

From Samarqand to Istanbul:
Astronomy and Scientific Education in the Commentaries of Faḥallāh al-Shirwānī

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Abstract

This dissertation explores the intellectual career of Fathallāh al-Shirwānī (d. c. 1486), a 15th century astronomer, theologian, and teacher. Shirwānī trained at the Timurid sultan Ulugh Beg's observatory and madrasa in Samarqand, the most renowned 15th century Islamic scientific institution. Later, he was active in Cairo and Istanbul, dedicating texts to the Ottoman sultan Mehmed II, conqueror of Constantinople. His writings on the sciences, religious thought, and law, which have received little attention from historians, frequently took the form of commentaries.

This dissertation focuses primarily on Shirwānī's massive commentary on a seminal work in Islamic theoretical astronomy (*hay'a*), the *Tadhkira fī 'ilm al-hay'a* of Naṣīr al-Dīn al-Ṭūsī (d. 1274). In the process of commenting on the *Tadhkira*, Shirwānī produced a work that was suitable for a narrower and more advanced audience than the *Tadhkira* itself. I argue that Shirwānī's commentary demonstrates a high level of interest in the outstanding problems of theoretical astronomy, and reflects a sophisticated effort to assess, criticize, and build upon the work of Ṭūsī, Quṭb al-Dīn al-Shīrāzī (d. 1311), Nizām al-Dīn al-Nīsābūrī (d. 1330), and other astronomers in the *hay'a* tradition.

As part of his commentary on the *Tadhkira*, Shirwānī included multiple anecdotes from his time as a student in Samarqand. Shirwānī's descriptions of interactions with his teacher Qāḍīzāde al-Rūmī (d. c. 1441), Ulugh Beg, and other members of the madrasa, nearly all of which are translated here for the first time, provide exceptionally rare oral testimony from the inside of a premodern science classroom. In addition to these anecdotes, I discuss Shirwānī's explanatory comments, other rhetorical devices, and examples of applications of astronomical knowledge. These mark the commentary as a teaching text, and I argue that they reflect the kinds of questions Shirwānī's students would have raised during his career as a traveling scholar and teacher.

This close study of Shirwānī's life and works reveals features of scientific education and institutions distinctive to particular political and cultural settings, as well as complex processes of knowledge transmission from Central Asia to Anatolia in a time of increasing contact between Europe and the Ottoman empire.

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Chapter 1 – From Samarqand to Istanbul: The Commentary as a Challenge to Decline Theses

In 1435, a bright young man by the name of Faṭḥallāh al-Shirwānī arrived in Samarqand, an ancient city on the Silk Road in Central Asia. He had come not for commerce, but to further his studies of the mathematical sciences, theology, and law. Decades earlier the Turco-Mongol conqueror Timur had made Samarqand his capital, bringing craftsmen from far and wide to erect spectacular mosques, madrasas, and shrines. When Shirwānī reached Samarqand, the city was the seat of Timur's grandson, Sultan Ulugh Beg, and the site of an extraordinary astronomical observatory and madrasa staffed with some of the most prominent scholars of the age. Indeed, with its giant 40-meter sextant, Ulugh Beg's observatory was the leading Islamic scientific institution of the 15th century. The astronomical measurements and revised tables produced there were not surpassed until the work of Tycho Brahe over a century later. After leaving Samarqand, Shirwānī achieved prominence in Istanbul during the early Ottoman empire, dedicating scientific and religious texts to Sultan Mehmed II the Conqueror and one of his grand viziers. Despite his association with two of the major political and scientific centers of the 15th century Islamic world, Shirwānī and his writings have received little scholarly attention until now. This dissertation argues that Shirwānī's work contains valuable insights into the state of astronomical research and education practiced at Ulugh Beg's observatory and madrasa, whence scholars would soon influence the development of early Ottoman science and intellectual life.

At its most specific, then, this dissertation introduces the intellectual world of an important but neglected figure in the histories of Islamic astronomy, optics, jurisprudence and theology. At its most general, however, this study offers analyses of new material that is directly relevant to several large-scale historiographical debates (with corrections and revisions to be discussed below). The broadest contribution is surely to the much vexed problem of decline in science generally, and in “Islamic civilization” in particular.¹ Here I cast doubts on the premature demise of science in Islamic civilization, a judgment that other historians of Islamic science have strongly criticized but that nevertheless lives on in the wider literature. A significant facet of this reassessment derives from a reappraisal of the role of the much maligned commentary in premodern intellectual life, and in science in particular. To put it bluntly, the commentary as a genre is often treated as a symptom of decline. Although most pertinent to the 15th century in Islamic civilization, the conclusions of my analysis have implications for the value of the commentary as a research and teaching tool in other civilizations and eras. At the very least, it should encourage other scholars to read more commentaries and to base their judgments on a posteriori readings of individual works rather than a priori dismissals of the entire genre. As my research emphasizes, the fact that a text is called a “commentary” gives very little information about its contents or the author’s approach to the material. As we shall see, specific commentaries can be full of surprises.

1. My use of the phrase “Islamic civilization” or the adjective “Islamic” in reference to bodies of knowledge, social and cultural practices, etc., is meant merely as a necessary shorthand for the complex cultural phenomena that evolved in a variety of contexts over centuries across the Middle East, Central Asia, and North Africa, and in general should not be interpreted as referring to an essential or underlying Muslim basis. In this sense, Nestorian Christians like Ḥunayn ibn Ishāq, Jews like Moses Maimonides, and Sabians like Thābit ibn Qurra were all leading contributors to “Islamic civilization,” along with a host of Muslims.

Like most scholars of his day, Shirwānī left us his thoughts predominantly in commentaries. In exploring Shirwānī's intellectual biography and scholarly production, this dissertation therefore focuses primarily on one such work: Shirwānī's massive commentary on a seminal work in Islamic theoretical astronomy (*hay'a*), the *Tadhkira fī 'ilm al-hay'a* of Naṣīr al-Dīn al-Ṭūsī (d. 1274). This study demonstrates that commentaries could be a flexible, creative genre through which the author could accomplish multiple goals. Remarkably, in the process of commenting on the *Tadhkira*, Shirwānī produced a work that was suitable for a narrower and more advanced audience than the *Tadhkira* itself. I argue that Shirwānī's commentary demonstrates a high level of interest and competence in the outstanding problems of theoretical astronomy. In particular, it reflects a sophisticated effort to assess, criticize, and build upon the work of Ṭūsī, Quṭb al-Dīn al-Shīrāzī (d. 1311), Nizām al-Dīn al-Nīsābūrī (d. 1330), and other astronomers in the *hay'a* tradition. As part of his commentary on the *Tadhkira*, Shirwānī included multiple fascinating anecdotes from his time as a student in Samarqand. His descriptions of interactions with his teacher Qāḍīzāde al-Rūmī (d. c. 1441), Ulugh Beg, and other members of the madrasa, nearly all of which are presented and translated here for the first time, provide exceptionally rare first-hand oral testimony from the inside of a premodern science classroom. In addition to these anecdotes, I discuss Shirwānī's explanatory comments, other rhetorical devices, and examples of applications of astronomical knowledge. These mark the commentary as a teaching text. I argue that these pedagogical features reflect the kinds of questions Shirwānī expected a reader to raise at various points, or even questions posed to him by students during his career as a traveling scholar and teacher.

Finally, Shirwānī's commentaries drew connections between astronomy and the fields of optics, geography, and theology, most notably in his discussion of key aspects of Ibn al-

Haytham's revolutionary *Optics*, which had received little attention in the Arabic scholarly tradition after Ibn al-Haytham's death in 1040. One scholar's sophisticated treatment of a wide range of topics in a small collection of commentaries provides further evidence that post-classical Islamic commentaries represent a rich and largely unexplored resource for assessing changing trends and developments in later Islamic intellectual history, including science.

Narratives of “Decline” and “Innovation”

Shirwānī lived in the middle of the so-called “post-classical” period of Islamic history (from approximately 1200-1900 CE). Historians once commonly viewed these centuries as a period of sharp decline in scientific activity after a “golden age” stimulated by the translation of Greek, Persian, and Indian material into Arabic. The notion of a scientific decline paralleled a broader assessment of the decline of “Islamic civilization” in general – an assessment that remains popular among the public as well as many scholars of other disciplines, despite having been roundly rejected by specialists in the history of Islamic societies. Not only does the view of history as a series of cycles – expansion, flowering, decline – exert a powerful hold on the imagination, but it is part and parcel of legitimate historical analysis to make comparisons between human activities in different times and places. It should not come as a surprise, therefore, that from the appearance of Islam until the present day, historians and other observers of Islamic societies have attempted to assess the latter in relation to what was known about their past, as well as in relation to contemporary European standards. In scholarly treatises, sermons, and polemics in support of successive crusades, medieval Christians evaluated Muslims on the basis of their religious beliefs and practices and through the lenses of infidelity, conversion, and

holy war.² In the Renaissance and early modern period, however, religious scholars and philologists grew interested in Arabic and other Middle Eastern languages as sources for better understanding the Jewish and Christian scriptures.³ During the same time, the Ottoman conquest of Constantinople in 1453 and the threat of continued aggression against Christian lands shaped complex, often contradictory, perceptions of “the Turk” or “the Muslim” as a barbarous threat to (European) civilization who could nevertheless be a powerful ally, a valuable trading partner, or an object of great curiosity.⁴ Contacts with the great Muslim empires of the Ottomans, Safavids, and Mughals in this period helped shape a European sense of “self” and “other” and strengthened a perceived dichotomy between West and East.⁵

By the 19th century, with its expansion of European colonialism and the conquest of Islamic societies across North Africa and parts of the Middle East, Central and Southeast Asia, it seemed self-evident to many that European science and culture were superior to everything

2. For a overviews of pre-Renaissance Christian writers on Islam, see John Tolan, *Sons of Ishmael: Muslims Through European Eyes in the Middle Ages* (Gainesville: University Press of Florida, 2008) and Norman Daniel, *Islam and the West: The Making of an Image*, 2nd ed. (Oxford: Oneworld, 1960/1993).

3. See, for example, James Turner, *Philology: The Forgotten Origins of the Modern Humanities* (Princeton: Princeton University Press, 2014) and G.J. Toomer, *Eastern Wisedome and Learning: The Study of Arabic in Seventeenth-Century England* (Oxford: Clarendon Press, 1996).

4. On trade and diplomatic connections, see Kate Fleet, *European and Islamic Trade in the Early Ottoman State: The Merchants of Genoa and Turkey* (Cambridge: Cambridge University Press, 1999).

5. For a wide-ranging overview of Renaissance European views of the Ottomans and their impact on the idea of an essential conflict between East and West, see Nancy Bisaha, *Creating East and West: Renaissance Humanists and the Ottoman Turks* (Philadelphia: University of Pennsylvania Press, 2004).

found in “the Orient.”⁶ As a result, many scholars turned their attention to understanding the causes of the “decline” of Islamic societies from their previous political, economic, and intellectual peaks. The 14th c. North African historian Ibn Khaldūn and his *Muqaddima*, translated into French by de Slane in 1863-1868, offered a cyclical view of history in which the social solidarity of a conquering nomadic dynasty gradually weakened owing to the conquerors becoming used to the luxury of sedentary life.⁷ Many saw the Mongol invasions of Central Asia and the Middle East as bringing about the near-destruction of Islamic civilization and learning, most vividly symbolized by the sack of the great ‘Abbāsid capital of Baghdad in 1258. In fact, by this date, the classical Islamic caliphate had long been split into rival empires. Nevertheless, it was not surprising that many scholars would blame the Mongols for the near-destruction of Islamic civilization since contemporary sources on the Mongols were written by Muslim authors from the conquered lands, or by the Mongols’ Mamluk rivals in Egypt and Syria.⁸ Other analyses pointed to something inherent in the religion of Islam itself, or in its adherents, as the cause of decline, such as William Muir in his *The Caliphate, Its Rise, Decline and Fall* (1891). The racisms of Gobineau (d. 1882) and Renan (d. 1892), although perhaps more extreme than that of

6. One cannot avoid mentioning Edward Said, *Orientalism* (New York: Pantheon, 1978), although Said’s ideas of cultural representation and imperialism have been more influential in the fields of literary and post-colonial studies than among historians of Islamic societies. For a critique of Said’s book and more sympathetic treatment of many of the same figures, see Robert Irwin, *For Lust of Knowing: The Orientalists and Their Enemies* (London: Allen Lane, 2006).

7. Ibn Khaldūn, *The Muqaddimah, an Introduction to History*, ed. N. J. Dawood, trans. Franz Rosenthal (Princeton: Princeton University Press, 1969).

8. On the rivalry between the Mongols and Mamluks, see Reuven Amitai-Preiss, *Mongols and Mamluks: The Mamluk-Ilkhanid War, 1260-1281* (Cambridge: Cambridge University Press, 1995) and Anne Broadbridge, *Kingship and Ideology in the Islamic and Mongol Worlds* (Cambridge: Cambridge University Press, 2008).

many of their contemporaries, tied in with widely shared beliefs about the inherent characteristics of various “nations” and their relative contributions to human civilization.

Most scholars focused on the life of the Prophet Muhammad, the Arab conquests, and the classical caliphate, taking “Islam” as an essential, largely unchanging object of study. Others, however, took a more nuanced approach. Ignaz Goldziher (d. 1921) studied collections of *ḥadīth* (sayings of the Prophet transmitted orally through chains of authorities), demonstrating that many of them were in fact invented centuries after the Prophet’s death. As a result, he argued they could be used as sources for analyzing the evolution of Islamic ideas about religious practice, law, and social issues at various times and places.⁹ As we will see below, Goldziher argued that religious factors had played a role in the decline of science in the Islamic world. Nevertheless, he opposed colonialism and believed that Islamic societies had the potential to revive themselves. A generation later, H.A.R. Gibb (d. 1971) followed Goldziher in focusing on the later evolution of Islamic societies, and adopted a cyclical model of rise and decline in his influential essay “An Interpretation of Islamic History.”¹⁰ His contemporary, Gustave von Grunebaum (d. 1972), also wrote prolifically on the history of “Classical Islam,” Arabic poetry, and cross-cultural comparisons between Islamic and other societies. Grunebaum, too, held a pessimistic view of later Islamic civilization as “arrested in its growth during the eleventh

9. See his essay “Ueber die Entwicklung des Ḥadīth (On the development of ḥadīth)” collected in Ignaz Goldziher, *Muhammedanische Studien*, 2 vols. (Halle: Niemeyer, 1889-1890).

10. H.A.R. Gibb, “An Interpretation of Islamic History,” *Cahiers d’Histoire Mondiale/Journal of World History* 1, (1953): 39-62.

century...it stagnated in self-inflicted sterility,” a civilization that had inherited the legacy of the Hellenistic world and merely passed it on to the West.¹¹

One of the earliest scholars to challenge this notion of “decline” was Marshall Hodgson (d. 1968), whose pioneering work *The Venture of Islam* has had a profound influence on the study of Islamic history.¹² Hodgson emphasized the important effects of climate and the physical environment on social and economic development, along with the contributions of non-Arab ethnic and cultural groups to Islamic societies. As a result, he argued that, rather than being a period of decline, the “Middle Period” of Islamic history (from the late 10th c. to the 16th c.) was a time of social, intellectual, and economic growth and vitality. In large part, this dynamic “Islamicate”¹³ high culture was made possible by the breakdown of political and cultural barriers that had previously separated Europe, Asia, and Africa. More recently, historians such as Kenneth Pomeranz have followed a similar approach to explore how shifting trade routes and access to resources may help explain the growth of 19th c. European empires relative to their rivals in the Middle East and Asia, rather than presuming that those rivals necessarily went into decline.¹⁴

11. Gustave von Grunebaum, *Medieval Islam* (Chicago: University of Chicago Press, 1946, 1954), 322.

12. Marshall Hodgson, *The Venture of Islam: Conscience and History in a World Civilization*, 3 vols. (Chicago: University of Chicago Press, 1974).

13. Hodgson preferred the terms “Islamdom” and “Islamicate” culture (by analogy with “Italianate”), intending them to refer to “the complex of social relations” and “the social and cultural complex historically associated with Islam and the Muslims, both among Muslims themselves and even when found among non-Muslims” respectively, rather than a geographic territory or the religion of Islam itself. Hodgson, *The Venture of Islam: Conscience and History in a World Civilization*, Vol. 1, 59.

14. Kenneth Pomeranz, *The Great Divergence: China, Europe, and the Making of the Modern World Economy* (Princeton: Princeton University Press, 2000).

For many historians, the “decline” thesis was supported by what appeared to be a rejection of science after an initial “golden age” sparked by translations of Greek (and Persian and Indian) material into Arabic. Thus, in an 1883 lecture at the Sorbonne under the title “Islam and Science” Renan allowed that, for around 500 years, “the Muslim world had been superior in terms of intellectual culture to Christendom.” However, he argued that “the really fruitful element of all of this came from Greece.” Islam itself “has constantly persecuted science and philosophy. It ended by stifling it.” He went on to claim that

what is, in fact, essentially distinctive of the Mussulman [sic] is his hatred of science, his persuasion that research is useless, frivolous, almost impious – the natural sciences, because they are attempts at rivalry with God; historical sciences, because, since they apply to times anterior to Islam, they may revive ancient heresies.¹⁵

To be fair to Renan, his language echoed condemnations made by prominent but not necessarily representative, medieval Muslim authors themselves, such as Ibn Taymiyya (d. 1328) and Taj al-Dīn al-Subkī (d. 1370). Goldziher made similar observations, although in less strident terms, in his influential essay on the attitude of “Orthodox Islam” toward science.¹⁶

Along with critiques of the notion of an overall decline in Islamic civilization came efforts to push back against the idea of a decline in Islamic science in particular. Recent work by historians of Islamic science has done much to provide a more nuanced view of both the quality

15. Ernest Renan, “Islamism and Science,” in *Readings in Orientalism*, Volume 1, ed. Bryan Turner, (New York: Routledge, 2000), 199-217.

16. Ignaz Goldziher, “Stellung der alten islamischen Orthodoxie zu den antiken Wissenschaften,” *Abhandlungen der Königlich Preussischen Akademie der Wissenschaften* 8, (1916): 3-46, translated into English as Ignaz Goldziher, “The Attitude of Orthodox Islam Toward the ‘Ancient Sciences’,” in *Studies on Islam*, ed. Merlin Swartz, (New York: New York University Press, 1981), 185-215.

of scientific activity and the relationships between “science” and “religion” in Islamic societies. Rather than portraying an inherent opposition on the part of Muslim scholars toward science, their research has argued that many devout Muslims saw no conflict between their religion and the study of the natural world. In a famous programmatic essay, A.I. Sabra criticized his predecessors’ “marginality thesis,” to the effect that science was the concern of a fringe within Islamic societies. To the contrary, although he accepted the claim of a certain amount of decline in scientific activity, he argued it was the result of Muslim scholars “naturalizing” Hellenistic thought in an assimilation process that influenced broader Islamic intellectual trends.¹⁷ He suggested that future research focus on the development of science in particular locations and times, and that madrasas, princely courts, and other institutions were all potential sites from which a fuller picture of Islamic scientific activity could be drawn.¹⁸ In addition to Sabra’s own research, the work of Jamil Ragep, George Saliba, and Robert Morrison has produced a great deal of evidence that, at least with respect to astronomy, the post-classical period witnessed sustained efforts to evaluate, criticize, and improve Greek models for the motions of the heavens.¹⁹

17. A. I. Sabra, “The Appropriation and Subsequent Naturalization of Greek Science in Medieval Islam: A Preliminary Statement,” *History of Science* 25, (1987): 223-243.

18. Sabra elaborated on these themes in an address to the History of Science Society in 1995, published as A. I. Sabra, “Situating Arabic Science: Locality versus Essence,” *Isis* 87, no. 4 (1996): 654-670. Similarly, Sonja Brentjes has forcefully argued for a more contextualized approach to the study of science in Islamic societies in Sonja Brentjes, “On the Location of the Ancient or “Rational” Sciences in Muslim Educational Landscapes (AH 500-1100),” *Bulletin of the Royal Institute for Inter-Faith Studies* 4, no. 1 (2002): 47-71 and Sonja Brentjes, “The Prison of Categories – ‘Decline’ and Its Company,” in *Islamic Philosophy, Science, Culture, and Religion: Studies in Honor of Dimitri Gutas*, ed. Felicitas Opwis and David Reisman, (Leiden: Brill, 2012), 131-156.

19. To cite only a few items, George Saliba, *A History of Arabic Astronomy: Planetary Theories During the Golden Age of Islam* (New York: New York University Press, 1994), F. Jamil Ragep, *Naṣīr al-Dīn al-Ṭūsī’s Memoir on astronomy = al-Tadhkira fī ‘ilm al-hay’a*, 2 vols. (New York:

Specialists in other fields have also sought to critique the decline narrative. With respect to Islamic intellectual history, Frank Griffel's recent study of al-Ghazālī (d. 1111) rejects the notion that al-Ghazālī's writings on theology and cosmology brought an end to the study of philosophy among post-classical scholars, and demonstrates the profound impact many of Ibn Sīnā's views had on al-Ghazālī's theology.²⁰ Historians of economics and material culture have pointed out the remarkable flowering of new forms of artistic expression accompanying the rise of international trade centers around the Mediterranean, as well as across Central Asia after the Mongol conquests.²¹

Nevertheless, although historians of medieval European and Islamic societies alike have worked to stamp out the rhetoric of the "Dark Ages" in favor of a more nuanced analysis of social, political, and intellectual developments, the "decline" narrative has recently gained new life for reasons unrelated to the historical situation four or more centuries ago. Particularly in the popular and educated non-specialist imagination, books and editorials after "9/11" and more recent attacks have focused attention on the violent wing of modern Islamist fundamentalism, encouraging a highly selective interpretation of Islamic history, and renewing debates over

Springer-Verlag, 1993), and Robert Morrison, *Islam and Science: The Intellectual Career of Nizām al-Dīn al-Nīsābūrī* (New York: Routledge, 2007).

20. Frank Griffel, *Al-Ghazālī's Philosophical Theology* (Oxford: Oxford University Press, 2009).

21. See, for example, David Morgan's overview of recent work on the cultural history of the Mongols in David Morgan, "Mongol Historiography Since 1985: The Rise of Cultural History," in *Nomads as Agents of Cultural Change: The Mongols and Their Eurasian Predecessors*, ed. Reuven Amitai and Michael Biran, (Honolulu: University of Hawai'i Press, 2015), 271-282; Anna Contadini, "Sharing a Taste? Material Culture and Intellectual Curiosity Around the Mediterranean, From the Eleventh to the Sixteenth Century," in *The Renaissance and the Ottoman World*, ed. Anna Contadini and Claire Norton, (Burlington, VT: Ashgate, 2013), 23-61; Thomas Allsen, *Culture and Conquest in Mongol Eurasia* (New York: Cambridge University Press, 2001); and Sheila Blair, *Text and Image in Medieval Persian Art* (Edinburgh: Edinburgh University Press, 2014).

“Islam and Science.” The Nobel physicist Stephen Weinberg’s comment in 2007 that “alas, Islam turned against science in the twelfth century... after al-Ghazzali, there was no more science worth mentioning in Islamic countries,” could easily have been written 70 or more years ago.²²

Opposing the persistence of the “decline” narrative is a more recent alternative narrative of “Islamic innovation.” This narrative, which has problems of its own, is exemplified by the acclaimed exhibition *1001 Inventions: The Enduring Legacy of Muslim Civilization* and its accompanying books, films, live performances, and educational materials for use in the classroom. *1001 Inventions* is a visually stunning international phenomenon, with shows in London, Istanbul, New York, Los Angeles, and Washington DC, as well as nearly a dozen other major cities worldwide. The ideological message behind most of the exhibition and associated “edutainment” products is made clear by the program’s organizer: the Foundation for Science, Technology and Civilization lists among its goals “To foster an accurate understanding of the thousand years of exceptional advances...within the Muslim world” and “To promote science and learning as an alternative to negative or extremist behaviour.”²³ *1001 Inventions* has been criticized by some historians of Islamic science for uncritically translating medieval science into modern terminology, implying that these inventions were later attributed to Westerners, and suggesting that many Western institutions were shaped by Muslim influence, and for lacking significant discussion of scientific principles or assessments of how medieval science actually developed over time.²⁴ Nevertheless, the fact remains that there was a great deal more innovation

22. Stephen Weinberg, “A Deadly Certitude: On God, Christianity and Islam,” *Times Literary Supplement*, January 17, 2007.

23. Foundation for Science, Technology and Civilization (FTSC), “Aims & Mission,” <http://www.fstc.org.uk/aims-mission> (accessed March 2, 2015).

24. Taner Edis and Sonja Brentjes, “A Golden Age of Harmony? Misrepresenting Science and

than decline in the post-classical period. More importantly, the existence of ongoing, ill-informed debates with such participants as prominent figures like Weinberg or the *1001 Innovations* exhibit indicates that, clearly, a better understanding of the post-classical Islamic period is essential for rewriting grand narratives about how the modern world took shape.

Rethinking commentaries

Such an understanding cannot take emerge in the abstract. This dissertation originated in a desire to explore the complex – and largely unstudied – middle ground between such extreme generalizations of “decline” and “innovation,” and from a sense that overarching narratives about Islamic intellectual history and science are incomplete, if not flawed, without taking into account the large number of texts that remain unstudied. The overwhelming majority of scholarly texts in the post-classical period of Islamic civilization appeared in the form of commentaries. One finds commentaries written on subjects ranging from grammar and linguistics, through poetry, philosophy and other sciences, to scriptural exegesis, the sayings of the Prophet Muhammad, and interpretations of the law. A commentary, by its very nature, is part of a textual tradition tied to an earlier work. In writing the narratives of decline discussed above, scholars often took the existence of these commentaries as evidence for a lack of originality. In other words, the genre itself was proof that the material in its constituent specimens could only be a stale repetition of the known, with nothing new to offer. They assumed that commentaries were a means of transmitting only established interpretations, and therefore spent little effort bothering to investigate the commentaries on their own terms. This assessment of the commentary genre as a

History in the 1001 Inventions Exhibit,” *Skeptical Inquirer* 36, no. 6 (2012): 49-53.

dull, dry, wasteland fed all too smoothly into colonial narratives about the decline and stagnation of pre-modern Islamic societies in general.

Happily, recent scholarship has begun to take a serious interest in commentaries as potential sites of intellectual engagement and growth.²⁵ Focused, technical analyses of individual texts, themes, and authors are preliminary requirements for a broader reinterpretation of the period.²⁶ As recently as 2013, the journal *Oriens* devoted an entire issue to the topic of incorporating commentaries into the study of Islamic intellectual history.²⁷ As the editors point out, the urgent task is to provide new frameworks internal to the Islamic intellectual tradition in order to assess it on its own terms. If we actually take the time to look at the contents of these commentaries, we might begin to understand why so many scholars chose this specific genre to discuss science. Commentaries can illustrate how they engaged with their predecessors, transforming knowledge through their writing and teaching in ways that fall somewhere between pure “innovation” or “decline,” as most modern work in the sciences arguably continues to do.

25. For example, Saliba called for a focus on astronomical commentaries in George Saliba, “Writing the History of Arabic Astronomy: Problems and Differing Perspectives,” *Journal of the American Oriental Society* 116, no. 4 (1996): 709-718.

26. For the rational sciences and philosophy, see Robert Wisnovsky, “The Nature and Scope of Arabic Philosophical Commentary in Post-classical (ca. 1100-1900 AD) Islamic Intellectual History: Some Preliminary Observations,” in *Philosophy, Science and Exegesis in Greek, Latin and Arabic Commentaries*, Vol. 2, ed. Peter Adamson et al., (Institute of Classical Studies, University of London, 2004), 149-191; Asad Ahmed, “Systematic Growth in Sustained Error: A Case Study in the Dynamism of Post-Classical Islamic Scholasticism,” in *The Islamic Scholarly Tradition: Studies in History, Law, and Thought in Honor of Professor Michael Allan Cook*, ed. Asad Ahmed et al., (Leiden: Brill, 2011), 343-378; and Jon McGinnis and Asad Ahmed, “Rationalist Disciplines in Post-Classical (ca. 1200–1900 CE) Islam,” *Oriens* 42, (2014): 289-291. Nahyan Fancy has begun similar studies of commentaries on medical texts, such as his recent book, Nahyan Fancy, *Science and Religion in Mamluk Egypt: Ibn Al-Nafīs, Pulmonary Transit and Bodily Resurrection* (New York: Routledge, 2013).

27. Asad Ahmed and Margaret Larkin, “The *Hāshiya* and Islamic Intellectual History,” *Oriens* 41, (2013): 213-216.

For example, Saliba has argued that scientific commentaries were a premodern analogue of modern scientific journals, providing a means of disseminating one's ideas in a specialized literature that drew the attention of the most interested readers.²⁸ In other cases, a commentary may play a crucial role as the main (or only) access to a particular text, preserving the intellectual and sometimes even the pedagogical content.²⁹ Part of the challenge is also that "commentary" is a broad genre encompassing numerous ways of engaging with an older text, the constructedness of which reflects cultural influences. As Glenn Most pointed out in his introduction to a collection of essays on commentaries in the field of Classical Studies, "there is nothing natural about the general form of commentary itself, and no matter how natural a particular form of commentary may seem to its own practitioners in any one place and time, it need not seem at all natural to other practitioners."³⁰ In other words, although *we* may associate the commentary with a lack of originality, we cannot blithely assume that past commentators did so as well. To the contrary, commentaries belong to a genus in evolution: to understand how they worked and what they meant, they must be studied in their own historical context.

Through close readings and technical analyses of post-classical Islamic commentaries, such as those written by Shirwānī, we can begin to reconnect these texts to localized understanding of disciplines and disciplinary boundaries, institutions, and a broader social and

28. Saliba, "Writing the History of Arabic Astronomy: Problems and Differing Perspectives," 714.

29. For example, Shirwānī's interaction with Ibn al-Haytham's seminal work in the field of optics, discussed here in Chapter 5, in all likelihood would not have been possible without the existence of a comprehensive 13th c. commentary. See A. I. Sabra, "The 'Commentary' that Saved the Text: The Hazardous Journey of Ibn al-Haytham's Arabic *Optics*," *Early Science and Medicine* 12, (2007): 117-133.

30. Glenn Most, ed., *Commentaries = Kommentare*, Aporemata Bd. 4 (Göttingen: Vandenhoeck & Ruprecht, 1999), vii.

cultural history of science in Islamic societies. Like many scholars in his day, Shirwānī was a polymath – not a dilettante but an expert in numerous fields that today we treat as radically separate disciplines. His work stands out in particular because he addresses three key topics in Islamic science: criticisms of Ptolemaic astronomy and new physical planetary models; the rediscovery of Ibn al-Haytham’s revolutionary project in optics; and the utility of science in understanding God’s creation, a fundamental question for every Muslim intellectual engaged in the study of the natural world. I argue that Shirwānī’s texts, which have scarcely been studied by modern historians, not only provide a fascinating example of the complexity and scope of interactions between scientific and religious thought in late medieval Islamic intellectual history, but also are critical for understanding the transmission of scientific knowledge from Central Asia to Istanbul, and possibly to Latin Europe as well.

Shirwānī’s life and works³¹

Since our scholar is scarcely a household name, a brief overview of his career is necessary. His full name was Faṭḥallāh ibn Abū Yazīd ibn ‘Abd al-‘Azīz ibn Ibrāhīm al-Shābarānī al-Shirwānī al-Shamākhī.³² He was born around 1417, likely in the city of Shābarān in the region of Shirwān

31. Since many of Shirwānī’s writings are still unstudied, with the existing manuscripts held in archives around the world, a complete intellectual biography would be impossible to write at this stage. Given his importance as a commentator in the sciences and theology, he is mentioned in several biographical dictionaries. For an English-language overview of Shirwānī’s life and education, see İhsan Fazlıoğlu, “Shirwānī: Faṭḥallāh ibn Abū Yazīd ibn ‘Abd al-‘Azīz ibn Ibrāhīm al-Shābarānī al-Shirwānī al-Shamākhī,” in *Biographical Encyclopedia of Astronomers*, ed. Thomas Hockey et al., (New York: Springer, 2007), 1055-1056, and İhsan Fazlıoğlu, “The Samarqand Mathematical-Astronomical School: A Basis for Ottoman Philosophy and Science,” *Journal for the History of Arabic Science* 14, (2008): 3-68, which rely on Cemil Akpınar, “Fethulla es-Sirvani,” in *Türkiye Diyanet Vakfı İslam Ansiklopedisi*, (Istanbul: Türkiye Diyanet Vakfı, 1995), 463-466.

32. In Shirwānī’s commentary on the *Tadhkira*, his teacher referred to Shirwānī as “Faṭḥallāh b. Abū Yazīd al-Shābarānī al-Shirwānī,” Topkapı Ahmet III MS 3314, fol. 16b. The contemporary

(in modern Azerbaijan). We know little about his family, but when Shirwānī left Samarqand after completing his education, he mentions that his father traveled with “an emissary from the ruler of Shirwān (*bi-rusūl min qibal malik al-Shirwān*)” to bring him home, indicating they were relatively prominent.³³ According to later biographers, Shirwānī began his education at home under the tutelage of his father before traveling to Sarakhs and Ṭūs. In Ṭūs, he studied with the Shī‘ī scholar al-Sayyid Abū Ṭālib, mastering a major text on theoretical astronomy, namely al-Sayyid al-Sharīf al-Jurjānī’s commentary on Naṣīr al-Dīn al-Ṭūsī’s 13th-century treatise *Tadhkira fi ‘ilm al-hay’a* (*Memoir on astronomy*). Ṭūsī’s *Tadhkira* was a seminal text in Islamic theoretical astronomy, laying out the principles for a program for both reforming Hellenistic models of planetary motion and resolving outstanding difficulties in astronomy. The *Tadhkira*

scholar and historian al-Sakhāwī (d. 1497) mentioned the grandfather ‘Abd al-‘Azīz and great-grandfather Ibrāhīm in his entry on Shirwānī in al-Sakhāwī, *al-Daw’ al-lāmi ‘li-ahl al-qarn al-tāsi’* (Cairo: Maktabat al-Qudsī, 1935-36), 166. Shābarān/Shabrān and Shamākhī were two of the major cities in the Shirwān region of the eastern Caucasus under the control of the Shirvanshah dynasty, and Shirwānī referred to Shamākhī as “our city” when, in his commentary on the *Tadhkira*, he demonstrated how to determine the *qibla* (direction to Mecca) for a given location, Topkapı Ahmet III MS 3314, fol. 347a. Fazlıoğlu’s English-language references to Shirwānī in the *Biographical Encyclopedia of Astronomers* and his article on the “Samarqand Mathematical-Astronomical School” transliterated the Turkish spelling of the city, Şamahı, as Shamāhī. Fazlıoğlu, “Shirwānī, Faṭhallāh,” and Fazlıoğlu, “The Samarqand Mathematical-Astronomical School” Ragep, in his list of *Tadhkira* commentators, referred to Shirwānī as “Faṭḥ Allāh b. ‘Abd Allāh al-Shīrwānī (al-Rumī al-Ḥanafī)” following Baghdādī (d. 1920), but Baghdādī’s information cannot be correct since he stated Shirwānī studied with Taftāzānī (d. 1390) and Jurjānī (d. 1413), and died in 1453. Ismail Paşa Bağdatlı, “Faṭhallāh al-Shirwānī,” in *Hadiyyat al-‘Arifin: Asmā’ al-Mu’allifin wa Āsrār al-Muṣannifin*, (Istanbul: Milli Eğitim Basımevi, 1951), 815; Ragep, *Tadhkira*, 62. More detailed discussions of biographical sources on Shirwānī are found Rudolf Sellheim, *Materialien zur arabischen Literaturgeschichte*, 2 vols., Verzeichnis der orientalischen Handschriften in Deutschland; Bd. 17. Arabische Handschriften; Reihe A (Wiesbaden: Steiner, 1976-1987), 163-168 and Akpınar, “Fethulla es-Sirvani.”

33. Shirwānī, *Sharḥ al-Tadhkira*, Topkapı Ahmet III MS 3314, fol. 16b.

became the subject of many commentaries over the following centuries, and its influence may extend to Copernicus.³⁴

Shirwānī's encounter with the *Tadhkira* in Tūs marked the beginning of a lifelong interest in astronomy and mathematics. In 1435 he became a student at the Samarqand madrasa established under the patronage of the “scholar-sultan” Ulugh Beg.³⁵ Samarqand, an ancient city on the Silk Road, was the capital of the Timurid empire and home to the largest observatory in the world. The madrasa's head teacher (and Ulugh Beg's own tutor) was Qāḍīzāde al-Rūmī, a highly renowned mathematician and one of the administrators of Ulugh Beg's observatory. During Shirwānī's time at Samarqand, Qāḍīzāde and other astronomers under the direction of Ulugh Beg were engaged in a decades-long program of observations that would lead to the production of a new, more accurate star catalog and revised set of astronomical tables called the

34. Ragep identifies 11 authors who wrote on the *Tadhkira*, some of whom wrote multiple texts. Ragep, *Tadhkira*, 58-64. In 1950s and 60s, Edward Kennedy and his student Victor Roberts identified striking similarities between some of the mathematical devices in the work of the Damascene astronomer Ibn al-Shāṭir (d. c. 1375) and Copernicus, suggesting that Arabic astronomical models made their way to Europe by an as-yet-unknown means, a thesis which has been the subject of debate ever since. Among others, see E. S. Kennedy and Victor Roberts, “The Planetary Theory of Ibn al-Shāṭir,” *Isis* 50, no. 3 (1959): 227-235 and Victor Roberts, “The Solar and Lunar Theory of Ibn ash-Shāṭir: A Pre-Copernican Copernican Model,” *Isis* 48, (1957): 428-432, and Noel Swerdlow, “The Derivation and First Draft of Copernicus's Planetary Theory: A Translation of the Commentariolus with Commentary,” *Proceedings of the American Philosophical Society* 117, no. 6, Symposium on Copernicus (1973): 423-512. For more recent overviews of the arguments in favor of the “Copernican connection,” see F. Jamil Ragep, “Copernicus and his Islamic Predecessors: Some Historical Remarks,” *History of Science* 45, no. 1 (2007): 65-82; George Saliba, *Islamic Science and the Making of the European Renaissance* (Cambridge, MA: MIT Press, 2007); Robert Morrison, “A Scholarly Intermediary between the Ottoman Empire and Renaissance Europe,” *Isis* 105, no. 1 (2014): 32-57; and Joseph Ibn Naḥmias, *The Light of the World: Astronomy in al-Andalus*, ed. and tr. Robert Morrison, (Berkeley: University of California Press, 2016), 1-48.

35. On Ulugh Beg, see V.V. Barthold, *Four Studies on the History of Central Asia*. Vol. 2, *Ulugh-Beg* (Leiden: Brill, 1958) and Beatrice Forbes Manz, “Ulugh Beg,” in *The Encyclopaedia of Islam*, 2nd Edition, (Leiden: Brill, 1960-2007),

Zīj-i Sulṭāni.³⁶ These tables were prepared as an update to the 13th-century *Zīj-i Īlkhānī* that originated at the Marāgha Observatory under Naṣīr al-Dīn al-Ṭūsī. In addition, Qāḍīzāde was also the author of a famous commentary (to judge by the number of surviving manuscripts and references) on an introductory astronomical text, al-Jaghminī's (fl. late 12th/early 13th c.) *al-Mulakhkhaṣ fī al-hay'a al-basīṭa* (Epitome of simple astronomy).³⁷

We are fortunate to have several glimpses of scholarly life and the curriculum at Samarqand from letters written by Jamshīd al-Kāshī to his father, as well as Shirwānī's own descriptions of lectures attended by Ulugh Beg.³⁸ Both Shirwānī and Kāshī describe lectures and discussions focused on mathematical and astronomical texts produced several generations earlier by "Marāgha school" intellectuals Naṣīr al-Dīn al-Ṭūsī, Quṭb al-Dīn al-Shīrāzī, and their students (on which more below). Some of the core texts seem to have been Ṭūsī's recension of Euclid's *Elements*, his fundamental *Tadhkira* on theoretical astronomy, along with the commentary on it by Shīrāzī's student Nizām al-Dīn al-Nīsābūrī (d. ca. 1330), and Shīrāzī's *Nihāyat al-Idrāk fī Dirāyat al-Aflāk* (The highest attainment in comprehending the orbs) and *al-Tuhfa al-Shāhiya* (The imperial gift).

36. Aydin Sayili, *The Observatory in Islam* (New York: Arno Press, 1981), Chapter 7.

37. On Jaghmīnī and his text, see Sally Ragep, "Maḥmūd ibn Muḥammad ibn 'Umar al-Jaghminī's *al-Mulakhkhaṣ fī al-hay'a al-basīṭa*: An Edition, Translation, and Study" (PhD diss., McGill University, 2014), which is due out in book form in late 2016.

38. For Kāshī's letters, see E. S. Kennedy, "A Letter of Jamshīd al-Kāshī to his Father: Scientific Research and Personalities at a Fifteenth Century Court," *Orientalia* 29, (1960): 191-213, and Mohammad Bagheri, "A Newly Found Letter of al-Kāshī on Scientific Life in Samarkand," *Historia Mathematica* 24, no. 3 (1997): 241-256. Shirwānī included descriptions of lectures at Samarqand in his commentary on Ṭūsī's *Tadhkira*, this portion of the text was edited and translated in Fazlıoğlu, "The Samarqand Mathematical-Astronomical School," 41-49.

Shirwānī's education at Samarqand was not limited to mathematics and astronomy, however. Under Qāḍīzāde's direction, he also studied the linguistic sciences, theology, and Islamic law. Shirwānī wrote a commentary on a work of Islamic jurisprudence, al-Ardabīlī (d. 1396)'s *al-Anwār li-a 'māl al-abrār*, which he presented to Ulugh Beg.³⁹ In the field of *kalām*, he studied the influential text of dialectical theology *al-Mawāqif fī 'ilm al-kalām* by 'Adūd al-Dīn al-Ījī (d. 1355) with al-Sayyid al-Sharīf al-Jurjānī's commentary.⁴⁰ In addition, the works of al-Sayyid al-Sharīf al-Jurjānī (d. 1413), whose *Tadhkira* commentary Shirwānī had already studied before arriving at Samarqand, presented a broad framework for thinking about both rational and religious questions in his commentaries on astronomy, logic, and *kalām* (dialectical theology). This was a demanding curriculum, suitable for advanced students, and one that tackled key works in the mathematical sciences.

This sketch of the curriculum hints at the excellence of teaching and learning near the sultan's court. In addition, Shirwānī relates an episode when he corrected an error made by one of the lecturers in the presence of Ulugh Beg, which illustrates the high level of debate and discussion carried out at the Samarqand madrasa.⁴¹ When the text referred to a geometrical proposition, the sultan paused to ask the audience about the specific wording and conditions.

39. al-Sakhāwī, *al-Ḍaw' al-lāmi'*, 166; Brockelmann, *GAL* vol. 2, 255.

40. Ījī's text is still taught at al-Azhar in Cairo, the foremost institution for the study of Sunni theology and law in the Islamic world, A. I. Sabra, "Science and Philosophy in Medieval Islamic Theology: The Evidence of the Fourteenth Century," *Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften* 9, (1994): 1-42. In addition to Sabra's article, see the entry "al-Īdjī" in the *Encyclopaedia of Islam*, 2nd ed. for more on Ījī's life and works.

41. This episode, and other classroom anecdotes recorded in Shirwānī's commentary, will be discussed in Chapter 4. See also Fazlıoğlu, "The Samarqand Mathematical-Astronomical School," 41-49, and George Saliba, "Reform of Ptolemaic Astronomy at the Court of Ulugh Beg," in *Studies in the History of the Exact Sciences in honour of David Pingree*, ed. Charles Burnett et al., (Leiden: Brill, 2004), 810-824.

Shirwānī, who had been studying the text on his own, realized that the answer given by the lecturer was incorrect. When Shirwānī was able to explain the mistake and satisfactorily answer probing questions from Ulugh Beg, he was rewarded by being appointed as reader of the text at future assemblies.⁴²

After five years at Samarqand, Shirwānī completed his education in 1440 and received an authorization (*ijāza*) to go out into the world and teach the texts he had studied under Qāḍīzāde.⁴³ In the text of his *Sharḥ al-Tadhkira*, Shirwānī includes a detailed *ijāza* from Qāḍīzāde authorizing him to teach not only mathematics and astronomy, but also certain texts of jurisprudence and *kalām*.⁴⁴ Normally an *ijāza* was given for texts in the so-called “Islamic sciences” of Qur’ānic exegesis and the sayings of the Prophet Muhammad, jurisprudence, and Arabic grammar, so it is rare to find a surviving *ijāza* connected to the exact sciences. In particular, the *ijāza* mentions three texts in the fields of astronomy, jurisprudence, and theology,

42. The reader (Ar. *qāri*’) in a madrasa was responsible for reading the text aloud to the class, pausing when the instructor wished to elaborate on a point, begin a discussion or disputation, or simply ask questions to check that the students understood the text.

43. An *ijāza*, often translated as “diploma,” was means of guaranteeing the oral transmission of a (typically religious or legal) text through a chain of transmitters. It was the instructor’s official approval that the student had mastered a particular text or group of texts with that instructor, and was fully qualified to teach his (or occasionally, her) own students. See the entry “*Idjāza*” in *Encyclopaedia of Islam*, 2nd ed., as well as George Makdisi, *The Rise of Colleges: Institutions of Learning in Islam and the West* (Edinburgh: Edinburgh University Press, 1981), 140-152. Makdisi’s link between madrasa and *ijāza* has, however, been criticized as overly restrictive in Jonathan Berkey, *The Transmission of Knowledge in Medieval Cairo: A Social History of Islamic Education* (Princeton: Princeton University Press, 1992), 31-34, and Michael Chamberlain, *Knowledge and Social Practice in Medieval Damascus, 1190-1350* (Cambridge: Cambridge University Press, 1994), 87-90.

44. The *ijāza* is contained in three extant manuscripts of Shirwānī’s *Sharḥ al-Tadhkira*: Topkapı Ahmet III MS 3314, fol. 15b-17a; Süleymaniye Damad İbrahim Paşa MS 847, fol. 14b-16a; and University of Tehran *Majmū‘a-i Mishkāt* MS 493, fol. 11a-12a.

which illustrate the range of scholarship carried out in Samarqand under Qāḍīzāde and Ulugh Beg.⁴⁵

After Samarqand, Shirwānī first returned with his father to Shirwān and taught in that region for a few years. With Qāḍīzāde's encouragement he traveled to Anatolia. Qāḍīzāde was originally from a prominent family in Bursa, where his father was a scholar and his grandfather had been a judge.⁴⁶ Along with the contacts Qāḍīzāde possessed as head of Ulugh Beg's madrasa in Samarqand, such socio-intellectual connections may have helped Shirwānī start his career in Anatolia. He settled in Kastamonu in north-central Anatolia (now the capital of Kastamonu Province, Turkey) and was received by the local ruler İsmail Bey (r. 1443-1461) of the Isfandiyarid (Candarid) dynasty. He lectured there for several years, teaching from the *Tadhkira* and commentaries by Qāḍīzāde on other mathematics and astronomy texts, as well as lecturing on *kalām*.⁴⁷ The biographical sources report that he lectured at madrasas and taught many students.⁴⁸

45. For a translation and discussion of the *ijāza* see below, Chapter 4, and Fazlıoğlu, "The Samarqand Mathematical-Astronomical School," 40-49.

46. F. Jamil Ragep, "Qāḍīzāde al-Rūmī: Salāḥ al-Dīn Mūsā ibn Muḥammad ibn Maḥmūd al-Rūmī," in *Biographical Encyclopedia of Astronomers*, ed. Thomas Hockey et al., (New York: Springer, 2007), 942.

47. Taşköprüzade, *al-Shaqā'iq al-Nu'māniya fī 'ulāmā' al-Dawla al-Uthmāniya* (Beirut: Dar al-Kitāb al-Arabi, 1975), 65-66.

48. Taşköprüzade mentioned that his father's maternal uncle Muḥyī al-Dīn Muḥammad ibn Ibrāhīm al-Nīksārī studied the religious and mathematical sciences under Shirwānī, Taşköprüzade, *al-Shaqā'iq al-Nu'māniya*. Likewise Katib Çelebi mentioned that Kamāl al-Dīn Mas'ūd b. Ḥusayn al-Shirwānī, a famous scholar and author of works on logic and theology, was Shirwānī's student, Katib Çelebi, *Kashf al-Zunūn 'an asāmī al-kutub wa-l-funūn* (Istanbul: Maarif Matbaası, 1941).

Shirwānī also received patronage from the influential grand vizier Halil Paşa, to whom he dedicated a Qur'an commentary in 857/1453 in Bursa.⁴⁹ That same year Shirwānī accompanied Halil Paşa to the conquest of Constantinople (1453), and around this time he dedicated a treatise on music theory to Sultan Mehmed II the Conqueror.⁵⁰ Unfortunately for Shirwānī, after the conquest of Constantinople the sultan had Halil Paşa executed. Having lost his patron, it appears Shirwānī was unsuccessful in joining the sultan's circle and instead returned to Kastamonu where he continued teaching for several years. However, around 872/1467 while Shirwānī was lecturing at madrasas in Iraq and Mecca after making the hajj, Mehmed II wrote a letter inviting him to visit Istanbul after returning from Arabia.⁵¹ Shirwānī passed through Cairo, spending an unknown amount of time there on his way back to Anatolia.⁵² In 878/1473 he completed a supercommentary (*hāshiya*) on Qāḍīzāde's commentary on Jaghmīnī's *al-Mulakhkhaṣ fī al-hay'a al-basīṭa*, a famous elementary astronomical text. He dedicated his work to Mehmed II in a second, apparently unsuccessful, attempt to gain patronage from the sultan. It was in 879/1475 that Shirwānī completed his commentary on Ṭūsī's *Tadhkira*, which is the focus of chapters 2 and 3 below. This work does not contain a dedication but was probably given to Mehmed II as it was already held at the Topkapı Palace library during the reign of Mehmed's successor Bayezid

49. Shirwānī, *Tafsīr āyat al-Kursī*, Fatih MS 167 and Bayezıt State Library MS 628.

50. A facsimile of Istanbul Topkapı Sarayı Ahmet III MS 3449, with brief introduction by Fuat Sezgin, is available as Faṭḥallāh Shirwānī, *Majallah fī al-mūsīqī (Codex on Music)*, ed. Fuat Sezgin (Frankfurt: Institut für Geschichte der Arabisch-Islamischen Wissenschaften, 1986).

51. Mehmed II's letter to Shirwānī was published in *Fatih Devrine Ait Münşeat Mecmuası*, ed. Necati Lugal and Adnan Erzi (Istanbul: Istanbul Matbaası, 1956), XXIII. I am grateful to Abdurrahman Atçıl for bringing this letter to my attention.

52. On the dates of Shirwānī's *hajj* and activities outside Anatolia, see al-Sakhāwī, *al-Ḍaw' al-lāmi*, vol. 6, p. 166-167.

II.⁵³ A few years later, Shirwānī – by now approximately 60 years old – left Anatolia to return to Shirwān, likely the city of Shamākhī, where he remained until his death, about eight years later in 891/1486.⁵⁴

One wonders about the effect the search for patronage, and the institutional settings of madrasa and court, had on Shirwānī's career. He was educated at one of the most prominent intellectual centers of his period, the Timurid madrasa and observatory in Samarqand. That gathering of scholars, including noted theologians, astronomers, and mathematicians, was made possible through the direct support and involvement of the ruler Ulugh Beg. Ottoman biographical sources specifically link Shirwānī with Anatolian elites such as Ismail Bey in Kastamonu, the grand vizier Halil Paşa, and Sultan Mehmed II himself. Despite spending many years in Kastamonu, Shirwānī does not seem to have dedicated any texts to Ismail Bey. In contrast, although Shirwānī dedicated multiple texts to Mehmed II, he left Istanbul at various points in his career to teach and travel for extended periods of time, perhaps implying that he did not receive the benefits one might expect from such a patron. It is not even clear whether Shirwānī dedicated the texts to Mehmed II in an attempt to gain the sultan's favor, or whether he needed permission to use the sultan's name ahead of time. Shirwānī's later career may have been

53. In addition to the seal of Sultan Ahmet III (r. 1703-1730), Topkapı Ahmet III MS 3314 contains Bayezid II's seal on fols. 1a and 368b. The colophon mentions this manuscript was copied from an autograph.

54. Despite leaving Anatolia, Shirwānī's return to Shirwān did not necessarily mean he had faded into obscurity or was disconnected from networks of scholars and traders passing through Central Asia. The cities of Shabārān and Shamākhī both remained regional strongholds at this time. Giosafat Barbaro, the Venetian traveler and ambassador to "Persia," passed through Shirwān and the city of Shamākhī in 1476. In the account of his travels, he described "Sammachi" as "a very good city" of 3,000-5,000 houses, able to supply between 8,000–10,000 horsemen at a need, and producing silks, heavy cotton, and other goods. Josafa Barbaro and Ambrogio Contarini, *Travels to Tana and Persia* (London: Haklyut Society, 1873), 86.

marred by his former association with the executed grand vizier. After this tragic event, Shirwānī's contacts perhaps no longer had the connections necessary to bring him closer to the sultan's court.

Near-contemporary anecdotes about the careers of scholars under Mehmed II suggest another limiting factor. As part of a process of reforming the fiscal affairs of the Ottoman territories, Mehmed II claimed ownership of endowed properties (*awqāf*, sing. *waqf*) and redistributed lands and tax rights to the military. Most Ottoman scholars considered this dissolution of *awqāf* illegal, and it made the sultan's patronage practices highly controversial.⁵⁵ At the same time, Mehmed II himself was building new mosques and endowing madrasas in Istanbul in an effort to rebuild the city. His policies included forcibly resettling the inhabitants of conquered regions and strongly encouraging merchants from wealthy cities such as Bursa to relocate, efforts that were met with strong resistance.⁵⁶ All of this gave Mehmed and his closest advisors a large degree of control over the careers of scholars, by controlling appointments to positions as judges and madrasa lecturers.⁵⁷ For example, shortly before Shirwānī dedicated his commentary on the *Mulakhkhaṣ* to Mehmed II in 878/1473, Mehmed II had appointed one of Shirwānī's colleagues from Samarqand, 'Alī al-Qūshjī, as head of the Ayasofya madrasa in

55. Oktay Özel, "Limits of the Almighty: Mehmed II's 'Land Reform' Revisited," *Journal of the Economic and Social History of the Orient* 42, no. 2 (1999), 226-246.

56. For an impressive, detailed study of Mehmed II's efforts to reconstruct the city, and in so doing reorder social and public space, see Çiğdem Kafescioğlu, *Constantinopolis/Istanbul: Cultural Encounter, Imperial Vision, and the Construction of the Ottoman Capital* (University Park: Pennsylvania State University Press, 2009).

57. Anooshahr argues that Taşköprüzade's anecdotes reflect "an endemic pattern of dismissal and excessive royal interference in the affairs of the ulema" in Ali Anooshahr, "Writing, Speech, and History for an Ottoman Biographer," *Journal of Near Eastern Studies* 60, no. 1 (2010), 52.

Istanbul.⁵⁸ Given the available sources, it is difficult to determine the extent to which Shirwānī's career is representative of other scholars prior to the creation of a more centralized bureaucracy governing scholarly appointments in a hierarchy of legal and administrative positions.⁵⁹

Ultimately, further research is necessary to better understand the practices and obligations of scholars under the patronage of Mehmed II before we can effectively evaluate Shirwānī's career.

Finally, one should not forget that Mehmed II's reign was a time of turmoil in Anatolia, particularly in Istanbul. Plagues repeatedly afflicted the city, with contemporary chronicles describing massive outbreaks in 859/1455 and 872/1467 in which perhaps over 50% of the population died or fled. Smaller outbreaks occurred in the following decade. As a result, the sultan and his court had to remain in the Balkans after military campaigns rather than return to the city in 859/1455 and 872/1467. These outbreaks likely wiped out over a decade of efforts by Mehmed II to repopulate Istanbul.⁶⁰ When viewed in this context, Shirwānī's extended absences from Istanbul take on a different, less personal significance.

58. İhsan Fazlıoğlu, "Qūshjī: Abū al-Qāsim 'Alā' al-Dīn 'Alī ibn Muḥammad Qushċi-zāde," in *Biographical Encyclopedia of Astronomers*, ed. Thomas Hockey et al., (New York: Springer, 2007), 946-948.

59. For a biographical study of the early Ottoman *muftis*, see R. C. Repp, *The Müfti of Istanbul: A Study in the Development of the Ottoman Learned Hierarchy* (London: Ithaca Press, 1986). See also the engaging account of sixteenth century developments in Cornell Fleischer, *Bureaucrat and Intellectual in the Ottoman Empire: The Historian Mustafa Âli* (Princeton: Princeton University Press, 1986).

60. Heath Lowry, "Pushing the Stone Uphill: The Impact of Bubonic Plague on Ottoman Urban Society in the Fifteenth and Sixteenth Centuries," in *Defterology Revisited: Studies on 15th and 16th Century Ottoman Society*, (Istanbul: Isis Press, 2008), 17-50.

Shirwānī's manuscript commentaries

As is apparent from his biography, Shirwānī's scholarly interests included a range of different fields, from the mathematical sciences to jurisprudence and theology. Most of Shirwānī's extant manuscripts are in Arabic, although a couple of Persian texts are attributed to him as well. Aside from the texts mentioned above, he wrote a *ḥāshiya* (gloss or supercommentary) on al-Sayyid al-Sharīf al-Jurjānī's *Sharḥ al-Mawāqif* on *kalām* and a commentary on Taftāzānī's work on logic and *kalām*, *Tahdhīb al-Manṭiq wa-l-Kalām*. His most important works, however, were the two astronomical commentaries completed near the end of his career. The larger and more detailed of these was his commentary on Naṣīr al-Dīn al-Ṭūsī's *Tadhkira fī 'ilm al-hay'a*, and the second was a supercommentary or gloss on his teacher Qāḍīzāde's commentary on Jaghmīnī's *Mulakhkhaṣ*. All of Shirwānī's texts are unedited. In this dissertation, I have utilized the following manuscripts, which represent all of the extant copies of the texts known to me.

I. *Sharḥ al-Tadhkira* – Commentary on [Ṭūsī's] *Tadhkira*

1. Istanbul, Topkapı Museum Library, Ahmet III MS 3314. Contains 368 folios numbered with Indo-Arabic numerals, 23 lines/page, catchwords, and marginalia. A fine presentation copy with decorative heading and *bismillah* on first page in gold and rich colors, ruled borders throughout. Copied from an autograph, according to the colophon Shirwānī completed the text on the night of Thursday, 3 Ramadan 879 / 11 January 1475. The manuscript copy's date is unknown, but it bears the seal of Sultan Bayezid II (r. 1481-1512) on fol. 1a and 368b, indicating that the copy was completed within Shirwānī's lifetime or shortly after his death. References to *Sharḥ al-Tadhkira* throughout this dissertation are to this manuscript (unless

otherwise noted) due to defects with the other two extant manuscripts. I have relied on the other manuscripts to confirm or correct my reading of the text when necessary.



Figure 1.1 Ahmet III MS 3314, fol. 1b-2a

2. Istanbul, Süleymaniye Library, Damat Ibrahim Paşa MS 847. Contains 212 folios numbered with Indo-Arabic numerals as well as very faint Eastern Arabic numerals, 21 lines/page, catchwords, and extensive marginalia. Copied from an autograph in the year of Shirwānī's death, 891/1486. The extant folios have been numbered consecutively in its current binding. Prior to binding, however the manuscript appears to have been dropped or damaged at some point in its history and large groups of folios are missing. For example, fol. 124b ends in the middle of Book II, Chapter 8 and fol. 125a picks up toward the end of Book II, Chapter 11,

and much of the discussion of Mercury (II.8), the remaining planets (II.9), planetary latitudes (II.10), and the Tūsī Couple and solutions to “difficulties” with Ptolemaic models (II.11) is lost.

3. Tehran, Tehran University Central Library, Mishkāt MS 493.⁶¹ Contains 242 folios numbered with Eastern Arabic numerals, 25 lines/page, and catchwords. Date unknown. The manuscript is incomplete, ending in the middle of a chapter (*fāṣl*) Shirwānī added between Chapters 11 and 12 of Book III, drawing from Ibn al-Haytham’s optical work on shadows.

II. *Hāshiya ‘alā Sharḥ al-Mulakhkhaṣ – Gloss on [Qāḍīzāde’s] Commentary on [Jaghminī’s] Mulakhkhaṣ*

1. Istanbul, Topkapı Museum Library, Ahmet III MS 3294. Contains 99 folios numbered with Indo-Arabic numerals, 15 lines/page, and catchwords. Completed in 878/1473, a date encoded by the *abjad* system of letter values in the phrase: “*fī sana tārikhuhā wa yanşarak Allāhu naşr^{an} ‘azīz^{an}.*” Seal of sultan Bayezid II (r. 1481-1512) on fol. 1a, 90b. Waqf seal of sultan Ahmed III (r. 1703-1730) on fol. 1a. Copyist: Darwīsh Muḥammad b. Muḥammad.

61. I am grateful to İhsan Fazlıoğlu for sharing a microfilm copy of this manuscript.



Figure 1.2 Ahmet III MS 3294, fol. 1b-2a

III. *Tafsīr āyat al-Kursī* – Exegesis of the [Qurʾānic] Throne Verse

1. Istanbul, Süleymaniye Library, Fatih MS 167. Contains 97 folios numbered with Eastern Arabic numerals, 11 lines/page, and catchwords. An autograph copy produced on Sunday, 9 Şafer 857 / 18 February 1453, from the rough copy completed Sunday, 10 Muḥarram 857 / 20 January 1453, in Bursa. The word “waqf” and seal of Sultan Bayezid II (r. 1481-1512) are on fol. 1b, seal is also on fol. 97a.

