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## TRANSACTIONS

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# ON THE ERRORS WITH WHICH LOGARITHMIC COMPUTATIONS ARE AFFECTED.* 

By Carl Bremiker, Pi. D.

## INTRODUCTION.

The original of the following dissertation on the errors of logarithmic computation appeared in 1852 as an introduction to the first edition of Bremiker's six place tables. The great excellence of the paper has led many students of mathematics to read it, in spite of the fact that the edition of Bremiker's' tables which contains it is quite inaccessible, and although the difficulties are further increased because the original is in Latin.

Mr. P. E. Doudna wrote out a translation of the paper in 1894-95, at which time he was holding a fellowship in applied mathematics in the University of Wisconsin. Mr. Doudna's failing health required him to remove to Colorado, where he died in February, 1900. Mr. E. F. Chandler, who was fellow in applied mathematics in the University of Wisconsin in 1898-99, wrote out, at my suggestion, an independent translation of the paper. He was furnished with a copy of Mr. Doudna's translation in order to verify his work. We deeply regret that Mr. Doudna was not able to go over the present translation before its publication, as his judgment would have been of great value. It has seemed right, however, that the translation should appear under their joint names.

[^0]Mr. Chandler has recomputed all of the tables and illustrative numerical examples given by Bremiker. A few corrections are indicated in the foot notes. No changes have been made at any point except the correction of palpable misprints and the insertion of the numbers attached to the principal equations and formulas to facilitate reference.

Chas. S. Silichter.
Madison, April, 1901.

## CONCERNING THE ERRORS BY WHICH EOGARITHMIC COMPUTATIONS ARE AFFECTED.

## § 1.

Since the exact value of any logarithm is in general an incommensurable number, which can not be expressed except by an infinite number of decimal figures, and since in the logarithmic tables only the first decimal figures are given, which we use in computation instead of the exact value, it is evident that a result obtained by the use of logarithms is affected by a greater or less error. But whenever we are willing to use in the computation more decimal places than are called for by the accuracy of our data, this error arising from the inaccuracy of the logarithms may be disregarded, in comparison with that which arises from the inaccuracy of the data. If, however, in order to save useless labor, only as many decimal figures be used in the computation as are called for by the accuracy of the data; then it will be proper before the computation is commenced to consider the theory of the errors which can arise from the omitted decimals. The discussion of this theory which is attempted below will show whether five, six, or seven decimals ought to be used in the computation.

## § 2.

First, assume that the errors of all the logarithms used in the computation (i. e. their true values) are known. Then by the aid of the differential calculus the error of their resulting sum can easily be found. For this it is sufficient to use only the first differentials, since, in comparison with the true values of the logarithms, the errors can be regarded as infinitesimals, whose higher powers are of no weight in the computation. In this computation the following equations may be used:

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$$
\begin{align*}
d(\log x) & =\frac{d x}{x} \\
d(\log \sin x) & =\cot x \cdot d x \\
d(\log \cos x) & =-\tan x \cdot d x  \tag{1}\\
d(\log \tan x) & =\frac{2}{\sin 2 x} \cdot d x \\
d(\log \cot x) & =-\frac{2}{\sin 2 x} \cdot d x
\end{align*}
$$

Furthermore, the manner in which the errors of the number $a$ and of the arc $A$ are connected with the errors of the Briggs logarithms is shown by the following equations:

$$
\left.\begin{array}{rl}
f(\log a) & =\frac{m}{a} f(\alpha)+f^{\prime} \\
f(\log \sin A) & =\frac{m}{r} \cot A f(A)+f^{\prime} \\
f(\log \cos A) & =-\frac{m}{r} \tan A f(A)+f^{\prime} \\
f(\log \tan A) & =\frac{2 m}{r \sin 2 A} f(A)+f^{\prime} \\
f(\log \cot A) & =-\frac{2 m}{r \sin 2 A} f(A)+f^{\prime} \\
f(a)=\frac{a}{m} \cdot\left\{f(\log a)-f^{\prime}\right. \\
f(A) & =\frac{r}{m} \tan A \cdot\left\{f(\log \sin A)-f^{\prime}\right\} \\
& =-\frac{r}{m} \cot A \cdot\left\{f(\log \cos A)-f^{\prime}\right\} \\
= & \frac{r}{2 m} \sin 2 A \cdot\left\{f(\log \tan A)-f^{\prime}\right\} \\
= & \frac{r}{2 m} \sin 2 A \cdot\left\{f(\log \cot A)-f^{\prime}\right\}
\end{array}\right\}
$$

where an $f$ indicates the error accumulated in computation and transferred from the logarithm to the number or arc, or from the number or are to the logarithm, and $f^{\prime}$ denotes the error of a logarithm taken from the table; $m=0.43429$ is the modulus of the Briggs system and $r=206265$ is the length of the radius expressed in seconds of arc.

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These equations, if applied to any logarithmic computation whatsoever, show the error of the final result as a linear function of theparate errors. To illustrate this, take as an example the equation

$$
\sin ^{2} \frac{1}{2} c=\sin ^{2} \frac{1}{2}(a-b)+\sin a \sin b \sin ^{2} \frac{1}{2} C
$$

which expresses the third side $c$ of a spherical triangle in terms of the given sides $a$ and $b$ and the included angle $C$. First compute

$$
\cot \mu=\frac{\sin \frac{1}{2} C \sqrt{\sin a \sin b}}{\sin \frac{1}{2}(a-b)},
$$

then

$$
\sin \frac{1}{2} c=\frac{\sin \frac{1}{2} C \sqrt{\sin a \sin b}}{\cos \mu}
$$

Then if the errors of the logarithms of the table are arranged in order,

| in $\sin \frac{1}{2} C$ | the error $f_{1}$ |
| :---: | :---: |
| " $\sin a$ | " $f_{2}$ |
| " $\sin b$ | " |
| " $\sin \frac{1}{2}(a-b)$ | " " $f^{\prime}$ |
| $\cdots \cot \mu$ | " " $f$ |
| " $\cos \mu$ | " " |
| c $\sin \frac{1}{8} c$ | " " |

the error in the sum from which $\cot \mu$ is determined will be

$$
f_{1}+\frac{1}{2} f_{2}+\frac{1}{2} f_{3}-f_{4}
$$

whence there arises an error
in $\mu \quad$ equal to $-\frac{r}{2 m} \sin 2 \mu\left(f_{1}+\frac{1}{2} f_{2}+\frac{1}{2} f_{3}-f_{4}-f_{3}\right)$

$$
\begin{aligned}
& \text { in } \log \cos \mu \text { " " } \sin ^{2} \mu\left(f_{1}+\frac{1}{2} f_{2}+\frac{1}{2} f_{3}-f_{4}-f_{6}\right)+f_{0} \\
& \text { in } \log \sin j_{0} \text { " " } f_{1}+\frac{1}{2} f_{2}+\frac{1}{2} f_{3}-\sin ^{2} \mu\left(f_{1}+\frac{1}{2} f_{2}+\frac{1}{2} f_{3}-f_{4}-f_{6}\right)-f_{6} \\
& =\cos ^{2} \mu\left(f_{1}+\frac{1}{2} f_{3}+\frac{1}{2} f_{3}\right)+\sin ^{2} \mu\left(f_{4}+f_{6}\right)-f_{0}
\end{aligned}
$$

and in cequal to $\frac{2 r}{m} \tan \frac{1}{2} c\left\{\cos ^{2} \mu\left(f_{1}+\frac{1}{2} f_{2}+\frac{1}{2} f_{3}\right)+\sin ^{3} \mu\left(f_{4}+f_{6}\right)-f_{6}-f_{7}\right\}$

## § 3.

Thus the error of computation takes the form $=$

$$
\alpha_{1} f_{1}+\alpha_{3} f_{2}+\alpha_{3} f_{3}+\ldots .+\alpha_{v} f_{v}
$$

in which the coefficients $\alpha$ are known, but in which only the limits of the quantities $f_{1}, f_{2}$, etc., are determined.

Assume each $f$ to have this limiting value, and add the products

$$
\alpha_{1} f_{1}+\alpha_{2} f_{2}+\alpha_{3} f_{3}+\ldots .
$$

the signs + or - being disregarded, and the result is the greatest possible error

$$
=\Sigma(\alpha f) ;
$$

the extreme limits within which the error of the computation must necessarily be included are

$$
-\Sigma(\alpha f) \text { and }+\Sigma(\alpha f) .
$$

But the error will never reach these extreme limits, since each error included within the expression frequently diminishes to an infinitesimal, so that the maximum error as given above is not a suitable test by which to measure the accuracy of the computation. If for each error lying within these limits a proportional number could be determined, which would exhibit the relation that holds between the number of errors of that magnitude and the total number of errors of every magnitude, or which shows how many errors of a certain magnitude there would be among all the errors possible, we should have enough proportionals to test the accuracy of any formula.

Such numbers may be found if first we give to each $f$ all the different values that are possible, then substitute in the sum

$$
\alpha_{1} f_{1}+\alpha_{2} f_{2}+\alpha_{3} f_{3}+\ldots+\alpha_{v} f_{v}
$$

all the combinations of the different quantities; in this way, we may observe how often the sum will equal zero, and how often it will be equal to any other given number within its extreme limits.

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## § 4.

If $\gamma$ denote the extreme limit of the error by which the logarithms of a given table can be affected, all the values which $\boldsymbol{f}$ can have are shown by the series

$$
-\frac{n}{n} \gamma,-\frac{n-1}{n} \gamma,-\frac{n-2}{n} \gamma, \ldots . \quad 0 \ldots+\frac{n-1}{n} \gamma,+\frac{n}{n} \gamma
$$

in which $n$ is an infinite number. The total number of these values is $2 n+1$. And it is a necessary assumption that each one of these values will occur once, and only once, since there is no cause at work which will give predominance to one value rather than another.

Then if we form a series of the values of $\alpha_{1} f_{1}$, this series, which is merely the previous one multiplied by $\alpha_{1}$, will be

$$
-\frac{n}{n} \alpha_{1} \gamma,-\frac{n-1}{n} \alpha_{1} \gamma,-\frac{n-2}{n} \alpha_{1} \gamma \ldots . \quad 0 \ldots+\frac{n-1}{n} \alpha_{1} \gamma,+\frac{n}{n} \alpha_{1} \gamma .
$$

We shall have a similar series for each $\alpha f$; the number of terms is the same in all, but the intervals are not the same. In place of these, and for the sake of simplicity, let the various series be so written that the terms have a common interval. It is evident that this is legitimate, since it is merely so distributing the total interval between $-\alpha_{1} \gamma$ and $+\alpha_{1} \gamma$ that the number of small intervals is changed. Thus instead of $\alpha_{1} f_{1}, \alpha_{2} f_{2}, \ldots$. we write the series

$$
\begin{array}{cc}
-\frac{n \alpha_{1}}{n} \gamma,-\frac{n \alpha_{1}-1}{n} \gamma \ldots & 0 \ldots+\frac{n \alpha_{1}-1}{n} \gamma,+\frac{n \alpha_{1}}{n} \gamma \\
-\frac{n \alpha_{2}}{n} \gamma,-\frac{n \alpha_{2}-1}{n} \gamma \ldots & 0 \ldots+\frac{n \alpha_{2}-1}{n} \gamma,+\frac{n \alpha_{2}}{n} \gamma \\
\ldots \ldots & \ldots . \\
-\frac{n \alpha_{v}}{n} \gamma,-\frac{n \alpha_{\nu}-1}{n} \gamma \ldots & 0 \ldots+\frac{n \alpha_{\nu}-1}{n} \gamma,+\frac{n \alpha_{v}}{n} \gamma
\end{array}
$$

in which the difference between the terms is everywhere the same, namely, $\frac{\gamma}{n}$, but the number of terms is respectively

$$
2 n \alpha_{1}+1, \quad 2 n \alpha_{2}+1 \ldots 2 n \alpha_{V}+1
$$

Furthermore, since it is required to find the number of combinations which, taken from the terms of the series, have a sum $p$, we form the series
and multiply these together. The coefficient of the term $x^{p}$ will be the required number.

To facilitate the multiplication, let us first divide the series by

$$
x^{-\alpha_{1} \gamma}, x^{-\alpha_{2} \gamma} \ldots x^{-\alpha_{\gamma} \gamma}
$$

respectively, and put

$$
x^{\frac{\gamma}{n}}=z, \quad 2 n \alpha_{1}+1=u_{1}, \quad 2 n \alpha_{2}+1=u_{2} \ldots . \quad 2 n \alpha_{\nu}+1=u_{\nu}
$$

Thus, (having thrown out the factors

$$
\left.x^{-\alpha_{1} \gamma}, x^{-\alpha_{2} \gamma}, \text { etc., }\right)
$$

there results the series

$$
\begin{gathered}
1+z+z^{2}+\ldots+z^{u_{1}-1} \\
1+z+z^{2}+\ldots+z^{u_{2}-1} \\
\ldots \\
1+z+z^{2}+\ldots+z^{u_{v}-1}
\end{gathered}
$$

in place of which can be written the expressions

$$
\frac{1-z^{u_{1}}}{1-z}, \frac{1-z^{u_{z}}}{1-z}, \quad \cdots \cdot \frac{1-z^{u_{v}}}{1-z}
$$

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The product of these

$$
\frac{\left(1-z^{u_{1}}\right)\left(1-z^{u_{2}}\right) \ldots\left(1-z^{u} v\right)}{(1-z)^{v}}
$$

is to be changed into a series of powers of $z$. 'And so instead of the divisor

$$
(1-z)^{v}
$$

we assume the series of ascending powers of $\boldsymbol{z}_{\text {, }}$,

$$
1+A_{1} z+A_{2} z^{2}+\ldots . A_{t} z^{t}+\ldots
$$

of which the general term is

$$
A_{t} z t=\frac{\nu(\nu+1)(\nu+2) \ldots(\nu+t-1)}{1.2 .3} \ldots{ }^{2} z_{0} .
$$

In place of the numerator, if we multiply its factors, we have the series

$$
\begin{aligned}
& 1-\left(z^{u_{1}}+z^{u_{2}}+\ldots+z^{u_{v}}\right)+\left(z^{u_{1}+u_{2}}+z^{u_{1}+u_{3}}+\ldots+z^{u_{v-1}+u_{v}}\right) \\
& -\left(z^{u_{1}+u_{2}+u_{3}}+z^{u_{1}+u_{2}+u_{4}}+\ldots+z^{\left.u_{v-2}+u_{v-1}+u_{v}\right)}\right. \\
& +\ldots \ldots \ldots+(-1)^{\nu} z^{u_{1}+u_{2}+\ldots+u_{v}}
\end{aligned}
$$

and if this series be multiplied by the series for

$$
(1-z)^{-v}
$$

the coefficient of the term $z^{t}$ will be

$$
\begin{aligned}
& A_{t}-\left(A_{t-u_{1}}+A_{t-u_{2}}+\ldots .+A_{t-u_{V}}\right)+A_{t-u_{1}-u_{2}}+A_{t-u_{1}-u_{3}} \\
& \quad+\ldots \ldots+A_{t-u_{v-1}-u_{V}-\left(A_{t-u_{1}-u_{2}-u_{3}}+\ldots .\right)+\ldots}
\end{aligned}
$$

In this expression we are to retain only those terms which have positive indices.

The binomial coefficient

$$
A_{t}=(\nu+t-1)_{t}=(\nu+t-1)_{\nu-1}
$$

in this last form of the numerator arises from a finite number $(v-1)$ of factors, of which the first is $v+t-1$, the last $t+1$. Furthermore, since it must be assumed that all the exponents $t, u_{1}, u_{2}$, . . . finally become infinite, we may write instead of

$$
(\nu-\vdash t-1)_{\nu-1}
$$

the expression

$$
\frac{t^{\nu-1}}{(\nu-1)!},
$$

where $(\nu-1)$ ! is the product $1.2 .3 \ldots(\nu-1)$. This change is merely the neglect of the lower powers of $t$ in comparison with higher. At the same time the coefficients

$$
A_{t-u_{1}}, A_{t-u_{2}}
$$

etc., are changed in value. Thus if $s \mu u$ denotes the sum of $\mu$ elements of the series $u_{1}, u_{2}, u_{3} \ldots u_{v}$ the coefficient of the term $z^{t}$ wlll be

$$
\frac{1}{(\nu-1)!}\left\{t^{\nu-1}-\Sigma\left(t-s_{1} u\right)^{v-1}+\Sigma\left(t-s_{2} u\right)^{\nu-1}-\ldots\right\}
$$

in which $\Sigma$ signifies the sum of all the similar values, and all the sums $s_{\mu} \iota$ which are greater than $t$ are omitted.

In order that the coefficient of the term $x^{p}$ may be found, its value, expressed as a function of $x$, must be substituted for $z$, and also the multiplication by the factors neglected above,

$$
x^{-\alpha_{1} \gamma}, x^{-\alpha_{2} \gamma}, \ldots x^{-\alpha_{\nu} \gamma},
$$

must be performed, so that the power $z^{t}$ is changed into

$$
x^{\frac{t}{n} \gamma-\left(\alpha_{1}+\alpha_{2}+\ldots+\alpha_{\nu}\right) \gamma}=x^{p} .
$$

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Thus we must put

$$
t=\left(\frac{p}{\gamma}+\alpha_{1}+\alpha_{2}+\ldots \alpha_{v}\right) n
$$

in order that everything may be included in the quantity $p$. Furthermore, if we put for brevity

$$
1 / 2\left(\frac{p}{\gamma}+\alpha_{1}+\alpha_{3}+\ldots .+\alpha_{v}\right)=m
$$

and instead of $u_{1}^{-}, u_{2}, \ldots u_{\nu}$, write the corpesponding values $2 n \alpha_{1}+1,2 n \alpha_{3}+1, \ldots 2 n \alpha_{\nu}+1$ we have

$$
\begin{aligned}
& t-u_{1}=2 n\left(m-\alpha_{1}\right) \\
& t-u_{2}=2 n\left(m-\alpha_{3}\right)
\end{aligned}
$$

and in general

$$
t-s_{\mu} u=2 n\left(m-s_{\mu} a\right)
$$

in which $s_{\mu}{ }^{\alpha}$ denotes the sum of $\mu$ clements in the series

$$
\alpha_{1}, \alpha_{2}, \alpha_{3}, \ldots \alpha_{\nu}
$$

By these substitutions we have

$$
\frac{(2 n)^{\nu-1}}{(\nu-1)!}\left\{m^{\nu-1}-\Sigma\left(m-s_{1} \alpha\right)^{\nu-1}+\Sigma\left(m-s_{2} \alpha\right)^{\nu-1}-\ldots .\right\}
$$

as the coeficient of the powcr

$$
x\left\{2 m-\left(\alpha_{1}+\alpha_{2}+\ldots \alpha_{\nu}\right)\right\} r_{;}
$$

this then is the number that indicates how many times among the total number of combinations,

$$
(2 n)^{V} \alpha_{1} \alpha_{8} \alpha_{8} \ldots \alpha_{V}
$$

that one is found whose sum is

$$
\left\{2 m-\left(\alpha_{1}+\alpha_{2}+\ldots .+\alpha_{v}\right)\right\} r
$$

Lastly, if we divide by the total number of combinations, we have the probability of the error

$$
\begin{gathered}
\left\{2 m-\left(\alpha_{1}+\alpha_{2}+\ldots .+\alpha_{v}\right)\right\} \gamma \text { viz: } \\
\frac{1}{2 n(\nu-1)!a_{1} \alpha_{2} \ldots \alpha_{v}}\left\{m^{\nu-1}-\Sigma\left(m-8_{1} \alpha\right)^{\nu-1}+\Sigma\left(m-8_{2} \alpha\right)^{\nu-1}-\ldots\right\}[5]
\end{gathered}
$$

Therefore the probability of any particular error approaches zero as $n$ is increased without limit.

## § 5.

If it is required to find the formula which expresses the number of combinations within certain limits, it is only necessary to choose a series of exponents in the product of the series [3] of §4 and sum the coefficients. For this purpose let us return to [4] and take the coefficient of the term $z^{t}$,

$$
\frac{1}{(v-1)!}\left\{t^{v-1}-\Sigma\left(t-s_{1} u\right)^{v-1}+\Sigma\left(t-s_{2} u\right)^{v-1}-\ldots .\right\}
$$

and for $t$ write in order all its values from $t=1$ to $t=t$ and sum them, which will give the expression required. First $t^{v}-1$ is changed into the series

$$
1+2^{v-1}+3^{v-1}+4^{v-1}+\ldots .+t^{v-1} ;
$$

this series is equivalent to

$$
t^{v}\left\{\left(\frac{1}{t}\right)^{v-1}+\left(\frac{2}{t}\right)^{v-1}+\left(\frac{3}{t}\right)^{v-1}+\ldots .+\left(\frac{t}{t}\right)^{v-1}\right\} \frac{1}{t}
$$

which, if $t$ is assumed infinite, can be expressed by the integral

$$
t^{\nu} \int_{0}^{1} x^{\nu-1} d x=\frac{t^{\nu}}{\nu}
$$

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By the same method,

$$
\left(t-u_{1}\right)^{v-1},\left(t-u_{2}\right)^{v-1}, \text { etc. }
$$

in which $t$ does not take values less than $u_{1}, u_{2}$, etc., become

$$
\frac{1}{v}\left(t-u_{1}\right)^{v}, \frac{1}{v}\left(t-u_{2}\right)^{v}, \text { etc. }
$$

The sum of these can be expressed in the form

$$
\frac{1}{\nu} \Sigma\left(t-s_{1} u\right)^{\nu}
$$

Similarly the powers

$$
\left(t-u_{1}-u_{3}\right)^{v-1},\left(t-u_{1}-u_{3}\right)^{v-1} \ldots\left(t-u_{v-1}-u_{\nu}\right)^{-1}
$$

in which $t$ has as its least values

$$
\left(u_{1}+u_{2}\right), \quad\left(u_{1}+u_{3}\right), \quad \ldots\left(u_{v-1}+u_{v}\right)
$$

can be cxpressed in the form

$$
\frac{1}{v} \Sigma\left(t-s_{2} u\right)^{\nu}
$$

Thus the sum of all the coefficients of the powers from $z^{0}$ to $z^{t}$ will be

$$
\frac{1}{v!}\left\{t^{\nu}-\Sigma\left(t-s_{1} u\right)^{\nu}+\Sigma\left(t-s_{2} u\right)^{\nu}-\cdots \cdot\right\}
$$

Then, substituting according to the same plan that was used in $\S 4$ and dividing by the sum of all the coefficients or (which is the same thing) by the number of all the combinations, we have

$$
\frac{1}{\nu!\alpha_{1} \alpha_{2} \alpha_{3} \ldots \alpha_{v}}\left\{m^{v}-\Sigma\left(m-s_{1} \alpha\right)^{v}+\Sigma\left(m-s_{2} \alpha\right)^{v}-\ldots\right\}
$$

which is the probability that the error is between the limits

$$
-\left(\alpha_{1}+\alpha_{3}+\ldots .+\alpha_{v}\right) \gamma \text { and }\left\{2 m-\left(\alpha_{1}+\alpha_{2}+\ldots+\alpha_{v}\right)\right\} \gamma
$$

The value of this will be equal to unity if we put

$$
m=\alpha_{1}+\alpha_{3}+\ldots+\alpha_{v}
$$

representing thus the certainty that the error must always be between

$$
-\left(\alpha_{1}+\alpha_{2}+\ldots+\alpha_{v}\right) \gamma \text { and }+\left(\alpha_{1}+\alpha_{2}+\ldots+\alpha_{v}\right) \gamma
$$

If, however, we had preferred to develop the serics in $\S 4$ according to decreasing powers instead of increasing, we should have reached the same formula for determining the probability of an error situated between

$$
\left(\alpha_{1}+\alpha_{2}+\ldots+\alpha_{\nu}\right) y \text { and }\left(\alpha_{1}+\alpha_{2}+\ldots .+\alpha_{\nu}-2 m\right) \gamma
$$

And if we put

$$
m=\frac{1}{3}\left(\alpha_{1}+\alpha_{2}+\ldots .+\alpha_{\nu}\right)
$$

it must be assumed that the value of this equals $1 / 2$, as indeed is. the case. But if we put

$$
m>\left(\alpha_{1}+\alpha_{2}+\ldots+\alpha_{\nu}\right)
$$

the formula will still give a value equal to unity; this and many other properties of the same formula, which can be proved by thetheory of finite sums and differences, it is needless to follow out here.

Lastly, if we designate the probability [6] found above as. $W_{m}$, and put

$$
\alpha_{1}+\alpha_{2}+\alpha_{3}+\ldots . \alpha_{\nu}=s
$$

then $W_{n}$ is the probability that the error lies between $-s y$ and $-(s-2 m) y$ and $1 / 2-W_{m}$ is the probability that it is between $-(s-2, n) y$ and 0 . And so, since the probability is the same if the limits are positive, the probability that the error lies between the limits $\pm(s-2 m) y$ is $1-2 W_{m}$, in which $m$ receives in order all the values from 0 to $1 / 2 s$.

## § 6.

To show by an example how the formula is to be used, let us take twenty logarithms from the tables and form their sum. All the logarithms taken from the table are affected by an error, of which the limit is one-half the last decimal place; it is required to find the error of the sum.

Here we take $\alpha_{1}=\alpha_{2}=\ldots=\alpha_{\nu}=1, v=20$ and $\gamma=1 / 2$ of the last decimal place. The sums $s_{1} \alpha$, of which there 20 , each equal 1 ; $s_{8} \alpha=2$, there being $\frac{20.19}{1.2}$ of these sums; $s_{3} \alpha=3$, and of these there are $\frac{20.19 .18}{1.2 .3}$; etc. So $W_{m}$ becomes according to [6]

$$
\begin{gathered}
\frac{1}{20!}\left\{m^{20}-2 \theta(m-1)^{20}+\frac{20 \cdot 19}{1.2}(m-2)^{20}-\frac{20.19 \cdot 18}{1.2 \cdot 3}(m-3)^{20}\right. \\
\left.+\frac{20 \cdot 19 \cdot 18 \cdot 17}{1.2 \cdot 3 \cdot 4}(m-4)^{20}-\ldots .\right\}
\end{gathered}
$$

The extreme limits of the error are -10 and +10 , and the probability that the error (without regard to sign) is between 0 and $10-m$ is $1-2 W m$. Then if instead of $m$ we put in turn 1 , $2,3, . . .9$, we have the probability of an error between the limits 0 and $9,8,7$, . . 1. Performing the subtraction and putting the total number of different errors equal to 10,000 , we have

| between | 0 and 1, | 5,586 errors. |  |
| :---: | :---: | :---: | :---: |
| 66 | $1 \times 2$, | 3,194 | 66 |
| 66 | 2 ، 3, | 1,025 | 6 6 |
| 6 | 3 6 4, | 178 | 66 |
| 66 | 4 ، 5, | 16 | 66 |
| 66 | $5 \times 6$, | 1 | 6 |
| 66 | 6 ' 6 10, | 0 | 66 |

that is to say, among 10,000 sums which are made by the addition of 20 logarithms taken from the table, 5,586 are affected by an error between the limits 0 and 1 in terms of the last place of decimals, and the error of no sum is greater than 6.

This computation can be tested to some extent by several trial additions of twenty logarithms. Take as an example the sum
of the first twenty incommensurable logarithms from a table of five decimal places, in which we find, by comparison with the sum of the same seven place logarithms, an error - 1.66 ; in other words the sum of the five place logarithms is 1.66 less in terms of the last decimal place than the sum of the seven place logarithms of the same numbers. Let us find in the same manner the error in the sum of each succeeding twenty logarithms (the rational logarithms of the numbers $1,10,100$, being omitted).

The errors are found to be:*

Sum of logarithms. | $2-22$ | error | $-1,66$ |
| ---: | :--- | :--- | :--- |
| $23-42$ | $"$ | $-2,16 \dagger$ |
| $43-62$ | $"$ | $-0,81$ |
| $63-82$ | $"$ | $-2,02$ |
| $83-103$ | $"$ | $+0,42$ |
| $10 \pm-123$ | $"$ | $+0,83$ |
| $124-143$ | $"$ | $-0,07$ |
| $144-163$ | $"$ | $+0,87$ |
| $164-183$ | $"$ | $+0,23$ |
| $154-203$ | $"$ | $+0,01$ |
| $204-223$ | $"$ | $-1,75 \dagger$ |
| $224-243$ | $"$ | $+3,23$ |
| $244-263$ | $"$ | $-0,12 \dagger$ |
| $244-283$ | $"$ | $-1,54$ |
| $234-303$ | $"$ | $+0,48 \dagger$ |
| $304-323$ | $"$ | $-0,12$ |
| $324-313$ | $"$ | $+0,19$ |
| $344-363$ | $"$ | $+0,75$ |
| $354-383$ | $"$ | -167 |
| $384-403$ | $"$ | $+0,80$ |

[^1]I also find several mistakes in signs which I have corrected, judging them to be misprints in the Latin edition. I did not correct the two given above, although it seems they should be as here given. If corrected, then the summary just below should read

|  | errors | twe |  | a |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5 | ، | " | 1 |  | ' | 2 |
| 2 | ، | ، | 2 |  | 6 |  |
| 1 | ، ${ }^{\text {d }}$ | ، |  |  | ' |  |

which agrees with the theoretical results given a few lines below better than those given by Bremiker.- E. F. C.
$\dagger$ I have changed the signs of these numbers.-E. F. C.

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so in these twenty sums selected at random there are

|  | ror | twe |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 4 | ' | '6 | 1 | ' |
| 2 | '6 | '6 | 2 |  |
| 1 | ، | ، |  |  |

We should compare these numbers wiih those found above, dividing of course by 500 since in this case there are only twenty sums, instead of 10,000 . After this division the probability is

| 11,2 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 6,4 | ، | ' | 1 |  |
| 2,0 | ، | '، | 2 |  |
| 0,4 | '6 | '، | 3 | ' |

so that there is as great an agreement as could be expected in so small a number of sums.

The smallness of the probability of the larger errors is seen both from this result and from the example above, since in 10,000 errors of this kind, only one exceeds five units; the extreme limit of ten units is so far remote from the range of errors which actually occur that errors of that sort are to be regarded as practically non-existent. For by actual computation the probability of an error between 9 and 10 is found to be 1: 2432900000000000000 . Moreover the extreme limit of error is of little value in logarithmic computations, as will appear below, where more accurate criteria are proposed for testing such computation.

## § 7.

In discussions of the probability of errors the probable error and the mean error must receive especial consideration. The limits within which exactly half of the total number of errors is situated is called the probable error. Thus the probable error, according to our formula, is $(s-2 m) \gamma$ if $m$ satisfies the equation $1-2 W_{m}=\frac{1}{2}$ or $W_{n}=\frac{1}{4}$. This equation cannot be solved by any direct method,

The mean error denotes the arithmetical mean of all possible

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erross, computed from the errors without regard to sign. We obtain it if we multiply the error $(s-2 m) \gamma$ by its probability $\frac{1}{2 n(\nu-1)!1_{1} \alpha_{9} \ldots \ldots \alpha_{\nu}}\left\{m^{\nu-1}-\Sigma\left(m-s_{1} \alpha\right)^{\nu-1}+\Sigma\left(m-s_{2} \alpha\right)^{\nu-1}-\ldots\right\}$
and double the integral of the product taken between the limits 0 and $1 / 2 s$. By this integration, either dircct or as in §5, we have
$\frac{4 \gamma}{(\nu+1)!\alpha_{1} \alpha_{2} \ldots \alpha_{\nu}}\left\{\left(\frac{s}{2}\right)^{\nu+1}-\Sigma\left(\frac{s}{2}-8_{1} \alpha\right)^{\nu+1}+\Sigma\left(\frac{s}{2}-\delta_{2} \alpha\right)^{\nu+1}-..\right\}[17]$
In this formula we are to take only those sums of powers that have positive exponents. If we take the same example as in $\S 6$, we have, according to this formula, the mean error equal to 1.03266. The probable error, however, is found to be slightly less than 1.

## § 8.

By the means proposed above we can determine the probability of an error within any assigned limits in any quantity computed by logarithms. Since, moreover, we have already shown how the probable error and the mean error can be found, the general problem may be considered as solved; unless there remain further difficulties in applying the formulas. For it happens that, whenever $\boldsymbol{v}$ is a large number, the separate terms of the series employed are formed from very large numbers which in turn cancel each other, so that it is necessary to use many more decimal places in the computation than are required in the completed result. Thus in the example in $\S 6$, eight place logarithms were used in order that the probability might be computed to four places. And for groater values of $v$ the difficulties are increased to such an extent that if $v=100$ the computation of the values can scarcely be undertaken.

In order to avoid these difficulties, theoretical accuracy must be to some extent sacrificed and resort had to approximate values, in order that we may express results in terms of the integral of $e^{-z^{2}}$, which is of fundamental importance in discussions of probability.

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$$
\text { § } 9 .
$$

For this purpose let us return to the series [3] of § 4 and pat $e^{-z_{V}-1}$ for $x^{\frac{\gamma}{n}}$, so that if $i$ is written instead of $\downarrow-1$ they are changed into

$$
\begin{gathered}
e^{-n \alpha_{1} z i}+e^{-(n-1) \alpha_{1} z i}+\ldots+e^{0}+\ldots+e^{(n-1) \alpha_{1} z i}+e^{n \alpha_{1} z i}=\varphi_{1}(z) \\
e^{-n \alpha_{2} z i}+e^{-(n-1) \alpha_{3} z i}+\ldots+e^{0}+\ldots+e^{(n-1) \alpha_{2} z i}+e^{n \alpha_{3} z i}=\varphi_{3}(z) \\
\ldots . \\
\ldots . \\
e^{-n \alpha_{\nu} z i}+e^{-(n-1) \alpha_{\nu} z i}+\ldots+e^{0} \ldots+e^{(n-1) a_{\nu} z i}+e^{n \alpha_{\nu} z i}=\varphi_{\nu}(z)
\end{gathered}
$$

By multiplying together these series we have, as before, a series whose coefficients show how many times the exponents can be combined into any given sum. We have accordingly.

$$
\varphi_{1}(z) \cdot \varphi_{z}(z) \ldots . \varphi_{\nu}(z)=\Sigma k_{t} e^{t z i}
$$

if $k_{t}$ is the coefficient of the power $e^{t z i}$ in which $\boldsymbol{t}$ receives all values from

$$
\left.-n_{1}^{\prime} \alpha_{1}+\alpha_{2}+\ldots+\alpha_{\nu}\right) \text { to }+n\left(\alpha_{1}+\alpha_{2}+\ldots a_{v}\right)
$$

If we multiply this equation by $e^{-t^{\prime} z i}$ and integrate between the limits $-\pi$ and $+\pi$ we shall have on the right hand side the sum of integrals of which each one, as

$$
k_{t} \int e^{\left(t-t^{\prime}\right) z i} d z
$$

becomes equal to zero if $t$ and $t^{\prime}$ have different values, excepting only the one in which $t=t^{\prime}$, which will equal $2 \pi k_{t}$ so that we have

$$
\begin{equation*}
\frac{1}{2 \pi} \int_{-\pi}^{\pi} \varphi_{1}(z) \cdot \varphi_{\mathfrak{2}}(z) \ldots . \varphi_{\nu}(z) e^{-t z i} d z=k_{t} \tag{S}
\end{equation*}
$$

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Then if we give in turn to $t$ all values from - $n c$ to $+n c$, instead of $e^{-t z i}$ we have the series

$$
\begin{aligned}
& e^{-n c z i+e-(n c-1) z i}+\ldots+e^{0}+\ldots+e^{(n c-1) z i}+e n c z i \\
= & c^{-n c z i}\left(1+e^{z i}+e^{2 z i}+\ldots+e^{(2 n c-1) z i}+e^{2 n c z i}\right) \\
= & e^{-n c z i} \frac{1-e^{(2 n c+1) z i}}{1-e z i} \\
= & e^{-n c z i} \cdot \frac{e^{\frac{2 n c+1}{2} z i}}{e^{\frac{z}{2}} i} \cdot \frac{\sin \left(\frac{2 n c+1}{2} z\right)}{\sin \frac{1}{\frac{1}{2} z}}=\frac{\sin \left(\frac{2 n c+1}{2} z\right)}{\sin \frac{1}{2} z}
\end{aligned}
$$

Similarly it is found that

$$
\varphi_{1}(z)=\frac{\sin \left(\frac{2 n \alpha_{1}+1}{2} z\right)}{\sin \frac{1}{2} z}, \quad \varphi_{2}(z)=\frac{\sin \left(\frac{2 n \alpha_{2}+1}{2} z\right)}{\sin _{\frac{1}{2} z} z}
$$

etc. By these substitutions the integral shown above is changed into

$$
\begin{aligned}
& \Sigma k_{t}[\text { from } t=-c \text { to } t=+c] \\
& =\frac{1}{2 \pi} \int \begin{array}{l}
+\pi \\
\frac{\sin \left(\frac{2 n \alpha_{1}+1}{2} z\right) \cdot \sin \left(\frac{2 n \alpha_{2}+1}{2} z\right) \ldots \sin \left(\frac{2 n \alpha_{v}+1}{2} z\right) \sin \left(\frac{2 n c+1}{2} z\right)}{-\pi} \cdot \frac{\left.\sin 1 / 2^{2}\right)^{v}}{\sin 1 / 2^{z}} d z
\end{array}
\end{aligned}
$$

or, if we put $n z=x$, into

$$
\frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{\sin \alpha_{1} x \cdot \sin \alpha_{2} x \ldots \cdot \sin \alpha_{v} x \sin c x}{\left(\sin \frac{x}{2 n}\right)^{r}} d x
$$

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But the function which is to be integrated here becomes a maximum if $x$ is taken equal to zero. As $x$ increases, the numerator repeatedly becomes zero, while the denominator continually increases, so that we have a sufficiently exact value for the integral if we take large finite quantities $-g$ and $+g$ in place of the infinite limits. By this substitution $\sin \frac{x}{2 n}$ is changed into $\frac{x}{2 n}$ for the quantity $n$ is assumed infinite. Thus after division by

$$
(2 n)^{\nu} \alpha_{1} \alpha_{2} \ldots \alpha_{\nu}
$$

we have the probability $V_{c}$ of an error between the limits - $e$ and $+c$.

$$
V_{c}=\frac{1}{\pi} \int_{-g}^{+\rho} \frac{\sin \alpha_{1} x \sin \alpha_{2} x}{\alpha_{1} x} \cdot \frac{\sin \alpha_{\nu} x}{\alpha_{2} x} \cdots \cdot \frac{\sin c x}{\alpha_{\nu} x} \cdot \frac{1}{x} d x
$$

Then if each sine is expressed as a series, so that

$$
\log \left(\frac{\sin \alpha_{1} x}{\alpha_{1} x}\right)=-\frac{1}{6} \alpha_{1}^{2} x^{3}-{ }_{r}^{1} \delta \alpha_{1} \alpha_{1}^{4} z^{4}-\ldots
$$

and similarly for

$$
\log \left(\frac{\sin \alpha_{2} x}{\alpha_{2} x}\right) \cdots \log \left(\frac{\sin \alpha_{\nu} x}{\alpha_{\nu} x}\right) .
$$

we have

$$
\log \left(\frac{\sin \alpha_{1} x}{\alpha_{1} x} \cdot \frac{\sin \alpha_{2} x}{\alpha_{2} x} \cdots \cdot \frac{\sin \alpha_{\nu} x}{\alpha_{\nu} x}\right)=-\frac{1}{\partial} \nu \sigma^{2} x^{2}-{ }^{18} \sigma^{2} \nu \sigma^{\prime} x^{4}-\ldots .
$$

if we put

$$
\alpha_{1}{ }^{2}+\alpha_{2}{ }^{2}+\ldots+\alpha_{\nu}{ }^{2}=\nu \sigma^{2}
$$

and

$$
\begin{equation*}
\alpha_{1}{ }^{4}+\alpha_{2}{ }^{4}+\ldots+\alpha_{v}{ }^{4}=\nu \sigma^{\prime 4} \tag{10}
\end{equation*}
$$

But this series can only be substituted if it is convergent, which is the case if a function of the variable $\nu$ of the form $k \nu^{-\lambda}$ is substituted for $x$. By this operation the sories is changed to

$$
-\frac{i^{2} \sigma^{2}}{6^{-\nu}} 1-2 \lambda-\frac{k^{4} \sigma^{\prime 4}}{150} v^{1-4 \lambda}-\ldots
$$

in which the first term increases, and all the others diminish, for increasing values of $v$ provided $\lambda$ is assumed to satisfy

$$
1 / 2>\lambda>1 / 4
$$

For then the exponent of $v$ will be positive in the first term, and negative in all the others. For example, let $\lambda=1 / 3$ and we obtain the series

$$
-\frac{1}{8} k^{2} \sigma^{2} \nu^{\frac{1}{3}}-1_{8}^{1} \delta k^{4} \sigma^{\prime} 4 \nu^{-\frac{1}{3}}-\ldots .
$$

Hence if in the integral writien above, we take the limits $g=k \nu-\frac{f}{s}$ instead of the limits - $\infty$ and $+\infty$, we shall approach the more nearly to the true value of the integral the greater the value of v. For we have

$$
\nabla_{c}=\frac{1}{\pi} \int_{-g}^{+g} e^{-\frac{1}{8} \nu \sigma^{2} x^{8}}\left(1-{ }_{1}^{\frac{1}{8}} \sigma^{\prime} \sigma^{\prime} s x^{4}\right) \frac{\sin c x}{x} d x
$$

and if we put $z=x \sqrt{\frac{\nu}{6}}$

$$
V_{c}=\frac{1}{\pi} \int e^{-\sigma^{2} z^{2}}\left(1-\frac{1}{5 \nu} \sigma^{\prime 4} z^{4}\right) \frac{\sin c z \sqrt{\frac{6}{\nu}}}{z} d z
$$

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The limits will be

$$
\pm l i v-\frac{1}{8} \cdot \sqrt{\frac{v}{6}}=\frac{k}{\sqrt{6}} v \frac{1}{3}
$$

in place of which $\infty$ may again be substituted since this limit becomes infinite with $r$. Moreover the function to be integrated remains the same whether $x$ has its positive or negative value, hence 0 may be put as the inferior limit and the integral doubled. Thus we have

$$
\begin{equation*}
\nabla_{0}=\frac{2}{\pi} \int_{0}^{\infty} e^{-\sigma^{2} z^{2}}\left(1-\frac{1}{5 \nu} \sigma^{\prime 4} z^{4}\right)-\sin c z \sqrt{\frac{6}{v}} z^{z} d z \tag{11}
\end{equation*}
$$

which can be considered as the sum of two integrals and computed by the aisistance of the ordinary formula

$$
\int_{0}^{\infty} e^{-\alpha^{2} x^{8}} \cos 2 \beta x d x=\frac{\sqrt{\pi}}{2 \alpha} e^{\frac{-\beta^{3}}{\alpha^{3}}}
$$

For by integration between 0 and $l$ with respect to $\beta$ this gives

$$
\int_{0}^{\infty} e^{-\alpha^{2} x^{2} \frac{\sin \Omega l}{x} d x=\frac{V}{\pi^{-}}} \int_{0}^{l} e^{-\beta^{\beta^{2}}} d \beta
$$

which is the desired reduction for the first part of the integral. Whence

$$
\nabla^{\prime} c=\frac{2}{\sigma \sqrt{\pi}} \int_{0}^{\frac{c}{2} \sqrt{\frac{6}{\nu}}} e^{-z^{2} d z=} \overline{\sqrt{\pi}} \int_{0}^{\frac{c}{2 \sigma} \sqrt{\frac{6}{\nu}}} e^{-z^{2} d z .[12]}
$$

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If, however, we differcatiate the formula three times with respect to $\beta$ we have

$$
\int_{0}^{\infty} e^{-\alpha^{2} x^{2}} x^{3} \sin 2 \beta x d x=\frac{\sqrt{\pi}}{2 \alpha} e^{\frac{-\beta^{2}}{\alpha^{2}}}\left(\frac{3 \beta}{2 \alpha^{4}}-\frac{\beta^{3}}{\alpha^{6}}\right)
$$

which we can employ in the reduction of the other part of the integral ; thus

$$
\begin{equation*}
V^{\prime}{ }_{c}=\frac{3 c \sigma^{\prime 4} \sqrt{\frac{6}{\pi}}}{20 \nu^{\frac{3}{2}} \sigma^{5}}-e^{-\frac{3 c^{2}}{2 \nu \sigma^{2}}\left(1-\frac{c^{2}}{\nu \sigma^{2}}\right)} \tag{13}
\end{equation*}
$$

Finally the equation

$$
\begin{equation*}
V=V^{\prime}-\nabla^{\prime \prime} \tag{14}
\end{equation*}
$$

shows the probability of an error lying between $-c z$ and $+c z$; the second part $V^{\prime \prime}$ either vanishes for large values of $v$ or is very small.

## § 10.

If $h y$ denotes the probable error, according to the definition $V_{h}$ will equal $1 / 2$. In order to find $h$ from this, first take $V^{\prime \prime}$ $=0$ and compute $h$ by means of the equation $V^{\prime}{ }_{h}=1 / 2$. Thus by the aid of a table which shows the value of the integral

$$
\frac{2}{\sqrt{\pi}} \int_{0}^{z} e^{-z^{2}} d z
$$

corresponding to any value of $z$ it is found that $z=0.476936$ if the integral itself equals $1 / 2$. This value of $z$ being denoted by $\rho$, we have from the equation $V^{\prime} c=1 / 2$

$$
\frac{c}{2 \sigma} \sqrt{\frac{6}{\nu}}=\rho \quad \text { or } \quad c=2 \sigma \rho \sqrt{\frac{\nu}{6}}
$$

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Thus we compute $c$ as a first approximation to $h$; then on putting $c+d c=h$ we shall have $d c$ from the equation

$$
d V^{\prime} c d c=V^{\prime} c \quad \text { or } \quad d c=\frac{V^{\prime}}{d V^{\prime}} *
$$

in which the value just found is to be substituted in place of a Thus, since

$$
d V^{\prime} c=\frac{\sqrt{\frac{6}{\nu}}}{\sigma \sqrt{\pi}} e^{-\frac{3 c^{2}}{2 v \sigma^{8}}},
$$

we shall have

$$
d c=-\frac{V^{\prime \prime}}{d \overline{V^{\prime}}}=\frac{3 \rho \sigma^{\prime 4}}{10_{\sqrt{ }} \overline{6 \nu \cdot \sigma^{3}}}\left(1-\frac{2}{8} \rho^{2}\right) .
$$

and

$$
\begin{equation*}
h=2 \sigma \rho \sqrt{\frac{\nu}{6}}+\frac{3 \rho \sigma^{\prime} 4}{10_{V} \overline{\sigma \nu} \cdot \sigma_{3}}\left(1-2 / 3 \rho^{2}\right) \tag{15}
\end{equation*}
$$

Then $h_{y}$ is the probable error; but the second term either vanishes for very large values of $v$ or is negligible.

The mean error, defined in $\S 7$, can in general be computed in the same way as the probable error was computei in § 9 . For we may start from the equation [8] which expresses the probability of the error $\frac{t}{n} \gamma$,

$$
k_{t}=\frac{1}{2 \pi} \int_{-\pi}^{+\pi} \varphi_{1}(z) \cdot \varphi_{\mathrm{s}}(z) \ldots . \varphi_{v}(z) e^{-t e t} d z \cdot \frac{1}{(2 n)^{v} \alpha_{1} \alpha_{2} \ldots \alpha_{v}}
$$

and substitute for $\varphi_{1}(z) \ldots \ldots \ldots$ the expressions involving sines, and substitute for the product of sines the function of the exponents. Then we shall have

$$
\frac{1}{x} \int_{0}^{\infty} e^{-\frac{1}{\delta} \gamma \sigma^{2} x^{2}}\left(1-\frac{v}{180} \sigma^{\prime} x^{4}\right) e^{-e x i} \frac{d x}{n}
$$

[^2]as the probability of an error $c y$. Then by putting $x \sqrt{\frac{\nu}{6}}=z$ and $\cos c x$ instead of $e^{-c x i}$ (since the imaginary part is cancelled) we shall have

Finally if we employ the reduction formulas

$$
\int_{0}^{\infty} e^{-\alpha^{2} z^{2}} \cos \beta z d z=\frac{\sqrt{\pi}}{2 \alpha} e^{-\frac{\beta^{2}}{4 \alpha^{2}}}
$$

and its fourth derivative

$$
\int_{0}^{\infty} e^{-\alpha^{9} z^{9}} \cos \beta z . z^{4} d z=\frac{\sqrt{\pi}}{2 \alpha} e^{-\frac{\beta^{3}}{4 \alpha^{3}}}\left(\frac{3}{4 \alpha^{4}}-\frac{3 \beta^{3}}{4 \alpha^{6}}+\frac{\beta^{4}}{16 \alpha^{8}}\right)
$$

we shall have

$$
\frac{1}{n} \frac{\sqrt{\frac{6}{v \pi}}}{2 \sigma} e^{-\frac{3 c^{2}}{2 \nu \sigma^{2}}}\left\{1-\frac{\sigma^{\prime 4}}{5 \nu}\left(\frac{3}{4 \sigma^{4}}-\frac{9 c^{2}}{2 \nu \sigma^{6}}+\frac{9 c^{4}}{4 \nu^{2} \sigma^{8}}\right)\right\}
$$

as the probability of an error $c_{\gamma}$.
The integral of this being taken between the limits $-c$ and $+c$, in which $\frac{1}{n}$ takes the place of $d c$, it is again necessary to avoid the avoid the formula for $V$ as found in $\S 9$. This is shown by the computation. The partial integration can be performed according to the formula

$$
\int e-b z^{2} z^{2 n+2} d z=\frac{2 n+1}{2 b} \int e^{-b z^{2}} z^{2 n} d z-\frac{1}{2 b} e^{-b z^{2} z^{2 n+1}}
$$

[家
and, if the factor 2 be inserted, the integrals may have zero $a 3$ lower limit.

But if we multiply by $2 c \gamma$ and integrate with respect to $c$ between the limits 0 and $s$ we shall have the mean error, or the aggregate of all the errors regardless of sign divided by the total number of them.

