Photography and a New Vision for Elementary Mathematics

By

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Dedication

To classroom teachers everywhere:
Thank you for doing the most difficult and most true work of the profession.
Your dedication and skills continue to inspire me, just as they continue to inspire your students.
May those who offer criticism or support have the opportunity to be in your shoes,
even for just one day,
so that they may better understand the sacrifices and joy that come with all that you do.

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Abstract

In this study, pre-K students were given opportunities to describe their thinking using manipulatives, their own photographs, and other content-related photographs while working to answer assessment questions in the areas of Quantifying, Understanding Spatial Relationships, and Understanding Shapes. Results with manipulatives-only were compared to results with photography. The theoretical framework stemmed from Activity Theory and components of the theory of Realistic Mathematics Education (RME). Ideas from each theory were combined to form a hybrid framework which placed RME at the center of the Activity Theory framework. Key findings from the study show that more students were better able to describe their thinking when referring to photographs than when referring to manipulatives. The mathematical skill with the greatest average number of improved responses with the use of photography was Quantifying, where students were able to give more detailed, more mathematically relevant, and more accurate responses when speaking from photographs as compared to manipulatives. The photography situation with the greatest average number of improved responses was when students were speaking from photographs that were shown to them but that they did not take themselves. For Understanding Spatial Relationships problems and Understanding Shapes problems, students were able to describe more spatial and shape attributes with the use of photography than with the use of manipulatives. Further, students showed fewer distractions with the use of photography in the Understanding Spatial Relationships tasks, and were able to recall more real-world shapes with the use of photography in the Understanding Shapes tasks.

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Chapter 1
Introduction

Photography and elementary mathematics are not necessarily entities that are commonly placed together in terms of classroom practice, educational research, or student assessment; however, I believe that the intersection of these fields holds great potential in terms of elementary mathematics teaching, learning, and communicating. Today more than ever, cameras are ubiquitous in that they are found on nearly every cell phone and tablet. These devices are not just understood and utilized by adults; Children are mastering the photo technologies available on cell phones and tablets at younger and younger ages. While photographs have had a home in the elementary classroom for many years in terms of the posters that cover walls and the colorful pictures that fill textbooks (especially science and literacy texts), I believe photographs and cameras are tools that are currently underutilized in the elementary classroom, especially in terms of students' abilities to make connections within and communicate about their mathematical understanding.

One enduring issue in mathematics education is that students have a difficult time making connections among mathematical facts, procedures, and concepts, while teachers have a difficult time encouraging and supporting classroom discussions that facilitate such mathematical understanding. Research suggests that even when students in the United States are presented with problems that have the potential to support this type of mathematical understanding, they are lacking in their ability to make mathematical connections, especially when compared to other countries who outperformed the U.S. on the Trends on International Mathematics and Science Study (TIMSS) mathematics assessments (Hiebert et al., 2003). Hiebert et al. found the following:

> In Australia and the United States, 8 percent and less than 1 percent, respectively, of making connections problems were solved by making connections. These percentages were smaller than in the other countries, which ranged on average from 37 to 52 percent. (p. 103)

Another enduring issue in mathematics education is that a focus on early childhood mathematics standards has created an increase in mathematics assessments at the early childhood level (Brown, 2007). These assessments are moving away from more traditional early childhood assessment practices such as observation toward practices such as problem solving or clinical interviews (Ginsburg & Seo, 1999). When young children come to both formal and informal assessment situations, there are a variety of problems that can arise, making it unclear from the assessments what mathematical knowledge the students actually possess (Wager, Graue, & Harrigan, 2015).

In my experience working with students in a whole-group instructional setting, as well as doing individual clinical interviews with young children, I have noticed four situations that occur when students are asked to solve math problems or perform mathematics assessment tasks, as illustrated in *Table 1*.

Table 1: Possible Mathematics Performance Result Situations

	Student is proficient in the concept or skill	Student is not proficient in the concept or skill
Student action or response shows the student as proficient	-Shows Proficient -Is Proficient (SPIP)	-Shows Proficient -Is Not-proficient (SPNP)
Student action or response shows the student as not proficient	-Shows Not-proficient -Is Proficient (SNIP)	-Shows Not-Proficient -Is Not-Proficient (SNIN)

Situations where students show they are proficient and they actually are proficient (SPIP), or where they show that they are not proficient and they actually are not proficient (SNIN) are less problematic than the other two situations because the results of the assessments are more accurately reflective of students' proficiency levels. In situations where students show they are proficient but they actually are not proficient (SPNP), I am assuming the problems lie more with the design of the assessment questions or tasks than anything else. For example, if a student is presented with a square and a cube, and is asked to point to the cube, there is a chance s/he could get the answer correct simply by guessing. While this situation is problematic, the situation that has the greatest potential to be helped by the use of photography is the SNIP situation, where students show that they are not proficient but actually are proficient (or at least more knowledgeable than the results reflect). This study sought to discover the affordances of photography specifically in these SNIP situations by examining student responses and demonstrated understanding of mathematics tasks and assessment questions both with and without the use of photography.

One central purpose of this study was to explore the potential affordances offered by the use of photography as a means for teachers to support a classroom of young children in making connections among mathematical facts, procedures, and concepts as they work through math problems and explain their thinking and processes when engaged in mathematical tasks. Another central purpose of this study was to explore the affordances offered by the use of photography in the individual assessment of the mathematical skills of young children. If photographs (either photographs that students take during mathematical tasks or photographs that are given to students in order to represent mathematical concepts) can serve as tools to help students better communicate their mathematical understanding surrounding a given assessment task or problem, then it is imperative that cameras and photographs are brought into the elementary mathematics classroom in order to offer a solution to the problem of students not being able to show what they truly know. Often in education, solutions either do not exist, or when they do, are not feasible for financial or practical reasons. Fortunately, bringing cameras and photographs into the elementary mathematics classroom can be done quickly, at a minimal cost, and at a minimal disruption to current classroom lessons and assessments.

In this study, pre-K students were given opportunities to describe their thinking using manipulatives, their own photographs, and other content-related photographs while working to answer assessment questions in the mathematical content areas of Quantifying, Understanding Spatial Relationships, and Understanding Shapes. These descriptions were used to compare how students communicated their mathematical understanding and ideas when they were speaking from photographs to when they were speaking from manipulatives. The idea of speaking ‘from photographs’ or ‘from manipulatives’ is similar to the idea of someone speaking ‘from their notes’ during a speech, or ‘from a PowerPoint’ during a presentation. The intention was not to examine what happened during the processes of using manipulatives or taking photographs, but instead to look at how students communicated their mathematical understanding when using photographs or manipulatives as platforms for such communication.

Research Questions

As stated earlier, teachers often have a difficult time encouraging and supporting classroom discussions where students are able to communicate what they know about mathematical tasks and the connections among mathematical concepts. Further, the growing reliance on clinical interviews to assess young children can be problematic, as this form of assessment also may not reveal what children truly understand. The need to address these concerns led me to the following research questions:

- What are the affordances offered by the use of photography with young children to make connections and explain their thinking when engaged in mathematical tasks?
    - Which mathematical content strand in this study offers the greatest affordances?
    - Which photography situation in this study offers the greatest affordances?
- What are the affordances offered by the use of photography in the individual assessment of the mathematical skills of young children?

Rational for Theoretical Framework

For this study, the theoretical framework stemmed from Activity Theory and components of the theory of Realistic Mathematics Education (RME). I incorporated ideas from each theory to form a hybrid framework which best represented the research goals and tasks in this study. This hybrid framework placed the RME framework at the center of the Activity Theory framework, and provided a model that was more complete than either of the first two frameworks on their own in relation to this study.

Chapter 2
Literature

In the first portion of this literature review, I discuss photography and the importance of sideways thinking in mathematics. This section includes research about general advantages of using photography with children, research advantages of using photography with children, and information about Photo Elicitation Interviews. In the second portion of this literature review, I discuss photography and the importance of abstracting and selecting in elementary mathematics. This section includes research about open-ended problem photographs, interactive problem photographs, and information about the Language Experience Approach. The next portion of this literature review includes research about photography and a new vision of realism for elementary mathematics. This section includes research about realistic visualizations, informational pictures, and concrete manipulatives. In the final portion of this literature review, I discuss the theoretical framework for the study. This section includes research about Activity Theory and the Theory of Realistic Mathematics Education, and then situates the study within a hybrid of the two theories.

Photography and the Importance of Sideways Thinking in Elementary Mathematics

Research in photography emphasizes the importance of the photographer being able to create some sort of order from the chaos that exists in the world (Rossbach, 2011). Expanding on this are the ideas of seeing and learning to see before a photograph is taken. The skills of seeing and learning to see in photography are collectively referred to as visualization, which in this context means the formation of a mental image (Sadler, 1993). People can often conceive of a picture before they take it, especially when the subject matter is concrete; however, working with more abstract subject matter, or subject matter with which the photographer is less familiar or comfortable, involves a willingness to try new approaches and techniques. In photography, finding new ways to see both concrete and abstract subject matter has been defined as thinking sideways (Patterson, 2004).

According to Patterson (2004), thinking sideways helps photographers not only to break away from their typical subject, styles, and settings, but also to see subject matter they may have overlooked or not observed carefully. It is a mindset that involves piecing a variety of images together in order to gain a more complete perspective. For example, imagine a photographer taking photographs of an accident scene. A photographer who is not thinking sideways might take a series of photos which show the entire scene from one vantage point, or take all of the photos at the same time of day/night. In contrast, a photographer who is thinking sideways might walk around the accident scene to photograph it from many angles, or take photographs at different times of the day/night. Even if there is not a definite plan or final concept in mind while shooting the individual pictures, piecing these pictures together in the end would give a more complete perspective than a series of photos of the accident scene shot from the same place or time.

The next sections will describe how many of the advantages of sideways thinking in photography can be applied to elementary mathematics teaching and learning. These sections will also show how using photography with children helps them to become sideways thinkers as they put the pieces of mathematical facts, procedures, and concepts together to help provide a more complete perspective. This more complete perspective is beneficial not only for their own learning of mathematics, but also for teachers and/or researchers who are attempting to gain information about children's mathematical understanding.

General Advantages of Using Photography with Children

According to Northcote (2011), one of the greatest advantages of handing over cameras to young children is that it can increase their level of ownership and interest in the process of learning about mathematics. These processes also promote collaborative learning, questions, and discussion among children about mathematics concepts, which in turn helps to increase participation in classroom activities and ensure that children are valued as active contributors to the classroom learning environment. In this way, children become “young ethnographers” who use cameras to record, share and extend their learning experiences (Richards, 2009).

A specific technique used to help develop mathematical problem-solving skills as well as encourage discussion among children about math problems as they work through them is called Thinking Aloud Pair Problem Solving (TAPPS) (Whimbey, 1984). This method involves students working in pairs, or a teacher working one on one with a student. One student works through the problem, and the other student or teacher does not participate in the problem-solving, but instead works to get the problem-solver to verbalize all of her/his thoughts and processes used in the problem-solving process. This method demands that the problem-solver justify all steps taken when working through the problem, including steps which may have led to a dead end or an incorrect conclusion. Beyond success being measured simply by obtaining a correct answer, a critical part of success involves the explaining of the logic of any step taken in solving the problem (Pestel, 1993).

TAPPS is mentioned here to stress that while students do have an easier time explaining their understanding with this process than without it, the process of explaining the steps taken to solve math problems remains difficult for children, especially when the topic or task is unfamiliar to them (Carpenter & Lehrer, 1999). According to Carpenter and Lehrer, "By struggling to articulate their ideas, especially with means like Mathematical symbols or models, students develop the ability to reflect on and articulate their thinking" (p. 23). Furthermore, difficulties in articulation can also exist if the models or tangible items they are using do not have meaning to them (Manches & O'Malley, 2012).

Student-produced photographs have the potential to serve as the tangible items needed to help in this process of explaining, as the familiarity of digital cameras and the photographs that children take with digital cameras can help to make mathematics more accessible and meaningful to young children (Northcote, 2011). Focusing lessons and activities around photographs taken by children provide them with a familiar context in which to situate their mathematics learning and understanding (Campbell & Scotellaro, 2009).

Another advantage of using photography with children involves the reduction of the cognitive load. The cognitive load theory is based on the idea that people have a limited amount of cognitive resources (Vredeveldt, Hitch, & Baddeley, 2011). Because photographs afford children the opportunity to work with familiar objects and situations, the cognitive load sometimes associated with children interpreting unfamiliar content can be reduced. This in turn, allows the mathematics to become the focus of lessons, and reduces the amount of time needed for children to get accustomed to unfamiliar items (Northcote, 2011).

Research Advantages of Using Photography with Children

In addition to having many advantages for children, photography is regarded as having many advantages for researchers who work with children. Cook and Hess (2007) describe the following key advantages of using photography with children when conducting research. First, taking photographs is quick, easy, something children are likely to enjoy, and something that is likely to engage and maintain their interest. Second, many children view taking pictures as easier and more fun than writing. Third, modern cameras produce acceptable results without children (or researchers) needing to be experts. Fourth, giving children cameras increases the children's power because they can make a choice and pick out things that are of importance to them. Fifth, once photographs have been taken, they can act as tangible representations of children's interests thereby enabling researchers to return to a topic at a future date for further discussion with the children using the photographs as a starting/reference point. Photographs provide an opportunity to have group discussions around a visual prompt which makes it easier than trying to talk about something in the abstract. When a child takes a photograph, the stimulus for discussion starts from their interest (MacDonald, 2012).

Photography also allows children to create and express meaning in ways that are useful to researchers. According to Einarsdottir (2005), "The interviews in which the children discussed and explained the pictures were of vital importance. There, the children's reality came into view as they explained things concerning the pictures that were not evident without their elucidations" (p. 538). Additionally, Fasoli (2003) highlighted the importance of how using a camera allows children to create meaning in a language other than written or verbal. When researchers are attempting to gain meaning from students using only verbal or written language, they may be missing critical components. In this way, allowing children to take and use photographs is a type of sideways thinking that ultimately will provide a more complete and meaningful perspective.

While teachers and researchers may embrace the benefits of photography in teaching and research with children, some show misgivings regarding children's competence with the technology involved in photography. Contrary to these concerns, research demonstrates that even very young children tend to come to school already having had experience using digital cameras (Byrnes & Wasik, 2009). Often, elementary students require little to no instruction as to how the digital cameras they are given for research purposes operate, even when they are given a different brand or model than the ones they have experience using. It is important to acknowledge the technical prowess of young children, because recognizing the strengths and capabilities of children is a step forward in addressing the potential power imbalances when conducting research with children (MacDonald, 2012).

By giving children opportunities to demonstrate their knowledge and experiences in different forms, researchers and teachers are offered insights into children's understandings about mathematics that are personalized and meaningful, and constructed within contexts that are familiar to the child. By gaining these insights, researchers are better equipped to find ways to help children build connections between the mathematics they encounter in school, at home, and in the community (MacDonald, 2012). These connections are essential in terms of helping teachers to identify students' mathematical strengths, and also helping students to gain meaning from mathematics in multiple contexts (Wager, 2012). Perhaps Cook and Hess (2007) summarize the ideas in this section best when they state, "It needs to be recognized that photographs are not an absolute representation of a given state, but a tool to help understandings develop" (p. 43).

Photo Elicitation Interviews

Photo elicitation has been used as a visual participatory research method since the 1950s, and continues to be used as a visual method in contemporary research (Prosser & Burke, 2008).

In the Photo Elicitation Interview (PEI), photographs are used as stimulus for further discussion. There are a variety of approaches to doing PEIs. Researchers must decide who will take the photographs. Some researchers choose to take the photographs, develop, organize, and present them to the interviewee. Others ask their interview participants to take their own photos, which will then be used later as interview stimuli. This second approach is sometimes called a photo-elicitation auto-driven interview (Clark-IbaNez, 2004).

There are many benefits to using the PEI, but it is first helpful to understand the meaning of photographs in this methodology. Harper (2002) presented the approach's three main uses of photographs. First, photographs can be used as visual inventories of objects, people, and artifacts (i.e., photos of someone's favorite vacation spots). Second, photographs can be used to depict events that are a part of a collective which together form the framework of a narrative (i.e., photos of every city someone has ever lived in). Third, photographs can be used to define intimate dimensions of the social (i.e., photos that show someone's connection to society, culture, or history).

Research stresses that in terms of PEIs, there need not be anything inherently interesting about the photographs themselves; instead, the photographs act as a medium of communication between the researcher and the participant (Clark-IbaNez, 2004). Further, the photographs need not represent empirical truths or reality. In this sense, photographs used in the PEI have a dual purpose. Researchers can use photographs as a tool to expand on questions, while participants can use photographs to provide a unique way to communicate dimensions of their lives.

When adapted for the purpose of interviewing children, the PEI becomes an ideal methodology to engage young people. Photos can improve the interview experience with children by providing them with a clear, tangible prompt, and empowering them as the experts. According to Clark-IbaNez (2004), "The most common experience conducting PEIs was that photographs spurred meaning that otherwise might have remained dormant" (p. 1513).

Interviews with children often include their families and sometimes even their neighbors and friends. The groups-setting serves several important functions (Schwartz, 1989). First, it shows that the children's photographs are capable of generating multiple meanings in the viewing process. Next, photographs can trigger discussions and reveal contrasts and tensions among the viewers. Finally, photographs can generate data that illuminate a subject invisible to the researcher but apparent to the interviewee.

