ESSAYS ON THE FINANCIAL INCOMPLETENESS AND ITS MACROECONOMIC EFFECTS

by

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ABSTRACT

The first chapter studies why the Great Recession came with a relatively large reduction in output growth given the size of the fall in total factor productivity (TFP) growth. We build a simple model with contractual frictions: limited commitment, limited liability and a corporate tax proportional to equity value. These constraints put restrictions on building up capital, but not on decreasing it. This asymmetry makes firms which have accumulated large capital stocks reduce them more in recessions. The model predicts that a relatively tranquil TFP history during the Great Moderation helped firms build up large capital stocks and generates up to a 60% larger output amplification in the Great Recession than in the 1973-1975 recession. The regression results confirm this history dependence of output amplification.

Most neoclassical models in macroeconomics generate recessions from news of an increase in future TFP, because positive wealth effects make agents work and save less from such good news. In the second chapter, we propose a new way of explaining business cycles from news shocks by adopting the incomplete enforcement of contracts. In the model, firms can default with borrowed capital but defaulting firms are excluded from the market. Lenders must consider this incentive to default. If a profitable future is expected from news of increasing future TFP, firms will not default today. The smaller incentive to default facilitates investment and creates a large substitution effect which overwhelms the positive wealth effect.

In the third chapter, we study the effects from the expansion in borrowing limit on output volatility in incomplete market. Extra borrowing capacity generates a decrease in total saving due to less precautionary saving motive, and an increase in interest rate. The rich saves more due to the higher interest rate, whereas the poor saves less due to the smaller precautionary saving motive. The rich is more susceptible to shocks in exogenous productivity than before, and hence counteracts fluctuations with more counter-cyclical labor supply. This effect dominates changes in the aggregate variables. The model can explain qualitative changes in aggregate volatility and wealth distribution during the Great Moderation.

1 INCOMPLETE LENDING CONTRACTS AND AGGREGATE OUTPUT FLUCTUATIONS

1.1 Introduction

During the Great Recession, output growth fell sharply given the size of the reduction in total factor productivity (TFP) growth. If we interpret the difference between the two growth rates as the amplification of TFP shocks to output, there was a large amplification in the Great Recession. However, the output amplification from negative TFP shocks has not always been large across recessions. For instance, the 1973–1975 recession had a relatively small output amplification. Figure 1.1 plots the U.S. growth rates of per capita real GDP and TFP.[1] Shaded areas correspond to the NBER recession dates. If we compare the Great Recession with the 1973–1975 recession, the falls in TFP growth rates during the two recessions are similar in size: -1.61% in the 1973–1975 recession and -1.53% in the Great Recession. However, the falls in output growth rates show a large difference. The lowest output growth rates are -2.16% in the 1973–1975 recession but -3.08% in the Great Recession.

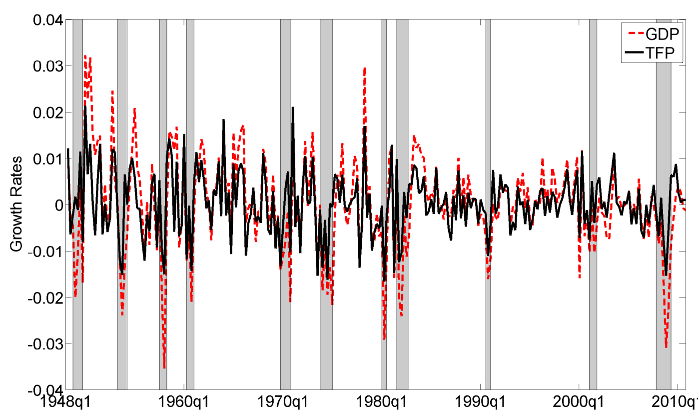


Figure 1.1: Growth Rates of TFP and GDP

[1]Data specifications and the method of calculating growth rates are in the Appendix.

What causes this difference in amplifications across recessions and why did output growth decrease so severely in the Great Recession? Without adopting other exogenous shocks, one distinct aspect of each recession is the different histories of TFP prior to them. In fact, the two recessions aforementioned have quite different histories of TFP. Within one year before the 1973–1975 recession, there was a growth in TFP of over 1%, whereas the TFP growth rates within one year before the Great Recession were particularly serene. They were very close to 0%.

In this paper, we argue that financial frictions generate variations in output amplifications, which depend on the history of TFP. As in Albuquerque and Hopenhayn (2004), limited commitment and limited liability hinder efficient lending between entrepreneurs and intermediaries. Borrowing capital is limited by the sum of expected future dividends—equity value—while dividends must be non-negative in any period. Thus, the equity value cannot grow too rapidly and neither can capital. We make two additional assumptions. First, capital and equity value are determined one period in advance, i.e., investment in this period affects next period's output. Second, we consider a corporate tax that is proportional to equity value. These assumptions prevent flexible changes in capital and consequently yield a strong growth restriction on capital.

Note that there is no restriction at all when diminishing capital; capital can fall freely. This asymmetric growth restriction on capital generates history dependence of output amplification. If there is a period of rapid growth in TFP before a recession, firms would want to build up large capital stocks. But since capital cannot catch up due to the growth restriction, proportionately little capital is accumulated, and the decrease in capital will be small in the following recession. However, without such a history of TFP, firms could accumulate large capital stocks. In this case, the reduction in capital will be large in a recession, and thus there will be more output amplification from the same negative TFP shock.[2] With a constant probability of death and birth of entrepreneurs, there are heterogeneous firms in terms of capital

[2]The model in Moscarini and Postel-Vinay (2013) generates a similar pattern for labor inputs. In their model, firms which accumulated more workers during expansions shed more workers in recessions. Their mechanism is very different, however, as their hiring constraint in expansions is generated by search frictions.

holdings. Hence, the composition of small and large firms in the economy will determine aggregate output amplifications.

From the viewpoint of our model, the different amplifications from the two recessions discussed earlier are caused by their different histories of TFP. For the 1973–1975 recession, TFP growth before the recession generated a firm size distribution dominated by small firms, and mild output amplifications ensued from the following negative shocks. In case of the Great Recession, the serene movement of TFP during the Great Moderation created a distribution with a lot of large firms, which was a favorable environment for large amplifications.

We test whether this history-dependent output amplification that our model predicts is borne out in the data. We regress output amplifications, defined as the difference between TFP growth and GDP growth, on lagged TFP growth. The result shows statistically significant history dependence of output amplifications. Previous TFP growth dampens amplifications from negative TFP shocks. To test our model, we run the same regression with simulation data from the model. We show that our mechanism helps explain the history-dependent output amplifications.

This strong history dependence of output amplification cannot be generated from Albuquerque and Hopenhayn (AH, 2004). If we remove even one of the two restrictions we added—the corporate tax or the different timing scheme in capital and equity value determination—strong history dependence would vanish. If the corporate tax is removed, for instance, large firms will hold extra equity value because there is no penalty for holding large equity values. Accordingly, large firms can easily increase capital holdings. This generates a stable fraction of large firms which have no growth restriction.

If we assume that the current capital and equity value are determined in the current period, full insurance arises against exogenous states. A one-sided no commitment optimal contract typically yields full insurance after certain periods: once an efficient level of capital for any exogenous state is achieved, the contract will immediately specify tailored equity values contingent on every exogenous state.[3] Therefore, similar to the case without the corporate tax, a constant fraction of

[3]See Chapter 19 in Ljungqvist and Sargent (2004).

large firms which have no growth restriction would arise. In both cases, large firms without the growth restriction make output amplifications much more stable and history dependence considerably weaker. Therefore, the AH model without the additional assumptions of this paper cannot generate significant history-dependent output amplification.

Another important characteristic that differentiates our model from others which adopt financial frictions is that large firms are more volatile than small firms in recessions. As noted recently by Kudlyak et al. (2010) and Moscarini and Postel-Vinay (2012), the same pattern is observed in data, in particular during recent recessions. In Table 1.1, we display the percentage decline from the peak to the trough of each variable in corresponding periods.[4] As shown here, short-term debt and sales fall more in large firms than in small firms in recessions on average. This is clearly identified in the Great Recession as well. These observations are contrary to the existing view of Gertler and Gilchrist (1994) whose analysis is based on tight money dates. Moreover, it challenges the literature on the macroeconomic effects of financial frictions, which generally relies on financial frictions to amplify exogenous shocks. If we consider that small firms face more financial frictions than large firms, small firms should reduce short-term debt and sales more in recessions, especially in the Great Recession. By contrast, our model is consistent with the empirical facts.

	Sales		Short-term Debt	
	Large	Small	Large	Small
The Great Recession	-14.94	-10.01	-18.62	-8.85
Average across Recessions	-9.85	-8.38	-22.54	-8.66
Tight Money Dates	-3.11	-6.44	-8.88	-11.12

Table 1.1: Change between Trough and Peak around the NBER Recessions and Tight Money Dates

The other contribution of our work is that we endogenize variables which are

[4]This Table is from Kudlyak et al. (2010). Row 3 is a replication of Gertler and Gilchrist (1994). Row 1 and 2 use the same data set and methodology. Actual peak quarters which are not always the same as the beginning of recessions are used. The 1969–1970 recession is the first recession from the sample. Tight money dates are 1968q4, 1974q2, 1978q3, 1979q4, 1988q4 and 1994q2.

used as exogenous shocks in other papers. First, the optimal contracts in the model set endogenous credit limits. The endogenous credit limit is a link in the model that builds a structural relationship between output amplifications and the history of TFP. Exogenous credit limits, or financial shocks such as in Nolan and Thoenissen (2009) and Jermann and Quadrini (2012), do not generate history dependence in general. Second, the model can generate a large dispersion of output growth among firms in recessions. The model predicts the largest interquartile range (IQR) of output growth for the Great Recession. This is one variable which has been used as a measure of uncertainty shocks by Schaal (2012) and Arellano et al. (2012). In the model, firms are heterogeneous in capital holding and show opposite reactions to changes in TFP according to their capital sizes. This yields a positive dispersion of output growth in recessions. If more firms decrease capital with a negative TFP shock, not only will this exacerbate the fall in output, but it will also create a more dispersed output growth distribution among firms. The observed large changes in these variables, treated as exogenous variables in the literature, is the result of our model and not the cause of the large amplification in the model.

Lastly, the model preserves the predictions on firm financing from AH. This follows because the structure of the optimal contract is very similar with the one in AH. If a contract starts with a small equity value, the optimal contract specifies zero dividends to increase the equity value of contracts and the size of capital. Once capital is equal to the efficient levels given the TFP level of that period, capital does not grow any more, and positive dividends ensue. This helps explain the stylized facts on firm financing and firm dynamics that AH emphasizes: the negative relationship between age and growth or size and growth, no dividends for growing firms, and positive dividends for large firms.

Relation with the Literature

This paper is related to the optimal contracting in firm financing. The most relevant papers are AH and Cooley et al. (2004). Both papers share the same constraints of contracts with this paper: contracts face a limited commitment constraint and a limited liability constraint of entrepreneurs when entering long-term contracts. But these papers assume different timing on the determination of capital and equity value: they are decided after the exogenous shock is realized. As a result, there is full insurance for certain firms, which eliminates the strong history dependence of output amplification. Furthermore, in Cooley et al. (2004), output fluctuations come from the changes in incentives to default for financially constrained (i.e. small) firms.

Another branch of related literature is that on real effects from the financial accelerator. Seminal works include Kiyotaki and Moore (1997) and Bernanke et al. (1999). The main mechanism of these papers is balance sheet effects. In Kiyotaki and Moore (1997), a positive productivity shock increases the value of collateral and the size of a loan, which leads to the amplification of the shock. In **?**, a positive productivity shock improves the quality of loans and reduces the external financing premium. This, in turn, leads to a decrease in agency costs and an increase in production. Carlstrom and Fuerst (1997) use the same mechanism to explain the auto-correlation output growth. In these papers, borrowing constraints are always binding. In our paper, however, borrowing constraints are occasionally binding. The composition of borrowing-constraints-binding firms and non-binding firms governs output amplifications. Financial constraints generate extra output fluctuations from negative TFP shocks in those papers, whereas financial constraints mitigate output amplification in this paper.

Finally, since the model can endogenize credit limits and the dispersion of sales growth, this paper relates to studies that focus on business cycles with exogenous shocks besides TFP. Papers such as Jermann and Quadrini (2012), Nolan and Thoenissen (2009) and Khan and Thomas (2011) adopt financial constraints as exogenous shocks to match aggregate data. There is a growing literature using uncertainty shocks at the firm level to explain business cycles properties. Bloom (2009), Bloom et al. (2012), Schaal (2012) and Arellano et al. (2012) use such firm-level uncertainty shocks.

The remainder of this paper is organized as follows. In the next section, we introduce the model. In Section 1.3, we study the characteristics of the model. We present the characteristics of an individual contract, and then those of the aggregate economy. The main results from the simulation of the benchmark model with calibrated parameters are shown in Section 1.4. In Section 1.5, we discuss a possible extension of the model towards agents' risk-aversion to address the volatility of other aggregate variables such as consumption. Section 1.6 concludes.

1.2 Model

Environment

The environment is similar to Cooley et al. (2004). Time is discrete and the time horizon is infinite $(t = 0, 1, 2, 3, \cdots)$. There are two types of agents: entrepreneurs and workers. The mass of workers and entrepreneurs are both one. Both of them are risk-neutral and share the same discount factor β. Workers live infinitely and maximize their expected utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t (c_t + c_t^h)$$

where c_t^h is consumption from home production and c_t is consumption from other income. Workers are endowed with one unit of labor. There is no disutility from labor. They can work at home or be hired by entrepreneurs. n_t is the working hour hired by entrepreneurs. The real wage is denoted by w_t. The workers' home production technology is $z_t A(1 - n_t)$, where z_t is the economy-wide TFP and A is a constant productivity. z_t follows the Markov chain. Home production is not counted as the market output and cannot be saved, i.e., $c_t^h = z_t A(1 - n_t)$.

Because the functions are linear, optimal choices are trivial. Since workers are risk-neutral, any consumption-saving choice would be optimal as long as the expected return on saving is equal to the inverse of a discount rate. Since the home production technology is linear, the labor supply for entrepreneurs is perfectly elastic. Thus, with the real wage equal to the productivity of home production $w_t = z_t A$, any labor choice would be optimal. Note that the real wage is pro-cyclical. This property of the real wage is not necessary for the logic of the paper but gives more realistic simulation results. Since the labor sector does not affect the main idea of the paper, we will drop the labor sector when we characterize the model analytically in Section 1.3.

Entrepreneurs face a constant probability of death, γ. Entrepreneurs also maximize their expected utility:

$$E_0 \sum_{t=0}^{\infty} (\beta(1-\gamma))^t c_t$$

Entrepreneurs' choices are limited to accepting or rejecting contracts offered by financial intermediaries. To replace the dead entrepreneurs, the exact same number of new entrepreneurs is born. Therefore, the measure of entrepreneurs is always the same at a given time.

Entrepreneurs are born without any funds, and their production technologies are individually endowed. An entrepreneur cannot access other entrepreneurs' technology or directly invest in them. If they do not start the project at their birth, their technology will be lost permanently, and hence they cannot wait to sign a contract. To start production, the entrepreneurs' production technology requires the initial investment cost, I_0. This becomes sunk cost. Since entrepreneurs need funds to start production, a financial transaction is required. The entrepreneurs' production technology uses capital and labor input. The production function is

$$y(z_t, k_t, n_t) = z_t k_t^{\theta} n_t^{1-\theta}$$

where k_t is capital and n_t is the amount of hired labor. The amount of capital is fixed one period in advance, as is usual in the business cycles literature. All entrepreneurs in the economy share the same TFP, z_t, but their amount of input can be different from each other. δ is the depreciation rate. Since agents' choices are trivial, the real actions happen from the contracts.

There are two types of agencies in this economy: financial intermediaries and the government. Financial intermediaries take savings from workers and finance entrepreneurs by entering long-term contracts. They maximize their profits. With a competitive financial market, financial intermediaries make zero profit. The government only collects taxes from output. The after tax output is $(1 - T_t)y_t$, where T_t is a function of the tax rate.

Individual Contracts

An entrepreneur and a financial intermediary can go into a long-term contract. Consider a newborn entrepreneur at time zero. The competitive financial intermediary will offer the contract to the entrepreneur. The contract includes the one-time lenders' payments for the initial investment, I_0 and the initial capital, k_1. Every period t thereafter, conditional on survival, the contract also states the amount of labor for which the entrepreneur should hire n_t, the amount of capital advancement k_{t+1}, the dividend flow to the entrepreneur each period d_t, and the flow to the financial intermediary $(1 - T_t)y_t + (1 - \delta)k_t - w_t n_t - d_t - k_{t+1}$. These variables are contingent on history, $h_t = \{k_s, n_{s-1}, d_{s-1}, z_{s-1}; k_1, z_0\}_{s=2}^{t}$.

We can summarize the timing of events as follows. For each period t, the TFP level is realized. Some entrepreneurs die. The entrepreneurs who survive can default with the borrowed capital or go on to production. According to the contracts, entrepreneurs hire labor and produce output. The output and the undepreciated capital are distributed to entrepreneurs, workers, financial intermediaries and the government. New entrepreneurs are born. Intermediaries and the new entrepreneurs sign contracts. Intermediaries advance capital to entrepreneurs for the next period's production. Note that capital at $t + 1$ is not contingent on the exogenous state at $t + 1$ because it is determined at period t by the timing assumption.

Suppose that we measure the value of a contract right after the death of some entrepreneurs at period t. Since the contract specifies all future dividends, the expected value of future dividends can be calculated after the contract is initiated. We denote the value of the contract to the entrepreneur, the equity value, as v_t. It is defined by the sum of discounted future dividends from the contract:

$$v_t \equiv E_t[\sum_{s=t}^{\infty}(\beta(1-\gamma))^{s-t}d_s] \tag{1.1}$$

$$= d_t + \beta(1-\gamma)E_t[v_{t+1}] \tag{1.2}$$

All produced goods after tax are distributed to either the entrepreneur or the financial intermediary. Once $(1 - T_t)y_t, d_t, k_{t+1}$ are determined, the portion of output to the intermediary is also fixed. Thus the contract implicitly defines the value of the contract to the financial intermediary. Let b_t denote this value.

The government collects corporate taxes from the output of contracts. In practice, the corporate tax is levied on the income of a corporate company. However, the income of a contract is ambiguous in this model. Instead, we assume that the corporate tax rate is proportional to the equity value of a contract. This is inspired by the fact that the intermediaries' portion of output can be covered as cost. The corporate tax rate function is defined by

$$T_t \equiv \frac{v_t}{T_c}$$

where T_c is a constant.

We additionally impose that it is too costly for financial intermediaries to provide v_{t+1} contingent on z_{t+1}. At period t, additional information about z_{t+1} will not change the expected value of $E_t[v_{t+1}]$. So $E_t[v_{t+1}] = v_{t+1}$. From this and equation (1.2), we can write the law of motion for v as:

$$v_t = d_t + \beta(1-\gamma)v_{t+1} \tag{1.3}$$

Not only the next period capital (k_{t+1}), but also the next period equity value (v_{t+1}) do not depend on the exogenous state at $t + 1$.

Given this setting, we focus on contracts that satisfy two conditions. First, the payment to entrepreneurs, or dividends, cannot be negative. If we interpret a contract as a corporate firm, entrepreneurs represent stock holders. Stock holders have limited liability; their payment cannot be negative. The entrepreneurs' payments are bounded above by zero.

$$d_t \geq 0 \tag{1.4}$$

The second restriction is the limited commitment constraint. Entrepreneurs can walk away with borrowed capital, k_t. They would deliberately do so if the equity value of the contract is less than the value of default. Hence, the equity value should always be larger than or equal to the amount of the borrowed capital.

$$k_t \leq v_t \tag{1.5}$$

Pareto optimal contracts between entrepreneurs and financial intermediaries should maximize one's value given the other's value. Thus, the optimal contracts can be found by maximizing v_t given b_t or by maximizing b_t given v_t. Cooley et al. (2004) follow the former with the Lagrangian method while AH follows the latter with the promised utility variable method. In this paper, we follow the latter. Let us define the total value of the contract W_t as $W_t \equiv v_t + b_t$. Then, maximizing b_t given v_t is equivalent to maximizing W_t given v_t.

