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The Wisconsin engineer. Vol. 70, No. 3 December 1965

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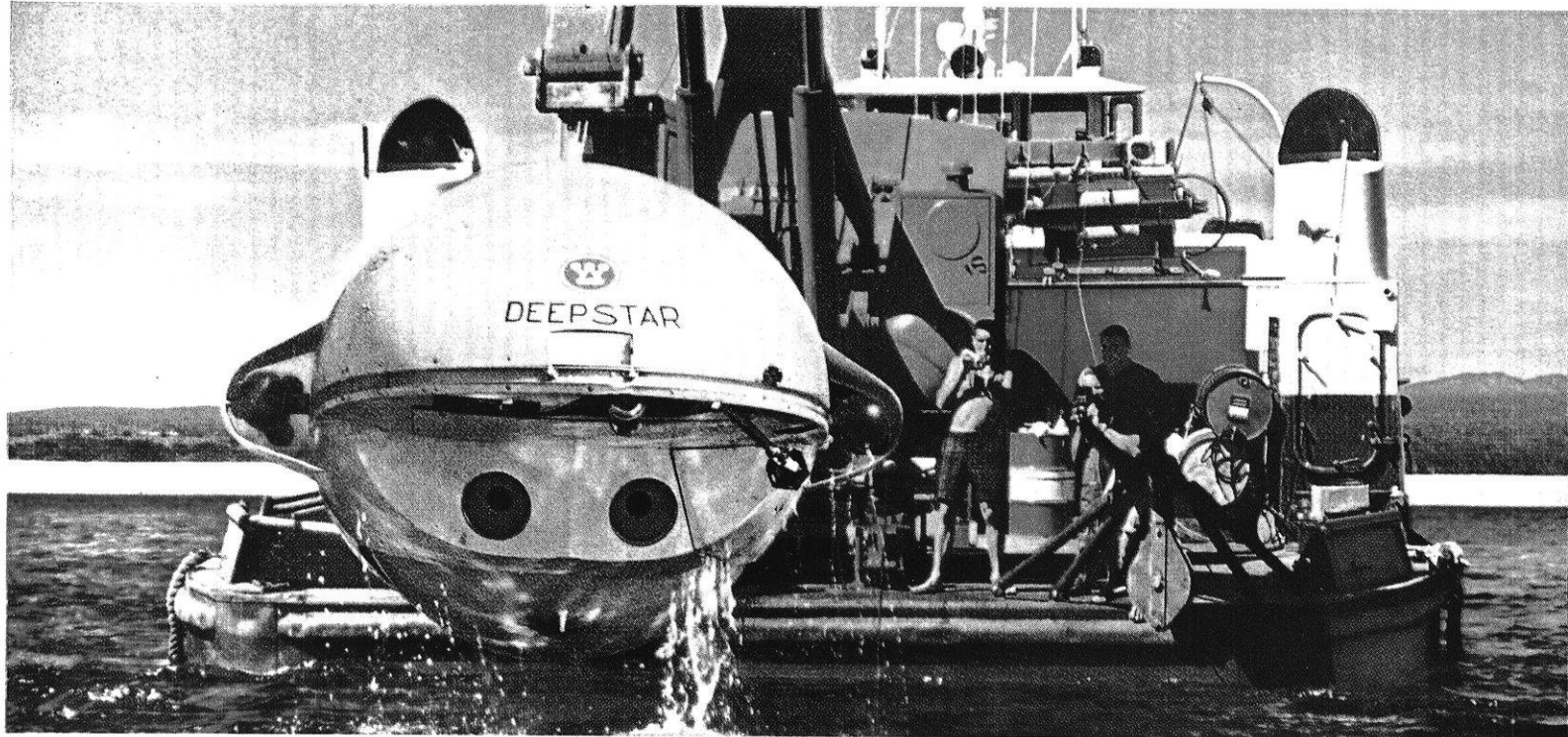
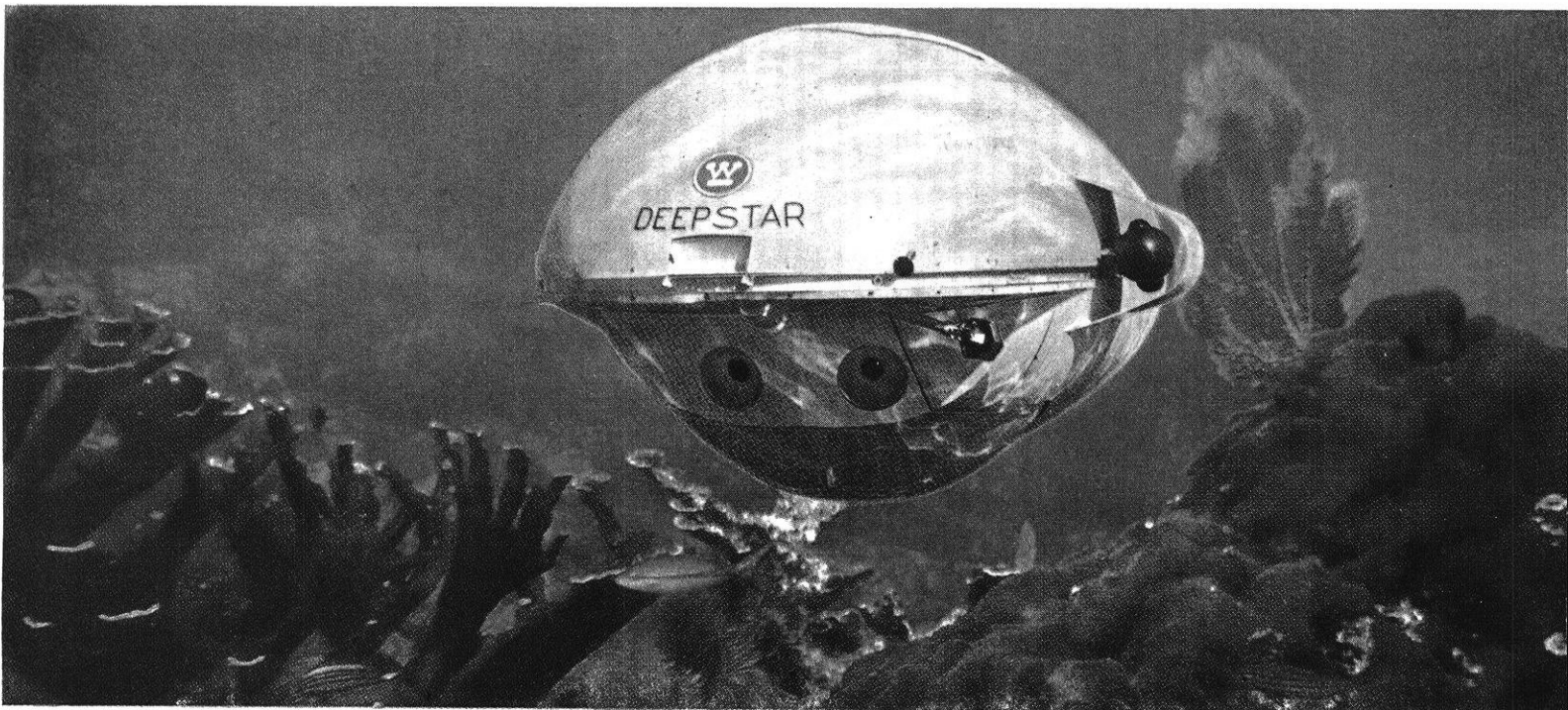http://rightsstatements.org/vocab/InC/1.0/

THE WISCONSIN ENGINEER

IRON-CARBON
PHASE DIAGRAMS—Page 14

Self-propelled Westinghouse undersea craft for world-wide charter service

It can explore, test, salvage and sample the ocean floor—4,000 feet down

Westinghouse has built a remarkable undersea exploration craft called *Deepstar 4000*.

It can dive to a depth of 4,000 feet without cables. It can rise, turn, back up at will. It is self-propelled and independent of surface craft while submerged.

Manned by a crew of three, the craft will explore the vast undersea world—sampling the ocean bottom, photographing specimens, setting and monitoring scientific instruments, salvaging lost cargo.
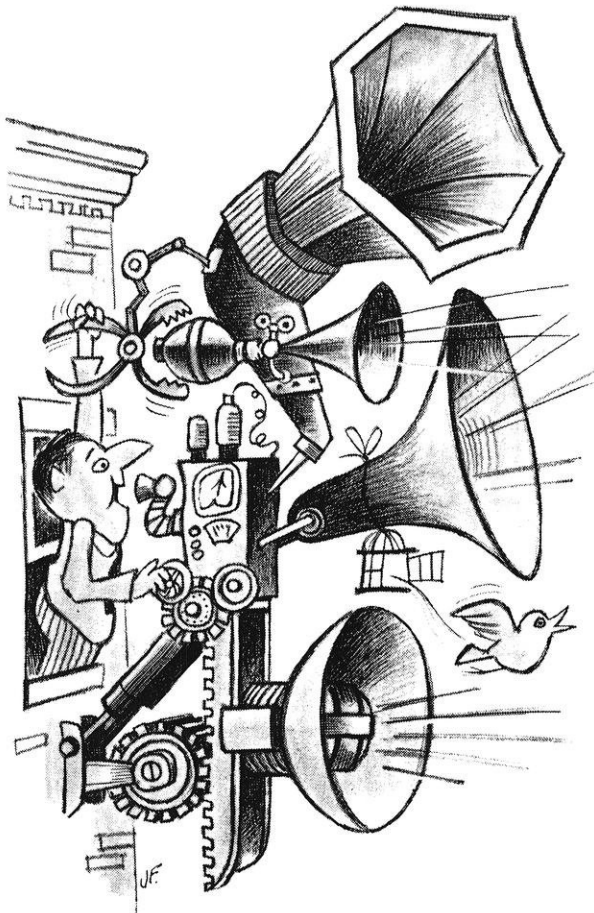
Deepstar 4000 is the first of a fleet of Westinghouse submersibles now being developed for depths up to 20,000 feet.

You can be <u>sure</u> if it's Westinghouse

Yell it out the window!

. . . or whisper it over his MOTRAN Radio*

THE PROBLEM: HOW CAN YOU BE

SURE YOUR MAN IN THE TRUCK MAKES THE SECOND STOP FIRST?

You might try the megaphone if you're not too concerned about the message. But if it's one that _has_ to get through . . . better you flip a switch and tell it to him over his Motorola MOTRAN 2-way radio.

As a matter of fact, for any 2-way communications problem, Motorola engineers more than likely have the solution . . . or they'll find it. They've been in the business since it began, and just about every significant development in 2-way radio since 1940 has been by Motorola.

It was no small! task to design a pocket-sized radio pager that could reach a doctor 20 miles away on a golf course.

Motorola did _that_ too!

*MOTRAN — Motorola Mobile 2-way Transistor Radio

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MOTOROLA

Engineering and Science at RCA

Traveling Wave Masers

RCA's recent maser research and development has yielded systems with outstanding low-noise microwave amplifier performance along with adaptability for field use. These amplifiers exhibit ultra-low noise temperature (8-10°K) and high gain (30-40 db) with extreme gain stability. Wide tunability (up to 50%) and large instantaneous bandwidth (up to 150 MHz) have been achieved.

Several technique areas involved with this work are of particular interest. Iron- and chromium-doped rutile (titanium dioxide) are employed as active paramagnetic materials, in a "meander-line" slow wave structure, providing wide bandwidth and high gain. Ferrite reverse isolators function to provide a high degree of gain unidirectionality. The requisite magnetic field is provided by a superconducting magnet within a cryogenic enclosure, and the entire system is operated by a closed-cycle refrigerator requiring no helium replenishment, so that field use in radar systems, satellite communications and radiometry is practical. Sectionalized magnet structures with independent controls permit "stagger-tuning" the maser, so that its very high gain can be traded for even greater bandwidth.

MEANDER LINE (FOLDED) — RUTILE — PUMP CAVITY
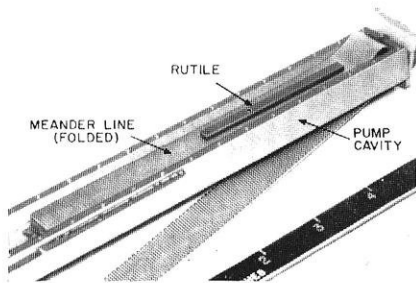
The illustration shows the active elements of a maser amplifier typical of such a high-performance system. The meander line, seen as a zig-zag conducting path on a flexible insulating sheet, goes down one side of the pump cavity, folds over, and returns on the other side. The cavity is the terminal portion of a waveguide assembly, with microwave pump energy being introduced at the other end. One of two rutile paramagnetic crystals is shown in close proximity to the meander line, the ferrite isolator being on the opposite side of the meander line and not visible. In operation, the entire structure shown in the photograph lies between pole faces of the superconducting magnet, which provides a precisely controlled and distributed transverse field, typically, of a few thousand gauss. The assembly including the magnet is enclosed in a chamber maintained at 4.2°K.

Amplifiers with performance as described above are by no means the end, however. New advances are in the offing through research in areas including optical inversion (pumping), operation at temperatures above 4.2°K, higher frequency operation, and the use of active materials in powder rather than single-crystal form.

References:

(a) L. C. Morris, "*A New Class of Traveling Wave Masers,*" International Conference on Microwave Circuit Theory and Information Theory, Tokyo, Sept. 11, 1964.

(b) L. C. Morris and D. J. Miller, "*Traveling Wave Masers Employing Iron-Doped Rutile,*" Proc. IEEE, Vol. 52, #4, p. 410, 1964.

Integrated Launch Control and Checkout Systems for Saturn Lunar Vehicles

Highly sophisticated Saturn automatic ground checkout and launch sequencing equipment has been under development by RCA since late 1960 for the National Aeronautics and Space Administration, Marshall Space Flight Center. The original Saturn Ground Computer System (SGCS) was used on the highly successful Saturn I program: an advanced version of the SGCS is currently being readied for the Saturn IB and Saturn V programs. The RCA 110 computer was the heart of the Saturn I SGCS: the RCA 110A is the heart of the Saturn IB and Saturn V SGCS.
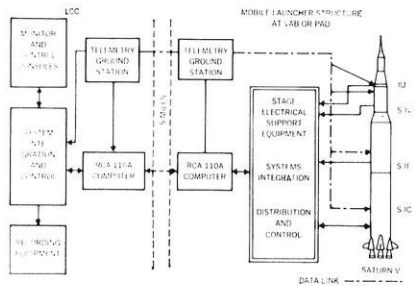
The block diagram shows the tandem, two computer configuration for Saturn V at Complex 39, the lunar program "space port" at NASA's Kennedy Space Center. Complex 39 is based on a mobile launch concept to gain high efficiency in launch operations. Vehicles are assembled in the Vehicle Assembly Building (VAB) on a Mobile Launcher structure. After the Saturn V with its Apollo Spacecraft is completely checked out, the vehicle in its Launcher is transported to one of three launch pads for a remotely controlled launch. The computer in the Launch Control Center (LCC) controls the activities of the "slave" computer in the Mobile Launcher via a 250 kilobit/sec digital data link. The configuration thus remains the same for both VAB and pad operations: only the length of the data link changes. The complex umbilical interface between the vehicle and ground support equipment remains undisturbed until launch. The LCC computer controls the sequence of checkout and launch countdown programs performed by the Mobile Launcher computer via commands transmitted over the data link. The "slave" computer in turn performs the detailed testing and sequencing, performs evaluation and data compression of test results, and transmits the data back to the LCC computer which relays it to the correct operator for display. LCC operators can override, via their console request keyboards, the predetermined sequence of programs stored in the Mobile Launcher computer or handle unusual test situations.

