Essays on Macroeconomics and Financial Markets

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Abstract

The first chapter explores liquidity risks and analyzes the relationship between funding liquidity and market liquidity. Recent empirical studies have shown a comovement of liquidity across bond and asset markets, that the comovement is driven by common factors such as monetary shocks. The financial system is more fragile if this comovement is stronger. In the model, I assume that firms' capital plays two roles: a main input for production and the collateral for secured loans. Funding liquidity is measured by borrowing constraints of secured loans, which are determined by endogenous loan-to-value ratios (LTV). Market liquidity is measured by the trading frequency and the ease of traders' negotiations. Higher borrowing constraints allow firms to invest in more new capital. Thereby, firms produce more efficiently and negotiate harder on the asset market, which implies increased market liquidity. Higher trading efficiency increases the profitability of firms, and hence results in higher LTV ratios and a continuing cycle of increased funding liquidity. Because of the dynamic interactions between credit limits and trading frictions, the economy responds persistently to liquidity shocks. Pushing further, I calibrate the model quantitatively by minimizing the squared distance between the data and the model. The simulated liquidity moments of these mar-

kets move together and present business cycle property. Moreover, money has an essential role as the medium of exchange in the exchange process that impose a non-trivial monetary policy implication.

The second chapter studies the efficiency and default risk of long-term loans in bank lending. The model derives a long-term lender-borrower relationship with the presence of limited commitments and limited liabilities. With predetermined terms of the loan, the borrower has incentive to terminate the loan earlier either by prepayment or default. The incentive-compatible loan contract in favor of a lower default risk encourages earlier prepayments, and vice versa. On the event of default, bank suffers a loss of the loan that has not been covered by the liquidation of the collateral. If prepayment happens, bank loses the rest of the interest payment. If the loan market is frictional that it takes time for the bank to find another borrower, the interest loss on prepayment becomes severe. Under this condition, allowing default increases the efficiency of lending. Moreover, the bank would allow a higher default rate if the initial market interest rate is lower or the size of the loan is larger.

The third chapter studies the effect of recourse law on homeowners' behavior during the residential mortgage foreclosure process, and shows evidence from 7 counties of the Illinois state. We construct the dataset from a loan-level foreclosure and land lien database to capture the individual-level heterogeneity. Although the percentage of deficiency judgment granted is low (around 2%), we find that the fear of banks' recourse right affects homeowner's bankruptcy and private sale decision during foreclosure. We set the framework that the homeowner evaluates the probability of deficiency judgment and then respond to it accordingly. The estimation results show that, with a high hazard rate of deficiency judgment, the homeowner tends to 1) conduct private sale to cure the foreclosure or hand over the property in a more lender-friendly way; 2) claim bankruptcy chapter 7 to avoid the deficiency judgment. If there is no deficiency threat, the bankruptcy chapter 7 claims would be lower by 3%, the bankruptcy chapter 13 claims would go up by 10% and the probability of public sale would be increased by 4%.

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1. Funding Liquidity and Market Liquidity

1.1. Introduction

Liquidity risks impacts financial market prices, asset trading volume and credit limits. Liquidity also predicts expected returns on financial assets. Specifically, funding liquidity affects the borrowers' fund flow from the bond market, while market liquidity affects trading frequency and the traders' fund flow from the asset market. If the bond market interacts with the asset market, financial instability would be amplified by this comovement between funding liquidity and market liquidity. Evidence from the data suggests that this comovement exists and responds to monetary shocks. Hence, this paper models this comovement as well as the role played by money in it. Equiped with the theory framework, the paper also discusses the intended and unintended effect of monetary policies.

Recent empirical work examines the time series of liquidity across the bond and asset market and documents the commonality between liquidity and trading activities. It is a stylized fact that funding liquidity and market liquidity covary. For example, Fleming *et al.* (1998) measures liquidity by the volatility of return and show a strong linkage between funding liquidity and market liquidity. During a liquidity crisis, funding liquidity and market liquidity mutually reinforce one another. A small negative shock to the economy might be amplified through this mechanism and result in a sudden drying-up of liquidity. In the event of crises, ex-ante and ex-post policy interventions are expected to alleviate the liquidity crunch. Chordia *et al.* (2004) shows that the comovement of liquidity across bond and asset market is driven by common factors such as monetary shocks. Adrian and Shin (2008) documents the impacts of monetary policies on financial markets and financial stability. Cochrane and Piazzesi (2002) finds that an unexpected increase of federal funds rate will increase the transaction cost and thereby the trading friction of the stock market, hence lowering market liquidity, *and vice versa.* Although monetary policy does not usually target financial markets directly, it affects liquidity by changing borrowing costs or trading activity. Fed and other Central Banks adjusted interest rates (conventional monetary policy) in response to the recession. With the presence of the liquidity trap, central banks have recently embarked upon non-conventional monetary policy, a term which essentially encompasses all other types of policy aimed at increasing the flow of credit, to fight the recent financial crisis.

Establishing a theoretical link between monetary policy, fund flows and liquidity is desirable for studying the role of monetary policy in financial market liquidity. Current literature on liquidity has developed separately on funding liquidity and market liquidity to answer policy related questions. Bernanke and Gertler (1989) analyzes why balance sheet liquidity affects output dynamics and thereby business cycles.[1] Kyiotaki and Moore's seminal work (1997) shows persistent and amplified effects of shocks due to the strength of the dynamic interaction between credit limits and asset prices as a transmission mechanism. Since that paper, many economists have adopted this idea to study financial friction, production and the business cycle. Brunnermeier and Pedersen (2008) develops a theoretical framework to link funding liquidity and market liquidity. Similar to traditional asset market modeling, trade is competitive and frictionless where the matching of buyers and sellers is instantaneous and costless.

However, recent empirical evidence shows that the trading friction in asset markets is not trivial. A new body of research describes the trading friction and designs trading mechanisms to build a micro-foundation understanding money and asset exchange. This effort was pioneered by Kiyotaki and Wright (1991, 1993) in monetary theory and by Duffie, Garleanu, and Pedersen (2005, 2007) in finance. Under this framework, Rocheteau and Weil (2009) and Rocheteau and Wright (2010) study asset pricing, market liquidity and monetary policies.

Motivated by these studies, I model the comovement between funding liquidity and market liquidity in order to explain why they covary and what the determinants of the comovement are. In the model, firms obtain funding in two ways: borrowing from the bond market or trading in the asset market.[2] Funding liquidity is measured by the fund flow from the bond market, while market liquidity is measured by the fund flow from the asset market. The goal is to capture the financial frictions of each market. Bond market friction comes from the credit limit. Firms

[1] Bearing the same economic intuition, liquidity is a catch-all term that may refer to different concepts, for example: balance sheet liquidity or accounting liquidity. Price spreads, volatilities of return and market depths are frequently used as measures of liquidity.

[2] In the model, assets are firms' inventory which is modeled as a specific product or service.

have limited commitment, and hence are required to borrow through secured loans which require collateral. The size of the loan is thus restricted to a fraction of the value of the collateral. The matching of traders is time consuming and that creates trading friction in the asset market. Once matched, buyers bargain with sellers to decide asset prices.

Firms' capital is the key to the comovement between funding and market liquidity. Similar to Kiyotaki and Moore (1997), firms' capital is an input in the production function, but it can also be used as collateral for loans. The loan-to-value ratio (LTV) for the secured loan is endogenized by each firm's incentive compatibility condition. The default risk is then converted into the credit limit specified by the LTV and the capital being used as the collateral. To address the searching and matching friction of the asset market, a search-based model with fiat money is used to endogenize trading behavior; the prices are therefore determined by a standard bargaining process. Similar to Kiyotaki and Wright (1989), this model emphasizes the role of money in the exchange process. Applying the search theoretic approach convenes the analysis of monetary policy's effect on liquidity. With this setting, capital not only decides production efficiency, but also the bargaining power of firms over asset prices. This argument is similar to Rocheteau and Lagos (2009) on liquidity and over-the-counter (OTC) market. In their paper, the exchange of assets in an OTC market are not only affected by the current asset valuation, but also the valuation of holding the asset for a certain period. Here, capital holding is related to bond holding and hence enters the bargaining process in the asset market.

Under this model setting, monetary shocks are still neutral but not super-neutral.[3] Inflation hurts the intensive margin, in that the trading volume of the asset market decreases and the borrowing margin increases. Steady state equilibrium analysis shows that money growth has a trading opportunity effect.[4] This means that inflation may change the agents' probability of finding a successful match. When inflation is very low and households have very low bargaining power over prices, positive money growth may increase the participation ratio of households and firms, thus increasing the extensive margin.

This paper also suggests that an effective direct intervention changes both the levels of fund flow and the comovement between funding liquidity and market liquidity. This is consistent with empirical observations. For example, Adrian and Shin (2008b) shows that monetary policy has a direct impact on broker-dealer asset growth via short-term interest rates, yield spread and risk measures. Further, this paper finds that a one time money injection into the bond market does not have a persistent effect and is thus ineffective at increasing credit limits and boosting asset trading frequency. To test the model's predictions, I project the equilibrium outcome of the model to the moments of the real economy and calibrate the parameters to quantitatively analyze the results. I also discuss extended models which include different variations of the baseline model.

[3]See Kiyotaki and Wright (1989) and Lagos and Wright (2003) for detailed discussion.
[4]Defined in Shi(1995)

1.2. Model

1.2.1. Environment

Time is discrete and infinite. The economy is primarily populated by two types of continuum-measured agents: households and firms. Households are risk averse and discount the future by β; while firms are risk neutral and discount the future by $\frac{1}{R}$. I normalize the exogenous supply of households and firms to one. Besides households and firms, financial intermediaries and monetary authorities play roles in the economy. Financial intermediaries are risk neutral and create risk-free bonds with gross interest rate R^f. If agents buy one unit of a bond, they are paid R^f in the sub-sequential period. Assume $\frac{1}{R} = \beta$ to simplify the choice for the risk-free bonds, then $\frac{1}{R^f} = \beta$. In the bond market, households are lenders with bond position b and firms are borrowers with debt position b. Financial intermediaries choose a LTV to secure firms' loan payments. Then default risk is replaced by the borrowing constraint.

Every period, there are two sub-periods. A centralized market (CM) appears first, followed by a decentralized market (DM). CM is a settlement period with no payment friction. Both households and firms rebalance their bond position and involve in CM goods production and consumption. CM goods are non-storable general goods which can be consumed by both households and firms. CM goods are considered the numeraire. DM goods are non-storable special goods which are only consumed by households. Firms and households have bilateral meetings in DM with chances α_f and α_h respectively to find a match. Once matched, firms produce DM goods y and households pay d to trade y. In DM trade, only money can be used as a means of payment. This assumption emphasizes money's role as means of payment and makes fiat money essential. Following the standard assumptions of a money search model, no barter is likely to happen and money is universally valued and accepted.[5]

1.2.2. Households

Households work and consume in CM with quasi-linear utility $U(x) - AL$, where x is CM goods consumption and L is working hours. Households get income wL, and then spend x on CM goods. Households use the remainder to buy bonds or to bring money to DM to buy DM goods y. DM utility of consuming y is $u(y)$. Both $U(x)$ and $u(y)$ are strictly concave. Money and bonds roll over time, while both x and y are non-storable. Specifically, x can only be consumed in CM, and y can only be consumed in DM. This specification guarantees that households always eat all the consumption goods immediately.

In DM, households have the chance to meet pairwise with firms to trade bilaterally. Let n be the participation ratio of households over firms, such that $n = \frac{N_h}{N_f}$. When solving for equilibrium, firms always participate, hence $N_f = 1$ and n is simply the participation measure of households. Let σ be the probability of double-coincidence, which is the trading probability conditional on matching. In the baseline model, trades happen automatically once matched, such that $\sigma = 1$. Use standard matching technology, let $\alpha_h = \frac{\alpha(n)}{n}\sigma$ and $\alpha_f = \alpha(n)\sigma$. Matching function $\alpha(n)$ satisfies $\alpha^{'}(n) > 0$, $\alpha^{''}(n) < 0$, $\alpha(n) \leq min\{1, n\}$, $\alpha^{'}(0) = 1$ and $\alpha(\infty) = 1$.

[5] Some papers use a mixture of assets and money as payment in trade, which raises the issue of asset pricing with asymmetric information or limited commitment. Since this paper only considers trading friction as DM friction, it is concise to allow fiat money only for means of payment.

1.2.3. Firms

Firms produce consumption goods in this economy. The CM production uses a neoclassical concave technology $f(k, L)$ where k is capital and L is labor. After CM production, firms pay back the loan, b; while capital k depreciates by δ. Then, firms invest I units of output to build up capital stock such that $k^{'} = (1-\delta)k + I$; but they have to wait until DM to use $k^{'}$ as the production input. In CM, firms use $k^{'}$ as collateral to borrow a secured loan $b^{'}$ with gross interest rate R^f. The borrowing constraint is $b^{'} \leq \gamma^{'} q^{'} k^{'}$, where $q^{'}$ is next period's collateral value of the capital $k^{'}$ and $\gamma^{'}$ is the LTV. Since CM goods are numeraire, the no arbitrage condition dictates that the price of $k^{'}$ should be one. Financial intermediaries add collateral property to $k^{'}$, which is desirable to budget constrained firms. Thus the price of this collateral property, $q-1$, might be positive under two conditions: 1) financial intermediaries incur costs to convert $k^{'}$ to collateral; 2) the financial intermediaries' capacity to create collateral property is limited. In this paper, financial intermediaries can always convert $k^{'}$ to collateral for free, so $q > 1$ only happens off the equilibrium path when financial intermediaries' capacity to create collateral is limited. For the rest of the paper, I will stick with the equilibrium path where $q = q^{'} \equiv 1$.

In CM, firms' fund inflows are net output, $f(k, L) - I$, and debt, $\frac{b^{'}}{R^f}$; fund outflows are wage payment, wL, and debt payment, b. The difference is net profit, z, which is firms' CM consumption. In DM, the firm has a chance to sell DM good y to get payment d. DM good production is characterized by a cost production $c(y, k^{'})$ that $c_1(y, k^{'}) > 0$, $c_2(y, k^{'}) < 0$, $c_{11}(y, k^{'}) > 0$, $c_{22}(y, k^{'}) > 0$ and $c_{12}(y, k^{'}) < 0$.[6]

[6] I adopt the cost function setup from Aruoba *et al.* (2008). The cost function is strictly

Similar to households, firms' debt and capital roll over time, and outputs are non-storable. The matching technology in the DM is constant returns to scale. Similar to Kyiotaki and Wright (1989), DM trade must be bilateral and *quid pro quo*, therefore the price of DM goods may contain a bubble which leads to inefficient production.

1.2.4. Planners

As a benchmark, consider the planner's problem with perfect credit. In each period, the planner chooses general goods consumption x and z for households and firms, and special goods consumption y for households. The planner also chooses optimal working hours L and investment I. In DM, firms and households are matched randomly. The planner can not guarantee a household finds a match in DM. Hence, households consume y with matching probability α_h. Since households have quasi-linear utility and firms have linear utility, the planner's goal is to maximize the sum of households and firms' utility. Then the objective function for the planner is:

$$H(k) = \max_{x,y,z,L,I} \quad U(x) - AL + z + \alpha_f \sigma \left[u(y) - c(y, k^{'}) \right] + \beta H(k^{'}) \tag{1.1}$$

$$s.t. \quad x + z + I = f(k, L) \tag{1.2}$$

$$I = k^{'} - (1 - \delta) k \tag{1.3}$$

where (1.2) is the resource constraint and (1.3) is the capital evolution function.

increasing and convex in y and strictly decreasing and convex in $k^{'}$. Moreover, the cross term $c_{12} < 0$ guarantees that more capital always increases DM production.

When the current capital stock k is big enough, there is an interior solution. The optimal allocations are specified by the following first order conditions:

$$U^{'}(\tilde{x}) = \quad 1 \quad [\tilde{x}] \tag{1.4}$$

$$u^{'}(\tilde{y}) - c_1\left(\tilde{y}, \tilde{k}^{'}\right) = \quad 0 \quad [\tilde{y}] \tag{1.5}$$

$$-\alpha\sigma c_2(\tilde{y}, \tilde{k}^{'}) + \frac{1}{R} f_1(\tilde{k}^{'}, \tilde{L}) = \quad 1 - \beta(1-\delta) \quad \left[\tilde{k}^{'}\right] \tag{1.6}$$

$$f_2(k, \tilde{L}) = \quad A \quad \left[\tilde{L}\right] \tag{1.7}$$

and $\tilde{z} = f(k, L) - I - x \geq 0$. Here planner arranges the credit and the DM production once matched. Hence production in both CM and DM are socially optimal. Roughly speaking, in the decentralized economy, the credit limit causes $k^{'} < \tilde{k}^{'}$ and $L \leq \tilde{L}$, while the bargaining process causes inefficient DM production of $y < \tilde{y}$.

1.3. Equilibrium

Assume a monetary authority injects money into households to target a specific interest rate. Households get lump sum transfer τ at the beginning of every CM. Transfers depend on the aggregate money supply M^s from last period, $\tau = M^s_{t-1}(\nu - 1)$, where ν is the gross growth rate of M^s.

The bond market, production market and general goods market are competitive spot markets which locate in CM. Financial intermediaries announce LTV $\gamma^{'}$ in CM. In general, all the essential decisions are made in CM. Firms build up capital to increase productivity and induce households to accumulate money for DM consumption. Once households and firms are matched in DM, the exchange of money and DM goods is resolved by a standard bargaining process.

1.3.1. Households' problem

In period t, households have a quasilinear utility function in CM, $U(x_t) - AL_t$; and a concave utility function in DM, $u(y_t)$. Let $W^H(m,b)$ be the households' continuation value of CM and $V^H(m^{'},b^{'})$ be the households' continuation value of DM. m is households' money holding at the beginning of CM; $m^{'}$ is the money holding at the beginning of DM. b is the bond yield received at the beginning of CM; $b^{'}$ is bond investment which will be paid next CM. Households' optimization problem is

$$\begin{aligned} W^H(m,b) &= \max_{x,L,b^{'},m^{'}} U(x) - AL + V^H(m^{'},b^{'}) \\ &s.t \quad \phi(m^{'} - m - \tau) + x = wL - \frac{b^{'}}{R^f} + b \end{aligned} \tag{1.8}$$

where ϕ is the price of money in terms of the CM good, w is the wage rate. Household's DM continuation value is:

$$\begin{aligned} V^H\left(m^{'},b^{'}\right) &= \alpha_h\left[u(y) + \beta W^H(m^{'} - \frac{d}{\phi^{'}}, b^{'})\right] \\ &\quad + (1-\alpha_h)\,\beta W^H\left(m^{'},b^{'}\right) \end{aligned} \tag{1.9}$$

Assume CM goods consumption x always has interior solution, $U^{'}(x) = \frac{A}{w}$. Envelope conditions $W_1^H(m,b) = \frac{A\phi}{w}$ and $W_2^H(m,b) = \frac{A}{w}$ imply linearity of W on m

and b. Hence (1.9) can be written as

$$V^H(m^{'}, b^{'}) = \beta W^H(m^{'}, b^{'}) + \alpha_h \left[u(y) - \beta \frac{A}{w^{'}} d\right] \tag{1.10}$$

From the simplified DM value function (1.10), $u(y) - \beta \frac{A}{w^{'}} d$ is the gain from trade for households. Since $V_2^H\left(m^{'}, b^{H'}\right) = \beta \frac{A}{w^{'}}$, $b^{H'}$ has a mix solution if $\frac{A}{w^{'}}\beta = \frac{A}{w}\frac{1}{R^f}$; $b^{H'} = 0$ if $\frac{A}{w^{'}}\beta < \frac{A}{w}\frac{1}{R^f}$; $b^{H'}$ has constraint solution if $\frac{A}{w^{'}}\beta < \frac{A}{w}\frac{1}{R^f}$. From the quasi-linear utility, an accepted wage rate w is not less than the marginal disutility rate A.

1.3.2. Firms' problem

Firms consume general goods z with a linear utility function. Let $W(b, k)$ be firms' continuation value of CM and $V\left(b^{'}, k^{'}\right)$ of DM. b is firms' debt rollover from the last DM to the current CM. After paying off b, firms borrow new debt $b^{'}$ before entering DM. k is the capital stock at the beginning of CM, $k^{'}$ is the capital stock at the beginning of DM. Firms' optimization problem is

$$W(b, k) = \max_{z \geq 0, I \geq 0, L, b^{'}} z + V\left(b^{'}, k^{'}\right) \tag{1.11}$$

$$s.t. \quad z + I = f(k, L) - wL + \frac{b^{'}}{R^f} - b \tag{1.12}$$

$$k^{'} - (1 - \delta) k = I \tag{1.13}$$

$$b^{'} \leq \gamma^{'} k^{'} \tag{1.14}$$

I is the new investment of capital. Capital k follows neoclassical growth model style law of motion. Firms' DM continuation value is

$$V\left(b^{'},k^{'}\right) = \alpha_f\left[-c\left(y,k^{'}\right)+\frac{1}{R}W\left(b^{'}-d,k^{'}\right)\right] + (1-\alpha_f)\frac{1}{R}W\left(b^{'},k^{'}\right) \tag{1.15}$$

Since firms are entrepreneurs with linear utility, the borrowing constraint (1.14) is always binding when $R^f \equiv R$. Then the new debt position is $b^{'} = \gamma^{'}k^{'}$. Wage rate w is the marginal productivity of labor, $w = f_2(k, L)$, which is bounded by A. When $f_2(k, L) = A$, $L = \tilde{L}$, and other allocations are socially optimal. Later, the bargaining solution shows that d depends on households money holding $m^{'}$. Applying the envelope condition, $W_1(b, k) = -1$, (1.15) becomes

$$V\left(b^{'},k^{'}\right) = \frac{1}{R}W\left(b^{'},k^{'}\right)+\alpha_f\left[-c\left(y,k^{'}\right)+\frac{1}{R}d\right] \tag{1.16}$$

In CM, if firms invest one more unit of capital, they can borrow $\gamma^{'}\beta$ from the financial intermediaries. $1-\gamma^{'}\beta$ is then the borrowing margin for $k^{'}$, which is the marginal default cost for firms. To induce firms to always pay back the loan, financial intermediaries equalize the marginal default cost and the marginal gain from default to secure loans and maximize lending. Since firms lose collateral $k^{'}$ if they default, the gain from default is just the one-period capital gain from $k^{'}$. Hence $\gamma^{'}$ is determined by

$$\underbrace{1-\gamma^{'}\beta}_{Default\,Cost} = \underbrace{-\alpha_f c_2\left(y,k^{'}\right)+\frac{1}{R}f_1\left(k^{'},L^{'}\right)}_{Default\,Gain} \tag{1.17}$$

Although γ' depends on k', firms take γ' as given when optimize CM problem (1.8).