At period 1, the total value of the contract W_1 given k_1, v_1, z_1 can be found by:

$$W_1(k_1, v_1, z_1) = \max_{\{k_{t+1}, n_t, d_t\}_{t=1}^{\infty}} E_1 \sum_{t=1}^{\infty} (\beta(1-\gamma))^{t-1} [(1 - \frac{v_t}{T_c}) z_t k_t^{\theta} n_t^{1-\theta} + (1-\delta)k_t - w_t n_t - k_{t+1}]$$

subject to $\forall t$,

$$\begin{aligned} &v_t = d_t + \beta(1-\gamma)v_{t+1} \\ &d_t \geq 0 \\ &k_t \leq v_t \end{aligned}$$

Recursively, it can be expressed as

$$W(k,v,z) = \max_{k',n,d}(1-\frac{v}{T_c})zk^{\theta}n^{1-\theta} + (1-\delta)k - wn - k' + \beta(1-\gamma)E[W(k',v',z')]$$

subject to

$$\begin{aligned} &v = d + \beta(1-\gamma)v' \\ &d \geq 0 \\ &k' \leq v' \end{aligned}$$

Note that d appears only in the first and second restrictions, so they can be merged into $v \geq \beta(1-\gamma)v'$.

To find the initial conditions of newborn entrepreneurs, we introduce auxiliary variables. Let $\tilde{v}_{t-1}$ and $\tilde{b}_{t-1}$ be the values of contracts to entrepreneurs and intermediaries, respectively, right after entrepreneurs are born at period $t-1$. From the definition of $\tilde{v}$ and $\tilde{b}$ and non-contingency of v with respect to the exogenous state, we get:

$$\begin{aligned} \tilde{v}_{t-1} &= \beta(1-\gamma)v_t \\ \tilde{b}_{t-1} &= \beta(1-\gamma)E_{t-1}[b_t] \end{aligned}$$

If we define $\tilde{W}_t$ as $\tilde{W}_t \equiv \tilde{v}_t + \tilde{b}_t$, we can relate $\tilde{W}_{t-1}$ with W_t:

$$\tilde{W}_{t-1} = \beta(1-\gamma)E_{t-1}[W_t]$$

Consider a contract which is signed at period 0. With the competitive financial market, financial intermediaries would suggest the largest $\tilde{v}_0$ to entrepreneurs as long as the intermediaries break even, and k_1 should be the value which maximizes

$\tilde{b}_0$. The initial variables should satisfy

$$\tilde{v}_0 = \sup\{v : \tilde{b}_0(k_1, v, z_0) \geq I_0 + k_1\} \tag{1.6}$$

$$k_1 = \arg\max_k \tilde{b}_0(k, \tilde{v}_0, z_0) \tag{1.7}$$

In practice, the procedure of finding the initial condition is as follows. Once W is calculated from the dynamic programming problem, $\tilde{W}$ is immediate from the relation between W_t and $\tilde{W}_{t-1}$. From equations 1.6 and 1.7, we can get $\tilde{v}_0$ and k_1 given z_0. v_1 is derived from the relation between v_t and $\tilde{v}_{t-1}$.

1.3 Characteristics of the Model

In this section, we study the characteristics of the model. To simplify matters, we drop the labor sector and consider the case with two levels of the exogenous states throughout this section. Without the labor sector, we can find the optimal contracts by the following dynamic programming problem:

$$W(k, v, z) = \max_{k', v'} (1 - \frac{v}{T_c}) z k^\theta + (1 - \delta)k - k' + \beta(1 - \gamma)E[W(k', v', z')]$$

subject to

$$\lambda_1 : k' \leq v' \tag{1.8}$$

$$\lambda_2 : v \geq \beta(1 - \gamma)v' \tag{1.9}$$

where λ's are the Lagrange multipliers for restrictions and z is the productivity level which follows the Markov chain with two states $(z \in \{z^l, z^h\})$, $0 < z^l < z^h$. We assume that z is persistent. If we denote $p(z'|z)$ as the probability of state z' condition on z, the persistent exogenous states imply $p(z^h|z^h) > p(z^h|z^l)$ and $p(z^l|z^l) > p(z^l|z^h)$. Equation (1.8) is the limited commitment constraint and equation (1.9) is the limited liability constraint. The first order conditions are:

$$\lambda_1 = \beta(1-\gamma)E[W_1(k', v', z')] - 1 \tag{1.10}$$

$$E[W_2(k', v', z')] = \lambda_2 - \frac{\lambda_1}{\beta(1-\gamma)} \tag{1.11}$$

From the envelope theorem we get,

$$W_1(k, v, z) = (1 - \frac{v}{T_c})\theta z k^{\theta-1} + (1-\delta) \tag{1.12}$$

$$W_2(k, v, z) = \lambda_2 - \frac{1}{T_c} z k^{\theta} \tag{1.13}$$

We need Assumption 1.1 to fully characterize the optimal policy function of the problem.

Assumption 1.1. *Let* $\bar{k} = [\frac{\frac{1}{(\beta(1-\gamma))} - 1 + \delta}{z^h \theta}]^{\frac{1}{\theta-1}}$*. Then, parameters satisfy*

$$0 < T_c < \bar{T}_c \tag{1.14}$$

where $\bar{T}_c$ *is* $\frac{k}{[\frac{(1-\delta) - \frac{1}{\beta(1-\gamma)}}{\theta E[z'|z] k^{\theta-1}} + 1]}$ *for* $\forall k$ *such that* $0 < k < \bar{k}$ *.*

$\bar{k}$ is the optimal capital without any restrictions or distortionary taxes at the high productivity level. Capital cannot be larger than $\bar{k}$. Assumption 1.1 addresses the condition for the tax constant. It should be small enough so that the presence of tax matters. Given this assumption, optimal contracts always specify $k' = v'$. Since the corporate tax is proportional to equity value and is significant, optimal contracts do not recommend maintaining the extra equity value in excess of the next period's capital level. In other words, contracts specify maximum borrowing amount given equity values to maximize tax benefits from borrowed capital. This kind of effect from the corporate tax is common in the literature. (e.g. Jermann and Quadrini (2012)).

Proposition 1.1. *Under Assumption 1.1,* $k' = v'$*.*

Proof. The proof derives from the first order conditions. If $k' < v'$, λ_1 should be zero from the Kuhn-Tucker condition. The first order condition can thus be modified as below.

$$\beta(1-\gamma)[(1-\frac{v'}{T_c})E[z'|z]\theta k'^{\theta-1} + (1-\delta)] = 1$$

From Assumption 1.1, the above equation cannot hold with $k' < v'$. □

We have the following proposition from Assumption 1.1 and the decreasing returns to scale in capital in the production function.

Proposition 1.1. *If the restriction on the growth of v', equation (1.9), does not bind, the optimal contracts specify the unique level of capital. Given z, the optimal $k^*(z)$ satisfies the equation below.*

$$(1-\frac{k^*}{T_c})E[z'|z]\theta k^{*\theta-1} + (1-\delta) = \frac{1}{\beta(1-\gamma)} + \frac{E[z'|z]k^{*\theta}}{T_c}$$

Proof. If equation (1.9) does not bind, the problem can be transformed to

$$\max_{k'} \beta(1-\gamma)((1-\frac{v'}{T_c})E[z'|z]k'^{\theta} + (1-\delta)k') - k'$$

subject to

$$k' \leq v'$$

By deriving the first order conditions of this maximization problem, we can get the condition for the optimal k^*. □

In other words, if the dividend is zero, equity value will grow at its maximum rate and so will capital (Proposition 1.1). When the dividend is positive, the capital will be k^*. We will call $k^*(z)$ **the efficient level of capital at** z. Note that the efficient level of capital does not depend on the endogenous states. Because the utility function is linear, $k^*(z)$ only depends on the current exogenous state. Given the persistence of the exogenous shock, the expected value of the next period's productivity is larger when the current state is high, i.e., $E[z'|z = z^h] > E[z'|z = z^l]$. The efficient level of capital is an increasing function in z.

Since $k' = v'$ always holds, we need to characterize just one optimal policy function for the two endogenous variables. The optimal policy function can be found by following these steps: first, given k, v and z, ignore the growth restriction on v and find the optimal k'. It will be $k^*(z)$. Second, make sure that the restriction does not bind with $k^*(z)$. If the restriction does not bind, then the value for the optimal policy function is $k^*(z)$. Otherwise, the restriction is binding, so $k' = \frac{1}{\beta(1-\gamma)}k$.

As a result, if $k \geq \frac{1}{\beta(1-\gamma)}k^*(z)$, then $\lambda_2 = 0$ and the optimal policy function will be flat at $k' = k^*(z)$. However, if today's capital is small, a large increase to $k^*(z)$ will violate the growth restriction on v. In this case, the contract will specify the maximum value for v' under the growth restriction. If $k < \frac{1}{\beta(1-\gamma)}k^*(z)$, then $\lambda_2 > 0$, which implies $k' = \frac{1}{\beta(1-\gamma)}k$. Figure 1.2 depicts the resulting optimal policy function.

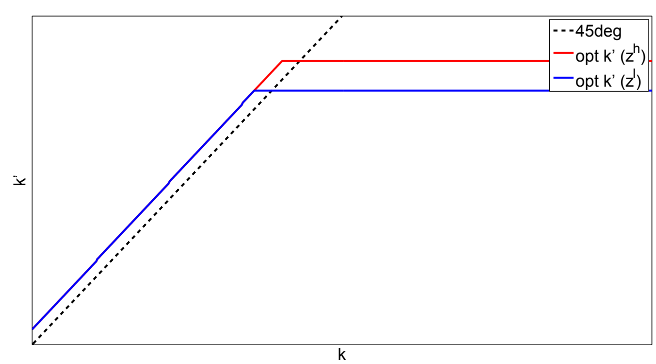


Figure 1.2: Optimal Policy Functions

Since we adopt two different levels of TFP as exogenous states, there are two efficient levels of capital: $k^*(z^l)$ and $k^*(z^h)$. One optimal policy function is eclipsed by the other up until capital reaches the efficient level of capital for the low productivity state, $k^*(z^l)$. However, beyond this value, the optimal policy function for the high productivity state keeps growing until it reaches the efficient level of capital for the high productivity state, $k^*(z^h)$, while the optimal policy function for the low productivity state becomes a flat line.

Characteristics of Individual Contracts

Dynamics of Firm Growth and Firm Financing

Note that the growth restriction on v is related to the limited liability constraint. Entrepreneurs cannot have negative flow from contracts and thus, they cannot invest actively once the contracts are signed. The only way to increase their value in contracts is to forgo their portion of dividends. Hence, the growth rate of equity values is limited by the discount rate and the death probability of entrepreneurs.

Let us assume that newborn entrepreneurs start with small initial capital. As time passes, these small and young firms will increase capital at their maximum rate and no dividend will be paid during these periods. Once these firms achieve the efficient level of capital, there will be no more growth in capital or equity value unless the exogenous state changes, and the dividends are strictly positive.

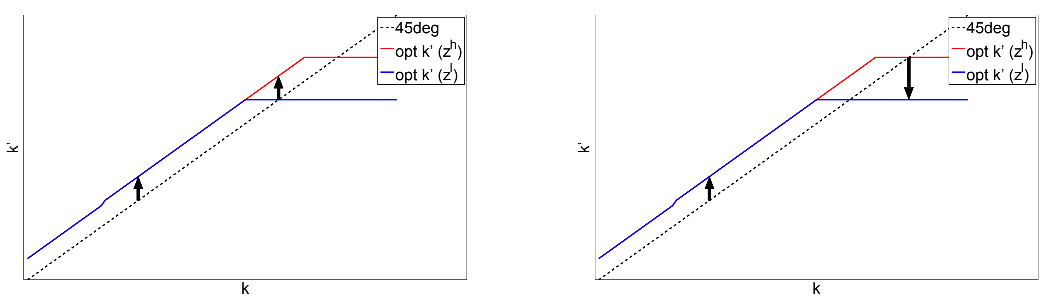
These characteristics can explain some stylized facts about the dynamics of firm growth. Evans (1987) and Dunne et al. (1989) noted that there is a negative relationship between the growth rate of firms and firms' ages and sizes. The optimal contract implies that the growth rate of firms decreases as firms' ages and sizes increase.

Second, Barclay and Smith (1995) find that the ratio of long term debt to short term debt decreases as a firm's market-to-book ratio increases. In our setting, W and I_0 can be considered as the market value of a firm and the book value of a firm, respectively. Then, $\frac{W}{I_o}$ is the market-to-book ratio. Also, since b is the value of a firm to a financial intermediary, it can be treated as long-term debt, whereas k represents short-term debt. $\frac{b}{k}$ then corresponds to the ratio of long-term debt to short-term debt.

As capital (k) grows to the efficient level, the total value of the contract (W) will increase. Under assumption 1.1, this increase in capital will raise the equity value. The contract value to financial intermediary (b) will decrease because the total contract value (W) is concave in equity value (v). That yields the negative relationship between the Q ratio $(\frac{W}{I_0})$ and the duration of the loan $(\frac{b}{k})$. Therefore, as in AH, our model explains the stylized facts about the dynamics of firm growth and firm financing.

Heterogeneous Reactions to Negative TFP Shocks

Let us compare two situations. In the first scenario, there is a positive change in TFP, and it lasts for two consecutive periods $(z_1 = z^l, z_2 = z^h, z_3 = z^h)$. In the second scenario, there is a negative change in TFP, and it lasts for two consecutive periods $(z_1 = z^h, z_2 = z^l, z_3 = z^l)$. We assume that it requires at least two periods to advance from the efficient level of capital for the low productivity, $k^*(z^l)$, to the efficient level of capital for the high productivity, $k^*(z^h)$. From the determination timing of capital and equity value, the reactions of contracts to changes in the exogenous state from period 1 to period 2 are realized in period 2. Figure 1.3 displays the reactions of different capital sizes of contracts to such changes in the exogenous states.



(a) Reactions to a Positive Shock

(b) Reactions to a Negative Shock

Figure 1.3: Reactions in Period 2

In the first scenario, since the first period exogenous state was low, all contracts will be located in the interval between 0 and $k^*(z^l)$. In period 2, all contracts will raise their capital since there is no contract which attained the efficient level of capital for z^h, $\forall k < k^*(z^h)$. The sizes of the increases in capital are proportional to contracts' capital level at period 2, because they increase capital at their maximum rate $\frac{1}{\beta(1-\gamma)}$.

In the second scenario, if a contract has a capital level smaller than $k^*(z^l)$ at period 2, the optimal policy function specifies that it should increase capital for period 3 even with the negative TFP shock. However, if a contract has capital larger than $k^*(z^l)$ at the beginning of period 2, it reduces its capital for period 3.

Due to the different reactions of contracts regarding a negative shock, different output amplifications across firms may arise from a negative TFP shock of the same size. Small firms will dampen output responses from a negative TFP shock by increasing capital. However, large firms will amplify output reactions from a negative TFP shock by decreasing capital. The amplification from a negative TFP shock is maximized when a firm achieves the efficient level of capital, $k^*(z^h)$, at the beginning of period 2.

These scenarios show that the optimal contract in this paper is able to explain the recent empirical findings that large firms are more cyclical than small firms in recessions. Gertler and Gilchrist (1994) use data from the Quarterly Financial Report for Manufacturing Corporations (QFR) to study the responses of small and large firms to monetary shocks. They report that small firms reduce sales, inventories, and short-term debt more than large firms in Romer dates. However, Kudlyak et al. (2010) find that large firms are more volatile around NBER recession dates on average, especially in recent recession dates. Other evidence can be found in Moscarini and Postel-Vinay (2012). Using the Business Dynamic Statistics (BDS), which includes longitudinal links, they show that large firms hire more workers when the unemployment rate is low and shed more workers when the unemployment rate is high than small firms do. In our model, the size of reduction in capital in recessions decreases as firms become smaller. Hence, large firms are more cyclical, particularly during recessions.

History Dependence of Output Amplification

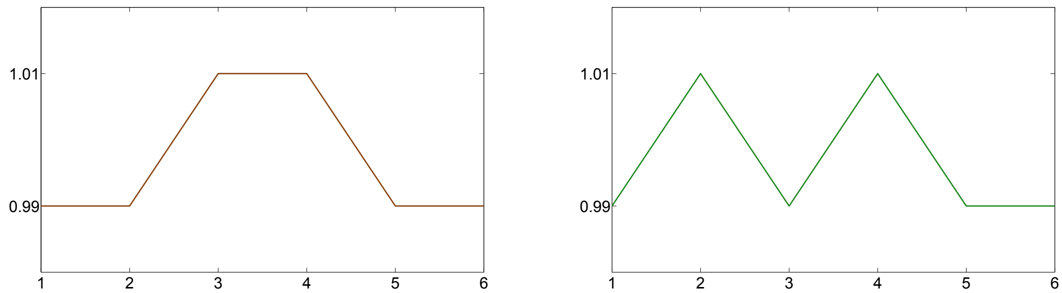
The previous section suggests that the size of capital relative to the efficient level of capital is important in measuring output amplification from a negative TFP shock. As a contract attains capital closer to $k^*(z^h)$ at the beginning of period 2, the amplification from a negative shock increases.

In which situation can a contract accumulate capital close to its efficient level? The answer to this question involves the history of variables, particularly the history of exogenous shocks, and hence a general answer is difficult to give. We provide one condition which can hinder a contract from amassing a relatively large capital stock.

The positive relationship between the relative size of capital to its efficient level and shock amplification can alternatively be expressed as the inverse relationship between the distance from capital to the efficient level and the intensity of the output response to a negative TFP shock. To shorten the distance, a contract needs enough time to build capital because its growth rate is limited by the risk-free interest rate ($\frac{1}{\beta(1-\gamma)}$). The efficient levels of capital depend on the levels of the exogenous variable, TFP. If TFP growth is rapid right before a negative shock hits, it is impossible for a contract to accumulate capital and shorten the distance between its capital and the efficient level. But if the TFP history before a recession was composed of serene movements or continuous decreases, a contract may attain the efficient level of capital right before the negative shock, which will result in a big decrease in output.

As a simple exercise, let us compare the results from two different histories of TFP. Figure 1.4 depicts two TFP series. In both cases, firms face a negative shock of the same size at period 5, and the low state continues into period 6. However, histories of TFP before the negative shock are different. There is growth in TFP right before the negative shock at period 5 in case II, but none in TFP in case I.

Figure 1.5 shows the capital and output. In terms of capital, a firm starts at $k^*(z^l)$ in both cases. Given the setting of the exercise, a firm needs two periods to achieve $k^*(z^h)$ from $k^*(z^l)$. In case I, the TFP series was constant at the high level

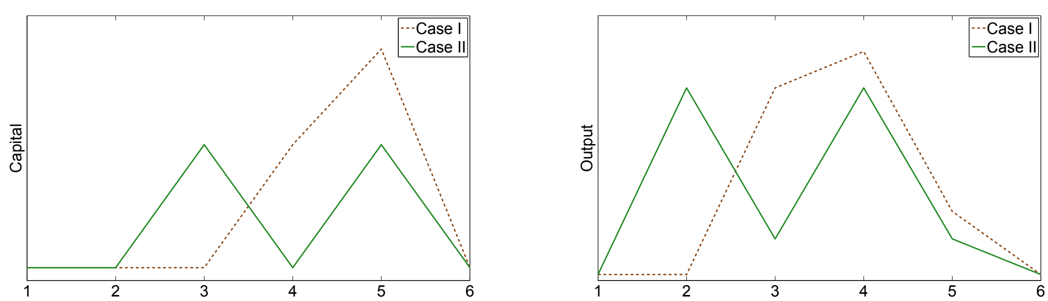


(a) TFP Level in Case I (b) TFP Level in Case II

Figure 1.4: Two cases

during periods 3 and 4 preceding the negative shock at period 5. Therefore, at the beginning of period 5, a contract attains the efficient level of capital $k^*(z^h)$. In case II, however, TFP grows between periods 3 and 4. A firm cannot achieve the efficient level of capital. As a result, there is a smaller decrease in capital from period 5 to 6, and a smaller output amplification ensues.

From period 4 to period 5, a firm in case I does not grow at its maximum growth rate because it is already somewhat large at period 4. However, in case II, a firm grows at its maximum rate. Therefore, we observe more output amplification from period 4 to period 5 in case I.



