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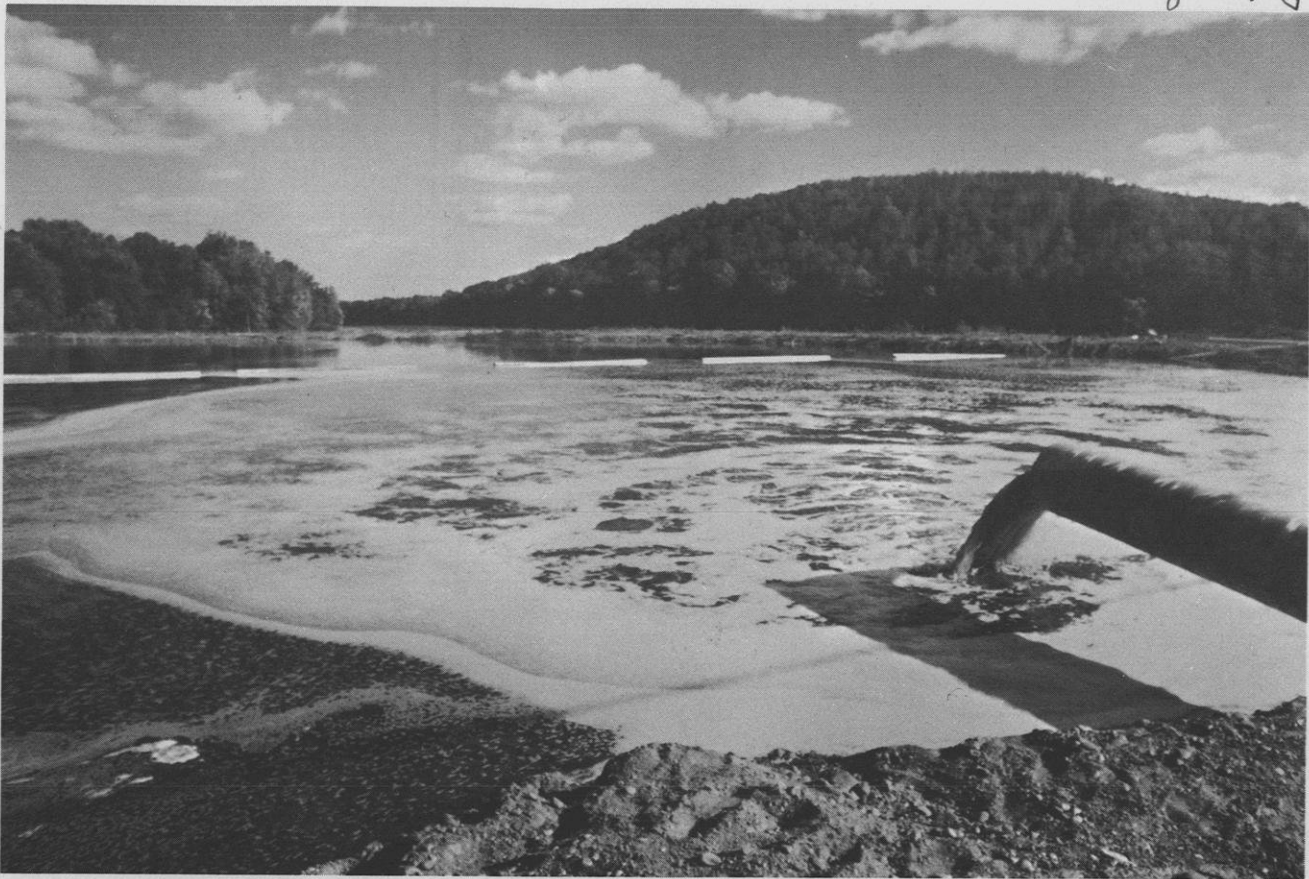
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# ALTERNATIVE METHODS OF ESTIMATING POLLUTANT LOADS IN FLOWING WATER

Technical Bulletin No. 133  
Wisconsin Department of Natural Resources  
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## ABSTRACT

A comparison is provided of three principal methods of estimating pollutant loads in flowing water: integration, composite, and stratified random sampling, enhanced with ratio estimation. Of these three, integration and composite sampling have already received extensive use, and are presented briefly here for descriptive and comparative purposes. Stratified random sampling, however, is a relatively new method of estimating pollutant loads, and is discussed in greater detail. One section of the report is devoted to a description of the method, another to the computations involved in using it, and a third to the reliability of the method under a wide range of sampling conditions. Finally, recommendations for choosing between integration, composite, and stratified random sampling are given, based on the relative attributes of each technique, and on the needs and limitations of the individual investigator.

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**ALTERNATIVE METHODS OF ESTIMATING POLLUTANT LOADS  
IN FLOWING WATER**

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# INTRODUCTION

This report provides a comparison of the three principal methods of estimating pollutant loads: integration, composite, and stratified random sampling. Its purpose is to assist water resource investigators in their selection of the most appropriate of these three alternatives. The first two are traditional, nonstatistical methods, while stratified random sampling is a non-traditional, statistical method. All rely upon periodic sampling of water quality and measurement of flow, and can be significantly upgraded with more extensive (continuous) flow monitoring.

The report is an outgrowth of analyses performed for the experimental design for the Wisconsin Department of Natural Resources Nationwide Urban

Runoff Project (NURP). In this project, stormwater runoff from paired (test and control) watersheds was compared to evaluate the effectiveness of an urban stormwater management practice. The experimental design was directed toward answering three questions: is stratified random sampling appropriate for the NURP analyses, and if so, how many samples would be required at each station for each season, and how should the samples be collected? It was difficult to answer these questions because very little had been written about estimating pollutant loads using stratified random sampling. An evaluation of this sampling method was needed along with a comparison of it to other sampling methods.

The three methods are discussed in some detail in this report, including a description of the techniques, the analytics involved in using them, sampling considerations and constraints, an evaluation of what happens in the event samples are not taken, and a critique of advantages and disadvantages. An extensive section is devoted to a presentation and evaluation of the stratified random sampling method since it is a new and largely untried method of estimating pollutant loads.

