ESSAYS ON MACROECONOMICS

by

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Abstract

This dissertation consists of two self contained essays on the applications of search in banking, monetary policy and labor, organized in the form of two chapters.

The first chapter develops a framework under which banks remove some of the frictions present in the trading process and insure agents against the risks of excess liquidity, cost of holding liquidity and bargaining frictions. This chapter shows that banks are equivalent to the case where agents can re-balance their money holdings if they learn about their type shocks before the Centralized Market closes. Therefore, banks act as an institution that transfers liquidity to those who need it, taking care of the cost of holding excess or undesired money balances (excess liquidity). Banks can also take care of bargaining frictions if the agents are allowed to deposit money balances and/or return borrowings without penalty in the decentralized market, and hence banks also insure agents against bargaining frictions. However, I also find that banks do not insure agents who are involved in trade against the inter-temporal costs of holding money (the Friedman Wedge). A bigger range of nominal interest rates can support a monetary equilibrium in the presence of banks, and that banks are able to achieve superior outcomes of trade because of their insurance mechanism against frictions. I am also able to show that demand for money is more inelastic in the presence of banks, and for sufficiently lower levels of nominal interest rate, lesser money is demanded.

Therefore, banks are complements to money and not substitutes.

In the second chapter, I develop a search based multi-agent model to analyze the search and matching decisions of a firm that can employ two types of labor. The labor types can be substitutes or complements of each other, depending on relative productivity levels of the two types. In this chapter, I find that firms needing the services of both types of labor may search sequentially instead of searching for both types at the same time even in the presence of random search. Employment and unemployment levels of labor are non-monotonic and may decrease in own productivity, and are also discontinuous. Aggregate welfare and value of search are also non-monotonic. It was also found that unemployment level of a particular type may respond to changes in the output of the other type, even if its own productivity does not change. This model can explain cyclical unemployment levels, and can be used as a framework for many areas like outsourcing, real estate, theory of team formation and finance.

Chapter 1

Financial Intermediation in a Search Model of Money

1.1 Introduction

If money is used as a medium of exchange, efficient quantities are not traded in equilibrium because of the presence of some frictions in the trading process. Even in the presence of assets, efficient quantities might not be traded for two reason: imperfect recognizability and/or divisibility leads to some transaction costs, and secondly, there might not be sufficient assets in the economy that can be used as an instrument of exchange (Lagos Rocheteau, 2008).

In all, there are four frictions in the trading process that the traders face:

1. Uncertainty about shock in the DM, and lack of insurance to hedge against the shock
2. Friedman Wedge (Inter temporal cost of holding money)
3. Bargaining frictions

4. Transaction costs associated with using the fruit bearing imperfectly verifiable/divisible asset during trades.

The first friction is the uncertainty associated with not knowing the type the agent would be in the decentralized market. Agents receive the shock of whether they would be buyers or sellers or inactive during the search process in decentralized markets. Money is useful only if agents become buyers in the decentralized market, as they would be able to use it in the exchange process. However, if they become sellers or are inactive, then money is not useful, and if it is costly to hold money, agents might not hold enough money due to the risk of them being sellers or inactive during the decentralized exchange. Cost of holding money takes us to the second friction which is the Friedman wedge. This wedge arises because agents discount the future, and if agents carry money that does not give them return commensurate with their discounting costs then agents do not carry enough money in the DM to trade efficient quantities. If there is a cost of holding money, then efficient quantities may not be traded in the presence of bargaining frictions, and in fact, even in the absence of the Friedman wedge, efficient quantities might not be traded[1]. The third friction is due to bargaining frictions. Since buyers incur the cost of holding money, they must be given all the surplus of trade[2], otherwise, buyers will carry smaller amount of money than desired to hedge themselves against the potential of being a buyer and subject to undesirable end of bargaining frictions. This is the classic hold-up problem that arises because buyers have already invested in carrying money balances, and hence have already incurred the inter-temporal cost of carrying a non-interest bearing asset. Therefore, when the buyers meet the sellers, the sellers know this fact and can exploit the buyers. To partially hedge against this risk, agents would

[1]This is bargaining protocol dependent. For example, it holds true if the protocol is Nash Bargaining solution, but is not true if bargaining protocol is Kalai Bargaining

[2]See Hosios (1990)

also shade down their money holdings when deciding on the amount of money to carry from the CM to DM. The fourth friction, which is not related to fiat money, is the cost associated with verification of the asset if the asset is not perfectly verifiable, and costs associated with making the asset divisible (for example, legal costs of transferring a partial claim of the asset to the seller, or searching and then exchanging the asset for money with an agent having excess liquidity).

In the environment without banks, agents cannot totally insure against all the 4 frictions presented in the previous section. However, if agents use fruit bearing assets as medium of exchange, then they can insure themselves against all the frictions (except perhaps bargaining frictions, depending on the bargaining method used in the model), if sufficient quantities of assets is present in the economy to trade efficient quantities. However, using such assets entails incurring transactions costs. If such transaction costs are absent, then holding fruit bearing assets as a medium of exchange can act as an insurance mechanism for agents entering DM from CM. Since it is a stylized fact that such assets are barely used as a medium of exchange, this device as an insurance mechanism is absent if agents intend to carry a medium of exchange in the DM. In this paper, I will show that Financial Intermediaries (FIs or banks) act as an insurance mechanism for agents, in the sense that they can take care of some of the frictions mention above, and hence partially insure the agents carrying money. In this paper I will show that banks can only insure agents against type shocks if they are closed when the actual trade takes place. However, banks also insure agents against bargaining frictions in addition to the uncertainty of types if they remain open during the trade process. However, in both scenarios, banks do not insure agents against the Friedman wedge.

Banks have been introduced in this stream of literature before. However, my paper is the first one to develop the model to study the insurance mechanism (or lack of insurance) of the bank with regards to the three frictions (4th one regarding the use of assets is just introduced in this paper, and full treatment is the subject of a separate project). In this paper, I have identified and separated the effects of the three frictions, and found that timing on when the banks open and close matters for the insurance function of banks against various frictions. For example, if the banks are closed when quid quo pro trading process takes place (Decentralized market), then banks do not insure agents against bargaining frictions, but they insure agents against type shocks of whether they would be buyers or sellers. They do not insure agents against the inter-temporal cost of holding money, as if the banks are closed and buyers have already borrowed, then they are going to incur the interest cost even if the trade does not take place. It is this cost that the buyers incur that causes hold-up problem in the bargaining process[3]. However, if the banks are allowed to remain open during the trading process, then buyers and sellers, who are agents of measure 0 and take the borrowing rate and the savings rate as given, get a better alternative in the form of depositing their money in the banks and earning interest on them. This increases the bargaining position of buyers, and hence the sellers are no longer able to hold-up the buyers. The banks therefore insure buyers against the hold-up problem as now the buyers can always threaten to walk away from trade. However, banks are still not able to remove the Friedman wedge from the trading process, and hence trade is still not done at first best quantities.

The model presented in this paper is closely related to Berentsen, Camera, Waller (2007). However, their paper does not answer the question related to the role of banks in alleviating the problems of bargaining frictions present in the trading process. He, Huang and Wright (2005, 2008) also introduced banks in a search model of money, but their models mainly consider the check accepting role of the banks. Bencivenga and Camera (2011) introduce

[3]The hold-up problem arises because the total trade surplus which is the subject of the bargaining process is split in such a way that is not commensurate with the relative cost that each of the parties incurs.

banks with capital in a search model, but they studied the impact on welfare in response to changes in the cost structure of banks. Gu et. al. (2013) also introduce banks in a search based model in which they develop a model of banks that accepts deposits and advance loans. They also show that banks are able to achieve superior allocations, but their paper also does not explicitly answer the question related to the role of banks in alleviating frictions in decentralized markets. My paper is the first one that studies the frictions separately, and identifies the contribution of different frictions in the trade process.

Before continuing further, I would like to mention here that the role of banks also depends on the terms of trade mechanism used, and might differ slightly with the protocol used. For example, terms of trade could be determined by an axiomatic approach (for example generalized Nash bargaining solution, or Kalai bargaining solution), or through a mechanism design approach (Hu, Kennan and Wallace, 2009). The insurance function of banks against the bargaining frictions would only be present in an environment where the terms of trade are determined through an axiomatic approach. However, it is worth mentioning that banks would perform the same function for both cases as far as the transformation of liquidity (insuring sellers against excess, costly liquidity and insuring buyers against liquidity shortage). It would be interesting to study the conditions under which we can prove equivalence of the role of banks using both terms of trade protocols (This project is work in progress). In this project, I will just consider an axiomatic approach for determining the terms of trade. Introducing banks with mechanism design approach of terms of trade is also an interesting topic which has been left for future work. My conjecture is that in a mechanism design approach, banks would relax the incentive compatibility constraints of sellers, that would result in higher quantities traded.

In the text that follows, in section 2, I will lay out the basic structure and environment of the model. In section 3, I will consider a scenario without banks in which agents know their types well before the trading process, so they can accumulate/deaccumulate money according to their own type. In section 4, I will add banks but will not allow them to open during the decentralized trade process. In section 5, I will allow banks to remain open during the trade process, and section 6 concludes.

1.2 Model

The model is based on Kiyotaki Wright (1991, 1993) and Lagos Wright (2005), hereinafter referred to as LW. Time is discrete and continues forever. There is a continuum of agents of measure 1 in the economy. Each period is divided into two sub periods. In the first sub-period, agents consume a non-storable specialized good in a decentralized market (DM). In this decentralized market, agents are anonymous i.e. once the match is broken, they meet again with probability 0. This setting is similar to Kiyotaki-Wright model. We can also call this the KW market. Because of anonymity, in the absence of any monitoring device, credit cannot exist and all trades are quid-pro quo. Therefore, agents need a medium of exchange that is recognized by both sides for any trade to take place. All agents in the second sub-period supply labor and trade in a frictionless centralized market (CM). Agents in CM derive a utility $U(X)$ from the consumption of a non-storable general good and incur a dis-utility H of working H amount of time. Therefore, agents derive a net utility of $U(X) - H$ in CM. We can also call this frictionless centralized market as the AD. Clearly, agents have a quasilinear utility in the AD. Price of the CM good is normalized to 1.

In addition to supplying labor and consuming the general good in the CM, agents also accumulate a portfolio of assets:

- A non-dividend paying object m called money. Price in terms of CM good is ϕ_m.

- Claims a to a tree bearing fruit δ that ripens at the beginning of the CM. A holder of the claim to the tree at the beginning of the CM gets to consume the fruit. Price of a claim in terms of the CM good is ϕ_a .

Both money and claims to the tree are portable to the DM without any cost. However, agents can easily counterfeit claims to the tree at 0 cost any time during the CM or DM because it is not perfectly recognizable. They cannot counterfeit money as money is perfectly recognizable (We can always say they can only counterfeit money at a cost of κ which is very big). Agents discount between the two periods at a rate β.

At the start of each decentralized market, agents receive a shock of whether they are buyers, sellers or inactive during the DM. Agents are buyers with probability σ, which can be interpreted here as the fraction of agents who found a trading partner during the search process in the decentralized market. It can also be referred to as the liquidity shock that agents get after the closure of centralized market. I am taking the former motivation here. Buyers get a utility $u(q)$ from the consumption of q units of KW good. Agents are sellers with probability σ i.e. all buyers are matched with the sellers. Sellers incur a dis-utility $c(q)$ from producing q units of KW good for the buyers. Usual conditions apply for $u(q)$ and $c(q)$ (i.e. $u'(q) > 0$, $u''(q) < 0$, $c'(q) > 0$, $c''(q) > 0$). There also exists $q*$ such that $u'(q*) = c'(q*)$. $q*$ is the efficient quantity traded. Agents are inactive during the DM with probability $1 - 2\sigma$ i.e. they do not find a trading partner in the search market and are neither buyers nor sellers.

Since trade is quid pro quo, agents need some form of exchange mechanism in KW. Buyers can either use their money holdings, or their claims to the trees. If buyers use their asset holdings, they need to incur a utility cost d to verify that the asset is not a counterfeit. The model that includes a utility cost is discussed in a companion paper. For this paper, we may assume that this cost is significantly high which prohibits agents from using fruit bearing assets as a medium of exchange. Money is easily recognizable, and there is no cost associated with using money during the transaction. However, since money does not bear any dividends, and agents discount at an inter-temporal rate β, agents incur an inter-temporal cost of carrying money (Friedman Wedge), the magnitude of which depends on β and the growth rate of money determined by the Central Bank and exogenous to the agents.

I will add banks to the model to allow me to analyze how, and to what extent do the FIs alleviate the lack of insurance problem faced by the traders. Once agents leave the centralized market, the shocks are realized (depending on outcome of the search process) as to whether they are buyers, or sellers, or would remain inactive in the following decentralized market. I will consider two separate scenarios: One in which the bank opens after CM but closes before DM trade process takes place. Second, in which bank opens after CM but remains open until the trades are finalized. However, before introducing banks, a couple of definitions are in order, and then I will consider a scenario where the agents know their shocks before entering the decentralized market, and hence can re-balance their money holdings in the centralized market, and then compare it with a scenario with banks.

Definition: DM Trading process DM trading process is defined as the actual event when the seller provides the service/goods to the buyer, and the buyer pays the seller money or any other instrument of exchange deemed acceptable by the seller in the decentralized market.

Definition: Borrowing Borrowing means claims on money that banks have on the agents in the centralized market. The associated interest rate is called the borrowing rate.

Definition: Saving Saving means claims on money that the agents have on banks in the centralized market. The associated interest rate is called the savings rate.

1.3 Certainty of types in DM

By certainty of types, I mean that when agents enter decentralized market and find a trading partner, they still have access to the feature of the centralized market that allows them to re-balance their asset portfolios. However, they do not have access to technology from centralized market that allows them to keep memory about trades. Therefore, money is still essential. After realizing their shocks, agents can always re-balance their portfolios. By re-balancing, I mean agents who will be buyers in DM can accumulate more money by selling their fruit bearing assets and agents who will be sellers or inactive can buy fruit bearing assets in exchange for their money holdings. Value functions of agents in CM, after incorporating the budget constraint, can then be written as:

Agents who will become buyers:

$$W_b(m) = \max_X\{U(X) - X\} + \phi_m m + T + \max_{m'}\{\beta V_b(m') - \phi_m m'\}$$

Agents who will become sellers:

$$W_s(m) = \max_X\{U(X) - X\} + \phi_m m + T + \max_{m'}\{\beta V_s(m') - \phi_m m'\}$$

Agents who will be inactive during DM:

$$W_n(m) = \max_X \{U(X) - X\} + \phi_m m + T + \max_{m'} \{\beta W(m') - \phi_m m'\}$$

Where

$$W(m) = \sigma W_b + \sigma W_s + (1 - 2\sigma) W_n$$

I am omitting assets here as a state variable here as they are not used as a medium of exchange. I will re-introduce them later. Here X is the amount of centralized market goods that agents consume, and $U(X)$ is the utility associated with the consumption of CM goods. V_i is the value function of the agents in next period decentralized market, and T are the lump-sum transfers by the government.

First order condition for buyers, sellers and inactive agents respectively give:

$$\beta \frac{\partial V_b}{\partial m'} \leq \phi_m \qquad = if\ m' > 0$$

$$\beta \frac{\partial V_s}{\partial m'} \leq \phi_m \qquad = if\ m' > 0$$

$$\beta \frac{\partial W_b}{\partial m'} \leq \phi_m \qquad = if\ m' > 0$$

Value functions in the DM are:

$$V_b = u(q) + W(m - p_m)$$

Where p_m is the payment made to the sellers.

Since buyers use all their money holdings in the DM,

$$V_b = u(q) + W(0)$$

Similarly for sellers,

$$V_s = -c(q) + W(m + p_m)$$

From the first order conditions,

$$\beta[u'(q')\frac{\partial q'}{\partial m'}] = \phi_m$$

For sellers, since the amount they produce does not depend on their own money holdings, $\frac{\partial q_s}{\partial m} = 0$. Therefore, $\frac{\partial V_s}{\partial m'} = 0$, and hence sellers do not carry any money in the decentralized market.

Similarly, for non-active agents, $m' = 0$ as long as Friedman rule is not followed as a policy tool.

In decentralized market, the Kalai bargaining solution in the absence of banks, using linearity of W, and using the fact that agents spend all their money holdings can be characterized by:

$$(q, p_m) \in \arg\max_{q,d_m} u(q) - \phi_m m \tag{1.3.1}$$

Subject to

$$-c(q) + \phi_m m \geq (1 - \theta)[u(q) - c(q)]$$

Solution to the bargaining solution gives

$$\phi_m m = \theta c(q) + (1-\theta)u(q) \equiv z(q)$$

Using the first order condition for buyers and the bargaining solution,

$$\frac{\phi_{m,t}}{\beta\phi_{m,t+1}} = \frac{u'(q_{t+1})}{z'(q_{t+1})} \tag{1.3.2}$$

If the agents do not know their shocks before the CM closes, then

$$\frac{\phi_{m,t}}{\beta\phi_{m,t+1}} = \sigma[\frac{u'(q_{t+1})}{z'(q_{t+1})} - 1] + 1 \tag{1.3.3}$$

This solution is different from the case discussed in LW, because in this case, since only the people in need of liquidity are carrying the money, there are no frictions associated with agents carrying unnecessary liquidity. These frictions are present if agents do not know their type before DM trading process.