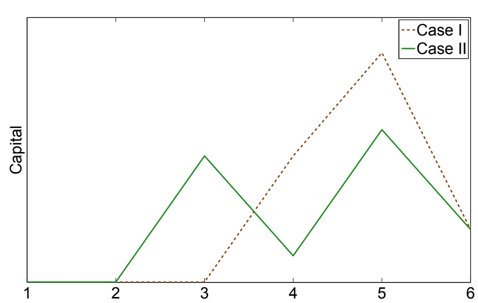
(a) Capital (b) Output

Figure 1.5: History Dependence Output Amplification: Individual Firm

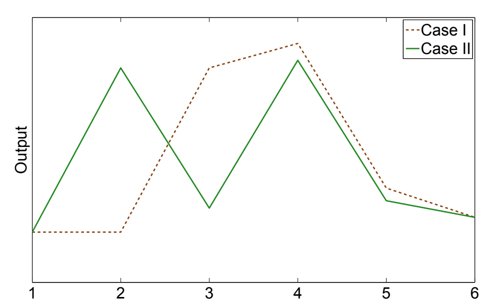
History Dependent Output Amplification in Different Models

Now we will examine the aggregate behavior of the model. To study the effects of the additional constraints relative to AH—corporate taxes and the non-contingency restriction—we compare our model with two different auxiliary models. First, we will only remove the corporate tax. Then we will remove non-contingency restrictions, as well as the corporate tax. The model without both constraints is the AH model with idiosyncratic births and deaths of entrepreneurs. To find out whether the history dependence of output amplification can exist in these setups, we feed in the same histories of TFP in Figure 1.4 to each model. For all cases, the economy starts with an invariant steady-state distribution of firms with the exogenous state, z^l.

Figure 1.6 presents the result of the exercise from our model at the aggregate level. By comparing Figure 1.6 and Figure 1.5, we can see that the history dependence of aggregate output amplification exists in the economy level as well as in the individual firm level. Comparing case I and case II, output decreases more in case I. The same reasoning can be applied here as in the individual contract setup. The difference is that now the composition of firms changes such that in case I, there are more large firms in the economy due to the serene history of TFP before the recession. This results in the additional output drop in case I.



(a) Capital



(b) Output

Figure 1.6: Capital and Output from the Model

The Model without the Corporate Tax

To study the role of each assumption, we compare our model with one without the corporate tax. However, we keep the assumption of timing with regards to capital and equity value, which are determined one period in advance. This economy can be summarized by the following equations:

$$W(k,v,z) = \max_{k',v'} zk^{\theta} + (1-\delta)k - k' + \beta(1-\gamma)E[W(k',v',z')]$$

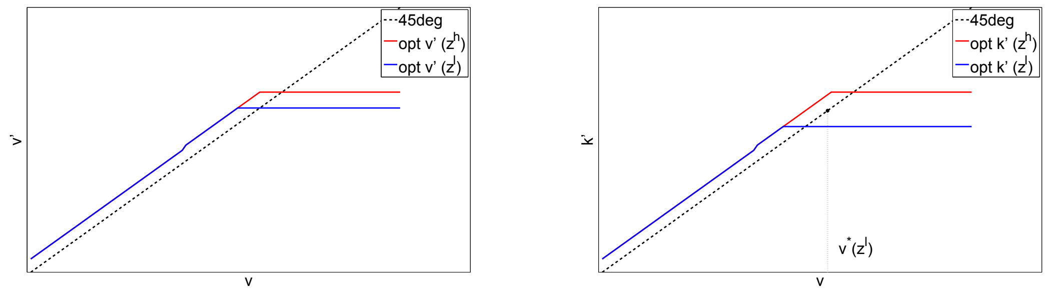
subject to

$$\lambda_1 : k' \leq v' \tag{1.15}$$

$$\lambda_2 : v \geq \beta(1-\gamma)v' \tag{1.16}$$

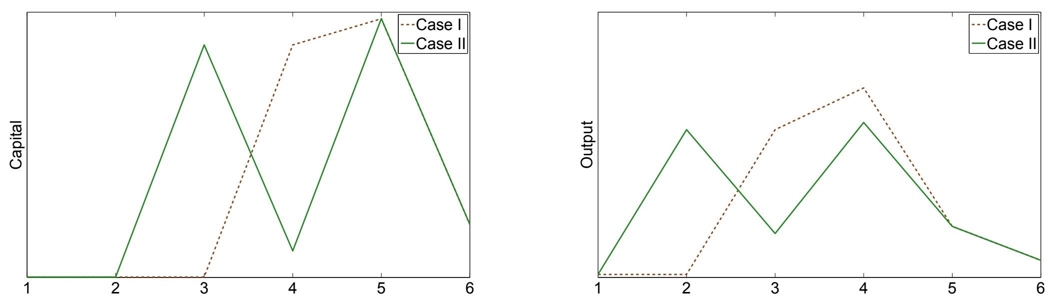
Figure 1.7 shows the optimal policies in this model. Note that the optimal policy functions do not depend on k. An important difference between our model and this model is that k is not always equal to v, as optimal contracts specify v' larger than k' for $v > k^*(z^l)$. Because there is no penalty for having a higher bound, as there is no corporate tax, optimal contracts specify higher equity value even if today's exogenous state is low in order to prepare for a future high state. In other words, contracts generate partial insurance in terms of v. This partial insurance allows contracts to adjust to changes in the exogenous state more flexibly. As a result, from $v^*(z^l)$, it only takes one period to reach $k^*(z^h)$.

Figure 1.8 presents output and capital from this model given the two different histories of TFP, analogous to Figure 1.4. In terms of output, there exists an additional decrease in case I compared with case II. However, the extra reduction occurs only from period 4 to period 5, because capital and equity value should be determined one period earlier. There is no history dependence beyond one period. This is clearly observed from the change in capital (Panel (a)), as there is no additional reduction from period 5 to period 6.



(a) Equity Value (b) Capital

Figure 1.7: Optimal Policy Functions from the Model without the Corporate Tax



(a) Capital (b) Output

Figure 1.8: Capital and Output from the Model without the Corporate Tax

AH Model: Contingent Contracts without the Corporate Tax

Next, we compare our model with the model suggested by AH, tailored to our model economy. In AH, contracts are contingent in terms of both capital and equity value. Working capital is assumed so contracts can change the current capital within a period. The version of the AH model we consider can be summarized as the following:

$$W(v, z) = \max_{k, v'(z')} \beta(1-\gamma)[zk^{\theta} + (1-\delta)k - (1+r)k + E[W(v'(z'), z')]]$$

subject to

$$\lambda_1 : k \leq v$$
$$\lambda_2 : v > \beta(1-\gamma)\sum_{z'} p(z'|z)v'(z') \tag{1.17}$$

where $r = \frac{1}{\beta(1-\gamma)}$.

Figure 1.9 shows the optimal policy functions from the AH model. We need policy functions of v only because now that working capital is adopted, capital is always the same as equity value. We have four policy functions with two exogenous states because the optimal policy is contingent on the exogenous states. We can see that the shapes of the policy functions are indeed similar to those in our model, and hence the characteristics of firm growth and firm financing are also comparable.

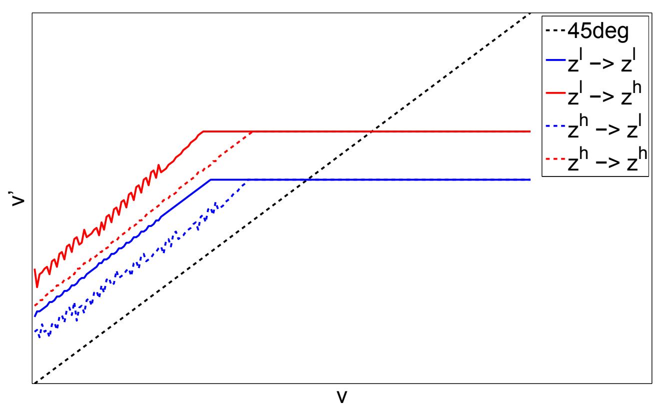


Figure 1.9: Optimal Policy Function from AH

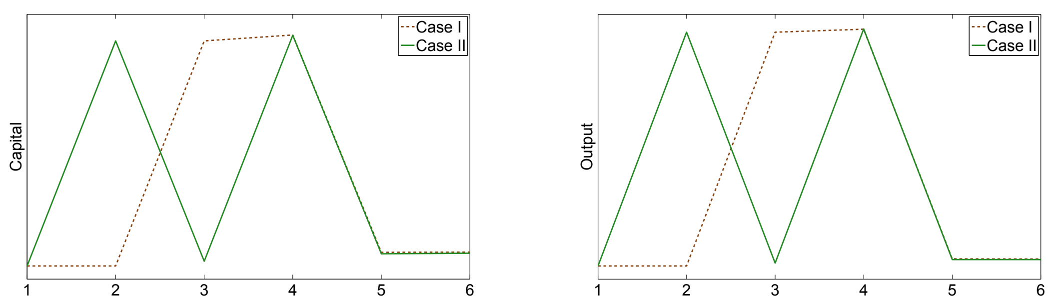
One notable point in AH is that the growth of equity value is faster when the current state is low productivity. This derives from the persistence of exogenous variables. Contracts want to allocate more capital to a high productivity state than a low productivity state. To support that, contracts allocate more equity value to a high state.

Suppose that the current state is high. Given the persistence of exogenous shocks, it is likely to be a high state in the next period as well. From equation (1.17), we know that it is more costly to assign a large equity value for the future high productivity state when the current state is also high productivity.

Despite the similarity of individual optimal policies between AH and our model, the characteristics of aggregate output are entirely different. In the model from AH, if an equity value is larger than a certain threshold value, there is full insurance from the change in exogenous states. Thus, the distribution of firms in this economy is very stable relative to our model.

For example, if the exogenous productivity increases quickly enough, the measure of firms which attain efficient levels of capital will be zero in our model. However, full insurance enables a positive measure of firms to have efficient levels of capital in the model from AH. If there were firms with efficient levels of capital before an increase in productivity, those firms immediately boost their capital level from $k^*(z^l)$ to $k^*(z^h)$.

This stability of the distribution of firms in the contingent contracts case hinders a strong dependence of amplification on the history of the exogenous variables. Figure 1.10 presents the results of the exercise from the AH model at the aggregate level. The changes in output and capital are identical between the two cases.



(a) Capital (b) Output

Figure 1.10: Capital and Output from AH

In sum, the timing of capital and equity value, and the corporate tax, are key assumptions for generating history dependence of output amplification. On top of limited commitment and limited liability constraints, these assumptions put more restrictions on changes in equity values and capital and make them less flexible to possible changes in the exogenous variable. Without such additional assumptions, contracts easily prepare equity values for future exogenous states, and this lessens the effects of growth restriction from limited liability.

1.4 Simulation of the Aggregate Economy

In this section, we study the characteristics of an economy with calibrated parameters. It is straightforward to aggregate heterogeneous firms because the price of capital and labor does not depend on the distribution of firms. The interest rate is fixed from the discount rate and the death rate, and wage depends only on the exogenous state alone.

Calibration

We estimated the TFP process from the Solow residuals as is conventional in the literature. Our method is similar with Ríos-Rull and Santaeulalia-Llopis (2010).[5] The period is from 1948 I to 2010 IV. We assume that the TFP process follows AR(1). The OLS estimation results is

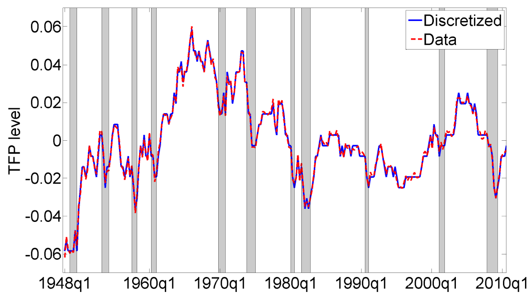
$$\log(z_t) = 0.9371 \log(z_{t-1}) + \epsilon_t$$

where

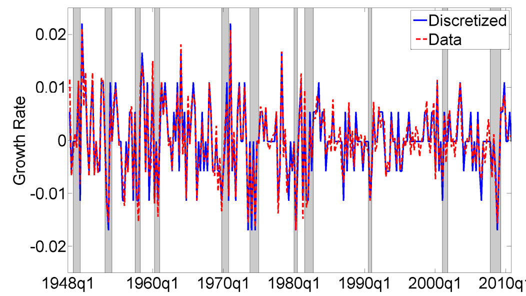
$$\epsilon_t \sim N(0, 0.0068^2)$$

Then, we discretized the process with 22 shock realizations $(z^1 < z^2 < \cdots < z^{22})$ by the methods of Tauchen (1986). As in Figure 1.11, 22 levels of TFP correspond with 9 states of TFP growth rates. From 22 states in levels, we can get a close approximation in growth rates.

[5] Detailed explanations about the method and data source are in the Appendix.



(a) TFP



(b) Growth Rate of TFP

Figure 1.11: Discretization of TFP

We choose 0.995 for the value of the discount rate, β and 0.01 for the entrepreneur's death rate per quarter (γ). The death rate corresponds to the exit rate of a firm. A 1% quarterly exit rate becomes an exit rate of 21.43% over six years. Evans (1987) reported 21.95% over the periods from 1976 to 1982. With these values of γ and β, the implied risk-free interest rate $(\frac{1}{\beta})$ is 2.0%, and the lending interest rate $(\frac{1}{\beta(1-\gamma)})$ is 6.2%, annually. A real risk-free rate is a nominal risk-free rate minus an expected inflation rate. If we replace the expected inflation rate with the realized inflation rate, CPI, the average of the real risk-free rates over the sample periods is 0.8%. On the other hand, the return on capital is 4.1%. Since our model does not distinguish the two, we choose a value in between the two. The value of 0.995 for β is not unusual in the business cycles literature.

We follow the methods in Ríos-Rull and Santaeulalia-Llopis (2010) to calibrate the capital share and the labor share. We calculate the fraction of unambiguous capital income from total unambiguous income for sample periods. We used the average of the fraction and got results similar to the literature. The depreciation rate, δ, is calibrated from the ratio of consumption of fixed capital to capital.

The coefficient A is related to the level of labor supply. The ratio of market work time to home production time is 0.54 for an average married couple in the Michigan Time Use Survey.[6] The corresponding value from the simulation gives 0.53. More

[6]This ratio is from Table 4.1 in Gronau (1986), which is based on the data presented by Hill

importantly, A affects the ratio of the corporate tax revenue to output. Because capital and labor are complementary when producing output, a small A increases not only labor but also capital. Given $k = v$, a small A will increase the tax rate, and the ratio of the total corporate tax revenue to output would be large. It turns out that T_c alone cannot change that ratio, because the change of T_c will affect not only k but also y. With $A = 1.7$ and $T_c = 550$, the average corporate tax revenue to output ratio is 2.4%. The Office of Management and Budget reports the average ratio of corporate tax revenue to GDP as 2.8% for the same periods.

Finally, the size of initial investment (I_0) determines the required capital for a starting firm. If the initial investment is small, financial intermediaries can offer a large equity value to new entrepreneurs, and this will lead to a large amount of initial capital. In the data, the ratio of the average size of new firms to firms more than 26 years old is 0.086. However, firms grow very rapidly in the first year in the data, unlike in the model. If we exclude age 0 firms, the value is 0.196 in the data. We choose $I_0 = 0.41$ so that the ratio of the capital of firms 1 year old to the average capital of firms which are more than 26 years old would be 0.17. In the benchmark model, it takes around 36 years for firms to have efficient levels of capital.

Table 1.2 summarizes the calibrated parameters used in our simulation of the benchmark model and Table 1.3 presents the calibrated moments. Given the parameter values and the estimated shock process above, $k' = v'$ holds for every state. Hence, the heterogeneity of firms will be based on k only. A more detailed explanation about the method of calibration and data source is in the Appendix.

Results

Figure 1.12 plots the firm size distribution from the benchmark model. It shows the invariant steady-state distribution given a constant exogenous state $(z = z^1)$. The horizontal axis is the size of capital relative to the largest capital size. A firm begins with a small capital stock and it will increase its capital level as time passes. However, a firm faces a constant probability of death, so the measure of firms

(1985)

Variables	Value	Description
β	0.995	Discount Rate
θ	0.324	Capital Share
δ	0.014	Depreciation Rate
γ	0.010	Death Rate of Entrepreneurs
A	1.7	Home Production Productivity
T_c	550.0	Tax Constant
I_0	0.41	Initial Investment

Table 1.2: Calibrated Parameters for the Benchmark Model

Moments	Empirical Values	Simulated Values
Risk-Free Interest Rate	0.8%	2.0%
Capital Share	0.324	0.324
Depreciation Rate	0.014	0.014
Exit Rate of Firms	21.95%	21.43%
Ratio of Market Work Time to Home Production Time	0.54	0.53
Corporate Tax Revenue to GDP	2.8%	2.4%
The Size of 1 Year Old Firms to Firms Older than 26 Years	0.196	0.17

Table 1.3: Calibrated Moments for the Benchmark Model

decreases as the size increases. To focus on output fluctuations from changes in aggregate TFP, we assume that all entrepreneurs share the same aggregate TFP. This results in a mass on the right end. Obviously, a more realistic assumption on idiosyncratic TFP could relax this, but that exercise will not be covered in this paper. The distribution is skewed to the right, which is consistent with the empirical observation of Angelini and Generale (2008) and other papers.

To highlight the benchmark case, we compare the simulation of the benchmark model with three other simulations, along with the corresponding data. One is from the same economy without the corporate tax. Second, we provide results from the first best economy where there is no financial constraint at all. The behavior of this economy would be a lot like the behavior from the model in AH. The last

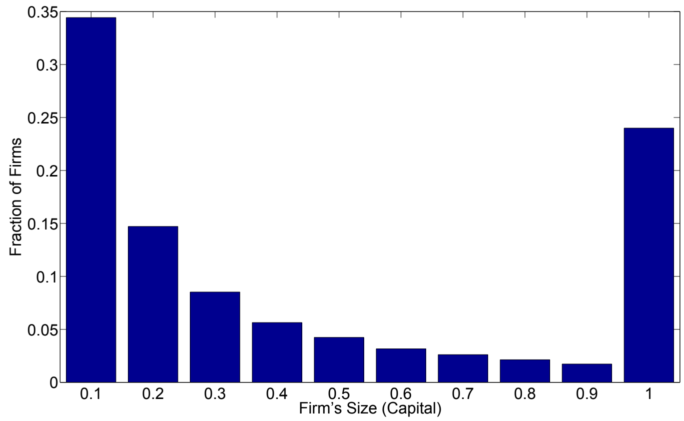


Figure 1.12: Firm Size Distribution

model is the standard real business cycles (RBC) model, which we include for comparison.[7]

We can check the history dependence of output amplification by regressing output amplification on lagged TFP growth. We measure output amplification by the difference between output growth and TFP growth, $gY - gTFP$. Table 1.4 displays the results of five regressions with different output series. Since they are time series regressions, there might be heteroscedasticity and auto-correlation in error terms. The OLS estimator for coefficients is consistent even in this environment. However, a usual standard error estimation might result in incorrect statistical significance of coefficient estimation. To address this concern, we report Newey-West standard errors for all regressions.

From the first column, we find that the coefficients of lagged TFP growth are positive and statistically significant up to lag 4. The standard errors are small enough so that their p-values are less than 0.01. Positive coefficients mean that output growth at period t is larger given the TFP growth at period t if there is positive growth of TFP in the preceding periods: $t-1$, $t-2$, $t-3$ and $t-4$.

TFP history affects output amplifications asymmetrically. Suppose that we have

[7]The specification of each model in simulations is provided in Appendix

a positive shock at period t. In this case, positive TFP growth history amplifies output growth. However, if there is a negative shock at period t, such history dampens output amplification. Therefore, the response of output growth will be small in a recession if TFP growth was positive before a recession. In other words, without a positive TFP growth history, more output amplification is expected in a recession.