The ideas raised in this section support the use of photography in elementary mathematics in several ways. When children take pictures, they take greater ownership of their learning. The process of taking photographs helps to engage students in mathematics, and the process of discussing the photographs helps to create meaning for students. It is important for children to be able to articulate their thinking in mathematics, and student-created photographs are tools that can help students with this articulation. Furthermore, many of the advantages of photography between a researcher and child also exist in the math classroom, especially in terms of students creating meaning from their photographs, and using their photographs to express that meaning to others. Photographs help students to make connections between mathematics concepts, but also to make connections between math in different contexts and environments. Finally, in terms of sideways thinking, an added benefit in photography is that it can be practiced at any time – not just when making photographs (Patterson, 2004). Similarly, in mathematics, many of the skills mentioned above can be used and practiced at any time – not just when children are in their math classrooms.

Photography and the Importance of Abstracting and Selecting in Elementary Mathematics

Research in photography emphasizes the importance of the photographer being able to abstract and select (Patterson, 2004). In photography, abstracting, or separating the parts from the whole, is recognizing both the basic form of something and the elements that make up that form. It is an important skill in making good photographs, because it helps photographers to recognize the visual elements that are common to all subject matter. Once photographers have abstracted the visual elements most essential to a person, place, or thing, they have to select. Selecting is choosing those parts of the subject matter that will best express the character of the person, place, or thing. Abstracting and selecting are important skills in photography, because they help the photographer to be able to say more by showing less (Douglis, 2011).

Photography is very much a matter of identifying the basic elements and knowing how to put them together in various expressive combinations (Patterson, 2004). Abstraction is a fundamental process of mathematics as well (Ferrari, 2003). With regard to abstraction, Ferrari states, "Mathematical practice requires one to develop the ability to focus on what is important, without completely getting away from the contexts" (p. 1227). For example, imagine that students are asked to solve the following problem:

> "There are 13 girls and 11 boys in Miss Ibarra's class. Miss Ibarra wants to take her class on a trip, and she will use a van company for transportation. Each van has a company driver, and then room for 10 more people. How many vans should Miss Ibarra order? Explain your reasoning."

In this case, abstracting (according to the ideas in photography) would be recognizing the basic elements of the stated problem, while selecting (according to the ideas in photography) would be recognizing which of those elements are needed to solve it. One element identified through abstraction might be the fact that there were 13 girls and 11 boys in the class; however, it would be in the selection process when students would realize that gender was not relevant to the problem. While students could ignore this part of the context, they would still need to pay attention to other parts of the context, such as the fact that one cannot order parts of a van.

Similar to sideways thinking, abstracting is something that can be practiced at any time – not just when making photographs, or not just when children are in their math classrooms. However, whereas thinking sideways involves chance and encourages completely new ways of looking at things, abstracting brings an order and a structure to seeing. Abstracting and selecting help to make clear expression possible (Patterson, 2004), and photography is a tool that can help develop these processes in elementary classrooms, especially when it is used with certain problem types such as open-ended and interactive problems.

Open-Ended Problem Photographs

According to Wu (1994), "Open-ended problems have become a popular tool in Mathematics education in recent years" (p. 115). Open-ended problems, compared to closed problems, present students with varied approaches or multiple solutions to a problem (Bragg & Nicol, 2011). Research suggests that using open-ended problems in the classroom is an effective teaching strategy for establishing, consolidating, extending, reinforcing and reflecting on mathematical concepts (Busatto, 2004). Through open-ended problems, students are presented with opportunities to explore varied strategic approaches and encouraged to think flexibly about mathematics.

While the process of developing open-ended problem photos can be challenging, it can ultimately enhance one's ability to connect with mathematics and to see math differently. Bragg and Nicol (2011) suggest that one method of building students' and teachers' awareness of the beauty and complexity of the mathematics around them is through activities that incorporate photography with open-ended problem posing, thereby creating an open-ended problem picture. An open-ended problem picture is a photograph of an object, scene or activity that is accompanied by one or more open-ended mathematical word problems based on the context of the photo. Educators and students can collect their own photographic images and design open-ended problems based on these images.

This can be done by starting with a problem, or starting with a photograph (Bragg & Nicol, 2011). By starting with a problem, the problem-poser begins by considering a math concept and then creates open-ended questions with an image in mind. The problem-poser then searches for photos that capture ideas or concepts that correspond to the questions. Problem-posers may find that there are many possible photos that would make sense to use for a certain question. Through this approach, the mathematics is at the forefront and drives the abstraction of the mathematical concepts and the selection of the image. Alternatively, the photos may be staged to fit the requirements of the open-ended problem. The inspired use of resources in the home environment can assist in the development of a series of questions that could stimulate students' interests thereby increasing motivation (Ainley, 2004).

A second approach to designing open-ended problem pictures is starting with a photo. This approach first involves the exploration of one's environment with a camera in hand. Immersion in the environment heightens the problem-poser's awareness of the potential for mathematics in everyday images (Bragg & Nicol, 2011). Research involving open-ended problem pictures is important because the success of creating and using open-ended problem pictures depends on students' abstracting of the mathematical concepts, as well as the selection of the content of the photographs.

Interactive Problem Photographs

When considering photographs used in math problems, an important distinction exists between an interactive problem photograph and an illustrative problem photograph. A problem is considered interactive if the accompanying photograph is essential to complete the problem, and is considered illustrative if the accompanying photograph is a visual enhancement or motivational device but unnecessary for solving the task (Bragg & Nicol, 2011). For example, students need to use the photo to respond to the question, "Describe the shapes in this playground photo." If the question was posed as "Describe the shapes you might find on a playground" instead, the photograph would act as a catalyst for the problem and be considered illustrative rather than interactive. Therefore, while adopting illustrative questions may have educational merit, the central stimulus of the photo lacks function and purpose. The opportunity for building connections with the surrounding environment may be lost due to the insignificance of the photo in the problem solving process (Ainley, 2004).

An exciting aspect of students developing open-ended problem photos is their awareness of the mathematics in the 'real-world,' and their proactive approach to creating meaningful mathematics tasks (Bragg & Nicol, 2011). However, there is evidence that suggests that some children who successfully perform mathematical problems in the ‘real-world’ are unable to solve word problems in a classroom context (Nunes, Schliemann, & Carrere, 1993). As Ainley (2004) suggested, “Interest opens the individual to new experience and brings them in direct contact with knowledge and experience that goes beyond their current level of achievement. In achievement settings positive activating emotions serve the function of maintaining connection with learning activities” (p. 7).

In order to bridge interests, classroom mathematics, and real-world mathematics, research emphasizes the importance of asking students to explain their responses at the end of each open-ended problem photo (Bragg & Nicol, 2011). This puts less importance on the correct answer, and more importance on the processes involved in solving the problem. Bragg and Nicol give the following warning:

> It is important to avoid the misconception that any response to an open-ended problem is acceptable. Requesting that students explain or illustrate their responses will provide the necessary evidence to determine the complexity of their cognitive processing of the problem. (p. 8)

The (Mathematical) Language Experience Approach

Because the research described up to this point has emphasized the importance of photographs serving as tangible prompts to encourage more meaningful discussion, it is interesting to consider other academic areas where this type of system has been used, and the affordances it has allowed. The Language Experience Approach (LEA) draws upon the important link between experience and education; it extends the practice of scribing a child’s discussion to using the child's narrative as the text for reading instruction (Wurr, 2002). Using these stories, a teacher can engage the child in discussion about important text features as well as invite the child to reread and possibly revise the stories (Marinak, Strickland, & Keat, 2010). LEA supports children's concept development and vocabulary growth while offering many opportunities for meaningful reading and writing. Important conversations with teachers and child-produced photographs can extend children's knowledge of the world around them while building a sense of classroom community (Marinak et al., 2010).

While the Language Experience Approach provides advantages for all children, it is especially useful as a method with dual language learners. When dual language learners described their photographs, teachers saw the power of those photographs to strengthen each child's voice (Keat, Strickland, & Marinak, 2009). According to Keat et al.:

> If voice is the capacity to convey a message from one person's mind to another's, then the child-taken photographs provided the dual language learners with microphones that enhanced their ability to have their messages understood. Teachers heard the children's diverse voices in many ways. (p. 35)

Another example of the advantages with dual language learners is provided by Marinak et al. (2010) as they describe a familiar scene that takes place in diverse classrooms:

> The teacher is working hard to bring a child who is a dual language learner into the class conversation. She attempts to engage the child, and the child responds with a smile and silence. The teacher's intent to care for and nurture the whole child is there, but something is in the way. Often the assumption is that the child's limited language skills

> are getting in the way of the teacher connecting with that child. However, is that the only obstacle? (p. 36)

Marinak et al. (2010) explored this question, and concluded that while spending one-on-one time with children is always important, it was photo-narrations and LEA processes that helped teachers connect with dual language learners. The aforementioned studies are particularly significant in that they provide practiced-based evidence of how photographs elicited more meaningful information from students than questioning, writing, or interviewing alone.

Photography and a New Vision of Realism for Elementary Mathematics

Patterson (2004) discusses the capacity of photography to render detail with a precision no other visual medium can match. When people examine photographs, they see many things that their eyes normally miss. In this way, a photograph becomes an aid to visual discovery. The faithfulness to detail and the objectivity of the camera counteract normal human subjectivity and force people to look at physical objects more carefully. This characteristic of photography demonstrates photography's intrinsic connection with realism (Benovsky, 2012).

The notion of 'real-life' mathematics is a recurring theme in the literature surrounding elementary mathematics, as well as the literature surrounding the use of photography in education (Bragg, Pullen, & Skinner, 2010; Sparrow, 2008). Research describes how creating opportunities that are real and relevant to the students provide possibilities to generate authentic engagement in mathematics (Bragg et al., 2010; Van den Heuvel-Panhuizen, 2000). Furthermore, in addition to the mathematical tasks being meaningful, "The tools teachers provide to support problem solving should be meaningful and, where possible, link to the representations students use in non-school settings and how those representations are used" (Wager, 2012).

Many mathematics educators support the view that we should make mathematics real for students and this is reflected in national and local curriculum documents (Sparrow, 2008). As Gerofsky (1996) noted, some mathematics educators have taken this challenge to mean creating higher quality and more varied word problems, the view being that making the connections to real-life situations is undertaken through students engaging in word problems. A logical conclusion might be that the more realistic or true-to-life mathematical visualizations, pictures, and manipulatives are, the better they will serve the students, teachers, or researchers who view or use them. Research however, disputes this conclusion in several areas. The following section examines three different forms of mathematical representations and highlights different affordances and challenges of using realistic representations.

Realistic Visualizations

Elementary school math textbooks and secondary school math textbooks have some key differences in terms of the purpose of the visual representations they contain. Most elementary school textbooks include representational pictures such as photographs or line drawings that illustrate the overall theme of the text and situate learning (Schroeder et al., 2011). Their major function is not to communicate information, but to keep students interested and to assist their comprehension (Carney & Levin, 2002). In secondary education, in contrast, the major function of instructional pictures is to convey additional information not provided by the text. The pictures can be realistic or exhibit various degrees of abstraction (i.e., photographs, tables, graphical representations). The interpretation of logical pictures – highly schematized pictures that do not look like the things they represent but are to be interpreted in some conceptual or logical way – is a case in point (Schroeder et al.). Similar to complex realistic pictures, logical pictures require high cognitive demands of learners. Students are required to actively invest the mental effort needed to process the picture (Scheiter, Gerjets, Huk, Imhof, & Kammerer, 2009).

Scheiter et al. (2009) have summarized the different arguments that have been brought forward for why one or the other type of representational format may be better suited to support knowledge acquisition. Proponents of realistic visualizations have argued that learning will be more complete as the number of cues in a learning situation increases. On the other hand, opponents of realistic visualizations have suggested that learning will be hindered by realistic visualizations because of the high demands with regard to visual attention. Excessively realistic cues may be distracting or possibly even evoke responses in opposition to the desired learning.

Informational Pictures

Picture books are one way that young students gain exposure to mathematical concepts and ideas. Harland (1990) noted that children's literature has the potential to motivate children to ask questions, investigate problem situations, and communicate their thinking and understanding (as cited in Elia, Van den Heuvel-Panhuizen, & Georgiou, 2010, p. 127). Furthermore, Griffiths and Clyne (1991) noted that picture books may help children develop positive attitudes towards mathematics (as cited in Elia, Van den Heuvel-Panhuizen, & Georgiou, 2010, p. 127).

Elia, Gagatsis, and Demetriou (2007) proposed a categorization of pictures based on their function in the context of arithmetic problem-solving, including a decorative, a representational, an organizational, and an informational function. Decorative pictures do not provide any form of problem-relevant information. Representational pictures illustrate a part or the entire content of the problem, but are not necessary for the understanding or the solution of the problem.

Organizational pictures provide directions for the organization of the problem's information, or drawn or written work that support the solution. Similar to the representational pictures, they are not essential for the solution of the problem. Informational pictures provide information that is essential for the solution of the problem, because the content of the problem is based on the picture.

Seemingly contrary to the findings of Bragg and Nicol (2011) that were discussed earlier, which claimed that interactive problem photos afforded the opportunity for building connections with the surrounding environment when the photo is significant in the problem solving process, Elia et al. (2007) found the informational pictures had a detrimental effect in solving arithmetic problems. This was attributed to the switching between information in the two different sources (text and picture) and the combination of these streams of information, which entail additional increase in the cognitive load of the task (Berends & Van Lieshout, 2009). A difference to keep in mind however is that Bragg and Nicol are referring to photographs only, while Elia et al. are considering other forms of visual representations when they refer to pictures, such as drawings and illustrations. Also, Bragg and Nicol focused primarily on pre-service teachers in their research, while Elia et al. focused primarily on kindergarten students.

In general, when illustrations and photographs are considered together, the story-related and mathematics-related components with a representational function elicited mathematical thinking to a greater extent than those with an informational function (Elia et al., 2010). Whereas the components with a representational function are an alternative 'description' that is additional to the text, in the case of the components with an informational function, the mathematical information can be obtained only from the picture as the content of the text is not sufficiently informative. This suggests that combining text and pictures of a similar content has a greater power to mathematically engage children than combining text and pictures of different content.

It is interesting to note that even the pictures whose mathematical components have a representational function but are not congruent with the mathematical content of the text may have the potential to yield stimulating cognitive activity to children, especially to those children who understand the relation between the picture and the text. Research has demonstrated that a picture of this kind elicited meaningful mathematical thinking to children, as it motivated them to compare the mathematical content of the text (Elia et al., 2010).

Concrete Manipulatives

The idea that concrete materials benefit children's learning has a long history in developmental psychology and education (McNeil & Uttal, 2009). McNeil and Uttal state, "Too often, however, educators and researchers may use these historical roots to give concrete materials a blanket endorsement. Educators seem to assume that children are making conceptual progress as long as they are busy working with concrete materials" (p. 137). However, just as adults' thinking cannot be labeled as inherently abstract, children's thinking cannot be labeled as inherently concrete. Also, it cannot be assumed that students will automatically construct abstract knowledge from interactions with concrete materials.

Sarama and Clements (2009) suggest that what matters is not whether a learning material is concrete or abstract but whether and how students gain insight into the meaning or purpose of what they are learning. They posit that there are two different types of concrete knowledge. Students with sensory-concrete knowledge need sensory material to make sense of a concept or procedure. Integrated-concrete knowledge is knowledge that is connected in special ways. What ultimately make mathematical ideas integrated-concrete are not their physical characteristics but how connected to other ideas and situations they are. Furthermore, good manipulatives are those that aid students in building, strengthening, and connecting various representations of mathematical ideas (Uttal, O'Doherty, Newland, Hand, & DeLoache, 2009).

Sarama and Clements (2009) suggest, "An overarching but underemphasized reason for the positive effects of computer manipulatives in various studies is that they provide unique affordances for the development of integrated-concrete knowledge" (p. 147). These affordances include bringing mathematical ideas and processes to conscious awareness, encouraging and facilitating complete, precise explanations, and supporting mental actions on objects (Sarama & Clements, 2009). Other researchers have looked at the affordances of virtual manipulatives as well. For example, Triona and Klahr (2003) examined how virtual manipulatives could provide educational benefits to children without the physical interaction of virtual manipulatives, and found improvements in learning when graphical representations are linked with symbolic representations. Further, Manches and O'Malley (2012) suggest, "Virtual representations may offer pedagogical advantages such as flexibility and control over interaction as well as more pragmatic benefits such as costs and the ability to share resources" (p. 417). While these affordances are situated in research involving virtual manipulatives, a purpose of this study is to examine whether similar affordances can be found through the use of photography in elementary mathematics.

An essential problem for mathematics instruction is to help children to understand, and to manipulate, symbolic representations. Manipulating concrete objects is certainly important, particularly in the early stages of learning, but children must be able to connect concrete and more symbolic representations. The concept of dual representation can shed light on this fundamental problem (Uttal et al., 2009). The central tenet of this concept is that all symbolic objects have a dual nature; they are simultaneously objects in their own right and representations of something else. To use a symbolic object effectively, one must focus more on what the symbol is intended to represent and less on its physical properties. In terms of the student-generated photograph, the photo itself is an object, but it is also a representation of a mathematical concept or problem component.

Sarama and Clements' (2009) research with computer-based manipulatives also highlights the need to think carefully about the design and use of concrete manipulatives. They note that computer-based manipulatives may make generalization and transfer easier than concrete manipulatives do. Using computer-based manipulatives reduces the demands of dual representation, enabling children to focus less on the on-screen objects themselves and more on the connections between the manipulatives and mathematical representations.

Other studies also provide evidence that concrete manipulatives may be distracting for students, although they differ in their reasoning about why this was the case. Kaminski, Sloutsky, and Heckler (2009) suggest that realistic concrete materials convey superficial information that interferes with learning. For example, a child counting apples may be distracted by the shape or color of the apples and, as a result, may be less likely to focus on how many apples are present. In this case, the attributes of the apple (shape or color) are irrelevant and they distract learners from the information that educators intend to share (number). In other words, the distracting or misaligned object features themselves hinder learning. Sarama and Clements

(2009) reinforce this idea by pointing out that physical manipulatives, in particular, can be distracting because they often have properties that are irrelevant to the target concept.