1.4 Banks Open After CM, Close before DM

Now I am going to introduce banks into the model, while now assuming that agents do not know their types when they leave the CM (they do not have access to any CM technology). Once the type shocks are realized, another market called the Diamond Dybvig market opens. This market allows the agents to re-balance their portfolios. The match between the agents (buyers and sellers) remains intact during this process. Buyers (also called borrowers), who are in need of liquidity can go to this market and borrow some money, and return their borrowings with interest determined by equilibrium at the beginning of CM. Likewise agents

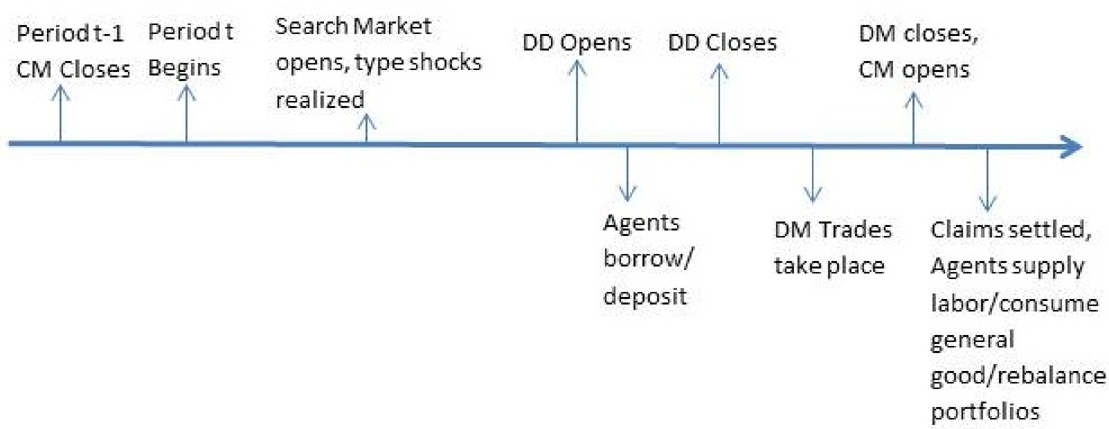


Figure 1.4.1: Time-line of Events if Banks Close Before DM Trade Process

who have excess liquidity (also called savers) can deposit their money holdings to earn interest. FI knows the ID of each agent. This is crucial for FI, as if this is not the case, then this market breaks down as borrowers will not return their borrowings in CM. Once agents have borrowed/deposited, DD market closes, and agents return to trading partners to trade. The match breaks after trade takes place. See **??** for time-line of events. In this model, agents discount between CM and DD at rate β. The probability of being a buyer is σ. I am also assuming here that banks are not subject to any frictions (Reserve requirements, Informational frictions), and that banks are perfectly competitive and earn zero profits.

1.4.1 CM

Let a be the assets an agent carries forward from previous decentralized market into the current centralized market. Similarly, let m be the money balances and b be the borrowings carried into the current centralized market. Therefore, a, m, b are the state variables. The value function of an agent in the centralized market, with banks, can be written as:

$$W(a, m, b; k) = \max_{a', m', H, X} U(X) - H + \beta D(a', m') \tag{1.4.1}$$

subject to

$$\phi_m m' + \phi_a a' + X = H + \phi m - \phi B(1 + i_k) + (\phi_a + \delta)a + T$$

Here $k \in \{buyer(b), seller(s)\}$. The budget constraint in this case is different from the case in the absence of banks, as now agents have to subtract their borrowing from their sources of funds (Note: savings are negative borrowings).

1.4.2 Diamond Dybvig Market

Agents realize a shock whether they are going to be buyers or sellers in the next sub-market, after the centralized market has closed. After the realization of the shock, the Diamond Dybvig market (abbreviated as DD) opens which allows the agents to re-balance their portfolios. Buyers who borrow in this market have to pay interest i_b in the centralized market. Sellers and non-active agents have no use for cash. They deposit their money balances and earn an interest i_s on their deposits.

Therefore, for buyers (borrowers),

$$D_b(a, m) = \max_{b} V_b(a, m, b) \tag{1.4.2}$$

First order conditions give:

$$\frac{\partial V_b(a,m,b)}{\partial b} = 0 \tag{1.4.3}$$

Here $V_b(a,m,b)$ is the value function of the buyers (henceforth called borrowers) in the Kiyotaki-Wright market (see below for value functions of borrowers in KW market)

Similarly, for sellers,

$$D_s(a,m) = \max_b V_s(a,m,b) \tag{1.4.4}$$

subject to

$$m + b \geq 0$$

First order conditions for sellers give

$$\frac{\partial V_s(a,m,b)}{\partial b} = 0 \tag{1.4.5}$$

In the above formulation, $V_s(a,m,b)$ is the value function for sellers (henceforth savers)

Envelope conditions for both buyers and seller give (where $i \in \{b,s\}$):

$$\frac{\partial D_i(a,m)}{\partial a} = \frac{\partial V_i(a,m,b)}{\partial a} \tag{1.4.6}$$

$$\frac{\partial D_i(a,m)}{\partial m} = \frac{\partial V_i(a,m,b)}{\partial m} \tag{1.4.7}$$

For non-active agents:

$$D_n(a,m) = \max_b W(a,m,b) \tag{1.4.8}$$

subject to

$$m + b \geq 0$$

Non-active agents are also savers, as they deposit their money in the bank.

Before leaving CM, expected value function in DD of an agent is:

$$D(a, m) = \sigma D_B(a, m) + \sigma D_S(a, m) + (1 - 2\sigma)D_n(a, m) \tag{1.4.9}$$

After substituting H from the budget constraint, the expression in 1.4.1 can be written as:

$$W(a, m, b) = \max_X \{U(X) - X\} + T + \phi_m m - \phi b(1 + i) + (\phi_a + \delta)a + \max_{a', m'} \{\beta D(a', m') - \phi_m m' - \phi_a a'\} \tag{1.4.10}$$

First order conditions give

$$\beta \frac{\partial D(a', m')}{\partial a'} = \phi_a \tag{1.4.11}$$

$$\beta \frac{\partial D(a', m')}{\partial m'} = \phi_m \tag{1.4.12}$$

and

$$U'(X) = 1 \tag{1.4.13}$$

Envelope conditions are the same as in the case without banks, except now we have borrowing as a state variable. Envelope conditions with respect to borrowing give:

$$\frac{\partial W(a,m,b)}{\partial b} = -\phi_m(1+i_k) \tag{1.4.14}$$

where $i_k \in \{i_b, i_s\}$, depending on whether the interest is being paid to a saver or being charged to a borrower.

1.4.3 KW

$$V_b(a,m,b) = u(q) + W(a - p_a, m + b_b - p_m, b_b) \tag{1.4.15}$$

$$V_s = -c(q) + W(a + \widetilde{p_a}, m + \widetilde{p_m} + b_s, b_s) \tag{1.4.16}$$

In the expressions above, b is the borrowing from the bank by the borrower, p_m is the payment made by the buyer to the seller and $\widetilde{p_m}$ is the payment received by the seller. The seller receives the payment and provides the goods/services as a quid pro quo arrangement. Buyers get a utility $u(q)$ from the provision of goods/services, whereas sellers incur a cost $c(q)$.

From the first order conditions for buyers in the DD,

$$\frac{\partial V_b}{\partial b} = u'(q)\frac{\partial q}{\partial b} - \phi(1+i_b) = 0 \tag{1.4.17}$$

This equation gives the demand for borrowings by the borrowers.

Differentiating V_s wrt b (assuming $i_s > 0$)

$$\frac{\partial V_s}{\partial b} = -\phi(1+i_s) + \lambda = 0$$

Here λ is the Lagrange multiplier on the savings (or deposits). Since the constraint binds

($\lambda = \phi(1 + i_s) > 0$), $b_s = -m$, which means that sellers deposit all their money in the bank in the form of savings (or negative borrowing).

Similarly, the constraint for non-active agents also binds and they also deposit all their money in the bank in the form of savings.

1.4.4 Agents use money and borrowings without collateral

Proposition: Monetary equilibrium with banks exists if nominal interest rate is not high enough Proof: See appendix and the section on incentive compatibility of buyers that follows for the definition of high enough nominal interest rate.

To find the equilibrium quantities traded, we need to find the terms of trade through which quantities and money exchanged are determined. As in the previous section, I will use the Kalai bargaining solution which can be characterized as:

$$\max_{q,b,d_b} u(q) + W(a, m + b_b - p_m, b_b) - W(a, m + b_b, b_b) \tag{1.4.18}$$

subject to

$$-c(q) + W(a, p_s, -m) - W(a, 0, -m) \geq [1 - \theta][u(q) - c(q)] \tag{1.4.19}$$

$$p_m \leq m + b_b$$

The bargaining solution is characterized in a way that the DD closes before the KW (DD) trade takes place. Therefore, once buyers enter the KW, they are stuck with their borrowing. Therefore, even if the trade does not take place, buyers have to pay interest on their borrowing in the CM, and their threat point in the above characterization depicts this fact. Using the linearity of $W(.)$ and the fact that for maximization both constraints bind (otherwise the buyers would not be maximizing),

$$\phi(m+b) = \theta c(q) + (1-\theta)u(q) \equiv z(q) \tag{1.4.20}$$

Differentiating the solution to the Kalai Bargaining solution (1.4.20) wrt b,

$$z'(q)\frac{\partial q}{\partial b} = \phi \tag{1.4.21}$$

Using the envelope conditions wrt m (1.4.7) and value functions in KW (1.4.15, 1.4.16), and inserting them into the first order condition (1.4.12) and simplifying (using time subscripts now)

$$\phi_t = \beta\phi_{t+1}[\sigma\frac{u'(q_{t+1})}{z'(q_{t+1})} + (1-\sigma)(1+i_{s,t+1})] \tag{1.4.22}$$

Consider a constant growth in money supply. Planner transfers the revenues raised by increased money supply lump-sum to the agents. Let me consider a stationary case, where next period money supply is a factor γ of the current period money supply. Then

$$M' = \gamma M$$

Under the assumption of stationarity,

$$\phi_m M = \phi_m' M'$$

Therefore,

$$\gamma\phi_m' = \phi_m$$

Using this policy, and the fact that $\gamma(1+r) = 1+i$ (where i is the nominal interest rate that agents take as given as it is being driven by the central bank policy, and ignoring the time sub-scripts), (1.4.22) can be simplified as:

$$\frac{u'(q)}{z'(q)} = \frac{1+i-(1-\sigma)(1+i_s)}{\sigma} = 1+i_s+\frac{i-i_s}{\sigma} \tag{1.4.23}$$

The first order condition (1.4.17) gives:

$$\frac{u'(q)}{z'(q)} = 1+i_b \tag{1.4.24}$$

Solving these two equations to eliminate $\frac{u'(q)}{z'(q)}$, we get

$$\sigma i_b + (1-\sigma)i_s = i \tag{1.4.25}$$

The interest rates i_b and i_s and total borrowing in the DD are given by the market clearing condition in the DD and zero profitability condition in CM and equation (1.4.24).

The total supply of funds in the DD market is $\sigma m + (1-2\sigma)m = (1-\sigma)m$. This is the total stock of funds deposited by savers in the bank. Since there are no reserve requirements, all these funds are lent by the bank to the borrowers. Therefore, $\sigma b = (1-\sigma)m \Rightarrow b = \frac{1-\sigma}{\sigma}m$.

Since banks make zero profits in the model, total revenue raised by the banks in the CM must be equal to their total interest expense. This implies:

$$\sigma b i_b = (1-\sigma)m i_s$$

Solving the market clearing condition and zero profit condition,

$$i_s = i_b$$

Therefore,

$$i_s = i_b = i$$

Total money carried by the agents is then given by the Kalai Bargaining solution:

$$\phi_m(m + b) = z(q) \Rightarrow \phi_m m = \sigma z(q)$$

Where the quantities traded are given by (1.4.24).

Formal proof of existence is given in appendix. Notice, that for a given buyer bargaining power, the range of nominal interest rate for which an equilibrium exists is bigger in the presence of banks that without banks. (See appendix). In other words, at a given nominal interest rate, equilibrium can be supported by a lower buyer bargaining power in the presence of banks.

Notice than unlike the scenario without banks, where $\frac{u'(q)}{z'(q)} = 1 + \frac{i}{\sigma}$, liquidity is available to those who need it, and hence a smaller gap between $u'$and z' which means the quantities traded are higher than those traded in the absence of banks, and are closer to optimal. This is the liquidity transformation function of the bank. Banks transfer liquidity from the agents who do not need it to the agents who need it. In the process, by paying interest to depositors, banks hedge agents against the risk of not having any use of money. Therefore, in the presence of this liquidity insurance mechanism, the quantities traded are more than what would have been in the absence of banks as agents carry more real balances than in the environment without banks. Simplifying (1.3.2) (equation that gives us quantities traded if agents can re-balance their portfolios before the CM closes):

$$\frac{u'(q_{t+1})}{z'(q_{t+1})} = 1 + i \tag{1.4.26}$$

This is exactly the same as we derived for the case where banks are present. Therefore, we have the following result:

Result 1 If we introduce banks in a Lagos-Wright framework, such that they open after the CM closes and close before the DM trade takes place, then they give the same result in terms of quantities traded if we keep the CM open until shocks are realized and allow the agents to re-balance their portfolios, as long as savings rate and borrowing rate equals the nominal interest rate.

From the above result, we can deduce that the banks take care of friction 1 mentioned above i.e. they remove the uncertainty about shock in the DM, and lack of insurance to hedge against the shock. However, bargaining frictions and the Friedman wedge are still present, and that is why, despite the presence of the liquidity transformation function of banks, efficient quantities are still not traded. The nominal interest rate is exogenous here. It is determined by central bank policy. However, savings rate and borrowing rate are endogenous to the model. Savings rate is determined by the measure of claims agents have on the banks in the CM, and by the measure of claims banks have on agents in CM, together with the zero profit condition.

Using thee bargaining solution, (1.4.26) can be written as:

$$u'(q) = \left(1 + \frac{i}{\theta - (1-\theta)i}\right) c'(q) \tag{1.4.27}$$

$\frac{i}{\theta-(1-\theta)i}$ here represents the wedge that causes inefficient trade i.e. $q < q^*$. This wedge is present due to both bargaining frictions and Friedman wedge. In this formulation, presence of θ represents bargaining frictions whereas presence of i represents the Friedman wedge.

Liquidity Premium By the definition of liquidity premium

$$l_b(i) \equiv \frac{u'(q)}{z'(q)} - 1 = i$$

In the absence of banks,

$$l(i) = \frac{i}{\sigma}$$

The liquidity premium in presence of FI is less than without FIs. This is because now buyers are getting more goods by borrowing from the bank. Therefore, the value of money at the margin declines.

1.4.5 Incentive Compatibility of Buyers

Borrowers who have obtained loans from the banks have to involve in more effort in centralized market to produce and pay-off their debt. Therefore, there is always an incentive for the borrowers to default. Since banks who IDs of each agent, they can devise a mechanism where they can punish the defaulting borrowers by excluding them from the financial inter-mediation in future. This means borrowers can now only use money for trade in the decentralized market. Let me denote the deviation quantities as q_D, which would be the same quantities traded in the absence of the banks (from the optimality conditions). However, since money holdings of seller do not influence trade, therefore a deviating agents will continue to produce

$q_b$if he becomes a seller in DM. Agents will not deviate i.e. pay off their debt if:

$$u(q_b) - z(q_b) + \sigma \frac{u(q_b) - c(q_b)}{r} - \phi_m b(1+i) \geq u(q_b) - z(q_b) + \sigma \frac{u(q_D)}{r} - \sigma \frac{c(q_b)}{r}$$

This inequality means that borrowers will not deviate i.e. they will pay back their loans to the bank in the next CM if the total lifetime value of paying back their debt and enjoying credit facilities from the bank is more than the autarky payoff in future while enjoying higher quantities today by borrowing and then reneging on their payments. Simplifying the above expression,

$$1 + i \leq \frac{\sigma}{1-\sigma} \frac{u(q_b) - u(q_D)}{rz(q_b)}$$

After examining this inequality, we can observe that increasing the interest rates have two effects on the right hand side of inequality: One effect is to decrease the quantities traded, and hence the utilities that the buyers get from consuming the DM good also decreases. This increases the incentive to deviate. However, the elasticity of quantities traded without credit facilities is greater than the elasticity of quantities traded in the presence of banks. Therefore, agents are more effected by increasing interest rates without banks. This reduces their incentives to deviate. The net effect of whether this constraint is tightened or laxed depends on the elasticities of quantities traded with respect to the nominal interest rate. These elasticities depend on the risk-aversion of buyers, trade surplus that they get from trades and the frequency of trades in case of quantities traded without banking. However, if interest rate is sufficiently high, then monetary equilibrium without banking does not exist, and in that case, the only affect is due to the reduction in quantities traded which is negative for the right hand side. In that case, the constraint is tightened.

1.4.6 Assets as Collateral

In my model, fruit bearing assets do not play any role as a medium of exchange. However, in the presence of banking, banks can use borrower's assets as collateral in case borrowers deviate in equilibrium from paying their debt. Therefore, assets can still play a role as collateral if deviations from repaying the loan exist.

If the incentive constraint as discussed above does not hold, then in the absence of any collateral, the FI will shut down. Assets can still play a role here in maintaining Financial Inter-mediation. As discussed in the companion paper, agents do not use both assets and money at the same time. However, agents can use both money and assets, in the sense that using assets allows buyers to get a "line of credit", and hence assets can still be used in addition to money in the presence of banks.

1.4.6.1 Agents are constrained to borrow only by using assets as collateral

If deviations as discussed above (in eq **??**) exist, then the maximization problem of an agent becomes

$$D_b(a, m) = \max_b V_b(a, m, b) \tag{1.4.28}$$

subject to

$$b \leq \frac{1-\sigma}{\sigma} m$$

and

$$\phi_m b \leq \frac{\phi_a + \delta}{1 + i_b} a$$

The first constraint is the market clearing constraint, which has been discussed previously. If this constraint binds, then agents have sufficient assets with them, and the banks, if they cannot issue their own inside money, are "funds" constrained. If the second constraint binds, meaning there are not sufficient pledge-able assets in the economy, then buyers are "collateral" constrained, and banks are stuck with excess liquidity in the form of excess savings by savers. If the second constraint binds, then the agent borrows up to the value of his asset portfolio.

In that case,

$$V_b = u(q) + W(a, m + \frac{\phi_a + \delta}{\phi_m(1 + i_b)} a - p, \frac{\phi_a + \delta}{\phi_m(1 + i_b)} a)$$

Where p is the payment made, which equals the money holding of the borrower, including his borrowing. The bargaining solution gives:

$$\phi_m m + \frac{\phi_a + \delta}{1 + i_b} a = z(q)$$

A point to notice here is that since the borrowers are credit constrained, therefore, not all the funds deposited are lent to the borrowers. For zero profit condition of banks, there is a wedge between the interest rate charged to borrowers and the interest paid to depositors. The zero profitability condition of banks gives:

$$i_b = \frac{(1 - \sigma)\phi_m m i_s}{\sigma(\phi_a + \delta)a - (1 - \sigma)\phi_m m i_s}$$

Notice, for equilibrium to exist, $i_s < \frac{\sigma(\phi_a + \delta)a}{(1 - \sigma)\phi_m m}$. The upper bound on the savings rate is the degree of excess funds with the bank, measured by the term on right hand side.