By partial integration according to the formula
all parts of the integral can be reduced to the form

$$
\int_{0}^{s} e^{-b x^{2} x d x=\frac{1}{2 b}\left(1-e-b s^{2}\right)}
$$

from which we obtain

$$
\begin{equation*}
2 \gamma \sigma \sqrt{\frac{\nu}{6 \pi}}\left(1+\frac{\sigma^{\prime 4}}{20 v \sigma^{4}}\right) \tag{16}
\end{equation*}
$$

if the terms are omitted which have $e^{-\frac{3 s^{2}}{2 \nu \sigma^{2}}}$ as a factor. And this is the mean error.

$$
\S 11 .
$$

In formula [6] of § 5 the probability of an error between the limits $\pm(s-2 m) \gamma$ was found to equal $1-2 W$; and in $\S 9$ this is shown to equal $V_{c}$ for limits $\pm c \gamma$. So if $c$ be taken equal to $s-2 m, 1-2 W$ and $V$ must necessarily express ti.e same probability. But $V$ is only an approximation, since, in deriving it, $v$ was assumed very large. In order that this agreement may be subjected to some numerical test, let us employ the

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example of $\S 6$. In this all values of $a$ are put equal to 1 ; $\nu=20 ; \gamma=1 / 2$; so that

$$
\begin{aligned}
s & =\alpha_{1}{ }^{2}+\alpha_{2}{ }^{2}+\ldots+\alpha_{\nu}{ }_{v}=20 \\
\sigma^{2} & =\frac{1}{\nu}\left(\alpha_{1}{ }^{2}+\alpha_{2}{ }^{2}+\ldots+\alpha_{\nu}{ }^{2}\right)=1 \\
\sigma^{\prime 4} & =\frac{1}{\nu}\left(\alpha_{1}{ }^{4}+\alpha_{2}{ }^{4}+\ldots+\alpha_{\nu}{ }^{4}\right)=1 \\
V^{\prime} & =\frac{2}{\sqrt{\pi}} \iint^{\frac{c}{2}} \sqrt{\frac{3}{10}} \\
0 & e^{-z^{2}} d z \\
V^{v} & =\frac{3}{400^{c} \cdot} \sqrt{\frac{3}{10 \pi}} e^{-\frac{3}{40} c^{2}}\left(1-\frac{c^{2}}{20}\right) \\
& =0,0023177 c e^{-\frac{3}{4 v} c^{2}}\left(1-\frac{c^{2}}{20}\right) \\
m & =10-\frac{1}{2} c
\end{aligned}
$$

Furthermore if instead of $c$ are put the values $0,2,4, \ldots$. . up to the limit 20 , the corresponding values of

$$
c \gamma, m, \frac{c}{2} \sqrt{\frac{3}{10}}, \frac{3}{40} c^{2}, 1-\frac{c^{2}}{20}
$$

will be as follows:*

| $c \gamma$ | $c$ | $m$ | $\frac{c}{2} \sqrt{\frac{3}{10}}$ | $\frac{3}{40^{2}} c^{2}$ | $1-\frac{c^{2}}{20}$ |
| ---: | ---: | ---: | :---: | :---: | :---: |
| 0 | 0 | 10 | 0 | 0, | $+1,0$ |
| 1 | 2 | 9 | 0,547725 | 0,3 | $+0,8$ |
| 2 | 4 | 8 | 1,095450 | 1,2 | $+0,2$ |
| 3 | 6 | 7 | 1,643175 | 2,7 | -08 |
| 4 | 8 | 6 | 2,190900 | 4,8 | $-2,2$ |
| 5 | 10 | 5 | 2,738625 | 7,5 | $-4,0$ |
| 6 | 12 | 4 | 3,286350 | 10,8 | $-6,2$ |
| 7 | 14 | 3 | 3,834075 | 14,7 | $-8,8$ |
| 3 | 16 | 2 | 4,381800 | 19,2 | $-11,8$ |
| 9 | 18 | 1 | 4,929525 | 24,3 | $-15,2$ |
| 10 | 20 | 0 | 5,477250 | 30,0 | $-19,0$ |

[^3]Then according to the formulas [11], [12] and [13] the approximate values $V^{\prime}, V^{\prime \prime}, V$ are easily derived which are compared in the following table with the corresponding values 1 $2 W$ computed according to the rigorous formulas [6] of § 5. These are indeed the same as those which were shown in § 6 giving the number of differences between 0 and 1, and 1 and 2, etc., for $c y$ with regard to integers only.

| $c y$ | $V$ | $V^{\prime}$ | $V=V^{\prime}-V^{\prime \prime}$ | $\mathbf{1 - 2 W}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{0}$ | 0 | 0 | 0 | 0 |
| $\mathbf{1}$ | 5814,2 | $+27,5$ | 5586,7 | 5586,0 |
| $\mathbf{2}$ | 878,6 | +56 | 8781,0 | 8780,8 |
| $\mathbf{3}$ | 9793,6 | $-7,5$ | 9806,1 | 9806,3 |
| $\mathbf{4}$ | 9980,5 | $-3,4$ | 9983,9 | 9983,9 |

It is needless to continue this farther; for the values which $1-2 W$ and $V$ receive evidently agree closely and this happens because the quantity $r$ which we have assumed equal to 20 is sufficiently large. For values $v=2$ or $v=3$ they would agree less exactly.

Accordin:g to formula [15] of $\S 10$, the probable error is

$$
h y=2 \gamma \sigma \rho \sqrt{\frac{\nu}{6}}+\frac{3 \gamma \rho \sigma^{\prime 4}}{10 \sqrt{6 v} \sigma^{3}}\left(1-2 / 3 \rho^{2}\right)
$$

or, if we substitute for $\gamma, \sigma, \sigma^{\prime}$, and $\nu$ the same values that they had above, and 0.476936 for $\rho$

$$
h y=0,87076+0,00554=0.87630 .
$$

The corresponding value of $m$ is

$$
m=\frac{20-h}{2}=9,12370
$$

If this value of $m$ is substituted in $W_{m}$ its value (according to $\S 7$ ) ought to be $1 / 4$. By making the substitution we actually obtain $W m=0.25002$, which may be considered perfect agreement.

The approximate value of the mean error, which we have found to equal 1.03266 by the exact formula [7] of $\S 7$, will be

$$
2 y \sigma \sqrt{\frac{\nu}{6 \pi}}\left(1+\frac{\sigma^{\prime 4}}{20 \nu \sigma^{4}}\right)
$$

from the formula of $\S 10 \mathrm{or}$, if the values found above are substituted,

$$
=\sqrt{\frac{10}{3 \pi}}\left(1+\frac{1}{4 \sigma \sigma}\right)=1,03263,
$$

A more exact agreement could not be asked.
Furthermore it is seen that if $v=20$ the terms which depead on $\sigma^{\prime}$ or on the fourth power of $\alpha$ are negligible in comparison with the first term. But since, as will appear below, the quantity $v$ is usually still larger or in a minor computation is not far different from that value, all the terms depending on $\sigma^{\prime}$ can be neglected altogether without fear that the result will vary far from the truth. So we can assumo

$$
\begin{equation*}
\frac{2}{\sqrt{\pi}} \int_{0}^{\frac{c}{2 \sigma}} V^{\frac{6}{\nu}} e^{-z^{2} d z} \tag{17}
\end{equation*}
$$

as the probability of an error between the limits $\pm c \boldsymbol{c}$,

$$
{ }^{2 \gamma \sigma \rho} \sqrt{\frac{\gamma}{6}} \text { will be the probable error, }
$$

and

$$
\begin{equation*}
{ }^{2 y \sigma} \sqrt{\frac{\nu}{6 \pi}} \text { the mean error. } \tag{19}
\end{equation*}
$$

Furthermore, if $\gamma=1 / 2$ expressed in terms of the last decimal place, and we put $\zeta=c y$, then

$$
\frac{2}{V^{\pi}} \int_{0}^{\frac{\zeta}{\sigma}} V^{\frac{6}{\nu}} e-z^{8} d z
$$

will be the probability of an error within the limits $\pm 5$
${ }^{\sigma \rho} \sqrt{\frac{\dot{\nu}}{6}}$ will be the probable error,
and

$$
{ }^{\sigma} \sqrt{\frac{\nu}{6 \pi}} \text { the mean error, }
$$

expressed in terms of the last decimal place. Or, if there be written in place of $\nu \sigma^{2}$ its value $\alpha_{1}{ }^{3}+\alpha_{2}{ }^{2}+\ldots .+a_{v}=\Sigma \alpha_{2}$

$$
\rho \sqrt{\frac{\Sigma \alpha^{2}}{6}} \text { will be the probable error, }
$$

and $\sqrt{\frac{\Sigma_{6}\left(x^{2}\right.}{\sigma^{2}}}$ the mean error.

## § 12.

Before we use these formulas, let us examine more carefully the general nature of logarithmic computations. First, it must not be assumed that every time we pass over from logarithm to number or from number to logarithm an error arises whose limit is the half of unity, since we may have added to the tabular logarithm a proportional part. Also, the probability of error in different intervals is not the same, since many sources of error exist. Lastly, the limits and the probabilities of error vary according as the transition is made from number to $\log$ arithm or from logarithm to number. Hence it will be necessary to modify equations [1] and [2] of § 2 , in which it was assumed that there was only a single source of error arising with each single transition. Therefore in order that everything may be correctly expressed by the formulas, let us more carefully consider the transition from number to logarithm.

If it is required to find the logarithm corresponding to a certain number, it is commonly done by adding to the tabular logarithm a proportional part from the table of differences. So
if $L_{1}$ and $L_{2}$ are the tabular logarithms between which the interpolation is to take place and $\varepsilon$ is the fraction by which the difference $\dot{\Delta}=L_{2}-L_{1}$, is to be multiplied, according to the ordinary rule (the basis of which is to be considered here)

$$
L_{1}+\varepsilon \triangle
$$

will be the required logarithm. In this of course $\varepsilon \Delta$ is accurately computed, but is expressed only in integral units of the last decimal place.

If it is assumed that $f_{1}$ and $f_{2}$ are the errors of the tabular logarithms $L_{1}$ and $L_{2}$, it is then known that since $L_{1}+f_{1}$ and $L_{2}+f_{2}$ are the exact values of the logarithms, the error of the $\operatorname{logarithm} L_{1}+\varepsilon \Delta$ will be

$$
f_{1}+\varepsilon\left(f_{2}-f_{1}\right)+f_{3}
$$

if $f_{3}$ denotes the error which arises from the curtailment of the product $\varepsilon \Delta$, but $f$ is considered to be any one of the equally probable values which lie between $+\gamma$ and $-\gamma$. In each particular case the quantity $\varepsilon$ (which lies between 0 and 1) will be fixed and definite; nor would it be difficult to introduce into the computation, if expressed numerically, all the ; values which $\varepsilon$ receives during the computation. But since we desire to investigate the general case, we shall attempt to find a mean value for $\varepsilon$; or else we may treat all the values of $\varepsilon$ as remaining indefinite, and finally find a probability which will show within what limits the mean error and the probable error lie.

We shall omit the consideration of the last method although it would add to the completeness and elegance of this investigation; but the final result which we seek would be helped but little.

So let us first inquire what values the sum

$$
(1-\varepsilon) f_{1}+\varepsilon f_{2}+f_{3}
$$

can receive if instead of each $f$ we write all the equally probable values between - $1 / 2$ and $+1 / 2$. The formula [6] of §5 serves best for this purpose. For if we substitute in it $\alpha_{1}=1-\varepsilon, \quad \alpha_{3}=\varepsilon, \alpha_{3}=1, \nu=3, \quad \gamma=\frac{1}{2}$ we shall have $s=2,(s-2 m) \gamma=1-m$.

Since $m$ can receive all values from 0 to $s$, the extreme limits are -1 and +1 . By substituting these values in $W_{m}$ we shall have,

$$
\frac{1}{6 \varepsilon(1-\varepsilon)}\left\{\begin{array}{c}
m^{3}-(m-1+\varepsilon)^{3}+(m-1)^{3}-\ldots \\
-(m-\varepsilon)^{3}+(m-2+\varepsilon)^{3} \\
-(m-1)^{3}+(m-1-\varepsilon)^{3}
\end{array}\right\}
$$

But in this formula several terms in which the quantities are negative are to be omitted since $m$ assumes only those values which lie between 0 and 1 ; and thus it is changed into

$$
W_{m}=\frac{1}{6 \varepsilon(1-\varepsilon)}\left\{m^{3}-(m-1+\varepsilon)^{3}-(m-\varepsilon)^{3}\right\}
$$

in which $m$ and $\varepsilon$ receive all values between 0 and 1 , and all powers of negative quantities are necessarily excluded. So $1-2 W_{m}$ will be the prubability that the sum $(1-\varepsilon) f_{1}+\varepsilon f_{2}+f_{3}$ lies between the limits $\pm(1-m)$. Below are the values which $1-2 W_{m}$ takes for each decimal division of the values of $m$ and $\varepsilon$ indicated in the table:

| $m$ | $(2-2 m) \gamma$ | $\varepsilon=0,1$ | $\varepsilon=0,2$ | $\varepsilon=0,3$ | $\varepsilon=0,4$ | $\varepsilon=0,5$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0,9 | 0,1 | 0,196 | 0,198 | 0,198 | 0,198 | 0,193 |
| 0,8 | 0,2 | 0,374 | 0,384 | 0,338 | 0,338 | 0,390 |
| 0,7 | 0,3 | 0,530 | 0,546 | 0,557 | 0,562 | 0,564 |
| 0,6 | 0,4 | 0,662 | 0,684 | 0,700 | 0,712 | 0,714 |
| 0,5 | 0,5 | 0,774 | 0,796 | 0,514 | 0,823 | 0,834 |
| 0,4 | 0,6 | 0,861 | 0,834 | 0,900 | 0,914 | 0,914 |
| 0,3 | 0,7 | 0,930 | 0,916 | 0,938 | 0,902 | 0,964 |
| 0,2 | 0,8 | 0,974 | 0,982 | 0,986 | 0,933 | 0,990 |
| 0,1 | 0,9 | 0,996 | 0,998 | 0,938 | 0,998 | 0,998 |

There is no need to extend the table further, since for $\varepsilon=0.6$, 0.7 , etc., $1-2 W_{m}$ takes on the same values that it has for $\varepsilon=0.4, \varepsilon=0.3$, etc. If we let $\varepsilon=0$ or 1 we shall have $W_{m}=1 / 2 m^{2}$ and so if

| $(2-2 m) \gamma=0,1$ then $1-2 W_{m}=0,190$ |  |  |
| :---: | :---: | :---: |
| 0,2 | '، | 0,360 |
| 0,3 | ' | 0,510 |
| 0,4 | ، | 0,610 |
| 0,5 | ، | 0,750 |
| 0,6 | ، | 0,840 |
| 0,7 | '، | 0.910 |
| 0,8 | '، | 0,960 |
| 0,9 | '، | 0,990 |

and the probable error and the mean crror are found to bo

| for $\varepsilon$ | $=0,0$ |
| ---: | :--- |
| 0,1 | 0,2 |
| 0,3 | 0,4 |
| probable error | $=0,293$ |
| mean error | $=0,233$ |
| m | 0,318 |
| 0,270 | 0,307 |
| 0,263 | 0,298 |
| 0,262 | 0,293 |
| 0,292 |  |

Hence it appears that it matters little what value $\varepsilon$ has, since the values which the mean error and the probable error receive for various values of $\varepsilon$ differ among themselves but little. In order to adopt a single average value and thus arrive at a more convenient form for computation, let us put $\varepsilon=\frac{1}{}$, for which the probable error will be 0.270 and the mean error 0.307 .

Therefore we may put the error of a logarithm found by interpolation as

$$
\begin{equation*}
f^{\prime}=\frac{4}{5} f_{1}+\frac{1}{6} f_{2}+f_{3} \tag{20}
\end{equation*}
$$

If the transition is from logarithm to number or arc, the error which is to be assumed as occurring in the logarithm has the form

$$
-(1-\varepsilon) f_{1}-\varepsilon f_{2} .
$$

For there are given, the $\log a$ rithm $L=\log \varphi(a)$, the tabular logarithms $L_{1}$ and $L_{2}$ between which the interpolation is to be performed, the difference $L_{2}-L_{1}=\Delta$ and $L-L_{1}=\delta$. In order that the argument $a$ corresponding to the logarithm $L$ may be found, the fraction $\frac{\delta}{\Delta}$ is computed and added to the argument of the logarithm $L_{1}$. Thus if $L_{1}+f_{1}, L_{2}+f_{2}$, are the true values of the logarithms, and therefore

$$
\delta-f_{1} \text { and } \Delta+f_{2}-f_{1}
$$

are the true differences, we have

$$
\frac{\delta-f_{1}}{\Delta+f_{2}-f_{1}}=\frac{\delta}{\Delta}-\frac{1}{\Delta} f_{1}-\frac{\delta}{\Delta^{2}}\left(f_{2}-f_{1}\right)+\ldots
$$

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Then, $\frac{\delta}{. J}$ being put equal to $\varepsilon$, the error by which $a$ is affected will be

$$
-\frac{1}{4}\left\{(1-\varepsilon) f_{1}+\varepsilon f_{2}\right\}
$$

But since

$$
d(\log \varphi(a))=\frac{d \varphi(a)}{\varphi(a)} m d a=\Delta d a^{*}
$$

the error of the logarithm from which $a$ is found by means of the table may be written

$$
-(1-\varepsilon) f_{1}-\varepsilon f_{2}
$$

Again, by examination of the various values of $\varepsilon$ it aprears that the value $\varepsilon=\frac{1}{5}$ holds amost the mean place. Therefore in the change from any particular logarithm to the number or to the are the error of the logaritm nay be assumed as

$$
\begin{equation*}
f^{\prime \prime}=-\frac{4}{5} f_{1}-\frac{1}{5} f_{2} \tag{21}
\end{equation*}
$$

Lastly, if it is ncecssary to pass from the logarithm of one trigonometric function $\varphi(a)$ to the logarithm of another trigonometric function $\varphi^{\prime}(a)$ by the aid of the tables, the error attaching to $\log \varphi^{*}(a)$ is

$$
f\left[\log \varphi^{\prime}(\alpha)\right]=\frac{d \log \varphi^{\prime}(\alpha)}{d \log \varphi(a)} f^{\prime \prime}+f^{\prime}
$$

and it does not matter whether $a$ is first found from $\log \varphi(a)$ and then from this $\log \varphi^{\prime}(a)$, or whether the transition is made directly from $\log \varphi(a)$ to $\log \varphi^{\prime}(a)$ without reference to the argument $a$.

[^4]Accordingly the formulas [1] and [2] of § 2 are slightly modified, thus:

$$
\begin{aligned}
f(\log a) & =\frac{m}{a} f(a)+f^{\prime} \\
f(\log \sin A) & =\frac{m}{r} \cot A \cdot f(A)+f^{\prime} \\
f(\log \cos A) & =-\frac{m}{r} \tan A \cdot f(A)+f^{\prime} \\
f(\log \tan A) & =\frac{m}{r \sin A \cdot \cos A} \cdot f(A)+f^{\prime} \\
f(\log \cot A & =-\frac{m}{r} \frac{1}{\sin A \cdot \cos A} \cdot f(A)+f^{\prime} \\
f(a) & =\frac{a}{m}\left\{f(\log a)+f^{\prime \prime}\right\} \\
f(A) & =\frac{r}{m} \tan A\left\{f(\log \sin A)+f^{\prime \prime}\right\} \\
& =-\frac{r}{m} \cot A\left\{f(\log \cos A)+f^{\prime \prime}\right\} \\
& =\frac{r}{m} \sin A \cos A\left\{f(\log \tan A)+f^{\prime}\right\} \\
& =-\frac{r}{m} \sin A \cos A\left\{f(\log \cot A)+f^{\prime}\right\}
\end{aligned}
$$

Furthermore in the transition from $\log \tan A$ and $\log \operatorname{cct} A$ to $\log \sin A$ and $\log \cos A$ the following formulas are used:

$$
\begin{aligned}
& f(\log \sin A)=\cos ^{2} A \cdot f(\log \tan A)+\cos ^{2} A \cdot f^{\prime \prime \prime}+f^{\prime} \\
& f(\log \cos A)=-\sin ^{2} A \cdot f(\log \tan A)-\sin ^{2} A \cdot f^{\prime \prime}+f^{\prime} \\
& f(\log \sin A)=-\cos ^{2} A \cdot f(\log \cot A)-\cos ^{2} A \cdot f^{\prime \prime}+f^{\prime} \\
& f(\log \cos A)=\sin ^{\prime} A \cdot f(\log \cot A)+\sin ^{2} A \cdot f^{\prime \prime}+f^{\prime} \\
& f(\log \sin A)=-\cot ^{2} A \cdot f(\log \cos A)-\cot ^{2} A \cdot f^{\prime \prime}+f^{\prime}
\end{aligned}
$$

where $f^{\prime}$ and $f^{\prime \prime}$ are to be taken according to formulas [20] and [21]

$$
\begin{aligned}
& f^{\prime}=\frac{4}{8} f_{1}+\frac{1}{8} f_{2}+f_{3} \\
& f^{\prime \prime}=-\frac{4}{8} f_{4}-f_{5}
\end{aligned}
$$

in which all the quantities $f_{1}, f_{2}$ etc., are entirely independent, so that each one denotes any one of the equally probable values between $-1 / 2$ and $+1 / 2$, whose number is infinite.

## § 13.

The formulas set forth above are sufficient to indicate the probable error and the mean error of the result of any computation according to any formula whatever, and to show how great is the likelihood that the error is contained within any assigned limits. In order to show this by an example, we shall examine more carefully some formulas according to which the third side in a spherical triangle may be found if two sides and the included angle are given.
A. First let us consider the formulas by means of which the third angle is computed by means the auxiliary angle $\mu$ where we put

$$
\cot \mu=\frac{\sin \frac{3}{2} C \sqrt{\sin a \sin b}}{\sin \frac{1}{2}(a-b)}
$$

and find

$$
\sin \frac{1}{3} c=\frac{\sin _{\frac{1}{2}}^{2} C \sqrt{\sqrt{\sin a \sin , b}}}{\cos \mu}=\frac{\sin \frac{1}{2}(\alpha-b)}{\sin \mu}
$$

From this formula $\log$ cot $\mu$ is computed by the addition of three logarithms, from the sum of which one is subtracted. Each one of these is assumed to be affected by an error $f^{\prime}$ (which in $\log \sin a$ and in $\log \sin b$ is in fact diminished a half on account of the division by 2 , if account is taken of the half-units arising in the division of the last decimal place). So we have the equation

$$
f(\log \cot \mu)=f_{1}^{\prime}+\frac{1}{2} f^{\prime}{ }_{2}+\frac{1}{2} f_{s}^{\prime}-f_{4}^{\prime}
$$

in which the subscripts indicate that the various quantities f? are independent.

Then according to the equations set forth in § 12 we shall have

$$
\begin{aligned}
& f(\log \cos \mu)=\sin ^{2} \mu\left(f^{\prime}{ }_{1}+\frac{1}{8} f^{\prime}{ }_{2}+\frac{1}{8} f^{\prime}{ }_{3}-f^{\prime}{ }_{4}\right)+\sin ^{2} \mu f^{\prime}{ }_{1}+f^{\prime}{ }_{5} \\
& f(\log \sin \mu)=-\cos ^{2} \mu\left(f^{\prime}{ }_{1}+\frac{1}{8} f^{\prime}{ }_{8}+\frac{1}{3} f^{\prime}{ }_{8}-f^{\prime}{ }_{4}\right)-\cos ^{2} \mu f^{\prime \prime}{ }_{2}+f^{\prime}{ }_{8} \\
& f\left(\log \sin \frac{1}{2} c\right)=f\left(\log \sin \frac{1}{2} C \sqrt{\sin a \sin b}\right)-f(\log \cos \mu) \\
& =f^{\prime}{ }_{1}+\frac{1}{2} f^{\prime}{ }_{2}+\frac{1}{2} f^{\prime}{ }_{3}-\sin ^{2} \mu\left(f^{\prime}{ }_{1}+\frac{1}{2} f^{\prime}{ }_{2}+\frac{1}{2} f^{\prime}{ }_{3}-f^{\prime}{ }_{4}\right) \\
& -\sin ^{2} \mu f^{\prime \prime}{ }_{1}-f^{\prime}{ }_{s}
\end{aligned}
$$

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Then we have

$$
f(c)=\frac{2 r}{m} \tan \frac{1}{8} c\left\{f\left(\log \sin \frac{1}{8} c\right)+f^{\prime \prime} 3\right\}
$$

If next the value found for $f(\log \sin 1 / 2 c)$ is substituted in these equations and if every $f^{\prime}$ is changed into the form ${ }^{\frac{4}{5}} f_{1}+\frac{1}{5} f_{2}+f_{3}$, and every $f^{\prime \prime}$ into the form - ${ }^{\frac{4}{5}} f_{1}-\frac{1}{5} f_{2}$, and then if, for simplicity, $f_{1}, f_{2}$, etc., be denoted by (1), (2), etc., so that we may write

$$
\begin{aligned}
& J^{\prime}{ }_{1}=\frac{4}{5}(1)+\frac{1}{5}(2)+(3) \\
& f^{\prime}{ }_{2}=\frac{4}{5}(4)+\frac{1}{5}(5)+(C) \\
& f^{\prime}{ }_{3}=\frac{4}{5}(7)+\frac{1}{5}(8)+(9) \\
& f^{\prime \prime \prime}=\frac{4}{5}(10)+\frac{1}{5}(11)+(12) \\
& J^{\prime \prime}{ }_{1}=-\frac{4}{5}(1.3)-\frac{1}{5}(14) \\
& \left.f^{\prime}{ }_{5}=\frac{4}{5}(15)+\frac{1}{5} 16\right)+(17) \\
& f^{\prime \prime}{ }_{2}=-\frac{4}{5}(18)-\frac{1}{5}(19) \\
& f^{\prime}{ }_{6}=\frac{4}{5}(20)+\frac{1}{5}(21)+(22) \\
& f^{\prime \prime}{ }_{8}=-\frac{4}{5}(23)-\frac{1}{5}(24)
\end{aligned}
$$

## we shall have

$$
f(c)=\frac{2 r}{m} \tan \frac{1}{2} c \times\left\{\begin{array}{c}
\cos ^{2} \mu\left\{\begin{array}{c}
\frac{4}{5}(1)+\frac{1}{5}(2)+(3) \\
+\frac{4}{10}(4)+\frac{1}{10}(5)+\frac{1}{2}(6) \\
+\frac{4}{10}(7)+\frac{1}{10}(8)+\frac{1}{2}(9)
\end{array}\right\} \\
+\sin ^{2} \mu\left\{\begin{array}{c}
\frac{4}{5}(10)+\frac{1}{5}(11)+(12) \\
+\frac{4}{5}(13)+\frac{1}{5}(14)
\end{array}\right\} \\
-\frac{4}{5}(15)-\frac{1}{5}(16)-(17) \\
-\frac{4}{5}(23)-\frac{1}{5}(24)
\end{array}\right\}
$$

Therefore the probable error by which $c$ is affected, if six place logarithms are used, is

$$
w F=10-6 \frac{\rho}{\sqrt{6}} \sqrt{\Sigma\left(\alpha^{2}\right)}
$$

in which it is necessary to write the coefficients of the quan ${ }^{\text {ities }}$

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(1), (2), etc., in place of $\alpha$. By this substitution we shall have

$$
\sqrt{\sum\left(\alpha^{2}\right)}=\frac{2 r}{m} \tan \frac{1}{2} c_{2} \sqrt{2,52 \cos ^{4} \mu+2,36 \sin ^{4} \mu+2,3 \mathrm{~J}}
$$

and, after writing for $r, m$, and $\rho$ the valucs

$$
\begin{aligned}
r & =206265 \\
m & =0,43122 \\
\frac{\rho}{v \overline{5}} & =0,1917
\end{aligned}
$$

we have

$$
w F(c)=0,15105 \tan 12 \sqrt{2} 2, \overline{25 \cos ^{4} \mu+2,36 \sin ^{4} \mu+2,36}
$$

expressed in seconds.
When $\mu$ comes out $>45^{\circ}$, another form is more convenient.*

$$
\sin \frac{1}{2} c=\frac{\sin \frac{1}{2}(a-b)}{\sin \mu}
$$

so that we have this condition:

$$
w F(\theta)=0,1 8 4 9 5 \operatorname { t a n } \frac { 1 } { 2 } \cdot \sqrt { 3 } 3 \longdiv { 2 0 \operatorname { c o s } ^ { 4 } \mu + 1 , 6 8 \operatorname { s i n } ^ { 4 } \mu + 2 , 3 6 }
$$

From these formulas, the first for values of $\mu<45^{\circ}$, the last for $\mu>45^{\circ}$, these values of $w \cdot F$ arise:

| $\mu$ | $w F(c)$ |  |
| :---: | :---: | :---: |
| $0^{\circ}$ | 0,"408ā7 | $\tan \frac{1}{6}$ |
| 10 | 0. 40235 |  |
| 20 | 0,38607 | ، |
| 30 | 0, 36641 | ، |
| 40 | 0,35241 | '6 |
| 50 | 0,34526 | '6 |
| 60 | 0,34626 | '6 |
| 70 | 0,35628 | ، |
| 80 | 0,36727 | '6 |
| 90 | 0,37175 | " |

[^5]Since the coefficients of $\tan 1 / 2 c$, which depend on the quantity $\mu$, do not vary greatly, the quantity $0.37^{*}$, which is scarcely $10 \%$ different from the extreme values, may be written as a mean value, so that we have:

| $c$ | $w F(c)$ |
| :---: | :---: |
| $0^{\circ}$ | $0,{ }^{\circ} 00$ |
| 10 | 0,03 |
| 20 | 0,07 |
| 30 | 0,10 |
| 40 | 0,14 |
| 50 | 0,17 |
| $c 0$ | 0,21 |
| 70 | 0,26 |
| 80 | 0,31 |
| 90 | 0,37 |

There is no need to continue this table, which shows approximately the probable error corresponding to any particular value of $c$, since for any value $c<90^{\circ}$ another equation

$$
\cos ^{2} \frac{1}{2} c=\cos ^{2} \frac{1}{2}(a+b)+\sin a \sin b \cos ^{2} \frac{1}{8} C
$$

is employed, the solution of which is the same and which has this same probable error for values of $c$ from $c=90^{\circ}$ to $c=180^{\circ}$.

The probability $W$ of an error within certain limits, or of an error within $\pm 1$ second, is computed from the equation [17].

$$
W=\underset{V \sqrt{\pi}}{2} \int_{0}^{\frac{\zeta}{\sigma} \sqrt{\overline{6}}} \bar{\nu}
$$

[^6]Now, since

$$
\frac{\zeta}{6} \sqrt{\frac{6}{v}}=\frac{\zeta \rho}{w F}
$$

let the limit be put for $\zeta$ and the probable error for $u F$, and the value of $W$ corresponding to the argument $\frac{\zeta}{2 \boldsymbol{F}}$ may be sought in a table of this integral.

If for example the greatest value found above ( 0.37 ) is put for $w F$, and $\zeta=1$, then the tables of the integral show for the argument $\frac{1}{0,37}$ the corresponding value $W=0.93$, or, in other words, under the most unfavorable condition ( $c=90^{\circ}$ ) there are 93 errors out of 100 which are less tha a one second.
B. As another example take the equation which is commended in many mathematical works,

$$
\sin ^{2} 1 / 2 c=\sin ^{2} 1 / 2(a+b)-\sin a \sin b \cos ^{2} 1 / 2 C
$$

Its solution is as follows: first from 'ne equation:

$$
\cos \mu=\frac{\cos 1 / 2 C \frac{1}{\sin a \sin b}}{\sin 1 / 2(a+b)}
$$

the auxiliary angle $\mu$ is compute $\bar{y}$, then $c$ from the equation

$$
\sin 1 / 2 c=\sin 1 / 2(a+b) \sin \mu
$$

Since $\log \cos \mu$ is computed by means of four logarithms taken singly from the tables, we shall have

$$
f(\log \cos \mu)=f_{1}^{\prime}+1 / 2 f_{8}^{\prime}+1 / 2 f_{3}^{\prime}-f^{\prime}
$$

in which $f^{\prime}{ }_{4}$ belongs to $\sin 1 / 2(a+b)$. Thence it follows that

$$
\begin{gathered}
f(\log \sin \mu)=-\cot ^{2} \mu\left(f^{\prime}{ }_{1}+1 / 2 f^{\prime}{ }_{8}+1 / 2 f^{\prime}{ }_{3}-f^{\prime}{ }_{4}\right)-\cot ^{2} \mu f^{\prime}{ }_{1}+f^{\prime}{ }_{5} \\
f(\log \sin 1 / 2 c)=\frac{1}{\sin ^{2} \mu} f^{\prime}{ }_{4}-\cot ^{2} \mu\left(f^{\prime}{ }_{1}+1 / 2 f_{2}+1 / 2 f_{3}^{\prime}+f^{\prime}{ }_{1}\right)+f_{5}^{\prime} \\
f(c)=\frac{2 r}{m} \tan 1 / 2 c .\left\{f(\log \sin 1 / 2 c)+f^{\prime}{ }_{2}\right\}
\end{gathered}
$$

Then if, instead of $f^{\prime}$ and $f^{\prime \prime}$ we put

$$
\frac{8}{5}(1)+\frac{1}{5}(2)+(3) \text { and } \frac{4}{5}(4)+\frac{1}{5}(5)
$$

we shall have ninetsen independent sources of error and find that

$$
\sqrt{\Sigma\left(\alpha^{2}\right)}=\frac{2 r}{m} \tan 1 / \curvearrowright c \sqrt{\left(1,68 \frac{1}{\sin ^{4} \mu}+3,20 \cot ^{4} \mu+2,36\right)}
$$

Lastly by inserting the factor

$$
10^{-6} \frac{\rho}{\sqrt{b}}
$$

we shall obtain the probable error by which $c$ is affected, as expressed in the following table:

| $\mu$ | $w F(c)$ |  |
| :---: | :---: | :---: |
| $0^{3}$ | $\infty$ |  |
| 10 | 13,"286 $\tan 1 / 20$ |  |
| 20 | 3, 243 | ، |
| ¢0 | 1,409 | ، |
| 40 | 0, 7*9 | ، 6 |
| co | 0.519 | ، |
| co | 0,442 | '، |
| 70 | 0,395 | ، |
| 80 | 0,377 | ، |
| 90 | 0، 372 | ' |

Since $w F$ approaches infinity as $\mu$ approaches zero, this formula is eidently unsuitable for general use.
C. The Gaussian equations for computing the third side $\boldsymbol{c}$ furnish another example. They are the following four, if we put

$$
1 / 2(A-B)=\mu \text { and } 1 / 2(A+B)=\nu
$$

viz:

$$
\begin{aligned}
\sin 1 / 2 c \sin \mu & =\sin 1 / 2(a-b) \cos 1 / 2 C \\
\sin 1 / 2 c \cos \mu & =\sin 1 / 2(a+b) \sin 1 / 2 C \\
\cos 1 / 2 c \sin \nu & =\cos 1 / 2(a-b) \cos 1 / 2 C \\
\cos 1 / 2 c \cos v & =\cos 1 / 2(a+b) \sin 1 / 2 C .
\end{aligned}
$$

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The order in which the logarithms are written is as follows:

1. $\sin 1 / 2(a-b) \cos 1 / 2 C$
2. $\cos 1 / 2(a-b) \cos 1 / 2 C$
3. $\sin 1 / 2(a+b) \sin 1 \frac{1}{2} C$
4. $\cos 1 / 2(a+b) \sin 1 / 2 C$

5. $\frac{\sin \nu}{\cos \frac{112}{1 / 2^{c}}}$

Lastly from the difference of the logarithms of 4. and 8. arises $\log \tan 1 / 2 c$, by means of which the side $c$ is found. But since the angles $u$ and $v$, which are used here only as auxiliaries, have but a small influence upon the probable error by which $c$ is affected, as already has appeared in example A, the formula may be simplified. As the transition has taken place in angle $\mu$ from tangent to cosine, in angle $v$ to sine, we shall write in place of them $0^{\circ}$ and $90^{\circ}$ respectively. Nevertheless we could pursue a different method without causing any essential difference in the final formula.

Furthermore, if the errors by which $\log \cos 1 / 2 C$ and $\log \sin$ $1 / 2 C$ are affected be denoted by $f^{\prime}{ }_{1}$ and $f^{\prime}{ }_{2}$, and the errors of the other logarithms, which are used only once in the computation, by $f^{\prime}, f^{\prime}$, etc., we shall have

$$
\begin{aligned}
& f(\log \tan \mu)=f^{\prime}{ }_{1}-f^{\prime}{ }_{2}+f^{\prime}{ }_{8}-f^{\prime}{ }_{4} \\
& f(\log \cos \mu)=-\sin ^{2} \mu\left(f_{1}^{\prime}-f^{\prime}{ }_{3}+f^{\prime}{ }_{8}-f^{\prime}{ }_{4}\right)-\sin ^{2} \mu f^{\prime \prime}{ }_{1}+f^{\prime}{ }_{5} \\
& f(\log \sin 1 / 2 c)=f^{\prime}{ }_{2}+f^{\prime}{ }_{4}-f(\log \cos \mu)
\end{aligned}
$$

then

$$
\begin{aligned}
& f(\log \tan v)=f_{1}^{\prime}-f^{\prime}{ }_{2}+f^{\prime}{ }_{6}-f^{\prime}{ }_{7} \\
& f(\log \sin v)=\cos ^{3} v\left(f^{\prime}{ }_{1}-f_{2}^{\prime}+f^{\prime}{ }_{6}-f^{\prime}{ }_{7}\right)+\cos ^{2} v f^{\prime}{ }_{2}-f^{\prime} 8 \\
& f\left(\log \cos 1 / 1_{2} c\right)=f_{1}+f^{\prime}{ }_{6}-f^{(\log \sin v)}
\end{aligned}
$$

## lastly

$$
f\left(\log \tan \frac{1}{2} c\right)=-f^{\prime}{ }_{1}+f^{\prime}{ }_{2}+f^{\prime}-f^{\prime}{ }_{0}-f(\log \cos \mu)+f(\log \sin v)
$$

whence it follows that

$$
f(c)=\frac{r}{m} \sin c\left\{f(\log \tan 1 / 2 c)+f^{\prime}{ }_{s}\right\}
$$

If in this formula we substitute the values already found, and put $\mu=0^{\circ}, \nu=90^{\circ}$, and for $f^{\prime}$ and $f^{\prime \prime}$ the values found in $\S 12$, it will be seen that $f(c)$ depends on twenty sources of error and that

$$
\sqrt{\sum\left(\alpha^{2}\right)}=\frac{r}{m} \sin c \sqrt{10.76}
$$

This value being substituted in

$$
w F(c)=10-\frac{\rho}{\sqrt{\bar{B}}} \sqrt{-\zeta\left(\alpha^{2}\right)}
$$

it follows that

$$
w F(c)=0, " 3033 \sin c
$$

or $w(c)$ will have the values shown in the following table:

| $c$ | $w F(c)$ |
| :---: | :---: |
| 00 | $0, " 00$ |
| 10 | 0,05 |
| 20 | 0,10 |
| 30 | 0,15 |
| 40 | 0,20 |
| 50 | 0,23 |
| 60 | 0,26 |
| 70 | 0,29 |
| 80 | 0,30 |
| 90 | 0,30 |

Thus it appears from this computation that the probable error is a little less than in the formula proposed under A, but the computation itself is longer on account of the auxiliaries, which moreover are in this case of no further use.
D. Lastly it is advisable to examine some equations in which functions of the entire angles are used in computing triangles.

Let us begin with the equations

$$
\begin{aligned}
\sin c \sin A & =\sin a \sin C \\
\sin c \cos A & =\cos a \sin b-\sin a \cos b \cos C \\
\cos c \quad & =\cos a \cos b+\sin a \sin b \cos C
\end{aligned}
$$

and put

$$
\begin{aligned}
\sin a \cos C & =m \cdot \sin M . \\
\cos a & =m \cdot \cos M .
\end{aligned}
$$

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Then we shall have

$$
\begin{aligned}
\sin c \sin A & =\sin a \sin C \\
\sin c \cos A & =m \sin (b-A) \\
\cos c \quad & =m \cos (b-M)
\end{aligned}
$$

Hence in order to find $c$ there must be computed

$$
\begin{aligned}
& \tan M=\tan a \cos C \\
& \tan A=\frac{\sin M}{\sin (b-M)} \tan C \\
& \tan c=\frac{\tan (b-M)}{\cos A}
\end{aligned}
$$

Accordingly we have

$$
\begin{aligned}
& f(\log \tan M) \quad=f^{\prime}{ }_{1}+f^{\prime}{ }_{2} \\
& f(M) \quad=\frac{r}{m} \sin M \cos M\left(f^{\prime}{ }_{1}+f^{\prime}{ }_{2}+f^{\prime \prime}{ }_{1}\right) \\
& f(\log \sin M) \quad=\cos ^{2} M\left(f^{\prime}{ }_{1}+f^{\prime}{ }_{2}+f_{1}{ }^{\prime \prime}\right)+f^{\prime}{ }_{3} \\
& f(\log \sin [b-M])=-\cot (b-M) \sin M \cos M\left(f^{\prime}{ }_{1}+f^{\prime}{ }_{2}+f^{\prime \prime \prime}{ }_{1}\right)+f^{\prime}{ }_{6} \\
& f\left(\log \tan [(b-M])=-\frac{\sin M \cos M}{\sin (b-M) \cos (b-M)}\left(f^{\prime}{ }_{1}+f^{\prime}{ }_{2}+f^{\prime \prime}{ }_{1}\right)+f^{\prime}{ }_{5}\right. \\
& f(\log \tan A) \quad=f(\log \sin M)-f(\log \sin [b-M])+f^{\prime} . \\
& f(\log \cos A) \quad=-\sin ^{9} A\left\{\begin{array}{c}
f(\log \sin M)-f(\log \sin (b-M) \\
+f^{\prime}+f^{\prime} 9
\end{array}\right\}+f_{8}^{\prime} \\
& f(\log \tan c) \quad=-\frac{\sin M \cos M}{\sin (b-M) \cos (b-M)}\left(f^{\prime}{ }_{1}+f^{\prime}{ }_{2}+f^{\prime}{ }_{1}\right)+f^{\prime}{ }_{B} \\
& +\sin ^{2} A \cos ^{2} M\left(f^{\prime}{ }_{1}+f^{\prime}{ }_{2}+f^{\prime \prime}{ }_{1}\right)+\sin ^{2} A \cdot f^{\prime}{ }_{8} \\
& +\sin ^{2} A \cot (b-M) \sin M \cos M\left(f^{\prime}{ }_{1}+f^{\prime}{ }_{2}+f^{\prime \prime}{ }_{1}\right) \\
& -\sin ^{2} A \cdot f^{\prime}{ }_{4}+\sin ^{2} A \cdot f^{\prime}{ }_{6}+\sin ^{2} A \cdot f^{\prime \prime},-f^{\prime}{ }_{7}
\end{aligned}
$$

finally,

$$
f(c)=\frac{r}{2 m} \sin 2 c\left\{f(\log \tan c)+f^{\prime}:\right\}
$$

Then if we put the side $b=90^{\circ}$ in order to simplify the computation and to obtain a representative value from which the others differ but little, the formula $f(c)$ is changed as follows:
$f(c)=\stackrel{r}{2}-\stackrel{r}{2} \sin 2 c\left\{\begin{array}{l}-\cos ^{2} A\left(f_{1}^{\prime}+f^{\prime}{ }_{2}+f^{\prime \prime}{ }_{1}\right)+f^{\prime}{ }_{5}-f^{\prime}{ }_{7}+f^{\prime \prime}{ }_{3} \\ +\sin ^{2} A\left(f_{3}-f^{\prime}{ }_{4}+f^{\prime}{ }_{6}-f^{\prime \prime}{ }_{2}\right)\end{array}\right\}$

Then, after we have written for the various quantities $f^{\prime}$ and $f^{\prime \prime}$, their values according to $\S 12$ there are seen to be 27 sources of error, and

$$
\sqrt{\Sigma\left(\alpha^{2}\right)}=\frac{r}{2 m} \sin 2 c \sqrt{4,04 \cos ^{4} A+5,72 \sin ^{4} A+4,04}
$$

whence it follows that

$$
w F(c)=10-6 \frac{\rho}{\sqrt{6}} \sqrt{\Sigma\left(\alpha^{2}\right)}
$$

The values of $w F(c)$ corresponding to different values of $A$ will be

| $A$ | $w F(c)$ |  |
| :---: | :---: | :---: |
| $0^{\circ}$ | $0,{ }^{\circ} 1314$ | $\sin 2 c$ |
| 10 | 0,1293 | $،$ |
| 20 | 0,1245 | $،$ |
| 30 | 0,1193 | $،$ |
| 40 | 0,1170 | $،$ |
| 50 | 0,1196 | $،$ |
| 60 | 0,1276 | $،$ |
| 70 | 0,1353 | $،$ |
| 80 | 0,1419 | $،$ |
| 90 | 0,1444 | $،$ |

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If instead of the factor which depends on $A$ we put the constant 0.1314 we shall have

| c | $w F(c)$ |
| :---: | :---: |
| $0^{\circ}$ | 0,00 |
| 10 | 0,"04 |
| 20 | 0,"08 |
| 30 | 0,"11 |
| 40 | 0,"13 |
| 50 | 0,"13 |
| 60 | 0,"11 |
| 70 | 0,08 |
| 80 | 0,04 |
| 90 | 0,"00 |

The maximum value of the probable error will here be $\ddagger$ of a second, while in the formulas considered under A and C it was $1 / 3$ of a second. Furthermore, as long as $c$ has a value less than $45^{\circ}$, formulas A, C, and D are affected with nearly the same error, but for greater values of $c$ the last equations are evidently to be preferred.

These examples may suffice to show the use of the formulas proposed above. Space does not permit the continuation of this subject in this place, but its further study may be strongly urged upon those who are required to make frequent use of formulas, as in astronomy. In general it will appear that a slight change in a formula will sometimes bring about a very great difference in precision, as cxamples A and B show; and furthermore, that the computation in which the functions of whole angles are used, such as was proposed in D, usually affords greater precision than if functions of the half angles were used, although usually at the cost of a greater expenditure of time. The dotermination of the probable error ought never to be omitted in actual practice since the following formulas:

$$
\begin{aligned}
\tan M & =\tan a \cos C \\
\cos c & =\cos a \frac{\cos (b-M)}{\cos M}
\end{aligned}
$$

which are recommended in many mathematical books instend of those which we have examined under D, ought not to be used
except with great circumspection and within proper limits. These limits having been found by an investigation of the probable error, such a formula, although useless elsewhere, can be used conveniently in the cases and within the limits where it offers the advantage of brevity and still affords sufficient precision.

## PERSONAL NAMES:

## THEIR SIGNIFICANCE AND HISTORICAL ORIGIN.

## JAMES DAVIE BUTLER, LL. D.

"Go to pot, I tell you, Sir, go to pot!" These words were my greeting from the head of the Boston public library when I began to seek there for the significance of personal names. His language was brusk and would have sounded contemptuous to a stranger. But he had been my classmate a decade before and had taken as many jokes as he had given. His meaning was; The best book in Boston to tell you what your name means is a volume by Augustus Frederick Pott. Its full title is; Die Personennamen; insbesondere die Familien-namen und ihre Entstehungarten, auch unter Berücksichtigung der Ortsnamen.

This work published in 1853 is by no means antiquated. It is larger than any one of the twenty volumes on the subject in our Historical Library,-and though each of those more recent works has points of superiority,-no better advice can now be given to a beginner in patronomatology than "Go to Pott, I tell you, Sir, go to Pott!"

My friend's jocular order, however, led me to tell him a trifle of my earliest experience in the Paris police-office when passports were a daily necessity. Ushered into a long hall where a score of clerks were writing on each side, I walked up to the nearest one, passport in hand, wishing to get it vised. He glanced up at me from his desk, and said O Booh! paying me no further attention. I passed on to another writer who also said O Booh! and nothing more. I turned across the room but heard no other salutation. O Booh to right of me, O Booh to
left of me, O Booh behind me, volleyed and thundered. At last it dawned on my darkness that no insulting English vocables were in the minds of the vociferators but that they were only informing me that I must go to the end of the office, au bout!

No theme so comes home to all men's business and bosoms as personal names. Some savor of their significance pervades all literature sacred and profane-downward from the Bible and Homer. The earliest family quarrel we read of was when an Attic farmer who never spent anything had taken a wife who counted nobody respectable that did not keep a horse. In naming their first boy he insisted on a name with the element saving in it, while she would hear of none which lacked the syllable horse. By way of compromise the name Pheidippides, that is son of a saving horse was invented and adopted.

Personal names being then of such varied interest cosmopolitan, pre-historic, post-historic, con-historic, sub-historic, no one should essay to treat the general subject in fewer pages than the thousand of Pott, indeed were Pott now writing and in regard to our American poly-glott and panti-glott directories his book would become doubly ponderous.

Broadly speaking personal names may be said to be derived from three sources, namely, first, some characteristic, actual, imagined or ascribed, secondly, one's occupation, and thirdly, his place of abode. Before tracing a name, however, to any one of these sources we must often ascertain the meaning of obsolete words or forms of words in our own tongue, or study foreign languages, or become conversant with many varieties of industry now no longer known or carried on by new processes.

In the following article the names chosen for illustrating the subject are of persons well known in Madison, but those of people in any other town or those of authors on the backs of books in any library would have been equally serviceable for the writer's purpose.

The earliest personal names were naturally given in view of some personal characteristic.

Hence originated the first Fairchild and the first Brown. Morris, that is Moorish, is another name for dark brown.

Adams and Reid-both mean red-the one being Hebrew and the other Scotch. Russell as a diminutive of red we may define as ruddy. So Julius as well as La Follette means soft hair, and Favill, as some think, is yellow hair.

Greeks called one man Eustace that is well-put-together, or Andrew meaning manly, Jews called him Asahel, that is Godmade, Saxons called him Charles and Irish Bryant, both meaning stout. A similar idea of physical prowess led to such names as Storm Bull, Buell also meaning bull, Hoyt, nimble, Bjorn, a bear, Oscar, an agile warrior, Martyn, Mars-like, Knapp, a striker.
Other names were indicative of mental qualities. Hugh was a thinker, Hutchinson and McKee both signify sons of Hugh. Hubert was bright Hugh, and so Albert is all-bright. Robert fame-bright and Herbert, army-bright, Gilbert, bright servant whose son became Gibson, Gib being a contract of Gilbert. So Aubertine was white-bright or perhaps self-bright. Hobbs and Hobbins are possibly allied to Robert, i. e., fame-bright.
Such of our personal names as do not show location or characteristics, are descriptive of occupation.
Among the most ancient of this class is Therese. This name is Greek and means either huntress or female harvester. Birge if not birch, may be French for shepherd, (Berger) in pastoral poets, a rustic lover. Birge is, however, more probably a contract from early English Her-berg-ere, meaning harborer, that is innkeeper. George is Greek for earthworker, that is farmer, which in German is Myers, in old English it is Bower and Burr. Gregory is Greek for watchman, not unlike Ward in English. Edward is a guard of goods, and Woodward of trees, and akin to Forster [forester]. The London Bunhill was named from a cake, and the proper name Bunn would seem to mean the maker or seller of that dainty.

Among occupational names which are plain of themselves we have Mason, Carpenter, Turner, and Bowman. Homer is set down as helmeter, that is helmet-maker,
Smith, is a smiter, and his monosyllabic name may be a survival of some compound, as Arrowsmith, Bowsmith, Blacksmith, Whitesmith, Gunsmith, Goldsmith, Silversmith, Song-
smith [poet]. It is owing to countless defunct compounds that Smiths are multitudinous. Baird is a poet, being Scotch for bard.

All patronymics-or names showing men's fathers, are in a loose sense characteristic. Such are all names ending in son, as Carson, son of Carr; many that end in $s$ as Hobbs, all that begin with Mc, and some that begin with B. Thus, Bolivar is the son of Oliver, and Breese the son of Reese, or Rice. This Welsh word Rhys is of cognate origin with the Latin rex, a king. Characteristic names when given to children betokened qualities not always possessed-but attributed by parental affection, or hoped for. Rasmus, contracted from the Greek Erasmus,in Latin Desiderius-one longed for and hence loved, is identical in significance with David and Davie in Hebrew (with Taffy as a variant), and Cary in Trish. A similar feeling led Leah at the birth of her first-born to exclaim, "Reuben!"-that is behold a son! Where primogeniture was law it was common to call the oldest boy Barnes, that is the bairn-the child by way of eminence as the predestined heir. In astrological eras as the hour of birth was held to have a life-long influence, and those born at dawn were destined to good luck, a boy born then was named Lucius or Lucien, that is, belonging to light, and so of good omen. Hence arose the Latin proverb Nomen omen.

Parental appreciations-sometimes exaggerations-of babes in the house, are to be credited with such names as Theodore, i, e., gift of God; John and Jones, God's grace, the most pervasive name in Christendom, leaving no corner of it untouched. On the same lines were formed Margaret-a pearl, Rosellaa little rose, Romanzo-a fairy tale, Anna-condescending, Augustus-dignified, Stevens-crowned, Emil-emulous, Ed-win-a lucky winner, Willard-strong, will, Elizabeth-devout, Delia-chaste from a name of the Grecian Artemis, Gold-the most precious of all metals and as too many think of all things-Stearns-a star, though some interpret it "stern, from the natural disposition of the first bearer."

Other names characterize still less flatteringly. Thus the Irish called Cassoday [Cassidy] an accuser or caviler, Jacob as well as its variant James is a heeler, heel-catcher, because the
patriarch at first seized his brother by the heel (Gen., 25, 26), and afterward tripped him up; and Favill in old English was a liar or at least a fabulist. Says Chaucer:
"Looke on the luft-hond, and see wher he stondeth Both fals and fauvel, and all his hole meyne."

Names suggested by temperament are frequent. Thus, a sanguineous daughter was called Ella-that is, sunny; Allen, if it was not at first a hound, All-win, as if he had won all, meant cheerful; Isaac meant laughter, for he was the cause of it; Mary-in Latin, Maria-was tearful, and so was well named bitter. The same sense I find assigned to the first Hervelin who was born blind, and so to a bitter life.

