

Supporting Teachers' Use of Virtual Manipulatives

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Abstract

Technology integration is a critical and longstanding issue in mathematics education. As access to various technology resources increases, so too has the expectation for teachers to use technology to enhance student engagement and understanding. Despite the potential benefits to integrating technology into teachers' instructional practices, teachers often teach *near* as opposed to *with* technology. *Teaching near technology* describes teachers using technology in a manner that does not promote opportunities for students to communicate, reflect, and connect mathematical representations. *Teaching with technology*, on the other hand, refers to using technology to promote opportunities for students to communicate and reflect on the mathematics involved, as well as use and connect mathematical representations. Which leads me to the question, *How can teachers be supported to teach with (as opposed to near) technology?* More specifically, *How can teachers be supported to teach with virtual manipulatives and activities?*

Virtual manipulatives (VMs) are one technology tool that has gained increased attention by teachers, researchers, and organizations as a tool for potentially increasing student engagement and understanding. Although VMs have been used in classrooms for quite some time, limited resources exist to support teachers' efforts to teach *with* this technology tool. Therefore, the following study investigated a professional development (PD) opportunity for secondary mathematics teachers aimed promoting teachers use of VMs and activities. A task analysis framework and a set of guiding questions were developed and refined during the PD in an effort to support teachers' efforts to teach *with* virtual manipulatives. Teachers' appropriation of practices related to implementing VMs and activities in their classrooms as well as tensions related to their implementation efforts were also explored.

A professional development model, set of guiding questions, and task analysis framework are presented as resources to support teachers' efforts to teach *with* virtual manipulatives. It is

posited that these resources could be adapted for future endeavors aimed at supporting pre-service and in-service teachers to teach *with* technology-based instructional tasks.

Part 1: Supporting Teachers' Use of Virtual Manipulatives

Introduction

I would like you to take a walk with me down a hallway of Algebra 1 teachers.

Stepping into Ms. Johnson's classroom, we see that for her lesson on properties of linear functions, she projects the PowerPoint lesson from the textbook publisher using the SMART board. Occasionally, in response to a student question, she writes an additional explanation on the whiteboard. At the end of the lesson, Ms. Johnson projects the online student textbook using the whiteboard.

Down the hall, Mr. Rodriguez also projects his lesson onto a SMART board. However, during the lesson, he uses a virtual manipulative on the board to explore how changing the slope and y -intercept of an equation of a line changes its graph. Students and Mr. Rodriguez propose possible changes to the equation and the effect the changes will have on the graph. After the exploration, Mr. Rodriguez continues with notes on the lesson. When needed, he goes back to the virtual manipulative to address a student question.

Next door, Ms. Kahn has designed an activity for students to work in groups investigating properties of linear functions using a virtual manipulative. Working in groups, students compare how changes in the equations affect the resulting graphs of the functions. They then use the virtual manipulative to test their hypotheses relating equations to specific properties they see in their graphs.

These vignettes demonstrate relatively different and typical ways teachers use technology. Additionally, they highlight the difference between teachers teaching *near* technology as opposed to teaching *with* technology. Adapted from Leatham (2002)'s description of a teacher *teaching near technology*, in this study, teaching near technology describes teachers using technology in a manner that does not promote opportunities for students to communicate, reflect, and connect mathematical representations (e.g., using an interactive whiteboard for projection only). *Teaching with technology*, on the other hand, refers to teachers using technology to promote opportunities for students to communicate, reflect on the mathematics involved, as well as use and connect mathematical representations.

Due to her projector-like use of the interactive whiteboard, Ms. Johnson is an example of a teacher teaching *near* technology—students are not encouraged to share and then test conjectures about the relationship between the equation and resulting graph of a linear function.

The responsibility for making connections between representations rests on the PowerPoint lesson and Ms. Johnson, since explanations are only coming from these two sources. In contrast, Mr. Rodriguez and Ms. Kahn are both examples of teachers teaching *with* technology. Although Mr. Rodriguez also projects his lessons on the interactive whiteboard, his use of the virtual manipulative (VM) promotes students in making and testing conjectures about the relationship between an equation and its graph. Opportunities exist for students to reflect and build on possible relationships shared by themselves and their classmates and, through the use of the VM, dynamically see the resulting graphs when the equations are changed. Ms. Kahn's VM investigation activity, which enabled students to dynamically interact with mathematical representations, places more responsibility on the students to test and reflect on the conjectures of their group members compared to Mr. Rodriguez's activity where he was the one manipulating the VM.

As a teacher interested in integrating technology within my own classroom, and while working with other teachers, I often wondered why teachers integrated technology differently. Why did it appear easier for some teachers to integrate technology into their classroom practices compared to others? Such technology integration differences also often occurred with teachers whom had attended the same trainings and taught the same content course(s). While some teachers embraced technology as a tool to support students in developing a deeper conceptual understanding of the content (e.g., Ms. Kahn), others (e.g., Ms. Johnson) used technology to support traditional methods of instruction (i.e., using an interactive whiteboard for projection only). Which leads me to the question: *How do we support teachers to teach with technology as opposed to teaching near technology?*

I focus on technology, specifically virtual manipulatives, because due to having certain concepts or *skills* built in, technology can transform the skills and concepts that need to be concentrated on in the mathematics curricula from focusing on calculations to modeling, making predictions, reflecting on the power and uses of mathematics, etc. (Romberg, 1998). Additionally, technology can provide opportunities for students to engage in conceptual conversations that foster students' understandings and intuitions (Knuth & Hartmann, 2005) and provide the means for deeper student reflections (Heid & Blume, 2008b). Furthermore, Dunham and Hennessy (2008) note that recent studies suggest that technology can be used to address issues of inequity amongst students and make higher levels of mathematics more accessible to all students. For example, due to the potential to off-load calculations to the technology tool so that students can then focus their attention on the content and development of conceptual understanding. Additionally, the interactive (sometimes "game-like") environment and potential to receive immediate feedback can promote student exploration and perseverance for students who disengage or get frustrated by paper and pencil investigations (Moyer-Packenham & Westenskow, 2013).

Despite the potential benefits to integrating technology into teachers' instructional practices (e.g., see Dunham & Hennessy, 2008), studies have found that when technology is used, it is not usually used to support student-centered instruction—instruction that is believed to be the most effective for supporting student learning (Ertmer & Ottenbreit-Leftwich, 2010). Additionally, many teachers do not effectively integrate technology (e.g., teaching *near* rather than *with* technology) into their instructional practices (Dunham & Hennessy), a critical and longstanding issue in mathematics education. Furthermore, teachers often state that they are not prepared to effectively integrate technology in their instruction (Albion, Tondeur, Forkosh-

Baruch, & Peeraer, 2015). Thus, if we accept that effective technology integration (i.e., teaching *with* technology) is *essential* to student learning and a necessary component of effective teaching (Ertmer & Ottenbreit-Leftwich), then technology can no longer be used solely as a *supplemental* teaching tool.

Using technology effectively in teaching mathematics requires a deeper knowledge of mathematics and technology, as well as of the ways in which teaching with technology transforms mathematics instruction (Wilson, 2008). Furthermore, Wilson suggests that teachers need to learn how to distinguish between mundane uses of technology (e.g., teaching *near* technology) and powerful instructional uses of technology (e.g., teaching *with* technology). Often, professional development (PD) is considered an integral component to teacher learning when changes to instructional practices and knowledge are sought (e.g., Borko, 2004; Driskell et al., 2016; Martin et al., 2010). However, over the past several decades minimal studies investigating PD aimed at promoting teachers use of VMs exist (Driskell et al., 2016). Therefore, the following study investigated a PD opportunity for secondary mathematics teachers that aimed to promote teachers' effective use of VMs as a means to enhance student learning (i.e., to help teachers move from teaching *near* technology to teaching *with* technology). The following questions guided the development of the study:

RQ1: *What instructional practices related to implementing virtual manipulatives to enhance student understanding are appropriated by teachers in a professional development opportunity?*

RQ2: *What systemic tensions influence the appropriation of these practices?*

Virtual manipulatives

Originally, VMs were online manipulatives modeled after physical objects. More specifically, Moyer-Packenham and colleagues defined a VM as “an interactive web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge” (Moyer, Bolyard, & Spikell 2002). By focusing on the interactive capability of a VM, the definition given excludes sites whereby a computer gives an answer in symbolic or visual form as the result of a user pointing or clicking. Although initially java-based applets, VMs are now found in a variety of platforms (e.g., HTML5, flash, cdf, etc.) and as apps for tablets (e.g., iPad and Android). Although many current VMs are based on physical manipulatives (e.g., algebra tiles), others do not have physical counterparts (e.g., scatterplots) (see Moyer-Packenham & Westenskow, 2013). Additionally, I consider pre-constructed dynamic geometry objects (e.g., dynamic objects constructed with GeoGebra and published to GeoGebra tube) as VMs due to the interactive nature of these dynamic objects.

Virtual manipulatives have been implemented in classrooms for quite some time. In fact, Clements & Batista reported in 1989 on a study with 48 third graders using LOGO (as cited in Moyer-Packenham et al., 2014). Since then, many studies have been conducted investigating VMs and student achievement related to VMs. Due to the visual nature and the ability to be manipulated (e.g., see Reimer & Moyer, 2005; Suh, Moyer, & Heo, 2005), several affordances of VMs have been highlighted. For example, well-designed and implemented VMs can increase the diversity of problems that students are able to think about and solve (Goldenberg, 2000), as well as help students become more aware of mathematical concepts and support the development of abstract representations (Sarama & Clements, 2000). Additionally, many VMs provide opportunities for students to receive immediate feedback (e.g., Edwards Johnson, Campert, Gaber, & Zuidema, 2012). Moyer-Packenham and colleagues (2014) claim that VMs can provide

equal access for students to learn content by reducing effects of students' demographics (e.g., SES and ELL status) as predictors of achievement. More recently, Bouck, Flanagan, and Bouck (2015) suggest that VMs benefit students with learning disabilities in their performance (i.e., the number of correctly solved problems), their confidence, and possibly the development of conceptual understanding. Furthermore, VMs can be used as a tool for differentiating instruction (e.g., Bouck, et al., 2015; Shin et al., 2017) for students who are struggling with the content as well as for students that need to be challenged more. Finally, the prevalence of interactive whiteboards, 1-1 classrooms¹, widespread availability of VMs (many are free) and unlimited access and supply of the manipulatives (Moyer et al., 2002), as well as the expectation for teachers to use technology are additional reason why I focused on VMs.

VMs are one form of technology that is used by teachers to enhance their instruction. Adapting the description of effectiveness of teaching from *Adding it Up: Helping Children Learn Mathematics* (National Research Council, 2001), effectiveness of a technology is dependent on the enactment of technology, meaning the “mutual and interdependent interaction” (p. 9) of technology with the teacher, the students, and the mathematical content. Like any tool, instructional enhancement occurs not due to the properties of the technology itself, but how the technology is used within the classroom by the teacher (and students) and the knowledge and meanings that students develop through their interaction with the technology (Meira, 1998). In the context of this study, effective use of VMs promote students' development of conceptual understanding of mathematics through reflection and communication (Hiebert et al., 1997), as well as through using and connecting mathematical representations (NCTM, 2014) (referred to earlier as teaching *with* technology).

¹ In 1-1 classrooms, each student has access to a technology device (e.g., computer, laptop, tablet, PDA, etc.).

Early repositories of VMs include NCTM Illuminations, Shodor Interactivate, and the National Library of Virtual Manipulatives. However, in recent years, new companies and collections have also become available. For example, The Concord Consortium's SmartGraphs (promoting graphical literacy), PhET Interactive Simulations from the University of Colorado Boulder (science and mathematics simulations), ExploreLearning's Gizmos (science and mathematics), Flash & Math, and various personal collections (my own as well as collections developed by other people).

Teachers can find VMs and activities through an internet search, Pinterest², or knowing about various collections. However, taking and implementing a VM or activity that is found online does not necessarily mean that it meets the needs of the students in the classroom or that the activity promotes opportunities to develop students' conceptual understanding. Unfortunately, very few studies have been conducted investigating how teachers critique, modify, or develop the activities based on their students' needs. Rather, current studies focus on how or why teachers use VMs, and/or the effects of using VMs. Additionally, very few resources exist for supporting teachers in critiquing and modifying activities to support student understanding and engagement with VMs.

Task Analysis Framework

Given the lack of studies or resources focused on teachers' critique, modification, and development of VM activities³, I developed a task analysis framework (see Table 1) to help

² Pinterest has become especially relevant as a source of curricula materials for "millennial" teachers (Wessman-Enzinger & Hertel, 2017).

³ A VM activity refers to a VM and all accompanying instructional materials (e.g., prompts and directions) whether on screen or in printed form. I use the word activity because the exploration could include more than one task (Sinclair, 2003) focused on investigating a particular concept (e.g., through alternative exploration paths), but it may include only one task. Note that "activity" in "VM activity" is not the same as "activity" in "Activity Theory" discussed later on.

Table 1. *Task Analysis Framework*

Affordances	Descriptions
N/A	Task is primarily a technology task with no focus on mathematics.
N/A	Virtual manipulative does not have mathematical fidelity required to respond to the prompts.
A	Task prompts students to recall a mathematical fact, rule, formula, or definition.
B	Task prompts students to report information from the virtual manipulative or consider mathematical concepts, processes, or relationships in the current display. The student is not expected to provide an explanation.
C	Task provides opportunities for students to explain the mathematical concepts, processes, or relationships in the current display.
D	Task provides opportunities for students to make predictions and then test their predictions using the virtual manipulative.
E	Task provides opportunities for students to connect multiple representations of a mathematical concept (e.g., graphical, algebraic, and tabular representations of a relation).
F	Task provides opportunities to check students' understanding of mathematical concepts, processes, or relationships. Task may provide minimal feedback to the student based on specific errors.
G	Task provides opportunities for students to go beyond the current display by considering multiple examples to generalize mathematical concepts, processes, or relationships.
H	Task supports students' exploration through manipulation of the display that may surprise one exploring the relationships represented or cause one to refine thinking based on themes within the surprise (e.g., addressing a common student misconception).

support and guide teachers as they critiqued, modified, and developed VM activities for use with their students that promoted the development of conceptual understanding. The task analysis framework was meant to help teachers distinguish between mundane uses of technology and powerful instructional uses (Wilson, 2008). Portions of the framework were adapted from frameworks by Trocki (2014, 2015) and Sinclair (2003), both of which were designed for supporting teachers' development of tasks involving dynamic geometry software⁴.

The affordances in the framework are not necessarily in hierarchical order, though there is some hierarchy built in. When teachers applied or used the framework, they were to apply affordances to the activity as a whole. Meaning, any given activity may have multiple affordances applied to it. Drawing from Gibson (1979, 1982 as cited in Pea, 1993, p. 51) and Pea affordances “refer to the perceived and actual properties of an” activity, specifically, the “functional properties that determine” how the activity may be used by the students (i.e., how students may engage in the activity and the opportunities the activity has for promoting students' conceptual understanding through communication, reflection, as well as using and connecting mathematical representations).

The various affordances in the framework can be separated into three groups. If either of the first two prompts applied (i.e., primarily a technology task without a focus on mathematics or the VM does not have mathematical fidelity), then it was suggested that teachers may not want to use the VM or do the activity. Affordances A and B are “lower level” affordances (i.e., prompting students to recall or report information), however, they may be helpful when familiarizing students with the VM, activating prior/background knowledge, or launching an activity. Affordances C through H are meant to provide opportunities for students to create

⁴ Trocki's task analysis framework aimed at supporting teachers' efforts to develop tasks in Geometer's Sketchpad whereas Sinclair focused on Cabri.

explanations, make predictions, connect multiple representations, receive feedback, generalize, and potentially address common student misconceptions. Activities that have affordances primarily from this third category may be more beneficial for a teacher to use in her classroom due to the potential of the activity to promote students' development of conceptual understanding.

Theoretical Grounding

Rather than focusing on only an individual's participation and the ways in which the individual participates in a community (as in socio-cultural or cultural-historical theory), this study draws from the third strand of activity theory developed and promoted by Engeström (1987, 1999). Engeström's inclusion of community and other mediating factors (see Figure 1) provided insight to investigate teachers' participation in the PD (i.e., their interactions with each other, the activities in the PD, various tensions, etc.), possible transformations in their instructional practices around using VMs (i.e., the goal or outcome of the PD) as well as how the PD transformed to meet the needs of the teachers involved. In this third strand of activity theory, the focus of analysis is joint-mediated activity (i.e., how individuals and groups participate within a community and assumes that their activity is mediated by their experiences, how they use tools and how the tools are transformed through activity, the context of the activity, etc.) (Daniels, Cole, & Wertsch, 2007). Therefore, when investigating teachers' practices related to using VMs and activities, I acknowledge that teachers' practices are not isolated but rather are mediated by several factors (e.g., student needs, curriculum, other teachers, available tools, school/district initiatives, etc.).

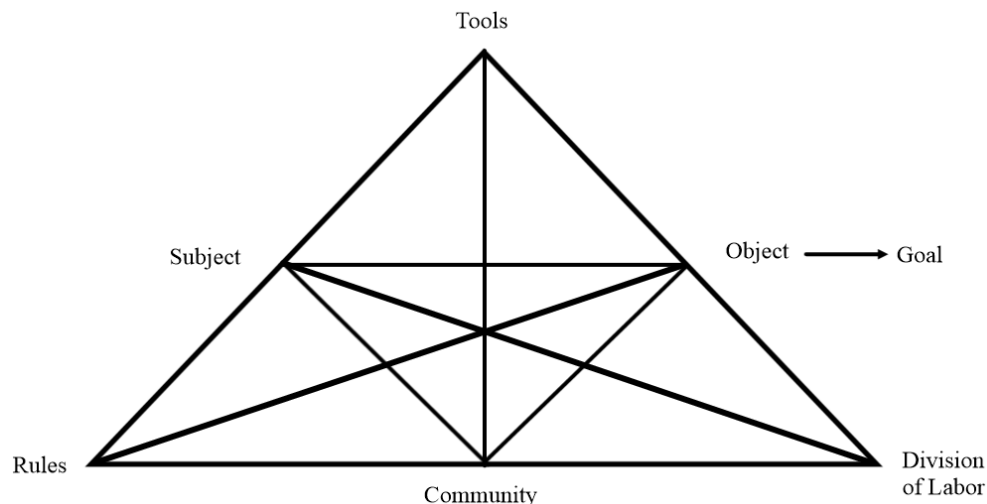


Figure 1. Structure of an activity learning system (adapted from Engeström, 1987, 1999).

Although the unit of analysis is an activity, an activity consists of a subject, object, and actions, all of which form the context of the activity. The *subject*, in this case the teachers in the PD, is the person or people engaged in the activity. Nardi (1996) describes the *object* (as in “objective” and can be thought of as the “object of the game”) as being the motivator for the activity and gives the activity specific direction. In this study, the object are teachers’ practices related to implementing VM activities (i.e., instructional practices include implementing modified VM activities to fit the particular needs of students and providing opportunities for students to reflect and communicate with others as they develop conceptual understanding of a concept). *Actions*, are “goal-directed processes that must be undertaken to fulfill the object” (Nardi, 1996, p. 37). In this study, the actions are how teachers engage with the VM activities, including their processes for finding, critiquing, and modifying/developing VM activities for their classroom. Due to the dynamic nature of the activity system, overtime actions may become operations (i.e., teachers’ practices may become routinized and subconscious). Other aspects of this activity system include the tools/mediating artifacts (e.g, the VMs and activities, the task analysis framework), rules (e.g., curriculum, instructional style), community (e.g., other teachers

in the school), and division of labor (e.g., do teachers work primarily individually or in collaboration with others). Figure 2 is a posited model of an activity system in this study.

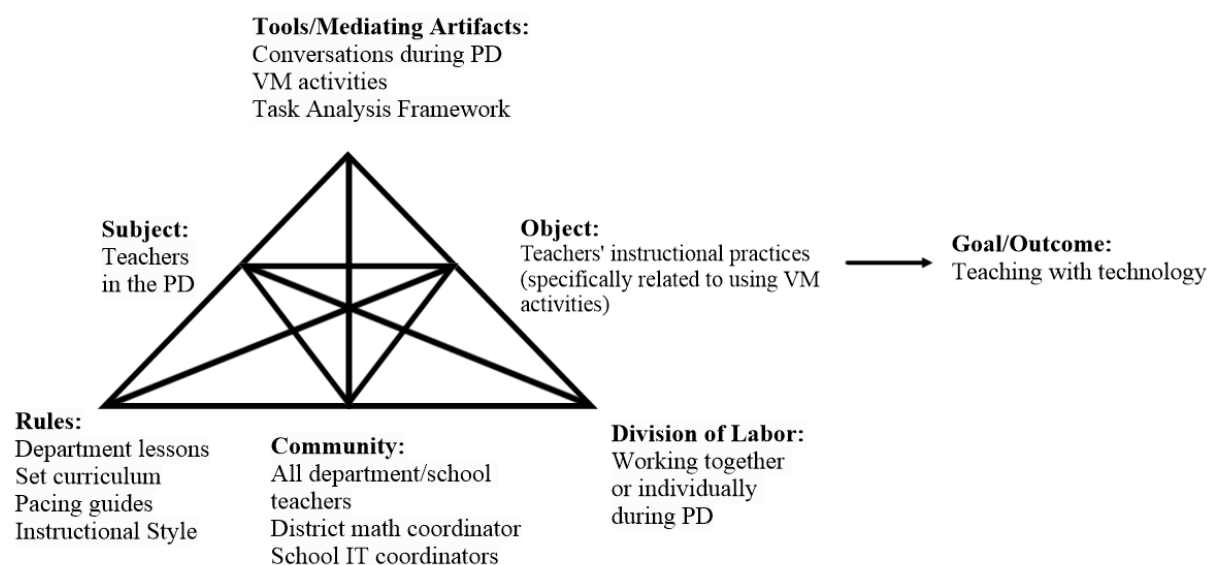


Figure 2. Posited structure of an activity learning system for this PD (adapted from Engeström, 1987, 1999).