First order conditions wrt b, given λ as the Lagrange multiplier on incentive constraint of borrowers, is:

$$\frac{u'(q)}{z'(q)} = 1 + i_b + \frac{\lambda}{\phi_m}$$

Using the envelope conditions in the CM along with the bargaining solution,

$$\beta[\sigma \frac{u'(q_{t+1})}{z'(q_{t+1})} \frac{\phi_{a,t+1} + \delta}{1 + i_b} + (1 - \sigma)(\phi_{a,t+1} + \delta)] = \phi_{a,t}$$

Simplifying, and using the fact that under the assumption of stationarity and for the prices to be non-explosive, $\phi_{a,t+1} = \phi_{a,t}$

$$\sigma(\frac{u'(q)}{z'(q)} \frac{1}{1 + i_b} - 1) + 1 = \frac{\phi_a}{\beta(\phi_a + \delta)}$$

Here the term $\sigma(\frac{u'(q)}{z'(q)} \frac{1}{1+i_b} - 1)$ is the pledgibility premium of the assets. Assets are priced above the fundamental value because they also serve a role as collateral, and hence have a premium. Notice if there are sufficient assets in the economy such that banks are able to lend all their funds with the collateral, then the pledgibility premium of the assets goes away, and in that case $\frac{u'(q)}{z'(q)} = 1 + i_b$ which is the equation derived above in which no pledgable assets were required.

From the first order conditions of money holdings,

$$\frac{u'(q)}{z'(q)} = \frac{1 + i - (1 - \sigma)(1 + i_s)}{\sigma} = 1 + i_s + \frac{i - i_s}{\sigma} \tag{1.4.29}$$

Substituting $\frac{u'(q)}{z'(q)}$

$$(1+i_b)\frac{\phi_a-\beta(1-\sigma)(\phi_a+\delta)}{\beta(\phi_a+\delta)}=i-i_s+\sigma(1+i_s)$$

Using the above equation along with the zero profit condition,

$$i_b=\frac{\beta(\phi_a+\delta)[\phi_m m(1+i)-\sigma(\phi_a+\delta)a]+\beta(\phi_a+\delta)(1-\sigma)\phi_m m-\phi_a\phi_m m}{\phi_m m[\phi_a-(1-\sigma)\beta(\phi_a+\delta)]}$$

$$i_s=\frac{\sigma(\phi_a+\delta)a-(1-\sigma)\phi_m m}{(1-\sigma)\phi_m m}$$

The spread is then given by:

$$i_b-i_s=\frac{(\phi_a+\delta)[\beta\phi_m m(1+i)(1-\sigma)-\sigma\phi_a a]}{(1-\sigma)\phi_m m[\phi_a-(1-\sigma)\beta(\phi_a+\delta)]}>0$$

Notice that all these equilibrium objects depend on the amount of pledgable assets a present in the economy.

1.5 Banks Remain Open During DM

Proposition: Monetary equilibrium with banks exists if nominal interest rate is not high enough Proof: See appendix and the section on incentive compatibility of buyers that follows for the definition of high enough nominal interest rate.

Proposition: If banks are allowed to remain open during the DM trade process, then banks can insure agents against bargaining frictions. Proof: See following discussion

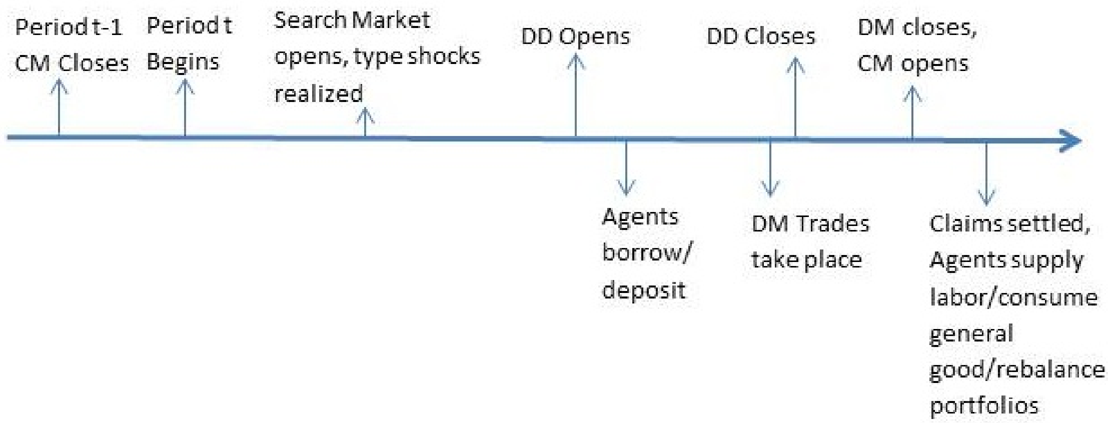


Figure 1.5.1: Time-line of Events if Banks Remain Open During DM Trade Process

Consider a case where now the banks remain open during the DM. This means that the DD market does not close after accepting deposits and advancing the loans. During the bargaining between buyer and seller, the buyer can always walk away from the bargaining process and threaten to deposit his/her money back in the bank, and earn interest on his/her savings instead of trading with the seller. This is a credible threat, as both the individual buyers and sellers are of measure 0 and they cannot influence the interest rates and/or supply of money balances. The DD market closes after trade takes place but before sellers can deposit money received from buyers. See figure 2 **??** for a graphical depiction of the time-line of events.

The bargaining solution in this case can be written as:

$$\max_{q,b,d_b} u(q) + W(a, m + b_b - p_m, b_b) - W(a, 0, -m) \tag{1.5.1}$$

subject to

$$-c(q) + W(a, p_s, -m) - W(a, 0, -m) \geq [1-\theta][u(q) - c(q) - \phi_m b i_b - \phi_m m i_s] \quad (1.5.2)$$

$$p_m \leq m + b_b$$

Solving the bargaining solution

$$\phi_m(m+b) = \theta c(q) + (1-\theta)[u(q) - \phi_m m i_s - \phi_m b i_b] \equiv y(q,i) = z(q) - (1-\theta)(\phi_m m i_s + \phi_m b i_b)$$

The difference between this case and the previous case is that due to a bigger threat point, now the buyer shades his payment by $(1-\theta)\phi_m(m i_s + b i_b)$ which is the total opportunity cost of interest (that includes interest that he can earn on his real balances and the interest he would not have to pay if the trade does not take place, adjusted for his share of the bargain).

Differentiating the bargaining solution wrt b and m

$$\frac{\partial q}{\partial m} = \frac{[1 + (1-\theta) i_s]\phi_m}{z'(q)}$$

$$\frac{\partial q}{\partial b} = \frac{[1 + (1-\theta) i_b]\phi_m}{z'(q)}$$

First order conditions wrt m give

$$\frac{u'(q)}{z'(q)} = 1 + \frac{i - i_s + \sigma\theta i_s}{\sigma[1 + (1-\theta)i_s]} \tag{1.5.3}$$

First order conditions wrt b give

$$\frac{u'(q)}{z'(q)} = 1 + \frac{\theta i_b}{1 + (1-\theta)i_b} \tag{1.5.4}$$

These two equations, together with zero profits and market clearing conditions give us

$$i_b = i_s = i \tag{1.5.5}$$

The real balances exchanged, after imposing the equilibrium conditions and zero profit condition can be written as:

$$\phi_m(m+b) = \frac{z(q)}{1+(1-\theta)i} = \frac{\theta c(q) + (1-\theta)u(q)}{1+(1-\theta)i}$$

The first order conditions, together with zero profit condition then give

$$\frac{u'(q)}{z'(q)} = 1 + \frac{\theta i}{1 + (1-\theta)i} \leq 1 + i \tag{1.5.6}$$

Rearranging and simplifying,

$$u'(q) = (1+i)c'(q)$$

Formal proof of existence is given in the appendix. Equilibrium in this case can be supported for any nominal interest rate. The relation gives the quantities traded in equilibrium for $\theta \geq 0$. Notice now the quantities traded do not depend on θ, which represents the bargaining frictions. If banks are open during the process of decentralized trade, then agents (buyers) are fully insured against bargaining frictions in the sense that now the terms of trade do not depend on the relative bargaining power of the buyer with respect to the seller. The quantities traded are exactly the same as they would have been traded had the banks been closed and buyers had all the bargaining power. However, there is still a wedge of nominal interest i, which is basically the Friedman wedge. Therefore, even if banks are open during the trade process, they are unable to insure buyers against the Friedman wedge (unless $i = 0$). The reason is that buyers still have to incur the cost to trade in DM. This cost is the inter-temporal cost of holding money, or in the case of borrowing, the interest payments that the buyer has to make in next CM. However, even in the presence of these costs, sellers are unable to hold up the buyers as now buyers can always threaten to walk away from the trade and deposit their money balances in the bank to earn interest.

If $\theta = 1$, then quantities traded in both scenarios (banks closed and banks remain open during the process of trade) are the same. The reason is that with $\theta = 1$, agents are already getting all surplus from trade, and hence keeping the banks open does not result in any difference in quantities traded. However, as θ falls below 1 , quantities traded decrease if banks are closed because of bargaining frictions. However, quantities traded are not affected if banks remain open during the trading process, and this gives us the following result:

Result: If banks remain open during the DM trade process, then quantities traded are weakly greater (strictly greater if buyer does not have all bargaining power) than what would have been traded if banks are closed during the DM trade process.

Proof: Quantities traded would be bigger if the wedge between marginal utility and marginal cost is smaller. Therefore, for quantities traded with banks open to be bigger, we need:

$$i \leq \frac{i}{\theta - (1-\theta)i}$$

For this inequality to hold, we need $\theta - (1-\theta)i \leq 1$. This is exactly the necessary and sufficient condition for equilibrium to exist in the case with banks closed during DM trade process (see appendix). QED.

This configuration of banks supports the biggest range of nominal interests for which a monetary equilibrium exists. We have the following proposition:

Proposition: If the banks remain open during the process of trade, then the monetary authority has the biggest range of nominal interest rates available at its disposal without losing monetary equilibrium, followed by the case in which the banks close during the process of trade.

Proof: See appendix.

1.5.1 Incentive Compatibility of Buyers

Incentive constraint for this case can be written in the same way as for that case when banks close during the trade process. Let me denote q_{bo} as the quantities traded with banks open and q_D as deviation quantities. Agents will not deviate i.e. pay off their debt if:

$$1 + i \leq \frac{\sigma}{1-\sigma} \frac{u(q_{bo}) - u(q_D)}{rz(q_{bo})}$$

This inequality is looser than in the case with the banks closed, as $q_{bo} > q_b$. It is also worth noticing that quantities traded with banks open is least elastic (or most inelastic) with respect to changes in the interest rate.

1.5.2 Social Surplus and Trade Surplus

Definition: Trade surplus here is defined as the sum of the surplus that accrues both to an individual buyer and a seller during a particular trade.

Definition: Social surplus is defined as the aggregate surplus from all the trades. Social surplus can mathematically be written as:

$$S(q) = \sigma[u(q) - c(q)]$$

Total trade surplus, adjusted for measure of trade is

$$S^{TS}(q) = \sigma \frac{u(q) - c(q)}{1 + i - \theta i}$$

See figure 1 for quantities traded with presence and absence of banks, and with different configurations of banks.

1.6 Conclusion

This paper developed a model which allowed me to study the conditions under which banks remove some of the frictions present in the trading process and insure agents against the risks of excess liquidity, the cost of holding liquidity and bargaining frictions. The paper showed that banks are equivalent to the case where agents can re-balance their money holdings if they

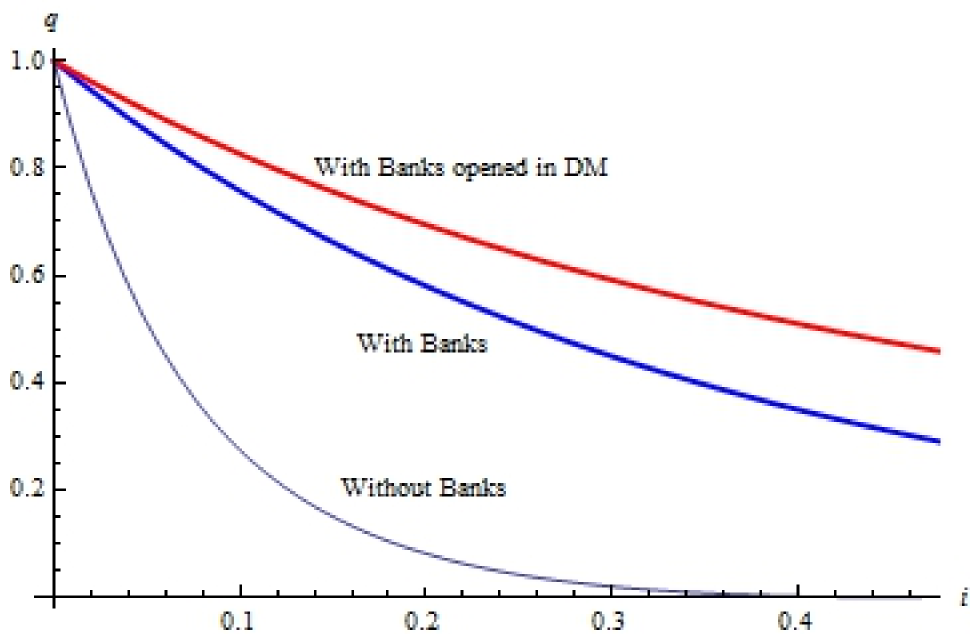
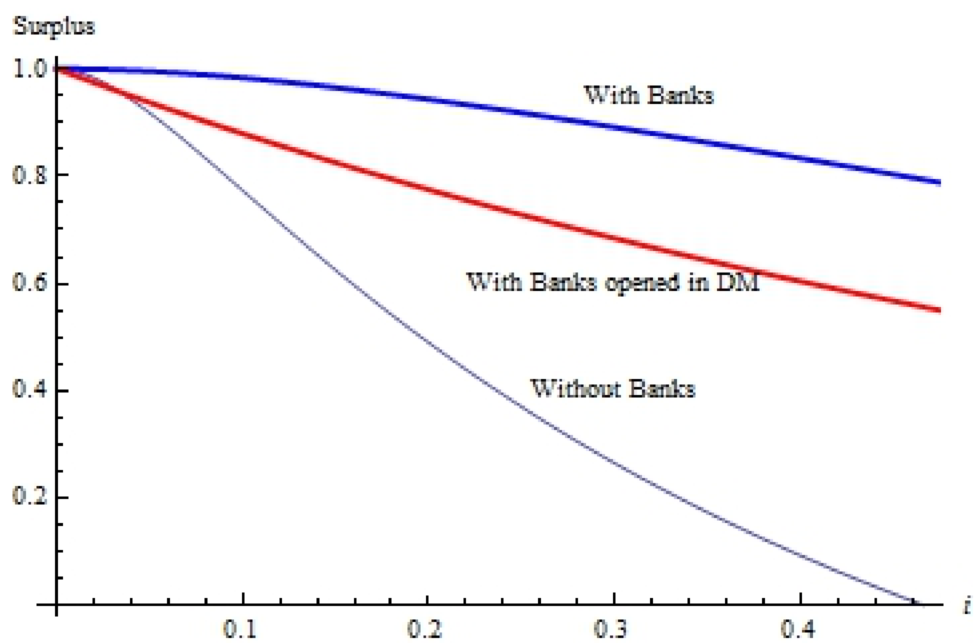


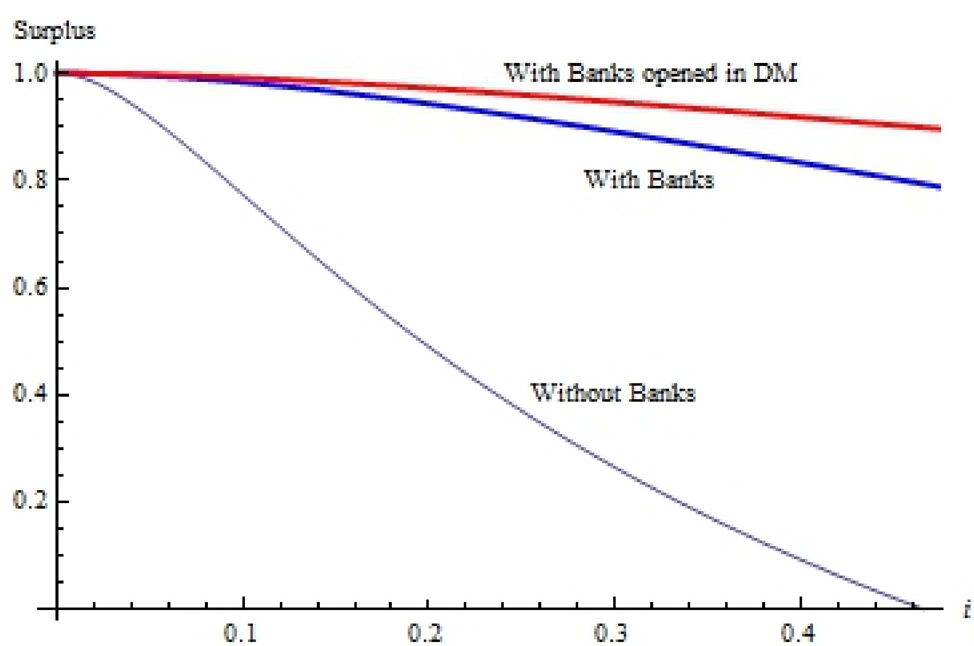
Figure 1.5.2: Quantities Traded

learn about their type shocks before the centralized market closes. The paper also showed that banks can also take care of the bargaining frictions if the agents are allowed to deposit money balances and/or return borrowing without penalty in the decentralized market, and hence banks not only insure against excess liquidity, but banks also insure agents against bargaining frictions, a feature of the banks not studied explicitly before. However, banks are unable to insure agents against the Friedman wedge.

The model of banks presented here is a very simple one, and more work needs to be done (that is left as future work to be done by the author) to understand the insurance role and the stability of the economy with banks in the presence of frictions, like information frictions and reserve requirements by the central bank. These frictions cannot be captured in a model with a representative agent. Therefore, heterogeneity needs to added to the model to capture these frictions, and understand the role of these frictions specifically in the context



(a) Individual Trade Surplus



(b) Social Surplus

Figure 1.5.3: Surplus

of monetary policy and the cost of inflation. Moreover, banks here also do not create a credit multiplier effect. This stems from the fact that agents are homogeneous within their types, and every agent gets access to the CM after every DM where the agent can re-balance their portfolios. Adding uncertainty in terms of access to CM for both borrowers and savers can generate a credit multiplier effect, which is a work in progress by the author. The main issue in this regard is the tractability of the model.

