ESSAYS ON LIQUIDITY AND MONETARY POLICY

by

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Abstract

This dissertation consists of two self-contained essays on liquidity and monetary policy, organized in the form of two chapters.

In the first chapter I study the effects of monetary policy, in the sense of fully anticipated inflation, on economic growth through entrepreneurship. In the model, money is needed because production costs need to be paid before revenue is received. I demonstrate how the existence of a borrowing constraint gives entrepreneurs an incentive to hold money, that is, to reduce the missed profit opportunity due to liquidity constraints when receiving favorable productivity shocks. Then I show that when agents are allowed to make occupational choices of being either workers or entrepreneurs, long-run inflation, at low levels, can increase growth by encouraging entrepreneurship. This channel is novel and provides a theoretical rationale for the empirical findings that growth can increase with long-run inflation for low inflation rates.

The second chapter studies economies where houses, in addition to providing utility as shelter, may also facilitate credit transactions, since home equity can be used as collateral. We document there were big increases in home-equity-backed consumption loans coinciding with the start of the house price boom, and suggest an explanation. When it can be used as collateral, housing can bear a liquidity premium. Since liquidity is endogenous, even when fundamentals are constant and agents fully rational house prices can display complicated equilibrium paths resembling bubbles. Our framework is tractable, with exogenous or with endogenous supply, yet captures several salient features of housing markets. Numerical work shows the model can capture may features of the data. The effects of monetary policy are also discussed.

Part I

Inflation, Entrepreneurship and Growth

1 Introduction

Long-run economic growth is one of the most important topics in macroeconomics and long-run inflation is found to be one of the most important and robust factors affecting long-run growth[1]. A classic question is how does inflation influence growth. Economists have proposed various mechanisms. Tobin (1965), Sidrauski (1967a, 1967b), and Stockman (1981) debated on whether inflation encourages or discourages the accumulation of physical capital. Subsequent studies, such as Chari, Jones and Manuelli (1995), studied how inflation distort the accumulation of physical and human capital. See Gillman and Kejak (2005) for an extensive survey of this literature. More recently, Berentsen, Breu and Shi (2009) and Chiu, Meh and Wright (2011) developed models where inflation discourages decentralized trades of either "innovation goods" or "new ideas", which are growth-enhancing. This paper contributes to the study of inflation and growth by investigating a novel channel: entrepreneurship, that is, people's occupational choices of either being an entrepreneur or a worker.

The idea that economic growth is driven by entrepreneurial innovations can be traced back to Schumpeter (1934). There is an extensive theoretical literature linking

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[1] See Durlauf, Kourtellos and Tan (2009).

economic growth to entrepreneurship. See Quadrini (2009) for a survey of the literature on entrepreneurship. These studies tend to share a common implication: policies or institutional changes that encourage entrepreneurial activities lead to higher growth. For example, King and Levine (1993) have argued that financial development can enhance economic growth by funding potential entrepreneurs with technology advancing projects. Aghion, et al. (2009) proposed a theory in which higher domestic saving help local entrepreneurs form cooperative ventures with foreign investors so they can adopt frontier technologies faster, generating faster economic growth. If entrepreneurial activity is one of the driven force for economic growth, then it makes sense to ask whether and how does inflation affect entrepreneurship when one is interested in the relationship between inflation and growth. I will try to address this question theoretically in this paper.

I will take the ability to innovate for each entrepreneur as given and focus on how inflation affects agents' occupational choices: a potential entrepreneur can either be a worker or an entrepreneur. In each period, every worker supplies one unit of labor and due to the limited commitment between entrepreneurs and workers, the former uses borrowed or own money to buy labor inputs. One can interpret this as a working capital requirement for entrepreneurs. Furthermore, entrepreneurs will receive idiosyncratic innovation shocks, which boosts their productivity for the period. Successful innovations of the entrepreneurs enter the public domain at the beginning of next period. This captures the externality of entrepreneurial activity. Economic growth will depend on the number of successful innovations, which will in turn depend on the number of entrepreneurs.

As other papers that studies long run inflation (fully anticipated steady state inflation), inflation is a tax on transactions that requires money. Since entrepreneurs need money to finance production, inflation would seemingly increase the cost of being an entrepreneur and discourage entrepreneurship. However, with working capital requirement, inflation tax is passed onto workers in the form of lower real wages. In other words, long-run inflation increases the cost of carrying money across periods and the nominal interest rate, thus lowering the demand for workers. To clear the labor market, real wages must fall. Inflation thus encourages entrepreneurship (and potentially growth) by lowering the payoff of the alternative: being an worker. This result depends on the working capital requirement and it is robust to the following changes in the assumption: (1) adding borrowing constraints for entrepreneurs; and (2) adding a cash-in-advance constraint for consumption good.

