Essays on Financial Markets

by

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Abstract

Chapter 1. Improving Access to Information Through Market Design (with Marzena Rostek) Availability of information to market participants is often seen as essential to the functioning of markets. We consider a market-design based alternative to improve access to information. We examine designs allowing traders to condition their demands on simultaneously determined prices in venues in which they do not participate. Facilitating access of information is neutral in competitive markets. Designs allowing cross-venue conditioning can be superior in markets with large traders who have price impact. Our results highlight the role of trader heterogeneity for design and regulation.

Chapter 2. Dynamic Market Choice In practice, many assets are traded in both transparent centralized markets and opaque decentralized markets. To explain traders' market choices, we develop a model of dynamic learning and market selection between the centralized and decentralized markets. With heterogeneous trader value correlations, we find that when asset payoff sensitivity or volatility is sufficiently low, traders prefer the decentralized market; when asset sensitivity or volatility is intermediate, switching between centralized and decentralized markets is the optimal market choice; when asset values are sensitive to volatile fundamentals, assets are traded only in the centralized market. The model's predictions are supported by empirical evidence from the Chinese corporate bond market. Our research uncovers new welfare implications for various market designs with endogenous market choices.

Chapter 3. The Impact of Leverage Ratio Regulation on Bond Market Liquidity While post-crisis regulations on banks' leverage ratio aim to build financial market resilience, it remains debatable whether they have adverse consequences on market liquidity. This paper explores the impact of leverage ratio regulations on bond market liquidity. Empirically, we find that leverage regulation has heterogeneous effects on the market liquidity across traders and assets during the March 2020 corporate bond market turmoil. In particular, the leverage regulation has a higher impact on the transaction cost of safe assets than risky assets. We build a decentralized market model with leverage ratio regulation to explain the findings. Then we quantify the welfare during the market turmoil and consider counterfactuals of adjusting the leverage regulation and market design solutions. We find that the adjustment of risk-weight on corporate bonds can potentially decrease welfare, and exempting the Treasury from calculating leverage exposure can improve welfare. Introducing a central clearing party and an all-to-all market can attenuate the unintended consequence of leverage regulation on market liquidity without hurting financial stability.

Dedication

To everyone who supported me.

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Chapter 1

Improving Access to Information Through Market Design

1.1 Introduction

Nowadays financial markets are fragmented. In 2021, financial assets are traded across about 500 execution platforms in the EU[1]. As of 2022, in the United States, equities alone are traded across 16 public exchanges[2], around 85 alternative trading systems (ATS) [3] and numerous broker-dealer networks[4]. The market fragmentation has created obstacles for traders to access cross-exchange information and have an overview of market condition in a timely manner. While the electronic trading has made trading more latency-sensitive and reliant on market data, the access to cross-exchange information is

[1]The statistics can be found in Fact Sheet: 2021 CMU package, consolidated tape: https://ec.europa.eu/info/sites/default/files/business_economy_euro/banking_and_finance/documents/211125-capital-markets-union-package-consolidated-tape-factsheet_en.pdf

[2]The statistics can be found in U.S. Equities Market Volume Summary: www.cboe.com/us/equities/market_share/.

[3]The statistics can be found in FINRA: list of ATS equity firms: www.finra.org/filing-reporting/otc-transparency/ats-equity-firms. Among 90 ATS firms registered with the Securities and Exchange Commission (SEC), 85 firms are operating as of July 31st 2022.

[4]According to 2022 FINRA financial industry snapshot, 3,394 broker-dealer firms are registered with FINRA in 2021.

significantly slower than the proprietary data provided by each venue.[5] As a response to that, the European Commission proposed the introduction of an EU consolidated tape, a system that consolidates as close to real-time as possible information on transactions taking place on trading platforms across the EU. [6] The U.S. Securities and Exchange Commission has also made further enhancement to the Regulation National Market System (RegNMS) in disseminating the national best bid and offer in 2020.[7]

Motivated by the policy concern, in this paper, we explore the market designs to improve access to information in latency-sensitive fragmented markets. We build on the recently introduced model which relaxes the assumption of full demand conditioning in the standard double-auction model based on the uniform-price mechanism (e.g. Wilson, 1979; Klemperer and Meyer, 1989; Kyle, 1989; Vives, 2011; Rostek and Weretka, 2015) for strategic traders in the quadratic-Gaussian setting. We examine the effects of cross-venue conditioning in markets with $N \geq 2$ exchanges. We consider the following *decentralized market designs*:

- No cross-venue conditioning: traders have access to price in their own venue and submit demands contingent on that.
- Full cross-venue conditioning: traders submit demands contingent on the price from all venues, including those where they do not participate.

The EU consolidated tape and RegNMS in US stock exchanges *de facto* induce cross-venue conditioning; however, analogous rules do not apply in markets for other asset classes or stock markets abroad (see Budish et al. (2019)). The data infrastructure may also be partially implemented, e.g., dark pools may access price information from lit exchanges, but lit exchanges do not have price information from dark pools. Therefore,

[5]In press release "SEC Adopts Rules to Modernize Key Market Infrastructure Responsible for Collecting, Consolidating, and Disseminating Equity Market Data", Washington D.C., Dec. 9, 2020, said Director Brett Redfearn of the SEC, "The content of national market system data for equities and the consolidation and dissemination of that data have lagged meaningfully behind the technologies and data content widely used for proprietary data products offered by exchanges."

[6]Details can be found in Fact Sheet: 2021 CMU package, consolidated tape.

[7]Details can be found in SEC Adopts Rules to Modernize Key Market Infrastructure Responsible for Collecting, Consolidating, and Disseminating Equity Market Data. www.sec.gov/news/press-release/2020-311.

we also consider the *decentralized market design* where the cross-venue conditioning is implemented in some but not all exchanges.

- Partial cross-venue conditioning: traders in some but not all venues have access to all prices and submit demands contingent on that; traders in the other venues have access to price in their own exchanges only.

A more radical proposal to increase information access is to create a centralized exchange through traders' full participation. In MiFID II, the European Securities and Markets Authority (ESMA) has put forward regulations to ensure nondiscriminatory market access, e.g., trading venues should not require minimum trading activity, and venues should not impose restrictions on the number of counterparties that a participant can interact with.[8] Motivated by the proposal, we also consider the *centralized market design*,

- Centralized market: all traders participate in all venues and submit demands contingent on all prices. This design functions like a single exchange.

We compare the welfare across the four designs. The comparison highlights two effects which jointly determine the optimal design, the liquidity effect and the learning effect. The *liquidity effect* corresponds to the welfare gain associated with market liquidity; the *learning effect* captures the welfare gain from learning with private signals and prices.

Accounting for the imperfect competition is critical to optimal design. We find that in competitive markets, facilitating trader access to information from exchanges in which they do not participate is neutral to equilibrium and welfare. This is because, in a competitive market, (i) there is no *liquidity effect*, and (ii) the *learning effect* is invariant to changes in demand conditioning. Intuitively, the gains from trade depend on the expected difference between the private value and the market price, conditional on the private signal and the price information. As traders within the same exchange have equal access to

[8]See Questions and Answers On MiFID II and MiFIR market structures topics: https://www.esma.europa.eu/sites/default/files/library/esma70-872942901-38_qas_markets_structures_issues.pdf.

price information across exchanges, the expected value and the market price increase by the same amount. Thus providing additional price information does not increase the learning effect.

When traders have price impact, allowing cross-venue conditioning is no longer welfare-neutral. In fact, allowing cross-exchange conditioning in the decentralized market is always weakly liquidity-improving and welfare-improving. This is because it changes the *liquidity effect*. To see why, recall that by definition, the price impact corresponds to the marginal price change as a response to a trader's deviation in demand. Suppose a trader increases the demand by a unit. The price impact can be decomposed into two parts. The first part is due to risk-aversion. The marginal valuation of the residual market increases as their holdings decrease. The second part is due to traders' inference. Given trader i in exchange n has deviated, the residual market attributes the price change to a different realization of the average signal in exchange n, and adjusts their conditional expectations accordingly. We find that when traders have cross-exchange price information, the price impact due to inference is lower. This is because traders' inference weight on the price in their own exchange is smaller when they have access to more information.

By comparing the decentralized market designs and the centralized market designs, we find that all designs can be Pareto-optimal, depending on the traders' value correlation. When the traders' values are equally correlated with traders in their own exchange and traders in the other exchange, the centralized market is always the most liquid and has the highest welfare. This result holds regardless of the number and size of exchanges. The prediction that allowing more conditioning is Pareto-improving extends to the fully integrated, centralized market when value correlations are symmetric. However, turning the decentralized market into a centralized market can decrease welfare with heterogeneous correlations. This is due to both the liquidity effect and the learning effect. First, when the average correlation in the centralized market is sufficiently larger than the decentralized market, the learning effect in the centralized market can also be lower. Although traders have an increase in information gain with a larger market size, the residual market's inference from price is sufficiently strong to counterbalance the size effect. Second, when the cross-exchange correlation is sufficiently large, the liquidity can worsen when we turn the decentralized market into the centralized market. This is when the residual market assigns a higher weight on the price, and increases the price impact by more due to inference.

Finally, we consider the price informativeness of the designs. We define the price informativeness as the extent to which the price information available to traders improves their inference. It is obvious that prices in the decentralized market with full-conditioning are more informative than those in the fully decentralized market, as the inference precision increases with access to cross-exchange prices. The two designs have the same price informativeness only when the cross-exchange correlations are all zero, i.e., traders' values are independent across exchanges. We also find that the decentralized market with full-conditioning has weakly higher price informativeness than the centralized market. Intuitively, the centralized market price is just a weighted average of signals in each exchange, and it can be expressed as a linear combination of decentralized market prices. So the centralized market price can not be more informative than that in the decentralized market with full conditioning.

To summarize, our findings have the following implications to the design of consolidated tape and RegNMS: (i) Improving access to cross-venue information is neutral when the market is competitive. It is liquidity-improving and welfare-improving when the market is imperfectly competitive. (ii) Creating a centralized market can yield lower or higher than providing cross-venue information alone, depending on traders' value correlation. (iii) Improving access to cross-venue information in the decentralized market weakly increases the price informativeness compared with other designs in this paper.

Our paper contributes to the market design literature by allowing for flexible expression of preferences over multiple assets or objects, e.g., Lahaie and Parkes (2004); Cramton et al. (2004). It is closely related to recent studies on trading with uncontingent schedules in imperfectly competitive markets (Chen and Duffie, 2021; Rostek and Yoon, 2021;

Wittwer, 2021).[9] The existing works compare the uncontingent and contingent demand schedule with traders' full participation. To the best of our knowledge, we are among the first to consider a model allowing traders to condition on simultaneously-determined prices in venues in which they do not participate.

This paper is also related to the literature of information spillover. Huangfu and Liu (2022) contemporaneously studies a model of the decentralized market with a focus on the price information spillover across exchanges. Traders within one exchange have equal value correlation but asymmetric access to cross-venue price information. Our paper takes a market design approach where traders have equal access to cross-venue prices, complements their paper by allowing heterogeneous value correlation, and provides a comparison with the centralized market design.

1.2 Model

Market Structure Consider a market of one divisible risky asset, and one risk-free asset as a numéraire, $N \geq 2$ exchanges, and I traders. Each exchange clears separately. We index exchanges by n and traders by i. An exchange $n \in N$ is identified with a subset of traders $I(n) \subseteq I$. Denote the number of traders in exchange n as I_n. Assume that each trader can only participate in one exchange, i.e., $I(m) \cap I(n) = \emptyset, \forall m \neq n$. Assume that each exchange has at least two traders, $I_n \geq 2$, $\forall n \in N$. The market is a double auction in a linear-normal setting. Trader i has a quasi-linear and quadratic utility function.

$$U^i\left(q^i\right) = \theta^i q^i - \frac{\alpha}{2}(q^i)^2,$$

where q^i is the quantity of traded asset and $\alpha > 0$. Traders are uncertain about the asset value. The uncertainty is captured by the randomness in the intercepts of the marginal utility functions $\{\theta^i\}_{i \in I}$, referred to as *values*.

Information structure Prior to trading, each trader observes a noisy signal about his true

[9] Cespa (2004) examined equilibrium in competitive uncontingent markets with two assets and noise traders, and characterized how uncontingent trading affects price informativeness.

value θ^i, $s^i = \theta^i + \varepsilon^i$. We adopt an affine information structure: Random vector $\left\{\theta^i, \varepsilon^i\right\}_{i \in I}$ is jointly normally distributed; noise ε^i is mean-zero i.i.d. with variance $\sigma_\varepsilon^2 > 0$ and the expectation $\mathbb{E}\left[\theta^i\right]$ and the variance σ_θ^2 of θ^i are the same for all i. The variance ratio $\sigma^2 \equiv \sigma_\varepsilon^2/\sigma_\theta^2$ measures the relative importance of noise in the signal.

The $I \times I$ variance-covariance matrix of the joint distribution of values $\left\{\theta^i\right\}_{i \in I}$, normalized by variance σ_θ^2, specifies the matrix of correlations,

$$\mathcal{C} \equiv \begin{pmatrix} 1 & \rho^{1,2} & \dots & \rho^{1,I} \\ \rho^{2,1} & 1 & \dots & \rho^{2,I} \\ \vdots & \vdots & \ddots & \vdots \\ \rho^{I,1} & \rho^{I,2} & \dots & 1 \end{pmatrix}.$$

Definition 1 (With-exchange Correlation and Cross-exchange Correlation)**.** *Define the average within-exchange correlation for trader* $i \in I(n)$ *as* $\bar{\rho}_n^i \equiv \frac{1}{I_n - 1} \sum_{j \neq i, j \in I(n)} \rho^{i,j} \in [-1, 1]$. *Define the average cross-exchange correlations between trader* $i \in I(n)$ *and traders in* $I(m)$ *as* $\bar{\rho}_{nm}^i \equiv \frac{1}{I_m} \sum_{j \in I(m)} \rho^{i,j} \in [-1, 1]$.

Assume that the within-exchange correlations and cross-exchange correlations equalize for all $i \in I(n)$ for any exchange $n \in N$, i.e., $\bar{\rho}_n^i = \bar{\rho}_n$, $\bar{\rho}_{mn}^i = \bar{\rho}_{mn}$, $\forall i \in I(n)$ for any $n \in N$. As $\rho^{i,j} = \rho^{j,i}$, it is easy to see that $\bar{\rho}_{nm} = \bar{\rho}_{mn}, \forall m, n \in N$.

With these assumptions, we can define the average correlation matrix,

$$\overline{\mathcal{C}} = \begin{bmatrix} \frac{1+(I_1-1)\bar{\rho}_1}{I_1} & \dots & \bar{\rho}_{1N} \\ \vdots & \ddots & \vdots \\ \bar{\rho}_{N1} & \dots & \frac{1+(I_N-1)\bar{\rho}_N}{I_N} \end{bmatrix} \in \mathbb{R}^{N \times N}, \tag{1.1}$$

where the $(n, m)^{th}$ element is the average value correlation between traders in exchange n and m.

Double Auction We study double auctions based on the canonical uniform-price mechanism. Traders submit strictly downward-sloping (net) demand schedules. The part of a bid with negative quantities is interpreted as a supply schedule.

The market clearing prices $\{p_n\}_n^*$ are such that aggregate demand all exchanges equal to zero, $\sum_{i \in I(n)} q^i(\cdot) = 0, \forall n \in N$. Trader $i \in I(n)$ obtains the quantity determined by his bid evaluated at the equilibrium price $(q^i)^*$, for which he pays $p_n^* \cdot (q^i)^*$. Trader payoff is given by $U^i\left((q^i)^*\right) - p_n^* \cdot (q^i)^*$.

Equilibrium We study the Bayesian Nash equilibrium in linear demand schedules (hereafter, equilibrium).

Definition 2 (Equilibrium). *A profile of (net) demand schedules $\left\{q^i(\cdot)\right\}_i$ is a Bayesian Nash equilibrium if, for each $i, q^i(\cdot)$ maximizes the expected payoff given the schedules of other traders $\left\{q^j(\cdot)\right\}_{j \neq i}$ and market clearing $\sum_{i \in I(n)} q^i(\cdot) = 0$ for all n.*

1.3 Equilibrium characterization

In this section, we will characterize the Bayesian Nash equilibrium of the four designs: 1. Decentralized market where traders have limited participation and access to price information in their own exchanges only. 2. Decentralized market with cross-venue conditioning where traders have limited participation but access to price information in all exchanges. 3. Decentralized market with partial conditioning where traders in a subset of exchanges have cross-venue conditioning. 4. Centralized market where traders have full participation and full conditioning.

In particular, we focus on the case of decentralized exchange with full conditioning and partial conditioning, as (i) they are two market designs new to the literature and (ii) they are relevant for policy designs like consolidated tape and RegNMS.

1.3.1 Decentralized Market

In this part, we consider a fully decentralized market without cross-venue information to model the market before the implementation of consolidated tape and RegNMS.

Design 1 (Decentralized Market) We first consider a design where the traders only know the price in their own exchanges, and submit demand contingent on that. We use superscript d to denote equilibrium in this design.

In this setting, the trader $i \in I(n)$ faces the following optimization problem:

$$\max_{q^i} \mathbb{E}[\theta^i q^i - \frac{\alpha}{2}(q^i)^2 - p_n^d q^i | p_n^d, s^i]. \tag{1.2}$$

A profile of (net) demand schedules $\left\{q^i(p_n^d)\right\}_i$ is a Bayesian Nash Equilibrium if for each trader $i \in I(n)$, (i) demand schedule satisfies the first-order condition given price impact $\lambda^{i,d}$: $q^i(p_n^d) = \frac{1}{(\alpha+\lambda^i)}\left(\mathbb{E}\left[\theta^i | s^i, p_n^d\right] - p_n^d\right)$, where the posterior expectation $\mathbb{E}[\theta^i | s^i, p_n^d] = c_{\theta,n}^d \mathbb{E}[\theta^i] + c_{s,n}^d s^i + c_{p,n}^d p_n^d$ is determined by the projection theorem given the equilibrium price distribution, and price impact such that (ii) price impact equals the slope of i's residual inverse supply: $\lambda^{i,d} = -\left(\sum_{j \neq i} \frac{\partial q_n^j(p_n^d)}{\partial p_n^d}\right)^{-1} \forall i \in I(n)$.

Proposition 1 (Decentralized Market Equilibrium)**.** *A linear Bayesian Nash equilibrium exists and is unique. The closed-form solution is characterized as below, and the existence condition can be found in the proof, (i) For trader $i \in I(n)$, the conditional expectation $\mathbb{E}[\theta^i | s^i, p_n^d] = c_{\theta,n}^d \mathbb{E}[\theta^i] + c_{s,n}^d s^i + c_{p,n}^d p_n^d$, where coefficients as functions of the primitives:*

$$\begin{aligned} c_{s,n}^d &= \frac{1-\bar{\rho}_n}{1-\bar{\rho}_n+\sigma^2}, \\ c_{p,n}^d &= \frac{(2-\gamma_n)\bar{\rho}_n}{1-\gamma_n+\bar{\rho}_n} \frac{\sigma^2}{1-\bar{\rho}_n+\sigma^2}, \\ c_{\theta,n}^d &= 1 - c_{s,n}^d - c_{p,n}^d. \end{aligned} \tag{1.3}$$

where $\gamma_n \equiv 1 - \frac{1}{I_n - 1} \in [0,1]$ is the index of exchange size.

(ii) The equilibrium price impact for traders in exchange $I(n)$ is

$$\lambda_n^d = \frac{1-\gamma_n}{\gamma_n - c_{p,n}^d}\alpha. \tag{1.4}$$

(iii) The equilibrium bid for trader i in exchange $I(n)$ is

$$q_n^{i,d} = \frac{c_{s,n}^d}{(\alpha+\lambda_n^d)}(s^i - \bar{s}_n). \tag{1.5}$$

(iv) The equilibrium prices satisfy

$$p_n^d = \mathbb{E}[\bar{\theta}_n|\bar{s}_n] = \mathbb{E}[\theta] + \frac{cov(\bar{\theta}_n, \bar{s}_n)}{Var(\bar{s}_n)}(\bar{s}_n - \mathbb{E}[\theta]). \tag{1.6}$$

An observation to the equilibrium is that the realized prices differ across exchanges. This motivates the design to enhance the access to information in the other exchange. In the following sections, we discuss designs which allow all or some of the traders to submit their demand schedule contingent on prices in other exchanges.

1.3.2 Decentralized Market with Full Conditioning

The EU consolidated tape and RegNMS in US stock exchanges *de facto* induce cross-venue conditioning. In this part, we will discuss the design of the decentralized market with full conditioning.

Design 2 (Decentralized Market with Full Conditioning) In this design, we allow all traders to see prices from all exchanges and submit demand contingent on that. We use superscript dc to denote equilibrium in this case.

Trader $i \in I(n)$ faces the following optimization problem:

$$\max_{q_n^i} \mathbb{E}[\theta^i q_n^i - \frac{\alpha}{2}(q_n^i)^2 - p_n^{dc} q_n^i | \boldsymbol{p}^{dc}, s^i]. \tag{1.7}$$

where q_n^i is the quantity submitted by trader i in exchange n, p_n^{dc} is the price in exchange $I(n)$, s^i is the signal received by trader i, $\boldsymbol{p}^{dc} = (p_n^{dc})_n \in \mathbb{R}^N$.

The first order condition is

$$\mathbb{E}[\theta^i | \boldsymbol{p}^{dc}, s^i] - \alpha q_n^i = p_n^{dc} + \lambda_n^i q_n^i, \tag{1.8}$$

where $\lambda_n^i = \frac{\partial p_n}{\partial q_n^i}$. It is the $(n,n)^{th}$ element of price impact matrix $\boldsymbol{\Lambda}^i \equiv \frac{\partial \boldsymbol{p}^{dc}}{\partial \boldsymbol{q}^i} =$

$$\begin{bmatrix} \frac{\partial p_1}{\partial q_1^i} & \cdots & \frac{\partial p_N}{\partial q_1^i} \\ \vdots & \ddots & \vdots \\ \frac{\partial p_1}{\partial q_N^i} & \cdots & \frac{\partial p_N}{\partial q_N^i} \end{bmatrix} \in \mathbb{R}^{N\times N} \text{ and } \boldsymbol{q}^i = (q_n^{i,dc})_n \in \mathbb{R}^N.$$

In equilibrium, (i) for trader $i \in I(n)$, the demand schedule $q_n^i = (\alpha+\lambda_n^i)^{-1}(\mathbb{E}[\theta^i|\boldsymbol{p}, s^i]-p_n)$. $\mathbb{E}[\theta^i|\boldsymbol{p}^{dc}, s^i] = c_{\theta,n}^{dc}\mathbb{E}[\theta] + c_{s,n}^{dc}s^i + \sum_m c_{p_m,n}^{dc}p_m^{dc}$, where coefficients are determined by the Projection Theorem given the equilibrium price distribution, and (ii) the price impact is correct, i.e.,

$$\boldsymbol{\Lambda}^i = -\begin{bmatrix} \sum_{j\neq i}\frac{\partial q_1^j}{\partial p_1} & \cdots & \sum_{j\neq i}\frac{\partial q_N^j}{\partial p_1} \\ \vdots & \ddots & \vdots \\ \sum_{j\neq i}\frac{\partial q_1^j}{\partial p_N} & \cdots & \sum_{j\neq i}\frac{\partial q_N^j}{\partial p_N} \end{bmatrix}^{-1}. \tag{1.9}$$

Proposition 2 (Decentralized Market with Full Conditioning Equilibrium). *A linear Bayesian Nash equilibrium exist and is unique. The closed-form solution is characterized as below, and the existence condition can be found in the proof. When traders can submit demand schedules conditional on prices from all exchanges:*

(i) Inference coefficients in $\mathbb{E}[\theta^i|\boldsymbol{p}^{dc}, s^i] = c_{\theta,n}^{dc}\mathbb{E}[\theta] + c_{s,n}^{dc}s^i + \sum_m c_{p_m,n}^{dc}p_m^{dc}$ *satisfy*

$$\begin{aligned} c_{s,n}^{dc} &= \tfrac{1-\bar\rho_n}{1-\bar\rho_n+\sigma^2}, \\ c_{p_m,n}^{dc} &= \tfrac{\sigma^2}{1-\bar\rho_n+\sigma^2}\mathbf{1}_{n=m} - \tfrac{(1-\bar\rho_n)\sigma^2(\bar{\mathcal{C}}^{-1})_{nm}}{(1-\bar\rho_n+\sigma^2)I_n}, \\ c_{\theta,n}^{dc} &= 1 - c_{s,n}^{dc} - \textstyle\sum_m c_{p_m,n}^{dc}. \end{aligned} \tag{1.10}$$

where $\bar{\mathcal{C}}$ *is the average correlation matrix.*

(ii) Trader $i \in I(n)$ *has price impact* [10]

$$\lambda_n^{dc} = \frac{1-\gamma_n}{\gamma_n - 1 + 1/(\boldsymbol{M}^{dc})_{nn}}\alpha. \tag{1.11}$$

where $\gamma_n = 1 - \frac{1}{I_n-1}$ *and* $\boldsymbol{M}^{dc} = (Id - \boldsymbol{C}_p^{dc})^{-1}$,$\boldsymbol{C}_p^{dc} = (c_{p_m,n}^{dc})_{nm} \in \mathbb{R}^{N\times N}$, $(\boldsymbol{M}^{dc})_{nn}$ *is the* $(n,n)^{th}$ *element of* $\boldsymbol{M}^{dc}$.

[10] Note that the off-diagonal elements of the price impact matrix Λ^i do not affect the equilibrium bid and welfare. So we only report the diagonal elements of the matrix here for simplicity.

(iii)The equilibrium bid at the equilibrium price for trader $i \in I(n)$ is

$$q_n^{i,dc} = \frac{1}{\alpha + \lambda_n^{dc}} c_{s,n}^{dc}(s^i - \bar{s}_n) \quad \forall i \in I(n), n = 1, \ldots, N. \tag{1.12}$$

(iv) The equilibrium prices satisfy

$$\boldsymbol{p}^{dc} = \mathbb{E}[\bar{\boldsymbol{\theta}}|\bar{\boldsymbol{s}}] = \mathbb{E}[\boldsymbol{\theta}] + cov(\bar{\boldsymbol{\theta}}, \bar{\boldsymbol{s}}') \cdot cov(\bar{\boldsymbol{s}}, \bar{\boldsymbol{s}}')^{-1} \cdot (\bar{\boldsymbol{s}} - \mathbb{E}[\boldsymbol{\theta}]). \tag{1.13}$$

There are several interesting observations of the equilibrium.

First, while prices in Design 1 are determined by signals within one exchange, prices in Design 2 are a function of the average signal from all exchanges. As the traders form expectations based on prices from all exchanges, the intercept of the demand schedule contains information from the other exchanges. The price adjusts with the cross-exchange information to clear the market.

Second, the equilibrium bid is not affected by the off-diagonal elements of the price impact matrix. As the traders pay for their bids at the prices in their own exchanges, their bids only respond to the price impacts in their own exchanges.[11]

Third, inference coefficients on signals remain the same as those in Design 1.

Corollary 1 (Inference Weight on Signals)**.** *Traders assign the same inference weight on private signals in Design 1 and Design 2.* $c_{s,n}^{dc} = c_{s,n}^{d}$.

Intuitively, the coefficient c_s is the marginal change in the conditional expectation of θ^i as a response to the change in signal s^i while holding the price constant. It reflects the additional information provided by s^i beyond what prices can provide. With a symmetric market structure, prices are functions of the average signals. In Design 1, price p_n^d is informationally equivalent to $\bar{s}_n$. In Design 2, price vector $\boldsymbol{p}^{dc}$ is informationally equiva-

[11]Despite the fact that the off-diagonal price impacts (or we can say, the cross-exchange price impacts), do not affect equilibrium bid and welfare, they are in general non-zero due to traders' inference. As trader $i \in I(n)$ submits the demand schedule q^i, it changes the price in his own exchange, p_n. As traders in the other exchanges form their expectations based on p_n, they also change their demand schedules, which in turn changes the prices in the other exchanges. This cross-exchange inference creates non-zero cross-exchange price impacts.

lent to $\{\bar{s}_n\}_n$. As the covariance between s^i and any $\bar{s}_m, m \neq n$ conditional on $\bar{s}_n$ is zero, the cross-exchange information does not change the additional information provided by s^i beyond the prices, and does not change how traders weigh the private signals.

Fourth, the correlation matrix is an important determinant for inference coefficients on prices $c^{dc}_{p_m,n}$. In particular, the cross-exchange correlations, which have no impact on equilibrium in Design 1, affect equilibrium in Design 2. The following example shows how within-exchange correlations and cross-exchange correlations affect the equilibrium.

Example 1 (Two Decentralized Exchanges with Full Conditioning)**.** *Consider two equal-sized exchanges with the number of traders $I_1 = I_2 = \frac{I}{2}$. The within-exchange correlations are $\bar{\rho}_1 = \bar{\rho}_2 = \rho_n$, and across-exchange correlations are $\bar{\rho}_{12} = \bar{\rho}_{21} = \rho_{nm}$. As the two exchanges are symmetric, we have $\lambda^{dc}_1 = \lambda^{dc}_2$. Without loss of generality, we focus on the price impact in exchange 1.*

The price impact in the decentralized exchange with full conditioning is

$$\lambda^{dc}_1 = \frac{1-\gamma_n}{\gamma_n - c^{dc}_{p_1,1} - \eta_2 c^{dc}_{p_2,1}}\alpha, \tag{1.14}$$

where $\gamma_n = 1 - \frac{1}{I/2-1}$, the inference coefficients on trader's own exchange is $c^{dc}_{p_1,1} = c^{dc}_{p_2,2} = \frac{\sigma^2(2-\gamma_n)\big(\rho_n(1-\gamma_n+\rho_n)-\rho^2_{mn}(2-\gamma_n)\big)}{(1-\rho_n+\sigma^2)\big((1-\gamma_n+\rho_n)^2-\rho^2_{mn}(2-\gamma_n)^2\big)}$, the inference coefficients on the other exchange $c^{dc}_{p_2,1} = c^{dc}_{p_1,2} = \frac{\sigma^2(2-\gamma_n)(1-\gamma_n)(1-\rho_n)\rho_{mn}}{(1-\rho_n+\sigma^2)\big((1-\gamma_n+\rho_n)^2-\rho^2_{mn}(2-\gamma_n)^2\big)}$, $\eta_2 = \frac{c^{dc}_{p_1,2}}{1-c^{dc}_{p_2,2}}$.

From **Example 1** we can see that the within-exchange correlations ρ_n and cross-exchange correlations ρ_{nm} determines the inference weights on prices and price impacts. In this example, the price impact λ^{dc}_n firstly increase with cross-exchange correlations ρ_{nm} for $\rho_{nm} \leq 0$, and decreases with cross-exchange correlations ρ_{nm} for $\rho_{nm} \geq 0$. [12]

[12] In general, the derivative of λ^{dc}_n with respect to cross-exchange correlation $\bar{\rho}_{nm}$ is

$$\frac{d\lambda^{dc}_n}{d\bar{\rho}_{nm}} = -\frac{(I_n-1)(\lambda^{dc}_n)^2}{\alpha}\frac{1-\bar{\rho}_n}{1-\bar{\rho}_n+\sigma^2}\frac{\sigma^2}{I_n}\frac{2cov(\bar{s}_n, \bar{\boldsymbol{s}}'_{-n})\left(cov(\bar{\boldsymbol{s}}_{-n}, \bar{\boldsymbol{s}}'_{-n})^{-1}\right)_m}{cov(\bar{\theta}_n, \bar{s}_n|\bar{\boldsymbol{s}}_{-n})^2},$$

where $\left(cov(\bar{\boldsymbol{s}}_{-n}, \bar{\boldsymbol{s}}'_{-n})^{-1}\right)_m$ is the column m of matrix $cov(\bar{\boldsymbol{s}}_{-n}, \bar{\boldsymbol{s}}'_{-n})^{-1}$. When there are two exchanges, $\frac{d\lambda^{dc}_n}{d\bar{\rho}_{nm}} \geq 0$ if $\bar{\rho}_{mn} \leq 0$, and $\frac{d\lambda^{dc}_n}{d\bar{\rho}_{nm}} \leq 0$ if $\bar{\rho}_{mn} \geq 0$.

1.3.3 Decentralized Market with Partial Conditioning

The data infrastructure to improve access to cross-venue information may be partially implemented. For example, dark pools can have access to price information from lit exchanges, but lit exchanges do not have price information from dark pools. Besides, data infrastructures like EU consolidated tape and RegNMS do not apply in markets for other asset classes or stock markets abroad. We may wonder about the welfare implications of a partial implementation. In this part, we will discuss partial conditioning, where only a subset of traders can see prices from all exchanges and submit demand contingent on that, while the others cannot.

Design 2' (Decentralized Market with Partial Conditioning) Assume that only traders in a subset of exchanges $I(1),\ldots,I(K)$,$K<N$ can see prices from all the exchanges and submit demand schedules contingent on that. We use superscript pc to denote equilibrium in this case.

Traders in $I(K+1),\ldots,I(N)$ can only see prices in their own exchanges. Therefore, the equilibrium for these traders is the same as that in Design 1.

For traders in $I(1),\ldots,I(K)$, the first order condition to their optimization problem remains the same as that in Design 2

$$\mathbb{E}[\theta^i|\boldsymbol{p}^{pc},s^i]-\alpha q_n^i=p_n^{pc}+\lambda_n^i q_n^i, \tag{1.15}$$

where p_n^{pc} is price in exchange $I(n)$, $\boldsymbol{p}^{pc}=(p_n^{pc})_n\in\mathbb{R}^N$, $\lambda_n^i=\frac{\partial p_n}{\partial q_n^i}$ is the $(n,n)^{th}$ element of price impact matrix $\boldsymbol{\Lambda}^i=\frac{\partial \boldsymbol{p}^{dc}}{\partial \boldsymbol{q}^i}$, $\boldsymbol{q}^i=(q_n^{i,dc})_n\in\mathbb{R}^N$.

In equilibrium, (i) the demand schedule $q_n^i=(\alpha^i+\lambda_{nn}^i)^{-1}(\mathbb{E}[\theta^i|\boldsymbol{p}^{pc},s^i]-p_n^{pc})$, where $\mathbb{E}[\theta^i|\boldsymbol{p}^{pc},s^i]$ is determined by the Projection Theorem given the equilibrium price distribution, and (ii) the price impact is correct.

Proposition 3 (Partial Conditioning Equilibrium)**.** *A linear Bayesian Nash equilibrium exists and is unique. The closed-form solution is characterized as below, and the existence condition can be found in the proof. When traders in a subset of exchanges $I(1),\ldots,I(K)$,$K<N$ can condition*

their demand on price from the other exchanges while the others cannot,

(i) The inference coefficients on price under partial conditioning for trader $i \in I(n)$, $n = 1, \ldots, K$,

$$\begin{aligned} c^{pc}_{s,n} &= \tfrac{1-\bar{\rho}_n}{1-\bar{\rho}_n+\sigma^2}, \\ c^{pc}_{p_m,n} &= \begin{cases} \tfrac{\sigma^2}{1-\bar{\rho}_n+\sigma^2}\mathbf{1}_{n=m} - \tfrac{(1-\bar{\rho}_n)\sigma^2(\bar{\mathcal{G}}^{-1})_{nm}}{(1-\bar{\rho}_n+\sigma^2)I_n} & \text{if } m = 1, \ldots, K. \\ -\tfrac{(1-\bar{\rho}_n)\sigma^2(\bar{\mathcal{G}}^{-1})_{nm}}{W_m(1-\bar{\rho}_n+\sigma^2)I_n} & \text{if } m = K+1, \ldots, N. \end{cases} \\ c^{pc}_{\theta,n} &= 1 - c^{pc,n}_s - \textstyle\sum_m c^{pc}_{p_m,n}. \end{aligned} \tag{1.16}$$

where $\bar{\mathcal{G}} = \frac{cov(\bar{\mathbf{s}},\bar{\mathbf{s}}')}{\sigma^2_\theta} - diag(\frac{\sigma^2}{I_n}\mathbf{1}_{n=1,\ldots,K})_n \in \mathbb{R}^{N\times N}$, and $W_m = \frac{c^d_{s,m}}{1-c^d_{p,m}} = \frac{1+(I_m-1)\bar{\rho}_m}{1+\sigma^2+(I_m-1)\bar{\rho}_m}$.

For trader $i \in I(n)$, $n = K+1, \ldots, N$,

$$\begin{aligned} c^{pc}_{s,n} &= c^{d,n}_s, \\ c^{pc}_{p_m,n} &= c^{d,n}_p \cdot \mathbf{1}_{m=n}, \\ c^{pc}_{\theta,n} &= c^{d,n}_\theta. \end{aligned} \tag{1.17}$$

(ii) The price impact for trader $i \in I(n)$

$$\lambda^{pc}_n = \lambda^{dc}_n \cdot \mathbf{1}_{n=1,\ldots,K} + \lambda^d_n \cdot \mathbf{1}_{n=K+1,\ldots,N}. \tag{1.18}$$

(iii) The equilibrium bid for trader $i \in I(n)$

$$q^{i,pc} = q^{i,dc} \cdot \mathbf{1}_{n=1,\ldots,K} + q^{i,d} \cdot \mathbf{1}_{n=K+1,\ldots,N}. \tag{1.19}$$

(iv) The equilibrium price for trader $i \in I(n)$

$$p^{pc}_n = p^{dc}_n \cdot \mathbf{1}_{n=1,\ldots,K} + p^d_n \cdot \mathbf{1}_{n=K+1,\ldots,N}. \tag{1.20}$$

It is trivial that $\lambda^{pc}_n = \lambda^d_n$ for $n = K+1, \ldots, N$, as traders in these exchanges do not have access to cross-exchange prices in both designs. The intuition that $\lambda^{pc}_n = \lambda^{dc}_n$ for $n = 1, \ldots, K$ is as follows.

Consider the case with $N = 2, K = 1$. As traders in $I(1)$ can submit demand conditioning on prices from two exchanges in both Design 2 and Design 2′, these traders can learn the weighted average signals in two exchanges $\bar{s}_1$ and $\bar{s}_2$. In Design 2, prices $\{p_1, p_2\}$ are linearly independent combination of average signals in two exchanges $\bar{s}_1$ and $\bar{s}_2$. In Design 2′, price p_2 deterministically reveals average signals $\bar{s}_2$, and price p_1 is linear combination of average signals $\bar{s}_1$ and $\bar{s}_2$. Thus in both cases, the traders in $I(1)$ learn from prices the exact change in average signals in two exchanges.

In a counterfactual analysis, suppose a trader i in exchange $I(1)$ deviates unilaterally from the Bayesian equilibrium by submitting a higher demand. In Design 2′, p_2 will not change as traders in $I(2)$ are not allowed to submit demand contingent on p_1. In Design 2, both p_1 and p_2 will change. Comparing the two designs, the remaining question is why the price impact for traders in exchange $I(1)$ does account for the additional change in p_2. This is because the traders in $I(1)$ can learn the average signals from p_1, p_2 by our previous argument. They can always decompose the change in p_2 into two parts, the part due to the change in $\bar{s}_1$ and the part due to the change in $\bar{s}_2$. Therefore, despite that p_2 changes with p_1 in Design 2, traders in $I(1)$ can detect such changes in p_2 are completely due to the change in the average signal in their own exchange. Therefore, given a unit deviation of q_1^i, the change in residual demand in $I(1)$ and the change in p_1 in Design 2 and Design 2′ are the same, i.e., $\lambda_1^{dc} = \lambda_1^{pc}$.

An observation of Design 2 and Design 2′ is that the realized prices are generally different across exchanges. To be clear, having access to information in the other exchanges does not eliminate the price differences, as the traders cannot participate in the other exchanges and arbitrage due to market segmentation. In the next section, we allow the traders to have full conditioning as well as participate in all exchanges. All exchanges clear simultaneously and will have the same price.

1.3.4 Centralized Market

A more radical proposal to increase information access is to create a centralized exchange through traders' full participation. In this section, we will discuss the centralized market design.

Case 3 (Centralized market) Now consider allowing the traders to submit contingent demand as well as participate in all exchanges. All exchanges clear jointly. This design works as if consolidating the decentralized exchanges into one centralized market. We use superscript c to denote equilibrium in this case. The trader i faces the following optimization problem:

$$\max_{q^i} \mathbb{E}[\theta^i q^i - \frac{\alpha^i}{2}(q^i)^2 - p^c q^i | p^c, s^i],$$

In equilibrium, (i) given $\lambda^{i,c}$, the price impact of trader i in the centralized market, the first order condition of the maximization problem is

$$q^i = (\alpha^i + \lambda^{i,c})^{-1}(\mathbb{E}[\theta^i | s^i, p^c] - p^c),$$

where the posterior expectation $\mathbb{E}[\theta^i|s^i, p^c] = c_\theta^c \mathbb{E}[\theta^i] + c_s^c s^i + c_p^c p^c$ given the affine information structure. The coefficients c_θ^c, c_s^c and c_p^c determined by the price distribution in the equilibrium. (ii) price impact equals the slope of i's residual inverse supply: $\lambda^{i,c} = -\left(\sum_{j \neq i} \frac{\partial q_n^j(p_n^c)}{\partial p_n^c}\right)^{-1} \forall i \in I(n)$.

Proposition 4 (Centralized Market Equilibrium)**.** *In a centralized exchange, the equilibrium inference coefficients and price impacts are jointly determined by the following equations:*

$$\begin{aligned}
c_s^{i,c} &= \frac{\left(\sum_i (\delta^{i,c})^2(1+\sigma^2) + \sum_i \sum_{j\neq i} \delta^{i,c}\delta^{j,c}\rho_{ij}\right) - \left(\delta^{i,c}(1+\sigma^2) + \sum_{j \neq i}\delta^{j,c}\rho_{ij}\right)\left(\delta^{i,c} + \sum_{j\neq i}\delta^{j,c}\rho_{ij}\right)}{(1+\sigma^2)\left(\sum_i (\delta^{i,c})^2(1+\sigma^2) + \sum_i \sum_{j\neq i} \delta^{i,c}\delta^{j,c}\rho_{ij}\right) - \left(\delta^{i,c}(1+\sigma^2) + \sum_{j\neq i}\delta^{j,c}\rho_{ij}\right)^2}, \\
c_p^{i,c} &= \left(\sum_i \frac{1 - c_p^{i,c}}{\alpha + \lambda^{i,c}}\right) \frac{\sigma^2 \sum_{j \neq i}\delta^{j,c}\rho_{ij}}{(1+\sigma^2)\left(\sum_i (\delta^{i,c})^2(1+\sigma^2) + \sum_i \sum_{j\neq i} \delta^{i,c}\delta^{j,c}\rho_{ij}\right) - \left(\delta^{i,c}(1+\sigma^2) + \sum_{j\neq i}\delta^{j,c}\rho_{ij}\right)^2}, \\
\lambda^{i,c} &= \left(\sum_{j\neq i}(\lambda^{j,c} + \alpha)^{-1}(1 - c_p^{j,c})\right)^{-1}.
\end{aligned} \tag{1.21}$$

where $\delta^{i,c} \equiv \frac{c_s^{i,c}}{\alpha + \lambda^{i,c}}$.

The equilibrium bid is

$$q^{i,c}(p) = \frac{c_s^{i,c}}{\alpha + \lambda^{i,c}}(s^i - \bar{s}), \tag{1.22}$$

where $\bar{s} = \frac{1}{I}\sum_i s^i$.

The equilibrium price is

$$\hat{p}^c = \left(\sum_i \frac{1 - c_p^{i,c}}{\alpha + \lambda^{i,c}}\right)^{-1}\left(\sum_i \frac{c_s^{i,c}\hat{s}^i}{\alpha + \lambda^{i,c}}\right). \tag{1.23}$$

A direct observation is that the realized prices in general differ from exchange to exchange in previous cases, but there is only one price in the centralized market.

For simplicity, in most following analysis, we will focus on a special case where all traders in the centralized market have the same average correlation with the others, i.e., $\frac{1}{I-1}\sum_{j\neq i}\rho_{ij} = \bar{\rho}$, $\forall i$. Note that this put an assumption on within-exchange and cross-exchange correlation, $\frac{(I_n-1)\bar{\rho}_n + \sum_{k\neq n} I_k\bar{\rho}_{nk}}{\sum_k I_k - 1} = \bar{\rho}$, $\forall n = 1, \ldots, N$.

Example 2 (Equicommonal Market (Rostek and Weretka, 2015))**.** *Assume that in the centralized market with* I *traders, each trader's average correlation with other traders in the market is equicommonal, i.e.*$\frac{1}{I-1}\sum_{j\neq i}\rho_{ij} = \bar{\rho}$, $\forall i$. *A linear Bayesian Nash equilibrium exists and is unique,*

$$\begin{aligned} c_s^c &= \frac{1-\bar{\rho}}{1-\bar{\rho}+\sigma^2}, \\ c_p^c &= \frac{(2-\gamma)\bar{\rho}}{1-\gamma+\bar{\rho}}\frac{\sigma^2}{1-\bar{\rho}+\sigma^2}, \\ c_\theta^c &= 1 - c_s^c - c_p^c. \end{aligned} \tag{1.24}$$

where $\gamma \equiv 1 - \frac{1}{I-1} \in [0,1]$ *is the index of market size.*

The equilibrium price impact

$$\lambda^{c,i} = \lambda^c = \frac{1-\gamma}{\gamma - c_p^c}\alpha, \quad \forall i \tag{1.25}$$

The equilibrium price

$$p^c = \mathbb{E}[\bar{\theta}|\bar{s}] = \mathbb{E}[\theta] + cov(\bar{s},\bar{\theta})cov(\bar{s},\bar{s})^{-1}(\bar{s} - \mathbb{E}[\theta]), \tag{1.26}$$

where $\bar{s} = \frac{1}{I}\sum_i s^i$,$\bar{\theta} = \frac{1}{I}\sum_i \theta^i$.

We can see the price impact λ^c is monotonically increasing in $\bar{\rho}$ in Example 2. So cross-exchange correlation also affects price impact in the centralized market, keeping everything else constant,λ^c is monotonically increasing $\bar{\rho}_{nm}$.

1.4 Welfare Comparison

The first practical concern is whether access to cross-exchange information can increase the market liquidity and allocation efficiency.[13] In this section, we compared the welfare in the four designs. We use the ex-ante expected utility of trader i to measure the welfare. For illustration purpose, we assume that the centralized market is equicommonal, i.e., $\frac{(I_n-1)\bar{\rho}_n + \sum_{k\neq n} I_k \bar{\rho}_{nk}}{\sum_k I_k - 1} = \bar{\rho}, \forall n = 1, \ldots, N$. We calculate the ex-ante expected utility in decentralized markets with different degrees of conditioning, and decompose it into liquidity effect and learning effect. The liquidity effect is only determined by the price impacts, and the learning effect is only affected by the difference between the trader's expected value conditional on price and private signal and the market price.

Design 1 Decentralized exchange In Design 1, trader $i \in I(n)$ has ex-ante expected utility

$$\mathbb{E}\left[u^{i,d}\right] = \frac{1}{2\alpha}\underbrace{\frac{\alpha\left(\alpha+2\lambda_n^d\right)}{\left(\alpha+\lambda_n^d\right)^2}}_{\text{liquidity effect}}\underbrace{\mathbb{E}\left[\left(\mathbb{E}\left[\theta^i|p_n^d, s^i\right] - p_n^d\right)^2\right]}_{\text{learning effect}} = \frac{\left(\alpha+2\lambda_n^d\right)}{2\left(\alpha+\lambda_n^d\right)^2}\frac{(1-\bar{\rho}_n)^2}{1-\bar{\rho}_n+\sigma^2}(1-\frac{1}{I_n})\sigma_\theta^2, \quad \forall i$$

Design 2 Decentralized exchange with Full Conditioning In Design 2, trader $i \in I(n)$ has ex-ante expected utility

$$\mathbb{E}\left[u^{i,dc}\right] = \frac{1}{2\alpha}\underbrace{\frac{\alpha\left(\alpha+2\lambda_n^{dc}\right)}{\left(\alpha+\lambda_n^{dc}\right)^2}}_{\text{liquidity effect}}\underbrace{\mathbb{E}\left[\left(\mathbb{E}\left[\theta^i|\boldsymbol{p}^{dc}, s^i\right] - p_n^{dc}\right)^2\right]}_{\text{learning effect}} = \frac{\left(\alpha+2\lambda_n^{dc}\right)}{2\left(\alpha+\lambda_n^{dc}\right)^2}\frac{(1-\bar{\rho}_n)^2}{1-\bar{\rho}_n+\sigma^2}(1-\frac{1}{I_n})\sigma_\theta^2, \quad \forall i$$

[13] The study on the creation of an EU consolidated tape, final report by European Commission in 2020 gives case studies of how consolidated tape could potentially improve trade outcome including better liquidity and allocation.

Design 2′ Decentralized exchange with Partial Conditioning In Design 2′, trader $i \in I(n)$ has ex-ante expected utility

$$\mathbb{E}\left[u^{i,pc}\right] = \mathbb{E}\left[u^{i,dc}\right] \cdot \mathbf{1}_{n=1,\ldots,K} + \mathbb{E}\left[u^{i,d}\right] \cdot \mathbf{1}_{n=K+1,\ldots,N}. \tag{1.27}$$

Compare the above cases, we find that the learning effect is the same for Design 1, Design 2, and Design 2′. The liquidity effect is the only determinant of welfare ranking among the three cases. Therefore, given the same set of parameters $\{\gamma_n, \bar{\rho}_n, \bar{\rho}_{nm}, \alpha, \sigma^2\}$, the price impacts are the sufficient statistics for the welfare comparison in these cases.

Lemma 1 (Welfare Ranking and Price Impact Ranking)**.** *Given the same set of parameters $\{\gamma_n, \bar{\rho}_n, \bar{\rho}_{nm}, \alpha, \sigma^2\}$, the ranking of welfare of Design 1, Design 2 and Design 2′ is inversely related to the ranking of price impacts of Design 1, Design 2 and Design 2′.*

We then calculate the welfare with non-zero price impact for the centralized market.

Design 3 Centralized market Under equicommonality assumption, in Design 3, trader i's ex-ante expected utility is [14]

$$\mathbb{E}\left[u^{i,c}\right] = \frac{1}{2\alpha} \underbrace{\frac{\alpha\left(\alpha + 2\lambda^c\right)}{\left(\alpha + \lambda^c\right)^2}}_{\text{liquidity effect}} \underbrace{\mathbb{E}\left[\left(\mathbb{E}\left[\theta^i | p^c, s^i\right] - p^c\right)^2\right]}_{\text{learning effect}} = \frac{\left(\alpha + 2\lambda^c\right)}{2\left(\alpha + \lambda^c\right)^2} \frac{(1-\bar{\rho})^2}{1-\bar{\rho}+\sigma^2}(1-\frac{1}{I})\sigma_\theta^2, \quad \forall i \tag{1.28}$$

The welfare ranking of Design 3 may be different from its price impact ranking, as the learning effect is different from Design 1, Design 2, and Design 2′. In Design 3, welfare changes as the correlation and the exchange size have changed. When the average correlation $\bar{\rho}$ is small, the centralized market always has the highest welfare. When $\bar{\rho}$ is large enough, the centralized market can yield lower welfare than the other three market designs.

To highlight how the presence of price impact changes the welfare ranking between the different cases, we first provide the welfare ranking in a competitive market where price impact equals to zero as a benchmark.

[14]Without equicommonality assumption, by Proposition 4, $\mathbb{E}\left[u^{i,c}\right] = \sum_i \frac{(\alpha+2\lambda^{i,c})}{2(\alpha+\lambda^{i,c})^2}(c_s^{i,c})^2 Var(s^i - \bar{s})$.

Theorem 2 (Competitive Market Benchmark)**.** *In competitive markets with all price impacts equal to zero,* $\mathbb{E}[u^{i,d}] = \mathbb{E}[u^{i,pc}] = \mathbb{E}[u^{i,dc}] \leq \mathbb{E}[u^{i,c}]$ *if* $\bar{\rho} \in [-1, \bar{\rho}^{+}(\bar{\rho}_n, I_n, \sigma)]$ *for* $i \in I(n)$*; and* $\mathbb{E}[u^{i,d}] = \mathbb{E}[u^{i,pc}] = \mathbb{E}[u^{i,dc}] > \mathbb{E}[u^{i,c}]$ *if* $\bar{\rho} \in (\bar{\rho}^{+}(\bar{\rho}_n, I_n, \sigma), 1]$ *for* $i \in I(n)$*.*

The intuition for Theorem 2 is as follows.

As there is no price impact and traders do not participate in other exchanges, the prices affect traders' ex-ante utility only through the learning effect, i.e. the difference between the expected value and the price.[15] Indeed, price information from other exchanges allows traders to infer about the average signals in other exchanges. However, as traders within an exchange have equal access to cross-exchange information, it changes the expected value for all traders as well as the price by the same constant. So the difference between expected value and price does not change. Therefore, the welfare in Design 1, Design 2, and Design 2' is the same.

The above result implies that access to cross-venue information does not increase welfare when the market is competitive. However, we find it is welfare-improving in imperfectly competitive markets. In the following analysis, we will see how the existence of price impact changes the above welfare ranking.

A side note here is that in an imperfectly competitive market, despite that the ex-ante expected prices are equal to the competitive ones, the ex-ante expected welfare is not the same. In the presence of price impact, the ex-ante expected bid is only a fraction of that in a competitive market. Therefore, the expected allocations are not at their efficient level. In general, the welfare can differ in each exchange and design with non-zero price impacts.

In the following subsections, we will firstly discuss the liquidity effect in detail, especially for the first three cases where the price impact is a sufficient statistic for welfare ranking; we will then discuss the learning effect, especially for the case of the centralized

[15]Note that the limited participation of traders is what distinguish the results of this paper from Rostek and Yoon (2021) where traders have full participation in all exchanges. In Rostek and Yoon (2021), price information from other exchanges can affect traders' expected trades of other assets. Inference error emerges when asset payoffs are correlated and traders can only submit uncontingent demand schedules. And therefore allowing cross-venue conditioning is not neutral to welfare. In this paper, traders have limited participation, and there is no inference error.

market.

1.4.1 Liquidity Effect

In this section, we compare the liquidity effect, or equivalently, the price impacts of the designs.[16] First, we analyze the welfare ranking with general value correlations where within-exchange correlation can be different from cross-exchange correlation. In this part, we find that allowing more conditioning can increase welfare in a Pareto sense. We also find that we can have lower price impacts for some traders in the decentralized market than in the centralized market. Second, we analyze the special case when within-exchange correlation is equal to cross-exchange correlation, $\bar{\rho}_n = \bar{\rho}_{nm}$. In this part, we find that the centralized market always has the lowest price impact among all four cases, and the decentralized market with conditioning has lower price impacts than the fully decentralized market.

1.4.1.1 Liquidity and Welfare in Decentralized Designs

In this part, we consider the general case where the within-exchange and across-exchange correlations can be different, i.e., $\bar{\rho}_{nm} \neq \bar{\rho}_n$, for $m \neq n$.

We begin by comparing the price impacts of three designs of the decentralized market with different degrees of conditioning. We will first compare price impacts in the decentralized market with no conditioning and those with full conditioning. Then we will establish the welfare ranking of the decentralized market designs. The results show that more conditioning is always weakly liquidity-improving and welfare-improving. And the full conditioning case (Design 2) is the optimal among the three decentralized designs.

Lemma 2 (Liquidity With Cross-Venue Conditioning)**.** *If the Bayesian equilibria for Design 1 and Design 2 exist, $\lambda_n^{dc} \leq \lambda_n^d$ for all $n = 1, \ldots, N$. $\lambda_n^{dc} = \lambda_n^d$ is taken if and only if cross-exchange*

[16] In this paper, we focus on non-negative price impacts. The purpose of this restriction is to show that our findings on welfare comparison hold even without relying on negative price impacts. Theoretically, it is possible to have negative price impacts in this model with a certain set of parameters.

correlations $\bar{\rho}_{mn} = 0$ *for any* $n \neq m$*.*

The intuition of **Lemma 2** is as follows. Suppose trader $i \in I(n)$ deviates by increasing the bid by $\delta > 0$, then the residual market $\{j\}_{j \in I(n), j \neq i}$ has to give up δ unit of the asset. The residual market has two incentives to increase the price. First, as the traders are risk-averse, the residual markets' marginal value increases when their holding decreases. This incentive increases the price and does not vary with designs. Second, the residual market can infer about the private value from the price p_n. When trader i deviates and the price p_n increases, the residual market will infer that the average signal realizations increase and update their expected value accordingly. The magnitude of price change due to inference depends on the inference weights on the price p_n. In Design 2, as the traders have access to cross-exchange price information, they assign a lower inference weight on prices in their own exchange than that in Design 1, [17]

$$c^{dc}_{p_n,n} = \frac{\sigma^2}{1-\bar{\rho}_n+\sigma^2}\left(1 - \frac{(1-\bar{\rho}_n)((\bar{\mathcal{C}})^{-1})_{nn}}{I_n}\right) \leq c^{d}_{p,n} = \frac{\sigma^2}{1-\bar{\rho}_n+\sigma^2}\left(1 - \frac{1-\bar{\rho}_n}{I_n(\bar{\mathcal{C}})_{nn}}\right). \tag{1.29}$$

The residual market's expected values increase by less with any price increase in their own exchange in Design 2 than in Design 1. This implies that the price impact for trader i will be lower.

A special case is when the cross-exchange correlations are zero, the price impacts in the two designs equalize. Intuitively, as the cross-exchange correlations are zero, the average values in traders' own exchanges are not correlated with the average signals from other exchanges. Therefore, traders cannot learn any new information about their values from cross-exchange prices. Allowing for conditioning makes no difference.

Lemma 1 and **Lemma 2** provide a foundation for welfare comparison between Design 1, Design 2 and Design 2′. As $\lambda_n^{pc} = \lambda_n^{dc}\mathbf{1}_{n=1,\ldots,K} + \lambda_n^{d}\mathbf{1}_{n=K+1,\ldots,N}$, we can easily get the following result.

Theorem 3 (Conditioning is Welfare-Improving)**.** *If the Bayesian equilibria for Design 1,*

[17] The inequality follows from $((\bar{\mathcal{C}})^{-1})_{nn} \geq ((\bar{\mathcal{C}})_{nn})^{-1}$.

Design 2 and Design 2′ exist, $\lambda_n^{dc} \leq \lambda_n^{pc} \leq \lambda_n^{d}, \forall n = 1, 2, \ldots, N$*, equivalently,* $\mathbb{E}[u^{i,dc}] \geq \mathbb{E}[u^{i,pc}] \geq \mathbb{E}[u^{i,d}]$ *for all traders* $i \in I$ *. The equality is taken if and only if cross-exchange correlations* $\bar{\rho}_{mn} = 0$ *for any* $n \neq m$*.*

Theorem 3 shows that allowing more conditioning always improves welfare for the decentralized market. In particular, compared with the competitive benchmark (c.f. **Theorem 2**) in which allowing access to price information in other exchanges does not improve welfare, it is no longer the case once we account for the price impacts. By argument in **Theorem 2**, the learning effect does not change with conditioning. By **Lemma 2**, market liquidity improves with more conditioning. Therefore, allowing access to price information from exchanges that traders do not participate in weakly improves welfare in the Pareto sense, and full conditioning is the optimal.

1.4.1.2 Liquidity in the Centralized Design vs. Decentralized Designs

In the following part of this section, we compare the price impacts of the centralized market (Design 3) with the three decentralized designs. [18] We allow traders' value correlation to be heterogeneous.

The purpose of the comparison below is to show how the liquidity effect can change with correlations and market size. The conventional wisdom is that the centralized exchange is always more liquid than the decentralized one. Surprisingly, we find that splitting the centralized market into the decentralized market can increase liquidity. This happens when the cross-exchange correlation is sufficiently large. This result resonates with that of Rostek and Weretka (2015), who showed that increasing market size can lower liquidity if the correlation in traders' value is sufficiently higher in the large market.

Theorem 4 (Splitting Market Can Increase Liquidity for Some Exchanges)**.** *If the Bayesian equilibria for all cases exist, there exists a set of parameters* $\{\gamma_n, \bar{\rho}_n, \bar{\rho}_{nm}, \sigma^2\}$ *such that splitting the market can increase liquidity for some exchange* n*.*

[18] Note that the learning effect in Design 3 is different from that in the three decentralized designs, thus, the welfare ranking between Design 3 and the other designs need not coincide with the liquidity ranking.

When we consolidate decentralized exchanges into a centralized exchange, there are two forces that can affect price impacts. The first one is the increase in exchange size. For any given positive deviation in trader i's demand, as the number of traders increases, each trader $j \neq i$ gives up fewer assets, so the residual market's marginal valuation of the asset increases by less. This size effect decreases the price impact. The second one is the change in interdependence between traders' values. This effect can increase the price impact when the traders' value correlations are sufficiently heterogeneous. If cross-exchange correlation is large enough such that traders' values are sufficiently more correlated in the centralized exchange, then the residual market assigns a higher weight on the price, and increases the price impact by more due to inference.

1.4.1.3 Equal Correlation

We have discussed the general setting where the within-exchange correlations and the cross-exchange correlations can be different. Now let's consider a special case that the correlations are equal, i.e. $\bar{\rho}_n = \bar{\rho}_{nm} = \rho$. We find that with equal correlations, the centralized market always has the lowest price impact.

Theorem 5 (Price Impact Ranking With Equal Correlation)**.** *With $\bar{\rho}_n = \bar{\rho}_{nm} = \rho$ for any n, m, $\lambda_n^c \leq \lambda_n^{dc} \leq \lambda_n^{pc} \leq \lambda_n^d$ for all $n = 1, 2, \ldots, N$ if the equilibria in these cases exist.*

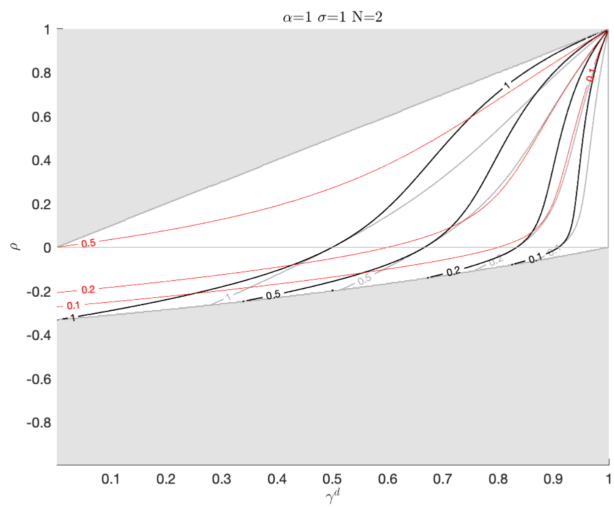
The intuition for **Theorem 5** is as follows. First, the price impact ranking with the decentralized case follows from **Theorem 3**. Second, with equal correlation, the residual market does not have a stronger inference in the centralized design than the decentralized designs. Third, as $I > I_n$ always holds, the increase in exchange sizes further reduces the price impacts in Design 3. Note that **Theorem 5** holds regardless of exchange size and the number of exchanges. [19]

Figure 1.1 gives an example of two exchanges with equal correlations and equal sizes. For any given exchange sizes and correlations, allowing cross-venue conditioning can

[19] Appendix Figure 1.4 gives an illustration of price impacts with respect to the number of exchanges. Appendix Figure 1.5 shows the price impacts with respect to exchange size.

decrease price impacts, and combining the two exchanges into one will further decrease price impacts.

Figure 1.1: Equal Correlations and Equal Size Exchanges



Note: The figure shows the relationship between λ^d, λ^{dc} and λ^c with $\alpha = 1, \sigma = 1$ and $N = 2$, $\bar{\rho}_1 = \bar{\rho}_2 = \bar{\rho}_{12} = \rho$, $\gamma_1 = \gamma_2 = \gamma^d$. Note that when correlations are equal and exchanges are of equal size, $\lambda_n^d = \lambda^d, \lambda_n^{dc} = \lambda^{dc}$ for all $n = 1, 2 \ldots, N$. The x-axis is the size of each exchange. The y-axis is the correlation ρ. The white region is where Bayesian Nash equilibria in Design 1, Design 2, Design 2' and Design 3 exist. The gray lines are the contour lines for λ^d. The black lines are the contour lines for λ^{dc}. The red lines are the contour lines for λ^c.

1.4.2 Learning Effect

In this section, we will discuss the learning effect. In particular, we focus on how the learning effect in the centralized market can be lower than that in the decentralized markets.

If we look more closely into the ex-ante utility, we can further decompose the learning effect into inference effect and size effect. In Design 2 (or Design 1, Design 2'), the decomposition is:

$$\underbrace{\mathbb{E}\left[\left(\mathbb{E}\left[\theta^i|\boldsymbol{p}^{dc}, s^i\right] - p_n^{dc}\right)^2\right]}_{\text{learning effect}} = \underbrace{(c_s^{dc})^2 Var(s^i - \bar{s}_n)}_{\text{learning effect}} = \underbrace{\frac{(1-\bar{\rho}_n)^2}{1-\bar{\rho}_n+\sigma^2}}_{\text{inference effect}} \underbrace{(1-\frac{1}{I_n})\sigma_\theta^2}_{\text{size effect}}. \tag{1.30}$$

In Design 3, we have the following decomposition:

$$\underbrace{\mathbb{E}\left[\left(\mathbb{E}\left[\theta^i|p_c,s^i\right]-p^c\right)^2\right]}_{\text{learning effect}}=\underbrace{(c_s^c)^2Var(s^i-\bar{s})}_{\text{learning effect}}=\underbrace{\frac{(1-\bar{\rho})^2}{1-\bar{\rho}+\sigma^2}}_{\text{inference effect}}\underbrace{(1-\frac{1}{I})\sigma_\theta^2}_{\text{size effect}}. \tag{1.31}$$

Intuitively, the learning effect stems from the gain from information provided by the private signal beyond the average signal, which can be learnt by the residual market from the prices. When the market is large, the private signal and average signal are less correlated, which implies that the information from the private signal beyond the market price is larger. So the welfare gain size effect increases with market size. However, the correlation between private signal and average signal decays at a slower rate when the market size is larger, so the potential gain through the size effect from turning the decentralized market into the centralized market decreases as individual exchange size grows. [20] Suppose we turn the decentralized market into a centralized market. Compare equations (1.30) and (1.31), as $I > I_n$, the welfare gain from size effect has increased. And as the exchange size I_n increases, such welfare gain from the size effect decays. [21]

As we turn the decentralized market into a centralized market, the change in inference effect is ambiguous. If $\bar{\rho} < \bar{\rho}_n$, the inference effect will increase; and if $\bar{\rho} > \bar{\rho}_n$, the inference effect will decrease.[22] Intuitively, when the value correlation increases, the private signal is more correlated with the average signal, so the gain from the private signal is lower. Therefore, welfare associated with learning decreases.

Note that $\bar{\rho}$ is a function of within-exchange correlation $\bar{\rho}_n$, cross-exchange correlation

[20]The correlation between private signal s^i and the average signal $\bar{s}$ is $\bar{\rho}+\frac{1-\bar{\rho}+\sigma^2}{I}$, which is decreasing in I, and the decreasing rate is increasing.

[21]This can be seen if we take the ratio of size effects in centralized design and decentralized design:

$$\frac{\text{size effect in centralized market}}{\text{size effect in decentralized market}}=1+\frac{\sum_{m\neq n}I_m}{(I_n-1)(I_n+\sum_{m\neq n}I_m)}. \tag{1.32}$$

This ratio is decreasing in I_n.

[22]To see this point, let's take the derivative of inference effect $\frac{(1-\rho)^2}{1-\rho+\sigma^2}$ with respect to ρ, we get

$$\frac{\partial\text{ inference effect}}{\partial\rho}=\frac{(\rho-1)\left(2\,\sigma^2-\rho+1\right)}{\left(\sigma^2-\rho+1\right)^2}\leq 0. \tag{1.33}$$

So the inference effect is monotonically decreasing in ρ.

$\bar{\rho}_{nm}$, and exchange size:

$$\bar{\rho} - \bar{\rho}_n = \frac{\sum_{m \neq n} I_m (\bar{\rho}_{nm} - \bar{\rho}_n)}{\sum_m I_m - 1} \quad \forall n = 1, \ldots, N, \tag{1.34}$$

From equation (1.34) we can see that the difference between $\bar{\rho}$ and $\bar{\rho}_n$ is dependent on the difference between cross-exchange correlation $\bar{\rho}_{nm}$ and within-exchange correlation $\bar{\rho}_n$. As the cross-exchange correlation increases, the welfare due to the inference effect can decrease if we turn the decentralized market into the centralized market. When $\bar{\rho}_{nm}$ is sufficiently larger than $\bar{\rho}_n$, the welfare reduction in inference effect can be large enough to counterbalance the increase in size effect. In this case, the learning effect can be lower in the centralized market design than in decentralized market designs.

1.4.3 Pareto-Optimal Design

In previous sections, we have shown that the welfare ranking of Design 1, Design 2, and Design 2′ is determined by the liquidity effect alone. We find that allowing access to price information in other exchanges always reduces price impacts and improves welfare in the Pareto sense. Full conditioning (Design 2) yields the highest welfare.

We are left with the welfare comparison between the decentralized designs and the centralized design. We have shown that both the liquidity effect and the learning effect can be welfare-decreasing for Design 3 compared with the three decentralized designs. Combining the liquidity effect and the learning effect, both centralized market and decentralized design (Design 2) can be Pareto-optimal.

Theorem 6 (Pareto-optimal Design)**.** *Decentralized market with contingent demand (Design 2) and the centralized market design (Design 3) can be the Pareto-optimal design among Design 1, Design 2, Design 2′, and Design 3.*

Let's consider again an example of two equal size exchanges, with $\bar{\rho}_1 = \bar{\rho}_2$, $\bar{\rho}_{12} = \bar{\rho}_{21}$ (Example 1 for decentralized market, and Example 2 for centralized market). Figure 1.2 plots the welfare-maximizing design with respect to exchange size (horizontal-axis γ_n)

and cross-exchange correlation (vertical-axis $\bar{\rho}_{12}$). We have several observations from the figure:

1. Given a large enough exchange size, as the cross-exchange correlation increases, the optimal design changes from the centralized market to the decentralized market with full conditioning. As the difference between the cross-exchange correlation and with-exchange correlation increases, the learning effect starts to decrease for the centralized market (see equations (1.30), (1.31), and (1.34)). Despite that the centralized market always have lower price impact than Design 2 in this setting, [23] the welfare gain from liquidity effect from consolidating decentralized markets into centralized one is decreasing when the cross-exchange correlation is large: We learn from Example 1 that λ_n^{dc} is decreasing in the cross-exchange correlation when it is positive; at the same time, we learn from Example 2 that the price impact in the centralized market is increasing with cross-exchange correlation; so the potential liquidity improvement is lower. To summarize, if we turn the decentralized exchange into a centralized one, the welfare loss due to the learning effect can dominate the welfare gain due to the liquidity effect when the cross-exchange correlation is sufficiently large.

2. Given cross-exchange correlation, as the exchange size increases, the optimal design changes from the centralized market to the decentralized market with full conditioning. As the market size increases, the difference in size effect between the centralized market and decentralized market becomes smaller (see equation 1.32). The reduction in size effect makes it more likely for Design 2 to be the optimal design.

The heterogeneous correlation is crucial for the decentralized market with contingent

[23] In this special case when there are only two equal-size exchanges, and $\bar{\rho}_1 = \bar{\rho}_2$, $\bar{\rho}_{12} = \bar{\rho}_{21}$, the centralized market always has the lowest price impact. This is because $\frac{(1-\gamma_n)(1-\bar{\rho}_n+\sigma^2)}{(1-\bar{\rho}_n)}\left(cov(\bar{\boldsymbol{\theta}}, \bar{\boldsymbol{s}}')cov(\bar{\boldsymbol{s}}, \bar{\boldsymbol{s}}')^{-1}\right)_{nn} \geq \frac{(1-\gamma)(1-\bar{\rho}+\sigma^2)}{(1-\bar{\rho})}cov(\bar{s}, \bar{\theta})cov(\bar{s}, \bar{s})^{-1}$ for $n = 1, 2$, for any positive semi-definite value correlation matrix. By Theorem 4, $\lambda^c \leq \lambda_n^{dc}$ for $n = 1, 2$. The argument does not hold when $\bar{\rho}_1 \neq \bar{\rho}_2$.

demand schedules to be the optimal design. When correlations are equal, the centralized market is always optimal. This result holds regardless of the number and size of exchanges.

Theorem 7 (Optimal Design with Equal Correlation)**.** *When within-exchange correlations and cross-exchange correlations are equal, i.e., $\bar{\rho}_n = \bar{\rho}_{mn} = \rho$, the centralized market is the Pareto-optimal design among Design 1, Design 2, Design 2′, and Design 3.*

When the correlations are equal, **Theorem 5** implies that the centralized market has highest liquidity. Besides, the inference effect is the same for all cases, and the gain from the size effect is always higher in the centralized market. So the learning effect is also the highest in the centralized market.

1.5 Price Informativeness

Besides allocation efficiency, another goal of the policy designs like the consolidated tape and RegNMS is to improve the information efficiency.[24] In this section, we compare the price informativeness of the above designs. We define the price informativeness as

$$\psi_n \equiv \frac{Var(\theta^i|s^i) - Var(\theta^i|s^i, \mathcal{I}^i)}{Var(\theta^i|s^i)}. \tag{1.35}$$

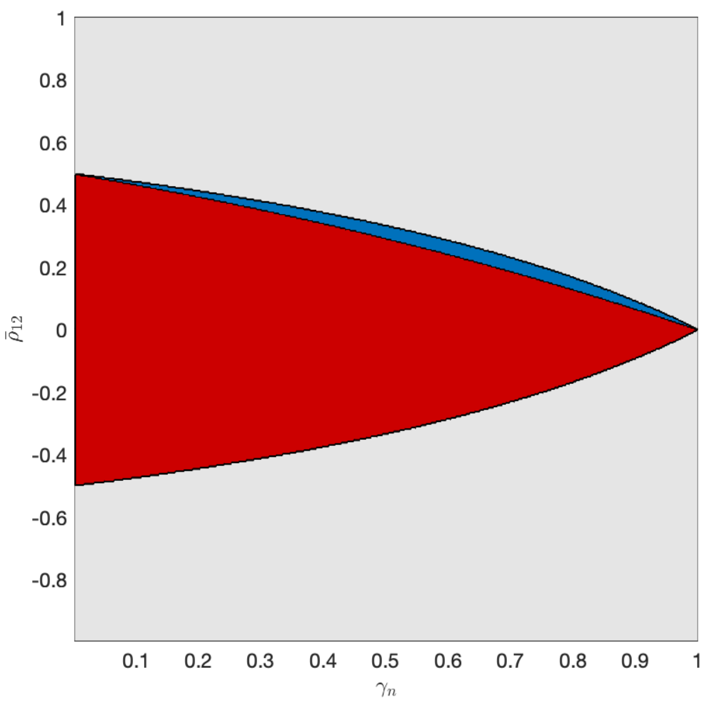
The index ψ_n quantifies how much the market price information available to a trader contributes to his inference about his value. [25] In a decentralized exchange where traders cannot access cross-exchange price information (Design 1), $\mathcal{I}^i = \{p_n^d\}$; in a decentralized exchange with cross-exchange information, $\mathcal{I}^i = \{\boldsymbol{p}^{dc}\}$; and in a centralized exchange, $\mathcal{I}^i = \{p^c\}$. [26] Note that traders within exchange n has the same price informativeness.

[24]In The study on the creation of an EU consolidated tape, final report by the European Commission in 2020, the consolidated tape is proposed to provide the benefit of "more accurate pricing, valuation and benchmarking".

[25]This definition of price informativeness is an overall price informativeness instead of the informativeness for the price in a given exchange. We focus on overall price informativeness as it is more policy-relevant. The informativeness in a given exchange in Design 2 may be lower than Design 1 and 3, when the value correlations are not sufficiently heterogeneous. However, traders in Design 2 have access to all prices. These prices overall are weakly more informative than the prices in Design 1 and 3.

[26]The price informativeness in Design 2′ for traders in exchange with access to cross-venue prices is the

Figure 1.2: Welfare Comparison

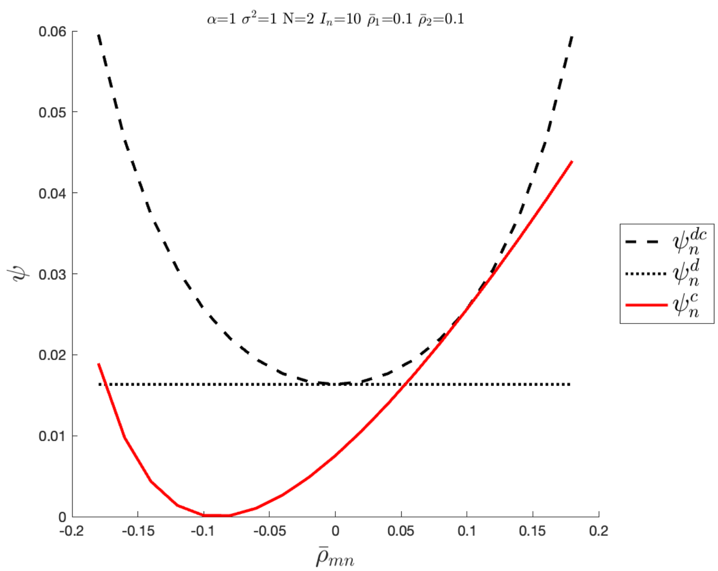


Note: The figure shows welfare ranking with $\alpha = 1, \sigma = 1, N = 2$. We assume that the two exchanges have the same size, $\gamma_1 = \gamma_2 = \gamma_n$, and the within-exchange correlation $\bar{\rho}_1 = \bar{\rho}_2 = 0$. The x-axis is the size of the two exchanges. The y-axis is the cross-exchange correlation. The gray region is where Bayesian Nash equilibria do not exist in all cases. The red region indicates the parameters where the Design 3 centralized market is Pareto optimal. The blue region indicates the parameters where Design 2 decentralized market with contingent demand is Pareto optimal.

This is due to the assumption that traders within one exchange have the same within-exchange correlation and cross-exchange correlations.

Figure 1.3 shows the price informativeness of the designs. In this symmetric two-exchange example, we find that Design 2 always has the highest price informativeness, and that Design 1 can have higher welfare than Design 3.

Figure 1.3: Price Informativeness



Note: This figure shows the price informativeness in Design 1, 2 and 3 with respect to cross-exchange correlations ρ_{mn}. There are two symmetric exchanges. Each has $I_n = 10$ traders. Within exchange correlation $\rho_1 = \rho_2 = 0.1$. Traders have risk-aversion $\alpha = 1$. Noise to value variance ratio $\sigma^2 = 1$.

In fact, the following theorem characterizes the price informativeness ranking of the designs.

Theorem 8 (Price Informativeness)**.** *When the equilibria for all designs exist,*

1. *The price informativeness in the decentralized market with cross-venue conditioning (Design 2) is weakly higher than the price informativeness in the decentralized market (Design 1), the decentralized market with partial conditioning (Design 2'). The equality is taken when all cross-exchange correlations are zero, i.e.,* $\bar{\rho}_{mn} = 0, \forall m \neq n$.

same as that of Design 2, and the price informativeness for those without cross-venue prices is the same as that of Design 1.

2. *The price informativeness in the decentralized market with cross-venue conditioning (Design 2) is weakly higher than that in the centralized market (Design 3).*

3. *The price informativeness in the decentralized market (Design 1, Design 2, Design 2′) can be higher than that in the centralized market (Design 3). The price in the centralized market (Design 3) is more informative than the fully decentralized market (Design 1) when the correlation matrix is sufficiently heterogeneous.*

The price informativeness in the decentralized market with cross-venue conditioning (Design 2) is always weakly higher than that without cross-venue conditioning (Design 1), as the traders can only learn about the average signal in their own exchanges in Design 1, but the average signals from all exchanges in Design 2. Design 2 has the same price informativeness as Design 1 if and only if the cross-exchange correlations are zero. Intuitively, this is when traders' average values are independent across exchanges, so allowing cross-venue conditioning does not change inference.