Methods

The aim of the following study was to design, implement, and investigate a PD designed to promote teachers' effective use of VMs. I posited that due to systemic tensions (e.g., teachers' need to adhere to their textbooks, knowledge of and comfortability with technology, etc.) and mediating factors, teachers might participate differently in the PD. Furthermore, I posited that teachers' participation, might influence what instructional practices they appropriated during the PD and how their instruction may begin to be transformed. Teachers' work, conversations, and reflections during the PD were used to investigate what teachers instructional practices related to effectively using VMs (**RQ1**) as well as how systemic tensions influenced teachers' appropriation of advocated instructional practices (**RQ2**). Additionally, interviews and classroom observations of focus teachers were used to further investigate **RQ1** and **RQ2**, specifically focusing on their instructional practices and systemic tensions. Detail information about

participants is included to support readers' interpretations of findings as well as future iterations and revisions of the PD opportunity (Lawless and Pellegrino, 2007).

Design of the Professional Development

In an effort to promote secondary mathematics teachers' effective use of VMs (i.e., to teach *with* technology), the design of this PD was based on evidence-based decisions. I began by reviewing research investigating technology focused PD opportunities for in-service teachers (e.g., Bicer & Capraro, 2016; Driskell et al., 2016; Martin et al., 2010; Matzen & Edmunds, 2007; Mouza, 2009; Roschelle et al., 2010; Walker et al., 2012; Wells, 2007; Yamagata-Lynch, 2003; Yamagata-Lynch & Haudenschild, 2009) and components of effective PD (Borko, 2004; Desimone, 2009, 2011; Desimone, Porter, Garet, Yoon, & Birman, 2002; Garet, Porter, Desimone, Birman, & Yoon, 2001; Guskey, 2002; Lawless & Pellegrino, 2007; Penuel, Fishman, Yamaguchi, & Gallagher, 2007; Webster-Wright, 2009). Drawing from similarities across the field, the following core components for effective PD guided the design of this PD (e.g., see Desimone, 2009, 2011; Desimone et al., 2002): content focus (mathematics), active learning (teachers used and then critiqued activities to be used in their classrooms), coherence (with the district/school goals to use technology to transform teaching and learning), duration (at least 20 hours distributed across the school year), and collective participation to build a learning community among a group of teachers implementing the same curriculum (teaching pairs).

Originally, it was thought that some teachers may use VMs primarily for demonstration purposes whereas others may use them for investigation. Due to the prevalence of the 1-1 classrooms⁵, the district leaders and teachers in the PD, primarily were looking for opportunities

⁵ In this district, all 6-9 students had their own Chromebooks assigned by the district for the 2015/2016. Additionally, at least one high school math teacher had a classroom set of Chromebooks. All teachers in the district were assigned Chromebooks for the 2015/2016 school year.

to better integrate Chromebooks into their instruction. According to district leaders, although all middle school students had Chromebooks for the past two years, the math teachers were not using the Chromebooks (unlike the other content areas) within their instruction. Hence the focus during the PD became using VMs for exploration activities rather than solely for demonstrations.

Table 2 gives a timeline and activities for the PD. Over the course of the PD, teachers used and critiqued various VMs and activities related to their content area (i.e., how the VM and/or activity could be used with their students, what may be gained/lost by the implementation, affordances for student learning, etc.). As the PD progressed, teachers took on the responsibility for finding, designing, and modifying VMs and activities to be used in their classrooms, thus strengthening the link between the PD and teachers' practice (Wilson, 2008). Furthermore, this process supported the expansive cycle of the activity system (Engeström, 1999). Conversations and reflections were facilitated through examples of how VMs and activities have been modified and implemented to meet different instructional needs of teachers and students.

Activity theory does not focus only on “doing,” but instead focuses on activity referring to “doing in order to transform something” (Barab, Barnett, Yamagata-Lynch, Squire, & Keating, 2002, p. 78). In the context of this study, the purpose of the PD was not only to introduce teachers to VMs, but to provide teachers with opportunities to interact with VMs and begin thinking about ways to incorporate VMs and activities that met the needs of their students for promoting conceptual understanding into their classroom practices. Teachers explored VMs from the collections discussed previously and many others during the PD. For a complete list of the collections used by the teachers in the PD, see Appendix A.

Through collaboration with others, the goal was for teachers to find, critique, and design/modify activities that promoted student exploration whereby students were encouraged to

communicate, reflect on the mathematics involved, and draw connections between the onscreen representations and the mathematical concepts. Additionally, a secondary goal was for teachers to use or develop activities that encouraged and taught students how to explore mathematical concepts through “black box⁶” and prediction tasks (Laborde, 2001).

Table 2. *Timeline of Professional Development Activities and Purpose*

Time	Activity	Purpose
Thursday, 22 Oct. 2015 (8:00-12:00)	<ul style="list-style-type: none"> • Introduce PD and resources to be used during PD • Conversations and reflections about role and use of technology currently in teachers’ classrooms • Teachers complete, critique, and compare 3 VM activities for solving two-step equations (i.e., Solving and Modeling Two-Step Equations from Explore Learning, Algebra Tiles from Illuminations, and Virtual Algebra Tiles from Michigan Virtual University) • Administer online background survey (see Appendix B) 	Introduce VMs and what they offer as a means for enhancing student understanding. Gather information about how teachers currently use and think about technology. Provide common experience for teachers using and critiquing online activities with their students in mind and applying the task analysis framework.
29 Oct. 2015 & 4 Nov. 2015 18-19 Nov. 2015	<ul style="list-style-type: none"> • Initial semi-structured interview with each focus teacher (4 teachers from PD) (see Appendix C) • Classroom observations of each focus teacher during a lesson of their choice demonstrating typical use of technology 	Gain information regarding teachers’ beliefs and instructional practices. Determine alignment between teachers’ observed instructional practices and practices advocated for in the PD and whether VMs are already being used.
Monday, 9 Nov. 2015 (8:00-12:00) Mondays (7 Dec, 11 Jan, 15 Feb, 14 March, 4 April, 2 May) 3:30-5:30	Teachers explore VMs based on their content, provide time to work with others to modify activities and discuss modifications. In November, teachers worked through two or three different activities based on the topics of their choice. By February, teachers were given suggestions for activities to explore. By April, teachers chose the activities on their own (with support from me and drawing from PD resources).	Conversation and reflection focuses on: affordances for student learning, what is gained/lost by implementing the activity, how they may be used in their classrooms (including potential modifications), etc.
31 March through 10 May 2016	<ul style="list-style-type: none"> • 1-2 classroom observation(s) of focus teachers implementing VM activities 	Gather information about teachers’: experiences in the PD (e.g., what did (not)

⁶ Laborde (2001) describes an example of a “black box” task as when students are presented with a diagram on the computer screen (e.g., a point P and its image P’) and students must ask themselves (or be asked) questions to figure out the properties of the unknown transformation that mapped P to P’.

	developed in PD or lessons of teachers' choice <ul style="list-style-type: none"> • Final semi-structured interview with each focus teacher (see Appendix C) 	work well for them), instructional practices, process for finding, modifying, and implementing VM activities in their classrooms.
May 2016	Administered online survey (all participants)	

Participants

Due to location convenience, I approached a suburban district in the Midwest about offering the PD for their secondary mathematics teachers. District leaders then encouraged but did not require their teachers to participate in the PD. Initially, the district leaders suggested targeting the PD for middle school teachers (about 16 teachers spread between two middle schools serving 6th/7th grades and one middle school serving 8th/9th grades) due to all students in the middle schools having Chromebooks (i.e., the middle schools are considered 1-1 technology schools). Therefore, the district math leader communicated information about the PD to all middle school teachers. The district math leader and myself then met with teams of interested teachers (14 in total) explaining the PD in more detail. The district math leader also contacted three high school teachers whom he thought may be interested in the PD due to their students receiving Chromebooks by the 2016/2017 school year. One 5th grade also requested to be part of the PD. The final PD schedule was created by the district math leader after finalizing which teachers wanted to participate and their availability due to other commitments (e.g., coaching, committees, family obligations, etc.). About half of the remaining interested teachers wanted the PD during the school day (which was the preference of the district leaders) and the other half wanted the PD after school (so that they did not have to miss class). Teachers received *professional development hours*⁷ based on the number of after school PD sessions attended.

⁷ Per district policy, teachers needed to acquire 50 professional development hours each school year.

Fourteen teachers (see Table 3) decided to participate in the PD. Teachers taught 5th grade through AP Calculus and had 2-20 years of teaching experience (Mean: 12.86 years, Median: 13.5 years) before the start of the 2015/2016 school year. Teachers represented five schools in the district. The 5th grade and three high school teachers decided to participate. The remaining 10 teachers, three of whom were intervention teachers, taught grades 6-8. Besides the fifth-grade teacher, all teachers taught only mathematics. Of the fourteen teachers in the PD, four teachers volunteered to be focus teachers (meaning there were additional interviews and classroom observations). The focus teachers had 15-20 years of teaching experience.

Although during the 2015/2016 school year the school district was investigating new curricular options to be implemented for the 2017/2018 school year, the current curricula used in the district was more explorative rather than traditional. Everyday Mathematics was used in the elementary schools (K-5), Connected Mathematics (more specifically CMP3) was used in the middle schools, and College Prep Mathematics (CPM) was used in the high school.

All teachers participated in the initial two PD sessions which were held during the school day as two four-hour sessions held in the morning, one in October and one in November. The district provided substitute teachers. The remaining PD sessions were six two-hour sessions that were held once a month afterschool December through May. Two teachers did not attend any of the afterschool PD sessions, the other teachers attended between two and five of the afterschool sessions (see Table 3). The afterschool sessions had between four to eight teachers (Mean: 6.33 attendees, Median: 6.5 attendees). All PD sessions were held at the district administration building.

Table 3. *Summary of Professional Development Participants*

Pseudonym	School	Grade Taught	After School PD Sessions Attended
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Kelly	Prospect ES	5th	4
Josh	Wetlands MS	6th	2
Mark	Wetlands MS	6th	5
Tracy	Wetlands MS	6th/7th Interventionist	4
Curt	Plains MS	7th	3
Karen	Plains MS	7th	0
Randy	Plains & Summit MS	6th/7th/8th Interventionist	2
Erin	Summit MS	8th	4
Stan	Summit MS	8th	0
Mari	Summit MS	8th	2
Pam	Summit MS	8th Interventionist	3
Daron	HS	Geometry, AP-Stats	3
Jake	HS	AP-Calc, Pre-Calc, Consumer Math	3
Mike	HS	AP-Clac, Pre-Calc, Trans. to Col. Math	3

Note. Bold entries denote focus teachers.

Data Sources and Analysis

Data collection occurred October through May (see Table 4). Data included an online background survey⁸ (see Appendix B for survey questions), and audio- and video-recordings of PD sessions, including teacher conversations, reflections, and work during the sessions. Additionally, semi-structured interviews and classroom observations were conducted with the focus teachers. Focus teachers were interviewed (see Appendix C for semi-structured interview questions) and observed during the first few weeks of the PD. Each focus teacher chose a lesson that demonstrated his or her typical use of technology for the observation. During the latter half of the PD, focus teachers were observed one or two times implementing activities developed during the course of the PD. They were also interviewed once again. Data collected via the online background survey and teacher conversations/reflections as well as via the initial teacher interviews and classroom observations of the focus teachers were used to gain information about

⁸ The background survey is adapted from the survey I used in my second-year project. Some of the questions used in the survey are taken/adapted (with permission) from surveys developed by others investigating teachers' beliefs about mathematics and/or technology in general (i.e., Ambrose et al., 2004; Hernandez-Ramos, 2005; Wozney, Venkstesh, & Abrami, 2006).

teachers' beliefs about VMs and their instructional practices (related to **RQ1** and **RQ2**). Recordings of PD sessions, including teachers' conversations/reflections/work, and researcher reflections were used to investigate: (i) what instructional practices related to using VMs effectively did teachers appropriate (**RQ1**), and (ii) what systemic tensions exist and how the tensions influence teachers' instructional practices (**RQ2**). It was hypothesized that components of the PD that appear to be the most meaningful to the teachers' learning and/or beliefs (based on their reflections and/or change(s) in their conversations/work) may be applicable for future PD. Final online surveys and interviews, along with classroom observations of focus teachers, were used to investigate how the professional development may have contributed to teachers' transformed instructional practices (**RQ1**).

Table 4. *Summary of Data Sources*

Participants	Data
All Teachers	Initial Survey Video and audio recorded PD sessions Teacher reflection documents Teacher developed VM activities Final Survey
Focus Teachers	Pre and Post Semi-structured interviews (audio recorded, field notes) Classroom observations (2-3 each) (video and audio recorded, field notes) Reflections after the observations VM activities Student work
Researcher	Reflection Journals

Audio recordings from initial and final interviews with focus teachers, and video and audio recordings of all PD sessions were transcribed. During the transcription process, along with teachers' work and screenshots of VMs, themes and initial analysis thoughts were added. Since activity theory implies that an individual's actions are always mediated by various factors, the context that surrounds an individual's actions (including her reflections) must be considered at all times. Pervasive or systemic tensions identified within the activity system helped to frame

the analysis as it was hypothesized that these tensions may have a role in an individual's participation in the PD and possible changes in her instructional practices. Engeström (1999) defines practice as learning and learning as practice. Therefore, teachers' learning within the PD was investigated through their engagement with the VM activities presented and their processes for finding, modifying, and developing activities (i.e., their actions).

Transcripts and teacher work were coded using NVivo. I began coding the transcripts using what Saldaña (2013) describes as an "initial" or "open coding" approach. Meaning, I was open to all theoretical directions that may arise as I was reading through the data. This coding approach allowed me to deeply reflect on the subtleties and contents of the data. Some codes rose organically from the data (e.g., tensions described by the teachers) and others were informed by literature (e.g., teachers' descriptions of their instructional practice and/or how they used technology).

Using the constant comparative method (Glaser & Strauss, 1967), I progressed further into data analysis-comparing coded data within categories, re-reading transcripts, and writing memos. Code definitions were developed from review of the literature as well as commonalities and themes between data within categories. Memos were used to reflect more deeply on the codes and data, as well as to expound on emerging themes (Birks, Chapman, & Francis 2008; Glaser & Straus, 1967; Saldaña, 2013). I then proceeded with focused coding, more specifically focused on teachers' appropriated activities and how the PD may have supported these activities in being appropriated. Through sorting codes vis-à-vis components of the activity system (e.g., division of labor, community, tools, tensions), looking more thoroughly at the data within a particular code, as well as linking portions of the data (e.g., teachers' conversations during the PD along with their work), patterns emerged that gave insight into how teachers' actions may

have been mediated and possibly transformed during the course of the PD. Triangulation (Creswell, 2013) of data sources was used to support (or corroborate) these findings.

Findings and Discussion

This section focuses on specific practices that were advocated during the PD (i.e., appropriated practices and implementation of VMs and activities) and how teachers appropriated these practices during the course of the PD. The findings begin by focusing on the practices teachers appropriated during the PD, namely, their use of guiding questions and the task analysis framework to support their processes for finding and critiquing/modifying activities, and their use/development of instructional guides. Also discussed are teachers' efforts for finding VMs and activities. Next, findings related to teachers' practices and their implementation of VMs and activities (i.e., the object of the activity system) are presented. In this sub-section, why teachers implemented VMs and activities, the instructional role of VMs and activities within teachers' instruction, as well as specifics about their implementation efforts are addressed. Tensions are discussed in relation to how they influenced teachers' appropriation of advocated practices and implementation of VM activities.

Appropriated Practices

The following sub-section focuses on three appropriated practices of teachers in this PD (i.e., need to critique/modify activities, use/development of instructional guides, and finding VMs and activities on their own). Appropriation of a practice was determined by whether the practice became the norm (i.e., whether a teacher consistently demonstrated the practice) regarding his or her process for integrating VMs and activities in his or her instruction. Each of the three appropriated practices were demonstrated by 10-12 teachers who attended the afterschool PD sessions. To support teachers' effective use of VMs and activities, teachers were

encouraged to critique/modify activities as opposed to taking activities as is and to use/develop instructional guides that would support students' exploration using the VM. Additionally, through the gradual release of responsibility, teachers learned how to find VMs and activities for use in their classrooms, thus supporting the expansive learning cycle. The following section discusses the findings in relation to teachers' appropriation of these practices as well as teachers' processes and tool use for finding VMs and activities (i.e., their actions).

Need to critique/modify activities. The first appropriated practice presented is teachers' practice of needing to critique/modify activities as opposed to automatically implementing activities as they found them, assuming the activities would be effective in promoting students' development of conceptual understanding. Drawing from teachers' practices deciding which VM activity to use, as well as finding their own VMs and activities, all teachers who attended the afterschool PD sessions (i.e., 12 teachers) appropriated the need to critique activities. Additionally, all but one teacher demonstrated the need to modify activities based on his learning goal and/or student needs. While interacting with VMs and activities, teachers consistently talked about how their learning goal(s), student needs, and possible modifications. Initially, this section discusses teachers' general change in instructional practice relating to intentionally critiquing and modifying VMs activities prior to implementing them with students. The discussion then focuses on two tools (i.e., guiding questions and the task analysis framework) that helped to support teachers' appropriation of this practice.

For each PD session, teachers were provided a link to an individual Google Document that contained links for VMs to explore, guiding questions to respond to after interacting with the VM, and questions that encouraged them to apply the task analysis framework. When teachers were presented with a VM or activity, they initially went through the activity as though they

were a student. They thought about their own students (e.g., how they might struggle with the activity or how they might benefit), the types of questions that were asked (e.g., yes/no or questions that prompted students to explain their reasoning), how some of their students might respond (e.g., whether they would click through the activity without making connections), opportunities for feedback and the type of feedback given (e.g., did feedback go beyond yes/no), and how they might use the VM or activity in their classroom.

Working through the activity provided a context for teachers to then critique the activity. It also made them more aware of where their students may struggle in the activity as well as possible modifications to make so that students' explorations were focused on the development of conceptual understanding as opposed to trouble shooting unnecessary challenges. Guiding questions as well as the task analysis framework supported teachers in appropriating the instructional practice of needing to critique/modify activities as opposed to taking activities as is or using them because they "looked cool."

During Jake's final interview in May, he expressed how the PD changed his instructional practices regarding finding and implementing technology based activities. Jake said,

I think that the PD helped me kind of think through some things. As opposed to just thinking like, "*Ahh well, we'll try it and see what happens, and then we'll kind of modify afterwards,*" as opposed to, like, being a little more thoughtful about how this is going to be used and what type of questions should be asked. Or do I need to modify this, this kind of worksheet that goes along with this, so that it's going to help beforehand as opposed to like, "*Oh, well that didn't go the way I really wanted it to go.*" And then you're doing it after the fact (*italicized portions represent change in tone*).

Jake's reflection highlights how he and the other 10 teachers changed their instructional practices related to finding and implementing technology based tasks. Rather than initially implementing the activity as is with students and then determining what

modifications may need to be done in subsequent iterations of the activity, Jake discussed how the PD helped him to think more critically about an activity and modifying it proactively rather than retroactively.

Although requested multiple times, teachers typically did not share activities that were finished or modified outside of the PD. Therefore, the findings below are based only on teachers' reflections, interview/survey responses, work completed during the PD, and the activities that teachers did share with me.

Guiding questions. To support the appropriation of this first practice, the first part of teachers' reflection documents specifically encouraged them to think about how the VM activity supported students' engagement related to the learning goal. Teachers' conversations during the PD sessions as well as their responses to the guiding question prompts on their reflection documents demonstrate how teachers began to critique activities and think about possible modifications. Modification categories and examples of teachers' work are presented below.

The guiding questions evolved during the course of the PD based on how teachers responded to the questions. By the December PD session, the guiding questions had been reduced from five questions to four questions (see Figure 3). However, in an effort to promote teachers in thinking critically about their modifications, asking teachers "Why would this modification help your students engage in the essential question?" was added beginning with the March PD session. Moving forward, it is suggested that "would" be changed to "might" in the guiding questions.

1. What is an essential question you would use this activity to explore?
2. Where do you think your students would struggle in this activity?
3. How would your students benefit from engaging in this exploration?
4. Thinking about the essential question, what is one modification you would make so that the exploration better fit the needs of your students? Why would this modification help your students engage in the essential question?

Figure 3. Guiding questions from a March teacher reflection document.

Table 5 highlights the different types of modifications that teachers implemented during the course of the PD. Although the number of data excerpts related to each category are included in the table, these numbers are for descriptive purposes only and should be treated with caution. Due to multiple data sources (e.g., teachers' conversations, individual responses on reflection documents when working in pairs, teacher modified instructional guides) and some modifications fitting into more than one category the numbers are only included to give readers a descriptive understanding of the relevance of various types of modifications. However, as displayed in the table, most of the modifications that teachers suggested related to helping students engage in the learning goal, creating an instructional guide, and addressing student needs.

Table 5. Categories, Definitions, and Examples of Modifications

Definition	Number of data excerpts	Example
Assessment: Modifications related to using the activity as an assessment tool (at least in part).	6	Daron (DecReflecDoc): I would also like to see some assessment questions to see if they understood the point of the applet.
Curricula: Modifications related to teachers' curriculum.	13	Josh (JanReflecDoc): We would eliminate the Lateral Area and put more focus on exploring what surface area is and how it is found (thinking about adding the area of all faces).
Instruction: Modifications related to how the activity would be implemented.	19	Curt (JanReflectDoc): Walking through a unit rate to help with writing ratios and labeling measures.

		This will help with labeling a unit when setting up proportions.
		Having them check in with the teacher after Activity A, #7.
Instructional Guides: Modifications related to developing an instructional guide.	33	Jake (NovReflecDoc): Create an activity guide
Learning Goal^a: Modifications related to the learning goal.	51	Curt (talking with Karen in Nov about a modification): Add the question “How do the image and preimage compare when the scale factor is one?”
Miscellaneous: Modifications did not fit into one of the other defined categories (e.g., related to technology constraints or non-specific modifications).	8	Josh and Mark (after Obs3): We would make changes based on the constraints (#3), but that can't be done to our knowledge.
None: Teachers said they would not modify the activity.	8	Erin (FebReflecDoc): I wouldn't change it, I think it fits the student's needs.
Practical Details: Modifications related to reformatting an instructional guide (e.g., to reduce space on a page or adding screenshots), clarifying directions, or correcting an error.	15	Mari (talking with Pam in Nov about a modification): Maybe we would just edit and only have two (pause) examples and save trees.
Student Needs: Modifications related to student needs.	25	Jake (NovReflecDoc): More basic functions. There were some pretty crazy functions that showed up that I wouldn't want my students to be getting frustrated with.

