Essays on Structural Behavioral Economics

By

Emilio Cuilty

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The dissertation is approved by the following members of the Final Oral Committee:

Alan Sorensen, Professor, Economics
Lorenzo Magnolfi, Assistant Professor, Economics
Daniel Quint, Associate Professor, Economics
Xiaoxia Shi, Associate Professsor, Economics
Justin Sydnor, Associate Professor, Business School

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# Abstract:

The first chapter extends the standard discrete choice framework to estimate demand in markets with differentiated products when consumers display reference dependent (R-D) utility. In industries with a common reference product, R-D preferences generate novel substitution patterns that cannot be rationalized by many of the standard models, making the R-D model testable. Such substitution patterns are also present with relatively low heterogeneity among consumers' reference product. As the level of heterogeneity increases, the difference between the R-D predictions and standard predictions decreases. When the true data generating process comes from R-D preferences, simulations show that standard models deliver biased estimates of elasticities. Finally, using consumer scanner data, I reject the hypothesis that the facial tissue industry contains no R-D consumers, and estimate that 25 percent of Kleneex's market share derives from its status as the reference brand.

The second chapter studies present bias and self control in a real mortgage market. I look at a novel dataset from a Mexican mortgage institution, where I individually follow state workers repayment decisions across 15 years. By applying Fan and Wang (2016) methodology, I am able to estimate the long term discount factor $\delta$, the quasi-hyperbolic discount factor $\beta$ and the degree of naivety $\tilde{\beta}$ the state worker has. I find that the state worker suffers from present bias ($\beta = 0.35$), and is not aware of it ($\tilde{\beta} = 1$). I show that the mortgage debt could be repaid faster if individuals could behave either as exponential discounters, or as sophisticated present biased discounters. These finding suggest that the risk of default is greater than the one estimated under traditional exponential models.

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# Chapter 1

# Discrete Choice Models of Demand with Reference Dependent Utility

## 1.1 Introduction

One of the most robust findings of the behavioral economics literature over the past 30 years is the importance of reference dependence: the idea that people evaluate outcomes by comparing them to a reference point, caring more about losses than gains.[1] This concept applies quite naturally in product markets, but little work has been done to incorporate reference dependence into empirical models of consumer demand. One reason for this may be that conventional models are believed to be flexible enough to accommodate behavioral biases like reference dependence, even without explicitly modeling these biases.[2] This paper shows this is not always true. Standard models, such as logit, nested logit, and in some cases the random coefficient model, cannot rationalize the substitution patterns generated by consumers with reference-dependent preferences. Thus ignoring consumers' biases might lead the empirical economist to wrong conclusions.

[1]See the surveys of : Camerer et al (2011), DellaVigna (2009), Kahneman (2003) or Mullainathan and Thaler (2000).

[2]See the discussions presented in: Camerer and Malmendier (2011), Ellison (2006) and Hendricks (2006).

My main goal is to extend the conventional discrete choice framework to account for reference dependent (R-D) preferences in markets with product differentiation. To pursue this goal, I incorporate the R-D preferences theory of Köszegi and Rabin (2006) into a broad class of discrete choice models originated by the work of McFadden (1974) and later extended by Berry et al (1995, 2004) (henceforth BLP). In this class of models, a consumer's indirect utility depends only on product characteristics and individual tastes. In the framework I propose, I also allow consumers to obtain asymmetric payoffs as functions of gains and losses with respect to a reference point.

Experimental evidence suggests that consumers may form reference points in accordance to expectations, aspiration levels, publicity, norms, and social comparisons (Baucells et al 2011). Incorporating reference points into the analysis of demand helps explain consumer unwillingness to change products after experiencing a quality decrease (status quo bias). Since R-D preferences induce loss aversion, the product's own price elasticities might be larger for a price increase than for a price decrease, a discrepancy that affects how firms choose their pricing strategies. Given that reference points can be generated by publicity, firms have incentives to advertise in order to place their product as the reference in the industry.

Previous research in industrial organization has shown that models that account for consumer heterogeneity (such as BLP) are flexible enough to accommodate rich substitution patterns.[3] This class of models is a key element in merger simulations and in many policy counterfactuals. In two different scenarios, I show that standard models could fail to predict important substitution patterns, and thus could generate opposite predictions than those generated under the R-D model. The first scenario (Proposition 1) assumes that consumers share a common reference product in the industry; the second scenario (Proposition 2) relaxes this assumption and studies the possibility of heterogeneity in the reference points among consumers.

[3]For examples, see Nevo (2001) and Petrin (2002).

Assuming a common reference point implies that consumers agree on which product serves as the standard for comparing other products. It is not a mere simplification but rather a possible description for several industries. For example the marketing literature identifies the following industries: Smartphones (iPhone), laundry detergent (Tide), cream cheese (Philadelphia), eReaders (Kindle) or live action cameras (GoPro).[4] From the researcher's point of view it could be challenging to specify which product is the reference product. As a first step, I provide a simple way for testing that the data generating process contains a common reference product.

Despite some obvious examples, for most industries a common reference product among consumers does not exist[5]. It is possible to model R-D preferences in this context if we are willing to assume a source that reveals which is the reference product for each consumer. In this case, I show that the model can still produce novel substitution patterns as long as a reference product has enough reference dependent consumers.

After establishing that the R-D model is non-parametrically testable, I direct attention on the linear random coefficient model to provide a practical and estimable version of an R-D model. I then present several simulations, comparing the performance of the R-D model against standard models. I show that in a simulated world with no reference dependence, the R-D model performs as well as BLP. Nevertheless the R-D model outperforms BLP whenever reference products exist in the simulated world. As shown with the non-parametric model, when the reference product becomes similar to a competing product in my simulation, BLP predicts (wrongly) a decrease in the market share of the latter, R-D correctly predicts an increase. In addition, the simulation shows that the linear BLP model overestimates consumer's sensitivity to prices, which is consistent with the theory.

Finally, using consumer scanner data I analyze the facial tissue industry, where I use the basic Kleenex as reference product. I estimate and test the R-D model by taking advantage

[4]See Amaldoss and He (2017) and Zhou (2011).

[5]For example, in the automobile industry it is unlikely to expect that all consumers consider the Lexus as the reference car.

of a long panel, and the available micro-data provided in the IRI academic dataset. My estimates are consistent with reference dependence and the idea that consumers exhibit loss aversion. I estimate that loss aversion provides the basic Kleenex 25 percent of its market share.

This research makes contributions in several fields. First it contributes to the industrial organization literature by providing a suitable model to estimate demand for industries that contain reference dependent consumers. This is also a contribution to the growing literature of structural behavioral economics, in particular by testing reference dependence in a real market setting.[6] By accounting for the endogeneity problem of prices, this research should appeal to the marketing readers where the idea of reference price has been studied in great detail. The rest of the paper is organized as follows: section 1 places this research in the context of previous literature; the R-D model and its testing results are presented in section 2; section 3 provides simulations to compare the performance of the model; section 4 describes the data for the empirical application; section 5 presents the results of it; and section 6 concludes with a discussion.

## 1.2 Relationship with previous work

In this paper R-D preferences are understood as in the model of Közsegi and Rabin (2006) (henceforth the KR model). This theoretical model generalizes previous behavioral models of R-D utility into a parsimonious theory. Loosely speaking, R-D preferences can be thought as a phenomenon when consumers care not only about the characteristics of their purchase, but also how those characteristics differ from a particular product, called the reference. This section explains the key lessons from the KR model, and how this model emerged from early work. It also positions my research in the context of the current literature, providing a behavioral economics background for those applied economists less familiar with this literature.

[6] See the Survey from DellaVigna (2017)

Kahneman and Tversky (1979) introduced R-D preferences to explain regularities in the experimental evidence of choice under uncertainty. With respect to a reference point, agents perceive differently gains (if the payoff is greater than the reference point) and losses (if the payoff is lower than the reference point). In their model (Prospect theory) the pain of losing $x$ dollars is greater than the pleasure of winning $x$ dollars, i.e., agents are loss averse. Over the following years the research on loss aversion, and particularly in the R-D expected utility, became extremely popular, yet it took 21 years to be extended into the theory of riskless choice. To that end, Tversky and Kahneman (1991) model the reference point as a combination of multiple characteristics. That is, any attribute of a product is evaluated separately as a gain or as a loss with respect to the reference product.[7] Usually the reference product was thought of as the initial or previous endowments consumers had. The theory explains common behavioral biases outside the uncertain world, such as the endowment effect and the status quo bias.

In reality, however, there are several scenarios where there is no endowment to act as the reference point. A researcher that expects to get published suffers more from rejection than one that expects to be rejected. In a more general sense reference points can be generated by expectations, aspiration levels, publicity, norms, and social comparisons. To account for that, the the KR model extends the notion of a reference point to beliefs that consumers have over possible outcomes. For example, a consumer might believe that with probability 1/2 the price of a beer will be 10 dollars, and with probability 1/2 the price will be 5 dollars. Then a beer that is 7 dollars feels like a gain with respect to the possibility of paying 10 dollars, but as a loss to the possibility of paying 5 dollars. The KR model considers the theory of Tversky and Kahneman (1991) as a special case when the beliefs assign all the probability to one particular product. By considering beliefs or expectations as the reference

[7] Suppose that you booked a hotel room with a twin bed and no air conditioner. The manager offers you two alternative rooms, one with a queen bed and no air conditioner, and one with a full bed with air conditioner. Any offered alternative seem better than your previous room. Suppose that you are indifferent between them. Another day you are assigned to a room with a twin bed but with air conditioner. The manager offers you again the alternatives to switch rooms as before. Loss aversion implies that you are more likely than before to choose full bed with air conditioner.

point it is possible to explain previous facts that seem to contradict the original findings of R-D preferences, for example why agents that expect to trade do not suffer from the endowment effect.

Most of the evidence that supports that agents have reference dependent preferences can be found in randomized controlled experiments.[8] This literature started with simple mug exchanges and developed later into more complex environments like sealed-bid auctions. The classical mug exchange experiment can be found in Kahneman et al (1990), where concepts such as endowment effect and status quo bias were tested for the first time. In the mug exchange experiments half of the subjects are randomly endowed with a good (usually mugs) to be kept or traded with the rest of the subjects. Neoclassical theory posits that half of the goods should be traded, however in most of these papers the authors show that only a few goods change hands (less than 10 percent of the mugs in Kahneman et al (1990)). In this kind of experiments the endowment acts as the reference point. The presence of loss aversion implies that to give up this endowment additional compensation is required, and so the willingness to pay is less than the willingness to accept. Testing for R-D preferences when the endowment is the reference point has been also documented in works of Isoni et al (2011) and in a field experiment by List (2004)

Recent experimental research tests the KR model by exogenously modifying agents' expectations. For example Yu et al (2017) varied the information shared with consumers regarding the waiting time in a call center. By structurally estimating an optimal stop model the authors show that the waiting cost increases once the real waiting time surpasses the expected one. In the same vein the experiment in Abeler et al (2011) manipulated the expected payment subjects could earn in an effort choice experiment. In their experiment subjects decided how much time to work on a tedious task. Once the subjects deiced to stop an observed lottery assigned them to a payment condition. In the first condition subjects were paid a previously announced fixed rate, in the second condition subjects payment depended

[8]The experimental literature regarding this topic is enormous and I will only mention a few papers.

on how much time they worked. The authors show that as the probability of being paid in a fixed rate increases the subjects decide to work significantly less.

Using expectations as reference points but in the context of auctions Banerji and Gupta (2014) show how loss aversion changes biding behavior. In their experiment students submit a sealed bid that plays against a random bid drawn from a known uniform distribution. If the student bid is greater than the number drawn by a computer, the student wins the auction but pays the random number. As in any second price auction agents should submit their valuations. However, the authors show in their model that loss aversion implies some shading of it. This is verified by observing how agents change their bids when the observed expected bid made by the computer varies.

Outside the experimental framework the complications of testing reference dependence magnifies, mostly due to the lack of information about the reference point. Camerer et al (1997) finds negative supply elasticities for taxi drivers in New York city. The authors suggests that cab drivers work as if they have target or reference income. Working more after reaching the target becomes extremely costly and thus most of the cab drivers stop. By modeling target income inside a supply model Farber (2008) tests the reference dependent hypothesis in a structural way. The author concludes that the target income varies too much across days to be consistent with a “status quo” kind of reference point, hence disregarding the previous work. Nevertheless Crawford and Meng (2011) revisit Farber’s work considering reference income as a set of beliefs that account for the probabilistic demand, i.e., as in the KR model. The authors successfully explain why target income varies across days while finding consistent evidence of R-D preferences.

The role of R-D preferences inside the study of markets has been limited mostly to theoretical models. Heidhues and Köszegi (2008) modifies the Salop (1979) model to introduce R-D preferences in the same way as in the KR model. The authors show that even if firms face different marginal costs, a focal price equilibrium arises under a wide variety of conditions. Intuitively, when a firm decides to charge a higher price than the one consumers expect, loss aversion implies losing a considerable amount of the market share. If on the other hand a firm decides to charge a lower price, the winnings in market share are small compared to the markup loss. In this case R-D preferences help to explain price stickiness, and moreover why different firms tend to choose a similar price. The focal price equilibrium however requires that the reference point comes from consumers rational expectations. Forming a reference point with rational expectations seems possible in industries where consumers constantly make purchasing decisions.

Instead of using expectations, Zhou (2011) considers the case where the reference point is a product in the industry. By considering a duopoly that competes à la Hotelling, the author shows that the unique equilibrium is when the reference firm plays a high\low price mixed strategy, and the non-reference firm uses a medium price. The mixed strategy equilibrium depends heavily on the duopoly assumption. Amaldoss and He (2017) extend the model to an arbitrary number of firms by modeling competition in a spoke network. In this case the equilibrium price strategy depends on consumers' intrinsic valuation of the product. For low valuations the reference product is priced lower than the competing products, on the other hand for high valuations the reference product is priced higher than the competition. Having a reference product in the industry might be the result of publicity or advertisement.

In empirical IO the role of reference dependence is still very new. There exists some skepticism on the value added by modeling R-D preferences inside a flexible enough choice model such as BLP. This research sheds some light of this value and presents situations where BLP falls short. In marketing, on the other hand, several choice models have been considered to explain reference prices. Reference price models incorporate loss aversion only in prices and not in product characteristics.[9] The reference point is defined by the researcher as the the previous price or as the expectation of all available prices. For example Mazumdar and Papatla (1995) conduct a simple logit model to estimate the price loss aversion parameter with scanner data on detergent and margarine. In a attempt to add heterogeneity, Kopalle

[9]The early work of Hardie et al (1993) considers also a reference quality, yet due to the hardships of measuring quality, subsequent papers focused on prices only.

et al (2012) uses the nested logit approach with scanner data on Cola beverages. Erdem et al (2001)**?** ] builds a random utility model with normal correlated errors across brands on ketchup, peanut butter and tuna. All these papers find evidence of reference prices, nevertheless none of them takes into consideration the classical endogeneity problem of prices and unobservables. As noted in Berry (1994) failing to account for endogenity might result in wrong estimates of the price coefficients. Finally from the IO perspective there is not a good reason to limit the research of R-D preferences only to the price domain, The model I propose allow for endogeneity and the reference point contains both prices and multiple product characteristics.

## 1.3 The R-D model

This section will first develop a general framework to introduce R-D preferences into discrete choice demand models. In pursuing comparability with BLP-type models, I impose assumptions on the utility function's substitution patterns that mimic the properties of standard discrete choice models. Once the general non-parametric model is presented, I show natural simplifications that are practical in empirical applications. Each of these specific models will be accompanied by propositions where some classical models fail to explain possible substitution patterns.

Building in the tradition of Berry et al (1995, 2004) a consumer $i$ in market $t$ chooses a good $j$ from $\mathcal{J}_t = \{0, 1, ...J_t\}$ available products. Each product $j$ in market $t$ is described by a triplet $(p_{jt}, x_{jt}, \xi_{jt})$, where $p_{jt} \in \mathbb{R}_+$ is the price of the product; $x_{jt} \in \mathbb{R}^k$ is a vector of $k$ observable characteristics; and $\xi_{jt} \in \mathbb{R}$ summarizes all unobserved characteristics that are relevant to the consumer's utility. As is common in industrial organization, prices may be correlated with $\xi$.[10] I label the outside good for all markets $t \in T$ with $j = 0$. Also, for exposition simplicity, I suppress the market index $t$ and bring it back when necessary.

[10] $\xi_{it}$ could be also correlated with all or some of the components of $x_{ij}$, see Gandhi and Houde (2016).

Consumers are identified by a triplet $(z_i, \zeta_i, G_i)$, where $\zeta_i$ captures unobservable tastes; $z_i \in \mathbb{R}^m$ is a vector of observable characteristics; and $G_i$ is the reference point. As in the KR model, the reference point will capture beliefs or expectations that consumers have over the product space. In particular let the reference point $G_i$, be a probability measure over the observable product space $(p_j, x_j)$. The indirect utility of consumer $i$ that is obtained by choosing product $j$, is given by:

$$U(z_i, \zeta_i, x_j, p_j, \xi_j; G_i) = \int u(\zeta_i, x_j, p_j, \xi_j; x_r, p_r) dG_i(x_r, p_r) \tag{1.1}$$

where $u(s_i, \zeta_i, x_j, p_j, \xi_j; x_r, p_r)$ is the indirect utility for a fixed reference product $r$. In this way, the model is flexible enough to capture perceptions of loss and gains with respect to different available products, previously available products, or even unavailable but expected features. Agents derive utility from two main parts: the consumption of the good, and the comparison of their purchase with respect to the reference point. This is captured by the following assumption.

**Assumption 1 (Separability)**: Conditional on a given reference product $r$, the indirect utility of subject $i$ that purchases good $j$ is given by

$$u(z_i, \zeta_i, x_j, p_j, \xi_j; x_r, p_r, \xi_r) = \upsilon(z_i, \zeta_i, x_j, p_j, \xi_j) + \eta(x_j, p_j; x_r, p_r) \tag{1.2}$$

where the first term $\upsilon(z_i, \zeta_i, x_j, p_j, \xi_j)$ captures the intrinsic utility of consuming good $j$; the second term $\eta(x_j, p_j; x_r, p_r)$ models the "gain-loss" utility with respect to a reference product $(x_r, p_r)$.

The separation made by assumption 1 not only brings tractability to the model, but also says that the effects of a purchase not only depend on the reference point and the consumer still enjoys the product's own features in the usual way. Naturally if $\eta(x_j, p_j; x_r, p_r) = 0$ the model simplifies to a standard discrete choice model; and so the R-D model nests the standard models.

The next step is to allow for different perceptions of gains and losses. For example, when making purchase a consumer might experience a loss in price yet at the same time a gain in quality. Thus, as in the KR model and in accordance with Tversky and Khaneman (1992), each component of the "gain and loss" function should be evaluated separately to be considered as a gain or a loss. The following assumption establishes the different categories where the model allows consumers to have different perceptions.

