Correspondence and drafts re: unpublished paper, "Some underfit streams in central Illinois". 1953-1958

Thwaites, F. T. (Fredrik Turville), 1883-1961
[s.l.]: [s.n.], 1953-1958

1 Dec., 1958

Dr. Luna B. Leopold, Chief Hyrdraulic Engineer
U. S. Geological Survey
Washington 25, D. C.   4000 0001

Dear Dr. Leopold:

Thank you for yours of the 21st. November and the manuscript.

Since first writing on it nearly30 years have elapsed and naturally my ideas have changed greatly. Things which were accepted doctrine then are now doubted. The subject boils down to as I see it ~~three~~ four problems:

(1) Are meander scars valid evidence of a former larger stream than now exists?

(2) If they are or some of them are then how best to measure the size of the former stream?

(3) What controls the size of meanders, volume alone or volume combined with other factors?

(3) If change in volume occured then what formulae will permit the degree of change to be estimated?

I am far from sure that any of these problems can be answered at present. he geomorpholgists of 30 years ago certainly never dreamed of them.

I will as opportunity offers overhaul the manuscript and may if you desire send it to you again.

Sincerely yours,

UNITED STATES
DEPARTMENT OF THE INTERIOR
GEOLOGICAL SURVEY
WASHINGTON 25, D. C.

IN REPLY REFER TO:
4000 0001

November 21, 1958

Dr. F. T. Thwaites
41 N. Roby Road
Madison 5, Wisconsin

Dear Professor Thwaites:

It is very gratifying that a man as experienced as you should pay attention to some of the work we have done on river channels. I am afraid that our equations are not as helpful for the solution of the problems to which you have addressed yourself as I would wish.

Be that as it may there is one fundamental problem in your manuscript which, unresolved, would leave the subsequent computations open to serious question. I know this fundamental problem very well because I ran into the same difficulty when I tried to do something similar to what you are attempting in the present manuscript.

The problem is this. The circular scarps which line the valley of the Kaskaskia conceivably could be formed by a reach of river which in itself has many meander loops but which forms a large bend. An example is the reach which intrudes itself into Section 36 of Town 8N., Range 1 E., and again on the Embarrass River, the large bend which exists in Section 36, near Newcomb. It seems conceivable to me then that a series of meanders which in themselves form a large loop could intrude themselves into Section 7, Town 7N., Range 10E. You will notice in the adjacent Section 8 there is a suggestion of a shorter wave length which might have been formed by a river having meanders no larger than those of the present Embarrass.

Thus I do not know whether it is possible to guarantee ourselves that these large reentrants which occur in Sections 32, 7 25 and 35 on the Kaskaskia can really be considered the outside of a single meander loop of a large river. The problem here was called to my particular attention when I started to map a stream in Wyoming, Squaw Creek, a tributary to the Popo Agie. As soon as I began to map I realized that the reentrants in the valley sides could quite well be explained by taking a short reach of river and allowing it to eat into the bank, thus forming an apparent meander scar of much larger radius than the meanders on the stream cutting the scar.

It would be perfectly possible for you to admit this possibility and go on to say that if your conjecture is correct then one would approach the problem in the following manner and proceed with your analysis. Even if you did that however, may I suggest that you provide a little table of measured values of width, length, and radius of curvature, perhaps identifying the location by section, town and range in order to support your statement on page 5 where you say that the old meanders were seven times as large as those of today. Since you are doing some of the other work quantitatively I believe the reader would be better satisfied if you would give him some idea of how many observations made up this value of seven times the present size.

A further suggestion is that I consider slope more or less a dependent variable and I believe I would eliminate your computation of discharge using the slope parameter. I believe that width is by far the best of the parameters to use for computation. Width gives the most consistent relation with discharge and is much less affected by size and type of sediment than some of the other parameters.

May I suggest further than on page 3 you use the exponents for the downstream relations rather than at-a-station, because when a river which is large is compared with one which is small the relationship should be expressed by the downstream comparison and will be somewhat different than if a given cross section is compared at high flow versus low flow. Therefore, on page 3 the exponent to use in the width equation is the square root value.

I would further suggest that you would be on sound ground to assume that the same exponent which I called "a" in the width-discharge relation should be kept the same for the original as compared with the present day stream. These coefficients are undoubtedly related to sediment size among other things and thus in turn to bank resistance as you suggest. From the streams I have seen, and I gather your observations are similar, there is nothing to differentiate the size of the material from the postulated early stream being different from the material now being carried by the present river. constant?

In summary, I do not mean to discourage you from publishing this paper, but I believe you would strengthen it by admitting the possibility which I described earlier, that your radii of curvature and wave-length parameters may be spurious. Secondly, I would concentrate the discharge computations using the equation relating width to discharge, and the width should be carried with the exponent of five tenths.

I might point out to you that Dury, who has shown me his work in South England, is continuing his work along the same line with considerable success. I have seen some new data which he is publishing in The Geographical Review summarizing his work. You might wish to send your final manuscript to him and enter into correspondence with him about your mutual problems. His address is:

Dr. G. H. Dury
Department of Geology
Birkbeck College
University of London
London, England

I will be happy to reread your manuscript at a later date if you pursue this work further and if you desire any further assistance which I can give you.

Sincerely,

Luna B. Leopold

Luna B. Leopold
Chief Hydraulic Engineer

41 N. Roby Road,
Madison 5, Wis.
14 Nov., 1958

Dr. L. B. Leopold,
Water Resources Branch, U. S. Geological Survey
Washington 25, D. C.

Dear Dr. Leopold:

Enclosed is the manuscript about which I spoke when I saw you in St. Louis. I am far from satisfied about it but maybe it is as good as is possible with present knowledge. I have made several assumptions but will appreciate your criticism, for you are one of the few qualified to make such. My feeling at present is that size of meanders is the result of the part of the total energy of a stream which is directed against the bank. The result must take into account the resistance of the bank to erosion as well as the length of time that erosion has been taking place. Thus we have a complex relation with a number of factors unknown. The conclusion that discharge alone is responsible is I think unjustified. If this is so, I could have been attempting the impossible in trying to estimate former discharge.

If you think the paper is worth publishing where do you advise sending it? I already have papers in the h ands of the Wisconsin Academy, Journal of Geology, and G. S. A. This makes it best to use some other publication. The little photostat is the only small copy of the big tracing which I have. My compitations probably need checking as well as the underlying theories. You will note that the paper has been delayed from the 30's to now. The Illinois Survey did not want to publish it. But most of the work was not done in that state.

In replying please note that I never got a PhD. This is too long a story to repeat here.

Sincerely yours.

F. T. Thwaites

STATE GEOLOGICAL SURVEY DIVISION

JOHN C. FRYE, CHIEF
121 NATURAL RESOURCES BUILDING
UNIVERSITY OF ILLINOIS CAMPUS
URBANA

June 18, 1955

Professor F. T. Thwaites
Department of Geology
University of Wisconsin
Madison 6, Wisconsin

Dear Fred:

I have your letter of May 31 and the short manuscript involving drainage along the Kaskaskia River in Illinois. I have read the manuscript and I think it is a very interesting contribution. However, in reviewing our various publications series, we are not set up to handle papers as short as this in an adequate manner. Generally speaking, we would use the Illinois Academy of Science or the Short Notes section of the Journal of Geology or GSA if this particular paper had originated here. I wonder if that is not the best thing for you to do with it. I am sending it back to you along with the photostat of the one illustration and have made a couple of very minor suggestions in the first paragraph. The main objective of the suggestions was to get the Geological Survey's name into it as having been the sponsor of the road materials study out of which this little paper grew.

In other words, in my opinion it is a nice contribution and I think it should be published, but our publications series is not adapted to this short a paper and therefore you have our complete approval to publish it elsewhere. Thanks a lot.

Cordially yours,

John C. Frye
Chief

May 31, 1955

Dr. John C. Frye, Chief
Illinois State Geological Survey
Natural Resources Building
Urbana, Illinois

Dear Dr. Frye:

I am sorry you were unable to join us at the Forest Bed on May 14. I made a second trip with two who missed the regular one on May 21 but failed to write you. It was too late when we decided to go. Exposures are faif at the present time unless we get a strong east wind which would sweep away the talus. No stumps in place were seen but lots of driftwood and much folding of the pre-Valders beds. Exposures at Valders were good. If you care to go up some time during the summer I would be glad to show both places to you, preferably after July 8.

After compiling all the radiocarbon results I am confident there is something wrong with the method. This is replacement of part of the Carbon 12 by Carbon 14 from modern ground water which contains carbon dioxide derived from modern plants. We know, despite the claims of some atomic physicists, that wood is replaced by silica, iron oxide, manganese oxide and other minerals. If this is the case specimens exposed to ground water should be unreliable. This is in my report now in New York.

I am enclosing a short report which was started when working for the Illinois Survey years ago, and which I feel should be submitted to you first. Perhaps it is not worth publishing, but I would be glad of your opinion. If not interested, please return the photostat of the map because I have no other copy, only a big original.

Sincerely yours,

F. T. Thwaites

SOME UNDERFIT STREAMS IN CENTRAL ILLINOIS.

F. T. Thwaites

Introduction. The term "underfit" is applied to rivers which have present-day meanders on a floodplain which are much smaller than older meanders which are displayed in the form of the adjacent hills. During the course of a road material survey of central Illinois for the Illinois Geological Survey ~~during~~ in the field seasons of 1929 and 1930 the writer observed this phenomenon in both Kaskaskia and Embarrass rivers. Figure 1 was traced from the drainage survey and shows the bottoms of the Kaskaskia above Vandalia where evidences are best shown. The topography of the surrounding country was later surveyed in the Ramsey and St Elmo quadrangles of the U. S. Geological Survey. These maps are on a much smaller scale and show far less detail than did the older survey of 1908 to 1911. The older meanders are displayed in the striking meander cusps of the eastern bluffs. One of the meanders cut through a spur and captured a small tributary stream. Only one old meander was discovered on Embarrass River just southeast of Newton. If there are other examples, the writer has not discovered them.

Hypotheses. Similar underfit streams have been described by a number of physiographers (Davis, Dury). These earlier students of the problem looked no farther for a cause of the change of size of meanders than the fact that the discharge of the stream has decreased. As a cause of this change Davis suggested diversion of the headwaters by stream capture whereas Dury thought that only a climatic change could account for the result. Later Davis suggested seepage through the alluvial fill which is callled "Lehmann's principle."
A study of the material of the fill and the use of formulas published by Slichter demonstrate that such underflow is entirely inadequate for an explanation unless the normal discharge of the surface stream is very small.

(1) validity of large scar
(2) competency of larger volume
(3) control of size of meanders
(4) estimate of change


KASKASKIA
RIVER
R.1 E.
R.2 E.
R.2 E.
R.3 E.
T.8 N.
T.7 N.
T.7 N.
T.6 N.
1 MILE
EMBARRASS
RIVER
NEWTON
T.8 N.
T.7 N.
R.9 E.
R.10 E.
1 MILE

Modern knowledge shows, however, that there are other factors involved than discharge only. Both slope and material of the bed are factors which cannot be overlooked. An alluvial fill in an older valley must in most localities reduce the slope from that of the older valley and at the same time introduce material which is in most instances of less resistance to erosion. Friedkin showed by experiment that both the width and length of bends, that is size of meanders, increase with slope. It is evident that the valley of Kaskaskia River has received an alluvial fill since it was first eroded, but the writer has found no data on how much the slope was altered, ~~because~~ the thickness of fill is not known. The present floodplain is silty clay whereas most of the older valley was eroded in glacial till which presumabley offers somewhat greater resistance to erosion. Just how to reconcile these known changes with a probable change in discharge due to the disappearance of the glacier is difficult to decide. It is presumed that the fill resulted not from change of level of the land or change of climate but from blocking of the outlet of Kaskaskia River with outwash of a later glaciation. We must realize that the same phenomenon of a change to small meanders on a floodplain has also been noted in the Driftless Area (Bates).

Formulas. When the writer was in the field and first began to write up the results, there was little knowledge of the problems involved and no formulas had been suggested by which the amount of possible change in discharge could be estimated. Now the studies of Leopold, Maddock, Wolman, and Friedkin offer possible solutions. The first three worked out a number of equations which show the relation of various dimensions of streams to discharge. The equations are what is known as empirical, that is they tell nothing of the physical relations which caused the changes. Rational equations differ in that they are based directly on the forces which are involved; whereas empirical equations were derived by plotting the data for the most part on logarithmic coordinates. When points are plotted, they approximate a straight line showing that there is a mathematical relationship. Failure to fall ~~xxxx~~ exactly on a line is termed scatter. It is due not only to errors of measurement but also to the neglect of other factors than those which were considered. Throughout all hydraulic phenomena it is known that several factors may enter into any relationship. When the plotting is on logarithmic coordinates, a straight line indicates a <u>power</u> <u>function</u> and the slope of the straight line shows the value of the exponent. The work of Leopold and his associates shows that most relationships are power functions. Different dimensions are referred to discharge (Q). The sum of the exponents of the dimensions width, depth, and velocity must equal unity, for discharge is the product of these three. For observations at a fixed locality or station the average they report is that width is related to the 0.26 power of discharge, depth to the 0.~~14~~ 40 power, and velocity to the 0.34 power. The units employed are British Engineering Units,(feet, and seconds.) The exponents of discharge tell of relationships when discharge changes. Another set of results gives conditions downstream where the discharge at a given stage increases in that direction. Here width is the 0.5 (square root) power of discharge, depth the 0.4 power, and velocity the 0.1 power. It seems probable that the ~~first~~ second set will apply best to the problem now in hand. Besides the exponent every equation also involves a constant. These constants ~~xxxxx~~ vary greatly and are not given by Leopold and Wolman. It is not known just what they are related to but probably a number of factors are involved. Lacking knowledge of the values of the constants we can derive each one by substituting the modern values of other parts of the equations. This method undoubtedly involves error, for conditions of a stream vary when discharge and bed material vary. Besides the formulas given above Leopold and his associates also offer expressions for slope and for wave length of meanders. Slopes are given both as feet per mile and as feet per foot, which is generally a very small quantity. Wave length is the distance along the channel in which there is a complete reversal of direction. Slope (S) is related to the 0.49 power of discharge which is so close to the square root that the difference may be ignored. $S = (3.97 \times 10^{-6}) / Q^{\frac{1}{2}}$ where slope is given in feet per foot. Two expressions for wave length are $36\ Q^{\frac{1}{2}}$ and $6.5\ w^{1.1}$ (w = width). From these it is thought that width is related to the 0.9 power of any linear dimension of the meanders. A dimension considered by the writer but not by others is the radius of curvature of meanders (r). This is because the force directed against the bank by unit mass of water is the angular acceleration of this unit mass. Text books of elementary physics show that this acceleration is proportional to the square of the velocity and inverse to the radius. It did not prove difficult to measure the radius of the circle which approximates to the form of any meander at any particular spot. Force is accleration of unit mass. From Manning's formula it will be seen that with other things equal the velocity squared of a stream of water is proportional to the slope. Hence, the formula for force exerted by unit mass of water on the bank is proportional to slope divided by radius (S/r).

Data. Data are more complete for Kaskaskia River than for the Embarrass. The reports of the U. S. Geological Survey place the mean discharge of the Kaskaskia at 1505 cubic feet per second (second feet) based on observations over a period of 42 years. The maximum recorded flood is given as 52,000 cubic feet per second. There is no statement as to the mean width or mean depth at the gauging stations. Width is scaled from the drainage maps as 135 feet. This corresponds to the width of the normal channel or "bank-full" stage. The same maps place the average channel slope at 1 foot in 5740 feet or 1.74×10^{-4}. The radius of the present meanders is about 0.1 mile and that of the old meanders shown in the bluffs is about 0.7 mile. There is no infomation on the depth of alluvium to glacial till or bedrock. The bluffs rise to a maximum height of about 100 feet above the present floodplain.

The problem. From the discussion given previously all that we can conclude definitely on change of size of meanders is that meandering is controlled by the portion of the total energy of a stream which is directed against the banks in relation to the resistance of the banks to erosion. Energy is related to discharge, velocity of the stream which is controlled by slope, size of channel, and nature of the bottom. In this complex problem it is clear that discharge is only one factor

However, an attempt will be made to compute possible changes in mean discharge which might have resulted either from meltwaters of the Tazewell substage of the Wisconsin stage of glaciation or from a more rainy climate than that of the present. None of the formulas tells anything of the effect of bank material on size of meanders.

Slope. Leopold's equation relating slope to discharge is an attractive means to solve this problem. He gives the constant as 3.97×10^{-6} when slope is measured in feet per foot. If we use the present-day value for discharge, however, we find a constant of 0.89×10^{-6} instead. If we use Leopold's constant for an estimated discharge of 50,000 cubic feet per second we obtain a slope of 1.77×10^{-6}.

This raises the question of the proper value of the constant under former conditions. It is almost impossible to find the slope of the channel when the old large meanders were formed. It is certain the present scars were not formed all at once. If we conclude that the fill of modern alluvium thickens downstream the old valley eroded into the till banks was steeper than the present channel. The writer suggests that meanders grow until the force against the banks is balanced by the resistance of the banks to erosion. That is the smaller meanders of today represent a state of equilibrium of a less powerful stream against weaker material. However, we lack information by which to solve this problem. A tentative resoration of the course of Kaskaskia River when it made the large meanders gives 1.43×10^{-4} slope. This is a very rough estimate but shows a less slope than the present. We can solve the equation $Q = (k/S)^2$ where Q is mean discharge, k a constant, and S the slope. We will use Leopold's value for k expressed as 397×10^{-4} and this estimated value for S. Then $Q = ((397 \times 10^{-4}) / (1.43 \times 10^{-4}))^2$ which is 277^2 or about 77,000 cubic feet per second which is more than the present-day flood maxiumum which has been recorded. No great weight can be attached to this result.

Width. We may next estimate the probable width of channel when the large meanders were formed. Since the old meanders were 7 times as large of those of today we might think that the channel width was 7 times as large or about 950 feet. There is no reason to think that the material of the floodplain between the meanders was much differeent than today.

The question is were the meanders of former times 7 times as large as now or should this be reduced to the 0.9 power as suggested by one of Leopold's formulas for wave length of meanders? If we take the former idea, the width may be estimated at 950 feet but under the second idea this figure is reduced to 478 feet. The formula is width (w) = a $Q^{.41}$ which when solved for Q is $((w/a))^{2.44}$. Next we must solve for the constant, a, by using modern values for Q and w. Let Q = 1505 and w = 135 then a = 135/19.95 which is 6.77. Two possible solutions for the former discharge are offered. Calling w = 950 we would have $Q = (950/6.77)^{2.44}$ or about 170,000 cubic feet per second. If we use the lower figure, for width this becomes $(480/6.77)^{2.44}$ or only about 32,000 cubic feet per second. The uncertainty of the figure for width is hence very important in the result because it is raised to a fairly high power. The constant a may also be incorrect because of the difference in the channel in former time. Leopold favored width

Wave length of meanders. The wave length of the meanders is not easy to measure on the map especially with the older meanders whose course is unknown. This quantity may be computed from Leopold and Wolman's formulas, or estimated from the values for radii given above. A ~~minimum~~ tentative value results from multiplying the radius by 2 pi. For radius 0.7 mile the result is 0.7 x 6.28 x 5280 = 23,200 feet. Leopold's formula is wave length = 6.5 $w^{1.1}$ feet. For width 480 feet this would make the result about 5800 feet. For width 950 the result is 12, 400 feet. This second approach should be the more accurate but yields surprisingly small results for meanders with a radius of 3696 feet.

Discharge from wave length. Discharge may be estimated from wave length in two ways. First, we may use the formula that discharge (Q) = $((w.l.)/36)^2$ Second we can equate Leopolds two formulas and eliminate wave length. Then $36\ Q^{\frac{1}{2}} = 6.5\ w^{1.1}$ and $Q = (.181\ w^{1.1})^2$. ~~which is exactly what was used above and hence yields the same result~~. For width 480 this yields about 25,800 cubic feet per second and for width 950 about 118,200 cubic feet about 26,000 cubic feet per second and for width 950 feet about 118,000 cubic feet per second. The other method gives with 5800 feet for wave length ~~with~~ a discharge of about 26,000 cubic feet per second. For wave length of 12,400 feet we obtain about 118,000 cubic feet per second discharge. The two methods are based on channel width and hence agree closely.

Summary. We have shown that the available formulas all indicate a much larger discharge in former times than now occurs but tell nothing of why this was a fact. In this glaciated region in which Kaskaskia River is situated the presumption is strong that this increase in discharge was due to floods of meltwater but we have not excluded the possibility of a moister climate. More will have to be known of glacial climate to decide this problem. The effect of change in slope on the total energy of the river is also unknown. The same remark may be made about the effect of a possible change in material of the banks although this factor seems a rather remote possibility. In evaluating the hypothesis of control before of meander size by discharge it has long been noted that small streams make small meanders and large streams make large meanders. Unfortunately this conclusion rests chiefly on map study and neglects both slope and bank material. The fact that a similar reduction in size of meanders occurred in the Driftless Area shows that glacial meltwater floods are not the answer in all localities.

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SOME UNDERFIT STREAMS IN CENTRAL ILLINOIS.

F. T. Thwaites

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A study of the material of the fill and the use of formulas published by Slichter demonstrate that such underflow is entirely inadequate for an explanation unless the normal discharge of the surface stream is very small.

Modern knowledge shows, however, that there are other factors involved than discharge only. Both slope and material of the bed are factors which cannot be overlooked. An alluvial fill in an older valley must in most localities reduce the slope from that of the older valley and at the same time introduce material which is in most instances of less resistance to erosion. Friedkin showed by experiment that both the width and length of bends, that is size of meanders, increase with slope. It is evident that the valley of Kaskaskia River has received an alluvial fill since it was first eroded but the writer has found no data on how much the slope was altered because the thickness of fill is not known. The present floodplain is silty clay whereas most of the older valley was eroded in glacial till which presumabley offers greater resistance to erosion. Just how to reconcile these known changes with a probable change in discharge due to the disappearance of the glacier is difficult to decide. It is presumed that the fill resulted not from change of level of the land or change of climate but from blocking of the outlet of Kaskaskia River with outwash of a later glaciation. We must realize that the same phenomenon of a change to small meanders on a floodplain has also been noted in the Driftless Area (Bates).

Formulas. When the writer was in the field and first began to write up the results there was little knowledge of the problems involved and no formulas had been suggested by which the amount of possible change in discharge could be estimated. Now the studies of Leopold, Maddock, Wolman, and Friedkin offer possible solutions. The first three worked out a number of equations which show the relation of various dimensions of streams to discharge. The equations are what is known as __empirical__, that is they tell nothing of the physical relations which caused the changes. __Rational__ equations differ in that they are based directly on the forces which are involved whereas empirical equations were derived by plotting the data for the most part on logarithmic coordinates. When points are plotted, they approximate a straight line showing that there is a mathematical relationship. Failure to fall ~~xxx~~ exactly on a line is termed __scatter__. It is due not only to errors of measurement but also to the neglect of other factors than those which were considered. Throughout all hydraulic phenomena it is known that several factors may enter into any relationship. When the plotting is on logarithmic coordinates, a straight line indicates a power function and the slope of the straight line shows the value of the exponent. The work of Leopold and his associates shows that most relationships are power functions. Different dimensions are referred to discharge (Q). The sum of the exponents of the dimensions width, depth, and velocity must equal unity for discharge is the product of these three. For observations at a fixed locality or station the average they report is that width is related to the 0.26 power of discharge, depth to the 0.40 power, and velocity to the 0.34 power. The units employed are British Engineering Units, (feet and seconds.) The exponents of discharge tell of relationships when discharge changes. Another set of results gives conditions downstream where the discharge at a given stage increases in that direction. Here width is the 0.5 (square root) power of discharge, depth the 0.4 power, and velocity the 0.1 power. It seems probable that the first set will apply best to the problem now in hand. Besides the exponent every equation also involves a constant. These constants vary greatly and are not given by Leopold and Wolman. It is not known just what they are related to but probably a number of factors are involved. Lacking knowledge of the values of the constants we can derive each one by substituting the modern values of other parts of the equations. This method undoubtedly involves error, for conditions of a stream vary when discharge and bed material vary. Besides the formulas given above Leopold and his associates also offer expressions for slope and for wave length of meanders. Slopes are given both as feet per mile and as feet per foot, which is generally a very small quantity. Wave length is the distance along the channel in which there is a complete reversal of direction. Slope (S) is related to the 0.49 power of discharge which is so close to the square root that the difference may be ignored. $S = (3.97 \times 10^{-6}) / Q^{\frac{1}{2}}$ where slope is given in feet per foot. Two expressions for wave length are $36\ Q^{\frac{1}{2}}$ and $6.5\ w^{1.1}$ (w = width). From these it is thought that width is related to the 0.9 power of any linear dimension of the meanders. A dimension considered by the writer but not by others is the radius of curvature of meanders. This is because the force directed against the bank by unit mass of water is the angular acceleration of this unit mass. Text books of elementary physics show that this acceleration is proportional to the square of the velocity and inverse to the radius. It did not prove difficult to measure the radius of the circle which approximates to the form of any meander at any particular spot. Force is accleration of unit mass. From Manning's formula it will be seen that with other things equal the velocity squared of a stream of water is proportional to the slope. Hence, the formula for force exerted by unit mass of water on the bank is proportional to slope divided by radius (S/r).