Compared with the existing endogenous growth theories that study the effect of long-run inflation, the mechanism in this paper is very different. In general, these theories assume some trades that are good for growth involves money. For example, inflation lowers consumption of cash good thus lowers the return of human capital as in Chari, et al. (1995); or inflation discourages trade of innovation goods as in Berentsen, et al. (2009) and of "ideas" as in Meh, et al. (2011). Inflation discourages such trades thus negatively affects growth. Here, the trade that requires money (labor inputs) is not directly related to growth. Inflation affects growth by influencing occupational choice.

Another important difference is that this paper rationalizes the stylized fact that inflation can enhance growth at low inflation rates. While theoretical studies, such as Shi (1999) and Laing, Li and Wang (2007) can generate positive effect of inflation on steady state output, the endogenous growth models generally predict that inflation negatively affects growth at all inflation levels[2][3]. On the empirical side, it is indeed widely accepted that too much long-run inflation is harmful for economic growth. However, it has also been advanced that the relationship between economic growth and inflation might depend on the inflation level. For example, Sarel (1996) and Ghosh and Philips (1998) found the "threshold effect", that is, the correlation between long-run inflation and economic growth is only negative above some threshold and insignificant or even slightly positive below the threshold. Benhabib and Spiegel (2009) documented a significant positive relationship between inflation and growth in ranges of moderate to negative inflation while Lopez-Villavicencio and Mignon (2011) found similar patterns for advanced countries and a non-significant correlation for emerging ones[4]. The above evidences suggest there must be some channel through which inflation can enhance growth and thus counter-balance those channels mentioned above at low inflation levels.

To incorporate a negative channel through which inflation discourages growth I add a variation of the model in Section 5 to investigate the R&D process conducted by the corporations. The future plan of this paper would be to see if the positive entrepreneurial channel and the negative corporate R&D channel will together reproduce the inverted-U shape relationship of inflation and growth.

The rest of the paper is organized as follows. In Section 2, the basic environment is introduced. Section 3 contains a version of the model with perfect credit and Section 4 studies versions of the model with imperfect bank credit. Section 5 consider

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[2] In Chari, Jones and Manuelli (1995), the correlation of long-run inflation and growth can be positive (only) at extremely high inflation levels.

[3] In the "social status" literature, such as Chang, et al. (2000), the positive correlation between inflation and growth could be generated by putting capital in the utility function, which is a very non-standard way to model growth.

[4] The thresholds for advanced countries and emerging ones are different: 2.7% and 17.5% respectively.

two variation of the model: (1) adding a CIA for consumption good and (2) adding R&D by corporate sector. Section 6 concludes with a discussion of potential extensions.

2 Environment

2.1 Basic Environment

There is a unit measure of agents in the economy. Time is discrete and agents live forever. Agents are of two types: $\bar{n}$ potential entrepreneurs and $1-\bar{n}$ others; the former can either be entrepreneurs or workers and the latter can only be workers. This is to capture the idea that entrepreneurial talent is scarce, or to say, not every agent can become an entrepreneur. In each period, let n be the actual number of entrepreneurs and $1-n$ be that of workers. Clearly $n \leq \bar{n}$ in every period. There are two kinds of nonstorable consumption goods, a market good and an endowment good. The market good is produced by entrepreneurs using labor inputs and the endowment good can be thought of as home produced by every agent. At the end of every period, each agent receives ZY units of the endowment good, where Z is aggregate productivity. The period utility of agents is $log(x)+y$, where x is consumption of the market good while y is consumption of the endowment good. Utility is linear in y, which will make agents risk neutral. Agents maximize the sum of discounted periodic utility with discount factor $\beta \in (0,1)$.

In each period, each worker produces one unit of the intermediate good at nominal price w_t in the market, and each entrepreneur operates a project, which needs intermediate inputs to produce the market good. Entrepreneurs do not produce intermediate goods themselves. What entrepreneurs do is operate their projects and look for innovations that can increase the productivity of their projects. The friction in this environment is that entrepreneurs need to use money to pay their production costs before they receive the subsequent revenue from production.

More specifically, at the beginning of a period, all agents hold money that is carried from the previous period and entrepreneurs realize whether they find innovations or not. The realized production function for entrepreneur i is $\eta_i Z f(\ell_i)$, where Z is aggregate productivity, ℓ_i is the intermediate inputs entrepreneur i uses and η_i is his idiosyncratic innovation shock drawn at the beginning of the period[5]. To make things simple, assume $\eta \in \{1, \bar{\eta}\}$, where $\bar{\eta} > 1$. Further assume that η is iid across entrepreneurs and time and an entrepreneur draws $\bar{\eta}$ with σ probability each period. One can think of what an entrepreneur does is first search for a good business idea and then acquire resources to implement the idea. The reason each period starts with the realization of the innovations is to capture the idea that when opportunities come, entrepreneurs can only acquire production inputs using the liquid funds that he or she owns or the money he or she can borrow from the financial market.