**Assumption 2 (Categorization):** Conditional on a given reference product $r$, the gain and loss utility can be decomposed as:

$$\eta(x_j, p_j; x_r, p_r) = \eta_p(p_j; p_r) + \eta_x(x_j; x_r) \tag{1.3}$$

Note that assumption 2 implies that consumers evaluate prices, and observable characteristics separately and possibly in different ways. However this assumption does not restrict the possibility of evaluating some or all observable characteristics at once. The reason behind this comes from the differences between horizontal and vertical attributes. Note that from the researcher point of view, vertical characteristics' values can be easily categorized as losses or as gains. A higher price than the expected reference price is a loss. However, horizontal characteristics depend on the consumer's ideal taste, and for that reason horizontal characteristics require special treatment. For simplicity, we can assume that all observed characteristics are horizontal, noting that vertical characteristics can be modeled in a similar fashion as prices. With this consideration I characterize loss aversion in the following two assumptions.

**Assumption 3 (Loss Aversion in Vertical Characteristics):** The loss and gain function associated with vertical characteristics (prices for exposition simplicity) can be written as follows:

$$\eta_p(p_j; p_r) = \mu_p(p_r - p_j)$$

Where $\mu_p : \mathbb{R} \to \mathbb{R}$ satisfies the following conditions

- $\mu_p(x)$ is continuous for all $x$, and $\mu_p(0) = 0$
- $\mu_p(x)$ is weakly increasing
- If $y > x > 0$ then $\mu_p(y) + \mu_p(-y) < \mu_p(x) + \mu_p(-x)$

Assumption 3 says that consumers weight the gain or loss in price of their purchase. Negative values or losses have more weight than positive values or gains. That is, individuals are loss averse. Assumption 3 is a weaker version of the assumptions made on the KR model.[11]

**Assumption 4 (Loss Aversion in Horizontal Characteristics):** The loss and gain function associated with observable characteristics is given by:

$$\eta_x(x_j; x_r) = \mu_x(-d(x_j, x_r))$$

where $d(\cdot, \cdot) : \mathbb{R}^k \times \mathbb{R}^k \to \mathbb{R}$ is a distance function and $\mu_x(\cdot)$ satisfies the same assumptions imposed to $\mu_p(\cdot)$.

Assumption 4 says that consumers pay a utility cost when buying something that differs in observable characteristics from their reference product. Loss aversion here is extreme, as there can be no additional gains from differentiation with respect to the reference product. Assumptions 1-4 provide the structure of the non-parametric model.

With assumptions 1 to 4, the non-parametric model generates the same results of proposition 1 in Közsegi and Rabin (2006), and thus the R-D model translates their theory to the discrete choice framework. In this context, market shares (or demands) can be obtained if we aggregate the individual behavior over the population distribution. Let the vector of individual characteristics be $\boldsymbol{Y_i} = (z_i, \zeta_i, G_i)$, with distribution $P_{\boldsymbol{Y}}$, and define the set

$$A_j(x_j, p_j, \xi_j) = \{\boldsymbol{Y} : U(z_i, \zeta_i, x_j, p_j, \xi_j; G_i) > U(z_i, \zeta_i, x_n, p_n, \xi_n; G_i) \, \forall n \in J\}$$

[11] Note that in this model the choice set is not stochastic. It is not necessary to impose the rest of the assumptions presented in the KR model.

the market shares implied by the model are given for every $j \in \mathcal{J}_t$ as:

$$\sigma_j(p, x, \xi) = \int_{A_j(x_j, p_j, \xi_j)} P_{\boldsymbol{Y}}(d\boldsymbol{y}) \tag{1.4}$$

While the general model seems too ambitious to be taken to data, there exist natural simplifications that allow us to do so.

### 1.3.1 The common reference product case

The first case simplifies the model to accommodate markets that have a unique reference product. As mentioned above, there exist several industries that fall into this category, and thus highlight the importance of R-D preferences in the real world. To see how to model this case, consider the following two assumptions

**Assumption 5 (Degenerate Measure)**: There exists an $r \in \mathcal{J}$ such that the measure $G_i$ puts all its mass on $(x_r, p_r)$.

**Assumption 6 (Common Reference Point):** for every $i$ ,$G_i = G$.

Assumption 5 imposes the restriction that the reference point for each consumer must be a single product in the product space. It still allows for situations heterogeneity among consumers. An example of this type of industries would be industries with two leading brands. On the other hand, assumption 6 imposes a common reference product. With these assumptions, the indirect utility reduces as follows:

$$\begin{gathered} U(z_i, \zeta_i, x_j, p_j, \xi_j;\, x_r, p_r) = \\ \upsilon(z_i, \zeta_i, x_j, p_j, \xi_j) + \mu_x(-d(x_j, x_r)) \quad + \mu_p(p_r - p_j) \end{gathered} \tag{1.5}$$

Even in this simpler form, the model can produce novel substitution patterns. Before proceeding to the formal proposition, consider the following simple but illustrative example.

Suppose a market has three products, labeled $a$, $b$ and $r$. These products are described by one characteristic, $x$. For simplicity, suppose that half of the population prefers products that have a large amount of $x$; the other half of the population prefers goods that have a smaller amount of $x$. Also assume that product $a$ has more of $x$ than product $b$. Let $r$'s $x$ be the average of those from $a$ and $b$. Without reference dependence and same prices consumers will most likely select $a$ and $b$ in similar proportions, and they will almost never select $r$. Unsurprisingly, if we allow for reference dependence with respect to $r$, the market share of $r$ will come from stealing loss averse consumers, taken equally from $a$ and $b$. Now consider the case where $r$ increases the amount of $x$ such that it gets closer to $a$. Without reference dependence the market share of $a$ is expected to weakly decrease; nevertheless, loss aversion with respect to the distance to $r$ makes $a$ more attractive than before. At the same time product $b$ is less appealing and its market share will weakly decrease. Note that this change cannot be produced by most standard discrete choice models. From this simple example it is natural to expect that changes in the reference product across markets (time) helps to identify the gain and loss components of the utility function of the R-D model.

Even though the previous example imposed strong assumptions on the tastes of the population, it turns out that the same logic works for a broad class of tastes and observable characteristics $(\zeta_i, z_i)$. Consider the set

$$\tilde{A}(x_j, p_j, \xi_j) = \{\boldsymbol{Y}_i : \upsilon(z_i, \zeta_i, x_j, p_j, \xi_j) > \upsilon(z_i, \zeta_i, x_n, p_n, \xi_n) \, \forall n \in J\}$$

The set $\tilde{A}(x_j, p_j, \xi_j)$ contains those individuals that derives more indirect intrinsic utility from product $j$ than from any other product. In the same way let $\tilde{\sigma}(x_j, p_j, \xi_j) = \int_{\tilde{A}(x_j, p_j, \xi_j)} P_{\boldsymbol{Y}}(d\boldsymbol{y})$. Note that $\tilde{\sigma}(x_j, p_j, \xi_j)$ are the market shares that traditional models will generate (i.e. when reference dependence does not matter). The following assumption restricts tastes $(\zeta_i, z_i)$ in a way that is natural for most of the applied models within the differentiated product world.

**Assumption 7 (Intrinsic Weak Substitution):** If two vectors $x_j$ and $x'_j$ satisfy

- $x_{jk} = x'_{jk}$ for all $k$ but one
- $d(x'_j, x_m) < d(x_j, x_m)$

- $\Delta\tilde{\sigma}_j = \tilde{\sigma}_j(p, x, \xi) - \tilde{\sigma}_j(p, x', \xi) > 0$

then $\Delta\tilde{\sigma}_m \leq 0$

Even when at first glance assumption 7 might seem restrictive, it is not. Assumption 7 says that the industry consists of products which have a non-trivial degree of substitution, that is to say, products that are closer in the characteristic space might share the same consumers. Assumption 7 is present in a a broad class of standard IO of models, the following lemma shows this .

*LEMMA 1: Generalized extreme value models satisfy assumption 7.*

Indeed, assumption 7 is satisfied by McFadden's Logit (1974) and Berry's Nested Logit (1994). Since mixed multinomial logit (MML) models are weighted average of individual logit probabilities, assumption 7 will be satisfied for those MML models that contain characteristics weighted by random coefficients with large enough mass on the positive support, or with large enough mass on the negative support. In particular, lemma 2 provides a sufficient condition for the linear MML models with a standard normal mixing distribution.

*LEMMA 2: If* $\upsilon(\zeta_i, x_j, p_j, \xi_j) = \sum_k x_{jk}\tilde{\beta}_{ik} - \tilde{\alpha}_i p_j + \xi_j + \epsilon_{ijt}$, *with* $\tilde{\beta}_{ik} = \bar{\beta}_k + \beta_k^u \nu_{ik}$, *and* $\nu_{ik} \sim N(0, 1)$ *for all* $k$.

*The model will satisfy assumption 7 if* $\frac{|\bar{\beta}_k|}{\beta_k^u}$ *is sufficiently large.*

In other words, Lemma 2 says that BLP might satisfy assumption 7, nevertheless it does not provide a necessary condition, and thus it remains an open question whether in general assumption 7 is satisfied in BLP. Having discussed several standard models that satisfy assumption 7, we can stay the main proposition of the paper.

*PROPOSITION 1: There exist preferences* $U$ *and distributions* $P_{\boldsymbol{Y}}$ *satisfying assumptions 1-7 and two vectors* $x_r$ *and* $x'_r$ *with* $d(x'_r, x_j) < d(x_r, x_j)$, *such that* $\Delta\sigma_j > 0$ *and* $\Delta\sigma_r > 0$

Since in previously discussed standard models it is always the case that $\Delta\sigma_j \leq 0$ whenever $\Delta\sigma_r > 0$ and $d(x'_r, x_j) < d(x_r, x_j)$, proposition 1 motivates the importance of accounting for reference dependent preferences within the field of industrial organization. With R-D preferences the evaluation of counterfactual could result in predictions of changes in market shares that have the opposite sign that those generated by the standard models. The real challenge of the reference product case might be to argue which is the actual reference product, which I will discuss later on.

### 1.3.2 Second scenario, heterogeneity on the reference point

The second scenario removes assumption 6, and so allows for different reference products among the population. While this case relates to a broader set of industries, it introduces additional concerns. First, the empiricist needs to obtain individual information that allows him to assign to each consumer a reference product. Second, having multiple reference products diminishes the importance of R-D preferences on the market structure, and therefore raises the question on when and how it matters.

Depending on the application, the first concern can be addressed by using the information on previous purchases made by the observed consumers. For example, it is natural to expect that a consumer will use her current car to evaluate gains and losses on her next car purchase. Using the previous purchase as the reference point or price can be found in the work of Hardie et al (1993). On the other hand, in markets where there exist a few leader brands and many followers, some statistics that account for the whole history of observed purchases (such as the mode) can help detect the reference product, Baucells et al (2011). As we should expect, heterogeneity in reference points requires more from the data in question, and so limits its application to its availability.

To answer to what extent having multiple reference products matters for the industry study consider the following example. Suppose that in a given industry there exist 4 products, two of which are reference products. Denote as before the non-reference ones as $a$ and $b$,

Table 1.1: Consumer Preferences

| | Prefers products that have large amounts of $x$ | Prefers products that have small amounts of $x$ |
|---|---|---|
| Reference $r_1$ | $\frac{1}{4}$ | $\frac{1}{4}$ |
| Reference $r_2$ | $\frac{1}{4}$ | $\frac{1}{4}$ |

and the reference ones as $r_1$ and $r_2$. Each product is defined by a characteristic $x$ . In terms of $x$ the products are ordered as follows: $a > r_1 > b > r_2$. Half of the consumer population prefers goods that contain large amounts of $x_1$ , the other half prefers goods that contain small amounts of $x_1$. Moreover suppose that half of the population that likes large amounts of $x$ has $r_1$ as reference product, and the other half has $r_2$. The same occurs for the part of the population that smaller amounts of $x$. Table 1.1 summarizes this information.

Without reference dependence and with similar prices, consumers will select $a$ and $r_2$ in similar proportions, and they will almost never select $r_1$ or $b$. With reference dependence, the market shares from $a$ will decrease due to loss averse consumers that have either $r_1$ or $r_2$ as their reference product. Similarly, the market share from $r_2$ will decrease due to loss averse consumers that have $r_1$ as their reference. Suppose that $r_1$ increases its amount of $x$ such that it gets closer to $a$. Without reference dependence, the market share of $a$ should weakly go down, since it has a closer substitute. Nevertheless reference dependence implies that the market share of $a$ should weakly increase, this increment comes from the$\frac{1}{4}$ of the population that has $r_1$ as their reference, and prefer large amounts of $x$, and not from the $\frac{1}{4}$ of the population that likes large amounts of $x$ but has the reference product $r_2$. Similarly the market share of $b$ or $r_2$ could go down since now they are farther away from $r_1$. While this example says that a reference product can produce contrary substitution patterns, it also suggests that to find them, the share of consumer's that have as reference the product that produces the novel patterns, should be large enough. The following proposition formalizes this idea

*PROPOSITION 2: If the proportion of consumers that has reference product $r$ is large enough, there exist preferences $U$ and distributions $P_{\boldsymbol{Y}}$ satisfying assumptions 1-5 and 6 and two vectors $x_r$ and $x'_r$ with $d(x'_r, x_j) < d(x_r, x_j)$, such that $\Delta\sigma_j > 0$ and $\Delta\sigma_r > 0$*

Proposition 2 says that the model could be testable in many situations, however it imposes the warning that the researcher might lose the ability to distinguish the R-D model from the standard modal if as the number of different reference products grow the proportion of of each reference product decreases. Note that even if the model allows for heterogeneity in the reference points, the model does not allow for heterogeneity on the degree of dependence the individual has, which as noted by Bodur and Arora (2014) is important when targeting consumers.

### 1.3.3 Stochastic Reference Product

Before moving to the estimation section it is important to discuss what happens if assumption 5 is removed, that is, when the model allows for a stochastic reference product. Having a stochastic reference product has two main advantages. The first one consists in letting the individuals to have an ideal product that is not necessarily available in the market. An individual could have a reference price equal to the average one of all available products, and at the same time expect some basic features (any smartphone should have a camera and should be touch screen). The second advantage comes from the researcher's point of view, there is no need to preemptively take a stand on which one is the reference product-rather the researcher allows the data answer it.

While these advantages seem interesting, the implications on the market structure are less clear. All previous arguments to distinguish the model and test the theory rely on observing movements in the reference product(s),which in this case are not directly observable. Note however that the model still predicts unexpected substitution patterns. To see this just replace the reference product(s) for fictional ones in the previous the examples. We should expect that in data sets which span long periods, the standard of comparison changes, hence producing substitution patterns that are hard to rationalize with standard models. Nevertheless additional research is required to formally state this intuition and thus leaves this question to a future agenda.

## 1.4 Estimation and Simulations

When taking the model to the data it is necessary to introduce a parametric version of it. In particular this section discusses how to estimate a linear random coefficient model for the case of a common reference product among consumers (the extension to the case with heterogeneous reference products is almost direct). Among possible parametric forms, I choose the linear form to make a direct comparison to the classical BLP model. Such comparisons will be later discussed in this section through the use of simulated data. Specifically, I show how each model performs when either the true data generating process has no reference dependence, and when it actually does. The following assumption provides such desired structure to the model.

**Assumption 8 (Linear Parametric Model)**: For all markets $t$ and all products $j \in \mathcal{J}_t \backslash \{0\}$ the indirect utility of consumer $i$ is given as follows:

$$u\left(z_i, \zeta_i, x_{jt}, p_{jt}, \xi_{jt}; \theta\right) = \sum_k x_{jkt} \tilde{\beta}_{ik} - \tilde{\alpha}_i p_{jt} + \xi_{jt} + \epsilon_{ijt} - \lambda(p_{jt} - p_{rt}) \mathbb{I}\left\{p_{jt} > p_{rt}\right\} - \gamma ||x_{jt} - x_{rt}||_2 \tag{1.6}$$

with

$$\begin{aligned} \tilde{\beta}_{ik} &= \bar{\beta}_k + \sum_r z_{ir} \beta^o_{kr} + \beta^u_k \nu_{ik} \\ \tilde{\alpha}_i &= \bar{\alpha} + \sum_r z_{ir} \alpha^o_r + \alpha^u \nu_{ip} \end{aligned} \tag{1.7}$$

and for all markets $t$, the utility of the outside good is $u(z_i, \zeta_i, x_{0t}, p_{0t}, \xi_{0t}) = \epsilon_{i0t}$

where $\nu_i = (\nu_{i1}, ..., \nu_{ik}, \nu_{ip})$ is a random vector that models the unobserved individual heterogeneity for each observable characteristic and price, $\epsilon_{it} = (\epsilon_{i0t}, \epsilon_{i1t}, ..., \epsilon_{iJ_t})$ is a mean zero vector of individual idiosyncratic shocks, and $|| \cdot ||_2$ is the $L^2 - Norm$.

In this particular model specification, the $\lambda$ coefficient measures the consumers' loss aversion when purchasing a product that is more expensive than the reference product; in a similar way $\gamma$ measures the loss aversion regarding the horizontal attributes. As in any parametric discrete choice model, an assumption over the consumers' unobservable features is required to predict market shares, and thus to estimate the model.

**Assumption 9 (Tastes Distributions):** In any market $t$, consumer $i$'s unobservables have the following underlying distributions:
1) $\nu_i$ is distributed joint normal with mean zero and identity covariance matrix
2) $\epsilon_{it}$ is distributed type-1 extreme value

The assumption 9 distributional forms are not a strict requirement, in fact $\epsilon_{it}$ could be replaced with a joint normal distribution, however assuming type-1 extreme value brings two advantages: first, it has a closed form that assists the exposition of the paper, second, it imposes less computational burden, and in fact the market shares can be written as follows:

$$\sigma_{jt=} \int \frac{exp\left(\sum_k x_{jtk}\tilde{\beta}_i - \alpha p_{jt} + \xi_{jt} - \lambda(p_{jt}-p_{rt})\mathbb{I}\{p_{jt}>p_{rt}\} - \gamma||x_{jt}-x_{rt}||_2\right)}{1+\sum_{n=1}^{Jt} exp\left(\sum_k x_{ntk}\tilde{\beta}_i - \alpha p_{nt} + \xi_{nt} - \lambda(p_{nt}-p_{rt})\mathbb{I}\{p_{nt}>p_{rt}\} - \gamma||x_{nt}-x_{rt}||_2\right)} dF(\nu)d\bar{F}(z) \tag{1.8}$$

where $\bar{F}(z)$ and $F(\nu)$ are the distribution of observable characteristics and the distribution of unobservabales. In practice the integral with respect to the unobservable characteristics is evaluated by Monte Carlo simulations by taking $M$ draws from the distribution $F(\nu)$.

Before estimating the parameters $\theta = (\tilde{\beta}, \tilde{\alpha}, \lambda, \gamma)$, we need to address the endogeneity in prices that comes from the fact that firms observe $\xi$. Following the same ideas presented in Berry et al (1995, 2004), let $w_{jt}$ be a vector of instruments that are correlated with the price, but uncorrelated with the unobservables $\xi$; that is, when such instruments satisfy $E[\xi_{jt}|w_{jt}, x_{jt}] = 0$. This relationship allows us to form $C$ moment conditions $E[\xi_{jt}h(w_{jt}, x_{jt})] = 0$ (where $h$ are some known vector valued functions), and thus we can follow a GMM approach. To that end, let $s_{jt}$ be the observed market share of product $j$ in market $t$, and let $s_t = (s_{jt})_{j=0}^{J}$ be the collection of all market shares for each market. Since the implied market shares from the model have to match the observed ones ($\sigma_j(p, x, \xi; \theta) = S_j$), it is possible to use the MPEC approach to minimize a GMM criterion function. That is, we can follow the algorithm proposed in Dubé et al (2012), and hence write the estimation problem as:

$$\begin{gathered} \min_{\theta,\xi} \; g(\xi)'\Omega g(\xi) \\ \text{subject to } \sigma(p, x, \xi; \theta) = S \end{gathered} \tag{1.9}$$

where $g(\xi) = \frac{1}{T}\sum_{t=1}^{T}\sum_{j=1}^{J}\xi_{jt}h(z_{jt}, x_{jt})$ and $\Omega$ is a GMM weighting matrix.