In addition to conventional serial computer functions, special parallel input/output capabilities are included for control of 1008 discrete (relay driver) outputs, monitoring of 1512 discrete (contact closure) inputs, a wide range of DC and AC analog outputs (72 in quantity), a wide range of DC and AC analog inputs (300 in quantity), telemetry interface, 3 internal interval timers, several external clock inputs, and an interface with the spaceborne computer.

In line with the developmental nature of the total Saturn program, the role of RCA's Saturn Ground Computer System is continuing to expand in factory and static testing, as well as launch operations, as automation techniques are applied to other Saturn subsystems.

Reference—J. E. Sloan and J. F. Underwood, "Systems Checkout for Apollo"—Astronautics and Aerospace Engineering, March 1963.

A Light Detector That Makes Laser Communications Practical

RCA has developed a photoconductive device that operates on an alternating current that can sense up to 100 million changes in light intensity per second. This is sufficient to distinguish as many as 25 separate television programs, all carried on a single laser beam. This major breakthrough in light detection is extremely fast, enormously sensitive and is responsive to the whole range of optical frequencies, ranging from infra-red through the visible spectrum to ultra-violet.

By contrast, previous means of detecting laser light employed photoconductors operated by direct current, photoelectric cells, semiconductor photodiodes and electron photomultiplier tubes. The major drawbacks were that these methods were either too slow, too insensitive, or too limited to the portions of the electromagnetic spectrum where most lasers operate poorly, if indeed, at all.

The laser is, to state it simply, a high frequency transmitter with the capacity to carry a fantastic amount of information. The real problem has been to develop a receiver both fast and sensitive enough to detect and process incoming information. This new device has the sensitivity, speed and frequency range that can make possible a practical system for laser communications.

This radical new detector is a tiny speck-sized piece of photoconductive material mounted in a small cavity continuously bathed in microwaves oscillating at 10 billion cycles per second.

When a laser beam bearing information in the form of intensity variations enters the cavity, it strikes the photoconductor and frees electrons. They, in turn, begin to oscillate rapidly up and down within the material, in direct response to the alternating electric field inherent in the surrounding microwaves. These electron oscillators control the amount of microwave power that leaves the cavity. The variations in the incoming light are then converted to intensity variations in the outgoing microwaves. Conventional microwave techniques make it possible to process these variations. These techniques are similar to those used in modern radar and commercial television systems.

Reference—H. S. Sommers, Jr. and E. K. Gatchell, presented at Annual Meeting, Optical Society of America, Philadelphia, October 5-8, 1965, Paper WE-1.
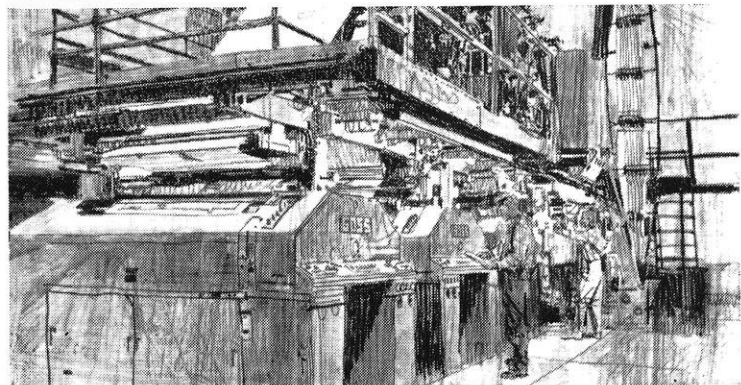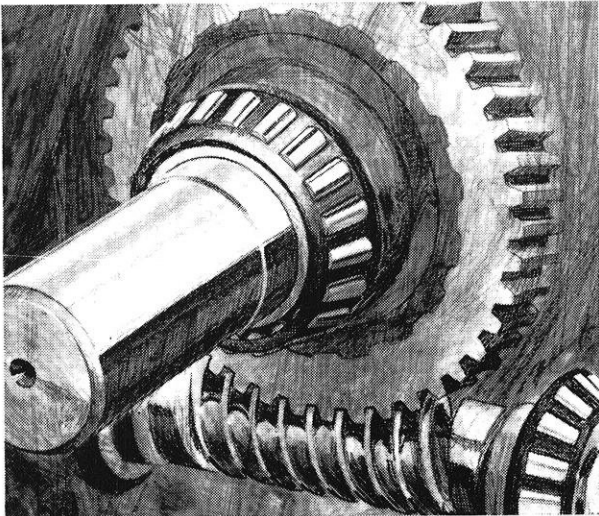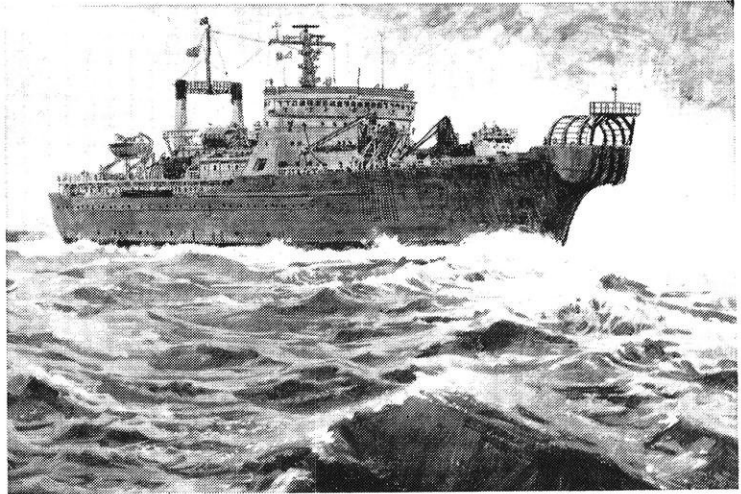
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WHEELS AND SHAFTS. Whatever you build—wheelbarrows to steel rolling mills—Timken tapered roller bearings can help it perform better, longer. They're precision-made of nickel-rich steel by: The Timken Roller Bearing Company, Canton, Ohio. Also makers of Fine Alloy Steel and Rock Bits.

WORLD-WIDE CABLES. The C.S. Long Lines, new Bell System cable-laying ship, is on the high seas. In its wake, thousands of miles of cable, paid out at eight knots. 2,816 Timken bearings keep the ship's linear cable engine operating.

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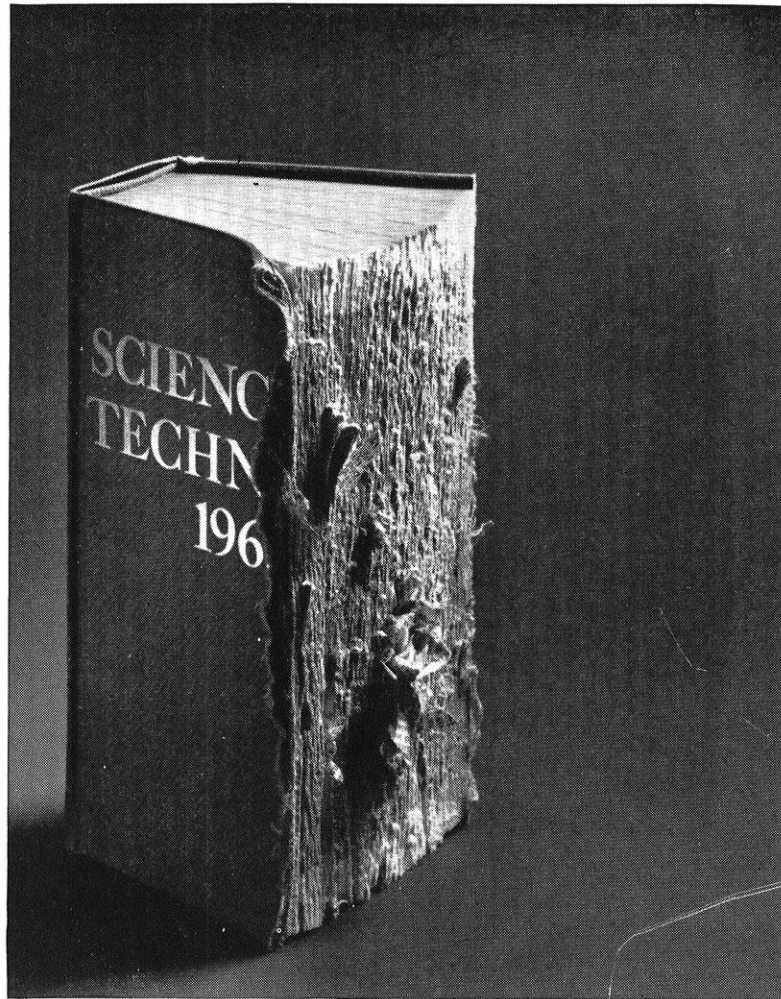
Where will you stand 10 years from today, when half of what you now know becomes obsolete?

Right now you're steeped in the latest technologies. But 10 years from now half of this knowledge will be obsolete. And half of what you will need to know isn't even available today.

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Who creates the "star" that dances?

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With every change of light, the star in the Linde created Star Sapphire moves, shimmers, and dances—matching the brilliance and beauty of the fabulous star rubies and sapphires of the Far East.

At Union Carbide we grow these and other crystals every day —not only for use in jewelry, but also for use in electronic equipment. And for the past several years we've been growing crystals for the heart of the laser beam, the miracle light that can vaporize diamonds, perform eye surgery, and slice through metal.

We're growing in lots of directions. Our carbon floors for blast furnaces have reached the point of wide acceptance by the steel industry. Our activated carbon is being increasingly used to purify and deodorize air in hospitals, homes and factories. And Glad Wrap, our new polyethylene wrapping for home use, is rapidly gaining popularity as a superior way to keep foods fresh longer.

To keep bringing you these and many other new and improved products, we'll be investing half a billion dollars on new plant construction during the next two years.

ARTICLES

THE WISCONSIN ENGINEER

The Student Engineer's Magazine Founded in 1896

FEATURES

THIS MONTH'S COVER

Iron-Carbon phase diagrams are supposed to be well understood by most engineers. We feel that often this is not the case. For a quick lesson, look at our cover, ponder, and then turn to page 14. The cover artwork was done by staff men who think they understand what it is all about.
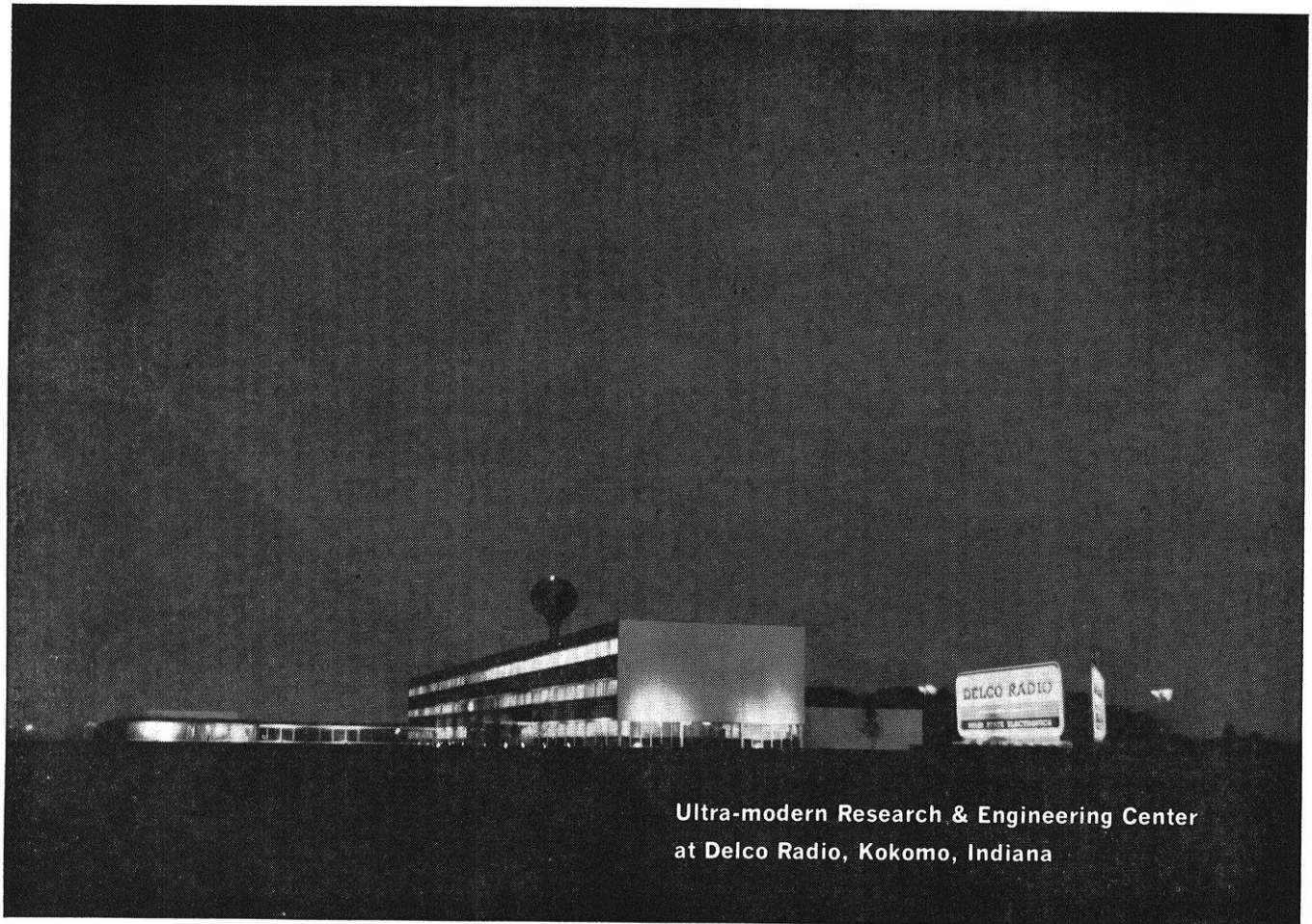
Ultra-modern Research & Engineering Center at Delco Radio, Kokomo, Indiana

To Continue To Learn And Grow...

... is a basic management philosophy at Delco Radio Division, General Motors Corporation. Since its inception in 1936, Delco Radio has continually expanded and improved its managerial skills, research facilities, and scientific and engineering team.

At Delco Radio, the college graduate is encouraged to maintain and broaden his knowledge and skills through continued education. Toward this purpose, Delco maintains a Tuition Refund Program. Designed to fit the individual, the plan makes it possible for an eligible employee to be reimbursed for tuition costs of spare time courses studied at the university or college level. Both Indiana University and Purdue University offer educational programs in Kokomo. In-plant graduate training programs are maintained through the off-campus facilities of Purdue University and available to

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College graduates will find exciting and challenging programs in the development of germanium and silicon devices, ferrites, solid state diffusion, creative packaging of semiconductor products, development of laboratory equipment, reliability techniques, and applications and manufacturing engineering.

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What's Tom Milling's secret?

The formula for the polymers for a unique new synthetic fiber. A fiber that might have a profound effect on more than one industry.

Tom is helping design a new process system for making the monomers—with Amoco Chemicals Corporation, our sister company. The system must be right, and right on time to meet an accelerating development deadline.

That may seem like quite an assignment for a 23-year-old chemical engineer. Less than a year from his B.S. at the University of Illinois. But not around here—at the Amoco Chemicals and American Oil Research

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Rambling with the Editor

"To 'G.I. Joe'"

Dear Joe,

There you are and here I am. That trite phrase makes for a number of differences between us. I'm here in Madison, Wisconsin, wearing Levis and a sweater, carrying books around and worrying about finals. You're in a jungle in Viet Nam, wearing dirty utilities, lugging your M14 around, and worrying about where the next VC is going to pop up. When you have the time, you probably wonder just what is going on around here, with all the ''Peaceniks'' running around, etc. I just wanted to drop you a line, to reassure you, more than anything else, that we're supporting you and the flag you're defending.

