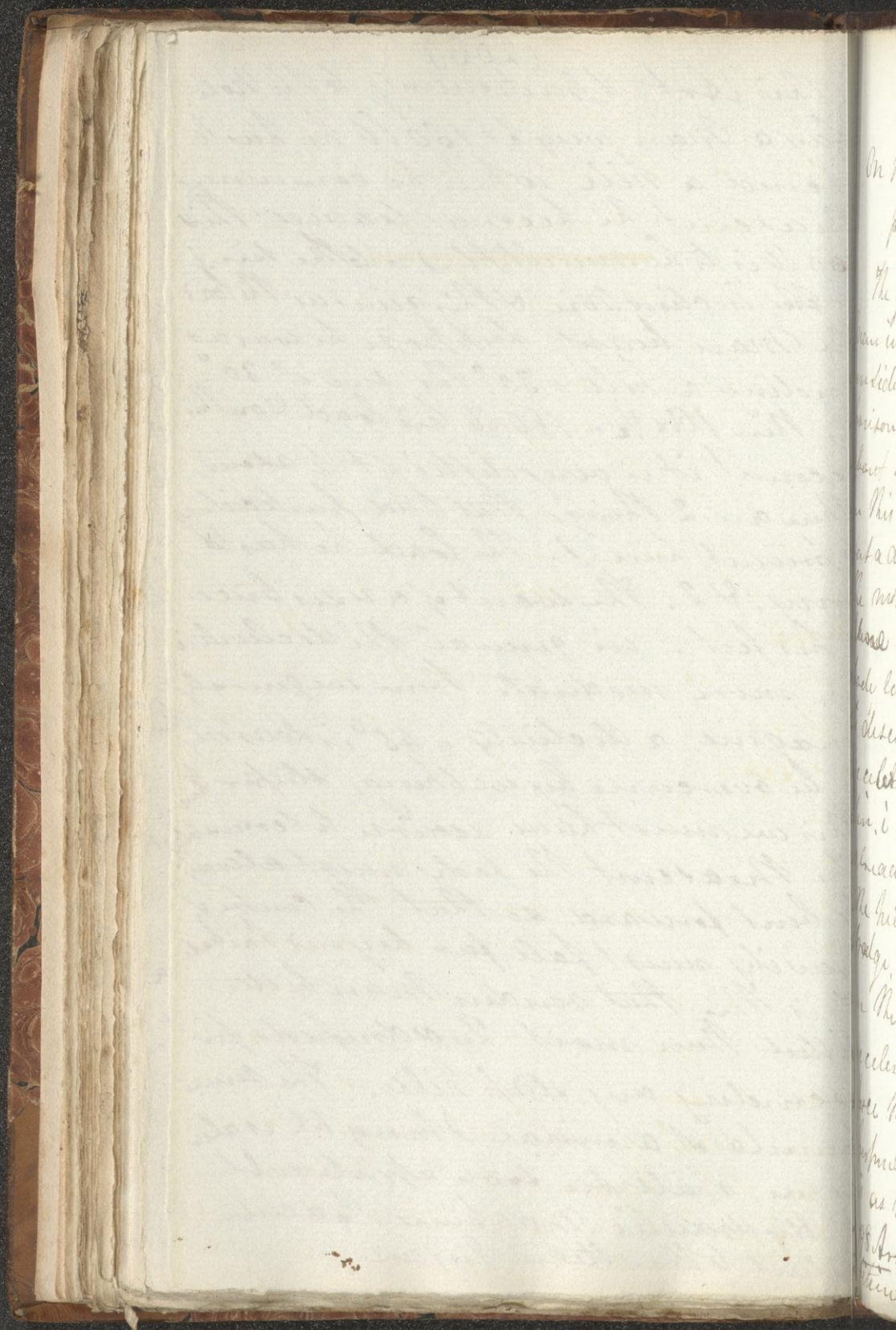
[illegible]

$v = c(1 - \sqrt{\frac{W}{P}})$, $W = P(1 - \frac{v}{c})^2$ is the formula for one horse, for 2 we have $W = 2 \cdot P(1 - \frac{v}{c})^2$ when v is given in both cases then the work done by 2 horses is the above doubled. If you go in a light Carriage with four horses, it has no advantage over an equally light with 2 except in going up hill, the carriage however must be very light, & no luggage. The experiment has been tried between this & London, the one with the 4 horses only arrived 2 hours before. There are various ways & attitudes in which man can do most work - such is the attitude of pulling a Pull, there seems no better than the rowing of a boat, a person can work in this situation almost longer than in any other, he has to assist him, the weight of his body, the muscular action, this feat of a pulcrum, Coulomb thought that ^{work} ~~can~~ be more advantageously done than by making a man work by his own weight & hardly know work done so well as the Coal heavers in London, which is done ^{almost} ~~completely~~ by the man's own weight.

183. Art - I wish to direct your attention

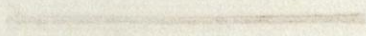
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to this Art - & particularly to the Note.
 Say a Man weigh 160 lb he has to
 ascend a hill, when he commences
 the ascent he becomes loaded; this
 160 lb is to be multiplied into the sine of
 the inclination & this gives us the load
 the Man bears - Suppose he was as-
 cending a hill = 30° the sine of 30°
 $\frac{1}{2}$ then $160 \times \frac{1}{2} = 180$ lb his load & on this
 account it is very difficult to ascend,
 there are 2 things that tend particularly
 to prevent him. 1. The load he has to
 carry, & 2. The want of a resistance
 to his feet - in general the declivities
 are more moderate than we generally
 imagine, a declivity = 45° is scarcely
 to be overcome by walking, steeper than
 this we must have recourse to descending.
 In the ascent the body must always
 be bent forward so that the centre of
 gravity must fall far beyond the feet.
 It is this that renders Man better
 fitted than most quadrupeds for
 ascending any steep hill. The same
 formula of animal strength is also
 as we shall see soon applicable
 to hydraulic machines & to a certain
 extent to the Steam Engine.



On the descent of heavy bodies on
plane & curved surfaces

The greatest power that is exerted by
 men in any subject we have already
 considered, is in mere nothing in com-
 parison to this uniform motion we are
 about to treat. We must on entering
 on this Problem proceed cautiously
 but at a distance we must begin with
 the motion of a body on an inclined
 plane See the 1st Art. 197 - Any
 body laid upon the plane has a tendency
 to descend let f be the force that
 accelerates it the $f : g$ the gravity ::
 $\sin. i : R$. Then $f = g \cdot \sin. i$ This has
 already been demon. strated under
 the head friction inclined planes
 &c. &c. Hence a body descending
 on this inclined plane is uniformly
 accelerated, but with a more feeble
 force than when falling down the
 perpendicular, & the space passed down
 is as the square of the times
 198 Art This is a mathematical propo-
 sition. The body falling from the D.



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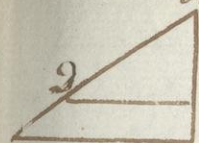
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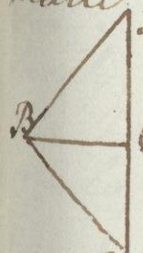
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From A to E will leave
when it arrives at D
an uniform velocity. This
 $v^2 = 2g \cdot S$ & $v^2 = 2g \cdot \sin i$
 $\times AD$ This is the square of the velocity
of the body descending on the
inclined plane; call the perpendicular
AE i . Then $v^2 = 2g \times AE = 2g \times AE \cdot \sin i$
 $= 2g \cdot \sin i \times AD$. now this is one of
the most remarkable propositions
of falling bodies -

199. Art - is another of very much
the same kind, we want now to
find the time of its descending
down the Inclined Plane. Suppose

I draw a perpendicular to the Inclined
Plane A



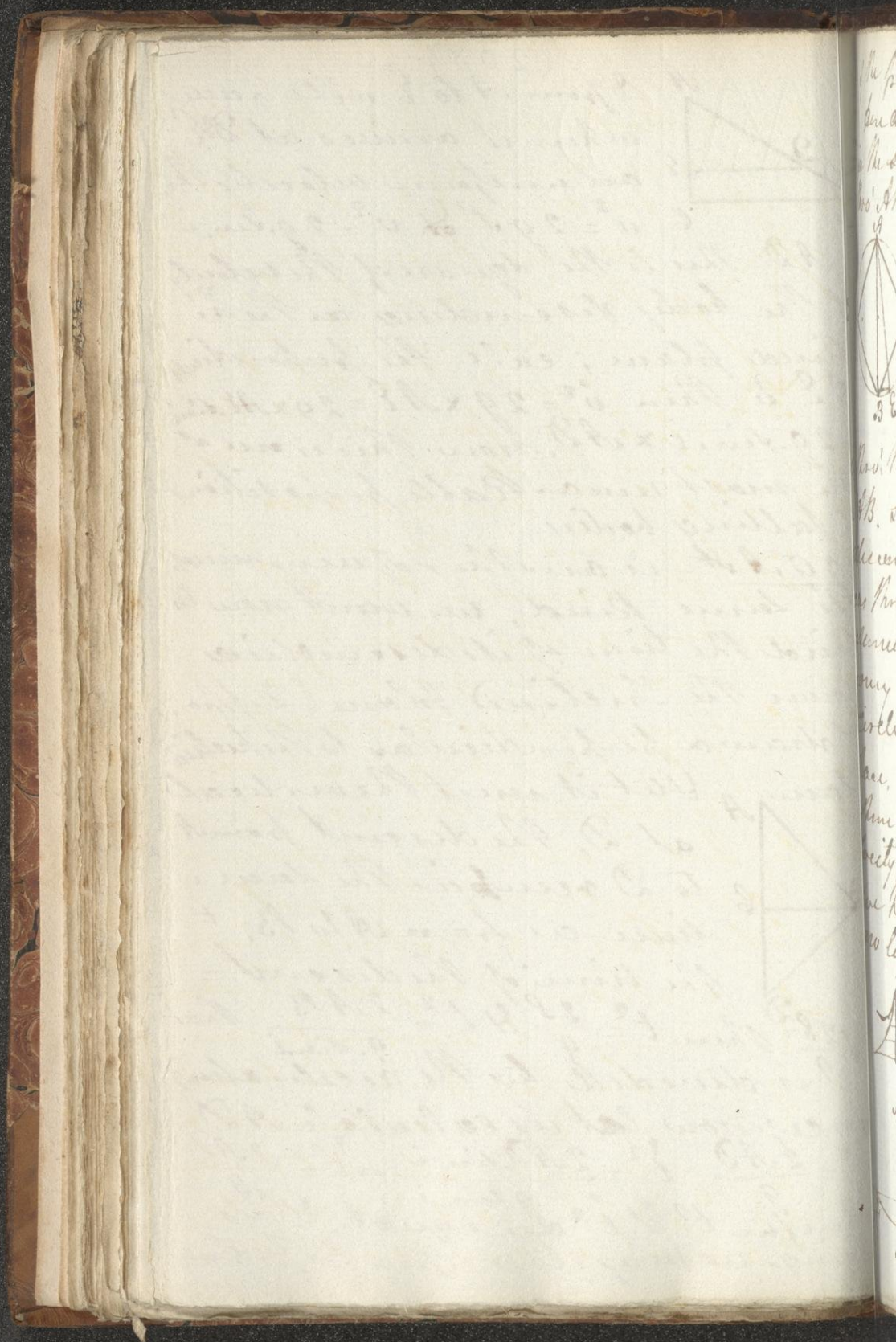
Let it meet the vertical
at D, the descent from A
to D occupies the same
time as from A to B; &
the time of the descent =

$$\sqrt{\frac{2S}{g}} \text{ then } t^2 = \frac{2S}{g} \text{ & } t^2 = \frac{2AB}{g \cdot \sin i} \text{ that}$$

is divided by the accelerating
force. now let us calculate AD

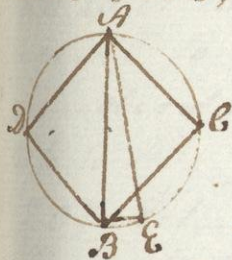
$$t^2 = \frac{2AD}{g}, \quad t^2 = \frac{2AD \cdot \sin i}{g \sin i} \text{ or } t^2 = \frac{2AD}{g}$$

Therefore t^2 & t^2 are equal, this
brings us very close upon the property



of the pendulum (209)

See a body will fall thro' CB in the same time another will fall thro' AB , for complete the rectangle



now we know it will fall

thro' AD from the last position in the same time

as it will thro' AB , but AD

= BC therefore it will fall

thro' BC in the same time it will

AB . So all the chords of a circle are

descended thro' in the same time

as thro' the diameter, thro' AE in the

same time as thro' AB &c & This is a

very remarkable property of the

circle if you have a crooked sur-

face, if there are angles at each of

them the body will lose a certain ve-

locity, but if instead of a poly gon

we have a curve there will be

no loss. This is a natural property

of all curved surfaces they must

however be uniformly curved.

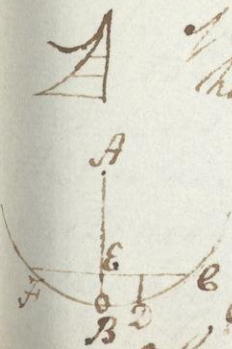
Suppose from the point

A there is suspended a

weight, take it to the height

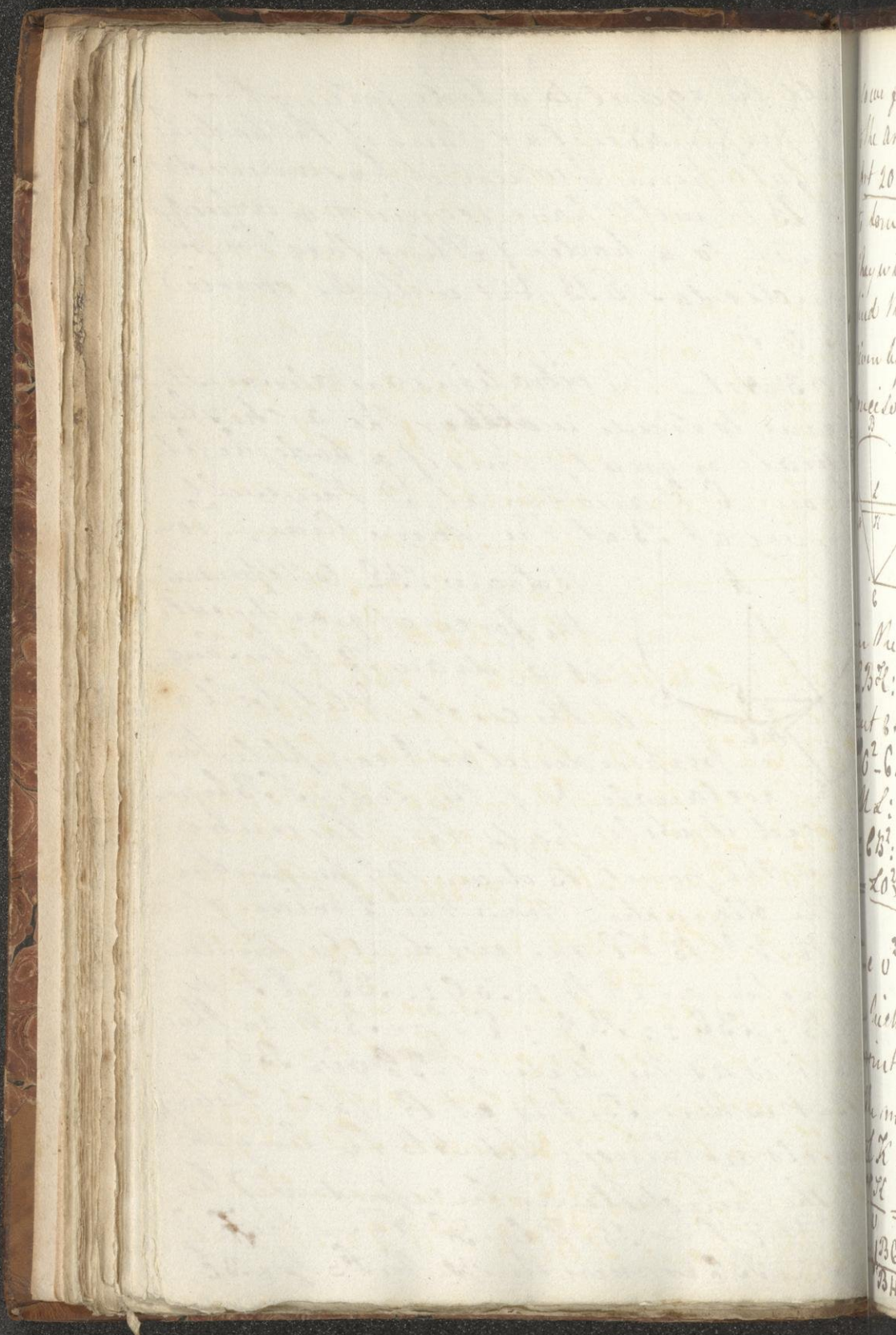
BC & let it fall & it will describe

a circle, at any point the velocity



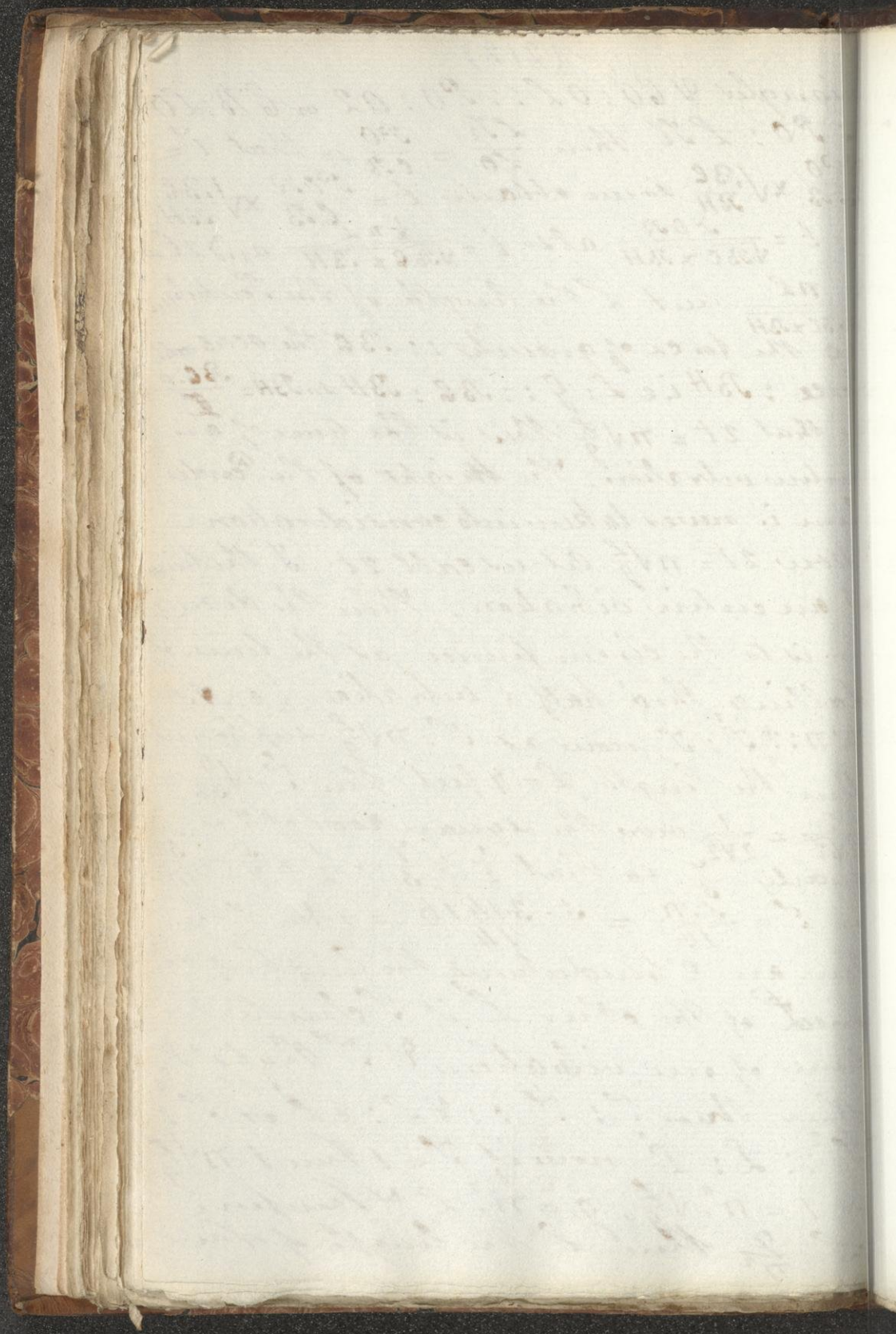


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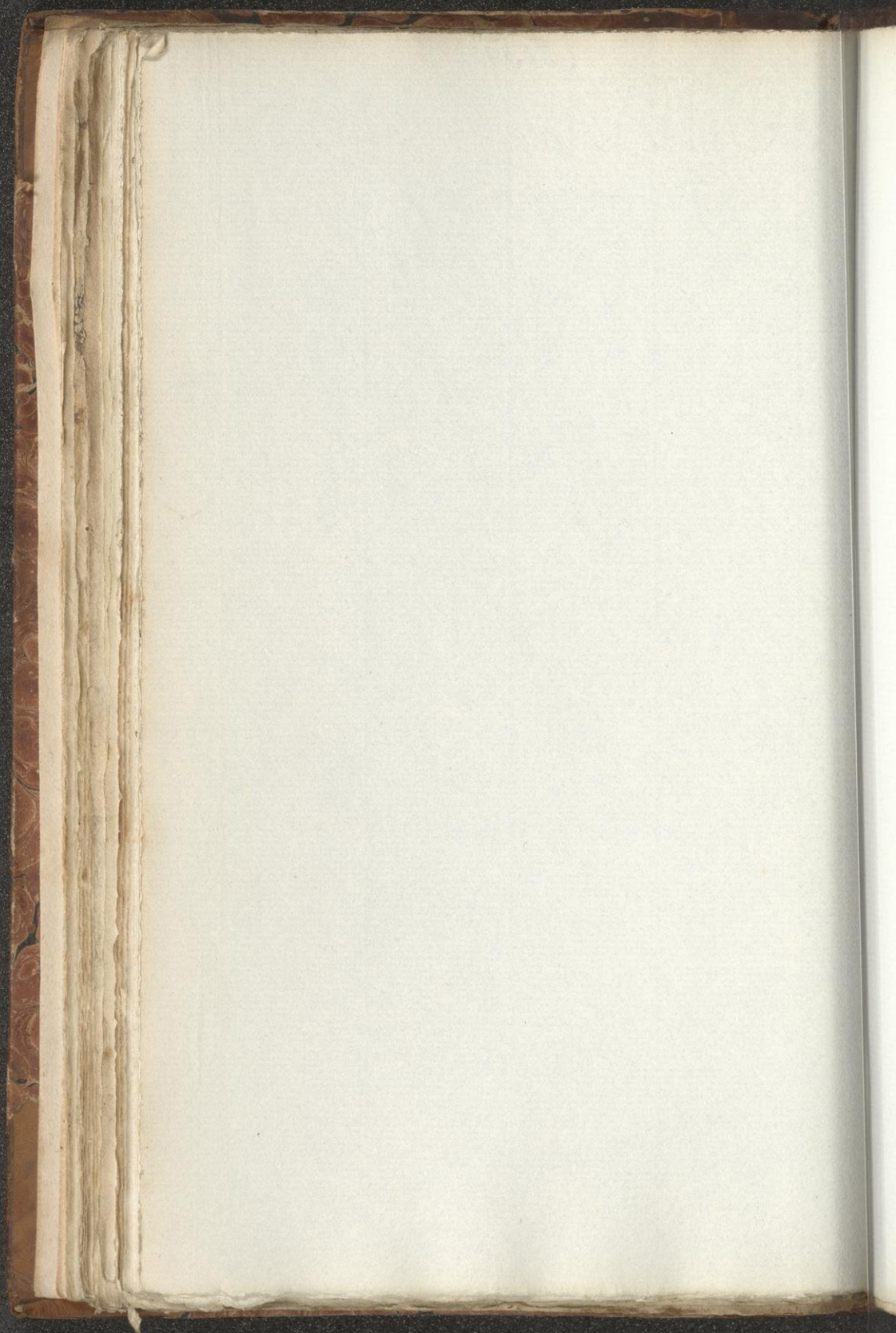


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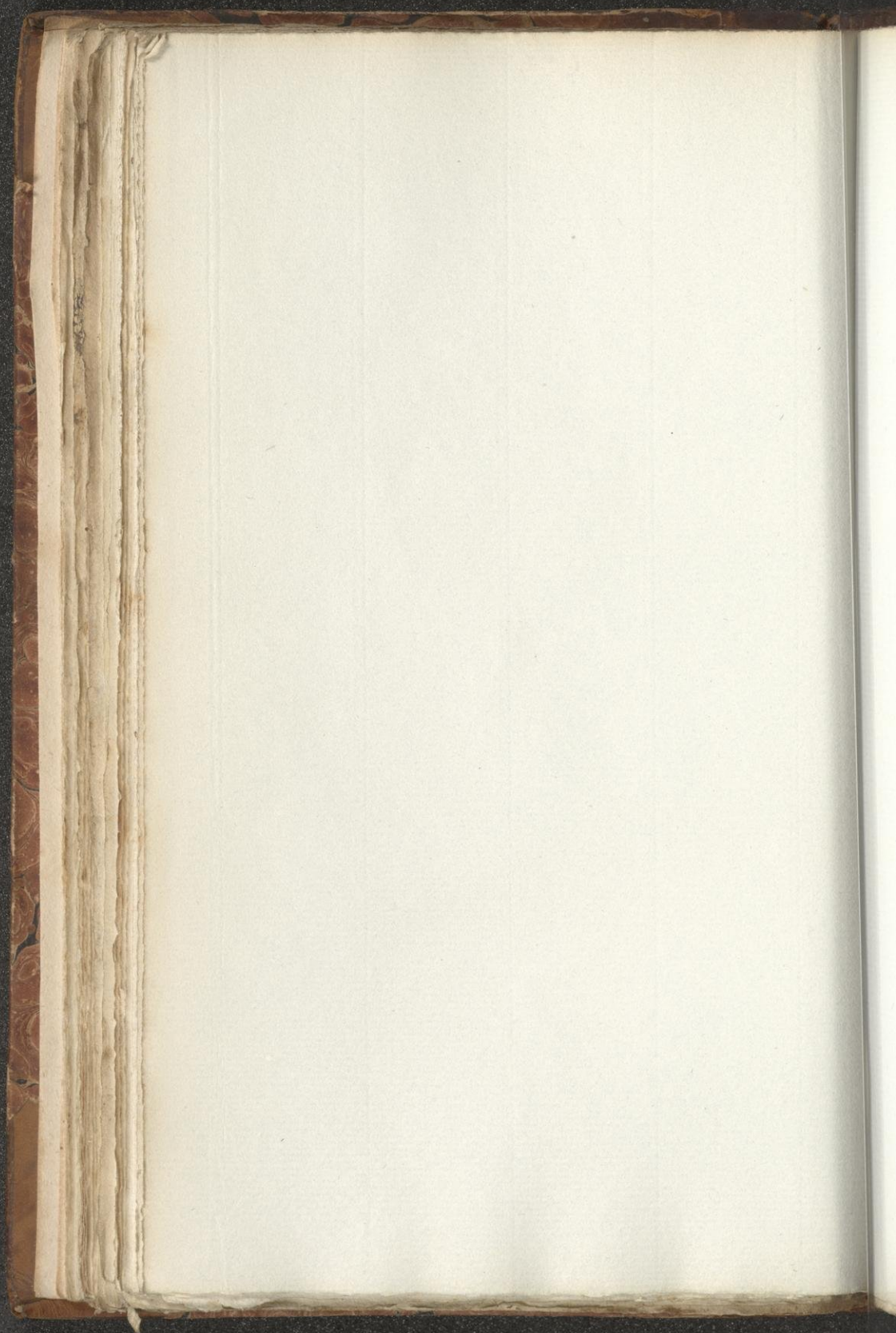
triangles $\triangle CO:OL::PO:OZ$ or $\triangle B:LO$
 $:: PO: LK$ then $\frac{LK}{LO} = \frac{PO}{LB}$ so that $t =$
 $\frac{PO}{LB} \times \sqrt{\frac{BC}{BH}}$ so we obtain $t = \frac{PO \cdot R}{LB} \times \sqrt{\frac{BC}{BH}}$
 or $t = \frac{PO \cdot R}{\sqrt{BC \times BH}}$ also $t = \frac{\frac{1}{2} \pi L}{\sqrt{BC \times BH}}$ and $2t =$
 $\frac{\pi L}{\sqrt{BC \times BH}}$ but L the length of the Pendulum
 $: g$ the force of gravity $:: BC$ the acceleration
 force $: BH$ i.e. $L: g :: BC: BH$ so $BH = \frac{BC \cdot g}{L}$
 so that $2t = \pi \sqrt{\frac{L}{g}}$ this is the time of an
 entire vibration. The weight of the Pendu-
 lum is never taken into consideration -
 Now $2t = \pi \sqrt{\frac{L}{g}}$ let us call $2t = T$ the time
 of an entire vibration. Then the diam-
 eter is to the circumference as the time of
 falling thro' half a vibration i.e. π
 $1: \pi :: T: T$ now as $T = \pi \sqrt{\frac{L}{g}}$ supposing
 then the length $L = 4$ feet then $T = \sqrt{\frac{4}{32}} =$
 $\frac{2}{4\sqrt{2}} = \frac{1}{2\sqrt{2}}$ now the square root of 2 is
 nearly $\frac{7}{5}$ so that $\frac{1}{2} \div \frac{7}{5}$ or $\frac{1}{2} \times \frac{5}{7} = \frac{5}{14}$
 or $T = \frac{5 \cdot \pi}{14} = \frac{5 \cdot 3.1416}{14} = 1.107$ Then
 there are 2 pendulums the length of the
 one L of the other L' if T denote the
 time of one vibration & T' that of the
 other then $T: T' :: \sqrt{L}: \sqrt{L'}$ or $T^2:$
 $T'^2 :: L: L'$ now if $T = 1$ then $1 = \pi \sqrt{\frac{L}{g}}$
 or $1 = \pi^2 \cdot \frac{L}{g}$, $g = \pi^2 \cdot L$ & therefore
 $L = \frac{g}{\pi^2}$ then L the length of the