Data. Data are more complete for Kaskaskia River than for the Embarrass. The reports of the U. S. Geological Survey place the mean discharge of the Kaskaskia at 1505 cubic feet per second (second feet) based on observations over a period of 42 years. The maximum recorded flood is given as 52,000 cubic feet per second. There is no statement as to the mean width or mean depth at the gaging stations. Width is scaled from the drainage maps as 135 feet. This corresponds to the width of the normal channel or "bank-full" stage. The same maps place the average channel slope at 1 foot in 5740 feet or 1.74×10^{-4}. The radius of the present meanders is about 0.1 mile and that of the old meanders shown in the bluffs is about 0.7 mile. There is no infomation on the depth of alluvium to glacial till or bedrock. The bluffs rise to a maximum height of about 100 feet above the present floodplain.

The problem. From the discussion given previously all that we can conclude definitely on change of size of meanders is that meandering is controlled by the portion of the total energy of a stream which is directed against the banks in relation to the resistance of the banks to erosion. Energy is related to velocity of the stream which is controlled by slope, size of channel, and nature of the bottom. In this complex problem it is clear that discharge is only one factor

However, an attempt will be made to compute possible changes in mean discharge which might have resulted either from meltwaters of the Tazewell substage of the Wisconsin stage of glaciation or from a more rainy climate than that of the present. None of the formulas tells anything of the effect of bank material on size of meanders.

Slope. Leopold's equation relating slope to discharge is an at ractive means to solve this problem. He gives the constant as 3.97×10^{-6} when slope is measured in feet per foot. If we use the present-day value for discharge, however, we find a constant of 0.89×10^{-6} instead If we use Leopold's constant for an estimated discharge of 50,000 cubic feet per second we obtain a slope of 1.77×10^{-6}.

This raises the question of the proper value of the constant under former conditions. It is almost impossible to find the slope of the channel when the old large meanders were formed. It is certain the present scars were not formed all at once. If we conclude that the fill of modern alluvium thickens downstream the old valley eroded into the till banks was steeper than the presnet channel. The writer suggests that meanders grow until the force aggainst the banks is balanced by the resistance of the banks to erosion. That is the smaller meanders of today represent a state of equilibrium of a less powerful stream against weaker material. However, we lack information by which to solve this problem. A tentative resoration of the course of Kaskaskia River when it made the large meanders gives 1.43×10^{-4} slope. This is a very rough estimate but shows a less slope than the present. We can solve the equation $Q = (k/S)^2$ where Q is mean discharge, k a constant, and S the slope. We will use Leopold's value for k expressed as 397×10^{-4} and this estimated value for S. Then $Q = ((397 \times 10^{-4}) / (1.43 \times 10^{-4}))^2$ which is 277^2 or about 77,000 cubic feet per second which is more than the present-day flood maxiumum which has been recorded. No great weight can be attached to this result.

Width. We may next estimate the probable width of channel when the large meanders were formed. Since the old meanders were 7 times as large of those of today we might think that the channel width was 7 times as large or about 950 feet. There is no reason to think that the material of the floodplain between the meanders was much differeent than today.

However, we will attempt to compute possible changes in mean discharge which might be due either to cessation of meltwaters from the Tazewell substage of the Wisconsin stage of glaciation or to a change from a more rainy climate. None of the formulas tells us anything of change in material into which meanders were eroded.

Slope. Leopold's equation relating slope to discharge is attractive as a means to solve the above problem. Hix constant averaged from streams he considered is 3.97×10^{-6} when slope is measured in feet per foot. If we use the present value of mean discharge we obtain a somewhat higher value of 4.47×10^{-6} for the constant. which is not enough different to alarm us.

The question is were the meanders of former times 7 times as large as now or should this be reduced to the 0.9 power as suggested by one of Leopold's formulas for wave length of meanders? If we take the former idea the width may be estimated at 950 feet but under the second idea this figure is reduced to 478 feet. The formula is width (w) = a $Q^{.41}$ which when solved for Q is $((w/a))^{2.44}$. Next we must solve for the constant, a, by using modern values for Q and w. Let Q = 15~~2~~05 and w = 135 then a = 135/19.95 which is 6.77. Two possible solutions for the former discharge are offered. Calling w = 950 we would have Q = $(950/6.77)^{2.44}$ or about 170,000 cubic feet per second. If we use the lower figure for width this becomes $(480/6.77)^{2.44}$ or only about ~~17,500~~ 32,000 cubic feet per second. The uncertainty of the figure for width is hence very important in the result because it is raised to a fairly high power. The constant a may also be incorrect because of the difference in the channel in former time.

Wave length of meanders. The wave length of the meanders is not easy to measure on the map especially with the older meanders whose course is unknown. This quantity may be computed from Leopold and Wolman's formulas, or estimated from the values for radii given above. A minimum value results from multiplying the radius by 2 pi. For radius 0.7 mile the result is 0.7 x 6.28 x 5280 = 23,200 feet. Leopolds formula is wave length = 6.5 $w^{1.1}$ feet. For width 480 feet this would make the result about 5800 feet. For width 950 the result is 12, 400 feet. This second approach should be the more accurate but yields surprisingly small results for meanders with a radius of 3696 feet.

Discharge from wave length. Discharge may be estimated from wave length in two ways. First, we may use the formula that discharge (Q) $= ((w.l.)/36)^2$ Second we can equate Leopolds two formulas and eliminate wave length. Then $36\,Q^{\frac{1}{2}} = 6.5\,w^{1.1}$ and Q = $(.181\,w^{1.1})^2$. ~~which is exactly what was used above and hence yields the same result~~. For width 480 this yields about

about 26,000 cubic feet per second and for width 950 feet about 118,000 cubic feet per second. The other method gives with 5800 feet for wave length ~~with~~ o discharge of about 26,000 cubic feet per second. For wave length of 12,400 feet we obtain about 118,000 cubic feet per second discharge. The two methods are based on channel width and hence agree closely.

Summary. We have shown that the available formulas all indicate a much larger discharge in former times than now occurs but tell nothing of why this was a fact. In this glaciated region in which Kaskaskia River is situated the presumption is strong that this increase in discharge was due to floods of meltwater but we have not excluded the possibility of a moister climate. More will have to be known of glacial climate to decide this problem. The effect of change in slope on the total energy of the river is also unknown. The same remark may be made about the effect of a possible change in material of the banks although this factor seems a rather remote possibility. In evaluating the hypothesis of control of meander size by discharge it has long been noted that small streams make small meanders and large streams make large meanders. Unfortunately this conclusion rests chiefly on map study and neglects both slope and bank material. The fact that a similar reduction in size of meanders occurred in the Driftless Area shows that glacial meltwater floods are not the answer in all localities.

25,800 cubic feet per second and for width 950 about 118,200 cubic feet per second. The other method gives for wave length 5800 feet 26, 300 cubic feet per second, for wave length 12400 feet about 118,000 cubic feet per second. This shows that the two methods agree closely.

Summary. Working on the hypothesis that size of meanders is closely related to discharge only we have obtained a variety of results for mean discharge many of them greater than any recorded flood of modern times. All of them are much above the present mean discharge. This is in line with map study of rivers which shows clearly that big streams have big meanders and small streams small meanders. Most comparisons do not include slope or material so that too much weight should be given to this ~~conclusion~~ map study. The possible climate of Pleistocene times of glaciation is little understood. Rainfall could have been greater than now south of the glacier although this does not agree with the idea of winds descending from the ice cap. Meltwaters seem a more logical cause for a larger discharge.
Further study is needed to connect these large meanders with the zone of outwash deposition nearer to the ice front.

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SOME UNDERFIT STREAMS IN CENTRAL ILLINOIS.

Introduction. The term "underfit" is applied to rivers which show modern floodplain meanders which are much smaller than older meanders which left traces in the adjacent bluffs. During the course of a survey of road materials in central Illinois for the Illinois State Geological Survey during the field seasons of 1929 and 1930 the writer observed the above phenomenon along both Kaskaskia and Embarrass rivers. Figure 1 was traced from the drainage surveys of ~~these~~ Kaskaskia River made by the U. S. Geological Survey in 1908-1911. These maps are much more detailed than the later Ramsey and St. Elmo quadrangles which show the area along ~~Kaskaskia~~ this River. The meander cusps of the eastern bluffs of Kaskaskia River are very striking and instances occur of cutting through of ~~such~~ spurs which resulted in capture of a tributary stream. Only one example of an old large meander was found on Embarrass River just southeast of Newton. Both of these rivers carried meltwater from the Wisconsin glaciers which passed through the area of pre-Wisconsin drift. There may be other examples of the same type but the writer has not discovered them.

Hypotheses. Similar underfit streams have been described by a number of students of physiography. (Davis, Dury) It is recognized that the size of meanders is related to the discharge of a stream ~~in most instances~~ although other factors ~~may~~ also enter the picture. Most students have looked no further than change in discharge and have suggested diversion of headwaters by stream capture, (Davis) change of climate, (Dury) or seepage through the alluvial fill of the valley (Davis). The last has been termed Lehmann's principle but a study of ground waters (Slichter) shows that such subsurface flow is small in comparison with channel discharge even where the alluvial fill is very permeable. Unless the surface stream were very small it is evident that such underground flow ~~could~~ would be of no importance. Both slope and material of bed and banks are factors which cannot be overlooked. An alluvial fill in an older valley normally reduces the slope of the modern stream compared with that of its predecessor. Friedkin has shown by experiment that both width and length of bends that is size of meanders, increases with slope. Although we must keep the above fact in mind because the bed of Kaskaskia River, and probably other streams, has undoubtedly been filled since the maximum erosion of the beds it seems probably that the major factor in decreasing the size of meanders has been a diminished mean discharge. No subsurface data are available to the writer on how much the slopes have been diminished by this alluvial fill. The valleys of the streams observed by the writer were eroded into the Illinoian till plain. Later glaciation obstructed the outlets of these rivers with outwash which caused aggradation not only to the level of the obstruction but upstream. The same phenomenon also occurred in streams which head in the Driftless Area and hence carried no outwash. (Bates).

Formulas. When the writer visited central Illinois in 1929 and 1930 no means existed by which an estimate could be made of the possible maximum discharge of the rivers, ~~when the volume was greater~~. Now the studies of Leopold, Maddock, and Wolman give a clue to this problem. These students of hydraulic phenomena have developed a number of equations showing the relation of various dimensions of streams to discharge. The equations are what is known as empirical, that is they tell nothing of the physical relations which cause the variations. It is on such causes that rational equations are based. Empirical equations result from plotting the quantities on logarithmic coordinates. When this is done the points fall so ~~in such a distribution~~ that a straight line can be drawn through them, the slope of which shows the exponent of a power function which gives ~~shows~~ the relationship. In ~~all~~ such studies the points fail to all fall exactly in one straight line, that is there is scatter. In part this is due to inaccuracies of the data and in part to the fact that other factors were negleced. For instance the formula which shows the mean velocity of water in an open channel depends ~~mainly~~ on three major factors, ~~xx~~ slope, size of the channel, and nature of the bottom. Other minor controls such as temperature are neglected. Leopold and Maddock give the following: at a given location (station) mean width is related to discharge to the 0.26 power, depth to the 0.41 power and velocity to the 0.34 power of discharge. All measurements

are given in British engineering units, feet and seconds. For points downstream width is rlealted to t he 0.5 (square root) power of discharge, depth to the 0.4 power of discharge and velocity to the 0.1 power of the same quantity. These exponents in a downstream direction could be of some value in making comparisons of conditions with a former larger discharge in a given stream. It must be remembered that there is another quantity in each equation, ~~namely~~ a constant. This constant ~~is variable~~ varies in different streams and is not given in Leopold's papers. E xpressions for wave length of meanders are given in the paper by Leopold and Wolman. Wave length is defined as the distance along the channel is which a complete circle is traversed so that the direction of the current is reversed. It is not easy to measure this quantity on maps because of the many irregularities of stream course. The two expressions are 36 $Q^{\frac{1}{2}}$ and 6.5 $w^{1.1}$
where Q = discharge and W = width and hence width ÷ $WL^{.9}$
Both were derived in the same way by platting on logarithmic coordinates. In another paper Leopold gives the expression for slope as $S = (3.97 \times 10^{-6}) / Q^{-.49}$ The latter quanity is so close to $Q^{\frac{1}{2}}$ that the difference can be ignored. The mean velocity of a stream is given by Manning's formula. In this v (ft/sec.) = ~~is~~ $(1.5/n)R^{2/3}S^{\frac{1}{2}}$ where n is a factor which varies from 0.05 down depending on the nature of the bottom, R is hydraulic radius (cross ection in square feet divided by wetted width of channel, and S is slope in feet per foot. In wide streams R is closely equivalent to mean depth, d. Rotational force of unit mass of water as exerted on the outside of a curve is v^2/r where r is radius of the circle which approximates to the shape of the bank at the given point and v = velocity. Since with other dimensions equal, velocity^2 is proportional to S this equation may be approximated as S/r.

<u>Data</u>. For Kaskaskia River we have the reports of the U. S. Geological Survey which put the mean discharge at 1505 cubic feet per second based on measurments over a period of 42 years. The maximum flood discharge is given as 52,000 cubic feet per second. The width of the normal channel or "bank-full" stage was scaled from the drainage surveys at 135 feet. Slope of the modern stream is found from the same source to be 1 foot in 5740 feet or as ordinarily expressed as the tangent of the angle of slope, 1.74×10^{-4} Radius of the present meanders is about 0.1 mile

and that of the older meanders shown in the bluffs 0.7 mile. Width of the present meander belt is about 0.5 mile and of the older meander belt about 3.5 miles. Nothing is known of the thickness of recent alluvium in the Kaskaskia ~~at~~ bottoms nor of the relative resistance to erosion of ~~kx~~ the silty clay of the modern bottoms compared to the clay till of the bluffs which rise 4o to nearly 100 feet above the prewent day floodplain.

<u>The problem</u>. The problem now arises whether the change in size of meanders was due to diminished volume and hence diminished energy or to reduction of slope with the same result, or to a change in erodibility of the bank material from till to silty clay? In seeking an answer on the dominance of one of these three possi bilities we will explore in turn the several equations which deononstrate relationships.

<u>Slope</u>. The equation $S = k/Q^{\frac{1}{2}}$ gives an empirical relation (Leopold). K represents a constant for which Leopold gives the value of 3.97×10^{-6} However, if we solve this equation for k using the present day values for the Kaskaskia River we will find that the that $k = S\,Q^{\frac{1}{2}}$ ~~becomes k~~ or $= 1.74 \times 10^{-4} \times 38.8$ Solving for k the result is 0.68×10^{-6} or much less than the average value given above. Is it correct to use the average value or the present value with the modern small stream? We cannot answer this question at present. We may also consider the fact that formation of the present small meanders is due to reduction in power of the stream, that is ~~in~~ its work within unit time. Can this be explained either by change in material or by reduction of slope or must we turn, as most have, to reduction in volume? It is very hard to restore a former course of Kaskaskia River when it was flowing in the large meanders and its fall is undeterminable. It may be presumed that when the width of the meander belt was greater than it now is the channel slope, not the slope on the center line of the valley, was less than it now is. If, however, the reduction in power of the stream were due to change of slope only, then its former slope ought to have been greater than it now is. This can be the case for the slope of the center line of the valley and agrees with the probable thinning of the alluvial fill upstream.

A similar train of reasoning also affects the hypothesis that meanders grew until the rotational force of the current, v^2 / r is equal to the resistance of the bank to erosion. In this case the small meanders of today in soft material require either a lesser slope or a reduced discharge. When we solve the slope equation to $Q = (k/S)^2$ it is exceedingly diffcult to supply any reasonable values. For k we may take the ~~larger~~ value given by Leopold namely $.397 \times 10^{-4}$ and take a tentative expression for slope derived by assumed fall and course when the large meanders were eroded. Then we have $Q = ((.397 \times 10^{-4}) / 1.43 \times 10^{-4}))^2$ This is 277^2 or 77000 cubic feet per second. No great weight can be attached to this result.

<u>Width.</u> Because we have no information on either mean depth or mean velocity we have to turn to width for a possible solution. ~~Since width is a linear quanity~~ We may tentatively assume that the width when the large meanders were eroded was 7.9 times that of the present because the meanders and meander belt were 7.9 times as large as they now are. The equation is w (width) = $aQ^{.41}$ We can solve this for a, the constant, by using present values. a = 135/19.95 or 6.77 Now if we take the former width as ~~950~~ ~~575~~ 480 feet the discharge, Q, figures out from $Q = (\cancel{950}\ \cancel{575}\ 480/a)^{2.44}$ as $(\cancel{950}\ \cancel{575}\ 480 / 6.77)^{2.44}$ or ~~169,800~~ ~~51,300~~ 18,840 cubic feet per second. The objection to this conclusion is that the constant, a, may not have had the same value when the discharge, Q was larger than it now is.

<u>Wave length of meanders.</u> The wave length of meanders is not easy to measure on the map and ~~it~~ is impossible ~~to do so for~~ with the older large meanders. It may be computed from one of Leopold's formulas which is related to width or by computing the circumferance of a circle with the measured radius. The first method uses the formula that the wave length = $6.5\ w^{1.1}$ Assuming again that w in former times was ~~950~~ 575 feet, its 1.1 power is ~~1884~~ 1096 feet. Multiplying this by 6.5 the result is ~~12250~~ 7154 feet. By the second method it is 2 Pi x .7 mile x 5280 feet. This figures out to 23, 200 feet, much larger than appears from the first method. Using the first value and substituting in the formula that ~~wave length = 36 $Q^{\frac{1}{2}}$ we obtain 116500~~ cubic feet per second.

$$Q = \left(\frac{WL}{36}\right)^2 = \left(\frac{7154}{36}\right)^2 = 200^2 = 40000$$

The other value for wave length gives 413449 cubic feet per second which is out of line.

Another solution is to equate Leopold's two formulas for wave length. Then $36\ Q^{\frac{1}{2}} = 6.5\ w^{1.1}$ Soulution ~~of~~ in this yields $Q = (6.5\ w^{1.1} / 36)^2$ x 1096 $\doteq \left(\frac{7154}{36}\right)^2 = 200^2$

stet ~~and substiution of the value of w given above naturally gives the same value for discharge or Q as above.~~ The above solutions avoid the problem of correct constant for they are based on streams of different discharges. On the other hand the assumed width at the time of the large meanders is far from certain.

Summary. The cause of change in size of meanders cannot be explained by a single ~~cause~~ factor. Discharge, slope, and bank material all enter the question. We cannot determine the slope of the channel at the time the big meanders occured. The compartive dimensions of the river at that time are also uncertain. The results of computations depend too largely on an assumed width. On the whole, it is simpler to conclude that a change of discharge was the major factor. To explain a former increased discharge two possible hypotheses can be advanced. First, the climate could have been much wetter than it now is, possibly during glaciation of the area to the north. We do not know ~~about~~ enough about glacial climate to decide on this hypothesis which is that of Dury. Second, although the Kaskaskia drained only about 25 miles of the ice front of the Tazewell substage of the Wisconsin Stage of glaciation, it is plausible to suppose that meltwaters swelled the discharge and that meanders developed below the zone of accumulation of coarse glacial outwash. Map study failed to show that the traces of big meanders extend up to the ice front on either Kaskaskia or Embarrass rivers. In most rivers the waters from the retreating ice front eroded the outwash into terraces. Not enough is known of the relations in Illinois ~~to~~ of ~~related~~ outwash terraces to the alluvial fill in the district of big meanders. The writer tentatively concludes that diminution of discharge due to the ~~melting~~ disappearance of the ice sheet accounts for the observed phenomena although similar underfit streams also exist in the Driftless Area on streams which never carried glacial meltwaters. It should be noted that on the floodplain of Mississippi River ~~the~~ small tributaries have small meanders compared to those of the Mississippi.

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SOME UNDERFIT STREAMS IN CENTRAL ILLINOIS.

<u>Introduction</u>. The term "underfit" is applied to rivers which show floodplain meanders which are much smaller than older meanders which left traces in adjacent bluffs. During the course of a survey of road materials in central Illinois for the Illinois State Geological Survey during the field seasons of 1929 and 1930 the writer observed the above phenomenon along both Kaskaskia and Embarrass rivers. Figure 1 was traced from the drainage surveys of these rivers made by the U. S. Geological Survey in 1908-1911. These maps are much more detailed than the later Ramsey and St. Elmo quadrangles which show the area along Kaskaskia River. The meander cusps of the eastern bluffs of Kaskaskia River are very striking and instances occur of cutting through of such spurs which resulted in capture of a tributary stream. Only one example of an old large meander was found on Embarrass River just southeast of Newton. Both of those rivers carried meltwater from the Wisconsin glaciers which passed through the area of pre-Wisconsin drift. There may be other examples of the same type but the writer has not discovered them.

<u>Hypotheses</u>. Similar underfit streams have been described by a number of students of physiography (Davis, Dury). It is recognized that the size of meanders is related to the discharge of a stream ~~in most instances~~ although other factors ~~may~~ also enter the picture. Most students have looked no further than change in discharge and have suggested diversion of headwaters by stream capture (Davis), change of climate (Dury), or seepage through the alluvial fill of the valley (Davis). The last has been termed Lahmann's principle but a study of ground waters (Slichter) shows that such subsurface flow is small in comparison with channel discharge even where the alluvial fill is very permeable. Unless the surface stream were very small it is evident that such underground flow could be of no importance. Both slope and material of bed and banks are factors which cannot be overlooked. An alluvial fill in an older valley normally reduces the slope of the modern stream compared with that of its predecessor.

Friedkin has shown by experiment that both width and length of bends that is size of meanders, increase~~s~~ with slope. Although we must keep t. fact in mind because the bed of Kaskaskia River and probably other streams, has undoubtedly been filled since the maximum erosion of the beds it seems probable that the major factor in decreasing the size of meanders has been a diminished mean discharge. No data are available to the writer on how much the slopes have been diminished by this alluvial fill. The valleys of the streams observed by the writer were eroded into the Illinoian till plain. Later glaciation obstructed the outletsof these rivers with outwash which caused aggradation not only to the level of the obstruction but upstream. The same phenomenon also occurred in streams which head in the Driftless Area and hence carried no outwash. (Bates) .