Uttal et al. (2009) suggest that realistic concrete materials hinder learning because children must deal with the demands of dual representation. According to this view, realistic concrete materials hinder learning because they have features that draw children's attention to the objects themselves rather than to the abstract concepts they represent. In dual representation, the individual features of the concrete objects hinder learning only to the extent that they pull attention toward the objects.

Martin (2009) provides an entirely different framework for understanding why realistic concrete materials may hinder learning. According to Martin, Physically Distributed Learning (PDL) is the coevolution of children's actions and ideas over time. In order for PDL to occur, learners need to interact with the environment in ways that allow them to construct stable, generalizable concepts for themselves. The problem of realistic concrete materials is that they may sometimes do too much of the work for learners. What this research suggests is that whether a manipulative is concrete or abstract, if a given set of materials provides children with a correct interpretation from the start, children may not engage in the active process of adapting to and reinterpreting the environment, and learning will be shallow. Photography has the potential to avoid this problem since it involves the photographer learning to think sideways, as well as abstracting and selecting subject matter from the environment. In other words, a photograph itself cannot provide a child with a correct interpretation; only the child can provide that interpretation.

Theoretical Framework

This portion of the literature review discusses the theoretical framework for the study. It includes research about Activity Theory and the Theory of Realistic Mathematics Education, and then situates the study within a hybrid of the two theories.

Activity Theory

Vygotsky's view of human learning challenged traditional views of mathematics as value-free, objective, and divorced from everyday personal concerns (Crawford, 1996). According to Vygotsky, the development of mathematical knowledge had a socio-cultural component in that it was mediated by the use of cultural tools, such as mathematical symbols, and also by interpersonal relations (Cobb, Zhao, & Visnovska, 2008). In mathematics education, this social view of the construction of mathematical knowledge means that learning mathematics is seen as a process of enculturation in which classroom interaction is a central element (Sfard, Nesher, Streefland, Cobb, & Mason, 1998). In this theory, learners are seen as active constructors of knowledge (Van den Heuvel-Panhuizen & Van den Boogaard, 2008). This "activity" of constructing knowledge serves as the foundation to Engeström's Activity Theory (Engeström, 1999).

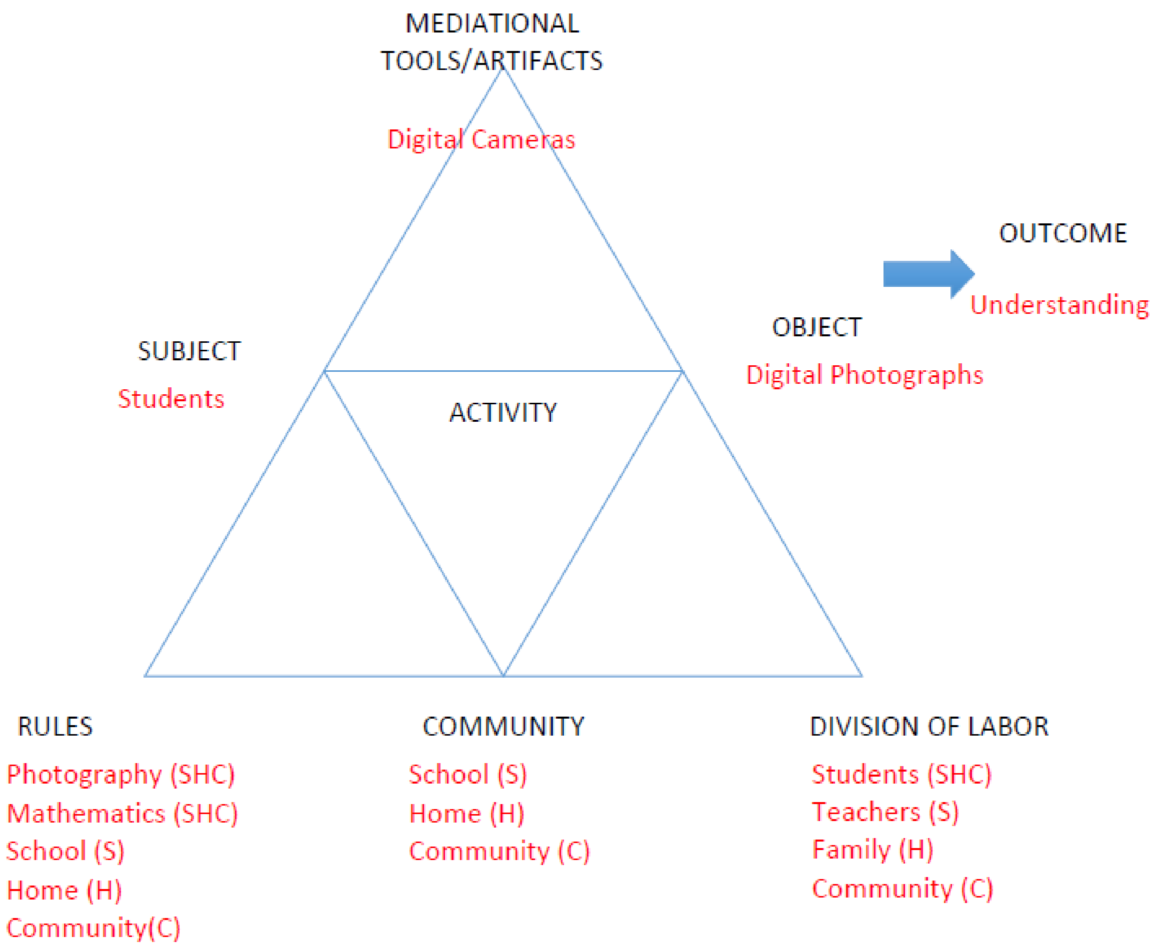
There are three generations of Engeström's Activity Theory (Yamagata-Lynch, 2010). In the first generation of this theory, subjects (individuals) use mediational means (tools) to produce objects (outcomes). In the case of this study, the subjects would be the students in the study, the mediational mean could be the digital camera if the produced object was the digital photographs, or the mediational mean could be the photographs themselves if the produced object was the mathematical outcome, which ideally consists of increased understanding and increased communication about mathematical understanding. In this model, the focus is more on the individual components rather than the relationships between and among the components.

In the second generation of this theory, the focus shifts from the individual components to the relationships between and among the components. The original Vygotskian triangle consisting of subjects, mediational means, and objects is now considered an activity system that is influenced by and interacts with elements such as the community, rules, and the division of labor (Engeström, 1999). In the case of this study, the community could consist of the environment in school, in the home, or in the community. The rules therefore, both photographic and mathematical in nature, would depend on which community is being considered, as would the division of labor, which determines the extent to which learning is passive or active. While this generation seems more complete than the first, it seems incomplete in that it does not simultaneously take into account multiple environments.

It is the third generation of Engeström's Activity Theory that seems to best account for these multiple environments. In this generation, the activity systems, or systems involving each environment and the norms and division of labor from those environments, work together to form a network (Yamagata-Lynch, 2010). This generation seems to best represent the components of this study because it simultaneously considers the activity systems of multiple environments in the network. This is important when looking at a tool that a child can use to get a desired outcome, especially when the child has the ability to use the tool in multiple environments, each having a unique set of rules and norms. It is also important to consider how these systems interact. For example, a student who has access to a cell phone camera at home will have a different experience and outcome using digital photography in the classroom than a student who does not have such access at home.

Figure 1 highlights how Activity Theory fits with the components of this study. The students use the digital camera to produce digital photographs, which are then used to communicate mathematical understanding. The rules of these activities are somewhat dependent on the environment in which they take place (school for the case of this study). There are also rules surrounding mathematics and photography that are in effect in any environment. An example of a photographic rule is that students will not be able to produce a clear image if they hold the camera too close or too far from the object that they are photographing. An example of a mathematical rule is that while a ball may have some properties of a circle, it is actually a sphere. Further, in each environment, there is a different division of labor that students must negotiate. Pre-K school assessments depend on the interaction, and especially the communication, between teachers and students. As teachers work to get information from students, and as students work to give information to teachers, there is specific work that each must do, and this study examines how the student photographs can help students with that work.

Figure 1: Activity Theory Framework (adapted from Engeström, 1999)

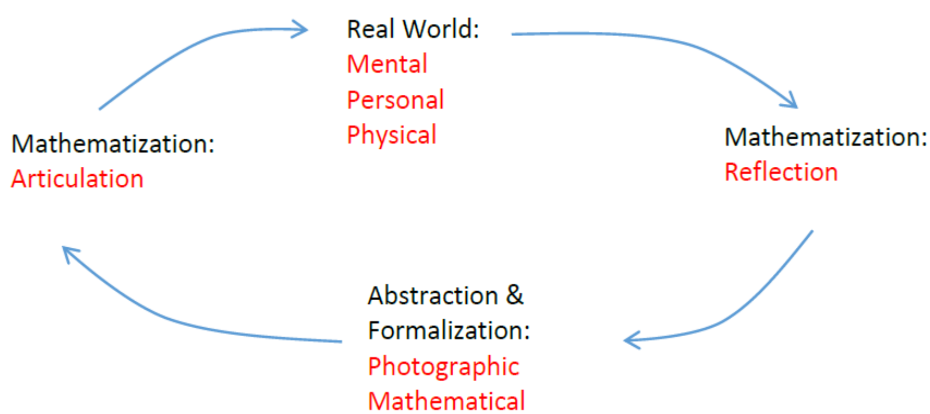


Theory of Realistic Mathematics Education

Realistic Mathematics Education (RME) is a teaching and learning theory in mathematics education developed by Freudenthal and his colleagues at the former IOWO, the oldest predecessor of the Freudenthal Institute (Van den Heuvel-Panhuizen, 2000). RME is similar to Activity Theory in that it views mathematics as a human activity, but it also views mathematics as something that must be connected to reality (Cobb et al., 2008). Reality in this case refers not just to the connection with the real-world, but also refers to problem situations which are real in the student's mind. Activity Theory, especially as described in the third generation, does serve as a strong model for this study; however, it seems to be missing this explicit component regarding the importance of mathematics being connected to reality. RME perceives mathematics as an integral part of the human experience. Furthermore, it sees children as active participants in the learning process, assigns great importance to giving children the opportunity to share and discuss ideas for solutions, and attaches high value to providing meaningful contexts from which context-based mathematical knowledge can emerge that serves as a basis for reaching more general and formal levels of understanding (Van den Heuvel-Panhuizen & Van den Boogaard, 2008). A central goal for this study was that the process of students taking photographs and then using those photographs to explain their understanding would provide this more meaningful context for them to share and discuss their ideas, as well as make the problem situations more real in the students' minds.

Cobb et al. (2008) describe two types of mathematization central to RME: horizontal and vertical. In horizontal mathematization, the students come up with mathematical tools which can help to organize and solve a problem located in a real-life situation. On the other hand, vertical mathematization is the process of reorganization within the mathematical system itself. In RME tasks, the starting point of an instructional sequence should be experientially real to students, and their engagement in the math activity should be personally meaningful in some way. For the designer, the immediate goal is that students' interpretations and solutions should lead to the development of informal ways of speaking, symbolizing, and reasoning across a range of instructional activities. This is the essence of horizontal mathematization (Cobb et al., 2008). Thompson (1992) reinforces the significance of this process by stating the importance of students becoming engaged in thinking about the ways that math relates to their real-world experiences. Further, it is important that this type of reflection occur both in and out of school contexts. *Figure 2* shows the RME framework (adapted from Lange, 1996).

Figure 2: RME Framework (adapted from Lange, 1996)



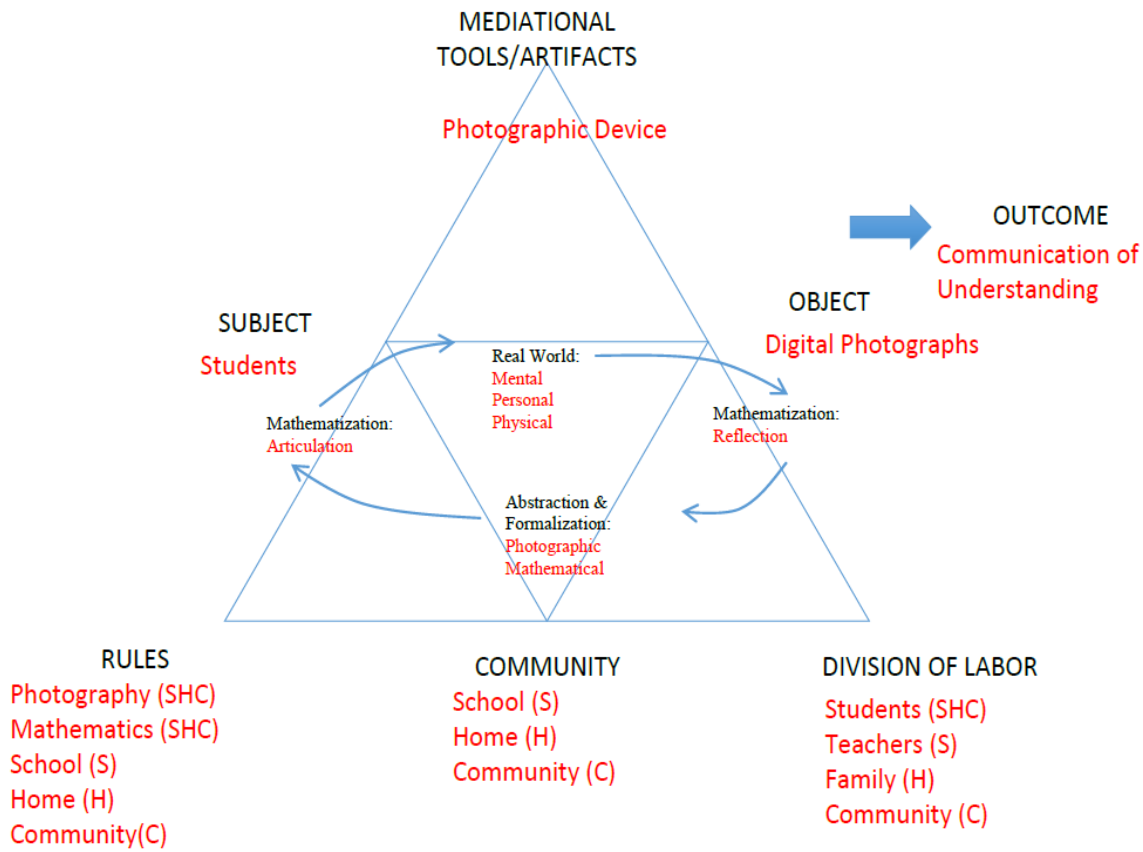
Activity Theory and RME Hybrid

Ball (1993) articulated a central issue with which mathematics educators continue to struggle when she asked:

> How do I [as a Mathematics teacher] create experiences for my students that connect with what they now know and care about but that also transcend the present? How do I value their interests and also connect them to ideas and traditions growing out of centuries of Mathematical exploration and invention? (p. 375)

I believe it is a hybrid model of the third generation of Engeström's Activity Theory, combined with the aforementioned components of Realistic Mathematics Education that can best address this issue. The Activity Theory RME hybrid model places the RME framework at the center of the Activity Theory framework, as seen in *Figure 3*. For this study, the hybrid model is more complete than either of the first two frameworks on their own in that it: 1) considers the network of activity systems within the school, home, and community along with the corresponding rules and division of labor, and 2) outlines the process of how students use digital cameras to produce digital photographs, which in turn helps students to communicate their understanding. This is done through a repeating pattern of vertical mathematization (the reorganization of ideas within mathematics), leading to abstraction (both photographic and mathematic), leading to horizontal mathematization (the reorganization of ideas within the real world), leading to different representations and understandings of the real world (the real world including the mental, the personal, and the physical).

Figure 3: Hybrid Activity Theory/RME Framework



Notice that in this hybrid model, the more outward and visible activity of students taking photographs, and then using those photographs to communicate their understanding is occurring simultaneously with the cycle described in the RME framework. The goal of this entire process is to give students a tool (their photographs) to help them better articulate their thinking. Further, this hybrid model suggests that the advantages of student photography can extend beyond just the articulation phase. Moving on from how students articulate their thinking, they can now choose additional items to photograph, and make decisions (based on mental math constructs as well as the physical environment) about how to photograph those items. This process and these images lead to further mathematization as the students reflect on their images in relation to the tasks and questions presented to them. And this reflection in turn leads to the abstraction and formalization of mathematical ideas, and ultimately to further mathematization as students again articulate their ideas and understanding. The immediate goal of this study was to examine how student photography for pre-K students could help them better communicate their mathematical understanding; however, the implications for instruction could reach far beyond mathematics as well as pre-K this when this hybrid model is considered.

Chapter 3
Methods

Participants and Location

The pre-K students who were asked to participate in this study attended the Growing Flowers Learning Center, and the Sun Shines Learning Center, both located in towns that are close to one another in the same Midwestern state. These sites were identified by consulting with a pool of early childhood teachers and other professionals who were in a professional development graduate course through a local Midwestern university. Pre-K teachers were identified who a) were not teaching for schools that were part of the major metropolitan public school district in the area, and b) had wrap-around care, which means the students attend an academic portion of pre-K during the morning, and then childcare wraps around through the afternoon. Wrap-around care was important for this study because it ensured that students were able to participate in this study during the afternoon daycare or non-academic portions of their mornings rather than the academic portions of their day. The students who participated in this study did so by their own choosing, and had the formal consent of a parent or guardian.