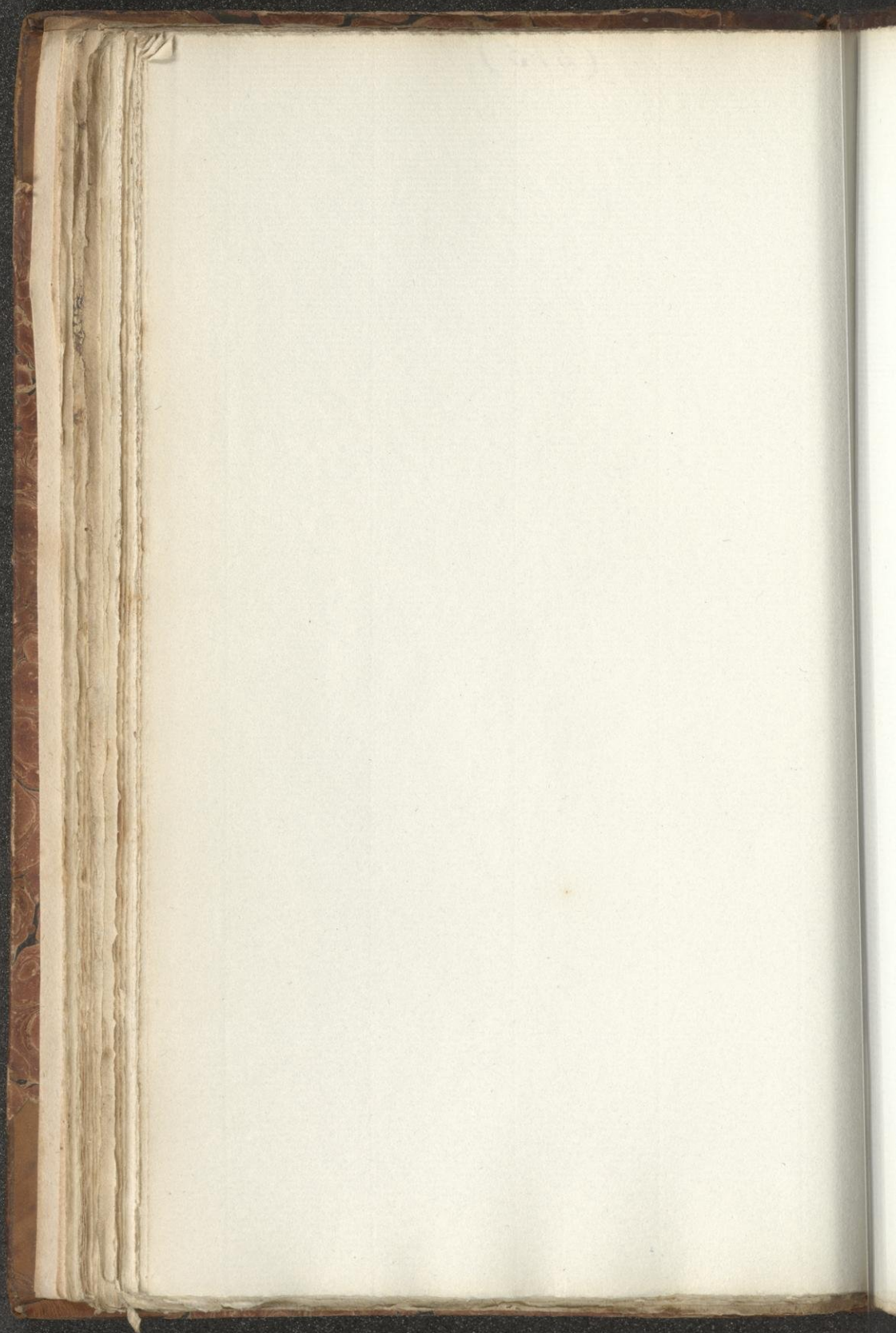
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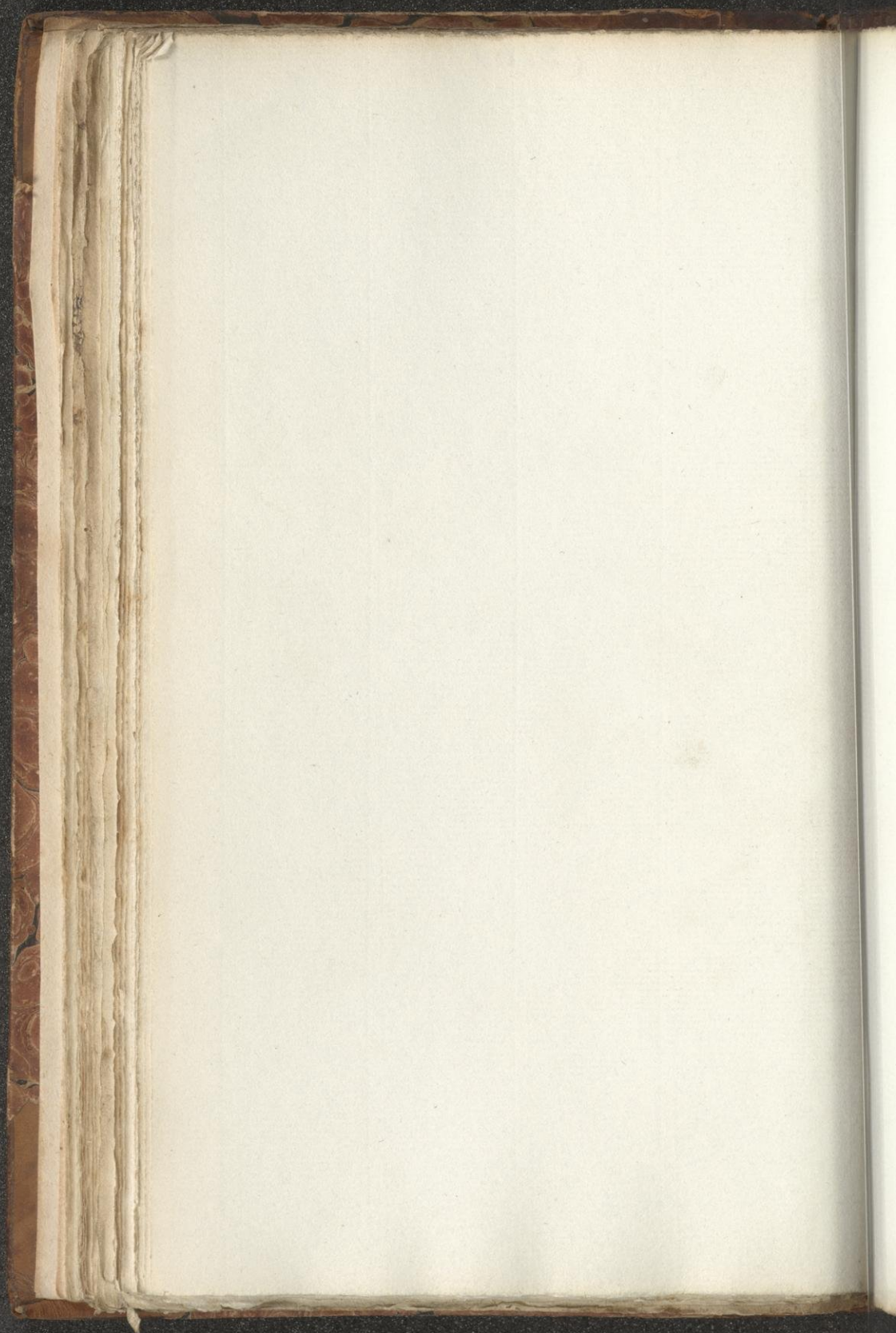
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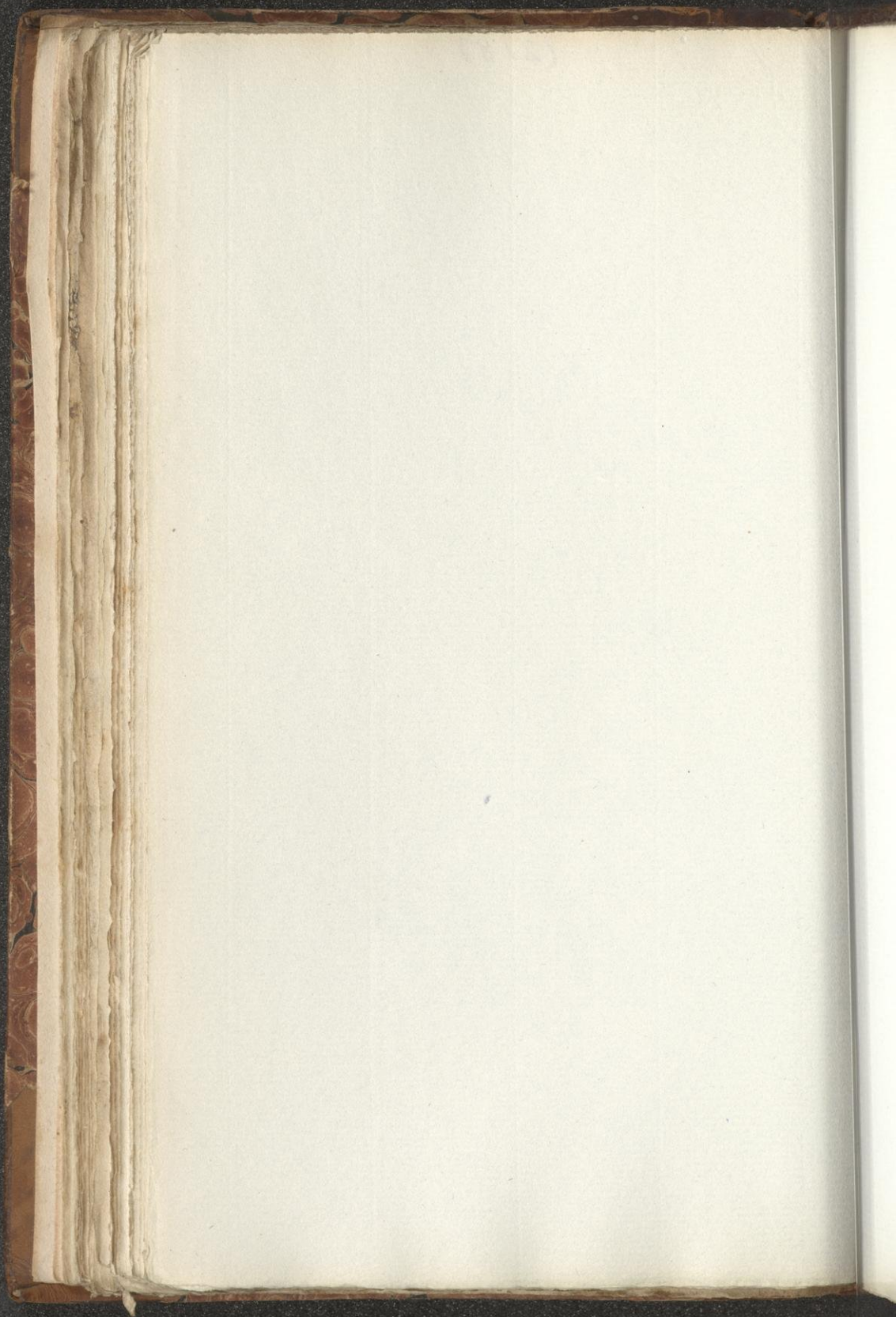
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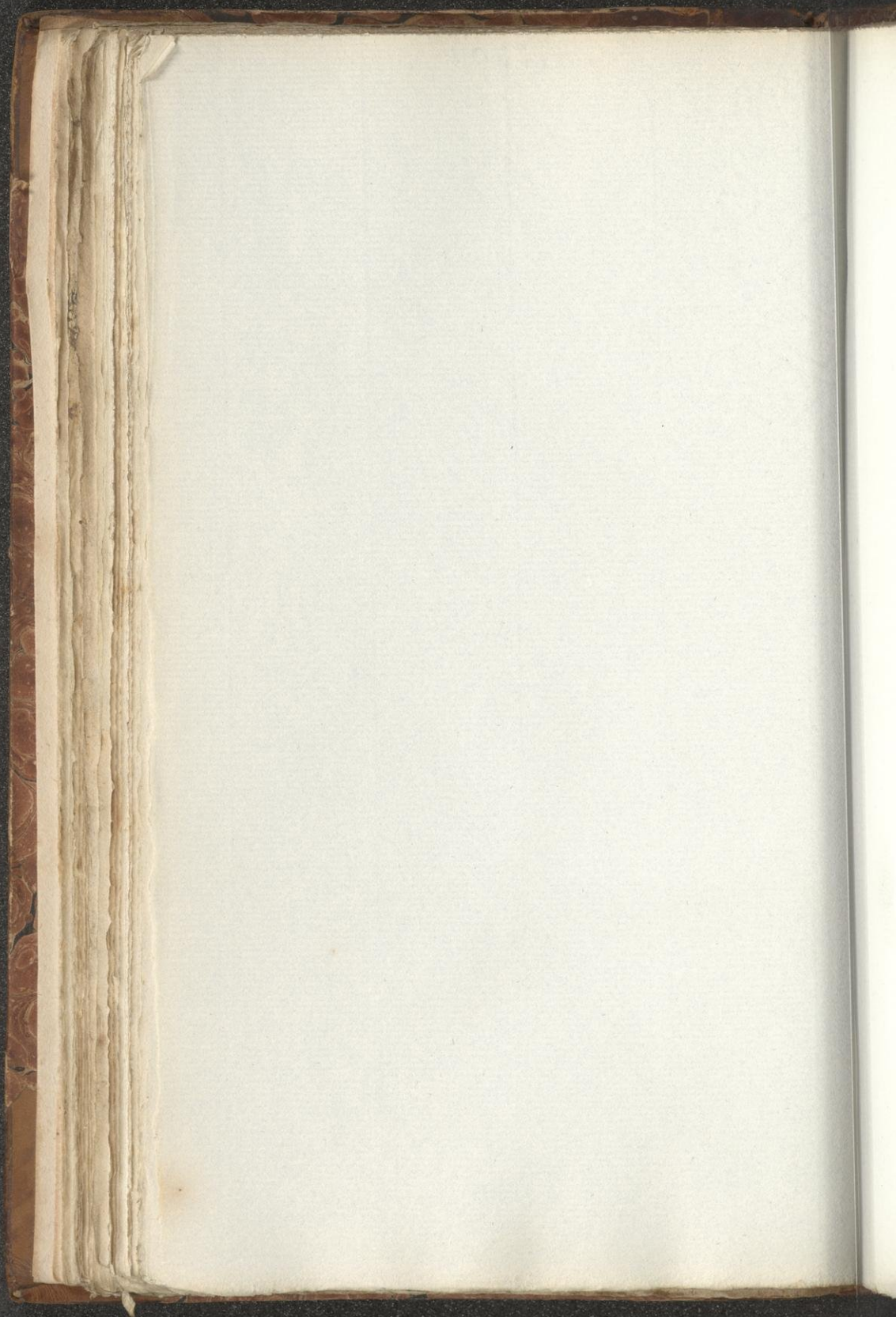
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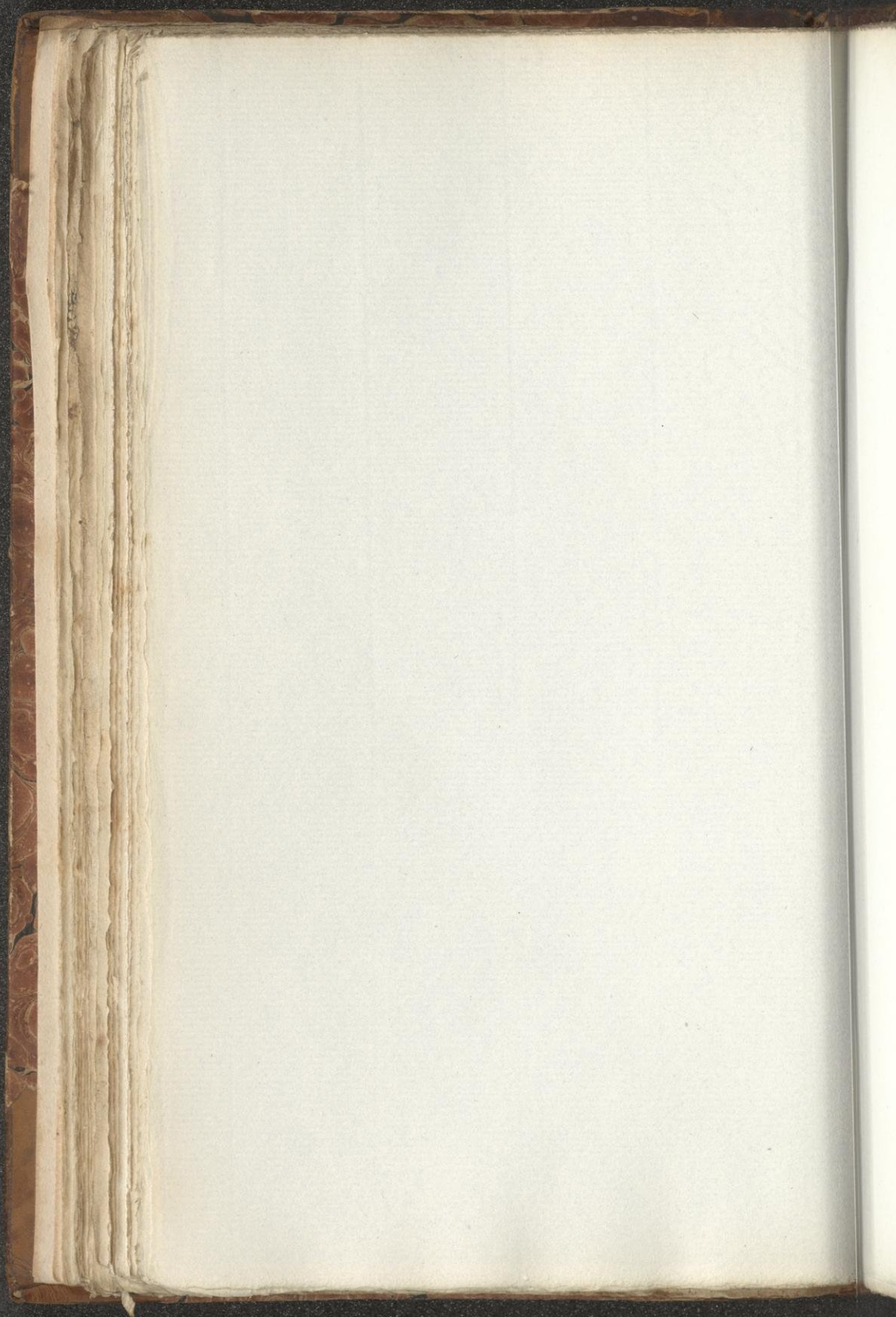
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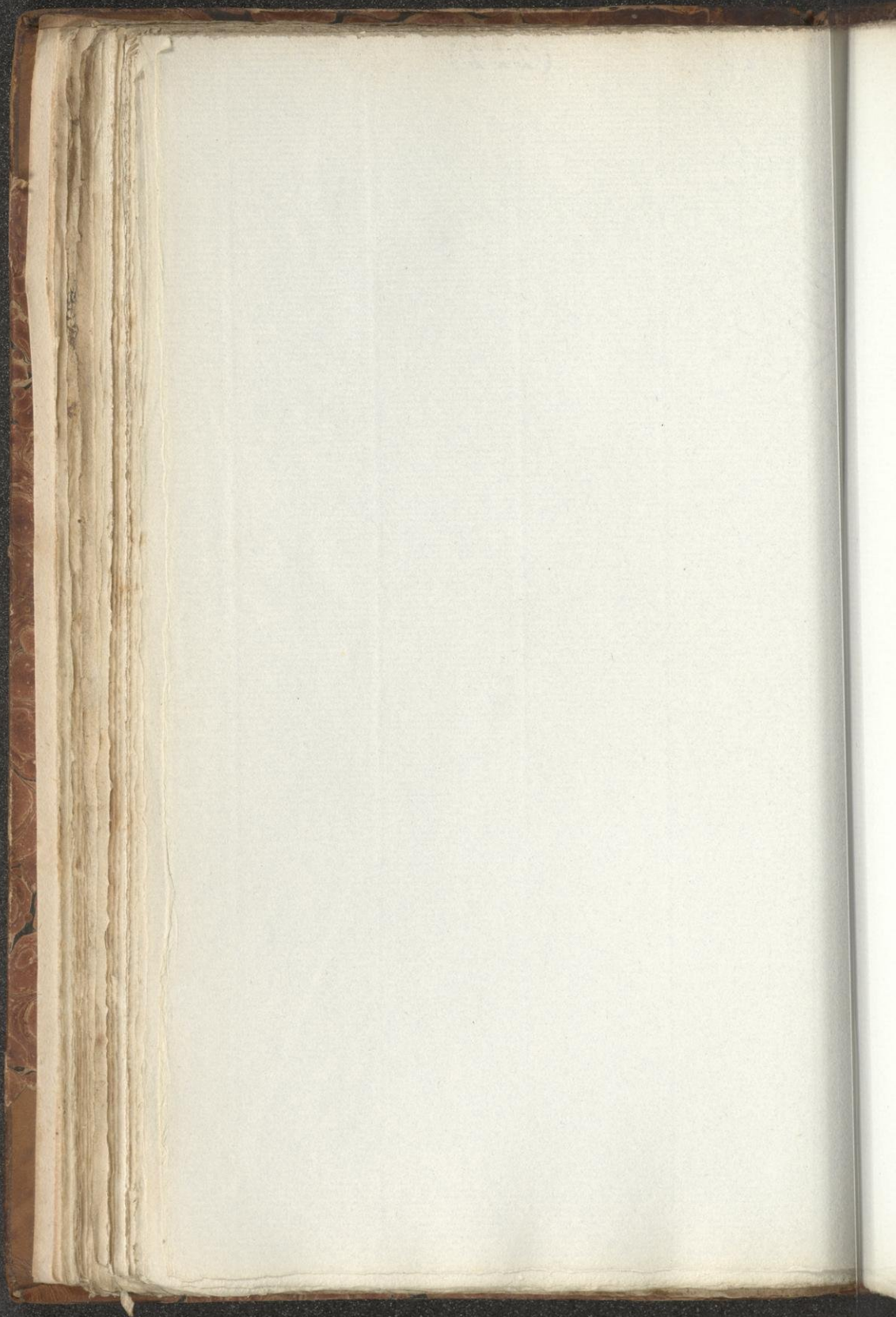
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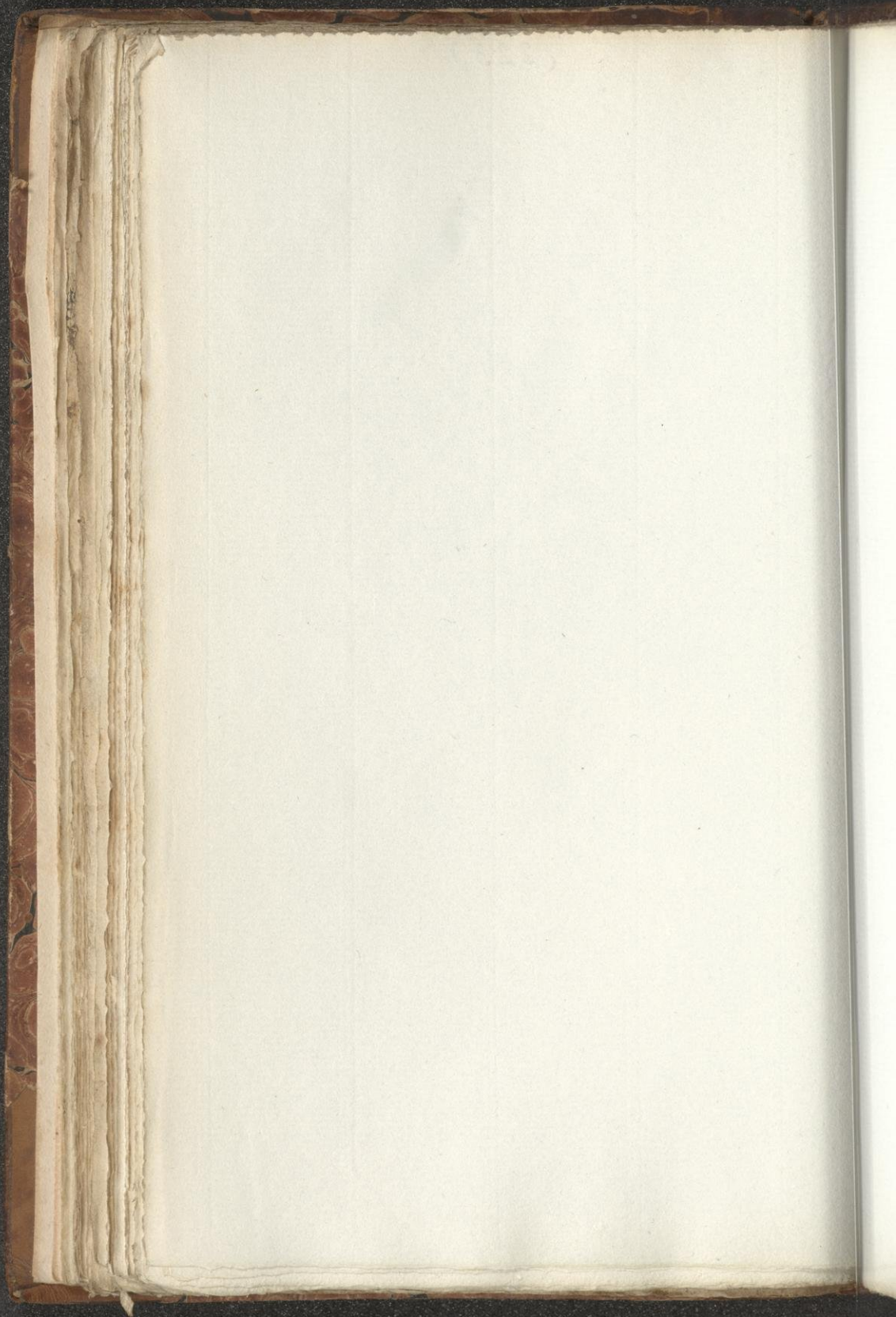
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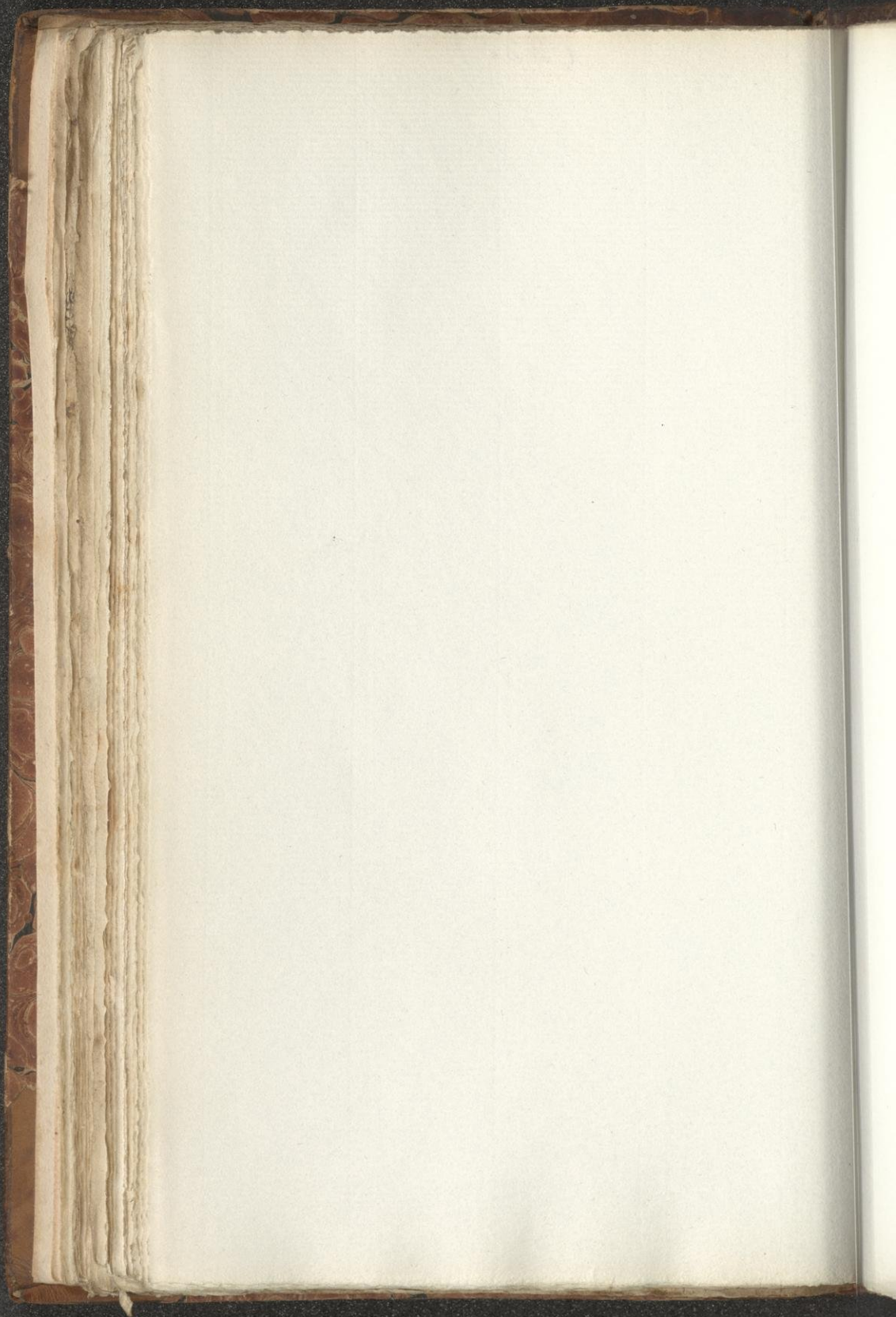
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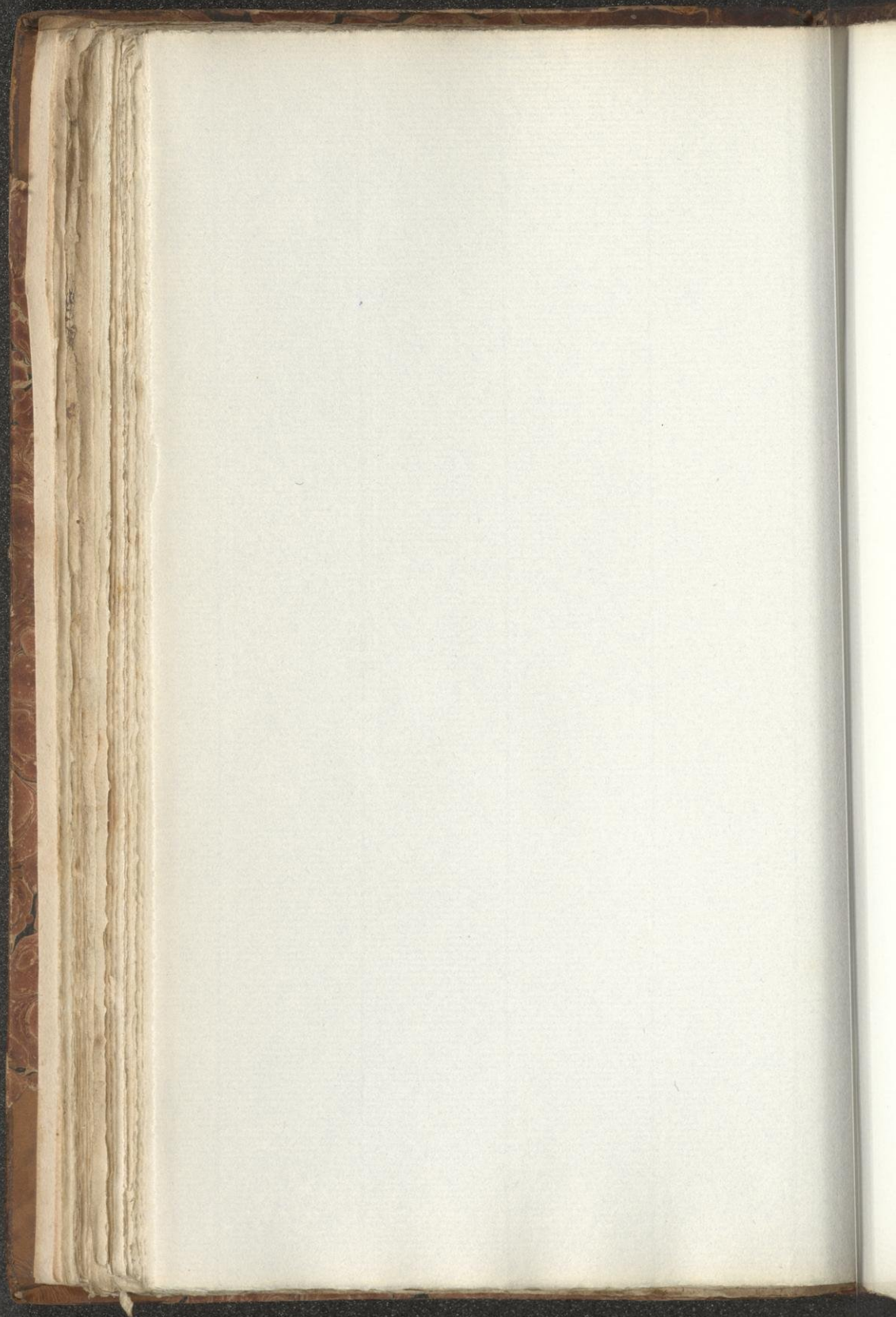
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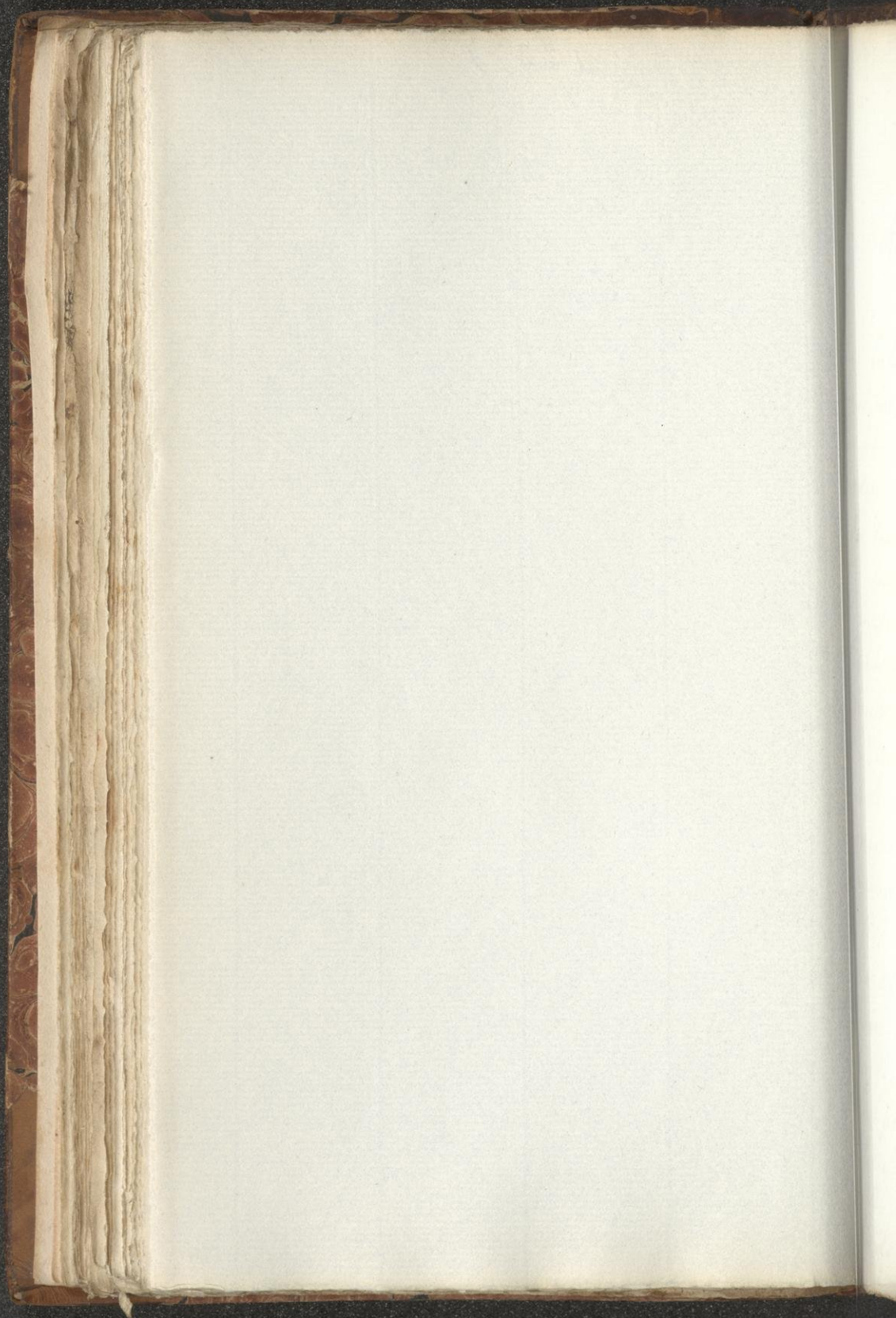
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Sect: I