Sure, a lot of unbathed types are carrying signs and burning draft cards, but newspapers make money on sensationalism, so let's allow them to make a buck—we all know that the commonplace events rate two inches next to the want-ad page.

You may be wondering just why there is opposition to American involvement over there. There are numerous reasons, some logical, others ridiculous. We've got a lot of weird people running around here, most of them so infatuated with their own intelligence that they are in a world of their own.

No guts? This might be the case once in a while, but I think the big problem is due to the lack of information, often ''voluntary.'' And, many of these

misguided individuals band together, forming a prime target for some kook subsidized by Uncle Ho and buddies. These non-student pawns have their puppets spread the word that it is cool to be in the minority and have your civil rights violated all the time.

Admittedly, there are some sound reasons for your _not_ being over there, but is any policy perfect? Nobody likes killing (VC excepted), but what should be done in reprisal for some of the unprintable VC atrocities? I wish more Americans would not expect a victory per se, but rather realize that we are attempting to give these people a choice they've never had. Quoting a great comic and person who just visited you, ''If we don't stop them over there, we'll be fighting them in Lodi.'' If we were fighting them in Lodi, Joe, we wouldn't be trudging up the Hill every day. We're darn proud of you and your buddies, constantly thankful that we can export law and order to that troubled land via fellows like you. Keep up the good work; many of us will be seeing you next year!

P.S. The next time you hear about the anti-war types around here, remember that this is the same school where last spring, in four days, over 6,000 signatures were placed on a petition supporting U.S. Viet Nam policy (they were subsequently delivered to the White House), the same campus where several thousand dollars was recently collected to provide CARE food packages for Vietnamese families.

If you still think glass is just glass,

ask Rover.

Ask about the Rover-B.R.M. A radical car with a rear-mounted gas turbine engine. One of the 51 cars that started at Le Mans this year. One of only 14 with the stamina to finish.

Ask Rover about the engine's unique heat exchanger that doubled miles per gallon of fuel. They'll tell you it was made of a Corning-developed glass-ceramic trademarked CERCOR®.

Noel Penny, the engine's designer, states: "The low coefficient of expansion, low thermal conductivity, high specific heat and economic feasibility of CERCOR glass-ceramic made it the logical choice for the Rover, with a much better prospect for future gas turbine engines than any metal counterpart."

Today glass can do jobs that metals can't—and more. Much more. It can withstand the awesome pressures at the ocean's floor, maintain constant electrical properties at missile speeds, save weight without sacrificing strength, bring the stars close to an astronomer's eye, bend, not bend, break, not break. It's the most versatile basic material in the world today.

For solutions to their materials problems, industry and government are coming to Corning. Because Corning is *the* glassmaster. It's a broad, international company, with one of the most daring, expert and imaginative research and engineering staffs in the world. Plus, a marketing principle that commits them to developing products only in areas where a need exists and no product does.

If you know a company with a technological problem, or a bright engineer looking for a future, mention Corning.

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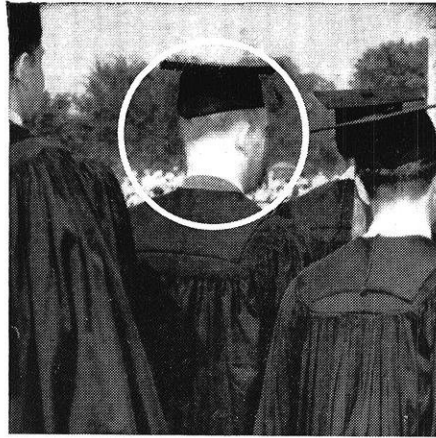
Tom Huck sought scientific excitement
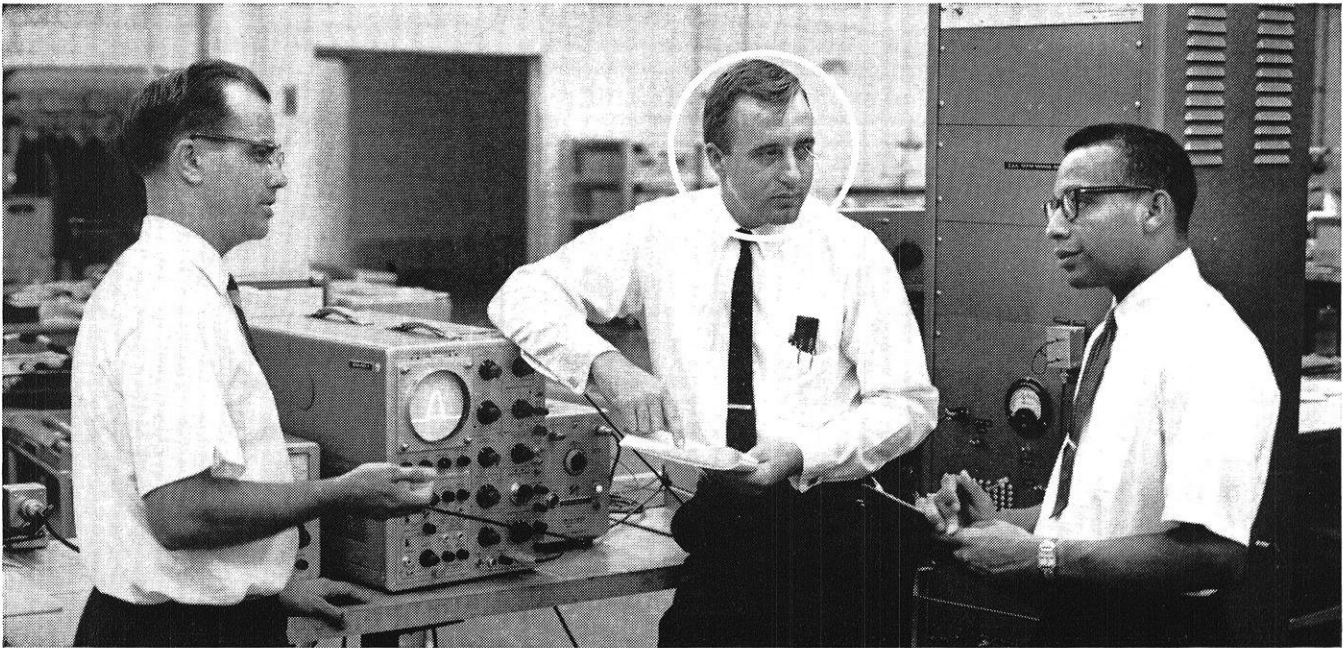
He's finding it at Western Electric

Ohio University conferred a B.S.E.E. degree on C. T. Huck in 1956. Tom knew of Western Electric's history of manufacturing development. He realized, too, that our personnel development program was expanding to meet tomorrow's demands.

After graduation, Tom immediately began to work on the development of electronic switching systems. Then, in 1958, Tom went to Bell Telephone Laboratories on a temporary assignment to help in the advancement of our national military capabilities. At their Whippany, New Jersey, labs, Tom worked with the Western Electric development team on computer circuitry for the Nike Zeus guidance system. Tom then moved on to a new assignment at Western Electric's Columbus, Ohio, Works. There, Tom is working on the development of testing circuitry for the memory phase of electronic switching systems.

This constant challenge of the totally new, combined with advanced training and education opportunities, make a Western Electric career enjoyable, stimulating and fruitful. Thousands of young men will realize this in the next few years. How about you?

If responsibility and the challenge of the future appeal to you, and you have the qualifications we are looking for, talk with us. Opportunities for fast-moving careers exist now for electrical, mechanical and industrial engineers, and also for physical science, liberal arts and business majors. For more detailed information, get your copy of the Western Electric Career Opportunities booklet from your Placement Officer. And be sure to arrange for an interview when the Bell System recruiting team visits your campus.

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Iron ⇌ Carbon Equilibrium Diagrams

At last a completely understandable explanation
of the nemesis of several courses

By ROBERT ALLEN, ME4

FOR more than three-quarters of a century, the demands of industry for materials with which it can fashion tools, machines, and structures have been growing by leaps and bounds. These demands have been responsible for advances of great magnitude in every avenue of engineering.

One of the contributing factors toward such tremendous progress has been the development of the science of the constitution, or internal structure, of metals and alloys, known as metallography or physical metallurgy.

This article is intended to provide interesting and informative material to the reader possessing an engineering and/or metallurgical background. It will be concerned with a portion of physical metallurgy, that of equilibrium (or phase) diagrams. Specifically, the Iron-Carbon Phase Diagrams will be developed and explained. In addition, the use of the lever rule for determining the relative amounts of phases by graphical means will be discussed.

BASICS OF EQUILIBRIUM DIAGRAMS

Solubility Curves

Before the complicated Iron-Carbon system is introduced, it is desirable that the reader comprehend the more general equalibrium systems.
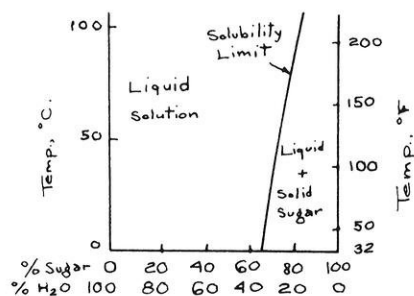

Figure 1.—Solubility of sugar in water.

Figure 1 shows the *solubility* of ordinary sugar in water; the curve is a *solubility curve*. All compositions shown to the left of the curve will form only one phase, because all the sugar is dissolved in the liquid phase. With the higher percentages of sugar shown to the right of the curve, however, it is impossible to dissolve the sugar completely, with the result that there is a mixture of two phases, solid sugar and liquid "syrup." This example shows the change of solubility with temperature and also demonstrates a simple method for plotting temperature (or any other variable) as a function of composition. From left to right, the abscissa of Figure 1 indicates the percentage of sugar. The percentage of water may be read directly from right to left, since the total of the components must, of course, equal 100 per cent.

A two-component system of more engineering significance than that of the first example is shown
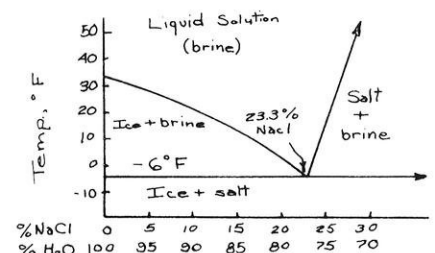

Figure 2.—Solubility of NaCl salt in brine and solubility of ice in brine.

in Figure 2. Here, the extremes of the abscissa indicate 100 per cent H_2O and 30 per cent NaCl. It should be noted from the figure that

(1) the solubility of NaCl in a brine solution decreases with decreasing temperature;

(2) the solubility of H_2O in a brine solution also decreases with decreasing temperature; and

(3) intermediate compositions have melting temperatures lower than those of either pure ice (32°F) or pure salt (1473°F).

The ideas expressed in (1) and (3) above are well-known facts. The concept in (2), the less well-known limited solubility of ice in the aqueous liquid, can be verified by a simple experiment. A salt-and-water solution, that is, sea water with 1.5 per cent NaCl, can be cooled to less than 32°F, and according to Figure 2, it will still

be entirely liquid until 30.5°F is reached. This is in agreement with observations made of any arctic saline sea. (There can be a slight variation if the salt content is not exactly 1.5 per cent.) When such a salty liquid is cooled below 30.5°F, ice crystals will form, and, because the solution cannot contain more than 98.5 per cent H_2O at that temperature, these crystals must separate from the liquid. At 0°F, the maximum amount of H_2O possible in a brine solution is 79 per cent, as can be varified by making a slush at 0°F and separating the ice from this liquid. The ice will be pure H_2O, and the remaining liquid will be saltier (i.e., lower in H_2O) than the original brine solution.

Another example of engineering importance is a mixture of H_2O and $CaCl_2$ (Figure 3). Calcium Chloride rather than NaCl is used to remove ice from highways in very cold climates. The reason is
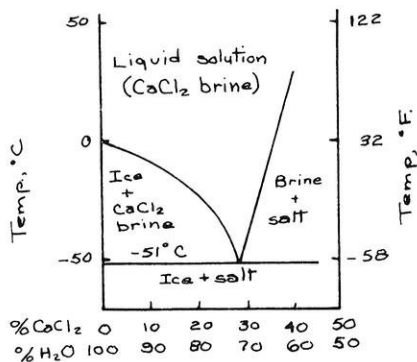

Figure 3.—Solubility of CaCl₂ and ice in CaCl₂ brine.

apparent from a comparison between Figures 2 and 3. An aqueous solution of calcium chloride remains liquid at temperatures as low as —60°F, while a similar solution of NaCl will freeze at —6°F. The lowest temperature at which a solution will remain completely liquid is called the *eutectic temperature*, and the solution which possesses this lowest freezing point is called the *eutectic composition*. The eutectic composition for the H_2O–NaCl system of Figure 2 is 76.7 per cent H_2O and 23.3 per cent NaCl. From Figure 3, the eutectic composition for H_2O–$CaCl_2$ is 71 per cent H_2O and 29 per cent $CaCl_2$. The intersection of the solubility curves in such diagrams marks the eutectic composition for the two components in a liquid solution.
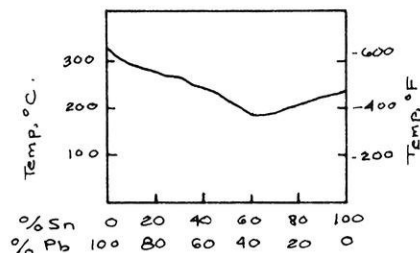

Figure 4.—Solubility of Pb and Sn in molten solders.

The Lead–Tin Equilibrium System

Now, a simple equilibrium diagram will be developed. The melting-freezing relationships discussed above are quite common in all types of two-component combinations. Figure 4 shows the solubility curves for lead and tin. The lowmelting "60–40" alloy is used in many solders because this eutectic composition permits the formation of metal-bonded joints with a minimum of heating.
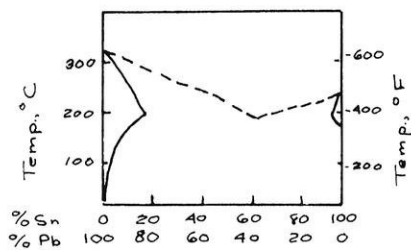

Figure 5.—Solid solubility.

Figure 5 shows the solubility curves for tin in the *solid* lead structure and for lead in the *solid* tin structure. In these particular alloys, the temperature at 360°F is the eutectic temperature and represents

(1) the lowest temperature at which any liquid in the series can exist;

(2) a temperature above and below which the solid solubility decreases; and

(3) a temperature above which any excess over the solid solubility limit is liquid, and below which any excess over the solid solubility limit is solid.

Figure 6 is a completed *phase (equilibrium) diagram* for the lead-tin system. This diagram can be used as a chart from which the phases present at any particular temperature of composition can be read if the alloy is at equilibrium.

For example, at 50 per cent tin and 100°C, the chart indicates two solid phases. α is a solid solution of lead with some dissolved tin. β is almost pure tin with very little dissolved lead. At 200°C, an alloy 10 per cent tin and 90 per cent lead lies in an area which is entirely in the phase. It is a solid solution of lead with some tin dissolved in it. At the same temperature, but for 30 per cent tin and
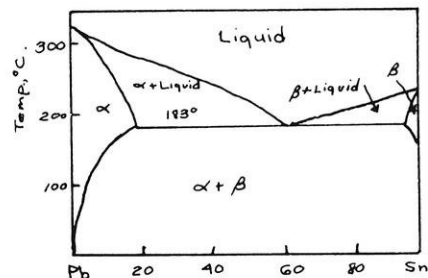

Figure 6.—Pb-Sn diagram.