Another area that is being worked on as a separate project is the cost of inflation. The cost of inflation varies from about 1% (Lucas, 2000) to 5% (Lagos and Wright, 2005) and (Craig and Rocheteau, 2008). However, with banks, as we observed, the cost may be different (actually less that 5%) because the agents are hedged against the cost of holding money. Therefore, the quantities traded are bigger and as noted, the elasticity of quantities with change in the nominal interest rate is smaller. Therefore, this would result in inflation cost estimates lesser than 5%. Another point to note here is that the cost varies among agents, depending on whether the agent has access to inter-mediated funds market or not. It comes as no surprise that countries with better developed financial markets incur a lower cost of inflation. Moreover, within a country, agents who have access to financial inter-mediation incur lesser cost of inflation than those who do not have access to such markets.

This project opens up new areas for quantitative as well as theoretical work that would enhance our understanding of financial markets, more specifically financial inter-mediation in conjunction with monetary policy.

Appendix

Optimality Conditions without Banking

The first order conditions, without banking can be written as (see Nosal and Rocheteau, 2012 for explanation):

$$\max_q\{-iz(q)+\sigma[u(q)-z(q)]\}$$

Rearranging and using the fact that $z(q)=(1-\theta)u(q)+\theta c(q)$

$$\max_q\{[\sigma-(1-\theta)(\sigma+i)]u(q)-(\sigma+i)\theta c(q)\}$$

Since u is concave and c is convex and $-(\sigma+i)\theta c(q)$ is concave., the necessary and sufficient conditions for a solution to exist is $\sigma-(1-\theta)(\sigma+i)>0$, which translates to

$$i<\frac{\theta}{1-\theta}\sigma$$

Therefore, the set i for which monetary equilibrium exists is:

$$\Omega^M(i)=\{i:i<\frac{\theta}{1-\theta}\sigma\wedge i\geq 0\}$$

Optimality Conditions for Banking

From (1.4.2) and (1.4.7), we can write $D(a,m)=V(0)+V(a,m)$. From the bargaining solution and market clearing condition, $\phi_m m=\sigma z(q)$ and $\phi_m b=(1-\sigma)z(q)$. From the Central Bank policy, $\phi_m m'=\frac{\phi_m}{\phi'_m}\phi'_m m'=\gamma\phi'_m m'=\gamma\sigma z(q_{t+1})$

The first order condition can then be written as (after inserting the value of $D(a,m)$ and $\phi_m m$) and imposing $i_s = i$:

$$\max_{q_{t+1}}\{\theta[u(q) - c(q)] - z(q)i\}$$

The above condition can be rewritten as:

$$\max_{q}\{[\theta - i(1-\theta)]u(q) - [i+\theta]c(q)\}$$

Since $c(q)$ is convex and $u(q)$ is concave, the first order conditions are necessary and sufficient if

$$\theta - i(1-\theta) > 0 \Rightarrow i < \frac{\theta}{1-\theta}$$

Therefore, the set i for which monetary equilibrium exists is:

$$\Omega^B(i) = \{i : i < \frac{\theta}{1-\theta} \wedge i \geq 0 \wedge 1 + i \leq \frac{\sigma}{1-\sigma}\frac{u(q_b) - u(q_D)}{rz(q_b)}\}$$

Since $\sigma < 0.5$, it is clear that $\Omega^M(i) \subset \Omega^B(i)$.

Optimality Conditions for Banking when opened in DM

Using similar steps as were done for the case above, but using bargaining solution when the banks are open in DM, the first order condition can be written as (using the fact that $c(q)$ does not depend on seller money holdings):

$$\max_q\{\sigma[u(q) - \frac{1}{1+(1-\theta)i}z(q)] + (1-\sigma)[\frac{i}{1+(1-\theta)i}z(q)] - \frac{i}{1+(1-\theta)i}z(q)\}$$

The above condition can be rewritten as:

$$\max_q\{u(q) - [1+i]c(q)\}$$

Since $c(q)$ is convex and $u(q)$ is concave, the first order conditions are necessary and sufficient for any $i \geq 0$

$$\Omega^{BO}(i) = \{i : i \geq 0 \wedge 1 + i \leq \frac{\sigma}{1-\sigma}\frac{u(q_{bo}) - u(q_D)}{rz(q_{bo})}\}$$

Therefore, $\Omega^M(i) \subset \Omega^B(i) \subset \Omega^{BO}(i)$.

Chapter 2

A Three Sided Model of Search

2.1 Introduction

Search theory in Economics, specifically in Labor Economics is more than 4 decades old, with earlier contributions by McCall (1970), Mortensen (1970 and Burdett (1977). However, since the seminal studies in this stream of literature, researchers have shied away from considering environments involving non-single agents[1]. Most of the models studied had two sides in the search environment (e.g a firm and a worker). Even though single-agent models (models without heterogeneity) have been very popular in this area of research, we do not find much work done in this area except some work done earlier by Burdett and Mortensen (1978). In their work they consider a two person search problem. They encouraged future work on the topic in their paper. Quoting them from their paper:

A number of avenues for future research suggest themselves. First, the multi-

[1]By environments having single agent I mean environments where there are two sides of search process, but agents on both sides are homogeneous. By non-single agent or multi-agent environments, I mean at-least one side has heterogeneity, or there are more than two sides of search.

> person version of the model needs to be elaborated further............. Second, one can easily incorporate multidimensional differences in job characteristics...........

This paper explores the decisions of the firms on when and how to search for labor when labor types are heterogeneous. For example if we consider two types of labor, then firms can make three types of products using these two types of labor. Product 1 can be produced using type 1 labor as input, product 2 can be produced using type 2 labor as input, where as product 3 can only be produced using both types of Labor. The heterogeneity arises because of different skill sets of the workers. Firm cannot produce product 3 without hiring both of them. For example this can be an assembly plant that requires an operations manager and an inventory manager: without having raw materials inventory, the operations manager cannot work and there is no need for an inventory manager if there are no operations. A firm searching for such managers would not stop search after matching with one of the types: It will continue to search for the second type. We can also consider one type of 'worker' as capital e.g. a specialized goods transport company that buys specialized trucks from 2nd hand market, and also requires the services of a driver as a labor input to operate the machine. The products here may not just be interpreted just as goods traded in the good market. These product types can be different products, or they can also refer to the change in productivity levels with having different labor types. This paper develops a search based model with heterogeneity in labor types to analyze decisions of the firms faced with such a scenario.[2]

[2]Same model can be developed for the workers. However, it would not be qualitatively different. We can think of this model as a three sided model of search e.g. Three friends, who are lost in Times Square in New York and are searching for each other in order to spend time together to have more fun. So the search process of all the three friends would not be qualitatively different from each other. The search model in this paper is a little bit more complicated, but the same logic applies here.

In my paper, I consider two types of labor that search for jobs. They have different productivity levels, job separation rates and reservation wages. This feature introduces heterogeneity in my model. There are also firms that are operating in a random search environment, and can search for two different types of labor, either sequentially or at the same time. All the agents search independently. This gives a feature of a multi-dimensional search in my model, and hence the title "A three sided model of search". Upon successful match with one of the types, the firm has the option to continue search for the other type.

This paper has contribution towards two strands of literature. The first strand consists of papers on matching. In Becker's Models (Becker, 81, 82), Becker considers sorting where the best matches with the best. However, in my model matching can take place even if search results in a worker that is not very productive. The match results because a firm that already matched with one type can continue to search for the other type, and upon matching, can benefit from the complementarity between the two types[3]. Burdett and Coles also have a similar paper, but they consider sorting in the marriage market. Search in their model also stops after the first successful match (No polygamy!). Durlauf and Sheshadri (2003) develop conditions under which complementarities between individual agents imply assortative matching is efficient. But these complementarities are between the two sides of a particular match (i.e. there are only two types). Their model also does not involve search. My model not only uses search as a mechanism to bring parties together to decide on a match, but also considers complementarities between firms and two types of workers.

The second strand of literature consists of papers that use search as an endogenous mechanism to bring together parties for a match. This work may be related to the celebrated work of Mortensen and Pissarides (1994), but they only consider bilateral search, instead

[3]Of course, if the complementarity is low, then either the firms would not search for the second type or first match would not be successful in the first place!

of a trilateral search considered in this paper. Burdett and Malueg (81) consider search for multiple goods. The individuals search for multiple goods and then match with entire vector of goods at the same time. In my model, agents (firms) do not have to search and match for the two types together as a bundle[4]. Lentz and Mortensen (2010) consider heterogeneous firms and workers but there are no complementarities. They focus on efficiency gains through reallocation and sorting. Burdett, Coles (2010) consider heterogeneous firms, but the workers are homogeneous. Their focus is on wage/tenure contracts, and not on the complementarities of the match. Shi (2002) has both skilled and unskilled workers, and hi tech and low tech firms, but each type can employ only one type. Neal (1999)[5] considers an environment with sequential search. However, in his model the first step (search for career by employers) is not reversible, in the sense that once this search is successful, agents may only search for jobs within the matched careers. Agents cannot search for careers while they search for the jobs. They have to leave their careers and then first search for a careers before searching for a job if they wish to search for jobs in a different career. Model presented in this paper does not impose these restrictions, and the firms are free to search for any type of labor in any sequence (or both at the same time) that is value maximizing. Guler, Guvenen and Violante (2012) also consider a search model in which couples search jointly for jobs. Their model considers a joint search by couples. In their model, both spouses or one of them might search for jobs depending on their employment status. Search process of the couples in their model is not independent of each other, and their model does not consider complementarities but focus more on the pooling of wages among the couple, and their search decisions depend on their pooled wage.

[4]In fact, as can be seen later in the paper, given the model considers events in continuous time, the probability that the firm gets matched to both types at the same instant is 0

[5]I am thankful to John Kennan for suggesting this reference

As this paper is about the search and matching decisions of the firms, it would be appropriate if I am explicit here in what I mean by matching in my model. In a random search environment, firms searching for a particular type of labor might come across another type. Even though the firm "searched" the other type, the match will not be successful as the firm has no intention of making the match successful, as that type does not give the firm a positive value. This mis-match is another type of friction in my model.

With multidimensional (multi-agent) search, I find that complementarities among the labor force is a driving force in determining the search and matching strategy of the firms. I find that based on the search decisions of the firms, unemployment of a labor type is non-monotonic and exhibits discontinuity in its own productivity level. The unemployment might increase as the productivity increases in some range of productivity levels. I also found that unemployment of a labor type also does not stay constant even if its own productivity levels are constant and the productivity levels of other type are changing. Welfare, defined as the aggregate production, might also decrease as the productivity levels of a particular type increase. The aggregate value of search, defined as the aggregate value of the firms both that are searching and that are matched, might also decrease as the productivity increases.

Section 2 below discusses some of the applications of my model, where as section 3 sets up the model. Section 4 discusses the equilibrium and section 5 solves the model quantitatively and provides a quantitative assessment. Section 6 concludes.

2.2 Applications

The model presented in this paper can be applied in two broad categories. The first category is the application of this model in a macro context. This model can help explain changes in unemployment levels in response to productivity shocks that affect different skill sets differently. Firms might land in a different regime (different firms' search decisions) with different relative productivity shocks, which changes the equilibrium unemployment for both types. More work needs to be done in this regard, as shocks in relative productivity determine the regime shifts. Shocks to absolute productivity may change the regime only if the shock affects relative net productivity levels of managers and workers differently. Nonetheless, this is a good starting point for discussing issues related to unemployment and business cycles. Another application would be to answer the questions related to complementarities changing because of some exogenous reasons. For example, questions about employment and unemployment related to outsourcing of jobs to other countries. The complementarities between high skilled and low skilled workers might get reduced significantly if firms are able to outsource low skilled jobs abroad. In that case, employment of not only low skilled workers but high skilled individuals might also change. A quantitative assessment of the impact of outsourcing in light of the model presented in this paper would give some insights into the economics of outsourcing in developed countries.

The second broad category relates to applications in the micro-context. Some examples of where this model can be applied are:

- Theory of team formation: This model can help explain the dynamics of team formation within a firm in firms that depend on heterogeneous agents for output. The model presented in this paper can be used for any empirical research that attempts to address issues related to this subject

- Search in over the counter markets for different assets. Different Assets may be complements or substitutes of each other (for example because of hedging needs or liquidity needs).

- Search in Real Estate Literature. This application related to the decision of an agent on searching for more than one unit of real estate.

In the next section, I will lay the foundations of the model.

2.3 Model

The economy under discussion is specified in continuous time and is inhabited by two types of workers (say workers and managers)[6]. There is a continuum of L_w workers and L_m managers, normalized such that $L_w + L_m = 1$. The preferences of both types of labor within their respective groups are homogeneous. Skill-set of workers and managers are different but homogeneous within their own groups. Firms can produce three types of products, product "$W0$", product "$M0$" and product "WM". Production of product "$W0$" requires input only from workers, whereas production of product "$M0$" requires inputs from managers, and production of product "WM" requires input from both types of labor (managers and workers). Product "$W0$" sells for an exogenous price P_{w0}. Similarly, exogenous price of product "$M0$" is P_{m0} and that of product "WM" is P_{wm}. Firms have perfect information about the productivity in different states i.e. they know the productivity in different states before they employ a worker (or a manager).

There are f_{00} firms that do not have a worker or a manager yet, and are searching for both types (or one type if it is not profitable to search for the other type). We say that the firm is in state $'00'$. Since the arrival distributions of both managers and workers are

[6] From this point onwards, instead of calling them type 1 and type 2 workers, I will call them workers and managers. These are just labels, and as already discussed above, this model can be applied to a broad scale of environments

assumed to be atom-less and independent, so probability that the firm comes across both worker types at the same instant is 0. There are f_{w0} firms that have employed workers and are producing product $W0$. In this case, the firm can be referred to as in state $w0$. In this state, the firms also have the option to search for managers. There are F_{wm} firms that have employed both types of labor, workers first, and are producing product 2 (firm is in state wm). Likewise, there are F_{mw} firms that have employed both types of labor, but managers first, and are producing product WM (firm is in state mw). There are f_{m0} firms that only have managers, and do not have workers in their workforce, but the firm has an option to search for the workers (firm is in state $m0$). The total unemployment in the economy for workers is μ_w, and that for managers is μ_m. Since firms can only employ a maximum of one of each type, the Labor market clearing conditions can be written as:

$$L_w = \mu_w + f_{w0} + F_{wm} + F_{mw} \tag{2.3.1}$$

$$L_m = \mu_m + f_{m0} + F_{wm} + F_{mw} \tag{2.3.2}$$

Note that f_{w0} is also the total measure of workers employed in a firm without a manager, f_{m0} is the total measure of managers employed in a firm without a worker and $f_{wm} = F_{wm} + F_{mw}$ is also the total measure of workers and managers employed in a firm with full employment. The cost of search is solely borne by the firms and workers/managers do not incur a cost to search.

Based on this setup, there can exist four forms of market segmentation:

- A single market for all labor types and firms. All firm types and labor types go to this market for search (see figure 1).

- A segmented market based on labor types, which means two markets exist - one each for workers and managers. Firms who do not have both worker and manager can go to either the market for workers or the market for managers, whereas firms not employing a manager but employing a worker go to market for managers and firms not employing a worker but employing a manager go to market for workers.

- A segmented market based on firm types. Based on set-up of this paper, 3 such markets exist, one where the firm has no worker or a manager, second where the firm has a worker but no manager, and third where the firm has a manager but no worker. Firms that have both managers and workers do not search. Workers searching for jobs can go to either market 1 or 3, whereas managers can go to markets 1 or 2.

- Complete segmentation: 4 such markets exist: Firm without any worker or manager looking for a worker, firm without any worker or manager looking for a manager, firm without any worker but employing a manager looking for a worker, and firm employing a worker but without a manager looking for a manager.

This paper only considers random search setting i.e. a single market for firms and labor types. The other three setting are discussed in a companion paper.

2.3.1 Match Process

The frequency with which the workers (or managers) meet the firms can be characterized by a homogeneous of degree one function, called the match function. In this model, since firms do not have ex-ante information about the type of labor they meet, all the firms searching for any type of worker create "congestion" effects for other types of firms in addition to firms of their own type. In this model, it is assumed that once the firm has employed the type(s) it is searching for, it leaves the market, and hence does not create congestion effects for other firms still searching. Analogously, workers also do not have the ex-ante information about the types of firms they are meeting. Therefore, they also create a congestion effects on each other[7]. Considering these facts, a single match function can be written for a market that takes as input the total number of firms searching for labor, and total labor searching for jobs, and gives the total number of matches in a unit time. If there is a measure F of firms looking for a labor type, and a measure L of labor of both types in the market, then the market tightness can be defined as $\gamma = F/L$. Let $m = m(F, L)$ be the match function, which gives the total number of matches in a unit time. Using constant returns to scale assumption, we can define $\alpha(\gamma) = m(F, L)/F = m(1, L/F) = m(1/\gamma)$, which is the average rate at which firms meet potential partners in a market with tightness γ. Using these definitions, average rate at which firms meet workers would then be $\alpha(\gamma)\frac{\mu_w}{\mu_w+\mu_m}$ and the average rate with which firms meet managers is $\alpha(\gamma)\frac{\mu_m}{\mu_w+\mu_m}$. Analogously, $m(F, L)/L = m(F/L, 1) = m(\gamma, 1) = \gamma\alpha(\gamma)$ represents the average rate at which labor types meet potential firms in a market with tightness γ. The function $\alpha(\gamma)$ is a decreasing function of γ, and its elasticity is strictly between -1 and 0[8].

2.4 A Single Market for Three Sided Search

There is a measure μ_w of workers and measure μ_m of managers searching for a job in this market, and measure f_{00} of firms searching for both workers and managers, measure of f_{w0} firms that are searching only for workers[9] and measure f_{m0} firms that are searching only for managers. The measure of firms that have employed both types is f_{wm}. The market tightness

[7] This is characterization of random matching

[8] The elasticity is dependent on the relative intensities with which the two parties search.