vol. II

Astronomy

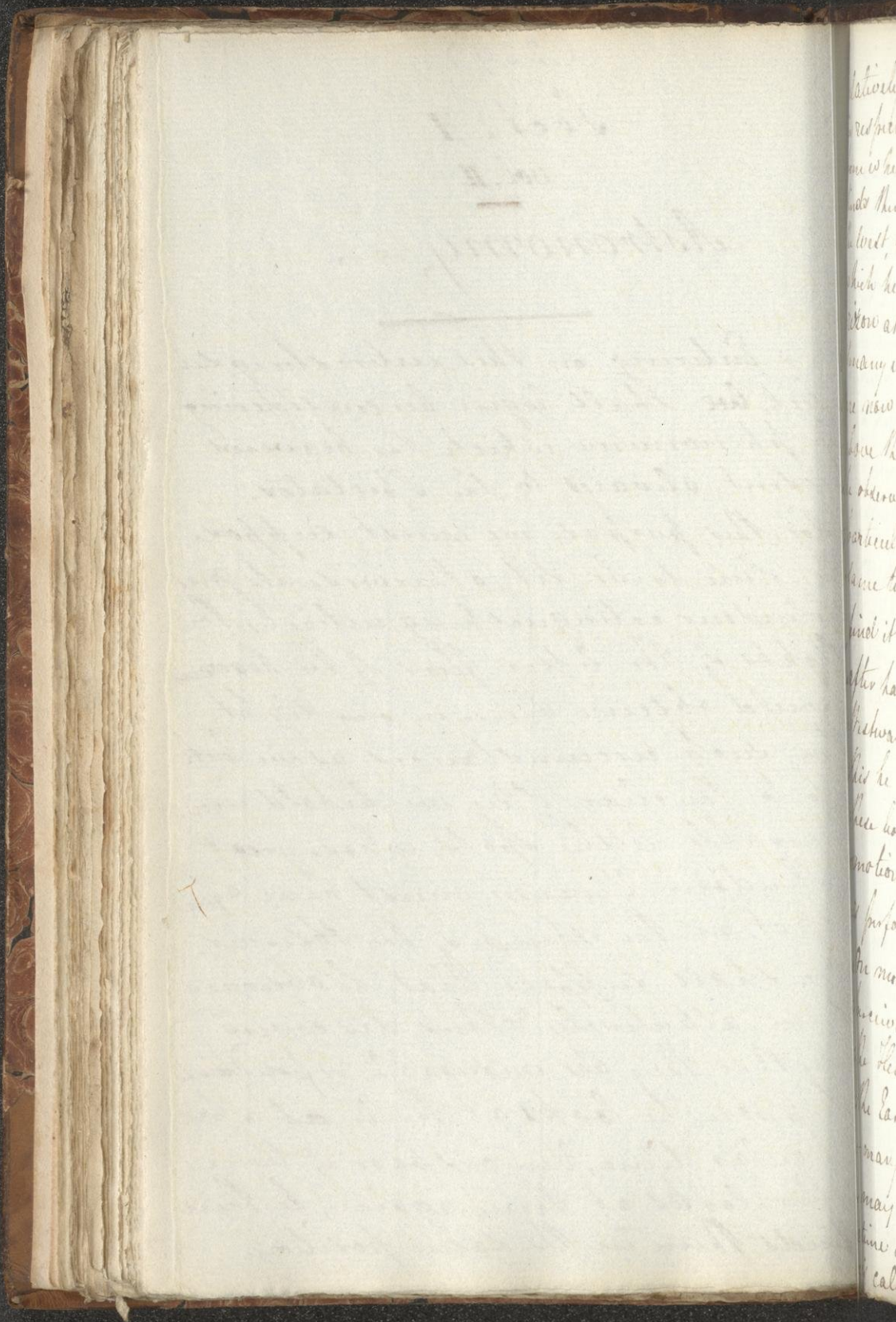
Entering on this interesting subject, we shall begin by considering the phenomena which the heavens present always to the Spectator.

For this purpose we must suppose the Sun to be set, otherwise its great splendour extinguishing entirely the lights of the lesser fires of the heavens, would obscure them in our sight.

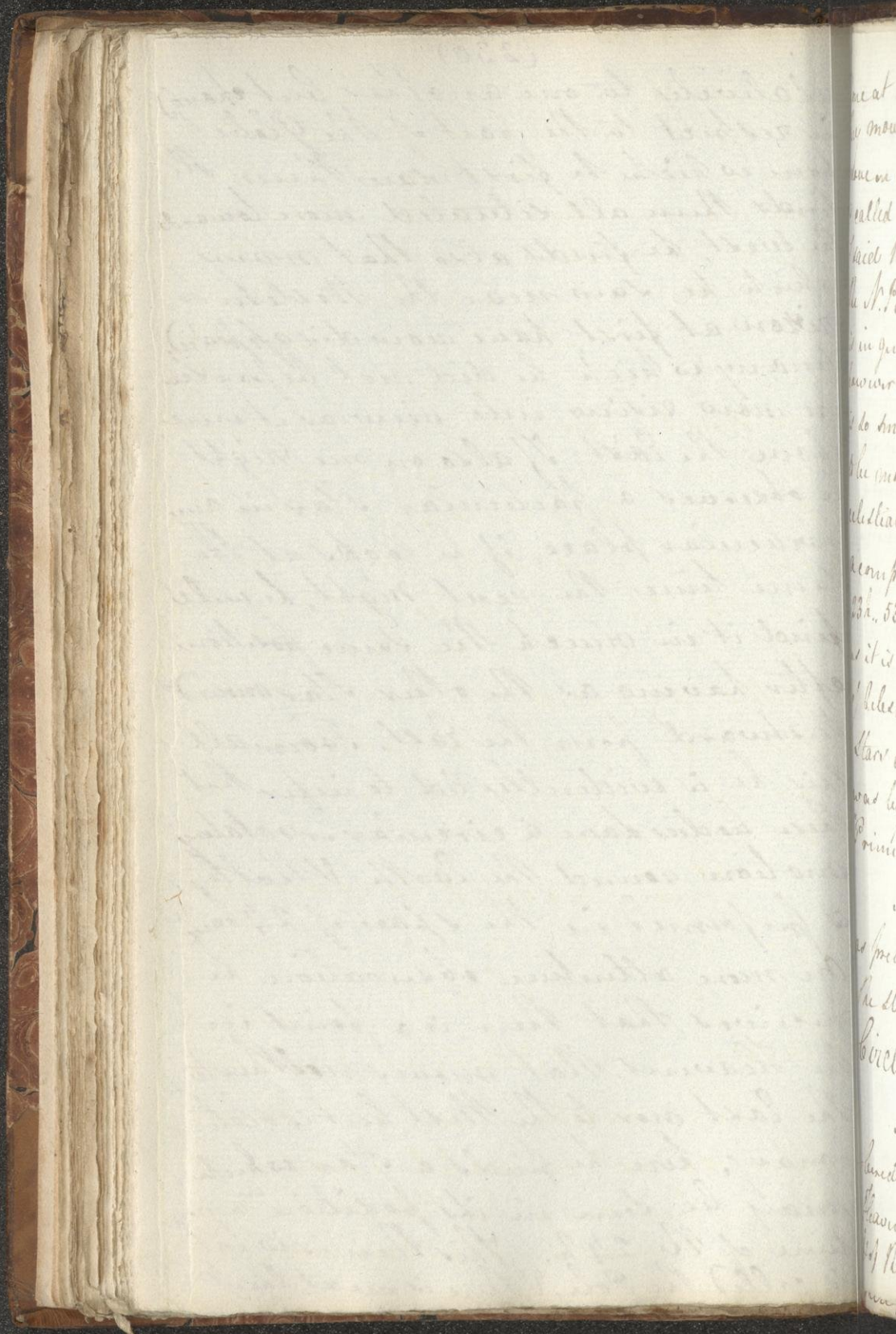
In such circumstances as we look up to the clear Sky, we behold innumerable little spots, whose great splendour & energy must have a fine effect on the mind of the Observer.

We shall suppose that he observes them attentively & thus discovers whether they are moveable or fixed points.

Suppose he looks at them at a particular time, & in a space of time after looks at them again, he then finds them in the same position



relatively to one another but change
 in respect to the part of the Globe
 from which he first saw them. He
 finds them all situated more towards
 the west, he finds also that many
 which he saw near the Western hor-
 -izon at first have now disappeared.
 Many which he did not before see
 are now rising into view as it were
 above the East. If also on one night
 he observes a particular Star in any
 particular place, if he looks at the
 same time the next night, he will
 find it in much the same position
 after having as the other Star moved
 Westward from the East. From all
 this he is evidently led to infer that
 these bodies have a circular or rotating
 motion round the Earth & that this
 is performed in the space of 24 hours.
 On more attentive observation he
 perceives that there is a point in
 the Heavens that moves neither
 the East nor to the West but is station-
 -ary, here he finds a Star which
 may be seen in its position ^{in any}
 time of the 24 h. This then is what
 is called the Pole, & there is one at the



(231)
Come at the South. The Heavens as
they move on there are supposed to
move on a line joining them, this line
is called the Axis of the Heavens.
I said that there is a small star at
the N. Pole which marks it; this star
is in general stationary, it does move
however rather in a circle, but this
is so small that it is not supposed
to be moveable. This whole sphere of
celestial bodies are observed to perform
a complete revolution in the space of
23h. 53'. 24" or in about 24 hours
as it is in general called. This sphere
of celestial bodies in which the
Stars are placed revolve round
was by the Ancients called the
"Primum mobile"

I now proceed to examine
as preliminary to our entering on
the study the Ideal.

Circles of the Celestial Sphere

These circles it must be remem-
bered do not in reality exist in the
Heavens, but are only Ideal or Imaginary.
Of these the first we shall consider is the
celestial equator

[Faint, illegible handwriting on the main page of the manuscript.]

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1. Horizon. This is a plane extending indefinitely, & bounding the utmost limit of our view. At sea only we can observe a perfect horizon, nothing then intercepts our view, & the circle bounding it all around is called the Horizon. In this situation the point directly over our heads is termed the Zenith, that directly under our feet the Nadir, derived from the Arabic.

2. Meridian. A circle passing thro' the Zenith & Nadir, & at the same time thro' the Poles of the World is called the Meridian of that place. This circle divides the Globe into 2 equal parts the Eastern & Western Hemisphere. When the Sun is on this circle in the Day, it is said to be midday, if at night ^{Mid} night. All circles that pass merely thro' the Zenith & Nadir & of course perpendicular to the Horizon are called vertical circles or Azimuths. The ^{one} remarkable

3. Prime vertical. This is a circle passing thro' the Zenith & Nadir, but at the same time dividing each of the 2 Hemispheres the Eastern & Western into 2 equal parts. These points of intersection are the true East

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West, so that the Meridian & Prime-
-meridian divide the Horizon into 4 equal
parts. The North, South, East & West.
All these circles are moveable & vary
according to the position of the Spectator;
but we now come to one which never
varies, but is always fixed

4 Equator. Art. 7. This is a fixed cir-
-cle at an equal distance from each
Pole, with its plane perpendicular to
the Axis of the World. When spoken of
as abiding to the Earth it is called the
Equinoctial line, but as regarding
the Heavens it is termed the Equator. It
divides the World into Northern & Southern
Hemispheres.

Art. 4. A little attention soon shows
us that the Earth we inhabit is of a
globular form. Thus when a Ship ap-
-proaches the Land her topmasts are
first visible. But it is not necessary to
insist on this innumerable other
similar proofs that might be adduced
since the frequent voyages of Men round
the Earth puts it beyond any doubt.

The Globe is generally divided
into 360° as being the most convenient

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division. Each of the Quarters therefore or quadrants contain 90 of these degrees. To each degree we reckon 69.04 miles, so that the whole circumference comes to be about 25,000 miles & the diameter about nearly 8,000 miles.

If we suppose the planes of these various circles - the Meridians, the Horizon, Equator &c to cut the surface of the Earth, then the same circles which are supposed to exist in the heavens may likewise be conceived to exist on the surface of the Earth. Also that point directly under the N. Pole of the Heavens on the Earth, is likewise termed the N. Pole of the Earth - so with the S. Pole.

By observation of the Stars, we divide the days, or rather time into days which are called Sidereal days. This is different from the Solar day. When for example we observe a particular Star, the first Star of Aries for instance, in any position of the heavens then the time which such a star takes to revolve & return into the same position again is termed a sidereal day.

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Now as the Stars move at the rate of 15° in an hour, a Clock called (Cositis) a Siderial clock is on this principle constructed, so that by referring to the Clock which commences at 12 o'clock or 0 h; we may know the position of any star. If then we observe a particular star at 12 o'clock, or any other time, then at 2 hours & $\frac{1}{2}$ afterwards, we shall find it has moved its position in respect to us $37^\circ 30'$. So in $1^h 20'$ it is $= 20^\circ$ of Arch.

Art: 20. The Right Ascension of any star is its distance from the first point of Aries on the Equator, its position being determined by a line of declination passing thro' the Body. The Declination is its distance from the Equator, & its distance from the pole on that side of the Equator is its Polar distance. The right Ascension is found by means of a Clock regulated for Siderial time; which marks beginning at 0^h at the first point of Aries. The declination which corresponds to latitude on the Earth is equal to the Elevation of the Pole above the Horizon. In the diagram (following) $EZ = PH$ so that if we know PH then we know



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ZE, viz by observation we find SO, then we have E the latitude, or declination of the star for it is = ZE-SO. The Polar star P is not quite stationary but may vary in the small

In circle FG, in which case we find $\frac{HL + PG}{2} = \frac{PZ}{2}$ the elevation of the Pole. The Declination & Right Ascension of any star, correspond exactly to the latitude & longitude of a place on the surface of the earth. As the latter gives any place on the earth, so the former gives any point on the celestial sphere.

3d Art. Thus it is that catalogues of all the stars are made out. The first as far as we know who formed a catalogue of the stars was Hipparchus of Rhodes, which according to the Elder Pliny he had "audus rem deo dignam". When we look at the stars on a starry night we think them quite innumerable, but more attentive observation soon shows us that they are easily numbered, that looking at them with the naked eye we cannot observe above 1000 at a time. Hipparchus



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did in reality number 1022 Stars to which he also added certain names. A catalogue of the Stars was also published by Ptolemy which used to be styled *μεγάλη σφαιρίδις*. A catalogue containing 977 Stars in a most accurate order was published by Ticho a Danish Astronomer. This was exact within 1 minute that of Lipparchus 1022 came within about 10'. A very accurate catalogue was published not long since in this Country containing about 3000 Stars. The last that has been published is by Noe Chausseur.