70 per cent lead, the chart indicates a mixture of liquid and solid solution. If this latter composition were heated to a temperature of 300°C, it would become all liquid.

Liquidus and Solidus

The term *liquidus* is the locus of temperatures above which all compositions are liquid, and *solidus* is the locus of temperatures above which all compositions are solid.
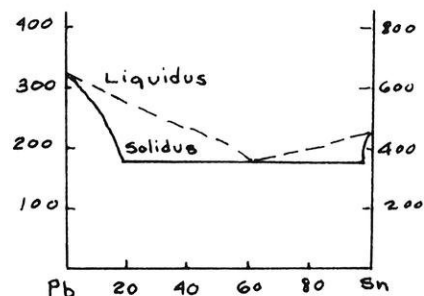

Figure 7.—Liquids and solids.

Every phase diagram for two or more components must show a liquidus and solidus boundary and an intervening freezing range (Fig. 7). Whether the components are metals or nonmetals, there are certain locations on the phase diagram where the liquidus and solidus meet. For a pure component, this point lies at the edge of the diagram. When it is heated, a pure material will remain solid

until its melting point is reached, and it will then change entirely to liquid before it can be raised to a higher temperature.

The solidus and liquidus must also meet at the eutectic. In Figure 6, the liquid solder composed of 61.9 per cent tin and 38.1 per cent lead is entirely solid below the eutectic temperature and entirely liquid above it.

THE LEVER RULE

After a study of the equilibrium diagram for two metallic constituents, it is learned, by a simple process of reasoning, that it reveals some things not suspected on first consideration. There may be determined, for instance, by simple measurement, the relative quantities of two phases (for example, solid and liquid) that are present in a given mixture at a given temperature.
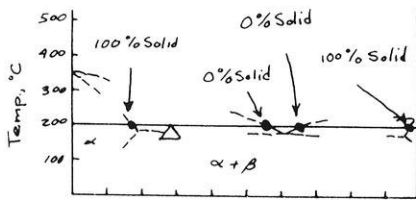
Figure 8.—200°C Pb-Sn Isotherm.

In the lead-tin diagram of Figure 8, at the 200°C isotherm, there is only one phase between the lead-rich end and the 18 per cent tin composition. At the 18 per cent level, there is no liquid present, only solid. Increasing the tin content beyond 18 per cent at 200°C will produce increasing amounts of liquid, until only liquid exists at 56 per cent tin. Between 18 per cent and 56 per cent tin, the amount of liquid solution increases from zero to 100 per cent, and the amount of solid solution decreases from 100 per cent to zero. Subtracting 18 from 56 indicates that the addition of 38 percentage units of tin makes a complete change in the amount of each of the two phases. At 30 per cent tin, which is 12 units of tin beyond the point where there is no liquid, there will be 12/38, or 31.6 per cent liquid. Similarly, with 30 per cent tin and 70 per cent lead at 200°C, (56–30)/38, or 68.4 per cent solid is to be expected.
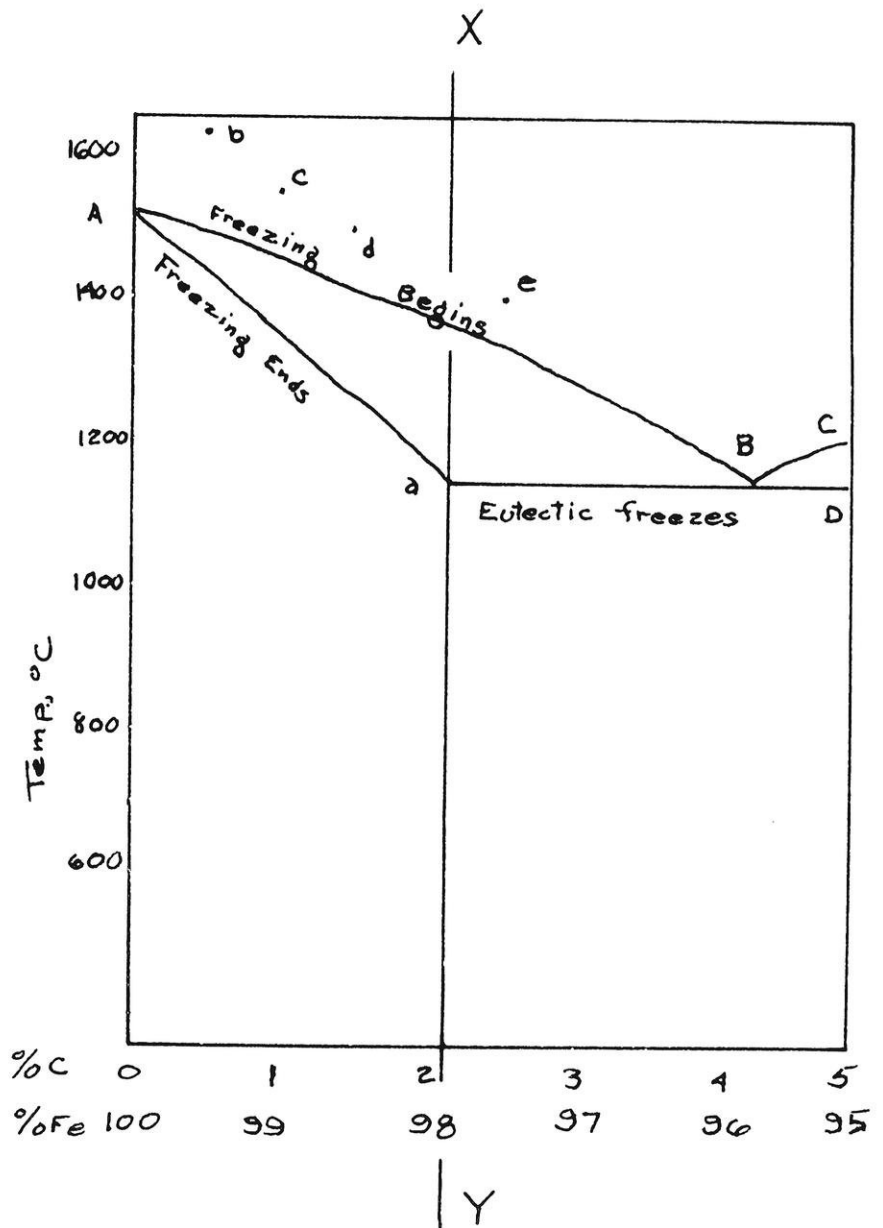
Figure 9.—The freezing of alloys of iron and carbon.

The so-called *lever rule* is a useful tool for calculating relative amounts of phases. For example, in the Pb-Sn diagram of Figure 8, the relative amounts of solid and liquid in an alloy of 30 per cent tin and 70 per cent lead at 200°C can be calculated by considering the 30 per cent tin point at the fulcrum of a lever. The greater amount of solid present, which would in this case contain 18 per cent tin, is necessarily greater than the amount of liquid, which contains 56 per cent tin and 44 per cent lead. In the analogy, the amount of solid is proportional to the distance from the fulcrum to the end of the lever marking the liquid composition. Conversely, the amount of liquid is proportional to the distance from the fulcrum to the other end, which marks the solid composition. This *inverse* relationship, which places the fulcrum and total composition at the "center of gravity" between the phases, serves as a simple rule for calculating the relative amounts of the equilibrated phases.

THE IRON–CARBON DIAGRAM

Progressing from the lead-tin system, the more complicated iron-carbon diagram will next be discussed. It should be noted that iron and carbon form a hard, crystalline compound, known metallographically as cementite, whose composition is represented by the formula Fe_3C. The theoretical carbon content of this compound is

6.67 per cent. Interest in the iron-carbon system is confined to alloys containing less than this amount of carbon; that is why the diagram goes up to only 6.67 per cent carbon.

The discussion of the diagram will be divided into two parts: One will concern the freezing curves; the other, the solid decomposition curves.

The Freezing of Iron and Steel

When the iron and steel alloys are liquid, they are composed of liquid carbon dissolved in liquid iron, and their freezing-point curves are shown in Figure 9. It will be noticed that the lines in this figure bear a close resemblance to those of the lead-tin system. This similarity is real, and the laws governing the freezing of this series of alloys are very similar to those governing the freezing of the lead tin alloys. There is a eutectic of this series when the line A B crosses the line C B, and the components of this eutectic are 95.7 per cent iron and 4.3 per cent carbon. In the study of the iron-carbon alloys, however, there must be taken into account a solid solution that forms. That is to say, it must be recalled that iron, if it contains as much as 2.2 per cent of carbon, never separates from the liquid state without carrying 2.2 per cent of carbon with it in solid solution. This solid solution may consist of carbon dissolved in the iron, or of iron carbide dissolved in the iron. If it contains less than 2.2 per cent carbon, it retains all of it in solid solution.

It will be remembered that when lead separated from the liquid solution, it carried with it a small amount of tin as an impurity, and that when tin separated, it carried with it a small amount of lead as an impurity. However, for the sake of simplicity in outlining the laws of solutions, these traces of impurity were disregarded.

In the case of iron-carbon alloys, however, the carbon carried out with the iron in solid solution, substantially as an impurity, is too important in its effect upon the material to permit neglect.

The line X Y divides the diagram in Figure 9 into two parts. Everything to the left of this line freezes as a solid solution. Everything to the right of the line X Y freezes selectively, according to the same laws as those given for the lead-tin alloys. It is because of this difference in the freezing of the alloys that the line X Y is arbitrarily considered as the dividing line between steel and cast iron. That is to say, all the alloys with less than 2.2 per cent carbon are defined as steel, and all with more than 2.2 per cent carbon are defined as cast iron.

The Freezing of Steel

All the steels freeze as solid solutions. A solution of 99.5 per cent iron and 0.5 per cent carbon at 1650°C will be considered. This will be represented by the point b in Figure 9. This cools until it meets the line A B, and then it commences to solidify. For a few degrees of temperature, it is part liquid and part solid, but by the time it has fallen to a temperature where it meets the line Aa, it has become entirely solid, and it is now a homogeneous solution of 0.5 per cent of carbon in iron. There will next be considered a solution containing 99 per cent of iron and 1 per cent of carbon, at the point c in Figure 9. When this cools to the temperature where it crosses the line A B, it commences to solidify, and it is in a partly liquid and partly solid condition until it crosses the line Aa, upon which solidification is completed, and it now becomes a homogeneous solution of 1 per cent carbon in iron.

The same actions occur with an alloy containing 98.5 per cent iron and 1.5 per cent carbon (the point d in Figure 9) and also in the case of 98 per cent iron and 2 per cent carbon. In all these alloys, there is finally realized a solid solution of carbon in iron. To this solid solution, the name of "austenite" is given, and this name applies no matter how much or how little carbon is in solid solution. In other words, all steels are in the condition of austenite as soon as their solidification is complete.

The Freezing of Cast Iron

The freezing of cast iron is shown by the diagram to the right of the line X Y, and if this part should be considered as a separate diagram, it would be similar to the freezing of the lead-tin alloys. There is one difference to be borne in mind, however. Along the line A B in Figure 7, there was a selective precipitation of lead; along the line A B in Figure 9, there is a selective precipitation, not of pure iron, but of iron containing 2.2 per cent of carbon. In other words, this entity, consisting of a solid solution of iron with 2.2 per cent of carbon, behaves as if it were an elemental substance. It is sometimes called "2.2 per cent austenite." Suppose there is, for example, a liquid solution of 2.5 per cent of carbon in iron at a temperature of 1400°C. This will be represented by the point e in Figure 9. The liquid solution will cool until it reaches a temperature of about 1320°C. At this point, there will begin to precipitate the entity of which mention has been made, namely, iron containing 2.2 per cent carbon in solution. This precipitation will

Robert Allen will receive his B.S. in Mechanical Engineering in January, at which time he will begin his career with a hitch in the Army, with a commission obtained through the ROTC program. Bob is a native of Milwaukee and has attended UW–M. He tells us that he became interested in Iron-Carbon diagrams in ME 530.
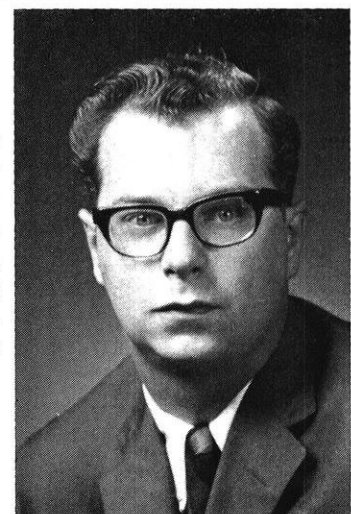
cause the liquid solution to be impoverished in iron, and it will consequently move to the right in the diagram as the temperature falls; or, in other words, it will travel down the line A B. By the time the temperature of 1135°C is reached, a large amount of 2.2 per cent austenite will have precipitated, and the small amount of liquid solution remaining will be at the point B, that is, the eutectic point, where there is 4.3 per cent carbon. With further cooling, the eutectic will cross the point B, and therefore it will complete its precipitation and its freezing. It decomposes into crystals, a portion of which are of tiny flakelets of the austenite before mentioned and the other part, crystals of graphite—i.e., carbon.

A similar result will be obtained in the case of a 3 per cent liquid solution of carbon in iron. This will cool until it reaches 1280°C, where the 2.2 per cent austenite will preceiptate, decreasing the residual solution in iron so that it travels down the line A B and finally reaches the point B, after which this eutectic solution at the point B will precipitate as before.

The preceding two paragraphs may be summed up by saying that any solution of iron containing more than 2.2 per cent and less than 4.3 per cent of carbon will consist, after freezing, of a eutectic together with a certain amount of previously precipitated austenite (consisting of iron with 2.2 per cent of carbon in solid solution).

One question that may arise concerns what will occur in case the solution contains more than 4.3 per cent of carbon. (It is very seldom that solutions contain more than 4.3 per cent of carbon, because this much carbon does not readily disolve in iron.) Take, for example, a liquid solution containing 4.7 per cent of carbon at a temperature of 1200°C. This will cool until a temperature of about 1170° is reached, and the line C B is met. As cooling proceeds to lower temperatures, carbon (i.e., graphite) precipitates out, and the liquid solution remaining moves to the left in the diagram. That is to say, it travels down the line C B. When the temperature 1135° is reached, so much graphite is precipitated that the residual solution is now of

the eutectic proportions. Further cooling results in the eutectic breaking up as previously, consisting thereafter of crystals of graphite and of austenite with 2.2 per cent of carbon.

All the solutions of iron and carbon containing less than 2.2 per cent of carbon will consist, after solidification, of a solid solution of iron and carbon having the same chemical composition as the original liquid solution and being a homogeneous solution of one in the other. There can be no eutectic form if there is not more than 2.2 per cent carbon. All the solutions with more than 2.2 per cent of carbon will consist, after solidification, of a eutectic together with a certain amount of previously precipitated graphite or of the previously precipitated 2.2 per cent austenite mentioned.
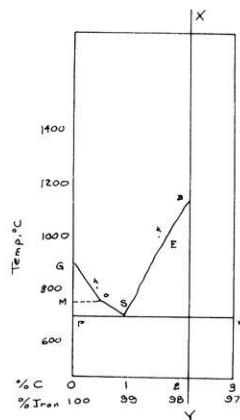

Figure 10.—Decomposition curves of the solid solutions of iron and steel.