The Growing Flowers Learning Center is a state licensed group childcare center. They provide full-time care for children aged six weeks through pre-K (age 4-5). The Sun Shines Learning Center is affiliated with a branch of the YMCA. They provide pre-K and kindergarten programs for four- and five-year-olds. Both sites focus on a child-centered, play-based curriculum for pre-K, and have teaching staff who are certified teachers. The Growing Flowers Learning Center and the Sun Shines Learning Center classrooms that I worked with had approximate enrollments of 15 and 25 pre-K students respectively, and most of those students attended the learning centers throughout the summer. The exact number of participants for the study was 24 (8 students from the Growing Flowers Learning Center, and 16 students from the Sun Shines Learning Center). Due to the nature of the research tasks in this study, all participants who wished to participate and who gained consent were able to participate this study, with the exception of one student who ended participation in the summer program early.

Task Selection

Tasks for this project were based upon the pre-K mathematics objectives stated in the Teaching Strategies GOLD Objectives for Development & Learning program (Heroman, Burts, Berke & Bickart, 2010). Although this curriculum was not used at the research cites, it was chosen because it is used in the major metropolitan public school district in the area. Research in choosing and refining tasks for this project stemmed from two key areas - the cognitive demands of mathematical tasks and the Cognitively Guided Instruction problem types (Carpenter, Fennema, Franke, Levi, & Empson, 1999). According to Smith, Stein, Arbaugh, Brown, & Mossgrove (2004), all tasks are not created equal. Different tasks require different levels and kinds of thinking from students. Low-level tasks often require students to perform a memorized procedure in a routine manner with little cognitive engagement. For example, imagine that students are asked to count out loud verbally to ten. This process demands little engagement with concepts and does not stimulate students to make purposeful connections to meaning or relevant mathematical ideas (Clements, 2001). In contrast, imagine that students are asked to describe a group of ten items (some red and some blue). Even though the mathematical content is similar in both examples (both fall under the objective of using number concepts and operations), they require different types of thinking from students.

A task like that in the second example meets the requirements of what Stein, Smith, Henningsen, & Silver (2009) refer to as a high-level task. High-level tasks focus on students' reasoning and explanations about their cognitive processes, and require considerable cognitive effort by students. They demand engagement with concepts and stimulate students to make purposeful connections to meaning or relevant mathematical ideas (Smith et al., 2004). This is in contrast to low-level tasks, which focus more on the answer than the students' cognitive processes involved, and require little cognitive effort by students (Stein et al., 2009).

Research about Cognitively Guided Instruction (Carpenter et al., 1999), specifically the problem types that are most challenging for students, was also considered. A goal of Cognitively Guided Instruction (CGI) is that young children become independent problem solvers who are able to approach and solve word problems using their own methods, and not those told to them by a teacher.

According to Carpenter et al. (1999), a number of factors influence what makes some problems more challenging for students than others. First, a problem that can be acted out is easier for a child to solve than one that cannot be acted out. Second, when the quantities given in a problem refer to a complete physical set of physical objects or amounts, the problem can be modeled directly. When a word problem can be directly modeled, that is, represented in some concrete way on fingers, with tally marks, drawings, or by manipulating counters, the problem is easier. Next, when first learning to solve word problems, young children approach them in the order in which they hear them. They do not begin at the end of the problem and work backward. Finally, because young children solve problems in the order that they hear them, problems that are worded in such a way so that the unknown quantity is located at the end are easier to solve. Problems with the missing quantity in the middle or at the beginning are more difficult.

Because the literature on CGI stresses the usefulness of students being able to work with physical manipulatives as they work through and model problems, the tasks that were selected for this project are those that can be modeled, solved, and/or explained with physical manipulatives. Since the literature on using photography with children suggests that photography might be useful to produce a manipulative (the photograph) which could serve to help students think through various problems, it was imperative to have tasks enacted with the physical manipulatives as well as with the student photographs and given photographs in order to see both the general affordances of using photographs as well as the specific affordances of using certain types of photographs in different ways.

Carpenter and Lehrer (1999) identify two main types of reflection discussed in mathematics education research: reflection by students about what they are doing and why, and reflection about tasks and their solutions after the tasks have been completed. Students are more likely to be reflective while solving problems if they know they will be asked to explain how they solved the problem. Reflection can also be encouraged by the type of questions posed to students while they are solving problems. In this sense, Herbel-Eisenmann and Breyfogle (2005) distinguish between funneling and focusing questions. Funneling occurs when questions lead students to a desired end. Here, more of the cognitive work is done by the teacher than the student. Focusing on the other hand, occurs when questions lead students to explain what they are thinking in a clear and articulate fashion. Here, more of the cognitive work is done by the student than the teacher. Questions like “What are you doing? Why are you doing that? and

How will that help you solve the problem?" encourage reflection. Being asked such questions on a regular basis helps students internalize them, so that they will ask themselves the same questions as they think about a given task (Carpenter & Lehrer, 1999).

The ability to communicate or articulate one's ideas is an important goal of education, and it also is a benchmark of understanding (Carpenter & Lehrer, 1999). Articulation involves the communication of one's knowledge, either verbally, in writing, or through some other means like pictures, diagrams, or models. In order to articulate our ideas, we must reflect on them in order to identify and describe critical elements (Carpenter & Lehrer, 1999). Furthermore, when dialogue becomes the central medium of teaching and learning, students can become active rather than passive participants in their education (Means & Knapp, 1991).

Consent

Three types of consent were needed for this study, in addition to the UW IRB approval. First, the directors of the Growing Flowers Learning Center and the Sun Shines Learning Center each gave consent by signing the Site Director Consent Form (see Appendix A). Second, consent was needed from parents and/or guardians of the students, as well as from the students themselves. Parents and/or guardians, and students, gave consent by signing the Parent/Guardian Consent Form (parents/guardians) and the Student Assent Form (students). Both of these forms are on the same document (see Appendix B). These forms were either given out to students to take home to their parents/guardians, or were given directly to the parents/guardians by the pre-K teachers at the Growing Flowers Learning Center and the Sun Shines Learning Center. Students or parents/guardians brought the signed forms back to the teachers, who then gave them to me. A second round of forms were given to those students (or their parents/guardians) who did not return their initial forms, but after that, it was assumed that those who did not return the forms did not wish to participate in this study.

Data Collection

The data for this study was collected during the summer of 2014. As stated in a previous section, the N for this study was 24. Students were at these sites Mondays-Fridays, with the mornings dedicated to their pre-K academics, and wrap-around daycare programs provided in the afternoons. All data was collected either during the afternoon daycare portions of the day, or during the non-academic portions of the morning (i.e., snack time). Before students took any photographs, they were read an IRB approved script of instructions (see Appendix C).

Students took their photos on a secure iPad, and used the free version of the drawing app You Doodle to mark their photos as they spoke about them. If multiple useable photos for a task were taken, students were allowed to choose the one that they wanted to save. There were two copies of each photo saved: 1) the original photo before the students spoke about them, and 2) the final photo after the students spoke about them. These final photos had their drawing marks saved to them, if indeed any drawing marks were made; otherwise, the final photos look identical to the original photos. The entire interaction with each student was audio recorded, and the student photographs were securely stored.

Each student interviewed went through tasks for the following two objectives from the GOLD curriculum: Uses number concepts and operations, and explores and describes spatial relationships and shapes. These objectives and specific skills within these objectives were selected because they rely more heavily on students' explanations and descriptions of ideas rather than just showing that they can do something or not (for example, can the student verbally count to 10). If a student had wished to stop at any point before we got through all of the tasks/questions, we would have stopped for that day, and they would have had the opportunity to resume on a different day. This was not necessary however, and all students were able to complete their tasks in a single session.

Table 2 below shows all of the objectives and skills for pre-K mathematics in the GOLD curriculum, and the shaded portions are those that were selected for this study.

Table 2: Objectives and Skills for Pre-K Mathematics in the Teaching Strategies GOLD Objectives for Development & Learning Program

Objective: Uses number concepts and operations		
Skill: Counts		Verbally counts to 20; counts 10-20 objects accurately; knows the last number states how many in all; tells what number (1-10) comes next in order by counting
Skill: Quantifies	Recognizes and names the number of items in a small set (up to five) instantly; combines and separates up to five objects and describes the parts	Makes sets of 6-10 objects and then describes the parts; identifies which part has more, less, or the same (equal); counts all or counts on to find out how many
Skill: Connects numerals with their quantities	Identifies numerals to 5 by name and connects each to counted objects	Identifies numerals to 10 by name and connects each to counted objects
Objective: Explores and describes spatial relationships and shapes		
Skill: Understands spatial relationships		Uses and responds appropriately to positional words indicating location, direction, and distance
Skill: Understands shapes		Describes basic two- and three-dimensional shapes by using own words; recognizes basic shapes when they are presented in a new orientation
Objective: Compares and measures		
Skill: Compares and measures	Compares and orders a small set of objects as appropriate according to size, length, weight, area, or volume; knows usual sequence of basic daily events and a few ordinal numbers	Uses multiples of the same unit to measure; uses numbers to compare; knows the purpose of standard measuring tools
Objective: Demonstrates knowledge of patterns		
Skill: Demonstrates knowledge of patterns	Copies simple repeating patterns	Extends and creates simple repeating patterns

For each of the three skills selected for this study, there were three different sets of questions presented to students. The Manipulatives & Their Photo questions first sought to determine how students respond when using only manipulatives (M), and then sought to determine how students respond when using a photo they took of those manipulatives (MP). The Their Photo Only (P) questions sought to determine how students respond when using a photo they took of manipulatives without first responding using the manipulatives. The Given Photo (G) questions sought to determine how students respond when using a content-related photo that was presented to them, but that they did not take themselves. If students were assigned to the Manipulatives & Their Photo (MP) questions for the first set of questions within each skill, then they were assigned to the Their Photo Only (P) questions for the second set of questions within each skill, and vice versa. Therefore, for each skill, 12 students were assigned to the Manipulatives & Their Photo (MP) questions, and a different set of 12 students were assigned to the Their Photo Only (P) questions. All students were asked the Given Photo (G) questions for both sets of questions within each skill.

For each skill, there were minimal changes between the two question sets. In the Quantifying tasks, one question set involved two different numbers of checkers, while the other question set involved an equal number of checkers (see Appendix D1 for this interview protocol). In the Understanding Spatial Relationships tasks, the position of the materials varied for each set, but the same materials were still used (see Appendix D2 for this interview protocol). In the Understanding Shapes tasks, the actual shapes used varied for each set, but were still kept to basic 2-dimensional and 3-dimensional shapes with different attributes such as size and color (see Appendix D3 for this interview protocol). The goal was to make sure the content addressed the appropriate skill while changing it slightly so that each student could do both question sets for each skill.

The Given Photo (G) portions of each task varied among the three skills. In the Quantifying tasks, the given photos were generally similar to the task and used all of the same materials, but positioned the checkers in a way that might promote interesting noticings and comments. In the Understanding Spatial Relationships tasks, the given photos were nearly identical to the task and used all of the same materials. In the Understanding Shapes tasks, the given photos related to the shapes used in the task, but were of actual objects. The goal was to be able to compare what happened when students were speaking from photos that were not their own when those photos were 1) similar to but not exactly like their photos and prior situations with manipulatives, 2) exactly like their photos and prior situations with manipulatives, and 3) completely different from their photos and prior situations with manipulatives. *Figure 4* below shows the Given photographs for each math content area: Quantifying, Understanding Spatial Relationships, and Understanding Shapes.

Figure 4: Given Photographs for Each Math Content Area

Data Analysis

To begin the data analysis process for this study, the audio recordings were transcribed, and an open coding system was used first to identify general categories of student responses for each content area, and then to code the responses. Open coding was selected so that categories and subcategories of events, actions, and interactions could be placed into categories and subcategories depending on what the data showed, and not what categories were predicted ahead of time (Corbin & Strauss, 1990). Research has shown that a key advantage of the open coding process is that the concepts emerge from the raw data and can later be grouped into conceptual categories (Khandkar, 1998). The goal was to build a descriptive, multi-dimensional preliminary framework for later comparison of the photography situations to the manipulative-only situations. Furthermore, I didn't want to set up codes ahead of time based on what I might have expected, since there would likely be categories of results that I could not predict ahead of time.

My first research question asked what the affordances are of using photography with young children to make connections and explain their thinking when engaged in mathematical tasks. To address this question, as well as to provide a more quantitative comparison between the content areas and photography situations, I structured my data analysis as follows: 1) For each content area, I recorded results from the Manipulative-only situation (M), and then compared the results from the photography situations to this baseline. I noted when the results improved as compared to the Manipulative-only situation, and 2) These results could then be used to compare students' performances across all three content areas, as well as to compare the performances across the photography situations.

My second research question asked what affordances are offered by the use of photography in the individual assessment of the mathematical skills of young children. To address this question, I used the previously recorded data and looked at the implications for each individual student as compared to the group of students as a whole. Putting these pieces together helped to form a layered narrative for individual students which had implications about the affordances of photography for these tasks. Examining the results as a whole and individually was critical as the affordances often differed greatly moving from the group data to the individual data.

The choices about data analysis were also made in order to get an understanding of how the components of the Activity Theory RME hybrid framework work together to ensure that the division of labor was weighted towards the students being active in the learning process and the creation of mathematical knowledge, instead of being passive recipients. The open coding of the transcribed data allowed for the examination of the students' thinking in terms of vertical mathematization (the reorganization of ideas within mathematics), abstraction (both photographic and mathematic), horizontal mathematization (the reorganization of ideas within the real world), and different representations and understandings of the real world (the real world including the mental, the personal, and the physical). Next, I will describe the specific data analysis process for each of the math content areas in this study.

First, for the Quantifying tasks, students were asked the open-ended question of "What can you tell me about these groups of checkers?" after they sorted the checkers by color (black and red). Student responses fell into the following 3 categories: 1) They didn't respond without a more specific prompt, 2) They did respond, but their responses were not mathematical in content (i.e., they focused on the colors of the checkers but not the amounts), and 3) Their responses were mathematical in content (i.e., they focused on the amounts of the checkers).

When looking at the results, I distinguished responses that had room for improvement (numbers 1 and 2 above) from responses that did not have room for improvement (number 3 above). I did this in order to keep the focus on SNIP situations (situations where students show they are less proficient than they actually are), and to see how photography improved results in these cases.

For the Quantifying tasks, students were also asked the closed question of "Are there more black checkers or red checkers?" after they sorted checkers by color. Student responses fell into the following 3 categories: 1)They didn't answer the question, or answered it incorrectly, 2)They answered the question correctly, but had no explanation or a non-mathematical explanation as to how they knew (i.e., there are more black checkers because black is my favorite color), and 3)They answered the question correctly, and had a mathematical explanation as to how they know (i.e., I counted both groups of checkers, and there were 4 checkers in each group, so that's the same amount). When looking at the results, I again distinguished responses that had room for improvement (numbers 1 and 2 above) from responses that did not have room for improvement (number 3 above).

Second, for the Understanding Spatial Relationships tasks, students had opportunities to speak about manipulatives that were set up in a very specific way. These items included a clear plastic container, a green marble, a pink eraser, a yellow checker, and a blue checker. Students were asked to describe what they saw, and the results were recorded as follows. First, the number of spatial words was recorded. These included terms such as next to, inside of, farther, closer, by, in between, under, over, and any other spatial word or variation of these words. Second, the number of total words was recorded. These included the spatial words, but also other non-spatial words such as the color of the objects, the name of the objects, or other attributes of the objects (i.e., the soft eraser). Next, the number of spatial words was divided by the number of total words to get a spatial word ratio. The idea of this is that the higher the ratio, the better the result in that students were more focused on spatial relationships vs. other non-spatial attributes. Finally, the number of distractions was recorded. These included drawing distractions that came about from students using the drawing app on their iPads (for example, if they started drawing happy faces instead of focusing on the math task at hand), and non-drawing distractions that came about from students answering a question like "Where is the green marble?" with a response of "It's right there!" or pointing at the marble.

For the Understanding Spatial Relationships tasks, there was no distinction made between the responses that did or did not have room for improvement, because there was always a chance for students to say more spatial words and more total words. The exceptions to this were the spatial word ratios (a ratio of 1 was as high as it could be) and the number of distractions (there couldn't be fewer than 0 distractions).

Third, for the Understanding Shapes Tasks, students had opportunities to speak about 6 different shape manipulatives (a large red rectangle, a large red circle, a small yellow square, a large yellow square, a small blue cube, and a small blue triangle). Students were asked to describe what they saw, and were also asked to name items that were similar to the shapes they were exploring, and the results were recorded as follows. First, the number of shape words was recorded. These included terms such as the names of the shapes, the number of sides or corners, the straightness or roundness of the sides, the idea of a shape being "flat" or 2D, or "puffy" or 3D, and any other shape word or variation of these concepts. Second, the number of total words was recorded. These included the shape words, but also other non-shape words such as the color of the objects, the size of the objects, or other attributes of the objects (i.e., the smooth square). Next, the number of shape words was divided by the number of total words to get a shape word ratio. The idea of this is that the higher the ratio, the better the result in that students were more focused on shape understanding vs. other non-shape attributes. Finally, the number of real-world shapes that students were able to name was recorded. These included items that were visible to students in the immediate surrounding (i.e., that box looks like a rectangle), and items that were not visible to students in the immediate surrounding (i.e., there are triangles in a book I have at home).

For the Understanding Shapes tasks, there was no distinction made between the responses that did or did not have room for improvement, because there was always a chance for students to say more shape words, more total words, and name more real-world shapes. The exception to this was the shape word ratios (a ratio of 1 was as high as it could be).

Limitations

There were three primary limitations of this research study. First, the relatively small N of 24 limits the generalizability of the results. Next, as this study was only conducted in two environments, it was a limitation that other environments and corresponding norms were not examined. Finally, no educational or other background information was known about the research participants, including the math levels of the students or any mathematical strengths. While this information did not necessarily affect the processes of the research tasks, it might have been interesting to know how the results were correlated with such information, as well as how teachers might have felt about the results.