Art. 33. The Stars are all separated into various clusters, which since the earliest times have been designated by various representations of Animals &c. These are called Constellations. This Artifice is of very ancient date. They were at first about 48 in number, but by various additions are now about 70 in all. This is merely meant to assist the imagination & memory. How long ago it is since Celestial Globes were used we know not, but it certainly must have been at some very distant time. We are told

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That Chyrou who accompanied the Argos-
 mants had with him a Globe on which
 were designated these constellations. Among
 the curiosities preserved at Rome one that
 struck me most, was a Stone Globe on
 which were rude figures of the constellations.
 Art: 32. Another distinction by which
 the Stars are known is their Magnitude.
 They are all divided into various orders
 there are stars of the 1st 2nd 3rd &c. to the
 6th order, after which they are called
 Telescopii Stars, not being discernable
 by the naked eye. All the first order
 are easily known by their extreme bright-
 -ness. The Polar Star belongs to the second
 order. Of the first order there are about
 13 or 14 & of the 2nd between 50 & 60. It
 is a curious remark by Kepler that
 in a Sphere only 13 points can be placed
 equally distant from each other. At
 present I do not consider the real
 Magnitude of the Stars, that ~~shall~~
 to be considered afterwards. I now only
 consider their apparent Magnitude.
 Art: 35 Much about the year 1600
 Kepler walking home one Evening
 observed a star which he never before

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seen or heard of, sometime after it disappeared, & he could never afterwards succeed in seeing it again. Some of the Stars when closely examined appear double, as is the case with the Polar Star, others appear triple. Thos' Herschel's glass I myself saw a triple Star. Some appear red others bluish. The Milky way a curious appearance in the Heavens was first discovered by the -gins, to be a cluster of Stars. Some of the Stars are observed to be nebulous resembling comets that the fiery tail of a Comet.

Herschel by his numerous instruments discovered many thousand Stars in a fine clear night. These were all grouped in various parts forming the constellations. The different Stars in these constellations are distinguished by the Greek letters, as for instance the α the β or the γ of Aries.

As yet we have only considered the fixed Stars which change their positions merely in relation to the Earth. Let us now consider those termed the Planets which have motions peculiar

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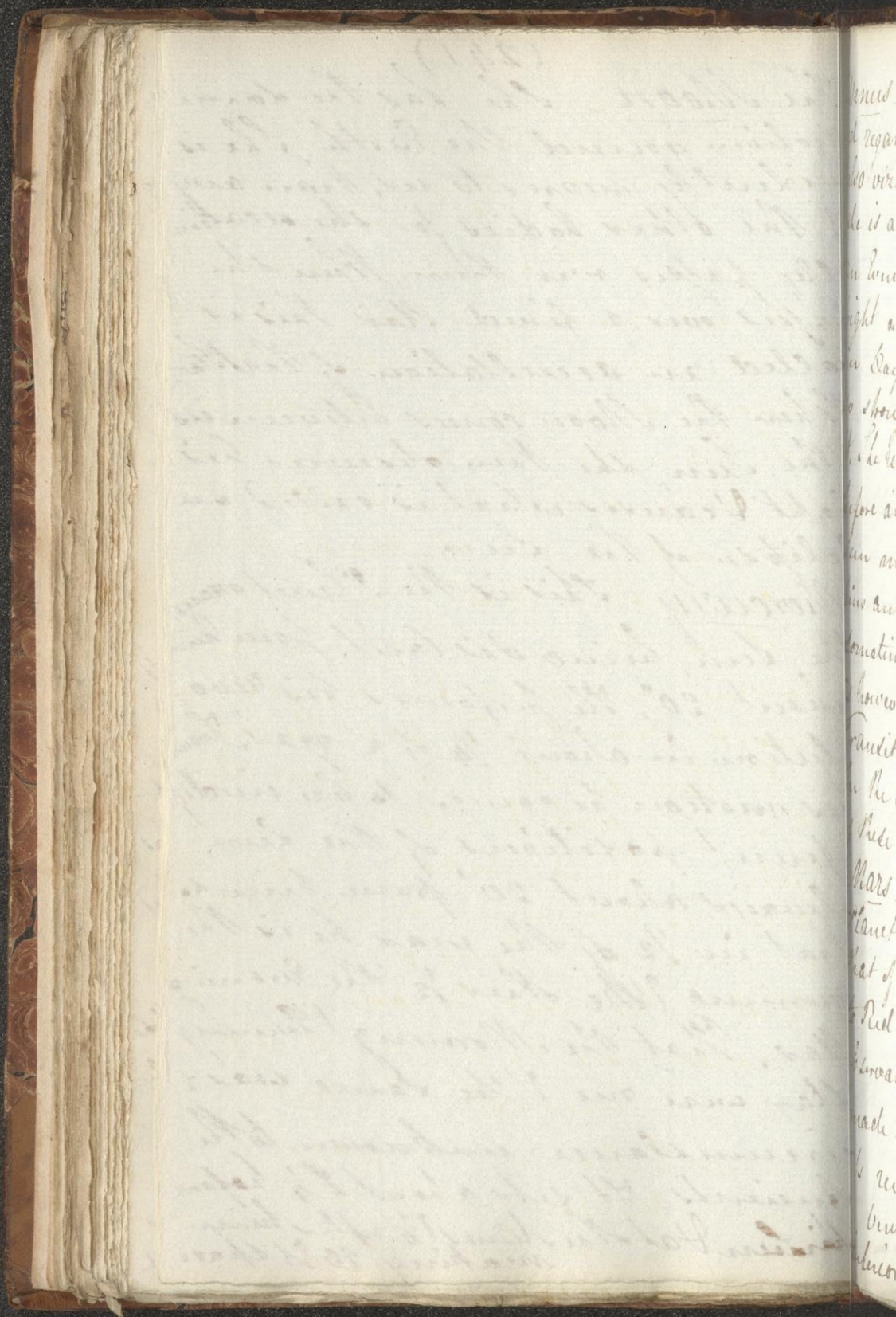
to themselves independant of the various
 Bodies around them. These are the Sun,
 Moon, Mercury, Venus, Mars, Jupiter,
 Saturn, Uranus, Ceres, Pallas, Juno,
 Vesta. Of these the first 7 have been
 known for the remotest ages, the other 4
 have been more recently discovered.
 Uranus was discovered in 1781 by Dr.
 Herschel, Ceres was discovered on the
 first day of the present Century by
 Piazzi. Pallas a few years afterwards.
 Juno & Vesta were since discovered & are
 very small in size.

Sun. When the Sun comes on our
 Meridian it is 12 o'clock & directly
 opposite him it is 12 at night &
 midnight. This motion is apparent
 round the Earth, & if viewed in a
 particular part of the Heavens at
 any time, he will not be again
 found in this very exact same posi-
 tion for 365 days & 6 hours, which
 is called a Solar year. This position
 may be determined by marking the
 Star which is directly opposite to it
 at any particular time. it will
 then be 365 days & 6 hours before he can
 be in exactly the same situation with
 the Star.

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The Moon - She has the same motion round the Earth. She is evidently nearer to us, than any of the other bodies for she occasionally passes over them. When she passes over a fixed star this is called an occultation of that star. When the Moon comes between us & the Sun, she then obscures his light & causes what is called an Eclipse of the Sun.

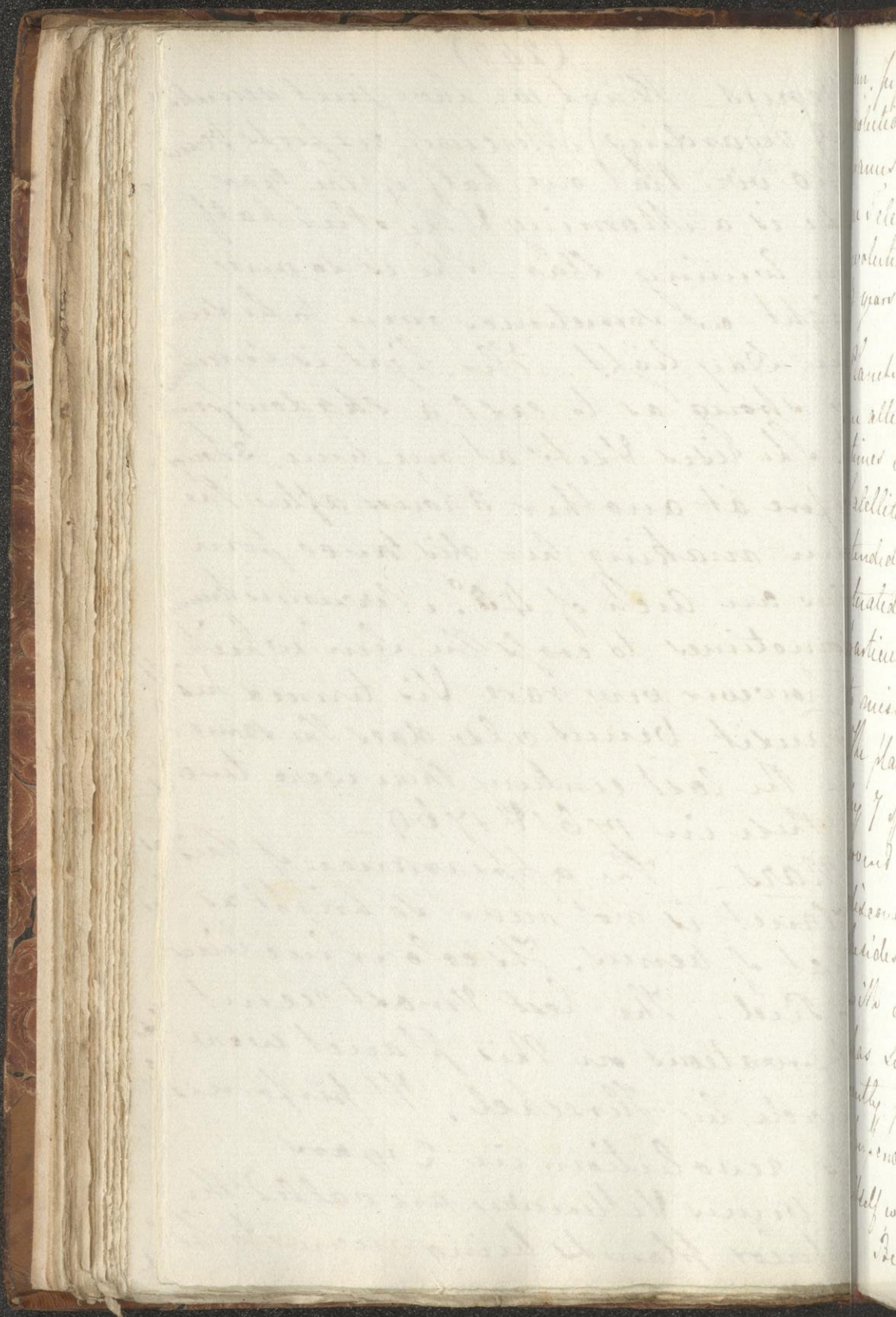
Mercury - This is the Planet nearest the Sun, being distant from him about 20° . He performs his revolution in about $\frac{1}{4}$ of a year. From his motions he comes to be in different positions of the Sun always about 20° from him so that in $\frac{1}{2}$ of the year he is the Morning Star & in the other $\frac{1}{2}$ the Evening Star. That the Morning & Evening Star were one & the same was a circumstance unknown to the Ancients. It sets about $\frac{1}{4}$ before the Sun & rises $\frac{1}{4}$ after him making $\frac{1}{2}$ of space.



Venus - What we have just mention^d
ed regarding Mercury, respects Venus
also viz. that one half of the year
she is a Morning & the other half
an Evening Star. She is so very
bright at sometimes even to be seen
in Day light. Her light is sometimes
so strong as to cast a shadow from
it. She rises & sets at one time, 3 hours
before at another 3 hours after the
Sun making her distance from
him an Arch of 45° . Mercury is known
sometimes to cross the Sun which
is however very rare. He turned his
Transit. Venus also does the same.
In the last century there were two
of these in 1761 & 1769 -

Mars - The appearance of this
Planet is not near so bright as
that of Venus. The colour inclines
to Red. The best & most recent
observations on this Planet were
made by Herschel. It performs
its revolution in 2 years

Venus & Mercury are called the
interior Planets being nearest to the



Sun. Jupiter performs his annual revolution in 12 years. Saturn in 29. Uranus takes 83. The other four stars are Telescopie, they perform their revolutions Ceres & Pallas in about 4 years.

Such then are the different Planets of our system. Some of them are attended by smaller bodies sometimes called Secondary planets or Satellites. Jupiter is generally attended by four of these thus situated:  with respect to him, particular attention is required not to mistake one Satellite for another. The planet Saturn is accompanied by 7 of these Satellites, 5 were discovered by Huygens, the others were discovered afterwards.  besides these, he is also surrounded with a luminous ring. Uranus has five, but situated very differently from any of the former, being perpendicular,  these with Uranus itself were discovered by Herschel.

Besides the primary & Secondary

See page. 285.

See page 285.

Planets already enumerated there are
 their bodies which revolve round the Sun
 have called by Astronomers by the names of
comets. These occasionally are seen
 in every part of the Heavens they sur-
 prise us by the singular appearance
 of a train or tail which accompanies
 them. This part of Astronomy is rather
 imperfect as till lately these appearances
 were merely considered as Meteors and
 little care was taken to examine them.
 They are supposed of the most intense
 heat. Sir Isaac Newton computed the
 heat of a comet which appeared in 1680
 when nearest the Sun to be 2000 times
 hotter than red hot Iron.

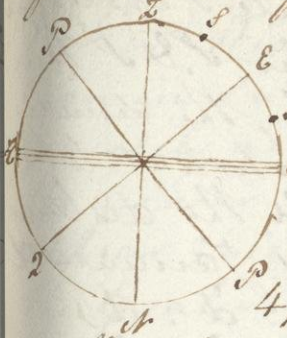
The Planets which we have enu-
 merated are known to Astronomers
 by different signs which are prefixed
 to each. Mercury is denoted thus ☿
 Venus by the Sign ♀. Mars by ♂.
 Ceres by ♄. Pallas ♁. Jupiter by ♃.
 Saturn by ♄. Uranus ♅. The Roman
 Naturalist Pliny also describes
 the Planets by their Colour.

The discovery of the obliquity of
 the Elliptic added greatly to the progress

This motion has been found to be as follows
between the 10th & 11th of Jan^{ry} he has advanced
in his own orbit $1^{\circ} 1' 8''$ in Feb^{ry} $1^{\circ} 0' 25''$ now
in March it is at the medium. on the 10th
July it is the slowest $0^{\circ} 54' 12''$

(245)

of Astronomy. In this figure $\angle P' - \angle P = 20^\circ$
 or = twice the distance of the
 Equator to the Sun's greatest
 height in the Ecliptic,



Therefore $\frac{\angle P' - \angle P}{2} = 0$ now it
 was found that $\angle P' - \angle P =$
 47° so $\frac{\angle P' - \angle P}{2} = 23^\circ 30'$ which
 is the obliquity of the Ecliptic by observing
 the Sun's highest & lowest place in the
 heavens. This is termed by Astronomers
 the Obliquity of the Ecliptic.

This depends upon a simple
 proposition or property in Spherical
 Trigonometry. Thus if in the triangle
 ASD then be given $\angle D$ the $\angle A$ &
 declination & the $\angle$ at A or O as it may
 be called. Then it is required to find
 $\angle S$. As Line A or O : Line D :: Line DS : $\angle S$
 or As $\angle O$: Rad. :: $\angle D$: $\angle S$ Therefore
 Line $AS = \frac{\text{Line } D}{\text{Line } O}$ -

In Winter the Diameter of the
 Sun is apparently greater than in
 the summer. About the 10th of Jan^y
 he moves swiftest & his semidiameter
 appears to be about $16''$ $17''$ 8 when how-
 ever it appears the least that is about 10th

Call the angular velocities of the Sun
 $\dot{\phi}$ & $\dot{\phi}'$ the Semidiameter S & S' then
 $\dot{\phi} : \dot{\phi}' :: S^2 : S'^2$ or $\frac{\dot{\phi}}{\dot{\phi}'} = \frac{S^2}{S'^2}$, the semidi-
 ameter must be proportional to the
 distance, so let us call the distance
 in the first of these cases January d
 in July d' , then $S : S' :: d : d'$
 $S^2 : S'^2 :: d^2 : d'^2$, then $\dot{\phi} : \dot{\phi}' :: d^2 : d'^2$
 the angular velocity therefore multiplied
 into the square of the Sun's distance
 $\dot{\phi} \times d^2 = \dot{\phi}'^2 \times d'^2$ is always the same

(246)

of July it is $11^{\circ} 45' 5''$. The reason of this is that in Winter we are nearer the Sun than in Summer. The distance is thus inversely as the Magnitude, that is the greater the distance the less will he appear vice versa the less the distance the greater he will appear. This Diurnal motion does not follow the same law. This diurnal motion at present as ours if (as will afterwards be proved) we are the moving body is $1^{\circ} 1' 12''$ when the motion is slowest it is $59' 3''$.

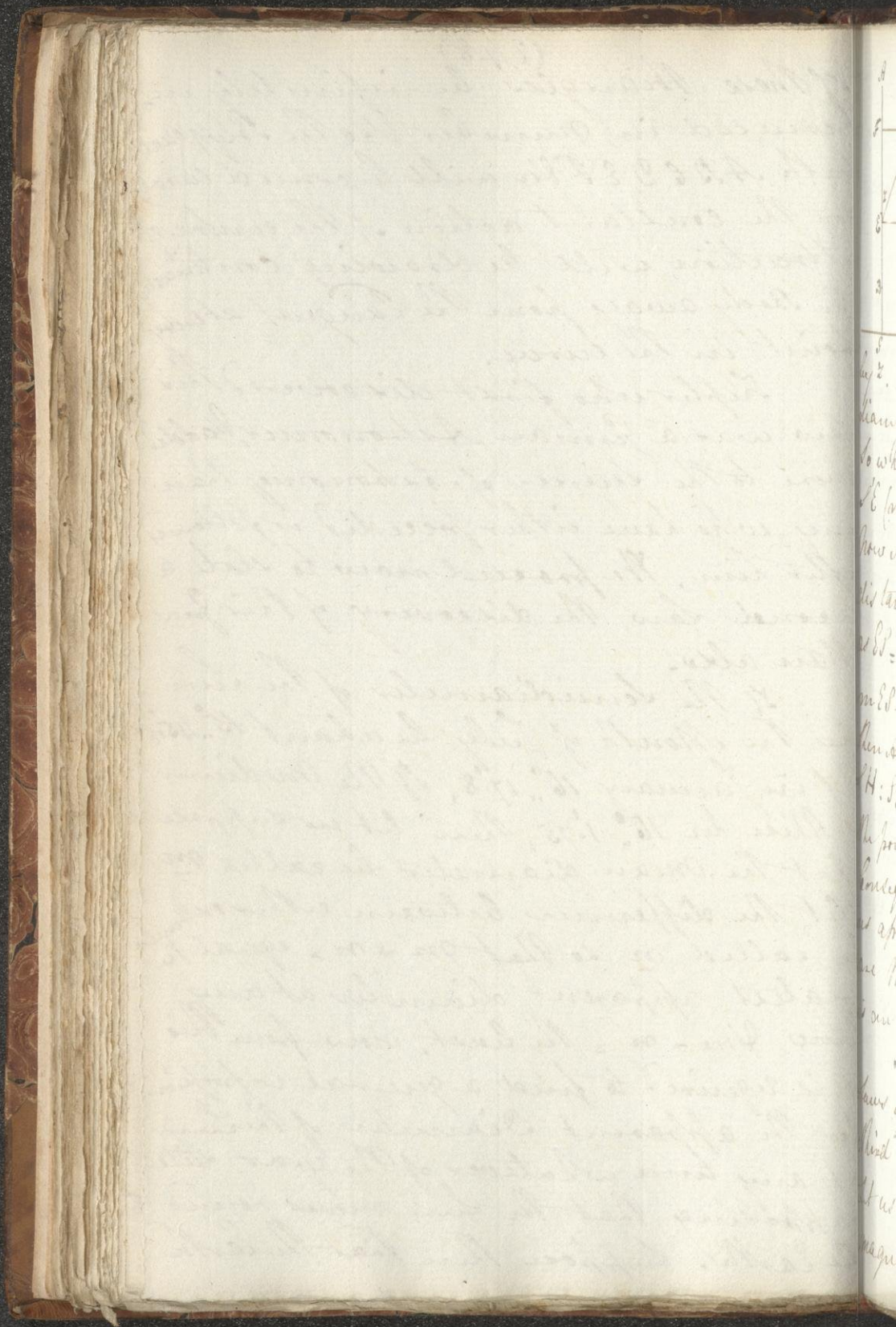
It is a fundamental Law in Nature none of those established by the unassisted Senses that "In the Orbit described by a body revolving in a certain time with a certain velocity in a nonresisting medium about a centre of attraction, the areas which are described by a straight line connecting the centre of attraction & the revolving body lie always in the same plane & are always proportional to the times in which they are described." Imagine the time to be divided into equal particles that a body round the centre of attraction it runs over the space AB

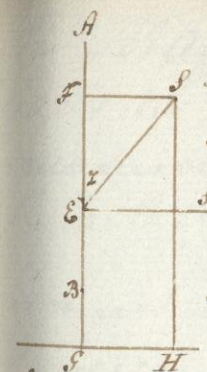
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If these Triangles be infinitely increased in number the Polygonal path $A B C D E F$ &c will become a curve for the constant action of the centre of attraction will be drawing continually the Body away from the Tangent at every point in the curve.

Kepler who first discovered this Law was a German Astronomer; & added more to the Science of Astronomy than any who have either preceded or followed after him. We proceed now to state a second Law, the discovery of this great Man also.

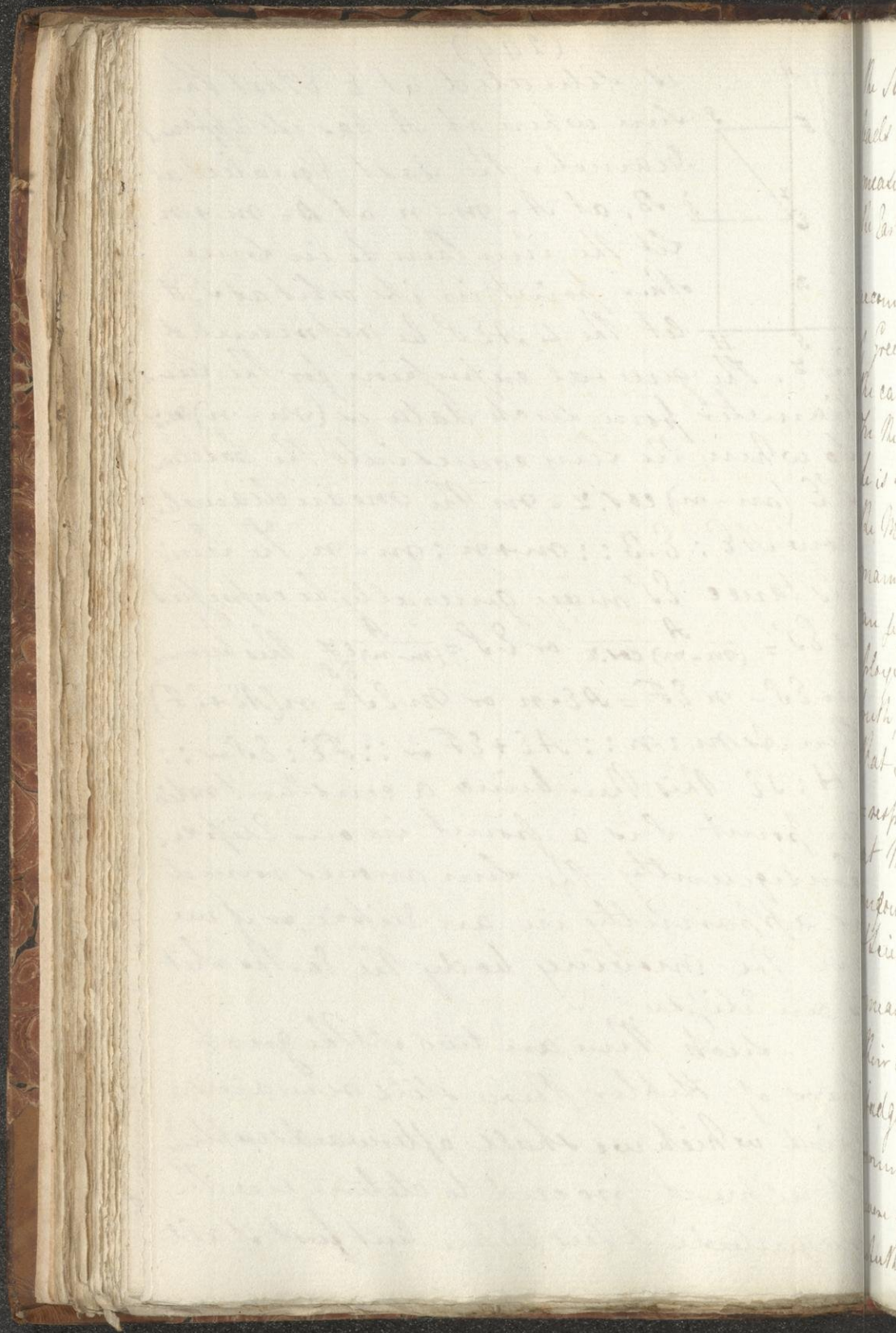
If the Semidiameter of the Sun in the Month of July be about $15''.45.5$ & in January $16''.17.8$. If the Medium of these be $16''.1.75$, then let us suppose that the Mean diameter be called m & that the difference between either extreme be called n so that $m + n =$ equal the greatest apparent diameter at any time & $m - n =$ the least, now from this it is required to find a general expression for the apparent Diameter of the Sun at any time whatever of the year. Still supposing that the Sun moves round the Earth. Suppose then that the Earth





is situated at E & that the Sun when at A has its apparent Diameter the least & greatest at $\frac{1}{2}$ B, at A = $m - n$ at B = $m + n$. Let the Sun then be in some other point in its orbit as S, & let the $\angle AES$ be represented by z . The general expression for the Sun's diameter from such data is $(m - n) \cos z$. So when the Sun comes into the position = S'E $(m - n) \cos z = m$ the mean diameter. Now $AE : EB :: m + n : m - n$. The Sun's distance ES may generally be expressed as $ES = \frac{A}{(m - n) \cos z}$ or $ES = \frac{A}{(m - n) \frac{ES}{A}}$ This becomes $m ES - n EF = AE \cdot n$ or $m ES = n(AE + EF)$ then as $m : n :: AE + EF :: FE : ES$ or $SH : SE$ This then being a constant ratio the point S is a point in our Ellipse. Consequently the Sun moves round us apparently in an Ellipse, or if we are the moving body the Earth's orbit is an Ellipse.

Such then are two of the great laws of Kepler there still remains a third which we shall afterward explain. Let us next proceed to determine the magnitude of this Ellipse but first of all



The Magnitude of the Earth. This leads us to consider the methods of measuring degrees on the surface of the Earth.

The first of whom we have any account was Eratostratus a native of Greece, & the second entrusted with the care of the Alexandrian Library. In the neighbourhood of Alexandria he is said to have measured a degree of the Meridian, but in a very rude manner. He however, as far as we can judge from the measures he employed, was not very far from the truth, making it nearly 69 miles that is to say measures which corresponded to this. The reign of Ptolemy at the commencement gave a little encouragement given to literature & science. About this time several measurements were taken, but of their accuracy we cannot positively judge as the stadia (which was the common measure of the ancients) were of different kinds and as no author or calculator thought proper

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to specify the particular one which he mentions or employs. One of them was the Alexandrian stadium, there were other 2 employed variously by the Greeks & a fourth which was employed by the Eastern Nations. Having in this rude manner determined the length of a degree of the Meridian they thus ascertained the circumference of the Earth altogether. In the reign of the Emperor Caliph Almanon a degree of the Meridian was measured with pretty considerable accuracy by those very Barbarians who burnt the Alexandrian Library. This was measured in the plains of Mesopotamia.

A considerable while ago a Mr Norwood of London measured the distance between that City & York with greater accuracy than any measurement of the kind before. Of the 4 different stadia among the Ancients, the 1st consisted of 114 toises, the 2nd $85\frac{1}{2}$, the 3rd $68\frac{1}{3}$, the 4th $51\frac{1}{6}$ of the French toises. These stadia

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(252)
being thus very various Has the Am^{er}
over thought of mentioning w^hat salaries
each employed, so we are much at a
loss to have a precise knowledge of
their estimates.