	(1)	(2)	(3)	(4)	(5)
VARIABLES	Data	Benchmark	No_tax	FBest	RBC
L.gTFP	0.322***	0.682***	1.143***	0.215	0.0117
	(0.0569)	(0.0520)	(0.0243)	(0.197)	(0.0634)
L2.gTFP	0.279***	0.163***	0.0558**	0.143	0.00273
	(0.0592)	(0.0334)	(0.0242)	(0.178)	(0.0642)
L3.gTFP	0.271***	0.0913**	-0.00600	-0.312**	-0.147***
	(0.0433)	(0.0359)	(0.0207)	(0.155)	(0.0518)
L4.gTFP	0.129***	0.0134	-0.00147	-0.185	-0.0928
	(0.0480)	(0.0328)	(0.0183)	(0.184)	(0.0664)
L5.gTFP	0.0682	0.0481	0.00465	-0.0653	-0.0458
	(0.0463)	(0.0334)	(0.0242)	(0.187)	(0.0667)
L6.gTFP	0.0924**	0.0424	0.0293	0.00738	-0.0332
	(0.0451)	(0.0324)	(0.0236)	(0.151)	(0.0548)
L7.gTFP	0.0734	-0.0210	-0.0299	-0.129	-0.0724
	(0.0468)	(0.0300)	(0.0213)	(0.179)	(0.0636)
L8.gTFP	0.0809*	-0.0114	-0.000964	0.0530	-0.0155
	(0.0480)	(0.0337)	(0.0189)	(0.192)	(0.0646)
Constant	-0.000303	0.000620***	0.000123	0.000419	6.09e-05
	(0.000480)	(0.000186)	(0.000104)	(0.00111)	(0.000356)
Observations	243	243	243	243	243
P-value (F)	5.73e-10	0	0	0.236	0.0249

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 1.4: Output Amplification from the History of TFP

The regression with simulated data reveals that output amplification in the benchmark model also depends on the TFP growth rates (col. 2). The coefficients are positive and maintain statistical significance at 5% up to lag 3. Similar patterns can be found between the results from the data and the benchmark model. Both coefficients and the statistical significance decrease as lags increase. However, the coefficients and significance vanish more quickly in the benchmark model than in the real data.

If we remove the corporate tax (col. 3), the history dependence is concentrated in lag 1. The coefficient of lag 1 is much larger than in col. 2. Also, the coefficients and significance diminish even more quickly than the benchmark case. In the case of the first best (col. 4) and the standard RBC model (col. 5), we cannot find any history dependence of output amplification in general. There is only one statistically significant lag for each regression, but it has a negative sign. Furthermore, the hypothesis that all coefficients are zero cannot be rejected by F-tests in both cases. We can compare these results with the exercise in Section 1.3. In that exercise, the economy without the corporate tax shows some history dependence of output amplification, but the effects are weaker than the benchmark model, while the AH model did not produce any significant history dependence.

The regression results of the benchmark model seem to be robust to initial conditions. For the benchmark model, the initial condition refers to the initial distribution of firms over the capital grid. Note that the simulation starts from the smallest exogenous state (z^1) at 1948q1. Therefore, no matter what the initial distribution is, no firm will hold capital larger than $k^*(z^1)$ at 1948q2. This generates a similar distribution in 1948q2 from different initial distributions. We used the invariant steady state distribution with $z = z^1$ as the initial distribution of firms.

Since TFP growth and output growth are positively correlated, we can additionally check whether the benchmark model can address the well known problem of the standard RBC model: auto-correlation of output growth. Cogley and Nason (1995) argued that the standard RBC model cannot account for the fact that output growth is positively auto-correlated, in particular for lags 1 and 2.

Figure 1.13 shows the simulation results for each model. As in Cogley and Nason (1995), the data shows a positive correlation for lags 1 and 2, but the RBC model and the first best economy cannot. Overall, the benchmark model matches the data well by generating the largest auto-correlation for lags 1 and 2 among the models. The model without the corporate tax has a large positive correlation only for lag 1, which is similar to what we observed in Table 1.4.

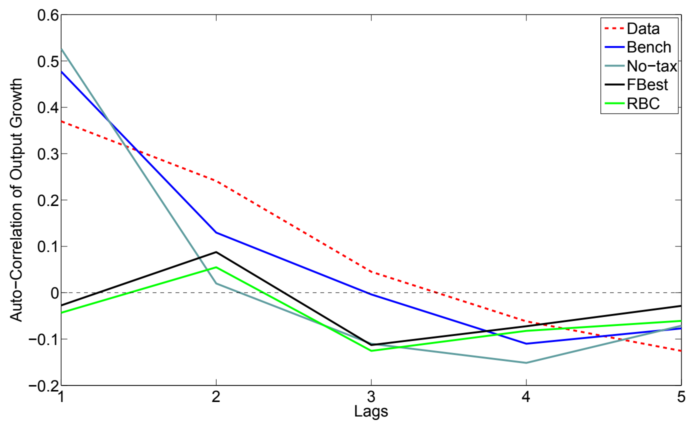
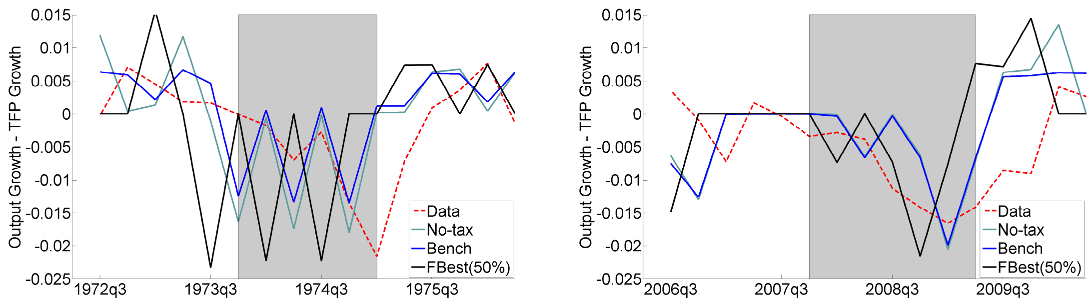


Figure 1.13: Auto-Correlation of Output Growth

Carlstrom and Fuerst (1997) also address the auto-correlation of output growth using a model with an endogenous financial constraint. In their setting, there is financial friction between entrepreneurs who make investment goods and lenders. Entrepreneurs are financially constrained and their borrowing size depends on their net worth. A positive productivity shock will lead to a gradual increase in entrepreneurs' net worth, and this will cause a hump-shaped response of investment and output. However, their model relies on the fact that more binding financial constraints generate more output reduction in a recession. Also, as denoted by **?**, financial friction in producing investment goods will not generate as much output amplification as financial friction in producing output goods.

To understand the output amplification mechanism behind Table 1.4, we compare the simulation results for two recessions: the 1973–1975 recession and the 2007–2009 recession. The two recessions were hit by a similar size of decrease in TFP growth but generated a large difference in output amplification. Figure 1.14 shows simulation results from each model over periods from these recessions. We scaled down the amplification of the first best by 50% to locate it in the same axis with the other graphs.



(a) the 1973 Recession (b) the 2007 Recession

Figure 1.14: Comparison of Output Amplifications from each Model

The benchmark model generates varying output amplification between these recessions. It exhibits a large output amplification in the Great Recession and a small amplification in the 1973–1975 recession. If we compare two quarters, 1973q4 and 2009q1, in which the same size of decrease in TFP growth hit, the benchmark model gives 60% more output amplification in the case of the Great Recession. This difference in amplification stems from the different TFP histories. There was a positive TFP growth right before the 1973–1975 recession, but we can observe tranquil TFP growth, close to 0, during the Great Recession.

There is some degree of history-dependent output amplification in the case of the model without the corporate tax as well. However, the intensity is less stark than the benchmark case. This point is in accord with previous results. Output amplification from the first best case is trivial. It is merely proportional to the size of the increase or decrease of TFP. Output amplification decreases almost exactly down to -0.02 between the two recessions. Since a firm in the first best economy always attains the efficient levels of capital, amplification only depends on the current exogenous states.

Table 1.5 reports the difference in output amplifications between the two recessions generated by each model. We calculate the total output amplification from each recession and calculate the difference between the two for each model and the data. First, we only choose quarters that were in recessions. However, since the first best economy reacts to the initial negative TFP shock before the 1973–1975 recession, calculation from this time window undervalues the difference from the first best economy. If we extend sample quarters to 16 quarters, as in Figure 1.14, the performance of the model without the corporate tax is better than the first best economy. In any case, the benchmark model can capture the difference between the two recessions better than other models.

Difference in Amplification	Data	Benchmark	No-tax	Fbest
In Recessions	-0.0195	-0.0041	0.0117	0.0082
Around Recessions	-0.0519	-0.0477	-0.0255	0.0004

Table 1.5: Difference in Amplification Between the Two Recessions

The output amplification in the benchmark model depends on the more longer-term history as well. The varying output amplifications are related to the distribution of firms. More specifically, the portion of firms that are mature enough to decrease capital levels upon impact of negative TFP shocks is critical in determining the size of amplifications. One way to measure the sensitivity from a negative shock is to find the fraction of firms which possess the efficient levels of capital. Figure 1.15 presents the 8 quarter moving average of the fraction of firms that hold efficient levels of capital from the benchmark model and the model without the corporate tax.

In the benchmark model, positive growth of TFP decreases the fraction of firms with efficient levels of capital. Note that during the 1960s, TFP was fast growing and increased the efficient level of capital. Since firms' capital growth is limited, the measure of firms with efficient levels of capital does not catch up quickly. However, as the Great Moderation began in 1984, TFP growth became tranquil and therefore more firms attained efficient levels of capital. As a result, not many firms attained efficient levels of capital before the 1973–1975 recession, but a large number of firms gained efficient levels of capital before the Great recession.

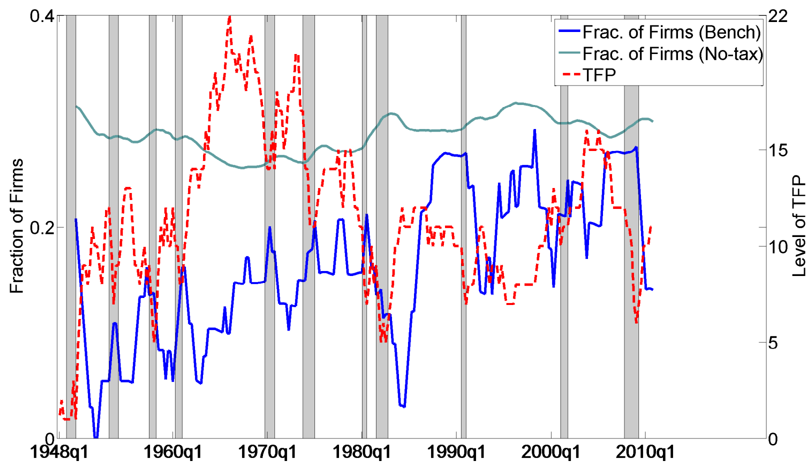


Figure 1.15: Fraction of Efficient Firms and the Level of Discretized TFP

This mechanism can be found in the model without the corporate tax, but Figure 1.15 displays a stark contrast between the models. The model without the corporate tax shows a much more stable fraction of firms with efficient level of capital. Compared with this difference observed in Figure 1.15, Figure 1.14 may seem like it is not showing enough of a contrast between the benchmark model and the model without the corporate tax. Among firms that decrease capital with a negative shock, not all firms have efficient levels of capital. As long as firms have more capital than the new efficient level of capital, they will reduce capital. Hence, the fraction of firms which possess efficient levels of capital is not a direct measure of amplification from negative TFP shocks.

Finally, Figure 1.16 presents the output amplification of the benchmark model for the whole sample period. Over the whole sample period, the benchmark model outperforms the other models in terms of matching the output amplification. Since output amplification is measured by the difference between output growth and TFP growth, this also means that the benchmark performs better than the other models in terms of output growth. To get a measure of the model's fit, we calculate the difference between two graphs: one is from simulation, and the other is from data. The values of the average absolute deviations are summarized in Table 1.6.

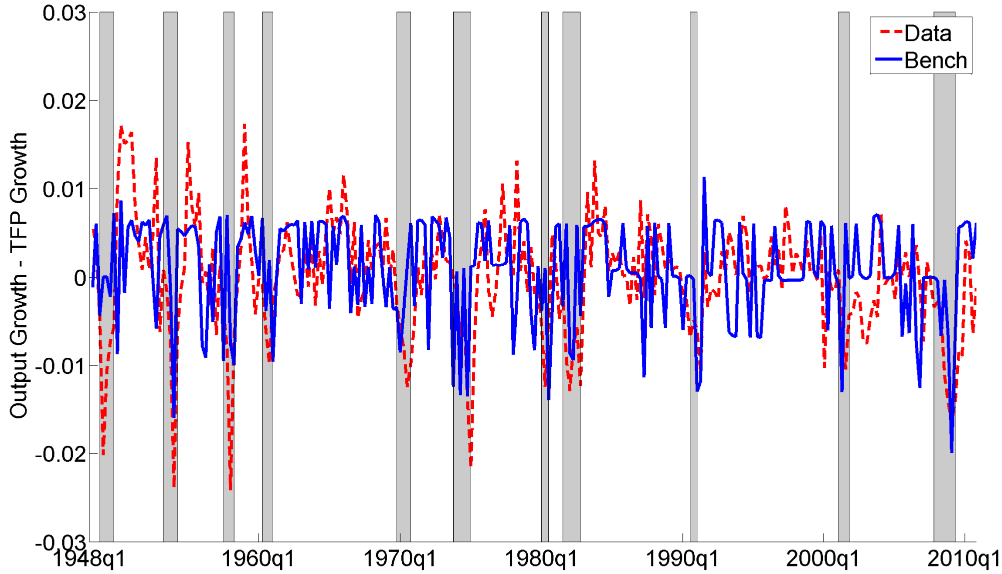


Figure 1.16: Output Amplification of the Benchmark Model and Data

	Benchmark	No_tax	First Best (50%)	RBC
Absolute Deviation	$5.54 \text{ x } 10^{-3}$	$6.68 \text{ x } 10^{-3}$	$8.21 \text{ x } 10^{-3}$	$6.675 \text{ x } 10^{-3}$

Table 1.6: Performance Comparison Between Models

Relation with Other Shocks

Papers such as Jermann and Quadrini (2012) and Nolan and Thoenissen (2009) find that fluctuations in borrowing size are helpful in explaining business cycles, in particular output fluctuations. They consider exogenous financial shocks which govern borrowing size. In the financial shocks literature, recessions with large output fall come with both TFP shocks and financial shocks. Our model shares the common characteristics with those papers in that, more amplifications of output are generated by a large reduction in the capital input. The amount of borrowing decreases more in severe recessions like the Great Recession from our model and also from papers with financial shocks.

The borrowing limit is given exogenously in the financial shocks papers whereas it is endogenous in this paper. In general, exogenous financial constraints do not generate history dependence of output amplification. Moreover, the borrowing limits are more binding in recessions and make firms reduce output more with aggregate negative financial shocks in those papers. However, in our paper, the financial constraints bind more in expansions and financially unconstrained firms will decrease output more in recessions.

Although it is not a direct measure, firm size is considered to be a proxy of capital market access. There are many empirical papers which discover a strong relationship between the two. For example, Angelini and Generale (2008) report that financial constrained firms are small firms on average. Beck et al. (2008) also find that financial development disproportionally benefits small firms. If small firms can be considered as more financially constrained firms, this paper can be supported by these recent empirical findings: as noted in Section 1.3, large firms are more cyclical than small firms.

More interestingly, our model is related to models of uncertainty shocks. Recently, studies such as Bloom et al. (2012), Schaal (2012) and Arellano et al. (2012), developed models using uncertainty shocks to explain the aggregate behavior of recessions. In the uncertainty shocks literature, more uncertainty decreases output, in particular, during the Great Recession. In these papers, increase in the dispersion of firms' sales growth is used as an evidence for the uncertainty shock. In our model, the increased dispersion of firms' output growth accompanies a large decrease in output growth during the Great Recessions.

Figure 1.17 shows the cross-sectioned interquartile range (IQR) of annual sales growth and the cross-sectioned IQR of firms' output growth from the benchmark model. The cross-sectioned IQR of annual sales growth is calculated using SALEQ in Compustat North America Fundamentals Quarterly. Most firms in the Compustat data are publicly traded, so they are highly selective.[8] Furthermore, the number of firms in Compustat changes dramatically as more firms are included in the data. The number of observations peaked in 1988q4. We include all data available to calculate growth rates but choose to display the series after 1989q1. We follow Arellano et al. (2012) to compute the growth rates of sales. We deflated sales data by the consumer price index (CPI), and then the growth rate is computed by $(sales_t - sales_{t-4})/0.5(sales_{t-1} + sales_t)$. We drop firms with less than 100 observations.

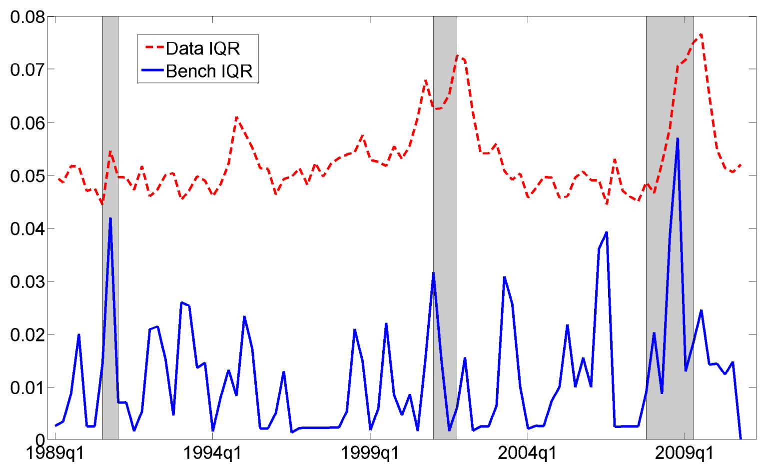


Figure 1.17: Standard Deviation of Firms' Output Growth

There are three recessions in the sample period. Both the benchmark model and the data generate increases in the IQR during recessions, with the largest increase occurring during the Great Recession. The observed uncertainty is countercyclical. Bloom et al. (2012) also discover the same pattern.

[8]For example, Axtell (2001) showed that the number of firms monotonically decreases as the firms' sizes increase in the data from the Census Bureau, but it is not true in Compustat. Davis et al. (2007) compare publicly traded firms, which are mostly in Compustat and privately held firms in the Longitudinal Business Database (LBD) from the Census Bureau, and show that there is a lot of difference between the two groups in terms of volatility and the dispersion of business growth rates.

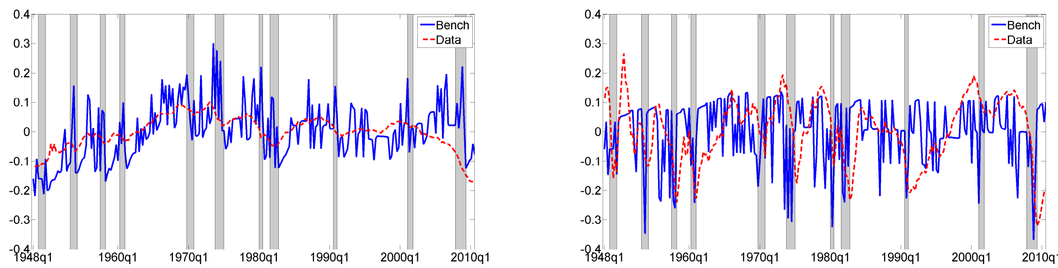
In the benchmark model, some firms increase capital while others decrease it with a negative TFP shock. This naturally leads to a positive level of growth dispersion among firms. Given the calibrated values of parameters in the benchmark case, the number of firms which decrease capital is smaller than that of firms which increase it. If we observe more amplification of output from the same negative shock, it comes together with greater dispersion of firms' output growth. Therefore, the large output growth dispersion in the Great Recession can also be a characteristic of our model with more amplification from a negative shock.

This explanation can be applied to other recessions in our model. In the 1990–1991 recession, output growth rates decreased more than in the 2001 recession. Hence, the model generates more dispersion for the 1990–1991 recession. However, the model cannot match the relative size of IQR in those two recessions. One possible reason can be the lack of idiosyncratic shocks in our model.

1.5 Extension

Figure 1.18 shows the percentage deviations of consumption and investment from trends. Because investment was too volatile, we scale the series down by 50%. The benchmark model generates large consumption and investment volatility since agents are risk-neutral. Moreover, there are overshoots of consumption at the moments of the arrivals of negative TFP shocks. The lack of consumption smoothing motives make mature firms decrease investment a lot with negative TFP shocks. A large fall in investment results in an increase in consumption, and, in particular, in the consumption of entrepreneurs. This increase in consumption during a recession can also be found in other heterogeneous agents model with financial frictions. For example, Carlstrom and Fuerst (1997) report a decrease in the consumption of entrepreneurs with positive TFP shock. However, the relative sizes of decreases in investment across recessions are in line with the data. In particular, the largest fall in investment is observed during the Great Recession from the benchmark model.