### 1.4.1 Evidence from Simulations

The following Monte Carlo simulations are intended to show the performance of the linear random coefficients R-D model vs the linear BLP model under different data generating processes. For that purpose, I consider a market structure based on $T = 30$ markets with $J = 10$ products. Each product is defined as the combination of $K = 3$ observable characteristics, its price $p_{jt}$ and the unobservable $\xi_{jt}$. The simulated data comes from random draws of the following distributions:

$$\begin{pmatrix} x_j^1 \\ x_j^2 \\ x_j^3 \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & -0.8 & 0.3 \\ -0.8 & 1 & 0.3 \\ 0.3 & 0.3 & 1 \end{pmatrix}\right)$$

$$\xi_{jt} \sim i.i.d.\, N(0,1), \text{ and } p_{jt} = |0.5\xi_{jt} + e_{jt} + x_j^1 + x_j^2|$$

where $e_{jt} \sim N(0,1)$. For each product $j$ in market $t$, I generate a vector of instruments $w_{jt}$ of dimension $D = 6$. Each component of this vector of instruments is obtained as follows, $w_{jtd} = u + \frac{1}{4}(e_{jt} + 1.1\sum_{k=1}^{3} x_{kjt})$, where $u \sim Uniform\,(0,1)$. In addition, the following polynomial functions are generated $z_{jtd}^2$, $z_{jtd}^3$, $x_{jk}^2$, $x_{jk}^3$, $\prod_{d=1}^{6} z_{jtd}$, $\prod_{k=1}^{3} x_{jk}$, $z_{jtd}x_{j1}$ and $z_{jtd}x_{j2}$.

With the same data, I generate the observed market shares in two scenarios. The first scenario is a world without R-D preferences, where the true utility function of the individual $i$ is given by equation (1.6) with the following parameters: $\bar{\beta} = (1\,1.5\,.5)$, $\beta^u = (.70\,.70\,.70)$, $\bar{\alpha} = -3$, $\alpha^u = .44$, $\lambda = 0$ and $\gamma = 0$. In total there exist 6 parameters to estimate with 42 instruments.

Table 1.2 contains the results from estimating several models: The logit model, the BLP, the R-D logit and, the random coefficients logit R-D model. As expected the BLP model does a great job at estimating the parameters, nevertheless the differences between the BLP model and the full R-D model are statistically negligible. This result might not be surprising since the BLP model is nested in the R-D model. In fact, given the parametric structure imposed in the gain and loss function for the price comparison, it is possible to construct a hypothesis test that fails to reject the absence of a common reference product in the data generating process.

To see this in each of the $t$ markets re-index the products in an ascending way with respect to their own price, call this new index $\tau$. With this new indexing it is possible to write the mean utilities for each product $\tau$

$$\delta_\tau = x_\tau\beta - \alpha p_\tau \mathbb{I}\{\tau \leq r\} - \phi p_\tau \mathbb{I}\{\tau > r\} - \gamma d(x_\tau, x_r) + \lambda p_r \mathbb{I}\{\tau > r\} + \xi_r$$

where $\phi = \alpha + \lambda$ and $r$ is a possible reference product. This setting allow us to use the findings from the structural break literature, in particular adapting the ideas in Andrews (2003). Let $\varsigma = [r_1, .., r_2]$ be the set of possible reference products, where $r_1 > 1$ and $r_2 < J_t$.

Let $\Psi_r$ be the Wald Statistic for testing

$$H_0 : \begin{pmatrix} \alpha - \phi \\ \gamma \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

when the possible reference product is $r$ . Then it is possible to construct the supremum statistic $Sup\Psi = \underset{r \in \varsigma}{sup} \Psi_r$, and test the null hypothesis of a data set without a common reference product using the critical values provided by Andrews (2003). The last line of Table 1.2 presents the value of the relevant information of this hypothesis where, as shown, I cannot reject the null. In other words, there is no evidence of a reference product in the data.

The second simulation contains preferences that are actually reference dependent. In particular, the parameters associated with the gain and loss functions take the following values $\lambda = -5$ and $\gamma = -4$. The results of estimating the same models as before are presented in Table 1.3. As expected the full R-D model does the best job at estimating the parameters, however now the results from BLP and Logit are way off for most of coefficients. In particular we can see that the mean price sensitivity is overestimated. This is not a particular result of the simulation, but rather a robust finding about the form of bias in this parametric model. Note that when estimating the BLP model, the price coefficient adds together its own sensitivity coefficient and the coefficient of loss aversion for those products that have a higher price than the reference. This is particularly important if demand estimates are inputs to additional models, such as merger evaluations or price policy counterfactuals. Note that the loss aversion coefficients are significant, implying that we reject the null hypothesis of non-reference dependence regarding that product.

As mentioned in the previous section, predictions regarding changes in market shares could go in opposite directions between the models that include R-D preferences and traditional ones. Table 1.4 presents such a scenario in my simulation. In detail, I exogenously changed the characteristics of the reference product to make it identical to a competing

Table 1.2: The True DGP has No Reference Dependence

| | True Value | Logit | BLP | R-D Logit | R-D |
|---|---|---|---|---|---|
| $\bar{\alpha}$ | -3.0 | -4.33 | -3.11 | -2.24 | -3.10 |
| | | (0.2182) | (0.0082) | (0.0153) | (0.0080) |
| $\alpha^u$ | 0.44 | - | 0.48 | | 0.475 |
| | | | (0.0088) | | (0.0024) |
| $\bar{\beta}_1$ | 1.0 | 0.65 | 0.91 | 1.64 | 0.945 |
| | | (0.0345) | (0.0240) | (0.0023) | (0.0243) |
| $\beta_1^u$ | 0.70 | - | 0.44 | | 0.49 |
| | | | (0.0153) | | (0.0139) |
| $\bar{\beta}_2$ | 1.5 | 1.13 | 1.31 | 2.1 | 1.30 |
| | | (0.0003) | (0.0235) | (0.0011) | (0.0238) |
| $\beta_2^u$ | 0.70 | - | 0.91 | | 0.97 |
| | | | (0.0099) | | (0.0093) |
| $\bar{\beta}_3$ | .50 | 0.66 | 0.51 | 0.68 | 0.50 |
| | | (0.1243) | (0.0156) | (0.0141) | (0.0158) |
| $\beta_3^u$ | 0.70 | - | 0.48 | | 0.71 |
| | | | (0.0088) | | (0.0086) |
| $\lambda$ | | - | | 0.58 | 0.06 |
| | | | | (1.311) | (0.0067) |
| $\gamma$ | | - | | 0.00 | -0.03 |
| | | | | (1.22) | (0.0013) |
| sup $\Psi$ | 12 | $<$ | Critical Value | 27.53 | Cannot Reject $H_o$ |

Standard Errors are reported in brackets. The results come fro evaluating the integral 1000 times. Several initial values were provided to MatLab with Knitro.
Source: Simulated data

Table 1.3: The True DGP has Reference Dependence

| | True Value | Logit | BLP | Logit-R-D | R-D |
|---|---|---|---|---|---|
| $\bar{\alpha}$ | -3.0 | -13.71 | -18.17 | -2.83 | -3.09 |
| | | (0.1233) | (0.0713) | (0.0010) | (0.0006) |
| $\alpha^u$ | 0.44 | - | 3.36 | | 0.47 |
| | | | (0.0180) | | (0.0002) |
| $\bar{\beta}_1$ | 1.0 | 6.36 | 1.92 | 1.60 | 0.92 |
| | | (1.221) | (0.1051) | (0.0415) | (0.0018) |
| $\beta_1^u$ | 0.70 | - | 0.00 | | 0.477 |
| | | | (0.3634) | | (0.0010) |
| $\bar{\beta}_2$ | 1.5 | 13.85 | 8.97 | 2.10 | 1.28 |
| | | (1.001) | (0.0541) | (0.1457) | (0.0017) |
| $\beta_2^u$ | 0.701 | - | 0.00 | | 0.94 |
| | | | (0.0934) | | (0.0007) |
| $\bar{\beta}_3$ | .50 | -0.85 | -1.30 | 0.64 | 0.49 |
| | | (0.0124) | (0.0735) | (0.0472) | (0.0012) |
| $\beta_3^u$ | 0.701 | - | 0.17 | | 0.67 |
| | | | (0.0933) | | (0.0006) |
| $\lambda$ | -5.0 | - | - | -4.45 | -4.92 |
| | | | | (0.0001) | (0.0005) |
| $\gamma$ | -4.0 | - | - | -3.99 | -4.16 |
| | | | | (0.0011) | (0.0002) |

Standard Errors are reported in brackets. The results come from evaluating the integral 1000 times. Several initial values were provided to MatLab with Knitro.
Source: Simulated data

Table 1.4: Predictions in Market Shares Changes

| | Before $d(x_j, x_r) = 1.76$ | After $d(x_j, x_r) = 0$ | | |
|---|---|---|---|---|
| | True Value | True Value | Prediction BLP | Prediction R-D |
| $\sigma_j$ | 0.06 | 0.24 | 0.02 | 0.30 |

The results come from evaluating the integral 1000 times. Several initial values were provided to MatLab with Knitro.
Source: Simulated data

product $j$. As shown, the market share of this product went up in reality, and the prediction of the R-D model captures it. However BLP wrongly predicts that given this change the market share should go down.

## 1.5 Empirical Application: Data Description

The remaining sections of the paper discuss an empirical application of the R-D model. Since the main goal is to show that R-D preferences matter when estimating demand, I focus on an industry with a common reference product –the facial tissue industry. To that end, I use retail scanner data from IRI academic dataset households panel. This panel follows consumers in two U.S. cities, Eau Claire, Wisconsin and Pittsfield, Massachusetts. The information was recorded weekly, starting in January 2001 and ending in December 2012. Besides recording purchasing decisions, the panel contains household demographics such as income, education, family size, and race. Table 1.5 presents demographics for the whole panel. On average, each region is similar in terms of income and percentage of white households. However, some differences exists in the number of education years in the household as well as family size.

The main goal of using micro-data is to allow for observed heterogeneity among households. Figure 1.1 histogram shows the spread of income brackets in the year 2006. As desired, each bracket has a considerable amount of density. Moreover the distribution shape resembles a typical income distribution for the United States population.

Table 1.5: Average Demographics

| | 2001 | 2006 | 2012 |
|---|---|---|---|
| | | Eau Claire | |
| Income | \$35,000 to \$44,999 | \$35,000 to \$44,999 | \$35,000 to \$44,999 |
| White | 97 Percent | 98 Percent | 98 Percent |
| Education | 12 Years | 12 Years | 13 Years |
| Family Size | 3 Persons | 2 Persons | 3 Persons |
| Households | 2,117 | 2,192 | 1,366 |
| | | Pittsfield | |
| Income | \$35,000 to \$44,999 | \$35,000 to \$44,999 | \$45,000 to \$54,999 |
| White | 94 Percent | 98 Percent | 96 Percent |
| Education | 12 years | 14 years | 15 Years |
| Family Size | 3 Persons | 3 Persons | 4 Persons |
| Households | 2,908 | 2,020 | 1,228 |

Income is the combined family yearly income before taxes.
Source: IRI Academic Data Set.

### 1.5.1 The Facial Tissue Industry

The western facial tissue industry started when a company in Neenah, Wisconsin introduced a disposable tissue in 1924, known as Kleenex. Originally tissues were meant to remove makeup and were not substitutes for the common handkerchief. Around that time some research suggested that the common handkerchief transmitted diseases, including the cold virus. By advertising this fact, Kleenex became the perfect safe substitute of the handkerchief. The company was bought by Kimberly-Clark in 1955, and since then they have been the market leaders in the industry. The brand Kleenex became so popular in western countries that the word Kleenex is accepted as a synonym for facial tissue.[12] Therefore Kleenex is a perfect candidate for a reference product.

During the period studied in this research, facial tissue industry composition contained a small number of firms that produced a large number of differentiated products. Table 1.6 presents the number of products offered by firms over time. As shown, this number changes

[12]For example, The Royal Academy of the Spanish Language accepts Clínex (which is how Kleenex sounds like in Spanish) as a correct word for facial tissue.

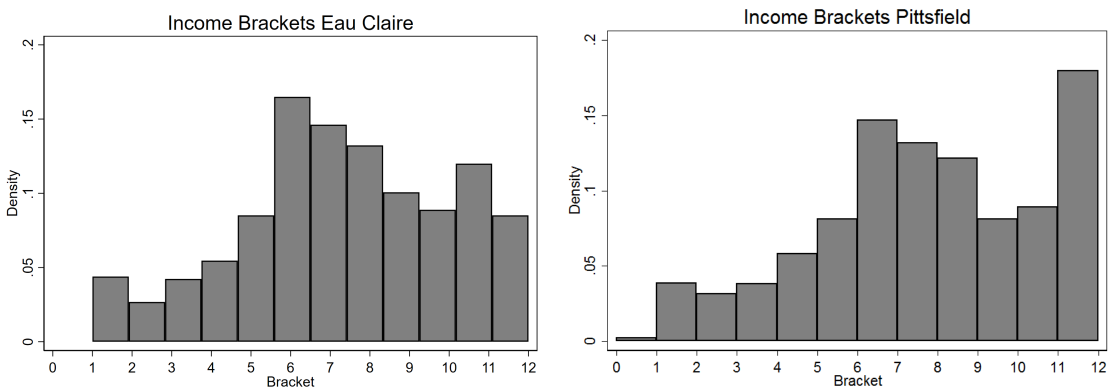


Figure 1.1: Income Brackets for 2006
The income brackets are defined as follows: 1=0 to $ 9,999; 2 = $10,000 to $11,999; 3 = $12,000 to $14,999; 4 = $15,000 to $19,999; 5 = $20,000 to $24,999; 6 = $25,000 to $34,999; 7 = $35,000 to $44,999; 8 = $45,000 to $54,999; 9 = $55,000 to $64,999; 10 = $65,000 to $74,999; 11 = $75,000 to $99,999; 12 = $100,000 and greater.
Source: IRI Academic Data Set

Table 1.6: Number of Products

| | Pittsfield | | | Eau Claire | | |
|---|---|---|---|---|---|---|
| | 2001 | 2006 | 2012 | 2001 | 2006 | 2012 |
| Kimberly-Clark (Kleenex) | 19 | 18 | 22 | 21 | 24 | 21 |
| Procter and Gamble (Puffs) | 14 | 17 | 14 | 14 | 22 | 13 |
| Irving Tissue Converters (Scotties) | 9 | 9 | 11 | 7 | 5 | 5 |
| Private Label | 11 | 14 | 22 | 8 | 14 | 13 |
| Other | 6 | 7 | 10 | 7 | 8 | 4 |
| Total | 59 | 65 | 79 | 57 | 73 | 56 |

The Other category contains combined information of firms with less than 1 percent of market share.
Source: IRI academic dataset.

across years and cities, however Kimberly-Clark dominates the industry in terms of number of products. In a given store, each firm only produces one brand. Figure 1.2 shows the market shares by top brands over time. As seen in the graph, Kleenex, on average, has more than 40 percent of the market, and in Eau Claire more than 60 percent in the first years. In early years, the brand Puffs was the closest competitor to Kleenex, whereas in later years the brands produced in retail stores became more popular. Scotties has consistently come in third place, and less than 1 percent of the market is divided among other brands.

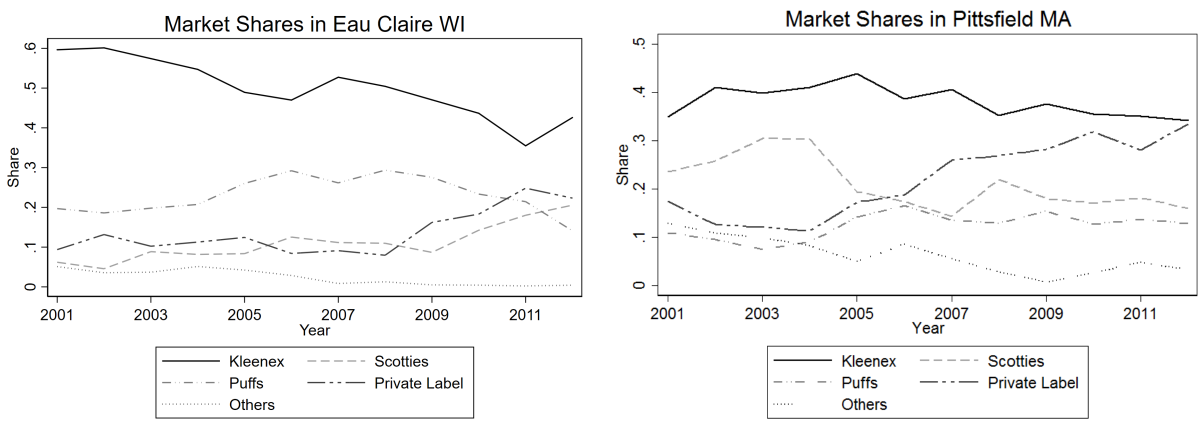


Figure 1.2: Market Shares Across Time
The Other category contains combined information of firms with less than 1 percent of the market share. Source: IRI academic dataset.

Table 1.7: Online Reviews

| | The Brand Mentions Kleenex | Kleenex Mentions the Brand |
|---|---|---|
| | Amazon | |
| Puffs | 5 percent out of 639 reviews | .1 Percent out of 1,225 reviews |
| Scotties | 2 percent out of 62 reviews | 0 Percent of 1,225 reviews |
| Nice! | Unavailable | Unavailable |
| | Walgreen's | |
| Puffs | 7 percent out of 1186 reviews | 2 Percent out of 823 reviews |
| Scotties | unavailable | unavailable |
| Nice! | 20 percent out of 23 reviews | 0 Percent out of 823 reviews |

The reviews were obtained as the time of writing the paper, and I consider the product that has more reviews.

Earlier in the discussion I suggested Kleenex as a reference brand. Since the reference product acts as a standard for comparison, it would be natural to observe that Kleenex as a brand is mentioned often when consumers review products. Unfortunately the IRI academic dataset does not ask households to review products. As a proxy for the missing reviews, Table 1.7 presents online reviews made in Amazon and Walgreen's. Note that in the Amazon's case no private label exists. Walgreen's, on the other hand, sells Nice! but does not sell Scotties. In each category Kleenex is mentioned more often than competition, supporting the idea of a reference brand.

Table 1.8: Shares of Product Characteristics

| | Average on All Markets | Standard Deviation |
|---|---|---|
| White | 0.87 | 0.3383 |
| 2 Plies | 0.83 | 0.3329 |
| Top Dispenser | 0.81 | 0.3927 |
| # of Sheets | 166.40 | 104.9780 |
| Aloe, Lotion or Antiviral | 0.12 | 0.3310 |
| Rectangular Box | 0.92 | 0.2592 |
| Cubic Box | 0.07 | 0.2504 |
| Plastic Wrap | 0.01 | 0.0718 |

All variables are dummy variables, except the number of sheets in the product
Source: IRI Academic Data Set.