The price informativeness in the decentralized market with cross-venue conditioning (Design 2) is weakly higher than the centralized market (Design 3). Prices in Design 2 reveal the average signals $\{\bar{s}_n\}_n$ in each exchange. The price in Design 3 is just a linear combination of these average signals and therefore is informationally equivalent to a weighted average of prices in Design 2. So the price in Design 3 cannot be more informative than that in Design 2. [27]

The comparison between the fully decentralized market (Design 1) and the centralized market (Design 3) is ambiguous. The price in the centralized market is more informative than the fully decentralized market when the correlations are sufficiently heterogeneous, but not otherwise. This is because, when the cross-exchange correlation is sufficiently large (or small), the average correlation between trader values in the centralized exchange is larger than that in the decentralized exchanges. Therefore, traders can

[27] Note that in Design 2′, for the traders in exchanges with access to cross-exchange information, the price informativeness is the same as Design 2. And for traders in exchanges without access to cross-exchange information, the price informativeness is the same as Design 1. The comparison of Design 2′ and the other designs then follows from the above argument.

infer more from the price in the centralized market.

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Appendices

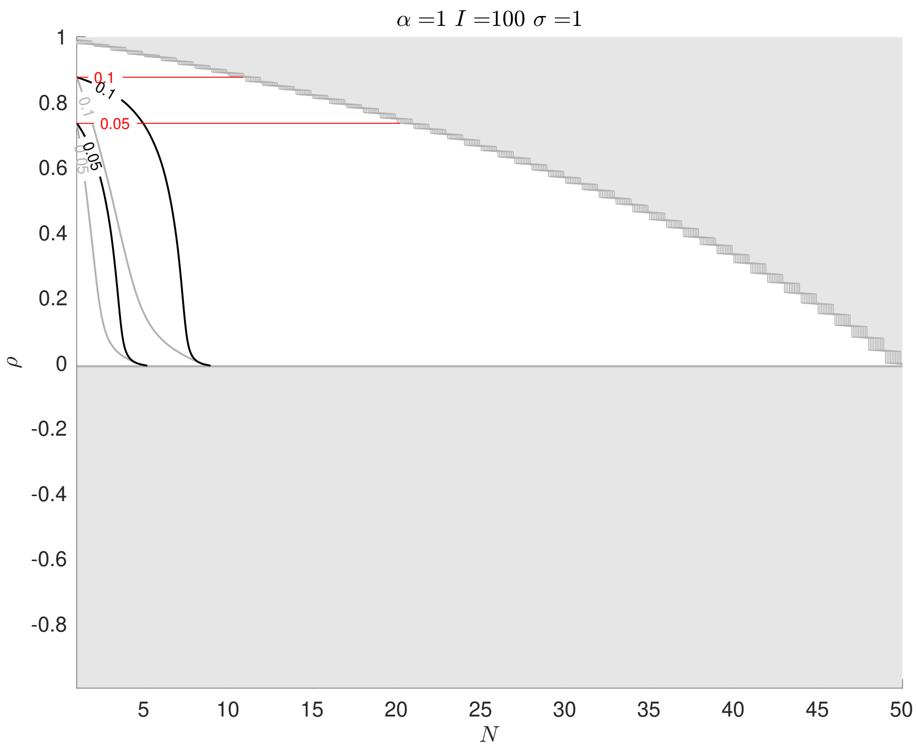
Appendix A. Additional Figures

Appendix B. Proofs

Appendix C. Robustness Checks

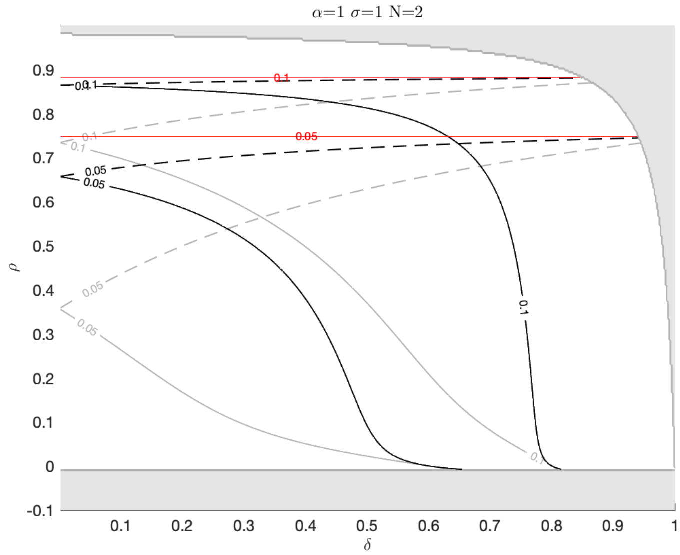
Appendix A. Additional Figures

Figure 1.4: Multiple Exchanges with Symmetric Correlation and Symmetric Size

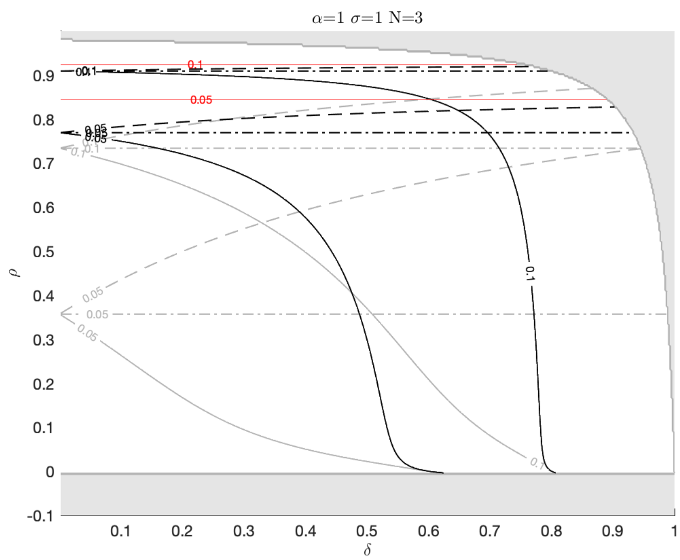


Note: The figure shows the relationship between λ_n^d, λ_n^{dc} and λ^c with $\alpha = 1, \sigma = 1$ and the total number of traders $I = 100$. The horizontal axis N is the number of exchanges. The white region is where Bayesian Nash equilibria in Design 1, 2 and 3 exist. The gray lines are the contour lines for λ_n^d. The black lines are the contour lines for λ_n^{dc}. The red lines are the contour lines for λ^c.

Figure 1.5: Symmetric Correlation and Asymmetric Exchange



(a)



(b)

Note: The figure shows the relationship between λ_n^d, λ_n^{dc} and λ^c with $\alpha = 1, \sigma = 1$. The horizontal axis δ measures the asymmetry in exchange size. $\delta = \frac{I_1 - I_2}{I_1 + I_2}$. $I_1 + I_2 = 104$. Panel (a) shows $N = 2$ exchanges, and Panel (b) shows $N = 3$ exchanges with $I_3 = 52$. The white region is where Bayesian Nash equilibria in Design 1, 2 and 3 exist. The gray lines are the contour lines for λ_n^d. The black lines are the contour lines for λ_n^{dc}. The red lines are the contour lines for λ^c. The solid line indicates the price impact in exchange $I(1)$, the dashed line indicates the price impact in exchange $I(2)$, the dashed-dot line indicates the price impact in exchange $I(3)$.

Appendix B. Proofs

Proof of Proposition 1. **Inference coefficients:** Using market clearing condition, the equilibrium price $p_n^d = \frac{1}{I_n}\sum_{i\in I(n)} \mathbb{E}[\theta^i|s^i, p_n^d]$. Combine it with $\mathbb{E}[\theta^i|s^i,p^d] = c_{\theta,n}^d\mathbb{E}[\theta] + c_{s,n}^d s^i + c_{p,n}^d p_n^d$, for $i \in I(n)$, the equilibrium price can be written as

$$p_n^d = \frac{c_{\theta,n}^d\mathbb{E}[\theta]}{1-c_{p,n}^d} + \frac{c_{s,n}^d}{1-c_{p,n}^d}\bar{s}_n, \tag{1.36}$$

where $\bar{s}_n = \frac{1}{I_n}\sum_{i\in I(n)} s^i$ is the average signal in exchange $I(n)$.

We can determine the inference coefficients as a function of the primitives (and in closed form). Random vector (θ^i, s^i, p_n^d) is jointly normally distributed

$$\begin{pmatrix} \theta^i \\ s^i \\ p_n^d \end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix} \mathbb{E}[\theta] \\ \mathbb{E}[\theta] \\ \mathbb{E}[\theta] \end{pmatrix}, \begin{pmatrix} \sigma_\theta^2 & \sigma_\theta^2 & cov\left(\theta^i, p_n^d\right) \\ \sigma_\theta^2 & \sigma_\theta^2+\sigma_\varepsilon^2 & cov\left(s^i, p_n^d\right) \\ cov\left(p_n^d, \theta^i\right) & cov\left(p_n^d, s^i\right) & Var\left(p_n^d\right) \end{pmatrix}\right],$$

where covariances are given by

$$cov\left(\theta^i, p^d\right) = \tfrac{1}{I_n}\tfrac{c_{s,n}^d}{1-c_{p,n}^d}(1+(I_n-1)\bar{\rho}_n)\sigma_\theta^2,$$
$$cov\left(s^i, p^d\right) = \tfrac{1}{I_n}\tfrac{c_{s,n}^d}{1-c_{p,n}^d}\left((1+(I_n-1)\bar{\rho}_n)+\sigma^2\right)\sigma_\theta^2,$$

and

$$Var\left(p^d\right) = \frac{1}{I_n}\left(\frac{c_{s,n}^d}{1-c_{p,n}^d}\right)^2\left((1+(I_n-1)\bar{\rho}_n)+\sigma^2\right)\sigma_\theta^2.$$

Applying the Projection Theorem and the method of undetermined coefficients yields the inference coefficients c_s^c and c_p^c . We can determine coefficients c_s^c and c_p^c and c_θ^c as functions of the primitives:

$$\begin{aligned} c^d_{s,n} &= \frac{1-\bar{\rho}_n}{1-\bar{\rho}_n+\sigma^2}, \\ c^d_{p,n} &= \frac{(2-\gamma_n)\bar{\rho}_n}{1-\gamma_n+\bar{\rho}_n}\frac{\sigma^2}{1-\bar{\rho}_n+\sigma^2}, \\ c^d_{\theta,n} &= 1-c^d_{s,n}-c^d_{p,n}. \end{aligned} \tag{1.37}$$

where $\gamma_n \equiv 1-\frac{1}{I_n-1} \in [0,1]$ is the index of exchange size.

Price impacts: By symmetry, the price impacts for traders in the same exchange are equal, i.e. $\lambda^i = \lambda^d_n, \forall i \in I(n)$. The correct price impact satisfies:

$$\lambda^d_n = -(\sum_{j\neq i, j\in I(n)} \frac{\partial q^j}{\partial p_n})^{-1}.$$

Plug in that $\frac{\partial q^j}{\partial p_n} = \frac{-(1-c^d_{p,n})}{\alpha+\lambda^d_n}$, we can solve the equilibrium price impact for traders in exchange $I(n)$ is

$$\lambda^d_n = \frac{1-\gamma_n}{\gamma_n - c^d_{p,n}}\alpha.$$

Note that the price impact is uniquely determined in equilibrium. As traders in the same exchange are identical, $-(\sum_{j\neq i,j\in I(n)} \frac{\partial q^j}{\partial p_n})^{-1}$ is a linear function of price impact.

Bids: The equilibrium bid for trader i in exchange $I(n)$ is

$$q^{i,d}_n = \frac{1}{(\alpha+\lambda^d_n)}(c^d_{\theta,n}\mathbb{E}[\theta] + c^d_{s,n}s^i + (c^d_{p,n}-1)p^d_n), \quad \forall i \in I(n).$$

By the fact that $p^d_n = \frac{c^d_{\theta,n}\mathbb{E}[\theta]}{1-c^d_{p,n}} + \frac{c^d_{s,n}}{1-c^d_{p,n}}\bar{s}_n$,

$$q^{i,d}_n = \frac{1}{(\alpha+\lambda^d_n)}c^d_{s,n}(s^i - \bar{s}_n), \quad \forall i \in I(n).$$

Prices: Plug in the inference coefficients in equation (1.37) into equation (1.36), we can solve for the equilibrium price in exchange $I(n)$

$$\hat{p}_n^d = \frac{c_{s,n}^d}{1 - c_{p,n}^d}\hat{s}_n = \frac{cov(\bar{\theta}_n, \bar{s}_n)}{Var(\bar{s}_n)}\hat{s}_n = \frac{1 - \gamma_n + \bar{\rho}_n}{(1-\gamma_n)(1+\sigma^2) + \bar{\rho}_n}\hat{s}_n.$$

where $\hat{p}_n^d = p_n^d - \mathbb{E}[\theta], \hat{s}_n = \bar{s}_n - \mathbb{E}[\theta]$.

Existence Conditions: For equilibrium to exist, the parameters have to satisfy the following conditions:

1. For the stochastic process that generates the joint distribution of value to exist $\rho_n > -(1-\gamma_n), \forall n$.

2. For the demand schedule to be downward-sloping, $\lambda_n^d > 0$.

■

Proof of Proposition 2. **Inference coefficients:** For $i \in I(n)$, $\mathbb{E}[\theta^i|\boldsymbol{p}^{dc}, s^i] = c_{\theta,n}^{dc}\mathbb{E}[\theta] + c_{s,n}^{dc}s^i + \boldsymbol{c}_{p,n}^{dc} \cdot \boldsymbol{p}^{dc}$ where $\boldsymbol{c}_{p,n}^{dc} = (c_{p_m,n}^{dc})_m \in \mathbb{R}^{1\times N}$ is a row vector. Combining this with market clearing condition $\sum_{i\in I(n)} q^i\left(\boldsymbol{p}^{dc}\right) = 0$, we have

$$\boldsymbol{p}^{dc} = \boldsymbol{c}_\theta^{dc}\mathbb{E}[\theta] + \boldsymbol{C}_s^{dc}\bar{\boldsymbol{s}} + \boldsymbol{C}_p^{dc}\boldsymbol{p}^{dc},$$

where $\boldsymbol{c}_\theta^{dc} = (c_\theta^{dc,n})_n \in \mathbb{R}^N, \boldsymbol{C}_s^{dc} = diag(c_s^{dc,n})_n \in \mathbb{R}^{N\times N}$, $\boldsymbol{C}_p^{dc} = (c_{p_m,n}^{dc})_{nm} \in \mathbb{R}^{N\times N}$ and $\bar{\boldsymbol{s}} = (\bar{s}_n)_n \in \mathbb{R}^N$ where $\bar{s}_n = \frac{1}{I_n}\sum_{i\in I(n)} s^i$. Combining the market-clearing condition and linear posterior expectation, we can solve

$$\boldsymbol{p}^{dc} = (Id - \boldsymbol{C}_p^{dc})^{-1}(\boldsymbol{c}_\theta^{dc}\mathbb{E}[\theta] + \boldsymbol{C}_s^{dc}\bar{\boldsymbol{s}}).$$

We can determine the inference coefficients as a function of the primitives (and in closed form). Random vector $\left(\theta^i, s^i, \boldsymbol{p}^{dc\prime}\right)$ is jointly normally distributed

$$\begin{pmatrix} \theta^i \\ s^i \\ \boldsymbol{p}^{dc} \end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix} \mathbb{E}[\theta] \\ \mathbb{E}[\theta] \\ \mathbb{E}[\boldsymbol{\theta}] \end{pmatrix}, \begin{pmatrix} \sigma_\theta^2 & \sigma_\theta^2 & cov\left(\theta^i, \boldsymbol{p}^{dc\prime}\right) \\ \sigma_\theta^2 & \sigma_\theta^2+\sigma_\varepsilon^2 & cov\left(s^i, \boldsymbol{p}^{dc\prime}\right) \\ cov\left(\boldsymbol{p}^{dc}, \theta^i\right) & cov\left(\boldsymbol{p}^{dc}, s^i\right) & cov\left(\boldsymbol{p}^{dc}, \boldsymbol{p}^{dc\prime}\right) \end{pmatrix}\right],$$

where

$$\begin{aligned} cov(\boldsymbol{p}^{dc}, \theta^i) &= (Id - \boldsymbol{C}_p^{dc})^{-1}\boldsymbol{C}_s^{dc} cov(\bar{\boldsymbol{s}}, \theta^i), \\ cov(\boldsymbol{p}^{dc}, s^i) &= (Id - \boldsymbol{C}_p^{dc})^{-1}\boldsymbol{C}_s^{dc} cov(\bar{\boldsymbol{s}}, s^i), \\ cov(\boldsymbol{p}^{dc}, \boldsymbol{p}') &= (Id - \boldsymbol{C}_p^{dc})^{-1}\boldsymbol{C}_s^{dc} cov(\bar{\boldsymbol{s}}, \bar{\boldsymbol{s}}')(\boldsymbol{C}_s^{dc})'((Id - \boldsymbol{C}_p^{dc})^{-1})'. \end{aligned} \tag{1.38}$$

where $cov(\bar{\boldsymbol{s}}, \theta^i) \in \mathbb{R}^N$,for $i \in I(\ell)$, $cov(\bar{\boldsymbol{s}}, \theta^i)_n = (\bar{\rho}_n + \frac{1}{I_n}(1-\bar{\rho}_n))\sigma_\theta^2$ if $n = \ell$. $cov(\bar{\boldsymbol{s}}, \theta^i)_n = \bar{\rho}_{n\ell}\sigma_\theta^2$ if $n \neq \ell$. $cov(\bar{\boldsymbol{s}}, s^i) \in \mathbb{R}^N$, for $i \in I(\ell)$, $cov(\bar{\boldsymbol{s}}, s^i)_n = (\bar{\rho}_n + \frac{1}{I_n}(1-\bar{\rho}_n+\sigma^2))\sigma_\theta^2$ if $n = \ell$. $cov(\bar{\boldsymbol{s}}, s^i)_n = \bar{\rho}_{n\ell}\sigma_\theta^2$ if $n \neq \ell$. And $cov(\bar{\boldsymbol{s}}, \bar{\boldsymbol{s}}') \in \mathbb{R}^{N\times N}$ is a matrix whose element $cov(\bar{\boldsymbol{s}}, \bar{\boldsymbol{s}}')_{nm} = \begin{cases} (\bar{\rho}_n + \frac{1}{I_n}(1+\sigma^2-\bar{\rho}_n))\sigma_\theta^2 & \text{if } n = m \\ \bar{\rho}_{nm}\sigma_\theta^2 & \text{otherwise} \end{cases}$.

Applying Projection Theorem,

$$\mathbb{E}[\theta^i|\boldsymbol{p}^{dc}, s^i] = E(\theta) + (\sigma_\theta^2 \quad cov(\theta^i, \boldsymbol{p}^{dc\prime}))\begin{pmatrix} \sigma_\theta^2+\sigma_\varepsilon^2 & cov(s^i, \boldsymbol{p}^{dc\prime}) \\ cov(\boldsymbol{p}^{dc}, s^i) & cov(\boldsymbol{p}^{dc}, \boldsymbol{p}^{dc\prime}) \end{pmatrix}^{-1}\begin{pmatrix} s^i - E(\theta) \\ \boldsymbol{p}^{dc} - E(\boldsymbol{\theta}) \end{pmatrix}.$$

Applying the method of undermined coefficients, we have

$$\boldsymbol{c}_\theta^{dc} = (Id - \boldsymbol{C}_p^{dc} - \boldsymbol{C}_s^{dc}) \cdot \boldsymbol{1}_N.$$

and

$$(c_s^i \quad \boldsymbol{c}_p^i) = (\sigma_\theta^2 \quad cov(\theta^i, \boldsymbol{p}'))\begin{pmatrix} \sigma_\theta^2+\sigma_\varepsilon^2 & cov(s^i, \boldsymbol{p}') \\ cov(\boldsymbol{p}, s^i) & cov(\boldsymbol{p}, \boldsymbol{p}') \end{pmatrix}^{-1}, \forall i.$$

If we plug $cov(\boldsymbol{p}, \boldsymbol{p}')$, $cov(\boldsymbol{p}, s^i)$ and $cov(\boldsymbol{p}, \theta^i)$ in equation (1.38) into the above equation, we will have

$$c_s^i cov(s^i, \bar{s}_n) + \boldsymbol{c}_p^i (Id - \boldsymbol{C}_p)^{-1} \boldsymbol{C}_s cov(\bar{\boldsymbol{s}}, \bar{s}_n) = cov(\theta^i, \bar{s}_n), \quad \forall i, n. \tag{1.39}$$

and

$$c_s^i (1 + \sigma^2) \sigma_\theta^2 + \boldsymbol{c}_p^i (Id - \boldsymbol{C}_p)^{-1} \boldsymbol{C}_s cov(\bar{\boldsymbol{s}}, s^i) = \sigma_\theta^2. \tag{1.40}$$

Summing equation (1.40) over all $i \in I(n)$ gives

$$c_s^i (1 + \sigma^2) \sigma_\theta^2 + \boldsymbol{c}_p^i (Id - \boldsymbol{C}_p)^{-1} \boldsymbol{C}_s cov(\bar{\boldsymbol{s}}, \bar{s}_n) = \sigma_\theta^2. \tag{1.41}$$

Combing equations (1.39) and (1.41), for $i \in I(n)$,

$$c_s^i = \frac{cov(\theta^i, \bar{s}_n) - \sigma_\theta^2}{cov(s^i, \bar{s}_n) - (1 + \sigma^2)\sigma_\theta^2} = \frac{1 - \bar{\rho}_n}{1 - \bar{\rho}_n + \sigma^2}.$$

Therefore,

$$\boldsymbol{C}_s^{dc} = diag\left(\frac{1 - \bar{\rho}_n}{1 - \bar{\rho}_n + \sigma^2}\right)_n.$$

Consolidating equation (1.39) into matrix form gives

$$\boldsymbol{C}_s^{dc} cov(\bar{\boldsymbol{s}}, \bar{\boldsymbol{s}}') + \boldsymbol{C}_p^{dc} (Id - \boldsymbol{C}_p^{dc})^{-1} \boldsymbol{C}_s^{dc} cov(\bar{\boldsymbol{s}}, \bar{\boldsymbol{s}}') = cov(\bar{\boldsymbol{\theta}}, \bar{\boldsymbol{s}}').$$

By rearranging terms, we get

$$\boldsymbol{C}_p^{dc} (Id - \boldsymbol{C}_p^{dc})^{-1} = \Big(cov(\bar{\boldsymbol{\theta}}, \bar{\boldsymbol{s}}') - \boldsymbol{C}_s^{dc} cov(\bar{\boldsymbol{s}}, \bar{\boldsymbol{s}}')\Big) cov(\bar{\boldsymbol{s}}, \bar{\boldsymbol{s}}')^{-1} (\boldsymbol{C}_s^{dc})^{-1}. \tag{1.42}$$

Plug in $\boldsymbol{C}_s^{dc} = diag(\frac{1-\bar{\rho}_n}{1-\bar{\rho}_n+\sigma^2})_n$, we can solve

$$\boldsymbol{C}_p^{dc} = diag(\frac{\sigma^2}{1 - \bar{\rho}_n + \sigma^2})_n - diag(\frac{(1 - \bar{\rho}_n)_n \sigma^2}{(1 - \bar{\rho}_n + \sigma^2) I_n}) \cdot \bar{\mathcal{C}}^{-1},$$

where $\bar{\mathcal{C}} = cov(\bar{\boldsymbol{s}}, \bar{\boldsymbol{s}}')/\sigma_\theta^2 - diag(\frac{\sigma^2}{I_n})_n = \begin{bmatrix} \frac{1+(I_1-1)\bar{\rho}_1}{I_1} & \cdots & \rho_{1N} \\ \vdots & \ddots & \vdots \\ \rho_{N1} & \cdots & \frac{1+(I_N-1)\bar{\rho}_N}{I_N} \end{bmatrix} \in \mathbb{R}^{N\times N}$.

To summarize, for trader $i \in I(n)$, the inference coefficients satisfy:

$$\begin{aligned} c_{s,n}^{dc} &= \tfrac{1-\bar{\rho}_n}{1-\bar{\rho}_n+\sigma^2}, \\ c_{p_m,n}^{dc} &= \tfrac{\sigma^2}{1-\bar{\rho}_n+\sigma^2}\mathbf{1}_{n=m} - \tfrac{(1-\bar{\rho}_n)\sigma^2(\bar{\mathcal{C}}^{-1})_{nm}}{(1-\bar{\rho}_n+\sigma^2)I_n}, \\ c_{\theta,n}^{dc} &= 1 - c_{s,n}^{dc} - \textstyle\sum_m c_{p_m,n}^{dc}. \end{aligned}$$

Price impact The correct price impact satisfies

$$\boldsymbol{\Lambda}^i = \begin{bmatrix} \frac{\partial p_1}{\partial q_1^i} & \cdots & \frac{\partial p_N}{\partial q_1^i} \\ \vdots & \ddots & \vdots \\ \frac{\partial p_1}{\partial q_N^i} & \cdots & \frac{\partial p_N}{\partial q_N^i} \end{bmatrix} = - \begin{bmatrix} \sum_{j\neq i} \frac{\partial q_1^j}{\partial p_1} & \cdots & \sum_{j\neq i} \frac{\partial q_N^j}{\partial p_1} \\ \vdots & \ddots & \vdots \\ \sum_{j\neq i} \frac{\partial q_1^j}{\partial p_N} & \cdots & \sum_{j\neq i} \frac{\partial q_N^j}{\partial p_N} \end{bmatrix}^{-1}.$$

Using first order condition from trader's optimization problem, we can calculate that for $j \in I(n)$, $\frac{\partial q_n^j(\boldsymbol{p})}{\partial p_m} = \begin{cases} (\alpha+\lambda_{nn})^{-1}(\boldsymbol{C}_p^{dc})_{nm} & \text{if } n \neq m \\ (\alpha+\lambda_{nn})^{-1}((\boldsymbol{C}_p^{dc})_{nm} - 1) & \text{if } n = m \end{cases}$, for $j \notin I(n)$, $\frac{\partial q_n^j(\boldsymbol{p})}{\partial p_m} = 0$.

We can write trader i's price impact matrix as

$$(\boldsymbol{\Lambda}^i)' = (Id - \boldsymbol{C}_p)^{-1} diag(\frac{\alpha+\lambda_{nn}}{I_n - \mathbf{1}_{i\in I(n)}})_n.$$

For trader $i \in I(n)$, the $(n,n)^{th}$ element of $\boldsymbol{\Lambda}^i$ satisfies

$$\lambda_{nn}^i = \frac{\alpha+\lambda_{nn}}{I_n - 1}(\boldsymbol{M}^{dc})_{nn},$$

where $\boldsymbol{M}^{dc} = (Id - \boldsymbol{C}_p)^{-1}$, $(\boldsymbol{M}^{dc})_{nn}$ is the $(n,n)^{th}$ element of $\boldsymbol{M}^{dc}$.

Note that $\frac{\alpha+\lambda_{nn}}{I_n-1}(\boldsymbol{M}^{dc})_{nn}$ is linear in the price impact. So if the price impact λ_{nn}^i exists, it is uniquely determined in equilibrium. Denote λ_{nn}^i as λ_n^{dc} for $i \in I(n)$,

$$\lambda_n^{dc} = \frac{1-\gamma_n}{\gamma_n - 1 + 1/(\boldsymbol{M}^{dc})_{nn}}\alpha,$$

where $\gamma_n = 1 - \frac{1}{I_n - 1}$.

Bids: The demand schedule for trader i satisfies

$$q_n^i = \frac{1}{\alpha + \lambda_n^{dc}}(c_{\theta,n}^{dc}\mathbb{E}[\theta] + c_{s,n}^{dc}s^i - p_n + \sum_m c_{p_m,n}^{dc}p_m), \quad \forall i \in I(n), n = 1, \ldots, N.$$

By the market clearing condition $\sum_{i\in I(n)} q_n^i = 0$,

$$q_n^i = \frac{1}{\alpha + \lambda_n^{dc}} c_{s,n}^{dc}(s^i - \bar{s}_n) \quad \forall i \in I(n), n = 1, \ldots, N.$$

Prices: The equilibrium price satisfies

$$\boldsymbol{p}^{dc} = \boldsymbol{c}_\theta^{dc}\mathbb{E}[\theta] + \boldsymbol{C}_s^{dc}\bar{\boldsymbol{s}} + \boldsymbol{C}_p^{dc}\boldsymbol{p}^{dc}.$$

As $\boldsymbol{c}_\theta^{dc} = (Id - \boldsymbol{C}_p^{dc} - \boldsymbol{C}_s^{dc}) \cdot \mathbf{1}_N$, we can rewrite the above equation into

$$\boldsymbol{p}^{dc} - \mathbb{E}[\boldsymbol{\theta}] = \boldsymbol{C}_s^{dc}(\bar{\boldsymbol{s}} - \mathbb{E}[\boldsymbol{\theta}]) + \boldsymbol{C}_p^{dc}(\boldsymbol{p}^{dc} - \mathbb{E}[\boldsymbol{\theta}]).$$

Define $\hat{\boldsymbol{p}}^{dc} = \boldsymbol{p}^{dc} - \mathbb{E}[\boldsymbol{\theta}]$,$\hat{\boldsymbol{s}} = \bar{\boldsymbol{s}} - \mathbb{E}[\boldsymbol{\theta}]$, we have

$$\hat{\boldsymbol{p}}^{dc} = \boldsymbol{C}_s^{dc}\hat{\boldsymbol{s}} + \boldsymbol{C}_p^{dc}\hat{\boldsymbol{p}}^{dc}.$$

As $\boldsymbol{C}_s^{dc} = diag(c_s^n)_n$, this equation has a good interpretation. The deviation of price from the fundamental value is a weighted sum of the innovation in signals received by traders in this exchange and deviations of price from the fundamental value in all exchanges.

$$\hat{\boldsymbol{p}}^{dc} = (Id - \boldsymbol{C}_p^{dc})^{-1}\boldsymbol{C}_s^{dc}\hat{\boldsymbol{s}}. \tag{1.43}$$

Plug $\boldsymbol{C}_s^{dc}$ and $\boldsymbol{C}_p^{dc}$ into (1.43) gives

$$\hat{\boldsymbol{p}}^{dc} = cov(\bar{\boldsymbol{\theta}}, \bar{\boldsymbol{s}}') \cdot cov(\bar{\boldsymbol{s}}, \bar{\boldsymbol{s}}')^{-1} \cdot \hat{\boldsymbol{s}}.$$

Existence Conditions: For equilibrium to exist, the parameters should satisfy the following conditions:

1. For the stochastic process that generates the joint distribution of value to exist $\rho_n > -(1-\gamma_n), \forall n$.

2. There are at least two traders in each decentralized exchange, i.e. $I_n \geq 2$, $\gamma \geq 1 - \frac{1}{2N-1}$.

3. The demand schedule for each trader is downward sloping. It requires $c_{p_n}^{dc,n} < 1$.

4. The second order condition requires that $\lambda_n^{dc} \geq -\frac{\alpha}{2}$.

■

Proof of Corollary 1. In Design 1 and Design 2, the inference coefficient on s^i is characterized by the joint distribution of $\left(\theta^i, s^i\right)$ conditional on the remaining conditioning variables:

$$
\begin{aligned}
c_{s,n}^{d} &= \frac{\operatorname{Cov}\left(\theta^i, s^i | p_n^d\right)}{\operatorname{Var}\left(s^i | p_n^d\right)} = \frac{\operatorname{Cov}\left(\theta^i, s^i | \bar{s}_n\right)}{\operatorname{Var}(s^i | \bar{s}_n)}, \\
c_{s,n}^{dc} &= \frac{\operatorname{Cov}\left(\theta^i, s^i | \mathbf{p}^{dc}\right)}{\operatorname{Var}\left(s^i | \mathbf{p}^{dc}\right)} = \frac{\operatorname{Cov}\left(\theta^i, s^i | \bar{\boldsymbol{s}}\right)}{\operatorname{Var}(s^i | \bar{\boldsymbol{s}})}.
\end{aligned}
$$

The second equality in each line holds by the symmetry of traders within each group $I(n)$, or equivalently, by the informational equivalence between p_n^d and $\bar{s}_n$, and the informational equivalence between $\boldsymbol{p}^{dc}$ and $\bar{\boldsymbol{s}}$.

$c_{s,n}^{d} = c_{s,n}^{dc}$ holds if $\operatorname{Var}\left(s^i \mid \bar{s}_n\right) = \operatorname{Var}\left(s^i \mid \bar{\boldsymbol{s}}\right)$, $\operatorname{Cov}\left(\theta^i, s^i \mid \bar{s}_n\right) = \operatorname{Cov}\left(\theta^i, s^i \mid \bar{\boldsymbol{s}}\right)$. By the Projection Theorem, we have

$$
\begin{aligned}
\operatorname{Var}\left(s^i \mid \bar{\boldsymbol{s}}\right) &= \operatorname{Var}\left(s^i \mid \bar{s}_n\right) - \frac{\operatorname{Cov}(s^i, \bar{\boldsymbol{s}}_{-n} | \bar{s}_n) \operatorname{Cov}(\bar{\boldsymbol{s}}_{-n}, s^i | \bar{s}_n)}{\operatorname{Var}(\bar{\boldsymbol{s}}_{-n} | \bar{s}_n)}, \\
\operatorname{Cov}\left(\theta^i, s^i \mid \bar{\boldsymbol{s}}\right) &= \operatorname{Cov}\left(\theta^i, s^i \mid \bar{s}_n\right) - \frac{\operatorname{Cov}(\theta^i, \bar{\boldsymbol{s}}_{-n} | \bar{s}_n) \operatorname{Cov}(\bar{\boldsymbol{s}}_{-n}, s_i | \bar{s}_n)}{\operatorname{Var}(\bar{\boldsymbol{s}}_{-n} | \bar{s}_n)},
\end{aligned}
$$

where $\bar{\boldsymbol{s}}_{-n} \in \mathbb{R}^{N-1}$ which contains all $\bar{s}_m$, $m \neq n$.

By equations above, we can see that $\operatorname{Var}\left(s^i \mid \bar{s}_n\right) = \operatorname{Var}\left(s^i \mid \bar{\boldsymbol{s}}\right)$, $\operatorname{Cov}\left(\theta^i, s^i \mid \bar{s}_n\right) = \operatorname{Cov}\left(\theta^i, s^i \mid \bar{\boldsymbol{s}}\right)$ if $\operatorname{Cov}\left(s^i, \bar{s}_m \mid \bar{s}_n\right) = 0$, i.e., for any $i \in I(n)$ and $m \neq n$, s^i is uncorrelated with $\bar{s}_m$ conditional on $\bar{s}_n$.

Formally, this can be proved as follows,

$$
\begin{aligned}
\operatorname{Cov}\left(s^i, \bar{s}_m \mid \bar{s}_n\right) \qquad & = \operatorname{Cov}\left(s^i, \bar{s}_m\right) - \frac{\operatorname{Cov}\left(s^i, \bar{s}_n\right)\operatorname{Cov}\left(\bar{s}_m, \bar{s}_n\right)}{\operatorname{Var}\left(\bar{s}_n\right)} \\
& = \bar{\rho}_{nm}\sigma_\theta^2 - \frac{\frac{1}{I_n}\left(1+\sigma^2+(I_n-1)\bar{\rho}_n\right)\cdot\bar{\rho}_{nm}}{\frac{1}{I_n}\left(1+\sigma^2+(I_n-1)\bar{\rho}_n\right)}\sigma_\theta^2 = 0.
\end{aligned}
$$

For all traders $j \in I(n)$, only the common value component in $\left\{s^j\right\}_{j\in I(n)}$, i.e., $\bar{s}_n$, is correlated with $\bar{s}_m$, but not the private value component in each s^j. Note that the symmetry across traders within each exchange is key to $c_{s,n}^d = c_{s,n}^{dc}$. Intuitively, as the additional information provided by s^i beyond $\bar{s}_n$ cannot be learned from $\bar{s}_m, m \neq n$, traders in Design 1 and Design 2 apply equal weight on the private signal. ■

Proof of Proposition 3. **Inference coefficients** Traders in $I(K+1), \ldots, I(N)$ cannot submit their demand schedule conditioning on prices from the other exchanges. Therefore, as in Design 1, prices in exchange $I(K+1), \ldots, I(N)$ satisfy

$$
p_n^{pc} = c_{\theta,n}^{pc}\mathbb{E}[\theta] + c_{s,n}^{pc}\bar{s}_n + c_{p_n}^{pc,n}p_n^{pc}, \tag{1.44}
$$

where $\bar{s}_n = \frac{1}{I_n}\sum_{i\in I(n)} s^i$.

For $i \in I(K+1), \ldots, I(N)$, their expectation $\mathbb{E}[\theta^i|\boldsymbol{p}, s^i] = c_{\theta,n}^{pc}\mathbb{E}[\theta] + c_{s,n}^{pc}s^i + \sum_m c_{p_m,n}^{pc}p_m$. Combining this with market clearing condition $\sum_{i\in I(n)} q^i\left(\boldsymbol{p}\right) = 0$, we have

$$
p_m^{pc} = c_{\theta,n}^{pc}\mathbb{E}[\theta] + c_{s,n}^{pc}\bar{s}_n + \sum_m c_{p_m,n}^{pc}p_m^{pc}, \tag{1.45}
$$

where $\bar{s}_n = \frac{1}{I_n}\sum_{i\in I(n)} s^i$.

The inference coefficients on price under partial conditioning can be written in matrix form

$$
\boldsymbol{p} = \boldsymbol{c}_\theta^{pc}\mathbb{E}[\theta] + \boldsymbol{C}_s^{pc}\bar{\boldsymbol{s}} + \boldsymbol{C}_p^{pc}\boldsymbol{p}, \tag{1.46}
$$

where $\boldsymbol{c}_\theta^{pc} = (c_{\theta,n}^{pc})_n \in \mathbb{R}^{K\times K}, \boldsymbol{C}_s^{pc} = diag(c_{s,n}^{pc})_n \in \mathbb{R}^K$, $\bar{\boldsymbol{s}} = (\bar{s}_n)_n \in \mathbb{R}^K$, and $\boldsymbol{C}_p^{pc} =$

$\begin{pmatrix} \tilde{\boldsymbol{C}}_p^{pc} \\ \boldsymbol{C}_{p^d}^{pc} \end{pmatrix}$, where $\tilde{\boldsymbol{C}}_p^{pc} \in \mathbb{R}^{K\times N}$ is the first K rows of $\boldsymbol{C}_p^{pc}$, $\boldsymbol{C}_{p^d}^{pc} = [\boldsymbol{0} \quad diag(c_{p_n,n}^{pc})_n] \in \mathbb{R}^{(N-K)\times N}$ is the last $N-K$ rows of $\boldsymbol{C}_p^{pc}$.

For trader $i \in I(1)\ldots,I(K)$, random vector $\left(\theta^i, s^i, (\boldsymbol{p}^{pc})'\right)$ is jointly normally distributed

$$\begin{pmatrix} \theta^i \\ s^i \\ \boldsymbol{p}^{pc} \end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix} \mathbb{E}[\theta] \\ \mathbb{E}[\theta] \\ \mathbb{E}[\boldsymbol{\theta}] \end{pmatrix}, \begin{pmatrix} \sigma_\theta^2 & \sigma_\theta^2 & cov\left(\theta^i, (\boldsymbol{p}^{pc})'\right) \\ \sigma_\theta^2 & \sigma_\theta^2+\sigma_\varepsilon^2 & cov\left(s^i, (\boldsymbol{p}^{pc})'\right) \\ cov\left(\boldsymbol{p}^{pc}, \theta^i\right) & cov\left(\boldsymbol{p}^{pc}, s^i\right) & cov\left(\boldsymbol{p}^{pc}, (\boldsymbol{p}^{pc})'\right) \end{pmatrix}\right].$$

For trader $i \in I(K+1)\ldots,I(N)$, random vector $\left(\theta^i, s^i, p_n^{dc}\right)$ is jointly normally distributed

$$\begin{pmatrix} \theta^i \\ s^i \\ p_n^{pc} \end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix} \mathbb{E}[\theta] \\ \mathbb{E}[\theta] \\ \mathbb{E}[\theta] \end{pmatrix}, \begin{pmatrix} \sigma_\theta^2 & \sigma_\theta^2 & cov\left(\theta^i, p_n^{pc}\right) \\ \sigma_\theta^2 & \sigma_\theta^2+\sigma_\varepsilon^2 & cov\left(s^i, p_n^{pc}\right) \\ cov\left(p_n^{pc}, \theta^i\right) & cov\left(p_n^{pc}, s^i\right) & Var\left(p_n^{pc}\right) \end{pmatrix}\right].$$

By the Projection Theorem, we have

$$(c_s^i \quad \boldsymbol{c}_p^i) = (\sigma_\theta^2 \quad cov(\theta^i, (\boldsymbol{p}^{pc})')) \begin{pmatrix} \sigma_\theta^2+\sigma_\varepsilon^2 & cov(s^i, (\boldsymbol{p}^{\boldsymbol{pc}})') \\ cov(\boldsymbol{p}^{pc}, s^i) & cov(\boldsymbol{p}^{pc}, (\boldsymbol{p}^{pc})') \end{pmatrix}^{-1} \quad \forall i \in I(n), n = 1, \ldots, K.$$

and

$$(c_s^i \quad c_p^i) = (\sigma_\theta^2 \quad cov(\theta^i, p_n^{pc})) \begin{pmatrix} \sigma_\theta^2+\sigma_\varepsilon^2 & cov(s^i, p_n^{pc}) \\ cov(p_n^{pc}, s^i) & Var(p_n^{pc}) \end{pmatrix}^{-1} \quad \forall i \in I(n), n = K+1, \ldots, N.$$

This gives

$$c_s^i cov(s^i, \bar{\boldsymbol{s}}') + \boldsymbol{c}_p^i (Id - \boldsymbol{C}_p^{pc})^{-1} \boldsymbol{C}_s^{pc} cov(\bar{\boldsymbol{s}}, \bar{\boldsymbol{s}}') = cov(\theta^i, \bar{\boldsymbol{s}}'), \quad \forall i \in I(n), n = 1, \ldots, K. \quad (1.47)$$

$$c_s^i cov(s^i, \bar{s}_n) + \frac{c_p^i}{1-c_p^i} c_s^i cov(\bar{s}_n, \bar{s}_n) = cov(\theta^i, \bar{s}_n), \quad \forall i \in I(n), n = K+1, \ldots, N. \tag{1.48}$$

$$c_s^i (1+\sigma^2)\sigma_\theta^2 + \boldsymbol{c}_p^i (Id - \boldsymbol{C}_p)^{-1} \boldsymbol{C}_s cov(\bar{\boldsymbol{s}}, s^i) = \sigma_\theta^2 \quad \forall i \in I(n), n = 1, \ldots, K. \tag{1.49}$$

and

$$c_s^i (1+\sigma^2)\sigma_\theta^2 + \frac{c_p^i}{1-c_p^i} c_s^i cov(\bar{s}_n, s^i) = \sigma_\theta^2 \quad \forall i \in I(n), n = K+1, \ldots, N. \tag{1.50}$$

Combining equations (1.47), (1.48), (1.49), and (1.50), we can solve

$$c_s^i = \frac{cov(\theta^i, \bar{s}_n) - \sigma_\theta^2}{cov(\theta^i, \bar{s}_n) - (1+\sigma^2)\sigma_\theta^2} = \frac{1-\bar{\rho}_n}{1-\bar{\rho}_n + \sigma^2} \quad \forall i \in I(n), n = 1, \ldots, N.$$

Therefore,

$$\boldsymbol{C}_s^{pc} = diag(\frac{1-\bar{\rho}_n}{1-\bar{\rho}_n+\sigma^2})_n.$$

Plug in c_s^i into equation (1.48), we can solve

$$c_p^i = \frac{I_n}{1+(I_n-1)\bar{\rho}_n} \frac{\sigma^2}{1-\bar{\rho}_n+\sigma^2} = \frac{(2-\gamma_n)\bar{\rho}_n}{1-\gamma_n+\bar{\rho}_n} \frac{\sigma^2}{1-\bar{\rho}_n+\sigma^2} \quad \forall i \in I(n), n = K+1, \ldots, N.$$

Observe that for $i \in I(n), n = K+1, \ldots, N$, the inference coefficient on price is the same as $c_p^{d,n}$. We can write equation (1.47) into matrix form

$$\tilde{\boldsymbol{C}}_s^{pc} cov(\bar{\boldsymbol{s}}, \bar{\boldsymbol{s}}') + \tilde{\boldsymbol{C}}_p^{pc} \cdot (Id - \boldsymbol{C}_p^{pc})^{-1} \boldsymbol{C}_s^{pc} cov(\bar{\boldsymbol{s}}, \bar{\boldsymbol{s}}') = cov(\tilde{\boldsymbol{\theta}}, \bar{\boldsymbol{s}}').$$

where $\tilde{\boldsymbol{C}}_s^{pc}$ is the first K rows of $\boldsymbol{C}_s^{pc}$, $\tilde{\boldsymbol{\theta}} = (\bar{\theta}_1, \ldots, \bar{\theta}_k)'$.

$$\tilde{\boldsymbol{C}}_p^{pc} \cdot (Id - \boldsymbol{C}_p^{pc})^{-1} = \Big(cov(\tilde{\boldsymbol{\theta}}, \bar{\boldsymbol{s}}') - \tilde{\boldsymbol{C}}_s^{pc} cov(\bar{\boldsymbol{s}}, \bar{\boldsymbol{s}}') \Big) cov(\bar{\boldsymbol{s}}, \bar{\boldsymbol{s}}')^{-1} (\boldsymbol{C}_s^{pc})^{-1}.$$

Note that the RHS of the above equation is the same as the first K rows of equation (1.42) in the case with full contingency, the LHS equals to the first K rows of $\boldsymbol{C}_p^{dc}(Id -$

$\boldsymbol{C}_p^{dc})^{-1}$. By using the fact that $Id + \boldsymbol{C}(Id - \boldsymbol{C})^{-1} = (Id - \boldsymbol{C})^{-1}$, we have

$$[(Id - \boldsymbol{C}_p^{pc})^{-1}]_{K\times N} = [(Id - \boldsymbol{C}_p^{dc})^{-1}]_{K\times N}. \tag{1.51}$$

Plug in $\boldsymbol{C}_p^{pc} = \begin{pmatrix} \tilde{\boldsymbol{C}}_p^{pc} \\ \boldsymbol{C}_{p^d}^{pc} \end{pmatrix}$, we can solve for $n = 1, \ldots, K$,

$$c_{p_m,n}^{pc} = \begin{cases} \frac{\sigma^2}{1-\bar{\rho}_n+\sigma^2}\mathbf{1}_{n=m} - \frac{(1-\bar{\rho}_n)\sigma^2(\bar{\mathcal{G}}^{-1})_{nm}}{(1-\bar{\rho}_n+\sigma^2)I_n} & \text{if } m = 1, \ldots, K. \\ -\frac{(1-\bar{\rho}_n)\sigma^2(\bar{\mathcal{G}}^{-1})_{nm}}{W_m(1-\bar{\rho}_n+\sigma^2)I_n} & \text{if } m = K+1, \ldots, N. \end{cases}$$

where $\bar{\mathcal{G}} = \frac{cov(\bar{\boldsymbol{s}},\bar{\boldsymbol{s}}')}{\sigma_\theta^2} - diag(\frac{\sigma^2}{I_n}\mathbf{1}_{n=1,\ldots,K})_n \in \mathbb{R}^{N\times N}$ and $W_m = \frac{c_{s,m}^d}{1-c_{p,m}^d} = \frac{1+(I_m-1)\bar{\rho}_m}{1+\sigma^2+(I_m-1)\bar{\rho}_m}$.

To summarize, for trader $i \in I(n)$, $n = 1, \ldots, K$,

$$\begin{aligned} c_{s,n}^{pc} &= \frac{1-\bar{\rho}_n}{1-\bar{\rho}_n+\sigma^2}, \\ c_{p_m,n}^{pc} &= \begin{cases} \frac{\sigma^2}{1-\bar{\rho}_n+\sigma^2}\mathbf{1}_{n=m} - \frac{(1-\bar{\rho}_n)\sigma^2(\bar{\mathcal{G}}^{-1})_{nm}}{(1-\bar{\rho}_n+\sigma^2)I_n} & \text{if } m = 1, \ldots, K. \\ -\frac{(1-\bar{\rho}_n)\sigma^2(\bar{\mathcal{G}}^{-1})_{nm}}{W_m(1-\bar{\rho}_n+\sigma^2)I_n} & \text{if } m = K+1, \ldots, N. \end{cases} \\ c_{\theta,n}^{pc} &= 1 - c_{s,n}^{pc} - \sum_m c_{p_m,n}^{pc}. \end{aligned}$$

For $n = K+1, \ldots, N$,

$$\begin{aligned} c_{s,n}^{pc} &= c_s^{d,n}, \\ c_{p_m,n}^{pc} &= c_p^{d,n}\mathbf{1}_{m=n}, \\ c_{\theta,n}^{pc} &= c_\theta^{d,n}. \end{aligned}$$

Price impacts: For traders in $I(K+1), \ldots, I(N)$, the price impact satisfies $\lambda^i = -(\sum_{j\neq i}\frac{\partial q_n^j}{\partial p_n})^{-1}$. From the first-order condition of the trader $i \in I(n)$ optimization problem, $\frac{\partial q_n^j}{\partial p_n} = \frac{1-c_{p_n,n}^{pc}}{\alpha+\lambda_n^{pc}}$,

$$\lambda_n^{pc} = \frac{1-\gamma_n}{\gamma_n - c_{p_n,n}^{pc}}\alpha.$$

As $c_{p_n}^{pc,n} = c_p^{d,n}$ for $n = K+1, \ldots, N$,

$$\lambda_n^{pc} = \lambda_n^d \quad \forall n = K+1, \ldots, N.$$

For traders in $I(1),\dots,I(K)$, the correct price impact satisfies $\mathbf{\Lambda}^i = ((-\sum_{j\neq i}\frac{\partial q^j(\boldsymbol{p})}{\partial \boldsymbol{p}})^{-1})'$, i.e.

$$\mathbf{\Lambda}^i = \begin{bmatrix} \frac{\partial p_1}{\partial q_1^i} & \cdots & \frac{\partial p_K}{\partial q_1^i} \\ \vdots & \ddots & \vdots \\ \frac{\partial p_1}{\partial q_K^i} & \cdots & \frac{\partial p_K}{\partial q_K^i} \end{bmatrix} = -\begin{bmatrix} \sum_{j\neq i}\frac{\partial q_1^j}{\partial p_1} & \cdots & \sum_{j\neq i}\frac{\partial q_K^j}{\partial p_1} \\ \vdots & \ddots & \vdots \\ \sum_{j\neq i}\frac{\partial q_1^j}{\partial p_K} & \cdots & \sum_{j\neq i}\frac{\partial q_K^j}{\partial p_K} \end{bmatrix}^{-1},$$

Plug in that $\frac{\partial q_n^j(\boldsymbol{p})}{\partial p_m} = \begin{cases} (\alpha+\lambda_{nn})^{-1}(\boldsymbol{C}_p^{pc})_{nm} & \text{if } n\neq m \\ (\alpha+\lambda_{nn})^{-1}((\boldsymbol{C}_p^{pc})_{nm}-1) & \text{if } n=m \end{cases}$ for $j\in I(n)$, $\frac{\partial q_n^j(\boldsymbol{p})}{\partial p_m}=0$ for $j\notin I(n)$,

$$(\mathbf{\Lambda}^i)' = (Id - [\boldsymbol{C}_p^{pc}]_{K\times K})^{-1} diag(\frac{\alpha+\lambda_{nn}}{I_n - \mathbf{1}_{i\in I(n)}})_n.$$

We can solve the price impact for $n=1,\dots,K$,

$$\lambda_n^{pc} = \frac{1-\gamma_n}{\gamma_n - 1 + 1/(\boldsymbol{M}^{pc})_{nn}}\alpha, \tag{1.52}$$

where $\gamma_n = 1-\frac{1}{I_n-1}$ and $\boldsymbol{M}^{pc} = (Id-\boldsymbol{C}_p^{pc})^{-1}$, $(\boldsymbol{M}^{pc})_{nn}$ is the $(n,n)^{th}$ element of $\boldsymbol{M}^{pc}$.

Observe that $(\boldsymbol{M}^{pc})_{nn} = (\boldsymbol{M}^{dc})_{nn}, \forall n=1,\dots,K$ by equation (1.51),

$$\lambda_n^{pc} = \lambda_n^{dc} \quad \forall n=1,\dots,K.$$

Bids: From the first-order condition of the traders

$$q_n^i = \frac{1}{\alpha+\lambda_n^{pc}}(c_{\theta,n}^{pc}\mathbb{E}[\theta] + c_{s,n}^{pc}s^i + \sum_m c_{p_m,n}^{pc}p_m),$$

Using the market clearing condition $\sum q_n^i = 0$,

$$q_n^i = \frac{1}{\alpha+\lambda_n^{pc}}c_{s,n}^{pc}(s^i - \bar{s}_n).$$

With $\lambda_n^{pc} = \lambda_n^{dc}\cdot\mathbf{1}_{n=1,\dots,K} + \lambda_n^d\cdot\mathbf{1}_{n=K+1,\dots,N}$ and $c_{s,n}^{pc} = c_s^{dc,n} = c_s^{d,n}$ We can solve the equilibrium bid for trader $i \in I(n)$,

$$q^{i,pc} = q^{i,dc} \cdot \mathbf{1}_{n=1,\dots,K} + q^{i,d} \cdot \mathbf{1}_{n=K+1,\dots,N}.$$

Prices: Substitute $\boldsymbol{c}_\theta^{pc} = (Id - \boldsymbol{C}_p^{pc} - \boldsymbol{C}_s^{pc}) \cdot \mathbf{1}_N$ into equation (1.46) gives

$$\hat{\boldsymbol{p}}^{pc} = (Id - \boldsymbol{C}_p^{pc})^{-1} \boldsymbol{C}_s^{pc} \hat{\boldsymbol{s}},$$

where $\hat{\boldsymbol{p}}^{pc} = \boldsymbol{p}^{pc} - \mathbb{E}[\theta]$, $\hat{\boldsymbol{s}} = \bar{\boldsymbol{s}} - \mathbb{E}[\theta]$.

Plug $\boldsymbol{C}_s^{pc}$ and $\boldsymbol{C}_p^{pc}$,

$$\hat{p}_n^{pc} = \begin{cases} \hat{p}_n^{dc} = [cov(\bar{\boldsymbol{\theta}}, \bar{\boldsymbol{s}}') \cdot cov(\bar{\boldsymbol{s}}, \bar{\boldsymbol{s}}')^{-1} \cdot \hat{\boldsymbol{s}}]_n & \text{if } n = 1, \dots, K. \\ \hat{p}_n^{d} = \frac{cov(\bar{\theta}_n, \bar{s}_n)}{Var(\bar{s}_n)} \hat{s}_n & \text{if } n = K+1, \dots, N. \end{cases}$$

Existence Conditions: For the equilibrium to exist, the parameters should satisfy the following conditions:

1. For the stochastic process that generates the joint distribution of value to exist $\rho_n \geq -(1-\gamma_n)$, $\forall n$.
2. There are at least two traders in each decentralized exchange, i.e., $I_n \geq 2$.
3. The demand schedule for each trader is downward sloping, for $n = 1, \dots, K$, $c_{p_n}^{pc} < 1$. This also implies $\lambda_n^{pc} > 0$ for $n = K+1, \dots, N$.
4. The second order condition of the trader's optimization problem is satisfied, $\lambda_n^{pc} \geq -\frac{\alpha}{2}$.

■

Proof of Proposition 4. Using market clearing condition, the equilibrium price $\sum_{i \in I} (\alpha^i + \lambda^{i,c})^{-1} (\mathbb{E}[\theta^i | s^i, p] - p) = 0$. Combine it with $\mathbb{E}[\theta^i | s^i, p^c] = c_\theta^{i,c} \mathbb{E}[\theta] + c_s^{i,c} s^i + c_p^{i,c} p^c$ if $i \in I(n)$, the equilibrium price can be written as

$$p^c = \left(\sum_i \frac{1-c_p^{i,c}}{\alpha+\lambda^{i,c}}\right)^{-1} \left(\sum_i \frac{c_\theta^{i,c}\mathbb{E}[\theta] + c_s^{i,c} s^i}{\alpha+\lambda^{i,c}}\right).$$

Random vector $\left(\theta^i, s^i, p^c\right)$ is jointly normally distributed

$$\begin{pmatrix} \theta^i \\ s^i \\ p^c \end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix} \mathbb{E}[\theta] \\ \mathbb{E}[\theta] \\ \mathbb{E}[\theta] \end{pmatrix}, \begin{pmatrix} \sigma_\theta^2 & \sigma_\theta^2 & cov\left(\theta^i, p^c\right) \\ \sigma_\theta^2 & \sigma_\theta^2 + \sigma_\varepsilon^2 & cov\left(s^i, p^c\right) \\ cov\left(p^c, \theta^i\right) & cov\left(p^c, s^i\right) & Var\left(p^c\right) \end{pmatrix}\right].$$

where

$$\begin{aligned} cov(p^c, \theta^i) &= \left(\sum_i \frac{1-c_p^{i,c}}{\alpha+\lambda^{i,c}}\right)^{-1} \left(\frac{c_s^{i,c}}{\alpha+\lambda^{i,c}} + \sum_{j\neq i} \frac{c_s^{j,c}}{\alpha+\lambda^{j,c}}\rho_{ij}\right)\sigma_\theta^2, \\ cov(p^c, s^i) &= \left(\sum_i \frac{1-c_p^{i,c}}{\alpha+\lambda^{i,c}}\right)^{-1} \left(\frac{c_s^{i,c}}{\alpha+\lambda^{i,c}}(1+\sigma^2) + \sum_{j\neq i} \frac{c_s^{j,c}}{\alpha+\lambda^{j,c}}\rho_{ij}\right)\sigma_\theta^2, \\ Var(p^c) &= \left(\sum_i \frac{1-c_p^{i,c}}{\alpha+\lambda^{i,c}}\right)^{-2} \left(\sum_i (\frac{c_s^{i,c}}{\alpha+\lambda^{i,c}})^2(1+\sigma^2) + \sum_i \sum_{j\neq i} \frac{c_s^{i,c}}{\alpha+\lambda^{i,c}} \frac{c_s^{j,c}}{\alpha+\lambda^{j,c}}\rho_{ij}\right)\sigma_\theta^2. \end{aligned}$$

By the Projection Theorem and method of undermined coefficients, we can determine the inference coefficients as a function of the primitives,

$$\begin{pmatrix} c_s^{i,c} & c_p^{i,c} \end{pmatrix} = \begin{pmatrix} \sigma_\theta^2 & cov(\theta^i, p^c) \end{pmatrix} \begin{pmatrix} \sigma_\theta^2 + \sigma_\varepsilon^2 & cov\left(s^i, p^c\right) \\ cov\left(p^c, s^i\right) & Var\left(p^c\right) \end{pmatrix}^{-1},$$

from which we can solve

$$\begin{aligned} c_s^{i,c} &= \frac{\left(\sum_i (\delta^{i,c})^2(1+\sigma^2) + \sum_i \sum_{j\neq i} \delta^{i,c}\delta^{j,c}\rho_{ij}\right) - \left(\delta^{i,c}(1+\sigma^2) + \sum_{j\neq i}\delta^{j,c}\rho_{ij}\right)\left(\delta^{i,c} + \sum_{j\neq i}\delta^{j,c}\rho_{ij}\right)}{(1+\sigma^2)\left(\sum_i (\delta^{i,c})^2(1+\sigma^2) + \sum_i \sum_{j\neq i} \delta^{i,c}\delta^{j,c}\rho_{ij}\right) - \left(\delta^{i,c}(1+\sigma^2) + \sum_{j\neq i}\delta^{j,c}\rho_{ij}\right)^2}, \\ c_p^{i,c} &= \left(\sum_i \frac{1-c_p^{i,c}}{\alpha+\lambda^{i,c}}\right) \frac{\sigma^2 \sum_{j\neq i}\delta^{j,c}\rho_{ij}}{(1+\sigma^2)\left(\sum_i (\delta^{i,c})^2(1+\sigma^2) + \sum_i \sum_{j\neq i} \delta^{i,c}\delta^{j,c}\rho_{ij}\right) - \left(\delta^{i,c}(1+\sigma^2) + \sum_{j\neq i}\delta^{j,c}\rho_{ij}\right)^2}, \end{aligned} \tag{1.53}$$

where $\delta^{i,c} \equiv \frac{c_s^{i,c}}{\alpha+\lambda^{i,c}}$.

The equilibrium price impact satisfies

$$\lambda^{i,c} = \left(\sum_{j \neq i} (\lambda^{j,c} + \alpha)^{-1} (1 - c_p^{j,c}) \right)^{-1},$$

The fixed point for the price impact is $\lambda^{i,c} = \alpha \beta^i$. where $\beta^i = \frac{2 - c_p^{i,c} - \psi + \sqrt{(2 - c_p^{i,c} - \psi)^2 + 4\psi}}{2\psi}$, $\psi = \sum_i \frac{1 - c_p^{i,c}}{1 + \beta_i}$.

Equilibrium Bid: The equilibrium inference coefficients $c_p^{i,c}, c_s^{i,c}$ and price impact $\lambda^{i,c}$ satisfies equation (1.53) and (1.21).

The equilibrium bid is

$$q^{i,c}(p) = \frac{1}{\alpha + \lambda^{i,c}} (c_\theta^{i,c} \mathbb{E}[\theta] + c_s^{i,c} s^i + (c_p^{i,c} - 1) p^c).$$

Combine it with market clearing condition $\sum_i q^{i,c} = 0$, we can rewrite the equilibrium bid as

$$q^{i,c}(p) = \frac{c_s^{i,c}}{\alpha + \lambda^{i,c}} (s^i - \bar{s}),$$

where $\bar{s} = \frac{1}{I} \sum_i s^i$.

Equilibrium Price: By projection theorem

$$c_\theta^i = 1 - c_p^i - c_s^i.$$

Therefore, the equilibrium price satisfies

$$\hat{p}^c = \left(\sum_i \frac{1 - c_p^{i,c}}{\alpha + \lambda^{i,c}} \right)^{-1} \left(\sum_i \frac{c_s^{i,c} \hat{s}^i}{\alpha + \lambda^{i,c}} \right), \tag{1.54}$$

where $\hat{p}^c = p^c - \mathbb{E}[\theta]$, $\hat{s}^i = s^i - \mathbb{E}[\theta]$.

Existence Conditions: For equilibrium to exist, the parameters should satisfy the following conditions:

1. For the stochastic process that generates the joint distribution of value to exist,

$Var(\frac{1}{I}\sum_i \theta_i) \geq 0$. This implies $\sum_i \sum_{j\neq i} \rho_{ij} \geq -I, \forall n$.

2. The demand schedule for each trader is downward sloping. It requires $\lambda^{i,c} > 0$. This also satisfies the second-order condition for each trader's optimization problem.

■

The following proofs of theorems and corollaries are under the condition that equilibria exist.

Proof of Lemma 1. We can calculate the ex-ante utility for the traders in Design 1, 2 and 2' as follows.

Design 1 Decentralized exchange In Design 1, trader $i \in I(n)$ has ex-ante expected utility

$$\mathbb{E}\left[u^{i,d}\right] = \frac{1}{2\alpha} \underbrace{\frac{\alpha\left(\alpha + 2\lambda_n^d\right)}{\left(\alpha + \lambda_n^d\right)^2}}_{\text{liquidity effect}} \underbrace{\mathbb{E}\left[\left(\mathbb{E}\left[\theta^i | p_n^d, s^i\right] - p_n^d\right)^2\right]}_{\text{learning effect}} = \frac{\left(\alpha+2\lambda_n^d\right)}{2\left(\alpha+\lambda_n^d\right)^2} \frac{(1-\bar{\rho}_n)^2}{1-\bar{\rho}_n+\sigma^2}(1-\frac{1}{I_n})\sigma_\theta^2, \quad \forall i.$$

Design 2 Decentralized exchange with Full Conditioning In Design 2, trader $i \in I(n)$ has ex-ante expected utility

$$\mathbb{E}\left[u^{i,dc}\right] = \frac{1}{2\alpha} \underbrace{\frac{\alpha\left(\alpha + 2\lambda_n^{dc}\right)}{\left(\alpha + \lambda_n^{dc}\right)^2}}_{\text{liquidity effect}} \underbrace{\mathbb{E}\left[\left(\mathbb{E}\left[\theta^i | \boldsymbol{p}^{dc}, s^i\right] - p_n^{dc}\right)^2\right]}_{\text{learning effect}} = \frac{\left(\alpha+2\lambda_n^{dc}\right)}{2\left(\alpha+\lambda_n^{dc}\right)^2} \frac{(1-\bar{\rho}_n)^2}{1-\bar{\rho}_n+\sigma^2}(1-\frac{1}{I_n})\sigma_\theta^2, \quad \forall i.$$

Design 2' Decentralized exchange with Partial Conditioning In Design 2', trader $i \in I(n)$ has ex-ante expected utility

$$\mathbb{E}\left[u^{i,pc}\right] = \mathbb{E}\left[u^{i,dc}\right] \cdot \mathbf{1}_{n=1,\ldots,K} + \mathbb{E}\left[u^{i,d}\right] \cdot \mathbf{1}_{n=K+1,\ldots,N}.$$

By comparing the ex-ante utility, we can see that the welfare ranking of the designs is inversely related to the ranking of price impacts. ■

Proof of Theorem 2. With price impacts equal to zero, the ex-ante expected utility for Design 1, Design 2 and Design 2′ equalize:

$$\mathbb{E}[u^{i,d}] = \mathbb{E}[u^{i,dc}] = \mathbb{E}[u^{i,pc}] = \frac{1}{2\alpha}\frac{(1-\bar{\rho}_n)^2}{1-\bar{\rho}_n+\sigma^2}\left(1-\frac{1}{I_n}\right)\sigma_\theta^2.$$

With equicommonality assumption, the ex-ante expected utility for Design 3 is

$$\mathbb{E}[u^{i,c}] = \frac{1}{2\alpha}\frac{(1-\bar{\rho})^2}{1-\bar{\rho}+\sigma^2}\left(1-\frac{1}{I}\right)\sigma_\theta^2.$$

$\mathbb{E}[u^{i,c}] \geq \mathbb{E}[u^{i,d}] = \mathbb{E}[u^{i,dc}] = \mathbb{E}[u^{i,pc}]$ for $i \in I(n)$ if and only if

$$\frac{(1-\bar{\rho})^2}{1-\bar{\rho}+\sigma^2} \geq \frac{(1-\bar{\rho}_n)^2}{1-\bar{\rho}_n+\sigma^2}\frac{\left(1-\frac{1}{I_n}\right)}{\left(1-\frac{1}{I}\right)}. \tag{1.55}$$

Denote $g(\bar{\rho}) \equiv \frac{(1-\bar{\rho})^2}{1-\bar{\rho}+\sigma^2}$. $g(\bar{\rho})$ is continuous. $\frac{\partial g(\bar{\rho})}{\partial \bar{\rho}} \leq 0$ for $\bar{\rho} \in [-1,1]$, the LHS of equation (1.55) is monotonically decreasing in $\bar{\rho}$. As $g(\bar{\rho}_n)$ is greater than the RHS of equation (1.55), $g(1)$ is no greater than the RHS of equation (1.55), we can find threshold $\bar{\rho}^+(\bar{\rho}_n, I_n, I)$, such that $\bar{\rho} \leq \bar{\rho}^+(\bar{\rho}_n, I_n, I)$ is equivalent to equation (1.55).

Similarly, we can show that $\mathbb{E}[u^{i,c}] < \mathbb{E}[u^{i,d}] = \mathbb{E}[u^{i,dc}] = \mathbb{E}[u^{i,pc}]$ for $i \in I(n)$ if and only if $\bar{\rho} \in (\bar{\rho}^+(\bar{\rho}_n, I_n, I), 1]$. ■

Proof of Lemma 2. For trader $i \in I(n)$, his price impact $\lambda_n^d = \frac{1-\gamma_n}{\gamma_n - c_{p,n}^d}\alpha$, $\lambda_n^{dc} = \frac{1-\gamma_n}{\gamma_n - 1 + 1/(\boldsymbol{M}^{dc})_{nn}}\alpha$ where $\gamma_n = 1 - \frac{1}{I_n - 1}$, $\boldsymbol{M}^{dc} = (Id - \boldsymbol{C}_p^{dc})^{-1}$. We can see that $\lambda_n^{dc} \leq \lambda_n^d$ if and only if

$$(\boldsymbol{M}^{dc})_{nn} \leq 1/(1-c_{p,n}^d).$$

By equation (1.37), $c_{p,n}^d = \frac{(2-\gamma_n)\bar{\rho}_n}{1-\gamma_n+\bar{\rho}_n}\frac{\sigma^2}{1-\bar{\rho}_n+\sigma^2}$. By equation (1.42), $(Id - \boldsymbol{C}_p^{dc})^{-1} = cov(\bar{\boldsymbol{\theta}}, \bar{\boldsymbol{s}}')cov(\bar{\boldsymbol{s}}, \bar{\boldsymbol{s}}')^{-1}(\boldsymbol{C}_s^{dc})^{-1}$ where $\boldsymbol{C}_s^{dc} = diag(\frac{1-\bar{\rho}_n}{1-\bar{\rho}_n+\sigma^2})$. Plug in $(Id - \boldsymbol{C}_p^{dc})^{-1}$ and $c_{p,n}^d$ into above equation, we can get

$$\left(cov(\bar{\boldsymbol{\theta}}, \bar{\boldsymbol{s}}')cov(\bar{\boldsymbol{s}}, \bar{\boldsymbol{s}}')^{-1}\right)_{nn} \leq cov(\bar{\theta}_n, \bar{s}_n)cov(\bar{s}_n, \bar{s}_n)^{-1}. \tag{1.56}$$

We claim that equation (1.56) holds for any $n = 1, 2, \ldots, N$.

To begin, we have

$$\begin{aligned}\left(cov(\bar{\boldsymbol{\theta}}, \bar{\boldsymbol{s}}')cov(\bar{\boldsymbol{s}}, \bar{\boldsymbol{s}}')^{-1}\right)_{nn} &= cov(\bar{\theta}_n, \bar{s}_n|\bar{\boldsymbol{s}}_{-n})Var(\bar{s}_n|\bar{\boldsymbol{s}}_{-n})^{-1} \\ &= \frac{cov(\bar{\theta}_n, \bar{s}_n) - cov(\bar{\theta}_n, \bar{\boldsymbol{s}}'_{-n})cov(\bar{\boldsymbol{s}}_{-n}, \bar{\boldsymbol{s}}'_{-n})^{-1}cov(\bar{\boldsymbol{s}}_{-n}, \bar{s}_n)}{cov(\bar{s}_n, \bar{s}_n) - cov(\bar{s}_n, \bar{\boldsymbol{s}}'_{-n})cov(\bar{\boldsymbol{s}}_{-n}, \bar{\boldsymbol{s}}'_{-n})^{-1}cov(\bar{\boldsymbol{s}}_{-n}, \bar{s}_n)}.\end{aligned}$$

As $cov(\bar{s}_n, \bar{\boldsymbol{s}}_{-n}) = cov(\bar{\theta}_n, \bar{\boldsymbol{s}}_{-n})$, we have

$$\left(cov(\bar{\boldsymbol{\theta}}, \bar{\boldsymbol{s}}')cov(\bar{\boldsymbol{s}}, \bar{\boldsymbol{s}}')^{-1}\right)_{nn} = \frac{cov(\bar{\theta}_n, \bar{s}_n) - cov(\bar{s}_n, \bar{\boldsymbol{s}}'_{-n})cov(\bar{\boldsymbol{s}}_{-n}, \bar{\boldsymbol{s}}'_{-n})^{-1}cov(\bar{\boldsymbol{s}}_{-n}, \bar{s}_n)}{cov(\bar{s}_n, \bar{s}_n) - cov(\bar{s}_n, \bar{\boldsymbol{s}}'_{-n})cov(\bar{\boldsymbol{s}}_{-n}, \bar{\boldsymbol{s}}'_{-n})^{-1}cov(\bar{\boldsymbol{s}}_{-n}, \bar{s}_n)}.$$

Let's denote $cov(\bar{\theta}_n, \bar{s}_n)$ as A, $cov(\bar{s}_n, \bar{s}_n)$ as B, $cov(\bar{s}_n, \bar{\boldsymbol{s}}'_{-n})cov(\bar{\boldsymbol{s}}_{-n}, \bar{\boldsymbol{s}}'_{-n})^{-1}cov(\bar{\boldsymbol{s}}_{-n}, \bar{s}_n)$ as C. As $cov(\bar{s}_n, \bar{s}_n) - cov(\bar{\theta}_n, \bar{s}_n) = \frac{\sigma^2}{I_n} > 0$, we have $A < B$. As $Var(\bar{s}_n|\bar{\boldsymbol{s}}_{-n})^{-1} > 0$, we have $B > C$. As $cov(\bar{s}_n, \bar{\boldsymbol{s}}'_{-n})cov(\bar{\boldsymbol{s}}_{-n}, \bar{\boldsymbol{s}}'_{-n})^{-1}cov(\bar{\boldsymbol{s}}_{-n}, \bar{s}_n)$ has quadratic form and $cov(\bar{\boldsymbol{s}}_{-n}, \bar{\boldsymbol{s}}'_{-n})$ is positive definite, we have $C \geq 0$, $C = 0$ if and only if $cov(\bar{s}_n, \bar{\boldsymbol{s}}'_{-n})$ is zero vector.

Let's leverage the lemma that

$$\frac{A - C}{B - C} \leq \frac{A}{B} \quad \text{if } A < B, B > C, C \geq 0.$$

The equality is taken if and only if $C = 0$.

By the above lemma, we have

$$\left(cov(\bar{\boldsymbol{\theta}}, \bar{\boldsymbol{s}}')cov(\bar{\boldsymbol{s}}, \bar{\boldsymbol{s}}')^{-1}\right)_{nn} \leq cov(\bar{\theta}_n, \bar{s}_n)cov(\bar{s}_n, \bar{s}_n)^{-1}.$$

This holds for any $n = 1, 2, \ldots, N$. Therefore $\lambda_n^{dc} \leq \lambda_n^d, \forall n = 1, 2, \ldots, N$. The equality is taken if and only if $cov(\bar{s}_n, \bar{\boldsymbol{s}}'_{-n}) = \mathbf{0}$ for all n, which holds if and only if cross-exchange correlations $\bar{\rho}_{mn} = 0$ for any $n \neq m$. ∎

Proof of Theorem 3. By Lemma 2, $\lambda_n^{dc} \leq \lambda_n^d, \forall n = 1, 2, \ldots, N$. As $\lambda_n^{pc} = \lambda_n^{dc}\mathbf{1}_{n=1,\ldots,K} + \lambda_n^d\mathbf{1}_{n=K+1,\ldots,N}$. This implies $\lambda_n^{dc} \leq \lambda_n^{pc} \leq \lambda_n^d, \forall n = 1, 2, \ldots, N$. By Lemma 1, $\mathbb{E}[u^{i,dc}] \geq$

$\mathbb{E}[u^{i,pc}] \geq \mathbb{E}[u^{i,d}]$ for all traders $i \in I$. The equality is taken if and only if cross-exchange correlations $\bar{\rho}_{mn} = 0$ for any $n \neq m$.

■

Proof of Theorem 4. If there is $\{\gamma_n, \bar{\rho}_n, \bar{\rho}_{nm}, \sigma^2\}$ such that $\lambda_n^d < \lambda_n^c$ under the equicommonal market assumption, then there exists set of parameters $\{\gamma_n, \bar{\rho}_n, \bar{\rho}_{nm}, \sigma^2\}$ such that $\lambda_n^d < \lambda_n^c$.

For equicommonal market,

$$\lambda_n^c = \frac{1-\gamma}{\gamma - c_p^c}\alpha = \frac{1}{-1 + \frac{1-c_p^c}{1-\gamma}}\alpha,$$

where $\gamma = 1 - \frac{1}{\sum_k I_k - 1}$, $c_p^c = \frac{(2-\gamma)\bar{\rho}}{1-\gamma+\bar{\rho}}\frac{\sigma^2}{1-\bar{\rho}+\sigma^2}$, $\bar{\rho} = \frac{(I_n-1)\bar{\rho}_n + \sum_{k\neq n} I_k \bar{\rho}_{nk}}{\sum_k I_k - 1}$.

$$\lambda_n^d = \frac{1-\gamma_n}{\gamma_n - c_p^d}\alpha = \frac{1}{-1 + \frac{1-c_p^d}{1-\gamma_n}}\alpha,$$

where $\gamma_n = 1 - \frac{1}{I_n - 1}$, $c_p^d = \frac{(2-\gamma_n)\bar{\rho}_n}{1-\gamma_n+\bar{\rho}_n}\frac{\sigma^2}{1-\bar{\rho}_n+\sigma^2}$.

Plug γ, γ_n, c_p^c, and c_p^d into the above equations, we get $\lambda_n^d < \lambda_n^c$ if and only if

$$\frac{(1-\gamma_n)(1-\bar{\rho}_n+\sigma^2)}{(1-\bar{\rho}_n)} cov(\bar{s}_n, \bar{\theta}_n) cov(\bar{s}_n, \bar{s}_n)^{-1} < \frac{(1-\gamma)(1-\bar{\rho}+\sigma^2)}{(1-\bar{\rho})} cov(\bar{s}, \bar{\theta}) cov(\bar{s}, \bar{s})^{-1}. \tag{1.57}$$

By Theorem 3, $\lambda_n^{dc} \leq \lambda_n^{pc} \leq \lambda_n^d < \lambda_n^c$ if and only if equation (1.57) holds.

Similarly,

$$\lambda_n^{dc} = \frac{1-\gamma_n}{\gamma_n - 1 + 1/(\boldsymbol{M}^{dc})_{nn}}\alpha$$

, where $\gamma_n = 1 - \frac{1}{I_n - 1}$, $\boldsymbol{M}^{dc} = (Id - \boldsymbol{C}_p^{dc})^{-1}$. We have $\lambda_n^{dc} < \lambda_n^c$ if and only if

$$\frac{(1-\gamma_n)(1-\bar{\rho}_n+\sigma^2)}{(1-\bar{\rho}_n)} \left(cov(\bar{\boldsymbol{\theta}}, \bar{\boldsymbol{s}}') cov(\bar{\boldsymbol{s}}, \bar{\boldsymbol{s}}')^{-1}\right)_{nn} < \frac{(1-\gamma)(1-\bar{\rho}+\sigma^2)}{(1-\bar{\rho})} cov(\bar{s}, \bar{\theta}) cov(\bar{s}, \bar{s})^{-1}. \tag{1.58}$$

We remark that condition for equation (1.58) to hold is less stringent than that for equation (1.57) as $\left(cov(\bar{\boldsymbol{\theta}}, \bar{\boldsymbol{s}}')cov(\bar{\boldsymbol{s}}, \bar{\boldsymbol{s}}')^{-1}\right)_{nn} \leq cov(\bar{s}_n, \bar{\theta}_n)cov(\bar{s}_n, \bar{s}_n)^{-1}$ for all n.