The Decomposition of Solid Solutions of Iron and Steel

The second set of curves for the iron-carbon diagram will now be discussed. The curves of decomposition of the solid solutions are shown in Figure 10. The line G O S is the line upon which there is selective precipitation of pure iron. To this pure iron the name of "ferrite" has been given.

There is first considered a solid solution containing 0.40 per cent of carbon at a temperature of 800°C. This will be at the point h in Figure 10. It will cool until it reaches a temperature of about 780°C, at which point ferrite will begin to precipitate. As the temperature continues to fall, more and more ferrite precipitates, which impoverishes the solid solu-

tion in iron and causes it to travel down the line O S. By the time the temperature has reached 690°, the solid solution has reached the point S, corresponding to 0.90 per cent carbon.

There is next considered an alloy containing 1.60 per cent carbon at 1000°C; this will be at the point k. It will cool until it reaches a temperature of about 970°, at which carbide of iron (Fe_3C) will begin to precipitate. This precipitation continues as the temperature falls, constantly decreasing the amount of carbon in the solid solution, which therefore travels down the line E S, until, at 690°, it reaches the point S, where there is 0.90 per cent carbon.

A similar precipitation will occur with all of the solid solutions of iron and carbon. If they contain less than 0.90 per cent carbon, they will begin to precipitate out ferrite when they fall to the line G O S. If they contain more than 0.90 per cent carbon, they will begin to precipitate carbide of iron when they meet the line E S. In either case, the residual solid solution will travel down the line G O S, or else E S, until it reaches the point S, when the temperature has fallen to 690°C. There will then be left some solid solution containing 0.90 per cent carbon and, mixed with this, some previously precipitated ferrite or cementite, as the case may be.

The Complete Iron–Carbon Diagram

The diagrams of Figures 9 and 10 may now be combined into one diagram that shall represent all the changes discussed. This diagram is illustrated in Figure 11.

THE MICRO-CONSTITUENTS OF STEEL

Ferrite, or α–Iron

The structural modification of pure iron at room temperature is called either α-iron or ferrite. Ferrite is quite soft and ductile; in the purity which is encountered commercially, its tensile strength is less than 45,000 psi. It is a ferromagnetic material at temperatures under 1414°F.

Austenite, or γ–Iron

The face-centered modification of iron is called austenite, or γ-iron.

It is the stable form of pure iron at temperatures between 1670°F and 2550°F. Making a direct comparison between the mechanical properties of austenite and ferrite is difficult, because they must be compared at different temperatures. However, at its stable temperatures, austenite is soft and ductile and consequently is well suited to fabrication processes. Most steel forging and rolling operations are performed at 2000°F or above, when the iron is face-centered cubic. Austenite is not ferromagnetic at any temperature.

δ–Iron

Above 2550°F, austenite is no longer the most stable form of iron, since then the crystal structure changes back to a body-centered-cubic phase called δ-iron. δ- is the same as α-iron except for its temperature range, and so it is commonly called δ-ferrite. The solubility of carbon in δ-ferrite is small, but it is appreciably larger than in α-ferrite, because of the higher temperature.

Because of the relative free energies (free energy is the energy which can enter into a chemical reaction) of ferrite and of austenite, ferrite has two temperature ranges of stability, which makes it unique among common engineering materials. The most stable form of any material is the form with the minimum free energy. Below 1670°F and above 2550°F, the body-centered-cubic structure has a lower free energy than the face-centered-cubic structure.

Cementite, or Iron Carbide

In iron-carbide alloys, carbon in excess of the solubility limit must form a second phase, which is often iron carbide (cementite). Iron carbide has the chemical composition of Fe_3C. This does not mean that iron carbide forms molecules of Fe_3C, but simply that the crystal lattice contains iron and carbon atoms in a three-to-one ratio. Fe_3C has an orthorhombic unit cell, with 12 iron atoms and 4 carbon atoms per cell, and thus has a carbon content of 6.67 per cent.

As compared with austenite and ferrite, cementite is very hard. The presence of iron carbide with fer-

THE FINAL PRODUCT
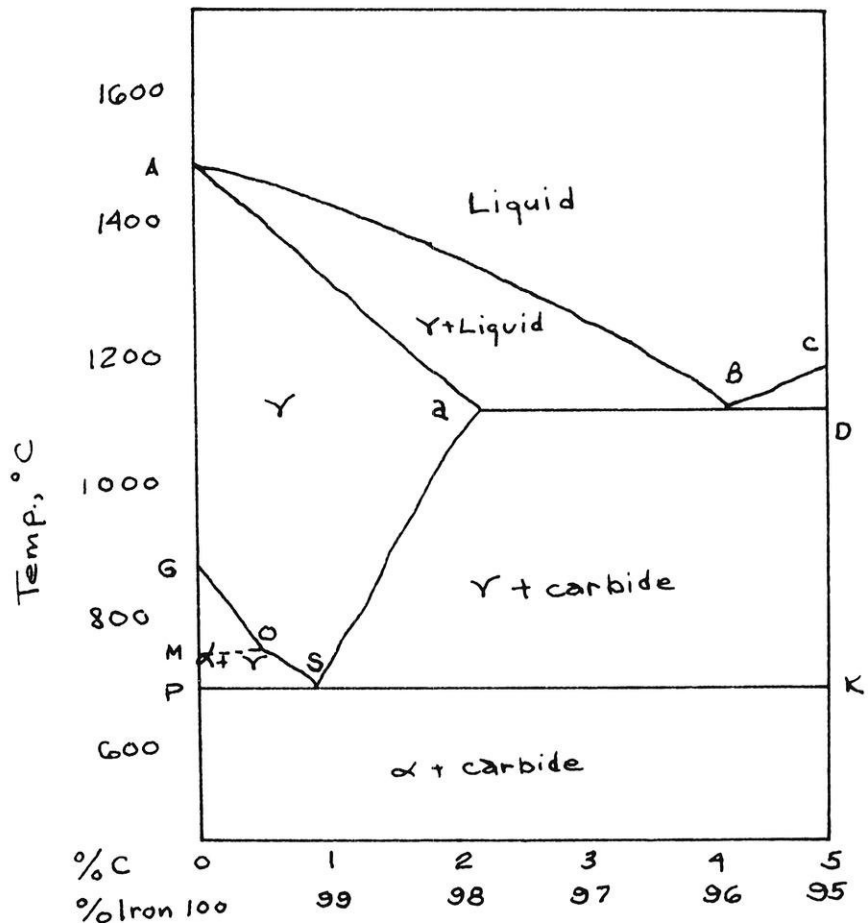
Figure 11.—The freezing and solid decomposition curves of the iron alloys.

rite in steel greatly increases the strength of the steel. However, because pure iron carbide is nonductile and therefore relatively weak by itself, it cannot adjust to stress concentrations.

SUMMARY

(1) Before the iron-carbon diagram was developed, simple solubility curves were constructed to show the basic melting-freezing relationships of two-component systems.

(2) The term *liquidus* is the locus of temperatures above which all compositions are liquid, and *solidus* is the locus of temperatures above which all compositions are solid.

(3) The lowest temperature at which a solution will remain completely liquid is called the *eutectic temperature,* and the amount of the liquid constituents is called the *eutectic composition.*

(4) An inverse relationship called the *lever rule* can be used for determining the relative quantities of two phases that are present in a given mixture at a given temperature.

(5) The basic melting-freezing relationships were employed to develop the iron-carbon diagram.

(6) The construction of the iron-carbon diagram consists mainly of two parts: One is the freezing curves; the second is the solid decomposition curves.

END

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THE WEIBULL
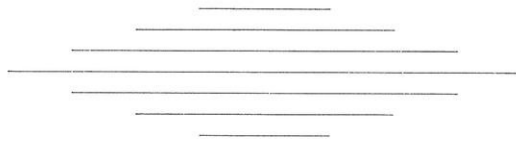
DISTRIBUTION

By

ROBERT ZIMMERLY

IN PLOTTING empirical data, one invariably finds distributions that cannot be described satisfactorily with usual methods such as normal distribution or exponential distribution. The Weibull distribution was discovered about 1950 by Woloddi Weibull of Stockholm, Sweden, in an effort to find a distribution that would correctly describe the pattern set by empirical data that did not follow other known distribution patterns. The Weibull distribution, because its three parameters act as built-in adjustments, can be fitted to distributions of many shapes. For this reason the distribution has been applied successfully to a wide range of applications.

It is the purpose of this article to present, to those readers who have an occasion to plot data, a method for describing the pattern of empirical data which cannot be described by other methods, in the hope that it may prove useful.

It is important to note that the Weibull distribution is not a cure-all nor is it intended to replace other distribution-fitting methods where they apply. It is instead intended as an additional tool to be used where other methods have failed to give satisfactory results.

Included are details on the various characteristics of the Weibull distribution and its parameters. Also presented are illustrations to help demonstrate these characteristics and an appendix which includes a step-by-step method of finding the parameters.

THEORY

The Weibull distribution is a cumulative distribution that has wide application possibilities, because of the built-in flexibility provided by the three parameters it uses. The origin of the distribution and methods of finding the parameters are included in this section. Emphasis is put on the graphical method of finding the parameters because the other methods are better handled using a computer and would require more space than is available in this paper to explain adequately.

Background

A cumulative distribution function of some variable X can be defined as $F(x) = n/N$

where

n = No. of individuals with $X \leqq x$

N = Total No. of individuals in the population.

The following simple example further demonstrates how a distribution function is determined:

Example 1. Given a sample of twenty steel specimens whose yield strength is plotted as follows:
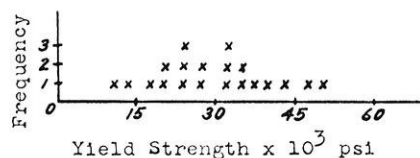
Figure 1.—Frequency vs. yield strength for twenty steel specimens.

If $F(x) = n/N$

Then $F(15) = 2/20 = 0.1$
$F(30) = 10/20 = 0.5$
$F(45) = 18/20 = 0.9$
$F(60) = 20/20 = 1.0$

or

$F(15) = 10\%$
$F(30) = 50\%$
$F(45) = 90\%$
$F(60) = 100\%$

A cumulative distribution function can also be represented by an equation of the form:

$$F(x) = 1 - e^{-\lambda(x)} \quad \dots\dots\dots [1]$$

Weibull took equation [1] and defined the function $\lambda(x)$ so that it is always positive, nondecreasing, and vanishing at the value γ. He found the simplest possible function to fulfill these conditions to be:

$$\lambda(x) = \frac{(x - \gamma)^\beta}{a}$$

Making this substitution in equation [1] gives us the Weibull distribution equation:

$$F(x) = 1 - e^{-\frac{(x - \gamma)^\beta}{a}} \quad \text{where } x \geqq \gamma \quad \dots\dots\dots [2]$$

It should be noted that there is no theoretical method of obtaining the Weibull distribution equation. It is just the simplest expression possible that fulfills the necessary conditions.

Weibull Distribution Parameters

The Weibull parameters are what give the Weibull distribution its flexibility. There are three parameters in all and they have been given names for the effect is a list of these parameters followed by a graphical representation:

1. a = Scale parameter, any positive real number.
2. β = Shape parameter, any positive real number.
3. γ = Location parameter, any real number which may or may not be equal to zero, such that $x \geqq \gamma$.

From Figure 2 it can be seen that the function may be shaped, scaled, and shifted to fit many distributions.

The simplest method of estimating the parameters, and the method of most interest, is the
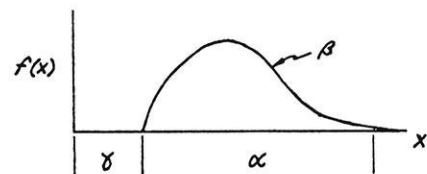

Figure 2.—Graphical representation of Weibull parameters.

graphical solution. This method transforms the Weibull distribution function, equation [2], into a form suitable for plotting as a straight line on special Weibull Probability Paper. Once this is accomplished the three parameters can then be estimated by using the procedure outlined in the adjacent box.

There are four other methods of determining the Weibull parameters available:

1. The method of least squares for transformed data.
2. The method of maximum likelihood for ungrouped data.
3. The method of maximum likelihood for grouped data.
4. The method minimized chi-squares.

These four methods can be adapted to computer solution and are far too involved to include in this article. Those interested in examining them more fully are referred to references 2 and 4.

GRAPHICAL ESTIMATION OF PARAMETERS

The simplest method of estimating the parameters, and the method of most interest, is the graphical solution. This may be accomplished by taking the Weibull distribution function, equation [2], and rearranging it into the following form:

$$1 - F(x) = e^{-\frac{(x-\gamma)^\beta}{a}}$$

inverting

$$\frac{1}{1 - F(x)} = e^{\frac{(x-\gamma)^\beta}{a}}$$

taking the ln of both sides

$$\ln\left(\frac{1}{1 - F(x)}\right) = \frac{(x-\gamma)^\beta}{a}$$

taking in the ln of both sides again

$$\ln\ln\left(\frac{1}{1 - F(x)}\right) = -\ln a + \beta \ln(x - \gamma) \quad \ldots\ldots\ldots [6]$$

Notice that this equation is the straight line equation of the form $y = a + mx$ where $-\ln a$ is the intercept and β is the slope. Using Weibull Probability Paper (WPP), which has a ln ln ordinate and a ln abscissa, equation [6] will appear as a straight line from which the parameters can be estimated. Below is a step-by-step procedure to follow in estimating the parameters and makes reference to Figure 3.

Step 1. Plot $F(x)$ vs. x, where $F(x) = n/N$ as shown previously in example 1. Suppose this gives curve "A".
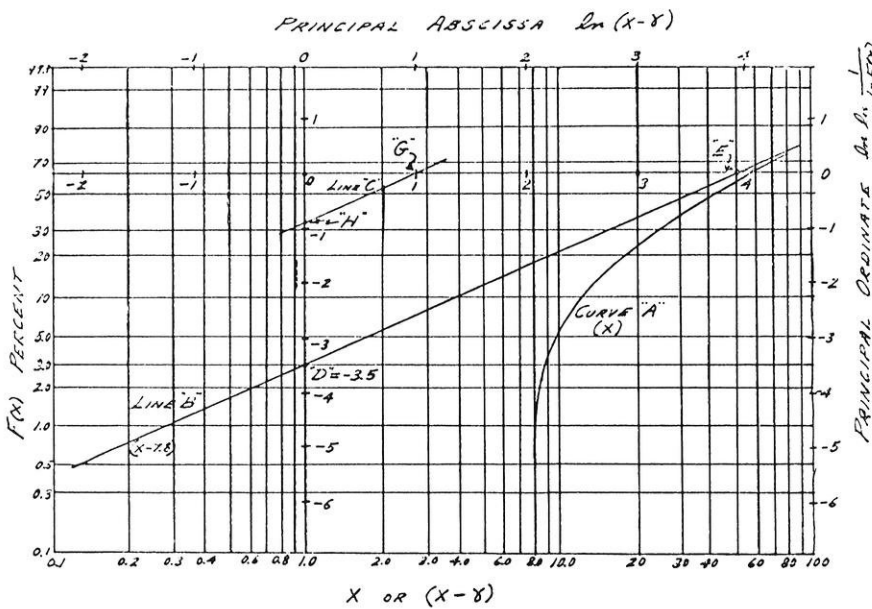
Figure 3.—Weibull probability paper showing the scales and method of plotting for a simple Weibull distribution.

SIMPLE WEIBULL DISTRIBUTIONS

The simple Weibull distribution is probably the most commonly used form of the distribution. It arises in cases where all the members of a population conform to one distribution and therefore only one set of parameters is needed to fully define the function.