Formulas. When the writer visited central Illinois in 1929 and 1930 no means existed by which an estimate could be made of the possible maximum discharge of the rivers, ~~when the volume was greater~~. Now the studies of Leopold, Maddock and Wolman give a clue to this problem. These students of hydraulic phenomena have developed a number of equations showing the relation of various dimensions of streams to discharge. The equations are what is known as empirical, that is they tell nothing of the physical relations which cause the variations. It is on such causes that rational equations are based. Empirical equations result from plotting the quantities on logarithmic coordinates. When this is done the points fall so ~~in such a distribution~~ that a straight line can be drawn through them, the slope of which shows the exponent of a power function which gives ~~shows~~ the relationship. In all such studies the points fail to all fall in one straight ~~striaght~~ line, that is there is scatter. In part this is due to inaccuracies of the data and in part to the fact that other factors were negleced. For instance the formula which shows the mean velocity of water in an open channel depends ~~mainly~~ on three major factors, ~~vz~~ slope, size of the channel, and nature of the bottom. Other minor controls such as temperature are neglected. Leopold and Maddock give the following:

at a given location (station) mean width is related to discharge to the 0.26 power, depth to the ~~0.24~~ .41 power and velocity to the 0.34 power. All measurements are given in British engineering units, feet and seconds. For points downstream width is rlealted to t he 0.5 (square root) power of discharge, depth to the 0.4 power of discharge and velocity to the 0.1 power of the same quanitity. These exponents in a downstream direction could be of some value in making comparisons of conditions with a former larger discharge in a given stream. It must be remembered that there is another quantity in each equation, namely a constant. This constant is variable in different streams and is not given in Leopold's papers. E xpressions for wave length of meandrs i are given in the paper by Leopold and Wolman. Wave length is defined as the distance along the channel is which a complete circle is traversed so that the direction of the current is reversed. It is not easy to measure this quantity on maps because of the many irregularities of stream course. The two expressions are $36\ Q^{\frac{1}{2}}$ and $6.5\ w^{1.1}$ Both were derived in the same way by platting on logarithmic coordinates. In another paper Leopold gives the expression for slope as $S = (3.97 \times 10^{-6}) / Q^{-.49}$ The latter quanity is so close to $Q^{\frac{1}{2}}$ that the difference can be ignored. The mean velocity of a stream is given by Manning's formulaX In this v (ft/sec.) = ~~is~~ $(1.5/n)\ R^{2/3} S^{\frac{1}{2}}$ where n is a factor which varies from 0.05 down depending on the nature of the bottom, R is hydraulic radius (cross ection in square feet divided by wetted width of channel, and S is slope in feet per foot. In wide streams R is closely equivalent to mean depth, d. Rotational force of unit mass of water as exerted on the outside of a curve is v^2/r where r is radius of the circle which approximates to the shape of the bank at the given point. Since with other dimensions equal velocity^2 is proportional to S this equation may be approximated as S/r

<u>Data</u>. For Kaskaskia River we have the reports of the U. S. Geological Survey which put the mean discharge at 1505 cubic feet per second based on measurments over a period of 42 years. The maximum flood discharge is given as 52,000 cubic feet per second. The width of the normal channel or "bank-full" stage was scaled from the drainage surveys at 135 feet. Slope of the modern stream is found from the same source to be 1 foot in 5740 feet or as ordinarily expressed as the tangent of the angle of slope, 1.74×10^{-4} Radius of the present meanders is about 0.1 mile

and that of the older meanders shown in the bluffs 0.7 mile. Width of the present meander belt is about 0.5 mile and of the older meander belt about 3.5 miles. Nothing is known of the thickness of recent alluvium in the Kaskaskia ~~bi~~ bottoms nor of the relative resistance to erosion of ~~ak~~ ~~e~~ the silty clay of the modern bottoms compared to the clay till of the bluffs which rise 40 to nearly 100 feet above the present day floodplain.

<u>The problem</u>. The problem now arises whether the change in size of meanders was due to diminished volume and hence diminished energy or to reduction of slope with the same result, or to a change in erodibility of the bank material from till to silty clay? In seeking an answer on the dominance of one of these three possibilities we will explore in turn the several equations which deononstrate relationships.

<u>Slope</u>. The equation $S = k/Q^{\frac{1}{4}}$ gives an empirical relation. K represents a constant for which Leopold gives the value of 3.37×10^{-6} However, if we solve this equation for k using the present day values for the Kaskaskia River we will find that the that $k = S\,Q^{\frac{1}{4}}$ ~~becomes~~ is $k = 1.74 \times 10^{-4} \times 38.8$ Solving for k the result is 0.68×10^{-6} or much less than the average value given above. Is it correct to use the average value or the present value with the modern small stream? We cannot answer this question at present. We may also consider the fact that formation of the present small meanders is due to reduction in power of the stream, that is in work within unit time. Can this be explained either by change in material or by reduction of slope or must we turn, as most have, to reduction in volume? It is very hard to restore a former course of Kaskaskia River when it was flowing in the large meanders and its fall is undeterminable. It may be presumed that when the width of the meander belt was greater than it now is the channel slope, not the slope on the center line of the valley, was less than it now is. If, however, the reduction in power of the stream were due to change of slope only, then its former slope ought to have been greater than it now is. This can be the case for the slope of the center line of the valley and agrees with the probable thinning of the alluvial fill upstream.

A similar train of reasoning also affects the hypothesis that meanders grew until the rotational force of the current, v^2 / r is equal to the resistance of the bank to erosion. In this case the small meanders of today in soft material require either a lesser slope or a reduced discharge. When we solve the slope equation to $Q = (k/S)^2$ it is exceedingly difficult to supply any reasonable values. For k we may take the ~~lower~~ value given by Leopold namely 397×10^{-4} and take a tentative expression for slope derived by assumed fall and course when the large meanders were eroded. Then we have $Q = ((397 \times 10^{-4}) / 1.43 \times 10^{-4}))^2$ This is 277^2 or 77000 cubic feet per second. No great weight can be attached to this result.

Width. Because we have no information on either mean depth or mean velocity we have to turn to width for a possible solution. Since width is a linear quantity we may tentatively assume that the width when the large meanders were eroded was 7 times that of the present because the meanders and meander belt were 7 times as large as they now are. The equation is w (width) = $aQ^{.41}$ We can solve this for a, the constant by using present values. a = 135/19.95 or 6.77 Now if we take the former width as 950 feet the discharge, Q, figures out from $Q = (950/a)^{2.44}$ as $(950/ 6.77)^{2.44}$ or 169,800 cubic feet per second. The objection to this conclusion is that the constant, a may not have had the same value when the discharge, Q was larger than it now is.

Wave length of meanders. The wave length of meanders is not easy to measure on the map and ~~it~~ is impossible ~~to do so~~ for the older large meanders. It may be computed from one of Leopold's formulas which is related to width or by computing the circumferance of a circle with the measured radius. The first method uses the formula that the wave length = $6.5\ w^{1.1}$ Assuming again that w in former times was 950 feet, its 1.1 power is 1884 feet. Multiplying this by 6.5 the result is 12250 feet. By the second method it is 2 Pi x .7 mile x 5280 feet. This figures out to 23, 200 feet, much larger than appears from the first method. Using the first value and substituting in the formula that wave length = $36\ Q^{\frac{1}{2}}$ we obtain 116500 cibic feet per second.

Another solution is to equate Leopold's two formulas for wave length. Then $36\,Q^{\frac{1}{2}} = 6.5\,w^{1.1}$ Solution of this yields $Q = (6.5\,w^{1.1}/36)^2$ and substiution of the value of w given above naturally gives the same value for discharge or Q as above. The above solutions avoid the problem of correct constant for they are based on streams of different discharges. On the other hand the assumed width at the time of the large meanders is far from certain.

Summary. The cause of change in size of meanders cannot be explained by a single cause. Discharge, slope, and bank material all enter the question. We cannot determine the slope of the channel at the time the big meanders occured. The compartive dimensions of the river at that time are also uncertain. The results of computations depend too largely on an assumed width. On the whole, it is simpler to conclude that a change of discharge was the major factor. To explain a former increased discharge two possible hypotheses can be advanced. First, the climate could have been much wetter than it now is, possibly during glaciation of the area to the north. We do not know ~~about~~ enough about glacial climate to decide on this hypothesis which is that of Bury. Second, although the Kaskaskia drained only about 35 miles of the ice front of the Tazewell substage of the Wisconsin tage of glaciation it is plausible to suppose that meltwaters swelled the discharge and that meanders developed below the zone of accumulation of coarse glacial outwash. Map study failed to show that the traces of big meanders extend up to the ice front on either Kaskaskia or Embarrass rivers. In most rivers the waters from the retreating ice front eroded the outwash into terraces. Not enough is known of the relations in Illinois ~~to~~ of ~~relate~~ outwash terraces to the alluvial fill in the district of big meanders. The writer tentatively concludes that diminution of discharge due to the melting of the ice sheet accounts for the observed phenomena although similar underfit streams also exist in the Driftless Area on streams which never carried glacial meltwaters. It should be noted that on the floodplain of Mississippi River ~~the~~ small tributaries have small meanders compared to those of the Mississippi.

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AGU
1515 Mass Ave NW
Washington 5 D C

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Some Underfit Streams of Central Illinois

Underfit rivers have floodplain meanders which are much smaller than the meanders cut into the adjacent bluffs. During the course of a study of sources of road materials in central Illinois for the Illinois State Geological Survey during the years 1929 and 1930, the writer noted this phenomenon along both the Kaskaskia and Embarrass rivers. The accompanying map was traced from the drainage surveys along Kaskaskia River of the U. S. Geological Survey made from 1908 to 1911. These show several striking meander cusps, mainly along the eastern bluffs (Fig. 1), which suggest a stream several times the volume of the present river. Attention should be directed also to several cases of intercision where such meanders cut through spurs and captured tributaries. A good example of a large meander was also noted on Embarrass River just southeast of Newton.

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Similar underfit streams have been described by several physiographers. Davis thought that the cause was diminution of volume, for it has long been recognized that the radius of meanders is related to the discharge of streams. Loss of volume has been ascribed either to diversion by stream capture, to change of climate, or to seepage into the valley filling. The latter is an inadequate explanation because the underflow even in relatively coarse material is very small compared to the dischage of most streams (Slichter). In the case of these Illinois rivers the cause is probably cessation of drainage from the Wisconsin ice sheet. The localities are below the locus of coarse outwash deposits. Here the glacial waters had become integrated into a single meandering stream unlike the braided complex above where active deposition took place.

In the early 1930's no information was available which could supply a reasonable estimate of the change in volume of Kaskaskia River from glacial times to the present. Now the studies of Leopold and Maddock supply some ground for such a surmise. The area has been mapped on the Ramsey and St. Elmo quadrangles. The radii of the meanders of the present river are about 1/10 mile whereas those shown by the scars in the banks of the bluffs average about 7/10 mile, seven times as much. Elementary physics shows that the lateral component of force of the river due to flowing in a segment of a circle is expressed by the formula, mass X velocity ^{2}/ radius. If we assumed that both sets of meanders reached equilibrium with the resistance of the banks we then have two problems: (a) how much more resistance did the older bluffs, 40 to nearly 100 feet high, ~~than~~ have in comparison to the banks of the recent meanders which average about 10 feet in height? and (b) how did the velocities of the rivers differ? To make an estimate of velocities it is necessary to know the hydraulic radius, R, (or mean depth, d, in feet,) the slope s, of the rivers involved in feet per foot, and the nature of the bottom (n). The formula involved is: $v = 1.5/n\ R^{2/3}\ s^{1/2}$. The slope is the least difficult to estimate. The present day Kaskaskia drops 10 feet in about 57,400 feet distance along the channel making a slope of 1 in 5740 or ~~1.75~~ 1.74 x 10^{-4} feet per foot. It is hard to restore the course of the glacial river, for the scars were certainly not all made at the same time. A tentative restoration suggests a drop of 30 feet in 40 miles which is 1 in 7030 or 1.43 x 10^{-4}. Although this is reasonable because a big river has a less slope than a small one, it is difficult to tell how much filling occurred on the old floodplain. The difference in slope may not be enough. Since the linear dimensions of the glacial and modern Kaskaskia bear a ratio of about 7 to 1, it might possibly be

have in comparison to

assumed that the glacial gradient was of a seventh of the present valve, u
about .25 X 10^{-4} feet per foot. This ratio is concluded from the width of the present meander belt which is about .5 mile and that of the glacial meander belt which is about 3.5 miles which checks with the radii of meanders given above. The present average width of the Kaskaskia is roughly 135 feet from which a glacial mean width of about 950 feet is deduced.

The studies of Leopold and Maddock provide a possible basis for quenitative comparison with glacial conditions. These authors find that the dimensions of a stream are power functions of the discharge (Q). Since we know the mean discharge (1505 cu. ft/sec) of the Kaskaskia over a period of 42 years, quantitative estimates may be attempted. The weakest point of these empirical equations is the fact that there is a constant, the value of which varies greatly in the case of different streams, probably reflecting difference in bed and banks. Two equations suggest applicability to the present problem. Slope, s,= k $Q^{-.49}$ where k is a constant and Q = discharge in cubic feet per second. This may also be
$K = S \times Q^{1/2}$
written S = k/$Q^{\frac{1}{2}}$. For present day western streams Leopold gives the value of K as 3.97 X 10^{-6} feet per foot. Solving for the modern mean discharge of the Kaskaskia, which is 1505 cu. ft/sec., we obtain a value K
OK of .68 x 10^{-6}. It is questionable that this is applicable to glacial conditions, because of the difference in nature of the bed and banks.

Solving the above slope equation for quanity of discharge it is evident that Q = $(K/S)^2$. By substituting the values for k at present and
OK
the minimum value assumed for the slope S, we find Q = (68 X 10^{-4} / .25 x 10^{-4}) 2
68^2
which works out to a little less than 74000 cu. ft/sec. If we use instead Leopold's original value of k and the higher estimate of slope of the glacial river, the result is 75,600 cu. ft./sec.

$\left(\frac{3.97 \times 10^{-4}}{1.43 \times 10^{-4}}\right)^2$

$\frac{39.7 \times 10^{-4}}{1}$

$(277)^2$

77000

The second equation involves mean width, w, $w = aQ^{.41}$ Substituting modern values, 135 feet = a x 20.04 (19.95) or a = 135/ 20.04 = 6.72 Using glacial values $950 = 6.72 \times Q^{.41}$ Solving above for Q, gives $Q = (950/6.72)^{2.44}$ which is 103000 cu ft./sec. Again the applicability of the constant, a, is debatable.

The above methods do not agree very closely but serve to give some sort of an idea of the mean discharge of the glacial Kaskaskia which drained not over 25 miles of the front of the Wisconsin ice sheet. It is perhaps not so large as some have imagined, for the maximum modern flood was estimated at slightly above 52,000 cu. ft./sec.

A recent paper by Leopold and Wolman gives two other formulas by which volume may be estimated. These relate to lamida, the wave length of the meanders which is the distance in feet of complete reversal of direction. It may be estimated as 2 pi times the radius of curviature. Formulas are lamida = $Q^{\frac{1}{2}}$36 and lamida = $6.5\ w^{1.1}$ Here we estimate lamida at 23.5 X 10^3 feet for the glacial stream. Solving the first equation which was derived empirically from a large number of observations the mean or "bank-full" discharge Q = lamida^2/ 36^2 Lamida squared is 538 X 10^6 Hence Q = 538 x 10^6/ 1296 410000 cu ft. per second, a result out of line with other computations. If we equate the two expressions for wave length we find that $Q = 6.5^2\ w^{2.2} / 36^2$ Taking w at 950 feet, the 2.2 power is 3.214 X 10^6 $6.5^2/ 36^2$ is .0325 which yields a result of 165000 cubic feet per second which is much closer to the earlier estimates.

Another value for λ = 12,250 based on w = 950

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$K = \frac{5}{Q^{1/2}}$

68 x 10-4

68 x 10-4

135 / 20.8

especially the contents which undoubtedly have changed

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Similar underfit streams have been described by several physiographers. Davis thought that the cause was diminution of volume, for it has long been recognized that the radius of meanders is related to the discharge of streams. Loss of volume has been ascribed either to diversion by stream capture, to change of climate, or to seepage into the valley filling. The latter is an inadequate explanation because the underflow even in relatively coarse material is very small compared to the dischage of most streams (Slichter). In the case of these Illinois rivers the cause is probably cessation of drainage from the Wisconsin ice sheet. The localities are below the locus of coarse outwash deposits. Here the glacial waters had become integrated into a single meandering stream unlike the braided complex above where active deposition took place.

In the early 1930's no information was available which could supply a reasonable estimate of the change in volume of Kaskaskia River from glacial times to the present. Now the studies of Leopold and Maddock supply some ground for such a surmise. The area has been mapped on the Ramsey and St. Elmo quadrangles. The radii of the meanders of the present river are about 1/10 mile whereas those shown by the scars in the banks of the bluffs average about 7/10 mile, seven times as much. Elementary physics shows that the lateral component of force of the river due to flowing in a segment of a circle is expressed by the formula, mass X velocity ^{2}/ radius. If we assumed that both sets of meanders reached equilibrium with the resistance of the banks we then have two problems: (a) how much more resistance did the older bluffs, 40 to nearly 100 feet high, than the banks of the recent meanders which average about 10 feet in height? and (b) how did the velocities of the rivers differ? To make an estimate of velocities it is necessary to know the hydraulic radius, R, (or mean depth, d, in feet,) the slope s, of the rivers involved in feet per foot, and the nature of the bottom (n). The formula involved is: $v = 1.5/n \, R^{2/3} S^{1/2}$. The slope is the least difficult to estimate. The present day Kaskaskia drops 10 feet in about 57,400 feet distance along the channel making a slope of 1 in 5740 or 1.75×10^{-4} feet per foot. It is hard to restore the course of the glacial river for the scars were certainly not all made at the same time. A tentative restoration suggests a drop of 30 feet in 40 miles which is 1 in 7030 or 1.43×10^{-4}. Although this is reasonable, because a big river has a less slope than a small one, it is difficult to tell how much filling occurred on the old floodplain. The difference in slope may not be enough. Since the linear dimensions of the glacial and modern Kaskaskia bear a ratio of about 7 to 1 it might possibly be appears to be about 135 ~~150~~ feet so that given similar depth-width relations in the glacial river a width of about 250 ~~1000~~ feet is suggested. This ratio of 7 to 1 agrees with the radii of meanders and the ratio of meander belts. The latter is 0.5 mile for the present and 3.5 miles for the glacial Kaskaskia. These figures suggest a possible change in hydraulic radius of 7 to 1, but since we have no data on the present radius the computation of velocity was not carried through. The value of n, the roughness of the bottom, may also have changed with reduced volume. The average discharge of the Kaskaskia over 42 years is 1505 cu. ft./sec. Leopold gives a formula for slope $S = K\ Q^{1/2}$. The square root of 1505 is 38.8 where K is a constant which he gives as 3.97×10^{-6}. Solving the above for K gives a value of 5.5×10^{-2} with present slope. Evidently Leopold must have dealt with streams with different bed and banks. Using the slope formula $S = K\ Q^{.49}$, transposition to find Q gives approximately $(S/K)^2$. Assuming the value of S for glacial times as 14.3×10^{-3} this divided by $5.5 \times 10^{-2} = 260$. 260 squared gives 67500 cu. ft./sec. for the glacial discharge. Taking another empirical formula for width; $w = a\ Q^{.41}$ from which $Q = (w/a)^{2.44}$. To find a we take the present width estimated at 150 feet and present discharge of 1505 cu. ft./sec., of which the .41 power is 20.18. From this the value of a is 7.43. To compute the glacial discharge we take the glacial width at 1000 feet which divided by 7.43 gives 134.5. Raising this to the 2.44 power gives Q, the discharge, at about 83000 cu. ft./sec. The two results contain many unknowns but will serve to show that the glacial Kaskaskia had an average discharge many times greater than the mean flow of the present successor. The maximum recorded modern flood discharge is over 52000 cu. ft./sec. The glacial Kaskaskia appears to have drained about 25 miles of Wisconsin ice front.

assumed that the glacial gradient was of a seventh of the present valve, about $.25 \times 10^{-4}$ feet per foot. This ratio is concluded from the width of the present meander belt which is about .5 mile and that of the glacial meander belt which is about 3.5 miles which checks with the radii of meanders given above. The present average width of the Kaskaskia is roughly 135 feet from which a glacial mean width of about 950 feet is deduced.

The studies of Leopold and Maddock provide a possible basis for quanitative comparison with glacial conditions. These authors find that the dimensions of a stream are power functions of the discharge (Q). Since we know the mean discharge (1505 cu. ft/sec) of the Kaskaskia over a period of 42 years quantitative estimates may be attempted. The weakest point of these empirical equations is the fact that there is a constant, the value of which varies greatly in the case of different streams, probably reflecting difference in bed and banks. Two equations suggest applicability to the present problem. Slope, $s = k\,Q^{-.49}$ where k is a constant and Q = discharge in cubic feet per second. This may also be written $S = k/Q^{\frac{1}{2}}$. For present day western streams Leopold gives the value of K as 3.97×10^{-6} feet per foot. Solving for the modern mean discharge of the Kaskaskia, which is 1505 cu. ft/sec., we obtain a value of $.68 \times 10^{-6}$. It is questionable that this is applicable to glacial conditions, because of the difference in nature of the bed and banks.

Solving the above slope equation for quanity of discharge it is evident that $Q = (K/S)^2$. By substituting the values for k at present and the minimum value assumed for the slope S, we find $Q = (68 \times 10^{-4} / .25 \times 10^{-4} x\ 2)$ which works out to a little less than 74000 cu. ft/sec. If we use instead Leopold's original value of k and the higher estimate of slope of the glacial river, the result is 75,600 cu. ft./sec.

The second equation involves mean width, w, $w = aQ^{.41}$ Substituting modern values, 135 feet = a x 20.04 or a = 135/ 20.04 = 6.72 Using glacial values $950 = 6.72 \times Q^{.41}$ Solving above for Q, gives $Q = (950/6.72)^{2.44}$ which is 103000 cu ft./sec. Again the applicability of the constant, a, is debatable.

The above methods do not agree very closely but serve to give some sort of an idea of the mean dischargeof the glacial Kaskaskia which drained not over 25 miles of the front of the Wisconsin ice sheet. It is perhaps not so large as some have imagined for the maximum modern flood was estimated at slightly above 52,000 cu. ft./sec.

A recent paper by Leopold and Wolman gives two other formulas by which volume may be estimated. These relate to lamida, the wave length of the meanders which is the distance in feet of complete reversal or direction. It may be estimated as 2 pi times the radius of curviature. Formulas are lamida = $Q^{\frac{1}{2}}36$ and lamida = $6.5\, w^{1.1}$ Here we estimate lamida at 23.2×10^{5} feet for the glacial stream. Solving the first equation which was derived empirically from a large number of observations the mean or "bank-full|" discharge Q = lamida^2? 36^2 Lamida squared is 538×10^{6} Hence $Q = 538 \times 10^{6} / 1296$ 415000 cu. ft. per second, a result out of line with other computations. If we equate the two expressions for wave length we find that $Q = 6.5^2 \times w^{2.2} / 36^2$ Taking w at 950 feet, the ~~xquxxxxxks~~ 2.2 power is 3.214×10^{6} $6.5^2 / 36^2$ is .0325 which yields a remult of 105000 cubic feet per second which is much closer to the earlier estimates.

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Jenbron

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No quantitative estimate of the loss of ~~discharge~~ volume appears possible, with existing knowledge of the hydraulics ~~principles~~ of streams. Most ~~estimates~~ studies have attempted only to relate width of meander belt to width of low water channel. Bates found ~~that~~ the width of the ~~meanders~~ belts of floodplain streams are roughly 11 1/2 times that of the stream bed itself. ~~at normal discharge.~~ ~~but that~~ This ratio seems to decrease with increase of channel width.