■

Proof of Theorem 5. By **Theorem 2**,

$$\lambda_n^{dc} \leq \lambda_n^{pc} \leq \lambda_n^{d} \quad \forall n = 1, \ldots, N.$$

With $\bar{\rho}_n = \bar{\rho}_{nm} = \rho$ for any n, m,

$$\lambda_n^c = \frac{1-\gamma}{\gamma - c_p^c}\alpha = \frac{1}{-1 + \frac{1-c_p^c}{1-\gamma}}\alpha \quad \forall n = 1, \ldots, N.$$

where $\gamma = 1 - \frac{1}{\sum_k I_k - 1}$, $c_p^c = \frac{(2-\gamma)\rho}{1-\gamma+\rho}\frac{\sigma^2}{1-\rho+\sigma^2}$.

$$\lambda_n^{dc} = \frac{1}{-1 + \frac{1}{(1-\gamma_n)(\boldsymbol{M}^{dc})_{nn}}}\alpha,$$

To prove that $\lambda_n^c \leq \lambda_n^{dc}$, we need to show $\frac{1-c_p^c}{1-\gamma} \geq \frac{1}{(1-\gamma_n)(\boldsymbol{M}^{dc})_{nn}}$. Equivalently, we need to show

$$(\boldsymbol{M}^{dc})_{nn} \geq \frac{I_n - 1}{\sum_k I_k - 1}\frac{1-\rho+\sigma^2}{1-\rho}\frac{1 + (\sum_k I_k - 1)\rho}{1 + (\sum_k I_k - 1)\rho + \sigma^2}.$$

As $(\boldsymbol{M}^{dc})_{nn} = ((Id - \boldsymbol{C}_p^{dc})^{-1})_{nn} = \left(cov(\bar{\boldsymbol{\theta}}, \bar{\boldsymbol{s}}')cov(\bar{\boldsymbol{s}}, \bar{\boldsymbol{s}}')^{-1}(\boldsymbol{C}_s^{dc})^{-1}\right)_{nn}$, where $\boldsymbol{C}_s^{dc} = diag(\frac{1-\rho}{1-\rho+\sigma^2})$.

$$(\boldsymbol{M}^{dc})_{nn} \;=\; \frac{(1-\rho)(1+(\sum_k I_k-1)\rho)+(1+(I_n-1)\rho)\sigma^2}{(1-\rho+\sigma^2)(1+(\sum_k I_k-1)\rho+\sigma^2)}\frac{1-\rho+\sigma^2}{1-\rho}.$$

Note that $1+(\sum_k I_k - 1)\rho > 0$ for λ_n^c to exist. If $\rho \geq 0$, then $1+(\sum_k I_k-1)\rho \geq 1+(I_n-1)\rho$,

$$\begin{aligned}(\boldsymbol{M}^{dc})_{nn} &\geq \frac{(1+(I_n-1)\rho)}{(1+(\sum_k I_k-1)\rho+\sigma^2)}\frac{1-\rho+\sigma^2}{1-\rho} \\ &\geq \frac{I_n-1}{\sum_k I_k-1}\frac{1-\rho+\sigma^2}{1-\rho}\frac{1+(\sum_k I_k-1)\rho}{1+(\sum_k I_k-1)\rho+\sigma^2}\end{aligned}$$

If $\rho < 0$, then $1 + (\sum_k I_k - 1)\rho < 1 + (I_n - 1)\rho$,

$$\begin{aligned}(\boldsymbol{M}^{dc})_{nn} &\geq \frac{(1+(\sum_n I_n-1)\rho)}{(1+(\sum_k I_k-1)\rho+\sigma^2)}\frac{1-\rho+\sigma^2}{1-\rho} \\ &\geq \frac{I_n-1}{\sum_k I_k-1}\frac{1-\rho+\sigma^2}{1-\rho}\frac{1+(\sum_k I_k-1)\rho}{1+(\sum_k I_k-1)\rho+\sigma^2}\end{aligned}$$

■

Proof of Theorem 6. For simplicity, in this proof we assume that the centralized exchange is equicommonal. The proof is restricted to parameters that make equilibria in the four cases exist. Note that this assumption restricts the parameters. If we showed that each case can be optimal under the equicommonal market assumption, then it holds in more general cases.

By Lemma 1, the welfare ranking of Design 1, Design 2 and Design 2′ is determined by the ranking of price impact, and Design 2 has the highest welfare among these decentralized market designs. We are left to show their relationship with Design 3.

We give the condition for each case to be optimal under equicommonality assumption below.

1. By **Theorem 3**, $\mathbb{E}[u^{i,dc}] \geq \mathbb{E}[u^{i,pc}] \geq \mathbb{E}[u^{i,d}]$ for all trader i. For Design 2 to be Pareto-optimal, $\mathbb{E}[u^{i,dc}] \geq \mathbb{E}[u^{i,c}]$ for all trader i. This holds if and only if

$$\frac{(\alpha+2\lambda^c)}{2\,(\alpha+\lambda^c)^2}\frac{(1-\bar{\rho})^2}{1-\bar{\rho}+\sigma^2}(1-\frac{1}{I})\sigma_\theta^2 \leq \frac{\left(\alpha+2\lambda_n^{dc}\right)}{2\,(\alpha+\lambda_n^{dc})^2}\frac{(1-\bar{\rho}_n)^2}{1-\bar{\rho}_n+\sigma^2}(1-\frac{1}{I_n})\sigma_\theta^2 \quad \forall n=1,\ldots,N. \tag{1.59}$$

   An example is when two equal-size exchanges with size $I_n = 3$, $\sigma = 1$, $\bar{\rho}_1 = \bar{\rho}_2 = 0$, $\bar{\rho}_{12} = \bar{\rho}_{21} = 0.3$, the above equation holds. We further confirm the existence of such parameters to satisfy these conditions with the numerical simulation (see Figure 1.2).

2. For Design 3 to be optimal, $\mathbb{E}[u^{i,c}] \geq \mathbb{E}[u^{i,dc}] \geq \mathbb{E}[u^{i,pc}] \geq \mathbb{E}[u^{i,d}]$ for all trader i. This holds if and only if

$$\frac{(\alpha+2\lambda^c)}{2\,(\alpha+\lambda^c)^2}\frac{(1-\bar{\rho})^2}{1-\bar{\rho}+\sigma^2}(1-\frac{1}{I})\sigma_\theta^2 \geq \frac{\left(\alpha+2\lambda_n^{dc}\right)}{2\,(\alpha+\lambda_n^{dc})^2}\frac{(1-\bar{\rho}_n)^2}{1-\bar{\rho}_n+\sigma^2}(1-\frac{1}{I_n})\sigma_\theta^2 \quad \forall n=1,\ldots,N.$$

   An example is when traders have independent values, i.e. $\rho^{i,j} = 0$ for all i, j, $\lambda^c < \lambda_n^{dc}$, and $\mathbb{E}[u^{i,c}] \geq \mathbb{E}[u^{i,dc}]$, for all i. Design 3 is Pareto-optimal.

■

Proof of Theorem 7. With equal correlation, for Design 3 to be optimal, $\mathbb{E}[u^{i,c}] \geq \mathbb{E}[u^{i,dc}] \geq \mathbb{E}[u^{i,pc}] \geq \mathbb{E}[u^{i,d}]$ for any trader i. This holds if and only if

$$\frac{(\alpha+2\lambda^c)}{2\,(\alpha+\lambda^c)^2}\frac{(1-\rho)^2}{1-\rho+\sigma^2}(1-\frac{1}{I})\sigma_\theta^2 \geq \frac{\left(\alpha+2\lambda_n^{dc}\right)}{2\,(\alpha+\lambda_n^{dc})^2}\frac{(1-\rho)^2}{1-\rho+\sigma^2}(1-\frac{1}{I_n})\sigma_\theta^2, \quad \forall n=1,\ldots,N.$$

By **Theorem 5**, we know the price impact is the lowest in the centralized market $\lambda^c \leq \lambda_n^{dc}$ for all n. As $I \geq I_n$, the above equation always holds. ■

Proof of Theorem 8. The price informativeness as

$$\psi_n \equiv \frac{Var(\theta^i|s^i) - Var(\theta^i|s^i, \mathcal{I}^i)}{Var(\theta^i|s^i)},$$

where

$$Var(\theta^i|s^i) = Var(\theta^i) - \frac{cov(\theta^i, s^i)Cov(s^i, \theta^i)}{Var(s^i)} = \frac{\sigma^2}{1+\sigma^2}\sigma_\theta^2,$$

and

$$Var(\theta^i|s^i, \mathcal{I}^i) = Var(\theta^i) \\ - \left(cov(\theta^i, s^i), cov(\theta^i, (\mathcal{I}^i)')\right)\begin{pmatrix} Var(s^i) & cov(s^i, (\mathcal{I}^i)') \\ cov(\mathcal{I}^i, s^i) & Var(\mathcal{I}^i)\end{pmatrix}^{-1}\begin{pmatrix} cov(s^i, \theta^i) \\ cov(\mathcal{I}^i, \theta^i)\end{pmatrix}.$$

Design 1: DM In a fully decentralized market, $\mathcal{I}^i = \{p_n^d\}$ for trader $i \in I(n)$.

$$\psi_n^d = \frac{\bar{\rho}_n^2\sigma^2}{(1-\bar{\rho}_n+\sigma^2)((1-\gamma_n)(1+\sigma^2)+\bar{\rho}_n)}.$$

Design 2: DC In decentralized market with cross-venue conditioning, $\mathcal{I}^i = \boldsymbol{p}^{dc}$.

$$Var(\theta^i|s^i, \mathcal{I}^i) = \sigma_\theta^2 - (c_{s,n}^{dc}, \boldsymbol{c}_{p,n}^{dc})\begin{pmatrix} \sigma_\theta^2 \\ cov(\boldsymbol{p}^{dc}, \theta^i)\end{pmatrix},$$

where $cov(\boldsymbol{p}^{dc}, \theta^i) = cov\left(\bar{\boldsymbol{\theta}}, \bar{\boldsymbol{s}}'\right) \cdot cov\left(\bar{\boldsymbol{s}}, \bar{\boldsymbol{s}}'\right)^{-1} cov(\bar{\boldsymbol{s}}, \theta^i)$.

Therefore,

$$Var(\theta^i|s^i, \mathcal{I}^i) = \left(\frac{\sigma^2}{1-\bar{\rho}_n+\sigma^2}\frac{(1-\bar{\rho}_n)}{2-\gamma_n} + (\mathcal{X})_{nn}\right)\sigma_\theta^2,$$

where $\mathcal{X} = (diag(\frac{I_n}{\sigma^2})_n + \bar{\mathcal{C}}^{-1})^{-1}$.

Therefore,

$$\psi_n^{dc} = 1 - \frac{1+\sigma^2}{1-\bar{\rho}_n+\sigma^2}\frac{(1-\bar{\rho}_n)}{2-\gamma_n} - \frac{1+\sigma^2}{\sigma^2}(\mathcal{X})_{nn}$$

Design 2′: PC In a decentralized market with partial conditioning, the price informativeness in the exchange with cross-venue conditioning is the same as that in Design 2, and the price informativeness in the exchange without conditioning is the same as that in Design 1.

$$\psi_n^{pc} = \psi_n^{dc} \cdot \mathbf{1}_{n=1,\ldots,K} + \psi_n^{d} \cdot \mathbf{1}_{n=K+1,\ldots,N}.$$

Design 3: CM In centralized market with $\mathcal{I}^i = p^c$. So the conditional variance of θ^i on s^i and p^c is

$$Var(\theta^i|s^i, p^c) = (1-c_s^{i,c})\sigma_\theta^2 - c_p^{i,c} cov(p^c, \theta^i),$$

where $cov(p^c, \theta^i) = \left(\sum_i \frac{1-c_p^{i,c}}{\alpha+\lambda^{i,c}}\right)^{-1}\left(\frac{c_s^{i,c}}{\alpha+\lambda^{i,c}} + \sum_{j\neq i}\frac{c_s^{j,c}}{\alpha+\lambda^{j,c}}\rho^{ij}\right)\sigma_\theta^2$.

Therefore the price informativeness for all $i \in I(n)$ is

$$\psi_n^c = \frac{\sigma^2\left(\sum_{j\neq i}\frac{c_s^{j,c}}{\alpha+\lambda^{j,c}}\rho^{ij}\right)^2}{(1+\sigma^2)\left(\sum_i\left(\frac{c_s^{i,c}}{\alpha+\lambda^{i,c}}\right)^2(1+\sigma^2) + \sum_k\sum_{j\neq k}\frac{c_s^{k,c}}{\alpha+\lambda^{k,c}}\frac{c_s^{j,c}}{\alpha+\lambda^{j,c}}\rho^{kj}\right) - \left(\frac{c_s^{i,c}}{\alpha+\lambda^{i,c}}(1+\sigma^2) + \sum_{j\neq i}\frac{c_s^{j,c}}{\alpha+\lambda^{j,c}}\rho^{ij}\right)^2}.$$

In equicommonal CM,

$$\psi^c = \frac{\bar{\rho}^2\sigma^2}{(1-\bar{\rho}+\sigma^2)((1-\gamma)(1+\sigma^2)+\bar{\rho})}.$$

Claim 1. *$\psi_n^{dc} \geq \psi_n^{pc} \geq \psi_n^d$, $\forall n$, and the equality is taken if and only if $\bar{\rho}_{mn} = 0$ for all $m \neq n$.*

To prove Claim 1,

$$\psi_n^{dc} - \psi_n^d = \frac{1-\gamma_n}{2-\gamma_n} \frac{\left(\sigma^2+1\right)\left(\bar{\rho}_n + 1 - \gamma_n\right)}{\left(\bar{\rho}_n + (1-\gamma_n)(1+\sigma^2)\right)} - \frac{1+\sigma^2}{\sigma^2}(\mathcal{X})_{nn}.$$

Note that

$$(\mathcal{X})_{nn} = \left(\frac{I_n}{\sigma^2} + \left(\frac{1+(I_n-1)\bar{\rho}_n}{I_n} - \frac{1}{\sigma_\theta^2} cov(\bar{\theta}_n, \bar{\boldsymbol{\theta}}'_{-n})(cov(\bar{\boldsymbol{\theta}}_{-n}, \bar{\boldsymbol{\theta}}'_{-n}))^{-1} cov(\bar{\boldsymbol{\theta}}_{-n}, \bar{\theta}_n)\right)^{-1}\right)^{-1},$$

where $\bar{\boldsymbol{\theta}} = (\bar{\theta}_m)_m \in \mathbb{R}^{N-1}$ is the vector of average value of traders in exchange $m \neq n$. As $(cov(\bar{\boldsymbol{\theta}}_{-n}, \bar{\boldsymbol{\theta}}'_{-n})$ is positive-semidefinite

$$cov(\bar{\theta}_n, \bar{\boldsymbol{\theta}}'_{-n})(cov(\bar{\boldsymbol{\theta}}_{-n}, \bar{\boldsymbol{\theta}}'_{-n}))^{-1} cov(\bar{\boldsymbol{\theta}}_{-n}, \bar{\theta}_n) \geq 0.$$

where the equality is take if and only if $cov(\bar{\theta}_n, \bar{\boldsymbol{\theta}}'_{-n})$ is a zero vector. This implies that

$$(\mathcal{X})_{nn} \leq \sigma^2 \frac{1-\gamma_n}{2-\gamma_n} \frac{(\bar{\rho}_n + 1 - \gamma_n)}{(\bar{\rho}_n + (1-\gamma_n)(1+\sigma^2))}.$$

Therefore, $\psi_n^{dc} - \psi_n^d \geq 0$. $\psi_n^{dc} = \psi_n^d$ if and only if $cov(\bar{\theta}_n, \bar{\boldsymbol{\theta}}'_{-n})$ is a zero vector, i.e. $\bar{\rho}_{mn} = 0$ for all $m \neq n$.

Given that $\psi_n^{dc} \geq \psi_n^{pc} \geq \psi_n^d$, we have proved Claim 1.

Claim 2. $\psi_n^{dc} \geq \psi_n^c$, $\forall n$.

We can prove Claim 2 by showing that for any given primitives, we can find a constant w_0 and a weight vector $\boldsymbol{w} \in \mathbb{R}^{1\times N}$, such that $p^c = w_0 + \boldsymbol{w} \cdot \boldsymbol{p}^{dc}$. If such weight vector exists, then $Var(\theta^i|s^i, p^c) = Var(\theta^i|s^i, w_0 + \boldsymbol{w} \cdot \boldsymbol{p}^{dc}) \geq Var(\theta^i|s^i, \boldsymbol{p}^{dc})$, and therefore $\psi_n^{dc} \geq \psi^{i,c}$, $\forall n, i \in I(n)$.

$$\boldsymbol{p}^{dc} = \mathbb{E}[\theta] + cov\left(\bar{\boldsymbol{\theta}}, \bar{\boldsymbol{s}}'\right) \cdot cov\left(\bar{\boldsymbol{s}}, \bar{\boldsymbol{s}}'\right)^{-1} \cdot (\bar{\boldsymbol{s}} - \mathbb{E}[\theta]),$$

and

$$p^c = \mathbb{E}[\theta] + \left(\sum_n I_n \frac{1-c_{p,n}^c}{\alpha + \lambda_n^c}\right)^{-1} \left(\sum_n I_n \frac{c_{s,n}^c\left(\bar{s}_n - \mathbb{E}[\theta]\right)}{\alpha + \lambda_n^c}\right).$$

With the method of undermined coefficients, we can find $\boldsymbol{w} = \boldsymbol{a} \cdot cov\left(\bar{\boldsymbol{s}}, \bar{\boldsymbol{s}}'\right) cov\left(\bar{\boldsymbol{\theta}}, \bar{\boldsymbol{s}}'\right)^{-1}$, $\boldsymbol{a} = (a_n)_n \in \mathbb{R}^{1\times N}$ where the n^{th} element is $\left(\sum_n I_n \frac{1-c^c_{p,n}}{\alpha+\lambda^c_n}\right)^{-1} \frac{I_n c^c_{s,n}}{\alpha+\lambda^c_n}$, and $w_0 = (1 - \boldsymbol{w} \cdot 1)\mathbb{E}[\theta]$, such that $p^c = w_0 + \boldsymbol{w} \cdot \boldsymbol{p}^{dc}$.

Claim 3. *There exists a set of parameters* $\left\{\gamma_n, \bar{\rho}_n, \bar{\rho}_{nm}, \sigma^2\right\}$ *such that* $\psi^{dc}_n \geq \psi^{pc}_n \geq \psi^d_n \geq \psi^c_n$, $\forall n$.

By Claim 1, $\psi^{dc}_n \geq \psi^{pc}_n \geq \psi^d_n$, $\forall n$. To show that there exists a set of parameters $\left\{\gamma_n, \bar{\rho}_n, \bar{\rho}_{nm}, \sigma^2\right\}$ such that $\psi^d_n \geq \psi^c_n$, for simplicity, we assume that the centralized exchange is equicommonal. Note that this assumption puts bounds on the parameters. If we showed that Claim 3 holds under the equicommonal market assumption, then we establish the existence of such set of parameters $\left\{\gamma_n, \bar{\rho}_n, \bar{\rho}_{nm}, \sigma^2\right\}$ without the assumption.

Under the equicommonality assumption, $\psi^d_n \geq \psi^c_n$, $\forall n$ if and only if

$$\frac{\bar{\rho}^2\sigma^2}{(1-\bar{\rho}+\sigma^2)((1-\gamma)(1+\sigma^2)+\bar{\rho})} \leq \frac{\bar{\rho}^2_n\sigma^2}{(1-\bar{\rho}_n+\sigma^2)((1-\gamma_n)(1+\sigma^2)+\bar{\rho}_n)}, \quad \forall n.$$

This is equivalent to

$$\bar{\rho} \in [\rho^-_n, \rho^+_n] \quad \forall n,$$

where

$$\rho^-_n = \frac{\bar{\rho}_n\left(\gamma\bar{\rho}_n - \sqrt{\gamma^2\bar{\rho}^2_n + 4(1-\gamma)\gamma_n\bar{\rho}_n\left(\sigma^2+1\right) + 4(1-\gamma)(1-\gamma_n)\left(\sigma^2+1\right)^2}\right)}{2(1-\gamma_n)\left(\sigma^2+1\right) + 2\gamma_n\bar{\rho}_n},$$

and

$$\rho^+_n = \frac{\bar{\rho}_n\left(\gamma\bar{\rho}_n + \sqrt{\gamma^2\bar{\rho}^2_n + 4(1-\gamma)\gamma_n\bar{\rho}_n\left(\sigma^2+1\right) + 4(1-\gamma)(1-\gamma_n)\left(\sigma^2+1\right)^2}\right)}{2(1-\gamma_n)\left(\sigma^2+1\right) + 2\gamma_n\bar{\rho}_n}.$$

We show that there exists a set of parameters $\left\{\gamma_n, \bar{\rho}_n, \bar{\rho}_{nm}, \sigma^2\right\}$ such that $\bar{\rho} \in [\rho^-_n, \rho^+_n], \forall n$ by construction. Consider an example where N exchanges have the same size, $\gamma_n = \bar{\gamma}$, $\forall n$. Then the size index of the centralized market is $\gamma = \frac{N(2-\bar{\gamma})-2(1-\bar{\gamma})}{N(2-\bar{\gamma})-(1-\bar{\gamma})}$. There exist $\bar{\rho}_{nm} = -\frac{\bar{\rho}}{(N-1)(2-\bar{\gamma})} + \frac{N}{N-1}\frac{\bar{\rho}^2_n\gamma}{2((1-\bar{\gamma})(1+\sigma^2)+\bar{\gamma}\bar{\rho}_n)}$, $\forall m \neq n$, such that $\bar{\rho} = \frac{1}{2}(\rho^-_n + \rho^+_n) \in [\rho^-_n, \rho^+_n]$

for all n. ■

Appendix C. Robustness Check

In this section, we examine the robustness of Theorem 3 with multiple assets, heterogeneous risk-aversions, and heterogeneous signal precisions.

Assume there are N exchange, each exchange is indexed by n. There are K risky assets, and a riskless asset as a numéraire. Each trader has private value about the risky assets $\boldsymbol{\theta}^i \in \mathbb{R}^K$, $\boldsymbol{\theta}^i \sim \mathcal{N}(\mathbb{E}[\boldsymbol{\theta}], \boldsymbol{\Sigma}_\theta)$. We allow traders to have correlated value. The within-exchange correlation for $i \in I(n)$ is $\bar{\rho}_n \in \mathbb{R}$. For $i \in I(n)$, the covariance $cov(\boldsymbol{\theta}^i, (\frac{1}{I_n-1}\sum_{j\neq i}\boldsymbol{\theta}^j)') = \bar{\rho}_n\boldsymbol{\Sigma}_\theta$. The cross-exchange correlation for $i \in I(n)$ and traders in exchange m is $\bar{\rho}_{nm} \in \mathbb{R}$. For $i \in I(n)$, the covariance $cov(\boldsymbol{\theta}^i, (\frac{1}{I_m}\sum_{j\in I(m)}\boldsymbol{\theta}^j)') = \bar{\rho}_{nm}\boldsymbol{\Sigma}_\theta$. It is easy to see that $\bar{\rho}_{nm} = \bar{\rho}_{mn}$ by definition. Before they trade, traders receive a noisy signal vector about their value of the assets in their own exchange, $\boldsymbol{s}^i = \boldsymbol{\theta}^i + \boldsymbol{\varepsilon}^i$. The noises $\boldsymbol{\varepsilon}^i$ are independently normally distributed across all traders and assets, and are independent of any other random variables. For trader $i \in I(n)$, $\boldsymbol{\varepsilon}^i \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_{\varepsilon,n})$, $\boldsymbol{\Sigma}_{\varepsilon,n} = \sigma^2_{\varepsilon,n}\boldsymbol{Id}_K$, where $\boldsymbol{Id}_K$ is $K \times K$ identity matrix. Denote the noise-value variance ratio as $\boldsymbol{\Sigma}_n \equiv \boldsymbol{\Sigma}_{\varepsilon,n}\boldsymbol{\Sigma}_\theta^{-1}$. We allow $\boldsymbol{\Sigma}_n$ to be heterogeneous across exchanges. Assume that traders in exchange n have risk-aversion α_n.

In Design 1, traders submit demand schedule contingent on the prices in their own exchanges. In Design 2, traders submit demand schedule contingent on the prices from all exchanges. In Design 2′, traders in exchange $I(1), \ldots, I(\tilde{N})$, $\tilde{N} < N$ can submit demand schedule contingent prices from all the exchanges, whhile the others do not. We find that Theorem 3 still holds with multiple assets, heterogeneous risk-aversions, and heterogeneous signal precisions (see Theorem 9).

Theorem 9 (Conditioning with Multiple Assets). *If the Bayesian equilibria for Design 1, Design 2 and Design 2′ exist,* $\boldsymbol{\lambda}_n^{dc} \preceq \boldsymbol{\lambda}_n^{pc} \preceq \boldsymbol{\lambda}_n^{d}, \forall n = 1, 2, \ldots, N$*, equivalently,* $\mathbb{E}[u^{i,dc}] \geq \mathbb{E}[u^{i,pc}] \geq \mathbb{E}[u^{i,d}]$ *for all traders* $i \in I$ *. The equality is taken if and only if cross-exchange correlations* $\bar{\rho}_{mn} = 0$ *for any* $n \neq m$.

Proof of Theorem 9. Let's first solve for the equilibrium of Design 1 and 2 with multiple assets, heterogeneous risk-aversions, and heterogeneous signal precisions, and compare the liquidity and welfare in Design 1 and 2. We will then solve for the equilibrium of Design 2′, and show that in Design 2′, the traders in exchange $I(1), \ldots, I(K), K < N$ have the same outcome as those in Design 2, and the traders in $I(K+1), \ldots, I(N)$ have the same outcome as those in Design 1. Therefore, the comparison between Design 1 and 2 can be easily applied to comparison of all three designs.

Design 1. DM: The trader $i \in I(n)$ maximize utility

$$\mathbb{E}\left[(\boldsymbol{\theta}^i)'\boldsymbol{q}^i - \frac{1}{2}\alpha_n(\boldsymbol{q}^i)'\boldsymbol{\Sigma}_\theta \boldsymbol{q}^i - \boldsymbol{p}_n'\boldsymbol{q}^i | \boldsymbol{s}^i, \boldsymbol{p}_n\right].$$

The first order condition with respect to $\boldsymbol{q}^i$ given price impact $\boldsymbol{\lambda}_n^i \equiv \frac{d\boldsymbol{p}_n}{d\boldsymbol{q}^i}$ is

$$\boldsymbol{q}^i = \left(\alpha_n \boldsymbol{\Sigma}_\theta + \boldsymbol{\lambda}_n^i\right)^{-1}\left(\mathbb{E}[\boldsymbol{\theta}^i | \boldsymbol{s}^i, \boldsymbol{p}_n] - \boldsymbol{p}_n\right).$$

By symmetry, let's parameterize the expected utility in a linear form, $\mathbb{E}[\boldsymbol{\theta}^i|\boldsymbol{s}^i, \boldsymbol{p}_n] \equiv \boldsymbol{c}_{0,n}\mathbb{E}[\boldsymbol{\theta}] + \boldsymbol{c}_{s,n}\boldsymbol{s}^i + \boldsymbol{c}_{p,n}\boldsymbol{p}_n$, where $\boldsymbol{c}_{0,n}, \boldsymbol{c}_{s,n}, \boldsymbol{c}_{p,n} \in \mathbb{R}^{K \times K}$.

The equilibrium price satisfies

$$\boldsymbol{p}_n = (\boldsymbol{Id}_K - \boldsymbol{c}_{p,n})^{-1}(\boldsymbol{c}_{0,n}\mathbb{E}[\boldsymbol{\theta}] + \boldsymbol{c}_{s,n}\bar{\boldsymbol{s}}_n),$$

where $\bar{\boldsymbol{s}}_n = \frac{1}{I_n}\sum_{i \in I(n)} \boldsymbol{s}^i$.

We can determine the inference coefficients as a function of the primitives (and in closed form). Random vector $\left(\boldsymbol{\theta}^i, \boldsymbol{s}^i, (\boldsymbol{p}^{dc})'\right)$ is jointly normally distributed

$$\begin{pmatrix} \boldsymbol{\theta}^i \\ \boldsymbol{s}^i \\ \boldsymbol{p}^{dc} \end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix} \mathbb{E}[\boldsymbol{\theta}] \\ \mathbb{E}[\boldsymbol{\theta}] \\ \mathbb{E}[\boldsymbol{\theta}] \end{pmatrix}, \begin{pmatrix} \boldsymbol{\Sigma}_\theta & \boldsymbol{\Sigma}_\theta & cov\left(\boldsymbol{\theta}^i, (\boldsymbol{p}_n)'\right) \\ \boldsymbol{\Sigma}_\theta & \boldsymbol{\Sigma}_\theta + \boldsymbol{\Sigma}_{\varepsilon,n} & cov\left(\boldsymbol{s}^i, (\boldsymbol{p}_n)'\right) \\ cov\left(\boldsymbol{p}_n, (\boldsymbol{\theta}^i)'\right) & cov\left(\boldsymbol{p}_n, (\boldsymbol{s}^i)'\right) & cov\left(\boldsymbol{p}_n, (\boldsymbol{p}_n)'\right) \end{pmatrix}\right],$$

where

$$\begin{aligned}
cov\left(\boldsymbol{p}_n, (\boldsymbol{\theta}^i)'\right) &= (\boldsymbol{Id}_K - \boldsymbol{c}_{p,n})^{-1}\boldsymbol{c}_{s,n} cov(\bar{\boldsymbol{s}}_n, (\boldsymbol{\theta}^i)'), \\
cov\left(\boldsymbol{p}_n, (\boldsymbol{s}^i)'\right) &= (\boldsymbol{Id}_K - \boldsymbol{c}_{p,n})^{-1}\boldsymbol{c}_{s,n} cov(\bar{\boldsymbol{s}}_n, (\boldsymbol{s}^i)'), \\
cov\left(\boldsymbol{p}_n, (\boldsymbol{p}_n)'\right) &= (\boldsymbol{Id}_K - \boldsymbol{c}_{p,n})^{-1}\boldsymbol{c}_{s,n} cov(\bar{\boldsymbol{s}}_n, \bar{\boldsymbol{s}}_n')\boldsymbol{c}_{s,n}' \left((\boldsymbol{Id}_K - \boldsymbol{c}_{p,n})^{-1}\right)'.
\end{aligned}$$

By Projection Theorem, we have

$$(\boldsymbol{c}_{s,n}, \quad \boldsymbol{c}_{p,n}) = \left(\boldsymbol{\Sigma}_\theta, \quad cov\left(\boldsymbol{\theta}^i, (\boldsymbol{p}_n)'\right)\right) \begin{pmatrix} \boldsymbol{\Sigma}_\theta + \boldsymbol{\Sigma}_{\varepsilon,n} & cov\left(\boldsymbol{s}^i, (\boldsymbol{p}_n)'\right) \\ cov\left(\boldsymbol{p}_n, (\boldsymbol{s}^i)'\right) & cov\left(\boldsymbol{p}_n, (\boldsymbol{p}_n)'\right) \end{pmatrix}^{-1}, \forall i,$$

and

$$\boldsymbol{c}_{0,n} = \boldsymbol{Id}_K - \boldsymbol{c}_{s,n} - \boldsymbol{c}_{p,n}.$$

We can solve for the inference coefficients on the private signal,

$$\begin{aligned}
\boldsymbol{c}_{s,n} &= \left(\boldsymbol{\Sigma}_\theta - cov(\boldsymbol{\theta}^i, \bar{\boldsymbol{s}}_n')\right)\left(\boldsymbol{\Sigma}_\theta + \boldsymbol{\Sigma}_{\varepsilon,n} - cov(\boldsymbol{s}^i, \bar{\boldsymbol{s}}_n')\right)^{-1} \\
&= (1-\bar{\rho}_n)\left((1-\bar{\rho}_n)\boldsymbol{Id}_K + \boldsymbol{\Sigma}_n\right)^{-1}. \qquad (1.60)
\end{aligned}$$

From the projection theorem, we have

$$\boldsymbol{c}_{s,n}(\boldsymbol{\Sigma}_\theta + \boldsymbol{\Sigma}_{\varepsilon,n}) + \boldsymbol{c}_{p,n}(\boldsymbol{Id}_K - \boldsymbol{c}_{p,n})^{-1}\boldsymbol{c}_{s,n} cov(\bar{\boldsymbol{s}}_n, (\boldsymbol{s}^i)') = \boldsymbol{\Sigma}_\theta.$$

Plug $\boldsymbol{c}_{c,n}$ can solve for the inference coefficients on the prices,

$$\boldsymbol{c}_{p,n} = \frac{I_n\bar{\rho}_n}{1+(I_n-1)\bar{\rho}_n}\left((1-\bar{\rho}_n)\boldsymbol{\Sigma}_n^{-1} + \boldsymbol{Id}_K\right)^{-1} \qquad (1.61)$$

To summarize the solution for inference coefficients,

$$\begin{aligned}
\boldsymbol{c}_{s,n} &= (1-\bar{\rho}_n)\left((1-\bar{\rho}_n)\boldsymbol{Id}_K + \boldsymbol{\Sigma}_n\right)^{-1}, \\
\boldsymbol{c}_{p,n} &= \frac{I_n\bar{\rho}_n}{1+(I_n-1)\bar{\rho}_n}\left((1-\bar{\rho}_n)\boldsymbol{\Sigma}_n^{-1} + \boldsymbol{Id}_K\right)^{-1} \\
\boldsymbol{c}_{0,n} &= \boldsymbol{Id}_K - \boldsymbol{c}_{s,n} - \boldsymbol{c}_{p,n}.
\end{aligned}$$

The price impact for trader i satisfies,

$$\boldsymbol{\lambda}_n^i = \left(-\sum_{j\neq i, j\in I(n)} \frac{d\boldsymbol{q}^j}{d\boldsymbol{p}_n}\right)^{-1} = \left(\sum_{j\neq i}\left(\left(\alpha_n\boldsymbol{\Sigma}_\theta + \boldsymbol{\lambda}_n^j\right)^{-1}\left(\boldsymbol{Id}_K - \boldsymbol{c}_{p,n}\right)\right)'\right)^{-1}.$$

By symmetry, $\boldsymbol{\lambda}_n^i = \boldsymbol{\lambda}_n$ for all $i \in I(n)$. We can solve that

$$\boldsymbol{\lambda}_n = \left((I_n-2)\boldsymbol{Id}_K - (I_n-1)\boldsymbol{c}_{p,n}\right)^{-1}\alpha_n\boldsymbol{\Sigma}_\theta$$

Note that $\boldsymbol{\lambda}_n$ is symmetric.

Price: By the market clearing condition, the price in exchange n is

$$\boldsymbol{p}_n = \mathbb{E}[\boldsymbol{\theta}] + (\boldsymbol{Id} - \boldsymbol{c}_{p,n})^{-1}\boldsymbol{c}_{s,n}\left(\bar{\boldsymbol{s}}_n - \mathbb{E}[\boldsymbol{\theta}]\right) = \mathbb{E}[\boldsymbol{\theta}] + cov(\bar{\boldsymbol{\theta}}_n, \bar{\boldsymbol{s}}_n')cov(\bar{\boldsymbol{s}}_n, \bar{\boldsymbol{s}}_n')^{-1}\left(\bar{\boldsymbol{s}}_n - \mathbb{E}[\boldsymbol{\theta}]\right)$$

Before we analyze the equilibrium of Design 2, let's first introduce some notations as they will appear several times in the analysis.

- $(\boldsymbol{A})_{nm}$ is the submatrix corresponding to the $(n-1)K+1^{th}$ to nK^{th} row and $(m-1)K+1^{th}$ to mK^{th} of the matrix $\boldsymbol{A}$.
- $diag(\boldsymbol{A}_n)_n$ is a blockwise diagonal matrix with submatrix $\boldsymbol{A}_n$ at its $(n-1)K+1^{th}$ to nK^{th} rows and columns.

Design 2. **DC:** With cross-venue price information, the trader $i \in I(n)$ maximize utility

$$\mathbb{E}\left[(\boldsymbol{\theta}^i)'\boldsymbol{q}^i - \frac{1}{2}\alpha_n(\boldsymbol{q}^i)'\boldsymbol{\Sigma}_\theta\boldsymbol{q}^i - \boldsymbol{p}_n'\boldsymbol{q}^i|\boldsymbol{s}^i, \{\boldsymbol{p}_n\}_n\right].$$

The first order condition is

$$\boldsymbol{q}^i = \left(\alpha_n\boldsymbol{\Sigma}_\theta + \boldsymbol{\lambda}_n^i\right)^{-1}\left(\mathbb{E}[\boldsymbol{\theta}^i|\boldsymbol{s}^i, \{\boldsymbol{p}_n\}_n] - \boldsymbol{p}_n\right).$$

By symmetry, let's parameterize the expected utility in a linear form, $\mathbb{E}[\boldsymbol{\theta}^i|\boldsymbol{s}^i, \boldsymbol{p}_n] \equiv \boldsymbol{c}_{0,n}\mathbb{E}[\boldsymbol{\theta}] + \boldsymbol{c}_{s,n}\boldsymbol{s}^i + \sum_{m=1}^{N}\boldsymbol{c}_{p,n,m}\boldsymbol{p}_m$, where $\boldsymbol{c}_{0,n}, \boldsymbol{c}_{s,n}\boldsymbol{c}_{p,n,m} \in \mathbb{R}^{K\times K}$.

By market-clearing condition, the equilibrium price satisfies

$$\boldsymbol{P} = (\boldsymbol{Id}_{NK} - \boldsymbol{C}_p)^{-1}(\boldsymbol{C}_0\mathbb{E}[\boldsymbol{\Theta}] + \boldsymbol{C}_s\bar{\boldsymbol{s}}),$$

where $\boldsymbol{P} \in \mathbb{R}^{NK}$ is the price vector for all assets, where the $(n-1)K+1^{th}$ to nK^{th} elements is the price vector $\boldsymbol{p}_n$; $\bar{\boldsymbol{s}} \in \mathbb{R}^{NK}$ is the average signal vector, where the $(n-1)K+1^{th}$ to nK^{th} elements is the average signal in exchange n, $\frac{1}{I_n}\sum_{i\in I(n)}\boldsymbol{s}^i$; $\mathbb{E}[\boldsymbol{\Theta}] \in \mathbb{R}^{NK}$ is the expected value vector. $\boldsymbol{C}_p \in \mathbb{R}^{NK\times NK}$, where the submatrix corresponding to $(n-1)K+1^{th}$ to nK^{th} row and the $(m-1)K+1^{th}$ to mK^{th} column is $\boldsymbol{c}_{p,n,m}$. $\boldsymbol{C}_0 \in \mathbb{R}^{NK\times NK}$, where the diagonal $(n-1)K+1^{th}$ to nK^{th} submatrix is $\boldsymbol{c}_{0,n}$; $\boldsymbol{C}_s \in \mathbb{R}^{NK\times NK}$, whose diagonal $(n-1)K+1^{th}$ to nK^{th} submatrix is $\boldsymbol{c}_{s,n}$.

We can determine the inference coefficients as a function of the primitives (and in closed form). Random vector $\left((\boldsymbol{\theta}^i)', (\boldsymbol{s}^i)', \boldsymbol{P}'\right)$ is jointly normally distributed

$$\begin{pmatrix}\boldsymbol{\theta}^i\\ \boldsymbol{s}^i\\ \boldsymbol{P}\end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix}\mathbb{E}[\boldsymbol{\theta}]\\ \mathbb{E}[\boldsymbol{\theta}]\\ \mathbb{E}[\boldsymbol{\Theta}]\end{pmatrix}, \begin{pmatrix}\boldsymbol{\Sigma}_\theta & \boldsymbol{\Sigma}_\theta & cov\left(\boldsymbol{\theta}^i, \boldsymbol{P}'\right)\\ \boldsymbol{\Sigma}_\theta & \boldsymbol{\Sigma}_\theta + \boldsymbol{\Sigma}_{\varepsilon,n} & cov\left(\boldsymbol{s}^i, \boldsymbol{P}'\right)\\ cov\left(\boldsymbol{P}, (\boldsymbol{\theta}^i)'\right) & cov\left(\boldsymbol{P}, (\boldsymbol{s}^i)'\right) & cov\left(\boldsymbol{P}, \boldsymbol{P}'\right)\end{pmatrix}\right],$$

where

$$\begin{aligned} cov\left(\boldsymbol{P},(\boldsymbol{\theta}^i)'\right) &= (\boldsymbol{Id}_{NK}-\boldsymbol{C}_p)^{-1}\boldsymbol{C}_s cov(\bar{\boldsymbol{s}},(\boldsymbol{\theta}^i)'), \\ cov\left(\boldsymbol{P},(\boldsymbol{s}^i)'\right) &= (\boldsymbol{Id}_{NK}-\boldsymbol{C}_p)^{-1}\boldsymbol{C}_s cov(\bar{\boldsymbol{s}},(\boldsymbol{s}^i)'), \\ cov\left(\boldsymbol{P},\boldsymbol{P}'\right) &= (\boldsymbol{Id}_{NK}-\boldsymbol{C}_p)^{-1}\boldsymbol{C}_s cov(\bar{\boldsymbol{s}},\bar{\boldsymbol{s}}')\boldsymbol{C}_s'\left((\boldsymbol{Id}-\boldsymbol{C}_p)^{-1}\right)' \end{aligned} \tag{1.62}$$

By Projection Theorem, for any exchange n, $\forall i \in I(n)$,

$$\boldsymbol{c}_{s,n}\left(\boldsymbol{\Sigma}_\theta+\boldsymbol{\Sigma}_{\varepsilon,n}\right)+\sum_m \boldsymbol{c}_{p,n,m} cov(\boldsymbol{p}_m,(\boldsymbol{s}^i)') = \boldsymbol{\Sigma}_\theta \tag{1.63}$$

$$\boldsymbol{c}_{s,n} cov(\boldsymbol{s}^i,\boldsymbol{P}')+\sum_m \boldsymbol{c}_{p,n,m} cov(\boldsymbol{p}_m,\boldsymbol{P}') = cov\left(\boldsymbol{\theta}^i,\boldsymbol{P}'\right) \tag{1.64}$$

and

$$\boldsymbol{c}_{0,n}=\boldsymbol{Id}_{NK}-\boldsymbol{c}_{s,n}-\sum_m \boldsymbol{c}_{p,n,m}.$$

If we plug the covariances in equation (1.62) into equation (1.64), we can simplify the equation as

$$\boldsymbol{c}_{s,n} cov(\boldsymbol{s}^i,\bar{\boldsymbol{s}}')+\sum_m \boldsymbol{c}_{p,n,m} cov(\boldsymbol{p}_m,\bar{\boldsymbol{s}}') = cov\left(\boldsymbol{\theta}^i,\bar{\boldsymbol{s}}'\right), \quad \forall n, i\in I(n) \tag{1.65}$$

Averaging equation (1.63) over all $i \in I(n)$, we have

$$\boldsymbol{c}_{s,n}\left(\boldsymbol{\Sigma}_\theta+\boldsymbol{\Sigma}_{\varepsilon,n}\right)+\sum_m \boldsymbol{c}_{p,n,m} cov(\boldsymbol{p}_m,\bar{\boldsymbol{s}}_n') = \boldsymbol{\Sigma}_\theta \quad \forall n. \tag{1.66}$$

Subtracting equation (1.66) from the $(n-1)K+1^{th}$ to nK^{th} rows of equation (1.65), we can solve that

$$\begin{aligned} \boldsymbol{c}_{s,n} &= \left(\boldsymbol{\Sigma}_\theta - cov(\boldsymbol{\theta}^i,\bar{\boldsymbol{s}}_n)\right)\left(\boldsymbol{\Sigma}_\theta+\boldsymbol{\Sigma}_{\varepsilon,n}-cov(\boldsymbol{s}^i,\bar{\boldsymbol{s}}_n)\right)^{-1} \\ &= (1-\bar{\rho}_n)\left((1-\bar{\rho}_n)\boldsymbol{Id}_K+\boldsymbol{\Sigma}_n\right)^{-1}. \end{aligned} \tag{1.67}$$

We have the following equation by averaging equation (1.65) over all i and writing it in matrix form,

$$(\boldsymbol{Id}_{NK} - \boldsymbol{C}_p)^{-1} = cov(\bar{\boldsymbol{\theta}}, \bar{\boldsymbol{s}}')cov(\bar{\boldsymbol{s}}, \bar{\boldsymbol{s}}')^{-1}\boldsymbol{C}_s^{-1} \tag{1.68}$$

Plug the solved $\boldsymbol{c}_{s,n}$ into equation (1.68), then we can solve $\boldsymbol{c}_{p,n}$,

$$\boldsymbol{C}_p = \boldsymbol{Id}_{NK} - \boldsymbol{C}_s - \boldsymbol{C}_s diag(\frac{1}{I_n}\boldsymbol{\Sigma}_n)_n \bar{\boldsymbol{\mathcal{C}}}^{-1}$$

where $\bar{\boldsymbol{\mathcal{C}}} = \bar{\mathcal{C}} \bigotimes \boldsymbol{Id}_K \in \mathbb{R}^{NK \times NK}$, where the $(n-1)K+1^{th}$ to nK^{th} row and $(m-1)K+1^{th}$ to mK^{th} submatrix equals to $\bar{\rho}_{nm}\boldsymbol{Id}_K$ if $n \neq m$, and equals to $\left(\frac{1}{I_n} + \frac{I_n - 1}{I_n}\bar{\rho}_n\right)\boldsymbol{Id}_K$ if $n = m$.

To summarize, the inference coefficients are

$$\begin{aligned}
\boldsymbol{c}_{s,n} &= (1-\bar{\rho}_n)\left((1-\bar{\rho}_n)\boldsymbol{Id}_K + \boldsymbol{\Sigma}_n\right)^{-1} \\
\boldsymbol{c}_{p,n,m} &= \left((1-\bar{\rho}_n)\boldsymbol{\Sigma}_n^{-1} + \boldsymbol{Id}_K\right)^{-1}\left(\boldsymbol{Id}_{K,n=m} - \frac{(1-\bar{\rho}_n)}{I_n}(\bar{\boldsymbol{\mathcal{C}}}^{-1})_{nm}\right), \\
\boldsymbol{c}_{0,n} &= \boldsymbol{Id}_K - \boldsymbol{c}_{s,n} - \sum_m \boldsymbol{c}_{p,n,m}.
\end{aligned}$$

where $\boldsymbol{Id}_{K,n=m}$ is an $K \times K$ identity matrix if $n = m$ and is zero matrix otherwise, $(\bar{\boldsymbol{\mathcal{C}}}^{-1})_{nm}$ is the submatrix corresponding to the $(n-1)K+1^{th}$ to nK^{th} row and $\sum_{\ell<m} K(\ell) + 1^{th}$ to $\sum_{\ell<m} K(\ell) + K(m)^{th}$ of $\bar{\boldsymbol{\mathcal{C}}}^{-1}$.

The equilibrium price impact satisfies

$$\boldsymbol{\Lambda}^i = \begin{bmatrix} \frac{\partial \boldsymbol{p}_1}{\partial \boldsymbol{q}_1^i} & \cdots & \frac{\partial \boldsymbol{p}_N}{\partial \boldsymbol{q}_1^i} \\ \vdots & \ddots & \vdots \\ \frac{\partial \boldsymbol{p}_1}{\partial \boldsymbol{q}_N^i} & \cdots & \frac{\partial \boldsymbol{p}_N}{\partial \boldsymbol{q}_N^i} \end{bmatrix} = -\begin{bmatrix} \sum_{j\neq i}\frac{\partial \boldsymbol{q}_1^j}{\partial \boldsymbol{p}_1} & \cdots & \sum_{j\neq i}\frac{\partial \boldsymbol{q}_N^j}{\partial \boldsymbol{p}_1} \\ \vdots & \ddots & \vdots \\ \sum_{j\neq i}\frac{\partial \boldsymbol{q}_1^j}{\partial \boldsymbol{p}_N} & \cdots & \sum_{j\neq i}\frac{\partial \boldsymbol{q}_N^j}{\partial \boldsymbol{p}_N} \end{bmatrix}^{-1}.$$

where $\frac{\partial \boldsymbol{q}_m^j}{\partial \boldsymbol{p}_\ell} = \left((\alpha_m \boldsymbol{\Sigma}_{\theta,m} + \boldsymbol{\lambda}_m^j)^{-1}(\boldsymbol{c}_{p,m,\ell} - \boldsymbol{Id}_{K,m=\ell})\right)'$. By symmetry, $\boldsymbol{\lambda}_n^i = \boldsymbol{\lambda}_n$ for all $i \in I(n)$, Therefore,

$$\left(\boldsymbol{\Lambda}^i\right)' = (\boldsymbol{Id}_{NK} - \boldsymbol{C}_p)^{-1} diag\left(\frac{1}{I_n - \boldsymbol{1}_{i\in I(n)}}(\alpha_n \boldsymbol{\Sigma}_\theta + \boldsymbol{\lambda}_n)\right)_n$$

We can solve for the diagonal submatrix $\boldsymbol{\lambda}_n^i$ of $\boldsymbol{\Lambda}^i$,

$$\boldsymbol{\lambda}_n = \left((I_n - 1)((\boldsymbol{M})_{nn})^{-1} - \boldsymbol{Id}_K\right)^{-1} \alpha_n \boldsymbol{\Sigma}_\theta$$

where $\boldsymbol{M} = (\boldsymbol{Id}_{NK} - \boldsymbol{C}_p)^{-1}$,and $(\boldsymbol{M})_{nn}$ is its diagonal submatrix corresponding to $(n-1)K + 1^{th}$ to nK^{th} rows and columns. Note that $\boldsymbol{\lambda}_n$ is symmetric.

Prices: By the market clearing condition, the equilibrium prices satisfy

$$\boldsymbol{P} = \mathbb{E}[\boldsymbol{\Theta}] + (\boldsymbol{Id} - \boldsymbol{C}_p)^{-1}\boldsymbol{C}_s(\bar{\boldsymbol{s}} - \mathbb{E}[\boldsymbol{\Theta}]).$$

Liquidity Comparison: DM v.s. DC

We denote the equilibrium variables in DM with superscript d, and the equilibrium variables in DC with superscript dc.

For traders in $I(n)$, the price impact in Design 1 is $\boldsymbol{\lambda}_n^d = \left((I_n - 2)\boldsymbol{Id}_K - (I_n - 1)\boldsymbol{c}_p^d\right)^{-1} \alpha_n \boldsymbol{\Sigma}_\theta$, and the price impact in Design 2 is $\boldsymbol{\lambda}_n^{dc} = \left((I_n - 1)((\boldsymbol{M})_{nn})^{-1} - \boldsymbol{Id}_K\right)^{-1} \alpha_n \boldsymbol{\Sigma}_\theta$. Therefore, $\boldsymbol{\lambda}_n^{dc} \preceq \boldsymbol{\lambda}_n^d$ if and only if

$$\boldsymbol{Id}_K - \boldsymbol{c}_p^d \preceq ((\boldsymbol{M})_{nn})^{-1}$$

By equations (1.60) and (1.61), $(\boldsymbol{Id} - \boldsymbol{c}_p^d)^{-1} = cov(\bar{\boldsymbol{\theta}}_n, \bar{\boldsymbol{s}}'_n)cov(\bar{\boldsymbol{s}}_n, \bar{\boldsymbol{s}}'_n)^{-1}(\boldsymbol{c}_{s,n}^d)^{-1}$. By equation (1.68) , $(\boldsymbol{M})_{nn} = \left((\boldsymbol{Id} - \boldsymbol{C}_p)^{-1}\right)_{nn} = \left(cov(\bar{\boldsymbol{\theta}}, \bar{\boldsymbol{s}}')cov(\bar{\boldsymbol{s}}, \bar{\boldsymbol{s}}')^{-1}\right)_{nn}(\boldsymbol{c}_{s,n}^{dc})^{-1}$. As $\boldsymbol{c}_{s,n}^d = \boldsymbol{c}_{s,n}^{dc} = (\boldsymbol{Id}_K + \frac{\boldsymbol{\Sigma}_n}{1-\bar{\rho}_n})^{-1}$, $cov(\bar{\boldsymbol{\theta}}_n, \bar{\boldsymbol{s}}'_n)cov(\bar{\boldsymbol{s}}_n, \bar{\boldsymbol{s}}'_n)^{-1} = \left(\boldsymbol{Id}_K + \frac{\boldsymbol{\Sigma}_n}{1+(I_n-1)\bar{\rho}_n}\right)^{-1} = \left(\boldsymbol{Id}_K + \frac{(\bar{\mathcal{C}})_{nn}^{-1}\boldsymbol{\Sigma}_n}{I_n}\right)^{-1}$, and $\left(cov(\bar{\boldsymbol{\theta}}, \bar{\boldsymbol{s}}')cov(\bar{\boldsymbol{s}}, \bar{\boldsymbol{s}}')^{-1}\right)_{nn} = \left(\boldsymbol{Id}_K + \frac{(\bar{\mathcal{C}}^{-1})_{nn}}{I_n}\boldsymbol{\Sigma}_n\right)^{-1}$, we can further simplify the equation as

$$\left((\bar{\mathcal{C}}^{-1})_{nn} - (\bar{\mathcal{C}})_{nn}^{-1}\right)(\boldsymbol{\Sigma}_n^{-1} + \frac{1}{1-\bar{\rho}_n}\boldsymbol{Id}_K)^{-1} \succeq \boldsymbol{0},$$

which always holds, as $(\bar{\mathcal{C}}^{-1})_{nn} - (\bar{\mathcal{C}})_{nn}^{-1} \geq 0$ (whose equality is taken if and only if $\bar{\rho}_{nm} = \boldsymbol{0}$ for any $m \neq n$) and $(\boldsymbol{\Sigma}_n^{-1} + \frac{1}{1-\bar{\rho}_n}\boldsymbol{Id}_K)^{-1} \succ \boldsymbol{0}$. Therefore, $\boldsymbol{\lambda}_n^{dc} \preceq \boldsymbol{\lambda}_n^d$. The equality is taken if and only if $\bar{\rho}_{nm} = \boldsymbol{0}$ for any $m \neq n$.

Welfare Comparison: DM v.s. DC

We denote the equilibrium variables in DM with superscript d, and the equilibrium variables in DC with superscript dc.

In Design 1. DM, trader $i \in I(n)$ has ex-ante expected utility

$$\mathbb{E}[u^{i,d}] = \mathbb{E}\left[\underbrace{\left(\mathbb{E}[\boldsymbol{\theta}^i|\boldsymbol{s}^i,\boldsymbol{p}_n^d] - \boldsymbol{p}_n^d\right)'}_{\text{learning effect}}\underbrace{\left(\frac{1}{2}\alpha_n\boldsymbol{\Sigma}_\theta + \boldsymbol{\lambda}_n^d\right)\left(\alpha_n\boldsymbol{\Sigma}_\theta + \boldsymbol{\lambda}_n^d\right)^{-2}}_{\text{liquidity effect}}\underbrace{\left(\mathbb{E}[\boldsymbol{\theta}^i|\boldsymbol{s}^i,\boldsymbol{p}_n^d] - \boldsymbol{p}_n^d\right)}_{\text{learning effect}}\right].$$

In Design 2. DC, trader $i \in I(n)$ has ex-ante expected utility

$$\mathbb{E}[u^{i,dc}] = \mathbb{E}\left[\underbrace{\left(\mathbb{E}[\boldsymbol{\theta}^i|\boldsymbol{s}^i,\boldsymbol{P}^{dc}] - \boldsymbol{p}_n^{dc}\right)'}_{\text{learning effect}}\underbrace{\left(\frac{1}{2}\alpha_n\boldsymbol{\Sigma}_\theta + \boldsymbol{\lambda}_n^{dc}\right)\left(\alpha_n\boldsymbol{\Sigma}_\theta + \boldsymbol{\lambda}_n^{dc}\right)^{-2}}_{\text{liqudity effect}}\underbrace{\left(\mathbb{E}[\boldsymbol{\theta}^i|\boldsymbol{s}^i,\boldsymbol{P}^{dc}] - \boldsymbol{p}_n^{dc}\right)}_{\text{learning effect}}\right].$$

As $\boldsymbol{c}_{s,n}^{dc} = \boldsymbol{c}_{s,n}^{d} = (\boldsymbol{Id}_K + \frac{1}{1-\bar{\rho}_n}\boldsymbol{\Sigma}_n)^{-1}$, the learning effects in the two designs equalize,

$$\mathbb{E}[\boldsymbol{\theta}^i|\boldsymbol{s}^i,\boldsymbol{P}^{dc}] - \boldsymbol{p}_n^{dc} = \boldsymbol{c}_{s,n}^{dc}(\boldsymbol{s}^i - \bar{\boldsymbol{s}}) = \boldsymbol{c}_{s,n}^{d}(\boldsymbol{s}^i - \bar{\boldsymbol{s}}) = \mathbb{E}[\boldsymbol{\theta}^i|\boldsymbol{s}^i,\boldsymbol{p}_n^d] - \boldsymbol{p}_n^d.$$

Therefore $\mathbb{E}[u^{i,dc}] \geq \mathbb{E}[u^{i,d}]$ if and only if,

$$\left(\frac{1}{2}\alpha_n\boldsymbol{\Sigma}_\theta + \boldsymbol{\lambda}_n^{dc}\right)\left(\alpha_n\boldsymbol{\Sigma}_\theta + \boldsymbol{\lambda}_n^{dc}\right)^{-2} \succeq \left(\frac{1}{2}\alpha_n\boldsymbol{\Sigma}_\theta + \boldsymbol{\lambda}_n^{d}\right)\left(\alpha_n\boldsymbol{\Sigma}_\theta + \boldsymbol{\lambda}_n^{d}\right)^{-2} \tag{1.69}$$

in Loewner order.[28]

For traders in $I(n)$, the price impact in Design 1 is $\boldsymbol{\lambda}_n^d = \left((I_n-2)\boldsymbol{Id}_K - (I_n-1)\boldsymbol{c}_p^d\right)^{-1}\alpha_n\boldsymbol{\Sigma}_\theta$, and the price impact in Design 2 is $\boldsymbol{\lambda}_n^{dc} = \left((I_n-1)((\boldsymbol{M})_{nn})^{-1} - \boldsymbol{Id}_K\right)^{-1}\alpha_n\boldsymbol{\Sigma}_\theta$. Therefore, $(\alpha_n\boldsymbol{\Sigma}_\theta + \boldsymbol{\lambda}_n^d)^{-1} = \alpha_n^{-1}\boldsymbol{\Sigma}_\theta^{-1}(\boldsymbol{Id} - \frac{1}{I_n-1}(\boldsymbol{Id} - \boldsymbol{c}_p^d)^{-1})$, $(\alpha_n\boldsymbol{\Sigma}_\theta + \boldsymbol{\lambda}_n^{dc})^{-1} = \alpha_n^{-1}\boldsymbol{\Sigma}_\theta^{-1}(\boldsymbol{Id} - \frac{1}{I_n-1}(\boldsymbol{M})_{nn})$. We can simplify equation (1.69) as

[28]Let A and B be two Hermitian matrices of order n. $A \succeq B$ if $A - B$ is positive semidefinite.

$$(\boldsymbol{Id}-\boldsymbol{c}_p^d)^{-2} \succeq ((\boldsymbol{M})_{nn})^2.$$

By equations (1.60) and (1.61), $(\boldsymbol{Id}-\boldsymbol{c}_p^d)^{-1} = cov(\bar{\boldsymbol{\theta}}_n, \bar{\boldsymbol{s}}_n')cov(\bar{\boldsymbol{s}}_n, \bar{\boldsymbol{s}}_n')^{-1}(\boldsymbol{c}_{s,n}^d)^{-1}$. By equation (1.68) , $(\boldsymbol{M})_{nn} = \left((\boldsymbol{Id}-\boldsymbol{C}_p)^{-1}\right)_{nn} = \left(cov(\bar{\boldsymbol{\theta}}, \bar{\boldsymbol{s}}')cov(\bar{\boldsymbol{s}}, \bar{\boldsymbol{s}}')^{-1}\right)_{nn}(\boldsymbol{c}_{s,n}^{dc})^{-1}$. As $\boldsymbol{c}_{s,n}^d = \boldsymbol{c}_{s,n}^{dc}$, $cov(\bar{\boldsymbol{\theta}}_n, \bar{\boldsymbol{s}}_n')cov(\bar{\boldsymbol{s}}_n, \bar{\boldsymbol{s}}_n')^{-1} = \left(\boldsymbol{Id}_K + \frac{1}{1+(I_n-1)\bar{\rho}_n}\boldsymbol{\Sigma}_n\right)^{-1} = \left(\boldsymbol{Id}_K + \frac{(\bar{\mathcal{C}})_{nn}^{-1}}{I_n}\boldsymbol{\Sigma}_n\right)^{-1}$, and $\left(cov(\bar{\boldsymbol{\theta}}, \bar{\boldsymbol{s}}')cov(\bar{\boldsymbol{s}}, \bar{\boldsymbol{s}}')^{-1}\right)_{nn} = \left(\boldsymbol{Id}_K + \frac{(\bar{\mathcal{C}}^{-1})_{nn}}{I_n}\boldsymbol{\Sigma}_n\right)^{-1}$, we can further simplify the equation as

$$\left((\bar{\mathcal{C}}^{-1})_{nn} - (\bar{\mathcal{C}})_{nn}^{-1}\right)^{-1}\left(((\bar{\mathcal{C}}^{-1})_{nn} + (\bar{\mathcal{C}})_{nn}^{-1})\boldsymbol{Id} + 2I_n\boldsymbol{\Sigma}_n^{-1}\right) \succeq 0 \tag{1.70}$$

As $((\bar{\mathcal{C}}^{-1})_{nn} + (\bar{\mathcal{C}})_{nn}^{-1})\boldsymbol{Id} + I_n\boldsymbol{\Sigma}_n^{-1} \succ \mathbf{0}$ and $(\bar{\mathcal{C}}^{-1})_{nn} - (\bar{\mathcal{C}})_{nn}^{-1} \geq 0$ (whose equality is taken if and only if $\bar{\rho}_{nm} = \mathbf{0}$ for any $m \neq n$), equation (1.70) always holds. Therefore, $\mathbb{E}[u^{i,dc}] \geq \mathbb{E}[u^{i,d}]$. The equality is taken if and only if $\bar{\rho}_{nm} = \mathbf{0}$ for any $m \neq n$.

Design 2': PC: With partial conditioning, traders $i \in I(\tilde{N}+1), \ldots, I(N)$ can see prices in their own exchanges only, and intuitively they have the same outcome as the traders in Design 1. The trader $i \in I(1), \ldots, I(\tilde{N})$ can see prices from all exchanges. They submit demand schedule to maximize utility

$$\mathbb{E}\left[(\boldsymbol{\theta}^i)'\boldsymbol{q}^i - \frac{1}{2}\alpha_n(\boldsymbol{q}^i)'\boldsymbol{\Sigma}_\theta\boldsymbol{q}^i - \boldsymbol{p}_n'\boldsymbol{q}^i | \boldsymbol{s}^i, \{\boldsymbol{p}_n\}_n\right].$$

where $\boldsymbol{p}_n = \mathbb{E}[\boldsymbol{\theta}] + cov(\bar{\boldsymbol{\theta}}_n, \bar{\boldsymbol{s}}_n')cov(\bar{\boldsymbol{s}}_n, \bar{\boldsymbol{s}}_n')^{-1}\left(\bar{\boldsymbol{s}}_n - \mathbb{E}[\boldsymbol{\theta}]\right)$ for $n = \tilde{N}+1, \ldots, N$.

The first order condition for trader $i \in I(1), \ldots, I(\tilde{N})$ is

$$\boldsymbol{q}^i = \left(\alpha_n\boldsymbol{\Sigma}_\theta + \boldsymbol{\lambda}_n^i\right)^{-1}\left(\mathbb{E}[\boldsymbol{\theta}^i|\boldsymbol{s}^i, \{\boldsymbol{p}_n\}_n] - \boldsymbol{p}_n\right).$$

By symmetry, let's parameterize the expected utility in a linear form, $\mathbb{E}[\boldsymbol{\theta}^i|\boldsymbol{s}^i, \boldsymbol{p}_n] \equiv \boldsymbol{c}_{0,n}\mathbb{E}[\boldsymbol{\theta}] + \boldsymbol{c}_{s,n}\boldsymbol{s}^i + \sum_{m=1}^N \boldsymbol{c}_{p,n,m}\boldsymbol{p}_m$, where $\boldsymbol{c}_{0,n}, \boldsymbol{c}_{s,n}\boldsymbol{c}_{p,n,m} \in \mathbb{R}^{K\times K}$.

By market-clearing condition, the equilibrium price satisfies

$$\boldsymbol{P} = (\boldsymbol{Id}_{NK} - \boldsymbol{C}_p)^{-1}(\boldsymbol{C}_0\mathbb{E}[\boldsymbol{\Theta}] + \boldsymbol{C}_s\bar{\boldsymbol{s}}),$$

where $\boldsymbol{P} \in \mathbb{R}^{NK}$ is the price vector for all assets, where the $(n-1)K+1^{th}$ to nK^{th} elements is the price vector $\boldsymbol{p}_n$; $\bar{\boldsymbol{s}} \in \mathbb{R}^{NK}$ is the average signal vector, where the $(n-1)K+1^{th}$ to nK^{th} elements is the average signal in exchange n, $\frac{1}{I_n}\sum_{i\in I(n)} \boldsymbol{s}^i$; $\mathbb{E}[\boldsymbol{\Theta}] \in \mathbb{R}^{NK}$ is the expected value vector. $\boldsymbol{C}_p \in \mathbb{R}^{NK\times NK}$, where the submatrix corresponding to $(n-1)K+1^{th}$ to nK^{th} row and the $(m-1)K+1^{th}$ to mK^{th} column is $\boldsymbol{c}_{p,n,m}$. $\boldsymbol{C}_0 \in \mathbb{R}^{NK\times NK}$, where the diagonal $(n-1)K+1^{th}$ to nK^{th} submatrix is $\boldsymbol{c}_{0,n}$; $\boldsymbol{C}_s \in \mathbb{R}^{NK\times NK}$, whose diagonal $(n-1)K+1^{th}$ to nK^{th} submatrix is $\boldsymbol{c}_{s,n}$. The $(\tilde{N}+1)K^{th}$ to NK^{th} row of $\boldsymbol{C}_p$ is $[\boldsymbol{0}, diag(\boldsymbol{c}^d_{p,n})_n]$, the $(\tilde{N}+1)K^{th}$ to NK^{th} row of $\boldsymbol{C}_0$ is $[\boldsymbol{0}, diag(\boldsymbol{c}^d_{0,n})_n]$, the $(\tilde{N}+1)K^{th}$ to NK^{th} row of $\boldsymbol{C}_s$ is $[\boldsymbol{0}, diag(\boldsymbol{c}^d_{s,n})_n]$.

We can determine the inference coefficients as a function of the primitives (and in closed form). For any trader $i \in I(n)$, $n = 1,\dots,\tilde{N}$, random vector $\left((\boldsymbol{\theta}^i)', (\boldsymbol{s}^i)', \boldsymbol{P}'\right)$ is jointly normally distributed

$$\begin{pmatrix} \boldsymbol{\theta}^i \\ \boldsymbol{s}^i \\ \boldsymbol{P} \end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix} \mathbb{E}[\boldsymbol{\theta}] \\ \mathbb{E}[\boldsymbol{\theta}] \\ \mathbb{E}[\boldsymbol{\Theta}] \end{pmatrix}, \begin{pmatrix} \boldsymbol{\Sigma}_\theta & \boldsymbol{\Sigma}_\theta & cov\left(\boldsymbol{\theta}^i, \boldsymbol{P}'\right) \\ \boldsymbol{\Sigma}_\theta & \boldsymbol{\Sigma}_\theta + \boldsymbol{\Sigma}_{\varepsilon,n} & cov\left(\boldsymbol{s}^i, \boldsymbol{P}'\right) \\ cov\left(\boldsymbol{P}, (\boldsymbol{\theta}^i)'\right) & cov\left(\boldsymbol{P}, (\boldsymbol{s}^i)'\right) & cov\left(\boldsymbol{P}, \boldsymbol{P}'\right) \end{pmatrix}\right],$$

where

$$\begin{aligned} cov\left(\boldsymbol{P}, (\boldsymbol{\theta}^i)'\right) &= (\boldsymbol{Id}_{NK} - \boldsymbol{C}_p)^{-1}\boldsymbol{C}_s cov(\bar{\boldsymbol{s}}, (\boldsymbol{\theta}^i)'), \\ cov\left(\boldsymbol{P}, (\boldsymbol{s}^i)'\right) &= (\boldsymbol{Id}_{NK} - \boldsymbol{C}_p)^{-1}\boldsymbol{C}_s cov(\bar{\boldsymbol{s}}, (\boldsymbol{s}^i)'), \\ cov\left(\boldsymbol{P}, \boldsymbol{P}'\right) &= (\boldsymbol{Id}_{NK} - \boldsymbol{C}_p)^{-1}\boldsymbol{C}_s cov(\bar{\boldsymbol{s}}, \bar{\boldsymbol{s}}')\boldsymbol{C}_s'\left((\boldsymbol{Id} - \boldsymbol{C}_p)^{-1}\right)' \end{aligned} \tag{1.71}$$

By Projection Theorem, $\forall i \in I(n)$, $n = 1,\dots,\tilde{N}$,

$$\boldsymbol{c}_{s,n}\left(\boldsymbol{\Sigma}_\theta + \boldsymbol{\Sigma}_{\varepsilon,n}\right) + \sum_m \boldsymbol{c}_{p,n,m} cov(\boldsymbol{p}_m, (\boldsymbol{s}^i)') = \boldsymbol{\Sigma}_\theta \tag{1.72}$$

$$\boldsymbol{c}_{s,n} cov(\boldsymbol{s}^i, \boldsymbol{P}') + \sum_m \boldsymbol{c}_{p,n,m} cov(\boldsymbol{p}_m, \boldsymbol{P}') = cov\left(\boldsymbol{\theta}^i, \boldsymbol{P}'\right) \tag{1.73}$$

and

$$\boldsymbol{c}_{0,n} = \boldsymbol{Id}_{NK} - \boldsymbol{c}_{s,n} - \sum_m \boldsymbol{c}_{p,n,m}.$$

If we plug the covariances in equation (1.71) into equation (1.73), we can simplify the equation as

$$\boldsymbol{c}_{s,n} cov(\boldsymbol{s}^i, \bar{\boldsymbol{s}}') + \sum_m \boldsymbol{c}_{p,n,m} cov(\boldsymbol{p}_m, \bar{\boldsymbol{s}}') = cov\left(\boldsymbol{\theta}^i, \bar{\boldsymbol{s}}'\right) \tag{1.74}$$

Averaging equation (1.72) over all $i \in I(n), \forall n = 1, \ldots, \tilde{N}$, we have

$$\boldsymbol{c}_{s,n}\left(\boldsymbol{\Sigma}_\theta + \boldsymbol{\Sigma}_{\varepsilon,n}\right) + \sum_m \boldsymbol{c}_{p,n,m} cov(\boldsymbol{p}_m, \bar{\boldsymbol{s}}'_n) = \boldsymbol{\Sigma}_\theta \tag{1.75}$$

Subtracting equation (1.75) from the $(n-1)K+1^{th}$ to nK^{th} rows of equation (1.74),$\forall n = 1, \ldots, \tilde{N}$ we can solve that

$$\begin{aligned} \boldsymbol{c}_{s,n} &= \left(\boldsymbol{\Sigma}_\theta - cov(\boldsymbol{\theta}^i, \bar{\boldsymbol{s}}_n)\right)\left(\boldsymbol{\Sigma}_\theta + \boldsymbol{\Sigma}_{\varepsilon,n} - cov(\boldsymbol{s}^i, \bar{\boldsymbol{s}}_n)\right)^{-1} \\ &= (1 - \bar{\rho}_n)\left((1 - \bar{\rho}_n)\boldsymbol{Id}_K + \boldsymbol{\Sigma}_n\right)^{-1}, \forall n = 1, \ldots, \tilde{N}. \end{aligned} \tag{1.76}$$

We can see from equation (1.76) that in the partial conditioning case, $\boldsymbol{c}_{s,n} = \boldsymbol{c}^{dc}_{s,n}$ for $n = 1, \ldots, \tilde{N}$.

Averaging equation (1.73) over all $i \in I(n), n = 1, \ldots, \tilde{N}$ and write that into matrix form,

$$\tilde{\boldsymbol{C}}_{s,n} cov(\bar{\boldsymbol{s}}, \bar{\boldsymbol{s}}') + \tilde{\boldsymbol{C}}_p(\boldsymbol{Id}_{NK} - \boldsymbol{C}_p)^{-1}\boldsymbol{C}_s cov(\bar{\boldsymbol{s}}, \bar{\boldsymbol{s}}') = cov(\tilde{\boldsymbol{\theta}}, \bar{\boldsymbol{s}}') \tag{1.77}$$

where $\tilde{\boldsymbol{C}}_s \in \mathbb{R} \in \mathbb{R}^{\tilde{N}K \times NK}$ is the first $\tilde{N}K$ rows of $\boldsymbol{C}_s \in \mathbb{R}^{\tilde{N}K \times NK}$, $\tilde{\boldsymbol{C}}_p$ is the first $\tilde{N}K$ rows of $\boldsymbol{C}_p$, and $\tilde{\bar{\boldsymbol{\theta}}} \in \mathbb{R}^{\tilde{N}K}$ is the average value $\bar{\boldsymbol{\theta}}_n$ of the first $\tilde{N}$ exchanges. By rearranging the terms in equation (1.77), we have

$$\tilde{\boldsymbol{Id}}_{NK}\left(\boldsymbol{Id}_{NK} - \boldsymbol{C}_p\right)^{-1} = cov(\tilde{\boldsymbol{\theta}}, \bar{\boldsymbol{s}}')cov(\bar{\boldsymbol{s}}, \bar{\boldsymbol{s}}')^{-1}\boldsymbol{C}_s^{-1}, \tag{1.78}$$

where $\tilde{\boldsymbol{Id}}_{NK} \in \mathbb{R}^{\tilde{N}K \times NK}$ is the first $\tilde{N}K^{th}$ row of $\boldsymbol{Id}_{NK}$. We can observe that the first $\tilde{N}K$ rows of $(\boldsymbol{Id} - \boldsymbol{C}_p)^{-1}$ in the partial conditioning case coincide with that in Design 2.

The equilibrium price impact satisfies,

$$\boldsymbol{\Lambda}^i = \begin{bmatrix} \frac{\partial \boldsymbol{p}_1}{\partial \boldsymbol{q}_1^i} & \cdots & \frac{\partial \boldsymbol{p}_N}{\partial \boldsymbol{q}_1^i} \\ \vdots & \ddots & \vdots \\ \frac{\partial \boldsymbol{p}_1}{\partial \boldsymbol{q}_N^i} & \cdots & \frac{\partial \boldsymbol{p}_N}{\partial \boldsymbol{q}_N^i} \end{bmatrix} = - \begin{bmatrix} \sum_{j \neq i} \frac{\partial \boldsymbol{q}_1^j}{\partial \boldsymbol{p}_1} & \cdots & \sum_{j \neq i} \frac{\partial \boldsymbol{q}_N^j}{\partial \boldsymbol{p}_1} \\ \vdots & \ddots & \vdots \\ \sum_{j \neq i} \frac{\partial \boldsymbol{q}_1^j}{\partial \boldsymbol{p}_N} & \cdots & \sum_{j \neq i} \frac{\partial \boldsymbol{q}_N^j}{\partial \boldsymbol{p}_N} \end{bmatrix}^{-1}.$$

Given the solved inference coefficients, we can solve for the $\boldsymbol{\lambda}_n^i$, for $n = 1, \ldots, \tilde{N}$,

$$\boldsymbol{\lambda}_n = \left((I_n - 1)((\boldsymbol{M})_{nn})^{-1} - \boldsymbol{Id}_K\right)^{-1} \alpha_n \boldsymbol{\Sigma}_\theta$$

where $\boldsymbol{M} = (\boldsymbol{Id}_{NK} - \boldsymbol{C}_p)^{-1}$,and $(\boldsymbol{M})_{nn}$ is its diagonal submatrix corresponding to $(n-1)K + 1^{th}$ to nK^{th} rows and columns. From equation (1.78), we can see that $(\boldsymbol{M})_{nn}$ in Design 3 coincide with that in Design 2 for $n = 1, \ldots, \tilde{N}$. Therefore, $\boldsymbol{\lambda}_n^{pc} = \boldsymbol{\lambda}_n^{dc}$ for $n = 1, \ldots, \tilde{N}$.

Given that inference coefficients $\boldsymbol{c}_{s,n} = \boldsymbol{c}_{s,n}^{dc}$ and price impact $\boldsymbol{\lambda}_n^{pc} = \boldsymbol{\lambda}_n^{dc}$ for $n = 1, \ldots, \tilde{N}$, we can easily solve for the equilibrium bids for trader $i \in I(1), \ldots, I(\tilde{N})$ that

$$\boldsymbol{q}^i = (\alpha \boldsymbol{\Sigma}_\theta + \boldsymbol{\lambda}_n)^{-1} \boldsymbol{c}_{s,n}(\boldsymbol{s}^i - \bar{\boldsymbol{s}}_n) = \boldsymbol{q}^{i,dc}$$

And by equation (1.78), the equilibrium prices for $n = 1, \ldots, \tilde{N}$,

$$\boldsymbol{p}_n = \mathbb{E}[\boldsymbol{\theta}] + [\boldsymbol{Id}_{NK} - \boldsymbol{C}_p]_n \boldsymbol{C}_s(\bar{\boldsymbol{s}} - \mathbb{E}[\boldsymbol{\theta}]) = \boldsymbol{p}_n^{dc}$$

where $[\boldsymbol{Id}_{NK} - \boldsymbol{C}_p]_n$ is the $(n-1)K + 1^{th}$ to nK^{th} rows of $[\boldsymbol{Id}_{NK} - \boldsymbol{C}_p]_n$.

Given that the equilibrium bids and prices in the Design 2′, the ex-ante utility of each trader i is the same as Design 2, if $i \in I(1), \ldots, I(\tilde{N})$ and is the same as Design 1, if $i \in I(\tilde{N}), \ldots, I(N)$. We can easily extend the welfare comparison results of Design 1 and 2 to Design 2′. ■

Chapter 2

Dynamic Market Choice

2.1 Introduction

In practice, many assets are traded in both transparent centralized and opaque decentralized markets. For instance, equities are mostly traded on transparent centralized exchanges, but they can also be traded in more opaque and decentralized markets, such as dark pools and over-the-counter (OTC) markets.[1] Bonds are available for trading on exchanges, OTC, or both.[2] Traders' market choices change over time. For example, during periods of high volatility, the volume of equity transactions in dark pools compared to that in exchanges tends to be lower.[3] Traders' active participation in the opaque decen-

[1]A dark pool is a type of alternative trading system (ATS) that allows institutional investors to trade securities without publicly revealing their intentions during the search for a buyer or seller. They emerged in the 1980s when the Securities and Exchange Commission (SEC) allowed brokers to transact large blocks of shares. See `https://www.investopedia.com/terms/d/dark-pool.asp`. In the U.S., the OTC equity markets include OTC QX, OTC QB, and OTC Pink Marketplace with different financial standards and regulations (Ang et al., 2013; Brüggemann et al., 2018).

[2]For example, in China, bonds are available for trading on exchanges, over-the-counter (OTC), or both. Mutual funds, insurance companies, and security firms can trade corporate bonds dual-listed on both exchange and OTC markets. See Section 2.6 for more institutional details about the Chinese corporate bond markets. In the U.S. and most European countries, bonds are predominantly traded in OTC markets, with a recent rise of more transparent electronic trading (Nagel, 2016; Bessembinder et al., 2020; O'Hara and Zhou, 2021b). Before World War II, corporate bonds and municipal bonds were actively traded in centralized exchanges (Biais and Green, 2019).

[3]See Investors Flee Dark Pools As Market Volatility Erupts, The Wall Street Journal, Sept. 2, 2011, and "Dark Pools" Draw More Trading Amid Low Volatility, The Wall Street Journal, May 3, 2019. Traders' trading places are also different for assets with different payoff sensitivities. Securities whose payoffs are designed to be less sensitive to issuers' fundamentals, such as bonds, are primarily traded in over-the-counter markets. Those more sensitive to fundamentals, like equities, are traded both in centralized markets and dark pools. The most sensitive securities,for instance, options are predominantly traded in centralized markets.

tralized markets has attracted policy debates on whether to introduce more transparency or shut down the decentralized markets. However, a fundamental question remains unclear. What determines traders' market choices? Understanding their choices can help policymakers design better markets.