Cumulative Distribution Function

The simple Weibull cumulative distribution function (c.d.f.) is defined by the equation:

$$F(x) = 1 - e^{-\frac{(x-\gamma)^\beta}{a}} \quad \begin{array}{l} x \geqq \gamma \\ a, \beta \text{ Positive} \end{array}$$
$$\ldots\ldots\ldots\ldots [2]$$

This equation is shown plotted in Figure 5 below for various values of β, to demonstrate the versatility built into the function by the parameters.

Notice that this graph is plotted with $F(x)$ vs. $(x - \gamma)$. The same graph would result if the distribution being plotted had for its location parameter a value of $\gamma = 0$ and were plotted as $F(x)$ vs. x. The scale parameter (a) has the effect of increasing or decreasing the length of the curves to fit the particular data being plotted.

Probability Density Function

A common method of representing a distribution is by its probability density function. The simple Weibull probability density function (p.d.f.) is obtained by taking the derivative of the cumulative distribution function and is defined by the equation:

$$f(x) = \frac{d(F(x))}{dx} =$$

$$\frac{\beta(x-\gamma)^{\beta-1}}{a} e^{-\frac{(x-\gamma)^\beta}{a}} \quad \begin{array}{l} x \geqq \gamma \\ a, \beta \text{ Positive} \end{array}$$

$$[3]$$

Plotting this equation for $f(x)$ vs. $(x - \gamma)$ will give curves similar to those shown in Figure 6.

Notice how the curve for $(\beta = 1)$ closely resembles an exponential distribution, and the curve for $(\beta = 3)$ resembles a normal distribution. Ordinarily, the Weibull distribution would not be used to represent an expo-
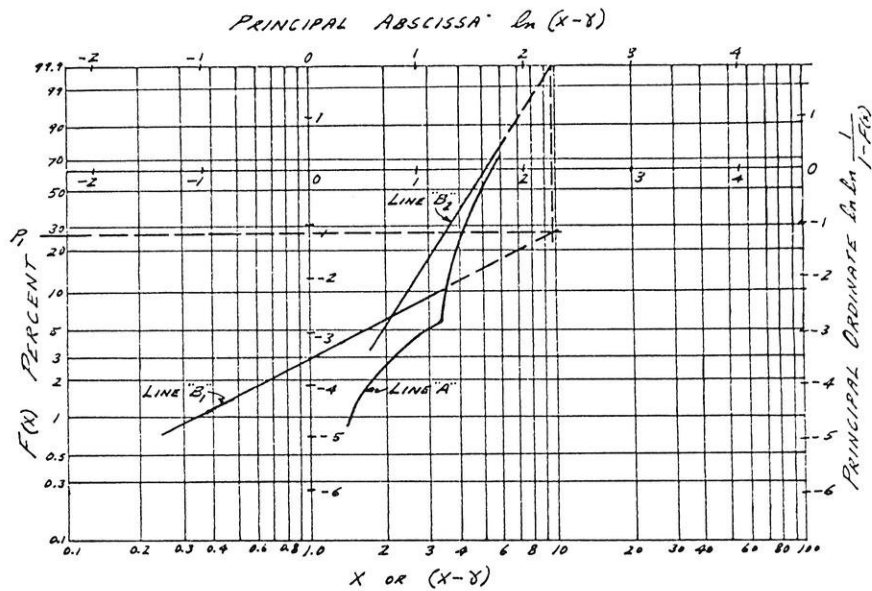
Figure 4.—Weibull probability paper showing the scales and method of plotting for a mixed Weibull distribution.

Step 2. By trial and error find a value for γ such that $(x - \gamma)$ for all points will give a straight line. For this example, $\gamma = 7.8$ will give the straight line "B". Therefore, the location parameter estimate $\hat{\gamma} = 7.8$ in this case.

Step 3. At point "D" the ln $(x - \gamma) = 0$, therefore the $\ln a =$ $-\ln \ln \dfrac{1}{1 - F(x)}$ and at point "E" the $\ln \ln \dfrac{1}{1 - F(x)} = 0$ which gives a value for $\ln(x - \gamma)$. From the definition of a slope the shape parameter estimate $\hat{\beta}$ can be found by the equation:

$$\hat{\beta} = \frac{\left(-\ln \ln \left(\dfrac{1}{1 - F(x)}\right)\right) @ \text{Pt. "D"}}{(\ln(x - \gamma)) @ \text{Pt. "E"}}$$

Therefore

$$\hat{\beta} = \frac{-(-3.5)}{3.9} = +0.9$$

Step 3. Alternate. By letting ln $(x - \gamma) = 1$ and plotting this point as "G" on the graph, and a line "C" drawn through "G" parallel to line "B" will give the $\hat{\beta}$ estimate directly as

$$\hat{\beta} = \frac{-\ln \ln \left(\dfrac{1}{1 - F(x)}\right)}{1}$$

$$= -(-0.9) = +0.9$$

and shown on the graph at point "H".

Step 4. At point "D" on line "B" it can be seen that:

$$\ln a = -\ln \ln \left(\frac{1}{1 - F(x)}\right)$$

$$a = e^{-\ln \ln \left(\frac{1}{1 - F(x)}\right)}$$

The scale parameter estimate a can then be determined and for this example is found to be:

$$a = e^{-(-3.5)} = e^{3.5} = 33.12$$

Substituting these estimates into equation [2] gives the Weibull distribution function for this equation as:

$$F(x) = 1 - e^{-\frac{(x - 7.8)^{0.9}}{33.12}} \quad x \geq 7.8$$

The mixed Weibull distribution is actually a summation of various simple Weibull distributions. In this case a separate set of parameters are needed for each subdistribution. These may be found by an extension of the procedure just used for finding the simple Weibull distribution parameters.

Begin by plotting F(x) vs. x on Weibull Probability Paper just as before. It is possible, that up to this point it is still unknown whether the distribution is simple or mixed, but by plotting F(x) vs. x, the type of distribution will soon become apparent. Unlike the simple Weibull distribution, which appears as one smooth curve, the mixed Weibull distribution will appear as a series of smooth curves connected by an abrupt transition as shown by curve "A" in Figure 4. These transitions are an indication of subdistributions and therefore indicate a mixed Weibull distribution.

Repeat For Each Segment

Once F(x) vs. x has been plotted the procedure outlined previously, for a simple Weibull distribution, is repeated for each section of smooth curve. This lives a set of parameters for each subdistribution which can then be applied to equation [4] to define the complete function.

An additional step can be taken to get an estimate of the proportionality constant P_1 by extending the lines B_1 and B_2 in the manner shown in Figure 8 by the dotted lines. The constant P_1 is then read from the F(x) scale. Since $\overset{k}{\underset{i=1}{\Sigma}} P_i = 1$, P_2 is found by subtracting P_1 from 1.
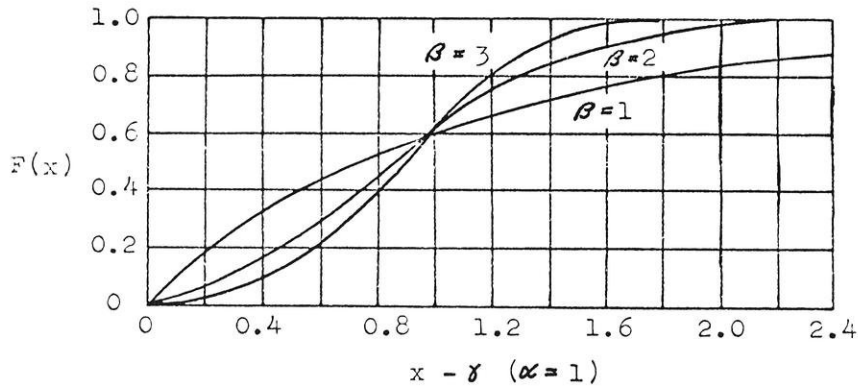
Figure 5.—Sample plot of F(x) vs. (x−γ) showing the effect of varying the parameter β.

nential or a normal distribution because of the increased work involved in estimating the parameters. However, the graph demonstrates again the wide versatility of the Weibull distribution.

MIXED WEIBULL DISTRIBUTION

The mixed Weibull distribution is very useful in cases where a sample contains a combination of two or more subdistributions. The resulting Weibull distribution is simply a summation of the various subdistributions. A separate set of parameters is then required for each subdistribution in order to fully define the function. The method for finding the different sets of parameters is explained in the box on page.......

Cumulative Distribution Function

The mixed Weibull cumulative distribution function (c.d.f.) is defined by the following equation:

$$F(x) = P_1F_1(x) + P_2F_2(x) + \cdots + P_kF_k(x)$$

This can be written as:

$$F(x) = \sum_{i=1}^{k} P_iF_i(x) \quad \ldots\ldots\ldots [4]$$

$$0 \leq P_i \leq 1, \sum_{i=1}^{k} P_i = 1$$

and

$$F_i(x) = 1 - e^{-\frac{(x-\gamma_i)^{\beta_i}}{a_i}}$$

Equation [4] is shown plotted in Figure 7 for the case where K = 2.

The above graph demonstrates how two subdistributions are added to make up the mixed Weibull c.d.f. It should be noted that, theoretically any number of subdistributions could be added in

this manner. However, from a practical standpoint, because of the difficulty in estimating the parameters, the mixed Weibull distribution is probably limited to three subdistributions.

Probability Density Function

The probability density function (p.d.f.) for the mixed Weibull distribution is obtained by the same method used in finding the c.d.f. and is defined by the equation:

$$f(x) = \sum_{i=1}^{k} P_if_i(x) \quad \ldots\ldots\ldots\ldots [5]$$

where

$$f_i(x) = \frac{\beta_i(x-\gamma_i)^{\beta_i-1}}{a_i} e^{-\frac{(x-\gamma_i)^{\beta_i}}{a_i}}$$

An example of equation [5], where K = 2, is plotted in Figure 8 below.

This graph demonstrates how the mixed Weibull p.d.f. can be obtained for any value of x even though more than one subdistribution is involved. It also shows for

what values of x the various subdistributions become important to the overall distribution.

CONCLUSION

The Weibull distribution is very flexible and because of this can be very useful in plotting data. The flexibility of the distribution is provided by its three parameters which act as built-in adjustments that can be used to shape the distribution to fit the particular data at hand. This paper is presented with the intention of providing those individuals who are involved with plotting data with information on how to apply the Weibull distribution, with the hope that the distribution may prove useful to them.

END

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4. Kao, John H. K.,—"A New Life-Quality Measure for Electron Tubes," Institute of Radio Engineers Transactions on Reliability and Quality Control, pp. 1–11, PGRQC—7, April, 1957.
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Figure 6.—Sample plot of f(x) vs. (x−γ) showing the effect of varying the parameter β.

$$\alpha_1 = \alpha_2 = 1 \ , \quad \beta_1 = \tfrac{1}{2} \ , \quad P = 0.2 \ , \quad \gamma_1 = 0$$
$$\beta_2 = 3 \qquad Q = 0.8 \qquad \gamma_2 = 0.4$$
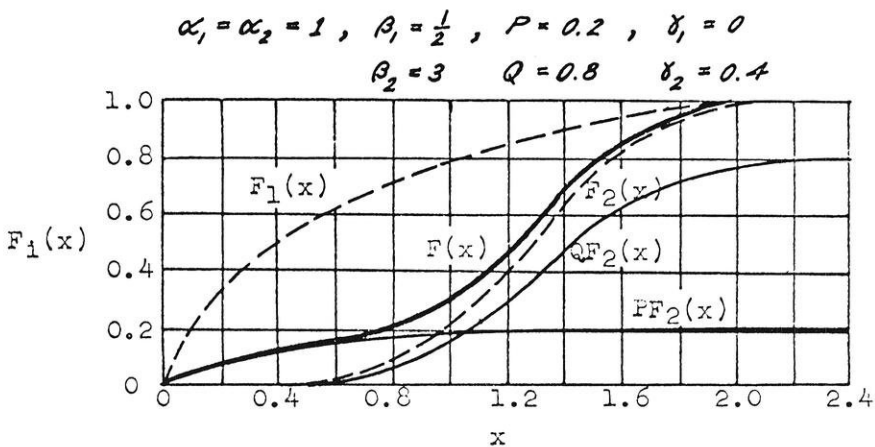
Figure 7.—Sample plot of $F_1(x)$ vs. x for the mixed Weibull c.d.f.

$$\alpha_1 = \alpha_2 = 1 \ , \quad \beta_1 = \tfrac{1}{2} \ , \quad P = 0.2 \ , \quad \gamma_1 = 0$$
$$\beta_2 = 3 \qquad Q = 0.8 \qquad \gamma_2 = 0.4$$
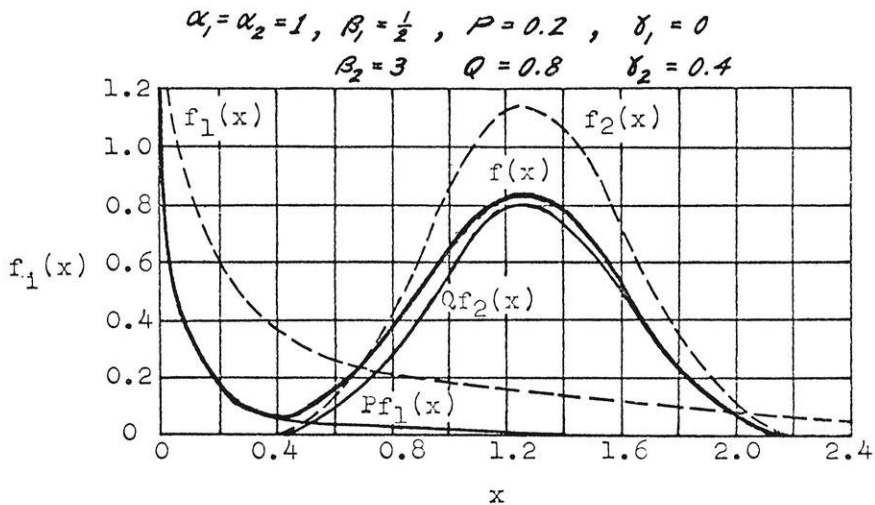
Figure 8.—Sample plot of $f_1(x)$ vs. x for the mixed Weibull p.d.f.
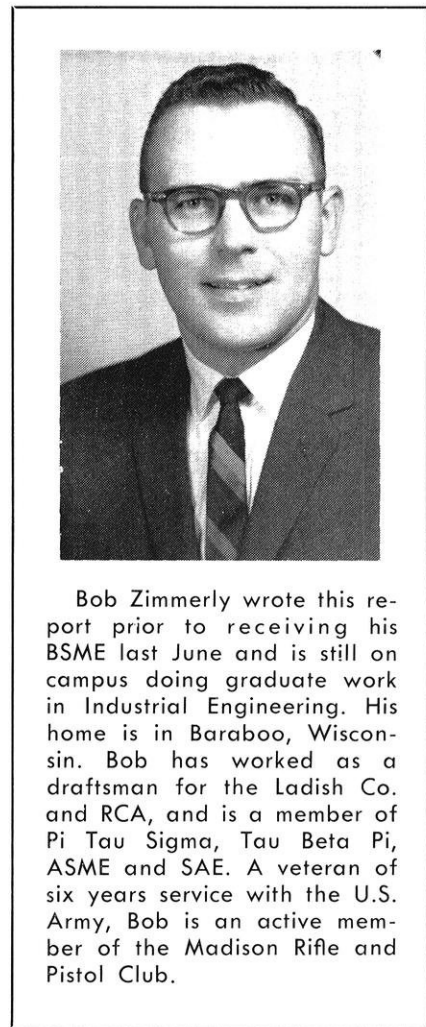
Bob Zimmerly wrote this report prior to receiving his BSME last June and is still on campus doing graduate work in Industrial Engineering. His home is in Baraboo, Wisconsin. Bob has worked as a draftsman for the Ladish Co. and RCA, and is a member of Pi Tau Sigma, Tau Beta Pi, ASME and SAE. A veteran of six years service with the U.S. Army, Bob is an active member of the Madison Rifle and Pistol Club.