Chapter 4
Results

There are three key findings in this study that support the use of photography with students during mathematical tasks. First, for the Quantifying tasks, when there was room for improvement in students' responses, 80% of the students were able to give more detailed and more mathematically relevant responses with the use of photography as compared to the use of manipulatives alone, and 64% of the students were able to give more accurate responses with the use of photography. Second, for the Understanding Spatial Relationships tasks, 63% and 83% of the students in this study were able to describe more *spatial attributes* and more *total attributes*, respectively, with the use of photography as compared to the use of manipulatives alone, and 38% of the students showed a decrease in the number of distractions they had with the use of photography. Third, for the Understanding Shapes tasks, 38% and 42% of the students in this study were able to describe more *shape attributes* and more *total attributes*, respectively, with the use of photography as compared to the use of manipulatives alone, and 54% of the students showed an increase in the number of real-world shapes they were able to recall with the use of photography.

For this section, the results for each of the three mathematical content areas will first be presented individually. Next, the content results will be summarized in order to highlight data that would be useful for considering the affordances of photography in a whole-group setting. This includes an examination of the differences among the three mathematical content areas, as well as the differences among the photography situations. Finally, the content results will be used to highlight data that would be useful for considering the affordances of photography for individual student assessments and learning. This includes an examination of how the layers of results for these tasks come together to form a photography narrative for individual students.

Quantifying

For these tasks, students had the opportunity to speak from manipulatives (checkers) when asked the open-ended question “What can you tell me about these groups of checkers?” after they sorted them by color (black and red). These results (M) were compared to the results when students spoke from 1) their own photo of the sorted checkers after they spoke from the manipulatives (MP), 2) their photo of the sorted checkers without first speaking from the manipulatives (P), and 3) given photos of groups of black and red checkers (G). Out of 24 initial responses to this open-ended question, 15 responses had room for improvement, meaning students either gave no response without a prompt, or they gave a response but the response was not mathematical in content (for example, if they said things like black is their favorite color, or that the position of the checkers looked like a flower, or even that they liked to play checkers at their friend’s house). Of these 15 students, 12 showed an improved response in at least one of the photography situations (with 6 students showing an improvement in two photography situations, and 2 students showing an improvement in all three photography situations).

Students also had the opportunity to speak from manipulatives (checkers) when asked the closed question “Are there more black checkers or red checkers?” after they sorted checkers by color (black and red). These results (M) were compared to the results when students spoke from 1) their own photo of the sorted checkers after they spoke from the manipulatives (MP), 2) their photo of the sorted checkers without first speaking from the manipulatives (P), and 3) given photos of groups of black and red checkers (G). Out of 24 initial responses, 14 responses had room for improvement, meaning students either got the answer wrong, or they got it right but were not able to explain in any meaningful mathematical sense how they knew. Of these 14 students, 9 showed an improved response in at least one of the photography situations (with 7 students showing an improvement in two photography situations, and 1 student showing an improvement in all three photography situations).

There are several other interesting results that come from the Quantifying data. First, of the 29 possible times there could have been improvements in each of the photography situations when compared to the Manipulative-only (M) situation, the Given Photo (G) situation showed the greatest number of improved responses with 19 (next was the Their Photo Only (P) situation with 16, and last was the Manipulatives & Their Photo (MP) situation with 5). Next, there were more improvements in photography situations for the open-ended questions than for the closed questions (improvements were shown 49% of the time vs. 43% of the time respectively). Finally, there were 6 students who went from a level 1 response with manipulatives (no response or an incorrect response, and no explanation or a mathematically irrelevant explanation) to a level 3 response with photography (a correct response accompanied by a mathematically relevant explanation). Full results for Quantifying can be seen in Appendix E1.

Understanding Spatial Relationships

For these tasks, students had opportunities to speak from manipulatives that were set up in a very specific way (a clear plastic container, a green marble, a pink eraser, a yellow checker, and a blue checker). Students were asked to describe what they saw when looking at these items, and these results (M) were compared to the results when students spoke from 1) their own photo of the items after they spoke from the manipulatives (MP), 2) their photo of the items without first speaking from the manipulatives (P), and 3) given photos of the items (G). First, for the number of spatial words, 15 of the 24 students showed an increased number of spatial words in at least one of the photography situations (with 4 students showing an improvement in two photography situations, and 7 students showing an improvement in all three photography situations). Second, for the number of total words, 20 of the 24 students showed an increased number of total words in at least one of the photography situations (with 6 students showing an improvement in two photography situations, and 6 students showing an improvement in all three photography situations). Next, for the spatial word ratios, 15 of the 24 students showed an improved spatial ratio in at least one of the photography situations (with 6 students showing an improvement in two photography situations, and 4 students showing an improvement in all three photography situations). Finally, for the number of total distractions, 9 of the 24 students showed a decreased number of total distractions in at least one of the photography situations (with 5 students showing an improvement in two photography situations, and 2 students showing an improvement in all three photography situations).

There are several other interesting results that come from the Understanding Spatial Relationships data. First, 6 students showed improvements for all four categories (spatial words, total words, spatial word ratios, and distractions) when using photography. Next, there were 33 improvements for spatial words, 38 improvements for total words, 29 improvements for spatial word ratios, and 18 improvements for distractions. Finally, when a student had an increase in the number of spatial words or total words with a photography situation (which happened 71 times in total), the number of words with photography was at least double what it was with manipulatives alone in 29 of those instances. Full results for Understanding Spatial Relationships can be seen in Appendix E2.

Understanding Shapes

For these tasks, students had opportunities to speak about 6 different shape manipulatives (a large red rectangle, a large red circle, a small yellow square, a large yellow square, a small blue cube, and a small blue triangle). Students were asked to describe the shapes while using the manipulatives, and then these results (M) were compared to the results when students spoke from 1) their own photo of the shapes after they spoke from the manipulatives (MP), and 2) their photo of the shapes without first speaking from the manipulatives (P). Students were also asked to name real-world items that were similar to the shapes they were exploring. First, for the number of shape words, 9 of the 24 students showed an increased number of shape words in at least one of the photography situations (with 2 students showing an improvement in both photography situations). Second, for the number of total words, 10 of the 24 students showed an increased number of total words in one of the photography situations. Next, for the shape word ratios, 14 of the 24 students showed an improved shape ratio in at least one of the photography situations (with 9 students showing an improvement in both photography situations, and 4 students showing an improvement in all three photography situations). Finally, for the number of real-world shapes students were able to name, 13 of the 24 students showed an increased number of named shapes in at least one of the photography situations (with 7 students showing an improvement in both photography situations).

There are several other interesting results that come from the Understanding Shapes data. First, 2 students showed improvements for all four categories (shape words, total words, shape word ratios, and real-world shapes) when using photography. Next, there were 11 improvements for shape words, 10 improvements for total words, 23 improvements for shape word ratios, and 20 improvements for real-word shapes. Finally, when a student had an increase in shape words or total words (which happened 21 times in total), the number of words with photography was at least double what it was with manipulatives alone in 7 of those instances. Full results for Understanding Shapes can be seen in Appendix E3.

Results Among the Various Mathematical Skills and Photo Situations

The results in the previous sections are important as they highlight what is significant *within* the various mathematical skills and photo situations. However, when discussing the results from this study, it is also important to consider the following: 1) What do the results signify *among* the various mathematical skills and photo situations? 2) What do the results mean in terms of individual students?

First, consider these results *among* the various mathematical skills and photo situations. *Table 3* indicates how many times improved responses were seen for each photo situation and skill when compared to the Manipulative-only situation (M). For example, for the Quantifying tasks, there were 29 possible times in this study when the students' responses could have improved in the Manipulatives & Their Photo (MP) situation when compared to the Manipulative-only (M) situation. The results show that the responses improved in 5 of those cases; therefore, the improvement ratio in this case is 0.17. The rest of the results can be seen in the table below, in addition to the averages for each skill set type and each photography situation type.

Table 3: Results Among the Various Mathematical Skills and Photo Situations

	MP	P	G	Average
Quantifying	5/29 = 0.17	16/29 = 0.55	19/29 = 0.66	0.46
Understanding Spatial Relationships	41/96 = 0.43	35/96 = 0.36	42/96 = 0.44	0.41
Understanding Shapes	29/96 = 0.30	35/96 = 0.36	N/A	0.33
Average	0.30	0.42	0.55	

In this study, the mathematical skill with the greatest average number of improved responses with the use of photography was Quantifying. Across all of the photography situations for Quantifying, improvements were shown 46% of the time as compared to the Manipulative-only situations. The skill of Understanding Spatial Relationships had the next highest average number of improved responses with the use of photography (improvements were shown 41% of the time as compared to the Manipulative-only situations). And even the skill of Understanding Shapes, which showed the smallest average number of improved responses with the use of photography, still showed improvements 33% of the time as compared to the Manipulative-only situations.

The photography situation with the greatest average number of improved responses was when students were speaking from photographs that were shown to them but that they did not take themselves (G). Across all of the mathematical content areas for these given photos (G), improvements were shown 55% of the time as compared to the Manipulative-only (M) situations. The photography situation when students were speaking from photographs that they took without speaking from the manipulatives first (P) had the next highest average number of improved responses (improvements were shown 42% of the time as compared to the

Manipulative-only situations). And even the photography situation when students were speaking from photographs that they took after speaking from the manipulatives first (MP), which showed the smallest number of improved responses, still showed improvements 30% of the time as compared to the Manipulative-only (M) situations.

Individual Student Results

Table 4 shows the results for each individual student. For example, Student 1 had 2 chances to show improved responses for the Quantifying problems in the situation where s/he spoke from her/his photo after s/he spoke from the manipulatives (MP). Out of these 2 chances, s/he showed an improved response 1 of those times, thus the 1/2 in that cell. Cells that show improvements exactly half of the time are shaded in light grey. Cells that show improvements greater than half of the time are shaded in dark grey. And cells that show improvements less than half of the time are left unshaded.

Table 4: Individual Student Results

Student 1	MP	P	G
Quantifying	1/2	1/2	2/2
Spatial	1/4	4/4	4/4
Shapes	1/4	1/4	N/A

Student 2	MP	P	G
Quantifying	0/1	0/1	1/1
Spatial	1/4	2/4	1/4
Shapes	1/4	1/4	N/A

Student 3	MP	P	G
Quantifying	0/1	1/1	1/1
Spatial	3/4	3/4	0/4
Shapes	1/4	1/4	N/A

Student 4	MP	P	G
Quantifying	N/A	N/A	N/A
Spatial	1/4	1/4	1/4
Shapes	1/4	2/4	N/A

Student 5	MP	P	G
Quantifying	0/1	0/1	0/1
Spatial	3/4	2/4	2/4
Shapes	1/4	1/4	N/A

Student 6	MP	P	G
Quantifying	0/2	0/2	2/2
Spatial	3/4	0/4	2/4
Shapes	2/4	0/4	N/A

Student 7	MP	P	G
Quantifying	1/2	1/2	1/2
Spatial	2/4	0/4	0/4
Shapes	2/4	3/4	N/A

Student 8	MP	P	G
Quantifying	0/2	1/2	1/2
Spatial	2/4	3/4	3/4
Shapes	1/4	1/4	N/A

Student 9	MP	P	G
Quantifying	0/1	0/1	0/1
Spatial	1/4	0/4	2/4
Shapes	2/4	1/4	N/A

Student 10	MP	P	G
Quantifying	0/1	1/1	1/1
Spatial	1/4	0/4	1/4
Shapes	1/4	2/4	N/A

Student 11	MP	P	G
Quantifying	0/2	2/2	2/2
Spatial	3/4	3/4	3/4
Shapes	2/4	0/4	N/A

Student 12	MP	P	G
Quantifying	0/1	0/1	0/1
Spatial	1/4	0/4	1/4
Shapes	0/4	0/4	N/A

Student 13	MP	P	G
Quantifying	1/2	1/2	0/2
Spatial	1/4	3/4	4/4
Shapes	0/4	0/4	N/A

Student 14	MP	P	G
Quantifying	0/2	1/2	1/2
Spatial	1/4	1/4	1/4
Shapes	0/4	1/4	N/A

Student 15	MP	P	G
Quantifying	2/2	2/2	2/2
Spatial	0/4	0/4	0/4
Shapes	1/4	2/4	N/A

Student 16	MP	P	G
Quantifying	0/2	1/2	0/2
Spatial	4/4	3/4	4/4
Shapes	1/4	1/4	N/A

Student 17	MP	P	G

Quantifying	0/1	1/1	1/1
Spatial	2/4	2/4	4/4
Shapes	3/4	1/4	N/A

Student 18	MP	P	G
Quantifying	0/1	0/1	1/1
Spatial	0/4	1/4	2/4
Shapes	2/4	2/4	N/A

Student 19	MP	P	G
Quantifying	0/2	2/2	2/2
Spatial	1/4	1/4	0/4
Shapes	2/4	1/4	N/A

Student 20	MP	P	G
Quantifying	N/A	N/A	N/A
Spatial	1/4	1/4	1/4
Shapes	0/4	2/4	N/A

Student 21	MP	P	G
Quantifying	0/1	1/1	1/1
Spatial	1/4	0/4	0/4
Shapes	2/4	4/4	N/A

Student 22	MP	P	G
Quantifying	N/A	N/A	N/A
Spatial	3/4	2/4	3/4
Shapes	2/4	4/4	N/A

Student 23	MP	P	G
Quantifying	N/A	N/A	N/A
Spatial	3/4	1/4	1/4
Shapes	1/4	1/4	N/A

Student 24	MP	P	G
Quantifying	N/A	N/A	N/A
Spatial	2/4	2/4	2/4
Shapes	0/4	3/4	N/A

In this study, there are several students who stand out as having benefitted the most from the use of photography. Looking at the various photography situations, students 7 and 22 showed improvements at least half of the time for all of the MP situations, where they spoke from their photograph of the manipulatives after first speaking from the manipulatives. Students 22 and 24 showed improvements at least half of the time for all of the P situations, where they spoke from their photograph of the manipulatives without first speaking from the manipulatives. And students 1, 6, 8, 11, 17, 18, 22, and 24 showed improvements at least half of the time for all of the G situations, where they spoke from a photograph that was given to them but that they did not take themselves.

Looking at the various math content skills in this study, students 1, 7, and 15 showed improvements at least half of the time for all of the Quantifying tasks. Students 5, 8, 11, 16, 17, 22, and 24 showed improvements at least half of the time for all of the Understanding Spatial Relationships tasks. And students 7, 18, 21, and 22 showed improvements at least half of the time for all of the Understanding Shapes tasks.

Looking at this data combined, we see that there are certain students who showed improvements at least half of the time in more than 1 content area and/or more than 1 photography situation. For example, student 7 showed improvements at least half of the time for both Quantifying tasks as well as Understanding Shapes tasks. Student 7 also showed improvements at least half of the time when s/he spoke from her/his own photo of the manipulatives after s/he first spoke from the manipulatives (MP). Student 24 showed improvements at least half of the time when sh/e spoke from her/his own photo of the manipulatives without first speaking from the manipulatives (P), and also when s/he spoke from photos that were given but that s/he did not take (G). In addition, student 24 showed improvements at least half of the time for the Understanding Spatial Relationships tasks.

Student 22 (let's call him Jabrel), had results that were even more significant. He appears in 5 of the 6 categories above, which is the highest of any student in this study. In the Quantifying tasks, Jabrel was able to discuss the groups of checkers he sorted by counting the checkers in each group, and coming to the correct conclusion that there were more black checkers. Because with the manipulatives only, he was able to give a mathematical response to the open-ended question without needing a prompt, and was also able to answer the closed question correctly, there was not room for improvement in the Quantifying tasks for him with the use of photography.

In the Understanding Spatial Relationships tasks, Jabrel showed improved responses at least half of the time for spatial words, spatial word ratios, and distractions (the only category not showing improved responses more than half of the time was total words). First, he used more spatial words in every photography situation than he gave in the manipulative-only situation. Next, he had an improved spatial word ratio in every photography situation except for the G situation (where he was speaking from a photo that was given to him but that he did not take himself). Finally, he had fewer distractions in every photography situation except for the P situation (where he was speaking from a photo that he took of the manipulatives without speaking from the manipulatives first).

In the Understanding Shapes tasks, Jabrel showed improved responses at least half of the time for shape words, total words, shape word ratio, and real-world shapes (therefore, in every category that was looked at for these tasks). First, he used more shape words and total words in the P situation (where he was speaking from a photo that he took of the manipulatives without speaking from the manipulatives first) than he used in the manipulative-only situation. Next, he had an improved shape word ratio in both photography situations. Finally, he was able to use more real-world shape examples in both photography situations. A student like Jabrel makes it clear how the use of photography can help us understand young children's thinking, especially in SNIP cases like this where students are not showing all that they know or can do when relying only on questioning and/or concrete manipulatives.

Chapter 5
Discussion

In this section, I will first discuss how the general results of this study are situated within several key ideas in the literature surrounding the use of photography with children, including ideas from my theoretical framework. Then, I will use these key ideas to help to answer my initial research questions:

- What are the affordances offered by the use of photography with young children to make connections and explain their thinking when engaged in mathematical tasks?
    - Which mathematical content strand in this study offers the greatest affordances?
    - Which photography situation in this study offers the greatest affordances?
- What are the affordances offered by the use of photography in the individual assessment of the mathematical skills of young children?

How the Results are Situated within the Literature

With this study, I sought to discover the affordances of photography specifically in SNIP situations, where students show that they are not proficient but actually are proficient (or at least more knowledgeable than the results reflect). I did this by examining student responses and demonstrated understanding of mathematics tasks and assessment questions both with and without the use of photography. Since students in this study were photographing concrete manipulatives only, they generally did not have to visualize or form a mental image before taking their photos (Sadler, 1993). The design of this study was purposeful in not allowing students to have much creativity in their photographs, not because this kind of visualization could not be helpful to students, but because a main purpose of this study was to look at the affordances of using photographs as platforms for mathematical communication as compared to using concrete manipulatives for such communication. One might wonder why there would be any difference at all when students use a set of concrete manipulatives compared to when they use a photo of those exact same manipulatives to describe their thinking. However, the results show that using photography (both student-generated and teacher-generated) with children in this manner gives students the opportunity to communicate a more complete perspective and understanding of the mathematics in which they were engaged, and the literature surrounding the use of photography with children supports these findings as well. In photography, this idea is known as sideways thinking (Patterson, 2004).