All three methods are applicable to flowing waters in either pipes or natural stream channels. The discussion in this report, however, focuses primarily on runoff event monitoring in streams.

## MONITORING METHODS

### INTEGRATION

#### General Description

Integration analysis is a straightforward and traditional means of estimating pollutant loads in a flowing body of water. It has been the principal tool of the U.S. Geological Survey (USGS) in their estimates of tributary suspended solids loads (Porterfield 1972, USGS 1975).

Integration requires that each event or period of flow be extensively monitored with a series of discrete samples and measured flows. Flow values are usually monitored continuously. By interpolating between points of measured concentrations, an estimated concentration can be paired with every measured flow value. Effectively, between every two adjoining flow measurements (such as every five minutes if using a five-minute punch stage recorder) the average of the two flows is multiplied by the average of their respective concentrations (usually estimated) to get a pollutant loading rate for the incremental period of measure. The sum of these many small incremental loads will then give a total pollutant load for the period or event of

concern.

Depending on the variability of the concentrations, integration may give an accurate estimate of the total pollutant load when an event or period is well sampled. However, the extensive analyses desired for integration can rapidly deplete an analytical budget. Then too, samplers do not always function as we would have them. If, because of sampler failure or a restrictive analytical budget, an event is poorly sampled, the number of samples may not be adequate to reflect changing conditions and concentrations, and hence, integration loses its accuracy. A method to provide supplemental data for these events, and for events that are not sampled at all, is necessary for the systematic use of integration.

Faced with this problem, the USGS provides such supplemental data by estimating concentrations at periodic intervals when they have not been measured (Porterfield 1972). These estimated points of concentration are then used as a basis from which to further interpolate to other concentrations. In making these estimates, the USGS uses curves relating past flow vs. concentration values from each monitoring station. These curves may be developed seasonally, depending upon seasonal variations in concentrations.

Further, concentrations are differentiated on the ascending vs. descending limbs of the hydrograph. For an event without any samples, or without enough samples to fully characterize the hydrograph, concentrations are estimated at periodic intervals along the hydrograph based upon the season of occurrence, peak flow, instantaneous flow rate, and ascending or descending limb of the hydrograph. The process is not entirely objective, and to a degree is dependent upon the interpreter's experience and skill.

#### Advantages

Integration's main advantage is that it is simple and straightforward, requiring less analytical effort than stratified random sampling, but more than composite analysis.

#### Disadvantages

Unfortunately, integration has a host of disadvantages. Most readily apparent is its high cost given the extensive number of analyses desired. Secondly, as samplers periodically

malfunction, estimates of concentration will periodically have to be used rather than actual samples. This in itself has a couple of drawbacks: estimates have some unknown disparity from actuality, and they are based on and limited by the availability of past data. Without a good history of water quality information, estimates of concentrations can only be tenuous at best. Therefore, integration, insofar as it relies on past sampling, is least appropriate for use at a new sampling station. Similarly, when a pollutant source (area or industry) has undergone some change that has affected the resultant water quality, an investigator cannot rely upon past data to make estimates of present water quality.

Lastly, integration loading estimates lack an error term. Even if all of the flows are extensively sampled, there is no way of knowing what error is associated with interpolating concentrations between points of measurement. Under very extensive sampling conditions, or relatively static concentrations, this error is likely small. With unmonitored or poorly monitored events, or with fluctuating concentrations, the error may be quite large. Without variance estimates, comparisons of any kind are highly problematic.

## COMPOSITE SAMPLING

### General Description

Composite sampling is probably the simplest and least costly method of estimating pollutant loads in flowing water. Simplicity and low cost stem from the ability to represent a large volume of water with only a single composited sample. Further, if the subsamples are collected frequently, composite sampling may give very precise results.

Composite sampling entails combining many discrete flow-weighted subsamples into one composite sample. Flow weighting means that either the frequency of subsample collection, or the size of the subsamples collected, is proportional to the flow. In this manner, concentrations at higher flows are effectively given proportionally more weight than are those at lower flows when combined in the composite sample. The composite sample represents the flow-weighted concentration of the entire flow. The concentration of the composite sample multiplied by the total flow then yields a total pollutant load for that flow.

For greatest ease, subsamples are collected flow proportionally, i.e., a subsample is collected every time a

predetermined volume of water has passed the sampling point (varied time, constant volume, or  $T_v V_c$ ). Samples are collected more frequently at higher flows and vice versa, so that each sample represents a unit amount of water. The subsamples may be deposited directly into a single large container, or into a number of separate bottles and subsequently combined. In either case, the sample must be reduced to a manageable size for analysis. The recently developed USGS Cone Splitter is extremely useful for this purpose.

Alternatively, subsamples may be collected at a uniform time interval (constant time, varied volume, or  $T_c V_v$ ). In this case, each subsample must be deposited into an individual bottle. Subsequently a volume of subsample, proportional to either the instantaneous flow at the time of subsampling or to the total volume of flow since the previous sample, can be extracted from each bottle and combined into the composite sample. Note that a simple composite using equal time increment subsampling and equal volume subsamples ( $T_c V_c$ ) will not yield accurate results under varying flows and concentrations.

Shelley and Kirkpatrick (1975) compared hypothetical composite sample concentrations obtained using the alternative sample collection schemes ( $T_v V_c$ ,  $T_c V_v$  and  $T_c V_c$ ) to the actual flow-weighted concentrations for various flow and concentration functions. When both flows and concentrations increased then decreased as a function of time ( $\sin \pi t$ ), the ratio of the composite sample concentration to the actual flow-weighted concentration was 0.97 for  $T_v V_c$ , 1.01 for  $T_c V_v$  with subsample volumes proportional to the instantaneous flow rate, 0.98 for  $T_c V_v$  with subsample volumes proportional to the total flow since the last sample, but only 0.80 for  $T_c V_c$ .

A major hurdle in establishing a composite sampling program is deciding what volume of flow should pass between points of subsampling (using flow-proportional sampling) or what time interval should be used (using equal time interval sampling). The answer to this question depends upon the nature of the flows and pollutant concentrations being sampled.