^aIn this district, the *learning goal* was referred to as the *essential question*.

Often, teachers' modifications spanned more than one category. For example, as an intervention teacher, many of Tracy's students struggled with reading as well as mathematics (Vukovic, 2012). Therefore, Tracy's modification for the VM activity that she was critiquing in December was to, “read each portion to the kids and have them answer before moving on. Give the kids screenshots of what each portion should look like. Possibly adjust the exploration guide to eliminate some of the language to make it a little more user friendly.” These modifications align with the instruction (i.e., reading each portion to the students), practical details (i.e., adding the screenshots), and student needs (i.e., eliminate some of the language to make it more user friendly) categories defined earlier.

In the following excerpt from the December PD session, Mari and Erin demonstrate how some (i.e., at least eight) teachers began to think about the types of modifications and whether the modifications were more focused on the students and their engagement with the learning goal or focused on a stylistic modification or a modification related to implementation (modifications that were more for them as teachers).

Erin: What's a modification we could make? Changing the initial values to smaller numbers?

Mari: (long pause) Umm, (long pause) I had, in the *how could you use this exploration*, I said break it up. [**Erin:** yeah.] And make it less of a time commitment, so that you could integrate it into pre-made lessons more easily. But I guess that's more of a modification for us. For kids (pause)

Erin: Well it says, "Think about what's one modification that you could make so that the exploration would be a better fit." Would better fit the needs of our students.

Mari: I don't think...(long pause)

Erin: Oh, umm...

Lindsay: (walking up to Erin and Mari) Do you want me to try and explain that? (*Then I go on to explain about different types of modifications.*)

On their reflection document, Erin and Mari decided that a modification they would make would be to change the initial values (see Figure 4). Changing the initial values was a modification based on the needs of their students and how they thought their students may struggle during the activity due to having to enter numbers in a different scale.

4. Where do you think your students would struggle in this activity?

Students may struggle in this activity when they are asked to put numbers into the gizmo, but the numbers are 24 instead of 24000. The initial value on the gizmo only goes up to 160. It might be confusing for some students to see the connection between the two.

5. How would your students benefit from engaging in this exploration?

Students would benefit with the sliding aspect of the gizmo. seeing the function change as the y-intercept changes or as the growth/decay rate changes.

6. Thinking about the essential question, what is one modification you would make so that the exploration better fit the needs of your students?

Change the initial values to a smaller number in the first activity so that the students can see how the Gizmo works before they have multiply the initial values by 100 or 1000.

Figure 4. Erin and Mari's responses to guiding questions on their reflection document in December.

Task analysis framework. To further support teachers appropriating the practice of critiquing and modifying activities, the second part of the teachers' reflection documents encouraged teachers to apply the task analysis framework (see Table 1). Although it was a goal of mine for teachers to use the framework, I did not know whether teachers would use the framework or find it helpful. The following sub section describes how teachers applied the framework (e.g., how they justified whether affordances applied) and how they found the framework useful for their practice. Although some teachers (i.e., five) mentioned using the framework to help them determine which activity may be more productive to use with their students (i.e., based on the number of higher order affordances) when comparing two activities, teachers primarily used the task analysis framework on their own to modify or develop instructional guides rather than as a tool to critique activities.

By including prompts in the second part of the reflection document specifically asking teachers to first apply the framework to the given activity, and then apply the framework after any modifications had been made (see Figures 6 and 7), the PD further supported the teachers in appropriating the action of needing to critique/modify VMs and activities, as opposed to implementing them as is or because they were something new and fun. Similar to the revision the guiding questions went through (discussed previously), the prompts that encouraged teachers to use the framework were also revised slightly as the PD progressed. The intent of the revision was to encourage teachers to use the task analysis framework both in critiquing an activity as well as in helping them to modify an activity.

Now, I want you to use apply the framework that we have been talking about (i.e., the **Task Analysis Framework_v2**). The updated framework is posted in the Google Folder.

1. Which prompts of the framework apply to the **Student Exploration: Exponential Growth and Decay** activity? What from the activity supports your claim(s)?
2. What is one modification you would make to the activity to better meet your learning goals? The prompts in the framework may help you to begin thinking about possible modifications.

Figure 6. Part II of a teacher reflection document in December.

Now, use the framework that we have been talking about (i.e., the [Task Analysis Framework_v3](#)) to help you think about how to modify the activity. The updated framework is posted in the Google Folder.

1. Which prompts of the framework apply to the activity you chose? **What from the activity supports your claim(s)?**

Now, use the framework to help you modify the activity to better fit the needs of your students. Please email or share with me the document that you create. You may want to shorten the activity or modify some of the prompts to better align with your learning goals and the needs of your students. Or, maybe you will need to create an exploration guide.

2. After modifying the activity, which prompts of the framework apply to the activity you chose? **What from the activity supports your claim(s)?**

Figure 7. Part II of a teacher reflection document in March.

Teachers' approaches to justifying which affordances applied in the framework varied greatly. Figure 8 depicts the common approaches teachers took when providing justifications for their framework application (note, responses represent three different framework versions). Some teachers (e.g., Jake) copied over the framework and added their justifications within the affordance description. Other teachers (e.g., Erin and Mari) simply listed the letters of applicable affordances without providing justifications on their reflection document. However, upon reviewing their conversation during the PD as they applied the framework, they discussed whether affordances applied, often referring back to the activity to corroborate their claim. Other teachers (e.g., Curt) listed specific prompts from the instructional guide to support whether an affordance applied. Finally, other teachers (e.g., Kelly) spoke in more general terms about why

an affordance applied to a particular activity. During the course of the PD, any given teacher's responses could be characterized as using two to four of the approaches displayed in Figure 8.

C	Task requires students to consider the mathematical concepts, processes, or relationships in the current display. <i>Scrolling along the curve and relating how the curve is changing to the actual value of the slope.</i>
D	Task requires students to explain the mathematical concepts, processes, or relationships in the current display. <i>The relationships of how the curve is changing to the value of the slope of the line.</i>
E	Task requires students to go beyond the current display and generalize mathematical concepts, processes, or relationships. <i>Identifying the relationship between how the curve is steepest to the high/low value of line.</i>
H	Task requires the consideration of multiple examples from which one can generalize. <i>Multiple slopes along the line to generalize the shape of the derivative curve.</i>
I	Task requires students to connect multiple representations of a mathematical concept (e.g., graphical, algebraic, and tabular representations of a relation). <i>Graphical and tabular.</i>

Jakes's framework application in October

1. Which prompts of the framework apply to the Student Exploration: Exponential Growth and Decay activity? What from the activity supports your claim(s)?

A, D, E, H

Erin and Mari's framework application in December

1. Which prompts of the framework apply to the activity you chose? **What from the activity supports your claim(s)?**

A -Converting problems from the warm-up

B- #1, give the unit given

C-Activity A, #1A

D- Activity A, #1C

E- Activity A, #6

F - Activity F, any problem after you click submit

G - - None

H - - None

I - - The ability to use the Flip Tile button to cancel (Activity A, #5)

Curt's framework application in January

1. Which prompts of the framework apply to the activity you chose? **What from the activity supports your claim(s)?**

A.) multiplication facts, simplifying fractions, fraction as division

B.) Parts of the exploration guide that are asked to be modeled and put into the gizmo with the calculations on.

C.) parts of the exploration that ask students to look at the model and explain relationships

E.) As students check their thinking with the gizmo and with calculations turned on, they are able to see the number model that is modeled in the images of the gizmo. This helps them connect the images to the number models and think about how they can transfer that understanding to other problems. Also in the parts where they are expected to find the missing number in the equations, in which they are supposed to use their mathematical understanding to solve.

Kelly's framework application in March

Figure 8. Teachers' approaches for justifying their framework application.

Whether including justifications is an automatic tendency of math teachers or not, teachers began including justifications for applicable affordances when they applied the framework during the October PD session, before we discussed how to justify the affordances. Granted, there were some instances where teachers agreed that an affordance applied but did not provide a justification as to why they thought this affordance applied. However, not supporting claims occurred minimally and only characterized a teacher's entire framework application four times during the entire PD (all four times occurred when the teacher was working by him/herself). Not justifying a claim occurred most often when the teachers thought an affordance "obviously" applied due to their experience with the activity.

Teachers' conversations throughout the PD and final interviews highlight how they found the framework helpful in supporting their efforts to modify and critique activities. For example, during the March PD session, Daron said,

I am looking at the framework and seeing what I want to change based on that. Umm, *I think the framework does help you* focus on like, different levels and (long pause) and just, I think sometimes too, just like the background knowledge, to ask questions, I think doing some of the guides that I have-to activate that background knowledge and get them thinking about what they already know. And then trying to, push them, push that forward too.

Daron used the framework to critique an activity to find out if it asked questions that activated background knowledge (e.g., through affordances A and/or B) but then pushed students forward by asking questions relevant to some of the "higher" affordances. Furthermore, according to Tracy, the framework helped them (i.e., the teachers), "look at-CRITIQUE them more critically and look for their VALUE versus just, a fun thing to try" (*capital letters used to signify emphasis in Tracy's speech*). Tracy's reflection demonstrates how she (and others) were able to progress

from using an activity because it looked cool to using an activity because it provided opportunities to promote students' development of conceptual understanding. During their final interviews, all four focus teachers independently claimed that the framework was one of their big takeaways from the PD.

As the PD progressed, for at least three teachers, their use of the framework became operationalized (i.e., a subconscious action). For example, during Josh's final interview, he said,

Yeah, the framework thing. I mean, in the beginning, it was like, "*Well, yeah, I guess that helps you think about is it worth DOING or not.*" And then by the time we did the, you know the last few, we didn't even, it was already in your HEAD. That, "*how-wh-what would I change to make it more worthwhile or is it fine the way that it is.*" 'Cause you kind of already have that down. But initially, it was confusing. I thought, I think it helped kind of pick out what was kind of, what was needed (*italicized portions represent change in tone, capital letters represent emphasis when spoken*).

Josh's reflection highlights how some teachers found the framework challenging to apply in the beginning of the PD (e.g., due to overlapping affordances, not understanding an affordance, etc.). In fact, during the January PD session, Josh stated that he finds it more useful to do the activity and think about the material covered rather than applying the framework. However, through conversations with others and repeatedly using the framework to critique different activities, by the end of the PD, it became second nature to apply the framework.

Although the framework was intended to provide a means to analyze activities, at least two teachers began to use the framework to inform other areas of their instruction. For example, Kelly found the framework helpful to know how to modify her instruction. During the January session, Kelly said

I think for me, the framework helps ME decide what do I need to supplement my lesson in, that they're not getting in using the tools. So that they can get ALL the well-rounded understanding of the whole piece. So, "*Okay, it's not in there. So what can I do with them, when I'M with them to make it better.*"

Curt found the framework helpful for modifying instructional tasks in general. Specifically, in March Curt said, “I have used that (i.e., the framework) on a lot of the modifying of tests for like the students to, step them through so that I don’t have to hold their hand through, a test. It’s worked really well.”

Use/development of instructional guides. Although not an initial action promoted in the PD, teachers quickly developed the desire (or need) to have instructional guides of some type to accompany VMs. Therefore, the second practice teachers appropriated during the PD was using or developing instructional guides that accompanied VMs. Beginning with the second PD session, all teachers who attended the afterschool PD sessions either developed/modified printed instructional guides or implemented VM activities that had on-screen prompts. One teacher, Daron, created a Google Form containing questions his students responded to as they interacted with the VM. Instructional guides were primarily in printed form as opposed to in some type of an on-screen format. The following subsection discusses why teachers gravitated towards using instructional guides and how they found themselves developing guides when ones did not already exist. Tensions related to using and developing instructional guides are discussed at the end of this subsection.

During the October PD session, after engaging with one VM activity that had an instructional guide and one that did not, the teachers very quickly gravitated towards activities that had some type of an instructional guide (or at a minimum had some type of exploration questions). Therefore, as the PD progressed, focused support regarding how to construct/modify instructional guides was given (e.g., potential questions to add, specific guides to look at, structural modification suggestions, etc.).

Tracy had used one of the gizmos from ExploreLearning during the prior year, however, she did not realize that gizmos have instructional guides. Upon learning about the instructional guide, she then became quite vocal with her table mates as well as the group at large regarding her thoughts about instructional guides and how beneficial it would have been for her students.

Tracy said,

I just think that it is more GUIDED and that makes it a little more REAL to them where they are actually having to do what they would have to do on a math test, you know. Like, that they are practicing those skills and, writing it down and maybe making it a little more concrete than just, click, click, click, click, click, click, click. And, see if we get an answer eventually that says, *“Hey, you’re right!”* (chuckles) So, I just think that it is more guided for them.

Already in the first PD session, teachers commented that instructional guides were helpful because: they were more guided, gave students something to refer back to, helped to keep students focused on the learning goal rather than “playing” (i.e., clicking through without making any connections), etc. In fact, during later PD sessions, teachers often asked whether a VM had an accompanying instructional guide before they began working with it.

As mentioned previously, teachers’ modifications to activities often consisted of creating an instructional guide if one did not already exist for a particular VM. In January, Daron talked about using the task analysis framework to make an instructional guide or putting together some questions so that the activity was meaningful for the students and promoted opportunities for developing understanding. He said that he used the framework to help him write a guide that was intentional and meaningfully moved students to the levels of understanding that he wanted as opposed to throwing questions out. Specifically, Daron said,

Some of the ones that I had been looking at don’t really have a guide, so that’s like, *“yeah, this,”* [the framework] I kind of keep these things [the affordances in the framework] in mind as I try to write a guide or put together some questions. For these, to try to make sure as many of these were covered (i.e., referring to the

affordances) so it was a meaningful activity for them as well. [**Lindsay:** Okay] Where otherwise you might just throw the questions out, you just, I think it allows you to think more like, “*How do I move them up to those levels that we’re trying to get them to?*”

Final interviews with the focus teachers support how valuable teachers thought instructional guides were. For example, Mark said,

The thing that I got a LOT out of and it-I thought this was early on, I think we talked about it was just having a study guide that kind of hits prior knowledge and then kind of guides the kids specifically through activities. And then kind of releases them to do, to solve problems based on what they what they did with the virtual manipulative. The way the Gizmos work. I I really like that progression. And so that’s why I think it’s easy to pick those, ‘cause you know you’re going to get that. Whereas other ones, you have to, you have to make stuff up. And it’s not like that’s super hard, once we’re working together on things.

Mark’s reflection also highlights why it was *easy* to implement a gizmo from ExploreLearning due to an instructional guide already existing rather than having to develop a guide (e.g., if the VM did not have an accompanying guide). For Tracy and Josh, one of their big takeaways from the PD was seeing the value of instructional guides for promoting students’ development of conceptual understanding.

Tracy: I think it also helped us see the value of an exploration guide.

Josh: OH yeah. (Tracy laughing) Big time.

Tracy: You know, with the framework and we’re like, “*Does it have an exploration guide or doesn’t it?*” You know, and when it doesn’t, let me tell ya, they’re [the students] just, it’s just click click click and there’s no VALUE to the math that they’re doing. Which is OK at times, BUT, you know. I think the exploration guides were HUGE and found myself even MAKING exploration guides when there WASN’T one.

As an intervention teacher, Tracy often looked for *alternative* means for having students engage in mathematics. However, her main goal was for students to willingly engage with mathematics as opposed to fearing it. Due to the scaffolds that could be incorporated in an instructional guide,

as well as questions that could promote students' development of conceptual understanding, Tracy found herself making guides when they did not exist.

Although teachers found the instructional guides to be beneficial, at times tensions existed regarding the development and use of instructional guides. For example, discussed periodically during the PD was that the use/creation of instructional guides automatically influences how students might engage with a VM (e.g., exploration paths, conjectures made and tested, etc.) (Sinclair, 2003). Although teachers found the task analysis framework helpful for guiding their efforts in creating instructional guides, they also needed to keep their learning goal and the needs of the students in mind so that the instructional guide was not too restrictive (thus limiting students' exploration) or too open ended (thus potentially providing opportunities for students to focus their attention on certain properties/features using the technology rather than the intended learning goal (Lobato & Ellis, 2002)). Students' reading levels were also a factor in the development and modification of instructional guides.

Another tension related to developing and using instructional guides related to the limits of Chromebooks when opening Word documents with images in Google Drive (i.e., images do not appear). Therefore, teachers either spent large amounts of time trying to modify the guides (e.g., change/add questions or create images from PDFs to add to the guides) during the PD sessions or they left the PD session so that they could use a non-Chromebook computer. Due to the difficulty modifying instructional guides on the Chromebooks, teachers' modifications were often completed outside of the PD session and not sent to me. Knowing the difficulty for modifying some instructional guides also influenced which activities teachers decided to use (i.e., choosing to use activities that needed "easy" or no modifications).

Finding VMs and activities. The third appropriated practice was teachers finding VMs and activities on their own. This appropriated practice was a goal of the PD to promote the use of VMs and activities beyond the PD (Guskey, 2002; Mouza, 2009). By the end of the March PD session, at least seven teachers demonstrated that they were able to find their own VMs and activities. As the PD progressed, these teachers continued to find their own VMs and activities. By the end of the PD, 10 out of 12 of the teachers attending the afterschool sessions demonstrated that they were able to find their own VMs and activities. This subsection discusses the tools that supported teachers' appropriated action. The subsection concludes by discussing teachers' experiences finding VMs and activities on their own as well as suggestions for how the tools could be used to further support teachers' efforts in appropriating this practice.

Before the PD, based on teachers' background survey responses, teachers had limited knowledge of the variety of VMs that existed. Therefore, to support teachers' efforts in finding resources, teachers were introduced to an annotated list of VM collections (see Figure 9). This list grew as the PD progressed and more collections were found. Additionally, VMs and activities related to teachers' requested content topics were organized in a Google Drive folder that was shared with all of the teachers (see Figure 9). The progression of teachers being given specific activities to explore, then suggested activities, then finding activities on their own was productive in not only exposing them to the variety of activities but also supported the expansive learning cycle (Engeström, 1999) through the gradual release of responsibility. By April, at least 9 teachers attending the PD sessions were able to move beyond the repository of resources to find their own activities, often starting from the annotated list of VM resources.

The Concord Consortium (<http://concord.org/stem-resources/subject/mathematics>)
These HTML5 resources are modules students can work through. Teacher lesson plans and student assessments are also provided. Additionally, teachers can create their own modules for students.

Euclid's Elements (<http://aleph0.clarku.edu/~djoyce/java/elements/elements.html>)
Java applets demonstrating each of the Elements.

Explore Learning Gizmos (<https://www.explorelearning.com>)
Gizmos are interactive math and science simulations for grades 3-12. Over 400 Gizmos aligned to the latest standards help educators bring powerful new learning experiences to the classroom. Gizmos can be searched by standard, grade & topic, or textbook. Explore Learning is working to update the Gizmos to be compatible with HTML5.
o You can sign-up for a free 30 day trial per email address. Otherwise, you can access | each Gizmo for 5 minutes per day per computer.

Flash & Math (<http://www.flashandmath.com/mathlets/index.html>)
Flash & Math has a large and growing collection of self-contained learning and teaching modules for mathematics - math applets (i.e. mathlets). Mathlets can be used by instructors for classroom demonstration, and by students for self-paced study and exploration. They are all programmed in ActionScript and run in the Flash Player.

Interactivate (<http://www.shodor.org/interactivate/>)
Offers free math and science lessons and activities.

Larry Green's Applet Page (<http://www.itconline.net/green/java/index.html>)
Contains numerous applets within basic math, beginning algebra, intermediate and college algebra, statistics, and calculus. Some applets require Java while others use flash.

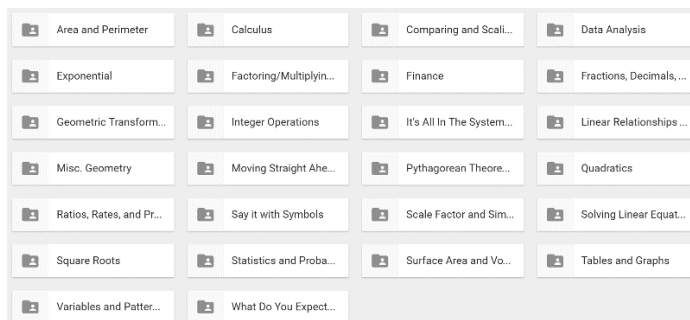


Figure 9. Images of the annotated VM list and the Google Drive folder of teachers' requested activities.

At least three teachers, were able to move beyond the suggested activities by January, however, the other teachers needed more support in their efforts to find relevant VMs and activities. For the two teachers who did not appropriate this practice during the course of the PD, rather than using the annotated list of VM resources to find their own activities, they relied on the suggested activities found in the Google Folders (see Figure 9). However, neither of these two teachers were present for at least the last two PD sessions when teachers were finding their own VMs and activities. Had they been present, though, there is no indication in the data that they would have been able to find their own activities. For at least two teachers, as the PD progressed and the list of resources grew, the wealth of resources became overwhelming. For example, during Jake's final interview, he said,

What worked was like, it helped when I had, when I was given one or two like, "Here, look at these." It was hard toward the end of the year to try and figure out what do, I want to look at, because you just-there was so much information. That I was kind of like, "Which one do I want to look at? And I want to look at ALL of them to kinda see which one do I want to explore deeper." And sometimes I spent too much time looking at like, what time, you know, which one I'd want to look at.

During the last PD session, Jake and Mark mentioned feeling overwhelmed about where to begin searching for VMs and activities. However, upon directing them to the annotated list, and telling them two or three collections that may be appropriate for their topic, both were able to find activities to implement in their classrooms. Therefore, the repository of resources, which initially served as a tool to introduce teachers to a smaller data set, was used later in the PD to help to mitigate this tension. Teachers were more productive when they had a particular learning goal in mind as opposed to “looking at what’s out there.” Thus, when supporting teachers’ use of VMs and activities, teachers should use their learning goal to concentrate their search efforts rather than trying to match a learning goal to a VM and/or activity that they may find. Additionally, until teachers become more knowledgeable about the wealth of activities and finding activities on their own, it may be helpful to focus their attention on a limited number of activities (e.g., two or three) and/or repositories.