2. Istanbul, Beyazıt State Library, MS 628. Contains 72 folios numbered with Eastern Arabic numerals, 13 lines/page, and catchwords. Copy made from an autograph, with marginal glosses (*taʿlīqāt*) taken from Shirwānī’s instructions, completed Friday, 11 Rabīʿ al-Akhar 873 / 28 October 1468. The manuscript has been trimmed over its lifetime and some catchwords are

cut through or entirely gone. Some pages have marginalia that extend onto part of the page beyond the current dimensions, frequently this was cut through as well. Decorative heading on first page and borders on first two pages. Waqf stamp on fol. 1a, 72a. Purple library stamp on fol. 2a, 43a, 55b, 62b, 69b, 71b. I have not been able to decipher the stamp text beyond the year 1301 [1883/84]. Copyist: Yūsuf b. Umar b. Muhammed b. ‘Umar b. Abī Bakr b. Muḥammad b. Aḥmed b. Muḥammad b. ‘Abd al-Qāhir b. Hibbat Allāh Ibn <al-Naṣabī> [final name is undotted]. I have generally used this manuscript rather than the earlier autograph since this one incorporates Shirwānī’s minor revisions and was copied directly from an autograph.



Figure 1.3 Beyazıt State Library, MS 628, fol. 1b-2a

Shirwānī and the Science of the Stars

Chapters 2 and 3 of this dissertation offer a detailed look at the technical content of Shirwānī’s massive commentary on Ṭūsī’s *Tadhkira*. I explore the means by which Shirwānī made his

commentary useful for an audience of trained astronomers and advanced students. He inserted geometric proofs, explicitly stated hidden assumptions and explored their validity, and linked his analysis to ongoing criticisms and debates about astronomical models in the works of contemporary scholars.

To understand the analyses that follow, it is important to know that various terms signify the science and study of the stars and celestial phenomenon in medieval Arabic:

- *‘ilm al-nujūm* (“the science of the stars” in the broadest sense that could include mathematical astronomy and astrology)
- *‘ilm aḥkām al-nujūm* (“the science of the judgments of the stars” or celestial influence in the form of astrology in particular)
- *‘ilm al-falak* (the science of the orbs or physical celestial spheres that structured the cosmos)
- *‘ilm al-hay’a* (“the science of the configuration [of the orbs/spheres]” referring to mathematical and natural philosophical models of the Earth and the cosmos)

These terms refer to a broad attempt to understand the various celestial phenomena and they approach the understanding of those phenomena with a variety of tools.⁶² Some scholars made mathematical attempts simply to calculate predictions of celestial phenomena in abstraction from the physical characteristics presumed to underlie them. (In other words, they treated the motions of dots of light against the background of the fixed stars). More concretely, others attempted to explain the motions of the heavens using models that could be constructed physically (e.g., they assumed that the mathematical models were representations of physical bodies – spheres, etc. – carrying the planets or stars). In addition, yet others took philosophical approaches to

62. For an overview of the study of the heavens in Islamic societies, with an emphasis on *hay’a*, see F. Jamil Ragep, “Astronomy,” in *Encyclopaedia of Islam*, 3rd Edition, (Leiden: Brill, 2007-),

understanding the natures and properties of the elements and celestial spheres in a broader framework that was inherited from the Hellenistic period (Aristotelian and Neoplatonic thought were the main influences in this approach). Remarkably, in Shirwānī's commentary on the *Tadhkira fī 'ilm al-hay'a* we find an approach that integrates all of these strands.

As understood by Shirwānī's Islamic predecessors Ibn al-Haytham and Naṣīr al-Dīn al-Ṭūsī, *'ilm al-hay'a* was the science of the “configuration” or “arrangement” (*hay'a*) of celestial orbs that produced the apparently-irregular motions of the Sun, Moon, and planets thanks to uniform motions. Moreover, it was a mathematical study of these spheres as simple physical bodies without consideration of their (Aristotelian) essential natures. Jamil Ragep has argued that one of the key shifts in the development of *hay'a* out of late Hellenistic astronomy was an increasing concern with physical cosmology and the “relationship between the physical principles and the mathematical models.”⁶³ In the Arabic *hay'a* tradition, astronomers mounted a sustained critique of Ptolemy's equant, a mathematical hypothesis employed to produce uniform motion along a circle with respect to a point other than the circle's center.⁶⁴ The *hay'a* astronomers argued that the equant violated Ptolemy's own commitment to uniform circular motion and that it was a physical impossibility. Their goal was to maintain or improve the predictive accuracy of Ptolemy's models, but with a system in which the mathematical models describing the planetary motions could correspond to three-dimensional physical bodies, i.e., could be constructed.⁶⁵ Ibn al-Haytham, one of the earliest proponents of such a physical

63. Ragep, *Tadhkira*, 47.

64. The equant and other mathematical hypotheses will be discussed in detail in Chapter 2.

65. A. I. Sabra, “Configuring the Universe: Aporetic, Problem Solving, and Kinematic Modeling as Themes of Arabic Astronomy,” *Perspectives on Science* 6, no. 3 (1998): 288-330.

cosmography, praised Ptolemy for his mathematical achievement but noted in *Shukūk 'alā Baṭlamyūs* (Doubts concerning Ptolemy) that there were “dubious passages...and contradictory meanings.”⁶⁶ He went on to reject Ptolemy’s justification that the equant was necessary to match the planetary models to the observed data, stating that “this statement does not excuse his assumption of false configurations whose existence is impossible...for it is not permissible that the actual motions of the planets [be stipulated] by a configuration whose existence is impossible.”⁶⁷ Although Ibn al-Haytham did not himself provide a resolution to most of these troublesome points, his work inspired later astronomers to invent models in which each motion within the model was explained in terms of a physical body.⁶⁸

In the 13th century, new political dynasties took shape as Mongol armies swept through much of China, Central Asia, and into the Middle East. In 1259 Hulegu Khan (grandson of Genghis Khan) established a large observatory at Marāgha in modern-day Uzbekistan.⁶⁹ To staff the observatory he gathered astronomers and mathematicians, including Naṣīr al-Dīn al-Ṭūsī and

66. Ibn al-Haytham, *al-Shukūk 'alā Baṭlamyūs (Doubts about Ptolemy)*, ed. A. I. Sabra and Nabil Shehaby (Cairo: National Library Press, 1971), 4. See also A. I. Sabra, “An Eleventh-Century Refutation of Ptolemy’s Planetary Theory,” in *Science and History: Studies in Honor of Edward Rosen* (Studia Copernicana XVI), (Wrocław: Polish Academy of Sciences Press, 1978), 117-131.

67. Ibn al-Haytham, *al-Shukūk 'alā Baṭlamyūs*, 38, my translation.

68. In another text, commonly attributed to Ibn al-Haytham, he seems to follow the Ptolemaic cosmology more closely. See Ibn al-Haytham, *Ibn al-Haytham’s On the Configuration of the World*, ed. and tr. Y. Tzvi Langermann, (New York: Garland, 1990). Rashed has argued that the 13th-century bio-bibliographers mistakenly identified two different scholars named Ibn al-Haytham as the same individual, most recently in Roshdi Rashed, “*The Configuration of the universe: A book by al-Ḥasan ibn al-Haytham?*,” *Revue d’histoire des sciences* 60, (2007): 47-63. For a counter-argument, see also A. I. Sabra, “One Ibn al-Haytham or Two? An Exercise in Reading the Bio-Bibliographical Sources,” *Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften* 12, (1998): 1-50 and A. I. Sabra, “One Ibn al-Haytham or Two? Conclusion,” *Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften* 15, (2002/03): 95-108.

69. On the Marāgha Observatory, see Sayili, *The Observatory in Islam*, 187-223.

Mu‘ayyid al-Dīn al-‘Urḍī (d. c. 1266). Prior to his arrival at Marāgha, Ṭūsī had produced new planetary models relying entirely on combinations of uniform circular motions of spheres, an innovation named the “Ṭūsī Couple” by modern historians.⁷⁰ Similarly, ‘Urḍī had developed a mathematical principle now called the “‘Urḍī lemma” to eliminate the equant.⁷¹ These innovations led to the development of multiple non-Ptolemaic planetary models by successive generations of Islamic astronomers in the 13th and 14th centuries. Discussions of these models and efforts in the broader project to reform Ptolemaic astronomy were largely carried out within the genre of commentaries on Ṭūsī’s *Tadhkira* which were used as teaching texts for advanced students of astronomy. As George Saliba and other historians have argued, such sustained, vibrant, and creative activities force us to reject the long-held belief that science and other areas of intellectual life in the Middle East and Central Asia had entered into a deep decline in the aftermath of the 13th-century Mongol invasions.⁷² In Chapters 2 and 3, I explore Shirwānī’s *Tadhkira* commentary, which points out his own views while simultaneously drawing on generations of previous scholarship in the *Tadhkira* commentary genre. As I argue, through the process of commenting on the *Tadhkira*, Shirwānī produced a work that was suitable for a narrower and more advanced audience than a more introductory work like the *Tadhkira*. He

70. On the Ṭūsī Couple, see E. S. Kennedy, “Late Medieval Planetary Theory,” *Isis* 57, no. 3 (1966): 365-378 and F. Jamil Ragep, “The Persian Context of the Ṭūsī Couple,” in *Naṣīr al-Dīn al-Ṭūsī: Philosophe et Savant du XIIIe siècle*, ed. N. Pourjavady and Ž. Vesel, (Tehran: Institut français de recherche en Iran/Presses universitaires d’Iran, 2000), 113-130.

71. For the ‘Urḍī Lemma, see the various papers by George Saliba collected in his Saliba, *A History of Arabic Astronomy: Planetary Theories During the Golden Age of Islam*, particularly George Saliba, “The Original Source of Qutb al-Dīn al-Shīrāzī’s Planetary Model,” *Journal for the History of Arabic Science* 3, (1979): 3-18.

72. Saliba forcefully critiques the “classical narrative” about late medieval scientific decline in his recent book Saliba, *Islamic Science and the Making of the European Renaissance*.

restored missing geometric proofs and introduced the reader to cutting-edge developments in astronomy that were being actively discussed in Samarqand.

Scientific education

In Chapter 4, I argue that features of Shirwānī's commentary on the *Tadhkira* mark it as a practical teaching text in addition to an exposition of theoretical astronomy. His commentary reveals new insights about the process of transmitting scientific knowledge in the post-classical period. In exploring these features of Shirwānī's commentary, I emphasize the importance of paying attention to diversity among educational practices in Islamic societies rather than constructing a single model of "Islamic education." The specific Samarqand milieu provides a set of examples of the kinds of differences that emerge when one views education in this light.

Shirwānī related several fascinating anecdotes about his experiences as a student in the lecture hall at Samarqand, describing his interactions with Sultan Ulugh Beg, his teacher Qāḍīzāde, and other members of the madrasa. These anecdotes, nearly all of which are translated and studied here for the first time, provide an invaluable account of what actually happened orally in a 15th-century classroom. I address important institutional questions about education in the exact sciences in Islamic societies and the transmission of knowledge from Central Asia to Anatolia. I challenge George Makdisi's thesis that the sciences were excluded from madrasa curricula, analyzing Shirwānī's commentary as a teaching text.⁷³ The evidence of multiple works such as Shirwānī's, on a variety of technical fields, provides strong evidence for the continued transmission of expertise in the exact sciences within a tradition of financial and institutional support for primarily religious scholarship. I argue that, in the Islamic context, the structure of

73. Makdisi, *Rise of Colleges*, 77-78.

commentaries reflected the structure of education itself, which centered more on the transmission of texts and the teacher-student relationship than on institutions. As I will show, Shirwānī's quotations from what is ostensibly Qāḍīzāde and Ulugh Beg's direct speech mirror his quotations from written texts, emphasizing the central role of the oral and the personal in the educational encounter. Furthermore, through close attention to Shirwānī's explanatory comments and other rhetorical devices, I argue that we can get the "feel" for the kinds of questions his students would commonly raise in the course of their education, as well as the variety of attitudes they displayed towards the philosophical and theological tensions associated with study of the natural world. Finally, Shirwānī's inclusion of a variety of supplementary material related to theoretical astronomy, such as information about the fixed stars, calendar systems used by various cultures, and methods for determining the *qibla* or direction to face during Muslim prayers, taught the reader how astronomical knowledge could be applied.

The diverse contents of commentaries

A close study of Shirwānī's commentaries reveals that his interests encompassed a range of issues in medieval Islamic intellectual history. Astronomy was closely connected to a number of other scientific disciplines, and moreover it touched on practical concerns in Islamic societies. In Chapter 5, I explore Shirwānī's work in three major areas: the rediscovery of Ibn al-Haytham's *Optics* and its application to astronomy, his extensive discussion of the geographic coordinates of various locales, and the relationships between astronomy, natural philosophy, and theology.

Even if he had done nothing else, Shirwānī would be a critical figure in the curious story of the fate of optics in Islamic societies. Ibn al-Haytham's *Optics* (*Kitāb al-Manāẓir*), which first synthesized the best parts of earlier theories and provided a correct solution to the problem of

coherent vision, was unquestionably the most significant and influential work of optics before Johannes Kepler (d. 1630). However, it seems to have had almost no impact in the Arabic-speaking world for two centuries following Ibn al-Haytham's death in 1040. It was not until the 13th century that Quṭb al-Dīn al-Shīrāzī, a scholar at the Marāgha Observatory and former student of Ṭūsī, told his student Kamāl al-Dīn al-Fārisī that as a youth he had seen a large work on optics attributed to Ibn al-Haytham. Sometime later, Shīrāzī obtained a copy of the *Optics* "from a distant land" for Fārisī, who proceeded to write a commentary and analysis entitled *Tanqīḥ al-Manāẓir*.⁷⁴ Thereafter, Ibn al-Haytham's *Optics* received a very small number of further analyses. These include the one contained in Shirwānī's commentary and a single manuscript written by the astronomer Taqī al-Dīn ibn Ma'rūf, who headed the short-lived Istanbul Observatory in the late 16th century.⁷⁵ In short, Shirwānī's interaction with Ibn al-Haytham's work is a landmark in the history of Arabic optics. In the first section of Chapter 5, I offer a preliminary analysis of Shirwānī's account of optics and its relation to the work of Ibn al-Haytham and Kamāl al-Dīn al-Fārisī, as well as Shirwānī's assessment of other scholars who worked on astronomy and optics, such as Quṭb al-Dīn al-Shīrāzī and Nizām al-Dīn al-Nīsābūrī. The rediscovery of Ibn al-Haytham's optics had significant consequences for the practice of astronomy, as it dealt with errors in visual perception and pointed to solutions to observational

74. The fate of Ibn al-Haytham's *Optics* in the Islamic Arabic and Persian worlds is outlined in Sabra, "The 'Commentary' that Saved the Text".

75. Recently, Bilal Ibrahim has demonstrated that Fakhr al-Dīn al-Rāzī (d. 1210) at the very least had access to Book II, Chapter 3 of the *Optics* and incorporated aspects of Ibn al-Haytham's theory of perception and sensible phenomena. See Bilal Ibrahim, "Freeing Philosophy From Metaphysics: Fakhr Al-Dīn Al-Rāzī's Philosophical Approach to the Study of Natural Phenomena" (PhD diss., McGill University, 2013), Ch. 6 and Bilal Ibrahim, "Fakhr ad-Dīn al-Rāzī, Ibn al-Haytham and Aristotelian Science: Essentialism versus Phenomenalism in Post-Classical Islamic Thought," *Oriens* 41, (2013): 379-431.

challenges such as the refraction of light through the atmosphere, especially near the horizon. My research explores Shirwānī's discussion of these issues and explains how he brought these "new" optical theories to bear on the study of the heavens.

The *hay'a* tradition of Islamic astronomy did not focus exclusively on the "configuration" of the celestial orbs. Astronomers also paid attention to the astronomical phenomena that depended on the observer's location on the surface of the Earth, the sizes and distances of the Earth and celestial bodies, and astronomically determined divisions of the Earth. In the second section of Chapter 5, I present a study of Shirwānī's extended discussion of geographic coordinates for various locales across the inhabited world in another of his commentaries. Shirwānī wrote this supercommentary on his teacher Qāḍīzāde's commentary on the elementary astronomical treatise *al-Mulakhkhaṣ* shortly after leaving Samarqand, and dedicated it to the grand vizier of the Ottoman sultan Mehmed II. I argue that Shirwānī's inclusion of these coordinates suggests the traditions of Arabic and Persian geographic texts continued into fifteenth-century Anatolia, alongside the Ptolemaic geographic texts and maps that Mehmed II was obtaining from Europe.

Shirwānī's scholarship in a variety of fields provides further evidence that historians must reject the "science versus religion" paradigm widely prevalent in Western intellectual circles in favor of a nuanced approach to how ideas about the physical world and theology evolved in dialogue both with, and against, one another. Shirwānī worked within a tradition that had grappled with Hellenistic natural philosophy and attempted to interpret it in light of religious doctrines. By the time Shirwānī began his studies, various schools of Islamic speculative and dialectic theology (*kalām*) had come to presume a basic familiarity with the methods and principles of sciences such as astronomy. In the third section of Chapter 5, I argue that what was

still at stake for scholars such as Shirwānī was the validity of astronomy's methods and the extent to which it invoked notions of causality, the reliability of human rationality as a means to obtaining knowledge, and physical principles that could not be harmonized with *kalām* theories. As we will see, in both his commentary on the *Tadhkira* and a commentary on the Throne Verse of the Qur'an, Shirwānī took issue with some of his predecessors and defended the value of science, and astronomy in particular, as a means of understanding the role of intermediate causes in God's creation. As I will argue, Shirwānī attempted to maintain a middle ground position on these issues. Although he followed Ṭūsī's direction in emphasizing that *hay'a* was the mathematical study of physical bodies, Shirwānī nevertheless displayed a renewed interest in natural philosophy as an essential component of astronomy and criticized the attempts of contemporary scholars, such as his teacher Qāḍīzāde al-Rūmī, to focus exclusively on the mathematical (as opposed to physical) concerns. Shirwānī provides insight into the ongoing debates about physical cosmology, the relative value of mathematical and natural philosophical approaches to the study of the heavens, and the status of Aristotelian principles centuries after their supposed rejection by Muslim theologians.

Chapter 2 – Astronomy in Shirwānī’s commentary on the *Tadhkira*, I: The Basic Hypotheses and the Sun

Introduction

Although a commentary, Shirwānī’s *Sharḥ al-Tadhkira* was first and foremost a textbook of mathematical astronomy. Ṭūsī’s *Tadhkira* itself had presented a summary of the results of the *Almagest* for nonspecialists. Ṭūsī had omitted the observational data and the geometric proofs that Ptolemy had used to construct his models, and adapted those models to versions that could be manifested in the motions of physical orbs. However, despite Ṭūsī’s desire to simplify, the attempt to explain the celestial motions in terms of solid orbs was hindered by unresolved problems in the *Almagest* models. Throughout the *Tadhkira*, Ṭūsī had highlighted several of the “difficulties” (*ishkālāt*) that arose when the *Almagest* models seemed to violate the principles believed to govern motion in the heavens, and attempted to solve them through the innovative use of the Ṭūsī Couple, as it has now been dubbed. In his preface to *Sharḥ al-Tadhkira*, Shirwānī stated that he wished to provide a more detailed and comprehensive text than the *Tadhkira* commentaries by either Nizām al-Dīn al-Nīsābūrī or al-Sayyid al-Sharīf al-Jurjānī, which were two of the main texts Shirwānī read as a student in Samarqand.¹

This chapter, along with the subsequent chapter, focuses on Shirwānī’s commentary on Book II of the *Tadhkira* - “The Configuration of the Celestial Bodies,” the largest of its four books of the *Tadhkira* and the one devoted to discussion of the *Almagest* program of celestial

1. Faṭḥallāh al-Shirwānī, *Sharḥ al-Tadhkira*, Topkapı Ahmet III MS 3314, fol. 1b-2a.

motions and models. Shirwānī's commentary closely followed the *Tadhkira* with explanations and comments between sentences, phrases, or even individual words. This approach gave him the opportunity to insert clarifying remarks and relevant quotations from other scholars, along with his own critiques of both them and Tūsī when necessary. Following the analysis of some of the technical topics in Shirwānī's commentary on Book II, in Chapter 4 we will return to this material to focus specifically on the pedagogical aspects of Shirwānī's presentation and the ways in which his commentary sheds light on 15th c. Islamic scientific educational practices.

Although one often thinks of a commentary as simplifying, rather than complicating, the material it treats, this chapter demonstrates that in the process of commenting on the *Tadhkira* Shirwānī produced in a textbook suitable for use by a narrower audience than that of the *Tadhkira* itself, namely specialists and students who already possessed a background in the mathematical sciences. Shirwānī's work does not appear to be unique in this regard; rather, it is an example of a category of commentary through which the commentator engaged critically with the text in order to present new ideas and material. In this sense, Shirwānī's *Tadhkira* commentary bears strong resemblances to that of Nīsābūrī, as well as to the works of the later Islamic astronomer Shams al-Dīn al-Khafīrī (d. 1550), who wrote highly technical commentaries on the *Tadhkira* and Quṭb al-Dīn al-Shīrāzī's *Nihāyat al-idrāk*.² Shirwānī restored many of the geometric proofs that must be worked through in order to develop a concrete understanding of

2. As with most of the Islamic astronomers mentioned in this dissertation, Khafīrī's texts remain unedited and largely unstudied. However, George Saliba has produced preliminary surveys that attest to the quality of Khafīrī's work. Among others, see George Saliba, "A Sixteenth-Century Arabic Critique of Ptolemaic Astronomy: The Work of Shams al-Dīn al-Khafīrī," *Journal for the History of Astronomy* 25, (1994): 15-38 and George Saliba, "The Ultimate Challenge to Greek Astronomy: *Ḥall mā lā yanḥall* of Shams al-Dīn al-Khafīrī (d. 1550)," in *Sic Itur Ad Astra: Studien zur Geschichte der Mathematik und Naturwissenschaften; Festschrift für den Arabisten Paul Kunitzsch zum 70. Geburtstag*, ed. Menso Folkerts and Richard Lorch, (Wiesbaden: Harrassowitz, 2000), 490-505.

how the models functioned, and situated the models in the context of the aims and methods of observational and theoretical astronomy.

Shirwānī also introduced the reader to the most recent developments in astronomy through frequent discussion and analysis of more advanced *hay'a* texts written after the *Tadhkira*. He repeatedly referenced Jurjānī's commentary on the *Tadhkira*, Nīsābūrī's commentaries on both the *Tadhkira* and Ṭūsī's recension of the *Almagest*, his teacher Qāḍīzāde's lectures at Samarqand, and most importantly Quṭb al-Dīn al-Shīrāzī's *Nihāyat al-Idrāk* and *al-Tuhfa al-Shāhīya*.³ In the preface, Shirwānī praised Jurjānī and Nīsābūrī, calling their *Tadhkira* commentaries unsurpassed in the investigation of *hay'a*. "And yet," he continued in poetic rhyme:

"having read them completely...
I found in their commentaries
some things that struck me,
for all their excellence, as missteps;
for all their learning, as mistakes;
and for all their rigor, as errors."⁴

Shīrāzī's works, technically not commentaries on the *Tadhkira*, were not mentioned in the preface, but in the commentary itself Shirwānī discussed them even more frequently than the commentaries of either Jurjānī or Nīsābūrī.

3. On Shīrāzī's life and works, see Kaveh Niazi, *Quṭb Al-Dīn Shīrāzī and the Configuration of the Heavens: A Comparison of Texts and Models* (New York: Springer, 2014).

4. Shirwānī, *Sharḥ al-Tadhkira*, fol. 1b-2a.

لكني وجدت في شرحيهما - وقد قرأتها تما على مثليهما - بعض ما يضرب عنده لكل جواد كبرة ولكل عالم هفوة ولكل صارم نبوة

Relationship between Shirwānī's commentary and *hay'a basīṭa* genre

Ṭūsī's *Tadhkira* came to exemplify a genre of Islamic astronomical writing known as *hay'a basīṭa* (plain or simplified astronomy). In his edition and translation of the *Tadhkira*, Jamil Ragep has highlighted several key points that characterize Ṭūsī's work and situate it within the broader tradition of theoretical astronomy. Ṭūsī himself emphasized that the *Tadhkira* was a summary and narrative presentation of the contents of the *Almagest*, although – as Ragep noted – it is in fact neither a commentary nor a mere summary. Ṭūsī went beyond the *Almagest* in focusing on bodies, rather than mathematical circles, as the subject of astronomy. As he stated in II.5 [10], “one who attempts to understand the principles (*mabādi'*) of the motions must know the configuration (*hay'a*) of the bodies.”⁵ Moreover, according to Ragep, Ṭūsī's goal was to provide a comprehensive account of how the various models of the planets fit into a complete and unified whole, a task that Ptolemy undertook in a separate work (*Planetary Hypotheses*) rather than in the *Almagest*.⁶ However, Ṭūsī and other Islamic astronomers criticized the models in the *Planetary Hypotheses* for using equatorial sections of spheres (Ar. *manshūrāt*), rather than the whole spheres, which they believed to be the only legitimate building blocks for physical models.⁷

5. Quoted by Ragep in his discussion of these issues, F. Jamil Ragep, *Naṣīr al-Dīn al-Ṭūsī's Memoir on astronomy = al-Tadhkira fī 'ilm al-hay'a*, 2 vols. (New York: Springer-Verlag, 1993), 25.

6. Only the first part of Book I survives in the original Greek, the majority of the text is known through Arabic translation. See Bernard Goldstein, “The Arabic Version of Ptolemy's Planetary Hypotheses,” *Transactions of the American Philosophical Society* 57, no. 4 (1967): 3-55 and Elizabeth Hamm, “Ptolemy's Planetary Theory: An English Translation of Book One, Part a of the *Planetary Hypotheses* With Introduction and Commentary” (PhD diss., University of Toronto, 2011).

7. Ragep, *Tadhkira*, 216.

Ragep argued that the genre of *hay'a basīṭa* can be defined based on five crucial features found in the *Tadhkira*.⁸ Shirwānī's commentary, however, differed from such *hay'a basīṭa* texts in significant ways that made it suitable for study by more advanced readers. First, the *Tadhkira* was structured in four books:

- I. an introduction to the mathematical and physical principles required for astronomy;
- II. a cosmological section on the configuration of the celestial spheres and the motions of the planets, including Ṭūsī's attempts to resolve many of the problems he identified in the Ptolemaic planetary models;
- III. a geographical section on the Earth, explaining the astronomically determined divisions of the Earth's surface (the equator, latitude/longitude, etc.) and astronomical phenomena that depend on the latitude of the observer;
- IV. a final section on the sizes and distances of the Earth and celestial bodies.

Shirwānī's commentary, being a line-by-line commentary rather than a general discussion of the *Tadhkira*, followed Ṭūsī's structure explicitly. Within that structure, however, he inserted additional material in the places that seemed most appropriate to him. These significant additions included:

- the lengthy Appendix drawing on Kamāl al-Dīn al-Fārisī's recension of Ibn al-Haytham's *Optics*, which he placed in Book I with the principles adopted from other sciences
- an overview of how to construct a celestial globe and other knowledge of the fixed stars, which Ṭūsī had designated in Book II as "a separate discipline"

8. Ragep, *Tadhkira*, 36-41.

- an extended discussion of the calendars and epochs used by other cultures, which Ṭūsī had also left out of the *Tadhkira* with the statement “understanding the details of that [other cultures’ calendars] does not pertain to this science.”⁹

The second and third features of *hay’a basīṭa* texts were that they did not contain geometric proofs, as they were intended to summarize astronomy for nonspecialists.¹⁰ Ṭūsī claimed to provide only a narrative overview of astronomy and a discussion of the results of the *Almagest*, supplemented with his critiques of the Ptolemaic models and his efforts to correct many of their flaws. His presentation included only a minimum of detail, such that a reader had no way to understand the methods astronomers used to carry out observations, transform the observational data into parameters and motions, or construct models to account for those motions. Ṭūsī did break from this tendency on a few occasions, such as when he provided geometric proofs in conjunction with his introduction of new models for planetary motion in II.11, but he almost apologized for doing so by stating “even though it was not our intention to provide geometrical proofs...”¹¹

Shirwānī’s commentary departed most radically from the *Tadhkira*’s presentation with regard to these two features. The simplest way of characterizing the way his commentary differed from the *Tadhkira* would be to say that Shirwānī intentionally restored the geometric

9. Shirwānī’s Appendix on Optics appears at the end of Book I, and will be discussed in detail in Chapter 4. The discussion of the celestial globe device is at II.4, and the discussion of epochs and calendars is at III.10.

10. In the introduction, Ṭūsī stated, “The scientific exposition that we wish to undertake will be a summary account...presented in narrative form. The details are expounded and proofs of the validity of most of them are furnished in the *Almagest*. Indeed, ours would not be a complete science if taken in isolation from the *Almagest* for it is a report of what is established therein.” Ragep, *Tadhkira*, 92.

11. Ragep, *Tadhkira*, 198.

foundations of astronomical modeling, in all of their technical detail, just what Ṭūsī had eliminated in favor of a simplified, narrative presentation. Instead of writing a *hay'a basīṭa* text for nonspecialists, Shirwānī wrote a detailed astronomy textbook for readers who were already trained in geometry and had read an elementary introduction to astronomy elsewhere. This level of geometrical explication may have been driven by a desire to supplant Nīsābūrī's commentary in the “textbook market,” since Shirwānī had stated he wanted to write a text that was more complicated than Nīsābūrī's. As we will see, he did not hesitate to dive into extended proofs that required the reader to follow complicated geometric diagrams across multiple folios. Throughout the commentary he expanded on Ṭūsī's terse statements of fact to explain to specialists, or rather specialists-in-training, the methods that Ptolemy and other astronomers had used to derive their results and how components of the models functioned.

The fourth and fifth aspects of the *Tadhkira* that Ragep identified as characteristic of *hay'a basīṭa* texts was that the simple bodies were taken as the subject of study, but *hay'a* should only deal with their external aspects. By “simple bodies,” Ṭūsī meant the material basis for everything in the cosmos – the four classical sublunar elements (fire, air, water, earth) and the celestial aether. The cosmological vision of *hay'a basīṭa* thus included the configuration of the heavens as well as the sublunar/terrestrial realm. However, the restriction to study of the *external* aspects of these bodies differentiated astronomy as a discipline from physics or natural philosophy.

The move to define *hay'a* astronomy in more “metaphysically neutral” terms was partly a response to wide-ranging theological debates and discourse about Hellenistic natural philosophy

and metaphysics, particularly in the post-classical period.¹² According to Ṭūsī, *hay'a* was concerned with “gaining knowledge of these bodies in and of themselves, of their shapes, of the manner of their arrangement and motions, of the amounts of their motions and distances, and of the reasons for changes in position.”¹³ In premodern divisions of the sciences such as those propounded by Aristotle and Ibn Sīnā, investigation into the external aspects and motions of the simple bodies was a discipline separate from the knowledge of the essential natures of the bodies found in natural philosophy. Both astronomy and natural philosophy studied the same phenomena, but differed in the questions they asked, the evidence deemed acceptable for use in proofs, and the relative status of the knowledge that each discipline produced. In this division, an astronomer sought to describe accurately the cosmological structure, without an investigation into causes, and to derive models to produce the planetary motions on the basis of observational data and mathematics. Conversely, a natural philosopher used physical and metaphysical arguments to demonstrate that the natures of the bodies explained why the cosmos was (necessarily) structured in a particular configuration and why the bodies moved in particular ways. In keeping with his tendency to simplify details and merely present the results of mathematical astronomy, Ṭūsī largely avoided natural philosophical issues outside of his brief summary (in I.2) of the principles that needed to be accepted in astronomy from natural philosophy. Since we will discuss Shirwānī's commentary on such topics in a later chapter, for now it is sufficient to note that Shirwānī generally followed Ṭūsī's approach. However, in certain ways his commentary went beyond the *hay'a basīṭa* genre since it is clear Shirwānī felt it was

12. F. Jamil Ragep, “Freeing Astronomy from Philosophy: An Aspect of Islamic Influence on Science,” *Osiris* 2nd Series, no. 16, *Science in Theistic Contexts: Cognitive Dimensions* (2001): 49-64+66.

13. Ragep, *Tadhkira*, 90.

important for his readers to have a better understanding of the philosophical principles that Ṭūsī merely summarized, he did not hesitate to discuss philosophical principles and draw on philosophical arguments raised by other scholars.

Shirwānī and the *Almagest* program

Despite the doubts and criticisms of Ptolemaic models raised by Ṭūsī and other Islamic astronomers, and despite the complexities involved in translating the *Almagest*'s two-dimensional models into corporeal, three-dimensional orbs, the *hay'a* project of understanding and predicting the motions of the celestial bodies adopted many of the same methods for conducting observations and calculating celestial positions that Ptolemy had utilized. The reader of Shirwānī's commentary on the *Tadhkira* soon realizes that one of Shirwānī's goals was to flesh out Ṭūsī's summary of the *Almagest* and to go into greater depth by explaining some of the general principles and methods Ptolemy used to derive his models.

The starting point for any study of the heavens, whether for the purposes of timekeeping, calendar reform, or astrology, was an awareness of the variety of phenomena and apparent motions that the stars and planets displayed. In an Earth-centered cosmos, the entirety of the heavens seems to rotate around the stationary Earth in a daily motion of rising and setting from east to west. However, by observing the stars at the horizon before sunrise or after sunset, it quickly becomes apparent that the Sun and the fixed stars do not exactly rotate around the Earth at the same speed. Indeed, the Sun appears to move slowly from west to east against the background of the fixed stars, at slightly less than 1° per day, making a complete circuit through the fixed stars in one year. The great circle that the Sun traces against the background of the fixed stars over the course of the year is known as the ecliptic. The ecliptic is inclined to the

celestial equator at an angle of about 23.5° because the Sun appears to move higher in the sky relative to the celestial equator for half of the year and lower in the second half, which creates seasonal variations in climate on the earth. Likewise, the Moon completes its own west-to-east circuit through the fixed stars in just over 27 days and passes through each of its phases in slightly more than 29.5 days.

Like the Sun and Moon, the planets each have a different slow “backwards” (west-to-east) motion from relative to the fast east-to-west rotation of the sphere of the fixed stars. Mercury is the fastest and Saturn the slowest, taking 30 years to make one west-to-east circuit through the fixed stars. The planets’ motions generally keep them near the path of the Sun, but the Moon gradually wanders the most from the ecliptic, staying, however, within a band of about 6° above and below it. For reference purposes, the Babylonians divided this band of stars through which the ecliptic runs, known as the zodiac, into 12 signs of 30° each roughly corresponding to twelve constellations.¹⁴ Furthermore, once one begins to pay attention to them, the Sun, Moon, and planets display a remarkable amount of irregularity in their motions. The Sun and Moon change speed in the course of their paths around the ecliptic, and from time to time the five planets individually perform “retrograde” loops or zigzags in which they slow down before coming to a stop, reverse their direction for a while to travel from east to west, then stop again before returning to their original direction from west to east against the background of the stars.

The challenge that Greek astronomers set themselves was to account for this collection of irregular motions by means of a system of uniform circular motions, the only motion considered

14. The signs of the zodiac were initially named for the constellations located at those regions, starting from the spring equinox point. Owing to the slow precession of the equinoxes, however, the actual constellations and the 30° regions of the zodiac no longer coincide (the equinox point has “moved”).

appropriate to the perfect, eternal celestial bodies. In the Hellenistic and Islamic *hay'a* conceptions of the cosmos, there was an essential difference between the celestial realm where the heavenly bodies moved in combinations of perfect, eternal, uniform, circular motions, and the constantly changing terrestrial region of growth and decay. Aristotle explained this difference by proposing the existence of different elements in each realm: terrestrial earth, water, air, and fire below the sphere of the Moon, but with a fifth element, *aether*, in the heavens since the latter did not behave like the familiar terrestrial elements. For Aristotle, concentric spheres of *aether* rotating uniformly about the center of the cosmos carried the planets and stars. Since his concerns were not quantitative, this schema was adequate. Although such homocentric models were rejected by most later astronomers concerned with quantitative prediction, the principle of using uniform circular motions as the building blocks for planetary models remained the basis for later attempts to model the celestial phenomena and make accurate predictions.

Ptolemy's *Almagest* (2nd c.), drawing on the innovations of Apollonius of Perga (3rd c. BCE), Hipparchus (2nd c. BCE) and the rich trove of Babylonian observational data, was a masterpiece of mathematical modeling of these planetary motions.¹⁵ Because of its predictive accuracy, it was also the standard against which later models were judged. Ptolemy's models were based on a small group of mathematical hypotheses,¹⁶ each of which represented an

15. G.J. Toomer, *Ptolemy's Almagest* (Princeton: Princeton University Press, 1998) and Olaf Pedersen, *A Survey of the Almagest, With Annotation and New Commentary by Alexander Jones* (New York: Springer Verlag, 2010). See also Otto Neugebauer, *A History of Ancient Mathematical Astronomy* (New York: Springer-Verlag, 1975), 21-343.

16. Ṭūsī, Shīrāzī, and many other *hay'a* astronomers used the Arabic word *aṣl* (pl. *uṣūl*) to refer to the mathematical building blocks used in the planetary models to account for observed irregular motions. *Aṣl* has the meaning of "foundation, basis, principle, etc." such as in the phrase *uṣūl al-fiqh* (roots/principles of jurisprudence, i.e., the sources and rules used in deducing Islamic law). Here I follow Robert Morrison, "Quṭb al-Dīn al-Shīrāzī's Hypotheses for Celestial Motions," *Journal for the History of Arabic Science* 13, (2005), 23-24 in translating *aṣl* as

increasing deviation from the principle of uniform circular motion about the center of the cosmos (see Figure 2.1). This deviation was required, however, in order to account for observed irregularities in the speed and direction in the course of the planets' passages through the fixed stars. In the first hypothesis, known as the eccentric, variation in the planet's speed could be accounted for by assuming that the Earth was some distance away from the center of the planet's circular path. In this way, although the planet itself continued to perform uniform circular motion about the center of the eccentric, its apparent motion as viewed from the eccentric Earth was irregular (Ptolemy used this model for the Sun). A second hypothesis, the epicycle-on-deferent, had the planet moving along a small circle (the epicycle), the center of which was in turn carried by a larger circle centered on the Earth. Depending on the relative sizes of these circles and the speed at which they rotated, the planet could be made to produce retrograde loops. Ptolemy noted that the epicycle-on-deferent could also produce the same apparent non-uniform speed as the eccentric within a given planetary model, as Apollonius had demonstrated. In many of the *Almagest* models, these two hypotheses were combined into an epicycle with an eccentric deferent.

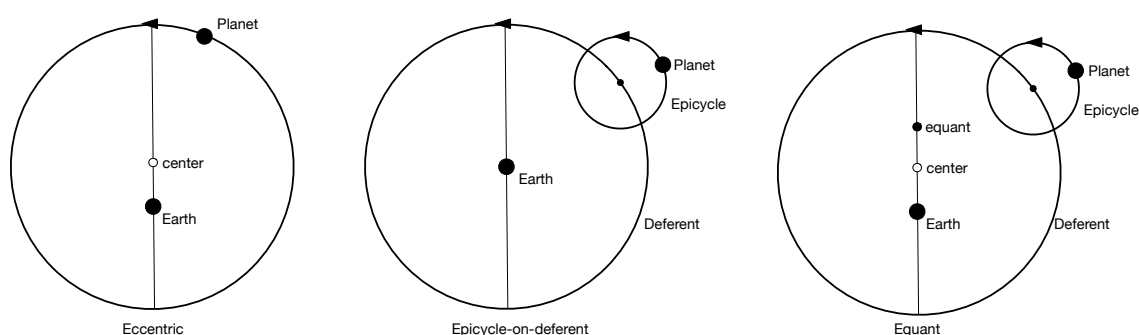


Figure 2.1 Eccentric, Epicycle, and Equant Hypotheses

“hypothesis” and using “model” to refer to the overall configuration of orbs to produce a planet's motion.

The third mathematical hypothesis, which became the focus of Islamic criticisms and spurred efforts to replace Ptolemaic models, was the equant. Although the other two were based on circles (or orbs, in the *hay'a* versions) revolving uniformly around their own centers, even when combined into an epicycle-on-eccentric deferent version they were not sufficient for Ptolemy to construct models that matched the planets' observed motions. In order to match the observational data more precisely, Ptolemy felt forced to model the fact that, as the epicycle center moved along the deferent, its motion was uniform not with respect to the deferent center, but instead with respect to a different point on the line connecting the center of the Earth and the center of the eccentric deferent, subsequently called the equant point (see Figure 2.1). Although the incorporation of the equant hypothesis into his models allowed Ptolemy to predict planetary motion accurately, the existence of circles revolving uniformly with respect to so-called equant points instead of their own centers was a violation of the principle of uniform circular motion.¹⁷ In the *Almagest* Ptolemy used elaborate combinations of all of these hypotheses to account for the motions of each of the seven classical planets in the form of separate mathematical models.¹⁸ If one wished to translate these mathematical hypotheses into three-dimensional spheres that

17. In Book IX, 2, Ptolemy noted that he had been compelled “to make some basic assumptions which we arrived at not from some readily apparent principle, but from a long period of trial and application...” Toomer, *Ptolemy's Almagest*, 422.

18. It should be noted that in the *Planetary Hypotheses*, Ptolemy attempted to arrange the separate geometric models of the *Almagest* into a unified collection of celestial bodies using on tambourine-like “sawed-off sections” of orbs, however this approach was not followed by many later astronomers. Only the first part of Book 1 (out of two) still exists in Greek, but the entire work was translated into Arabic. See Goldstein, “The Arabic Version of Ptolemy's Planetary Hypotheses”

could produce the appropriate motions, however, the equant represented a major stumbling block: a sphere cannot rotate uniformly about an axis that is not one of its diameters.¹⁹

In the *Almagest* (and therefore in the *Tadhkira* and Shirwānī's commentary), the order in which Ptolemy presented the topics was not arbitrary. Rather, they were set forth followed a sequence that, broadly speaking, reflected the fact that observations of certain phenomena relied on having accurate models for other phenomena. The solar model, which is the simplest, is constructed first after a general description of celestial phenomena.²⁰ One might then expect Ptolemy to proceed to discuss the models for the superior planets, as they involve the least additional theoretical complexity and would be an appropriate place to illustrate how an epicycle-on-deferent hypothesis, previously introduced in *Almagest* III.3, was employed to account for certain aspects of planetary motion. In fact, however, the next model Ptolemy discussed was the complex theory of the Moon's motion, which contained unique features not seen in the subsequent models for the planets.²¹ The reason for placing the lunar model immediately after the model for the Sun was due to the fact that the position of the Moon was used in determining the position of the fixed stars, and subsequently the planets. The procedure worked by measuring the distance between the Moon and Sun before sunset, and later measuring

19. Historians have disagreed on details of the equant problem, and whether it should be interpreted primarily as a violation of uniform circular motion, or as a physical absurdity arising when the mathematical description of a sphere's motion could not be carried out by a physical sphere. For representatives of either side in this debate, and the equant problem in the broader scope of Islamic astronomy, see A. I. Sabra, "Configuring the Universe: Aporetic, Problem Solving, and Kinematic Modeling as Themes of Arabic Astronomy," *Perspectives on Science* 6, no. 3 (1998): 291-295 and George Saliba, "Arabic versus Greek Astronomy: A Debate over the Foundations of Science," *Perspectives on Science* 8, no. 4 (2000): 328-341.

20. The solar model was based simply on observations of the times of solstices and equinoxes.

21. Since the full-blown lunar model was based on eclipse observations, it had to come after the solar model.

the distance between the Moon and the desired star, which was not visible until after sunset. The two measurements were combined to calculate the star's longitude, and its latitude could be measured directly. Since the Moon moves rather quickly, however, one needed an accurate model to account for the change in its position between the two measurements.²² Therefore, for observational reasons, the lunar model and all its complications, including motion in both longitude and latitude, necessarily came before the planetary models.

Shirwānī's desire to convey a sense of how astronomers worked and thought is reflected in his frequent discussion of the observations or unstated assumptions behind Ṭūsī's remarks, or their consequences. For example, when discussing how to account for the Moon's speed, which changed over the course of one monthly circuit and which did not return to the same relative maximum or minimum from circuit to circuit, Shirwānī explained that these motions cannot be based on either an eccentric without an epicycle, or on an epicycle carried on a concentric deferent, because those produce slowness and speed in the same [respective] parts.²³ Elsewhere, Shirwānī explained the fact that Mercury was united with the Sun during the middle of both its periods of direct and retrograde motion indicated that it had an epicycle whose center was carried by a deferent that moved with the same motion as the center of the Sun.²⁴ In a more complicated example that will be discussed below, Ṭūsī mentioned that the latitudes of the superior planets (i.e., their positions above or below the ecliptic) varied in two ways, whereas the inferior planets had a third component of latitudinal variation. In discussing this passage, Shirwānī described the

22. See Pedersen, *A Survey of the Almagest*, 239-240.

23. Shirwānī, *Sharḥ al-Tadhkira*, fol. 192b.

24. Shirwānī, *Sharḥ al-Tadhkira*, fol. 210b.

specific observations that were carried out to determine that this third variation did not apply to the superior planets.²⁵

Throughout the commentary, Shirwānī also made repeated references to data from the *Almagest*, summarized calculations that Ptolemy had used to derive parameters for the planetary models (whereas Ṭūsī had merely reported the final results,) and provided more precise data as a supplement to Ṭūsī's approximations. For example, in his overview of the motions and orbs of the Sun in *Tadhkira* II.6, Ṭūsī did not state any values for solar phenomena, such as the length of the year. In the following chapter on the Moon, he mentioned an approximate value for the mean Sun as 0;59° per day. In contrast, Shirwānī supplied a value for the Sun's mean motion as 0;59,8,20° per day. Moreover, he explained how the value was calculated by dividing the length of the solar circuit (360°) by the duration it took for the Sun to traverse the complete circuit starting from the spring equinox (approximately 365 and 1/4 days).²⁶ In another case, in *Tadhkira* II.7 when Ṭūsī stated the motion of the lunar nodes was “three minutes plus a fraction,” Shirwānī clarifies it was 0;3,11°. ²⁷ In the discussion in II.13 of lunar eclipses and the sections of the lunar circuit where it was possible for them to occur, when Ṭūsī stated “the eclipse limits have been measured as 12°” from either of the lunar nodes, Shirwānī provided a more precise

25. Shirwānī, *Sharḥ al-Tadhkira*, fol. 231a.

26. Shirwānī, *Sharḥ al-Tadhkira*, fol. 188a. There is a scribal error in the Ahmet III manuscript, stating the length of the solar circuit as 365° rather than 360°. The Tehran University and Damad Ibrahim manuscripts have the correct value. Shirwānī's value is an approximation of Ptolemy's value from the *Almagest*, see Toomer, *Ptolemy's Almagest*, 140.

27. Shirwānī, *Sharḥ al-Tadhkira*, fol. 195a.

value of $12;12^\circ$, referred the reader to *Almagest* VI where the value was derived, and described the relevant parameters and trigonometric calculations to carry out in order to derive this limit.²⁸

It is significant that Shirwānī supplied these additions, clarifications, and references to the *Almagest*, as well as other foundational texts including Euclid's *Elements* and *Optics*, that Ṭūsī explicitly stated would be left out of the *Tadhkira*. This material did not merely save the reader the trouble of looking up the proofs in these other texts. More importantly, it filled in the intellectual and programmatic gaps in the presentation of the models and helped the reader understand how they operated. By walking the reader through the calculations of specific astronomical values, or supplying geometric diagrams for the reader to consult alongside proofs of Ṭūsī's statements, the commentary went beyond providing a summary of the results of astronomers' efforts: it started to introduce readers to the actual methods and practical knowledge required to become astronomers themselves.

Incorporation of material from advanced *hay'a* texts

The references and demonstrations Shirwānī provided from the *Almagest* and other “ancient” sources already provide strong evidence that he was making his commentary into a more advanced astronomical textbook. Perhaps more important, he also integrated discussion, criticisms, and alternative ideas from more “modern” advanced *hay'a* texts. As we will see below, Shirwānī used these advanced texts for multiple purposes. For technical topics, they often provided a particularly clear explanation of Ṭūsī's point or the reasoning behind it. They were also a useful source of proofs for claims that Ṭūsī merely stated in the *Tadhkira*. In other situations, the authors debated Ṭūsī's claims or suggested modifications, which Shirwānī

28. Shirwānī, *Sharḥ al-Tadhkira*, fol. 257a.

repeated. And, as Shirwānī noted in his preface, errors and misstatements in these other sources became the subject of Shirwānī's own discussions, clarifications, and counter-proofs. Altogether, Shirwānī used these sources to introduce the reader to the ongoing investigations and debates in the science of *hay'a*. They provided opportunities to bridge the gap between the introductory, yet still technical, material of the *Tadhkira* and the more specialized discussions in the later commentaries on it and other texts. The reader soon understands that *hay'a* was not only the collection of impressive results and predictive methods built up over centuries, but remained a discipline characterized by active research and careful attention to both theoretical underpinnings and precise observations.

Preliminaries – The motion of the Earth

The first four chapters of *Tadhkira*, Book II, covered a range of preliminary observations of the collective and individual motions of the celestial bodies, as well as definitions of the circles astronomers used as references to discuss and measure these motions. In chapter one, Ṭūsī laid out a number of arguments for the configuration of the Earth and the heavens, providing evidence for the sphericity of the heavens and Earth, the Earth being in relation to the heavens as the center of a sphere to its circumference, and the Earth being completely stationary. This section generally corresponds to *Almagest* I, Chapters 3-7. Ṭūsī's comments on the topic of the Earth's rotation, however, gave rise to a sophisticated commentary by Shirwānī as part of an extended debate among Islamic scholars about just such a possibility.

Shirwānī began by noting that some Indian scholars did attribute the daily motion of the stars to the Earth. They reasoned from the analogy of a ship passing by the shore: we imagine (*yatakhayyal*) that the shore is moving in the opposite direction from the ship, and likewise our

perception of the motions of the celestial bodies around the Earth could be the effect of the Earth's rotation in place. Shirwānī explained that such scholars viewed the stars as moving in two directions simultaneously – the first being the “accidental” swift motion toward the west (the impression of daily movement actually caused by the Earth's rotation) and the second being the star's “essential” slow motion toward the east relative to the “fixed” points where the Sun crossed the celestial equator (the equinoxes).²⁹

Shirwānī rejected this theory and agreed with Ṭūsī that it is not possible to attribute the daily movement of the heavens to the Earth. However, if we dig deeper into the basis for his rejection of the Earth's motion, we quickly realize that Shirwānī was picking up on a longstanding debate stretching back to Ptolemy's *Almagest* about the role of evidence and observation in determining whether the Earth was in fact in motion or at rest. In Book I of the *Almagest*, Ptolemy noted that on the basis of the celestial phenomena alone, one could imagine that the Earth's rotation produces the impression of celestial motion. However, physical arguments quickly led Ptolemy to reject this notion. Shirwānī referred to Ptolemy's arguments, which can be summarized as follows:

1. The Earth would have to move extremely rapidly, and “the result would be that all objects not actually standing on the Earth would appear to have the same motion, opposite to that of the Earth.”

2. Or if we assumed that the air was carried around with the Earth at the same speed, “objects in the air would none the less always seem to be left behind by the motion of both [Earth and air].”

29. Shirwānī, *Sharḥ al-Tadhkira*, fol. 69b.

3. Or if we assumed that the objects “were carried around, fused, as it were, to the air, then they would never appear to have any motion either in advance or rearwards; they would always appear still...”

Since we do not observe any of these phenomena, but rather plainly see objects moving in all manner of ways through the air, Ptolemy concluded that the Earth is not in motion.³⁰

In the *Tadhkira*, Ṭūsī rejected this line of argumentation, stating that “the part of the air adjacent to the [Earth] could conceivably conform (*yushāyi* ‘*u*’) to the Earth’s motion along with whatever is joined to it, just as the [level of fire in the upper atmosphere] conforms to the [lunar] orb as evidenced by the comets, which move with its motion.” Jamil Ragep has analyzed Ṭūsī’s remarks in comparison with a similar passage written by Copernicus in *De Revolutionibus*, arguing that the similarities strengthen the conclusion that Copernicus was influenced by the tradition of medieval Islamic astronomy.³¹ Regardless of the potential impact on Copernicus, this passage in the *Tadhkira* was taken up by many subsequent Islamic astronomers as a means of discussing three issues: the relevance of comets for discussing the Earth’s rotation; the issue of observational tests in general; and the debate over whether it was proper or necessary for mathematical astronomy to use principles taken from natural philosophy.

30. Toomer, *Ptolemy’s Almagest*, 43-45.

31. F. Jamil Ragep, “Ṭūsī and Copernicus: The Earth’s Motion in Context,” *Science in Context* 14, no. 1-2 (2001): 145-163. Moreover, ‘Alī al-Qūshjī, Shirwānī’s colleague from Samarqand, went so far as to claim that the Earth’s motion was equally possible, rejecting principles of Aristotelian physics, see Ragep, “Freeing Astronomy from Philosophy” But see also Michael Shank, “Setting up Copernicus? Astronomy and Natural Philosophy in Giambattista Capuano da Manfredonia’s *Expositio* on the *Sphere*,” in *Evidence and Interpretation in Studies on Early Science and Medicine: Essays in Honor of John E. Murdoch*, ed. Edith Sylla and William Newman, (Leiden: Brill, 2009), 290-315 for evidence of pre-Copernican Latin discussions of these topics.

What Ṭūsī meant was that the bizarre phenomena predicted by Ptolemy would not necessarily occur – objects in the air could be carried in such a way that an observer would not be able to determine whether or not he or she was in motion. The reference to comets follows from Aristotle, who located comets in the sublunar realm of the four elements rather than in the heavens. To explain the apparent daily motion of the comets along with the heavens, therefore, Aristotle claimed that the highest level of fire underneath the Moon’s orb moved in conformity with the heavens, carrying the comets along.

Shirwānī addressed the topic of the air’s motion in conformity with the Earth by quoting Quṭb al-Dīn al-Shīrāzī’s *al-Tuḥfa al-Shāhīya*, which rejected both Ṭūsī and Aristotle on this point. Shīrāzī stated “if the comets did move in conformity with the celestial orbs, then they would always move parallel to the celestial equator” (*law kānat bi-l-mushāya ‘a li-mā zālat ‘an muwāzāt al-mu ‘addil*).³² In fact, however, we observe comets moving from north to south and vice versa, which Shīrāzī attributed to their having a soul governing their movement.

Commenting on this argument, Shirwānī stated that he accepted the possibility of conformity in general, but in this case he agreed with Shīrāzī that the motion of the comets was not explained by conformity with the motion of the celestial orbs. However, he went on to reject Shīrāzī’s next argument against the air’s motion in conformity with the Earth. Shīrāzī had claimed that if the air was in motion along with the Earth, then a large and a small stone thrown along the meridian would land at different places because the large stone would be less affected by the air than the small stone. Shirwānī criticized this argument, stating explicitly “We do not accept that...” and appealing to Ibn Sīnā’s remarks in *al-Shifā’* IV.13 to argue that both stones would be affected similarly regardless of the motion of the air. Ibn Sīnā had explained that the upper atmosphere

32. Quoted in Shirwānī, *Sharḥ al-Tadhkira*, fol. 70a-b. لو كانت بالمشايعة لما زالت عن موازاة المعدل

moved in conformity with the motion of the sphere of the Moon, not as the result of a force, but as an accidental motion with respect to the atmosphere's natural position relative to the (moving) sphere of the Moon.³³ Similarly, Shirwānī argued, when thrown on a rotating Earth, the two stones would move in conformity with their natural place (the Earth), and thus both stones would be projected in an identical manner.³⁴

Thus, Shirwānī supported Ṭūsī's view (against Ptolemy) that the hypothetical motion of the Earth could not be determined on the basis of astronomical or physical observations. As a result, in this instance astronomers were forced to rely on the conclusions of the natural philosophers, who used deductive reasoning to establish the fact that the Earth was stationary. Ṭūsī briefly explained that since the element of earth has the principle of rectilinear motion, it could not also have a natural circular motion of rotation in place.

Preliminaries – Precession, Obliquity of the ecliptic, Trepidation

Attention to the fixed stars reveals that, as a whole, they possess two separate motions. The first is the apparent daily rotation of the heavens from east to west around the Earth. The second motion, which is only detectable through careful observation over very long periods of time, is what we call the precession of the equinoxes. Discovered by Hipparchus, this very slow rotation

33. Ibn Sīnā, *The Physics of the Healing: A Parallel English-Arabic Text in Two Volumes*, ed. and tr. Jon McGinnis, (Provo: Brigham Young University Press, 2010), 499.

34. Shirwānī, *Sharḥ al-Tadhkira*, fol. 70b.

سلمنا المشايعة ثمة لكن لا مشايعة ههنا والا لا وقع الحجران المختلفان في الصغر والكبر المرميان في الهواء من سمت خط واحد على الأرض كخط من خطوط انصاف النهار على ذلك الخط لان تحريك الهواء للكبير يكون اقل من تحريكه للصغير بل كان يجب ان يقع الكبير في الجانب الغربي من الصغير والوجود بخلافه هذا كلامه. والجواب عن منع مشايعة الاثير وقد ذهب اليها الشيخ أيضا في الفصل الثالث عشر من المقالة الرابعة من طبيعيات الشفاء ان جواز مشايعة الهواء كافية لنا في نفي التعليل المذكور سواء كان للأثير مشايعة او لا وانما ذكرها لتقريب مشايعة الهواء الى الافهام وعن دليل ابطال مشايعة الهواء ان لا نسلم ان تحريك الهواء للكبير يكون اقل لان مشايعته للأرض مع شيء يتصل به يستلزم يحرك ذلك الشيء بقدر حركته بل بقدر حركة الأرض لان مشايع المشايع لشيء مشايع لذلك الشيء فلا فرق في ذلك بين الكبير والصغير.

of the fixed stars at an angle to the first motion and in the opposite direction (from west to east) occurs at a rate of 1° in somewhere between 66 and 100 years, according to different groups of premodern astronomers. Thus, in the *hay'a* tradition, two separate orbs were employed to represent these primary, contrary motions. The orb that accounted for the daily rotation was called the equinoctial orb, and its equator is the celestial equator. The orb of the second motion was called the orb of the ecliptic or the orb of the fixed stars, for its equator is the ecliptic, the circle traced out by the Sun as it moves through zodiacal constellations over the course of a year. The two points where these equators intersect are the vernal and autumnal equinoxes, marking the start of spring and autumn respectively. For this reason, the second, slower motion is often referred to as the “precession of the equinoxes” because the fixed stars were gradually moving parallel to the ecliptic, creating the appearance that the location of the equinoxes was shifting with respect to any given constellation. As a result, over the approximately 2,000 years between Ptolemy’s day and our own, the vernal equinox appears to have shifted from the constellation Aries to Pisces.

Since these two orbs represented the most fundamental motions in the heavens, the angle between them, known as the “obliquity of the ecliptic” or simply “obliquity,” was a key parameter in astronomical models and calculations. Ptolemy’s value was $23;51,20^\circ$ and Ṭūsī stated that the value was somewhere between 24° and $23;30,3^\circ$.³⁵ According to Shirwānī, an alternative minimum value for the obliquity was found by al-Khujandī (d. c. 1000), who measured the ecliptic by means of an instrument called “the Fakhrī sextant.” This device was a sixth of a meridian circle with a huge diameter of 20 cubits [approx. 43 m], built with the patronage of Fakhr al-Dawla, the Buyid ruler of Rayy. Through observations with the sextant,

35. Ragep, *Tadhkira*, 120.

Khujandī determined the obliquity to be $23;32,21^\circ$. However, Shirwānī noted that al-Bīrūnī rejected this value. It appears that the building in which the sextant was constructed settled between successive observations at the summer and winter solstices, which affected the position of the aperture through which the Sun’s light fell upon the arc.³⁶ Shirwānī also copied Nīsābūrī’s report that after completing the *Tadhkira*, Ṭūsī made new observations at Marāgha and found the obliquity to be $23;30^\circ$.³⁷

In the *Tadhkira* itself, Ṭūsī stated that most astronomers accepted the value of $23;35^\circ$. However, the obliquity was found to have different values based on different observations by Islamic astronomers over the centuries. Moreover, the value for the obliquity of the ecliptic seemed to be decreasing over time. This led some to propose that the obliquity was actually changing, perhaps varying between extreme values, which led to investigations as to how it would be possible for the equators of the primary motions to be gradually moving towards one another.

Regarding this point, Shirwānī first argued that the basis for the differences in measurements of the obliquity supporting this claim of movement of one of the two equators toward the other was actually

36. Shirwānī, *Sharḥ al-Tadhkira*, fol. 99a. On Khujandī, see Glen van Brummelen, “Khujandī: Abū Maḥmūd Ḥāmid ibn al-Khiḍr al-Khujandī,” in *Biographical Encyclopedia of Astronomers*, ed. Thomas Hockey et al., (New York: Springer, 2007), 630-631. Bīrūnī discussed Khujandī’s instrument and the flawed observations in *Taḥdīd al-Amākin (Determination of the Coordinates of Cities)*. The value Khujandī actually reported for the obliquity of the ecliptic was $23;32,19^\circ$, Shirwānī’s report of Khujandī’s value as $23;32,21^\circ$ may be taken from Nīsābūrī’s *Tawḍīḥ al-Tadhkira*, it is also the value reported by Bīrūnī in *al-Qānūn al-Mas’ūdī*, see Robert Morrison, *Islam and Science: The Intellectual Career of Nizām al-Dīn al-Nīsābūrī* (New York: Routledge, 2007), 270 n.66.

37. Shirwānī, *Sharḥ al-Tadhkira*, fol. 99a.

differences in the observational instruments and defects in constructing their circularity, or [defects in] their sizes or divisions/markings, or [defects] in installing them correctly in the plane of the meridian, rather than the motion of one of the equators toward the other and approaching it. Otherwise, the decrease in obliquity would be found to grow regularly over time, which is not the case. However, this only indicated that the basis of the variation was not due to [the equators] drawing nearer [alone], without differences in the instruments.³⁸

Thus, Shirwānī concluded, it was possible for these divergent measurements to be “due to the equators drawing nearer along with the existence of differences in the instruments, and the absence of regularity in the different [values] was due to the differences in the instruments.”