Using flow-proportional sampling, the maximum subsampling frequency (i.e., the smallest volume of water to pass between points of subsampling) is limited by either (1) the estimated maximum flow expected as the real time sampling frequency may be limited by the cycling time of the sampler, or (2) the estimated total volume of flow as it relates to either the total number of subsample bottles available or the total volume of the single com-

posite container (depending on the type of containers being used).

Using equal time interval sampling, the maximum subsampling frequency is limited by either (1) the cycling time of the sampler or, more likely, (2) the maximum duration of flow as it relates to the maximum number of subsample bottles available.

In either sampling scheme, the minimum sampling frequency (i.e., the maximum volume of water or time interval between points of subsampling) is dictated by the fluctuations in concentrations and the relative accuracy desired. The more variable the concentration, the more important it is to subsample frequently. The more frequently a flow is subsampled, the more accurate and representative will be the composite sample.

The inherent conflict with composite sampling lies between wanting to subsample more frequently for greater accuracy, while risking overextending the sampler cycling time or the sample bottle capacity, or sampling less frequently to ensure complete subsampling of larger events while sacrificing accuracy.

### Advantages

When flows are extensively subsampled, composite sampling can give a very accurate representation of the flows that are monitored. In NURP, the DNR will be using samplers that have both flow-proportional and equal time interval sampling capacity. Additionally, they have 24 one-liter bottles, each of which can hold from one one-liter subsample to ten 100-ml subsamples, or some variation in between. Therefore, using flow-proportional sampling, as many as 240 100-ml samples could be taken in a very large event. If the samplers were slightly modified to deposit into a single large container, the number of potential subsamples would only be limited by the size of the container.

Another advantage, of course, is composite sampling's low cost. Each event can be represented with only one lab analysis. This can present a huge cost saving compared to the extensive sampling for integration analyses.

Composite sampling also allows for excellent inter-event analysis, i.e., for comparison of pollutant loads between events at one station or between stations for one event, albeit again without the use of variance estimates.

Lastly, the personnel and/or computer time required to estimate pollutant loads using composite sampling are less than that required for integration or stratified random sampling.

## Disadvantages

Composite sampling is not without its disadvantages. One significant drawback involves difficulties associated with equipment malfunctions. One EPA study found that on the average, 16% of the potential water quality sampling data is lost due to equipment malfunctions (Harris and Keffer 1974). Since there is only one analysis per event, there may not be enough sampled events to make good statistical estimates about unsampled events. In NURP, the DNR is attempting to calculate seasonal total pollutant loads. Long-term averages indicate there will be about 15 events per season of concernable size. Should the situation arise, the error associated with estimating loads for two events (approximately 13%) based on only 13 numbers may be large.

Another problem relates to the inability to estimate the precision of the pollutant loading estimates of monitored events. Even though we assume, given a high frequency of subsampling, that a single analysis for each event is quite accurate, there is no way of placing confidence limits on either the loading estimates from the events that are sampled, or on the cumulative series of events as a whole.

Another disadvantage is that composite sampling, while it allows for excellent inter-event analysis, does not allow for intra-event analysis. There is no way of knowing how pollutant concentrations change over time within an event.

Lastly is the aforementioned problem relating to sample collection frequency. It is an issue that may not be readily resolved.

## STRATIFIED RANDOM SAMPLING

### General Description

Stratified random sampling is a statistically based sampling method that can be applied to estimating pollutant loads. Unlike integration and composite sampling, it can be used to estimate both a pollutant load or mean loading rate, as well as the reliability of the loading estimates. And with continuous flow monitoring, it enables the inclusion of unsampled events into these calculations while still generating replicable estimates.

While stratified random sampling has been used by investigators in many fields, it has seldom been used for estimating pollutant loads in flowing

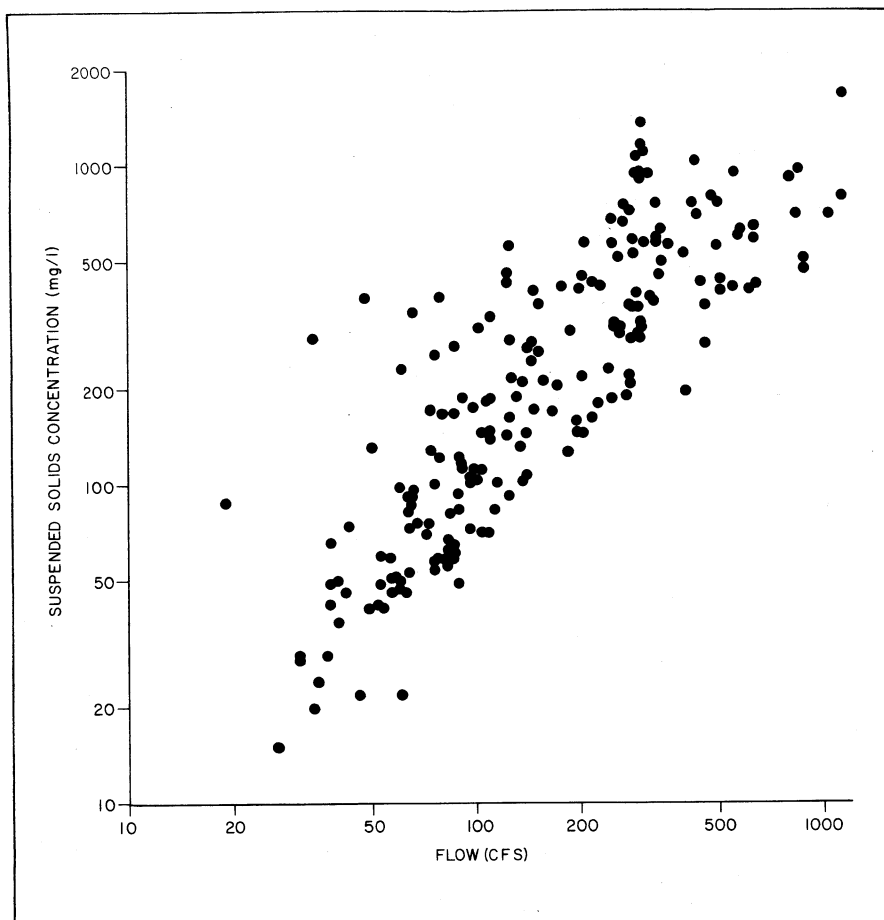


FIGURE 1. Suspended solids concentration vs. flow at 70th Street.