Implementing VMs and Activities

Teachers’ appropriation of the three practices (i.e., their actions) discussed previously supported their efforts to implement VMs and activities (i.e., the object of the activity system). Drawing from teachers’ conversations and final surveys at least 13 out of the 14 teachers in the PD implemented VMs and/or activities in their classroom at least once during the PD. At least three teachers implemented over 12 activities during the school year. One teacher started using the VMs and activities with students whom he tutored. As discussed later, the actual number of VMs and/or activities for each teacher is not known. In this section, findings regarding how teachers thought their students might benefit from using a VM or activity, the instructional role VMs and activities had for various teachers in the PD, and their implementation efforts are presented.

Why teachers implemented VMs and activities. Teachers implemented VMs and activities because they thought their students might benefit and/or their learning would be enhanced. The following subsection discusses the categories and examples of how teachers thought their students may benefit from interacting with VMs and activities.

Table 6 highlights the different types of potential benefits teachers described related to why (or if) they use VMs and activities. Once again, the number of data excerpts related to each category are included in the table for descriptive purposes only. However, as displayed in the table, teachers identified the visual nature of VMs (e.g., see Reimer & Moyer, 2005; Suh, Moyer, & Heo, 2005), opportunities for student exploration, and assist in differentiating instruction (e.g., see Shin et al., 2017; Bouck et al., 2015) as the most common benefits to instruction and student understanding. Consistent with the field, teachers also cited students benefiting through the use of VMs and activities due to the ability for students to offload procedural aspects of a task (Romberg, 1998), receive feedback (e.g., Edwards Johnson et al., 2012), practice problems (including extending beyond the VM), connect multiple representations, and encourage student engagement (Roberts & Stephens, 1999) in the lesson and learning goal.

Table 6. *Categories, Definitions, and Examples of Student Benefits When Using VMs and Activities*

Definition	Number of data excerpts	Example
Connect Multiple Representations: opportunities for students to connect multiple representations	8	Mark (DecReflecDoc): The connection between the area model, the measurements, and the formula for area and perimeter of rectangles makes this a meaningful lesson.
Differentiation: opportunities for personalized learning by differentiating instruction based on student needs.	16	Josh (MarchPDSummary): Josh shares his thoughts on the assessment questions at the end (i.e., Activity B) one of the questions he would get about 50% of the students and Q5, he will get about 2% of the students, which would be perfect. Josh says it would push the students in different directions.
Engagement: promotes student engagement.	9	Mark (FinalSurvey): A larger number of students are engaged. All students have an opportunity to interact with the problem(s) they

		are attempting to solve.
Exploration: promotes opportunities for students to explore concepts through manipulation, interaction, etc.	27	Jake (FinalSurvey): One advantage is, kids, you—they can manipulate. I mean, they can move things around, they can change parameters, they can change...
Feedback: provides opportunities for students to receive some type of feedback.	11	Jake (FinalSurvey): ...some of them they can get feedback instantly or very close to instantly. So—it allows, when you are doing something, if they are getting that feedback, that you don't have to be at, 20 different 20 different people or 20 different desks or whatever.
Modeling: provides opportunities for students to use or develop models.	8	Kelly (NovReflecDoc): Being able to model division situations and having a better understanding of how remainders happen and what they mean.
Offload: due to built in features (e.g., calculations, drawing graphs) of the VM, students can focus on the content rather than the <i>menial</i> or <i>tedious</i> components of a task.	12	Pam (MarchReflecDoc): The students would benefit from this because it's less time-consuming than creating the intersecting graphs with a pencil and paper. They would be able to practice with more solid examples.
Practice: provided opportunities for students to practice problems.	11	Tracy (OctPD): It's is a—it's got a lot of sequential steps. I mean, it starts with the very very basic of just learning how to build them. And then gets you going, like you can keep clicking to the next level to get more difficult, more difficult, more difficult. So, umm, it's great practice for kids who, just don't even understand what Algebra tiles are.
Visual: provides a visual of the concept being investigated.	30	Mari (DecReflecDoc): Students would benefit with the sliding aspect of the gizmo. seeing the function change as the y-intercept changes or as the growth/decay rate changes.

Teachers also identified other student benefits to why or if they chose to use VM activities that are not included in the table due to the low occurrence of these benefits being mentioned. The benefits included: communication (i.e., promoted students' discussion about what they were doing and learning), conceptual understanding (i.e., promoted opportunities to develop students' conceptual understanding), concrete (i.e., potential for making an abstract concept more concrete), make and test conjectures (i.e., provides opportunities for students to make and test conjectures), and using technology (i.e., using technology is a benefit in and of itself). These benefits occurred six or fewer times in the dataset. Although not included in the

table, these benefits are still noteworthy and may contribute to why other teachers consider integrating VM activities in their instruction.

Instructional role of VMs and activities. Teachers implemented VMs and activities for a variety of purposes. The following subsection discusses the instructional role that teachers reported VMs and activities had within their classrooms over the course of the PD. The findings and discussion presented next are based on the classroom observations and reflections of the focus teachers implementing VM activities, along with their final interviews. Additionally, reflections and conversations with teachers during the PD sessions regarding their use of VMs and activities as well as final survey responses informed the uses that are presented.

Table 7 highlights the different instructional roles teachers described related to their implementations of VMs and activities. Teachers primarily used VMs and activities to enhance their student learning, as a re-teaching tool, and as a tool for enabling students to receive in the moment feedback as well as a tool for collecting information about student learning that could be used to inform future instruction. Unfortunately, as described in more detail in the next subsection, teachers did not often share with me about their implementation efforts (i.e., they implemented VMs and activities more frequently than depicted in the data based on their final interviews and surveys).

Table 7. *Categories, Definitions, and Examples of Instructional Role of Virtual Manipulatives and Activities*

Definition	Number of data excerpts ^a	Example
Enhance: used to enhance students' understanding of concepts and skills through practice, communication, feedback, connecting representations, etc.	13	Josh(FinalInt): I mean more to enhance some of the skills we're working on. We've used a lot of the Gizmos... at the end of that particular unit where we've talked about adding and subtracting fractions. And then it's, " <i>Well, let's do this Gizmo and see how it looks in a different light. ...So more to enhance and follow-up and build on some of the stuff that that they've already been learning about in class.</i> "

Feedback: provide feedback to students and/or teachers during the lesson	8	Stan (OctPD): Well, we're getting feedback too. I think that's the MOST important.
Re-teach: used as a tool to re-teach a particular concept in a different way	11	Tracy (FinalInt): I've use them with my Math Plus kiddos, more as another way to learn, another re-teaching. Because it's hard for them to, in their SECond math class of the day, to have to sit and do things pencil and paper. So, I try to use it more as some sort of instructional technology where they can see it. More, you know, just a different way...so it's different from what they did in their first math class of the day.
Solidify understanding: used as a tool to help solidify student understanding by having students interact with different representations.	7	Josh (JanPD): Kids are used to it, they like it. . .we've been using it more to solidify some of the things that we are doing in class [Kelly: Umm hmm]. When we did this activity, you know look on the Gizmo and try it on this.
Supplement curriculum: used as a tool to supplement curriculum by providing models of word problems, investigations, etc.	7	Kelly (JanPDSummary): Kelly said that her current textbook (i.e., Everyday Mathematics) does not have a lot of examples for what they are doing (i.e., adding fractions with unlike denominators). If they had the word problems in a ConceptuaMath module, then they could use the tools to model the problems, it would be helpful. Once they know how to use the tool, then it could be used to show different things.

^aNumber of excerpts is included for descriptive purposes only.

Other instructional uses of VMs and activities exist that are not included in the table. For example, teachers also used VMs to launch/preview and review particular concepts and units. However, these instructional uses only occurred three and four times in the dataset. Additionally, using VMs for demonstration purposes was not a role that teachers identified. This lack of use is most likely due to the prevalence of technology in the classroom (e.g., due to most students having Chromebooks) as well as a focus in the PD for using VMs and activities to promote student exploration as opposed to only for demonstration purposes. However, for teachers with more limited access to technology, using VMs for demonstrations (e.g., similar to Mr. Rodriguez in the opening vignette) may be a means for enhancing student understanding and/or engagement.

Teachers' curriculum and emphasized standards may influence the instructional role of VMs and activities. For example, some middle school teachers (e.g., Stan and Erin) found it challenging to integrate VMs and activities within their instruction due to their curriculum investigations building on each other. Replacing an investigation in one part of the unit with a VM could lead to additional investigations needing to be modified due to students not having the background information/context from previous investigations. Therefore, there was an emphasis on using VMs to supplement or enhance teachers' curriculum rather than replace investigations. At times, some teachers (e.g., Curt, Kelly, and Mike) explicitly looked for opportunities where textbook problems and/or investigations could be done using the VM (e.g., see the example for *supplement curriculum* in Table 6). For example, during the March PD session, Curt talked about using an ExploreLearning gizmo in place of the textbook investigation because the gizmo provided opportunities to extend students' thinking (i.e., comparing the *fairness* of three spinners and make spinners that satisfied conditions) beyond the original textbook investigation (which only had students compare two spinners). Curt said, "Because that one [the gizmo] had the three spinners, and then you have to put different sign names on them. I think that was a nice problem. That's really taking it to the next level. Most of the times we don't hit on that at the end of the investigation, like this one." Therefore, the manner in which textbook problems and/or investigations build on each other may influence how teachers decide to use VMs and activities.

Additionally, due to previous professional development experiences, some teachers were limited in the ways they thought about and used VMs and activities (i.e., VMs and activities could only be used as supplements and not to replace investigations). The 8th grade teachers were the largest team of teachers in the PD. During the November PD session, when discussing how and when a VM activity could be used within their current unit (e.g., replacing an investigation,

as a review, etc.), Mari wanted to use the activity to replace an investigation, but was not able to do so because all 8th grade math teachers (i.e., Pam, Mari, Stan, Erin, and an additional teacher not part of the PD) were supposed to be teaching the same thing and Stan and Erin did not want to eliminate the proposed investigation. Mari then suggested using the activity with her lowest students as a pre-teaching tool, but got push back from Stan. Stan was adamant that it could not be used to replace an investigation. He drew from his experience with the previous K-12 math leader who had said that you need to “trust and stick to the curriculum” even when students are struggling with understanding the investigation.

Implementation efforts. The following subsection describes teachers’ implementation efforts during the course of the PD. The findings and discussion are based on teachers’ discussions and reflections during the PD sessions, observations and reflections of focus teachers, as well as teachers’ responses to the final survey and final interview with focus teachers.

During the January PD session, Daron brought up his experience taking his AP Statistics students to the computer lab to do the Binomial Probability gizmo from ExploreLearning (gizmo was not compatible with Chromebooks). Two days later, when he pulled the gizmo up during a class discussion, he and the class realized that it had now been converted to HTML5 (therefore compatible with Chromebooks). According to Daron, after doing the gizmo, students had less questions about the binomial theorem compared to years past. Additionally, they were not relying on their calculators (rather they were directly substituting into the formula) and were not making some of the common errors that previous students had made (e.g., leaving off the beginning part of the binomial formula). After the PD session, Daron emailed the following thoughts related to his implementation of the Binomial Probabilities gizmo:

I think students were much more engaged in learning the binomial pattern. They were able to see where the pattern was coming up from the most basic situation to more complex examples. I think students were able to see the pattern in the factorials and pascals triangles quickly, then were able to adapt them to more difficult problems. I asked for student feedback and they said they really liked using the gizmo and it helped them to see the pattern clearly.

Daron's thoughts demonstrate how both he and his students found the VM to be helpful for students' understanding.

The final survey at the end of the PD and final interviews with the focus teachers brought to light how teachers were implementing VMs and/or activities. For example, in May, I found out that Karen had been implementing VM activities during the school year even though she did not attend any of the afterschool PD sessions. Through collaborations with her partner teacher (i.e., Curt) who attended some of the after-school sessions, and having access to the resources, she was able to benefit from the PD even though she was no longer attending the sessions. Yet another example relates to two of the focus teachers in the PD. Based on the PD sessions, I thought Mark and Josh had implemented about four or five VM activities with their students. However, during Josh's final interview he shared that they had implemented at least 10 different gizmos alone. In fact, students came to expect that they would regularly be using their Chromebooks in math class (which had not been the norm). In hindsight, it would have been helpful if I asked teachers how many VMs and/or activities they had implemented during the course of the year.

At times, all teachers found it challenging to implement VMs and activities due to limits of the Chromebooks (e.g., needing to be HTML5 friendly) as well as being part of a larger team of teachers with differing beliefs regarding adherence to the curriculum (related to the *Rule* and *Community* factors of the activity system). For example, twice, Pam and Mari created VM activities to be implemented with their students within the week following the PD session.

However, the activities did not get implemented after taking them back to the rest of the team members. The lack of implementation may have been related to limits on instructional time (e.g., not having the time due to not being able to replace a curriculum investigation), the fact that the other team teachers (three of them) had more experience on the 8th grade team than Pam and Mari (2015/2016 was Pam and Mari's second year as 8th grade teachers and members of the team), or something else entirely. Had Mari and Pam been on a team together or individual teachers, it is posited that both would have implemented more VMs and activities during the school year.

Implications

The design of this study relied on the central tenets of activity theory. Since tensions are paramount to understanding motivations behind particular actions, understanding how the tensions exist within the activity system is needed to support and understand continual innovation of the activity system (Barab et al., 2002). That is to say, by better understanding the systemic tensions that exist related to how teachers are supported in modifying VM activities to fit the needs of their students in particular, teachers more generally can be supported to teach *with* technology as well as how to modify instructional activities. Furthermore, this study aimed to address Webster-Wright (2009)'s call to investigate teacher learning from a holistic-situated perspective whereby the context, learner, and learning are interwoven rather than studied separately.

Similar to Yamagata-Lynch and Haudenschild (2009) teachers in the PD viewed the PD opportunity as an impetus to improve their teaching practices and professional growth. For example, Kelly requesting to be in the PD and some teachers (e.g., Josh, Matt, and Tracy)

making the PD part of your Professional Development Plan⁹. Furthermore, teachers' experiences with VMs and implementing them in their classrooms extended beyond the PD sessions, more than I realized. For example, Mike shared that he had designed a project to use after the AP Calculus exam whereby his students would search for "and find a few manipulatives that may have helped them. Then see if they can create their own." During the 2016/2017 school year, at least four teachers shared on their own that they were still using the resources from the PD and implementing additional activities into their instruction. Three teachers also requested to share the resources and activities with additional teachers at their school, thus promoting the use of VMs and activities to teachers outside of the PD.

The findings suggest that when teachers are supported in learning about VMs and how VMs can be used to promote students' development of conceptual understanding (i.e., teaching *with* technology), teachers were able to appropriate specific practices advocated during the PD. Although the findings and outcomes of this study are activity specific (i.e., specific to this PD), it is hoped that the findings can inform future efforts aimed at promoting teachers' use of technology-based instructional activities whereby teachers are teaching *with* as opposed to *near* technology.

An important aspect of the PD was providing teachers with the time and space to investigate VMs and activities, as well as to support their collaboration and efforts to modify activities. Encouraging teachers to fully explore a VM or activity, as well as using guiding questions that focused teachers to use their learning goal as a lens to critique activities (i.e., active learning), helped teachers to focus where students might struggle in an activity and how

⁹ Each educator is required to complete a Professional Development Plan (PDP) to renew their license. The PDP is a process based on intended professional growth and evidence regarding how student growth was impacted due to the educator's professional growth.

students might benefit by engaging in the exploration. Additionally, a few teachers began being able to distinguish between modifications that might help students engage in the learning goal versus modifications that were more stylistic (e.g., breaking an exploration into smaller components).

Teachers found the task analysis framework helpful for critiquing VMs and activities as well as modifying or developing instructional guides, thus helping them to distinguish between mundane and powerful uses of technology (Wilson, 2008). Additionally, some teachers used the task analysis framework to help think about and transform their instruction more generally. Having teachers provide justifications for why affordances applied may have further supported their ability to operationalize this action. Finally, during the course of the PD, teachers were able to focus on using virtual manipulatives due to their potential for promoting conceptual understanding as opposed to because the VMs were cool or because they had to (i.e., due to student, parent, administration expectations). Based on my experience working with pre-service secondary mathematics teachers, the use of the guiding questions and task analysis framework could be helpful in promoting their efforts to teach *with* as opposed to *near* technology as well.

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Appendix A: Annotated List of Virtual Manipulative Resources

The following annotated list of VM resources was a Google Document used by teachers during the PD.

Analyze Math (http://www.analyzemath.com/mobile_math/HTML5_applets.html)

“This site contains online html5 interactive web applications (suitable for iPads and tablets as well as laptops and desktops) to explore topics in mathematics and mathematical objects such as graphs of equations and functions, angles and trigonometric functions, inverse functions. These apps would be useful in group work, outside class activities, exploration for deep understanding.”

Applets for Calculus (<http://www.sfu.ca/~jtmulhol/calculus-applets/html/appletsforcalculus.html>)

These applets are GeoGebra files related to various Calculus topics. Many of the clips are visualizations of common problems (e.g., Squeeze Theorem, Related Rates, Riemann Sums, etc.). The page is maintained by Jamie Mulholland from the Department of Mathematics at Simon Fraser University.

Calculus Applets using GeoGebra

(<http://webspace.ship.edu/msrenault/GeoGebraCalculus/GeoGebraCalculusApplets.html>)

This site contains multiple GeoGebra applets for Calculus that are suitable for class demonstrations and student explorations. The site a project by Marc Renault and supported by Shippensburg University.

Calculus Java Applets (<https://community.plu.edu/~heathdj/java/calc2/Shell.html>)

This site has a collection of Java applets illustrating various calculus concepts by Deej Heath at Pacific Lutheran University. Animations developed in Mathematica and playable with QuickTime can be found at http://www.calculus.org/Heath/maple_anim.html. He has also included the code for these animations.

Calculus on the Web (<http://cow.math.temple.edu/~cow/cgi-bin/manager>)

COW is an internet utility for learning and practicing calculus. It was designed at Temple by two members of the Temple University Mathematics Department, Gerardo Mendoza and Dan Reich. The main purpose of COW is to provide users with the opportunity to learn and practice problems in calculus. This site can be useful for providing students with additional practice along with some feedback on errors.

Calculus with GeoGebra (<http://tube.geogebra.org/student/b77409>)

This link contains several GeoGebra applets created by Sangeeta Gulati. Many of these applets are good for demonstrations, but some include exploration questions as well.

Conceptua Math (<https://www.conceptuamath.com/app/tool-library#AddingUD>)

This website contains various applets and lessons teachers and students can use to explore fractions. A free account enables the user to access the 16 applets and 2 lessons. To access more resources, requires a subscription.

The Concord Consortium (<http://concord.org/stem-resources/subject/mathematics>)

These HTML5 resources are modules students can work through. Teacher lesson plans and student assessments are also provided. Additionally, teachers can create their own modules for students.

Desmos: Classroom Activities (<https://teacher.desmos.com/>)

Contains a variety of activity “bundles” that include activities focused on essential questions.

Euclid’s Elements (<http://aleph0.clarku.edu/~djoyce/java/elements/elements.html>)

Java applets demonstrating each of the Elements.

Explore Learning Gizmos (<https://www.explorelarning.com/>)

Gizmos are interactive math and science simulations for grades 3-12. Over 400 Gizmos aligned to the latest standards help educators bring powerful new learning experiences to the classroom. Gizmos can be searched by standard, grade & topic, or textbook. Explore Learning is working to update the Gizmos to be compatible with html5.

- You can sign-up for a free 30 day trial per email address. Otherwise, you can access each Gizmo for 5 minutes per day per computer.

Flash & Math (<http://www.flashandmath.com/mathlets/index.html>)

Flash & Math has a large and growing collection of self-contained learning and teaching modules for mathematics - math applets (i.e. mathlets). Mathlets can be used by instructors for classroom demonstration, and by students for self-paced study and exploration. They are all programmed in ActionScript and run in the Flash Player.

Interactivate (<http://www.shodor.org/interactivate/>)

Offers free math and science lessons and activities.

Larry Green’s Applet Page (<http://www.ltconline.net/green/java/index.html>)

Contains numerous applets within basic math, beginning algebra, intermediate and college algebra, statistics, and calculus. Some applets require Java while others use flash.

Math Applets at Saint Louis University (SLU)-Below Calculus

(<http://www.slu.edu/classes/maymk/AppletsSLUBelowCalc.html>)

This page is a collection of applets organized by topics (e.g., college Algebra, triangles in Geometry, vectors, advanced graphing, etc.). Some of the applets require Java, while others use GeoGebra or Geometer’s Sketchpad. These applets would be good for demonstrations, and possibly student exploration.

Math Applets for Calculus at Saint Louis University (SLU)

(<http://www.slu.edu/classes/maymk/MathApplets-SLU.html>)

This page is a collection of applets organized by topics (e.g., graphs, continuity, differentiation,

integration, etc.). Some of the applets require Java, while others use GeoGebra or Geometer's Sketchpad. These applets would be good for demonstrations, and possibly student exploration.

Math Playground (http://www.mathplayground.com/math_manipulatives.html)

This site includes virtual manipulatives and math games for several elementary and lower middle school topics (e.g., fraction bars, spinners, a protractor, etc.).

Mathlets: Java Applets for Math Explorations

(<http://cs.jsu.edu/mcis/faculty/leathrum/Mathlets/>)

Written by Tom Leathrum, these interactive applets cover many topics from pre-calculus through calculus.

MathTools (<http://mathforum.org/mathtools/>)

A collection of lessons and activities to support the learning and teaching of mathematics. It is part of the Math Forum at Drexel University. It includes exercises, lessons, discussions, virtual manipulatives, activities, etc. sorted by content topics.

National Library of Virtual Manipulatives (<http://nlvm.usu.edu/en/nav/vlibrary.html>)

Manipulatives are organized by grade band and content focus. **Note, these manipulatives require Java.** There is an app that you can purchase to access the manipulatives offline. The app is about \$30.

NCTM Illuminations (<http://illuminations.nctm.org/>)

Provides lessons, games, brain teasers, etc. organized by grade band and content focus.

PhET Interactive Simulations (<http://phet.colorado.edu/en/simulations/category/new>)

The University of Colorado Boulder has produced interactive simulations for math and science. The link above is for the HTML5 simulations. Simulations requiring Java and Flash are located at (<http://phet.colorado.edu/>). Many of the simulations aim to help students make connections between math/science and real life phenomena. All simulations are free.