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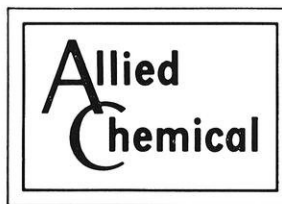
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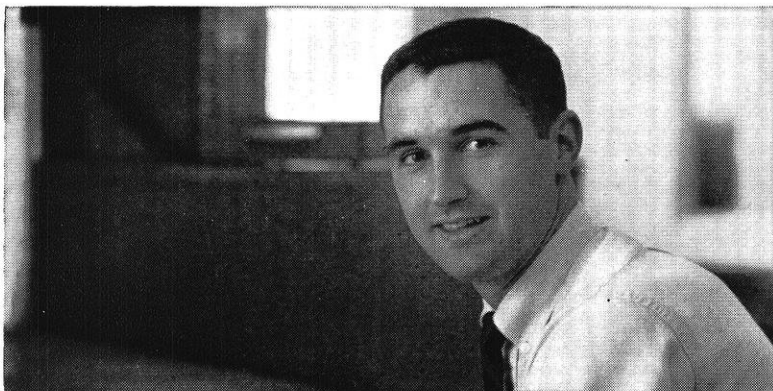
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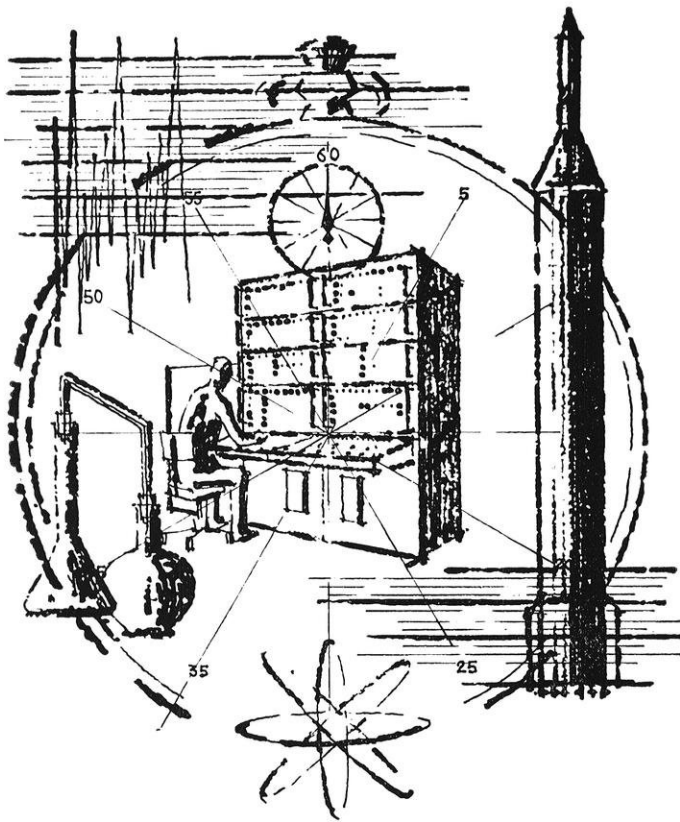
SCIENCE HIGHLIGHTS

A brief resume of new developments in government and industry, compiled by the *Wisconsin Engineer* staff

U.S.–U.S.S.R. REACH UNDERSTANDINGS ON SPACE COOPERATION

Representatives of the U.S. National Aeronautics and Space Administration and the U.S.S.R. Soviet Academy of Sciences reached two satisfactory understandings in discussions on space cooperation in New York during October.

The first understanding reaffirms the existing agreement for the exchange of weather satellite data between Washington and Moscow. It is now understood by representatives of both sides that satellite data is expected to be available on a continuing basis from both sides within a few months. Meanwhile, the present daily and useful exchange of conventional weather data will continue.

The second step is a new agreement for the preparation and publication of a joint review of research in space biology and medicine in the two countries. This agreement provides for a joint Editorial Board and for full cooperation by both sides in the preparation of materials available in the two countries, the selection of authors, and the publication of their work, probably in two or more volumes. The agreement goes into effect automatically in the absence of objection from either side within two months. A text of the agreement will be made available only after the conclusion of the two-month waiting period.

The negotiators were Academician A. A. Blagonravov and Professor A. A. Nichiporovich for the U.S.S.R. and Dr. John W. Townsend, Deputy Director, Goddard Space Flight Center, and Arnold W. Frutkin, NASA Assistant Administrator for International Affairs for the U.S.

SE WISCONSIN BOASTS NEW TYPE NATURAL GAS PLANT

The first commercial liquefied natural gas storage plant to become operational in the nation was dedicated recently at Oak Creek, Wisconsin by Wisconsin Natural Gas Company.

The $2,000,000 "peak shaving" plant is designed to liquefy and store a huge reserve of natural gas for very cold days when consumer demand is highest. During these peak use periods, the liquefied na-
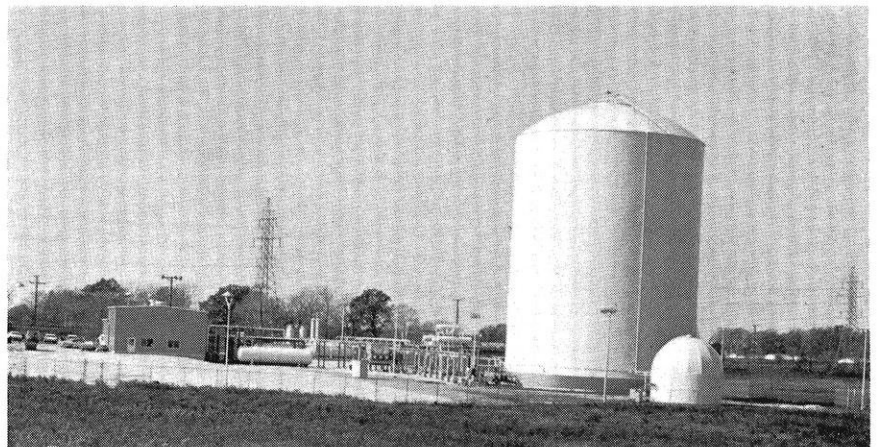
First commercial LNG storage plant to become operational in the nation, this is Wisconsin Natural Gas Company's $2,000,000 facility at Oak Creek, Wisconsin. Keystone of the peak shaving facility is the 3,000,000-gallon cryogenic LNG storage "tank-within-a-tank" (center). The structure's inner vessel is the world's largest aluminum tank for cryogenic service. Small tank at right is a "breather" vessel for nitrogen-purged insulation space between shells of the large tank. At the left is the natural gas liquefaction plant.

tural gas will be re-vaporized and injected into the distribution system to supplement the normal supply.

Filling of the cryogenic facility's three million gallon aboveground metal storage tank was commenced September 25 by Chicago Bridge & Iron Company, prime contractor for design, construction and start-up of the complete plant.

Located on a 36-acre site, the new plant will provide improved, economical service to more than 100,000 customers in the Racine-based company's seven-county operating area.

Keystone of the LNG plant is the double wall tank—125 feet tall and 81½ feet in diameter overall—with insulating material between the inner and outer vessels. The inner tank is the largest aluminum vessel in the world for cryogenic service. With a shell height of 101 feet and diameter of 71½ feet, it will hold three million gallons of gas liquefied at about 260 degrees below zero, the equivalent of 250 million cubic feet of natural gas. The outer tank is built of carbon steel.

A five-stage cooling process shrinks the natural gas storage ratio from 630 cubic feet to one cubic foot in liquid form.

The LNG plant liquefies and stores gas received from a nearby company gas main during periods when weather is mild and the demand for gas is low. Then, on very cold days when demand is at a peak the stored liquid natural gas will be vaporized, warmed to normal temperature of about 50 degrees and returned to the gas main.

One million cubic feet of natural gas is liquefied each day during normal operation. The plant will be able to supply up to 50 million cubic feet of gas a day to company mains on extremely cold days.

In addition to the outdoor storage tank, liquefaction unit and vaporizing equipment, the plant includes a one story building to house compressors and auxiliary equipment as well as a control room. The plant operator monitors a pneumatic control board which records gas levels and pressures at all points in the plant. Two identical vapor systems provide 100 per cent standby capacity.
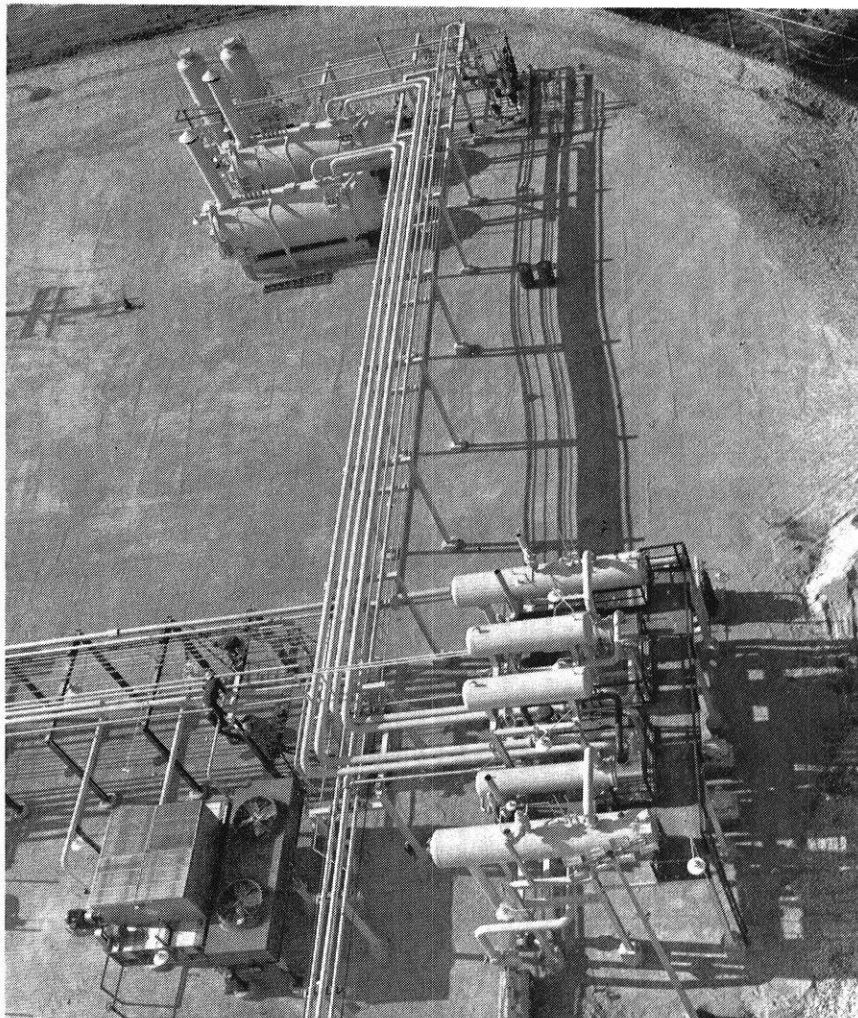
Looking down on liquefied natural gas vaporizing unit at LNG peak shaving plant. Here, water heated in boilers (background) is circulated to heat exchangers (foreground) to vaporize LNG pumped from the double wall storage tank, send it out to consumers at main pressure. Maximum send-out capacity is 50 million standard cubic feet a day. Normal operating rate is 25 million standard cubic feet daily with a 100 per cent standby capability.
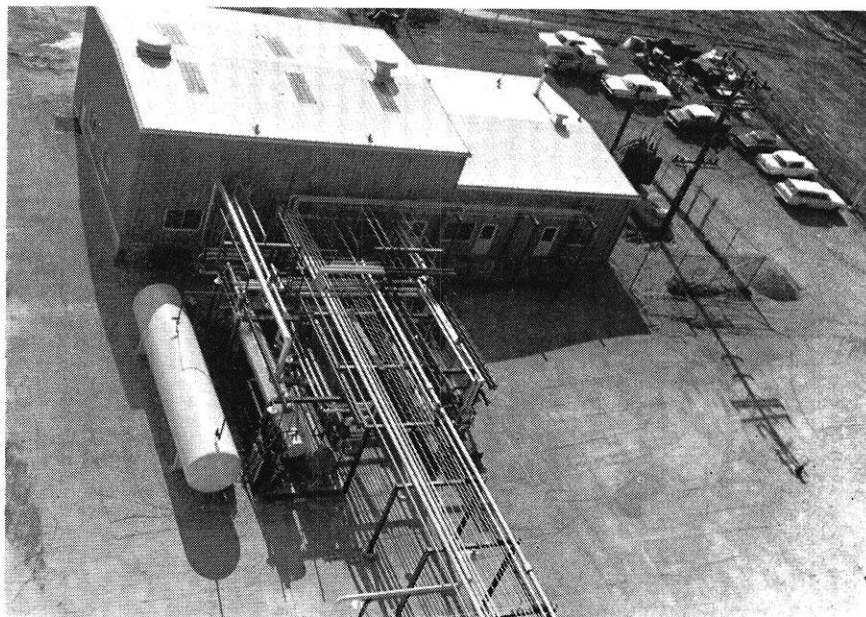
Skid mounted outdoor liquefaction plant (center) and building which houses compressors, control panel and auxiliary equipment at the LNG plant. Long tank at immediate left of liquefaction plant is an ethylene expansion tank, part of the "cascade" refrigerating system which utilizes propane, ethylene and natural gas to achieve the LNG's minus 260 degree storage temperature.
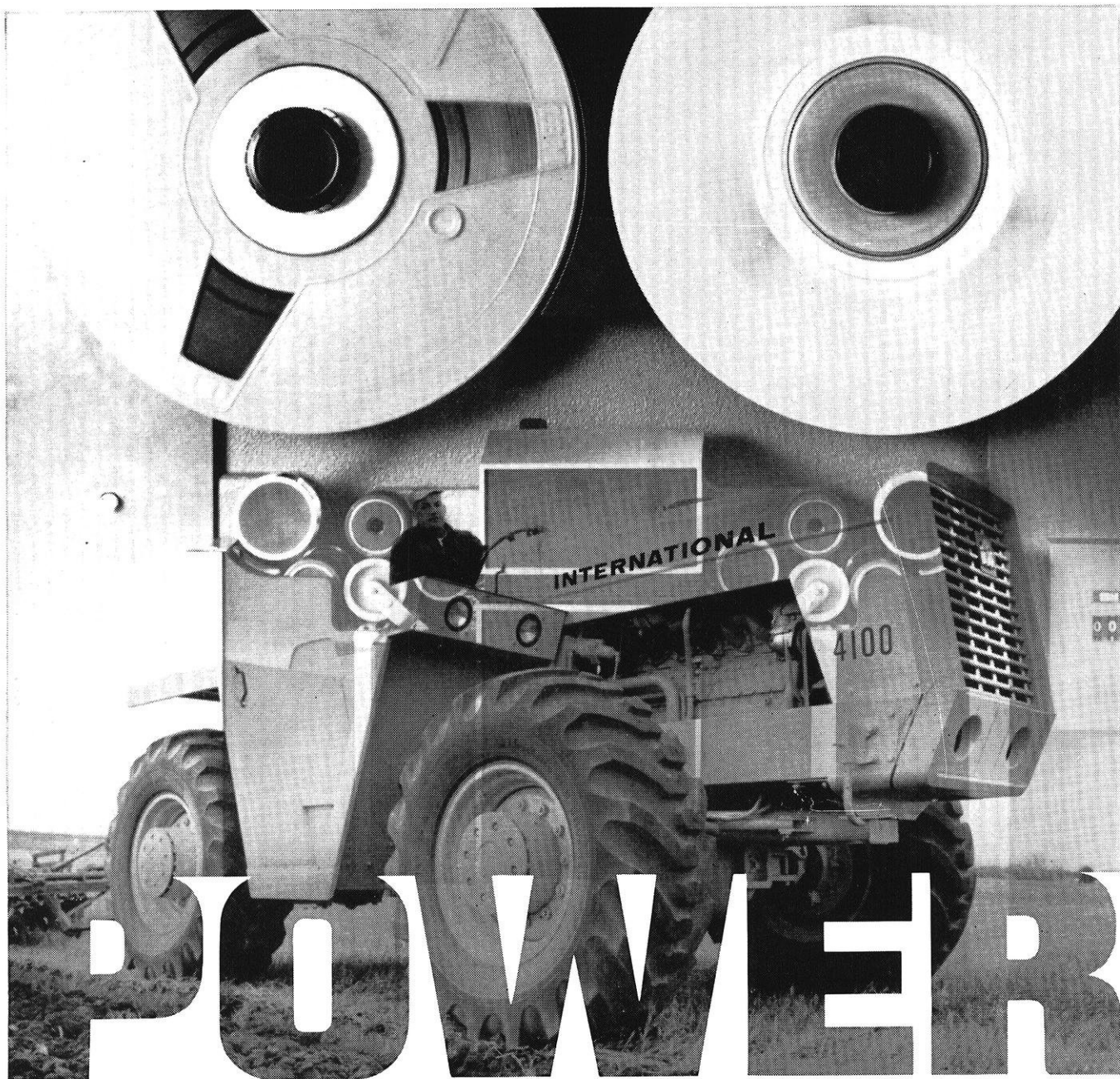
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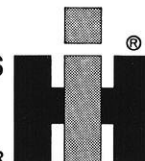
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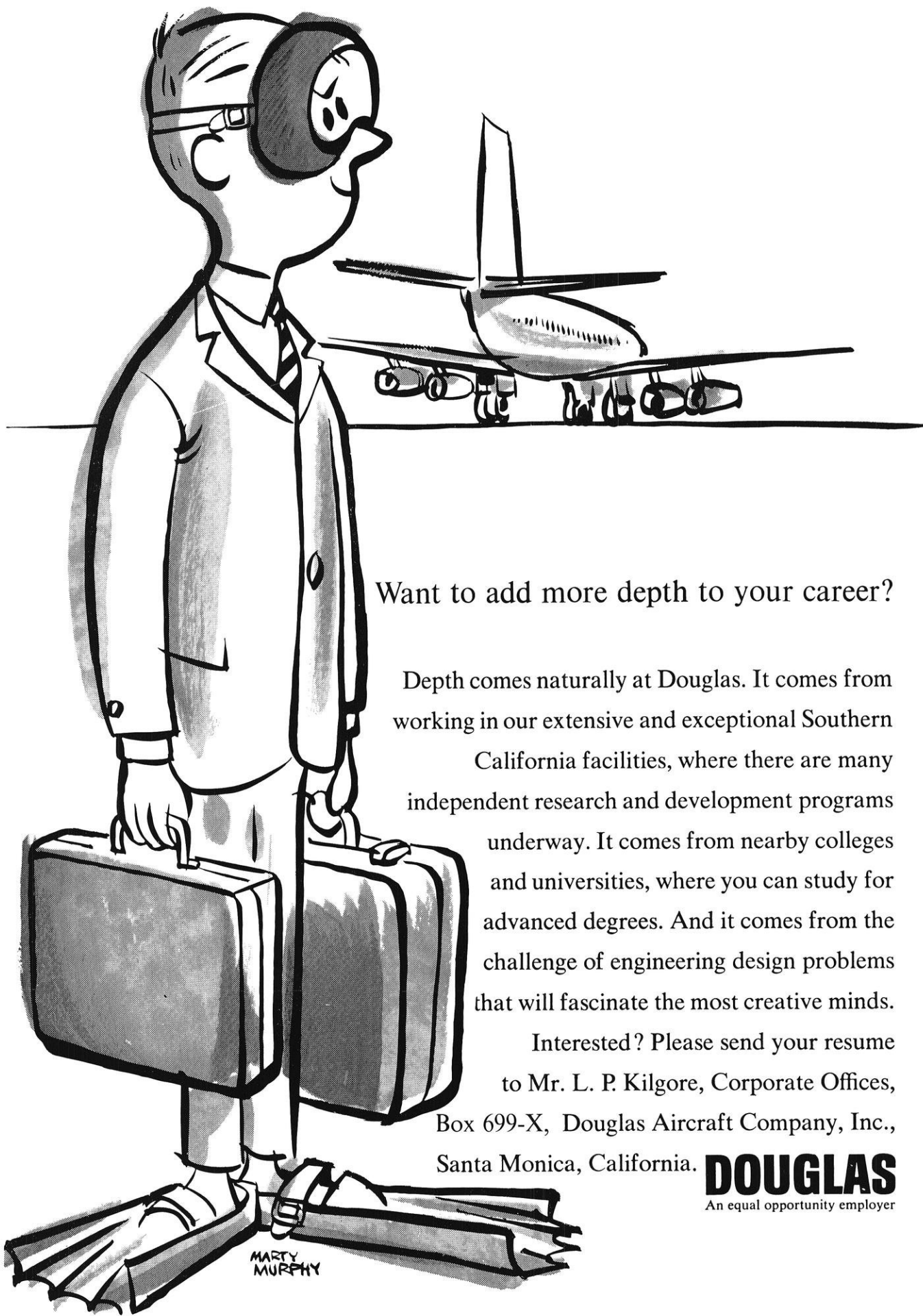
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Index to Articles, 1965