water. The application presented here was first proposed by John Clark, International Joint Commission (IJC) statistician (Whitt 1977). It was subsequently used for estimating pollutant loads from the Menomonee River and its tributaries in the Menomonee River Pilot Watershed Study (Bannerman et al. 1978). Clark's presentation is modeled after a discussion of stratified random sampling techniques by Cochran (1963). Clark's formal and somewhat lengthy term for it is "a stratified random sampling method enhanced with a ratio estimator". For ease of reference, it will simply be called stratified random sampling. The following paragraphs introduce a number of concepts inherent to stratified random sampling.

#### (1) Stratification

The purpose of stratification is to gain a better knowledge of a population by grouping it into subpopulations, or strata, of similar characteristics. By examining subpopulations as a unit, a better estimate of the population as a whole can be made.

A simple example will help illustrate this point. Suppose one wanted to estimate the average weight of 1,000 adults from a population of 650 women

and 350 men. Suppose further that only 100 people could be weighed. If the 100 people were considered as one sample population, the average weight would have relatively high variance associated with it. This same high variance would be reflected in the confidence placed on the estimate of the average weight of the entire population. If instead, the 100 people were considered as two separate sample populations of men and women, the average of each group would have considerably less variance. From a further knowledge of how many men and women were in the entire population, an estimate of the average weight of the population could be made with greater confidence in the accuracy of the estimate.

Similarly, the use of strata can reduce the variance associated with pollutant loading estimates. Division of a population into strata is done in a way that will minimize the concentration variation within each stratum while maximizing the variation between strata. To the extent that each stratum is adequately sampled and the samples are homogeneous, the estimates of the individual and combined strata will be both accurate and reliable.

Strata may be defined either prior

**TABLE 1.** Various strata and suspended solids loading estimates at 70th Street.

Number of Event Strata	Flow Cutoff (s) (cfs)	Suspended Solids Load (kg/ha)	95% Confidence Interval (+)	Mean Square Error
2	100	94.1	16.8	8.5
2	250	86.9	10.9	5.4
3	225, 700	83.6	8.6	4.2

**TABLE 2.** Notation for stratified random sampling equations.

Variable	
L	Mean pollutant load, L (kg), or loading rate $L_d$ (kg/day)
Q	Mean flow ( $M^3/day$ )
q	Mean flow at times of sampling, ( $M^3/day$ )
l	Mean flow-weighted loading rate (kg/day)
B	Bias correction factor
n	Number of samples
$X_i$	Discrete flow measurement (l/sec)
$C_i$	Discrete concentration measurement (mg/l)
$Y_i$	Discrete loading measurement, $X_i * C_i$ , (mg/sec)
MSE	Mean-square-error
N	Duration (days)
edf	Effective degrees of freedom of combined strata
CI	Confidence interval (+)
Subscript	
h*	Stratum identifier
d**	Daily estimate

\*If variables L, MSE, CI or N have an (h) subscript, they are an estimate (L, MSE, or CI) or a measurement (N) of the individual stratum value. Without the subscript, they are an estimate of the combined strata value.

\*\*If variables L, MSE or CI have a (d) subscript, they are daily estimates of stratum (h) or the combined strata. Without the subscript, they are estimates for the entire duration of stratum (h) or the combined strata.

to or after sampling, provided that there is a rational basis for their formation and that care is taken not to artificially deflate variance estimates. When extensive sampling is first begun at a particular station, careful exploratory analysis of the hydrograph, and possibly of seasonal effects, will be required to determine how concentrations group under different conditions. Once patterns are discerned, stratification should be possible prior to further sampling.

In the Menomonee River study it was assumed that concentrations would vary seasonally and with flow, so strata were defined prior to sampling into four quarterly seasons and each season into low flow and event flow. Since flow and concentration are generally correlated, the seasonal event stratum at each station was further subdivided, following sampling, into two or three strata delineated by flow cutoffs. These cutoffs were selected af-

ter observing a plot of flow vs. concentration for each seasonal event stratum. Again, the primary strata selection criterion was to maximize concentration variation between strata while minimizing it within each stratum. The cutoff process required some trial and error, since with each increase in the number of strata, there is a concomitant decrease in the number of samples per stratum, which can offset the reduced variance associated with a smaller range of concentrations.

For each stratum, the mean loading rate (kg/day), total loads (kg), and 95% confidence intervals were calculated. The seasonal event strata were combined to obtain seasonal event-related loads. They were further combined with the seasonal low flow strata to obtain seasonal total loads, and seasonal total loads were combined to obtain annual total loads.

To illustrate the process of selecting strata, an example of figurative data

and associated loading calculations is provided. These data are also used subsequently in this report. Figure 1 depicts the relationship of flow and suspended solids concentrations at the 70th Street monitoring site on the Menomonee River. These 209 concentrations were collected over 12 extensively sampled events in the summer of 1977. Stratified random sampling was employed to estimate the suspended solids loads associated with these events. Different flow cutoffs, or strata, were used to derive various loading estimates and associated error terms (Table 1). Based on the respective error terms, the best loading estimate is  $83.6 \pm 8.6$  kg/ha.

## (2) Flow-Weighted Loads and Ratio Estimator

The mean flow-weighted loading rate is a simple average of a number of discrete flow times concentration measurements. This rate or its counterpart forms the basis of integration, composite, and stratified random sampling analyses. With integration analysis, a concentration value, either measured or interpolated, is coupled with each flow measurement. In this manner the total loading is actually the sum of a series of flow-weighted loads. With composite sampling, concentrations are flow-weighted by taking either more, or larger subsamples when flows are higher. The estimated total load is simply the average composite concentration times the total flow. With either integration or composite sampling, the mean loading rate is the total load divided by the total time of flow.