When the paragraphs above ~~was~~ were written in the early 1930's no information was available which could supply a reasonable estimate of the change in volume of Kaskaskia River from glacial times to the present. Now the studies of Leopold and Maddock supply some ground for such a surmise. The area has been mapped on the Ramsey and St Elmo quadrangles. The radii of the meanders of the present river ~~is~~ are about 1/10 mile whereas those shown by the scars in the banks of the bluffs average about 7/10 mile, 7 times as much. Elementary physics shows that the lateral component of force of the river due to flowing in a segment of a circle is expressed by the formula mass. velocity^2/ radius. If we assumed that both sets of meanders reached equilibrium with the resistance of the banks we then have two problems: (a) how much more resistance did the older bluffs 40 to nearly 100 feet high than the banks of the recent meanders which average about 10 feet in heigth? and (b) how did the velocities of the rivers differ? To make an estimate of velocities it is necessary to know ~~both~~ the hydraulic radius (or mean depth) ~~and~~ the slope of the rivers involved and the nature of the bottom. The formula involved is $v = 1.5/n\ R^{2/3} S^{1/2}$

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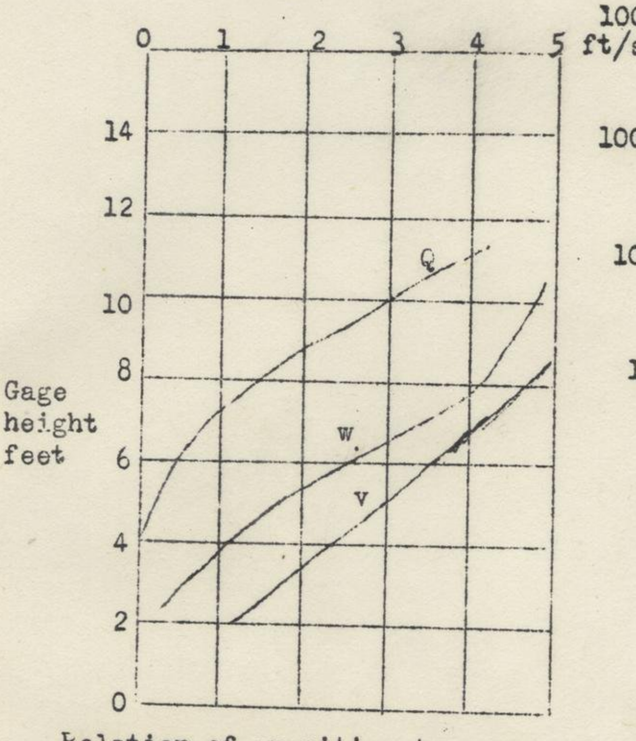
Dimensions and competence of running water. Supplement I, 1953, p. 1

<u>Introduction</u>. Three papers have appeared on the subject of running water which appear to show marked progress in understanding of some problems. Two of these not only clarify some of the basic points of the physics of streams but also point the way to solution of many important problems of sediment transport. The third, deals with particle size distribution on an alluvial fan.

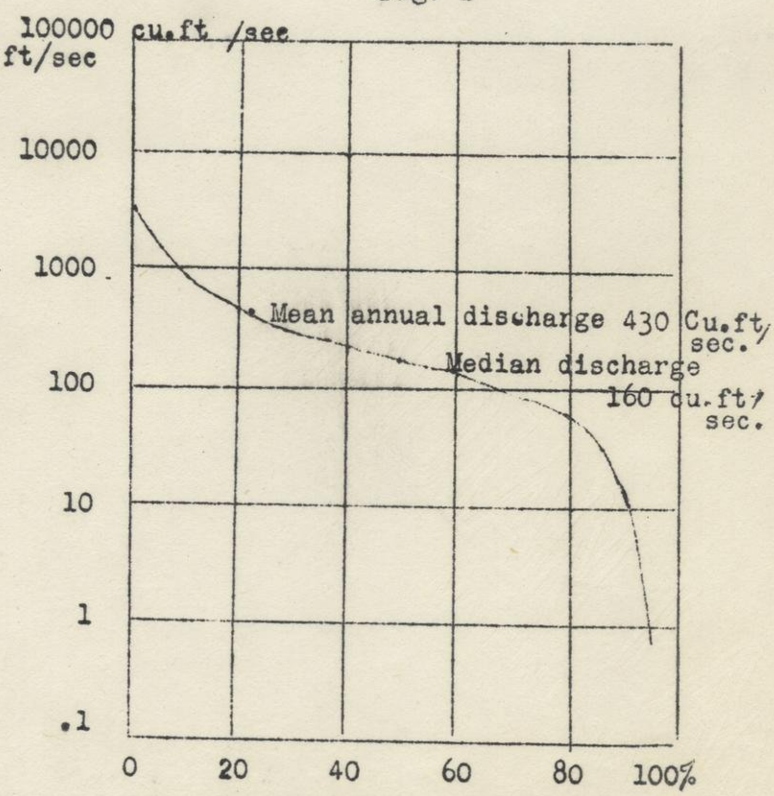
<u>Discharge of streams</u>. The fundmental quantity measured by hydraulic engineers is the discharge of streams. To find this figure they first discover a suitable cross section of the channel. This is subdivided into segments of known dimensions, then the average velocity of flow is found in each segment giving its discharge and the final sum of the segments is the Discharge (Q) = average width of channel (w) X average depth (d), X average velocity, (v) or Q = w.d.v. British engineering units are employed, cubic feet per second, and feet. Since the discharge of all rivers varies constantly it is necessary to connect each actual measurment to the gauge reading of water level in the river at that time. Most discharge determinations are read from a curve (Fig. 1) which indicates this relationship. Next a curve (Fig. 2) must be prepared which shows the percent of days that any given discharge is equalled or exceeded. The <u>mean</u> discharge is also computed as the aritmetical average of all recorded daily discharges. This quantity is generally larger than the <u>median</u> discharge which is equalled or exceeded exactly 50% of the time.

Fig. 1



Relation of quanities to gage reading in feet.
0 200 400 600 800 ft. width
0 10000 20000 30000 40000 cu.ft./sec.

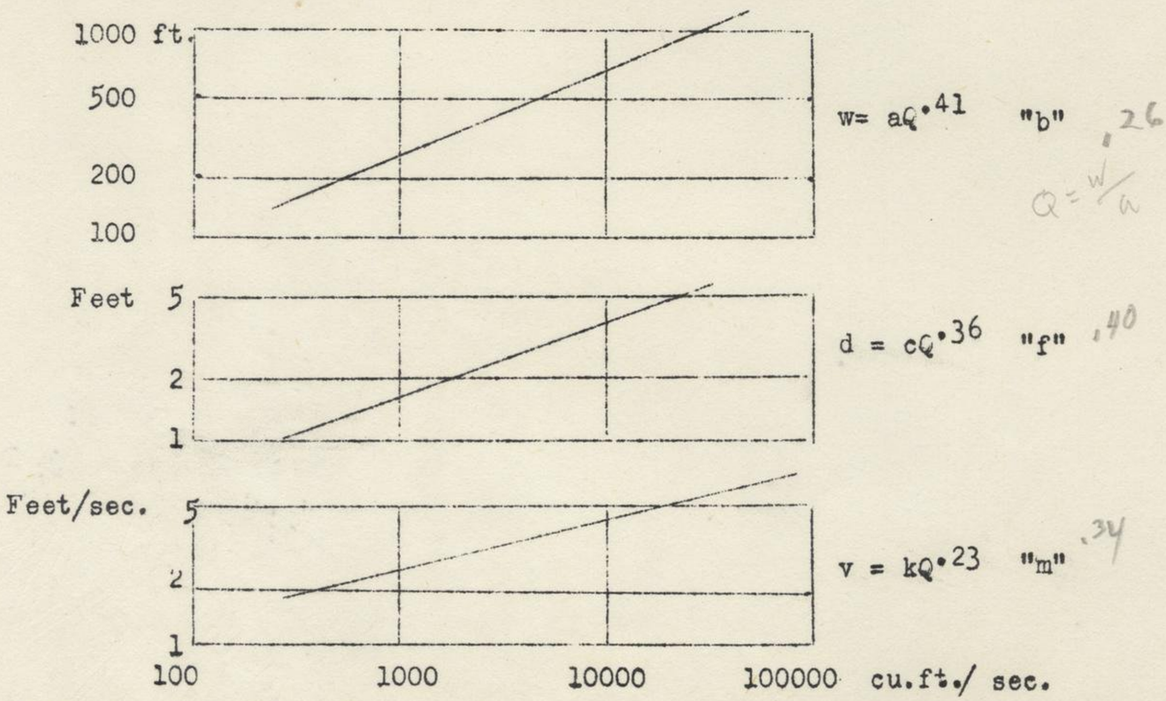
Fig. 2



Typical frequency or flow-duration curve of a river

Inter-relations of quanities. Platting on log-log paper demonstrates, as shown in Fig. 3, that w, d, and v are simple power functions of Q, the primary determination. In mathematical expressions $Q = wdv = aQ^b \times cQ^f \times kQ^m = ackQ^{(b+f+m)}$ From this it is evident that the sum of the exponents of Q must be unity and the product of the numberical constants must be the same. An average of 20 river sections studied gave b = 0.26, f = 0.40, m = 0.34 but the values of the constants varies much more widely than do the exponents. Evidently the values are related to the materials of the stream beds and possible to other factors. The limits of variation are unknown. Depth increases with discharge faster than does width.

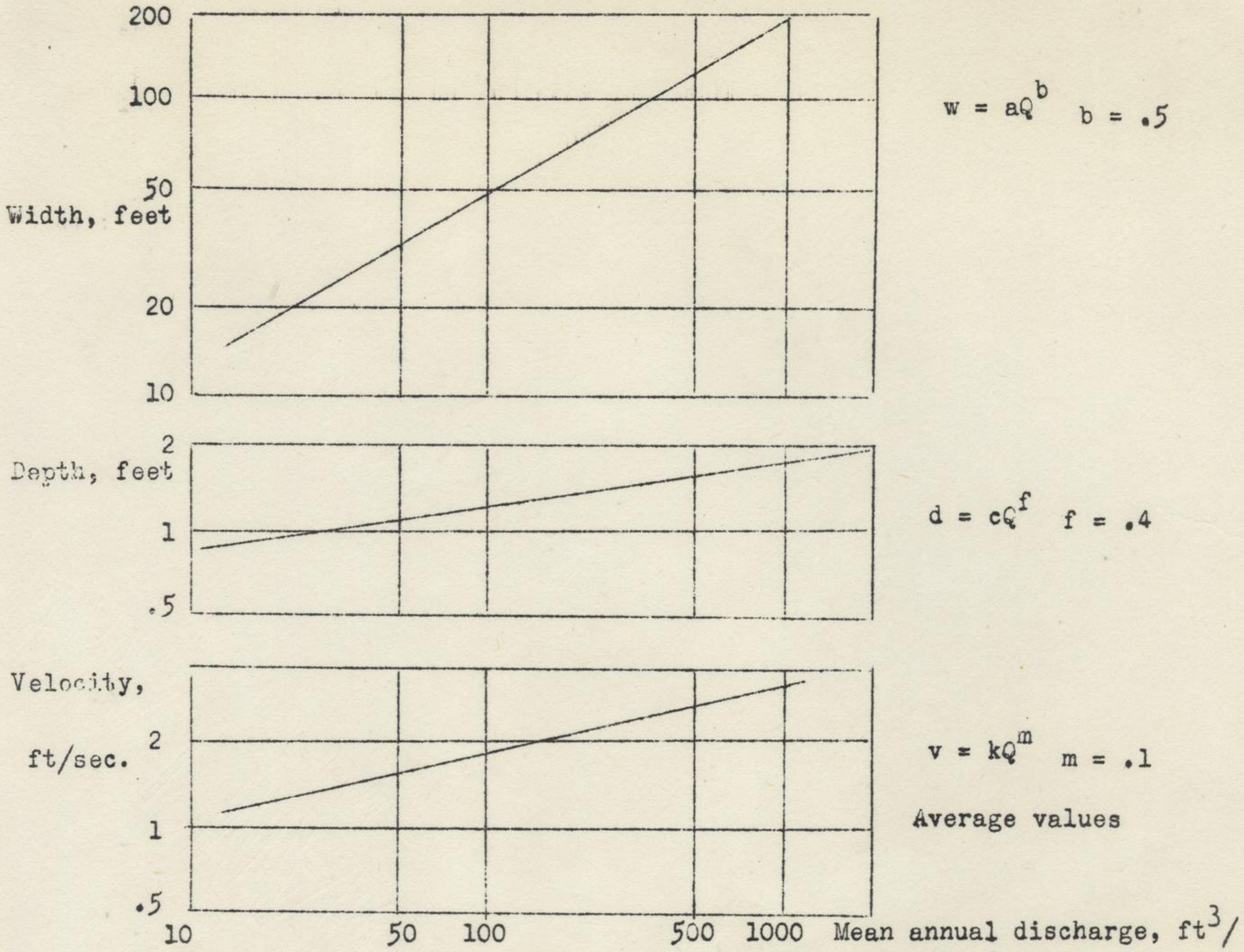
Fig. 3



Relations of width, depth and velocity to discharge as plotted on log-log paper. Scatter of points not shown.

Downstream variations in channel shape. In computing the relations of dimensions of stream channels in a downstream direction it is evident that all comparisons must be made for a specified discharge at every station. Most of the log-log plats were made for mean annual discharge which occurs or is exceeded on the average about one day in every four. In almost all rivers discharge increases downstream. Some were made for flows which occur less frequently.

Fig. 4



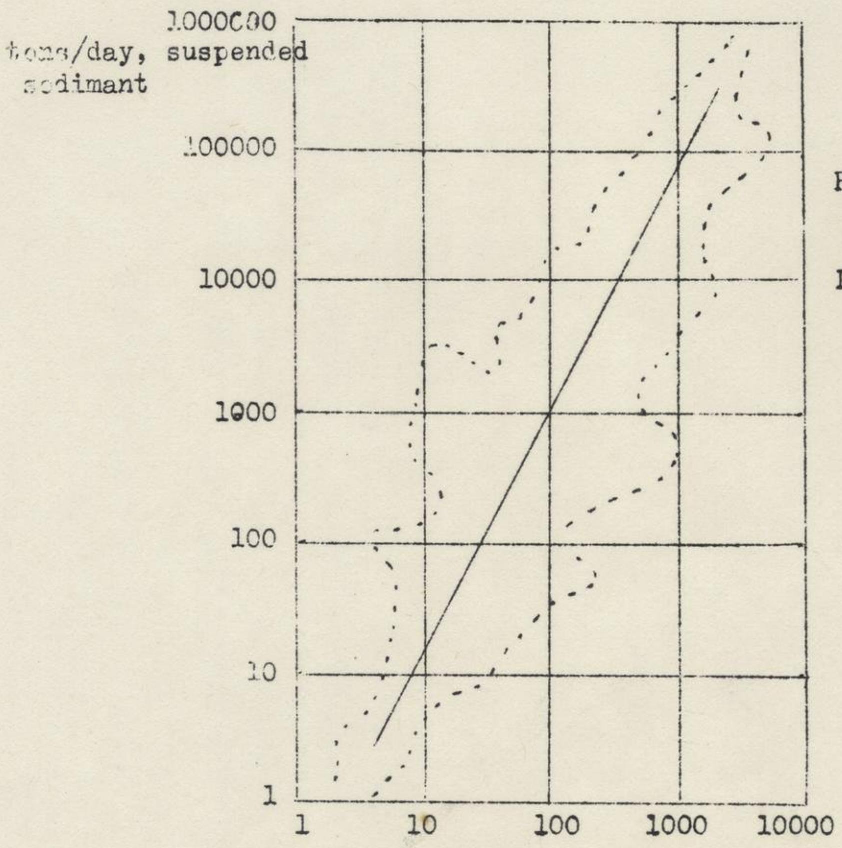
Downstream changes in dimensions of streams in Wyoming and Montana. Points not shown

Despite the expectable "scatter" of points when platted, there is a remarkable agreement in results. Using the notation above, $w = aQ^b$, $d = cQ^f$, and $v = kQ^m$, the average values are $b = 0.5$, $f = 0.4$, and $m = 0.1$ This shows that for increase in discharge downstream all quantities including velocity increase. Increase in velocity is least and this quantity may be almost constant in some streams. Even in the headquaters however, the conclusion is demonstrable. It is contrary to what nearly everyone formerly thought and hence demands some explaination. To do this we will restate Mannings formula for velocity of a stream with turbulent flow: mean velocity (v) ft/sec $= \frac{1.5\ d^{2/3}\ S^{\frac{1}{2}}}{\text{roughness } (n)}$ (dimensions in feet) Note that for wide stream mean depth (d) replaces hydraulic radius (R or cross section area divided by width.) From this it may be seen that most geomorphologists have ignored both depth of water and roughness of the bed. Together these overcompensate for the fact seen in the field that slope of the water surface almost everywhere decreases downstream. Slope (s) in feet per foot mile $= 0.021Q^{-0.49}$ on the average.

Sediment transport. Streams carry sediment in two ways, (a) as bed load or bed-material, and (b) as in suspension or wash load. The two may change in proportion with alterations of the stream so that what is suspended at one time may be a portion of the bed and vice versa. The mathematical relations of the two are only vaguely known for there is at present no accurate method of determining transport of material on a stream bottom. Any mechanical device to catch such load introduces changes in the currents which render the results valueless. Suspended load can be and is being measured at a number of localities. Possibly data on the filling of reservoirs may eventually supply some of the missing information. The following discussion is almost wholly on suspended load.

Suspended load. Platting of the weight of suspended load in given time against discharge of a stream shows at once (Fig. 5) that, despite scattering of points, the amount of sediment increases with discharge as a power function with with an exponent between 2 and 3, thus demonstrating an increase in more than direct proportion to discharge.

Fig. 5

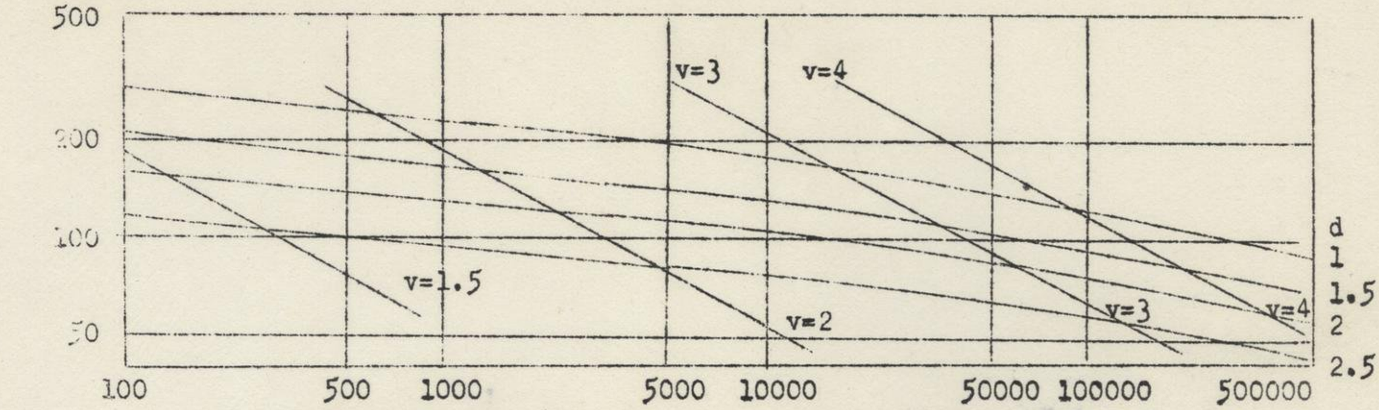


Relation of suspended sediment load to discharge, Powder River, Wyoming
Limit of scatter of determinations shown by dotted line

The cause of this rapid increase is known only in general terms. Factors are: (a) infiltration rate and storage of rain in puddles is greatest at start of a rain, (b) raindrop erosion increases with wetting of soil, (c) long duration of rainfall increases depth of and erosion by sheet wash, (d) increase in velocity of large streams enhances both scour of bottom and undercutting of banks, (e) changes in channel shape during a flood are caused by the suspended load, and (f) suspended sediment concentration may be considered as an independent variable on which both velocity and depth depend. Despite the known alteration of banks by floods, the conclusion of Leopold and Maddock is "that the observed increase in sediment concentration results primarily from erosion of the watershed rather than from scour of the bed of the main stream in the reach where the measurment is made." They found that there are not enough observations to permit of direct conclusions on changes in concentration downstream.

It appears to be slight so far as known for it is observed that increase in sediment with increase of drainage area is less for large basins than for small. It is possible to present a graph such as Fig. 6 showing the relations of width, depth and velocity to total suspended sediment load.

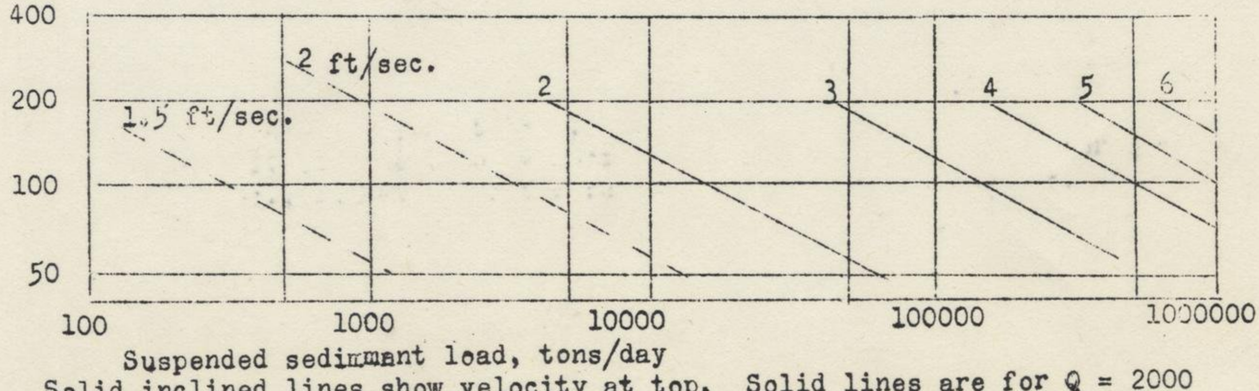
Fig. 6



Widths and depths in feet. Velocity in feet/second

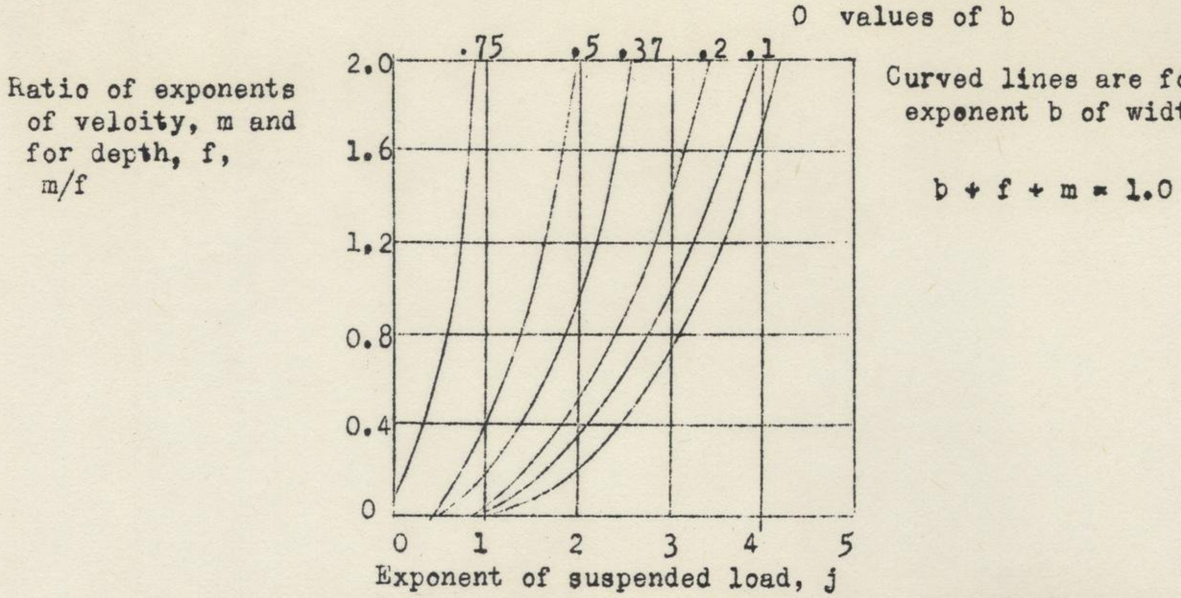
General conclusions. (1) If discharge and width are constant increase in velocity means increase in total suspended sediment and a decrease in depth. (2) With velocity constant, increase in width decreases both the suspended sediment load and depth. (3) Both decreasing width with constant velocity and increasing velocity at constant width increase capacity for suspended load at constant discharge. (4) A wide river carries less suspended load than a narrow river with the same velocity and discharge. (5) Two rivers of equal width and discharge load of suspended solids is larger in that having the higher velocity. Suspended sediment transport with variable discharge. Due to fact that Q = wdv the sum of the exponents b + f + m must be unity as explained above. Hence if two of these exponents are known the third can be computed and from this fact some deductions may be made. First we draw Fig. 7 showing relation of suspended sediment to velocity, width, and discharge.