Mr Norwood who I have mentioned
 measured very accurately, proceeded
 in his measurement in the following
 manner. He went to determine the
 distance of York from London direct
 North. As he measured along the
 road which was of course often
 very crooked, he rectified it in the
 following manner. He has a line
 of 40 perches due North & South.
 He sets out from one end of the line
 from L Vincennes to A & the L A,

* And a certain other party to the Polar
regions, these accomplished their mission
more quickly than those sent to Peru
having their base on the ice of a frozen
River, but notwithstanding this they
err'd extremely having a very great
excess, Yet was not until very lately
when some Schonomers from Stockholm
undertook the same thing that it was
rectified

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proceeded measuring 6° of winding
68, so now he had 2° the distance
due North, he thus proceeds till he
finds 2° the distance of York due
North from London. This he found to
be 905751 feet or $2^{\circ} 28' = 367196$
yards which is a little more than
200 miles, & which considering the
rude manner in which he performed
his measurement, is by no means
very far from the truth. After this
more accurate measurements were
made. Sometime afterwards a degree
of the Meridian was measured in
the latitude $66^{\circ} 20'$ in Lapland, where
after a most accurate measurement
it was found to be equal to $57185\frac{1}{2}$
toises. After this an united body of
academicians from Paris were
sent out to Peru to measure accu-
rately a degree of the Meridian
where it was found to be equal to
 5653 toises being less than the former.
Thus was the prediction of Newton fulfilled
that a degree at the Equator wd be found less
than nearer the Poles.

X Newton has hinted & formed as his
opinion that the longer diameter of
the Earth is shorter as 231:230. In France
it was soon discovered that the degrees
were not all be equal - but it was asked
what would be the nature of the difference
If say's some the Earth be of an elliptical
form the degree ought to be less at
the poles, being nearer the centre -
We find many able & honourable
supporting this opinion as La Caille
but the reasoning certainly is simple
but very superficial, & it generally
shakes most people on first considera-
-ing it - but you must observe what
constitutes the degree - the tendency
of the plummet - it is always perpen-
-dicular to the ellipse, that is to a
Tangent to it, & therefore to the centre
of the curvature - therefore the degree
is in the inverse ratio of the curva-
ture - the circle at the pole being
greater than at the Equator the
degree also there is greater -

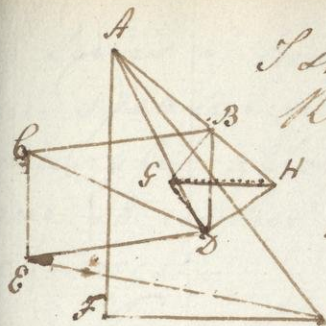
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After this a very extensive measurement took place in France extending from Dunkirk to the South-east of the Balearic Isles being an extent of 12° . Here on accurate computation the Medium degree was found to be about 57012. toises. The Latitude of Dunkirk is $51^{\circ} 2' 4''$ that of the Southernmost Isle $38^{\circ} 39' 58''$ so that the difference on the distance measured was just $12^{\circ} 22' 13''$. From all these circumstances we find that the degrees of the Meridian increase as we approach the Poles. The Globe therefore is flattened towards the Poles. The Medium radius of the Globe is 3955.4 English Miles & the Poles are the 330th part of this, nearer the centre of the Earth, than the surface at the Equator, i.e. about 11 Miles.

A Trigonometrical Survey is at present going on in England on a very extensive scale & as it is a very useful as well as interesting enquiry.



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I shall very shortly explain
the method of proceeding.

First then let there be
a Meridian line as
A¹F. Then suppose

there are a number
of elevated stations A, B, C, D, E, F & that
these are severally joined. All the
angles about each are determined
by means of Instruments. Then with
the utmost accuracy measure the
base G¹H of 6 or 8 miles on a level plain.
Having G¹H in the triangle G¹H¹D
Call the angles we easily find D¹H¹
in B¹C¹H having G¹H & the angles we easily
find B¹H. Now in the triangle B¹H¹D
having B¹H & D¹H and (by observation)
the included angle we easily obtain
B¹D. In the triangle B¹A¹D having B¹D
Call the angles (by observ.) we easily
obtain A¹D & having the angles in the
triangle A¹D¹F we easily find A¹F. In the
right angled triangle A¹F¹F as Radius
line F :: A¹F : FA This is the line that
was required. The accuracy attending
some of these operations is remarkable
a line of 40 miles found to be so by a



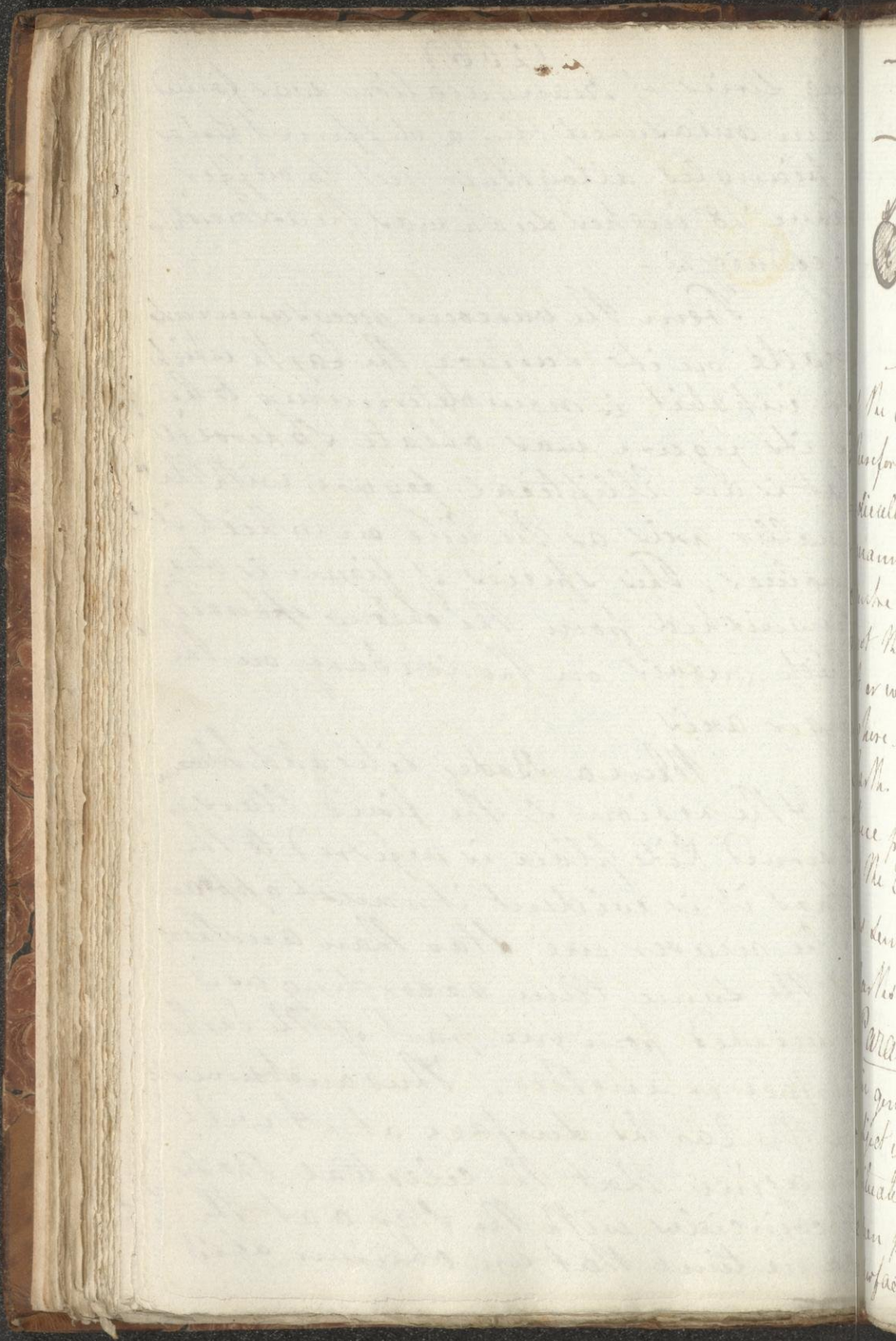
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long series of triangulation was found when measured by a different series of triangles altogether, not to differ, above 18 inches such was the astonishing accuracy!!

From the various measurements made on its surface, the Earth which we inhabit is now determined to be, in its figure was oblate spheroid, that is an Elliptical figure with the smaller axis as the line on which it revolves. This species of figure is distinguished from the oblong spheroid which moves on the contrary on the longer axis.

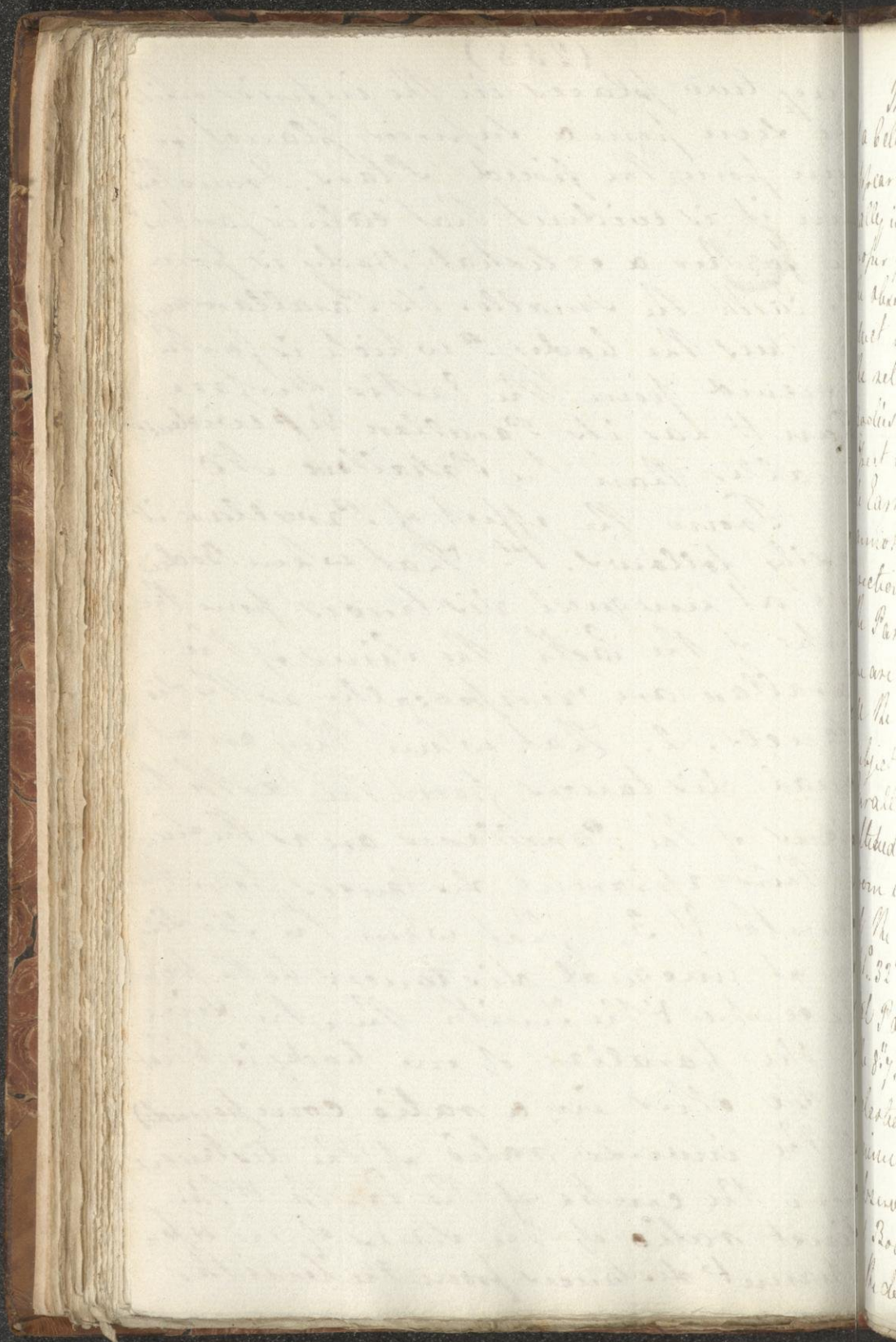
Thus a Body situated between us & the region of the fixed Stars, is observed this place is referred to the Stars it is evident it must appear to be nearer one Star than another, at the same time according as it is viewed from one part of the Earth's surface or another. Thus an observer on the Earth's surface at A will imagine that the celestial Body coincides with the Star c at the same time that an observer at B will



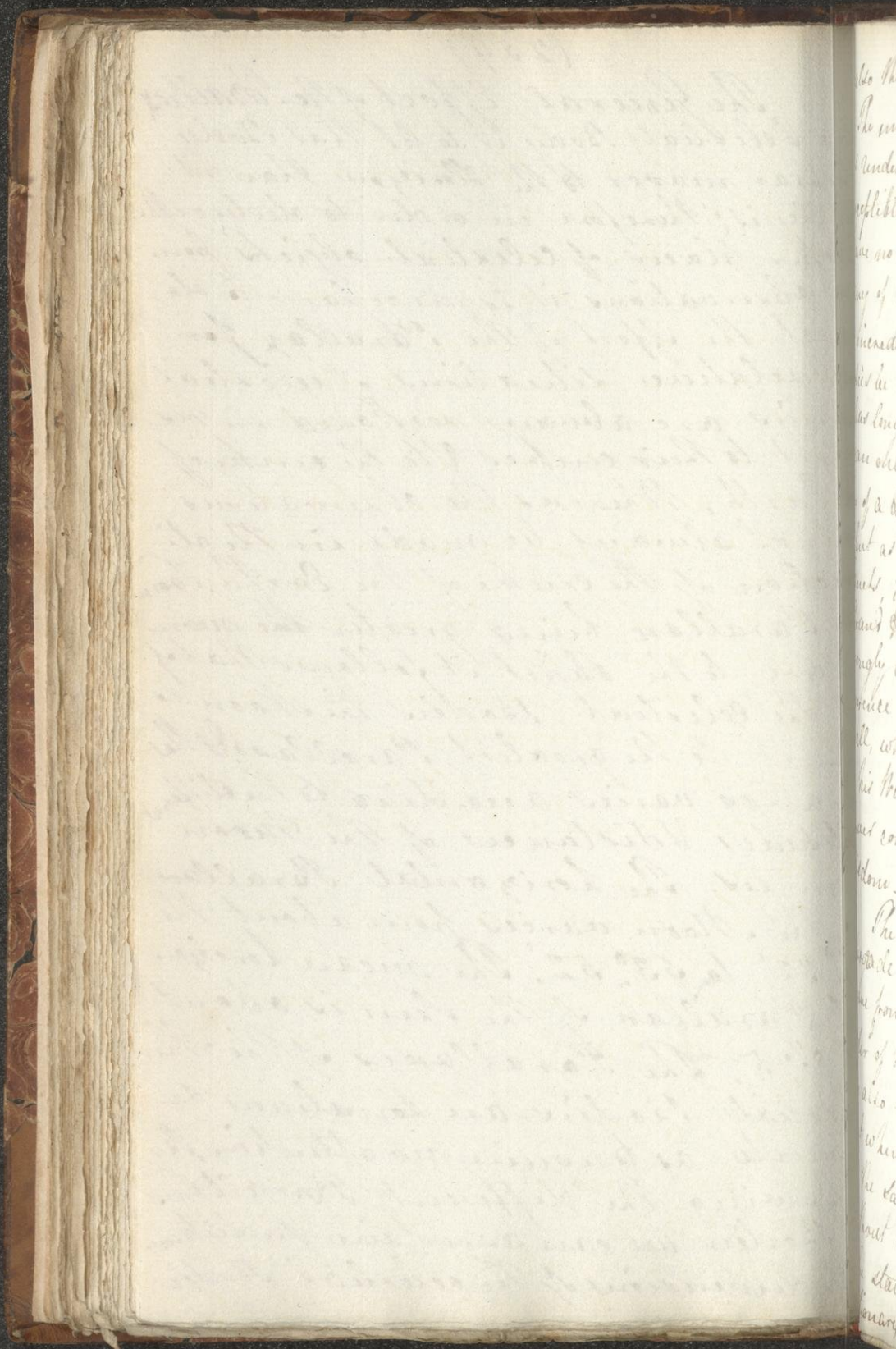


any two places in the inferior orbits are seen from a superior planet or even from the fixed Stars. From this then it is evident that "eatenis paribus" the farther a celestial Body is from the Earth the smaller its Parallax must be. Thus the body P which is farther removed from the Earth's surface than H has its Parallax p evidently smaller than the Parallax m .

From the effect of Parallax it easily follows. 1st That when Bodies are at unequal distances from the centre of the Earth the Sines of the Parallax are reciprocally as the Distances. 2. That when they are at equal distances from the Earth the Sines of the Parallax are as the Sines of their apparent distances from the Zenith. 3. That when the Bodies are at unequal distances both from the centre & the Zenith then the Sine of the Parallax of one body is to that of the other in a ratio compounded of the inverse ratio of the distances from the centre of the Earth & the direct ratio of the Sines of the apparent distances from the Zenith.



The General Effect of the Parallax of a Celestial Body is to let that Body appear nearer to the Horizon than it really is; therefore in order to deduce the proper places of celestial objects from the observations it is necessary to deduct the effect of the Parallax for the relative situations of celestial Bodies are always reckoned in respect to their centres & to the centre of the Earth. Whereas the observations cannot always be made in the direction of the centre of the Earth. From the Parallax being greater the nearer we are to the object it follows that of all the celestial Bodies the Moon is subject to the greatest Parallax & the Parallax varies according to the different altitudes & distances of the Moon from us. The horizontal Parallax of the Moon varies from about the $61^{\circ} 32'$ to $53^{\circ} 52'$. The mean horizontal Parallax of the Sun is about the $8''.75$. The Parallaxes of the other celestial Bodies are sometimes so minute as to require no attention. By observing the different Parallaxes of Bodies we can ascertain the distances & the dimensions of the celestial Bodies.



as also the dimensions of their orbits.

The amazing distance of the fixed stars renders their Parallax almost imperceptible indeed most of them seem to have no Parallax at all. With ourselves many of them really have none. What an incredible distance then must their Bodies be to what are we to suppose them. It has long been an idea (for an Idea it can only be) that they are each the Sun of a different system, & that our Sun is but as it were a fixed star to the Planets, if there be such systems what a Grand Sublime Idea this is, & how strongly does it excite in us the utmost reverence for the power of the Great Creator of all, whose ways are as unsearchable as his Works are the effects of infinitely power combined with the most infinite Wisdom.

The Planets are said to have a retrograde motion when they appear to move from East to West - contrary to the order of the Signs - In this case they are also said to be in Antecedentia. But when a Planet appears to remain in the same place for some time without changing its place it is said to be stationary. Saturn appears to be stationary for 8 Days, Jupiter for 4.

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Mars. 2, Venus $1\frac{1}{2}$ V. Mercury for 12h.
 In regard to retrograde motions Saturn
 appears retrograde for 140 days, Je-
 -piter for 120, Mars 73, Venus 42,
 Mercury for 22 days.

All this system of Stars & Planets
 Moons &c was by the Ancients de-
 -nominated the firmum mobile.
 The Stars were all supposed to be stuck
 in a sort of crystalline or transparent
 sphere in which they all moved
 round. We however know what has
 all along been said can have nothing
 of the truth of the Copernican system
 so far as relates to the motion of our earth
 in its orbit. There is one circumstance
 however remains to be mentioned viz
 how do we come to be insensible of our
 motion which is at the rate of 96 millions
 of miles in the course of a year. This was
 an objection long maintained ag^t the
 true system of the World, but must
 be entirely removed by consideration.
 When we move we move as part of
 the globe of the Earth, in which case
 like persons in the deck of a ship we
 have relative motion but not absolute.
 The Copernican system at first made
 but slow advancement. The apparent

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system of the Universe for some time
 strenuously opposed it. In Germany it
 was keenly supported by Kepler who
 was very successful in its behalf. He
 demonstrated on clear Mathematical
 principles that many of the most im-
 portant phenomena were according
 to the system of Copernicus. In Italy
 Galileo eagerly caught at this new
 system in which however he was less
 intimate than Kepler. No sooner had
 he published the true system of the
 World, than the Church was up in arms
 agt. him. From some of the texts in
 Scripture it was found (as they imagined)
 to be contrary to the holy writings for which
 he was called before the Inquisition &
 was to the everlasting infamy & disgrace
 of Humanity obliged before the torture
 to retract all he had published & thus
 to deny the true doctrine of the grand
 system of the Universe. He afterwards
 published a book of Dialogues in which
 he explained & demonstrated the sim-
 plicity & truth of the Copernican Sys-
 tem. This book quickly found its
 way into Germany & afterwards returned
 back into Italy where it being discovered
 that Galileo was the Author this un-
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Man was called a second time before
 the Inquisition & cast into Prison.
 Next to the Motion of the Earth in its
 orbit is its motion round its Axis. This
 it performs in 24 hours. Now how very
 simple is this when we think of the
 Ptolemaic system which made all
 the heavenly Bodies move round the
 Earth. The Stars are at very different
 distances & the fixed Stars at such an
 incredible distance that they appear
 to have no parallax. Now to conceive these
 Bodies to move round the Earth wd be
 conceiving a motion even far more
 rapid than that of Light which is
 quite impossible; whereas if we but
 suppose the Earth to revolve on its Axis
 the whole phenomena are simply
 accounted for. According to late ob-
 servations the Planets each revolve
 on their own Axis. According to Dr.
 Herschel the Planet Mars makes one
 revolution in the
 space of one day & of 39'. Jupiter in
 10h. Saturn in 10h 16' so that it is
 a general Law that the Planets daily
 revolve on their Axis. This Axis

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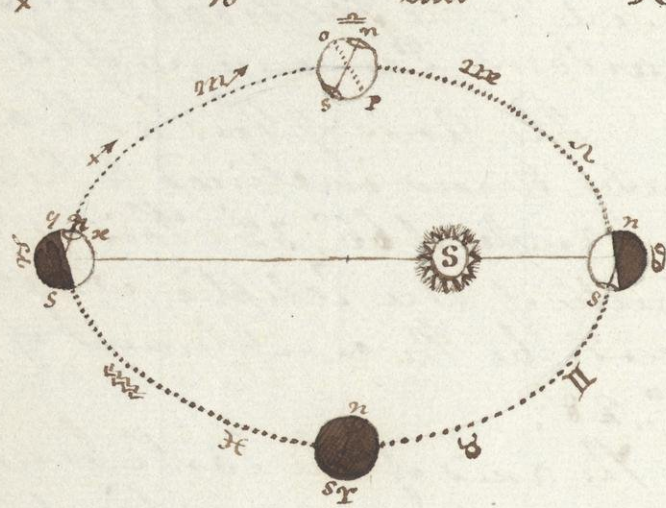
which is but an imaginary line extending from Pole to Pole is always parallel to itself in the orbit of the Planet which accounts for the variety of Seasons the change of Night & Day &c after discussing and also some observations on astronomical Instruments we shall proceed to the Physical causes of the various Phenomena of the Universe.

The Axis of the Earth as you already know inclined to the Ecliptic at an angle of $66^{\circ} 32'$. This is the obliquity of the Ecliptic which necessarily of course be the complement of this or $23^{\circ} 28'$.

The axis of the Earth besides retaining the same inclination to the plane of the Ecliptic does also remain always directed to the same Star; or in other words if a line parallel be drawn to that axis while the Earth is in any part of its orbit the axis will always be parallel to that line excepting a regular & very small vibration.

Now the various Seasons of the Year & the different Lengths of the day

Aries ♈	Taurus ♉	Gemini ♊	Cancer ♋
Leo ♌	Virgo ♍	Libra ♎	Scorpio ♏
Sagittarius ♐	Capricornus ♑	Aquarius ♒	Pisces ♓



in the Globe are owing to the above mentioned inclination of the Axis to the orbit & to that Axis moving round the Ecliptic in a direction parallel to a line nearly immovable. The effects are just the same as have been explained already when the phenomena were described on the supposition that the Earth stood immovable & the celestial objects moved round it. Let us represent this by a Diagram. Here then you observe that the Earth in either of these 4 points has its axis pointed in one direction that is always parallel to itself. The cause of the inequalities of the days & nights & the variety of the seasons will be plainly perceived by inspecting this figure, for they both arise from the inclination of the Earth's orbit to the plane of the Ecliptic. 1. In the Spring when the Earth is in that part of its orbit in which a spectator in the Sun wd see the Earth coincide with the sign Libra of the Ecliptic the inhabitants of the Earth see the Sun in the direction of series v. At that time the circle of illumination of the light & dark keeps parallel

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thro' the poles or $V S$ Therefore the Earth in its diurnal rotation round its axis at this time, has every part of its surface as long in the light as in the shade viz: the Days & Nights are equal the Sun at that time being successively vertical to the Equatorial parts of the Earth.

2. As the Earth proceeds in its Orbit from West to East along the Signs III & IV & the Sun is seen to advance along the Signs $VIII$ & IX & gradually becomes vertical to those parts of the Earth which are on the North of the Equator, So that when the Earth is in VS the Sun appears in IX & is perpendicular to those parts of the Earth which are under the tropics viz about $23^{\circ} 28'$ North of the Equator. Therefore the Inhabitants of the Northern Hemisphere will enjoy Summer on account of the Solar rays falling more perpendicularly, & they will have their days longer than the nights, in proportion as they are more distant from the Equator, but those whose latitude exceeds $66^{\circ} 32'$ will have constant Day & of course will see the Sun at all hours; for by inspecting the figure that $x y$ within the Polar circle in that part of its surface illuminated by the Sun.