With stratified random sampling, however, loading values are estimated from a relatively small number of concentration measurements coupled with a full knowledge of the flows. The mean flow-weighted loading rate is the average of a number of flow times concentration values. It is calculated independently for each stratum.

Since the average of the flows at the time the samples are taken (q) may be different than the overall mean flow (Q), the ratio estimator (Q/q) compensates for this difference. By multiplying the mean flow-weighted loading rate by the ratio estimator (and incorporating the bias correction factor), a better estimate of the true mean loading rate is derived. By multiplying this by the total duration of flow, an estimate of the total load is obtained. Further discussion of the use of ratio estimators is given by Cochran (1977), Kendall and Stewart (1968), and Tin (1965).

## (3) Bias

Simple ratio estimation is inherently biased. Much of the bias can be removed by adjusting the ratio (Q/q)



by an appropriate factor. The ratio estimator used in IJC work is from Beale (1962) and employs the sample estimate of the covariance between load and flow and the sample estimate of flow variance in the adjusting factor. This particular ratio estimator has been evaluated by extensive sampling experiments with real tributary mouth data sets and appears to perform well.

#### (4) Confidence Intervals

One of the primary assets of stratified random sampling is that, through an analysis of the variability of concentration, it enables the investigator to test hypotheses and to place confidence estimates on the calculated loading values. Knowledge of precision would seem critical in most water resource investigations, for example, estimating runoff loads from a watershed or in monitoring compliance with a pollution discharge permit. It is especially important in comparing pollution discharges from two sources, or from one source over time, or in evaluating the effectiveness of a management alternative. Of integration, composite, and stratified random sampling, only stratified random sampling enables investigators to place confidence limits on their estimates.

### Advantages

Stratified random sampling has two very strong assets. It enables an estimate of confidence limits to be placed upon the loading estimates, and it enables unsampled flows to be included in the loading estimates with some knowledge of the uncertainties involved. Knowledge of confidence, or reliability, is essential to most water resource investigations. Stratified random sampling enables confidence intervals to be placed upon the loading estimates, even when those estimates include loads from periods of unmonitored events.

In addition, stratified random sampling can probably give accurate loading estimates with considerably fewer samples than would normally be required with integration.

Lastly, stratified random sampling will give better estimates than other simple statistical calculations. Dolan et al. (1981) found that under biweekly sampling conditions, stratified random sampling gave better annual total phosphorus load estimates than either calculations of loads based on the means of concentrations over time, or calculations of loads based on the use of log-linear regression to estimate concentrations over time.

### Disadvantages

Stratified random sampling is not without its disadvantages. If the number of samples is larger than would be required for composite analyses, it would be proportionately more expensive.

Another problem relates to choosing the appropriate number of samples. If the required number of samples is underestimated, it will result in larger error terms than were desired. If the number of samples is overestimated, it will result in greater analytical costs than were necessary. Where the variations in pollutant concentration are more predictable, this is less of a concern.

### Computations for Estimating Pollutant Loads and Confidence Intervals

The Department of Natural Resources has developed a computer program, written in Fortran for a UNIVAC system, that makes the computations for estimating pollutant loads. The program accepts inputs of time, flow, concentration, and strata delineation, and outputs individual stratum and total daily loading estimates, standard errors, and 95% confidence intervals. This program is nonproprietary, and is available for a nominal charge to offset storage, handling and documentation expenses. However, export to a non-UNIVAC system would necessitate extensive modification. (A more readily exportable Pascal program is planned for 1983.) For further information regarding acquisition of the program and accompanying documentation, contact: Chief, Nonpoint Source Section, Bureau of Water Resources Management, Wisconsin Department of Natural Resources, P.O. Box 7921, Madison, WI 53707.

Although computations by hand are quite tedious, the method is presented below for reference purposes, with equations (A-N) shown on the facing page.

1. Calculating daily loading rates for each stratum (see Table 2 for explanation of symbols).

The formula for calculating the mean daily pollutant loading rate for each stratum (h) is: A. And where, assuming the population is very large (here it is the number of potential points of sampling), the bias-correction factor for each stratum (h) is: B.

The mean-square-error for the daily loading rate of each stratum may be

expressed to terms of the order  $n_h^{-2}$  by the equation: C.

This equation assumes that the population is very large, i.e., that there are a great many moments which could be sampled.

Confidence intervals for the stratum estimated daily loading rate may be found in the usual way, using the square root of the mean-square-error and the desired t value from the tabulated t-distribution: D.

2. Calculating total loads for each stratum.

The estimated total load associated with each stratum is the estimated daily loading rate ( $L_{hd}$ ) multiplied by the total amount of time that flows were within that stratum ( $N_h$ ): E.

The estimated total mean-square-error of each stratum ( $MSE_h$ ) is the estimated daily mean-square-error ( $MSE_{hd}$ ) multiplied by the square of  $N_h$ : F.

The  $\pm$  confidence interval for the estimated total load of each stratum is: G.

3. Calculating daily loading rates for all strata combined.

Following calculations for each stratum, the combined total load, mean-square-error, effective degrees of freedom and corresponding confidence intervals for the pooled strata can be made.

The estimated total daily loading rate for combined strata is: H.

The estimated total daily mean-square-error of the combined strata is: I.

The effective degrees of freedom for the combined strata is: J.

The  $\pm$  confidence interval for the estimated daily loading rate of the combined strata is: K.

4. Calculating total loads for the combined strata.

The estimated total load for the combined strata is: L.

The estimated total mean-square-error of the combined strata is: M.

And finally, the  $\pm$  confidence interval for the estimated total load is: N.

### Evaluation

Stratified random sampling is a relatively new approach to estimating pollutant loads. It is, perhaps, not as intuitively obvious or straightforward as integration or composite sampling. To objectively evaluate the stratified random sampling method, better understand the limits of its application, and facilitate its general acceptance, a series of tests were designed and implemented.