To address this question, we develop a model with endogenous dynamic market choices. The main insight from the analysis is that asset payoff sensitivity and volatility influence price history informativeness, which in turn affects dynamic market choices. To grasp the intuition behind traders' dynamic market choices, let us start with the classic static trade-off discussed in the literature (e.g. Rostek and Weretka, 2012; Yoon, 2017; Rostek and Wu, 2021; Babus and Parlatore, 2022). The centralized market is larger and can be more liquid than the decentralized market. However, traders can trade exclusively with the best counterparty with the most opposite trading needs in the decentralized market, while they face competitors in the centralized market.[4] In the dynamic model, this trade-off between liquidity and competition will change as traders not only learn from the current price as in the static models, but also learn from the price history. As traders learn from the price history, the adverse selection in the market is lower. Consequently, market liquidity improves with access to the price history. This improvement is more pronounced in the decentralized market due to its smaller size relative to the centralized market.[5] What is unique to the dynamic market is that the price history and market choices evolve endogenously over time. More informative past prices lead traders to favor the decentralized market over the centralized one. Asset payoff sensitivity and volatility determine the growth and decay rate of the informativeness of past prices, thus shaping dynamic market choices.

The dynamic model features short-lived traders arriving each period to trade a risky

[4]Here we focus on the trade-offs when traders are indifferent between the centralized and decentralized markets. When traders in the centralized market have highly correlated values, even the liquidity can be lower in the centralized market than in the decentralized markets due to adverse selection. Decentralized markets will be the obvious optimal market choice. (e.g. Rostek and Weretka, 2012; Yoon, 2017; Rostek and Wu, 2021).

[5]In fact, this intuition applies to any additional public signals. See Rostek and Wu (2024) for more discussions on the equilibrium existence conditions and properties for bilateral double auctions with public signals.

asset. The *asset properties* can be summarized with asset payoff sensitivity and volatility. The asset value changes across time with AR(1) shocks. *Asset payoff sensitivity* measures how much asset value changes with shocks. *Asset volatility* measures the innovations in the AR(1) shocks. In each round, traders choose between a centralized market with a double auction for all participants, or a decentralized market where they find the best counterparties for bilateral double auctions that give them the highest expected utility. Traders have private values with varying degrees of correlation, representing different hedging needs or disagreements in asset value. From the traders' perspective, the centralized and decentralized markets differ in market size, correlation with counterparties, and transparency. In the decentralized market, traders exclusively engage with their best counterparties - those with the lowest value correlation. In the centralized market, traders interact not only with their best counterparties but also with competitors who have more correlated values. The decentralized market is opaque; traders cannot see past prices from this market. However, traders can observe and learn about their values from past centralized market prices. After they choose the market, traders receive private signals about the asset, submit demand schedules and the market clears.[6]

Different dynamic market choices can naturally emerge as price history evolves endogenously over time. In this model, the impact of past prices on the current market choice can be summarized by a single sufficient statistic: *price history informativeness*. It measures how much traders can learn about their values from price history. Higher price history informativeness improves liquidity and increases traders' expected utility, with this improvement being more pronounced in the decentralized market.

Two asset properties determine the evolution of *price history informativeness* and therefore traders' market choices. The first is the asset's payoff sensitivity to shocks in fundamentals. When an asset is insensitive to shocks, its value remains relatively stable over time. Consequently, the price history becomes more informative and decays slowly. If

[6] The assumption that traders receive private signals after they choose the market follows Yoon (2017). It ensures that comparative statics are not affected by the realization of the signals. It is not crucial for the results.

the sensitivity is sufficiently low, the first round centralized market price is sufficiently informative for traders to remain in the decentralized market afterwards. For assets more sensitive to value changes across rounds, traders switch between the decentralized and centralized markets. Traders tend to prefer the decentralized market as price history accumulates. However, once they have chosen the decentralized market, its opaque nature causes price history informativeness to decay – the price history gradually becomes stale and uninformative as new shocks change the asset values. As price history informativeness decreases, the decentralized market becomes illiquid. It drives traders back to the centralized market when the price history is sufficiently informative. For the most sensitive assets, price history informativeness is sufficiently low for traders to always stay in the centralized market.

Asset volatility also affects market choices. If volatility is sufficiently low, traders remain in the decentralized market starting from the second round. An intermediate level of volatility makes price history informativeness decay faster, leading traders to switch between centralized and decentralized markets. High volatility diminishes the informativeness of past prices, compelling traders to remain in the centralized market.

Only when traders' value correlations are highly homogeneous will they choose to stay in the centralized market for all rounds. When value correlations are very heterogeneous, traders consistently opt for the decentralized market. This choice stems from the significant benefit each trader gains by matching bilaterally with the counterparty having the lowest correlation, and thus the most diverse values. In this case, the advantage of heterogeneous values outweighs the reduced liquidity of a decentralized market. Conversely, when value correlations are highly homogeneous, traders invariably choose the centralized market. This preference arises because similar correlations across traders diminish the advantage of exclusive trading with a specific counterparty in the decentralized market. The higher liquidity attracts traders to the centralized market.

We test model predictions on asset properties and market choices in the Chinese corporate bond market. In China, two bond markets co-exist: an over-the-counter (OTC) market and a centralized exchange market. Non-bank financial institutions, such as mutual funds or insurance companies, can choose to trade in either market. We focused on traders' market choices for corporate bonds dual-listed in both markets, collecting daily transaction data from January 1 to May 31, 2018. Consistent with the model predictions, we find that bonds with higher sensitivity to shocks to fundamentals are more likely to be traded in the centralized market; and that assets with higher volatility, as measured by greater price volatility in the last 30 days, are more likely to be traded in the centralized market.

We then directly test the key mechanism regarding price history informativeness and dynamic market choices using the same dataset. Our model predicts that as price history accumulates, traders tend to shift from the centralized market to the decentralized market. Empirically, we find that bond traders are more likely to switch from the centralized market to the over-the-counter market when the bond trades more frequently.

Besides evidence from the Chinese corporate bond market, we also find evidence in support of the model prediction on volatility and market choices in the U.S. equity market. In Appendix 2.11, we test the model predictions in the U.S. equity market.[7] Menkveld et al. (2017) and Buti et al. (2022) find evidence with U.S. equity data in support of our model prediction, i.e., the share of equities traded in dark pools decreases when the asset volatility is high. These empirical findings, consistent across various asset classes and market structures, strongly support the model's key implications about what drives traders to choose between centralized and decentralized trading venues.

The dynamic market choice we explore has new policy implications for market designs. In particular, we highlight a novel trade-off between the current round decentralized market efficiency and future traders' utilities. Given this trade-off, designs that improve efficiency in the decentralized market without post-trade transparency may hurt

[7] We collect data for equities traded in exchanges, alternative trading systems (ATS), and over-the-counter (OTC) markets during 2019-2022 from the Financial Industry Regulatory Authority (FINRA) and Wharton Research Data Service (WRDS). We classify the centralized exchanges such as Nasdaq and NYSE as centralized markets, ATS and OTC as decentralized markets. We use the standard deviation of prices in the last 100 trading days for each stock as a proxy for volatility. We find a negative correlation between price volatility and the proportion of transaction volume traded in the ATS and OTC.

the overall welfare. This trade-off is absent in static market designs. For instance, while a static model might suggest that introducing pre-trade transparency in the decentralized market improves efficiency, it may decrease welfare in a dynamic context where the decentralized market lacks post-trade transparency. This occurs because price history informativeness decreases for future traders, as the more efficient pre-trade transparent decentralized market attracts more trades while simultaneously making prices unobservable. In scenarios with sufficient trading rounds and stable asset values due to low sensitivity or volatility, the long-term effect of diminished price history informativeness outweighs the efficiency gains in the decentralized market, ultimately leading to decreased welfare.

The trade-off between current-round decentralized market efficiency and future traders' utilities also informs market structure design. The prevalent coexistence of centralized and decentralized markets has sparked policy debates about mandating all trades in the centralized market. In a static version of this model, we might conclude that such a mandate deprives traders of the ability to trade with the best counterparty when beneficial. However, in the dynamic model, while shutting down the opaque decentralized market may decrease traders' utility in the current round, it can improve overall welfare by increasing price history informativeness for future traders.

Literature: This paper is closely related to the literature studying endogenous market structure. One strand of literature focuses on the endogenous formation of core-periphery trading networks (Chang and Zhang, 2015; Glode and Opp, 2016; Wang, 2016; Farboodi et al., 2018; Babus and Parlatore, 2022; Hugonnier et al., 2022; Sambalaibat, 2022; Farboodi, 2023; Farboodi et al., 2023). This paper is more closely related to the literature on the endogenous formation of coexisting centralized and decentralized markets (Pagano, 1989; Rust and Hall, 2003; Yoon, 2017; Vogel, 2019; Seppi, 1990; Desgranges and Foucault, 2005; Bolton et al., 2016; Lee and Wang, 2018; Huang and Xu, 2021; Dugast et al., 2022). While the literature focuses on static endogenous market choices, this paper studies dynamic market choices. In terms of underlying frictions giving rise to decentralized markets, the papers closest to this paper are Yoon (2017) and Babus and Parlatore (2022), which focus on the trade-off between the larger market size of the centralized market and counterparty with a lower correlation (i.e. more disagreement in the word of Babus and Parlatore (2022)) in the decentralized markets. While Yoon (2017) and Babus and Parlatore (2022) focus on traders' role in determining market structures, e.g. value correlation across traders and private signal precision, this paper highlights the impact of asset properties on traders' market choices.[8] We show that high price history informativeness due to low asset sensitivity and volatility is a new mechanism. That explains why some traders choose to trade in decentralized markets. By incorporating the impact of price history on traders' market choices, this paper endogenizes the dynamic evolution of price history informativeness, traders' beliefs, price impacts and market choices.

Second, this paper is related to the literature studying the welfare implication of coexisting centralized and decentralized markets dynamically (e.g. Miao, 2006; Antill and Duffie, 2021; Blonien, 2023). Papers using a static approach to analyze the welfare of coexisting centralized and decentralized markets include Zhu (2014), Buti et al. (2017), Malamud and Rostek (2017), Liu et al. (2018), and also the papers mentioned above that endogenize this market structure. Existing papers assume exogenous timing for traders to participate in decentralized markets. This paper contributes to the literature by endogenizing the time for traders to choose between the decentralized and the centralized market.

Finally, this paper is related to papers on transparency designs in financial markets (Duffie et al., 2017; Asriyan et al., 2017; Ollar et al., 2021; Back et al., 2020; Kakhbod and Song, 2020, 2022; Glebkin et al., 2023; Cespa and Vives, 2023; Vairo and Dworczak, 2023; Rostek and Wu, 2024; Rostek et al., 2024). The existing literature usually considers transparency designs without allowing traders to choose the venues. In this paper, we show transparency designs with endogenous dynamic market choices. Among this strand of literature, two existing papers (Rostek and Wu, 2024; Rostek et al., 2024) are closest to this paper. Rostek and Wu (2024) provided the existence condition for bilateral double

[8] The asset sensitivity and volatility are properties related to the asset's value correlation across rounds, as opposed to the value correlation across traders.

auction which this paper builds on. Rostek et al. (2024) explores which assets should be traded over the counter by jointly analyzing market structure and transparency design. Instead, this paper examines the dynamic endogenous choice of venues with a new trade-off between the current round decentralized market efficiency and future traders' utilities.

2.2 Model

Market Structure Consider a market of one divisible risky asset and one risk-free asset as a numéraire. The market has T rounds, and $I \geq 4$ even number of short-lived traders arrive each round. In each round before they trade, and conditional on the history of prices that they observe, traders first choose the market structure $\mathcal{M} = \{CM, DM\}$ that gives them the higher expected utility.[9] The traders can choose the market that can either be one *centralized market* (CM) where all traders participate in the same exchange, or *decentralized markets* (DM) where traders are matched with a counterparty according to an algorithm a la Irving (1985). The matching is pairwise stable in the sense that no trader wants to leave the current counterparty and form a new pair. We assume that trades only choose the DM if they strictly prefer it to CM. This prevents our results from being driven by the indeterminacy of a tie-breaking rule in the case that the DM and CM lead to the same utility.

Information structure Each trader i's value of the risky asset is given $\theta_{i,t} \equiv d_t + e_{i,t}$. The common value part is given by $d_t = u + \xi f_t$, where f_t are shocks to the asset fundamentals, u are macro-level risks unrelated to the asset fundamentals such as interest rate risk, and ξ measures the asset's value sensitivity to the asset fundamentals relative to the macro-

[9] Please refer to the information structure for the details of the price history. Note that we do not allow traders to split orders, i.e. to submit demand schedules to DM and CM simultaneously. Order splitting is proved to be equivalent to CM in our set-up, given that the price and price impact will equalize between DM and CM and the two markets operate as if they are one centralized market (Malamud and Rostek, 2017; Rostek and Wu, 2021). In practice, order splitting is often imperfect because traders are unable to observe prices in both the DM and CM at the same time and cannot execute their orders based on both prices simultaneously due to various regulations and market frictions. We find empirical evidence that traders choose the market instead of splitting orders in the Chinese corporate bond market (see Section 2.6).

level risks. The higher ξ, the more sensitive the asset payoff is to shocks. Without loss of generality we normalize d_t to have a standard normal distribution, $d_t \sim \mathcal{N}(0,1)$, $u \sim \mathcal{N}(0, \frac{1}{1+\xi^2})$ and $f_t \sim \mathcal{N}(0, \frac{1}{1+\xi^2})$.[10] f_t is time-varying given the growth of the underlying asset, e.g. firm issuers. It follows an AR(1) process $f_t = \kappa f_{t-1} + y_t$, where $\kappa \in [0,1]$, $y_t \sim \mathcal{N}(0, (1-\kappa^2)\frac{1}{1+\xi^2})$ is the innovation independent of any other random variables. κ measures the autocorrelation of the shocks across rounds. $e_{i,t} \sim \mathcal{N}(0, \epsilon^2)$ captures the heterogeneity of traders' value. $e_{i,t}$ is independent of u and f_t. By assumption the mean of $\theta_{i,t}$ is normalized as $\mathbb{E}[\theta_{i,t}] = 0$. Denote the variance of $\theta_{i,t}$ as $\sigma_\theta^2 \equiv 1 + \epsilon^2$. We allow $e_{i,t}$ to be correlated across traders, such that $\{\theta_{i,t}\}_i$ has the joint correlation matrix at round t,

$$\mathcal{C}_t \equiv \begin{pmatrix} 1 & \rho_{1,2,t} & \cdots & \rho_{1,I,t} \\ \rho_{2,1,t} & 1 & \cdots & \rho_{2,I,t} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{I,1,t} & \rho_{I,2,t} & \cdots & 1 \end{pmatrix}.$$

To simplify the analysis, we assume that for any trader i, there is only one trader $j \neq i$ whose value correlates ρ_ℓ with trader i, and any other traders $k \neq j, i$ has value correlation $\rho_{i,k} > \rho_\ell$ with trader i. Later in Section 2.3, we will see that this assumption ensures unique pairwise matching a la Irving (1985).

Following Rostek and Weretka (2012), the market is equicommonal by assumption, i.e., the average correlation between any trader i and the residual market is the same, $\frac{1}{I-1}\sum_{j\neq i} \rho_{i,j,t} = \bar{\rho}_t$.

Traders are uncertain about the asset value $\theta_{i,t}$ and do not observe u, $\{f_t\}_t$ and $\{e_{i,t}\}_{i,t}$. After they choose the market and before trading, each trader observes a private noisy signal about his true value $s_{i,t} = \theta_{i,t} + \varepsilon_{i,t}$, where $\varepsilon_{i,t} \sim \mathcal{N}(0, \sigma^2\sigma_\theta^2)$ and σ^2 measures the relative importance of noise in the signal. We assume σ is sufficiently large, $\sigma \geq$

[10] This normalization ensures that the comparative statics with ξ does not change anything else other than the relative sensitivity to risks. In particular, it does not change the traders' value variances. The normalization ensures the comparative analysis is not affected by the magnitude of risk but the sensitivity, but it is not necessary to generate all the results in the paper. With the normalization, u can also be seen as a numéraire for payoff sensitivity to shocks.

$((\frac{2(I-1)}{I})^{1/3}-1)^{-1/2}$.[11]

Traders can observe the current market price and submit demand contingent on that. Besides the private signals and the current market price, traders also observe past prices in the CM. Traders cannot observe prices in the DM other than the price in their current pair. We define the observed price history at round t as $\mathcal{H}_t \equiv \{p_s^{CM}\}_{s<t}$. Given the symmetric market assumption, if the optimal market choice of all traders at round t is the CM, then the price history updates as $\mathcal{H}_{t+1} = \mathcal{H}_t \cup \{p_t\}$, otherwise, $\mathcal{H}_{t+1} = \mathcal{H}_t$.

Preferences: The market is a double auction in a linear-normal setting. After the traders choose the market, they submit a demand schedule $q_{i,t}$, to maximize the payoffs conditioning on the history of past round price information $\mathcal{H}_t$, the current round signal $s_{i,t}$, and the current round price p_t,

$$\mathbb{E}[U_{i,t}|\mathcal{H}_t, s_{i,t}, p_t] = \mathbb{E}[\theta_{i,t}q_{i,t} - \frac{\alpha}{2}(q_{i,t})^2 - p_t q_{i,t}|\mathcal{H}_t, s_{i,t}, p_t].$$

The linear-quadratic utility function form follows the literature (Kyle, 1989; Vives, 2011; Rostek and Weretka, 2012; Yoon, 2017), where α is traders' risk aversion.

The centralized market clears with p_t when $\sum_i q_{i,t}(p_t) = 0$. In the decentralized market, after the traders are matched bilaterally in $N = \frac{I}{2}$ pairs, each pair $n \in N$ clears independently with $p_{t,n}$ when $\sum_{i \in I(n)} q_{i,t}(p_{t,n}) = 0$. This set-up of a decentralized market has a close mapping to markets in reality. In terms of the bond market, we use this decentralized trading mechanism to model the bilateral trade in the over-the-counter (OTC) market. In terms of the stock market, we use this decentralized market trading mechanism to model the dark pools operating as continuous non-displayed limit order books. This is the type of dark pool with the largest market share (around 70 percent) of total U.S. dark pool volumes in 2011 according to Tabb Group (2011). It includes many dark pools owned by major broker-dealers.[12]

[11] This assumption is a sufficient but not necessary condition to generate all the results in the paper. This is to avoid the nonmonotonicity of utility to σ, and to simplify the proof of Lemma 5. See Vives (2011) for a discussion of the nonmonotonic impact of σ.

[12] There are other two types of dark pools, one derives price from the lit venues, and the other acts like fast electronic market maker (Tabb Group, 2011; Zhu, 2014).

Timing: We summarize the timing of each round with Figure 2.1.

Figure 2.1: Timing

2.3 Equilibrium

As the traders are short-lived, trading is static with a time-varying information set. Therefore, we are subject to solving the model round by round forwardly given price history $\mathcal{H}_t$. In each round, the problem is solved with backward induction. First, we solve the trading strategy given the market structure. Then, we solve each trader's optimal market structure choice, by comparing each trader's expected utility in CM and DM. We apply the tie-breaking rule of choosing CM when CM and DM give the trader the same utility. We focus on the symmetric Nash equilibrium where each trader in the same round will have the same market choice and ex-ante expected utility. Given the optimal market choice, we can determine the evolution of the price history.

2.3.1 Second Stage Trading Equilibrium

Denote the chosen market structure as $\mathcal{M}^*$. By symmetry, choosing the market structure is equivalent to choosing the number of traders in the market $I_{t,\mathcal{M}^*}$ and the average correlation across traders $\rho_{t,\mathcal{M}^*}$. It is easy to see that $\mathcal{M}^* = CM$, the number of traders in the exchange is $I_{t,\mathcal{M}^*} = I_t$ with an average correlation between any trader and the residual market $\rho_{t,M^*} = \bar{\rho}$. If $\mathcal{M}^* = DM$, the number of traders in each pair is $I_{t,\mathcal{M}^*} = 2$ and every pair clears independently. Without solving the ex-ante expected utility, we will not be able to know each trader's choice of counterparty. For now, let us assume that the correlation within each pair (i, j) is $\rho_{t,\mathcal{M}^*} = \rho_{i,j}$ and solve the bilateral equilibrium. With the equilibrium strategy solved in the second stage, we can write the ex-ante utility as a function of $\rho_{i,j}$ in the first stage, and the trader $j \neq i$ that gives the trader i the highest ex-ante utility will be the trader i's counterparty.

Given the market structure $\mathcal{M}_t^*$, at round t, traders submit a demand schedule $q_{i,t}$ to maximize the utility

$$\max_{q_{i,t}} \mathbb{E}[\theta_{i,t} q_{i,t} - \frac{\alpha}{2}(q_{i,t})^2 - p_t q_{i,t} | \mathcal{H}_t, s_{i,t}, p_t]$$

By taking first order condition with respect to $q_{i,t}$, we can solve the trader i's demand schedule,

$$q_{i,t} = \frac{\mathbb{E}[\theta_{i,t}|\mathcal{H}_t, s_{i,t}, p_t] - p_t}{\alpha + \lambda_{i,t}} \tag{2.1}$$

where $\lambda_{i,t} \equiv \frac{dp_t}{dq_{i,t}}$ is the price impact. By symmetry, the price impacts are the same for all traders in the same round $\lambda_{i,t} = \lambda_t, \forall i \in I_{t,\mathcal{M}^*}$. We can parameterize $\mathbb{E}[\theta_{i,t}|\mathcal{H}_t, s_{i,t}, p_t] = c_{\mathcal{H},i,t}\mathcal{H}_t + c_{s,i,t}s_{i,t} + c_{p,i,t}p_t$. By symmetry, the inference coefficients are the same for all traders in the same round, $c_{\mathcal{H},i,t} = c_{\mathcal{H},t}$, $c_{s,i,t} = c_{s,t}$ and $c_{p,i,t} = c_{p,t}$.

In equilibrium, by market clearing condition, λ_t is equal to the inverse of the slope of the residual demand,

$$\lambda_t = (-\sum_{j\neq i} \frac{dq_{j,t}}{dp_t})^{-1} = \frac{\alpha + \lambda_t}{(I_{t,\mathcal{M}^*} - 1)(1 - c_{p,t})}$$

Given the parameterization, the equilibrium price is,

$$p_t = (1 - c_{p,t})^{-1}(c_{\mathcal{H},t}\mathcal{H}_t + c_{s,t}\bar{s}_t) \tag{2.2}$$

where $\bar{s}_t = \frac{1}{I_{t,\mathcal{M}^*}} \sum_i s_{i,t}$ is the average signal in the exchange (for DM, it is the average signal in each pair).

The trader i's value $\theta_{i,t}$, the equilibrium price p_t given equation (2.2), the history $\mathcal{H}_t$ and the private signal $s_{i,t}$ are joint normally distributed. By the projection theorem, the inference coefficients $c_{\mathcal{H},t}$, $c_{s,t}$, and $c_{p,t}$ can be determined given the joint distribution of

$(\theta_{i,t}, s_{i,t}, \mathcal{H}_t, p_t)$.

Theorem 10 (Trading Equilibrium)**.** *The equilibrium price impact is*

$$\lambda_t = \frac{\alpha}{(I_{t,\mathcal{M}^*} - 1)(1 - c_{p,t}) - 1}, \quad \forall i$$

where $c_{p,t} = \frac{I_{t,\mathcal{M}^*}(\rho_{t,\mathcal{M}^*} - \eta_t)\sigma^2}{(1-\rho_{t,\mathcal{M}^*}+\sigma^2)(1+(I_{t,\mathcal{M}^*}-1)\rho_{t,\mathcal{M}^*} - I_{t,\mathcal{M}^*}\eta_t)}$. $\eta_t = \frac{var(\theta_{i,t}) - var(\theta_{i,t}|\mathcal{H}_t)}{var(\theta_{i,t})} = \frac{\boldsymbol{\tau}_t'(\boldsymbol{\Upsilon}_t)^{-1}\boldsymbol{\tau}_t}{\sigma_\theta^2} \in \mathbb{R}$*,* $\boldsymbol{\tau}_t \equiv cov(\mathcal{H}_t, \theta_{i,t}) \in \mathbb{R}^{|\mathcal{H}_t|}$*, and* $\boldsymbol{\Upsilon}_t \equiv cov(\mathcal{H}_t, \mathcal{H}_t') \in \mathbb{R}^{|\mathcal{H}_t| \times |\mathcal{H}_t|}$*.*

The utility conditional on $\mathcal{H}_t$ *for trader* i *is*

$$\mathbb{E}[U_{i,t}^{\mathcal{M}^*}|\mathcal{H}_t] = \frac{\alpha + 2\lambda_t}{2(\alpha+\lambda_t)^2} \frac{I_{t,\mathcal{M}^*} - 1}{I_{t,\mathcal{M}^*}} \frac{(1-\rho_{t,\mathcal{M}^*})^2}{1-\rho_{t,\mathcal{M}^*}+\sigma^2}, \quad \forall i$$

2.3.2 First Stage Market Choice

Given the trading equilibrium in Theorem 10, we can obtain the ex-ante utility of the traders. By comparing the ex-ante utility of traders in DM and CM, we can determine the optimal market choice.

Ex-ante Utility in CM: If the market structure is CM, the ex-ante utility for trader i is

$$\mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_t] = \frac{\alpha + 2\lambda_t^{CM}}{2(\alpha+\lambda_t^{CM})^2} \frac{I-1}{I} \frac{(1-\bar{\rho})^2}{1-\bar{\rho}+\sigma^2} \quad \forall i \in I \tag{2.3}$$

where $\lambda_t^{CM} = \frac{\alpha}{(I-1)(1-c_{p,t})-1}$, $c_{p,t} = \frac{I(\bar{\rho}-\eta_t)\sigma^2}{(1-\bar{\rho}+\sigma^2)(1+(I-1)\bar{\rho}-I\eta_t)}$.

Ex-ante Utility in DM: For traders in the DM, we will need to first determine the trader i's counterparty a la Irving (1985). The trader j that gives trader i the highest utility is matched with trader i. Given that the traders $j \neq i$ are ex-ante identical except for their correlation with trader i, equivalently, this optimal choice of counterparty can be framed as the optimal choice of $\rho_{i,j}$ among the pairwise correlations $\{\rho_{i,j}\}_{j\neq i}$,

$$\max_{\rho_{i,j}|j\in I, j\neq i} \mathbb{E}[U_{i,t}^{DM}(\rho_{i,j})|\mathcal{H}_t]$$

Lemma 3 (Ex-ante Utility with Correlation Across Traders)**.** *Keeping everything else constant, the ex-ante utility* $\mathbb{E}[U_{i,t}^{\mathcal{M}}(\rho)|\mathcal{H}_t]$ *is decreasing in the correlation* ρ.

By Lemma 3, the trader j with lowest correlation with i is matched as i's counterparty. By assumption, only one trader j has the lowest correlation ρ_ℓ with trader i, so the algorithm a la Irving (1985) generates a unique matching result. Given the matching result, trader i's ex-ante utility in DM is

$$\mathbb{E}[U_{i,t}^{DM}|\mathcal{H}_t] = \frac{\alpha + 2\lambda_t^{DM}}{4(\alpha+\lambda_t^{DM})^2}\frac{(1-\rho_\ell)^2}{1-\rho_\ell+\sigma^2} \quad \forall i \in I \tag{2.4}$$

where $\lambda_t^{DM} = \frac{\alpha}{-c_{p,t}}$, $c_{p,t} = \frac{2(\rho_\ell-\eta_t)\sigma^2}{(1-\rho_\ell+\sigma^2)(1+\rho_\ell-2\eta_t)}$.

2.3.3 Price History Informativeness

One observation from Theorem 10 is that the impact of the price history $\mathcal{H}_t$ on the market choice can be summarized by a sufficient statistic, the informativeness of the price history to the traders $\eta_t = \frac{var(\theta_{i,t})-var(\theta_{i,t}|\mathcal{H}_t)}{var(\theta_{i,t})}$.[13] It measures how much traders can learn about their values from the price history.

Given the above observation that the informativeness of the price history η_t governs the impact of the past market choices on the current market choice, we will first discuss the impact of η_t on the market choice before we analyze the dynamics. We have the following comparative statics results for η_t.

Lemma 4 (Comparative Statics With Price History Informativeness η (Rostek and Wu, 2024))**.** *Keeping everything else constant, when* η *increases, the price impact* λ_t *decreases; and the ex-ante expected utility for any trader* i *increases.*

Lemma 4 implies that higher price history informativeness improves liquidity and utility for both centralized and decentralized market. To understand the intuition of Lemma 4, we can decompose the trader i's utility into two parts, the liquidity effect and

[13]When $\mathcal{H}_t$ is a scalar, η_t is the square of the correlation between the price history $\mathcal{H}_t$ and any trader i's value $\theta_{i,t}$.

the gain from heterogeneous values,

$$\mathbb{E}[U_{i,t}^{\mathcal{M}^*}|\mathcal{H}_t] = \underbrace{\frac{\alpha + 2\lambda_t}{2(\alpha+\lambda_t)^2}}_{\text{liquidity effect}} \underbrace{\mathbb{E}[(\mathbb{E}[\theta_{i,t}|\mathcal{H}_t, s_{i,t}, p_t] - p_t)^2|\mathcal{H}_t]}_{\text{gain from heterogeneous values}}$$

First, price impact decreases as price history informativeness increases. A more informative price history reduces adverse selection by providing additional information to calibrate the asset value. Specifically, given a more informative price history, the residual market (the counterparty) requires a smaller price increase to sell an additional unit to the trader, thus lowering the price impact.

Second, the gain from heterogeneous values remains constant regardless of price history informativeness. This is because price history only informs about the value that remains constant across rounds, not the idiosyncratic shocks that determine the gain from heterogeneous values. Since the liquidity effect increases with price history informativeness while the gain from heterogeneous values remains constant, traders' expected utilities rise as price history becomes more informative.

Given heterogeneous trader values, when the price history is sufficiently informative, the traders' expected utility can be higher in DM than in CM.

Lemma 5 (Optimal Market Choice at Round t)**.** *Given $\mathcal{H}_t$, at round t,*

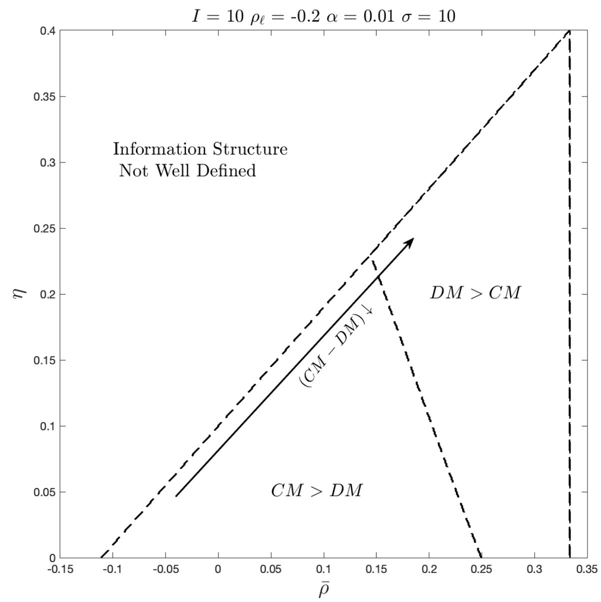
1. $\mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_t] - \mathbb{E}[U_{i,t}^{DM}|\mathcal{H}_t]$ *is decreasing in η_t;*

2. *if $\bar{\rho} \leq \bar{\rho}^* \left(I, \rho_\ell, \sigma^2\right)$, then any trader i will choose CM;*

3. *if $\bar{\rho} > \bar{\rho}^* \left(I, \rho_\ell, \sigma^2\right)$, there exists $\tilde{\eta} \left(I, \bar{\rho}, \rho_\ell, \sigma^2\right)$ the any trader i will choose CM if $\eta_t \leq \tilde{\eta}$, and otherwise if $\eta_t > \tilde{\eta}$.*

Figure 2.2 serves as an example for Lemma 5. It shows the comparison of current round utility in CM versus DM with respect to trader value correlation and past price informativeness. We can see that when $\bar{\rho}$ is sufficiently low, the utility in CM is always higher than DM. When $\bar{\rho}$ is high, there exists $\tilde{\eta}$ such that DM gives traders higher utility than CM. As η increases, the utility difference between CM and DM decreases.

Figure 2.2: Utility in CM vs. DM With Traders' Value Correlation and Price History Informativeness



Note: This figure shows the comparison of utility (welfare) in CM versus DM with traders' value correlation and price history informativeness η. When the price history informativeness is higher or when the value correlation across all traders is higher, the difference between utilities in DM and CM becomes larger.

To understand Lemma 5, we can start with the intuition in static models without price history (Rostek and Weretka, 2012; Yoon, 2017; Rostek and Wu, 2021; Babus and Parlatore, 2022). The horizontal axis of Figure 2.2 represents the static case $T = 1$. The trade-off between CM and DM is between market size and the value correlation with counterparty. CM always has a higher liquidity effect than the bilateral DM given a larger market size.[14] DM is better than CM in terms of counterparty. The gain from heterogeneous values reflects the highest potential gain from trade without price impacts with the residual market. In DM, the residual market is the best counterparty in DM - the one with the lowest correlation ρ_ℓ. In CM, traders not only interact with their best

[14]While empirically transaction volume is widely used to measure liquidity, we want to clarify that the liquidity effect in this paper refers merely to the effect due to price impact but not the transaction volume. Equation (2.1) implies that the transaction volume is determined by both liquidity and gain from heterogeneous values. Ex-ante, the variance of the transaction volume can be higher in DM than in CM when the gain from heterogeneous values is sufficiently large. Therefore, we may see higher transaction volume ex-post with lower liquidity (high price impact) in the DM than in the CM.

counterparty but also compete with $I-2$ other counterparties with correlation ρ_h. The residual market in CM can be viewed as $I-1$ average counterparties with correlation $\bar{\rho}$. When the best counterparty is sufficiently different from the average counterparty, i.e. the correlation ρ_ℓ is sufficiently lower than $\bar{\rho}$, the gain from heterogeneous value is lower in CM compared to DM.

In a dynamic model, having access to price history changes the static trade-off. Figure 2.2 shows that price history informativeness η expands another dimension that is independent of the value correlation $\bar{\rho}$. Price history enhances utility in DM more than that in CM. Higher price history informativeness improves liquidity in both markets, as demonstrated by Lemma 4, with the improvement being more pronounced in the DM. This is because the CM already boasts high liquidity, leaving less room for enhancement. With sufficiently heterogeneous correlation, i.e., $\bar{\rho} > \bar{\rho}^* \left(I, \rho_\ell, \sigma^2\right)$, the benefit from trading exclusively with the best counterparty is substantial. With high enough price history informativeness η, the loss of liquidity in DM is marginal. Consequently, traders tend to favor DM, prioritizing the optimal counterparty even if it means a slight sacrifice in liquidity.

Lemma 5 implies that, given sufficiently heterogeneous value correlation, when price history is sufficiently informative, the traders have incentives to switch to DM. This intuition applies to the analysis of dynamic market choices. In each round, we only need to compare the correlation $\bar{\rho}$ with the threshold $\bar{\rho}^* \left(I, \rho_\ell, \sigma^2\right)$ and the price history informativeness with its threshold $\tilde{\eta} \left(I, \bar{\rho}, \rho_\ell, \sigma^2\right)$ to determine the market choice.

2.3.4 Dynamic Equilibrium

Given the expected utility in DM and CM characterized by equations (2.3) and (2.4), if $\mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_t] \geq \mathbb{E}[U_{i,t}^{DM}|\mathcal{H}_t]$, then the optimal market choice at round t is CM and the price history $\mathcal{H}_{t+1} = \mathcal{H}_t \cup \{p_t\}$, otherwise, the optimal market choice at round t is DM and $\mathcal{H}_{t+1} = \mathcal{H}_t$. We have the following recursive algorithm to generate the equilibrium of dynamic market choice through updates of $\mathcal{H}_t$,

Theorem 11 (Algorithm for Dynamic Market Choice Equilibrium)**.** *The Bayesian Nash equilibrium is a set of price history* $\{\mathcal{H}_t\}_t$*, a sequence of market choice* $\{\mathcal{M}_t^*\}_t$*, and a set of inference coefficients* $\{c_{s,t}, c_{p,t}, \boldsymbol{c}_{\mathcal{H},t}\}$ *that is characterized forwardly recursively.*

1. *Initialize with* $t = 1$*,* $\mathcal{H}_1 = \emptyset$*.*

2. *Given* $\mathcal{H}_t$*, the equilibrium inference coefficients* $\{c_{s,t}, c_{p,t}, \boldsymbol{c}_{\mathcal{H},t}\}$ *is characterized in Theorem 10 with* $\rho_{t,\mathcal{M}^*} = \rho_\ell \; I_{t,\mathcal{M}^*} = 2$ *if* $\mathcal{M}^* = DM$*, and* $\rho_{t,\mathcal{M}^*} = \bar{\rho} \; I_{t,\mathcal{M}^*} = I$ *if* $\mathcal{M}^* = CM$*.*

3. *Given inference coefficients* $\{c_{s,t}, c_{p,t}, \boldsymbol{c}_{\mathcal{H},t}\}$*, If (i)* $\bar{\rho} \leq \bar{\rho}^* \left(I, \rho_\ell, \sigma^2\right)$*, or (ii)* $\bar{\rho} > \bar{\rho}^* \left(I, \rho_\ell, \sigma^2\right)$ *and* $\eta_t \leq \tilde{\eta} \left(I, \bar{\rho}, \rho_\ell, \sigma^2\right)$*, then* $\mathcal{M}_t^* = CM$*,* $\mathcal{H}_{t+1} = \mathcal{H}_t \cup \{p_t\}$*; otherwise,* $\mathcal{M}_t^* = DM$*,* $\mathcal{H}_{t+1} = \mathcal{H}_t$*. Repeat Steps 2-3 with the next* t*, until t=T.*

The proof of Theorem 11 is immediate from the above analysis.

2.4 Dynamic Market Choice

In this section, we will explore how market choices evolve dynamically. In particular, we are interested in how trader value correlations and asset properties affect the dynamic market choice.

2.4.1 Constant Market Choice

In this part, we discuss sufficient conditions for traders to choose only one market structure in all rounds.

Homogeneous Correlation: First, let us consider a simple case where the traders' value correlations are homogeneous. In this case, traders will always choose CM.

Proposition 5 (Homogeneous Correlation)**.** *When the traders value correlations are sufficiently homogeneous* $\bar{\rho} \leq \bar{\rho}^* \left(I, \rho_\ell, \sigma^2\right)$*, traders will always stay in the CM.*

The proof of Proposition 5 directly follows from Lemma 5, as no price history informativeness η will allow traders to choose DM. When the lowest correlation and the correlation with all other traders are similar, the benefit of trading with one counterparty with lower adverse selection in the DM is dominated by the loss of lower liquidity, regardless of the price history. Thus the traders have no incentive to choose DM in any rounds.

Sufficiently Heterogeneous Correlation: On the opposite side, when the traders' value correlation is sufficiently heterogeneous, traders will always choose DM. If traders choose DM in the first round, they will choose DM for all rounds.

Lemma 6 (DM persisitency)**.** *Keep everything else constant, if $\mathcal{M}_1^* = DM$, then $\mathcal{M}_t^* = DM$ for all t.*

Proof. Suppose traders choose DM over CM in round 1. It is easy to see that $\eta_1 = 0$. Given that in each round primitives $(I, \bar{\rho}, \rho_\ell, \sigma^2)$ are the same, this means they prefer DM over CM if $\eta_t = 0$. As DM is opaque, if $\eta_t = 0$ and traders choose DM at round t, then $\eta_{t+1} = 0$. This implies traders will always choose DM by forward induction. ■

Given Lemma 6, if we find sufficiently heterogeneous correlation makes traders choose DM in the first round, then they will stay in DM for all rounds.

Proposition 6 (Sufficiently Heterogeneous Correlation)**.** *There exists $\underline{\rho}_\ell < 0$ and $\bar{\bar{\rho}} > \bar{\rho}^*$ such that for any $\rho_\ell < \underline{\rho}_\ell$ and $\bar{\rho} > \bar{\bar{\rho}}$, traders will stay in the DM for all rounds.*

Propositions 5 and 6 are consistent with Yoon (2017), where the traders' heterogeneous value correlation is crucial for traders to choose DM. Note that the condition for Propositions 6 is the same in both static ($T = 1$, Yoon (2017)) and dynamic models ($T \geq 2$) given Lemma 6. However, the threshold $\bar{\rho}^*$ for Proposition 5 in the dynamic model ($T \geq 2$) is lower than that in the static model ($T = 1$). This is because correlation should be more homogeneous to keep traders in a centralized market with a longer price history. Appendix Section 2.9.3 provides simulations of the market choices as the number of rounds increases.

2.4.2 Alternating Market Choices

The equilibrium becomes more interesting when we the traders' value correlations are neither too homogenous nor too heterogeneous. Alternating between CM and DM can emerge endogenously as the optimal market choice. We find that the asset properties, including asset sensitivity to shocks to fundamentals ξ, and the fundamental value volatility inversely related to autocorrelation κ, are crucial for the market choices.

Proposition 7 (Heterogeneous Correlation and Asset Sensitivity)**.** *With heterogeneous correlation $\bar{\rho} > \bar{\rho}^* \left(I, \rho_\ell, \sigma^2\right)$, $\rho_\ell \geq 0$, $\epsilon < \bar{\epsilon}(\sigma^2, I)$, $3 \leq T < \bar{T}(\sigma^2, I, \epsilon)$, and $\kappa < \bar{\kappa}(\sigma^2, I, \epsilon)$, there exists $\underline{\xi}$ and $\bar{\xi}$ such that such that traders will choose CM in the first round, and*

1. *When the asset sensitivity to shocks to fundamentals is sufficiently low $\xi \in [0, \underline{\xi})$, the traders shift to DM in the second round and stay there.*
2. *When the asset sensitivity to shocks to fundamentals is intermediate $\xi \in [\underline{\xi}, \bar{\xi})$, the traders will alternate between CM and DM.*
3. *When the asset sensitivity to shocks to fundamentals is sufficiently high $\xi \in [\bar{\xi}, \infty)$, the traders will always stay in the CM.*

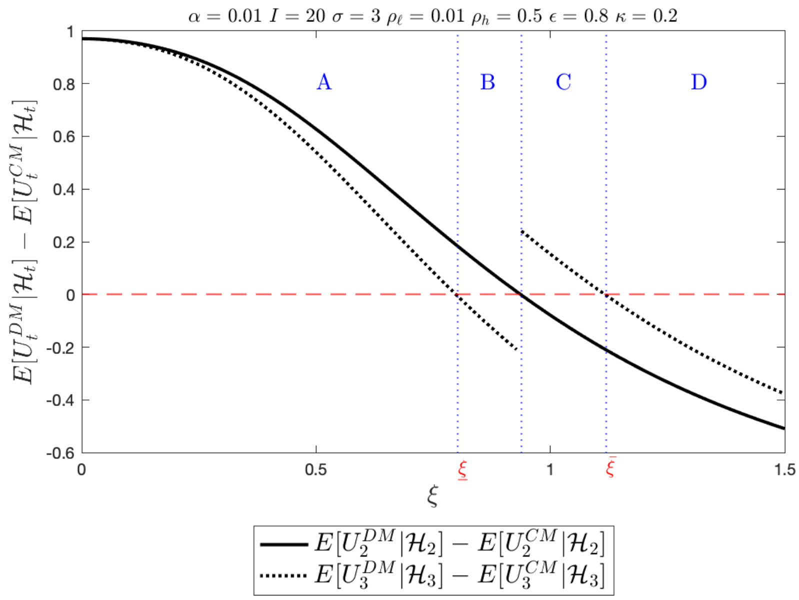
Intuitively, when traders' value correlations are heterogeneous, i.e., when $\bar{\rho}$ and $\bar{\rho}_\ell$ are sufficiently different, the traders have incentives to shift to DM by our previous analysis. To further understand this result with respect to asset sensitivity, let us consider the following three-round example.

Example 3 (Three-round Market)**.** *We consider a market with $T = 3$. Assume that $\rho_\ell > 0$, such that in the 1st round the DM does not exist and the traders will always choose CM. Assume also that any pair of traders that do not have correlation ρ_ℓ have value correlation $\rho_h > \rho_\ell$.*

Figure 2.3 shows the market choice of the traders in the 2nd and 3rd round. In the Appendix, we also provided the price history informativeness in the 2nd and 3rd rounds with respect to ξ.

***Region A**: When the asset sensitivity is low, this implies the asset value is less susceptible to shocks to fundamentals and more correlated across rounds. This also implies that the price history*

Figure 2.3: Dynamic Market Choice with Asset Sensitivity ξ in $T = 3$ Market



Note: The black solid line plots the difference between the ex-ante expected utility of DM and that of CM in the 2nd round, and the black dotted line plots that difference in the 3rd round. The red dashed line is a reference line of 0. When the black solid(dotted) line is above the reference line, then the traders choose DM in the 2nd round(3rd round), and if it is below the reference line, the traders choose CM in the 2nd round(3rd round). The jump in the difference of utility in CM vs. DM in the third round comes from the difference in the second-round choice.
In region A, i.e. $\xi \in [0, \underline{\xi})$, traders choose DM in both 2nd and 3rd round. In region B, i.e. $\xi \in (\underline{\xi}, \bar{\xi}]$ and in the lower partition, traders choose DM in the 2nd round and CM in the 3rd round. In region C, $\xi \in (\underline{\xi}, \bar{\xi}]$ and in the higher partition, traders choose CM in the 2nd round and DM in the 3rd round. In region D, $\xi \in (\bar{\xi}, 1]$, traders choose CM in both the 2nd round and the 3rd round.

is more informative to the traders. A more informative price history can lower the price impact and increase liquidity effect and utility. Such an increase is higher for DM than CM, as CM is already very liquid thus leaving less room for liquidity improvement. The higher liquidity improvement in DM can decrease the loss of liquidity effect for choosing DM and be dominated by the gain in heterogeneous values. This gives rise to a shift to DM in the 2nd round.

When the asset sensitivity is sufficiently low, $\xi \in [0, \underline{\xi})$, traders will continue to stay in DM in the 3rd round, as the 1st round price is still informative enough for them to trade with better counterparty at just a bit higher price impact in DM.

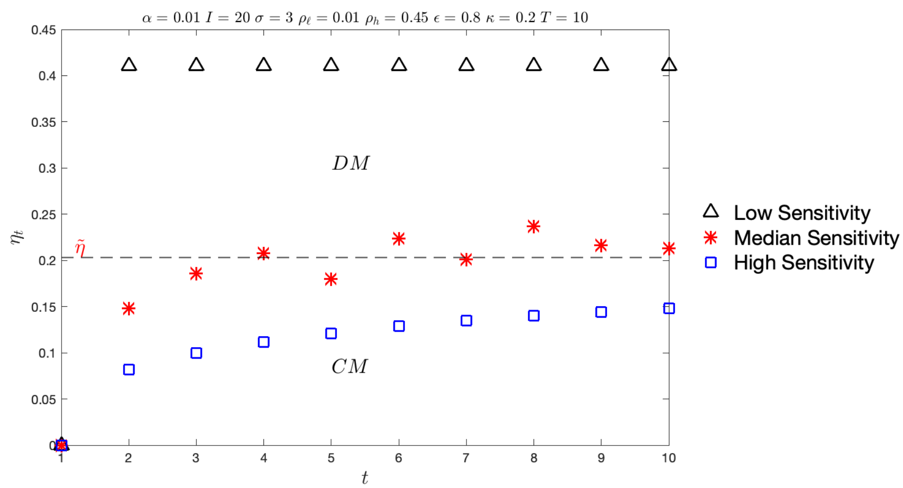
***Region B**: However, when the asset sensitivity is not sufficiently low, i.e., $\xi \in [\underline{\xi}, \bar{\xi})$, traders* *will alternate between DM and CM. traders will choose DM in the second round and choose CM in the 3rd round. As the asset value is not that stable across time and traders do not know the DM price, the price in the 1st round becomes stale and not informative enough for the 3rd round values. The liquidity difference in CM and DM again becomes large, making traders shift back to CM for liquidity improvement.*

***Region C**: When the asset sensitivity is high but not high enough, i.e.,* $\xi \in [\underline{\xi}, \bar{\xi})$ *and higher than that in Case 2, traders will still alternate between DM and CM. Traders will choose CM in the first two rounds and shift to DM in the last round. This is because the asset sensitivity is not low enough such that traders will choose CM in the second round for higher liquidity. However, the asset sensitivity is low enough such that the price history is sufficiently informative in the 3rd round when traders see both the 1st round and 2nd round prices in CM. In the 3rd round, the liquidity improvement in DM is sufficiently higher than that in CM. The traders shift to DM in the 3rd round for a better counterparty.*

***Region D**: When the asset sensitivity is sufficiently high, i.e.,* $\xi \in [\bar{\xi}, \infty)$*, traders will stay in the CM for both the 2nd and 3rd round. This is because the value of assets changes frequently across time, making price history not informative enough to largely boost the liquidity in the DM. Therefore, the liquidity difference between the DM and CM remains large, preventing the traders from choosing DM for the benefit of the best counterparty.*

To summarize, Example 3 shows how the asset sensitivity affects the informativeness of the price history η, and then affects the liquidity effect and therefore current market choice. Past prices in the CM increase η and lower price impact. Better counterparties attracts traders to the DM. However, DM opacity can lower future price history informativeness η and push traders back to the CM. These intuitions from Example 3 can be extended to more than 3 rounds. Figure 2.4 shows the evolution of price history informativeness and market choice with $T = 10$ with respect to different levels of asset sensitivities. When the marker is above (below) the reference line $\tilde{\eta}$ which is defined by Lemma 5 and calculated according to trader value correlations, then traders choose DM (CM).

Figure 2.4: Evolution of Price History Informativeness for Different Asset Sensitivities



Note: This figure shows the evolution of price history informativeness for different levels of asset sensitivity ξ for $T = 10$. The black dashed line is a reference line of threshold $\tilde{\eta}$. When the marker is above the reference line, then the history informativeness in that round is higher than $\tilde{\eta}$ and traders choose DM. If the marker is below the reference line, then the history informativeness in that round is lower than $\tilde{\eta}$ and traders choose CM.

We would like to clarify that the mechanism for alternating market choice does not come from the tie-breaking rule which we do not impose any indeterminacy. It also differs from the mechanism as in Yoon (2017) where (i) traders in the DM do not access CM price; and (ii) marginal trader's (weak) indifference between DM and CM gives rise to coexistence. In this paper, traders choose DM when DM gives them a strictly higher utility than CM. DM emerges endogenously as a result of learning from price history, and fades endogenously when the price history becomes uninformative.

Proposition 7 is consistent with our real life observations. Securities that are designed to be insensitive to issuers' fundamentals, like bonds, are firstly traded in the centralized primary market and then mostly traded in the secondary over-the-counter market. Securities that are relatively more sensitive to issuers' fundamentals, like equities, are mostly traded in the centralized market, sometimes traded in dark pools. Securities that by design are most sensitive to issuers' fundamentals, like options, are only traded in the centralized market.

Proposition 8 (Heterogeneous Correlation and Autocorrelation)**.** *With heterogeneous correlation* $\bar{\rho} > \bar{\rho}^*\left(I, \rho_\ell, \sigma^2\right)$*,* $\epsilon < \bar{\epsilon}(\sigma^2, I)$*,* $3 \leq T < \bar{T}(\sigma^2, I, \epsilon)$*, and* $\xi > \underline{\xi}(\sigma^2, I, \epsilon)$*, there exists* $\underline{\kappa}$ *and* $\bar{\kappa}$ *such that traders will choose CM in the first round, and*

1. *When the autocorrelation is sufficiently low* $\kappa \in [0, \underline{\kappa}]$*, the traders will always stay in the CM.*

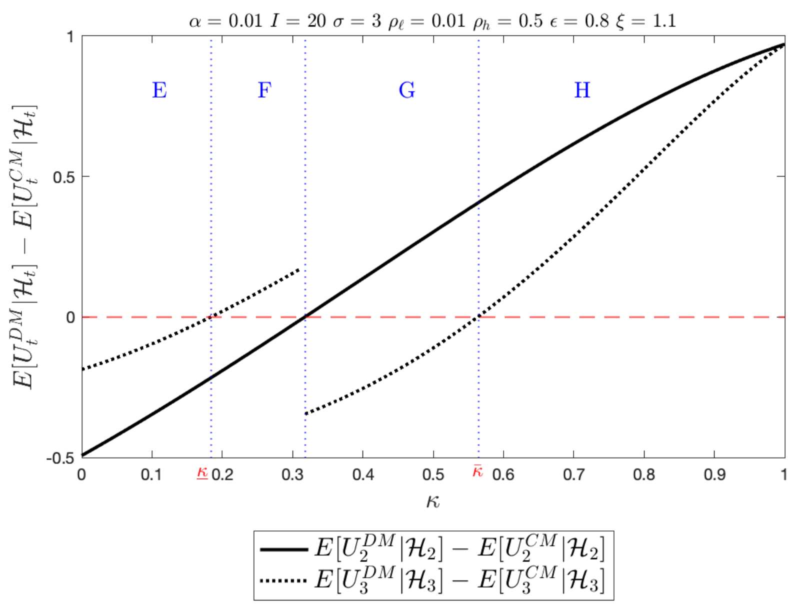
2. *When the autocorrelation is intermediate* $\kappa \in (\underline{\kappa}, \bar{\kappa}]$*, the traders will alternate between CM and DM.*

3. *When the autocorrelation is sufficiently high* $\kappa \in (\bar{\kappa}, 1]$*, the traders will choose DM over CM in the second round and never choose CM again.*

Figure 2.5 shows the market choice of the traders in the 2nd and 3rd round in Example 3 with respect to autocorrelation κ. In the Appendix, we also provided the price history informativeness in the 2nd and 3rd rounds with respect to κ. Similar to the analysis of Proposition 7, the intuition for Proposition 8 also works through the dynamics of the price history informativeness η. The price history informativeness η is increasing in autocorrelation κ. When autocorrelation is higher, this means the values are less volatile across rounds, the price history is more informative, and the traders are more likely to shift to DM. The intuition of Example 3 also applies to a market with more rounds. Figure 2.6 shows the evolution of price history informativeness and market choice for a $T = 10$ round market with respect to different levels of autocorrelation. When the marker is above (below) the reference line $\tilde{\eta}$ which is defined by Lemma 5 and calculated according to trader value correlations, then traders choose DM (CM).

The autocorrelation κ captures the volatility of the fundamentals. Proposition 8 implies that when the shocks are less volatile, i.e. high κ, then the traders are more likely to trade in DM. The implication of Proposition 8 is consistent with some existing empirical literature. Both Menkveld et al. (2017) and Buti et al. (2022) find that the market share of the dark pools (corresponding to DM in our model) relative to the lit venues (corresponding to CM in our model) decreases when the market is more volatile.

Figure 2.5: Dynamic Market Choices with Autocorrelation κ in $T = 3$ Market



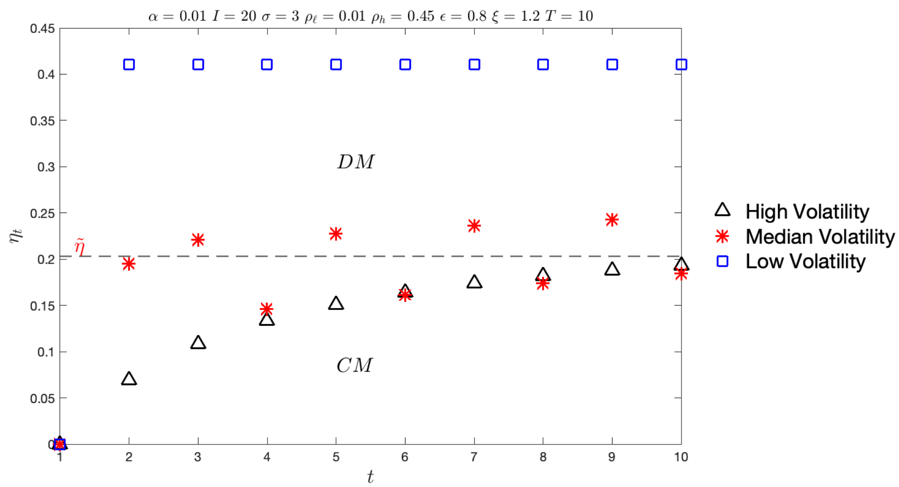
Note: The black solid line plots the difference between the ex-ante expected utility of DM and that of CM in the 2nd round, and the black dotted line plots that difference in the 3rd round. The red dashed line is a reference line of 0. When the black solid (dotted) line is above the reference line, then the traders choose DM in the 2nd round (3rd round), and if it is below the reference line, the traders choose CM in the 2nd round (3rd round). The jump in the difference of utility in CM vs. DM in the third round comes from the difference in the second-round choice.
In region E, i.e. $\kappa \in [0, \underline{\kappa}]$, traders choose CM in both 2nd and 3rd round. In region F, i.e. $\kappa \in (\underline{\kappa}, \bar{\kappa}]$ and in the lower partition, traders choose CM in the 2nd round and DM in the 3rd round. In region G, $\kappa \in (\underline{\kappa}, \bar{\kappa}]$ and in the higher partition, traders choose DM in the 2nd round and CM in the 3rd round. In region H, $\kappa \in (\bar{\kappa}, 1]$, traders choose DM in both the 2nd round and the 3rd round.

We also want to clarify the difference between asset sensitivity and volatility. Even if the issuer's fundamental has high volatility, it is possible for the issuer to design securities that have low asset sensitivity to be traded in the DM.

2.4.3 Discussions and Extensions

General Private Information Precision: In the baseline model, traders receive private signals with constant precision. Appendix Section 2.9.1 extends the model to include varying precision of private information. We find that traders are more likely to choose CM after rounds with less precise private signals. Additionally, we demonstrate that

Figure 2.6: Evolution of Price History Informativeness For Different Autocorrelations



Note: This figure shows the evolution of price history informativeness for different levels of autocorrelation κ for $T = 10$. The black dashed line is a reference line of threshold $\tilde{\eta}$. When the marker is above the reference line, then the history informativeness in that round is higher than $\tilde{\eta}$ and traders choose DM. If the marker is below the reference line, then the history informativeness in that round is lower than $\tilde{\eta}$ and traders choose CM.

traders can still alternate between CM and DM, even with post-trade transparent DM, when precise signals are infrequent.

Non-movers: In the baseline model, we focus on the behavior of traders who move across venues without frictions. This generates a pattern where all traders either choose CM or DM. In practice, some traders only have access to one type of the markets. For example, the retail traders in the U.S. equity market usually do not trade in the dark pools, and the bank dealers are prohibited from trading in the centralized bond market in China. Appendix Section 2.9.2 provides an extension to accommodate the non-movers in the market without changing the mechanisms and qualitative results in the baseline model.

Proportion of Time in CM: Appendix Section 2.9.3 provides the analysis on the proportion of time traders choose CM over DM with sensitivity and volatility. Consistent with our intuition for Propositions 7 and 8, we find the proportion of time traders choosing

CM is positively correlated with asset sensitivity and volatility.

Alternative Tie-breaking Rule: In this paper, we do not allow traders to choose DM and CM in the same round. Following Yoon (2017), we can allow the traders to choose DM or CM until no trader would like to deviate to the other market. Note that our results still hold qualitatively with this new tie-breaking rule, as price history informativeness η can still evolve endogenously with traders' past market choices and in turn determines traders future choices.

2.5 Market Designs with Endogenous Market Choices

In previous sections, we endogenize the traders' market choices given their value correlations and asset properties. Most of the literature focuses on comparing market designs with a fixed number of traders in each market. We may wonder how to improve market efficiency taking into account the flow of traders across venues. In this section, we will revisit some popular market designs given the endogenous market choice.

2.5.1 Transparency

So far, we have assumed that DM is opaque, i.e. future traders cannot see prices in DM and traders in DM cannot see prices in other pairs. In this section, we will consider introducing transparency designs in DM.

It is of policy interest to discuss the impact of transparency on market structures and welfare. In reality, traders have post-trade transparency in some decentralized markets, e.g. TRACE in the bond market, and blockchain technology in the crypto market. Some decentralized trading mechanism allows pre-trade transparency, e.g. request-for-quote. However, some decentralized markets are relatively opaque, e.g. dark pools for equities. The lack of transparency in dark pools has received critique and policy attention. However, the impact of introducing transparency to dark pools remains unclear. Our dynamic model allows us to explore the impact of transparency designs on traders' market choices and welfare.

2.5.1.1 Post-trade Transparency

In this section, we will consider introducing post-trade transparency to DM, i.e., prices in DM will enter the price history and affect future market choices. This definition of post-trade transparency follows Vairo and Dworczak (2023) and Rostek et al. (2024).

It is easy to see that Theorem 10 still applies to equilibrium with post-trade transparency. Denote the number of trading pairs in the DM as $N = \frac{I}{2}$, and each trading pair as n. We can slightly modify the price updating rule in Theorem 11 to characterize the new equilibrium.

Theorem 12 (Algorithm for Dynamic Market Choice Equilibrium with Post-trade Transparency)**.** *The Bayesian Nash equilibrium is a set of price history $\{\mathcal{H}_t\}_t$, a sequence of market choice $\{\mathcal{M}^*_t\}_t$, and a set of inference coefficients $\{c_{s,t}, c_{p,t}, \boldsymbol{c}_{\mathcal{H},t}\}$ that is characterized forwardly recursively.*

1. *Initialize with $t = 1$, $\mathcal{H}_1 = \emptyset$.*

2. *Given $\mathcal{H}_t$, the equilibrium inference coefficients $\{c_{s,t}, c_{p,t}, \boldsymbol{c}_{\mathcal{H},t}\}$ is characterized in Theorem 10 with $\rho_{t,\mathcal{M}^*} = \rho_\ell$ $I_{t,\mathcal{M}^*} = 2$ if $\mathcal{M}^* = DM$, and $\rho_{t,\mathcal{M}^*} = \bar{\rho}$ $I_{t,\mathcal{M}^*} = I$ if $\mathcal{M}^* = CM$.*

3. *Given inference coefficients $\{c_{s,t}, c_{p,t}, \boldsymbol{c}_{\mathcal{H},t}\}$, If $\frac{\alpha+2\lambda_t^{CM}}{2(\alpha+\lambda_t^{CM})^2} \frac{I_t-1}{I_t} \frac{(1-\bar{\rho}_t)^2}{1-\bar{\rho}_t+\sigma^2} \geq \frac{\alpha+2\lambda_t^{DM}}{4(\alpha+\lambda_t^{DM})^2} \frac{(1-\rho_\ell)^2}{1-\rho_\ell+\sigma^2}$, then $\mathcal{M}^*_t = CM$, $\mathcal{H}_{t+1} = \mathcal{H}_t \cup \{p_t\}$; otherwise, $\mathcal{M}^*_t = DM$, $\mathcal{H}_{t+1} = \mathcal{H}_t \cup \{p_{n,t}\}_n$, where $p_{n,t}$ is the equilibrium price of bilateral trading pair n. Repeat Steps 2-3 with the next t, until t=T.*

The proof of Theorem 12 follows the analysis in Section 2.3.

First, we explore the impact of post-trade transparency on traders' optimal market choice. Perhaps surprisingly, we find that with post-trade transparency, traders will stay in DM once they have chosen it. This is because the price history informativeness η never decays, attracting traders to stay in DM.

Proposition 9 (Post-trade Transparency: Once DM, Always DM)**.** *With post-trade transparency, if $\mathcal{M}_t^* = DM$, then $\mathcal{M}_\tau^* = DM, \forall \tau \geq t$.*

By Proposition 9, the potential dynamic market choices will be (i) choosing DM for all rounds; (ii) choosing CM at first and DM thereafter; and (iii) choosing CM for all rounds. Alternating back and forth between DM and CM is no longer an optimal dynamic market choice. Note that this result is different from Lemma 6 which only describes one possible market choice, i.e., DM persists when traders choose DM in the first round. Proposition 9 implies that if we introduce post-trade transparency in dark pools, the traders will not return to the centralized market. Note that Proposition 9 holds only when the private signals and trading rounds arrive regularly, which may not hold in reality. If we slightly modify the model by allowing precise private signals to arrive less frequently, then the price history informativeness naturally decays during these long no-trade or uninformed-trade periods, and traders can return to CM from DM even with post-trade transparency (see Appendix Section 2.9.1).

Still, regardless of its impact on the market choice, post-trade transparency in DM weakly increases overall welfare.

Proposition 10 (Post-trade Transparency Improves Welfare)**.** *Post-trade transparency weakly improves welfare regardless of market choices.*

Post-trade transparency in DM does not affect the utility of traders when they choose CM, but can weakly increase welfare when they choose DM. The intuition is as follows. η_t^{post} with post-trade transparency will always be weakly higher than η_t without post-trade transparency, as the DM prices are informationally equivalent to the average signal of each bilateral pair, which is at least as informative as the centralized market price in the same round if traders choose CM without post-trade transparency. Given $\eta_t^{post} \geq \eta_t$, any market choice without post-trade transparency will not give traders higher utility than DM with post-trade transparency.

2.5.1.2 Pre-trade Transparency

In this section, we will consider introducing pre-trade transparency in DM. The definition of post-trade transparency follows Rostek et al. (2024). We allow traders in each pair to not only submit demand schedules contingent on their price but also the prices in other pairs. Their demand schedule in DM at round t will be $q_{i,t}(\boldsymbol{p}_t) : \mathbb{R}^N \to \mathbb{R}$, where $\boldsymbol{p}_t \in \mathbb{R}^N$ is the vector all prices in all pairs whose n^{th} element is the price in pair n at round t, $p_{n,t}$. For tractability, besides that each trader will have a correlation ρ_ℓ with only one trader, we further assume that each trader has a correlation ρ_h with all other traders in the same round.

Equilibrium Characterization: It is easy to see that given history $\mathcal{H}_{i,t}$, the trading equilibrium in CM will not be affected by the pre-trade transparency in DM. We can still apply Theorem 10 to characterize CM equilibrium. We need to solve for the new trading equilibrium for DM.

With pre-trade transparency, traders in the DM will have access to prices from other pairs and submit demand schedules contingent on them. Trader $i \in I(n)$ submit demand schedule $q_{i,t}(\boldsymbol{p}_t) : \mathbb{R}^N \to \mathbb{R}$ to maximize the expected utility conditional on the history $\mathcal{H}_t$, private signal $s_{i,t}$, and

$$\max_{q_{i,t}(\boldsymbol{p}_t)} \mathbb{E}[\theta_{i,t}q_{i,t} - \frac{1}{2}\alpha q_{i,t}^2 - p_{n,t}q_t | \boldsymbol{p}_t, \mathcal{H}_t, s_{i,t}]$$

trader i 's first-order condition as

$$q^i(\boldsymbol{p}_t) = \frac{\mathbb{E}[\theta_{i,t}|\boldsymbol{p}_t, \mathcal{H}_t, s_{i,t}] - p_{n,t}}{\alpha + \lambda_{i,t}}$$

where $\lambda_{i,t}$ is the trader i's price impact within pair n. Trader i also has cross-pair price impact as traders from other pairs will change their bids when price p_n changes with i's bid. Trader i's price impact over all pairs can be described with a price impact matrix $\boldsymbol{\Lambda}_{i,t} = (\frac{d\boldsymbol{p}}{d\boldsymbol{q}_{i,t}}) \in \mathbb{R}^{N\times N}$, where the n^{th} diagonal elements is $\lambda_{i,t}$. Each trader i 's price impact matrix is equal to the transpose of the Jacobian of trader i's inverse residual supply:

$$(\boldsymbol{\Lambda}_{i,t})' = \left(-\sum_{j\neq i}\frac{d\boldsymbol{q}_{j,t}}{d\boldsymbol{p}_t}\right)^{-1}$$

We can parameterize $\mathbb{E}[\theta_{i,t}|\boldsymbol{p}_t, \mathcal{H}_t, s_{i,t}] = \boldsymbol{c}_{\mathcal{H},i,t}\mathcal{H}_t + c_{s,i,t}s_{i,t} + \boldsymbol{c}_{p,i,t}\boldsymbol{p}_t$. $\boldsymbol{c}_{\mathcal{H},i,t} \in \mathbb{R}^{1\times|\mathcal{H}_t|}$, $c_{s,i,t} \in \mathbb{R}$, and $\boldsymbol{c}_{p,i,t} \in \mathbb{R}^{1\times N}$. Given symmetry within each pair, $\boldsymbol{c}_{\mathcal{H},i,t} = \boldsymbol{c}_{\mathcal{H},n,t}$, $c_{s,i,t} = c_{c,n,t}$, $\boldsymbol{c}_{p,i,t} = \boldsymbol{c}_{p,n,t}$ and $\lambda_{i,t} = \lambda_{n,t}$.

Given the market clearing condition, $\sum_{i\in I(n)} q_{i,t}(\boldsymbol{p}_t) = 0$, and trader symmetry within exchanges, we have the equilibrium price in all pairs in vector form,

$$\boldsymbol{p}_t = (\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1}\left(\boldsymbol{C}_{\mathcal{H},t}\mathcal{H}_t + \boldsymbol{C}_{s,t}\bar{\boldsymbol{s}}_t\right),$$

where $\boldsymbol{C}_{s,t} = \text{diag}\left(c_{s,n,t}\right)_n \in \mathbb{R}^{N\times N}$, $\boldsymbol{C}_{\mathcal{H},t} = \left(\boldsymbol{c}_{\mathcal{H},n,t}\right)_n \in \mathbb{R}^{N\times|\mathcal{H}_t|}$, $\boldsymbol{C}_{p,t} = \left(\boldsymbol{c}_{p,n,t}\right)_n \in \mathbb{R}^{N\times N}$. $\bar{\boldsymbol{s}}_t \in \mathbb{R}^N$ is the average signals for all pairs, where the n^{th} element is the average signal in pair n.

Given that value $\theta_{i,t}$, private signal $s_{i,t}$, prices $\boldsymbol{p}_t$, and price history $\mathcal{H}_t$ are jointly normally distributed, we can solve the inference coefficients $\boldsymbol{C}_{s,t}$, $\boldsymbol{C}_{\mathcal{H},t}$ and $\boldsymbol{C}_{p,t}$ through the projection theorem.

Theorem 13 (DM Trading Equilibrium with Pre-trade Transparency)**.** *The price impact for trader i in pair n is*

$$\lambda_{n,t} = \left(\left(\left((\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1}\right)_{nn}\right)^{-1} - 1\right)^{-1}\alpha.$$

where $(A)_{nn}$ is an operator that gives the n^{th} diagonal element of matrix A.
$\boldsymbol{C}_{p,t} = \text{diag}\left(\frac{\sigma^2}{1-\rho_{n,t}+\sigma^2}\right)_n\left(\boldsymbol{Id} - \text{diag}\left(\frac{1-\rho_{n,t}}{2}\right)(\bar{\mathcal{C}} - \boldsymbol{1}\boldsymbol{1}'\eta_t)^{-1}\right)$. $\eta_t = \frac{\boldsymbol{\tau}_t'\boldsymbol{\Upsilon}_t^{-1}\boldsymbol{\tau}_t}{\sigma_\theta^2}$ *is price history informativeness.* $\bar{\mathcal{C}} = \frac{cov(\bar{\boldsymbol{\theta}}_t,\bar{\boldsymbol{\theta}}_t')}{\sigma_\theta^2} \in \mathbb{R}^{N\times N}$ *is the correlation of pairwise average values across all pairs, where $\bar{\boldsymbol{\theta}}_t \in \mathbb{R}^N$ is the vector of average value per trading pair where the n^{th} value is* $\bar{\theta}_{n,t} = \sum_{i\in I(n)}\theta_{i,t}$

The expect utility for trader i in pair n conditional on the price history is

$$\mathbb{E}[U_{i,t}^{DM}|\mathcal{H}_t] = \frac{\alpha + 2\lambda_t}{2(\alpha+\lambda_t)^2}\mathbb{E}[(\mathbb{E}[\theta_{i,t}|\mathcal{H}_t, s_{i,t}, \boldsymbol{p}_t] - p_{t,n})^2|\mathcal{H}_t] = \frac{\alpha + 2\lambda_{n,t}}{2(\alpha+\lambda_{n,t})^2}\frac{1}{2}\frac{(1-\rho_{n,t})^2}{1-\rho_{n,t}+\sigma^2}$$

Thoerem 13 shows that the price history's impact on the current round utility is still through the price impact, and can be summarized by the sufficient statistic, price history informativeness η_t.

Our next question is, will pre-trade transparency change the matching results in DM? We find that the expected utility is still monotonic in the correlation $\rho_{n,t}$ (see Lemma 7). Therefore, each trader will be matched with the counterparty that has the lowest correlation ρ_ℓ, same as the matching results in Section 2.3.

Lemma 7 (Monotonicity of Utility with Pre-trade Transparency)**.** *With pre-trade transparency, $\mathbb{E}[U_{i,t}^{DM}|\mathcal{H}_t]$ is monotonically decreasing in $\rho_{n,t}$.*

Given that introducing pre-trade transparency does not change the matching results in DM, and the price history update rule remains the same, we can still apply Theorem 11 to characterize the equilibrium.

Pre-trade Transparency and Welfare: With the equilibrium characterization, we would be able to discuss the impact of pre-trade transparency on market choice and welfare.

First, we find that given price history $\mathcal{H}_t$, introducing pre-trade transparency always weakly increases the utility for all traders in DM.

Lemma 8 (Pre-trade Transparency Increases DM Utility)**.** *Given price history $\mathcal{H}_t$, introducing pre-trade transparency weakly increases the utility for all traders in DM.*

Given Lemma 8, it is intuitive that, holding everything else constant, it is more likely for traders to choose DM over CM as the threshold of history informativeness $\tilde{\eta}$ for traders to opt for DM is weakly lower.

Proposition 11 (Pre-trade Transparency Precipitates DM)**.** *With pre-trade transparency, (i) the first time for traders to choose DM is no later than without transparency; (ii) if the round when traders first choose DM is the same as the round when traders first choose DM without pre-trade transparency, then they stay in DM for weakly longer.*

The fact that pre-trade transparency can make traders choose DM earlier creates nuances in terms of welfare. By Lemma 8 we know that transparency increases utility for traders in DM given the price history. However, choosing DM earlier and staying longer can potentially decrease the price history informativeness and welfare in later rounds. Pre-trade transparency can bring down welfare when the loss of history informativeness dominates the benefit in DM.

Proposition 12 (Pre-trade Transparency and Welfare)**.**

1. *For sufficiently heterogeneous trader value $\rho_\ell < \underline{\rho}_\ell < 0$ and $\bar{\rho} > \bar{\bar{\rho}}$, pre-trade transparency weakly improves welfare.*

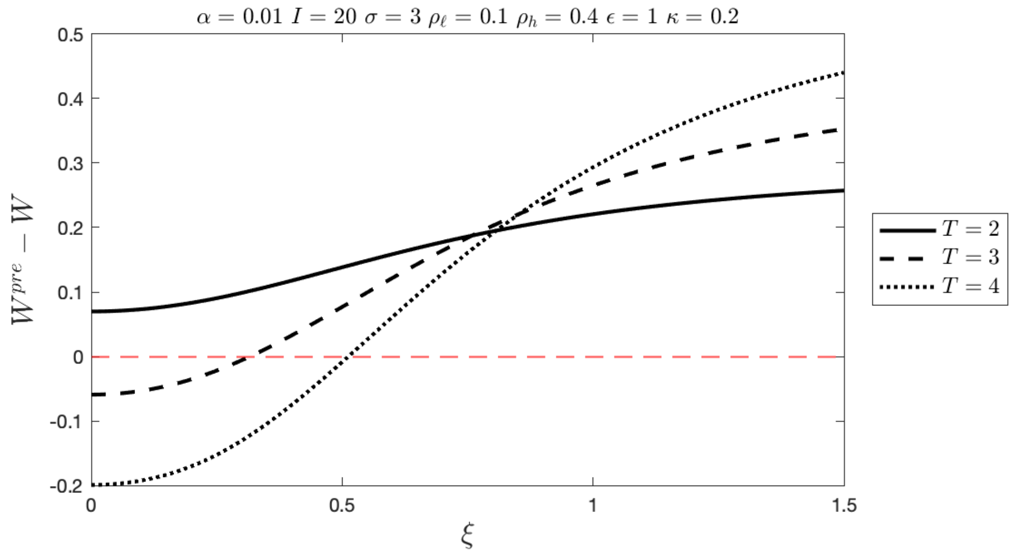
2. *For sufficiently homogenous trader value, $\bar{\rho} < \bar{\rho}^{*,pre}(I, \rho_\ell, \sigma^2)$, pre-trade transparency does not change welfare.*

3. *When traders' value correlations are neither sufficiently heterogeneous nor sufficiently homogenous, pre-trade transparency can decrease welfare when the number of rounds T is sufficiently large, asset sensitivity ξ is low, or autocorrelation κ is high.*

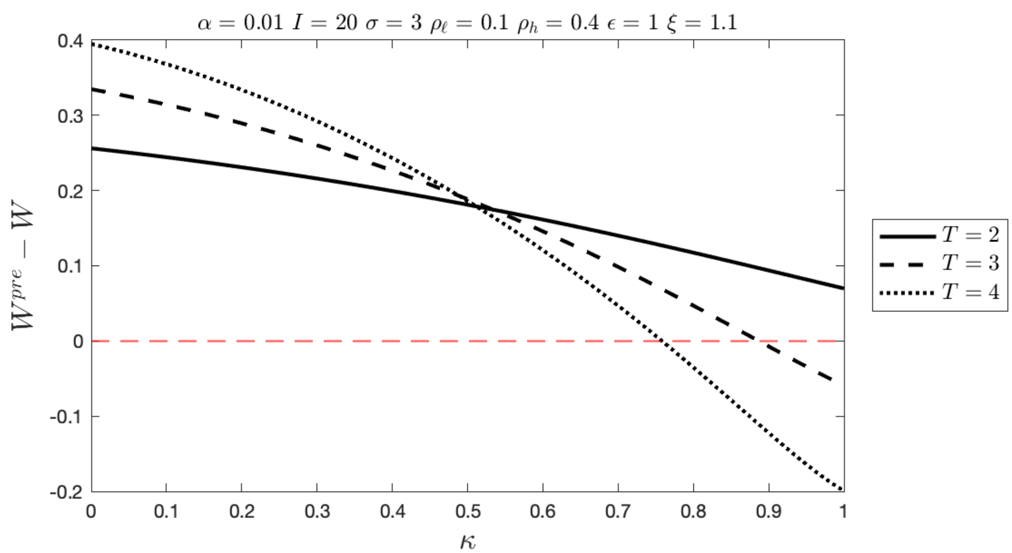
Intuitively, Proposition 12.1 corresponds to the constant DM choices both with and without pre-trade transparency, and given Lemma 8, pre-trade transparency should always weakly increase welfare. Proposition 12.2 corresponds to the constant CM choices both with and without pre-trade transparency. As traders do not choose DM, pre-trade transparency does not change welfare.

The impact of pre-trade transparency on welfare is ambiguous when traders' value correlation heterogeneity is of intermediate level (Proposition 12.3). This is when traders can have alternating market choices in the benchmark model. Despite that DM utility is higher with pre-trade transparency, price history informativeness may be lower as traders are more likely to choose DM earlier with pre-trade transparency. Figure

Figure 2.7: The Difference Between Welfare With Pre-trade Transparency and Welfare With Opaque DM



(a) Asset sensitivity ξ



(b) Autocorrelation κ

Note: Each black line plots the welfare with pre-trade transparency minus the welfare with opaque DM. The red dash line is a reference line of 0. If the black line is higher than (or at) the reference line, then pre-trade transparency (weakly) improves welfare, otherwise, it decreases welfare.