These two issues - too much volatility and overshooting - require different modifications. To reduce volatility, we could introduce a consumption smoothing



(a) Consumption

(b) Investment

Figure 1.18: The Simulation Results of the Benchmark Model

motive by making agents risk-averse. In particular, since financial intermediaries are making consumption-investment choices for maximizing workers' utility, we need risk-averse workers. The overshooting is due to a sudden increase in the consumption of entrepreneurs upon the impact of a negative TFP shock. The entrepreneurs' consumption (dividends) can be controlled by changing default incentives. Dividends are the difference between the equity values of yesterday and today. By making the size of the dividend small relative to workers' consumption, we can make aggregate consumption decreases in recessions.[9]

We now show how to extend the model to address these issues. Let us consider the situation where there is no birth or death of entrepreneurs. Entrepreneurs and workers are composed of representative agents. We modify workers utility function to $U(c,n) = \log(c) + A(1-n)$, where A is the coefficient for leisure. We assume that entrepreneurs get only a fraction ϕ of borrowed capital when they default. The dynamic programming problem of optimal contracts is then:

$$W(k,v,z) = \max_{n,d,k',V'} \log(c) + A(1-n) + d + \beta E[W(k',v',z')]$$

[9]This is similar in spirit with Carlstrom and Fuerst (1997). They resolve this issue by the relatively small portion of entrepreneurs' consumption. An entrepreneurial labor share is 0.0001.

subject to

$$
\begin{aligned}
c &= (1-\frac{v}{T_c})e^z k^\theta n^{1-\theta} + (1-\delta)k - k' - d \\
v' &\geq \phi k' \\
v &\geq d + \beta v' \\
d &\geq 0
\end{aligned}
$$

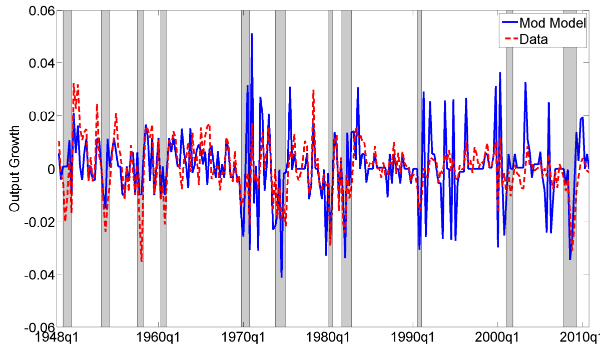
With parameter values given in Table 1.7, the simulation results of the modified model are shown in Figure 1.19. With such modifications, the volatility of consumption and investment are dampened and there is no more overshooting of consumption. Furthermore, this example shows that the main effect of the model survives with the risk-averse utility function as well. From 1948 to 1970 and from 2000 to 2005, we know that a representative firm is growing since investment tends to increase. During those periods, the decrease in output growth is dampened.

β	A	δ	θ	ϕ	T_c
0.996	2.4	0.019	0.321	0.5	200.0

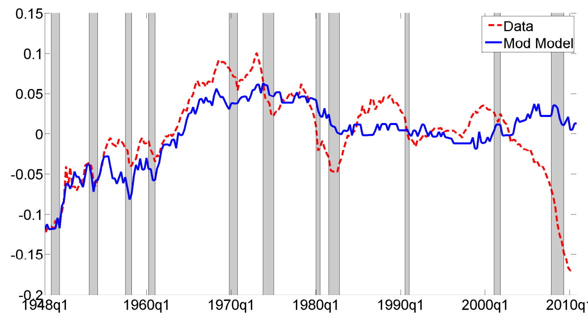
Table 1.7: Parameters

The model is difficult to solve numerically if we add individual death shocks to entrepreneurs with risk-averse agents. With the same utility function above, let us denote p as the marginal utility of consumption $p \equiv \frac{\partial U(c,n)}{\partial c}$. Disutility from labor is $\frac{\partial U(c,n)}{\partial n} = -A$. Then the optimal wage should be $w = \frac{A}{p}$. Let us call the distribution of firms in the economy as μ. Now, we can characterize the individual optimal policy by dynamic programming similar to Khan and Thomas (2003).

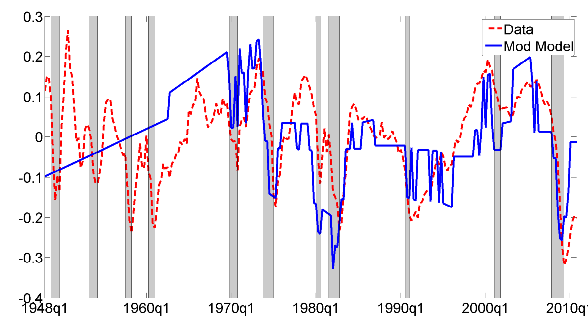
$$
W(k,v;z,\mu) = \max_{n,d,k',v'} ((1-\frac{v}{T})zk^\theta n^{1-\theta} - wn + (1-\delta)k - k' - d)p + d + \beta E[W(k',v';z',\mu')] \tag{1.18}
$$



(a) Output Growth



(b) Consumption



(c) Investment

Figure 1.19: Simulation Results of Modified Model

with the following restrictions

$$v' \geq k'$$
$$v = d + \beta v'$$
$$d \geq 0$$

Note that the marginal utility of consumption is not constant anymore; rather, it is a function of the state variables of an economy. That is, we have

$$p' = \hat{p}(z, \mu)$$
$$\mu' = \hat{\mu}(z, \mu)$$

where $\hat{p}$ and $\hat{\mu}$ denote the corresponding functions for each variable.

This problem includes a distribution as an endogenous state. A famous method was suggested by Krusell and Smith (1998) for solving such a problem. This method uses the first moment to represent the whole distribution. The method provides accurate approximation when the optimal policy function of dynamic programming is linear or close to linear. However, as we have seen above, the optimal policy function of this problem is non-linear.

A slightly different approach from Krusell and Smith is suggested by Den Haan and Rendahl (2010). They approximate optimal policy functions with polynomials and deduce the law of motion of aggregate variables from the approximation. This method provides one of the most efficient ways to calculate such a problem.[10] When we apply this method, we need at least up to the fourth moment to get a reasonably accurate approximation. However, we were unable to obtain satisfactory numerical results.

Heterogeneous Reactions and Volatility

The benchmark model with risk-neutral agents always generates smaller output amplification responses than that from the first best. This result is obvious because, with a negative TFP shock, small firms are increasing capital and mature firms are decreasing capital, but not by more than the firms in the first best case. Then it is legitimate to question whether the mechanism in our model constantly generates smaller responses with risk-averse agents than the RBC model since the RBC model

[10] Den Haan (2010) compares various techniques and found that the Den Haan and Rendahl method is one of the most efficient choices in terms of accuracy and time cost.

can be considered as the first best with risk-averse agents. However, this analogy breaks down with the heterogeneous reactions of the firms.

In this section, let us exclude the labor sector and dividends for convenience. We assume a log utility function. In equation (1.18), p represents the marginal utility of consumption. p and p' with homogeneous firms can be represented by:

$$p = \frac{1}{c} = \frac{1}{zk^{\theta} + (1-\delta)k - k'}$$
$$p' = \frac{1}{c'} = \frac{1}{zk'^{\theta} + (1-\delta)k' - k''}$$

Consider a situation where there is a fall in z so that agents want to decrease k'. However, if they decrease k' too much, it will increase c and decrease p. At the same time, it will decrease c' and increase p'. This decrease in current marginal utility and increase in future marginal utility make current consumption less attractive but future consumption more attractive. Therefore, too much reduction of k' generates a force that increases k'. This makes the RBC model with risk-averse agents generate much more dampened responses from TFP shocks than the first best with risk-neutral agents.

In the benchmark model, there are small firms and large firms which give heterogeneous responses of k' to the same TFP shocks. Let us assume that there are two firms: firm A is small so that it increases k' with a negative TFP shock, and firm B is large so that it decreases k'. Then, p and p' can be represented by:

$$p = \frac{1}{c} = \frac{1}{\sum_i zk_i^{\theta} + (1-\delta)k_i - k_i'}$$
$$p' = \frac{1}{c'} = \frac{1}{\sum_i zk_i'^{\theta} + (1-\delta)k_i' - k_i''}$$

where $i \in \{A, B\}$.

Again, consider a situation with a fall in z. Since firm A is increasing k_A' from a negative TFP shock, a decrease in k_B' will not decrease p or increase p' as much as in a homogeneous case. This makes firm B decrease more k_B' than in the homogeneous case. Therefore, it is not certain whether the benchmark model with risk-averse agents would generate a more dampened response than the RBC model.

1.6 Conclusion

In this paper, we present a model which can explain the varying output amplifications across recessions and the large amplification in the Great Recession. We argue that the history dependence of output amplification provides an explanation for these observations. For example, in the 1973–1975 recession, the growth of TFP right before the recession generates a dampened response of output to a negative TFP shock. On the other hand, the tranquil TFP history during the Great Moderation results in a large output amplification in the Great Recession.

The source of this history dependence is the growth restriction on capital. The optimal contracts with contractual frictions such as limited commitment and limited liability restrict efficient lending. The corporate tax and non-contingency of the next period's capital and equity value impose a strong growth restriction even for large firms.

The model has several other implications. First, it can explain the recent empirical observations: large firms are more cyclical than small firms in recessions. Second, it can endogenize the variables that are used to support financial shocks and uncertainty shocks. Finally, it helps explain stylized facts on firm financing and firm dynamics.

Since the model is highly tractable and only one exogenous variable is adopted, it could be used for various applications. For instance, the model can be used for policy evaluations. Once risk-averse agents are adopted, it would be interesting to study welfare effects from tax policies. Since the corporate tax tends to decrease aggregate volatility, we may see non-trivial welfare effects from tax policies. Furthermore, since the financial friction in our model does not depend on the balance sheet effects, there may be new implications for the role of monetary policy. These research topics are left for the future.

2 LIMITED CONTRACT ENFORCEABILITY AND NEWS SHOCKS

2.1 Introduction

Recent literature discovered the importance of news shock or anticipated shock in studying business cycles. Beaudry and Portier (2006) found out the prominent role of anticipated shock in explaining the variance of forecast error in consumption, output and labor time series in their Vector Auto-Regression (VAR) system. Schmitt-Grohé and Uribe (2012) argue that the anticipated shocks account for two thirds of variance of macroeconomic variables such as output, consumption, investment and labor.

However, as noted by previous research such as Beaudry and Portier (2004), Cristiano and Fitzgerald (1998) and Jaimovich and Rebelo (2009), hereafter JR, the neoclassical models in macroeconomics generate a recession from news of increasing future TFP. Such good news have positive wealth effects. So, agents work and save less and that decreases output and investment.

To generate increasing output, it is critical to have a large enough substitution effect (from wage increase) which overwhelms the wealth effect. For this purpose, Jaimovich and Rebelo (JR, 2009) propose three features to the standard dynamic stochastic general equilibrium model. Those are variable capital utilization, adjustment costs to investment and preference with low wealth effect. Although they succeed in hindering recession from good news shock, their features do not have microeconomic foundations and the magnitudes of responses are comparably small.

In this paper, We propose a new way of explaining the business cycles from news shock by adopting the decision of firms about default and market exclusion. If firms have default option, the lending party must consider this incentive to default. But if profitable future is expected and there is strict market exclusion after default, firms will not default today. And it can help to explain output increase from news shock.

Since this paper makes use of financial friction, it is related to the contract theory literature. With limited enforcement of contracts, there is an incentive for borrowers not to honor their obligation – a default. To prevent default, the optimal contracts in this setting include various incentive schemes to exclude default. As stated in Ljungqvist and Sargent (2004), "stick and carrot" are used in this environment to deter default.

For example, in Cooley et al. (2004), financial intermediaries use next period capital to prevent default of entrepreneurs. Because firms can divert capital, the opportunity to borrow a lot of capital next period means more profit when they stick to the existing contract. So, it acts like "carrot" which prevents default.

The use of next period capital is similar in this paper too. It is also used as "carrot" because firms can divert borrowed capital in this model. However, next period capital is the result of optimal contracts in their paper. It is under the control of financial intermediaries which offer contracts. But, in this paper, it is determined by the expected technology level and supply and demand of capital. Financial intermediaries do not command directly the level of next period capital.

Instead, intermediaries control the interest rate controls default. Firms are more likely to default under a high interest rate, and a low interest rate decreases default. Since interest rate also determines the profit of intermediaries, they charge the highest interest rate without the default of firms. With respect to controlling default, this paper is close to Stiglitz and Weiss (1981). In their paper, a bank cannot increase interest rate because only risky firms apply for loans. In my model, intermediaries also face a trade-off where high interest rate increases profit and the incentive to default.

In this paper, We focus on the role of market exclusion as a "stick". If a firm will be out of market after default, default decision should include the expected profit in the future. If big profit is expected in the future, not default today is more attractive. If positive technology shock implies increased profit of the firm, good news shock of future technology level can decrease default. In the literature, Cooley et al. (2004) study the role of market exclusion after default. They found that no market exclusion after default brings more volatility in output from temporary technology shock and less growth from permanent technology shock.

Besides, We found that the benchmark model which was intended for news shock is useful to explain the relationship between contract enforcement and GDP per capita. Each country has different degrees of contract enforceability. Data shows that there is a strong positive connection between two variables across countries. The benchmark model could generate a positive relation between steady state output and contract enforcement. In this paper, the financial friction requires high level of marginal productivity of capital so that it only allow the economy to maintain limited amount of capital. That level of capital brings out small output.

This paper is organized in the following way. In section 2.2, I briefly introduce the logic of the model with a simple example. Section 3.2 contains the description of the full model. In section 2.4 the simulation result is provided. Section 2.5 is about the relationship between contract enforcement and output. Section 2.6 concludes.

2.2 The Logic Behind The Model

Single Period Example

Before we study the model, let us look at the problem of firms in a single period. Let us assume full depreciation. Since all firms will not have any capital at the beginning of every period, projects must be funded by banks. The financial contracts are made between banks and firms. They include the amount of funds and the lending interest rate. Let us denote the amount of loan as k_i and the interest rate as r_i for firm i. When the contracts are made all information about production including the technology level, is known to both parties.

After the financial decision, firms decide whether they will involve in production or not. If production occurs, banks can observe the output of the project, $y_i = A_i k_i^{\alpha} l_i^{1-\alpha}$ and retrieve their investment and interest. If the output is less than the size of repayment and labor cost, a firm can claim default. Then its obligation to repay vanishes. But the default firms never get the chance to be financed in the future. If a production doesn't occur, the loan can be hoarded. Then output is zero and the profit is just, k_i. In this case, default follows.

So given r_i and k_i, the net profit from each project at t is

$$\begin{aligned} &\max[\sup_{l_i} A_i k_i^{\alpha} l_i^{1-\alpha} - (1+r_i)k_i - wl_i, k_i] \\ =&\max[A_i k_i^{\alpha} l_i^{*1-\alpha} - (1+r_i)k_i - wl_i^*, k_i] \\ =&\max[\pi_i^*, k_i] \end{aligned}$$

where w is the real wage, l_i^* satisfies $(1-\alpha)A_i k_i^{\alpha} l_i^{*-\alpha} = w$, and π_i^* is the profit under execution.

Given the above firms' behavior, banks will maximize their profit. That is, banks should charge the highest interest rate. In this setting, however, a too high interest rate results in default of the firms. Banks should set their interest rates so that the firms get at least a profit of k_i. Therefore, the interest rate will be determined by:

$$\max r_i \qquad s.t.\ \pi_i^* = k_i$$

The optimal interest rate can be denoted by:

$$r_i^* = \frac{A_i k_i^{\alpha} l_i^{*1-\alpha} - wl_i^* - 2k_i}{k_i}$$

In the sense that high interest rate results in risky behavior of the firms, this situation is similar to the one in Stiglitz and Weiss (1981). Note that since this case is just a single period, there is no role of market exclusion after default.

Two Periods Example

Now, let us assume two periods. In this situation, firms' behavior in period 1 will be affected by the expectation of period 2. Let us assume that high technology is associated with more investment. Suppose that there is news in period 1 about positive technology shock in period 2. Since firms' problem in period 2 will be the same as above, the profit of period 2 is $k_{i,2}$. A better technology in period 2 means the profit of the firm will be higher than the previous example, i.e. $k_{i,2} > k_i$.

Given this, firms' problem in period 1 can be denoted by:

$$\max[\sup_{l_{i,1}} A_{i,1}k_{i,1}^{\alpha}l_{i,1}^{1-\alpha} - (1+r_{i,1})k_{i,1} - wl_{i,1} + k_{i,2}, k_{i,1}]$$

Here, the firm will be out of market if it is defaulted in period 1. The firm can enjoy $k_{i,2}$ only after not defaulting in period one, thus $k_{i,2}$ appears in the first part only. Then the bank will set their interest rate to solve the following:

$$\max r_{i,1} \qquad s.t.\ \pi_i^* = k_{i,1}$$

The solution can be described by:

$$r_{i,1}^* = \frac{A_{i,1}k_{i,1}^{\alpha}l_{i,1}^{*1-\alpha} - wl_{i,1}^* - 2k_{i,1} + k_{i,2}}{k_{i,1}}$$

Note that now, banks can charge much higher interest rates compared with the single period case; $r_{i,1}^* > r_i^*$. The reason is that firms fear the market exclusion from default because there will be positive profit in period 2, which makes firms behave less riskily in period 1. Banks make use of this fact and raise the interest rate to extract maximum profit from firms.

2.3 The Model

There is a continuum of agents and firms each on the unit interval $[0, 1]$. Let us assume that all firms' technology levels are the same in a given period. Agents' portfolio is all the same with the same amount of stock for each firms. In other words, each agent has measure 0 stock for each firm. Therefore, no one controls firm's behavior and there is no stock trading in equilibrium. Also, there is no individual shock for firms and agents. We can therefore consider agents and firms as representative entities.

The representative agent maximizes her expected utility

$$\max E_0[\sum_{t=0}^{\infty}\beta^t U(c_t, l_t)]$$

where c_t is consumption and l_t is the amount of labor supply. The budget constraint for the representative consumer is

$$(1 + r_t^d)s_t + w_t l_t + \pi_t = c_t + s_{t+1}$$

where s_t is the saving of each agent, r_t^d is the deposit interest rate, w_t is the real wage, and π_t is the profit from the stock holding.One thing to note here is that the deposit interest rate from t to $t + 1$ is unknown at t. Labor market is competitive, so the real wage is determined by the marginal production of labor.

In this economy, there is only one channel that links saving and investment — a bank. The representative firm starts with no capital because it distributes profits to shareholders at the end of each period. The firm must borrow all the capital from the bank to produce. The firm's production function is $A_t k_t^{\alpha} l_t^{1-\alpha}$.

After borrowing, the firm has a choice between production and hoarding. If it chooses to produce, its outcome will be monitored and the bank will retrieve their investment and interest. If it chooses not to produce, the firm can hoard the loan and its profit will just be k_t. But after default, the firm will be out of market forever, which means zero profit afterwards. The firm is determined to maximize its profit even if it means default.

The representative firm implies that there will be no default in the equilibrium as long as there is output. The competitive banking industry leads to zero profit for the bank. The lending interest rate in equilibrium will be the same as the deposit interest rate, $r_t = r_t^d$. Therefore, the profit of the bank is always zero not only ex ante but also ex post. Since the measure of firms and agents are the same, s_t becomes not only saving, but also the amount of capital to the representative firm (s_t=k_t).

The Determination of the Interest Rate

During the borrowing process, the bank make a take-it-or-leave-it offer to firms. The bank writes down the interest rate which is determined to maximize the bank's profit given the amount of deposit since the bank should loan out all its deposit. As we saw in a 2 periods example, we need to consider the sum of the future profits in each period from $t+1$ to find out the interest rate at period t for this multi-periods economy. Let the current value of all future profits from $t+1$ be V_{t+1}. Here, I assumed that the discount rate of the firm is the same with the agent's discount rate. Then the interest rate is determined by

$$\max r_t \tag{2.1}$$

$$s.t. A_t k_t^{\alpha} l_t^{1-\alpha} - w_t l_t - (1+r_t)k_t + (1-\delta)k_t + \beta E_t[V_{t+1}] \geq k_t \tag{2.2}$$

In other words, the bank will charge a firm the highest interest rate as long as a firm's current value of all future profit from t does not fall below k_t. Otherwise, the firm will default and the bank can not recover their investment.