Seeing Math (http://seeingmath.concord.org/sms_interactives.html)

These interactive tools aim to clarify key mathematical ideas in middle and high school mathematics. Instantaneous updates in representations (i.e., symbolic, graphical, etc.) promote students' connecting multiple representations.

**These interactives use Java. You can download the interactives to use offline (they download as a zip file).

Teaching Resources from the Freudenthal Institute

(<http://www.uu.nl/onderzoek/freudenthal-instituut/onderwijs/leermiddelen>)

These are math and science activities from the Dutch Freudenthal Institute. Note, you will need to translate the page (unless your Dutch is pretty good). Activities are sortable based on Flash, html5, iOS, Java, and Modules.

- **WisWeb** (<http://www.fi.uu.nl/wisweb/en/>)

Java applets for students in grades 7-12. Subjects include number sense, number

and estimation, measurement, algebra and calculus, geometry, discrete math, statistics and probability.

TeacherLED (<http://www.teacherled.com/all-interactive-whiteboard-resources/>)

TeacherLED is a site containing interactive lessons and resources for teachers. The applets can be done on interactive whiteboards, computers, or tablets.

University of Illinois: Mathematics, Science, & Technology Education (MSTE) Online Resource Catalog (<http://mste.illinois.edu/resources/>)

Contains math and science resources sorted by major standard strands (e.g., Number & Operations, Algebra, Geometry, etc.). Many of the applets require Java, but some have been updated to HTML5.

University of Chicago School Mathematics Project (UCSMP) Interactive Demos (<http://ucsmp.uchicago.edu/secondary/curriculum/>)

This site includes the interactive demos for the UCSMP textbooks. You can select a textbook and view/use the interactive demos. Some applets (e.g., Pan Balance with UCSMP Algebra) allow you to print student work.

Virtual Algebra Tiles (Michigan Virtual University)

(http://media.mivu.org/mvu_pd/a4a/homework/applets_applet_home.html)

This site contains an assortment of activities involving algebra tiles. Activities range from modeling and simplifying algebraic expressions to multiplying and factoring polynomials. Also included is an activity for completing the square.

Visual Calculus (<http://archives.math.utk.edu/visual.calculus/index.html>)

Visual Calculus consists of modules that can be used for studying and teaching calculus.

Visual Fractions (<http://www.visualfractions.com/>)

This site, developed by Richard Rand, is an online tutorial with instruction and interactive practice for students in identifying, renaming, comparing, and operating on fractions, including mixed numbers (add, subtract, multiply, and divide). All examples are modeled with number lines or circles. Target audience are students in grades 3-8.

Walter Fendt html5 Mathematics Apps (<http://www.walter-fendt.de/html5/men/>)

Walter Fendt has produced various apps for math and physics. The link above is for the html5 mathematics apps only. He does have additional apps available on his homepage. **Note, this page may also need to be translated from German.**

Wolfram Demonstrations Project (<http://demonstrations.wolfram.com/>)

Demonstrations are organized by subjects and searchable by grade bands, keywords, and CCSSM. After selecting the demonstration, you will need to download the demonstration file from the link on the right-hand side. You can also download the author's source code. **Note, you will need to download the free CDF player to play/interact with the demonstrations.**

Appendix B: Background Survey Questions

Survey was administered through Qualtrics (an online survey platform).

	Question(s)	RQ Alignment
Background Questions	1. Including this year, how long have you been teaching?	Background
	2. Please select the math course(s) you are currently teaching, be as specific as possible.	Background
	3. Preferred teaching methodology (choose only one) <ul style="list-style-type: none"> • Largely teacher-directed (i.e. teacher-led discussion, lecture) • More teacher-directed than student-centered • Even balance between teacher-directed and student-centered activities • More student-centered than teacher-directed • Largely student-centered (i.e. cooperative learning, discovery learning) 	RQ2
	4. Actual teaching methodology (choose only one)	
	5. If your preferred teaching methodology <u>does not match</u> your actual teaching methodology, please describe why that is the case. (e.g., lack of support, lack of resources, lack of experience, not enough time, collaborative/department lesson plans, etc.)	
	6. If your preferred teaching methodology <u>does match</u> your actual teaching methodology, what enables you to teach in the manner you prefer?	
	7. Instructional technology refers to technology that is used for instruction (e.g. graphing calculators and probes, interactive whiteboards, tablets and slates, virtual manipulatives, dynamic geometry software, Internet, computer, etc.). Technology that is used for record keeping (e.g., taking attendance on the computer) or communication only (e.g., email) is not considered instructional technology for the purpose of this survey. Describe your ideal use, if any, of instructional technology.	RQ1
	8. How well does your current curriculum lend itself to technology integration? <ul style="list-style-type: none"> • Very Difficult • Difficult • Somewhat Difficult • Somewhat Easy • Easy • Very Easy 	RQ2
	9. Please explain your answer above.	
Experience with Technology	10. In a typical week, how often do you integrate instructional technology in your teaching activities? <ul style="list-style-type: none"> • Never • once • twice • three times • four or more times 	RQ2
	11. In a typical week, how often would you prefer to integrate instructional technology into your teaching activities? <ul style="list-style-type: none"> • Never • once • twice • three times • four or more times 	

	<p>12. If your typical frequency of instructional technology use does not match your preferred use, please describe why that is the case.</p> <p>13. If your typical frequency of instructional technology use does match your preferred use, what enables you to use instructional technology as frequently as you prefer?</p>	
	<p>14. Please indicate your level of proficiency with each of the instructional technology tools listed below.</p> <p>Selection Options:</p> <ul style="list-style-type: none"> • I have no experience with the tool and do not want to know more about the tool. • I have no experience with the tool, but want to learn more about the tool. • I have used the tool, but do not feel comfortable integrating it into my instruction without further experience or support. • I have used the tool and feel comfortable integrating it into my instruction. • I am comfortable with the tool and try integrating it as much as possible into my instruction. <p>Tools:</p> <ul style="list-style-type: none"> • Graphing Calculators • Dynamic Geometry Software (e.g., Geometer's Sketchpad, GeoGebra, Cabri, etc.) • Dynamic Statistics Software (e.g., Fathom, Excel, SPSS, Tinkerplots, Systat) • Dynamic Graphing Software (e.g., Mathematica, Maple, Desmos, GeoGebra, etc.) • Explore Learning's Gizmos • Virtual Manipulatives (i.e., online manipulatives and activities that are modeled after physical manipulatives) • Tablets • Data collection devices (e.g., CBR or CBL with graphing calculator, "clickers," etc.) • Other: _____ (please describe) 	Background
	<p>15. Please read the following descriptions of the proficiency levels a user has in relation to instructional technology. Select the level that best describes you currently.</p> <ul style="list-style-type: none"> • Unfamiliar—I have no experience with instructional technology. • Newcomer—I have attempted to use instructional technology, but I still require help on a regular basis. • Beginner—I am able to perform basic functions in a limited number of instructional technology applications. • Average—I demonstrate a general competency in a number of instructional technology applications. • Advanced—I have acquired the ability to competently use a broad spectrum of instructional technology. • Expert—I am extremely proficient in using a wide variety of instructional technology. <p>16. Select the level that best describes where you would like to be within the next 1-3 years.</p> <p>17. If you would like to be at a different level than what you are currently at, what support and/or resources would you need to help you achieve your desired level?</p>	Background
	<p>18. Over the past seven years, what is the total amount of training you have received to date on using instructional technology in the classroom:</p> <ul style="list-style-type: none"> • None • A full day or less • More than a full day and less than a one-semester course • A one-semester course 	Background

	<ul style="list-style-type: none"> • More than a one-semester course 	
	<p>19. Please select the extent to which disagree or agree to the statements listed below.</p> <ul style="list-style-type: none"> • My pre-service teacher certification program included teaching with technology methodologies that were beneficial to me. • I believe teachers need release time to collaborate with technology support staff to design effective lessons that integrate technology. • Whenever I have a question about using technology in the classroom, I contact a trusted colleague who is an experienced user. • Teachers should feel comfortable letting their students teach them about anything related to technology. 	Background
Process of Integration	<p>20. Please read the descriptions of the six stages related to the process of integrating instructional technology. Select the stage that best describes where you are in the process.</p> <ul style="list-style-type: none"> • Awareness—I am aware that technology exists, but have not used it—perhaps I’m even avoiding it. I am anxious about the prospect of using instructional technology. • Learning—I am currently trying to learn the basics. I am sometimes frustrated using instructional technology and I lack confidence when using them. • Understanding—I am beginning to understand the process of using technology and can think of specific tasks in which it might be useful. • Familiarity—I am gaining a sense of self-confidence in using technology for specific tasks. I am starting to feel comfortable using instructional technology. • Adaptation—I think about instructional technology as a tool to help me and am no longer concerned about technology. I can use many different forms of instructional technology. • Creative application—I am able to apply what I know about technology in the classroom. I am able to use it as an instructional aid and have integrated instructional technology into the curriculum. 	Background
Instructional Scenarios	<p>An interactive whiteboard was recently installed in Ms. Johnson's classroom. Ms. Johnson projects the PowerPoint lessons from the textbook publisher using the whiteboard. Every once in a while, in response to a student question, she may write an additional explanation on the whiteboard during the lesson. At the end of the lesson, Ms. Johnson projects the online student textbook using the whiteboard.</p> <p>21. Drag the slider to indicate how well you think Ms. Johnson effectively integrates the interactive whiteboard within her instruction.</p> <p>22. You answered that Ms. Johnson (<i>at least somewhat effectively/ at least somewhat ineffectively/ neither effectively nor ineffectively</i>) integrates the interactive whiteboard into her instruction. What would you say to support your position if a friend of yours disagreed with you?</p>	RQ1
	<p>The principal at Memorial Middle School has recently purchased classroom sets for graphing calculators for each mathematics classroom. Mr. Patel, one of the teachers, is excited to use the calculators in his classroom and to finally integrate technology into his classroom. He assigns a calculator to each student and allows students to use the calculators whenever they feel necessary during the class period. In addition, Mr. Patel has a few personal graphing calculators that he allows students to check out for one night. Eventually, Mr. Patel would like to use the calculators more, but for now is satisfied with his technology integration by making the calculators available to the students.</p> <p>23. What are the strengths in how Mr. Patel is using the graphing calculators? 24. What are the weaknesses in how Mr. Patel used technology?</p>	RQ1

<p>Mr. Rodriguez projects his lessons onto a SMART board. During the lesson, he runs a virtual manipulative on the board to explore how changing the slope and y-intercept of an equation of a line changes its graph. Students and Mr. Rodriguez propose possible changes to the equation and the effect the changes will have on the graph. After the exploration, Mr. Rodriguez continues with notes on the lesson. If needed, he goes back to the virtual manipulative to address a student question.</p> <p>Ms. Kahn has designed an activity for students to work in groups investigating properties of rational functions using a virtual manipulative. Working in groups, students compare how changes in the equations affect the resulting graphs of the functions. They then use the virtual manipulative to test their hypothesis relating equations to specific properties they see in their graphs.</p> <p>25. What is similar about these teachers' instructional practices? 26. What is different about these teachers' instructional practices?</p>	RQ1
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Appendix C: Semi-Structured Interview Protocol

Initial Interview Questions (Focus Teachers Only)

The initial interview will be semi-structured; therefore, questions may evolve based on teacher responses. Overall, the questions will focus on teachers' beliefs about instruction and technology as well as their current instructional practices. **Teachers will be told that they should not reveal any highly personal, sensitive, or identifiable information about themselves or others.** The questions given below are representative of the themes that will be investigated during the interview. Some questions may be removed or added depending on teachers' responses.

Stimulus & backup	Expected Themes	Follow-ups & Probes	Stimuli for extended narrations	Comments
<p>(RQ1) Please describe your general instructional style.</p> <p>Please describe a typical day's lesson.</p> <p>Tell me about a successful lesson that you taught? What made it successful?</p> <p>Tell me about a lesson that did not go as well as you had planned. What would you change if you were to teach that lesson again?</p>	<p>Teacher directed Mixed Student centered</p> <p>Instructional Practices</p> <p>Types of Technology</p>	<p>Do you ever do/use classroom demonstrations?</p> <p>What are the student roles in your classroom?</p> <p>How often do you have inquiry-based activities in your lesson?</p> <p>Is there anything you really want to try in your lessons that you don't have the time to learn, plan, or don't know how to do? (RQ2)</p> <p>Do you have any desire to integrate technology into your lessons? Can you give me an example of when you used technology within your instruction?</p>	<p>If so, when? Why?</p> <p>Can you give me an example of inquiry-based activities?</p> <p>Tell me about a successful lesson that you taught? What made it successful? Tell me about a lesson that did not go as well as you had planned. What would you change if you were to teach that lesson again?</p>	

<p>(Background) How do you describe instructional technology?</p> <p>Can you give examples of things you consider to be technology and things that you do not consider to be technology?</p>	<p>Specific tool</p> <p>Frustration/emotion</p> <p>Range of knowledge or experience</p>	<p>Please describe your ideal use of technology?</p> <p>What is your familiarity with technology?</p>	<p>Are there things that keep you from using technology? (RQ2)</p>	
<p>(RQ2) Can you tell be about your typical lesson planning process?</p> <p>When you are developing a new lesson, what do you think about?</p>	<p>Collaborate</p> <p>Individual</p> <p>Pacing Guide</p> <p>Pre-made lessons</p>	<p>How do you decide if you are going to keep an old lesson plan or develop a new one? Or tweak an existing one?</p> <p>Does your school provide common planning?</p> <p>How comfortable do you feel modifying lessons?</p>	<p>Do you think about specific students when designing your lessons?</p> <p>Do you plan with the same person/people?</p> <p>Tell me about the last time that you modified a lesson.</p>	
<p>(Background) Please describe your experience with instructional technology.</p> <p>What kind of technology have you used in your classroom?</p>	<p>Teacher education/training</p> <p>Frustration</p> <p>Types of technology</p>	<p>What was your educational experience as a student with technology?</p> <p>What was your education experience as a student teacher with technology?</p> <p>How accessible is technology in your school? (RQ2)</p> <p>What are your experiences with virtual manipulatives (VM)?</p> <p>What type of support do you need to integrate technology? (RQ2)</p>	<p>Do you get support from your school in terms of using technology? (RQ2)</p>	

(RQ1) How do you think technology should be used within mathematics instruction?	Learning Mathematics Specific tools	What would you say to a friend that disagreed with you? (RQ1, RQ2) Do you think certain subjects lend themselves more easily to technology than others? (RQ2)	Why? Do you think your curriculum lends itself to technology?	
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Present teachers with (some) of the instructional scenarios below (or similar scenarios):

Ms. Johnson projects the PowerPoint lessons from the textbook publisher using the SMART board. Every once in a while, in response to a student question, she writes an additional explanation on the whiteboard. At the end of the lesson, Ms. Johnson projects the online student textbook using the SMART board.

Mr. Rodriguez also projects his lessons onto a SMART board. During the lesson, he runs a virtual manipulative on the board to explore how changing the slope and y-intercept of an equation of a line changes the graph. Students and Mr. Rodriguez propose possible changes to the equation and the effect it will have on the graph. After the exploration, Mr. Rodriguez continues with notes on the lesson. If needed, he goes back to the VM to address a student question. At the end of the lesson, Mr. Rodriguez projects the online student textbook using the SMART board and makes the VM available to students who want to do further explorations together.

Ms. Kahn has designed an activity for students to work in groups investigating properties of rational functions (i.e., asymptotes, holes, and roots). Parth, Melissa, and Andi are using a virtual manipulative to compare how changes in the equations affect the resulting graphs of the functions. After comparing their equations, tables, and graphs of the functions with the VM, the threesome develop a hypothesis relating equations to specific properties they see in their graphs. They then modify their given equations and graph their functions to test their hypothesis.

The principal at Memorial Middle School has recently purchased classroom sets for graphing calculators for each mathematics classroom. Mr. Patel, one of the teachers, is excited to use the calculators in his classroom and to finally integrate technology into his classroom. He assigns a calculator to each student and allows students to use the calculators whenever they feel necessary during the class period. In addition, Mr. Patel has a few personal graphing calculators that he allows students to check out for one night. Eventually, Mr. Patel would like to use the calculators more, but for now is satisfied with his technology integration by making the calculators available to the students.

At the end of a geometry unit on surface areas of polyhedrons, teachers at Basset High School introduce students to JavaGami. JavaGami "is a software environment for designing paper models using polyhedra and user-created variants of polyhedra" (<http://goo.gl/2uufn>). Using JavaGami, students must design and construct an original polyhedra. As part of this project, they must also find the surface area of their polyhedra.

Prompts:

- How well you think _____ effectively integrates the (*technology tool*) within her/his instruction?
- What would you say to support your position if a friend of yours disagreed with you?
- What are the strengths in how _____ is using (*the technology tool*)? Why?
- What are the weaknesses in how _____ is using (*the technology tool*)? Why?

Final Interview Questions (Focus Teachers Only)

The final interview will be semi-structured; therefore, questions may evolve based on teacher responses and knowledge gained through their participation in the professional development. **Teachers will be told that they should not reveal any highly personal, sensitive, or identifiable information about themselves or others.** The intent of the final interview is to gather information regarding teachers’: experiences in the PD (e.g., what did (not) work well for them), beliefs, instructional practices, process for finding, modifying, and implementing VM activities in their classrooms.

The questions given below are representative of the themes that will be investigated during the interview. Some questions may be removed or added depending on teachers’ responses and their participation in the PD. The initial questions are the same to help track changes that may have occurred due to teachers’ participation in the PD.

Stimulus & backup	Expected Themes	Follow-ups & Probes	Stimuli for extended narrations	Comments
<p>(RQ1) Please describe your general instructional style.</p> <p>Please describe a typical day's lesson.</p> <p>Tell me about a successful lesson that you taught? What made it successful?</p>	<p>Teacher directed</p> <p>Mixed</p> <p>Student centered</p> <p>Instructional</p>	<p>Do you ever do/use classroom demonstrations?</p> <p>What are the student roles in your classroom?</p>	<p>If so, when? Why?</p>	

<p>Tell me about a lesson that did not go as well as you had planned. What would you change if you were to teach that lesson again?</p>	<p>Practices</p> <p>Types of Technology</p>	<p>How often do you have inquiry-based activities in your lesson?</p> <p>Is there anything you really want to try in your lessons that you don't have the time to learn, plan, or don't know how to do? (RQ2)</p> <p>Do you have any desire to integrate technology into your lessons? Can you give me an example of when you used technology within your instruction?</p>	<p>Can you give me an example of inquiry-based activities?</p> <p>Tell me about a successful lesson that you taught? What made it successful?</p> <p>Tell me about a lesson that did not go as well as you had planned. What would you change if you were to teach that lesson again?</p>	
<p>(Background) How do you describe instructional technology?</p> <p>Can you give examples of things you consider to be technology and things that you do not consider to be technology?</p>	<p>Specific tool</p> <p>Frustration/emotion</p>	<p>Please describe your ideal use of technology?</p>	<p>Are there things that keep you from using technology? (RQ2)</p>	
<p>(RQ2) Can you tell me about your typical lesson planning process?</p> <p>When you are developing a new lesson, what do you think about?</p>	<p>Collaborate</p> <p>Individual</p> <p>Pacing Guide</p> <p>Pre-made lessons</p>	<p>How do you decide if you are going to keep an old lesson plan or develop a new one? Or tweak an existing one?</p> <p>Does your school provide common planning?</p> <p>How comfortable do you feel modifying lessons?</p> <p>Do you intentionally look for opportunities to integrate VMs?</p>	<p>Do you think about specific students when designing your lessons?</p> <p>Do you plan with the same person/people?</p> <p>Tell me about the last time that you modified a lesson.</p> <p>When? How? Why?</p>	

(RQ1, RQ2) What kind of technology do you currently use in your classroom?	Frustration Types of technology	How accessible is technology in your school? What type of support do you need to integrate technology? (RQ2)	How have you used VMs in your instruction? Do you get support from your school in terms of using technology? (RQ2) Did the PD support you and your integration efforts? (RQ1, RQ2) Please give an example of how the PD supported your efforts.	
(RQ1, RQ2) How do you think technology should be used within mathematics instruction?	Learning Mathematics Specific tools	What would you say to a friend that disagreed with you? Do you think certain subjects lend themselves more easily to technology than others?	Why? Do you think your curriculum lends itself to technology? What about VMs specifically?	
(RQ1, RQ2) Please describe your experience participating in the professional development opportunity.	What worked What did not work Specific examples relevant to instruction	What could have been done to better meet your needs? Have you started implementing VM activities into your instruction? How has your instruction changed over the course of the year regarding technology?	How? Why? Can you given an example of how your instruction has changed? Why do you think your instruction has changed?	

Present teachers with (some) of the instructional scenarios below (or similar scenarios):

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Prompts:

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What would you say to support your position if a friend of yours disagreed with you?

What are the strengths in how _____ is using (*the technology tool*)? Why?

What are the weaknesses in how _____ is using (*the technology tool*)? Why?

Part 2: Supporting Teachers to Teach WITH Virtual Manipulatives

Introduction

During a recent professional development opportunity, teachers were engaged in a conversation about why they use technology. Some of the responses included: *It's cool. Keep students engaged. My principal says I have to. Students (and parents) expect it. I need a break sometimes. Differentiate instruction. Make the math "come alive." I want to prepare my students for life beyond the classroom. Save time. My students don't have basic skills.* Although some of these responses focus on powerful uses of technology related to improving instruction and student learning (e.g., differentiating instruction), other responses (e.g., save time) may be focusing on the more mundane uses of technology (Wilson, 2008).

The role of technology in the classroom has received much attention during the past several decades, in the way it can: transform the classroom, increase student interest level and motivation (Roberts & Stephens, 1999), support complex thinking skills and cognitive skill development (Moersch, 1999, as cited in Rakes, Fields, & Cox, 2006), and provide the means for students to construct personal meaning, as well as learn from peers and experts (Judson, 2006). However, some (e.g., Hixon & Buckenmeyer, 2009) claim that technology has minimally influenced the education process stating that "technology integration is not synonymous with technology access, or even technology use" (p. 132). Additionally, many mathematics teachers find it challenging to effectively integrate technology into their instructional practices (Dunham & Hennessy, 2008), a critical issue in mathematics education. Therefore, the question becomes, *what support is necessary for all teachers to effectively use technology* (Blume & Heid, 2008)?