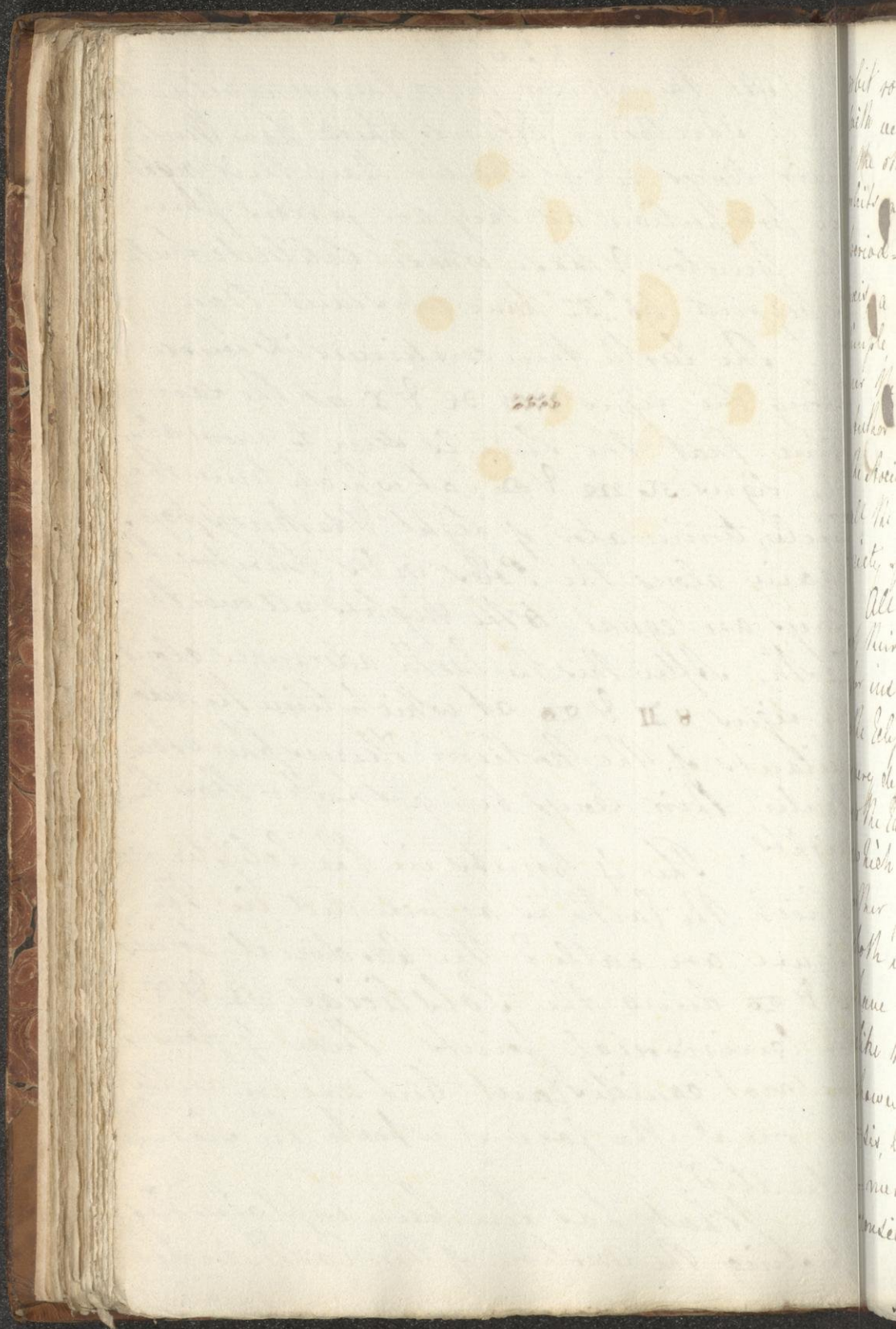
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At the same time the Inhabitants of the Southern Hemisphere have Winter their Days being shorter than their Nights in proportion as they are farther from the Equator & those whose latitude south is beyond $66^{\circ} 32'$ have constant Day.

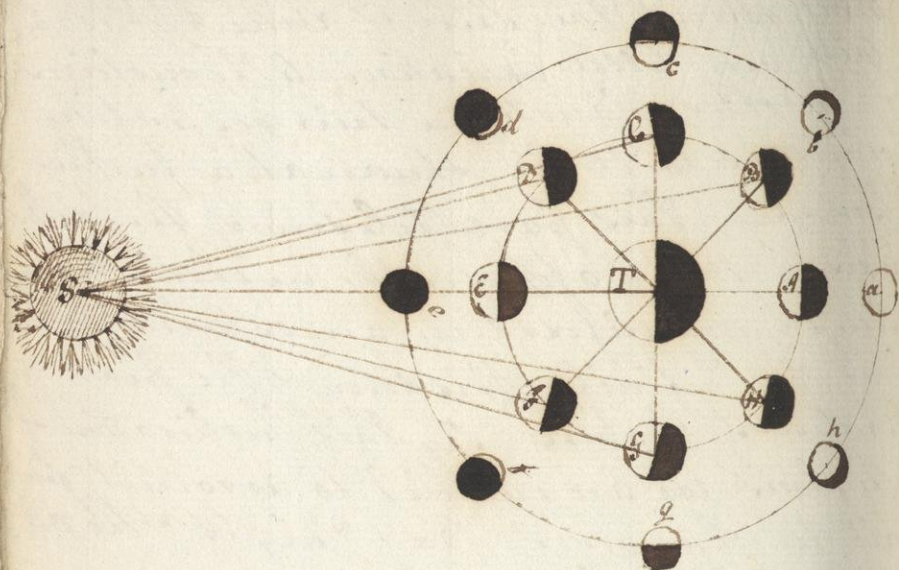
The Earth then continues its course along the signs ~~xxx~~ π & γ at the same time that the Sun is seen to move along the signs α π & ϖ ; at which time the Circle, terminator of light & darkness passes again along the Poles on & so therefore the Days are equal to the Nights all over the Earth. After this the Earth advances along the signs ν π & φ at which time the Inhabitants of the Northern Hemisphere have Winter their Days being shorter than their Nights. The 4 points in the Ecliptic in which the Earth is represented in the figure are called the Cardinal points, π & φ being the Solsticial, α & γ the Equinoctial points. These 4 points are not equidistant but are in an Ellipse in one of the foci of which the Sun is situated.

What has here been explained respecting the motion of the Earth in her



orbit round the Sun is alike applicable with very little variation to the motions of the other planets in their respective orbits as will be shown at a further period - This parallelism of the Earth's axis, a thing apparently in itself so very simple, manifests in an especial manner the Infinite Wisdom of the Great Author of Nature - By this inclination of the Axis we are enabled to account for all the changes of Day & Night & the variety of the Seasons.

All the Planets have peculiar motions of their Axes. The Axis of the planet Venus for instance is said to be in the plane of the Ecliptic which produces in her changes very different from those which belong to the Earth. From the great analogy which subsists between the Earth & the other planets of this Solar system both in their figure & motions, Men have been led to conclude that they like the Earth are inhabited. This however can be but a mere hypothesis, but it is one which from innumerable reasons carries with it considerable probability -

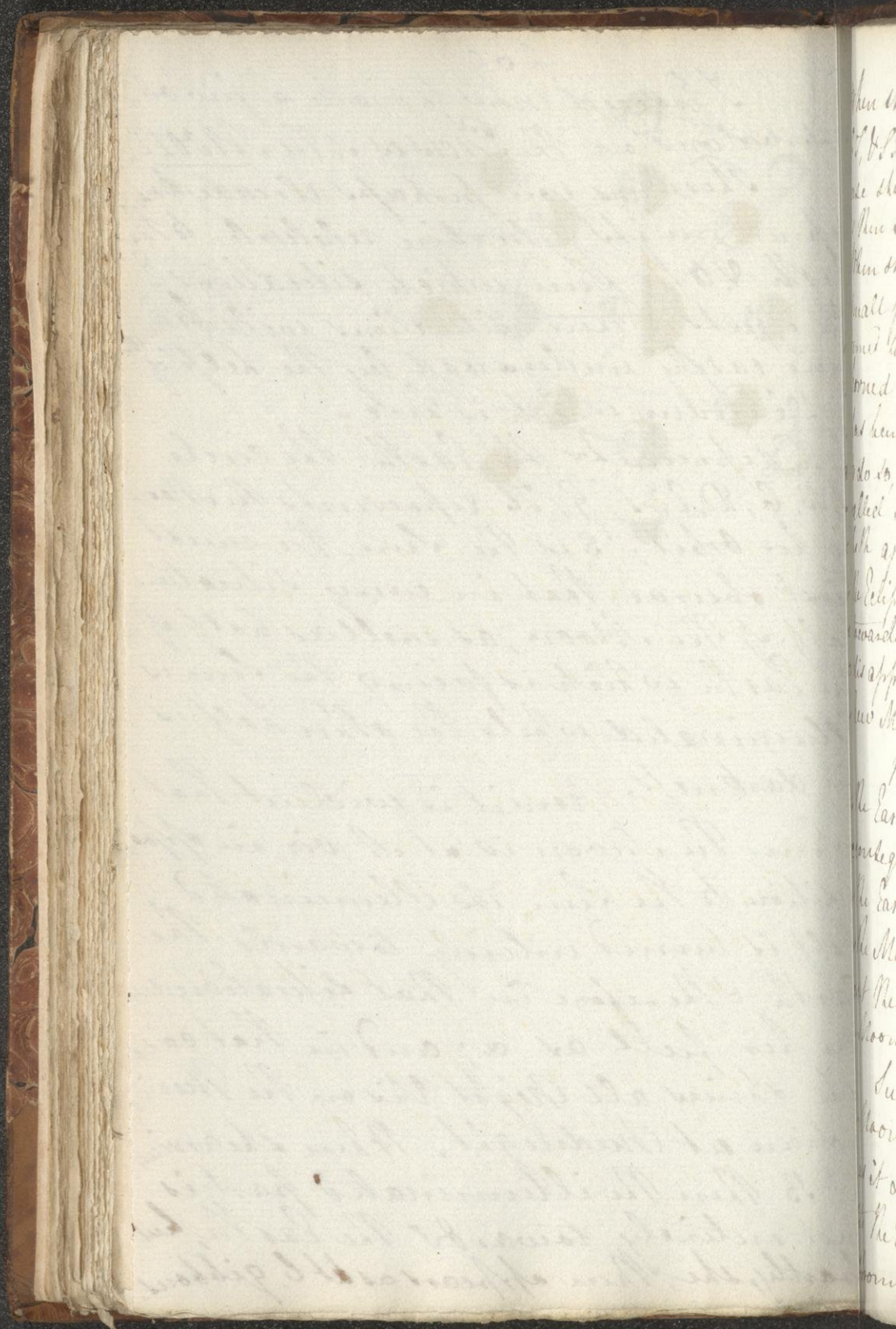


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I proceed now to make a few observations on the Phases of the MOON.

These as you perhaps already know depend on its situation relative to the Earth & to the Sun which situations & the effects of their situations, will be much more easily understood by the help of a Diagram, which is here -

I represent the Earth the circle A, B, C, D, E, F, G, H represents the Moon in her Orbit, S is the Sun. We must first observe that in every situation half of the Moon, as well as half of the Earth which is facing the Sun is illuminated while the other half is in darkness. Now it is evident that when the Moon is at A or in opposition to the Sun, its illuminated half is turned entirely towards the Earth therefore in that situation we see her full as a; and in that case she shines all Night & is on the Meridian at Midnight. When she arrives at B then the illuminated part is not entirely towards the Earth, but partly, she then appears as at b. gibbous.



When she appears at C then the line
 C T, & S T form a right Angle, in which
 case she appears as at c bisected, she
 is then said to be in her last quarter -
 When she has arrived at D then but a
 small part of her illuminated side is
 turned to the Earth, we then see her
 horned as at d. All this time the Moon
 has been waning or decreasing & continues
 to do so, till it arrives at E, & which is
 called its conjunction with the Sun they
 both appearing in the same point in
 the Ecliptic. The dark part is now entirely
 towards the Earth & of course the Moon
 disappears, at which time it is called
 New Moon.

When the Moon is at A, then
 the Earth is between it & the Sun, in
 consequence of which the shadow of
 the Earth covers the whole disc of
 the Moon & an Eclipse is the consequence
 at the same time a Spectator in the
 Moon wd lose sight of the Sun.

Such then are the Phases of the
 Moon, & such their cause. Depending
 as it does on its situation relatively
 to the Earth & Sun, the whole arises
 from the simple property in Optics

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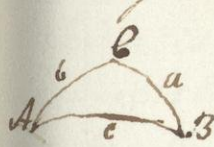
 $b = \frac{1}{2}$
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That the Angle of Reflection is equal to the angle of Incidence viz the Angle which the Sun makes with the Moon in reflection is equal to the angle made by the Incidentat Line from the Moon to the Earth.

Before proceeding to the explanation of the Physical causes of the Motion of the Celestial Bodies, I shall mention a property in spherical Trigonometry which it is necessary to keep in mind!

In a Triangle having the 3 sides to determine the Angles of the Spherical Triangle.



Let the sides of the spherical triangle ABC be denoted by the Letters a, b, c

Then to find $\angle C$ we have

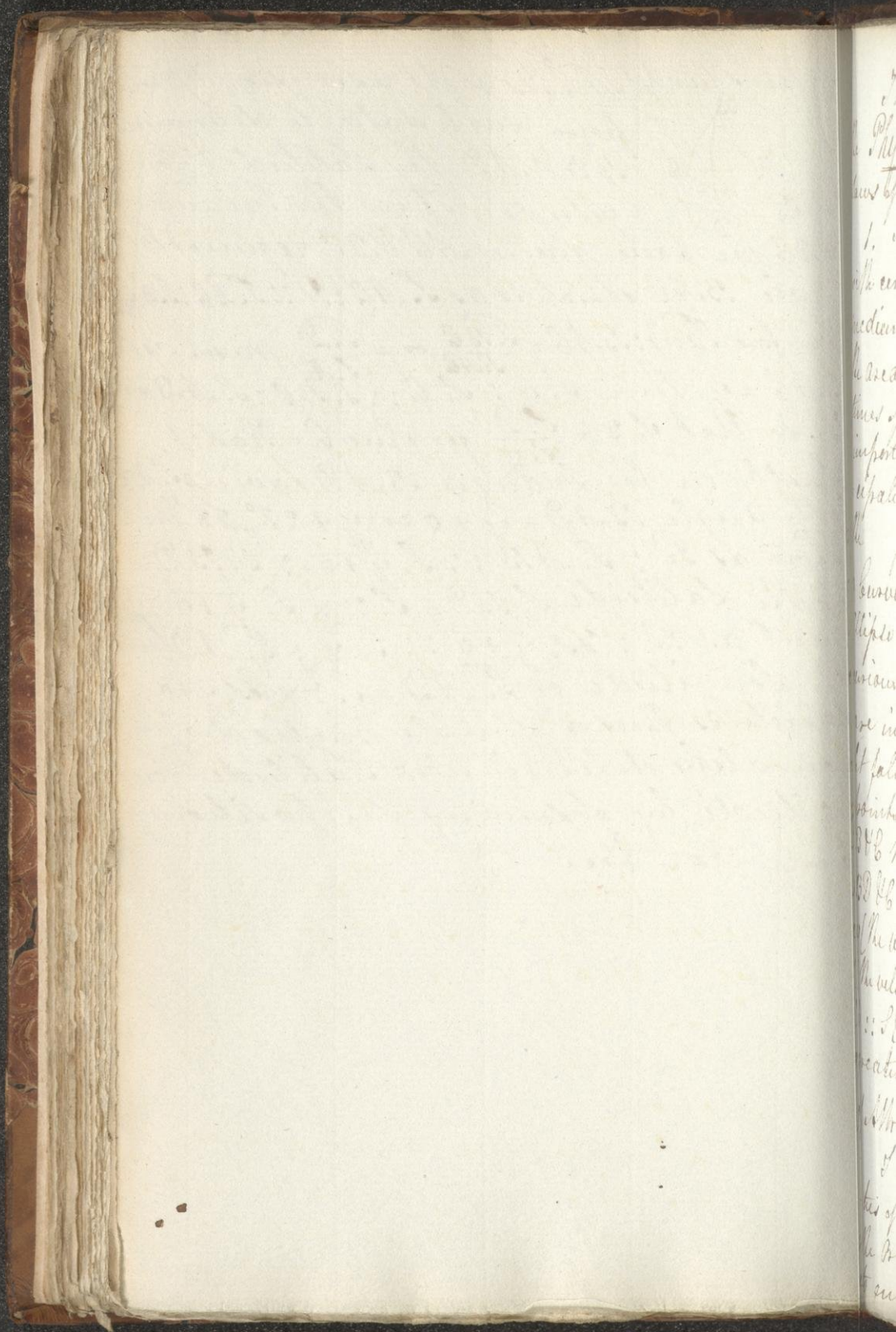
$$\frac{1}{2}C = \sqrt{\frac{\text{Sine } \frac{1}{2}(c+b-a) \times \text{Sine } \frac{1}{2}(c-b+a)}{\text{Sine } a \times \text{Sine } b}}$$

From this simple formula we obtain a method calculating the Latitude and Longitude by have the declination & right ascension given.

Thus suppose AC (over leaf) the Ecliptic AD the Equator, B any Star, BD is the Latitude, AD the Longitude. If then we have given BC the Declination

[illegible]

of the Star V. A. C. its right ascension required
 from such data to determine
 B. D. & A. D. the Latitude & Lon-
 = gitude. If in the Triangle
 A. B. C. we have given A. C. & B. C. required the
 angle B. A. C. which is as $S. A. C. :: R. :: T. B. C. : T. B. A.$
 Therefore $Tang. : B. A. C. = \frac{T. B. C.}{T. A. C.}$ or $= \frac{T. D.}{T. A.}$ next calc
 $B. A. C. = y$. Then $S. y : S. C. :: S. B. C. : S. A. B.$ or
 x so that $S. x = \frac{T. D.}{T. y}$ for sine $C = Rad. : 1$.
 Lastly in the Triangle B. A. D. having $A. B.$
 & the angle B. A. D. $= y + 0$ or $y + 23^{\circ} 28'$ &c
 Then as $R. : S. A. B. :: S. y + 0. : S. B. D.$
 so the Latitude $S. B. D. = S. x \times S. y + 0$
 next as $R. : Cos. y + 0. :: T. x : T. A. D.$ then
 the Longitude or $T. A. D. = T. x \times Cos. y + 0.$
 which is thus a simple method of ac-
 curately determining Latitude & Lon-
 = gitude, by observing the position of
 any Star &c



I now proceed to consider minutely
the Physical Causes of the 3 great
Laws of Kepler.

1. For the Orbits of Bodies moving
with certain velocities in non-resisting
mediums about a centre of Attraction
the areas which are described in equal
times of the revolution are equal. This
important proposition has been ante-
cipated & demonstrated already in
the page of these lectures. This becomes
a Curve returning into itself forms an
ellipse. In this there is also another
curious property viz That the velocities
are inversely as the perpendiculars
let fall on the Tangents of the two
points of the Curve. If at
B & C the 2 points Tangents D
BD & CE be drawn then $SD \times C$
(the velocity at B) = $SE \times C$
(the velocity at C) therefore $v :$
 $\delta :: SE : SD$ so that the velocity is always
greater the nearer we are to S, the centre
of Attraction.



I shall now mention a few propo-
sitions of this Curve the Ellipse which is
the Orbit of all the Planets previous
to entering into the investigation of this



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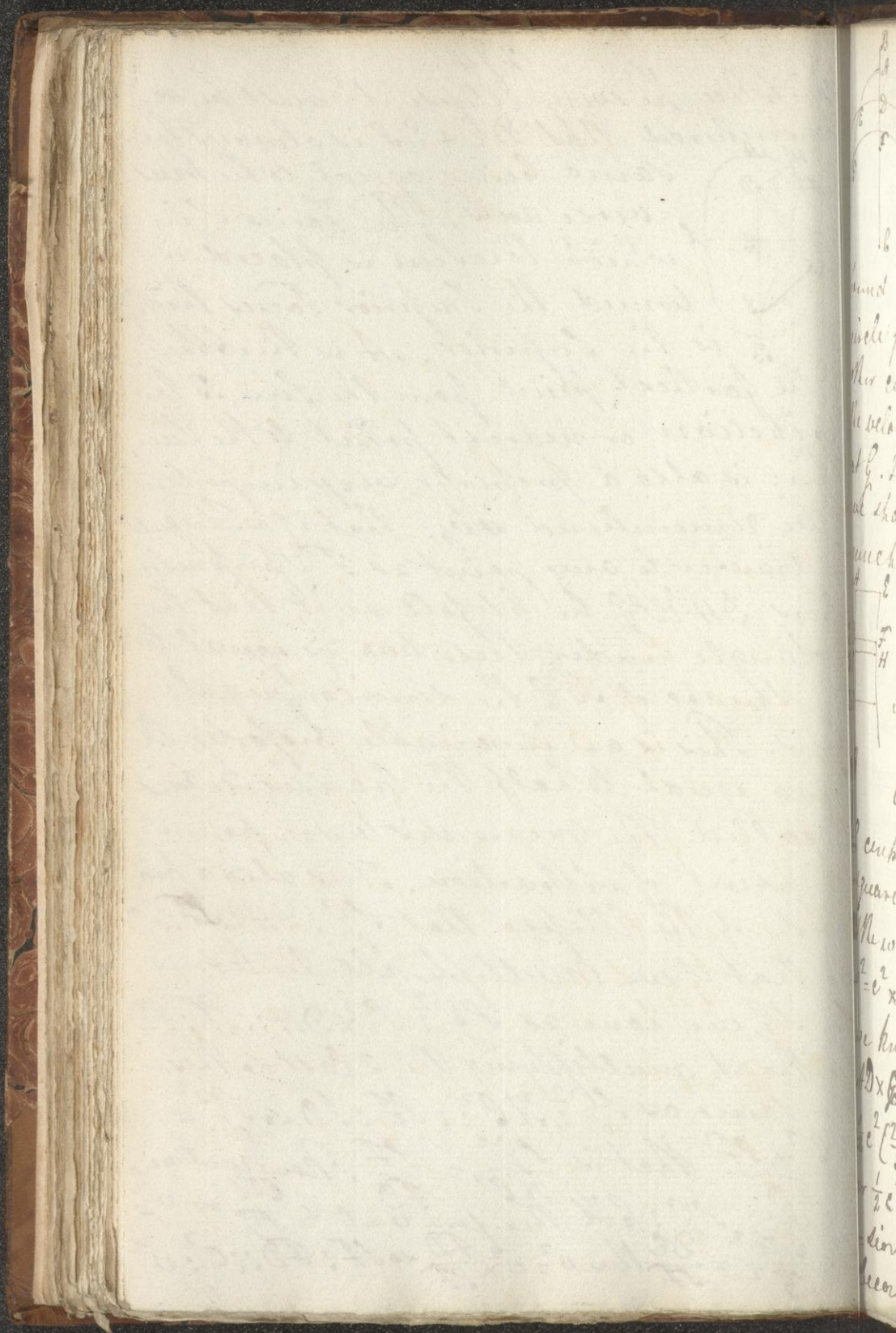
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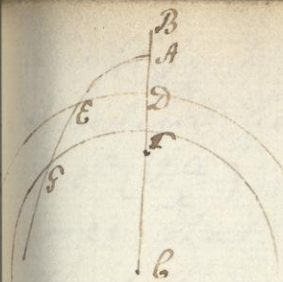
Orbits &c. In every Ellipse it must be re-
 membered that $DE + ES$ is always the
 same being equal to the trans-
 verse axis. The Focus S in
 which the Sun is placed is
 termed the Superior Focus the other
 is the Inferior. A is the Aphelion
 or the farthest point from the Sun, B the
 Perihelion or nearest point to the Sun.

There is also a property very important
 to be remembered viz. that if a Tangent
 be drawn to any point as F & perpendiculars
 GH & GC be let fall on it, that the
 rectangle under these two is equal to
 the square of HC the semiconjugate
 axis. This is an invariable property & HC
 being equal to half the transverse axis
 is called the mean distance from S
 the point of Attraction. It is also a pro-

perty of the Ellipse that $SG : DH :: PF : FD$
 so that if we multiply all the terms
 by SG we have as $SG^2 : SG \times DH :: PF : FD$
 without multiplying the 2 last so this is
 the same as $SG^2 : HC^2 :: PF : FD$ so $SG^2 =$
 $HC^2 \times PF$ that is $\frac{SG^2}{HC^2} = \frac{PF}{FD}$. Now we know

$c : v :: SG : CH$ therefore $v = c \times \frac{SG}{CH}$ or $v =$
 $c \times \frac{CH^2}{SG^2} = \frac{DE}{SF}$ Now $v^2 = c^2 \times \frac{FD}{PF}$ so $PF : FD :: c^2 : v^2$





If the velocity of any body in falling to D for example in this figure be equal to its velocity in the curve at E which 2 points are found by describing a portion of a circle from the centre C, then let any other circle as GF be described and the velocity at F will be equal to that at E. This proposition being known we shall proceed to the next with much greater facility.

Let AB be any line directed downwards in B, take any other point in AB as D. Then call the velocity at B = C of D = v & suppose $v = c \times AD$, or $v^2 = c^2 \times \frac{AD}{CD}$. Then the centrifugal force is inversely as the square of the distance for call CD = x & the whole line 2a then the expression $v^2 = c^2 \times \frac{AD}{CD}$ becomes $v^2 = c^2 \times \frac{2a - x}{x}$ but we know that $v^2 = 2 \cdot AD \cdot DE$ then $\frac{1}{2} v^2 = AD \times DE$ so that $AD \times C = \frac{1}{2} c^2 \times \frac{2a - x}{x} = \frac{1}{2} c^2 \left(\frac{2a}{x} - 1 \right)$ then $AG \times C = \frac{1}{2} c^2 \left(\frac{2a}{x - \frac{1}{2}} - 1 \right)$ or $\frac{1}{2} c^2 \left(\frac{2a}{x - \frac{1}{2}} - \frac{2a}{x} \right)$ which is the expression for the space DGH; this becomes $\frac{1}{2} c^2 \left(\frac{2ax - 2ax + 2ax}{x^2 - x\frac{1}{2}} \right)$ or to

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simplify it farther $\frac{1}{2} c^2 \left(\frac{2ax}{x^2} \right) = DF \times x$
 dividing both sides by x it becomes
 $\frac{1}{2} c^2 \left(\frac{2a}{x} \right) = DF$ but $DF =$ the accelerating
 force that acted at D so $= \frac{1}{2} c^2 \left(\frac{2a}{x} \right)$
 Thus then $DF = \frac{ac^2}{x}$ which will be
 found to be the very result of the pre-
 =ceding proposition that the centripetal
 =tal = the accelerating force at such
 a point in the curve.

In this figure before us we have
 v^r the velocity at the point G is equal
 to c^2 the mean velocity multi-
 =plied into $\frac{2a}{x}$. b^r then $\frac{2a}{x}$
 $= c^2 \times \frac{2a}{x}$ or $= \frac{2ac^2}{x}$ which is



the same expression for the body
 moving in an Ellipse as that of a body
 descending in a straight line. This
 then is the great Law by which the
 Planets are preserved in their orbits
 viz. that the forces that act on the body
 so as to preserve it in its Elliptical
 orbit are as the Squares of the distances
 of the body from the centre of Attraction.

The general expression for Gravity
 is $\frac{c^2}{x}$ at any point x The expression
 for the Gravity is $\frac{ac^2}{x^2}$ This is one
 another

X from this page on to p. 288, where this
subject is continued; as far as the 297 page.
Return to the consideration of the figure
of the Earth.

in terms of c is $\frac{2\sqrt{1-e^2}}{e^2}$ from which 2
 expressions it is evident that $ac = m^3$
 $\& c^2 = \frac{m^3}{a}$ & therefore $c = \frac{m^{3/2}}{a^{1/2}}$. It is a
 curious property which I may here
 remark that in the last figure AB
 $\& C.B$ or the semi-conjugate axis multi-
 -plied into the semi-transverse, & that
 into the area of the Ellipse is equal to
 the rectangle under the square of the
 Radius & the Area of the Circle; which
 shows strongly the great connection
 subsisting between the circle & the Ellipse.

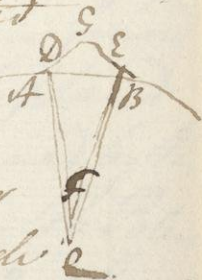
This leads us to consider The Figure of The Earth

This is not quite spherical but is
 a spheroid & an oblate spheroid. It
 resembles exactly the figure of a
 well proportioned orange. This disparity
 in its figure, its extension at the
 Equatorial parts & its compression
 at the poles is evidently owing to the
 centrifugal force. And by observation
 it has been found to be 330^{th} part
 flattened at the Poles. Also that the

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centrifugal force at the Equator is to the force of gravity as $1:289$ or $f:g::1:289$ so that $f = \frac{g}{289}$ & by Mr. Laplace's it has been found that were the Earth homogeneous the centrifugal force would be $= 289 \times \frac{5}{4}$. It is known that at the radius at the Equator: f that at the pole $1:231:230$. If the Earth were homogeneous.