With probability σ, an entrepreneur comes up with an innovative idea which gives him/her a productivity boost over the current aggregate productivity ($\eta = \bar{\eta}$). With probability $1 - \sigma$, the entrepreneur receive a mediocre idea and he or she uses the publically known technology ($\eta = 1$). The reason to assume η is iid across time is to capture the idea that when an entrepreneur gets a good idea, he gets a boost of productivity in the short run and then the idea gets into the public domain

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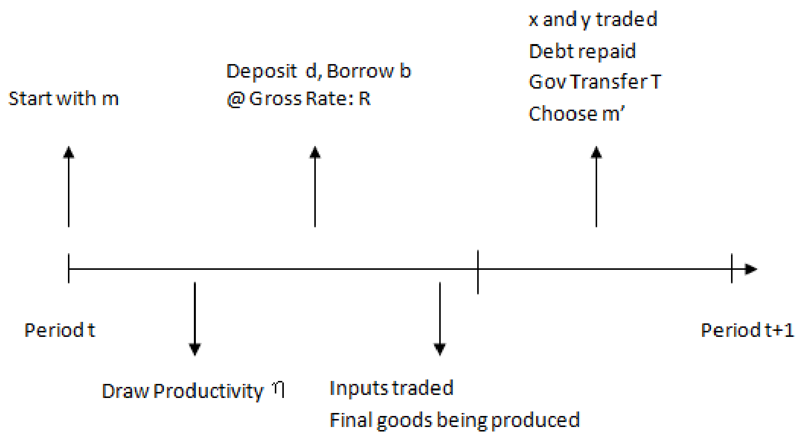
[5] $\ell_{i,t}$ not only includes the workers "employed" by entrepreneur i, but also includes other inputs used by the entrepreneur. For simplicity, all intermediate inputs are produced directly by workers and all of this cost needs to be paid before an entrepreneur receives revenue.

Figure 1: Time Line



and everyone can use it in the long run. The iid assumption also gives analytical tractability together with quasilinear utility. After realizing the productivity shocks, all agents have access to a competitive financial market where they borrow or lend money at gross nominal interest rate R. Then entrepreneur i pays his production cost $w\ell_i$, using the money he has on hand, including the money he borrows from the financial market. At the end of the period the two consumption goods are traded at nominal price p (market good) and p_y, debts are repaid, government impose a lump sum transfer/tax and agents choose money holdings for next period. Later on, when I endogenize entrepreneurship, I will also let agents choose their occupation for next period.

Next let me introduce the evolution of aggregate productivity. Let N be the total number of innovations in each period. Aggregate productivity in the next period is given by the following formula: $Z' = G(N)Z$. In general I assume $G'(N) > 0$. Here I give an example of the evolution of aggregate productivity:

$$Z' = \rho[N\bar{\eta}^{\varepsilon} + (1 - N)]^{1/\varepsilon} Z$$

where ρ is an exogenous component and ε is a parameter capturing the substitutability of individual innovations in generating aggregate knowledge. These innovations get aggregated into the public knowledge and every agent knows the technology Z' next period. As special cases: if $\epsilon = 1$, then aggregate productivity next period is just an average of the technology everyone knows this period, if $\epsilon = +\infty$, then it is given by the maximum productivity; and $\epsilon = -\infty$ implies it is given by minimum productivity. This formulation is the same as the one used in Chiu, Meh and Wright (2011). The difference is: there N is determined by the trade between innovators and entrepreneurs and here entrepreneurs get ideas themselves so N depends directly on the total number of entrepreneurs: since I assume each entrepreneur gets innovation with probability σ, so $N = n\sigma$. Therefore $G'(n) > 0$.

This formulation captures the idea that entrepreneurs seek innovations that generate short-term profits for themselves end up causing positive externalities to aggregate productivity. Using this setup, inflation can enhance growth if it can induce more people to become entrepreneurs.

2.2 On the Timing of the Environment

In this subsection I will discuss three assumptions about timing in the model: first, the timing of innovation shocks; second, the timing of the lump-sum transfer; third, the timing of debt repayment.

First, I assume that the iid innovation shocks happens at the beginning of each period. The reason for this assumption is because I am mainly interested in the short-term needs of liquidity of entrepreneurs. This captures the idea that when an entrepreneur needs liquidity (or more specifically money), he/she needs to work with whatever liquidity he/she has on hand plus whatever liquidity he/she can borrow from the credit market (or banks). This is exactly what makes the question interesting: because of the uncertainty of the innovation shocks, an entrepreneur will anticipate this liquidity need and choose his/her money holding accordingly.