2.7 shows the difference between welfare with pre-trade transparency and welfare with opaque DM with respect to asset sensitivity ξ, volatility $(1-\kappa^2)$, and the number of rounds T. When the number of rounds T is large, and the asset value is stable either due to low sensitivity ξ or low volatility (high κ), low price history informativeness has a persistent and long-run impact. With these conditions, the loss of price history informativeness dominates the utility gain in DM, making pre-trade transparency welfare-decreasing.

This welfare result contrasts Vairo and Dworczak (2023) where they find pre-trade transparency always improves welfare. The key difference is that they focus on the impact of transparency given the decentralized market structure, but we endogenize the impact of pre-trade transparency on dynamic market choice and highlight the loss in price history informativeness.

2.5.2 Coexisting DM&CM vs. CM only

Besides lack of transparency, another concern on DM is market fragmentation. For example, the decentralized corporate bonds market in the U.S. has raised policy concerns about its lack of efficiency. There is an ongoing debate on whether to introduce a centralized market to the decentralized corporate bond market (e.g. Plante, 2017; Kutai et al., 2022; Allen and Wittwer, 2023a).

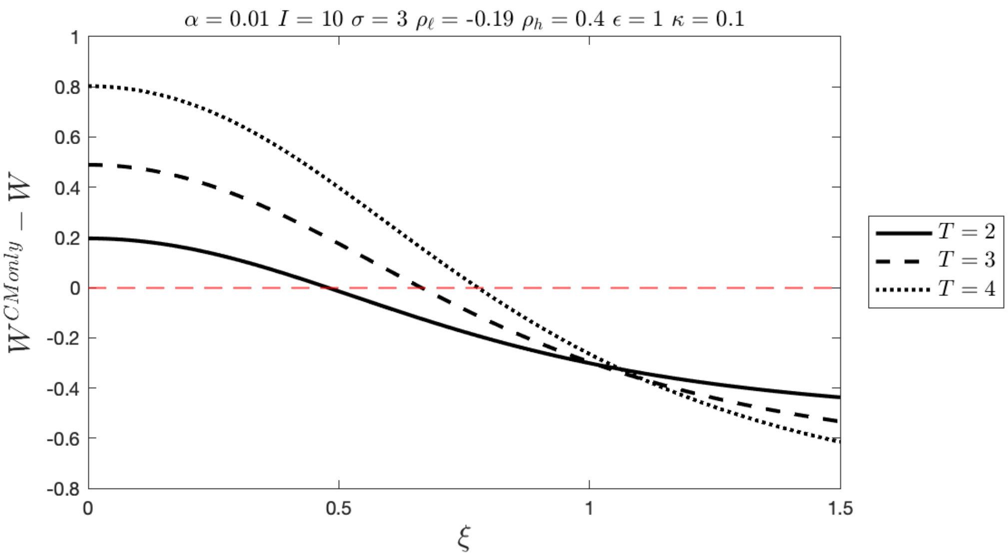
We first consider the welfare impact of providing traders in DM with the option to trade in CM. In our model, we will compare the welfare under the baseline model with both opaque DM and CM and the welfare of the opaque DM market only. We find that introducing CM alongside DM weakly increases welfare (see Proposition 13). [15]

Proposition 13 (Welfare Comparison: DM vs. DM&CM)**.** *Compared with DM only, introducing CM weakly improves welfare.*

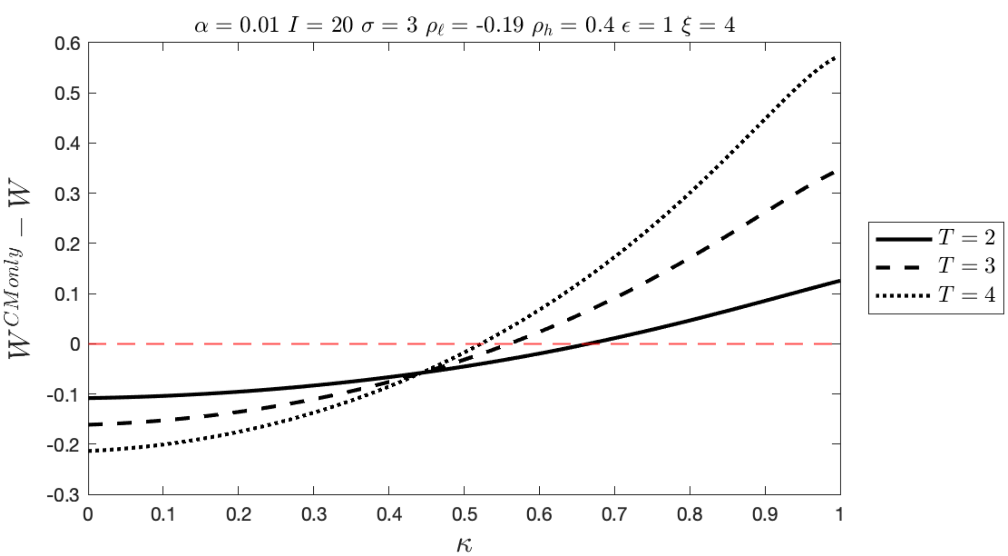
A more radical market design is to move all traders to the centralized market. We model this design as a design where traders no longer have the option to trade in DM.

[15] To allow a more interesting comparison, the DM-market-only case can be relaxed to allow traders to access both CM and DM in the first round but only DM later. Proposition 13 still holds. See proof of Proposition 13 for details.

Figure 2.8: The Difference Between Welfare With Centralized Market Only and Welfare With Parallel Markets with Sufficiently Heterogeneous Correlation



(a) Asset sensitivity ξ



(b) Autocorrelation κ

Note: Each black line plots the welfare with CM only minus the welfare with both DM and CM with sufficiently heterogeneous correlation $\rho_\ell < \underline{\rho}_\ell < 0$ and $\bar{\rho} > \bar{\bar{\rho}}$. The red dash line is a reference line of 0. If the black line is higher than (or at) the reference line, then centralizing the DM improves welfare, otherwise, it decreases welfare.

We find that compared with having access to both CM and DM, centralizing DM can decrease welfare when the value correlations are sufficiently heterogeneous, or when the asset sensitivity and volatility are sufficiently high with trading rounds $T < \bar{T}$. Intuitively, with sufficiently heterogeneous value correlation, traders' gain from trading with the lowest correlation trader is higher than the loss from the less deep market in the DM with no price history, so they prefer DM over CM in the first round. Over time, the transparent CM increases the price history informativeness and the CM welfare. If the asset sensitivity and volatility are sufficiently high with short trading horizons, the improvement in price history informativeness in CM is not large enough to offset the welfare loss in the beginning. Conversely, with sufficiently heterogeneous correlation, when the asset sensitivity and volatility are sufficiently low, with long enough trading horizons, the CM can improve welfare upon the coexisting markets (see Figure 2.8).

Proposition 14 (Welfare Comparison: CM vs. DM&CM)**.** *Compared with the parallel markets of CM and DM,*

1. *Centralizing DM does not change welfare, (i) if values are sufficiently homogenous,* $\bar{\rho} < \bar{\rho}^{*,pre}(I, \rho_\ell, \sigma^2)$ *or (ii) if traders' value correlations are neither sufficiently heterogeneous nor sufficiently homogenous, and the number of rounds* $T < \bar{T}$*, centralizing DM does not change welfare if the asset sensitivity is sufficiently high* $\xi \in [\bar{\xi}, \infty)$*, or volatility is sufficiently high* $\kappa \in [0, \underline{\kappa}]$.

2. *Centralizing DM decreases welfare if trader values are sufficiently heterogeneous* $\rho_\ell < \underline{\rho}_\ell < 0$ *and* $\bar{\rho} > \bar{\bar{\rho}}$*, the asset sensitivity is sufficiently high* $\xi \in [\bar{\xi}^{CM\ only}, \infty)$ *and when the volatility is sufficiently high* $\kappa \in [0, \underline{\kappa}^{CM\ only})$ *with finite rounds* $T < \bar{T}$.

3. *Centralizing DM improves welfare if trader value correlations satisfy* $\bar{\bar{\rho}} < \bar{\rho} < \tilde{\bar{\rho}}$*,* $\underline{\underline{\rho}}_\ell < \rho_\ell < \underline{\rho}_\ell < 0$*, and the number of rounds* $T > \tilde{T}$ *is sufficiently large.*

2.6 Empirical Evidence

The model provides us with some testable predictions. Proposition 7 implies that the assets with higher sensitivity (higher ξ) to shocks to fundamentals are more likely to be traded in CM. Proposition 8 implies that assets with higher volatility (lower κ) are more likely to be traded in CM. The model also predicts a dynamic market choice as price history informativeness grows. Lemma 5 implies that traders are more likely to switch to DM from CM with a more informative price history.

We collect and analyze the Chinese corporate bond market data to test the predictions.

2.6.1 Institutional Background

Before delving into the data, it is beneficial to offer an overview of the distinctive institutional framework of Chinese bond markets.

Parallel OTC and CM: In China, there exist two concurrent bond markets: the interbank market and the exchange market. The interbank bond market, established in 1997, operates similarly to the U.S. interbank bond market and is an over-the-counter (OTC) market. On the other hand, the exchange bond market, inaugurated in 1990, functions as part of the Shanghai and Shenzhen Stock Exchanges and operates as a centralized market. Both markets comprise a cash bond market for primary issuance and secondary trading, as well as a repo market.

Participants. Participants in the two bond markets exhibit some variation, but largely share most non-bank institutional investors. The interbank bond market caters to qualified institutional investors, including commercial banks, mutual funds, insurance companies, and security firms, functioning as a wholesale market. Conversely, the exchange-based bond market operates as a retail market, permitting non-bank institutions, corporate investors, and retail investors to engage in bond investments. Commercial banks' involvement in the exchange market is minimal due to restrictions on repo transactions.

However, many non-bank financial institutions, such as mutual funds, insurance companies, and security firms actively participate in both markets. According to Chen et al. (2023), the non-bank financial institutions take up 76 percent and 57 percent of aggregate enterprise bond holdings, over 80 percent and nearly 50 percent of enterprise bonds' spot transactions on the exchange and interbank markets respectively by the end of 2014.

Bond Products. Bonds traded in the exchange market typically exhibit smaller sizes compared to those in the interbank market. Nonetheless, certain bond products, particularly some enterprise bonds and government bonds, are dual-listed, being traded in both markets. Enterprise bonds are corporate bonds issued by state-owned enterprises or entities with substantial state ownership. Access to enterprise bonds in the exchange market was limited until 2005 when the National Development and Reform Commission (NDRC) granted non-public-listed state-owned enterprises entry to the exchange market. Since then, dual-listed enterprise bonds have experienced significant growth. In 2018, over 28 percent of outstanding enterprise bonds were dual-listed. We will focus on these dual-listed corporate bonds in the following analysis.

Regulators and Clearing Houses. The two markets are overseen by different regulatory bodies. The People's Bank of China (PBOC) serves as the primary regulator of the interbank bond market, while the China Securities Regulatory Commission (CSRC) regulates the exchange market. In the interbank market, trading occurs via the China Foreign Exchange Trade System (CFETS), with clearing services provided by the Shanghai Clearing House (SHCH) and China Central Depository & Clearing Co. Ltd (CCDC), which exclusively offers custodial services. Conversely, in the exchange market, investor bids are consolidated in electronic order books, with the exchange acting as the central clearing house, and all matched trades are settled through the China Securities Depository & Clearing Co. Ltd (CSDC).

Limited Same-day Arbitrage: Several obstacles hinder a trader's ability to trade in both markets on the same day. Firstly, as outlined by Chen et al. (2023), the transfer of bonds between the interbank market (CCDC) and the exchange market (CSDC) took approximately 3-4 working days in 2014, with even longer durations (about 4-6 working days) required to move bonds from the exchange market to the interbank market. Although the transfer process has become faster in recent years, it still entails a significant waiting period. Secondly, transferring funds from the exchange market to the interbank market encounters settlement delays. While the interbank market operates on a "T+0" settlement basis, the exchange market follows a "T+1" settlement model, necessitating a day's wait for fund transfers. Transferring funds from the interbank market to the exchange market also involves time constraints. Typically, if a transaction concludes in the interbank market in the morning, settlement occurs in the afternoon. Given that the exchange market closes at 3:00 pm, executing same-day arbitrage between the two markets becomes nearly infeasible. Thirdly, the settlement fee is relatively high compared to the potential gains from cross-market arbitrage opportunities. Finally, shorting bonds is prohibited in bond markets in China. Therefore traders cannot apply a long-short strategy across the two markets to arbitrage. As a result of these barriers, it is unusual to see traders trading the same bonds in both markets on the same day.

2.6.2 Asset Properties and Market Choices

We obtain daily prices and transaction volume of corporate bonds in China from WIND. We focus on enterprise bonds dual-listed in both the interbank market and the exchange market. The sample period is January 1st 2018 to May 31st, 2018.[16] The observations are at the bond $\times$ day level. Hereafter, we will refer to the interbank market as DM and the exchange market as CM.

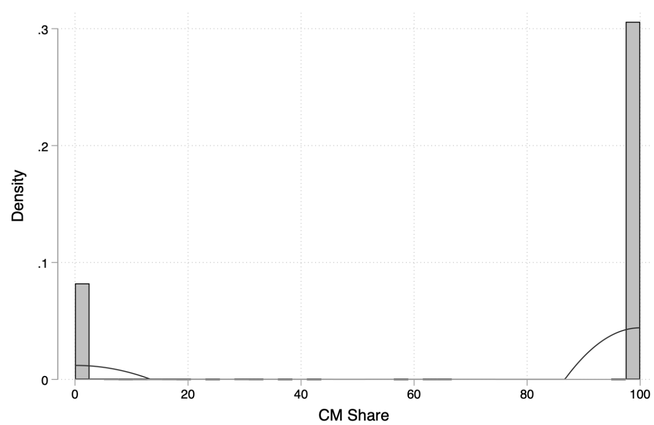
In this section, we tested the relationship between asset properties and market choices

[16]We chose this sample period as there were no collateral-based monetary policies or any changes in bond pledgeability during this period. These policies can create a large divergence in the prices of the same bond in the two markets (see Fang et al. (2020) and Chen et al. (2023)), thus may introduce noises in the measurement of asset value volatility.

(Propositions 7 and 8).

We define CM share as the daily transaction volume in CM as a percentage of the total daily transaction volume at bond × day level. Figure 2.9 shows the distribution of CM share. We can see that most of the mass is distributed on either 0% or 100%. Table 2.1 shows the count of observations by their markets. We find that only around 3% of observations are bonds traded in both markets on the same day. This is largely consistent with our model assumption where traders choose the market before they bid instead of submitting orders to both markets. We define two bond choice dummies, the indicator CM is 1 if the bond is traded in CM on that day and 0 otherwise, and the indicator DM is 1 if the bond is traded in DM on that day and 0 otherwise.

Figure 2.9: Distribution of Centralized Market Share



Note: This figure shows the density of centralized market share for dual-listed corporate bonds traded between Jan. 1st, 2018 and May 31st, 2018.

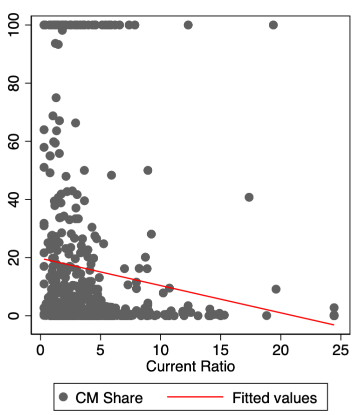
Table 2.1: Count of Observations by Their Markets

	Not traded in DM ($DM = 0$)	Traded in DM ($DM = 1$)	Total
Not traded in CM ($CM = 0$)	-	1,653	1,653
Traded in CM ($CM = 1$)	7,098	299	7,397
Total	7,098	1,952	9,050

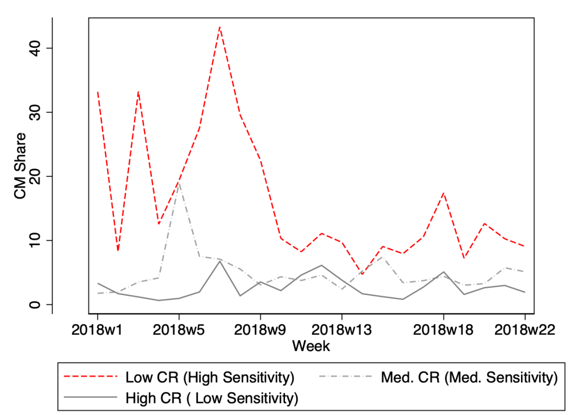
Note: This table shows the number of observations by traders' market choice of the bond on that day.

We use default risk to proxy for the sensitivity of assets to the issuing firms' fundamental value. Given the hockey-stick-like bond payoff structure, we would expect when the default risk is higher, the bond payoff is more sensitive to the issuer's fundamentals. Therefore, we collect the 2017 year-end current ratio for each bond issuer as the proxy for their asset sensitivity.[17] The current ratio (CR) is defined as the ratio of the issuer's current assets to its current liabilities. When CR is lower, the default risk is higher and the bond sensitivity is higher. We winsorize these ratios at 1%.[18] Figure 2.10 (a) shows the scatter plots of average centralized market share across the sample period for each bond with respect to their current ratios. Figure 2.10 (b) shows the average centralized market share for each bond with high (above 75 percentile), median (25-75 percentile), and low (below 25 percentile) current ratios across time. We find that when the current ratio is lower, the bond has a larger overall centralized market share. This is consistent with the prediction of Proposition 7.

Figure 2.10: Current Ratio and the Centralized Market Share



(a) Current Ratio and CM Share



(b) Current Ratio and CM Share Over Time

Note: This figure shows the correlation between the current ratio and the centralized market share of the dual-listed corporate bonds traded during Jan. 1st, 2018 - May 31st, 2018. Panel (a) shows the scatter plot of CM share against the current ratio of the bond issuer with a fitted linear curve. Panel (b) shows the time series of current ratios with respect to low, median, and high levels of current ratios.

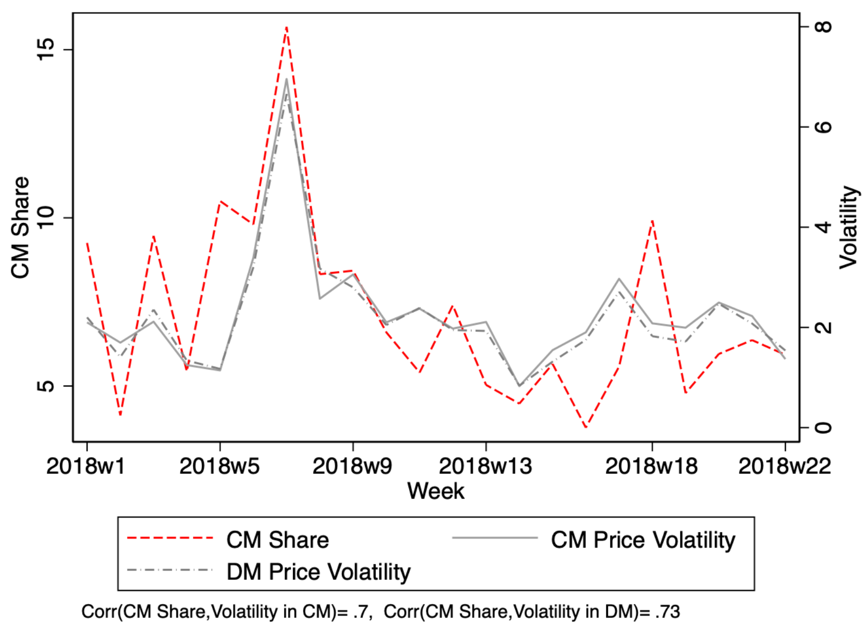
We use the standard deviation of prices between the $t-30$ to $t-1$ trading days as a proxy for the volatility at day t for each bond. Given that we have two markets,

[17] We also use the 2017 year-end debt-to-asset ratio, cash interest ratio of the issuers, and the bond credit rating to proxy the default risk in robustness check.

[18] The regression results are robust without winsorization.

we construct the price volatility for both CM and DM. Figure 2.11 shows the correlation of the average centralized market share and weekly average price volatility across bonds weighted by their transaction volume. We can see that the centralized market share moves in tandem with the volatility measures. The correlation between the centralized market share and the volatility in the CM (DM) market is 0.7 (0.73).[19] Figure 2.11 is consistent with the prediction of Proposition 8, that traders are more likely to choose CM with higher asset volatility.

Figure 2.11: Asset Volatility and the Centralized Market Share



Note: This figure shows the time series of asset price volatility and the centralized market share of the dual-listed corporate bonds traded during Jan. 1st, 2018 - May 31st, 2018. The correlation between the CM price volatility and the CM share is 0.70, and the correlation between the DM price volatility and d the CM share is 0.73.

Panel A of Table 2.2 provides the summary of statistics for the variables used in this section.

We then formally tested Propositions 7 and 8 in a regression framework. We first

[19] There may be concerns that the positive correlation is driven by the high volatility in week 7 of 2018. As a robustness check, we drop the observations in week 7 and calculate the correlations between the weekly average CM share and price volatility measures weighted by bond transaction volume. These correlations remain positive without observations in week 7. The correlation between the centralized market share and the volatility measures in the CM (DM) market is 0.3 (0.39).

Table 2.2: Summary of Statistics

VARIABLES	(1) N	(2) Mean	(3) Std. Dev.	(4) Min	(5) Max
Panel A: Full Sample					
CM Share	9,050	78.54	40.97	0	100
CM	9,050	0.817	0.386	0	1
DM	9,050	0.216	0.411	0	1
Current Ratio	9,050	3.575	3.442	0.291	24.44
CM Price Volatility	9,050	2.992	3.997	0	17.53
DM Price Volatility	9,050	2.820	4.118	0	18.00
Panel B: Switching Sample					
Pr(From CM to DM)	848	0.508	0.500	0	1
Trade Frequency	848	0.586	0.155	0.0769	0.971

Note: This table shows the summary of statistics for the dual-listed corporate bonds traded between Jan. 1st, 2018 and May 31st, 2018. Panel A is statistics of the full sample for regression equations (2.5) and (2.6). Panel B is statistics of the sample where traders switch markets for the bond in the next round for regression equation (2.7).

test the model predictions using a probit regression, with the indicator dummy CM (or DM) as the dependent variable and the current ratio and the volatility measures as independent variables. Note that 3% of the observations are the same bonds traded in both CM and DM within a day, the coefficients in the following two probit regressions are not exactly opposite.

$$\begin{aligned} Pr(CM_{it} = 1) &= \Phi(\beta_0 + \beta_1 \text{Volatility}_{it} + \beta_2 \text{Current Ratio}_i) \\ Pr(DM_{it} = 1) &= \Phi(\beta_0 + \beta_1 \text{Volatility}_{it} + \beta_2 \text{Current Ratio}_i) \end{aligned} \tag{2.5}$$

where i is the index for bonds and t is the index for dates. Table 2.3 shows the regression results of equation (2.5). We find that higher price volatility and asset sensitivity increase the probability of traders choosing the centralized market, consistent with the patterns in Figures 2.10 and 2.11.

Table 2.3: Market Choice and Asset Properties

VARIABLES	(1) Pr(CM=1)	(2) Pr(CM=1)	(3) Pr(DM=1)	(4) Pr(DM=1)
Current Ratio	-0.0443***	-0.0442***	0.0399***	0.0398***
	(0.00411)	(0.00410)	(0.00404)	(0.00403)
CM Price Volatility	0.0403***		-0.0371***	
	(0.00428)		(0.00402)	
DM Price Volatility		0.0353***		-0.0309***
		(0.00401)		(0.00378)
Constant	0.964***	0.982***	-0.833***	-0.854***
	(0.0244)	(0.0239)	(0.0236)	(0.0231)
Observations	9,050	9,050	9,050	9,050
Pseudo R-squared	0.0245	0.0223	0.0197	0.0171

Note: This table shows the impact of asset properties on trader's market choice from probit regression equation (2.5). We focus on dual-listed corporate bonds in China. The sample period is Jan. 1st, 2018, and May 31st, 2018. We include robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

Instead of running regressions, we can also use CM share as the dependent variable to test Propositions 7 and 8, with the asset properties as the independent variable, controlling for the date fixed effects,

$$\text{CM Share}_{it} = \beta_1 \text{Current Ratio}_{it} + \beta_2 \text{Volatility}_{it} + \text{Date FE} + \varepsilon_{it} \tag{2.6}$$

where i is the index for bonds and t is the index for dates, and ε are robust standard errors.

Table 2.4 shows the regression results of equation (2.6). We find that higher price volatility and asset sensitivity increase centralized market share, consistent with our previous findings and model predictions.

2.6.3 Price History and Market Choices

In this section, we directly test the mechanism of the model that when the price history is more informative, traders are more likely to choose DM (Lemma 5). This implies that bonds with more price history are more likely to be traded in DM.

Table 2.4: CM Share and Asset Properties

VARIABLES	(1) CM Share	(2) CM Share
Current Ratio	-1.339***	-1.338***
	(0.142)	(0.142)
CM Price Volatility	0.847***	
	(0.118)	
DM Price Volatility		0.689***
		(0.111)
Date FE	Yes	Yes
Observations	9,050	9,050
R-squared	0.068	0.066

Note: This table shows the impact of asset properties on the CM share of the bond from regression equation (2.6). We focus on dual-listed corporate bonds in China. The sample period is Jan. 1st, 2018, and May 31st, 2018. We include robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

The difficulty in testing this prediction is that we do not know the threshold $\tilde{\eta}$ and therefore the exact time for traders to switch from CM to DM. The price history informativeness η may be above the threshold, such that past CM trades lead to DM trades. It is also possible for η to be below the threshold, such that past CM trades still lead to more CM trades. Therefore, simply running a probit regression with the number of past trades as the independent variable and the current round market choice as the dependent variable is not feasible.

To test Lemma 5, we can focus on the sample when traders change their market choice. Lemma 5 implies that conditional on traders changing their market, more (less) past trades incentivize traders to switch from CM to DM (from DM to CM).

We drop the observations when the same bond is traded in the same market for two consecutive trades, and focus on the observations that the next trading market differs from the current market. We define the dependent variable $FromCMtoDM_{it}$ as a dummy which takes 1 if the bond i is traded in CM on trading day t but in DM the next time when it is traded, and 0 otherwise. We use $TradeFrequency \equiv \frac{TradeCount}{TotalTradeCount}$ to proxy for price history informativeness. $TradeCount$ is the number of days traded in

DM and CM for each bond during the last 60 trading days, i.e. $[t-60, t-1]$ for bonds traded on trading day t. The $TotalTradeCount$ is the total number of trades during the sample period from Jan. 1, 2018, to May 31, 2018, for each bond. Panel B of Table 2.2 provides the statistics of the variables used in this section.

We then tested the relationship between price history and traders' market choices with the following probit regression,

$$Pr(FromCMtoDM_{it}) = \Phi(\beta_0 + \beta_1 TradeFrequency) \tag{2.7}$$

Table 2.5 column (1) shows the regression results of equation (2.7). We find that consistent with the model prediction, traders are more likely to shift from CM to DM when they observe more past trades.

Table 2.5: The Impact of Past Trading Frequency on Market Choices

	(1)
VARIABLES	Pr(From CM to DM)
Trade Frequency	1.046***
	(0.284)
Constant	-0.593***
	(0.172)
Observations	848
Pseudo R-squared	0.0118

Note: This table shows the impact of past trade frequency on the probability for traders to switch from CM to DM from regression equation (2.7). We focus on dual-listed corporate bonds in China. The sample period is Jan. 1st, 2018, and May 31st, 2018. We include robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

2.6.4 Other Empirical Results

Amplification Effects We also examine whether the volatility could amplify the impact of asset sensitivity on traders' market choices. Table 2.6 shows the regression results of equation (2.5) with an additional interaction term of current ratio and volatility. We find that higher volatility together with higher sensitivity (i.e. lower current ratio) further increases the probability for traders to choose CM, and decreases the probability for traders to choose CM. Table 2.7 shows that the amplification effect, despite being consistent in the signs, is not statistically significant in regression equation (2.6) with the interaction term.

Robustness Check 1. Other Default Risk Measures One may wonder whether our results hold robustly to other default risk measures. To verify this, we include the 2017 year-end debt-to-asset ratio and cash interest ratio of the bond issuers, and the bond credit rating on each day in regressions (2.5) and (2.6) as a robustness check.[20] Table 2.8 Panel A shows the statistics of these additional default risk measures. Tables 2.9 and 2.10 show that the coefficients on the current ratio and the volatility measure do not differ much from the baseline regressions. The coefficients on the 2017 year-end debt-to-asset ratio and cash-interest ratio of the bond issuers are not significant. The coefficients on the bond credit rating AA and AA+ are negative, mostly driven by the fact that retail traders in CM trade more AAA bonds than those bonds with lower ratings.

Robustness Check 2. Alternative Trade Frequency Measure We include alternative trade frequency measure as a robustness check. We measure $TradeCount$ with the number of days traded in DM and CM for each bond in the last 30 instead of 60 trading days. Table 2.8 Panel B shows the statistics of this alternative trade count measure and other default risk measures for equation (2.7). Table 2.11 shows the regression results. We find with the new measure, the results are still consistent with the model prediction that more trade history makes traders more likely to switch from CM to DM.

Additional Evidence from the U.S. Markets: We provided additional empirical evidence for Proposition 8 with the U.S. equity market data (see Appendix 2.11). Also, by comparing the average centralized market share of bonds in China and the equities in

[20]The the 2017 year-end debt-to-asset ratio, and cash interest ratio of the bond issuers are winsorized at 1% to avoid extreme values. There are three credit ratings for bonds in the sample, AA, AA+, and AAA. The AAA corporate bonds are taken as the reference group in the regression.

the U.S. we obtain an additional piece of evidence in support of Proposition 7: The assets more sensitive to shocks to fundamentals such as equities generally have higher centralized market share than those with lower sensitivity such as bonds.

2.7 Conclusion and Discussions

This paper presents a model examining the dynamic market choice between centralized and decentralized markets, where arriving traders must decide between a centralized market and a bilaterally matched decentralized market in each period. The emergence of dynamic market choice is observed as a consequence of learning from the centralized market price history. Optimal market choices, influenced by asset properties, include switching between centralized and decentralized markets when traders' value correlation is moderately heterogeneous. In cases where asset values are insensitive to shocks or shocks are predictable, traders switch between centralized and decentralized markets or stay in the decentralized market after one round in the centralized market. Conversely, when asset values are sensitive to unpredictable fundamentals, traders choose to stay in the centralized market.

It is interesting to see that learning from price history alone generates rich dynamic market choices. It is also important to recognize that we abstract away from the inventory held by traders by assuming short-lived traders. Adding dynamic inventory significantly reduces tractability in the linear-quadratic double auction setting like this paper. Inventory management across rounds is also an important aspect of trading strategies. We believe dynamic market choice with both dynamic inventory and learning warrants future research.

The empirical results can also be explained by alternative theories. For instance, the positive correlation between centralized market share and volatility measures might be attributed to retail traders, who typically trade in the centralized market, increasing their trading activity during periods of higher volatility. However, we cannot empirically rule out such alternative explanations since our dataset does not identify individual traders.

These possibilities remain open for investigation in future studies.

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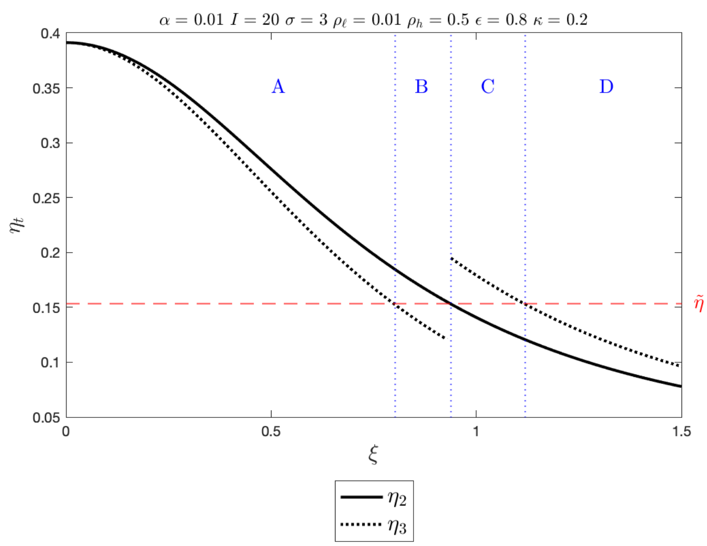
Appendices

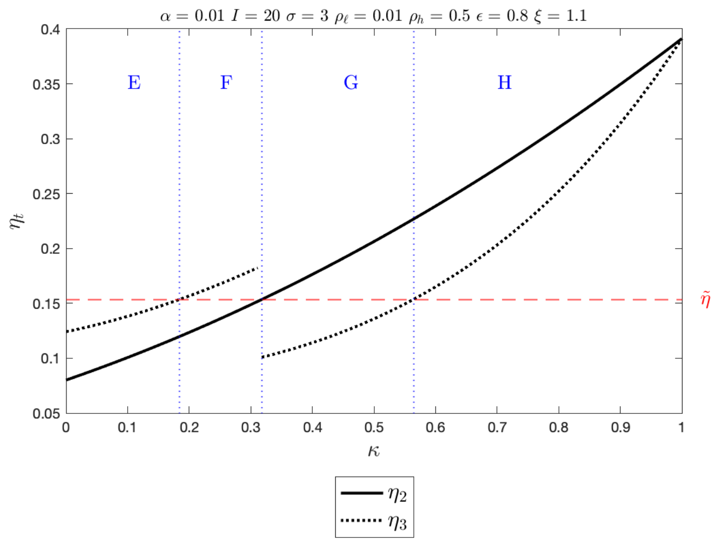
2.8 Additional Tables and Figures

Figure 2.12: Price History Informativeness with Asset Sensitivity ξ in $T = 3$ Market



Note: $\eta_1 = 0$. The black solid line plots η_2, and the black dotted line plots η_3. The red dashed line is a reference line of $\tilde{\eta}$. When the black solid (dotted) line is above the reference line, then the traders choose DM in the 2nd round (3rd round), and if it is below the reference line, the traders choose CM in the 2nd round (3rd round). The jump in the difference of utility in CM vs. DM in the third round comes from the difference in the second-round choice.
In region A, i.e. $\xi \in [0, \underline{\xi})$, traders choose DM in both 2nd and 3rd round. In region B, i.e. $\xi \in (\underline{\xi}, \bar{\xi}]$ and in the lower partition, traders choose DM in the 2nd round and CM in the 3rd round. In region C, $\xi \in (\underline{\xi}, \bar{\xi}]$ and in the higher partition, traders choose CM in the 2nd round and DM in the 3rd round. In region D, $\xi \in (\bar{\xi}, 1]$, traders choose CM in both the 2nd round and the 3rd round.

Figure 2.13: Price History Informativeness with Autocorrelation κ in $T = 3$ Market



Note: $\eta_1 = 0$. The black solid line plots η_2, and the black dotted line plots η_3. The red dashed line is a reference line of $\tilde{\eta}$. When the black solid (dotted) line is above the reference line, then the traders choose DM in the 2nd round (3rd round), and if it is below the reference line, the traders choose CM in the 2nd round (3rd round). The jump in the difference of utility in CM vs. DM in the third round comes from the difference in the second-round choice.

In region E, i.e. $\kappa \in [0, \underline{\kappa}]$, traders choose CM in both 2nd and 3rd round. In region F, i.e. $\kappa \in (\underline{\kappa}, \bar{\kappa}]$ and in the lower partition, traders choose CM in the 2nd round and DM in the 3rd round. In region G, $\kappa \in (\underline{\kappa}, \bar{\kappa}]$ and in the higher partition, traders choose DM in the 2nd round and CM in the 3rd round. In region H, $\kappa \in (\bar{\kappa}, 1]$, traders choose DM in both the 2nd round and the 3rd round.

Table 2.6: Amplification Effect: Market Choice and Asset Properties with Interaction Term of Volatility Measures and Current Ratio

	(1)	(2)	(3)	(4)
VARIABLES	Pr(CM=1)	Pr(CM=1)	Pr(DM=1)	Pr(DM=1)
Current Ratio	-0.0389***	-0.0401***	0.0353***	0.0366***
	(0.00473)	(0.00452)	(0.00467)	(0.00446)
CM Price Volatility	0.0535***		-0.0476***	
	(0.00718)		(0.00673)	
Current Ratio × Exchange Volatility	-0.00321**		0.00264**	
	(0.00139)		(0.00134)	
DM Price Volatility		0.0474***		-0.0397***
		(0.00682)		(0.00640)
Current Ratio × Interbank Volatility		-0.00293**		0.00217*
		(0.00132)		(0.00128)
Constant	0.943***	0.966***	-0.816***	-0.842***
	(0.0260)	(0.0250)	(0.0251)	(0.0242)
Observations	9,050	9,050	9,050	9,050
Pseudo R-squared	0.0251	0.0229	0.0201	0.0174

Note: This table shows the impact of asset properties on the CM share of the bond from regression equation (2.5) with an additional interaction term between the current ratio and the volatility measure. We focus on dual-listed corporate bonds in China. The sample period is Jan. 1st, 2018, and May 31st, 2018. We include robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

Table 2.7: Amplification Effect: CM Share and Asset Properties with Interaction Term of Volatility Measures and Current Ratio

VARIABLES	(1) CM Share	(2) CM Share
Current Ratio	-1.314***	-1.321***
	(0.156)	(0.154)
CM Price Volatility	0.895***	
	(0.170)	
Exchange Volatility × Current Ratio	-0.0129	
	(0.0377)	
DM Price Volatility		0.725***
		(0.167)
Interbank Volatility × Current Ratio		-0.00970
		(0.0371)
Observations	9,050	9,050
R-squared	0.068	0.066
Date FE	Yes	Yes

Note: This table shows the impact of asset properties on the CM share of the bond from regression equation (2.6) with an additional interaction term between the current ratio and the volatility measure. We focus on dual-listed corporate bonds in China. The sample period is Jan. 1st, 2018, and May 31st, 2018. We include robust standard errors in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.1$.

Table 2.8: Robustness Check: Summary of Statistics for Other Default Risk Measures

VARIABLES	(1) N	(2) Mean	(3) Std. Dev.	(4) Min	(5) Max
Panel A: Full Sample					
Debt-to-Asset Ratio	9,050	54.92	11.97	26.03	77.36
Cash Interest Ratio	9,050	-21.91	211.8	-1,935	385.9
AA	9,050	0.429	0.495	0	1
AA+	9,050	0.344	0.475	0	1
Panel B: Switching Sample					
Trade Frequency (in 30 days)	1,193	0.336	0.159	0.0294	0.969
Pr(From CM to DM)	1,193	0.490	0.500	0	1

Note: This table shows the summary of statistics for risk measures other than the current ratio. We focus on dual-listed corporate bonds in China. The sample period is Jan. 1st, 2018, and May 31st, 2018. Panel A is statistics of the full sample for regression equations (2.5) and (2.6). Panel B is statistics of the sample where traders switch markets for the bond in the next round for regression equation (2.7), where the trade frequency is measured in the last 30 days instead of the last 60 days.

Table 2.9: Robustness Check: Market Choices with Other Default Risk Measures

VARIABLES	(1) Pr(CM=1)	(2) Pr(CM=1)	(3) Pr(DM=1)	(4) Pr(DM=1)
Current Ratio	-0.0395***	-0.0410***	0.0357***	0.0373***
	(0.00508)	(0.00487)	(0.00500)	(0.00480)
CM Price Volatility	0.0533***		-0.0477***	
	(0.00729)		(0.00684)	
Current Ratio × Exchange Volatility	-0.00323**		0.00267**	
	(0.00140)		(0.00135)	
DM Price Volatility		0.0475***		-0.0400***
		(0.00699)		(0.00655)
Current Ratio × Interbank Volatility		-0.00294**		0.00219*
		(0.00133)		(0.00129)
Debt-to-Asset Ratio	-0.000610	-0.000823	0.000559	0.000739
	(0.00147)	(0.00147)	(0.00140)	(0.00140)
Cash Interest Ratio	8.44e-05	8.21e-05	-9.55e-05	-9.18e-05
	(6.74e-05)	(6.73e-05)	(6.74e-05)	(6.72e-05)
AA	-0.0338	-0.0385	0.0520	0.0529
	(0.0420)	(0.0425)	(0.0405)	(0.0409)
AA+	-0.0989**	-0.108**	0.130***	0.137***
	(0.0418)	(0.0419)	(0.0403)	(0.0404)
Constant	1.030***	1.070***	-0.918***	-0.957***
	(0.0980)	(0.0982)	(0.0940)	(0.0940)
Observations	9,050	9,050	9,050	9,050
Pseudo R-squared	0.0260	0.0239	0.0215	0.0190

Note: This table shows the impact of asset properties on the CM share of the bond from regression equation (2.5) with (1) an additional interaction term between the current ratio and the volatility measure; and (2) default risk measures other than the current ratio, Debt-to-Asset Ratio, Cash Interest Ratio and bond rating dummies (AA and AA+). We focus on dual-listed corporate bonds in China. The sample period is Jan. 1st, 2018, and May 31st, 2018. We include robust standard errors in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.1$.

Table 2.10: Robustness Check: CM Share with Other Default Risk Measures

VARIABLES	(1) CM Share	(2) CM Share
Current Ratio	-1.332***	-1.344***
	(0.164)	(0.162)
CM Price Volatility	0.922***	
	(0.173)	
Current Ratio × Exchange Volatility	-0.0147	
	(0.0384)	
DM Price Volatility		0.766***
		(0.172)
Current Ratio × Interbank Volatility		-0.0122
		(0.0379)
Debt-to-Asset Ratio	-0.0368	-0.0389
	(0.0414)	(0.0415)
Cash Interest Ratio	0.00245	0.00236
	(0.00187)	(0.00187)
AA	-3.268**	-3.262**
	(1.349)	(1.356)
AA+	-5.584***	-5.751***
	(1.235)	(1.240)
Date FE	Yes	Yes
Observations	9,050	9,050
R-squared	0.070	0.069

Note: This table shows the impact of asset properties on the CM share of the bond from regression equation (2.6) with (1) an additional interaction term between the current ratio and the volatility measure; and (2) default risk measures other than the current ratio, Debt-to-Asset Ratio, Cash Interest Ratio and bond rating dummies (AA and AA+). We focus on dual-listed corporate bonds in China. The sample period is Jan. 1st, 2018, and May 31st, 2018. We include robust standard errors in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.1$.

Table 2.11: Robustness Check: Market Choice and Alternative Measure of Price History

	(1)
VARIABLES	Pr(From CM to DM)
Trade Frequency	0.460**
	(0.228)
Constant	-0.179**
	(0.0849)
Observations	1,193
Pseudo R-squared	0.00246

Note: This table shows the impact of past trading frequency on the probability for traders to switch from CM to DM from regression equation (2.7) with past trading frequency measured by the bond's trade count in the last 30 trading days over the total trade counts. We focus on dual-listed corporate bonds in China. The sample period is Jan. 1st, 2018, and May 31st, 2018. We include robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

2.9 Extensions

2.9.1 General Private Information Precision

In the baseline model, traders receive private signals with constant precision. In this section, we provide an extension with general private information precision. We find that traders are more likely to choose CM after rounds with less precise private signals. We show that the traders can still switch back and forth between CM and DM, even with post-trade transparent DM (as opposed to Proposition 9), when the precise signals do not arrive frequently.

We generalize the private signals' precision to accommodate different information arrival frequencies. Assume trader will receive a private signal $s_{i,t} = \theta_{i,t} + \varepsilon_{i,t}$, where $\varepsilon_{i,t} \sim \mathcal{N}(0, \sigma_{i,t}^2 \sigma_\theta^2)$, $\sigma_{i,t} \geq ((\frac{2(I-1)}{I})^{1/3} - 1)^{-1/2}$. Note that $\sigma_{i,t} \to \infty$ is equivalent to traders not receiving private signals.

With the above generalization, we can solve the trading equilibrium as follows:

Proposition 15 (Trading Equilibrium with General Private Information)**.** *Given the price history $\mathcal{H}_t$ and the market structure $\mathcal{M}_t^*$, the equilibrium at round t can be characterized by a fixed point of inference coefficients,*

$$c_{s,t} = \frac{1 - \rho_{t,\mathcal{M}^*}}{1 - \rho_{t,\mathcal{M}^*} + \sigma_{i,t}^2}$$

$$\boldsymbol{c}_{\mathcal{H},t} = \frac{(1 - \rho_{t,\mathcal{M}^*})\sigma_{i,t}^2}{\left(1 - \rho_{t,\mathcal{M}^*} + \sigma_{i,t}^2\right)(1 + (I_{t,\mathcal{M}^*} - 1)\rho_{t,\mathcal{M}^*} - I_{t,\mathcal{M}^*}\eta_t)} \boldsymbol{\tau}_t' \boldsymbol{\Upsilon}_t^{-1}$$

$$c_{p,t} = \frac{I_{t,\mathcal{M}^*}(\rho_{t,\mathcal{M}^*} - \eta_t)\sigma_{i,t}^2}{\left(1 - \rho_{t,\mathcal{M}^*} + \sigma_{i,t}^2\right)(1 + (I_{t,\mathcal{M}^*} - 1)\rho_{t,\mathcal{M}^*} - I_{t,\mathcal{M}^*}\eta_t)}$$

where $\eta_t = \frac{var(\theta_{i,t}) - var(\theta_{i,t}|\mathcal{H}_t)}{var(\theta_{i,t})} = \frac{\boldsymbol{\tau}_t'(\boldsymbol{\Upsilon}_t)^{-1}\boldsymbol{\tau}_t}{\sigma_\theta^2}$*,* $\boldsymbol{\tau}_t \equiv cov(\mathcal{H}_t, \theta_{i,t}) \in \mathbb{R}^{|\mathcal{H}|}$*, and* $\boldsymbol{\Upsilon}_t \equiv cov(\mathcal{H}_t, \mathcal{H}_t') \in \mathbb{R}^{|\mathcal{H}| \times |\mathcal{H}|}$*.*

The equilibrium price impact is

$$\lambda_t = \frac{\alpha}{(I_{t,\mathcal{M}^*} - 1)(1 - c_{p,t}) - 1}, \quad \forall i$$

The utility conditional on $\mathcal{H}_t$ for trader i is

$$\mathbb{E}[U_{i,t}^{\mathcal{M}^*}|\mathcal{H}_t] = \frac{\alpha + 2\lambda_t}{2(\alpha+\lambda_t)^2}\mathbb{E}[(\mathbb{E}[\theta_{i,t}|\mathcal{H}_t, s_{i,t}, p_t] - p_t)^2|\mathcal{H}_t] = \frac{\alpha + 2\lambda_t}{2(\alpha+\lambda_t)^2}\frac{I_{t,\mathcal{M}^*} - 1}{I_{t,\mathcal{M}^*}}\frac{(1-\rho_{t,\mathcal{M}^*})^2}{1 - \rho_{t,\mathcal{M}^*} + \sigma_{i,t}^2}, \quad \forall i$$

The dynamic market equilibrium is characterized as follows:

Theorem 14 (Algorithm for Dynamic Market Choice Equilibrium with General Private Information). *The Bayesian Nash equilibrium is a set of price history $\{\mathcal{H}_t\}_t$, a sequence of market choice $\{\mathcal{M}_t^*\}_t$, and a set of inference coefficients $\{c_{s,t}, c_{p,t}, \boldsymbol{c}_{\mathcal{H},t}\}$ that characterized forwardly recursively.*

1. *Initialize with $t = 1$, $\mathcal{H}_1 = \emptyset$.*

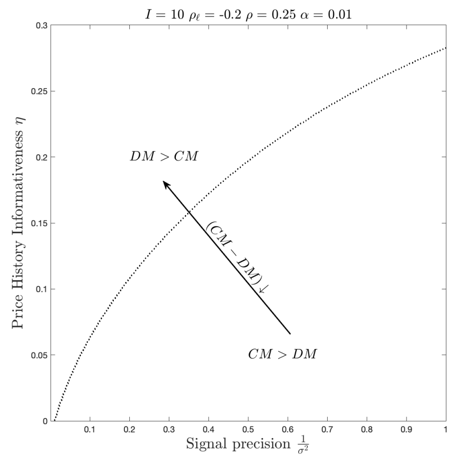
2. *Given $\mathcal{H}_t$, the equilibrium inference coefficients $\{c_{s,t}, c_{p,t}, \boldsymbol{c}_{\mathcal{H},t}\}$ is characterized in Proposition 15 with $\rho_{t,\mathcal{M}^*} = \rho_\ell$ $I_{t,\mathcal{M}^*} = 2$ if $\mathcal{M}^* = DM$, and $\rho_{t,\mathcal{M}^*} = \bar{\rho}$ $I_{t,\mathcal{M}^*} = I$ if $\mathcal{M}^* = CM$.*

3. *Given inference coefficients $\{c_{s,t}, c_{p,t}, \boldsymbol{c}_{\mathcal{H},t}\}$, If $\frac{\alpha+2\lambda_t^{CM}}{2(\alpha+\lambda_t^{CM})^2}\frac{I_t-1}{I_t}\frac{(1-\bar{\rho}_t)^2}{1-\bar{\rho}_t+\sigma_{i,t}^2} \geq \frac{\alpha+2\lambda_t^{DM}}{4(\alpha+\lambda_t^{DM})^2}\frac{(1-\rho_\ell)^2}{1-\rho_\ell+\sigma_{i,t}^2}$, then $\mathcal{M}_t^* = CM$, $\mathcal{H}_{t+1} = \mathcal{H}_t \cup \{p_t\}$; otherwise, $\mathcal{M}_t^* = DM$, $\mathcal{H}_{t+1} = \mathcal{H}_t$. Repeat Steps 2-3 with the next t, until $t = T$.*

Differences Compared to the Baseline Model: There are two major differences in the model with general private signals compared with the baseline model.

First, the traders are more likely to go to DM in rounds with less precise private information. The threshold of price history informativeness to choose DM over CM in rounds with noisy private signals is lower than the threshold with precise private signals. Figure 2.14 compares the utility of each trader in CM versus DM. We can see that when the private signals become less precise, the threshold of price history informativeness for traders to choose DM over CM becomes lower.

Figure 2.14: Utility in CM vs. DM With Private Signal Precision and Price History Informativeness



Note: This figure shows the comparison of utility (welfare) in CM versus DM with private signal precision $\frac{1}{\sigma^2}$ and price history informativeness η. When the price history informativeness is higher given precision, or when the signal precision is lower (i.e. σ is higher) given price history informativeness, the difference between utilities in DM and CM becomes larger.

Second, the traders are more likely to go to CM after rounds without private signals, as price history informativeness decreases. Intuitively, the price aggregates private signals, and it is less informative when the private signals are less precise. The price history informativeness η_t will decrease after a round with extremely noisy signals. Traders will choose CM if the price history informativeness falls below the threshold by Lemma 4. In rounds traders do not receive private signals ($\sigma_{i,t} \to \infty$), there will be no trade in those rounds, and the price history informativeness decays with time.[21]

With Post-trade Transparency: When DM is post-trade transparent, different from Proposition 9 where traders stay in DM once they choose DM, traders can switch back and forth with the general information structure. By Lemma 4, we would expect the traders to choose CM if the price history informativeness is below the threshold $\tilde{\eta}$ after sufficient

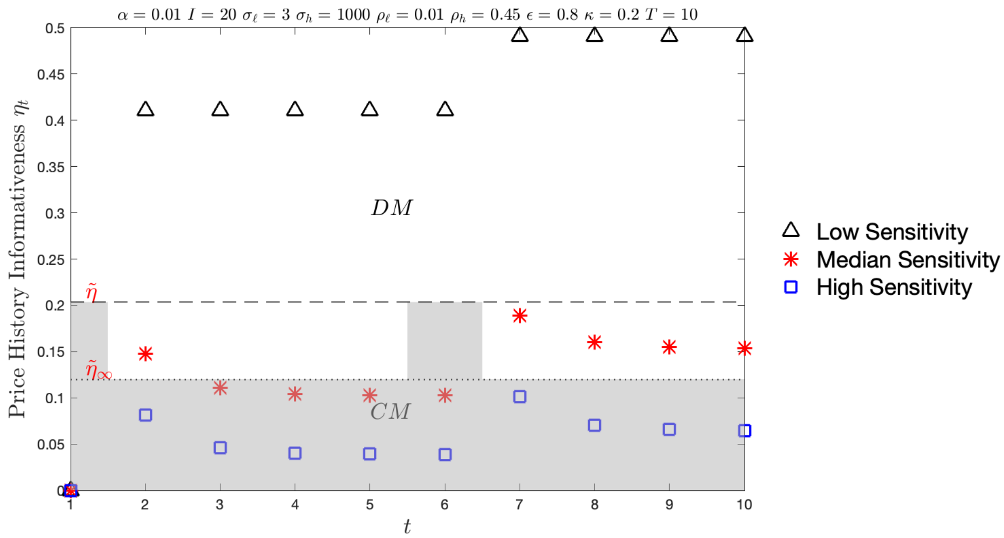
[21]The price is a linear combination of past prices in rounds without private signals. $\mathcal{H}_t$ is a sufficient statistic for p_t when $\sigma \to \infty$. Even if traders can observe the price, they do not learn from it conditional on $\mathcal{H}_t$.

rounds of extremely noisy signals, regardless of post-trade transparency.

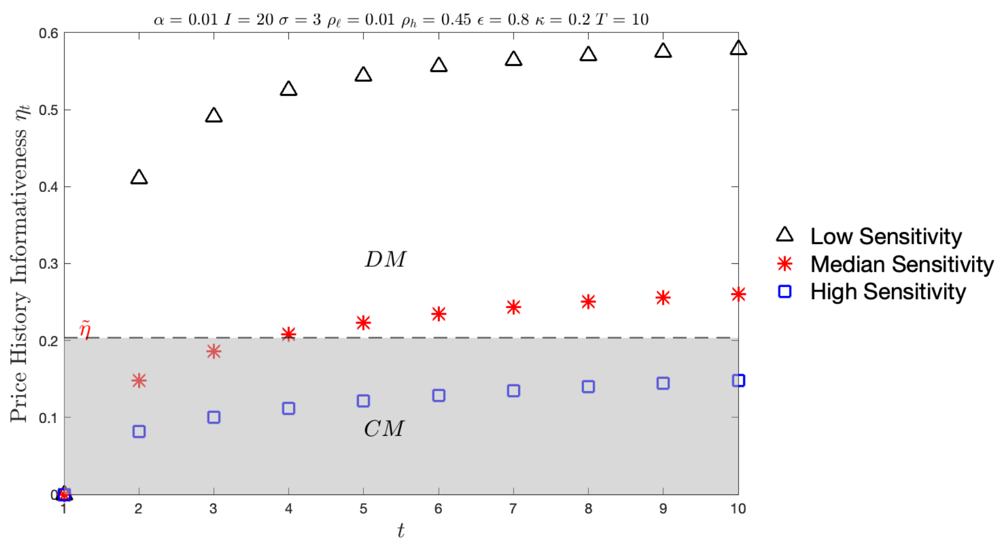
Example 4 (Infrequent Arrival of Private Signals)**.** *We consider a $T = 10$ market where DM is post-trade transparent. Each trader will receive a private signal $s_{i,t} = \theta_{i,t} + \varepsilon_{i,t}$, where $\varepsilon_{i,t} \sim \mathcal{N}(0, \sigma_{i,t}^2 \sigma_\theta^2)$, $\sigma_{i,t} = \sigma_\ell$ in rounds $t = 5k + 1$, $k \in \mathbb{N}$, and $\sigma_{i,t} = \sigma_h \gg \sigma_\ell$ otherwise. This setup captures a case where traders receive precise private signals in rounds 1 and 6, and extremely noisy signals in other rounds.*

Figure 2.15 shows the simulated evolution of price history informativeness for different asset sensitivities. Panel (a) shows the price history informativeness and corresponding traders' market choices with the infrequent arrival of precise private signals. In panel (b) we include an example as the baseline model with post-trade transparent DM for comparison. Panel (a) indicates that when the sensitivity is neither too high nor too low and precise private signals arrive infrequently, the traders switch back and forth between CM and DM. In this example, traders choose DM in rounds 2, 7, 8, 9, and 10 and CM in other rounds. In contrast, with private signals being precise in every round (Panel (b)), traders choose to stay in DM once they enter DM (see Proposition 9). Figure 2.16 shows a similar pattern for the evolution of price history informativeness with different levels of asset volatilities (autocorrelations).

Figure 2.15: Evolution of Price History Informativeness For Different Asset Sensitivities with Post-trade Transparent DM



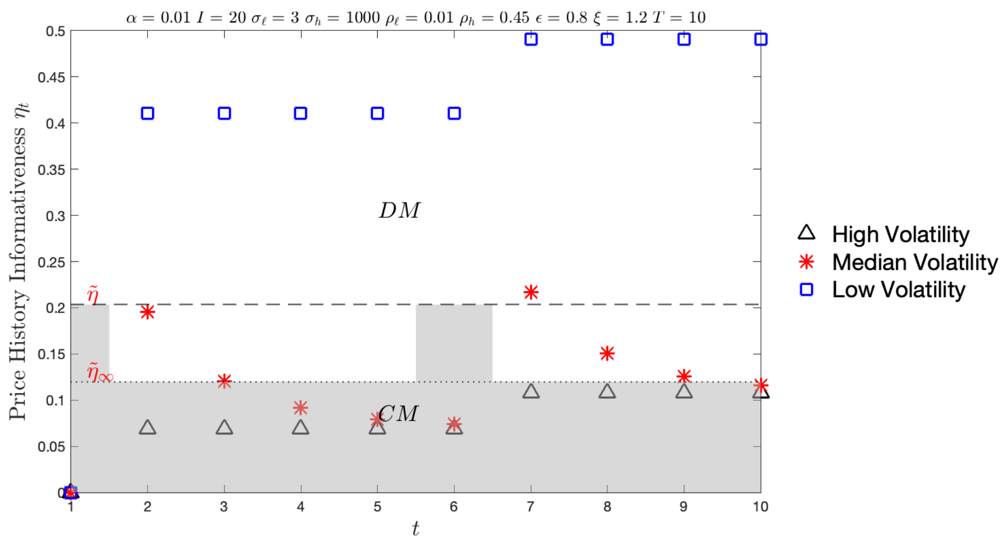
(a) Infrequent Precise Private Signals (Example 4)



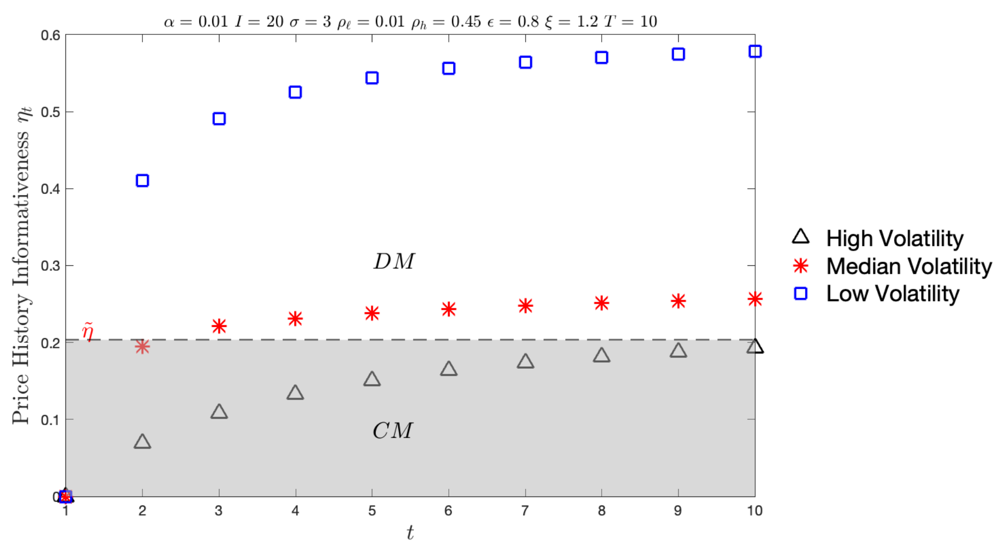
(b) Frequent Precise Private Signals

Note: This figure shows the evolution of price history informativeness for different levels of asset sensitivity ξ for $T = 10$ when DM is post-trade transparent. The black dashed line is a reference line of threshold $\tilde{\eta}$ with private signals. The black dotted line is a reference line of threshold $\tilde{\eta}_\infty$ with noisier private signals. When the marker is in the shaded area, the history informativeness in that round is lower than $\tilde{\eta}$ (or $\tilde{\eta}_\infty$ in rounds with noisier private signals) and traders choose CM. When the marker is in the unshaded area, the history informativeness in that round is higher than $\tilde{\eta}$ (or $\tilde{\eta}_\infty$ in rounds with noisier private signals) and traders choose DM.

Figure 2.16: Evolution of Price History Informativeness For Different Autocorrelations with Post-Trade Transparent DM



(a) Infrequent Precise Private Signals (Example 4)



(b) Frequent Precise Private Signals

Note: This figure shows the evolution of price history informativeness for different levels of autocorrelation κ for $T = 10$ when DM is post-trade transparent. The black dashed line is a reference line of threshold $\tilde{\eta}$ with private signals. The black dotted line is a reference line of threshold $\tilde{\eta}_\infty$ with noisier private signals. When the marker is in the shaded area, the history informativeness in that round is lower than $\tilde{\eta}$ (or $\tilde{\eta}_\infty$ in rounds with noisier private signals), and traders choose CM. When the marker is in the unshaded area, the history informativeness in that round is higher than $\tilde{\eta}$ (or $\tilde{\eta}_\infty$ in rounds with noisier private signals) and traders choose DM.

2.9.2 Non-movers and Trading Volumes

The baseline model in the paper models traders who move without frictions across decentralized and centralized markets. However, in practice, there are non-movers who trade only in one market due to regulations, lack of information, or entry cost. For example, banks are not allowed to trade in the centralized Exchange Bond Market in China, and retail traders hardly trade in dark pools in the United States. We can extend the model by adding non-movers in both the centralized and decentralized markets.

Let us add to the baseline model $I_d = 2N_d$ traders that always trade in the decentralized market. These traders are ex-ante identical to the traders in the baseline model except that they are non-movers. We also have a continuum of non-strategic retail traders who always trade in the centralized market with exogenous demand, such that the market clearing price is $p_t = A_t + \frac{B_t}{I} \sum_i q_{i,t}$, where A_t and B_t are constant known to all traders. Intuitively, if DM is opaque, adding these non-movers does not affect the price informativeness and movers' choice of venues, but the price levels and total trading volumes in CM and DM. If DM is post-trade transparent, more non-movers in DM increases the price history informativeness, and thus future traders' incentive to choose DM.

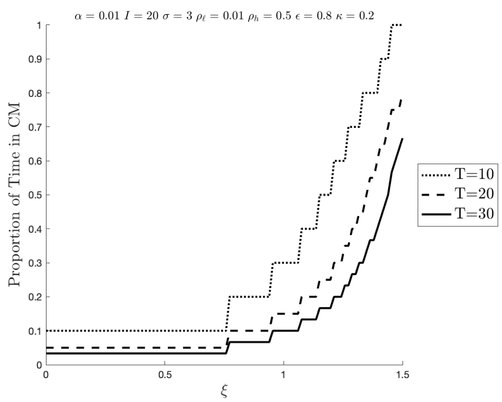
2.9.3 Proportion of Time in CM

Figure 2.17 shows the proportion of time when traders choose CM with respect to asset sensitivity ξ and volatility $(1 - \kappa^2)$. Consistent with our intuition for Propositions 7 and 8, we find that the proportion of time in CM increases with ξ, and decreases with κ.

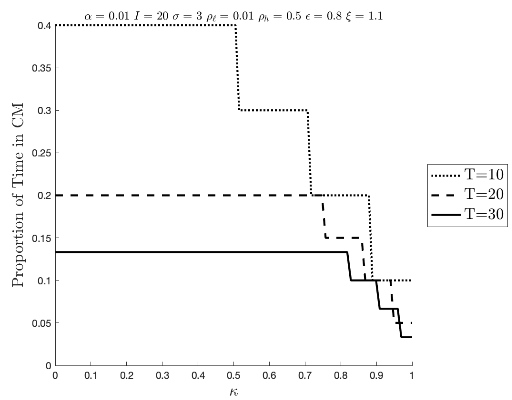
Figure 2.17 also shows the proportion of time when traders in CM with respect to rounds T. Numerically, we find that the choice of alternating markets between DM and CM is generally more prevalent as the trading round T increases. Note that with a small probability the proportion of time in CM with a smaller T can be lower than that with larger T, this is because the last round can end at different stages of an alternating cycle.

Intuitively, with longer T the price history informativeness η increases as its length accumulates, and it is more likely for traders to choose DM over CM. This implies assets with shorter terms are more likely to be traded in the centralized market, e.g. most options are less than 90 days. Assets with the longer term are more likely to be traded in the decentralized market or alternating market structure, e.g. bonds have maturities as long as 30 years, and equities usually do not have maturity.

Figure 2.17: Proportion of Time in CM



(a) Asset sensitivity ξ



(b) Autocorrelation κ

Note: This figure shows the proportion of time in CM for (a) asset sensitivities and (b) autocorrelation. The solid, dashed, and dotted lines plot the proportions of rounds in CM with total rounds $T = 30$, $T = 20$, and $T = 10$ respectively.

2.10 Proofs

Proof of Theorem 10. Given the market structure $\mathcal{M}^*$, at round t, traders submit a demand schedule $q_{i,t}$ to maximize the utility

$$\max_{q_{i,t}} \mathbb{E}[\theta_{i,t}q_{i,t} - \frac{\alpha}{2}(q_{i,t})^2 - p_t q_{i,t}|\mathcal{H}_t, s_{i,t}, p_t]$$

By taking first order condition with respect to $q_{i,t}$, we can solve the trader i's demand schedule,

$$q_{i,t} = \frac{\mathbb{E}[\theta_{i,t}|\mathcal{H}_t, s_{i,t}, p_t] - p_t}{\alpha + \lambda_{i,t}}$$

where $\lambda_{i,t} \equiv \frac{dp_t}{dq_{i,t}}$ is the price impact. By symmetry, the price impacts are the same for all traders in the same round $\lambda_{i,t} = \lambda_t, \forall i \in I_{t,\mathcal{M}^*}$. We can parameterize $\mathbb{E}[\theta_{i,t}|\mathcal{H}_t, s_{i,t}, p_t] = \boldsymbol{c}_{\mathcal{H},i,t}\mathcal{H}_t + c_{s,i,t}s_{i,t} + c_{p,i,t}p_t$, where $\boldsymbol{c}_{\mathcal{H},i,t} \in \mathbb{R}^{1\times|\mathcal{H}_t|}$, $c_{s,i,t} \in \mathbb{R}$, and $c_{p,i,t} \in \mathbb{R}$. By symmetry, the inference coefficients are the same for all traders in the same round, $\boldsymbol{c}_{\mathcal{H},i,t} = \boldsymbol{c}_{\mathcal{H},t}$, $c_{s,i,t} = c_{s,t}$ and $c_{p,i,t} = c_{p,t}$.

In equilibrium, by market clearing condition, λ_t is equal to the inverse of the slope of the residual demand,

$$\lambda_t = (-\sum_{j\neq i}\frac{dq_{j,t}}{dp_t})^{-1} = \frac{\alpha}{(I_t - 1)(1 - c_{p,t}) - 1}$$

Given the parameterization, the equilibrium price is,

$$p_t = (1 - c_{p,t})^{-1}(\boldsymbol{c}_{\mathcal{H},t}\mathcal{H}_t + c_{s,t}\bar{s}_t) \tag{2.8}$$

where $\bar{s}_t = \frac{1}{I_t}\sum_i s_{i,t}$ is the average signal in the exchange (for DM, it is the average signal in each pair).

(Step 1: Inference Coefficients) The trader i's value $\theta_{i,t}$, the equilibrium price p_t given equation (2.8), the history $\mathcal{H}_t$ and the private signal $s_{i,t}$ are jointly normally distributed. By projection theorem, the inference coefficients $c_{\mathcal{H},t}$, $c_{s,t}$, and $c_{p,t}$ can be determined given the joint distribution of $(\theta_{i,t}, s_{i,t}, \mathcal{H}_t, p_t)$,

$$\begin{pmatrix} \theta_{i,t} \\ s_{i,t} \\ \mathcal{H}_t \\ p_t \end{pmatrix} \sim \mathcal{N} \left[\begin{pmatrix} \mathbb{E}[\theta] \\ \mathbb{E}[\theta] \\ \mathbb{E}[\boldsymbol{\theta}] \\ \mathbb{E}[\theta] \end{pmatrix}, \begin{pmatrix} var(\theta_{i,t}) & cov(\theta_{i,t}, s_{i,t}) & cov(\theta_{i,t}, \mathcal{H}'_t) & cov\left(\theta_{i,t}, p'_t\right) \\ cov(s_{i,t}, \theta_{i,t}) & var(s_{i,t}) & cov(s_{i,t}, \mathcal{H}'_t) & cov\left(s_{i,t}, p'_t\right) \\ cov\left(\mathcal{H}_t, \theta_{i,t}\right) & cov\left(\mathcal{H}_t, s_{i,t}\right) & cov\left(\mathcal{H}_t, \mathcal{H}'_t\right) & cov\left(\mathcal{H}_t, p'_t\right) \\ cov\left(p_t, \theta_{i,t}\right) & cov\left(p_t, s_{i,t}\right) & cov\left(p_t, \mathcal{H}'_t\right) & cov\left(p_t, p'_t\right) \end{pmatrix} \right]$$

where

$$\begin{aligned} cov\left(p_t, \theta_{i,t}\right) &= (1 - c_{p,t})^{-1} \left(c_{s,t} cov\left(\bar{s}_t, \theta_{i,t}\right) + \boldsymbol{c}_{\mathcal{H},t} cov(\mathcal{H}_t, \theta_{i,t})\right) \\ cov\left(p_t, s_{i,t}\right) &= (1 - c_{p,t})^{-1} \left(c_{s,t} cov\left(\bar{s}_t, s_{i,t}\right) + \boldsymbol{c}_{\mathcal{H},t} cov(\mathcal{H}_t, s_{i,t})\right) \\ cov(p_t, \mathcal{H}'_t) &= (1 - c_{p,t})^{-1} \left(c_{s,t} cov\left(\bar{s}_t, \mathcal{H}'_t\right) + \boldsymbol{c}_{\mathcal{H},t} cov(\mathcal{H}_t, \mathcal{H}'_t\right) \\ cov\left(p_t, p'_t\right) &= (1 - c_{p,t})^{-1} \left(c_{s,t} var(\bar{s}_t) + c_{s,t} cov\left(\bar{s}_t, \mathcal{H}'_t\right) \boldsymbol{c}'_{\mathcal{H},t} + \boldsymbol{c}_{\mathcal{H}} cov(\mathcal{H}_t, \mathcal{H}'_t)\, \boldsymbol{c}'_{\mathcal{H}}\right) \end{aligned}$$

By projection theorem, we have

$$[c_{s,t}, \boldsymbol{c}_{\mathcal{H},t}, c_{p,t}] \begin{pmatrix} cov(s_{i,t}) & cov(s_{i,t}, \mathcal{H}'_t) & cov\left(s_{i,t}, p_t\right) \\ cov\left(\mathcal{H}_t, s_{i,t}\right) & cov\left(\mathcal{H}_t, \mathcal{H}'_t\right) & cov\left(\mathcal{H}_t, p_t\right) \\ cov\left(p_t, s_{i,t}\right) & cov\left(p_t, \mathcal{H}'_t\right) & cov\left(p_t, p'_t\right) \end{pmatrix} = [cov(\theta_{i,t}, s_{i,t}), cov(\theta_{i,t}, \mathcal{H}'_t), cov(\theta_{i,t}, p_t)] \tag{2.9}$$

From equation (2.9), we have the following equations,

$$cov(c_{s,t} s_{i,t} + \boldsymbol{c}_{\mathcal{H},t} \mathcal{H}_t + c_{p,t} p_t, s_{i,t}) = cov(\theta_{i,t}, s_{i,t}) \tag{2.10}$$

$$cov(c_{s,t} s_{i,t} + \boldsymbol{c}_{\mathcal{H}_t,t} \mathcal{H}_t + c_{p,t} p_t, \mathcal{H}'_t) = cov(\theta_{i,t}, \mathcal{H}'_t) \tag{2.11}$$

$$cov(c_{s,t} s_{i,t} + \boldsymbol{c}_{\mathcal{H}_t,t} \mathcal{H}_t + c_{p,t} p_t, p'_t) = cov(\theta_{i,t}, p_t) \tag{2.12}$$

Given that $p_t = (1 - c_{p,t})^{-1}(\boldsymbol{c}_{\mathcal{H},t} \mathcal{H}_t + c_{s,t} \bar{s}_t)$, subtracting $\boldsymbol{c}_{\mathcal{H}}$ times equation (2.11) from equation (2.12) gives us

$$cov(c_{s,t}s_{i,t} + \boldsymbol{c}_{\mathcal{H},t}\mathcal{H}_t + c_{p,t}p_t, \bar{s}_t) = cov(\theta_{i,t}, \bar{s}_t) \tag{2.13}$$

Averaging equation (2.10) over i in the same exchange gives

$$c_{s,t}(1+\sigma^2)\sigma_\theta^2 + cov(\boldsymbol{c}_{\mathcal{H},t}\mathcal{H}_t + c_{p,t}p_t, \bar{s}_t) = \sigma_\theta^2 \tag{2.14}$$

Comparing equation (2.13) and (2.14), we have

$$c_{s,t} = \frac{cov(\theta_{i,t}, \bar{s}_t) - \sigma_\theta^2}{cov(s_{i,t}, \bar{s}_t) - (1+\sigma^2)\sigma_\theta^2} = \frac{1-\rho_{t,\mathcal{M}^*}}{1-\rho_{t,\mathcal{M}^*}+\sigma^2} \tag{2.15}$$

where $\rho_{t,\mathcal{M}^*}$ is the correlation of traders given market structure $\mathcal{M}^*$.

Given equation (2.15), we can rewrite equation (2.11) as

$$(1-c_{p,t})^{-1}\left(c_{s,t}cov(s_{i,t}, \mathcal{H}_t') + \boldsymbol{c}_{\mathcal{H},t}cov(\mathcal{H}_t, \mathcal{H}_t')\right) = cov(\theta_{i,t}, \mathcal{H}_t') \tag{2.16}$$

and equation (2.13) as

$$(1-c_{p,t})^{-1}\left(c_{s,t}var(\bar{s}_t) + \boldsymbol{c}_{\mathcal{H}}cov(\mathcal{H}_t, \bar{s}_t)\right) = cov(\theta_{i,t}, \bar{s}_t) \tag{2.17}$$

Given that $cov(\mathcal{H}, \theta_i) = cov(\mathcal{H}, s_i) = cov(\mathcal{H}, s_j), \forall j \neq i$, and c_s in equation (2.15) , we can solve the term $\boldsymbol{c}_{\mathcal{H},t}$ and $c_{p,t}$ by equation (2.16) and equation (2.17),

$$\boldsymbol{c}_{\mathcal{H},t} = \frac{(1-\rho_{t,\mathcal{M}^*})\sigma^2}{(1-\rho_{t,\mathcal{M}^*}+\sigma^2)\left(1+(I_{t,\mathcal{M}^*}-1)\rho_{t,\mathcal{M}^*} - I_{t,\mathcal{M}^*}\eta\right)}\boldsymbol{\tau}_t'\boldsymbol{\Upsilon}_t^{-1}$$

$$c_{p,t} = \frac{I_{t,\mathcal{M}^*}(\rho_{t,\mathcal{M}^*}-\eta)\sigma^2}{(1-\rho_{t,\mathcal{M}^*}+\sigma^2)\left(1+(I_{t,\mathcal{M}^*}-1)\rho_{t,\mathcal{M}^*} - I_{t,\mathcal{M}^*}\eta\right)}$$

where $\eta = \boldsymbol{\tau}_t'(\boldsymbol{\Upsilon}_t)^{-1}\boldsymbol{\tau}_t$, $\boldsymbol{\tau}_t \equiv \frac{cov(\mathcal{H}_t,\theta_{i,t})}{\sigma_\theta^2} \in \mathbb{R}^{|\mathcal{H}|}$, and $\boldsymbol{\Upsilon}_t \equiv \frac{cov(\mathcal{H}_t,\mathcal{H}_t')}{\sigma_\theta^2} \in \mathbb{R}^{|\mathcal{H}|\times|\mathcal{H}|}$.

The equilibrium price impact is

$$\lambda_t = \frac{\alpha}{(I_{t,\mathcal{M}^*} - 1)(1 - c_{p,t}) - 1}, \quad \forall i$$

The ex-ante utility for trader i is

$$\mathbb{E}[U_{i,t}|\mathcal{H}_t] = \tfrac{\alpha+2\lambda_t}{2(\alpha+\lambda_t)^2}\mathbb{E}[(\mathbb{E}[\theta_{i,t}|\mathcal{H}_t, s_{i,t}, p_t] - p_t)^2|\mathcal{H}_t] = \tfrac{\alpha+2\lambda_t}{2(\alpha+\lambda_t)^2}\tfrac{I_{t,\mathcal{M}^*}-1}{I_{t,\mathcal{M}^*}}\tfrac{(1-\rho_{t,\mathcal{M}^*})^2}{1-\rho_{t,\mathcal{M}^*}+\sigma^2}, \quad \forall i$$

■

Proof of Lemma 3. We leave out the subscripts t and $\mathcal{M}^*$ to ease the notation. Taking the derivative of welfare over ρ, we have

$$\frac{d\mathbb{E}[U_i|\mathcal{H}]}{d\rho} = -\frac{\alpha + 2\lambda}{2(\alpha+\lambda)^2}\frac{I-1}{I}\frac{(1-\rho)\left(1-\rho+2\,\sigma^2\right)}{\left(1-\rho+\sigma^2\right)^2} - \frac{\lambda}{(\alpha+\lambda)^3}\frac{I-1}{I}\frac{(1-\rho)^2}{1-\rho+\sigma^2}\frac{d\lambda}{d\rho} < 0.$$

Keep everything else constant,

$$\frac{d\lambda}{d\rho} = \lambda^2 \frac{I\,\sigma^2\,(I-1)\left(I(\eta - \frac{(I-1)\rho+1}{I})^2 + \frac{I-1}{I}(1-\rho)^2 + (1-\eta)\sigma^2\right)}{\alpha\,\left(1-\rho+\sigma^2\right)^2\,\left(1+(I-1)\rho - I\eta\right)^2} > 0.$$

Thus $\frac{d\mathbb{E}[U_i|\mathcal{H}]}{d\rho} < 0$. The traders' welfare decreases with trader value correlation ρ. ■

Proof of Lemma 4. We leave out the subscripts t and $\mathcal{M}^*$ to ease the notation. Keep everything else constant,

$$\frac{d\lambda}{d\eta} = -\lambda^2 \frac{I\,\sigma^2\,(I-1)\,(1-\rho)}{\alpha\,(1-\rho+\sigma^2)\,\left(1+(I-1)\rho - I\eta\right)^2} < 0.$$

Therefore the price impact decreases with price history informativeness η.

The expected utility of any trader i is

$$\mathbb{E}[U_i|\mathcal{H}] = \frac{\alpha+2\lambda}{2(\alpha+\lambda)^2}\frac{I-1}{I}\frac{(1-\bar{\rho})^2}{1-\bar{\rho}+\sigma^2}.$$

Taking the derivative of welfare over η, we have

$$\frac{d\mathbb{E}[U_i|\mathcal{H}]}{d\eta} = \frac{\sigma^2}{\alpha} \frac{\left(1-\rho+\sigma^2\right)\,(1+(I-1)\rho - I\,\eta)}{(I-1)\,(1+(I-1)\rho+\sigma^2-I\,\eta)^3} > 0.$$

Therefore the traders' welfare increases with price history informativeness η. ■

Proof of Lemma 5. We leave out the subscript t to ease the notation.