1.3.3. Nash Bargaining

Let θ be the households' bargaining power and $(1-\theta)$ be the firms' bargaining power. The gain from trade is $u(y)-\beta\frac{A}{w'}d$ for the household, and is $-c(y,k')+\frac{1}{R}d$ for the firm. The terms of trade, (d,y), solve the following problem

$$\begin{aligned} \underset{y,d}{max} \quad & \left(u(y)-\beta\tfrac{A}{w'}d\right)^{\theta}\left(-c(y,k')+\tfrac{1}{R}d\right)^{1-\theta} \\ s.t. \quad & d \leq \phi' m' \end{aligned}$$

Households' gain from trade depends on the wage and working disutility ratio $\frac{w}{A}$, consider the stationary condition that $w=w'$, then $u(y)-\beta\frac{A}{w'}d=\frac{w}{A}\left(u(y)-\beta d\right)$. The total gain from trade on the equilibrium path is then $\frac{w}{A}u(y)-c(y,k')$. When $\frac{w}{A}u(y)-c(y,k')\geq 0$, a bargaining solution exists. Let y^* solve $\frac{w}{A}u'(y^*)-c_1(y^*,k')=0$, which is the efficient production level. Let m^* solve

$$\beta\phi' m^* = \theta c(y,k') + (1-\theta)u(y)$$

Let $\hat{\beta}=\frac{\beta A}{w'}$, given k', define $\hat{y}$ as determined by the following implicit function

$$d = \frac{\theta u'(\hat{y})c(\hat{y},k') + (1-\theta)c_1(\hat{y},k')u(\hat{y})}{\theta\frac{1}{R}u'(\hat{y}) + (1-\theta)\hat{\beta}c_1(\hat{y},k')}$$

Then the solution to this Nash Bargaining problem is

$$d = \begin{cases} \phi' m^* & if\, m' \geq m^* \\ \phi' m' & if\, m' < m^* \end{cases}$$

$$y = \begin{cases} y^* & if\, m' \geq m^* \\ \hat{y} & if\, m' < m^* \end{cases}$$

Since utility function $u\left(y\right)$ is strictly concave, and cost function $c\left(y, k'\right)$ is strictly increasing and convex in y and strictly decreasing and convex in k', $y'_d = \frac{1}{g_1\left(y, k'\right)} > 0$ when $m' < m^*$.[7] Following Lagos and Wright (2003), the standard bargaining mechanism can not achieve first-best, then $m' < m^*$ in a monetary equilibrium. Thus $d = \phi' m'$, $y = \hat{y}$.

Definition 1.1. Bargaining Solution. Given bargaining power θ and CM optimization, $d = \phi' m'$ and $\beta d = g(y, k')$ where

$$g\left(y, k'\right) = \frac{\theta u'(y) c(y, k') + (1-\theta) c_1(y, k') u(y)}{\theta u'(y) + (1-\theta)\frac{A}{w} c_1(y, k')}$$

Households' surplus share is $\frac{\frac{w}{A}u - g}{\frac{w}{A}u - c}$ while firms' surplus share is $\frac{g-c}{\frac{w}{A}u - c}$. Agents' bargaining power is positively correlated with their share of the surplus. Firms' capital stock k' decreases firms' surplus share and increases households' surplus share. When firms are able to build large amount of k', their debt positions on the bond market are also high. Similar to Lagos and Rocheteau (2011), it is more costly to hold a higher bond position, and hence firms have stronger incentive

[7]See appendix for proof.

to trade in DM. As a result, the firms give in during the bargaining process to attract households. Meanwhile, a higher $k^{'}$ increases the total gain from the trade, $\frac{w}{A}u\left(y\right)-c\left(y,k^{'}\right)$.

1.3.4. Equilibrium

Corollary 1.2. *PARTICIPATION. n is determined by households' participation constraint*

$$\left\{-i\frac{A}{w}g(y,k^{'})+\alpha_h\left[u(y)-\frac{A}{w}g(y,k^{'})\right]\right\} \geq 0 \tag{1.18}$$

when firms optimize.

When (1.18) is binding, the household is indifferent between participating and not, hence n is pinned down by the binding condition that

$$\alpha_h\left(n\right)=\frac{i\frac{A}{w}g(y,k^{'})}{u(y)-\frac{A}{w}g(y,k^{'})}$$

When (1.18) is not binding, households always participate, hence participation ratio is binding that $n=1$.

Assumption 1.3. $\frac{u^{'}(y)}{g_y(y,k)}$ *is strictly decreasing in y $\forall k$, $\lim\limits_{y\to 0}\frac{u^{'}(y)}{g_y(y,k)}=\infty$.*

Given a bargaining solution, the marginal contributions of DM state variables are

$$\begin{aligned}
\frac{\partial V\left(b^{'},k^{'}\right)}{\partial k^{'}} &= \frac{1}{R}\left[(1-\delta)+f_1(k^{'},L^{'})\right]-\alpha_f\left(n\right)c_2\left(y,k^{'}\right)\\
\frac{\partial V^H\left(m^{'},b^{'}\right)}{\partial m^{'}} &= \hat{\beta}\phi^{'}+\alpha_h(n)\phi^{'}\left[\frac{\beta u^{'}(y)}{g_1(y,k^{'})}-\hat{\beta}\right]
\end{aligned}$$

and $\frac{\partial V^H\left(m^{'},b^{'}\right)}{\partial b^{'}} = \hat{\beta}$. Combined with FOCs of (1.8) and (1.12), the optimal conditions of $k^{'}$and $m^{'}$ are determined.

With the preparation of the above assumptions and bargaining solution definition, I can define the equilibrium as follows.

Definition 1.4. EQUILIBRIUM. Given states $\{m, b; k, b\}$, households' choices $\{L, x, m^{'}, b^{'}\}$ solve households' CM problem (1.8), and firms' choices $\left\{L, z, k^{'}, b^{'}\right\}$ solve firms' CM problem (1.12). Markets clear that $\int_h b^{'} = \int_f b^{'}$, $\int_h L = \int_f L$. DM terms of trade $\{d, y\}$ solve the bargaining problem, which is given by the definition of the bargaining solution. $\phi^{'}$ comes from the resource constraint condition $\int x + \int z = \int [f(k, L) - I]$. n is binding when the participation constraint is not binding; otherwise, n is pinned down by the binding condition. LTV ratio is pinned down by the no-default IC condition (1.17).

Unconstrained equilibrium solutions of $k^{'}$ and y solve

$$1 - (1-\delta)\beta = \beta f_1(k^u, L^{'}) - \alpha_f(n) c_2(y, k^u) \tag{1.19}$$

$$\frac{\phi}{\beta\phi^{'}} \frac{w^{'}}{w} - 1 = \alpha_h \left[\frac{w^{'}}{A} \frac{u^{'}(y)}{g_1(y, k^u)} - 1 \right] \tag{1.20}$$

In CM, firms need to borrow $I - f(k, L) + wL + b \triangleq b^I$ to support investment and various payments. If $b^I \leq \beta\gamma^{'} k^u$, firms are able to choose the unconstrained solution for new capital $k^{'}$. If $b^I > \beta\gamma^{'} k^u$, unconstrained solutions are restricted by the LTV ratio limitation. Under this condition, $k^{'}$ has the constrained solution that

$$k^c = \frac{(1-\delta)k + f(k,L) - wL - b}{1-\beta\gamma'} \tag{1.21}$$

Then (1.21) and (1.20) define constrained equilibrium solutions of k' and y.

Lemma 1.5. *Optimal* $\left(k'\right)^* = min\{k^u, k^c\}$

k^c is the bound of feasible capital considering the borrowing constraint. If $k^c < k^u$, it means that firms are not able to optimize k' to maximize profit; then firms have to choose the second best k^c. If $k^c \geq k^u$, then firms optimize k'. LTV γ' achieves the optimal ratio $1-\delta$ in an unconstrained equilibrium, and $\gamma' < 1-\delta$ in a constrained equilibrium.

Proposition 1.6. *EXISTENCE. A monetary equilibrium exists if and only if*

$$\max_y \left\{-i\frac{A}{w}g(y,k') + \alpha_h\left[u(y) - \frac{A}{w}g(y,k')\right]\right\} > 0$$

where $k' = min\{k^u, k^c\}$.

In steady state equilibrium, real money demand $\phi M'$ is constant, hence $\frac{\phi}{\phi'} = \nu = 1+\pi$. The Fisher equation implies that $\frac{\nu}{\beta} = (1+\pi)(1+r) = 1+i$. Given n^s, y^s and k^s, solve

$$\frac{i}{\alpha_h} = \frac{w^s}{A}\frac{u'(y^s)}{g_1(y^s,k^s)} - 1 \tag{1.22}$$

$$k^s = min\{k^u, k^c\} \tag{1.23}$$

where k^u and k^c are determined by

$$1 - \beta\left(1-\delta\right) \quad = \quad \beta f_1(k^u, L^s) - \alpha_f(n^s) c_2(y^s, k^u) \tag{1.24}$$

$$wL^s - \alpha_f d^s = f\left(k^c, L^s\right) + \left(\beta\gamma' - \gamma' - \delta\right) k^c \tag{1.25}$$

Figure Fig. 1.1 illustrates the dynamic of $\{k_t\}$. The left graph shows that only the unconstrained equilibrium A^u is supported as a steady state equilibrium; the right graph shows that only the constrained equilibrium B^c exists in steady state. In both equilibria, market liquidity and funding liquidity move together, although the predicted relationship is defined differently.

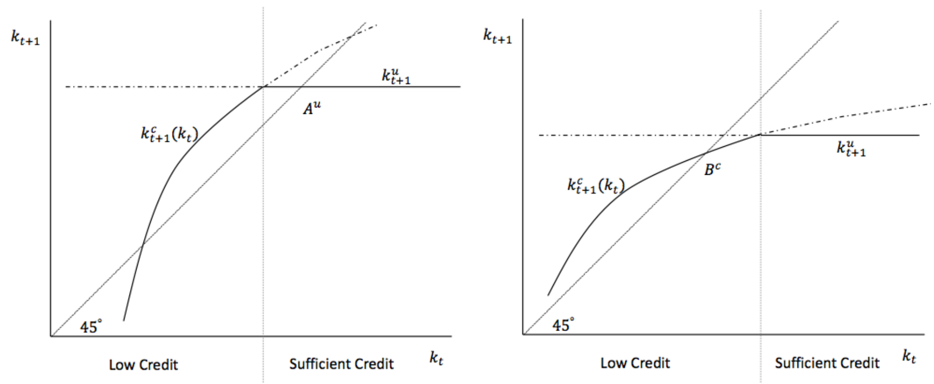


Figure 1.1.: Dynamic Evolution Of Capital

Comparing to previous literature, the comovement of funding and market liquidity does not only rely on firms' constrained investment with binding borrowing constraint; it also depend on agents' interdependent decisions across the bond and DM goods market. Firms' investment decision in the CM period affects the marginal cost of the DM goods and hence its fundamental value. The trading efficiency of DM goods enters the collateral value of firms' capital.

1.4. Quantitative Analysis

1.4.1. Preliminaries

With equilibrium results, I measure liquidity risks by the markups of the risk free bond and DM goods to check the comovement of funding liquidity and market liquidity. In DM, one dollar is worth $\frac{A}{w}\beta\phi'$ utils and the marginal cost in terms of dollars is $c_y\frac{w}{A}\frac{1}{\phi'\beta}$. Since the dollar price of DM goods y is $\frac{m'}{y}$, the markup in DM is defined by

$$1+\mu_d=\frac{\frac{m'}{y}}{c_y\frac{w}{A}\frac{1}{\phi'\beta}}=\frac{\frac{A}{w}g\left(y,k'\right)}{yc_1\left(y,k'\right)} \tag{1.26}$$

The markup of the bond market depends on the borrowing margin and excess debt supply z

$$1+\mu_b=\frac{1-\beta\gamma'}{1-\beta\left(1-\delta\right)}-\frac{z}{k'} \tag{1.27}$$

Definition 1.7. Define the funding illiquidity by the markup μ_b and the market illiquidity by the markup μ_d.

Markup μ_d and μ_b reflect financial market frictions. Since $\frac{\partial k'}{\partial \mu_d}\leq 0$ and $\frac{\partial \mu_b}{\partial k'}<0$, the markups in the two markets move together.[8]

[8]See Appendix A.3 for the proof.

The real GDP in this model is $Y = \int (f(k,L) + \alpha_f d)$, and real money balance in the model is $M^d = \int m' = \int \frac{d}{\phi'}$, then $\frac{M^d}{P} = \phi \left(M^d\right)^{ss} = n\nu d$. The money demand from the model is

$$L(i) = \frac{M^d}{PY} = \frac{n\nu d}{f(k,L) + \alpha_f d} \tag{1.28}$$

The labor share of income is $LS = \frac{wL}{f(k,L)+\alpha_f d}$, and the capital output ratio $\frac{K}{Y} = \frac{K}{f(k,L)+\alpha_f d}$. Let households' utility functions be $U(x) = \lambda log x$ in CM and $u(y) = \frac{y^{1-\rho}}{1-\rho}$ in DM, where $\lambda > 0$ and $0 < \rho < 1$. I adopt the method and production specification in Aruoba *et al.* (2008), the CM production follows Cobb-Douglas such that $f(k,L) = k^a L^{1-a}$, and the DM cost function is $c(y,k) = y^{(1-\chi)} k^{\chi}$, where $0 < a < 1$ and $\chi < 0$. To pin down A, I use relative working hours $H = 0.33$ from the literature.[9] Since the utility of leisure is $\hat{A}(1-H)$, then $H = \frac{AL}{\alpha_h \left[u'(y) - \frac{A}{w}\beta d\right]} \triangleq \frac{AL}{\hat{A}}$.

Interest rate elasticity of money is $\eta_m = \frac{\partial \phi' m'}{\partial i} \frac{i}{\phi' m'}$. Since $\beta \phi' m' = g\left(y, k'\right)$, $\eta_m = \left(g_y \frac{\partial y}{\partial i} + g_k \frac{\partial k'}{\partial i}\right) \frac{i}{g}$. In unconstrained equilibrium,

$$\eta_m^u = \frac{g_y^2 \left(g_y/\alpha_h + g_k n c_{yk}\right)}{\left[\left(\beta f_{kk} - \alpha_f c_{yk}\right)\left(u_{yy} g_y - u_y g_{yy}\right) - u_y g_{yk} \alpha_f c_{yk}\right]} \frac{i}{g}$$

In constrained equilibrium, $\eta_m^c = g_y y_i \frac{i}{g}$.

From the model, measures of funding liquidity are excess margin requirement $\frac{1-\beta\gamma'}{1-\beta(1-\delta)} - 1$ and the volume of the debt b'; measures of market liquidity are the DM markup μ_d and the DM trading volume.

[9] See Benhabib, Rogerson and Wright (1990) for the discussion of H. Aruoba *et al.* (2008) also adopts the value $H = 0.33$.

1.4.2. Calibration

I calibrate the model parameters by minimizing the squared distance between the data and the model, and then comparing the liquidity moments generated from the model to the corresponding moments from Chordia *et al.* (2003).[10] Ideally, liquidity measures generated from the model should fit liquidity measures of a asset-backed security market and the corresponding underline asset market. Limited by the data availability, I will loosely compare with the bond and stock market liquidity. The discount factor $\beta = \frac{1}{R}$ is pinned down by the real interest rate. Gross growth rate of money supply, ν, is pinned down by the inflation rate. Capital depreciation rate, δ, is matched to the investment and capital ratio $\frac{I}{K} \approx 0.07$. Without loss of generality, let $\lambda = 1$ in the baseline model. Table Tab. 1.1 shows the specifications of these parameters.

Table 1.1.: Simple parameters

Parameter	R	ν	δ	λ
Data (1991-1997)	4.35%	2.99%	0.070	1

To pin down preference parameters ρ and A, production parameters a and χ, and bargaining power θ, the plan is to match the following targets: 1) The money demand $L(i)$ is 14.6 percent on average over the period of 1991-1997. 2) Labor's share of income is used to determine the production parameter a. The numbers are

[10]In Chordia *et al.* (2003), OIB is measured by the percentage difference of sellers and buyers' shares of asset. QSPR is measured by daily time-weighted average quoted bid-ask spread per 100 dollars. QSPRB is QSPR of the bond market and QSPRS is QSP of the stock market. Similarly, OIBB is for the bond market and OIBS is for the stock market.

from to St. Louis Fed's estimation of total compensation to national income ratio. 3) Use the capital stock and GDP ratio $\frac{K}{Y}$ to pin down DM production parameter χ. The number comes from Aruoba *et al.*(2008). 4) A comes from relative working hours H as discussed above. 5) To get θ, I will use money elasticity. Table Tab. 1.2 summarizes the targets from the data and the calibrated moments from two specifications of the model, bargaining and price posting. The parameters ρ, A, a, χ and θ are decided simultaneously. The calibrated values are showed in Table Tab. 1.3.

Table 1.2.: Target moments

Targets	$L(i)$	H	LS	K/Y
Data (1991-1997)	0.1464	0.33	0.7	2.32
Bargaining	0.1464	0.33	0.7	2.32

Table 1.3.: Calibrated Parameters

Targets	ρ	A	a	χ	θ
Bargaining	0.7971	0.7406	0.2464	-4.4950	0.7593

Using calibrated parameters to predict liquidity risks in the period of 1982 to 2011, the simulated markups of the bond market and the DM are showed in the figure Fig. 1.2. Since markup defines illiquidity, figure Fig. 1.2 shows that market liquidity and funding liquidity move together across time and present business cycle property. As indicated by the vertical dash line, 1989 to 1991 is the period of

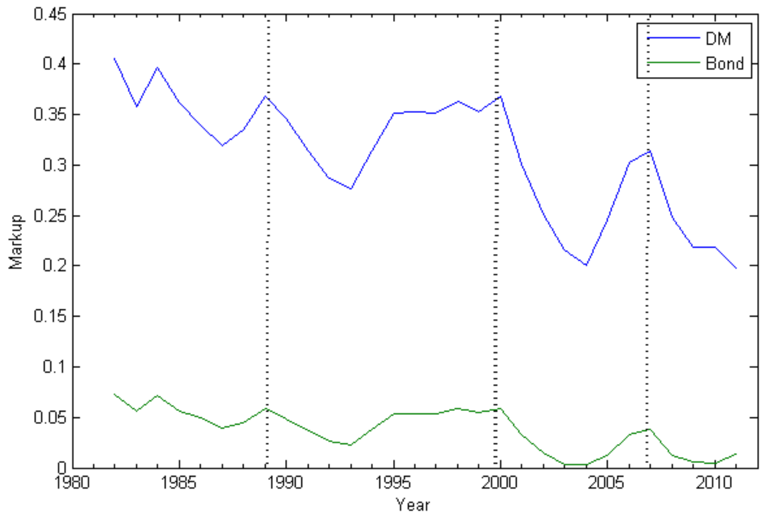


Figure 1.2.: Predicted Markups from 1982 -2011

US Saving & Loan Crisis; preceded by the late 2000s recession the dot-com bubble bursted around 2000; the recent financial crisis started from 2007.

1.5. Monetary Policy

When the nominal interest rate i increases, money becomes more costly for households to carry to the DM since the real balance of money decreases if households do not get the chance to trade DM goods. The real balance effect decreases households money holding at the beginning of the DM. Since every household decreases its money holding $m^{'}$, the trading opportunities become larger because of the spillover effect. Figure Fig. 1.3 illustrate the effect of inflation on $\left(y, k^{'}\right)$.The following analysis will stick to the stationary equilibrium conditions.

Proposition 1.8. $\frac{\partial k^{'}}{\partial \tau} < 0$ *and* $\frac{\partial y}{\partial \tau} < 0$. *Inflation hurts the intensive margin of DM trade* y *and investment* $k^{'}$.

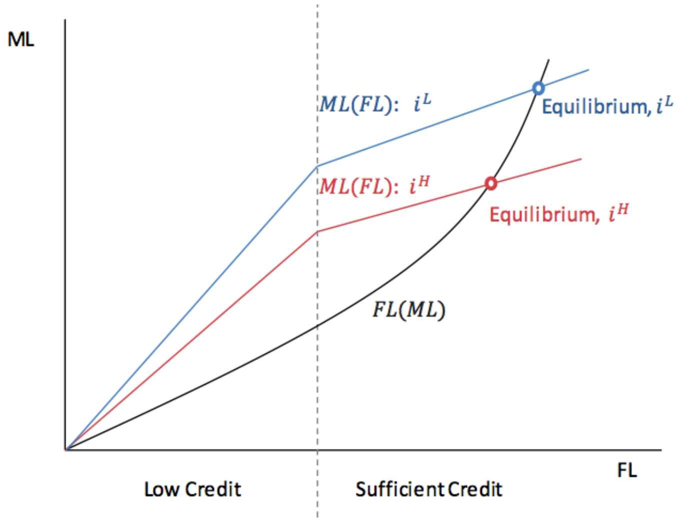


Figure 1.3.: Funding and market liquidity with inflation shock

According to Proposition 2, fund flow from the bond market, $\gamma' k'$, always decreases with inflation since γ' is constant in unconstrained equilibrium and positively correlated with k' in constrained equilibrium. Firms' revenue from DM trade $Rg\left(y, k'\right)$ may increase or decrease since $g_y > 0$ and $g_k < 0$. Inflation in this context has both real balance effect and trading opportunity effect. When the trading opportunity effect dominants, a mild inflation may stimulate DM trade volume. It happens when households have very small bargaining power and the nominal interest rate is low.

Let $y^{\theta i}$ be the optimal DM goods production with nominal interest rate i and bargaining power θ for households. Then $\frac{w}{A}u'(y^{\theta i}) = \left(\frac{i}{\alpha_h} + 1\right) g_1(y^{\theta i}, k')$. The inefficiency caused by the bargaining process is $y^{1i} - y^{\theta i} \geq 0$, while the efficiency loss from inflation is $y^{\theta 0} - y^{\theta i} \geq 0$. The efficiency loss of DM production is also positvely related to the inefficient capital investment k' which comes from the comovement property. When $i = 0$ and $\theta = 1$, the DM production is more efficient that $y^{10} \geq y^{\theta i}$. To achieve socially optimal $\tilde{y}$ that $u^{'}(\tilde{y}) = c_1(\tilde{y}, \tilde{k})$, $k^{'} = k^u$ and $w = A$ are required. However, it can be shown that firms' budget constraint is always binding when $w = A$, $\theta = 1$ and $i = 0$. Hence $k^{'} = k^c$ and $y^{\theta i} \leq y^{10} < \tilde{y}$.

Besides, compare $k^{'}$ in the decentralized economy with the socially optimal $\tilde{k}$ from (1.6). Given y, $k^c < k^u < \tilde{k}$. The efficiency loss $k^u - k^c$ increases with the credit limit which is the main friction of the bond market. The inefficiency caused by the comovement, $\tilde{k} - k^u$, increases with the DM market friction. When the nominal interest rate is zero, the decentralized equilibrium is still worse than the planners' allocations. When using standard bargaining, the Friedman rule does not work since the credit limit friction is independent with the monetary shock.

Proposition 1.9. *The Friedman rule is the optimal monetary policy in terms of y and $k^{'}$, but efficiency allocations can not be achieved. $k^{'} < \tilde{k}$ and $y < \tilde{y}$.*

I also consider unconventional monetary policies which affect the bond market directly, such as quantitative easing (QE). Suppose the monetary authority invest Γ secured loan using risk-free treasury bonds. Then $\int_f b^{'} = \int_h b^{'} + \Gamma$ where $\Gamma > 0$. In the constrained equilibrium, firms' demand of the bond market is constrained by $\gamma^{'} k^{'}$ where the LTV ratio $\gamma^{'}$ is inefficient. A larger Γ leads to higher $k^{'}$ and $\gamma^{'}$, hence increases market liquidity. In the unconstrained equilibrium, the LTV ratio $\gamma^{'} = 1 - \delta$ is efficient. A larger Γ crowds out households' bond investment $\int_h b^{'}$.

Proposition 1.10. *In constrained equilibrium, $\frac{\partial \gamma^{'}}{\partial \Gamma} \geq 0$, $\frac{\partial k^{'}}{\partial \Gamma} \geq 0$*

A proportional injection Γ^p, such that $\int_f b^{'} = \int_h b^{'} (1 + \Gamma^p)$, has similar effect with Γ. When there is unconstrained equilibrium in the economy, a larger Γ increases funding liquidity and market liquidity. When there is constrained equilibrium in the economy, a larger Γ increases funding liquidity more effectively but decreases market liquidity. Specifically, Γ may decrease the participation ratio n.