Second, I assume that lump-sum transfers take place at the end of the period. Other possibilities include: (a) after agents access financial market and (b) at the beginning of the period before agents realize their innovation shocks and access the financial market. (a) is obviously not a good choice because once the transfer happens agents will immediately want to access the financial market again, effectively changing (a) into (b). (b) is not too sensible because this effectively forces workers to put the money they receive as lump-sum transfers in the financial market. In this case lump-sum transfers not only change the money stock and thus price levels, but also affect the credit market in an artificial way. This being said, (b) is worthwhile considering from a purely theoretical point of view.

Third, I assume debts are repaid within the period. We should notice m' is the money holdings for agents before they realize their innovation shocks and access the financial market in the next period. As long as m' is higher than dR, that is, as long as the (targeted) money holdings at the beginning of next period is higher than the principle and interest on current deposits, then whether the debt is repaid at the end of the period or at the beginning of the next period before innovation shocks are realized is irrelevant.

3 Market Economy with Perfect Bank Credit

3.1 Exogenous Entrepreneurship

In this section I will show how the model works with perfect bank credit, which means anyone can borrow as much as he/she wants. Moreover, I will take the number of entrepreneurs as fixed at n in this section. First look at the profit maximization problem faced by an entrepreneur with money holding m and realized idiosyncratic innovation shock η, while W^e is his end-of-period wealth from market activity:

$$\begin{aligned} W^e(m,\eta;M,Z) &= \max_{\ell,d,b} \eta Z p f(\ell) + (d-b)R && (1) \\ s.t. \quad & w\ell = m - d + b && (2) \\ & d \leq m, && (3) \end{aligned}$$

where M is the aggregate money stock at the beginning of the period; d and b are the entrepreneur's deposit and borrowing from the financial market. Constraint (2) says the entrepreneur needs to pay his/her production cost before realizing revenue and (3) simply requires the deposit is no greater than the initial money holding at the beginning of the period. Because with perfect bank credit there is no restriction on b, we can plug in ℓ from (2) into (1), then we have

$$W^e(m,\eta;M,Z) = \max_{\ell} \eta Z p f(\ell) + (m - w\ell)R. \quad (4)$$

So the first order condition for ℓ is

$$Z\eta p f_{\ell}(\ell) = wR. \tag{5}$$

The LHS is the marginal nominal benefit of purchasing an additional unit of labor input while the RHS is the marginal cost of purchasing an additional unit of labor input. The reason why the marginal cost is wR is because instead of paying an additional unit of input at the beginning of the period, entrepreneurs can deposit w in the financial market and get R at the end of the period.

Similarly, the end-of-period wealth from market activity for a worker is given by:

$$W^{w}(m, 1; M, Z) = w + mR, \tag{6}$$

where a worker does not receive any innovation shocks so the second argument is 1. This says that a worker's end-of-period wealth from market activity is just wage plus his/her principal and interest from deposit. This is because a worker does not need to use the money brought into the current period and will deposit all of it in the financial market. Notice (4) and (6) imply that

$$\partial W^{i}(m, \eta; M, Z)/\partial m = R. \tag{7}$$

Given the two wealth functions, the value function of an agent is given by

$$\begin{aligned} V^{i}(m, \eta; M, Z) &= \max_{x,y,h} log(x) + y + \beta E V^{i}(m', z'; Z) \\ & s.t. \quad xp + yp_y + m' = ZYp_y + W^{i}(m, \eta; M, Z) + T, \end{aligned} \tag{8}$$

where i could either be e or w, standing for entrepreneur or worker respectively. T is the lump sum transfer or tax imposed by the government. Specifically we will let $T = \tau M$, where τ is the growth rate of the money supply. Solve for interior solutions for $y > 0$ and eliminate y, then

$$\begin{aligned} V^i(m, \eta; M, Z) &= \max_x \{log(x) - x\frac{p}{p_y}\} + ZY + \frac{1}{p_y}[W^i(m, \eta; M, Z) + \tau M] \\ &\quad + \max_{m'}\{-\frac{1}{p_y}m' + \beta EV^i(\eta', m'; M', Z')\}. \end{aligned} \tag{9}$$

The first order condition for x is

$$\frac{1}{x} = \frac{p}{p_y}. \tag{10}$$

It is clear from (12) that consumption of the market good x does not depend on either the occupation or individual state variables of an agent[6]. Despite this, y could vary depending on their occupations and state variables. The Euler equation for money holding is

$$\frac{1}{x}\frac{1}{p} = \beta\frac{1}{x'}\frac{1}{p'}W^i(m', \eta'; M', Z')/\partial m'. \tag{11}$$