Monotonicity: The difference between trader i's utility in the CM and DM is

$$\mathbb{E}[U_i^{CM}|\mathcal{H}] - \mathbb{E}[U_i^{DM}|\mathcal{H}] = \frac{\alpha+2\lambda^{CM}}{2(\alpha+\lambda^{CM})^2}\frac{I-1}{I}\frac{(1-\bar{\rho})^2}{1-\bar{\rho}+\sigma^2} - \frac{\alpha+2\lambda^{DM}}{2(\alpha+\lambda^{DM})^2}\frac{1}{2}\frac{(1-\rho_\ell)^2}{1-\rho_\ell+\sigma^2}.$$

Taking its derivative over the public informativeness η, we have

$$\frac{d(\mathbb{E}[U_i^{CM}|\mathcal{H}]-\mathbb{E}[U_i^{DM}|\mathcal{H}])}{d\eta} = \frac{\sigma^2}{\alpha}\left(\frac{\left(1-\bar{\rho}+\sigma^2\right)(1+(I-1)\bar{\rho}-I\,\eta)}{(I-1)\,(1+(I-1)\bar{\rho}+\sigma^2-I\,\eta)^3} - \frac{\left(1-\rho_\ell+\sigma^2\right)(1+\rho_\ell-2\,\eta)}{(1+\rho_\ell+\sigma^2-2\,\eta)^3}\right) < 0.$$

given that $\sigma \geq ((\frac{2(I-1)}{I})^{1/3}-1)^{-1/2}$, $\bar{\rho} > \rho_\ell$, and $\eta \leq \frac{1+(I-1)\bar{\rho}}{I} \leq \frac{1+\rho_\ell}{2}$, for the joint correlation matrix of values to be positive semidefinite.

CM vs. DM: The lowest possible η is ρ_ℓ for equilibrium existence in the DM. $\lim_{\eta\to\rho_\ell}\lambda^{DM} = \infty$ and $\lim_{\eta\to\rho_\ell}\mathbb{E}[U_i^{DM}|\mathcal{H}] = 0$, therefore

$$\lim_{\eta\to\rho_\ell}(\mathbb{E}[U_i^{CM}|\mathcal{H}] - \mathbb{E}[U_i^{DM}|\mathcal{H}]) = \lim_{\eta\to\rho_\ell}\mathbb{E}[U_i^{CM}|\mathcal{H}] > 0. \tag{2.18}$$

Given $\frac{(\rho_\ell+1)}{2} - \frac{1+(I-1)\bar{\rho}}{I} \geq 0$ for the joint correlation matrix of values to be positive semidefinite, the maximum η is $\frac{1+(I-1)\bar{\rho}}{I}$,

$$\lim_{\eta\to\frac{1+(I-1)\bar{\rho}}{I}}(\mathbb{E}[U_i^{CM}|\mathcal{H}] - \mathbb{E}[U_i^{DM}|\mathcal{H}]) = \frac{1}{2\alpha}\frac{I-1}{I}\frac{(1-\bar{\rho})^2}{1-\bar{\rho}+\sigma^2} - \frac{\alpha+2\lim_{\eta\to\frac{1+(I-1)\bar{\rho}}{I}}\lambda_n}{4(\alpha+\lim_{\eta\to\frac{1+(I-1)\bar{\rho}}{I}}\lambda_n)^2}\frac{(1-\rho_\ell)^2}{1-\rho_\ell+\sigma^2}.$$

where $\lim_{\eta\to\frac{1+(I-1)\bar{\rho}}{I}}\lambda_n = \frac{\alpha\left(1-\rho_\ell+\sigma^2\right)(\frac{1+\rho_\ell}{2}-\frac{1+(I-1)\bar{\rho}}{I})}{(\rho_\ell-\frac{1+(I-1)\bar{\rho}}{I})\sigma^2}$. There exists unique $\bar{\rho}^*$ as a function of (I,ρ_ℓ,σ^2) such that $\lim_{\eta\to\frac{1+(I-1)\bar{\rho}}{I}}(\mathbb{E}[U_i^{CM}|\mathcal{H}] - \mathbb{E}[U_i^{DM}|\mathcal{H}]) = 0$ if $\bar{\rho} = \bar{\rho}^*(I,\rho_\ell,\sigma^2)$.

If $\bar{\rho} > \bar{\rho}^*$,

$$\lim_{\eta \to \frac{1+(I-1)\bar{\rho}}{I}} (\mathbb{E}[U_i^{CM}|\mathcal{H}] - \mathbb{E}[U_i^{DM}|\mathcal{H}]) < 0. \tag{2.19}$$

Given that the difference between the ex-ante utility of the centralized market and that of the decentralized market is continuous and monotonically decreasing in η, by equations (2.18) and (2.19), if $\bar{\rho} > \bar{\rho}^*(I, \rho_\ell, \sigma^2)$, there exist $\tilde{\eta}(I, \bar{\rho}, \rho_\ell, \sigma^2)$ such that the centralized market has equal welfare as the decentralized market if $\eta = \tilde{\eta}$, the centralized market has higher welfare than the decentralized market if $\eta < \tilde{\eta}$, and otherwise if $\eta \geq \tilde{\eta}$.

If $\bar{\rho} \leq \bar{\rho}^*(I, \rho_\ell, \sigma^2)$,

$$\lim_{\eta \to \frac{1+(I-1)\bar{\rho}}{I}} (\mathbb{E}[U_i^{CM}|\mathcal{H}] - \mathbb{E}[U_i^{DM}|\mathcal{H}]) > 0. \tag{2.20}$$

Given that the difference between the utility of the centralized market and that of the decentralized market is continuous and monotonically decreasing in η, by equation (2.20) the utility in the centralized market is always higher than the utility in the decentralized market regardless of η. ∎

Proof of Proposition 5. The proof of Proposition 5 directly follows from Lemma 5, as no price history informativeness η will allow traders to choose DM. ∎

Proof of Proposition 6. By Lemma 3, the expected utility $\mathbb{E}[U_i^{CM}|\mathcal{H}]$ decreases with $\bar{\rho}$, $\mathbb{E}[U_i^{DM}|\mathcal{H}]$ decreases with ρ_ℓ, if $\mathbb{E}[U_i^{CM}(\bar{\bar{\rho}})|\mathcal{H}] - \mathbb{E}[U_i^{DM}(\underline{\rho}_\ell)|\mathcal{H}] < 0$, then $\mathbb{E}[U_i^{CM}(\bar{\rho})|\mathcal{H}] - \mathbb{E}[U_i^{DM}(\rho_\ell)|\mathcal{H}]$ for any $\rho_\ell < \underline{\rho}_\ell$ and $\bar{\rho} > \bar{\bar{\rho}}$.

By Lemma 6 we are subject to find $\underline{\rho}_\ell$ and $\bar{\bar{\rho}}$ that makes $\mathbb{E}[U_i^{CM}(\bar{\bar{\rho}})|\mathcal{H}] - \mathbb{E}[U_i^{DM}(\underline{\rho}_\ell)|\mathcal{H}] < 0$ when $\eta = 0$. It is easy to see $\underline{\rho}_\ell < 0$ for DM to exist. And by Lemma 5, $\bar{\bar{\rho}} > \bar{\rho}^*$ given there exists η for traders to choose DM over CM.

When $\eta = 0$, the trader's utility in the CM is

$$\mathbb{E}[U_i^{CM}] = \frac{\alpha + 2\lambda^{CM}}{2(\alpha + \lambda^{CM})^2} \frac{I-1}{I} \frac{(1-\bar{\rho})^2}{1-\bar{\rho}+\sigma^2} \quad \forall i \in I$$

where $\lambda^{CM} = \frac{\alpha}{(I-1)(1-c_p^{CM})-1}$, $c_p^{CM} = \frac{I\bar{\rho}\sigma^2}{(1-\bar{\rho}+\sigma^2)(1+(I-1)\bar{\rho})}$.

The trader's utility in the DM is

$$\mathbb{E}[U_i^{DM}] = \frac{\alpha + 2\lambda^{DM}}{4(\alpha + \lambda_1^{DM})^2} \frac{(1-\rho_\ell)^2}{1-\rho_\ell+\sigma^2} \quad \forall i \in I$$

where $\lambda^{DM} = \frac{\alpha}{-c_p^{DM}}$, $c_p^{DM} = \frac{2\rho_\ell \sigma^2}{(1-\rho_\ell+\sigma^2)(1+\rho_\ell)}$.

For the correlation matrix to be well-defined (positive-semidefinite), the maximum $\bar{\rho}$ as a function of ρ_ℓ is $\frac{\frac{I(1+\rho_\ell)}{2}-1}{I-1}$.

$$\lim_{\rho_\ell \to -1} \lim_{\bar{\rho} \to \frac{\frac{I(1+\rho_\ell)}{2}-1}{I-1}} \mathbb{E}[U_i^{DM}|\mathcal{H}] - \mathbb{E}[U_i^{CM}|\mathcal{H}] = \frac{1}{\alpha(2+\sigma^2)} - \frac{1}{2\alpha} \frac{I}{(I+(I-1)\sigma^2)} > 0$$

Given $\mathbb{E}[U_i^{CM}]$ decreases with $\bar{\rho}$, $\mathbb{E}[U_i^{DM}]$ decreases with ρ_ℓ, and $\mathbb{E}[U_i^{DM}] - \mathbb{E}[U_{i,1}^{CM}]$ is continuous in $\bar{\rho}$ and ρ_ℓ, there exists $\underline{\rho}_\ell < 0$ and $\bar{\bar{\rho}} > \bar{\rho}^*$ such that $\mathbb{E}[U_i^{DM}] = \mathbb{E}[U_i^{CM}]$, $\mathbb{E}[U_i^{DM}] - \mathbb{E}[U_i^{CM}] > 0$ if $\rho_\ell < \underline{\rho}_\ell$ and $\bar{\rho} > \bar{\bar{\rho}}$. ■

Proof of Proposition 7. With $\rho_\ell \geq 0$, the DM equilibrium does not exist due to extreme adverse selection. Traders will choose CM in the first round.

Step 1. Less (More) history, lower (higher) η_t: $\eta_t = \frac{var(\theta_{i,t}) - var(\theta_{i,t}|\mathcal{H}_t)}{var(\theta_{i,t})}$. To see this point, consider $\tilde{\eta}_t$ derived from $\tilde{\mathcal{H}}_t$. $\tilde{\mathcal{H}}_t$ is a strict subset of the price history $\tilde{\mathcal{H}}_t \subset \mathcal{H}_t$. $\tilde{\eta}_t = \frac{var(\theta_{i,t}) - var(\theta_{i,t}|\tilde{\mathcal{H}}_t)}{var(\theta_{i,t})}$. As $\tilde{\mathcal{H}}_t$ is a sub-sigma-algebra of $\mathcal{H}_t$, $var(\theta_{i,t}|\mathcal{H}_t) \leq var(\theta_{i,t}|\tilde{\mathcal{H}}_t)$. Thus $\tilde{\eta}_t \leq \eta_t$. This result tells us to keep everything else including the market choices in other rounds constant, if the trader chooses DM (CM) instead at round t, the informativeness in any round $\tau > t$ decreases (increases).

Step 2. Higher ξ, lower η_t: With symmetric market assumption, the price history is a linear combination of the past average signals in the CM. Let $\mathcal{H}_t = \boldsymbol{L}\bar{\boldsymbol{s}}_{\tau<t}^{CM}$, where $\bar{\boldsymbol{s}}_{\tau<t}^{CM} \in \mathbb{R}^{|\mathcal{H}_t|}$ is the vector of the average signals in past rounds where CM is the optimal market

choice, and $\boldsymbol{L} \in \mathbb{R}^{|\mathcal{H}_t|\times|\mathcal{H}_t|}$ is a linear operator. We have the following equivalence:

$$\begin{aligned}\eta_t &= \frac{cov(\theta_{i,t}, \mathcal{H}_t)cov(\mathcal{H}_t, \mathcal{H}_t')^{-1}cov(\mathcal{H}_t, \theta_{i,t})}{\sigma_\theta^2} \\ &= \frac{cov(\theta_{i,t}, \boldsymbol{L}'(\bar{\boldsymbol{s}}_{\tau<t}^{CM})')cov(\boldsymbol{L}\bar{\boldsymbol{s}}_{\tau<t}^{CM}, (\bar{\boldsymbol{s}}_{\tau<t}^{CM})'\boldsymbol{L}')^{-1}cov(\boldsymbol{L}\bar{\boldsymbol{s}}_{\tau<t}^{CM}, \theta_{i,t})}{\sigma_\theta^2} \\ &= \frac{cov(\theta_{i,t}, (\bar{\boldsymbol{s}}_{\tau<t}^{CM})')cov(\bar{\boldsymbol{s}}_{\tau<t}^{CM}, (\bar{\boldsymbol{s}}_{\tau<t}^{CM})')^{-1}cov(\bar{\boldsymbol{s}}_{\tau<t}^{CM}, \theta_{i,t})}{\sigma_\theta^2}\end{aligned}$$

We only need to compute the joint distribution of $\{\bar{s}_\tau\}_{\tau<t}$ and $\{\theta_{i,t}\}_i$ to obtain the η_t given the above equivalence.

Given the primitive, we have $cov(\bar{s}_\tau^{CM}, \bar{s}_\tau^{CM}) = \frac{1+(I-1)\bar{\rho}+\sigma^2}{I}\sigma_\theta^2$, $cov(\bar{s}_\tau^{CM}, \bar{s}_{\tau'}^{CM}) = \frac{1+\xi^2\kappa^{|\tau'-\tau|}}{(1+\xi^2)(1+\epsilon^2)}\sigma_\theta^2$, $cov(\bar{s}_\tau^{CM}, \theta_{i,t}) = \frac{1+\xi^2\kappa^{t-\tau}}{(1+\xi^2)(1+\epsilon^2)}\sigma_\theta^2$ for $\tau < t$. Fixing the past market choice, we have the following comparative static:

$$\frac{d\eta_t}{d\xi} = \frac{1}{\sigma_\theta^2}\frac{dcov(\theta_{i,t}, (\bar{\boldsymbol{s}}_{\tau<t}^{CM})')cov(\bar{\boldsymbol{s}}_{\tau<t}^{CM}, (\bar{\boldsymbol{s}}_{\tau<t}^{CM})')^{-1}cov(\bar{\boldsymbol{s}}_{\tau<t}^{CM}, \theta_{i,t})}{d\xi} < 0$$

which means, the price history informativeness is decreasing in asset sensitivity ξ given past market choice.

Step 3. Existence of $\underline{\xi}$: By Lemma 5, to show the existence of $\underline{\xi}$, we will need to check if there exists ξ that $\eta_t \geq \tilde{\eta}$, $\forall t$. By Step 1 the lowest possible η_t over t and all possible market choices is the $\underline{\eta}_T$ with price history set including only p_1^{CM}. Given $\frac{d\eta_t}{d\xi} < 0$, we are subject to check if the smallest ξ makes $\underline{\eta}_T \geq \tilde{\eta}$.

$$\lim_{\xi\to 0}\underline{\eta}_T = \frac{1}{(1+\epsilon^2)^2}\frac{I}{1+(I-1)\bar{\rho}+\sigma^2}$$

To show that $\underline{\eta}_T \geq \tilde{\eta}$, we are subject to show $\lim_{\xi\to 0}\mathbb{E}[U_i^{CM}(\underline{\eta}_T)|\mathcal{H}] - \mathbb{E}[U_i^{DM}(\underline{\eta}_T)|\mathcal{H}] < 0$.

$\lim_{\xi\to 0}\mathbb{E}[U_i^{CM}(\underline{\eta}_T)|\mathcal{H}] - \mathbb{E}[U_i^{DM}(\underline{\eta}_T)|\mathcal{H}] = \frac{\alpha+2\lambda^{CM}}{2(\alpha+\lambda^{CM})^2}\frac{I-1}{I}\frac{(1-\bar{\rho})^2}{1-\bar{\rho}+\sigma^2} - \frac{\alpha+2\lambda^{DM}}{2(\alpha+\lambda^{DM})^2}\frac{1}{2}\frac{(1-\rho_\ell)^2}{1-\rho_\ell+\sigma^2}$.

$\frac{d\lim_{\xi\to 0}\mathbb{E}[U_i^{CM}(\underline{\eta}_T)|\mathcal{H}]-\mathbb{E}[U_i^{DM}(\underline{\eta}_T)|\mathcal{H}]}{d\epsilon} < 0$. There exist $\bar{\epsilon}(\sigma^2, I)$ such that for any $\epsilon < \bar{\epsilon}(\sigma^2, I)$,

$\lim_{\xi \to 0} \mathbb{E}[U_i^{CM}(\underline{\eta}_T)|\mathcal{H}] - \mathbb{E}[U_i^{DM}(\underline{\eta}_T)|\mathcal{H}] < 0$. Given $\frac{d\eta_t}{d\xi} < 0$ and η_t is continuous in ξ, and $\epsilon < \bar{\epsilon}(\sigma^2, I)$, there exists $\underline{\xi}$, such that for any $\xi \in [0, \underline{\xi})$, traders will stay in the DM since the 2nd round.

Step 4. Existence of $\bar{\xi}$: By Lemma 5, to show the existence of $\bar{\xi}$, we will need to check if there exists ξ that $\eta_t \le \tilde{\eta}$, $\forall t$. By Step 1 the highest possible η_t over t and all possible market choices is $\bar{\eta}_T$ when all past market choices are CMs and all past prices are available. Therefore, we are subject to check a hypothetical $\bar{\eta}_T$ that is generated with the history of all past CM prices. Given Step 2, $\frac{d\eta_t}{d\xi} < 0$ and η_t is continuous in ξ, we are subject to check if the highest ξ makes $\bar{\eta}_T \le \tilde{\eta}$.

$$\lim_{\xi \to \infty} \bar{\eta}_T < (\frac{\kappa}{1+\epsilon^2})^2 \frac{I(T-1)}{1+(I-1)\bar{\rho}+\sigma^2}$$

There exists $\bar{\kappa}$ such that for $\kappa < \bar{\kappa}(\sigma^2, I, \epsilon)$, $3 \le T < \bar{T}(\sigma^2, I, \epsilon)$, $\frac{d \lim_{\xi \to 0} \mathbb{E}[U_i^{CM}(\bar{\eta}_T)|\mathcal{H}]}{dI} - \frac{d\mathbb{E}[U_i^{DM}(\bar{\eta}_T)|\mathcal{H}]}{dI} > 0$. Given $\frac{d\eta_t}{d\xi} < 0$, η_t is continuous in ξ, and $\kappa < \bar{\kappa}(\sigma^2, I, \epsilon)$, there exist $\bar{\xi}$, for any $\xi \in [\bar{\xi}, \infty)$, traders will stay in the CM.

Step 5. Summarize: Given $3 \le T < \bar{T}(\sigma^2, I, \epsilon)$, $\epsilon < \bar{\epsilon}(\sigma^2, I)$ and $\kappa < \bar{\kappa}(\sigma^2, I, \epsilon)$, there exists $\underline{\xi}$ and $\bar{\xi}$ such that traders will choose CM in the first round, and

1. When the asset sensitivity to shocks to fundamentals is sufficiently low $\xi \in [0, \underline{\xi})$, the traders shift to DM in the second round and stay there.

2. When the asset sensitivity to shocks to fundamentals is intermediate $\xi \in [\underline{\xi}, \bar{\xi})$, the traders will alternate between CM and DM. This is because, for $\xi \in [\underline{\xi}, \bar{\xi})$, there exists t such that $\eta_t > \tilde{\eta}$, and there also exists t such that $\eta_t < \tilde{\eta}$.

3. When the asset sensitivity to shocks to fundamentals is sufficiently high $\xi \in [\bar{\xi}, \infty)$, the traders will always stay in the CM.

■

Proof of Proposition 8. With $\rho_\ell \ge 0$, the DM equilibrium does not exist due to extreme adverse selection. Traders will choose CM in the first round.

Step 1. Less (More) history, lower (higher) η_t: See proof of Proposition 7.

Step 2. Higher ξ, lower η_t: The derivation of η as a function of the joint distribution of signals and values follows from the proof of Proposition 7. Fixing the past market choice, we have the following comparative static:

$$\frac{d\eta_t}{d\kappa} > 0$$

which means, the price history informativeness is decreasing in autocorrelation κ given past market choice.

Step 3. Existence of $\bar{\kappa}$: By Lemma 5, to show the existence of $\bar{\kappa}$, we will need to check if there exists κ that $\eta_t \geq \tilde{\eta}$, $\forall t$. By Step 1 the lowest possible η_t over t and all possible market choices is the $\underline{\eta}_T$ with price history set including only p_1^{CM}. Given $\frac{d\eta_t}{d\kappa} > 0$, we are subject to check if the highest κ makes $\underline{\eta}_T \geq \tilde{\eta}$.

$$\lim_{\kappa \to 1} \underline{\eta}_T = \frac{1}{(1+\epsilon^2)^2} \frac{I}{1+(I-1)\bar{\rho}+\sigma^2}$$

There exist $\bar{\epsilon}(\sigma^2, I)$ such that for any $\epsilon < \bar{\epsilon}(\sigma^2, I)$, $\lim_{\kappa \to 0} \mathbb{E}[U_i^{CM}(\underline{\eta}_T)|\mathcal{H}] - \mathbb{E}[U_i^{DM}(\underline{\eta}_T)|\mathcal{H}] < 0$. Given $\frac{d\eta_t}{d\kappa} > 0$ and η_t is continuous in κ, and $\epsilon < \bar{\epsilon}(\sigma^2, I)$, there exists $\bar{\kappa}$, such that for any $\kappa \in (\bar{\kappa}, 1]$, traders will stay in the DM since the 2nd round.

Step 4. Existence of $\underline{\kappa}$: By Lemma 5, to show the existence of $\underline{\kappa}$, we will need to check if there exists κ that $\eta_t \leq \tilde{\eta}$, $\forall t$. By Step 1 the highest possible η_t over t and all possible market choices is the $\bar{\eta}_T$ when all past market choices are CMs and all past prices are available. Therefore, we are subject to check a hypothetical $\bar{\eta}_T$ that is generated with the history of all past CM prices. Given Step 2, $\frac{d\eta_t}{d\kappa} > 0$ and η_t is continuous in κ, we are subject to check if the highest κ makes $\bar{\eta}_T \leq \tilde{\eta}$.

$$\lim_{\kappa \to 0} \bar{\eta}_T < \left(\frac{1}{(1+\xi^2)(1+\epsilon^2)}\right)^2 \frac{I(T-1)}{1+(I-1)\bar{\rho}+\sigma^2}$$

There exists $\underline{\xi}$ such that for $\xi > \underline{\xi}(\sigma^2, I, \epsilon)$, $3 \leq T < \bar{T}(\sigma^2, I, \epsilon)$, $\frac{d \lim_{\xi \to 0} \mathbb{E}[U_i^{CM}(\bar{\eta}_T)|\mathcal{H}]}{dI} -$

$\frac{d\mathbb{E}[U_i^{DM}(\bar{\eta}_T)|\mathcal{H}]}{dI} > 0$. Given $\frac{d\eta_t}{d\kappa} > 0$, η_t is continuous in κ, and $3 \leq T < \bar{T}(\sigma^2, I, \epsilon)$, $\xi > \underline{\xi}(\sigma^2, I, \epsilon)$, there exist $\underline{\kappa}$, for any $\kappa \in [0, \underline{\kappa}]$, traders will stay in the CM.

Step 5. Summarize: Given $\epsilon < \bar{\epsilon}(\sigma^2, I)$ and $\xi > \underline{\xi}(\sigma^2, I, \epsilon)$, $3 \leq T < \bar{T}(\sigma^2, I, \epsilon)$, there exists $\underline{\kappa}$ and $\bar{\kappa}$ such that traders will choose CM in the first round, and

1. When the autocorrelation is sufficiently low $\kappa \in [0, \underline{\kappa}]$, the traders will always stay in the CM.

2. When the autocorrelation is intermediate $\kappa \in (\underline{\kappa}, \bar{\kappa}]$, the traders will alternate between CM and DM, as there exists t such that $\eta_t > \tilde{\eta}$, and there also exists t such that $\eta_t < \tilde{\eta}$.

3. When the autocorrelation is sufficiently high $\kappa \in (\bar{\kappa}, 1]$, the traders will choose DM over CM in the second round and never choose CM again.

■

Proof of Proposition 9. The proof of Proposition 9 is simple and intuitive. By the first monotonicity result in Lemma 5, if $\mathcal{M}_t^* = DM$ for η_t, and price history informativeness increases $\eta_{t+1} \geq \eta_t$, then $\mathcal{M}_{t+1}^* = DM$. We are subject to show that $\eta_{t+1} \geq \eta_t$ if traders choose DM at round t. $\eta_t = \frac{var(\theta_i) - var(\theta_i|\mathcal{H}_t)}{var(\theta_i)}$. If traders choose DM at round t, then $\mathcal{H}_{t+1} = \mathcal{H}_t \cup \{p_{n,t}\}_n$, and $\eta_{t+1} = \frac{var(\theta_i) - var(\theta_i|\mathcal{H}_{t+1})}{var(\theta_i)}$. Given that $\mathcal{H}_t \subset \mathcal{H}_{t+1}$, $var(\theta_i|\mathcal{H}_{t+1}) \leq var(\theta_i|\mathcal{H}_t)$, and therefore $\eta_{t+1} \geq \eta_t$. ■

Proof of Proposition 10. If traders always stay in CM. Post-trade transparency has no impact on welfare.

If traders have ever chosen DM, denote the round that traders first choose DM as t^*. For $t \leq t^*$, post-trade transparency has no impact on traders' utility. For $t > t^*$, denote the price history informativeness as $\eta_t^{post} = \frac{var(\theta_i) - var(\theta_i|\mathcal{H}_t^{post})}{var(\theta_i)}$. By symmetry, the price history is a linear combination of the average signal in the market and is informationally equivalent to the average signal per exchange. Thus $\eta_t^{post} = \frac{var(\theta_i) - var(\theta_i|\mathcal{S}_t^{post})}{var(\theta_i)}$, where $\mathcal{S}_t^{post} \equiv \{\bar{s}_\tau\}_{\tau < t^*}, \{\bar{s}_{n,\tau}\}_{n, t^* \leq \tau < t})$. Without post-trade transparency in DM, $\eta_t =$

$\frac{var(\theta_i)-var(\theta_i|\mathcal{S}_t)}{var(\theta_i)}$, where $\mathcal{S}_t \subset \{\bar{s}_\tau\}_{\tau<t}$. filtration generated by $\mathcal{S}_t$ is a sub σ-algebra of filtration generated by $\mathcal{S}_t^{post}$, therefore, $var(\theta_i|\mathcal{S}_t^{post}) \leq var(\theta_i|\mathcal{S}_t)$, and $\eta_t^{post} \geq \eta_t$, $\forall t$.

If the traders choose DM at round t without post trade transparency, given that $\eta_t^{post} \geq \eta_t$, $\mathbb{E}[U_{i,t}^{DM,post}|\mathcal{H}_t^{post}] \geq \mathbb{E}[U_{i,t}^{DM}|\mathcal{H}_t]$.

If the traders choose CM at round t without post-trade transparency, $\mathbb{E}[U_{i,t}^{DM,post}] > \mathbb{E}[U_{i,t}^{CM,post}|\mathcal{H}_t^{post}] \geq \mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_t]$, the first inequality follows from the fact that traders prefer DM over CM at round t given proof of Proposition 9, the second equality follows from $\eta_t^{post} > \eta_t$. ■

Proof of Theorem 13. **(Step 1: Optimization)** Let the cross pair price information be $\boldsymbol{p}_t \in \mathbb{R}^N$, whose n^{th} element is the price in pair n at round t, $p_{n,t}$. Trader $i \in I(n)$ submit demand schedule $q_{i,t}(\boldsymbol{p}_t) : \mathbb{R}^N \to \mathbb{R}$ to maximize the expected utility conditional on the history $\mathcal{H}_t$, private signal $s_{i,t}$, and

$$\max_{q_{i,t}(\boldsymbol{p}_t)} \mathbb{E}[\theta_{i,t}q_{i,t} - \frac{1}{2}\alpha q_{i,t}^2 - p_{n,t}q_t|\boldsymbol{p}_t, \mathcal{H}_t, s_{i,t}]$$

trader i 's first-order condition as

$$q^i(\boldsymbol{p}_t) = \frac{\mathbb{E}[\theta_{i,t}|\boldsymbol{p}_t, \mathcal{H}_t, s_{i,t}] - p_{n,t}}{\alpha + \lambda_{i,t}}$$

where $\lambda_{i,t}$ is the trader i's price impact within pair n. Trader i also has cross-pair price impact as traders from other pairs will change their bids when price p_n changes with i's bid. Trader i's price impact over all pairs can be described with a price impact matrix $\boldsymbol{\Lambda}_{i,t} = (\frac{d\boldsymbol{p}}{d\boldsymbol{q}_{i,t}}) \in \mathbb{R}^{N\times N}$, where the n^{th} diagonal elements is $\lambda_{i,t}$. Each trader i 's price impact matrix is equal to the transpose of the Jacobian of trader i 's inverse residual supply:

$$(\boldsymbol{\Lambda}_{i,t})' = \left(-\sum_{j\neq i}\frac{d\boldsymbol{q}_{j,t}}{d\boldsymbol{p}_t}\right)^{-1}$$

We can parameterize $\mathbb{E}[\theta_{i,t}|\boldsymbol{p}_t, \mathcal{H}_t, s_{i,t}] = \boldsymbol{c}_{\mathcal{H},i,t}\mathcal{H}_t + c_{s,i,t}s_{i,t} + \boldsymbol{c}_{p,i,t}\boldsymbol{p}_t$. $\boldsymbol{c}_{\mathcal{H},i,t} \in \mathbb{R}^{1\times|\mathcal{H}_t|}$, $c_{s,i,t} \in \mathbb{R}$, and $\boldsymbol{c}_{p,i,t} \in \mathbb{R}^{1\times N}$. Given symmetry within each pair, $\boldsymbol{c}_{\mathcal{H},i,t} = \boldsymbol{c}_{\mathcal{H},n,t}$, $c_{s,i,t} =$

$c_{c,n,t}$, $\boldsymbol{c}_{p,i,t} = \boldsymbol{c}_{p,n,t}$ and $\lambda_{i,t} = \lambda_{n,t}$.

Given the market clearing condition, $\sum_{i \in I(n)} q_{i,t}(\boldsymbol{p}_t) = 0$, and trader symmetry within exchanges, we have the equilibrium price in all pairs in vector form,

$$\boldsymbol{p}_t = (\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1} (\boldsymbol{C}_{\mathcal{H},t}\mathcal{H}_t + \boldsymbol{C}_{s,t}\bar{\boldsymbol{s}}_t),$$

where $\boldsymbol{C}_{s,t} = \operatorname{diag}(c_{s,n,t})_n \in \mathbb{R}^{N \times N}$, $\boldsymbol{C}_{\mathcal{H},t} = (\boldsymbol{c}_{\mathcal{H},n,t})_n \in \mathbb{R}^{N \times |\mathcal{H}_t|}$, $\boldsymbol{C}_{p,t} = (\boldsymbol{c}_{p,n,t})_n \in \mathbb{R}^{N \times N}$. $\bar{\boldsymbol{s}}_t \in \mathbb{R}^N$ is the average signals for all pairs, where the n^{th} element is the average signal in pair n.

(Step 2: Inference Coefficients) We determine the inference coefficients as a function of the primitives (and in closed form). Random vector $(\theta_{i,t}, s_{i,t}, \mathcal{H}_t, \boldsymbol{p}_t)$ is jointly normally distributed:

$$\begin{pmatrix} \theta_{i,t} \\ s_{i,t} \\ \mathcal{H}_t \\ \boldsymbol{p}_t \end{pmatrix} \sim \mathcal{N} \left[\begin{pmatrix} \mathbb{E}[\theta] \\ \mathbb{E}[\theta] \\ \mathbb{E}[\boldsymbol{\theta}] \\ \mathbb{E}[\boldsymbol{\theta}] \end{pmatrix}, \begin{pmatrix} var(\theta_{i,t}) & cov(\theta_{i,t}, s_{i,t}) & cov(\theta_{i,t}, \mathcal{H}_t') & cov(\theta_{i,t}, \boldsymbol{p}_t') \\ cov(s_{i,t}, \theta_{i,t}) & var(s_{i,t}) & cov(s_{i,t}, \mathcal{H}_t') & cov(s_{i,t}, \boldsymbol{p}_t') \\ cov(\mathcal{H}_t, \theta_{i,t}) & cov(\mathcal{H}_t, s_{i,t}) & cov(\mathcal{H}_t, \mathcal{H}_t') & cov(\mathcal{H}_t, \boldsymbol{p}_t') \\ cov(\boldsymbol{p}_t, \theta_{i,t}) & cov(\boldsymbol{p}_t, s_{i,t}) & cov(\boldsymbol{p}_t, \mathcal{H}_t') & cov(\boldsymbol{p}_t, \boldsymbol{p}_t') \end{pmatrix} \right]$$

where

$$\begin{aligned}
cov(\boldsymbol{p}_t, \theta_{i,t}) &= (\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1} (\boldsymbol{C}_{s,t} cov(\bar{\boldsymbol{s}}_t, \theta_{i,t}) + \boldsymbol{C}_{\mathcal{H},t} cov(\mathcal{H}_t, \theta_{i,t})) \\
cov(\boldsymbol{p}_t, s_{i,t}) &= (\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1} (\boldsymbol{C}_{s,t} cov(\bar{\boldsymbol{s}}_t, s_{i,t}) + \boldsymbol{C}_{\mathcal{H},t} cov(\mathcal{H}_t, s_{i,t})) \\
cov(\boldsymbol{p}_t, \mathcal{H}_t') &= (\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1} \left(\boldsymbol{C}_{s,t} cov\left(\bar{\boldsymbol{s}}_t, \mathcal{H}_t'\right) + \boldsymbol{C}_{\mathcal{H},t} cov(\mathcal{H}_t, \mathcal{H}_t')\right) \\
cov\left(\boldsymbol{p}_t, \boldsymbol{p}_t'\right) &= (\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1} \left(\boldsymbol{C}_{s,t} cov\left(\bar{\boldsymbol{s}}_t, \bar{\boldsymbol{s}}_t'\right) (\boldsymbol{C}_{s,t})' + \boldsymbol{C}_{\mathcal{H},t} cov(\mathcal{H}_t, \mathcal{H}_t') \boldsymbol{C}'_{\mathcal{H},t}\right. \\
&\left. + \boldsymbol{C}_{\mathcal{H},t} cov(\mathcal{H}_t, \bar{\boldsymbol{s}}_t') \boldsymbol{C}'_{s,t} + \boldsymbol{C}_{s,t} cov(\bar{\boldsymbol{s}}_t, \mathcal{H}_t') \boldsymbol{C}'_{\mathcal{H},t}\right) \left((\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1}\right)'
\end{aligned}$$

By Projection Theorem, we have

$$[c_{s,n,t}, \boldsymbol{c}_{\mathcal{H},n,t}, \boldsymbol{c}_{p,n,t}] \begin{pmatrix} var(s_{i,t}) & cov(s_{i,t}, \mathcal{H}_t') & cov\,(s_{i,t}, \boldsymbol{p}_t') \\ cov\,(\mathcal{H}_t, s_{i,t}) & cov\,(\mathcal{H}_t, \mathcal{H}_t') & cov\,(\mathcal{H}_t, \boldsymbol{p}_t') \\ cov\,(\boldsymbol{p}_t, s_{i,t}) & cov\,(\boldsymbol{p}_t, \mathcal{H}_t') & cov\,(\boldsymbol{p}_t, \boldsymbol{p}_t') \end{pmatrix}$$
$$= [cov(\theta_{i,t}, s_{i,t}), cov(\theta_{i,t}, \mathcal{H}_t'), cov\,(\theta_{i,t}, \boldsymbol{p}_t')] \tag{2.21}$$

From equation (2.21), we have the following equations,

$$cov(c_{s,n,t}s_{i,t} + \boldsymbol{c}_{\mathcal{H},n,t}\mathcal{H}_t + \boldsymbol{c}_{p,n,t}\boldsymbol{p}_t, s_{i,t}) = \sigma_\theta^2, \tag{2.22}$$

$$cov(c_{s,n,t}s_{i,t} + \boldsymbol{c}_{\mathcal{H},n,t}\mathcal{H}_t + \boldsymbol{c}_{p,n,t}\boldsymbol{p}_t, \mathcal{H}_t) = cov(\theta_{i,t}, \mathcal{H}_t'), \tag{2.23}$$

and

$$cov(c_{s,n,t}s_{i,t} + \boldsymbol{c}_{\mathcal{H},n,t}\mathcal{H}_t + \boldsymbol{c}_{p,n,t}\boldsymbol{p}_t, \boldsymbol{p}_t') = cov(\theta_{i,t}, \boldsymbol{p}_t'). \tag{2.24}$$

Given that $\boldsymbol{p}_t = (\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1}(\boldsymbol{C}_{\mathcal{H},t}\mathcal{H} + \boldsymbol{C}_{s,t}\bar{\boldsymbol{s}}_t)$, subtracting $\boldsymbol{C}_{\mathcal{H},t}$ times equation (2.23) from equation (2.24) gives us

$$cov(c_{s,n,t}s_{i,t} + \boldsymbol{c}_{\mathcal{H},n,t}\mathcal{H}_t + \boldsymbol{c}_{p,n,t}\boldsymbol{p}_t, \bar{\boldsymbol{s}}_t') = cov(\theta_{i,t}, \bar{\boldsymbol{s}}_t'). \tag{2.25}$$

Averaging equation (2.22) over $i \in I(n)$ gives

$$c_{s,n,t}(1+\sigma^2)\sigma_\theta^2 + cov(\boldsymbol{c}_{\mathcal{H},n,t}\mathcal{H}_t + \boldsymbol{c}_{p,n,t}\boldsymbol{p}_t, \bar{s}_n) = \sigma_\theta^2, \quad \forall n. \tag{2.26}$$

Comparing equation (2.25) and (2.26), we have

$$c_{s,n,t} = \frac{cov(\theta_{i,t}, \bar{s}_n) - \sigma_\theta^2}{cov(s_{i,t}, \bar{s}_n) - (1+\sigma^2)\sigma_\theta^2} = \frac{1-\rho_{n,t}}{1-\rho_{n,t}+\sigma^2}. \tag{2.27}$$

where $\rho_{n,t}$ is the correlation for traders in pair n.

Given $\boldsymbol{C}_{s,t} = \mathrm{diag}(c_{s,n,t})$ solved in equation (2.27), we can rewrite equation (2.23) in matrix form,

$$(\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1} \left(\boldsymbol{C}_{s,t}\mathbf{1}\boldsymbol{\tau}_t' + \boldsymbol{C}_{\mathcal{H},t}\boldsymbol{\Upsilon}_t\right) = \mathbf{1}\boldsymbol{\tau}_t'. \tag{2.28}$$

and equation (2.25) as

$$(\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1} \left(\boldsymbol{C}_{s,t} cov(\bar{\boldsymbol{s}}_t, \bar{\boldsymbol{s}}_t') + \boldsymbol{C}_{\mathcal{H},t}\boldsymbol{\tau}_t\mathbf{1'}\right) = cov(\bar{\boldsymbol{\theta}}_t, \bar{\boldsymbol{s}}_t'), \tag{2.29}$$

where $\boldsymbol{\tau}_t = cov(\mathcal{H}_t, \theta_{i,t})$, $\boldsymbol{\Upsilon}_t = cov(\mathcal{H}_t, \mathcal{H}_t')$.

We can solve the term $\boldsymbol{C}_{\mathcal{H},t}$ and $\boldsymbol{C}_{p,t}$ by the above two equations,

$$\begin{aligned}
\boldsymbol{C}_{p,t} &= \boldsymbol{Id} - \boldsymbol{C}_{s,t} - \boldsymbol{C}_{s,t}\,\text{diag}\left(\frac{\sigma^2}{I_n}\right)_n (\bar{\mathcal{C}} - \mathbf{11'}\eta_t)^{-1} \\
&= \text{diag}\left(\frac{\sigma^2}{1-\rho_{n,t}+\sigma^2}\right)_n \left(\boldsymbol{Id} - \text{diag}\left(\frac{1-\rho_{n,t}}{2}\right)(\bar{\mathcal{C}} - \mathbf{11'}\eta_t)^{-1}\right).
\end{aligned}$$

and

$$\boldsymbol{C}_{\mathcal{H},t} = (\boldsymbol{Id} - \boldsymbol{C}_{p,t} - \boldsymbol{C}_{s,t})\mathbf{1}\boldsymbol{\tau}_t'\boldsymbol{\Upsilon}_t^{-1} = \text{diag}\left(\frac{(1-\rho_{n,t})\sigma^2}{2(1-\rho_{n,t}+\sigma^2)}\right)_n (\bar{\mathcal{C}} - \mathbf{11'}\eta_t)^{-1}\mathbf{1}\boldsymbol{\tau}_t'\boldsymbol{\Upsilon}_t^{-1}.$$

$\eta_t = \frac{\boldsymbol{\tau}_t'\boldsymbol{\Upsilon}_t^{-1}\boldsymbol{\tau}_t}{\sigma_\theta^2}$ is price history informativeness. $\bar{\mathcal{C}} = \frac{cov(\bar{\boldsymbol{\theta}}_t, \bar{\boldsymbol{\theta}}_t')}{\sigma_\theta^2} \in \mathbb{R}^{N\times N}$ is the correlation of pairwise average values across all pairs, where $\bar{\boldsymbol{\theta}}_t \in \mathbb{R}^N$ is the vector of average value per trading pair where the n^{th} value is $\bar{\theta}_{n,t} = \sum_{i\in I(n)} \theta_{i,t}$.

(Step 3: Price impacts) In equilibrium, each trader i 's price impact is equal to the transpose of the Jacobian of trader i 's inverse residual supply:

$$(\boldsymbol{\Lambda}_{i,t})' = \left(-\sum_{j\neq i}\frac{d\boldsymbol{q}_{j,t}}{d\boldsymbol{p}_t}\right)^{-1} = (\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1}\,\text{diag}\left(\frac{\alpha+\lambda_{n,t}}{2-\mathbf{1}_{i\in I(n)}}\right)_n.$$

From the last equation, we can solve for the within-exchange price impact for all $i \in I(n)$,

$$\lambda_{n,t} = \left(\left(\left((\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1}\right)_{nn}\right)^{-1} - 1\right)^{-1}\alpha.$$

where $(A)_{nn}$ is an operator that gives the n^{th} diagonal element of matrix A. Denote the matrix of within-exchange price impacts by $\hat{\mathbf{\Lambda}}_t \equiv \operatorname{diag}(\lambda_{n,t})_n$. In equilibrium,

$$\hat{\mathbf{\Lambda}}_t = \left(\left(\left[(\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1} \right]_{nn} \right)^{-1} - \boldsymbol{Id} \right)^{-1} \alpha,$$

where $[A]_{nn}$ is an operator that gives the diagonal elements of matrix A while setting all off-diagonal elements to zero.

In this paper, we focus on nonnegative price impacts such that the residual supply curve is downward-sloping, i.e., $\lambda_n \geq 0$, for all n. This is satisfied under the following conditions:

$$((\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1})_{nn} \leq 1$$

$\left((\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1}\right)_{nn} = (1+\frac{\sigma^2}{1-\rho_{n,t}}) \left(1 - \frac{\sigma^2}{2}(\frac{1+\rho_{n,t}+\sigma^2}{2} - \eta_t - A_t)^{-1}\right)$, where $A_t = (\frac{cov(\bar{\theta}_{n,t}, \bar{\boldsymbol{\theta}}'_{-n,t})}{\sigma_\theta^2} - \mathbf{1}'\eta_t)(\frac{cov(\bar{\boldsymbol{\theta}}_{-n,t}, \bar{\boldsymbol{\theta}}'_{-n,t})}{\sigma_\theta^2} - \mathbf{1}\mathbf{1}'\eta_t)^{-1}(\frac{cov(\bar{\boldsymbol{\theta}}_{-n,t}, \bar{\theta}_{n,t})}{\sigma_\theta^2} - \mathbf{1}\eta_t)$. Therefore, the following conditions are needed for equilibrium existence,

$$\eta_t + A_t \geq \rho_{n,t} \quad \forall n$$

The second-order condition for the trader i 's optimization problem is, $\lambda_n \geq -\frac{1}{2}\alpha$, and is trivially satisfied with nonnegative price impacts.

(Step 4: Utility) Given the inference coefficients and price impacts solved in the previous section, the expected utility conditional on price history is

$$\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t] = \frac{\alpha + 2\lambda_t}{2(\alpha + \lambda_t)^2}\mathbb{E}[(\mathbb{E}[\theta_{i,t}|\mathcal{H}_t, s_{i,t}, \boldsymbol{p}_t] - p_{t,n})^2|\mathcal{H}_t] = \frac{\alpha + 2\lambda_{n,t}}{2(\alpha + \lambda_{n,t})^2}\frac{1}{2}\frac{(1-\rho_{n,t})^2}{1-\rho_{n,t}+\sigma^2}$$

■

Proof of Lemma 7. Taking derivative of $U_{i,t}^{DM}$ with respect to $\rho_{n,t}$, we have

$$\frac{d\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t]}{\rho_{n,t}} = -\underbrace{\frac{\alpha + 2\lambda_{n,t}}{4(\alpha + \lambda_{n,t})^2}\frac{(1-\rho_{n,t})(1-\rho_{n,t}+2\sigma^2)}{(1-\rho_{n,t}+\sigma^2)^2}}_{>0} - \underbrace{\frac{\lambda_{n,t}}{2(\alpha + \lambda_{n,t})^3}\frac{(1-\rho_{n,t})^2}{1-\rho_{n,t}+\sigma^2}}_{>0}\frac{d\lambda_{n,t}}{d\rho_{n,t}}$$

.

The derivative of price impact to correlation $\frac{d\lambda_{n,t}}{d\rho_{n,t}}$ is

$$\frac{d\lambda_{n,t}}{d\rho_{n,t}} = \frac{\lambda_{n,t}^2}{\alpha}\left(\left((\boldsymbol{Id}-\boldsymbol{C}_{p,t})^{-1}\right)_{nn}\right)^{-2}\frac{d\left((\boldsymbol{Id}-\boldsymbol{C}_{p,t})^{-1}\right)_{nn}}{d\rho_{n,t}}$$
$$= \frac{\lambda_{n,t}^2}{\alpha\left(\left((\boldsymbol{Id}-\boldsymbol{C}_{p,t})^{-1}\right)_{nn}\right)^2}$$
$$\cdot\left(\frac{\sigma^2\left((\boldsymbol{Id}-\boldsymbol{C}_{p,t})^{-1}\right)_{nn}}{(1-\rho_{n,t})(1-\rho_{n,t}+\sigma^2)} + (1+\frac{\sigma^2}{1-\rho_{n,t}})\frac{\sigma^2}{4}(\frac{1+\rho_{n,t}+\sigma^2}{2}-\eta_t-A_t)^{-2}\right)$$
$$> 0$$

given that $\left((\boldsymbol{Id}-\boldsymbol{C}_{p,t})^{-1}\right)_{nn} = (1+\frac{\sigma^2}{1-\rho_{n,t}})\left(1-\frac{\sigma^2}{2}(\frac{1+\rho_{n,t}+\sigma^2}{2}-\eta_t-A_t)^{-1}\right)$, where $A_t = (\frac{cov(\bar{\theta}_{n,t},\bar{\boldsymbol{\theta}}'_{-n,t})}{\sigma_\theta^2}-\boldsymbol{1}'\eta)(\frac{cov(\bar{\boldsymbol{\theta}}_{-n,t},\bar{\boldsymbol{\theta}}'_{-n,t})}{\sigma_\theta^2}-\boldsymbol{1}\boldsymbol{1}'\eta)^{-1}(\frac{cov(\bar{\boldsymbol{\theta}}_{-n,t},\bar{\theta}_{n,t})}{\sigma_\theta^2}-\boldsymbol{1}\eta)$, and $\bar{\boldsymbol{\theta}}_{-n,t}\in\mathbb{R}^{N-1}$ is the vector of average values in pairs $m\neq n$. The last inequality follows from the fact that $\frac{1+\rho_{n,t}}{2}-\eta_t-A_t>0$ give positive-semidefinite joint correlation matrix of $\bar{\theta}_{n,t}$, $\bar{\boldsymbol{\theta}}_{-n,t}$ and history $\mathcal{H}_t$.

Therefore, $\frac{d\mathbb{E}[U_{i,t}^{DM}|\mathcal{H}_t]}{\rho_{n,t}}<0$. The expected utility in DM is decreasing in $\rho_{n,t}$. ■

Proof of Lemma 8. To show that given $\mathcal{H}_t$ (and therefore given η_t) the utility for any trader i weakly increases, we are subject to show that the expected utility $\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t]\geq\mathbb{E}[U_{i,t}^{DM}|\mathcal{H}_t]$. Comparing $\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t]$ in Theorem 13 and $\mathbb{E}[U_{i,t}^{DM}|\mathcal{H}_t]$ in Theorem 10, we find if and only if $\lambda_{n,t}^{DM,pre}\leq\lambda_{n,t}^{DM}$, then $\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t]\geq\mathbb{E}[U_{i,t}^{DM}|\mathcal{H}_t]$.

$\lambda_{n,t}^{DM,pre}\leq\lambda_{n,t}^{DM}$ if and only if

$$\left((\boldsymbol{Id}-\boldsymbol{C}_{p,t})^{-1}\right)_{nn}\leq\frac{1}{1-c_p^{DM}} \tag{2.30}$$

Following the proof of Lemma 7,

$$\left((\boldsymbol{Id}-\boldsymbol{C}_{p,t})^{-1}\right)_{nn} = (1+\tfrac{\sigma^2}{1-\rho_{n,t}})\left(1-\frac{\sigma^2}{2\left(\frac{1+\rho_{n,t}+\sigma^2}{2}-\eta_t-A_t\right)}\right)\leq(1+\tfrac{\sigma^2}{1-\rho_{n,t}})\frac{\frac{1+\rho_{n,t}}{2}-\eta_t}{\frac{1+\rho_{n,t}+\sigma^2}{2}-\eta_t}$$

as $A_t=(\frac{cov(\bar{\theta}_{n,t},\bar{\boldsymbol{\theta}}'_{-n,t})}{\sigma_\theta^2}-\boldsymbol{1}'\eta_t)(\frac{cov(\bar{\boldsymbol{\theta}}_{-n,t},\bar{\boldsymbol{\theta}}'_{-n,t})}{\sigma_\theta^2}-\boldsymbol{1}\boldsymbol{1}'\eta_t)^{-1}(\frac{cov(\bar{\boldsymbol{\theta}}_{-n,t},\bar{\theta}_{n,t})}{\sigma_\theta^2}-\boldsymbol{1}\eta_t)\geq 0$ given it has a quadratic form and $\frac{cov(\bar{\boldsymbol{\theta}}'_{-n,t},\bar{\boldsymbol{\theta}}'_{-n,t})}{\sigma_\theta^2} - \mathbf{1}\mathbf{1}'\eta_t = \frac{cov(\bar{\boldsymbol{\theta}}_{-n,t},\bar{\boldsymbol{\theta}}'_{-n,t}|\mathcal{H}_t)}{\sigma_\theta^2}$ is positive semidefinite. By Theorem 10, $\frac{1}{1-c_p^{DM}} = (1+\frac{\sigma^2}{1-\rho_{n,t}})\frac{\frac{1+\rho_{n,t}}{2}-\eta_t}{\frac{1+\rho_{n,t}+\sigma^2}{2}-\eta_t}$. Therefore, equation (2.30) holds, $\lambda_{n,t}^{DM,pre} \leq \lambda_{n,t}^{DM}$ and $\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t] \geq \mathbb{E}[U_{i,t}^{DM}|\mathcal{H}_t]$. ■

Proof of Proposition 11. **First Time Choosing DM:** Let the threshold to choose DM without pre-trade transparency by $\tilde{\eta}$ (see Lemma 5), and the threshold to choose DM with pre-trade transparency by $\tilde{\eta}^{pre}$. Suppose $\bar{\rho} > \bar{\rho}^*$, for any $\mathcal{H}_t$ generating $\eta_t \geq \tilde{\eta}$, given results of Lemma 8, $\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t] \geq \mathbb{E}[U_{i,t}^{DM}|\mathcal{H}_t] \geq \mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_t]$. This implies (i) the threshold to choose DM without pre-trade transparency is at least as low as $\tilde{\eta}$, $\tilde{\eta}^{pre} \leq \tilde{\eta}$; (ii) and the first round that traders choose DM with pre-trade transparency is no later than without pre-trade transparency, i.e. if $t_1^{DM} \equiv \min_t\{\mathcal{M}_t^* = DM\}$, then $t_1^{DM,pre} = \min_t\{\mathcal{M}_t^{*,pre} = DM\} \leq t_1^{DM}$.

First Time Stay in DM: If traders choose DM with pre-trade transparency in the same round as with opaque DM, i.e., $t_1^{DM} = t_1^{DM,pre}$, then the length of stay in DM when traders first choose DM is (weakly) longer with pre-trade transparency. This is because, given that they enter the DM at the same round, the evolution of η_t is the same before they first exit the DM after the first time they choose DM. And $\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t] \geq \mathbb{E}[U_{i,t}^{DM}|\mathcal{H}_t]$ implies, the first time traders exit DM with pre-trade transparency is no earlier than the first time when they exit the opaque DM. Thus, the length of stay in DM when traders first choose DM is (weakly) longer with pre-trade transparency.

We are not sure about the following rounds of choosing DM, as the evolution of η_t will not be the same with and without pre-trade transparency, except for the $\tilde{\eta} = 0$ special case. If $\tilde{\eta} = 0$ then $\tilde{\eta}^{pre} = 0$, trader will always choose DM.

■

Lemma 9. *If A and $A+B$ are invertible, and B has rank 1, then let $g = \operatorname{trace}\left(BA^{-1}\right)$. Then $g \neq -1$ and*

$$(A+B)^{-1} = A^{-1} - \frac{1}{1+g}A^{-1}BA^{-1}.$$

Proof of Proposition 12. **Constant CM regardless of pre-trade transparency:** It is trivial that when traders choose CM for all rounds with or without pre-trade transparency, then pre-trade transparency should not have any impact on welfare. Given that $\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t] > \mathbb{E}[U_{i,t}^{DM}|\mathcal{H}_t]$ for any η, choosing CM constantly implies $\mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_t] - \mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t] \geq 0$ for any η_t. We know that if $\bar{\rho} = \rho_h = \rho_\ell$,

$$\begin{aligned}\left((\boldsymbol{Id} - \boldsymbol{C}_{p,t})^{-1}\right)_{nn} &= (1 + \frac{\sigma^2}{1-\rho_\ell})\left(1 - \frac{\sigma^2}{2}(\frac{1+\rho_\ell+\sigma^2}{2} - \eta_t - A_t)^{-1}\right) \\ &> \frac{1}{(I-1)(1-c_{p,t}^{CM})} = \frac{1}{I-1}(1+\frac{\sigma^2}{1-\rho_\ell})(\frac{1+(I-1)\rho_\ell}{I} - \eta_t)(\frac{1+(I-1)\rho_\ell+\sigma^2}{I} - \eta_t)^{-1}\end{aligned}$$

where $A_t = (\frac{cov(\bar{\theta}_{n,t}, \bar{\boldsymbol{\theta}}'_{-n,t})}{\sigma_\theta^2} - \mathbf{1}'\eta_t)(\frac{cov(\bar{\boldsymbol{\theta}}'_{-n,t}, \bar{\boldsymbol{\theta}}'_{-n,t})}{\sigma_\theta^2} - \mathbf{1}\mathbf{1}'\eta_t)^{-1}(\frac{cov(\bar{\boldsymbol{\theta}}_{-n,t}, \bar{\theta}_{n,t})}{\sigma_\theta^2} - \mathbf{1}\eta_t) = (\rho_h - \eta_t)\frac{g}{1+g}$, $g = N\frac{\rho_h - \eta_t}{\frac{1+\rho_\ell}{2} - \rho_h}$ by Lemma 9, and $\bar{\boldsymbol{\theta}}_{-n,t} \in \mathbb{R}^{N-1}$ is the vector of average values in pairs $m \neq n$. Therefore, $\lambda_{n,t}^{CM} < \lambda_{n,t}^{DM,pre}$ and

$$\mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_t] = \frac{\alpha + 2\lambda_{n,t}^{CM}}{2(\alpha+\lambda_{n,t}^{CM})^2}\frac{I-1}{I}\frac{(1-\rho_\ell)^2}{1-\rho_\ell+\sigma^2} > \mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t] = \frac{\alpha+2\lambda_{n,t}^{DM,pre}}{4(\alpha+\lambda_{n,t}^{DM,pre})^2}\frac{(1-\rho_\ell)^2}{1-\rho_\ell+\sigma^2}$$

Given that $\mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_t]$ is decreasing in $\bar{\rho}$, there exists $\bar{\rho}^{*,pre}(\rho_\ell, I, \sigma^2)$ such that if $\bar{\rho} < \bar{\rho}^{*,pre}(\rho_\ell, I, \sigma^2)$, $\mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_t] - \mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t] \geq 0$ for any η. So if $\bar{\rho} < \bar{\rho}^{*,pre}(\rho_\ell, I, \sigma^2)$, traders always choose CM with and without pre-trade transparency.

Transparency changes market choice: Without pre-trade transparency, when traders' value correlations are neither sufficiently heterogeneous nor sufficiently homogeneous, alternating market choice can be optimal according to Proposition 7 and Proposition 8. With pre-trade transparency, as the utility in DM is higher given the same parameters, traders are more likely to choose DM (weakly) earlier (see Proposition 11). And this can potentially decrease the price history informativeness and welfare. We can find a non-trivial set of parameters such that the pre-trade transparency can decrease welfare. A most intuitive case is a set of parameters such that (i) traders always choose CM or alternate between CM and DM with $\eta_t > 0$ for $t > 1$ without pre-trade transparency; (ii) traders always choose DM with pre-trade transparency, resulting $\eta_t = 0$ for all t; (iii) the total welfare over all rounds is higher without pre-trade transparency. We provide proof of the existence of such parameters below.

To satisfy condition (i), the traders' expected utility in CM is higher than the expected utility in opaque DM when $\eta_t = 0$, i.e. $\mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_t]|_{\eta_t=0} \geq \mathbb{E}[U_{i,t}^{DM}|\mathcal{H}_t]|_{\eta_t=0}$.

To satisfy condition (ii), we require the traders in DM with pre-trade transparency to have higher utility than in CM given $\eta_t = 0$. When $\eta_t = 0$, the expected utility in CM is

$$\mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_t]|_{\eta_t=0} = \frac{\alpha + 2\lambda^{CM}}{2(\alpha+\lambda^{CM})^2} \frac{I-1}{I} \frac{(1-\bar{\rho})^2}{1-\bar{\rho}+\sigma^2} \quad \forall i \in I$$

where $\lambda^{CM} = \frac{\alpha}{(I-1)(1-c_p)-1}$, $c_p = \frac{I\bar{\rho}\sigma^2}{(1-\bar{\rho}+\sigma^2)(1+(I-1)\bar{\rho})}$. And when $\eta_t = 0$, the expected utility in DM with pre-trade transparency is

$$\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t]|_{\eta_t=0} = \frac{\alpha + 2\lambda^{DM,pre}}{2(\alpha+\lambda^{DM,pre})^2} \frac{1}{2} \frac{(1-\rho_\ell)^2}{1-\rho_\ell+\sigma^2}$$

where $\lambda^{DM,pre} = (1+\frac{\sigma^2}{1-\rho_\ell})(\frac{1+\rho_\ell}{2} - A_0)\left(\frac{\sigma^2}{2} - (\frac{\sigma^2}{1-\rho_\ell})(\frac{1+\rho_\ell}{2} - A_0)\right)^{-1}\alpha$,
$A_0 = \frac{cov(\bar{\theta}_{n,t}, \bar{\boldsymbol{\theta}}'_{-n,t}) cov(\bar{\boldsymbol{\theta}}_{-n,t}, \bar{\boldsymbol{\theta}}'_{-n,t})^{-1} cov(\bar{\boldsymbol{\theta}}_{-n,t}, \bar{\theta}_{n,t})}{\sigma_\theta^2} \geq 0$.

Given that $\mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_t]|_{\eta_t=0}$ is decreasing in $\bar{\rho}$ and $\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t]|_{\eta_t=0}$ is decreasing in ρ_ℓ given Lemma 3, and $\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t]|_{\eta_t=0} > \mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_t]|_{\eta_t=0}$ when $\rho_\ell = 0$ and $\bar{\rho} = \frac{\frac{I}{2}-1}{I-1}$, there exists $0 \leq \rho_\ell < \bar{\rho}_\ell$ and $\bar{\rho} > \bar{\bar{\rho}}^{pre}\left(I, \rho_\ell, \sigma^2\right)$ such that $\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t]|_{\eta_t=0} \geq \mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_t]|_{\eta_t=0}$.

To satisfy condition (iii), we require the total welfare over all rounds is higher without pre-trade transparency than with pre-trade transparency, i.e. $W^{pre} \equiv IT\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t]|_{\eta_t=0} \leq W \equiv I\sum_{t=1}^{T} \mathbb{E}[U_{i,t}^{\mathcal{M}^*}|\mathcal{H}_t]$. Given the same market choice, the expected utility weakly increases with the length of price history, i.e. $\mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_\tau] \geq \mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_t]$ for $\tau > t$. For sufficiently small ξ and sufficiently large κ, there exists $\underline{t}$, such that $\eta_{\underline{t}}$ is sufficiently large, $\mathbb{E}[U_{i,\underline{t}}^{CM}|\mathcal{H}_{\underline{t}}] > \mathbb{E}[U_{i,\underline{t}}^{DM,pre}|\mathcal{H}_{\underline{t}}]|_{\eta_t=0}$. Therefore, we can rewrite the difference between welfare without and with pre-trade transparency as

$$(W - W^{pre})/I = \sum_{t=1}^{T} \mathbb{E}[U_{i,t}^{\mathcal{M}^*}|\mathcal{H}_t] - T\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t]|_{\eta_t=0}$$

$$\geq (T - \underline{t}) \underbrace{(\mathbb{E}[U_{i,t}^{CM}|\mathcal{H}_t] - \mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t]|_{\eta_t=0})}_{>0} + \underbrace{\sum_{t=1}^{\underline{t}} \mathbb{E}[U_{i,t}^{\mathcal{M}^*}|\mathcal{H}_t] - \underline{t}\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t]|_{\eta_t=0}}_{>-\underline{t}\mathbb{E}[U_{i,t}^{DM,pre}|\mathcal{H}_t]|_{\eta_t=0}}$$

Easy to see $W - W^{pre}$ is increasing in T. Given that the second part is bounded below, $\lim_{T\to\infty} W - W^{pre} > 0$. This implies we can find sufficiently small ξ and sufficiently large κ and sufficiently large T, such that condition (iii) is satisfied. ∎

Proof of Proposition 13.

1. Introducing CM does not change welfare when traders choose DM in all rounds. This is the case when trader values are sufficiently heterogeneous $\rho_\ell < \underline{\rho}_\ell < 0$ and $\bar{\rho} > \bar{\bar{\rho}}$. If traders are allowed to have access to CM in the first round, the welfare also does not change when conditions of Proposition 7.1 and Proposition 8.3 are satisfied.

2. Introducing CM weakly improves welfare for any other cases (where there could be no trade with DM only). This is because, (1) when traders have access to both CM and DM, traders choose CM when the expected utility conditional on the history in CM is weakly higher than that in DM, and (2) choosing CM weakly increases price history informativeness and weakly increase utility for all future traders. In math,

$$\mathbb{E}[U_{i,t}^{\mathcal{M}^*}|\mathcal{H}_t] \geq \mathbb{E}[U_{i,t}^{\mathcal{M}^*}|\mathcal{H}_t^{\text{DM only}}] \geq \mathbb{E}[U_{i,t}^{\text{DM only}}|\mathcal{H}_t^{\text{DM only}}], \quad \forall t.$$

   The first inequality comes from the fact that $\mathcal{H}_t^{\text{DM only}} = \emptyset \subseteq \mathcal{H}_t$, $\eta_t \geq \eta_t^{\text{DM only}} = 0$ and that the expected utility is increasing in η_t (Lemma 4). The second inequality comes from the optimal market choice $\mathcal{M}^*$.

∎

Proof of Proposition 14.

1. For sufficiently homogeneous value, $\bar{\rho} < \bar{\rho}^{*,pre}(I, \rho_\ell, \sigma^2)$, by

Proposition 5 traders will always choose CM, and therefore centralizing DM does not change welfare.

When traders' value correlations are neither sufficiently heterogeneous nor sufficiently homogeneous, and the number of rounds $T < \bar{T}$, by Proposition 7 and 8, if the asset sensitivity is sufficiently high $\xi \in [\bar{\xi}, \infty)$, or volatility is sufficiently high $\kappa \in [0, \underline{\kappa}]$, traders will always choose CM, and therefore centralizing DM does not change welfare.

2. For sufficiently heterogeneous value $\rho_\ell < \underline{\rho_\ell} < 0$ and $\bar{\rho} > \bar{\bar{\rho}}$, by the proof of Proposition 6, traders will always choose DM if they have the choice of choosing either CM or DM and $\mathbb{E}[U_{i,1}^{CM}] < \mathbb{E}[U_{i,1}^{DM}]$. If we centralize the market, the traders will be worse off in the first round, as they will prefer DM with zero price history informativeness. Given that $var(\theta_i|\mathcal{H}_t^{\text{CM only}}) \geq var(\theta_i|\mathcal{H}_{t+1}^{\text{CM only}})\ \forall t$, $\eta_t^{\text{CM only}} = \frac{var(\theta_i) - var(\theta_i|\mathcal{H}_t^{\text{CM only}})}{var(\theta_i)}$ is monotonically increasing in t. By Lemma 4, $\mathbb{E}[U_{i,t}^{\text{CM only}}|\mathcal{H}_t^{\text{CM only}}]$ is monotonically increasing in t. As rounds increase, the price history informativeness increases in CM as the history accumulates, the utility increases in CM with $\eta > 0$, and may exceed the utility in DM with $\eta = 0$ ($\mathcal{H}_t = \mathcal{H}_1 = \emptyset$, $\eta_t = \eta_1 = 0$ with parallel market). The difference in welfare in parallel markets and welfare in CM is,

$$W^{\text{CM only}} - W = I(\sum_{t=1}^{T} \mathbb{E}[U_{i,t}^{\text{CM only}}|\mathcal{H}_t^{\text{CM only}}] - T\mathbb{E}[U_{i,1}^{DM}])$$

Differentiate $W^{\text{CM only}} - W$ over ξ, we can see that it is monotonically decreasing in ξ,

$$\frac{dW^{\text{CM only}} - W}{d\xi} = I \sum_{t=1}^{T} \underbrace{\frac{d\mathbb{E}[U_{i,t}^{\text{CM only}}|\mathcal{H}_t^{\text{CM only}}]}{d\eta_t}}_{>0} \underbrace{\frac{d\eta_t}{d\xi}}_{<0} < 0 \tag{2.31}$$

where the first term in the summation is positive by Lemma 4, and the second term in the summation is negative as $\frac{d\eta_t}{d\xi} = \frac{1}{\sigma_\theta^2} \frac{dcov(\theta_{i,t}, (\bar{\mathbf{s}}_{\tau<t}^{CM})')cov(\bar{\mathbf{s}}_{\tau<t}^{CM}, (\bar{\mathbf{s}}_{\tau<t}^{CM})')^{-1}cov(\bar{\mathbf{s}}_{\tau<t}^{CM}, \theta_{i,t})}{d\xi} < 0$. Intuitively, when the asset sensitivity is higher, then CM price informativeness is lower and therefore the welfare with CM only is lower.

Similarly, Differentiate $W^{\text{CM only}} - W$ over κ, we can see that it is monotonically increasing in κ,

$$\frac{dW^{\text{CM only}} - W}{d\kappa} = I \sum_{t=1}^{T} \underbrace{\frac{d\mathbb{E}[U_{i,t}^{\text{CM only}} | \mathcal{H}_t^{\text{CM only}}]}{d\eta_t}}_{>0} \underbrace{\frac{d\eta_t}{d\kappa}}_{>0} > 0 \tag{2.32}$$

where the first term in the summation is positive by Lemma 4, and the second term in the summation is negative as $\frac{d\eta_t}{d\kappa} = \frac{1}{\sigma_\theta^2} \frac{dcov(\theta_{i,t},(\bar{\mathbf{s}}_{\tau<t}^{CM})')cov(\bar{\mathbf{s}}_{\tau<t}^{CM},(\bar{\mathbf{s}}_{\tau<t}^{CM})')^{-1}cov(\bar{\mathbf{s}}_{\tau<t}^{CM},\theta_{i,t})}{d\kappa} > 0$. Intuitively, when the asset volatility is higher, then CM price informativeness is lower and therefore the welfare with CM only is lower.

When $\xi = \infty$ and $\kappa = 0$, then $\eta_t = 0$ for all t, $W^{\text{CM only}} - W < 0$ given Proposition 6. By equations (2.31) and (2.32), with finite rounds $T < \bar{T}$, there exists $\bar{\xi}^{\text{CM only}}$ and $\underline{\kappa}^{\text{CM only}}$, such that when the asset sensitivity is sufficiently high $\xi \in [\bar{\xi}^{\text{CM only}}, \infty)$ and the volatility is sufficiently high $\kappa \in [0, \underline{\kappa}^{\text{CM only}}]$, $W^{\text{CM only}} - W < 0$.

3. For sufficiently heterogeneous value $\rho_\ell < \underline{\rho}_\ell < 0$ and $\bar{\rho} > \bar{\bar{\rho}}$, by the proof of Proposition 6, traders will always choose DM if they have the choice of choosing either CM or DM and $\mathbb{E}[U_{i,1}^{CM}] < \mathbb{E}[U_{i,1}^{DM}]$.

   Given Lemma 4, $\mathbb{E}[U_{i,T}^{\text{CM only}} | \mathcal{H}_T^{\text{CM only}}]$ is increasing in the price history informativeness η. So for $T > 1$, $\mathbb{E}[U_{i,T}^{\text{CM only}} | \mathcal{H}_T^{\text{CM only}}] > \mathbb{E}[U_{i,1}^{\text{CM only}} | \mathcal{H}_1^{\text{CM only}}] = \mathbb{E}[U_{i,1}^{CM}]$.

   As $\lim_{\rho \to \bar{\bar{\rho}}, \rho_\ell \to \underline{\rho}_\ell} \mathbb{E}[U_{i,1}^{CM}] = \lim_{\rho \to \bar{\bar{\rho}}, \rho_\ell \to \underline{\rho}_\ell} \mathbb{E}[U_{i,1}^{DM}]$ following the proof of Proposition 6, $\lim_{\rho \to \bar{\bar{\rho}}, \rho_\ell \to \underline{\rho}_\ell} \mathbb{E}[U_{i,\infty}^{\text{CM only}} | \mathcal{H}_\infty^{\text{CM only}}] > \lim_{\rho \to \bar{\bar{\rho}}, \rho_\ell \to \underline{\rho}_\ell} \mathbb{E}[U_{i,1}^{CM}] = \lim_{\rho \to \bar{\bar{\rho}}, \rho_\ell \to \underline{\rho}_\ell} \mathbb{E}[U_{i,1}^{DM}]$.

   There exist correlation $\bar{\bar{\rho}} < \bar{\rho} < \tilde{\bar{\rho}}$ and $\underline{\underline{\rho}}_\ell < \rho_\ell < \underline{\rho}_\ell < 0$ and $\bar{T}$ such that for any $T \geq \bar{T}$,

$$\mathbb{E}[U_{i,T}^{\text{CM only}} | \mathcal{H}_T^{\text{CM only}}] > \mathbb{E}[U_{i,T}^{DM}] = \mathbb{E}[U_{i,1}^{DM}] \tag{2.33}$$

Given equation (2.33), for $T > \bar{T}$ the total welfare can be decomposed as

$$W^{\text{CM only}} - W = \quad I(\sum_{t=1}^{\bar{T}} \mathbb{E}[U_{i,t}^{\text{CM only}} | \mathcal{H}_t^{\text{CM only}}] - \bar{T}\mathbb{E}[U_{i,1}^{DM}]) + I \sum_{\bar{T}}^{T} \underbrace{\left(\mathbb{E}[U_{i,t}^{\text{CM only}} | \mathcal{H}_t^{\text{CM only}}] - \mathbb{E}[U_{i,1}^{DM}]\right)}_{>0}$$

where the first part is finite, and the second part is positive. As T increases, $W^{\text{CM only}} - W$ increases. When $T \to \infty$, $\lim_{T\to\infty} W^{\text{CM only}} - W > I(\sum_{t=1}^{\bar{T}} \mathbb{E}[U_{i,t}^{\text{CM only}} | \mathcal{H}_t^{\text{CM only}}] - \bar{T}\mathbb{E}[U_{i,1}^{DM}]) + I \lim_{T\to\infty}(T - \bar{T}) \left(\mathbb{E}[U_{i,\bar{T}}^{\text{CM only}} | \mathcal{H}_t^{\text{CM only}}] - \mathbb{E}[U_{i,1}^{DM}]\right) \to \infty$.

Therefore, there exists $\tilde{T}$, such that when $T > \tilde{T}$, $\bar{\bar{\rho}} < \bar{\rho} < \tilde{\bar{\rho}}$ and $\underline{\underline{\rho}}_{\ell} < \rho_{\ell} < \underline{\rho}_{\ell} < 0$ and $\bar{T}$, centralizing the decentralized market improves the welfare $W^{\text{CM only}} - W > 0$.

■

2.11 Evidence from the U.S. Equity Markets

The model provides us with testable predictions. It shows that a higher asset autocorrelation can lead to market fragmentation (Proposition 8). To test this prediction, we collected data for U.S. equities traded in exchanges, alternative trading systems (ATS), and over-the-counter (OTC) markets.

We obtain the ATS weekly summary of transaction volumes from FINRA and Exchange and OTC equity prices and transaction volumes from Wharton Research Data Service (WRDS). Our sample period is 2019-2022. We classify the lit exchanges as CM, e.g., Nasdaq, and NYSE. We classify ATS (e.g., Credit Suisse Crossfinder, Instinet) and OTC as DM.[22] We consider two samples for the regression and construct variables for each sample respectively. The first sample is the full sample that includes all equities traded in all venues. There is a concern that some equities may be restricted to be traded only in CM or DM due to regulations, preventing traders from changing their venues as the model assumption. To mitigate the influence of the market restrictions on our identification, we consider another sample, which includes those equities that have ever been traded in both CM and DM during 2019-2022. We drop singleton observations of equities with only one-week transactions in both samples.

We construct the dependent variable $DMshare_{i,t}$, which is the transaction volume of equity i in DM as a proportion of the total transaction volume of equity i in all venues in week t. Given that lower κ implies higher volatility in values, we use the price volatility in the last 100 days $Volatility^{[d-100,d]}$ as a proxy for κ, which is constructed as follows. We first calculate the standard deviation of the close price $p_{i,d}$ in the last 100 trading days $[d-100,d]$, and then take the weekly average of it for each equity i and week t.[23] We winsorize the top and bottom 1% to avoid the impact of extreme values.

We use the following regression to test the model prediction in Proposition 8 with

[22] Please refer to FINRA equity ATS Firms and SEC Form ATS-N Filings and Information for a complete list and more detailed information of current and past ATS for equities.

[23] As some OTC equities are not traded frequently, not all trading days have close prices. We use the midpoint of the best bid and ask prices on each trading day as the close price.

both the full sample and a smaller sample of equities traded in both DM and CM,

$$DMshare_{i,t} = \beta Volatility_{i,t}^{[d-100,d]} + \delta_i + \gamma_t + \varepsilon_{i,t} \tag{2.34}$$

where δ_i are equity fixed effects, γ_t are week fixed effects, and $\varepsilon_{i,t}$ are robust standard errors.

One concern is that traders' market choices may affect the price fluctuations. It can cause reverse causality and weaken our identification results. Therefore, we construct the lagged price volatility $Volatility_{i,t}^{[d-200,d-101]}$ as an instrumental variable (IV). It is the weekly average of the standard deviation of the close price $p_{i,d}$ between trading day $[d-200, d-101]$ for each equity i in week t. We winsorize the top and bottom 1% to avoid the extreme value.

Table 2.12 shows the summary of statistics of the variables. The average proportion traded in DM is 57.27% over the full sample and 10.94% for equities ever traded in both DM and CM. The price volatility and its IV on average are 4.190 and 4.164 respectively for the full sample. For equities traded in both DM and CM, the price volatility and its IV on average 4.652 and 4.709 respectively.

Figure 2.18 shows the average price volatility for each equity by their DM share. We can see that the average volatility is the highest for equities only traded in CM, lower for the equities traded in both DM and CM, and lowest for equities traded in DM only.

Table 2.13 shows the regression results of equation (2.34). Panel A shows the OLS regression results, where we can see that for both the full sample and the restricted sample with equities traded in CM and DM, the volatility is negatively correlated with the proportion of transaction volume in DM. Panel B-D shows the two-stage least-square (2SLS) regression results using the $Volatility^{[d-200,d-101]}$ as an IV. Panel B shows the reduced-form results with the IV as the independent variable. Panel C shows the first stage of 2SLS regression which indicates the IV is strongly correlated with $Volatility^{[d-100,d]}$. Panel D shows the second stage of 2SLS regression. We find that the volatility significantly decreases the proportion of transaction volume in DM, and the magnitude is larger than the

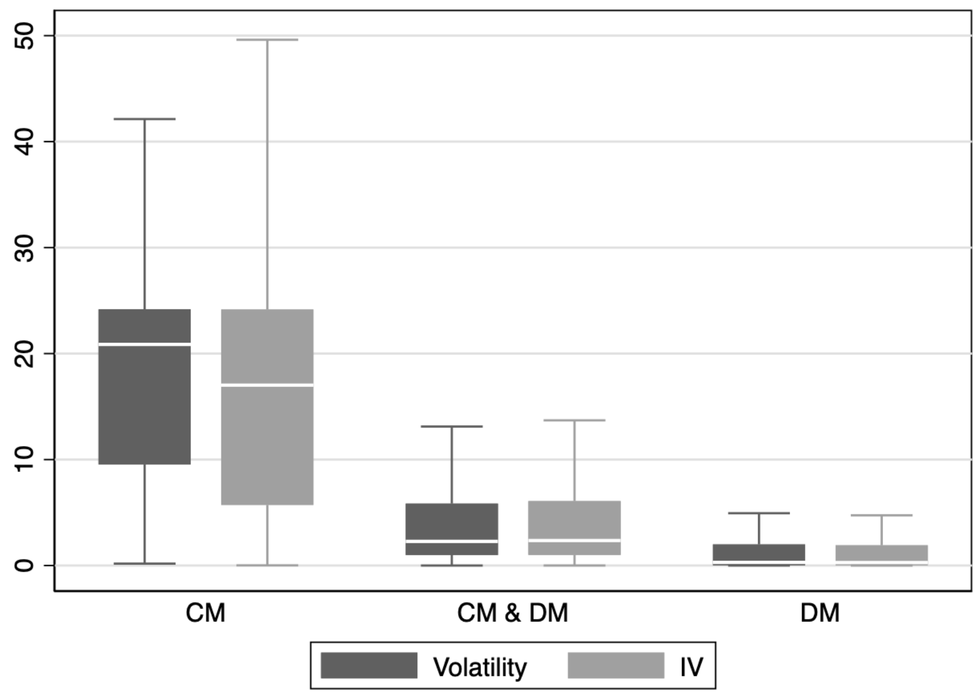
OLS regression results.