1.6. Extensions

1.6.1. Price Posting Equilibrium

Previous analysis shows that the bargaining process itself causes inefficiency. In the first extended model, I use price posting instead of bargaining as the DM pricing mechanism. Price posting is an efficient process which is consistent with Hosio's condition. Terms of trade, contracts, can be posted by firms, households or market makers. I adopt the market makers version, where market makers design and post a number of contracts (y, d) before the DM meeting. Agents are able to direct their search towards favorable terms of trade. The market tightness of a submarket with contract (y, d) is $n = \frac{h}{f}(y, d)$, where h is the measure of households applying the contract and f is the measure of firms posting the contract. h and f are constrained by the exogenous supply of households and firms. Define the matching function $\mathcal{M}(h, f)$, then the matching probabilities $\alpha_f = \frac{\mathcal{M}(h,f)}{f} = \mathcal{M}(n, 1)$ and $\alpha_h = \frac{\mathcal{M}(h,f)}{h} = \frac{\mathcal{M}(n,1)}{n}$. Use the previous results for households' and firms' optimization problems. The households' problem is simplified as

$$W^H(m, b) = U_1 + \max_{m',b'} \left[\left(\beta \frac{A\phi'}{w'} - \frac{A\phi}{w} \right) m' + \alpha_h \left[u(y) - \beta d \right] \right] \tag{1.29}$$

where $U_1 = \frac{A\phi}{w}(m + \tau) + \frac{A}{w} b + U(x^*) - \frac{A}{w} x^* + \beta W^H(0, 0)$.

The firms' problem is rearranged as

$$W(b,k) = \Pi_1 + \max_{b',k'} \left\{ -k' + \beta(1-\delta)k' + \beta f\left(k', L\right) + \alpha_f(n)\left[-c(y,k') + \beta d\right] \right\} \tag{1.30}$$

where $\Pi_1 = -b + (1-\delta)k + f(k, L^*) - AL^* + \frac{1}{R}W(0,0)$.

Market makers solve the price posting problem to maximize firms' continuation value $W(b,k)$ while guaranteeing households a reservation utility $\bar{U}$

$$\begin{aligned} \max_{\{n,y,d\}} \quad & W(b,k) \\ s.t. \quad & W^H(m, b^H) = \bar{U} \end{aligned} \tag{1.31}$$

Plug (1.30) and (1.29) into (1.31), and the market makers' problem becomes

$$\max_{\{n,y,d,k'\}} \left\{ \left[\beta(1-\delta) - 1\right]k' + \beta f\left(k', L\right) + \alpha_f\left[-c(y,k') + \beta d\right] \right\} \tag{1.32}$$

$$s.t. \max_{m'} \left\{ \left(\beta \frac{A\phi'}{w'} - \frac{A\phi}{w} \right) m' + \alpha_h \left[u(y) - \beta \frac{A}{w'} d \right] \right\} = \bar{U} - U_1 \tag{1.33}$$

In equilibrium, households bring just enough money $\phi' m' = d$ to the DM market for trade. Define $\bar{U} - U_1 \triangleq \hat{U}$, then households' individual rationality constraint (1.33) becomes $\left(\beta \frac{A}{w'} - \frac{A\phi}{w\phi'} - \alpha_h \frac{A}{w'}\beta\right) d + \alpha_h u(y) = \hat{U}$. Plug the individual rationality constraint back to (1.32); the price posting problem solves

$$\underset{\{n,y,k'\}}{max} \{[\beta(1-\delta)-1]k' + \beta f\left(k', L\right) - \alpha_f c(y,k')$$
$$+\alpha_f \frac{\hat{U} - \alpha_h u(y)}{\frac{A}{w'} - \frac{A\phi}{w\phi'\beta} - \alpha_h \frac{A}{w'}}\}$$

Let η_h and η_f be the matching elasticities of households' and firms' contribution. Then $\eta_h = \frac{\Delta\mathcal{M}/\mathcal{M}}{\Delta H/H} = \frac{\alpha_f'(n)}{\alpha_h}$, $\eta_f = \frac{\Delta\mathcal{M}/\mathcal{M}}{\Delta F/F} = -\frac{n\alpha_h'(n)}{\alpha_h}$ and $\eta_f + \eta_h = 1$. Using the first order conditions with respect to n and y, the equilibrium condition of DM trade is similar to the bargaining solution that

$$\beta d = \frac{\eta_h c(y,k')u'(y) + \eta_f c_1(y,k')u(y)}{\eta_h u'(y) + \eta_f \frac{A}{w'} c_1(y,k')} \triangleq \tilde{g}(y,k') \tag{1.34}$$

Definition 1.11. Competitive Search Equilibrium. Given $\bar{U}$, $\{k', n, y, d\}$ solves the market makers problem. The DM market clears such that $\int nFirm = 1$.

The market tightness $n(\hat{U})$ is strictly decreasing in $\hat{U}$. When $\hat{U} = 0$, the household is indifferent between participating in the DM and not. n is constrained by the ratio of the outside supply of household and firms where $n \leq 1$. If $n(0) > 1$, n has a constraint solution where $n = 1$ as the ratio of the exogenous supply of households to firms, $\hat{U} = n^{-1}(1)$ and $\bar{U} = \hat{U} + U_1$. A monetary equilibrium always exists such that $n > 0$ and $y > 0$. If $n(0) \leq 1$, then $\hat{U} = 0$ and $\bar{U} = U_1$.

Claim 1.12. Firm's IR Constraint. A monetary equilibrium exists if and only if the optimal y and k' are greater than zero when $n = n(0)$

$$\underset{\{n,y,k'\}}{max} \left\{ [\beta(1-\delta)-1]k' + \beta f\left(k', L\right) + \alpha_f \left[-c\left(y, k'\right) + \frac{\hat{U} - \alpha_h u(y)}{\frac{A}{w'} - \frac{A\phi}{w\phi'\beta} - \alpha_h \frac{A}{w'}} \right] \right\} > 0$$

where $n = n(0)$.

Bargaining power θ is endogenous in price posting equilibrium and it always has the efficient level η_h. In steady state, $k^{'}$, y and n are decided by (1.23) and the following conditions

$$i = \alpha_h(n^s)(\frac{w}{A}\frac{u^{'}(y^s)}{c_1(y^s, k^s)} - 1) \tag{1.35}$$

$$\frac{\alpha_h(n^s)u(y^s) - \hat{U}}{i + \alpha_h(n^s)} = \tilde{g}(y^s, k^s) \tag{1.36}$$

Proposition 1.13. *When the nominal interest rate is zero, the Friedman rule achieves a second best equilibrium, that the DM production y achieves the secondary efficient level y^0.*

This is obvious from DM optimization condition (1.35) that $\frac{w}{A}u^{'}(y^0) = c_1(y^0, k)$ if $i = 0$. The equilibrium conditions are similar to the previous equilibrium with bargaining, except the bargaining power θ is replaced by the endogenous matching elasticity η_h, which is convenient for calibrating the model. The calibrated parameters of this model are more robust than the bargaining model.

Table 1.4.: Target moments

Targets	$L\,(i)$	H	LS	K/Y
Data	0.1464	0.33	0.7	2.32
Price Posting	0.1462	0.36	0.7	2.31

Table 1.5.: Calibrated Parameters

Targets	ρ	A	a	χ
Price Posting	0.7993	1.1688	0.5317	-0.8313

1.6.2. Price posting with endogenous search intensity

Until now, DM trades have happened once households and firms are matched such that $\sigma = 1$. In this model, I assume the double-coincidence probability is related to search effort $\hat{z}$ such that $\sigma(z)$ is the probability of trading once traders meet in DM. The double-coincidence probability $\sigma(z)$ is higher if firms decide to invest more on σ in the CM. Firms now choose whether to consume more z in CM or invest more $\hat{z}$ to increase DM trading probability σ. The cost function of σ is $\hat{z}(\sigma)$, which is strictly convex such that $z_\sigma > 0$, $z_{\sigma\sigma} < 0$. Assume firms always enter such that $z(\sigma_0) = 0$ where $\sigma_0 > 0$. The trading mechanism is price posting. Firms post contracts (y, d, σ) which depend on n before DM meeting where n is the market tightness of the submarket with contract (y, d, σ), $n = \frac{h}{f}(y, d, \sigma)$. Firms' continuation value in CM becomes

$$
\begin{aligned}
W(b,k) \quad &= \quad \max_{z,\hat{z},k',b'} z + V_\sigma\left(b', k'\right) \qquad (1.37)\\
s.t. \quad & z + \hat{z}(\sigma) + k' - (1-\delta)k = f(k,L) - wL + \frac{b'}{R^f} - b \\
& b' \leq \gamma' k'
\end{aligned}
$$

Then envelope conditions $W_b = -1$, $W_k = 1 - \delta + f_1(k, L)$, then the firms' DM problem is

$$V_\sigma\left(b', k'\right) = \alpha_f(n)\,\sigma\left(-c(y, k') + \beta d\right) + \beta W\left(b', k'\right) \tag{1.38}$$

Similarly, $b' = \gamma' k'$, rewrite (1.37)

$$W(b,k) = \Pi_1 + \underset{\sigma,k'}{max}\left\{(\beta + \beta\delta - 1)\,k' + \beta f\left(k', L\right) - \hat{z}(\sigma) + \alpha_f(n)\,\sigma\left[-c\left(y, k'\right) + \beta d\right]\right\}$$

where $\Pi_1 = -b + (1-\delta)k + f(k, L) - wL + \beta W(0, 0)$

Households' problem is similar to the previous model, except the trading opportunity becomes $\alpha_h(n)\sigma$. Equilibrium requires that no submarket makes some firms better off without making households worse off. Market makers choose (n, y, d, σ) to maximize firms' continuation value W such that households get the promised utility $\bar{U}$. The price posting problem solves

$$\begin{aligned} \underset{\{n,y,d,\sigma\}}{max} \quad & W(b,k) \\ s.t. \quad & W^H(m,b) = \bar{U} \end{aligned} \tag{1.39}$$

Solving the equilibrium by the same manor, households' individual rationality constraint becomes

$$\bar{U} - U_1 \triangleq \hat{U} = \left[\beta\frac{A}{w'} - \frac{A\phi}{w\phi'} - \alpha_h(n)\,\sigma\beta\frac{A}{w'}\right]d + \alpha_h(n)\,\sigma u(y)$$

Then $d = \frac{\hat{U} - \alpha_h(n)\sigma u(y)}{\hat{\beta} - \frac{A\phi}{w\phi'} - \alpha_h(n)\sigma\hat{\beta}}$, market makers choose (n, y, d, σ) to solve

$$\max_{\{n,y,d,\sigma,k'\}} \{(\beta + \beta\delta - 1) k' + \beta f \left(k', L\right) - \hat{z}(\sigma) + \alpha_f(n)\sigma \left[-c\left(y, k'\right) + \beta \frac{\hat{U} - \alpha_h(n)\sigma u(y)}{\hat{\beta} - \frac{A\phi}{w\phi'} - \alpha_h(n)\sigma\hat{\beta}}\right]\}$$

Similarly, let $\eta_h = \frac{\Delta\mathcal{M}/\mathcal{M}}{\Delta H/H} = \frac{\alpha_f'(n)}{\alpha_h}$, $\eta_f = \frac{\Delta\mathcal{M}/\mathcal{M}}{\Delta F/F} = -\frac{n \cdot \alpha_h'(n)}{\alpha_h}$ be the matching elasticities of households' and firms' contributions. The first order conditions for y and n imply a similar form of solution where

$$\beta d = \beta\phi' m' = \frac{\eta_h c(y, k')u'(y) + \eta_f c_1(y, k')u(y)}{\eta_h u'(y) + \eta_f \frac{A}{w'} c_1(y, k')} \triangleq \tilde{g}(y, k')$$

The optimization condition for σ is $\hat{z}_\sigma \leq \alpha_f(n)\left(-c(y, k') + \beta d\right)$. If

$$\hat{z}_\sigma < \alpha_f(n)\left[-c(y, k') + \beta d\right]$$

$\forall\sigma$, then $z = 0$

$$\hat{z} = f(k, L) - wL + \frac{b'}{R^f} - b - k' + (1 - \delta)k$$

In this model, both firms' profitability and capital investment affect DM trade. With a constrained equilibrium, the main channel that connects funding and market liquidity is capital investment; with an unconstrained equilibrium, the capacity of the investment $\hat{z}$ to increase the DM matching probability affects the DM friction. Compared to the previous model, this setting explicitly models the relationship between funding and market liquidity when the economy has an unconstrained equilibrium.

1.6.3. Planner's problem

Consider the planner's problem with an endogenous σ. Since the planner also has to respect the existence of matching friction, investment $\hat{z}$ is still required to increase σ.

$$\begin{aligned} H(k) &= \max_{x,L,k',y,z,\sigma} U(x) - AL + z + \alpha\sigma\left[u(y) - c(y,k')\right] + \beta H(k') \\ & \quad s.t. \quad z + \hat{z}(\sigma) = f(k,L) - x - k' + (1-\delta)k \end{aligned}$$

The first order conditions are the same, except the one for σ, that

$$-\hat{z}_{\sigma} + \alpha\left[u(y) - c\left(y,k'\right)\right] = 0 \tag{1.40}$$

Proposition 1.14. *The Friedman rule applies if capital stock is abundant. When (1.40) is satisfied and investment is unconstrained, allocations achieve socially efficiency when the money supply falls at the discount rate.*

1.7. Conclusion

In the liquidity literature, the time-series properties of the comovement between funding liquidity and market liquidity and common factors that drive this comovement have remained largely unexplored. This paper's main effort is to characterize this comovement by connecting the bond market and the asset market through firms' capital. Since capital is both collateral for secured loan and input factor for consumption goods production, funding liquidity and market liquidity are related through the value and volume of firms' capital. Credit limits or borrowing constraints on secured loans depend on the pledge-ability, or profitability, of the collateral and volume of the collateral. Firms are able to build a larger capital stock if capital has a higher degree of pledge-ability. A higher pledge-ability comes from higher pledge-able income, which is affected by DM market frictions.

Market liquidity is measured by firms' fund flow from the DM where firms trade special goods for money with households. Capital plays a strategic role interacting with market liquidity. Since capital is used as collateral, it is positively correlated with firms' debt position. Higher debt position brings abundant fund flows from the bond market, but increases the cost of holding debt over time which weakens firms' stance in the bargaining process. In other words, firms try harder to attract households to DM trades with larger transaction volume by lowering asking price. DM productivity also allows interpretation of the other effect of capital on market liquidity; higher capital k' decreases the average cost of DM production, which increases the total gains from trade.

As a result, the construction of the model links funding liquidity and market liquidity together. Regardless of whether the equilibrium has a constrained or unconstrained solution, liquidity of the bond and DM markets covary because the fundamental financial market frictions are correlated. With constrained capital investment, the comovement between funding and market liquidity is easier to observe, while the comovement is less obvious with unconstrained investment. The quantitative analysis portion of this paper uses standard methods to calibrate the model. Although I set up the model with explicit trading frictions, the calibration can handle non-conventional model parameters, such as the bargaining power θ. In the extended models, a price posting mechanism endogenizes the bargaining power θ, and the market tightness n depends on households' reservation utility $\hat{U}$. One can apply the extended models to the counterfactual policy experiment and estimate welfare gains or losses accordingly.

Finally, this model explains the effect of monetary shocks on financial market liquidity. When monetary authorities inject money continuously into households, it causes inflation, which affects liquidity and welfare. Inflation hurts at the intensive margin, in that trading volume y decreases and the capital investment k' decreases. However, for certain specifications, mild inflation increases the extensive margin n, in that the trading opportunity increases. Injecting money into the bond market or restricting the terms of trade within the bond market requires precise evaluation of economic conditions. As discussed in section 5, liquidity injection increases funding liquidity; it also changes traders' relative asset positions and hence the negotiation over DM goods. The impact of liquidity injection on market liquidity depends on agents' preferences and specifications of the economy, which may bring unintended consequences such as inflation or higher trading frictions.

2. Efficiency and Default in Bank Lending

2.1. Introduction

This paper studies the default rate of a long-term loan in bank lending. Default in long-term loan is defined as failure to make all the period payments. Defaults may occur if debtors are either unwilling or unable to pay back their debts such as, debtors have missed scheduled payment or have violated loan covenants of debt contracts. In many contract theory papers, default is usually a threat of deviation and not exists in equilibrium if the contract is perfect. In practice, default is allowed in non-recourse loan and happened quite often, especially during financial crises. A long-term loan may not be perfect for many reasons, such as moral hazard, asymmetric information or non-contingent restriction. Quadrini (2004) argues that the liquidation of the firm may arise as the outcome of the optimal contract in a long-term contract with moral hazard. In a standard Principle-Agent model, liquidation seems to be never observed in a renegotiation-proof contract, unless driven by a deterioration of the firm technology or market opportunities.

But in a long-term contract, even if default is not free from renegotiation, it may exist in the renegotiation-proof contract. Albuquerque and Hopenhayn (2002) studies optimal lending with dynamic of borrowing constraint. They also point out the importance of studying default. DeMarzo and Fishman (2007) models a long-term financial contracting relationship by constructing a long-term debt, a line of credit, and equity as optimal securities. The optimal debt-equity ratio is history dependent, but debt and credit line terms are independent of the amount financed. The main friction of their paper is the borrowers' ability to divert cash flows. The goal of studying default is not to design a perfect default-free contract, but to understand the dynamic of variables that move together with the default rate.

In this paper, default happens due to the non-contingent feature of the long-term contract that the payment schedule and interest rate is usually predetermined. Although a contingent contract might prevent potential default, it would be as attractive to borrowers as fixed term contract. Bank as the lender is aware of the possibility of default and able to change the term of the contract to influence the default probability. Meanwhile, bank also concerns about its profitability, which is related to the "efficiency" issue. Since bank collects money/fund from depositors, bank pays for the interest incurred from using the fund. The "efficiency" of bank's performance is better if the rate of return from investing the fund is higher. Since it takes time for bank to find a qualified borrower to invest, the efficiency of using the fund dose not only depend on the investment rate of return, but also the utilization rate of the fund. This efficiency of using the fund is higher if bank requests more payments from the same amount of lending, or bank spends less time to find a borrower. To describe the friction of bank lending out money and collecting money from depositors, I adopt the search and matching friction and develop a model from labor search and on-the-job search framework.

In the model, bank has chance to meet with an entrepreneur and establish a long-term borrowing and lending relationship. The entrepreneur is willing to borrow money once he/she comes with an opportunity to start a project. Initial fund is required to start the project, which drives the entrepreneur to the bank. I assume that the entrepreneur has limited commitment and can not collect fund by itself to initiate the project. This assumption emphasize bank's role as the financial intermediary. But this assumption is not indispensable as long as it's easier for the entrepreneur to borrow the initial fund from the bank than directly from the investor. Once the project starts, the entrepreneur runs the project and gains income over time. While running the project, the entrepreneur has randomly coming opportunity to pay back the loan earlier and increase his/her lifetime utility to a higher level, which is called a "jump" in this paper. This "jump" represents the potential market value appreciation of this ongoing project. The entrepreneur decides when to materialize the "jump". The value of the jump is a random variable, but the distribution of the jump is a common knowledge to everyone. To be more concrete about the jump, we can consider a number of heterogeneous buyers who can potentially run the ongoing project better than the entrepreneur. While the entrepreneur runs the project, he/she meets potential buyers randomly. In the meeting, the buyer make an offer to buy the project, and the entrepreneur decides whether to accept or reject the offer. If the entrepreneur accepts the price, he/she sells the project to the buyer and pays back the rest of his loan. If the entrepreneur defaults on his/her debt, the project will be forfeited as the punishment.

This business pattern is similar to venture capital firms or investment banks that lend money to starter firms or growing firms. If a starter firm has a promising future, the investment banks offer loan contracts in hopes of the gain from the future Merger and Acquisition (M&A) opportunities besides the firms' own profitability. If the entrepreneur is able to issue bond after he/she starts the project, the jump can also be considered as a randomly coming opportunity to issue the corporate bond at a price drawn from a known distribution. Therefore, the entrepreneur expects to profit from running the project as well as gaining from the potential jump.

For the bank, such a long-term loan contract bears two types of risks: early termination risk and default risk. Early termination risk is also called prepayment risk, which happens if the entrepreneur terminates the loan earlier. On the event of prepayment, there would be no default on the loan, but it also incurs costs. First, bank loses the interest payment of the loan for the rest of the loan period. Then bank has to spend time and effort to search for new loans more frequently, which generate an opportunity cost of letting bank's fund sit idle since bank has to pay interest to depositors as a compensate of using their fund. The search cost is inevitable if bank can not change its liquidity position freely. On the event of entrepreneur defaulting, bank loses the rest of the loan payments and forfeit the project. Prepayment would be delayed if bank induces the entrepreneur to behave more aggressive that the entrepreneur becomes more picky about the acceptable price of the project. However, the risk of default would increase if the entrepreneur behave more aggressively. On one side, bank wants to make more profit from lending and to use the fund/deposit more efficiently; on the other side, bank cares about the default risks. Bank will balance between these two concerns depending on the investment opportunity and search frictions of the fund and lending market. The result of the model shows that, depending on the environment of the economy, bank may choose to suffer a higher default risk in return for a more efficient use of the fund and a higher profit margin. If all the banks adjust their strategy to accommodate the change of the macroeconomy which leads to a higher default probability, an negative externality would be expected. Hence the result of this paper is implicative to the policy maker.

2.2. Environment

2.2.1. Agents

The economy is populated by banks and entrepreneurs living in continuous time. Bank is risk neutral and performs as financial intermediary that the bank can commit on financial contracts. On one side, bank can issue risk free bond to issue fund/deposit from households; on the other side, bank lends money to entrepreneurs to make investment profit. After the termination of the loan, bank has chances

to either change its liquidity position with rate λ, or find another loan with rate α. The cost of waiting C^w decreases with λ, α.

Entrepreneurs are risk averse with limited commitment. They specialize in innovation, operating projects and searching for jumps. In the model, I assume the preference of the entrepreneur follows a CARA utility $u(c) = -e^{-\sigma c}$, where $\sigma > 0$. CARA has some features to simplify the algebra, but CARA is not necessary for the result of the model. First, the marginal utility of CARA is linear in the level of utility that $u'(c) = -\sigma u(c)$. Besides, $u(c_1 - c_2) = -\frac{u(c_1)}{u(c_2)}$ and $u(c_1 + c_2) = -u(c_1)\, u(c_2)$. This property avoids possible wealth effect on the entrepreneur's decision that helps me to get a close form solution of the equilibrium. This assumption on functional form of utility does not hurt the generality of this model much.

2.2.2. Project

The entrepreneurs come up with a project at a Poisson rate α^e. A typical project requires initial investment C and generates a continuous income stream b. The entrepreneur has to finance the initial investment by acquiring a loan to start the project. Afterwards, there is no explicit operation cost to run the project. If an entrepreneur successfully start a project, he will randomly meet potential buyers at Poisson arrival rate α. During a meeting, the entrepreneur can only meet one potential buyer and decide whether to accept the buyer's offer on the project or not. The potential buyer offers a price P to buy the project that P is drawn from a distribution F. F is common knowledge to both the entrepreneur and bank, but bank does not know the realization of the offer P on the project during the meeting.

Let r be an exogenous interest rate that bank pays for depositors' fund. Define the entrepreneur's operation value of the project $P^e = \frac{b}{r}$. The project is indivisible. If the entrepreneur accepts a price P, he sells the whole project to the buyer.

The value of the project jumps from P^e to P. If the entrepreneur defaults on his loan, the project will be forfeited by the bank and liquidated at $P^l = \gamma^l \frac{b}{r}$. Moreover, let μ be the mean value of P, that $\mu = E(P)$. The distribution F can be interpreted as the environment of the macro economy, that a higher μ implies a more promising market of the project and the change in μ reflects the change of the market expectation.