Other characteristic appellations allude to some exploit historic, or prophetic, or hoped for. Thus, Daniel is divine judge; Uriah, light of God; Joseph, augmentation; Jairus, enlightener; Alexander, helper of men; William, helmet of many; Louis, bold warrior; Pringle, pilgrim. Oliver-so called from complexion, or as living by an olivet-long meant peaceful, with a reference to the olive branch of Noah's dove. But becoming confused with Olaf, the old Norse fighter, it came to mean a champion whom no one but Roland could match. Hence the phrase, Give a Roland for an Oliver. Henry means home-ruler; Hanks, Haskins, and Hendrick are some of its variants, though Lower says Hanks means house-wolf (p. 146). Frederick is peace-ruler; Richard, great heart; Willard, strong will; Sarah is mistress. Noble explains itself, and also Eugene, which is Greek for well-born. Freeman describes a man who first among a crowd of serfs broke his birth's invidious bar, and so was known by way of distinction as the freeman.

Another class of personal names denotes location-the place of origin or abode.

Jastrow comes from a Polish town so called in the present province of Posen. Brandenburger is a name given to one who originated in that German region of which Berlin is the capital, and Frankenburger denoted one who came from an impe rial circle now in Bavaria. The name Lincoln showed that a man had to do with the English county of that name. Orville
and Favill, if French, may mean: the one a town of gold, and the other a suburb. Ely and Ramsay indicate islands, the one of eels and the other of rams, as Shelly points to an isle of shells. Pinney is isle of pins, a word which of old included both pegs and underpinning. The name of old was spelled with a y; so was pin, which also had two n's. Such place-names were often first given by outsiders when natives migrated, and in their new homes were thought of in reference to their provenance.

Several names came from connection with trees. Grover, one dwelling by a grove, is similar to Atwood. Analogous is Silas, that is, woodman, being a contraction of silvanus, which is woodman in Latin. So Birge, if English, is probably birch wood. Thwaites is a clearing in a wood,-a place where the trees have been thwacked. Other names show connection with valleys. Comstock is the clan in a comb-comb in old English meaning a valley or the ridge which hems it in. The word honeycomb still shows the ancient meaning. Dalzel is the white dell, and Kendall the dale of kindred.

Newton-new town-is self-explaining. Stanley is a field of stone. Lansing-the field of a lancer-reminds of a Westernism, namely, tomahawk right. Sheldon is hill of shells, and Conover, a cow-yard.

Several local names were given in reference to water. Moore is a marsh; so is Kerr, so is Van Hise-Hise being a corruption of ness, which is Dutch for a swamp, and akin to the German nass.

Bashford is the passage of a stream; the first syllable may be bas as in Bascom-i. e., lower valley, or is it rather a variation of bush?* Olin, unless his lineage runs back to the Norse Ole, may denote a pool,-and especially, one not shallow. Lincoln is a colony beside a pool. Upham is upper hamlet, and Updike probably upon the dike.

The name Wright is the same word as worker, the letter r being transposed. It was in usage restricted to a mechanical worker, as cartwright, etc. The name Wayne, the old English form of wain, or wagon, is still found in the compound, Wain-

[^7]wright. The name Cole is collier abbreviated, and Mills is miller. McMynn is Scotch for son of milner [miller]; Stuart, that is, Stow-ward, is guardian of what is stowed in a house. Reeve is a similar term, but with a wider meaning. Thus Chaucer says, Bards, 223 :

> "His lordes shepe, his nete and his deirie, His swine, his hors, his store and his pultrie Were wholly in this reves governing."

It is agreed that Butler means cup-bearer, but it has been doubted whether his name came from the bottle he uncorks or from the butt out of which bottles are filled. The name King, apparently signifying the highest of occupations, may really have been an adjunct of the lowest-King's groom, scullion or factotum, as King John said (iv., 2, 222) of such tools,

> "Fellows by the hand of nature marked Quoted and signed to do a deed of shame."

It seems clear, on the whole, that personal names may all be reckoned to have been at first significant of characteristics, locations, or occupations. As time went on, however, those names were often bestowed without any regard to their original meaning. Christian names were from early ages so given, a fact too little dwelt on by writers on the specialty of names. Church calendars had their beginnings as early as the fourth century. They grew rapidly so that, though every dog has his day, many a saint must be content with none, or at most with a part of one. Each saint was viewed as the tutelar genius-or guardian of all children born on his own day-and so they were naturally often called by his name, with no care for their location, occupation or characteristics. All old almanacs-whatever they lacked-showed at least one saint for every day in the year. Such a list was invaluable. It taught the most ignorant mother in a moment what name it was predestinated her child should bear. This saintly calendar appeared in the first volume of the world-famous Gotha Almanac issued in 1776, and it has been repeated every year since. The adoption of names in accordance with the days of saints did not end at the Refor-
mation even in Protestant Europe. The Gotha roster is threefold: one column presents Greek saints, the second Catholic, and the third, equally long, the saints recognized by the Reformers. As often as the Lutherans rejected one of the Catholic band, they filled the gap by inserting another saint, and that usually from the Old Testament. Thus Genevieve was thrown out and Enoch was substituted. This fact shows where many hard Hebrew vocables came from which were fathered upon the Puritans, but have been proved to be older than they, and also prevalent in the church of England. Adam in the Reformed calendar stands for the Catholic Delphine, who was the tutelar of Dec. 24th.

Whatever then the proximate occasion of our being called by the names we bear, it would seem that not a few of them ultimately came to us from the calendar of the saints, often because we were namesakes of god-fathers. Tracing the possible lineage or descent of one single name may shed a side-light on that of others. The name George, as has been stated, means farmer and was derived from occupation. It was doubtless common in a region between the Black and Caspian seas which was called Georgia, or land of farmers, to distinguish it from the Nomads or pastoral tribes around. Near there in Cappadocia a certain George slew a dragon and saved an imperilled virgin. This exploit, as some hold rather prosaically, lay in delivering the ground from weeds and enabling it to yield crops. In the view of others, George vanquished the persecutors of the church, of whom the serpent Satan was chief. At all events, George became a saint, one of the seven champions of Christendom, and the patron of knights, and of several countries. In England, his royal chapel-still the finest in the kingdom-at Windsor, was completed four centuries ago. It was in 1349 that St. George had been taken by Edward III., at the siege of Calais, as the national saint of England. His name became the English battle-cry, so in Shakespeare Talbot exclaims: "Saint George and victory, fight, soldiers, fight!" A hundred and sixty-two parish churches have been ascertained to bear his name. His day was the 23d of April, but we may be sure that many a boy born on other days was christened with
this favorite's name. King George the First, who came from Hanover, getting his own name from the saint, passed it on to three of his successors. All the royal Georges unawares spread the name among unroyal scions, as when Calhoun took snuff all Carolina sneezed. Among such plebeian namesakes was George Washington, to whom more Americans proximately owe their first name than to any king or saint.

Several others of our names go back ultimately to notables whom they strive to keep in mind. Alexander is a memorial of a monarch who in 1286 was the Washington, or at least the Lincoln, of Scotland. Lucien was suggested to many a mother by the career of the most irreproachable brother of the first Napoleon. So was Eugene, by Napoleon's step-son Beauharnais, and Bolivar, meaning as already stated, son of Oliver, who, more than any other man, was the liberator of South America. His name has been linked with our greatest name by Byron, who sang:
> "The prophets of young freedom summoned far, From climes of Washington and Bolivar."

It is no wonder that political sympathies multiplied his name, Bolivar, among us. Veneration for a most heroic missionary whose sun went down at noon, has given us many a Henry Martin, by adopting his whole name. In remembrance of the first American chief magistrate who died in office, and that on its threshold, more than one cradling was then named William Henry. If Abraham as well as Lincoln appears in the name of a citizen, it is clear that his mother's heart was a shrine of our first presidential martyr. Homer says the infant Astyanax was like a star, and so, no doubt, many an English boy was in the eyes of his mother. Then she had no hesitation about a name for her earth-treading star. Many a Grecian mother must have lulled her babe to sleep with an epigram of Plato's:
> "Thy looks are heavenward to the starry rays, Were I that heaven all stars on thee should gaze!"

So she would name him Star, or its variant Sterns. Should that brightest of stars in her eyes have proved too good to live, she had a still better epitaph, also from Plato:

> "Thou wert the morning star to all the living Ere thy young life had sped, Now, like the evening star, thou 'rt giving New lustre to the dead."

Some names represent the ecstasies of young mothers and perpetuate them. One of those blessed women beholding in her infant the best of what she had read in romances realized, called him her romance-Romanzo. Another, admiring the Old Testament knight,-most without fear and without reproach, the Chevalier Fayard of the Bible-called her manchild the light of God, Uriah. We cannot see the name degraded in the Uriah Heep of Dickens without feeling again as we did when the flag of the Union was hauled down and dragged in the dust. Milton, however, had glorified the name in its variant, Uriel, beyond degradation :
> "The archangel Uriel-one of the seven Who in God's presence nearest to his throne, Stand ready at command, and are his eyes That run through all the heavens, or down to the earth."

St. Rasmus-the Christian Castor and Pollux-showing at the mast-head electric flashes welcome to sailors as a light-house, gave Norwegian mothers of sea-farers a name that was above all other names. That corposant lighted vikings to many dark deeds. In the Italian Rasmus is spelled St. Elmo, and gives name alike to Neapolitan sailor boys and to the high-gleaming castle at Naples which is their land-mark, or sea-mark.

This lucubration began with Dr. Pott, and it may well end with him. I repeat it then: Go to Pott! Pott, starting with his own family name, passed on to the study of all personal names and then to comparative linguistic research, rising still from high to higher until he became one of the most illustrious philologists of his time, and some would maintain of all time; as Germans say, a path-breaker and epoch-maker.

We may naturally gain some analogous impulses from the
study either of our own names or those of those with whom we are most familiar. The habit of letting no word pass from us without analysis will give us always and everywhere food for thought and speech. It will satisfy us that in properly studying words we must study things. Botany turns in the eyes of its votaries every weed into a flower. Linguistics transform every vocable, no matter how trite or trivial. They show our commonest phrases to be carrier-doves flying through all ages and continents bearing on their wings a flavor of poetry, history, philosophy, religion. Thus they become veritable birds of paradise regained.

# DEVELOPMENT AND STRUCTURE OF THE SWARMSPORES OF HYDRODICTYON. 

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Of the various large groups of plants the green Algae, with the possible exception of Spirogyra, have perhaps been the least investigated from the modern standpoint of cell structure and by aid of the newer technique. The special problems that need careful attention are connected with the method of cell division, the structure and division of the nuclei and the development and structure of the swarm spores.

Until very recently the prevailing accounts of cell division accompanying spore formation in many of the coenocytic Fungi and green Algae agreed that the protoplasm is divided at once into uninucleate segments. But the researches of Harper (11, 12) upon Synchitrium, Pilobolus, Sporodinia and Fuligo showed that many stages of cleavage in these forms had been entirely overlooked by previous observers. Harper showed that in the forms enumerated instead of the cleavage being simultaneous it is progressive in that the protoplast becomes divided into large multinucleate masses that are further divided into uninucleate ones. There is, moreover, quite a variation in the manner in which this process of progressive cleavage may be accomplished. It may be done by means of constriction furrows in the plasma membrane alone, as for example, in Sporodinia or by means of constriction furrows which fuse with angular vacuoles on the inside of the protoplast as is the case in Pilobolus. Analogous processes may be expected to occur in the spore forming cells of the coenocytic Algae. Klebs (16) has, in fact, described a process of progressive cleavage
in Hydrodictyon but, as will appear later, without giving a full or accurate account of the important details. Klebahn (15) has also briefly described a similar process in Sphaeroploea annulina. In this case irregular clefts are said to arise in the protoplast and by further growth and branching to separate it into the final cleavage products. The origin of the clefts Klebahn does not describe.

To Schmitz (21) and Strasburger (23) is due the credit for establishing the presence and permanency of numerous nuclei in many lower Algae, especially such forms as Hydrodictyon, Cladophora, etc., as well as many of the lower Fungi, but the structure and division of the nuclei so far as the coenocytic green Algae are concerned, are practically unknown with the exception of Fairchild's (8) work on Valonia and more recently that of Klebahn onSphaeroploea.
In Fairchild's account of nuclear division in Valonia, there is described and plainly figured for the first time stages showing very clearly that the mitotic division in that plant is similar in its essential features to the nuclear division in the higher plants and animals.

The chromatin collects into a thread and segments into chromosomes that are collected into an equatorial plate. The daughter chromosomes are drawn to the poles of the spindle where they form the daughter nuclei in a manner similar to that observed in most cells. The method of spindle formation was not made out in detail, but at the time of the formation of the equatorial plate the fibres were seen to converge at two points on opposite sides of the nucleus. Whether centrosomes are present Fairchild could not definitely determine, but his figures show quite well defined bodies at the poles of the spindle. Around these bodies numerous short rays radiate forming apparently quite typical asters.

The nuclear membrane persists, according to Fairchild, until the anaphases in which it is drawn out into a long sac like structure between the two daughter nuclei. Fairchild saw no trace of the central spindle fibres in connection with this sac like structure. The fact that, in Valonia, amitotic nuclear division may occur in the same cell with the mitotic divisions
seems to have also been clearly established by Fairchild's researches.

Klebahn's figures of Sphaeroploea show by the presence of a well defined spindle and distinct chromosomes that in that alga the nuclei divide by a typical mitotic process.

Golenkin (10) has recently studied the nuclei of various green algae and has attempted to show that the nuclei in such forms as Hydrodictyon and Sphaeroploea correspond in structure to those in Spirogyra in that in the resting stage the chomatin material is collected in the middle of the nucleus in the form of a large nucleole. Between this body and the nuclear membrane the space is filled with a formless hyaline substance. During the division stages the bodies corresponding to the chromosomes of higher plants are differentiated out of the central nucleole-like mass. Golenkin thinks that this type of nucleus is of a primitive character as indicated by its occurrence in these lower forms, but it is quite probable that Golenkin worked with poorly prepared material in which the nuclei were distorted. Such a result can easily happen in connection with forms whose nuclei are so small as those of Hydrodictyon. But my figures show that the nucleole in this plant is the same organ in structure and occurrence as it is in the nuclei of the higher organisms.
The discovery of special cilia forming organs in the antherozoids of various Gymnosperms and Pteridophytes, by Webber (33), Ikeno (14), Hirase (13), Belajeff (3), and Shaw (22) and the attendant discussion as to their homology has brought into prominence the question as to the homologies of cilia bearing organs in swarm spores and other ciliated cells.
In a paper on the Chlamydomodineae, Dangeard (6) discusses at some length the question of the structure of the cilia bearing organ in the swarm spores of those plants. He thinks that the cilia are attached to a small basal nodule just within the plasma membrane. There extends from this nodule toward the nucleus, according to Dangeard, a fine protoplasmic fibre, that may or may not reach the nuclear membrane. The basal nodule is thought by Dangeard to be equivalent to the blepharoplast of the antherozoids of the Gymnosperms and ferns.

Strasburger (25) has recently discussed very fully the question as to the structure and homology of the cilia bearing organs of the swarm spores of such forms as Cladophora, Vaucheria and Oedogonium. He thinks that there is a simple swelling of the plasma membrane at the point of insertion of the cilia, and that this is neither a centrosome nor the equivalent of the blepharoplast. Strasburger contends that the entire "mouth piece" of the swarm spore of Oedogonium is to be regarded as the homologue of the cilia bearing band of the antherozoids of the Gymnosperms and ferns.

Dangeard (7) has attempted to establish homologies between the structure of the swarm spores and gametes of the Chlamydomodineae and Polytoma uvella, and the spermatozoa of the higher animals. But he fails to show that the development of the structures that he thinks are homologous is in any way similar.

The main features in the morphology and reproduction of Hydrodictyon have become well known through the researches of Vaucher (3), Areschoug (1), Braun (4), Pringsheim (20), Suppanetz and others. A very complete historical account of these researches is given in Artari's (2) paper referred to below, so that I shall take account of those papers only that have a direct bearing upon the problems here investigated. The most accurate and complete account given by the earlier observers of the cell structure and method of spore formation was that of Braun (6).

According to this observer the protoplasmic contents of the cell consist of three distinct layers: 1. The "primordial membrane," a thin, somewhat opaque layer appearing finely punctate which is drawn away from the cell membrane by the action of acids. 2. An outer "mucilaginous" layer, which is thicker than the preceding, but thinner than the following layer. It appears to be irregular on the outer and inner surfaces and contains numerous "mucilage granules" (nuclei ?) which are somewhat large and irregular in outline. 3. "Inner mucilaginous layer"-the thickest layer of the three containing the chlorophyl which in young or poorly developed cells appears in irregular rows of small granules forming a network,
but in old, well developed cells is evenly distributed. This layer also contains starch vesicles (pyrenoids) that project into the central vacuole on the inside.

While the above description lacks many important details and is in error as to the differentiation of the protoplast into layers, it is notable that later observers using the same methods as Braun, have made very little advance beyond the results he obtained as to the structure of the cell contents.

The cell wall is also, according to Braun, composed of three layers as follows: 1. An outer, thin cuticle. 2. A middle thick layer, that swells strongly in sulphuric acid and stains blue with iodine. 3. An inner layer that swells so as to become the thickest of the three when treated with sulphuric acid and to present wavelike folds on the inner surfaces.
In the account of the spore formation Braun described only some stages that belong to the later phases of the cleavage of the whole protoplasmic mass. He determined, however, that preliminary to the beginning of the process the pyrenoids (starch vesicles) disappear. The first stage of cleavage consists, as he described it, in the appearance in the protoplasm, of numerous light spots equidistant from each other between which the granules of chlorophyl arrange themselves in rows. This stage is followed by one in which the granules retreat toward the clear spots, leaving transparent lines that mark off hexagonal areas on the surface. Each mass thus formed finally becomes a spore. The transparent lines are supposed to be of some substance that separates the hexagonal masses, and is dissolved during the subsequent process of the complete separation of the spores, when the lines disappear except for triangular spaces between the rounded masses.

After a period of rapid movement inside the mother cell wall the swarm spores come to rest and form a new net. The pyrenoids begin to appear immediately after the swarm spores have come to rest. Braun held that their origin was entirely de novo and that they were never multiplied by fission.

In some cases swarm spores were observed which had two or more pairs of cilia. This was accounted for by the non appearance of the light cleavage lines in an earlier stage. It is to
be noted that Braun failed to distinguish the nuclei as such. It is quite probable, of course, that the light spots that he speaks of were the nuclei, but it would be hard to always distinguish them in living material.

In material fixed in alcohol and stained with haematoxyline, Strasburger (23) was able to distinguish, in the protoplasmic layer of the cell, numerous small nuclei which he thought divided just prior to cleavage. Strasburger bases his account of the rest of the process of cleavage entirely upon the observations of Braun and Cohn (5) as their descriptions agreed well with his account of simultaneous cleavage in many other coenocytic cells of the Algae and Fungi.

In 1890 Artari (2) attacked the special problem of the significance of the nuclei in the process of cleavage and also gave considerable attention to the structure of the chromatophore. He thought that the chromatophore is in the young cell an irregular plate like body with long projections similar to the chromatophore of Draparnaldia. During the growth of the cell the projections bend over and fuse so as to form a net which by increase in extent of its parts becomes a perforated plate on the inner side of which the nuclei lie. In material fixed in picric acid and mounted in glycerine or Canada balsam Artari was able to make out that each nucleus contained a prominent central nucleole. Farther than this, however, no details of nuclear structure were described. Cleavage begins, according to Artari, by the division of the chromatophore into irregularly hexagonal areas each of which contains a single nucleus. These areas are separated by a transparent plasma corresponding to the light lines of Braun.
It will be seen from the above paragraph that Artari's account of the cleavage process differs in no important particular from that of Strasburger. His description of the development of a distinct chromatophore is, as I have pointed out in another place, based upon inadequate means of observation, due to his methods of treating the material. The apparent similarity of the chromatophores of such a form as Draparnaldia to the chlorophyl containing cytoplasm in the young cells of $H y$ drodictyon might easily lead one to the conclusion that chro-
matophores of similar structure were being observed in the two cases provided only surface views were studied. Artari does not attempt to solve the problem as to the mechanism by which the so-called chromatophore is divided. His account of the cleavage process is no doubt influenced by the conception of the Vollzellbildung of Strasburger in connection with numerous other forms as noted above. That he observed only the later stages in the process is evident from the work of Klebs and the figures that I have shown in this connection.

Klebs (16) gave a very complete account of the appearance of the Hydrodictyon cell under various conditions and described the process of cleavage as taking place in a manner entirely different from that described by previous observers. Klebs, however, like his predecessors, limited his observations to surface views of the material for the most part in the living condition. He confirms Braun's statement that there are three distinct layers of protoplasm in the cell, but identifies the outer as the hautschicht and the inner as the vacuolar membrane or tonoplast of De Vries, leaving the middle layer occupied by the chromatophore and the layer of protoplasm containing the nuclei. Klebs' account of the structure of the chromatophore is essentially the same as that of Artari. He thinks that while this is a distinct chromatophore it is of a low order of organization since it is merely passive during the process of cleavage and shows at no time the initiative power of reproduction. The nuclei are said by Klebs to occupy no definite position with reference to the pyrenoids, but to appear in the openings through the chromatophore as if lying deeper in the protoplasm. They are described as vesicular structures each having a single large central nucleole. The nuclei, like the pyrenoids, are connected by special strands of protoplasm forming a network with the nuclei at the angles of the meshes.

The process of cleavage begins, according to Klebs, by the formation of angular, in some cases almost canal like, vacuoles in the middle layer of protoplasm. The vacuoles fuse with one another so as to cut the protoplasm, including the chromatophore, into large irregular pieces which, by constriction or by further branching of the vacuoles, are cut into smaller pieces
until the whole protoplast is divided into uninucleate bodies whose mutual pressure gives to each a hexagonal shape. The hexagonal bodies appear to be separated by light lines. This appearance is caused, as Klebs rightly observes, by the two limiting layers of protoplasm being pressed together. This is the condition described by Strasburger and Artari as the earliest stage in the division of the cell. At no time, according to Klebs, do the cleavage vacuoles cut through either the plasma membrane (hautschicht) or the vacuolar membrane of the mother cell. Klebs thinks that the cleavage is never complete but that the swarm spores always remain attached to one another by thin strands of protoplasm, which only disappear after the spores have come to rest and the new walls are formed around the young cells.

From the fact that in many cells the number of nuclei is greater than the number of swarm spores that could be formed from the same cells, Klebs suggests that there must be a fusion of the nuclei in such cases prior to the completion of the cleavage. The observations of Braun and others of the disappearance of the pyrenoids prior to cleavage is confirmed by Klebs.

I have published elsewhere a brief resume of some of the main facts described in the following pages. (28) The material with which I have worked was collected in the vicinity of Madison, Wisconsin, from small, slow flowing streams or in protected portions of the lakes surrounding the city. The plants were generally brought into the laboratory in large quantities and placed in aquarium jars in well lighted places. No attempts were made to use special culture conditions for the production of swarm spores but it was often found that they were produced within a few days after the collection of the material. This phenomenon, however, was by no means constant and very frequently material was kept in the laboratory for many days without showing any signs of spore formation. Very often cultures growing under diverse conditions of light and temperature would all be producing swarmspores at the same time. I have not succeeded in finding any material producing swarmspores when collected, but in some

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lots new nets apparently under twenty-four hours old were observed.

The difficulties in getting well fixed material of such alga cells as those of Hydrodictyon possibly help to account for the fact that so few investigations have been carried on in this group of plants by means of modern cytological methods. The relatively thick and impervious cell walls and thin layers of cytoplasm combined with a large central vacuole present a set of conditions under which fixation without shrinkage and consequent distortion of the plasma contents is hard to accomplish. A solution must be found that will readily penetrate the cell wall and plasma membrane and thus fix the cells without distortion. The two solutions that I have found to best answer these conditions have been Merkel's platinum chloride chromic acid mixture and a mixture of iridium chloride and acetic acid. Two formulas were used for this latter combination; one that of Eisen consisting of 100 parts, five-tenths per cent iridium chloride in distilled water and 1 part glacial acetic acid; and a stronger solution consisting of 100 parts of $1 \%$ iridium chloride in distilled water and 3 parts glacial acetic acid. Very little difference could be detected in the effects of the two iridium-chloride-acetic acid mixtures, although the stronger was generally more reliable for fixing the finer details of structure, especially in the nuclei.

As between the Merkel's solution and the iridium chloride acetic acid mixture, the latter is to be preferred for cases where very delicate structures are to be dealt with, such, for example, as the comparatively young cells of Hydrodictyon where the layer of protoplasm is very thin and its finer structures very easily destroyed. Still both solutions are very satisfactory and I have tried them on various other Algae, including Spirogyra and Vaucheria, with gratifying results. They are to be further recommended on account of their adaptability to various stains. I have found that the triple stain of Flemming, the Fuchsin-Iodine green of Zimmerman and Ironhaematoxylin all give good results.

Flemming's chromo-osmo-acetic acid mixture was tried for fixing but abandoned owing to the blackening of the tissue
caused apparently by the action of the killing fluid on the chlorophyl and the frequent distortion of the structure of the protoplasm. Solutions containing mercuric chloride were generally unsatisfactory since nearly all details of structure were lost in material killed in them.

The best stain for delicacy of differentiation was Flemming's well known triple stain, although very good results were obtained by the use of Zimmerman's Fuchsin-Iodine green. This brought out the figures of the dividing nuclei very well, but was of little value to aid in studying the pyrenoids and other structures.

I have discussed in another connection (29) the general structure of the cell and have showed that there is no distinct chromatophore in Hydrodictyon. In that place I pointed out that the pyrenoids and nuclei were distributed in such a way as to preclude the possibility of the protoplasm between the plasma and the vacuolar membranes being differentiated into distinct layers. We may then pass directly to a consideration of the structure of the cytoplasm and nuclei and the process of cleavage. The cytoplasm varies in appearance from an almost homogeneous finely granular mass with angular vacuoles placed quite wide apart, to a decidedly foamy structure whose typically rounded alveolae differ much in size ranging from very minute, scarcely perceptible openings to quite large vacuoles whose diameter extends nearly the whole depth of the plasma layer. Neither of these appearances is confined to any one stage in the development of the cell. They occur in the recently formed swarm spores as well as in the mature cell, and it is quite probable that they simply represent different conditions of metabolism, etc. The foamy appearance is however much the more frequent of the two in all stages. In many cells having this alveolar structure, there is a tendency for the small alveoli to be arranged in rows giving the appearance of furrows or tubes through the cytoplasm, but in nearly all cases strands or lamellae could be detected cutting off rounded alveoli (Figs. 3,21 ). During cleavage there is often a noticeable tendency for the larger vacuoles to be aggregated along the cleavage furrows (Figs. 24-26) in a manner much resembling the arrangement in Synchitrium.

Whether the smaller alveoli are really vacuoles in the same sense as the larger openings may be a matter of doubt. But the fact that all grades in size can be found from the very small alveoli to the larger ones would seem to show that there is no difference between them. In many sections the smaller vacuoles appear to be fusing with larger ones. All stages may be made out from that in which the two adjacent vacuoles have nearly their original spherical form to those in which the fusion is almost complete and one appears as a slight protuberance upon the other. The larger vacuoles above described often appear in turn to fuse with the central vacuole so that their membrane becomes continuous with that of the central vacuole (Fig. 2). It is of course possible that the appearance of fusion of the vacuoles just described is due to slight distortion in fixation, for it can be readily seen that if two vacuoles each surrounded by a slight film, are lying very close together; a very slight disturbance in the protoplasm might cause a break in the films so as to give the appearance of stages in the fusion of the vacuoles. Still there seems to be no doubt that in the growth of the cells from swarm spores the central vacuole is the result of the fusion of two or more' smaller vacuoles of the young cell.

While the above description of the relations of the larger and smaller vacuoles seems to agree in many respects with what Wilson (35) has recently described for some Echinoderm eggs, I have not been able to make out here any such morphological series consisting of granules, microsomes, alveoli and vacuoles as Wilson described. Wherever the alveoli and vacuoles can be made out they seem to be quite distinct from the other cell contents, and I am therefore inclined to think of them as distinct cell organs perhaps in some respects coordinate with such structures as the pyrenoids. Still if this view be correct the question as to the origin of the vacuoles is a perplexing one. The fact that smaller vacuoles may fuse to form larger ones and that as a result of cleavage the large central vacuole entirely disappears, would seem to strongly negative the doctrine of De Vries (31) and Went (34) that the vacuole is a permanent cell organ reproduced by division of a preexisting vacuole. To be sure the vacuolar
membrane is retained as a portion of the plasma membranes of the new cells so that it is in a sense a permanent structure. But it performs an entirely new role and the identity of the vacuole is as completely lost as if the membrane itself were destroyed.

The production of artificial vacuoles by Pfeffer (19) and more lately by Nemec (18) throws little light on the real problem as to the origin of the vacuole, for it is not at all certain that the vacuoles that were apparently formed anew were not already existing, except in those cases described by Pfeffer where the so-called food vacuoles in some of the slime molds may arise as invaginations of the outer plasma membrane. This identity in character of vacuolar and plasma membranes so shown, is important when taken in connection with the fact shown in various Ascomycetes and Phanerogams that the latter membrane is formed by a direct metamorphosis of the kinoplasmic fibres. In various Phanerogamic cells I have shown that the young cell plate splits and the halves become partly separated before the plate has reached the mother cell membrane (27). This fact may furnish a suggestion as to the possible method of origin of vacuoles, for the cleft in the cell plate is essentially a flat vacuole surrounded entirely by the protoplasm of the mother cell, and enclosed by a membrane derived from the spindle fibres. The fact that its membrane finally forms a part of the plasma membrane of either daughter cell may be compared to the history of the membrane of the central vacuole in the cleavage of Hydrodictyon and in those fungus sporanges in which the cleavage is partly accomplished by means of numerous small angular vacuoles. To be sure the vacuole formed by the splitting of the cell plate may be considered a very special structure without any relation to the other vacuoles of the cell. But the analogy just pointed out may prove suggestive as to a possible line of investigation in connection with the question of the origin of vacuoles in other cases.

The structure and division of the nuclei of Hydrodictyon is in the main features the same as that of the nuclei of the higher plants. In general there are one or more fairly prominent nucleoles that take a bright red color in the triple stain, and a
blue stained chromatin network that can always be clearly seen in good preparations. The nuclear membrane is quite sharply differentiated and sometimes stands out as a deeply stained thick boundary.
While the above general statement is true for all of the nuclei there is a remarkable individual variation both in appearance of contents and in form and size. In a typical resting stage the nuclei are generally quite small spherical bodies with the chromatin arranged in a very fine slightly stained, but very clearly visible network. At various places in the net the chromatin appears to be more or less collected into lumps. Very frequently in cells showing great vegetative activity, as for example, in the growth of the young nets or in cases in which starch is being rapidly formed the nuclei become quite large and vesicular in appearance (See Figs. 41 and 42. Cf. also Fig. 1 in my paper on starch formation in Hydrodictyon). In these cases the chromatin frequently appears as very fine lines radiating from the prominent central nucleole to the periphery. Unless the preparations are well stained the chromatin in such cases might be easily overlooked, but in no stages of cell life have I been unable to demonstrate a clear differentiation of the nuclear contents into chromatin and nucleole.

Spherical nuclei with the contents arranged as above described are always found in cells showing no signs of reproductive activity. In all cases, however, in which cleavage is taking place the chromatin is collected into denser roughly elongated masses taking a deeper stain and connected by fine threads as of linin (Figs. 24-32).

The nucleole becomes much less sharply defined in these than in the resting stages just described and in some cases is so surrounded by the chromatin as to be almost indistinguishable. The whole structure of the nucleus is in fact identical in appearance with the early prophases or late anaphases of division.

The nuclei vary in shape from the spherical forms just described to those with very irregular outlines, some of which are quite sharply angular, while others have blunt rounded extensions resembling the pseudopods of an amoeba. In many cases, especially in cells in process of cleavage, the nuclei are
slightly elongated with one end quite small and drawn out into a blunt point, that is directed toward the cleavage plane (Figs. 29 and 30 ). In other cases the nuclei are more elongated and the two ends are of the same size. Such forms may be constricted in the equatorial region so as to suggest strongly that direct division is taking place. (Figs. 21a, 22, 23.) Both of the above mentioned forms of nuclei generally occur together in the same cell. (Figs. 21-23). They may in turn be associated with those that are more definitely amoeboid or angular in outline. The size of such nuclei varies very strikingly (Compare Figs. 6, 21, 34 and 44). In figures 6, 34 and 35 are shown the extremely large size that the elongated amoeboid and angular nuclei may reach. Such unusually large nuclei may occur in apparently any stage of cell life, although my preparations have not shown them in ciliated swarm spores. Still they occur in the cells in which the cleavage is complete (Figs. 34, 35), as well as in the cells of very young nets (Fig. 44). By comparing figures 43 and 44 a very good idea may be obtained as to the relative size that such nuclei may attain. Both figures are drawn according to the same scale of magnification.

In all of the irregular shaped nuclei the contents generally present the same appearance as in the other nuclei in the cells in which cleavage is taking place.

The distribution of the nuclei presents some interesting phenomena. Those described above as typical resting nuclei are generally quite evenly distributed throughout the cytoplasm at points equidistant from one another (Fig. 3). The number of such nuclei that may occur in a given area of the protoplasm varies considerably. As Klebs pointed out, cells are often seen in which the nuclei are much too numerous to have swarm spores of the normal size formed in the cell with a single nucleus to each spore. But the assumption that a fusion of nuclei occurs in such cases before or during cleavage is not at all necessary, for there is no evidence that such cells are going into cleavage stages without further growth. Even if they are in the same net in which some of the cells are forming spores they may not, as I shall show later, themselves undergo
such a process, but are just as likely to live for some time vegetatively. The only case in which an excessive number of nuclei need to lead to the assumption of fusion, would be to find such a numerical excess in connection with the later stages of cleavage, and this relation I have not found in any of my preparations. It is quite probable that the large number of nuclei is the result of a period of nuclear division just completed that is to be followed by a period of cell growth.

In cells in which cleavage is in progress the nuclei are very frequently arranged in pairs, the two nuclei of a pair often almost touching one another. (Figs. 25-30.) This arrangement is very clearly the result of the process of nuclear division as I shall describe it farther on. Figure 21 shows a very striking arrangement of the smaller irregular nuclei into groups consisting of from two to eight individuals. How such an arrangement came about I have not been able to learn. The forms of the nuclei might suggest that they have moved together from various parts of the cell, but other evidence in support of such a suggestion is entirely lacking. On the other hand it is quite possible that each group represents the product of a series of successive nuclear divisions, and that the individual nuclei will later be distributed more equally throughout the cell. The fact that the dividing nuclei sometimes appear in groups and that the daughter nuclei when first formed are frequently quite irregular in shape, being thus identical in form and structure with some of the grouped nuclei in the conditions shown in Fig. 21, adds weight to this latter hypothesis.

Owing to the small size of the nuclei it is impossible to make out clearly all of the details of karyokinesis, but enough stages stand out sharply to show that the process is essentially the same as in the higher organisms. A very distinct loosely coiled thread is formed that apparently becomes segmented into chromosomes (Figs. 7 and 8). In a few cases I have been able to make out quite clearly that there are ten segments; but usually it was almost impossible to distinguish the individual chromosomes accurately enough to count them. When this stage is reached the nuclear membrane has generally disappeared so that the peripheral parts of the chromosomes lie in im-
mediate contact with the cytoplasm, which has not, however, penetrated to the interior of the nuclear cavity. The nucleole disappears so far as I have been able to observe before the equatorial plate stage is reached.

The history of the spindle in these early stages I have not been able to make out at all. When the equatorial plate stage is reached the spindle is distinct. It usually ends in two sharply defined poles, at either of which there is a small spherical, densely stained body; but there are no indications of polar radiations to form an aster (Fig. 9). The spindle fibres generally come to a sharp point apparently just at the surface of the body so as to give the appearance of a distinct body lying rather in contact with the end of the spindle than forming a part of it.

Owing to the impossibility of observing the early stages of spindle formation the origin of the above described bodies, as well as their relation to spindle development, could not be made out. During the period of the reconstruction of the daughter nuclei when the spindle disappears, the bodies also become indistinguishable. Whether they are the homologues of centrosomes is, in view of the scarcity of data in connection with their history, of course, not evident, still the constancy with which they appear in the equatorial plate stages and early metaphases indicates that they bear the same relation to the process of division as the centrosomes in other cells. I shall apply the name centrosomes to them in the subsequent discussion. In many cases the whole spindle seems to lie in a clear cavity, as if the nuclear membrane persisted throughout the greater part of the division process, but the boundary of this cavity is always quite irregular and I am inclined to think that it is the product of the fusion of the vacuoles of the cytoplasm surrounding the nucleus. (Figs. 9, 11, 12.) In other cases the cytoplasm is apparently in immediate contact with the spindle. (Fig. 10.) The chromatin material in the equatorial plate forms so compact a mass as to render it almost impossible to distinguish the individual chromosomes, and consequently to make out the method of the separation of the daughter chromosomes.

In the metaphases the daughter chromosomes go back to their respective poles in dense groups, all the individuals of each
group lying practically in the same plane. (Fig. 10.) During these stages the spindle is frequently much elongated so that at the beginning of the anaphases the central spindle is drawn out into a fine thread in its middle part. (Fig. 12.) During the anaphases the chromosomes form at first somewhat rounded dense masses that later form the daughter spirems. New nuclear membranes are formed and the daughter nuclei are completely organized. While the formation of the daughter nuclei is taking place they underge a remarkable change of position. With the disappearance of the central spindle the nuclei approach each other so that by the end of the anaphases they lie almost in contact. (Figs. 12-14.) The explanation of this change in position is not at all evident. I have described a similar phenomenon in connection with the formation of the cell plate in the onion and larch and suggested that it might be due to the mechanical pressure of the cytoplasm upon the daughter nuclei which pushes them into the space left vacant when the central spindle disappears. The same explanation may possibly hold here although the small number of fibres constituting the central spindle would seem to render it less probable. The juxtaposition of the daughter nuclei thus brought about gives a striking appearance to the cells where all of the nuclei are frequently in pairs. This position of the nuclei, as previously stated, may persist in the cleavage stages. Fig. 20 presents a curious condition in which it appears as if the chromosomes, instead of going back to the poles in compact bodies, as above described, are here more or less strung out along the spindle fibres either singly or in irregular clumps. The explanation of these figures is not apparent. It is possibly due to some peculiar effect of the fixing fluid although the other cell contents show no abnormalities.

Interesting variations in the size and form of the spindle are shown in figures 17-19. In figures 18 and 19 is shown very clearly that two spindles lying closely adjacent to one another may vary as much in size as do the nuclei in some cases in which division is not clearly in progress. That the apparent difference in size is not due to a possibly somewhat flattened spindle viewed in different aspects is shown in polar views of the equa-
torial plates in which the chromatin mass always appears, so far as my preparations show, to be nearly equal in its length and breadth. The equatorial plate in the larger spindle consists of a greater amount of chromatin than that of the smaller one. Whether the number of chromosomes varies in the two cases is an important point that I could not determine, as I was unable to find the stages of division in which the chromosomes could be distinguished from each other. It is quite probable that the different sized nuclear figures are derived from nuclei that differed in the same respect before division commenced, though I have not yet succeeded in finding the stages that would confirm this suggestion. The largest spindles are somewhat multipolar with the chromatin arranged in such a way as to form a branched equitorial plate' (Figs. 17a and 19a), so that the entire figure has the appearance of the well known cases of polycentric nuclear figures in cells treated with certain poisons or subjected to other unusual stimuli. But there was no other evidence of abnormality either in the structure of the cytoplasm or that of the nuclear figures themselves.

So far as I could determine there was no particular time during the day when nuclear division was most likely to take place. Material in which I have found it was in part killed at various hours during the forenoon and in part in the afternoon. In a small amount of material killed at different times during one night, I was unable to find any indications of division, but of course so small an amount of material would not show conclusively that it never takes place at night. Still it seems quite evident that there is no regular daily period to which division is confined. This is made the more certain by the fact that in a single net one cell alone may show nuclear division while in the other cells the nuclei seem to be in a resting condition. That, however, the divison depends upon conditions common to the whole cell is shown by all of the nuclei of a single cell dividing at the same time though without being in the same stage. Very often all stages from the early prophases to the late anaphases can be found in a single cell in a more or less regular succession from one end of the cell to the other. (Figs. 15 and 16.) This fact is in-

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teresting when taken in connection with the fact that in the process of cleavage the same order of events is frequently observed.

As is known, any cell of a net may form spores without the accompanying cells being visibly affected. It generally happens, however, that a number of the cells are in some stage of spore formation at the same time. I have frequently found in my material single nets of which the majority of the cells were in a vegetative condition while one might show stages of cleavage, another ciliated spores, and in still another there would be a newly formed net. The relation of these different cells to another so far as their position in the net is concerned, I could not make out, since in imbedded and sectioned material the general arrangement is difficult to determine. I have not made any special observations as to the age of the nets whose cells produce swarm spores, although such an investigation would be of great value. Klebs has shown that a number of special conditions of nutrition, etc., will cause spore formation, but the age of the cells thus experimented upon was not accurately determined.

The first indication of the approach of cleavage consists of the disappearance of the pyrenoid and increase in thickness of the protoplasm. This latter phenomenon is very clearly shown in Fig. 32, which shows sections of two adjacent cells, in one of which cleavage has just begun, while in the other there is no indication of it. In the account of the process of starch formation in Hydrodictyon I described the disappearance of the starch and pyrenoids as usually occurring before cleavage begins, but showed that such a process is not necessarily preliminary to cleavage since it sometimes happens that some of the starch and the pyrenoids may persist through all the stages even to the spores and young cells formed from them.

Cleavage itself is, as Klebs pointed out, a progressive process; but it is accomplished entirely by means of two sets of surface constrictions instead of, as Klebs thought, by means of intraplasmic vacuoles. In the first stages in the process short furrows that have no apparent special orientation with reference to one another or to the nuclei appear here and there through
the cytoplasm. (Fig. 22.) Seen in a surface section these furrows appear as single lines and thus might easily be taken for cell plates formed in the protoplasm without the help of visible spindle fibres; but if a vertical section is studied, the appearance of furrows becomes manifest. (Figs. 31-32.)

In these latter sections it is also clearly shown that the cleavage furrows thus formed cut through at right angles to the surface. In none of my material have I found the furrows forming appreciable oblique angles with the surface, although such a phenomenon might of course occur in large cells in which the protoplasmic layer is much thicker, as for example, in such a cell as that shown in Fig. 2. I have not succeeded in getting the stages of cleavage in these larger cells. During the succeeding stages the furrows become branched and increase in length so as to soon intersect with one another and thus block out very irregular multinucleate areas on the surface of the protoplasm. Concurrently with the development of the cleavage furrows on the outer surface similar ones are formed by the vacuolar membrane on the inner side of the protoplasm (Fig. 33.) The two furrows from opposite sides finally meet in the interior of the protoplasm and fuse, thereby completely cutting through the entire layer. The process of growth and branching of the cleavage furrows continues until the entire protoplast is cut into unincleate pieces, which later round up into the swarmspores.

While the two sets of cleavage furrows generally seem to accomplish the complete division by coming together from opposite places on the two surfaces, it frequently happens that one furrow may reach nearly through the entire layer of protoplasm before there is any indication of a corresponding furrow being formed on the opposite side. Still I have not found any cases where one furrow cut clear through without meeting a constriction from the opposite side. The deeper constriction may, however, be from either the inner or outer surface. In the early stages of cleavage the constrictions are irregular and wavy in a surface section, while in the later stages they seem to become straighter and their intersections form sharper angles. (Fig. 27.)

As before stated the process of cleavage may show a succession of stages from one end of a cell to another. Figures 26,27 , and 28 bring this out very clearly, since they are all taken from the same cell at different regions thmrough its length. The process of cleavage above described seems to go on in its early stages at least, entirely independently of the nuclei, but the result is always, with an exception to be presently noted, the formation of uninucleate segments. In cells in which the cleavage planes cut between recently divided nuclei, so that the nuclei lie quite close to the cleft, the nuclei frequently have the elongated pointed shape previously described. (Figs. 28, 30.) The smaller end of the nucleus lies nearest the cleavage plane. In some cases a distinct granule can be seen lying next to the newly formed membrane at the point nearest the smaller end of the nucleus. Whether this granule has anv special significance could not be determined. Its occurrence is by no means constant, and the cases where it was observed are quite possibly accidental ones, the granule itself being but one of the numerous small bodies frequently found in other parts of the cytoplasm as well.

When the cleavage has reached the uninucleate stage the segments begin to separate from one another and to round up into distinct bodies. The sides of the cleavage furrows are drawn away from each other so as to leave between them irregular quite broad clefts through the protoplasm. (Fig. 30.) So far as I have observed the clefts do not usually appear until the uninucleate stage is reached, but in some cases the separation may take place before cleavage is complete with the result that large binucleate masses form spores directly. In these spores there is a pair of cilia connected with each nucleus. (Fig. 39.) These giant spores were, as previously stated, observed by Braun and his conclusion that they represent cases of incomplete cleavage is undoubtedly correct. The clefts thus produced are perhaps what Klebs thought were intraplasmic vacuoles, and it is quite possible that he failed to see the earlier stages showing the cleavage furrows first formed. It would certainly be very easy to overlook such stages in material poorly fixed and stained and viewed only from the surface of the entire cell.

An explanation of the mechanics of the process of cleavage described above is by no means obvious. The best known cases of cell formation from large coenocytic cells in plant tissues are those of the formation of a layer of endosperm cells from a large multinucleate mother cell and those cases of spore formation in certain fungus sporanges as described recently by Harper. The Hydrodictyon cell has a striking superficial resemblance to the multinucleate endosperm mother cell in that in both cases there is a relatively thin layer of protoplasm containing numerous nuclei and surrounding a large central vacuole. There is also somewhat of a resemblance in the process of cleavage in that a somewhat regular succession of stages may be observed in passing from one end of the cell to the other. (See Strasburger, 23.) But at this point the resemblance ceases, and it may serve to bring out more sharply the problem to be solved in case of Hydrodictyon if the differences in the actual process of cleavage in the two cases are pointed out. In the first place in the embryo sac the cleavage is very clearly in direct connection with the nuclei the position of any one part of the cleavage plane being determined jointly by the two nuclei that are connected by the fibres in which that part of the cell plate is being formed. To be sure the planes may not always be so arranged as to cut out uninucleate pieces as Strasburger has shown for Corydalis and other forms, but in these cases it is none the less true that where the division does take place the cleavage plane is just ass distinctly determined by the nuclei in pairs. In Hydrodictyon as previously shown, no such relation of cleavage planes to nuclei is at all evident, with the possible exception of those cases where the constriction cuts in between two recently formed nuclei. Again, in the embryo sac the process is clearly initiated and practically completed in the midst of the protoplasm, the two boundary membranes being simply divided into portions that form the outer and inner portions of the membranes of the new cells, the lateral membranes being formed entirely anew from the spindle fibres, while in Hydrodictyon the process is clearly indicated on either surface by the limiting membranes and its completion depends
upon their further growth from the surface inward, so that the new cells are entirely surrounded by the portions of the two original membranes. The cleavage in the two cases, then, so far as the mechanics of the process is concerned seems to belong to totally different categories. Still if the previous suggestion that the cleft formed by the splitting of the cell plate is comparable to a vacuole prove correct it would become more nearly possible to find a similarity in the two processes, but even in that case it must be kept clear that the substance for the increase in extent of the vacuole in the one case is furnished by the spindle fibres for whose formation the nucleus is probably a metabolic center while in the other there is not the slightest direct evidence of any such connection between the nucleus and the growth of the membrane forming the cleavage furrow. To be sure there may be, as Harper has suggested for Synchitrium, a diffusion of kinoplasmic material from the nucleus to the plasma membrane, but here, as in Synchitrium direct evidence for such diffusion is entirely wanting. On the whole the process of cleavage in Hydrodictyon seems to correspond most closely to that of the fungus sporanges in which there is a progressive and complete cleavage, as for example in the formation of the protospores in Synchitrium decipiens and Pilobolus where the cleavage continues to the ultimate formation of the uninucleate cells.

A point of much significance in connection with the cleavage in Hydrodictyon is the fact that the two membranes which take part in the cleavage are entirely discontinuous, although they lie parallel to each other with a relatively thin layer of protoplasm between them. But the two cleavage furrows produced by these independent membranes from opposite sides regularly meet in the midst of the protoplasm. This would seem to render necessary the assumption that the impulse for division is seated in the protoplasm between the two membranes rather than in the membranes themselves. Such an assumption need not involve the conception that the beginning of the process takes place in the protoplasm between, but merely that the stimulus to which the cleavage is a response is an internal stimulus acting upon the two membranes at
the same time, a conception that, however, in no way helps to a mechanical explanation of the process. The conditions here must be kept sharply distinct from those cases of the bipartition of a single coenocytic cell by simple constriction, as for example cell division in Cladophora. In this latter case one may readily imagine that the stimulus arises in the membrane itself and is either external or internal since it is a case of the construction of a single continuous membrane.

When the spores are fully formed it is quite evident that they are entirely separated from one another. It is not improbable that what Klebs took for connecting strands of protoplasm was the slime which is sometimes found on the inside of the central vacuole lying next to the vacuolar membrane.

The general structure and form of the spores I have found to be identical with that described by previous observers. The young spores are generally almost spherical bodies with the nucleus occupying a position near the plasma membrane and connected with a pair of cilia which are attached to the cell at the point nearest the nucleus. (Fig. 36.) In older stages the portion to which the cilia are attached becomes somewhat elongated and filled with hyaline protoplasm, that extends down into the cell as far as the nucleus. (Figs. 37 and 38.) In connection with the structure of the swarm spore most interest now attaches to the locomotor apparatus. This consists in all of the normal uninucleate spores of a pair of cilia attached to a small spherical basal body lying generally in contact with, but quite clearly independent of the plasma membrane. (Fig. 38.) Connecting the basal body with the nucleus there are two or more protoplasmic fibres. Owing to the small size of the above mentioned organs, it is extremely difficult to make out any details as to their structure or origin. The basal body generally stains quite densely, especially in the triple stain. In the best stained preparations it frequently appears to be of a dark red color, a fact that shows quite clearly that it is entirely distinct from the plasma membrane, which, in the same preparations has a blue color. In nearly all cases too it can be seen that the contour of the basal body is clearly distinct from the plasma mem-
brane. (Fig. 38.) It is not impossible that the basal body is the granule previously described, that lies against the cleavage furrow near the nucleus in the latest stages of cleavage; but I could not find such a body during the stages of separation and rounding up of the spores.

The threads connecting the basal body with the nucleus are very fine and are not always easy to distinguish from the surrounding cytoplasm; but in most cases they are clearly enough differentiated to enable one to trace them from one body to the other. They are colored in the various staining methods used about the same way as the spindle fibres or cilia, and in general appearance resemble very much such structures. There is no evidence, however, that any portion of the spindlé ever remains after nuclear division is complete which could be identified as these threads. At the point where a connecting thread is in contact with the nucleus there is no indication of any distinct body or even a swelling of the nuclear membrane. I have sometimes found three such threads quite plainly visible, but in most spores there are only two. I have found no cases in which it was clearly evident that there was but a single thread.

The cilia appear as single fine threads resembling in staining capacity and structures the spindle fibres. In spores in which the contour of the basal body is clearly distinct from the plasma membrane, the attachment of the cilia to the basal body itself can be clearly seen. (Fig. 38.) The points of attachment of the two cilia to the basal body may either lie at some distance from each other or they may be in immediate contact. (Figs. 36-39.) As previously indicated, the protoplasm in the forward end of the spore has in the mature spore a hyaline homogeneous appearance. This fact makes the differentiation of the basal body and connecting threads much clearer than it otherwise would be. In the giant swarmspores there is a complete locomotor apparatus consisting of cilia, basal body and connecting threads connected with each nucleus. (Fig. 39.)