Fig. 7



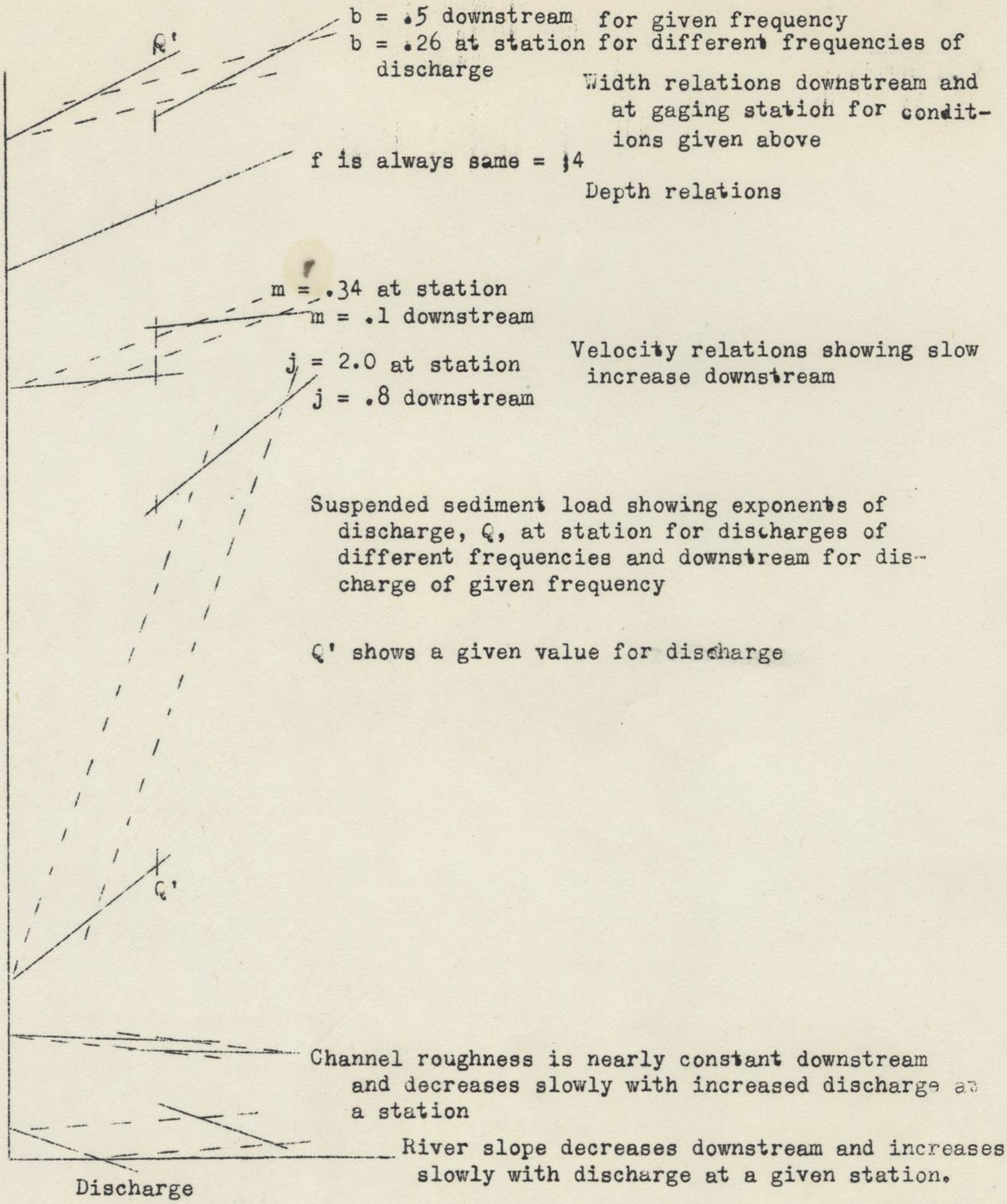
Solid inclined lines show velocity at top. Solid lines are for Q = 2000 cu.ft/sec.; dashed lines toward left are for 500 cu.ft./sec.

Fig. 8



From this it is possible to draw curves showing values of j, the exponent of Q for suspended sediment, in terms of both b, and the ratio of m to f . The ratio between increase of velocity with discharge and increase of depth with discharge is, therefore, related to amount of suspended sediment. For the average cross section of a river m/f is 0.85, b = 0.26, and j = 2.3 This is in line with the statement that sediment concentration should decrease slightly downstream. (Fig. 8) Comparisons of different river cross sections indicate that: suspended sediment load varies: (1) directly with as a function of velocity, (2) directly as a function of depth, (3) inversely as a function of width, (4) as a large power of velocity, and (5) as small powers of depth and width.

Fig. 9

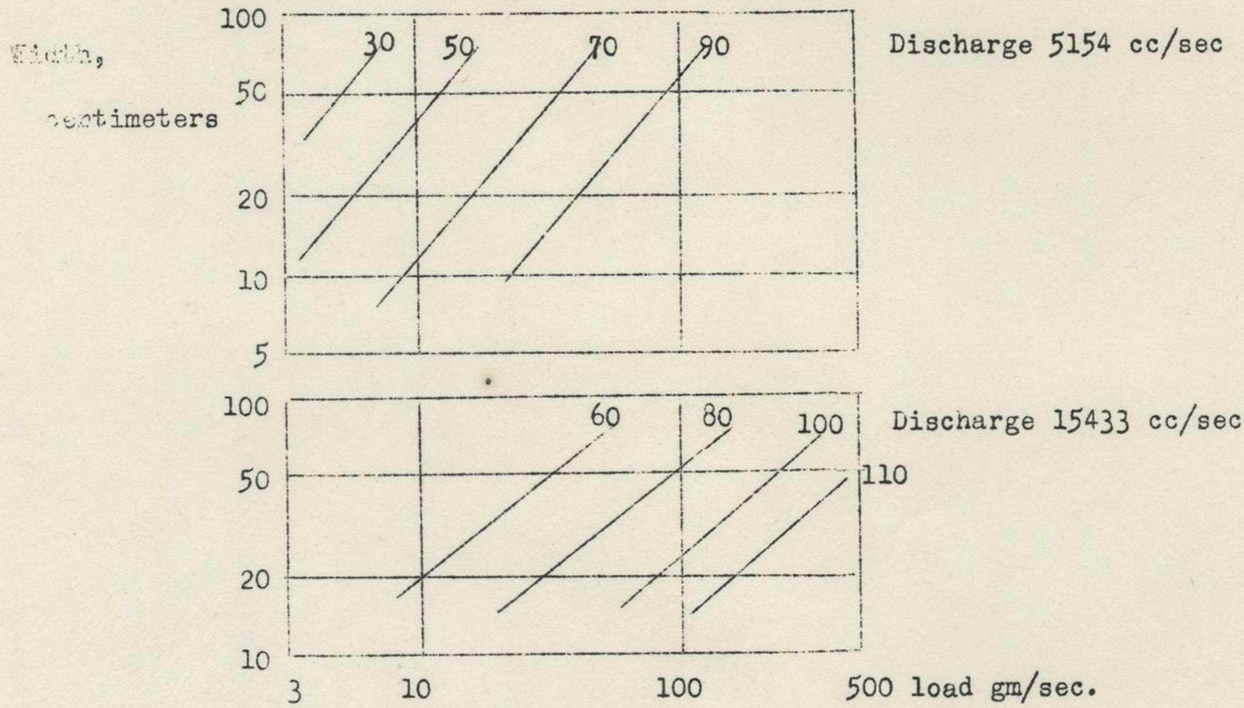


Solid lines for slope on exponent on log-log plat to show downstream change for Q of given frequency Dashed lines for changes at station.

Fig. 9 summarizes the information by showing the comparative changes at a station and downstream by giving the proper slopes of the lines which display the values of the exponents of discharge in log-log platting. We may say that for given width and discharge increase in suspended sediment requires increase in velocity and reduction in depth. The quantities involved are adjusted to the nature of the drainage basin so that they are independent of the channel system.

Bed load. Since there is little information of bed load transport in natural streams recourse must be had to the experiments of Gilbert in wooden troughs here restated in the C. G. S. system. Fig. 10 shows at once that the relation of the lines of equal velocity is exactly opposite to those of Fig. 6 for suspended sediment. Data are given for two different discharges both with same kind of sand. Tentative conclusions are: (1) with constant discharge and width increased velocity increases both bed load and suspended sediment, (2) with constant velocity and discharge increase of width decreases suspended load and increases bed load, (3) broad shallow channels are needed to transport a large bed load.

Fig. 10



Inclined lines show velocity in cm/sec.

Changes of channel form. At some gaging stations measurments have been made of changes in channel form during floods. Some places at the start of a flood, when concentration of suspended sediment is high, display a rise in level of the bottom. This is followed, when sediment decreases, by scour and lowering of the bed. Obviously the latter causes a lower velocity when less velocity is needed for transport. At other places erosion begins at once with the rise of discharge with high sediment concentration and later filling takes place during fall of water level. It has been noted that the spring floods of melted snow in western rivers lower river beds whereas later season floods due to rain result in fill. Filling often occurs during times of increasing velocity.

Roughness of channel. At constant width and discharge it is obvious that the product of v. d must be constant. Hence any increase in velocity requires a decrease in depth. From the usual velocity formula it is evident that for any increase in velocity and decrease in depth the factor $\left(\frac{S^{\frac{1}{2}}}{n}\right)$ must increase.

The two equations: $d = cQ^f$ and $v = kQ^m$ make it possible to set up another. $kQ^m = 1.5\ (cQ^f)^{2/3}\ S^{\frac{1}{2}}/n$ where the constants c and K vary. Hence $Q^m : Q^{2/3 \cdot f} (S^{\frac{1}{2}}/n)$ Where S and n are constant with discharge then $m = 2/3\ f$ or $m/f = 2/3$ From this it follows that if $S^{\frac{1}{2}}/n$ increases with discharge m/f is more than 2/3 and when this ratio decreases with discharge then m/f is less than 2/3. Now at a given station the average ratio of m/f is 0.85 whereas downstream this is only 0.25

From this it appears that $S^{\frac{1}{2}}/n$ increases with discharge at a given station and decreases downstream. It has also been observed that in the downstream direction roughness (n) remains about constant so that slope must decrease to preserve the above relations. Observation has also disclosed that an increase in suspended load decreases channel resistance and hence increases velocity. Possibly this is really related to decreasing turbulence. Increased values of sediment concentration are associated with decreased values of n. At a given station, however, the slope does not change very much so that the alteration of n must be considerabl with change in concentration of sediment. Changes in velocity-depth relations might be attributed to change in sediment concentration where an increase diminishes the roughness, n, of the bottom. A check consists of the behavior of Colorado River after the completion of Boulder (Hoover) Dam which caught much of the sediment leaving clear water below. This is the same as a lake in the course of a river. Alterations below the dam consist of (1) increase in depth in spite of a lowering of surface elevation, (2) decrease in width due to reduction of flood volume, (3) decrease in mean velocity, (4) increase in roughness of bottom, apparently a result not of change in type of material but of decrease in suspended load, (5) reduction of bed load in the narrowed channel, (6) increase in capacity for suspended load due to change in velocity and discharge, (7) no appreciable change in slope.

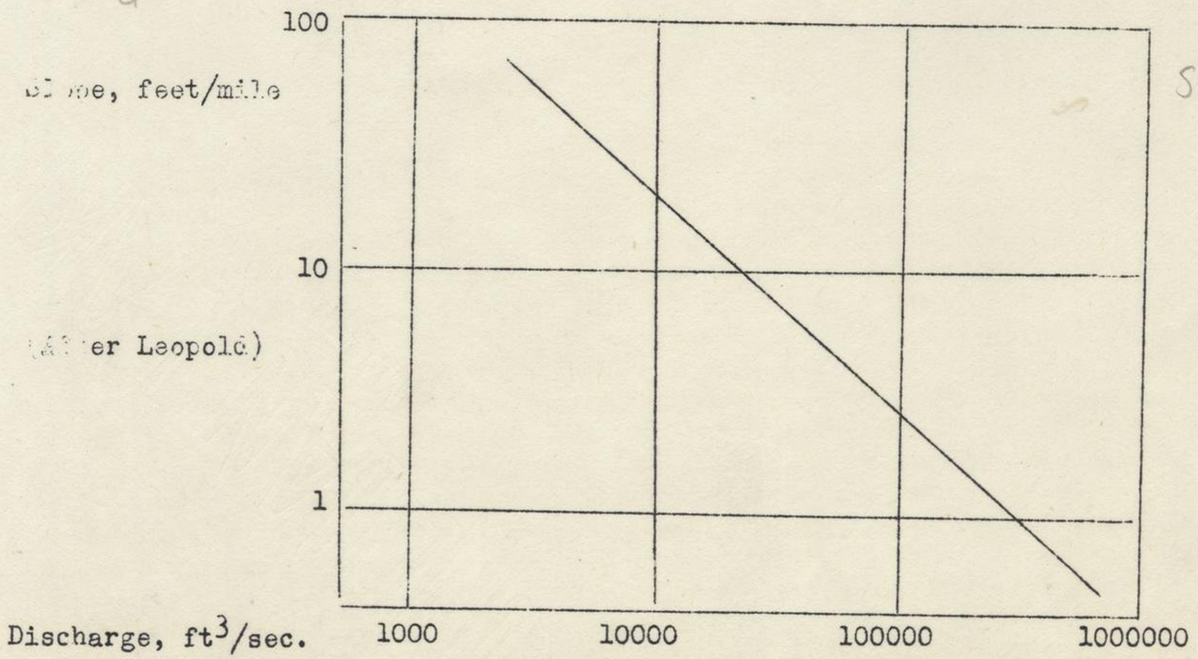
Factors of channel roughness. Channel roughness is due to (1) particle size, (2) bed configuration, and (3) sediment load. It is commonly observed that the material of most stream beds diminishes in size of particles downstream although from this it does not necessarily follow that decrease in slope is directly attributable to this phenomenon. Waves and ripples on the stream bed are very important factors in roughness, although they are not permanent. Increased bed roughness decreases velocity in respect to depth hence affecting the capacity for load. These waves or ripples vary in nature with different kinds of sediment. They pass with increasing discharge from smooth bottom through successive forms into antidunes which travel upstream. For fixed slope and discharge decreased particle size tends to increase roughness. Bottom material is most important in the headwaters of streams where the bed consists of boulders, cobbles, and pebbles. Under this condition, downstream decrease in size of particles decreases roughness. The Powder River, Wyoming, has a value of n on gravel of .087 which falls to .017 on silt farther downstream. However, in other streams the value of n is about the same downstream despite marked differences in nature of bottom. There it must be that bottom configuration is dominant. In summary, it is clear that slope is the dependent factor which the stream is able to change. As noted above it is common to find at a given station that suspended load of streams increases rapidly with discharge. This requires a relatively rapid increase in velocity compared to depth, that is a high value of m/f. Such is accomplished primarily be an increase in the value of n which is related to increase in concentration of suspended load. However, in a downstream direction load does not keep pace with discharge and the concentration of suspended sediment decreases slightly. To do this depth must increase with discharge faster than does velocity so that the m/f ratio must be low. Hence $s^{\frac{1}{2}}/n$ must decrease downstream. With roughness about constant this can be done only by decreasing the slope.

Graded streams. By definition a graded stream can over a period of time just transport the amount of sediment furnished it. Engineers have constructed many irrigation canals which do exactly this, that is they neither erode nor silt up. Some rules were derived by experiment which used perimeter, P, instead of width and hydraulic radius, R, instead of mean depth. A sediment factor, F, is also introduced. The basic equations are: $P = 2.67\ Q^{\frac{1}{2}}$ and $V_{mean_1} = 1.15F^{\frac{1}{2}}R^{\frac{1}{2}}$. Note that in the studies of Leopold and Maddock they found that $w = aQ^{\frac{1}{2}}$ (downstream). By combining the relations $d = cQ^f$ and $v = kQ^m$ we find that $(d/c)^{1/f} = (v/k)^{1/m}$ or $v{:}d^{m/f}$. In natural streams this ratio of m to f downstream is only 1/4 whereas in the canals it was 1/2. But we must recall that canals for irrigation are not like streams because they loose discharge downstream as it is dispersed into laterals. They can have no change in suspended sediment concentration hence the value of j cannot be above 1.0. If b = .5 and j = 1.0 this means that m/f would be 0.43 or not far from that value already given. This suggests that j must in practice be less than 1. In summary, Maddock and Leopold conclude that with available data it is not possible to discriminate graded from ungraded sections of a river.

Longitudinal profile of rivers. It has long been assumed that the profile of a river bed is directly related to the maximum particle size of sediment in its bed. It has also been assumed that wear of the load results in a downstream reduction of size of particles. The latter can be checked in the field, although it is hard to distinguish material derived from tributaries and cut banks and not brought far downstream. Now if the velocity of flow really increases downstream how can competence of the current be the controlling factor of river profiles? Some have derived equations to substantiate this assumption but the issue is confused by several phenomena. (1) Decrease of particle size increases roughness by promoting ripples; (2) roughness is also related to concentration of suspended sediment and, (3) in practice roughness does not vary much downstream. Hence to preserve the required velocity-depth relations to transport the load the slope of a normal stream must decrease downstream. Leopold gives the empirical equation that slope, $S = 0.021\ Q^{-0.49}$, that is slope is approximately inverse to the square root of discharge. ft/m

Fig. 11

$S = \frac{.021}{Q^{.49}}$



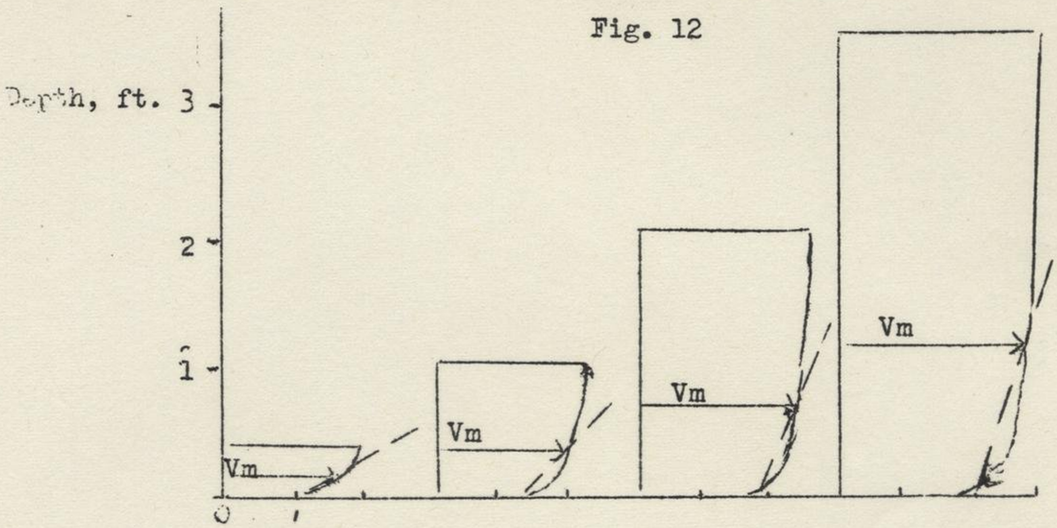
$S = \frac{.021}{Q^{.49}}$ $\frac{.021}{5280^2}$

3.97×10^{-6}

root of discharge. We cannot, however, construct a longitudinal profile of a river from this without knowing how the discharge varies in a downstream direction. This is commonly in direct proportion to drainage area not to distance along the channel.

<u>Vertical velocity distribution</u>. It has long been known that in rivers which are relatively wide in proportion to depth, that is where the banks are readily erodable, the vertical distribution of velocity is approximately proportional to the logarithm of distance from the bottom, z. Such being the case the rate of increase of velocity with respect to distance from the bed is inverse (see any text book of Calculus). Now this rate of change in velocity upward from the bed determines the shear or rate of energy transfer from the stream to the bottom. Since in most streams depth increases downstream as a power function of discharge the slope of the line representing rate of velocity (dv/d2) change near to the bed must decrease with increase in total depth.

Fig. 12



Velocity, ft/sec.
Inclined dashed lines show downstream decrease in shear

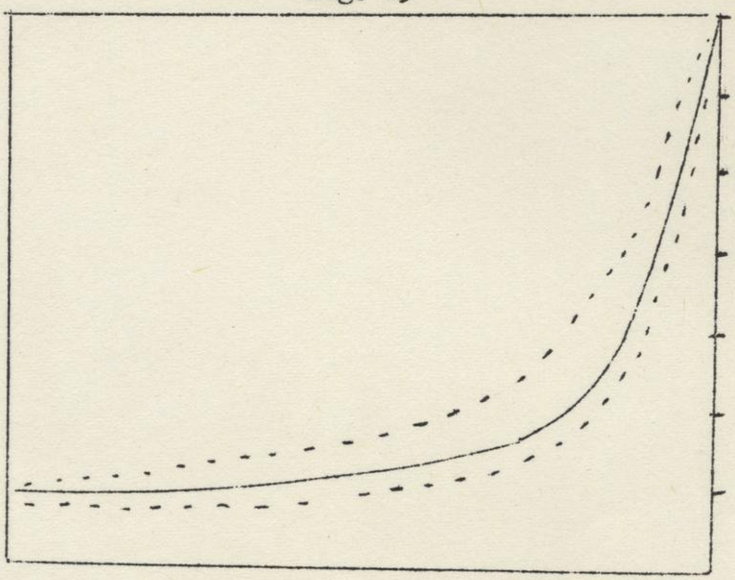
Another factor is that total force on the bed is proportional to depth times slope. As brought out above, depth increases on the average at the 4/10th power of discharge whereas slope decreases at approximately the square root of that quantity. Hence the product DS must decrease slowly downstream at about the minus 1/10th power of discharge.

<u>Summary</u>. Although the old idea that river slopes are directly related to velocity which decreases downstream thus decreasing competence must be abandoned, it is clear that there is a downstream decrease in competence. The details of just how this comes about are not simple. The vertical velocity profile and shear on the bed are interrelated and depend not only on mean velocity but also on depth, and on roughness of bottom. This shear also affects the intensity of turbulence which is necessary to keep material off the bed. Downstream decrease in roughness may diminish both shear and turbulence despite increase in mean velocity. Leopold lists the variables which enter into this problem: discharge, width, depth, velocity, slope, roughness, load, and size of particles in transit. These constitute eight simultaneous equations whose solution is at present impossible. Of them only the flow equation (Q=wdv) and Mannings formula for velocity are accepted by common use. The others comprise relation of load to nature of basin, rate of particle size change downstream, width-depth ration in relation to nature of the bed and banks, change in value of n, the roughness factor, with depth, material, discharge, and slope, and relation of n to sediment concentration. The interdependence of these factors is evident and it is clear that the stream is capable of adjusting its slope to fit the requirements of the others. The cross section of a stream is adjusted so as to equalize shear on both bed and banks. The form of the bed can be changed so as to alter roughness. All of these factors are much more complex than we were led to believe

by the pioneer students of geomorphology who did not employ quanitative methods even if they were correct in general principle.

<u>Change of particle size downstream</u>. As explained above it is generally impracticable to measure the downstream reduction of size of particles transported by a river. On alluvial fans, however, all the debris is derived above the apex and reasonable success has been attained in comparing the maximum particle size with distance from the source. An article by Blissenbach based on fans in Arizona shows (Fig. 13) that despite considerable scatter a definite

Fig. 13



After Blissenbach Dotted lines show scatter of points

relationship does hold. From the known fact that diameter of pebbles is related to the square of velocity of transporting water it could then be concluded that the ratio of mean depth (or hydraulic radius) to bottom roughness must remain reasonably constant. On alluvial fans this might be expected for all the water is derived from the head so that the individual streams on the fan do not vary widely in size despite some loss by evaporation and perhaps by seepage. Roughness, which should decrease with smaller particles downward on the fan, could be maintained by more ripples in the bed on lower slopes. The log-log. platting (taking slope as directly proportion to degrees measured) of the diagrams published show that slope is approximately inverse to the square root of horizontal distance from apex. Fall must, therefore (see integral calculus) be in proportion to the square root of distance from apex. The same paper also presents some data on relationship of maximum particle size to angle of slope (on steeper slope the degrees do not correspond directly to the technical definition of slope which is tangent of the angle) whic seem to confirm the determinations of Fair in South Africa. In the case of the Black Hills terrace gravels there is rough agreement of slope to logarithm of geometric mean size of stones. All of the above data is inconclusive for no attention has been paid to mean particle size of entire deposit and it is known that there is much finer material along with these maximum particles. On a table does the average or medium size control the coefficient of friction?