[1]

2.2.3. Financial Contract

Consider a financial contract between one bank and one entrepreneur that both sides optimize. In practice, an entrepreneur may have multiple choices over lenders and vice versa. The one to one contract can explain the economy as long as the financial market is not perfect competitive. Bank finances the project by lending initial investment C to the entrepreneur. Bank prefers to charge a higher markup to maximize its profit. But bank can not directly control entrepreneur's decision on whether to accept a specific offer or not. If the lending contract provided by the bank can induce the entrepreneur wait for a higher price offer and benefit from a larger jump, the entrepreneur's expected utility from borrowing with the same markup of the loan would be higher. Therefore, bank is able to charge a higher markup to promise the same level of utility to the entrepreneur.

The entrepreneur optimizes his/her consumption and selling decision based on the prediction of different continuation values. If bank requires a higher markup of

[1] C is not essential now, it will be used as the threat point in the bargaining problem. In the current stage, C is nothing more than a constant.

the loan, the burden from the debt would increase the entrepreneur's reserve price to sell the project. Then the entrepreneur is more likely to wait longer to sell the project, which leads to a lower prepayment risk and a higher default risk. If bank loan's markup is low, the entrepreneur is more likely to sell the project fast, which leads to a higher prepayment risk and lower default risk. By varying the term of the loan contract, bank can control the cutoff of the transaction price, hence affects the expected timing of the jump and the range of the jump

The contract would be perfect if bank can observe the realization, make terms of the loan contingent on the price offer and control the entrepreneur's consumption directly. Here the lending contract is optimized with restrictions and limitations. In the model, the entrepreneur is financed by a combination of a long-term non-recourse loan and a credit line scheme. This setup is similar to DeMarzo&Fishman (2006), that it constructs the optimal contract by a combination of a long-term debt, credit line and equity. Besides, both long-term debt and credit line scheme exist in real life, which makes it easier to justify the model and provide economic intuition. Allowing the entrepreneur to draw money from a credit line gives necessary liquidity and flexibility to the entrepreneur, that he/she is able to optimize the consumption while searching for potential buyers. But the potential hidden saving/debt of the entrepreneur increases the uncertainty of fulfilling the loan contract. I also propose a contingent contract in the last chapter to extend my analysis. It is hard to implement state contingent contract, because the realization of P_t is not observable by the bank.

In the baseline model, the long-term debt contract has fixed period payment τ and maturity T. The credit line contains credit limit $\left\{a_t^L\right\}$and interest rate $\tilde{r}$. The accumulated credit debt level at time t, a_t, is revolving based on the interest rate $\tilde{r}$. The entrepreneur can borrow and save as long as the credit debt level does not exceed the credit limit $\left\{a_t^L\right\}$. The total float payment from the entrepreneur is the sum of fixed payment τ and interest payment $\tilde{r}a_t$. Let L_t be the loan balance at time t, the loan balance follows law of motion

$$\dot{L} = rL - \tau$$

then

$$\begin{aligned} L_t &= \frac{\tau}{r}\left[1 - e^{-r(T-t)}\right] \ 0 \leq t \leq T \\ &= L_0 \frac{1 - e^{-r(T-t)}}{1 - e^{-rT}} \end{aligned}$$

2.2.4. Default

If the entrepreneur terminates the loan before the maturity T, it's defined as early termination. *Default* happens if the entrepreneur commits an early termination without making up the payments for the rest period. Early termination also happens out of prepayment that the entrepreneur pay back the rest of the loan before maturity. Default happens if and only if the following two conditions happened before the expiration date T: 1) Insolvent: even if entrepreneur consumes zero, he can not pay τ without intra-period borrowing; 2) Borrowing Constraint: the entrepreneur reaches or exceeds the borrowing constraint at time t.

On the event of default, bank loses L_t on the loan and a_t on the credit. Consider a borrowing constraint associated with the loan to value ratio (leverage) γ, such that, $L_t - a_t \leq \gamma \frac{b}{r}$. If the entrepreneur defaults, he will lose the project and have zero consumption forever. Liquidation value of the project for the bank is $P^l = \gamma^l \frac{b}{r}$. As a result, bank's loss from default is $L_t - a_t - \gamma^l \frac{b}{r} \approx \left(\gamma - \gamma^l\right) \frac{b}{r}$. it can be shown that optimal consumption $\tilde{c}$ is always achieved if $\tilde{c} - r\bar{P} \geq -r\gamma P_0$. To simplify the algebra, here I assume the interest rate of the credit line $\tilde{r}$ equals to the interest rate r.

The project will be continue if the liquidation value of the project P^l is less than or equal to the continuation value of the project. In an optimal contract, one can set the credit limit as $a_t^L = \left(\gamma - \gamma^l\right) \frac{b}{r}$. If the credit limit is very generous, default is less likely to happen, but the credit bank would suffer more once default happens.[2]

Suppose the initial loan-to-value ratio $\gamma = \frac{L_0}{P^e}$ is exogenously given. The initial loan is the difference between the initial investment and entrepreneur's initial asset that $L_0 = C - a_0$. These variables are exogenous for now. A more ambitious extension of this model should introduce a general equilibrium framework, internalizing the supply of loan L_0 and interest rate r.

2.3. Optimal Loan Contract

2.3.1. Entrepreneur's problem

The entrepreneur optimizes his/her consumptions and choose the timing to materialize the opportunity to increase his/her continuation value. There are three options for the entrepreneur after he/she starts the project. First, the entrepreneur

[2] Credit limit will effect the equilibrium, but it is not essential to the key results.

can accept the potential buyer's offer on the project, terminate the loan out of prepayment and transfer the project. Second, the entrepreneur can default on his/her debt. Last, the entrepreneur can just continue running the project and make payments on time. Define the value of jump V_t^s, which depends on the timing t and the realization of the jump; the value of default is a constant V^D ; and the value of continuing without jump and default is V_t.

Since potential jump comes randomly and follows an iid distribution, the entrepreneur chooses the timing to capitalize the jump. The buyer with a higher price offer is always more preferred. The entrepreneur will set a reservation price, $\bar{P}$, that the trade would happen if the price offer is above this line $P \geq \bar{P}$. Although bank can not observe P, it can control $\bar{P}$ by varying terms of the loan contract. Roughly speaking, bank's gain from the jump of the project depends on $\bar{P}$, and entrepreneur's gain from the jump depends on $P - \bar{P}$.

Consider a loan contract with fix payment τ from time 0 to time T. The loan balance at time t is

$$L_t = \begin{cases} \frac{\tau}{r}\left(1 - e^{-r(T-t)}\right) & 0 < t \leq T \\ 0 & t > T \end{cases} \tag{2.1}$$

If the entrepreneur sells the project at price P_t, he makes prepayment L_t and transfers the project. Since the entrepreneur's optimization problem after selling the project is static, the value of jump

$$V_t^s = \frac{1}{r} u\left(c_t^s\right) \tag{2.2}$$

$$c_t^s = r\left(P_t - L_t + a_t\right) \tag{2.3}$$

where a_t is the cumulative saving/credit at time t. If the entrepreneur chooses to continue, the HJB equation of his/her optimization problem is

$$rV\left(L_t, a_t\right) = \max_{c_t} u\left(c_t\right) + \dot{V} + \alpha \int max\left(V_t^s - V_t, 0\right) dF\left(P\right) \tag{2.4}$$

$$s.t.\ \dot{a}_t = b - \tau + ra_t - c_t \tag{2.5}$$

The entrepreneur chooses to default if $V_t < V^D$. With CARA utility, close form solution is available for this optimization problem.

Lemma 2.1. *Entrepreneur's optimization after maturity ($t > T$)*

If the project is still running by the entrepreneur after the maturity of the loan, the equilibrium is:

- The entrepreneur accepts price offers $P \geq \bar{P}$, where $\bar{P}$ is determined by the following equation:

$$\frac{r\sigma}{\alpha}\left(r\bar{P} - b\right) = \int_{\bar{P}} \left[u\left(rP - r\bar{P}\right) + 1\right] dF\left(P\right) \tag{2.6}$$

- The continuation value of running the project is

$$V\left(a_t\right) = \frac{1}{r} u\left(r\bar{P} + ra_t\right) \tag{2.7}$$

- The optimal consumption is

$$\tilde{c}_t = r\bar{P} + ra_t \tag{2.8}$$

From (2.6), we can see that the cutoff price does not depends on the entrepreneur's debt a_t and loan balance L_t. If parameters r, σ, α and the distribution of potential buyers' price offers F are given, there exist unique solution to (2.6) which is the cutoff price.

Lemma 2.2. *Entrepreneur's optimization during the loan contract period (*$0 \leq t \leq T$*)*

The equilibrium defines the entrepreneur's optimal consumption $\tilde{c}_t$ *and his/her decision on jump, default or continue*

- At time t, the entrepreneur sells the project if $P \geq \bar{P}_t$ where $\bar{P}_t$ solves

$$\frac{r\sigma}{\alpha}\left(r\bar{P}_t - \dot{\bar{P}}_t - b\right) = \int_{\bar{P}_t}\left[u\left(rP - r\bar{P}_t\right) + 1\right] dF\left(P\right) \tag{2.9}$$

and the terminal condition for $\bar{P}_t$ is $\bar{P}_T = \bar{P}$

- The continuation value of running the project is

$$V\left(a_t\right) = \frac{1}{r} u\left(r\bar{P}_t + ra_t - rL_t\right) \tag{2.10}$$

- The optimal consumption is

$$\tilde{c}_t = r\bar{P}_t + ra_t - rL_t \tag{2.11}$$

Related proof for the above lemmas on entrepreneur's optimizations are given in the Appendix. Since the optimal consumption $\tilde{c}_t$ is equivalent to jump at the cutoff price, I can use guess and verify to solve the problem. Given the matching set, the value function together with optimal consumption path solves the HJB equation (2.4), then we verify the initial guess.

Notice that $\dot{\tilde{c}}_t = -\frac{\alpha}{r\sigma}\int_{\bar{P}_t}\left[u\left(rP - r\bar{P}_t\right) + 1\right]dF\left(P\right) < 0\ \forall t < T$. It means that the entrepreneur's consumption is decreasing over time until the expiration of the loan contract. The optimal consumption can also be expressed as $\tilde{c}_t = r\bar{P}_t - rL_0 - r\int_0^t\left(\bar{P}_s - P^e\right)ds$, hence the cutoff price $\bar{P}_t$ is also decreasing over time until the maturity date T. Once P_T is decided, the path of cutoff prices can be calculated by backward induction.

The distribution $F\left(P\right)$ also affects cutoff prices. Rearrange (2.9) and integral both sides from 0 to t, then

$$\frac{\sigma}{\alpha}\bar{P}_t = \frac{r\sigma}{\alpha}\int_0^t\left(\bar{P}_s - P\right)ds - \int_0^t\int_{\bar{P}_s}\left[u\left(P - \bar{P}_t\right)\right]dF\left(P\right)ds + \frac{\sigma}{\alpha}\bar{P}_0 \tag{2.12}$$

From (2.11), (2.8), and (2.5), the accumulated credit debt $\{a_t\}$ in equilibrium is

$$a_t = \int_0^t \dot{a}_s ds = \begin{cases} \int_0^t\left(rP - r\bar{P}_s\right)ds - \frac{\tau}{r}e^{-rT}\left(e^{rt} - 1\right) & t \leq T \\ \int_0^t\left(rP - r\bar{P}_s\right)ds - \frac{\tau}{r}\left(1 - e^{-rT}\right) & t \geq T \end{cases} \tag{2.13}$$

Then $L_t - a_t = \frac{\tau}{r}\left(1 - e^{-rT}\right) + \int_0^t \left(\bar{b}_s - b\right) ds$.

2.3.2. Optimal Contract

Using the above optimization solutions, the entrepreneur's expected continuation value from time $t^{'}$ onward for a given contract $\left\{\tau, T, \left\{a_t^L\right\}\right\}$ is

$$U(t') = \int_{t'}^{\infty} e^{-\int_{t'}^{t}\left(r+\alpha\left(1-F\left(\bar{P}_s\right)\right)\right)ds}\left(u(\tilde{c}_t) + \alpha\int_{\bar{P}_t}\frac{u(rP - rL_t + ra_t)}{r}dF(P)\right)dt \tag{2.14}$$

where $\left\{\tilde{c}_t, \bar{P}_t, a_t\right\}$ are optimal solutions from the previous section. Bank's problem is to maximize its expected revenue from the loan contract while promise a certain level of utility to the entrepreneur. Then bank's problem is

$$\begin{aligned} \pi|L_0 \quad &= \max_{\tau,T,\bar{P}} \quad \int_0^{T^*} e^{-\int_0^t\left\{r+\alpha\left[1-F\left(\bar{P}_s\right)\right]\right\}ds}\left\{\tau + \alpha\left[1 - F\left(\bar{P}_t\right)\right]L_t\right\}dt \qquad (2.15)\\ &\qquad\qquad -\Gamma e^{-rT^*}\left(\gamma - \gamma^l\right)P_0 + a_0 \\ &s.t. \qquad \frac{1}{r}u\left(r\bar{P}_0 - rL_0 + ra_0\right) \geq v_0 \end{aligned}$$

where Γ is the probability of default. T^* is the termination time of the debt contract, which is either the first time the entrepreneur exceeds the credit limit or the duration of the debt contract T. $U_0\left(\tau, T, \left\{a_t^L\right\}\right)$ is the lifetime expected utility based on the entrepreneur's best response to the financial contract $\left\{\tau, T, \left\{a_t^L\right\}\right\}$, and v_0 is the promised utility to the borrower.

$$U_0(\tau, T) = \int_0^\infty e^{-\int_0^t \left\{r+\alpha\left[1-F\left(\bar{P}_s\right)\right]\right\}ds}[u(\tilde{c}_t) + \tag{2.16}$$

$$\alpha \int_{\bar{P}_t} \frac{u(rP - rL_t + ra_t)}{r} dF(P)]dt \tag{2.17}$$

If the contract is optimal, $U_{t'} = V_{t'}$ and $U_0 = \frac{1}{r}u\left(r\bar{P}_0 - rL_0 + ra_0\right)$. On the equilibrium path, if the financial contract is optimal and the promised utility is v_0, the probability of default is

$$\Gamma(v_0) = 1 - \int_0^{T^*} e^{\int_0^t -\alpha\left(1-F\left(\bar{P}_s\right)\right)ds} \alpha\left(1 - F\left(\bar{P}_t\right)\right) dt$$

$$= e^{\int_0^{T^*}\left[-\alpha\left(1-F\left(\bar{P}_s\right)\right)\right]ds} \tag{2.18}$$

It means that the entrepreneur fulfills his/her debt payment with probability $1-\Gamma$, and pays part of his loan and then defaults with probability Γ. The revenue from optimal contract can be expressed as

$$\Pi(v_0) = L_0[1 - \Gamma(v_0)] + \frac{\tau}{r}\left(1 - e^{-rT*}\right)\Gamma(v_0) \tag{2.19}$$

2.3.3. Static Analysis

In the bank's problem (2.15), the promised utility constraint should be binding; if not, bank can increase τ a little bit to increase the revenue while the constraint still holds. In the Kuhn-Tucker's problem, the shadow price for v_0 will be negative, which means a higher promised utility will encroach the bank's profit.

Proposition 2.3. *In the optimal contract, keeping everything else constant,* $\Pi(v_0)$ *is decreasing in* v_0*.*

From (2.18), default rate $\Gamma(v_0) = e^{\int_0^{T^*}\left[-\alpha\left(1-F\left(\bar{P}_s\right)\right)\right]ds}$. It depends on the endogenous termination time T^* and cutoff prices $\left\{\bar{P}_t\right\}$, that a larger T^* and lower levels of $\left\{\bar{P}_t\right\}$ imply a lower default rate in equilibrium. Intuitively, if the promised utility v_0 increases, bank can not gain as much profit as before; then the optimal contract will favor the one inducing lower default rate as a compensate.

Proposition 2.4. *On the equilibrium path, the default rate of the optimal contract,* $\Gamma(v_0)$ *, is decreasing in the promised utility,* v_0*.*

If the entrepreneur requires higher level of promised utility, bank would prefer to offer more flexible contract with longer maturity and lower period payment; on the other hand, the entrepreneur would behave more conservative in response to this type of loan. Within all the feasible loan contracts inducing the same level of promised utility, the contract with higher profit will have higher default rate. While inducing a higher promised utility, the optimal loan contract also increase the probability of prepayment, which lead to a lower default rate at the same time.

Proposition 2.5. *If the distribution* $F(P)$ *changes to* $G(P)$*, and the mean value* $\mu_F < \mu_G$*. Under the following conditions, the default rate increases that* $\Gamma_F(v_0) < \Gamma_G(v_0)$*.*

1. $\frac{r}{\alpha}$ is small enough, $P_G = P_F + \epsilon$, where $P_F \sim F(\cdot), P_G \sim G(\cdot)$, and $\epsilon > 0$ is a constant;

2. $G(\cdot) \underset{FSD}{\succ} F(\cdot)$, $\frac{r}{\alpha}$ is small enough that $G\left(\bar{P}_t\right) > F\left(\bar{P}_t\right)$.

Low interest rate r implies low opportunity cost of holding the deposit/fund, which also means an easy credit condition. The arrival rate α measures the market intensity of the project market. If the ratio $\frac{r}{\alpha}$ is small enough, the default rate of loan may go up with an upward shift of the market condition; in the mean time, the expect profit Π will also go up, but the increase would be lower than the increase in the market with a higher $\frac{r}{\alpha}$.

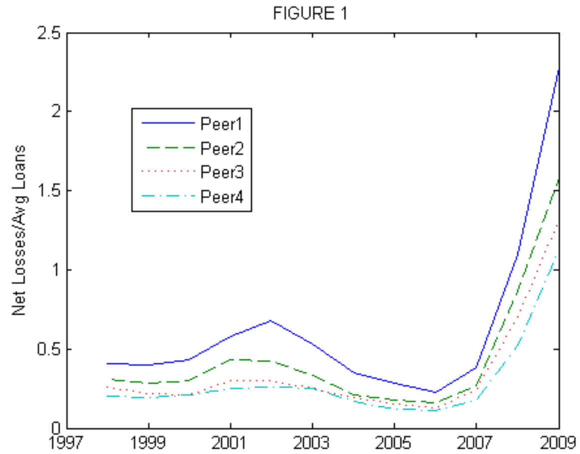
2.3.4. Bargaining Solution

The result of this paper also tends to provide an explanation to different default rates of different sized banks. Figure Fig. 2.1 illustrates various sized banks' default rates from 1998 to 2009. Numbers on the vertical axis are percentages of loans with delayed or missed payments, which are proportional to default rates of loans. Each line represents loan delinquency rates of a certain Peer Group. According to the data from the Bank Holding Company (BHC), Peer 1 banks have the largest asset holding. From Peer 1 to Peer 4, the asset holding levels decrease. [3]

From Figure Fig. 2.1, larger banks (Peer 1 banks) always have higher default rates than smaller banks. Also, the default rate increases significantly during financial crisis and decreases during boom; and the default rate is stable periods between financial crisis if the economy recovers from the crises and performs normally. The paper discusses the heterogeneity between big banks and small banks in terms of different bargaining powers over the price of loans.

[3] Peer groups are classified by the Consolidated Asset Size at the end of the quarter. Peer 1 group banks hold more than \$10 billion asset; Peer 2 group has \$3 billion to \$10 billion; Peer 3 group has \$1 billion to \$3 billion; and Peer 4 group has \$500 million - \$1 billion.

Figure 2.1.: Bank Delinquency Rates



Given entrepreneur's reservation utility U^d and bank's cost, π^d, of holding the liquidity position, let's consider a bargaining solution between the bank and the entrepreneur. Assume larger banks have higher bargaining power and smaller banks have relatively lower bargaining power. Given the promised utility, bank can choose the optimal contract based on the entrepreneur's best response. Let $\pi(c)$ be the expected revenue from the optimal loan contract; $U(c) = \frac{u(c)}{r}$ be the continuation utility of the entrepreneur, where c is the equivalent consumption. Use θ to represent the bargaining power of the bank, and $(1-\theta)$ for the bargaining power of the entrepreneur. The generalized Nash bargaining solves

$$\begin{aligned} &\underset{c}{max} \left[\pi(c) - \pi^d\right]^\theta \left[U(c) - U^d\right]^{1-\theta} \\ &s.t. \pi(c) \geq \pi^d \\ &\quad U(c) \geq U^d \end{aligned}$$

where π^d and U^d are thread points. Let $U^d = U(0)$ and π^d proportional to the mean value of P. If $\theta = 1$, bank has absolute bargaining power over the entrepreneur, that it can offer take-it-or-leave-it offer to the entrepreneur. Then $c^* = 0$, $\pi(c) = \pi(0)$. If $\theta = 0$, entrepreneur can maximize his utility as long as the bank breaks even. Then $\pi(c^*) = \pi^d$, $c^* = \pi^{-1}\left(\pi^d\right)$. If $\theta \in (0,1)$, the bargaining solution is $\pi(c^*) = \left[\pi(0) - \pi^d\right]\theta + \pi^d$, $c^* = (1-\theta)\,\pi^{-1}\left(\pi^d\right)$.

From the bargaining solution, higher bargaining power corresponds to a lower promised utility to the entrepreneur. According to Proposition 2.4, lower promised utility leads to a higher default rate; hence higher bargaining power implies higher default rate. As a result, large banks intend to provide contracts attracting borrowers with low promised utilities, and small banks intend to offer contracts to attract high promised utility borrowers more. The compositional difference of borrowers from different banks causes a higher default rate to large banks. This result corresponds to the stylized fact depicted by Figure Fig. 2.1.

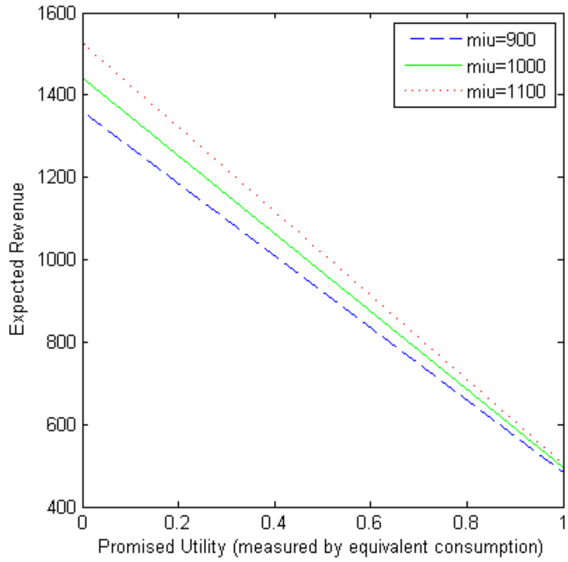
2.4. Numerical Example

Let $b = 1$, $\alpha = 1$, $\sigma = 1$, $r = 0.001$, and price draw follows a truncated normal distribution $rP \sim N(\mu, Var)$, $\mu = 1$, $Var = 0.3$.

2.4.1. Price Draw Distribution

Vary the mean value of the price distribution, $\mu_1 = 0.9$, $\mu_2 = 1$, $\mu_3 = 1.1$. The expect revenue of optimal contract is approximately linear in promised utility (I use equivalent consumption in the graph to represent promised utility).