Districts have spent significant amounts of money (Borko, 2004; Hixon & Buckenmyer, 2010; Hokanson & Hooper, 2011) responding to calls from legislation (e.g., No Child Left Behind (Watson, 2006)) and professional organizations (e.g., National Council of Teachers of Mathematics [NCTM], 2008) encouraging the use of technology to enhance student learning. For

example, one local district posts the following statement on the webpage for their Information and Technology Plan:

Technology will empower students, staff and families to engage in continuous learning through discovery, collaboration and creation. Accessible, flexible and differentiated technology for every student at every school with ongoing professional learning for staff and support for families will strengthen high-quality instruction and personalize learning to allow every student to graduate college, career and community ready (MMSD, 2014).

Yet, there is a gap between technology that is available for teachers and their use of technology (Hixon & Buckenmyer, 2010; Kopcha, 2012). Studies have found that it is still uncommon to find teachers using technology to increase higher order thinking and create student centered learning environments (e.g., Ertmer & Ottenbreit-Leftwich, 2010). Teachers often use technology in ways that support existing and often traditional teaching practices (Palak & Walls, 2009), or may use technology primarily for classroom management tasks (Gray, Thomas, & Lewis, 2010; Littrel, Zagumny, & Zagumny, 2005). Although studies have shown that technology can be used to promote student centered learning environments, why are these student-centered classrooms the exception and not the norm?

Over the past few decades, the role of technology in the classroom has collected much interest in the manner in which it supports students in taking a much greater role in their learning. The 1989 release of the NCTM Standards suggested that computer software be used to help students “become actively involved in the learning process through exploration and testing of conjectures” (Roberts & Stephens, 1999, p. 23). Technology integration was seen as a means to support student learning by providing opportunities that would enhance students’ reasoning abilities and problem solving skills. Through the years, technology has gained greater acceptance in mathematics education as it “influences the mathematics that is taught and enhances students’ learning” (NCTM 2000, p. 24). In fact, Knuth and Hartmann (2005) claim that technology can be

used to provide opportunities for students to engage in “conceptual conversations¹⁰” that foster students’ understandings and intuitions.

Previous research primarily focused on external or environmental factors (i.e. access to technology, training, and issues related to equipment: technical problems and malfunctions (Mueller et al., 2008)) that contributed negatively to teachers’ technology integration efforts. However, since many of those factors are no longer considered significant barriers (Ertmer et al., 2012), recent research (e.g., Ertmer, Ottenbreit-Leftwich, Sadik, Sendurur, & Sendurur, 2012; Ottenbreit-Leftwich, Glazewski, Newby, & Ertmer, 2010; Teo, Chai, Hung, & Lee, 2008), has begun to focus on the individual differences in attitudes, beliefs, and skills that affect the integration of technology in the classroom. The shift in focus has come about as teachers are seen as having a greater role in the learning process (Cross, 2009; Wilson, 2008) and as the key decision makers in how technology is integrated into the classroom (Heid & Blume, 2008b; Zbiek & Hollebrands, 2008). For example, teachers often rely on their own experiences with technology to decide what supports students need to use the technology (e.g., whether they need step-by-step instructions) (Zbiek & Hollebrands, 2008). Additionally, Zbiek and Hollebrands also note that teachers’ perceptions of student ability influence how they engage students with technology. Therefore, there is a need to further investigate teachers’ role in how technology is integrated (Albion, Tondeur, Forkosh-Baruch, & Peeraer, 2015).

The following article explores one technology tool (i.e., virtual manipulatives (VMs)) that teachers use in their classrooms and current supports that exist for supporting teachers’ efforts to use VMs effectively (i.e., to teach *with* as opposed to *near* technology). I begin by

¹⁰ Knuth and Hartmann (2005) define “conceptual conversations” as conversations that have “a diminished emphasis on technique and procedures and an increased emphasis on relationships, images, and explanations (Thompson, 1996)” (p. 151).

defining key terms (i.e., TPACK, technology, teaching *with* technology, virtual manipulatives, and professional development) before reviewing literature related to the importance of technology by discussing how technology can transform instruction as well as how technology has been used to respond to various calls for reform in mathematics instruction. Two main supports for supporting teachers' efforts to teach with technology (i.e., guides and professional development(PD)) are then discussed. Building from the literature about instructional task frameworks and components of effective PD, a task analysis framework and PD model are suggested for supporting teachers' efforts to teach *with* VM activities¹¹ (i.e., to use VMs to promote opportunities for students to communicate, reflect, as well as use and connect mathematical representations). The PD model relies on the central tenets of activity theory and aims to account for teachers' practices related to how they use technology being mediated by several factors (e.g., student needs, curriculum, other teachers, available tools, school/district initiatives, etc.). The task analysis framework and PD model aim address the question: *How can secondary mathematics teachers be supported in implementing technology-based instructional activities that both challenge and support student learning?*

Conceptual Framework

Technological, Pedagogical, and Content Knowledge (TPACK)

Building on Shulman's (1987) pedagogical content knowledge (PCK), Koehler and Mishra (2005) introduced technological pedagogical content knowledge (now known as Technology, Pedagogy, and Content Knowledge [TPACK]) as a knowledge framework describing the knowledge base for teachers to effectively teach with technology (see Figure 1).

¹¹ A VM activity refers to a VM and all accompanying instructional materials (e.g., prompts and directions). I use the word activity because the exploration could include more than one task (Sinclair, 2003) focused on investigating a particular concept (e.g., through alternative exploration paths), but it may include only one task.

They claim that TPACK is an evolving form of knowledge that transcends knowledge of technology, pedagogy, and content.

Instead, TPACK is the basis of effective teaching with technology, requiring an understanding of the representation of concepts using technologies; pedagogical techniques that use technologies in constructive ways to teach content; knowledge of what makes concepts difficult or easy to learn and how technology can help redress some of the problems that students face; knowledge of students' prior knowledge and theories of epistemology; and knowledge of how technologies can be used to build on existing knowledge to develop new epistemologies or strengthen old ones (Koehler & Mishra, 2009, p. 66)

The authors go on to say that to teach successfully with IT, teachers must continually create, maintain, and re-establish the evolving equilibrium among all components of the TPACK framework.

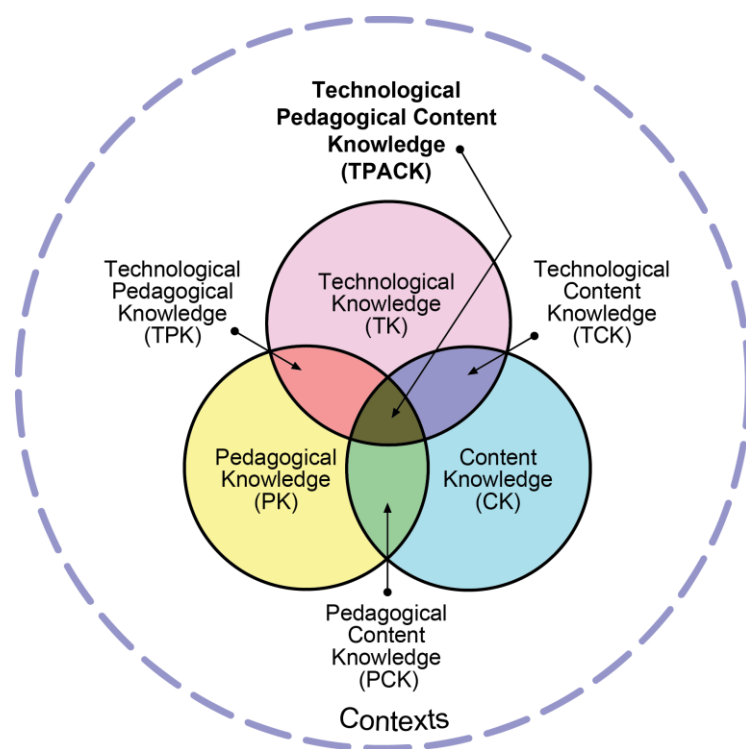


Figure 1. Graphical representation of the TPACK framework and its knowledge components. Reproduced by permission of the publisher, © 2012 by tpack.org.

However, this framework (i.e., the TPACK framework) is not without its shortcomings (Walker et al., 2012). First of all, some argue (e.g., Archambault & Barnett, 2010; Graham,

2011) that while the framework is useful from an organizational standpoint, the seven knowledge constructs may be impossible to actually separate and measure; thus calling into question whether the constructs exist in practice. Furthermore, using factor analysis, Archambault and Barnett (2010) claim that the only knowledge domain that does distinguish itself from the others is the technology domain (as opposed to the technology, pedagogy, and content knowledge domains). Graham (2011) claims that while numerous studies use TPACK for their theoretical framing, little theoretical building of the model has actually occurred. Additionally, TPACK has been criticized for building on Shulman's PCK model (due to the lack of clarity in Shulman's model) and having imprecise definitions of the seven knowledge components (Graham). Although Graham suggests that the TPACK framework has the potential to provide theoretical guidance in approaching technology training opportunities (e.g., PD opportunities), recent studies (e.g., Mouza, 2009; Walker et al., 2012) focus on specific knowledge components (i.e., CK, PK, and PCK) recommendations from literature (e.g., Lawless & Pellegrino, 2007) and available means to investigate the knowledge components individually.

The main types of knowledge discussed in literature (and often the focus of various PD opportunities are i) content knowledge (CK), ii) pedagogical knowledge (PK), and iii) pedagogical content knowledge (PCK). Content knowledge refers to teachers' knowledge related to the subject matter (in this case mathematics) that is to be learned or taught. Drawing from Shulman (1986), Koehler and Mishra (2009) note that this knowledge includes knowledge of concepts, theories, what counts as evidence, established approaches and practices towards developing the knowledge, etc. Pedagogical knowledge refers to teachers' knowledge about the methods of teaching, including understanding how students learn, lesson planning, classroom management, etc. Finally, pedagogical content knowledge refers to the knowledge related to

teaching the specific content. It includes knowledge in promoting student learning through connections between pedagogy, curriculum, and assessment.

Technology

In the context of this article, technology refers to tools that are used to enhance instruction, produce positive learning outcomes, and promote effective methods of instruction. Although this definition of technology includes paper and pencil as well as other physical manipulative tools (e.g., algebra tiles, unifix cubes, etc.), this study focuses on *digital technologies* (also known as *information and communication technologies* (ICT)), such as computers, software, and other devices of interactive display (e.g., graphing calculators and probes, interactive whiteboards, tablets and slates, virtual manipulatives, dynamic geometry software, etc.). Specifically, this study focuses on virtual manipulatives and tasks. Technology that is used for record keeping (i.e., taking attendance on the computer) or communication only (i.e., email) is not considered technology.

Teaching *with* as opposed to *near* technology. Teachers use technology in multiple ways and for multiple purposes, some uses are more powerful than others (Wilson, 2008). In this study, I adapt Leatham (2002)'s description of a teacher *teaching with technology* to refer to teachers using technology to promote opportunities for students to build conceptual understanding through communication and reflection (Hiebert et al., 1997), as well as using and connecting mathematical representations (NCTM, 2014). *Teaching near technology* then refers to teachers using technology in ways that do not promote opportunities for students to build conceptual understanding. Effective use of technology refers to teachers teaching *with* technology. However, adapting the description of effectiveness of teaching from *Adding it Up: Helping Children Learn Mathematics* (National Research Council, 2001), effectiveness of technology is dependent on the enactment of technology, meaning the “mutual and

interdependent interaction” (p. 9) of technology with the teacher, the students, and the mathematical content.

Virtual manipulatives. Virtual manipulatives (VMs) are online manipulatives originally modeled after physical manipulatives (e.g., algebra tiles or based ten blocks), however some current VMs do not have physical counterparts (e.g., graphical VMs) (Moyer-Packenham & Westenskow, 2013). More specifically, Moyer-Packenham and colleagues define a VM as “an interactive web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge” (p. 373, Moyer, Bolyard, & Spikell 2002). By focusing on the interactive capability of a VM, the definition given excludes sites whereby a computer gives an answer in symbolic or visual form as the result of a user pointing or clicking. Initially, many VMs were Java-based applets; however, as tools have transformed, additional platforms are now available (e.g., HTML5, Shockwave, GeoGebra, Flash, apps for tablets, etc.). I also consider pre-constructed dynamic geometry objects (e.g., dynamic objects constructed with GeoGebra and published to GeoGebra tube) as VMs due to the interactive nature of these dynamic objects. Examples of various collections include: National Library of Virtual Manipulatives (NLVM), NCTM Illuminations, Shodor Interactivate, Flash & Math, ExploreLearning, PhET Interactive Simulations, etc. Many VMs are free and can be accessed anywhere Internet is available.

Professional Development

Professional development opportunities take on a variety of forms and occur in a myriad of contexts (e.g., Borko, 2004; Desimone, 2009). These opportunities range from brief hallway conversations with a fellow teacher to more formal professional development workshops or courses. In this article, I adopt Guskey (2000)’s definition of professional development as “those

processes and activities designed to enhance the professional knowledge, skills, and attitudes of educators so that they might, in turn, improve the learning of students” (p. 16, as cited in Driskell et al., 2016). This definition was chosen due to encompassing the myriad of professional development experiences that occur supporting teachers’ efforts to improve student learning.

Importance of Technology

The following section focuses on the need for integrating technology into teachers’ instructional practices—a need that reflects both technology’s potential to transform instruction and student learning as well as calls of reform initiatives. For example, technology can change expectations regarding how students engage during mathematical lessons as well as the type of understanding that is promoted (i.e., procedural vs. conceptual). Additionally, technology has contributed to the need for curricula reform, emphasized standards and mathematical practices, as well as expectations for mathematics teacher support and preparation.

Transforming Instruction

Technology can transform mathematics instruction and student learning of mathematics (see Dunham & Hennessy, 2008 for a summary). For example, technology can provide opportunities for students to engage in conceptual conversations which foster students’ understandings and intuitions (Knuth & Hartmann, 2005) and provide the means for deeper student reflections (Heid & Blume, 2008b). Additionally, technology may provide the means for students to construct personal meaning, as well as learn from each other and experts (Judson, 2006). When technology is used effectively, it can “reinforce higher cognitive skill development and complex thinking skills such as problem solving, reasoning, decision making, and scientific inquiry” (Moersch, 1999, as cited in Rakes et al., 2006, p. 411). Furthermore, Dunham and

Hennessey note that recent studies suggest that technology can be used to address issues of inequity amongst students and make higher levels of mathematics more accessible to all students.

Together, technology and students form a powerful functional cognitive system. Similar to a slide rule that has concepts *built in*, technology can be used to reduce the cognitive demand of a task by integrating easier concepts into the tool (Heid & Blume, 2008b; Huthchins, 1995). More attention can then be given to developing conceptual understanding, rather than focusing on the more tedious components of a task (e.g. calculations). When used effectively, technology is not an amplifier of cognition, but instead transforms the functional organization and hence what students actually do in an activity (Pea, 1993). Consequently, technology allows concepts to be more deeply explored to further students' conceptual understanding.

When teachers teach *with* technology, classrooms can be transformed “into dynamic centers of purposeful and experiential learning that intuitively move students from awareness to authentic action” (Moersch, 1998 as cited in Rakes et al. 2006, p. 411). Effectively designed and implemented technology allows a domain's tools to be used in authentic activity. Brown, Collins, and Duguid (1989, p. 34) claim that in order for students to really learn subjects, as opposed to just learning about them, they must be “exposed to the use of the domain's conceptual tools in authentic activity.” Technology as a tool can enhance instruction; it is not something that replaces instruction or mathematics itself. Instead, it is an example of a tool that is an *instrument of access* to the practices, activities, and knowledge of a particular group (Lave & Wenger, 1991). Like any tool, instructional enhancement occurs not due to the properties of the technology itself but how the technology is used within the classroom by the teacher (and students) and the knowledge and meanings that students develop through their interaction with the IT (Meira, 1998).

Brown and Campione (1994) suggest that academic activities should be contextualized and situated within authentic practices. Technology can be used to ground curricula in the experiences from which it has been abstracted. In doing so, motivation for learning is inherently developed rather than externally forced upon the student. As Dewey (1902) stated,

When the subject-matter has been psychologized, that is viewed as an outgrowth of present tendencies and activities, it is easy to locate in the present some obstacle, intellectual, practical, or ethical, which can be handled more adequate if the truth in question be mastered. This need supplies the motive for learning. An end which is the child's own carries him on to possess the means of its accomplishment. But when the material is directly supplied in the form of a lesson to be learned as a lesson, the connecting links of need and aim are conspicuous for the absence (pg. 203).

Furthermore, Romberg (1998) claims that in order for students to be able to use mathematics to solve real-world problems, and reflect on the use of mathematics in doing so; students must actually study the applications of mathematics through mathematical modeling. Technology can assist students with mathematical modeling through the development of visual representations, comparing predictions, and exploring consequences (CCSSI, 2010). Additionally, technology can support the development of a greater variety of representations and support students in more flexibly moving between the representations (Koehler & Mishra, 2009).

Responses to Calls for Reform

Besides the benefits to student learning already mentioned, there are also reasons related to calls for reform to consider when choosing why to integrate technology. First of all, due to having certain concepts or *skills* built in, technology transforms the skills and concepts that need to be concentrated on in the mathematics curricula from focusing on calculations to modeling, making predictions, reflecting on the power and uses of mathematics, etc. (Romberg, 1998). Tom Romberg, while addressing writers in the first writing session of what lead to the NCTM's 1989 *Principles and Standards of School Mathematics*, claimed that technology has changed the focus of mathematics from that of procedural knowledge, to students needing to understand when and

why procedures are needed and used (McLeod, Stake, Schappelle, Mellissinos, & Gierl, 1996). Romberg (1992) argued that computers and calculators profoundly changed the world of mathematics and that the emphasis in school mathematics “must be on the social nature of mathematics” (p. 772). Whereby classrooms “become discourse communities where conjectures are made, arguments are presented, and strategies discussed,” (Romberg, 1998, p. 12). In addition, technology should be used to provide opportunities for group problem solving (Romberg, 1992).

Additionally, the U.S. Department of Education claims that effective technology use can improve education and was consequently a major part of No Child Left Behind (NCLB) initiatives (Watson, 2006). Technology is also one of the five Essential Elements supporting NCTM’s first Guiding Principle (i.e., Teaching and Learning) for School Mathematics (NCTM, 2014). Echoing their 2008 position statement regarding *The Role of Technology in the Teaching and Learning of Mathematics* (NCTM, 2008), NCTM describes technology as an essential resource “to help students learn and make sense of mathematical ideas, reason mathematically, and communicate their mathematics” (NCTM, 2014, p. 4). The Common Core State Standards for Mathematics (CCSSM) (CCSSI, 2010) describe using technology in one of the eight Standards for Mathematical Practice (i.e., Use appropriate tools strategically). It states that mathematically proficient students will use technology in mathematical modeling and are able to use technology to strengthen their understanding of various concepts (CCSSI, 2010). Finally, the Association of Mathematics Teacher Educators (AMTE) state that “[w]ell-prepared beginning teachers of mathematics are proficient with tools and technology designed to

support mathematical reasoning and sense making, both in doing mathematics themselves and in supporting student learning of mathematics” (p. 11, AMTE, 2017).

Summary

In summary, technology can be used to make mathematics more accessible to all students. Technology not only increases students’ interest level and participation, but when used effectively, can support complex thinking skills and cognitive skill development. Furthermore, technology may also provide the means for students to have conceptual conversations, construct personal meaning, provide opportunities for deeper student reflections, as well as learn from peers and experts. Furthermore, there are also reason related to reform initiatives that motivate the integration of technology. Effectively using technology in the classroom was a focus of NCLB. Moreover, NCTM states that technology is vital for today’s students learning mathematics. They have been calling for the use of technology to make mathematics more accessible for all students for over twenty-five years. In addition, CCSSM state that students should use technology in mathematical modelling and for strengthening their understanding of various concepts. Recent standards for preparing teachers of mathematics also state that mathematics teachers are proficient with technology and how it can be used to support student learning.

Guides Supporting Teachers’ Use of Technology-based Tasks

As mentioned previously, the enhancement to student learning that can occur through effective technology use does not occur due to features of the technology alone but how students, teachers, curricula, etc. interact and the meanings that are developed as a result of such interactions. Therefore, teaching mathematics *with* technology requires a deeper knowledge of mathematics and technology, as well as how teaching with technology transforms mathematics

instruction (Wilson, 2008). In an effort to support teachers in teaching *with* technology, several guides have been developed; in some cases the guides are in the form of key components and/or questions for teachers to consider as they critique or develop an instructional task (e.g., Sinclair, 2003); while in other cases the guides are in the form of frameworks aiming to support the same purpose (e.g., Sherman & Cayton, 2015; Trocki, 2014).

Pre-constructed Dynamic Geometry Sketches and Accompanying Materials

Drawing from her investigation of the limitations and benefits of using pre-constructed, web-based, dynamic geometry sketches, Sinclair (2003) reports on her findings regarding the relationship between the design and use of materials accompanying the sketches and students' exploration process. Building from the literature on how to support students' exploration process using dynamic geometry software (e.g., Laborde, 2001) and strategies for promoting student understanding through exploration (e.g., Towers, 1999 as cited in Sinclair), Sinclair identified components of a task that support student learning through exploration. Specifically, Sinclair suggests that a sketch must provide the opportunity for students to investigate the following questions and instructions. In general,

1. When a question aims to focus student attention, the sketch must provide the visual stimulus. It must *draw attention* through colour, motion, and markings.
2. When a statement prompts action, such as asking students to drag, observe or deduce, the sketch must contain the necessary provisions. It must *provide affordances* so that the student can take the required steps.
3. Questions that invite exploration are open-ended. In order to explore uncharted territory, the student requires a sketch that allows options. Thus, when a question invites exploration, the sketch must *provide alternate paths*.
4. A question can *surprise* – which may lead to further exploration; however, the teacher is not necessarily there to correct any misinterpretation. Thus, the sketch must *support experimentation* to unmask the confusion. It must be flexible enough to help students examine cases, yet constrained enough to prevent frustration.
5. Questions that check understanding are important parts of any learning situation. In the study tasks, the checking involved students looking together for the answer. Although peer-interactions were not discussed in this article,

study results showed that the sketch aided this process by *providing a shared image* for students to consider and discuss (p. 312).