The LHS is the marginal cost of bringing one unit of currency into the next period while the RHS is the marginal benefit. The term $W^i(m', \eta'; M', Z')/\partial m'$ is the marginal benefit of one unit of currency to the end-of-period nominal wealth in the next period. Because of (7), this Euler equation does not depend on any individual

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[6] As in models following Lagos and Wright (2005), the quasilinear utility together with iid shocks simplify the analysis.

information, so everyone is indifferent of holding any amount of money. Therefore the money holding distribution is indeterminate. Assume for simplicity that all entrepreneurs hold m_e and all workers hold m_w, then labor market clearing and credit market clearing requires that

$$n\sigma f_l^{-1}(\frac{wR}{\bar{\eta}Zp}) + n(1-\sigma)f_l^{-1}(\frac{wR}{Zp}) = 1-n, \tag{12}$$

$$n\sigma[m_e - wf_l^{-1}(\frac{wR}{\bar{\eta}Zp})] + n(1-\sigma)[m_e - wf_l^{-1}(\frac{wR}{Zp})] + (1-n)m_w = 0. \tag{13}$$

Then, using the fact that $nm_e + (1-n)m_w = M$, we have that[7]

$$M = (1-n)w. \tag{14}$$

Lastly we need market clearing for the market good, that is

$$x = n\sigma Z\bar{\eta}f[f_l^{-1}(\frac{wR}{\bar{\eta}Zp})] + n(1-\sigma)Zf[f_l^{-1}(\frac{wR}{Zp})]. \tag{15}$$

We are interested in balanced growth equilibrium, which is defined as prices $\{p, p_y, w\}$, nominal interest rate $\{R\}$ and allocations $\{x, y, \ell, d, b, m'\}$ that satisfy (a) utility maximization; (b) profit maximization; (c) markets clearing; and (d) constant growth of aggregate state variables.

These conditions require that the following objects will be constant: $G(n)$, τ, $\frac{w}{Zp}$, R, and $\frac{x}{Z}$.

First, to solve for R, we will notice using $p_y = xp$, (14) and the fact that w/Zp

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[7] Actually for any money holding distribution we will end up with (14).

is constant, we know $p'_y/p_y = (1+\tau)$, which is the gross growth rate of the money supply. Then, using (7), we can rewrite the Euler equation as[8]

$$(1+\tau)\frac{1}{\beta} = R \tag{16}$$

The LHS can be seen as the gross nominal interest rate of an illiquid bond sold at the end of a period that matures at the end of the next period, and R is the nominal interest rate of deposits/loans after agents realize their idiosyncratic shocks. When there is no commitment problem between banks and agents, these two interest rates are the same. If this is the case, even workers will be willing to hold money. In this case the money holding decision is arbitrary, and any person is happy to hold any amount. There is only one thing different for entrepreneurs and workers: entrepreneurs sometimes are borrowers, but workers are never borrowers. In this environment an increase in τ will increase R one-for-one.

Lemma 1. *With exogenous entrepreneurship (fixed n), equilibrium exists and is unique. The productivity adjusted real wage is decreasing in the growth rate of money, τ.*

Proof: The equilibrium is characterized by (12) and (16). The two unknowns in the system are R and w/Zp, or the real wage in terms of the market good adjusted for productivity. Once we get R from (12), because now n is fixed, we can find w/Zp from the inputs market clearing condition (12). Since (12) is decreasing in both R and w/Zp, we are sure that the equilibrium exists and is unique.

Because the LHS of 12 is decreasing function of wR/Zp, we know that given n, wR/Zp is a constant. Therefore an increase in τ will increase R from 16 and thus lower the value of w/Zp. QED.

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[8] The Euler equation can also be written as $(1+g)(1+\pi) = \beta R$, where π is the inflation rate for market good x.

3.2 Endogenous Entrepreneurship

In this subsection, I will allow agents to choose between two occupations: entrepreneurs and workers. Specifically, at the end of the period, each agent, regardless of current occupation, can choose for next period whether to become a worker or an entrepreneur. Now the agents' problem is given by

$$\begin{aligned} V^i(m,\eta;M,Z) &= \max_{x,y} log(x) + y + \max_{m'}\{\beta EV^e(m',\eta';M,Z),\ \beta V^w(m',1;M,Z)\} \\ &s.t.\ \ xp + yp_y + m' = ZYp_y + W^i(m,\eta;M,Z) + \tau M. \end{aligned} \tag{17}$$

In addition to the analysis in the previous subsection, we have the following free entry condition for entrepreneurship:

$$Zp\{\sigma\{f[f_l^{-1}(\frac{wR}{\bar\eta Zp})] - wRf_l^{-1}(\frac{wR}{\bar\eta Zp})] + (1-\sigma)[f[f_l^{-1}(\frac{wR}{Zp})] - wRf_l^{-1}(\frac{wR}{Zp})\}\} = w \tag{18}$$