**A**  $L_{hd} = \frac{q_h}{q_h} * 1_h * B_h$   
 where  $1_h = \frac{\sum Y_{hf}}{n_h}$

**B**

$$B_h = \frac{1 + \frac{1}{n_h} * \frac{S_{xy_h}}{q_h 1_h}}{1 + \frac{1}{n_h} * \frac{S_{x_h^2}}{q_h^2}}$$

and where  $S_{xy_h} = \sum X_{hf} Y_{hf} - n_h * q_h * 1_h$  and  $S_{x_h^2} = \frac{\sum Y_{hf}^2 - n_h * q_h^2}{n_h - 1}$

$$MSE_{hd} = 1_h^2 * \left[ \frac{1}{n_h} * \left( \frac{S_{x_h^2}}{q_h^2} + \frac{S_{y_h^2}}{1_h^2} - \frac{S_{xy_h}}{q_h 1_h} \right) + \frac{1}{n_h^2} * \left( 2 * \frac{S_{x_h^4}}{q_h^4} - \frac{4 S_{x_h^2}}{q_h^2} * \frac{S_{xy_h}}{q_h 1_h} + \frac{S_{xy_h}^2}{q_h^2 1_h^2} + \frac{S_{x_h^2}}{q_h^2} * \frac{S_{y_h^2}}{1_h^2} \right) \right]$$

**C**

where  $S_{y_h^2} = \frac{\sum Y_{hf}^2 - n_h * 1_h^2}{n_h - 1}$

**D**  $CI_{hd} = \sqrt{MSE_{hd}} * t(a/2, n_h - 1)$

**E**  $L_h = L_{hd} * N_h$

**F**  $MSE_h = MSE_{hd} * N_h^2$

**G**  $CI_h = \sqrt{MSE_h} * t(a/2, n_h - 1)$

**H**  $L_d = \sum \left( L_{hd} * \frac{N_h}{N} \right)$

**I**  $MSE_d = \sum \left( \frac{N_h^2 * MSE_{hd}}{N^2} \right)$

**J**  $edf = \frac{(N_h^2 * MSE_d)^2}{\sum \frac{N_h^4 * MSE_{hd}}{n_h - 1}}$

**K**  $CI_d = \sqrt{MSE_d} * t(a/2, edf)$

**L**  $L = L_d * N = \sum (L_{hd} * N_h)$

**M**  $MSE = MSE_d * N^2 = \sum (N_h^2 * MSE_{hd})$

**N**  $CI = \sqrt{MSE} * t(a/2, edf)$

**TABLE 3. Seasonal loading values: integration vs. stratified random sampling.**

Station Name	Station Size (ha)	Number of Events	Total Days of Event Flow	Total Runoff (M <sup>3</sup> /ha)	Number of Samples	Susp. Solids Load (kg/ha) (Based on Integration)	Susp. Solids Load (kg/ha) (Based on Stratified Random Sampling)	95% C.I. (Based on Stratified Random Sampling)
Noyes Creek	550	6	8.29	5,581	70	243	230	104
Honey Creek	2,800	10	9.21	330	172	155	158	24
70th Street	32,200	12	22.62	216	209	68	84	9

For these analyses, data from the Menomonee River study were made available. During this study, runoff from 17 primarily urban watersheds, ranging in size from 64 to 34,400 ha, was continuously recorded and extensively sampled over a three-year period. This sampling program generated a data base with hundreds of samples at each station. Because the summer of 1977 was an extremely wet season, records from that period were selected for analysis. All of the tests performed used suspended solids data to calculate suspended solids loads.

The first task was to test stratified random sampling under ideal conditions, i.e., over a series of flows, all of which were very well sampled. This test required comparing the loading estimates obtained with stratified random sampling to a known quantity. This involved the concept of a "perfectly known season", i.e. one in which all of the events were very well sampled. By using only extremely well-sampled events, loads could be calculated using both integration and stratified random sampling. Assuming that integration would give accurate results under these conditions, stratified random sampling estimates could then be compared to the integration estimates.

For this first test, three stations were selected that were very heavily sampled. After an examination of the summer of 1977 hydrographs from these stations, those events that were not extremely well sampled were discarded from further consideration. After selecting the "perfectly known season" for each station, a staff person from the U.S. Geological Survey calculated the associated loads using integration. The same flows and concentrations were likewise used for stratified random sampling calculations. The results of this test are given in Table 3. For two stations, the respective estimates are nearly identical. At 70th Street they are quite close. The confidence limits are high at Noyes Creek where one storm, yielding approximately one-third of the total water load, generated 85% of the total pollutant load. Concentrations in this

**TABLE 4. Seasonal loading values using samples from half the events at 70th Street.**

Events With Samples Included in Test	Number of Samples	Est. Susp. Solids Loading (kg/ha)	95% C.I. (+)
All events (1-12)	209	84	9
2,4,6,7,11,12	68	66	7
1,4,6,8,10,12	121	80	12
2,3,4,6,9,12	100	83	10
2,5,6,7,9,12	66	58	8
3,4,6,9,10,11	86	88	11
3,6,8,9,10,11	97	88	11

**TABLE 5. Seasonal loading values using samples from half of the events at Honey Creek.**

Events Included in Test	Number of Samples	Est. Susp. Solids Loading (kg/ha)	95% C.I. (+)
All events (1-10)	170	158	24
1,5,7,8,9	58	160	31
1,2,7,8,10	73	196	45
4,5,7,8,10	70	160	30
1,2,3,4,6	114	164	34
1,2,3,8,9	88	210	36
2,5,6,8,10	103	162	32

**TABLE 6. Seasonal loading values using a reduced number of samples from all of the events.**

Number of Samples	Est. Susp. Solids Loading (kg/ha)	95% C.I. (+)
<b>70th Street</b>		
209*	84	9
59	87	15
45	83	17
28	86	36
<b>Honey Creek</b>		
172*	158	24
59	140	34
45	146	41
29	137	46

\*Original number.

storm ranged from less than 500 mg/l to more than 3,000 mg/l, and the associated error term reflects this variability. This first test demonstrates that stratified random sampling gives good loading estimates under extensive sampling conditions.