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(17 1/2)

~~This~~ The ratio is considerably greater where the meanders are eroded in bedrock. These studies were in large part based on inaccurate maps where the streams were shown much wider than they actually are. No ~~studies~~ investigations based on aerial photographs have come to the notice of the writer. Bates concluded that the misfit relation of Kickapoo River, ~~Wiscons~~ Wisconsin, is due to the development of a floodplain within the rock valley on which meanders do not grow as large as where confined between rock wall. Although these central Illinois stream valleys have doubtless been ~~filled~~ aggraded as a result of Wisconsin glaciation it hardly seems possible that the change in width of their meander belt could have be due to this factor alone. The bluffs are ~~either~~ almost entirely till ~~or soft Pennsylvanian shales~~ ~~neither of~~ which is ~~notably~~ not very much more resistant to erosion than is the silt of the floodplain. It may, therefore, be concluded that the misfit relation of the Kaskaskia and Embarrass rivers is due to the ending of glacial drainage.

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GEOLOGY 109

GEOMORPHOLOGY

Supplements, 1954, pt. IV

Papers on Work of Running Water

Dury, G. H., Contribution to a general theory of meandering valleys: Am. Jour. Sci. 252: 193-224, 1954.

The paper by Dury of the University of London, England, takes up the long-disputed ideas on underfit or misfit rivers where there are two sizes of meanders, one of the valley and the other on a valley filling. One of the most important lines of the new approach was the study of the valley fills along a number of river in the south of England by means of auger borings to bed rock.

Older theories. Dury promptly discards the long-disproved hypothesis of a lessened volume by reason of headwater diversion, disappearance of glacial meltwater, diversion to underflow, erosion of the larger curves during flood stages, and influence of different formations of bed rock during downcutting. All of these are plainly inapplicable to the streams which he studied. He also briefly dismisses Wright's idea that the meanders of rock-bound streams grow much larger than those in alluvial deposits, as well as Bates' suggestion of a change of meander size due to aggradation of a valley.

Description. Meanders are carefully described. The "deeps" at the outside of bends are called swales, the shallows maintain the usual name of crossings, and the depositional features inside the curves are termed scrolls. It is admitted that meanders may be initiated on quite steep slopes and hence are not everywhere the result of a low stream velocity. The author's conclusion is that "if a straight channel becomes more sinuous, the hydraulic radius and mean velocity increase, while the wetted perimeter is reduced. Thus a deeper channel with more stable banks can be maintained and a more efficient runoff occur than with a straight course." The fact that meanders maintain themselves is thus explained despite the fact that in weak rocks the entrenched meanders which survive in firmer rocks are missing.

Formulas. Next, meanders are considered in relation to stream size. The variables considered include width, W, depth, D, wetted perimeter P, (all in feet) slope S, catchment area M, in m^2, width of meander belt M_b, and meander wave length M_l, (both in feet) velocity V, and discharge, Q (in ft3/sec). On the authority of others Dury presents several empirical formulas to relate M_b and M_l to W. M_b is from 14 to 17.38 times W and M_l is 6.06 W. The relation to discharge is $M_b = 84.7\ Q^{\frac{1}{2}}$, $P = 2.67\ Q^{\frac{1}{2}}$. W can also be approximated by the formula W = Beta $(CRM^{.6})$ where R is the annual runoff in inches, M, the catchment area is in m^2, and C the runoff coefficient. Beta varies from 0.3 to 0.375. Dury had not seen Leopold and Maddock's work on the mathematical relation of W, D, and V to Q. They found that W = constant $.Q^{.41}$. It may well be doubted that any of these equations give due weight to the nature of bottom and banks of a stream in regulating its width in relation to Q. No mention is made either of the inaccurate maps used by some of these investigators, or the fact that some were working with irrigation canals. Full data are tabulated for 6 streams and 9 localities. These include m, R. C, W. observed and computed, P observed and computed, width of filled channel W_f, its ratio to that of the present channel, Q at present from the formula $Q = (P/2.67)^2$ in ft^3/sec, and last rainfall intensity, i, necessary to fill the larger channel. The last is derived from a "rational formula" where Q_{max} = 640 CiM where i is rainfall in inches per hour and other quantities are given above. Hence $i = Q_{max}/640$ M. Dury concludes that an annual rainfall of 300 to 400 inches would be needed to fill the buried channels, with i equal to .20 to .33 inches per hour. The result could have been obtained if rainfall intensities, which now occur rather seldom, were once much more common. 300 inches per year could fall in 900 hours at a rate of .35 inches per hour.

Discussion. Dury concluded that the change in size of meanders is due primarily to a reduction in annual rainfall since the Pleistocene. He states that it is very difficult to compute radii of curviture which may account for neglect of the force directed against the outer bank due to a curved course. As all text books of physics demonstrate acceleration is proportional to V^2/r where r is the radius of curviture at the point under consideration. Since the formula is for acceleration, if force is desired the mass of water in the river which affects the outer bank must be considered. Since this is the location of greatest depth and highest turbulence, it is evident that the entire mass passing in unit time, Q, must be considered. However, it is clear that the force against the outer bank is only the lateral component of the total energy of the stream. In estimating this component rate of curvature must be considered. The formula gives this for it shows the portion of total kinetic energy which is necessary to keep the water flowing in a curved path. From the formula it is also evident that the inverse relation of force to radius is a factor which must limit the size of meanders at the point where resistance of the bank to erosion equals force directed against it. Another self-limiting factor is the obvious fact that meandering increases the length of a stream which at the same time decreases its slope. Now, other things being equal, velocity of a stream is related to square root of slope. Hence for V^2 we can substitute S and obtain the final result that force against outer bank is proportioned to slope divided by radius of the curve, multiplied by the mass of water passing unit length of bank in unit time. Although we have definitely shown that meanders themselves limit their size and that only big rivers can make big meanders we are met with an apparent contradiction. How is it that entrenched meanders which cut into bed rock are so much bigger than floodplain meanders in relatively soft material? Before we can answer this we must first consider three problems. First, what caused the deposition of the alluvial fill in a former rock-bound stream valley; second, what determines the wave length of meanders; and third, what effect does change in bank material have upon dimensions of the channel with the same discharge.

Causes of valley filling. Most text books ascribe the widening of a valley to lateral erosion of the stream when it has reached grade. If this were true, the thickness of the alluvial fill should be small and streams with entrenched meanders should develop wide flood plains. This condition does exist in some places; but in most parts of the world, valley filling is due to a change in level of the outlet of a stream. This change may arise (a) from change in sea level, (b) filling of an enclosed basin, (c) deposition of glacio-fluvial or other stream deposits at the outlet, or (d) lengthening of a stream by delta building . A change of climate is possible, as it also earth movement, both of which can affect the slope of a stream. In all of the cases outlined above, the slope of the stream is necessarily changed. The

Kickapoo River, Wisconsin, studied by Bates, had its outlet into Wisconsin River raised more than 150 feet by glacial outwash. Rivers of the Atlantic Coastal Plain and the British Isles all show drownding. Many rivers which flow into the Great Lakes show a similar feature due to tilting of the region which caused a rise in lake level. Thus, without any necessary change in runoff, the slope of the valley floor is changed. Streams aggrade, or degrade, their courses until a stable condition is reached in sediment transportation. Streams like the Kickapoo had a complex Pleistocene history. The aggraded to meet the ponding of the outlet by glacial outwash, then eroded when glacial meltwaters removed a part of this fill and are now aggrading apparently due to the increased supply of sediment since cultivation of the surrounding hills. Streams of the Great Plains which display terraces have undergone a complex combination of climatic changes, and tilting of the land. It is clear, therefore, that we must not ascribe all changes of stream slope to climatic change alone.

Wave length of meanders. We can consider meanders like the phenomena of a ball rolling down a flat-bottomed trough. If the ball is started straight, it may roll the entire length of the trough without ever striking the sides. The higher the velocity, the more likely this is to happen. But if the motion has a lateral component, collison is inevitable. On this happening the ball is reflected across, strikes the other side, is again reflected and so on. The angles of incidence and reflection alone affect the distance between collisions with the sides, the wave length. A stream behaves in much the same way except that it cannot be reflected as sharply. The wider the stream the harder it is to turn it. Another similie is the vibration set up in a hanging rope when struck which forms stationary waves. Hence it is easy to understand the effort of Nemenyi to liken meandering to some form of wave motion or rhythm. In nature, however, variation in material of the banks plus effect of tributaries make it difficult to determine a wave length. In the Vicksburg experiments Friedkin does not report it but used instead, length of bends from one shoal to the next and width of the bends, distance between line tangent to bends and parallel to axis of stream. The first comes closer to wave length.

Effect of bank material on channel dimensions. The quantitative results reported by Leopold and Maddock were derived from about 20 rivers in Western United States and hance do not represent all conditions. In Mississippi River it has long been noted that the finer and more compact the bank material the straighter and deeper the channel. On the other hand the Vicksburg experiments demonstrated that very soft erodible banks do not permit the formation of any meanders but result in a braided course. In Dury's examples there is everywhere a wide difference between observed channel dimensions and those computed from the formulas used. From this it is clear that none of the formulas can be relied upon except with the bank materials where they were derived. It is also evident that bank material has a profound influence upon meander size.

Vicksburg experiments. The experiments with model streams at the Vicksburg laboratory reported by Friedkin are almost the only ones with controlled conditions. Friedkin reports the following:

a) Length of bends is in direct proportion to discharge.

b) Width of bends increases at less than direct proportion with increase of discharge.

c) When slope was altered with discharge constant length of bends was almost in direct proportion to slope.

d) Under condition of (c) width of bends increases at less than direct proportion to slope.

e) The initial angle of attack, where the stream was deflected, is inverse to length of bends. This is exactly the same as with the ball rolling down an open trough.

f) Considering width of bends, the angle of attack is almost in direct proportion.

g) Turning to increase in length of the stream compared to original airline distance (sinuosity), the increase is almost in direct proportion to discharge.

h) Sinuosity increases at much less than direct proportion to slope.

i) In a meandering river shoaling or deepening takes place at any given spot depending on the relation of sand entering the area to the ability of the stream to carry such sediment.

j) The slope of a river is changed with change in level of its bed to bring about an adjustment between bank erosion and rate of sand movement.

k) Shape of the cross section of a channel is changed by the erodibility of the banks; the original form makes no difference to that fixed by flow, banks, slope, and alignment; slope is changed by cross section of channel.

l) There are three interrelated variables: discharge and channel form which regulate sand transport, amount of sand to be moved, and rate of bank erosion. No set formulas are possible. Stability involves a wide shallow stream which neither erodes its banks nor forms meanders.

m) The only reason an alluvial river does not erode it bed is the load of sand which it is carrying.

n) Although bank erosion causes the outside of a bend to be eroded back deposition builds up the inside of each curve thus reducing the channel area with sand eroded from the bend above. Width remains fairly constant.

o) What limits the size of bends is the formation of <u>chutes</u> across the points on the insides of the curves. Chutes form when a bend becomes too sharp and when the alignment upstream changes the direction of the current.

p) Variability of material on the floodplain, which is common in nature, disturbs regular growth of meanders producing dissimilar bends.

q) Meanders normally move downstream (<u>sweep</u>) and natural <u>cutoffs</u> across the neck only occur when a downstream meander is slowed up by variation of material.

r) Braided streams are often called "overloaded" and occur with steep slopes. The tests showed the primary cause is very soft bank material.

Three types of valleys can be distinguished: (1) resistant banks = deep narrow channel with low slope. (2) slowly eroding banks = meandering, fairly deep channel with fairly low slope. (3) easily eroded banks = stream with fairly steep slope and shallow meandering channel.. (4) extremely soft banks = braided stream with extremely high slope. Intermediate between (3) and (4) is a stream with any straight shallow wide reaches, islands and bars.

Applying the above to the Mississippi Valley one begins with the last alluviation probably associated with postglacial or lateglacial rise of sea level. Deposition took place until the stream was able to carry its load. Sediments decrease in particle size downstream. Subsequent development follows the above laws. A secondary reason for the downstream decrease in slope is that less velocity is required to transport the finer sediments (See Leopold and Maddock, "Dimension and competence of running water"). The easy erodibility of the sediments in the upper part of the valley is counteracted by the wide shallow bed of the river so that meandering is no more rapid than below. We must remember that natural rivers do not have constant discharge. Adjustment to bank conditions is never complete.

Entrenched meanders. With meandering valleys the problem arises as to whether the curves follow directly those acquired on a former floodplain or whether they have grown larger during downcutting. The latter are what Rich termed ingrown meanders. Leaving this question aside, it is evident that with meandering valleys we must in general have a stream bottom which is gravelly or sandy with abundant rock outcrops. Although such bends do migrate downstream more rapidly in soft than in resistant formations, it is clear that downward erosion occured so rapidly that no floodplain originated. Cutoffs and chutes could not be formed, although some cutoffs through caves are reported in the Ozarks. Very much elongated bends are common and it has long been observed that the ratio betweeen width of bends and width of channel is much higher than is the case on floodplains. The gravelly nature of much of the bottom means less easy erosion so that it may be presumed that the cross section of the streams during erosion was on the whole much deeper in proportion to width than is the case in soft sandy alluvium. It is also safe to assume that the slope during erosion was considerably more than on floodplains since more and coarser load was being transported.

Floodplain meanders. Floodplain meanders after the filling of a meandering valley with relatively soft alluvial deposits presented an entirely different problem to the stream. The slope was also decreased over much of its length. Following the laws discovered, above the bends should then be smaller and the stream channel wider and shallower, the latter counteracting to some extent the softer material of the banks. Shortening by chutes and cutoffs could occur readily. Shallow wide reaches should be more abundant.

Conclusion. With the above listed changes in controls other than a decrease in average discharge it is evident that Dury's conclusion of a climatic change should not be regarded as established beyond doubt.

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In a paper by Frey and Leonard, some of the practical problems of terrace correlation are discussed. Errors include mistaking a rock terrace for a depostional feature, mistaking a colluvial wash deposit from the valley wall for a terrace of material brought down the stream, concealment of a high terrace by loess, and confusion of terraces with flanking pediments. The effect of a resistant bedrock formation on the grade, both of stream and terraces, is also pointed out. Dissection of old terraces by stream valleys makes it difficult to discriminate between post-terrace erosion and original surface irregularities due to stream work. Miscorrelation of terraces along the stream valley is made possible by these chances of error.

Gilbert, G. K., The transportation of debris by running water: U. S. Geol. Survey Prof. Paper 86, 1914.

Introduction. Gilbert commenced his studies of movement of material by running water in a purely qualitative manner. When he began a study of the movement of the debris from hydraulic mining in California mines the need for quantitative knowledge became apparent and a study was begun at the University of California in Berkeley. The experiments, which almost all used straight wooden troughs, failled to discover a simple law but nevertheless are the most elaborate ever carried out. When sand is added to water flowing in a trough, it builds up the bottom until the slope is adjusted to that needed to transport the load. If the rate of feeding is not above a certain limit the bottom is stable, but if it exceeds that limit of capacity then the bottom is built up by the debris which cannot be carried forward. In experiments the several variables, slope, ratio of depth to width, discharge, nature of debris, and depth can be kept constant and hence the effects separated. These variables are not independent. We must recall two fundamental equations: discharge, Q = width times depth times velocity, V, and Manning's formula where R = hydraulic radius and S = slope, $V = \text{constant } R^{2/3} S^{\frac{1}{2}}$, where the constant involves the nature of the bottom. Furthermore we should remember that a natural stream can adjust the form of its cross section to the discharge and debris in transport whereas the trough is fixed unless the sides are changed purposely. Flume transport is different in that no debris was allowed to accumulate on the bottom.

Natural streams. In natural streams those which are supplied with debris to less than their capacity erode their beds and bedrock is exposed in places and at certain times, these are corrading streams. When the supply of debris equals or exceeds capacity the stream bed is wholly composed of this debris although there may be some rock banks. These are rock-walled streams. Streams which have so much available debris that the entire bed is composed of loose material are termed alluvial. A given stream may have segments of its course in different catagories. Streams adjust their beds to meet the condition imposed by the local supply of debris. This process is termed gradation. Most alluvial streams are aggrading and have a flood plain. Gilbert regarded meandering streams as confined to low slopes which are underlain by fine material. Most river channels curve and on a bend the deepest and swiftest water is on the outside instead of in the middle of the channel. This develops an asymmetric cross section where the outside of the bend is eroding and the inside being built up thus preserving a nearly constant width. Shoals, called crossings, are built up diagonally across the channel from the inside of one bend to the inside of the next. These bars are built up at highwater and eroded down at low stages when the deep places are filled up. It is flood stages which perform a large part of modification of the channel to fit the river. With change in stage go changes in velocity and hence in the limits of competence. This is why the shoals come to be surfaced by large particles which cannot be moved at the lower stages. When the channel of a river is fully adjusted to discharge the same load will be transported in every section but the relative amounts in suspension and bed load may change with local conditions. On the whole the ratio of mean depth to width is much less in natural channels than in the optimum ratio for capacity found in the experiments. A complicating feature of natural streams is the nature of the debris supplied to them in reference to their competence. This load may bear no relation to capacity. Suspended load influences velocity in three ways: (1) its mass increases the mass of the stream and hence its energy, (2) suspended particles are always settling toward the bottom and work is required to keep them from doing this, (3) the load affects viscosity. An empirical formula derived from the experiments is that Mean velocity, $V_m = Q^{.25} S^{.3}$ times a constant, from which it concluded that addition of a suspended load increases velocity slowly with slope and inversely to the discharge. From quantitative comparison of the work needed to keep material off the bottom with its addition of energy of the stream it was concluded that the former is greater and hence the velocity of the stream is retarded by suspended load. Increased viscosity also retards the flow of the stream. Retardation by viscosity may reach 15%. Gilbert had difficulty in finding any retardation of velocity due to bed load. (tractional load). It is possible that such is related to the load, slope and velocity. He did find that a load influences the vertical distribution of velocity in a stream. Gilbert concluded that there is an automatic separation of suspended and traction loads. Were the Mississippi deprived of its bed load, he thought it would shoal the channel and reduce its slope until part of the load would be carried on the bottom. On the other hand he concluded that removal of the suspended load would increase velocity and lift some of the material now carried on the bottom. Checks are difficult because of the lack of measurements of bed load.

Application of laboratory results. Slope does not enter into computations at a given locality but applies to streams over a longer distance. In nature there is more variation than in the laboratory. Discharge must be measured to represent equal phases of stream work. The problem is complicated by simultaneous changes in nature of material carried, as well as by changes in velocity and competence. During low stages traction is confined to the tops of the crossings or bars. Turning to the ratio of depth to width, this is related to the resistance of the banks. In general bank resistance of a natural stream should be greater than that of the smooth wall of the laboratory trough. In general the difficulty in extending laboratory formulas to real streams is that they are empirical and not rational. Models do not have the same relations between dimensions that are present in the originals. A glaring example is the size of particles used for transportation. Gilbert simplified his experiments by using several size ranges of sand and fine gravel which were designated by letters. Some tests involved mixtures. Of his sizes the three smallest, .304 mm, .374 mm, and .508 mm average diameter are well below the point where competence of the current changes from the law that linear dimensions of particles are related to the square of velocity. It is not clear that this was recognized by Gilbert. For the sake of simplicity it was tried to set up equations for capacity which are power functions of some one of the variables. In each case a threshold value of that variable at which sand movement first is noticed must be subtracted before applying the exponent. On account of the change in competence with size of grains it is evident that this procedure would have to be changed at the critical point of about 1 mm diameter. When slope alone was used in the above manner, the exponent varied, from .93 to 2.37 and was found to be an inverse function of both discharge and coarseness of debris. When discharge was used the exponent was from .81 to 1.24 suggesting a nearly direct ratio, although the values are an inverse function of slope and size of debris. With fineness, the exponent varied from .5 to .62 suggesting a square root relationship. Values were inverse functions of slope and discharge. Capacity may be made to reach 0 either by making the stream very wide and shallow or very deep and narrow. The optimum ratio of depth to width varied from .5 to .04, inverse to slope, discharge, and fineness. Velocity, which many have thought of as the sole variable, could only be measured as mean velocity. With slope constant the average exponent was 3.2. With constant discharge it was 4.0, and with constant depth, 3.7, seemingly an inverse function of slope, discharge, and fineness. When a mixture of sizes was used the movement was more free. With addition of fine particles to coarse, the movement of the coarse was increased. In the case of changes in depth, results varied when other factors were held constant. With constant discharge velocity increases so that capacity is inverse to depth; However, when slope is held constant depth is related to discharge and capacity varies with depth. Depth is related to the .62 power of discharge. If velocity is held constant both direct and inverse relations were found. The average, considering sign, was -.54 suggesting an inverse square root relationship, but it is evident that depth is a dependent variable and cannot be used alone in a formula. The form ratio or ratio of depth to width has two zeros of capacity, one for a very high value, the other for a very low value.

Flume transportation. The conditions of a flume with no debris left on the bottom may occur in segments of natural streams. This condition leads to an increase of capacity because rolling is more important than jumping. With such motion, capacity is largest for coarse particles; whereas with leaping particles, the reverse is true. Capacity is reduced by roughness of the bottom.

Criticism. It seems clear from the above that any rational formula for capacity (1) must include several variables, possibly all of them, (2) must consider the change in competence with size of particles, (3) should include the relationship to both shearing force on the bed and to degree of turbulence resulting from that, (4) must include the form of the bed, rippled, smooth, or antidunes, all of which occur in succession with velocity increase. Some of these things have been discussed in other supplements and hence will not be here repeated. See especially "concavity of slopes" and "Dimensions and competence of running water."

Little, J. M., Erosional topography and erosion, a mathematical treatment, A. Carlisle and Company, San Francisco, 1940.

Little's book of 1940 appears to be one of the first, if not the first, attempts to find the mechanical features of erosion and the resulting topography. The primary approach of the author was to tie in erosional geomorphology with hydraulics and hydrology. Second to this, he desired to obtain an "erosional rating" for given slopes, soil and cover of vegetation which would be a basis for classification of lands for human use. Both flow in sheets and in channels was considered. He fully recognized the complexity of the problem and stated that some conclusions would have to wait for, or be modified by, the collection of more data. Fundamental assumptions included basing "erosive power of flow" on "some velocity to depth relationship that is exponential" (A power function as here defined).

Types of flow and energy of flow. Little dismisses laminar flow as having no erosive power and little or no coarse silt transporting power. He considers only turbulent flow and shooting (plunging) flow, which occurs when turbulence is excessive. Little concluded that "erosive power of turbulent flow is a function of velocity and depth and not of velocity alone," of which the exact nature under different conditions is undknen. He assumed that the relationship is V^2/D where the side walls of channel do not interfere. The exponent 2 was considered tentative. V = velocity and D = depth. He realized that any attempt to relate erosion to total force exerted on the stream bed in direction of flow is useless because turbulent flow is needed to raise material off the bed. In this he used the Schmidt computation of intensity of turbulence which related it to the total potential energy of a column (or prism) of water divided by the rate of change of velocity at the base. Since the rate of velocity change with depth is at a maximum near the bed of a stream, it is there that turbulence is greatest. "Since turbulence is proportional to kinetic energy of flow, $V^2/2g$, its intensity varies as V^2." Note that this expression is for kinetic energy of unit weight of water in British Engineering Units where mass is obtained by dividing by g, the acceleration of gravity. "The influence of D on turbulence in proximity to the bed is inverse." From this, the fact the relationship of erosive power, E, to V^2/D was deduced. As a check it was noted that this corresponds to loss of head in pipes but no mention was made that this is the same as slope, s, in an open channel. The expression was to apply to scouring and silt transportation by both suspension and bed movement. The equation $V^2/gD = 1$ was then set up with the value of unity expressing erosive power at critical flow, the passage from ordinary turbulent flow to plunging flow. It was concluded that in "the interaction of two materials, liquid and solid,

V^2/gD is a measure of the intensity-distribution of the internal forces which ultimately destroy turbulent flow--without regard to what actual values are assigned to V and D. Neglect of total pressure on the bed is accounted for because "loss of head and internal friction (related to turbulence) are independent of pressure, for the reason that viscosity and density of water are independent of pressure."