As is well known, the swarmspores of IIydrodictyon do not normally escape from the cell wall within which they are formed but after swimming about for some time in the cavity
come to rest, lose their cilia, become surrounded by cellulose walls, and become attached to one another at various points so as to form: a young net that is later set free by the breaking down of the mother cell wall. The changes that occur in the cell contents during growth are of importance as throwing some light upon the structures found in mature cells. In the very young cells I have looked carefully for the first indications of the formation of the pyrenoids but without getting much light as to the nature of the process. As Braun long ago showed, these organs are formed shortly after the spores come to rest. The youngest stages in which I could clearly distinguish structures that were undoubted pyrenoids showed them as small spherical red stained bodies enclosed by a hyaline region sharply bounded off from the surrounding cytoplasm (Figs. 41 and 42). The difficulty of identifying the earliest stages in the formation of such structure lies in the fact that any of the numerous granules so frequently found in the cell may be mistaken for a young pyrenoid, and I have not discovered any reaction by which to determine whether such bodies are or are not the young pyrenoids. It is interesting, however, in this connection, to note that the pyrenoids and nuclei are generally quite closely associated in the young cells, and that in many cases the number of nuclei is the same as that of the pyrenoids. But this equality is lost in the older stages and the number of nuclei is frequently in excess of the number of pyrenoids. The reverse condition, however, is often observed. See especially Fig. 44, where the number of pyrenoids is greatly in excess of the number of nuclei, a relation that holds quite generally in the case of these very large nuclei.

I did not find any stages of the first nuclear division, but a comparison of Figs. 40 and 41 shows clearly that a large increase in the size of the cell may occur before any nuclear divisions take place. (See also Fig. 43.) It is important to note, however, that the increase in size of the cell is out of proportion to the increase in the amount of protoplasm. Before a cell has grown very much, fairly large vacuoles begin to appear in either end apparently leaving a very thin layer of
protoplasm between themselves and the cell wall (Fig. 41). The nucleus and pyrenoid are generally situated in the central region of the cell where the protoplasm still extends between the vacuoles clear through the diameter of the cell, as is clearly shown in a cross section (Fig. 42). In these early stages in living cells, the chlorophyl can only be seen in the thicker portions of protoplasm as the middle part in which the nucleus and pyrenoid lie and the ends where the vacuoles do not press out to the cell wall. But this appearance is probably due to the fact that the layer of cytoplasm between the cell and vacuoles is so very thin as to render the color indiscernible when viewed from the surface. There is certainly no indication in sections of any differentiation of the cytoplasm to form a chromatophore in this or any other stage of the cell life. The arrangement described above is undoubtedly what led to Artari's (2) statement that in the earlier stages the chromatophore is a branched structure similar to the chromatophore of Draparnaldia. But if Artari had had better prepared material he would have seen that there is no trace of any differentiation of the protoplasm into an organized chromatophore.

As the cell becomes larger the vacuoles fuse into one large central vacuole and the protoplasm, now containing numerous nuclei and pyrenoids, becomes evenly distributed in a layer next to the cell wall (Fig. 1).

If we attempt now to apply the facts described in the foregoing pages to the problems suggested in the introduction, several points of general interest are at once apparent. Notwithstanding the very small size of the nuclei the structure and essential features of mitotic divisions are identical with those of the higher plants. Hydrodictyon agrees in this respect with all of the carefully investigated cases of nuclear structure and division in the other thallophytes with the possible exception of Spirogyra, in which it seems quite probable according to the recent researches of Mitzkewitsch and Wissenlingh that the nuclear structure is different from that of most other forms. But in that case the peculiar structure of the nucleus probably represents a highly specialized type and
is not to be considered in any way more primitive than the ordinary type of nucleus found in other thallophytes and the higher plants. The existence of a body so closely resembling a centrosome in Hydrodictyon is in accord with the facts as worked out in other lower forms, especially among the Fungi and brown Algae.

The significance of the variation in size of the nuclei is not at all clear. The very large nuclei probably represent cases of unusual growth of the smaller ones. Such a variation in size of the nuclei is not uncommon in other plants at different stages of the life history. For example, in the uninucleate plant of Synchitrium the nucleus is many times as large as the nuclei of the multinucleate stage or the single nuclei of the spores. The striking fact in Hydrodictyon is, however, that the very large nuclei may occur in practically any stage in the life history. Whether these large nuclei give rise to smaller ones by division, as is the case in Synchitrium, is not clearly evident; but the variation in the size of the spindle indicates that they do.

The question as to whether direct nuclear division takes place in Hydrodictyon is as yet unsettled. While many of the figures indicate quite strongly stages in such division, the evidence is not at all conclusive. As Hertwig very justly remarks, the mere fact that a nucleus may be elongated and constricted in its equatorial region is not to be taken as positive evidence that direct division is in progress, for in many cells the nuclei are capable of assuming a great variety of forms without showing any other indication of division. In Hydrodictyon the capacity of the nuclei to assume such variations in form is very marked and it is quite probable that the elongated nuclei that appear to be constricted in the middle are simply forms of resting nuclei coordinate with the more angular or pointed forms.

As the facts at present stand, there are two distinct methods of cell division in plant cells, peculiar to the thallophytes on the one hand, and the cormophytes on the other. In the former types we have such coenocytic cells as the fungus sporanges and the Hydrodictyon cell where there is a progressive
cleavage by means of surface constrictions. In cell division in Cladophora, conidia formation in the mildews, and the cutting off of the gametes in Sporodinia we have simple constriction, as Harper has pointed out. All of these cases may be grouped together as examples of cell division by constriction and thereby be clearly distinguished from the case of the cormophytes where cell division is accomplished by means of the formation of a cell plate.

These two types of cell division are most sharply differentiated when compared as to their relation to the nuclei. In the case of division by constriction there is no visible relation of the nuclei to the process so far as the mechanism involved is concerned, while in the case of cell plate formation the nuclei are apparently the active controlling centers for the process by means of the spindle fibres, many of which, and sometimes all, may be formed for the express purpose of cell division.

The relation of the processes of cell division in certain brown Algae to the two types above described is not at all clear. In all of these forms so far described division is said to take place by means of a cell plate that is formed without being directly connected with the nuclei. (See $9,17,24$, 26.) Owing to the difficulty of getting numerous stages of division in these forms it is quite possible that some of the most important phases have been missed.

The structure of the swarm spore of Hydrodictyon compared to the structure of other ciliated cells presents many interesting problems. The most thoroughly investigated cases of the development of such cells is that of the spermatozoa of the higher animals and the antherozoids of some Pteridophytes and Gymnosperms. While there are some differences as to detail, zoologists seem to be in accord as to the main facts concerning the development of the spermatozoon ( See , Wilson, 36.) The axial filament of the tail grows out from a spermatid centrosome that may persist as a distinct body ("end knob") at the base of the fibre or may enlarge to form a part of the middle piece. This fact of the growth of the axial filament from the centrosome forms a possible basis of comparison be
tween the spermatozoon and the swarmspore. The cilia of the swarmspore clearly bear the same relation to the basal body as does the axial filament to the middle piece or more particularly to that part of the middle piece derived from a centrosome. But whether the basal body really corresponds to the middle piece must be left unsettled until its origin is determined.

Discussion of the question as to whether the basal body in Hydrodictyon corresponds to the blepharoplast of some Pteridophytes and Gymnosperms, is for the present quite premature. That question is also quite distinct from the further one as to the homology of the blepharoplast itself.

The development of the swarm spores in such forms as Cladophora and Oedogonium needs to be carefully investigated. Strasburger's (24) conclusion that the individual nodule at the base of each cilium is merely a swelling of the plasma membrane was based entirely, so far as his descriptions and figures indicate, upon the study of the fully formed spores. It is quite possible that early stages in the development of the spores would show that the origin of both the cilia and the nodules at their bases is quite different from what Strasburger supposed it to be.

University of Wisconsin, November, 1901.

## LIST OF PAPERS REFERRED TO.

1. Areschoug: Ueber die Vermehrungsart des Wassernetzes. Linnaea, 1842.
2. Artari, A.: Zur Entwickelungsgeschichte des Wassernetzes. Bull. de la Soc. Imp. des Nat. Moscou, 4: 269, 1890.
3. Belajeff, W.: (Various papers on spermatogenesis. Ber. d. deut. bot. Ges. 16, 1898 and 17, 1899.)
4. Braun, A.: Betrachtungen über die Erscheinungen der Verjüngung in der Natur. 1849-1850. Eng. Ed. in Trans. Ray. Soc. 1853.
5. Cohn, F.: Untersuchungen über die Entwickelungsgeschichte der Algen und Pilzen. Nova Acta Acad. Leop. Carol. 24.
6. Dangeard, P. A.: Memoire sur les Chlamydomonadinees ou l' Historie d'une cellule. Le Botaniste 6: 65, 1899.
7. -: Etude sur la structure de la cellule et ses functions. Le Polytoma uvella. Le Botaniste 8:5, 1901.
8. Fairchild, D. G.: Ein Beitrag zur Kenntniss der Kerntheilung bei Valonia utricularia. Ber. d. deut. bot. Ges. 12: 331, 1894.
9. Farmer, J. B. and Williams, J. L.: Contributions to our knowledge of the Fucaceae. Philos. Trans. of the Roy. Soc. London. Ser. B. 190: 623, 1898.
10. Golenkin, M.: Ueber die Befruchtung bei Spaeroploea annulina und üb. die Structur die Zellkerne bei einigen grünen Algen . Bull. d. la Soc. imp. d. Nat. Moscou 1899, p. 343.
11. Harper, R. A.: Cell division in Sporangia and Asci. Annals of Botany, 13: 467, 1899.
12. -: Nuclear and Cell division in Fuligo varians. Botanical Gazette $30: 217,1900$.
13. Hirase, S.: Etude sur la fecondation et l'embryogenie du Gingko biloba. Journ. Coll. Sci. Tokio, 7: No. 2.
14. Ikeno, S.: Untersuchungen über die Entwickelung der Geschlechtsorgane und den Vorgang der Befruchtung bei Cycas revoluta. Jahrb. für wissen. Bot. 32:557, 1898.
15. Klebahn, H.: Die Befruchtung von Sphaeroploea annulina, Ag. Festschrift für Schwendener, p. 81, 1899.
16. Klebs, G.: Fortpflanzungszellen bei Hydrodictyon utriculatum, Roth. Botanische Zeitung 49:789, 1891.
17. Mottier, D. M.: Nuclear and Cell Division in Dictyota dichotoma. Annals of Botany 14:163, 1900.
18. Nemec, B.: Ueber experimentelle erzielte neubildung von Vacuolen in hautumkleideten Zellen. Sitzungsb. d. Kön. böhm. Ges. d. Wiss. Math.—Naturwiss. Classe, 1900, No. 5.
19. Pfeffer, W.: Zur Kentniss der Plasmahaut und der Vacuolen. Leipzig, 1890.
20. Pringsheim, N.: Ueber die Dauerschwärmer des Wassernetzes, Berliner Akad., 1860.
21. Schmitz, Fr.: Ueber die Zellkerne der Thallophyten. Sitzungsb. d. Niederrh. Gesellsch. f. Nat. u. Heilk. 1879, p. 345.
22. Shaw, W. R.: Ueber die Blepharoplasten bei Onoclea u. Marsilia. Ber. d. deut. bot. Ges. 16:1898.
23. Strasburger, E.: Zellbildung u. Zelltheilung, 3 Aufl. 1880.
24. -: Kerntheilung und Befruchtung bei Fucus. Jahrb. f. wissen. Bot. 30: 251, 1897.
25. -_: Ueber Reduktionstheilung Spindelbildung Centrosome und Cilienbildner in Pflanzenreiche. Histologische Beiträge 6. Jena, 1899.
26. Swingle, W. T.: Zur Kenntniss der Kern und Zelltheilung bei den Sphacelariaceen. Jahrb. f. wissen. Bot. 30: 297, 1897.
27. Timberlake, H. G.: The Development and Function of the Cell Plate in Higher Plants. Botanical Gazette 30: 73, 1900.
28. -: Swarm Spore Formation in Hydrodictyon utriculatum, Roth. Botanical Gazette 31: 203, 1901.
29. -: Starch Formation in Hydrodictyon utriculatum. Annals of Botany 15: 619, 1901.
30. Vaucher: Histoire des Conferves. Gènéve', 1803.
31. Vries, H. de: Plasmolytische studien über die Wand der Vacuolen. Jahrb. f. wissen. Bot. 16: 463, 1885.
32. Webber, H. J.: Peculiar structures occurring in the pollen tube of Zamia. Botanical Gazette 23: 453, 1897.
33. -_: The development of the Antherozoids of Zamia. Botanical Gazette 24 : 16, 1897.
34. Went, F. A. F. C.: Die Entstehung der Vacuolen im Fortpflanzungszellen der Algen. Jahṛ. f. wissen. Bot. 21: 299, 1890.
35. Wilson, E. B.: On protoplasmic structure in the eggs of Echinoderms and some other animals. Journal of Morphology 15: 1. Supplement.
36. -_: The Cell in Development and Inheritance. New York, 1900.

# Timberlake-Swarm-Spores of Hydrodictyon. 519 

PLATE XXIX.

## EXPLANATION OF PLATE XXIX.

All figures were drawn with the aid of the Abbe camera lucida and with the Zeiss apochromatic objective 2 mm . aperture 1.30, and compensating oculars 2, 4, 12, and 18. They have been reduced in reproduction to $7 / 10$ the size of the original drawings.

Fig. 1, Vertical longitudinal section of an entire cell, x 175.
Fig. 2, Small portion of section similar to that shown in Fig. 1, showing distribution and structure of cell contents, x 600.
Fig. 3, Portion of tangential section, x 1500.
Fig. 4, Portion of tangential section, showing large spherical nuclei, x 1500.
Fig. 5, Procleavage stage, after disappearance of pyrenoids, x 1500.
Fig. 6, Vegetative stage with large nuclei, irregular in outline, x 1500 .
Figs. 7-14, Successive stages of nuclear division, x 2250.
Figs. 15-16, Different parts of a single cell, showing successive stages of nuclear division from one end of the cell to the other, x 600.
Figs. 17-19, Spindles of unequal size, a. Multipolar spindles, x 1500 .
Fig. 20, Unusual forms of figures of nuclear division, x 1500.
Fig. 21, Irregular forms of nuclei, collected into groups, $x$ 1500.

Figs. 22-23, Early cleavage stages. Many nuclei with elongated or pointed shapes, x 1500 .


# Timberlake-Swarm-Spores of Hydrodictyon. 521 

## EXPLANATION OF PLATE XXX.

Figs. 24-25, Successive stages of cleavage, x 1500.
Figs. 26-28, Successive stages of cleavage as found in different parts of the same cell, $\times 1500$.
Fig. 29, Late stage of cleavage. Nuclei arranged in pairs with cleavage planes cutting in between the adjacent nuclei, x 1500 .
Fig. 30, Beginning of the separation of the uninucleate segments, x 1500 .
Fig. 31, Cross section of cell in early stage of cleavage with the cleavage furrows formed by both plasma and vacuolar membranes, x 600 .
Fig. 32, Portion of vertical section of two adjacent cells, one of which is undergoing cleavage, x 1500.
Fig. 33, Portion of vertical longitudinal section of cell in late cleavage stage, x 1500 .
Figs. 34-35, Uninucleate segments containing large irregular nuclei, x 1500.
Fig. 36, Young swarm spores with cilia, x 1500.
Fig. 37, Mature swarm spores, x 1500.
Fig. 38, Anterior end of mature swarm spore, showing locomotor apparatus, x about 3000 .
Fig. 39, Giant swarm spores. Mostly binucleate, x 1500.
Fig. 40, Longitudinal sections of cells of very young net, $x$ 1500.

Fig. 41, Longitudinal sections of somewhat older cells. Each cell still contains a single nucleus and a single pyrenoid, x 1500.
Fig. 42, Cross sections of cells in same net as those in Fig. 41, x 1500.
Fig. 43 , Binucleate stage of young cell, x 1500.
Fig. 44, Section of a quite large cell containing but two large nuclei, x 1500 .


## AXIAL BIFURCATION IN SNAKES.

## ROSWELL HILL JOHNSON.

This paper contains descriptions and skiagraphs of thirteen two-headed snakes, a recapitulation of others previously described, and a concluding general treatment of this abnormality.

In the belief that the skeleton should be the basis of descriptions of cases of axial bifurcation, I have studied as many of these abnormalities as could be obtained from American museums. It seemed to me that skiagraphy, offering an opportunity for such skeletal study, might reveal some possible laws of this abnormality. The numerical data for the thirteen individuals is given in Tables I and II.

Case I. This is the case recorded as "a small double-headed snake from South America," No. 856, in the descriptive Catalogue of the Anatomical Museum of the Boston Society for Medical Improvement. This museum is now a part of that of the Harvard Medical School. This specimen [Plate XXXII, Fig. 1] was found to have scales impermeable to Roetgen rays, the only one of the snakes in which this was the case. The snake is in so bad a condition as to render specific determination difficult. It is probably Tropidonotus fasciata fasciata Linn., the range of which is the Austroriparian Region, or the Southern States. A normal specimen of this species, however showed the scales to be permeable to the Roentgen rays. The arrangement of the shields upon the dorsal aspect of the head is unusually irregular as Plate XXXIII, Figure 1a, shows. The angle of the sagittal planes of the skulls is $90^{\circ}$, that of the frontal planes is about $145^{\circ}$ with the angle facing dorsally. Measurements upon the two divisions show that the left is less than a millimeter shorter and slightly broader than the right division.

Table I.
Skulls united.

|  | Left Skull. |  | Right Skdll. |  | Condyle to TAIL. |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Length. | Length to Union. | Length. | Length to Union. | Length. |
| 1.................... | 10 | 7 | 10 | 7 | 170 |
| 2 .................... | 13 | 7 | 13 | 7 | 250 |
| $11 . . . . . . . . . . . . . . . . . .$. | 8 | 3 | 8 | 3 | 123 |

Skulls separate.

|  | Length <br> Left Skull. | Length <br> Right <br> Skull. | Length <br> Left Neck. | Number of <br> Vertebrae. | Length <br> Right <br> Neck. | Number of <br> Vertebrae. | Length <br> Re- <br> maining. |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $3 \ldots \ldots$ | 14 | 14 | 5 | 9 | 5 | 10 | 215 |
| $4 \ldots \ldots$ | 11 | 11 | 21 | 26 | 19 | 25 | 225 |
| $5 \ldots$. | 9 | 9 | 3 | 10 | $\pm$ | $6+$ | 162 |
| $6 \ldots .$. | 14 | 13 | 9 | 11 | 7 | 8 | 289 |
| $7 \ldots$. | 11 | 11 | 11 | 18 | 11 | 16 | 251 |
| $8 \ldots$. | 9 | 9 | 17 | 22 | 18 | 22 | 190 |
| $9 \ldots \ldots$ | 13 | 13 | 16 | 22 | 17 | 20 | 210 |
| $10 \ldots$. | 14 | 14 | 72 | 67 | 85 | 72 | 245 |
| $12 \ldots$. | 10 | 9 | 15 | 25 | 11 | 20 | 119 |
| $13 \ldots$ | 9 | 9 | 3 | 6 ap. | 3 | 6 ap | 165 |
| $14 \ldots$. | 10 | 11 | 2 | 9 | 3 | 10 | 151 |
| $\ldots$ |  |  |  |  |  |  |  |

Case II. This is a Bascanium constrictor Linn. [Plate XXXII, Figs. 2 and 2a, Plate XXXIII, Figs. 2b and 2c], from the collection of the Buffalo Society of Natural Sciences. It was without label but was undoubtedly brought in from the vicinity of Buffalo, where the species is occasionally seen. The point of division of the heads of this snake is close behind the eyes. Plate XXXIII, Figs. 2b and 2c, show the arrangement of the plates on the heads. The angle of the sagittal planes is $75^{\circ}$. The two frontal planes coincide. Here also the left head is slightly shorter and broader than the right.

Table II.


Case III. This is an Ancistrodon piscivorus Lacépède [Plate XXXIV, Fig. 3], lent from the collection in the Washington and Jefferson University. The heads are equal in size and normal in appearance except that there is an additional supralabial upon the right side of the right head. The angle of the sagittal planes is $105^{\circ}$, that of the frontal planes about $150^{\circ}$, facing ventrally.

Case IV. This Ophibolus getulus getulus Linn. [Plate XXXIV, Figs. 4 and 4a], is No. 7276 in the Smithsonian collection. It was found by Miss Marshall at Point Tobacco, Maryland, and was described in the American Naturalist, Vol. XII, p. 470 , by H. C. Yarrow. The heads are equal. The angle of the frontal planes is a trifle less than $90^{\circ}$. The vertebral column of the right division is in less intimate connection with the common vertebral column than that of the left division.

Case V. This specimen [Plate XXXV, Fig. 5], lent through the kindness of Mr. Whiteaves by the Geological Survey of Canada, bears this label: "Shore of Moira Lake, near Modoc, Aug.
1886. E. Coote." It is a Eutainia sirtalis sirtalis Linn. The light dorsal stripe divides caudad to the point of division of the vertebral column. The angle presented by the frontal planes of the two heads is nearly a right angle, that of the sagittal planes is about $70^{\circ}$. The right head is slightly longer and broader than the left one.

Case VI. This Pityophis catenifer Blainv. [Plate XXXV, Fig. 6] lent by the California Academy of Sciences was collected by J. W. A. Wright in 1878. It was later described in the American Naturalist, Vol. XII, p. 264. The angle presented by the frontal planes of the two heads is about $115^{\circ}$, that between the sagittal planes about $35^{\circ}$. A ventral fold of skin which extends forward from the point of division in the plane of the gastrosteges is present similar to that in the Ophibolus, No. 7276 of the Smithsonian collection.

Case VII. This is another Ophibolus getulus Linn. [Plate XXXVI, Figs. 7 and 7a] lent by the Smithsonian Institute. It was collected by J. M. Spainhorer in Lenoir, N. C., and bears the number 14540. The frontal planes make an angle of about $90^{\circ}$. Their longitudinal axes are almost parallel, however, for the necks curve toward the plane of the common partition and are connected by skin for an unusual distance cephalad. The heads are equal.

Case VIII. This is a Thamnophis elegans lineolata Cope [Plate XXXVI, Fig. 8] belonging to the University of California. The data given are "Marin Co. (?) California. 1879. C. A. Allen, collector." An additional peculiarity in this snake is the presence of a sharp angle in the common vertebral column, as shown in the skiagraph. The light dorsal stripe, prominent in this specimen, divides sharply 5 mm . cephalad to this angle and so is considerably caudad to the external division as well as to the vertebral division. There are twenty-one longitudinal rows of scales, which is the normal number for this species, from the sharp angle to a point about two centimeters caudad to it, where two rows end. Cephalad from where the dorsal stripe divides, many new rows begin. The right neck is shorter and the right head slightly shorter and narrower than the left.

Case IX. This is the young Bascanium constrictor Linn. [Plate XXXVI, Fig. 9] described by Wyman in 1863. It was obtained from the Warren Museum of the Harvard Medical College. When the necks are as long as in this specimen, it is impossible to determine the angle of the frontal planes. However, as the snake is now, this angle is $90^{\circ}$. The heads and necks are nearly equal. From the point of external division to the point of division of the vertebral column, the dorsal blotches are in two rows.

Case X. This is the snake in the United States National Museum bearing the catalogue number 25398 [Plate XXXVII, Figs. 10 and 10a.] It was collected by Miss M. Desha; the locality is not given. It belongs to the genus Pityophis, but its specific position in this difficult genus is less plain. If, however, the species Pityphis sayi Schlegel be made to include Pityophis bellona Bd. and Gird, following Cope, it may be called Pityophis sayi, since it seems intermediate between these two forms. This specimen is remarkable for the extreme length of the anterior doubled portion, which is much longer than in any case ever described, with the exception of that of Redi. The skiagraph reveals the division of the vertebral column very much farther back than that of the bodies proper, although the color markings show irregularities over the portion underlaid by the two vertebral columns.

Case XI. This small snake embryo [Plate XXXVIII, Fig. 11] has been so bady dessicated that a determination of its species is impossible. Together with cases XII and XIII it was purchased from a dealer, Mr. N. L. Wilson of 170 Tremont St., Boston, Mass., who was unable to furnish data concerning any of his specimens. The point of division is posterior to the eyes. The angle of the sagittal planes is approximately $80^{\circ}$, and there is no appreciable angle of the frontal planes.

Case XII. This is an embryo of Eutainia sirtalis Linn, still without color [Plate XXXVIII, Fig. 12.] It is remarkable for the considerable inequality in the length of the two necks and the deficiencies in structure. The right head on the shorter neck is singularly deficient. The lower jaw is represented by a mere stump and the dorso-ventral thickness of the
head is much reduced. The eyes are absent on this right head, but two slits possibly representing orbits are found lateral to the parietal plates. The left side of the lower jaw of the left head is also deficient.

Case XIII. This is either an embryonic or very young Tropidonotus fasciata sipedon Linn. [Plate XXXVIII, Fig. 13]. The inner side of the left head is deficient, there being no eye. A sharp angle is noticeable in the vertebral column resembling very much that of Case VIII. Somewhat farther caudad is a loop with the ventral surfaces inward and mutually coalesced. Cephalad to this loop is a median deficiency in the gastrosteges making an opening into the abdominal cavity. The number of longitudinal rows of scales is variable, for rows begin and end at several points along the body. The angle of the frontal planes is about $90^{\circ}$.

It might be well at this point to put on record other cases of two-headed snakes which were described in the popular press or which were seen by the author but were not made the subject of detailed study.

Mr. Outram Bangs had a young Tropidonotus fasciata sipedon Linn. which he had collected at Stoneham, Mass. In degree of bifurcation it resembled the individual described above as Case II.

In the Scientific American of December 5, 1896, is an article describing a double-headed snake and illustrated by a cut from a photograph. It is said to be a Heterodon simus Linn. from Central America. But since this locality is outside the range of the species as given by Cope, it is probably wrong. From the cut the degree of bifurcation seems to approximate that of Case VI. in this paper.

A two-headed snake was advertised on exhibition in a cigar store in New York in 1896 or 1897. The owner, Mr. H. C. Somers, told me that after the snake died, it had been destroyed. It was found in the Catskills, though attributed in an advertisement to South America. I believe that this was the snake described in the Scientific American.

A tro-headed snake was found near the New York Zoological Park in 1901. The New York Tribune said it was a
milk snake ten inches long. Judging from the photograph reproduced in the paper, the degree of bifurcation was most nearly like that in Case XII.

Dr. J. S. Kingsley in describing an abnormal frog in the American Naturalist, Vol. XII, pp. 694-695, states that a two-headed snake is in the collection of the Lyceum of Natural History at Williams College. Prof. S. F. Clark informs me, however, that the snake probably referred to is one which merely simulates this condition from having been crushed.

Mr. Samuel Garman tells me that this abnormality may be found frequently in embryoes of Tropidonotus fasciata sipedon Linn.

## RESUME OF ALL CASES DESCRIBED.

Table III records the more important data gathered from previous descriptions of two-headed snakes which are sufficiently complete.
Table III.

| Author. Date. | Species. | Degree of Bifurcation. | Angle. | Relative size of divisions. |
| :---: | :---: | :---: | :---: | :---: |
| Aldrovandi. 1640.... | (From Bologne ) | From the cut the common body was 32 cm . long. long. The right division was 5 and the left division 4 cm . |  | Left division 1 cm . longer. |
| Redi. 1684............ |  | Divisions about 22 per cent. of whole length. Spinal cord double to middle of back. Two oesophagi lungs, stomachs, hearts, and livers. One intestine and set of genital organs. |  | Equal. <br> The right heart and right lung larger. |
| Lacépède. 1789.... .. | " Perhaps Vipère Fer deLance." Lachesis lanceolatus Lacép. | One neck with two heads. | $50^{\circ}+$ | Equal. |
| Edwards. $1751 . . . . .$. | "Yellow snake of Barbadoes." | Union just anterior to corners of mouth but mouths length. separate. 10 cm . to point of union. 26.5 cm . total | $19^{\circ} \mathrm{ap}$. Frontal planes at acute angle. | Equal. |
| Edwards. 1751. ..... | "Common English snake." | Two heads quite separate. Necks unite about an inch from the heads. |  |  |
| Bancroft. 1769. ...... | " Probably Tropidonotus fusciata sipedon Linn." <br> Lake Champlain. | short. <br> Point of union behind cephalic shields but necks very short. | $90^{\circ} \mathrm{ap}$. |  |
| Mitchill. 1826.. ...... | Bascanium constrictor Linn. | Necks very short. | Frontal <br> planes at <br> right angle. | The distance to tip of left head is 3 mm . less than to tip of right head. |


| Mitchill. 1826. ....... | Bascanium constrictor Linn. Same brood. | Heads separate at eyes. Three eyes. |  | Equal. |
| :---: | :---: | :---: | :---: | :---: |
| Mitchill. 1826........ | Bascanium constrictor Linn. Same brood. | Three eres. Single lower jaw. Heads separate at corner of month. Body single only 18 mm . of its length. 94 cm . to tip of right tail. 110 cm . to tip of |  | Left anterior part of the head larger. Left tail 16 cm . longer. |
| Wyman. 1853......... | Tropidonotus fasciata sipedon Linn. | Each head supported by a distinct neck. The tail also double for about an inch. At middle of body size is increased and vertebral column is double and provided with a double set of ribs. |  | Equal. |
| Dusseau. 1865........ | Tropidonotus natrix Linn. | Two distinct necks. | Acute angle. |  |
| Dorner. 1873. ...... | Pelias berus Merr. | Each head on its owu short neck. Two hearts. Two lungs wholly separate. Vertebral columns united 16 mm . from the tip, or 103 per cent. of total length of 155 mm . |  | Left head a mere trifle larger. Right heart heart. |
| Boettger. 1890.... | Telamis bicolor Band. | Four nasal plates instead of two. Four nasal openings. | - | Equal. |
| Von Braun.... ...... | Tropidonotue natrix Linn. Embryo 15 mm . long. | The point of division lies in the region of medulla. | $130^{\circ}$ | Equal. |
| Borgert. 1897......... | Peliäs berus Merr. | See case XIV in other tables. | $60^{\circ}$ | Right division slightly longer. |

Species. One must expect more of these abnormalities to be found in those species native to regions where most attention is paid to the fauna and also in the most abundant species. Yet after making these corrections, it is probable that this axial bifurcation is more frequent in some species than in others.

Tabulating the data, we find: Tropidonotus.

3 T. fasciata sipedon Linn.
2 T. natrix Linn.
1 T. fasciata fasciata Linn.
5 Bascanium constrictor Linn.
3 Eutainia.
2 E. sirtalis Linn.
1 E. elegans lineolata Cope.
2 Ophibolus getulus Linn.
2 Pelias berus Merr.
2 Pityophis.
$1 P$. catenifer Blainv.
$1 P$. sayi Schlegel.
1 Ancistrodon piscivorus Lacépède.
1 Pelamis bicolor.
1 Lachesis lanceolatus (?)
1 Heterodon simus Linn (?)
1 "Yellow snake of Barbadoes."
This result together with Mr. Garman's statement concerning Tropidonotus would seem to make it advisable to examine gravid snakes of the genus Tropidonotus and eggs of Bascanium if in search of this abnormality.

Type of Bifurcation. The great majority of cases are those of cephalic bifurcation, the catadidyma of Fisher. None show merely caudal bifurcation, the anadidyma of Fisher. But three cases have been described where both cephalic and caudal bifurcation existed, the anacatadidyma of Fisher. These are the case of Wyman and two of the three described by Mitchill. The former's specimen is further remarkable for a median duplicity of the vertebral column,
which is the only case of the kind on record and which lies without Fisher's system of classification.

Degree of Bifurcation. The data as regards degree of bifurcation aside from the thirteen cases here described and that of Borgert have little quantitative value, for frequently the descriptions are no more specific than merely "two-headed." Fortunately, however, the fact is sometimes noted whether there are two heads on separate necks, or two skulls in organic fusion. Classifying on this basis we have:

8 dipropi:
7 near the eye.
1 in the region of the nose.
23 dicephali:
15 "short neck."
8 "lang neck."
In Tables I and II, and Plate XXXI, the degrees of bifurcation of the thirteen snakes here described and the one of Borgert (No. 14) are represented. Only by such precise criteria can the question of degree of bifurcation be properly studied. Three criteria were employed as a discontinuity might be shown by one criterion which would not be shown by the others. These were the number of vertebrae, the lengths of the divisions in terms of proportion of the total length and in terms of vertebral units. The lengths of the divisions were measured to the cephalic end of the skeletal division, not the external division. The vertebral unit was chosen, i. e., the length of one vertebra where the vertebræ are largest, because it was found that occasionally in the divisions there was a very large number of abnormally small vertebro. In each case, both classification by the longer division and by the average division was used.

Even the few cases here described are so discontinuous in their distribution as to degree that I believe it is safe to say that axial bifurcation is more likely to occur in some degrees than in others, and that the most frequent degree is from about 6 to $13 \%$ of the length from the cephalic end.
Angle of Bifurcation. In the case of dicephali, the general rule may be made that when the vertebral column is double for a very short distance, the angle of divergence of the proximal
ends of the double vertebral column is a large one, and it decreases as the degree of bifurcation increases. This applies only to the bifurcation of the vertebral column. The apparent angle made by the divisions does not seem to conform to any rule.

In the cases of dipropi the rule is the reverse,-the greater the degree of bifurcation, the greater the angle.

The frontal planes of the two heads do not always coincide, but may meet at an angle. In all the cases of dicephali, this angle was directed downward. Of the three cases of dipropi, the angle was directed upward in one, and in the other two there was no appreciable angle.

Accompanying Abnormalities. In the cases of dicephali, the necks are likely to be of different lengths, the difference being greater as the necks are longer, though with the data in hand, it cannot be said that the difference would be greater in comparison.

Abnormalities of the shields on the head are not noticeable in the cases of dicephali, except where there is marked deficiency, but they are evident in the cases of dipropi, as would be expected. These irregularities, however, were usually of proportion and of outline, rarely of absence or of redundancy.

Deficiencies were noted in three cases: VIII, XII, and XIII, enough to lead one to believe that they are frequently correlated with axial bifurcation. Another peculiarity which seems to be correlated in some way is the presence of sharp angles in the vertebral column in Cases VIII and XIII.

Especial attention has been given in this paper to the point of divergence of the vertebral column. This, however, does not fully express the degree of bifurcation of the color markings and of the internal organs. I think it may be said that when the color markings show a longitudinal arrangement, they will be affected caudad to the point of division in the axis.

Data concerning the internal anatomy of these anomalies are very uncommon, the only instances of adequate descriptions being those of Redi, Dorner, and Borgert. The evidence there given certainly shows that in these cases there was a doubling of internal organs caudad to the vertebral bifurcation.

## SUMMARY AND CONCLUSION.

1. Thirteen two-headed snakes are described by means of skiagraphs.
2. The previous descriptions of two-headed snakes are reviewed.
3. It is concluded that this abnormality is more abundant in some species than in others and that the point of bifurcation is most likely to occur in the cephalic half of the snake, between 6 and $13 \%$ of the entire length from the cephalic end.
4. The point of bifurcation of the vertebrae is more posterior than would be supposed from external examination. The skulls frequently appear united externally when in reality they are not.

I am much indebted to Prof. C. B. Davenport for kind advice and assistance, to Mr. Samuel Garman for help in determination of species, to Mr. Outram Bangs, Mr. F. K. Mixer, Mr. J. F. Whiteaves, Prof. Edwin Linton, Mr. F. W. True and to Prof. W. E. Ritter for loans of material, and to Dr. W. McM. Woodworth for assistance with the bibliography and for the loan of books.

Anatomical Laboratory, University of Wisconsin, November 21, 1901.

## BIBLIOGRAPHY.

Aldrovandi, Ulyssis. 1640. Historiæ Serpentum et Draconum, Libri Duo, Bononiæ. p. 60.
Bancroft, Edward. 1769. Natural History of Guiana. London. p. 214.
Bateson, Wm. Materials for the Study of Variation. London. 1894. p. 561.
Bœttger, O. 1890. Ber. üb d. Senck. nat. Ges. in Frankf. a. M. p. LXXIII.

Borgert, H. Uber eine Zweiköpfige Kreuzotter. Verhandlungen des Naturwissenschaftlichen Vereins in Hamburg (3). Band IV. 1896.

College of Surgeons Museum Teratological Catalogue, 1872, Nos. 24-27.
Dorner, H. Eine Kreuzotter mit zwei Köpfen. Zoologische Garten, 1873, XIV. p. 407.
Edwards. Natural History of Birds, \&c., Pt. IV. London. 1751. p. 207.

Fisher, G. J. Diploteratology. Albany. 1868.
Geoffroy-St. Hilaire. Histoire des Anomalies. Ed. 1838, II. p. 197.

Jackson, J. B. S. Descriptive Catalogue of the Anatomical Museum of the Boston Society for Medical Improvement. Wm. D. Ticknor \& Co., Boston. 1847.
Klaussner, Ferdinand. Mehrfachbildunger bei Wirbelthieren. Eine Teratologishe Studie. Rieger, München. 1890.
Lacépède. Histoire naturelle des Serpens, II. 1789 ; p. 482.
Lanzoni. Miscell. curios., 1690, Obs. CLXXI, p. 318, Fig. 36.
Mitchill, S. L. American Journal of Science. X. 1826. p. 48.

Redi. Osserv. int. agli anim. vienti, \&c. 1778. p. 2, Tav. 1.
Schenck, J. G. Monstrorum historia memorabilis, monstrosa humanorum partuum etcetera. Francofurti.
Valentin, M. B. Amphiteatrum Zootomicum. Francofurti ad Moenum. 1704. Ibidem 1720. Pars. II. p. 172.
Wright, J. W. A. Mining and Scientific Press, San Francisco, Feb. 16, 1878.
Wyman, J. Proceedings Boston Society of Natural History, 1862, IX. p. 193.
Yarrow, H. C. American Naturalist, Vol. XII. p. 470.

## EXPLANATION OF PLATES.

The figure numbers in each plate correspond to the number of the case.

The skiagraphs were taken with dorsal surfaces next the plate. The sides are not reversed therefore in the figures, as may be seen by comparing the photograph and skiagraph in Plate XXXVII.

Figures 12 and 13 are tracings from skiagraphs since the plate was broken before good prints were obtained.

Figure 11 is a drawing substituted by the engraver for an obscure skiagraph.

$\prod_{x}^{\varepsilon} \prod_{x}^{\varepsilon} \prod_{11}^{c}$





Fig. ia.


Fig. 2l.


Fig. 2 c.


.


$\%$



FIG. 11


FIG. 12


FIG. 13

## ON THE RELATION BETWEEN HEAT CONDUCTIVITY and density in some of the common woods.

LOUIS W. AUSTIN AND C. W. EASTMAN.

In 1828 De la Rive and De Candolle, ${ }^{1}$ after examining the conductivities of five varieties of woods, announced the interesting fact that along the fibers the thermal conductivities of the different woods were approximately proportional to their densities. Some observations made by C. G. Stangel ${ }^{2}$ in 1899 in connection with his work on the effect of moisture on the heat conductivities of woods and rocks called attention again to the relations between the conductivities and densities of woods. As far as has been ascertained no other work has been done on the subject since that of De La Rive and De Candolle, the extensive work of Tyndall ${ }^{3}$ on woods being confined to the rapidity of propagation of the heat wave, or, diffusivity.

In our own experimental work a method was made use of which was presented in outline by Voigt ${ }^{4}$ in 1898, the general plan of which is as follows: The specimens whose heat conductivities are to be compared are cut in the form of right angled triangles and joined along the hypotenuse, as shown in the figure. If one of the edges AB be heated until a steady flow of heat is established so that the temperature at each point becomes constant, the isothermal lines in the lower specimen will be parallel to the base $A B$, while in the upper specimen they will be bent upward or downward depending on whether the conductivity is greater or less than in the lower. Accord-

[^8]ing to the tangential law of refraction of a disturbance crossing the boundary of two media in which it is propagated with conductivities $\mathrm{k}_{1}$ and $\mathrm{k}_{2}$,
$$
\frac{\mathrm{K}_{1}}{\mathrm{~K}_{2}}=\frac{\tan \cdot \varphi_{1}}{\tan \cdot \varphi_{2}} .
$$

The determination of the relative conductivities then resolves itself into the measurement of the angles $\phi_{1}$ and $\phi_{2}$. In practice the isothermal line is determined by the melting of a thin layer of wax spread over the specimen.


In applying this method to determining the relative conductivities of woods the following arrangement was used: Blocks of wood about 8 cm . square and 2 cm . thick were cut in triangular form and placed in a wooden frame with their fibers parallel to AC, as shown in the figure. These were covered on the bottom and on the sides AC and BD with thick pieces of asbestos to cut down the loss of heat. A uniform distribution of heat along AB was obtained by pressing a strip of iron of about the same dimensions as the edge of the block against the wood, and heating this by means of a row of small gas flames. The wax used to show the isothermal line was composed of paraffin mixed with a little turpentine. This mixture had a melting point of about $40^{\circ}$ and gave a sharp line of demarcation between the melted and unmelted portions. To prevent the wax soaking into the wood and also to insure a more uniform radiation both blocks were covered with a sheet of tinfoil on which the melted wax was painted with a flat brush. This de-
vice was first used by C. G. Stangel in his work on heat conductivity to which reference has already been made.

It was found that there were two chief sources of error to be guarded against, the first of which is the direct effect on the wax of radiation from the source of heat. To prevent this the metal from which heat is communicated to the wood must not be allowed to extend above the edge of the wood, this being much more effective than any system of screens used with a larger heating plate. The second possible source of error lies in the difficulty in obtaining a perfectly good contact between the specimens. To insure this as far as possible, two strips of tinfoil were placed between and the blocks pressed tightly together by means of wedges. It was also found necessary in order to preserve the contact, to plane the blocks frequently as they became warped by the heat. With these precautions very uniform results were obtained.

In our experiments only the conductivities parallel to the fibers were tested. The piece next to the source of heat (position 1 in the figure) was in all cases the same specimen of white oak, the others being placed in position for comparison with it. The densities were determined partly by weighing and measuring the specimens and partly by the method of immersion. Five sets of angles were taken for each wood and the average of these used in computing the conductivity. These measurements were estimated to be correct to within two per cent. Some errors are undoubtedly introduced into the results by the fact that in the second specimen the flow of heat is not in general strictly parallel to the fibers, and since the conductivity at right angles to the fibers is smaller, the true values of the conductivities of some of the lighter woods may be slightly greater than those given. In order to reduce the relative conductivities at least approximately to absolute units, the value for walnut parallel to the fibers, found by Péclet ${ }^{1}$ is assumed to be correct for our specimen and the conductivities of all the other woods are expressed in the same terms.

[^9]In the following table are given the conductivities and densities of the twelve woods examined:

| Wood. | Density. | Cond'ctivity. | Wood. | Density. | Cond'ctivity. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Sequoia... | 0.380 | 0.000342 | Gum | 0.559 | 0.000458 |
| Batternut. . | 0.394 | 372 | Walnut. | 0.609 | 480 |
| Pine....... | 0.406 | 358 | White oak. | 0.615 | 472 |
| Linden. | 0.408 | 391 | Brown ash. | 0.649 | 539 |
| White wood. | 0.506 | 426 | Georgia pine. | 0.657 | 540 |
| Cherry | 0.534 | 451 | Red birch | 0.711 | 528 |

The results of the work seem to indicate that the law of proportionality between heat conductivity and density, as announced by De la Rive and De Candolle is at least very approximately obeyed. It is true that in our results slight exceptions to this are shown but it is quite possible that all the variations observed are due to errors of observation. It seems more probable, however, that slight variations really do exist, as the difference in form of the cells in different woods must produce a difference in the distribution of material which would in itself, at least in some degree, affect the conductivity.

The University of Wisconsin, December, 1900.

# ECONOMIC AND SOCIAL DEVELOPMENT OF KENOSHA and la fayette counties. 

ROBERT HUGH DOWNES, B. L., AND KATHERINE PATRICIA REGAN, B. L., WITH AN INTRODUCTION BY ORIN GRANT LIBBY, PH. D.

## INTRODUCTION.

The two studies in local history following this introduction are part of a year's seminary work on a new field in Wisconsin history. The purpose of this seminary has been to lay an adequate statistical foundation for later work of a somewhat different character. Owing to the fact that accurate and detailed state records are not published to any considerable extent by any state it became necessary to compile carefully a large body of statistics from the original records before any really valuable research work could be carried on in Wisconsin local history. In the two senior theses published under this cover the complete record, on which a part of the discussion is based, are to be found in the appendix. Smaller tables in more concise form and maps of several kinds appear in the papers and make clear what is sought to be proved.

The work so far accomplished has proceeded along certain definite lines. A special county has been studied, not as a whole but with reference to its units, the towns. The boundary changes have been in most cases worked out and there has been a brief survey of the geological condition and the main topographical features in so far as they bear upon the question of soil and general productive power. Following this the distribution of the population is studied, and the nativity of the whole population is worked out for every town. This supplies a basis for a still further study of the per capita wealth, occupation, etc., of all who have entered the state since 1850. Thus
in this way we are able to compare the productive power of the various nationalities, to discover where they go and what occupations they engage in. To some degree also we may be able to trace the movement of emigration across the continent by the birth places of the various members of a family who in the end settle in this state. In one of the theses the study was carried to the political conditions and the various nationalities were considered not only as immigrants and producers of wealth, but also as voters. One not unimportant result of the study so far as carried on, is the means it has afforded for the correction or verification of the published census records of the state and United States. The count of the actual population for 1850, 1860 and 1870 has enabled the students to discover substantial errors in the published reports.

It is my purpose in the end to use this work of the seminary for a series of years as the foundation and starting point for an economic history of Wisconsin. The great problems of immigration, of industrial change and development, and of the influence of foreigners on our social, economic and political life can never be fairly discussed without some such careful study as is indicated above. It is no slight task to undertake the examination and arrangement of the hitherto unpublished records of the state. But the task has been attempted with the hope that the results obtained will be of permanent value to the student of statistics and political economy, as well as to the student of sociology and of history.

I desire to acknowledge my indebtedness to the Secretary of State for his courtesy in loaning to the State Historical Society, at my request, the copies of the original United States census returns for 1850,1860 and 1870, on file in his office. This rendered these very valuable unpublished materials more accessible to the students of my seminary for the present year and made the labor of compiling statistical tables less burdensome. These census reports seem to have been entirely ignored heretofore by students of Wisconsin history and it is to be hoped that they will not again be lost sight of by students of local history in the state.
O. G. Libby.

University of Wisconsin, November 28, 1901.

# ECONOMIC AND SOCIAL DEVELOPMENT OF KENOSHA COUNTY. ${ }^{1}$ 

## ROBERT HUGH DOWNES.

## Chapter I.

## FORMATION OF THE COUNTY AND TOWNS.

Kenosha county is located in the southeastern part of the state of Wisconsin. It is bounded on the east by Lake Michigan and on the south by the state of Illinois, and covers an area of 268.04 sq . miles. At the territorial formation of the present state of Wisconsin in 1836, the state was divided into four sounties-Crawford, Iowa, Brown, and Milwaukee. The egion north of the parallel $46^{\circ} 31^{\prime}$ north latitude was yet unorganized. The southeastern part of the state was included in Milwaukee county. ${ }^{2}$ An act passed by the legislative assembly in 1836 , provided that the townships number one, two, three, and four north, of ranges nineteen, twenty, twenty-one, twenty-two, and twenty-three east of the fourth principal meridian be erected into a separate county, named Racine.

By this act what is now Kenosha county was included in Racine county and it remained a part of Racine county until 1850, when by an act approved January 30,1850 , it was enacted that all of Racine county within the boundaries, commencing at the southwest corner of township one, range nineteen east and running thence east on the state line to the center of lake Michigan; thence northerly along the eastern boundary line of Wisconsin to the township line between townships two and three; thence west on said township line to the range line between ranges

[^10]nineteen and twenty east; thence south on said range line to the section line between sections twenty-four and twenty-five, in township two, range nineteen east; thence west on said section line to the range line between ranges eighteen and nineteen east; thence south on said range line to the place of beginning, shall be erected into a separate county called Kenosha. ${ }^{1}$

The boundary of the county as determined by the above act has never been altered, but since that time there have been several changes in the town boundaries for no special reason other than local advantage or convenience. The accompanying maps, I and II, show Kenosha county and the towns as erected by the act approved Jan. 30, 1850, and as they are today. By consulting these maps one will readily see the nature of the changes that have been made. The legislature, Jan. 2, 1838, created the towns of Salem, Pleasant Prairie, and Southport. ${ }^{2}$ Salem and Pleasant Prairie have remained unchanged to the present time, while Southport by an act of the Board of Supervisors of Kenosha county, Feb. 26, 1853, was vacated and divided. The parts, except so much of the town as was embraced in the city of Kenosha by the act of the legislature of Jan., 1850, incorporating the city, were added to the towns of Somers and Pleasant Prairie. That part of Southport lying in town one north, range twenty-three east, was annexed to the town of Pleasant Prairie, and that part lying in town two north, range twentythree east, was annexed to the town of Somers. ${ }^{3}$ The towns of Bristol and Paris were created by the action of the County Board of Supervisors of Racine county, Jan. 11, 1850, and neither have been changed since that time. It was impossible to ascertain the dates when the towns of Brighton, Pike, and Wheatland were created, but all three existed when Kenosha was formed. Brighton has remained unchanged, while the name of Pike was changed to Somers early in 1851.4 The town of Wheatland was divided by the action of the Board of Supervisors of Kenosha county in 1860 by a line commencing at the

[^11]northeast corner of section thirteen, town one, range nineteen east, and extending west on the north line of sections thirteen, fourteen, fifteen, sixteen, seventeen, and eighteen to the range line, and the town of Randall created. ${ }^{1}$ This was the last change made in the boundaries of any of the towns. ${ }^{2}$

## Chapter II.

GEOLOGY, SOILS AND TOPOGRAPHY.
The surface features of the county are simple and undulating in character with a gentle slope to the southeastward. The height of the land above the level of Lake Michigan gradually increases, with the exception of a small region in the western part of the town of Pleasant Prairie and the eastern part of the town of Bristol, to the westward, reaching its greatest height in the extreme northwestern part of the town of Wheatland.

The important rivers in the county are the Fox, Des Plaines, and the Pike. These are located in the western, central, and northeastern parts of the county, respectively. The current in these rivers and their tributaries is not swift enough to furnish any power for manufacturing purposes. ${ }^{3}$ The slight rise in the height of the land in the eastern part of the county which extends through the central part of the town of Pleasant Prairie, and then northwestward through the towns of Somers and Paris is notable because it makes the streams which flow into Lake Michigan short and abrupt, and forms a continuation of the watershed between the St. Lawrence and the Mississippi basins.

In its geological formation Kenosha county is divided into four distinct north and south sections by the deposit of four different varieties of coarse drift by the glaciers upon the Niagara limestone, which forms the bed of the entire county. These four different varieties of drift differ from each other

[^12]in color, degree of coarseness, time of deposit, and chemical composition. The section in the western part of the county, including the greater part of the towns of Randall and Wheatland, is the region of Kettle Range Moraine Kames. This section composes the extreme southeastern part of the Kettle range. This region is more undulating than any other part of the county, in fact some of the hills are quite abrupt. In this same region and also in the town of Salem there are numerous small moraine lakes, which add much to the beauty of the land. The most important of these lakes are Powers, Elizabeth, Mary, Silver, and Camp.

Adjoining this section to the eastward, and including the eastern part of the towns of Randall and Wheatland, all of Brighton and Salem, and the western part of Paris and Bristol, is the region of bowlder clay. This region is of an earlier formation than that just described and consists of a commingled mass of clay, sand, gravel, and bowlders variously arranged with reference to each other and spread out irregularly over the surface of the limestone bed below.

The formations of the other two sections are known as the light colored pebble clay and the beach. The light colored clay covers the greater part of the towns of Paris, Bristol, Somers, and Pleasant Prairie. This formation is very similar to the bowlder clay, and only differs from it in that the bowlders are smaller and the clay is of a lighter color. The beach formation covers the eastern part of the towns of Somers and Pleasant Prairie, extending on an average about one mile inland from the lake shore. At the surface there is a deposit of sand and gravel, with a varying, but subordinate admixture of clayey and marly material. The gravel averages about ten feet in thickness, and is usually fine, and interstratified with sand and occasionally with clay.