Each brand produces different products. For the purpose of the model, a product is defined by a combination of observable characteristics, brand, and unobservables. Table 1.8 summarizes the available observable characteristics in the dataset, and also shows how often a product with that characteristics has been purchased. As expected the most popular characteristics are the rectangular box and the color white for the tissues. On average the number of sheets is 166, although considerable dispersion exists. As noted in previous research, price variation is essential when estimating demand systems. Figure 1.3 shows how prices vary in my data. This observed variation was obtained by defining the market as a quarter of a year and a city. This is the same market specification used in Nevo (2001), and will be used for estimating the model.

So far I have considered Kleenex as the reference brand, however it is necessary to choose which Kleenex product is the reference one. One possibility is to choose the basic product that the company offers. The other possibility is to choose the most popular. Fortunately in this case the basic product coincides with the most popular one. This product is the rectangular Kleenex box with a top dispenser which contains plain white tissues formed by 2 plies. This product has been offered containing different number of sheets. It contained 160 sheets from the years 2001-2005 (65 percent of the all sizes were 160). The company decided to eliminate the 160 sheets in 2006 and introduce a product with 200 sheets instead.

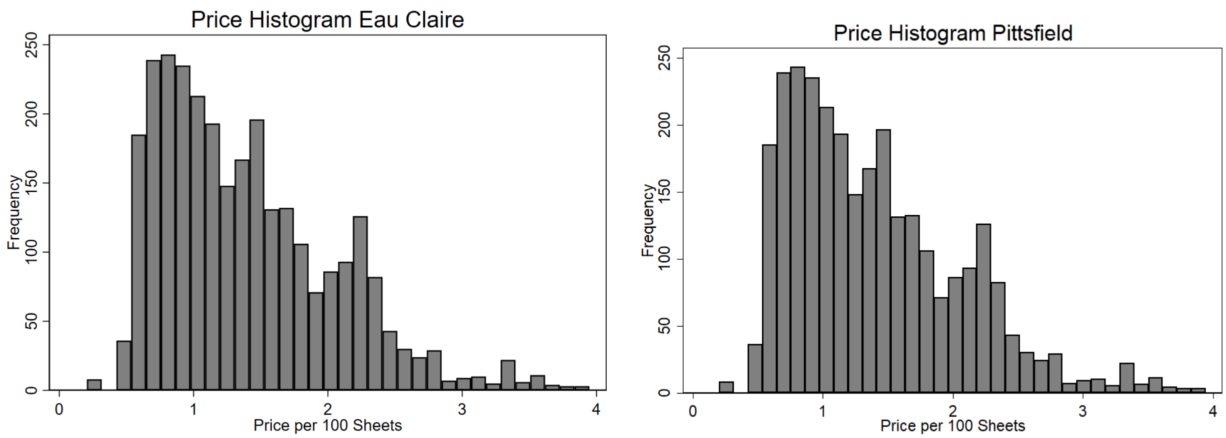


Figure 1.3: Price Variation
Quarterly observed prices, from January 2001 to December 2012.
Source: IRI Academic Dataset.

Table 1.9: Correlation between market shares and the reference product distance

| | All Brands | Excluding Kleenex |
|---|---|---|
| Correlation | -0.3153 | -0.2277 |

The distance is the Euclidean distance with respect to observable characteristics of the reference product.
Source: IRI Academic Data Set.

As seen in the identification section, such changes allows me to identify the R-D model. Before estimating the model, I present in Table 1.9 the correlations between the market shares and the distance with respect to the reference product. Although we cannot asses a causal direction, it shows evidence that being far from the reference product is correlated with having lower market shares.

## 1.6 Empirical Application: Results

The results presented in this section come from estimating several demand models across 2 cities and 48 quarters (such that, in total, the model considers 98 markets). Overall 4,833 inside products exist, with only 5 brands. Each of these models use BLP instruments to account for the endogeneity problem of prices.[13] Since the reference product's characteristics enter in all other products indeirect utility, BLP instruments need to be modified to satisfy the exogeneity retriction. To do so, the characteristics of the reference product are excluded as instruments, both in the sum of the characteristics of other products being produced by the firm, and for the sum of the characteristics produced by other firms. Moreover, as suggested by Nevo (2001), brand dummies are included for all models to account for the time fixed endogenety.

Table 1.10 starts the discussion by presenting the Logit results. Even when the Logit models capture unrealistic substitution patterns, we can still see the previously derived insights from the R-D model. Both OLS and Logit models estimate a larger price coefficient (in absolute terms) than their R-D counterparts. Even when on average the own price elasticities are similar for both type of models (1.71 IV and 1.75 IV Logit), their predictions drastically differ with respect to the reference product price position. In particular, the average elasticity for those products that are less expensive than the reference is -0.33 for the R-D Logit, and -0.70 for the IV Logit model. For products that are more expensive, the predicted elasticities are -2.10 and -1.97.

The estimated values for the full R-D model and the BLP model are presented in the following two tables. Table 1.11 contains the results for the mean and random coefficients. At first glance, it is possible to see that most of the mean utilities have the same direction and similar magnitude in both models. However, as in the Logit model, the mean price coefficient ($\bar{\alpha}$) is larger in BLP. This fact is also present in the estimated values of the random coefficient $\alpha^u$. Since the R-D model has an additional degree of freedom to accommodate changes in prices, it is conceivable to expect that BLP will rely on greater unobserved heterogeneity to accommodate the same data. Note that the coefficient which captures loss aversion with respect to the distance is significant and its magnitude is comparable to the the rest of the characteristics magnitude. Even when the reference product comes in a rectangular box,

[13] The instruments on BLP are the sum over the characteristics produced by the firm, and the sum on the characteristics produced by the competition.

Table 1.10: Logit Results

| | OLS | OLS R-D | IV | IV R-D |
|---|---|---|---|---|
| Price per 100 sheets | -0.5381 | -0.2530 | -1.022 | -0.5043 |
| | (0.0212) | (0.056) | (0.0531) | (0.2972) |
| Size | -0.0495 | -0.0218 | -0.1396 | -0.1503 |
| | (0.0124) | (0.003) | (0.0954) | (0.001) |
| White | -0.3814 | -0.1362 | 0.1396 | 0.1025 |
| | (0.1680 ) | (0.0785) | (0.0954) | (0.1022) |
| 2 Plies | 0.2056 | 0.3029 | 0.4479 | 0.3899 |
| | (0.0549) | (0.0563) | (0.0642) | (0.0715) |
| Top Dispenser | 0.3985 | 0.4301 | 0.4128 | 0.3496 |
| | (0.0586 ) | (0.0604) | (0.0627) | (0.0687) |
| Aloe | -0.3991 | -0.2562 | -0.3350 | -0.2908 |
| | (0.0591) | (0.0624) | (0.0654) | (0.0710) |
| Cubic Box | 0.3883 | 0.8650 | 0.3696 | 0.3550 |
| | (0.1010) | (0.0980) | (0.1251) | (0.1447) |
| Rectangular Box | 0.6200 | 0.8419 | 0.3844 | 0.3053 |
| | (0.0841) | (0.0797) | (0.1058) | (0.1247) |
| Loss Aversion Price ($\lambda$) | | -0.76648 | | -0.5878 |
| | | (14.250) | | (0.3670) |
| Loss Aversion Characteristic Space ($\gamma$) | | -0.2586 | | -0.1088 |
| | | (0.0460) | | (0.0497) |

BLP instruments. Brand dummy variables included.
Source: IRI academic dataset.

Table 1.11: Random Coefficient Models

| | Mean Parameters | | Deviation Parameters | |
|---|---|---|---|---|
| | R-D | BLP | R-D | BLP |
| Price per 100 sheets | -1.4380 | -3.8512 | 2.531 | 7.152 |
| | (0.4720) | (0.6891) | (1.987) | (2.187) |
| Price Loss Aversion ($\lambda$) | -1.07 | - | - | - |
| | (0.1412) | - | - | - |
| White | -0.1201 | -0.2201 | 0.278 | 0.3045 |
| | (0.0004) | (0.0010) | (0.003) | (0.0954 ) |
| 2 Plies | 0.3210 | 0.8580 | 0.132 | 0.1868 |
| | (0.1336) | (0.0729) | (0.0126) | (0.0234) |
| Top Dispenser | 0.4256 | 0.5560 | 2.142 | 1.1616 |
| | (0.1853) | (0.0921) | (0.5013) | (0.0159 ) |
| Size | -0.2514 | -0.1719 | 1.62 | 0.2009 |
| | (0.0011) | (0.3067) | (0.0520) | (0.4847) |
| Aloe | -0.3727 | -0.2227 | 3.195 | 1.3505 |
| | (0.211) | (0.2127) | (0.0150) | (0.0319) |
| Cubic Box | 1.324 | 0.461 | 0.199 | 1.7392 |
| | (0.2142) | (0.0273) | (0.0219) | (0.7370) |
| Rectangular Box | 0.818 | 0.9009 | 0.2812 | 0.7422 |
| | (0.0285) | (0.0095) | (0.0376) | (0.9952) |
| Loss Aversion ($\mu$) | -0.3520 | - | - | - |
| | (0.0151) | - | | |
| Sup $\Psi$ | 55 | > | Critical Value | 14.23 |

BLP instruments. Brand dummy variables included. Integral simulated with 1000 draws. Tests based on critical values provided by Andrews (2003).
Source: IRI academic dataset.

premiums exist for the cubic box. The last line of Table 1.11 presents the value of the structural break hypothesis. As expected, the data rejects the null of a common reference product absence.

Table 1.12 contains the results for the interaction terms with micro-data. Roughly the same findings are maintained in both models, noting that higher income is correlated with less disutility on prices. Wealthier families also tend to prefer tissues with add-ons such aloe, and more plies. Family size has a negative relationship with respect to the size and prices.

Table 1.12: Micro data Interaction Terms

| | Interaction Income | | Interaction Family Size | | Interaction Education | |
|---|---|---|---|---|---|---|
| | R-D | BLP | R-D | BLP | R-D | BLP |
| Price per 100 sheets | 22.050 | 15.050 | -0.851 | -0.851 | - | - |
| | (3.5212) | (2.1215) | (0.0121) | (0.1014) | - | - |
| White | - | - | -0.5214 | -0.3321 | - | - |
| | - | - | (0.021) | (0.0156) | - | - |
| 2 Plies | -0.002 | -0.001 | - | - | - | - |
| | (0.0014) | (0.0016) | - | - | - | - |
| Top Dispenser | - | - | - | - | -0.2832 | 0.5321 |
| | - | - | - | - | (0.0142) | (0.0425) |
| Size | -0.21 | -0.15 | 2.55 | 1.07 | - | - |
| | (0.1210) | (0.1210) | (0.7251) | (0.5151) | - | - |
| Aloe | 1.00 | 1.25 | - | - | -0.112 | -0.1842 |
| | (0.0663) | (0.3343) | | | (0.0043) | (0.5521) |
| Cubic Box | 2.03 | 2.03 | - | - | - | - |
| | (1.1820) | (.9988) | | | - | - |
| Rectangular Box | -0.021 | -0.0093 | - | - | - | - |
| | (0.0174) | (0.0186) | | | | |

BLP instruments. Brand dummy variables included. Tests based on critical values provided by Andrews (2003). Integral simulated with 1000 draws.
Source: IRI academic dataset.

Table 1.13: Counterfactual

| | Change Market Share | Difference in Characteristics |
|---|---|---|
| Kleenex reference product | -25% | None |
| Kleenex closest competing product | -10% | Size |
| Kleenex farthest competitor | 2% | All but brand |
| Puffs closer competitor | -5% | Brand |
| Puffs farthest competitor | 35% | All |
| Scotties closer competitor | 2% | Brand |
| Scotties farthest competitor | 5% | All |
| Private Label closest competitor | -33% | Brand |
| Private Label farthest competitor | -2% | All |

Counterfactual assuming $\lambda = \gamma = 0$. Integral simulated with 1000 draws.
Source: IRI academic dataset.

So far, I have shown evidence that R-D preference matters for the study of the facial tissue industry. Now I plan to show evidence for how much it does. To that end, I predict what the changes in market shares would be if consumers did not have reference dependent utility, that is if $\mu = 0\,\gamma = 0$. Table 1.13 presents the percentage change of a subset of products. Note that the reference product loses 25% of its market share, but it is also the case that close competing products lose market share. The biggest loser is the identical private label, which before was gaining market share by offering a similar product to the reference, but cheaper. Puffs, on the other hand, seems to enjoy this change.

## 1.7 Discussion

This research has presented a model that allows the empirical economist to estimate discrete choice models of demand with reference dependent utility. Accounting for reference dependent utility is extremely relevant in industries where a common reference product exists among consumers. In these industries, ignoring R-D preferences can bias the estimation results in two ways. First, the estimates may exaggerate how consumers react to prices. This phenomenon occurs because standard models will confound the price sensitivity with loss aversion. Identifying these effects separately is important when trying to model firm's price decisions, or when conducting policy counterfactuals. Second, and perhaps most important, standard models assume that market shares are hindered when facing close substitutes. On the other hand, the R-D model permits competing products to enjoy benefits for being closer to the reference product. This fact might lead the researcher to opposite conclusions when trying to predict changes in market shares. This is how the R-D model provides richer substitution patterns, which as shown in the identification results, can be tested.

An example of an industry with a common reference product is the facial tissue industry. A consumer that purchase a product that is different from the basic Kleenex product pays an additional utility cost. This cost provides Kleenex an advantage, and in fact, my estimates suggest that without it their market share would go down by 25 percent. Since R-D preferences benefit the reference product's close substitutes, competing products would be also affected if the loss and gain component disappeared. With respect to that, my estimates suggest that private labels that imitate Kleenex would lose market share as well (33 percent). Puffs, on the other hand, could have a larger market share if consumers were not reference dependent. These findings suggest that firms have incentives to advertise to position themselves as the reference product. In fact, accounting for R-D preferences when modeling supply decisions could be possible avenues for new applied research.

It is natural to expect that most industries contain heterogeneity among consumers with respect to their reference product. While this research has pointed out that the R-D model still produces richer substitution patterns, the difference with respect to standard model becomes less important as the heterogeneity among consumers increases. In other words, we shouldn't be too worried on missing substitution patterns in industries like the automobile or the ready-to-eat cereal. However, interesting insights might result from estimating an R-D model in the soft drinks industry, or in the over-the-counter painkillers.

Finally, the general R-D model allows for the possibility of a stochastic reference product. While more research is needed to provide a formal identification argument, its an appealing model for several reasons. One of them is relaxing the burden on the researcher to specify the reference product in an industry. Second, it incorporates the well-documented notion that consumers form beliefs on prices with respect to previous purchases. Most likely a consumer that pays $ 500 dollars for a domestic flight will experience a loss sensation if she had previously only booked cheaper flights.

The tools provided in this paper are first steps into building connections between the findings from behavioral economics and those from the structural study of markets. It opens a research agenda that should complement both fields.

# Chapter 2

# Present Bias and Self Control: Structural Estimation from a Mortgage Market in Mexico

It is not surprising that long term decisions require deeper levels of rationality by individuals. Retirement savings, mortgages, and credit loans are examples where agents wrongly estimate their own future payoffs. From a behavioral economics perspective, this phenomenon has been explained by bounded rationality, reweighting of outcome probabilities, loss aversion, present bias and lack of self-control.[1] Although most of these theories have been tested in different applications, either in an experimental framework or in the field, there is a shortage of non experimental empirical work to credit and financial markets. Therefore, some implications might not be applicable for policy making, regulation or forecasting market behavior. It is no coincidence that there is a lack of non-experimental research for credit markets, as noted by de Clippel and Rozen (2014), as testing theories that violate some rationality assumptions require large data sets of outstanding quality, which for most cases

[1] A good survey can be found in DellaVigna (2009), Oshry (2006) or Robson and Samuelson (2009) .

will require individual proprietary data. Nevertheless, the growing availability of individual data by state owned firms presents a good opportunity for doing so.

By looking into a real credit market, the present paper aims to test one of the most popular theories on present bias and self-control: Quasi-Hyperbolic Discounting (QHD).[2] An individual that exhibits QHD places additional weight on present payoffs over long run payoffs, i.e. he is present biased. Consequently, naive present biased individuals borrow excessively and often fail to repay debt [**?** ]. If banks or any other credit institutions fail to take the effects of QHD into consideration, they might underestimate the risk of default. In developing countries where the credit market is relatively small this could be extremely problematic.

To test QHD I analyze the debt repayment behavior of state workers in one of Mexico's public mortgage institutions. Despite the fact that Mexico's mortgage market contains private institutions (such as banks and a small number of housing providers), the public institutions account for 70 percent of the market share (SHF 2014). Among the public sector there exist only two institutions, one for state workers (20 percent of the market share) and one for non-state formal workers (50 percent of the market share). One notable difference between these public agencies is their repayment schemes; in particular their definition of mortgage debt. The state workers have to pay a fixed amount of minimum wages whereas the non-state workers have to pay a fixed amount of Mexican pesos. Facing a mortgage debt measured in minimum wages implies that the real debt increases every time there is a minimum wage increment.[3] Thus a state worker has additional incentives to repay sooner rather than later, but present bias might preclude this behavior.

In this novel data I observe a random sample of state workers over a long period of time, in which it is possible to see how they decide to repay their mortgage. Every worker faces a mortgage contract that fortnightly discounts 30 percent of her base salary as payment, and

[2]While others theories might be interesting as well, QHD has been studied deeper in theoretical and experimental models since its first appearance in 1997 in Laibson's paper.

[3]In Mexico the minimum wage is raised every year, during my sample period said raise was above the inflation rate

she has the option of making extra contributions in order to repay her debt faster. This option of making additional payments allows me to build a dynamic discrete choice model, in which the worker decides every period whether to make an additional contribution or not. It is worth mentioning that I am assuming that the amount of an extra payment is optimal, in this way I only consider the decision of making an extra payment.

Applying the methodology designed by Fang and Wang (2015) I am able to identify present bias, and in particular, if the agents are aware of having this bias. This is achieved by structurally estimating three discount factors (long run, quasi-hyperbolic, and degree of naiviety). Identifying all discount factors relies on using exclusion variables. These variables affect the transition probabilities but not the difference of instantaneous payoffs.[4] Distinguishing if individuals are sophisticated or naive is important when making public policy recommendations. For example, a sophisticated agent with present bias who has access to an illiquid asset will be able to smooth consumption [**?** ], whereas the naive agent might struggle to do so. Although the present bias hypothesis can be studied from a reduced form approach (see Kuchler (2015)), the structural estimation allows me to conduct simulations of several public policies after recovering the relevant parameters.

For the purpose of this paper Mexico is quite interesting, since it is not yet a fully developed nation and the credit market is relatively small. To my knowledge this is the first paper that uses mortgages to recover the structural parameters of a dynamic discrete choice model. Therefore, this research contributes to the literature of behavioral economics, especially to that which uses structural empirical methods, and it also provides public policy recommendations for Mexico's case. The rest of the paper is organized as follows: Section 1 relates my research to the current literature; the mortgage contract and data are described in Section 2; Section 3 introduces the model; Section 4 explains the econometric strategy as well as the identification argument; the main results and simulations are presented in Section 5; and a closing discussion is provided in Section 6.