[9] Firms might not search for managers either because they already have employed a manager, or it might not be profitable for them to search for one

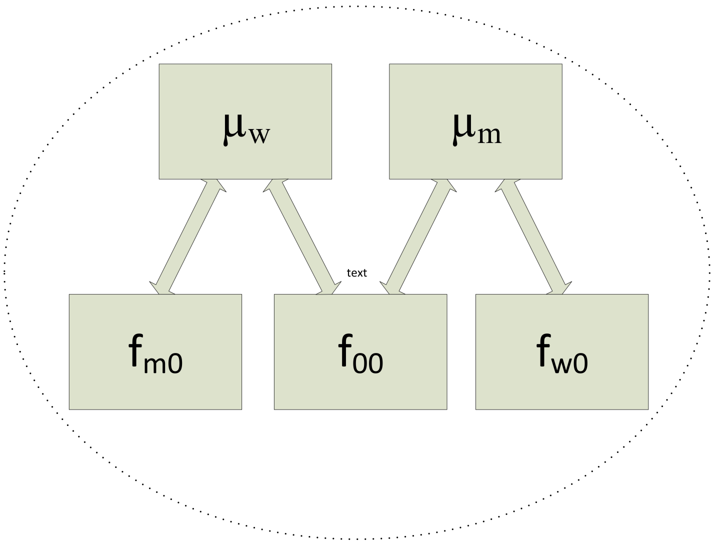


Figure 2.4.1: Non-Segmented Search Market

in this single market is $\gamma = \frac{f_{00}+f_{w0}+f_{m0}}{\mu_w+\mu_m}$ if all types are searching. It is assumed that the firms not searching exit the market, and hence do not create congestion effects on the other firms.

2.4.1 Job Creation and Value Functions of Firm

Firms *'create jobs'* when a firm with a vacant job and an agent meet and start producing (see Mortensen and Pissarides, 1994). If a job is hit by an idiosyncratic shock, it exits, and the job lies vacant once again, and the firm starts the matching process once again to fill up the vacancy. There is also a cost of recruiting, and in this model it is modeled as per unit cost of maintaining vacancy, c_i, where $i \in \{w, m, wm\}$, referring to the cost of recruiting managers (c_m) or workers (c_w), or both (c_{wm}). I assume $c_{wm} > c_w$ and $c_{wm} > c_m$ to make the problem interesting. This cost may arise due to advertising costs, hiring costs (e.g. interviewing, processing applications etc), or may also refer to the cost of training a worker or a manager for his/her job.

The steady state flow Bellman equation of the firm in state 00, in which it has no worker or manager, and is searching for both types (or one type) can be written as:

$$\begin{aligned} rV_{00} &= \alpha(\gamma)\frac{\mu_w}{\mu_w+\mu_m}(V_{w0}-V_{00})\tau_{w0}+\alpha(\gamma)\frac{\mu_m}{\mu_w+\mu_m}(V_{m0}-V_{00})\tau_{m0} \\ &\quad -\tau_{w0}\tau_{m0}c_{wm}-\tau_{w0}(1-\tau_{m0})c_w-\tau_{m0}(1-\tau_{w0})c_m \end{aligned}$$

Here V_{00} is the value function for the firm in state 00, V_{w0} and V_{m0} are the value functions in state $W0$ and $M0$ respectively, r is the exogenous risk free real interest rate, and $\alpha(\gamma)$ is the match process with market tightness γ, where $\gamma = \frac{f_{00}+f_{w0}+f_{m0}}{\mu_w+\mu_m}$. τ_{w0} is an indicator function, which is equal to 1 when both the matched firm and the matched worker want to make it a successful match. Firms would only search for a worker if $V_{w0} - V_{00} \geq 0$ (for search decision of workers, see below). τ_{w0} is 0 if either of these or none of these want to make the match successful. The interpretation of this indicator function is that the firms would search for workers and gain from a match only if the continuation value of having a worker is at least equal to the continuation value of not having the worker. Otherwise, the firm would opt not to search for the worker, and $\tau_{w0} = 0$ captures this behavior of the firm. Same holds for τ_{m0}, which is the managers' counterpart of τ_{w0} for workers.

For firm in state $w0$, in which it has already created a job for a worker and is searching for a manager, the steady state flow Bellman equation can be expressed as:

$$rV_{w0} = P_{w0} - w_{w,0} - c_{w0}\tau_{mw} + \alpha(\gamma)\frac{\mu_m}{\mu_w+\mu_m}(V_{wm}-V_{w0})\tau_{mw} + \delta_w(V_{00}-V_{w0}) \qquad (2.4.1)$$

In the above equation, P_{w0} is the productivity, and hence the real value of goods produced by employing a worker, w_{w0} is the wage that the firm is paying to the worker (first letter of subscript w denotes that the wage is that of the worker, whereas second letter of subscript 0 denotes that this wage is paid to the worker when no manager has been employed by the firm), and c_m is the cost of vacancy of the manager to the firm. τ_{mw} is an indicator function which is equal to 1 if both parties (firms and managers) want to form a match, which in case of firms, translates to $E[V_{wm} - V_{w0}] > c_m$ and τ_{mw} is 0 if either or none of the parties do not want to make the match successful.

Similarly, for firm in states $M0$, WM and MW, the flow Bellman equations can be expressed as:

$$rV_{m0} = P_{m0} - w_{m,0} - c_w\tau_{wm} + \alpha(\gamma)\frac{\mu_w}{\mu_w + \mu_m}(V_{mw} - V_{m0})\tau_{wm} + \delta_m(V_{00} - V_{m0}) \tag{2.4.2}$$

$$rV_{wm} = P_{wm} - w_{w,wm} - w_{m,wm} + \delta_w(V_{0m}\tau_{0m} + V_{00}(1-\tau_{0m}) - V_{wm}) + \delta_m(V_{w0}\tau_{w0} + V_{00}(1-\tau_{w0}) - V_{wm}) \tag{2.4.3}$$

$$rV_{mw} = P_{wm} - w_{w,mw} - w_{m,mw} + \delta_w(V_{0m}\tau_{0m} + V_{00}(1-\tau_{0m}) - V_{mw}) + \delta_m(V_{w0}\tau_{w0} + V_{00}(1-\tau_{w0}) - V_{mw}) \tag{2.4.4}$$

P_{m0} and P_{wm} is the value of the real output to the firm when only manager produces and when both workers and manager produce respectively. Notice that since the model is setup in continuous time, and the cdfs of job separation are assumed to be atom-less, so the probability that both the worker and the manager are separated from the firm at the same instance is 0. τ_{wm} has the same interpretation as τ_{mw}, except that it is equal to 1 if $V_{mw} - V_{m0} > 0$ and 0 otherwise. Note that depending on wage determination mechanism,

V_{wm} may not be the same as V_{mw}, as $w_{w,wm}$ may not be equal to $w_{w,mw}$. Same goes for managers. ($w_{w,wm}$ means wage of a worker when worker was the first type hired, whereas $w_{w,mw}$ means wage of a worker when worker was the second type hired. Same goes for the manager)

This model allows for free entry of firms. Therefore, In equilibrium, all profit opportunities from jobs are taken, driving rents from vacant jobs to 0 i.e. $V_{00} = 0$.

2.4.2 Value Functions of Workers and Managers

The steady state flow Bellman equations for the workers in both employed and unemployed states can be written as

$$rU_w = b_w + \gamma\alpha(\gamma)\frac{f_{00}}{f_{00} + f_{w0} + f_{m0}}(T_{w0} - U_w)\tau_{w0} + \gamma\alpha(\gamma)\frac{f_{m0}}{f_{00} + f_{w0} + f_{m0}}(T_{w,mw} - U_w)\tau_{wm} \tag{2.4.5}$$

$$rT_{w0} = w_{w0} + \delta_w(U_w - T_{w0}) + \alpha(\gamma)\frac{\mu_m}{\mu_m + \mu_w}(T_{w,wm} - T_{w0})\tau_{mw} \tag{2.4.6}$$

$$rT_{w,wm} = w_{w,wm} + \delta_w(U_w - T_{w,wm}) + \delta_m(T_{w0} - T_{w,wm}) \tag{2.4.7}$$

$$rT_{w,mw} = w_{w,mw} + \delta_w(U_w - T_{w,mw}) + \delta_m(T_{w0} - T_{w,mw}) \tag{2.4.8}$$

U_w is the value function of the worker in unemployed state. b_w represents the benefits that the worker gets while being unemployed. This might include unemployment insurance benefit, or the income that the worker might be able to earn by doing odd or irregular jobs in a secondary sector of the economy if such a sector exists. b_w also includes the imputed real return from any unpaid leisure activities, such as home production or recreation (see Pissarides: Equilibrium Unemployment Theory). b_w is assumed to be constant, and independent of market returns. In addition to unemployment benefits, the worker also searches for the job, and in one unit time, he expects to move into employment with probability $\gamma\alpha(\gamma)\frac{f_{00}}{f_{00}+f_{w0}+f_{0m}}$ into a firm that has no manager, and with probability $\gamma\alpha(\gamma)\frac{f_{0m}}{f_{00}+f_{w0}+f_{0m}}$ into a firm that already created a job for a manager. Notice that with probability $\gamma\alpha(\gamma)\frac{f_{w0}}{f_{00}+f_{w0}+f_{0m}}$ a worker meets a firm which already has a worker, and hence match is not successful. τ_{w0} and τ_{wm} are the same indicator functions that were discussed in the section above. τ_{w0} is equal to 1 if $V_{w0} - V_{00} \geq 0$ and $T_{w0} - U_w \geq 0$, and 0 otherwise. τ_{wm} is equal to 1 if both $V_{wm} - V_{w0}$ and $T_{wm} - U_w$ are greater than 0, and 0 otherwise. T_{w0} is the value function of the worker in employed state in a firm without manager, in which the worker earns a wage w_{w0}. In this state, the worker is also at the risk of being separated from the job and become unemployed with an exogenous probability δ_w, which follows a Poisson process. In addition, the firm might continue to search for a manager and upon a successful match with a manager, the state of the worker would change from being employed alone to employed with a manager. $T_{w,wm}$ is the value function of the worker in employed state in a firm in which manager was employed after the worker (In the notation, subscript w preceding the comma refers to the fact that this is the value function of a worker. The two letters after the comma refer to the sequence in which worker/manager was hired. For example, $T_{w,wm}$ means value function of a worker when worker was employed before the manager). In this state, the worker earns a wage $w_{w,wm}$. Similarly, $T_{w,mw}$ is the value function of the workers in a state when the managers were employed before the workers.

Steady state flow Bellman equations for managers can be defined analogously:

$$rU_m = b_m + \gamma\alpha(\gamma)\frac{f_{00}}{f_{00}+f_{w0}+f_{m0}}(T_{m0}-U_m)\tau_{m0} + \gamma\alpha(\gamma)\frac{f_{w0}}{f_{00}+f_{w0}+f_{m0}}(T_{m,wm}-U_m)\tau_{mw} \quad (2.4.9)$$

$$rT_{m0} = w_{m0} + \delta_m(U_m - T_{m0}) + \alpha(\gamma)\frac{\mu_w}{\mu_w+\mu_m}(T_{m,mw}-T_{m0})\tau_{wm} \quad (2.4.10)$$

$$rT_{m,wm} = w_{m,wm} + \delta_m(U_m - T_{m,wm}) + \delta_w(T_{m0} - T_{m,wm}) \quad (2.4.11)$$

$$rT_{m,mw} = w_{m,mw} + \delta_m(U_m - T_{m,mw}) + \delta_w(T_{m0} - T_{m,mw}) \quad (2.4.12)$$

We can use an axiomatic bargaining mechanism to calculate the wages (e.g. Kalai Proportional Bargaining, Generalized Nash Bargaining solution). In what follows, the wages are determined by firm making a take it or leave it offer, then $w_{w,wm} = w_{w,mw} = w_{w0} = b_w$ and $w_{m,wm} = w_{m,mw} = w_{m0} = b_m$. In this case, the sequence in which the managers or workers are matched does not matter for value function of firm in state in which both labor types are employed. In this case, $V_{wm} = V_{mw}$, and $T_{w0} = T_{w,wm} = T_{w,mw} = U_w = \frac{b_w}{r}$ and $T_{m0} = T_{m,wm} = T_{m,mw} = U_m = \frac{b_m}{r}$. This also allows me not to keep track of both F_{wm} and F_{mw} separately.

2.4.3 Job Flows

Job creation occurs when worker or manager and firm meet and agree on a contract which specifies the wages and productivity of the firm. Since labor types are homogeneous within groups, so the firm productivity in a particular state is the same across all firms. Since the purpose of this paper is not to to look at the information problems, so to simplify analysis, it is assumed that the firm has perfect information about the productivity of the agent i.e. firm knows if the agent is a worker or a manager after they meet. Job creation and subsequent hiring of the agent by the firm results in the flow from unemployment to employment. Flow from employment to unemployment results because of the job separations. These job separations occur due to exogenous idiosyncratic shocks. Together these two flows i.e. flows into and out of unemployment constitute the job flow equation.

In steady state, flows in and out of a particular state for both workers and managers are equal. This results in the following job flow equations.

$$\alpha(\gamma)(f_{00} + f_{m0})\frac{\mu_w}{\mu_w + \mu_m} = \delta_w(f_{w0} + f_{wm} + f_{mw}) \tag{2.4.13}$$

Similarly, writing the other flow equations

$$\alpha(\gamma)f_{00}\frac{\mu_w}{\mu_w + \mu_m} + \delta_m(f_{wm} + f_{mw}) = \alpha(\gamma)f_{w0}\frac{\mu_m}{\mu_w + \mu_m} + \delta_w f_{w0} \tag{2.4.14}$$

$$\alpha(\gamma)f_{w0}\frac{\mu_m}{\mu_w + \mu_m} = (\delta_m + \delta_w)f_{wm} \tag{2.4.15}$$

$$\alpha(\gamma)f_{m0}\frac{\mu_w}{\mu_w + \mu_m} = (\delta_m + \delta_w)f_{mw} \tag{2.4.16}$$

Similarly for managers,

$$\alpha(\gamma)(f_{00} + f_{w0})\frac{\mu_m}{\mu_w + \mu_m} = \delta_m(f_{m0} + f_{wm} + f_{mw}) \tag{2.4.17}$$

$$\alpha(\gamma)f_{00}\frac{\mu_m}{\mu_w + \mu_m} + \delta_w(f_{wm} + f_{mw}) = \alpha(\gamma)f_{m0}\frac{\mu_w}{\mu_w + \mu_m} + \delta_m f_{m0} \tag{2.4.18}$$

$$\alpha(\gamma) f_{w0} \frac{\mu_m}{\mu_w + \mu_m} = (\delta_m + \delta_w) f_{wm} \tag{2.4.19}$$

$$\alpha(\gamma) f_{m0} \frac{\mu_w}{\mu_w + \mu_m} = (\delta_m + \delta_w) f_{mw} \tag{2.4.20}$$

Here δ_i, $i \in \{w, m\}$ is the exogenous probability of job separation that follows a Poisson process.

2.4.4 Best Responses of Firms

This section discusses the conditions as to when it is profitable for firms to search for a particular type of labor, and given the firm (and workers/managers) search, the equilibrium conditions governing the search. The firms and workers/managers participate in search if they get a non-negative payoff from participating. Since the workers are not incurring any cost of search, so workers (and managers) participate and accept any match if they are given at least their reservation utility, b_i. In the sections that follow, the scenarios where firms makes a take it or leave it offer are discussed. In this case, the workers and managers are given their reservation utilities in all states, and further, it is assumed that managers and workers accept to make a successful match if they are paid reservation utilities[10].

Assuming free entry, following proposition summarizes the correspondence between the indicator functions and value functions:

Proposition

- τ_{w0} is 1 when $V_{w0} - V_{00} \geq 0$

[10] The motivation for workers and managers searching for a job even if they are only given their reservation wages might be that they would lose their unemployment insurance benefits, or the firms may pay ϵ above the respective reservation wages etc. Since the paper does not model this, so we may abstract away from answering why do the managers and workers search and focus more on the search and matching process.

- τ_{m0} is 1 when $V_{m0} - V_{00} \geq 0$
- τ_{mw} is 1 when $\alpha(\gamma)\frac{\mu_m}{\mu_w+\mu_m}(V_{wm} - V_{w0}) \geq c_m$
- τ_{wm} is 1 when $\alpha(\gamma)\frac{\mu_w}{\mu_w+\mu_m}(V_{mw} - V_{m0}) \geq c_w$

Proof: The first two parts are trivial. If the value of a worker (or a manager) to firms is less than without them, then the optimal behavior of firms is to leave the search market. The next two parts are related to the search decision of firms when they have already matched with a particular type. Firms would only search for the other type if the expected value of search outcome is bigger than the search costs and this is exactly what is depicted in the last two inequalities. QED

These $\tau's$ are another way to represent the max operator. Since firms have perfect information about the productivity of different agent types ex-ante (but they do not know which type they are meeting when they meet), so the firms take into account their best responses in other states when they decide on making a match successful in a particular state. In the text that follows, equilibrium will be discussed, based on different values of $\tau_{ij}'s$. Also note that this discussion is based on wage determination mechanism where the firm makes a take it or leave it offer. But before discussing that, I will define some terms:

Definitions:

Complements: Type A is complement of Type B if net productivity (their total productivity minus their wages) of both of them alone is very low, but net productivity of them working together is sufficiently high.

Substitutes: Type A is substitute of Type B if net productivity (their total productivity minus their wages) of both of them alone is sufficiently high, and sufficiently close to each others' net productivity. Their net productivity working together is also close to their individual net productivity levels.

Net Productivity: Net productivity is defined as the total productivity of a labor type, subtracted by the wages firm is paying to that type. For example, net productivity of a worker is $P_{w0} - b_w$.

Regime, or decisiob rule: Regime, or a decision rule means a particular combinations of τ_{ij} that the firms are following, given productivity levels P_{w0} and P_{m0}. Decision values $\tau_{w0} = 1, \tau_{m0} = 0, \tau_{wm} = 0, \tau_{mw} = 0$ is a regime, whereas $\tau_{w0} = 0, \tau_{m0} = 1, \tau_{wm} = 0, \tau_{mw} = 0$ is another regime.

2.4.5 Equilibrium

Now lets discuss equilibrium for all the regimes that firms can follow.

2.4.5.1 $\tau_{w0} = 1$, $\tau_{m0} = 0$, $\tau_{wm} = 0$, $\tau_{mw} = 0$

This is the case where the firms search only for a worker if they are in state 00 and the firms would not search for a manager even after a match with worker was successful. Putting the values of τ_{ij} and solving the corresponding value functions of the firms using free entry condition $V_{00} = 0$,

$$\alpha(\gamma)\frac{\mu_w}{\mu_w + \mu_m}\frac{P_{w0} - b_w}{r + \delta_w} = c_w \tag{2.4.21}$$

From this equation, we can observe that P_{w0} needs to be strictly bigger than w_{w0} due to search frictions, and due to the cost incurred during the search. These search frictions create a wedge between the wages and the productivity, captured by the second term on the right hand side of the above equation. Here, the market tightness $\gamma = \frac{f_{00}}{\mu_w + L_m}$, as all the managers are unemployed and they all contribute to the congestion effects on the workers.