If the relationship between the bank and the firm is as above, the current value of all future profits in period t will just be k_t ($V_t = k_t$) no matter what the firm's profit is in each future period. By updating one period, we can write the future profit V_{t+1} as k_{t+1}. Therefore, the interest rate will be given by:

$$r_t^* = \alpha A_t k_t^{\alpha-1} l_t^{1-\alpha} - \delta + [\beta \frac{E_t[k_{t+1}]}{k_t} - 1] \tag{2.3}$$

The Role of the Modified Interest Rate

The effect of modified interest rate can be well understood when we compare two different steady states. At the steady states, $k_t = k_{t+1}$. So the modified interest rate is equal to the interest rate in the standard DSGE model with additional term, $1 - \beta$.

Let us consider the situation where a permanent and positive technology shock at $t+p$ is anticipated at t. Once the actual shock arrives at $t+p$, the jump of technology level induces a new steady state. Let us denote k^*_{t+p} as the steady state capital level after the actual shock arrives and k^*_t as the steady state capital level before any shock.

We know that as A increases, the level of steady state capital increases too: $k^*_{t+p} > k^*_t$. When there is a difference of capital level across time, the modified interest rate rule, (2.3), makes interest rate higher than the standard rule. As we explained before, k represents the current value of all future profit. So expected increased profit leads banks to set higher interest rate today, given that firms will not default to seize the profit in the future.

This high interest rate induces more saving from agents. Since saving is investment in this economy, it will lead to increased investment. That raises marginal productivity of labor, which is real wage. As agents put in more labor, output increases from news shock.

Equilibrium

With this new interest rate, we have the following competitive equilibrium.

Definition 2.1. *The competitive equilibrium consists of the prices $\{r_t, w_t\}_{t=0}^{\infty}$ and the quantities $\{c_t, l_t, y_t, k_t\}_{t=0}^{\infty}$ such that*

1. *Given r_t and w_t, agents maximize their expected utility by choosing c_t, l^s_t, and k_{t+1}.*
2. *Given r_t and w_t, firms maximize their expected profit by the decision about default and choosing l^d_t .*
3. *Banks maximize their profit for each period by choosing r_t such that $r_t = \alpha A_t k_t^{\alpha-1} l_t^{1-\alpha} - \delta + [\beta \frac{E_t[k_{t+1}]}{k_t} - 1]$.*
4. *Goods, capital and labor markets clear.*

The first-order conditions and market clearing conditions are

$$\begin{aligned}
U_c(c_t, l_t) &= \beta E_t[U_c(c_{t+1}, l_{t+1}) r_{t+1}] \\
U_c(c_t, l_t) w_t &= U_l(c_t, l_t) \\
k_{t+1} &= A_t k_t^{\alpha} l_t^{1-\alpha} + (1-\delta)k_t - c_t \\
r_t &= \alpha A_t k_t^{\alpha-1} l_t^{1-\alpha} - \delta + [\beta \frac{E_t[k_{t+1}]}{k_t} - 1] \\
w_t &= (1-\alpha) A_t k_t^{\alpha} l_t^{-\alpha}
\end{aligned}$$

And the shock process is

$$lnA_t = \rho lnA_{t-1} + e_{t-p} - u_t$$

This process is from Christiano et al. (2011). In this way, e shock acts as a news shock. Agents will know that there will be an increase in technology level in p periods after e shock. Furthermore, we can make false news shock by manipulating u shock. For example, let us assume that e shock came in at period 0. By giving a u shock at period p, we can counteract the effect of e shock. Since agents can not predict u shock, e shock becomes the false news shock in the end.

2.4 A Quantitative Evaluation of the Model

Impulse Responses from News Shock

We solve the model by log-linearizing the equations around the steady state. We use the parameter values below for our benchmark case. Table 2.1 summarizes the parameters of the benchmark model.

The first parameter (σ) is the inverse of elasticity of intertemporal substitution in consumption. It is set to give high elasticity compared with other news shock papers. For example, JR set it as 1. β and α are from JR. The last parameter, depreciation rate δ, is from King et al. (1988).

Parameters	Values
σ	0.4
α	0.36
β	0.985
δ	0.025

Table 2.1: Parameters

Since we are considering the case of permanent technology shock, ρ will be set to 0.999. p is set to 2 as in JR. So, the news shock is announced at period 1. The actual shock arrives at period 3.

In other news shock papers such as JR and Beaudry and Portier (BP, 2004), the main element which hinders the model from generating a boom is the wealth effect. The positive wealth effect from the expected increase of technology makes agent work less and generates a recession. So we will first consider the utility function of King, Plosser and Rebelo (KPR, 1988), which makes labor supply sensitive to wealth effect.

The following impulse responses are from the shock of 0.01 increase of technology level.

KPR utility function

The utility function in KPR takes the form of

$$U(c_t, l_t) = \frac{(c_t(1-l_t))^{1-\sigma} - 1}{1-\sigma}$$

Figure 2.1 is the impulse responses of variables from the benchmark model. All vertical axises in Figure 1 are percentage deviation from steady states. Here, labor supply and investment are increasing between the news shock and the actual shock. So output is increasing. There is a boom from news shock.

This effect comes from two facts. First, the interest rate jumps up in this model when there is news shock. The actual shock, which is realized at period 3, makes the steady state value of k higher. That affects the current interest rate via the modified interest rate rule. Therefore, although the shock is not realized yet, the interest rate goes up.

Then agents invest more capital. It increases marginal productivity of labor. So agents spend more time at work.

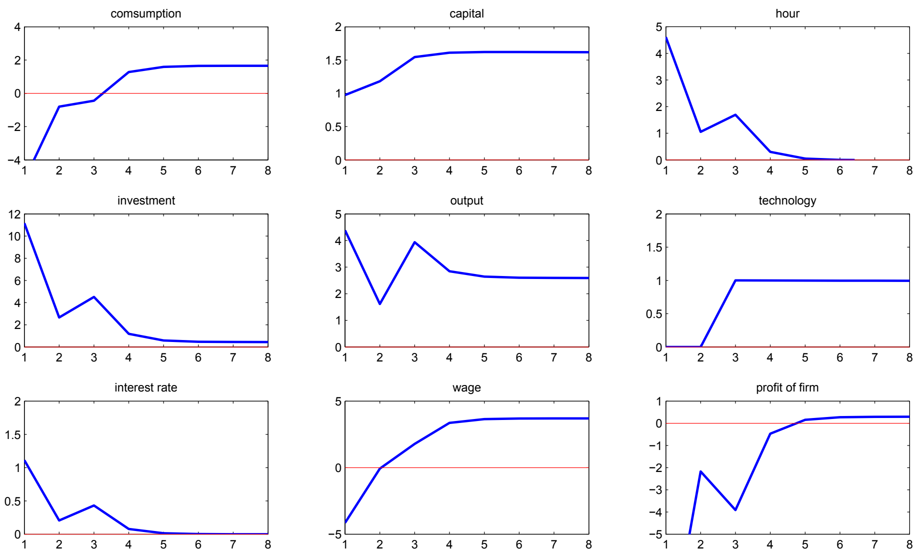


Figure 2.1: Benchmark Case

Second, between the news shock and the actual shock, the modified interest rate is higher than marginal productivity of capital. Since the Cobb-Douglas production function is assumed, this means that the current profit of firms diminishes. Because the profit of firms is distributed to agents every period, it causes negative wealth effects. Hence, agents put in more labor.

This channel can be also described as the distortion of economy from news shock. This distortion from the modified interest rate makes agents invest too much and prevents the economy from the efficient allocation. This inefficiency causes negative wealth effect and makes agents work more.

Unlike other papers with news shocks, there are two different wealth effects here. One from the news of advanced technology level makes positive wealth effects, decreasing labor supply. Whereas, the distortion of adjusting from news shock makes negative wealth effects and induces more labor supply.

It is noticeable that the size of deviation is greater when the news hit the economy than when the actual shock arrives. That is particular feature of the benchmark case because previous papers such as JR and BP do not generate such movements. However, this is sensitive to the parameter choices. Table 2.2 shows the range of the elasticity of intertemporal substitution which produces such results. More responses from news shocks than from actual shocks require large elasticity.

	Value of σ
The minimum value which satisfies Blanchard - Kahn condition	0.33
The maximum value which generates bigger impacts of news shock	0.41
The maximum value which increases output from news shock	1.44

Table 2.2: Robustness of σ

In case of benchmark, consumption decreases at the impact of news shock. When consumption decreases, it is less costly for agents with KPR utility function to increase labor supply. Because consumption and labor are compliments. Therefore, with increased elasticity, there will be more decrease of consumption from news shock. That will make it easier for agents to supply even more labor.

In Figure 2.2, the impulse responses of the unit ρ case is provided. Here, the responses of the news shock are mitigated. The responses from actual shock outweigh the responses of news shock.

There is an ongoing debate in the literature about this elasticity. Hall (1988) estimates it as 0.1. But with that value, Weil (1989) finds that the plausible range of risk free interest rate can not be achieved.

We could not generate the same directional changes of consumption and investment in this benchmark. As in Beaudry and Portier (2007), typical Cobb-Douglas

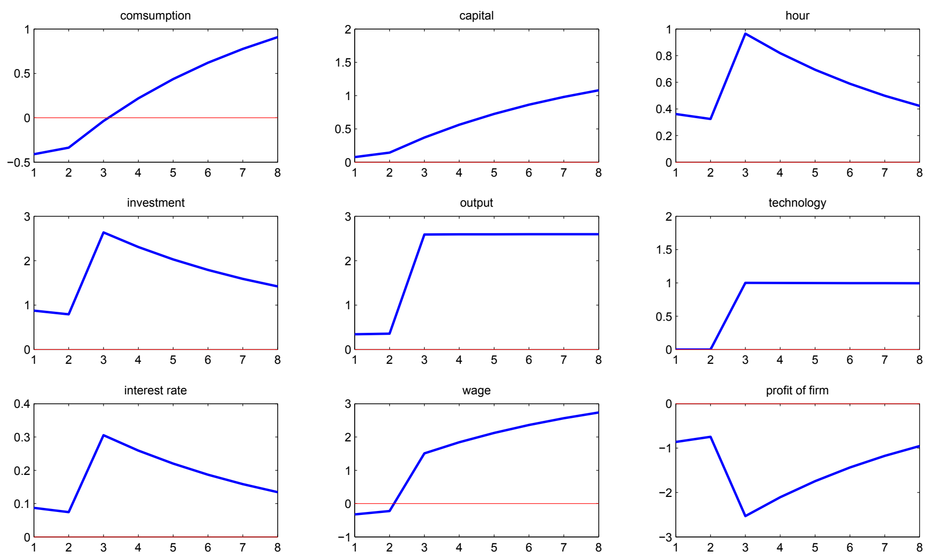


Figure 2.2: Unit σ Case

production function and market clearing condition can not generate the same directional change of consumption, labor and investment from a change in expectation.

We provide the impulse responses from the standard model in Figure 2.3. The standard model has one different equation for its first order conditions from the benchmark; r_t is just marginal product of capital. As we can see in Figure 2.3, news shock is producing recession before the actual shock arrives in benchmark model because the wealth effect from future income is bigger than the substitution effect of wage increase.

GHH utility function

Now, consider the utility function as in Greenwood et al. (1988).

$$U(c_t, l_t) = \frac{(c_t - \frac{1}{1+\theta} l_t^{1+\theta})^{1-\sigma} - 1}{1-\sigma}$$

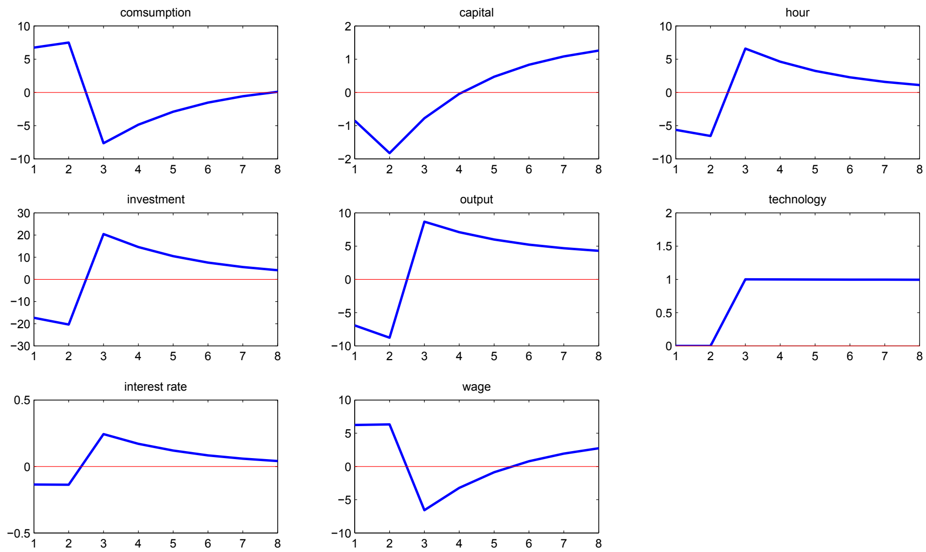


Figure 2.3: Standard Model Case

This utility function in Greenwood, Hercowitz and Huffman (GHH, 1988) generates labor supply which is less susceptible to wealth effect compared with KPR utility function. We chose $\theta = 0.4$ as suggested in JR. The impulse responses of GHH case are in Figure 2.4.

The directions of impulse responses in Figure 2.4 are the same as in Figure 2.1. Labor is strictly increasing and investment is increasing, too. So, output is increasing when there is a news shock. But consumption is still decreasing.

By comparing hour in Figure 2.1 and Figure 2.4, we can check the role of wealth effects to labor supply in benchmark case with KPR utility function. Since labor supply from GHH utility function is only affected by real wage, it is independent from the wealth effect. Figure 2.5 compares between the two.

The solid line is from KPR utility function and the dashed line is from GHH case. In Figure 2.5, the increase of labor supply before the realization of news shock is smaller in GHH case than in KPR case. Therefore, the wealth effect in KPR case

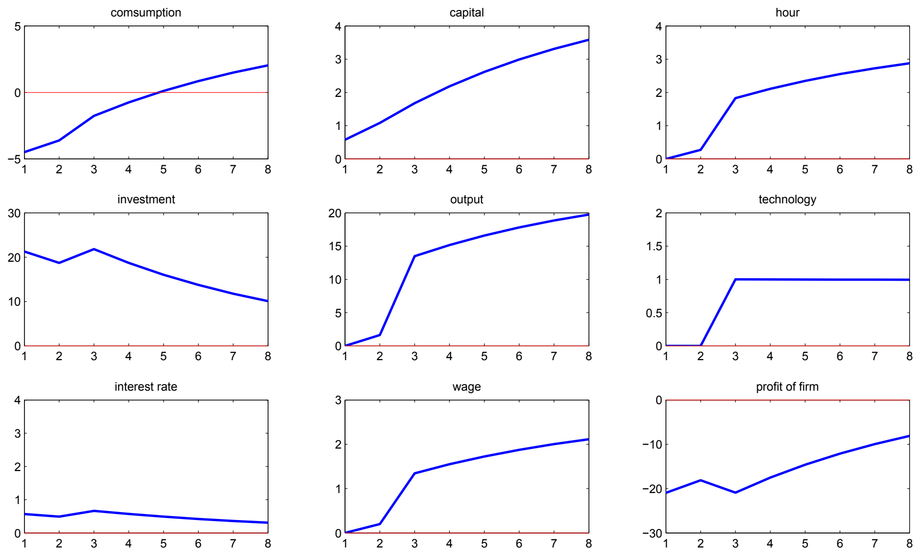


Figure 2.4: GHH Utility Function Case

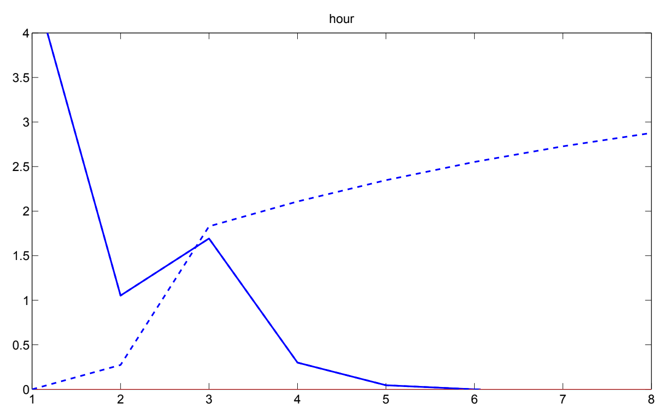


Figure 2.5: Hour from KPR vs GHH

actually helps to increase labor supply. In other words, the negative wealth effect from diminished dividends is greater than the positive wealth effect from expected future productivity. This result is opposite from the previous literature which mainly generates positive wealth effect from news shock.

Pigou Cycles

A line of research in news shock literature including BP and JR investigates the ability of the model to produce sizeable recessions in a situation where agents learn about the positive news but the actual shock never comes. This explanation for a recession has a long history, dated back up to Arthur Cecil Pigou. So, it is called Pigou cycles. Recessions from misleading expectation about the future are considered to exist in the real world. For example, the recession of 2001 right after dot come boom is believed to come from the wrong expectation about the future.

Figure 2.6 shows the comparison between the impulse response from the model and the average time series during recession periods for the U.S. economy. Solid line is from the benchmark model with KPR utility function. In this case, the agents receive news about the 0.07 increase in technology 2 periods later at period 1. But at period 3, it turns out to be a wrong expectation. The technology level remains constant. These solid lines are impulse responses from that process. The vertical axis means the deviations from the steady states.

Dashed lines are from the U.S. economy. They are the average of the deviations from HP-filtered trend. Period 2 represents the peak dates of the National Bureau of Economic Research (NBER) dates. So, period 3 will be the beginning of the recessions. GDP is real GDP. Consumption represents the real consumption of non durables and services. Investment is real private investment. These variables are from the Bureau of Economic Analysis. Hour means non-farm business hour from the Bureau of Labor Statistics. 11 recessions from 1948 are included.

The benchmark model can mimic the behavior of recessions except consumption. Especially, the diminishing magnitude of deviations between the news shock and the actual shock looks similar with the recession data before the peak dates. But data suggests more prolonged recessions while the benchmark gives a short-run recession.

Here, We do not argue that the news stock is the prominent reason of U.S.

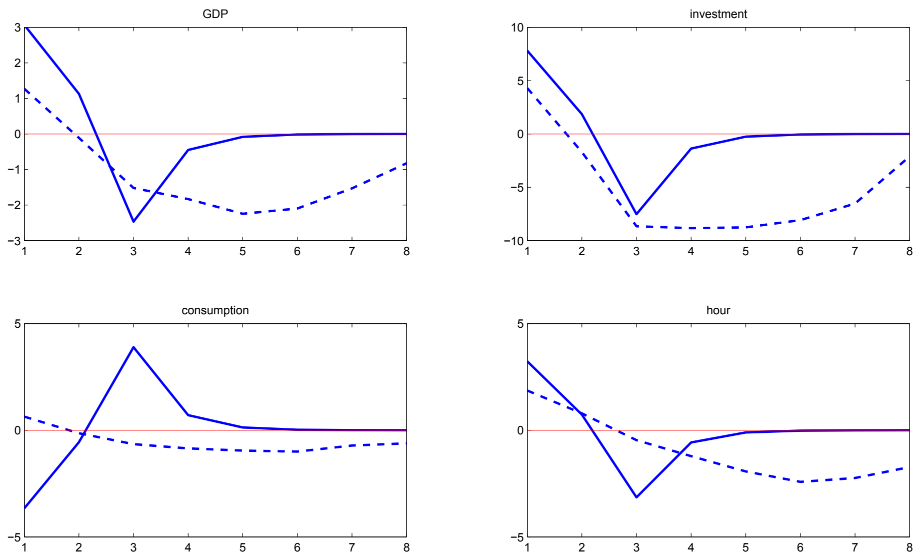


Figure 2.6: Impulse Response From False News Shock vs Average movements of Variables in Recession

recessions. But it is possible to capture some characteristics of U.S. recession by false news shock.