[4]For example, gender affects the probability of getting a raise, but it doesn't affect the difference of utilities between making a contribution or not.

## 2.1 Relationship with the Literature

The first model with present bias captured by QHD's preferences was introduced in Laibson (1997). Some of the key features of the QHD consumption-savings model are: aggregate consumption tracks income, present biased individuals save less than dynamically consistent individuals, and in some situations, defaulting might be optimal. An important prediction of this model is that if consumers have access to an illiquid asset (i.e., a commitment device) present biased individuals are able to smooth consumption.[5] Without modeling QHD explicitly, this theoretical feature has been well documented in several experiments. For example, by offering an illiquid saving program, Thaler and Benartzi (2004) observed an increase of 10 percent in individual savings. Similarly but varying the intensity of the commitment, Dupas and Robinson (2013) offered savings accounts to poor people in Kenya. By making available safe accounts, the authors observed an increase in savings. Moreover health emergencies were covered thanks to iliquid characteristics of some of the accounts. When social pressure was added to the mix, the saved amount increased substantially. Further discussion and classification of commitment devices can be found in Bryan et al. (2010). The authors remark that psychological punishments, enormous penalties fees, and blocking access to accounts might have different effects on how individuals allocate their resources. Importantly the mortgage contract analyzed in this research, discounts 30 percent of the agents base salary, imposing some sort of commitment.

The first empirical research that modeled QHD explicitly comes from representative agent models with aggregate variables. For instance Angelatos et al. (2001) showed that a QHD model approximates aggregate data patterns better than an exponential discount model. By calibrating the discount parameters, the authors explain why households have very little liquid assets and at the same time maintain a substantial amount of debt in their credit cards. As shown in the work of Laibson et al. (2017), the former findings are robust to modern

[5] While Laibson's model only considers illiquid assets as commitment devices, it is possible to find several other devices with the same effect.

econometric techniques such as the simulated method of moments. Moreover the authors show that a QHD model can replicate additional stylized facts from the U.S. data, such as retirement wealth accumulation. Combining both aggregate and individual data, Dellavigna and Paserman (2005) tested theoretical implications of present bias in a job search model. Most of the unemployment data facts are only consistent with the predictions of their QHD model, rejecting the exponential hypothesis. As it is common in structural work, aggregate data questions the identification of the relevant parameters. It will be shown later that having individual data provides a cleaner way of estimating the discount parameters.

As is common in behavioral economics several papers conducted experiments to explore present bias and in particular QHD. In a university experiment Ariely and Wertenbroch (2002) found that present bias explains why individuals fail to finish their homework on time. It is not just about procrastination, present bias matters in credit markets. Meier and Sprenger (2010) found that present biased individuals are more likely to borrow more than dynamically consistent individuals. This finding comes by combining information from a choice experiment (to identify time preferences) and real credit card usage data. Jones and Mahajan (2015) carried out a field experiment in which they were able to identify both the long run discount and the present bias discount in a QHD structural model. Their experiment varied exogenously the time and amount of money an individual would receive in a real tax refund. Such variation can hardly appear outside the experimental framework, and therefore identification of structural parameters requires additional assumptions.

Among the papers that use non-experimental individual data, the reduced form approach seems to be more popular. Using a gym attendance data set, DellaVigna and Malmendier (2006) tested some predictions of a model with standard time preferences and a model with present bias. They found that standard intertemporal preferences do not capture the behavior of the data, while QHD models do. The research of Kuchler (2015) analyzed credit card data at an individual level. She had access to a software designed to help individuals repay their credit card debt. By observing individual transactions as well as the individual's desired level of debt, she was able to distinguish between individuals that were not present biased from those who were. Among those who had present bias, she distinguished between those individuals that believed they discounted in an exponential way from those that were aware of being present biased (i.e., naive and sophisticated agents). She rejected exponential discount and found that naive present biased individuals are very likely to default. One drawback of the reduced form approach is the lack of counterfactuals, and thus the inability of simulating public policies.

The structural models were avoided due to the complexity of identifying the relevant discount parameters. Rust (1994) showed that discount factors in a standard dynamic discrete choice model are generically not identified. By introducing exclusion restrictions Magnac and Thesmar (2002) were able to identify the exponential discount factor. In the same way as Hotz and Miller (1993) , the paper of Fang and Wang (2015) solves the identification issue by using exclusion restrictions similar to those by Magnac and Thesmar (2002). The present paper closely follows Fang and Wang (2015) and, like them, I am able to identify the parameter that characterizes the long run discount, the QHD, and the agent's naivety degree.

Several structural papers that rely on parametric assumptions or some exogenous variations are also worth mentioning. Fang and Silverman (2009) implemented a finite horizon dynamic discrete choice model and analyzed a welfare program. Their identification results rely on using a large panel data set with a clear final period. They showed that there are no observationally equivalent set structures with exponential discounting that yield QHD predictions.[6] However they could not identify the naivety degree. Paserman (2008) analyzed a job search model in which identification is achieved by the large variation in unemployment spells and accepted wages. Although he rejected exponential discounting, his results depend on the model's specific structure and on the functional form of the wage distribution.

[6]They used National Longitudinal Surveys from 1979.

Finally some axiomatic derivations to test present bias have been made in the paper of Echenique et al. (2017). Using experimental data their test showed that half of their sample was consistent with exponential discounting and only a few more were consistent with QHD. The test consists in a non-parametric revealed preference approach, nevertheless to implement it several choice decisions need to be observed. This escapes the nature of the data used in this paper.

## 2.2 Background and Data description

The data come from a de-centralized Mexican public mortgage institution, *Fondo de la Vivienda del Instituto de Seguridad y Servicios Sociales para los Trabajadores del Estado* (State Worker's Social Security and Services Housing Fund), hereafter FOVISSSTE. FOVISSSTE was created in 1972 in order to meet the housing credit demand for most of Mexico's state workers (approximately 2.3 millions workers). The state workers affiliated to FOVISSSTE are either federal, state and municipal governments, as well as public universities and local agencies. FOVISSSTE offers programs with different credit schemes, I will use a sample from the most important one, the "Traditional Credit Scheme".[7]

The Traditional Credit Scheme works like this: Every year FOVISSSTE allows workers to register in a lottery in which some fixed number of mortgages are randomly assigned (in 2013, 45,000 mortgages were given). If the worker is selected, she has to accept a mortgage contract that offers no more than U.S. $58,852.[8] The mortgage's value depends on the worker's base salary, and it should be fully repaid in 30 years. After signing the contract the worker chooses a house from a menu of external companies affiliated to FOVISSSTE. Once she selects a house, FOVISSSTE transfers the money to the external company, and when she receives her house she starts repaying the mortgage. Each fortnight FOVISSSTE

[7]The Traditional Credit Scheme comprehends somewhere around 62 percent of their total mortgages.

[8]As of the writing, the exchange rate is approximately $16 pesos per dollar.

Table 2.1: Time invariant statistics

| Variable | Mean | Standard Deviation | Min | Max |
|---|---|---|---|---|
| Gender (Female==1) | 0.61 | 0.4872 | - | - |
| Age at 2000 | 38.05 | 7.8345 | 21 | 62 |
| Interest Rate | 5.42 | 1.4566 | 4 | 6 |
| Mexico City | 0.43 | 0.4959 | - | - |

Random sample of 410 individuals that started to repay their debt in Jan 2000.

charges a fixed interest rate. The assigned interest rate depends on her base salary (it could go from 4 percent to 6 percent yearly), and it is applied to the current outstanding debt.

I consider a random sample of 410 workers that start repaying their mortgage from the first week of January 2000 to the last week of June 2015. Every two weeks I observe the transactions that both the individuals (payments) and FOVISSSTE (charges) have made. Therefore I have a panel of $T = 348$ periods with potentially 142,000 observations.[9] If the worker is employed I observe her base salary, otherwise I assume the worker has no income. It's worth mentioning that I am not able to find other sources of income which potentially can bias my results, hereafter I will use wage, salary and income indistinctly.

Time invariant statistics of my sample are presented in Table 2.1. When the individuals enter the panel their age ranges from 21 to 62 years old, where the average worker is 38 years old. Among the workers, 43 percent live in Mexico City and 61 percent of my sample are women. The average interest rate is 5.42 percent.

The repayment scheme works like this: FOVISSSTE fortnightly deducts 30 percent of the worker's base salary. If the worker is unemployed, it is possible to make contributions to the mortgage, however in the data this is a rare scenario.[10] Every two months the agency where she works makes an extra 5 percent contribution of her base salary. Additional payments can be done by the worker every fortnight, these payments can be of any magnitude. Recall that the mortgage is measured in minimum wages, whenever there is an increment in the

[9]My data set consists on 110,356 observations, since any agent who repays the the full mortgage exits the panel.

[10]Only .3 percent of my sample made a contribution when unemployed.

Table 2.2: Mortgage Debt Statistics

| Year | Mean | Median | S.D. | State Workers |
|---|---|---|---|---|
| 1 | 23,069 | 21,952 | 6,605 | 410 |
| 7.5 | 16,581 | 15,142 | 6,063 | 370 |
| 15 | 9,989 | 8,548 | 7,014 | 316 |

Random sample of 410 individuals that started to repay their debt in Jan 2000. Mortgage debt is measured in real U.S. dollars, in which 1 Mexican peso equals 16 dollars. The year 2015 is used as base year.

minimum wage the oubstanding real debt increases. In my sample period every January the minimum wage increased on average 7 percent.[11]

### 2.2.1 Mortgage Debt

Before analyzing some insightful characteristics of the mortgage debt it would be useful to introduce its law of motion. Let $S \subset T$ be the set of fortnights when the additional 5 percent contribution occurs, and let $W \subset T$ be the set of fortnights in which the minimum wage increment takes place. Then in every fortnight $t \in T$, whenever $m_{t-1} > 0$ the law of motion of the mortgage debt is captured by:

$$m_t = [1 + \Delta_t \mathbf{1}\{t \in W\}]\,[(1+r)m_{t-1}] - .3y_t - a_t - .05y_t \mathbf{1}\{t \in S\} \tag{2.1}$$

where $m_t$ is the value of the mortgage at time $t$, $y_t$ is the base salary of the worker at time $t$, $r$ is the interest rate , $a_t$ is the additional contribution and $\Delta_t$ is the minimum wage's increment.

Table 2.2 presents mortgage debt statistics at different periods of my sample. On average both the mean and median debt decreased over time, which is expected given the nature of how the debt is repaid. Nevertheless it is possible to observe that some workers finished paying their mortgage within 15 years. Also note that the variance is relatively higher as time passes. In other words my sample is heterogeneous regarding debt repayment. One

[11] From January 2000 to January 2015 the minimum wage has increased 82 percent.

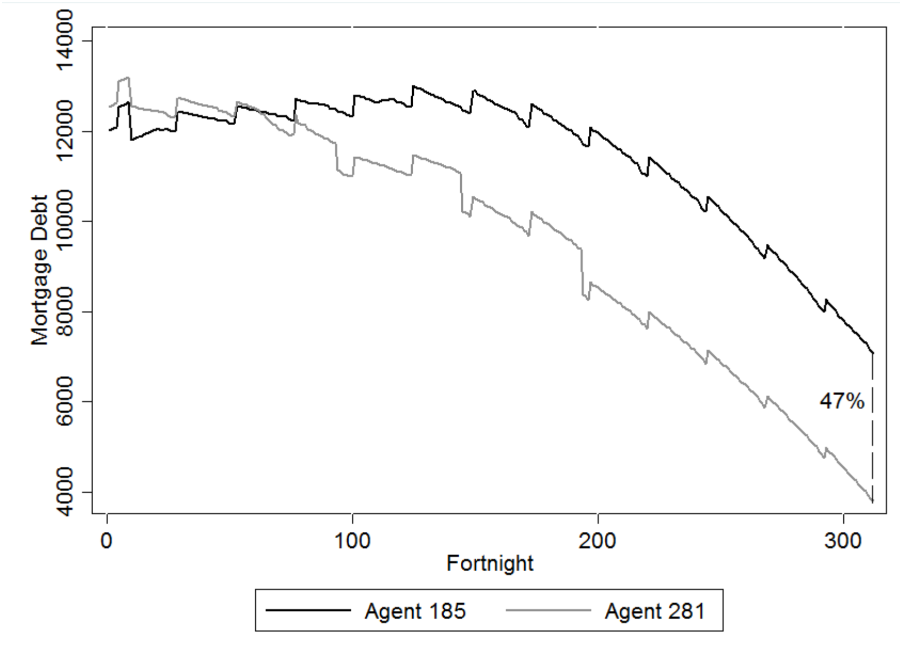


Figure 2.1: Individual Mortgage Comparison
Individual mortgage debt across time. All variables are measured in U.S. dollars, where $16 Mexican pesos equals $1 dollar.

possible explanation is that some workers experienced meaningful increments in their wage over time, it also could be that some workers decided to pay more than just 30 percent of their income.

To illustrate how different repayment behavior changes the mortgage debt over time I show the comparison between two state workers in figure 2.1. Both state workers started with very similar mortgage debt, yet there is a 47 percent difference after 15 years. In this figure it is possible to observe both the minimum wage increments (cyclical upward jumps) and some additional contributions (downward jumps), the workers' income is captured by the slope of the series. My hypothesis is that because the debt increases every year and, since it is possible to lose the job in such a long period span, a worker has incentives to repay as soon as possible. Nevertheless present bias can preclude this behavior. Thus if both workers in the series have similar income patterns, the 47 percent difference might be explained by present bias.

Table 2.3: Annual Base Salary Statistics

| Year | Mean | Median | S.D. | State Workers |
|---|---|---|---|---|
| 1 | 5,056 | 4,635 | 2,880 | 410 |
| 7.5 | 7,136 | 5,336 | 6,039 | 370 |
| 15 | 7,043 | 5,536 | 5,847 | 316 |

Random sample of 410 individuals that started to repay their debt in Jan 2000. The annual base salary is measured in U.S. dollars, where 1 Mexican peso equals 16 dollars. The year 2015 is used as a base year.

### 2.2.2 Income

Some standard economic theories predict that income grows when we consider a long period span. Table 2.3 depicts the former fact in the first 7 years, both the median and the average worker experienced real income growth. On the other hand the last 8 years of my sample remain statistically the same. Therefore the hypothesis that individuals should make additional contributions as soon as possible might not be true for the first years, since the worker might be expecting wage increments. Still if income remains constant (as for the last 8 years) , or if it is possible to experience a job loss, an exponential discounter should make early additional contributions.

Figure 2.2 presents a comparison of the same state workers as in last figure, but now comparing income. First note that real wage is considerably stable over time. We can observe some temporary income changes due to penalization at work (i.e., late arrivals or missing days) or small bonus due to extra hours worked that fortnight. Note that both workers have a very similar income pattern with a correlation of .89. In other words, the 47% difference observed in the mortgage debt is not coming from income shocks but from additional payments.

Table 2.4 shows how income varies across different groups. On average male state workers earn more than women, previous studies have shown that Mexico suffers from gender discrimination in terms of wages [**?** ]. Predictably workers older than 40 earn more than young workers. Living in Mexico City, on the other hand, implies having less

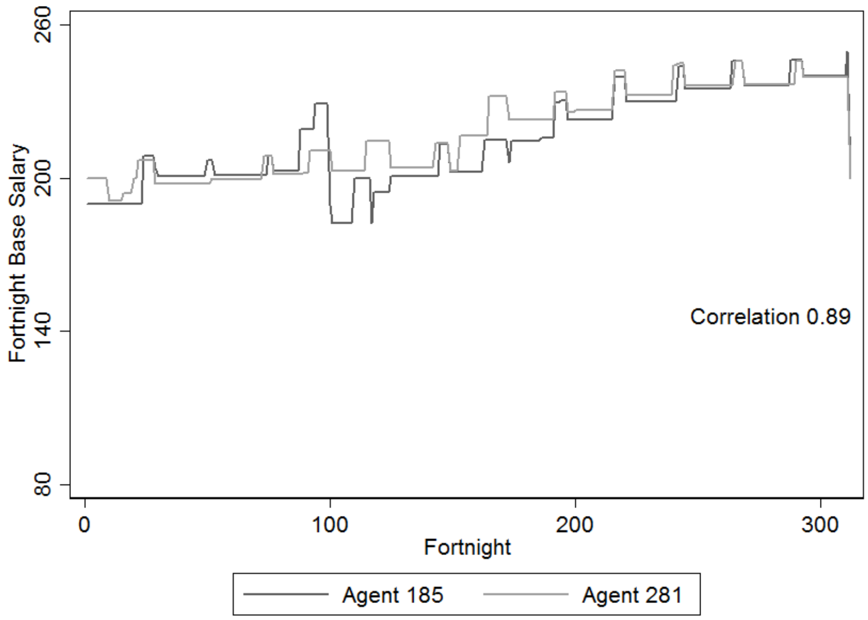


Figure 2.2: Individual Base Salary Comparison
Income across time. U.S. dollars, where 1 Mexican peso equals 16 dollars. Income is considered to be the fortnight base salary.

Table 2.4: Average Income Across Groups

| | Income mean | Standard Deviation |
|---|---|---|
| Female | 6,140 | 6,893 |
| Male | 6,893 | 9,232 |
| Mexico City | 6,051 | 5,958 |
| Rest of the Country | 6,698 | 7,538 |
| Age<40 | 5,636 | 5,617 |
| Age>40 | 7,423 | 8,188 |

Random sample of 410 individuals that started to repay their debt in Jan 2000. All variables are measured in U.S. dollars, where 1 Mexican peso equals 16 dollars. Income is considered to be the base salary a worker earned in a year. The year 2015 is used as a base year.

Table 2.5: Annual Additional Payments Statistics

| Year | Mean | S.D. | Mean Excluding Zero | S.D. Excluding Zero | State Workers | Workers with Extra Payment |
|---|---|---|---|---|---|---|
| 1 | 185.12 | 504.90 | 703.09 | 864.43 | 410 | 95 |
| 7.5 | 135.60 | 604.68 | 1,570.83 | 1,868.91 | 370 | 34 |
| 15 | 158.28 | 956.74 | 2,278.41 | 3,307.59 | 316 | 19 |

Random sample of 410 individuals that started to repay their debt in Jan 2000. Annual payment is measured in U.S. dollars, where 1 Mexican peso equals 16 dollars. The year 2015 is used as base year.

income than in the rest of the country. These variations across groups will be relevant at the time of estimating transition probabilities, which will be detailed later in the econometric specification.

### 2.2.3 Additional Payments

As discussed earlier additional payments can have a large impact on how fast the mortgage debt is repayed. Table 2.5 presents the average additional payment across different years in my sample. Since several workers never made an additional contribution, I also provide the mean that excludes those workers. Overall the largest payments were made during the first years. These early payments might lead some workers to exit my sample. Note that when excluding the zero additional payment the average contribution increases over time. The opposite effect occurs with the frequency of workers. In other words, as time goes we have less workers who decided to contribute, and among them, the payment is higher. As hypothesized earlier, present bias might preclude workers from making early contributions. To have the same effect on the mortgage debt, due to minimum wage increments, latter payments need to be greater than early payments. Workers that never supply additional contributions might not be able to finish paying their mortgage in 30 years, and so, they will enter into default.