Furthermore, Shirwānī pointed out that “it was not possible for the approaching of the two equators to be due to the motion of the equinoctial toward the ecliptic, since that would require the individual lands on the earth to have varying latitudes, and for the earth’s equator to be at a different latitude at each time, which no one had observed, let alone suggested.”³⁹

However, some astronomers proposed that the ecliptic moved toward the equinoctial, which Ṭūsī indicated with a brief listing of eight possibilities for how the ecliptic might move relative to the equinoctial and some consequences that might be observed in each case. These were not models, but rather an analytical division of possible motions. For example, the ecliptic might move through an entire cycle or it might reach a particular limit and then turn back.

38. Shirwānī, *Sharḥ al-Tadhkira*, fol. 99b.

والظاهر أنّ أصل هذا الاختلاف أنّما هو بسبب اختلاف الآلات الرصدية واختلافها في صنعة استدارتها أو في مقدارها أو في قسمتها أو في نصبها في حقيقة سطح نصف النهار لا لتحرك إحدى المنطقتين إلى الأخرى وتقاربها. والا لوجد تناقص الميل بتزايد الأيام على ترتيب ونظام وليس كذلك كما تبين مما ذكرنا. لكن بطلان التالي إنما يدل على أن أصل الاختلاف ليس للتقارب من غير اختلاف الآلات.

39. Shirwānī, *Sharḥ al-Tadhkira*, fol. 99b.

ويكون عدم النظام والترتيب بين الاختلافات لاختلاف الآلات ولما امتنع أن يكون تقارب المنطقتين لحركة المعدل إلى منطقة البروج لاستلزامه اختلاف عرض كلّ بلد عمّا هو عليه ويكون خطأ الاستواء في كلّ زمان مكانا آخر لم يذهب إليه أحد.

Shirwānī noted that he was working with the later “Baghdad” revision of the *Tadhkira* listing these eight possibilities; the earlier “Marāgha” version of the text had only stated four.⁴⁰

Shirwānī’s brief additions to this section were mainly to clarify some of Ṭūsī’s statements or to point out exceptions to the consequences Ṭūsī predicted in the extreme case of an observer at 90° latitude (whose zenith was the pole of the ecliptic). Ṭūsī’s use of the word *za‘ama* (“maintain, allege”) and Shirwānī’s repetition of Ṭūsī’s phrase “if this were true (*wa-in kāna dhālika ḥaqq^{an}*)” suggested that additional observations were required in order to settle the question of whether the obliquity was changing or not.

As for precession, Shirwānī acknowledged it as a verifiable motion with important ramifications for collecting data about celestial phenomena. Since the tropical year used by astronomers was defined by the time it took for the Sun to return to the vernal equinox, precession meant that the tropical year differed slightly from the sidereal year, defined by the Sun’s return to a particular star. Moreover, because the vernal equinox was used as the 0° point for measurements of longitude (around the ecliptic circle), precession needs to be taken into account when comparing recent observations with much earlier ones, and an incorrect value for the rate of precession could lead to erroneous conclusions. In *Almagest* VII, Ptolemy quoted Hipparchus’s explanation of how he discovered precession when he found differences between his observations of the star Spica and those of Timocharis, an earlier astronomer. Based on his own observations Ptolemy determined the rate of precession to be 1° in 100 years.⁴¹ Ṭūsī

40. Shirwānī, *Sharḥ al-Tadhkira*, 101a. On the different versions of this section, and its importance for dating the *Tadhkira*, see Ragep, *Tadhkira*, 70-75. Unlike Shirwānī, Nīsābūrī used the earlier Marāgha version of the *Tadhkira* for his commentary, see Morrison, *Islam and Science*, 183.

41. Toomer, *Ptolemy’s Almagest*, 327-329.

reported this valued found by the “ancients” (*al-qudamā*), but noted that most “modern” (*al-muḥdathūn*, i.e. Islamic) astronomers had adopted the rate $1^\circ/66$ years, while “a reliable group” of the modern astronomers (*qawm min muḥaqqiqīhim*) found it to be $1^\circ/70$ years.

Shirwānī noted that this latter value was the one “used in our time, on account of its being established by the new observations carried out by the author [Ṭūsī] at Marāgha.”⁴² Shirwānī also mentioned that another astronomer associated with Marāgha, Muḥyī al-Dīn al-Maghribī (d. 1283), claimed to have carried out a number of observations and found the rate to be $1^\circ/66$ years.⁴³ However, Shirwānī followed this remark with an extended quotation from a treatise on disagreements among scholars by Abū Naṣr Yaḥyā b. Jarīr (d. 1103/4) which pointed out that errors could enter into the observations in many ways. Chief among these were flaws in constructing the instruments: they might not be perfectly flat circular rings, or they might not be aligned precisely with the local meridian.⁴⁴ Furthermore, errors could arise in the process of converting from local observations carried out on the surface of the Earth, because the local horizon was not the celestial equator and the instrument was removed from the center of the cosmos by the distance of the Earth’s radius. Finally, Yaḥyā pointed out that measurements of the Sun’s position (or other observations) had to be accepted from earlier astronomers, and whether they agreed with or differed from more recent observations, one could not know if the

42. Shirwānī, *Sharḥ al-Tadhkira*, fol. 101a.

43. See George Saliba, “An Observational Notebook of a Thirteenth-Century Astronomer,” *Isis* 74, no. 3 (1983), 399.

44. On Yaḥyā b. Jarīr, see Herman Teule and Mark Swanson, “Yaḥyā Ibn Jarīr,” in *Christian-Muslim Relations. A Bibliographical History. Volume 3 (1050-1200)*, ed. David Thomas and Alexander Mallett, (Leiden: Brill, 2011), 280-286. According to Ibn Abī Usaybi‘a, Yaḥyā wrote a work on astrology which may be the text Shirwānī refers to, although the authorship of this text is disputed. See Fuat Sezgin, *Geschichte des arabischen Schrifttums* (Leiden: Brill, 1967-), VII, 19-20.

earlier results were reliable. These criticisms were similar to the ones Shirwānī raised against concluding that the obliquity of the ecliptic varied. By highlighting a disagreement between two prominent contemporary astronomers as to the correct rate of precession, Shirwānī was hinting that careful attention to detail was crucial for astronomical work. Astronomers could not function without relying on their predecessors' results. Even if those observations might be flawed, Shirwānī's remarks indicated that although he favored observational methods as the way to resolve outstanding questions in astronomy wherever possible, rather than turning to principles from natural philosophy, he was not shy of critically examining observational methods as a source of error.

As in the case of the obliquity of the ecliptic, the fact that astronomers calculated different rates for precession led some to propose that the fixed stars were not moving at a constant rate parallel to the ecliptic, but were in fact changing speed over time. According to the hypothesis known as "trepidation," the ecliptic orb moved forward (*iqbāl* or "accession") at a rate of $1^\circ/80$ years for 640 years, moving through 8° before reversing direction (*idbār* or "recession") at the same rate for an equal amount of time.⁴⁵ Ṭūsī attributed the theory of trepidation to some "practitioners of the art of talismans," who, Shirwānī explained, were Greek magicians who intermingled the active celestial powers with terrestrial recipients to bring about

45. The first extant discussion of a linear zig-zag function causing this oscillation of 8° amplitude appears in Theon of Alexandria's *Small Commentary* on Ptolemy's *Handy Tables*, for a translation and analysis of this discussion see Neugebauer, *HAMA*, 631-634. On Islamic discussions of trepidation predating the *Tadhkira*, see F. Jamil Ragep, "Al-Battānī, Cosmology, and the Early History of Trepidation in Islam," in *From Baghdad to Barcelona: Studies in the Islamic Exact Sciences in Honour of Prof. Juan Vernet*, ed. Josep Casulleras and Julio Samsó, (Barcelona: 1996), 267-298.

wondrous and strange effects.⁴⁶ Tūsī stated that, if trepidation in fact existed, it would require an additional orb to bring about the motion.

Shirwānī, however, flatly rejected trepidation, stating “[you should] know that this conjecture is false” due to the fact that the motions entailed by the trepidation models had never been observed. Shirwānī argued first on philosophical grounds – a simple body would not produce this kind of “incomplete” circuit. On the other hand, if we assume that a combination of simple motions produced a rate of trepidation of $1^\circ/80$ years, the precessional motion of the stars could be greater or less than, or equal to, this rate. Shirwānī explained that any of those situations would lead to an observable shift in the positions of the stars, and provided numerical examples to illustrate this point. For instance, if precession was $1^\circ/66$ years, then the orb would move a little more than $1;45^\circ$ in the first phase of trepidation and $0;10,30^\circ$ in the second half. If the rate of precession was equal to trepidation ($1^\circ/80$ years), then in just 40 years the combined motions in the first phase of trepidation would cause the stars to move by 1° and the motions would negate each other in the second half of the trepidation cycle. Since none of these phenomena had been observed (*lam yūjad shay' minhā*) over the duration of time from Hipparchus to the present, which was more than enough time for an entire cycle of trepidation to have elapsed, Shirwānī concluded that trepidation ought to be rejected.⁴⁷

46. At this point, Shirwānī quotes Fakhr al-Dīn al-Rāzī for a brief classification of different ways one might seek to bring about effects, ranging from sleight-of-hand to sorcery and talismans, as well as the hidden properties of terrestrial objects such as medicines. Shirwānī, *Sharḥ al-Tadhkira*, 101b.

47. Shirwānī, *Sharḥ al-Tadhkira*, 102b-105a.

واعلم أنّ هذا الظنّ باطل ليس بحق لبطلان الحركة الاقبالية والادبارية التي نباه عليها لأنها ان كانت بسيطة أي صادرة عن جرم واحد بسيط لزم عدم إتمام الدور في السماويات وان كانت مركبة أي صادرة عن بسائط فوق واحدة فالحركة الثانية ان كانت أكثر منها كما في مذهبي السبعين والست والستين لزم على المذهب الاول ان يقطع في كلّ سبعين سنة في نوبة الاقبال او الادبار ما مرّ وعلى الثاني ان يقطع في كلّ ستّ وستين سنة اقبالية درجة وأكثر من ثلاثة ارباع درجة وفي الادبارية عشر دقائق وثلثين ثانية. وان كانت أقلّ منها كما في مذهب المائة لزم ان يقطع في كلّ ثمانين سنة في نوبة الاقبال درجة وأربعة أخماس درجة وان لا يحس بها في نوبة الادبار بل يحسّ بالادبار جُمس درجة. وان كانت مساوية لها فرضاً لزم ان يقطع في كلّ

Finally, Ṭūsī mentioned that some astronomers had proposed a model that accounted for both supposed divergent motions, the variation in the obliquity of the ecliptic and the rate of precession, through the addition of a single mover. As Ragep described it, Ṭūsī's brief description of the model is "misleading at best, utter nonsense at worst."⁴⁸ In this model, the additional mover supposedly causes the ecliptic orb to move in such a way that every point on it moves in a small circle.⁴⁹ Shirwānī's attempt to explain what Ṭūsī might have meant followed largely from Nīsābūrī's commentary, and he supplied geometric diagrams and proofs to illustrate how the various orbs were meant to interact.⁵⁰ However, as with the previous analyses of possible variations in the obliquity and rate of precession, Shirwānī ultimately rejected this proposal on the basis of observational evidence. The model predicted ranges of values for the obliquity and rates of precession that simply were not observed to occur.

The basic hypotheses: Eccentrics and Epicycles

As Ṭūsī reminded the reader at the beginning of *Tadhkira* II.5, apparent irregularities in celestial motion required a hypothesis according to which that motion was analyzed into components that each exhibited uniform circular motion. Shirwānī defined irregular motion as that which makes, in equal times, unequal angles with respect to the center of the ecliptic orb and unequal arcs

أربعين سنة في نوبة الاقبال درجة وان لا يحس بها ولا بالادبارية في نوبة الادبار. واللوازم كلها باطلة اما الاول فلاستحالته واما البواقي فلأنه لم يوجد شيء منها من زمن ابرخس الى زماننا هذا وهو أكثر من ضعف مدة كل واحد من الاقبال والادبار.

48. Ragep, *Tadhkira*, 402.

49. For a discussion of the history of this model, which Shirwānī and other commentators attributed to Ibrāhīm b. Sinān, grandson of Thābit b. Qurra, see Ragep, *Tadhkira*, 400-408.

50. Shirwānī, *Sharḥ al-Tadhkira*, 102b-105a. For a discussion of Nīsābūrī's (and Shirwānī's) analysis of this model for trepidation and variable obliquity, see Morrison, *Islam and Science*, 185-187.

along it. Thus, astronomers sought a hypothesis such that it produced equal angles in equal times with respect to some other point and irregularity with respect to an observer at the center of the World. For, as Shirwānī explained, irregular motion does not arise from the celestial bodies owing to their simplicity (*li-basāṭatihā*). Commenting on the title of this chapter, Shirwānī followed Shīrāzī's *Nihāya* in listing six categories (*aqsām*) of irregular motion.⁵¹

1. Speed, slowness, and mean motion.
2. Direct motion, retrograde motion, and stationary points.
3. Motion of a mobile being uniform around a point that is not the center of the path of its mover, or the motion being uniform around a point that is not the center of its path when [the two motions] derive from one thing.
4. Motion being uniform around a point regardless of the nearness/distance of the mobile [from that point].
5. Not preserving the alignment of the diameter of a sphere moving along the boundary of a sphere with the center of that sphere (‘*adam inḥifāẓ muḥādhāt quṭr al-kura al-mutaḥarrika ‘alā muḥīṭ kura li-markaz tilka al-kura*).⁵²
6. Not completing a revolution in the celestial motions (‘*adam itmām al-dawra fī al-ḥarakāt al-samāwiya*).

He also listed the nine hypotheses (*uṣūl*, a reference to Ṭūsī's title for this chapter) that Shīrāzī described to account for these irregular motions. As for Ṭūsī, Shirwānī explained that, in

51. See Morrison, “Quṭb al-Dīn al-Shīrāzī's Hypotheses for Celestial Motions” 30-32.

52. This relates to the problem of the Moon's prosneusis point, which will be discussed in Chapter 3, as well as the fact that the diameter of the epicycle in the model for the upper planets is aligned with the equant point, not with the center of the deferent.

this chapter, he limited himself to the 1st and 2nd irregularities and their hypotheses: the eccentric and epicycle.

In the case of the eccentric (*al-khārij al-markaz*), the planet was assumed to move uniformly along a circle surrounding the Earth but not centered on it. Ṭūsī explained that this caused the appearance of irregular motion, with slower motion in the segment farther from the Earth and faster motion in the near segment, because a given arc would appear smaller when it was farther away. Following his approach of supplying many geometric proofs that Ṭūsī had left out for simplicity's sake, Shirwānī gave not one but two different demonstrations for the apparent motions produced by the eccentric hypothesis, with references throughout to propositions from Euclid's *Elements*.⁵³ He followed these demonstrations with a discussion and proof of Ṭūsī's statement that the planet's mean motion would occur at the two points where a line through the Earth, perpendicular to the line connecting the planet's farthest and nearest distances, intersected the eccentric.⁵⁴

53. Shirwānī, *Sharḥ al-Tadhkira*, fol. 167a-168a.

54. Shirwānī, *Sharḥ al-Tadhkira*, 168b-170b.

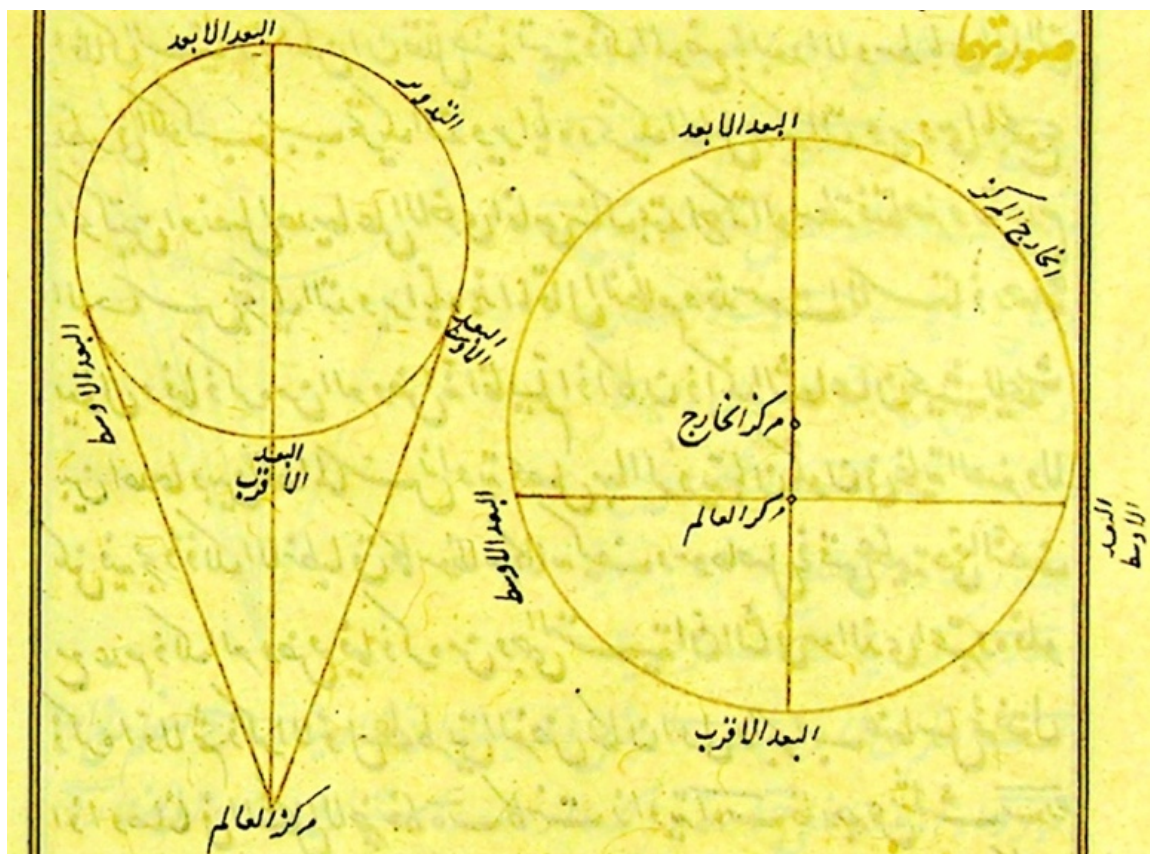


Figure 2.2 Nearest, farthest, and mean distances on epicycle and eccentric
Ahmet III MS 3314, fol. 172b

For the epicycle (*al-tadwīr*), Tūsī first treated the case of a planet moving around an epicycle that was fixed at a given point relative to the Earth. In this situation, equal arcs would again vary with respect to an observer on the Earth. In addition, and unlike the case of the eccentric, the planet would be seen to have direct motion in one segment of the epicycle and retrograde motion in the other, the two segments being divided by lines from the Earth tangent to the epicycle on either side. Shirwānī again chose to walk the reader through geometric proofs of these features. Furthermore, he clarified what he identified as a misstatement by Nīsābūrī regarding the relationship between the change in the planet's apparent size over different segments of the epicycle. The relationship between increasing size and increasing speed that

applied in the case of the eccentric does not hold for the case of an epicycle alone, contrary to what Nīsābūrī had stated. For this, Shirwānī supplied a proof that he attributed to Qāḏīzāde.⁵⁵

An epicycle carried on another circle (the deferent) centered on the Earth could be made to produce the same apparent planetary motion as an eccentric, provided that certain conditions were fulfilled.⁵⁶ In this situation, Ṭūsī stated the planet would describe (via the compounding of the motions of the epicycle and deferent) an “eccentric circuit” that is the same as the eccentric. As Shirwānī explained, this meant that “in any given period of time the planet’s center will pass through similar arcs of the circumference of the eccentric and the circumference of this circuit, that is, [chords] subtending equal angles at their centers.”⁵⁷ To demonstrate the equivalence of the eccentric and epicycle-on-deferent hypotheses from the perspective of an observer on the Earth, Shirwānī introduced two different geometric proofs (see Figure 2.3 for an example). Through these proofs, Shirwānī clarified to the reader that the paths of the planet in the two hypotheses were not necessarily identical, in the sense that the planet’s distance from the Earth in one hypothesis did not have to match its distance in the other.⁵⁸ Nevertheless, from the

55. Shirwānī, *Sharḥ al-Tadhkira*, 171b-172b.

وقد ظهر من هذا البيان أنّ تعاضم جرم الكوكب بهبوطه من البعد الابدع يستتبع البطو على خلاف ما ذكره النظام مقتفيا اثر ابي الريحان من ان تصاغر اجرام الكوكب في المنظر يستتبع البطو وتعاضمها يستتبع السرعة نعم ذلك كلى في الخارج المركز لا في التدوير.

56. The conditions were: the ratio of the deferent radius to the epicycle radius had to equal the ratio of the eccentric radius to the eccentricity; the motions of the deferent and eccentric were equal and in the same direction; and the epicycle moved with the same rate motion as the deferent and eccentric such that it moved in the opposite direction from the deferent in the far segment and in the same direction as the deferent in the near segment.

57. Shirwānī, *Sharḥ al-Tadhkira*, 173b.

على معنى أنّ ايّ زمان نفرض يقطع فيه مركز جرم الكوكب من محيط الخارج ومحيط هذا المدار قوسين متشابهتين أي موترتين لزاويتين متساويتين على مركزيهما.

58. In the diagram for the second proof, Shirwānī arbitrarily chose the radius of the eccentric to be greater than the combination of the radii of the epicycle and deferent.

perspective of the observer, in both cases the planet traveled through the same angles in equal times.⁵⁹

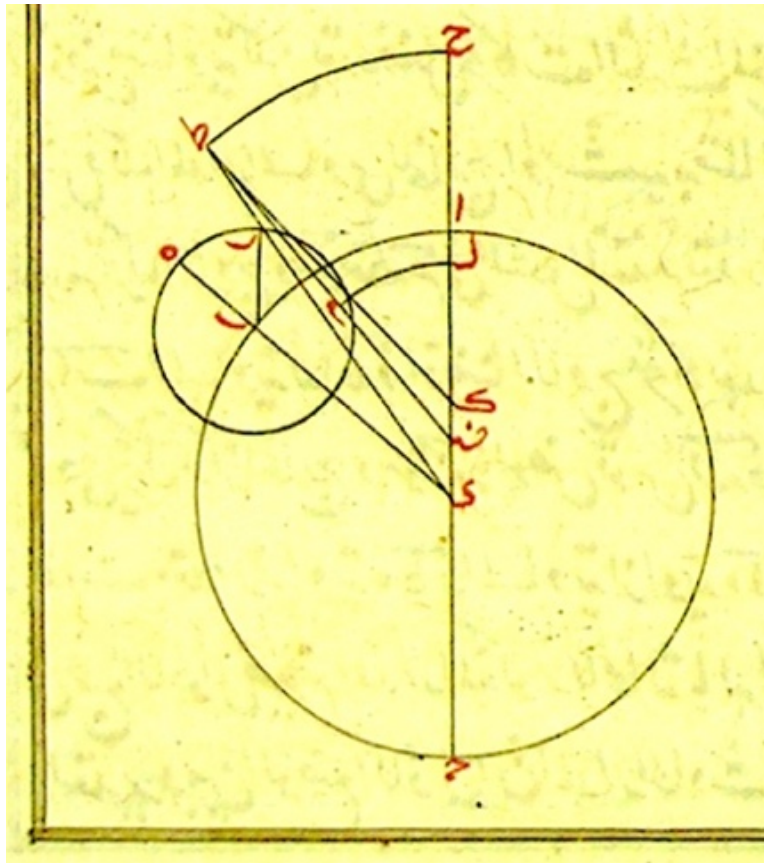


Figure 2.3 Angular equivalence of epicycle and eccentric
Ahmet III MS 3314, fol. 174a

Shirwānī explained that Ptolemy had chosen to work with the case where the eccentric circuit produced by the epicycle-on-deferent equaled the eccentric. But in principle, as he noted that al-Bīrūnī had pointed out, if the eccentric was either larger or smaller than the epicycle's eccentric circuit, then in both situations there were three cases, because the eccentric and the epicycle's eccentric circuit could either (1) intersect, or (2) touch at a point, or (3) be entirely separate. Shirwānī also drew the reader's attention here to a subtle remark (*nukta sharīfa*) mentioned by Nīsābūrī in the context of the solar model, that the eccentric apogee, if assumed to

59. Shirwānī, *Sharḥ al-Tadhkira*, 173b-174b.

move in the same direction as the motion of the eccentric, would require an additional mover.

Therefore, this necessitated that another orb be present in the case of the equivalent epicycle-on-deferent hypothesis as well, in order to move the deferent with this motion.⁶⁰

The main differences between the eccentric and epicyclic hypotheses were that the eccentric required one motion, while the epicycle had two, and the epicycle entailed an eccentric circuit while the eccentric did not entail an epicycle. According to Ṭūsī, Ptolemy considered the eccentric to be simpler and chose it, rather than the equivalent epicycle, as the basis for his solar theory. If an additional orb were necessary to account for the motion of the apogee, Shirwānī noted that the eccentric still required fewer motions (two for the eccentric instead of one, and three for the epicycle instead of two). With this comment, Shirwānī was making a theoretical analysis of all of the possibilities for the benefit of the reader, rather than any recommendations as to immediate utility. Ultimately, both eccentrics and epicycles were used to account for various celestial motions.

After introducing the two hypotheses, Ṭūsī proceeded to explore how they could produce some of the more complicated motions observed in the heavens. In this section, Shirwānī wanted to ensure that his readers understood the kinematics of the moving components in general, so that these would already be familiar when his readers encountered them in the specific celestial models Ṭūsī described in the subsequent chapters. As an example, discussing the conditions for producing retrograde motion using either an epicycle-on deferent or an eccentric with an additional concentric mover, Shirwānī followed the original “Marāgha” version of the *Tadhkira*,

60. Shirwānī, *Sharḥ al-Tadhkira*, 174b.

وقد ذكر النظام ههنا نقطة شريفة وهي أنّ أوج الخارج لو فرض متحركاً بحركة ما إلى جهة حركة الخارج وجب أن يفرض الحامل متحركاً أيضاً بفلک آخر قدر تلك الحركة بالعرض والحركة الذاتية للخارج والحامل والتدوير تفرض متساوية لكن يقدر فضل الحركات الثلاث المفروضة لها أولاً على حركة الأوج ليلزم مثل ذلك المدار المساوي للخارج والشبيه.

See also Morrison, *Islam and Science*, 173-174.

which assumes the proportions (eccentric radius : eccentricity) and (deferent radius : epicycle radius) are equal, and he corrected Ṭūsī's mistake in setting equal the motion of the two concentric deferents. In fact, as Shirwānī correctly explained, the motion of the eccentric's mover must be equal to the combined motion of epicycle's concentric deferent and the motion of the planet on the epicycle.⁶¹ He noted that this correct condition is given in both the *Tuhfa* and *Almagest* XII, but he did not comment on why Ṭūsī might have made this mistake or on the origin of the proposed, but also incorrect, change found in the later "Baghdad" version of the *Tadhkira*. Shirwānī explained that if one wanted to do so, one could use the proportions in the *Almagest* instead, namely the proportion of radius of the eccentric/epicycle to the connecting line can be smaller/equal/greater than the proportion of the motions of the concentrics to their [i.e. eccentric or epicycle's] motions, because they are equivalent (*fā-innahumā mutalāzimān*).⁶² We then compare (A) the proportion of the motion of the eccentric or epicycle to the motion of its respective deferent, with (B) the proportion of the line joining the center of the concentric to the nearest distance on either the eccentric or the epicycle to the radius of the eccentric or epicycle, respectively. The ratio of the motions (A) could either be smaller than, equal to, or greater than the ratio of the distances (B), which determined the planet's motion on the near and far segments of the epicycle or eccentric as well as the possibility of station(s) and retrograde motion.

61. Shirwānī, *Sharḥ al-Tadhkira*, fol. 176b. The revised "Baghdad" version of the *Tadhkira* uses a proportion that does not accomplish the desired result, and most of the commentators ignored it.

وهذا سهو منه اذ يجب لما سيجي ان يجعل حركة موافق الخارج مساوية لمجموع حركتي التدوير وموافقه إلى جهة كما في التحفة موافقا لما في المجسطي في المقالة الثانية عشرة.

For Ragep's discussion on this point, see Ragep, *Tadhkira*, 412-414.

62. Shirwānī, *Sharḥ al-Tadhkira*, fol. 176b-177a.

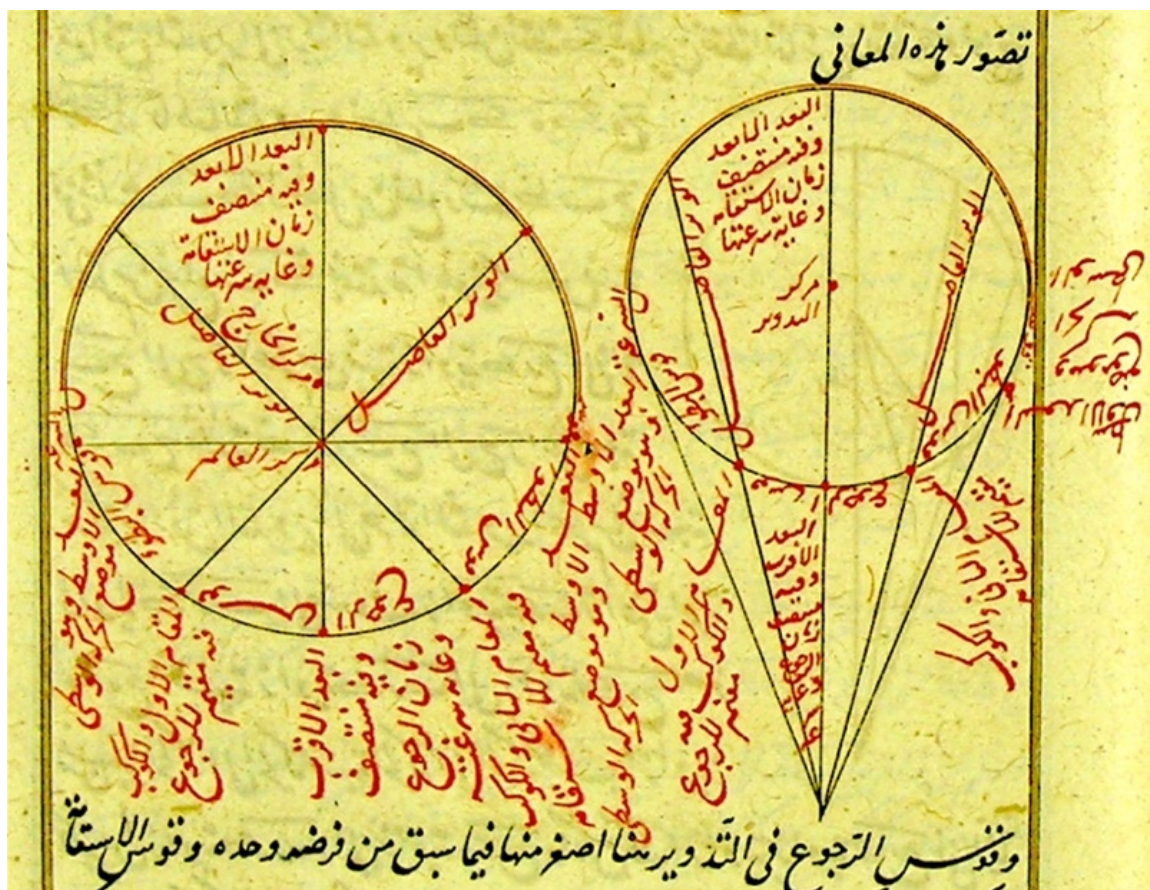


Figure 2.4 Motion on near/lower and far/upper segments of eccentric (left) and epicycle (right)
Ahmet III MS 3314, fol. 181a

1. If (A) is smaller than (B), the planet will only undergo faster motion in the far segment and slower motion in the near segment. When Ṭūsī mentioned that the change in apparent motion for the eccentric is due to the difference in the apparent sizes of the arcs, Shirwānī clarified that the arcs are in reality equal (*ma'a tasawīhumā fī nafs al-amr*).⁶³ As for the epicycle, the planet's motion in this case is due to the addition or subtraction of the epicycle's motion from the motion of the deferent.

63. The philosophical concept of *nafs al-amr* or “the fact of the matter” and the existence (whether mental or external) of mathematical entities will be discussed in Chapter 4. The place of this concept in Shirwānī's thought is an important topic for future research. See İhsan Fazlıoğlu, “Between Reality and Mentality: Fifteenth Century Mathematics and Natural Philosophy Reconsidered,” *Nazariyat Journal for the History of Islamic Philosophy and Sciences* 1, no. 1 (2014), 24-30.

2. If the ratios (A) and (B) are equal, at the midpoint of the slower motion, the planet will stop, which is at the point of nearest distance. Shirwānī provided the proofs and diagrams for Apollonius's lemma used in Ptolemy's demonstration of stations and retrograde motion from *Almagest* XII, 1.⁶⁴ After Apollonius's lemma, he returned to the topic of this case, namely the occurrence of stationary points in a planet's motion. He paraphrased Nīsābūrī, explaining that there is a moment (*al-ān*) at which the planet's apparent motion stops due to the opposing motions becoming equal, nevertheless the planet is seen to be stationary for a few days because of the small difference in the excess of the two motions.⁶⁵ He also noted Nīsābūrī's claim that a station is also possible in the case when (A) is less than (B), not only when they are equal, because the possibility of a station is based on the assumption that the ratio of motions is not greater, regardless of whether the ratios are equal or smaller. Shirwānī stated that Nīsābūrī's claim is contrary to what was stated in the *Tadhkira*, and also mentioned comments that Qāḍīzāde had made in class about Nīsābūrī's claim. Qāḍīzāde had stated that:

the proof did not demonstrate the absence of a station in the case when the ratio of motions is smaller than the ratio of distances. However, [Ṭūsī] asserted it intuitively because when he demonstrated that the ratio being greater necessitates retrogradation (as will be shown) it was because the backward motion exceeded the forward motion. When the ratios are equal it necessitates a station at the point when the motions are equal. Thus [Ṭūsī] intuited that in the case when the ratio is smaller, it did not necessitate [a station] because the backward motion in this case is slower than the case when the ratios are equal [and thus, the motions cannot be equal which would cause a station].⁶⁶

64. Shirwānī's discussion and proof of Apollonius' lemma is on fol. 177b-178b, and his discussion of how Ptolemy uses the lemma in the cases of the eccentric and epicycle are fol. 179a-179b. See Toomer, *Ptolemy's Almagest*, 558-562.

65. Shirwānī, *Sharḥ al-Tadhkira*, fol. 178b.

وعلى تقدير التكافؤ يرى الكوكب أياما معدودة واقفا لقلة تفاضل الحركتين.

66. Shirwānī, *Sharḥ al-Tadhkira*, fol. 178b-179b.

3. If (A) is greater than (B), the planet will undergo retrogradation in the near segment between the two stations. Shirwānī's comments were only basic clarifications, as the real substance of Ṭūsī's discussion of this case occurs in the following section. Referring to the mean speed occurring at the two mean distances, Ṭūsī said that this was the motion of the concentric alone. To be more precise, the mean speed was due just to the motion of the concentric in the case of the epicyclic model, but not for the eccentric, because in the eccentric model (as Shirwānī correctly explained), the mean motion depends on the difference between the motions of the eccentric and the concentric deferent. Shirwānī commented that some copies of the *Tadhkira* did not discuss the epicycle here, and pointed out that Shīrāzī chose this version “as if he did not notice its lack of validity in [the case of] the eccentric.”⁶⁷ Based on this and other statements throughout his commentary, it is clear that Shirwānī had collected multiple manuscripts of the *Tadhkira* and was concerned about textual variants or corruptions.

Next, Shirwānī supplied the missing geometric proofs that retrograde motion occurs in the near segment between the two stations.⁶⁸ Following the proofs, he quoted Shīrāzī's *Tuhfa* as follows:

وقد سمعت الأستاذ رحمه الله يقول البرهان وإن لم يدل على عدم الوقوف على تقدير الأصغرية لكن المصنّف جزم به حدساً لأنه لما دل على أنّ أعظمية النسبة توجب الرجعة كما سيجيئ لغلبة الحركة إلى خلاف التوالي الحركة إلى التوالي. ومساواتها توجب إمكان الوقوف لإمكان تساوي الحركتين. فقد حدس منه أنّ أصغريتها لا توجب الإمكان لأن الحركة إلى خلاف التوالي على هذا التقدير تكون أبطأ منها على تقدير المساواة.

67. Shirwānī, *Sharḥ al-Tadhkira*, fol. 181b.

ولم يوجد لفظه «في التدوير» في بعض النسخ واختار هذه النسخة صاحب النهاية والتحفة وكأنه لم ينتبه لما فيها من عدم صحتها في الخارج.

Ragep notes that the explicit phrase “in the case of the epicycle” appears to be a late insertion, Ragep, *Tadhkira*, 414.

68. Shirwānī, *Sharḥ al-Tadhkira*, fol. 181b-183a. See Toomer, *Ptolemy's Almagest*, 559-562.

it must be known that there is no difference between the principle of the eccentric and the epicycle, taking into account the [given] conditions and proportions, and the principle of the concentric deferent with the epicycle. Therefore, Ptolemy demonstrated with the second [principle, i.e. epicycle], as we mentioned, and employed the first [i.e., eccentric], as will be clear when picturing/visualizing the orbs of the planets, God willing.⁶⁹

Shīrāzī went on to explain that the reason for this was that observation of the planets' retrograde arcs showed them to be equal to the assumed motions of the planet on the epicycle and epicycle center. He then restated the principle of equivalence between the two hypotheses

[which is] due also to the variation in the largeness and smallness of the two circles in different places and the variation of the two motions in quickness and slowness [in different places]. The difference in the ratios between the two motions ... is due to the difference in the distances of the planets and sizes of the epicycles, not through the proportionality of the arcs of retrogradation to their times.⁷⁰

For a proof of these remarks, Shirwānī referred the reader to Nīsābūrī's commentary on the *Almagest*.

Next, Tūsī noted that if the motions of the epicycle and eccentric hypotheses fulfilled certain [different] conditions, the situations of the near and far segments (see Figure 2.3, above) would be reversed. Shirwānī clarified what "reversed" meant in terms of which motions would occur in which segments, based on the relative ratios - i.e., slower motion could arise in the near

69. Shirwānī, *Sharḥ al-Tadhkira*, fol. 183a-183b.

قال صاحب التحفة ويجب ان يعلم ان حكم الخارج مع التدوير إذا روعيت الشرائط وحفظت النسب هو حكم الحامل الموافق مع التدوير من غير فرق. ولهذا برهن بطلميوس على الثاني كما ذكرنا واستعلم الأول كما سيظهر عند تصوّر افلاك المتحيرة ان شاء الله العزيز.

70. Shirwānī, *Sharḥ al-Tadhkira*, fol. 183a-183b.

كان حكم الخارج حكم الموافق ولاختلاف هاتين الدائرتين بالعظم والصغر في الاماكن المختلفة واختلاف الحركتين بالسرعة والبطء فيها أيضا واختلاف النسب التي بين الحركتين..لاختلاف أبعاد الكواكب ومقادير التدوير لا بتناسب قسي الرجوعات مع ازمنتها.

See also Morrison, "Qutb al-Dīn al-Shīrāzī's Hypotheses for Celestial Motions," 50.

segment rather than the farther segment, retrograde motion could be produced in the far segment rather than the near segment, and so forth. He noted that Shīrāzī and Nīsābūrī had both complained about how Ṭūsī phrased the necessary conditions for different phenomena, say for having retrograde motion occur in the far segment. Shirwānī was not as concerned as the earlier authors, however, because Ṭūsī's intent was merely to mention the general rules whose proofs were in the *Almagest*. Unlike Shīrāzī and Nīsābūrī, Shirwānī opted not to discuss in detail proofs for this “reversed” scenario. Such proofs are not actually mentioned in the *Almagest*, for they are unnecessary: the planets' retrograde and direct motions do not occur according to this (hypothetical) set of ratios between the motions of the epicycle and eccentric, but rather according to the original conditions Ṭūsī had mentioned previously.⁷¹ Fortunately for the reader, apparently Shirwānī's desire to provide a more comprehensive and advanced treatment of planetary kinematics did not compel him to discuss every possible configuration, only the ones that actually occur in the *Almagest* and *hay'a* models. This choice reflected Shirwānī's broader commentarial approach, which involved detailed analysis of the models as presented by Ptolemy, Ṭūsī, and others along with a careful eye for hidden assumptions and possible examples that could sharpen excessively general statements.

Following his discussion of the *Almagest*'s eccentric and epicyclic hypotheses, Ṭūsī reminded the reader that the hypotheses and rules were only stated here; their geometric proofs were given in the *Almagest*.⁷² Shirwānī explained that Ṭūsī had qualified the proofs as geometric

71. Shirwānī, *Sharḥ al-Tadhkira*, 183b-184a.

ولمّا رأيتُ أن مقصود المص ذكر القوانين التي براهينها مذكورة في المجسطي تركت ذكر ذلك البرهان على ذلك المدّعي لعدم ذكرهما في المجسطي بالفعل مع أنّه لا حاجة إليه إذ ليس في الكواكب رجوع واستقامة على هذا الوجه بل على الوجه السابق.

72. Despite his proof of the equivalence of the eccentric and epicycle models, in *Almagest* XII, 1, Ptolemy made the confusing assertion that the eccentric model “is a viable hypothesis only for the three [superior planets].” In fact, however, it applies to the inferior planets as well. Toomer, *Ptolemy's Almagest*, 555. In the 15th century, Regiomontanus proved the equivalence of the two

(i.e., using lines) because many of the *Almagest* proofs are stated using planes and circles, and so this science is called “incorporeal astronomy” (*hay’a ghayr mujassama*, lit. “incorporeal configuration,” we might say two-dimensional) and is part of the pure mathematical sciences (*min al-‘ulūm al-riyāḍīya al-ṣarfa*). As Ṭūsī noted, however, “one who attempts to understand the principles of the motions must know the configuration of the bodies (*hay’at al-ajsām*), which move with these motions in such a way that these motions manifest themselves on their equators.” Thus, this science is called “corporeal astronomy” (*hay’a [al-aflāk al-]mujassama*, “configuration of corporeal [orbs]”) and according to Shirwānī, draws in part on natural philosophy (*lahā’irq min al-‘ilm al-ṭabī‘ī*).⁷³

models for all planets in his *Epitome of the Almagest*, and as Noel Swerdlow has shown, his models would soon convert easily into the Tychonic and Copernican planetary systems. Noel Swerdlow, “The Derivation and First Draft of Copernicus’s Planetary Theory: A Translation of the Commentariolus with Commentary,” *Proceedings of the American Philosophical Society* 117, no. 6, Symposium on Copernicus (1973): , 471-478. Curiously, Regiomontanus did not draw attention to the fact that he was correcting Ptolemy’s error, a fact that Michael Shank has explored in Michael Shank, “Regiomontanus on Ptolemy, Physical Orbs, and Astronomical Fictionalism: Goldsteinian Themes in the “Defense of Theon against George of Trebizond”,” *Perspectives on Science* 10, no. 2 (2002): 179-207, Michael Shank, “Regiomontanus as a Physical Astronomer: Samplings from *The Defence of Theon against George of Trebizond*,” *Journal for the History of Astronomy* 38, (2007): 325-349, and elsewhere. In contrast, Jamil Ragep has argued that Regiomontanus’ proof derived from Qūshjī in F. Jamil Ragep, “Alī Qūshjī and Regiomontanus: Eccentric Transformations and Copernican Revolutions,” *Journal for the History of Astronomy* 36, (2005): 359-371.

73. Shirwānī, *Sharḥ al-Tadhkira*, fol. 184b. The Arabic word *‘irq* has the connotation of the roots of a tree, or parental/ancestral descent. Compare with Regiomontanus’s similar criticisms of the *Almagest* for using circles rather than bodies, Shank, “Regiomontanus on Ptolemy, Physical Orbs, and Astronomical Fictionalism,” 192-201.

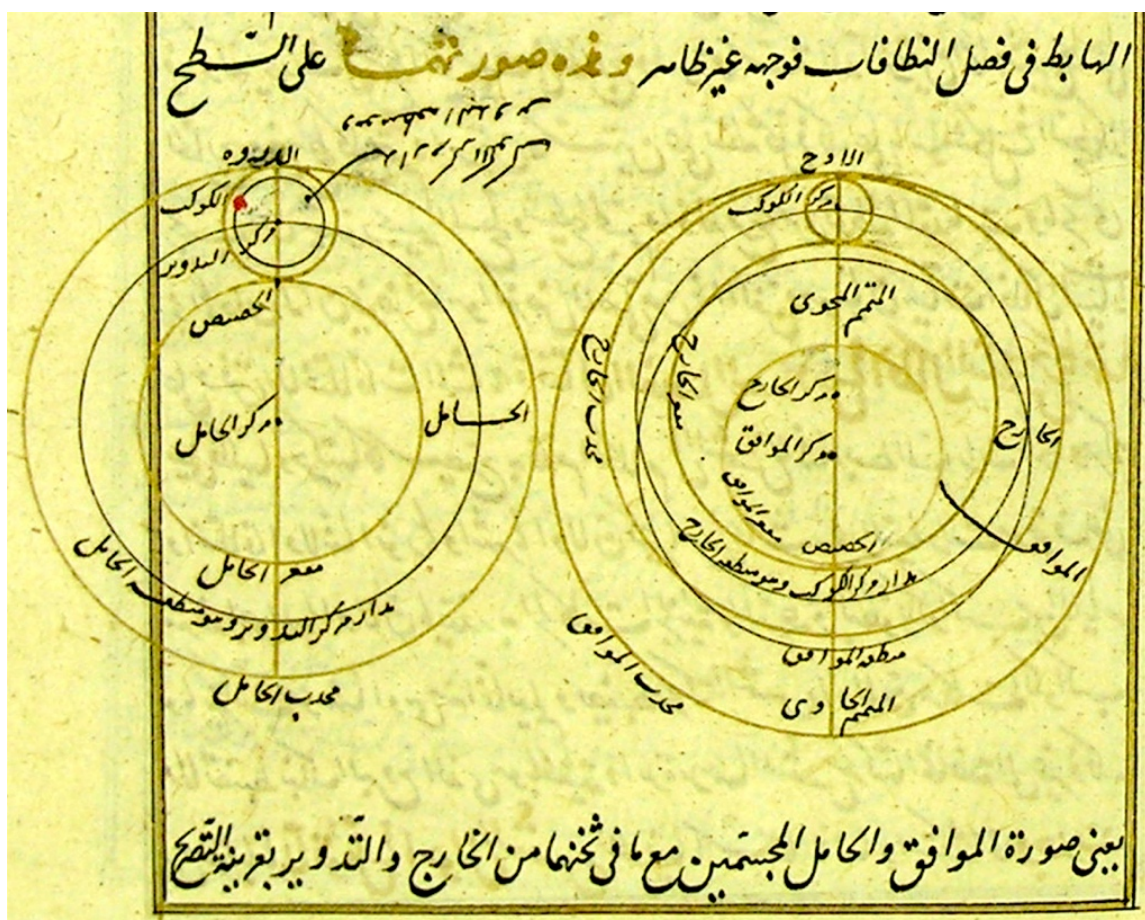


Figure 2.5 Eccentric (right) and Epicyclic (left) Hypotheses as Orbs
Ahmet III MS 3314, fol. 186a

Here, then, we find the key transition from circles to three-dimensional orbs. The configuration of bodies that Ṭūsī assembled to produce the motions of the eccentric and epicycle consisted of a concentric orb around the center of the cosmos, containing within its thickness a second, eccentric orb. The eccentric orb's thickness, in turn, was such that it could contain another orb, representing the epicycle, in which the planet was embedded (*markūz fīhi*). In this way, the necessary motion was generated by the epicycle's rotation carrying the planet as it in turn was carried by the eccentric orb. The spaces remaining if one removed the eccentric from concentric orb were filled by two "complementary bodies," curved solid bodies, thick in the middle and tapering to a point opposite their maximum thickness. Shirwānī explained that these "complementary bodies" were necessary because without one to fill each space between the

eccentric and concentric there would be a void (*lawlāhu lazama al-khalā*'), from which the reader would recall that according to the principles of natural philosophy described in *Tadhkira* Book I, there could be no void in the cosmos.

Shirwānī noted that Ṭūsī refers to the “complementary bodies” as “curved bodies” rather than “spheres” as they are called in Jaghmīnī’s *Mulakhkhaṣ*.⁷⁴ They are called “complementary” (*mutammim*) in the sense that they complete, or make whole, the concentric by being added to the eccentric. Shirwānī reported that Shīrāzī’s *Tuhfa* and *Nihāya*, as well as Jurjānī’s and ‘Ubaydī’s commentaries on the *Tadhkira*, explained they were so named because the bodies complete the eccentric in such a way that the combination moves around the concentric center. However, he reminded the reader that in fact the eccentric is complete without these bodies – rather the purpose of the complementary orbs was to complete the concentric orb. He went on to say that each of the complementary bodies is inside the boundary of the orb with respect to [the orb’s] concave surface. Thus a total of four bodies made up the epicycle-on-eccentric deferent model: the eccentric, the concentric (containing the complementaries), the epicycle, and the planet itself.⁷⁵

Shirwānī criticized Ṭūsī’s description of the epicycle as a sphere (*kura*) rather than a kind of orb (*falak*) in this discussion of the configuration of the bodies. Ṭūsī had stated that “the orb of

74. On Jaghmīnī’s terminology in the *Mulakhkhaṣ*, see Sally Ragep, “Maḥmūd ibn Muḥammad ibn ‘Umar al-Jaghmīnī’s al-Mulakhkhaṣ fī al-hay’ a al-basīṭa: An Edition, Translation, and Study” (PhD diss., McGill University, 2014), 176-178, 313.

75. Shirwānī, *Sharḥ al-Tadhkira*, fol. 185b-186a.

ط: «ويسميان المتممين» لأنهما يتممان الموافق بانضمامهما إلى الخارج. وفي النهاية والتحفة وشرحي السيّد والعبيدي لأنهما يتممان الخارج فيصير المجموع الموافق المركز ولا يخفي عليك أنّ الخارج تام بدونهما وإنما يتم بهما الموافق. وإعلم أنّ كلا من المتممين داخل في حدّ الكرة باعتبار محدبه فلا بعد في عدّ الخارج مع الموافق أربع كرات لأنّ التدوير يعدّونه كرة بالاتفاق مع أنه بعد انفصال الكوكب عنه مثل المتمم وأمّا أنه لا يجوز أن يكون في الأجرام السماوية جرم مختلف الثخن بالغلظ والرقّة فقد عرفت جوابه.

the epicycle is a sphere... (*wa-falak al-tadwīr kura*).” Shirwānī was referring back to Ṭūsī’s definitions in Book I of a sphere as a solid and an orb as a solid spherical shell bounded by two concentric surfaces.⁷⁶ Shirwānī remarked “it is evident [he should have] said ‘the epicycle is an orb’ because it stands in the place of the embodiment of circles in the incorporeal configuration, and the name here is ‘epicycle,’ as you know, not ‘orb of the epicycle.’ Accepting this name for the orb in this regard is wrong.” However, Shirwānī conceded, Ṭūsī did not say the epicycle is “an orb” given that his definition of “orb” in Book I arguably did not apply to the epicycle.⁷⁷

What did it mean for the planet to be “embedded” in the epicycle? This appears to be a subject on which the physicality and mechanical nature of the models becomes somewhat fuzzy. Ṭūsī’s terminology becomes confusing at this point for despite imagining that the planet is inside the epicycle, he stated that the planet was “embedded in [the epicycle] in such a way that its outer surface is tangent to the convex (*muḥaddab*) surface of the epicycle at a point.” Shirwānī explained that this point was shared between the two at the midpoint of the epicycle’s poles,

for if that were not the case, either [the planet] would be protruding from the surface or sunk into it rather than touching. The first would require a hole (*kharq*) or void (*khalāʾ*), and the second would require an excess in the epicycle that is unnecessary. It is more suitable to say its surface touches the epicycle’s surface or its convex [surface touches the epicycle’s] convex [surface].⁷⁸

76. See Ragep, *Tadhkira*, 96-99. In the case of the epicycle, the concave inner surface was rarely taken into account

77. Shirwānī, *Sharḥ al-Tadhkira*, fol. 185a.

كان الظاهر أن يقول «والتدوير فلكا» لأنه في مقام تجسم الدوائر التي في الهيئة الغير المجسّمة والاسم فيها هو التدوير كما عرفت لا فلك التدوير والتجوّز في اسم الفلك في هذا المحلّ قبيح. وإنما لم يقل فلكا نظرا إلى أنّ التعريف المذكور لا يشمل التدوير.

78. Shirwānī, *Sharḥ al-Tadhkira*, fol. 185a-b.

إذ لو لم يكن بتلك الحيثية لكان إمّا نائنا عنه أو مغرّقا فيه غير مماس والأوّل يستلزم الخرق أو الخلاء والثاني يستلزم فضلا في التدوير لا يحتاج إليه. وكان الأولى أن يقول يماس سطحه سطح التدوير أو محدبّه محدبه.

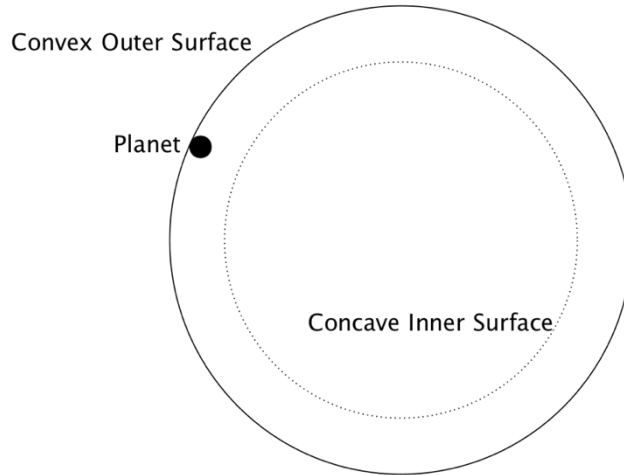


Figure 2.6 Planet embedded in epicycle

The reason for referring to the convex surfaces of the epicycle and planet, despite the planet being positioned within the epicycle, was that, according to Ṭūsī's definition of an orb as a spherical shell, the concave surface was the inner surface of the shell parallel to the convex outer surface. Ṭūsī went on to state that for neither the planet nor the epicycle was their inner concave surface taken into account. Shirwānī explained that indeed the planet does not have an inner surface at all. This remark was clarifying that the planet is in the thickness of the epicyclic orb just as the epicycle is in the thickness of the eccentric. Thus, Ṭūsī may have feared that referring to the concave side of the outer convex surface would mislead the reader into thinking the planet was deeper inside the epicycle at the parallel concave inner surface, rather than just tangent to the outer surface. Shirwānī's explanation – that Ṭūsī's statement “indicated that the epicycle has a non-parallel concave [inner surface] that is the planet's place” – supports this interpretation.⁷⁹

Shirwānī had a great deal to say about this chapter of the *Tadhkira*, a sign of how important it was to understand the various hypotheses before progressing to the actual planetary

79. Shirwānī, *Sharḥ al-Tadhkira*, fol. 185a-b.

وإنّما عوّل المص في فهم تقيّد مقعر التدوير بذلك على قوله «والكوكب مركز فيه الخ» فأنه يدل على أنّ للتدوير مقعرا غير موازيا هو مكان الكوكب.

models in the subsequent chapters of Book II. His comments reveal that the finer details of the mathematical hypotheses and principles, as well as the consequences of embodying them in the motions of physical spheres, remained the subject of active investigation and debate both in the commentary tradition as well as among the scholars at Samarqand.

Solar theory

Following the order of topics in the *Almagest*, Tūsī had moved from his discussion of the cosmological preliminaries and the epicyclic and eccentric hypotheses to the case of the Sun without, however, explaining why this should be the proper starting point.⁸⁰ Shirwānī, in contrast, explained the many significant reasons why Tūsī began his discussion of the celestial orbs with the Sun:

Of all the planets, its model, motion, and anomaly are the simplest. It is the brightest and best-known, moreover, many investigations of the other planets require knowing the circumstances of the Sun. Measurements of the particular motions and speed and slowness of the planets over days, and what is composed from them [i.e., months and years], are from [knowledge of the Sun], for they are known and made precise by the motion of the Sun. Indeed, they are measured along the ecliptic, which in truth is the (great) circle the Sun traces with its own motion.⁸¹

80. However, previously in II.2 [4] he had mentioned that Sun's orb was located in a medial position among the other orbs because the other 6 were "connected to it" - the upper planets in one way, the lower planets in another, and the Moon in yet another. Ragep, *Tadhkira*, 110.

81. Shirwānī, *Sharḥ al-Tadhkira*, fol. 186b.

قدّم الكلام في الشمس لأنها أبسط السيارات فلما وحركة واختلافاً أو لأنها أنورها وأشهرها أو لأن كثيراً من أبحاث سائر السيارات موقوف على معرفة أحوالها أو لأن ما يقدر به الحركات الجزئية والسرعية والبطء للكواكب من الأيام وما يتركّب هو منها أو هي منه إنما يعلم ويضبط بحركة الشمس أو لأن حركات الكواكب إنما تضبط بفلك البروج الذي هو بالحقيقة دائرة ترسمها الشمس بحركتها الخاصة.

Observation of the Sun reveals that it has two irregular motions: a variation in speed over the course of a year (slower in a certain half and faster in the other), and a gradual motion of its apogee. Shirwānī stated that the variation in speed or “the anomaly in longitude” is known by [observing that] the time for the Sun to pass from the vernal equinox to the autumnal equinox is longer than the time required for the second half of the ecliptic, so its motion is slower in the first half. On the other hand, the Sun is always found along the ecliptic equator, so it has no anomaly in latitude. Shirwānī noted that these phenomena were known to both the “ancients” (*al-mutaqaddimīn*) and the “moderns” (*al-muta’akhhirīn*). Moreover, Ṭūsī reported that by careful observation of solar eclipses, the “moderns” had determined that the solar body appears smaller during the middle of the slower motion. In connection with the apparent variation in solar size, Shirwānī mentioned two notable observations by two Islamic astronomers. Muḥammad b. Ishāq al-Sarakhsī had perceived the smaller size of the Sun during an eclipse visible in the middle of the period of slower solar motion, whereas Abū al-‘Abbās al-Īrānshahrī had witnessed a luminous ring remaining from the Sun encircling the Moon in the middle of the period of rapid solar motion, despite the fact that the Moon was the same distance from the Earth at both of these times.⁸² In contrast, Shirwānī noted, the “ancients” had not found a sensible difference in the solar body connected to its change in speed, and if there were a variation in its distance from the earth, it would necessitate solar parallax, so they judged the solar diameter to be the same at all distances.

82. Shirwānī, *Sharḥ al-Tadhkira*, fol. 187a. Although he did not cite any references here, al-Bīrūnī must have been the source for Shirwānī’s remarks. Bīrūnī mentioned Īrānshahrī’s observation of an annular eclipse, which reportedly took place on 28 July 873 in Nīsābūr, in his *al-Qānūn al-Mas’ūdī*, Ragep, *Tadhkira*, 461. Bīrūnī referred to Sarakhsī in several of his works; see E. S. Kennedy, “A Survey of Islamic Astronomical Tables,” *Transactions of the American Philosophical Society* 46, no. 2 (1956), 131.

As for the motion of the solar apogee, it “is approximately the movement of the fixed stars due to the second motion [i.e., precession].” Shirwānī explained the procedure for determining the existence and magnitude of this motion through observations, and repeated his report from the discussion of precession that most of the “moderns” determined the motion of the solar apogee to be 1° in 66 years and another group (including Ṭūsī himself as part of the observations at Marāgha) found it to be 70 years.⁸³ Ṭūsī mentioned that Ptolemy did not find this motion, which Shirwānī explained by quoting from *Almagest* III.4. There Ptolemy had provided Hipparchus’s observations of equinoxes and solstices to determine the lengths of the seasons, which matched the lengths that Ptolemy himself calculated based on his own observations, leading him to conclude that the Sun’s eccentric orb always maintained the same position relative to the solstitial and equinoctial points.⁸⁴ Shirwānī also quoted Bīrūnī’s explanation that the agreement between Hipparchus’ and Ptolemy’s findings was an argument for the immobility of the solar apogee. In contrast, the fact that his findings differed from those of the “moderns,” whose observations all agreed that the apogee was no longer where Ptolemy reported, was an even stronger argument for its motion. According to Shirwānī, the efforts of the moderns determined that the increase over Ptolemy’s apogee was greater than 15° (as he put it, $1/4$ of $1/6$ th of the [solar] circuit). Indeed, in his *al-Qānūn al-Mas‘ūdī*, Bīrūnī was concerned with investigating the motion of the apogee in order to determine a more accurate value for the length

83. Shirwānī, *Sharḥ al-Tadhkira*, fol. 187b.

بأنّ رصد مقدار سيرها في جزء معين من منطقة البروج بعد مفارقتها الاعتدال الربيعي وقبل بلوغها غاية أبطائها إلى أن جاوزتها ووصلت إلى مثل الحالة الأولى نعلم أن الأوج على منتصف القوس التي بين الحالتين والحضيض على مقابله. ثم رصد موضع الأوج بعد برهة فوجد منتقلا عن الموضع الأول إلى موضع آخر على التوالي فقسمت القوس التي بين الموضعين من منطقة البروج على الزمان الذي بين الرصدين فخرجت حركته في كلّ ستّ وستين سنة بحسب أرصاد أكثرهم وفي كل سبعين سنة بحسب أرصاد قوم من المحققين طابقتها رصد المص درجة واحدة.