The LHS is the nominal payoff for an entrepreneur from market activity, the RHS is the nominal payoff for a worker. This condition can be modified as

$$\frac{Zp}{wR}\{\sigma\{f[f_l^{-1}(\frac{wR}{\bar\eta Zp})] - wRf_l^{-1}(\frac{wR}{\bar\eta Zp})] + (1-\sigma)[f[f_l^{-1}(\frac{wR}{Zp})] - wRf_l^{-1}(\frac{wR}{Zp})\}\} = \frac{1}{R} \tag{19}$$

Proposition 2. *With perfect credit and endogenous entrepreneurship, τ increases R, wR/Zp and n.*

Proof: The LHS of (19) is decreasing in wR/Zp while the right hand side is decreasing in R. From the Euler equation (16), an increase in τ will increase R and thus wR/Zp. Further, the labor market clearing condition (12) can be rewritten as

$$\sigma f_l^{-1}(\frac{wR}{\bar{\eta}Zp}) + (1-\sigma)f_l^{-1}(\frac{wR}{Zp}) = \frac{1-n}{n} \tag{20}$$

The LHS is decreasing in wR/Zp and the RHS is decreasing in n. Thus n is increasing in wR/Zp and thus in τ. QED.

Because $G'(n) > 0$, we have $\partial G/\partial\tau > 0$ as well. In the next section we will consider adding a borrowing constraint for entrepreneurs. The proof of similar propositions there will be more complicated. But the qualitative results remain the same.

4 Imperfect Bank Credit

4.1 Exogenous Entrepreneurship

Now we come back to look at the cases where bank credit is not perfect. Specifically, we will focus our attention on the case where sometimes entrepreneurs are going to be constrained. Now the end-of-period wealth from market activity for an entrepreneur is given by

$$\begin{aligned} W^e(m,\eta;M,Z) &= \max_{\ell,d,b} \eta Zpf(\ell) + (d-b)R && (21)\\ s.t.\quad & w\ell = m-d+b && (22)\\ & bR \leq \bar{b}Zp && (23)\\ & d \leq m. && (24) \end{aligned}$$

Plug in the constraint (22), the wealth function becomes

$$W^e(m,\eta;M,Z) = \max_{l,d,b} \eta Zpf(\frac{m-d+b}{w}) + (d-b)R \tag{25}$$

subject to (23) and (24). Now, solving for d and b, we get

$$\begin{cases} d = 0 \; and \; b = \frac{\bar{b}Zp}{R}, & if \; \eta Zpf_l(\frac{m}{w} + \frac{\bar{b}Zp}{wR}) > wR \\ d > 0, \; and \; b < \frac{\bar{b}Zp}{R}, & otherwise \end{cases} \tag{26}$$

An entrepreneur will borrow to his limit if $\eta Zpf_l(\frac{m}{w} + \frac{\bar{b}Zp}{wR}) > Rw$. If this is not the case, then we will have interior solutions for d and b then $d - b = m - wl$. The end-of-period wealth from market activity for workers is the same as given by (6). Now the Euler equation (11) for entrepreneurs becomes

$$\frac{1+\tau}{\beta} = \sigma\bar{\eta}\frac{Zp}{w}f_\ell(\frac{m}{w} + \frac{\bar{b}Zp}{wR}) + (1-\sigma)R \tag{27}$$

As before the LHS is the marginal cost of carrying one more unit of money into the period in nominal terms, which is increasing in the growth rate of the money supply and decreasing in the discount factor. The RHS is the marginal benefit of one additional unit of money in terms of end-of-period nominal wealth: with $(1-\sigma)$ probability an entrepreneur gets no innovation shock, so he is unconstrained and just deposits the money into the bank while with σ probability an entrepreneur receives an innovation shock so he is constrained and $\sigma\bar{\eta}\frac{Zp}{w}f(\frac{m}{w} + \frac{\bar{b}Zp}{wR})$ is the nominal benefit of an additional unit of money for a constrained entrepreneur.

Lemma 3. *Workers do not hold cash across periods if some entrepreneurs are constrained.*

Proof: As long as some entrepreneurs are constrained, from the first order condition (26) we know $\bar{\eta}\frac{Zp}{w}f_\ell(\frac{m}{w}+\frac{\bar{b}Zp}{wR}) > R$, so from (27) we know $R < (1+\tau)/\beta$. On the other hand, for workers to be willing to hold cash into the next period, we will need $R = (1+\tau)/\beta$, which cannot be satisfied. QED.