A second test involved eliminating the samples from one-half of the events while still including the flows from those events in the analysis. Because of the anomalies at Noyes Creek, this station was not included in this phase of the evaluation. The events at

each of the other two stations were numbered sequentially, and with each repeat of the test, those events with samples to be included in the analysis were selected at random. The tests were repeated six times. Results of these tests are given in Tables 4 and 5. Even with half of the events not sampled, stratified random sampling usually gave estimates very similar to those using samples from all of the events.

A third test involved randomly reducing the number of samples across all of the events. The results of these tests are given in Table 6. While the loading estimates are now still well within the 95% confidence intervals of the original loading estimates, the confidence intervals associated with the new loading estimates rose appreciably as the number of samples was reduced.

Two modifications of the second and third tests were again run on the 70th Street data, only this time three events with a combined total of only 13 samples were excluded from further analysis. This left nine events with 196 samples, with an associated loading estimate of  $62.4 \pm 7.6$  kg/ha. The estimated load from the USGS integration calculations was 60 kg/ha. Interestingly, it had been the three least well-sampled events that had caused the original disparity between the integration and stratified random sampling figures (68 vs.  $84 \pm 9$  kg/ha, respectively).

In the first iteration, the second and third tests were combined to examine the effects of a reduced number of samples from a reduced number of events. For the first such test, six events were randomly selected from the nine total events on six different occasions. Then 36 samples were randomly selected out of each group of six events. The six runs, each using 36 samples from six events to extrapolate to all nine events, yielded an average loading estimate of  $62.1 \pm 13.3$  kg/ha (Table 7).

**TABLE 7.** Seasonal loading values using a reduced number of samples from reduced number of events, 70th Street.\*

Events Included	Est. Susp. Solids Loading (kg/ha)	95% C.I. (+)
<b>36 Samples From 6 Events</b>		
1,2,4,6,8,9	59.6	18.6
2,3,4,5,7,9	59.3	11.9
1,2,3,5,7,8	55.7	8.9
1,2,3,5,6,9	66.3	12.8
1,3,4,5,6,9	65.5	12.5
3,4,5,6,7,9	66.3	14.9
Average	62.1	Average 13.3
<b>18 Samples From 3 Events</b>		
5,7,9	59.8	19.9
3,4,6	65.2	16.9
1,2,9	66.2	27.4
1,3,8	61.1	16.8
4,5,7	48.9	12.9
1,4,5	51.9	17.9
Average	58.8	Average 18.6

\*The estimated suspended solids loading using 196 samples from all nine events is  $62.4 \pm 7.6$  kg/ha.

**TABLE 8.** Seasonal loading values using a reduced number of samples from nine events, 70th Street.

Number of Samples	Est. Susp. Solids Loading (kg/ha)	95% C.I. (+)
196 (all samples)	62.4	7.6
54	62.7	12.3
45	62.7	13.5
36	55.7	13.2
27	55.9	12.2
18	58.0	13.4

**TABLE 9.** Seasonal loading values from two small urban watersheds.

Station Name	Station Size (ha)	Number of Events	Total Days of Event Flow	Total Runoff ( $M^3$ /ha)	Number of Samples	Calculated Susp. Solids Loads Using Stratified Random Sampling (kg/ha)	95% C.I. (+)
Stadium Interchange	64	11	.69	391	57	197.0	56.5
Brookfield Square	61	7	.61	250	27	36.1	17.0

For the second such test, three events were randomly selected out of the total nine events on six different occasions. Then 18 samples were randomly selected out of the three events. The six runs, each using 18 samples from three events to extrapolate to all nine events, yielded an average loading estimate of  $58.8 \pm 18.6$  kg/ha (Table 7).

Another test was run using a reduced number of samples randomly taken from all nine events. The repeated runs used 54, 45, 36, 27 and 18 samples, respectively. The greatest deviation from the original estimate of  $62.4 \pm 7.6$  kg/ha (using 196 samples) was  $55.7 \pm 13.2$  kg/ha (using 36 samples) (Table 8).

As demonstrated by these analyses, stratified random sampling is able to generate accurate loading estimates based upon a relatively limited number of samples. However, the watersheds used in this evaluation are considerably larger (2,800 and 32,200 ha) than those watersheds that were going to be monitored in the Nationwide Urban Runoff Project, which are typically around 50 ha. Following these analyses, the tests were extended to include two smaller urban watersheds of 61 and 64 ha. Because of their size, these watersheds were more appropriate for the experimental design of the urban runoff project. However, since they had not been monitored as extensively as the larger watersheds, their data base

was considerably more limited.

Again, events from the summer of 1977 were selected for analysis. Because of their small size, their extensive impervious areas coupled with efficient curb and gutter drainage, and because of the short intensive nature of midwestern summer showers, event runoff at these two stations typically lasted only an hour or two. As with the earlier analyses, only those events that were extensively monitored were included in the evaluation, which at one station included 11 events with 57 samples over a total runoff time of 16.5 hours, and at the other station included 7 events with 27 samples over a total of 14.5 hours (Table 9).

After calculating the respective loads, the associated confidence intervals were quite high, equalling 28 and 48% of the respective loading estimates. These high confidence intervals resulted from the widely ranging concentrations found over the entire range of flows.

In an attempt to reduce these error terms, another basis for stratification was sought. One possibility was to stratify prior to and following the peak flow, as concentrations are usually higher in the early stages of an event. But, where that seemed to work reasonably well on an event-by-event basis, when events of all different magnitudes were combined the pre-peak and post-peak stratum each had a wide range of concentrations. Again, that

yielded a high confidence interval.

Another possibility was to stratify as a function of peak flow, i.e., each event would be subdivided into respective strata using cutoff values that were a predetermined percentage of the peak flow. Again, on an individual event basis it seemed to work reasonably well, but when a number of events of varying peak flows were combined there was a wide range of concentrations in each stratum. This did not yield better confidence estimates than did stratification based simply on flow.