84. Toomer, *Ptolemy’s Almagest*, 153-154.

of the tropical year, since if the solar apogee moved then the starting point from which one measured a tropical year would affect the observed length of the year. Bīrūnī laid out theorems proving this point, and determined the motion of precession to be 1° in 68 years, 11 months. Even combined with his own observations, however, the data available to him was not accurate enough to determine whether the solar apogee had its own motion separate from precession, so he assumed (as a hypothesis), that the motion of the apogee was identical with precession.⁸⁵

Shirwānī also cited Abū Naṣr Yaḥyā b. Jarīr (d. 1103/4), who reported that scholars disagreed about the solar apogee but agreed on the planetary apogees, for Ptolemy and those who came before him did not establish that the solar apogee moved. But as for the Christians, they argued that if the apogee moved, it would cause great events to occur in the world, which Yaḥyā described at great length.⁸⁶ One might wonder why Shirwānī included this report, which can be read in support of the side arguing that the solar apogee does not move separately from the general precession of the equinoxes. In the following discussion, both Shirwānī and Ṭūsī referred to ongoing debate about the motion of the solar apogee, although Shirwānī included it as one of the four “discoveries” or phenomena that must be accounted for in the solar model. Recall that Ṭūsī had stated that the motion of the solar apogee was “approximately” equal to the rate of precession. The fundamental question by this time was whether the solar apogee’s motion, although real, was separate from the precession of the equinoxes. If the solar apogee was taken to

85. On Bīrūnī’s investigation of the tropical year and solar apogee, see Willy Hartner and Matthias Schramm, “Al-Bīrūnī and the Theory of the Solar Apogee: An Example of Originality in Arabic Science,” in *Scientific Change*, ed. A.C. Crombie, (New York: Basic Books, 1963), 206-218.

86. Shirwānī, *Sharḥ al-Tadhkira*, fol. 187b.

وقال أبو نصر يحيى بن جرير اختلف العلماء في أوج الشمس واتفقوا في أوجات الكواكب فبطلميوس ومن قبله من علماء هذه الصناعة لم يثبتوا الأوج الشمس حركة والناصرين لهم احتجوا بأن هذا الأوج لو تحرك لوجب أن يحدث في العالم أحداثا عظيمة ثم شرحها بما فيه طول.

be a point in the heavens, then it would not be surprising for it to gradually precess along with the equinoxes and other “fixed” points. In contrast, if one claimed that the solar apogee had its own separate motion, then in principle it should be detectable with enough time and careful observation. Shirwānī’s reference to Yaḥyā indicated some participants in the debate were also willing to think about the movement of the solar apogee in terms of astrology. Moreover, the fact that Shirwānī included Yaḥyā’s remarks alongside the arguments of authoritative astronomers and thinkers like Ptolemy and Bīrūnī suggests he believed astrological concepts merited some modest consideration. At the very least, he did not dismiss them out of hand.

Ṭūsī next presented two configurations for the Sun’s orb, using the eccentric and epicyclic models respectively. Ultimately, as mentioned previously, Ptolemy had chosen the eccentric model for the Sun “because it is simpler,” in that it requires one motion, and one circle, rather than two. In his comments on this section, Shirwānī clarified for the reader which aspects of each model accounted for each of the solar phenomena listed above. He also introduced some of the parameters and calculations related to the Sun’s motion. Surprisingly, Ṭūsī by this point had not mentioned a value for the length of the (tropical) year or, equivalently, the Sun’s mean daily motion. In the following chapter, on the lunar theory, Ṭūsī would casually mention the Sun’s mean motion as $0;59^\circ$ daily, and in Book III he approximated the length of the year as “365 and $1/4$ days minus a fraction.”⁸⁷ These approximate values sufficed to give a general working sense of the motions of the heavens, but were clearly inadequate for constructing

87. In his commentary on III.10. Shirwānī provided three more precise values for the fractional amount to be subtracted from 365 and $1/4$ days (represented in sexagesimal fractions as $365;15$): the Ptolemaic value ($1/300$ of a day or 12 seconds, giving a solar year as $365;14,48$ days), the value generally accepted by “the moderns” or Islamic astronomers ($1/150$ of a day) and the value of $1/120$ of a day calculated by Muḥyī al-Dīn al-Maghribī, Shirwānī, *Sharḥ al-Tadhkira*, fol. 326b.

accurate predictions or astronomical tables. In contrast, Shirwānī reported the Sun's mean motion as $0;59,8,20^\circ$ per day.⁸⁸ This value is somewhat more precise, and is the same value used in Jaghmīnī's *Mulakhkhaṣ*, but it falls well short of the level of precision Ptolemy claimed was necessary for an accurate solar model.

To produce an eccentric model out of physical orbs, Ṭūsī followed the principles previously set forth in II.5, establishing a concentric orb (known as the parecliptic orb) whose thickness contains the eccentric, and two complementary bodies to fill the remaining space. Shirwānī, as was his practice, interrupted his exposition to answer a rhetorical question chosen to emphasize the physicality of the planetary orbs and their arrangement in one unified system: Why didn't Ṭūsī mention only the parecliptic and eccentric orbs, without the complementary bodies? Shirwānī replied that what was being established here was not merely the motions, and if the model did not have the complementary bodies, a void would occur. He was thinking about the physical relationship between the planetary models, wherein the Sun's orb is situated between the orbs of Mars and Venus. Shirwānī explained that the concave [inner surface] of Mars's parecliptic orb was not allowed to touch the convex [outer surface] of the eccentric in the way the concave surface of the containing complementary body touches [the eccentric]. Likewise, neither was the convex [outer surface] of Venus's parecliptic orb allowed to touch the concavity of the eccentric in the way the convex surface of the contained complementary body touches [the eccentric]. Otherwise (i.e., if they were directly touching the eccentric), the two parecliptic orbs would not be parallel, which is absurd. Therefore, in consideration of the

88. Shirwānī, *Sharḥ al-Tadhkira*, fol. 187b. Ptolemy claimed it was sufficient to carry out the calculation to 6 sexagesimal places, and reported a value of $0;59,8,17,13,12,31^\circ$ in the *Almagest*. Likewise, Bīrūnī's *al-Qānūn al-Mas'ūdī* gives $0;59,8,12,7,56,33^\circ$. See Toomer, *Ptolemy's Almagest*, 140; Kennedy, "A Survey of Islamic Astronomical Tables," 158; and Ragep, "Jaghmīnī's al-Mulakhkhaṣ" 321.

parallelism of the solar orb's two [inner and outer] surfaces, the amount (otherwise) occupied by the complementary bodies would be void. Furthermore, he pointed out, if the orbs of Mars and Venus were assumed to be non-parallel and to touch the eccentric, then the motion of the solar apogee and perigee would cause the eccentric to break through the orbs of Mars and Venus.⁸⁹

The parecliptic orb, a component of each planetary model ensuring that they are all concentric to the center of the World, generally contained the planet's other orbs and moved with the motion of the fixed stars (the situation for the Moon is slightly more complicated). According to Ṭūsī, in the eccentric model for the Sun, the parecliptic moved the solar apogee and perigee. However, in the epicyclic model for the Sun, Ṭūsī made the confusing statement that the eighth orb (the orb of the fixed stars, responsible for precession) moved the solar apogee and perigee “since it moves everything below it” and thus the epicycle's deferent was its parecliptic. As Ragep points out, this is a puzzling and somewhat contradictory statement because in the eccentric model, the parecliptic is also “below” the eighth orb — so why does the eighth orb not move the eccentric parecliptic in the same way? Or alternatively, since it was generally assumed that each orb moves with its own motion, why should any of them be moved by the eighth orb?⁹⁰ According to Shirwānī, the reason why, in the eccentric model, the parecliptic moves [with its own proper motion] and moves the apogee/perigee, despite the eighth orb also being sufficient

89. Shirwānī, *Sharḥ al-Tadhkira*, fol. 188b.

طوسي: ويلزم على أصل الخارج المركز إثبات فلك موافق المركز يكون الخارج المركز في ثخنه ويفضل عليه بتمميمه. شرواني: فإن قلت «فهلا قال أما خارج مركز وحامل منطقتاهما في سطح منطقة البروج؟» قلت: لأن لزوم إثباته ليس لاقتضاء ذلك بل لأنه لو لاه لزم الخلاء لأنه لا يجوز أن يماسّ مقعر ممثّل المريخ محدّب الخارج كما ماسّه مقعر المتمم الحاوي ولا محدّب ممثّل الزهرة مقعر الخارج كما ماسّه محدّب المتمم المحوى والا لم يكن الممثلان متوازي السطحين وهو باطل لا اعتبار التوازي في سطحي الفلك فيخلو قدر ما يشغله المتممان من الجسم. وإن جوّزت المماسّة كذلك على تجويز عدم التوازي لزم الخرق من حركة الأوج والحضيض.

90. Ragep mentions that Khafri and Jurjānī wanted to have the parecliptics in each model move with their own motions rather than have them depending on the eighth orb, and proposed an alternative for the solar model that avoided Ṭūsī's dilemma, Ragep, *Tadhkira*, 415-416.

[to move them], was that the parecliptic must not be left idle or without purpose. But the reply to or refutation of that argument is that the parecliptic in that case does not move because of the motion of the eighth orb despite its being contained in “everything below [the eighth orb].” So, Shirwānī continued, it might be better to say that the parecliptics, aside from the lunar parecliptic, are self-moved (*mutaḥarrika bi-dhawātihā*) with a motion like the motion of the eighth orb, which could be the meaning of Ṭūsī’s statement that the motion of the eighth orb “moves everything below it.” Then Mars’s parecliptic would be responsible for moving the solar apogee. Thus, the eighth orb did not move any of the parecliptics (i.e., does not directly cause them to move). By defining the issue this way, Shirwānī preserved the principle of each orb being self-moved and (somewhat awkwardly) explained away Ṭūsī’s statement about the eighth orb “moving” lower orbs.

As a more likely alternative reading, Shirwānī proposed that the parecliptics all move with accidental motion (*mutaḥarrika bi-l-‘ard*, i.e. rather than being self-moved) because of the motion of the eighth orb. This would not leave the orbs without purpose, since although their self-motions are unnecessary the orbs fulfilled other roles in the models. Shirwānī’s alternative reading was a more faithful literal reading of the *Tadhkira*, and had the appeal of economy – the same motion that occurs in the system for each planet is brought about by a single mover. Presumably he meant that, in the eccentric model for the Sun, the parecliptic would still have its own proper motion to account for the motion of the apogee, and in the eccentric model the proper motion of Mars’s parecliptic would move the solar apogee.⁹¹

91. Shirwānī, *Sharḥ al-Tadhkira*, fols. 188b-189a.

طوسي: إذ هو محرّك لجميع ما دونه. شرواني: وإنما فرضوا الممثل متحركا للأوج والحضيض على أصل الخارج مع كفاية الثامن عليه أيضا في تحريكهما كيلا يلزم تعطيله ويرد على هذا أنّ الممثل حينئذ لا يتحرك بحركة الثامن مع شموله لجميع ما دونه. فالأولى أن يقال الممثلات سوى ممثل القمر متحركة بذواتها مثل حركة الثامن وهذا معنى شمول حركته لجميع ما دونه وممثل المريخ يحرك أوج الشمس أيضا على أصل التدوير. ولا يحرك الثامن حينئذ شيء من الممثلات. أو يقال هي متحركة

The Sun has a single anomaly, which Ṭūsī defined as the difference between the Sun's observed motion and mean motion; its greatest value occurs at the two mean distances⁹² and depends on the amount of the eccentricity. He reported a value for the solar eccentricity as 2;30 parts (with the radius of the eccentric being defined as 60 parts) according to Ptolemy, and close to 2;05 parts according to more recent observers. The position of the solar apogee was $24\frac{1}{2}^\circ$ ahead of the summer solstice point according to Ptolemy, but according to the moderns it varies. Shirwānī added that Qāḍīzāde obtained a value of “2;01[parts] and a fraction” for the solar eccentricity based on observations at Samarqand. Shirwānī also noted that the maximum value for the anomaly was $2;23^\circ$ if the eccentricity is 2;30 parts, and $1;59^\circ$ if the eccentricity is 2;05 parts.⁹³ As for the position of the apogee, Shirwānī noted that different values were reported in the *zijes* depending on the date of the *zīj* since the apogee moved gradually over time. According to Shirwānī, Ṭūsī mentioned in his *Zīj-i Īlkhānī* that on the 1st of year 701 Yazdigird the apogee was at $27;50,4^\circ$ Gemini.⁹⁴

Ṭūsī concluded with a summary of the standard Ptolemaic model for the motion of the Sun. The sum of the motion of the solar apogee and the motion of the Sun around the eccentric produced the “mean” or average position of the Sun (i.e., the Sun's position relative to the

بالعرض بتحريك الثامن ولا محذور في تعطيلها بل يمكن ترجيحه إذ حركتها فضل لا يحتاج إليه. وإذا كان المحرك لهما على أصل التدوير هو الفلك الثامن.

92. Of which there are two definitions, see II.6 [5].

93. Shirwānī, *Sharḥ al-Tadhkira*, fol. 189b.

وقد سمعت من الأستاذ قحمة الله أنه وجده برصد سمرقند بـ ا وكسراً. وغاية الاختلاف على أن ما بين المركزين بـ ل يكون بـ كج وعلى أنه بـ ه يكون ا نط.

94. Shirwānī, *Sharḥ al-Tadhkira*, fol. 189b.

وقد ذكر المص في زيجه أنه في أول سنة إحدى وسبعمائة يزددجردية في سبع وعشرين درجة وخمسين دقيقة وأربع ثوان من الجوزاء.

eccentric center). To this one added or subtracted the value of the solar anomaly to determine the Sun's true position. Thus, the Sun was ordered via two orbs (the parecliptic and eccentric) and two motions (the motion of the apogee and the motion of the eccentric). Shirwānī explained that the Sun's true position was therefore the result of combining the mean position with the angle of the solar anomaly. Thus, the true position will be less than the mean by the amount of the anomaly when the Sun is descending from the apogee toward the perigee, which can be seen on the following figure since the endpoint of the line from the center of the world to the Sun is closer to the apogee than the endpoint of the line from the eccentric center. The true and mean Sun will be the same at perigee and apogee, and the true position will be greater than the mean by the amount of the anomaly on the ascending half of the eccentric.

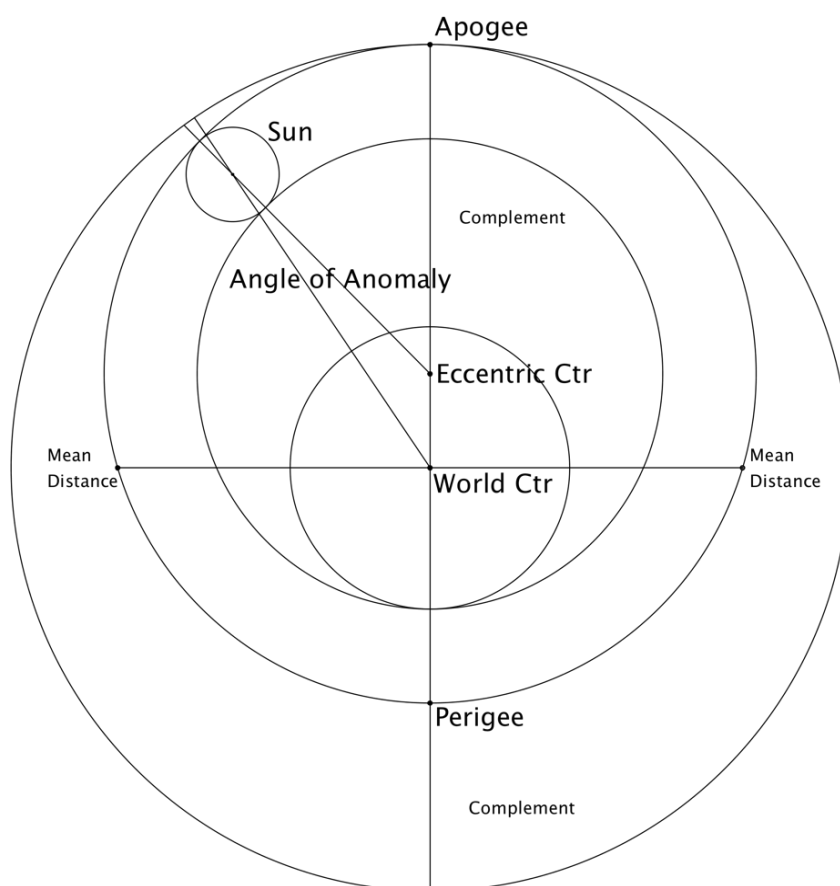


Figure 2.7 Orbs of the Sun

Conclusion

The basic hypotheses of the eccentric, epicycle, and equant, along with the solar model, set the stage for the remaining planetary models. We will analyze Shirwānī's comments on these topics, and make some general conclusions, in the following chapter. For now, it will suffice to point out that even when dealing with the introductory material of this chapter, Shirwānī took the opportunity to guide the reader through a number of technical issues that lay behind Ṭūsī's remarks, while simultaneously introducing significant criticisms and clarifications.

Chapter 3 – Astronomy in Shirwānī’s commentary on the *Tadhkira*, II: The Moon, Planets, and Ṭūsī Couple

Introduction

This chapter completes the analysis of technical material in Shirwānī’s massive commentary on Ṭūsī’s *Tadhkira*. The topics of the previous chapter, which discussed the basic mathematical hypotheses (*uṣūl*) along with Ṭūsī’s solar model, were a foundation upon which the models for the Moon and other planets were based. Since these later models were much more complex than that of the Sun, Shirwānī had numerous opportunities to offer clarifications and criticisms. Most importantly, Shirwānī brought the discussion “up to date” by quoting or referring to criticisms and alternatives that had been proposed in later texts, or that had been raised in the classroom at Ulugh Beg’s Samarqand madrasa.

Lunar theory

The Moon’s motion is significantly more complicated to describe than that of the Sun. In the *Almagest*, Ptolemy presented a series of three lunar models, each representing an increasingly complex refinement of the previous model. However, a reader of the *Tadhkira* would not necessarily be aware of these different models, since Ṭūsī limited his presentation to the observed lunar phenomena and the final Ptolemaic model. Moreover, the fact that Ṭūsī’s own lunar model would soon supersede that of the *Almagest* complicates citations of Ptolemy in Shirwānī’s commentary (and in the *Tadhkira* itself).

Commenting on the title of this section, Shirwānī noted that the Moon has motions in both longitude and latitude, so the title of this chapter (“On the Orbs and Motions of the Moon”) was not limited to longitude, unlike the titles of the two following chapters (“On the Orbs and Longitudinal Motions of Mercury,” “On the Orbs and Longitudinal Motions of the Remaining Planets”). Shirwānī noted that the motion of the Moon resembles that of the Sun in certain ways, neither luminary stops or moves with retrograde motion, and both the Sun and Moon determine the months and years. The Moon moves along a circuit that is inclined relative to the ecliptic, intersecting the ecliptic at two points diametrically opposite each other (the nodes) and giving the Moon a variation in latitude (moving above and below the ecliptic on either half of its circuit around the Earth). Not only does the Moon have an irregular speed along this circuit, but in addition its relative maximum and minimum speeds vary from revolution to revolution around the Earth. Ṭūsī described this as returning to a “comparable” speed after the completion of a lunar revolution.¹ Furthermore, its distance from the Earth (and thus, its perceived size) varies over the course of its circuit. Therefore, in *Tadhkira* II.7, Ṭūsī laid out an account of four Ptolemaic orbs and motions that accounted for most of the observations before listing some of the problems or difficulties (*ishkālāt*) that he would attempt to resolve in II.11.

The configuration for the Moon consisted of:

1. The parecliptic orb (*al-falak al-mumaththal bi-falak al-burūj*), centered on the Earth, nested inside and contiguous with Mercury’s parecliptic;
2. The inclined orb (*al-falak al-mā’il*), centered on the Earth, inside the parecliptic and surrounding the sphere of Fire. It was called “inclined” because its equator has a fixed inclination

1. Ṭūsī was referring to the “anomalistic month” (the period of the Moon’s return to its apogee) being longer than the “tropical month” (the Moon’s return to the same point on the ecliptic).

to the parecliptic equator whose maximum value as found by observation is 5° – this accounted for the Moon’s variation in latitude.

3. The eccentric orb, contained within the thickness of the inclined orb, with its equator in the plane of the inclined orb’s equator.

4. The epicycle in the thickness of the eccentric, which is its deferent. The Moon is embedded in the epicycle and adheres always to its equator (also in the plane of the eccentric equator).

Furthermore, the two intersections at opposite points of the parecliptic and the inclined orbs’ equators were called the nodes, and are the locations near which solar and lunar eclipses may occur. Ṭūsī and Shirwānī would discuss eclipses later, in separate chapters.

Shirwānī reminded the reader that one can account for the Moon’s variation in speed using either an eccentric or an epicycle, but its return to a “comparable” speed from circuit to circuit could be explained by neither an eccentric without an epicycle, nor an epicycle on a concentric deferent, because those arrangements produce slowness and speed in the same [respective] parts. Therefore the lunar theory required an epicycle in the thickness of an eccentric deferent, with another “inclined” orb moving its apex around the center of the World and causing slowness and speed in all parts of the ecliptic.² In II.7 [10], Ṭūsī made the problematic statement that motion of the eccentric was constant about the center of the World, and repeated the statement shortly thereafter in II.7 [12]: because the three motions he had previously listed (the motions of the parecliptic, inclined, and eccentric orbs) are all “about the center of the World,

2. Fathallāh al-Shirwānī, *Sharḥ al-Tadhkira*, Topkapı Ahmet III MS 3314, fol. 192b.

والاختلاف بالبطء والسرعة وحده يمكن إسناده إما إلى خارج وحده وإما إلى تدوير وحامل موافق كما في الشمس لكنه مع الانتقال والعود المذكورين لا يمكن إسناده إلى خارج وحده أعني بلا تدوير ولا إلى تدوير في ثخن حامل موافق كما في الشمس لاقتضاء كل منهما كون البطء والسرعة في أجزاء بأعيانها فأسندوه إلى تدوير في ثخن حامل خارج يحرك أوجه فلك آخر حول مركز العالم هو المائل ليقع السرعة والبطء في جميع أجزاء البروج.

they are all uniform with respect to us.” This is surprising, of course, because in the original conception of an eccentric model, the motion is uniform with respect to the eccentric center, whereas, from the center of the World, the motion is observed as nonuniform (this is the very point of the model). Shirwānī mentioned that “this calls for study (*wa-fīhi naẓar*) because if the motion of the Moon’s eccentric is uniform around the center of the World [i.e., making equal angles in equal times at the Earth, rather than the center of the eccentric], then how can it be correct to base [the Moon’s] variation on it?”³

In *Almagest* IV-V, as Ptolemy worked through three increasingly complex models in order to account for the observed motions of the Moon, he replaced the eccentric with a kind of equant. In the final model, the center of the Moon’s epicycle is carried along a circle containing, but not centered on, the Earth. However, by using a “crank mechanism” to move the eccentric center in a circle around the Earth, the motion of the epicycle center is made to be uniform with respect to the Earth, effectively placing the Earth at an equant point.⁴ This behavior is what Ṭūsī described in his overview of the Moon’s observed motions and the Ptolemaic attempt to model them, although he did not describe the additional circle Ptolemy used to move the center of the eccentric around the Earth. In his comment in II.7 [10] and again in II.7 [12], Shirwānī merely stated that the phrases “around the center of the World” and “uniform” with respect to the center of the World both mean equal angles in equal times from the perspective of the Earth. He made it clear that he was aware of the true complexity of the Ptolemaic system and that the Moon’s longitudinal motion was a problem to be addressed, but for the moment he seemed to follow Ṭūsī

3. Shirwānī, *Sharḥ al-Tadhkira*, fol. 192b.

وفيه نظر لان حركة خارج القمر متشابهة حول مركز العالم فكيف يصح إسناد ذلك الاختلاف إليه؟

4. G.J. Toomer, *Ptolemy’s Almagest* (Princeton: Princeton University Press, 1998), 220ff.

in presenting the basic components and motions of Ptolemy's system that needed to be accounted for, without touching on the aspects that would be eliminated or reformed in Ṭūsī's new system presented in *Tadhkira* II.11.

Shirwānī briefly mentioned some positions in this debate about the roles of the eccentric and the epicycle in shifting the position of the Moon's relative speeds. He reported that some (unspecified) astronomers attributed the Moon's change in speed to an eccentric, and attributed the shift in the Moon's return to a "comparable" speed to an epicycle carried by the eccentric in such a way that the Moon's motion on the epicycle was less than the mean motion. Shirwānī doubted this model (*wa fīhi naẓar*) because, as mentioned previously, the motion of the Moon's eccentric was posited to make equal angles in equal times at the center of the World, and therefore could not account for the change in speed. He reported that Qāḍīzāde objected as well, but for a different reason – although the epicycle's motion was less than the mean motion, that alone did not account for the change in return to a "comparable" speed. Rather, one also needed to take into account the motion of the lunar apogee. According to Shirwānī, "the answer to this will be forthcoming, God willing." He then mentioned that in the *Nihāya*, Shīrāzī had objected similarly to the common inference that the comparable speeds in different parts of the Moon's circuit could not occur without an epicycle. Instead, Shīrāzī argued, on account of the rapid motion of the lunar apogee, it was possible for the Moon to speed up and slow down in all parts of the ecliptic on the basis of an eccentric alone.⁵

5. Shirwānī, *Sharḥ al-Tadhkira*, fol. 192b.

واعترض الأستاذ رحمه الله بوجه آخر وهو أنّ سياق كلامهما موافقا لكلام المص في حركة التدوير يدلّ على أنّ سبب عود كلّ اختلاف لا إلى مثله بعينه هو كون حركة التدوير أقلّ من حركة الوسط وليس كذلك لأنّ الأوج متحرّك إلى خلاف التوالي أقلّ من حركة الوسط إلى التوالي فيلزم عدم عود كلّ اختلاف إلى مثله وأن كانت حركتا التدوير والوسط متساويتين نعم لو كان الأوج ساكنا لتعيّن لسببية ذلك كون حركة التدوير أقلّ من حركة الوسط وسيجيئ الجواب عنه إن شاء الله تعالى. واعترض صاحب النهاية مثل هذا على استدلال الجمهور باختلاف حركته بالبطؤ والسرعة في أجزاء لا بأعيانها على أنّه لا يكون إلّا

Turning to the four motions in Ptolemaic lunar theory, the first is “the motion of the parecliptic with the motion of the nodes.” Shirwānī pointed out that, unlike the parecliptics of the other planets, that of the Moon does not move at the same rate as the precession of the fixed stars; rather, the observed motion of the nodes is due to the fact that their proper motion is faster than precession and in the opposite direction. Ṭūsī had listed this motion as “three minutes plus a fraction” daily in the counter-sequence of the zodiac, which Shirwānī clarified as $0;3,11^\circ$. He next provided an overview of *Almagest* V.9 and Ptolemy’s method for determining the Moon’s mean motion in latitude (i.e., motion along the inclined orb’s equator, measured with respect to the ascending node) using pairs of lunar eclipse observations that met certain specific conditions.⁶ Assuming the conditions are met, Shirwānī explained that the Moon will have gone through its cycle in latitude an integer number of times between the two eclipses, allowing one to calculate the daily mean motion in latitude. Shirwānī reported the Ptolemaic value of $13;13,45^\circ$. Next, he explained that, if we subtract the Moon’s daily mean motion in longitude (i.e., motion along the ecliptic with respect to the vernal equinox), which is $13;10,34^\circ$, the remainder ($0;3,11^\circ$) is the motion of the Moon’s parecliptic.⁷

Since all of the Moon’s orbs move with this motion, Ṭūsī referred to it as the motion of the nodes. It is indistinguishable from precession (since the two motions are around the same axis), but Ṭūsī noted that other authors had mistakenly claimed that the reason the difference is imperceptible was because the ratio between precession and this faster motion is so small. In

بسبب التدوير فقال ليس ذلك بحسن عندي إذ مع وجود الحركة السريعة لأوجه يجوز أن يسرع ويبطئ في جميع أجزاء البروج وحده.

6. Toomer, *Ptolemy’s Almagest*, 205-207.

7. Shirwānī, *Sharḥ al-Tadhkira*, fol. 195a.

fact, if the two motions were around different axes, then even a small difference would add up over time and become manifest in the prediction of eclipses. Shirwānī identified al-Kharaqī (d. 1138), author of the popular elementary astronomy text *al-Tabṣira fī 'ilm al-hay'a*, as one who made this false claim.⁸ Shirwānī explained that eclipses can only occur within a certain range around the nodes, about 12° on either side for a lunar eclipse and 18° for a solar eclipse. He walked the reader through a calculation of what would happen to a predicted lunar eclipse after 1000 years under this false assumption, demonstrating that new locations of the vernal equinox and the nodes lead to the prediction that the eclipse would take place approximately 25° away from the node!⁹

The second Ptolemaic motion is the motion of the inclined orb counter-sequentially, about the center of the World, of 11;9° daily (and 7" more, Shirwānī added with reference to *Almagest* V.2). The eccentric, and its center, are moved by this motion, which is called the "motion of the apogee." Shirwānī pointed out that because the eccentric's center describes a small circle around the center of the World by this motion, one scholar ascribed the latter to a 5th orb (Abū al-Mafākhir in his *Kitāb al-Hidāya*).¹⁰ He noted Shīrāzī also established a 5th lunar

8. al-Kharaqī (or al-Khiraqī) was the author of two works on astronomy, the *Tabṣira* and a longer, less popular work entitled *Muntahā al-idrāk fī taqāsīm al-aflāk*. Neither text has been published or studied by modern scholars, although German translations of the introductions to both works can be found in Eilhard Wiedemann and K. Kohl, "Einleitung zu Werken von al-Charaḳī," *Sitzungsberichte der Physikalisch-medizinischen Sozietät in Erlangen* 58-59, (1926/1927): 203-218.

9. Shirwānī, *Sharḥ al-Tadhkira*, fol. 195b.

فلو حسبنا خسوفا أو كسوفا يقع بعد ألف سنة على بعد عشر درجات من المائل قبل إحدى العقدتين ولنفرضها حينئذ في أول الحمل بحسب الحساب حتى يكون الوقوع في آخر العشرين من الحوت تقريبا لأنّ تعديل النقل في الكسوفات لا يزيد على دقيقتين فبعد ألف سنة وحركة الثوابت يرتقي فيها إلى قريب من نصف برج لا يمكن وقوعه على ذلك القول لنحرك العقدة بحركة الثوابت في تلك المدة إلى قريب الخامسة عشر من الحمل وصيرورة البعد بين تلك العقدة وموضعي النيرين من الحوت خمسا وعشرين درجة وهذا أكثر من حدود الخسوفات والكسوفات.

10. I have been unable to identify this scholar or his book.

orb, but not for this motion.¹¹ He also summarized Shīrāzī's procedure for determining the apogee's speed, but criticized it, saying that in both the *Tuhfa* and *Nihāya*, Shīrāzī stated that the motions of the apogee and parecliptic and inclined orbs were determined thus:

when they found the Sun at the quadratures located between the apogee and the epicycle center, they reckoned it to be similar in all situations, and therefore subtracted the mean Sun from the mean Moon. The remainder was the distance of the epicycle center from the Sun, which was equal to the distance of the Sun from the apogee, so they subtracted from it the mean Sun and the known motion of the nodes, thus the resulting remainder is the motion of the apogee.¹²

In the *Nihāya* Shīrāzī also added:

it is apparent from what we have said that knowledge of the mean Moon is preliminary to knowing the motion of the apogee, rather than vice versa as is said [that] if the Sun is between them [i.e., the apogee and epicycle center] and the motion of the apogee is known then the mean Moon is [determined as a result].

Shirwānī noted this was subject to debate (*fīhi naẓar*), because the opposite was apparent from Ṭūsī's later discussion of the mean Moon.¹³ By that I take Shirwānī to mean that, according to the order in which these values were presented in the *Tadhkira*, the motion of the lunar apogee came prior to, and was used to derive, the mean motion of the Moon. However, Shīrāzī's description of the relationship between these values more accurately reflected the fact that the

11. He did not provide further details on Shīrāzī's 5th lunar orb at this point.

12. Shirwānī, *Sharḥ al-Tadhkira*, fol. 196a.

قال صاحب التحفة فيها وفي النهاية إنما عرف مقدار هذه الحركة بأنهم لما وجدوا الشمس في الترتيبين متوسطاً بين الأوج ومركز التدوير حسبوها كذلك في جميع الأوضاع ولذلك نقصوا من وسط القمر وسط الشمس فبقي بُعد مركز التدوير عن الشمس معلوماً وهو مساو لبعد الشمس عن الأوج فنقص منه وسط الشمس الجوزهر المعلوم فبقي الباقي وهو حركة الأوج معلوماً.

13. Shirwānī, *Sharḥ al-Tadhkira*, fol. 196a.

وقال في النهاية فظاهر مما قلنا إن معرفة وسط القمر مقدّمة لمعرفة حركة الأوج لا بالعكس كما قيل إذا كانت الشمس متوسطاً بينهما وحركة الأوج معلومة فإذن وسط القمر كذا. وفيه نظر لأن العكس ظاهر ممّا سيأتي في بيان وسط القمر وإن لم يكن كما قيل فيلزم من الأصل دور.

mean lunar motion was a fundamental parameter in all of the lunar models Ptolemy had presented in the *Almagest*. In contrast, the motion of the lunar apogee did not enter into discussion until the introduction of the “crank mechanism” to move the center of the eccentric in Ptolemy’s second, more complex, lunar model.

The third lunar motion was the sequential motion of the eccentric, about the center of the World, of $24;23^\circ$ daily. Shirwānī explained that the motion of the eccentric being around the center of the World meant it described equal angles in equal times, and noted that this eccentric but uniform motion around the center of the World raised a problem (*ishkāl*) that would be resolved later. Shirwānī also mentioned that *zījes* did not include an “equation of the eccentric” for the Moon, therefore, because the eccentric’s motion was already uniform. Tūsī called this third motion the “motion of the center” because the eccentric carried the center of the Moon’s epicycle. Shirwānī next explained the procedure that Hipparchus used to determine the Moon’s periods and mean motions, as related in the *Almagest*, which required establishing long intervals of time in which the various motions went through a whole number of cycles. He stated that “[the ancients] determined the uniformity of these motions around the center of the World by finding that the center of the epicycle crosses the ecliptic orb, in a period of 4267 [synodic] months called the ‘circuit period’ (*al-zamān al-dawrī*) of the Moon, 4612 times [tropical months] less 3° , not $7\frac{1}{2}^\circ$ as is stated as such in the *Almagest*, which is an error that is apparent through calculation, as shown in the *Tuhfa*.¹⁴

14. Shirwānī, *Sharḥ al-Tadhkira*, fol. 196b.

وإنما عرف تشابهها حول مركز العالم من حيث أنهم وجدوا مركز التدوير يقطع فلك البروج في كل أربعة آلاف ومائتين وسبعة وستين شهرا وهو المسمى بالزمان الدوري للقمر أربعة آلاف وستمائة واثنين عشرة مرة الا ثلاثة أجزاء لا الا سبعة أجزاء ونصف وان وقع في المجسطي كذلك فإنه غلط يظهر بالحساب كذا في التحفة.

This is the Hipparchan period of 4267 synodic months = 4573 anomalistic months $\approx$ 4612 tropical months (4612 revolutions - $7\frac{1}{2}^\circ$). Shirwānī’s comment that 3° is the correct amount to subtract from 4612 revolutions relates to difficulties in reconciling Ptolemy’s derivation of these

This discussion of lunar periods sent Shirwānī off on a slight digression to talk about the periods of the planets, and the idea of a great “period of the world” attributed to the scholars of India and Persia, by which they meant the duration required for all of the planets to revolve and return to their original circumstances at the beginning of time. He reports a few staggeringly huge numbers before referring the reader to consult the *Kitāb al-Adwār* of Abū al-Wafā’ al-Būzjānī (d. 997/998).¹⁵ Furthermore, he stated

I found in a number of *zīj*es that the days of the world, according to the Persian scholars and some the wise men of al-Hind, [has a] duration of 360,000 years. This is the duration from the conjunction of all the planets at the vernal equinox until their second conjunction. What they mean by ‘days of the world and its periods’ is the days of its [accidental] existence and its cycles, and the Day of Judgment will bring about their termination. Then the world will start anew, and that will be unending.¹⁶

values and determining which values were calculated rather than observed by Hipparchus or Babylonian astronomers. The lag of 3° in longitude is the result one obtains by calculating the motion of the Sun during the Hipparchan period using Ptolemy’s value for the motion of the mean Sun, and this must be the calculation in the *Tuhfā* Shirwānī mentions. In contrast, the value of 7 1/2° reported by Ptolemy is the result of dividing the Hipparchan period by the length of the Babylonian year. See Toomer, *Ptolemy’s Almagest*, 174-179 and Olaf Pedersen, *A Survey of the Almagest, With Annotation and New Commentary by Alexander Jones* (New York: Springer Verlag, 2010), 161-164, 466-468.

15. Shirwānī did not refer to Abū Ma’shar (d. c. 886), who was often the source of later authors’ information about these pre-Islamic cosmic cycles. See David Pingree, *The Thousands of Abū Ma’shar* (London: Warburg Institute, 1968). On Būzjānī, see A. P. Youschkevitch, “Abū’l-Wafā’ al-Buzjani, Muhammad ibn Muhammad ibn Yahya ibn Isma’il ibn al-‘Abbas,” in *Dictionary of Scientific Biography*, (New York: Scribner’s Sons, 1972), 39-43 and Behnaz Hashemipour, “Būzjānī: Abū al-Wafā’ Muḥammad ibn Muḥammad ibn Yaḥyā al-Būzjānī,” in *Biographical Encyclopedia of Astronomers*, ed. Thomas Hockey et al., (New York: Springer, 2007), 188-189.

16. Shirwānī, *Sharḥ al-Tadhkira*, fol. 196b-197a.

ووجدتُ في بعض الزيجات أنَّ أيام العالم عند علماء الفرس وبعض حكماء الهند مدَّتها ثلثمائة وستون ألف سنة وهي مدَّة اجتماع السيارات في نقطة أول الحمل إلى اجتماعها ثانياً. ومرادهم بأيام العالم وأدواره أيَّام بقائه وأدواره ويقوم القيامة بانقضائها ثم يستأنف العالم باستئنافها وهكذا إلى غير نهاية.

For an examination of this heretical doctrine, he referred the reader to Shīrāzī's *Sharḥ Hikmat al-Ishrāq* (*Commentary on the Philosophy of Illumination* [of al-Suhrawardī]).

According to the Ptolemaic model as described by Ṭūsī thus far, the center of the Moon's epicycle moves with the combination of these previous motions: $11;12^\circ$ counter-sequentially (on account of the parecliptic and inclined orbs) and $24;23^\circ$ in the sequence of the zodiac (on account of the eccentric), producing a net difference of $13;11^\circ$ daily toward the east, called the "mean motion of the Moon."

The fourth motion is that of the epicyclic orb, the Moon's "proper motion" (*ḥarakatuhu al-khāṣṣa*) of $13;4^\circ$ per day such that it is counter-sequential in the upper half of the epicycle. Shirwānī reminded the reader that this is not merely taken from Ptolemy's account of the Moon, but it is also known from direct consideration of observations that the duration of the Moon's faster motion is less than the duration of its slower motion (*wa-innamā 'alama dhālik bi-kawn zamān sur'atihi aqṣar min zamān buṭ'ihī*). He mentioned that this motion along the epicycle was also called the "motion of the anomaly" (*ḥarakat al-ikhtilāf*), a name that will be used for the motion of each planet's epicycle. Shirwānī was referring to the epicycle's role in producing variable (i.e., anomalous) motion around the average motion of its eccentric deferent.

Ṭūsī had stated that because¹⁷ the ratio of the Moon's proper motion to its mean motion is less than the ratio of the line joining the center of the World and the epicyclic perigee to the epicyclic radius, the Moon does not have a station or retrogradation. Rather, its motion will be slower in the apex half of the epicycle and faster in the perigeal half. Furthermore, Ṭūsī had stated, because the motion of the epicycle is less than the mean motion, the positions of the

17. The Arabic here is *li-kawn*, so the sentence might also be translated as "the ratio of ... being less than the ratio of ..., [the Moon] will not have a station or retrogradation" rather than expressing a causal relationship about the physical components of the model.

slower and faster motions shift; the return to the same anomaly will occur after the return to the same part of the ecliptic orb. Shirwānī reminded the reader that in fact this return is to approximately the same speed (*ilā ikhtilāf...bi-‘aynihi taqrīb^{an}*) at nearly the same part of the ecliptic (*ba‘d al-‘awd ilā juz’ bi-‘aynihi min falak al-burūj bi-zamān qalīl*), referring back to what had been stated previously about the return to “comparable” speeds. Indeed, he continued, the relative speed varies because the epicycle center will be at a different distance from the center of the World when the Moon returns to a given position on the epicycle. As a result, arcs along the epicycle that are in fact equal will appear unequal to observers on the Earth, which Shirwānī demonstrated with a geometric proof and accompanying figure.¹⁸



Figure 3.1 Apparent change in a given epicycle arc as viewed from Earth
Ahmet III MS 3314, fol. 199b

18. Shirwānī, *Sharḥ al-Tadhkira*, fol. 199a-b.

Ṭūsī had stated that “because the radius of the epicycle varies in size with respect to the center of the World due to its [i.e., the epicycle’s] varying distance from it *in the two orbs* [my italics], the rates of the slowest and fastest speeds are not always the same but will also vary.” Ṭūsī’s reference to “in the two orbs” here is unclear, for if he were simply talking about the varying distance of the epicycle [center] from the center of the world, the eccentric would suffice. In his edition of the *Tadhkira*, Ragep translates the first pronoun in Ṭūsī’s statement as referring to the epicycle’s varying distance from the center of the world, which does seem to be the correct reading. In contrast, Shirwānī read the pronoun as referring to the epicycle’s radius (or rather, diameter), and interpreted the difference in size in the “second” orb as the difference between the epicycle’s diameter and the chord connecting the points of the Moon’s mean distance on the epicycle from the eccentric center. Shirwānī also mentioned possible interpretations from Shīrāzī and Jurjānī, while dismissing ‘Ubaydī (who suggested that Ṭūsī meant the inclined orb for the second orb). In the *Tuhfa*, Shīrāzī read these two orbs as the epicycle’s changing distance from the center of the world along with the changing position of the Moon along the epicycle, while Jurjānī suggested that the phrase “in the two orbs” be deleted and mentioned that in some manuscripts of the *Tadhkira* it was, thus avoiding the problematic phrase entirely.¹⁹

In part, the confusion here (and elsewhere in the *Tadhkira* and commentaries on it) stemmed from the lack of a precise vocabulary for discussing speed, particularly instantaneous

19. Shirwānī, *Sharḥ al-Tadhkira*, fol. 200a.

طوسي: منه في الفلكين. شرواني: إما في الخارج المركز فظاهر وإما في التدوير فلتبذل نقطة البعد الأوسط باختلاف أبعاده من مركز العالم في الخارج وتبذلها يستلزم اختلاف أبعاد طرفه المذكور منه. وصاحب التحفة قال لاختلاف أبعاده منه في الخارج وأوضاعه منه في التدوير ولعل مراده ما ذكرنا. وأما السيد فقال لفظة في الفلكين زائدة ولذلك ضرب عليها في بعض النسخ وفيه اظهار لعجز الضارب عن توجيه معناها. وأما العبيدي ففسر الفلكين بالمائل والخارج ولا يخفي ما فيه.

See also F. Jamil Ragep, *Naṣīr al-Dīn al-Ṭūsī’s Memoir on astronomy = al-Tadhkira fi ‘ilm al-hay’a*, 2 vols. (New York: Springer-Verlag, 1993), 418-419.

and maximum speeds. The Arabic word Ṭūsī used – *ikhtilāf* – carried multiple meanings. It is usually translated as “anomaly” in the astronomical sense of departure from the overall pattern of uniform circular motion. For example, in *Tadhkira* II.6, the difference between the Sun’s observed motion and its mean motion is called the Sun’s *ikhtilāf*, and it is measured by the angle at the center of the Sun to the centers of the earth and the eccentric (see Figure 2.7 at the end of Chapter 2). However, *ikhtilāf* and other related words from the same Arabic root (*kh-l-f*) also had the more general senses of difference, variation, or disagreement. In the case of the lunar epicycle’s changing maximum and minimum speeds is more appropriate to translate *ikhtilāf* as “anomalous speed” rather than a mathematically defined function because Ṭūsī was talking about variation from circuit to circuit.²⁰

The concept of speed, and indeed motion itself, was notoriously challenging for ancient and medieval astronomers and philosophers. In Arabic, one could speak of motion (*ḥaraka*), quickness and slowness (*sur‘a, buṭ‘*), quantity of motion (*miqdār al-ḥaraka*), or mean motion (*ḥarakat al-wasaf*), for example, or employ a less precise term such as *sayr* that had the sense of walking, progress, or pace. Generally, however, the concept of motion was tied up with the Aristotelian/Avicennan attempts to analyze motion as a philosophical category, define the causal relationships between the body in motion and its mover, and conceptualize how motion as a state existed in relationship to space and time.²¹ Islamic astronomers were able to sidestep these

20. In the glossary to his edition of the *Tadhkira*, Ragep translates *ikhtilāf* as variation, change, divergence, anomaly, or anomalous speed depending on the context. The term also appears in compound phrases such as difference in longitude (*ikhtilāf al-ṭūl*) and parallax (*ikhtilāf al-manẓar*, lit. “difference in sight”), Ragep, *Tadhkira*, 597.

21. The literature on physics and the philosophical analysis of motion in the Islamic tradition is vast. See, for example, the following works and the sources cited therein: Jon McGinnis, “A Medieval Arabic Analysis of Motion at an Instant: The Avicennan Sources to the *forma fluens/fluxus formae* Debate,” *British Journal for the History of Science* 39, no. 2 (2006): 189-205; Ibn Sīnā, *The Physics of the Healing: A Parallel English-Arabic Text in Two Volumes*, ed.

debates, in large part, by focusing on continuous, uniform motion that could be expressed in relationship to intervals of time such as degrees of arc per day. However, when an occasion arose where it was necessary to talk about speed at a particular moment or determine extreme values for variable speed, they had neither the physical or mathematical tools to do so in a precise manner, as in the case of the Moon's variable "maximum" or "minimum" speeds.

Having provided an overview of the Moon's observed motions, Ṭūsī followed his pattern of explaining the anomalies (*ikhtilāfāt*) that resulted from these motions. The first anomaly is the "independent equation" (*al-ta'dīl al-mufrad*), the angle occurring at the center of the World between the epicycle center and the lunar body [also called the angle of anomaly or *zāwiyat ikhtilāf*]. Its maximum is based on the radius of the epicycle being observed at its two mean distances with a size of $5 \frac{1}{4}$ parts, where the radius of the inclined orb is 60. Shirwānī referred to this size as *al-miqdār al-naḥs al-amrī*, implying that although the epicycle radius does not exist in physical reality, it nevertheless has a level of existence as a mathematical entity beyond the imaginings of the mind, and thus can be used as the basis to obtain true knowledge about the world. He walked the reader through a simple explanation of why the anomaly is added to the mean motion when the Moon is ascending the epicycle, subtracted from the mean when it is descending, and disappears at the apparent apex and perigee. Shirwānī pointed out that this first anomaly is sometimes referred to as the "first equation" but cautioned that in most *zīj*es it is instead called the "second equation" because in practice it appears after the equation of proper

and tr. Jon McGinnis, (Provo: Brigham Young University Press, 2010); Ruth Glasner, *Averroes' Physics: A Turning Point in Medieval Natural Philosophy* (New York: Oxford University Press, 2009); and Paul Lettinck, *Aristotle's Physics and Its Reception in the Arabic World: With an Edition of the Unpublished Parts of Ibn Bajja's Commentary on the Physics* (Leiden: Brill, 1994).

motion related to the prosneusis point, which is thus referred to in these *zījes* as the “first equation.”²²

Ṭūsī explained that the second lunar anomaly results from an increase in the previous one when the epicycle is at a distance other than the farthest distance, called the “anomaly of the nearest distance” (*ikhtilāf al-bu‘d al-aqrab*). Its maximum occurs when the epicycle is at the two quadratures (i.e. at the perigee) and it is $2\frac{2}{3}^\circ$ for this epicycle radius if the first anomaly is at its maximum. It decreases accordingly as the first anomaly decreases, and is additive or subtractive in the same arcs as the first anomaly. Shirwānī explained that, as the epicycle moves closer to the center of the world, a given arc on the epicycle will subtend a larger angle with respect to the observer, and reminded the reader that this value will have its maximum at perigee, the Moon’s closest point to the Earth. He noted that the more precise maximum value was $2\frac{2}{3}^\circ - 1'$ according to Shīrāzī’s *Tuhfa*. As with the first anomaly, Shirwānī pointed out that the treatment of this anomaly in the *zījes* varied and that one group, the practical astronomers (*ahl al-‘amal*), merely calculated the maximum value at perigee and included it in their tables. Then instead of calculating the precise values at intermediary points, they decreased the value proportionally in order to simplify the calculation.²³ Shirwānī also drew attention to a number of errors in the figure Nīsābūrī provided during his own commentary on this anomaly, such as stating that certain arcs were equal when they were not. Shirwānī neglected to reproduce the figure despite

22. Shirwānī, *Sharḥ al-Tadhkira*, fol. 200b-201a.

طوسي: ويسمى التعديل المفرد. شرواني: لانفراده في الوجود عن غيره وعدم افتقاره في النقصان والزيادة إلى أن يخلط بغيره. والتعديل الأول لوجوده قبل غيره كما سمي بالاختلاف الأول لذلك ويسمى في الزيجات بالتعديل الثاني لتأخره بحسب العمل عن الاختلاف الثالث المسمى فيها تعديلا أول.

23. Shirwānī, *Sharḥ al-Tadhkira*, fol. 201a-202b

وفي كلّ من الاصطلاحين رعاية دقيقة وهي أنّ أهل العمل استخرجوا تلك الزيادات عند كون المركز في الحضيض ووضعوها في الجدول فلها سَمَوها باسم وأما الزيادات في سائر المنازل فلم يستخرجوها ولم يضعوها في الجدول لجزء جزء إلا غاياتها فإنها مستخرجة مسماة بدقائق الحِصص ودقائق النسب لسهولة تظهر في العمل فلها لم يسموا تلك الزيادات باسم.

referring to Nīsābūrī's labels for the figure's points, arcs, and triangles. Unless this was an oversight, Shirwānī apparently expected the reader to have a copy of Nīsābūrī's commentary to read alongside his own, his point here perhaps drawing directly on his experience teaching Nīsābūrī's commentary.²⁴

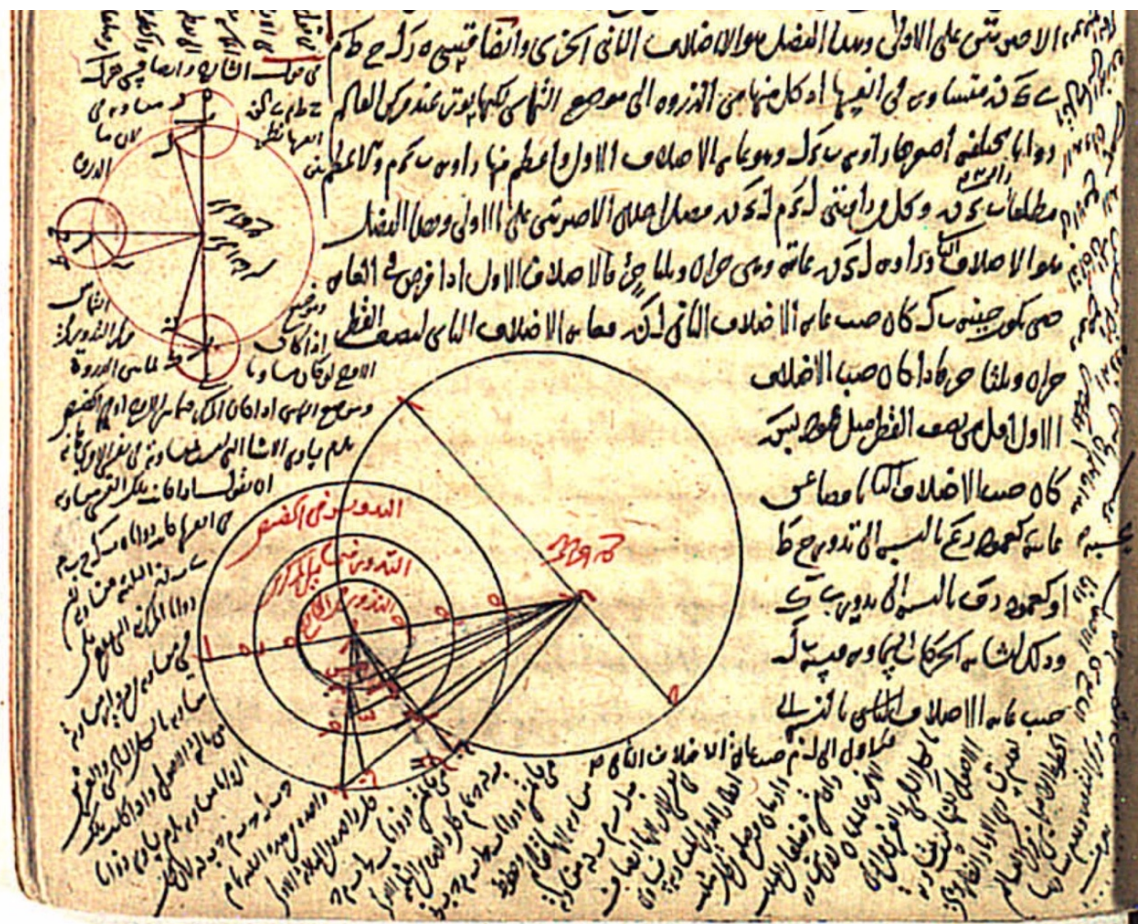


Figure 3.2 Diagram from Nīsābūrī's *Tawḍīḥ al-Tadhkira*
Fatih MS 3397, fol. 44a

24. Shirwānī, *Sharḥ al-Tadhkira*, fol. 201a-202b.

واعلم أنّ في الشكل الذي ذكره النظام لتسهيل هذين الاختلافين مؤاخذات وأنا أنبّه عليها تبرعاً وأن توقف التنبّه على النظر فيه. منها أنّ قوله قسي هـ ز ل ح ط م د ك ن متساوية في أنفسها غير صحيح وقوله إذ كلّ منها من الذروة إلى موضع التماس لا يُثبت لتبديل النقطة التماس بتقارب مركز التدوير من مركز العالم. ومنها إنّ قوله الاختلاف الأول إذا فرض في الغاية حتى يكون جيبه ب ل كان جيب غاية الاختلاف الثاني ل ن مبني على ما توهمه من مرور خط ب ل م ن على نقطة التماس كلّها وفيه ما فيه. ومنها أنّه رسم عمود ز ع تحت عمود ز ف وحقه أن رسم فوقه لأننا نفرض نقطة تقاطع ز ف ي ط في فنقول في مثلث ز ع ق زاوية ق منفرجة لكون زاوية د ق ف حادة فلو كانت زاوية ز ع ق قائمة يلزم وقوع قائمة ومنفرجة في المثلث.

The figure in question can be found in Nīsābūrī's *Tawḍīḥ al-Tadhkira*, Süleymaniye Fatih MS 3397, fol. 44a.

The third lunar anomaly is the “equation of proper [motion]” (*ta ‘dīl al-khāṣṣa*). It concerns the fact that the diameter of the epicycle (the diameter connecting the epicycle’s apex and perigee) is always aligned with a point that is in the direction of the eccentric perigee. The distance of this eccentric perigee from the center of the World is the same as the distance of the eccentric center, which is in the direction of the apogee, from the center of the World. This point in the direction of the eccentric perigee is called the “point of alignment” (*nuqṭat al-muḥādhā*) [Ptolemy’s prosneusis point; Ṭūsī implied that the diameter might be expected to remain aligned with either the eccentric center or the center of the World, but this only occurs when the epicycle center is at apogee or perigee because those points are on the same line as the prosneusis point]. The distance between the center of the World and either the eccentric center or the prosneusis point is 10;19 parts (where the radius of the inclined orb is 60). Shirwānī noted that Ptolemy calculated these distances from the center of the world in *Almagest* V, chapters 4-5, and that because the eccentric center was moved by the inclined orb, the prosneusis point moved with it, traveling along a small circle around the center of the world.²⁵ To further clarify this point, he also included a more detailed discussion of the third anomaly drawn from Ṭūsī’s recension of the *Almagest*.²⁶

There is an additional fourth anomaly in the sense of the Moon’s distance from the nodes being different when measured along the parecliptic or inclined equators. Shirwānī pointed out that this was important to take into account in order to determine the times of solar and lunar eclipses. As he had mentioned earlier, an eclipse can only occur within a certain proximity to a node, and he noted that the practical handbooks (*kutub al-‘amal*) refer to the process of

25. Shirwānī, *Sharḥ al-Tadhkira*, fol. 202b.

26. Shirwānī, *Sharḥ al-Tadhkira*, fol. 203b.

converting between the positions on the inclined or parecliptic equators as “moving the Moon from the inclined to the ecliptic” (*naql al-qamar min al-mā'il ilā al-burūj*). He reported when the Moon was between a node and the extreme point in latitude, its distance on the inclined orb to the closest node was always greater than its respective distance on the parecliptic (he provided Nīsābūrī's diagram and proof of this statement). He also reported the maximum difference was 7', whereas the maximum possible value that still allowed for a solar eclipse was 2'.²⁷

Planetary theory

Ṭūsī's presentation of the planetary theory spanned three chapters, one specifically for Mercury's orbs and motion in longitude, another for the longitudinal motions of the remaining planets, and a third for the planets' motions in latitude. In large part, the models discussed in these chapters incorporated the same orbs as the solar and lunar models, with appropriate modifications to sizes and parameters to reflect the particular motions of the planets. However, since the orbs had already been exhaustively discussed in the earlier chapters, Shirwānī's comments were no longer as detailed and frequently consisted of either rephrasing what Ṭūsī had said or pointing the reader to the earlier explanation in a previous chapter. We will only touch on a few examples of Shirwānī's more significant comments here. They concern two crucial aspects of planetary motion: retrograde motion, which does not occur for the Sun or Moon; and the “difficulty” of the equant point.

Ṭūsī began the chapter on Mercury with an overview of some basic observations and the phenomena Mercury exhibits. These include a general eastward motion in longitude that is near, but not precisely along, the ecliptic (it can be seen both north and south of the ecliptic), and a

27. Shirwānī, *Sharḥ al-Tadhkira*, fol. 204b-205b.

variation in speed such that it overtakes the Sun before slowing to a stop and moving backward. Then after the Sun has passed it, Mercury speeds up again in its original eastward direction. Thus, it is in conjunction with the Sun in the middle of its periods of direct and retrograde motion. Ṭūsī noted that Mercury's maximum elongation from the Sun was found to be no more than 27° . In addition, if one compared similar motions at different times, such as comparing one instance of retrograde motion to another retrogradation, or one faster motion to another faster motion, etc., they are not identical but vary in both size and duration at different parts of the ecliptic.

Shirwānī elaborated on these observations by highlighting what they entailed in terms of motions and orbs, reminding the reader of how tools used in the solar and lunar models could be applied, with modifications, to Mercury. For Mercury's motion near the ecliptic, he explained that its variation above and below the ecliptic indicated Mercury, like the Moon, moved along a circuit that was inclined to the path of the Sun. However, he cautioned that, unlike that of the Moon, Mercury's inclination was not fixed at a particular angle (a full accounting of planetary latitudes would be forthcoming in II.10). The fact that Mercury was united with the Sun during the middle of both its periods of direct and retrograde motion indicated that Mercury had an epicycle whose center was carried by a deferent that moved with the same motion as the center of the Sun, so that the centers of the Sun and Mercury's epicycle were along the same line from the center of the World. Then the radius of Mercury's epicycle determined the maximum distance it could be found preceding or following the Sun.

Shirwānī also provided examples of what observers of Mercury might see if they compared similar motions at different times, and how various astronomers interpreted these phenomena. For instance, Mercury's retrograde motions were found to be $7;54,22^\circ$ in 21 days in

one part of the ecliptic, 12;16,10° in 22 1/2 days in another part of the ecliptic, and 15;12,46° in 23 days in a third part of the ecliptic, according to *Almagest*, XII.²⁸ This indicated “the fact of the matter” was that Mercury’s retrograde arcs along its epicycle at these points were different (*mukhtalifa fī nafs al-amr*), for if they were the same, then their durations would not be different. As Shirwānī explained, this phenomenon required that the center of the epicycle’s deferent orb be eccentric from the center of the world. However, according to Shirwānī, Nīsābūrī and Jurjānī argued that these observations indicated Mercury’s retrograde arc along its epicycle was the same (*wāḥida fī nafs al-amr*) at all parts of the ecliptic, and that the apparent variation was due to its changing distance from the earth along the eccentric, making the [identical] arcs appear larger or smaller at closer or farther distances, respectively.²⁹ I find it difficult to understand why they would have made that claim, because in the *Almagest* Ptolemy had investigated the retrograde motions of each of the planets at their mean, nearest, and farthest distances, demonstrating that the retrograde arcs were truly different (in the sense that the planet moved around different arcs on its own epicycle) and not merely an appearance due to their taking place at different distances from an observer on the earth. This conclusion was a result of Ptolemy’s adaptation in Book XII of Apollonius’s lemma relating the occurrence of a stationary point in the planet’s circuit to the motion of the planet along the epicycle, the motion of the epicycle around the eccentric, and the

28. Toomer, *Ptolemy’s Almagest*, 578-581. Shirwānī’s second value of 12;16,10° should be 12;17,10° according to the *Almagest*, a likely scribal error at some point along the way because the *abjad* representations of 16 and 17 are easy for a scribe to mistake: ١٦ versus ١٧, where the dot was frequently left off the letter ز.