## 2.3 The Model

In this section I present a dynamic discrete choice model that substantiates the optimization problem of a state worker with a FOVISSSTE mortgage. Both the model and the identification argument will use the results developed in the paper of Fang and Wang (2015), extensive derivations and proofs can be found in their paper. Before appealing to the discrete choice framework it will be useful to present a continuous choice model. This underlying model motivates how state variables will be defined in the discrete choice approach.

### 2.3.1 The Underlying Model: Continuous Choice

Assume that time horizon is infinite, indexed by $t = 1, 2, 3...$, where every period represents a fortnight. The state worker's preferences are defined over a sequence of consumption and random unobserved preference shocks $U_t : \{c_t,\, \varepsilon_t\}_{t=1}^{\infty} \to \mathbb{R}$. Such time preferences have the same $(\beta, \delta)$-preferences as Laibson (1997).

$$U_t(u_t, u_{t+1}, ...) = u^*(c_t,\, \varepsilon_t) + \beta \sum_{k=t+1}^{\infty} \delta^{k-t} u^*(c_k,\, \varepsilon_t) \tag{2.2}$$

Where $u^*(\cdot)$ is the instantaneous utility, $\beta \in (0, 1]$ captures present bias, referred also as the present bias discount or QHD discount, and $\delta \in (0, 1]$ is the long run discount. Note that if $\beta = 1$ the model becomes the classic exponential discount model.

The state worker faces a mortgage contract that deducts automatically 30 percent of her income. Every period the worker chooses how much to consume and implicitly how much additional payment to make, there are no extra savings in the model. Therefore the budget constraint that the state worker faces every fortnight $t$ is defined by:

$$c_t + a_t \mathbf{1}\{m_t > 0\} \leq .7y_t + .3y_t \mathbf{1}\{m_t = 0\} \tag{2.3}$$

The budget constraint depends on the value of the mortgage debt $m_t$ , and so the worker needs to consider its law of motion defined in equation (2.1). Note that the law of motion relies on the increment of the minimum wage $\Delta_t$, hence the worker is uncertain about the value of her mortgage debt in some periods. In particular let the stochastic process of the minimum wage increment be described as

$$\Delta_t = \Delta_{t-24} + \Omega_t \tag{2.4}$$

Where $\Omega_t$ is an i.i.d. random shock.

The worker is uncertain about her future income as well. Inspired by the former section, the random component will be related to her invariant features. She knows that the stochastic income process is defined as follows:

$$y_t = y_{t-1} + \gamma_t \tag{2.5}$$

where $\gamma_t$ is a random variable that depends on the worker's characteristics, such as gender, age and where the worker lives.

Taking into account equations (2.1), (2.2), (2.3), (2.4) and (2.5) the problem of the state worker can be understood as

$$\underset{\{a_t\}_{t=1}^{\infty}}{Max}\, u^*(.7y_t - a_t - \varepsilon_t) + \beta \sum_{k=t+1}^{\infty} \delta^{k-t} E\left[u^*(.7y_k - a_k - \varepsilon_t) | m_{k-1},\, y_{t-1}\right]$$

$$s.t.$$

$$m_t = \left[1 + \Delta_t \mathbf{1}\left\{t \in W\right\}\right]\left[(1+r)m_{t-1} - .05y_{t-1}1\left\{t \in S\right\}\right] - .3y_t - a_t$$

$$a_t \geq 0$$

The solution to this problem will yield an optimal sequence of consumption and additional payments $\{c_t^*, a_t^*\}_{t=0}^{\infty}$. At the end how much utility the worker gets when making an additional contribution will depend on the values of state variables such as: income $y_t$, the random preference shock $\varepsilon_t$ and the outstanding debt after charges, $M_t \equiv [1 + \Delta_t \mathbf{1}\{t \in W\}]\left[(1+r)m_{t-1} - .05y_{t-1}1\{t \in S\}\right]$. These state variables, and the way they relate to invariant characteristics, motivate the construction of the discrete choice model.

### 2.3.2 The Discrete Choice Model

The final goal of the paper is to obtain estimates of the discount parameters. To do so I will rely on the discrete choice framework. Instead of considering the value of $a_t$, assume that the agent decides whether to make an optimal additional payment or no additional payment at all, $i \in \mathcal{I} = \{0, \mathbf{1}\{a_t = a_t^*\}\}$. In this way the problem is transformed into a dynamic discrete choice model. Now it is possible to define the preferences over the discrete choice $i \in \mathcal{I}$, state variables $x_t \in \mathcal{X}$, and a choice specific preference vector shock $\boldsymbol{\varepsilon_t} = (\varepsilon_{0t}.\varepsilon_{1t})$.

$$U_t(u_t, u_{t+1}, ...) = u_i^*(x_t, \boldsymbol{\varepsilon}_t) + \beta \sum_{k=t+1}^{\infty} \delta^{k-t} u_i^*(x_k, \boldsymbol{\varepsilon}_k) \tag{2.6}$$

Motivated by the underlying model the set of state variables $\mathcal{X}$ contains: the outstanding debt, her income level and some personal characteristics such as age, gender and where does the state worker lives. The choice specific vector $\boldsymbol{\varepsilon_t}$ changes how the state worker feels if she makes an additional contribution. For example if on a given fortnight $t$ the worker has access to a better quality consumption bundle, her happiness level when making an additional contribution will be less than when she has access to her regular consumption bundle.[12] For econometric purposes it is useful to make the following assumption:

**Assumption 1** ***(Additive separability)***: For each $i \in \{0, 1\}$, the instantaneous utilities are given by,

[12] A better consumption bundle could be sales on given products, access to rare products or events that increase the enjoyment of consumption.

$$u_i^*(x_t, \boldsymbol{\varepsilon}_t) = u_i(x_t) + \varepsilon_{it} \tag{2.7}$$

Where $u_i(x_t)$ is the utility's deterministic component from choosing $i$ at $x_t$, and $(\varepsilon_{0t}, \varepsilon_{1t})$ has a joint distribution $G_t$ which is absolutely continuous with respect to the Lebesgue measure in $\mathbb{R}^2$.

It is well known that the $(\beta, \delta)$-preferences may generate dynamic inconsistency. A common approach to account for it is to represent the problem as an interpersonal game. The set of players of the interpersonal game consists in the period-$t$ selves of the same state worker. In accordance with her current utility $U_t(u_t, u_{t+1}...)$, each period-$t$ self decides whether to make or not an optimal additional payment, while her future selves command her subsequent decisions. In this context a Markovian strategy profile for all selves is $\boldsymbol{\sigma} = \{\sigma_t\}_{t=1}^{\infty}$, where $\sigma_t : \mathcal{X} \times \mathbb{R}^2 \to \mathcal{I}$ for all $t$, i.e, this is a collection of when to make additional payments in all possible states and random shocks. Let $\boldsymbol{\sigma_t^+} \equiv \{\sigma_k\}_{k=t}^{\infty}$ be the continuation strategy profile from period $t$ on, and define the expected continuation utility under her long discount $V_t(x_t, \boldsymbol{\varepsilon}_t; \boldsymbol{\sigma_t^+})$ by

$$V_t(x_t, \boldsymbol{\varepsilon}_t; \boldsymbol{\sigma_t^+}) = u_{\sigma_t(x_t,\varepsilon_t)}(x_t) + \boldsymbol{\varepsilon}_{\sigma_t(x_t,\varepsilon_t)t} + \delta E[V_t(x_{t+1}, \boldsymbol{\varepsilon}_{t+1}; \boldsymbol{\sigma_{t+1}^+})|x_t, \sigma_t(x_t, \varepsilon_t)] \tag{2.8}$$

where $\sigma_t(x_t, \varepsilon_t) \in \mathcal{I}$ is the choice specified by $\sigma_t$ and the expectation is taken over the future states $x_{t+1}$ and future random shocks $\boldsymbol{\varepsilon_{t+1}}$.

As discussed before it is important to consider the possibility that the state worker ignores her own present bias. Like in Stroz (1955), Phelps and Pollak (1968), and O'Donoghue and Rabin (1999) I define a partially naive state worker if her period-$t$ self with present bias $\beta$ believes her future selves have present bias $\tilde{\beta} \in [\beta, 1]$. In particular if $\tilde{\beta} = \beta$, the period-$t$ self is sophisticated, and if $\tilde{\beta} = 1$ period-$t$ self is completely naive.

The equilibrium will be defined for the partially naive state worker. To that end it is necessary to introduce how the partially naive state workers perceive her future selves will behave. (O'Donoghue and Rabin (1999)) A perception continuation strategy profile for a partially naive agent is a strategy profile $\tilde{\boldsymbol{\sigma}} \equiv \{\tilde{\sigma}_t\}_{t=1}^{\infty}$ such that for all $t = 1, 2, ...,$ all $x_t \in \mathcal{X}$, and all $\boldsymbol{\varepsilon}_t \in \mathbb{R}^2$,

$$\tilde{\sigma}_t(x_t, \boldsymbol{\varepsilon}_t) = \underset{i \in \{0, \mathbf{1}\{a_t = a_t^*\}\}}{arg\,max} \left\{ u_i(x_t) + \varepsilon_{it} + \tilde{\beta}\delta E[V_{t+1}(x_{t+1}, \boldsymbol{\varepsilon}_{t+1}; \boldsymbol{\sigma}_{\mathbf{t+1}}^{+})|x_t, i] \right\}$$

Note that definition 1 is not observed in the data but influences the true decision of the state worker. As in Fang and Wang (2015), the equilibrium for a partially naive agent can be defined as a perception-perfect strategy profile. A perception-perfect strategy profile for a partially naive agent is a strategy profile $\boldsymbol{\sigma}^* \equiv \{\sigma_t^*\}_{t=1}^{\infty}$ such that for all $t = 1, 2, ...,$ all $x_t \in \mathcal{X}$, and all $\boldsymbol{\varepsilon}_t \in \mathbb{R}^2$,

$$\sigma_t^*(x_t, \boldsymbol{\varepsilon}_t) = \underset{i \in \{0, \mathbf{1}\{a_t = a_t^*\}\}}{arg\,max} \left\{ u_i(x_t) + \varepsilon_{it} + \beta\delta E[V_{t+1}(x_{t+1}, \boldsymbol{\varepsilon}_{t+1}; \tilde{\boldsymbol{\sigma}}_{t+1}^{+})|x_t, i] \right\}$$

Definition 1 and definition 2 fully characterize the equilibrium of the interpersonal game. Note that when $\tilde{\beta} = \beta$ (i.e., the agent is sophisticated), $\tilde{\boldsymbol{\sigma}} = \boldsymbol{\sigma}^*$. To apply the identification argument developed in the paper of Fang and Wang (2015) the following assumptions on the data generating process have to be made.

**Assumption 2.** ***(Stationarity)***: The observed choices are generated under the stationary perception perfect strategy profile of the infinite horizon game played among different selves of the state workers.

**Assumption 3.** ***(Conditional Independence)***: The transition probabilities satisfy the following:

$$\pi(x_{t+1}, \boldsymbol{\varepsilon}_{t+1}|x_{t,}\boldsymbol{\varepsilon_t}, i_t) = q(\boldsymbol{\varepsilon}_{t+1}|x_{t+1})\pi(x_{t+1}|x_t, i_t)$$

$$q(\boldsymbol{\varepsilon}_{t+1}|x_{t+1}) = q(\boldsymbol{\varepsilon})$$

**Assumption 4.** ***(Extreme Value Distribution)***: $\boldsymbol{\varepsilon_t}$ is i.i.d. extreme value distributed.

Imposing stationarity could be unrealistic, in particular if we considering that income grows faster in the first years, and then it slows down in the later years. A different approach is to estimate the transition probabilities for every period. Note that consistent estimates of the transition probabilities for every fortnight require a large sample of state workers. Since this is not the data's case, I will exploit the long periodicity to estimate the stationary transition probabilities. This an important limitation of the paper, and potentially can bias the results.

Both assumption 3 and assumption 4 are commonly used in the dynamic discrete choice models. Fang and Wang (2015) remark that it is possible to obtain similar identification results assuming another distribution of the unobservables, however without making any assumption on the distribution the model is not identified. In that sense, making an assumption of $G_t$ imposes a weaker version of semi parametric identification.

At this point it is possible to characterize the decision of the state worker using value functions. Let $W_i(x)$ be the deterministic current choice-specific value function, defined as:

$$W_i(x) = u_i(x) + \beta\delta \sum_{x' \in \mathcal{X}} V(x')\pi(x'|x, i) \tag{2.9}$$

Where $\pi(x'|x, i)$ is the transition probability from state $x$ to $x'$ when action $i$ is taken, and $V(x)$ is the perceived long run value function, defined as expected value over the stationary value function defined in (2.8) under the continuation strategy $\tilde{\boldsymbol{\sigma}}$. In some sense equation (2.9) dictates the behavior of the period-$t$ self.

In the same way let $Z_i(x)$ capture the deterministic perceived choice-specific value function of the future self by the current self as:

$$Z_i(x) = u_i(x) + \tilde{\beta}\delta \sum_{x' \in \mathcal{X}} V(x')\pi(x'|x, i) \tag{2.10}$$

equation (2.10) regulates how the current state worker $t$ self believes her future self will behave.

Hence by definition 2 the probability of observing in the data that an additional payment has been made at state $x$ is

$$P_1(x) = Pr[\sigma_t^*(x, \boldsymbol{\varepsilon}) = 1] = Pr[W_1(x) + \varepsilon_1 \geq W_0 + \varepsilon_0] \tag{2.11}$$

and by assumption 4 it becomes

$$P_1(x) = \frac{exp[W_1(x)]}{exp[W_0(x) + W_1(x)]} \tag{2.12}$$

Analogously by definition 1 the probability of making an additional contribution by the next period state worker as perceived by the current period state worker at state $x$ is

$$\tilde{P}_1(x) = Pr[\tilde{\sigma}_t(x, \boldsymbol{\varepsilon}) = 1] = Pr[Z_1(x) + \varepsilon_1 \geq Z_0 + \varepsilon_0] \tag{2.13}$$

and so with assumption 4 it becomes

$$\tilde{P}_1(x) = \frac{exp[Z_1(x)]}{exp[Z_0(x) + Z_1(x)]} \tag{2.14}$$

Recall that in the data it is possible to observe $P_i(x)$, but $\tilde{P}_i(x)$ is not observable. Finally to fully characterize the model, an expression of $V(\cdot)$ is missing. For this propose let $V_i(x)$ be the perceived choice-specific long run value function as follows:

$$V_i(x) = u_i(x) + \delta \sum_{x' \in \mathcal{X}} V(x')\pi(x'|x, i) \tag{2.15}$$

Thus using equations (2.10), (2.15) and the assumptions on $V(x) = E_{\varepsilon}[V_{\tilde{\sigma}(x,\varepsilon)}(x) + \varepsilon_{\tilde{\sigma}(x,\varepsilon)}]$, it is possible to get

$$V(x) = ln\left\{exp(Z_0(x) + Z_1(x))\right\} + (1-\tilde{\beta})\delta\sum_{j=0}^{1}\left[\tilde{P}_j(x)\sum_{x'\in\mathcal{X}}V(x')\pi(x'|x,j)\right] \tag{2.16}$$

In summary, the dynamic discrete choice model consists on $\{V_i(x), W_i(x), Z_i(x) : x \in \mathcal{X}\}$ as defined by equations (2.15), (2.9) and 2.10), where $W_i(x)$ is the value function that dictates the state worker's behavior in period $t$; $Z_i(x)$ is what she perceives from her future self choices; and $V_i(x)$ is an auxiliary value function that evaluates payoffs from the choices that the current self perceives will be made by her future selves.

## 2.4 Econometric specification and Identification

In this section I present the econometric specification and some intuition that leads to the identification of the discount parameters $\left\langle \delta, \beta, \tilde{\beta} \right\rangle$. A formal proposition and proof can be found in Fang and Wang (2015).

### 2.4.1 Identification

Denote the structure of the model by $\theta$, identified by the following parameters (Fang and Wang 2015):

$$\theta = \{\left\langle \delta, \beta, \tilde{\beta} \right\rangle,\ G,\ \langle\{u_i(x),\ Z_i(x'),\ V_i(x') : i \in \mathcal{I},\ x \in \mathcal{X},\ x' \in \mathcal{X}\}\rangle\}$$

Where $G(\cdot)$ is a type-I extreme value distribution by assumption 4, and $Z_i(x')$ and $V_i(x')$ satisfy equations (2.10) and (2.15), respectively.

Two structures $\theta, \theta' \in \Theta$ are observationally equivalent if the predicted probabilities of making and additional payment by the model are equal, i.e. $\hat{P}_i(x;\theta) = \hat{P}_i(x;\theta')\ \forall i \in$

$\mathcal{I}$ and $x \in \mathcal{X}$. The model is identified if, and only if, for any $\theta, \theta' \in \Theta$, $\theta = \theta'$ if they are observationally equivalent. In other words, there exists no other specification for the utility function, the discount parameters, and the value functions such that they generate the same probabilities. Most of the previous structural present bias modeling requires to assume a parametric form of the utility function, which allows the Rubinstein critique to kick in.[13] The flexibility in the Fang and Wang methodology is robust enough the withstand the Rubinstein critique, at least in observables.

The identification argument of Fang and Wang (2015) consists in two steps. First they show that $\langle\{u_i(x),\ Z_i(x'),\ V_i(x') : i \in \mathcal{I},\ x \in \mathcal{X},\ x' \in \mathcal{X}\}\rangle$ is identified given values of $\left\langle \delta, \beta, \tilde{\beta} \right\rangle$, then they provide conditions to identify the discount parameters. Since the identification argument suggest how to estimate the parameters, I present the basic idea of how each step works.

**First Step**

The first step consists on fixing values of $\left\langle \delta, \beta, \tilde{\beta} \right\rangle$ to identify

$$\langle\{u_i(x),\ Z_i(x'),\ V_i(x') : i \in \mathcal{I},\ x \in \mathcal{X},\ x' \in \mathcal{X}\}\rangle$$

Such identification will be accomplished by solving a system of equations for all states $x \in \mathcal{X}$ and both decisions $i \in \mathcal{I}$. This system of equations will be generated by two blocks.

For the first block let $Q$ be a mapping from $\boldsymbol{W}(x) = (W_0(x), W_1(x))$ to $\boldsymbol{P}(x) = (P_0(x), P_1(x))$. By normalizing one $W_i(x)$ Hotz and Miller (1993) showed that it is possible to invert $Q$. Using assumption 4, the inverse of this normalized map is $D(x) = W_1(x) - W_0(x) = ln\frac{P_1(x)}{P_0(x)}$. When combining equations (2.9) and (2.10) with $D(x)$, it is possible to find a relationship that captures the perceived difference of making an additional payment by

[13] Rubinstein (2003) shows that choices that are consistent with QHD under a parametric utility function might also be constistent with exponential discounting under a different parametric utility function.

future selves:

$$Z_1(x) - Z_0(x) = \frac{\tilde{\beta}}{\beta} ln \frac{P_1(x)}{P_0(x)} + \left(1 - \frac{\tilde{\beta}}{\beta}\right) [u_1(x) - u_0(x)] \tag{2.17}$$

The first block consists in equation (2.17) for all different states $x \in \mathcal{X}$.