Experiments on this point were made on Mount Schiehallion in Perthshire by Br. Maskelyne of an interesting nature. The great attraction which exists in the elevated parts of the Earth draws the plummet entirely from its proper perpendicular position; that is drawn from that direction which if produced would pass thro' the Earth's centre. He experimented on both sides of this Mountain & found the plummet always drawn from its perpendicular so that the 2 lines on each side instead of meeting if produced, at C, they would meet about F, supposing that the Attraction of the Mountain



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he found Thus $1784:1:39$: A from
which he found that the density of
the whole Earth was as 4.4815
Water as 1. This however was a very
coarse estimate for the density of
Mount Shehalore instead of 2.5 (the
estimate of Dr M) is 2.78 as deter-
mined by a series of Experiments
in which I had the satisfaction to
assist. Newton foreseeing this, came
to the conclusion that the Radius
at the Equator: that at the pole::
 $331:330$.

The degrees of the Meridian as
we have already remarked vary
much in their magnitude. By
observation it has been ascertained
that the degree under the Equator
is just equal to 60.435 fathoms.
In Latitude 45° it has been found
to be 60.757 fathoms. The Lapland
where a degree was measured by the
French it was found under the Polar
circle in $66^\circ 20'$ to be 60.952 fathoms.

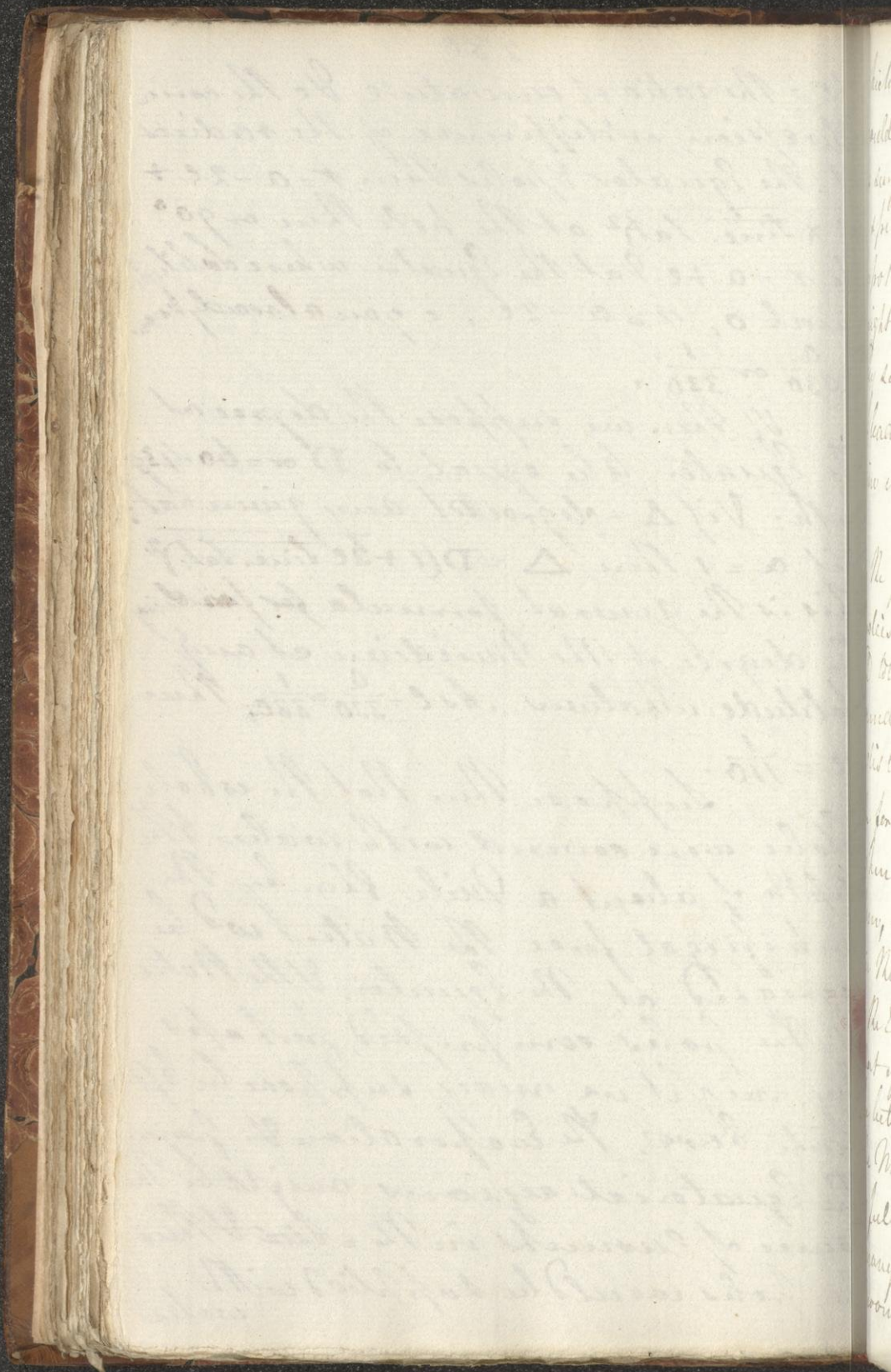
If then we wish to ascertain
precisely the radius of curvature
suppose a to be equal to the semidiameter

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$r =$ The ratio of curvature & the compression or difference of the radii at the Equator & poles then $r = a - 2c + 3c \times \text{Time. Lat}^2$ at the pole then or 90° the $r = a + c$ & at the Equator where latitude equal 0, $r = a - 2c$, c you already know
 $= \frac{a}{330}$ or $\frac{1}{330}$.

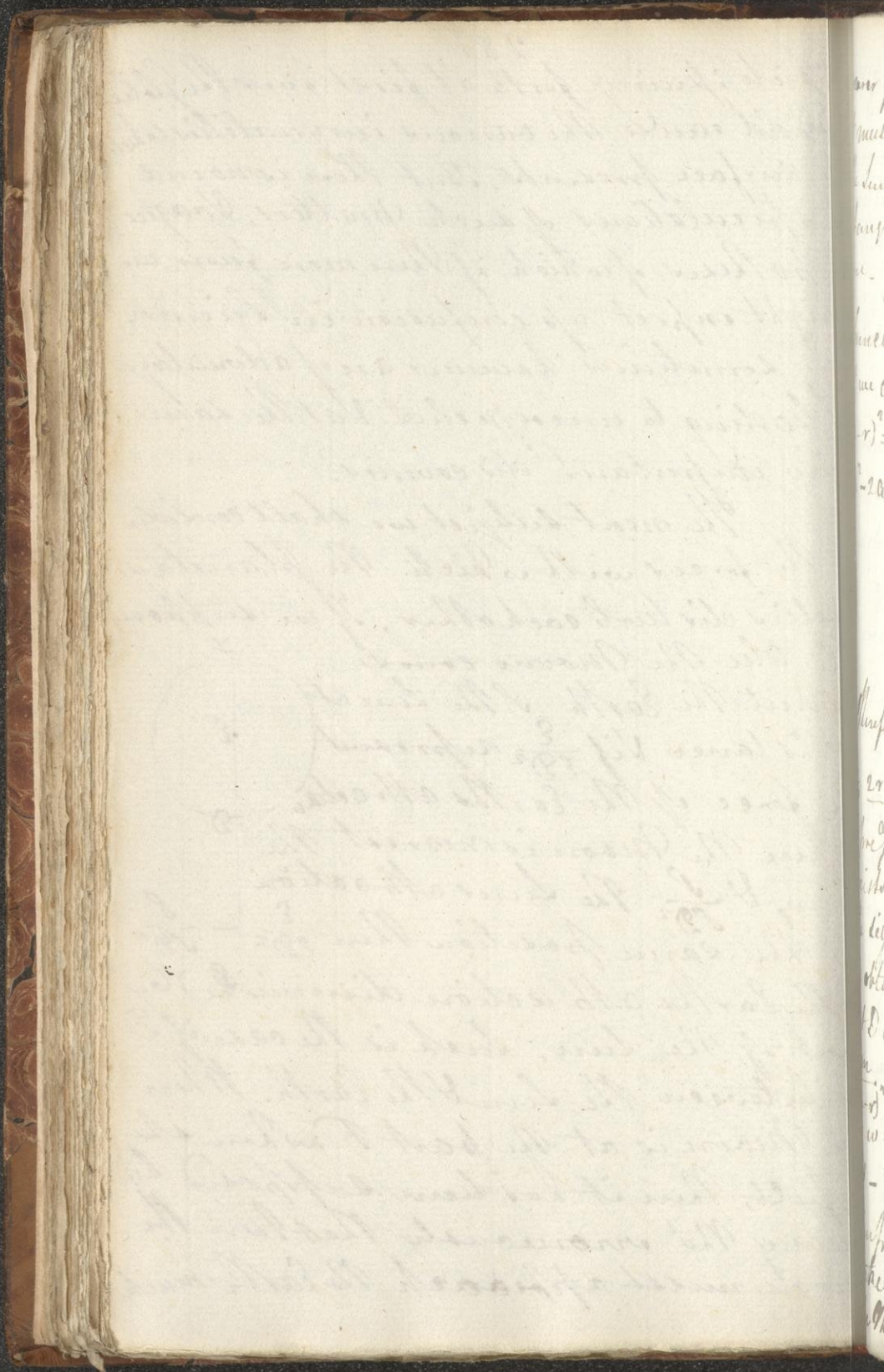
If then we suppose the degree at the Equator to be equal to $D = 60.438$ fath: Vis Δ - degrees at any given Lat: Vis $a = 1$ then $\Delta = D(1 + 3c \text{ Time. Lat}^2)$
 This is the general formula for finding the degree of the Meridian at any latitude whatever. As $c = \frac{a}{330}$ or $\frac{1}{330}$. Then $3c = \frac{1}{110}$.

Suppose then that the whole Globe were covered with water & the depth of about a mile then by the centrifugal force the water wd be increased at the Equator, & the water at the poles compressed, perhaps they might we may suppose be left quite bare. The Evaporation & from the Equatorial regions might be the source of currents in the Sea & thus the poles would be supplied with water



which issuing forth at first over the globe
would cause the various inequalities which
its surface presents. But there is no end
to speculations of such matters, Vague
hypotheses of which if there were fewer we
might expect less confusion in Science.
They sometimes however are of advantage
as leading to unexpected & at the same
time important discoveries.

The next subject we shall consider
is the forces with which the planetary
bodies disturb each other. If we suppose
FD to be the Moon's course
round the Earth & the Sun at
a distance $\frac{E}{ED^2}$ represent
the force of the Earth's attraction
when the Moon is nearest the
Sun, & $\frac{S}{SD^2}$ the Sun's attraction
in the same position then $\frac{E}{ED^2} - \frac{S}{SD^2}$
= the Earth's attraction diminished by
that of the Sun. Such is the case if D
be between the Sun & the Earth. When
the Moon is at the part F when she
is full, then it has been supposed by
many tho' erroneously that then the
Moon must approach the Earth much



nearer for it is farther from the Sun, but it must be remembered that now the Sun acts power on Earth in changing us it were from the Moon here - At the new Moon the distance

is thus expressed $\frac{m}{a-r}$ now if we actually perform this division

$$(a-r)^2 = a^2 - 2ar + r^2 \text{ now}$$

$$\frac{a^2 - 2ar + r^2}{m} = \frac{a^2}{m} - \frac{2ar}{m} + \frac{r^2}{m}$$

$$\frac{a^2}{m} = \frac{a^2}{m} + \frac{2ar}{m} - \frac{r^2}{m}$$

$$\frac{2ar}{m} = \frac{2ar}{a^2} + \frac{r^2}{a^2}$$

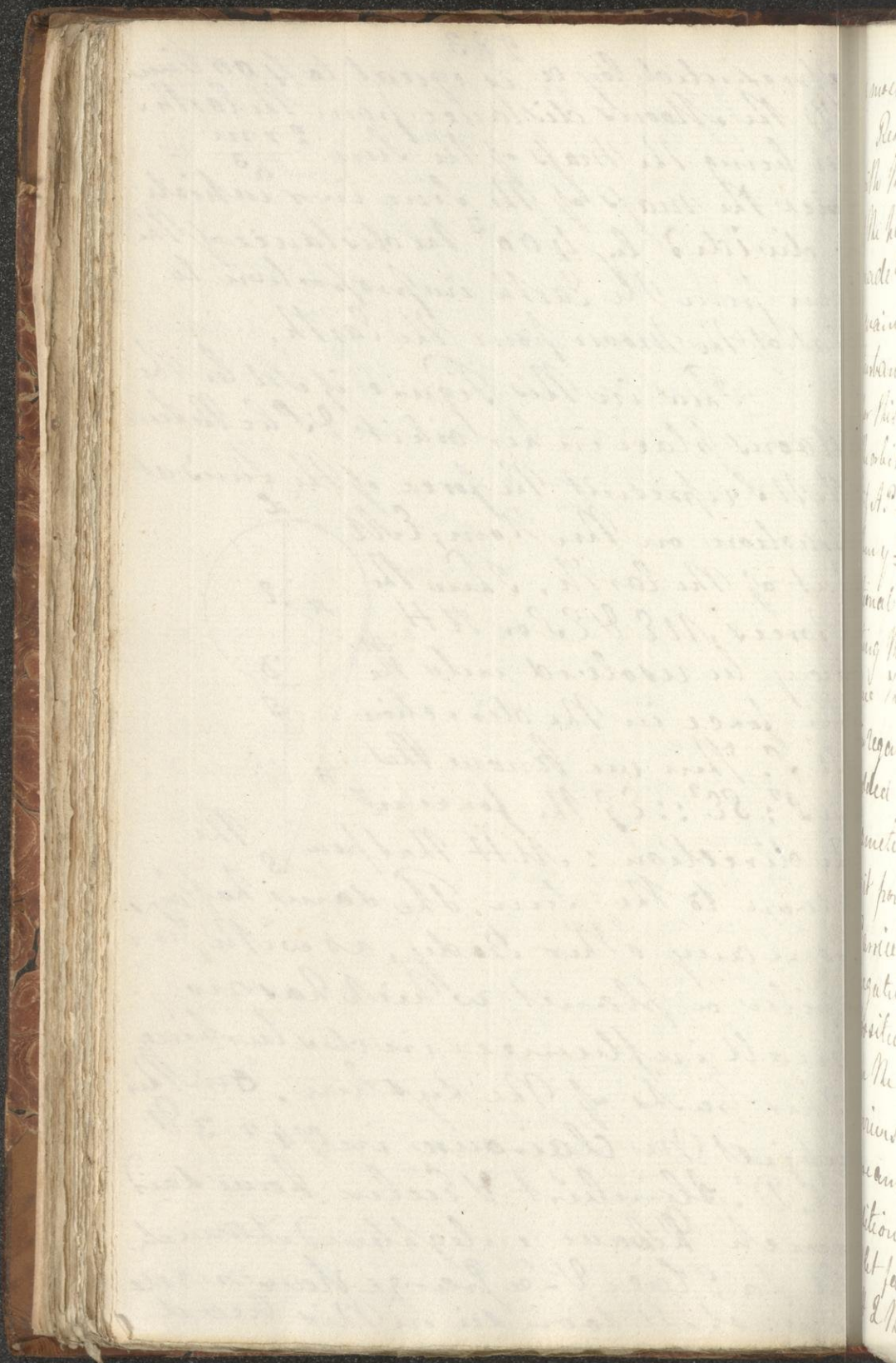
$$\frac{2ar}{a^2} = \frac{4ar}{a^3} \text{ see}$$

Therefore $\frac{m}{(a-r)^2} = \frac{m}{a^2} + \frac{2ar}{a^3}$ or ordinarily to $\frac{2ar}{a^3}$ which is then the proper correction for the distance which exists at new Moon. At full Moon the signs are changed, the same result is obtained with a negative sign. So at D in the last figure the distance $\frac{m}{(a-r)^2}$ is expressed by $\frac{2ar}{a^3}$ at E when the Moon is full the distance is $-\frac{2ar}{a^3}$. This comes to be a very important proposition, when we take into consideration the tides in the last figure the distance ED

represented by a is equal to 400 times
 ED the Moon's distance from the Earth.
 In being the mass of the Sun $\frac{27m}{a^3} =$
 twice the mass of the Sun in r which
 is divided by 400^3 the distance of the
 Sun from the Earth in proportion to
 that of the Moon from the Earth.

Now in this Figure if M be the
 Moon's place in her orbit & S the Medium
 of M represent the force of the Sun's at-
 traction on the Moon; EM
 that of the Earth. Now the
 2 forces ME & ES or HH
 may be resolved into the
 one force in the direction
 MS . Now we know that
 $MS^2 : SE^2 :: ES$ the force in
 the direction: MS that from S the
 Moon to the Sun, The same happens
 from any other Body, as with Ju-
 piter or planet which has no
 small influence in disturbing
 other parts of the system. On this
 subject Mr Clairaut in 1742-3 &
 M. D'Alembert & Euler have said
 much & have enlightened it much.
 M. La Place & Le Jeune have made
 as we shall soon see on this head





a most sublime discovery.

Respecting the different disturbances with the Planets on each other a discovery of the highest importance has of late been made by which we are enabled to explain the nature of these disturbances. The expression

for this in any point of P

The orbit as P is $m \times P \cdot A \cdot P$

If $A \cdot P$ be expressed by y , 2

then $y = At$ (as are propor-

tional to the time) $+ m \times \text{Line } x$ (x deno-

tating the Sine arc) $+ m' \times \text{Line } x' + m'' \times \text{Line } x''$



These additions vary considerably in regard to the Moon &c. if there are added one after another. Draw the diameter AD & let fall PE perpendicular to it from this point in the orbit. When P arrives at P then the sine becomes negative. So there is thus as on any positive signs on one side as negative on the other so that when the Body arrives again at A all the disturbances are annihilated is in the exact position as at first. Let $AD = 2AP$ & let fall EP perpendicular on AD At 2 the disturbance will be doubly great.

See page. 244

The period in which Saturn commences
Vannihilates at length his different dis-
turbances from Jupiter is 900 years in
which time he finds himself exactly
situated as at that period before.

None of the grandest effusions either of
the Poet or Orator can possibly describe
in higher terms the infinite Wisdom
& power of the Almighty Creator of all
than this simple algebraical equation
does. If instead of $m \times \text{line} \times$ it were $m \times$
 $\times$ Tangent &c then the greatest difference
would happen & the end of Creation wd
be invaded - How vain then must be
the attempts of those who endeavour to
impute to necessity the mighty works of
Creation -

From all this it may be asked,
What account can we give of
Comets - These are found by Mi-
nute inspection to be Bodies extremely
rare so much so indeed that Stars
have been distinctly seen between
them. This move in general in very
irregular orbits & fly off to immense
distances, & probably come in contact
with other Systems & are thus driven
out of their course.

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of this there was an instance not long ago. It is a curious fact that the disturbance of the Planets on one another was known to Newton, but the Laws by which these were brought about & their returning again & annihilating their disturbance was a fact he knew not, nor was he to be expected to know it, he supposed that the hand of the Author of Nature was at times requisite to correct his irregularity. This Leibnitz the adversary of the Newtonian Philosophy caught at & considered it as an opinion highly inconsistent with the Supreme Wisdom which conclusion has since been verified. But the great opinion of Newton that there are disturbances among the Planets was first demonstrated. Thus then so far both were right & the cause of this disturbance annihilating itself in process of time was the grand discovery of Laplace.

The orbit of our Earth is at present constantly diminishing as is easily seen

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from comparing modern observations with those of the most ancient Indians of whom we have any account - Thus then all these irregularities, as well as the disturbances just mentioned, and that of the obliquity of the Ecliptic have received from La Place's discovery a most complete & satisfactory explanation. All are as the vibrations about a certain mean point to which they always return - Thus the system of creation is not like a human legislature, carrying decay in its very elements. Time has no effect on this - It will endure to eternity, whole and unimpaired, unless the Will of its great Author put an period to its never fading existence.

It is curious to remark that without the aid of astronomical knowledge we could be very little acquainted if indeed at all with the Geography of our Earth. The first of the Ancients who zealously applied himself to the study of the Stars for this end was Hipparchus who first produced a regular catalogue of Stars. He was the first who determined the position of any place by Latitude & Longitude.

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Latitude was so termed ~~from~~ ^{from} its extending over the breadth as it were of the Earth, Longitude was the contrary direction, both which being ascertained they determined the position of a certain point. Latitude is easily found being reckoned from a part in the Heavens which never varies but is common to all parts of the World. The other Longitude is however very different, and — continued page 297

From p 277 — continuation
indeed strictly speaking the latter is but an extension of the circle — Call the periodic time p . Then $\frac{2\pi ab}{p} = p$ or $p = \frac{2\pi a}{c}$ but $c = \frac{m^{\frac{3}{2}}}{a^{\frac{1}{2}}}$ so that substituting this value of c we have the $p = \frac{2\pi a}{\frac{m^{\frac{3}{2}}}{a^{\frac{1}{2}}}}$ or $p = 2\pi a \times \frac{a^{\frac{1}{2}}}{m^{\frac{3}{2}}}$ & $p^2 = \frac{4\pi^2 a^3}{m^3}$ p being the periodic time of one planet, p' that of another do with a the distance then $p^2 : p'^2 :: a^3 : a'^3$ with regard to the Moon then we say $p^2 : p'^2 :: 8 : 1$ therefore $p'^2 = \frac{p^2}{8}$ & cause $p'^2 = \frac{p^2}{2\sqrt{2}}$ that of $p' = \frac{p}{4\sqrt{2}}$. The Moons periodic time is just $29^d \ 12^h \ 43^m$ Now we know that $p^2 = \frac{a^3}{m^3}$ and $p = \frac{a^{\frac{3}{2}}}{m^{\frac{3}{2}}}$ const.

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on $\frac{3}{2} = \frac{a^{\frac{3}{2}}}{p}$ whence it evidently follows
 that $m^{\frac{3}{2}} : m'^{\frac{3}{2}} :: \frac{a^{\frac{3}{2}}}{p} : \frac{a'^{\frac{3}{2}}}{p'}$ & therefore
 $m^3 : m'^3 :: \frac{a^3}{p^2} : \frac{a'^3}{p'^2}$ which confirms
 most satisfactorily the preceding conclu-
 sion viz that the squares of the periodic
 times are as the cubes of the distances, the
 great Law discovered by Kepler, forming
 as it does the very basis of Physical Astronomy.

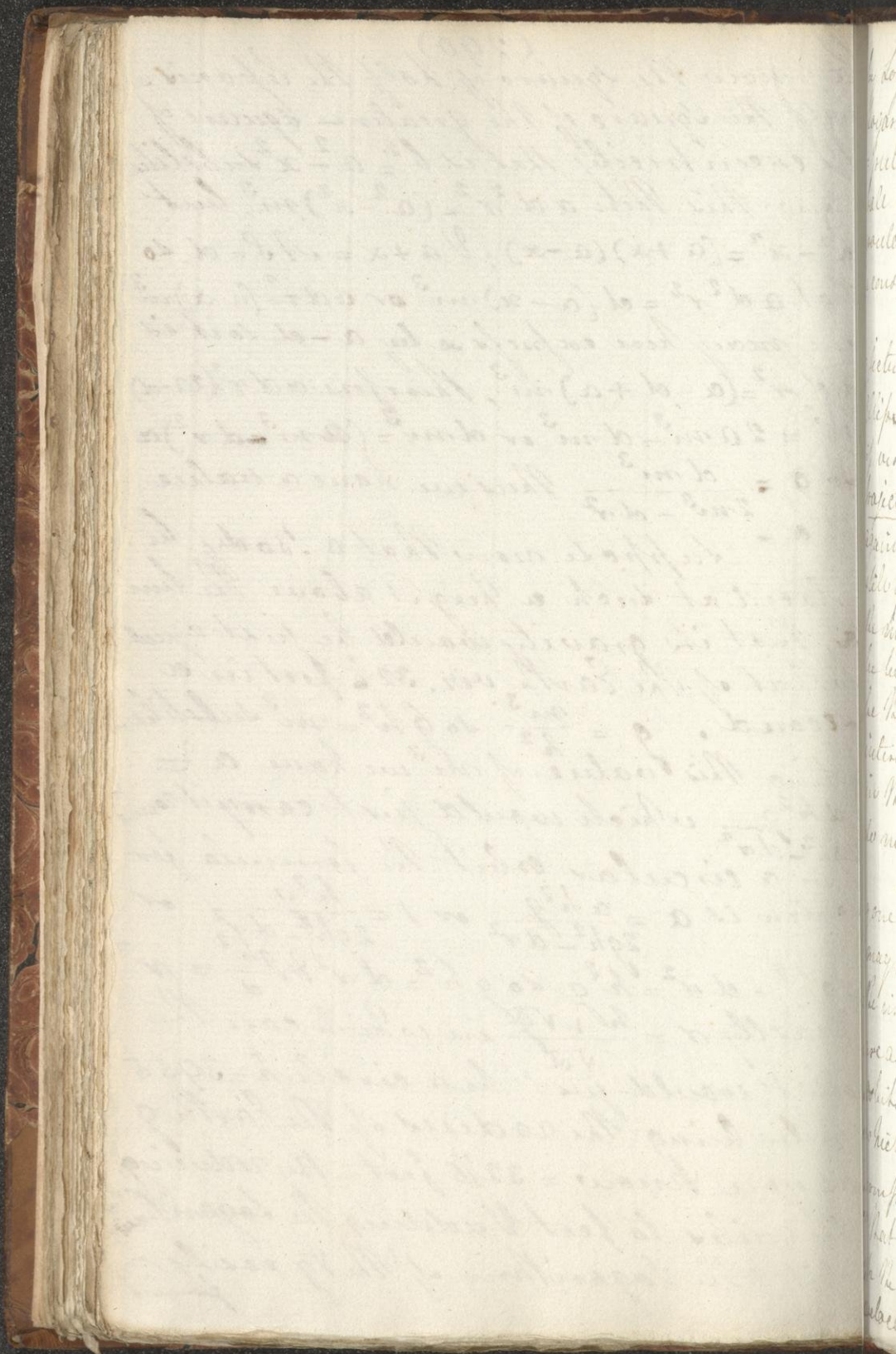
Let us now apply the foregoing pro-
 position to the solution of a very im-
 portant problem in Physical Astro-
 nomical Astronomy - Having given
 the projection & the initial velocity
 of any Planet it is required to deter-
 mine the orbit of that Planet.