This is different from the perfect banking credit case. Another way to interpret this result is by observing: here the nominal interest rate in the credit market is depressed by the financial frictions, that is, lower than what the nominal interest rate would be when we have perfect bank credit. For entrepreneurs, they sometimes are constrained and have higher marginal benefit higher than the nominal interest rate, so they are willing to hold money. But for workers, the benefit of holding money is just the nominal interest rate which is now not enough to compensate for the cost of carrying money across periods.

Another question is whether some financial institutions can take deposits at the end of a period and make loans to the entrepreneurs at the beginning of the next period after they receive their innovation shocks. This cannot happen either: because workers will only be willing to deposit at the end of a period if financial institutions offer an interest rate R_d that is equal to $(1+\tau)/\beta$. But then the credit market interest rate is only $R < R_d$. Therefore no financial intermediary will borrow at R_d and lend at R. To sum up, in this environment only entrepreneurs will carry money and they will deposit and borrow in the credit market depending on their periodic productivity shocks.

Next we look at the two market clearing conditions. Inputs market clearing requires:

$$n\sigma(\frac{m}{w}+\frac{\bar{b}Zp}{wR}) + n(1-\sigma)f_l^{-1}(\frac{wR}{Zp}) = 1-n. \tag{28}$$

Credit market clearing (divided both sides by w) requires:

$$(1-\sigma)[m - wf_l^{-1}(\frac{wR}{Zp})] = \sigma\frac{\bar{b}Zp}{R} \tag{29}$$

Proposition 4. With exogenous entrepreneurship, when the borrowing constraint $bR \leq \bar{b}Zp$ is binding for the entrepreneurs with innovation shocks, there exists an unique equilibrium with $dR/d\tau > 0$ and $d(w/Zp)/d\tau < 0$.

Proof: To solve for equilibrium, we will notice from (28), when n is fixed, there exists a unique wR/Zp. Now rewrite the Euler equation in the following way:

$$\frac{1+\tau}{\beta} = R[\sigma\bar{\eta}\frac{Zp}{wR}f_\ell(\frac{m}{w} + \frac{\bar{b}Zp}{wR}) + (1-\sigma)]. \tag{30}$$

Because wR/Zp is constant and $m/w = (1-n)/n$ is also constant, therefore an increase in τ induces a one-to-one change in R. Because wR/Zp is constant, an increase in R means a decrease in w/Zp. QED.

The intuition for this result is that when τ increases, the cost of carrying money from period to period increases. This lowers the entrepreneurs' demand for labor inputs. In order to maintain labor market clearing, we will need w/Zp to decrease. When this happens, total output of the market good does not change. Inflation acts as a transfer between workers and entrepreneurs[9].

Corollary 5. *With exogenous entrepreneurship, inflation transfers wealth from workers to entrepreneurs.*

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[9] This is of course because we held the labor supply in the formal sector constant. Otherwise inflation will decrease labor supply thus total output could change. I do not consider that here because if we put a disutility from working in the market for workers, then we need to say something about disutility from being an entrepreneur. There is no standard way to model entrepreneurs' disutility. This will be further discussed in the last section.

Proof: We can look at the total wealth, including the lump-sum transfer for agents. Because we have shown that consumption of the market goods is the same for all agents, from (8) we can find consumption of the endowment good for the agents with different occupations:

$$y = ZY - \frac{xp}{p_y} + \frac{1}{p_y}[-m' + W^i(m, \eta; M, Z) + \tau M]. \tag{31}$$

Since workers will set their $m' = 0$, and $W^w(m, \eta; M, Z) = w$. Now from (36), $xp = p_y$, so for workers

$$y_w = ZY - \frac{xp}{p_y} + \frac{Z}{x}\frac{w}{Zp} + \tau\frac{Z}{x}\frac{w}{Zp}\frac{M}{w}. \tag{32}$$

Because total production of market goods is unchanged with respect to τ, so Z/x is constant. The third term is workers' wage in terms of the market good adjusted for productivity, and the fourth term is the value of the government transfer in terms of the endowment good. Though we know w/Zp is decreasing in τ, but it is not straightforward to see whether the fourth term is increasing or decreasing in τ. But if we rewrite the expression:

$$y_w \;=\; ZY - \frac{xp}{p_y} + \frac{Z}{x}\frac{w}{Zp}n + \frac{Z}{x}\frac{wR}{Zp}\frac{1+\tau}{R}(1-n) \tag{33}$$

Because x, Z/x, wR/Zp and $(1+\tau)/R$ are constant, and w/Zp is decreasing in τ so we know y_w is decreasing in τ. We know that the total production of market goods and the total endowment good are unchanged. So inflation acts just as transfer from workers to entrepreneurs. QED.