There are two general advantages of using photography with children that are important to revisit with respect to the results of this study. First, the process of explaining mathematical thinking and reasoning remains difficult for children in group learning situations and individual assessment situations (Carpenter & Lehrer, 1999). This can be especially true when the models or manipulatives that children are working with do not have meaning to them (Manches & O'Malley, 2012). Therefore, a photograph is a way to give those models or manipulatives meaning they would not otherwise have for children (Northcote, 2011). This helps to explain why there were improved results in this study when students used their photographs of manipulatives as compared to when they used only the manipulatives to describe their thinking.

Second, using photographs may help students to reduce their cognitive load, thus allowing the mathematics to be the focus more than other non-mathematical attributes. The cognitive load theory is based on the idea that people have a limited amount of cognitive resources (Vredeveldt et al., 2011). Once an image is captured, certain potentially distracting features of the manipulatives are negated (for example, they cannot be picked up and repositioned). While I am not making an argument here for or against the value of physically manipulating objects in order to increase or solidify mathematical understanding, I am saying that in terms of communicating mathematical understanding, there are potentially distracting features of physical manipulatives that can get in the way. This helps to explain why there were improved results when students spoke from photographs of manipulatives as compared to when they spoke from the manipulatives only. It also helps to explain why there were greater improvements when students spoke from the given photographs as compared to the photographs that they took, since the given photographs contained a minimal amount of distracting information that may have been left in student photographs (for example, there were not the superfluous materials in the frame that sometimes were captured in student photos).

There are also two research advantages of using photography with children that are important to revisit with respect to the results of this study. First, the students in this study generally seemed to enjoy this process (as evidenced by several students asking to do more problems and stay longer after they had completed their tasks-something I have not heard often when doing more traditional mathematics assessments with children). Further, all of the students in this study were able to successfully use the technology. This was true of students who said they used iPads in their classrooms or at home on a regular basis, as well as for students who said that they had never used an iPad before, or had little experience taking digital photographs. This is important because it shows that bringing in this type of technology does not necessarily create a power imbalance even when an imbalance exists in terms of prior experience with or current access to such technologies, since the students are able to master the technologies so quickly and easily (MacDonald, 2012).

Second, unlike manipulatives, these photographs could be used at a later date as a starting point for further discussion either in a group learning setting, or in an individual assessment situation (Cook & Hess, 2007). These photographs could also be useful when having discussions with parents about students' mathematical understandings and progress, as it would provide something beyond what is verbal or written, and serve as a platform for the students' voices in such discussions.

It is also important to consider the function of the photographs used with children to examine why photography worked in this study to help students communicate their mathematical understanding. Traditionally, photographs used with elementary students are used not to communicate information, but to keep students engaged (Carney & Levin, 2002). Elementary curricular materials tend to be full of bright, colorful, and fun photographs that serve more to maintain students' interest than to convey important information. In contrast, photographs used with secondary students tend to convey additional information, thus requiring extra mental effort to process the photographs (Scheiter et al., 2009).

In these terms, the photographs in this study provided the best of both worlds in that they simultaneously were able to engage students and convey information. While the photographs were not inherently interesting or engaging in themselves, this was never the point. For the student-created photographs, the engagement piece came because they were the ones who created the photographs. And for the student-created photographs as well as the teacher-created photographs, the mathematical content was there, but it was not in a form that overloaded them with extra information. In fact, the photographs helped to remove some of the extra stimuli, thus leaving them with a greater focus on the mathematical content. By doing the opposite of what photographs normally do in an elementary environment, we see a new way that photographs can be used with young children.

The theoretical framework for this study is represented by a hybrid model of the third generation of Engeström's Activity Theory, combined with components of the Realistic Mathematics Education (RME) framework. This Activity Theory RME hybrid model places the RME framework at the center of the Activity Theory framework. It is important to understand that together, the group of tasks involved in this study make up only a small part of this model; however, the model as a whole needs to be considered in order to see the full value and purpose of this study.

The overarching process in this model in relation to this study is that the subjects (the students) use a mediational tool/artifact (the photographic device which in this case was an iPad) to produce an object (the digital photograph) which produces an outcome (the communication of mathematical understanding). This is what was seen in the general results of this study. However, this process is part of a cycle that would not end here if students were able to continue to use photography in their mathematics learning.

To look at this more closely, let's take an example of a given Quantifying photograph where there were in fact more black checkers than red checkers (6 black checkers and 5 red checkers), but since the black checkers were clumped together and the red checkers were spread out over a larger space, students sometimes incorrectly concluded that there were more red checkers, which is a classic Piaget finding (Baroody & Wilkins, 1999). Students were able to use this photograph to communicate their understanding and reasoning about the situation, often citing the fact that the red checkers took up more space as their reason as to how they knew there were more red checkers. What would come next, according to this hybrid model, is that students would enter into a repeating cycle of vertical mathematization (the reorganization of ideas within mathematics) and horizontal mathematization (the reorganization of ideas within the real world) as they worked towards a different representation and understanding of the mathematics involved in real-world situations.

Students with the idea that the amount of space the checkers take up can have something to do with the amount or quantity of checkers are not entirely incorrect. Given that the checkers are the same size, more of them do take up more actual space. However, spreading a given amount of checkers out over a larger space does not mean that the amount of checkers increases or changes in any way. This is a mathematical idea which would need to be reorganized. In order for this to happen, students need opportunities to reflect on this concept using real-word situations. For example, if a child had 5 books to place onto a bookshelf at home, hopefully s/he would understand that whether s/he spread them out on the shelf, or placed them right next to each other, the amount of books would not change. The only way to get more books is to add more books (spreading them out does not increase the number of books, having larger books does not increase the number of books, etc.). Taking various photographs of the books on the bookshelf and then speaking from those photographs could help this child reflect on these ideas, and then better articulate the related mathematical concepts. Further, providing this child photographs of books in various arrangements could also help in terms of reflection and articulation. This cycle would repeat as more and more real-world examples are found, and as the child has opportunities to use photography to help communicate her/his understanding about the mathematics involved in these real-word situations.

Recall also that this hybrid model considers the network of activity systems within the school, home, and community along with the corresponding rules and division of labor. For example, in the community, a student might notice this mathematical phenomenon with groups of children in a park; however, there may be certain photography norms about whether it's ok to take photographs of those children without the consent of their parents and/or guardians. Every situation has a setting with specific "rules" or norms that affect what can be done with photography, just as there are certain mathematical "rules" or norms that exist for each situation. Further, for each situation, individuals have different roles which bring with them different amounts of power and influence. For this Activity Theory/RME hybrid model, photography isn't just a way to record the activity, it *is* the activity central to the reflection and articulation of mathematical ideas for young children.

How the Results are Situated within the Research Questions

My first research question was "What are the affordances offered by the use of photography with young children to make connections and explain their thinking when engaged in mathematical tasks?" In addition to the general affordances aforementioned earlier in this discussion, what the results of this study offer to anyone who works with these students in a group-setting, especially in a mathematical teaching and learning context, is a guide as to what content areas might be most affected in a positive way by the use of photography, as well as what types of photography situations might best be used for students to communicate their mathematical understanding. For example, a teacher of this particular group of students would know by these results that in a class or whole-group setting, showing photographs to students in Quantifying tasks could give the greatest improvements in responses as compared to using manipulatives alone. And just as the results are important to inform a teacher when it might make the most sense to use photography, the results are also important to inform a teacher when, for a certain group of students, it might make the most sense to have students use and speak from manipulatives instead of photographs. For example, when students spoke from their photographs after they spoke from the manipulatives in the Quantifying tasks, they only showed improved responses 17% of the time as compared to using manipulatives alone. This suggests to a teacher that for Quantifying tasks in a class or whole-group setting for this group of students, there perhaps is not as much to be gained from having students take photographs and speak from those photographs after they use and speak from the manipulatives.

My second research question was "What are the affordances offered by the use of photography in the individual assessment of the mathematical skills of young children?" In addition to the general affordances aforementioned earlier in this discussion, what the results of this study offer to anyone who works with these students individually, especially in a mathematical assessment context, is a guide as to what content areas might be most affected in a positive way by the use of photography, as well as what types of photography situations might best be used for students to communicate their mathematical understanding. Students 5, 6, 15, 16, and 21 all showed clear improvements (improved half of the time or more) across one of the math content areas *or* across one of the photography situations. Students 1, 8, 11, 17, and 18 all showed clear improvements (improved half of the time or more) across one of the math content areas *and* across one of the photography situations. Student 7 only had two areas (out of eight) where improvements were not seen more than half of the time, and student 24 only had one area (out of five) where improvements were not seen more than half of the time. And recall that student 22 (Jabrel) made improvements at least half of the time in every single content area and photography situation where it was possible to make improvements. Finally, it is important to note that even though certain students are not highlighted in this section, every single student in this study exhibited improved responses in at least two areas through the use of photography.

When thinking about the different mathematical content strands and the different photography situations in this study, it is important to go beyond *what* the results were and consider *why* the results came out as they did. Recall that *Table 3* showed the results among the various mathematical skills and photo situations.

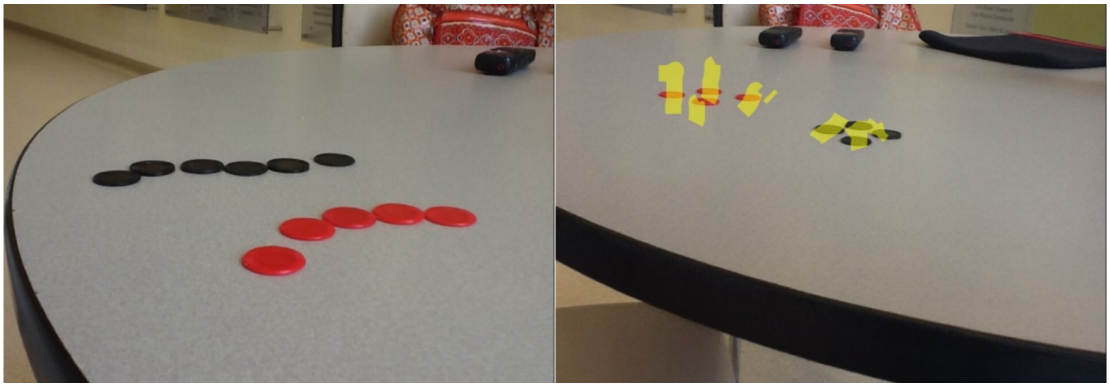
Table 3: Results Among the Various Mathematical Skills and Photo Situations

	MP	P	G	Average
Quantifying	5/29 = 0.17	16/29 = 0.55	19/29 = 0.66	0.46
Understanding Spatial Relationships	41/96 = 0.43	35/96 = 0.36	42/96 = 0.44	0.41
Understanding Shapes	29/96 = 0.30	35/96 = 0.36	N/A	0.33
Average	0.30	0.42	0.55	

From column to column, row to row, or cell to cell, what accounts for the variations in these results? Let's start by looking at the MP situation for Quantifying. This situation showed the smallest amount of improvement from when students spoke from the manipulatives-only (M) to when they spoke from their photos of the manipulatives after they spoke from the manipulatives first (MP). In the Quantifying strand, students had to physically handle the manipulatives (i.e., the checkers) in order to sort them by color. In doing this, the manipulatives were serving as useful tools in relation to the task (for example, several students counted as they were laying the checkers down into their respective color groups). When they were asked to describe the groups of checkers, these students tended to focus more on the amounts or quantities of the checkers over other non-mathematical attributes, such as color or position. When the photographic element was introduced, there was therefore less work for it to do. Students' photographs still did, however, contribute to the improvement we see in this cell (both by giving meaning to the manipulatives and by reducing the cognitive load), just not as much as they perhaps could have if so much of the work had not already been done by the manipulatives.

In the examples below in *Figure 5*, the student on the left counted as she sorted the checkers by color. With just the manipulatives, she was able to state that there were more black checkers than red checkers because "this is 5 and this is 6" (she said this as she pointed to the red line and the black line respectively). The student on the right also counted as he sorted the checkers by color. He first yelled out "Oh! It's the same amount…4!" And then he counted "1,2,3,4,5…1,2,3,4." When I asked him "Are there more black checkers or red checkers?" he replied "both." When he spoke from his photograph of the manipulatives, he again counted and marked his photo as he did so. When asked if there were more black or red checkers, he replied "both…same amount…same amount." These examples highlight how once the manipulatives have helped the students show what they know, there is not as much work left for the photos to do.

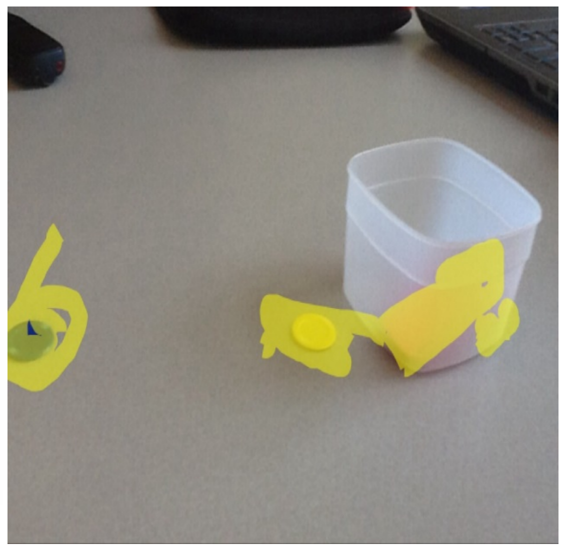
Figure 5: Examples of Counting While Grouping Checkers



If this explanation is sound, then we would expect to see a reverse pattern when manipulatives were not serving as useful tools in relation to the task. Consider the MP situation for Understanding Spatial Relationships. In this strand, students did not handle the manipulatives at all, as the items were already set up and students were asked to simply describe what they saw. In this case, the manipulatives did virtually no work, and therefore it was the student photographs that came in to help students better show what they know. The photographs did this both by giving meaning to the manipulatives, and because students were able to mark them. This ability to mark the photographs helped students to keep track of what they had described and what still needed to be described, which led them to say more in general and also to say more spatial words specifically.

In the example below in *Figure 6*, the student first had described the set-up with manipulatives only (the left photo shows what this set-up looked like). Here she used the spatial term “inside” to describe 2 items only. She stated that the eraser was “inside of the container,” and that the marble was “inside of the container”, and no other spatial terms were used. When she spoke from her photo of the manipulatives however (the right photo shows her photo) and marked the photo as she spoke, she added 2 additional location words. First, she added that the yellow checker was "next to the container." Second, she added that the marble was "under the eraser." This was an especially interesting addition because it showed her ability to give more than one spatial attribute to a single item (i.e., the marble was both inside of the container, and it was under the eraser).

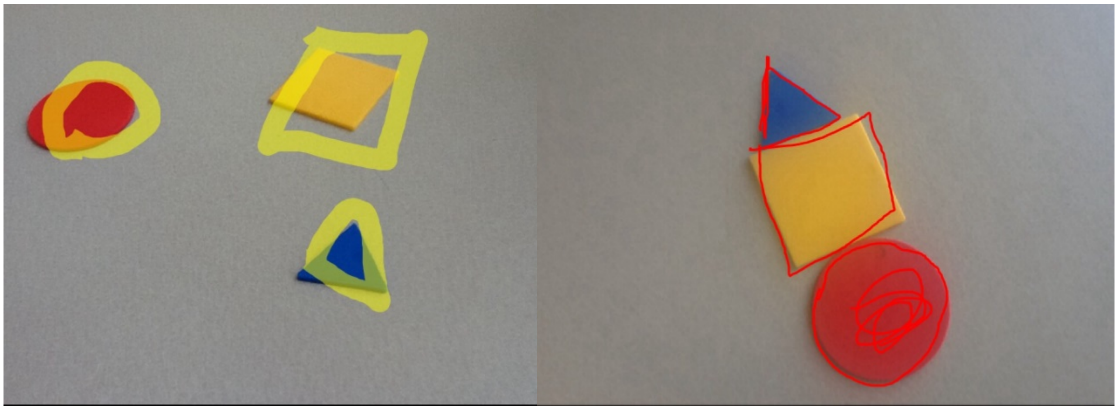
Figure 6: Example of Marking While Describing Items



So indeed, we did see a reverse pattern when manipulatives were not serving as useful tools in relation to the task. For the Quantifying tasks, when students physically handled the manipulatives, the improvement in the photo situation (MP) to the manipulatives-only situation (M) was 0.17, while in the Understanding Spatial Relationships tasks, the improvement was 0.43. For the Understanding Shapes tasks, students could touch and pick up the shape manipulatives if they wanted to, but this wasn't required. So in this case, we might expect results that are in-between those from the other strands, and that is indeed what we saw (a 0.30 improvement). Further, for the Understanding Shapes tasks, students marking their photographs helped them to focus on shape attributes (for example, several students counted the sides as they traced the shapes on their photos, but they did not do this with the manipulatives).