29. Shirwānī, *Sharḥ al-Tadhkira*, fol. 211a.

فدلّ ذلك على أنّ قسي الرجوع من التدوير في تلك الأبعاد مختلفة في نفس الأمر وإلا لم يختلف أزمنتها وذلك مقتضى خروج مركز حامل التدوير عن مركز العالم. وقال النظام فدلّ ذلك على أنّ قوس رجوعه التي هي واحدة في نفس الأمر تبعد عنا تارة غاية البعد فتري أقلّ قدرا وتقرب أخرى غاية القرب فتري أكثر قدرا وفيما بين البعدين متوسط حالها بين الحالين وذلك مقتضى ذلك وتبعه السيد فيما قال.

distance of the epicycle from the center of the world. To resolve the dispute in favor of the position that the retrograde arcs along the epicycle could not actually be the same size (*li-ẓuhūr man ‘ waḥdatihā fī nafs al-amr*), Shirwānī introduced Qāḍīzāde’s argument, as follows. If the arcs were identical, they would necessarily equal the smallest observed arc which occurred in the smallest observed time. But then the arcs should appear smaller at some times and larger at other times [than is actually observed], for after one learns that an increase in retrograde arc along the epicycle occurs according to an increased duration of retrograde motion, the large disparity in the three arcs Ptolemy mentioned and the small differences in the times show that the arcs on the epicycle are not equal.³⁰

Shirwānī’s brief discussion and the not entirely clear passage from Qāḍīzāde barely hint at the difficulties in analyzing retrograde motion and the various situations in which it could occur, part of a broader investigation among Islamic astronomers of how different models could account for retrograde motion. Nīsābūrī had explored the equivalence of using eccentrics and epicycles to account for either retrograde motion or mean motion, and he had discussed his confusion with his teacher Shīrāzī. Their investigations of the way retrograde motion could be generated at the apogee or perigee of an epicycle, or by introducing a subsidiary epicycle that varied in size, are detailed in multiple texts and commentaries.³¹ Understandably, Shirwānī did

30. Shirwānī, *Sharḥ al-Tadhkira*, fol. 211a. Shirwānī attributed Qāḍīzāde’s remarks to an unnamed *ḥāshiya* or supercommentary/gloss, most likely his supercommentary on Nīsābūrī’s commentary on Ṭūsī’s *Tahrīr al-Majisī*.

ولظهور منع وحدتها في نفس الأمر قال الأستاذ رحمه الله في الحاشية يعني أنّ قوس الرجوع من التدوير التي هي متحدة في نفس الأمر في جميع الأحوال وهي ما تقطعه الكوكب من محيط التدوير في أقلّ الأزمنة الموجودة للرجوع ترى تارة أقلّ وأخرى أكثر يدل على ذلك كثرة التفاوت بين المقادير المذكورة من فلك البروج في الأحوال الثلاث وقلته بين أزمنتها على ما يظهر بعد أن يعرف أنّ ازدياد قوس الرجوع في التدوير يكون بحسب ازدياد زمان الرجوع بأدنى تأمل.

31. In addition to Ṭūsī’s own recension of the *Almagest* (*Tahrīr al-Majisī*) these works include Nīsābūrī’s *Sharḥ Tahrīr al-Majisī*, his *Kashf-i Haqā’iq*, and his *Tawdīḥ al-Tadhkira*, as well as Shīrāzī’s *Fa’alta fa-lā talum* and Kamāl al-Dīn al-Fārisī’s short commentary on retrograde

not delve into the technical arguments at this point in his commentary; instead, he signaled to the reader that this was a point for future study.

Shirwānī also mentioned the equant point as a complicating factor in determining the location and duration of retrograde motion. He used the rhetorical device of question-and-answer to ask why it was not obvious that the largest retrograde arc would occur at the apogee, since that is the point at which the epicycle center moves the slowest. He stated that, as a student, he had even suggested this point to Qāḍīzāde, who noted that one needed to take into account the equant and the fact that the epicycle center's motion was fixed relative to the equant rather than the Earth. In that model, the retrograde arc would therefore occur during a different amount of time and length of arc than would be predicted in the simpler epicycle-on-eccentric-deferent model.³²

As we saw in Chapter 2, for *hay'a* astronomers, the equant point itself was one of the most problematic aspects of the Ptolemaic planetary models. Ṭūsī defined the equant as the point about which the motion of the epicycle center and the motion of the deferent were uniform, despite the fact that this point was not the center of the deferent. Ṭūsī also explained that another way of thinking about the equant was to imagine a line, which Shirwānī referred to as the “dirigent line,” extending from the equant center to the center of the epicycle, so as to move the epicycle with uniform motion around the equant. Shirwānī explained that this line did not have a

motion in the *Tadhkira*, see the discussion in Robert Morrison, *Islam and Science: The Intellectual Career of Nizām al-Dīn al-Nīsābūrī* (New York: Routledge, 2007), 174-177.

32. Shirwānī, *Sharḥ al-Tadhkira*, fol. 211a-b.

فإن قلت الظاهر أنّ زمان الرجوع في الأوج أكثر ما يكون لا أقله لأنّ حركة التدوير في الأوج أكثر غلبة على حركة مركزه في غير الأوج لشدة بطء حركة مركزه في الأوج فيجب أن تقع الرجوع في الأوج قبل أن يصل الكوكب إلى موضع الرجوع في غيره فيكون قوس الرجوع في الأوج أعظم فيكون زمانه أكثر. قلت عرضت ذلك على الأستاذ رحمه الله حال القراءة فقال نعم ذلك هو الظاهر نظرا إلى ما ذكرته لكنه كما زال في الأوج شيء من مانع غلبة حركة التدوير بزيادة بطؤ حركة مركزه فقد حدث مانع آخر وهو زيادة بطؤ حركة التدوير بالقياس إلى مركز العالم أعني الخاصة المعدلة فلعلّ الحادث أقوى من الزائل فيكون قوس الرجوع فيه أصغر فيكون زمانه أقلّ ما يكون.

fixed length, but rather extended and contracted because the epicycle center moves farther and closer relative to the equant center. Therefore, it did not trace out a circle that could be called the “equant circle.”³³ He went on to clarify that the reason one might state that the epicycle center described equal arcs on the circumference of an “equant circle” in equal times while not being located on such a circumference was that “the epicycle center is aligned with parts of the circumference of the equant, and that is sufficient for those segments, because the point where the “turning line” intersects the [equant] circumference describes equal arcs.”³⁴ Shirwānī was imagining something like an arm revolving uniformly around the equant, but with a sliding part of the arm connected to the epicycle. Even as the end of the arm slid in and out to adjust the distance of the epicycle, as a whole the arm (and therefore, the epicycle) rotated through equal angles in equal times.

Ṭūsī Couple

In Book II, Chapter 11 of the *Tadhkira*, Ṭūsī introduced his attempts to resolve some of the difficulties arising from the Ptolemaic models for planetary motion. The title of this chapter reads: “An indication (*al-ishāra ilā*) of the solution - of that which is amenable to a solution - of the difficulties (*al-ishkālāt*) referred to previously that arise from the aforementioned motions of

33. Shirwānī, *Sharḥ al-Tadhkira*, 215b.

ويسمى هذا الخطّ الخطّ المدير وهو يطول ويقصر لأنّ مركز التدوير يبعد ويقرب من مركز معدّل المسير فلا يرتسم به دائرة تسمى معدّل المسير كما توهمه بعضهم بل الحقّ أنّه ما ذكره المص.

34. Shirwānī, *Sharḥ al-Tadhkira*, 215b.

والجواب عمّا قيل كيف يقطع مركز التدوير من محيط معدّل المسير قسماً متساوية في أزمنة متساوية وليس على محيطه إنّ مركز التدوير مسامت لأجزاء محيط معدّل المسير وذلك كاف في ذلك القطع لأنّ نقطة تقاطع الخطّ المدير مع محيطه تقطع منه وحكم مركز التدوير حكم تلك النقطة لكونه على الخطّ المدير وكذا حكم جميع النقط التي عليه قسماً متساوية

the planets.”³⁵ This phrase signaled to the reader that Ṭūsī was recommending a certain approach to resolving these problems, not producing a definitive solution. Indeed, his work spurred “an on-going – even systematic – response to the unresolved problems...of astronomy.”³⁶

The key component of Ṭūsī’s solutions was his introduction of the now-famous “Ṭūsī couple,” a device that utilized the combined uniform rotations of two circles (or orbs) to produce an oscillating linear motion. In the basic statement of the device, the smaller circle has half the radius of the larger circle, and is contained in the larger circle tangent to it. We imagine a line connecting the centers of the two circles with the initial point of tangency, which Ṭūsī referred to as the diameter of the larger circle. However, since both circles will soon rotate, it is better to imagine this line as a fixed reference. If the larger circle rotates in one direction, and the smaller circle rotates twice as quickly in the opposite direction, then the point on the smaller circle that was originally tangent to the larger circle will oscillate along the fixed reference line, returning to its original position after the larger circle has completed one rotation (and the smaller circle has rotated twice).

To illustrate this motion and follow Ṭūsī’s proof that the chosen point always lies on the reference line, refer to the following figure (3.3).³⁷ The larger circle is AGB around center D, and the smaller circle is GED around center Z. Initially, let diameter GD coincide with line AD, and points G and E both coincide with point A. Then let circle GED rotate in the direction GE and let point E be moved along due to its motion until it has described an arc, say arc GE. At the same time, let circle AGB rotate in the direction AG with half the former’s motion, and let the

35. Ragep, *Tadhkira*, 194.

36. Ragep, *Tadhkira*, 427.

37. This figure and paraphrase of Ṭūsī’s proof are taken from Ragep, *Tadhkira*, 198-199.

endpoint of diameter DG be moved along until it has described arc AG, which will then be equal to half of arc GE. Now connect E to Z and E to D. Then angle GZE is twice angle GDA on account of the two motions. It is also twice this [i.e., angle GDA, but see below] because it is an exterior [angle] of triangle EZD and equal to the two interior [angles] ZED and ZDE, which are equal because the two legs ZE and ZD are equal. Thus the two angles GDE and GDA are equal, and line DE coincides with line DA. Therefore, point E is on diameter BA, not deviating from it, and the same will be the case in all other positions.

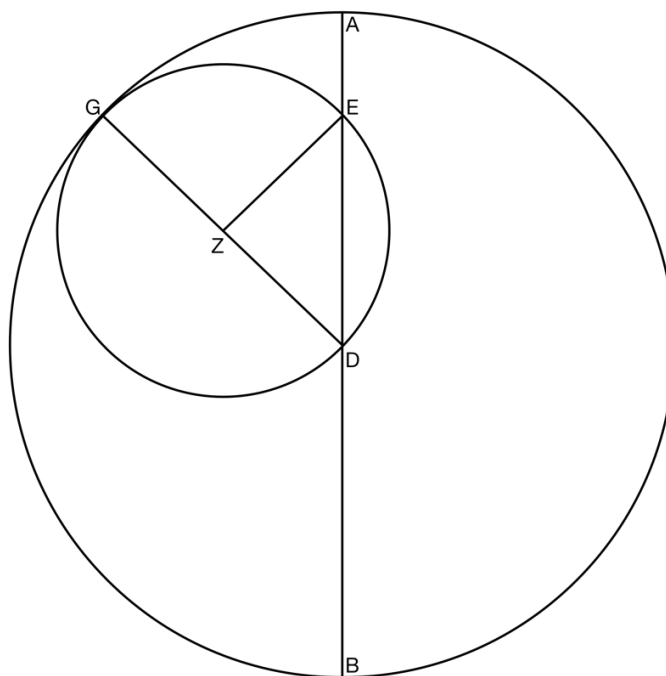


Figure 3.3 Tūsī Couple

Tūsī's proof of the basic lemma (for circles in a plane) was straightforward, although there is one ambiguous pronoun that created a major flaw. As Ragep explains, when Tūsī stated that the exterior angle of the triangle was equal to the two interior angles, he phrased it in a way that he assumed what he was attempting to prove, namely that the point E was on line DA. Shirwānī corrected the proof by stating that the pronoun referred to the correct angle GDE rather than the angle GDA. That is by no means a straightforward reading of the Arabic, however: since

angle GDE had not been mentioned previously, logically it could not be the antecedent of the pronoun. However, this emendation is the step necessary for the proof to work, because it shows angles GDE and GDA are each half of angle GZE for separate reasons, and therefore they are equal to one another, placing point E on line BA. Aside from this correction to Ṭūsī's proof, the rest of Shirwānī's comments consisted of additional phrases making explicit the justifications for Ṭūsī's claims, with reference to specific propositions of Euclid's *Elements* and the postulated properties of the two circles.³⁸

Ṭūsī subsequently explained how these motions could be implemented by solid orbs, rather than two-dimensional circles, with the oscillating point becoming the center of the planet's epicycle. Using these orbs, he modified the Ptolemaic models to replace non-uniform motions around equant points with an epicycle center moving along a circuit generated by the motions of uniformly rotating orbs. Ṭūsī discussed the modifications in detail for the lunar model, and stated that a similar collection of orbs could replace the equant "difficulty" in the models for Venus and the superior planets, although a solution to Mercury's motion eluded him.

In his discussion of the revised lunar model, Ṭūsī noted that the path of the epicycle center "resembled" a circle but it was not, in fact, a circle because of the epicycle center's position with respect to the center of the World oscillated. However, he claimed that calculations showed the difference between his model and the Moon's "true position" (as defined by the Ptolemaic lunar model with equant) was less than $1/6^\circ$ (i.e., 10 minutes of arc), with the maximum occurring at the midpoints between conjunction, opposition, and the quadratures – an "imperceptible" (*ghayr maḥsūs*) difference at those points.³⁹

38. Shirwānī, *Sharḥ al-Tadhkira*, fol. 233b-234b.

39. Seeing as how the angular resolution of the naked eye is about $1'$, under the right conditions one would assume a $10'$ difference to be entirely perceptible and it is difficult to understand why

Shirwānī's commentary on this difference between Ṭūsī's revised lunar model and the Ptolemaic model reveals once again that his audience was technical specialists, and that the question of observational evidence for theoretical models remained foremost on the mind of the astronomers at Samarqand. Ṭūsī did not actually provide calculations to support his claim that the maximum difference was less than $1/6^\circ$, but Nīsābūrī had attempted to investigate the issue in his commentary. Shirwānī signaled that there were flaws and mentioned that he would describe the criticisms of Qāḍīzāde and Ulugh Beg after presenting Nīsābūrī's calculations. Nīsābūrī's mistake had been to use a linear approximation to calculate the change in distance between the epicycle center and Earth, which would have been valid if the epicycle center was following a circular path. But since the effect of the Ṭūsī Couple was to flatten out the path of the epicycle center as the epicycle reached perigee, this approximation placed the epicycle center farther from the earth than the model actually predicted. Nevertheless, as Robert Morrison has shown, although Nīsābūrī's mistakes actually had the effect of overstating the maximum difference between the two models, this (erroneous) difference was still an acceptably small value of less than seven minutes of arc.⁴⁰

In his criticisms of Nīsābūrī's remarks, Shirwānī first noted out that Nīsābūrī had mistaken the source of the difference between the models, which he claimed was the equation of the proper motion. However, as Shirwānī pointed out, Ṭūsī specified (in II.7) that the maximum equation of the proper motion occurred when the center of the epicycle was at the sextiles or

Ṭūsī claimed it was not. Ragep calculated the maximum difference between the models as $8'$ and emphasized that Ṭūsī was not saying one could not observe the Moon to within $10'$ of arc in general. As Ragep pointed out, since Ṭūsī was the director of an observatory, his statement "should be taken into account in any future studies on medieval observations." Ragep, *Tadhkira*, 443-445.

40. Morrison, *Islam and Science*, 179-181.

trines with respect to the Sun, whereas the maximum difference between his lunar model and the Ptolemaic model occurred at the octants.⁴¹

In order to follow Qāḍīzāde's and Ulugh Beg's criticisms, it will be helpful to refer to Nīsābūrī's diagram (Figure 3.4), which Shirwānī included in his commentary.

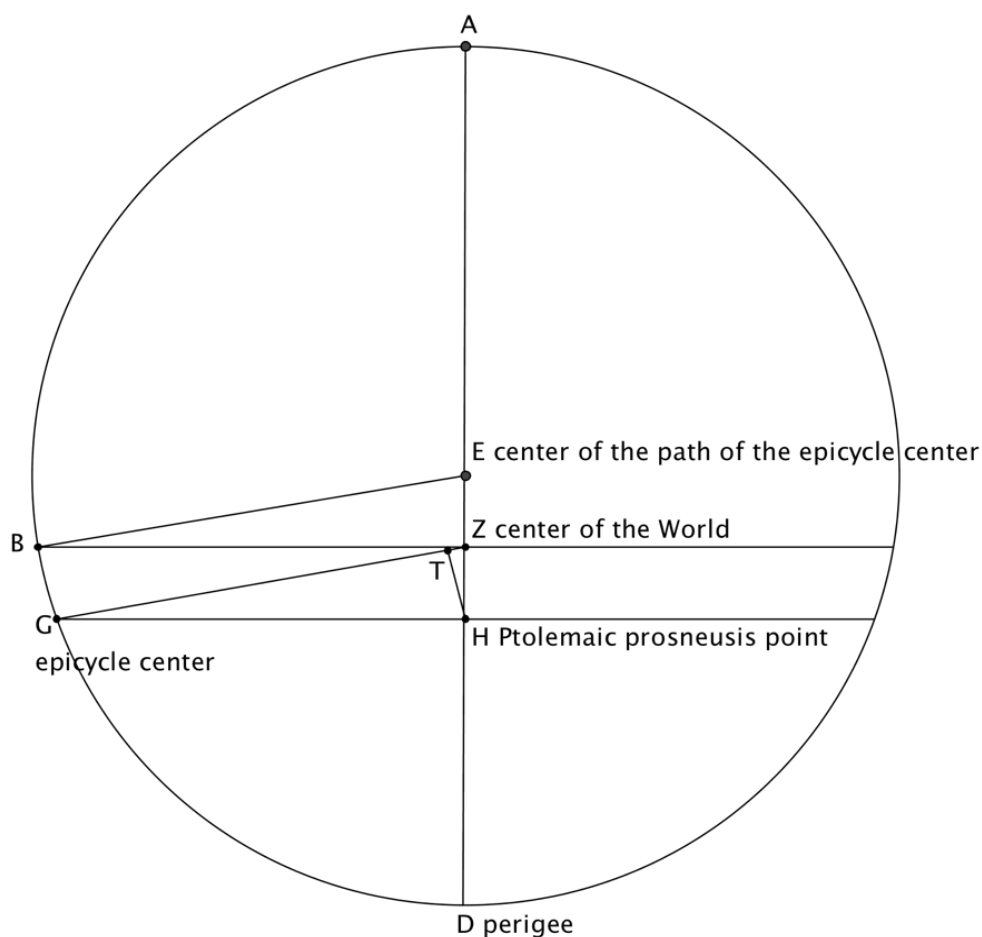


Figure 3.4 Diagram from Nīsābūrī's error analysis for *Tadhkira* lunar model
Motion of the epicycle center along its deferent

Nīsābūrī stated that arc ABG was 114° , a value that could be criticized on two grounds. For the first criticism, Qāḍīzāde noted

41. Shirwānī, *Sharḥ al-Tadhkira*, fol. 239a.

ويرد على قوله «الظاهر أنه يريد بذلك التفاوت الواقع بسبب تعديل الخاصة» أن ظاهر قول المص «وغايته يكون في منتصف الأرباع» مع قوله في تعديل الخاصة «غايته عند كون مركز التدوير على تسديس الشمس أو تثليثها» يأبى هذه الإرادة.

the arc must be considered either along the deferent or the inclined orb. If along the deferent, then Nīsābūrī's statement that arc $AB = 90^\circ$ is not correct because it would indeed be thus if it were along the inclined orb. And if it was considered along the inclined orb, then there is a difficulty (*ishkāl*) in arc ABG being 114° since arc AB in that case is 90° ; but BG is not 24° because the alternating angles BZG , ZGH are equal. The origin of the error here is that Ptolemy put the maximum value of this equation at 114° , and the Commentator [Nīsābūrī] alleged that there the epicycle center was at the end of the perpendicular line passing through the point of alignment [prosneusis point H]. Thus [the error] occurred. We have indicated the true configuration in the glosses [supercommentaries on Nīsābūrī's commentary] on the *Almagest*.⁴²

As for the second criticism of arc ABG being 114° , Shirwānī pointed out that Nīsābūrī's value differed from what Ptolemy stated in the "famous configuration" (i.e., Ptolemaic lunar model) – that the arc between the farthest distance and the endpoint of the aforementioned perpendicular line is 120° .⁴³

Shirwānī next reported an additional criticism of Nīsābūrī's error analysis in the form of a geometrical proof and diagram from Ulugh Beg regarding the effect of the Tūsī Couple (the "large" and "small" spheres in the quotation below) on the motion of the lunar epicycle's center. At one point in the calculations, Nīsābūrī stated that a particular ratio was the same as the ratio of 24° to the unknown value. However, Ulugh Beg identified a flaw in that claim due to the

42. Shirwānī, *Sharḥ al-Tadhkira*, fol. 239a.

و[يرد] على قوله «قوس ا ب ج قيد» شيان. أحدهما قال الأستاذ رحمه الله وفيه بحث لأن هذه القوس لا يخلو من أن يعتبر من الحامل أو من المائل فإن اعتبرت من الحامل لا يصح قوله وقوس ا ب ص لأنها إنما تكون كذلك لو كانت من المائل وإن اعتبرت من المائل ففي كون قوس ا ب ج قيد إشكال لأن ا ب حينئذ ص لكن ب ج لكون متبادلي ب ز ج ز ج ح متساويين لا يكون ك د. ومنشأ الخطأ ههنا أن بطلميوس وضع غاية هذا التعديل في قيد وزعم الشارح أن مركز التدوير عند ذلك في طرف العمود المار بنقطة المحاذاة فوق فيما وقع وقد أشارنا إلى حقيقة الحال في حواشي المجسطي.

43. Shirwānī, *Sharḥ al-Tadhkira*, fol. 239a.

والثاني أنه يخالف ما ذكره في الهيئة المشهورة من أن ما بين البعد الأبعد وطرف العمود المذكور مائة وعشرة.

epicycle center's non-uniform motion. Shirwānī presented the proof as follows (refer to Figure 3.5 below):

Indeed, that would be the case [i.e., Nīsābūrī's ratios would be equal] if the descent of the center of the epicycle along the diameter of the large [sphere] were uniform. But that is not the case, since when the small [sphere] moves a quarter and the large [sphere] an eighth, we connect the centers of the small [sphere] and the epicycle, making triangle ABG with right angle B, so hypotenuse AG is longer than the radius of the small [sphere] (i.e., AB). Thus the amount DA by which the epicycle center descends along the radius of the large [sphere] is shorter than the radius of the small [sphere].

When the small [sphere] moves another quarter, the epicycle center reaches G, the center of the large [sphere], so the descent along this quarter is faster. The same explanation [applies] to the third quarter with triangle GEZ, since GZ is longer than ZH, which [the epicycle center] describes in the fourth quarter of the small [sphere]. Therefore, the descent in the second and third quarters of the small [sphere]'s rotation, I mean segment AZ of the diameter of the large [sphere], is faster than [the descent] in the other two quarters, I mean segments DA, ZH.

The Sultan, may God have mercy upon him, mentioned this demonstration during my reading [i.e., while Shirwānī read this section of Nīsābūrī's *Tadhkira* commentary for the audience of Qāḍīzāde's lectures at Samarqand].⁴⁴

44. Shirwānī, *Sharḥ al-Tadhkira*, fol. 239a-b.

وعلى قوله كنسبة كد إلى المجهول أنه إنما يكون كذلك لو كان نزول مركز التدوير على قطر الكبيرة متشابهًا وليس كذلك لأنه إذا تحركت الصغيرة ربعًا والكبيرة ثمنًا ووصلنا بين مركزي الصغيرة والتدوير حدث مثلث ا ب ج وزاوية ب منه قائمة فوترها وهو ا ج أطول من نصف قطر الصغيرة. فيكون المقدار الذي نزل مركز التدوير من قطر الكبيرة وهو د ا أقصر من نصف قطر الصغيرة. وإذا تحركت الصغيرة ربعًا آخر وصل مركز التدوير إلى ج مركز الكبيرة فيكون النزول في هذا الربع أسرع وكذا البيان في الربع الثالث في مثلث ج ه ز فإن ج ز أطول من ز ح الذي يقطعه في الربع الرابع من الصغيرة فالنزول في الربعين الثاني والثالث من دور الصغيرة أعني ا ز من قطر الكبيرة أسرع منه في الربعين الآخرين أعني د ا ز ح. ذكر هذا البرهان السلطان رحمه الله حال قراتي.

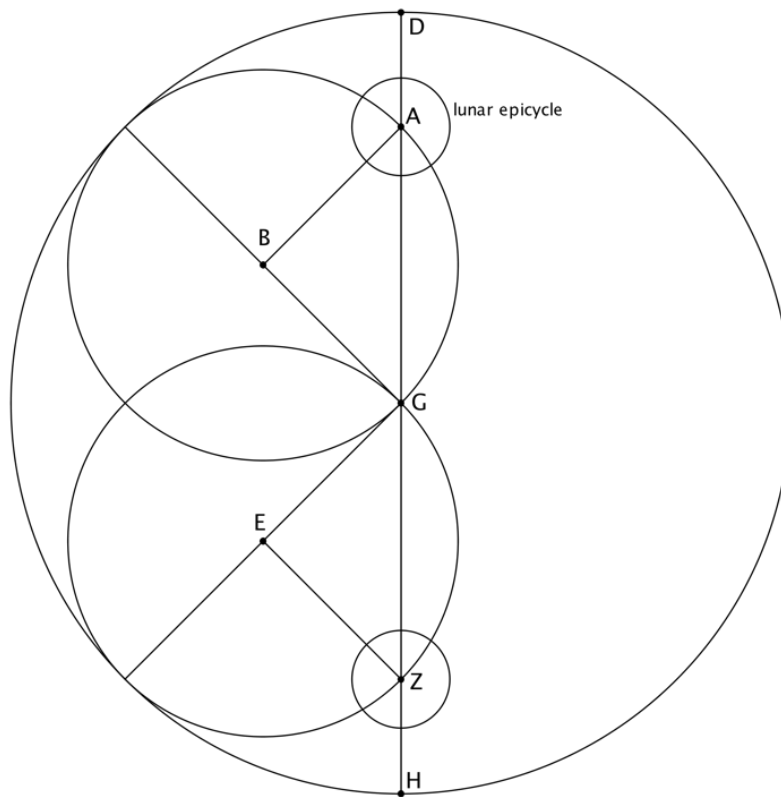


Figure 3.5 Ulugh Beg's diagram for descent of lunar epicycle with Tūsī Couple

Shirwānī went on to make additional criticisms relating to the finer details of Nīsābūrī's error analysis (sides of triangles and sizes of angles) that followed from the more fundamental errors described above. The attention to detail that Ulugh Beg, Qāḍīzāde, and Shirwānī displayed in the classroom and in this commentary reminds us once again that these astronomers were working on the cutting edge of research, and were certainly not content merely to study and pass along the results of their predecessors. The reader of Shirwānī's commentary would benefit from the knowledge of flaws in Nīsābūrī's *Tadhkira* commentary, and at the same time receive an illustrative lesson about the importance of verifying assumptions at the heart of these complex models.

Eclipse limits

Eclipses occur when the Moon passes through the Earth's shadow at opposition (lunar eclipse) or when the Moon passes between the Earth and the Sun at conjunction (solar eclipse). Predicting eclipses, therefore, required accurate models for the motions of the Sun and Moon. Conversely, observations of solar and lunar eclipses provided precise knowledge of the Moon's position that could be used to refine existing models. Moreover, the fact that several Qur'ānic verses mentioned eclipses provided additional justification for studying them carefully. Although the Moon passes the Sun approximately every 29.5 days, eclipses are much less frequent because the Moon's circuit is inclined to the ecliptic and thus it is often not in the same plane as the Earth and the Sun.

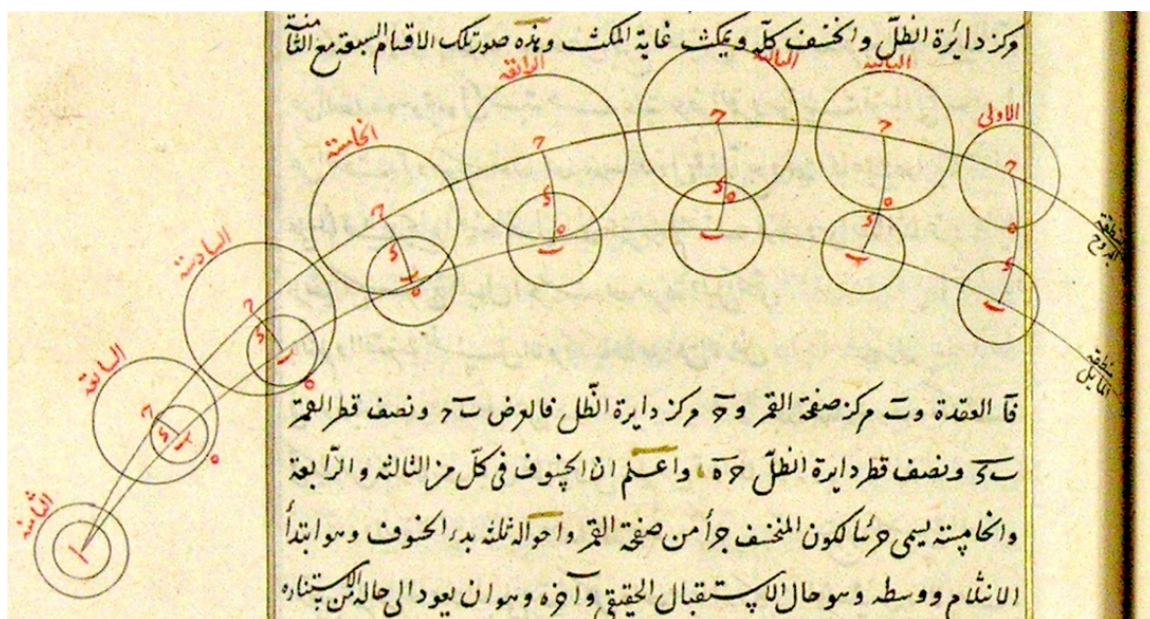


Figure 3.6 Possible configurations for Moon (lower) and shadow circle (upper)
Ahmet III MS 3314, fol. 257a

Ṭūsī explained that astronomers had calculated certain ranges, or eclipse limits, around the nodes in which the latitude of the Moon was small enough to bring it within the Earth's

shadow and cause a partial or total lunar eclipse.⁴⁵ He stated that the eclipse limits had been calculated as 12° for the Moon's distance from one of the nodes. Shirwānī supplied the missing demonstration, explaining how to derive the precise lunar eclipse limit of $12;12^\circ$ from a node using trigonometry, taking the inclination of the lunar circuit to the ecliptic to be 5° , and quoting values for the maximum radii of the apparent lunar area and Earth's shadow cone from *Almagest*, VI. Shirwānī explained that the observation of many total eclipses in different locations on the Earth led to the conclusion that the shadow circle is significantly larger than the apparent lunar surface. Ṭūsī stated its diameter is 2 and $3/5$ times larger than the lunar diameter, but Shirwānī noted that this value was taken from *Almagest*, V.14 and VI.4, where it was calculated based on the Moon being at perigee/closest distance. Thus, he reminded the reader to be aware that the use of this value at other locations was merely approximate and for ease of calculation (he noted Nīsābūrī provided a calculation that the value remained approximately 2 and $3/5$ in his *Sharḥ al-Zīj*).⁴⁶

As a final example of the kinds of calculations Shirwānī supplied to demonstrate Ṭūsī's statements, we turn to the question of how frequently eclipses might occur. Ṭūsī summarized the results of Ptolemy's investigations into the possible intervals of time between two consecutive lunar or solar eclipses. Ṭūsī had stated the values for solar eclipse limits (which depend on the observer's latitude, due to lunar parallax) in the 4th clime as 18° north of the ecliptic ("after the

45. Recall that the lunar nodes are the two opposite points on the Moon's circuit around the Earth that intersect the ecliptic, or yearly path of the Sun. The farther the Moon traveled from a node, the greater its latitude above or below the ecliptic, reaching a maximum latitude when it was 90° away from the nodes.

46. Shirwānī, *Sharḥ al-Tadhkira*, fol. 257a-b.

فَعَلِمَ أَنَّ النِّسْبَةَ الَّتِي ذَكَرَهَا الْمُصَنِّفُ تَقْرِيبِيَّةٌ وَأَنَّ أَحْكَامَ بَكُونِهَا مُحْفُوظَةٌ فِي جَمِيعِ الْأَبْعَادِ مَبْنِي عَلَى التَّسَاهُلِ وَالتَّقْرِيبِ وَيَدُلُّ عَلَى الثَّانِي بَرَهَانٌ هِنْدَسِيٌّ أَيْضًا ذَكَرَهُ النِّظَامُ فِي شَرْحِ الزِّيَجِ.

node of the head or before the node of the tail”) and 7° south of the ecliptic (“before the node of the head or after the node of the tail”).⁴⁷ Shirwānī mentioned the parameters used to derive these limits, although not the methods of calculation:

The greatest lunar parallax during a solar eclipse is $64'$, and the combination of the solar and lunar radii does not exceed $34'$. If the latitude is to the north, one must subtract the parallax to arrive at the apparent [value]; thus if the northerly latitude is $98'$ and the lunar parallax is at maximum, then the apparent latitude is $34'$, equal to the radii. Therefore, the northern eclipse limit must be where the latitude is $98'$ and that is after the head and before the tail by 18° . As for the south, parallax is added to obtain the apparent [latitude]. The greatest southern latitude at which a solar eclipse is possible is $34'$ which is before the head and after the tail by 7° . The Moon’s inclined [equator] divides into four sections by solar eclipse limits as well, but they are different. Those in which a solar eclipse is possible are 25° , but 18° of it are to the north and 7° are to the south, and those in which a solar eclipse is not possible are 144° in the north and 166° in the south.⁴⁸

Given these limits on solar eclipses, Ṭūsī had noted that it is possible for two eclipses to occur at a 5-month interval. Shirwānī explained the conditions for this to occur, with the first eclipse taking place shortly after the node of the head (ascending), and the second shortly before the node of the tail (descending). As we just saw, the interval between the two northern limits in which an eclipse is not possible is 144° , and since the Sun passes through nearly 150° in 5 months it will enter the second limit and move into opposition with the second node. He also

47. See Ragep, *Tadhkira*, 461-463 for a discussion of how Ṭūsī may have calculated these values, and a warning against taking these numbers too seriously given the (known) modifications of the tables since Ptolemy’s day and the (unknown) improvements in techniques used by Islamic astronomers for calculating solar eclipse limits.

48. Shirwānī, *Sharḥ al-Tadhkira*, fol. 261b. For the following discussion on lunar and solar eclipse limits and durations between eclipses, see Toomer, *Ptolemy’s Almagest*, 282-294.

reported Nīsābūrī's comment regarding the "true motion in latitude"⁴⁹ over 5 months, if it were the mean [motion] it would not be less than $145;32^{\circ}$ ⁵⁰ so the recurrence of an eclipse is possible after 5 mean months close to the latter limit, which is before the node of the tail (descending). If the duration is greater, then the recurrence will be earlier because the true argument of latitude due to the Sun [i.e., longitudinal component of motion along the inclined equator necessary to overtake the Sun] rises to $159;5^{\circ}$ and this greatly exceeds the arc on which a solar eclipse is not possible. Shirwānī clarified that Nīsābūrī said "due to the Sun" as shorthand for the intended meaning that the argument of latitude is the combination of the motion of the Sun and the motion of the node.

Similarly, two solar eclipses may occur 7 months apart if the first occurs before the node of the tail (descending) and the latter after the node of the head (ascending). Rather than repeating an analogous discussion from the case of eclipses after 5 months, here Shirwānī pointed out a flaw in Jurjānī's use of Shīrāzī's explanation in the *Nihāya*:

this is because if the Sun is at the first conjunction before the node of the tail close to the boundary of the eclipse limit and is eclipsed, then in 7 months it moves 205° and thus will have exceeded the head by 7° . In this same time the head will have moved 11° counter-sequentially, so the distance between the Sun and the head is approximately 18° and thus the Sun will not yet have exited the eclipse limit.⁵¹

49. By the Moon's "motion in latitude" Nīsābūrī and Shirwānī (and Ptolemy) were referring to what modern astronomers call the moon's "argument of latitude," which is the Moon's angular distance in its orbit from the ascending node. The values reported are the Moon's increase in longitude as a component of its motion along the inclined equator. Hereafter I will refer to this as the "argument of latitude."

50. This is the *Almagest*, VI value for the Sun's greatest true motion over 5 months, Toomer, *Ptolemy's Almagest*, 288.

51. Shirwānī, *Sharḥ al-Tadhkira*, fol. 262a-b.

According to Shirwānī, Jurjānī's statement required two corrections:

First, the amount by which the Sun exceeds the head should be 7° and a fraction, which is approximately the edge of the limit. Second, the distance between them becomes 18° and that same fraction. Therefore, [the Sun] exits the solar eclipse limit. [Or, if] we grant that it is at the limit there is no eclipse but rather [the Sun and Moon] touch. So what [Jurjānī] mentioned did not show the possibility of two eclipses at the end of 7 months. [Shīrāzī] did not address this because he stated that the northern limit is $18 \frac{1}{2}^\circ$, as you know, and [Jurjānī] followed the text as if it were 18° only.⁵²

As for Ṭūsī's remark that there was no doubt about the possibility of two solar eclipses occurring after 6 months, Shirwānī explained that the conditions and restrictions from the cases of 5 months or 7 months did not apply here, and in general after six months the Sun would be found within the next eclipse limit. Indeed, Shirwānī reported that, according to Qāḍīzāde, it was more suitable to demonstrate [the conditions under which] a solar eclipse would not recur after 6 months than to demonstrate when it would recur.⁵³

Likewise, Ṭūsī noted there was no doubt that a lunar and a solar eclipse could occur during a consecutive pair of oppositions and conjunctions, and Shirwānī reported Nīsābūrī's

قال السيد بعبارة صاحب النهاية لان الشمس إذا كانت في الاجتماع الأول قبل الذنب على قرب من طرف الحدّ وقد انكسفت ففي سبعة أشهر يتحرك مائتين وخمس درجات فيكون قد جاوزت الرأس بسبع درجات والرأس في هذه المدة قد يحرك إلى خلاف التوالي إحدى عشرة درجة فيصير البعد بين الشمس والرأس حينئذ ثمانى عشرة درجة تقريبا فلم يكن الشمس خارجة عن حدّ الكسوف بعد.

52. Shirwānī, *Sharḥ al-Tadhkira*, fol. 262a-b.

وفيه أنّ اللازم من قوله أولا أن يكون المجاوزة بسبع وكسر هو قدر القرب من طرف الحدّ ومن قوله ثانيا أن يصير البعد بينهما ثمانى عشرة وكسرا هو ذلك القدر فيكون خارجة عن حدّ الكسوف سلّما كونها في حدّه لكن لا كسوف فيه بل فيه تماسّ فلم يظهر ممّا ذكره إمكان كسوفين على طرفي سبعة أشهر. ولا يرد ذلك على صاحب النهاية لأنه صرح فيها بأن الحدّ الشمالي يمانى عشرة درجة ونصف كما عرفت والسيد جرى على ظاهر المتن من أنه ثمانى عشرة درجة فقط.

53. Shirwānī, *Sharḥ al-Tadhkira*, fol. 262b.

فلا يقع استنباه في إمكانه على طرفيها حتى أن الأستاذ رحمه الله كان يقول إمكان عدم عود الكسوف بعد ستة أشهر أولى بالبيان من إمكان عوده.

calculation illustrating the necessary motions. However, Ṭūsī stated it is not possible for two lunar eclipses to be one month apart, and neither was that possible for two solar eclipses unless they occurred in two locations with different directions in latitude. “As you know from what came previously,” Shirwānī explained that Ṭūsī had pointed out these limitations here to distinguish them from the previously mentioned phenomena. Shirwānī then analyzed how latitude affected the possibility of two solar eclipses occurring a month apart:

Regarding the lack of possibility [of two eclipses] in a location and in two locations in the same direction of latitude, it is because the combination of the two solar eclipse limits at the nodes does not exceed 25° and in a lunar month the Sun traverses nearly 30° and exits the eclipse limit. As for their possibility in two locations with different directions in latitude of the same amount⁵⁴ such that the first eclipse occurs at one of them and the second at the other, it is because the determination for southern latitude in the southern regions is like the determination of northern latitudes for the northern, so at a southern location with latitude like the latitude of the middle of the 4th clime, the solar eclipse limit south of the ecliptic is like its limit north of the ecliptic, that is, 18° – so the combination of the two limits around the sides of the head [=of the dragon, i.e. the ascending node], for example, with respect to these locations is 36° . It is not possible for the Sun to traverse this in one month, so if an eclipse occurred at this southern location before the head close to the limit at conjunction, it is possible for another eclipse to occur after the head at this northern location at the subsequent conjunction.⁵⁵

54. Due to parallax, the eclipse limits vary with the observer’s latitude. Shirwānī chose to discuss an example where two locations had equivalent latitudes so that the mathematics was simpler – he could just double the eclipse limit instead of having to deal with different limits corresponding to locations with different latitudes.

55. Shirwānī, *Sharḥ al-Tadhkira*, fol. 262b-263a.

أما عدم إمكانهما في بقعة وفي بقعتين متفتحتي جهة العرض فلان مجموع حدي الكسوف عن جنبتي عقدة لا يزيد على خمس وعشرين درجة والشمس في مدة شهر قمرى تقطع قريبا من ثلاثين درجة ويخرج عن حد الكسوف. وأما إمكانهما في بقعتين مختلفتين في جهة العرض متفتحتين في قدره بان يقع أحدهما في أحدهما والآخر في الأخرى فلان حكم العرض الجنوبي في البقاع الجنوبية كحكم الشمالي في الشمالية ففي بقعة جنوبية عرضها مثل عرض وسط الإقليم الرابع يكون حد الكسوف بعد الذنب وقبل الرأس مثل حدّه بعد الرأس وقبل الذنب أعني في عشرة درجة فيكون مجموع الحدّين عن جنبتي الرأس مثلا بالنسبة

As with the previous sections, in discussing the topic of eclipse limits Shirwānī went beyond Ṭūsī's report of the *Almagest* results to supply additional details and explain the process by which those results had been calculated. It is a typical example of his thoroughness, desire for clarity in technical discussions, and interest in nailing down the last strand of an issue.

Conclusion

Along with the preceding chapter, this chapter's overview of some of the technical content in Shirwānī's commentary on Book II of the *Tadhkira* has barely scratched its surface.

Nevertheless, it should be clear that Shirwānī's commentary was not pitched at a wide audience of general readers who might have been seeking a simplified overview of astronomy. Rather, as I have shown, Shirwānī deepened Ṭūsī's overview of the *Almagest* and created a text that would be useful for intermediate and advanced-level astronomy students. His supplementary clarifications and justifications for Ṭūsī's statements helped readers understand the astronomical principles that lay behind components of the mathematical models. Shirwānī's inclusion of detailed geometric proofs from the *Almagest*, and more importantly, other sources that had advanced beyond Ptolemy, allowed the reader to gain familiarity with the methods for turning observations and data into mathematical models. In principle, they now had the tools to criticize Ptolemy as well. Furthermore, Shirwānī's frequent quotations from more advanced sources such as Nīsābūrī's commentary on the *Tadhkira*, Shīrāzī's *Nihāya* and *Tuhfa*, and lectures by

إلى تلك البقعتين ستًا وثلاثين درجة ولا يمكن أن يقطعها الشمس في شهر فإن وقع كسوف في تلك البقعة الجنوبية قبل الرأس على قرب الحد في اجتماع أمكن أن يقع كسوف آخر بعد الرأس في هذه البقعة الشمالية في اجتماع يلي ذلك الاجتماع.

Qāḍīzāde and Ulugh Beg at Samarqand situated the commentary in an active tradition at the cutting edge of astronomical research and debate.

Chapter 4 – Education in Samarqand and beyond: Shirwānī as a student and teacher

Introduction

Beyond their value as sources for the development of technical fields such as astronomy, post-classical Islamic commentaries like Shirwānī's contain precious clues for identifying pedagogical practices employed by authors within scientific texts, as well as by teachers in face-to-face encounters with their students. Completed toward the end of a long career as a traveling scholar, Shirwānī's commentary on the *Tadhkira* reflected his experiences both as a student at the Samarqand observatory and madrasa under Qāḍīzāde and Ulugh Beg, and as a teacher in his own right. It is important to emphasize that, whereas Shirwānī's wanderings as a teacher represent more of a random sample, his student days in Samarqand were spent in one of the stellar gatherings of scholars and students in the mathematical sciences of Islamic civilization. After providing an overview of scholarship on education in Islamic societies, this chapter explores features of Shirwānī's commentary that mark it as a practical teaching text and not merely an exposition of theoretical astronomy. Anecdotes from his student days in Samarqand brought the reader inside the classroom and modeled the behavior a good student should emulate, even as rhetorical devices helped recreate the classroom experience as the reader worked through the text. In addition, supplementary material connected theoretical astronomy to related disciplines and provided examples of how knowledge from the *Tadhkira* could be put into practice.

Madrasas and “Islamic education”

In attempting to understand the transmission of knowledge in the medieval Middle East, historians have generally taken “Islamic education” as their primary object of study. To do so, they typically have focused on one group (the ‘*ulamā*’ or scholars of Islam as a religion and body of law) and one institution (the madrasa or law college). Such studies have explored the kinds of people included in the category of ‘*ulamā*’, their social backgrounds, goals, relationships and interactions with other broadly defined social groups. Others studies have investigated the organization, funding, curricula, and social roles of the madrasa. In most cases, scholars have treated these two entities as well-defined, stable components of Islamic society, and have attempted to explain and understand the functioning of Islamic society through the study of its system of education. Two related problems emerge in this sort of historical analysis. The first is that the chosen unit of analysis is “Islamic” society in the strictly religious sense, focusing exclusively on religious education in an institutional setting. This approach ignores the other forms of education in a society comprised of both Muslims and non-Muslims seeking knowledge in a range of fields beyond the exclusively religious domain.¹ Secondly, it treats the ‘*ulamā*’, madrasa, Islamic education, indeed even Islamic society in general as unchanging, largely static objects to be understood in terms of a monolithic, essentialized Islam.

In contrast with the limitations of adopting a single model of “Islamic education,” much can be gained by attention to difference. An expanded definition of “education” would encompass the pursuit of knowledge and skills in all their forms. These would include, but not be

1. For this reason, it is unfortunate that the term “Islamicate,” coined by Marshall Hodgson by analogy with “Italianate” to describe the broader culture of the medieval Middle East, has not been adopted by historians. Marshall Hodgson, *The Venture of Islam: Conscience and History in a World Civilization*, 3 vols. (Chicago: University of Chicago Press, 1974), 57-60.

restricted to, the practical education of scribes and bureaucrats; fields of study requiring intellectual and physical training, such as medicine, architecture, and the fine arts; the rational sciences of philosophy, mathematics, and astronomy; as well as the traditional “Islamic” sciences of theology, jurisprudence, and grammar. It would also encourage historians of the medieval Middle East to narrow their focus from “Islamic society” to specific regions and time periods in order to study the dynamics of education in the diversity of their particular contexts, such as Ulugh Beg’s Samarqand madrasa and the education Shirwānī received there.

The standard description of “Islamic education” as training in theology and law begins with the study of the Arabic language, the Qur’an, and collections of *ḥadīth* (reports of the words and actions of the Prophet Muhammad) with commentaries, which were then applied in the study of theology, jurisprudence, and the *sharī‘a* (Islamic legal framework encompassing law and social practice in a variety of forms). This religious education began at a young age. Children studied Arabic and memorized part or all of the Qur’an from family members or local instructors at a *kuttāb* (primary school). After the initial years of training, those who would become members of the *‘ulamā’* began studying and memorizing *ḥadīth*. Travel in the pursuit of *ḥadīth* was quite common, as the pious-minded scholars sought to insert themselves into living chains of individuals who transmitted the Prophet's words to future generations. Students would collect *ḥadīth* from different authorities and obtain their teacher's authorization to transmit their knowledge to others. If a student wished to pursue a higher education in topics such as theology or law, he² would attend classes at an institution devoted to these topics, the madrasa. There the students would listen to professors recite and comment on authoritative texts. They would study

2. Scholars tended to be male, but women also played important roles in the transmission of religious knowledge to varying degrees over the medieval period. Asma Sayeed, *Women and the Transmission of Religious Knowledge in Islam* (Cambridge: Cambridge University Press, 2013).

a curriculum of logical reasoning, disputation, and legal practice, culminating in their recognition as qualified practitioners of the law. At the end of the educational process, the student could hope to assume a position among the ‘*ulamā*’ as a teacher, judge, or jurist. According to this description of Islamic education, the entire process was designed to produce the legal scholars and jurists necessary for the well-being of a proper Muslim society in which all questions related to law and social life were resolved within the framework of the *sharī‘a*. Thus sketched, this picture of Islamic education appears clear. Upon closer inspection, however, it does not reveal much about the pursuit and transmission of knowledge in medieval Middle Eastern societies.

Nearly every part of the preceding description of Islamic education can be challenged (and indeed, has been challenged) on a number of grounds. George Makdisi’s *The Rise of Colleges: Institutions of Learning in Islam and the West* (1981) examined this narrative of Islamic educational processes through an analysis of the madrasa as an institution, in comparison with the Latin European college or university.³ His claim that Islamic educational institutions influenced European universities, based on parallels in the disciplines studied, the use of lecture and disputation, licensing systems for granting authority to teach, and what he considered to be other structural similarities, has been widely challenged and will not be discussed here.⁴ Makdisi based the project on a study of *waqfiyas*, legal documents establishing a charitable endowment (*waqf*) that provided support for the madrasas, and *fatwas*, scholarly opinions on matters of Islamic law relating to their administration. The laws of *waqf* allowed individuals to set aside

3. George Makdisi, *The Rise of Colleges: Institutions of Learning in Islam and the West* (Edinburgh: Edinburgh University Press, 1981).

4. For a partial overview of this debate, see Tim Geelhaar, “Did the Medieval West Receive a “Complete Model” of Education From Classical Islam? Reconsidering George Makdisi and His Thesis,” in *Cultural Transfers in Dispute. Representations in Asia, Europe and the Arab World Since the Middle Ages*, ed. Jörg Feuchter et al., (Frankfurt: Campus Verlag, 2011),

land, income, or other resources for the support of some charitable goal. The rules governing the madrasa and management of the *waqf* were instituted by the madrasa's founder and recorded in a *waqfiya*, detailing the conditions for appointing instructors, disbursing stipends, and other legal matters relating to the madrasa's day-to-day functioning. Makdisi analyzed these documents to understand the madrasa's internal structure. Each founder decided which courses would be taught. Makdisi claimed that by specifying these courses in a *waqfiya*, the founder in effect established his own curriculum and program of study. Based on these documents, he argued that the rational sciences were largely excluded from the madrasas, which instead focused on Islamic law and jurisprudence.

Responding to some of Makdisi's early work on madrasas, Tibawi pointed out that education under Islam had always been an informal process, and the appearance of institutions such as the madrasa did not alter this informality. Criticizing attempts to generalize across the entire Islamic world, he urged an approach based on the study of individual institutions. In addition, he cautioned against making judgments about the place of non-religious subjects in the madrasas, arguing instead that instruction covered a variety of subjects for an audience that included students and lay members of the community.⁵

Jonathan Berkey's study of education in medieval Cairo paints a more detailed picture of the educated community and the interactions between the 'ulamā' and the Mamluk elites.⁶ By studying a large number of Cairo *waqfiyas* along with biographical dictionaries, Berkey documented the great diversity in educational practices and environments that had disappeared

5. A.L. Tibawi, "Origin and Character of *al-Madrasah*," *Bulletin of the School of Oriental and African Studies* 25, no. 1 (1962): 225-238.

6. Jonathan Berkey, *The Transmission of Knowledge in Medieval Cairo: A Social History of Islamic Education* (Princeton: Princeton University Press, 1992).

when Makdisi subsumed it all in the term “madrasa.” As a matter of practical concern, the mosque and madrasa were indistinguishable for medieval Cairenes. Indeed, Berkey pointed to “several institutions dating from the end of the Mamluk period that were specifically known by contemporaries as madrasas, yet which made no provisions at all for classes in any of the religious sciences.”⁷ The schools were founded by a wide range of social groups, ranging from sultans and *amīrs* to bureaucrats, scholars, and merchants. Furthermore, the buildings served as places of prayer as well as instruction, and provided community meeting space. The list of individuals deriving income from a *waqf* often went beyond the teachers and students to include religious officials, prayer leaders, and maintenance workers. Berkey placed the student-teacher relationship at the core of the educational process, observing that the biographical dictionaries list an individual's instructors, but significantly they do not mention the locations in which he studied.⁸ A student seeking to become educated would be advised to choose the best teacher, not the best school. The *ijāza*, which Makdisi had claimed functioned as a license to teach, symbolized the importance of individual connections and personal contact. The benefit of studying with an individual teacher was not merely to acquire a certain body of knowledge (which one could, in principle, learn elsewhere), but also to benefit from that individual's personal authority over the texts. Since Berkey drew on biographical dictionaries and literary materials in addition to the *waqf* endowment deeds, he was able to supplement the formal institutional picture Makdisi had presented. His account described the informal style of education and instruction on the personal level, noting the frequent use of terms like *ṣuḥba* and *mulāzama*

7. Berkey, *Transmission of Knowledge in Medieval Cairo*, 17.

8. Berkey, *Transmission of Knowledge in Medieval Cairo*, 23.

that indicated notions of “companionship” or “discipleship,” and the use of parental metaphors to describe the relationship between teacher and student.⁹

These informal relationships extended into the pursuit of employment and appointment to positions within a madrasa. Berkey argued that while the *ijāza* served as the mark of a student's ability to teach a text, the decision of whether someone would be allowed to teach in a madrasa was based on other considerations. In Cairo, there were many more people “licensed” to teach than there were positions in the madrasas, but that did not prevent them from teaching. By analyzing the patronage relationships between Mamluk military and merchant elites who established madrasas and the scholars who sought to benefit and profit from control of endowment funds and access to stipends and teaching positions, he noted the frequent occurrence of personal ties between the individual Mamluk who undertook to establish a madrasa and the scholars who were appointed as professors. Beyond this initial appointment, personal connections continued to be the most effective means of obtaining a position. In practice, control over these appointments was unclear and contested, due to disagreement among legal scholars about the jurisdiction of political and religious authorities. Moreover, scholars took advantage of the system to exercise their own forms of patronage. In many cases, individual scholars would make recommendations about who was best qualified for a particular post. Beyond this, it was common for a professor to select, or even officially to install, his own successor when he was about to retire. The histories describe this by saying someone “resigned a post in favor of,” “left it to him,” or “in favor of him.” Such appointments often involved close family or teacher-student bonds.

9. Berkey, *Transmission of Knowledge in Medieval Cairo*, 33-38.

Informal patronage and family connections even extended to the founding of madrasas themselves. Members of the *'ulamā'* and military elites all established madrasas for related reasons. Within the competitive arena of Mamluk politics, establishing an institution provided political legitimacy, emphasizing and commemorating the power and “symbolic capital” of the benefactor. Moreover, there were also financial motivations for wealthy individuals to establish a madrasa. The founders usually controlled the *waqf* endowment during their lifetime, after which control of the endowment passed to their children or heirs. Moreover, the funds and properties held in a *waqf* were sheltered from confiscation by the sultan or administration officials. Thus, Berkey argued, “the protection of personal and family wealth lay close to, if not absolutely at the front of, a founder’s list of priorities.”¹⁰

Berkey’s study has provided much needed insight into the social context of the madrasas and Islamic education generally, arguing that the influence of education and institutions was felt more strongly in the broader urban society than merely among the academic elite. The role of educational institutions thus went well beyond education; they also served as communal spaces for prayer, worship, and legal proceedings. Religious figures such as prayer leaders and preachers associated with a madrasa belonged to the world of education, even if they did not participate in the acts of scholarship and teaching. They thus helped extend networks of social ties from the *'ulamā'* to the rest of the population.

Similarly, Daphna Ephrat’s study of the *'ulamā'* of 11th c. Seljuk Baghdad explored the “link between the cultural and social practices involved in the process of the transmission of

10. Berkey, *Transmission of Knowledge in Medieval Cairo*, 137.

Islamic learning, and the construction of social bonds and identities...”¹¹ Employing a sociological approach, she described the educational system and scholarly affiliations in terms of “vertical chains” linking teachers and transmitters of knowledge through generations, and “radial networks” of students around their shared teacher.¹² According to Ephrat, the study circle, or *ḥalqa*, centered on a particular teacher and was the defining practice of Baghdadi education. The individual teacher had great authority over all matters pertaining to the *ḥalqa*: “wherever the *ḥalqa* was convened — be it the floor of a mosque, a private living room, a shop, a riverbank, or a chamber of the madrasa — it was apparently the teacher who determined the acceptance of students to his study group, the frequency of its assembly (*majlis*), as well as the sequence and method of all instruction.”¹³ Accordingly, the madrasas provided no more than an organizing framework for individual cultural practices in education that had existed previously. The Baghdad *‘ulamā’* used patronage, primarily through networks of personal ties, to acquire paid positions at madrasas for their favored students. As a result, the seemingly fluid and open world of academic positions was in fact controlled by a small group of elites.

Michael Chamberlain’s study of the military and academic elites of Damascus in the 13-14th centuries drew similar conclusions about the informal nature of education, although he argued that the madrasas in general should be understood in terms of political economy and competition, rather than as a “system of education.”¹⁴ Chamberlain took the elite household as

11. Daphna Ephrat, *A Learned Society in a Period of Transition: The Sunni ‘ulama’ of Eleventh-Century Baghdad* (Albany: State University of New York Press, 2000), 6.

12. Ephrat, *A Learned Society in a Period of Transition*, 76-80.

13. Ephrat, *A Learned Society in a Period of Transition*, 79.

14. Michael Chamberlain, *Knowledge and Social Practice in Medieval Damascus, 1190-1350* (Cambridge: Cambridge University Press, 1994).

his unit of analysis, and explored how the interests of military and scholarly households intersected in the madrasa, since both groups used madrasas and their associated financial devices to acquire status and preserve their own power for generations. For the military elites, endowing a madrasa meant that they could preserve their capital in the form of a *waqf* and control its use by appointing the founder or one of his relatives as director. As in Berkey's study of Cairo, the madrasas of Damascus also served social and political purposes. Foreign-born military households could attach members of the '*ulamā*' to themselves by founding a madrasa. The building itself often functioned as a tomb, making a symbolic political statement about the founder and their family's role in the life of the city. For the learned elites, madrasas provided a prestigious source of income for students and teachers alike through stipendiary positions (*manṣabs*).¹⁵

In rejecting the formal terminology and structures laid out by Makdisi, Chamberlain pointed out that the sources used to construct Makdisi's "system of education" more likely described an ideal than reflected the reality of educational practice or institutional structure.¹⁶ Whereas the madrasas of Baghdad, Cairo, and Damascus may have provided space for teaching, prayer, and social interaction in addition to valuable sources of income and prestige, they were not specialized institutions of higher education. Instead, students would study a variety of topics with different instructors at multiple locations. The key site for the transmission of knowledge remained the *ḥalqa*, which could convene just as frequently outside a madrasa as within one. Chamberlain drew attention to the breadth of knowledge praised in biographical dictionaries, as opposed to a specialized knowledge of law. The same lecturers who taught law were also

15. Chamberlain, *Knowledge and Social Practice*, 91-107.

16. Chamberlain, *Knowledge and Social Practice*, 69-90.

philosophers, Sufis, or scholars of the rational sciences and medicine. While jurists may have wished that instructors teach only law in the madrasa, Chamberlain pointed to the abundant evidence that other subjects were taught both in madrasas as well as study circles throughout the city. Ultimately, scholars taught what they knew, and students learned from multiple teachers. Their goal was to acquire a breadth of knowledge. The literary sources described scholars as “complete men” who united various fields or “mastered” subjects, but Chamberlain reminds us that “in high medieval Arabic there is no word for ‘curriculum,’ ‘list of books,’ or ‘program of study’.”¹⁷ Thus, the goal of education was not certification to teach in a madrasa, but rather to acquire the “quality of exemplarity” embodied in the person of the pious, learned shaykh.¹⁸

As the remainder of this chapter demonstrates, Shirwānī’s experiences as a student at Ulugh Beg’s Samarqand madrasa, along with aspects of the *Tadhkira* commentary he wrote after a long teaching career, provide examples of the kinds of differences that emerge when one pays attention to diversity in the history of education in Islamic societies, rather than attempting to describe a single model of “Islamic education.” Despite this call for attention to difference, certain continuities remind us that we are not dealing with atomized phenomena – for example, the *ijāza* is evidence that practices could be very long-lived even when they were used for distinctive purposes.

Anecdotes from Samarqand

Together, the preceding studies offer a useful framework for approaching Ulugh Beg’s observatory and madrasa at Samarqand, one of the most famous scientific institutions of the

17. Chamberlain, *Knowledge and Social Practice*, 85-87.

18. Chamberlain, *Knowledge and Social Practice*, 106-107.

premodern Islamic world. It was established in order to carry out a program of new observations that would be used to generate updated tables of astronomical data used by practicing astronomers and astrologers.¹⁹ In addition to Shirwānī's teacher Qāḍīzāde al-Rūmī, several other significant individuals studied there or were employed as instructors and astronomers. The most notable of these were Qāḍīzāde's predecessor as director of astronomical observations, Jamshīd al-Kāshī (d. 1429), and 'Alī al-Qūshjī (d. 1474), who succeeded Qāḍīzāde. Two previously published letters from Kāshī to his father provide vivid descriptions of the early stages of scientific work carried out at the observatory, as well as a strong sense of Kāshī's personality and ego.²⁰ Shirwānī, who arrived at Samarqand several years after Kāshī's death, has left us new illustrations of student life through personal recollections scattered throughout his commentary on the *Tadhkira*. Like his commentary as a whole, nearly all of Shirwānī's classroom anecdotes have so far remained unstudied by historians.²¹ Although no doubt idiosyncratic in their detail, they portray an environment of active intellectual engagement with challenging material. Students were expected not only to read and memorize the text but also to be prepared to demonstrate their knowledge. This they did by responding to questions not only from their teachers, but also from the scholar-sultan Ulugh Beg himself, and by participating in disputations with other students.