In this case, $f_{m0} = f_{wm} = f_{mw} = 0$ and $\mu_m = L_m$. From the worker flow equation,

$$\gamma\alpha(\gamma)\mu_w = \delta_w f_{w0} \tag{2.4.22}$$

The measure of unemployed workers, employed workers (and hence firms with workers) and the measure of firms looking for workers can be pinned down by solving (2.4.21,2.4.22) and market clearing condition for workers $L_w = \mu_w + f_{w0}$. This situation arises when a manager is neither a complement nor a substitute for the workers. (Manager would be a “near perfect” substitute if a firm would search for both, and would stop searching if it found one of them. Manager would be a complement if the firm would continue to search for a manager after matching with a worker).

Comparative Statics: Keeping everything else the same, the measure of unemployed workers, μ_w is decreasing in Productivity the worker brings to the firm and is increasing in b_w, c_w, L_w L_m, δ_w, and the real interest rate, r. The measure of employed workers, and of firms with a worker, f_{w0} is exactly the opposite of the comparative statics of μ_w, except that in the case of L_w, where it is increasing. f_{00} is increasing in b_w, c_w, L_m, δ_w and r, whereas it is decreasing in productivity, P_{w0}.

2.4.5.2 $\tau_{w0} = 0,\ \tau_{m0} = 1,\ \tau_{wm} = 0,\ \tau_{mw} = 0$

This is the case symmetric to case discussed above where $\tau_{w0} = 1$, rest all 0. Deviations are also symmetric.

2.4.5.3 $\tau_{w0} = 1,\ \tau_{m0} = 1,\ \tau_{wm} = 0,\ \tau_{mw} = 0$

This is the case where a firm searches for both a manager and a worker, but when match with one of them is successful the firm stops further search. After the firm hires a worker (or a manager), it discontinues its search, and hence does not incur the flow search costs. This case arises when these two labor types are substitutes for the firm. The increased cost (c_{wm}) that the firm incurs than it would have incurred if it had searched only for a worker or a manager alone is balanced out by the fact that in this case, match is always successful when a firm meets with someone with whom it would like to from a match. From the flow equations,

$$\mu_w = \frac{\delta_w}{\gamma\alpha(\gamma) + \delta_w} L_w \tag{2.4.23}$$

$$\mu_m = \frac{\delta_m}{\gamma\alpha(\gamma) + \delta_m} L_m \tag{2.4.24}$$

$$f_{w0} = L_w - \mu_w \tag{2.4.25}$$

$$f_{m0} = L_m - \mu_m \tag{2.4.26}$$

These equations, together with free entry condition (see below) determine the equilibrium values of μ_w, μ_m, f_{w0}, $f_{m0}$and f_{00}.

$$\alpha(\gamma)\frac{\mu_w}{\mu_w+\mu_m}\frac{P_{w0}-b_w}{r+\delta_w}+\alpha(\gamma)\frac{\mu_m}{\mu_w+\mu_m}\frac{P_{m0}-b_m}{r+\delta_m}=c_{wm} \tag{2.4.27}$$

In this case, the market tightness is $\gamma=\frac{f_{00}}{\mu_w+\mu_m}$.

Comparative Statics: The measure of unemployed workers is increasing in total workers and job separation rate δ_w. With increase in total workers, there are more workers who are looking for a job. Some of them get employed increasing the number of employed workers, but search frictions also cause the measure of unemployed workers to go up. The measure of employed workers (and hence firms with workers) is increasing in total number of workers, and is decreasing in job separation rate. Same comparative statics hold for managers.

2.4.5.4 $\tau_{w0}=1$, $\tau_{m0}=0$,$\tau_{wm}=0$,$\tau_{mw}=1$

This is the regime where firms decide to search for a worker only if they are in state 00, and conditional on the firms having a successful match in state 00, search for a manager. In this case, since V_{m0} is less than V_{00}, so if the worker is separated, then the manager is also separated, as firms endogenously break the existing match with the manager and go to state 00 and start to search for a worker only. Therefore, though δ_m is exogenous, but the actual separation of managers is correlated with the separation of the workers, as the separation rate of managers in this case becomes $\delta_w+\delta_m$. These conditions mean that worker alone is worth more to firm than the manager alone, and the combined productivity of the manager and the worker is higher than the wages it is paying to the manager and the value it is foregoing by moving to the state WM from state $W0$. This would happen if a manager complements the productivity of a worker, while being very unproductive alone.

Solving relevant steady state flow Belman equations after putting relevant values of τ_{ij}, the free entry condition is:

$$c_w = \alpha(\gamma)\mu_w \frac{(r+\delta_w+\delta_m)(P_{w0}-b_w-c_m)+\alpha(\gamma)\frac{\mu_m}{\mu_m+\mu_w}(P_{wm}-b_w-b_m)}{(r+\delta_w)((\mu_w+\mu_m)(r+\delta_w+\delta_m)+\alpha(\gamma)\mu_m)} \tag{2.4.28}$$

where $\gamma = \frac{f_{00}+f_{w0}}{\mu_w+\mu_m}$. The flow equations for this case are:

$$\gamma\alpha(\gamma)\frac{f_{00}}{f_{00}+f_{w0}}\mu_w = \delta_w f_{w0} + \delta_w f_{wm} \tag{2.4.29}$$

$$\alpha(\gamma)\frac{\mu_m}{\mu_m+\mu_w} f_{w0} = (\delta_w+\delta_m) f_{wm} \tag{2.4.30}$$

These equations, together with market clearing conditions, $L_w = \mu_w + f_{w0} + f_{wm}$and $L_m = \mu_m + f_{wm}$ determine the equilibrium values of μ_w, μ_m, f_{00}, f_{w0} and f_{wm}.

2.4.5.5 $\tau_{w0} = 0$, $\tau_{m0} = 1$,$\tau_{wm} = 1$,$\tau_{mw} = 0$

This case is symmetric in parameters to the above case.

2.4.5.6 $\tau_{w0} = 1$,$\tau_{m0} = 1$,$\tau_{wm} = 0$,$\tau_{mw} = 1$

In this case, firms without any type search for both workers and managers. If its first successful match is with a worker, then it continues to search for a manager. This implies that $\alpha(\gamma)[V_{wm} - V_{w0}] > c_m$. On the other hand, if its first successful match is with a manager, then it stops searching for a worker. This implies that $\alpha(\gamma)[V_{wm} - V_{m0}] < c_w$. This means that expected surplus of searching for a worker is less than the search cost of the worker after firms match with a manager.

2.4.5.7 $\tau_{w0} = 1, \tau_{m0} = 1, \tau_{wm} = 1, \tau_{mw} = 0$

In this case, firms without any type of labor search for both workers and managers. In this case if firms' first successful match is with a manager, they continue to search for a worker. This implies that $\alpha(\gamma)[V_{wm} - V_{m0}] > c_w$. On the other hand, if their first successful match is with a manager, then they stop searching for a worker. This implies that $\alpha(\gamma)[V_{wm} - V_{w0}] < c_m$.

2.4.5.8 $\tau_{w0} = 1, \tau_{m0} = 1, \tau_{wm} = 1, \tau_{mw} = 1$

This is the case where firms search for both workers and managers, form a match with one they meet first. After their first successful match, they continue the search process until the second type is also matched. In this case, $V_{w0} > V_{00}$, $V_{m0} > V_{00}$, $E[V_{wm} - V_{w0}] > c_m$ and $E[V_{wm} - V_{m0}] > c_w$. Invoking the free entry condition and solving the steady state flow Bellman equations, the free entry condition gives:

$$
\begin{aligned}
c_{wm} \quad = \quad & \alpha(\gamma)\frac{\mu_w}{\mu_w + \mu_m}\frac{((r+\delta_m)(r+\delta_m+\delta_w+\alpha(\gamma)) - \alpha(\gamma) r \frac{\mu_m}{\mu_w+\mu_m})(P_{w0} - b_w - c_m)}{(r+\delta_w)(r+\delta_m)(r+\delta_w+\delta_m+\alpha(\gamma)) + \alpha(\gamma)^2 r \frac{\mu_w\mu_m}{(\mu_w+\mu_m)^2}} \\
& +\alpha(\gamma)\frac{\mu_m}{\mu_w + \mu_m}\frac{((r+\delta_w)(r+\delta_m+\delta_w+\alpha(\gamma)) - \alpha(\gamma) r \frac{\mu_w}{\mu_w+\mu_m})(P_{m0} - b_m - c_w)}{(r+\delta_w)(r+\delta_m)(r+\delta_w+\delta_m+\alpha(\gamma)) + \alpha(\gamma)^2 r \frac{\mu_w\mu_m}{(\mu_w+\mu_m)^2}} \\
& +\alpha(\gamma)^2\frac{\mu_m\mu_w}{(\mu_w + \mu_m)^2}\frac{(r+\delta_m+\delta_w)(2r+\delta_m+\delta_w+\alpha(\gamma))(P_{wm} - b_m - b_w)}{(r+\delta_w)(r+\delta_m)(r+\delta_w+\delta_m+\alpha(\gamma)) + \alpha(\gamma)^2 r \frac{\mu_w\mu_m}{(\mu_w+\mu_m)^2}}
\end{aligned}
$$

In this scenario, $\gamma = \frac{f_{00}+f_{m0}+f_{w0}}{\mu_w+\mu_m}$. The first term on the right had side of the above equation is the expected value of being matched with a worker. Second term is the expected value of being matched with a manager. The third term is expected value of getting matched with second type, conditional on getting matched with the first type. Free entry condition requires the expected gain be equalized with the flow costs of search. The flow equations are:

$$\alpha(\gamma)\frac{\mu_w}{\mu_w+\mu_m}(f_{00}+f_{m0})=\delta_w(f_{w0}+f_{wm}) \tag{2.4.31}$$

$$\alpha(\gamma)\frac{\mu_m}{\mu_w+\mu_m}(f_{00}+f_{w0})=\delta_m(f_{m0}+f_{wm}) \tag{2.4.32}$$

$$\frac{\alpha(\gamma)}{\mu_w+\mu_m}(\mu_w f_{m0}+\mu_m f_{w0})=(\delta_w+\delta_m)f_{wm} \tag{2.4.33}$$

These equations, together with market clearing conditions, $L_w = \mu_w + f_{w0} + f_{wm}$ and $L_m = \mu_m + f_{m0} + f_{wm}$ determine the equilibrium values for μ_w, μ_m, f_{00}, f_{w0}, f_{m0}, and f_{wm}.

2.4.6 Finding Deviations and Regimes

Checking for each of the deviations is a complicated and cumbersome process. Deviations need to be checked for a regime to all other regimes. Since there are 8 possible regimes, this results in checking $\frac{8*9}{2} = 36$ deviations. Another way to identify the regime is to check the measure of firms in each regime for a particular productivity level. This comes as a direct result of the following lemma:

Lemma:

Different regimes at given productivity levels can be ranked in order of measure of firms satisfying the free entry condition. The regime with highest number of firms is followed in equilibrium.

Proof:

Suppose there are two regimes A and B , and the measure of state 00 firms are f_A and f_B respectively. Let $g : R^2 \to R$ be a function that maps the value functions of the firms in state

00 and the measure of f_{00} firms to a real number, the rents or profits of the firms. $g(V, f)$ is increasing in V (value function) and decreasing in f (measure of firms). Notice depending on regime, V might also be dependent on f and if so, it is decreasing in f . Using the free entry condition, we would have equilibrium if $g(V, f) = 0$. Suppose $g(V_A, f_A) = 0$ for regime A . If $g(V_B, f_A) > 0$ then existing firms would switch regime to V_B as there are positive profits to be made and new firms would enter the search market until profits go down to 0, and equilibrium with free entey $g(V_B, f_B) = 0$ is restored with $f_B > f_A$. On the other hand, if $g(V_B, f_A) < 0$ then firms do not have incentive to switch regimes, as equilibrium would only be restored if some firms leave the search market, that is with $f_B < f_A$.

2.5 Feasible Productivity Ranges - A quantitative Assessment

Please see Figure **??** for the feasible productivity ranges for the cases discussed above, with P_{w0} (on y-axis) and P_{m0} (on x-axis), both normalized by P_{wm}. It can be observed that for, say significantly higher productivity level of P_{w0} , firms do not search for managers: they only search for workers. This happens because the "complement" effects of searching for a manager are very small and coupled with the fact that b_m is significant, firms do not search for the managers. The managers are also not substitutes for workers, as their net productivity is not high enough for the firms to search for them instead of the workers. The upper left/upper portion of the graph refers to this regime.[11] If P_{w0} is not too high, then managers are complement to workers, provided P_{m0} is also low enough, as workers and managers together can produce more than they can produce alone. In such a situation, firms may search for

[11] Note that size of the region depends on b_w(and other parameters).

both, either sequentially or both at the same time. Sequential search here means that firms first search for a particular type, and if other type is found during search, the firms do not make the match successful and instead keep on searching for the type they were searching. Upon successful match with the desired type, firms search for the other type. The bottom left portion refers to these regimes. If the net productivity of workers is relatively higher than those of managers, then the firms would first search for a worker, and upon a successful match, would search for a manager, and vice versa for the case where net productivity of managers is relatively higher than those of workers. For the case where net productivity levels of both are low and close to each other, the firms do not search sequentially, and instead search for both. Firms hire the type that meets them first, and then firms continue to search for the other type. If the net productivity of managers is very high relative to the workers' net productivity, then firms search for a manger only. In this case, workers are neither a complement nor a substitute. The lower left region on the graph corresponds to this regime. If the productivity of both managers and workers is very high then both are substitutes of each other. Firm search for both, but stop search and exit the search market once one of them has been matched with them. Regimes in which firms search for both, and stop search when one particular type is matched but continue to search if the other type is matched (cases $\tau_{w0} = 1, \tau_{m0} = 1, \tau_{wm} = 0, \tau_{mw} = 1$ and $\tau_{w0} = 1, \tau_{m0} = 1, \tau_{wm} = 1, \tau_{mw} = 0$) are not followed in equilibrium.

The measure of firms in state f_{00} is continuous in net productivity levels (though it might not be differentiable at points where regimes change), even if a slight change in net productivity results in a regime switch (see proof in appendix). However, the measure of all other states of firms is continuous within a regime with changes in net productivity. However, there is a discontinuity in μ_w, μ_m with the regime shifts. There are also discontinuities in

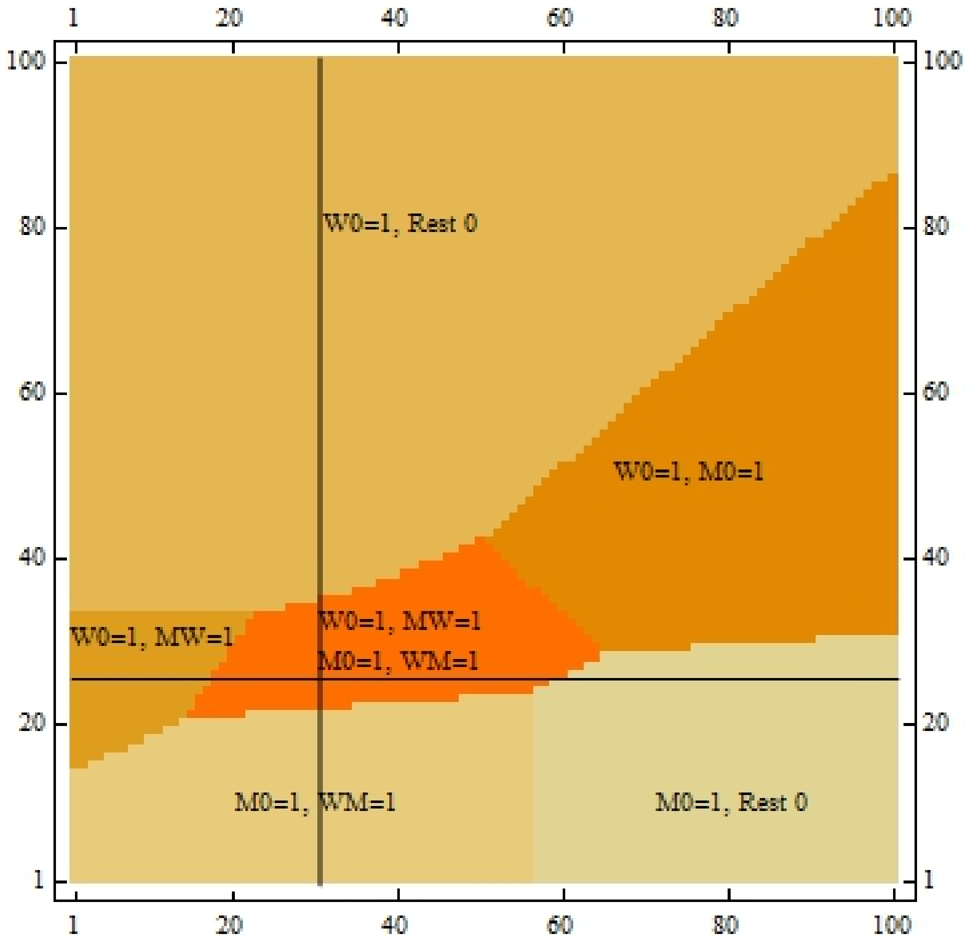


Figure 2.5.1: Feasible Productivity Ranges

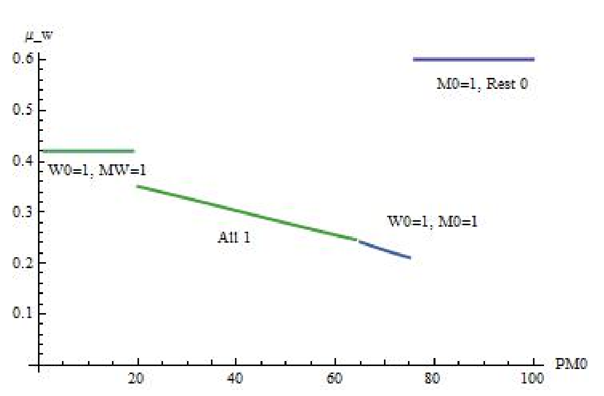
f_{w0}, f_{m0}, f_{wm} with regime shifts.