<u>Turbulent flow in rectangular channels</u>. In working out the laws of erosion in rectangular channels with water sides, i. e. vertical sections of a wide stream, substitution of Q (quantity in c.f.s.) = DV yields the conclusion that $D = Q^{2/3}/3.18$ and $V = 3.18\ Q^{1/3}$. Mannings formula for velocity is also substituted with some interesting results in solving, by various simple algebraic transformations, for the several quantities involved. Assuming that E (erosive power) = 1 and the roughness coefficient, n = .04, then $S = .03427\ Q^{-2/9}$ which may be compared with the empirical conclusion of Leopold, $S = .021\ Q^{-.49}$. Also, $Q = V^3/gE$.

<u>Turbulent flow in trapezoidal channels</u>. For channels with a flat bottom and sloping sides like many irrigation ditches, the development of formulas for critical flow when E = 1 simply modify results by the ratio of cross sectional area to area of a rectangle. Little then produces the result that for n =.04, $S = .4364\ Q^{-2/15}$ which departs widely from the observed result of Leopold.

<u>Effect of roughness coefficient, n.</u> By taking Mannings formula and substituting Q/D for V and solving for n, it is possible to give a formula by which n can be found experimentally by sprinkling a plot of land with fixed slope. It also appeared that E varies inversely as the 9/5 power of n.

<u>Sheet runoff</u>. It is with sheet runoff that the major relationship to geomorphology was found. Little suggested that the passage from sheet erosion to rill formation is a "point of breakdown or failure--analogous to the failure of any material in the testing laboratory." This he related to the attainment of unity as the value of E. Since he was concerned primarily with land use practices it was then necessary to introduce a "rainfall equation" to give quantity of rainfall. He chose R (rate in inches per hour)=8 T (duration in minutes)$^{-.25}$ A coefficient for relation of runoff to rainfall is needed. Q (runoff) = C (fixed coefficient) x area, A, x rainfall rate, R. Area is a direct proportion to horizontal distance from summit, h. C was placed at .7. By algebraic transformations using formulas given previously, Little arrived at $V = .3662\ h^{2/5} S^{3/10}\ t^{-.1}$ where t is in seconds. To relate observed concave slopes of hills to such erosion, Little concluded that purely convex profiles are typical of young topography which is the result of channel and not sheet erosion. Sheet erosion, in conjunction with removal of material at the bottoms of the slopes by streams, might result in uniform slopes. In older topography he observed that the common relationship is the compound reverse curve, convex above and concave below. Such regular curves develop only on homogeneous material and in nature there are always irregularities along any slope which can produce gulley erosion. He concluded that the work of man in cultivating the soil has increased this tendency and reduced the areas of uniform sheet wash.

<u>Relation to coordinates of slope</u>. In order to relate erosional power to coordinates of a slope, it was necessary to eliminate t and to give other quantities in terms of horizontal distance from divide, h, and to fall, f. Now to eliminate t, it was necessary to find two values for $t^{1/4}$ and then equate them. The first value was derived by equating two expressions for Q. $Q = V^3/gE$ is put as equal to $Q = Ch\ t^{-\frac{1}{4}}$ or $= .00035847h\ t^{-\frac{1}{4}}$ from 1 square foot, and this is

solved for $t^{1/4}$ which equals 1404.5 $S^{9/2}$ h E^{-5}. Further algebraic transformations give $V = .3662\ h^{2/5}\ S^{3/10}\ t^{-1/10}$. By setting V = dh/dt and multiplying both sides by dt/dh it appears that with S constant, $.3662\ S^{3/10} \int dt \cdot t^{1/10} = \int dh \cdot h^{-2/5}$. Integrating this and solving for $t^{\frac{1}{4}} = 1.479 h^{1/6} S^{-1/12}$. Now the two values of $t^{\frac{1}{4}}$ are equated eliminating that term. Solving for S, substituting df/dh for S, and multiplying both sides by dh yields $df = .2229\ E^{12/11}\ h^{-2/11}$. Integrating, it is clear that $f = .2724\ E^{12/11}\ h^{9/11}$. This equation represents fall in feet for horizontal distances in feet, a concave slope. E is supposed to be constant so does not enter into the result. Tables were presented for different values of E, some of them above unity. The basic idea is that slope wash forms the slope until the condition of constant E obtains. C is taken at .7 and n as .04, but the tables also show conditions for other values of these qualities.

Conclusions. Considerable discussion was devoted to the problem of the proper rainfall equation but none seems to be applied to an average over geologic time. No attention was given to the problem of erodibility of different sizes of particles or of mixtures of different sizes. Relation of fineness of particles to age of soil was also considered. Little stressed the idea that hill slopes are formed by sheet erosion only in the later stages of the erosion cycle starting by "gouging" at the bottoms of slopes next to streams. Old ridges should then be narrow and flanked with concave slopes. Little concluded that convex slopes must have a value of E which increases away from the divide whereas concave slopes have a constant E rating. When undisturbed by man a balance between erosion and soil resistance was approached but never quite attained. He desired to obtain E ratings of soils on different slopes by experiments with sprinkled plots rather than by physical tests of the soil. By the development of the concave slopes of fixed E rating the line of division between convex summits and concave lower slopes progresses uphill. It was recognized, however, that some convex divides survive in quite old topography. It was suggested that vegetation which retained rainfall on divides plus laminar flow there might account for this suggestion of Horton's "belt of no erosion." H, total horizontal distance from divide to stream channel, must remain constant during development of erosional topography; whereas F, the total fall, would decrease. A ratio of F to H might express maturity of development. Mass movement due to weight of water, swelling of soil and frost aiding gravity was recognized but not regarded as important. It was stated that "a prominent feature of top soil occurrence is its continuity and its proneness to maintain a uniform thickness on profiles." It was concluded that soil formation follows upon the development of slopes and is not important in forming them. The final conclusion "obviously implies that erosion has been, in general, the dominant process in geologic denudation."

Einstein, H. A., The bed-load function for sediment transportation in open channel flows, U. S. Dept. of Agriculture, Soil Conservation service, Tech. Bull. 1026, 1950.

Under the above title an attempt was made to reexamine an old problem, namely the rate of transportation of the bed load in streams. This problem is not only of scientific interest but is of great practical importance for at present it is difficult to predict just what changes in the bed of a stream will take place when one of the variables is altered by man. Such change upsets the equilibrium of nature which adjusted the size, shape, and slope to the amount of and variation of discharge. Einstein's solution is evidently intended to be a "new look" and he admits that it is not final. Although the title does not so indicate both suspended and bed loads are considered. He admits that no positive answer can now be given as to "what bed composition can be expected from a known sediment load in a known flow."

The general approach is highly mathematical which makes for very slow reading. However, the main points are explained in the text except where they were covered in previous publications. There are two pages of symbols and abbreviations.

From one of the earlier papers it is stated that velocities in the downstream direction vary with the logarithm of distance from the bed or the top of an inferred layer of laminar flow next to the bed. Hence a factor based on roughness of the bed in introduced. The importance of ripples on roughness is considered. Turbulence is considered under the idea of three components in the three primary directions. When Einstein states that all of these have a 0 time average, it is difficult to see how there could be any net downstream velocity! He states that velocity is variable and that a graph at a given level would show a very irregular line, although it would not reach 0 at any time.

Suspension. The primary idea of suspension as support of particles by water motion above their settling velocities is conventional. However, the discussion of distribution of concentration of solids in reference to depth is most unconvincing. His formulas are complex and involve the integral calculus and the fact seems to be ignored that turbulence must distribute suspended load fairly well. A hypothetical example is worked out which process takes three pages. Einstein's result is a tenth of what it should be because a decimal point was misplaced in computing the dry weight of sediment in a cubic foot of water, also his result is described as "per second foot," when it should have been per foot of stream width. The corrected answer is 3.29 pounds per foot width. However, if we compute by Mannings formula the velocity of his hypothetical stream as 3.5 feet per second, take the dry weight per foot as .0642 pounds, and multiply this by velocity, and by depth, 15 feet, to obtain total sediment, the result is 3.27 pounds per foot width. It is then obvious that we have only another example of a "hard way to do an easy thing."

Bed load. Einstein then turns to the particles which slide, roll or hop along the bed of a stream. He works out his theory on the basis of the probability of movement of particles of a given size with a bed load equation to show equilibrium between particles in motion and those at rest. He then evolves the ratio between lifting force and weight of particles. This involves a complex formula relating the submerged weight of a given particle size to the hydraulic radius, slope and square of the velocity at the bed. From this evolves a dimensionless figure for the intensity of transport of this given grain size. From this an actual example was worked out in 44 steps including an estimate of how the different sized particles of the real load affect the theory. Last, this bed load is combined with the suspended load to show the total sediment discharge of the stream. He does not tell how to check this result in the field!

Fisk, H. N., Geological Investigation of the Atchafalaya Basin and the problem of Mississippi River diversion, U. S. Army, Corps of Engineers, Mississippi River Commission, Waterways Experiment Station, 1952.

Introduction. It is a well-known fact that many rivers change their courses where they flow on an alluvial plain above a delta. Among the better known examples may be mentioned the Rhone, Po, Ganges, Yellow (of China) and Colorado. Although some of these events were quite well investigated and dated, in no case was the wealth of detail available that Fisk had for the study of the Atchafalaya distributary of the Mississippi. Thousands of logs of borings, many made especially for the study, detailed maps, air photographs, and measurements of stream discharge and sediment load were all provided. Although possibly the example may not be typical of streams which carry a more heavy sediment load than does the Mississippi it is thought that valuable lessons may be learned from it. The study was undertaken to ascertain if there is serious danger that the Atchfalaya will divert the Mississippi, leaving New Orleans without a major river.

History of the river. Fisk briefly reviews the recent history of the Mississippi River as worked out by him in his 1944 report. He notes the effect of eustatic change in sea level so that he discriminates the sediments of the last filling from those previously laid down, eroded and weathered when the Gulf was lower than it now is. These recent sediments are mainly of postglacial age. Fisk devised them into the sandy substratum which is overlain by a much finer-grained topstratum. River scour in many places extends through the topstratum undermining the more coherent material. The top stratum may be divided into: (a) deposits of natural levees, silt and fine sand; (b) point bar deposits on the insides of meander loops which are made of sand; (c) channel fillings in abandoned courses which are dominantly tight clay; (d) backswamp deposits also fine clay; and (e) deltaic deposits which are in part lacustrine and in part those of brackish water. Material derived from the Red River may be distinguished by its color from the deposits of the Mississippi. The different types of deltaic deposits are illustrated by mechanical analyses. No evidence of recent earth movement other than that due to compaction could be demonstrated.

Past changes in course. , Several distinct courses of the Mississippi River, all occupied since the filling was essentially completed are described. The Teche-Mississippi was far to the west of the present course from a point well above Vicksburg and formed a delta slightly west of south of New Orleans. Traces of this former meandering course now form Teche Bayou. Next the LaFourch-Mississippi was diverted near the course of Red River to essentially the present river course as far south as Donaldsonville. Previously this was the course of the Yazoo. Here the river flowed more to the south forming its delta in essentially the same location as the Teche delta. The date of this change Fisk estimates at 300 to 400 A. D. Next followed another diversion near Vicksburg into the course of the Yazoo which joined the earlier phase of LaFourche Mississippi. This second diversion of the LaFourche route is thought to have occurred about 1000 to 1100 A. D. After this, the modern Mississippi route was formed by a diversion near Donaldsonville in 1100 to 1200 A. D. At first this route was more to the north than the present course below New Orleans and formed the St. Bernard delta well north of the present one. The present route below New Orleans Fisk concludes was formed about 1500 to 1600 A. D., probably not long before white men first saw the river. DeSoto discovered the river near Natchez in 1541 but the mouth was not used until later. It is now forming a delta closer to deep water than were any of the others.

Sediment load of Mississippi River. Fisk shows that the lower Mississippi is now anywhere nearly as highly loaded with sediment as many other rivers. At a discharge of 1,065,000 ft^3/sec. the Mississippi carries only about 871 parts per million of sediment. The Yellow River of China carries on the average about 50,000 p.p.m. at a discharge only a fifth that of the Mississippi, although this load may be quadrupled at times. The Colorado River carries 6,000 p.p.m. Sediment concentration is not wholly due to the hydraulic characteristics of the channel but mainly to the contributions of tributaries.

Cause of diversion. Diversion of a stream naturally occurs in order to secure a more favorable slope to the sea, which an more readily transport the load. In very heavily sediment-laden streams the bed may be built up so high that a break-through of a natural levee forms a permanent channel. Then diversion is sudden and the change of slope will in a short time affect the original channel above the point of diversion. Below this point the old channel is rapidly blocked by alluvial deposits. In the case of the Mississippi such break-throughs or crevasses have often been observed. The original channel is not high enough above the backswamp, which is mostly densely timbered, to make a permanent channel. Instead sand and silt are deposited in a braided channel which is abandoned when the flood subsides. Despite its nearness, the Mississippi has never succeeded in breaking through to Lake Ponchatrain above New Orleans. It is evident that here we must have a different process. Long ago the meandering Mississippi intersected the course of Red River. The northern arm of this loop was soon plugged with alluvial debris, but the south one was open. The Atchafaylaya was blocked with a "raft" of driftwood when whitemen first navigated the river. Absence of Indian mounds on the Atchafaylaya suggests it was not an important route prior to this. The southern arm is called Old River. The interconnection was both freed of driftwood and dredged by white men. Instead of being an outlet of the Red, together with its southward continuation of the Atchafaylaya, it is now a distributary. The diverted waters are steadily increasing in percentage of total Mississippi flow. Extrapolation of the curve indicates that by 1971 about 40% of the annual flow will go through the Atchafaylaya. Then there is grave danger of blocking of the course below with sediment leaving New Orleans without any river. The Atchafaylaya channel has also been much widened and deepened by the increased flow. The process has been probably accelerated by the building of artificial levees along the higher banks of the Atchafaylaya, but retarded by the building of delta in Grand Lakes farther down the course. Obviously some artificial control of the flow is urgently needed.to prevent the consumation of this type of gradual diversion.

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2 carbons

Some Underfit Streams of Central Illinois

Underfit rivers have floodplain meanders which are much smaller than the meanders cut into the adjacent bluffs. During the course ~~of exploration for road materials~~ of a study of sources of road materials in central Illinois for the Illinois State Geological Survey during the years 1929 and 1930, the writer noted this phenomenon along both the Kaskaskia and Embarrass rivers. The accompanying map was traced from the drainage surveys along ~~Kaskaskian~~ Kaskaskia River of the U. S. Geological Survey made from 1908 to 1911. These show several striking meander cusps, mainly along the eastern bluffs (Fig. 1), which suggest a stream several times the volume of the present river. Attention should be directed also to several cases of intercision where such meanders cut through spurs and captured tributaries. A good example of a large meander was also noted on Embarrass River just southeast of Newton.

Similar underfit streams have been described by several physiographers. Davis thought that the cause was diminution of volume, for it has long been recognized that the radius or wave length of meanders is related to the discharge of streams. Loss of volume has been ascribed either to diversion by stream capture, to change of climate, or to seepage into the valley filling. The latter ~~apparently~~ is an inadequate explanation because the underflow even in relatively coarse material is very small compared to the discharge of most streams (Slichter). In the case of these Illinois rivers the cause is probably cessation of drainage from the Wisconsin ice sheet. The localities are below the locus of coarse outwash deposits. Here the glacial waters had become integrated into a single meandering stream unlike the braided complex above where active deposition took place.

~~When the paragraphs above were written~~ in the early 1930's no information was available which could supply a reasonable estimate of the change in volume of Kaskaskia River from glacial times to the present. Now the studies of Leopold and Maddock supply some ground for such a surmise. The area has been mapped on the Ramsey and St. Elmo quadrangles. The radii of the meanders of the present river are about 1/10 mile whereas those shown by the scars in the banks of the bluffs average about 7/10 mile, seven times as much. Elementary physics shows that the lateral component of force of the river due to flowing in a segment of a circle is expressed by the formula, mass ~~x~~ x velocity^2/ radius. If we assumed that both sets of meanders reached equilibrium with the resistance of the banks we then have two problems: (a) how much more resistance did the older bluffs, 40 to nearly 100 feet high, than the banks of the recent meanders which average about 10 feet in height? and (b) how did the velocities of the rivers differ? To make an estimate of velocities it is necessary to know the hydraulic radius, R, (or mean depth, d, in feet,), the slope s, of the rivers involved, in feet per foot, and the nature of the bottom (n). The formula involved is: $v = 1.5/n\ R^{2/3}\ S^{1/2}$. The slope is the least difficult to estimate. The present day Kaskaskia drops 10 feet in about ~~47000~~ 57,400 feet distance along the channel making a slope of 1 in ~~4700~~ 5740 or ~~2.13×10^{-4}~~ 1.75×10^{-4} feet per foot. It is hard to restore the course of the glacial river for the scars were certainly not all made at the same time. A tentative restoration suggests a drop of 30 feet in 40 miles which is 1 in 7030 or 1.43×10^{-4}. Although this is reasonable, because a big river has a less slope than a small one, ~~but~~ it is difficult to tell ~~about~~ how much filling occurred of the old floodplain. ~~The present average width of the Kaskaskia~~ no ¶

The difference in slope ~~does~~ may not ~~appear to~~ be enough. Since the linear dimensions of the glacial and modern Kaskaskia bear a ratio of about 7 to 1 it might possibly be assumed that the glacial gradient was or about $.25 \times 10^{-4}$ feet per foot, ~~that is~~ a seventh of the present ~~condition~~ value. The's ratio is concluded from the width of the present meander belt which is about .5 mile and that of the glacial meander belt which is about 3.5 miles, which ~~This~~ checks with the radii of meanders given above. The present average width of the Kaskaskia is roughly 135 feet from which a glacial mean width of about 950 feet is deduced ~~on this basis~~.

~~Imx At the yime of the road material survey there was no reasonable quanitative method known by which the volume of the glacial Kaskaskia could be determined. Since then~~ The studies of Leopold and Maddock provide ~~such~~ a possible basis for quantitative comparison with glacial conditions. These authors find that the dimensions of a stream are power functions of the discharge (Q). Since we know the mean discharge ~~(Q)~~ (1505 cu ft/sec) of the Kaskaskia over a period of 42 years quantitative estimates may be attempted. The weakest point of these empirical equations is the fact that there is a constant, ~~preceeding the power function~~ the value of which varies greatly in the case of different streams, probably reflecting difference in bed and banks. Two equations suggest applicability to the present problem. Slope 'S' ~~'x'~~ $= k\,Q^{-.49}$ where k is a constant and Q = discharge in cubic feet per second. This may also be written $S = k/Q^{.49}$ Now the difference between this exponent and the square root (.5 power) is so slight that for all practical purposes it may be neglected and we can state that $S = k/Q^{\frac{1}{2}}$ For present day western streams Leopold gives the value of K as 3.97×10^{-6} feet per foot. Solving for the modern mean discharge of the Kaskaskia which is 1505 cu. ft/sec., we obtain a ~~much different~~ value of $.68 \times 10^{-6}$ ~~and~~ It is questionable that this is applicable to ~~the~~ glacial conditions, because of the difference in nature of the bed and banks.

Solving the above slope equation ~~for slope~~ for quanity of discharge it is evident that $Q = (K/S)^2$ By substituting the values for k at present and the minimum value assumed for the slope S, we find $Q = (68 \times 10^{-4} / .25 \times 10^{-4})^2$ which works out to a little less than 74000 cu. ft/sec. If we use instead Leopold's original value of k and the higher estimate of slope ~~measurement~~ of the glacial river, the result is 75,600 cu. ft./sec.

The second equation ~~is~~ involves mean width, W, = $aQ^{.41}$ Substituting modern values, 135 feet = a x 20.04 or a = 135/ 20.04 = 6.72 Using glacial values, 950 = 6.72 x $Q^{.41}$ Solving above for Q, gives Q = $(950/6.72)^{2.44}$ ~~This becomes~~ which is 103000 cu ft./sec. Again the applicability of the constant, a, is debatable. ~~but there does not appear to be any other value known.~~

The above methods do not agree very closely but ~~will~~ serve to give some sort of an idea of the mean discharge of the glacial Kaskaskia which drained not over 25 miles of the front of the Wisconsin ice sheet. It is prehaps not so large as some have imagined for the maximum modern flood was estimated at slightly above 52,000 cu. ft./sec.

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Nevertheless, we will attempt to compute possible changes in discharge which are presumed to have come from the cessation of meltwater flow from the glacier of the Tazewell substage of the Wisconsin stage.

Slope. Leopold's equation which gives the relation between slope and discharge is attractive as a means to estimate possible change in discharge. He gives the constant as $3,97 \times 10^{-6}$ for slope in feet per foot. However, if we solve the expression for the constant, k with modern values, we do not obtain this result $1,74 \times 10^{-4}$. ~~x 38.~~8 ~~or~~ but 0.68×10^{-6} which is much less than Leopold's value. This makes it somewhat doubtfull which value we should use for the glacial floods of meltwater. In attempting to find the slope for the time of formation of the larger meanders it is almost impossible to restore the exact channel for any particular time. Meanders are not made all at once. We may presume that the slope of the old valley eroded into the till was steeper than that of the modern channel. This agrees with the hypothesis that meanders grow until the rotational force equals the resistance of the bank to further enlargement. The small meanders of today are apparently in softer material than were the old large ones and hence record less energy of the stream on a reduced slope. A tentative restoration of the former slope of Kaskaskia River is 1.43×10^{-4} which is admittedly a very rough estimate. We can now proceed to solve the equation $Q = (k/S)^2$ where Q is discharge, k, a constant, and S the slope. We may express Leopolds value for k as 397×10^{-4} and use the above value for S. then $Q = ((397 \times 10^{-4}) / 1.43 \times 10^{-4}))^2$ which is 277^2 or about 77,000 cubic feet per second, somewhat more than the maximum flood recorded. Owing to the great uncertainty in the values of the constant and slope, no great weight can be attached to this estimate.

WidthWidth. The problem is to estimate the probable width of channel when the large meanders were formed. It has been shown above that the old meanders were close to 7 times as large as those of the present but we have no idea of the bank materials when thse huge loops formed. The outer banks were till, but there may have been a floodplain of softer material in the middle of the valley.

However, we will attempt to compute possible changes in mean discharge which might be due either to cessation of meltwaters from the Tazewell substage of the Wisconsin stage of glaciation or to a change from a more rainy climate. None of the formulas tells us anything of change in material into which meanders were eroded.

Slope. Leopold's equation relating slope to discharge is attractive as a means to solve the above problem. Hix constant averaged from streams he considered is 3.97×10^{-6} when slope is measured in feet per foot. If we use the present value of mean discharge we obtain a somewhat higher value of 4.47×10^{-6} for the constant. which is not enough different to alarm us.

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Nevertheless, we will attempt to compute possible changes in discharge which are presumed to have come from the cessation of meltwater flow from the glacier of the Tazewell substage of the Wisconsin stage.

is

Slope. Leopold's equation which gives the relation between slope and discharge is attractive as a means to estimate possible change in discharge. He gives the constant as $3{,}97 \times 10^{-6}$ for slope in feet per foot. However, if we solve the expression for the constant, k with modern values, we do not obtain this result for $1{,}74 \times 10^{-4}$ (1505.5). ~~x 38.8 or~~ is ~~DJF~~ 4.47 0.68×10^{-6} which is ~~much less~~ more than Leopold's value. This makes it somewhat doubtfull which value we should use for the glacial floods of meltwater. In attempting to find the slope for the time of formation of the larger meanders it is almost impossible to restore the exact channel for any particular time. for Meanders are not made all at once. We may presume that the slope of the old valley eroded into the till was steeper than that of the modern channel. This agrees with the hypothesis that meanders grow until the rotational force equals the resistance of the bank to further enlargement. The small meanders of today are apparently in softer material than were the old large ones and hence record less energy of the stream on a reduced slope. A tentative restoration of the former slope of Kaskaskia River is 1.43×10^{-4} which is admittedly a very rough estimate. We can now proceed to solve the equation $Q = (k/S)^2$ where Q is discharge, k, a constant, and S, the slope. We may express Leopold's value for k as 397×10^{-4} and use the above value for S. then $Q = ((397 \times 10^{-4}) / 1.43 \times 10^{-4}))^2$ which is 277^2 or about 77,000 cubic feet per second, somewhat more than the maximum flood recorded. Owing to the great uncertainty in the values of the constant and slope, no great weight can be attached to this estimate.

not? 9

is

channel

=?