There appears no better means of stratifying these events to reduce the error terms. The alternative, of course, is to increase the number of samples. While the error terms here are high, they are at least known. Using integration, the associated error would not only be similarly high, but would be unknown. Using composite sampling, because of the ability to subsample more frequently, the error term would likely be smaller, but again would be unknown.

The results of these analyses demonstrate that stratified random sampling gives accurate loading estimates even when many of the events are not sampled, and it gives error terms that reflect the variability in the concentrations, which gives the investigator an indication of the reliability of the estimate.

## CONCLUSIONS AND RECOMMENDATIONS

### INTEGRATION

Where integration works well, it does so only because the flows are extensively sampled and hence it is very expensive. It does not enable an error term to be placed on the loading estimates, and it has limited applicability at new stations. Integration costs more money than composite sampling and yields a less useful product than stratified random sampling.

### COMPOSITE SAMPLING

When events are well sampled, composite sampling is inexpensive and,

depending on the variability of concentrations, may be very accurate. Further, it also allows for excellent inter-event analyses. The major drawback is a lack of an error term. The true error is a function of the variability of concentrations and of the number of samples taken. While one can increase the number of samples to improve the precision of the loading estimate, the variability among the samples, and the actual precision of the estimate, remain lost in the composite sample.

Composite sampling should possibly be used under two conditions: when inter-event analyses are critical or when limited by a tight analytical budget. It should not be used when knowledge of confidence intervals associated with the loading estimates are required.

### STRATIFIED RANDOM SAMPLING

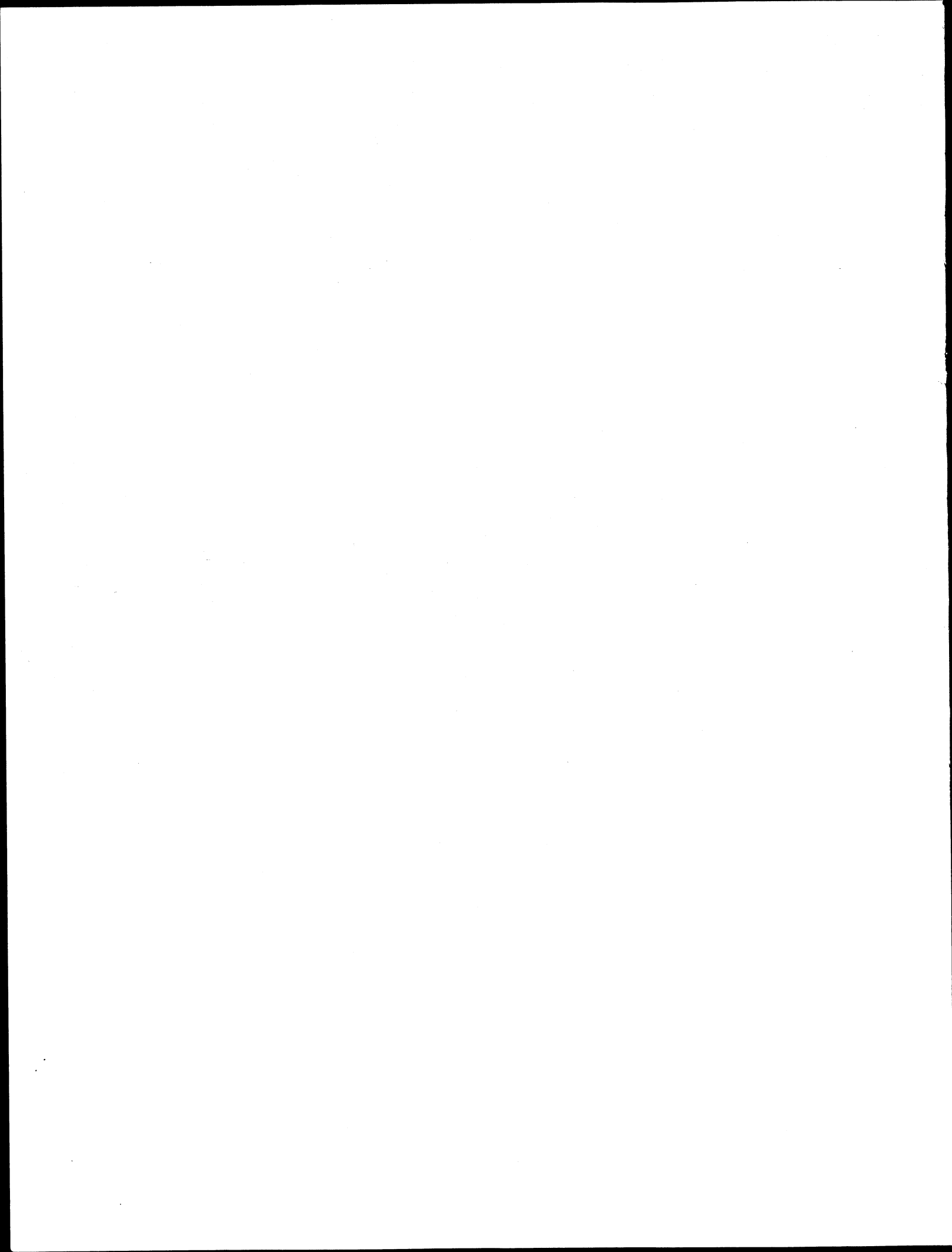
Stratified random sampling is capable of accurately estimating pollutant loads. Depending on the variability of concentrations and on the number of samples taken, confidence intervals on the loading estimate can range from narrow to wide, but an estimate of the error of measurement will be determined. Stratified random sampling also allows for the inclusion of unmonitored events into the total calculations in an objective, replicable manner. Lastly, it can frequently allow for a relatively small number of samples while still generating reasonable confidence intervals.

Because of these important assets, stratified random sampling could and should be used frequently in water resource monitoring. It should probably be used whenever inter-event analyses are not essential, and when the sam-

pling budget does not restrict the analysis to composite sampling. Stratified random sampling has many advantages that promise to make significant contributions to water resource investigations.

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