Table 2.12: Summary of Statistics

Full Sample					
Variable	Obs	Mean	Std. Dev.	Min	Max
$DMshare(\%)$	3,451,675	57.27	45.39	0	100
$Volatility^{[d-100,d]}$	3,451,675	4.190	13.20	6.70e-05	111.7
$Volatility^{[d-200,d-101]}$	3,451,675	4.164	12.65	4.61e-05	105.0
Equities Traded in CM & DM					
Variable	Obs	Mean	Std. Dev.	Min	Max
$DMshare(\%)$	1,651,680	10.94	12.83	0	100
$Volatility^{[d-100,d]}$	1,651,680	4.652	9.033	6.70e-05	111.7
$Volatility^{[d-200,d-101]}$	1,651,680	4.709	9.038	4.61e-05	105.0

Note: This table presents the summary of statistics for U.S. equity traded in exchanges, ATS, and OTC market during 2019-2022.

Figure 2.18: Volatility of Each Equity by DM Share



Note: This figure shows the average volatility for each equity during 2019-2022 by their average DM share. We classified the lit exchanges as CM, and ATS or OTC as DM. The dark box plots the volatility between $[t-100, t]$. The lighter box plots the IV, volatility between $[t-200, t-100]$. The lower and the upper end of the box are values at the 25th and 75th percentile. The white line in the box indicates the median value. And the lower and upper end of whiskers are lower and upper adjacent values.

Table 2.13: The Impact of Equity Volatility on DM Volume Share

Panel A. OLS		
Dependent Variable: $DMshare$	Full	CM&DM
$Volatility^{[t-100,t]}$	-0.00372***	-0.0172***
	(0.000397)	(0.00189)
Week FE	Yes	Yes
Equity FE	Yes	Yes
Observations	3,451,675	1,651,680
R-squared	0.982	0.546
Panel B. Reduced		
Dependent Variable: $DMShare$	Full	CM&DM
$Volatility^{[t-200,t-101]}$	-0.00291***	-0.0129***
	(0.000419)	(0.00173)
Week FE	Yes	Yes
Equity FE	Yes	Yes
Observations	3,451,675	1,651,680
R-squared	0.982	0.546
Panel C. First Stage of 2SLS		
Dependent Variable: $Volatility^{[t-100,t]}$	Full	CM&DM
$Volatility^{[t-200,t-101]}$	0.154***	0.290***
	(0.00277)	(0.00344)
Week FE	Yes	Yes
Equity FE	Yes	Yes
Observations	3,451,675	1,651,680
R-squared	0.743	0.775
Panel D. Second Stage of 2SLS		
Dependent Variable: $DMshare$	Full	CM&DM
$Volatility^{[t-100,t]}$	-0.0189***	-0.0447***
	(0.00274)	(0.00599)
Week FE	Yes	Yes
Equity FE	Yes	Yes
Observations	3,451,675	1,651,680
R-squared	0.982	0.546
Cragg-Donald Wald F statistic	3100	7097

Note: This table shows the impact of equity volatility on the proportion of volume traded in the DM versus CM. Robust standard errors are included in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

Chapter 3

The Impact of Leverage Ratio Regulation on Bond Market Liquidity

3.1 Introduction

The disruption of the U.S. bond market in March 2020 has raised regulatory concerns on whether some post-crisis regulation, in particular the supplemental leverage ratio (SLR) regulation, has contributed to market inefficiency. Following the bond market turmoil due to COVID-19, the Federal Reserve Board announced a one-year temporary waiver to SLR on April 1st, 2020, aiming to ease the strains in the Treasury market.[1] Although the temporary exemption expired on March 31, 2021, the Federal Reserve is still seeking comments on measures to adjust the SLR for potential improvement.[2] This paper aims to explore how the SLR, or more generally, the leverage ratio constraint has affected the bond market efficiency, both theoretically and empirically.

[1] This temporary change excludes U.S. Treasury securities and deposits at Federal Reserve Banks from calculating the SLR rule for holding companies.

[2] Jan 11th, 2022, Reuter, "Fed's Powell sees tweaks to the key leverage ratio, climate analysis on agenda going forward." https://www.reuters.com/markets/rates-bonds/feds-powell-sees-tweaks-key-leverage-ratio-climate-analysis-agenda-going-forward-2022-01-11/

We first examined the impact of leverage regulation on market liquidity through an event study of the March 2020 U.S. corporate bond market turmoil. Using a difference-in-difference method, we estimated changes in liquidity measures during the crisis. Risky principal trades, which require dealers to adjust their balance sheets for intermediation, served as the treatment group. Agency trades and riskless principal trades, which don't affect dealers' balance sheets, acted as the control group. Following O'Hara and Zhou (2021a), we used *transaction cost*, the price difference between interdealer and dealer-customer market prices, as a measure of market liquidity. Our findings revealed that the leverage regulation's impact on transaction costs varied across trading directions and underlying assets. Transaction costs rose significantly for dealer buy orders but slightly decreased for dealer sell orders. These changes were more pronounced in investment-grade (IG) bonds compared to high-yield (HY) bonds.

We build a model to explain the empirical findings. The model captures three features of the U.S. bond market. First, the bond market is highly decentralized. Assets are not cleared jointly, and prices can be different for the same asset across different traders. Second, the bond market is imperfectly competitive. Traders have market power and trade strategically. Third, bank-affiliated dealers are subject to leverage ratio regulation. Due to the constraint on balance sheet sizes, dealers are limited to providing balance sheet space for corporate bonds.

Theoretically, we demonstrate that leverage regulation has a heterogeneous impact on liquidity measures, aligning with our empirical findings. As dealers' inventory increases and the constraint tightens, they intermediate lower volumes. Consequently, the transaction cost for a customer selling to a dealer rises, while it falls for a customer buying from a dealer. Interestingly, the transaction cost for trading IG bonds can fluctuate more than that for HY bonds when the risk-weight on IG bonds is sufficiently high. This occurs when IG bonds contribute more to the balance sheet size than HY bonds, compelling dealers to demand higher compensation for intermediating IG bonds.

Next, we calibrated the model using U.S. corporate bond market transaction data from the March 2020 market turmoil. We first evaluated the welfare loss due to leverage ratio regulation by considering the counterfactual of completely removing it. Our findings show that dealers benefit from lifting the regulation. However, the impact on customers is mixed. Customers of dealers associated with global systemically important banks (GSIBs) benefit from relaxed leverage regulation, while other customers are slightly worse off.

We then consider two adjustments to the leverage ratio regulation: (i) modifying the risk weights for investment grade (IG) and high yield (HY) corporate bonds, and (ii) exempting Treasury positions from the leverage exposure. Our findings show that lowering the risk weight for IG bonds while increasing it for HY bonds can actually decrease overall welfare. This occurs because when the risk weight on safer assets decreases, dealers tend to intermediate more safe assets and fewer risky ones when the constraint is binding. This shift can potentially worsen risk-sharing between customers and dealers. Interestingly, we discover that exempting Treasury positions from the calculation improves welfare for both dealers and customers, effectively mimicking the outcome of removing leverage regulation entirely.

Adjusting the leverage ratio raises concerns about potentially decreasing financial stability, and balancing financial stability with market liquidity is challenging. To address this, we explored market design solutions that could mitigate unintended consequences without compromising stability. Our findings show that both introducing a central clearing party (CCP) and centralizing the market can increase overall welfare. Introducing a CCP has an effect similar to removing leverage regulation. The centralized market design offers the highest allocation efficiency among the options considered. We estimate that introducing a CCP would increase overall welfare by 42.4%, while centralizing the market could more than double overall welfare in the U.S. bond market during March 2020. However, it's important to note that while centralizing the market improves customers' welfare, it comes at the expense of dealers' welfare. Dealers experience a significant decrease in welfare as they lose profits from transaction costs.

Literature Review: This paper is related to the double auction literature in the tradition of Kyle (1989) (e.g. Vayanos, 1999; Vives, 2011; Rostek and Weretka, 2012; Ausubel et al., 2014; Malamud and Rostek, 2017). In particular, this paper builds a double auction model for decentralized market following Malamud and Rostek (2017) and Babus and Parlatore (2022). This model brings together the research on financial intermediaries and that on double auction, by introducing the intermediaries' leverage ratio constraint to a general framework of a decentralized market with strategic traders. We point out that market structure can amplify or attenuate the impact of post-crisis regulation on the bond market.

This paper is also related to literature that explores the effect of leverage ratio constraints of financial intermediaries on asset prices and market liquidity. (e.g. Brunnermeier and Pedersen, 2009; Garleanu and Pedersen, 2011; Adrian and Shin, 2010; Duffie, 2017a; Anderson and Stulz, 2017; He and Krishnamurthy, 2013; He et al., 2019; Chikis and Goldberg, 2021; He et al., 2022).

Among the existing literature, Allen and Wittwer (2022) are most closely related to this paper, which studies the capital-constrained traders in a centralized market in the style of Kyle (1989). Allen and Wittwer (2022) build a model of primary bond auction where leverage-constrained dealers trade strategically and empirically estimate how dealer capitalization under the leverage ratio regulation affects the demand and price of Canadian Treasuries. Similar to Allen and Wittwer (2022), this paper discusses how leverage ratio regulation affects market liquidity and prices. However, this paper is different from Allen and Wittwer (2022) in the following aspects. First, while Allen and Wittwer (2022) focus on a centralized market, this paper builds a model of a decentralized market with an endogenous dealer capital structure. It emphasizes how the dealer network, besides the dealers' market power, affects the market efficiency. Second, this paper complements Allen and Wittwer (2022) by exploring how leverage ratio regulation affects the secondary bond market.

This paper will provide a policy-relevant evaluation of SLR's impact on bond market liquidity. Existing literature found mixed results on how post-crisis regulation affects market liquidity. Some research found that there was no significant deterioration or even improvement in market liquidity with post-crisis regulation (e.g. Mizrach, 2015; Bessembinder and Maxwell, 2016; Anderson and Stulz, 2017; Adrian et al., 2017; Trebbi and Xiao, 2019). Some found that market liquidity worsened with leverage regulation (e.g. Adrian et al., 2017; Dick-Nielsen and Rossi, 2019; Choi et al., 2023). Recent literature has implied that the dealer's unwillingness to supply liquidity during the bond market turmoil during March 2020 might be caused by the leverage regulation (e.g. Haddad et al., 2021; He et al., 2022). It is puzzling why the impact of leverage regulation on market liquidity is mixed. This paper can provide an explanation with a decentralized bond market model.

Finally, this paper is related to the literature on market design solutions to bond market illiquidity. Enhancing bond market liquidity by redesigning the market has attracted attention from academia, industry, and regulators (Duffie, 2017b; Malamud and Rostek, 2017; Plante, 2017; Blackrock, 2020; Duffie, 2020; Liang and Parkinson, 2020; Allen and Wittwer, 2021; Fleming and Keane, 2021; Kutai et al., 2021; Cheshire and Embry, 2023). Among the few papers that quantify the improvement of the market designs during the crisis, Fleming and Keane (2021) shows that expanded central clearing of all outright trades of Treasury securities would have lowered dealers' daily gross settlement obligations 70% during the March 2020 sell-off, and Kutai et al. (2021) shows that spreads in government bond markets without exchanges would have been 30%–60% lower if there had been an exchange during the March 2020 crisis. This paper documents a 42.4% increase in overall welfare if there were a central clearing party, and more than 100% increase in overall welfare if the market is centralized in the U.S. bond market during March 2020.

3.2 Institutional Background

3.2.1 Post-Crisis Leverage Regulation

In the aftermath of the financial crisis in 2008, regulators established Basel III, aiming to strengthen the regulation, supervision, and risk management of banks. A core reform of Basel III is a set of leverage regulations to prevent excessive risk-taking by banks and other financial entities.

As part of the U.S. implementation of the Basel III reforms, on Sept. 3, 2014, U.S. regulators announced the adoption of the supplementary leverage ratio (SLR) for large bank holding companies, aiming to strengthen the risk management of the banking sector. The effective date of the SLR is January 1, 2018. Advanced approaches banking organizations must calculate and disclose their SLR beginning in the first quarter of 2015.[3] The SLR is the ratio of Tier 1 capital banks must hold relative to their total leverage exposure.[4] It is 3% for large US banks, and 5% for top-tier bank holding companies. When calculating SLR, all corporate bonds are included in the assets at 100% of their market value. The SLR, which does not distinguish between assets based on risk, is conceived as a backstop to existing risk-weighted capital requirements such as the Tier 1 capital ratio. SLR is cited as an important factor limiting banks and bank-affiliated dealers' ability to expand their balance sheets for bond market intermediation (Duffie, 2017a).

Prior to SLR, Basel III also implemented a risk-weighted capital ratio regulation. The Tier 1 capital ratio is the ratio of Tier 1 capital to the total risk-weighted assets. For investment grade corporate bonds, the tier 1 capital requirement assigns 6.25% risk-weight for bonds with 6 months or less remaining contractual maturity, 25% to those with 6-24

[3]There are some important changes to SLR following the bond market turmoil due to the COVID-19 pandemic. On April 1st, 2020, the Federal Reserve Board announced a temporary change to SLR. The change would exclude U.S. Treasury securities and deposits at Federal Reserve Banks from the calculation of the rule for holding companies. This temporary exemption expired on March 31, 2021. See "Federal Reserve Board announces temporary change to its supplementary leverage ratio rule to ease strains in the Treasury market resulting from the coronavirus and increase banking organizations' ability to provide credit to households and businesses", https://www.federalreserve.gov/newsevents/pressreleases/bcreg20200401a.htm.

[4]The total leverage exposure includes on-balance sheet assets, derivative assets, repo-style transaction exposures, and other off-balance sheet exposures. See "Agencies adopt supplementary leverage ratio final rule". https://www.federalreserve.gov/newsevents/pressreleases/bcreg20140903b.htm

months maturity, and 50% to those with more than 24 months maturity. For high-yield (HY) corporate bonds, the tier 1 capital requirement assigns a 150% risk-weight.[5] Tier 1 capital ratio is the basis for the Basel III international capital and liquidity standards devised after the financial crisis. All U.S. banks are required to have a minimum Tier 1 capital ratio of 6%. Advanced approaches banks holding companies are required to hold 2.5% capital surcharge, and global systemically important banks (G-SIBs) are required to hold a 1-4% more capital surcharge.

3.2.2 U.S. Corporate Bond Market Structure

The corporate bond market is highly fragmented and imperfectly competitive.

Most bond market participants are institutional traders. Major customers include pension funds, mutual funds, hedge funds, insurance companies, and sovereign wealth funds. Major bond dealers are subsidiaries of investment banks, commercial banks, or investment companies.

In the dealer-customer market, customers can negotiate and trade with dealers through traditional voice methods, or request-for-quote (RFQ) from dealers in electronic platforms such as MarketAxess. Most of the trades are still bilateral.

Dealers have two ways to trade, *principal trades*, and *agency trades*. In a *principal trade*, dealer commit their own capital and absorb the customer order with their own inventory. In an *agency trade*, a dealer finds a counterparty for the customer. The dealer does not need to use their capital or inventory to meet the customers' needs.

The SLR imposes a constraint on the balance sheet of bank-affiliated dealers. In particular, the SLR can limit the dealer to use *principal trades* to buy from customers as they increase the dealer's balance sheet size. The fact that the principal trades affect balance sheet size while the agency trades do not allows us to take a difference-in-difference approach in the event study.

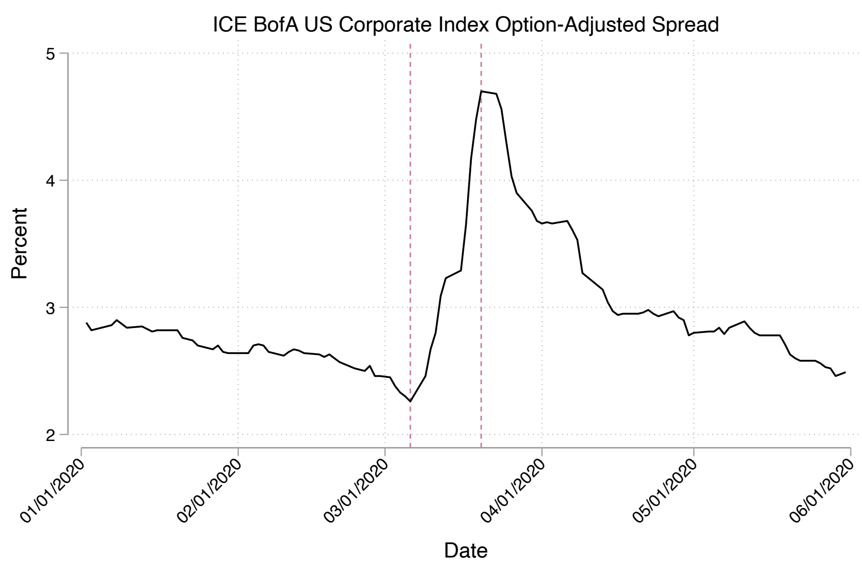
[5]See 12 CFR Part 217 Subpart F for risk-weight asset calculation of market risk. Using the Standardized measurement method for specific risk of corporate debt position, the risk-weighted value of the corporate debt position is the fair value × risk-weighting factor × 12.5.

3.3 Event Study

To examine whether the leverage regulation has an impact on market liquidity, we conduct an event study of the March 2020 bond market turmoil using Academic TRACE corporate bond transaction data. We follow Dick-Nielsen and Poulsen (2019) to clean the dataset.

Our sample period covers Feb. 1, 2020 - Mar. 19, 2020. We classified our sample into two periods, the normal period Feb. 1 - Mar. 5, and the crisis period Mar. 6 - Mar. 19, following O'Hara and Zhou (2021a). According to the trend in the ICE BofA US Corporate Index Option-Adjusted Spread (see Figure 3.1), the spread started to increase on March 6. The spread did not decrease until the Primary Dealer Credit Facility (PDCF) started operating on March 20. We focus on the period before PDCF was implemented.

Figure 3.1: ICE BofA US Corporate Index Option-Adjusted Spread



We apply the difference-in-difference method to estimate the impact of leverage regulation on the market outcomes during the March 2020 bond market turmoil. We follow Choi et al. (2023) to define the dealer-customer principal trade that is followed by an interdealer trade of the same asset of the same volume in a different direction in 15 minutes as riskless principal trades.[6] We will take agency trades and riskless principal trades as the controlled group, as it does not affect the dealer's balance sheet size. The principal trades, excluding riskless principal trades, are defined as *risky principal trades*. We will take the risky principal trades as the treatment group, as it changes dealers' balance sheet size and therefore the leverage ratio constraint.

We follow O'Hara and Zhou (2021a) to measure liquidity using the dealer's transaction cost for each transaction,

$$Cost_{ijkt} = \ln\left(p_{ijkt}/BenchmarkPrice_{ijkt}\right) \times TradeSign_{ijkt}$$

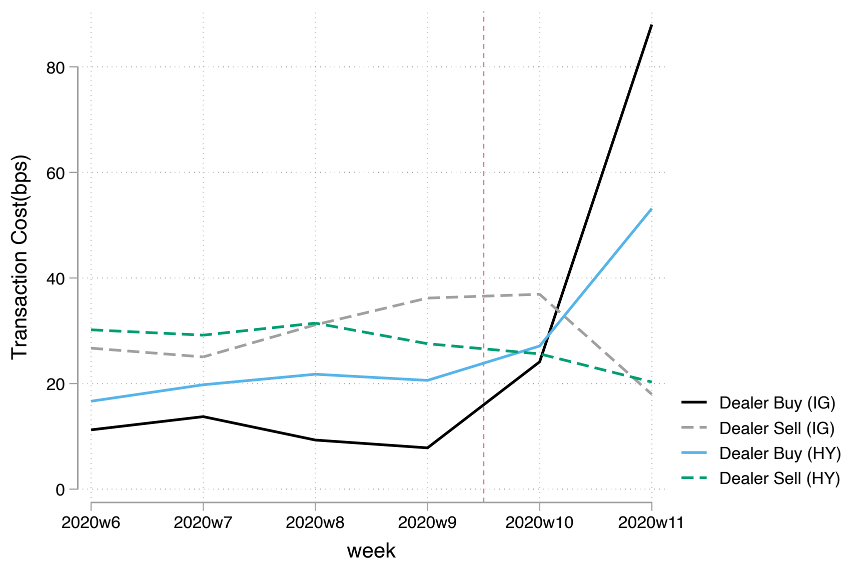
where $p_{ijkd\tau}$ is the transaction price; $BenchmarkPrice_{ijkd\tau}$ is the transaction price of the prior trade in that bond in the interdealer market; $Sign_{ijkd\tau}$ is 1 for a dealer sell order, and -1 for a dealer buy order. We multiply it by $10,000$ to compute transaction cost in basis points of value. The benchmark price may be stale due to infrequent transactions, creating extreme values. To avoid such noises, we winsorize the top and the bottom 1% across the sample. $Cost_{ijkdt}$ captures the daily average price difference between the interdealer trades and the dealer-customer trades.

Figure 3.2 shows the difference between the average transaction cost of principal trades and agency trades before and after the bond market disruption. We can see that the difference in the transaction cost remains stable from 10 to 30 bps before the market turmoil. The difference increased dramatically for the trades where dealers buy from the customers, while it slightly decreased for the trades where dealers sell to the customers during the crisis. We can also see that the change is larger for the IG bonds than the HY bonds. The difference between principal trades and agency trades in the transaction cost for dealers to buy IG bonds from customers was around 10 bps before the crisis, but reached over 80 bps during the crisis. The difference in the transaction cost for dealers to

[6] Dealers are required to report their transactions to FINRA within 15 minutes. If a dealer has a dealer-customer principal trade that is followed by an interdealer trade of the same asset of the same volume in a different direction in 15 minutes, it's possible that the dealer did not provide balance sheet space for the customer but facilitated a match of the customer with the next dealer.

buy HY bonds from customers was 10-20 bps before the crisis and less than 50 bps during the crisis.

Figure 3.2: The Difference of Transaction Costs Between Principal and Agency Trades Before and During the March 2020 Market Turmoil



Note: Each line is the difference of weekly average transaction cost between principal and agency trades for dealers to buy/sell IG/HY bonds, respectively. The vertical dashed line indicates the week of March 6, 2020, when the market disruption happens.

Baseline Regression: We use the following baseline regression to estimate the impact of the leverage constraint on market liquidity:

$$Cost_{ijkdt} = \beta Principal_d \times Post_t + Maturity_{kt} + \eta_k + \zeta_d + \delta_t + \varepsilon_{ijkdt} \quad (3.1)$$

where i indicates the buyer, j indicates the seller, k indicates the bond, t indicates the week, d indicates trade type ($d \in \{\text{principal, agency}\}$). $Principal_d$ is a dummy that indicates if the trade is principal trading. $Post_t$ is a dummy that equals 1 if the trade happens during the crisis period (Mar. 6, 2020 - Mar. 19, 2020) and 0 if it happens during the normal period (Feb. 1, 2020 - Mar. 5, 2020). We control the term-to-maturity of the traded bonds $Maturity_{kt}$. η_k, δ_t, and ζ_d are fixed effects for the bond, the transaction date, and the trade type, respectively. ε_{ijtkd} are standard errors clustered at the bond and date level. We focus on β, which measures the effect of leverage constraints on market liquidity.

We run the baseline regression with four different samples: (i) dealers sell IG bonds to customers, (ii) dealers buy IG bonds from customers, (iii) dealers sell HY bonds to customers, and (iv) dealers buy HY bonds from customers. Table 3.1 shows the baseline regression results. We can see two patterns of the leverage ratio regulation's impact on liquidity. First, the impact of leverage ratio regulation on liquidity is heterogeneous with different trading directions. The transaction cost for the dealer buy order significantly increased, but the spread for the dealer sell order decreased during the crisis. Second, the impact on liquidity is larger for safe assets than the risky assets. Investment grade (IG) bonds have a steeper increase(decrease) than the high yield (HY) bonds for a dealer buy (sell) order.

Table 3.1: The Impact of Leverage Constraint on Market Liquidity

	(1) dealers sell IG	(2) dealers buy IG	(3) dealers sell HY	(4) dealers buy HY
Principal × Post	-14.97**	63.08***	3.218	20.94**
	(5.089)	(2.996)	(3.886)	(7.940)
Maturity	-39.80**	19.39	-26.53**	-16.48
	(9.902)	(29.08)	(8.874)	(35.26)
Observations	297,446	316,115	107,300	112,394
R-squared	0.207	0.238	0.138	0.142
Trade Type FE	yes	yes	yes	yes
Bond FE	yes	yes	yes	yes
Date FE	yes	yes	yes	yes

*** $p<0.01$, ** $p<0.05$, * $p<0.1$

Regression by Dealer Size: To examine the impact of leverage ratio on different dealers, we split the sample into large and small dealers. We define the large dealers as those whose total trading volume with customers between January 1, 2020 and February 1, 2020 is at 95 percentile or higher among all dealers. And the other dealers are defined as small dealers. Note that the large traders account for over half of the dealer-customer transactions during the sample period.

Table 3.2 shows the regression results of equation (3.1) with the large and small dealers. We can see that the transaction cost significantly increases for the large dealers to buy IG bonds by 79.95 bps, while it increases by 48.27 bps for them to buy HY bonds. The transaction cost decreases for larger traders to sell IG bonds, and does not significantly change for them to sell HY bonds. In contrast, the transaction cost for small dealers to buy IG bonds only slightly increases by 11.05 bps, and it even decreases for the small dealers to buy the HY bonds. The transaction cost does not significantly change for small dealers to sell the bonds to customers.

Table 3.2: The Impact of Leverage Constraint on Market Liquidity by Dealer Size

	(1)	(2)	(3)	(4)
Panel A. Large Dealers	dealer sell IG	dealer buy IG	dealer sell HY	dealer buy HY
Principal × Post	-25.13***	79.95***	-3.466	48.27***
	(5.073)	(4.435)	(4.215)	(5.493)
Maturity	-60.03***	34.86	-33.31*	4.712
	(13.05)	(38.19)	(15.34)	(42.39)
Observations	170,920	194,166	54,650	62,034
R-squared	0.217	0.287	0.152	0.179
Trade Type FE	yes	yes	yes	yes
Bond FE	yes	yes	yes	yes
Date FE	yes	yes	yes	yes
	(1)	(2)	(3)	(4)
Panel B. Small Dealers	dealer sell IG	dealer buy IG	dealer sell HY	dealer buy HY
Principal × Post	-1.125	11.05**	14.32	-19.27*
	(3.187)	(3.691)	(7.922)	(8.183)
Maturity	-14.08**	-9.485	-16.14	-34.20
	(3.834)	(17.26)	(12.47)	(27.25)
Observations	124,995	120,285	52,414	50,056
R-squared	0.304	0.256	0.218	0.179
Trade Type FE	yes	yes	yes	yes
Bond FE	yes	yes	yes	yes
Date FE	yes	yes	yes	yes

*** $p<0.01$, ** $p<0.05$, * $p<0.1$

3.4 Model

In this section, we build a network model with leverage ratio regulations to explain the empirical findings. The empirical pattern in Section 3.3 is consistent with the intuition that the leverage ratio regulations can potentially hurt the dealers' ability to intermediate the market. In particular, non-risk-weighted leverage ratio regulation like SLR can make the dealers less willing to intermediate the relatively safe assets like IG bonds. It is also consistent with the fact that the large dealers – which are usually bank dealers – are subject to tighter leverage ratio regulations and therefore can be less willing to intermediate the market during the crisis.

3.4.1 Model Set-up and Equilibrium

The market has a core-periphery network structure. There are N dealers indexed by n. Each dealer can trade with $I(n)$ customer and with other dealers. Each customer in $I(n)$ can only trade with the dealer n. We denote the number of customers $I(n)$ as I_n. Traders can trade K risky assets with joint-normal payoffs $\mathcal{N}(\boldsymbol{\mu}, \boldsymbol{V})$ indexed by k, and 1 riskless asset with zero interest rate as the numéraire.[7]

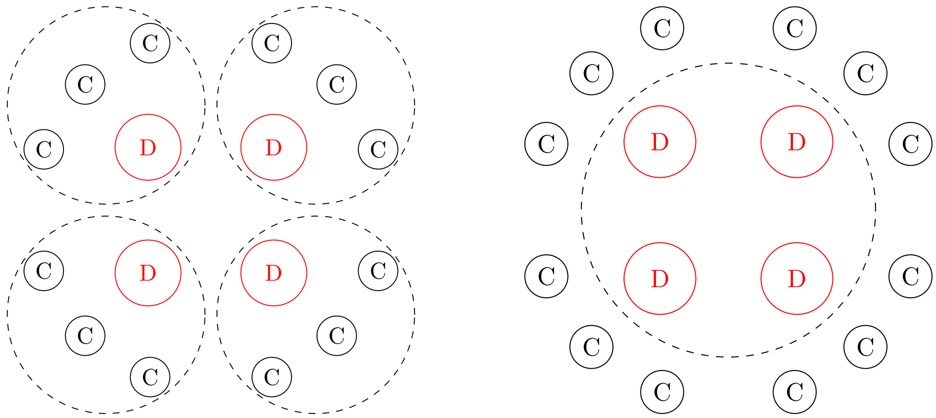
The market has three periods. At $t = 0$, customers receive endowment shocks. At $t = 1$, the dealer-customer market opens and customers trade with dealers (see Figure 3.3a). At $t = 2$, the interdealer market opens, and dealers trade with each other (see Figure 3.3b).

Endowment Shocks: At round $t = 0$, the dealer-customer market opens, each customer i receives exogenous endowment shock, $\boldsymbol{q}_0^i \sim \mathcal{F}(\boldsymbol{q}_0^i)$.

Dealer-Customer Market: At round $t = 1$, the dealer-customer market opens where customers trade with their dealers. For simplicity, we assume that the dealer has market

[7]We choose this model instead of the search model because (i) the traders have a linear demand instead of unit demand, which makes it possible to directly model the dealers' leverage ratio constraints with demand shocks from customers; (ii) we will later propose a centralized market design solution to the unintended consequence of the leverage ratio on bond market liquidity, and this framework provides a convenient comparison of the decentralized market and centralized market; and (iii) we want to model the friction due to the dealer's market power which is better captured by this model. Without search frictions, the model still captures price dispersions through market fragmentation.

Figure 3.3: Market Structure



(a) Dealer-customer Market (b) Interdealer market

power while customers do not, i.e., dealers trade strategically considering their price impact, while customers do not. Each customer i submits a demand schedule $\boldsymbol{q}^i_{dc}(\boldsymbol{p}^n_{dc}) : \mathbb{R}^K \to \mathbb{R}^K$ to maximize the utility,

$$v^i_{dc} = \boldsymbol{\mu}'(\boldsymbol{q}^i_0 + \boldsymbol{q}^i_{dc}) - \frac{1}{2}\kappa(\boldsymbol{q}^i_0 + \boldsymbol{q}^i_{dc})'\boldsymbol{V}(\boldsymbol{q}^i_0 + \boldsymbol{q}^i_{dc}) - (\boldsymbol{p}^n_{dc})'\boldsymbol{q}^i_{dc} \tag{3.2}$$

Without loss of generality, assume that the customers do not have market power while dealers have market power. Customers do not consider their price impact when submitting their bids.[8] By taking first order condition of v^i_{dc} with respect to $\boldsymbol{q}^i_{dc}$, we can solve for the demand schedule of customer i:

$$\boldsymbol{q}^i_{dc} = (\kappa\boldsymbol{V})^{-1}(\boldsymbol{\mu} - \kappa\boldsymbol{V}\boldsymbol{q}^i_0 - \boldsymbol{p}^n_{dc}) \tag{3.3}$$

Each dealer n submits a demand schedule $\boldsymbol{Q}^n_{dc}(\boldsymbol{p}^n_{dc}) : \mathbb{R}^K \to \mathbb{R}^K$ to maximize the utility given his price impact $\boldsymbol{\Lambda}^i_{dc}$:

$$u^n_{dc} = \mathbb{E}[u^n_{id}|\boldsymbol{Q}^n_{dc}] - (\boldsymbol{p}^n_{dc})'\boldsymbol{Q}^n_{dc} \tag{3.4}$$

[8] In Appendix 3.9, we provide a model with two-sided market power where both customers and dealers submit demand schedules given their price impact. The comparative statics results are not qualitatively different from this model with one-sided market power. The results do not quantitatively differ much from the current model.

subject to a leverage ratio constraint

$$\frac{\mathcal{E}^n}{\mathcal{A}^n + \boldsymbol{\mu}'\boldsymbol{W}\boldsymbol{Q}^n_{dc}} \geq \varphi^n \tag{3.5}$$

where $\mathcal{E}^n$ and $\mathcal{A}^n$ are dealer n's equity and risk-weighted inventory respectively. Balance sheet information ($\mathcal{E}^n$ and $\mathcal{A}^n$) is public information among the dealers. We use the asset value $\boldsymbol{\mu}$ to calculate the value of the traded assets.[9] In general, there are two ways to model the leverage ratio, constraints on debt (e.g. Brunnermeier and Pedersen, 2009; Adrian and Shin, 2010; Garleanu and Pedersen, 2011), or constraints on equity (e.g. He and Krishnamurthy, 2013; Allen and Wittwer, 2021). We use the latter one based on the fact that most bonds are used as collateral in the repo market to fund their purchase, so the debt level can change easily, while the equity cannot.

Equivalently, the dealer n chooses demand schedule $\boldsymbol{Q}^n_{dc}(\boldsymbol{p}^n_{dc})$ to maximize

$$u^n_{dc}(\boldsymbol{Q}^n_{dc}) \quad = \quad \mathbb{E}[u^n_{id}|\boldsymbol{Q}^n_{dc}] - (\boldsymbol{p}^n_{dc})'\boldsymbol{Q}^n_{dc} - r^n(\varphi^n\boldsymbol{\mu}'\boldsymbol{W}\boldsymbol{Q}^n_{dc} + \varphi^n\mathcal{A}^n - \mathcal{E}^n)$$

r^n is the shadow cost of the leverage ratio constraint. r^n satisfies complementary slackness that

$$r^n(\varphi^n\boldsymbol{\mu}'\boldsymbol{W}\boldsymbol{Q}^n_{dc} + \varphi^n\mathcal{A}^n - \mathcal{E}^n) = 0. \tag{3.6}$$

Without loss of generality, we assume that the impact of $\boldsymbol{Q}^n_{dc}$ on the expected utility $\mathbb{E}[u^n_{id}|\boldsymbol{Q}^n_{dc}]$, i.e. $\frac{d\mathbb{E}[u^n_{id}|\boldsymbol{Q}^n_{dc}]}{d\boldsymbol{Q}^n_{dc}}$ can be parameterized as $\boldsymbol{c}^n + \boldsymbol{C}^n\boldsymbol{Q}^n_{dc}$. By taking first order condition of u^n_{dc} with respect to $\boldsymbol{Q}^n_{dc}$, we can solve for the demand schedule of dealer n,

$$\boldsymbol{Q}^n_{dc} = \boldsymbol{S}^n\left(\boldsymbol{c}^n - \boldsymbol{p}^n_{dc} - \boldsymbol{\psi}^n_{dc}\right) \tag{3.7}$$

where $\boldsymbol{\psi}^n_{dc} = r^n_{dc}\varphi^n\boldsymbol{W}\boldsymbol{\mu}$ is the additional marginal cost of dealer n's demand in the dealer-customer market due to leverage regulation, $\boldsymbol{S}^n = \left(\boldsymbol{C}^n + \frac{1}{I_n}\kappa\boldsymbol{V}\right)^{-1}$ is the dealer n's

[9] Prices differ across different dealers in the decentralized market. To standardize the computation of dealers' balance sheet sizes using identical asset prices, we use the asset value $\boldsymbol{\mu}$ instead of their transaction prices to calculate the value of the traded assets. In the Appendix 3.9, we provide an extension of the model using market prices to calculate the value of traded assets following Allen and Wittwer (2023b).

demand elasticity (in absolute value) to the price $\boldsymbol{p}^n_{dc}$.

The market clears when $\boldsymbol{Q}^n_{dc} + \sum_{i \in I(n)} \boldsymbol{q}^i_{dc} = 0$ for all n. Given the market clearing condition, dealer n's price impact $\boldsymbol{\Lambda}^n_{dc} = \frac{1}{I}\kappa \boldsymbol{V}$.

Interdealer Market: At round $t = 2$, all dealers trade with each other in an imperfectly competitive centralized market. Each dealer n chooses the demand schedule $\boldsymbol{Q}^n_{id}(\boldsymbol{p}^n_{id}) : \mathbb{R}^K \to \mathbb{R}^K$ to maximize the utility given the dealer-customer trades $\boldsymbol{Q}^n_{dc}$ and his price impact $\boldsymbol{\Lambda}^n_{id}$,

$$u^n_{id} = \boldsymbol{\mu}'(\boldsymbol{Q}^n_{dc} + \boldsymbol{Q}^n_{id}) - \frac{1}{2}\alpha(\boldsymbol{Q}^n_{dc} + \boldsymbol{Q}^n_{id})'\boldsymbol{V}(\boldsymbol{Q}^n_{dc} + \boldsymbol{Q}^n_{id}) - (\boldsymbol{p}_{id})'\boldsymbol{Q}^n_{id}$$

subject to the leverage ratio constraint

$$\frac{\mathcal{E}^n}{\mathcal{A}^n + \boldsymbol{\mu}'\boldsymbol{W}\left(\boldsymbol{Q}^n_{dc} + \boldsymbol{Q}^n_{id}\right)} \geq \varphi^n$$

Equivalently, each dealer i chooses the demand schedule $\boldsymbol{Q}^n_{id}(\boldsymbol{p}^n_{id}) : \mathbb{R}^K \to \mathbb{R}^K$ to maximize the utility given his price impact $\boldsymbol{\Lambda}^n_{id}$,

$$u^n_{id} = \boldsymbol{\mu}'(\boldsymbol{Q}^n_{dc}+\boldsymbol{Q}^n_{id}) - \frac{1}{2}\alpha(\boldsymbol{Q}^n_{dc}+\boldsymbol{Q}^n_{id})'\boldsymbol{V}(\boldsymbol{Q}^n_{dc}+\boldsymbol{Q}^n_{id}) - \boldsymbol{p}'_{id}\boldsymbol{Q}^n_{id} - r^n_{id}(\varphi^n\boldsymbol{\mu}'\boldsymbol{W}(\boldsymbol{Q}^n_{dc}+\boldsymbol{Q}^n_{id}) + \varphi^n\mathcal{A}^n - \mathcal{E}^n)$$

By taking the first order condition with respect to $\boldsymbol{Q}^n_{id}$, we can solve for the optimal demand schedule

$$\boldsymbol{Q}^n_{id}(\boldsymbol{p}_{id}) = (\alpha\boldsymbol{V} + \boldsymbol{\Lambda}^n_{id})^{-1}(\boldsymbol{\mu} - \boldsymbol{p}_{id} - \alpha\boldsymbol{V}\boldsymbol{Q}^n_{dc} - \boldsymbol{\psi}^n_{id}) \tag{3.8}$$

where $\boldsymbol{\psi}^n_{id} = r^n_{id}\varphi^n\boldsymbol{W}\boldsymbol{\mu}$ is the additional marginal cost of dealer n's demand in the interdealer market due to leverage regulation, r^n_{id} satisfies the complementary slackness condition $r^n_{id}(\varphi^n\boldsymbol{\mu}'\boldsymbol{W}(\boldsymbol{Q}^n_{dc} + \boldsymbol{Q}^n_{id}) + \varphi^n\mathcal{A}^n - \mathcal{E}^n) = 0$.

Given the market clearing condition $\sum_n \boldsymbol{Q}_{id}^n = 0$, trader i's price impact satisfies:

$$\boldsymbol{\Lambda}_{id}^n = \left(-\sum_{n \neq \ell} \frac{d\boldsymbol{Q}_{id}^n}{d\boldsymbol{p}_{id}} \right)^{-1} \tag{3.9}$$

Given dealers' strategy at $t = 2$, we can write each dealer's utility u_{id}^n as a function of $\boldsymbol{Q}_{dc}^n$ and compute $\frac{d\mathbb{E}[u_{id}^n | \boldsymbol{Q}_{dc}^n]}{d\boldsymbol{Q}_{dc}^n} = \boldsymbol{c}^n + \boldsymbol{C}^n \boldsymbol{Q}_{dc}^n$. By matching the coefficients, we can obtain the parameters $\boldsymbol{c}^n$ and $\boldsymbol{C}^n$.

Theorem 15 (Equilibrium)**.** *The Beysian-Nash equilibrium is a profile of dealer's demand schedule and price impact, $\{\boldsymbol{Q}_{dc}^n, \boldsymbol{Q}_{id}^n, \boldsymbol{\Lambda}_{dc}^n, \boldsymbol{\Lambda}_{id}^n\}_n$ and a profile of customer's demand schedules $\{\boldsymbol{q}_{dc}^i\}_i$, and a profile of shadow cost of leverage constraint $\{r^n\}_n$ such that*

1. *In the dealer-customer market, each customer submits a demand schedule in equation (3.3); each dealer submits demand schedule in equation (3.7), given the shadow cost of leverage ratio r^n in equation (3.6) and the impact of inventory on the next round utility $\frac{d\mathbb{E}[u_{id}^n | \boldsymbol{Q}_{dc}^n]}{d\boldsymbol{Q}_{dc}^n}$.*

2. *In the interdealer market, each dealer submits a demand schedule in equation (3.8), given the price impact in equation (3.9).*

3.4.2 Comparative Statics with Inventory

Before the market disruption happened in March 2020, the bank-affiliated primary dealers' inventory had increased and pushed their leverage ratio towards the regulatory bound. The primary dealers have on average a $6.8 billion increase in their net positions from January 1st, 2020 to the week of March 6th, right before the bond market disruption. In this section, we consider the comparative statics of increasing inventory on market liquidity.

For tractability, in this section we assume that the market is symmetric, i.e. $I_n = I$ for all dealer n; the dealers have identical capital structure, i.e. $\mathcal{A}^n = A$, $\mathcal{E}^n = \mathcal{E}$; and the leverage requirements are the same for all dealers $\varphi^n = \varphi$. We assume there are two assets $K = 2$, a safe asset with mean payoffs μ_{safe} and variance σ_{safe}^2, and a risky asset with mean payoffs μ_{risky} and variance $\sigma_{risky}^2 > \sigma_{safe}^2$. The risk-weight on the safe asset is w_{safe}, and that on the risky asset is w_{risky}. We assume that for any customer of dealer n, $\boldsymbol{q}^i = \boldsymbol{\delta}^n$, $\boldsymbol{\delta}^n$ is either $\boldsymbol{\delta} < 0$ or $-\boldsymbol{\delta}$ with equal probability. $\boldsymbol{\delta}^n$ measures the imbalance of customers' endowment shocks across dealers and drives the intermediation need. Given these assumptions, without loss of generality, we can focus on the equilibrium for two dealers and their customers: (i) a dealer trading with customers whose endowment shocks make them natural buyers ($\boldsymbol{\delta}^n = -\boldsymbol{\delta} < 0$); and (ii) a dealer trading with customers whose endowment shocks make them natural sellers ($\boldsymbol{\delta}^n = \boldsymbol{\delta} > 0$). Hereafter, we refer to the first case *dealer-sell order*, and the second case *dealer-buy order*.

We construct the liquidity measure similar to the transaction cost in the event study. Let the transaction cost of dealer n be the price differences between the dealer-customer market and interdealer market, $Cost^n = |\boldsymbol{p}^n_{dc} - \boldsymbol{p}_{id}|$.

Proposition 16 (Dealer Buyer v.s. Dealer Seller)**.** *Given a symmetric market structure and identical capital structure, as the inventory included in the leverage exposure increases, i.e., $\mathcal{A}$ increases, the transaction cost $Cost^n$ increases for dealers to buy from customers, and decreases for dealers to sell to customers.*

As risk-weighed inventory $\mathcal{A}$ increases, the transaction cost measured by the price differences between the dealer-customer market and interdealer market increases for dealer-buy orders but decreases for dealer-sell orders. For a dealer-buy order, as the dealers' leverage ratio constraint becomes binding, it brings a shadow cost for dealers to hold the assets. Therefore, the marginal value of dealers decreases as they buy from the customers. In the interdealer market, as there is a limited supply of assets, the dealers can sell the assets at a higher price. Therefore, the transaction cost for the dealer to buy from customers increases. The price for dealers to sell to customers slightly increases as they expect the interdealer market price to increase, but not as much as the increase in the interdealer market. Therefore, the transaction cost for the dealer to sell to customers decreases.

Proposition 17 (Risky Asset v.s. Safe Asset)**.** *Given a symmetric market structure and identical capital structure, as the inventory included in the leverage exposure increases, i.e. $\mathcal{A}$ increases,* *the change in transaction cost ($d|\boldsymbol{p}_{dc} - \boldsymbol{p}_{id}|$) is larger for the safe asset than the risky asset when the risk-weight on the safe asset is sufficiently large, i.e.,* $w_{safe} \geq \bar{w} = \frac{\mu_{risky}}{\mu_{safe}} w_{risky}$.

As the risk weight on the safe asset increases, holding safe assets becomes more likely to tighten the balance sheet constraint than holding risky assets, and therefore, dealers require a higher transaction cost to compensate for the balance sheet cost of holding safe assets.

Note that we do not require the risk weight on the safe asset to be higher than the risk weight on the risky asset. We can have a larger change in transaction cost of the safe asset than that of the risky asset by applying the same weight on the safe and risky assets, just like the SLR. This is possible when the value of the risky asset is lower than the safe asset $\mu_{risky} < \mu_{safe}$. [10]

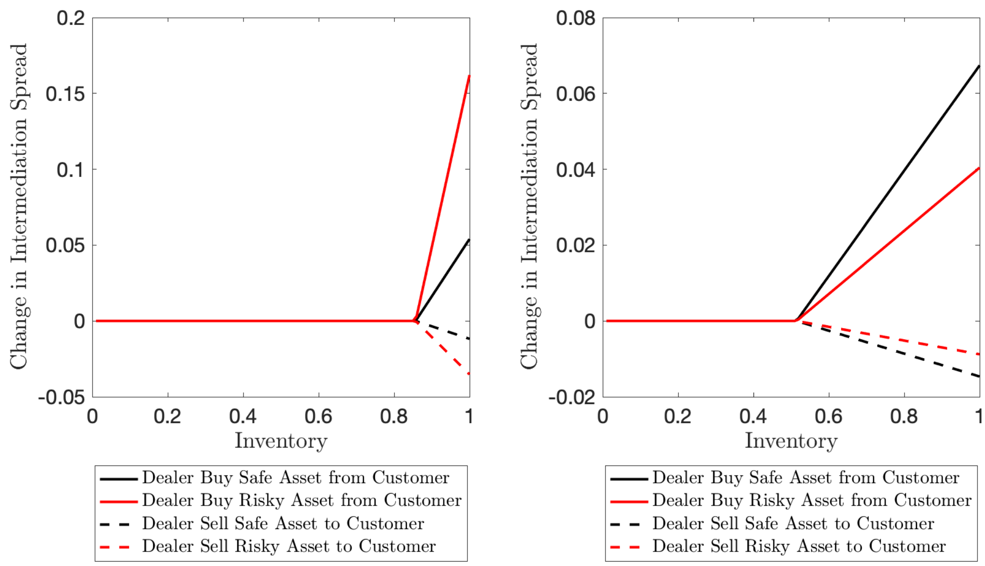
Figure 3.4 shows a simulation of the change in transaction cost with respect to inventory $\mathcal{A}$. Keeping everything else constant, when the risk weight on the safe asset is low, the change in the transaction cost of the risky asset is larger than that of the safe asset; when the weight on the safe asset is high, the patterns of change in transaction cost is consistent what we've found with the event study.

3.5 Quantitative Analysis

In this section, we will calibrate the model and estimate the impact of leverage ratio regulation on welfare. First, we consider the impact of the leverage ratio constraint on different types of dealers and their customers by considering the counterfactual of removing the leverage ratio regulation. Then we consider the counterfactual of different adjustments of the regulation, including adjusting the risk weight on corporate bonds and excluding Treasuries from the SLR calculation.

[10] When we extend the model by using market price instead of the value of the assets to calculate the balance sheet size of the dealers (see Appendix), $\mu_{risky} \leq \mu_{safe}$ is sufficient for us to find a set of primitive such that the change in transaction cost of safe asset to be larger than that of the risky asset with equal risk weight $w_{safe} = w_{risky}$. This is because dealers buy the risky asset with a lower price than the safe asset from customers as $\sigma^2_{risky} > \sigma^2_{safe}$. Given the same risk weight, the safe asset contributes more to the balance sheet constraint than the risky asset. The required compensation for the balance sheet cost is higher for holding safe assets than risky assets.

Figure 3.4: Change of Transaction Cost with Increasing Inventory



(a) $w_{safe} < \bar{w}$ (b) $w_{safe} > \bar{w}$

While adjustments of leverage ratio regulation can improve welfare, it may come at the cost of financial stability. These adjustments might be against the purpose of the leverage ratio regulation. To address such concerns, we will present market design solutions that can improve allocation efficiency without hurting financial stability by the end of this section. We consider two designs, introducing a central clearing party (CCP) and centralizing the corporate bond market. We find that both designs improve welfare.

3.5.1 Calibration

We estimate the general model allowing for heterogeneity across dealers. We allow three types of dealers differing in their capital structures ($\mathcal{A}$ and $\mathcal{E}$), market structure (I_n), and leverage requirements (φ): (i) G-SIB affiliated dealers; (ii) advanced-approach banks (AA banks) affiliated dealers; and (iii) other dealers.

Market Structure: We obtain the number of dealers from the cleaned Academic TRACE. Firstly, we constructed the trading network in 2019. We use the average number of active dealers per day as a proxy for the total number of dealers N. We use the 99 percentile of the number of transactions per dealer per day as a proxy for $I(n)$ of G-SIBs, the 95 percentile of the number of transactions per dealer per day as a proxy for $I(n)$ of advanced-approach banks, and the average number of transactions per dealer per day as a proxy for other dealers.

Asset Payoffs: We estimate the average prices for HY and IG bonds as the asset payoffs ($\boldsymbol{\mu}$) and covariances ($\boldsymbol{V}$) in the model. Note that each bond has a different coupon rate and maturity, therefore, we cannot aggregate the transaction prices directly. We first calculate the volume-weighted average yield of HY and IG bonds. Then we turn the weighted-average yields into the prices of representative IG and HY bonds with volume-weighted maturity and coupon rate. Finally, we calculate the average prices for the representative HY and IG bonds, respectively. We intend to use the CDS data to estimate the covariance matrix of the HY and IG bonds. For now, we use the covariance matrix in Table 3.3 (b).

Dealer Capital Structure: The balance sheet statistics of bank holding companies (BHC) are obtained from the Bank Regulatory Database of Wharton Research Data Service (WRDS). This dataset provides quarterly accounting data of detailed assets and liabilities, enabling the estimation of the bank's equity ($\mathcal{E}$), non-risk-weighted assets ($\mathcal{A}$, used for SLR), and risk-weighted assets ($\mathcal{A}$, used for Tier 1 capital ratio). To maintain the heterogeneity across banks, we estimated them by three different categories of banks: G-SIBs, large banks, and other banks. We assume that $log(\mathcal{A})$ and $log(\mathcal{E})$ are normally distributed, and present their mean and variance in Table 3.3. We calculate these measures using report data from 2019 Q4, the quarter before the bond market turmoil in March 2020. We also estimate the average primary dealer's change of total net positions of securities from Jan 1st, 2020 to Mar 6th, 2020 as inventory shocks to the G-SIBs during the crisis.

Leverage Regulation: The SLR requirement (ϕ) and risk weight ($\boldsymbol{W}$) are publicly available from the Federal Reserve. The SLR was established on September 3rd, 2014, with a 3% requirement for large US banks and a 5% requirement for top-tier bank holding companies. SLR assigns equal weight to all bonds for calculating total assets ($\boldsymbol{W} = \boldsymbol{Id}$).

Internal Calibration: We estimated the rest of the parameters using the two-step Simulated Method of Moments (SMM). The first set of parameters is risk-aversion of dealers (α), and risk-aversion of customers (κ). The second set parameter is stress capital buffer as an additional surcharge on the leverage ratio.[11] The third set of parameters describes the distribution of the demand shock to customers, including the mean and variance of the demand shock of IG/HY bonds, and the covariance of the demand shock of IG/HY bonds before and during the bond market turmoil. The targeted moments are the average daily customer buy/sell volume and transaction cost of HY and IG bonds in the normal period Feb. 1 - Mar. 5, and the crisis period, March 6 - March 19. Table 3.4 shows estimated parameters, the targeted moments, and the estimated moments from the model.

3.5.2 Welfare Impact of Leverage Regulation

In this section, we will discuss the welfare impact of leverage regulation on different types of dealers and their customers, by comparing the welfare during the March 2020 corporate bond market crisis and its counterfactual without leverage regulation. We measure the welfare as the sum of all traders' ex-ante utility,

$$W = \underbrace{\sum_n \mathbb{E}[u^n_{dc}]}_{\text{dealers' welfare}} + \underbrace{\sum_i \mathbb{E}[v^i_{dc}]}_{\text{customers' welfare}}$$

Figure 3.5 shows the welfare change for customers and dealers. Overall, the welfare improves for both dealers and customers without leverage regulation. The percentage improvement is 51.9% for dealers, 17.8% for customers, and 42.4% overall. The overall allocation is strictly better given that the dealers are not constrained to intermediate the

[11] Starting in 2013, the Federal Reserve's capital assessment of large banks consisted of two primary components: the Dodd-Frank Act Stress Test (stress test) and the Comprehensive Capital Analysis and Review (CCAR). The banks hold additional capital for the stress tests.

Table 3.3: External Calibration

(a) Market Structure

Parameter	Estimate
N: G-SIB	8
N: AA banks	16
N: others	366
$I(n)$: G-SIB	433
$I(n)$: large banks	51
$I(n)$: others	3

(b) Asset Payoffs

Parameter	Estimate
$\boldsymbol{\mu}$	$\begin{bmatrix} 1116.34 \\ 939.88 \end{bmatrix}$
$\boldsymbol{V}$ before crisis	$\begin{bmatrix} 1200 & 1000 \\ 1000 & 1300 \end{bmatrix}$
$\boldsymbol{V}$ during crisis	$\begin{bmatrix} 3600 & 3000 \\ 3000 & 3900 \end{bmatrix}$

(c) Leverage Regulation

Parameter	SLR
φ: G-SIB	5%
φ: AA banks	3%
φ: others	0%
$\boldsymbol{W}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(d) Capital Structure (in $\log_{10}$($Billion))

Parameter	Estimate	S.E.
$log(\mathcal{E})$: G-SIB	1.915	(0.436)
$log(\mathcal{E})$: AA banks	1.275	(0.292)
$log(\mathcal{A})$: G-SIB	3.114	(0.426)
$log(\mathcal{A})$: AA banks	2.474	(0.240)

(e) Inventory Shocks (in $Billion)

Parameter	Estimate
$\Delta\mathcal{A}$: G-SIB	6.815

Figure 3.5: The Welfare Change with Removing Leverage Regulation During March 2020 Corporate Bond Market Disruption

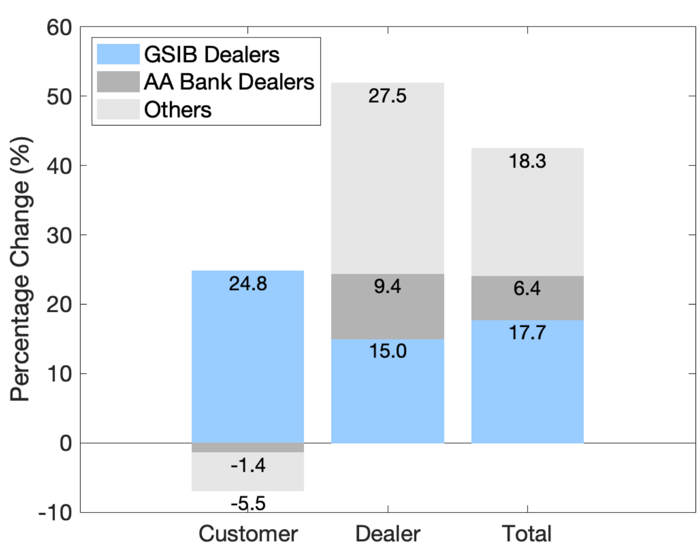


Table 3.4: Internal Calibration

(a) Risk Aversion

Parameter	Estimate
α ($\times 10^{-6}$)	1.2
κ ($\times 10^{-6}$)	1.3

(b) Capital Buffer

Parameter	Estimate
Capital Buffer:	1.291%

(c) Customer Demand Distribution

Parameter	Estimate
mean of IG $\boldsymbol{\delta}$ before crisis ($\times 10^3$):	0.6
mean of IG $\boldsymbol{\delta}$ during crisis ($\times 10^3$):	2
std. of IG $\boldsymbol{\delta}$ before crisis ($\times 10^3$):	45
std. of IG $\boldsymbol{\delta}$ during crisis ($\times 10^3$):	90
mean of HY $\boldsymbol{\delta}$ before crisis ($\times 10^3$):	0.5
mean of HY $\boldsymbol{\delta}$ during crisis ($\times 10^3$):	1
std. of HY $\boldsymbol{\delta}$ before crisis ($\times 10^3$):	40
std. of HY $\boldsymbol{\delta}$ during crisis ($\times 10^3$):	68
corr. of IG and HY $\boldsymbol{\delta}$ before crisis ($\times 10^3$):	-5
corr. of IG and HY $\boldsymbol{\delta}$ during crisis ($\times 10^3$):	-55

(d) Target Moments

Moments	Data	Model
Average daily dealer-sell volume of IG bonds before crisis ($billion)	4.28	4.25
Average daily dealer-buy volume of IG bonds before crisis ($billion)	4.52	4.26
Average daily dealer-sell volume of IG bonds during crisis ($billion)	5.79	5.81
Average daily dealer-buy volume of IG bonds during crisis ($billion)	5.62	5.67
Average dealer-sell orders' transaction cost of IG bonds before crisis (bps)	39.48	24.30
Average dealer-buy orders' transaction cost of IG bonds before crisis (bps)	14.42	43.34
Average dealer-sell orders' transaction cost of IG bonds during crisis (bps)	34.95	40.56
Average dealer-buy orders' transaction cost of IG bonds during crisis (bps)	114.20	92.78
Average daily dealer-sell volume of HY bonds before crisis ($billion)	4.15	4.18
Average daily dealer-buy volume of HY bonds before crisis ($billion)	4.09	4.01
Average daily dealer-sell volume of HY bonds during crisis ($billion)	5.23	5.49
Average daily dealer-buy volume of HY bonds during crisis ($billion)	5.08	5.13
Average dealer-sell orders' transaction cost of HY bonds before crisis (bps)	40.17	26.41
Average dealer-buy orders' transaction cost of HY bonds before crisis (bps)	28.66	55.62
Average dealer-sell orders' transaction cost of HY bonds during crisis (bps)	27.65	48.26
Average dealer-buy orders' transaction cost of HY bonds during crisis (bps)	94.49	86.60

market. Although the leverage ratio has a heterogeneous impact on the transaction cost, those transaction costs don't have a direct impact on overall welfare, as they are just transfers within the market.

There are some redistributions across customers of different dealers. GSIB dealers were constrained dealers during March 2020. The customers affiliated with GSIB dealers have higher welfare without leverage regulation. In a market under selling pressure, the customers of GSIB dealers pay a lot more transaction costs and sell less with leverage regulation than without leverage regulation. The customers affiliated with other dealers have a lower welfare without leverage regulation. The leverage regulation makes the unconstrained dealers buy more from the customers and sell less to the customers. In a market under selling pressure, the unconstrained dealers take more risk with leverage regulation than without. The allocation efficiency is worse for unconstrained dealers' customers without leverage ratio regulation.

The dealers all benefit from removing the leverage regulation, regardless of whether they are constrained or not during the 2020 market disruption.

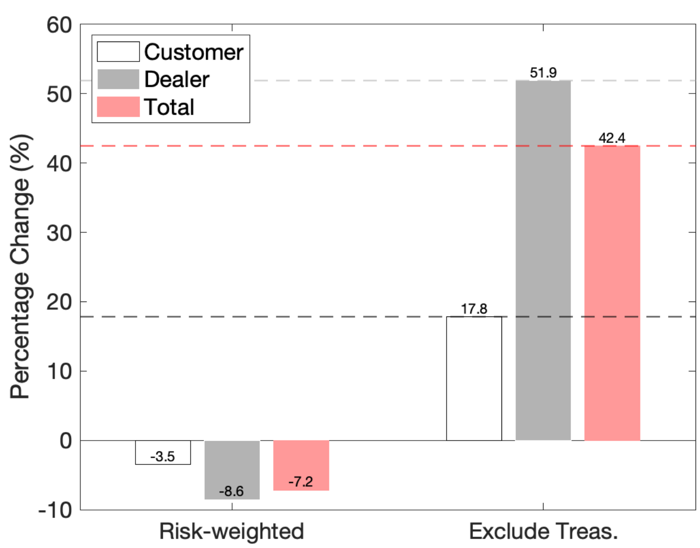
3.5.3 Adjustments of SLR

In this section, we consider the impact of leverage regulation on the welfare of customers and dealers, versus the following counterfactuals: (i) changing the risk-weight; (ii) excluding Treasuries from the total leverage exposure. Figure 3.6 shows the estimated welfare in these counterfactuals. The dashed line plots the welfare change of removing the leverage constraint as a benchmark.

We first consider changing the risk-weight $\boldsymbol{W}$ on the corporate bonds to the same risk-weight as Tier 1 Capital Ratio, i.e., 50% on IG bonds, and 150% on HY bonds. Figure 3.6 shows estimated welfare change. We find that changing the risk weight slightly decreases the welfare. It shows that the welfare of customers decreases by 3.5%, the welfare of dealers decreases by 8.6%, and the total welfare decreases by 7.2% compared with the welfare with the original leverage constraints.

We then consider the counterfactual of excluding Treasuries from the total leverage exposure. To ease strains in the Treasury market turmoil, the Fed temporarily modified the SLR to exclude U.S. Treasury securities and central bank reserves on April 1st, 2020. This policy was announced after the PDCF and SMMCF, and therefore, its impact on the corporate bond cannot be disentangled from the other policies using reduced-form methods. We use the primary dealers' average net Treasury positions during the week of Mar. 6, 2020, as the position of the G-SIB dealers. We simulated the model assuming that the net Treasury positions are excluded from the total leverage exposure. Figure 3.6 shows the estimated welfare change. It shows that the welfare of customers increases by 17.8%, the welfare of dealers increases by 51.9%, and the total welfare increases by 42.4% compared with the welfare with the original leverage constraints. The exemption of Treasuries from the total leverage exposure effectively relaxes the leverage constraints, and the simulated equilibrium results are the same as removing the leverage ratio regulation (see Figure 3.6).

Figure 3.6: Welfare Change with Adjustments of Leverage Regulation



3.5.4 Market Design Solutions

Despite that we find the leverage regulation may or may not decrease the welfare compared with existing capital requirements, we hesitate to give policy recommendations on adjusting the leverage ratio requirement, as changing leverage ratio regulation may hurt the financial stability of the banks. And it is always difficult to trade off the potential benefit from financial stability and the loss from allocation inefficiency. In this section, we consider two market design solutions that attenuate the unintended consequences of leverage regulation without hurting financial stability. We show that they can actually increase the welfare by more than relaxing the leverage constraints.

3.5.4.1 Central Clearing Party

One market design is to introduce a central clearing party (CCP) to trade with all traders, including both customers and dealers. The CCP will allow the netting of trades of the same asset among the same counterparties on the same settlement day. For now, the CCP is available in the Treasury market, and is only accessible to dealers and interdealer brokers, but is usually not accessible to customers and principal trading firms. [12]

Allowing all traders to net their trades at CCP can potentially reduce the usage of dealers' balance sheets when there is a daisy chain of intermediation between dealers and customers. For example, customer $C1$ sells a units of the asset to dealer $D1$, $D1$ sells b units to dealer $D2$, and dealer $D2$ sells c units of the asset to $C1$. Without CCP, the dealer $D1$'s balance sheet size can increase a units before $D1$ sells to $D2$, which can make the dealer's leverage ratio constraint binding such that $D1$ may not be willing to buy from $C1$. With CCP, the trades are netted so that CCP will give the customer $c - a$ units, dealer $D1$ $a - b$ units, and dealer $D2$ $b - c$ units. This reduces the dealer $D1$'s pressure of providing balance sheet space for intermediation.

In this model, allowing CCP to be accessible to all traders is equivalent to letting

[12] See SEC Proposes Rules to Improve Risk Management in Clearance and Settlement and to Facilitate Additional Central Clearing for the U.S. Treasury Market, Sept. 14, 2022.

the constraint be binding only after the interdealer market trades are settled. Dealers will no longer be constrained to take inventory from customers before they trade in the interdealer market, i.e., equation (3.5) is not binding for dealer-customer trades. In that case, at $t = 1$, dealer's demand schedule becomes

$$\boldsymbol{Q}_{dc}^{n} = (\boldsymbol{C}^{n} + \frac{1}{I_n}\kappa \boldsymbol{V})^{-1}(\boldsymbol{c}^{n} - \boldsymbol{p}_{dc}^{n})$$

Figure 3.7 shows that introducing CCP can increase welfare to the extent that as if dealers are not subject to the leverage ratio regulation. Both the customers' welfare and the dealers' welfare increase.

3.5.4.2 Centralized Market (All-to-all Market)

A more radical design is to centralize the bond market. Assume all dealers and customers trade in a centralized exchange that operates as a double auction. Each customer $i \in I$ submits demand schedule $\boldsymbol{q}_c^i(\boldsymbol{p}_c) : \mathbb{R}^K \to \mathbb{R}^K$ to maximize the utility,

$$v_c^i = \boldsymbol{\mu}'\boldsymbol{q}_c^i - \frac{1}{2}\kappa(\boldsymbol{q}_0^i + \boldsymbol{q}_c^i)'\boldsymbol{V}(\boldsymbol{q}_0^i + \boldsymbol{q}_c^i) - (\boldsymbol{p}_c)'\boldsymbol{q}_c^i$$

By taking first order condition with respect to $\boldsymbol{q}_c^i$ we can solve the customer i's demand schedule

$$\boldsymbol{q}_c^i = (\kappa \boldsymbol{V})^{-1}\left(\boldsymbol{\mu} - \boldsymbol{p}_c - \kappa \boldsymbol{V}\boldsymbol{q}_0^i\right) \tag{3.10}$$

Each dealer $n \in N$ the demand schedule $\boldsymbol{Q}_{id}^n(\boldsymbol{p}_{id}^n) : \mathbb{R}^K \to \mathbb{R}^K$ to maximize the utility given his price impact $\boldsymbol{\Lambda}_c^n$,

$$u_c^n = \boldsymbol{\mu}'\boldsymbol{Q}_c^n - \frac{1}{2}\alpha(\boldsymbol{Q}_c^n)'\boldsymbol{V}\boldsymbol{Q}_c^n - (\boldsymbol{p}_c)'\boldsymbol{Q}_c^n$$

subject to the leverage ratio constraint

$$\frac{\mathcal{E}^n}{\mathcal{A}^n + \boldsymbol{\mu}'\boldsymbol{W}\boldsymbol{Q}_c^n} \geq \varphi^n$$

By taking first order condition with respect to $\boldsymbol{Q}_c^n$ we can solve the dealer n's demand schedule

$$\boldsymbol{Q}_c^n = (\alpha \boldsymbol{V} + \boldsymbol{\Lambda}_c^n)^{-1} (\boldsymbol{\mu} - \boldsymbol{p}_c - \boldsymbol{\psi}_c^n) \tag{3.11}$$

where $\boldsymbol{\psi}_c^n = r_c^n \varphi^n \boldsymbol{W} \boldsymbol{\mu}$ is the marginal cost of holding the assets due to leverage ratio regulation.

The dealer n's price impact is equal to the inverse of the residual demand curve,

$$\boldsymbol{\Lambda}_c^n = \left(\sum_i \frac{d\boldsymbol{q}_c^i}{d\boldsymbol{p}_c} - \sum_{m \neq n} \frac{d\boldsymbol{Q}_c^m}{d\boldsymbol{p}_c} \right)^{-1}$$

The market clears when $\sum_i \boldsymbol{q}_c^i + \sum_n \boldsymbol{Q}_c^n = 0$, from which we can solve for the equilibrium centralized market price.

To understand how the centralized market can improve the welfare, let's compare the customer i's allocation (eq. (3.10)) with that in the decentralized market,

$$\boldsymbol{q}_{dc}^i = (\kappa \boldsymbol{V})^{-1} \left(\boldsymbol{\mu} - \boldsymbol{p}_{dc}^n - \kappa \boldsymbol{V} \boldsymbol{q}_0^i \right)$$

The differences between customer demand in the decentralized market and the centralized market are the dealer customer market prices. In the decentralized market, customers do not have access to the interdealer market price because of limited participation. The customers have to pay a transaction cost as rent extracted by the dealers' market power. The customer with positive endowment shock may face an even higher transaction cost when his dealer is constrained to buy from him, further worsening the customer allocation efficiency. In the centralized market, the transaction cost is eliminated. The customers face a deeper market as unconstrained bank dealers, non-bank dealers, and customers with opposite endowment shocks participate in the same exchange. So the customers receive a better price in the centralized market.

Now let's compare the dealer n's allocation in the centralized market (eq.(3.11)) with the dealer's allocation in the decentralized market,

$$\boldsymbol{Q}_{dc}^{n} + \boldsymbol{Q}_{id}^{n} = (\alpha \boldsymbol{V} + \boldsymbol{\Lambda}_{id}^{n})^{-1}(\boldsymbol{\mu} - \boldsymbol{p}_{id} + \boldsymbol{\Lambda}_{id}^{n}\boldsymbol{Q}_{dc}^{n} - \boldsymbol{\psi}_{id}^{n})$$

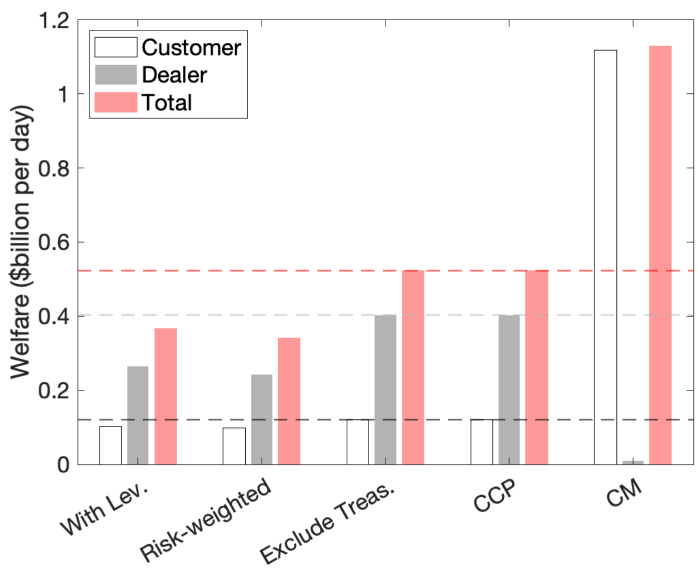
First, the price impact is lower in the centralized market as the dealer faces more elastic residual demand with the participation of customers. Second, in the decentralized market, the dealer firstly takes inventory from the customers but later is not able to trade away all inventory towards the target portfolio due to price impact, while in the centralized market, the inefficiency of holding inventory is eliminated. Third, as there's no need for the dealers to hold inventory for intermediation as in the decentralized market, the leverage ratio constraint is less binding, and the shadow cost of constraint is lower.

Still, even the the leverage constraints are less binding and allocation efficiency improves, centralizing the market does not necessarily bring Pareto improvements. It can decrease the welfare of the dealers. Figure 3.7 shows that the welfare increases for the customers but decreases for dealers during the crisis if the market is centralized. This is because dealers lose the profit from transaction costs between the interdealer market and the dealer-customer market.

3.6 Conclusion

While the post-crisis regulations targeting banks' leverage ratios aim to enhance financial market resilience, there is an ongoing debate about their potential adverse effects on market liquidity. This study delves into the influence of leverage ratio regulations on the liquidity of the bond market. In our empirical analysis, we observe varied effects of leverage regulation on market liquidity among different traders and assets, particularly during the corporate bond market turmoil in March 2020. Notably, the impact of leverage regulation is more pronounced on safe assets compared to risky assets. We construct a decentralized market model incorporating leverage ratio regulation to provide insights into these observations. Additionally, we assess welfare implications during market turmoil

Figure 3.7: Comparisons of Welfare with Adjusting Leverage Regulation vs. Market Designs



and explore counterfactual scenarios involving adjustments to leverage regulation and market design solutions. Our findings indicate that modifying the risk weight on corporate bonds could potentially reduce welfare, while excluding the Treasury from leverage exposure calculations may enhance welfare. The introduction of a central clearing party and an all-to-all market has the potential to mitigate the unintended consequences of leverage regulation on market liquidity without compromising financial stability.

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Appendix

3.7 Proofs

Proof of Theorem 15. At $t = 3$, given the dealer trades $\boldsymbol{Q}^n_{dc}$, each dealer i chooses the demand schedule $\boldsymbol{Q}^n_{id}(\boldsymbol{p}^n_{id}) : \mathbb{R}^K \to \mathbb{R}^K$ to maximize the utility given his price impact $\boldsymbol{\Lambda}^n_{id}$,

$$\begin{aligned} u^n_{id} &= \boldsymbol{\mu}'(\boldsymbol{Q}^n_{dc} + \boldsymbol{Q}^n_{id}) - \frac{1}{2}\alpha(\boldsymbol{Q}^n_{dc} + \boldsymbol{Q}^n_{id})'\boldsymbol{V}(\boldsymbol{Q}^n_{dc} + \boldsymbol{Q}^n_{id}) - (\boldsymbol{p}_{id})'\boldsymbol{Q}^n_{id} \\ &\quad - r^n_{id}(\varphi^n \boldsymbol{\mu}'\boldsymbol{W}(\boldsymbol{Q}^n_{id} + \boldsymbol{Q}^n_{dc}) + \varphi^n \mathcal{A}^n - \mathcal{E}^n) \end{aligned}$$

By taking the first order condition with respect to $\boldsymbol{Q}^n_{id}$, we can solve for the optimal demand schedule

$$\boldsymbol{Q}^n_{id}(\boldsymbol{p}_{id}) = (\alpha \boldsymbol{V} + \boldsymbol{\Lambda}^n_{id})^{-1}(\boldsymbol{\mu} - \boldsymbol{p}_{id} - \alpha \boldsymbol{V}\boldsymbol{Q}^n_{dc} - \boldsymbol{\psi}^n_{id}) \tag{3.12}$$

where $\boldsymbol{\psi}^n_{id} \equiv r^n_{id}\varphi^n \boldsymbol{W}\boldsymbol{\mu}$. r^n_{id} satisfies the complementary slackness condition

$$r^n_{id}(\varphi^n \boldsymbol{\mu}'\boldsymbol{W}\,(\boldsymbol{Q}^n_{id} + \boldsymbol{Q}^n_{dc}) + \varphi^n \mathcal{A}^n - \mathcal{E}^n) = 0.$$

Given the market clearing condition, trader i's price impact satisfies:

$$\boldsymbol{\Lambda}^n_{id} = \left(-\sum_{n \neq \ell} \frac{d\boldsymbol{Q}^n_{id}}{d\boldsymbol{p}_{id}}\right)^{-1} = \left(\sum_{\ell \neq n}(\alpha \boldsymbol{V} + \boldsymbol{\Lambda}^\ell_{id})^{-1}\right)^{-1} = \frac{1}{N-2}\alpha \boldsymbol{V}. \tag{3.13}$$

By the market clearing condition,

$$\boldsymbol{p}_{id} = \boldsymbol{\mu} - \alpha \boldsymbol{V}\bar{\boldsymbol{Q}}_{dc} - \bar{\boldsymbol{\psi}}_{id} \tag{3.14}$$

where $\bar{\boldsymbol{\psi}}_{id} \equiv \frac{1}{N}\sum_n r^n_{id}\varphi^n \boldsymbol{W}\boldsymbol{\mu}$.

So the demand schedule in the interdealer market can be rewriten as

$$\boldsymbol{Q}_{id}^n(\boldsymbol{p}_{id}) = (\alpha\boldsymbol{V} + \boldsymbol{\Lambda}_{id}^n)^{-1}(\alpha\boldsymbol{V}(\bar{\boldsymbol{Q}}_{dc} - \boldsymbol{Q}_{dc}^n) + \bar{\boldsymbol{\psi}}_{id} - \boldsymbol{\psi}_{id}^n) \tag{3.15}$$

Differentiating the dealer's third-round utility $\mathbb{E}[u_{id}^n|\boldsymbol{Q}_{dc}^n]$ with respect to $\boldsymbol{Q}_{dc}^n$, we have

$$\frac{d\mathbb{E}[u_{id}^n|\boldsymbol{Q}_{dc}^n]}{d\boldsymbol{Q}_{dc}^n} = \frac{\partial\mathbb{E}[u_{id}^n|\boldsymbol{Q}_{dc}^n]}{\partial\boldsymbol{Q}_{dc}^n} + \frac{\partial\mathbb{E}[u_{id}^n|\boldsymbol{Q}_{dc}^n]}{\partial\boldsymbol{Q}_{id}^n}\frac{d\boldsymbol{Q}_{id}^n}{d\boldsymbol{Q}_{dc}^n} \tag{3.16}$$

Given that $\frac{\partial\mathbb{E}[u_{id}^n|\boldsymbol{Q}_{dc}^n]}{\partial\boldsymbol{Q}_{id}^n} = 0$ by the first order condition in the second round, we can simplify the equation as

$$(\frac{d\mathbb{E}[u_{id}^n|\boldsymbol{Q}_{dc}^n]}{d\boldsymbol{Q}_{dc}^n})' = (\frac{\partial\mathbb{E}[u_{id}^n|\boldsymbol{Q}_{dc}^n]}{\partial\boldsymbol{Q}_{dc}^n})' = \boldsymbol{\mu} - \alpha\boldsymbol{V}(\boldsymbol{Q}_{dc}^n + \mathbb{E}[\boldsymbol{Q}_{id}^n|\boldsymbol{Q}_{dc}^n]) - \mathbb{E}[\boldsymbol{\psi}_{id}^n] = \boldsymbol{c}^n - \boldsymbol{C}^n\boldsymbol{Q}_{dc}^n \tag{3.17}$$

where $\boldsymbol{c}^n = \boldsymbol{\mu} - \frac{N-2}{N-1}\left(\frac{1}{N}\alpha\boldsymbol{V}\sum_{m\neq n}\mathbb{E}[\boldsymbol{Q}_{dc}^m] + \mathbb{E}[\bar{\boldsymbol{\psi}}_{id} - \boldsymbol{\psi}_{id}^n]\right) - \mathbb{E}[\boldsymbol{\psi}_{id}^n]$, and $\boldsymbol{C}^n = \frac{2}{N}\alpha\boldsymbol{V}$.

Each customer i submits demand schedule $\boldsymbol{q}_{dc}^i(\boldsymbol{p}_{dc}^n) : \mathbb{R}^K \to \mathbb{R}^K$ to maximize the utility,

$$v_{dc}^i = \boldsymbol{\mu}'(\boldsymbol{q}_0^i + \boldsymbol{q}_{dc}^i) - \frac{1}{2}\kappa(\boldsymbol{q}_0^i + \boldsymbol{q}_{dc}^i)'\boldsymbol{V}(\boldsymbol{q}_0^i + \boldsymbol{q}_{dc}^i) - (\boldsymbol{p}_{dc}^n)'\boldsymbol{q}_{dc}^i$$

By taking the first order condition,

$$\boldsymbol{q}_{dc}^i = -\boldsymbol{q}_0^i + (\kappa\boldsymbol{V})^{-1}(\boldsymbol{\mu} - \boldsymbol{p}_{dc}^n) \tag{3.18}$$

Given the market clearing condition that $\boldsymbol{Q}_{dc}^n + \sum_{i\in I(n)}\boldsymbol{q}_{dc}^i = 0$, we have

$$\boldsymbol{p}_{dc}^n = \boldsymbol{\mu} - \kappa\boldsymbol{V}\bar{\boldsymbol{q}}_0^n + \frac{1}{I_n}\kappa\boldsymbol{V}\boldsymbol{Q}_{dc}^n \tag{3.19}$$

where $\bar{\boldsymbol{q}}_0^n \equiv \frac{1}{I_n}\sum_{i\in I(n)}\boldsymbol{q}_0^i$.