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25,800 cubic feet per second and for width 950 about 118,200 cubic feet per second. The other method gives for wave length 5800 feet 26, 300 cubic feet per second, for wave length 12,400 feet about 118,000 cubic feet per second. This shows that the two methods agree closely.

Summary. Working on the hypothesis that size of meanders is closely related to discharge only, we have obtained a vairiety of results for mean discharge many of them greater than any recorded flood of modern times. All of them are much above the present mean discharge. This is in line with map study of rivers which shows clearly that big streams have big meanders and small streams small meanders. Most comparisons do not include slope or material so that too much weight should be given to this map study conclusion. The possible climate of Pleistocene times of glaciation is little understood. Rainfall could have been greater than now south of the glacier although this does not agree with the idea of winds descending from the ice cap. Meltwaters seem a more logical cause for a larger discharge.
Further study is needed to connect these large meanders with the zone of outwash deposition nearer to the ice front.

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Kaskaskia rule

radius

T_W | - present $\frac{1}{10}$ m

old $\frac{7}{10}$ m

Some 15

log 1505 = $\overline{2}$/3.1774

1,5887 =

38.80

bp = 514

499 500 at N line Fayette Co middle 9

480 middle ~~35~~, 8 N

31

470 " ~~11~~ 7 N S side 22 better

Vandalia 459 ff plan = 473

about 46'

Beardsbridge river Q = 1505 sec/ft

$\sqrt{1505} = 38.8$

ft/m

$S = .021 \times 38.8 =$

$\frac{-2}{-1\,2} = 0$

40
40
1600
40
200

K =

.00213 = K × 38.8

$K = \frac{.000213}{38.8} = .0000074$

S = .021 Q = 0.49 Leopold

old slope = $\frac{30''\ \text{mi}}{}$ ~~35~~ 40 m = $\frac{1}{7030'}$

modern slope $\frac{1}{4700'}$

.000213
2.13

S = -2.000143

$F = K\frac{V^2}{r}$

$V^2 = \frac{1.5}{n} d^{2/3} S^{1/2}$

$\frac{30 \times 2}{211000}$ (40 × 5280)

000213 = 000369 $Q^{1/2}$

$Q^{1/2} = \frac{000143}{0000074} = 19.5$

$19.5^2 = 380.00$

width old meanders 3.5 miles

new meanders .5 mile

or $\frac{000213}{021} = Q^{1/2}$

modern width $\frac{1}{30}$ m or less = 175'

better 150'

1050

old width

$\frac{7}{30}$ mile = 1230'

1050' better

21100.

7030

$\frac{000213}{\frac{000213}{Q^{1/2}}} = Q^{1/2}$

150
150
7500
150

$Q = \frac{W^{2.44}}{a}$

$W = aQ^{.41}$

$Q^{.41} = \frac{W}{a}$

$Q = \frac{W^{2.44}}{a}$

$1505 = \frac{150^{2.44}}{a}$

$a = \frac{150^{2.44}}{1505}$

$3.00 \times 2.44 = 7.35$

log 150 = 2.1761

× 2.44 = 5.300 = 199500

log 1000 = 3.

15 $\frac{199500}{1505} = 132.5$

$Q = \frac{1000^{2.44}}{132.5}$

$= \frac{22390000}{132.5}$

$-4 \quad +1 = 3$

$\frac{1505}{7}$
10535

= 168900. ft/sec

$000143^2 =$
00000548

$S = \text{const}\ Q^{1/2}$

$Q = \left(\frac{\text{const}}{S}\right)^2$

$K = \frac{.000213}{38.8} = 00000\ 548$

$\frac{0213}{.38.8}$

$\begin{matrix}-3 \\ -(+)2\end{matrix} = -5$

$-3 - (+2) = -5$
-1

$\frac{.213}{.0388} = .0548$

$\frac{.000213^2}{0548^2} = \frac{.000000\ 453}{.003} =$

$-3^+ - 3 = -6$

$-1+-1 = -2$

$-1 - (+4) = -5$

$-6 - -2$

$150 = a\ 1505^{.41}$ $\log 1505 =$ $3.1776 \times .41 = 1.305$

$= 2.018$

$a = \frac{150}{2.018} = 74.5$

~~1000 =~~ $Q = \frac{1000^{2.44}}{2.018}$

$\log 1000 = 3.0$
$3.0 \times 2.44 = 7.33$

2,138,000

5.02×10^{11}

wavelength = complete curve to renewal of direction = 2π radians
or linear distance between successive ripples

bankfull discharge

$\lambda = 36\, Q^{1/2}$ or $\left(\frac{\lambda \text{ feet}}{36}\right)^2 = Q$

$\lambda = 6.5\, W^{1.1}$

equate $36\, Q^{1/2} = 6.5\, W^{1.1}$

square both sides

$36^2 Q = (6.5)^2 W^{2.2}$

$Q = \left(\frac{6.5}{36}\right)^2 W^{2.2}$

now let $\lambda = 2\pi\, .7 \times 5280 = 6.28 \times .7 \times 5.28 \times 10^3$ feet $= 23.2 \times 10^3$ feet

$\lambda^2 = \cancel{5}38 \times 10^6$ $36^2 = 1296$

$\frac{538 \times 10^6}{1296} = .417 \times 10^6 = 4.17 \times 10^5 = 417{,}000$ sq ft

$6.5^2 = 42.25$ $\frac{42.25}{1296} = .03250$ $W = 950$ ft

log $950 = 2.9777$

$950^{2.2} = 6.509$ 6.28

$2-4 = -2+1 = -1$

log $42.25 = 1.6258$

log $1296 = 3.1129$

$-2.5129 = .03250$ ok "

$= 3214.00 \times .0325 =$

1043.0

or $\cancel{.03250}$ $1.4871 = 30.69$ ok

$= 3.214 \times 10^5$

$3.214 \times 10^5 \times .0365 =$

3199

3200×3

$3.214 \times 10^5 = 1.17 \times 10^5$

$.117 \times 10^5$

~~117000~~

$\frac{1}{\cancel{32.6}}$ $\frac{1}{30.69}$

or $\frac{321.400}{30.4} = 105000$

6-2

4

23^2

232

232

464

696

4641

538.24

$.7m = 3700$

$2\pi \times 3700 = 23200.0$

$\times 4$

1

33

6

198

$6.5 W^{1.1} = 36 Q^{1/2}$ λ $Q = \frac{\lambda^2}{36^2}$

$Q^{1/2} = \frac{6.5}{36} W^{1.1}$

$Q = \frac{42.25}{1296} W^{2.2}$

$= .0326 W^{2.2}$

$950^{2.2}$

3.1416

6.28

$\frac{1}{30.6}$

$(2-4)+1$

$-2+1=-1$

$\log 950 = 2.9777$

$\log 950^{2.2} =$

modern $\lambda = 2\pi \times .7m = 2\pi \cdot .7 \times 5280' = 4400'$

older $\lambda = 7 \times 2\pi \times .7 = 4.9 \times 2\pi \times 5280 = 30800$

$Q = \frac{\lambda}{36^2}$

$= \frac{\cancel{4400}}{36^2} = \frac{4400}{1296} = \frac{(30.8 \times 10^3)^2}{1296}$

2.9777
2.2
5.9554
59554
6.55094

antilog $= 4477000 = 4.477 \times 10^6$

$\frac{4.477 \times 10^6}{30.6} = 3.06 \times 10$

$= 1.565 \times 10^5$

156500

$\frac{90}{3}$

$2-(1+1) = 1$

30.6
30.6
2464
9240
9488 4.49×10^4

6.5
6.5
325
390
42.25

36
36
216
108
1296

95

$W = 950$

Dodgeville NO 4

Limestone 150-170

no signs of basal beds

1505

$\frac{1}{7} \times 10^{-3}$

$.14 \times 10^{-3}$

slope { 1 in 5740 = 1.75×10^{-4} modern S_m

1 in 7030 = 1.43×10^{-4} glacial S_g

$S = KQ^{-.49}$

$S = \frac{K}{Q^{.49}}$

$= (K/S)^2$

$K = \frac{SQ^{.49}}{\cancel{Q^{.49}}} = \frac{1.75 \times 10^{-4}}{\cancel{38.8}} \times 38.8 = \frac{68}{\cancel{40.8}} \times 10^{-4} = K$

$= \cancel{4.5 \times 10^{-6} = K}$

$Q = \left(\frac{S_g}{K}\right)^2 \quad \frac{143 \times 10^{-6}}{\cancel{4.5 \times 10^{-6}}\ 45 \times 10^{-4}} = \cancel{3.18} \times 10 = 31.8$

3.18×10

~~should be $318^2 = 100{,}000$ cusec~~

$w = aQ^{.41}$

$a = \frac{w}{Q^{.41}}$

$Q_m = 1505$ cusec

$Q_m^{.41} = 20.04$

$\left(\frac{68 \times 10^{-4} = K}{1.43 \times 10^{-4} = S_g}\right)^2$

$= \cancel{27.8}$

$\cancel{31.5}$

47.5^2

27^2

$.46 \times 10^{-4}$ .25 .53

~~$31.5^2 = 1000$~~

$(136)^2$

$47.5^2 =$

2,250

1.75

.265

$a = \frac{135}{20.04} = 6.72$

50
50
250
250
2750

$Q = \left(\frac{w}{a}\right)^{2.44} = \left(\frac{950}{6.72}\right)^{2.44} = 141.5^{2.44}$

17,500

21,500

74000

$.1 = 10^{-1}$ $.01 = 10^{-2}$

$141.5^{2.44} = 177800$ Q_g

$.021 = 21 \times 10^{-3}$

5280 5.28×10^{3}

3.97×10^{-6}

log 141.5 =

$Q_g = Q_m \times \left(\frac{W_g}{W_m}\right)^{2.44}$

50 x present

log 141.5 = 2.1508

$1.22^2 = 1.5$

log $141.5^{2.44}$ = ~~3.600~~

5.1900

= ~~4786~~

$Q_g = Q_m \left(\frac{S_m}{S_g}\right)^2$

$\left(\frac{1.75 \times 10^{-4}}{1.43 \times 10^{-4}}\right)^2$ (1.22)

1000

103,000

~~Final~~ solution

mean discharge 1505 cu ft/sec

$S = K_s Q^{-.49}$ or $S = \frac{K}{Q^{1/2}}$ or $Q = \left(\frac{K}{S}\right)^2$

now K (required) = 3.97×10^{-6} but solving $K = \frac{S}{Q^{1/2}}$ where Q = 1505

$Q^{1/2} = 38.8$ let glacial S = $.25 \times 10^{-4}$ modern S = 1.75×10^{-4} $\left(\frac{1}{5740}\right)$

1.43×10^{-4}

(glacial S might be $\frac{1}{7}$ this value) or tentative glacial S

modern K = $\frac{1.75 \times 10^{-4}}{38.8}$ $\frac{1.75 \times 10^{-2}}{38.8}$ = 4.50×10^{-2} =

now solving $Q = \left(\frac{K}{S}\right)^2 = Q = \left(\frac{4.50 \times 10^{-2}}{.25 \times 10^{-4}}\right)^2 = \frac{20.3 \times 10^{-4}}{.625 \times 10^{-8}} = 32.5 \times 10^4 =$

32500 cu ft/sec

or $Q = \left(\frac{3.97 \times 10^{-6}}{1.43 \times 10^{-4}}\right)^2 = (2.77 \times 10^{-2})^2 = 7.67 \times 10^{-4}$

.000174

$\frac{1}{5740}$

$W = a Q^{.41}$ modern W = 135 $a = \frac{W}{Q^{.41}}$ Q = 1505 log 1505 = 3.1776

$\frac{W}{Q^{.41}} = \frac{135}{19.95} = 6.77$ $Q^{.41} = 19.95$

$K = S Q^{1/2}$ $Q^{1/2} = 38.8$, $S = .25 \times 10^{-4} =$

$K = 9.7 \times 10^{-4}$

2.9912
29.912
329032

$Q = \left(\frac{K}{S}\right)^2$ $K = \frac{S}{Q^{1/2}}$

$\left(\frac{9.7 \times 10^{-4}}{.25 \times 10^{-4}}\right)$ = 10000

3.97×10^{-6}

$K = \frac{.25 \times 10^{-4}}{38.8}$

$\frac{.021}{5280}$ $\frac{21 \times 10^{-3}}{5.28 \times 10}$ $\frac{1.75}{38.8000}$ $\frac{9.7 \times 10^{-3}}{.25 \times 10^{-4}}$ $(3.88 \times 10)^2$

$15,1000 \times 100$
$15,100000$

18
18
144
18
324

$Q = \frac{S}{K}$

$\frac{.25 \times 10^{-4}}{4.5 \times 10^{-2}}$

2.9912
29912
29034

29777
29777
327547

1761
15
1776

$\frac{25 \times 10^{-2}}{4.5 \times 10^{-2}} = 5.55$

$Q = \left(\frac{K}{S}\right)^2$ $\frac{3.97 \times 10^{-6}}{.25 \times 10^{-2}} = (15.9 \times 10^{-4})^2$

254×10^{-8}

$K = \frac{S}{Q^{1/2}}$

$S = KQ^{-1/2}$ or $S = \frac{K}{Q^{1/2}}$ $\cancel{K = SQ^{1/2}}$ $K = SQ^{1/2}$

let $S = .25 \times 10^{-4}$

$Q^{1/2} = 38.8$

$K = \frac{.25 \times 10^{-4}}{38.8} = \frac{2.50 \times 10^{-1}}{38.8} = 6.45 \times 10 = 64.5$

68
4
272

68.1×10^{-4}

K potent is $(1.75 \times 10^{-4}) \times 38.8$

$Q = \left(\frac{K}{S}\right)^2 = \left(\frac{68 \times 10^{-4}}{.25 \times 10^{-4}}\right)^2 = 272^2 = 74000$

$K = SQ^{1/2}$ $S = \frac{K}{Q^{1/2}}$ $Q = \left(\frac{K}{S}\right)^2$

$K = SQ^{1/2}$ $1.73 \times 10^{-4}(38.8)$

$Q = \left(\frac{K}{S}\right)^2$ let $S = .25 \times 10^{-4}$

$K = 39.7 \times 10^{-5}$

$Q = \left(\frac{.25 \times 10^{-4}}{3.97 \times 10^{-6}}\right)^2 = \left(\frac{2.50 \times 10^{-5}}{3.97 \times 10^{-6}}\right)^2$

1.43

$= (6.3 \times 10^{1})^2$

39.69×10^4

$39{,}690$

189
378
39.69

63
63
189
189
2079

189
189
2079

$(1.74 \times 10^{-4})\,38.8 = 68.0 \times 10^{-4}$

$= \frac{14.3 \times 10^{-3}}{3.97 \times 10^{-6}} = 3.62 \times 10^3$

$= 13.10 \times 10^6$

$Q = \left(\frac{K}{S}\right)^2 = \frac{68 \times 10^{-4}}{.25 \times 10^{-4}} = 372$

$372^2 = 138000$

$\frac{14.3 \times 10^{-3}}{4.5 \times 10^{-2}}$ $1.3.18$

define wave length λ final

$\lambda = 36\, Q^{1/2}$ or $\frac{\lambda^2}{1296} = Q$ ~~$\lambda =$~~

$\lambda = 6.5\, W^{1.1}$

$36\, Q^{1/2} = 6.5\, W^{1.1}$

$Q^{1/2} = \frac{6.5}{36} W^{1.1}$

or equate $36\, Q^{1/2} = 6.5\, W^{1.1}$

$36^2 Q = (6.5)^2 W^{2.2}$ or $Q = \frac{42.25}{1296} W^{2.2}$ $Q = \frac{6.5^2}{36^2} W^{2.2}$

$\lambda = 2\pi \times .7 \text{ mi} \times 5280 = 6.28 \times .7 \times 5.28 \times 10^3 = 23.5 \times 10^3$ $\lambda^2 = \frac{532}{\times 10^6}$

23500

(4.15×10^5)

$\frac{\lambda^2}{36^2} = \frac{532 \times 10^6}{1296}$ $= Q = .410 \times 10^6 = 410000$ sec ft

~~4.1 [illegible]~~

$\frac{42.25}{1296} = .0325$ or $1/30.69$

$6.5^2 = 42.25$ $W = 950'$ $W^{2.2} = 3{,}214{,}000$

$3.214 \times 10^6 \times .0325 = .105 \times 10^6 = 1.05 \times 10^5 = 105000$

---

log 950 = 2.9777

22

log $950^{2.2}$ 59554

59554

6.55094

74000

75600

103000

410000

~~105000~~

116000

5 | 774600

av = 154900' average

5 | 774600 (1549

5

27

25

24

20

46

$\frac{3555000}{30.69} = 116000$

3.069

$\lambda = 6.5\, W^{1.1}$ Q

W = 950 log 950 = 2.9777

1.1

29777

29777

log $W^{1.1}$ 3.27547 $W^{1.1} = 1884.$

$\lambda = 12250$

$2\pi = 6.28$. $r = .7 \times 5280 = 3690\,ft = 23150\,ft = 2.315 \times 10^4$

3.69×10^3

$(10^4)^2 \quad 2.315^2 = 5.39 \times 10^8$

$\frac{5.39 \times 10^8}{1296} = \frac{539 \times 10^6}{1296} = .417 \times 10^6 = 4.17 \times 10^5$

417000

modern Q = 1505 $\quad \log 1505 = 3.1776$

$\log 1505^{.26} = .826$

$1505^{.26} = 6.699 \times c$

$c = 19+$

49 36
49

Final solution

$$\text{scale right } c_1 + c_2 - 1 = c_{final}$$ meet
$$\text{left } c_1 + c_2 = c_3$$
$$\text{right } (c_1 - c_2) + 1 = c_6$$ avoid
$$\text{left } (c_1 - c_2) .25 = c_4$$

Slope -

Present $S = 1.75 \times 10^{-4}$ estimated glacial slope $\frac{1}{7}$ or $.25 \times 10^{-4}$

$$S = KQ^{-.49} \text{ or } S = \frac{K}{Q^{+1/2}} \text{ or } Q = \left(\frac{K}{S}\right)^2 \text{ or } \cancel{K = \frac{S}{Q^{1/2}}}$$

modern $Q = \cancel{1505}$ cu ft/sec $\cancel{Q^{1/2} = 38.8}$

modern $K = \cancel{\frac{1.75 \times 10^{-4}}{38.8} = \frac{174 \times 10^{-2}}{38.8} = 4.5 \times 10^{-2}}$ assuming glacial same

glacial $Q = \cancel{\left(\frac{4.5 \times 10^{-2}}{.25 \times 10^{-4}}\right)^2 = \left(\frac{4.5}{.25} \times 10^2\right)^2 = (18 \times 10^2)^2 = 324 \times 10^4}$

$= 3240,000$ — no $\quad Q = \left(\frac{S}{K}\right)^2 = \left(\frac{.25 \times 10^{-4}}{3.97 \times 10^{-6}}\right)^2 = \left(\frac{25 \times 10^{-2}}{3.97 \times 10^{-6}}\right)^2$

$= (6.27 \times 10^4)^2$

$= 39.2 \times 10^8 = 392$

Leopold gives $\cancel{K = 3.97 \times 10^{-6}}$

$\cancel{Q = \left(\frac{3.97 \times 10^{-6}}{.25 \times 10^{-4}}\right)^2 = (16 \times 10^{-2})^2 = 256 \times 10^{?}}$

all wrong

Leopold gives $3.97 \times 10^{-6} = K$

---

width $\quad W = aQ^{.41}$ $\quad$ modern $Q = 1505$ cu ft/sec $\quad \log 1505 = 3.1776$

$\log 1505^{.41} = 1.3000$

$1505^{.41} = 19.95$

$W = a \times 19.95$ $\quad$ glacial $W = 950$ $\quad a = \frac{W}{Q^{.41}}$

modern $W = 135$

$a = \frac{19.95}{W} = \cancel{\frac{19.95}{135}} = \cancel{.148}$ $\quad a = \frac{135}{19.95} = 6.77$ ✓

$Q = \left(\frac{950}{6.77}\right)^{2.44} = (140)^{2.44}$ $\quad \log 140 = 2.1461$

$\log 140^{2.44} = 5.230 = \cancel{177800}$

169,800 OK

---

meander length $\quad \lambda = 6.5 \times W^{1.1}$ $\quad$ let $W = 950$ $\quad \log 950 = 2.\cancel{9772}$ 9777

$Q^{1/2} = \frac{\lambda}{36}$

$6.5 \times \cancel{1884} = 12250$ ft, $12,680$, $\cancel{12350}$

$2.9777 \times 1.1 = 3.275$ $\quad 3.590 = \cancel{950}, \cancel{3890}$, 1884

$Q = \left(\frac{\lambda}{36}\right)^2 = \left(\frac{12250}{36}\right)^2 = (352)^2$ $\quad$ 116500, 118000, 124000 OK

115500, 115500
124000, 116500
163800, 169800
74000, 77000
75600
39640, 92800
138000
OK, smaller

equation two $\quad 36\,Q^{1/2} = 6.5W^{1.1}$ $\quad$ let $W = 950$ $\quad W^{1.1} = \cancel{1950}$ 1884

$Q^{1/2} = \frac{6.5W^{1.1}}{36} = \frac{6.5 \times 1884}{36} = \frac{12250}{36} = 340$ $\quad$ mean 114500

$340^2 = 115500$

$Q = \left(\frac{6.5W^{1.1}}{36}\right)^2$ 4-27

$Q1 = \left(\frac{6.5 \times 1884}{36}\right)^2 = \left(\frac{6.5 \times 52.3}{6.5 \times 52.3}\right)^2 = (340)^2$

$340^2 = 116500$

$2\pi \times 3V = x\left(\frac{5280}{\ }\right)$

$6.28 \times .7 \times 5280$

$628 \times .7 \times 5.28 \times 10^3$

$44 \times 5.28 \times 10^3$

23.2×10^3

$23.2000 = \lambda \quad 12,250$

$Q = \left(\frac{\lambda}{36M}\right)^2 = \left(\frac{232 \times 10^2}{36}\right)^2$

$= (6.63 \times 10^2)^2 = (44 \times 10^4) = 440,000$

44

$\cancel{1000}$

$4^3 = 16$

$\begin{array}{r} 16 \\ 4 \\ \hline 64 \end{array}$ even

$\begin{array}{r} 36 \\ 6 \\ \hline 216 \end{array}$ $\begin{array}{r} 49 \\ 7 \\ \hline 343 \end{array}$ 3/2+1 1

$\begin{array}{r} 64 \\ 8 \\ \hline 512 \end{array}$ $\begin{array}{r} 81 \\ 9 \\ \hline 729 \end{array}$ $\begin{array}{r} 121 \\ 11 \\ \hline 1331 \end{array}$ 4-2 even

$\begin{array}{r} 144 \\ 12 \\ \hline 288 \\ 144 \\ \hline 1728 \end{array}$ even 4-2 $2 = \frac{1}{10}4+2$

invert scale

c=1 set 1 on right scale to figure in left scale look for coincidence

$cx = \frac{1}{3}(ca+2)$ set 1 on right scale to figure in right scale same $ca = \frac{1}{3}ca$ $ca = \frac{1}{3}(ca+1)$

c = 2 set 1 on right scale ~~$cx = \frac{1}{3}(ca$ or $\frac{1}{3}(ca+1)$~~

or 3 same

c = 4 set 1 on right scale to figure in right scale same or to left same

$ca = \frac{1}{3}(ca+2)$ $\frac{4+2}{3} = 2$

odd $\cancel{cx} = \frac{1}{3}(\cancel{ca}+2)$ or $(ca+1)$

even $cx - \frac{1}{3}ca$ or $\frac{1}{3}(ca+2)$

first set 1 on right scale to figure in either 2 scale and look for same values opposite on 2 lower scales either way you can get it

$\frac{a^{N}}{ra}x$ $x = \sqrt[3]{a}$ ~~when of a odd use left scale~~

" a even use right scale no

a above 100 " use left below 100 right

$cx = \frac{1}{3}(ca+1)$ $ca = \frac{1}{3}(ca+1)$