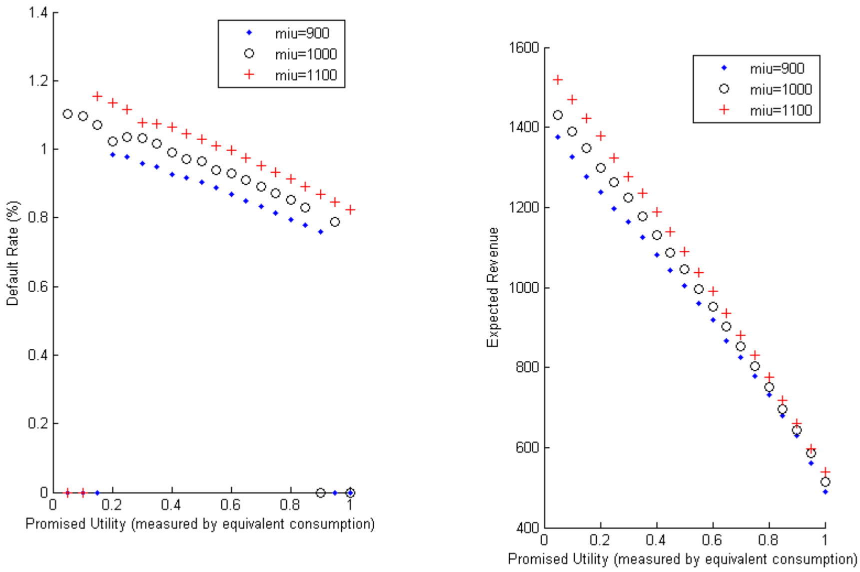
Figure 2.2.: Revenue and Utility



If fixing the contract and entrepreneur's choice over reservation prices and consumption plan, default rate would fall if μ increases. This can be proved by algebra. However, if μ increases, the entrepreneur sees a better perspective of the future, then the rational reaction for him/her is to behavior more aggressively. This will stimulate the entrepreneur's consumption and push up the reservation prices. This result can be shown numerically. Given the same contract, the entrepreneur's best response leads to a higher default rate and continuation utility.

We can see this result more intuitively from the following graph. The default rate and expected revenue is higher if the perspective of the project is better. Both default rate and expected revenue falls if the promised utility to the entrepreneur increases.

Figure 2.3.: Default Rates



2.4.2. Arrival rate and preference

If the arrival rate α increases, default rate increases, entrepreneur's utility increases, and bank's revenue decreases. The loan contract in favor of a lower default probability encourages earlier prepayments, and hence increases the waiting period and the expected waiting cost. If the preference parameter σ increases, entrepreneur prefers the short term contract more than the long term contract, and vice versa.

2.5. Conclusion and Extension

2.5.1. Contingent Contract

Considering a one time payment plan $\{d_t\}$, the entrepreneur has two options in this contract, he can either choose to sell the project at time t' and to pay d_t, or he can wait for a better opportunity and pay back later. The payment of the loan contract varies across time. Theoretically, there is no default in this kind of contract, because the entrepreneur can choose to pay back after a very long time.

2.5.1.1. Entrepreneur's problem

At first, I assume no liquidity for the borrower. If he sells the project, he will get P_t and pay d_t; if he rejects the price offer, he will receive b and consume all of it. Value of selling the project is

$$V^s = \frac{1}{r} u\left(c^s\right)$$

$$c^s = rP_t - rd_t$$

The continuation value of the entrepreneur at time t' is

$$U\left(t', \left\{\bar{P}_t\right\}, \{d_t\}\right) = \int_{t'}^{\infty} e^{-\int_{t'}^{t} r+\alpha\left(1-F\left(\bar{P}_s\right)ds\right.} \left(u\left(b\right) + \alpha \int_{\bar{P}_t} \frac{u\left(rP - d_t\right)}{r} dF\left(P\right)\right) dt$$

Given reservation prices $\left\{\bar{P}_t\right\}$, contract $\{d_t\}$, let $v\left(t\right) = U\left(t, \left\{\bar{P}_t\right\}, \{d_t\}\right)$, then

$$\dot{v}\left(t\right) = rv\left(t\right) - u\left(b\right) - \alpha \int_{\bar{P}_t} \left[\frac{u\left(rP - d_t\right)}{r} - v\left(t\right)\right] dF\left(P\right)$$

Now considering the optimal contract $\{d_t\}$. Let the bank choose reservation prices $\left\{\bar{P}_t\right\}$ to maximize its expected revenue, as long as $\left\{\bar{P}_t\right\}$ is incentive compatible that $U\left(t, \left\{\bar{P}_t\right\}, \{d_t\}\right) \geq U\left(t, \left\{\bar{P}'_t\right\}, \{d_t\}\right)$.

Claim 2.6. The incentive compatible constraint is equivalent to $v\left(t\right) = \frac{u(rP_t - rd_t)}{r}$

2.5.1.2. Optimal Financial Contract

Then, the bank's problem is

$$\pi_0 = \max_{\{\bar{P}_t\},\{d_t\}} \int_0^\infty e^{-\int_0^t r+\alpha\left(1-F\left(\bar{P}_s\right)\right)ds} \left[\alpha \int_{\bar{P}_t} d_t dF(P)\right] dt$$
$$s.t. U\left(0, \left\{\bar{b}_t\right\}, \{d_t\}\right) \geq v_0$$

From IC constraint and utility constraint binding, at optimal, $d_t = r\bar{P}_t - \frac{u^{-1}(rv)}{r}$; then π_t can be expressed as a function of reservation utility v. Then the HJB equation is

$$r\pi_t(v) = \max_{\bar{b}_t, d_t} \dot{\pi} + \alpha \int_{\bar{P}_t} (d_t - \pi_t)\, dF(P) \tag{2.20}$$

s.t.

$$U\left(t, \left\{\bar{P}_t\right\}, \{d_t\}\right) \geq v$$
$$U\left(t, \left\{\bar{P}_t\right\}, \{d_t\}\right) = \frac{u\left(\bar{P}_t - d_t\right)}{r}$$

Use $d = \frac{\bar{b} - u^{-1}(rv)}{r}$ and rearrange (2.20) as

$$r\pi(v) = \max_{\bar{b}} \pi'_v \left(rv - u(b) + \alpha v \int_{\bar{P}} \left[u\left(rP - r\bar{P}\right) + 1\right] dF(P)\right)$$
$$+ \alpha \int_{\bar{P}} \left(\frac{r\bar{P} - u^{-1}(rv)}{r} - \pi\right) dF(P)$$

Then the optimal contract solve the above HJB problem.

2.5.2. Conclusion

This paper proposes a model that allows the risk of default to be a choice variable. Optimal long-term lending contract may allows a positive expected default rate. In summary, bank would allow a higher expected default probability Γ if 1) the loan market is more frictional since the expected waiting period would be longer; 2) the size of the loan is bigger since the bank's holding cost of the current liquidity position would be larger and hence the waiting cost would be larger.

The model can be used to address a project such as, a small business, real estate asset, or other collateralized asset, and debt obligations such as, mortgages, loans, bonds, or promissory notes. An important assumption to this model is that, the fundamental value of the project is certain, but the market value of the project is a random variable. This assumption of the project is similar to the Merton Model (1974), but the mechanism to reconcile the conflict between the profit and the risk is different. Considering bank's tradeoff between efficiency of using the fund and default risk, the bank affects the distribution of the project's market value conditional on prepayment, the conditional distribution together with the promised utility to the entrepreneur affect the default risk of the loan. Then the risk of default is decided within the model.

Larger banks usually have larger capacity of lending and better technologies to diversify risk. Relying on these advantages, they don't usually scrutinize loan applications case by case. Instead, big banks prefer the standardized contracting procedure, meaning that they basically offer take-it-or-leave-it contracts to the borrowers. Small banks, on the other hand, are more willing to establish good relationships with local customers. They value customers' loyalty and rely on better negotiation skills. Moreover, a higher deposit position is accompanied with a higher holding cost to the bank. Hence large banks care more about the efficiency rather than default risks

In Proposition 2.5, the model suggests that the loan's payment and delinquency react to the revenue shock but the consequent reaction depends on market conditions. If there is a positive shock to the project market, if $\frac{r}{\alpha}$ is small enough, the default rate will have a mild increase, and the expected profit from the loan will also increase; if $\frac{r}{\alpha}$ is large, the default rate will decrease, and the expected profit will increase much more than the first case. On the contrary, hit by a negative shock, if $\frac{r}{\alpha}$ is not small enough, there will be a higher default rate and a lower profit from the loan. A lower r means a easier credit condition, and a higher α means a smaller trading friction. On a frictionless market with the access to the easy credit, the payment stream of the loan is expected to be stable, and the delinquency of the loan is predictable and small. If the trading friction is large and the cost of the credit is high, the illusion produced by the bubble is amplified. In this context, the model offers a new interpretation to the delinquency and default behavior with respect to the bubble. Considering the recent US housing bubble, from the year of 2002 to 2004, easy credit is accessible. Meanwhile, huge inflow of foreign capital has increases the trading frequency and price. These conditions imply a very small $\frac{r}{\alpha}$ in the model. Although the housing price increased during this period, the loan performance is stable. The easy credit condition ended in 2005, $\frac{r}{\alpha}$ increases rapidly afterwards. If the housing price continue to grow, the growth of the bubble accelerates instead of staying low under the new economic environment. if the price bubble burst after the peak of the housing price in the mid-2006, default rate started to climb up, and bank suffered severe loss on mortgage loan. To address the business cycle feature, one direction of the extension might be introducing dynamic of $\tilde{b}$ and aggregate shock to the model. To endogenies the distribution of market value $\tilde{b}$, one way is to use quadratic search to make the state variable tractable. Then it's possible to predict default rates under different variations.

3. Do Mortgage Recourse Limitations Matter?

3.1. Introduction

Mortgage banks may request deficiency judgments if a property in foreclosure is sold at a public sale for less than the loan amount that the underlying mortgage secures. The availability of a deficiency judgment depends on whether the lender has a recourse or non-recourse loan, which is largely a matter of state law. In forty one US states, lenders have recourse rights that they are able to collect on debt not covered by the proceedings from a foreclosure sale by obtaining a deficiency judgment.

Many papers suggest that US mortgages are generally "no recourse" loans since deficiency judgments have rarely been executed. While many argue that the recourse law affects both lenders and borrowers' decision making process. The results of this paper will support the latter that recourse law does matter. Foote *et.al* (2008) analyzes a rich dataset from Massachusetts and concludes that negative equity is not a sufficient condition for default. Ghent *et.al* (2011) finds that recourse law decreases borrowers' sensitivity to negative equity. It states that the effect of recourse is not reflected in the frequency of deficiency judgments but, instead, in the way the threat of recourse alters borrower behavior. Earlier works by Clauretie (1987), Jones (1993), and Deng *et.al* (2001) also empirically study the difference in default patterns across jurisdictions. Various conclusions are drawn. Clauretie (1987) analyzes the aggregate default rates in state-level and finds that whether or not a state allows deficiency judgment does not significantly affect the state's default rate. Jones (1993) looks at evidence from Alberta, which does not allow deficiency judgments, and British Columbia, which does permit them, and finds that defaults in Alberta are more likely to be deliberate, rather than caused by trigger events that is involuntary. Deng *et.al* (2001) includes a institutional variable to indicate the ease in which lenders can obtain deficiency judgments against defaulted borrowers. It studies the determinants of mortgage default using the sample of Federal Housing Administration (FHA) loans originated in 1989. The principal of FHA loans is guaranteed by the FHA, that FHA has a policy of not pursuing defaulted borrowers. Deng *et.al* suggests that borrowers may not know the policy well ex-ante, and hence is intimidated by the potential court judgments. Their results show that default rate is positively affected by this recourse variable. However, FHA loans may be particularly poorly suited to study the effect of recourse on default behavior.

Other related papers include Crawford and Rosenblatt (1995), Ambrose *et.al* (1997) and Corbae and Quintin (2010). Crawford and Rosenblatt (1995) finds that, conditional upon default occurring, loss severity is greater in non-recourse states. Ambrose *et.al* (1997) and Corbae and Quintin (2010) build theoretical framework to study the effect of recourse law on default and find that recourse deters default.

In this paper, we take a closer look at the effect of recourse law. Different from previous literature, we assemble a micro-level foreclosure dataset that also records all the follow-up granted deficiencies. The frequency of successful deficiency judgments observed in the data is 2-3%, which is low but far from “rarely been executed”. The estimation results show that the fear of deficiency judgment alters homeowner/borrower’s behavior.

3.2. Foreclosure and deficiency law in Illinois state

3.2.1. Illinois foreclosure statue

Illinois is known as a lien theory state where the property acts as security for the underlying loan. Private sale foreclosures are not permitted in Illinois. In Illinois, lenders go to court in what is known as a judicial foreclosure proceeding where the court must issue a final judgment of foreclosure. The property is then sold as part of a publicly noticed sale. The court with jurisdiction over a foreclosure is known as the Circuit Court. These Circuit Courts in Illinois are broken down by county. Usually, the Chancery Division of a Circuit Court handles foreclosures. A complaint is filed in Circuit Court along with what is known a lis pen-dens, which is a recorded document that provides public notice that the property is being foreclosed upon.

There are three types of foreclosures. The first one is Deed in lieu, which effectively deeds the property back to the lender. There would be no deficiency judgment if the lender accepts a Deed in lieu of foreclosure. The second is consent foreclosure. If legal action has already ensued, the property owner can request a Consent Foreclosure. It involves the court issuing a judgment of foreclosure and giving title to the property to the lender (with both the lender and borrower's consent). There is no sale and no deficiency judgment. The foreclosure process will be shorter. The last one, strict foreclosure, is more commonly observed in our dataset. Lenders foreclose by using the common law strict foreclosure method, in which case a deficiency judgment could be obtained. In the recent economy environment, strict foreclosures have been used more often by the lenders.

Illinois does not have a post-foreclosure sale statutory right of redemption, which would allow a party whose property has been foreclosed to reclaim that property by making payment in full of the sum of the unpaid loan plus costs. On residential properties, there is either (1) a 7 month right of redemption from the time the foreclosure complaint is filed or (2) 3 months from the time a final judgment of foreclosure is entered. A foreclosure sale cannot occur until these time periods expire. Reinstatement and redemption rights can only be exercised once every 5 years. Upon the foreclosure sale, some homeowners also have chances to claim bankruptcy chapter 13 to rearrange the mortgage payment with lenders and keep their home.

A deficiency judgment may be obtained when a property in foreclosure is sold at a public sale for less than the loan amount that the underlying mortgage secures. This means that the borrower still owes the lender the difference between the amount collected from the foreclosed property's auction and the amount of the original loan. Deficiency judgments are not permitted in cases of Consent Foreclosure or a Deed in lieu of foreclosure. The law that govern Illinois foreclosures is explained in 735 ILCS 5/Art XV (HB3789).[1] According to HB3879 bill (page 8-9), deficiency judgment should be included in the confirmation order of the sale. In any order confirming a sale pursuant to the judgment of foreclosure, the court shall also enter a personal judgment for deficiency against any party (i) if otherwise authorized and (ii) to the extent requested in the complaint and proven upon presentation of the report of sale in accordance with Section 15-1508. Although a wide variety of assets are exempted from deficiency judgments in Illinois, it is one of the most deficiency-friendly states among all recourse states.[2] The ease of pursing deficiency judgments also depends on the discretion of county court judges.

3.2.2. Example of deficiency case

The property is a single family residence in Kane county. The borrower started a conventional mortgage loan from Bank of New York Mellon in February 2007. The mortgage had 30 years term with 7.38% interest rate. The original mortgage amount is 191900. When the borrower foreclosed the mortgage in 2010, the complaint amount was 209294. Apparently the borrower had been behind the payment for a long time. In 2011, the foreclosed property was sold to the lender bank with 82500, which was less than half of the complaint amount. The lender filed deficiency judgment to request the gap. In the same year, lender was granted 140876 by the court.

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[1] http://www.ilga.gov/legislation/ilcs/ilcs4.asp

[2] The exemption includes 85% of their wages, life insurance, and most retirement accounts.

3.3. Data

The foreclosure and land lien data is from the Record Information Services (RIS). RIS collects and compile data from case files of county circuit court. The foreclosure database contains loan characteristic, homeowner and lenders' basic information and foreclosure results. The land lien database includes all the successful deficiency judgment lawsuits that associate with residential mortgage. We use the data in the period of July 30 of 2008 to July 30 of 2012. It covers seven counties, DeKalb, DuPage, Kane, Kendall, Lake, McHenry and Will. They are all peripheral areas of Chicago Metropolitan area. The zip-code level housing price indices are from Zillow.com. We also collect the population and income information from US census bureau and BLS.

3.3.1. Loan Variables

There are three types of underline property for residential mortgages: single family residence, townhouse and condominium. For now, we only focus on single family residence mortgage foreclosures which accounts for about 80% of our observed foreclosures. [3]

Table Tab. C.1 shows basic summary statistics of foreclosed loans. *Complaint amount* is the loan balance at the recorded time of foreclosure. Some foreclosure cases experience several revision during the process and hence have different complaint amounts after each revision. Under this condition, we only take the latest revision of the foreclosure. *Loan size* is the original amount of the mortgage which

[3]Condominium accounts for 12.4% and townhouse 6.8%.

is unique for each mortgage. *Interest* and *Term* are interest rate and term of the mortgage. *Age of loan* is the length of the time in day between the original date of the mortgage and the recording date of foreclosure. *Census age* and *Census income* are the average age and income in the census tract of the foreclosed home. Types of the mortgage include ARM, balloon, conventional, FHA, VA. Each type is treated as a dummy variable relative to conventional loans.

We use the Zip-code level housing price indices from Zillow.com to construct the percentage change of the property's market value, $\Delta M_{i,t,n_i}$, where n_i is the Age of Loan for observation i. Then

$$\Delta M_{i,t,n_i} = \frac{ZPI_{i,t}}{ZPI_{i,t-n_i}} - 1$$

where $ZPI_{i,t}$ is the Zillow price index of observation i's zip-code area at time t.

In Table Tab. C.1, the first column to the 4th column are basic statistic of all observations, the 5th column is of foreclosures with deficiency judgments, the 6th column is of foreclosures without deficiency judgment and the 7th to 8th column record all closed foreclosure cases. Comparing with no deficiency judgment, foreclosed loans with deficiency judgments has significantly larger loan balance, loan size, higher interest rate of the mortgage and younger age. Besides, deficiency judgments are more likely to be observed among ARM type loans.

3.3.2. Foreclosure and Bankruptcy Variables

A foreclosure sale is "public sale" if the public real estate auction is conducted. During the public sale, the foreclosed property is usually sold to a third party or sold back to the mortgage bank. A private sale in the data usually implies a short sale or redemption that the home owner buys back the property through a private channel. Homeowners are able to secure their home if the private sale is conducted successfully, in which case deficiency judgment is also avoided. In practice, lenders can still pursue deficiency judgment if the promised payment for a private sale is not fulfilled afterwards. In the estimation, the sale variable takes a value of 1 if the foreclosure is private and 0 if public.

Homeowners can claim bankruptcy chapter 7 or chapter 13 to default on their debt or rearrange the mortgage payment with mortgage banks. Variable *Bankruptcy Flag* is 1 if homeowners have bankruptcy claim on their record which is irrelevant to the current mortgage, and 0 otherwise. Variable *Bankruptcy* is 1 if there exists any bankruptcy claim with regard to the current mortgage. More specifically, a dummy BR7 indicates whether there is bankruptcy claim on chapter 7 and a dummy BR13 indicates whether there is bankruptcy claim on chapter 13. According to the recourse law, the homeowner can avoid deficiency judgment by claiming bankruptcy chapter 7.

Table Tab. C.2 shows that both bankruptcy claim and private sale effectively reduce the risk of deficiency judgment. For public sale, the deficiency frequency of no bankruptcy cases is more than double of the bankruptcy cases. The deficiency frequency of private sale foreclosures are almost the same across bankruptcy and no bankruptcy cases. These frequencies suggest that private sale and bankruptcy claim may reduce the recourse right of lenders. Since homeowners make their decisions conditional on their own situations, the absolute frequency is not enough to reflect the conditional decisions, which is the key issue we are gonna deal with later.

3.3.3. Deficiency judgment

The land lien database records informations of the plaintiff, defendant, the judgment amount granted and the association to the real estate property. Dummy variable *Def* refers to whether a judgment is linked to the foreclosure case. We also use *PDef* to indicate the probability of deficiency judgment for a certain foreclosure, which is independent with both lender's deficiency decision and homeowner's bankruptcy decision. Lender's deficiency decision depends on *PDef*, as well as the lender's recourse right and the lender's willingness to execute the recourse right. For example, if the lender's expected loss on foreclosure is higher the lender would be more likely to request deficiency judgment; if the homeowner has assets other than the foreclosed home or the homeowner has too high income to claim bankruptcy the lender would have more incentive to execute the recourse right. Under either condition, *Def* is more sensitive to *PDef*. If homeowner responds to the recourse law, we expect to see the bankruptcy decision reacting to the change of *PDef*.

We also construct variables *areadef* and *blockdef* to indicate the popularity and threat of the deficiency judgment in a certain area or block. The geographic areas and blocks follow the rule of real estate property pin number in each county. Every property has a unique 14 digits pin number, that the first four digits pin down an area in a given county and the first 7 digits pin down a block in a given county. Then we aggregate all the deficiency judgments within the area/block to get *areadef* and *blockdef*. These two deficiency regressors are only used in the preliminary test.

3.3.4. Sample

We merge the datasets extracted from the foreclosure database and the land lien database. To identify the corresponding foreclosure of a deficiency judgment case, we use the following criteria. After 2011 July 1st, the land lien database includes the associated foreclosure case number in the record. Hence the two databases are merged according to this unique foreclosure case-number. Before 2011 July 1st, the case number is not available. Some judgment cases record the pin numbers of foreclosed properties, hence we use pin numbers and "county" to identify the corresponding foreclosures. The judgment cases may also enter after the title of the foreclosed home been transferred, that the pin numbers can not be tracked. Under the last scenario, we use name and address information of the plaintiff, defendant to identify the related foreclosures. The address information includes street address, city, zip-code and county. Since there might be multiple homeowners with multiple foreclosures for the same property, we match the timeline of the foreclosure to rule out such situation.

Besides, we focus on residential mortgage foreclosure only. We drop observations with commercial, land and construction mortgages. We also drop open end cases that the foreclosure is still in process. Then we are left with 48553 observations that the unconditional deficiency judgment frequency is 2.35%.

3.4. The Threat of Recourse Right

Both Ghent *et.al* (2009) and Deng *et.al* (2001) agree that the way recourse affects default behavior depends on the property's deficiency judgment risk, which is positively related to the unpaid amount of the loan or the loan size. The low frequency of deficiency judgment needs not imply that the recourse right does not matter. The fear of deficiency judgment may impact people's decisions, that the homeowner foresees the risk of deficiency judgment and conducts private sale or claim bankruptcy to avoid deficiency judgment. Then the frequency of deficiency judgment observed becomes lower.

For preliminary tests, we don't differentiate different types of bankruptcy claim and conduct probit regressions to study what factors impact bankruptcy claim. Table Tab. C.3 shows probit regressions of bankruptcy claim, BR, on covariates X and area deficiency dummy. Dummy *areadef* is 1 if deficiency judgment has been conducted in the same area. The first to the fourth column use the whole sample, in which the first column does not include deficiency dummy, the second include *areadef* in level, the third include *areadef* in interactions with *Complaint amount (L), Loan size (C)* and % *Change of home value* ($\%\Delta M$), and the fourth includes *areadef* in level as well as in interaction with L, C and $\%\Delta M$. Using L, C and $\%\Delta M$, we can constructed the potential deficiency gap, which is the difference between the loan balance and the estimated market value of the foreclosed property.

$$gap = L - C\,(\%\Delta M + 1)$$

Intuitively, the bankruptcy claim should be positively related to this potential deficiency gap. The signs of variables L, C and $\%\Delta M$ are consistent with this intuition. The level of *areadef* is rarely significant. But the interactions of *areadef* and L, C and $\%\Delta M$ show that the threat of recourse right increases the home owner's incentive to claim bankruptcy conditional on the equity and this effect is significant. This result shows that the enforcement of the recourse law affects homeowners' decision, especially homeowners' sensitivity to the current estimated equity level. Bankruptcy claim is positively related to the term and age of the mortgage. One explanation is that younger homeowners are more likely to claim bankruptcy during foreclosure. BR is negatively related to the census income level which suggests that an unaffordable debt is not the single cause of bankruptcy. Some covariates have unexpected coefficients which need further investigation. For example, bankruptcy claim is less likely to happen with ARM type mortgage or FHA mortgage comparing to conventional mortgage. Higher interest rate is negatively related to the bankruptcy claim.