2.5 Consequences from Limited Enforceability

Recent development of literature using financial frictions in macroeconomics finds out the prominent role of the limited contract enforceability. The enforceability is expressed by two channels. One is the size of embezzlement when the borrower renege on the contract. If the economy can make the size of embezzlement small, the enforceability of legal institution can be considered as strong. The other is the degree of market exclusion for the reneger. As in Cooley et al. (2004) market exclusion after default can be interpreted as a higher degree of contract enforceability.

Figure 2.7 shows the relationship between contract enforcement and GDP per capita. The enforcement index is made by Business Environmental Risk Intelligence. It gives the measure of "whether the country's laws are efficiently and impartially enforced and whether governments tent to change the nature of contracts ex post" (Demirgüç-Kunt and Levine, 2004). As one can see, there is a strong positive relation between the degree of contract enforcement and GDP per capita.

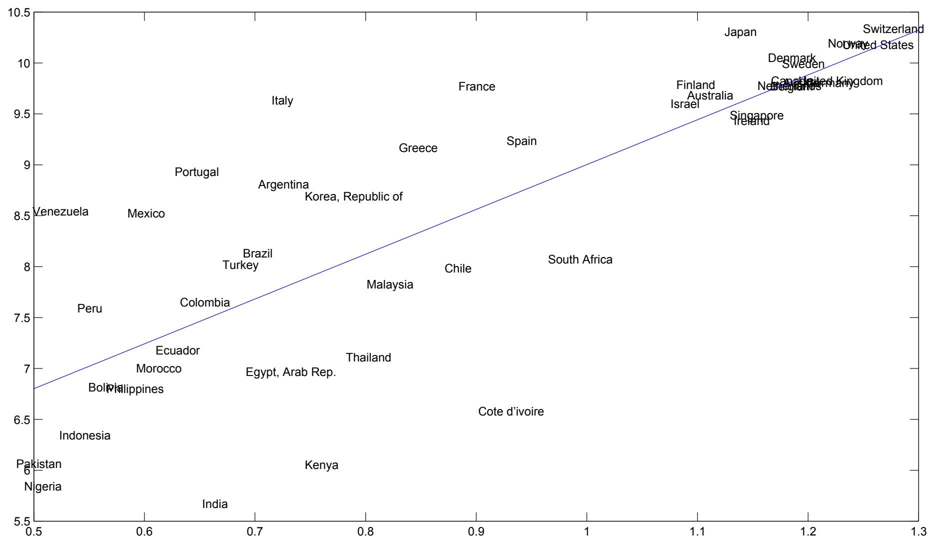


Figure 2.7: The relationship between Enforcement Index and GDP per capita

In a fashion of Buera et al. (2011), the size of embezzlement can be capture by the parameter ϕ. Let us assume that the default firms can keep fraction of 1-ϕ of the undepreciated capital. Then, we can consider ϕ as the strength of contract enforceability.

It is also available to express the degree of market exclusion in benchmark model. Suppose the default firms can enter the loan market again with γ probability in every future periods and after re-entering they are treated equally with never default firms. The probability of re-entering the market will be as below:

$$\gamma, (1-\gamma)\gamma, (1-\gamma)^2\gamma, ...$$

The maximization problem (2.2) will be changed as the below:

$$\begin{aligned} &\max r_t \qquad (2.4) \\ &s.t. \quad A_t k_t^{\alpha} l_t^{1-\alpha} - w_t l_t - (1+r_t)k_t + (1-\delta)k_t + \beta V_{t+1} \\ &\geq (1-\phi)k_t + \gamma\beta V_{t+1} + (1-\gamma)\gamma\beta^2 V_{t+2} + (1-\gamma)^2\gamma\beta^3 V_{t+3} + ... \end{aligned}$$

In stochastic model, it is hard to get the value for the sum of profits in all future periods such as V_{t+1}, V_{t+2} and so on. So I will consider the deterministic steady states only. In steady states, the sum of profits in all future periods from t should be the same with the one from $t+1$. So $V_t = V_{t+1} = V_{t+2} = ...$ We can write the following equation for the steady states from the fact that the right hand side of inequalitiy in maximization problem (2.4) should be V_t. Note that undated variables stand for steady state values.

$$V = (1-\phi)k + \gamma\beta V + (1-\gamma)\gamma\beta^2 V + (1-\gamma)^2\gamma\beta^3 V + ...$$

The resulting modified interest rule is

$$r = \alpha A k^{\alpha-1} l^{1-\alpha} - \delta - (1-\beta(1-\gamma))(1-\phi) \qquad (2.5)$$

The case of ϕ=1 is exception. It represents the full credit economy. Firms do not have an incentive to default since there is no way to embezzle. In this case, for all degree of market exclusion, $\gamma \in [0, 1]$, the interest rate rule becomes standard one.

$$r = \alpha A k^{\alpha-1} l^{1-\alpha} - \delta$$

which is the interest rate of the standard model, the marginal productivity of capital.

In this economy, to prevent default, firms should have profit as much as the size of possible embezzlement. As capital size decreases, its marginal productivity increases and the size of possible embezzlement decreases. Therefore, the financial frictions makes the economy choose less capital by adding additional term to the interest rate to limit the size of capital.

In case of $\gamma = 0$, default firms can not re-enter the market. Equation (2.5) becomes

$$r = \alpha A k^{\alpha-1} l^{1-\alpha} - \delta - (1-\phi) + \beta(1-\phi)$$

In case of $\gamma = 1$, firms are free to re-enter the market after default. Equation (2.5) becomes

$$r = \alpha A k^{\alpha-1} l^{1-\alpha} - \delta - (1-\phi)$$

As ϕ increases, there is less distortion in the economy in both cases. Figure 2.8 shows the relationship between the parameter ϕ and the steady state output for various values for γ.

In any case of γ, the steady state output increases as value of ϕ increases. Plus, if phi is equal to 1, then it does not matter whether re-entering is permitted or not. This Figure 2.8 is in accordance with Figure 2.7.

2.6 Conclusion

In this paper, we illustrate a novel model which can solve the puzzle of recessions from positive technology news shock. Both the opportunity of default and the market exclusion after default are key elements in the model. With those two assumptions, firms will not default with positive news shock which can bring more profit in the future. This creates a large substitution effect which can overwhelm wealth effects. Furthermore, with high elasticity of intertemporal substitution, I could generate the impulse responses which show larger responses on the arrival

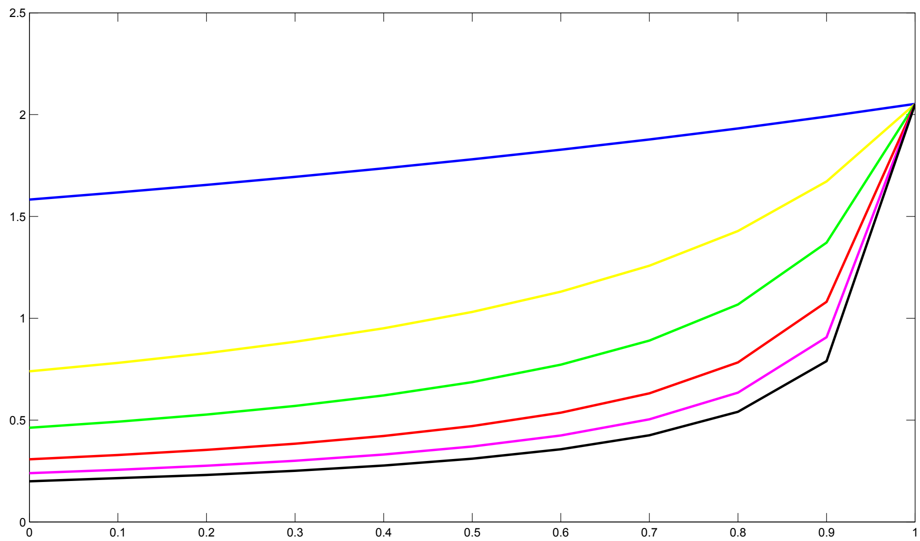


Figure 2.8: The relationship between the value of ϕ and the steady state output

of news shock than the impact of actual shock. This paper stands out among other papers in a way that this paper seeks microeconomic foundations — financial contracts between firms and financial intermediaries — to explain the reactions from news shock, rather than adopting ad hoc assumptions. However, unlike other variables such as output and hour, consumption decreases from news shock. To address this, one might need additional assumptions about the production process or other financial frictions.

To explore more about the fitness of the model, it is crucial to identify the news shock from available data. There is an approach using VAR to verify the news shock and its impulse responses to other economic variables. However, there is a discrepancy in this literature. Beaudry and Portier (2006) found that news shock discovered by S&P 500 increases output, consumption and hour. But Sims (2009) discovered that news shock identified from TFP decreases such variables. Future research might narrow this gap.

3 BORROWING LIMITS, HETEROGENEITY IN WEALTH, AND BUSINESS CYCLES

3.1 Introduction

During the two decades since the 1980s, output volatility has decreased significantly (the Great Moderation) and the distribution of wealth in the U.S. has become more polarized. For the same periods, there were series of innovations in the financial industry which contributed to the increase in the size of lending. Figure 3.1 shows the ratio of total consumer liability to nominal GDP.[1] Before the first quarter of 1984 (the dashed line), the ratio was stable around 60%. However, it steadily increases after that.[2]

In this paper, we study the effects of expansion in borrowing limits on output volatility in the incomplete market model. In the model, the expansion in borrowing limits causes an increase in wealth concentration. The aggregate asset holding decreases since there is less precautionary saving motive. This is particularly true for the poor as they are close to the borrowing limit. On the other hand, reduction in capital causes the interest rate to rise, which in turn facilitates saving of the rich. As a result, more wealth condensation arises.

In terms of capital, more wealth concentration brings more volatility. Rich agents change their asset holdings sharply compared with poor agents who are mainly hand-to-mouth. Fluctuations in capital become stronger as the portion of asset which is held by rich agents increases.

In terms of labor supply, however, more wealth concentration results in less volatility. When there is more wealth condensation, rich agents are more susceptible to changes in exogenous productivity shocks. They try to smooth their consumption

[1] Data is from Den Haan and Sterk (2011). The total consumer liability is defined by the sum of mortgages and consumer credit.

[2] There are many studies which denote early 1980s as the beginning of the Great Moderation. For example, Bernanke (2012) and Jermann and Quadrini (2012) use 1984q1 and Iacoviello and Pavan (2012) use 1983q1.

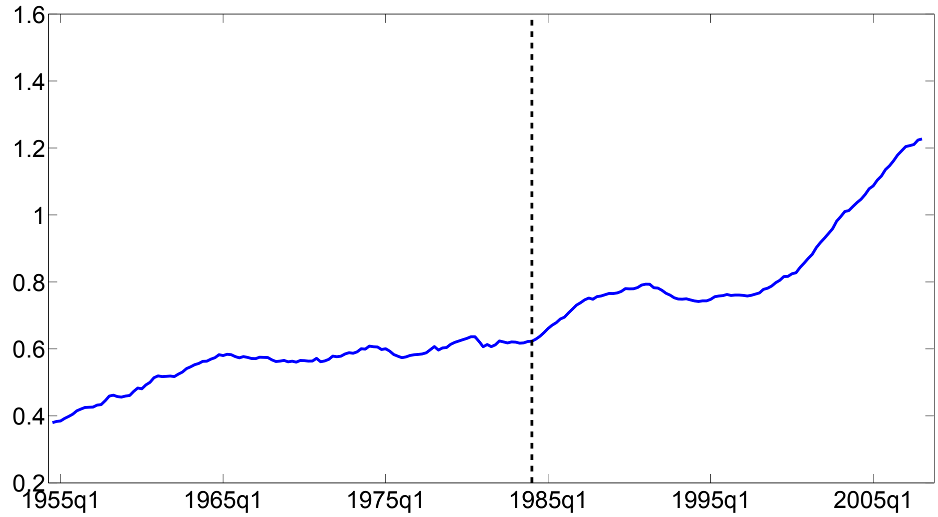


Figure 3.1: The Ratio of Total Consumer Liability to Nominal GDP

by counteracting the fluctuations in return rates of their assets from exogenous shocks with labor supply. They increase labor supply in recession and decrease it in expansion. This income effect on labor supply dominates the volatility of output in the main simulation given calibrated parameters to the U.S. economy. Therefore, the expansion in the borrowing limit causes reduction in output volatility.

Qualitatively, this can be an explanation of what we observe in data. There were increases in borrowing, increases in wealth concentration and decreases in output volatility during the Great Moderation. However, when we calibrate the model to the U.S. economy, it does not generate a sufficiently large decrease in output volatility. Quantitatively, a plausible expansion in the borrowing limit decreases output volatility only by 3% whereas there was a 40% decrease in output volatility in data.

Our paper studies the output volatility with wealth distribution. Particularly, our model is related with the incomplete market literature where there is only one asset to trade — a claim to physical capital. In the incomplete market literature, there are many papers which study the wealth inequality in the U.S. Castaneda et al. (2003) add taxes and social securities to the incomplete market model with the feature of the dynastic and the life cycle and match U.S. wealth distribution. Chang and Kim (2006) use indivisible labor and Cagetti and De Nardi (2006) introduce an occupational choice between workers and entrepreneurs to match wealth distribution in the incomplete market model.

The seminal work by Krusell and Smith (1998) study the aggregate fluctuations in the incomplete market where individual risk cannot be insured. Our analysis is based on Chang and Kim (2007) which add indivisible labor choice to the setting of Krusell and Smith (KS, 1998).

Compared with KS, the model in Chang and Kim (CK, 2007) is particularly useful in two folds. First, it can generate a realistic wealth distribution. This is important because the expansion in borrowing limit affects aggregate fluctuations through the channel of wealth distribution. Second, it includes labor supply as an endogenous choice of agents. We can study the total effects from changes in borrowing limits by including not only capital but also labor choices.

There are many papers which study the relationship between financial innovations and the great moderation. For example, Dynan et al. (2006) study possible links between them via various empirical tests. Guerron-Quintana (2009) show that heterogeneous- and sluggish-portfolio adjustment could match the data on interest semi-elasticities, and that more flexible portfolio could reduce volatility of output. Iacoviello and Pavan (2012) argue that higher individual income risk and lower down payment can bring reduction in output volatility. Recently, Den Haan and Sterk (2011) question the empirical evidence which supports this link. All these papers focus on the direct relationship between changes in the financial side and changes in output volatility. However, our paper studies the indirect relationship between the two via changes in wealth distribution.

The rest of the paper consists of the following sections. In Section 3.2, we introduce the model. Results are presented in Section 3.3. Section 3.4 concludes.

3.2 The Model

The model is based on CK. Time is discrete and infinite. There is a unit mass of agents in the economy. They maximize their expected utility. Their utility function is given by:

$$U(c,h) = \log(c) - B\frac{h^{1+\gamma}}{1+\gamma}$$

Their utilities depend on consumption (c) and labor supply (h). Labor supply is not divisible. Agents must choose between $\{0, \bar{h}\}$. Each agent has different labor productivity. x_t stands for effectiveness of labor. $x_t h_t$ is the total effective labor supply. x_t follows an AR(1) process.

$$\log x_t = \rho_x \log x_{t-1} + \epsilon_x$$

where ϵ_x follows a Normal distribution with mean 0 and standard deviation σ_x.

There is only one asset in this economy — a claim of physical capital. a_t denotes the amount of the asset at period t. Agents can save and borrow by trading the asset. The exogenous borrowing constraint is specified by $\bar{a}$. The budget constraint is given by:

$$\begin{aligned}(1+r_t)a_t + w_t x_t h_t - a_{t+1} &= c_t \\ a_{t+1} &\geq \bar{a}\end{aligned}$$

r_t is an interest rate and w_t is wage at period t.

There is a representative firm. The aggregate production technology requires capital and labor. The production technology is described by:

$$Y_t = \lambda_t K_t^{1-\alpha} L_t^{\alpha}$$

K_t is the total amount of capital and L_t is the total amount of effective labor supply.

In this economy, each agent is distinguished by asset holdings and effectiveness of labor. Let $\mu(a, x)$ be the distribution of agents over asset and productivity. Then, the amount of total capital and total effective labor are

$$K_t = \int a_t d\mu_t$$
$$L_t = \int x_t h_t d\mu_t$$

λ is total factor productivity and it follows an AR(1) process:

$$\log \lambda_t = \rho_\lambda \log \lambda_{t-1} + \epsilon_\lambda$$

where ϵ_λ follows a Normal distribution with mean 0 and standard deviation σ_λ. ϵ_x and ϵ_λ are independent.

The representative firm maximizes its profit. As a result, the marginal products of capital and labor are equal to the price of capital, interest rate, and the price of labor, wage.

$$\lambda_t(1-\alpha)K_t^{-\alpha}L_t^{\alpha} = r_t$$
$$\lambda_t(\alpha)K_t^{1-\alpha}L_t^{\alpha-1} = w_t$$

Since aggregate capital and aggregate labor depend on the distribution of agents, this implies that the calculation of interest rate and wage should consider the distribution of agents.

The value function for the agent is

$$V(a, x; \lambda, \mu) = \max\{V^E, V^N\}$$

where

$$V^E(a,x;\lambda,\mu) = \max \log(c) - B\frac{\bar{h}^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}} + \beta E[V(a',x';\lambda',\mu')]$$

subject to

$$\begin{aligned} c = w(\lambda,\mu)x\bar{h} + (1+r(\lambda,\mu))a - a' \\ a' \geq \bar{a} \\ \mu' = \mathcal{T}\mu \end{aligned}$$

and

$$V^N(a,x;\lambda,\mu) = \max \log(c) + \beta E[V(a',x';\lambda',\mu')]$$

subject to

$$\begin{aligned} c = (1+r(\lambda,\mu))a - a' \\ a' \geq \bar{a} \\ \mu' = \mathcal{T}\mu \end{aligned}$$

Here, $\mathcal{T}$ is a transition operator of the law of motion for the distribution of agents.

It is difficult to numerically implement continuous distribution as state variables. We adopt the method suggested by KS to approximate the distribution with moments. They showed that the first moment is enough to get a good approximation since individual policy function is almost linear in this type of models. Following that, we use these proxies for the law of motion for aggregate capital and the function of interest rate and wage.

$$\log K_{t+1} = co(1,1) + co(1,2)\log K_t + co(1,3)\log \lambda_t$$
$$\log w_t = co(2,1) + co(2,2)\log K_t + co(2,3)\log \lambda_t$$
$$\log r_t = co(3,1) + co(3,2)\log K_t + co(3,3)\log \lambda_t$$

Here, co is a 3x3 matrix which contains the coefficients of the equations above.

Calibration

The parameters in the simulation come from CK. Table 3.1 displays the parameters used in the simulation.

Parameters	Value	Description
α	0.64	Labor share in production function
β	0.98	Discount factor
γ	0.4	Individual labor-supply elasticity with divisible labor
B	166.3	Utility parameter
$\bar{h}$	0.33	Labor supply if working
ρ_x	0.929	Persistence of idiosyncratic productivity shock
σ_x	0.227	Standard deviation of innovation to idiosyncratic productivity
ρ_λ	0.95	Persistence of aggregate productivity shock
σ_λ	0.007	Standard deviation of innovation to aggregate productivity

Table 3.1: Parameters

To study the effect from the expansion of borrowing limit, we compare two cases. Each case has a different value of $\bar{a}$. In Model 1, we use -2.0 for $\bar{a}$ as in CK. In Model 2, we set it at -4.0. We use the same values for all other parameters.

	Model 1	Model 2
$\bar{a}$	-2.0	-4.0

Table 3.2: Borrowing Limits

To characterize the economy, we follow the numerical method in this literature (for example, KS, CK). First, we simulate the economy with $20,000$ agents and $2,000$ periods with individual optimal policy functions for asset holdings and labor choices which are calculated with the initial coefficients for the law of motion for aggregate capital and the functions of interest rate and wage. From the simulation results, we do the regression for coefficients and do the same simulation with updated coefficients. Repeat this procedure until the old coefficients and the new coefficients are close enough to each other. We used the steady state distribution of agents as the initial distribution in the simulation. We disregard the first 500 series to avoid possible effects from the initial distribution.

Specifically, the sizes of the grid for a and K are 660 and 10, respectively. We use the method of Tauchen (1986) for the discretization of continuous AR (1) shock processes. For both processes for the variables x and λ, we use 7 states. With this more sparse grid, we could get comparable results as in CK.