Let it be projected in a direction
 A.B at right angles to A.P.
 Let C be the mean distance
 that is half the transverse
 axis, which transverse axis
 we call a, let S.A = d, c express
 the velocity at any distance as the
 mean distance, & call the velocity
 of the projection v; the Mass of the Sun
 we express by m. Now $\frac{1}{2} d v = \frac{1}{2} b c$ or $d v = b c$
 be now as $c^2 = \frac{m^3}{a^3}$ & $c = \frac{m^{\frac{3}{2}}}{a^{\frac{1}{2}}}$ so $d v = b \times \frac{m^{\frac{3}{2}}}{a^{\frac{1}{2}}}$
 and $d^2 v^2 = b^2 \times \frac{m^3}{a}$ so $a d^2 v^2 = b^2 \times m^3$ call C =

1. $x^2 - 2x + 1 = (x-1)^2$
 2. $x^2 - 4x + 4 = (x-2)^2$
 3. $x^2 - 6x + 9 = (x-3)^2$
 4. $x^2 - 8x + 16 = (x-4)^2$
 5. $x^2 - 10x + 25 = (x-5)^2$
 6. $x^2 - 12x + 36 = (x-6)^2$
 7. $x^2 - 14x + 49 = (x-7)^2$
 8. $x^2 - 16x + 64 = (x-8)^2$
 9. $x^2 - 18x + 81 = (x-9)^2$
 10. $x^2 - 20x + 100 = (x-10)^2$
 11. $x^2 - 22x + 121 = (x-11)^2$
 12. $x^2 - 24x + 144 = (x-12)^2$
 13. $x^2 - 26x + 169 = (x-13)^2$
 14. $x^2 - 28x + 196 = (x-14)^2$
 15. $x^2 - 30x + 225 = (x-15)^2$
 16. $x^2 - 32x + 256 = (x-16)^2$
 17. $x^2 - 34x + 289 = (x-17)^2$
 18. $x^2 - 36x + 324 = (x-18)^2$
 19. $x^2 - 38x + 361 = (x-19)^2$
 20. $x^2 - 40x + 400 = (x-20)^2$
 21. $x^2 - 42x + 441 = (x-21)^2$
 22. $x^2 - 44x + 484 = (x-22)^2$
 23. $x^2 - 46x + 529 = (x-23)^2$
 24. $x^2 - 48x + 576 = (x-24)^2$
 25. $x^2 - 50x + 625 = (x-25)^2$
 26. $x^2 - 52x + 676 = (x-26)^2$
 27. $x^2 - 54x + 729 = (x-27)^2$
 28. $x^2 - 56x + 784 = (x-28)^2$
 29. $x^2 - 58x + 841 = (x-29)^2$
 30. $x^2 - 60x + 900 = (x-30)^2$
 31. $x^2 - 62x + 961 = (x-31)^2$
 32. $x^2 - 64x + 1024 = (x-32)^2$
 33. $x^2 - 66x + 1089 = (x-33)^2$
 34. $x^2 - 68x + 1156 = (x-34)^2$
 35. $x^2 - 70x + 1225 = (x-35)^2$
 36. $x^2 - 72x + 1296 = (x-36)^2$
 37. $x^2 - 74x + 1369 = (x-37)^2$
 38. $x^2 - 76x + 1444 = (x-38)^2$
 39. $x^2 - 78x + 1521 = (x-39)^2$
 40. $x^2 - 80x + 1600 = (x-40)^2$
 41. $x^2 - 82x + 1681 = (x-41)^2$
 42. $x^2 - 84x + 1764 = (x-42)^2$
 43. $x^2 - 86x + 1849 = (x-43)^2$
 44. $x^2 - 88x + 1936 = (x-44)^2$
 45. $x^2 - 90x + 2025 = (x-45)^2$
 46. $x^2 - 92x + 2116 = (x-46)^2$
 47. $x^2 - 94x + 2209 = (x-47)^2$
 48. $x^2 - 96x + 2304 = (x-48)^2$
 49. $x^2 - 98x + 2401 = (x-49)^2$
 50. $x^2 - 100x + 2500 = (x-50)^2$

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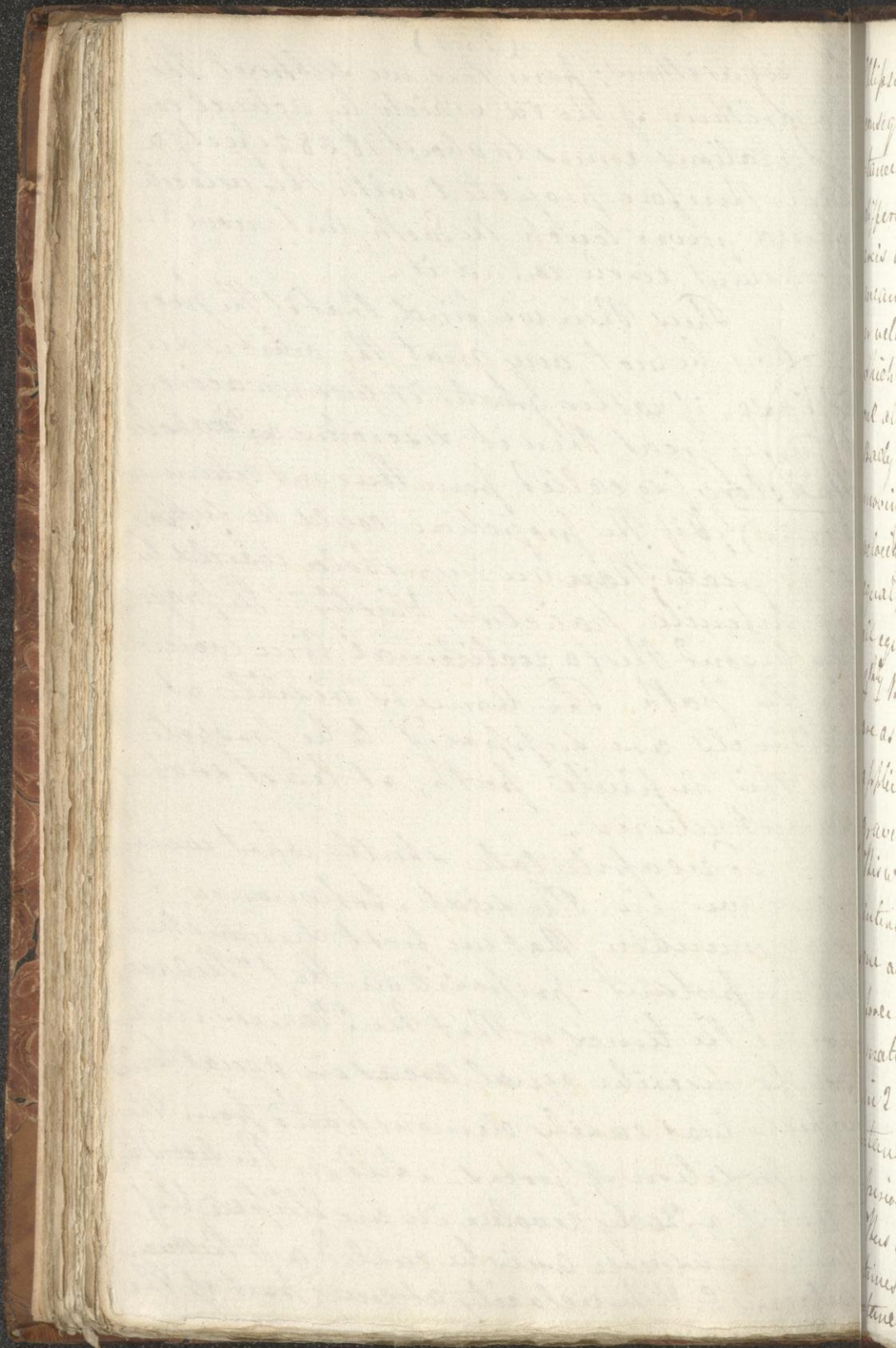
x ; now the square of half the latus rectum =
 half the square of the greater - square of
 the eccentricity that is $b^2 = a^2 - x^2$ substitute
 this then $a d^2 r^2 = (a^2 - x^2) m^3$, but
 $a^2 - x^2 = (a + x)(a - x)$; & $a + x = AP = d$ so
 that $a d^2 r^2 = d(a - x) m^3$ or $a d r^2 = (a - x) m^3$
 one may here express x by $a - d$ so it is
 $a d r^2 = (a - d + a) m^3$, therefore $a d r^2 = (2a - d) m^3$
 $m^3 = \frac{2a d r^2}{2a - d}$ or $d m^3 = (2a m^3 - d r^2) a$
 so $a = \frac{d m^3}{2m^3 - d r^2}$ Thus we have a value
 of a - Suppose now that a Body be
 placed at such a height above the Sun
 a , that its gravity would be just equal
 to that of the Earth viz. $32\frac{1}{8}$ feet in a
 second, $g = \frac{m^3}{h^2}$ so $g h^2 = m^3$ substitute
 this value of m^3 we have $a = \frac{d h^2 g}{2g h^2 - d r^2}$ which would just carry it round
 in a circular orbit the formula for
 which is $a = \frac{a h^2 g}{2g h^2 - d r^2}$ or $1 = \frac{h^2 g}{2g h^2 - d r^2}$ or
 $2g h^2 - d r^2 = h^2 g$ so $g h^2 = d r^2$ & $\frac{g h^2}{d} = r^2$
 & lastly $r = \frac{h \sqrt{g}}{\sqrt{d}}$ in which case the
 orbit would just be a circle $h = 3955$
 miles being the radius of the Earth g
 as you know = $32\frac{1}{8}$ feet - By reducing
 the miles to feet & adding the logarithm
 of it to the logarithm of the $\sqrt{g}$ easily found



by Logarithms; from this we subtract the Logarithm of the v which by actual beam = projections comes to about 18,382 feet; a ball therefore projected with this velocity would never touch the Earth, but move in a constant circle round it.

Thus then we find that if the projection be not very great the orbit is an ellipse, if rather greater it becomes a circle, if very great then it describes a Parabolic trajectory (so called from their not returning again); if the projection could be supposed still greater then a Hyperbola would be the infinite trajectory, Lastly if the projection be beyond this a rectilinear line would be the path. The Comets visible at intervals are supposed to be projected in this infinite path, at least such as do not return.

To recapitulate shortly what we have gone over in Physical Astronomy, I may mention, that we first demonstrated the important proposition that "Planets are as the times, or that the Planets in their orbits describe equal areas in equal times" which was easily demonstrated from the composition of forces. And in the second place "that if a Body revolve in an Ellipse & if in the transverse axis be called a & the mean velocity C & the velocity at any part of the



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Ellipse the $v^2 = c^2 \times \frac{2a-x}{x}$ or that $v = c\sqrt{\frac{2a-x}{x}}$
 consequently $x : 2a-x :: c^2 : v^2$ that is "the dis-
 tance from the point of attraction is to the
 difference between this & the transverse
 axis as the square of its velocity at the
 mean distance is to its present velocity,
 or velocity at that point." The 3^d proposition
 which we demonstrated was an important
 one also viz. "that if there be one force or
 body descending in a straight line & another
 moving in an elliptical orbit, then if the
 velocity of the one at an equal distance be
 equal, to that of the other, they will be so at
 all equal distances in future". We showed
 4th that the squares of the periodic times
 are as the cubes of the distances - This we
 applied to finding the masses & specific
 gravities &c in the other heavenly bodies.
 This was done by supposing we knew the
 intensity of the centrifugal force in the
 one as the Earth A; then the centrifugal
 force in A: that in B:: quantity of
 matter in A: that in B. Also the masses
 in 2 Planets are as the cubes of the dis-
 tances divided by the squares of the
 periodic times that is $m^3 : m'^3 :: \frac{a^3}{p^2} : \frac{a'^3}{p'^2}$
 Thus we may from knowing the periodic
 times of the planets, discover the mean dis-
 tances of them, thus we find in the following that

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(p denoting the periodic time, & the distance from the Sun or mean distance as it is commonly termed -

<u>Mercury</u>	= 1335.205 Days & = 3.871
<u>Venus</u>	= 224.701 D° = 7.233
<u>Earth</u>	= 365.256 D° = 10.0000
<u>Mars</u>	= 686.979 D° = 15.203
<u>Ceres</u>	= 1681.539 D° = 27.674
<u>Pallas</u>	= 1681.709 D° = 27.676
<u>Juno</u>	= 1590.998 D° = 26.871
<u>Vesta</u>	= 1335.205 D° = 23.730
<u>Jupiter</u>	= 4332.596 D° = 52.628
<u>Saturn</u>	= 10758.969 D° = 95.387
<u>Uranus</u>	= 30688.713 D° & = 191.833

From the same method we also discover the diameters of the Planets, which from very accurate observations may be said to be as follows -

<u>Sun</u>	= 109.93	<u>Mars</u>	= .555
<u>Mercury</u>	= .971	<u>Jupiter</u>	= 11.561
<u>Earth</u>	= 1.	<u>Saturn</u>	= 9.509
<u>Uranus</u>	= 4.263		

Now the bulks or masses are as the cubes of the diameters so if each of them be raised to the cube they will be to one another as the masses, now as the cube of 1 is 1 the mass of the Sun must be very great in proportion to the Earth -

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Having the Masses, we know that the Mass of the Earth is to the Mass of Jupiter or Saturn or Uranus, all of which have the Satellites as the cube of the Moon's distance from the Earth divided by the square of the periodic time to the cube of any of the Satellites of these Planets divided by their periodic time. - Thus the most remarkable viz: Jupiter the mean distance of the nearest or

$$1^{\text{st}} \text{ Satellite} = 5.813 \text{ } \mathcal{L} = 1.769 \text{ Days}$$

$$2^{\text{nd}} \text{ Satellite} = 9.248 \text{ } \mathcal{L} = 3.551 \text{ D.}$$

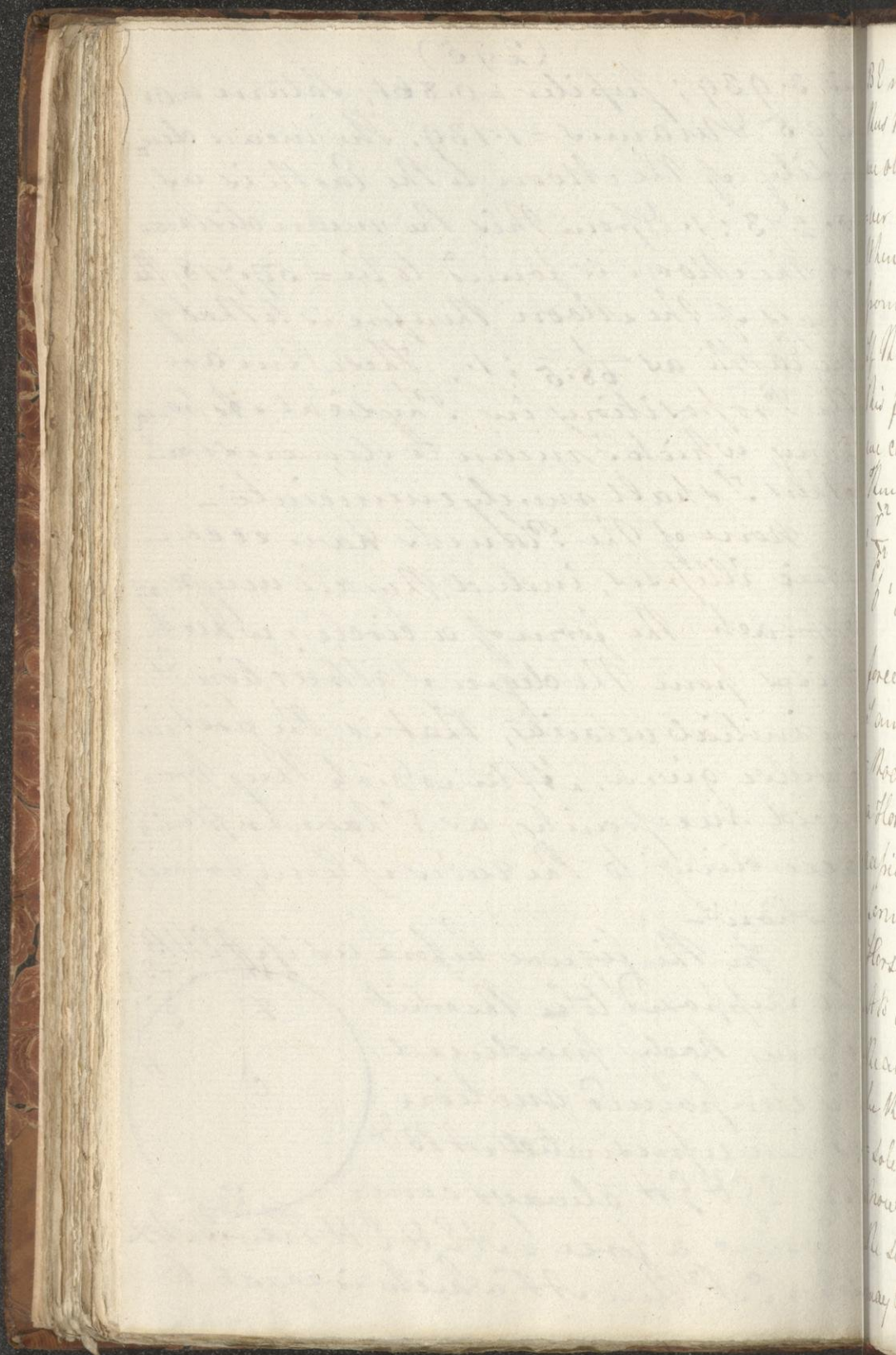
$$3^{\text{rd}} \text{ Satellite} = 14.752 \text{ } \mathcal{L} = 7.154 \text{ D.}$$

$$4^{\text{th}} \text{ Satellite} = \underline{25.946 \text{ } \mathcal{L}} = 16.688 \text{ D.}$$

If then we suppose the quantity of matter in the Sun to be 1. & in the Earth to be $\frac{1}{329630}$; then the quantity of Matter in Jupiter = $\frac{1}{1067.1}$; that in Saturn = $\frac{1}{3559}$; & in Uranus $\frac{1}{19.504}$. With regard to the other Planets which have Satellites they are so small that no certain observations have yet been made concerning them.

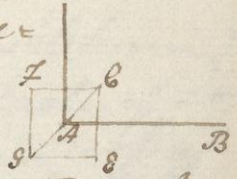
From these circumstances also we may easily compute the densities. If we suppose that of the Sun to be 1. then it has been found that, that of the Earth

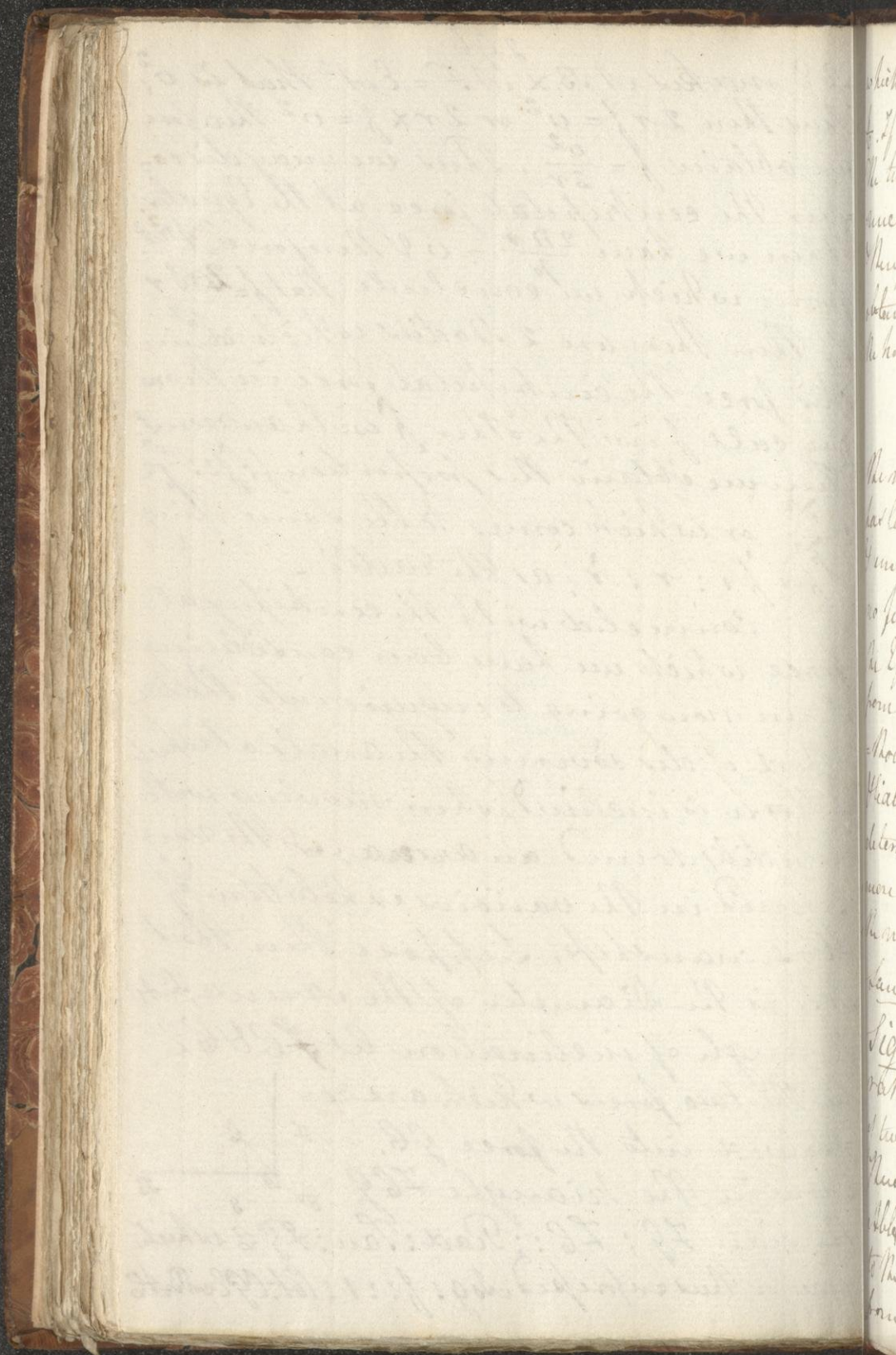
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BE makes $AB \times AF = EA^2$ that is v^2 ;
 Thus then $2rf = v^2$ or $2r \times f = v^2$ therefore
 we obtain $f = \frac{v^2}{2r}$, Thus we may disco-
 -ver the centripetal force at the Equator.
 When we have $\frac{2\pi r}{v} = v$ therefore $v = \frac{2\pi r}{v}$
 from which we conclude that $f = \frac{2\pi^2 r}{v^2}$
 If then there are 2 Bodies which receive
 this force the centripetal force in them
 we call f in the other f with an accent.
 Then we obtain this proportion $f : f' :: r^2 : r'^2$
 or which comes to the same thing
 $f : f' :: r : r'$ as the radii.

Connected with the centrifugal
 force which we have been considering
 I am now going to enquire into the Me-
 -thod of discovering the angle at which
 a Horse is inclined when moving with
 rapidity round an arena; as the ring
 formed in the various exhibition of
 Horsemanship. Suppose then that
 AB is the Diameter of the Arena, BAC
 the angle of inclination let $\angle GBC$
 be the two forces which are re-
 -solved into the force GC .
 Now in the triangle GBC
 the side $GC : CB :: Rad : Tan : \angle GBC$ which
 may be thus expressed $Log : f :: 1 : \cot. \angle GBC$



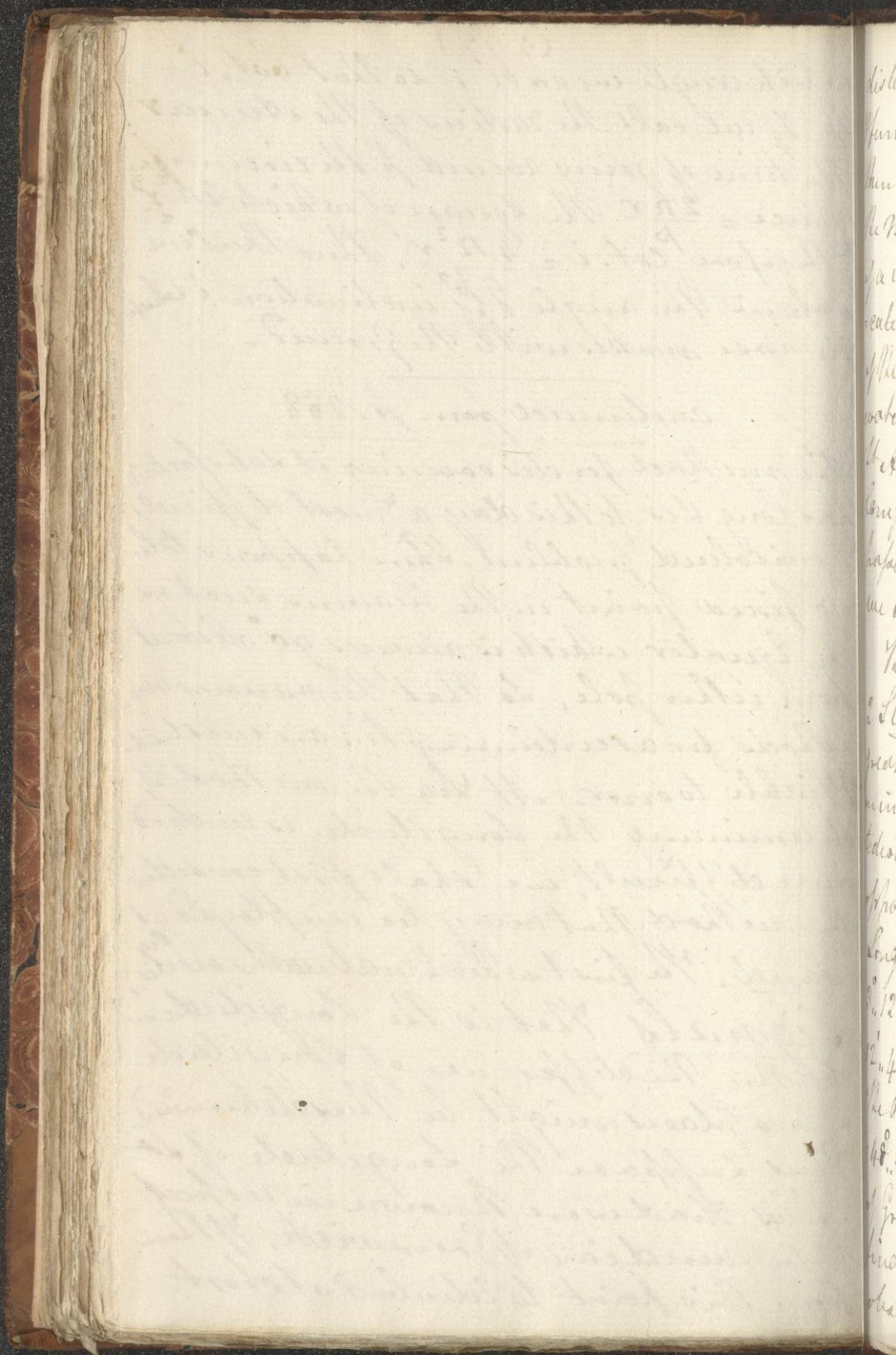


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which angle we call i so that $\cot. i = \frac{4}{3}$. If we call the radius of the circle the time of going round p the circumference $= \frac{2\pi r}{p}$ the square of which $\frac{4\pi^2 r^2}{p^2}$ & therefore $\cot. i = \frac{4\pi^2 r^2}{p^2}$. Thus thus we obtain the angle i of inclination which the horse makes with the ground.

continued from p. 288

The method for discovering it satisfactorily has long & is to this day a most difficult & involved problem. There happens to be no fixed point in the Heavens such as the Equator which is always 90 distant from either pole. So that the various methods for ascertaining this are necessarily liable to error. At sea the method of determining the Longitude is much more difficult, we shall first consider the method that may be employed at Land. The first which naturally occurs; Signals that is the Longitude rather the difference of Longitude of two places might be thus determined. Thus suppose the Longitude of London had been known in respect to the Meridian of Greenwich, & then from this point to Edinburgh at short



distances there be proper signal posts
 & furnished with good Telescopes. If
 then the instant the sun comes on
 the Meridian of St. Abb's head a signal
 of a very simple kind be communi-
 -cated to the next stand the movement
 of the hand for instance, this by careful
 watching might be communicated from
 St. Abb's head to Edinburgh in a second,
 Comparing this then with the truest
 paper over the Meridian of Edinburgh
 we obtain the Longitude of the latter.

The next method is by actual
 2 Survey after the manner of the
 great trigonometrical survey carrying
 on in this country - This however is a very
 tedious method - The Latitude of Edin^{burgh}
 opposite the College is $55^{\circ} 56' 42''$ & the
 Longitude West from Greenwich is
 $3^{\circ} 12' 15''$ which in time is expressed by
 $12^{\circ} 49'$. The Latitude of Greenwich is $51^{\circ} 28' 40''$
 The Observatory in Paris is in Latitude
 $48^{\circ} 54' 14''$. The Longitude is $2^{\circ} 20'$ East
 of Greenwich - The 3rd method of
 finding the Longitude is by
 observing the