The fifth and sixth column of Table Tab. C.3 uses sample with public sale only and private sale only. Both columns include *areadef* in levels and interactions. The bankruptcy claim responds to covariates X and *areadef* significantly different from public sale cases to private sale cases. Homeowners with private sale seem to be more sensitive to the deficiency threat.

3.5. Default

Homeowner/debtor usually faces two major threats in residential mortgage foreclosure: 1) losing the home; 2) being charged of the deficiency gap. As mentioned earlier, the deficiency threat of a specific foreclosed home depends on the properties of the mortgage, homeowner's financial status, loan payment delinquency and etc. During the foreclosure process, homeowners can take actions, such as private sale and bankruptcy claims, to reduce the impact of the above threats. These actions can either save their homes, or reduce banks' recourse right on the deficiency gap, or increase the winning probability of a deficiency judgment case and etc. If a homeowner prevents a public sale by buying back the foreclosed home or leading a short sale, this would stop banks' deficiency judgment unless the homeowner fails to fulfill the payment occurred in the private sale. If the homeowner satisfies certain income and debt requirements, a timely bankruptcy claim chapter 13 allows the homeowner and bank renegotiate terms of loan payments, which helps the homeowner to keep the home. There will be no deficiency judgment unless the homeowner fails to meet the new payment schedule. Bankruptcy claim chapter 7 allows the homeowner default on his/her debt. Ideally, successful bankruptcy claim should reduce lender's recourse right to zero. In practice, the situation is more complicate and noisy that the bankruptcy claim and deficiency judgment is observed in the same foreclosure without much surprise. For example, other members of the household may have asset to go after and not claim bankruptcy. This noise increases the difficulty of our estimation that simple probit regression is not feasible to identify the effect of the recourse law.

Hence we assume that homeowners make decision on "Private sale" and "bankruptcy claim" sequentially. "Private sale" increases the chance of keeping the home as well as decreasing the deficiency judgment probability, however the financial cost of going "Private sale" is higher. The frequency of deficiency judgment out of "private sale" is about 1%, and the frequency out of "Public sale" is more than 3%. Moreover, bankruptcy claim significantly reduces the execution of deficiency judgment in "Public sale", not so effective in "Private sale".

3.5.1. Model

Let i indicate observation, X_i be covariates of the observation i such as properties of the foreclosure, mortgage and homeowner. Deficiency event dummy variable D_i takes value 1 if deficiency judgment happens and zero otherwise. D_i depends on the unobserved variable *PDef* and homeowner's decisions on foreclosure sale and bankruptcy claim. $PDef$ is the exposure to deficiency judgment of foreclosure i, which is independent of the type of sale or homeowners' bankruptcy claim. Therefore, we assume the unobservable $PDef$ depends on covariates of the foreclosure i, such that $PDef_i = X_i\beta_1$. The execution of deficiency judgment depends on $PDef$ as well as foreclosure sale type and bankruptcy claim status since they would affect banks' effective recourse right and hence the deficiency amount banks can recover. We define a foreclosure sale dummy $S_i \in \{0, 1\}$ that $S_i = 0$ means "Public sale" and $S_i = 1$ means "Private Sale". Then the deficiency dummy

$$D = func\left(X\beta_1, S, br7, br13\right) + \varepsilon$$

where $\varepsilon \sim N\left(0, 1\right)$.

To estimate β_1, we assume that β_1 is same across all the observations and apply probit regression on a subset of the sample that $S = br7 = br13 = 0$. If the foreclosure sale goes through public sale and no bankruptcy claim is involved, the deficiency judgment would happen if the unobservable $PDef$ is greater than 0, that

$$D = \mathbb{I}\left[\beta_1 X \geq 0 | \left(S = 0, br7 = 0, br13 = 0\right)\right]$$

Using estimated coefficient $\hat{\beta}_1$, we generate the estimated deficiency judgment variable $\widehat{PDef}_i = X_i \hat{\beta}_1$ for all observations. Following the sequential decision theory, homeowner first makes decision on foreclosure sale's type S according to PS and $PDef$, where $PS = \beta_2 X$ is homeowner's willingness to keep the home, that

$$S = func\left(PS, \widehat{PDef}\right) + \epsilon$$

where $\epsilon \sim N(0, 1)$. Using estimated β_2 to generate $\widehat{PS}$, the generated variable PS indicates the home owner's willingness to keep the home and cure the foreclosure. Then homeowner makes decision on bankruptcy claim according to $\widehat{PDef}$ and $\widehat{PS}$. We use probit regressions for $br7$ and $br13$ on X, $\widehat{PDef}$ and $\widehat{PS}$ to estimate how the risk of deficiency judgment, the willingness to keep the home and other covariates X affect home owner's decision on bankruptcy claim.

$$br_k = func\left(X, \widehat{PDef}, \widehat{PS}\right) + \xi \tag{3.1}$$

where $br_k \in \{br, br7, br13\}$

In another specification, we use sales result to control home owner's willingness to keep the home, that

$$br_k = \gamma_{1s} X_i + \gamma_{2s} \widehat{PDef}_s + \gamma_{3s} \widehat{PS}_s + \xi_s \tag{3.2}$$

This specification assumes that the bankruptcy decision rule has a structural change with regard to different foreclosure sale types. The estimated parameter coefficients γ_{2s}, γ_{3s} and the coefficient for the interaction term of $\widehat{PDef}$ and $\widehat{PS}$ are of interest. Since PS indicates the homeowner' willingness to keep their home, $\gamma_{2s} > 0$ means that the homeowner is more likely to claim bankruptcy if they have stronger intention to keep their home, and vice versa. Similarly, $\gamma_{3s} > 0$ means that the homeowner is more likely to claim bankruptcy in response to a stronger threat of deficiency judgment.

3.5.2. Results

With the above setup, we can test whether homeowner responds to the deficiency threat $PDef$ during the foreclosure.

Conjecture 3.1. *Given X and $\widehat{PS}$, homeowner is more likely to claim bankruptcy if $\widehat{PDef}$ is higher. Specifically, given X and $\widehat{PDef}$ is higher, homeowner is more likely to claim $br13$ rather than $br7$ if the propensity of keeping the home $\widehat{PS}$ is higher; given X and $\widehat{PS}$, homeowner is more likely to claim $br7$ than $br13$ if the deficiency threat $\widehat{PDef}$ is higher.*

Bankruptcy Claim Chapter 7 allows the debtor to default on his debtor with a fresh start, which should be positively related to $\widehat{PDef}$ if others are the same. However, $br7$ does not help keep the home. Homeowner can claim Bankruptcy Chapter 13 to reorganize the debt and keep the home. Hence $br13$ is more sensitive to $\widehat{PS}$ than $br7$ and $br7$ should be negatively related to $\widehat{PS}$ given X and $\widehat{PDef}$. The number of foreclosure cases with different bankruptcy claims in Table Tab. C.2 shows that the ratio of $br7$ to $br13$ is around 4 in public sale and around 2 in private sale. This supports the conditional negative relationship between $br7$ and $\widehat{PS}$ since foreclosures with private sale have higher $\widehat{PS}$ than foreclosures with public sale on average. The relationship between br and $\widehat{PDef}$ is not as obvious since foreclosures with higher deficiency threat can not be identified by a single label. We need to get the estimated coefficient of $\widehat{PDef}$ conditional on X and $\widehat{PS}$ to prove the first conjecture.

The first specification of the statistical model generates estimation results showed in Table Tab. C.4. The first column is regression of $br7$, second of $br13$ and third of br. The interaction term of $\widehat{PDef}$ and $\widehat{PS}$ is significant in all three regressions and the sign of its coefficient is positive. if $\widehat{PS}$ is not too small, $\widehat{PDef}$ has positive effect on bankruptcy claim. If the homeowner has stronger willingness to keep the home, the effect of $\widehat{PDef}$ on $br7$, $br13$ and br would be stronger.

Moreover, the homeowner is more likely to claim bankruptcy if the loan balance is higher and the market value is smaller. Mortgage type also affects the bankruptcy claim that comparing with conventional loans, non-conventional loans such as ARM, BALLOON, FHA and VA reduces $br7$ but increases $br13$. Comparing column (1) and column (2), the decision making processes of $br7$ and $br13$ are quite different. The homeowner is more likely to claim bankruptcy chapter 7 if the loan is older; however, the loan age is not a significantly covariate for $br13$. $br7$ is less likely to happen with higher interest rate and longer term mortgage during foreclosure; but $br13$ is on the contrary. These evidences are not intuitive with common sense. Normally, we may assume that the homeowner claims bankruptcy if the debt is large. The debt on the mortgage is usually positively related to the interest rate and negatively related to the loan age. The regression results show that the financial gain from default on the mortgage debt is not enough to explain the reason of the bankruptcy claim.

Conjecture 3.2. *Given X and $\widehat{PS}$, homeowner is more likely to conduct private sale if $\widehat{PDef}$ is higher. The bankruptcy decision rule is different between foreclosures with private sale and public sale.*

Table Tab. C.5 shows regressions with public sale and private sale only. The probability of $br7$ is higher if $PDef$ and PS are larger; while the probability of $br13$ is higher if $PDef$ and PS are smaller. $br7$ is more sensitive to $PDef$ in public sale, and the effect of $PDef$ is not significant in private sale. In private sale, the threat of deficiency affects the homeowner's bankruptcy decision on Chapter 7 rather than Chapter 13. A sensible explanation is that, the homeowner with private sale claims bankruptcy since the mortgage is truly unaffordable.

3.5.3. Alternative Model

Consider another type of sales result that the foreclosure is cancelled or rescheduled. Let $\tilde{S} = 2$ indicates this type of sale. Assume ordered logistic regression for sales results $\tilde{S} \in \{0, 1, 2\}$, the linear prediction to determine the discrete choice $\tilde{S}$

is $PS = \tilde{\beta}_2 X + \tilde{\beta}_3 \widehat{PDef} + \epsilon$, where $\tilde{\epsilon} \sim logistic$. Then generate the estimated $\widehat{PS}$ using estimated coefficient $\hat{\tilde{\beta}}_2$ and $\hat{\tilde{\beta}}_3$. With constructed variables $\widehat{PDef}$ and $\widehat{PS}$, we estimate br_k as in previous model. Table Tab. C.6 shows the regression results with the alternative model.

3.6. Conclusion

In this paper, we use evidence from 7 counties of the Illinois state to show the effect of recourse law during residential foreclosure. We find out that the homeowner does care about the recourse law and respond to the threat of deficiency. The homeowner may have different decisions on foreclosure sale type and bankruptcy claim with high and low threat from deficiency. If we categorize homeowners in foreclosure according to their exposure to deficiency judgment, the group with higher exposure is more sensitive to their debts and more motivated to avoid deficiency judgment. A counterfactual example from the baseline model shows that, if there is no deficiency threat, the bankruptcy chapter 7 claims would be lower by 3%, the bankruptcy chapter 13 claims would go up by 10% and the probability of public sale would be increased by 4%.

We do not want to jump to the conclusion that the recourse law reduces foreclosure or helps with a healthy residential mortgage market. This piece of study on Illinois state foreclosure shows that the recourse right effectively increases the foreclosure cost of homeowners who are not truly insolvent. If the affordability of the home loan is defined as being able to maintain a minimum living with the loan, the recourse right will increase the cost of foreclosures out of affordable loans. In the long run, adverse selection may change the structure of mortgage. For example, homeowners will be more cautious about predatory lending since it becomes more risky. Recourse law also suppress strategic default. However, homeowners will be more sensitive to the expected market value of the house since real estate depreciation becomes fatal with the recourse law. As a consequence, a small negative shock to the economy might be magnified.

A. Appendix. Funding Liquidity and Market Liquidity

A.1. Baseline Model

Properties of $g\left(y, k^{'}\right)$

$g\left(y, k^{'}\right)$ is determined by Nash Bargaining Solution.

$$g\left(y, k^{'}\right) = \frac{\theta u_y c(y, k^{'}) + (1-\theta) c_y u(y)}{\theta u_y + (1-\theta)\frac{A}{w} c_y}$$

$$\frac{u}{g} = \frac{\theta u_y u + (1-\theta)\frac{A}{w} c_y u}{\theta u_y c(y, k') + (1-\theta) c_y u(y)} = \frac{A}{w} + \frac{\theta u_y u - \theta \frac{A}{w} u_y c}{\theta u_y c(y, k') + (1-\theta) c_y u(y)}$$

If the total surplus of trade is positive, $c\left(y,k^{'}\right) < g\left(y,k^{'}\right) < \frac{w}{A}u\left(y\right)$. Take first order derivative with regard to y and $k^{'}$

$$g_1\left(y,k^{'}\right) = \frac{\theta u_y c_y}{\theta u_y + (1-\theta)\frac{A}{w}c_y} + \frac{\theta\left(1-\theta\right)\left[u(y) - \frac{A}{w}c(y,k^{'})\right]\left(c_{yy}u_y - c_y u_{yy}\right)}{\left[\theta u_y + (1-\theta)\frac{A}{w}c_y\right]^2} \quad (A.1)$$

$$g_2\left(y,k^{'}\right) = \frac{\theta u_y c_k}{\theta u_y + (1-\theta)\frac{A}{w}c_y} + \frac{\theta\left(1-\theta\right)\left[u(y) - \frac{A}{w}c(y,k^{'})\right]u_y c_{yk}}{\left[\theta u^{'}(y) + (1-\theta)\frac{A}{w}c_y\right]^2} \quad (A.2)$$

Since $u_y > 0$, $u_y < 0$, $c_y > 0$, $c_{yy} > 0$, $c_k < 0$, $c_{kk} > 0$, $u(y) - \frac{A}{w}c(y,k^{'}) > 0$, $c_{yk} < 0$, then $g_1\left(y,k^{'}\right) > 0$ and $g_2\left(y,k^{'}\right) < 0$.

Since y is less than efficient level of output, $\frac{w}{A}u_y > c_y$. Then the first term of (A.1) is greater then c_y. Since the second term of (A.1) is positive, $g_y > c_y$. Similarly $g_k < c_k < 0$. Also, it can be showed that $\frac{\partial k^{'}}{\partial y} > 0$.

Proof of Proposition 1. Existence of Moneteary Equilibrium

Proof. In the bargaining solution, firms take $d = \phi^{'}m^{'}$ as given and choose capital investment $k^{'}$. For households, the choice of $m^{'}$ can be interpreted as the choice of y. Households take $k^{'}$ as given and maximize y as follows

$$\underset{y}{max} G\left(y;k^{'}\right) = \underset{y}{max}\left\{-ig(y,k^{'}) + \alpha_h\left[u(y) - \frac{A}{w}g(y,k^{'})\right]\right\} \quad (A.3)$$

Firms take y as given and $\left(\beta\gamma^{'}-1\right)k^{'}+f(k,L)+(1-\delta)k-wL-b\geq 0$

$$\underset{k^{'}}{max}\left[\beta f\left(k^{'},L\right)+\beta\left(1-\delta\right)k^{'}-k^{'}-\alpha_{f}c(y,k^{'})\right] \tag{A.4}$$

The solution of (A.3) and (A.4) is a Nash equilibrium. The first derivative of (A.3) is

$$G_{y}=-ig_{y}(y,k^{'})+\alpha_{h}\left[u_{y}-\frac{A}{w}g_{y}(y,k^{'})\right]$$

Rearrange it to

$$\frac{G_{y}}{u_{y}}=\alpha_{h}-\left(i+\alpha_{h}\frac{A}{w}\right)\frac{g_{y}(y,k^{'})}{u_{y}}$$

Since $\frac{g_y}{u_y}$ is strictly decreasing in y and $\frac{1}{u_y}$ is increasing in y, G_y is decreasing in y that $G_{yy}<0$. Given y, the second derivative of (A.4) is $\beta f_{kk}\left(k^{'},L\right)-\alpha_{f}c_{kk}(y,k^{'})<0$. The second derivatives of (A.3) and (A.4) show that objective functions are concative. From the Barganing solution, $y<y^{*}$ where y^{*} solves $\frac{w}{A}u^{'}(y^{*})-c_{1}(y^{*},k^{'})=0$. Capital investment is always less than socially optimal level that $k<\tilde{k}$. According to Nash's existence Theorem, the continuity and concavity of (A.3) and (A.4) imply the existence of Nash equilibrium $(y,k^{'})$. Moreover, $G(0;k^{'})=0$ and objective function (A.4) is zero for any $k^{'}$ if $y=0$. Then (A.3) and (A.4) are always non-negative. Since $G\left(y;k^{'}\right)>0\ \forall y>0$, $\underset{y}{max}G\left(y;k^{'}\right)>0$ implies that the maxizer is greater than zero. Besides, if $y>0$, than optimal $k^{'}>0$ from the optimization problems □

First order conditions of CM problem imply $U^{'}(x)=\frac{A}{w}$. Assume labor income is sufficient to support interior solution for x. This assumption is not essential for results, but it avoids lengthy analysis for corner solutions. Wage rate is equal to marginal productivity of labor that $w = f_2(k, L)$, and is bounded below by A that $w \geq A$. Then x increases with w.

Others are determined as follows. $d = Rg(y, k^{'})$, $m^{'} = \frac{d}{\phi^{'}}$, $z = f(k, L) - wL + \beta b^{'} - b - I$, $I = k^{'} - (1-\delta)k$, $\frac{1}{\phi^{'}} = \frac{\phi(m+\tau)+\alpha_f d}{\phi d}$, $L = \frac{1}{w}\left[\phi\left(m^{'} - m - \tau\right) + x + \beta b^{'} - b\right]$.

Proof of Proposition 2 that $\frac{\partial k^{'}}{\partial \tau} < 0$ and $\frac{\partial y}{\partial \tau} < 0$

Proof. In unconstrained equilibrium, $\frac{\partial y}{\partial i}$ and $\frac{\partial k^{'}}{\partial i}$ are determined by

$$\frac{1}{\alpha_h} = \frac{w^{'}}{A}\frac{u_{yy}g_y - u_y g_{yy}}{g_y^2}\frac{\partial y}{\partial i} - \frac{w^{'}}{A}\frac{u_y g_{yk}}{g_y^2}\frac{\partial k^{'}}{\partial i} + \frac{\partial w^{'}}{\partial i}\frac{u_y}{A g_y}$$

$$0 = (\beta f_{kk} - \alpha_f c_{kk})\frac{\partial k^u}{\partial i} - \alpha_f c_{yk}\frac{\partial y}{\partial i}$$

Then $\frac{\partial k^u}{\partial i}$ and $\frac{\partial y}{\partial i}$ have the same sign. In constrained equilibrium, $\frac{\partial y}{\partial i}$ is decided by the same equation, k^c is constrained by the borrowing margin $1 - \beta\gamma^{'}$ and hence depends on y. Again, $\frac{\partial k^u}{\partial i}$ and $\frac{\partial y}{\partial i}$ have the same sign. Since $w = f_{kL}$ implies $\frac{\partial w^{'}}{\partial i} = f_{kL}\frac{\partial k^{'}}{\partial i}$. The sign of $\frac{\partial y}{\partial i}$depends on

$$\frac{w^{'}}{A}\frac{u_{yy}g_y - u_y g_{yy}}{g_y^2} + \left(f_{kL}\frac{u_y}{A g_y} - \frac{w^{'}}{A}\frac{u_y g_{yk}}{g_y^2}\right)\frac{\alpha_f c_{yk}}{\beta f_{kk} - \alpha c_{kk}}$$

Let $B(y; i) = \frac{w^{'}}{A}\frac{u^{'}(y)}{g_1(y,k)} - 1 - \frac{i}{\alpha_h}$, $f(y; i) = 0$ solves the bargaining problem.

$$B_y\left(y;i\right)=\frac{w^{'}}{A}\frac{u_{yy}g_y-u_yg_{yy}}{g_y^2}-\left(f_{kL}\frac{u_y}{Ag_y}-\frac{w^{'}}{A}\frac{u_yg_{yk}}{g_y^2}\right)\frac{\partial k^{'}}{\partial y}$$

Since $\frac{w^{'}}{A}\frac{u_y}{g_y}$ is increasing in i and $\frac{w^{'}}{A}\frac{u_y}{g_y}=1$ if $i=0$, $lim_{i\to 0}B_y<0$. y^* satisfies $\frac{w^{'}}{A}\frac{u_y}{g_y}=1$. The bargaining solution y lies in $(0,y^*]$. Then $B_y\left(y;i\right)<0$ if $i>0$ and $y<y^*$. Therefore, $\frac{\partial k^{'}}{\partial i}<0$ and $\frac{\partial y}{\partial i}<0$. □

Proposition 4. In constrained equilibrium, $\frac{\partial \gamma^{'}}{\partial \Gamma}\geq 0$, $\frac{\partial k^{'}}{\partial \Gamma}\geq 0$

Proof. In constrained equilibrium, sincefootnotesize

$$k^C = \frac{\left(1-\delta\right)k^c+f\left(k,L\right)-wL-\int_h b^{'}-\Gamma}{1-\beta\gamma^{'}} \quad (A.5)$$

$$1-\beta\gamma^{'} = -\alpha_f c_k\left(y,k^C\right)+\beta f_k\left(k^C,L\right) \quad (A.6)$$

Then take partial derivative of (A.6) with respect to k^Cfootnotesize

$$\frac{\partial\gamma^{'}}{\partial k^C}=R\left[\alpha_f c_{kk}\left(y,k^C\right)-\beta f_{kk}\left(k^C,L\right)\right]\geq 0$$

Plug (A.6) into (A.5) and take derivative with respect to Γfootnotesize

$$\begin{aligned}\frac{\partial k^C}{\partial\Gamma} &= \left[1-\delta+\alpha_f c_k-\beta f_k+\left(\alpha_f c_{kk}-\beta f_{kk}\right)k^C+f_k\right]^{-1}\\ &\geq \left[\beta\gamma^{'}-\delta+\left(\alpha_f c_{kk}-\beta f_{kk}\right)k^C+f_k\right]\geq 0\end{aligned}$$

Hence $\frac{\partial k'}{\partial \Gamma} \geq 0$ and $\frac{\partial \gamma'}{\partial \Gamma} = \frac{\partial \gamma'}{\partial k^C}\frac{\partial k^C}{\partial \Gamma} \geq 0$. In unconstrained equilibrium, L^s, k^s and y are determined together

$$\int_h wL = n\alpha_h g\left(y, k'\right) + nx + (\beta - 1)\left(\gamma' k' - \Gamma\right) \tag{A.7}$$

where $w = max\left\{A, f_2\left(k, L\right)\right\}$

$$\frac{i}{\alpha_h} + 1 = \frac{w}{A}\frac{u'(y^s)}{g_1(y^s, k^s)} \tag{A.8}$$

$$1 - \frac{1-\delta}{R} = \frac{1}{R}f_1\left(k^u, L^s\right) - \alpha_f c_2\left(y^s, k^u\right) \tag{A.9}$$

Consider a $k^s - y^s$ space, the first equation does not shift with L^s, the second one shifts down (given k, y decreases with L), the third one shifts up with L^s (given k, y increases with L). In unconstrained equilibrium, k^s and y^s decreases with L^s; in constrained equilibrium, k^s and y^s increases with L^s, which means $\frac{\partial wL}{\partial \Gamma} = \beta$. Since w decreases with L given k', L increases with Γ given k'. Use steady state equilibrium (), k' increases with Γ. y increases with Γ in unconstrained equilibrium. But in constrained equilibrium y may decreases with Γ. Moreover, if participation constraint is binding, $n \propto \frac{1}{\alpha_h} = \frac{u}{i\frac{A}{w}g} - \frac{1}{i}$ decreases with Γ. Together with $\frac{\partial k'}{\partial \Gamma}$, borrowing margin $1 - \gamma'\beta$ decreases with Γ and $\frac{\partial \gamma'}{\partial \Gamma} \geq 0$. □

Properties of markups

The DM markup of one dollar $\mu_d = \frac{\frac{m'}{y}}{c_y \frac{w}{A} \frac{1}{\phi' \beta}} - 1 = \frac{\frac{A}{w} g\left(y,k'\right)}{y c_1\left(y,k'\right)} - 1$. According to A.1, $\frac{\frac{A}{w} g\left(y,k'\right)}{y} \geq \frac{A}{w} g_y > c_y$.

$$1 = \frac{\left[g_y y_k y c_y + g_k y c_y - g y_k c_y - g y_k y c_{yy} - g y c_{yk}\right]}{\left(y c_y\right)^2} \frac{A}{w} \frac{\partial k'}{\partial \mu}$$

Hence $\frac{\partial k'}{\partial \mu} \leq 0$. The markup of the bond market is measured by the borrowing margin $\mu_b = \frac{1-\beta\gamma'}{1-\beta(1-\delta)} - \frac{z}{k'}$. if $k' = k^c$, $\mu_b = \frac{1-\beta\gamma'}{1-\beta(1-\delta)}$ where $1 - \beta\gamma' = -\alpha_f c_k + \beta f_1$ and hence $\frac{\partial \mu_b}{\partial k'} < 0$. if $k' = k^u$, $\mu_b = -\frac{z}{k'}$ where $z = f(k,L) - x - \left[k' - (1-\delta)k\right]$. In steaty state, $\frac{\partial \mu_b}{\partial k'} < 0$.