Furthermore, an underlying theme to the sketch and prompts must be an aim to support students as they learn how to use changes in the sketch to investigate mathematical properties and expand their ability to interpret the visual representation.

Dynamic Geometry Task Analysis Framework

Building from work done by Sinclair (2003) as well as Smith and Stein (1998), Trocki (2014, 2015) developed a framework (see Table 1) for task analysis of dynamic geometry tasks. Trocki's framework addresses a task's allowance for mathematical depth as well as for types of technological actions. This framework was initially used by researchers as they coded tasks developed by teachers as they were beginning to use Geometer's Sketchpad (GSP) in their geometry classrooms (Trocki, 2014). The framework was later used by pre-service and in-service teachers as they wrote and revised tasks in GSP (Trocki, 2015). Teachers reported finding the framework helpful as something to reference regarding the mathematical practices to incorporate in a task as well as the role in having students explain and the types of interaction while exploring.

Table 1. *Dynamic Geometry Task Analysis Framework*

Allowance for Mathematical Depth	
Levels	Hierarchical Levels and Descriptions
N/A	Prompt requires a technology task with no focus on mathematics.
0	Sketch does not have mathematical fidelity required to respond to prompt.
1	Prompt requires student to recall a math fact, rule, formula, or definition.
2	Prompt requires student to report information from the construction. The student is not expected to provide an explanation.
3	Prompt requires student to consider the mathematical concepts, processes, or relationship in the current sketch.
4	Prompt requires student to explain the mathematical concepts, processes, or relationships in the current sketch.
5	Prompt requires student to go beyond the current construction and generalize mathematical concepts, processes, or relationships.
Types of Technological Action	
Affordances	Descriptions

N/A	Prompt requires no drawing, construction, measurement, or manipulation of current sketch.
A	Prompt requires drawing within the current sketch.
B	Prompt requires measurement within current sketch.
C	Prompt requires construction within current sketch.
D	Prompt requires dragging or use of other dynamic aspects of the sketch.
E	Prompt requires creation/consideration of multiple examples from which one can generalize.
F	Prompt requires manipulation of the sketch that allows for recognition of emergent invariant relationship(s) or patterns(s) among it within geometrical object(s).
G	Prompt requires manipulation of the sketch that may surprise one exploring the relationships represent or cause one to refine thinking based on themes within the surprise (adapted from Sinclair (2003), p. 312).

Note. Adapted from Trocki (2015).

Interactive Geometry Software (IGS) Framework

More recently, Sherman and Clayton have collaborated to develop a framework (see Table 2) guiding teachers' efforts to design and implement technology tasks in secondary classrooms (Cayton, Sherman, & Funsch, 2017; Sherman & Cayton, 2015). This framework stems from their independent work as well as builds from the design principles of dynamic geometry sketches and accompanying materials suggested by Sinclair (2003) and the work of Pea (1993) regarding how technology can be used as an amplifier or reorganizer to support student learning. They suggest that the IGS framework be used as a tool for investigating how the use of technology supports instructional goals and possible suggestions for potential task revisions aimed at obtaining specific goals more effectively.

Table 2. *Interactive Geometry Software (IGS) Framework*

Goals	Question	Technology Used As	
		Amplifier: Students could achieve the same goal without the technology.	Reorganizer: The mathematical goal of the task would be difficult to achieve without IGS.
Make mathematically meaningful observations; look for invariant relationships	Do the sketch and prompts use the dynamic affordances of the IGS in a way that would be difficult or impossible to replicate without it?	Students create multiple static examples, either by construction or dragging, and reason from those static examples. For example, students are prompted to make observations or generalizations based on a table or static	The sketch allows for continuous dragging, and students are guided to examine measurements or relationships dynamically. Students are required to make or explain observations or generalizations dynamically in terms of the sketch.

		measurements without reference to the sketch.	
Mathematical exploration; use appropriate tools strategically	How does technology support mathematical exploration?	Sketch and prompts guide students to investigate the same example or set of examples to explore mathematical connections or invariances. Freedom with respect to dragging does not provide alternative paths if students are all investigating the same example.	Sketch and prompts allow students to explore their individual observations of mathematical concepts, connections, or invariances within the sketch. The sketch supports students' mathematical exploration by providing alternate paths.
Make and test conjectures; modify thinking; foster curiosity	Does the sketch provide feedback? Do the prompts encourage or require students to use feedback?	Sketch is limited by restrictive construction or does not provide feedback to allow students to explore their conjectures. Prompts do not explicitly guide students to test conjectures.	Sketch provides feedback or allows students to test and refine conjectures. Prompts explicitly guide students to use the sketch to test conjectures.

Note. Adapted from Sherman and Cayton (2015).

These guides aim to support teachers' efforts to teach *with* technology as opposed to teaching *near* technology. As tools, they provide teachers with a reference for critiquing technology based tasks, specifically tasks involving dynamic geometry software, as well as guides as teachers revise tasks to better support their instructional goals and student learning. These guides can support teachers' efforts in using technology to transform their instruction and the mathematics involved.

Supporting Teachers' Use of Virtual Manipulatives

Virtual manipulatives are not new, in fact studies of VMs have been showing up in the literature for almost 30 years (e.g., see Moyer-Packenham et al., 2014). Additionally, studies have advocated for their use in mathematics classrooms for quite some time (e.g., Moyer et al., 2002). Presented in this next section is a discussion about how VMs can be used to support student understanding. Due to the lack of resources aimed at supporting teachers' efforts to critique and modify VM activities, a task analysis framework is suggested. This framework builds from the guides presented in the previous section.

Why Focus on Virtual Manipulatives?

Due to the visual nature and the ability to be manipulated (e.g., see Reimer & Moyer, 2005; Suh, Moyer, & Heo, 2005), several affordances related to student learning and use of VMs have been highlighted in studies. For example, well-designed and implemented VMs increase the diversity of problems that students are able to think about and solve (Goldenberg, 2000).

Additionally, VMs can help students become more aware of mathematical concepts and support the development of abstract representations (Sarama & Clements, 2000). VMs can also be used as a tool for supporting teachers' efforts for differentiating instruction based on student needs (e.g., Shin et al., 2017; Bouck, Flanagan, & Bouck, 2015). Furthermore, Moyer-Packenham and colleagues (2014) claim that VMs can provide equal access for students to learn mathematics by lessening effects of students' socioeconomic status or English Language Learner status as predictors of mathematics achievement. Additionally, Bouck and colleagues suggest that students with learning disabilities benefit from the use of VMs in relation to performance (i.e., the number of correctly solved mathematics problems), their confidence, and potentially their development of conceptual understanding.

VMs have instructional benefits as well. For example, many VMs provide opportunities for students to receive immediate feedback (e.g., Edwards Johnson, Campert, Gaber, & Zuidema, 2012) thus promoting student engagement as well as continued conjecturing. The unlimited access and supply of VMs can support teachers' efforts to encourage student investigations within the classroom and at home (Moyer et al., 2002). Finally, the prevalence of interactive whiteboards, increasing number of 1-1 classrooms¹², and widespread availability of VMs (many are free) are additional reasons why I focused on VMs.

¹² In 1-1 classrooms, each student has access to a technology device (e.g., computer, laptop, tablet, PDA, etc.).

Despite the benefits to student learning and the ease in access to virtual manipulatives, very few resources exist for supporting teachers' efforts in finding, critiquing, and modifying/designing VM activities that promote students' engagement in a particular learning goal and are based on student needs. In an effort to capitalize on the benefits of using VMs and supporting teachers to teach *with* VMs, the following task analysis framework (see Table 3) is proposed as a tool to support teachers in critiquing and modifying/designing VM activities.

Task Analysis Framework

Table 3 presents a task analysis framework aimed at supporting teachers' efforts to critique, modify, and design VM activities. As defined previously, an activity includes the VM itself as well as the materials (i.e., questions or instructions, also known as *prompts*) that accompany the VM in printed form or onscreen (Sinclair, 2003). Although Sinclair uses the word *task*, I use *activity* to signify that an activity may have more than one task included in an investigation (e.g., exploring a concept through more than one approach). The framework builds from the guides already presented for supporting teachers' efforts to teach *with* dynamic geometry software. However, this framework focuses on VM activities as opposed to only tasks involving dynamic geometry software. Additionally, the framework is based on the assumption that students are interacting with a VM as opposed to constructing a dynamic geometry sketch. It is posited that due to its *generic* nature and focus on promoting students' development of conceptual understanding, the framework may be helpful in supporting teachers' efforts to critique, modify, and develop instructional tasks in general.

Table 3. *Task analysis framework*

Affordances	Descriptions
N/A	Task is primarily a technology task with no focus on mathematics.
N/A	Virtual manipulative does not have mathematical fidelity required to respond to the prompts.
A	Task prompts students to recall a mathematical fact, rule, formula, or definition.
B	Task prompts students to report information from the virtual manipulative or consider mathematical concepts, processes, or relationships in the current display. The student is not expected to provide an explanation.
C	Task provides opportunities for students to explain the mathematical concepts, processes, or relationships in the current display.
D	Task provides opportunities for students to make predictions and then test their predictions using the virtual manipulative.
E	Task provides opportunities for students to connect multiple representations of a mathematical concept (e.g., graphical, algebraic, and tabular representations of a relation).
F	Task provides opportunities to check students' understanding of mathematical concepts, processes, or relationships. Task may provide minimal feedback to the student based on specific errors.
G	Task provides opportunities for students to go beyond the current display by considering multiple examples to generalize mathematical concepts, processes, or relationships.
H	Task supports students' exploration through manipulation of the display that may surprise one exploring the relationships represented or cause one to refine thinking based on themes within the surprise (e.g., addressing a common student misconception).

A Professional Development Model

Developing a task analysis framework to support teachers' efforts in teaching *with* VMs does not go far enough in supporting teachers' efforts as teachers should also be supported as they learn about the tool and how to use the tool to transform their instruction. After all, teachers can find it challenging to integrate new instructional approaches, tools, and resources (Walker, et al., 2012). Therefore, integrating research-based components of effective PD, a PD model is proposed aimed at further supporting teachers' efforts to teach *with* VMs. This model draws from studies investigating features of PD opportunities that have had demonstrated impacts on student learning and accounts for the situated nature of teachers' practices regarding technology integration (i.e., using tenets of Activity Theory and acknowledging the influence of various mediating factors on teachers' practices).

Components of Effective Professional Development

Supporting teachers as they embrace educational reform, in this case to teach *with* as opposed to *near* technology, can take many forms (Borko, 2004; Desimone, 2009). PD is an essential feature of reform efforts aimed at improving student learning (Borko, 2004; Driskell et al., 2016; Desimone, 2009; Guskey, 2002; Lawless & Pellegrino, 2007; Martin et al., 2010; Mouza, 2009). Across studies, the following features of effective PD programs have been identified: active learning, coherence, collective participation, content focus, duration, and a reform approach. Although these features have been written about in the field for quite some time, empirical studies linking effects of PD programs and student learning are more recent. The publishing of Garet, Porter, Desimone, Birman, and Yoon (2001)'s empirical study investigating what makes PD effective was the first study of its kind. Since then, numerous studies have built from, corroborated, and added to the findings of this seminal study. Additional features (e.g.,

opportunities for promoting reflection and addressing teachers' beliefs) have been found by some to also contribute to effective PD programs. The following components are briefly defined and discussed in relation to a PD model aimed at supporting teachers' efforts to teach *with* VM activities.

Active learning. Active learning in a PD takes on different forms (Borko, 2004; Desimone, 2009; Desimone, Porter, Garet, Yoon, & Birman, 2002; Driskell et al., 2016; Garet et al. 2001; Martin et al., 2010; Mouza, 2009; Penuel, Fishman, Yamaguchi, & Gallagher, 2007; Wells, 2007). For example, active learning may be providing opportunities for teachers to directly interact with VMs (Albion et al., 2015) and tasks as well as supported opportunities for teachers to critique and revise VM activities. Penuel et al., (2007) suggest that PD might be more effective in supporting new practices when it provides time for discussion, instructional planning, and consideration of fundamental principles of curriculum, thus contributing to teachers' CK, PK, and PCK. Active learning also includes providing opportunities for teachers to observe expert teachers (or be observed themselves) implementing VM activities, followed by discussion and feedback; reviewing student work from a VM activity; as well as discussing potential modifications of a VM activity and how the modifications may influence student learning.

Coherence. Similar to active learning, coherence also has different foci (Desimone, 2009; et al., 2002; Driskell et al., 2016; Garet et al. 2001; Martin et al., 2010; Penuel et al., 2007). Desimone (2009) defines coherence to relate to the alignment of the goals of the PD with teachers' knowledge and beliefs as well as school, district, and state initiatives. Penuel et al. (2007) claim that coherence is important because teachers filter what they learn from PD and how they integrate what they have learned from PD through their own interpretative frames.

Collective participation. Another component of effective PD is collective participation of a group of teachers from the same grade, school, department (Desimone, 2009; et al., 2002; Driskell et al., 2016; Garet et al. 2001; Martin et al., 2010). It is posited that teachers implementing the same curriculum within a district may also come together to collectively participate in PD, thus forming a professional learning community (e.g., Borko, 2004). Collective participation provides opportunities for teachers to collaborate with each other, discuss how VM activities may be revised and implemented, reflect on current lessons, etc. all of which can be powerful supports for teacher learning (Borko, 2004; Wells, 2007).

Content focus. Desimone (2009) contends that content focus may be the most important feature of effective PD stating that there is much evidence (e.g., see Borko, 2004; Garet et al., 2001; Desimone et al. 2002) linking “activities that focus on subject matter content and how students learn that content with increases in teacher knowledge and skills, improvements in practice, and, to a more limited extent, increases in student achievement” (p. 184). Therefore, beyond focusing on VMs or mathematics content separately, teachers varied contexts (Mishra & Koehler, 2006) should be considered when designing a PD opportunity. That is to say, the PD should provide opportunities for teachers to work with VMs and activities specifically related to their curriculum thus promoting teachers in considering how an activity may be integrated and how their instruction and opportunities for student learning may be transformed. Additionally, Martin et al. (2010) suggest that providing opportunities for teachers to work on specific instructional units during the PD (where they can receive additional supports) may be more beneficial than focusing on using technology for its own sake. Focusing on teachers’ content may also support teachers’ development of CK, PK, as well as PCK.

Duration. Although Yoon et al. (2007, as cited in Martin et al., 2010) suggest a minimum of 14 hours, Desimone and colleagues (Desimone, 2009, 2011; Desimone et al., 2002) suggest a minimum of 20 hours for a PD. Additionally, the PD should be spread out rather than condensed (Driskell et al., 2016; Garet et al., 2001; Lawless & Pellegrino, 2007; Levin & Wadmany, 2006; Mouza, 2009; Wells, 2007) to provide opportunities for teachers to engage in active learning, reflect on their instruction and possible transformations, observe others and be observed, etc. Desimone (2009) states that a specific number has not been determined by research, but suggests that there is some indication of PD occurring over a semester or possibly an intense summer opportunity with follow-up occurring the next semester is more effective than shorter implementations. A prolonged duration of PD provides opportunities for teachers to see improvements in student learning, which is essential for changes in teachers' instructional practices to endure (Guskey, 2002).

Reform approach. Research also suggests that PD efforts that have a reform approach (i.e., focused on the context of the teachers involved, provide opportunities for teachers to be immersed in and reflective about the technology tool involved) as opposed to a traditional approach (i.e., short PD opportunity focused on a specific technology tool (often with no connection to teachers' context) and offered over a short span of time) are more effective (Desimone, 2009; et al., 2002; Driskell et al., 2016; Garet et al. 2001; Martin et al., 2010; Mouza, 2009; Wells, 2007). Wells (2007) cites others to claim that PD designed and implemented from the traditional approach "results in learners who are not well versed on the new innovation at the conclusion of the PD event, and therefore unlikely to realize any lasting change in their practice" (p. 104). Penuel et al. (2007) suggest that reform oriented PD opportunities have a focus on

proximity to practice, supporting teachers' efforts in translating practices from the PD to their classroom practices.

Additional features. Some studies advocate for additional features beyond the six identified by Garet et al. (2001). For example, Mouza (2009) and Webster-Wright (2009) suggests that effective PD supports teachers in continually reflecting on their practice and how they are integrating technology to enhance student learning. Walker et al. (2012) suggest that effective technology-related PD opportunities have follow-up opportunities that continue to support teachers' efforts in teaching *with* technology after the PD has ended. Mouza also states that to support teacher learning beyond the PD, PD opportunities need to "engage teachers in planning and implementing *rigorous* technology-enhanced activities in their classrooms. Enactment of new activities and resulting student outcomes can help teachers gather concrete evidence on the importance of technology and foster reconsideration of beliefs" (p. 1237).

Although PD may in itself influence teachers' beliefs towards technology, teachers' beliefs influence how the PD is perceived and teachers' willingness to implement what is learned in the PD (Richardson, 1996; Speer, 2008). Therefore, teachers' beliefs (about mathematics, instruction, and technology) should be a component of PD opportunities before changes in instructional practices will occur (Cross, 2009). Unfortunately, many PD opportunities focus on a singular approach to technology integration rather than addressing teachers' beliefs as well as their varied contexts of teaching and learning (Koehler & Mishra, 2009).

Drawing together features of effective PD opportunities that have been shown to enhance student learning, the following PD model is proposed to promote teachers in teaching *with* as opposed to *near* technology. This model acknowledges how teachers' practices are mediated by various components within the activity system (Engeström, 1987, 1999; Nardi, 1996). The PD

aims to support teachers' efforts to use VM activities to promote opportunities for students to develop conceptual understanding. The PD promotes the use of the task analysis framework described earlier to further support teachers' efforts to teach *with* VM activities.

Professional Development Model to Promote Teaching *with* Virtual Manipulative Activities

Professional development model. The following PD model (see Figure 1) draws from literature about teacher learning and various PD opportunities that highlight specific features of effective PD (i.e., components of PD opportunities demonstrated to be important in contributing to student learning). This model highlights three phases of the PD aimed at supporting an expansive learning cycle (Engeström, 1999) of teachers learning how to teach *with* VM activities (i.e., as they internalize how to find, critique and modify/design VM activities to be implemented in their classrooms). The model is for a 20+ hour PD opportunity occurring over four to six months. The model may be applied to a shorter span (e.g., an intensive summer institute) but would require continual support during the school year as teachers implement the VM activities from the PD. To promote collaboration and supporting teachers' instruction, teams of teachers from the same school/department (or minimally working together to implement the same curriculum within a district) should enroll in the program.

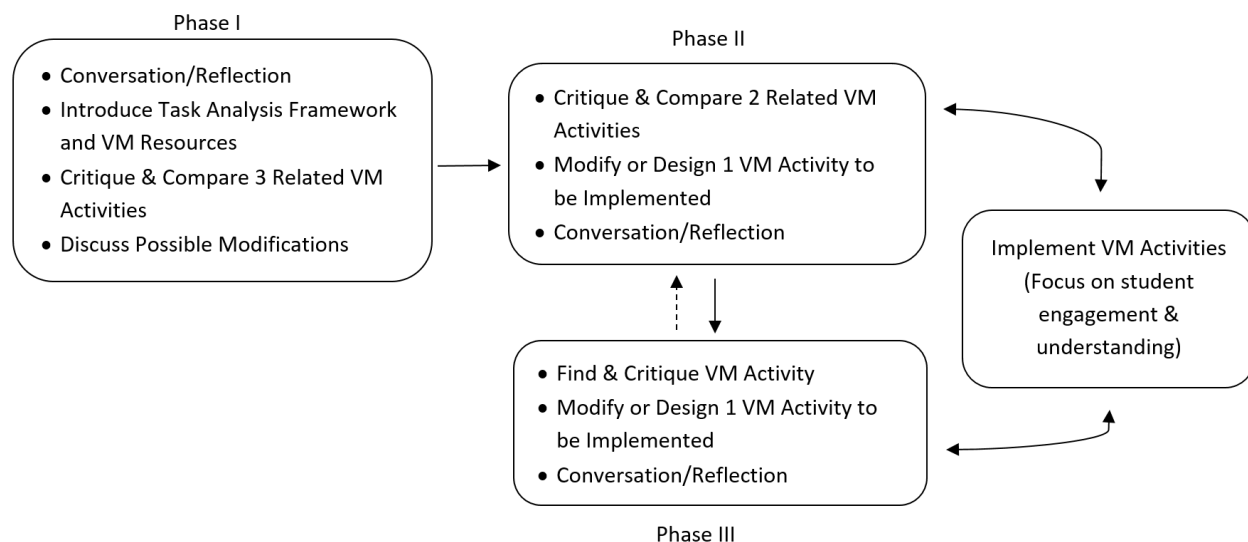


Figure 1. Professional development model promoting teaching *with* VM activities.

Phase I. The first phase of the model begins with teachers in the PD reflecting on the role of technology in the classroom and how they currently use technology in the classroom. This reflection may occur individually as well as through conversations with others in the PD (small groups or whole group). The task analysis framework (see Table 3) as a tool to support teachers as they critique and then modify/design VM activities and annotated list of VM resources are shared as teachers are introduced to VMs. Teachers then work through three related VM activities (i.e., activities that focus on the same content topic but may have different affordances) (e.g., Modeling and Solving Two-Step Equations from ExploreLearning, Algebra Tiles from NCTM Illuminations, and Solving Two-Step Equations Module from Michigan Virtual University). Teachers work with others to apply the task analysis framework and begin thinking about possible modifications. Guiding questions such as: *How might your students struggle in this exploration? How might your students benefit from engaging in this exploration?* and *Thinking about your learning goal, what is one modification you would make so that the exploration better fit the needs of your student? Why would this modification help your students engage in the learning goal?* may be helpful to guide teachers' reflections, conversations, and help them to connect the PD to their instructional goals. Phase I may take one to two PD sessions depending on the length of the session.