19. See Aydin Sayili, *The Observatory in Islam* (New York: Arno Press, 1981), 260-289.

20. See E. S. Kennedy, "A Letter of Jamshīd al-Kāshī to his Father: Scientific Research and Personalities at a Fifteenth Century Court," *Orientalia* 29, (1960): 191-213 and Mohammad Bagheri, "A Newly Found Letter of al-Kāshī on Scientific Life in Samarkand," *Historia Mathematica* 24, no. 3 (1997): 241-256.

21. İhsan Fazlıoğlu studied the first anecdote discussed here in İhsan Fazlıoğlu, "The Samarqand Mathematical-Astronomical School: A Basis for Ottoman Philosophy and Science," *Journal for the History of Arabic Science* 14, (2008): 3-68.

In this section, I will generally present the primary documents in all of their narrative freshness, allowing them to speak first in their own voice. In the first scene, Shirwānī's diligence in studying outside of class allowed him to correct a teacher's misstatement and earned him the position of reader for the class. This anecdote also includes Shirwānī's *ijāza* (authorization to teach) from Qāḍīzāde. In the second episode, while 'Alī al-Qūshjī was discussing Nīsābūrī's *Tadhkira* commentary on the motion of the lunar nodes and apogee, Shirwānī identified an error in the text. As a result, Ulugh Beg invited Shirwānī to join his private assembly. The third anecdote involved the question of how to interpret statements Nīsābūrī made to qualify or restrict comments in the *Tadhkira* about the Moon's motion in latitude, which surprised Qāḍīzāde because he had not thought of that question despite reading and teaching the text many times. Finally, in the fourth anecdote Shirwānī recounts how Ulugh Beg identified an error in Jurjānī's *Tadhkira* commentary related to geometry, optics, and the phases of the Moon.

Anecdote 1 - Parallel lines and a virtuous student²²

Early in his commentary, Shirwānī described an episode in which the sultan and patron Ulugh Beg visited his madrasa and interrupted the discussion to ask a question about a proposition from Euclid regarding parallel lines in different planes. When a teacher responded with an incorrect answer, and then repeated the incorrect answer after the sultan rephrased the question, Shirwānī spoke up with the correct answer. Ulugh Beg then tested him by asking if he was actually referring to a different proposition, but Shirwānī knew the second proposition as well and

22. Faṭḥallāh al-Shirwānī, *Sharḥ al-Tadhkira*, Topkapı Ahmet III MS 3314, fol. 15a-17a.

insisted that he was correct, which pleased the sultan. At the conclusion of the anecdote, Shirwānī inserted the text of his *ijāza* from Qāḍīzāde.²³ Shirwānī told the story as follows:

I was among the audience for Nizām's [i.e., Nīsābūrī's] commentary [on the *Tadhkira*] under my master and professor Qāḍīzāde al-Rūmī, the Imām whose like has not been sanctioned since the celestial sphere has turned, at Samarqand during the recitation by one of the members of that noble assembly²⁴ known for his intelligence and high status in the madrasa of the sultan, son of the sultan, Ulugh Beg Gurkān... One day the philosopher-sultan, who would attend the lecture one or two days per week, was attending the lecture. When the lesson reached this topic [about parallel lines], he asked, "Why did [Euclid] say, 'in a plane' and what is the purpose of restricting it [to a plane]?" One of the teachers of the madrasa, who would attend the lectures with the sultan, proceeded to answer, saying, "That is because Euclid, in his book, proved that parallel lines cannot be other than in a single plane."²⁵

With the help of God Most High, by that day I had finished studying and analyzing twelve books of that text, preparing them well. No one knew of my situation for I had studied them privately during the vacation. Therefore, I noticed his [the teacher's] mistake and that he spoke of that which he did not know. It was likely that the sultan was also aware of it, for he had prepared that book and indeed all of the mathematical sciences, their principles and derivations. However, I did not point out his error and reply to it, which would establish his error for the listeners. For the sultan's custom and habit was to return to the examination a second or third time, as if by doing so he was testing for himself those in his madrasa who were appointed to test the

23. The following translation is my own. A critical edition of the Arabic text of this episode and Shirwānī's *ijāza*, with English translation, has been published previously in Fazlıoğlu, "The Samarqand Mathematical-Astronomical School," 40-49. Saliba also summarized this anecdote in George Saliba, "Reform of Ptolemaic Astronomy at the Court of Ulugh Beg," in *Studies in the History of the Exact Sciences in Honour of David Pingree*, ed. Charles Burnett et al., (Leiden: Brill, 2004), 817-819.

24. A marginal note in Damad Ibrahim MS 847 identifies the reader as Khwāja Shihāb al-Dīn.

25. A marginal note in Damad Ibrahim MS 847 identifies the teacher as Mawlānā 'Alā' al-Shāshī.

students, who were arranged into advanced, intermediate, and lower [groups] with teachers having fifty or forty [students] each. For these excellent and honorable [teachers], doing wrong to those in their care would be like apostasy.

When the reader went on at length, having dwelled a while on the definition of parallel planes, the sultan returned to questioning and [the teacher, al-Shāshī] who had answered repeated what he said the first time. At this point I said, “However, the matter that is in the book is different from what you said.” The sultan turned toward me and the reader lowered his head since he was in the direct line of sight between myself and the sultan, close to him and Qāḍīzāde, who were each seated at the corner of a triangle whose sides were about one cubit. He asked, “How is that?” I said, “In proposition 9 of book XI, Euclid said that lines parallel to a [given] line, even if they are not all in one plane, are parallel. Then he demonstrated it.”

He turned to Qāḍīzāde, agreeing with me and praising what I said more than twice. Likewise, he turned to those in agreement, whether sitting or standing, for many of the eminent scholars who accompanied him would stand before him on the days when he attended [lecture]. The one who had answered hung his head, and when the sultan finished that gracious action, he [i.e., al-Shāshī] raised his head and said, “Perhaps what Euclid said in that book is that any two angles whose corresponding sides are parallel, which are not all in the same plane, are equal, and not what you reported.” I said, “That proposition comes after [the first] and follows from it.” Then the sultan returned to his gracious words, but became extremely short and so ended the discussion brusquely and departed. That day was the last day of study for the week, and that same week, the reader [Shihāb al-Dīn] was dismissed and I was appointed in his place, so at the beginning of the following week, I began [reciting] from the definition of parallel planes. The one who was dismissed [Shihāb al-Dīn] became very ill and departed for Herat, and the one who was treated brusquely [al-Shāshī] hid himself away for more than a month, then he reemerged and appeared lively.

When I silenced him [al-Shāshī], I saw Qāḍīzāde’s face had become as red as a newly opened rose. For, according to what I heard, he had in mind something of the false claims of the other side. [But] it was the characteristic of Plato, Aristotle, Ptolemy, and Euclid to be ashamed of error if there was a chance to avoid it. So after the sultan’s

departure, he returned and said in wonderment with a pleasant smile, “Have you also studied the solid geometry from Euclid’s book?” I replied, “Those available in the professor’s lecture room, but three books remain to be studied and I have not found a copy.” So he ordered a servant to bring his copy, and I copied the remainder and completed studying that book with the help of God Most High, and at the end of my copy I wrote about my amazing situation.

As for reading [Nīsābūrī’s] commentary, I followed the method that the wise professor chose for the sultan — little by little over a long period of time. By spending a great deal of time, the reading rewarded [me] with greater understanding through the increased number of explanatory examples. With the aid of God Most High, it was completed in nearly five years. Along with its completion came the attainment of the sciences through reading, listening, and careful study. At that time, at my request my father, may God most High provide him a noble place in the Gardens of delight, came to Samarqand with an emissary from the ruler of Shirwān, may God the Compassionate reward him with the reward of the enlightened ones. Then the sultan authorized me to return, and Qāḍīzāde gave written authorization in the sciences. And here I state this authorization as a blessing and proof:

[Qāḍīzāde] said: In the name of God, the All-Merciful. Praise be to God, whose custom is to open the doors of knowledge to those who strive to pursue its paths and plains. And blessings upon the unlettered Prophet, who commanded “seek [knowledge], even if it is in China,” and upon his family and Companions, casting out ignorance and bringing forth certainty. Among those who follow this example in acquiring [knowledge] and undergoing the greatest hardship to assess and improve it is this virtuous youth, indeed in the realm of the virtuous, he contends for superiority, the essence of the virtuous and exemplar of the intelligent, Mawlānā Faṭḥallāh b. al-mawlā al-nabīh al-farāḍī al-faqīh Abī Yazīd al-Shābarānī al-Shirwānī, may God facilitate their hopes and rectify their circumstances. For indeed in order to acquire [knowledge], he traveled from the West to the East, crossing plateaus and valleys and mountain peaks, until he reached us in the fortified city of Samarqand. He read *Sharḥ al-Tadhkira* with us, and since that time [i.e., his arrival] until now, he listened to all that took place in our lecture hall among those possessing the highest insight and intellect. He joined the most advanced

group of masters, those possessing virtue and excellence. He had an inner core but listened carefully and witnessed, noticing every riddle, even the most obscure, until he reached the end of his quest and aspiration (*muntahā al-su' l wa-l-amal*) through listening to the *Sharḥ al-mukhtaṣar fī 'ilmay al-uṣūl wa-l-jadal*.²⁶ He verified the intentions of the philosophers, and the doctrines of the scholars of Islam through attention to the *Sharḥ al-mawāqif* in the field of *kalām*.²⁷ He excelled and dazzled in all fields, and his reputation grew and spread among the senior [members of the school]. When he had reached this point, his father came to return him to that land [i.e., Shirwān], for returning is praiseworthy and a father has a right to his son. He sought my authorization, which pleased me, so I authorized him to transmit on my authority all he had heard from me, including all that which was corrected by him. He is permitted to transmit this in my name. I exhorted him to faith and piety, especially in transmitting [knowledge] and legal opinion (*al-fatwā*). May God preserve us, and him, from error and transgression in word and deed. I am one who is in need of God the Abundant, Mūsa b. Muḥammad b. Maḥmūd, known as Qāḍīzāde al-Rūmī. Written in the middle of the month of Rabī' al-ākhar in the year 844 [September 1440].

Anecdote 2 - Shirwānī joins Sultan Ulugh Beg's private assembly²⁸

As we saw in the previous chapter, Ṭūsī's description of the Ptolemaic lunar model involved four orbs: the parecliptic orb and the inclined orb, both centered on the Earth, and the eccentric with its epicycle, which were contained in the thickness of the inclined orb. The planes of the equators of the eccentric and the epicycle were aligned in the same plane as the inclined orb. Shirwānī reminded the reader that the orbs' equators were arranged that way since the Moon had no

26. al-Ījī's commentary on a work of jurisprudence by Jamāl al-Dīn 'Uthman b. 'Umar (d. 1249), see Fazlıoğlu, "The Samarqand Mathematical-Astronomical School," 49 n.206.

27. Jurjānī's commentary on al-Ījī's seminal work in theology, *al-Mawāqif fī 'ilm al-kalām*.

28. Shirwānī, *Sharḥ al-Tadhkira*, fol. 194a-b.

motion in latitude other than that due to the inclined orb. The equator of the inclined orb, so called because it was at a fixed angle to the ecliptic, intersected the ecliptic at two diametrically opposed points called the “nodes” — the (ascending) node where the Moon crossed the ecliptic from south to north was known as the “head” and the southern crossing point (the descending node) was called the “tail.”

Shirwānī described a classroom episode in which he corrected an error in Nīsābūrī’s commentary on *Tadhkira* II.7 on the relation between the inclined and eccentric orbs and the lunar nodes. As with his previous anecdote in Book I, this story brings the reader inside Ulugh Beg’s madrasa at Samarqand and paints a vivid picture of the kind of environment in which students would be reading Shirwānī’s own commentary. Furthermore, the future head of the Ayasofya madrasa in Istanbul, ‘Alī Qūshjī, is mentioned by name and features prominently in the story.²⁹ This anecdote establishes clearly that he and Shirwānī were acquaintances long before Qūshjī joined Mehmed II’s circle in Istanbul in 1472.³⁰ Shirwānī describes Qūshjī as a member of Ulugh Beg’s circle of scholars, who were visiting Qāḍīzāde’s class that day with the sultan. In fact, Qūshjī was the one expounding on Nīsābūrī’s statement about the motions of the lunar nodes and apogee when Shirwānī interjected to point out an error. Sometime thereafter, Shirwānī

29. Saliba briefly summarized this anecdote in a chapter on Qūshjī, Saliba, “Reform of Ptolemaic Astronomy,” 820-821.

30. Since their time at Samarqand overlapped, Shirwānī and Qūshjī must have known each other, but I am not aware of another text in which one of them mentioned the other. After the death of Ulugh Beg in 1449, Qūshjī moved to Herat. Years later he relocated to Tabrīz, which was under the control of Uzun Ḥasan, sultan of the Aq Qoyunlu (a rival dynasty to the Ottomans). Qūshjī was likely sent as an ambassador to Mehmed II at some point after 1469, and in 1472 he relocated to Istanbul where he was later made head of the Ayasofya madrasa. İhsan Fazlıoğlu, “Qūshjī: Abū al-Qāsim ‘Alā’ al-Dīn ‘Alī ibn Muḥammad Qushčī-zāde,” in *Biographical Encyclopedia of Astronomers*, ed. Thomas Hockey et al., (New York: Springer, 2007), 946-948.

was invited to join the sultan's private assembly, silencing those who had scolded him for speaking up in lecture and raising a problem with Nīsābūrī's commentary.

On the previous page, when Ṭūsī defined the inclined orb, Shirwānī mentioned that Nīsābūrī put the plane of the Moon's inclined equator in the plane of its eccentric equator (which is the opposite of how Ṭūsī refers to them, since the eccentric is carried by the inclined), "after imagining/visualizing its [the eccentric equator's] intersection with all of the orbs (*ba'd tawahhama qaṭ' ihi li-jamī' al-aflāk*).” After Ṭūsī defined the nodes and Shirwānī mentioned their etymology, he returned to Nīsābūrī's comment:

From the existence of a latitude for the Moon in the aforementioned manner, the plane of the eccentric equator — after imagining its intersecting all of the orbs — must be posited to intersect the orb of the ecliptic at two points facing one another called the nodes (or *jawzahars*) as well. Then we say, the plane of the equator of the eccentric's carrier — that is, the inclined orb — is either [A] in the aforementioned [eccentric] plane or [B] not. If it is not there, it must move the points of intersection by the same amount as the apogee, however the motion of the apogee, as will be shown, must be posited as multiple times the motion of the intersecting points. Thus, the plane of the equator of the deferent of the eccentric must be in that plane [of the eccentric]. For this reason, it is called 'inclined'.

After quoting Nīsābūrī, Shirwānī (who at this point was the reader for the lecture) began his anecdote saying:

Two days after I read this part of his commentary, the professor [Qāḍīzāde] announced that the sultan, God have mercy on them both, had come to the lecture. He said to me, "For today, read the lesson from the day before yesterday, since there is no problem (*ishkāḷ*) in the two subsequent lessons," the sultan being averse to lessons that did not contain problems. I had been deeply absorbed in this part after the reading, until an idea came to me against the commentator (*ḥattā sanaḥa lī fīhi shay' 'alā al-shāriḥ*). When the recitation arrived at [Nīsābūrī's] statement, "For if it is not there, it must move the points of intersection by the same amount as the apogee," which was the spot of [my]

idea, one of the sultan's disciples known as 'Alī al-Qūshjī accepted it as true while standing and discussing it.

Then I said, "On the contrary, it requires investigation (*bal fīhi baḥth*) because their being moved [i.e., motion of the points of intersection] by this amount is necessary if the plane of the equator of the deferent of the eccentric [the inclined orb] is in the plane of the ecliptic. But that negation [statement B above] does not require this conclusion [i.e., that the nodes move by the same amount as the apogee]. For if [the inclined] were between [the ecliptic] and the aforementioned plane [i.e., the eccentric], for example, [their] being set in motion would be less than that amount [i.e., the motion of the apogee]. This becomes apparent through imagining that the two intersection points of the aforementioned plane and the equator of the deferent of the eccentric are superimposed upon the nodes, then by the motion of the deferent of the eccentric [the inclined orb], the two points separate from the nodes [which] move slowly subsequently with the motion of the parecliptic.

Nevertheless, the nodes complete a circuit on the assumption that the maximum distance between the equator of the deferent of the eccentric [i.e., the inclined orb] and the ecliptic is less than the maximum distance between it [the ecliptic] and the aforementioned [eccentric] plane, since the assumption that they are equal necessarily negates the [existence of] the nodes (*yalzam intifā' al-'uqdatayn*), with the aforementioned plane coinciding with the ecliptic with the two intersections dividing the ecliptic equator in half. Assuming the opposite entails their [the nodes'] recession (*idbār*) when reaching the maximum distance of the aforementioned plane from the ecliptic to the ecliptic³¹ after [their] accession counter-sequentially slightly. But Nīsābūrī's requiring that the intersection points are moved with the motion of the apogee indicates both the lack of negation [of the nodes] and retreat [of the nodes] on the assumption he mentioned."

31. *Wa 'alā al-'aks yalzam idbārhimā 'inda wuṣūl ghāyat bu 'd al-saṭḥ al-madhkūr 'an minṭaqaṭ al-burūj ilā minṭaqaṭ al-burūj ba 'd iqbāl qalīl 'alā khilāf al-tawālī.* I think Nīsābūrī was trying to describe the nodes reaching a distance from the ecliptic equal to the maximum distance between the eccentric plane and ecliptic.

When I said this, all were silent and no one offered a response, neither the assenter [Qūshjī], the professor [Qāḏīzāde], the sultan, or any one of the outstanding teachers of that school or those listening to my lesson. Then the sultan rose and left the room and Qāḏīzāde followed him into the antechamber, then he sat on his bench and examined the most knowledgeable of his students in this art about what I said. He [Ulugh Beg] asked him, “What do you think about what that one said (*Kayf tarā mā qāla fulān*)?”

[The student] said, “I believe the matter is as he said (*aẓunn al-amr kamā qāla*).”

So he stood up, commanding careful contemplation on it, but when he mounted his horse, the students went overboard in scolding me, describing my remarks with coldness, and in pouring cold water on the lecture hall ablaze with the fire of investigation and explanation, until a day or two after the days of vacation, my attending his [Ulugh Beg’s] blessed assembly in his noble house was approved (i.e., Shirwānī joined the Sultan’s *ḥalqa* or private study circle). I saw my critics and others in attendance there, then when I took a seat, he said, “Did you contemplate that at another time (*hal ta ’ammalta fī dhālik tārāt^{an} ukhrā*)?”

I replied, “Indeed, I said it after exhausting my power in examination ’til there was nothing remaining in me of the possibility of additional examination (*innamā qultu dhālik ba ’d istifrāgh juhūdī fī al-ta ’ammul fa-lam yabqa fiya li-mazīd al-ta ’ammul fīhi masāgh*).”

Then he [Ulugh Beg] said, “Truly an excellent objection and response to Nizām [Nīsābūrī]!” Thus when we left the assembly, the matter and circumstances were reversed, and the critics began to scold themselves and praise the scolded one (i.e., Shirwānī) and they became like the adage: “he who criticizes others deserves to be criticized (*rabb la ’m malīm*).”

Anecdote 3 - Explaining the Moon's motion in latitude³²

Ṭūsī defined the Moon to be “ascending” as it moved from its maximum latitude south of the ecliptic to its maximum latitude north of the ecliptic, and “descending” on the other half of its circuit back down to its maximum southern latitude. Shirwānī added that in northern lands, “ascending” for the Moon meant drawing nearer to the “visible pole” and “descending” meant becoming distant from it. Modern readers may be confused here because this “pole” was not the star Polaris or the north celestial pole of the daily rotation of the heavens. Rather, since the ecliptic (the annual path of the Sun against the background of the fixed stars) was the astronomers’ frame of reference for celestial longitudes and latitudes, Shirwānī (and Ṭūsī) referred to the poles of the ecliptic orb as simply “the poles,” expecting that the reader would understand. For inhabitants of northern latitudes, therefore, the “visible pole” referred to the point in the heavens representing the north pole of the ecliptic orb. The opposite “invisible pole” would be hidden below the horizon for northerners and only “visible” to inhabitants of southern latitudes.

Shirwānī continued,

There are some who explain ‘ascent’ and ‘descent’ as moving closer to the zenith³³ or away from it, as if it were obvious. [But] that judgment (*dhālik al-ḥukm*) regarding this explanation does not generalize to all northern lands, because it is not correct in the northern lands in which the Moon reaches the zenith before it arrives at its maximum

32. Shirwānī, *Sharḥ al-Tadhkira*, fol. 206a-b.

33. The zenith is the point in the heavens directly above a given location. So, for example, the zeniths of Samarqand, Istanbul, and Mecca were all different.

northern latitude.³⁴ Al-Nizām [Nīsābūrī] identified the visible pole³⁵ of the ascending and descending [motion of the Moon] with the pole of the ecliptic, and in that there is something unspecified in some lands, due to the visible pole [of the Moon's motion in latitude] disappearing and the invisible becoming visible. Thus it needs to be restricted to most of the globe.

By this, I take Shirwānī to be referring to the fact that since the plane of the Moon's orbit is inclined to the ecliptic, and because the plane and the nodes precess around the ecliptic over approximately 18.6 years (the draconitic period), for observers at very low northern latitudes near the Earth's equator the north pole of the Moon's orbit could be hidden for periods of time and the south pole of the Moon's orbit would be the "visible" pole instead. Shirwānī continued:

[Nīsābūrī] said, "indeed we explained 'ascending' as the Moon drawing closer to the visible pole of the ecliptic and 'descending' as its becoming distant from it [=visible pole of the ecliptic] in order to generalize the judgment in all times. Since if we interpreted them as nearness to the zenith each day and as distance from it, by analogy with the motion of the Sun along its circuit, the judgment would not generalize." Thus he generalized his statement with the phrase "at all times" but without [the phrase] "in all northern lands" as in the *Nihāya*."

And in that [statement] there is something vague/obscure, which disappears in what I [i.e., Shirwānī] relate to you. When I did not find the variant reading he mentioned to be appropriate and clarifying to the base text, I examined Qāḍīzāde about it during the reading. So he concentrated deeply on it for a while as the members of the audience were silent, and after this careful study he said, "The variant reading was to explain the generalization of the judgment to the first interpretation [i.e., motion relative to the pole

34. This could occur in a northern latitude within about 5° of the equator, since the inclination of the Moon's circuit to the ecliptic is a little over 5°.

35. As with the "visible pole [of the ecliptic]," here "visible pole" of the Moon's motion in latitude for northern observers means the point in the heavens that is the north pole of the Moon's inclined orb.

of the ecliptic] without the second [i.e., motion relative to the zenith], not to clarify what is in the base text. There is no doubt this clarifies it, for it becomes manifest that there is a duration of the Moon passing from the zenith to the northern extreme [as it] approaches the visible pole of the ecliptic, and thus the judgment generalizes to the first explanation. And [in that duration the Moon is] not drawing closer to the zenith but rather retreating from it, so it does not generalize to the second explanation. And the excellence of his statement ‘at all times’ is clear as well.”

Then he considered his disciples who had studied Nīsābūrī’s commentary with him and said, “I am surprised we were not aware of this despite our having passed through this spot and its readings and interpretations many times!” Then he spoke kind words, saying, “They say that when ‘Izz al-Dīn al-Ḥelwānī was giving a lecture and a reader or listener raised an objection to a point in expectation of an answer, he approved, saying ‘Indeed, I read this book once and did not find an objection to this point.’”

Anecdote 4 - Optics and Astronomy³⁶

Ṭūsī stated that the illuminated and dark parts of the Moon are separated by a great circle, or nearly so. Shirwānī explained that, although according to our senses we perceive the illuminated part to be a full half of the Moon’s sphere, mathematical optics had demonstrated it to be slightly less, although Aristarchus showed that the ratio of the diameter of the illuminated circle to the diameter of the Moon itself was greater than 89/90.

Similarly, Ṭūsī stated “another great circle (or one nearly so) separates that which is apparent to the observer from that which the light of the eye does not reach.” According to Shirwānī, Ṭūsī’s reference (*iḥāla*) for this statement was Euclid’s *Optics*, Prop. 24: “When a sphere is viewed from any position, then that which is seen of it is less than a hemisphere, and that which is seen of it is enclosed by a circle.” Shirwānī argued that this was a better choice of

36. Shirwānī, *Sharḥ al-Tadhkira*, fol. 252b-253a.

reference than that chosen by Nīsābūrī and Jurjānī, who based their comments on Prop. 28 of Euclid's *Optics*, which demonstrates that if both eyes view a sphere and the distance between the eyes is less than the sphere's diameter, then less than half of the sphere will be seen. Propositions 26-28 deal with different cases of viewing a sphere when the distance between the eyes is equal, greater, or less than the sphere's diameter. The reason for preferring Prop. 24, according to Shirwānī, was that Ṭūsī had demonstrated flaws in these other propositions in his recension of Euclid's *Optics*.³⁷

Finally, Shirwānī noted that in the Arabic version of Ṭūsī's earlier Persian work *Zubdah-i hay'a*, Ṭūsī stated that the shared division between the illuminated and dark parts of the Moon was posited to be nearly a great circle because it was the base of the shadow cone. Similarly, the shared division between the visible and unseen parts was nearly a great circle, and the two circles are taken to be great circles because there is no sensible difference between each of them and a great circle. This was the best determination of the situation, according to Shirwānī (*wa hādha-l-taqrīr aḥsan*).

Ṭūsī went on to describe how these two circles intersect and overlap to varying degrees over the duration of the Moon's circuit around the Earth, producing the phases. The special cases of their coinciding at conjunction and opposition produced the new and full Moon, respectively. Apparently, at this point the class discussion referred to Jurjānī's commentary on the *Tadhkira*, for Ulugh Beg intervened to clarify a misunderstanding about whether the two circles truly coincided.³⁸

37. Euclid, *The Arabic Version of Euclid's Optics*, ed. and tr. Elaheh Kheirandish, (New York: Springer, 1999), 70-91.

38. For example, according to the Euclidean optical results mentioned above, since the Moon is closer to the Earth than the Sun, the "half" that we see at the full Moon (opposition) is very slightly smaller than the "half" that is illuminated. The more acute the visual angle subtending

When I was reading [this passage], the sultan, may God have mercy on him, said, “Do not say that verified coincidence [of the two circles] is inconceivable at opposition, for we say there is no difference between conjunction and opposition in terms of the coincidence of the two circles after they are posited to be great circles. For if one asks: How is it possible to conceive of approximate coincidence at conjunction and opposition? We say: Conjunction may be in such a way that the line extending from the eye to the Sun passes through the centers of the two luminaries, so in this case the coincidence is verified. [Or] it may not be in that way, so it would be approximate [coincidence]. Opposition may be in such a way that the line extending from the eye to the Moon coincides with the diameter of the ecliptic equator³⁹ whose endpoints are the positions of the two luminaries, so in this case the coincidence is verified. [Or] it may not be in that way, so it would be approximate [coincidence]. Thus the difference of their verified and approximate coincidences indeed relates to conjunction and opposition, not to the two circles taken to be great circles. And if they were not posited to be great circles, then this difference with respect to being verified or approximate would still occur.” The analysis of that philosopher [i.e., Ulugh Beg] showed that there was no need for what Jurjānī said following Shīrāzī’s *Tuhfa*, which took them to be small [circles].

Ulugh Beg’s comments suggest that two issues were being confused in the reading and discussion on that day. Whether the circle separating the light/dark surfaces of the Moon and the circle separating the observable/hidden surfaces of the Moon with respect to the Earth coincided was not just a question of their being great circles or not. It also depended on the relative configurations of the Earth, Moon, and Sun, since astronomers allowed the terms “conjunction”

the Moon, the greater the illuminated circle. Alternatively, the more acute the angle, the closer to 180 degrees is the angle between the normals to the tangents of the visual angle subtending the Moon, and the larger the visible circle (a maximum when the visual angle is infinitely acute).

39. Here, as elsewhere in the *Tadhkira* and Shirwānī’s commentary, the conceptualization of the models in terms of physical orbs meant that the authors referred to the Ptolemaic ecliptic circle as the “equator of the ecliptic orb (*minṭaqat falak al-burūj*)” or “ecliptic equator (*minṭaqat al-burūj*).”

and “opposition” to refer to configurations when the three bodies fell nearly along the same line, rather than limiting the definition to the precise moment of alignment. As the sultan pointed out, if the three were not all on the same line, then technically the circles would not coincide exactly, regardless of whether one assumed them to be great circles or not.⁴⁰

Discussion

Shirwānī included many more examples of interjections by Qāḍīzāde and Ulugh Beg during lectures, often in the form of shorter comments and clarifications to the other commentaries that were being read at the time, as well as to the *Tadhkira* itself. The long anecdotes translated here, along with the numerous shorter comments, offer precious insights into the activities undertaken in the Samarqand madrasa. It must be emphasized that these accounts are invaluable, for it is exceedingly difficult, in just about any culture, to find reliable historical information about what actually happened orally in the classroom or disputation hall. As Shirwānī mentioned in the episode about parallel lines, the preferred method of study was to work through a text slowly, reading it aloud section by section, with frequent pauses for discussion. The students were expected to continue studying the text on their own so they could raise questions or explain how a statement related to previous concepts. However, since hand copied manuscripts were time-consuming and expensive to produce, even a skilled and presumably well-off student such as Shirwānī may not have had complete personal copies of even basic texts, such as Euclid’s *Elements*, since he needed to borrow Qāḍīzāde’s copy to obtain the missing three books.

40. It should be noted that this issue of the Moon’s illuminated surface and visibility from Earth also depended crucially on the distances between the Sun, Moon, and observer, as well as the size of the Sun. The Sun is not a point source of light. Rather, illumination from points across the Sun’s body created a larger illuminated circle on the Moon than circle that would result if all of the Sun’s light came from a point source at the Sun’s center.

Equally important, Shirwānī's anecdotes also provide a model for how students should behave in the lecture hall and offer reflections of their values and reactions. After critical reading and even memorization, they should not be afraid to speak up with questions. On multiple occasions, Qāḍīzāde and Ulugh Beg praised Shirwānī for drawing attention to an unnoticed problem or for making a clarifying comment. The reactions from Shirwānī's classmates suggest that, in the competitive atmosphere of the classroom, this behavior was not always appreciated, especially by those who had misspoken, whether peers or lecturers. Nevertheless, the ideal was to arrive at a deeper understanding of the text through active discussion and debate. Based on their performance, students could ascend through the classes, which were divided into groups surrounding different teachers. Equally impressive is the class size of 40-50, which Shirwānī mentioned in passing: a large number, given the demands of the material.

Ulugh Beg, in particular, seems to have been interested in challenging the students and teachers by posing difficult questions that drew from his extensive knowledge of mathematics and the sciences. This action not only helped to draw attention to difficult aspects of the material, but could be used as a means of testing the qualifications of the teachers and advanced students. Shirwānī's descriptions of these interactions are consistent with those of al-Kāshī, the former director of Ulugh Beg's observatory, who recounted episodes in which the sultan requested solutions to mathematical puzzles or applications of astronomical theory. Those who attempted a solution, or who raised criticisms of a text during lecture, were expected to demonstrate their claims with geometric proofs or convincing arguments.⁴¹ The clever student was advised to prepare for such challenges, such as when Shirwānī commented on the configuration of orbs for

41. See Kennedy, "Letter of Jamshīd al-Kāshī" and Bagheri, "A Newly Found Letter of al-Kāshī on Scientific Life in Samarkand."

the planet Mars. As part of that discussion, Shirwānī highlighted the curious fact that Mars is closer to the Sun at opposition than at conjunction since its epicycle is much larger than the Sun's parecliptic, and said that one might ask this question as a test of someone's knowledge of astronomy.⁴² Not least, as a patron deeply engaged in learning, Ulugh Beg used the give-and-take of the classroom to select the students and scholars whose critical skills shone under pressure and whom he wished to promote and encourage. Here too one glimpses the way in which careers were made and unmade.

Ulugh Beg's influence and direct involvement in the classroom stands out especially in the second anecdote. His patronage of the Samarqand observatory and madrasa consisted of much more than financial support. Rather, his hands-on approach shaped the curriculum and experiences of scholars and students alike. After learning that the sultan would attend that day's lecture, Qāḍīzāde changed the lesson plan in order to focus on a problem (*ishkāl*). Ulugh Beg was not concerned with straightforward presentations of the astronomical models. Rather, his interest lay in the unresolved difficulties Ṭūsī and other astronomers had identified in the Ptolemaic models, only some of which Ṭūsī had been able to resolve. His regular participation in the lessons led to greater critical thinking on the part of the lecturers and encouraged students to join the debates.

The *ijāza* Qāḍīzāde granted to Shirwānī is noteworthy for the fact that it mentions a scientific text in addition to the "traditional" fields such as linguistics, theology, ḥadīth studies, or jurisprudence. Shirwānī inserted the *ijāza* after the first anecdote, which was an early display of competence in the mathematical sciences. The personal tone of the *ijāza* resembles a letter of recommendation rather than a diploma, with its references to Shirwānī's intellectual journey and

42. Shirwānī, *Sharḥ al-Tadhkira*, fol. 225a.

his brilliance in the classroom, along with an emphasis on personal connections (his father coming from Shirwān. It marks a general approval or satisfaction with Shirwānī's ability to teach specific texts, rather than the completion of a fixed curriculum. Furthermore, given our interest in the role of commentaries in Islamic intellectual history, it must be emphasized that all three of the texts mentioned by name in the *ijāza* were commentaries written within the last century and a half, and that a scientific text (Nīsābūrī's commentary on Ṭūsī's *Tadhkira*) preceded the texts on jurisprudence and kalām.

Pedagogical aspects of the commentary

Geometric proofs

Various features of Shirwānī's commentary on the *Tadhkira* reflect the kinds of activities described in his classroom anecdotes, and mark the commentary as a whole as a teaching text. One prominent pedagogical aspect of the commentary was the inclusion of detailed geometric proofs, whether they be proofs from the *Almagest* that Ṭūsī had intentionally left out, proofs from other commentators, or proofs that Shirwānī himself contrived and inserted into his text. It was one thing for Ṭūsī (or Ptolemy) to make a statement about how a particular combination of circular motions might produce the desired effect, but an entirely different matter to see, and understand, those motions explicitly represented in a geometric proof. By following through the steps of a proof and referring to the labeled diagrams that Shirwānī inserted alongside the text, the reader was able to develop a mental image of the motions and relations between models.

The development of such mental images, particularly for the most basic and fundamental components of the models, was a crucial prerequisite for moving on to the more complicated aspects of planetary theory later in the text. Indeed, even the modern reader of the *Almagest* or

Tadhkira is well-rewarded by devoting time to close attention to the geometric diagrams and proofs, rather than skimming over them and then attempting to understand how and why the models and supporting data were laid out in their particular sequence. Since these diagrams and proofs were largely left out of the *Tadhkira*, the few diagrams that remained were largely illustrations of the overall configuration of orbs for each celestial body, and difficult to understand without having previously worked through the individual components. For example, Ṭūsī's illustration of Mercury's orbs at the end of II.8 contains more than a dozen labeled points, parts, and angles.⁴³ However, by the time the reader of Shirwānī's commentary reached this illustration, he would have worked through demonstrations of how each piece functioned.

43. F. Jamil Ragep, *Naṣīr al-Dīn al-Ṭūsī's Memoir on astronomy = al-Tadhkira fī 'ilm al-hay'a*, 2 vols. (New York: Springer-Verlag, 1993), 174-175.

frequently represented by such prefixes, suffixes, and pronouns attached to the verb, rather than explicitly stated as separate nouns, especially when they have already been stated earlier in a paragraph. Since Shirwānī's comments were interwoven with the text of the *Tadhkira* itself, it was not uncommon for the pronoun and antecedent to be separated by several lines, sometimes even a page or more.⁴⁴ Therefore, to aid the reader, Shirwānī would often restate the antecedent noun of pronouns in the *Tadhkira* if there was any risk of misunderstanding.

Another basic form of clarifying remark was simply to point to information elsewhere. To remind the reader of a previously established result, Shirwānī would speak directly with general phrases like “as you know... (*kamā 'arafta...*)” or refer specifically to the location of the definition, explanation, or proof that had been presented earlier in the text or commentary. This is partly a matter of style, signaling to the reader that what is being discussed is not “new” information. However, in conjunction with the other rhetorical devices that appear in the commentary, one suspects Shirwānī's addressing the reader reflected the text's close connection to the teaching process. At other points in the commentary, especially when the discussion of an issue at hand began to touch on related topics, Shirwānī would allude to the fact that there was more to say but that this was not the appropriate time with phrases such as “as will come in... (*kamā sa-ya 'tī fī...*)” or “it will be clarified in... (*sa-yattaḍīḥ fī...*).”

More complex clarifications pointed out technicalities, highlighted significant consequences of a statement, or articulated explicitly assumptions that had been left unstated but were necessary for an argument to work. These types of clarifications were similar to the episodes Shirwānī described in his classroom anecdotes, where he impressed Qāḍīzāde and

44. The same situation necessarily occurs in my English translations – I have endeavored to clarify as best I can.

Ulugh Beg with his detailed knowledge of the premises and consequences of the topic at hand. For example, in *Tadhkira* II.13, when discussing eclipses and the cone of shadow cast by the Earth, Ṭūsī had stated “the center of the shadow cone is always on the [plane of the] ecliptic equator since the Sun is always on it and since the center of the Earth is the center [of this equator].”⁴⁵ There seems to have been disagreement about the definition of the “center” of the shadow cone. Shirwānī mentioned that in the *Tuhfa*, Shīrāzī said it was the center of the cone’s base, whereas Nīsābūrī interpreted it as the apex. Shirwānī went on to say that, in the *Nihāya*, Shīrāzī suggested that Ṭūsī’s statement would have been complete if the center of the Earth were the center of its shadow cone, but that is not the case in fact. Rather, it only seems so according to the senses. Shirwānī did not include the basis for Shīrāzī’s objection but this is likely what he meant: the base of the shadow cone does not pass through the center of the Earth because of the fact that the Sun is much larger than the Earth.⁴⁶ Shirwānī noted that Jurjānī accepted this suggestion and provided a different basis for the statement. However, Shirwānī argued that in fact Ṭūsī’s reasoning was complete even though the center of the Earth was not the center of the shadow cone (*wa-l-ḥaqq inna istidlāl al-muṣannif yatimm wa-in lam yakun markaz al-arḍ markaz makhrūṭ al-zill*). Nevertheless, he also admitted that the necessity of a particular inference did require an explanation. Namely, since the center of the earth is the center of the ecliptic, then the center of the cone is on the plane of the ecliptic as well, because the axis of the

45. Ragep, *Tadhkira*, 232.

46. This is a counterpart to the issue of the illuminated part of the Moon being slightly less than half according to geometric optics, as discussed in one of Shirwānī’s classroom anecdotes above.

“great cone” (of light from the Sun tangent to the surface of the earth) upon which the three centers [Sun, Earth, base of shadow cone] lie is in the plane of the ecliptic.⁴⁷

“Question & Answer”

Ranging from grammatical explanations to complicated discussions of astronomical principles, Shirwānī’s clarifying remarks served to aid the reader in understanding the text and stood in the place of a teacher, allowing students to follow the text and benefit from reading it on their own. Another rhetorical feature of the commentary served a similar function and even more explicitly recreated the classroom dynamic. Shirwānī frequently employed the pedagogical device of questions and answers (*masā’il wa-ajwiba*) in the form of “If you were to say...then I would say... (*in qulta...fa qultu...*),” to interrupt the commentary in order to answer relevant questions, clear up confusion, and resolve apparent contradictions. This topos was used across essentially all fields of knowledge in the Islamic scholarly tradition, whether as a form of dialectic and argumentation or, as in Shirwānī’s commentary, for the didactic presentation of knowledge.⁴⁸ In Shirwānī’s case, one assumes that many of the “questions” raised in the commentary reflect actual questions and difficulties that his students encountered in their studies of astronomy.

47. Shirwānī, *Sharḥ al-Tadhkira*, fol. 256a.

48. For an overview, see Hans Daiber, “Masā’il wa-Adjwiba,” in *The Encyclopaedia of Islam*, 2nd Edition, (Leiden: Brill), 6:636-639. Some historians have argued that Islamic disputations (especially in legal education) were the genesis of the *sic-et-non* and *summa* literature of medieval Europe, notably George Makdisi, “The Scholastic Method in Medieval Education: An Inquiry into Its Origins in Law and Theology,” *Speculum* 49, no. 4 (1974): 640-661 and Makdisi, *Rise of Colleges*. However, in the commentaries I have studied Shirwānī was not attempting to provide exhaustive treatments of multiple points of view, but rather provide a direct answer to a specific question that seemed likely to arrive at a particular point in the text.

Although most of the examples discussed above are complicated, it is important to remember that, at times, these questions could also be rather simple.⁴⁹ In *Tadhkira* II.6, the model for the Sun's motion can be thought of as based on two components, a "mean Sun" moving at a constant rate, and the "solar anomaly," a variable amount that is added to or subtracted from the mean to represent the fact that the true Sun does not move at a constant rate. Shirwānī explained that the Sun's true position is found by combining the position of the mean Sun with the angle of the solar anomaly. The true position will be less than the mean by the amount of the anomaly when the Sun is descending along the eccentric from the apogee toward the perigee; it will be equal to the mean at perigee and apogee; and finally, the true position will be greater than the mean by the amount of the anomaly on the ascending half of the eccentric. At this part of his commentary, Shirwānī paused to clarify a simple question about this calculation when it occurs around the vernal equinox, which was the 0° reference point for measuring celestial motion.

Question: The anomaly should be subtracted from the mean position when the Sun is descending, and added to it when the Sun is ascending, to obtain the true position. So if the first [point] of Aries [the vernal equinox] was at the midpoint of an anomaly of two degrees which is to be added, how does one handle the anomaly?

Answer: The mean in this case is a circuit less a degree (i.e., 359°), thus if the anomaly is added, then discard the circuit (360°) and the true position is the remainder, which is one degree.⁵⁰

This is a simple question if one is looking at an illustration of the model (i.e., just start over from 0 once you have moved all the way around the circle), but perhaps appropriate to ask

49. Granted, "simple" is a relative term when it comes to concepts in mathematical astronomy.

50. Shirwānī, *Sharḥ al-Tadhkira*, fol. 190b.

during a lecture to make sure the listeners understand the definitions of these crucial terms. It is a kind of consolidation question, which allows the instructor to check up on the coherence of the students' understanding. Other questions dealt with more complex situations, such as Mercury's retrograde motions. Recall that Mercury's retrograde arcs appear at different locations around the ecliptic, and vary in both duration and length. The question, which Shirwānī said he had asked of Qāḍīzāde, was: why was it not obvious that the largest retrograde arc would appear at Mercury's apogee, since that is the point at which the epicycle center moves the slowest? The answer reminded the reader that the size and duration of retrograde motions was due not merely to the motion of the epicycle. One needed to take into account the equant and the fact that the epicycle center's motion was fixed relative to the equant rather than the Earth. In the model for Mercury, therefore, the retrograde arc would occur during a different amount of time and through a different arc than would be predicted in a simple model that used only an epicycle on an eccentric-deferent.⁵¹

In a similar rhetorical move, Shirwānī often interjected with the pair of phrases “one should not say...for we say... (*lā yuqāl...li-annā naqūl*)” to clarify potential confusion or caution the reader against drawing erroneous conclusions and interpretations that might follow from a given topic. For example, in *Tadhkira* II.12, Ṭūsī was explaining how parallax affected the observation of the Moon and the planets closest to the Earth. He stated that a planet does not have parallax at the zenith because the two lines forming the angle of divergence⁵² are one and the same there. The divergence of that angle increases the closer it is to the horizon, with the

51. Shirwānī, *Sharḥ al-Tadhkira*, fol. 211a-b.

52. One line from the center of the Earth to the planet, and a second line from the observer on the Earth's surface to the planet.

maximum occurring at the planet's rising or setting. Shirwānī explained this point by analogy with what was determined in II.5. There, the issue concerned the maximum equation [of center] for the eccentric hypothesis occurring at the mean distances and then diminishing until it ends at the perigee. According to Shirwānī, "[Tūsī's statement] becomes clear if we imagine (*'idhā tawahhamnā*) the center of the Earth here as the center of the eccentric in that model, and the position of the observer as the center of the Earth [in the eccentric model], and the zenith point as the perigee, and the radius of the Earth as the eccentricity." However, he cautioned the reader:

One should not say: But at the true horizon [the angle of divergence] is equal to [the angle of divergence] at rising or setting [i.e., at the apparent horizon] because in both positions, the sine of the angle is the same, being the radius of the Earth.

For we say: At the true horizon, the sides [of the angle of divergence] are longer, so [the angle] is smaller there, although the divergence occurring below the apparent horizon should not be called parallax.⁵³

Shirwānī was warning against a potential misreading of the diagram associated with this part of the *Tadhkira*. At first glance, it *does* appear that the angles of divergence formed when the planet is at the apparent and true horizons are the same; indeed, they would be the same if the line between the points marking the apparent and true horizons were a straight line. However, those points are located along the circle representing an altitude circle on the ecliptic orb. Therefore, the hypotenuse of the triangle for the angle of divergence at the apparent horizon is the radius of the altitude circle, whereas the hypotenuse of the triangle for the angle of divergence at the true horizon is the distance from the observer to the true horizon, which is longer than the radius of the altitude circle. Therefore, the sines of the two angles are in fact different, and the angle at the

53. Shirwānī, *Sharḥ al-Tadhkira*, fol. 249a.

true horizon is smaller.⁵⁴

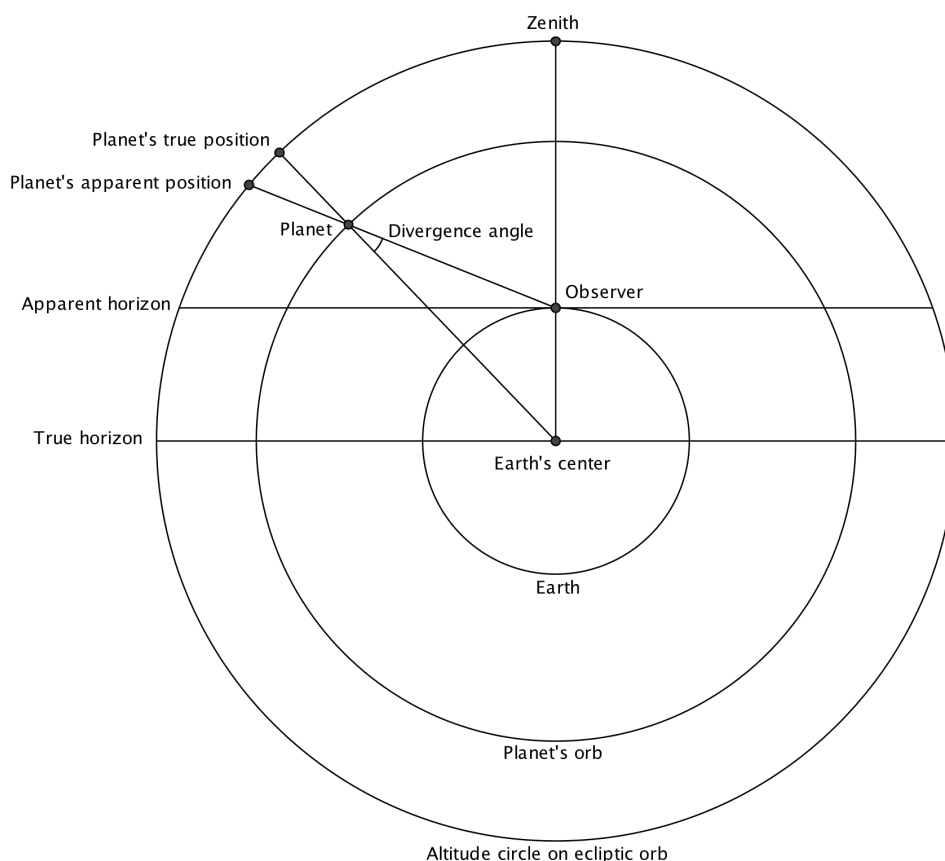


Figure 4.2 Planetary Parallax

Shirwānī's use of clarifying remarks, "question and answer," and other phrases to frame information and guide the reader through the text likely reflects the proximity of the text to the teaching process. He had studied two *Tadhkira* commentaries (Jurjānī, Nīsābūrī) incredibly closely by the time he left Samarqand, and completed his own commentary towards the end of a

54. According to the simple trigonometric definition, the sine of any angle in a right triangle is the ratio between the side opposite the angle and the hypotenuse. Since the opposite side for both of these angles of divergence is the radius of the earth, the sine of the angle at the true horizon will be smaller.

career as a teacher. Lecture notes compiled over decades of encounters with students no doubt formed the basis for much of his commentary. Recurring questions, false conclusions, and classroom debates over how best to interpret particularly troublesome passages would become natural candidates for inclusion in a comprehensive commentary and analysis of the text.

Supplementary & practical material

The final aspect of Shirwānī's commentary on the *Tadhkira* that distinguished it from other, more theoretical texts was the inclusion of various supplementary materials, sometimes in the form of practical information, sometimes simply for the edification of the reader. These discussions connected astronomy to related disciplines, or demonstrated for the reader how knowledge from the *Tadhkira* could be put to use.

Optics

One of the larger sections of supplementary information that Shirwānī added was an appendix on optics. This section, which will be discussed in detail in the following chapter, drew on Ibn al-Haytham's revolutionary work in optics as expounded in his *Kitāb al-Manāẓir (Optics)* and revised by Kamāl al-Dīn al-Fārisī. Shirwānī presented his discussion of optics at the end of his commentary on *Tadhkira* I, as an additional body of knowledge alongside geometry and natural philosophy from which astronomers drew established conclusions. By selectively discussing specific topics from the *Optics* and rearranging them in a manner more conducive to independent study, Shirwānī provided a manageable introduction to this discipline and highlighted specific principles and results that were especially relevant to making astronomical observations. It is one of the paradoxes of transmission and diffusion that Ibn al-Haytham's work was little-known in

the Islamic world at that time. Shirwānī's discussion of optical theories thus served to diffuse this material and also to correct erroneous statements in other texts that students would be reading at the time, such as Ṭūsī's recension of Euclid's *Optics*.

Knowledge of the fixed stars

The second major section that Shirwānī added as supplementary information concerned the stars and constellations. In *Tadhkira* II.4, Ṭūsī stated that the fixed stars “could not possibly be enumerated (*lā yumkin an tuḥṣā kathra^{tan}*),” but 1022 had been observed and their coordinates recorded.⁵⁵ He explained that astronomers imagined shapes in the heavens (constellations) that the stars were either on or near, so that the constellations could be used as a way of referring to the location of specific stars. For example, one might refer to a particular star as being on the head of this constellation or near the hand of another. Ṭūsī referred to 48 major constellations, listing the names of all but the 12 famous constellations of the zodiac. Shortly thereafter, he concluded the section with the statement “knowledge of the fixed stars and that which concerns them being a separate discipline, it is best that we confine ourselves to just what has been presented.”⁵⁶ However, for this section Shirwānī devoted over 50 folios to a comprehensive listing of all 48 major constellations and their fixed stars.⁵⁷ He believed that this information would be beneficial to his readers:

55. Ptolemy compiled a catalogue of 1022 fixed stars, arranged into or around constellations, in Books VII and VIII of the *Almagest*. See G.J. Toomer, *Ptolemy's Almagest* (Princeton: Princeton University Press, 1998), and C.H.F. Peters and Edward Knobel, *Ptolemy's Catalogue of Stars; a Revision of the Almagest* (Washington, D.C.: Carnegie Institute of Washington, 1915).

56. Ragep, *Tadhkira*, 128.

57. Shirwānī, *Sharḥ al-Tadhkira*, fol. 100b-166a.

Know that the author [Ṭūsī] limited the explanation of the 48 constellations to a brief summary because knowledge of the fixed stars and that which concerns them, despite its being part of this discipline, is almost a separate discipline like ophthalmology (*‘ilm al-kuḥl*) in medicine or inheritance (*al-farā’iḍ*) in the law. However, from this [discipline] we also mention a goodly amount from the tongue of its master and shaykh, Abu al-Ḥusayn al-Ṣūfī [d. 986], so that the reader of this commentary of ours may benefit from his great effort. We mention the number of stars for each constellation, their names, their titles according to the systems of the astronomers (*al-munajjimīn*) and the Arabs, and their magnitudes.⁵⁸

The subsequent lists and descriptions were taken from Ṣūfī’s *Book of the Fixed Stars*, which was itself a rendering of Ptolemy’s data that had been revised with new observations and updated to correct for the precession of the stars during the centuries separating Ṣūfī from Ptolemy.⁵⁹

However, Shirwānī did not include the tables of coordinates, only the verbal descriptions of the positions of the stars within each constellation.

Referring back to Ṭūsī’s statement that knowledge of the fixed stars was a separate discipline, Shirwānī ended the section with an overview of how to visualize the constellations and stars with the aid of a celestial globe:⁶⁰

58. Shirwānī, *Sharḥ al-Tadhkira*, fol. 112b-113a.

59. On Ṣūfī’s star catalogue, see Paul Kunitzsch, “The Astronomer Abū ’l-Husayn al-Ṣūfī and his Book on the Constellations,” *Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften* 3, (1986): 56-81, E. S. Kennedy, “al-Ṣūfī on the Celestial Globe,” *Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften* 5, (1989): 48-93, Paul Kunitzsch, “The Astronomer al-Ṣūfī as a Source for Ulugh Beg’s Star Catalogue (1437),” in *La science dans le monde iranien à l’époque islamique*, ed. Ž. Vesel et al., (Tehran: Institut Français de Recherche en Iran, 2004), 41-47, and Ihsan Hafez, “‘Abd al-Raḥmān al-Ṣūfī and His Book of the Fixed Stars: A Journey of Re-discovery” (PhD diss., James Cook University, 2010).

60. Recall that Ṭūsī’s stated goal was to present a summary of the configuration (*hay’a*) of physical models accounting for the motions of the heavens. Hence, he avoided going into detail about other subfields of astronomy or related disciplines whenever possible.

Whereas in the books of this discipline, both in texts and commentaries, we saw them limit the model explaining that which concerns the fixed stars, and avoid an investigation of that book which is precious due to its inclusion of difficult stellar depictions, [so] in this our commentary we present a summary of its contents, as you know, lest the reader be in the darkness about what they avoided. It is distinguished from the investigations in other books of this discipline by additional demonstrations of their conditions. It should be known that Ptolemy, after duly completing the description of the 48 constellations and the Milky Way, set forth a chapter on constructing a sphere designed to resemble the heavens, with the fixed stars and Milky Way drawn upon it in order to know their shapes and relative positions. This is the sphere that al-Šūfī mentioned when he accused Ptolemy of errors in the longitude and latitude of some of the stars of a constellation, or in one of them [longitude or latitude], due to its violating the arrangement of the heavens when placed on the sphere.⁶¹ So we saw that mentioning [this sphere] in our commentary would fulfill a need.⁶²

Shirwānī's description of how to construct a celestial globe was taken directly from the *Almagest*, where in Book VII Ptolemy gave instructions for turning a solid sphere into a representation of the night sky.⁶³ One took a sphere and colored it dark blue to represent the night

61. That is to say, using Ptolemy's coordinates for placing these stars on a celestial globe did not produce the same constellation image that we observe in the heavens. Throughout his book, Šūfī criticized the Ptolemaic coordinates as well as transcription/copying errors in translations of the *Almagest* and related star catalogues. Šūfī also criticized contemporary makers of celestial globes for not verifying that their correction for precession matched what was actually seen in the heavens. See the English translation of *Book of the Fixed Stars* in Hafez, "al-Šūfī and His Book of the Fixed Stars", as well as Emilie Savage-Smith and Andrea Belloli, *Islamicate Celestial Globes: Their History, Construction, and Use*, Smithsonian Studies in History and Technology, vol. 46 (Washington, D.C.: Smithsonian Institution Press, 1985), 22-23; Kunitzsch, "The Astronomer Abū 'l-Husayn al-Šūfī and his Book on the Constellations"; and Kennedy, "al-Šūfī on the Celestial Globe".

62. Shirwānī, *Sharḥ al-Tadhkira*, fol. 165a-b.

63. Shirwānī, *Sharḥ al-Tadhkira*, fol. 165b-166a. Toomer, *Ptolemy's Almagest*, 404-407.

sky and so the markings for the stars could be seen clearly. Next, one marked two diametrically opposite points to represent the poles of the ecliptic. A great circle perpendicular to a great circle passing through the poles represented the ecliptic itself. Ptolemy then instructed the globe maker to attach two intersecting semicircular rings, marked with fine divisions, to the globe for use in marking off equatorial and ecliptic coordinates. With the rings, and the data from the star catalog, one could proceed around the globe, marking off the stars with points of different sizes corresponding to their magnitudes, and perhaps with different colors, as appropriate. The shapes of the constellations could also be drawn in, faintly, so they would be seen but not obscure the stars themselves.

Timekeeping and calendars

As he had with the topic of the fixed stars, Ṭūsī declared at the end of III.10 (on the calculation of hours, days, months, and years according to celestial phenomena) that “every people has an epoch to which they refer the years of their history; understanding the details of that does not pertain to this science.”⁶⁴ Shirwānī replied, “however, nothing keeps the commentator from examining its details when the author [merely] examined it in general,”⁶⁵ taking this opportunity to supplement his commentary with an overview of the commonly used calendar systems of the premodern world.⁶⁶ Despite its being more recent than the others, Shirwānī discussed the

64. Ragep, *Tadhkira*, 302.

65. Shirwānī, *Sharḥ al-Tadhkira*, fol. 328a.

66. For an insightful discussion of calendars and their relation to politics, science, and religion in premodern Abrahamic societies, see Part I of Barbara Stowasser, *The Day Begins At Sunset: Perceptions of Time in the Islamic World* (New York: I. B. Tauris, 2014), and particularly for calendars in Antiquity see Sacha Stern, *Calendars in Antiquity: Empires, States, and Societies* (Oxford: Oxford University Press, 2012).

Islamic/*hijrī* calendar first due to it being “more noble and famous.” Shirwānī then summarized the Roman/Alexandrian, Persian/Zoroastrian, Ilkhānid, Jewish, and “Chinese-Uighur” Turkish⁶⁷ calendars. For each, he explained how they came to be, the event marking their starting dates, whether their months and years were solar or lunar, and how they accounted for the fractional day or month(s) to keep the calendar in alignment with the cycles of the Sun and/or Moon. He supplied various tables laying out the names of the months in different calendar systems and the number of days each contained, and mentioned that this knowledge was indeed relevant for astronomers and astrologers. For example, if one knew that a particular calendar system defined the day to begin at sunrise, rather than at sunset, celestial observations collected according to that system would need to be adjusted before they could be used as data for astronomical tables and models.

Method of determining the *qibla*

At the end of Book III, Ṭūsī had provided a simple way, given the local longitude, to determine the *qibla* (the direction toward Mecca that the faithful had to face while praying) for any location. It was based on the fact that the Sun transits the zenith of Mecca (i.e., passes directly overhead at noon) on two specific days each year (according to Ṭūsī, they are when the Sun is in 8° of Gemini and 23° of Cancer). The difference in longitude between Mecca and the given location is converted into a time difference, so that one could make an observation at the local time corresponding to Mecca’s noon on one of those days. At that moment, the shadow cast by a gnomon will point in the direction of Mecca. As Ragep pointed out, this method was simple and

67. See E. S. Kennedy, “The Chinese-Uighur Calendar as Described in the Islamic Sources,” *Isis* 55, no. 4 (1964): 435-443 and Yoichi Isahaya, “The Tārīkh-i Qitā in the Zīj-i Ilkhānī: The Chinese calendar in Persian,” *SCIAMVS* 14, (2013): 149-258.

theoretically exact.⁶⁸ However, there were only two chances each year to make the observation. More importantly, neither geographic coordinates nor methods of timekeeping were very precise, and small errors in the time of observation could lead to inaccurate measurements of the *qibla* direction compared to the more elaborate mathematical techniques that had already been developed. Nevertheless, the method provided a relatively accurate means of determining the *qibla* and was both simple to explain and put into practice.⁶⁹

In his comments on this section, Shirwānī supplied two additional methods for determining the *qibla* using an astrolabe instead of simple shadow observations.

1. Use the astrolabe and the difference in longitudes between Mecca and the observer to calculate the local position of the Sun when it will be noon in Mecca, and then observe when the Sun reaches that degree in the sky. At that moment, the shadow will point in the direction of the *qibla*. As Shirwānī mentioned, this method was approximately the same as the one Ṭūsī described, applying only on the two days when the Sun was at Mecca's zenith, but it had the benefit of using observations of the Sun rather than relying on a (potentially inaccurate) timekeeping device.

2. The second method involved using the differences in longitude and latitude between any two locations to construct a line that pointed in the *qibla* direction. Starting with a circle, mark off two perpendicular diameters that will represent the cardinal directions and mark 360° around the circumference.⁷⁰ Starting from either the north or south point, as appropriate, mark off

68. Ragep, *Tadhkira*, 306-308, 497-499.

69. For more on the *qibla*, see David King, *World-Maps for Finding the Direction and Distance to Mecca: Innovation and Tradition in Islamic Science* (Boston: Brill, 1999).

70. In practice, the method only used the quarter-circle corresponding to the general direction of Mecca.

the degree point on the circumference representing the difference in longitude: to the west if the longitude of Mecca is smaller; to the east, if it is larger. Then, starting from either the east or west point, as appropriate, mark off the degree point on the circumference representing the difference in latitude: to the south, if Mecca's latitude is smaller; and to the north, if it is larger. From the first point, draw a straight line parallel to the meridian line (north-south) and from the second point a line parallel to the east-west line, they will necessarily intersect. Draw a straight line between the center [of the circle] and the intersecting point, extending to the circumference, this is the line of the *qibla* direction.