A.2. Extended Models

Price Posting Equilibrium

First order conditions specify k', y and $n$footnotesize

$$\begin{aligned}
k': &\quad \left(\frac{1-\delta}{R} - 1\right) + \frac{f_1\left(k',L\right)}{R} - \alpha_f(n) c_2(y,k') &= 0 \\
y: &\quad \alpha_f(n) c_1(y,k') + \frac{1}{R}\alpha_f(n) \frac{\alpha_h(n) u'(y)}{\hat{\beta} - \frac{A\phi}{w\phi'} - \alpha_h(n)\hat{\beta}} &= 0 \\
n: &\quad -\alpha_f'(n) c(y,k') + \frac{1}{R}\alpha_f'(n) \frac{\hat{U} - \alpha_h(n) u(y)}{\hat{\beta} - \frac{A\phi}{w\phi'} - \alpha_h(n)\hat{\beta}} + \frac{1}{R}\alpha_f(n) \frac{\alpha_h'(n)\left[\hat{\beta}\hat{U} - \hat{\beta}u(y) + \frac{A\phi}{w\phi'} u(y)\right]}{\left[\hat{\beta} - \frac{A\phi}{w\phi'} - \alpha_h(n)\hat{\beta}\right]^2} &= 0
\end{aligned}$$

Combine the last twofootnotesize

$$\begin{aligned} d &= \phi^{'} m^{'} = \frac{-\alpha_f^{'}(n) c(y,k^{'}) u^{'}(y) + n\alpha_h^{'}(n) c_1(y,k^{'}) u(y)}{-\alpha_f^{'}(n)\frac{1}{R} u^{'}(y) + n\alpha_h^{'}(n)\hat{\beta} c_1(y,k^{'})} \\ &= \frac{\eta_h c(y,k^{'}) u^{'}(y) + \eta_f c_1(y,k^{'}) u(y)}{\eta_h u^{'}(y) + \eta_f \frac{A}{w^{'}} c_1(y,k^{'})} \end{aligned}$$

Detail about extended model

The unconstraint problem is footnotesize

$$\begin{aligned} & \underset{\{n,y,d,\sigma,k^{'}\}}{max} \{ (\beta - \beta\delta - 1) k^{'} + \beta f\left(k^{'}, L\right) - \hat{z}(\sigma) \\ & \quad + \alpha_f(n)\,\sigma \left[-c\left(y, k^{'}\right) + \beta \frac{\hat{U} - \alpha_h(n)\,\sigma u(y)}{\hat{\beta} - \frac{A\phi}{w\phi^{'}} - \alpha_h(n)\,\sigma\hat{\beta}} \right] \} \end{aligned}$$

Solve n, y, $k^{'}$ and σ by first order conditions

$$\begin{aligned} k^{'}: &\quad (\beta - \beta\delta - 1) + \beta f_1\left(k^{'}, L\right) - \alpha_f(n)\,\sigma c_2(y,k^{'}) &= 0 \\ \sigma: &\quad \alpha_f(n)\left[-c\left(y,k^{'}\right) + \beta d\right] + \alpha_f(n)\,\sigma\beta \tfrac{\partial d}{\partial \sigma} &\geq \hat{z}_\sigma \\ y: &\quad -\alpha_f(n)\,\sigma c_1(y,k^{'}) - \alpha_f(n)\,\sigma \tfrac{\alpha_h \sigma u^{'}(y)}{\frac{A}{w^{'}} - \frac{A\phi}{w\phi^{'}} R - \alpha_h \sigma \frac{A}{w^{'}}} &= 0 \\ n: &\quad -\alpha_f^{'}(n)\sigma c\left(y,k^{'}\right) + \alpha_f^{'}(n)\sigma\beta \tfrac{\hat{U} - \alpha_h(n)\sigma u(y)}{\hat{\beta} - \frac{A\phi}{w\phi^{'}} - \alpha_h(n)\sigma\hat{\beta}} + \alpha_f(n)\sigma\beta \tfrac{\alpha_h^{'}(n)\sigma u(y)\left[\frac{A\phi}{w\phi^{'}} + \alpha_h(n)\sigma\hat{\beta}\right]}{\left[\hat{\beta} - \frac{A\phi}{w\phi^{'}} - \alpha_h(n)\sigma\hat{\beta}\right]^2} &= 0 \end{aligned}$$

Let $\eta_h = \frac{\Delta\mathcal{M}/\mathcal{M}}{\Delta H/H} = \frac{\alpha_f'(n)}{\alpha_h(n)}$, $\eta_f = \frac{\Delta\mathcal{M}/\mathcal{M}}{\Delta F/F} = -\frac{n\cdot\alpha_h'(n)}{\alpha_h(n)}$ be the elasticities for HH and firm. $\eta_f + \eta_h = 1$. then

$$\beta d = \phi' m' = \frac{\eta_h c(y,k')u'(y) + \eta_f c_1(y,k')u(y)}{\eta_h u'(y) + \eta_f \frac{A}{w'} c_1(y,k')} \triangleq \tilde{g}(y,k')$$

So the FOC of y and n in SS can be rewritten as

$$i = \alpha_h(n^s)\sigma(\frac{w^s}{A}\frac{u'(y^s)}{c_1(y^s,k^s)} - 1)$$

$$\frac{\alpha_h(n)\,\sigma u(y) - \hat{U}}{i + \alpha_h(n)\,\sigma} = \frac{A}{w^s}\tilde{g}(y^s,k^s)$$

Friedman Rule

Planner's Problem satisfies

$$H(k) = \max_{x,L,k',y,z,\sigma} U(x) - AL + z + \alpha\sigma\left[u(y) - c(y,k')\right] + \beta H(k')$$

$$s.t.\; z + \hat{z}(\sigma) = f(k,L) - x - k' + (1-\delta)\,k$$

where $n = 1$.

$$H(k) = \max_{x,L,k',y,z,\sigma} U(x) - AL + f(k,L) - x - k' + (1-\delta)\,k - \hat{z}(\sigma)$$
$$+ \alpha\sigma\left[u(y) - c(y,k')\right] + \beta H(k')$$

Envelope condition implies $H^{'}(k) = f_1(k, L) + (1 - \delta)$. First order conditions are

$$\begin{aligned}
U^{'}(x) &= 1 \\
f_2(k, L) &= A \\
\beta f_1(k^{'}, L) - \alpha\sigma c_2(y, k^{'}) &= 1 - \beta(1 - \delta) \\
u^{'}(y) - c_1(y, k^{'}) &= 0 \\
-\hat{z}_\sigma + \alpha\left[u(y) - c(y, k^{'})\right] &= 0
\end{aligned}$$

In the extended model, if $i = 0$, then $\frac{w}{A}u^{'}(y) = c_1\left(y, k^{'}\right)$ and $\alpha_h(n)\,\sigma u(y) - \hat{U} = \frac{A}{w}\tilde{g}(y, k)\alpha_h(n)\,\sigma$. Plug into FOC of σ in A.7,

$$\alpha_f(n)\left[-c\left(y, k^{'}\right) + \beta\frac{u(y)}{\hat{\beta}}\right] \geq \hat{z}_\sigma$$

If the resource is abundant, such that $w = A$ and $\hat{z}$ is not binding. Then $\alpha_f(n)\left[-c\left(y, k^{'}\right) + u(y)\right] = \hat{z}_\sigma$, $u^{'}(y) = c_1\left(y, k^{'}\right)$.

B. Appendix. Efficiency and Default in Bank Lending

Proof. Claim 1 □

At search equilibrium, search value equals to the value matching with marginal partner, that $V_t = {}^{u(\tilde{c}_t)}/_r$ and optimal consumption $\tilde{c} = \bar{b}_t + ra_t$. Substitute into HJB equation (2.4)

$$u\left(\tilde{c}_t\right) = u\left(\tilde{c}_t\right) + u^{'}\left(\tilde{c}_t\right) \cdot \left(b + ra_t - \tilde{c}_t\right) + \frac{\alpha}{r}\int_{\bar{b}_t}^{\hat{b}}\left(u\left(\tilde{b} + ra_t\right) + u\left(\bar{b} + ra_t\right)\right) dF\left(\tilde{b}\right)$$

$$\sigma u\left(\tilde{c}_t\right)\left(b - \bar{b}_t\right) = \frac{\alpha}{r}\int_{\bar{b}_t}^{\hat{b}}\left(u\left(\tilde{b} + ra_t\right) + u\left(\tilde{c}_t\right)\right) dF\left(\tilde{b}\right)$$

$$\frac{r\sigma}{\alpha}\left(\bar{b} - b\right) = \int_{\bar{b}}^{\hat{b}}\left(u\left(\tilde{b} - \bar{b}\right) + 1\right) dF\left(\tilde{b}\right)$$

Use integration by parts,

$$
\begin{aligned}
RHS &= \left(u\left(\tilde{b}-\bar{b}\right)+1\right)dF\left(\tilde{b}\right)|_{\bar{b}}^{\hat{b}} - \int_{\bar{b}}^{\hat{b}} F\left(\tilde{b}\right)d\left(u\left(\tilde{b}-\bar{b}\right)+1\right) \\
&= \int_{\bar{b}}^{\hat{b}} \left(1-F\left(\tilde{b}\right)\right)du\left(\tilde{b}-\bar{b}\right)
\end{aligned}
$$

Proof. Claim 2 □

If matching with type $\tilde{b}$ at time t, the matching value is $W_t = \frac{1}{r}u\left(c_t^s\right)$ and the optimal consumption after matching is $c_t^s = \tilde{b}_t + ra_t - rPD_t$. Guess the value function of search and optimal consumption are $V_t\left(a_t\right) = {}^{u(\tilde{c}_t)}/_{r}$ and $\tilde{c}_t = \bar{b}_t + ra_t - rPD_t$. To verify my guess, substitute $\{\tilde{c}, V_t\left(a_t\right)\}$ into HJB equation (2.4). Rewrite (2.4) as

$$
\dot{V}_t\left(a_t\right) = rV_t\left(a_t\right) - u\left(\tilde{c}_t\right) - \alpha\int_{\bar{b}_t}^{\hat{b}}\left(W - V_t\left(a_t\right)\right)dF\left(\tilde{b}\right)
$$

then substitute the guess into the above HJB equation

$$
\begin{aligned}
LHS &= \frac{1}{r}u'\left(\tilde{c}_t\right)\frac{d\tilde{c}_t}{dt} \\
&= -\sigma u\left(\tilde{c}_t\right)\left(b - \bar{b}_t + \frac{1}{r}\dot{\bar{b}}_t\right)
\end{aligned}
$$

$$
\begin{aligned}
RHS &= -\alpha \int_{\bar{b}_t}^{\hat{b}} \left(W - V_t\left(a_t\right)\right) dF\left(\tilde{b}\right) \\
&= -\alpha u\left(\tilde{c}_t\right) \int_{\bar{b}_t}^{\hat{b}} \frac{1}{r}\left(u\left(c_t^s\right) - u\left(\tilde{c}_t\right)\right) dF\left(\tilde{b}\right) \\
&= \frac{\alpha}{r} u\left(\tilde{c}_t\right) \int_{\bar{b}_t}^{\hat{b}} \frac{1}{r}\left(u\left(\tilde{b} - \bar{b}_t\right) + 1\right) dF\left(\tilde{b}\right)
\end{aligned}
$$

Set $LHS = RHS$, the threshold type is determined by

$$
\frac{r\sigma}{\alpha}\left(\bar{b}_t - \frac{1}{r}\dot{\bar{b}}_t - b\right) = \int_{\bar{b}_t}^{\hat{b}} \left[u\left(\tilde{b} - \bar{b}_t\right) + 1\right] dF\left(\tilde{b}\right)
$$

Then the termination time T^* is determined by

$$
-\int_0^t \left(\bar{b}_s - b\right) ds - \frac{\mathcal{T}}{r} e^{-rT}\left(e^{rt} - 1\right) = a_t^L \tag{B.1}
$$

and optimal consumption can be expressed as

$$
\tilde{c}_t = \bar{b}_t - rPD_0 - r\int_0^t \left(\bar{b}_s - b\right) ds \tag{B.2}
$$

Proof. Proposition 4 □

First, I will show how to get (2.19) from (2.15)

$$
\begin{aligned}
\Pi\left(v_0\right) &= \int_0^{T^*} e^{-\int_0^t\left(r+\alpha\left(1-F\left(\bar{b}_s\right)\right)\right)ds}\left(\tau+\alpha\left(1-F\left(\bar{b}_t\right)\right)PD_t\right)dt \\
&= -\frac{\tau}{r}\int_0^{T^*} d\left[e^{-\int_0^t \alpha\left(1-F\left(\bar{b}_s\right)\right)ds}\left(e^{-rt}-e^{-rT}\right)\right] \\
&= -\frac{\tau}{r}\left[e^{-\int_0^t \alpha\left(1-F\left(\bar{b}_s\right)\right)ds}\left(e^{-rT*}-e^{-rT}\right)-\left(1-e^{-rT}\right)\right] \\
&= PD_0-\frac{\tau}{r}\Gamma\left[\left(1-e^{-rT}\right)-\left(1-e^{-rT*}\right)\right] \\
&= \left(1-\Gamma\right)PD_0+\Gamma\cdot\frac{\tau}{r}\left(1-e^{-rT*}\right)
\end{aligned}
$$

Consider

$$
\begin{aligned}
U_0\left(\tau,T,\left\{a_t^L\right\}\right) = \int_0^{T*} & e^{-\int_0^t\left\{r+\alpha\left[1-F\left(\bar{b}_s\right)\right]\right\}ds}[u\left(\tilde{c}_t\right) \\
&+\alpha\int_{\bar{b}_t}^{\hat{b}}\frac{u\left(\tilde{b}-rPD_t+ra_t\right)}{r}dF\left(\tilde{b}\right)]dt
\end{aligned}
$$

Since innovator's consumption will be zero after the termination of the project, I change the upper bound of the support to T^* to focus on the innovator's benefit.

$$
\begin{aligned}
U_0 &= \int_0^{T*} e^{-\int_0^t\left(r+\alpha\left(1-F\left(\bar{b}_s\right)\right)\right)ds}\left(u\left(\tilde{c}_t\right)-\alpha u\left(\tilde{c}_t\right)\int_{\bar{b}_t}^{\hat{b}}\frac{u\left(\tilde{b}-\bar{b}_t\right)}{r}dF\left(\tilde{b}\right)\right)dt \\
&= \int_0^{T*} e^{-\int_0^t\left(r+\alpha\left(1-F\left(\bar{b}_s\right)\right)\right)ds}u\left(\tilde{c}_t\right)\left(1+\frac{\sigma}{r}\dot{\tilde{c}}_t+\frac{\alpha}{r}\left(1-F\left(\tilde{b}\right)\right)\right)dt \\
&= -\frac{1}{r}\int_0^{T*} d\left[e^{-\int_0^t\left(r+\alpha\left(1-F\left(\bar{b}_s\right)\right)\right)ds}u\left(\tilde{c}_t\right)\right] \\
&= \frac{1}{r}\left[u\left(\tilde{c}_0\right)-e^{-rT^*}\Gamma u\left(\tilde{c}_{T^*}\right)\right]
\end{aligned}
$$

Rearrange both equation

$$\begin{aligned}\Pi(v_0) &= PD_0 - e^{-rT^*} PD_{T^*}\Gamma \\ rU_0 &= u(\tilde{c}_0) - e^{-rT^*} u(\tilde{c}_{T^*})\Gamma\end{aligned}$$

Proof. Proposition 5 □

If $\tilde{b}_G = \tilde{b}_F + \epsilon$, where $\tilde{b}_F \sim F(\cdot), \tilde{b}_G \sim G(\cdot)$, and $\epsilon > 0$ is a constant, $F\left(\tilde{b} - \epsilon\right) = G\left(\tilde{b}\right)$. According to (2.6), basic threshold type $\bar{b}$ increases that $\bar{b}_F < \bar{b}_G$. From (B.1), the termination time T^* decreases after the shift of the distribution, $T^*_F > T^*_G$. Then, I want to know how much the basic threshold type increases. $\bar{b}_F$ and $\bar{b}_G$ solve

$$\begin{aligned}\frac{r\sigma}{\alpha}\left(\bar{b}_F - b\right) &= \int_{\bar{b}_F}^{\hat{b}} \left(1 - F\left(\tilde{b}\right)\right) d\left[u\left(\tilde{b} - \bar{b}_F\right)\right] \\ \frac{r\sigma}{\alpha}\left(\bar{b}_G - b\right) &= \int_{\bar{b}_G}^{\hat{b}} \left(1 - G\left(\tilde{b}\right)\right) d\left[u\left(\tilde{b} - \bar{b}_G\right)\right]\end{aligned}$$

Let $\Delta\bar{b} = \bar{b}_G - \bar{b}_F$, subtract the above equations

$$\begin{aligned}\frac{r\sigma}{\alpha}\Delta\bar{b} &= -\sigma \int_{\bar{b}_G}^{\hat{b}+\epsilon} \left[1 - G\left(\tilde{b}\right)\right] u\left(\tilde{b} - \bar{b}_G\right) d\tilde{b} + \\ &\quad \sigma \int_{\bar{b}_F}^{\hat{b}} \left[1 - F\left(\tilde{b}\right)\right] u\left(\tilde{b} - \bar{b}_F\right) d\tilde{b} \\ \Rightarrow \frac{r}{\alpha}\Delta\bar{b} &= \int_{\bar{b}_F}^{\hat{b}} \left(1 - F\left(\tilde{b}\right)\right) u\left(\tilde{b} - \bar{b}_F\right) d\tilde{b} - \\ &\quad \int_{\bar{b}_F + \Delta\bar{b} - \epsilon}^{\hat{b}} \left(1 - F\left(\tilde{b}\right)\right) u\left(\tilde{b} - \bar{b}_F + \epsilon - \Delta\bar{b}\right) d\tilde{b}\end{aligned}$$

If $\Delta\bar{b} = \epsilon$, $LHS = \frac{r\epsilon}{\alpha} > 0, RHS = 0$, so $LHS > RHS$; if $\Delta\bar{b} = 0$, $LHS = 0$,

$$\begin{aligned}
RHS &= \int_{\bar{b}_F}^{\hat{b}} \left(1 - F\left(\tilde{b}\right)\right) u\left(\tilde{b} - \bar{b}_F\right) d\tilde{b} - \\
&\quad \int_{\bar{b}_F - \epsilon}^{\hat{b}} \left(1 - F\left(\tilde{b}\right)\right) u\left(\tilde{b} - \bar{b}_F + \epsilon\right) d\tilde{b} \\
&> u\left(\epsilon\right)[\int_{\bar{b}_F - \epsilon}^{\hat{b}} \left(1 - F\left(\tilde{b}\right)\right) u\left(\tilde{b} - \bar{b}_F + \epsilon\right) d\tilde{b} - \\
&\quad \int_{\bar{b}_F}^{\hat{b}} \left(1 - F\left(\tilde{b}\right)\right) u\left(\tilde{b} - \bar{b}_F\right) d\tilde{b}] \\
&= u\left(\epsilon\right)\int_{\bar{b}_F - \epsilon}^{\bar{b}_F} \left(1 - F\left(\tilde{b}\right)\right) u\left(\tilde{b} - \bar{b}_F + \epsilon\right) d\tilde{b} \\
&> 0 = LHS
\end{aligned}$$

Both LHS and RHS are monotone with respect to $\Delta\bar{b}$, so $0 < \Delta\bar{b} < \epsilon$, and $F\left(\bar{b}_F\right) = F\left(\bar{b}_G - \Delta\bar{b}\right) > F\left(\bar{b}_G - \epsilon\right) = G\left(\bar{b}_G\right)$. According to the evolvement of $\bar{b}_t$

$$\frac{r\sigma}{\alpha}\dot{\bar{b}}_t = \frac{r\sigma}{\alpha}\left(\bar{b}_t - b\right) - \int_{\bar{b}_t}^{\hat{b}} \left[1 - F\left(\tilde{b}\right)\right] d\left[u\left(\tilde{b} - \bar{b}_t\right)\right]$$

If $\tilde{b}_F$ changes to $\tilde{b}_G$, the path of $\left\{\bar{b}_t\right\}$ becomes steeper. This means at certain time $t^{'}$, the difference between threshold types will be ϵ, that $\epsilon = \bar{b}_G - \bar{b}_F$. Then, $G\left(\bar{b}_{tG}\right) > F\left(\bar{b}_{tF}\right)$ $\forall t < t^{'}$. From (Claim 2)

$$\tilde{c}_t = \bar{b}_t - rPD_0 - r\int_0^t \left(\bar{b}_s - b\right) ds$$

$$a_t = bt - \int_0^t \bar{b}_s ds - \frac{\tau}{r}e^{-rT}\left(e^{rt} - 1\right)$$

Suppose bank uses the same contract after the distribution shifting from F to G, promised utility to the entrepreneur will increase. To increase the profit while keeping the same promised utility, bank can increase τ a little while keep T the same. Threshold type $\left\{\bar{b}_G\right\}$ will be the same while a_t decreases faster, then T^* becomes smaller. If $\frac{r}{\alpha}$ is smaller, $\Delta\bar{b}$ will be closer to ϵ, $\left\{\bar{b}_{tG}\right\}$ becomes steeper, t' will be closer to T; especially if r is smaller, the increase of τ will have stronger effect to $\dot{a}_t$. According to (2.18), Γ decreases in T^* and increases in $F\left(\bar{b}_t\right)$; if $\frac{r}{\alpha}$ or r is small enough, default rate will increase that $\Gamma^G > \Gamma^F$.

First Order Stochastic Dominance. $\bar{b}$ is determined by (2.6) and (2.9), If $G(\cdot) \succ_{FOSD} F(\cdot)$, then

$$\left(1 - G\left(\tilde{b}\right)\right) d\left[u\left(\tilde{b} - \bar{b}\right)\right] > \left(1 - F\left(\tilde{b}\right)\right) d\left[u\left(\tilde{b} - \bar{b}\right)\right] \ \forall\tilde{b}$$

\hence, $\bar{b}_G > \bar{b}_F$. Let the increase of threshold type be $\Delta\bar{b} = \bar{b}_G - \bar{b}_F$, and the increase of mean be $\Delta\mu = \mu_G - \mu_F$. If $\frac{r}{\alpha}$ is larger, $\Delta\bar{b}$ may be much smaller than $\Delta\mu$. If $G\left(\bar{b}_G\right) \leq F\left(\bar{b}_F\right)$, higher threshold type will stimulate consumption, then increase τ a little while keep T the same will increase bank's profit while keeping the same promised utility. Then a_t will drop faster, hence touch the limit earlier. From equation (2.18), default rate will increase under such condition.