Phases II and III. Phases II and III are focused on teachers' instructional goals. In Phase II, to expose teachers to the variety of VMs and tasks, as well as support their learning in the expansive learning cycle, teachers are given two related VM activities to explore and critique based on their instructional goals. After critiquing the VM activities by applying the task analysis framework and justifying why affordances apply, teachers choose one VM activity to modify and

implement in their classrooms. If accompanying materials do not exist for a given VM, then teachers design appropriate materials to transform the VM into a VM activity. Teachers within a given team may find it helpful to consult teachers in other teams regarding possible modifications and how the modifications may help their students engage in the learning goal and promote the development of conceptual understanding. Guiding questions from Phase I are still used to support teachers in connecting the PD to their instruction. Examples of how other teachers have modified and implemented VM activities may be helpful to support teachers' learning. Additionally, talking with others about their implementations of previous activities (e.g., how students engaged in the VM activities from previous sessions, how student understanding was enhanced/constrained due to the activities, etc.) may be helpful for further connecting the PD to teachers' instructional practices as well as increasing teachers' PK, CK, and PCK. Phase II is based on individual teachers' needs, but will likely last three to five sessions.

When teachers are comfortable finding their own VMs and tasks with minimal support, they then move to Phase III. Note, it is possible (and likely) to have teachers in different phases during the same PD. Additionally, the dashed arrow between Phase III and II signifies that some teachers may find that they need more support before they are able to find their own VMs and tasks. The Tasks Analysis Framework and guiding questions mentioned previously are still used to support teachers' efforts to critique and modify/design VM activities. Additionally, teachers continue collaborating with others to modify/design VM activities as well as reflect/talk about their implementation efforts, including discussions related to student engagement and learning. Periodically small or large group conversations about the role of technology may be helpful in promoting teachers' efforts to teach *with* VM activities. As Phase III progresses, the role of the

PD facilitator in supporting teachers' efforts to find and modify/design VM activities should decrease. Phase III may last three to five sessions.

Activity Theory. From an activity theory perspective, the goal of the activity system (i.e., teachers teaching *with* technology) is mediated by several factors. Figure 2 represents the PD activity system for the PD model described above. This activity system acknowledges that teachers' instructional practices (i.e., the object of the activity system) are not isolated but rather are situated within a system that includes tools, rules, community, etc. To support the goal of teachers teaching *with* technology, the PD provides a task analysis framework as well as conversations and reflections that act as tools/mediating artifacts. Teachers beliefs, curricula, and school/district initiatives are intentionally embraced through conversations and designing VM activities that are tied to teachers' specific learning goals. Teachers' communities, which include other teachers that may not be part of the PD as well district resources personnel (e.g., math and/or instructional technology coaches), may act as supports or make it more challenging to implement VM activities (e.g., whether VM can replace or only supplement instructional units). Teachers' students may influence the design of VM activities based on how they engage in activities, student needs, and how teachers connect student understanding to students' experiences with the VM activities. How teachers collaborate with others may influence the design of VM activities and ideas for implementing the activities. Having teachers work in teams, the PD promotes opportunities for teachers to participate in "reflection on action" (Roth & Tobin, 2002) which may support transformation of beliefs and instructional practices related to implementing VM activities. Tensions within the activity system are embraced as opportunities to further support teachers' efforts to teach *with* technology. Creating lasting

changes in teachers' practices to teach *with* technology requires changes throughout the activity system (Roth & Tobin) rather than only in the object.

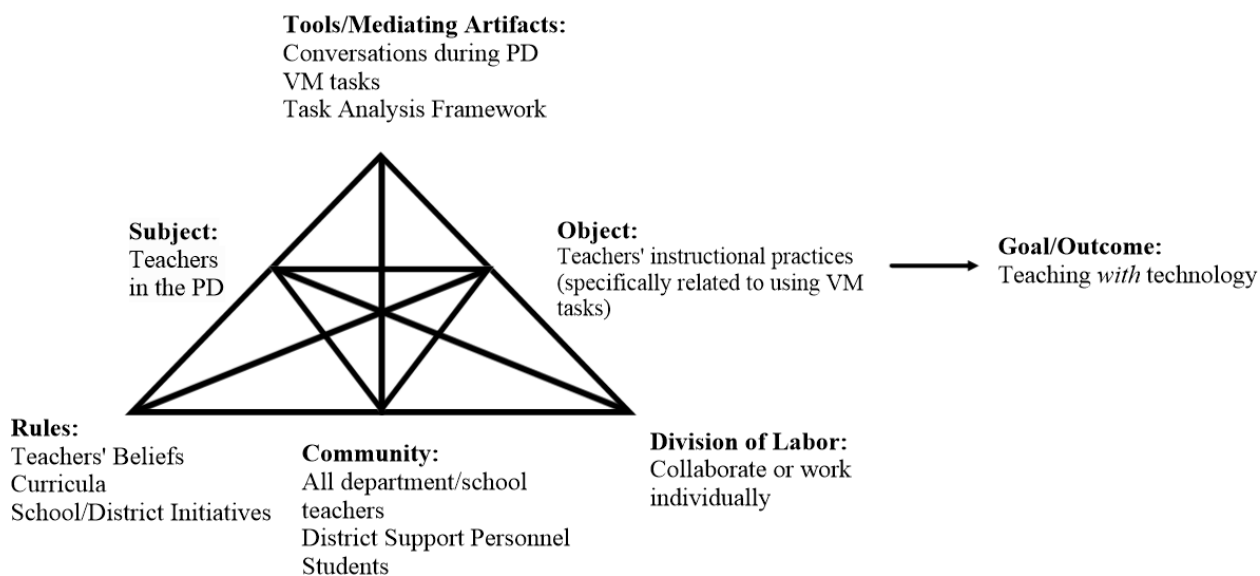


Figure 2. Professional development activity system.

Conclusion and Implications

Effectively integrating technology is more than simply learning how to use the technology. “It is also a process of reflecting on how technology-enhanced practices challenge assumptions about what and how to teach and how students can learn most effectively in today’s world” (Wiske, Fans, & Breit, 2005, p. 3). Using technology may require teachers to rethink how they design their instruction as well as the types of solutions and explanations that are accepted when using technology. When technology is solely used to deliver content, the only expected educational benefits can be improved access to technology and educational efficiency (e.g., using a calculator to perform calculations) (Hoakanson & Hooper, 2011). The difficult part, fostering actual understanding, occurs when students learn how to coordinate the operations (Hutchins, 1995). Therefore, “improved educational effectiveness requires using technology to enable

learners to explore, expand, and enhance their own capabilities, that is, to create their own knowledge” (Hoakanson & Hooper, p. 138).

Curricula must be adapted to account for the manner in which functional systems are transformed through the use of technology (Pea, 1987 as cited in Pea, 1993). For example, due to the treatment of mathematical representations with technology, algebra curricula can be designed to focus on functional conceptions as opposed to focusing on symbolic manipulations (Heid & Blume, 2008b). Technology does not take learning away from the students; rather it can enable students to use what they already know to develop more complex conjectures to test and refine as needed on a larger scale than what they would be able to do with paper and pencil alone. Additionally, technology may provide opportunities for students to more deeply reflect on their conjectures and actions in a different way (Heid & Blume) than they are able to without technology.

Learning how to teach mathematics *with* technology requires “a deeper knowledge of both mathematics and technology, as well as a transformed practice of mathematics teaching that incorporates technology” (Wilson, 2008, p. 416). Wilson further suggests that teachers need to learn how to distinguish between mundane uses of technology (i.e., using a calculator for computation only) and powerful instructional uses of technology (i.e., using technology to provide opportunities for students to interact with mathematical representations that promote the development of conceptual understanding). Teachers need to design “activities so that students act in ways that are potentially productive” (Bowers & Doerr, 2001, p. 131). However, redesigning activities requires changes in the manner in which teachers think about instruction.

Districts have spent countless monetary resources on technology, yet teachers often find it challenging to capitalize on these resources to enhance student understanding. Improvement in

teachers' efforts to teach *with* as opposed to *near* technology, is a process (not an event) and one that must be continually supported as teachers strive to improve student understanding (Guskey, 2002). When teachers are asked to transform their practice, they need: educational experiences that resemble reformed practices, guidance from people experienced in the reform practices, and opportunities to try out and adapt reformed practices and materials with colleagues to expand their new skills, beliefs, and knowledge (Romberg, 1992). The actual value of technology comes not simply from its use but instead comes from how it enables new instructional methods (Hoakanson & Hooper, 2011) and provides opportunities for conceptual conversations (Knuth & Hartmann, 2005).

Teachers often find it challenging to integrate new tools, resources, and instructional approaches, especially in relation to technology integration and the ability to develop tasks that promote conceptual understanding (Mouza, 2009; Walker et al., 2012). Building from previous studies, the task analysis framework (Table 3) and PD model (Figure 1) presented aim to support teachers' efforts to teach *with* VM activities. The presented framework and model were revised based on an iteration of a PD with secondary mathematics teachers. Future iterations, including a focus on investigating student understanding after VM activities are implemented, are needed in an effort to investigate the effectiveness of the PD and possible revisions to the framework and/or model (Borko, 2004; Driskell et al. 2016; Lawless & Pellegrino, 2007; Wells, 2007). Due to the increasing number of VMs and activities available to teachers, the potential for enhancing student learning, as well as the expectation for teachers to effectively teach *with* technology, supports are needed that promote teachers' effective use of this technology tool. In Driskell et al. (2016)'s review of studies published about technology related PD across several decades (i.e., 1980s onward), only four of the final 47 studies included a focus on applets (of which VMs

would be included). Therefore, there is a need to promote and investigate guides and PD opportunities aimed at supporting teachers' effective use of VM activities as a means to teach *with* as opposed to *near* technology.

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Part 3: Teaching WITH (not near) Technology

Introduction

From calculators to computers, mathematics teachers have been using technology in the classroom for decades. As access to various technology resources increases, so too has the expectation for teachers to use technology to enhance student engagement and understanding. Teachers are expected to prepare students to use technology in mathematical modeling and strengthen their understanding of various concepts (CCSSI, 2010). Additionally, the Association of Mathematics Teacher Educators (AMTE) Standards for Preparing Teachers of Mathematics state that “[w]ell-prepared beginning teachers of mathematics are proficient with tools and technology designed to support mathematical reasoning and sense making, both in doing mathematics themselves and in supporting student learning of mathematics” (AMTE, 2017, p. 11). Unfortunately, teachers often report that they are not prepared to use technology in an innovative manner and effectively in their instruction (Albion, Tondeur, Forkosh-Baruch, & Peeraer, 2015).

How do we (i.e., mathematics teacher educators, administration, instructional coaches, fellow teachers) support teachers to teach **WITH** as opposed to *near* technology? I define *teaching near technology* (Leatham, 2002) to describe teachers using technology in a manner that does not promote opportunities for students to communicate, reflect, and connect mathematical representations. *Teaching with technology*, on the other hand, refers to teachers using technology to promote opportunities for students to develop conceptual understanding through reflection and communication (Hiebert et al., 1997), and through using and connecting mathematical representations (NCTM 2014).

Supporting teachers to teach *with* technology goes beyond providing them with access to technology tools. Rather, it includes providing opportunities for teachers to interact with and try

out the technology tools integrated within their current curriculum. Additionally, it means providing teachers with resources that support their integration efforts and emerging understanding regarding how to teach with technology. For example, resources could include: professional development that is grounded in teachers' lesson goals and focus; opportunities to observe and critique other teachers using the technology tools; time to implement and reflect on how the tools can be used with their own students; frameworks that can guide the creation of technology based instructional tasks, etc.

Over the past three decades, virtual manipulatives (VMs) are one technology tool that has gained increased attention by teachers, researchers, and organizations as a tool for potentially increasing student engagement and understanding. Although VMs have been used in classrooms for quite some time (e.g., see Moyer-Packenham et al., 2014), limited resources exist to support teachers' efforts to teach *with* this technology tool. Therefore, to support teachers' efforts to teach *with* virtual manipulatives (VMs) and activities, this article provides a set of guiding questions and a task analysis framework. These tools were refined during a professional development (PD) for secondary mathematics teachers aimed at supporting their efforts to teach *with* VM activities. An example of how to apply the task analysis framework is also provided. Before introducing the two tools, VMs are briefly defined and then discussed in relation to some of the ways teachers' instruction and student learning can benefit when VMs and activities are used effectively.

Virtual Manipulatives

Moyer-Packenham and colleagues define a VM as “an interactive web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge” (Moyer, Bolyard, & Spikell 2002). VMs are not new, and in fact, teachers have been

using VMs from the National Library of Virtual Manipulatives, NCTM Illuminations, and Shodor Interactivate for the past two decades. Recently, as interactive whiteboards, tablets, Chromebooks, etc. become more prevalent in classrooms new VM repositories have become available: The Concord Consortium's SmartGraphs (promoting graphical literacy), ExploreLearning's Gizmos (science and mathematics), Flash & Math, PhET Interactive Simulations (science and mathematics simulations), and various personal collections (contact author or see the online resources for an annotated list of VM Resources).

Why use virtual manipulatives?

Due to the visual nature and the ability to be manipulated, VMs appeal to teachers for many reasons. For example, well-designed and implemented VMs can increase the diversity of problems that students are able to think about and solve (Goldenberg, 2000), support the development of abstract representations (Sarama & Clements, 2000), and may provide opportunities for students to receive immediate feedback (e.g., Edwards Johnson, Campert, Gaber, & Zuidema, 2012). Recently, studies suggest that VMs can provide equal access for students to learn content by reducing effects of students' demographics (e.g., SES and ELL status) as predictors of achievement (Moyer-Packenham et al., 2014) and can be used as a tool for differentiating instruction (e.g., Shin et al., 2017; Bouck, et al., 2015). Finally, the widespread availability of VMs (many are free) and unlimited access of the manipulatives make VMs attractive.

However, what is often missing are tools for supporting teachers in using VMs to promote opportunities for developing students' conceptual understanding through communication, reflection, and using and connecting multiple representations. Therefore, the following set of guiding questions and task analysis framework are tools that teachers in a recent

PD program found helpful in supporting their efforts to critique and modify/design VM activities (i.e., VMs and accompanying instructional materials). Teachers focused on VM activities (as opposed to only VMs) because the prompts/questions included in accompanying instructional materials were more guided, gave students something to refer back to, helped to keep students focused on the learning goal rather than clicking through the VM without making connections, etc. Therefore, teachers developed instructional guides to accompany VMs when ones did not already exist. The tools presented below became particularly useful in supporting their efforts to modify and design instructional guides.

Guiding Questions

Finding an activity online does not necessarily mean that the activity meets the needs of the students in one's classroom or that the activity promotes opportunities for students to develop conceptual understanding. Therefore, taking the time to go through an exploration from a student's perspective can provide insight into how an activity may promote opportunities for students to engage in the learning goal(s) and develop conceptual understanding. Teachers in the PD found the following questions helpful as they critiqued a VM or activity and thought about possible modifications.

- *What is your learning goal?*
- *How might your students struggle in this exploration?*
- *How might your students benefit from engaging in this exploration?*
- *Thinking about your learning goal, what is one modification you would make so that the exploration better fit the needs of your student? Why would this modification help your students engage in the learning goal?*

Using their learning goal as a lens (the first guiding question), teachers thought about how their students might struggle or benefit from engaging with an activity, how some of their students might respond (e.g., whether they would click through the activity without making connections), opportunities for feedback and the type of feedback given (e.g., did feedback go beyond yes/no), as well as how they might use the VM or activity in their classroom. Working through the activity provided a context for teachers to then critique the activity. Additionally, working through the activity also served to make teachers more aware of where their students may struggle in the activity as well as possible modifications to make so that the students' interaction with the exploration was focused on the development of conceptual understanding as opposed to troubleshooting unnecessary challenges. For example, Jake, a teacher in the PD, said

As opposed to just thinking like, "Ahh well, we'll try it and see what happens, and then we'll kind of modify afterwards," ... being a little more thoughtful about how this is going to be used and what type of questions should be asked. Or do I need to modify this, this kind of worksheet that goes along with this, so that it's going to help beforehand as opposed to like, "Oh, well that didn't go the way I really wanted it to go." And then you're doing it after the fact (italicized portions represent change in tone).

Jake's comment suggests that he now understands the importance of critiquing and modifying an activity proactively, as opposed to retroactively. Meaning, rather than finding an activity and implementing it as is, he now intentionally thinks about how the activity might be used with his students and whether the questions that are asked promote student engagement in the learning goal and opportunities for the development of conceptual understanding or whether modifications need to be made to the activity.

Task Analysis Framework

Table 1 presents the task analysis framework that teachers used during the PD to further guide their efforts to critique and modify/design VM activities; the framework builds upon other

tools aimed at supporting teachers in critiquing and designing dynamic geometry tasks (e.g., Sinclair, 2003; Trocki, 2014).

Table 1. *Task analysis framework*

Affordances	Descriptions
N/A	Task is primarily a technology task with no focus on mathematics.
N/A	Virtual manipulative does not have mathematical fidelity required to respond to the prompts.
A	Task prompts students to recall a mathematical fact, rule, formula, or definition.
B	Task prompts students to report information from the virtual manipulative or consider mathematical concepts, processes, or relationships in the current display. The student is not expected to provide an explanation.
C	Task provides opportunities for students to explain the mathematical concepts, processes, or relationships in the current display.
D	Task provides opportunities for students to make predictions and then test their predictions using the virtual manipulative.
E	Task provides opportunities for students to connect multiple representations of a mathematical concept (e.g., graphical, algebraic, and tabular representations of a relation).
F	Task provides opportunities to check students' understanding of mathematical concepts, processes, or relationships. Task may provide minimal feedback to the student based on specific errors.
G	Task provides opportunities for students to go beyond the current display by considering multiple examples to generalize mathematical concepts, processes, or relationships.
H	Task supports students' exploration through manipulation of the display that may surprise one exploring the relationships represented or cause one to refine thinking based on themes within the surprise (e.g., addressing a common student misconception).

Separated into three groups, the affordances in the framework are not necessarily in hierarchical order, though there is some hierarchy to the affordances. If either of the first two affordances applies (i.e., primarily a technology task without a focus on mathematics or the VM does not have mathematical fidelity), then it is suggested that teachers may not want to use the VM or do the activity. Affordances A and B are “lower level” affordances (i.e., prompting students to recall or report information); however, they may be helpful for familiarizing students with the VM, activating prior knowledge, or launching an investigation. Affordances C through H are meant to provide opportunities for students to create explanations, make predictions, connect multiple representations, receive feedback, generalize, and potentially address common student misconceptions. Activities that have affordances primarily from the third category may be more beneficial for a teacher to use in her classroom as they promote opportunities for students to develop conceptual understanding.

Tracy, a teacher in the PD, said the framework helped them (i.e., the teachers), “look at-CRITIQUE them more critically and look for their VALUE versus just, a fun thing to try” (*capital letters used to signify emphasis in Tracy’s speech*). Additionally, when talking about the framework, Josh another teacher in the PD, said

I mean, in the beginning, it was like, “*Well, yeah, I guess that helps you think about is it worth DOING or not.*” And then by the time we did the, you know the last few, we didn’t even, it was already in your HEAD. That, “*how-wh-what would I change to make it more worthwhile or is it fine the way that it is.*” ‘Cause you kind of already have that down. ... I think it helped kind of pick out what was kind of, what was needed (*italicized portions represent change in tone, capital letters represent emphasis when spoken*).

Therefore, for Josh, the framework not only helped him decide whether an activity was worth doing or not, but overtime it became ingrained so that as he was critiquing the activity he was thinking about what he “would change to make it more worthwhile.”

Applying the Task Analysis Framework

Figure 1 contains screenshots from ExploreLearning’s Addition of Polynomials gizmo, a VM activity used by some teachers in the PD. In this gizmo, students use algebra tiles to model given polynomials and then find the sum. The VM activity consists of the on-screen prompts in the gizmo, the exploration guide, and assessment questions. Students begin by building one polynomial and check it before moving on to the second polynomial. Minimal feedback is given when students check how they built the polynomial (**Affordance F**). Consistent with the teachers’ curriculum, students remove “zero pairs” and combine like terms to find the sum of the polynomials. After trying a variety of problems, students can respond to the assessment questions. The first question asks students to simplify a given problem, the second problem asks students which expression may be modeled with the given algebra tiles, thus supporting students in connecting multiple representations (**Affordance E**). The remaining three questions ask

students to simplify given algebraic expressions thus providing opportunities for students to move beyond the VM (**Affordance G**). Some feedback is given to students when they submit their responses for the assessment questions (**Affordance F**). When considering the prompts from the exploration guide, additional affordances apply (e.g., report information (**Affordance B**), create explanations (**Affordance C**), and make /test conjectures (**Affordance D**)).

Additionally, Activity B in the exploration guide encourages students to first sketch a model and then use algebra to find the sum, thus encouraging students to use and connect multiple representations (**Affordance E**).

Find the sum
 $(x^2 + 3x + 5) + (2x^2 - 4x - 1)$.

1.) Next, place tiles in the bottom box to model:

$2x^2 - 4x - 1$

2.) Then click 'Continue' to proceed.

Tools

The interface shows three stages of tile placement:

- Stage 1:** A large light green box contains one blue x^2 tile, three green x tiles, and five yellow 1 tiles.
- Stage 2:** A row of tiles is shown: one blue x^2 tile, one red $-x^2$ tile, one green x tile, and one yellow 1 tile. Below this, another row shows one red $-x$ tile and one red -1 tile.
- Stage 3:** A large light green box contains two blue x^2 tiles, four red $-x$ tiles, and one red -1 tile.

Buttons: **New** and **Continue**

Figure 1. Screenshots of Addition of Polynomials Gizmo from ExploreLearning.

Depending on the learning goal and student needs, possible modifications to the VM activity include: modifying vocabulary based on student reading levels and lesson goals, including additional prompts that encourage students to explain their process when *using algebra* to add polynomials, including problems that involve subtraction as an extension, etc.

Conclusion

The transforming potential of technology comes not simply from its use, but from how it promotes students in developing conceptual understanding through communication, reflection, as well as using and connecting multiple representations. Although teachers are often expected to teach *with* technology, they often are not provided the tools to do so. Therefore, the two tools presented in this article aim to address this need. Teachers in a PD found these tools helpful in their efforts to: critique an investigation for how it engaged students in a learning goal as opposed to using an activity because it was cool, modify activities proactively (as opposed to retroactively), and asking questions that activated background knowledge (i.e., affordances A

and/or B) but then pushed students forward by asking questions that embraced some of the “higher” affordances (i.e., promoted opportunities for students to develop conceptual understanding). It is believed that other teachers will find these tools helpful in supporting their efforts to teach *with* technology.

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