To further explain this second procedure, Shirwānī provided a worked example using the coordinates of his hometown in the region of Shirwān: "The longitude of our city Shamākhī, may God most High preserve it in the justice of its ruler (*baldatnā Shamākhī, ṣānahā Allāh ta'ālā fī 'aḍl wālīhā*), from the Fortunate Islands⁷¹ is 84;30° and its latitude is 40;50°." The difference between its longitude and that of Mecca is 7;17° since the minutes of Mecca's longitude are verified as 13'.⁷² The difference between their latitudes is 19;10°. So on the circumference of the circle, starting from the south point, we count off toward the west the amount of difference in their longitudes. And from the east point toward the south, [we count off] the amount of the

71. Ptolemy chose the Fortunate Islands (usually believed to be the Canary Islands) as the 0° meridian, but other cartographers and astronomers measured longitude from the "coast of the western sea" 10° east of the Fortunate Islands. Thus, it was important to specify your reference point when relating longitudinal coordinates. See Gerald Tibbetts, "The Beginnings of a Cartographic Tradition," in *History of Cartography*, Volume 2, Book 1: *Cartography in the Traditional Islamic and South Asian Societies*, ed. J. B. Harley and David Woodward, (Chicago: University of Chicago Press, 1992), and E. S. Kennedy and M. H. Kennedy, *Geographical Coordinates of Localities From Islamic Sources* (Frankfurt: Institut für Geschichte der Arabisch-Islamischen Wissenschaften, 1987), xi.

72. Tūsī had previously given Mecca's longitude as 77 1/6° [77;10°] from the Fortunate Islands and 67 1/6° [67;10°] from the coast of the western sea, which Shirwānī said was "approximate." Hence, here Shirwānī refers to a more accurate longitude of 77;13°.

differences in their latitudes. From the end of the first count, we extend a line parallel to the meridian, and from the end of the second, we count a line parallel to the east-west line. We connect the center and the point of their intersection with a straight line crossing the circumference. Thus its *qibla* [i.e., Shamākī's *qibla*] is determined." The two-page spread following this worked example of the *qibla* for Shamākī contains an illustration of the *qibla* construction, a large circle with the appropriate degree markings and lines representing the differences in longitude and latitude.

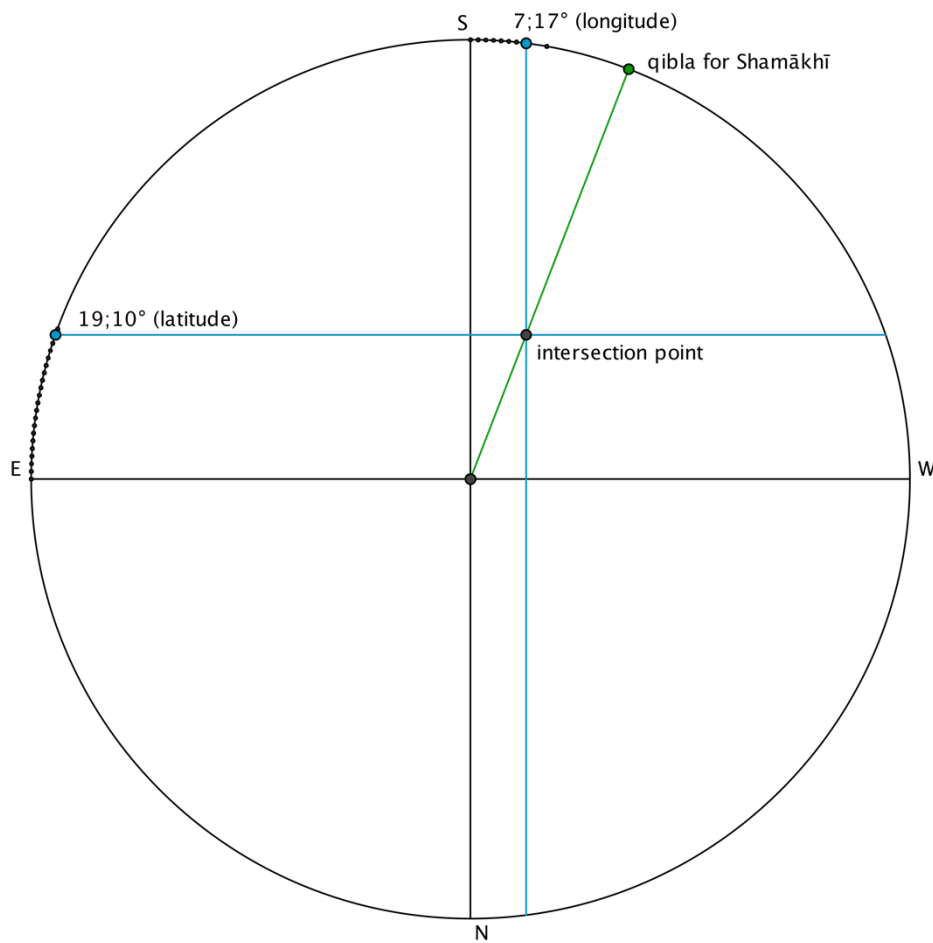


Figure 4.3 Shirwānī's determination of the *qibla* for Shamākī

Conclusion

As a whole, the features of Shirwānī's commentary analyzed in this chapter served to recreate key elements of the classroom experience: interacting with a teacher and fellow students, supplying theoretical, practical, and supplementary information all within the confines of a single text.⁷³ In doing so, the commentary replicated the oral and personal transmission of knowledge commonly found in both madrasas and study circles. The striking anecdotes and personal observations Shirwānī recounted for his readers set the scene for reading the rest of the commentary in a certain frame of mind. Like its author, the reader should strive to understand the text in all its complexity, paying attention to technical issues, illustrative examples, and hidden assumptions, as well as considering potential conclusions, generalizations, and consequences. Any of these could easily become the subject of an impromptu interrogation, or could be deployed in a disputation as a show of one's command of the material. In short, the frame of mind was one of critical engagement. The commentary raised no expectation that it represented the final word on any of the subjects discussed. On the contrary, it noted that previous commentators had been incomplete, and sometimes wrong. The implication seemed to be that the reader should emulate this approach.

Shirwānī's anecdotes reinforce the sense of education as a personal, participatory experience, as described by Berkey, Chamberlain, and Ephrat. Beyond these specific anecdotes, Shirwānī's presentation of what he heard in Samarqand in the form of quotations from Qāḍīzāde and Ulugh Beg was identical to his presentation of written sources. Throughout his commentary,

73. William Smyth has made similar points about commentaries on rhetoric and logic in William Smyth, "The Making of a Textbook," *Studia Islamica* 78, (1993): 99-115 and William Smyth, "Controversy in a Tradition of Commentary: The Academic Legacy of al-Sakkākī's *Miftāḥ Al-'Ulūm*," *Journal of the American Oriental Society* 112, no. 4 (1992): 589-597.

phrases such as “I heard from Qāḏīzāde” or “Ulugh Beg said” introduce these quotations of direct speech, just as the phrase “Nīsābūrī said” introduces a quotation from a written text. Whether in person or mediated by texts, orality was central to the process of knowledge transmission. The texts were read aloud, and the process of investigation and thinking through the text took place via discussion and debate in the classroom.

Thanks to the various pedagogical devices identified in this chapter, Shirwānī’s commentary stood in the place of an instructor. Geometric diagrams and step-by-step proofs helped the reader understand how the various components of the planetary models functioned separately, and eased the way to being able to visualize how their combined motions interacted in the final configuration of orbs. Shirwānī’s frequent use of clarifying comments and the rhetorical device of “question and answer” alerted the reader to difficult topics, cautioned against misunderstandings, and allowed the reader to benefit from Shirwānī’s knowledge and years of teaching experience. Moreover, supplementary material on related disciplines that Ṭūsī had chosen to leave out of his concise overview of theoretical astronomy showed the reader how astronomical knowledge could be utilized to practical effect. In short, far from being merely a dry rehearsal of topics addressed many times before, Shirwānī’s commentary sheds unanticipated light not only on the intellectual life of the classroom, but also on its social life and values.

Chapter 5 – Rounding out the mathematical sciences: Shirwānī on optics, geography, and scientific knowledge

Introduction

Both of Shirwānī's astronomical commentaries contain large sections devoted to a related field, optics in one case and geography in the other. In the first section of this chapter, I present Shirwānī's appendix on optics, which represents a rare example of an Islamic scholar demonstrating a profound understanding of Ibn al-Haytham's landmark discoveries in the centuries after his death. It points out how a correct understanding of light and human vision could reveal practical challenges to conducting astronomical observations, such as accurately measuring the magnitudes of distant objects or understanding how refraction through the atmosphere affected attempts to observe the heavens. His selection of topics for further elaboration also provides a glimpse of the most important and difficult concepts a student needed to understand at the intermediate level of study. The second section of this chapter analyzes Shirwānī's extended discussion of the cities and regions of the inhabited world, often including geographic coordinates of localities. I demonstrate that the traditions of Arabic and Persian geographic texts continued into fifteenth-century Anatolia and suggest that post-classical commentaries provide a way to trace routes of transmission and influence on the early Ottoman scientific tradition. Moreover, at various points in his commentaries, Shirwānī addressed topics that touched on astronomy, natural philosophy, and theology. In the third section of this chapter I explore some of these issues in an effort to understand how a practicing astronomer and trained

theologian situated his work within the context of rival traditions and competing claims about the proper way to obtain true knowledge about God's creation.

Ibn al-Haytham's *Optics* in Shirwānī's Commentary on the *Tadhkira*

In addition to his astronomical learning, Shirwānī is a significant figure in the history of optics in Islamic societies. For reasons that remain unclear, Ibn al-Haytham's revolutionary explanation of light and visual perception in the 11th century had a split legacy in the Latin and Arabic traditions.¹ His *Optics* (*Kitāb al-Manāẓir*) was translated into Latin near the end of the 12th century, and was an authoritative text for generations of scholars in the Latin *perspectiva* tradition.² It remained unsurpassed until Kepler's analysis of the retinal image in the early 17th century. However, Ibn al-Haytham seems to have had very little impact on optics in the Arabic-speaking world for centuries following his death. It was not until the late 13th century that

1. On the "revolutionary" aspect of Ibn al-Haytham's work, see A. I. Sabra, "Ibn al-Haytham's Revolutionary Project in Optics: The Achievement and the Obstacle," in *The Enterprise of Science in Islam: New Perspectives*, ed. J. Hogendijk and A. I. Sabra, (Cambridge: MIT Press, 2003), 85-118.

2. A. Mark Smith has produced critical editions of the Latin version of the *Optics* with English translation in *Transactions of the American Philosophical Society* from 2001-2010: A. Mark Smith, "Alhacen's Theory of Visual Perception: A Critical Edition, with English Translation and Commentary, of the First Three Books of Alhacen's *De Aspectibus*, the Medieval Latin Version of Ibn al-Haytham's *Kitāb al-Manāẓir*," *Transactions of the American Philosophical Society* 91, no. 4-5 (2001): 1-819; A. Mark Smith, "Alhacen on the Principles of Reflection: A Critical Edition, with English Translation and Commentary, of Books 4 and 5 of Alhacen's *De Aspectibus*, the Medieval Latin Version of Ibn al-Haytham's *Kitāb al-Manāẓir*," *Transactions of the American Philosophical Society* 96, no. 2 (2006): 1-697; A. Mark Smith, "Alhacen on Image-Formation and Distortion in Mirrors: A Critical Edition, with English Translation and Commentary, of Book 6 of Alhacen's *De Aspectibus*, the Medieval Latin Version of Ibn al-Haytham's *Kitāb al-Manāẓir*," *Transactions of the American Philosophical Society* 98, no. 1 (2008): 1-393; and A. Mark Smith, "Alhacen on Refraction: A Critical Edition, with English Translation and Commentary, of Book 7 of Alhacen's *De Aspectibus*, the Medieval Latin Version of Ibn al-Haytham's *Kitāb al-Manāẓir*," *Transactions of the American Philosophical Society* 100, no. 3 (2010): 1-550.

Kamāl al-Dīn al-Fārisī, student of Qutb al-Dīn al-Shīrāzī, wrote a detailed commentary and analysis of Ibn al-Haytham's *Optics*.³ Fārisī's *Tanqīḥ al-Manāẓir*, along with additional treatises by Ibn al-Haytham, formed the basis for a large appendix (*tadhniḥ*) on optics, light, and vision in Shirwānī's *Sharḥ al-Tadhkira*. Until the modern edition of Ibn al-Haytham's *Optics* in the 20th century, the *Tanqīḥ* itself and the occasional discussion (such as that found in Shirwānī's *Sharḥ*) represent some of the only premodern Arabic-language analyses of Ibn al-Haytham's discoveries.⁴ In short, Shirwānī's interaction with Ibn al-Haytham's work is a milestone in the history of Arabic optics. Therefore, Shirwānī's appendix on optics is essential for both an understanding of the transmission of optics in the Islamic context as well as a precondition for any sound attempt at comparing the Islamic tradition of optics with the *perspectiva* tradition of the Latin West.

Ibn al-Haytham's *Optics* is a massive text comprising 7 books that include lengthy discussions about the nature of light and color, the physiology of the eye, the physical and psychological processes of visual perception, reflection and images seen via different reflecting

3. For an overview of the development of Islamic optics up to Ibn al-Haytham and al-Fārisī, see Elaheh Kheirandish, "The Mixed Mathematical Sciences: Optics and Mechanics in the Islamic Middle Ages," in *The Cambridge History of Science*, Volume 2: *Medieval Science*, ed. David Lindberg and Michael Shank, (New York: Cambridge University Press, 2013), The fate of Ibn al-Haytham's *Optics* is outlined in A. I. Sabra, "The 'Commentary' that Saved the Text: The Hazardous Journey of Ibn al-Haytham's Arabic *Optics*," *Early Science and Medicine* 12, (2007): 117-133.

4. Bilal Ibrahim has recently argued that Fakhr al-Dīn al-Rāzī (d. 1210) was aware of Ibn al-Haytham's *Optics*, drawing on concepts from Book II to criticize Ibn Sīnā's reliance on the knowledge of essences in his theory of demonstrative science. Instead, Rāzī proposed a methodology based on a theory of phenomenal knowledge such as that obtained by the mind through inferences from the point-form image of light and color sensed by the eye. See Bilal Ibrahim, "Freeing Philosophy From Metaphysics: Fakhr Al-Dīn Al-Rāzī's Philosophical Approach to the Study of Natural Phenomena" (PhD diss., McGill University, 2013) and Bilal Ibrahim, "Fakhr ad-Dīn ar-Rāzī, Ibn al-Haytham and Aristotelian Science: Essentialism versus Phenomenalism in Post-Classical Islamic Thought," *Oriens* 41, (2013): 379-431.

surfaces, and the phenomena of refraction and its role in vision. Since the majority of Shirwānī's quotations from Ibn al-Haytham derive from Fārisī's *Tanqīh*, it seems unlikely that Shirwānī had access to a copy of Ibn al-Haytham's *Optics* itself.⁵ However, he also quotes Ibn al-Haytham's *On Seeing the Stars*, a treatise not present in the *Tanqīh*.⁶ A critical analysis of Shirwānī's appendix on optics and its specific relationships with the works of Ibn al-Haytham and Fārisī is beyond the scope of this chapter. Instead, building on the previous chapter's argument that Shirwānī intended his commentary on the *Tadhkira* to be a practical teaching text, we will provide an overview and analysis of the appendix's contents that suggests Shirwānī added it for pedagogical reasons.

Shirwānī placed the optical appendix (*tadhnīb*) early in his commentary on the *Tadhkira*, before any discussion of celestial bodies or elements, as a supplement to Book I on the principles of astronomy taken from geometry (I.1) and natural philosophy (I.2). Shirwānī viewed optics as an ancillary to the practice of astronomy. In language echoing Ṭūsī's introductions of geometry and natural philosophy, he stated “know that in this science [i.e., *hay'a*] one may require knowledge (*i'lam qad yuḥtāj fi hādihā al-'ilm ilā ma'arifat...*)” of a list of multiple topics and principles from the science of optics.⁷ Shirwānī's quotations from Ibn al-Haytham and Fārisī

5. In comparing the texts, I relied on Ibn al-Haytham, *The Optics of Ibn al-Haytham: Books I-III: On Direct Vision*, ed. and tr. A. I. Sabra, 2 vols., (London: Warburg Institute, 1989); Ibn al-Haytham, *The Optics of Ibn al-Haytham: Books IV-V On Reflection, and Image Seen by Reflection*, ed. A. I. Sabra, 2 vols. (Kuwait: National Council for Culture, Arts and Letters, 2002); Süleymaniye Fatih MS 3216 (*Optics*, Book VII), copied in 1084; and al-Fārisī's *Tanqīh al-Manāẓir*, Ahmet III MS 3340, copied 1316 CE.

6. A. I. Sabra and Anton Heinen, “On Seeing the Stars: Edition and Translation of Ibn al-Haytham's *Risāla fi Ru'yat al-kawākib*,” *Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften* 7, (1991-92): 31-72.

7. Faḥḥallāh al-Shirwānī, *Sharḥ al-Tadhkira*, Topkapı Ahmet III MS 3314, fol. 31b.

range throughout the texts, indicating that he actively reorganized the content to convey a clear summary. Even at 55 pages (almost as long as the previous sections on geometry and natural philosophy together), Shirwānī's appendix could not hope to provide even a cursory overview of all this material. Examining what he chose to include or omit therefore offers insight into the issues that he considered most important and necessary for the study of astronomy. He stated, "I elaborate on [the aforementioned topics] and related issues (*wa-absuṭu al-kalām fīhā wa fī ba'ḍ mā yata'allaq bihā*)" such that the reader "need not refer to the *Optics* (*tastaghni bihā 'an al-rujū' ilā al-Manāẓir*)."⁸ Shirwānī recognized that awareness of optical theories was crucial for the practice of astronomy. At the same time, he understood that the prevailing treatments of vision, namely the geometric analyses of Euclid or al-Kindī or the natural philosophical approach taken by Aristotle and Ibn Sīnā, were flawed. Fortunately, Ibn al-Haytham had provided a way to integrate the strengths of these competing views into a single unified theory.

The topics Shirwānī chose to discuss point to three overarching goals or reasons for including this optical material in a work of astronomy. The first was to explain Ibn al-Haytham's theory of vision as a combination of principles taken from the natural philosophers, the techniques of geometric analysis for rays of light, and the Galenic description of the eye's physiology [sections 1-3, 8]. Shirwānī's second goal was to differentiate between the phenomena of reflection and refraction [sections 4-7], which were confused or used interchangeably in many ancient and Islamic discussions of optics. His final goal was to demonstrate through detailed mathematical proofs how the principles of optics applied to specific issues in astronomy, such as the use of refraction in explaining the apparent difference in the magnitudes and relative

8. Shirwānī, *Sharḥ al-Tadhkira*, fol. 31b. The text is supplemented by many detailed figures. Although most are taken from Ibn al-Haytham or Fārisī, some seem to be original as they are not found in either.

distances of celestial objects at the horizon compared to the zenith, or in analyzing shadows based on the relative sizes of the luminous and opaque bodies [sections 9-11].

Shirwānī chose to begin his discussion of optics with an overview of the eye itself and the nerves connecting the eyes to the brain. In this section, he followed the traditional medical/Galenic physiological analysis of the eye's seven "tunics" and three "humors," describing their location in the eye and function in the process of visual perception. In particular, he emphasized the crystalline humor (*al-ruṭūba al-jalīdiya*) as the sensitive organ at the center of the eye, which receives the rays and senses the visible object.⁹

These sections of the appendix contain a lengthy discussion of the concept of "ray" and the "cone of vision" or "cone of visual rays" and a proper understanding of the process of visual perception. As with astronomy, the study of optics in the ancient and medieval world depended on several bodies of knowledge and should not be viewed exclusively as a mathematical discipline. Islamic scholars inherited from the Hellenistic traditions diverse opinions about the nature of light and the process of vision that drew on physical and mathematical principles. Generally speaking, opinions about the ray and process of vision could be grouped into three major traditions, some of which Shirwānī discussed and criticized in these sections of the appendix. Indeed, he began this section with a quotation from Shīrāzī's *Nihāya* in which Shīrāzī drew parallels between rays of light emitted by luminous objects and visual rays emitted by the eye.¹⁰ Shirwānī drew on Ibn al-Haytham's *Discourse on Light*, which criticized the erroneous views of the "mathematicians" (*aṣḥāb al-ta'ālīm*) and "philosophers" (*aṣḥāb al-ṭabī'a*), as well

9. Shirwānī, *Sharḥ al-Tadhkira*, fol. 31b-33b.

10. Shirwānī, *Sharḥ al-Tadhkira*, fol. 33b-34a.

as drawing on the *Optics* itself.¹¹ His first definition of “ray” was the light that is transmitted in transparent bodies along straight lines, and these straight lines are imaginary, not sensible. At this point, he did not indicate which direction the ray travels – it could be either emanating from the eye or entering into the eye.¹²

According to Ibn al-Haytham, the mathematicians applied the term “ray” to the rays of sight (*shu‘ā‘ al-baṣar*) by analogy with the ray of the sun or of fire, since early mathematicians believed that vision took place by means of a ray that emerged from the eye to the visible object, and that the ray is a luminous power of the genus of light. Their ray was a “visual power (*al-quwwa al-bāṣira*)” and it extended from the eye on straight lines whose origin is the center of the eye.¹³ This description referred to the work of Euclid (which was later extended by Ptolemy), who applied geometry to the problem of human perception by arguing that the eye emitted “visual rays” which could be studied mathematically as a cone of straight lines. According to this theory, known as “extramission,” questions such as the perceived size and position of visual objects at various distances from the eye could be treated as geometric problems. However, this approach did not lend itself well to explanations of the physiology of sight, the psychology of perception, or an understanding of color theory.¹⁴

11. For the *Discourse on Light*, see Ibn al-Haytham, *Majmū‘ Al-Rasā’il* (Hyderabad: 1938), no. 2; a parallel Arabic/German edition is J. Barmann, “Abhandlung über das Licht von Ibn al-Haytham,” *Zeitschrift der Deutschen Morgenländischen Gesellschaft* 36, (1882): 195-237; and a critical French edition is Roshdi Rashed, “Le «Discours de la lumière» d’Ibn al-Haytham (Alhazen). Traduction française critique,” *Revue d’histoire des sciences et de leurs applications* 21, no. 3 (1968): 197-224. Fārisī’s recension is Ahmet III MS 3340, fol. 204b-205b.

12. Shirwānī, *Sharḥ al-Tadhkira*, fol. 34a.

13. Shirwānī, *Sharḥ al-Tadhkira*, fol. 34a.

14. This summary of ancient theories of vision follows Chapter 1 of David Lindberg, *Theories of Vision from al-Kindi to Kepler* (Chicago: University of Chicago Press, 1976).

Another theory, known as “intromission,” argued that something must enter the eye from the visible object in order for the latter to be perceived. Shirwānī quoted Ibn al-Haytham’s description of a mathematical version of the intromission theory, stating:

[a]s for those who believe that vision takes place by means of a form (*ṣūra*) that comes [from the visible object] to the eye, they define the ray as that through which perception (*idrāk*) occurs. It is the light that extends from the object along the straight lines that meet at the center of the eye, out of all the light extending from the object to the eye along straight lines; for those who hold this view believe that the eye by nature (*maṭbūʿ*) senses the lights that reach it along these straight lines alone.¹⁵

The mathematicians defined “ray” as the light that extends on these imaginary straight lines meeting in the center of the eye, along with the lines themselves. Therefore the visual ray, according to them, was a kind of light that extended along straight lines, whether that light was the light of the stars, or of fire, or of sight.¹⁶

Proceeding to a more detailed level of analysis, Shirwānī discussed disagreements about the “cone of vision,” for according to the proponents of these theories, vision was carried out through a multitude of rays that formed a cone

whose apex is at the center of the eye and base is the circumference of the opening of the uvea. When the cone extends and widens from the eye, for each object within the cone the form of the object extends along the straight lines of the ray/cone and carries through the layers [of the eye] along straight lines.¹⁷

15. Ibn al-Haytham refers to the fact that, out of the infinity of straight lines connecting a given point on the object to the surface of the eye, the eye only “senses” the line connecting that point to the center of the eye.

16. Shirwānī, *Sharḥ al-Tadhkira*, fol. 34a. Shirwānī noted that he found variations between what Ibn al-Haytham said here (in *On Light*) and in the *Optics*, cf. *Optics* I.1 [4-5].

17. Shirwānī, *Sharḥ al-Tadhkira*, fol. 34a.

As with the previous discussion of the “ray,” Shirwānī provided a simplified overview of various ancient and Islamic arguments about the “cone of vision” and quoted Ibn al-Haytham’s comment that “investigation into the manner of visual sensation is composed of (*murakkab min*) the natural and mathematical sciences.”¹⁸

Since neither Ibn al-Haytham nor Shirwānī referred to the proponents of these theories by name, it will be helpful to put this summary into context. Several early Arabic authors paid particular attention to the medical and physiological aspects of vision. In his *Ten Treatises on the Eye*, Ḥunayn ibn Ishāq (d. 873), the famous physician and translator from Greek into Arabic, discussed the physical structure of the eye and the treatment of various diseases.¹⁹ The extramission theory was defended by the philosopher al-Kindī as part of his broader philosophy of nature. In his treatise *On Stellar Rays*, he argued that from everything in the world emanated rays of power that exerted influence on other objects. Just as rays from the Sun and stars act on the terrestrial world, so things like fire, music, images, and words have the power to affect our surroundings by radiation.²⁰ In this way, for al-Kindī, optics as the study of rays was the key to a mathematization of natural philosophy and to the general study of the radiation of power. In his mathematical treatment of optics, al-Kindī followed Euclid’s general principles even as he criticized him on a number of specific points. Al-Kindī argued, for example, that the visual cone emitted by the eye was not a collection of discrete rays, but rather a continuous body of radiation that “illuminated” the perceived object. Through geometric analysis al-Kindī attempted to

18. Shirwānī, *Sharḥ al-Tadhkira*, fol. 34b.

19. Ḥunayn ibn Ishāq, *The Book of Ten Treatises on the Eye Ascribed to Hunain ibn Is-haq (809-877 A.D.)*, ed. Max Meyerhof (Cairo: Government Press, 1928).

20. al-Kindī, *On the Stellar Rays*, ed. Robert Hand, trans. Robert Zoller (Berkeley Springs, WV: Golden Hind Press, 1993).

explain variations in sensitivity and clarity within the visual cone (e.g., we see objects more clearly at the center of our vision compared to the periphery). Furthermore, al-Kindī also drew on the Galenic tradition to argue that, as the visual rays emanated from the eye, their power transformed the medium.²¹

The most influential Islamic criticism of the extramission theory prior to Ibn al-Haytham was found in the work of Ibn Sīnā, who attacked it on the basis of Aristotelian physical and philosophical principles. Ibn Sīnā systematically refuted several variations of the Euclidean and Galenic extramission schemes. A central point in his criticism was that, in any version of the extramission theory, some sensitive power from the eye must traverse the space between the eye and the perceived object. No physical explanation of this process stood up to rational scrutiny. No substance emitted from the eye could fill up the vast space between it and a star, for example. If the eye were able to perceive the object through some sort of direct, physical contact, it would sense the object's true magnitude rather than following the laws of perspective that govern the appearance of distant objects. If the eye merely transformed the medium into an instrument of perception, as in the Galenic variant of extramissionism, then how can we explain the fact that multiple observers can perceive through the same medium simultaneously? The only solution was to follow Aristotle and argue that the transparent air, being in contact with both the object and the eye, acts on the eye by somehow transmitting the form of the object to it.²² Although the rejection of the various extramission theories left intromission as the only option, there remained many unanswered questions as to how intromission could account for all of the observed phenomena of human visual perception. For example, how do we coherently perceive individual

21. Lindberg, *Theories of Vision from al-Kindi to Kepler*, 18-32.

22. Lindberg, *Theories of Vision from al-Kindi to Kepler*, 43-52.

objects out of the jumble of forms that are transmitted to our eyes from throughout the visual field? How were distances and sizes perceived, and how could the “form” of something huge fit into the eye to be perceived?

Evaluated against the background of these competing theories of vision, Ibn al-Haytham’s synthesis is truly remarkable, and it rightly made a dramatic impression on later scholars such as Shirwānī. Ibn al-Haytham proposed a new analysis of radiation, light, and color that allowed him to combine the mathematical, physiological, and physical approaches to the problem of human perception into a united intromission theory. Rather than continue the debates of his predecessors, Ibn al-Haytham started afresh from first principles to build up a new science of vision on the basis of empirical observations and experimentation. The keys to Ibn al-Haytham’s synthesis were a mathematical analysis of light as the sole agent of vision through rectilinear propagation, reflection, and refraction, coupled with a mathematical and physiological explanation of how the eye could produce a coherent image out of the rays of light that it received from the visual field. After demonstrating that light radiated (or is reflected, in the case of opaque objects) in all directions from all points on the surface of an object, Ibn al-Haytham was faced with the problem that every point on the surface of the eye likewise encountered an incoherent collection of rays from all directions. However, most of these rays are refracted or bent as they crossed the surface of the eye, and are further refracted as they pass through the structures of the eye. Only the unique ray that entered the eye at a right angle to its surface would pass through unrefracted, and this collection of unique rays, one per point on the eye, mathematically reproduced the geometric visual cone of the Euclideans. Thus, Ibn al-Haytham restored a one-to-one correspondence of points on the eye with distinct points in the visual field,

producing an explanation for the coherence of the visual image that could be sensed and interpreted by the brain.²³

Returning to Shirwānī's overview of these debates, he followed Ibn al-Haytham's discussion of the visual cone. The starting point for rejecting the extramission theory was to recognize that if vision occurred through something that issued from the eye, then either it is a body or it is not. If it is a body, then when we look at the sky and the stars, there must emerge from our eye a body that fills up the space between the sky and the earth while nothing is lost from the eye. This is impossible. On the other hand, if that "something that issues from the eye" is not a body, then it cannot be said to sense anything, because animate bodies are the only objects that have the power of sensation or transmitting sensation. Therefore, having eliminated the extramission hypothesis on logical and physical grounds, Shirwānī reported that in *Optics* I.6, Ibn al-Haytham proved that the air and transparent bodies are sufficient to receive the form of the visible object and convey it to the eye, which receives it. Thus it becomes superfluous to suppose the existence of a visual ray as a means of sensation. Ibn al-Haytham would continue to use the mathematical concept of a (virtual) ray as an abstraction of the physical sphere of light propagating from any point on an illuminated object, stating that "they [the mathematicians] use in their reasoning and demonstrations imaginary "ray-like" lines, and we have shown that the eye cannot perceive anything except through these lines alone."²⁴ He concluded that this instrumentalist hypothesis of imaginary lines was the correct opinion for modeling visual perception using the tools of the mathematicians.

23. See Lindberg, *Theories of Vision from al-Kindi to Kepler*, Ch. 4; Ibn al-Haytham, *Optics, Books I-III*, Vol. 2, Introduction; and Sabra, "Ibn Al-Haytham's Revolutionary Project in Optics."

24. Shirwānī, *Sharḥ al-Tadhkira*, fol. 34b-35a.

The fact that Shirwānī quoted long sections from Ibn al-Haytham describing and refuting his predecessors suggests that he thought Ibn al-Haytham's criticisms bore repetition, or were not widely known to his readers in the 15th century. Shirwānī could not merely summarize Ibn al-Haytham's own synthesis without first criticizing the theories that his students would be reading in the course of their education. Later in the appendix, Shirwānī explained how Ibn al-Haytham's theory could account for how we perceive the size or magnitudes of visible objects, and why the same object appears larger when it is near the eye and smaller when it is more distant. In general, he explained, this perception follows a geometrical analysis of the visual cone – when the object is closer, the angle formed by a vertex at the center of the eye and lines drawn to the extremities of the object is larger than the angle required to contain the same object when it is far from the eye. However, the mathematicians erred when they said “size is perceived by the eye by the magnitude of this angle alone and nothing else.” As Ibn al-Haytham pointed out, there is a psychological component to perception as well: “along with (the angle), the eye must perceive the distance of the visible object and its relative position.”²⁵ For example, if we hold a small object close to our eye and compare it to a large object at some distance, such as a stone and a distant mountain, although they both may be contained within the same visual angle, the eye recognizes that the mountain is much farther away and therefore is, in reality, larger than the stone.

Having demonstrated the correct understanding of how a mathematical analysis of vision could be preserved, Shirwānī devoted the next several sections of the appendix to clarifying certain fundamental optical phenomena, particularly reflection and refraction. As with his overview of theories on the visual ray, Shirwānī's discussion of reflection (*Ar. in 'iṭāf*) and

25. Shirwānī, *Sharḥ al-Tadhkira*, fol. 40a-b.

refraction (Ar. *in 'ikās*) reveals a concern that his readers and students needed a correct exposition that would clear up misunderstandings and errors in other texts, or at least point his readers to the truth. Reflection and refraction were frequently confused or used interchangeably in Hellenistic and Islamic discussions of optics, going back to translations of Euclid's *Optics* and continuing down to the work of both Ṭūsī and Shīrāzī.²⁶ Ṭūsī had produced a recension of Euclid's *Optics* as part of his project of editing and revising older Arabic translations of several Greek mathematical and scientific texts, but neither he nor Shīrāzī were familiar with the contents of Ibn al-Haytham's *Optics*.²⁷ Indeed, one of the reasons Kamāl al-Dīn al-Fārisī gave for his dissatisfaction with previous scholars was their misunderstanding of refraction.²⁸ For example, Ṭūsī, in a treatise on "refraction" as well as in his recension of Euclid's *Optics*, claimed that a ray incident upon a smooth surface such as water led to the appearance of 4 equal angles, the incident ray, the reflected ray, the penetrating ray, and the "refracted" ray which actually bent backwards in the water in the direction of the source of the ray.²⁹ Likewise, Shīrāzī at times used the terms "reflection" and "refraction" interchangeably in his *Nihāyat al-Idrāk* and the *Tuhfa al-Shāhiya*.³⁰

26. Euclid, *The Arabic Version of Euclid's Optics*, ed. and tr. Elaheh Kheirandish, (New York: Springer, 1999), Vol. II, 54-59.

27. Euclid, *The Arabic Version of Euclid's Optics*, Vol. I, xxii.

28. Refraction, of course, played a crucial role in Fārisī's explanation of the rainbow. Roshdi Rashed, "Le modèle de la sphère transparente et l'explication de l'arc-en-ciel: Ibn al-Haytham, al-Farisi," *Revue d'histoire des sciences et de leurs applications* 23, no. 2 (1970): 109-140.

29. H. J. J. Winter and W. 'Arafat, "A Statement on Optical Reflection and 'Refraction' Attributed to Naṣīr ud-Dīn aṭ-Ṭūsī," *Isis* 42, no. 2 (1951): 138-142. Fārisī explicitly mentions these 4 rays in the introduction to the *Tanqīh*, Ahmet III MS 3340, fol. 3a.

30. Ibn al-Haytham, *Optics, Books I-III*, lxix-lxx, n.110.

In contrast, Shirwānī clearly distinguished between the reflection of a ray off of a smooth surface and the refraction of a ray that passed between media of different transparencies. For refraction, he explained that if the ray strikes the second medium perpendicularly, it passes through the latter unbroken; otherwise it is refracted and “the ray falling onto a smooth surface breaks into a different direction from the luminous object or towards the direction of the perpendicular, and from there proceeds along a straight line as well.”³¹ He supplemented this discussion with references to Shīrāzī and Nīsābūrī, and several explanatory remarks and illustrations from Fārisī’s *Tanqīh* to clarify how the angle of refraction varied according to the ray’s angle of incidence.³² Although the precise mathematical ratio for calculating the angle of refraction had not yet been discovered, Shirwānī’s students would nevertheless derive a clearer understanding of these two different phenomena than they could obtain from any other text.³³

Among other reasons, Shirwānī needed to clarify the phenomenon of refraction because it was a fundamental principle for Ibn al-Haytham’s final explanation of visual perception. Early in the *Optics*, Ibn al-Haytham had argued that the unique ray from a point on the visible object that fell perpendicularly on the eye was the only ray perceived by the eye. However, in Book VII, he admitted that non-perpendicular rays were refracted upon entering the eye and also played a role

31. Shirwānī, *Sharḥ al-Tadhkira*, fol. 35a.

32. Shirwānī, *Sharḥ al-Tadhkira*, fol. 35b-36b.

33. Ibn Sahl (fl. late 10th c.), in his treatise on burning instruments, applied a geometric relationship that can be rewritten trigonometrically as the sine law of refraction. On this basis, Roshdi Rashed has argued that Ibn Sahl discovered the law of refraction. However, as A.I. Sabra has pointed out, Ibn Sahl did not state it as a general law or discuss the idea of the index of refraction for a particular medium. Ibn al-Haytham, who was familiar with Ibn Sahl’s work, continued to discuss refraction in terms of inequalities of angles. See Roshdi Rashed, “A Pioneer in Anaclastics: Ibn Sahl on Burning Mirrors and Lenses,” *Isis* 81, no. 3 (1990): 464-491 and A. I. Sabra, review of *Géométrie et dioptrique au Xe siècle : Ibn Sahl, al-Qūhī et Ibn al-Haytham*, by Roshdi Rashed, *Isis* 85, no. 4 (1994): 685-686.

in perception.³⁴ Shirwānī explained that non-perpendicular rays were refracted through the transparent tunics of the eye before reaching the crystalline humor. If vision only occurred by means of the rays striking the eye orthogonal to its surface, then the field of vision would be strictly defined by that cone of rays with vertex at the eye's center, passing through the pupil. However, since we are also able to see objects outside of this cone (e.g., with peripheral vision), they must be perceived by means of refracted rays.³⁵

The principles of refraction and the psychology of perception came together in some of the final sections of Shirwānī's appendix, where he explicitly applied them to problems in astronomy. Shirwānī quoted at length from Ibn al-Haytham's treatise *On Seeing the Stars* to explain that "the stars appear larger through vapor than through clear air," and "they appear larger at the horizon than in the middle of the sky."³⁶ In both of these cases, Ibn al-Haytham was dealing with the so-called "Moon illusion," a subject of much debate since Antiquity and which most ancient astronomers explained as a physical phenomenon similar to the apparent magnification of an object under water.³⁷ In *On Seeing the Stars*, Ibn al-Haytham criticized the apparently contradictory explanations of this phenomenon in Ptolemy's *Optics* and *Almagest*. He identified two factors that contributed to a celestial object's apparent change in size. The first was the refraction of rays crossing from the celestial aether into the terrestrial air, and the second

34. A. I. Sabra, "Ibn Al-Haytham," in *Dictionary of Scientific Biography*, (New York: Scribner's Sons, 1972), 193-194.

35. Shirwānī, *Sharḥ al-Tadhkira*, fol. 50a-51b.

36. See Sabra and Heinen, "On Seeing the Stars."

37. A. I. Sabra, "Psychology Versus Mathematics: Ptolemy and Alhazen on the Moon Illusion," in *Mathematics and Its Applications to Science and Natural Philosophy in the Middle Ages: Essays in Honor of Marshall Clagett*, ed. Marshall Clagett et al., (Cambridge: Cambridge University Press, 1987), 217-247.

an additional layer of the “moist vapor” exhaled by the Earth near the horizon. In the basic case of refraction between the aether and the air, Shirwānī supplied Ibn al-Haytham’s entire geometric proof that the star’s magnitude will be perceived to be smaller than it actually is, but that the difference decreases as the star moves toward the zenith because the angle of refraction diminishes. On the other hand, if part of the air became dense and filled with the “moist vapors” (*al-bukhār*) believed to be exhaled from the Earth, Ibn al-Haytham argued that it was possible for the ray to be refracted yet again, producing a larger angle and causing the star to appear larger than it would if it were seen in the absence of the vapor. Again, this difference in apparent size decreases as the star rises from the horizon away from the moist air.³⁸

Shirwānī concluded with a discussion of shadows cast by bodies, analyzed geometrically in terms of rays of light. This section drew on Ibn al-Haytham’s treatise *On the Quality of Shadows*, where he introduced principles concerning the mathematical analysis of shadows based on the relative sizes of the luminous and opaque objects. In particular, these principles were relevant to observing eclipses, namely the existence of a secondary shadow (penumbra) around the lightless core shadow (umbra). He noted that the knowledge of shadows was a fundamental principle (*aṣl*) for astronomy, as it related to understanding the motions of the Sun and Moon, measuring time, measuring the position of the Sun throughout the day, and determining the magnitudes and times of eclipses.³⁹

38. Later, in *Optics* Book VII, Ibn al-Haytham also introduced a psychological explanation for the “Moon illusion” effect. As mentioned previously, Shirwānī was aware that there is a psychological component to perception of sizes and distances in general, but he did not explicitly refer to it in this section of the appendix, which seems to draw only from Ibn al-Haytham’s treatise, “On Seeing the Stars.” Further research is necessary to determine the extent to which Shirwānī understood the differences between Ibn al-Haytham’s argument in that treatise and in *Optics*, VII.

39. Shirwānī, *Sharḥ al-Tadhkira*, fol. 53b-58b.

From this brief discussion, it becomes clear that large portions of Shirwānī's appendix focused on problems associated with observing the heavenly bodies. These included the role of refraction and the "Moon illusion" in causing the appearance of increased magnitude and increased mutual distances between objects near the horizon, where many important observations were made after sunset and before sunrise. The placement of these topics at the end of the commentary on *Tadhkira* Book I, which was devoted to providing an overview of knowledge from other disciplines that was essential for further study of astronomy, indicates that Shirwānī considered optics to be just as important as geometry and natural philosophy.

Not least, Shirwānī's presentation of optics as a foundation for astronomy should remind us that in the Islamic world optics was studied as one of "mixed sciences" that drew on both mathematics and physics or natural philosophy.⁴⁰ Additional research on the incorporation of optics into astronomical commentaries promises to deepen our understanding of the development and transmission of this discipline. Although Shirwānī's *ijāza* did not mention any optical texts, his deep engagement with Ibn al-Haytham's work as mediated by Fārisī's *Tanqīh* makes it likely that the latter was a staple in the curriculum at Samarqand.⁴¹

Shirwānī drew on mathematical, physical, and psychological arguments to identify relevant problems in optics with for his readers, in some cases providing solutions and in others referring the reader to a fuller explanation in Ibn al-Haytham's work.⁴² In many ways, this

40. Kheirandish, "The Mixed Mathematical Sciences: Optics and Mechanics in the Islamic Middle Ages."

41. In his appendix on optics Shirwānī rarely mentioned Qāḍīzāde's oral comments or referred to his regular sources (Shīrāzī, Nīsābūrī, Jurjānī), especially as compared to the rest of the commentary. However, he did refer to a comment by Qāḍīzāde on the topic of refraction and the Earth's "vapor" at Shirwānī, *Sharḥ al-Tadhkira*, fol. 41b-42a.

42. As I mentioned at the beginning of this chapter, based on the numerous passages that I checked directly, it seems unlikely that Shirwānī had direct access to Ibn al-Haytham's *Optics*.

procedure was similar to his treatment of topics in the rest of his commentary on Ṭūsī's *Tadhkira*. In this appendix, however, Shirwānī was not writing a line-by-line commentary. He was therefore free to choose which topics from the *Optics* he wished to cover, and rearrange the material according to his desire to present a simplified introduction, presumably for readers who did not know about Ibn al-Haytham's work. For the most part, Shirwānī allowed the material to speak for itself, as the *Optics* was already a systematic presentation of the observations, experiments, and reasoning that had led Ibn al-Haytham to his revolutionary theories of light and vision. Considering Shirwānī's *Sharḥ al-Tadhkira* in its role as a teaching text, this inclusion of a broad overview of optical principles was intended to prepare readers for some of the complications they would encounter in the practice of astronomy, and served as a manageable introduction to the arguments and demonstrations that would be studied in more advanced texts such as Fārisī's *Tanqīḥ*. Here we have a bonus of the commentary tradition – an appendix about a different text and area of research that is necessary to understand the principal text. Yet another reason why scholars should actually look at commentaries rather than dismiss them. One could not have predicted that this material would show up here, and its appearance demands further research regarding the transmission of the *Optics/Tanqīḥ* and the evolution of education in mathematical sciences in the 15th c. Islamic world.

Rather, he was quoting from the *Optics* by way of Fārisī's *Tanqīḥ*. Nevertheless, Shirwānī almost always cited the *Optics*. In the future I will systematically compare all three texts to determine if any of the content could demonstrate that Shirwānī did indeed have direct access to the *Optics* independent of the *Tanqīḥ*.

Geography in Shirwānī's supercommentary on the *Mulakhkhaṣ*

As explained in the previous chapters, in the post-classical Islamic period, the focus of theoretical astronomy (‘ilm al-hay’a) was the broad attempt to understand the various celestial phenomena on the basis of observation and to develop mathematical and natural philosophical models of the cosmos that could exist in physical reality and accurately predict the motions of the heavens. The basic principles of astronomy could be studied in many texts, but the most popular introduction to the material was al-Jaghmīnī’s (fl. late 12th-early 13th c.) *al-Mulakhkhaṣ fī al-hay’a al-basīṭa* (Epitome of plain theoretical astronomy). The *Mulakhkhaṣ* can be found today in thousands of extant manuscript copies in Arabic and Persian. Jaghmīnī’s text provided a simplified overview of astronomical phenomena without geometric proofs, and was divided into two parts that summarized the configuration of the celestial orbs and the terrestrial world respectively.⁴³ It could be studied on its own as part of an elementary collection of texts in the sciences, although owing to the highly technical and complex astronomical material being discussed in rather concise terms, it was often supplemented by a commentary that made it useful for intermediate students, such as the commentary written by Qāḍīzāde al-Rūmī and dedicated to Ulugh Beg in 1412. Qāḍīzāde’s commentary on the *Mulakhkhaṣ* was brought to Anatolia by Shirwānī and others, where it eventually became a part of the Ottoman scientific curriculum and was still studied into the eighteenth century. So far, approximately 300 extant manuscript copies have been located in Istanbul libraries, including one in Qāḍīzāde’s own hand. Qāḍīzāde’s commentary on the *Mulakhkhaṣ* was in turn the subject of many supercommentaries, including the one by Shirwānī known as *al-Farā’id wa-’l-fawā’id fī tawḍīḥ Sharḥ al-Mulakhkhaṣ*, also

43. See Sally Ragep, “Maḥmūd ibn Muḥammad ibn ‘Umar al-Jaghmīnī’s *al-Mulakhkhaṣ fī al-hay’a al-basīṭa*: An Edition, Translation, and Study” (PhD diss., McGill University, 2014).

referred to in the biographies as a *ḥāshiya* or *ta' līqāt* on *Sharḥ al-Mulakhkhaṣ*.⁴⁴ I shall refer to it below as the *ḥāshiya*.

In the preface to the *ḥāshiya*, Shirwānī describes his early education and praises Qāḍīzāde's teaching, explaining that he based the commentary in part on notes from Qāḍīzāde's lectures in Samarqand. Shirwānī then traveled to Istanbul to present it as a gift (*tuhfa*) to Sultan Mehmed II, whom he praised effusively as a symbol of knowledge and wisdom.⁴⁵ However, an inspection of the manuscript and its contents reveals that it may not necessarily have been intended for the sultan's personal benefit, but perhaps was intended as a source of discussion among intellectuals at court.⁴⁶ Other commentaries contain the entirety of the base text divided into sentences, phrases, or individual words with the commentator's observations and additions interwoven throughout, and could therefore be read on their own as a self-contained text. Shirwānī's *ḥāshiya*, however, only refers to Jaghmīnī's and Qāḍīzāde's statements and does not quote them in their entirety. Shirwānī's text therefore presumes that the reader is either familiar with Qāḍīzāde's commentary or has a copy so that both can be read side by side. Indeed,

44. As mentioned in Chapter 1, I know of only one extant MS of Shirwānī's work: Topkapı Ahmet III MS 3294, which was completed in 878/1473 and presented to Sultan Mehmed II in Istanbul.

45. Fathallāh al-Shirwānī, *Ḥāshiya 'alā Sharḥ al-Mulakhkhaṣ*, Topkapı Ahmet III MS 3294, fol. 1a-3a.

46. On the other hand, Mehmed II read several languages (including Arabic) and was very engaged in intellectual matters, collecting manuscripts from multiple languages, commissioning translations, and sponsoring learned disputations at court. See, for example, Julian Raby, "Mehmed the Conqueror's Greek Scriptorium," *Dumbarton Oaks Papers* 37, (1983): 15-34; Anna Akasoy, "Mehmed II as a Patron of Greek Philosophy: Latin and Byzantine Perspectives," in *The Renaissance and the Ottoman World*, ed. Anna Contadini and Claire Norton, (Burlington, VT: Ashgate, 2013), 245-256; and Maria Mavroudi, "Translations From Greek Into Arabic At the Court of Mehmed the Conqueror," in *The Byzantine Court: Source of Power and Culture. Papers From the Second International Sevgi Gönül Byzantine Studies Symposium, Istanbul, Turkey, June 2010*, ed. A. Ödekan et al., (Istanbul: Koç University Press, 2013), esp. 204-205.

Qāḍīzāde's commentary already goes into great detail and stands as an intermediate textbook on its own. Shirwānī's supercommentary reads in many parts like a set of instructor's notes for clarifying difficult passages or providing a context and the background for the concepts in the base text. However, Shirwānī has also written long, detailed sections that expand on Qāḍīzāde's remarks and almost become an appendix or supplement. One of the latter is the geographic material on which we focus here.

In Part II of his *Mulakhkhaṣ*, Jaghmīnī related how, following Ptolemy, geographers divided the surface of the globe into quarters with the inhabited region of the Earth being one of the two northern quarters. By taking a set of lines parallel to the equator, this inhabited region can be subdivided into seven sections or "climes" in which the length of the longest day of the year is the same.⁴⁷ Qāḍīzāde supplemented Jaghmīnī's list of climes and their latitudes with lists of the major cities or regions in each clime. Altogether this occupied four pages in Qāḍīzāde's *Sharḥ*.⁴⁸ At this point, Shirwānī launched into a detailed description of the more than 130 regions and cities mentioned by Qāḍīzāde that required 58 folio pages – nearly a third of the entire manuscript.⁴⁹ An interesting feature of this section is the fact that Shirwānī reported longitude and latitude coordinates for many locales, which provides a means of comparing his data with earlier Arabic and Persian geographic texts and astronomical tables in order to uncover his

47. Various classical authors specified different sets of latitude lines for dividing the Earth into climes, but these seven were commonly attributed to Ptolemy. See Ragep, "Jaghmīnī's al-Mulakhkhaṣ," 345-347; cf. J. L. Berggren and Alexander Jones, *Ptolemy's Geography: An Annotated Translation of the Theoretical Chapters* (Princeton: Princeton University Press, 2000), 7-13, and G.J. Toomer, *Ptolemy's Almagest* (Princeton: Princeton University Press, 1998), Bk II [1-6].

48. Qāḍīzāde al-Rūmī, *Sharḥ al-Mulakhkhaṣ fī al-hay'a*, Süleymaniye Fatih MS 3403, fol. 50b-52a.

49. Shirwānī, *Hāshiya 'alā Sharḥ al-Mulakhkhaṣ*, fol. 56b-85a.

sources. This study of Shirwānī's geographic data is intended as a contribution to the broader investigation of routes of transmission and scientific influence in post-classical Islamic science, particularly the topic of *hay'at al-ard* (configuration of the Earth) in Islamic astronomy. As will be discussed below, Sultan Mehmed II had a particular interest in Greek manuscripts of Ptolemy's *Geography*, and he sought to obtain maps from Europe, especially Italy. Shirwānī's inclusion of similar data in a text dedicated to the sultan was likely an attempt to gain Mehmed II's favor by playing to his intellectual and political concerns.

The table below presents the longitude and latitude coordinates for over 60 locales listed in Shirwānī's *ḥāshiya*. Tabulating this data allowed for location-by-location comparison with coordinates from dozens of *zīj*es and geographic texts.⁵⁰ Allowing for minor scribal errors, approximately 3/4 of Shirwānī's values match those found in the geographic dictionary *Mu'jam al-Buldān* by Yāqūt al-Ḥamawī al-Rūmī (d. 1229).⁵¹ Further investigation reveals that Shirwānī's comments on many other locales without geographic coordinates are also taken from Yāqūt's text. Yāqūt, a scholar and biographer from Baghdad, spent most of his life traveling widely throughout the Middle East and Central Asia and gathering information for his celebrated works of geography, biography, and literary history.⁵² His *Mu'jam al-Buldān* is organized alphabetically and contains geographic descriptions in addition to histories of locales and

50. E. S. Kennedy and M. H. Kennedy, *Geographical Coordinates of Localities From Islamic Sources* (Frankfurt: Institut für Geschichte der Arabisch-Islamischen Wissenschaften, 1987).

51. Yāqūt al-Ḥamawī, *Mu'jam al-Buldān*, ed. F. Wüstenfeld, 6 vols. (Leipzig: Deutsche Morgenländische Gesellschaft, 1886-1873). See also Yāqūt al-Ḥamawī, *The Introductory Chapters of Yāqūt's Mu'jam al-Buldān*, ed. and tr. Wadie Jwaideh, (Leiden: Brill, 1959).

52. On Yāqūt's life and works, see S. M. Ahmad, "Yāqūt al-Ḥamawī al-Rūmī," in *Dictionary of Scientific Biography*, (New York: Scribner's Sons, 1973), 546-548 and Claude Gilliot, "Yāqūt al-Rūmī" in *The Encyclopaedia of Islam*, 2nd Edition (Leiden: Brill, 1960-2007).

	Shirwānī		Yāqūt		notes
	Long.	Lat.	Long.	Lat.	
Hadramaut	71°	12°	71°	12°	
Sabā'	64°	17°	64°	17°	
Hajar	63°	24° 15'	73°	24° 15'	Shirwānī's longitude is likely a scribal error.
Bahrayn	74°	24° 45'	74° 20'	24° 45'	Shirwānī's longitude is likely a scribal error dropping "+ 1/3"
Mansūra (M. al-Sind)	93°	22°	93°	22°	
Tanja	8°	35° 30'	8°	35° 30'	
al-Sūs	34°	-	34°	-	Attributed to Ptolemy in both texts, Shirwānī does not know which Sūs he meant.
Sūsa	34° 18'	32° 16'	34° 18'	32° 45'	Attributed to Ptolemy in both texts, Shirwānī's latitude "32 + 1/5 + 4" is a scribal error.
al-Qayrawān	31°	30° 40'	31°	30° 40'	Most other sources in Kennedy list latitude 31° 40'.
Misr (Cairo)	54° 40'	29° 15'	54° 40'	29° 15'	
Ṭabariya (Tiberias)	57° 45'	32°	57° 45'	32°	
al-Madā'in (Ctesiphon)	70° 20'	33° 20'	70° 20'	33° 20'	
Baghdād	70°	33° 20'	70°	33° 20'	Most other sources in Kennedy list latitude 33° 25'.
Wāsiṭ	71° 40'	33° 20'	71° 40'	33° 20'	
Basra	74°	31°	74°	31°	
al-Ahwāz	75°	32°	75°	32°	
Shīrāz (capital of Fārs)	78° 30'	29° 30'	78° 30'	29° 30'	
Qundahār	110°	30°	110°	30°	
Rhodes, Island of	50°	35° 30'	50°	35° 30'	
Qubrus (Cyprus)	61° 15'	35° 03'	61° 15'	35° 03'	No other source in Kennedy gives latitude ending in 03'.
Ṭarsūs	58° 30'	36° 22.5'	58° 30'	36° 15'	Shirwānī writes latitude 36 + 1/4 + 1/8, likely error writing 1/8 (<i>thumna</i>) for 'thumma.'
Atārāblus Shām (Tripoli)	60° 35'	34°	60° 35'	34°	
Antākiya (Antioch)	69°	35° 30'	69°	35° 30'	Attributed to Ptolemy in both texts.
Halab (Aleppo)	69° 30'	35° 25'	69° 30'	35° 25'	
Āmid	75° 40'	35° 15'	75° 40'	35° 15'	
Arzanjān (Erzincan)	64° 50'	39° 40'	-	-	Shirwānī's comments are from Yāqūt. Coordinates do not match any source in Kennedy, although latitude matches <i>Zīj-i Ilkhānī</i> which has 77° - 39° 40'.
Nasībīn	75° 20'	36° 12'	75° 20'	36° 12'	No other source in Kennedy gives latitude ending in 12'.
Moṣul	69°	34° 20'	69°	34° 20'	
Surra Man Rā (Samarra)	69° 40'	37° 10'	69° 40'	37° 10'	All sources in Kennedy other than Yāqūt give latitudes close to 34°.
Marāgha	-	-	73° 20'	37° 20'	Shirwānī mentions astronomical observations by Ṭūsī.
Tabriz	-	-	73° 10'	37° 30'	Shirwānī immediately proceeds to paraphrase Yāqūt's discussion of Azerbaijan.
Adharbayjan	73°	40°	73°	40°	
Hulwān	71° 45'	34°	71° 45'	34°	
Ardābil	73°	-	80°	36° 33'	Shirwānī quotes Yāqūt's description of location relative to Caspian Sea, Tabriz.
Nihāwand	72°	36°	72°	36°	
Hamadān/Hamadhān	73°	36°	73°	36°	
Sulṭāniya	71°	36°	-	-	Not mentioned by Yāqūt. Shirwānī describes it as a famous town (<i>balda</i>) 7 days from Tabriz, coordinates do not match any source in Kennedy.

Qazwīn	“same as Daylam”	75°	37°	Shirwānī quotes Yāqūt that it is 27 <i>farsakhs</i> from Rayy.
Daylam	75°	36° 10'	36° 10'	
Sāwah	77° 20'	35' (sic)	77° 50'	Shirwānī writes longitude as 77° + 1/3, likely missing + 1/2 from Yāqūt. For latitude, read <i>darajā</i> for <i>daqīqa</i> .
Qumm	74°	34° 40'	34° 40'	No other source in Kennedy gives longitude 74°, likely scribal error.
Amul	77° 20'	37° 45'	37° 45'	
Amul/Ammuya	85° 30'	37° 40'	37° 40'	Both texts locate it on the road from Bukhara to Marv. Shirwānī's longitude is likely a scribal error dropping “+ 1/4”
Asirābād	79° 50'	38° 45'	38° 45'	
Jurjān	80° 45'	38° 15'	38° 15'	Yāqūt gives alternate coordinates 86° 30' - 40°, Shirwānī mentions that “it is said its latitude is 40°.”
Nīsābūr	80° 45'	37°	37°	
Tūs	81°	37°	37°	
Harāh (Herat)	94° 20'	34° 30'	-	Shirwānī's comments are taken from Yāqūt, who does not give coordinates. Shirwānī's coordinates match <i>Zij-i Ilkhānī</i> .
Marv al-Shāhijān	84° 20'	37° 35'	37° 35'	
Anbār (Fallujah)	69° 30'	32° 40'	32° 40'	Shirwānī mentions there is another Anbār near Balkh in Juzjān region.
Balkh	101°	36° 40'	115° - 37° (K. <i>al-Malḥama</i>) 88° 35' - 38° 40' (<i>Abu 'Aun</i>)	Shirwānī's comments are taken from Yāqūt, but his coordinates nearly match the value 101° - 36° 41' from <i>Zij-i Ilkhānī</i> .
Tubbat (Tibet?)	130°	39°	130°	Shirwānī's comments are taken from Yāqūt, likely scribal error in latitude.
'Ammūriya	53°	37°	53°	Both Shirwānī and Yāqūt refer to Abu 'Aun's <i>zīj</i> for these coordinates.
Qaysāriya Rūm	67° 20'	41° 50'	67° 20'	
Bukhārā	87°	40° 15'	87° - 41° (K. <i>al-Malḥama</i>) 36° 50' lat. only (<i>Abu 'Aun</i>)	Shirwānī's latitude does not match any source in Kennedy, perhaps significant scribal error from Yāqūt's first value.
Kesh (Shahri Sabz)	99°	39° 30'	-	Some comments taken from Yāqūt, Shirwānī adds that the name Shahri Sabz is due to the mild climate and abundance of vegetation. Coordinates may be copied from <i>Zij-i Ilkhānī</i> which has 99° 30' - 39° 30'
Samarqand	99°	40°	89° 30'	Shirwānī's comments are not taken from Yāqūt. Coordinates match <i>Zij-i Ilkhānī</i> but not Ulugh Beg's <i>Zij Sulṭānī</i> which has 99° 16' - 39° 37'.
Isbjāb	98° 10'	39° 50'	98° 10'	
Tarāz	100° 30'	40° 25'	100° 30'	
Qusṭanṭīniya (Constantinople)	99° 50'	41°	59° 50' - 45° 56° 20' - 43° (K. <i>al-Malḥama</i>)	Yāqūt's value appears in <i>Zij-i Ilkhānī</i> , Shirwānī's longitude is likely a scribal error as all other sources report values near 49° or 59°, his comments are not taken from Yāqūt, and he notes some <i>zījes</i> report latitude 45° but this “is not in agreement with practice.”
al-Lān	80°	44°	-	Shirwānī's comments differ from Yāqūt on this land/people in the Caucasus. Coordinates may reflect a scribal error from 83° - 44° in <i>Zij-i Ilkhānī</i> .
Bulghār	90°	49° 30'	-	Shirwānī's comments are from Yāqūt. Coordinates match <i>zīj</i> of Muḥyī al-Dīn al-Maghribī (colleague of Tūsī at Marāgha), but not <i>Zij-i Ilkhānī</i> which has 87° - 49° 30'.
Shirwānī = <i>Hāshīya</i> on Qāḍīzāde's <i>Sharḥ al-Mulakhkhaṣ</i> , Ahmet III MS 3294 Yāqūt = Yāqūt al-Ḥamawī, <i>Mu'jam al-Buldān</i> Kennedy = Kennedy and Kennedy, <i>Geographical Coordinates of Localities From Islamic Sources</i>				

biographic details of prominent individuals. It was abridged in the fourteenth century by Ibn ‘Abd al-Ḥaqq under the title *Marāṣid al-iṭṭilā’ ‘alā asmā’ al-amkina wa-l-biqā’*.⁵³ However, that abridgement is not the source of Shirwānī’s data because it does not contain geographic coordinates. Shirwānī’s contemporary, Jalāl al-Dīn al-Suyūṭī (d. 1505) reportedly wrote an abridgement called *Mukhtaṣar Mu‘jam al-Buldān*, but there does not seem to be much information about this text. Katib Çelebi confused it with the abridgement by Ibn ‘Abd al-Ḥaqq, quoting most of the latter’s introduction but attributing it to al-Suyūṭī.⁵⁴ In the absence of evidence to the contrary, we will presume Shirwānī obtained the majority of his geographic data from Yāqūt’s *Mu‘jam al-Buldān*.