Given his price impact $\boldsymbol{\Lambda}_{dc}^n$, the dealer ℓ chooses $\boldsymbol{Q}_{dc}^n(\boldsymbol{p}_{dc}^n)$ to maximize

$$u_{dc}^n(\boldsymbol{Q}_{dc}^n) \quad = \quad \mathbb{E}[u_{id}^n|\boldsymbol{Q}_{dc}^n] - (\boldsymbol{p}_{dc}^n)'\boldsymbol{Q}_{dc}^n - r_{dc}^n(\varphi^n\boldsymbol{\mu}'\boldsymbol{W}\boldsymbol{Q}_{dc}^n + \varphi^n\mathcal{A}^n - \mathcal{E}^n)$$

r^n_{dc} is the shadow cost of the leverage ratio constraint. r^n_{dc} satisfies complementary slackness that

$$r^n_{dc}(\varphi^n \boldsymbol{\mu}' \boldsymbol{W} \boldsymbol{Q}^n_{dc} + \varphi^n \mathcal{A}^n - \mathcal{E}^n) = 0. \tag{3.20}$$

Taking first order condition with respect to $\boldsymbol{Q}^n_{dc}$, we can solve customer n's equilibrium demand schedule in the dealer-customer market,

$$\boldsymbol{Q}^n_{dc} = \boldsymbol{S}^n \left(\boldsymbol{c}^n - \boldsymbol{p}^n_{dc} - \boldsymbol{\psi}^n_{dc}\right) \tag{3.21}$$

where $\boldsymbol{S}^n = \left(\boldsymbol{C}^n + \frac{1}{I_n}\kappa \boldsymbol{V}\right)^{-1}$, $\boldsymbol{\psi}^n_{dc} = r^n_{dc}\varphi^n \boldsymbol{W}\boldsymbol{\mu}$ is the additional marginal cost of dealer m's demand in the dealer-customer market due to leverage regulation.

The market clears when $\boldsymbol{Q}^n_{dc} + \sum_{i\in I(n)} \boldsymbol{q}^i_{dc} = 0$. Given the market clearing condition, we can solve that the equilibrium price

$$\boldsymbol{p}^n_{dc} = \boldsymbol{b}^n - \boldsymbol{B}^n \bar{\boldsymbol{q}}^n_0, \tag{3.22}$$

where $\boldsymbol{B}^n = \left(\frac{1}{I_n}\boldsymbol{S}^n + (\kappa \boldsymbol{V})^{-1}\right)^{-1}$, $\boldsymbol{b}^n = \boldsymbol{B}^n \left(\frac{1}{I_n}\boldsymbol{S}^n(\boldsymbol{c}^n - \boldsymbol{\psi}^n_{dc}) + (\kappa \boldsymbol{V})^{-1}\boldsymbol{\mu}\right)$.

Now let's solve the coefficients $\boldsymbol{C}^n$ and $\boldsymbol{c}^n$. Plugging the equilbrium price in equation (3.22) into equation (3.21) and taking the expectation, we have

$$\mathbb{E}[\boldsymbol{Q}^m_{dc}] = \mathbb{E}[\boldsymbol{S}^m \boldsymbol{B}^m (\kappa \boldsymbol{V})^{-1} \left(\boldsymbol{c}^m - \boldsymbol{\psi}^m_{dc} - \boldsymbol{\mu} + \kappa \boldsymbol{V} \bar{\boldsymbol{q}}^m_0\right)] \tag{3.23}$$

We can solve $\boldsymbol{c}^n$ by plugging equation (3.23) into equation (3.17),

$$\boldsymbol{c}^n = \boldsymbol{\mu} - \tfrac{N-2}{N-1}\left(\tfrac{1}{N}\alpha \boldsymbol{V} \textstyle\sum_{m\neq n} \boldsymbol{S}^m \boldsymbol{B}^m (\kappa \boldsymbol{V})^{-1} \left(\boldsymbol{c}^m - \mathbb{E}[\boldsymbol{\psi}^m_{dc}] - \boldsymbol{\mu} + \kappa \boldsymbol{V} \mathbb{E}[\bar{\boldsymbol{q}}^m_0]\right) + \mathbb{E}[\bar{\boldsymbol{\psi}}_{id}]\right) - \tfrac{1}{N-1}\mathbb{E}[\boldsymbol{\psi}^n_{id}], \quad \forall n. \tag{3.24}$$

We can rewrite eq. (3.24) in the following form,

$$\vec{\boldsymbol{c}} = (\boldsymbol{Id} + \boldsymbol{M})^{-1}\left(\vec{\boldsymbol{\mu}} + \boldsymbol{M}(\mathbb{E}[\vec{\boldsymbol{\psi}}_{dc}] + \vec{\boldsymbol{\mu}} - \mathbb{E}[\vec{\boldsymbol{d}}]) - \frac{N-2}{N-1}\boldsymbol{G}\mathbb{E}[\vec{\boldsymbol{\psi}}_{id}] - \frac{1}{N-1}\mathbb{E}[\vec{\boldsymbol{\psi}}_{id}]\right) \tag{3.25}$$

where $\vec{\boldsymbol{c}} \in \mathbb{R}^{KN}$ whose $(n-1)K+1$ to nK elements are $\boldsymbol{c}^n$, $\vec{\boldsymbol{\mu}} \in \mathbb{R}^{KN}$ whose $(n-1)K+1$

to nK elements are $\boldsymbol{\mu}$, $\boldsymbol{M} \in \mathbb{R}^{KN\times KN}$ whose $(n-1)K+1$ to nK rows and $(m-1)K+1$ to mK columns are $\frac{N-2}{N(N-1)}\alpha \boldsymbol{V}\boldsymbol{S}^m\boldsymbol{B}^m(\kappa\boldsymbol{V})^{-1} = \frac{N-2}{N(N-1)}\alpha(\frac{2}{N}\alpha + \frac{2}{I_m}\kappa)^{-1}\boldsymbol{Id}$ if $m \neq n$ and zero otherwise, $\vec{\boldsymbol{\psi}}_{dc} \in \mathbb{R}^{KN\times 1}$ whose $(n-1)K+1$ to nK element are $\boldsymbol{\psi}^n_{dc}$, $\vec{\boldsymbol{d}} \in \mathbb{R}^{KN}$ whose $(n-1)K+1$ to nK elements are $\kappa\boldsymbol{V}\bar{\boldsymbol{q}}^m_0$, $\boldsymbol{G} \in \mathbb{R}^{KN\times KN}$ whose $(n-1)K+1$ to nK rows and $(m-1)K+1$ to mK columns are $\frac{1}{N}\boldsymbol{Id}$, $\vec{\boldsymbol{\psi}}_{id} \in \mathbb{R}^{KN}$ whose $(n-1)K+1$ to nK elements are ψ_{id}. $\vec{\boldsymbol{c}}$ is uniquely determined given $\vec{\boldsymbol{\psi}}_{dc}$ and $\vec{\boldsymbol{\psi}}_{id}$ (equivalently given $\{r^n_{dc}, r^n_{id}\}_n$) as $\boldsymbol{Id} + \boldsymbol{M}$ has full rank.

Given $\vec{\boldsymbol{c}}$, the marginal cost of leverage constraint in the interdealer market is also uniquely determined,

$$\boldsymbol{\psi}^n_{dc} = \max\left\{0, \boldsymbol{W}\boldsymbol{\mu}\frac{\mathcal{A}^n - \mathcal{E}^n/\varphi^n + \boldsymbol{\mu}'\boldsymbol{W}\boldsymbol{Q}^n_0 + \boldsymbol{\mu}'\boldsymbol{W}\boldsymbol{S}^n\boldsymbol{B}^n(\kappa\boldsymbol{V})^{-1}(\boldsymbol{c}^n - \boldsymbol{\mu} + \kappa\boldsymbol{V}\bar{\boldsymbol{q}}^n_0)}{\boldsymbol{\mu}'\boldsymbol{W}\boldsymbol{S}^n\boldsymbol{B}^n(\kappa\boldsymbol{V})^{-1}\boldsymbol{W}\boldsymbol{\mu}}\right\}$$

Given $\vec{\boldsymbol{c}}$ and $\vec{\boldsymbol{\psi}}_{dc}$, $\vec{\boldsymbol{\psi}}_{id}$ is uniquely determined,

$$\boldsymbol{\psi}^n_{id} = \max\left\{0, \boldsymbol{W}\boldsymbol{\mu}\frac{\mathcal{A}^n - \mathcal{E}^n/\varphi^n + \boldsymbol{\mu}'\boldsymbol{W}\boldsymbol{Q}^n_{dc} + \boldsymbol{\mu}'\boldsymbol{W}(\frac{N-1}{N-2}\alpha\boldsymbol{V})^{-1}(\alpha\boldsymbol{V}(\bar{\boldsymbol{Q}}_{dc} - \boldsymbol{Q}^n_{dc}) + \bar{\boldsymbol{\psi}}_{id})}{\boldsymbol{\mu}'\boldsymbol{W}(\frac{N-1}{N-2}\alpha\boldsymbol{V})^{-1}\boldsymbol{W}\boldsymbol{\mu}}\right\}$$

The expected $\boldsymbol{\psi}^n_{id}$ given $\boldsymbol{Q}^n_{dc}$ is

$$\mathbb{E}[\boldsymbol{\psi}^n_{id}] = \max\left\{0, \boldsymbol{W}\boldsymbol{\mu}\frac{\mathcal{A}^n - \mathcal{E}^n/\varphi^n + \frac{2}{N}\boldsymbol{\mu}'\boldsymbol{W}\boldsymbol{Q}^n_{dc} + \boldsymbol{\mu}'\boldsymbol{W}(\alpha\boldsymbol{V})^{-1}(\boldsymbol{\mu} - \boldsymbol{c}^n - \frac{1}{N-1}\mathbb{E}[\boldsymbol{\psi}^n_{id}])}{\boldsymbol{\mu}'\boldsymbol{W}(\frac{N-1}{N-2}\alpha\boldsymbol{V})^{-1}\boldsymbol{W}\boldsymbol{\mu}}\right\}$$

Given that $\boldsymbol{Q}^n_{dc} = (\frac{2}{N}\alpha + \frac{2}{I_n}\kappa)^{-1}(\boldsymbol{V})^{-1}\left(\boldsymbol{c}^n - \boldsymbol{\psi}^n_{dc} - \boldsymbol{\mu} + \kappa\boldsymbol{V}\bar{\boldsymbol{q}}^n_0\right)$, we can rewrite it in the matrix form,

$$\mathbb{E}[\vec{\boldsymbol{\psi}}_{id}] = \max\left\{\boldsymbol{0}, \vec{\boldsymbol{f}}_{id} + \boldsymbol{F}_{id}(2\boldsymbol{Q}(\vec{\boldsymbol{c}} - \vec{\boldsymbol{\psi}}_{dc} - \vec{\boldsymbol{\mu}} + \vec{\boldsymbol{d}}) + \frac{N-1}{N-2}(\vec{\boldsymbol{\mu}} - \vec{\boldsymbol{c}} - \frac{1}{N-1}\mathbb{E}[\vec{\boldsymbol{\psi}}_{id}])\right\}$$

where $\max$ is an elementwise operator, $\vec{\boldsymbol{f}}_{id} \in \mathbb{R}^{KN}$ where the $(n-1)K+1$ to nK elements are $\boldsymbol{W}\boldsymbol{\mu}\frac{\mathcal{A}^n - \mathcal{E}^n/\varphi^n}{\boldsymbol{\mu}'\boldsymbol{W}(\frac{N-1}{N-2}\alpha\boldsymbol{V})^{-1}\boldsymbol{W}\boldsymbol{\mu}}$, $\boldsymbol{F}_{id} \in \mathbb{R}^{KN\times KN}$ where the $(n-1)K+1$ to nK rows and columns are $\boldsymbol{W}\boldsymbol{\mu}\frac{\boldsymbol{\mu}'\boldsymbol{W}(\frac{N-1}{N-2}\alpha\boldsymbol{V})^{-1}}{\boldsymbol{\mu}'\boldsymbol{W}(\frac{N-1}{N-2}\alpha\boldsymbol{V})^{-1}\boldsymbol{W}\boldsymbol{\mu}}$ and are zeros for all the other elements, $\boldsymbol{Q} \in \mathbb{R}^{KN\times KN}$ where the $(n-1)K+1$ to nK rows and columns are $\frac{N-2}{N(N-1)}\alpha(\frac{2}{N}\alpha + \frac{2}{I_n}\kappa)^{-1}\boldsymbol{Id}$ and zero for all the other elements.

We can solve $\vec{\boldsymbol{c}}$, $\vec{\boldsymbol{\psi}}_{dc}$ and $\vec{\boldsymbol{\psi}}_{id}$ as a function of the primitives.

Welfare: The sum of utility of all customer i affiliated to dealer n is

$$\sum_{i\in I(n)} v^i_{dc} = \sum_{i\in I(n)} \boldsymbol{\mu}'(\boldsymbol{q}^i_0 + \boldsymbol{q}^i_{dc}) - \frac{1}{2}\kappa(\boldsymbol{q}^i_0 + \boldsymbol{q}^i_{dc})'\boldsymbol{V}(\boldsymbol{q}^i_0 + \boldsymbol{q}^i_{dc}) - (\boldsymbol{p}^n_{dc})'\boldsymbol{q}^i_{dc}$$

The gain from trader for dealer n is

$$u^n = \boldsymbol{\mu}'(\boldsymbol{Q}^n_{dc} + \boldsymbol{Q}^n_{id}) - \frac{1}{2}\alpha(\boldsymbol{Q}^n_{dc} + \boldsymbol{Q}^n_{id})'\boldsymbol{V}(\boldsymbol{Q}^n_{dc} + \boldsymbol{Q}^n_{id}) - (\boldsymbol{p}_{id})'\boldsymbol{Q}^n_{id} - (\boldsymbol{p}^n_{dc})'\boldsymbol{Q}^n_{dc}$$

The total welfare is

$$W = \textstyle\sum_i \boldsymbol{\mu}\boldsymbol{q}^i_0 - \sum_i \frac{1}{2}\kappa(\boldsymbol{q}^i_0 + \boldsymbol{q}^i_{dc})'\boldsymbol{V}(\boldsymbol{q}^i_0 + \boldsymbol{q}^i_{dc}) - \sum_n \frac{1}{2}\alpha(\boldsymbol{Q}^n_{dc} + \boldsymbol{Q}^n_{id})'\boldsymbol{V}(\boldsymbol{Q}^n_{dc} + \boldsymbol{Q}^n_{id})$$

The total gain from trade is

$$-\sum_i \frac{1}{2}\kappa\left((\boldsymbol{q}^i_0 + \boldsymbol{q}^i_{dc})'\boldsymbol{V}(\boldsymbol{q}^i_0 + \boldsymbol{q}^i_{dc}) - (\boldsymbol{q}^i_0)'\boldsymbol{V}\boldsymbol{q}^i_0\right) - \sum_n \frac{1}{2}\alpha(\boldsymbol{Q}^n_{dc} + \boldsymbol{Q}^n_{id})'\boldsymbol{V}(\boldsymbol{Q}^n_{dc} + \boldsymbol{Q}^n_{id})$$

■

Lemma 10. *Given symmetric market structure and symmetric leverage constraint, and that the endowment shock of customers has equal probability to either* $\boldsymbol{\delta} > 0$ *or* $-\boldsymbol{\delta}$*, the second round leverage constraint is not binding for any dealer* n*, i.e.* $r^n_{id} = 0, \forall n \in N$*.*

Proof. Given that customers' demand has equal probability to be either $\boldsymbol{\delta} > 0$ or $-\boldsymbol{\delta} < 0$, we are subject to analyze two types of dealers: a dealer who buys from customers with $\boldsymbol{\delta} > 0$ denoted with superscript b, and a dealer who sells to customers with $-\boldsymbol{\delta} < 0$ denoted with superscript s. We will prove the lemma by first calculating the demand schedules with $r^n_{id} = 0$ and then verifying that the constraints are not binding for all dealers in the interdealer market.

First, if $r^n_{id} = 0$, then $\boldsymbol{Q}^b_{dc} > 0$, $\boldsymbol{Q}^s_{dc} < 0$ and $\boldsymbol{Q}^b_{dc} + \boldsymbol{Q}^s_{dc} \leq 0$. Given symmetry, $\boldsymbol{c}^n = \boldsymbol{c}$. Intuitively, a dealer who sells to customers will not be leverage-constrained, $r^s_{dc} = 0$. If the dealers who buy from the customer are not constrained, then $\boldsymbol{c}^n = \boldsymbol{\mu}$, $\boldsymbol{Q}^b_{dc}+\boldsymbol{Q}^s_{dc} = 0$. If the dealers who buys from the customer are constrained, then the constrained demand $\boldsymbol{Q}^b_{dc}$ is lower than the counterfactual unconstrained demand $\tilde{\boldsymbol{Q}}^b_{dc}$ if the buyer is not subject to the regulation, i.e., $\boldsymbol{Q}^b_{dc} < \tilde{\boldsymbol{Q}}^b_{dc}$, as the unconstrained demand will violates the constraint $\varphi^b \boldsymbol{\mu}' \boldsymbol{W} \tilde{\boldsymbol{Q}}^b_{dc} + \varphi^b \mathcal{A}^b - \mathcal{E}^b > 0 = \varphi^b \boldsymbol{\mu}' \boldsymbol{W} \boldsymbol{Q}^b_{dc} + \varphi^b \mathcal{A}^b - \mathcal{E}^b$. As $\tilde{\boldsymbol{Q}}^b_{dc} + \boldsymbol{Q}^s_{dc} = 0$ given our previous argument, $\boldsymbol{Q}^b_{dc} + \boldsymbol{Q}^s_{dc} < \tilde{\boldsymbol{Q}}^b_{dc} + \boldsymbol{Q}^s_{dc} = 0$.

If $r^n_{id} = 0$, then the demand schedule for dealer n in the interdealer market is

$$\boldsymbol{Q}^n_{id} = \frac{N-2}{N-1}(\bar{\boldsymbol{Q}}_{dc} - \boldsymbol{Q}^n_{dc})$$

For a dealer who buys from the customer in the dealer customer market, $\boldsymbol{Q}^b_{id} < 0$ as $\boldsymbol{Q}^b_{dc} > \bar{\boldsymbol{Q}}_{dc} = \frac{1}{2}(\boldsymbol{Q}^b_{dc} + \boldsymbol{Q}^s_{dc})$. Given that $\boldsymbol{Q}^b_{id} < 0$, the total risk-weighted asset this dealer holds in the interdealer market is lower than that in the dealer-customer market, $\varphi^b \boldsymbol{\mu}' \boldsymbol{W}(\boldsymbol{Q}^b_{id} + \boldsymbol{Q}^b_{dc}) + \varphi^b \mathcal{A}^b - \mathcal{E}^b < \varphi^b \boldsymbol{\mu}' \boldsymbol{W} \boldsymbol{Q}^b_{dc} + \varphi^b \mathcal{A}^b - \mathcal{E}^b \leq 0$. So the constraint is not binding for the dealer buyer in the dealer-customer market.

For a dealer who sells to the customer in the dealer customer market, $\boldsymbol{Q}^s_{dc} + \boldsymbol{Q}^s_{id} \leq 0$, as $\boldsymbol{Q}^s_{dc} + \boldsymbol{Q}^s_{id} = \frac{N-2}{N-1}\bar{\boldsymbol{Q}}_{dc} + \frac{1}{N-1}\boldsymbol{Q}^s_{dc} = \frac{N-2}{N-1}\frac{1}{2}(\boldsymbol{Q}^b_{dc} + \boldsymbol{Q}^s_{dc}) + \frac{1}{N-1}\boldsymbol{Q}^s_{dc} < 0$. Therefore the constraint is not binding for the dealer who sells to the customer. ■

Proof of Proposition 16. Given the symmetric market structure, without loss of generality, we can focus on one buyer and one seller. Let's denote the buyer with superscript b and the seller with superscript s.

Dealer buyers' volume in the dealer-customer market:

Given symmetric market structure, $\boldsymbol{S}^n = \boldsymbol{S}$, $\boldsymbol{B}^n = \boldsymbol{B}$, $\forall n$, and by Lemma 10, we can simplify the demand of the dealer buyer as

$$\boldsymbol{Q}^n_{dc} = \boldsymbol{S}\left((\boldsymbol{Id} - \frac{1}{I}\boldsymbol{B}\boldsymbol{S})(\boldsymbol{c}^n - \boldsymbol{\psi}^n_{dc}) - \boldsymbol{B}((\kappa \boldsymbol{V})^{-1}\boldsymbol{\mu} - \bar{\boldsymbol{q}}^n_1)\right)$$

Taking derivative of $\boldsymbol{Q}^b_{dc}$ over $\mathcal{A}$, we can obtain that

$$\frac{d\boldsymbol{Q}^b_{dc}}{d\mathcal{A}} = \boldsymbol{S}(\boldsymbol{Id} - \tfrac{1}{I}\boldsymbol{BS})\left(\tfrac{1}{2}\left(1 + (\tfrac{N-2}{N(N-1)}\alpha)^{-1}(\tfrac{2}{N}\alpha + \tfrac{2}{I}\kappa)\right)^{-1} - 1\right)\varphi \boldsymbol{W}\boldsymbol{\mu}\frac{dr^b_{dc}}{d\mathcal{A}} \quad (3.26)$$

Given that $\boldsymbol{S}(\boldsymbol{Id} - \tfrac{1}{I}\boldsymbol{BS})\left(\tfrac{1}{2}\left(1 + (\tfrac{N-2}{N(N-1)}\alpha)^{-1}(\tfrac{2}{N}\alpha + \tfrac{2}{I}\kappa)\right)^{-1} - 1\right) < 0$, all elements of $\frac{d\boldsymbol{Q}^b_{dc}}{d\mathcal{A}}$ have the opposite sign as $\frac{dr^b_{dc}}{d\mathcal{A}}$.

If the leverage ratio constraint is binding, the equilibrium price in the dealer-customer market satisfy

$$\boldsymbol{\mu}'\boldsymbol{W}\boldsymbol{Q}^b_{dc} = \frac{\mathcal{E}}{\varphi} - \mathcal{A} \quad (3.27)$$

Taking the derivative of eq. (3.27), we have

$$\boldsymbol{\mu}'\boldsymbol{W}\frac{d\boldsymbol{Q}^b_{dc}}{d\mathcal{A}} = -1$$

Given all elements of $\frac{d\boldsymbol{Q}^b_{dc}}{d\mathcal{A}}$ have the same direction, the volume for customer-sell orders decreases with $\mathcal{A}$:

$$\frac{d\boldsymbol{Q}^b_{dc}}{d\mathcal{A}} < \boldsymbol{0} \quad (3.28)$$

and

$$\frac{d\mathbb{E}[r_{dc}]}{d\mathcal{A}} > 0, \frac{d\boldsymbol{c}}{d\mathcal{A}} > 0$$

Dealer buyers' price in the dealer-customer market: By eq. (3.19), $\frac{d\boldsymbol{p}^b_{dc}}{d\boldsymbol{Q}^b_{dc}} = \frac{1}{I}\kappa\boldsymbol{V}$. Applying chain rule to eq. (3.28), we have

$$\frac{d\boldsymbol{p}^b_{dc}}{d\mathcal{A}} = \frac{1}{I}\kappa\boldsymbol{V}\frac{d\boldsymbol{Q}^b_{dc}}{d\mathcal{A}} < \boldsymbol{0} \quad (3.29)$$

Dealer sellers' price and volume in the dealer-customer market: Therefore dealer m's dealer-customer market price increases as $\mathcal{A}$ increases for n,

$$\frac{d\boldsymbol{p}^s_{dc}}{d\mathcal{A}} = \frac{1}{I}\boldsymbol{BS}\frac{d\boldsymbol{c}}{d\mathcal{A}} > \boldsymbol{0} \quad (3.30)$$

and

$$\frac{d\boldsymbol{Q}_{dc}^{s}}{d\mathcal{A}} = \boldsymbol{S}\boldsymbol{B}(\kappa\boldsymbol{V})^{-1}\frac{d\boldsymbol{c}}{d\mathcal{A}} > \boldsymbol{0} \tag{3.31}$$

Note that as $\boldsymbol{Q}_{dc}^{s} < 0$, $\frac{d\boldsymbol{Q}_{dc}^{s}}{d\mathcal{A}}$ implies that the seller intermediates less assets.

Interdealer market price $\boldsymbol{p}_{id}$: Given that increasing $\mathcal{A}$ decreases dealer buyers' volume $\boldsymbol{Q}_{dc}^{n}$, the supply of asset in the interdealer market decreases, and the interdealer market price increases:

$$\frac{d\boldsymbol{p}_{id}}{d\mathcal{A}} = -\alpha\boldsymbol{V}\frac{d\bar{\boldsymbol{Q}}_{dc}}{d\mathcal{A}} = -\frac{1}{2}\alpha\boldsymbol{V}\left(\frac{d\boldsymbol{Q}_{dc}^{s}}{d\mathcal{A}} + \frac{d\boldsymbol{Q}_{dc}^{b}}{d\mathcal{A}}\right) > \boldsymbol{0}$$

as

$$\frac{d\boldsymbol{Q}_{dc}^{s}}{d\mathcal{A}} + \frac{d\boldsymbol{Q}_{dc}^{b}}{d\mathcal{A}} = -(\frac{N-2}{2N}\alpha\boldsymbol{V})^{-1}\frac{d\boldsymbol{c}}{d\mathcal{A}} < \boldsymbol{0} \tag{3.32}$$

Transaction Cost $|\boldsymbol{p}_{dc} - \boldsymbol{p}_{id}|$: Given that $\boldsymbol{p}_{dc}^{b} < \boldsymbol{p}_{id}$, $\frac{d\boldsymbol{p}_{id}}{d\mathcal{A}} > 0$ and $\frac{d\boldsymbol{p}_{dc}^{b}}{d\mathcal{A}} < 0$, $\frac{d|\boldsymbol{p}_{dc}^{b} - \boldsymbol{p}_{id}|}{d\mathcal{A}} > 0$, the transaction cost for a dealer buyer increases. For a dealer seller (customer buyer), $\boldsymbol{p}_{dc}^{s} > \boldsymbol{p}_{id}$, and

$$\frac{d|\boldsymbol{p}_{dc}^{s} - \boldsymbol{p}_{id}|}{d\mathcal{A}} = \left(\frac{1}{I}(\frac{2}{N}\frac{\alpha}{\kappa} + \frac{2}{I})^{-1} - \frac{N}{N-2}\right)\frac{d\boldsymbol{c}}{d\mathcal{A}} < 0.$$

So the transaction cost for a dealer seller (customer buyer) decreases. ■

Proof of Proposition 17. Given results in the proof of Proposition 16, we take the difference of the derivative of transaction cost with respect to inventory $\mathcal{A}$ of the safe asset and that of the risky asset

$$\frac{d|p_{dc,safe}^{b} - p_{id,safe}|}{d\mathcal{A}} - \frac{d|p_{dc,risky}^{b} - p_{id,risky}|}{d\mathcal{A}}$$
$$= \underbrace{\left(\frac{1}{2}\left(\frac{N}{N-2} - \frac{1}{I}\left(\frac{2}{N}\frac{\alpha}{\kappa} + \frac{2}{I}\right)^{-1}\right)\left(1 + \frac{N(N-1)}{N-2}\left(\frac{2}{N} + \frac{2}{I}\frac{\kappa}{\alpha}\right)\right)^{-1} + \frac{1}{I}\left(\frac{2}{N}\frac{\alpha}{\kappa} + \frac{2}{I}\right)\varphi\frac{dr_{dc}^{b}}{d\mathcal{A}}\right)^{-1}}_{>0}$$
$$\cdot(w_{safe}\mu_{safe} - w_{risky}\mu_{risky})$$

and

$$\begin{aligned}
&\frac{d|p^s_{dc,safe} - p_{id,safe}|}{d\mathcal{A}} - \frac{d|p^s_{dc,risky} - p_{id,risky}|}{d\mathcal{A}} \\
&= \left(\frac{1}{I}\left(\frac{2}{N}\frac{\alpha}{\kappa} + \frac{2}{I}\right)^{-1} - \frac{N}{N-2}\right)\left(\frac{dc_{safe}}{d\mathcal{A}} - \frac{dc_{risky}}{d\mathcal{A}}\right) \\
&= \underbrace{\left(\frac{1}{I}\left(\frac{2}{N}\frac{\alpha}{\kappa} + \frac{2}{I}\right)^{-1} - \frac{N}{N-2}\right)\frac{1}{2}\left(1 + (\frac{N-2}{N(N-1)})^{-1}(\frac{2}{N} + \frac{2}{I}\frac{\kappa}{\alpha})\right)^{-1}\varphi\frac{dr^b_{dc}}{d\mathcal{A}}}_{<0} \\
&\cdot(w_{safe}\mu_{safe} - w_{risky}\mu_{risky})
\end{aligned}$$

Easy to see that $\frac{d|p^b_{dc,safe}-p_{id,safe}|}{d\mathcal{A}} - \frac{d|p^b_{dc,risky}-p_{id,risky}|}{d\mathcal{A}}$ is increaing in w_{safe}, which means the dealer buyer's transaction cost of safe asset increase by more than that of the risky asset as a response to an increase of inventory if w_{safe} is large; and $\frac{d|p^s_{dc,safe}-p_{id,safe}|}{d\mathcal{A}} - \frac{d|p^s_{dc,risky}-p_{id,risky}|}{d\mathcal{A}}$ is decreasing in w_{safe}, which means the dealer seller's transaction cost of safe asset decrease by more than that of the risky asset with respsect to an increase of inventory if w_{safe} is large.

The absolute change in the transaction cost of the safe asset with respect to an increase in inventory is larger than that of the risky asset for dealer buyers and sellers if and only if

$$w_{safe} > \bar{w} = \frac{\mu_{risky}}{\mu_{safe}} w_{risky} \tag{3.33}$$

If $w_{safe} = w_{risky}$, equation (3.33) is satisfied if and only if $\mu_{risky} < \mu_{safe}$. ■

3.8 Model Extension with Change in Price Impact

In the benchmark model, we consider a leverage constraint where we use the exogenous asset value to calculate the total asset value. In that case, the dealers' price impacts stay constant with and without the leverage ratio regulation. In this section, we consider the impact of leverage regulation on the price impacts by considering a leverage ratio regulation that uses the market prices to calculate the total asset values.

3.8.1 Event Study with Price Impacts

Amihud Measure: We construct the Amihud measure as a proxy for price impact. To construct the Amihud measure, we first compute the price change of each trade,

$$r_{ijkd\tau} = \frac{p_{ijkd\tau} - l.p_{ijkd\tau}}{l.p_{ijkd\tau}}$$

where $p_{ijk\tau d}$ is the price of bond k at time τ. $l.p_{ijk\tau d}$ is the price of last trade of type d for bond k that happens within an hour.

We then construct the Amihud measure by dividing the absolute price change by the volume,

$$\lambda_{ijkdt} = \frac{|r_{ijkd\tau}|}{vol_{ijkd\tau}}$$

where vol_{ijktd} is the par volume of the trade and N_{ijkdt} is the number of trade of type d for bond k on day t. Finally, we winsorize the top and the bottom 1% to eliminate the extreme values.

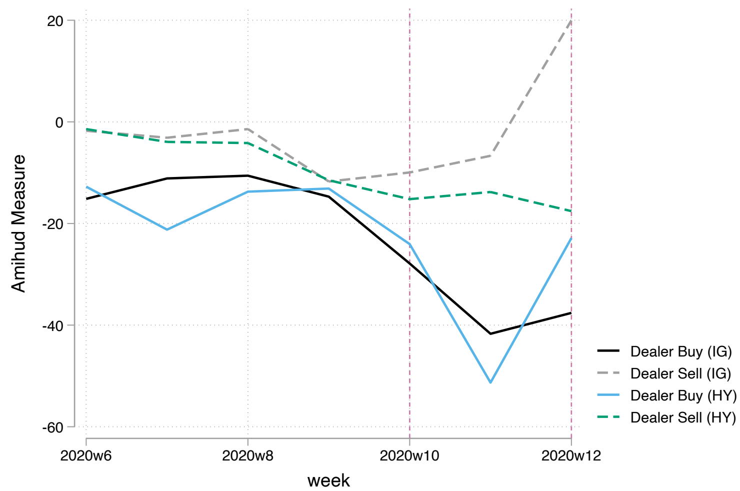
The Amihud measure proxies the impact of a customer's order on the price in the dealer-customer market. Figure 3.8 shows the difference in Amihud measures between the principal trades and agency trades before and after the crisis. It shows that the price impact decreased for a customer to sell to a dealer during the crisis, but it is not obvious for a customer to buy from the dealer during the market turmoil. This is also consistent with the model prediction.

3.8.2 Leverage Regulation and Market Liquidity

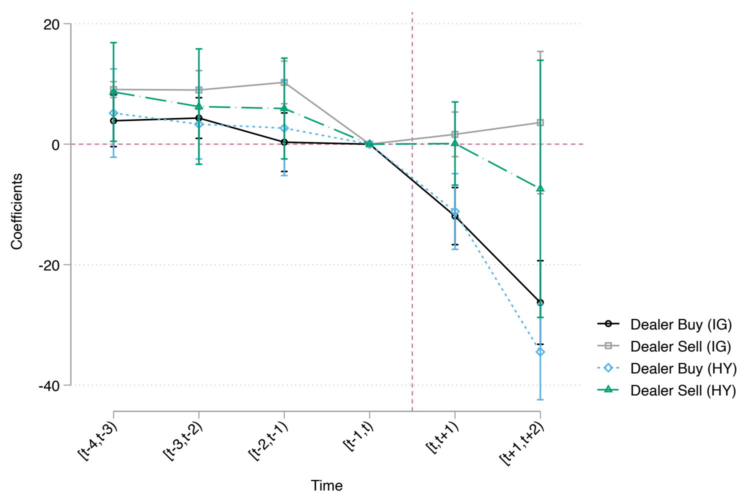
In this section, we will discuss the comparative statics of market liquidity with respect to leverage regulation under symmetric leverage regulation, where all dealers are subject to the same leverage constraint.

The SLR requirement can change three parameters in the model, (i) the weighted inventory $\mathcal{A}^n$, (ii) the risk-weight matrix $\boldsymbol{W}$, and (iii) the required leverage ratio φ^n. To understand the effect of each change on market liquidity, respectively, we provide a com-

Figure 3.8: The Difference of Price Impacts Between Principal and Agency Trades Before and During the March 2020 Market Turmoil



(a) Statistics



(b) Event-study

Note: Each line is the difference of weekly average price impacts between principal and agency trades for a customer to buy from or sell to dealers IG/HY bonds, respectively. The first vertical dash line indicates the week of March 6, 2020, when the crisis begins. The second vertical dashed line indicates the week when PDCF started.

parative statics analysis. For illustration purposes, we assume that the market is symmetric, i.e. $I_n = I$ for all dealer n; the dealers have identical capital structure, i.e. $\mathcal{A}^n = A$, $\mathcal{E}^n = \mathcal{E}$; and the leverage requirements are the same for all dealers $\varphi^n = \varphi$. We assume $K = 2$, a safe asset with non-zero payoff variance, and a risky asset with higher payoff variance. Without loss of generality, we assume that $\boldsymbol{\delta}^n$ is either $\boldsymbol{\delta} < 0$ or $-\boldsymbol{\delta}$ with equal probability so that we can focus on the equilibrium for two dealers and their customers: (i) a dealer trading with customers whose endowment shocks make them natural buyers ($\boldsymbol{\delta}^n = -\boldsymbol{\delta} < 0$); and (ii) a dealer trading with customers whose endowment shocks make them natural sellers ($\boldsymbol{\delta}^n = \boldsymbol{\delta} > 0$). Hereafter, we refer to the first case *dealer-sell order*, and the second case *dealer-buy order*.

We construct three liquidity measures similar to the event study, (i)quantity-based liquidity measured by the transaction volume of the dealer in the dealer-customer market ($\boldsymbol{Q}^n_{dc}$);(ii)price-based liquidity measure such as transaction cost measured by price differences between the dealer-customer market and interdealer market ($|\boldsymbol{p}^n_{dc} - \boldsymbol{p}_{id}|$); and (iii)price-impact-based liquidity measured by the customers' price impacts of in the dealer-customer market, which is defined as the inverse of the elasticity of the dealers they trade with ($(\boldsymbol{S}^n)^{-1}$).

Risk-weighted Inventory $\mathcal{A}$: The bank-affiliated dealers have a higher risk-weighted inventory $\mathcal{A}$ for SLR than for Tier 1 Capital Ratio. This is because, firstly, SLR assigns equal weights to the assets, which in general increases the weighted value of safe assets inventory, and secondly, SLR also includes off-balance sheet exposure and repo-style transaction exposure into the total leverage exposure. In this part, we will consider the comparative statics with respect to risk-weighted inventory $\mathcal{A}$, keeping everything else constant (see Figure 3.9).

Proposition 18 (Liquidity and Inventory $\mathcal{A}$)**.** *Given a symmetric market structure and identical capital structure, as the inventory included in the leverage exposure increases, i.e., $\mathcal{A}$ increases,*

1. *Dealers intermediate less volume ($\boldsymbol{Q}_{dc}$) in the dealer-customer market.*

2. *The transaction cost ($|\boldsymbol{p}_{dc} - \boldsymbol{p}_{id}|$) increases for customer sell orders, and decreases for customer buy orders.*

3. *The price impact of selling to dealers decreases. The price impact of buying from a dealer does not change.*

4. *The total welfare weakly decreases.*

First, we find that higher risk-weighted inventory decreases the transaction volume. Increasing risk-weighed inventory $\mathcal{A}$ tightens the leverage ratio constraint of the bank-affiliated dealers when they buy from customers. As more initial holdings of assets will increase the total leverage exposure, dealers' ability to hold the assets and resell them in the interdealer market is limited. For a dealer-sell order, the SLR does not tighten the leverage ratio constraint. However, as dealers expect that there are fewer assets to buy from the interdealer market due to limited supply from those constrained dealers, they sell less to the customers.

Second, as risk-weighed inventory $\mathcal{A}$ increases, the transaction cost measured by the price differences between the dealer-customer market and interdealer market increases for dealer-buy orders but decreases for dealer-sell orders. For a dealer-buy order, as the dealers' leverage ratio constraint becomes binding, it brings a shadow cost for dealers to hold the assets. Therefore, the marginal value of dealers decreases as they buy from the customers. In the interdealer market, as there is a limited supply of assets, the dealers can sell the assets at a higher price. Therefore, the transaction cost for the customer-sell order increases. For dealer-sell orders, the price for dealers to sell to customers slightly increases as they expect the interdealer market price to increase, but not as much as the increase in the interdealer market. Therefore, the transaction cost for the dealer-sell orders decreases.

The price impact decreases for the dealer-buy order as risk-weighted inventory $\mathcal{A}$ increases. As shadow cost brings the additional cost per unit of asset, the dealer's demand is more sensitive to price change, i.e., the effective demand curve is flatter. As the residual demand curve becomes flatter, the customers have a smaller price impact. For dealer-sell orders, the leverage constraint is never binding, so the slope of the dealer's demand curve does not change. Their customers' price impacts stay constant.

Despite the changes in the transaction cost being heterogeneous and the price impact decreasing, the welfare decreases as $\mathcal{A}$ increases and the leverage ratio constraint becomes binding for bank-affiliated dealers. This is because the limit on the intermediation decreases the allocation efficiency.

Risk Weight $\boldsymbol{W}$: Another major difference between SLR and existing leverage ratio regulation is that SLR assigns equal weight to all assets, while the existing leverage ratio assigns less weight to safe assets. In this part, we explore the impact of the risk weight $\boldsymbol{W}$ on the liquidity measures. Figure 3.10 provides a comparative static analysis with respect to the risk-weight on the safe asset while keeping everything else constant.

Proposition 19 (Liquidity and Risk Weight on Safe Assets w_1)**.** *As the risk weight on the safe asset increases, i.e., w_1 increases,*

1. *Dealers intermediate less safe asset ($\boldsymbol{Q}_{dc}$) in the dealer-customer market; the volume of intermediated risky assets is firstly decreasing in w_1 and can potentially be increasing in w_1 as w_1 is sufficiently large.*

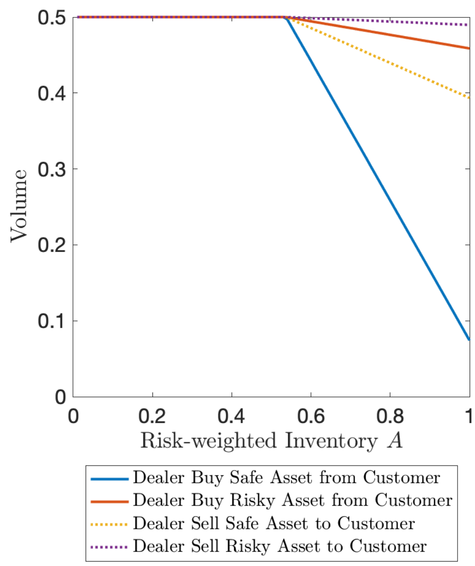
2. *The transaction cost ($|\boldsymbol{p}_{dc} - \boldsymbol{p}_{id}|$) of the safe asset increases for dealer buyers, and decreases for dealer sellers. The transaction cost ($|\boldsymbol{p}_{dc} - \boldsymbol{p}_{id}|$) of the risky asset increases for dealer buyers, and decreases for dealer sellers; as w_1 is sufficiently large, the transaction cost ($|\boldsymbol{p}_{dc} - \boldsymbol{p}_{id}|$) of the risky asset decreases for dealer buyers, and increases for dealer sellers.*

3. *The price impact of selling the safe asset to dealers decreases. The price impact of selling the risky asset to dealers firstly decreases, and can potentially increase if w_1 is sufficiently large. The price impact of buying from a dealer does not change.*

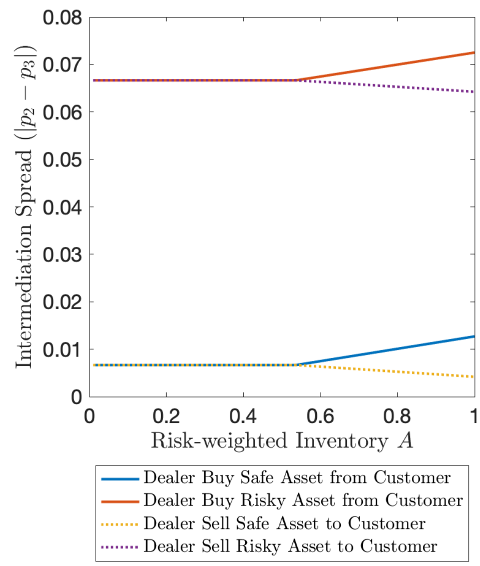
4. *The welfare is first decreasing and then increasing in w_1.*

We find that increasing the risk weight on the safe asset decreases the transaction volume, and it has a heterogeneous effect on the liquidity of risky and safe assets. As

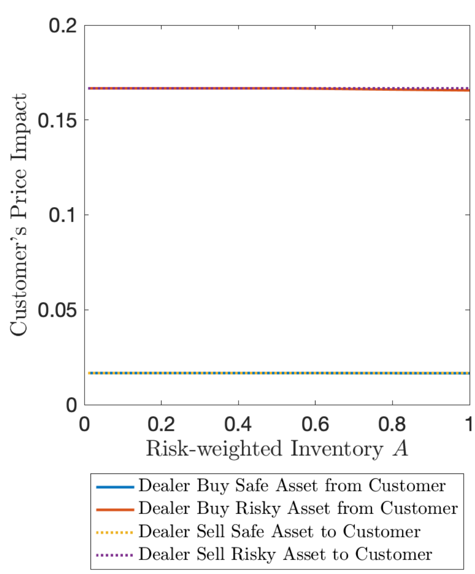
Figure 3.9: The Comparative Statics of Liquidity Measures with Risk-weighted Inventory $\mathcal{A}$



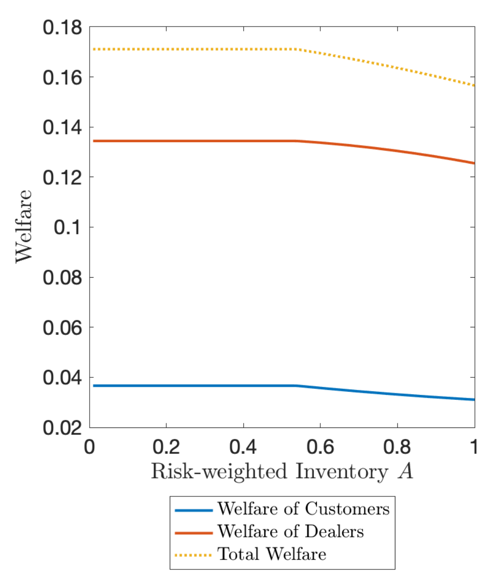
(a) Volume



(b) Transaction Cost



(c) Price Impact



(d) Welfare

the risk-weight on the safe asset increases, dealers first intermediate more safe assets than risky assets; however, when the risk-weight on the safe asset is sufficiently high, the traders intermediate more risky assets than safe assets. Intuitively, as the risk weight increases and the leverage ratio constraints become binding, dealers first reduce their demand for risky assets to reduce risk-aversion loss. However, as the risk weight on the safe asset becomes sufficiently high, dealers intermediate less safe assets than the risky assets as the cost of the leverage ratio constraint dominates the risk-aversion. The safe asset takes up too much balance sheet space while it provides a lower intermediate spread than the risky asset. So dealers choose to intermediate more risky assets.

We also find that the change in the transaction cost is more homogeneous for safe and risky assets, compared to their changes with respect to the risk-weighted inventory. This is because as the risk weight on the safe asset increases, holding safe assets becomes more likely to tighten the balance sheet constraint, and therefore, dealers require a higher transaction cost for safe assets.

Similarly, the difference between the change in price impact for the safe asset and risky asset is smaller when the risk weight on the safe asset becomes larger. The risky asset used to contribute more to the leverage constraint, and the slope of the dealers' demand curve becomes more sensitive to the risky asset price. The price impact for the risky asset decreases by more than the safe asset when the risk weight on the safe asset is low. As the risk weight on the safe asset increases, the safe asset contributes more to the leverage exposure, and the additional marginal cost due to the binding leverage constraint to buy safe assets becomes higher. So dealers become more sensitive to the price change in the safe asset. The slope of the dealers' demand curve for the safe asset becomes flatter, and the reduction in price impact of the safe asset is larger.

Finally, increasing risk-weight on the safe asset has an ambiguous effect on welfare. As the risk weight on the safe asset increases, firstly, the dealers' ability to intermediate both the safe asset and risky asset decreases, and the welfare decreases. Given that the leverage constraint is binding, when the risk weight on the safe asset increases from a median level to a high level, the dealer starts to intermediate more risky assets and fewer safe assets. It improves the risk-sharing between the dealers and customers, and the welfare increases.

Leverage Ratio φ: Compared with risk-weighted leverage ratio like CET1, SLR slightly decreases the required leverage ratio.[13] Now let's consider the impact of changing leverage ratio φ on the liquidity measures. Figure 3.11 shows the comparative statics analysis. We find that the patterns are similar to comparative statics of the risk-weighted inventory. A higher leverage ratio requirement tightens the leverage ratio constraint, and the impacts on the liquidity measures align with the comparative statics of the risk-weighted inventory.

Proposition 20 (Liquidity and Require Ratio φ)**.** *Given the symmetric market structure and identical capital structure, as the required leverage ratio increases, i.e., φ increases,*

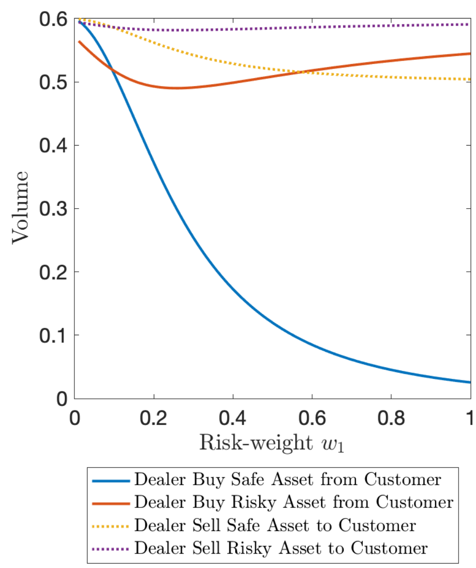
1. *dealers intermediate less volume ($\boldsymbol{Q}_{dc}$) in the dealer-customer market.*
2. *The transaction cost ($|\boldsymbol{p}_{dc} - \boldsymbol{p}_{id}|$) increases for dealer buyers, and decreases for dealer sellers.*
3. *The price impact of selling to a dealer decreases. The price impact of buying from a dealer does not change.*
4. *The welfare weakly decreases.*

3.8.3 Rise of Non-Bank Dealers

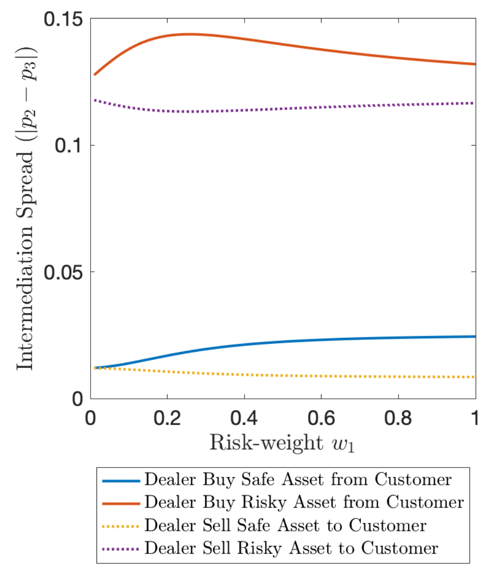
Non-bank-affiliated dealers such as principal trading firms (PTFs) are also important liquidity providers in the corporate bond market (Bessembinder and Maxwell, 2016). In this section, we will discuss the comparative statics of market liquidity with respect to leverage regulation under asymmetric leverage regulation, where some dealers are subject to the leverage constraint while others are not.

[13] Note that although SLR decreases the required leverage ratio, the stress test results show that SLR is still the most binding ratio (Duffie, 2017b), indicating that the overall increase in risk-weighted inventory and the risk-weight is large enough to offset the impact of a lower required ratio.

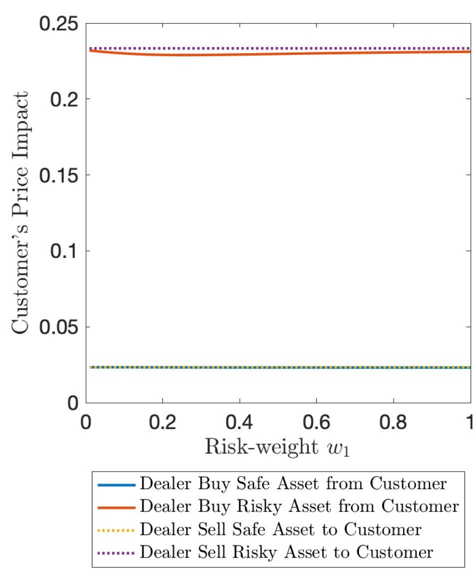
Figure 3.10: The Comparative Statics of Liquidity Measures with Risk Weight $\boldsymbol{W}$



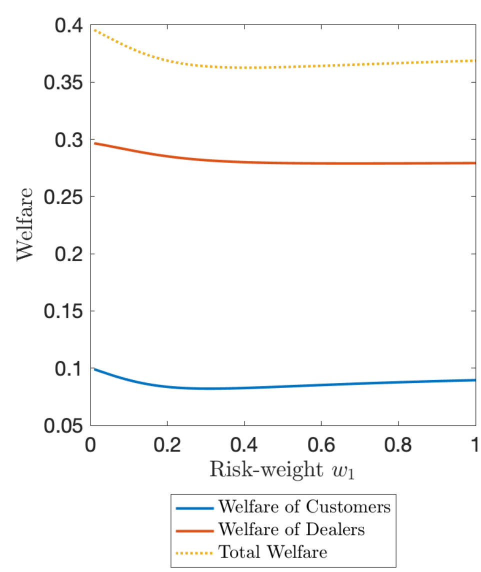
(a) Volume



(b) Transaction Cost

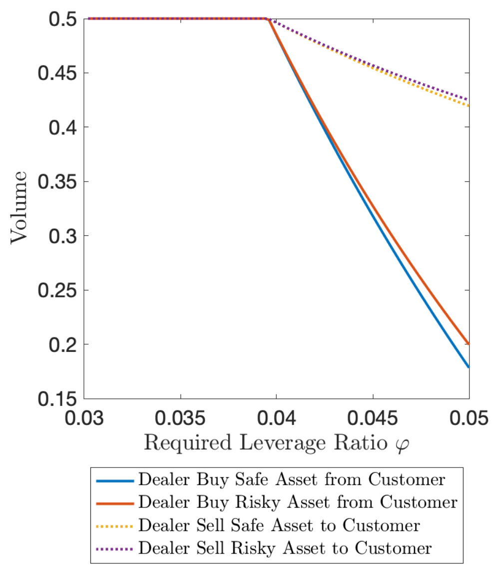


(c) Price Impact

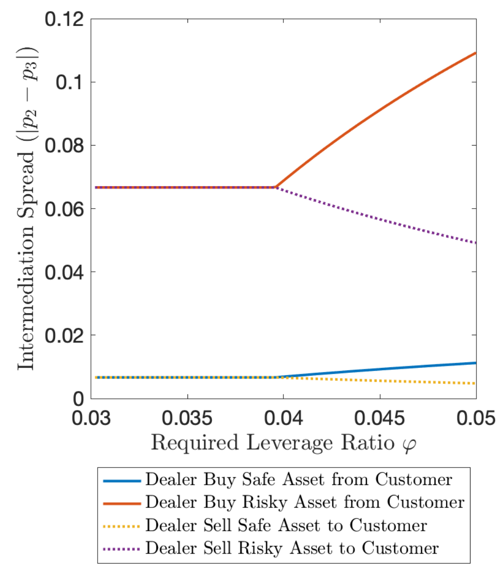


(d) Welfare

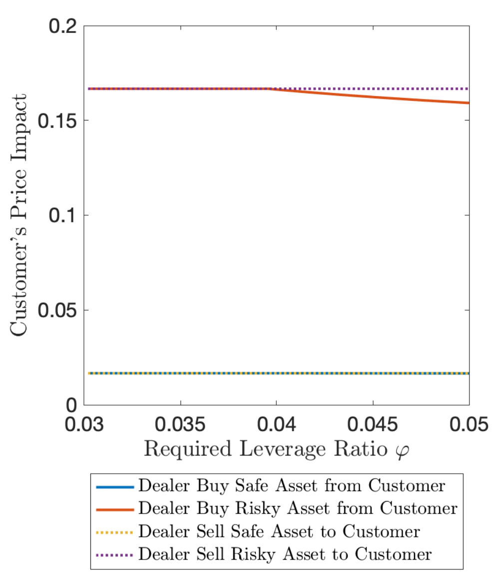
Figure 3.11: The Comparative Statics of Liquidity Measures with Leverage Ratio φ



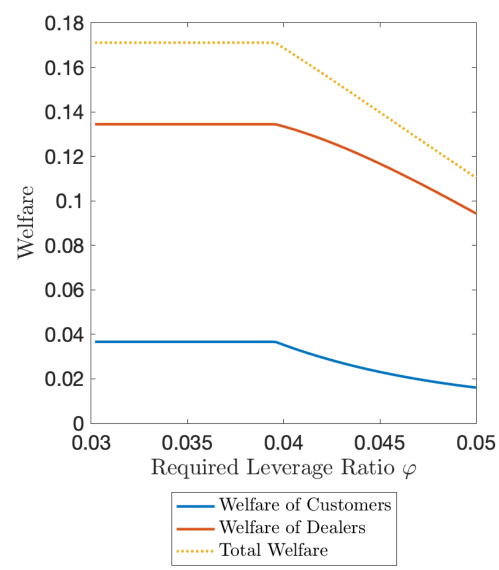
(a) Volume



(b) Transaction Cost



(c) Price Impact



(d) Welfare

For simplicity, we keep the symmetric market structure and symmetric capital structure assumption. Let some dealers be bank-affiliated dealers who are subject to the same leverage ratio regulation($\varphi^i = \varphi$), and the other dealers are not subject to any leverage ratio regulation($\varphi^i = 0$). Assume that customers affiliated with both groups of dealers have an equal probability of receiving customer demand shock $\boldsymbol{\delta}^n = \boldsymbol{\delta}$ and $\boldsymbol{\delta}^n = -\boldsymbol{\delta}$.

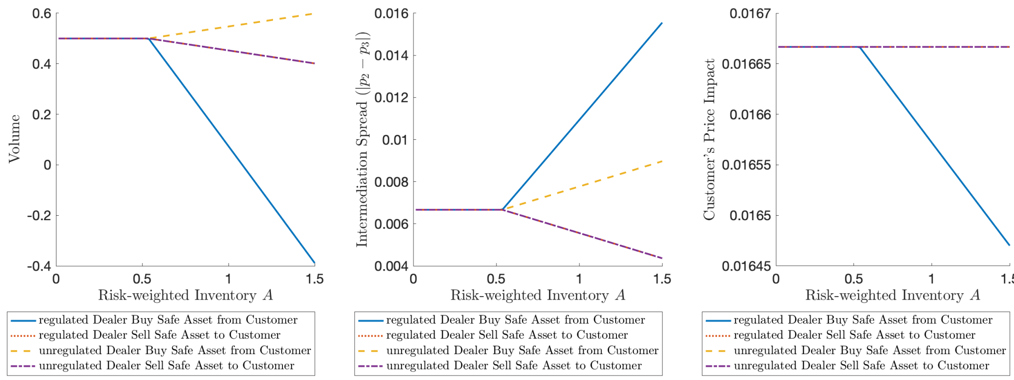
Figure 3.12 shows the comparative statics of liquidity measures with respect to the inventory $\mathcal{A}$. We find that the non-bank dealers, expecting that the bank-affiliated dealers may be constrained to sell in the interdealer market, will buy more from customers and sell in the interdealer market. The non-bank dealer buyers also increase the transaction cost, but slightly lower than the bank-affiliated dealer buyers, as they are not subject to the leverage constraint. The non-bank sellers decrease transaction costs. The demand elasticity of non-bank dealers is not affected by the leverage regulation, therefore, the price impact of customers trading with non-bank dealers does not change. The increased intermediation of non-bank dealers is consistent with the observation that non-bank financial institutions have increased their market share and capital commitment in the U.S. corporate bond markets (Bessembinder and Maxwell, 2016).

Proposition 21 (Rise of Non-Banks)**.** *Given the symmetric market structure and identical capital structure, as the inventory included in the leverage exposure increases, i.e., $\mathcal{A}$ increases for bank-affiliated dealers,*

1. *Non-bank dealer buyers intermediate more volume in the dealer-customer market. Non-bank dealer sellers intermediate less volume in the dealer-customer market.*
2. *Non-bank dealer buyers' transaction costs increase, and non-bank dealer sellers' transaction costs decrease.*
3. *The price impact of trading with non-bank dealers does not change.*

However, the decentralized market structure can limit the non-bank dealers' ability to intermediate the market, especially when the bank-affiliated dealers receive higher customer demands.

Figure 3.12: The Comparative Statics of Liquidity Measures with Risk-weighted Inventory $\mathcal{A}$ under Asymmetric Leverage Regulation



(a) Volume (b) Transaction Cost (c) Price Impact

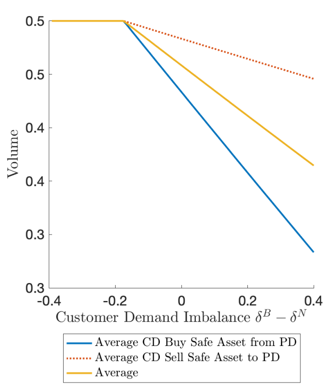
We keep the symmetric market structure and the identical capital structure assumption. Assume half of the dealers are bank-affiliated dealers subject to leverage ratio regulation $\varphi^i = \varphi$, and the others are non-bank dealers. Assume the customer demand shock for bank-affiliated dealers has an equal probability to be $\boldsymbol{\delta}^B > 0$ or $-\boldsymbol{\delta}^B$. Let the customer demand shock for non-bank dealers have an equal probability to be either $\boldsymbol{\delta}^N > 0$ or $-\boldsymbol{\delta}^N$. We define the difference $\boldsymbol{\delta}^B - \boldsymbol{\delta}^N$ as the imbalance in customer demand.

Figure 3.13 illustrates the change in liquidity with respect to the imbalance in customer demand. When the non-bank dealers have a larger market share, the bank-affiliated dealers are not subject to leverage ratio regulation. The leverage regulation is welfare-neutral. When the customer demand increases for the bank-affiliated dealers, the bank-affiliated dealer buyers are subject to binding leverage ratio constraints. The volume decreases, and transaction costs increase. Increasing customer demand for the bank dealers increases the average transaction cost and decreases the average volume.

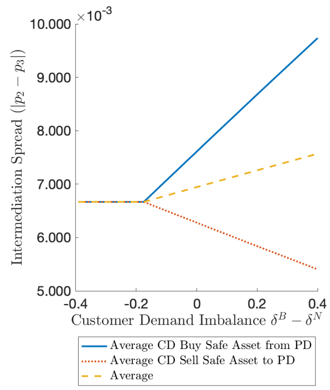
3.8.4 Summary and Remarks

The comparative statics shed light on the mechanisms of how leverage regulation changes the market liquidity.

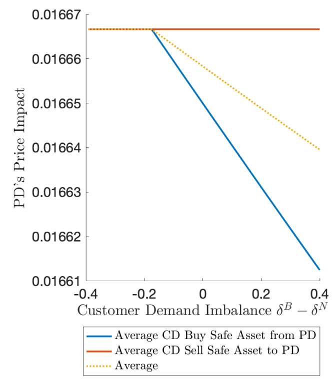
Figure 3.13: The Comparative Statics of Liquidity Measures with Risk-weighted Inventory $\mathcal{A}$ under Asymmetric Leverage Regulation



(a) Volume



(b) Transaction Cost



(c) Price Impact

First, even when the market is symmetric, the impact of leverage ratio regulation on liquidity can be heterogeneous. It decreases the leverage-constrained dealers' valuation for the assets as it increases the shadow cost of the leverage ratio constraint. The transaction volume is lower. However, at the same time, it also decreases dealers' market power, so the liquidity measured by the dealer's demand elasticity is higher. The transaction cost increases for a constrained dealer to buy and decreases for a dealer to sell. The SLR regulation can increase bank-affiliated dealers' risk-taking. Safe assets contribute more to the leverage exposure with SLR, making dealers more willing to intermediate risky assets than safe assets.

When the bank-affiliated dealers are subject to leverage regulation, the non-bank dealers will step into the market to intermediate the assets. However, the decentralized market structure can limit the non-bank dealers' ability to intermediate the market, especially when the bank-affiliated dealers receive higher customer demands.

The effect of SLR on the welfare is ambiguous. The SLR can decrease the allocation efficiency of the safe assets, however, it can potentially increase risk-sharing by incentivizing the dealers to intermediate more risky assets.

3.9 Model Extension with Two-sided Market Power

At $t = 3$, given the dealer trades $\boldsymbol{Q}^n_{dc}$, each dealer i chooses the demand schedule $\boldsymbol{Q}^n_{id}(\boldsymbol{p}^n_{id}) : \mathbb{R}^K \to \mathbb{R}^K$ to maximize the utility given his price impact $\boldsymbol{\Lambda}^n_{id}$,

$$u^n_{id} = \boldsymbol{\mu}'(\boldsymbol{Q}^n_{dc} + \boldsymbol{Q}^n_{id}) - \frac{1}{2}\alpha(\boldsymbol{Q}^n_{dc} + \boldsymbol{Q}^n_{id})'\boldsymbol{V}(\boldsymbol{Q}^n_{dc} + \boldsymbol{Q}^n_{id}) - (\boldsymbol{p}_{id})'\boldsymbol{Q}^n_{id}.$$

By taking the first order condition with respect to $\boldsymbol{Q}^n_{id}$, we can solve for the optimal demand schedule

$$\boldsymbol{Q}^n_{id}(\boldsymbol{p}_{id}) = (\alpha\boldsymbol{V} + \boldsymbol{\Lambda}^n_{id})^{-1}(\boldsymbol{\mu} - \boldsymbol{p}_{id} - \alpha\boldsymbol{V}\boldsymbol{Q}^n_{dc}). \tag{3.34}$$

Given the market clearing condition, trader i's price impact satisfies:

$$\boldsymbol{\Lambda}^n_{id} = \left(-\sum_{n\neq\ell}\frac{d\boldsymbol{Q}^n_{id}}{d\boldsymbol{p}_{id}}\right)^{-1} = \left(\sum_{n\neq\ell}(\alpha\boldsymbol{V} + \boldsymbol{\Lambda}^\ell_{id})^{-1}\right)^{-1} = \frac{1}{N-2}\alpha\boldsymbol{V}. \tag{3.35}$$

By the market-clearing condition,

$$\boldsymbol{p}_{id} = \boldsymbol{\mu} - \alpha\boldsymbol{V}\bar{\boldsymbol{Q}}_{dc}.$$

where $\bar{\boldsymbol{Q}}_{dc} = \frac{1}{N}\sum_\ell \boldsymbol{Q}^n_{dc}$.

Differentiating the dealer's third-round utility $\mathbb{E}[u^n_{id}|\boldsymbol{Q}^n_{dc}]$ with respect to $\boldsymbol{Q}^i_{dc}$, we have

$$\frac{d\mathbb{E}[u^n_{id}|\boldsymbol{Q}^n_{dc}]}{d\boldsymbol{Q}^n_{dc}} = \frac{\partial\mathbb{E}[u^n_{id}|\boldsymbol{Q}^n_{dc}]}{\partial\boldsymbol{Q}^n_{dc}} + \frac{\partial\mathbb{E}[u^n_{id}|\boldsymbol{Q}^n_{dc}]}{\partial\boldsymbol{Q}^n_{id}}\frac{d\boldsymbol{Q}^n_{id}}{d\boldsymbol{Q}^n_{dc}}$$

Given that $\frac{\partial\mathbb{E}[u^n_{id}|\boldsymbol{Q}^n_{dc}]}{\partial\boldsymbol{Q}^n_{id}} = 0$ by the first order condition in the second round, we can simplify the equation as

$$\left(\frac{d\mathbb{E}[u^n_{id}|\boldsymbol{Q}^n_{dc}]}{d\boldsymbol{Q}^n_{dc}}\right)' = \left(\frac{\partial\mathbb{E}[u^n_{id}|\boldsymbol{Q}^n_{dc}]}{\partial\boldsymbol{Q}^n_{dc}}\right)' = \boldsymbol{\mu} - \alpha\boldsymbol{V}(\boldsymbol{Q}^n_{dc} + \mathbb{E}[\boldsymbol{Q}^n_{id}|\boldsymbol{Q}^n_{dc}]) = \boldsymbol{c}^n - \frac{2}{N}\alpha\boldsymbol{V}\boldsymbol{Q}^n_{dc} \tag{3.36}$$

where $\boldsymbol{c}^n = \boldsymbol{\mu} - \frac{N-2}{N-1}\frac{1}{N}\alpha \boldsymbol{V} \sum_{m\neq n} \mathbb{E}[\boldsymbol{Q}^m_{dc}|\boldsymbol{Q}^n_{dc}]$.

Given his price impact $\boldsymbol{\Lambda}^n_{dc}$, the dealer ℓ chooses $\boldsymbol{Q}^n_{dc}(\boldsymbol{p}^n_{dc})$ to maximize

$$u^n_{dc}(\boldsymbol{Q}^n_{dc}) \quad = \quad \mathbb{E}[u^n_{id}|\boldsymbol{Q}^n_{dc}] - (\boldsymbol{p}^n_{dc})'\boldsymbol{Q}^n_{dc} - r^n(\varphi^n(\boldsymbol{p}^n_{dc})'\boldsymbol{W}(\boldsymbol{Q}^n_{dc}) + \varphi^n\mathcal{A}^n - \mathcal{E}^n)$$

r^n is the shadow cost of the leverage ratio constraint. r^n satisfies complementary slackness that

$$r^n(\varphi^n(\boldsymbol{p}^n_{dc})'\boldsymbol{W}(\boldsymbol{Q}^n_{dc}) + \varphi^n\mathcal{A}^n - \mathcal{E}^n) = 0. \tag{3.37}$$

Taking first order condition with respect to $\boldsymbol{Q}^n_{dc}$, we can solve customer n's equilibrium demand schedule in the dealer-customer market,

$$\boldsymbol{Q}^n_{dc} = (\frac{2}{N}\alpha\boldsymbol{V} + \boldsymbol{\Lambda}^n_{dc}\boldsymbol{\Gamma}^n)^{-1}(\boldsymbol{c}^n - \boldsymbol{\Gamma}^n\boldsymbol{p}^n_{dc}) \tag{3.38}$$

where $\boldsymbol{\Gamma}^n = \boldsymbol{Id} + r^n\varphi^n\boldsymbol{W}$.

Each customer i submits demand schedule $\boldsymbol{q}^i_{dc}(\boldsymbol{p}^n_{dc}) : \mathbb{R}^K \to \mathbb{R}^K$ to maximize the utility,

$$v^i_{dc} = \boldsymbol{\mu}'\boldsymbol{q}^i_{dc} - \frac{1}{2}\kappa(\boldsymbol{q}^i_0 + \boldsymbol{q}^i_{dc})'\boldsymbol{V}(\boldsymbol{q}^i_0 + \boldsymbol{q}^i_{dc}) - (\boldsymbol{p}^n_{dc})'\boldsymbol{q}^i_{dc}$$

By taking the first order condition,

$$\boldsymbol{q}^i_{dc} = (\kappa\boldsymbol{V} + \boldsymbol{\lambda}^i_{dc})^{-1}(\boldsymbol{\mu} - \kappa\boldsymbol{V}\boldsymbol{q}^i_0 - \boldsymbol{p}^n_{dc}) \tag{3.39}$$

Given the market clearing condition, dealer n's price impact satisfies:

$$\boldsymbol{\Lambda}^n_{dc} = \left(-\sum_{i\in I(n)} \frac{d\boldsymbol{q}^i_{dc}}{d\boldsymbol{p}^n_{dc}}\right)^{-1} \tag{3.40}$$

and customer $i \in I(n)$ has price impact

$$\boldsymbol{\lambda}^i_{dc} = \left(-\frac{d\boldsymbol{Q}^n_{dc}}{d\boldsymbol{p}^n_{dc}} - \sum_{j\neq i, j\in I(n)} \frac{d\boldsymbol{q}^j_{dc}}{d\boldsymbol{p}^n_{dc}}\right)^{-1} \tag{3.41}$$

The market clears when $\boldsymbol{Q}^n_{dc} + \sum_{i \in I(n)} \boldsymbol{q}^i_{dc} = 0$. Given the market-clearing condition, we can solve for the equilibrium price

$$\boldsymbol{p}^n_{dc} = \boldsymbol{b}^n - \boldsymbol{B}^n \sum_{i \in I(n)} (\kappa \boldsymbol{V} + \boldsymbol{\lambda}^i_{dc})^{-1} \kappa \boldsymbol{V} \boldsymbol{q}^i_0, \tag{3.42}$$

where $\boldsymbol{B}^n = \left((\frac{2}{N}\alpha \boldsymbol{V} + \boldsymbol{\Lambda}^n_{dc}\boldsymbol{\Gamma}^n)^{-1}\boldsymbol{\Gamma}^n + \sum_{i \in I(n)} (\kappa \boldsymbol{V} + \boldsymbol{\lambda}^i_{dc})^{-1} \right)^{-1}$,
$\boldsymbol{b}^n = \boldsymbol{B}^n \left((\frac{2}{N}\alpha \boldsymbol{V} + \boldsymbol{\Lambda}^n_{dc}\boldsymbol{\Gamma}^n)^{-1}\boldsymbol{c}^n + \sum_{i \in I(n)} (\kappa \boldsymbol{V} + \boldsymbol{\lambda}^i_{dc})^{-1} \boldsymbol{\mu} \right)$.

Now let's solve the intercept $\boldsymbol{c}^n$. Plugging the equilibrium price in equation (3.42) into equation (3.39) and taking the expectation, we have

$$\mathbb{E}[\boldsymbol{Q}^m_{dc}|\boldsymbol{Q}^n_{dc}] = (\frac{2}{N}\alpha \boldsymbol{V} + \boldsymbol{\Lambda}^m_{dc}\boldsymbol{\Gamma}^n)^{-1} \left(\boldsymbol{c}^m - \boldsymbol{\Gamma}^m(\boldsymbol{b}^m - \boldsymbol{B}^m \sum_{j \in I(m)} (\kappa \boldsymbol{V} + \boldsymbol{\lambda}^j_{dc})^{-1} \kappa \boldsymbol{V} \mathbb{E}[\boldsymbol{q}^j_0]) \right) \tag{3.43}$$

We can solve $\boldsymbol{c}^n$ by matching the coefficients of equation (3.36),

$$\boldsymbol{c}^n = \boldsymbol{\mu} - \tfrac{N-2}{N(N-1)}\alpha \boldsymbol{V} \textstyle\sum_{m \neq n} (\frac{2}{N}\alpha \boldsymbol{V} + \boldsymbol{\Lambda}^m_{dc}\boldsymbol{\Gamma}^m)^{-1} \left(\boldsymbol{c}^m - \boldsymbol{\Gamma}^m(\boldsymbol{b}^m - \sum_{j \in I(m)} \boldsymbol{B}^m (\kappa \boldsymbol{V} + \boldsymbol{\lambda}^j_{dc})^{-1} \kappa \boldsymbol{V} \mathbb{E}[\boldsymbol{q}^j_0]) \right) \tag{3.44}$$

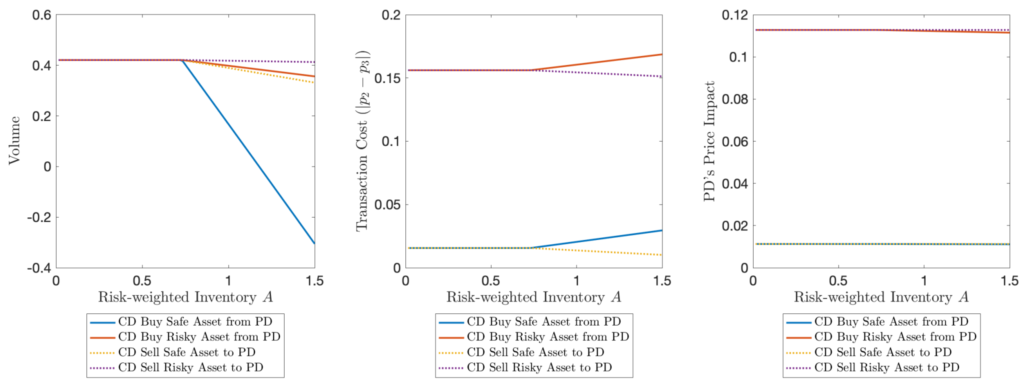
$$\boldsymbol{C}^n = \frac{2}{N}\alpha \boldsymbol{V} \tag{3.45}$$

Theorem 16 (Equilibrium)**.** *The equilibrium is a profile of dealer's demand schedule and price impact,* $\{\boldsymbol{Q}^n_{dc}, \boldsymbol{Q}^n_{id}, \boldsymbol{\Lambda}^n_{dc}, \boldsymbol{\Lambda}^n_{id}\}_n$ *and a profile of customer's demand schedules and price impacts* $\{\boldsymbol{q}^i_{dc}, \boldsymbol{\lambda}^i_{dc}\}_i$*, and a profile of shadow cost of leverage constraint* $\{r^i\}_i$ *such that*

1. *In the dealer-customer market, each customer submits a demand schedule in equation (3.39), given the price impact* $\boldsymbol{\lambda}^i_{dc}$ *in equation (3.41). Each dealer submits a demand schedule in equation (3.38), given the price impact* $\boldsymbol{\Lambda}^n_{dc}$ *in equation (3.40), the shadow cost of leverage ratio* r^n *in equation (3.37) and the impact of inventory on the next round utility* $\frac{d\mathbb{E}[u^n_{id}|\boldsymbol{Q}^n_{dc}]}{d\boldsymbol{Q}^n_{dc}}$ *parameterized by* $\boldsymbol{c}^n$ *(eq.(3.44)) and* $\boldsymbol{C}^n$ *(eq.(3.45)).*

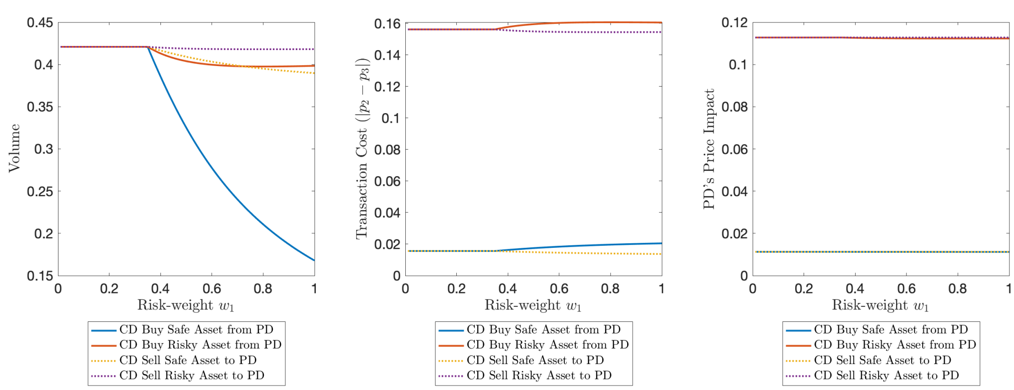
2. *In the interdealer market, each dealer submits a demand schedule in equation (3.34), given the price impact in equation (3.35).*

Figure 3.14: The Comparative Statics of Liquidity Measures with Risk-weighted Inventory $\mathcal{A}$ in Model with Two-sided Market Power



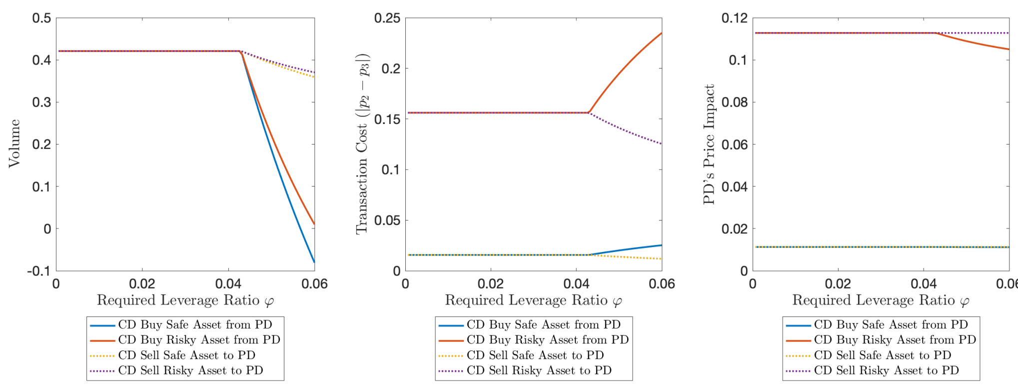
(a) Volume (b) Transaction Cost (c) Elasticity

Figure 3.15: The Comparative Statics of Liquidity Measures with Risk-weight on the Safe Asset w_1 in Model with Two-sided Market Power



(a) Volume (b) Transaction Cost (c) Elasticity

Figure 3.16: The Comparative Statics of Liquidity Measures with Required Leverage Ratio φ in Model with Two-sided Market Power



(a) Volume

(b) Transaction Cost

(c) Elasticity