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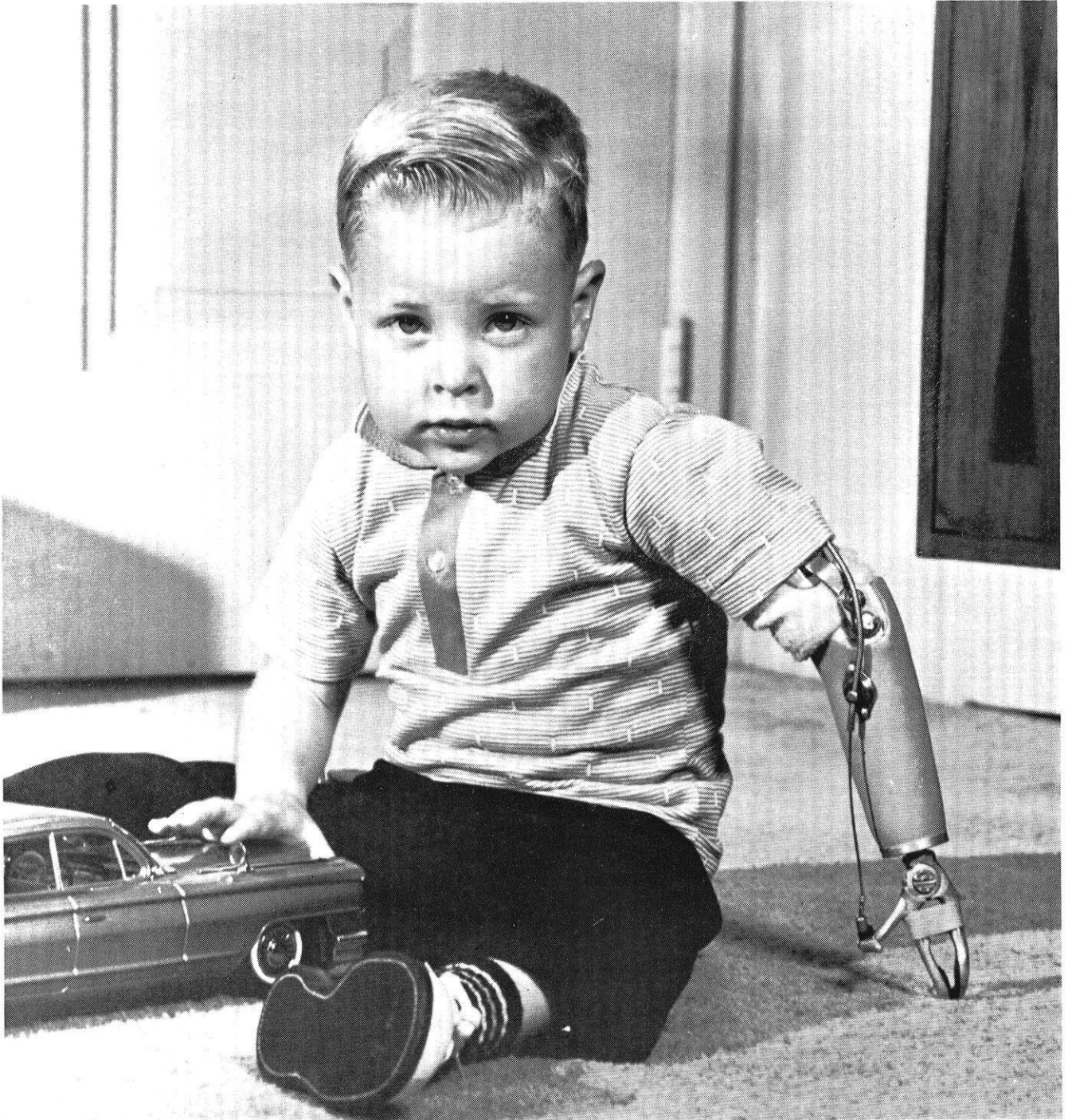
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A little Jewish boy was trying to talk a little Catholic girl to go swimming with him.

She: "But I don't have a swimming suit."

He: "We don't need any, we can swim in our birthday suits."

Finally the little Catholic girl agreed and they took off their clothes to go swimming.

The little Jewish boy looked at her and said, "Now, I see what the difference is between the Catholics and the Jews."

* * *

"Lips that touch liquor shall never touch mine."

"Your lips?"

"No, my liquor."

* * *

Then there was the janitor who worked in the girls' dorm and was entrusted with a pass-key to every room in the building. The following week the dean ran across him and asked, "Why didn't you come around Friday for your pay, John?"

"What! Do I get wages, too?"

* * *

Truck driver stopped beside a stalled Volkswagen on highway: "What's the matter buddy, need a new flint?"

* * *

"I don't like Bill," confided a coed to her roommate. He knows too many naughty songs."

"Does he sing them to you?" asked her friend.

"Well, no — but he whistles them."

And then there was the Arts student who let his EE roommate fix him up for a blind date with Allis-Chalmers.

* * *

"Elderly man to beautiful young girl: "We're made for each other! You're stacked and I'm loaded!"

* * *

The bright student looked long and thoughtfully at the second examination question, which read: "State the number of tons of coal shipped out of the United States in any given year." Then his brow cleared and he wrote:

"1492—none."

* * *

An adult is a person who has stopped growing at both ends and started growing in the middle.

* * *

"Next case," yelled the magistrate
    as I walked up to the bar.
For a man who liked to have a
    good time
The judge said I'd gone too far.
    But he believed my story,
And thought it quite a feat.
    For I'd been in a theater
When a girl said, "Hold my seat."

* * *

My sister-in-law received this thank-you note from a recent bride:

"Thank you for the lovely gift. George and I will use it when entertaining our friends."

Her gift to them? Sheets.

A liquor salesman, a food salesman and a mattress salesman were sitting around and chatting.

"Y'know I hate to see a woman drink alone," the liquor salesman said.

The food salesman countered with, "I hate to see a woman eat alone."

The mattress salesman said, "Say, what do you think of the weather we've been having lately?"

* * *

These days the necessities of life cost you about three times what they used to, and half the time you find they aren't even fit to drink.

* * *

C.E.: "Boy I didn't get a bit of sleep last night."

Ag.E.: "Why was that?"

C.E.: "The shade was up."

Ag.E.: "Why didn't you pull it down?"

C.E.: "What did you expect me to do, reach across the street."

* * *

Have you heard about the girl they call turnpike because there isn't a curve in sight?

* * *

"What shall I do?" said the sweet young thing. "I'm engaged to a man who says he just can't bear children?"

"Well my dear," said grandma, with a twinkle in her eye, "You shouldn't expect too much of a husband."

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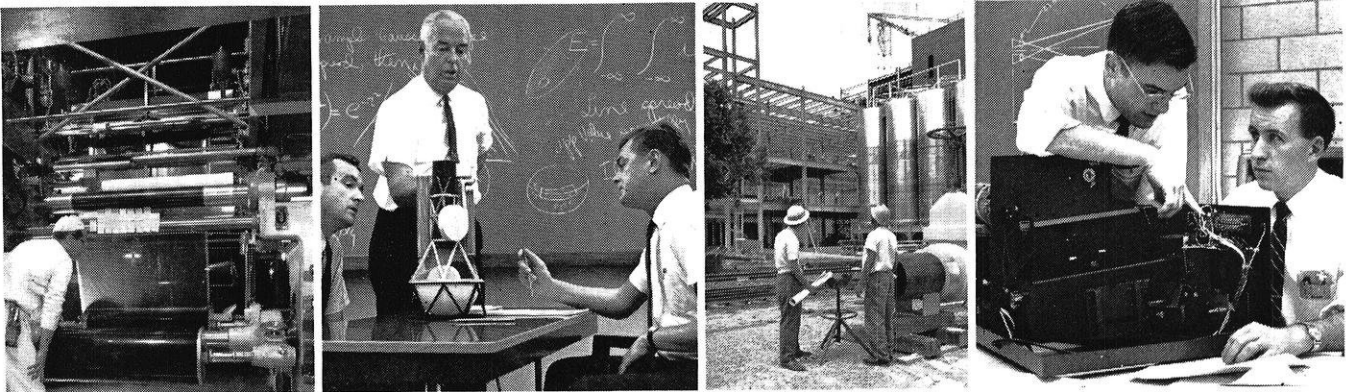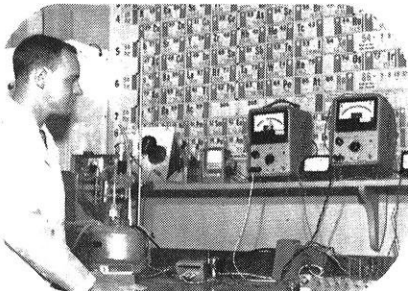

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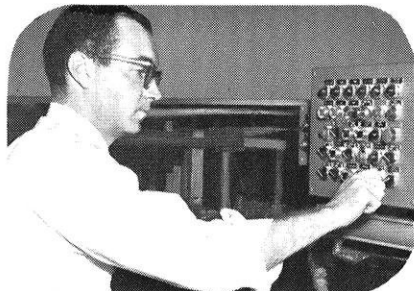
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OUTPUT VOLTAGES from nickel-cadmium cells are examined by engineer John Bliven, BSEE, Union College '63 on assignment at G.E.'s Battery Business Section.

PRODUCT RELIABILITY of electric slicing knife components is the responsibility of Mike Reynolds, BSME, New Mexico State, a recent Manufacturing Training Program graduate.

PRICE AND DELIVERY information on nickel-cadmium batteries is supplied by Bob Cook, BSME, Univ. of Florida '65 on a Technical Marketing Program assignment in Gainesville.

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