The essential features of a drift formation of this sort are its industrial value. The accumulation, deposit, powdering, and commingling of a vast variety of materials by the glacial forces must inevitably result in producing a sub-soil rich in variety of minerals and well suited to give a secure and permanent founfation to agricultural industries.

Distribution of Soils


지 Prairie lorm
HEAVIER marly clay
國 hUMUS (PEAT) SILICEOUS SANDY LOAM
$\square$ LIGHTER MARLY CLAY

Boundary Changes


Taking up now the prevailing sub-soils, we find that there are four in the county. They are known as prairie loam, lighter marly clay, humus (peat) heavier marly clay, and silicious sandy loam. Map II shows the location of each. The prairie loam is found chiefly in the central and eastern part of the county. It predominates in the towns of Paris and Somers, while in the four counties in the west there are only small scattered areas. This is a rich, black soil and very responsive to proper fertilizers. Its chief ingredient is silica, with which is associated a variety of soluble mineral substances which exist in abundance in the limestone and drift deposits beneath. These soluble minerals constitute excellent food for plant life.

The lighter marly clay is the predominating soil in the county. It prevails in the same general region as the prairie loam, its areas being interwoven with them. It is a reddish soil and is very durable and fertile. The chief ingredients are calcium and magnesium, and it contains enough of sandy material to make it loamy and easy to work.

The heavier marly clay is a heavier sub-soil than any of the other soils in the county. This soil with the silicious sandy loam constitute the poorer soils of the county, and fortunately they only form a small part of the soil when compared with the area of prairie loam and the lighter marly clay. The chief characteristics of this soil are that it contains a notable amount of lime, magnesia, and silica. The surface is frequently strewn with bowlders, chiefly "hard heads," while cobble stones and pebbles mingle more or less with the soil. The silicious sandy loam is found along the lake shore and is a result of the beach deposit already described. It is a sterile, silicious soil, but when mingled with clay it produces a rich fertile soil well adapted to certain kinds of plant life.

The humus (peat) is found along the Fox and Des Plaines It is a result of the decay of peat and swamp muck, upon which various grasses flourish. ${ }^{1}$

From this study it seems evident, that the soil and contour of Kenosha county are admirably suited for agricultural

[^13]purposes; that the vast supply and variety of minerals, in the underlying strata, constitute excellent plant food, and insure a permanent foundation for agricultural industries, and that the wealth of the county must depend primarily upon agricultural and dairy pursuits. ${ }^{1}$

## Chapter III.

THE PEOPLING OF THE COUNTY.
We now come to the study of the peopling of Kenosha county. During the period of 1836 and the few years just preceding that rivers and in small areas in the low lands throughout the county. time, the great West, its boundless natural resources and its many advantages for the acquiring of wealth was the subject of absorbing attention throughout the Middle and Eastern states. In December, 1834, a resident of the town of Hannibal, Oswego county, N. Y., gave a dinner party to a number of friends. At this dinner the dominant theme of discussion was the West, its beautiful prairies, productive soil, and remarkable possibilities. Members of the party related marvelous tales and glowing descriptions which they had heard from travelers who had explored the country west of the great lakes. The enthusiasm during the evening became so great that before the party broke up those present had mutually resolved upon a plan to organize an association to settle a colony in the West, in which those becoming members should be of assistance to each other, and mutually share profits and losses in the enterprise. Shortly afterwards there was a general meeting of the people of the town called for the purpose of inviting the co-operation of all who desired to join such an enterprise. The meeting was well attended and the object under consideration met with more general favor than was anticipated. At a subsequent meeting, held Feb. 20, 1835, an organization was finally perfected, under the name of the "Western Emigration Company." Peter Woodin

[^14]and John Bullen, Jr., both of Hannibal, were elected president and secretary of the company. ${ }^{1}$

The constitution of the company proposed to raise a cash capital of $\$ 8,000$ by subscriptions of stock in shares of $\$ 10$ each, the funds so raised to be invested in real estate suitable for a town site, and the share-holders to be entitled to the proceeds arising from any increase in the value of the property. The stock of the company promised to be lucrative, and many people of small means, who desired to find a new home in the West, became share-holders. Old men and young men, and even unmarried women, who were employed as house servants, in some instances, appropriated from their earnings sufficient to purchase a share, in the hope of realizing large profits.

In the spring of 1835, the company appointed a committee to explore the distant, and then comparatively little known regions of the west. The explorers left Hannibal March 19, 1835. The account says the day of departure was one of considerable interest to the inhabitants of Hannibal. The instructions to the committee were explicit and reduced to writing. The committee was instructed to examine the country along the western shore of Lake Michigan with a view of finding an elegible situation for a commercial town, with lands in the vicinity adapted to agricultural pursuits. Milwaukee was fixed as the first point the committee was to visit-that being the only place then known between Chicago and Green Bay as being settled by white inhabitants. From Milwaukee they were directed to explore either north or south, along the shore, as they might judge best.

On reaching Milwaukee the committee learned that there were several points on the lake shore toward Chicago capable of being rendered of commercial importance, which were yet unoccupied. Thereupon the committee proceeded southward, exploring such points as they considered afforded a natural advantage for the construction of a harbor. Their first step was at the north of the Root river. The land here was claimed by other parties, but the committee entered into an agreement with

[^15]these parties by which they were to pay $\$ 2,700$ for the claim upon the land upon which the principal part of the city of Racine now stands.

Owing to some misunderstanding the committee was removed and John Bullen, Jr., was made sole agent for the company. On his arrival at Root river, the parties who made the agreement with the committee refused to abide by its terms. An attempt to enforce the agreement failed and Bullen, together with his party proceeded to examine the country further south. On the 6 th of June, 1835, the exploring party reached Pike Creek. They were struck by the depth and width of the creek and decided at once to build a town upon its shores. As soon as the news reached Oswego county, immediate preparations were made by stockholders to emigrate to the newly-selected home. About fifteen families, mostly from the town of Hannibal, came on during the summer and fall of 1835. A part of these were not members of the company, and on their arrival, made claims on land in the vicinity of Pike Creek for the purpose of pursuing the business of farming. This was the beginning of the village of Southport, the first village in the county, and which in 1850 became the city of Kenosha. ${ }^{1}$

The Western Emigration Company was dissolved in Dec., 1836. It proved a losing venture to most of the stockholders, but it nevertheless served as an intermediary between the people in New York who were about to emigrate and the new lands of Kenosha county. During the short existence of the company it advertised and made Pike Creek known to a large number of the people of New York. In so doing it must undoubtedly have directed a large part of those emigrating from New York to the West to this point. In our study of the nativity of the population of the county for the two decades from 1850 to 1870 , we may therefore expect to find a large percentage of the population born in New York. Table $1^{2}$ shows this to be true in every town in the county.

[^16]Table I.—Nativity by percentage.

| County and Towns. | 岗 |  | $\begin{aligned} & \text { 를 } \\ & \text { B } \\ & \text { B } \\ & \text { B } \end{aligned}$ |  |  |  |  |  | \|c|c|c |  | 第 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Kenosha county ... | 1850 1860 | $\begin{gathered} 11.9 \\ 7.8 \\ 4.8 \end{gathered}$ | $\left\{\begin{array}{l} 29.6 \\ 18.3 \\ 12.4 \end{array}\right.$ | $\begin{gathered} 2.2 \\ 1.5 \\ 1.5 \end{gathered}$ | $\begin{array}{l\|l\|} \hline 2 & 18.7 \\ 5 & 31.6 \\ 5 & 45.3 \end{array}$ | $\left[\begin{array}{c} 5.5 \\ 3.3 \\ 3.8 \end{array}\right.$ | $\begin{aligned} & .7 \\ & .7 \\ & \hline 8 \end{aligned}$ | $\begin{aligned} & 2.5 \\ & 1.6 \\ & 1.1 \end{aligned}$ | $\begin{array}{cc} 9.2 \\ \vdots & 7.5 \\ \hline & 7.5 \\ \hline .6 \end{array}$ | $\begin{gathered} 11.7 \\ 8.7 \end{gathered}$ | $\begin{aligned} & 7.9 \\ & \frac{1}{3} \\ & 3 \end{aligned}$ | ${ }_{8}^{8.6}$ |  |
|  | 5 |  |  |  |  |  |  |  |  |  |  |  |  |
| City of Kenosha ... |  | $\begin{gathered} 11.4 \\ 7.9 \\ 6 . \end{gathered}$ | $\begin{aligned} & 27.6 \\ & 15.4 \\ & 11.6 \end{aligned}$ | $\begin{aligned} & 1.5 \\ & 1.0 \\ & 1.1 \end{aligned}$ | $\begin{array}{l\|l\|l\|} 5 & 17.4 \\ 0 & 3.8 \\ \hline 1 & 44.8 \end{array}$ | $\begin{aligned} & 7.6 \\ & 4.6 \\ & 5.4 \\ & 5.3 \end{aligned}$ | $\begin{array}{\|l\|l\|} \hline 14 \\ \hline \end{array}$ | $\begin{array}{c\|c\|c} 2.5 & 7.3 \\ 1.9 \\ 1.6 & 5.1 \\ \hline .3 .3 \end{array}$ |  | 31 16.4 <br> 11 11.8 <br>   |  | - 5.5 . 6 | $\stackrel{.8}{6.9}$ |
|  | 1880 1870 |  |  |  |  |  |  |  |  |  |  |  |
| Brighton........... |  | $\begin{array}{r\|r\|} 5.5 \\ 1.9 \\ 5 \end{array} \mathbf{1}^{1}$ | $\begin{array}{r\|} 16.1 \\ 7.6 \\ 3.6 \end{array}$ |  |  |  |  |  |  |  |  |  |  |  |
|  | 1860 1870 |  |  |  |  |  |  |  |  |  |  |  |  |
| Bristol............ | 5 | $\begin{gathered} 20.8 \\ 10.3 \\ 10.3 \end{gathered}$ | $\begin{array}{\|l\|l} 3.8 .8 \\ 32.3 \end{array}$ | $\left.\begin{array}{\|c\|} \hline 3.3 \\ 1.3 \\ 1.4 \\ 3 \end{array} \right\rvert\,$ | $\begin{aligned} & 1.8 .8 \\ & \left.\begin{array}{l} 2.7 \\ 41.6 \end{array} \right\rvert\, \end{aligned}$ |  |  |  |  | 7.9 <br> 6.4 |  |  | ${ }^{5} 5$ |
|  | 50 |  |  |  |  |  |  |  | 1.4 |  | 1.9 <br> 1.0 <br> 1.0 <br> 5.3 |  |  |
| Paris. | 1850 |  | $\begin{array}{r} 28.7 \\ 55.4 \\ 6.5 \end{array}$ |  |  | $\begin{aligned} & 4.8 \\ & 1.8 \\ & 1.0 \end{aligned}$ |  |  | $\begin{array}{l\|l\|l} \hline 6 & 15.2 \\ .4 & 13.4 \\ .6 & 12.7 \end{array}$ | $\begin{array}{lll}7.3 & 11 \\ 5.9 \\ 4.1 & 4 \\ 5\end{array}$ |  | $\begin{array}{l\|l} 1.1 & 1.0 \\ 7.610 .5 \\ 4.9 & 12.5 \end{array}$ | 2.73.54.8 |
|  | 1880 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Pike | 1850 | 13.6 | 29.7 | 2 | $\begin{array}{ll}20.8 & 4.5\end{array}$ |  | . 5 | . 9 |  | 5.4 | 10. | . 1 |  |
|  | ${ }_{180}^{1860}$ |  |  |  |  |  |  |  |  |  |  |  |  |
| Pleasant Prairie | 1850 |  |  |  |  |  | $\begin{array}{l\|l} 2 & .1 \\ : 6 & .6 \\ \hline & 1.3 \end{array}$ | $\begin{array}{r}1.8 \\ .8 \\ .8 \\ \hline\end{array}$ |  | $\begin{array}{\|l\|} 19.6 \\ 11 . \\ 10.7 \end{array}$ | 3.5 <br> 4.5 <br> 2.9 <br> 2.9 <br> 8 |  | 1.213.12.4 |
|  | 1850 |  |  |  |  |  |  |  |  |  |  | 8 |  |
| Randall. |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 1880 1870 |  |  | $\begin{array}{r} 76 \\ 1.5 \end{array}$ |  | $\begin{array}{c\|c} \dddot{3} .6 \\ 6.1 & \cdots \\ 6.1 \end{array}$ |  | -1.2 |  | $\begin{array}{r} \cdots .2 \\ 2.6 \\ 5.1 \\ 8.6 \\ 8.6 \\ 6.6 \end{array}$ | $\begin{array}{rr\|r} \dddot{2.8} \\ .5 & i 13.5 \\ 13.5 \end{array}$ |  | $\stackrel{7}{7 .}$ |
| Salem ...... ....... |  | $\left\lvert\, \begin{gathered} 13 . \\ 10.4 \\ \hline 6.2 \end{gathered}\right.$ | $\begin{aligned} & 36.1 \\ & 24 \\ & \hline 18.1 \end{aligned}$ |  |  |  |  |  |  |  |  |  | 1.51.4 |
|  | 8 |  |  |  |  |  |  |  |  |  |  | -8 |  |
| Somers. |  |  | $\begin{gathered} 20.4 \\ 14.8 \\ 14.8 \end{gathered}$ | $\begin{array}{r} 1.6 \\ 1.2 \end{array}$ | $\left\lvert\, \begin{aligned} & 33.4 \\ & 45.5 \end{aligned}\right.$ | $\dddot{2.3} 2.5$ |  | $\left\|\begin{array}{r} \because .5 \\ 1.7 \end{array}\right\|$ | $\frac{10.8}{9.5}$ | $\begin{gathered} 4.4 \\ 1.8 \end{gathered}$ |  |  | - $\begin{array}{r}6.1 \\ 3.4 \\ 8\end{array}$ |
|  | $\begin{aligned} & 600^{6} \\ & \hline 001 \end{aligned}$ |  |  |  |  |  |  | $\begin{array}{c\|c} 4.3 \\ 4.4 & 80 \\ 10 \end{array}$ |  |  |  |  |  |
| Southport ......... |  |  | 34.9 | $\begin{array}{l\|l} 1.15 \\ \hdashline \ldots & \ldots \end{array}$ |  | 4.2. |  |  | . 3. |  |  | 5.5 . 5 |  |
|  |  |  | 8 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Wheatland |  |  |  | $\begin{gathered} 35.2 \\ \hline 1.2 \\ 4.4 \\ 4.5 \end{gathered}$ |  |  | $\begin{array}{l\|l}  & \begin{array}{l} 6 . \\ 3.3 \\ 7.3 \\ 2 . \\ 2 . \end{array} \end{array}$ | $\begin{array}{c\|c} 0 \\ 3 & 3 \\ 5 & \text { an } \end{array}$ | (1.5 | $\begin{array}{l\|l\|} \hline 8 & 4.6 \\ 8 & 14 \\ \hline 5 & 1.2 \end{array}$ | 4.2  <br> 3.0  <br> 8  <br>   | $\begin{array}{l\|l} 5.7 \\ \hline 5.58 \\ \hline 5.6 & 28.4 \\ 58.8 \end{array}$ |  | $\xrightarrow{4.5}$ |
|  | 1880 1870 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

Coming now to a more careful study of the results in Table 1, we find that as late as 1870 only two towns, Brighton and Paris, had a population of over $50 \%$ born in Wisconsin. Of the total population of the entire county in 1870 only $45.3 \%$ were born in Wisconsin. This shows plainly that the bulk of the population in 1870 and previously, was born either in other states or in foreign countries.

The population of the city of Kenosha in 1850 consisted mainly of those born in New York, Wisconsin, Ireland, New England and Great Britain. In 1860 and 1870 the percentages of the population furnished by these states and countries fall, while that of Wisconsin and Prussia rise. The rise in the percentage of the Wisconsin born is to be expected, ${ }^{1}$ but the rise of the percentage of those born in Prussia shows the coming in of a large number of Germans during the two decades. ${ }^{2}$ The large decline in the percentage of the New York element from $29.6 \%$ to $18.3 \%$ between 1850 and 1860 , and from $18.3 \%$ to $12.5 \%$ between 1860 and 1870 indicates very plainly that a large number of those born in New York left the city and went elsewhere. The falling off of the Irish element indicates the same thing, but not to such an extent as in the case of the New York element.

A large percentage of the population of the towns of Brighton, Paris and Wheatland in 1850 were from New York, New England, Ireland, and Great Britain. Between 1850 and 1870 a large part of the New York and New England population left the towns; especially is this true in the towns of Wheatland and Paris, where the New England element sank from 6.8\% to 3.8\% in Paris, and from $8.9 \%$ to $1 \%$ in Wheatland, and the New York element from $28.7 \%$ to $6.5 \%$ in Paris, and from $35.2 \%$ to $4.5 \%$ in Wheatland. The Englisi and Irish elements remained in the three towns and naturally declined a little each year, with the exception of the town of Brighton. The fall of the percentage of the Irish in this town from $18.9 \%$ to $10.4 \%$ between 1850 and 1860 shows that part of the Irish emigrated. The most important feature of Brighton and Wheatland is the rapid increase in the number of Germans, chiefly from Prussia. The number of Germans increased slightly in the town of Paris, but not to such a degree as in these other two towns. By 1870 these three towns were mainly settled by people of a foreign nationality. Wheatland was chiefly settled by Germans, Brighton by Germans, English and Irish, and Paris by Germans and English.

[^17]The town of Pleasant Prairie has a characteristic of its own. In 1850 the percentage of the population born in Ireland and Great Britain was greater than that of those born in New York and New England, while in 1860 those born in New York and New England were greater than those born in Ireland and Great Britain, and in 1870 they were about equal. The noticeable sinking in the percentages of the Trish and English elements from $19.6 \%$ to $11.1 \%$ and from $16.8 \%$ to $10.1 \%$, respectively, during the first decade makes it very plain that the Irish and English elements either settled in the town and then left, or that they settled in what was Pleasant Prairie in 1850 , and when the southern part of the town of Southport was annexed in 1851, the great predominance of the New York and New England elements in the district added, offset the large percentage of Irish and English which existed in Pleasant Prairie in 1850. By examining the actual number of the population born in New York, New England, Ireland, and Great Britain in the town of Pleasant Prairie and Southport in 1850, and in the town of Pleasant Prairie for 1860 and 1870 in Table 1 of the Appendix, the latter statement will be seen to be true. So it is evident that if any of the English and Irish elements emigrated from this town they did not do so in such great numbers as the New York and New England elements. The German element increased steadily during the period.

The towns of Bristol, Somers, formerly Pike, Salem, and Randall, are the ones which show the least thinning out of the New York and New England population, but the fall in the percentages, however, is greater than it would ordinarily be, providing no emigration of these elements had taken place. In all of these towns there was a small increase in the number of Germans. The town of Randall shows the greatest increase. From 1860 to 1870 the German element in this town rose from $1.5 \%$ to $13.5 \%$. The town of Bristol was settled chiefly by the New York and the New England population. In 1850 the New York element was $39 \%$ and the New England 20.8\%, making together $59.8 \%$ of the total population of the town. From 1850 to 1860 there was a slight increase in the number of Irish and from 1860 to 1870 a small increase in the number of Eng-
lish. The New York and New England elements nevertheless remained the predominating population of the town. Salem, like the town of Bristol, was settled by the New York and New England people, and they remained the prevailing elements in the town. During the first decade quite a number of Irish and English came in and settled. There was also a small but steady increase in the German population. The towns of Somers and Randall, although Randall was not formed until 1860, show a greater increase in the number of Germans than the other two towns. In Somers those born in New York, Germany, chiefly Prussia, Ireland, and Great Britain, were the greatest in number, while in Randall, those born in New York and Germany predominated.

To sum up the study of the accompanying Table 1 and Table 1 of the Appendix, we find: 1. That the nativity of the population of the county, according to relative importance, takes the following order: in 1850, New York, Wisconsin, New England, Ireland, Great Britain and Germany ; in 1860 Wisconsin, New York, Ireland, Prussia, New England and Great Britain; and in 1870, Wisconsin, New York, Prussia, Ireland, Great Britain and New England. 2. That the population furnished by New York and New England declined steadily and rapidly since 1850, not so much by natural decline, as by emigration from the county. Those that remained in the county massed in the towns of Bristol, Salem, Randall, Somers, and a large number in Pleasant Prairie. 3. That the Irish have steadily declined since 1850 . They scattered in the county, but settled chiefly in Pleasant Prairie, Brighton, and the city of Kenosha. They are the only foreign nationality that had a tendency to emigrate. From 1850 to 1860 a large number left the city of Kenosha and the town of Brighton, some leaving the county, and others moving into the towns of Bristol and Salem. 4. That the English in the county increased slightly from 1850 to 1860, and settled mainly in the city of Kenosha, Brighton, Paris, Pleasant Prairie, Salem and Somers. The only town in which there is an indication that a small number emigrated after settling is Brighton. 5. That the total number of Germans in the county, chiefly from Prussia, increased nearly 145\%
from 1850 to 1870 . They scattered all over the county, but massed principally in the towns of Brighton and Wheatland. 6. That the foreign nationality predominated in Brighton in 1860.

Table 2.-Nativity of county and town by percentage.

|  | Native. |  |  | Foreign Born. |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1850. | 1860. | 1870. | 1850. | 1860. | 1870. |
| Kenosha county | 68.2 | 65.6 | 69.1 | 31.8 | 34.4 | 30.9 |
| Kenosha city | 66.6 | 64.7 | 68.4 | 33.4 | 35.3 | 31.6 |
| Brighton | 50.2 | 49.7 | 59.3 | 49.8 | 50.3 | 40.7 |
| Bristol | 87.6 | 81.9 | 82.9 | 12.4 | 18.1 | 17.1 |
| Paris | 60.7 | 59.1 | 60.7 | 39.3 | 40.9 | 39.3 |
| Pike-Somers | 70.4 | 65.8 | 68.7 | 29.6 | 34.2 | 31.3 |
| Pleasant Prairie | 57.0 | 66.3 | 66.7 | 43.0 | 33.7 | 33.3 |
| Randall | .... | 78.9 | 78.3 | .... | 21.1 | 21.7 |
| Salem | 75.8 | 74.7 | 74.3 | 24.2 | 25.3 | 25.7 |
| Southport | 75.6 | .... | .... | 24.4 | $\ldots$ | .... |
| Wheatland | 73.7 | 54.9 | 60.8 | 26.3 | 45.1 | 39.2 |

Note.-For the total number of native and foreign born in the county and towns for the three years; see Table 2, Appendix.

To further show the distribution of the foreign and native population in the county and towns, Table 2 was made by considering all born within the United States as native, and all born outside as foreign, for the three years 1850,1860 and 1870. Taking the percentages of the native and foreign born in each town for each of the three years, and comparing them with the percentages of the native and foreign born of the entire county for the same years, one readily sees that in 1850 the towns of Bristol, Salem, Southport, and Wheatland were settled mainly by native born. In the towns of Somers, then Pike, and the city of Kenosha, the native born were about the same as the county average, the former being a little above and the latter a little below. In the towns of Brighton, Pleasant Prairie, and Paris, the percentage of native born falls below that of the entire county, which indicates strongly the presence of a large foreign element in these towns. In 1860 the per-
centages of the native and foreign born of these various towns bear about the same relation to that of the entire county as they did in 1850, except Pleasant Prairie and Wheatland. When Southport was divided and the southern part added to Pleasant Prairie, the evident predominance of the New York and New England elements in this district, had the effect of raising the percentage of the native born in this town from $57 \%$ in 1850 to $66.3 \%$ in 1860 , which is $.7 \%$ above the county average in 1860. The town of Randall, which was formed out of what was a part of Wheatland in 1850 , has a percentage of native born of $78.9 \%$, or $13.3 \%$ above that of the county, while the percentage of the native born of what is left of Wheatland, after the division, sinks to $54.9 \%$, or $10.7 \%$ below the county average. This is a strong indication that, while the town was not arbitrarily divided in 1850, it was nevertheless naturally divided by the settlement of the foreign elements in the northern part and the native in the southern part. This fact may have been what led to the division of Wheatland in 1860. In 1870 the percentage of the native born in Pleasant Prairie again sinks below the county average, while the percentage of the native and foreign born for the other towns maintain about the same relation to the county average as they did in 1850 and 1860.

The sub-joined maps in Plate XL. show very clearly the location of the native and foreign elements in the country in 1850, 1860 , and 1870 , and verify the conclusion drawn from Table 1 in regard to the location of the foreign elements.


COUNTY AVERAGE. NATIVE $69.1 \%$ FOREIGN $30.9 \%$

$\square$ about county average
BELOW "
ABOVE

## Cinapter IV.

## DENSITY OF POPULATION, VALUATION PER CAPITA AND OCCUPATION.

Taking up first the subject of population, it will be observed from Table 3 that the population per square mile from 1850 to 1900 for the entire county, including the city of Kenosha, has steadily increased, but with considerable variation. Between

> TAble 3.- Density of Population.

|  | Area in square miles. | Population Per Square Mile. |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 1850. | 1860. | 1870. | 1880. | 1890. | 1900. |
| Kenosha county .............. | 268.04 | 40.3 | 50.4 | 49.1 | 50.7 | 58.0 | 80.9 |
| City of Kenosha | 1.25 | 2831.2 | 3193.6 | 3447.2 | 4031.2 | 5225.6 | 9284.8 |
| Co. of Ken. (exclud'g city).. | 266.79 | 27.7 | 35.6 | 33.5 | 31.7 | 33.7 | 37.8 |
| Brighton | 35.7 | 24.6 | 32.9 | 33.2 | 28.7 | 25.7 | 23.8 |
| Bristol | 35.98 | 31.2 | 38.1 | 31.7 | 29.7 | 29.9 | 32.3 |
| Paris' .... | 35.67 | 26.8 | 34.2 | 28.4 | 28.9 | 24.6 | 22.4 |
| Pike-Somers | $\left\{\begin{array}{l}40.07 \\ 35.42\end{array}\right.$ | 19.2 | 31.9 | 33.9 ' | 36.4 | 40.7 | 50.1 |
| Pleasant Prairie | $\left\{\begin{array}{l} 43.41 \\ 35.93 \end{array}\right.$ | 26.7 | 32.2 | 31.7 | 32.0 | 37.9 | 40.9 |
| Randall | 21.59 |  | 30.5 | 24.7 | 20.9 | 30.5 | 36.3 |
| Salem ...... | 30.77 | 36.5 | 46.9 | 45.0 | 42.3 | 48.7 | 60.2 |
| Wheatland | $\left\{\begin{array}{l} 21.59 \\ 45.15 \end{array}\right.$ | 26.4 | 51.6 | 40.3 | 38.7 | 34.8 | 38.5 |

Note.-The area of Pike hefore changed to Siomers was 35.42 sq. miles. The area of Pleasant Prairie before the northern part of Southport was set off was 35.93 sq . miles, and the area of Wheatland before Randall was set off was 45.15 sa. miles.

1860 and 1890 there was only a very small increase of population, but from 1880 to 1900 there was an increase of 30.2 to the square mile. The largest and most consistent growth of population took place in the city of Kenosha. The population to the square mile for the county, not including the city of

Kenosha, varies greatly. Starting with 27.7 to the square mile in 1850 , it reaches 35.6 in 1860 , sinks to 31.7 in 1880 , and again increases to 37.8 in 1900. Somers is the only town in the county which has a steady and constant increase. The town of Pleasant Prairie attains its maximum density in 1900. Between 1860 and 1880 the population to the square mile slightly declined. Considering the inland towns, which have a predominance of foreign elements, namely, Brighton, Paris, and Wheatland, as Group 1, and those which have a predominance of native born, namely, Bristol, Salem and Randall, as Group 2, it will be seen that the population of Group 1 tends, with considerable variation, to decline, while that of Group 2 tends to steadily increase. The towns composing Group 1 attain their maximuc density of population in 1860 and 1870, while those in Group 2 reach their maximum density in 1890 and 1900. The fluctuation of the population of the town of Wheatland is more marked than any other town in the county. In 1850 the population to the square mile in this town was 26.4 , in 1860, 51.6 , in $1870,40.3$, and in $1900,38.5$.

Plate XLI shows those towns in which the population has declined, and Chart I, Plate XLII, shows graphically the rise and fall of population in each town from 1850 to 1890.

Table 4.-Valuations per capita.

| Group 2. | Year. | Cash value of farms. | Total valuation. |
| :---: | :---: | :---: | :---: |
| Towns having native-born population ............ | 1850 1880 1870 | $\$ 252.9$ 667.2 659.5 | $\begin{aligned} & \$ 80.89 \\ & \hline 87.8 \\ & 584.9 \end{aligned}$ |
| Group 1. <br> Towns having largest foreign-born population... | 1850 1880 1870 | 214.2 285.0 619.8 | 96.39 273.9 467.1 |

Note.-See Table 4, Appendix, for the cash value of farms and the total assessed valuation of the county and towns for 1850, 1860, and 1870.

Having seen that the population in the towns of Group 1 has declined, while that of the towns of Group 2 has increased, let us now compare the cash value of the farms per capita and the total valuation of the personal and real estate per capita of Group 1 for 1850,1860 , and 1870 , with that of Group 2 for the

same years. The figures in Table 4 show that the cash value of the farms per capita is greater in Group 2 than in Group 1 for the years under consideration. The same is true of the total valuation of personal property and real estate per capita, except that in 1850 that of Group 2 is less than that of Group 1. This exception may be partly accounted for by the fact that there seems to have been a great demand and rush for the farms in Wheatland and Brighton during the period from 1850 to 1860, due to the excellent quality and quantity of the wheat grown in this section. ${ }^{1}$ The fact that these valuations per capita are greater in Group 2 than in Group 1 may be regarded as an indication that the native born were more thrifty and made greater improvements on their farms than the foreign born.

This decline in the population in the towns of Group 1, and the fact that the cash value of the farms per capita is lower on the whole than that of Group 2, leads us to examine the products of the towns of the two groups. Chart II, Plate XLII, shows graphically the fluctuation of the chief products of the county, excluding the city of Kenosha, from 1850 to 1895. The most striking fact shown is the steady decline of the number of bushels of wheat per capita and the great increase in the number of pounds of butter and bushels of oats per capita. The result in Table 6 of the Appendix points out clearly that there was a strong tendency in the towns of Group 1 to depend upon the production of wheat, oats, and Indian corn, while the towns of Group 2 went into the dairy industry. In the study of the soils of the county it was pointed out that the prevailing soil in the western area of these two groups of towns is the lighter marly clay and in the eastern area, including the towns of Paris and Bristol, there is a considerable amount of prairie loam, especially in Paris. This fact leads irresistibly to the conclusion that the decline in the population and the smallness of the cash value of the farms and the total valuation per capita of Group 1, when compared with that of Group 2, is due not so much to the soil as to the nativity of the people. The Germans, English, and

[^18]Trish evidently lacked the ability to adapt themselves to the economic changes and agricultural improvements; on the other hand, the native born of Group 2 took advantage of such changes and improvements.

Table 5--Profession, occupation,or trade of the males over 15 years of age in Kenosha county, excluding the city, by total and percentage.

|  | Totals. |  |  | Percentage. |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1850. | 1860. | 1870. | 1850. | 1860. | 1870. |
| Farmers ..................... | 1,567 | 1,324 | 1,429 | 73.9 | 43 | 52.7 |
| Artisans and mechanics. ... | 133 | 104 | 112 | 6.3 | 3.08 | 4.07 |
| Small trades and profes. sions. ........................ | 44 | 42 | 58 | 2.01 | 1.3 | 2.1 |
| Petty tradesmen ............ | 10 | 4 | 8 | . 47 | . 1 | . 3 |
| Laborers..... | 336 | 806 | 907 | 15.8 | 26.4 | 32.8 |
| Professional men............ | 23 | 40 | 38 | 1.08 | 1.3 | 1.3 |
| Capitalists and merchants.. | 5 | 18 | 43 | . 23 | . 6 | 1.56 |
| Miscellaneous.. |  | 712 | 161 |  | 23.3 | 5.8 |

Note.-For the percentage of occupation in each town see Table 5, Appendix.
The principal occupation of the males of the county, excluding the city of Kenosha, is farming. In 1850 of the total number of males over fifteen years of age, as will be seen from Table 5, $73.9 \%$ were farmers, in $1860,43 \%$, and in 1870 , $52.8 \%$ The laborers in 1850 constituted $15.8 \%$ of the males, $26.4 \%$ in 1860 , and $32.8 \%$ in 1870 . The decrease in the number of farmer sfrom 1850 to 1870, and the increase in the number of laborers would seem to indicate the concentration of the farm lands into the hands of a few large and wealthy farmers, the many small farmers finding employment as laborers.

To conclude we find: 1. That the population of the county has steadily increased, but with a considerable variation. The greatest increase has taken place in the city of Kenosha, the coast towns, and the town of Salem. 2. That the towns which show a marked decline in population and a lower cash value of farms and a total valuation per capita, are the towns settled by foreign born, chiefly Germans, English, and Irish. The only

exception is the town of Pleasant Prairie, in which the foreign elements are mainly English and Irish. 3. That the decline in population and valuations per capita is due not so much to the soil as to the nativity of the occupants; and 4 , that the principal occupation of the county is farming.

## Chapter V.

## A STUDY OF THE POPCLATION BY NATIVITY AND OCCUPATION IN 1850.

The following tables were prepared in order to get an idea of the class of the people that had just settled in Kenosha county, or were on the move in 1850. Table 6 gives the number and per cent. of the males over fifteen years of age in each occupation. From the table it will be seen that a little over onehalf of the male population were farmers, while $18.1 \%$ were laborers, and $12.6 \%$ were artisans and mechanics.

Table 7 shows the percentage of males in each occupation, with and without wealth. The striking feature of this table is the excellent condition of the farmers. Slightly over one-half have more or less wealth. Aside from the capitalists and the merchants, the next best off in order are the artisans and mechanics and the smaller trades and professions.

Table 6.- Occupation, number, and per cent.

|  | Number. | Per cent. |
| :---: | :---: | :---: |
| Farmers. | 1,620 | 51.7 |
| Artisans and mechanics | 295 | 12.6 |
| Smaller trades and professions. | 221 | 7.0 |
| Laborers | 569 | 18.1 |
| Petty tradesmen | 118 | 3.7 |
| Professional men | 82 | 2.5 |
| Capitalists and merchants | 46 | 1.1 |
| No occupation | 82 | 2.5 |

Table 7.- Percentage in each occupation of those with and without wealth.

|  | With. | Without. |
| :---: | :---: | :---: |
| Farmers. | 50.7 | 493 |
| Artisans and mechanics | 40.7 | 59.3 |
| Smaller trades and professions | 37.1 | 62.9 |
| Laborers | 17.9 | 82.1 |
| Petty tradesmen.. | 31.3 | 68.7 |
| Professional men.. | 47.5 | 52.5 |
| Capitalists and merchants | 97.8 | 2.2 |

From Table 8 it is seen that the Middle States furnished more males with and without wealth than New England or the miscellaneous states. The number in each occupation without wealth, except the capitalists and merchant who came from the Middle States, is greater than those with wealth. On the other hand, those who came from the New England states are less in number, but those with wealth in each occupation is greater than those without, and the average wealth in each occupation on the whole is greater than that of the Middle States.

The most noticeable thing in Table 9 is that the average wealth in each occupation of the native born is much larger than that of the foreign born. The number of farmers born in foreign countries who have wealth is greater than those who have no wealth, while the number of native born farmers who have wealth is less than those who have no wealth.
The object in preparing these tables and pointing out a few of the facts which the figures in the table show, was simply to indicate that there is a possibility of reaching valuable conclusions as to the nature of the emigration movement in 1850, by studying the occupation, the nativity and wealth of those who first settled in the county, and taking them as typical of the emigration that moved further forward.

Table 8. - Number and wealth of those from the Eastern States.

|  | New England. |  |  | Middle States. |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Without. | With. | Average. | Without. | With. | Average. |
| Farmers ................. | 129 | 161 | \$3,337.6 | 355 | 319 | \$2,461.15 |
| Artisans and mechanics.. | 35 | 36 | 1,089.7 | 82 | 60 | 879.13 |
| Smaller trades and pro- fessions | 26 | 26 | 2,273.4 | 58 | 20 | 8,879.9 |
| Laborers .................. | 16 | 2 | 1,150. | 74 | 6 | 508.3 |
| Capitalists \& merchants.. | .. | 16 | 17,166.8 | . | 25 | 16,345.5 |
| Professional men ........ | 12 | 17 | 4,226.4 | 20 | 15 | 2,640. |
| Petty trades .............. | 18 | 11 | 747.27 | 28 | 7 | 1,712.8 |

Table 9.- Proportion by occupation of native and foreign-born and the wealth of the native and foreign-born of each occupation.

|  | Native. |  |  | Fortign. |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | With. | Without. | Average. | With. | Without. | A verage. |
| Farmers . ................. | 508 | 487 | \$1,144.4 | 289 | 334 | \$1,233.4 |
| Artisans and mechanics.. | 131 | 100 | 846.78 | 100 | 61 | 676.2 |
| Smaller trades and professions | 91 | 49 | 3,652.4 | 43 | 30 | 980. |
| Laborers ................... | 101 | 8 | 618.8 | 343 | 82 | 476.7 |
| Capitalists \& merchants.. | . | 42 | 11,925.6 | -• | 3 | 8,680. |
| Professional men ......... | 35 | 32 | 3,482.8 | 7 | 7 | 507.1 |
| Petty trades .............. | 48 | 20 | 1,087. | 33 | 18 | 715. |

## Chapter VI.

SOME CORRECTIONS IN THE UNITED STATES AND STATE CENSUS REPORTS FROM 1850 TO 1870.

It was discovered upon comparing the results obtained in this study with those of the United States and State Census returns that there were considerable variations. The following tables show the differences for the years 1850, 1860, and 1870. In column one the population is given which was obtained from the records already referred to. Columns 2 and 3 show these variations from the United States Census returns. The plus sign before the number indicates that the returns for the towns exceed and the minus sign that they are less than those found in the records. The blanks indicate that there is no variation.

In getting the results from the records a table similar to Table 11 was used. The number of persons on each page of the Census report from each separate state or country was counted and the result placed in a small square opposite the name of the state or country. When all the names on the page were classified according to nativity, the column was added and the result placed below. Then to make sure that no names were omitted, the names on the page were counted and placed above the column. This result had to correspond to the footing if the work was correct. When all the material for a town was examined the number from each state or country was added and the same placed in a column to the right. To still further avoid a mistake and to be positive that the results were correct, the horizontal and vertical columns of sums were added, and if these results were equal the work was considered correct.

Table 10.- Correction of the U. S. and State Census.

| Town. | Population. | United States Census. | Wisconsin Blue Book. |
| :---: | :---: | :---: | :---: |
| 1850: |  |  |  |
| Brighton | 880 |  |  |
| Bristol | 1,125 |  |  |
| Kenosha city | 3,539 | -84 | ............ |
| Paris | 956 |  |  |
| Pike ........... | 680 |  |  |
| Pleasant Prairie . | 959 | ................ | - |
| Salem | 1,123 |  |  |
| Southport . | 363 |  |  |
| Wheatland | 1,193 |  |  |
| 1860: |  |  |  |
| Brighton | 1,173 | -65 | - 3 |
| Bristol | 1,369 | -17 | -1 |
|  |  |  |  |
| Paris | 1,085 | -289 | -3 |
| Pleasant Prairie | 1,400 | . | -1 |
|  |  |  |  |
| Salem ..................... | 1,444 | -28 | -1 |
| Somers | 1,278 | - 1 | - 3 |
|  | 1,115 | $-20$ |  |
| 1870: |  |  |  |
| Brighton | 1,187 | - 2 | -2 |
| Bristol | 1,140 |  |  |
| Kenosha City | 4,307 | - 2 | -2 |
| Paris .... | 1,014 | $-1$ | -1 |
| Pleasant Prairie | 1,377 | ................ | ............ |
| Randall | 533 |  |  |
| Salem | 1,386 |  |  |
| Somers | 1,359 |  |  |
| Wheatland | 871 | -28 | -28 |

Table 11.-Method used in obtaining the accurate Population by nativity in county and towns of Kenosha.

Randall, 1860.

|  | 40 | 77 | 78 | 76 | 77 | 79 | 79 | 78 | 76 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| New York....... | 9 | 9 | 25 | 32 | 32 | 17 | 32 | 17 | 19 |  | $=192$ |
| Wisconsin ...... | 15 | 10 | 28 | 18 | 13 | 23 | 20 | 21 | 27 |  | $=175$ |
| Massachusetts.. | 3 | ...... | ...... | $\ldots$ | ...... |  | 2 | 2 | 4 |  | $=11$ |
| Connecticut..... |  |  | 1 | 3 | 3 | ...... |  |  | 4 |  | $=11$ |
| New Hampshire | ..... | .... | ..... | ...... | ...... | ...... | ...... |  | ...... |  | $=0$ |
| Vermont ......... | 1 | 4 | 2 | 1 | 6 | 6 | 2 |  | 1 |  | $=23$ |
| Pennsylvania ... |  |  | 9 | 1 | ...... | 14. | 8 | 2 | 1 |  | $=35$ |
| Ohio ............. | . |  | - | 2 | $\ldots$ | 2 | 2 |  | 4 |  | $=10$ |
| Illinois. | $\ldots$ | $\ldots$ | 2 | 2 |  |  | 2 | 4 | 1 |  | $=11$ |
| Canada.......... |  |  |  | ...... | 4 |  |  | 1 | 2 |  | $=7$ |
| England ........ | 5 | 4 | 3 | 1 | 4 | 2 | 5 |  |  |  | $=24$ |
| Ireland.......... | 6 | 5 | 4 | 2 | 6 | 4 |  | 11 | 3 |  | $=41$ |
| Germany ........ |  | 1 | 2 | 1 |  |  | 1 | 2 | ...... |  | $=7$ |
| Rhode Island... | 1 |  |  | ...... |  |  |  | 1 | 1 |  | $=3$ |
| Not given ....... |  | 39 | 2 |  | 3 |  | 1 |  | 1 |  | $=46$ |
| New Jersey . . . . |  | 3 |  |  | 1 |  | 2 |  | 3 |  | $=9$ |
| Holland......... |  |  |  | 1 | ...... | 1 |  |  |  |  | $=2$ |
| Saxony.......... |  |  |  | 7 | ..... |  | .... |  | 1 |  | $=8$ |
| Michigan . . . . . |  |  |  | 1 | 2 | ..... |  |  |  |  | $=3$ |
| Prussia |  |  |  | 2 | 1 | 1 | 2 | 1 | 3 |  | $=10$ |
| Baden........... |  |  | ...... | 2 | ... | - | ...... | ...... | . |  | $=2$ |
| N. Carolina..... |  |  |  |  | 1 |  |  |  |  |  | $=1$ |
| Brunswick...... |  | ...... |  | ..... | ..... | 1 |  |  |  |  | $=1$ |
| Isle of Man..... |  |  |  |  | ...... | 7 | . | 7 | . |  | $=14$ |
| Wittenburg..... |  |  |  |  |  | 1 |  |  | ...... |  | $=1$ |
| Wales . . . . . . . . |  |  |  |  | ..... |  |  | 9 |  |  | $=9$ |
| Luxemburg ..... |  |  |  |  |  |  |  |  | 1 |  | $=1$ |
| Scotland . |  | 2 |  |  | 1 |  |  |  |  |  | $=3$ |
|  | 40 | 77 | 78 | 76 | 77 | 79 | 79 | 78 | 76 | $=660$ | $=660$ |

## Chapter VII.

ANALYSIS OF THE EIEETION RETURNS FOR PRESIDENTIAL ELECTORS, MEMBERS OF CONGRESS, AND GOVERNORS OF THE STATE.

Since it has been shown in the preceding chapters that the towns of the county in which the native born predominate show an increase in population and wealth, while those towns settled by the foreign born, chiefly Germans, English and Irish, show a marked decline in population and a decrease in wealth, it becomes an interesting and an important feature of this study to investigate the political associations of the people of each town and group of towns.

To consider the votes of the towns for one or two elections would be inaccurate and useless, on account of the many local and personal considerations brought to bear in every election. These influences, however, may be eliminated by combining the

Table 12.-Combined vote for presidential electors, congressmen, and governors, 1859-1900.*

|  | Republican. |  | Democratic. |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Vote. | Per cert. of total. | Vote, | Per cent of total. |
| Kenosha county | 67,532 | 55 | 56,352 | 45 |
| Kenosha city | 25,188 | 49 | 25,941 | 51 |
| Brighton | 2,868 | 35 | 5,221 | 65 |
| Bristol .... | 7,273 | 75 | 2,360 | 25 |
| Paris | 3,740 | 49 | 3,910 | 51 |
| Pleasant Prairie | 7,135 | 58 | 5,035 | 42 |
| Randall | 3,329 | 66 | 1,699 | 34 |
| Salem | 8,311 | 66 | 4,335 | 34 |
| Somers | 8,011 | 68 | 3,662 | 32 |
| Wheatland | 2,677 | 38 | 4,084 | 62 |

${ }^{\circ}$ These figures are taken from the Wisconsin Blue Books.
votes for the candidates of the leading political parties, respectively, for a series of years. Table 12 shows the combined result of all the votes cast in the county, city of Kenosha, and in each town for the Republican and Democratic candidates for presidential electors, members of congress, and governors from 1859 to 1900, with the exception of a few elections of member of congress and governor, for which it was impossible to obtain the election returns by towns. From this table it is seen at once that the strength of the Republican party is in the towns of Bristol, Somers, Salem, and Randall, while that of the Democratic party is in the towns of Brighton, Wheatland, Paris, Pleasant Prairie, and in the City of Kenosha. The strongest republican town in the county is Bristol and the strongest democratic town is Brighton. Seventy-five per cent. of the combined vote of the former from 1859 to 1900 was cast with the Republican party, while 65 per cent. of the total vote for the same period in the latter town was cast with the Democratic party.

On referring to table 9 of the Appendix it will be seen that in not a single election did the town of Bristol go democratic or the town of Brighton republican. The towns of Salem and Somers have gone republican in every election, and the town of Wheatland has gone democratic in every election except in the last congressional election when the election was a tie, and in the last two presidential elections, in which the Republican candidate received a slight majority. These facts are very important when the measures advocated by these two parties during this series of years, are considered.

By combining the votes cast in the inland towns of the county into the same groups, which were used in the chapter on the

Table 13 - Vote for presidential electors, congressmen and governors, 1859-1900. ${ }^{1}$


[^19]density of population, valuation per capita, and occupation, we find from table 13 that of the total vote cast in Group I, 41 per cent. was for the Republican candidates and 59 per cent. for the Democratic. In Group II, 69 per cent. of the total vote cast was for the Republican and 31 per cent. for the Democratic candidates.

From these results we must conclude that there is a strong indication of a close connection between the political, social, and economic conditions of the people. Those towns in Kenosha County in which the foreign born predominate, and in which there has been a decline in population and a decrease in wealth, furnished the strength of the Democratic party, while the strength of the Republican party is in those towns in which the native born predominate and in which there has been a marked increase in both population and wealth.

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## APPENDIX.

Table 1.-Population by nativity for 1850, 1860 and 1870.
Native Born.

| County and Towns | Years. | New England. | New York. | Middle states excluding New York. | Wiscon- sin. | Northwest. | Miscellaneous states. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Kenosha county... | 1850 | 1,300 | 3,203 | 243 | 2,025 | 579 | 31 |
|  | 1860 | 1,016 | 2,483 | 210 | 4,286 | 457 | 100 |
|  | 1870 | - 646 | 1,640 | 203 | 5,959 | 504 | 116 |
| City of Kenosha... | 1850 | 419 | 965 | 55 | 615 | 268 | 16 |
|  | 1860 | 318 | 618 | 42 | 1,262 | 176 | 73 |
|  | 1870 | 259 | 501 | 49 | 1,908 | 230 | 40 |
| Brighton | 1850 | 49 | 143 | 36 | 187 | 24 | 3 |
|  | 1860 | 23 | 89 | 11 | 446 | 16 | - |
|  | 1870 | 6 | 36 | 7 | 645 | 6 | 4 |
| Bristol | 1850 | 234 | 439 | 38 | 213 | 62 |  |
|  | 1860 | 171 | 438 | 20 | 368 | 74 | 1 |
|  | 1870 | 117 | 254 | 10 | 475 | 73 | 16 |
| Paris | 1850 | 65 | 271 | 12 | 186 | 46 |  |
|  | 1860 | 65 | 168 | 19 | 371 | 15 | ${ }^{2}$ |
|  | 1870 | 40 | 68 | 9 | 476 | 11 | 10 |
| Pike | 1850 | 93 | 202 | 7 | 142 | 31 | 4 |
|  | 1860 | .......... |  |  |  |  |  |
|  | 1870 | ....... |  |  |  |  |  |
| Pleasant Prairie... | 1850 | 114 | 193 | 12 | 205 | 21 | 1 |
|  | 1860 | 128 | 218 | 13 | 415 | 22 | 9 |
|  | 1870 | 65 | 179 | 90 | 538 | 22 | 19 |
| Randall | 1850 |  |  |  |  |  |  |
|  | 1860 | 48 | 192 | 44 | 175 | 24 | 1 |
|  | 1870 | 16 | 109 | 8 | 248 | 33 |  |
| Salem | 1850 | 146 | 444 | 26 | 192 | 40 | 5 |
|  | 1860 | 150 | 362 | 18 | 452 | 73 | 9 |
|  | 1870 | 85 | 251 | 12 | 594 | 73 | 14 |
| Somers | 109 |  |  |  |  |  |  |
|  | 1860 | 76 | 260 | 8 | 429 | 30 | ${ }_{12}^{2}$ |
|  | 1870 | 49 | 202 | 17 | 618 | 34 | 12 |
| Southporf | 1850 | 73 | 126 | 4 | 56 | 15 |  |
|  | 181870 |  |  |  |  |  |  |
| Wheatland | 1850 | 107 | 420 | 53 | 229 | 72 | 1 |
|  | 18 ¢n | 37 | 128 | 35 | 368 | 27 | 3 |
|  | 1870 | 9 | 40 | 1 | 457 | 22 | 1 |

Foreign Born.

| County and Towns. | Years. | British America. | Ireland. | Rest of Great Britain. | Prussia. | Rest of Germany | Rest of Europe cellaneous. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Kenosha county... | 1850 | 272 | 1,227 | 1,004 | 36 | 801 | 97 |
|  | 1860 | 225 | 1,177 | 1,012 | 1,168 | 549 | 836 |
|  | 1870 | 150 |  | 777 | 1,622 | 428 | 322 |
| City of Kenosha... | 1850 | 90 | 574 | 257 | 22 | 229 | 29 |
|  | 1860 1870 | 77 | 471 320 | 144 | 277 476 | 197 | ${ }_{124}^{278}$ |
| Brighton ........... | 1850 | 28 | 167 | 144 |  | 94 | 5 |
|  | 1860 | 7 | 122 | 171 | ${ }_{267}^{234}$ | ${ }_{14}^{26}$ | 31 |
| Bristol | 1850 | 15 | 53 | 19 |  | 46 | 6 |
|  | 1860 | 25 | 108 | 19 | 29 | 13 | 103 |
|  | 1870 | 4 | 73 | 25 | 61 | 12 | 20 |
| Paris | 1850 | 6 | 75 | 152 | 10 | 106 | 26 |
|  | 1860 1870 | 5 | 64 41 | 146 128 | 1109 | 83 50 | 38 49 |
| Pike | 1850 | 6 | 37 | 89 | 1 | 68 |  |
|  | 1860 |  |  |  |  |  |  |
| Pleasant Prairie... | 1850 | 18 | 188 | 161 |  | 34 | 12 |
|  | 1860 | 13 | 156 | 142 | 37 | 63 | 184 |
|  | 1870 | 11 | 148 | 110 | 122 | 40 | 33 |
| Randall | 1850 |  |  |  |  |  |  |
|  | 1800 1870 | 8 1 | ${ }_{14}^{41}$ | 50 24 | 10 10 | 19 | $\begin{gathered} 48 \\ 5 \end{gathered}$ |
| Salem | 1850 | 91 | 58 | 99 |  |  |  |
|  | 1860 1870 | ${ }_{16}^{42}$ | 125 | 138 1,111 | 41 102 | 9 14 | 25 22 |
| Somers | 1850 |  |  |  |  |  |  |
|  | 1860 |  | 56 | 138 | 114 | 55 | 78 |
|  | 1870 | 23 | 24 | 128 | 144 | 61 | 47 |
| Southport | 1850 | 11 | 25 | 28 | 2 | 20 | 3 |
|  | 1870 |  |  |  |  |  |  |
| Wheatland | 1850 |  | 50 |  |  | 188 |  |
|  | 1860 | 16 | 34 | 5 | 317 | 84 | 51 |
|  | 1870 |  |  | 11 | 252 |  |  |

Note.-The figures in the above table are taken from copies of the original U. S. Census Returns, which are in the vault of the office of the Secretary of State.