Remark B.1. How to get HJB equation in 5.2.2

write

$$\pi(t) = \max_{\bar{P}_t, d_t} e^{-r\Delta}\left(\alpha\Delta + \circ(\Delta)\right) \int_{\bar{P}_{t'}}^{\infty} \left[d_{t'} - \pi(t')\right] dF(P) + e^{-r\Delta}\pi_{t'}$$

$$\frac{1-e^{-r\Delta}}{\Delta}\pi_t = \underset{\bar{P}_t,d_t}{max}\; e^{-r\Delta}\left(\frac{\pi_{t'}-\pi_t}{\Delta}\right)+$$
$$e^{-r\Delta}\frac{(\alpha\Delta+\circ(\Delta))}{\Delta}\int_{\bar{P}_{t'}}^{\infty}\left[d_{t'}-\pi(t')\right]dF(P)$$

take limit $\Delta \to 0$, get HJB as

$$r\pi_t = \underset{\bar{P}_t,d_t}{max}\; \dot{\pi}_t + \alpha\int_{\bar{P}_t}^{\infty}(d_t-\pi_t)\,dF(P)$$

$$r\pi_t(v) = \underset{\bar{b}_t,d_t}{max}\pi'_v\left[rv(t)-u(b)-\alpha\int_{\bar{b}_t}^{\hat{b}}\frac{u\left(\tilde{b}-d_t\right)}{r}-v(t)\,dF\left(\tilde{b}\right)\right]$$
$$+\alpha\int_{\bar{b}_t}^{\hat{b}}(d_t-\pi_t)\,dF\left(\tilde{b}\right)$$

$$s.t\; U\left(t,\left\{\bar{b}_t\right\},\{d_t\}\right) \;\geq\; v$$
$$U\left(t,\left\{\bar{b}_t\right\},\{d_t\}\right) \;=\; \frac{u\left(\bar{b}_t-d_t\right)}{r}$$

Use $d=\frac{\bar{b}-u^{-1}(rv)}{r}$ and rearrange it as

$$r\pi(v) = \underset{\bar{P}}{max}\;\pi'_v\left[rv-u(b)+\alpha v\int_{\bar{b}}^{\hat{b}}\left[u\left(\tilde{b}-\bar{b}\right)+1\right]dF\left(\tilde{b}\right)\right]+$$
$$\alpha\int_{\bar{b}}^{\hat{b}}\left[\frac{\bar{b}-u^{-1}(rv)}{r}-\pi\right]dF(P)$$

C. Appendix. Do Mortgage Recourse Limitations Matter?

C.1. Appendix. Summary Statistics

Table C.1.: Summary Statistics

			Mean	
	Mean	Std. Dev.	Def	No Def
Complaint Amount	230116.9	306875.3	280785.7	229345.9
Loan Size	231938.6	262803.4	274260.1	231295.5
ARM	0.31	0.46	0.38	0.31
Conventional	0.55	0.5	0.52	0.55
Interest	677.02	160.24	704.06	676.62
Term	28.98	5.74	28.26	28.99
Age of loan	1603.42	984.19	1324.35	1607.66
% change	-14.54%	15.49%	-10.31%	-14.61%
Census age	34.35	378.99	32.99	34.37
Census income	63784.36	18620.06	64019.05	63780.71
Number of foreclosure	90959	90959	1387	89572

Table C.2.: Deficiency frequency

Def	No BR	BR	BR7	BR13	Total
Public Sale	0.034	0.015	0.013	0.026	0.03
	23930	5565	4420	1145	29495
Private Sale	0.012	0.018	0.017	0.019	0.013
	15646	3412	2241	1171	19058
Total	0.025	0.016	0.014	0.022	0.024
	39576	8977	6661	2316	48553

C.2. Appendix. Regressions

Since PS and $PDef$ are generated regressors, bootstrap standard errors are clustered by foreclosure in brackets. All the regressions below use bootstrap with 200 repeatations.

Table C.3.: Preliminary Test

	(1)	(2)	(3)	(4)	(5)	(6)
Complaint	0.945	0.938	0.675	0.675	0.553	0.609
	(0.184)**	(0.184)**	(0.221)**	(0.223)**	(0.266)*	(0.484)
Loan Size	-0.949	-0.956	-0.658	-0.725	-0.519	-0.800
	(0.191)**	(0.191)**	(0.228)**	(0.233)**	(0.278)	(0.513)
$\%\Delta M$	0.391	0.381	0.126	0.166	0.036	0.457
	(0.052)**	(0.052)**	(0.085)	(0.087)	(0.115)	(0.221)*
ARM	-0.117	-0.116	-0.118	-0.117	-0.127	-0.066
	(0.016)**	(0.016)**	(0.016)**	(0.016)**	(0.021)**	(0.041)
BALLOON	0.008	0.007	0.004	0.003	-0.013	-0.024
	(0.060)	(0.060)	(0.060)	(0.060)	(0.078)	(0.141)
FHA	-0.082	-0.077	-0.081	-0.079	-0.088	-0.036
	(0.025)**	(0.025)**	(0.025)**	(0.025)**	(0.032)**	(0.074)
VA	-0.042	-0.038	-0.037	-0.036	-0.097	-0.077
	(0.077)	(0.077)	(0.077)	(0.077)	(0.099)	(0.218)
Term	9.155	9.266	8.955	9.046	10.064	1.731
	(1.593)**	(1.593)**	(1.596)**	(1.596)**	(2.062)**	(3.479)
Interest	-0.294	-0.289	-0.295	-0.294	-0.449	-0.099
	(0.051)**	(0.051)**	(0.051)**	(0.051)**	(0.067)**	(0.111)
Census Age	0.589	0.379	0.430	0.375	0.814	1.315
	(0.662)	(0.663)	(0.662)	(0.663)	(0.858)	(1.661)
Census Income	3.365	3.244	3.226	3.212	4.359	1.843
	(0.446)**	(0.447)**	(0.448)**	(0.448)**	(0.586)**	(1.090)
Loan Age	0.130	0.130	0.132	0.132	0.136	0.138
	(0.010)**	(0.010)**	(0.010)**	(0.010)**	(0.013)**	(0.027)**
$Areadef$		-0.066		-0.052	-0.062	-0.081
		(0.016)**		(0.028)	(0.035)	(0.072)
$Areadef$*Complaint			0.781	0.760	0.117	2.496
			(0.378)*	(0.379)*	(0.432)	(1.066)*
$Areadef$*Loan Size			-0.845	-0.703	-0.043	-2.504
			(0.386)*	(0.393)	(0.448)	(1.100)*
$Areadef$*$\%\Delta M$			0.371	0.312	0.239	0.040
			(0.097)**	(0.102)**	(0.135)	(0.262)
Constant	-1.235	-1.195	-1.220	-1.194	-1.133	-1.316
	(0.084)**	(0.084)**	(0.084)**	(0.085)**	(0.109)**	(0.201)**
N	41,951	41,951	41,951	41,951	25,631	8,341

* $p < 0.05$; ** $p < 0.01$

Table C.4.: Regression 1

	br7	br13	br
Complaint Amount	-0.169	1.014	0.403
	(0.317)	(0.674)	(0.337)
Loan Size	0.714	-1.161	-0.172
	(0.386)	(0.755)	(0.379)
Term	-8.310	10.483	3.132
	(3.528)*	(5.095)*	(3.473)
Interest	-0.921	0.700	-0.468
	(0.087)**	(0.122)**	(0.079)**
Census Age	7.763	-1.238	4.142
	(1.439)**	(1.845)	(1.173)**
Census Income	21.845	-2.028	11.951
	(2.947)**	(4.074)	(2.385)**
Loan Age	0.429	-0.090	0.254
	(0.047)**	(0.068)	(0.041)**
$\%\Delta M$	-2.425	2.032	-0.663
	(0.356)**	(0.483)**	(0.296)*
ARM	-0.064	-0.123	-0.124
	(0.029)*	(0.041)**	(0.028)**
BALLOON	0.091	-0.111	-0.003
	(0.075)	(0.135)	(0.068)
FHA	-0.406	0.056	-0.215
	(0.062)**	(0.079)	(0.057)**
VA	-0.234	0.142	-0.179
	(0.113)*	(0.145)	(0.110)
$PS * PDef$	27.879	36.716	27.248
	(5.871)**	(9.825)**	(5.780)**
PS	-9.096	0.187	-4.752
	(1.373)**	(2.001)	(1.172)**
$PDef$	-6.707	-11.030	-5.476
	(2.509)**	(3.207)**	(2.820)
$\%\Delta M$	137.521	8.799	97.867
	(87.996)	(114.684)	(76.305)
Constant	-0.089	-2.208	-0.672
	(0.218)	(0.290)**	(0.227)**
N	33,941	33,941	33,941

* $p < 0.05$; ** $p < 0.01$

Table C.5.: Regression2

	br7	br13	br7	br13
Complaint Amount	-0.144	0.562	0.006	2.828
	(0.299)	(0.663)	(0.784)	(1.683)
Loan Size	0.605	-0.654	0.463	-3.524
	(0.357)	(0.719)	(0.903)	(1.902)
Term	-1.887	11.316	-14.562	6.408
	(3.720)	(6.689)	(7.044)*	(11.601)
Interest	-0.934	0.651	-0.706	1.077
	(0.089)**	(0.139)**	(0.184)**	(0.361)**
Census Age	5.804	-1.425	8.268	-0.480
	(1.453)**	(2.073)	(3.247)*	(4.614)
Census Income	16.913	-0.687	22.684	-8.297
	(2.781)**	(4.320)	(6.834)**	(9.715)
Loan Age	0.349	-0.086	0.447	-0.103
	(0.046)**	(0.075)	(0.108)**	(0.157)
$\%\Delta M$	-1.905	2.035	-2.167	2.240
	(0.353)**	(0.514)**	(0.808)**	(1.118)*
ARM	-0.071	-0.151	-0.049	0.037
	(0.033)*	(0.052)**	(0.055)	(0.096)
BALLOON	0.066	-0.080	0.122	-0.339
	(0.093)	(0.135)	(0.161)	(0.197)
FHA	-0.313	0.026	-0.475	0.236
	(0.062)**	(0.105)	(0.143)**	(0.195)
VA	-0.181	0.090	-0.295	0.439
	(0.115)	(0.196)	(0.283)	(0.324)
PS	-6.038	-0.179	-10.076	3.329
	(1.378)**	(2.200)	(3.015)**	(4.391)
$PDef$	-4.226	-10.880	-10.483	-13.957
	(3.174)	(4.696)*	(5.526)	(7.541)
$PS * PDef$	16.785	41.549	42.752	18.130
	(7.875)*	(10.954)**	(12.954)**	(20.239)
Constant	-0.452	-2.203	-0.123	-2.537
	(0.234)	(0.427)**	(0.427)	(0.555)**
N	25,604	25,604	8,337	8,337

* $p < 0.05$; ** $p < 0.01$

Table C.6.: Alternative Regression 1

	br7	br13	br
Complaint Amount	1.211	0.905	1.300
	(0.251)**	(0.564)	(0.354)**
Loan Size	-1.555	-0.392	-1.378
	(0.272)**	(0.610)	(0.365)**
Term	11.639	-8.147	7.789
	(2.436)**	(3.360)*	(2.245)**
Interest	-1.334	1.701	-0.373
	(0.125)**	(0.168)**	(0.111)**
Census Age	-1.683	2.574	-0.164
	(0.804)*	(1.057)*	(0.735)
Census Income	-9.733	21.472	0.744
	(1.965)**	(2.496)**	(1.750)
Loan Age	-0.436	0.883	0.018
	(0.083)**	(0.107)**	(0.077)
$\%\Delta M$	1.902	-1.799	0.844
	(0.320)**	(0.405)**	(0.290)**
ARM	-0.245	0.236	-0.128
	(0.037)**	(0.050)**	(0.036)**
BALLOON	-0.260	0.539	-0.027
	(0.080)**	(0.118)**	(0.069)
FHA	-0.317	0.258	-0.142
	(0.038)**	(0.053)**	(0.035)**
VA	-0.497	0.996	-0.057
	(0.119)**	(0.156)**	(0.104)
$PDef$	-12.903	-32.396	-22.355
	(5.440)*	(7.529)**	(5.205)**
$PS * PDef$	25.051	19.172	28.876
	(7.251)**	(9.914)	(7.019)**
Constant	-5.617	6.212	-1.447
	(0.753)**	(0.988)**	(0.681)*
N	41,951	41,951	41,951

* $p < 0.05$; ** $p < 0.01$

Table C.7.: Regression Alternative 2

	br7	br13	br7	br13
Complaint Amount	0.923	0.061	1.089	1.210
	(0.227)**	(0.412)	(0.729)	(1.559)
Loan Size	-1.201	0.633	-1.327	-0.802
	(0.246)**	(0.443)	(0.772)	(1.612)
Term	13.483	-5.180	1.215	-18.668
	(3.155)**	(4.855)	(5.607)	(9.043)*
Interest	-1.469	1.814	-0.675	2.511
	(0.153)**	(0.244)**	(0.287)*	(0.385)**
Census Age	-1.311	3.821	-0.336	7.435
	(1.073)	(1.775)*	(1.809)	(2.637)**
Census Income	-8.979	25.024	-2.359	28.018
	(2.585)**	(3.719)**	(4.468)	(6.060)**
Loan Age	-0.435	0.977	-0.066	1.265
	(0.109)**	(0.158)**	(0.200)	(0.249)**
$\%\Delta M$	1.806	-2.150	0.956	-3.496
	(0.427)**	(0.608)**	(0.748)	(0.987)**
ARM	-0.263	0.225	-0.097	0.515
	(0.049)**	(0.078)**	(0.094)	(0.117)**
BALLOON	-0.279	0.522	-0.016	0.358
	(0.108)**	(0.166)**	(0.184)	(0.227)
FHA	-0.312	0.343	-0.214	0.469
	(0.052)**	(0.080)**	(0.111)	(0.155)**
VA	-0.514	0.882	-0.254	1.462
	(0.152)**	(0.235)**	(0.349)	(0.352)**
PS	9.994	-17.058	3.463	-21.279
	(1.830)**	(2.595)**	(3.401)	(4.438)**
$PDef$	6.030	-15.721	0.813	-29.672
	(2.579)*	(4.232)**	(4.726)	(5.342)**
Constant	-6.357	6.828	-2.948	9.422
	(1.012)**	(1.416)**	(1.884)	(2.484)**
N	25,631	25,631	8,341	8,341

* $p < 0.05$; ** $p < 0.01$

Bibliography

[AAD01] Brent W. Ambrose, Capone Charles A, and Yongheng Deng. Optimal put exercise: An empirical examination of conditions for mortgage foreclosure. *The Journal of Real Estate Finance and Economics*, 23(2): 213–34, 2001, http://EconPapers.repec.org/RePEc:kap:jrefec:v:23:y:2001:i:2:p:213-34.

[AH04] Rui Albuquerque and Hugo A. Hopenhayn. Optimal lending contracts and firm dynamics. *Review of Economic Studies*, 71(2): 285–315, 04 2004, http://ideas.repec.org/a/bla/restud/v71y2004i2p285-315.html.

[AS08] Tobias Adrian and Hyun Song Shin. Financial intermediaries, financial stability, and monetary policy. Staff Reports 346, Federal Reserve Bank of New York, 2008.

[AWW09] S. Boragan Aruoba, Christopher J. Waller, and Randall Wright. Money and capital: a quantitative analysis. Working Papers 2009-031, Federal Reserve Bank of St. Louis, 2009.

[Bar86] Cordes Joseph J. Yezer Anthony M. J. Barth, James R. Benefits and costs of legal restrictions on personal loan markets. *journal Journal of Law and Economics*, XXIX: 357–80, 1986.

[BG89] Ben Bernanke and Mark Gertler. Agency costs, net worth, and business fluctuations. *American Economic Review*, 79(1): 14–31, March 1989, http://ideas.repec.org/a/aea/aecrev/v79y1989i1p14-31.html.

[BP07] Markus K. Brunnermeier and Lasse Heje Pedersen. Market liquidity and funding liquidity. NBER Working Papers 12939, National Bureau of Economic Research, Inc, February 2007.

[CH90] Terrence M Clauretie and Thomas N Herzog. The effect of state foreclosure laws on loan losses: Evidence from the mortgage insurance industry. *Journal of Money, Credit and Banking*, 22(2): 221–33, May 1990, http://ideas.repec.org/a/mcb/jmoncb/v22y1990i2p221-33.html.

[CH06] Gina Luca Clementi and Hugo A Hopenhayn. A theory of financing constraints and firm dynamics. *The Quarterly Journal of Economics*, 121(1): 229–265, 02 2006, http://ideas.repec.org/a/tpr/qjecon/v121y2006i1p229-265.html.

[Cho05] Tarun Chordia. An empirical analysis of stock and bond market liquidity. *Review of Financial Studies*, 18(1): 85–129, 2005, http://ideas.repec.org/a/oup/rfinst/v18y2005i1p85-129.html.

[CP02] John H. Cochrane and Monika Piazzesi. The fed and interest rates: A high-frequency identification. NBER Working Papers 8839, National Bureau of Economic Research, Inc, March 2002.

[CR08] B. Craig and G. Rocheteau. State-dependent pricing, inflation, and welfare in search economies. *European Economic Review*, 52(3): 441–468, 2008.

[CSS03] Tarun Chordia, Asani Sarkar, and Avanidhar Subrahmanyam. An empirical analysis of stock and bond market liquidity. Staff Reports 164, Federal Reserve Bank of New York, 2003.

[DF07] Peter M. DeMarzo and Michael J. Fishman. Optimal long-term financial contracting. *Review of Financial Studies*, 20(6): 2079–2128, 2007, http://rfs.oxfordjournals.org/content/20/6/2079.abstract.

[DGS01] Pradeep Dubey, John Geanakoplos, and Martin Shubik. Default and punishment in general equilibrium. (1304R5), May 2001, http://ideas.repec.org/p/cwl/cwldpp/1304r5.html.

[Don10] Mei Dong. Inflation and variety. *International Economic Review*, 51(2): 401–420, 05 2010, http://ideas.repec.org/a/ier/iecrev/v51y2010i2p401-420.html.

[DQ11] Corbae Dean and Erwan Quintin. Mortgage innovation and the foreclosure boom. February 2011.

[FGGW08] Christopher L. Foote, Kristopher Gerardi, Lorenz Goette, and Paul S. Willen. Just the facts: An initial analysis of subprime's role in the housing crisis. *Journal of Housing Economics*, 17(4): 291–305, December 2008, http://ideas.repec.org/a/eee/jhouse/v17y2008i4p291-305.html.

[FGGW09] Christopher Foote, Kristopher Gerardi, Lorenz Goette, and Paul

Willen. Reducing foreclosures: No easy answers. (15063), June 2009, http://ideas.repec.org/p/nbr/nberwo/15063.html.

[FHP88] Steven M. Fazzari, R. Glenn Hubbard, and BRUCE C. PETERSEN. Financing constraints and corporate investment. *Brookings Papers on Economic Activity*, 19(1): 141–206, 1988, http://ideas.repec.org/a/bin/bpeajo/v19y1988i1988-1p141-206.html.

[FHW02] Scott Fay, Erik Hurst, and Michelle J. White. The household bankruptcy decision. *American Economic Review*, 92(3): 706–718, June 2002, http://ideas.repec.org/a/aea/aecrev/v92y2002i3p706-718.html.

[FKO98] Jeff Fleming, Chris Kirby, and Barbara Ostdiek. Information and volatility linkages in the stock, bond, and money markets1. *Journal of Financial Economics*, 49(1): 111–137, July 1998, http://ideas.repec.org/a/eee/jfinec/v49y1998i1p111-137.html.

[GG91] Mark Gertler and Simon Gilchrist. Monetary policy, business cycles and the behavior of small manufacturing firms. NBER Working Papers 3892, National Bureau of Economic Research, Inc, November 1991.

[GG93] Mark Gertler and Simon Gilchrist. The cyclical behavior of short-term business lending: Implications for financial propagation mechanisms. *European Economic Review*, 37(2-3): 623–631, April 1993, http://ideas.repec.org/a/eee/eecrev/v37y1993i2-3p623-631.html.

[GS10] Allen C. Goodman and Brent C. Smith. Residential mortgage default: Theory works and so does policy. *Jour-*

nal of Housing Economics, 19(4): 280–294, December 2010, http://ideas.repec.org/a/eee/jhouse/v19y2010i4p280-294.html.

[HB98] Richard Hynes and Jeremy Berkowitz. Bankruptcy exemptions and the market for mortgage loans journal of law and economic. (98-17), June 1998, http://ideas.repec.org/p/wop/pennin/98-17.html.

[Jon93] Lawrence D Jones. Deficiency judgments and the exercise of the default option in home mortgage loans. *Journal of Law and Economics*, 36(1): 115–38, April 1993, http://ideas.repec.org/a/ucp/jlawec/v36y1993i1p115-38.html.

[KM95] Nobuhiro Kiyotaki and John Moore. Credit cycles - (now published in journal of political economy, vol.105(2), april 1997, pp.211-248.). STICERD - Theoretical Economics Paper Series 285, Suntory and Toyota International Centres for Economics and Related Disciplines, LSE, 1995.

[KM12] Nobuhiro Kiyotaki and John Moore. Liquidity, business cycles, and monetary policy. NBER Working Papers 17934, National Bureau of Economic Research, Inc, March 2012.

[KW89] Nobuhiro Kiyotaki and Randall Wright. On money as a medium of exchange. *Journal of Political Economy*, 97(4): 927–54, August 1989, http://ideas.repec.org/a/ucp/jpolec/v97y1989i4p927-54.html.

[LRW09] Ricardo Lagos, Guillaume Rocheteau, and Pierre-Olivier Weill. Crises and liquidity in over-the-counter markets. NBER Working Papers 15414, National Bureau of Economic Research, Inc, October 2009.

[LRW11] Ricardo Lagos, Guillaume Rocheteau, and Pierre-Olivier Weill. Crises and liquidity in over-the-counter markets. *Journal of Economic Theory*, 146(6): 2169–2205, 2011, http://ideas.repec.org/a/eee/jetheo/v146y2011i6p2169-2205.html.

[Qua04] Vincenzo Quadrini. Investment and liquidation in renegotiation-proof contracts with moral hazard. *Journal of Monetary Economics*, 51(4): 713–751, May 2004, http://ideas.repec.org/a/eee/moneco/v51y2004i4p713-751.html.

[RW10] Guillaume Rocheteau and Randall Wright. Liquidity and asset market dynamics. Working Paper 1016, Federal Reserve Bank of Cleveland, 2010.

[RW11] G. Rocheteau and P. Weill. Liquidity in frictional asset markets. *Journal of Money, Credit and Banking*, 43(s2): 261–282, 2011.

[Shi99] Shouyong Shi. Search, inflation and capital accumulation. *Journal of Monetary Economics*, 44(1): 81–103, August 1999, http://ideas.repec.org/a/eee/moneco/v44y1999i1p81-103.html.

[SW06] Rafael Silveira and Randall Wright. Liquidity and the market for ideas. 2006 Meeting Papers 77, Society for Economic Dynamics, December 2006.

[SW08] Robert Shimer and Ivan Werning. Liquidity and insurance for the unemployed. *American Economic Review*, 98(5): 1922–42, December 2008, http://ideas.repec.org/a/aea/aecrev/v98y2008i5p1922-42.html.