These discontinuities have an important result in that they can explain pro-cyclical unemployment. Please see figure 3 to see plots of $\mu_w$and μ_m for a cross section where P_{w0} is held fixed and P_{m0} varies from 0 to P_{wm}. It can be observed that unemployment of managers has discontinuities as the regimes change. Also, unemployment is non-monotonic. As the regimes change, unemployment of managers jumps up and then decreases within a regime. Consider a cross section in the above plot in which P_{w0} is held fixed and P_{m0} varies. If the manager productivity is low, search is sequential (This cross section is taken from the point where P_{w0} is sufficiently high). In this regime, unemployment of managers is high. As the manager productivity crosses a certain threshold, there is a regime shift, where now all $\tau_{ij}' s$ are 1, meaning firms search for both workers and managers, and in any order. Because of this regime shift, now firms are also searching for managers, and hence measure of unemployed managers goes down. Unemployed managers are being searched by two types of firms (f_{00} and f_{w0}) in this case. This causes a discontinuity in the measure of unemployment of managers. Once manager net productivity becomes sufficiently high, there is another regime shift. Now, because of high productivity of both managers and workers, they are substitutes. Firms now search for both types, but discontinue their search after their first match. Therefore, despite an increase in manager net productivity in the neighborhood of regime shift, the manager unemployment goes up. As the manager productivity increases within the regime, unemployment goes down. With further increase in manager net productivity, it becomes closer to the net productivity of both managers and workers combined. Therefore, firms now search for managers only, as now the workers are neither complements nor substitutes for managers. Manager unemployment again goes up as now all the workers are unemployed, and the probability that the firms match with the workers goes up resulting in the complement probability, the firms matching with the managers, going down. The unemployment of managers decreases as the net productivity of managers goes up.

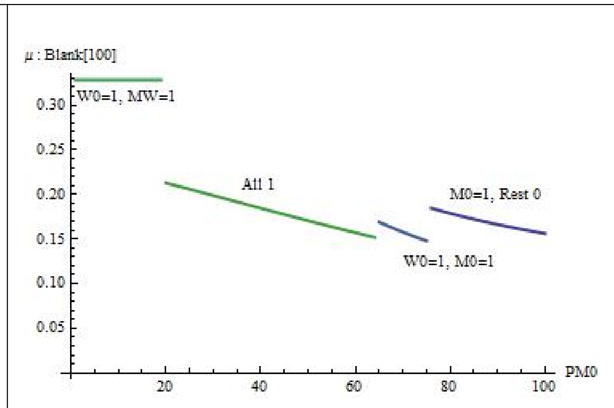
The evolution of unemployment of workers with the change in net productivity is more interesting. Notice that the plot for worker unemployment is plotted for the case where worker productivity stays constant. Starting from first regime (lowest manager productivity), the worker unemployment is very high. There are two reasons for that (refer to free entry condition for the regime). First, the worker productivity is low, and hence there are not many firms in the market. With lesser firms, the match rate is low and hence higher worker unemployment. The second reason is that even though managers have high complement effects, but the match rate for managers is even lower due to lesser firms in status f_{w0}, resulting in lower expected payoff. As the manager productivity increases, the regime shifts to one where the firms search for both types and in any order. This results in lower worker unemployment, as now workers are now being hired (matches being successful) by firms in state f_{m0} , in addition to the firms in state f_{00}. Notice workers unemployment decreases even though the worker net productivity is not increasing. As the manager net productivity further increases, we reach a regime where managers become substitutes for workers, and hence firms search for both types. Since the measure of searching firms is increasing, the worker unemployment further decreases. With the further increase in manager productivity, we reach a point where workers are not longer complements or substitutes. Therefore, firms stop searching for workers and all the workers are now unemployed.

Measure of firms evolve in a similar fashion. See figure 4 for the evolution of the measure of firms in various states, and figure 5 for $V_{ij}'s$ i.e. the values in various states.

Figures related to welfare (defined here as the aggregate production, net of wage bills,), average welfare (defined here as aggregate production divided by the total number of firms) and value of search (defined as the aggregate of value functions) are shown in figure 4. From these figures, it can be observed that Welfare is also not necessarily monotone, and decreases as regimes shift.

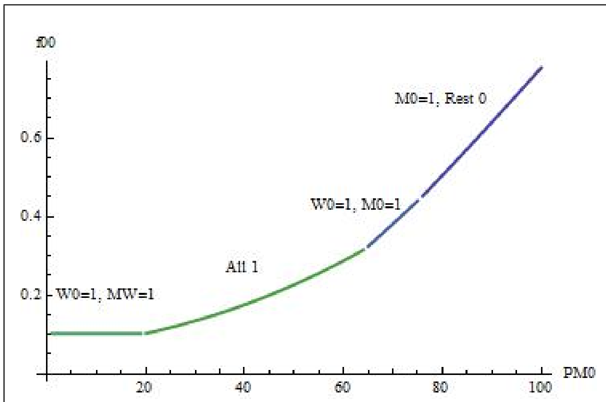


(a) μ_w

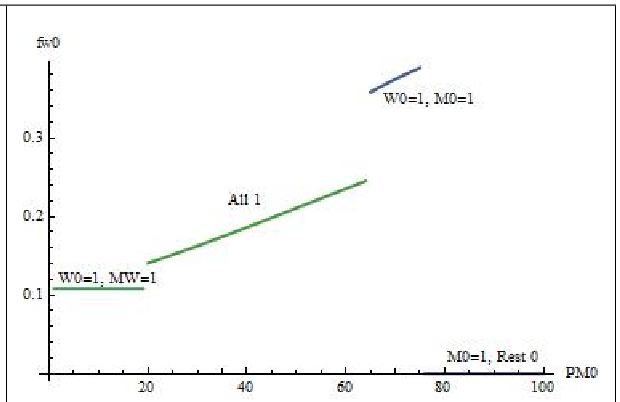


(b) μ_m

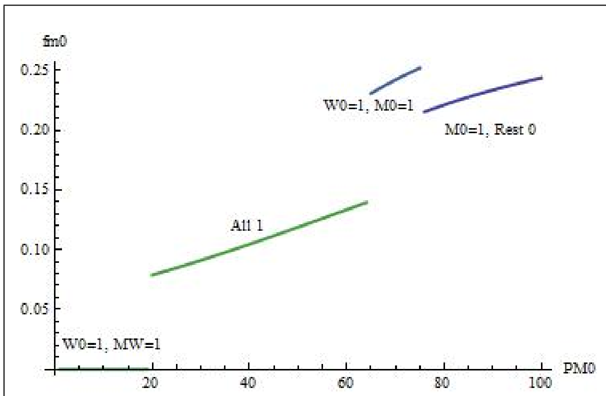
Figure 2.5.2: Unemployment



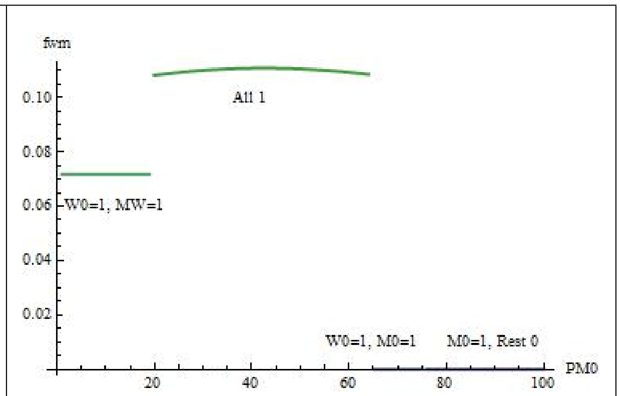
(a) Firms in State f_{00}



(b) Firms in State f_{w0}



(c) Firms in State f_{m0}



(d) Firms in State f_{wm}

Figure 2.5.3: Firms

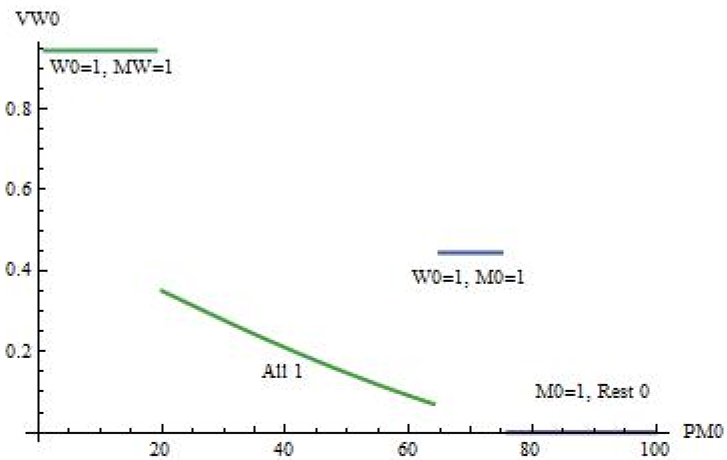


Figure 2.5.4: Value in State V_{w0}

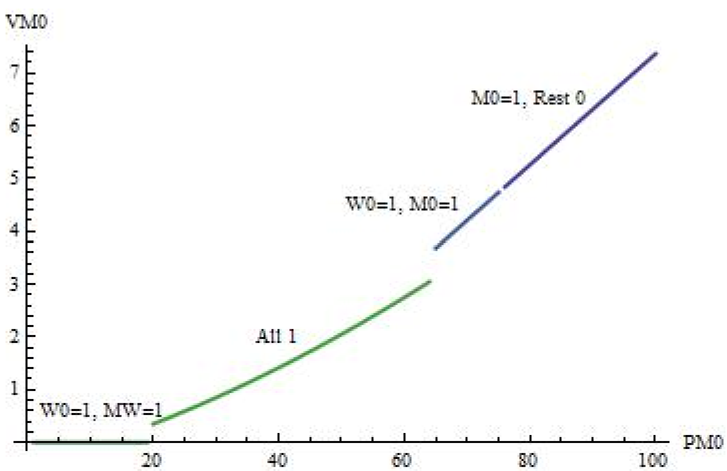


Figure 2.5.5: Value in State V_{m0}

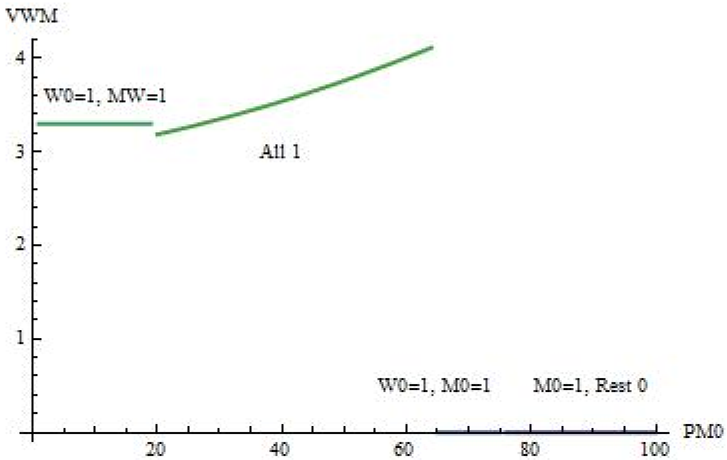
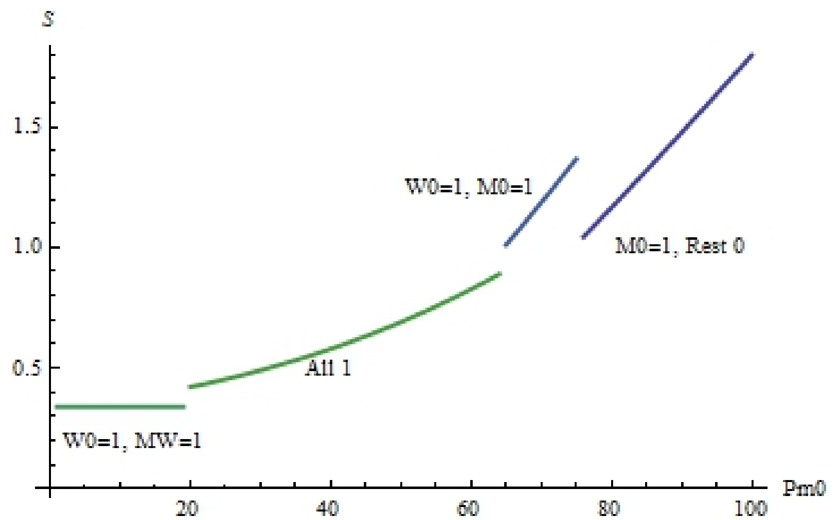


Figure 2.5.6: Value in State V_{wm}

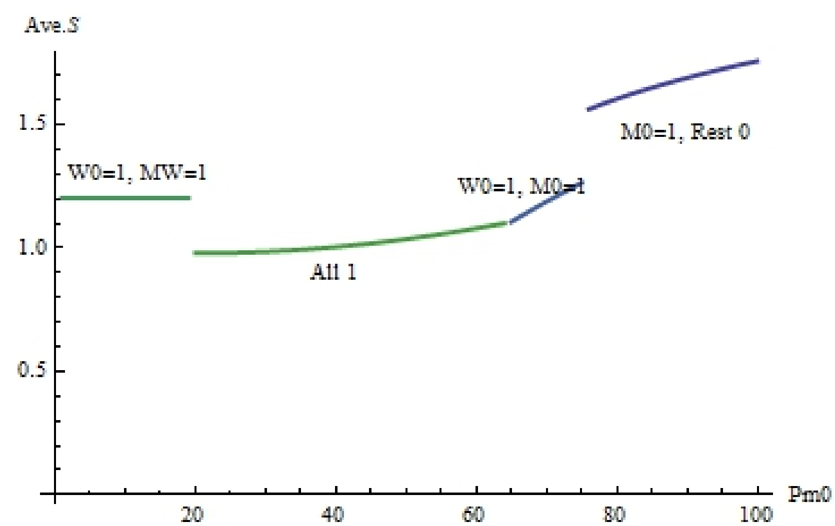
2.6 Conclusion

This paper has studied the job hiring decisions of a firm that needs two types of labor. As found in the paper, depending on the productivity of each type, they can be substitutes or complements of each other. This model can help explain cyclical unemployment levels, and can be used as a framework for many areas like real estate, theory of team formation and finance. It was also found that unemployment levels of a particular type may respond to changes in the output of the other type, even if its own productivity does not change.

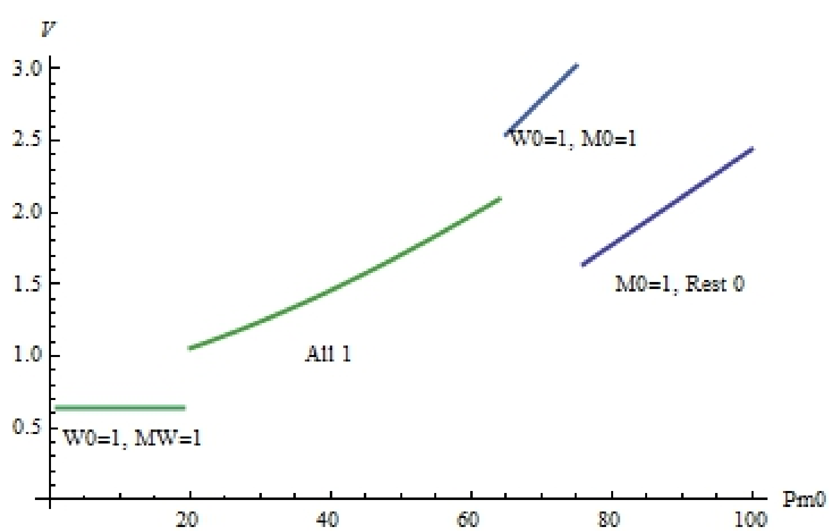
The wage mechanism that was assumed was that firms make a take it or leave it offer to the workers or managers, which meant that the entire surplus went to the firms. Alternate wage determination mechanisms will be explored in future projects. One of the mechanisms may be Nash Bargaining solution. One of the difficulty in this solution is that in this model there are three parties. Nash Bargaining Solution cannot be used to determine wages when three parties are involved. One way to counter this challenge is to model the bargaining between



(a) Total Surplus



(b) Average (per firm) Surplus



(c) Value of Search

Figure 2.5.7: Welfare

a firm that has no labor type that matches with a worker (manager). These two parties bargain for wages, and once the match is successful, the firm makes a wage commitment to the worker (manager) i.e. the wage of the worker (manager) would not be readjusted if subsequently the firms makes a successful match with a manager (worker). Therefore, when the firm with a worker (manager) meets a manager (worker), the firm takes the wage of its existing worker (manager) as given and bargains with the manager (worker). This is just a first step in this direction. There is still a lot of work that can be done. A bargaining solution with non zero bargaining power on both sides can be included in the model so analyze the effects of bargaining frictions. In addition to these wage determination mechanisms, one can also make wages endogenous by introducing a neo-classical production function.