3.3 Results

Steady State

In this section, we study the steady state of each model. Here, the steady state means the constant state of exogenous variables λ. We assume that λ is 1.

Compared with Model 1, the borrowing limit is expanded in Model 2. This creates two separate effects on saving. First, it reduces precautionary saving motivation. The borrowing limit is less binding now, so agents do not need to hold as much saving as in Model 1. Therefore, the aggregate capital is smaller than that in Model 1. However, this reduction of capital increases interest rate. The raise in interest rate generates an incentive for agents to save more.

The overall effects on saving from these two opposite effects are different across agents. For poor agents, the first effects dominates the second. The expansion of the borrowing limit is more important than the increase of interest rate because their asset holdings are close to the borrowing limit. Those agents reduce saving in

Model 2 than in Model 1.

However, rich agents increase saving. For them, increase in interest rate is more crucial than the extension of borrowing limit, because they have less chance to hit the borrowing limit. They increase their saving in Model 2 from Model 1. Table 3.3 and Table 3.4 summarize the property of steady states in each model. Since poor agents hold less asset and rich agents possess more asset as the borrowing limit expands, the wealth concentration in Model 2 is more severe than that in Model 1.

This increase in wealth concentration can be matched with the rise in the wealth condensation of the U.S. from the 1980's. We follow the method in CK to get the wealth distribution in the U.S. Data is from the Panel Study of Income Dynamics (PSID). Samples are restricted to households whose heads are high school graduates.[3] Table 3.5 shows the trend in wealth concentration. In data, the portion of the top 20% wealth has increased by 18.4% from 1983 to 2006. Between Model 1 and Model 2, the corresponding change is 5.8%.

	Model 1	Model 2
Aggregate Capital	12.59	12.02
Aggregate Efficient Labor	0.33	0.33
interest rate	1.00%	1.09%

Table 3.3: Steady State Values

	Top 20%	Next 20%	Next 20%	Next 20%	Last 20%
Model 1	63.8	25.8	10.0	2.8	−2.6
Model 2	69.6	26.7	9.0	0.5	−5.8

Table 3.4: Size Distribution of Wealth in Each Model

The interaction between the borrowing constraint and wealth distribution is studied in Cagetti and De Nardi (2006). In their paper, a relaxed borrowing limit results in more wealth concentration as well. There are entrepreneurs who require borrowing for efficient production. A relaxed borrowing limit increases capital

[3] Do not include those with higher education.

Year	Top 20%	Next 20%	Next 20%	Next 20%	Last 20%
1983	64.8	19.0	10.8	5.0	0.4
1988	70.2	18.2	8.7	2.9	0.0
1993	70.2	19.7	8.8	2.8	-1.5
1998	78.0	14.5	6.5	1.8	-0.8
2002	79.4	14.0	5.7	1.7	-0.8
2006	83.2	12.6	4.2	0.9	-0.9

Table 3.5: Size distribution of wealth in the United States

input in the entrepreneur sector, so the interest rate decreases in their model. However, in our case, the expansion in borrowing limit mainly affects precautionary saving motive, so it decreases capital and increases interest rate.

Business Cycles Moments

In this section, we study the effects from expanding the borrowing limit on business cycles moments. In Model 1, the law of motion for aggregate capital and the functions of wage and interest rate are

$$\begin{aligned}\log k_{t+1} &= 0.1203 + 0.9524 \log k_t + 0.1194 \log \lambda_t \quad R^2 = 0.9998\\ \log w_t &= -0.0847 + 0.3746 \log k_t + 0.8948 \log \lambda_t \quad R^2 = 0.9918\\ \log r_t &= -1.6644 - 0.6658 \log k_t + 1.1863 \log \lambda_t \quad R^2 = 0.9309\end{aligned}$$

In Model 2, the law of motion for aggregate capital and the functions of wage and interest rate are calculated by:

$$\begin{aligned}\log k_{t+1} &= 0.1147 + 0.9538 \log k_t + 0.1252 \log \lambda_t \quad R^2 = 0.9998\\ \log w_t &= -0.0941 + 0.3800 \log k_t + 0.9010 \log \lambda_t \quad R^2 = 0.9917\\ \log r_t &= -1.6478 - 0.6755 \log k_t + 1.1726 \log \lambda_t \quad R^2 = 0.9334\end{aligned}$$

This result is comparable to that of CK in terms of accuracy. In their paper, the

R^2 values for the law of motion for aggregate capital and the functions of wage and interest rate are $0.9999, 0.9977$ and 0.9887, respectively. Slight decreased accuracy from our simulation is due to the more sparse grid and the small number of agents.

In Model 2, the coefficient of λ in the law of motion for aggregate capital is larger than that in Model 1 $(0.1252 > 0.1194)$. This implies that aggregate capital is more volatile when the aggregate state, λ changes. This is due to more concentration of wealth in Model 2. In the incomplete market model like ours, it is well known that wealthy agents make more volatile changes in asset holdings. To smooth their consumption, wealthy agents decrease asset holdings with negative productivity shocks and increase them with positive productivity shocks. However, poor agents are mainly hand-to-mouth. Their asset holdings are mostly constant — close to zero — regardless of fluctuations in aggregate productivity. In Model 2, more capital is held by wealthy agents than in Model 1. Therefore, changes in aggregate capital are more volatile in Model 2.

Note that labor supply is determined by the reservation wage in our models. The reservation wage depends on the size of asset holdings. Wealthy agents have relatively large reservation wage as they can afford consumption without labor supply. Poor agents have small reservation wage so their participation rate is higher than the wealthy. This reservation wage is also affected by the aggregate productivity. The changes in aggregate productivity generate substitution and income effects on labor supply. Increased wage from high productivity facilitates labor supply whereas more returns from factor inputs discourage agents' drive to work.

Now, let us compare two models in terms of labor fluctuations. In Model 2, more concentrated wealth results in more volatile changes in capital. Since wealthy agents accumulate relatively more asset in Model 2 than in Model 1, they face greater income effects. As a result, they do not increase labor supply in Model 2 as much as in Model 1 when the aggregate productivity is high, and they tend to put more labor supply in Model 2 than in Model 1 when the aggregate productivity is low. However, poor agents do not show as a stark difference in labor supply as wealthy agents do between two models. Because they maintain small reservation wage regardless of changes in productivity.

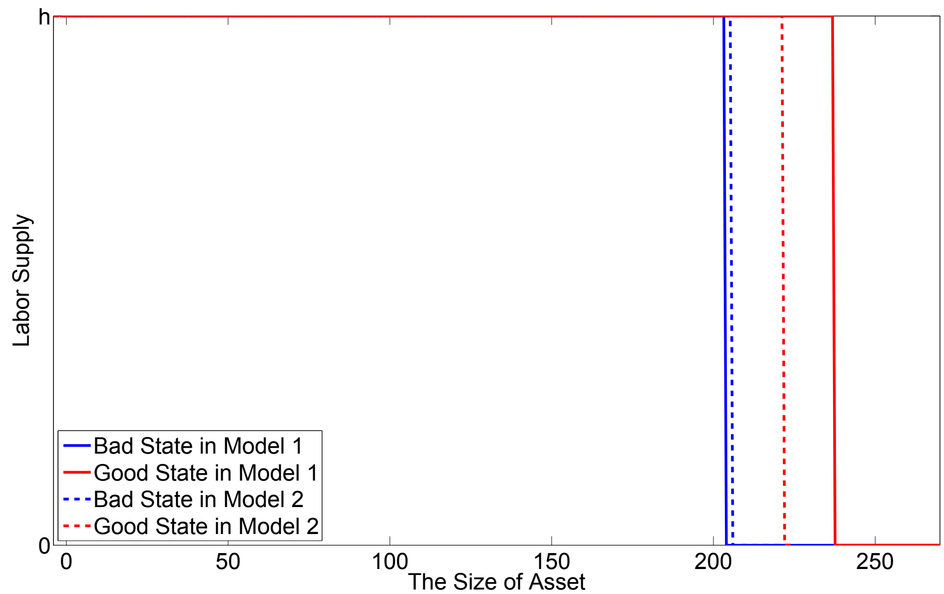


Figure 3.2: The Optimal Policy function of Labor Supply

Figure 3.2 depicts the labor supply decision of agents given the same individual productivity and aggregate capital. The solid lines are from Model 1 and the dotted lines are from Model 2. The difference between two lines within the same model represents the difference in labor supply as the aggregate productivity changes. The difference in Model 1 is wider than the difference in Model 2. Since the difference in Model 1 includes the whole difference in Model 2, the fluctuations of labor supply in Model 2 would be smaller than those in Model 2.

The changes in input factors which determine the output show opposite patterns between two models. Capital tends to increase volatility as the borrowing limit expands whereas changes in labor dampen output volatility. Overall, the effects from labor dominate the effects from capital.

Table 3.6 shows business cycles moments in each model. In Model 2, the standard deviation of output has decreased by 3.3% from Model 1. As we discussed above, this reduction in output volatility mostly comes from the labor side. The standard deviation of labor in Model 2 has been reduced by 6% from Model 1, whereas the reduction in standard deviation of investment is just 2.5%. In this table, the standard deviations are calculated by the standard deviations of hp-filtered (parameter, 1600) log values.

	Model 1	Model 2	$\frac{Model2}{Model1}$
σ_y	0.0273	0.0264	96.7%
σ_i	0.0570	0.0556	97.5%
σ_l	0.0083	0.0078	94.0%

Table 3.6: Business Cycles Moments from Simulation

Although the model makes qualitatively correct predictions, its quantitative impact is small. Table 3.7 shows some business cycles moments in data. The reductions in volatility of aggregate variables were quite large. Comparing two tables, expansion in borrowing limit can explain 8% of reduction in output volatility.

	Before 1984(B)	After 1984(A)	$\frac{A}{B}$
σ_y	0.0200	0.0118	59.1%
σ_i	0.0797	0.0530	66.5%
σ_h	0.0200	0.0146	68.0%

Table 3.7: Business Cycles Moments from Data

Updated Aggregate Productivity Process

In this section, we re-estimate the exogenous aggregate productivity process with sample periods after 1984 only. The parameters for updated shock process are 0.9405 and 0.0045 for ρ_λ and σ_λ. In Model 3, we use the updated process with a tight borrowing limit ($\bar{a} = -2.0$). In Model 4, both expanded borrowing limit ($\bar{a} = -4.0$) and the updated process are used in the simulation.

With updated process, the reduction in output volatility is more significant. With updated process alone, the output volatility decreases up to 8%. Loose bor-

	Model 1	Model 3(% to Model 1)	Model 4(% to Model 1)	Ratio in Data
σ_y	0.0273	0.0184(67.3%)	0.0177(64.8%)	59.1%
σ_i	0.0570	0.0432(75.7%)	0.0411(72.1%)	66.5%
σ_l	0.0083	0.0075(90.3%)	0.0068(81.9%)	68.0%

Table 3.8: Business Cycles Moments from Simulation with Updated Processes

rowing limit and updated process result in 11.5% decrease in output volatility. That corresponds to 28% of the observed reduction in data.

3.4 Conclusion

The expansion of borrowing limit in incomplete market can explain more unequal wealth distribution and reduction in output volatility during the Great Moderation periods. The enlarged borrowing limit generates less precautionary saving motive, which results in smaller aggregate capital and an increase in interest rate. This facilitates saving for the rich but poor people reduce their saving. As a result, more unequal wealth distribution arises. Rich agents are more susceptible to fluctuations from exogenous productivity changes as they are holding more assets. They counteract these changes with counter-cyclical labor supply. Overall, this effect dominates aggregate variables and reduces output volatility.

Although, the model makes correct predictions on wealth distribution and output volatility, its quantitative importance is not large. The expanded borrowing limit can explain about 8% of the observed reduction in data.

To capture the reduction in output volatility quantitatively, we may need other sources of variation. Furthermore, at the end of the sample period (the Great Recession), output responded a lot from changes in aggregate productivity as noted by Kim. For future research, one can address these additional two issues.

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A.1 Data

We use the following data for each variable.

Output NIPA Table 1.1.6. Gross Domestic Product

Capital Fixed Assets Accounts Table 1.2. Fixed assets and consumer durable goods

Employee All employees: Total nonfarm (PAYEMS, St. Louis Fed)

Hour After 1964 - Average weekly Hour of Production and Nonsupervisory Employees: Total Privat (AWHNONAG, St. Louis Fed)
Pre 1964 - Average weekly hour: Total Private (EEU00500005, BLS)

Population Civilian Noninstitutional Population (CNP16OV, St. Louis Fed)

Rental Income NIPA Table 1.12

Corporate Profit NIPA Table 1.12

Net Interests NIPA Table 1.12

Current Surplus of Government Enterprise NIPA Table 1.12

Consumption of Fixed Capital NIPA Table 1.7.5

Compensation of Employee NIPA Table 1.12

All variables are measured quarterly except capital and pre-1964 hours. To transform the annual series into quarterly series, we use a constant growth rate within a year in the case of capital and the same value for all four quarters in the case of employees.

Growth Rates

To measure the growth rate, we take a log and linearly detrended for economic variable X.

$$\log X_t = a + bt + \tilde{X}_t$$

where a is constant and b is coefficient for time variable, if we take the difference between $\tilde{X}_t$

$$\tilde{X}_t - \widetilde{X_{t-1}} = \log X_t - \log X_{t-1} - b$$

More precisely, we report the growth rate of a variable X minus its linear trend growth rate as the growth rate of X.

Solow Residuals

To calculate the Solow residuals, we take logs of output (Y), capital (K) and employee times hour (N) series and linearly detrend them with constants. The Solow residuals can be recovered by

$$SR_t = \tilde{Y}_t - \theta \tilde{K}_t - (1 - \theta)\tilde{N}_t$$

Calibration of θ, δ and I_0

The value of θ is calibrated by the average of $\frac{UCI+DEP}{UI}$ for the sample period. UCI means unambiguous capital income, and it is composed of rental income, corporate profit, net interest and a current surplus of government enterprises. DEP means the consumption of fixed capital. UI means unambiguous income, and it is composed of unambiguous capital income and the compensation of employees. Labor share is $1 - \theta$. The value of δ is calibrated by the average of $\frac{DEP}{nominalK}$.

Data about firm size and age is from Business Dynamics Statistics (BDS). We consider the period from 2003 to 2010 in which all values are reported. The firm size is categorized by the size of employment. We choose the smallest number for each category to represent the category of firm size. We treat the distribution as stable across time. We observe that there is consistently a large growth (over 100%) in the mean of firm size from age 0 to age 1. Other periods do not exceed even 30%. Therefore, we treat age 1 firms as newborn firms, rather than age 0 firms. We calculate the average size of age 1 firms and age 26 or more firms from 2003 to 2010. This series is pretty constant. The smallest one is 0.146 and the largest one is 0.226. The average is 0.195.

A.2 Specification of Other Models

The First Best Economy

The first best economy is where there are no constraints at all. Any firm in the first best economy will get the amount of capital, which maximizes the profit of a firm, immediately. The size distribution of firms will degenerate at one point for a given period. The first best economy can be characterized by the following problem:

$$\max_{k,n}(1-\frac{k}{T_c})zk^{\theta}n^{1-\theta}+(1-\delta)k-wn-rk$$

where r is the interest rate, $r=\frac{1}{\beta(1-\gamma)}$ and the wage is determined by $w=zA$ as in the benchmark model. To make enough concavity in production function, we leave a distortionary tax on capital. For the parameter values, we use the same ones in Table 1.2.

The Standard RBC

The specification of the standard RBC comes from the baseline in RÃ-os-Rull and SantaeulÃ lia-Llopis (2010). We choose a Hansen-Rogerson log-linear utility function. The model itself comes from Cooley and Prescott (1995), but the parameter values are updated. The standard RBC model can be characterized by the following dynamic programming problem:

$$V(k,z) = \max_{k',n} \log(zk^{\theta}n^{1-\theta} + (1-\delta)k - (1+\eta)(1+\lambda)k') + \kappa(1-n) + \beta(1+\eta)E[V(k',z')]$$

η is the population growth rate, and λ is the productivity growth rate.

The calibrated parameter values are summarized in Table A.1. Except θ, all parameters are from RÃ-os-Rull and SantaeulÃ lia-Llopis (2010).

β	κ	δ	θ	λ	η
0.988	2.92	0.019	0.324	0.0037	0.0044

Table A.1: Parameters

A.3 Additional Materials

Table A.2 shows the regression result from periods with negative TFP shocks only. The table shows more contrast between the benchmark and the first best.

Table A.3 shows the regression result from periods with positive TFP shocks only. The benchmark and the first best are similar in this case.

	(1)	(2)	(3)	(4)	(5)
VARIABLES	Data	Bench	No_tax	FBest	RBC
L.gTFP	0.279***	0.710***	1.108***	0.223	0.0226
	(0.0672)	(0.0601)	(0.0339)	(0.179)	(0.0669)
L2.gTFP	0.303***	0.0998**	0.0193	0.271	0.0277
	(0.0789)	(0.0485)	(0.0282)	(0.205)	(0.0821)
L3.gTFP	0.299***	0.0772	-0.0508	-0.279	-0.157**
	(0.0696)	(0.0497)	(0.0323)	(0.192)	(0.0751)
L4.gTFP	0.115	0.0206	-0.0614**	-0.0349	-0.0660
	(0.0756)	(0.0457)	(0.0272)	(0.225)	(0.0862)
L5.gTFP	0.136*	0.0309	-0.0443	0.242	0.0446
	(0.0722)	(0.0580)	(0.0356)	(0.233)	(0.0983)
L6.gTFP	0.226***	0.112**	0.0330	0.0114	0.0191
	(0.0760)	(0.0508)	(0.0343)	(0.200)	(0.0790)
L7.gTFP	0.0851	-0.0517	-0.0390	0.0548	-0.0410
	(0.0694)	(0.0482)	(0.0274)	(0.225)	(0.0862)
L8.gTFP	0.101	-0.00937	-0.0276	0.0189	-0.0196
	(0.0779)	(0.0487)	(0.0320)	(0.203)	(0.0769)
Constant	-0.00108**	0.000978***	0.000487**	-0.0129***	-0.00388***
	(0.000467)	(0.000340)	(0.000229)	(0.00139)	(0.000542)
Observations	118	118	118	118	118
R-squared	0.404	0.694	0.925	0.054	0.044
P-value (F)	6.42e-07	0	0	0.581	0.730

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table A.2: Output Amplification on the History of TFP with Negative Shocks

VARIABLES	(1) Data	(2) Bench	(3) No_tax	(4) FBest	(5) RBC
L.gTFP	0.419***	0.655***	1.193***	0.268	0.0371
	(0.0688)	(0.0549)	(0.0380)	(0.227)	(0.0888)
L2.gTFP	0.214**	0.227***	0.0902**	-0.287	-0.122
	(0.0882)	(0.0529)	(0.0364)	(0.201)	(0.0825)
L3.gTFP	0.263***	0.0753	0.0149	0.00962	-0.0424
	(0.0601)	(0.0561)	(0.0338)	(0.172)	(0.0663)
L4.gTFP	0.199***	0.00669	0.0565*	-0.0133	-0.0138
	(0.0633)	(0.0458)	(0.0287)	(0.182)	(0.0761)
L5.gTFP	0.0494	0.0751	0.0428	-0.164	-0.0671
	(0.0624)	(0.0481)	(0.0325)	(0.181)	(0.0685)
L6.gTFP	-0.0448	-0.00703	0.0109	-0.153	-0.126**
	(0.0610)	(0.0419)	(0.0311)	(0.159)	(0.0601)
L7.gTFP	0.0968	-0.0215	-0.0310	0.0647	-0.000104
	(0.0598)	(0.0408)	(0.0301)	(0.175)	(0.0676)
L8.gTFP	0.0959*	0.00556	0.00205	0.213	0.0370
	(0.0537)	(0.0422)	(0.0285)	(0.155)	(0.0596)
Constant	0.000493	0.000287	-9.72e-05	0.0131***	0.00391***
	(0.000468)	(0.000288)	(0.000220)	(0.00121)	(0.000478)
Observations	125	125	125	125	125
R-squared	0.409	0.704	0.918	0.060	0.062
P-value (F)	0	0	0	0.296	0.282

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table A.3: Output Amplification on the History of TFP with Positive Shocks