In the examples below, the student on the right did not count or mention the number of sides of the shapes with manipulatives-only. When he spoke from his photo of these shape manipulatives however, he counted the sides as he traced them. He was able to state that "the triangle has 1,2,3" sides, and that "the square has 1,2,3,4" sides. He did trace the circle too, but seemed to get stuck on what to say after he traced it. He paused for a moment, moving his finger around and around inside of the circle, but never did comment on any of its attributes. The student on the left also did not count or mention the number of sides of the shapes with manipulatives-only. When he spoke from his photo of these shape manipulatives, he also traced the shapes. However, instead of counting the sides as the student on the right had done, he instead made "woosh" type sounds that corresponded with each side. For the triangle, he made 3 "woosh" sounds as he traced, for the square he made 4 sounds, and for the circle he made 1 sound. These are interesting examples because they demonstrate students being able to show more of what they know using their photographs as compared to using the manipulatives alone. While the manipulatives could have been traced in a manner similar to how their photographs were traced, the drawing app is what ultimately provided this opportunity.

Figure 7: Examples of Marking While Describing Shapes



Let's move now from looking down the MP column to looking across from the MP cells to the P cells. For the Quantifying tasks, there was an increase from 0.17 improvement in the MP situation to 0.55 improvement in the P situation. In the MP situation, recall that much of the work had been done by the manipulatives, so when students spoke from their photographs after speaking first from the manipulatives, there wasn't as much work left to do. However, in the P situation, when students spoke from their photographs without speaking first from the manipulatives first, that work had been done but the ideas from it had not yet been expressed. What we are seeing in that first value (0.17) are the advantages offered by the fact that the photographs were meaningful to the students because they took them, and that the cognitive load was reduced in that the checkers could no longer be picked up and manipulated. We are still seeing these things in the second value (0.55), but in addition we are seeing the advantages of being able to use the manipulatives meaningfully in the task.

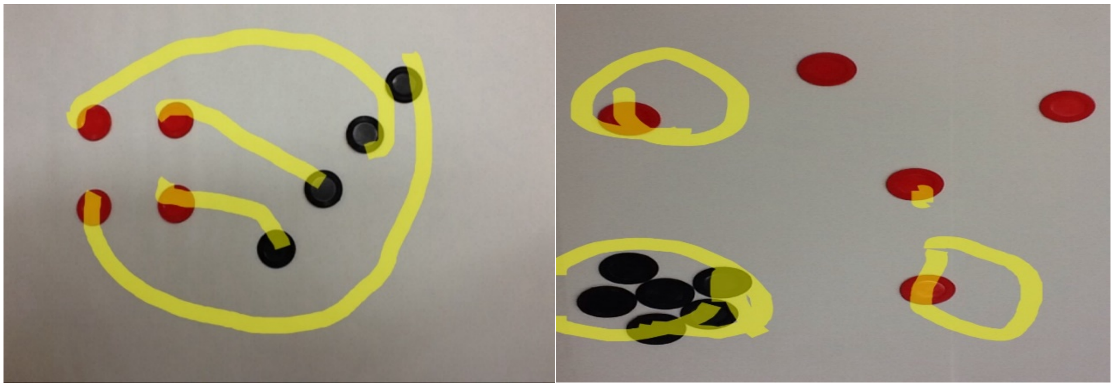
For the Understanding Spatial Relationships tasks, there was a decrease from 0.43 improvement in the MP situation to 0.36 improvement in the P situation. In the MP situation, recall that virtually none of the work had been done by the manipulatives, so when students spoke from their photographs after speaking first from the manipulatives, there was a lot of work left to do. I had expected these values to be similar, since in both cases, the student photographs offered meaning, and in both cases, drawing helped students to track what they were saying. Also, in both cases, the cognitive load was not reduced, since students never handled manipulatives in the first place. A possible reason as to why the MP value is higher than the P value is that in the MP situation, students had an opportunity to reflect on how the items were positioned in 3 dimensions before describing the items in 2 dimensions. In the P situation, students did not have the opportunity to do this. This issue came up in the Understanding Shapes tasks as well, but only for the cube since all other shapes were 2 dimensional.

For the Understanding Shapes tasks, there was an increase from 0.30 improvement in the MP situation to 0.36 improvement in the P situation. Recall that in the MP situation, little work had been done by the manipulatives (especially if students didn't pick up the shapes), so when students spoke from their photographs after speaking first from the manipulatives, there was a lot of work left to do. What we are seeing in that first value (0.30) are the advantages offered by the fact that the photographs were meaningful to the students because they took them, the cognitive load was reduced in that the shapes could no longer be picked up and manipulated, and the marking of the photos helped students to name shape attributes. We are still seeing these things in the second value (0.36), but in addition we are seeing the slight advantage of being able to use the manipulatives meaningfully in the task.

Another important difference between the mathematical content strands was the content of the given photographs (see *Figure 4*). For the Quantifying tasks, the given photographs were similar in content (i.e., still the same number of black and red checkers present in each), but different in that they were arranged in a meaningful way (one set showed the smaller set of checkers spread out over a larger area, and the other set showed a similar amount of checkers arranged in a square and in a line). For the Understanding Spatial Reasoning tasks, the given photographs were exactly the same as the original set-ups. And in the Understanding Shapes tasks, the given photographs were completely different from the original shape manipulatives (they still focused on real-life shapes, but were photographs of ice cubes and a bicycle). For Quantifying tasks, when the given photographs were set up in a meaningful way, we saw the greatest improvement as compared to the manipulatives-only situation (0.66).

In the examples below in *Figure 8*, there is something gained through the given photos that did not come through in the student photos or the manipulatives. In the example on the left, the student knew that there were “equal” amounts of black and red checkers because “they’re both 4!” When she was given the chance to mark the photo and asked how she knew, she drew lines connecting each black checker to a red checker. Although she didn’t use words to explain what she was doing, this marked photograph serves as a record of her understanding of this idea what when the amounts of black and red checkers are equal, for every checker of one color there must be a checker of the other color. In the example on the right, the student incorrectly said that there were more red checkers because “they’re separated.” In this case, the given photograph served to bring out a misunderstanding about quantity that the student held.

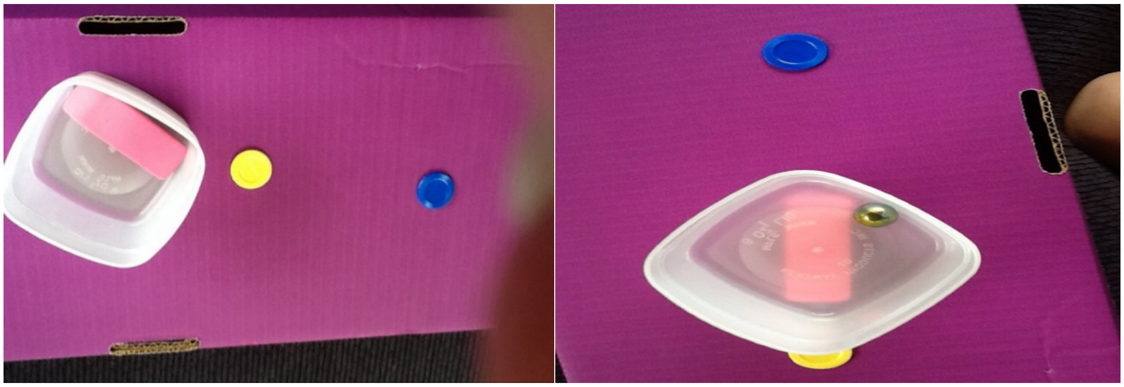
Figure 8: Examples of Quantifying Given Photos



For Understanding Spatial Relationships tasks, I had expected the G value to be similar to the P value because for both of these situations, students were not able to reflect on first on the 3 dimensional representation; however, it was instead similar to the MP value. A possible reason for this is that in the given photographs, all of the items were clearly visible and there was no distracting information in the photographs. In other words, student photographs in this case may have actually added to the cognitive load more than reduced it, in that 1) students now had to describe 3-dimensional objects from a 2-dimensional representation, 2) at times some of those items were no longer clearly visible in the photo, and 3) at times there was potentially distracting information in the photo.

In the examples below in *Figure 9*, the photographs were taken by the same student. In the photo on the left, you cannot clearly see every item, while in the photo on the right, you can clearly see each item. When speaking from the photo on the left, this student only improved in 1 of 4 possible areas when compared to speaking from manipulatives-only; however, when speaking from the photo on the right, this student improved in 3 of 4 possible areas.

Figure 9: Examples of Understanding Spatial Relationships Student Photos



In summary, in SNIP situations, there is a gap between what students know and what students show that they know. There are many factors that can help fill this gap-photography being one of them, and how much photography is able to help do this seems to be dependent somewhat on what the other factors (such as manipulatives) are able to offer students. When manipulatives do a significant portion of the work, there is little left for student photographs to do; however, when manipulatives do little of the work, this is where student photographs can best help to fill in this gap. Also important is whether photographs reduce the cognitive load, or increase it. There are situations where photographs may help to reduce the cognitive load in that the manipulatives can no longer be picked up or cause distractions. There are other situations however, when student photographs may actually increase the cognitive load (such as when a photo cuts out important information). Finally, for this study, the drawing app served as an additional tool that allowed students to keep track of their thinking and better communicate their understanding.

Chapter 6
Conclusion

While this study is situated primarily in the areas of photography and mathematics education, it also draws from educational psychology, international education, and educational departments other than mathematics such as literacy education. It is important to note that while some of the research discussed relates specifically to using photography with children and using photography in mathematics education, other areas were included to illustrate the potential affordances of combining elementary mathematics education and photography. For example, research on the Language Experience Approach suggests that similarly positive results may be found when using the same methods and principles in mathematics education instead of literacy education.

The most significant professional influence for this study came from examining the visual participatory research method know as photo elicitation. Photo elicitation has been used since the 1950s, and continues to be used as a visual method in contemporary research (Prosser & Burke, 2008). Recall that in the Photo Elicitation Interview (PEI), photographs are used as platforms for discussion. When PEIs are used with children, it increases the amount of engagement young people have in the tasks they are asked to perform. Photos can provide children with clear and meaningful entry points into discussions, which empowers them as the experts. According to Clark-IbaNez (2004), "The most common experience conducting PEIs was that photographs spurred meaning that otherwise might have remained dormant" (p. 1513).

The most significant personal influence for this study came through my love of photography, not just as an art, but as a way to communicate (not that those are or could ever be entirely separate entities). In my daily life, I consistently choose photographs to communicate ideas to others, and I prefer to speak from photographs instead of speaking from text. With photographs as a platform for communication, I tend to feel like there is less risk involved. Photographs tend to put the focus on the ideas I am trying to convey, and take the focus away from myself as a speaker. Therefore, with photographs, I end up saying more, and saying it more clearly, than I do without photographs. Also, when I come across a word I don't know, I will do a google image search instead of a definition search. Looking at the array of visual results seems to give me a quicker understanding of the meaning of the word than I would get with a written definition alone, and the definitions obtained in this manner tend to stay with me longer than would written definitions.

The ideas raised in this study support the use of photography in elementary mathematics in several ways. When children take pictures, they take greater ownership of their learning. The process of taking photographs helps to engage students in mathematics, and the process of discussing the photographs helps to create meaning for students. It is important for children to be able to articulate their thinking in mathematics, and student-created photographs are tools that can help students with this articulation. It is my hope that future research in this area can examine the advantages of using photography in mathematics education specifically with ELL students, and/or other students who traditionally underperform on mathematics assessments, or who may not have a strong voice in their mathematics classrooms.

Furthermore, many of the advantages of photography between a researcher and a child also exist in the math classroom, especially in terms of students creating meaning from their photographs, and using their photographs to express that meaning to others. Photographs help students to make connections between mathematics concepts, but also to make connections between math in different contexts and environments. Finally, in terms of sideways thinking, an added benefit in photography is that it can be practiced at any time – not just when making photographs (Patterson, 2004). Similarly, in mathematics, many of the skills involved in this study can be used and practiced at any time – not just when children are in their math classrooms.

Monk (2012) shares a famous quote from Sigmund Freud's 1923 book The Ego and the Id, "Thinking in pictures stands nearer to unconscious processes than does thinking in words, and is unquestionably older than the latter both ontogenetically and phylogenetically" (p. 42). In other words, there is something innate about the relationship between thinking visually and understanding. Monk also shares related ideas from philosopher Ludwig Wittgenstein. It was fundamental to Wittgenstein's philosophy that understanding relies on seeing connections between what can and cannot be put into words, since not everything that can be conceived of can be put into words. According to Monk, "For philosopher Ludwig Wittgenstein, to think, to understand, was foremost to picture" (p. 43).

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Appendices

Appendix A: Site Director Consent Form as Submitted to and Approved by the IRB

UNIVERSITY OF WISCONSIN-MADISON

Site Director Consent Form

I, ____, give my permission for Kelly Harrigan to

(please print name)

conduct research for the project described below in the Parent/Guardian consent form at

____ (name of center). I am the site director, and have authorization to give such consent.

____ ____________________

(signature) (date)

Appendix B: Parent/Guardian Consent Form/Student Assent Form as Submitted to and Approved by the IRB

UNIVERSITY OF WISCONSIN-MADISON

Letter Detailing Research Study

Title of the Study: Photography and Elementary Mathematics

Principal Investigator: Anita Wager (phone: 608-263-5142) (email: awager@wisc.edu)

Student Researcher: Kelly Harrigan (phone: 608-263-5141) (email: kharrigan@wisc.edu)

DESCRIPTION OF THE RESEARCH

Your daughter/son is invited to participate in a research study about photography and elementary mathematics. S/he has been asked to participate because s/he attends ____ (name of center). The purpose of the research is to examine the benefits of having children take photographs, view photographs, and use photographs to elicit questions and discussions as they work through Mathematics tasks.

Audio tapes will be made of your daughter's/son's participation. Only members of the study team will hear the audio recordings. They will be transcribed by the student researcher, and at no time will the children be identified. The audio tapes themselves will not be used in any papers or presentations; however, quotes from the audio tapes may be used. The photographs that your child takes may be used in future presentations or papers; however, there will be NO PHOTOGRAPHS OF YOUR CHILD.

WHAT WILL YOUR CHILD'S PARTICIPATION INVOLVE?

Participants in this research will be asked to do several mathematics tasks using the following set of procedures. First, they will be asked to complete some tasks using just physical manipulatives. Second, they will be asked to complete some tasks first with manipulatives, and then by taking pictures of the manipulatives or objects in the room and using those photographs to answer questions and describe their thinking. Third, they will be asked to answer some questions based only on their photographs of manipulatives or objects, and not on the actual manipulatives.

Participants will take photos of items or places at the research site that they feel represent various stages of the math problems they are working through. This might include things like math manipulatives, nearby desks, furniture, or objects in the surrounding area, windows as they look for examples of area, floor tile patterns as they look for examples of symmetry, etc. They will not be taking photographs of individuals who do not have consent to participate in this study.

ARE THERE ANY RISKS TO PARTICIPANTS?

There is the potential risk of breach of confidentiality. To mitigate this risk, all data will be stored on a password protected laptop and on secure servers. The laptop will be stored in locked facilities when not in use. The identities of all participants will be protected through the use of pseudonyms for individuals and sites. After students are given pseudonyms, any identifying or sensitive information will be deleted.

All research artifacts will be available only to the research team and will be securely stored as per standard social science procedures.

ARE THERE ANY BENEFITS TO PARTICIPANTS?

Although there are no direct benefits, possible benefits of participation include increased understanding of the mathematics involved in the tasks, as well as increased confidence in math.

HOW WILL PARTICIPANT CONFIDENTIALITY BE PROTECTED?

While there will probably be publications as a result of this study, children's names will not be used. Only group characteristics will be published. I would like to be able to quote participants directly without using names. If you agree to allow us to quote your child in publications, please initial the statement at the bottom of this form.

WHOM SHOULD I CONTACT IF I HAVE QUESTIONS?

You may ask any questions about the research at any time. If you have questions about the research you should contact the Principal Investigator Anita Wager at (608) 263-5142. You may also call the student researcher, Kelly Harrigan at (608) 263-5141. If you are not satisfied with the response of the research team, have more questions, or want to talk with someone about your rights, you should contact the Education Research and Social & Behavioral Science IRB Office at 608-263-2320. Participation in this study is completely voluntary. If participants begin participation and change their mind they may end participation at any time without penalty.

UNIVERSITY OF WISCONSIN-MADISON

Parent/Guardian Consent Form

Your signature on this consent form indicates that you have read the attached letter detailing this research study. It also indicates that you had an opportunity to ask any questions about your child's participation in this research, and that you voluntarily give consent for your daughter/son to participate. You will receive a copy of this form for your records.

____

Name of Participant (please print)

______________________________________ ______________

Parent/Guardian Signature Date

UNIVERSITY OF WISCONSIN-MADISON

Initial to give your permission for your child to be quoted directly in publications without using her/his name.

Student Assent Form

Your signature on this assent form indicates that you have discussed the attached letter detailing this research study with a parent or guardian. It also indicates that you had an opportunity to ask any questions about your participation in this research, and that you voluntarily consent to participate. You will receive a copy of this form for your records.

____

Name of Participant (please print)

____ _______________

Participant Signature Date

Initial to give your permission for you to be quoted directly in publications without using your name.

Appendix C: Script/Supplemental Information Page as Submitted to and Approved by the IRB

Script

This information will be told to students before their work on the math tasks begins, and before they take any photos:

Hello___,

I want to tell you that you will be taking photos of items or places here at -

____ (name of center) that you feel represent various stages of the math problems you will be working through. This might include things like math manipulatives, nearby desks, furniture, or objects in the surrounding area, windows as you look for examples of area, floor tile patterns as you look for examples of symmetry, etc. You will not be taking photographs of individuals who do not have consent to participate in this study.

To clarify this important point, I want to let you know that the photos you take should NOT include people. This includes other students who attend or are visiting your building, and it also includes adults who work at or are visiting your building. You should NOT take a picture of yourself either, even if your face is not visible in the photo. Now, someone might say it's ok for you to take a picture of them, but you still should NOT do this.

Do you have any questions about this? Do you understand these instructions?