2.7 Appendix

2.7.1 Sketch of Proof of Existence

- Proposition: The equilibrium for all the regimes discussed above exists
- Sketch of proof:
  - $\mu_w$and f_{w0} bounded above by L_w and below by 0
  - $\mu_m$and f_{m0} bounded above by L_m and below by 0
  - f_{wm} bounded above by $min(L_w, L_m)$ and below by 0
  - f_{00} bounded above by the free-entry condition and below by 0
  - Use Brouwer's fixed point theorem

2.7.2 Comparative Statics

2.7.2.1 No Segmentation

2.7.2.2 $\tau_{w0} = 1$, $\tau_{0m} = 0$, $\tau_{wm} = 0$, $\tau_{mw} = 0$

Define $DEN = \alpha^2(P_{w0} - b_w) - \alpha c_w(r + \delta_w) - \alpha'(\delta_w(P_{w0} - b_w) + c_w\gamma(r + \delta_w))$

$$\frac{\partial \mu_w}{\partial P_{w0}} = \frac{-\alpha\mu_w(\alpha + \alpha'\gamma)}{DEN} < 0$$

$$\frac{\partial f_{00}}{\partial P_{w0}} = \frac{\alpha\mu_w(\delta_w - \alpha'\gamma)}{DEN} > 0$$

$$\frac{\partial \mu_w}{\partial b_w} = \frac{\alpha\mu_w(\alpha + \alpha'\gamma)}{DEN} > 0$$

$$\frac{\partial f_{00}}{\partial b_w} = \frac{-\alpha\mu_w(\delta_w - \alpha'\gamma)}{DEN} < 0$$

$$\frac{\partial \mu_w}{\partial c_w} = \frac{(r+\delta_w)(\mu_w + L_m)(\alpha + \alpha'\gamma)}{DEN} > 0$$

$$\frac{\partial f_{00}}{\partial c_w} = \frac{-(r+\delta_w)(\mu_w + L_m)(\delta_w - \alpha'\gamma)}{DEN} < 0$$

$$\frac{\partial \mu_w}{\partial L_w} = \frac{-\alpha'\delta_w(P_{w0} - b_w)}{DEN} > 0$$

$$\frac{\partial f_{00}}{\partial L_w} = \frac{\delta_w((\alpha + \alpha'\gamma)(P_{w0} - b_w) - c_w(r+\delta_w))}{DEN} > 0$$

$$\frac{\partial \mu_w}{\partial L_m} = \frac{c_w(r+\delta_w)(\alpha + \alpha'\gamma)}{DEN} > 0$$

$$\frac{\partial f_{00}}{\partial L_m} = \frac{-c_w(r+\delta_w)(\delta_w - \alpha'\gamma^2)}{DEN} < 0$$

$$\frac{\partial \mu_w}{\partial \delta_w} = \frac{-\alpha'(L_w - \mu_w)(P_{w0} - b_w) + (\alpha + \alpha'\gamma)(L_m + \mu_w)c_w}{DEN} > 0$$

$$\frac{\partial f_{00}}{\partial \delta_w} = \frac{(L_w - \mu_w)((\alpha - \alpha'\gamma)(P_{w0} - b_w) - c_w(r+\delta_w)) - (\delta_w - \alpha'\gamma^2)(L_m + \mu_w)c_w}{DEN} >< 0$$

$$\frac{\partial \mu_w}{\partial \delta_w} = \frac{(\alpha + \alpha'\gamma)(L_m + \mu_w)c_w}{DEN} > 0$$

$$\frac{\partial f_{00}}{\partial \delta_w} = \frac{-c_w(r + \delta_w)) - (\delta_w - \alpha'\gamma^2)(L_m + \mu_w)c_w}{DEN} < 0$$

2.7.2.3 $\tau_{w0} = 1$, $\tau_{0m} = 1$, $\tau_{wm} = 0$, $\tau_{mw} = 0$

Define $M = \mu_w + \mu_m$, and notice that denominator is less than 0, and $s = |\frac{\alpha'\gamma}{\alpha}| < 1$.

Also define

$$\begin{aligned} DEN &= [\alpha\gamma + \delta_w]\,[c_{wm} f_{00} - \alpha\gamma M V_{M0}] \\ &+ [\alpha\gamma + \delta_m]\,[c_{wm} f_{00} - \alpha\gamma M V_{W0}] \\ &- s c_{wm} M^2\,[\alpha\gamma + \delta_w]\,[\alpha\gamma + \delta_m] \end{aligned}$$

$$\begin{aligned} \tfrac{\partial f_{00}}{\partial \delta_w} &= \{\alpha\gamma\mu_w M \tfrac{P_{w0} - b_w}{(r+\delta_w)^2}\left([\alpha\gamma + \delta_w]\,[\alpha\gamma + \delta_m]\,M^2 - f_{00}\,[2\alpha\gamma + \delta_w + \delta_m]\right) \\ &- M(L_w - \mu_w)\left(M\,[\alpha\gamma + \delta_m]\,[\alpha\gamma M V_{w0} - (1-s)c_{wm} f_{00}] + \alpha\gamma f_{00}\,[V_{m0} - V_{w0}]\right)\} \\ &\quad /(M * DEN) \\ &\quad < 0 \end{aligned}$$

$$\begin{aligned} \tfrac{\partial \mu_w}{\partial \delta_w} &= \{-\alpha\gamma\mu_w M \tfrac{P_{w0} - b_w}{(r+\delta_w)^2}\,[\alpha\gamma + \delta_w] \\ &+ [L_w - \mu_w]\,[f_{00} c_{wm} - s(\alpha\gamma + \delta_m) M^2 c_{wm} - \alpha\gamma M V_{m0}]\}/DEN \\ &\quad \lesseqgtr 0 \end{aligned}$$

$$\begin{aligned}
\frac{\partial \mu_m}{\partial \delta_w} &= \{-\alpha\gamma\mu_w M \tfrac{P_{w0}-b_w}{(r+\delta_w)^2}\left[\alpha\gamma+\delta_w\right] \\
&+ \left[L_w-\mu_w\right]\left[\alpha\gamma M V_{w0} - f_{00}c_{wm}\right]\}/DEN \\
&\gtreqless 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial f_{00}}{\partial P_{w0}} &= -\tfrac{\alpha\gamma\mu_w}{(r+\delta_w)}\left(\left[\alpha\gamma+\delta_w\right]\left[\alpha\gamma+\delta_m\right]M^2 - f_{00}\left[2\alpha\gamma+\delta_w+\delta_m\right]\right) \\
&/DEN \\
&> 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \mu_w}{\partial P_{w0}} &= \tfrac{\alpha\gamma\mu_w M}{(r+\delta_w)}\left[\alpha\gamma+\delta_m\right] \\
&/DEN \\
&< 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \mu_m}{\partial P_{w0}} &= \tfrac{\alpha\gamma\mu_w M}{(r+\delta_w)}\left[\alpha\gamma+\delta_w\right] \\
&/DEN \\
&< 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial f_{00}}{\partial b_w} &= \tfrac{\alpha\gamma\mu_w}{(r+\delta_w)}\left(\left[\alpha\gamma+\delta_w\right]\left[\alpha\gamma+\delta_m\right]M^2 - f_{00}\left[2\alpha\gamma+\delta_w+\delta_m\right]\right) \\
&/(M * DEN) \\
&< 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \mu_w}{\partial b_w} &= -\tfrac{\alpha\gamma\mu_w M}{(r+\delta_w)}\left[\alpha\gamma+\delta_m\right] \\
&/DEN \\
&> 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \mu_m}{\partial b_w} &= -\tfrac{\alpha\gamma\mu_w M}{(r+\delta_w)}\left[\alpha\gamma+\delta_w\right] \\
&\quad /DEN \\
&\quad > 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial f_{00}}{\partial r} &= \alpha\gamma M\left(\tfrac{\mu_w}{r+\delta_w}V_{w0}+\tfrac{\mu_m}{r+\delta_m}V_{m0}\right) \\
&* \left(\left[\alpha\gamma+\delta_w\right]\left[\alpha\gamma+\delta_m\right]M^2-f_{00}\left[2\alpha\gamma+\delta_w+\delta_m\right]\right) \\
&\quad /(M*DEN) \\
&\quad < 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \mu_w}{\partial r} &= -\alpha\gamma M\left(\tfrac{\mu_w}{r+\delta_w}V_{w0}+\tfrac{\mu_m}{r+\delta_m}V_{m0}\right)\left[\alpha\gamma+\delta_m\right] \\
&\quad /DEN \\
&\quad > 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \mu_m}{\partial r} &= -\alpha\gamma M\left(\tfrac{\mu_w}{r+\delta_w}V_{w0}+\tfrac{\mu_m}{r+\delta_m}V_{m0}\right)\left[\alpha\gamma+\delta_w\right] \\
&\quad /DEN \\
&\quad > 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial f_{00}}{\partial c_{wm}} &= f_{00}\left(\left[\alpha\gamma+\delta_w\right]\left[\alpha\gamma+\delta_m\right]M^2-f_{00}\left[2\alpha\gamma+\delta_w+\delta_m\right]\right) \\
&\quad /DEN \\
&\quad < 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \mu_w}{\partial c_{wm}} &= -f_{00}M\left[\alpha\gamma+\delta_m\right] \\
&\quad /DEN \\
&\quad > 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \mu_m}{\partial c_{wm}} &= -f_{00}M\left[\alpha\gamma + \delta_w\right] \\
&\quad /DEN \\
&\quad > 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial f_{00}}{\partial L_w} &= -\delta_w \left(M\left[\alpha\gamma + \delta_m\right]\left[\alpha\gamma M V_{w0} - (1-s)c_{wm}f_{00}\right] + \alpha\gamma f_{00}\left[V_{m0} - V_{w0}\right]\right) \\
&\quad /DEN \\
&\quad \gtreqless 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \mu_w}{\partial L_w} &= \delta_w \left[f_{00}c_{wm} - s(\alpha\gamma + \delta_m)M^2 c_{wm} - \alpha\gamma M V_{m0}\right] \\
&\quad /DEN \\
&\quad \gtreqless 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \mu_m}{\partial L_w} &= \delta_w \left[\alpha\gamma M V_{w0} - f_{00}c_{wm}\right] \\
&\quad /DEN \\
&\quad \gtreqless 0
\end{aligned}$$

2.7.3 Deviations

Deviations need to be checked for all regimes. Deviations are checked by evaluating the value functions at the deviating regimes, and using the current regime employment/unemployment levels:

2.7.3.1 $\tau_{w0} = 1$, $\tau_{0m} = 0$, $\tau_{wm} = 0$, $\tau_{mw} = 0$

- $\tau_{w0} = 0,\ \tau_{m0} = 1,\ \tau_{wm} = 0,\ \tau_{mw} = 0$

$$\alpha \frac{\mu_m}{\mu_w + \mu_m} V_{m0} > c_m$$

Notice these value functions are evaluated at the regime to which deviations are being checked, whereas the unemployment/employment are the levels for the current regime.

- $\tau_{w0} = 0,\ \tau_{m0} = 1,\ \tau_{wm} = 1,\ \tau_{mw} = 0$

$$\alpha \frac{\mu_m}{\mu_w + \mu_m} V_{m0} > c_m$$

- $\tau_{w0} = 1,\ \tau_{m0} = 0,\ \tau_{wm} = 0,\ \tau_{mw} = 1$

$$\alpha \frac{\mu_w}{\mu_w + \mu_m} V_{w0} > c_w$$

- $\tau_{w0} = 1,\ \tau_{m0} = 1,\ \tau_{wm} = 0,\ \tau_{mw} = 0$, $\tau_{w0} = 1,\ \tau_{m0} = 1,\ \tau_{wm} = 0,\ \tau_{mw} = 1$, $\tau_{w0} = 1,\ \tau_{m0} = 1,\ \tau_{wm} = 1,\ \tau_{mw} = 0$, $\tau_{w0} = 1,\ \tau_{m0} = 1,\ \tau_{wm} = 1,\ \tau_{mw} = 1$

$$\alpha \frac{\mu_w}{\mu_w + \mu_m} V_{w0} + \alpha \frac{\mu_m}{\mu_w + \mu_m} V_{m0} > c_{wm}$$

2.7.3.2 $\tau_{w0} = 1$, $\tau_{0m} = 1$, $\tau_{wm} = 0$, $\tau_{mw} = 0$

- $\tau_{w0} = 0,\ \tau_{m0} = 1,\ \tau_{wm} = 0,\ \tau_{mw} = 0$

$$\alpha \frac{\mu_m}{\mu_w + \mu_m} V_{m0} > c_m$$

- $\tau_{w0} = 0,\ \tau_{m0} = 1,\ \tau_{wm} = 1,\ \tau_{mw} = 0$

$$\alpha \frac{\mu_m}{\mu_w + \mu_m} V_{m0} > c_m$$

- $\tau_{w0} = 1,\ \tau_{m0} = 0,\ \tau_{wm} = 0,\ \tau_{mw} = 0$

$$\alpha \frac{\mu_w}{\mu_w + \mu_m} V_{w0} > c_w$$

- $\tau_{w0} = 1,\ \tau_{m0} = 0,\ \tau_{wm} = 0,\ \tau_{mw} = 1$

$$\alpha \frac{\mu_w}{\mu_w + \mu_m} V_{w0} > c_w$$

- $\tau_{w0} = 1,\ \tau_{m0} = 1,\ \tau_{wm} = 0,\ \tau_{mw} = 1$, $\tau_{w0} = 1,\ \tau_{m0} = 1,\ \tau_{wm} = 1,\ \tau_{mw} = 0,$ $\tau_{w0} = 1,\ \tau_{m0} = 1,\ \tau_{wm} = 1,\ \tau_{mw} = 1$

$$\alpha \frac{\mu_w}{\mu_w + \mu_m} V_{w0} + \alpha \frac{\mu_m}{\mu_w + \mu_m} V_{m0} > c_{wm}$$

Bibliography

[1] Wage/tenure contracts with heterogeneous firms. *Journal of Economic Theory*, 145(4):1408 – 1435, 2010.

[2] Daron Acemoglu and Robert Shimer. Holdups and efficiency with search frictions. *International Economic Review*, 40(4):827–849, 1999.

[3] S. Boragan Aruoba, Christopher J. Waller, and Randall Wright. Money and capital. *Journal of Monetary Economics*, 58(2):98–116, 2011.

[4] Gary S. Becker. A theory of marriage: Part i. *Journal of Political Economy*, 81(4):813, 1973.

[5] Gary S. Becker. A theory of marriage: Part ii. *Journal of Political Economy*, 82(2):11, 1974.

[6] Ken Burdett and Melvyn Coles. Equilibrium wage-tenure contracts. *Econometrica*, 71(5):pp. 1377–1404, 2003.

[7] Ken Burdett and Melvyn G. Coles. Marriage and class. *The Quarterly Journal of Economics*, 112(1):pp. 141–168, 1997.

[8] Kenneth Burdett and David A Malueg. The theory of search for several goods. *Journal of Economic Theory*, 24(3):362 – 376, 1981.

[9] Peter A. Diamond. Wage determination and efficiency in search equilibrium. *The Review of Economic Studies*, 49(2):pp. 217–227, 1982.

[10] R. Wright G. Rocheteau. Money in search equilibrium, in competitive equilibrium and in competitive search equilibrium. *Econometrica*, 73:175–202, 2004.

[11] Chao Gu, Fabrizio Mattesini, Cyril Monnet, and Randall Wright. Banking: A new monetarist approach. *Review of Economic Studies*, 80(2):636 – 662, 2013.

[12] Bulent Guler, Fatih Guvenen, and Giovanni L. Violante. Joint-search theory: New opportunities and new frictions. *Journal of Monetary Economics*, 59(4):352 – 369, 2012.

[13] Ping He, Lixin Huang, and Randall Wright. Money, banking, and monetary policy. *Journal of Monetary Economics*, 55(6):1013–1024, 2008.

[14] Arthur Hosios. On the efficiency of matching and related models of search and unemployment. *Review of Economic Studies*, 57:279–298, 1990.

[15] Wright R. Kiyotaki, N. On money as a medium of exchange. *Journal of Political Economy*, 97:927–954, 1989.

[16] Wright R. Kiyotaki, N. A search-theoretic approach to monetary economics. *American Economic Review*, 83:63–77, 1993.

[17] N. Kocherlakota. Money is memory. *J. Econ. Theory*, 81:232–251, 1998.

[18] Rocheteau G. Lagos, R. Money and capital as competing media of exchange. *Journal of Economic Theory*, 142:247–258, 2008.

[19] Rasmus Lentz and Dale T. Mortensen. Labor market models of worker and firm heterogeneity. *Annual Review of Economics*, 2(1):577–602, 2010.

[20] Benjamin Lester, Andrew Postlewaite, and Randall Wright. Information, liquidity, asset prices, and monetary policy. *Review of Economic Studies*, 79(3):1209 – 1238, 2012.

[21] Postlewaite A. Wright R. Lester, B. Information, liquidity, asset prices and monetary policy. 2010.

[22] X. Huangfu M. Faig. Competitive search equilibrium in monetary economies. *J. Econ. Theory*, 136(1):709–718, 2007.

[23] Dale T. Mortensen and Christopher A. Pissarides. Job creation and job destruction in the theory of unemployment. *The Review of Economic Studies*, 61(3):pp. 397–415, 1994.

[24] Wright R. Mortensen, D.T. Competitive pricing and efficiency in search equilibrium. *International Economic Review*, 43:1–20, 2002.

[25] Derek Neal. Industry-specific human capital: Evidence from displaced workers. *Journal of Labor Economics*, 13(4):653, 1995.

[26] Derek Neal. The complexity of job mobility among young men. *Journal of Labor Economics*, 17(2):237, 1999.

[27] Rocheteau G. Nosal, E. *Money, Payments, and Liquidity*. MIT Press, 2010.

[28] He Ping, Huang Lixin, and Wright Randall. Money and banking in search equilibrium. *International Economic Review*, 46(2):637 – 670, 2005.

[29] Christopher A. Pissarides. *Equilibrium Unemployment Theory*. Oxford Blackwell, 1990.

[30] Crampton G R. Labour-market search and urban residential structure. *Environment and Planning A*, 29(6):989–1002, 1997.

[31] N. Wallace R. Cavalcanti. Inside and outside money as alternative media of exchange. *J. Money, Credit, Banking*, 31(2):443–457, 1999.

[32] N. Wallace R. Cavalcanti. A model of private bank-note issue. *Rev. Econ. Dynam*, 2:104–136, 1999.

[33] G. Rocheteau R. Lagos. Inflation, output and welfare. *Int. Econ. Rev.*, 46:495–522, 2005.

[34] R. Wright R. Lagos. A unified framework for monetary theory and policy analysis. *J. Polit. Economy*, 113:463–484, 2005.

[35] Richard Rogerson, Robert Shimer, and Randall Wright. Search-theoretic models of the labor market: A survey. *Journal of Economic Literature*, 43(4):959 – 988, 2005.

[36] S. Williamson S. Aiyagari. Money and dynamic credit arrangements with private information. *J. Econ. Theory*, 91:248–279, 2000.

[37] S. Shi. Money and prices: a model of search and bargaining. *Journal of Economic Theory*, 67:467–496, 1995.

[38] S. Shi. Credit and money in a search model with divisible commodities. *Rev. Econ. Stud.*, 63:627–652, 1996.

[39] S. Shi. A divisible search model of fiat money. *Econometrica*, 65:75–102, 1997.

[40] S. Shi. A divisible search model of fiat money. *Econometrica*, 65:75–102, 1997.

[41] S. Shi. Search, inflation and capital accumulation. *Journal of Monetary Economics*, 44:81–104, 1999.

[42] Hu Tai-wei, John Kennan, and Neil Wallace. Coalition-proof trade and the friedman rule in the lagos-wright model. *Journal of Political Economy*, 117(1):116–137, 2009.

[43] Wright R. Trejos, A. Search, bargaining, money and prices. *Journal of Political Economy*, 103:118–141, 1995.

[44] Wright R. Williamson, S. Monetarist economics: Methods. *Federal Reserve Bank of St.Louis Review*, 92:265–302, 2010a.

[45] Wright R. Williamson, S. New monetarist economics: Models. In Woodford M. Friedman, B., editor, *Handbook of Monetary Economics*, pages 25–96. Elsevier, 2010b.