To build the second block take equation (2.16) and apply the normalization to get

$$V(x) = Z_0(x) + ln\left\{exp[Z_1(x) - Z_0(x)] + 1\right\} + (1-\tilde{\beta})\delta \sum_{j=1}^{1} \left[ \frac{exp[Z_j(x)]}{exp[Z_0(x) + Z_1(x)]} \sum_{x' \in \mathcal{X}} V(x')\pi(x'|x, j) \right] \tag{2.18}$$

Note that for each state $x \in \mathcal{X}$ and given$\{\left\langle \delta, \beta, \tilde{\beta} \right\rangle \langle Z_i(x) : i \in \mathcal{I},\, x \in \mathcal{X} \rangle\}$, equation (2.18) represents a system of $card(\mathcal{X}) = X$ linear equations with $X$ unknowns, namely, $\boldsymbol{V} \equiv [V(1), ..., V(X)]^T$.

It is possible and convenient to express $\boldsymbol{V}$ in matrix notation. For that purpose let $\boldsymbol{A} = Z_0(x) + ln\left\{exp[Z_1(x) - Z_0(x)] + 1\right\}$ be a vector of dimension $X \times 1$; let $\tilde{\boldsymbol{P}} \equiv \left[\tilde{\boldsymbol{P}}_0, \tilde{\boldsymbol{P}}_1\right]$ be the matrix of dimension $X \times 2X$ that contains the choice probabilities; and let $\boldsymbol{\Pi}$ be a matrix of dimension $2X \times X$ that captures the matrices of transition probabilities.[14] Then the system

of equations generated by equation (2.18) can be expressed as:

$$\boldsymbol{V} = \left[\boldsymbol{I} - \left(1 - \tilde{\beta}\right)\delta \tilde{\boldsymbol{P}}\boldsymbol{\Pi}\right]^{-1} \boldsymbol{A} \tag{2.19}$$

Combining system (2.19) with equation (2.10) for all states $x \in \mathcal{X}$ and $i \in \mathcal{I}$ results in the second block that consists of $2 \times X$ equations.

$$Z_i(x) = u_i(x) + \tilde{\beta}\delta\boldsymbol{\Pi_i}(x)\left[\boldsymbol{I} - \left(1 - \tilde{\beta}\right)\delta \tilde{\boldsymbol{P}}\boldsymbol{\Pi}\right]^{-1} \boldsymbol{A} \tag{2.20}$$

Without loss of generality normalize $u_0(x) = 0$ for all $x \in \mathcal{X}$. Hence the two blocks defined in (2.17) and (2.20) yield to $2 \times X$ values for $\{Z_i(x) : i \in \mathcal{I},\, x \in \mathcal{X}\}$ and $1 \times X$ values

[14] Where $\boldsymbol{\Pi} \equiv [\boldsymbol{\Pi_0}, \boldsymbol{\Pi_1}]^{\boldsymbol{t}}$ and $\boldsymbol{\Pi_i} = [\boldsymbol{\Pi_i(1)}, ..., \boldsymbol{\Pi_i(X)}]$, in which $\boldsymbol{\Pi_i(x)} = [\pi(1|x,i), ..., \pi(X|x,i)]^t$.

for $\{u_1(x) : x \in \mathcal{X}\}$. The number of unknowns is the same as the number of equations in the system, $3 \times X$. Solving this system while taking as given values of $\left\langle \delta, \beta, \tilde{\beta} \right\rangle$ identifies $\langle \{u_i(x),\ Z_i(x'),\ V_i(x') : i \in \mathcal{I},\ x \in \mathcal{X},\ x' \in \mathcal{X}\} \rangle$.

**Second Step**

In order to identify $\left\langle \delta, \beta, \tilde{\beta} \right\rangle$ the following assumption on the states variables has to be made.

**Assumption 5.** ***(Exclusion Restriction)***: There exists state variables $x_1 \in \mathcal{X}$ and $x_2 \in \mathcal{X}$ with $x_1 \neq x_2$, such that

1. For all $i \in \mathcal{I}$, $u_i(x_1) = u_i(x_2)$
2. But for at least one $i \in \mathcal{I}$, $\pi(x'|x_1, i) \neq \pi(x'|x_2, i)$

For notation simplicity divide the state variables as follows, $(x_r, x_e)$, where $x_r \in \mathcal{X}_r$ refers to the state variables that affect the instantaneous utility function $u_i(x_r)$, and $x_e \in \mathcal{X}_e$ refers to the state variable that satisfies assumption 5. Instead of presenting the formal argument, I will outline how the use of exclusion variables allows a distinction of $\delta$ from $\beta$ and $\beta$ from $\tilde{\beta}$.

To see how it is possible to separate $\beta$ from $\delta$, suppose that a state worker is an exponential discounter with discount factor $\hat{\delta}$. By plugging $\tilde{\beta} = 1$ in equation (2.18) her expected continuation payoff becomes

$$V(x) = Z_0(x) + ln\left\{exp[Z_1(x) - Z_0(x)] + 1\right\} \tag{2.21}$$

Which as aforementioned only depends on $ln\frac{P_1(x)}{P_0(x)}$. In other words the expected continuation payoff for an exponential discounter is completely determined by the observed choice probabilities.

Now consider a sophisticated present biased state worker. Her current self values differently the future payoffs than her perceived future self. This incongruence leads to an additional term,

$$(1-\beta)\delta\sum_{j\in\mathcal{I}}\left[\frac{exp[Z_j(x)]}{exp[Z_0(x)+Z_1(x)]}\sum_{x'\in\mathcal{X}}V(x')\pi(x'|x,j)\right]$$

in equation (2.18). Hence the expected continuation payoff for a sophisticated present biased state worker is more than just observed choice probabilities. As discussed in the first step continuation utilities will determine the identified values of $\{u_1(x) :, x \in \mathcal{X}\}$. By making $u_1(x)$ independent of $x_e$, it is possible to distinguish $\beta$ from $\delta$.

$\beta$ and $\tilde{\beta}$ can be distinguished as follows: Suppose that $\delta = 1$, and note that equation (2.17) contains $\frac{\tilde{\beta}}{\beta}$. If $\frac{\tilde{\beta}}{\beta} = 1$ then $Z_1(x) - Z_0(x) = ln\frac{P_1(x)}{P_0(x)}$. Hence $\{u_1(x) : \ x \in \mathcal{X}\}$ will be pinned down from the data, which could be refuted if the identified values of $u_1(x)$ do not satisfy the exclusion restriction. Hence $\beta, \tilde{\beta}$ are identified. Proposition 1 in Fang and Wang (2015) provides a formal argument.

### 2.4.2 Estimation

By the identification argument the estimator of $\left\langle \beta, \tilde{\beta}, \delta \right\rangle$ consists in a two step approach. The first step consists of estimating the choice probabilities (for all states) of making an additional contribution. It also requires estimating the transition probabilities $\pi(x'|x,i)$ for all $i \in \mathcal{I}$ and all $(x', x) \in \mathcal{X}^2$.

The second requires solving the following equation system for all $x \in \mathcal{X}$

$$\begin{aligned}
Z_1(x) &= u_1(x) + \tilde{\beta}\delta\boldsymbol{\Pi_1}(x)\left[\boldsymbol{I} - \left(1-\tilde{\beta}\right)\delta\tilde{\boldsymbol{P}}\boldsymbol{\Pi}\right]^{-1}\boldsymbol{A} \\
Z_0(x) &= \tilde{\beta}\delta\boldsymbol{\Pi_0}(x)\left[\boldsymbol{I} - \left(1-\tilde{\beta}\right)\delta\tilde{\boldsymbol{P}}\boldsymbol{\Pi}\right]^{-1}\boldsymbol{A} \\
Z_1(x) - Z_0(x) &= \tfrac{\tilde{\beta}}{\beta}ln\tfrac{P_1(x)}{P_0(x)} + \left(1-\tfrac{\tilde{\beta}}{\beta}\right)[u_1(x)]
\end{aligned}$$

The solution of the system yields values of $Z_1(x)$, $Z_0(x)$ and $u_1 = \hat{u}_1(x_r, x_e)$ for a given triple of $\left\langle \beta, \tilde{\beta}, \delta \right\rangle$ for all $x \in \mathcal{X}$. Such utility values need to satisfy assumption 5, hence it is important to impose the following restriction:

$$\hat{u}_1(x_r) = \frac{1}{|\{(x_r, \tilde{x}_e) : \tilde{x}_e \in \mathcal{X}_e\}|} \sum_{\{(x_r, \tilde{x}_e) : \tilde{x}_e \in \mathcal{X}_e\}} \hat{u}_1(x_r, \tilde{x}_e) \tag{2.22}$$

Given $\hat{u}_1(x_r)$ as defined in equation (2.22), it is possible to predict the choice probabilities $\hat{P}\left(x; \langle \beta, \beta, \delta \rangle\right)$ and then formulate a pseudo-likelihood from the observed data:

$$\mathcal{L} = \prod_{n \in N} \prod_{i=0}^{1} \prod_{x \in \mathcal{X}} \hat{P}\left(x; \left\langle \beta, \tilde{\beta}, \delta \right\rangle\right)^{\Phi_n(x)} \left[1 - \hat{P}\left(x; \left\langle \beta, \tilde{\beta}, \delta \right\rangle\right)\right]^{1 - \Phi_n(x)}$$

Where $n$ stands for an individual and $N$ stands for the number of individuals in the sample, and $\Phi_n(x)$ is a function that takes the value of 1 when the individual makes an extra contribution. I maximize the pseudo-likelihood function to estimate $\left\langle \beta, \tilde{\beta}, \delta \right\rangle$. As shown in Fang and Wang (2015), this estimator is consistent and asymptotically normal, with asymptotic variance given by

$$H = \sum_{x \in \mathcal{X}} Var \left( \frac{\Phi_n(x) - \hat{P}\left(x; \left\langle \beta, \tilde{\beta}, \delta \right\rangle\right)}{\hat{P}\left(x; \left\langle \beta, \tilde{\beta}, \delta \right\rangle\right) \left(1 - \hat{P}\left(x; \left\langle \beta, \tilde{\beta}, \delta \right\rangle\right)\right)} \frac{\partial \hat{P}(x)}{\partial \theta}(\theta) \right)$$

therefore standard errors are easily computed.

### 2.4.3 Econometric Specification

Now I proceed to describe how I perform the estimation. First note that a state in the model is defined as a combination of: the outstanding level of debt $M_t$, the level of income $y_t$, the gender of the worker, the age when the worker signed the contract and whether she lives in Mexico City. Thus estimating the choice probabilities can be done by a logistic regression through assumption 4. To perform this estimation I assume robust standards errors.

Then, in order to non-parametrically estimate the transition probabilities, I need to discretize the income level and the outstanding level of debt. I divide both variables into 6 equidistant groups, where income level ranges from 0 to 125 or more minimum wages and the outstanding debt goes from 0 to 7500 or more minimum wages. For simplicity purposes I also consider age to be a binary variable that captures if the worker was 40 years old or more when she signed the contract. Therefore I define a state to be a combination of $x = (M,\, y,\, Female, 40years,\, Mexico\,City)$, thus I can observe how many individuals in $x$ move to $x'$. Hence the estimator of the transition probabilities is

$$\hat{\pi}(x'|x,i) = \frac{n(x'|x)}{n(x)}$$

where $n(\cdot)$ is the counting function. In other words, it is the ratio between the number of workers at state $x'$ that were at $x$ and the total number of workers at state $x$.

In step two, I propose initial values of $\left\langle \beta, \tilde{\beta}, \delta \right\rangle$, then I solve the system. Solving the system could yield to one solution, multiple solutions, or no solution. If it is the case that there are multiple solutions, I choose the one that obtains the greater pseudo-likelihood value. If the system has no solution, I assign for the likelihood a negative number that is sufficiently large. Once a solution has been found, I impose the restriction (2.22) unto the utilities. To do so I define my exclusion variables to be $x_e = (Female,\, 40years,\, Mexico\,City)$. From the data description section we observed that these variables suggest different assignments of income levels, so they indeed affect the transition probabilities. Since those state variables are time invariant, it is reasonable to assume that they do not affect the difference in instantaneous payoffs of making or not an additional contribution. Once the values of $\hat{u}_i(x_r)$ are obtained, I predict the probabilities under a logistic distribution. This procedure is done recursively until the computer finds an optimum. To ensure global optimality, several initial values are provided, and different optimization methods are used.

## 2.5 Results and Policy simulations

As discussed in the previous section the estimation consists of two steps. This section summarizes the results of both steps and presents some comments and interpretations about them. The first step consists of estimating the choice probabilities and the transition probabilities.

### 2.5.1 First Step Results

The choice probabilities are estimated under a logistic regression. I present three different model specifications in which the exclusion variable set changes. All models consider the relevant payoff outcomes in the utility function: outstanding debt and income. Model 1 only includes gender as an exclusion variable; model 2 adds age above 40; model 3 also considers if they live in Mexico City or not. Table 2.6 summarizes the information on the three specifications and the results for each model. Note that income seems to be positively correlated with making an extra contribution, on the other hand the outstanding debt seems to be negative correlated. Although this relationship is not causal, it seems to be consistent with present biased behavior. As noted before a present biased individual will be more likely to make additional payments later in time, where on average the debt is lower and income is higher. All exclusion variables turn out to be significant, and all of them are negatively correlated with making an additional contribution. Since I am not interested in how much each variable affects the choice probability, but in how well I can predict them instead, the pseudo $R^2$ is reported. In all specifications the pseudo $R^2$ takes a value above .4.

The transition probabilities are estimated non-parametrically. In all models the states are defined by a combination of: one of the 6 levels of income, one of the 6 levels of outstanding debt, one combination of the exclusion state variables. Therefore for every $i \in \mathcal{I}$ the dimension of the transition probability matrix when only gender is included (model 1) is

Table 2.6: Logit Regression of Choice Probabilities

| | (I) | (II) | (III) |
|---|---|---|---|
| Outstanding debt | -0.0033*** | -0.0037*** | -0.0036*** |
| | (0.0003) | (0.0003) | (0.0003) |
| Income | 1.4300*** | 1.4932*** | 1.4852*** |
| | (0.2930) | (0.0306) | (0.0299) |
| Female | -0.2836*** | -0.2353*** | -0.2434*** |
| | (0.0488) | (0.0482) | (0.0484 ) |
| 40 years old | / | -0.7504*** | -0.8364*** |
| | / | (0.0534) | (0.0568) |
| Mexico City | / | / | -0.3795*** |
| | / | / | (0.0566) |
| Constant | -7.5694*** | -7.3553*** | -7.1472*** |
| | (0.1251) | (0.1214) | (0.1216) |
| Pseudo $R^2$ | 0.4041 | 0.4139 | 0.4162 |
| Observations | 110,356 | 110,356 | 110,356 |

Random sample of 410 individuals that started to repay their debt in Jan 2000. Income and outstanding debt are measured in minimum wages. Robust standard errors are presented in parenthesis. ***,**,* represent statistical significance at 1%, 5% and 10% respectively.

$72 \times 72$. When age above 40 is included (model 2) this number grows to $144 \times 144$. When adding if they reside in Mexico City (model 3) the matrix dimension grows to $288 \times 288$.

Figures 2.3 and 2.4 present a subsample of the estimated transition probabilities. In both figures I consider an individual that started with an outstanding debt between 100 and 150 minimum wages, his (her) fortnight income ranged between 2 and 3 minimum wages, when he (she) signed his (her) mortgage he (she) was at most 39 years old, and he (she) lives outside Mexico City. These figures show the next period probability for the individual to move to any other income-debt state. The only difference between figure 2.3 and 2.4 is that figure 2.3 considers a male state worker whereas 2.4 considers a female one. Since I have a total of 36 income level and outstanding debt combinations, I choose to present them by overlapping the 6 levels of income over the 6 outstanding debt cutoffs. For example in figure 2.3, the worker will most likely move to a state between 50 and 100 minimum wages of outstanding debt, and his income should remain constant. Making an additional payment

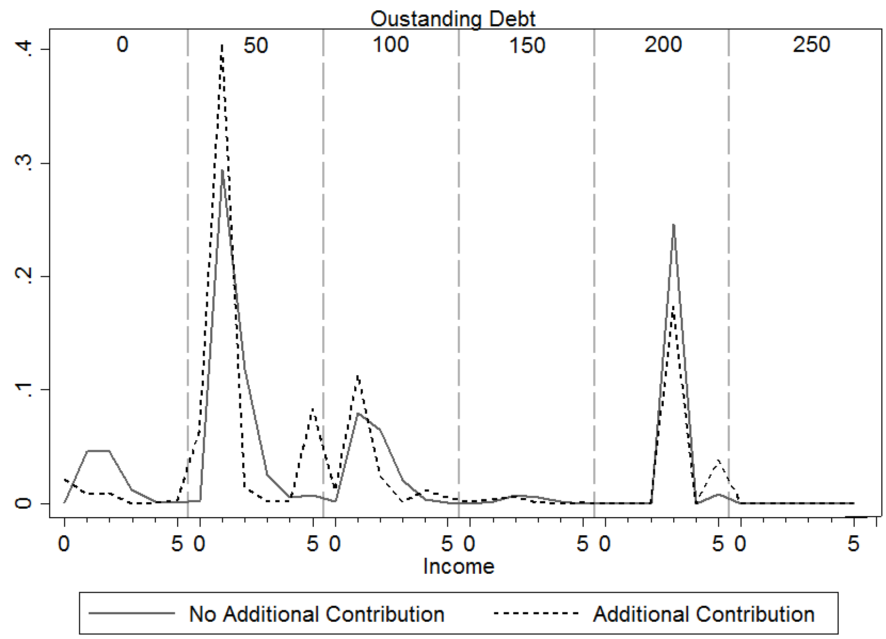


Figure 2.3: Transition Probabilities for a Male State Worker
Non parametric estimates of the transition probabilities

in both figures increases the probability of lowering the mortgage debt. Also a female state worker has more volatility regarding new income states.

### 2.5.2 Second Steps Results

Table 2.7 presents the estimates for $\left\langle \beta, \tilde{\beta}, \delta \right\rangle$ for all three sets of exclusion variables. The results show that the average worker with a FOVISSSTE mortgage is present biased $\beta \in (0.34, 0.56)$, has a high degree of naivety (in one specification is completely naive $\tilde{\beta} = 1$) and her exponential discount is similar to what the literature has found, $\delta \in (0.85, 0.90)$. Because the estimated value of $\tilde{\beta}$ lies on the parameter space boundary, the correction of Moran (1970) for standard errors is applied. Note that the confidence intervals for this estimation are very tight and all the estimated values are statistically different from zero.