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Tabce 2.-Nativity of county and towns.

|  | Native-Born. |  |  | Foreign-Born. |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1850. | 1860. | 1870. | 1850. | 1860. | 1870. |
| Kenosha county | 7,381 | 8,500 | 9,068 | 3,437 | 4,444 | 4,057 |
| City of Kenosha | 2,338 | 2,437 | 2,987 | 1,201 | 1,328 | 1,292 |
| Brighton .... | 442 | 585 | 704 | 438 | 590 | 483 |
| Bristol | 986 | 1,072 | 945 | 139 | 236 | 194 |
| Paris ........ | 581 | 640 | 614 | 375 | 443 | 397 |
| Pike-Somers .. | 479 | 805 | 932 | 201 | 420 | 423 |
| Pleasant Prairie | 546 | 805 | 913 | 413 | 433 | 456 |
| Randall |  | 484 | 414 |  | 130 | 115 |
| Salem | 853 | 1,064 | 1,029 | 270 | 364 | 356 |
| Southport | 274 |  |  | 89 |  |  |
| Wheatland | 882 | 608 | 530 | 311 | 500 | 341 |

Note.-The above figures were obtained by carefully going over the copy of the original manuscript of the census returns for the above years, which are in the vault of the office of the Secretary of State. The census taker neglected to give the birth of 575 in 1860 , and 49 in 1870 , consequently these numbers are not included in the table.

Table 3.- Total Population.

|  | Area in square mile. | 1850 | 1860 | 1870 | 1880 | 1890 | 1900 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Kenosha county... | 268.04 | 10,818 | 13,519 | 13,174 | 13,550 | 15,581 | 21,707 |
| City of Kenosha... | 1.25 | 3,539 | 3,992 | 4,307 | 5,039 | 6,532 | 11,606 |
| Kenosha Co., excl. city of Kenosha. | 266.79 | 7,279 | 9,527 | 8,867 | 8,511 | 9,049 | 10,101 |
| Brighton .... | 35.7 | 880 | 1,176 | 1,187 | 1,024 | 926 | 850 |
| Bristol .............. | 35.98 | 1,125 | 1,369 | 1,140 | 1,069 | 1,071 | 1,151 |
| Paris | 35.67 | 956 | 1,085 | 1,014 | 1,002 | 871 | 818 |
| Pike-Somers ....... | $\left\{\begin{array}{l}40.07 \\ 35.42\end{array}\right.$ | 680 | 1,278 | 1,359 | 1,458 | 1,632 | 2,044 |
|  |  |  |  |  |  |  |  |
| Pleasant Prairie... | $\left\{\begin{array}{l}43.41 \\ 35.93\end{array}\right.$ | 959 | 1,400 | 1,377 | 1,386 | 1,646 | 1,776 |
|  |  |  |  |  |  |  |  |
| Randall ............ | 21.59 |  | 660 | 533 | 451 | 658 | 784 |
| Salem ...... | 30.77 | 1,123 | 1,444 | 1,386 | 1,286 | 1,493 | 1,846 |
| Southport .......... | 12.14 | 363 |  |  |  |  |  |
| Wheatland | 21.59 | 1,193 | 1,115 | 871 | 853 | 752 | 832 |
|  | 45.15 |  |  |  |  |  |  |

Note.-The above figures for population were all taken from the U. S. Census Reports. Those for 1850 , 1860 and 1870 are as corrected. The area in square miles was obtained by getting the actual number of acres in the towns from the plats of the original U. S. Survey in $1835-36$ and dividing by 640 .

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Table 4.-Valuations.

|  | Cash Value of Farms. |  |  | Total Valdation. |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1800. | 1860. | 1870. | 1850. | 1860. | 1870. |
| Kenosha Co. | \$1,995,510 | \|\$4,494,499 | \|\$6,013,271 | \$1,065,956 | \$3,973,150 | \$5,047,434 |
| City of Kenosha |  |  |  | 348,317 | 1,040,093 | 439,111 |
| Brighton | 148,430 | 360,186 | 706,748 | 85,940 | 316,626 | 596,059 |
| Bristol | 277,964 | 1,646,93j | 792,821 | 95,543 | 413,933 | 854,697 |
| Paris | 245,445 | 465,690 | 823,801 | 90,955 | 373,553 | 661,276 |
| Pike-Somers | 312,637 | 529,521 | 1,050,690 | 100,732 | 538,048 | 873,197 |
| Pleasant Prairie | 299,990 | 582,492 | 1,094,836 | 100,071 | 479,135 | 516,723 |
| Randall |  | 88,225 | 457,050 |  | 221,156 | 260,864 |
| Salem | 226,092 | 576,370 | 704,344 | 90,615 | 358,457 | 671,553 |
| Southport ........ | 215,805 |  |  | 60,955 |  |  |
| Wheatland | 269,147 | 245,080 | 373,081 | 92,818 | 232,148 | 173,954 |

Note.-Cash value of farms taken from U. S. Census Returns, and the total valuation from the Assessment File in the vault of the office of the Sec. of State.

Table 5.-Occupation of Males over 15 yeors of age by percentage for the years 1860 and 18\%0.

|  | Years. | Per cent. of total population. | $\underset{\text { ers. }}{\text { Farm- }}$ | Capitalistis and Merch ants | Professional men. | $\begin{gathered} \text { Artisans } \\ \text { and } \\ \text { Mechan- } \\ \text { ics. } \end{gathered}$ | Laborers. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Kenosha County .... | 1860 1870 | 38.4 30.6 | ${ }_{37.06}^{33.2}$ | 1.8 2.8 | 2.3 | 7.5 | 25.5 31,01 |
| City of Kenosha .... | 1860 1870 | 27.3 23.1 | 4.3 3.7 | 5.3 5.7 | 5.03 4.5 | 19.02 | 23.1 26.8 |
| County without city | 1860 1870 | $\stackrel{32}{31.1}$ | $\stackrel{43}{52.7}$ | ${ }_{1.56}{ }^{\text {. }}$ | 1.3 | 3.08 4.07 | 26.4 32.8 |
| Brighton ............. | $\begin{aligned} & 1860 \\ & 1870 \end{aligned}$ | 31.6 30.2 | 41.2 60.5 | .3 | ${ }^{1} .5$ | 2.1 | 29.8 35.0 |
| Bristol .............. | 1860 1870 | 33.3 32.5 | $\stackrel{39}{54.8}$ | .9 3.7 | 2.1 | 4.6 | 30.6 29.9 |
| Paris ................ | 1860 1870 | 34.2 83.2 | 41.8 58.2 |  | 1.8 | 2.4 | 34.5 30.6 |
| Pleasant Prairie ... | 1860 1870 | 33.8 84.2 | 41.2 43.5 | $\stackrel{.2}{2}$ | 1.2 | 1.4 | 17.6 37.6 |
| Somers ............. | 1860 1870 | $\stackrel{32.1}{31.4}$ | 57.2 59.9 | . 7 | .9 1.6 | 2.6 3.9 | 14.6 27.2 |
| Randall .............. | $\begin{aligned} & 1860 \\ & 1870 \end{aligned}$ | 33.4 19.2 | 42.2 17.6 | . 9 | . 9 | 4.9 | 26.8 56.9 |
| Salem ............... | $\begin{aligned} & 1860 \\ & 1870 \end{aligned}$ | $\begin{aligned} & 30.7 \\ & 31.5 \end{aligned}$ | 39.9 41.9 | 1.12 2.7 | ${ }_{2}^{2.03}$ | 7.6 | 25.3 33.5 |
| Wheatland | 1860 1870 | 28.6 27.9 | $\begin{aligned} & 59.7 \\ & 58.9 \end{aligned}$ | 1.5 .4 | 1.25 2.07 | 4.4 4.14 | 36.2 26.9 |

Table 6.-Products.
1850.

|  | Value of farm per capita. | Bushels Per Capita. |  |  |  | Lbs. Per Cap. |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Wheat. | Indian corn. | Oats. | Barley. | Butter. | Cheese. |
| Kenosha county without city | 272.1 | 41.7 | 13.8 | 28.8 | 1.3 | 36.4 | 8.08 |
| Kenosha county with city.... | 184.4 | 28. | ${ }^{9.3}$ | 19.3 | 1.8 | 24.5 | 5.43 |
| Brighton | 168.5 | 33.9 | 8.66 | 17.5 | 1.13 | 44.3 | 1.58 |
| Paris................................. | 246.8 256.4 | 39.9 43.7 | 14.7 | 21.9 | 1.12 | 41.2 | 10.02 |
| Pleasant Prairie . ................ | 312.4 | 43.7 42.2 | 12.6 | 30.1 98.3 | . 6 | 40.8 42.5 | 3.7 |
| Pike............ | 460.4 | 50.7 | 14.7 | 57.6 | 1.78 | 42.5 29.9 | 12.1 8.3 |
| Salem...... | 200.4 | 34.7 | $\stackrel{14}{9.27}$ | 17.8 | 1.87 | 29.9 32.6 | 88.18 |
| Southport....................... | 594. | 6.52 | 9.35 | 8.76 |  | 24.2 | 38.7 |
| Wheatland.................... | 225.1 | 59.3 | 21.8 | 18.7 | 2.41 | 28.7 | 1.8 |

1860. 

| Kenosha county with city. | 332.1 | 21.9 | 11.8 | 15.15 | 2. | 24.2 | 2.9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Kenosha county without city | 472. | 31.1 | 16.8 | 21.5 | 2.9 | 34.5 | 4.2 |
| Brighton....................... . | 306.8 | 37.7 | 12.4 | 21.6 | 2.1 | 34.5 32.8 | 4.2 .3 |
| Bristol | 1,202.8 | 32.4 | 18.5 | 27.9 | 6.4 | 12.3 | 7.3 |
| Paris...... | 1, 431.8 | 54.5 | 14.3 | 31.9 | 4.8 | 45.7 | 3.09 |
| Pleasant Prairie | 416. | 28.9 | 13.6 | 18.1 | 2.1 | 48.5 | 3.09 |
| Randall........................ | 133.7 | 12.6 | 17.6 | 11.2 | . 2 | 13.5 | 3.1 |
| Somers.. | 413.8 | 29.2 | 15.5 | 17.2 | 3.3 | 52. | 10.9 |
| Wheatland | 220. | 22.5 | 15.9 | 15. | . 4 | 20.5 | . 7 |

1870. 

| Kenosha county with city .... | 5161 | 15.4 | 23.1 | 24.7 | 36.6 | 35.4 | 22.8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Kenosha ceunty without city. | 678. | 22.6 | 34.5 | 36.6 | 48.3 | 52.6 | 34. |
| Brighton. | 591.9 | 38.1 | 34.8 | 26.2 | 3.5 | 75.2 | 4.4 |
| Bristol | 694.1 | 19.4 | 34.8 | 45. | 9.1 | 56.2 | 27.8 |
| Paris. | 811.9 | 40.2 | 29.2 | 40. | 6.6 | 65.2 | 17.5 |
| Pleasant | 796.5 | 14.2 | 31.6 | 36.4 | 5.8 | 44.5 | 162.2 |
| Randa | 8771 | 37.2 | 47.5 | 54.2 | 9.3 | 51.9 | 6.9 |
| Somers | 506.2 782.5 | 12.3 | 23.5 | 29.2 | 2.9 | 40.5 | 11.5 |
| Wheatland | 782.5 427.5 | 14.7 | 20.1 | 40.6 | 4.9 | 50.8 | 1.7 |
| Wheatiand | 427.5 | 31.4 | 41.6 | 29.1 | 2.2 | 37.1 | . 6 |

1885. 

| Kenosha county with city.... | 426.9 | 3.6 | 43.8 | 45. | 5.9 | 51.4 | 10.5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Kenosha county without city. | 668. | 5.7 | 68.4 | 70. | 9.2 | 81.5 | 16.5 |
| Brighton........................ | 691. | 7.39 | 808 | 115.8 | 85 | 63.8 | 10.3 |
| Bristol | 647. | 2.2 | 52.2 | 66.3 | 8.4 | 127.2 |  |
| Paris.. | 672.5 | 7.6 | 49.8 | 111.9 | 17.4 | 66.9 |  |
| Pleasant Pra | 723 . | 7.3 | 45.7 | 54.9 | 6.5 | 126.1 | 5.8 |
| Randall | 7528 | 12.1 | 131.2 | 105.3 | 22.4 | 53.8 | 163.1 |
| Salem | 714. | 3.7 | 106.8 | 51.7 | 1.7 | 88.5 | 27.8 |
| Somers | 506. | 3.9 | 26.7 | 57.3 | 8.4 | 58.4 |  |
| Wheatland | 533.5 | . 47 | 109.8 | 38.7 | 11.5 | 23.9 | 14.5 |

1895. 

| Kenosha county with city.... | 423. | .523 | 23.85 | 42.1 | 1.6 | 83.4 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Kenosha county without city | 787.2 | . 981 | 44.5 | 78.4 | 3.1 | 155.1 |  |
| Brighton.... .................... | 879.9 | . 8 | 108.9 | 145. | 3.9 | 122.2 |  |
| Bristol | 826. | . 5 | 41.3 | 65.7 | 2.9 | 230.4 |  |
| Paris............ | 616.5 | . 2 | 90.8 | 173.9 | 2.3 |  | 22.9 |
| Pleasant Prairie | 1,062.5 | 1.3 | 20.8 | 51.1 | 4.1 | $135.2{ }^{\text {a }}$ |  |
| Randall. | ${ }^{1} 971.0$ | 3.2 | 43.7 | 92.1 | 8.1 | 546.9 |  |
| Salem. | 528.9 | 1.0 | 15.4 | 31.4 | 2.7 | 79.2 |  |
| Somers... | 779.2 | . 7 | 28.2 | 63.1 | 4.55 | 151.5 |  |
| Wheatland | 788. | 1.77 | 96.3 | 81.3 | 3.462 | . 304 |  |

Table 7.- Occupations, 1860-1870.

|  | $\begin{aligned} & \dot{\ddot{L}} \\ & \underset{\sim}{\tilde{0}} \\ & \stackrel{\rightharpoonup}{0} \end{aligned}$ |  |  |  | $\begin{aligned} & \text { Professional } \\ & \text { men. } \end{aligned}$ |  |  |  | $$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Kenosha county............ | 1860 | 4,141 | 1,371 | 76 | 95 | 312 | 189 | 71 | 1,059 | 968 |
|  | 1870 | 3,971 | 1,475 | 113 | 93 | 350 | 2¢6 | 99 | 1,233 | 342 |
| Kenosha city....... . .... | 1860 | 1,091 | 47 | 58 | 55 | 208 | 147 | 67 | 253 | 256 |
|  | 1870 | 1,215 | 46 | 70 | 55 | 238 | 208 | 91 | 326 | 181 |
| County without city....... | 1860 | 3,050 | 1,324 | 18 | 40 | 104 | 42 | 4 | 806 | 712 |
|  | 1870 | 2,756 | 1,429 | 43 | 38 | 112 | 58 | 8 | 907 | 161 |
| Brighton..................... | 1860 | 372 | 172 |  | 4 | 8 | 1 |  | 111 | 76 |
|  | 1870 | 357 | 216 | 1 | 2 | 5 |  | 1 | 125 | 7 |
| Bristol........................ | 1860 | 456 | 178 | 4 | 10 | 21 | 11 |  | 140 | 92 |
|  | 1870 | 371 | 203 | 14 | 6 | 22 | 12 |  | 111 | 3 |
| Paris | 1860 | 371 | 155 | ... | 3 | 9 | 3 | ..... | 128 | 58 |
|  | 1870 | 356 | 207 |  | 5 | 1.) | 2 | . | 109 | 18 |
| Pleasant Prairie., | 1860 | 473 | 195 | 1 | 6 | 7 | 6 |  | 82 | 176 |
|  | 1870 | 471 | 205 | 11 | 3 | 16 | 12 | 2 | 177 | 45 |
| Randall | 1860 | 220 |  |  |  |  | 2 | ...... | 59 | 66 |
|  | 1870 | 102 | 18 | 1 | i | ${ }^{5}$ | 2 | ...... | 58 | 19 |
| Somers . . . . . . . . . . . . . . . . . | 1860 | 411 | 235 | 3 | 4 | 11 |  | 1 | 59 | 94 |
|  | 1870 | 426 | 255 | 3 | 7 | 17 | 7 | 2 | 116 | 19 |
| Salem. | 1860 | 444 | 177 | 5 | 9 | 34 | 12 | 2 | 112 | 93 |
|  | 1870 | 437 | 183 | 12 | 9 | 27 | 19 | 1 | 146 | 40 |
| Wheatland.................. | 1860 | 318 | 119 | 5 | 4 | 14 | 3 | 1 | 115 | 57 |
|  | 1870 | 241 | 142) | 1 | 5 | 10 | 6 | 2 | 65 | 10 |

Note.-The above figures were taken from the U. S. census returns, in the office of the Secretary of State.

Table 8.- Populution and chief products of county and towns for 1850, 1860, 1870, 1885 and 1895.

1850 ,

| County and town. | Total population. | Cash value of farms. | $\begin{gathered} \text { Bush. } \\ \text { of } \\ \text { wheat. } \end{gathered}$ | Bush. of <br> Indian corn. | Bush of oats. | $\begin{gathered} \text { Bush. } \\ \text { of } \\ \text { barley. } \end{gathered}$ | $\begin{gathered} \text { Pounds } \\ \text { of } \\ \text { butter. } \end{gathered}$ | $\begin{aligned} & \text { Pounds } \\ & \text { of } \\ & \text { cheese. } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Kenosha county . | 10,818 | 1,995,510 | 303, 176 | 100, 294 | 209,790 | 9,459 | 265.023 | 58,923 |
| Brighton........... ... | 880 | 148,430 | 29, 812 | 7,616 | ? 5,427 | 974 | 38,980 | 1,390 |
| Bristol | 1,125 | 277, 56t | 44,545 | 16,642 | 23, 828 | 1, 258 | 46,340 | 11,273 |
| Paris. | 976 | 215,445 | 41,560 | 12,060 | 28,94x | 581 | 39, 120 | 3,540 |
| Pleasant | 959 | 299,990 | 40,331 | 14,032 | 27,140 | 429 | 40,760 | 11,570 |
| Pike | 680 | 312,637 | 34,470 | 10,05.9 | 39,136 | 1,235 | 20,130 | 5,647 |
| Salem | 1,123 | 226,09 | 39, 061 | 10,410 | 2i, 193 | 2,114 | 36, 645 | 9.191 |
| Southpo | 363 | 215, 805 | 2,363 | 3,393 | 3,179 |  | 8,780 | 14,060 |
| Wheatlan | 1,193 | 259, 147 | 70, 884 | 20, 082 | 2i,889 | 2,865 | 34, 266 | 2,250 |
| 1860. |  |  |  |  |  |  |  |  |
| Kenosha county | 13,519 | 4, 494, 199 | 297, 732 | 160,911 | 205,117 | 27,825 | 328, 739 | 40,239 |
| Brighton........ | 1,176 | , 360,186 | 44.069 | 14,678 | 30,092 | 2, 474 | 38,608 | -365 |
| Bristol. | 1,369 | 1,646, 935 | 44,346 | 23, 341 | 38,25i | 8,8:35 | 16,8×3 | 9,555 |
| Paris. | 1,085 | 465, 030 | 55, 159 | 15,553 | 32,416 | 5, 292 | 49, 600 | 3,355 |
| Pleasant Prairio | 1,400 | 582, 492 | 40,434 | 19, 03n | 25, 358 | 2,953 | 67,779 | 4,352 |
| Randall | 660 | 88, 225 | 8,359 | 11,644 | 7,394 | 143 | 8,930 |  |
| Salem. | 1,444 | 576, 370 | 39, 889 | 37, 108 | 30, 831 | 3,378 | 57,517 | 7,850 |
| Somers. | 1,278 | 529,521 | 37,333 | 19,779 | 21,979 | 4,275 | 66,427 $-2,965$ | 13,962 .792 |
| Wheatland ........... | 1,115 | 245, 080 | 25,111 | 17,753 | 16,790 | 475 | 22,965 | 792 |

1870. 

| Kenosha county ...... | 13,174 | 6,01?,271 | 205, 732 | 270, 919 | 325, 246 | 4.5, 870 | 466, 407 | 300,517 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Brighton........ | 1,187 | 706, 748 | 41,751 | 36,307 | 31, 185 | 4,203 | 89,003 | 5,200 |
| Bristol.. | 1,140 | 792, 821 | 22, 109 | 39,712 | 51,494 | 10,422 | 64,099 | 31,730 |
| Paris | 1,014 | 823, 801 | 40,849 | 29,610 | 40,634 | 6,774 | 66, 452 | 17, 830 |
| Pleasant Prairie | 1,377 | 1,094, $\times 36$ | 19,506 | 43,600 | 50, 226 | 8,069 | 61, 520 | 223,065 |
| Randall... | 53: | 457,050 | 16,971 | 25, 360 | 28, 997 | 4,972 | 27, 785 | 3,725 |
| Salem | 1,386 | 704,314 | 17,117 | 32,618 | 40,719 | 2,778 | 56, 190 | 16,025 |
| Somers | 1,359 | 1,060, 690 | 19, 92:3 | 27,346 | $\stackrel{26,250}{ }$ | 6,721 | 69,003 | 2,385 |
| Wheatland | 871 | 373,081 | 27,506 | 36,365 | 25, 391 | 1,931 | 32,350 | 557 |
| 1885. |  |  |  |  |  |  |  |  |
| Kenosha county | 14,137 | 6,032, 904 | 52,179 | 618,036 | 632,403 | 83, 125 | 726,096 | 149,572 |
| Brighton............... | , 961 | 854,780 | 7,118 | 77, 793 | 111,195 | 8,242 | 61, 460 | 9,920 |
| Bristol ................. | 1,134 | 734, 240 | 2,515 | 59, 215 | 75, 215 | 9, 998 | 144, 115 |  |
| Paris. | 991 | 665, 405 | 7,543 | 49,370 | 110,952 | 17, 264 | 66, 365 |  |
| Pleasant Pra | 1,494 | 1,078,350 | 10, 906 | -8,102 | 82,236 | 9,775 | 188,650 | 6,800 |
| Randall..... | , 489 | 367, 879 | 5,910 | 64,211 | 51,552 | 10,964 | 26, 281 | 79,812 |
| Salem | 1,382 | $9 \times 5,050$ | 5,203 | 147,340 | $71,4 \times 8$ | 2,245 | 122,365 | 3ミ,540 |
| Somer | 1,290 | 805, 000 | 6,299 | 42, 445 | 91, 130 | 13, 437 | 92,960 |  |
| Wheatlan | 993 | 532, 200 | 478 | 109,560 | 33,640 | 11, 350 | 2s,900 | 14,500 |
| 1895. |  |  |  |  |  |  |  |  |
| Kenosha county...... | 17,548 | 7.416,043 | 9,211 | 418,815 | 734, 658 | 29,403 | 1.460.856 | 17,900 |
| Brighton............... | 878 | 763, 143 | 773 | 95, 500 | 127,190 | ${ }^{8} 86$ | 19,550 |  |
| Bristol | 1,143 | 916,650 | 55 | 47,495 | -75, 255 | 3, 321 | 265,187 | 17900 |
| Paris.. | 883 | 544,275 | $\because 210$ | 80, 0.0 | 153, 5.56 | , 6,040 |  | 17,900 |
| Pleasant Prairie | 1,5:4 | 1,617,900 | 1,992 | 30,800 | 77, 825 |  | 206,450 | ....... |
| Randall. | 64:3 | 624,500 | 2,112 | 28, 075 | 59, 230 | 5,218 | $351,8: 0$ 149,040 | - |
| Salem | 1,887 | 99x, 605 | 1,139 | 23, 360 | 78, 210 | 1,165 | 149,040 191,293 |  |
| Somers. | 1,819 | 1,410,360 | 1,28t | 51, 050 | 114, ${ }_{5}$ | 8,262 | 1977, 280 |  |
| Wheatland ........... | 649 | 510,610 | 1,149 | 62,815 | 54, 785 | 2,245 | 277, 280 |  |

NOTE.-The above figures for 1850, 1860, and 1870 were taken from the United States census returns, which are in the office of the Secretary of State, and the rest from the state census reports.

Table 9.-Summary and percentage of Republican and Democratic votes for president, governor, and congressman.

|  | President, 1860-1900. |  |  |  | Governor, 1859-1500. |  |  |  | Congressman, 1866-1900. |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Repablican. |  | Democratic. |  | Republican. |  | Democratic. |  | Republican. |  | Democratic. |  |
|  | $\begin{aligned} & \dot{\text { ® }} \\ & \stackrel{0}{\circ} \end{aligned}$ |  | ¢ |  | $\stackrel{\text { ® }}{\stackrel{\circ}{\circ}}$ |  | $\stackrel{\dot{+}}{\stackrel{+}{\circ}}$ |  | $\stackrel{\circ}{\square}$ |  | $\stackrel{\square}{\circ}$ | 这 |
| Kenosha county .. | 20,101 | 58 | 15,900 | 44 | 27,943 |  | 24,510 | 47 | 19,488 | 55 | 19,942 | 45 |
| Kenosha city . | 7,100 | 52 | 7,030 | 48 | 10,387 |  | 11,266 | 52 | 7,70i |  | 7,645 | 48 |
| Brighton. | 869 | 35 | 1,584 | 65 | 1,208 |  | 2,283 | 66 | 791 |  | 1,349 | 69 |
| Bristol . | 2,139 |  | 678 |  | 3,150 |  | 1,030 | 25 | 1,984 |  | 652 | 25 |
| Paris.. | 1,169 | 51 | 1,122 |  | 1,558 |  | 1,703 | 52 | 1,013 |  | 1,085 | 52 |
| Pleasaut Prairie.. | 2,107 | 58 | 1,503 | 42 | 3,068 |  | 2,174 | 42 | 1,960 | 59 | 1,358 | 41 |
| Randall... | 1,025 | 67 | 491 | 33 | 1,409 |  | 711 | 34 | 895 | 64 | 497 | 36 |
| Salem..... | 2,446 | 68 | 1,253 | 32 | 3,617 | 66 | 1,855 | 34 | 2,248 | 65 | 1,227 | 35 |
| Somers . | 2,441 |  | 1,077 | 31 | 3,386 |  | 1,540 | 32 | 2,184 | 68 | 1,045 | 32 |
| Wheatland | 805 |  | 1,157 |  | 1,160 | 38 | 1,843 | 62 | 712 |  | 1,084 | 61 |

# ECONOMIC AND SOCIAL DEVELOPMENT OF LA FAYETTE COUNTY BETWEEN 1850 AND $1870{ }^{1}$ 

## KATHERINE PATRICIA REGAN.

## Chapter I.

## FORMATION OF THE COUNTY AND TOWNS.

La Fayette county, situated in the southwestern part of Wisconsin, was originally a part of Iowa county. But the discovery of the lead mines about 1820 resulted in so great an increase of population that in 1847 it was found necessary to divide Iowa county, the southern part of which containing the lead mines of the Wisconsin district was designated as La Fayette county.

The class of people attracted by mining interests, however, made no permanent settlements. They were of too migratory a nature, moving here and there as their interests directed them. And though there was a considerable seattered population soon after 1822, no permanent settlement was made before 1826. It is doubtful whether any would have been made even then, had not the inhabitants found it necessary to build a fort at Gratiot's Grove for protection against the Indians.

Certain lands north of what is known as the Ridge ${ }^{2}$ had been reserved for the Indians. But finding that these contained rich lead deposits, miners constantly crossed the line in utter disregard for any rights the Indians might have. Repeated protests on the part of the Indians resulting in no redress of their grievances, the matter finally concluded in the uprising commonly

[^20]knewn as the Winnebago War. Then it was that the fort known as Gratiot's Fort was built, and the first permanent settlement started. The prompt action of Governor Edwards of Illinois quelled the disturbance, however, and the fort was never used as such. This defeat had the effect of quieting the Indians, and the valuable lands held by them were thereafter open to settlement. ${ }^{1}$

The legal boundaries of La Fayette county are as follows: "That part of the county embraced in Iowa county designated as towns $1,2,3,4$ and 5 east, and the south half of town 4, ranges $1,2,3,4$, and 5 , shall be set off into a separate county named La Fayette." ${ }^{2}$ It was organized for judicial purposes after May 1, 1847.

Before its organization into towns La Fayette county consisted of five election districts,-Kendall, Willow Springs, Argyle, Gratriot and the Fever River District. On January 3, 1849, from the Kendall district the towns of Belmont and Kendall were organized; from Willow Springs district, Darlington (Centre) and Willow Springs; from Argyle, Fayette, Argyle and Wiota; from Gratiot, Waynie ${ }^{3}$ and Gratiot; from the Fever River district, Benton, New Diggings, Elk Grove, Shullsburg, White Oak Springs and Monticello. Blanchard was organized out of the northern part of the town of Argyle, and Seymour from Centre, in 1869. ${ }^{4}$

[^21]
## Chapter II.

## TOPOGRAPHY AND SOILS.

Before taking up a discussion of the population of this county, it will be necessary to state something of its geology: ${ }^{1}$ The county forms part of the great watershed passing from Madison to the confluence of the Wisconsin and Mississippi rivers. The western branch of this watershed separates the rivers that flow into the Fever river from those that flow into the Pecatonica. Entering at the town of Belmont, it passes through Shullsburg in a southeasterly direction, leaving the county through Monticello. The highest point on this divide is the Platte Mounds, in the northern part of Belmont.

La Fayette is well watered by the Pecatonica, East Pecatonica and Fever rivers and their branches. Springs also abound, especially in the towns of Willow Springs, Centre, Wiota, White Oak Springs, Shullsburg, Benton and New Diggings. They are scarce on the prairie lands of Kendall, Belmont, Elk Grove, Wayne, Gratiot and Monticello. ${ }^{2}$

The soil is well adapted to agricultural purposes and unusually large successive wheat crops have been raised with no regard to rotation. A belt of rich, black loam extends through the western and central parts, including the greater parts of Seymour, Shullsburg, Darlington, Gratiot, White Oak Springs and Monticello; also large parts of Elk Grove and Belmont. There is also a much narrower belt passing through the westcentral part of Fayette, which curving through the west-central parts of Wiota, enters the southern part of Argyle. The subsoil is clay underlaid with limestone, which forms in ridges along the larger streams, affording some valuable quarries. Sand is found in the eastern part to some extent and along the rivers. ${ }^{3}$

[^22]Boundary Changes


- Original Boundaries 1847
---- Principal Changes in Boundaries Since 1847
Distribution of Solls


Prairie Soil $\square$ Clay Soil

Chapter III.

## POPULATION.

A study of the population was made by taking the nativity and occupations of all inhabitants of the county from the census of the United States for 1850,1860 and $1870 .{ }^{1}$ The population was then considered as divided into two parts, the native and foreign born. ${ }^{2}$ The native-born were again grouped into sections, namely, New England, Middle States, Southern States, Southwestern States, Northwestern States and Western States. Wisconsin is given in the table separately. Under the heading, Western States, are included all the states not mentioned in the other sections.
In the foreign countries the following groups were made: British America, Great Britain, the rest of Europe ${ }^{3}$ and miscellaneous. Under the latter were grouped such as were born at sea, on the lakes, in the United States and the like.

Tables were made of the population of each section by nativities for the various towns. Comparisons can be made between the census for 1850,1860 and 1870 , for such towns only as did not change their boundary lines. These towns are Benton, New Diggings, Shullsburg, Wayne, White Oak Springs and Willow Springs.

By examining Table 1 in the Appendix it will be seen that in native population Wisconsin leads for the entire period considered, that is, from 1850 to 1870 ; so it will not be considered in the present comparison. Next to Wisconsin are the Northwestern States, the towns of Belmont, Wayne, Fayette, Centre, and Argyle leading. The Middle States are next, but with a percentage almost one-third less. The number from New York

[^23]alone differs but slightly from that of the remainder of the section of Middle States. In this Middle Section the town of Fayette contains about one-fifth of the entire number represented by both the Middle and Northwestern sections. The New England and Southern sections are represented about equally. In New England settlers Centre, Willow Springs Wayne, Fayette and Argyle are in the lead. In the Southern section the towns of Monticello, White Oak Springs, Kendall, Fayette and Belmont have the largest number.

In the foreign population Great Britain stands first, with Ireland second ; together these sections constitute nearly one-half of the entire foreign population of the county. Under Great Britain, Benton leads with Elk Grove, New Diggings, White Oak Springs and Shullsburg closely following. These are the towns, it will be remembered, that constitute the mining district. ${ }^{1}$ Ireland is represented chiefly in this district, also, in New Diggings, Shullsburg, Benton and Willow Springs and Kendall. The rest of the foreign population altogether constitutes about 5 per cent. of the entire foreign population. It is thus seen that the mining districts contain the greatest foreign population in 1850.

In 1860 the towns showing the highest percentage of population from the Northwestern section are Belmont, Fayette, Monticello, Gratiot and Wiota. ${ }^{2}$ New York in this census is represented by a greater percentage than the rest of the Middle States. Centre, Gratiot, Wayne, Argyle, Wiota and Kendall, the last two being equal, lead in population from this state. The rest of the Middle section is most largely represented in Monticello, Fayette, Wiota, Belmont and Gratiot.

In the foreign population Ireland leads Great Britain, but has decreased slightly since 1850, though the decrease in the percentage of Great Britain has been twice as great. The towns having the highest percentage of Irish are New Diggings, Kendall, Shullsburg, Willow Springs and Benton. These are the

[^24]same towns which showed the highest percentage of Irish in $1850 .{ }^{1}$

The towns having the highest percentage of population from Great Britain are Benton, White Oak Springs, New Diggings, Elk Grove and Shullsburg. These towns also lead in $1850 .^{2}$ The percentage from Great Britain and Ireland is more than twice as great as the percentages of the combined population of the rest of the foreign countries.

In 1870 the Northwestern States still have the greatest percentage of population, with the towns of Wayne, Monticello, Gratiot, Benton and White Oak Springs leading. ${ }^{3}$ The percentage of New York differs but slightly from that of the remainder of the Middle States, but both have decreased since 1860. New England and the South are about equally represented.

In the foreign population the percentages of Great Britain and Ireland are almost equal. The towns with the largest Irish population in 1870 are Kendall, Seymour, New Diggings, Shullsburg and Benton, the last three of which held the same relative position in 1860. The towns having the largest population born in Great Britain are New Diggings, Benton, White Oak Springs, Willow Springs and Seymour, three of which, New Diggings, Benton and White Oak Springs, had the greatest population from the same country in 1860.

To summarize:-The Wisconsin-born population has the largest number, and the Northwestern States stand next in all three censuses. New York has a greater population than any state in the East during the whole period. In 1850 the New England and Southern sections are about equal; in 1860, New England has not changed, but the South has decreased slightly; in 1870 they are again about equal.

In the foreign population in 1850, Great Britain leads with Ireland second ; in 1860, Ireland exceeds Great Britain, but the population from both these countries has decreased. In 1870

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Great Britain and Ireland are about equal. It was shown that the percentage of population of these two countries taken together was greater than the percentages of population of the rest of the foreign countries. The percentage of German population greately increased between 1850 and 1860.

Table 1.- Percentages of native and foreign born population.

|  | 1850. |  | 1860. |  | 1870. |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Native. | Foreign. | Native. | Foreign. | Native. | Foreign. |
| Argyle .................. | 72 | 28 | 65 | 34 | 64 | 36 |
| Belmont .............. | 86 | 14 | 77 | 23 | 68 | 32 |
| Benton ................... | 39 | 61 | 52 | 48 | 61 | 79 |
| Blanchard ............... | . | . | . | . | 60 | 40 |
| Darlington ............... | 78 | 22 | 76 | 24 | 80 | 20 |
| Elk Grove ..... | 57 | 43 | 56 | 44 | 65 | 35 |
| Fayette ............... | 85 | 15 | 79 | 21 | 76 | 24 |
| Gratiot | 79 | 21 | 82 | 18 | 78 | 22 |
| Kendall | 78 | 22 | 66 | 34 | 67 | 33 |
| Monticello .... | 64 | 36 | 71 | 29 | 76 | 24 |
| New Diggings ........... | 43 | 57 | 53 | 47 | 62 | 38 |
| Seymour ....... | .. | . | . | .. | 55 | 45 |
| Shullsburg ............... | 52 | 48 | 57 | 43 | 65 | 35 |
| Wayne | 92 | 8 | 92 | 8 | 88 | 12 |
| White Oak Springs...... | 64 | 36 | 66 | 34 | 71 | 29 |
| Willow Springs | 69 | 31 | 66 | 34 | 70 | 30 |
| Wiota | 74 | 26 | 71 | 29 | 72 | 28 |
| Average for county | 60 | 40 | 66 | 34 | 70 | 30 |

In Table 1 the percentages of the entire native and foreign born for each town and for each census are given. In 1850 the percentage of native born population for the entire country was 60 ; of the foreign born population, 40. By 1870, the native born population had increased to 70 per cent. ; the foreign born had decreased to 30 per cent.

Of the towns whose boundaries did not change, Benton had 39 per cent. native born in 1850, and 61 per cent. in 1870, a gain of 22 per cent. Of foreign born population, Benton had 61 per cent. in 1850, and 39 per cent. in 1870. New Dig-
gings in 1850 had a native born population of 43 per cent.; in 1870, 62 per cent., a gain of 9 per cent., while the foreign born population decreased from 57 per cent. in 1850 to 38 per cent. in 1870. The towns of Shullsburg, Wayne and White Oak Springs show a similar increase in the native born population and decrease in the foreign. Willow Springs, however, in 1850 had a native born population of 69 per cent.; in 1860, of 66 per cent., and in 1870 of 70 per cent., showing a total increase between 1850 and 1870, of 1 per cent. In 1850, Willow Springs had 5 per cent. more native born population than White Oak Springs, but in 1860 the percentages of both are the same. In 1870 White Oak Springs had increased 1 per cent. over Willow Springs.

## Chapter IV.

DENSITY, WEALTH AND OCCUPATION.

Table 2.-Per capita wealth, and population per square mile.

| Towns. | Years. | Nativity. |  | Property. |  | Population per squaremile. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Native. | Foreign. | Person'l. | Real estate. |  |
| Benton .......... | $\begin{aligned} & 1850 \\ & 1860 \\ & 1870 \end{aligned}$ | $\begin{aligned} & 39 \\ & 52 \\ & 61 \end{aligned}$ | $\begin{aligned} & 61 \\ & 48 \\ & 39 \end{aligned}$ | 11 11 32 | 34 62 168 | $\begin{aligned} & 81 \\ & 76 \\ & 62 \end{aligned}$ |
| New Diggings | $\begin{aligned} & 1850 \\ & 1860 \\ & 1870 \end{aligned}$ | 43 53 62 | $\begin{aligned} & 57 \\ & 47 \\ & 38 \end{aligned}$ | 6 20 33 | 46 71 155 | 65 63 68 |
| Shullsburg ......... | $\begin{aligned} & 1850 \\ & 1860 \\ & 1870 \end{aligned}$ | $\begin{aligned} & 52 \\ & 57 \\ & 65 \end{aligned}$ | $\begin{aligned} & 48 \\ & 43 \\ & 35 \end{aligned}$ | 10 45 64 | 66 79 217 | 47 69 73 |
| Wayne .............. | $\begin{aligned} & 1850 \\ & 1860 \\ & 1870 \end{aligned}$ | $\begin{aligned} & \mathbf{9 2} \\ & 92 \\ & 88 \end{aligned}$ | 8 8 12 | 13 40 53 | 65 216 143 | 9 18 18 29 |
| White Oak Springs | $\begin{aligned} & 1850 \\ & 1860 \\ & 1870 \end{aligned}$ | $\begin{aligned} & 64 \\ & 66 \\ & 71 \end{aligned}$ | 36 34 29 | 78 25 73 | 107 146 282 | 28 32 33 |
| Willow Springs .. | $\begin{aligned} & 1850 \\ & 1860 \\ & 1870 \end{aligned}$ | $\begin{aligned} & 69 \\ & 66 \\ & 70 \end{aligned}$ | 31 34 30 | 7 49 79 | 114 238 232 | 12 17 23 |

In Table 2 is given the per capita of real estate and personal property and the native and foreign born populations with the population per square mile for six towns of fixed boundaries. ${ }^{1}$ In this comparison the native born population increases in every town except Wayne, where a slight decrease is noted between 1860 and 1870. But this town also shows a less increase per capita for that period than any other town. Between 1860 and 1870 the population per square mile is doubled, the native and foreign born retaining the same percentage as in 1850, while the per capita is more than trebled. A greater increase in the per capita is shown in the towns where the native population increases. This may be shown more clearly, perhaps, by a comparison between the mining towns and farming towns of this table: New Diggings and Shullsburg composing the first

Thble 3.- Percentages of increase and decrease of density in popu- $_{\text {and }}$ lation.

|  |
| :--- |

class of towns and Wayne, White Oak Springs and Willow Springs, the second class. The per capita of the real estate and personal combined in the mining towns for the whole period between 1850 and 1870 is $\$ 64$, while a similar per capita of the farming towns is $\$ 224$. But the area of the latter class of towns is one-tenth greater than that of the former; this would leave the per capita of the towns of the farming class about

[^26]$\$ 200$. This is the more striking when we remember that White Oak Springs, classed with the farming towns, had large mining interests in an earlier period. We may thus see that although mining as a factor in the industrial life of the county had almost disappeared, its influence upon the economic life of the towns considered is still marked.

In comparing the percentages of increase and decrease between 1850 and 1870 of these same towns, in Table 3 we find Wayne increasing 214 per cent. and from 9 to 29 per square mile. White Oak Springs increases 19 per cent. and from 28 to 33 per square mile. The greatest increase in the first class is in Shullsburg, with 61 per cent., and from 47 to 73 per square per square mile.

Table 4.-Percentage of principal occupations.

|  | Farmers. |  |  | Miners. |  |  | Laborers. |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1850. | 1860. | 1870. | 1850. | 1860. | 1870. | 1850. | 1860. | 1870. |
| Argyle | 65.3 | 62 | 38.3 | 18.2 | ..... | 3.2 | 5.7 | 26 | 41 |
| Belmont | 84.8 | 72.4 | 30.8 | 3 |  | . 43 | 1.5 | 18 | 51.9 |
| Benton | 8.5 | 30 | 23.1 | 77.2 | 50.5 | 37.7 | 1.6 | 7.2 | 28.4 |
| Blanchard |  |  | 42.9 |  |  | 2.4 |  |  | 39.6 |
| Darlington, town |  |  | 55.81 |  |  |  |  |  | 38.6 |
| Darlington, town \& village | 76.6 | 56 |  | 7.8 | . 2 |  | . 6 | 18 |  |
| Darlington village |  |  | 9.8 |  |  | . 5 |  |  | 33.7 |
| Elk Grove | 73.6 | 69 | 43.8 | 11.1 | 5 | 1.1 | 3.5 | 17 | 47.8 |
| Fayette | 74 | 71 | 55 | 15.8 |  | 1.8 | 1 | 24 | 35 |
| Gratiot | 80 | 65 | 39.4 | 3 |  |  | 1.5 | 29 | 40.8 |
| Kendall | 84 | 78 | 50.1 |  | 3 | . 31 | 2.2 | 15.8 | 48 |
| Monticello | 52.9 | 48.8 | 42.7 | 2.9 | 3.8 |  |  | 41 | 52.6 |
| New Diggings | 7.5 | 16.2 | 70.5 | 73 | 50.4 | 47.6 | 2.2 | 21 | 26 |
| Seymour |  |  |  |  |  |  |  |  | 43 |
| Shullsburg | 12.3 | 24 | 23.4 | 61 | 35 | 32.2 | 4.5 | 15 | 25.8 |
| Wayne | 87.3 | 59.7 | 50 | 1.2 |  |  | 1.2 | 21 | 48 |
| White Oak Springs | 53 | 38.4 | 44.9 | 21 | 13 | 3.5 | 2 | 54.7 | 47.3 |
| Willow Springs ............. | 76.4 | 21.7 | 60.9 | 13.7 | ..... | . 99 | 3.3 | 55.4 | 29 |
| Wiota ........................ | 63 | 63 | 41.2 | 7:4 |  | 1.9 | 3.7 | 31.5 | 52 |

The mining class was kept separate since the southwestern part of the county is a mining district. Farmers, miners and laborers have the largest percentage. Table 4 shows the rela-
tive importance of these occupations in the census of 1850, 1860 and 1870.

The percentage of farmers seems to have greatly decreased between 1850 and 1870, but a fair estimate cannot be made since in the census of 1850 no distinction is made between farmers and farm laborers; that is, all are classed as farmers, irrespective of property. In the census of 1860 and 1870 this distinction is made. This fact will also explain, at least in part, the large increase in the laboring class between 1860 and 1870. In comparing the towns whose boundary lines remain fixed, we find that the percentage of farmers has increased in Benton, New Diggings and Shullsburg between 1850 and 1870, despite the fact that farm laborers are included in 1850. The percentage of artisans is but 1.9 less than the percentage of farmers in 1850 , while in 1870 the latter has increased 20.6 per cent. The mining class, on the contrary, has decreased. Wayne shows a decrease in the farming class and the mining class disappears after 1860, but the laboring class increases 41.8 per cent. White Oak Springs and Willow Springs show so slight a decrease between 1850 and 1870 in the farming class that we may conclude that had the census tabulations been made upon the same basis in all three censuses, an increase would have been shown. Mining declines in both these towns and the laboring class increases.

We see from the foregoing that La Fayette county is distinctly an agricultural region, though that particular industry would seem to be on the decline. We should note that in the towns of the mining district mining has given place to farming, but the influence of the former occupation is shown in the lower per capita of these towns. ${ }^{1}$ The constant shifting of the boundary lines of the various towns makes comparisons between all of the towns impossible for the different census periods, and hence the study has not been as extensive as we would like. It has been possible to compare an equal number of towns of the farming and mining regions, however, and these may be considered typical for the entire county.

[^27]
## APPENDIX.

Tables 1-5.
Showing Nativity of population of each town in La Fayette county for 1850, 1860 and 1870. Taken from copies of the original Census Returns in the office of the Secretary of State.

Table 1.

| 1850. | New England States. |  |  |  |  |  |  | Middle States. |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{aligned} & \text { 烒 } \\ & \text { 己 } \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ |  |  |  |  |  | ※ |  |  |  | - | - |
| Argyle ................ | 9 <br> 3 <br> 7 <br> 4 <br> 3 <br> 8 <br> 1 |  |   <br> $\ldots$  <br> $\ldots$  <br> $\ldots$ $\ldots$ <br> $\ldots$  <br> $\ldots$ $\ldots$ <br> $\ldots$  <br> $\ldots$ $\ldots$ <br> $\ldots$  | 5 <br> 3 <br> 3 <br> 32 <br> 2 <br> 16 <br> 16 <br> 8 <br> 1 <br> 1 <br> 1 <br> 6 <br> 19 <br> 3 <br> 10 <br> 10 <br> 3 <br> 11 <br> 123 | $r$ <br> 1 <br> 2 <br> 3 <br> 6 <br> $\ldots$ <br> $\ldots$ <br> 7 <br> 7 <br> 1 <br> 2 <br> 3 <br> 4 <br> 6 <br> 6 <br> 11 <br> $\ldots .$. <br> 4 <br> 1 <br> 51 | 4 <br> $\ldots$ <br> 5 <br> 6 <br> 6 <br> 2 <br> 9 <br> 9 <br> 9 <br> $\ldots .$. <br> 2 <br> 21 <br> 11 <br> 1 <br> 9 <br> 9 <br> 9 <br> 10 <br> 98 | 2513225574949245884250222723 $\|$ | 31 <br> 19 <br> 40 <br> 58 <br> 37 <br> 35 <br> 83 <br> 38 <br> 11 <br> 44 <br> 58 <br> 26 <br> 39 <br> 37 <br> 60 <br> 616 | 1 <br> 1 <br> 1 <br> 22 <br> 7 <br> 2 <br> 2 <br> 5 <br> $\ldots$ <br> $\ldots$ <br> 1 <br> 10 <br> 7 <br> 2 <br> 3 <br> 2 <br> 2 <br> 4 <br> 67 | 40 <br> 22 <br> 38 <br> 49 <br> 28 <br> 93 <br> 64 <br> 59 <br> 15 <br> 59 <br> 51 <br> 61 <br> 26 <br> 53 <br> 12 <br> 52 <br> 671 |  | 72 <br> 42 <br> 100 <br> 115 <br> 68 <br> 134 <br> 147 <br> 98 <br> 27 <br> 110 <br> 129 <br> 54 <br> 95 <br> 51 <br> 118 <br> 1,360 |
| Belmont ............... |  |  |  |  |  |  |  |  |  |  |  |  |
| Benton ................ |  |  |  |  |  |  |  |  |  |  |  |  |
| Centre ................. |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
| Elk Grove ............. |  |  |  |  |  |  |  |  |  |  |  |  |
| Fayette ............... |  |  |  |  |  |  |  |  |  |  |  |  |
| Gratiot ............... |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
| Kendall |  |  |  |  |  |  |  |  |  |  |  |  |
| Monticello | 1 |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
| Shullsburg | 6 |  |  |  |  |  |  |  |  |  |  |  |
| Wayne | 6 |  |  |  |  |  |  |  |  |  |  |  |
| Willow Springs | 8 |  |  |  |  |  |  |  |  |  |  |  |
| White Oak Springs | 3 |  |  |  |  |  |  |  |  |  |  |  |
| Wiota |  |  |  |  |  |  |  |  |  |  |  |  |
| Total | 67 |  |  |  |  |  |  |  |  |  |  |  |

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Table 1.- Continued.

| 1850. | Southern States. |  |  |  |  |  |  |  |  | SodthwesternStates. |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | - |  |  |  |  |  |  | - |  |  |  | - |
| Argyle ........... |  | 1 |  | 4 |  | 3 |  |  |  |  |  |  |  |
|  |  | 1 | 1 |  | ..... | 3 | 12 | 1 | 22 |  |  |  | 2 |
|  |  | 1 | 3 |  |  | 4 | 14 | 7 | 32 |  |  |  | 8 |
| Benton ........... |  | 4 | 4 | 4 |  | 15 | 38 | 7 | 73 | 55 |  | 1 | 56 |
| Centre |  |  | 3 | 2 |  | 8 | 33 | 6 | 53 | 2 |  |  | 2 |
| Elk Grove |  |  | 2 |  |  | 11 | 8 | 1 | 24 | 11 |  |  | 12 |
| Fayette |  |  | 5 |  |  | 13 | 11 | 22 | 58 | 7 |  |  | 7 |
| Gratiot |  | 3 | 6 | 1 |  | 3 | $1)$ | 2 | 17 | 2 |  |  | 2 |
| Kendall |  |  |  |  |  | 10 | 9 | 2 | 25 | 3 |  |  | 3 |
| Monticello |  |  |  | 1. |  | 9 | 3 |  | 25 |  |  |  | , |
| New Diggings .. |  | 1 |  | 6 |  | 10 |  |  |  |  |  |  |  |
| Shullsburg |  |  |  |  |  |  |  |  |  |  |  |  | 29 |
|  |  |  |  | 2 |  | 19 | 24 | 10 | 77 | 50 |  | 1 | 52 |
| Wayne |  | 2 |  | 2 |  | 6 | 4 | 13 | 28 | 4 |  |  | 4 |
| Willow Springs.. |  |  | 1 | 1. |  | 7 | 13 | 1 | 23 | 14 |  |  | 14 |
| White Oak Spr.. |  |  |  | 3 |  | 14 | 6 |  | 34 | 7 |  |  | 7 |
| Whita Oak Spr.. |  | 1 |  |  |  | 14 | 10 | 10 | 40 |  |  |  | 1 |
| Total ....... | 3 | 16 |  | 45 | 8 | 146 | 207 | 104 | 5791 | 199 | 2 | 4 | 205 |

Table 1.- Continued.