Yāqūt’s text was a geography, a travel guide, a dictionary, and a history all rolled into one. In the introduction, he explained that his aim was to provide the correct written forms of place names, their locations, and knowledge of their histories. Such knowledge would benefit the levying of taxes in conquered regions or investigating *ḥadīth* and histories, and would be useful for physicians and astrologers whose practices relied on the local environment, as well as poets and grammarians who could draw on illustrative examples.⁵⁵ Yāqūt cited works from a large number of ancient and Islamic authors, and Shirwānī copied these references in many of his comments.⁵⁶ For Shirwānī’s purposes in an astronomical commentary, however, the relevant

53. Ibn ‘Abd al-Ḥaqq, *Marāṣid al-iṭṭilā’ ‘alā asmā’ al-amkina wa-l-biqā’*, ed. T. W. Juynboll (Leiden: Brill, 1852-1864).

54. Yāqūt al-Ḥamawī, *The Introductory Chapters of Yāqūt’s Mu‘jam al-Buldān*, ix-x. However, Jwaideh notes that according to *GAL Supp.* I, 880 a MS copy is in the Āṣafīyah collection in Ḥaydarābād.

55. Yāqūt al-Ḥamawī, *The Introductory Chapters of Yāqūt’s Mu‘jam al-Buldān*, 4-9.

56. Yāqūt lists his predecessors and their works, some of which are no longer extant, Yāqūt al-Ḥamawī, *The Introductory Chapters of Yāqūt’s Mu‘jam al-Buldān*, 10-13.

information was only the mathematical geography (longitude and latitude coordinates as well as travel times between locales). He thus dispensed with most of Yāqūt's narratives and anecdotes. In effect, Shirwānī took a work of one genre (a geographic dictionary) and reconfigured it into a work of the other major post-classical geographic genre: the general geographic treatise organized by Ptolemaic climates.⁵⁷ Shirwānī's borrowing of merely the "facts" from Yāqūt's text, rather than the stories and personal accounts, perhaps explains why he did not refer to Yāqūt as the source of this information. Elsewhere in his commentaries, he was scrupulous about attributing arguments and explanations to sources such as Ptolemy, Ibn Sīnā, Ṭūsī, and Shīrāzī.

Of the remaining locales in Shirwānī's *ḥāshiya* whose coordinates do not match those found in *Mu'jam al-Buldān*, four have descriptions taken from Yāqūt with coordinates that are difficult to explain as simple scribal errors (Erzincan, Ardabil, Sulṭāniya, and Bukhara). An additional six locales (Herat, Balkh, Kesh, Samarqand, al-Lān, and Bulghār) have coordinates that match or nearly match values found in the group of Persian *zījes*, including the *Zīj-i Īlkhānī* from the Marāgha Observatory and Ulugh Beg's *Zīj-i Sulṭānī*. However, one curious oversight is the fact that Shirwānī did not provide coordinates for Marāgha at all, neither the value reported in Yāqūt's *Mu'jam al-Buldān* nor the one in the *Zīj-i Īlkhānī* itself. The *Zīj-i Sulṭānī* had copied the coordinates of Marāgha from the earlier *Zīj-i Īlkhānī* whereas the coordinates of Samarqand were presumably determined from scratch as part of the Ulugh Beg's program of re-deriving solar, lunar, and planetary parameters for use in calculating the new astronomical tables.

57. Gerald Tibbetts, "Later Cartographic Developments," in *History of Cartography*, Volume 2, Book 1: *Cartography in the Traditional Islamic and South Asian Societies*, ed. J. B. Harley and David Woodward, (Chicago: University of Chicago Press, 1992), 142-143. In theory, these coordinates could be used to assist in constructing maps or comparing astronomical observations in one locale with those recorded at another. However, given the difficulty in measuring longitudes or the distances between two locales with much accuracy it is perhaps not surprising that few Arab mapmakers relied on tables of coordinates for drawing their world maps.

Shirwānī's coordinates for Samarqand, however, match Ṭūsī's *Zīj-i Īlkhānī*. For Herat, Balkh, Kesh, and Bulghār, Shirwānī's comments largely came from *Mu'jam al-Buldān*. However, Shirwānī also included an explanation that Kesh is now more famous as Shahri Sabz (the "Green City") owing to its moderate climate and abundant vegetation, comments that do not appear in *Mu'jam al-Buldān*. Shirwānī's comments on Samarqand and al-Lān do not match Yāqūt's text either, indicating that he was including personal knowledge or quoting from an additional, as yet unknown source.

Shirwānī's entry for Constantinople⁵⁸ is unique in that it simultaneously differed from Yāqūt's description and can be definitively dated to within Shirwānī's lifetime. Whereas Yāqūt's comments include topics such as the history of Roman kings, the naming of the city, and descriptions of the city gates, Shirwānī referred to the fact that Constantinople was formerly part of the Christian lands but has now become one of the great cities in the lands of Islam.⁵⁹ Furthermore, Shirwānī noted that some *zījes* (including Yāqūt's *Mu'jam al-Buldān*, the *Zīj-i Īlkhānī*, and the *Zīj-i Sulṭānī*) report Constantinople's latitude as 45° but "this is not in agreement with practice" (*wa huwwa lā yakūn mūwāfiq^{an} bi-l- 'aml*). Shirwānī's phrase indicated that he had carried out his own astronomical observations of the city's latitude, having lived and worked in Constantinople after the Ottoman conquest. Indeed, Shirwānī reported a latitude of 41° in agreement with the modern value of approximately 41° 01'.

58. Shirwānī referred to the city by its Arabized name, *Qusṭanṭīniya*. Constantinople and Istanbul were both used as names for the city throughout the Ottoman period. "Constantinople" appeared more frequently in official/literary contexts, and "Istanbul" was generally used in unofficial/vernacular contexts. Çiğdem Kafescioğlu, *Constantinopolis/Istanbul: Cultural Encounter, Imperial Vision, and the Construction of the Ottoman Capital* (University Park: Pennsylvania State University Press, 2009), 227, n.2.

59. For Yāqūt's description, see Yāqūt al-Ḥamawī, *Mu'jam al-Buldān*, IV, 95-97. Shirwānī's comments are Shirwānī, *Ḥāshiya 'alā Sharḥ al-Mulakhkhaṣ*, 83b-84a.

One complicating factor when comparing coordinates from different sources is the fact that Hellenistic and Islamic astronomers used two different locations as the prime meridian from which to measure longitudes. One group followed Ptolemy in locating the 0° meridian at the Fortunate Isles (usually thought to be the Canary Islands), while another took the “western shore of the encompassing sea (Atlantic)” 10° east of the Fortunate Isles on the coast of Africa. As a result, longitudes from one set of tables generally differ by 10° from the other, although the occasional practice of measuring relative longitudes from a major city such as Baghdad introduced further variation in later tables.⁶⁰ Shirwānī was clearly aware of these different “standard” meridians because he discussed them in his *ḥāshiya* on the *Mulakhkhaṣ* as well as in his commentary on the *Tadhkira*.⁶¹ In addition, Yāqūt quoted al-Bīrūnī on the different systems for latitude in the introduction to *Muʿjam al-Buldān*, noting that “intelligence and skill are required in order to distinguish the one from the other.”⁶² Since Yāqūt drew on such a large collection of sources for his material, he did not claim that all coordinates were reckoned according to the same meridian, and noted that his practice in general was to report what he found and leave it to the reader to exercise judgment.⁶³ For example, the two longitudes he reported for Constantinople appear to be based on the Ptolemaic meridian at the Canary Islands.

60. See the discussions of prime meridians in Gerald Tibbetts, “The Beginnings of a Cartographic Tradition,” in *History of Cartography*, Volume 2, Book 1: *Cartography in the Traditional Islamic and South Asian Societies*, ed. J. B. Harley and David Woodward, (Chicago: University of Chicago Press, 1992), 102-104, and Kennedy and Kennedy, *Geographical Coordinates of Localities From Islamic Sources*, xi.

61. Shirwānī, *Ḥāshiya ʿalā Sharḥ al-Mulakhkhaṣ*, fol. 56b; Shirwānī, *Sharḥ al-Tadhkira*, fol. 282a-b.

62. Yāqūt al-Ḥamawī, *The Introductory Chapters of Yāqūt’s Muʿjam al-Buldān*, 60, quoting from Bīrūnī’s *Tafhīm*.

63. Yāqūt al-Ḥamawī, *The Introductory Chapters of Yāqūt’s Muʿjam al-Buldān*, 13-15.

For the most part, however, the longitudes in Yāqūt's text seem to be based on the Atlantic coast meridian adopted by the astronomers associated with the caliph al-Ma'mūn and by al-Khwārizmī in his *Kitāb Ṣūrat al-arḍ*. In contrast, the *zīj*es compiled at Marāgha and Samarqand were based on the Ptolemaic meridian. As a result, the longitudes Shirwānī added to his *ḥāshiya* for Herat, Balkh, Kish, Samarqand, al-Lān, and Bulghār are approximately 10° greater than the “theoretical” longitudes that might be calculated relative to the other locales in the *ḥāshiya*.⁶⁴

Although Shirwānī was a trained astronomer, it is perhaps unfair to expect him to have recognized this difference given the confusion surrounding this topic in his sources.

As Giancarlo Casale has pointed out, despite the assumptions of previous historians, it remains an open question as to how much access the early Ottomans had to the works of Arab and Persian mathematical geographers. Although Istanbul libraries currently hold large numbers of such manuscripts, much work remains to be done to trace their provenances and to determine whether they were obtained from Arab lands as the Ottoman empire expanded in the 1500s.⁶⁵

The texts known to have circulated in Anatolia in the late 15th c. were largely “wonders of creation” cosmographies, such as Qazwīnī's *ʿAjāʿib al-Makhlūqāt* or Ahmet Bican's *Dürr-i meknûn*, and contained only stylized world maps.⁶⁶ Mehmed II is also known to have sponsored the copying of a set of Arabic geographic manuscripts containing stylized, decorative maps,

64. For example, Shirwānī's reported value for Samarqand's longitude was 99°, matching the value from *Zīj-i Īlkhānī*, whereas Yāqūt reported 89°30'.

65. Giancarlo Casale, *The Ottoman Age of Exploration* (New York: Oxford Univ. Press, 2010), 15-22.

66. On the material and social history of *ʿajāʿib* literature, see Persis Berlekamp, *Wonder, Image, and Cosmos in Medieval Islam* (New Haven: Yale University Press, 2011). Bican's Ottoman Turkish text with critical apparatus and commentary in German is available as Ahmet Bican, *Dürr-i meknûn: kritische Edition mit Kommentar*, ed. Laban Kaptein (Asch: Laban Kaptein, 2007).

however they were so stylized that they would be largely useless for military or political purposes.⁶⁷ At least with respect to the Indian Ocean region, it is unlikely that detailed geographic information circulated among Ottoman scholars prior to the sixteenth century.⁶⁸ It is true that portolan maps used for navigation in the Mediterranean did spread from Europe to the Islamic world in the fourteenth and fifteenth centuries, but these did not include much detail beyond coastlines.⁶⁹ The other significant source of cartographic information during the reign of Mehmed II was the “rediscovery” of Ptolemy’s *Geography* by Italian humanists and subsequent production of new “world maps.” Although Ptolemy’s *Geography* had been known to Arabic scholars since the ‘Abbāsīd era, it stimulated rather more interest in mathematical geography than in the production of maps. Thus, rather than drawing on an extant tradition of Arabic cartography, in 1465 Mehmed II commissioned from George Amirutzes of Trebizond a translation of the *Geography* along with a single world map that would combine the text’s regional maps, of which he also sought to obtain Italian reproductions.⁷⁰ In light of the sultan’s interests, Shirwānī may have chosen to include of a vast trove of geographic information in his commentary, including approximate measurements of the area of different regions along with

67. Karen Pinto, “The Maps Are the Message: Mehmet II’s Patronage of an ‘Ottoman Cluster’,” *Imago Mundi* 63, no. 2 (2011): 155-179.

68. Casale, *The Ottoman Age of Exploration*, 17-18.

69. By the 16th c. at least, geographic information was also flowing in the opposite direction. For example, oral sources likely played a role in improving European maps of Anatolia and Western Asia. See Sonja Brentjes, “Giacomo Gastaldi’s Maps of Anatolia: The Evolution of a Shared Venetian-Ottoman Cultural Space?,” in *The Renaissance and the Ottoman World*, ed. Anna Contadini and Claire Norton, (Burlington, VT: Ashgate, 2013), 123-141.

70. Ahmet Karamustafa, “Military, Administrative, and Scholarly Maps and Plans,” in *History of Cartography*, Volume 2, Book 1: *Cartography in the Traditional Islamic and South Asian Societies*, ed. J. B. Harley and David Woodward, (Chicago: University of Chicago Press, 1992), 209-210.

coordinates and travel times between cities, because he believed it to be highly desirable information for Ottoman elites.⁷¹ Shirwānī's incorporation of material from Yāqūt's *Mu'jam al-Buldān* into his *ḥāshiya* on the *Mulakhkhaṣ* represents one piece of evidence about early Ottoman access to Arab geographic manuscripts prior to the sixteenth century, although we cannot determine whether Shirwānī owned a copy of Yāqūt's text or whether he took notes from it at some point during his travels. It is a fascinating example of the transmission of knowledge made possible by the movement of madrasa scholars through networks connecting Istanbul to centers such as Cairo and Samarqand.⁷²

Shirwānī's work between astronomy, philosophy, and theology

On a practical level, astronomy (solar and lunar theory) was necessary to determine the times for the five daily prayers (defined in terms of shadow lengths and the Sun's position throughout the day), predicting the first visible crescent after the new Moon that marked the beginning of a new month, and establishing an accurate calendar. These activities ensured that astronomy had a significant amount of social utility, and often provided employment for astronomers as official

71. However, as we know today the data was rather useless for making maps since accurate measurements of longitude were incredibly difficult in the premodern period. Furthermore, this analysis of Shirwānī's data supports David King's observation that many collections of geographic coordinates reflected "an uncritical handling of a superfluity of uncontrolled data, without regard to the reliability of the sources, which could not be checked anyway." David King, "Mathematical Geography in Fifteenth-Century Egypt: An Episode in the Decline of Islamic Science," in *Islamic Thought in the Middle Ages: Studies in Text, Transmission and Translation, in Honour of Hans Daiber*, ed. Anna Akasoy and Wim Raven, (Leiden: Brill, 2008), 340-341.

72. On the influence of Cairene scholars on Anatolian intellectual circles, see Louise Marlow, "A Thirteenth-Century Scholar in the Eastern Mediterranean: Sirāj al-Dīn Urmavī, Jurist, Logician, Diplomat," *al-Masāq* 22, no. 3 (2010): 279-313 and Sara Nur Yildiz, "From Cairo to Ayasuluk: Hacı Paşa and the Transmission of Islamic Learning to Western Anatolia in the Late Fourteenth Century," *Journal of Islamic Studies* 25, no. 3 (2014): 263-297.

“timekeepers” (*muwaqqit*) at the community mosque. The desire to reform the calendar and to produce more accurate astronomical tables to aid in future predictions helped convince rulers to support observatories staffed by astronomers and mathematicians at Baghdad, Marāgha, Samarqand, and Istanbul.

Another “practical” use of mathematical astronomy in the medieval world was astrology. For example, at Marāgha one of the reasons that the Ilkhanids may have been interested in sponsoring the observatory and efforts of scholars like Naṣīr al-Dīn al-Ṭūsī was the perceived similarity between Mongol shamanistic practices and the practice of astrology in the Hellenistic and Islamic traditions.⁷³ The influence of the heavens on the terrestrial region is undeniable, and arguments justifying astrology began in antiquity. We find an excellent summary of these ideas in Ptolemy’s *Tetrabiblos*, where he pointed out the clear evidence of the Sun’s effect on the weather and on plants, the Moon’s influence on the tides, and so forth.⁷⁴ Therefore it stood to reason that other objects in the heavens had perhaps more subtle, but no less genuine, effects on the inhabitants of the terrestrial region. These sorts of views were elaborated, challenged, and defended throughout the ancient and medieval Greek, Arabic, and Latin-speaking worlds. The astrologer’s actual ability to put these theories into practice was challenged throughout history as well, with critics frequently pointing out the differing fates of identical twins or the great

73. For a discussion of this Mongol “filtering” process of cultural appropriation, see Thomas Allsen, *Culture and Conquest in Mongol Eurasia* (New York: Cambridge University Press, 2001), 203-209. In contrast, George Saliba has argued that astrology played almost no role in the decision to establish the Maragha observatory, see George Saliba, “Horoscopes and Planetary Theory: Ilkhanid Patronage of Astronomers,” in *Beyond the Legacy of Genghis Khan*, ed. Linda Komaroff, (Leiden: Brill, 2006), 357-368.

74. Claudius Ptolemy, *Tetrabiblos*, ed. and tr. Frank Robbins, (Loeb Classical Library, 1940).

difficulty in determining the precise moment of conception.⁷⁵ Nevertheless, the idea of a relationship of “influence” (literally, influxes) from the heavens to the Earth was a powerful one, and rulers often patronized astrologers for their skill as advisors or for casting horoscopes to determine the most propitious moment for an event such as a battle or the foundation of a new city (e.g., Baghdad).⁷⁶ To be sure, critics of astrology among Islamic theologians and other scholars had additional arguments at their disposal. Shirwānī’s predecessors’ had carried on wide-ranging discussions of theological (*kalām*) and natural philosophical arguments about causality and God’s absolute power to determine what occurs in His creation. There were many questions and disagreements about the idea that the stars and planets could have any sort of causal power or ability to influence the course of events in the terrestrial region. This idea of influence was nonetheless supported with arguments: although God was the ultimate cause, some scholars defended the notion of secondary causality, in which God gave powers to celestial spheres, spirits, intellects, or other entities that themselves carried out causes and actions that ultimately were attributable to God. As Robert Morrison has shown, Nīsābūrī, the author of the widely read *Tadhkira* commentary that Shirwānī had studied at Samarqand, accepted the validity

75. See, for example, A.A. Long, “Astrology: Arguments Pro and Contra,” in *Science and Speculation: Studies in Hellenistic Theory and Practice*, ed. Jonathan Barnes et al., (Cambridge: Cambridge University Press, 1982), 165-192.

76. For example, several sources attest to the ‘Abbāsīd caliph al-Manṣūr’s (r. 754-775) interest in astrology, including choosing the day to lay the foundations of Baghdad based on the astrologer Nawbakht’s advice. See Dimitri Gutas, *Greek Thought, Arabic Culture: The Graeco-Arabic Translation Movement in Baghdad and early ‘Abbasid Society* (New York: Routledge, 1998), 28-34.

of astrology to a certain extent and argued that God chose to control terrestrial events through intermediaries.⁷⁷

Nīsābūrī discussed such issues of secondary causality and astrology in his theological texts and his commentary on the *Zīj-i Īlkhānī* (Ṭūsī's Marāghan astronomical tables), rather than in his *Tadhkira* commentary.⁷⁸ It is therefore not surprising that little evidence of Shirwānī's opinions on astrology or God's involvement in the motions of the heavens appears in his *Sharḥ al-Tadhkira*. However, a few suggestive hints do surface from time to time. In his comments on *Tadhkira* II.6, when discussing the question of whether the solar apogee moved separately from the precession of the fixed stars, one of the opinions Shirwānī cited was that of Abū Naṣr Yaḥyā b. Jarīr (d. 1103/4). Yaḥyā had argued that if the apogee moved, it would cause wondrous effects in the world. For Shirwānī, the fact that those effects were not observed to be occurring may have confirmed that the apogee had no separate motion.⁷⁹ Without such an assumption, it is difficult to account for his attention to Yaḥyā's remark.

However, astronomy could not answer every question about the motions and effects observed in the world. In Chapter 2, we analyzed Shirwānī's extended remarks on the possibility of attributing the daily rotation of the heavens to the Earth, with the Earth rotating around its axis once in approximately 24 hours in the midst of the stationary heavens. After discussing various proposals and counter-arguments from ancient and Islamic astronomers, Shirwānī ultimately endorsed Ṭūsī's statement that the question of the Earth's motion could not be settled on the

77. See Robert Morrison, *Islam and Science: The Intellectual Career of Nizām al-Dīn al-Nīsābūrī* (New York: Routledge, 2007), especially Ch. 6.

78. Morrison discusses the *Kashf-i Haqā'iq*, Nīsābūrī's commentary on Ṭūsī's *zīj*, in Morrison, *Islam and Science*, Ch. 4.

79. Shirwānī, *Sharḥ al-Tadhkira*, fol. 187b.

basis of observations. Instead, this issue became one of a handful in which astronomy needed to rely on natural philosophy. At this point, Shirwānī made some additional remarks, also drawing on arguments from natural philosophy, to dispute alternative justifications for the Earth's hypothetical motion. He rejected the possibility of the Earth's having a forced, rather than natural, circular motion, since forced motions come to an end. If the Earth were rotating, it must be an eternal motion. Furthermore he rejected the suggestion that the Earth could have a separate motion like the orbs, because the orbs' motions were voluntary (*irādīya*), not forced (*qasrīya*).⁸⁰

These references to natural philosophy led Shirwānī into a discussion of Jurjānī's remark that this issue of the Earth's motion was shared by the mathematical sciences and natural philosophy. Obviously astronomy dealt with topics that are both mathematical and physical. The broader debate, however, was over the extent to which the premises of astronomy were connected to, or dependent on, natural philosophy. The difference between these approaches was in their manner of proof. The mathematical sciences relied on "observation and testing (*al-raṣd wa-l-i'tibār*)" to establish their facts whereas natural philosophy produced demonstrative proofs (*al-burhān al-limmī*), the "why" of nature or the "reasoned fact." Shirwānī returned to this topic a few pages later when he discussed Ṭūsī's statement that the proofs in this section (which cover the sphericity of the heavens, Earth and water, and the Earth's centrality in the cosmos and imperceptibility relative to the celestial orbs) were proofs that convey existence (*hādhihi l-adilla inniya tufīd al-wuqū'*); those [proofs] that convey the necessity of that existence (*allati tufīd wujūb al-wuqū' min al-limmiyāt*) were given in Aristotle's *De caelo*. As Ragep has argued, Ṭūsī's use of these terms traced back to Aristotle's distinction in the *Posterior Analytics* I.13

80. Shirwānī, *Sharḥ al-Tadhkira*, fol. 70b. Shirwānī was following Ibn Sīnā (and Aristotle) in referring to the motion of the celestial spheres as "voluntary" rather than "forced."

between the fact and the reasoned fact, although Ṭūsī was using them in a somewhat different way from Aristotle because they were occurring in different disciplines.⁸¹ *Innī* proofs, from the Arabic word *inna* (*that*) = “proof of the that,” were used in astronomy to provide proofs of the configurations of the simple bodies based on observations. Conversely, *limmī* proofs, from the Arabic word *limā* (*why*) = “proof of the why,” were used in natural philosophy to provide proofs of why a particular configuration occurs and not some other.

Shirwānī’s discussion of these terms makes it clear that he and other Islamic astronomers were using this crucial distinction as a means of defining astronomy as a separate discipline from philosophy, to the extent that for the most part they use different terms from Islamic philosophy to distinguish between the kinds of proofs. An *innī* proof based on observations is called a *dalīl* which translates as “proof, indication, or sign” whereas a *limmī* proof based on deductions from *a priori* premises is referred to by the stronger term *burhān* or “demonstration, syllogistic proof.”

As mentioned in Chapter 1, Ragep has argued that the concern to define astronomy as a separate discipline, largely (although not entirely) built on its own proofs rather than on those of natural philosophy, was most likely a reaction to certain religious objections to Hellenistic metaphysics.⁸² But it is important to note that this influence of theology on science was not a simple case of opposition. Rather, in the best sense of constructive criticism, it led astronomers to rethink their premises and to explore alternatives. For example, Shirwānī’s contemporary in Samarqand and Istanbul, ‘Alī al-Qūshjī, rejected Aristotelian causality and natural philosophy, at

81. F. Jamil Ragep, *Naṣīr al-Dīn al-Ṭūsī’s Memoir on astronomy = al-Tadhkira fi ‘ilm al-hay’a*, 2 vols. (New York: Springer-Verlag, 1993), 38-41 and 386-388.

82. F. Jamil Ragep, “Freeing Astronomy from Philosophy: An Aspect of Islamic Influence on Science,” *Osiris* 2nd Series, no. 16, *Science in Theistic Contexts: Cognitive Dimensions* (2001): 49-64+66.

least in astronomy. As a result, although he agreed with Ṭūsī and Shirwānī that the Earth's rotation could not be determined on the basis of observations, there was no need to rule it out on the basis of principles from natural philosophy. Qūshjī thus concluded that “nothing false follows [from the assumption of a rotating Earth]” – as Ragep has pointed out, Qūshjī's position was quite similar to Copernicus's and if a more compelling physics could have been developed Qūshjī seemed prepared to accept a rotating Earth.⁸³

Turning to another example of Shirwānī's commentaries as a site of engagement with the broader intellectual and cultural tradition, let us consider his suggestion that science provided a way to better know and understand God's creation. The Āyat al-Kursī, or Throne Verse (Qur'ān 2:255), is one of the most famous Qur'ānic verses for its depiction of the unity of God and his power over the entire universe:

Allāh - there is no deity except Him, the Ever-Living, the Sustainer of [all] existence. Neither drowsiness overtakes Him nor sleep. To Him belongs whatever is in the heavens and whatever is on the earth. Who is it that can intercede with Him except by His permission? He knows what is [presently] before them and what will be after them, and they encompass not a thing of His knowledge except for what He wills. His Throne (*kursī*) extends over the heavens and the earth, and their preservation tires Him not. And He is the Most High, the Most Great.

Shirwānī wrote an entire commentary on this verse early in his career, after arriving in Anatolia from Samarqand, and dedicated it to the Ottoman grand vizier Halīl Paşa in 1453. It is a rather long commentary (around 140 pages) in which Shirwānī devotes approximately half of his

83. Ragep, “Freeing Astronomy from Philosophy,” 62. Recently, in a dissertation I have not read, Mustapha Kara-Ali has argued that Qūshjī developed a new constructivist epistemology as an alternative to Aristotle. Mustapha Kara-Ali, “Constructivism At the Birth of the Scientific Revolution: A Study of the Foundations of Qushji's Fifteenth Century Astronomy” (PhD diss., International Islamic University Malaysia, 2014).

discussion to the first phrase on the unity of God. Towards the end of that long discourse on the meaning of God's unity and various aspects of belief, Shirwānī discussed several sects, heresies, and variants of polytheism. Among them were the Sabians, a pre-Islamic religious group mentioned in the Qur'ān along with Christians and Jews. They were often considered to be "People of the Book," having their own scriptures like the Christians and Jews and eligible for special status under Islamic law, but their religious practices were interpreted as a form of star-worship. Shirwānī explained that the star worshippers held a number of false beliefs. They ascribed souls to each of the celestial orbs that rule or govern their motions. For example, they believed the outermost orb is moved by something they called the "Universal Soul" from which diverged the various other souls that moved the lower orbs. Beyond that, they imagined, for each degree of the heavens, souls that caused effects when the Sun reaches that point. Each day and minute is therefore affected, and all the parts of the earth, and plants and animals are governed by these souls.⁸⁴

Categorizing these concepts as heretical would appear to be a strong rejection of the principles of astrology (and by association, an astronomy that posited the existence of alternative causes for the motion of the spheres other than God's direct control), but in fact a more nuanced position allowed for the acceptance of astrology within an Islamic framework. Shirwānī distinguished between the belief that God exerted control over the world through the stars, and the heretical practice of star worship or the idea that one could make actionable forecasts based on celestial motions. Later, when commenting on the Qur'ānic phrases regarding God's

84. Faṭḥallāh al-Shirwānī, *Tafsīr Āyat al-Kursī*, Beyazıt State Library MS 628, fol. 39b-40a.

knowledge, Shirwānī explained that since God's knowledge was perfect and encompassed all of Creation, such actions must take place under his direct divine control.⁸⁵

Shirwānī returned to the theme of God's relationship to celestial motions the end of his commentary on the Throne Verse when he explained that God was known as the "Most High, Most Great" because He was the sole necessary existent in the Universe, beyond the heavens and the Earth, and all other things in the world occurred according to His determination, an interpretation that opened the route for accepting the principle that God used the celestial bodies to cause effects in the world. In addition, when commenting on Ṭūsī's statement in *Tadhkira* II.9 that the superior planets at the mean apices of their epicycles are always aligned with the mean Sun, Shirwānī added the remark, "at the midpoint of [the planets'] direct motions, by God's decree (*bi-taqdīrihi ta 'ālā*)."⁸⁶ This hidden regularity in the motions of the heavens could only have come about through God's will, and for Shirwānī it was evidence that astronomy provided a means to better know His creation.

In Shirwānī's discussion of the Qur'ānic phrase *To Him belongs whatever is in the heavens and whatever is on the earth*, he reconciled an apparent contradiction between references elsewhere in the Qur'ān to the "seven heavens" and the astronomers' statement that there were 9 celestial orbs. He explained that the astronomers' 8th and 9th orbs (which correspond respectively to the precession of the fixed stars and the daily [once per 24 hours] rotation of the fixed stars) were known by theologians and referred to in the Qur'ān as God's Throne (*'arsh*) and Footstool (*kursī*). According to Shirwānī's interpretation, the fact that the Qur'ānic verse listed the heavens before the Earth was an indication the heavens were nobler (*ashraf*) and there

85. Shirwānī, *Tafsīr Āyat al-Kursī*, fol. 59a-b.

86. Shirwānī, *Sharḥ al-Tadhkira*, fol. 224b.

should be a greater interest in the heavens because they demonstrated God's greatness (*dalla 'alā 'azmaṭihi ta 'ālā*). As explained above, Shirwānī's use of the term *dalla/dalīl* here signified evidence or indication, rather than philosophical proof.⁸⁷

Shirwānī returned to this line of interpretation when he listed various interpretations of the phrase *His Throne extends over the heavens and the earth*. Although the primary interpretation of the "throne" was of course metaphorical, symbolizing God's rulership over all of Creation, another meaning of the phrase referred to a great body encompassing the heavens and the Earth. Shirwānī explained that this is in fact the well-known equinoctial orb, or the sphere of the fixed stars. We can infer that Shirwānī upheld this potentially controversial belief because he added a remark to the reader that "many [scholars] affirm this [interpretation], as you know" (*wa qad jazama jam 'a bi-hādhā, kamā 'arafta*).⁸⁸

The theme of using astronomy to better understand God's creation also appeared in the introduction to Shirwānī's *Tadhkira* commentary, which was full of Qur'ānic verses referring to celestial phenomena.⁸⁹ To just list a few, Shirwānī quoted 21:33 – *It is He who created the night and the day and the sun and the moon; all [heavenly bodies] swimming in an orb (falak)*. He also quoted 10:5 – *It is He who made the sun a shining light and the moon a derived light and determined for it phases/mansions*. Shirwānī inserted the number 28 after mansions, a reference to the astronomers' method of dividing the lunar zodiac into segments corresponding to the moon's progress each day of the lunar month, and used as part of the calendar system. As mentioned in Chapter 3, Shirwānī added an extensive discussion of timekeeping and calendars,

87. Shirwānī, *Tafsīr Āyat al-Kursī*, fol. 55a.

88. Shirwānī, *Tafsīr Āyat al-Kursī*, fol. 61a.

89. Shirwānī, *Sharḥ al-Tadhkira*, fol. 1b.

including descriptions of the comparative calendar systems of various peoples and cultures, as a supplement to the *Tadhkira*.

Shirwānī referred as well to 65:12 – *It is He who has created seven heavens and the Earth*, 25:61 – *who has placed constellations in the sky*, and 37:6 – *adorned them with stars/planets*. He noted 50:6 – *[the heavens] have no rifts/flaws*. Indeed, Shirwānī referred to the Qur’ān when he explained that the science of astronomy (*hay’a*) provides the highest knowledge (a play on astronomy’s knowledge of the heavens and also God’s creation), quoting verse 3:191 – *[Those who] give thought to the creation of the heavens and the earth, [say] ‘Our Lord, You did not create this in vain.’*

Shirwānī filled his introduction with these Qur’ānic quotations not only to demonstrate his knowledge of the Qur’ān, but to make the not-so-subtle point that the kind of knowledge produced by astronomy was divinely sanctioned, and even encouraged. God created all of the celestial phenomena as signs for humans to interpret. It was God’s will that regulated their motions and accounted for their perfection, rather than simply natural properties as bodies of Aristotelian aether. For Shirwānī, understanding the rational principles underlying the apparently irregular motions of the heavens would lead the reader to a deeper appreciation and sense of wonder at God’s creation.

However, because non-astronomers in particular did not always perceive astronomy, astrology, and natural philosophy as separate disciplines, mathematical astronomy was vulnerable to the criticism that it rested on theologically questionable principles. The Arabic terms *falsafa* and *kalām*, which can be translated as “natural philosophy” and “dialectical theology,” respectively, represent two rival ontologies. Ibn Sīnā, perhaps the most famous representative of *falsafa* as a Hellenistic-inspired philosopher, developed a synthesis of

Aristotelian and Neoplatonic thought that he felt was compatible with, and provided solid support for, Islamic revelation. In explaining the composition of the natural world, Ibn Sīnā adopted the Aristotelian framework of the four elements (Earth, Water, Air, and Fire) and argued that their natural motions (down in the case of Earth and Water, up in the case of Air and Fire) were the result of inherent properties instilled in them by God. In contrast, *kalām* scholars who sought to preserve the absolute omnipotence of God were unwilling to concede that the elements had even this level of secondary causality. They developed an alternative “occasionalist” metaphysics in which the world consisted of attribute-less matter that God constantly provided with accidents at every instant.⁹⁰

As *kalām* practitioners developed more sophisticated theories, they began asking the same sorts of questions about the world and adopting terminology and methods from *falsafa* in their attempts to subject the revealed truths of Islam to rational inquiry. A.I. Sabra has described this transformation as part of the process by which the “foreign” rational sciences became “naturalized” in Islamic societies.⁹¹ *Kalām* criticisms of the epistemology presented by Islamic philosophers like Ibn Sīnā led some later scholars, particularly those associated with Ulugh Beg’s Samarqand madrasa, to reanalyze the relationship between external reality and mathematical concepts. They promoted what has been referred to as “mathematical humanism” –

90. For excellent summaries of these positions, see the chapters by Robert Wisnovsky (on Ibn Sīnā) and Michael Marmura (on al-Ghazālī) in Peter Adamson and Richard Taylor, *The Cambridge Companion to Arabic Philosophy* (Cambridge: Cambridge University Press, 2005) and Alnoor Dhanani, *The Physical Theory of Kalām: Atoms, Space, and Void in Basrian Mu‘tazilī Cosmology* (Leiden: Brill, 1994).

91. See in particular A. I. Sabra, “The Appropriation and Subsequent Naturalization of Greek Science in Medieval Islam: A Preliminary Statement,” *History of Science* 25, (1987): 223-243 and A. I. Sabra, “Science and Philosophy in Medieval Islamic Theology: The Evidence of the Fourteenth Century,” *Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften* 9, (1994), 15-23.

an epistemology based on human cognition and mathematical rather than physical and philosophical principles.⁹² The result was a hybrid of *kalām* metaphysics and mathematics as a way of understanding nature. In contrast with Ibn Sīnā, who argued that mathematical entities existed in the mind and were therefore only conjectural, Qāḍīzāde, ‘Alī al-Qūshjī, and other 15th c. scholars argued that mathematical entities and models nevertheless corresponded to reality in a way that provided true knowledge. According to İhsan Fazlıoğlu, these scholars developed the concept of *naḥs al-amr* (“the thing itself” or “the fact of the matter”) to encompass both the external and the mental, such that *naḥs al-amr* “was considered to encompass that which is true in both the external and mental worlds, thus being an objective world comprising ultimate reality.”⁹³

In the majority of his commentary on the *Tadhkira*, when philosophical topics could not be avoided Shirwānī tended to follow Ṭūsī in expressing Aristotelian/Avicennan principles, rather than following his teacher Qāḍīzāde’s more mathematical approach. However, the concept of *naḥs al-amr* appeared at times as well, and the extent to which this “mathematical humanism” played a role in Shirwānī’s understanding of astronomy is an intriguing question for additional research. For the time being it will suffice to mention a few instances of this concept in Shirwānī’s commentary. When Ṭūsī stated that astronomers assigned the daily rotation of the heavens to the highest orb because it was without stars, Shirwānī stated that it was in fact

92. These developments in post-classical Islamic intellectual history are the subject of İhsan Fazlıoğlu’s current research. For a stimulating discussion of his recent findings, see İhsan Fazlıoğlu, “Between Reality and Mentality: Fifteenth Century Mathematics and Natural Philosophy Reconsidered,” *Nazariyat Journal for the History of Islamic Philosophy and Sciences* 1, no. 1 (2014): 1-39.

93. Fazlıoğlu, “Between Reality and Mentality,” 28. Kara-Ali, “Constructivism At the Birth of the Scientific Revolution: A Study of the Foundations of Qushji’s Fifteenth Century Astronomy,” which I have not read, may also discuss the role of *naḥs al-amr* in Qūshjī’s thought.

possible for this orb to have stars in *naḥs al-amr*. In response to those who argued that if the orb had stars we would see them, he replied that the stars could be present but too small to be visible.⁹⁴ When discussing the Moon's epicycle, Shirwānī referred the size of its radius as “*al-miqdār al-naḥs al-amrī*” because it was calculated on the basis of observations to be $5\frac{1}{4}$ parts (where the radius of the inclined orb was 60 parts). Thus it was a mathematical model that nevertheless provided a true account of reality.⁹⁵ In II.10, which dealt with the latitudes of the planets, when Ṭūsī listed the maximum inclinations of each planet, Shirwānī clarified that the first sets of numbers were the greatest inclinations of the epicycles, stated as they are in *naḥs al-amr*, with respect to the epicycle center; rather than the inclinations we actually observe from Earth, which were generally smaller and which Ṭūsī listed in the following section.⁹⁶ Finally, commenting on Ṭūsī's statement that one does not perceive parallax for planets far from the Earth, Shirwānī noted that their parallaxes were investigated in *naḥs al-amr* and they had the same mathematical definition as the observable parallaxes for the near planets.⁹⁷ These passages suggest that mathematics and logic, perhaps as a supplement to the natural philosophical justifications set forth in the *Tadhkira*, played a role in strengthening Shirwānī's commitments to the knowledge produced by astronomy and the reality of the configuration of celestial orbs.

94. Shirwānī, *Sharḥ al-Tadhkira*, fol. 78b-79a.

95. Shirwānī, *Sharḥ al-Tadhkira*, fol. 200b.

96. Shirwānī, *Sharḥ al-Tadhkira*, fol. 230a.

97. Shirwānī, *Sharḥ al-Tadhkira*, fol. 249a.

Conclusion

This study of ancillary scientific material contained in Shirwānī's astronomical commentaries highlights the value in reconsidering commentaries as sources for new narratives about science, religion, institutions, and the circulation of knowledge in the post-classical Islamic period.

Shirwānī's engagement in his astronomical commentaries with important sources from related fields, such as Ibn al-Haytham's *Optics* and Yāqūt al-Ḥamawī's *Dictionary of Countries*, suggests that attention to other commentaries has the potential to reshape our understanding of the scientific interests of scholars and elites alike. His defense of astronomy as being in harmony with theological teachings, and indeed as providing a means to better understand God's creation, placed Shirwānī squarely in the camp of post-classical figures for whom scientific and religious scholarship coexisted peacefully. It must be emphasized that he was writing several centuries after al-Ghazālī (d. 1111), who according to many proponents of the "decline" narrative had brought all such thinking to an end and marked the end of notable scientific activity in Islam.⁹⁸ Shirwānī incorporated "new" material in these commentaries, made this material relevant to students of astronomy as well as a broader audience at the Ottoman court, and offered hints of new epistemological approaches. All this demonstrates that his commentaries, and presumably those of many other unstudied authors, could function as sites of intellectual engagement with the scientific tradition as well as broader social and cultural concerns.

98. For a detailed, convincing rebuttal to the widely-repeated claim that Ghazālī rejected the rational sciences, see Frank Griffel, *Al-Ghazālī's Philosophical Theology* (Oxford: Oxford University Press, 2009).

Conclusion

When Faṭḥallāh al-Shirwānī left Samarqand in 1440, little did he know his future travels would bring him to Cairo, Baghdad, and Mecca, or that he would witness the Ottoman conquest of Constantinople. Over the course of his career as a traveling scholar, he must have influenced the lives of scores, if not hundreds, of students who are now anonymous. It is amazing to map the distances he traveled in his lifetime, tally the mix of languages and cultures he encountered, and consider that, in many ways, countless other contemporary scholars, merchants, diplomats, and pilgrims shared similar travel experiences. We are fortunate to have some record of his experiences as a teacher, and as a student at Ulugh Beg's Samarqand madrasa, preserved in his astronomical commentaries.

We are now in a position to appreciate that Shirwānī's commentaries attest to a high level of scientific and intellectual activity in a period formerly dismissed as in "decline." Most importantly, they remind us that writing a commentary could be a transformative process. Rather than simply explicating the base text, a commentator of Shirwānī's caliber could incorporate and synthesize new material, and create a dialogue between the present and the past that the reader was invited to join. Shirwānī's commentary on Ṭūsī's *Tadhkira* aptly illustrates this transformative process. His writing brought into conversation several of the most significant contributors to the ongoing *hay'a* project of reforming Ptolemaic astronomical models, including Quṭb al-Dīn al-Shīrāzī, Nizām al-Dīn al-Nīsābūrī, al-Sayyid al-Sharīf al-Jurjānī, Qāḍīzāde al-

Rūmī, and – not least – Sultan Ulugh Beg. Shirwānī supplied alternative solutions, reported and assessed previous contributions, and drew comparisons between the ideas of these and other scholars. The technical additions found in Shirwānī’s commentary turned it into a significantly more advanced text, one that was aimed at an audience of trained experts and students of the mathematical sciences that was narrower than the readership of Ṭūsī’s original work.

Like many of his contemporaries, Shirwānī had developed a level of expertise that was not limited to a single field or discipline. This is amply indicated by his incorporation into the *Tadhkira* commentary of material from Ibn al-Haytham’s *Optics*. For Shirwānī, knowledge of optics was preliminary to the study and practice of astronomy, which, like optics, was a “mixed science” drawing from both mathematics and natural philosophy. As I have shown, Shirwānī’s purpose for including this discussion of optics was to present a “correct” understanding of light and vision rather than the presumably more-commonly known theories represented by authors such as Euclid, al-Kindī, and Ibn Sīnā. In addition, he highlighted areas in which optics directly related to astronomy, including the effect of refraction while making observations and the analysis of light and shadows during eclipses. Shirwānī’s engagement with Ibn al-Haytham’s *Optics* is a rare example of a premodern scholar in the Arabic tradition who was directly familiar with its contents and results, and raises new questions about the transmission of the *Optics* and its study elsewhere in the premodern Islamic world that await further research.

I have argued that Shirwānī’s commentaries reflect the educational practices employed at the Samarqand madrasa, as well as in many other premodern Islamic contexts. These practices included an emphasis on the mastery of particular texts and the teacher-student relationship, rather than an institutional curriculum or a set of formal requirements. Training in astronomy involved many of the same methods as the transmission of other forms of knowledge: close

reading of a text in a group, with discussion with and commentary by an authoritative teacher, and debates or formal disputations in the lecture hall. Its excellence and patronage by a scholar-sultan no doubt make the Samarqand madrasa a unique case in some ways, but its similarities with known educational practices elsewhere should remind us that astronomy and other sciences could easily be integrated into formal madrasas and informal study circles elsewhere.

Among the most remarkable idiosyncrasies in Shirwānī's commentaries are his rare "oral" testimonies about Ulugh Beg and Qāḍīzāde, two of the most important figures in post-classical Islamic science. His anecdotes emphasize the crucial role that Ulugh Beg played in shaping the day-to-day lessons at Samarqand, with a focus on technical details, criticisms, and argument, not abstract displays of erudition. The sultan himself participated directly in the analysis and criticism of astronomical models during Qāḍīzāde's lectures. This information adds a new dimension to our picture of Ulugh Beg and the Samarqand astronomers, who are most famous for their interest in observational astronomy and their production of a revised star catalog and new astronomical tables. However, Shirwānī's anecdotes reflect that fact that theoretical topics were of great interest, and attempts to develop new non-Ptolemaic astronomical models were the subject of intensive discussion at Samarqand.

Shirwānī's critical engagement with earlier works in the *hay'a* tradition, along with the wide range of other topics he discussed in his commentaries on the *Tadhkira* and other texts, provides compelling evidence that commentaries can reshape our understanding of historical developments in specific fields, and in Islamic intellectual history writ large. Once they are no longer viewed as symptomatic of intellectual decline, the vast number of post-classical Islamic commentaries currently sitting unread in archives around the world become a rich resource for analyzing a crucial period in world history.

Appendix – Arabic text of Shirwānī's classroom anecdotes

١ – وكنت من السامعين لشرح النظام على شيعي وأستاذي قاضي زاده الرومي ذلك الامام الذي لم يسمح بمثله الأدوار ما دار الفلك الدوار بسمرقند عند قراءة بعض أعالي مجلسه الشريف معروفا بالذكاء والعلاء في مدرسته وإليها السلطان بن السلطان ألغ بيك كوركان بعد أن كنت قرأت شرح السيد الشهاب الثاقب على السيد الأيد أبي طالب بمشهد الإمام علي الرضا رضي الله عنهم أجمعين.

فقال السلطان ذلك الفيلسوف يوما وقد حضر مجلس الدرس وكان يحضره في أسبوع يوما أو يومين ووصل الدرس إلى هذا المحل: «لم قال الكائنة في سطح وما الفائدة في التقييد به؟»

فيادر أحد المدرسين في مدرسته وكانوا يحضرونه مع حضوره إلى الجواب وقال: «لأن أفليدس برهن في كتابه على أن الخطوط المتوازية لا تكون إلا في سطح واحد.»

وكنت يومئذ فارغا بتوفيق الله تعالى من مطالعة اثنتي عشرة مقالة وحلها من ذلك الكتاب مستحضرا لها ولم يطلع على حالي ذلك أحد إذ كنت أطالعه في أيام العطلة مستخفيا فوقفت على خطائه ورجمه بالغيب. وكان الظن أن السلطان أيضا أطلع عليه إذ كان مستحضرا لذلك الكتاب بل لسائر فنون الرياضي أصوله وفروعه لكنني لم أبادر إلى تخطئته والرد عليه كي يتقرر خطأؤه في الأسماع إذ كان من ديدن السلطان ودأبه أن يعود إلى التفتيش ثانياً أو ثالثاً وكأنه في ذلك كان ممتحنا بنفسه لمن في مدرسته بعد أن كان عينَ لامتحان طلبتها المرتبين أعالي وأواسط وأداني لبعض المدرسين خمسين ولبعضهم أربعين جماعةً غيرهم من عدول الأفاضل وفحول الأمثال كان الحيف عندهم فيمن يدين كالكفر في الدين.

فلما أفاض القارئ بعد مكث في قراءة تعريف السطوح المتوازية أض السلطان إلى الفتش بعاد المجيب إلى ما بادر إليه أولاً فقلت حينئذ: «بل الأمر في ذلك الكتاب على خلاف ما قلت.»

فأشرب السلطان لي وطأ من القارئ رأسه إذ كان بيني وبين السلطان على سمت شعاع البصر قريبا منه ومن الأستاذ وكانوا يقدون على زوايا مثلث كل ضلع من أضلاعه ذراع تقريبا فقال: «كيف ذلك؟»

قلت: «قال أفليدس في تاسع الحادية عشرة من كتابه الخطوط المتوازية لخط وإن لم تكن جميعا في سطح فهي متوازية ثم برهن عليه.»

فتوجه إلى الأستاذ مصدقا لي ومستحسنا لمقالي فوق مرتين ثم توجه هكذا إلى كل من ارتضاه من قاعد أو قائم فإن كثيرا من فحول العلماء الملازمين له كانوا يوم حضوره قائمين بين يديه. وكان المجيب ساعته مطاطي الرأس ناكسه فلما فرغ السلطان من ذلك الفعل الجميل رفع هو رأسه فقال: «لعل ما قال أفليدس في تلك المقالة هو أن كل زاويتين توازت أضلاعهما النظائر ولم يكن الجميع في سطح فهما متساويتان لا ما نقلته.»

قلت: «هذا الشكل يلي ذلك ويتلوه.»

فأعاد السلطان جمليه فجاء إفحامه على أبلغ وجه فختم السلطان الكلام على الإفحام فقام. وكان ذلك اليوم آخر أيام التحصيل من الأسبوع فعزل في ذلك الأسبوع القارئ من القراءة ونصيني لها مكانه فابتدأت في أول الأسبوع التالي لذلك الأسبوع من تعريف السطوح المتوازية. فمرض المعزول كثيرا ثم جلا إلى هرة وتوارى المفحم أكثر من شهر ثم خرج فظهر الحيوية.

ورأيت وجه الأستاذ حين أفحمته قد تورّد كالورد في أول تفتحه إذ كان في رأسه على ما سمعت شيء من الباطل دعوى معاملة ذلك الجنب وكان من شأن أفلاطون وأرسطاطاليس وبطلميوس وأفليدس أن يستحيوا من ذلك لو كان لهم الإياب فقال بعد خروج السلطان ورجوعه من التشيع متعجبا على تبسم مليح: «أفلان قد طالعت أنت المجسمات أيضا من كتاب أفليدس؟»

قلت: «بمن مجلس الأستاذ لكن بقيت مطالعة ثلاث مقالات ولا أجد نسخة.» فأمر الفتوح من غلمانه بإحضار نسخته فانتسخت منها البقية وأتممت مطالعة ذلك الكتاب بتسديد الله تعالى وكتبت في آخر نسختي حالي العجب في حله.

وأما القراءة الشرح فكنت فيها على ما أختاره الأستاذ البرهان لأجل السلطان من قليل قليل في زمان طويل فيما نقصت طولا زادت بأضعافه عرضا وبما يسرت كما كثرت بأمثاله كيفاً وتمت بتيسير الله تعالى في خمس سنين تقريبا وتم بتمامها تحصيل العلوم قراءةً وسماعاً ومطالعةً وحينئذ ورد الوالد أوردته الله تعالى جنات النعيم في مقام كريم سمرقند برسول من قتل ملك الشروان جزاه الله الرحمن جزاء أهل العرفان في طلبه فأجازني السلطان في الإياب والأستاذ فيه وفي العلوم بالكتاب. وها أنا أذكره تبركاً وأجعله فيها تمسكاً:

قال بسم الله الرحمن الرحيم الحمد لله الذي جرى عادته على فتح أبواب العلم لأهله بعد ما جدَّ وجدَّ في سلوك حزن طريقه وسهله. والصلوة على النبي الأُمي الذي أمر بطلبه ولو كان بالصين. وعلى آله وأصحابه رجوم الجهل ونجوم اليقين. وبعد فإن ممن أمتثل هذا الأمر لتحصيله وارتكب مشاق الشقة في تقويمه وتعديله الشاب الفاضل بل الذي في مضممار الأفاضل فاضل زبدة الفضلاء وقودة الأذكياء مولانا فتح الله ابن المولى النبيه الفرصي الفقيه أبي يزيد الشايراني الشرواني بسّر الله أمالهما وأصلح أحوالهما فإنه سافر لاكتسابه من الجانب الغربي نحو الشرق وجاب النجد والغور وخرق الطود والخرق حتى نزل بمحروسة سمرقند لدينا وقرأ شرح التذكرة علينا وسمع كل ما جرى من ذلك الزمان إلى هذا الأوان في مجلس مدارسنا مع الأصحاب أولى البصائر والألباب فانتظم في سلك الأعالى من الموالى ذوي الفضائل والمعالي كان لي قلب أو القى السمع وهو شهيد فتنبه على كل رمزة وإن كانت من مكان بعيد حتى بلغ منتهى السؤل الأمل بسماعه شرح المختصر في علمي الأصول والجدل وحقّق مقاصد الحكماء وعقائد علماء الإسلام بالإصغاء إلى شرح المواقف في صناعة الكلام ومهر في سائر الفنون وبهر وأشتهر بين الأكابر صيته وانتشر فلما انتهى أمره إلى هذا الأمد وجاء والده ليرجع به إلى ذلك البلد فإن العود أحمد والوالد أحقّ بالولد استجازني فأجبتّه وأجزت أن يروى عنّي جميع ما سمع مني بل جميع ما صحّ عنده أنه يجوز روايته لي وأوصيته بالديانة والتقوى خصوصاً في الرواية والفتوى عصمنا الله وإياه عن الخطأ والزلل في القول والعمل. وأنا الفقير إلى الله الغني موسى بن محمد بن محمود المدعوّ بقاضي زاده الرومي. تحريراً في أواسط ربيع الآخر سنة أربع وأربعين وثمانمائة.

٢ - وقد قال النظام ولوجود عرض للقمر على الوجه المذكور وجب أن يفرض سطح منطقة الخارج بعد توهم قطعه جميع الأفلاك مقاطعاً لفلك البروج على نقطتين متقابلتين يسميان العقدتين والجوزهرين أيضاً ثم نقول سطح منطقة حامل الخارج يعني المائل إما أن يكون في السطح المذكور أو لا فإن لم يكن فيه أوجب تحريك التقاطعين بقدر تحريك الأوج لكن حركة الأوج كما يجيء ينبغي أن يفرض أضعافاً مضاعفة لحركة التقاطعين فوجب أن يكون سطح منطقة حامل الخارج في ذلك السطح فلهذا سمي مائلاً.

وبعد أن قرأت هذا المحلّ من شرحه بيومين إذ أخبر الأستاذ أنّ السلطان رحمهما الله توجّه إلى مجلس الدرس فقال لي: «اقرأ اليوم درس أمس الأمس.» إذ لا أشكال فيما بعده من الدرسين وكان السلطان ينقبض من درس لا أشكال فيه. وكنت قد تعمّقت فيه بعد القراءة حتّى سنح لي فيه شيء على الشارح فلما وصلت القراءة إلى قوله «فإن لم يكن فيه أوجب تحريك التقاطعين بقدر تحريك الأوج» وهو محلّ السنوح صدّقه فيه من تلامذة السلطان من كان يعرف بعليّ القوشجيّ وكان يتكلّم قائماً.

فقلت: «بل فيه بحث لأنّ تحريكهما بهذا القدر إنما يجب إذ كان سطح منطقة حامل الخارج في سطح منطقة البروج لكن ذلك النفي لا يستلزم هذا الإثبات فلو كان بينه وبين السطح المذكور مثلاً يكون التحريك بأقلّ من ذلك القدر يظهر ذلك عند تخيل انطباق تقاطعي السطح المذكور ومنطقة حامل الخارج على العقدتين فإنهما تفارقان بحركة حامل الخارج العقدتين المتحرّكتين على بطؤ عقيبهما بحركة الممثل.

ومع ذلك فإنّ العقدتين إنّما يتّمان الدورة على تقدير أن يكون غاية البعد بين منطقة حامل الخارج ومنطقة البروج أقلّ من غاية البعد بينها وبين السطح المذكور إذ على تقدير التساوي يلزم انتفاء العقدتين بانطباق السطح المذكور على منطقة البروج عند قطع التقاطعين نصف منطقة البروج وعلى تقدير العكس يلزم إدبارهما عند وصول غاية بعد السطح المذكور عن

منطقة البروج إلى منطقة البروج بعد إقبال قليل على خلاف التوالي لكن إطلاقه إيجاب تحريك التقاطعين بقدر تحريك الأوج
 مشعر بعدم الانتفاء والإدبار مطلقا على التقدير الذي ذكره.»
 فلما قلت ما قلت سكت الكل ولم يتعرّض للجواب أحد لا المصدّق ولا الأستاذ ولا السلطان ولا غيرهم من فحول
 المدرّسين في تلك المدرسة والسامعين لدرسي. فقام السلطان وخرج وشايحه الأستاذ إلى دهليزها ثم جلس على صُفّته ففتش أعلم
 تلامذته في هذا الفنّ عما قلت فقال: «كيف ترى ما قال فلان؟»
 قال: «أظنّ الأمر كما قال.»
 فقام أمرا له بحسن التأمل فيه فلما ركب فرسه ارتكب الطلبة لومي يصفون كلامي ذلك بالبرودة وتبريد المجلس
 المشتعل بنار البحث والبيان إلى أن اتّفق حضوري بعد يوم أو يومين من أيام العطلة مجلسه المبارك في داره الشريفة فرائيتُ
 لوامي وغيرهم حاضرين هناك فلما قعدتُ قال: «هل تأملت في ذلك تارة أخرى؟»
 قلت: «إنما قلت ذلك بعد استفراغ جهدي في التأمل فلم يبق فيّ لمزيد التأمل فيه مسأغ.»
 فقال: «نعم ذلك اعتراض على النظام وأرد.»¹
 فلما خرجنا من المجلس انقلب الأمر وانعكس الحال فصار اللائمون يلومون أنفسهم ويمدحون الموم فوقعوا في المثل
 السائر ربّ لائم ملئم.

٣ – ومنهم من فسّر الصعود والهبوط ههنا بالقرب من سمت الرأس والبعد عنه كما هو المتبادر ولا يطرد ذلك الحكم
 على هذا التفسير في جميع البلاد الشمالية لأنه لا يصح في البلاد الشمالية التي تصل القمر فيها إلى سمت الرأس قبل أن يصل
 إلى غاية عرضه الشماليّ.
 والنظام حمل القطب الظاهر المعتبر في الصعود والهبوط على قطب البروج وفيه إنّه غير متعيّن في بعض البلاد
 لصيرورة الظاهر فيه خفيا والخفيّ ظاهرا فيحتاج إلى التقييد بمعظم المعمورة. وقال «أنما فسّرنا الصعود بتقارب القمر من
 قطب البروج الظاهر والهبوط بتباعده عنه ليطرّد الحكم في جميع الاوقات إذ لو فسّرنا ههما بالقرب من سمت الرأس كلّ يوم
 وبالبعد عنه قياسا على حركة الشمس على مدارها لم يطرد الحكم.» فقيّد الاطراد بقوله «في جميع الاوقات» دون «جميع البلاد
 الشمالية» كما في النهاية. وفيه خفاء يزول بما أقصّه عليك.
 وإنّي لمّا لم أجد التشكيل الذي ذكره ههنا مناسباً للمتن وموضحا له فتّشتُ الأستاذ رحمه الله عند القراءة عن ذلك فأمعن
 في التأمل جدّا وأهل المجلس ساكتون وقال بعد الامعان «هذا التشكيل لبيان إطراد الحكم على التفسير الأوّل دون الثاني لا
 لإيضاح ما في المتن ولا شكّ أنه يتضح به ذلك إذ ظهر به أنّ مدّة صيرورة القمر من الرأس إلى الغاية الشمالية متقارب من
 قطب البروج الظاهر فيكون الحكم مطردا بالتفسير الأوّل وغير متقارب من سمت الرأس بل متباعد عنه فلا يكون مطردا
 بالتفسير الثاني وظهر أيضا حسن قوله «في جميع الاوقات.» ثمّ نظر إلى تلامذته الذين قرأوا عليه شرح النظام فقال
 «والعجب منا لم نطلع على هذا مع مرورنا بهذا المحلّ مرارا كثيرة مطالعه ومباحثه.» ثم ذكر لطيفة فقال «حكى عن المولى
 عز الدين الحلواني أنه كان يقول في مجلس درسه إذا اعترض قارئ أو سامع على محلّ متوقّعا لجواب جوز واجوزوا (!) فإنّي
 طالعت هذا الكتاب مرّة ولم أجد في هذا المحلّ اعتراضا.»

٤ – قال ذلك السلطان رحمه الله وقت قراءتي «لا يقال لا يتصور في الاستقبال التطابق الحقيقي لانا نقول لا فرق
 بين الاجتماع والاستقبال في تطابق الدائرتين بعد فرضهما عظيمتين فإن قيل فكيف يتصور بعده التطابق التقريبي في الاجتماع
 والاستقبال قلنا الاجتماع قد يكون بحيث يمرّ الخط الخارج من البصر إلى الشمس بمركزي النيرين فيكون التطابق حينئذ
 تحقيقيا. وقد لا يكون كذلك فيكون تقريبا. والاستقبال قد يكون بحيث ينطبق الخط الخارج من البصر إلى القمر على قطر منطقة

1. Damad Ibrahim MS 847 fol. 109a and Tehran Univ. Library MS 493 fol. 139a:

نعم ذلك اعتراض وأرد على النظام.

البروج طرفاه موضعا النيرين فيكون التطابق تحقيقا. وقد لا يكون بهذه الحثية فيكون تقريبا. فاختلاف تطابقهما بالتحقيق والتقريب إنما هو من جانب الاجتماع والاستقبال لا من جانب الدائرتين المأخوذتين عظيمتين. ولو لم تفرض الدائرتان عظيمتين لوقع الاختلاف بالتحقيق والتقريب من جانبهما أيضا. فقد ظهر من تحقيق هذا الفيلسوف أنه لا حاجة إلى ما ذكره السيد تابعا لصاحب التحفة الأخذ لهما صغيرتين.

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