Supplemental Information

If there is inadvertent capture of another child in a photograph, that photograph will be deleted immediately upon discovery from both the camera card memory as well as the computer memory. These deleted files will not be recoverable.

Appendix D1: Interview Protocol for Quantifying

Skill	Manipulatives & Photo	Their Photo	Other Photo	Manipulatives & Photo	Their Photo	Other Photo
<u>Quantify</u> Recognizes and names the number of items in a small set (up to five) instantly; combines and separates up to five objects and describes the parts; makes sets of 6-10 objects and then describes the parts; identifies which part has more, less, or the same (equal); counts all or counts on to find out how many	Do: Place 5 red and 6 black checkers on the table in a mixed pile. Say: Can you separate these checkers into groups by color, keeping the checkers flat on the table? Say: Now, can you tell me about these groups of checkers that you made? You can use your finger to point as you tell me about them if want to. (If these things don't come up, ask) "Are there more red checkers or black checkers on the table?" "How do you know?" Say: Now, can you take a picture of your groups of checkers? (The student can take several pictures until s/he has one that s/he is happy with).	Do: Place 5 red and 6 black checkers on the table in a mixed pile. Say: Can you separate these checkers into groups by color, keeping the checkers flat on the table? Say: Now, can you take a picture of your groups of checkers? (The student can take several pictures until s/he has one that s/he is happy with). Do: Clear the checkers from the table. Say: Now, let's take a	Do: On the iPad, show the photo of the 5 red checkers spread far apart and the 6 black checkers squished close together. Say: Now, let's take a look at this picture! Say: Can you tell me about these two groups of checkers? You can use your finger to touch the iPad and point as you talk if you want to. (If these things don't come up, ask) "Are there more red checkers or black checkers (on the table)?" "How do you know?"	Do: Place 4 red and 4 black checkers on the table in a mixed pile. Say: Can you separate these checkers into groups by color, keeping the checkers flat on the table? Say: Now, can you tell me about these groups of checkers that you made? You can use your finger to point as you tell me about them if you want to. (If these things don't come up, ask) "Are there more red checkers or black checkers on the table?" "How do you know?" Say: Now, can you take a picture of your groups of checkers? (The student can take several pictures until s/he has one that s/he is happy with).	Do: Place 4 red and 4 black checkers on the table in a mixed pile. Say: Can you separate these checkers into groups by color, keeping the checkers flat on the table? Say: Now, can you take a picture of your groups of checkers? (The student can take several pictures until s/he has one that s/he is happy with). Do: Clear the checkers from the table. Say: Now, let's take a	Do: On the iPad, show the photo of the 4 red checkers in the shape of a square and the 4 black checkers in the shape of a line. Say: Now, let's take a look at this picture! Say: Can you tell me about these two groups of checkers? You can use your finger to touch the iPad and point as you talk if you want to. (If these things don't come up, ask) "Are there more red checkers or black checkers (on the table)?" "How do you know?"

	Do: Clear the checkers from the table. Say: Now, let's take a look at your picture! Say: Can you tell me about these groups of checkers? You can use your finger to touch the iPad and point as you talk if you would like. (If these things don't come up, ask) "Are there more red checkers or black checkers (on the table)?" "How do you know?"	look at your picture! Say: Can you tell me about these two groups of checkers? You can use your finger to touch the iPad and point as you talk if you want to. (If these things don't come up, ask) "Are there more red checkers or black checkers (on the table)?" "How do you know?"		Do: Clear the checkers from the table. Say: Now, let's take a look at your picture! Say: Can you tell me about these groups of checkers? You can use your finger to touch the iPad and point as you talk if you would like. (If these things don't come up, ask) "Are there more red checkers or black checkers (on the table)?" "How do you know?"	look at your picture! Say: Can you tell me about these two groups of checkers? You can use your finger to touch the iPad and point as you talk if you want to. (If these things don't come up, ask) "Are there more red checkers or black checkers (on the table)?" "How do you know?"	

Appendix D2: Interview Protocol for Understanding Spatial Relationships

Skill	Manipulatives & Photo	Their Photo	Other Photo	Manipulatives & Photo	Their Photo	Other Photo
<u>Understands Spatial Relationships</u> Uses and responds appropriately to positional words indicating location, direction, and distance	Do: Set up the following: 1)a container with a marble inside of it, 2)the container is sitting on top of an eraser, 3)a yellow checker is close to the container (1 inch or so), 4)a blue checker is on the opposite side of the container from the yellow checker and farther from the container (8 inches or so). Say: Can you tell me what you see on the table, and where everything is? You can use your finger to point as you tell me about them if you would like. (If these things don't come up, ask) "Where is the green marble?" "Where is the pink eraser?" "Which checker is farther the container?" "Where is the	Do: Set up the following: 1)a container with a marble inside of it, 2)the container is sitting on top of an eraser, 3)a yellow checker is close to the container (1 inch or so), 4)a blue checker is on the opposite side of the container from the yellow checker and farther from the container (8 inches or so). Say: Now, can you take a picture of what you see on the table? (The student can take several pictures until s/he has one that s/he is happy with). Do: Clear the items	Do: On the iPad, show the photo of the following: 1)a container with a marble inside of it, 2)the container is sitting on top of an eraser, 3)a yellow checker is close to the container (1 inch or so), 4)a blue checker is on the opposite side of the container from the yellow checker and farther from the container (8 inches or so). Say: Now, let's take a look at this picture! Say: Can you tell me what you see (on the table), and where everything is? You can use your finger to	Do: Set up the following: 1)a container with a marble inside of it, 2)the eraser is inside of the container too with one side of it on top of the marble, 3)a yellow checker is close to the container (1 inch or so), 4)a blue checker is on the same side of the container from the yellow checker and farther from the container (4 inches or so). Say: Can you tell me what you see on the table, and where everything is? You can use your finger to point as you tell me about them if you would like. (If these things don't come up, ask) "Where is the green marble?" "Where is the pink eraser?"	Do: Set up the following: 1)a container with a marble inside of it, 2)the eraser is inside of the container too with one side of it on top of the marble, 3)a yellow checker is close to the container (1 inch or so), 4)a blue checker is on the same side of the container from the yellow checker and farther from the container (4 inches or so). Say: Now, can you take a picture of what you see on the table? (The student can take several pictures until s/he has one that s/he is happy with).	2bZ Do: On the iPad, show the photo of the following: 1)a container with a marble inside of it, 2)the eraser is inside of the container too with one side of it on top of the marble, 3)a yellow checker is close to the container (1 inch or so), 4)a blue checker is on the same side of the container from the yellow checker and farther from the container (4 inches or so). Say: Now, let's take a look at this picture! Say: Can you tell me what you see (on the table), and where

	yellow checker?" Say: Now, can you take a picture of what you see on the table? (The student can take several pictures until s/he has one that s/he is happy with). Do: Clear the items from the table. Say: Now, let's take a look at your picture! Say: Can you tell me what you see (on the table), and where everything is? You can use your finger to touch the iPad and point as you talk if you would like. (If these things don't come up, ask) "Where is the green marble?" "Where is the pink eraser?" "Which checker is farther the container?" "Where is the yellow checker?"	from the table. Say: Now, let's take a look at your picture! Say: Can you tell me what you see (on the table), and where everything is? You can use your finger to touch the iPad and point as you talk if you would like. (If these things don't come up, ask) "Where is the green marble?" "Where is the pink eraser?" "Which checker is farther the container?" "Where is the yellow checker?"	touch the iPad and point as you talk if you would like. (If these things don't come up, ask) "Where is the green marble?" "Where is the pink eraser?" "Which checker is farther the container?" "Where is the yellow checker?"	"Which checker is farther the container?" "Where is the yellow checker?" Say: Now, can you take a picture of what you see on the table? (The student can take several pictures until s/he has one that s/he is happy with). Do: Clear the items from the table. Say: Now, let's take a look at your picture! Say: Can you tell me what you see (on the table), and where everything is? You can use your finger to touch the iPad and point as you talk if you would like. (If these things don't come up, ask) "Where is the green marble?" "Where is the pink eraser?" "Which checker is	Do: Clear the items from the table. Say: Now, let's take a look at your picture! Say: Can you tell me what you see (on the table), and where everything is? You can use your finger to touch the iPad and point as you talk if you would like. (If these things don't come up, ask) "Where is the green marble?" "Where is the pink eraser?" "Which checker is farther the container?" "Where is the yellow checker?"	everything is? You can use your finger to touch the iPad and point as you talk if you would like. (If these things don't come up, ask) "Where is the green marble?" "Where is the pink eraser?" "Which checker is farther the container?" "Where is the yellow checker?"

				farther the container?" "Where is the yellow checker?"		

Appendix D3: Interview Protocol for Understanding Shapes

Skill	Manipulatives & Photo	Their Photo	Other Photo	Manipulatives & Photo	Their Photo	Other Photo
<u>Understands Shapes</u> Describes basic two- and three-dimensional shapes by using own words; recognizes basic shapes when they are presented in a new orientation	Do: Place a larger yellow square, a larger red circle, and a smaller blue triangle out on the table. Say: Can you tell me about this shape (point to the square)? You can use your finger to point as you tell me about it if you would like. Say: Can you tell me about this shape (point to the circle)? You can use your finger to point as you tell me about it if you would like. Say: Can you tell me about this shape (point to the triangle)? You can use your finger to point as you tell me about it if you would like. Say: Do you ever see any of these shapes in real life? Tell me about that! Say: Now, can you take a	Do: Place a larger yellow square, a larger red circle, and a smaller blue triangle out on the table. Say: Now, can you take a picture of these shapes? Do: Clear the items from the table. Say: Now, let's take a look at your picture! Say: Can you tell me about this shape (point to the square)? You can use your finger to touch the iPad and point as you talk if you would like. Say: Can you tell me about this shape (point to the circle)? You can use your finger to touch the	Do: On the iPad, show the photo of the bicycle. Say: Now, let's take a look at this picture! Can you tell me about the shapes you see in this picture?	Do: Place a larger red rectangle, a smaller yellow square, and a smaller blue cube out on the table. Say: Can you tell me about this shape (point to the rectangle)? You can use your finger to point as you tell me about it if you would like. Say: Can you tell me about this shape (point to the square)? You can use your finger to point as you tell me about it if you would like. Say: Can you tell me about this shape (point to the cube)? You can use your finger to point as you tell me about it if you would like. Say: Do you ever see any of these shapes in real life? Tell me about that!	Do: Place a larger red rectangle, a smaller yellow square, and a smaller blue cube out on the table. Say: Now, can you take a picture of these shapes? Do: Clear the items from the table. Say: Now, let's take a look at your picture! Say: Can you tell me about this shape (point to the rectangle)? You can use your finger to touch the iPad and point as you talk if you would like. Say: Can you tell me about this shape (point to the square)? You can use your finger	Do: On the iPad, show the photo of the ice cubes. Say: Now, let's take a look at this picture! Can you tell me about the shapes you see in this picture?

	picture of these shapes? (The student can take several pictures until s/he has one that s/he is happy with). Do: Clear the items from the table. Say: Now, let's take a look at your picture! Say: Can you tell me about this shape (point to the square)? You can use your finger to touch the iPad and point as you talk if you would like. Say: Can you tell me about this shape (point to the circle)? You can use your finger to touch the iPad and point as you talk if you would like. Say: Can you tell me about this shape (point to the triangle)? You can use your finger to touch the iPad and point as you talk if you would like.	iPad and point as you talk if you would like. Say: Can you tell me about this shape (point to the triangle)? You can use your finger to touch the iPad and point as you talk if you would like. Say: Do you ever see any of these shapes in real life? Tell me about that!		Say: Now, can you take a picture of these shapes? (The student can take several pictures until s/he has one that s/he is happy with). Do: Clear the items from the table. Say: Now, let's take a look at your picture! Say: Can you tell me about this shape (point to the rectangle)? You can use your finger to touch the iPad and point as you talk if you would like. Say: Can you tell me about this shape (point to the square)? You can use your finger to touch the iPad and point as you talk if you would like. Say: Can you tell me about this shape (point to the cube)? You can use your finger to touch the iPad and point as you	to touch the iPad and point as you talk if you would like. Say: Can you tell me about this shape (point to the cube)? You can use your finger to touch the iPad and point as you talk if you would like. Say: Do you ever see any of these shapes in real life? Tell me about that!	

	Say: Do you ever see any of these shapes in real life? Tell me about that!			talk if you would like. Say: Do you ever see any of these shapes in real life? Tell me about that!		

Appendix E1: Results for Quantifying

	Open-Ended Question Response				Closed-Ended Question Response			
	M	MP	P	G	M	MP	P	G
1	1	2	1	3	1	1	3	2
2	2	2	2	3	3	3	1	2
3	2	2	3	3	3	2	3	3
4	3	2	2	3	3	3	3	2
5	3	3	2	2	2	2	1	1
6	2	2	2	3	1	1	1	2
7	1	3	2	3	1	1	1	1
8	2	2	2	2	1	1	2	3
9	3	2	2	2	2	1	1	1
10	3	2	1	3	1	1	2	3
11	2	2	3	3	2	2	3	3
12	2	2	2	2	3	3	3	2
13	1	2	2	1	2	2	1	1
14	2	2	2	2	2	2	3	3
15	1	2	3	3	2	3	3	3
16	2	2	3	2	2	2	1	2
17	2	2	3	3	3	3	3	3
18	2	2	2	3	3	3	3	3
19	1	1	3	2	2	2	3	3
20	3	3	2	3	3	3	3	3
21	3	3	3	3	2	2	3	3
22	3	2	2	2	3	3	3	2
23	3	2	3	2	3	3	3	2
24	3	3	2	3	3	3	3	3

Appendix E2: Results for Understanding Spatial Relationships

	Spatial Words				Total Words				Spatial Word Ratio				Distractions			
	M	MP	P	G	M	MP	P	G	M	MP	P	G	M	MP	P	G
1	2	1	4	4	6	1	11	10	.3	1	.4	.4	3	4	1	2.5
2	2	0	4	1	2	3	8	4	1	0	.5	.3	1	6	2	2
3	2	3	8	3	5	6	15	8	.4	.5	.5	.4	0	0	0	.5
4	3	3	3	3	11	12	7	8	.3	.3	.4	.4	0	0	0	1
5	3	5	4	4	6	9	8	10	.5	.6	.5	.4	0	0	0	0
6	2	3	2	3	2	6	2	3	1	.5	1	1	1	0	2	6
7	0	1	0	0	4	2	1	0	0	.5	0	N/A	3	3	3	4.5
8	2	3	3	4	4	4	4	8	.5	.8	.8	.5	2	2	0	.5
9	4	4	0	5	7	8	4	10	.6	.5	0	.5	0	1	6	2.5
10	1	1	0	1	5	1	4	6	.2	1	0	.2	1	5	4	5
11	2	3	5	3	2	11	12	8	1	.3	.4	.4	2	0	0	.5
12	6	3	5	4	11	3	11	7	.5	1	.5	.6	0	0	0	.5
13	1	0	3	3	3	1	4	7	.3	0	.8	.4	1	0	1	0
14	3	3	3	3	3	5	4	4	1	.6	.8	.8	1	1	2	1
15	5	4	3	4	12	12	9	10	.4	.3	.3	.4	0	0	0	0
16	0	3	3	2	4	5	4	5	0	.6	.8	.4	4	1	1	2.5
17	0	1	1	4	7	3	6	8	0	.3	.2	.5	3	3	5	1
18	5	3	4	4	10	8	9	11	.5	.4	.4	.4	1	2	0	0
19	3	3	3	3	7	5	8	5	.4	.6	.4	.4	0	0	0	.5
20	4	3	3	4	5	10	11	10	.8	.3	.3	.4	0	0	0	0
21	5	3	2	4	9	3	6	7	.6	1	.3	.6	0	1	0	0
22	2	3	4	3	8	5	8	12	.3	.6	.5	.3	1	0	3	0
23	2	4	2	2	10	12	6	7	.2	.3	.3	.3	1	2	3	1.5
24	3	4	4	4	6	9	10	8	.5	.4	.4	.5	0	0	0	0

Appendix E3: Results for Understanding Shapes

	Shape Words			Total Words			Shape Word Ratio			Real-World Shapes		
	M	MP	P	M	MP	P	M	MP	P	M	MP	P
1	6	2	3	8	2	3	.8	1	1	4	1	2
2	3	3	3	5	3	3	.6	1	1	2	0	1
3	5	4	0	5	4	6	1	1	0	0	1	0
4	4	1	3	7	2	4	.6	.5	.8	0	11	2
5	6	3	2	9	4	5	.7	.8	.4	0	0	1
6	3	5	3	3	5	3	1	1	1	0	0	0
7	0	1	3	1	1	3	0	1	1	2	1	0
8	5	1	3	8	4	5	.6	.3	.6	0	2	1
9	5	3	0	7	3	3	.7	1	0	0	1	1
10	4	3	3	6	3	3	.7	1	1	0	0	1
11	5	6	3	5	6	4	1	1	.8	2	0	0
12	9	4	4	9	4	4	1	1	1	0	0	0
13	3	1	2	3	1	2	1	1	1	0	0	0
14	5	1	4	7	4	7	.7	.3	.6	0	0	2
15	5	4	6	5	4	6	1	1	1	0	1	0
16	5	5	3	6	5	3	.8	1	1	0	0	0
17	3	6	3	6	9	3	.5	.7	1	0	0	0
18	6	6	4	11	9	7	.5	.7	.6	0	1	4
19	4	2	4	7	3	7	.6	.7	.6	0	1	2
20	7	4	8	8	6	12	.9	.7	.7	0	0	0
21	4	6	10	10	9	11	.4	.7	.9	2	1	3
22	1	1	5	4	1	6	.3	1	.8	0	1	1
23	7	4	4	7	4	4	1	1	1	0	3	1
24	2	1	4	4	4	6	.5	.3	.7	2	0	1