Table 1.-Continued.


Table 1.- Continued.

| 1850. | Rest of Europe. |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\begin{aligned} & \dot{8} \\ & \underset{y}{\otimes} \\ & \text { \#n } \end{aligned}$ |  |  | 霛 | $\begin{aligned} & \dot{\tilde{0}} \\ & \stackrel{3}{B} \\ & \dot{0} \\ & \dot{Z} \end{aligned}$ | $\left.\begin{aligned} & \dot{\tilde{n}} \\ & \stackrel{\rightharpoonup}{n} \end{aligned} \right\rvert\,$ |  | (1) |  |
| Argyle |  |  | 2. |  |  |  | 77 |  |  | \|.... |
| Belmont |  |  |  |  |  |  |  |  | 3 |  |
| Benton |  |  | 110 |  |  |  | .... | 2 | 131 |  |
| Centre . |  |  | 10. |  |  |  | ... | 3 | 17 |  |
| Elk Grove |  |  | 16 |  |  |  |  | . $\cdot$ | 16 |  |
| Fayette |  |  | 12. |  |  | 6. |  |  | 18 |  |
| Gratiot |  |  | 2 |  |  |  |  | 2 | 12 |  |
| Kendall |  |  |  |  |  | 1 |  |  | 6 |  |
| Monticello |  |  | 7 |  |  |  |  |  | 11 |  |
| New Diggings |  | 5 | 14 |  |  | 2. |  |  | 21 |  |
|  |  |  |  |  |  |  |  |  |  |  |
| Shullsburg |  | 7 | 25 |  |  |  |  |  | $40 \mid$ |  |
| Wayne . |  |  |  |  |  | 18. |  |  | 18. |  |
| Willow Springs .... |  |  | 10. |  |  |  |  | 6 | 16 |  |
| White Oak Springs |  | 1 |  |  |  |  |  |  | 9 |  |
| Wiota ... |  |  |  |  |  |  |  | 1 | 87 |  |
| Total | 1 | 29 | $\overline{216}$ | 1 |  | 208 |  | 27 | 484 | 2 |

Table 2.

| 1860. | New England States. |  |  |  |  |  |  | Middle States. |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{aligned} & \dot{\tilde{0}} \mathbf{~} \\ & \text { 己 } \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ |  |  |  |  | + |  |  |  | 咢 |  |  |
| Argyle <br> Belmont <br> Benton $\qquad$ <br> Centre (Darlington). <br> Elk Grove $\qquad$ <br> Fayette $\qquad$ <br> Gratiot $\qquad$ <br> Kendall $\qquad$ <br> Monticello $\qquad$ <br> New Diggings ........ <br> Shullsburg $\qquad$ <br> Wayre $\qquad$ <br> Willow Springs ..... <br> White Oak Springs. <br> Wiota $\qquad$ <br> Total | $\begin{array}{r}14 \\ 6 \\ 3 \\ 42 \\ 7 \\ 9 \\ 13 \\ 2 \\ 2 \\ 2 \\ \ldots . . \\ 4 \\ 8 \\ 8 \\ \hline \ldots . \\ 2 \\ 4 \\ 4 \\ \hline 116\end{array}$ | $\begin{array}{r}7 \\ 2 \\ 5 \\ 50 \\ 2 \\ 6 \\ \hline 3 \\ 7 \\ \\ \hline 6 \\ \hline 2 \\ 9 \\ 3 \\ 4 \\ 0 \\ \hline 6\end{array}$ | 10 <br> 4 <br> 4 <br> 10 <br> $\ldots \ldots$ <br> 5 <br> $\ldots \ldots$ <br> 2 <br> $\ldots \ldots$ <br> 3 <br> 2 <br> 38 <br> $\ldots . .$. <br> 11 <br> 3 <br> 92 | 9 <br> 1 <br> $\ldots \ldots$ <br> 17 <br> 2 <br> 6 <br> 1 <br> 4 <br> $\ldots$ <br> $\ldots$ <br> 4 <br> 12 <br> 27 <br> 1 <br> 1 <br> 1 <br> 3 <br> 88 |  | 7 <br> 11 <br> 1 <br> 55 <br> 3 <br> 23 <br> 6 <br> 5 <br> 5 <br> 1 <br> 9 <br> 14 <br> 13 <br> 7 <br> $\ldots .$. <br> 5 <br> 160 | 4925131761450342035454979 | 109 | $\ldots .$. <br> 4 <br> 15 <br> 24 <br> 9 <br> 7 <br> 3 <br> 3 <br> 2 <br> 6 <br> 8 <br> 11 <br> 5 <br> 2 <br> $\ldots \ldots$ <br> 99 |  |  | $\begin{array}{r}180 \\ 89 \\ 57 \\ 450 \\ 135 \\ 151 \\ 225 \\ 157 \\ 79 \\ 69 \\ 69 \\ 149 \\ 110 \\ 71 \\ 40 \\ 40 \\ 199 \\ \hline, 161\end{array}$ |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  | 23 |  |  |  |  |
|  |  |  |  |  |  |  |  | 337 |  |  |  |  |
|  |  |  |  |  |  |  |  | 65 |  |  |  |  |
|  |  |  |  |  |  |  |  | 48 |  |  |  |  |
|  |  |  |  |  |  |  |  | 152 |  |  |  |  |
|  |  |  |  |  |  |  |  | 80 |  |  |  |  |
|  |  |  |  |  |  |  |  | 23 |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  | ,211 |  |  |  |  |

Table 2．－Continued．

| 1860. | Southern States． |  |  |  |  |  |  |  |  |  |  | SouthwestrenStates． |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | 范 | 哭 |  |  |  |  | $\begin{aligned} & \text { 䔍 } \\ & \stackrel{0}{1} \end{aligned}$ |  |  |  | ［ |
| Argyle ．．．．．．．．．．．．．． | 1 |  |  | ｜．．．｜ | $\ldots .$ | $\ldots$ |  | $\cdots$ |  |  | ${ }_{4}^{47}$ | 35231 |  |  | 3 |
| Belmont ． | 10 | 6 | 1 | 2 | 1 |  |  |  | 10 |  |  |  |  | ．． | 6 |
| Benton ． | 5 | 3 |  | 2 | 4 | 9 |  |  | 5 |  | 28. |  |  |  | 23 |
| Centre（Darlington） | 54 | 2 |  |  | 2 | 7 |  |  | 3 |  | 71 | 5 |  |  | 5 |
| Elk Grove | 5 |  |  |  | 5 | 9 |  |  | 2 |  | 21 | 9 |  |  | 9 |
| Fayette | 10 |  |  |  |  | 3 |  |  | 15 |  |  | 1 |  |  | I |
| tte |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Gratiot | 5 | 8 |  |  | 2 | 18 |  |  |  |  | 35 | 3 |  |  |  |
| Kendall ． | 11. |  |  |  |  | 6 |  |  | 1 |  | 19 | 4 |  |  | 4 |
| Monticello | 1 |  |  |  | 4 | 3 |  |  | 3 |  | 13 | 3 |  |  | 3 |
|  | 13 |  |  |  |  | 5 |  |  |  |  |  |  |  |  | 18 |
| New Diggings．．．．．．． |  |  |  |  |  |  |  |  |  |  |  | 16 |  |  | 18 |
| Shullsburg ．．．．．．．．． | 22 |  |  |  |  | 15 |  |  |  |  | 60 | 34 |  |  | 34 |
| Wayne ．．． | 4 |  |  |  |  |  |  |  | $5$ |  | $17!$ | 3 |  |  | 3 |
| Willow Springs．．．．．． | 12 |  |  |  |  |  |  | 1 | 2 |  | 161 | 2 |  |  | 2 |
| White Oak Springs | 1 | 3 |  |  | $5$ | $3$ |  |  | $3 \mid .$ |  | 17 | 1 |  |  |  |
| Wiota ． |  |  |  | ．． | 3 | 6 |  |  | 7 | 1 | 301 | 1 |  |  | 1 |
| Total | 164 | 29 | 31 |  |  |  |  | 1 | 63 |  |  | 113 | 3 | 1 | 117 |

Table 2．－Continued．

| 1860. | Northwestern States． |  |  |  |  |  |  | $\begin{aligned} & \dot{Z} \\ & \text { 弟 } \\ & \text { Z } \\ & 0 \\ & 0 \\ & 0 \\ & H \\ & B \end{aligned}$ | Miscellaneous． |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{aligned} & \dot{\oplus} \\ & \stackrel{\ddot{\theta}}{\ddot{Z}} \\ & \dot{\ddot{7}} \end{aligned}$ | $\begin{aligned} & \text { 品 } \\ & \text { 节 } \\ & \text { E. } \end{aligned}$ | $\begin{gathered} \text { ®i } \\ \underset{H}{-1} \end{gathered}$ |  | $\begin{aligned} & \text { ®i } \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & \text { E } \\ & \text { E } \\ & \text { B } \end{aligned}$ | $\stackrel{\circ}{0}$ | 永 |  |  |  |  | W゙す |
| Argyle | 26 | 8 | 1 | 5 |  | 60 | 100 | 378 |  |  |  |  |
| Belmont | 77 | 2 | 3 | 1 |  | 57 | 140 | 269 |  |  |  |  |
| Benton | 50 | 6 | 2 |  |  | 18 | 76 | 890 |  |  |  | 1 |
| Centre（Darlington）．． | 59 | 9 | 5 | 14 |  | 107 | 194 | 576 |  |  | 1 | 1 |
| Elk Grove | 44 | 5 | 3 |  |  | 25 | 77 | 484 |  |  |  |  |
| Fayette | 39 | 17 | 2 | 1 |  | 89 | 148 | 398 |  |  |  |  |
| Gratiot | 59 | 18 | 7 | 6 |  | 57 | 147 | 3791 |  |  |  |  |
| Kendall | 36 | 11 | 2 |  | 1 | 62 | 112 | 432 |  |  |  | 1 |
| Monticello | 39 | 4 |  | 1 |  | 28 | 72 | 157 |  |  |  |  |
| New Diggings ．．．．．．． | 21 | 1 |  | 2 | $1$ | 9 | 34 | 691 | 3 |  |  | 3 |
| Shullsburg | 90 | 18 | 11 | 6 | 1 | 29 | 155 | 966 |  |  |  |  |
| Wayne | 33 | 16 | 1 | 3 |  | 97 | 150 | 240 |  |  |  |  |
| Willow Springs ．．．．．． | 22 | 14 |  | 1 |  | 48 | 85 | 3571 |  |  |  |  |
| White Oak Springs．． | 34 | 8 | 1 | 1 |  | 4 | 48 | 219 |  |  |  |  |
| Wiota | 50 | 20 | 1 | 6 |  | 76 | 153 | 4391 |  |  |  |  |
| Total | 679 | 157 | 39 | 47 | 3 | 766 | 1，691 | 6，875 | ．．．．． |  |  | 6 |

Table 2.- Continued.


Table 2.-Continued.


Table 3.


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Table 3.- Continued.


Table 3.- Continued.


Table 3．－Continued．

| 1870. | British america． |  |  |  |  |  | Great Britain． |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 兑感 |  |  |  | $\begin{aligned} & \stackrel{7}{\square} \\ & \stackrel{y}{\circ} \end{aligned}$ |  | 茄 | 宫 | － |  | 盛 | \＃ّ ¢ － |
| Argyle | $\begin{array}{r} 1 \\ 8 \\ 16 \\ \ldots . \end{array}$ | $\begin{array}{r} 1 \\ \ldots \\ \ldots \end{array}$ | ． | ．．．． | ．．．．． | $1 \mid$ | 78 | 26 |  | ．．．．． | ．．． | ．．．．． | 121 |
| Belmont |  |  |  |  |  | 9 | 100 | 73 | 4 |  |  |  | 256 |
| Benton ．．．．．．．．．．． |  |  |  |  |  | 16 | 319 | 270 | 15 |  |  |  | 604 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Blanchard ．． |  |  |  |  |  |  |  | 42 |  |  |  |  | 49 |
| Darlington Vil．．． |  |  |  | 1 |  | 35 | 138 | 125 | 6 |  |  |  | 273 |
| Darlington town． | 34 |  |  |  |  | 9 | 46 | 134 |  |  |  |  | 188 |
| Elk Grove ．．． | 8 |  |  | 2 |  | 8 | 170 | 129 | 17 |  |  |  | 318 |
|  |  |  |  |  |  |  |  |  |  |  |  |  | 318 |
| Fayette ．． |  | 4. |  |  |  | 4 | －71 | 83 | 1 |  |  |  | 155 |
| Gratiot |  |  |  | 3 |  | 12｜｜ | 85 | 139 | 14 |  |  | 1 | 239 |
| endall |  |  |  |  |  |  |  |  | 3 |  |  |  | 295 |
|  | 8 |  |  |  |  |  |  |  |  |  |  |  |  |
| Monticello ．．．．．．．． | 12. |  |  |  |  | 12\｜ | 51 | 28 | 2 |  |  |  | 82 |
| New Diggings．．．． |  |  |  |  |  | 41 |  |  | 4 |  |  |  | 652 |
| Seymour |  |  |  |  |  | 11 II |  |  | 2 |  |  |  | 133 |
| Shullsburg ．．．．．． |  |  |  |  |  | 23｜｜ |  |  | 3 |  |  |  | 808 |
| yne |  |  |  |  |  | $12{ }^{11}$ |  |  | 2 |  |  |  |  |
| Wayne ．．．．．．． |  |  |  |  |  |  |  |  | 2 |  |  |  | 32 |
| Willow Springs．． | 12 |  |  |  |  | 1211 |  | 155 | 2 |  |  |  | 272 |
| Wiota ．．． |  |  |  |  |  | 911 |  |  | 81 |  |  |  | 163 |
| White Oak Spr． |  |  |  |  |  | 51 |  | 47 |  |  |  |  | 138 |
| Total | 169 | 1 | $1 \mid$ | 7 | 21 | 18011 | ，199＇ |  | 110 | 102 |  |  | 4，778 |

Table 3.- Continued.


Table 4.- Population of Lafayette County.

| Towns. | 1850. |  |  | 1860. |  |  | 1870. |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\left\|\begin{array}{c} \text { Na- } \\ \text { tive } \\ \text { born. } \end{array}\right\|$ | Foreign born. | Total. | $\begin{gathered} \text { Na- } \\ \text { five } \\ \text { born. } \end{gathered}$ | Foreign born. | Total. | $\begin{aligned} & \text { Na- } \\ & \text { tive } \\ & \text { born. } \end{aligned}$ | Foreign born. | Total. |
| Argyle. . | 305 | 116 | 421 | 711 | 884 | 1,098 | 755 | 424 | 1,178 |
| Belmont.. | 278 | 47 | 325 | 566 | 166 | 733 | 891 | 412 | 1,303 |
| Benton. | 870 | 1,353 | 2,223 | 1,088 | 1,004 | 2,092 | 1,052 | 666 | 1,718 |
| Blanchard .. |  |  |  |  |  |  | 271 | 182 | 453 |
| Centre... | 472 | 129 | 601 | 1,473 | 464 | 1,938 |  |  |  |
| $\text { Darlington }\left\{\begin{array}{l} \text { Village. . } \\ \text { Town ... } \end{array}\right.$ |  |  |  |  |  |  | 1,366 833 | 355 210 | 1,721 1,043 |
| Elk Grove. | 361 | 262 | 623 | 740 | 570 | 1,315 | 899 | 478 | 1,377 |
| Fayette. | 644 | 108 | 752 | 781 | 204 | 985 | 913 | 276 | 1,189 |
| Gratiot. | 398 | 121 | 519 | 824 | 182 | 1,006 | 1,349 | 367 | 1,716 |
| Kendall... | 260 | 73 | 333 | 745 | 383 | 1,128 | 758 | 373 | 1,131 |
| Monticello | 127 | 71 | 198 | 327 | 129 | 456 | 364 | 115 | 480 |
| New Diggings. | 751 | 990 | 1,741 | 891 | 800 | 1,692 | 1,112 | 677 | 1,791 |
| Seymour . |  |  |  |  |  |  | 228 | 190 | 418 |
| Shullsburg. | 878 | 795 | 1,673 | 1,418 | 1,073 | 2,492 | 1,751 | 947 | 2,701 |
| Wayne.... | 309 | 29 |  | 617 | 56 | 673 | 930 | 126 | 1,056 |
| Willow Springs....... | 424 | 191 | 615 | 552 | 281 | 836 | 779 | 345 | 1,124 |
| White Oak Springs... | 290 | 163 | 453 | 343 | 176 | 519 | 386 | 154 | 520 |
| Wiota. | 540 | 183 | 723 | 847 | 415 | 1,262 | 1,222 | 474 | 1,696 |
| Total. | 6,907 | 4,627 | 11,534 | 11,926 | 6,290 | 18,225 | 15,859 | 6,772 | 22,635 |

Note.-The above table was compiled from copies of the original returns in the office of the Secretary of State. The total for $18{ }^{2} 0$ is increased by 9 for whom no birthplace is recorded, and the total for 1870 is increased by four from the same source.

Table 5.- Areas, and population per square mile.

| La Fayette County. | Area in Square Miles. |  |  | Population Per Square Mile. |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1850. | 1860. | 1870. | 1850. | 1860. | 1870. |
| Argyle............ | 54 | 54 | 36 | 7 | 30 | 32 |
| Belmont ... | 36 | 36 | 42 | 9 | 20 | 31 |
| Benton................... | 27.4 | 27.4 | 27.4 | 81 | 76 | 62 |
| Darlington | 74 | 74 | 76 | 8 | 26 | 49 |
| Elk Grove.... | 54 | 54 | 36 | 11 | 25 | 38 |
| Fayette | 35 | 35 | 35 | 21 | 29 | 34 |
| Gratiot | 63 | 54 | 54 | 9 | 18 | 31 |
| Kendall...... | 54 | 54 | 48 | 6 | 20 | 23 |
| Monticello........... | 10 | 19 | 19 | 19 | 24 | 24 |
| New Diggings | 26.6 | 26.6 | 23.6 | 65 | 63 | 68 |
| Shullsburg..... | 36 | 36 | 36 | 47 | 69 | 73 |
| Wayne................. | 36 | 36 | 33 | 9 | 18 | 29 |
| White Oak Springs. | 16 | 16 | 16 | 28 | 32 | 33 |
| Willow Springs.. | 48 | 48 | 48 | 12 | 17 | 23 |
| Wiouta. | 60 | 60 | 60 | 12 | 19 | 28 |
| Blanchard . |  |  | 18 |  |  | 25 |
| Seymour |  |  | 36 |  |  | 11 |

Note.- The above table was compiled from the Plat Book for 1895, office of Secretary of State, and from maps in State Historical Library.

## MEMORIAL ADDRESSES.

## EDWARD ORTON.

The many American geologists who since the death of Professor Edward Orton have paid tribute to his memory ${ }^{1}$ lay much stress upon the beauty of his character. Wherever he was known, he was a valued friend, and from all his lovable nature and sterling worth called forth an affection but rarely seen between man and man. In the State of Ohio, where most of his life work was done and throughout which he traveled in pursuit of his investigations, he appears to have been looked upon as a great and good man whom it was an honor and a privilege to welcome and assist. Yet this lovable personality offers a striking instance of a man persecuted because the expansion of his intellect and the widening of his horizon of knowledge brought changes in his religious belief. Sensitive man that he was, these persecutions caused him much mental suffering.

[^28]

Edward On torn
By courtesy of the American Geologist.

So dominant is the note of praise for Edward Orton the man, that one unfamiliar with Edward Orton the geologist and educator might think that it is only as the man that his name will be remembered. Yet surely a man who was chosen President of the American Association for the Advancement of Science, President of the Geological Society of America, President of a large State University, and for many years State Geologist of Ohio, must have some claim to recognition as a scientist and educator.

Edward Orton was born at Deposit, Delaware County, New York, March 9, 1829. His father, who was a Presbyterian clergyman, soon after removed to Ripley, New York, an agricultural community. Prepared for college by his father, at the age of fifteen he entered the sophomore class of Hamilton College, and graduated in 1848. After a year of teaching at Erie, Pa., he entered Lane Theological Seminary (Presbyterian) to prepare for the ministry, but his eyesight failing he gave up study at the end of a year and took a position as clerk on a coasting vessel. In 1851 he was Instructor in Natural Sciences and German in the Delaware Literary Institute of Franklin, New York. The following year he spent at Harvard University in study of chemistry and botany, returning to Franklin for another year of teaching. He then resumed his preparation for the ministry, this time at Andover Theological Seminary. Licensed to preach in 1845 he shortly thereafter was ordained as pastor of the Presbyterian church of Downsville, Delaware County, New York.

Yet with his course of study there had arisen doubts respecting matters of belief which warred against the doctrines instilled in his early life. In 1856 he resigned his pastorate to become professor of Natural Sciences in the New York State Normal School at Albany. Here, though not compelled to do so, he avowed his change of belief from that of the Presbyterian church to essentially Unitarian doctrines. In the fifties, before the days of evolution, the church was controlled by narrow and illiberal ideas, and most educational institutions were dominated by the church. This avowal of Dr. Orton's, therefore, was regarded as so serious a matter as to lose him the position which he held. The succeeding six years were spent in an obscure
academy, but in 1865 he was called to Antioch College, Ohio, by the acting president, who had learned to know and respect him. Dr. Orton felt that here at last was his opportunity, and he said, "The prison doors are at last opened for me." Driven from his own state by danger of persecution he felt that here his search for truth would be untrammeled. While at Antioch he was appointed assistant to Dr. Newbury, then State Geologist of Ohio, in which capacity he won such golden opinions that when, under the Morrill Act of 1862, the new State Agricultural and Mechanical College was founded, he was chosen for its President. The difficulties of this position may be better imagined than described. Ohio, always foremost among the states in the number of its strongly denominational colleges, naturally regarded the new institution as an interloper, but Professor Orton's tact was only equalled by the necessity for it, and notwithstanding the unpromising conditions, he was able to make of the Agricultural and Mechanical College the State University of Ohio as it is today. Prof. I. C. White says of this work:
"The unceasing toil of eight years which Dr. Orton had given to its interests had not only allayed all opposition but built up for it a host of friends in every portion of the state, so establishing it in the hearts of the people that its continued growth and influence have been phenomenal. The Ohio State University is so largely the creation of Dr. Orton's personal efforts that he needs no other monument to perpetuate his name and fame, not only as a great teacher, but also as a consummate organizer, director, and promoter of educational forces. 1

President T. C. Mendenhall, who was associated with Dr. Orton for a period of thirty years, says of him:
"He believed that the character of an educational institution should be judged by the quality of its work rather than by the number of students enrolled in the annual catalogue, a principle which everybody admits and nearly everybody ignores. To stand up for it and do it, especially during the early struggling years of a college, demands a courage that few possess. That Dr. Orton did this, even under the most trying conditions, I set down as, on the whole, the most notable characteristic of his career as president. For I am thoroughly convinced that if he had chosen to do otherwise, if the doors had been opened wide, at both ends of the curriculum, the institution would have long since sunk into a deserved oblivion. ${ }^{2}$

[^29]Professor Orton's scientific work was very largely done in the state of Ohio in the capacity of assistant to Professor Newbury in 1869, as above described, and later as State Geologist, his activity as an officer of the state extending over a term of thirty years. The results are largely contained in the voluminous reports of the Ohio Geological Survey. Devoting himself largely to the problems of stratigraphy of the sedimentary formations and the superincumbent drift mantle, and to the economic resources of the state in coal, gas and oil, there is little that is spectacular or striking in his views, but the every-day problems which he studied were so thoroughly examined that his conclusions stand unchallenged. In his later years perhaps the best known specialist on natural gas in its relations to coal and oil in subterranean reservoirs, he is most widely known from the simple and adequate theory which he propounded to explain them.

Of his geological work, Mr. G. K. Gilbert says:
"As an investigator he generalized freely and did not shrink from the propounding of theories, but all his theories were so broadly founded upon, and so faithfully verified by, the phenomena of observation that they came to the world as demonstrations which could not be gainsaid."

Professor J. J. Stevenson says of him:
"The debt of geology to Edward Orton is very great, far greater than we are apt to think, for, in his writings he effaced himself and often gave credit to others for what was rightfully his own."
"We can lay a double tribute upon his grave, one to the man whom we loved and one to the geologist who solved so many perplexing problems."

Professor Orton died in 1899 after nine years of partial incapacity for work caused by a paralytic stroke. Only a short time wefore his death, however, he delivered an address as president of the American Association for the Advancement of Science. The Wisconsin Academy of Science, Arts, and Letters, in electing him to corresponding membership, chose a man whom all recognize as a worthy representative of American science, and who has been as widely known and loved as it is the privilege of a man of science to be.

## JOHN EUGENE DAVIES.

John Eugene Davies was born at Clarkstown, N. Y., on the 23d of April, 1839. Two years later his parents moved to the city of New York, where he was sent to the public schools until twelve years of age, when he was admitted on examination to the Free Academy, now known as the University of the City of New York. In 1855 he came with his parents to Wisconsin, where he continued his studies as best he could, while teaching in the winters and doing farm work in the summers. He entered the sophomore class of Lawrence University, at Appleton, Wis., in 1859, and graduated from that institution three years later, with honors, by reason of special attainments in pure and applied mathematics.

After receiving his baccalaureate degree, he entered at once, through the solicitation of a physician friend, upon the study of medicine, yielding the more readily as he knew such a course would keep him somewhat in touch with the sciences, for which he was already acquiring a taste. But his studies were soon interrupted. The news of the battle of Pittsburg Landing, and of President Lincoln's call for " 300,000 more," was too much for him to withstand, and he enlisted as a private in the 21st regiment, W. V. I.

His record as a soldier was varied, interesting, and highly creditable. He marched with his regiment, first to Covington, Kentucky, and afterward to Louisville, during General Bragg's approach from Chattanooga. At both places he served in the trenches and performed such other duties as fell to the lot of a private soldier. After General Bragg's retreat, he was, without solicitation, put on detail duty by Major-General Buell in one of the hospitals of Louisville. He was afterwards appointed Sergeant-Major of his regiment, and took part in the battles of Chickamauga and Mission Ridge. He served six months on picket duty on the top of Lookout Mountain, Tennessee; was with his regiment in all its fighting on the march to Atlanta, Georgia, and around that city, and on its backward march to Chattanooga, and he afterwards saw Atlanta burned. He was


JOHN EUGENE DAVIES.
By courtesy of the Wisconsin Alumni Magazine.
recommended for promotion at this time, but his commission as first lieutenant did not reach him until after the battle of Bentonville, and the army had entered Goldsboro, North Carolina. He finally accompanied Sherman on his famous "march to the sea," and returned home with his regiment by way of Richmond and Washington, having served three years without a day's furlough.

His war record was without a blemish. Entering the ranks as a private and from pure love of country, he served, it is true, in comparatively humble positions, but always with the highest courage and fidelity.

As soon as he was mustered out of service, he resumed his studies at the Chicago Medical College, and received the degree of Doctor of Medicine in the spring of 1868. He continued his attendance, however, at clinical lectures in Cook county hospital, and at Mercy hospital, until the end of August, 1868, when he came to Madison to enter upon his duties as professor of natural history and chemistry in the University of Wisconsin, to which chair he had been elected by the Board of Regents at their midsummer meetirg. He had already occupied a professorship in the Chicago Medical College for a year, giving lectures on organic and inorganic chemistry and toxicology. In connection with the regular work of his professorship, Dr. Davies taught the subject of astronomy also in the University, and in 1874 the title of his chair was changed to that of astronomy and physics. In 1878 his chair was made to include physics only, and in 1891 it was changed to that of electricity and magnetism and mathematical physics, which chair he continued to hold at the time of his death.

Dr. Davies was a charter member of the Wisconsin Academy of Sciences, Arts, and Letters, was for many years its general secretary, and was always one of the Academy's most sincere and devoted friends, sharing liberally in its best contributions to the cause of science. He was also an active and efficient co-laborer upon the United States coast survey, contributing many manuscript volumes to its records, and performing special and very valuable service in the triangulation, and on the general geodetic survey of Wisconsin. It was through his inter-
cession that the department of the coast survey prevailed upon the Board of Regents to establish a magnetic observatory upon the grounds of the University, which for some years was used very efficiently under the Professor's general supervision.

He was married in March, 1866, to Miss Anna Burt, of Chicago. One child was born as the fruit of this marriage, but died in infancy. He was married again, March 31st, 1891, to Miss Olive M. Thayer, of Madison, who also bore him one child, a son, of whom he was very fond and who, with the widow, still survives.

Dr. Davies was a devotee of science and a thorough mathematician. He loved especially the natural sciences, and seemed to revel in the very mathematical calculations their investigation involved. Few men have been more fully imbued with the true scientific spirit. He was also a scholarly man, and one who read much and read widely. His contributions not only show careful though upon the subjects they discuss, but they also show a broad range of scientific inquiry. He was a modest man - seemed almost diffident at times-but his modesty was such as usually graces the genuine scholar and investigator. When antagonized upon questions which he had thoroughly studied, and in regard to which he felt reasonably sure of his ground, he was always resolute and self-reliant.

As an instructor of college classes, he was subjected, as most instructors are, to some criticism; but no one, so far as I know, ever made complaint as to his grasp of the subject he was called upon to teach; as to his zeal and enthusiasm in his efforts to impart instruction, or as to his sympathetic nature, and earnest desire to render the best possible service to those under his charge. The criticisms sometimes made were half complimentary. It has been urged that his very familiarity with the subjects he usually had under consideration made it difficult for him to appreciate the limitations of his pupils, and that his eagerness to grapple at once with the most difficult questions connected with the matter in hand, and his intense enthusiasm in their handling, tended to carry his instruction over the heads of the average of his pupils. But it is more than doubtful whether any who entered his classes with suitable preparation


WILLARD HARRIS CHANDLER.
and with an earnest determination to do thorough work, ever failed to profit by his instruction or had any personal ground for complaint.

Dr. Davies was a man of kindly nature, of deep sincerity, and of warm and generous impulses. He was domestic in his life and habits, as gentle as a child, and as true as steel to the demands of honor and the claims of friendship. Coming to the University in his early manhood, he gave his maturer lifehis very best years-to its service. By that service he has won the lasting gratitude of every true friend of the University; and for his priceless qualities of heart and soul, his memory will always be warmly cherished by his neighbors and his colleagues, and especially by those who knew him longest and knew him best.

J. B. Parkinson.

## WILLARD HARRIS CHANDLER.

Willard H. Chandler was born in Brattleboro, Vermont, Nov. 18, 1830. He made the most of the educational advantages offered by his home school, until his fifteenth year, when he entered the office of the Vermont Phoenix, a weekly newspaper, where he remained, with the exception of a short period, until he came to Wisconsin in 1854. He first settled in Delavan, Walworth county. One year later he moved to Windsor, a few miles east of Madison, bought a piece of prairie land, and commenced to work out a farm. This farm he sold in 1868 and moved to Sun Prairie village, where he resided for twenty years. For the greater part of this time he was in the employ of the state, in some capacity or other. His liking for a farm home was so strong, however, that he purchased a fine tract of land near Sun Prairie village, which he improved and on which he resided at the time of his death, March, 1901.

His farm life was prosperous and characterized by the same energy, judgment and earnestness evinced by him in every undertaking.

His career as a servant of the state was most remarkable, especially when it is remembered that his school-days practically closed when he was fifteen years of age. But he was always a student, and his nine years of experience in every phase of work to be found in a newspaper office did much to make amends for the loss of school-room instruction in those early days. He was a thinker, and at all times honest and independent in his thinking. He was perhaps slow in reaching conclusions, but a conclusion once reached became a conviction to be lived up to. It may seem singular that one who had made no special preparation for educational work should have had so large a part in building up a great and connected public school system, and in directing the trend of educational work in a great state. Yet this is the case.

In 1856 he taught in an ungraded school near his home in Windsor; later he was made town superintendent of schools. When this system was abolished by legislative enactment, he was eiected to the office of county superintendent, a position which he held for four years. He was most efficient in the discharge of the duties imposed by these positions, and his work attracted most favorable notice. In 1861 he was elected to represent his district in the Assembly. He at once became a recognized leader on his side of the House. He proposed and was able to secure much helpful legislation along educational lines, during the two years he represented his district. In 1863 he was elected to the State Senate, and was a member of that body for four years. Part of this time he was president pro tem., and his experience and acknowledged ability enabled him to secure very important legislation relating to the organization of our state normal schools, the first of which was opened at Platteville, in 1866. In 1871 he was made a member of the Board of Regents of Normal Schools, a position which he held continuously until 1892. For nine years of this time, from 1881 to 1890, he also held the position of Assistant State Superintendent of Public Instruction, and in this capacity he was instrumental in securing helpful legislation for the common schools, as well as the inauguration of needed reforms. The labor of organizing and outlining the work to be carried on by the teachers' institutes fell
largely under his direction, and through this work he was largely instrumental in bringing the normal school and common school work into close and vital connection. On his resignation of the secretaryship of the Board of Regents in 1892, the following resolution was unanimously adopted by the board:
"Whereas, W H. Chandler, for many years a member and secretary of this board, has just retired therefrom, and tenders his resignation of the secretaryship,
"Resolved, That in accepting such resignation, it is the sense of the board that as such member and secretary the service of Mr. Chandler to the normal schools of this state, in their establishment, extension and maintenance, and in the improvement and supervision of the instruction given, and in his efforts which have materially contributed to make these schools efficient and prosperous to a degree which will bear favorable comparison with like schools elsewhere, as well as in moulding and directing the institute work of the state for many years, has been of inestimable value to this board and to the public school system of Wisconsin, and that such services merit and should receive the cordial recognition and grateful acknowledgement of the board and the friends of public education throughout the state."

As a speaker, Mr. Chandler was clear and forceful. His familiarity with the details of school management and instruction, his clear knowledge of the body of the law relating to schools, as well as his earnest and sympathetic nature, enabled him to secure and hold the confidence and esteem of all with whom he came in contact in the educational field. He possessed a judicial temperament, and though perhaps not brilliant in the general acceptance of the term, he was always dignified, logical, and fair in the exposition of any topic which he discussed, and never failed to secure attention. He was also a man of genial qualities and large benevolence. He was always ready to aid young men and women in securing education. It is safe to say that no worthy person with a meritorious purpose in view ever applied to Mr. Chandler for aid, without securing it.

He became a member of the Wisconsin Academy in 1872, and continued his membership until the time of his death. During the four years that he was inspector of high schools under the administration of State Superintendent Emery, it was my fortune and privilege to occupy a desk at his side, and, though he had ceased to take an active part in the proceedings
of the Academy, he was fond of discussing many of the topics presented at its annual meetings.

In closing his public labors, in January, 1899, he stated his intention of never again taking upon himself the burdens imposed by public office. He looked forward with pleasure to a season of rest and travel. The only journeys of any moment which proved possible for him to take, was one to the Pacific coast, where he spent a most delightful two months, and one to his old home in Vermont.

His three score years and ten were crowded full of high ideals and an unselfish purpose. His earnestness and ability have left an impress on this state that time will not efface and such as has been left by few other persons. The work which it was his fortune to do was done with courage and fidelity with all his heart, and with the mind and hand of a master.

Charles L. Harper.

## TRUMAN HENRY SAFFORD.

Truman Henry Safford ${ }^{1}$ was born January 6, 1836, at Royalton, Vermont. His parents belonged to the great middle class to which we are indebted for so many of our teachers and professional men. Even in his infancy he showed signs of that marvelous power to deal with numbers mentally, which he possessed to an unusual degree throughout his entire life. When he was six years of age he said to his mother that if he knew

[^30]the number of rods around his father's large meadow he could tell the measure in barley corns, and when he was told that it was 1,040 , he computed the result, 617,760 , mentally in a few minutes. Before he was ten years old he had computed a table of logarithms of numbers from 1 to 60 from the formula given in Hutton's mathematics, and had constructed an almanac which was published. Before he was eleven he had constructed four more almanacs. It is related that on one occasion when he was about ten years old he performed the astonishing feat of finding the square of $365,365,365,365,365$, giving the correct result, a number with thirty-six figures, in about one minute. This power of reckoning with large numbers and of discerning the divisors of large numbers he possessed to some extent throughout his life, but it was not nearly so marked in his later years.

He entered Harvard University at an early age and was graduated in 1854 at the age of 18, after having enjoyed the instruction of Benjamin Peirce, one of the foremost American mathematicians. After his graduation he remained for some years at Harvard as observer in the Harvard College Observatory under Professor Bond. In 1865 he came to Chicago as director of the old Dearborn Observatory, which position he held until the great fire of 1871 by which the observatory was destroyed. The people of the great city by the lake were too busy repairing the damage wrought by the fire to feel the need of a ? new observatory so the young astronomer had to seek employment in other fields. He found it in Wheeler's astronomical survey in the far west, and through this work became connected with various scientific bureaus at Washington. In 1876 he was made professor of Astronomy at Williams College, in which work he continued till his death June 12, 1901.

Professor Safford's real scientific work began in 1866 at Chicago, when he undertook the observation of one of the zones of the Astronomische Gesellschaft. This work was cut short, however, by the great fire of 1871. During the years between the great fire and his call to Williams College he seems to have been engaged principally in routine work, of which computation formed the greater part. While at Williamstown he took up the work of discussing the stars most suitable for the deter-
mination of geographical latitudes in the United States, and as a result of this labor constructed a catalogue of 2,018 stars, which was published by the Engineers' Department of the United States Army. This work was extended by a similar catalogue of 612 stars, which was published in 1898 as a part of the Mexican Boundary Commission's report. One of the latest papers published by him is a short paper entitled "Combinations of Pythagorean triangles as giving exercises in computation," which appeared in Vol. XII of the "Transactions" of the Wisconsin Academy. Perhaps his most striking work was the prediction in 1861 of the existence of the minute companion to Sirius, based on very small irregularities in the existing observation. The companion was found by Alvan Clark in the place indicated in 1862.

Professor Safford's life work was doubtless much influenced by the fact that when he went to Harvard the Harvard astronomers saw in him the making of a great computer, and his training was carried on with that side of astronomy always in view. He was a good astronomer and his work is of much value, but it is always the work of the patient observer and computer, and contains little of theoretical interest. Owing perhaps to the bent given him at Harvard, his work in pure mathematics is inconsiderable, although he undoubtedly possessed exceptional ability for such work. Indeed, from his writings and especially from his monograph on mathematical teaching one is led to believe that he looked upon mathematics rather as part of his physics than as a branch of knowledge to be cultivated for its own sake. He says, "The modern, and to my mind the true, theory is that mathematics is the abstract form of the natural sciences; and that it is valuable as a training of the reasoning powers not because it is abstract, but because it is the representation of actual things." For this reason he was not wholly in sympathy with "the new mathematical school-chiefly in certain branches of abstract higher algebra-recently established at the Johns Hopkins University."

As a teacher Professor Safford did not come into contact with a large number of students, but those who were instructed by him are unanimous in their testimony as to the value and the
inspiration of his work. In the article already referred to, Professor Harold Jacoby says of him, "Great as were his abilities as an astronomer, he was yet at his best as a teacher." The influence he exercised over a few bright students who resorted to Williamstown to profit by his instruction was undoubtedly great. On the other hand it has been said of him that his instruction was not adapted to the student not well advanced in his work. He himself probably realized this weakness as thoroughly as any one, for in the monograph on mathematical teaching, already mentioned, he insists strongly that we fail to recognize the difficulties that beset a beginner.

His range of information was very broad, not only along all lines of physical and natural science but in literature and in music as well. Says Professor Charles Forster Smith, "If, as a professor in Williams College, I became interested along any line of work whatever, I was sure of a cordial and sympathetic interest on the part of Professor Safford."

His home life is said to have been singularly happy. Careless and inattentive to the petty details of practical every day life, he gave these things over to the charge of his devoted wife, who took delight in relieving him from care and making it possible to give himself wholly to his books and his telescope. Professor Jacoby says, "He was a man of simple and genuine piety for whom the conflict of science and religion had no terrors. He knew that no such conflict exists." As a teacher, as an investigator and as a man he did honor to the college to which he gave the best years of his life.

## Ernest B. Skinner.

## LIST OF PUBLICATIONS OF THE LATE PROFESSOR SAFFORD.

The following list of Professor Safford's writings has been prepared through the kindness of Rev. Charles H. Burr, Librarian of Williams College. It is believed to be substantially complete:
Safford, Treman Henry. Catalogue of declinations of 532 stars near the zenith of the observatory of Harvard College. (From American Academy, Memoirs. N. S. Vol. 8). Camb. 1861. folio. Welch, Bigelow \& Co.

- Catalogue of the mean declination of 2,018 stars between $0^{\mathrm{h}}$ to $2^{\mathrm{h}}$ and $12^{\mathrm{h}}$ to $24^{\mathrm{h}}$ right ascension and $10^{\circ}$ and $70^{\circ}$ of north declination for Jan. 1, 1875. Wash. 1879. folio. Govt.
Catalogue of the mean declinations of 981 stars between twelve hours and twenty-six hours of right ascension and thirty degrees and sixty degrees of north declination for Jan. 1, 1875. Wash. 1873. folio. Govt.
A catalogue of standard polar and clock stars for the reduction of observations in right ascension. (Harvard College Observatory, Annals, vol. 4). Camb. 1863. folio. Welch, Bigelow \& Co.
Combinations of Pythagorean triangles as giving exercise in computation. (Wisconsin Academy of Sci., Arts, and Letters, Transactions, Vol. 12, pp. 505-8.) Madison. 1889. $8^{\circ}$.
- Comparison of Groombridge's and Struve's right ascensions of close circumpolar stars. (Royal Astron. Soc., Monthly Notices, Vol. 46, No. 2.) L. $1885.8^{\circ}$.
- The development of astronomy in the United States. A discourse read June 25, 1888, to commemorate the fiftieth anniversary of the dedication of the Hopkins Observatory of Williams College. Boston. 1888: $8^{\circ}$. T. R. Marvin \& Son.
- Mathematical teaching and its modern methods. Boston. 1887. $12^{\circ}$. D. C. Heath \& Co.

Mean right ascensions of 133 stars near the north pole, observed in 1882 and 1883 at the Field Memorial Observatory of Williams College. (Am. Academy of Arts and Sci., Proceedings, 1884.) $8^{\circ}$.

- Nebulae found at the Dearborn Observatory 1866-8. (In Chicago Astron. Soc., Annual reports 1885-6.) Chi. 1887. $8^{\circ}$.
- Notes upon certain doubtful star places. (Royal Astron. Soc., Monthly Notices, Vol. 43, No. 5.) L. 1883. $8^{\circ}$.
- Editor. Observations upon the great nebula of Orion by G. P. Bond. (Harvard College Observatory, Annals, Vol. 5.) Camb. 1867. folio. Riverside Press.
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The psychology of the personal equation. (Science, N. S., Vol. 6, pp. 784-88.) $8^{\circ}$.

Williams College catalogue of north polar stars, right ascension for 1885.0. Williamstown. 1888. $8^{\circ}$. College.
Williamstown (Mass.) Field Memorial Observatory. (Vierteljahrsschrift der Astron. Gesellschaft, Band 19, Heft 2.) $8^{\circ}$.
On Pistor and Martins's prismatic reflecting circle. (American Journal of Astronomy, Vol. 21, No. 7.) Cambridge. 1901. $4^{\circ}$.

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A. B., A. M. (DePauw); Ph. D. (Johns Hopkins). Professor of Latin, University of Wisconsin.
Slichter, Charles Sumner,
636 Francis St., Madison.
B. S., M. S. (Northwestern). Professor of Applied Mathematics, University of Wisconsin.
Smith, Erastus Gilbert, 649 Harrison Av., Beloit.
A. B., A. M. (Amherst) ; A. M., Ph. D. (Göttingen). Professor of Chemistry, Beloit College.
Smith, Howard Leslie, 222 Langdon St., Madison.
A. B., LL. B. (Wisconsin). Professor of Law, University of Wisconsin.
Smitif, Leonard Sewell, $\quad 939$ University Ave., Madison.
B. c. e., C. e. (Wisconsin). Assistant Professor of Topographic and Geo-
detic Engineering, University of Wisconsin.

Smith, Walter McMynn, 218 Park St., Madison.
A. B. (Wisconsin). Librarian, University of Wisconsin.
Snow, Benjamin Warner, 518 Wisconsin Ave., Madison. Ph. D. (Berlin). Professor of Physics, University of Wisconsin.

Sparling, Samuel Edward, 505 N. Carroll St., Madison.
A. B. (Indiana); Ph. D. (Wisconsin). Assistant Professor of Political Science, University of Wisconsin.
Starr, William J., 135 Marston Av., Eau Claire.
Ll. b. (Columbia). Member, Board of Commissioners of Fisheries, Wis-
consin; President of the Eau Claire Public Library.

[^33]| Weidman, Samuel, B. S., Ph. D. (Wiscons | $\begin{array}{r} 22! \\ \text { Geologist, Wis } \end{array}$ |
| :---: | :---: |
| Whitcomb, Annabell Cook (Mrs. Henry F.), |  |
|  | 721 Franklin |
| Whitson, Andrew <br> B. s. (Chicago) Profe |  |

# Wingate, Uranus O. B., 204 Biddle St., Milwaukee. <br> M. D. (Dartmouth). Professor of Diseases of the Mind and Nervous System, Wisconsin College of Physicians and Surgeons; Secretary of State Board of Health. 

Winkenwerder, Hugo August, 217 Murray St., Madison. Assistant in Biology, University of Wisconsin.
Wolcott, Edson Ray, 202 Langdon St., Madison.
B. S. (Wisconsin). Fellow in Physics, University of Wisconsin.
Wolff, Henry C., 225 State St., Madison. B. S., M. S. (Wisconsin). Instructor in Mathematies, University of Wis- consin.
Wous, Fritz Wilhelm, 424 Charter St., Madison.
B. S., Ph. B. (Christiania); M. S. (Wisconsin). Assistant Professor of Agricultural Chemistry and Chemist to the Agricultural Experiment Station, University of Wisconsin.
Zimmermann, Charles Frederick A., 622 Otjen St., Milwaukee.Ph. B. (Illinois Wesleyan); A. M. (Charles City). Principal, SeventeenthDistrict School.
Zimmermann, Oliver Bruner, 209 Brooks St., Madison.
B. M. E., M. E. (Wisconsin). Instructor in Machine Design and De- seriptive Geometry, University of Wisconsin.
CORRESPONDING MEMBERS.
Аввотт, Charles Conrad, Trenton, N. J.
M. D. (Pennsylvania).
Andrews, Edmund, 100 State St., Chicago, Ill.
A. B., A. M., M. D., LL. D. (Michigan). Professor of Clinical Surgery, Northwestern University; Surgeon, Mercy Hospital; Consulting Sur- geon, Michael Reese Hospital and Illinois Hospital for Women and Children.
Armsby, Henry Prentiss, State College, Pa.B. S. (Worcester Polytechnic); Ph. B., Ph. D. (Yale). Director of Experi-ment Station.
A. B., A. M. (Williams) ; D. D. (Iowa); LL. D. (Amherst, Williams). Professor of Political Science, Williams College.
Bennett, Charles Edwin, 1 Grove Place, Ithaca, N. Y. A. B. (Brown). Professor of Latin Language and Literature, Cornell University.
Bridge, Norman,
217 S. Broadway, Los Angeles, Calif.; Oct. and Nov. each year, Rush Medical College, Chicago, Ill.
A. M. (Lake Forest); M. D. (Northwestern, Rush). Emeritus Professor of Medicine, Rush Medical College.
Caverno, Charles,
Lombard, Ill.
A. B., A. M. (Dartmouth) ; LL. D. (Colorado). Clergyman, retired.

Coulter, John Merle, University of Chicago, Chicago, Ill. A. B., A. M., Ph. D. (Hanover); Ph. D. (Indiana). Head Professor of Botany, University of Chicago.
Crooker, Joseph Henry, 110 N. State St., Ann Arbor, Mich. D. D. (St. Lawrence, Nashville). Minister. Unitarian Church.

Davis, Floyd, 317 Iowa Loan and Trust Bldg., Des Moines, Ia.
Ph. B., C. E., E. M. (Missouri); Ph. D. (Miami). Analytical and Consulting Chemist.

Eckels, William Alexander,
Oxford, Ohio.
A. B., A. M. (Dickinson); Ph. D. (Johns Hopkins). Professor of Greek, Miami University.

Fallows, Samuel, 967 W. Monroe St., Chicago, Ill.
A. B., A. M., LL. D. (Wisconsin); D. D. (Lawrence). Presiding Bishop of the Reformed Episcopal Church; Chancellor of the University Association; President of Board of Managers, Illinois State Reformatory.

Hendrickson, George Lincoln, 5515 Woodlawn Av., Chicago. A. B. (Johns Hopkins). Professor of Latin, University of Chicago.

Higley, William Kerr, Lincoln Park, Chicago, Ill.
Ph. M. (Michigan). Secretary, Chicago Academy of Sciences; Editor, Birds and Nature.

Hodge, Clifton Fremont, 3 Charlotte St., Worcester, Mass.
A. B. (Ripon); Ph. D. (Johns Hopkins). Assistant Professor of Physiology and Neurology, Clark University.

Holden, Edward Singleton,
U. S. Military Academy, West Point, N. Y.
B. S., A. M. (Washington) ; S. D. (Pacific); LL. D. (Wisconsin and Columbia). Astronomer.

Holland, Frederic May, Main St., Concord, Mass. A. B. (Harvard).

Hoskins, Leander Miller, Palo Alto, Calif. B. S., M. S., B. C. E., C. E. (Wisconsin). Professor of Applied Mathematics, Leland Stanford Jr. University.
Iddings, Joseph Paxson, 5730 Woodlawn Av., Chicago, $11 l$. Ph. B. (Yale). Professor of Petrology, University of Chicago.
Kinley, David,
Urbana, Ill.
A. B. (Yale) ; Ph. D. (Wisconsin). Dean of the College of Literature and Arts, and Professor of Economics, University of Illinois.
Leterett, Frank,
Ann Arbor, Mich.
B. Sc. (Iowa Agricultural). Geologist, U. S. Geological Survey.

Lurton, Freeman Ellsworth,
Spring Valley, Minn. B. S., M. S. (Carleton); Ph. D. (Gale). Superintendent of Public Schools.

Luther, George Elmer,
266 S. College Av., Grand Rapids, Mich.
Chief Mortgage Clerk, Michigan Trust Co.; Treasurer of the Historical Society of Grand Rapids.
Marx, Charles David,
Palo Alto, Calif.
B. C. E. (Cornell); C. E. (Carlsruhe). Professor of Civil Engineering, Leland Stanford Jr. University.
McClumpha, Charles Flint,
Minneapolis, Minn.
A. B., A. M. (Princeton) ; Ph. D. (Leipzig). Professor of English Language and Literature, University of Minnesota.
Moorehouse, George Wilton, 39 Cutler St., Cleveland, O.
B. L., M. L. (Wisconsin) ; M. D. (Harvard). Physician to the Dispensary of Lakeside Hospital and Western Reserve University.
Peet, Stephen Denison, 5817 Madison Av., Chicago. A. M., Ph. D. (Beloit). Clergyman; Editor, American Antiquarian.

Ротter, William Bleecker, 1225 Spruce St., St. Louis, Mo. A. B., A. M., M. E. (Columbia). Mining Engineer and Metallurgist.

Power, Frederick Belding, 535 Warren St., Hudson, N. Y.
Ph. G. (Phila. Coll. of Pharm.) ; Ph. D. (Strassburg). Director of Wellcome Chemical Research Laboratories, London, Eng.
Raymond, Jerome Hall,
Chicago, Ill .
A. B., A. M. (Northwestern) ; Ph. D. (Chicago). Associate Professor of Sociology, University of Chicago.
Salisbury, Rollin D., University of Chicago, Chicago, Ill.
A. M. (Beloit). Profesosr of Geographic Geology and Dean of the Graduate School of Science, University of Chicago; Geologist, U. S. Geological Survey, State Geological Survey, New Jersey.