More interesting hypothesis tests are presented in table 2.8, where it is possible to see if the classical economic theory is refuted. I will focus in the second model since it is the one that obtains the largest pseudo-likelihood. The first test rejects the absence of present

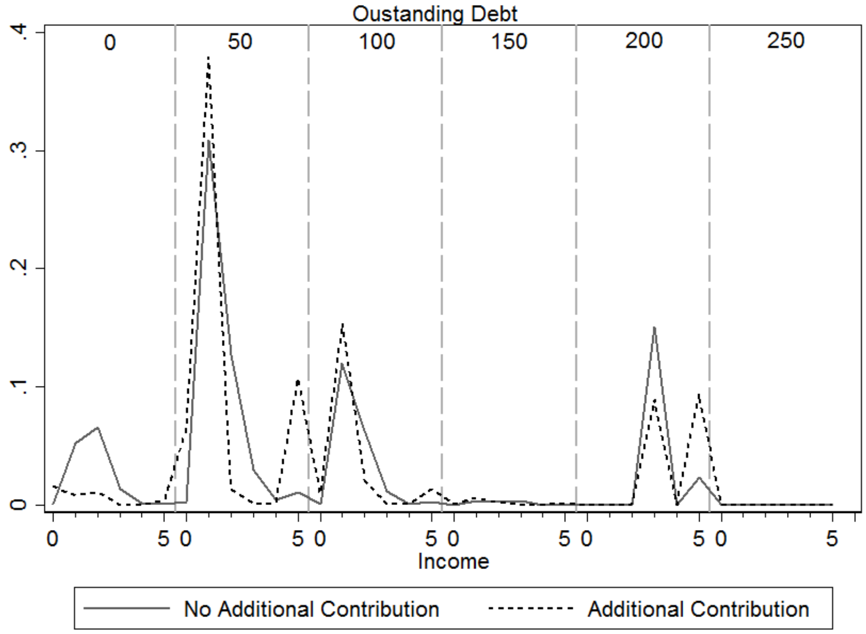


Figure 2.4: Transition Probability for a Female State Worker
Non parametric estimates of the transition probabilities

Table 2.7: Discount Parameters

| Parameter | (I) | (II) | (III) |
|---|---|---|---|
| $\delta$ | 0.8532** | 0.9960*** | 0.9023*** |
| | (0.0216) | (0.0001) | (0.0132) |
| | [0.7975 0.9089] | [0.9957 0.9962] | [0.8682 0.9363] |
| $\tilde{\beta}$ | 0.8010*** | 1.000*** | 0.9998*** |
| | (0.0131) | (0.0759) | (0.0563) |
| | [0.76720 0.83479] | [0.5236 1.4763] | [ 0.8548 1.1448] |
| $\beta$ | 0.5634*** | 0.3463*** | 0.4875*** |
| | (0.0010) | (0.0002) | (0.0007) |
| | [0.56087 0.5695] | [0.3452 0.3473] | [0.4869 0.4893] |
| LogLikelihood | 476.3201 | 564.3760 | 527.3210 |

Random sample of 410 individuals that started to repay their debt in Jan 2000. Income and outstanding debt are measured in minimum wages. Robust standard errors are presented in parenthesis. ***,**,* represent statistical significance at 1%, 5% and 10% respectively.

Table 2.8: Classic Theory Tests/ Under Specification 2

| Hypotheses | Decision | Wald-Statistic |
|---|---|---|
| Ho: $\beta = 1$ | Reject | 2455.55 |
| Ho: $\beta = \tilde{\beta}$ | Reject | 3284.44 |

Random sample of 410 individuals that started to repay their debt in Jan 2000. The Rejection of the null hypotheses is achieved at 99 percent of confidence.

bias, $\beta = 1$. This implies that the average worker will consume as much as possible in the first periods, in fact if the worker exits the public labor force her probability of default should be greater that the one considered under standard models. The second test rejects the possibility of being aware of her own present bias $\beta = \tilde{\beta}$. As aforementioned having an illiquid asset, such as a mortgage, helps smooth consumption. However when an agent ignores her own present bias weak commitment devices are not used, and thus, we observe less additional payments. So, would happen if the worker was not present biased? or How much would her behavior change if she were sophisticated? The next next subsection explores this scenarios.

### 2.5.3 Counterfactual Simulations

Table 2.9 describes the average predicted probability of making an additional contribution in several scenarios. The first row shows the predicted values for the estimated parameters in model 2. When comparing them with the case of the worker that discounts exponentially (second row), its possible to see an increase in the probability of making an additional contribution. There is also an increase on the average instantaneous utility of making a payment (across states). The same prediction occurs when we compare it to a sophisticated worker, nevertheless the change are smaller.

So far the economic models assume that individuals cannot transit from having quasi-hyperbolic preferences towards classic exponential ones. Nevertheless there is room for policy making to sophisticate individuals. How to do it escapes the scope of the present paper and further neuroeconomics research is needed. Since a sophisticated worker will have on average a greater probability of making an additional payment, FOVISSSTE could offer a contract that discounts more than 30 percent of their base salary as payment. And so, a better commitment device would be available. Finally, recall that some of the incentives to repay the debt faster come from the fact that the mortgage is measured in minimum wages, thus moving the scheme to one measured in Mexican pesos might lower the default probability for the naive present biased workers.

Table 2.9: Simulations of Counterfacutals

| Model | Predicted Probability | Instantaneous utility |
|---|---|---|
| Estimated | 0.03425 | 3.4222 |
| $\beta = \tilde{\beta} = 1$ | 0.08030 | 3.7009 |
| $\tilde{\beta} = \beta$ | 0.06419 | 3.4243 |

Random sample of 410 individuals that started to repay their debt in Jan 2000.

## 2.6 Discussion

In this paper I use novel data from a de-centralized Mexican public mortgage institution to estimate a dynamic structural model with QHD. Using the methodology in Fang and Wang (2015), I am able to identify and estimate the long run discount parameter, the present bias discount parameter and the parameter that captures the state worker's naivety degree. Identification is achieved by using exclusion variables that affect the transition probability of outstanding debt and income, but not the instantaneous payoffs of making an additional payment to their mortgage. To that end I used gender, age and whether the worker lives in Mexico City as exclusion variables. It is worth mentioning that I am exploiting the long periodicity of the panel to estimate the stationary transition probabilities. In order to relax the stationarity assumption more subjects are needed in the current data set.

Conditional on assuming that there are no other sources of income, I find that the average state worker has a standard long run discount parameter $\delta = 0.9960$, is present biased

$\beta = 0.3463$, and does not have perfect awareness of it $\tilde{\beta} = 1.0$. These findings are robust to distinct specifications of the exclusion variables set. I reject the hypotheses of the classical economic theory (exponential discounting), and I also reject that the average individual is sophisticated. I provided counterfactuals that suggest that a policy that educates the state worker will increase the probability of making an additional payment. How to do so remains an open question.

When policy makers ignore the fact that the mortgage market contains present biased individuals, the risk of default is underestimated. Since 2009 FOVISSSTE started to issue debt through the securitization of their mortgages. Under the assumption of time consistent individuals, and since there exist an automatic payment from the state worker, these securities are cataloged as risk free. Still over the last years their nonperforming mortgages have increased from 2 percent in 2002 to 8 percent in 2014 (SHF 2015).

To account for present bias, I suggest that FOVISSSTE should offer a contract that deducts more than 30 percent of their base salary. In this way there exists an option of a stronger commitment device. I also mentioned that lending Mexican pesos instead of minimum wages could reduce the incentives to repay their mortgage faster. This could be beneficial for those who want to repay faster, but who are also dissuaded from doing so due to their present bias. In 2015 FOVISSSTE announced their first credit scheme in Mexican pesos; however, it is too soon to see if that measure has been beneficial, and thus it opens an opportunity for research.

From a behavioral economics perspective this paper contributes to augment the literature on present bias, moreover it contributes to the short list of non experimental structural estimation on behavioral economics.

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# Appendix

## Proofs of Propositions of Chapter 1

**Lemma 1** *Generalized Extreme Value models satisfy assumption 7*

*Proof.* Let $G = G\left(e^{\upsilon(x_0,p_0,\xi_0)}, \ldots, e^{\upsilon(x_J,p_J,\xi_J)}\right)$, be a continous and positive function.

$G$ also satisfy the following properties:

1) $G \to \infty$ as $e^{\upsilon(x_j,p_j,\xi_j)} \to \infty$ for any $j \in \mathcal{J}$

2) $G$ is homogeneous of degree one

3) $G_j = \frac{\partial G(e^{\upsilon(x_0,p_0,\xi_0)},\ldots,e^{\upsilon(x_J,p_J,\xi_J)})}{\partial e^{\upsilon(x_j,p_j,\xi_j)}} \geq 0$ for all $j \in \mathcal{J}$

4) $G_{jm} = \frac{\partial G_j}{\partial e^{\upsilon(x_m,p_m,\xi_m)}} \leq 0$ for all $m \neq j$

Then the market share of good $j$ in any GEV model can be written as

$$\sigma_j = \frac{e^{\upsilon(x_j,p_j,\xi_j)} G_j}{G}$$

Consider the case when $\frac{\partial \sigma_j}{\partial x_{jk}} > 0$, that is when

$$\frac{\partial \sigma_j}{\partial x_{jk}} = \frac{\left(\frac{\partial \upsilon(x_j,p_j,\xi_j)}{\partial x_{jk}} e^{\upsilon(x_j,p_j,\xi_j)}\right)\left\{\left(G_j G + e^{\upsilon(x_j,p_j,\xi_j)} G_{jj} G\right) - e^{\upsilon(x_j,p_j,\xi_j)} G_j^2\right\}}{G^2} > 0$$

By applying Euler's Theorem we can restate the condition as

$$\frac{\partial \sigma_j}{\partial x_j} = \frac{\left(\frac{\partial \upsilon(x_j,p_j,\xi_j)}{\partial x_{jk}} e^{\upsilon(x_j,p_j,\xi_j)}\right)\left\{\left(G_j \sum_{i \neq j} e^{\upsilon(x_i,p_i,\xi_i)} G_i + e^{\upsilon(x_j,p_j,\xi_j)} G_{jj} G\right)\right\}}{G^2} > 0$$

Therefore, $\frac{\partial \sigma_j}{\partial x_{jk}} > 0$ implies that

$$\frac{\partial \upsilon(x_j,p_j,\xi_j)}{\partial x_{jk}} > 0$$

Consider now the change in good $m$

$$\frac{\partial \sigma_m}{\partial x_j} = \frac{\frac{\partial \upsilon(x_j,p_j,\xi_j)}{\partial x_j} e^{\upsilon(x_m,p_m,\xi_m)} e^{\upsilon(x_j,p_j,\xi_j)} \left(G_{jm} G - G_j G_m\right)}{G^2}$$

By the properties 4 and 3 of $G$, $G_{jm}G - G_j G_m \leq 0$

Since $\frac{\partial \upsilon(x_j,p_j,\xi_j)}{\partial x_{jk}} > 0$ , $\frac{\partial \sigma_m}{\partial x_j} \leq 0$ as desired □

**Lemma 2** *If* $\upsilon(\zeta_i, x_j, p_j, \xi_j) = \sum_k x_{jk}\tilde{\beta}_{ik} - \tilde{\alpha}_i p_j + \xi_j + \epsilon_{ijt}$,
*with* $\tilde{\beta}_{ik} = \bar{\beta}_k + +\beta_k^u \nu_{ik}$, *and* $\nu_{ik} \sim N(0,1)$ *for all* $k$.
*The model will satisfy assumption 7 if* $\frac{|\bar{\beta}_k|}{\beta_k^u}$ *is sufficiently large*

*Proof.* Consider the case when

$$\frac{\partial \sigma_j}{\partial x_j} > 0$$

then

$$\frac{\partial \sigma_m}{\partial x_{jk}} = -\int s_j(\nu) s_m(\nu)(\bar{\beta}_k + \beta_k^u \nu_k) dF(\nu)$$

we can re write the expression as follows

$$\frac{\partial \sigma_m}{\partial x_{jk}} = -\bar{\beta}_k \int s_j(\nu) s_m(\nu) dF(\nu) - \beta_k^u \int s_j(\nu) s_m(\nu) \nu_k dF(\nu)$$

If $\beta > 0$

$$\frac{\partial \sigma_m}{\partial x_{jk}} \leq -\bar{\beta} E[s_j(\nu) s_m(\nu)] - \beta_k^u \int_{\nu \leq 0} s_j(\nu) s_m(\nu) \nu_k dF(\nu)$$

Since $s_j(\nu)s_m(\nu) \in (0\,1/4)$, we can find a $C_m > s_j(\nu)s_m(\nu)$ s.t.

$$\int_{\nu \leq 0} s_j(\nu) s_m(\nu) \nu_k dF(\nu) < C \int_{\nu \leq 0} \nu_k dF(\nu)$$

thus

$$\frac{\partial \sigma_m}{\partial x_{jk}} \leq -\bar{\beta} E[s_j(\nu) s_m(\nu)] - \beta_k^u C \int_{\nu \leq 0} \nu_k dF(\nu)$$

And because of the normality assumption we can write

$$\frac{\partial \sigma_m}{\partial x_{jk}} \leq -\bar{\beta} E[s_j(\nu) s_m(\nu)] + \beta_k^u 2 \sqrt{\frac{2}{\pi}}$$

Therefore $\frac{\partial \sigma_m}{\partial x_{jk}} \leq 0$ only if

$$\frac{\bar{\beta}}{\beta^u} \geq 2 \sqrt{\frac{2}{\pi}} \frac{C_m}{E[s_j(\nu) s_m(\nu)]}$$

This can be done fore all products $m$, so as long as $\frac{\bar{\beta}}{\beta^u} \geq \frac{1}{2}\sqrt{\frac{2}{\pi}} \frac{1}{E[s_j(\nu)s^*(\nu)]}$, where $E[s_j(\nu)s^*(\nu)] = \underset{m}{Min} E[s_j(\nu)s_m(\nu)]$, we obtain the desired result

□

**Proposition 1** *There exist preferences $U$ and distributions $P_{\boldsymbol{Y}}$ satisfying assumptions 1-7 and two vectors $x_r$ and $x_r'$ with $d(x_r', x_j) < d(x_r, x_j)$, such that $\Delta\sigma_j > 0$ and $\Delta\sigma_r > 0$*

*Proof.* Start by conditioning on $(p, \xi)$.

Rewrite the set $A_j$ as follows:

$$A_j = \{\boldsymbol{Y} : U(\boldsymbol{Y}_i, x_j;\, x_r) > U(\boldsymbol{Y}_i, x_n;\, x_r) \;\; \forall n \in J\}$$

By using assumptions 1-6 it becomes the following set:

$$A_j = \{\boldsymbol{Y} : \upsilon(\boldsymbol{Y}_i, x_j) + \mu(-d(x_j, x_r)) > \upsilon(\boldsymbol{Y}_i, x_n) + \mu(-d(x_n, x_r)) \forall n \in J\}$$

In a similar way write the set $\tilde{A}_j$ as follows

$$\tilde{A}_j = \{\boldsymbol{Y} : \upsilon(\boldsymbol{Y}_i, x_j) > \upsilon(\boldsymbol{Y}_i, x_n) \, \forall n \in J\}$$

Take $x_r$ and $x'_r$ such that $d(x_j, x'_r) < d(x_j, x_r)$ and $\Delta\sigma_r > 0$, by assumption 7 it implies that

$$\int P_{\boldsymbol{Y}:}(d\boldsymbol{y}) \geq \int_{\tilde{A}'_j} P_{\boldsymbol{Y}:}(d\boldsymbol{y})$$

where $\tilde{A}'_j$ is the corresponding set when the new product space, $J'$, contains $x'_r$ instead of $x_r$. In other words, the former inequality implies that given a realization of $\boldsymbol{Y}_i$,

$$\underset{n \in J'}{Max} \{\upsilon(\boldsymbol{Y}_i, x_n)\} = \upsilon(\boldsymbol{Y}_i, x'_m) \geq \upsilon(\boldsymbol{Y}_i, x_m) = \underset{n \in J}{Max} \{\upsilon(\boldsymbol{Y}_i, x_n)\}$$

Take $\boldsymbol{Y}_i \in A_{j'} = \{\upsilon(\boldsymbol{Y}_i, x_j) + \mu(-d(x_j, x'_r)) > \upsilon(\boldsymbol{Y}_i, x'_m) + \mu(-d(x'_m, x'_r))\}$
By rearranging terms, it is easy to see that such $\boldsymbol{Y}_i$ satisfies

$$\mu(-d(x_j, x'_r)) - \mu(-d(x'_m, x'_r)) > \upsilon(\boldsymbol{Y}_i, x'_m) - \upsilon(\boldsymbol{Y}_i, x_j)$$

Since $\upsilon(\boldsymbol{Y}_i, x'_m) > \upsilon(\boldsymbol{Y}_i, x_m)$,$\boldsymbol{Y}_i$ also satisfies

$$\mu(-d(x_j, x'_r)) - \mu(-d(x'_m, x'_r)) > \upsilon(\boldsymbol{Y}_i, x_m) - \upsilon(\boldsymbol{Y}_i, x_j)$$

then any preferences $U$ that satisfy

$$\mu(-d(x'_m, x'_r)) + \mu(-d(x_j, x_r)) > \mu(-d(x_m, x_r)) + \mu(-d(x_j, x'_r))$$

implies that $\boldsymbol{Y}_i$ also satisfies

$$\mu(-d(x_j, x_r)) - \mu(-d(x_m, x_r)) > \upsilon(\boldsymbol{Y}_i, x_m) - \upsilon(\boldsymbol{Y}_i, x_j)$$

that is $\boldsymbol{Y}_i \in A_j$, and thus $A_j \subset A_{j'}$ and so

$$\sigma'_j = \int_{A'_j} P_{\boldsymbol{Y}}(d\boldsymbol{y}) > \int_{A_j} P_{\boldsymbol{Y}}(d\boldsymbol{y}) = \sigma_j$$

$\square$

**Proposition 2** *If the proportion of consumers that has reference product $r$ is large enough, there exist preferences $U$ and distributions $P_{\boldsymbol{Y}}$ satisfying assumptions 1-5 and 6 and two vectors $x_r$ and $x'_r$ with $d(x'_r, x_j) < d(x_r, x_j)$, such that $\Delta\sigma_j > 0$ and $\Delta\sigma_r > 0$*

*Proof.* Consider the case where there exists $n$ reference products.

Let $C_r \subset [0,1]$ be the set of consumers that have reference product $r$

The market share $\sigma_j$, is defined as follows

$$\sigma_j = \sum_{r=1}^{n} \left( \int_{A_{j_r}} P_{\boldsymbol{Y}}(d\boldsymbol{y}) \right) M(C_r)$$

Where $M(\cdot)$ is the Lebesgue measure and

$$A_{j_r} = \{\boldsymbol{Y}_i \in \boldsymbol{Y} : U(\boldsymbol{Y}_i, x_j; x_r) > U(\boldsymbol{Y}_i, x_n; x_r) \;\; \forall n \in J\}$$

.

Let $r^*$ indicate the set such that for all $r \in \{1, ..., n\}$ $M(C_{r^*}) > M(C_r)$

Let $\sigma_{j_{r^*}} = \left( \int_{A_{j_{r^*}}} P_{\boldsymbol{Y}}(d\boldsymbol{y}) \right)$

As shown in proposition 1, we can increase $x_{r^*}$ to $x'_{r^*}$ with $d(x_j, x'_{r^*}) < d(x_j, x_{r^*})$ such that

$$\sigma'_{j_{r^*}} > \sigma_{j_{r^*}}$$

Then by assumption 7 $\sigma'_{j_r} \leq \sigma_{j_r}$ for all $r \in \{1, .., n\} \setminus \{r^*\}$

As long as

$M(C_r) > \sum M(C_n) \frac{\left(|\sigma'_{j_n} - \sigma_{j_n}|\right)}{(\sigma'_{j_r} - \sigma_{j_r})}$

$$\sigma'_j > \sigma_j$$

$\square$