Sawyer, Wesley Caleb, Elm and Asbury Sts., San Jose, Calif. A. B., A. M. (Harvard); A. M., Ph. D. (G̈̈ttingen).
$\underset{\text { Shipman, Stephen Vaughn, }}{\substack{\text { Colonel, } \\ \text { Regiment, U. s. } \\ 269 \text { Wolunteers, Civil War; Architect. }}}$
Stone, Ormond, University Station, Charlottesville, Va.
m. A. (Chicago). Director, Leander McCormick Observatory and Professor of Practical Astronomy, University of Virginia.
Tatlock, John, Jr., $\quad 32$ Nassau St., New York, N. Y.
A. B., A. M. (Williams); F. R. A. S. Assistant Actuary, Mutual Life Insurance Co.
Tolman, Albert Harris, $\quad 5750$ Woodlawn Av., Chicago, Ill.
A. B. (williams); Ph. D. (Strassburg). Assistant Professor of English Literature, University of Chicago.

Tolman, Herbert Cushing,
Nashville, Tenn.
A. B., Ph. D. (Yale); D. D. (Nashville). Professor of Greek, Vanderbilt University.
Townley, Sidney Dean, 2023 Bancroft Way, Berkeley, Cal. B. S., M. S. (Wisconsin) ; S. D. (Michigan). Instructor in Practical Astronomy, University of California.
Trelease, William, Botanical Garden, St. Louis, Mo.
B. S. (Cornell); S. D. (Harvard); LL. D. (Wisconsin). Director of Missouri Botanical Garden and Henry Shaw School of Botany, Engelmann Professor of Potany, Washington University; Secretary, Academy of Science of St. Louis; Secretary, The Round Table, St. Louis; Honorary President, Engelmann Botanical Club, St. Louis.

Van de Warker, Ely, 404 Fayette Park, Syracuse, N. Y. M. D. (Albany Medical and Union). Surgeon Central New York Hospital for Women; Consulting Physician St. Ann's Maternity Hospital; Senior Surgeon Women's and Children's Hospital; Commissioner of Education, Syracuse; Member of the Holland Society.
Van Vleck, Edward Burr, Middletown, Ct.
A. B., A. M. (Wesleyan); Ph. D. (Göttingen). Professor of Mathematics, Wesleyan University.

Verrill, Addison Emory, 86 Whalley Av., New Haven, Ct. S. B. (Harrard); A. M. (Yale). Professor of Zoölogy, Yale University.

Winchell, Newton Horace,
113 State St., Minneapolis, Minn. A. M. (Michigan).

Young, Albert Adams, $\quad 531$ S. Claremont Av., Chicago, Ill.
A. B., A. M. (Dartmouth); D. в. (Andover). Clergyman.

# MEMBERS DECEASED. 

## INFORMATION OF WHOSE DECEASE HAS BEEN RECEIVED SINCE THE ISSUE OF VOLUME XII.

Orton, Edward, A. M., Ph. D., LL. D., Columbus, Ohio. Professor of Geology, Ohio State University, State Geologist of Ohio, 1899.

Davies, John Eugene, A. M., M. D., LL. D., Madison, Wis. Professor of Electricity and Magnetism and Mathematical Physics, University of Wisconsin; Jan. 22, 1900
Chandler, Willard Harris, Madison, Wis.
State Inspector of High Schools; Mareh 24, 1901.
Safford, Truman Henry, Ph. D., Williamstown, Mass. Field Memorial Professor of Astronomy, Williams College; June 12, 1901.
Bacon, Charles Alfred, A. B., A. M., Beloit, Wis.
Professor of Astronomy, Beloit College, and Director of Smith Observa- tory; Sept., 1901.
Steele, George McKendrae, D. D., LL. D., Chicago, Ill.President of Lawrence University, 1885-1879; Principal Wesleyan Academy,Wilbraham, Mass., 1879-1893; Jan., 1902.

Conrath, Adam, Ph. C., Pharmacist; Jan., 1901.
Goodiue, William Fletcher,
Civil Engineer; 1901.

## PROCEEDINGS.

## REPORT OF THE SECRETARY, 1900.

## Thirty-first Annual Meeting.

Milwaukee, Wis., Dec. 27-28, 1900.
All meetings of the Academy were held in Club Room No. 486 of the Plankinton House. The several sessions were carried out in accordance with the printed program, with the exception of some few changes in the order in which the papers were read. This program was as follows:
thursday morning, december 27.
Reports of officers and other general business.
Address in memory of the late Professor John E. Davies, by Professor J. B. Parkinson.

Reading of papers at $10: 15$ o'clock.

1. An example of a theoretical system of weight factors of ready application in the solution of observation equations. Albert S . Flint.
2. On an improved method of determining latent heat of vaporization. Louis Kahlenberg.
3. The plankton of Green Lake and Lake Winnebago. C. Dwight Marsh.
4. Some recent observations on the migration of birds. H. A. Winkenwerder.

THURSDAY AFTERNOON.
5. Determinism, decrees, and immutable law. Charles Caverno.
6. Personal names, their etymology. James D. Butler.
7. A campaign cry of 1844. H. J. Desmond.
8. Shakespeare's knowledge of criminal psychology. $\boldsymbol{F}$. C. Sharp.
9. Early political platforms in Wisconsin. John G. Gregory.

General business.
Reading of papers at 10:00 o'clock.
10. The cause of cleavage in rocks. C. K. Leith.
11. The supposed lessening of geyser activity in the Yellowstone National Park. D. P. Nicholson.
12. Harmonic curves of three frequencies. Second paper; with exhibition of stereograms drawn by E. A. Hook. Charles S. Slichter.
13. On repeating decimals. E. A. Hook.
14. On the thermal conductivity of the common woods. L. W. Austin and C. W. Eastman. (By title.)
15. The expansion of wood due to absorption of water. L. W. Austin, G. S. Cassels and W. H. Barber. (By title.)

## Friday Afternoon.

16. The orientation of stream channels as related to geological structure. William H. Hobbs.
17. The old tungsten mine at Trumbull, Ct. William H. Hobbs.
18. The future of the clay and cement industry in Wisconsin. Ernest R. Buckley.

At the various sessions matters of business were transacted as follows:

Thursday, Dec. 27.
Morning Session.
The meeting was called to order at $9: 30$ o'clock by President Slichter. On motion, the reading of the minutes of the 30 th annual meeting was dispensed with, on account of their publication during the intervening year in Vol. XII. of the "Transactions."

An informal report of the secretary was read. On account of the absence of the treasurer and librarian the reading of their reports was postponed till the meeting of Friday.

No further business was transacted during the day.

$$
\text { Friday, Dec. 28, } 1900 .
$$

The morning session was called to order by President Slichter. The report of the treasurer was read and accepted. The auditing committee, consisting of Messrs. Bruncken, Flint and

Leavenworth, previously appointed by the president, reported, approving the accounts of the treasurer.

The treasurer's report showed that the treasury contains $\$ 1,513.87$, of which $\$ 1,000.00$ are invested in a debenture bond, leaving the sum of $\$ 513.87$ uninvested.

It was voted that the president and treasurer be hereby instructed to invest $\$ 500.00$ of the above sum in interest bearing bonds, the said $\$ 500.00$ to be added to the permanent fund of the Academy.

The committee on membership then made its report. On its recommendation the following names were added to the list of active members of the Academy:
B. W. Snow, Madison.
J. B. Parkinson, Madison.
J. C. Monaghan, Madison.

Howard L. Smith, Madison.
R. W. Wood, Madison.
E. K. J. H. Voss, Madison.
F. W. Meisnest, Madison.

Otto E. Lessing, Madison.
Walter H. Smith, Madison.
A. A. Meggett, Madison.
W. G. Bleyer, Madison.
H. G. Timberlake, Madison.
H. A. Winkenwerder, Madison.
E. R. Wolcott, Madison.

Bernard M. Palmer, Madison.
E. A. Hook, Madison.
S. T. Smythe, Delafield.
R. H. Halsey, Oshkosh.
J. E. Lough, Oshkosh.
W. S. Leavenworth, Ripon.
W. H. Neilson, Milwaukee.
G. A. Chamberlain, Milwaukee.
C. E. Monroe, Milwaukee.
J. A. Merrill, West Superior.
W. S. Watson, Whitewater.
L. J. Freese, Waukesha.

Chancey Juday, Madison.

Lewis Sherman, Milwaukee.
R. W. Pringle, Appleton.
A. O. Greason, Appleton.
V. A. Suydam, Ripon.

Miss Mary Armstrong, Portage.
Miss Gertrude Anthony, Madison.
Miss F. Belle Stanton, Warren, Ill.
The committee further recommended that former president, C. Dwight Marsh, and former secretary, Albert S. Flint, be elected to life membership in the Academy in view of their great services to the Academy during their term of office. This recommendation was unanimously adopted.

The report of the librarian was then read. As this report contained the statement that the present librarian deemed it his duty to resign in order that the work could be put into the hands of some one better versed in library matters and the library properly cared for, the report was referred to the Council of the Academy with the request that they formulate some motion upon the subject in time to be acted upon at the afternoon session.

The afternoon session was called to order at $2: 45$ by the president. The following recommendations presented by the Council were severally adopted:
Resolved, That the Librarian, Dr. Kahlenberg, be authorized to delegate to some person selected by himself any of the powers now belonging to his office.

Resolved, That the books of the Academy when catalogued shall be given a label of distinctive shape.

Resolved, That a selection of the most important transactions shall be kept in the room of the Academy at Madison.

The Council further recommend for passage at the next annual meeting the following amendment to the constitution:
Article VI. Section 2 is hereby amended to read as follows: The library committee shall consist of five members, of which the librarian shall be ex-offcio chairman and of which a majority shall not be from the same city.

It was voted that this amendment be allowed to take its regular course. F. C. Sharp, Secretary.

REPORT OF THE SECRETARY, 1901.

## Thirty-second Annual meeting.

Milwaukee, Wis., Dec. 26-27, 1901.
The meetings of the Academy were held in Room B 5 of the Milwaukee Normal School Building. The various meetings were carried out in accordance with the printed program with a few changes in the order of papers presented and the addition of four papers, the titles of which reached the secretary too late for printing. The program was as follows:
thursday morning, dec. 26.
Reports of officers and other general business.
Reading of papers at 10:30 o'clock.

1. The origin of Wisconsin place-names. Henry E. Legler.
2. The economic and social development of Kenosha county. Robert H. Downes.
3. The economic and social development of LaFayette county. Katherine P. Regan.
4. A study in longevity. Charles H. Chandler.
5. Language forms known as negative. Edward T. Owen.
6. The problem of interrogation. Edward T. Owen.

THURSDAY AFTERNOON.
7. The axial bifurcation of snakes. Roswell H. Johnson.
8. On a new species of Cantho-camptus from Idaho. C. Dwight Marsh.
9. Further notes on the plankton of Green Lake and Lake Winnebago. C. Dwight Marsh.
10. The marching of the young caterpillars of the maia moth, Hemileuca maia. W. S. Marshall.
11. The oak pruner, Elaphidion villosum. W. S. Marshall.
12. The early prophases of the nuclear division in the pollen of the mother cells of Larix and Lilium. C. E. Allen.
13. The development and structure of the swarm-spores of Hydrodiction. H. G. Timberlake.
14. Cell structure and reproduction in Hymenomycetes. $R$. A. Harper.

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15. Some fungi parasitic on mushrooms. R. A. Harper.
16. Third supplementary list of parasitic fungi of Wisconsin. J. J. Davis.
17. Quinhydrones as plant pigments. I. W. Brandel and Edward Kremers.
18. A case of alum poisoning. W. W. Daniells.
19. The records left upon the soil of Wisconsin. Stephen D. Peet.

FRIDAY MORNING.
General business.
Reading of papers at 9:15 o'clock.
20. Migration velocities of the ions in solutions of silver nitrate in pyridine. Herman Schlundt.
21. Nitriles as solvents in molecular weight determinations. Louis Kahlenberg.
22. Instantaneous chemical reactions and the theory of electrolytic dissociation. Louis Kahlenberg.
23. A method of analyzing the inorganic acids. W. S. Leavenworth.
24. The action of selenic acid on gold. Victor Lenher.
25. Natural telluride of gold. Victor Lenher.
26. The action of tellurium and selenium on gold and silver salts. Roy D. Hall.
27. Note on the use of the terms "solvent" and "flux" for higher temperatures. A. J. Rogers.
28. Some complex nitro-compounds. H. W. Hillyer.

Memorial addresses:
Edward Orton, - - $\quad$ Wm. H. Hobbs.
Willard H. Chandler, - - C. L. Harper.
Truman H. Safford, - - Ernest B. Skinner.
Charles A. Bacon, - - E. G. Smith.
friday afternoon.
29. Evidence of the former extension of the Newark formation on the Atlantic slope. Wm. H. Hobbs.
30. Boulder trains in the Pomperaug valley, Connecticut. Wm. H. Hobbs.
31. The erosion history of southwest Wisconsin. Ellwood C. Perisho.
32. The physiographic features of central Wisconsin. Samuel Weidman.
33. The manner in which igneous rocks make their way to the surface. C. R. Van Hise.
34. Subterranean rivers. Charles S. Slichter.
35. The status of geological survey work in Missouri. Ernest R. Buckley.

Of the above papers Nos. 2, 3, 7, 8, 16, 21, 28, were read by title; No. 17 was presented by Professor Kremers, No. 22 by Dr. Schlundt, No. 24 by Professor Kremers, and No. 25 by Professor Van Hise.

Matters of business were transacted at the various sessions as follows:

## Thursday Morning, Dec. 26.

The Academy was called to order by President Slichter at 10:40 a. m. Owing to the fact that Mr. Legler was obliged to leave early in the forenoon, general business was postponed until after the reading of his paper. Mr. Legler's paper was discussed by Professor Chandler, Dr. Davis and others.

The report of the secretary was read. This report, besides giving the minutes of the meeting for 1900 , stated that since the last meeting Part I of Vol. XIII of the "Transactions" of the Academy had been issued and distributed, and Part II is ready for the press.

The minutes were approved and the report accepted.
The report of the librarian was read by the secretary, the librarian being absent. The report which appears in full in another place, was accepted and placed on file.

The secretary read a letter from the Société Nationale des Sciences Naturelles et Mathématiques de Cherbourg, announcing the celebration of the fiftieth anniversary of the founding of the Cherbourg Society. By vote of the Academy the secretary was directed to enter upon the record a minute indicating that greetings and congratulations had been sent to the Society at Cherbourg.

A letter addressed to President Slichter by the secretary of the University Club of Milwaukee, extending the privileges of the Club to visiting members of the Academy during the meeting, was read by the secretary. The secretary was directed to reply to the invitation and to thank the Club for its hospitality.

The reading of papers was then resumed. Paper No. 4 was discussed by Professor Slichter. Papers 5 and 6 were discussed by Messrs. Van Hise, Slichter and others. Seventeen persons were in attendance at the morning session.

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## Thursday Afternoon.

The meeting was called to order at $2: 40$ by President Slichter. The reading of papers was taken up immediately and the transaction of business was postponed till later in the day. Papers 7 to 19 inclusive were read. Papers 9 and 17 were discussed by Professor Harper.

Mr. Peet's paper was an attempt to interpret certain symbols found in the Indian mounds of Wisconsin, and he made a strong plea for the preservation of the records left by the Indians.

The report of E. R. Buckley, treasurer, was read and accepted. The president appointed Messrs. Hillyer, Chandler and Kremers as auditing committee.

At the close of the report Dr. Buckley handed in his resignation as treasurer, a step made necessary by his removal from the state.

## Friday Morning.

Professor Frank C. Sharp, secretary of the Academy, who has been absent in Europe for some months, handed in his resignation through President Slichter. This resignation and the resignation of the treasurer received on Thursday afternoon were accepted. President Slichter appointed as a committee to nominate officers to fill the vacancies, Professors Van Hise and Leavenworth and Dr. Schlundt.

The following amendment proposed at the annual meeting in December, 1900, was adopted:

Article VI, Section 2, is hereby amended to read as follows: The library commíttee shall consist of five members, of which the librarian shall be ex-officio chairman, and of which a majority shall not be from the same city.

The membership committee recommended the following named persons for active membership:

Paul H. Dernehl, Milwaukee.
Victor Lenher, Madison.
Roy Dykes Hall, Madison.
Roswell Hill Johnson, Madison.
E. C. Case, Milwaukee.
I. N. Mitchell, Milwaukee.

Samuel Edward Sparling, Madison.
William B. Cairns, Madison.
Charles Elmer Allen, Madison.
Andrew Robinson Whitson, Madison.
Alexander Rudolph Hohlfeld, Madison.
Ellwood C. Perisho, Platteville.
Irvin W. Brandel, Madison.
Robert Hugh Downes.
Katherine P. Regan.
William Christian Sieker, Milwaukee.
Richard C. Hughes, Ripon.
George Kirkpatrick, Ripon.
William Chase Bennett, Milwaukee.
Charles L. Harper, Madison.
By vote the secretary was instructed to cast the ballot for all the persons named. The ballot was cast and all were declared elected.

The committee further recommended that George P. Bacon of Beloit, a corresponding member who has returned to the state, be transferred from the list of corresponding members to the list of active members, and that G. A. Talbert of Oshkosh, formerly an active member, and having again taken up his residence in the state should be restored to active membership without payment of the initiation fee. The recommendations were adopted and both gentlemen put upon the list of active members.

The nominating committee reported, recommending that E. B. Skinner be chosen secretary to fill the unexpired term of F. C. Sharp, resigned, and H. W. Hillyer treasurer to fill the unexpired term of E. R. Buckley, resigned. The report was adopted and Messrs. Skinner and Hillyer were declared elected secretary and treasurer respectively to serve until after the annual meeting in 1902.

The reading of papers was then resumed. In the absence of the authors paper 22 was read by Dr. Schlundt, paper 24 by Professor Kremers and paper 25 by Professor Van Hise. No. 22 was discussed at some length by Professor Kremers, Dr. Schlundt and others, and No. 25 by Professors Van Hise and Hobbs.

Owing to lack of time the addresses in memory of W. H. Chandler, T. H. Safford and C. A. Bacon were omitted with the statement that they would appear in full in the "Transactions."

Attendance at morning session, about 45.

## Friday Afternoon.

The meeting was called to order at $2: 15$ o'clock by President Slichter.

The auditing committee reported that they had examined the treasurer's books and had found them correctly cast and vouched.

President Slichter appointed as additional members to serve on the library committee, in accordance with the amendment to Article VI, Section 2, of the constitution as adopted Dec. 27, 1901, Messrs. Marsh, Jegi and Davis.

The remainder of the session was taken up with the reading of papers 29 to 35 inclusive. No. 31 was discussed at some length by Professor Van Hise and others.

The Academy adjourned sine die.
E. B. Skinner, Acting Secretary.

## REPORTS OF THE LIBRARIAN.

## REPORT OF THE LIBRARIAN, 1900.

Since the last meeting of the Academy about thirty important serial publications have been added to the list of exchanges. The receipt of all publications sent to the Academy has been acknowledged and volume XII, part II, of the Academy's "Transactions," has been distributed.
During the month of September the entire library of the Academy was removed from the Capitol Building to the State Historical Library Building. The force of men that had just completed the moving of the library of the State Historical Society also transferred. the library of the Academy to the new building. The Academy is greatly indebted to Mr. Reuben G. Thwaites for thus placing at its disposal the services of these men, who accomplished the transfer of the whole collection without expense to the Academy. To Mr. W. M. Smith, who supervised the moving of the books and their disposal on the shelves of the stack room, the Academy also owes a debt of gratitude.

The Academy is to be congratulated that its valuable collection of books is now housed in the magnificent fire-proof building that also contains the libraries of the State Historical Society and the State University. The volumes of the Academy are placed in a separate part of the stack room, and they will henceforth be more secure and more accessible than ever before.

In the past, one of the most important duties of the librarian has been to endeavor to increase the size of the library by adding to the number of exchanges; while this will always remain an important duty, the number of volumes already in the collection, and the fact that they are now permanently placed in the
new building, call for a great deal of work in properly arranging and cataloguing the books so as to make them readily available. A good card catalogue should be prepared and kept at the library, and members should be furnished a new printed list of exactly what the library contains. Many of the volumes are still unbound. Before the work of binding can proceed, however, the numbers that are lacking to complete various sets must be secured. Besides looking over the entire library carefully, this will involve considerable work in the way of correspondence. The effort of the Academy should be to get its library into such condition that the treasures which it contains may be readily accessible to all.

To do the work here outlined requires time and skilled help; indeed the services of an expert librarian are indispensable for its accomplishment. Again, to issue promptly at any time such books as may be required necessitates the presence of the librarian in the library building. An opportunity has now happily presented itself to accomplish all that ought to be done with little or no expense to the Academy beyond what has usually been annually expended for assistance in the library. Since the books have been moved into the new library building Mr. W. M. Smith has kindly consented to act as my deputy in arranging the volumes, acknowledging the receipt of exchanges, etc. On becoming acquainted with the value of the library and its actual needs, he has kindly offered to enter into the work of securing additional exchanges, of completing the sets already at hand and preparing them for the bindery, and of arranging the books systematically on the shelves, and cataloguing them in a proper manner. He has offered further to issue promptly at all times such books as may be wanted, thus making the volumes of the Academy properly accessible for the first time. All this he has offered to do, as already stated, without expense to the Academy beyond what it has been accustomed to pay for clerical services. This work on the part of Mr. Smith would be simply a labor of love. The offer has been prompted solely by a sincere desire to make the library of use as it should be.

When I accepted the office of librarian I had but a faint idea of the true condition of the library and of the large amount of expert work that ought to be done upon it. I must confess
that I do not see how I could possibly give the library the amount of time that would be necessary to do the required work, even if I were sufficiently versed in library matters. It therefore seems to me that the Academy ought to accept Mr. Smith's offer gladly and to vest him with full authority to do the work here detailed. In order that this may be done, I deem it my duty to hereby tender my resignation as librarian of the Academy so that Mr. Smith may be elected for the unexpired term and may at once proceed vigorously with the work.

Louis Kahlenberg,
Librarian.
Madison, Wis., Dec. 27, 1900.

## REPORT OF THE LIBRARIAN, 1901.

The books constituting the library of the Wisconsin Academy of Sciences, Arts and Letters are now carefully arranged on the shelves of stack F of the Historical Library Building. The work of completing sets and binding volumes is in progress. About one hundred and fifty volumes have been sent to the bindery thus far.

Members of the Academy may at any time obtain books from the library without delay. If requests for books are sent directly to the office of the Librarian of the University, the desired volumes will be sent out on the same day that such request is received.

The Regents of the University have greatly aided the work connected with the Academy library by placing at the disposal of the Librarian of the Academy the library staff of the University under the direction of Librarian Smith. To the latter the Academy is especially indebted for the interest he has taken in the work.
'Louis Kahlenberg,
Librarian.
Madison, Wis., Dec. 24, 1901.

## REPORTS OF THE TREASURER.

## REPORT OF THE TREASURER

for the year ending December 31, 1900.
The treasurer would respectfully report that the total active membership of the Academy is 179.

Five members have been dropped from the list by request or for non-payment of dues. Of the total membership 136 have paid dues up to date or beyond; 16 are one year in arrears; 14 are two years in arrears; 4 are three years in arrears; 7 are four years in arrears; 2 are five years in arrears.

I would respectfully recommend that members now four years or more in arrears be dropped from the Academy roll in compliance with the resolution passed by the Academy at its annual meeting in 1892. If no objections are raised such names will be stricken from the list of members.

Accompanying this report is a statement of the finances of the Academy with vouchers for expenditures. There is now in the treasury one thousand five hundred and thirteen dollars and eighty-seven cents ( $\$ 1,513.87$ ). One thousand dollars is invested in a debenture bond bearing 5 per cent interest, payable January 1st, 1901. Five hundred and thirteen dollars and eighty-seven cents ( $\$ 513.87$ ) are in the general fund, an increase of seventy-five dollars and forty-four cents (\$75.44) over the balance of December 28th, 1899.

At the beginning of the year the treasurer was authorized to purchase a new set of books and transfer all the accounts. This was done and the books accompany this report for inspection.

Respectfully submitted,
E. R. Buckley,

Treasurer.
Treasurer's Report. ..... 661
Statement, 1899-1900.
Cash.
1899. Dr. Cr.
Dec. 28. To balance on hand ..... $\$ 43843$ 1900.
Dec. 27. To dues ..... 19750
Dec. 27. To interest on bond ..... 5000
Dec. 27. By printing ..... $\$ 7240$
(Vouchers Nos. 6, 8, 9, 11, 15, and 16.)Dec. 27. By clerical services and miscellane-ous supplies for librarian.......3347(Vouchers Nos. 1, 2, 3, 4, 7, 10,$12,17,18,21,24$.
Dec. 27. By clerical services and miscellane- ous supplies for secretary ..... 5056
(Vouchers Nos. 5, 13, 14, 22, and 23.)
Dec. 27. By clerical services and miscellane- ous supplies for treasurer. ..... 1563
(Vouchers Nos. 19 and 20.)
Dec. 27. Balance51387$\$ 68593$$\$ 68593$

The above report of the treasurer was duly examined by the auditing committee appointed by the chair, and found correct. Ernest Bruncken, W. S. Leavenworth, Albert S. Flint, Auditing Committee.

## REPORT OF THE TREASURER

for the year ending December 27, 1901.
During the last two years there has been a total of 219 names on the roll of active members. Eight of these are names of persons who were elected to membership but failed to qualify. Omitting this number, leaves a total of 211 active members during the years 1900 and 1901. Of this number one has died ;
two have been elected life members; two have been elected corresponding members; five have been dropped by request, dues having been paid up to current year; two have been dropped by request, dues having not been paid; and fourteen have been dropped for non-payment of dues for four consecutive years.

This makes a total of twenty-four names removed from the roll of active members, leaving a net active membership of one hundred and eighty-five (185).

Of the 185 active members there are thirty that are in arrears for payment of dues, as follows:

| 1 for 5 | years | \$500 |
| :---: | :---: | :---: |
| 5 for 3 | years | 1500 |
| 4 for 2 | years | 800 |
| 19 for 1 | year | 1900 |
| 1 for $1 / 2$ | year | '50 |
| - |  |  |
| 30 | Total | \$4750 |

Of the 155 members whose dues are paid to date, five, namely, the president, secretary, acting secretary, librarian and treasurer have had their dues remitted according to the by-laws of the Academy. This leaves a net total of 150 members paying dues for the year 1901. One of this number paid dues for five years; one for four years; six for three years; thirty-four for two years, and one hundred and eight for one year.
E. B. Copeland, S. A. Hooper and E. D. Jones are at present non-residents of Wisconsin and I would recommend that they be transferred to the list of corresponding members with the remission of such dues as may now be unpaid.

At the meeting of the Academy last year it was voted to transfer $\$ 500.00$ from the general to the permanent fund. The bonds, in which the $\$ 1,000$ permanent fund of the Academy were invested, became due January 1st, 1902. These bonds had been paying 5 per cent. interest, but owing to a reduction in interest rates, re-investment in the same bonds would now only return 4 per cent. The treasurer was instructed to reinvest in bonds the $\$ 1,000$, and also the $\$ 500$ transferred from the general to the permanent fund.

In accordance with these instructions application was made to the Clerk of the City of Madison for 15 street improvement
bonds. These bonds, which bear 6 per cent. interest, were obtained about the middle of May at a small premium. The treasurer invested $\$ 1,508.00$ in fourteen bonds, there not being sufficient money on hand to purchase the fifteenth. The additional bond, however, was taken up by President C. S. Slichter and will be turned over to the Academy as soon as there are sufficient funds to pay the same. These are long term bonds, extending from six to ten years, three being payable the first year and four each succeeding year. They will net the Academy a little over five per cent.

I would recommend that all moneys invested in these bonds, not already part of the permanent fund, be transferred to such fund ; that hereafter all interest accruing from the investment of the permanent fund be added to the permanent fund and invested from time to time as directed by the council.

The following is a recapitulation of the financial transactions of the Academy for the year 1900, and an itemized statement of the receipts and expenditures for the year 1901.

Statement, 1899-1900.
RECEIPTS.
1899.

Dec. 28. Balance on hand in general fund................... $\$ 43800$ 1900.

Dec. 27. Receipts from dues for year 1900................... 19700
Dec. 27. Interest on permanent fund, 1900
5000

Total
$\$ 68500$

## EXPENDITURES.

1900. 

Dec. 27. General and incidental expenses for $1900 . \ldots .$. . $\$ 17113$

Balance on hand in general fund Dec. 27, 1900.... \$5̄13 87
Permanent fund ......................................... 1,00000
Grand total . . . . . . . . . . . . . . . . . . . . . . . . . . . . . \$1,513 87

## Statement, 1900-1901.

RECEIPTS.
1900.
Dec. 27. Balance on hand in general fund ..... $\$ 51387$
1901.
Dec. 26. Receipts from dues for 1901 ..... 20600
Interest on permanent fund 6 mo . to Jan. 1, 1901 ..... 2500Interest on $\$ 1,000$ deposited in bank for 4 mo .at $2 \%$667
Total receipts $\$ 75154$
1901.
May 31. Transferred to permanent fund ..... $\$ 50800$
Balance in general fund ..... $\$ 24354$
EXPENDITURES.
No. of
sub-sub-

1. Dec. 31. Anna L. Moore ..... $\$ 100$
2. Jan. 15. Stamps (Treasurer) ..... 400
3. Jan. 23. Schwab Stamp Co. ..... 160
4. Jan. 14. Miss Beecroft ..... 829
5. Jan. 14. Miss Beecroft ..... 175
6. Jan. 15. Dr. Hobbs ..... 200.
7. Feb. 4. G. A. Mowry ..... 410
8. Mch. 4. Wm. J. Park \& Co. ..... 850
9. Mch. 16. Miss Beecroft ..... 30
10. Mch. 18. L. C. Burke ..... 3550
11. Feb. 4. H. C. Johnson ..... 540
12. Feb. 4. Stamps ..... 100
13. May 23. Schwab Stamp \& Seal Co ..... 205
14. June 15. W. Leonard ..... 1400
15. Aug. 20. Stamps (Secretary) ..... 200
16. Nov. 11. Stamps (Treasurer) ..... 200
17. Dec. 8. Stamps (Secretary) ..... 600
18. July 2. M. E. Yager ..... 1155
19. Aug. 3. Haswell \& Scholl ..... 4075
20. Aug. 6. L. C. Burke ..... 2625
21. Oct. 25. Democrat Ptg. Co. ..... 1950
22. Oct. 25. Tracy, Gibbs \& Co. ..... 1760
23. Dec. 5. E. D. Lorigan ..... 200
24. Dec. 5. Herman Schlundt ..... 80
25. Dec. 5. F. C. Sharp ..... 110
26. Dec. 5. Menges' Pharmacy ..... 80
27. Dec. 5. Capital City Paper Co ..... 300
28. Dec. 5. Philip Gross Hardware Co. ..... 350
Net receipts ..... \$243 54$\$ 22634$
Balance in general fund ..... $\$ 1720$
Balance in permanent fund ..... \$1,508 00
Grand total ..... \$1,525 20

In conclusion, I desire to express to the members my appreciation of their promptness in meeting the financial obligations which they have had toward the Academy. I regret exceedingly that it becomes necessary for me, at this time, to tender you my resignation as treasurer.

$$
\begin{aligned}
& \text { Respectfully submitted, } \\
& \text { E. R. Bucklex, } \\
& \text { Treasurer. }
\end{aligned}
$$

The Committee appointed to audit the accounts of the Treasurer of Wisconsin Academy of Sciences, Arts, and Letters report that they have examined his accounts and find them correctly cast and vouched.

H. W. Hillyer, Edward Kremers, Chas. H. Chandler, Auditing Committee.

Milwaukee, Wis., Dec. 27, 1901.

## EXTRACTS FROM THE CHARTER.

## An Act to incorporate the Wisconsin Academy of Sciences, Arts, and Letters.

The people of the state of Wisconsin, represented in senate and assembly, do enact as follows:

Section 1. Lucius Fairchild, Nelson Dewey, John W. Hoyt, Increase A. Lapham, * * *1 at present being members and officers of an association known the "The Wisconsin Academy of Sciences, Arts, and Letters," located at the city of Madison, together with their future associates and successors forever, are hereby created a body corporate by the name and style of the "Wisconsin Academy of Sciences, Arts, and Letters," and by that name shall have perpetual succession; shall be capable in law of contracting and being contracted with, of suing and being sued, of pleading and being impleaded in all courts of competent jurisdiction; and may do and perform such acts as are usually performed by like corporate bodies.
Section 2. The general objects of the Academy shall be to encourage investigation and disseminate correct views in the various departments of science, literature, and the arts. Among the specific objects of the Academy shall be embraced the following:

1. Researches and investigations in the various departments of the material, metaphysical, ethical, ethnological, and social sciences.
2. A progressive and thorough scientific survey of the state with a view of determining its minerai, agricultural, and other resources.
3. The advancement of the useful arts, through the applications of science, and by the encouragement of original invention.
4. The encouragement of the fine arts, by means of honors and prizes awarded to artists for original works of superior merit.
5. The formation of scientific, economic, and art museums.
6. The encouragement of philological and historical research, the collection and preservation of historic records, and the formation of a general library.
7. The diffusion of knowledge by the publication of original contributions to science, literature, and the arts.
[^34]Section 3. Said Academy may have a common seal and alter the same at pleasure; may ordain and enforce such constitution, regulations, and by-laws as may be necessary, and alter the same at pleasure; may receive and hold real and personal property, and may use and dispose of the same at pleasure; provided, that it shall not divert any donation or bequest from the uses and objects proposed by the donor, and that none of the property acquired by it shall, in any manner, be alienated other than in the way of exchange of duplicate specimens, books, and other effects, with similar institutions and in the manner specified in the next section of this act, without the consent of the legislature.

Section 4. It shall be the duty of the said Academy, so far as the same may be done without detriment to its own collections, to furnish, at the discretion of its officers, duplicate typical specimens of objects in natural history to the University of Wisconsin, and to the other schools and colleges of the state.

Section 7. Any existing society or institution having like objects embraced by said Academy, may be constituted a department thereof, or be otherwise connected therewith, on terms mutually satisfactory to the governing bodies of the said Academy and such other society or institution.

Approved March 16, 1870.

# EXTRACTS FROM THE WISCONSIN STATUTES. 

## STATUTES OF 1898. <br> TRANSACTIONS OF THE ACADEMY.

Section 341. There shall be printed by the state printer biennially in pamphlet form two thousand copies of the transactions of the Wisconsin Academy of Sciences, Arts, and Letters, uniform in style with the volumes heretofore printed for said society.

## CHAPTER 22.

of the distribution of public documents.
Section 365. The transactions of the Wisconsin Academy of Sciences, Arts, and Letters shall be distributed as follows: One copy to each member of the legislature, one copy to the librarian of each state institution; one hundred copies to the State Agricultural Society; one hundred copies to the State Historical Society; one hundred copies to the State University, and the remainder to said Academy.

Section 366. In the distribution of books or other packages, if such packages are too large or would cost too much to be sent by mail, they shall be sent by express or freight, and the accounts for such express or freight charges, properly certified to, shall be paid out of the state treasury.

STATUTES. OF 1901.

## CHAPTER 447.

binding of exchanges.
Section 1. Section 341 of the revised statutes of 1898 is hereby amended by adding thereto the following: The secretary of state may authorize the state printer to bind in suitable binding all periodicals and other exchanges which the Society shall hereafter receive, at a cost not exceeding one hundred and fifty dollars per annum. The secretary of state shall audit the accounts for such binding.

Note.-The Academy allows each author one hundred separates of his paper erom the Transactions without expense to the author, except a small charge for printed covers when desired.-Editor.

## CONSTITUTION

OF THE WISCONSIN ACADEMY OF SCIENCES, ARTS, AND LETTERS.

[As amended in Articles V, VI andIX at the regular meetings of December, 1899 and December, 1901.]

Article I.-Name and Location.
This association shall be known as the Wisconsin Academy of Sciences, Arts, and Letters, and shall be located at the city of Madison.

Article II.-Object.
The object of the Academy shall be the promotion of sciences, arts, and letters in the state of Wisconsin. Among the special objects shall be the publication of the results of investigation and the formation of a library.

## Article III.-Membership.

The Academy shall include four classes of members, viz.: life members, honorary members, corresponding members, and active members, to be elected by ballot.

1. Life members shall be elected on account of special services rendered the Academy. Life membership in the Academy may also be obtained by the payment of one hundred dollars and election by the Academy. Life members shall be allowed to vote and to hold office.
2. Honorary members shall be elected by the Academy and shall be men who have rendered conspicuous services to science, arts, or letters.
3. Corresponding members shall be elected from those who have been active members of the Academy, but have removed from the state. By special vote of the Academy men of attainments in science or letters may be elected corresponding members. They shall have no vote in the meetings of the Academy.
4. Active members shall be elected by the Academy and shall enter upon membership on the payment of an initiation fee of two dollars which shall include the first annual assessment of one dollar. The annual assessment shall be omitted for the president, secretary, treasurer, and librarian during their term of office.

## Article IV.-Officers.

The officers of the Academy shall be a president, a vice-president for each of the three departments, sciences, arts, and letters, a secretary, a librarian, a treasurer, and a custodian. These officers shall be chosen by ballot, on recommendation of the committee on nomination of offcers, by the Academy at an annual meeting and shall hold office for three years. Their duties shall be those usually performed by officers thus named in scientific societies. It shall be one of the duties of the president to prepare an address which shall be delivered before the Academy at the annual meeting at which his term of office expires.

## Article V.-Council.

The council of the Academy shall be entrusted with the management of its affairs during the intervals between regular meetings, and shall consist of the president, the three vice-presidents, the secretary, the treasurer, the librarian, and the past presidents who retain their residence in Wisconsin. Three members of the council shall constitute a quorum for the transaction of business, provided the secretary and one of the presiding officers be included in the number.

## Article VI.-Committees.

The standing committees of the Academy shall be a committee on publication, a library committee, and a committee on the nomination of members. These committees shall be elected at the annual meeting of the Academy in the same manner as the other officers of the Academy, and shall hold office for the same term.

1. The committee on publication shall consist of the president and secretary and a third member elected by the Academy. They shall determine the matter which shall be printed in the publications of the Academy. They may at their discretion refer papers of a doubtful character to specialists for their opinion as to scientific value and relevancy.
2. The library committee shall consist of five members, of which the librarian shall be ex officio chairman, and of which a majority shall not be from the same city.
3. The committee on nomination of members shall consist of five members, one of whom shall be the secretary of the Academy.

## Article VII.—Meetings.

The annual meetings of the Academy shall be held between Christmas and New Year, at such place as the council may designate; but all regular meetings for the election of the board of officers shall be held at Madison. Summer field meetings shall be held at such times and places as the Academy or the council may decide. Special meetings may be called by the council.

## Article VIII.-Publications.

The regular publication of the Academy shall be known as its Transactions, and shall include suitable papers, a record of its proceedings, and any other matter pertaining to the Academy. This shall be printed by the state as provided in the statutes of Wisconsin. All members of the Academy shall receive gratis the current issues of its Transactions.

## Article IX.-Amendments.

Amendments to this constitution may be made at any annual meeting by a vote of three-fourths of all the members present; provided, that the amendment has been proposed by five members, and that notice has been sent to all the members at least one month before the meeting

## RESOLUTIONS

## REGULATIVE OF THE PROCEEDINGS OF THE ACADEMY.

## THE TRANSACTIONS OF THE ACADEMY.

[By the Academy, December 28, 1882. Transactions, Vol. VI, p. 350.]
2. The secretary of the Academy shall be charged with the special duty of overseeing and editing the publication of future volumes of the Transactions.
3. The Transactions of the Academy hereafter published shall contain: (a) a list of officers and members of the Academy; (b) the charter, by-laws and constitution of the Academy as amended to date; (c) the proceedings of the meetings; and (d) such papers as are duly certified in writing to the secretary as accepted for publication in accordance with the following regulations, and no other.
6. In deciding as to the papers to be selected for publication, the committee shall have special regard to their value as genuine, original contributions to the knowledge of the subject discussed.
9. The Sub-Committee on Publication shall be charged with insisting upon the correction of errors in grammar, phraseology, etc., on the part of authors, and shall call the attention of authors to any other points in their papers, which in their judgment appear to need revision.
[By the Academy, June 2, 1892, Vol. IX, p. ii.]
The secretary was given authority to allow as much as ten dollars for the illustrations of a paper when the contribution was of sufficient value to warrant it. A larger amount than this might be allowed by the Committee on Publication.
[By the Academy, December 29, 1896, Vol. XI, p. 558.]
The secretary was directed to add to the date of publication as printed on the outside of author's separates the words, "Issued in advance of general publication."

FEES OF LIFE MEMBERS.
[By the Academy, July 19, 18\%0, Vol. I, p. 18\%.]
Resolved, That the fees from memivers for life be set apart as a permanent endowment fund to be invested in Wisconsin state bonds, or other equally safe securities, and that the proceeds of said fund, only, be used for the general purposes of the Academy.

ANNUAL DUES.
[By the Academy, December 29, 1892, Vol. IX, p. vi.]
Resolved, That the secretary and treasurer be instructed to strike from the list of active members of the Academy the names of all who are in arrears in the payment of annual dues, except in those cases where, in their judgment, it is desirable to retain such members for a longer time.

## ARREARS OF ANNUAL DUES.

[By the Council, December 29, 1897.]
Resolved, That the treasurer be requested to send out the notices of annual dues as soon as possible after each annual meeting and to extend the notice to the second or third time within a period of four months where required.

Note.-The Printing Commissioners of the State of Wisconsin now require all copy to be at hand ready for the printer before the permit for printing shall be issued by the Secretary of State. But, under a ruling of the Commissioners, made in response to a presentation by the Committee of the Academy appointed December 29, 1897, each volume of the Transactions may be issued in two consecutive parts; so that a publication may thus be issued each year covering the papers accepted after the previous annual meeting.

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Yarrow, H. C., in American Naturalist, cited, 525.
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Zygoballus, 322.


[^0]:    *Translated by P. E. Doudna A. M., late Asst. Professor of Mathematics in Colorado College, and E. F. Chandler, A. M., Instructor in Mathematics in the University of North Dakota.

[^1]:    * By performing the addition from two different sets of seven place tables (one of them Vega, Bremiker's 40th edition; Berlin 1887), I find consistently these rest $\bullet$, :

    $$
    \begin{array}{ccc}
    \text { Sum of logarithms } & 121-143 \text { error }-1.92 \\
    224-243 & +3.13
    \end{array}
    $$

[^2]:    * In the notation of the anthor, $d V^{\prime}$ here denotes the derivative of $V^{\prime}$ with respect to a. [IT. F. C.]

[^3]:    * I get for the first number after 0 in the fourth column 0.547723 and so on to the last 5.477226 , but it hardly seems worth while to correct such small errors.-E. F. C.

[^4]:    *According to the notation of the author, $d \varphi(\alpha)$ here denotes the derivative of $\varphi(a)$ with respect to $a$.-[E. F. C.]

[^5]:    * Those who are accustomed to computation use the following rale for computing $C^{2}=A^{2}+B^{2}$ by the aid of the auxiliary angle $\mu$. First, $\log A-\log B$ is taken, if

[^6]:    $A>B$, and this difference is sought in the column of the table marked cot. Then from the column marked cos the corresponding logarithm is taken, which subtracted from $\log A$ gives the required $\log C$. But if $B>A$ the differeace $\log B-\log \mathrm{A}$ is to be sought in the cot column and the corresponding logarithm from the cos column is to be subtracted from $\log B$. That difference is always taken which is the logarithm of a quantity greater than 1, and the corresponding logarithm from the cos column subtracted from the greater of the given logarithms. In the computation of the probable error the rule is so used that for $\mu>45^{\circ}$ the change is made from cotangent to sine.
    *0.38 in the original. E. F. C.

[^7]:    *In the British gazetteer I find three places now set down as bas-ford, and one as bas church.

[^8]:    1 De la Rive and De Candolle, Pogg. Ann., vol. 14, 1828, p. 590.
    ${ }^{2}$ C. G. Stangel, University of Wisconsin Thesis, 1899.
    ${ }^{3}$ Tyndall, Phil. Trans. Roy. Soc., vol. 143, 1853, p. 217.
    ${ }^{4}$ Voigt, Wied. Ann., vol. 64, 1898, p. 95.

[^9]:    ${ }^{1}$ C. G. S. System of Units, Everett, p. 128.

[^10]:    ${ }_{1}$ A thesis submitted to the faculty of the University of Wisconsin for the degree B. L., June, 1901.
    ${ }^{2}$ Formation of the counties of Wisconsin. A thesis by Herbert Scott Blake, '94, pages 3-4, ${ }^{\text {², }} 21$ - 2 .

[^11]:    ${ }^{1}$ Session Laws, Wis., 1850, p. 25. Southport Telegraph, Friday, Feb. 15, 1850.
    ${ }^{2}$ Hist. of Racine and Kenosha Counties, 1879, p. 308.
    ${ }^{3}$ Alterations of Towns, filed March 9, 1853. File Town Plats, "Old", Office of Sec. of State.
    ${ }^{4}$ Kenosha Telegraph, April 11, 1851, March 21, 1851.

[^12]:    ${ }^{1}$ Kenosha County Records.
    2 For changes in town boundaries see Plate XXXIX.
    ${ }^{3}$ Census taker of 1850 , remarked: "The only remarkable thing to be seen in this town (Bristol) is a flouring mill built upon a stream (Des Plaines) that it would require an 'observation' to tell which way the water runs."

[^13]:    ${ }^{1}$ This discussion of the Geology, soils and topography, is abridged from Geology of Wisconsin, vol. 2, part 2, 1873-1877.

[^14]:    ${ }^{1}$ For soils and topography see Plate XXXIX.

[^15]:    ${ }^{1}$ A communication printed in the Kenosha Leader, June 26, 1890, states that John Bullen was originator and organizer of the Western Emigration Co., and was the only individual at any time elected president of the association.

[^16]:    ${ }^{1}$ This account of the first settlement in Kenosha Co. is abridged from the History of Racine and Kenosha Counties, 1879, pages 331-340.
    ${ }^{2}$ This ta 0 le was made from copies of the original U. S. census returns for the state for 1850,1860 and 1870, which are in the vault of the office of the Sec. of State. For the actual number born in the different states and foreign countries see Table 1, Appendix.

[^17]:    ${ }^{1}$ Those born in Wiscousin rapidly increased, so the Wisconsin born is an important factor in each town.
    ${ }^{2}$ The percentage of the Germans who were born in Prussia is so great, that hereafter in this discussion the term German will mean those who were born in Prussia.

[^18]:    ${ }_{1}$ See Table 6, Appendix. The census taker of 1850 in his remarks on the town of Wheatland said that the town had a great reputation for wheat.

[^19]:    ${ }^{1}$ Vote of county, city and lake shore towns not included.

[^20]:    ${ }^{1}$ A thesis submitted to the faculty of the University of Wisconsin for the degree B. L., June, 1901.
    ${ }^{2}$ The Ridge is two miles north of the village of Shullsburg.

[^21]:    ${ }^{1}$ The above was taken from the History of La Fayette County, Wisconsin, 1881, and the Geography and Gazeteer of Wisconsin.
    ${ }^{2}$ These boundaries are copied from the thesis of B. M. Palmer, "The Lead Regions of Illinois." They became the legal boundaries Feb. 14, 1847, Session Laws of Wisconsin Territory for 1847, p. 57.

    3 Wayne is known as the "lost township" as the original survey of that town was lost on its way to the United States land office. In 1835 it was re-surveyed.
    ${ }_{4}$ See Plate XLIII.

[^22]:    ${ }^{1}$ Geology of Wisconsin, vol. II, part IV, 1873-1877.
    2 Taken from the United States Census for 1850, manuscript records, in the office of the Secretary of State.
    ${ }^{3}$ See Plate XLIII.

[^23]:    1 United States Census, manuscript records, office of Secretary of State.
    2 Those who were born in any part of the United States were considered native born,-the rest, foreign born.
    ${ }_{3}$ This division includes the countries not comprised in the other groups of foreign countries.

[^24]:    ${ }^{1}$ For further information on this subject see: Thesis of F. Belle Stanton, Lead Regions of Wisconsin, Univ. of Wis, 1901; Libby, An Economic and Sócial Study of Lead Regions in Iowa, Illinois and Wisconsin; Thwaites, Early Lead Mining in Illinois and Wisconsin.
    ${ }_{2}$ See Appendix, Table 2.

[^25]:    ${ }^{1}$ Kendall must be omitted in this comparison as its boundary line had been changed between 1850 and 1860 .
    ${ }^{2}$ Except Elk Grove, whose boundary line was changed between 1850 and 1860.
    ${ }^{3}$ See Appendix, Table 3.

[^26]:    ${ }^{1}$ In computing the population per square mile, some difficulties arose because of the changes in town boundary lines, the county history being inaccurate in this respect. After a comparison of the Plat book for 1895, an atlas for 1847 and various maps found in the office of the Secretary of State and the map room of the Historical Library, the areas given in Table 5 in the Appendix were decided upon as being approximately correct.

[^27]:    ${ }^{1}$ See Table 2.

[^28]:    ${ }^{1}$ Memoir of Edward Orton, by G. K. Gilbert, Bull. Geol. Soc. Am. Vol. 11 (1900), pp. 542-550. (Includes a bibliography of Professor Orton's scientific writings.)

    Edward Orton, by J. J. Stevenson. Jour. Geol. Vol. 8 (1900), pp. 205-213.

    Fdward Orton, by I. C. White. Am. Geologist. Vol. 25 (1900), pp. 197-210. (Contains a bibliography compiled by Lucy Allen, Ohio State University Library.)

    Edward Orton, Educator, by T. C. Mendenhall. Science, N. S. Vol. 11 (1900), pp. 1-6.
    Edward Orton, Geologist, by G. K. Gilbert. Ibid., pp. 6-11. (Not the article published in Bull. Geol. Soc. Am.)

    The above articles have been freely used in compiling this article.
    Note.-The Historical Collections of Ohio (Centennial Edition, 1899. Vol. 2, p. 59), contains a sketch of Dr. Orton's life which according to Gilbert is, though anonymous, clearly autobiographic.

[^29]:    11. c. p. 200 .

    21 c. pp. 3-4.

[^30]:    ${ }^{1}$ Arithmetical Prodigies, by E. W. Scripture, American Journal of Psychology, Vol. IV., No. 1.
    Truman Henry Safford, by Harold Jacoby, Science, N. S., Vol. XIV., No. 340.
    Truman Henry Safford, by H. P. Hollis, The Observatory, Vol. 24, No. 308.
    The Late Professor Safford, by E. Knobel, The Observatory, Vol. 24, No. 309. Reference is made in this article to an article on "The Boy Safford" in Chambers' Edinburgh Journal for Oct., 1847.
    Obituary Notice of T. H. Safford, by Arthur Earle, Astronomische Nachrichten, Bd. 157, No. 3749.
    The above articles have been freely drawn upon in compiling this notice. I have also to thank Professors G. C. Comstock and C. F. Smith of the University of Wisconsin for important data concerning Professor Safford's like and work.

[^31]:    *Deceased. **Deceased December 9, 1899. Professor Birge elected to fill unerpired term.

[^32]:    Davis, John Jefferson,
    504 Monument Sq., Racine. B. S. (Illinois) ; M. D. (Ifahnemann). Physician.

[^33]:    Stanton, Florence Belle, Lodi. B. L. (Wisconsin). Teacher in High School.

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    Viebahn, Charles Frederick, 703 Western Av., Watertown. Superintendent of Schools and Principal of High School.
    Voget, Guido Charles,Milwaukee.
    B. s. (Wisconsin).
    Voss, Ernst Karl Johann Heinrich,
    23 E. Johnson St., Madison.
    Ph. D. (Leipzig). Professor of German Philology, University of Wisconsin.
    Watson, Walter S.,Whitewater.M. S. (...........). Professor of Biology and German, State Normal

[^34]:    ${ }^{1}$ Here follow the names of forty others. Sections $5,6,8$, and 9 are omitted here as of no present interest. For the charter in full see Traresactions, vol. viii, r xi, or earlier volumes.