4.2 Endogenous Entrepreneurship

In this subsection, I will allow agents to choose between two occupations: entrepreneurs and workers. Specifically, at the end of the period, each agent, regardless of current occupation, can choose for next period whether to become a worker or an entrepreneur. Now the agents' problem is given by

$$\begin{aligned} V^i(m,\eta;M,Z) &= \max_{x,y} log(x) + y + \max_{m'}\{\beta EV^e(m',\eta';M,Z),\ \beta V^w(m',1;M,Z)\} \\ s.t. &\quad xp + yp_y + m' = ZYp_y + W^i(m,\eta;M,Z) + \tau M. \end{aligned} \tag{34}$$

Solve for interior solutions of y

$$\begin{aligned} V^i(m,\eta;M,Z) &= \max_{x}\{log(x) - x\frac{p}{p_y}\} + ZY + \frac{1}{p_y}[W^i(m,\eta;M,Z) + \tau M] \\ &\quad + \max_{m'}\{-\frac{1}{p_y}m' + \max\{\beta EV^e(m',\eta';M,Z),\ \beta V^w(m',1;M,Z)\}\}. \end{aligned} \tag{35}$$

Since Lemma W still holds here, the optimal money holding for a worker is zero. Now consider the case when $n < \bar{n}$, or to say, there are still potential entrepreneurs that choose to be workers. Then these agents must be indifferent about either being a worker or an entrepreneur[10]. Then we must have

$$\beta V^w(m',1;M',Z') \leq \max_{m'}\{-\frac{1}{p_y}m' + \beta EV^e(m',\eta';M',Z')\}, \tag{36}$$

where the inequality is binding if $n < \bar{n}$. Since x' and $\tau M'$ are independent of

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[10] One can add fixed costs or benefit of being an entrepreneur, which will not qualitatively change the analysis.

state variable and occupational choices, we have the following free entry condition (in terms of the previous period):

$$-\frac{1}{p_{y,-1}}m+\beta\frac{1}{p_y}E_{-1}W^e(m,\eta;M,Z)\geq\beta\frac{1}{p_y}w,\tag{37}$$

where the inequality is binding if $n<\bar{n}$. The expected wealth for market activity can be written as

$$E_{-1}W^e(m,\eta;M,Z)=\sigma[pZ\bar{\eta}f(\ell_H)-\bar{b}Zp]+(1-\sigma)[pZf(\ell_L)+(m-w\ell_L)R],\tag{38}$$

where $\ell=\frac{1-n}{n}+\frac{\bar{b}Zp}{\theta wR}$ and $\ell_L=f_\ell^{-1}(\frac{wR}{Zp})$. Notice $\sigma\bar{b}Zp$ is the amount the constrained entrepreneurs (borrowers) need to pay back at the end of the period, and $(1-\sigma)(m-w\ell_L)R$ is the amount the unconstrained entrepreneurs (lenders) receive at the end of the period. Notice from credit market clearing (29) that we know that these two terms cancel out in (38). This is simply because borrowers' debts are the deposits of the lenders. After some algebra, we arrive at the following simplified free-entry condition for entrepreneurship:

$$\sigma\bar{\eta}f(\ell_H)+(1-\sigma)f(\ell_L)-\frac{1+\tau}{\beta}\frac{1-n}{n}\frac{w}{Zp}\geq\frac{w}{Zp},\tag{39}$$

where the inequality is binding if $n<\bar{n}$. The endogenous variables are R, w/Zp and n. Now the equilibrium is characterized by three conditions: (28), (30), and (39).

Proposition 6. *With a binding borrowing constraint for entrepreneurs with innovation shocks and endogenous entrepreneurship, there exists a unique equilibrium with* $dR/d\tau>0$, $d(\frac{w}{Zp})/d\tau<0$, *and when* $n<\bar{n}$ *we have* $dn/d\tau>0$.

Proof: See Appendix A.

We can get some intuition for this proposition by looking at the simplified free-entry condition for entrepreneurship. The RHS of (39) is what a worker gets from market activity (adjusted for productivity). The LHS of (39) is the expected benefit of being an entrepreneur from market activity (adjusted for productivity): the first two terms are the expected revenue in terms of market goods (adjusted for productivity), and the third term is the expected cost of being an entrepreneur. The cost is increasing in w/Zp, the real wage in terms of market goods adjusted for productivity. It is also increasing in $(1-n)/n$, the expected measure of labor inputs each entrepreneur hires. Lastly, and most interestingly, the cost is increasing in $(1+\tau)/\beta$. The reason for this is that entrepreneurs need to sacrifice consumption in the previous period to prepare the liquidity/money needed for their business in the current period. Such sacrifice is increased when we have a higher growth rate of the money supply.

When n is fixed, then changes inτ do not change the LHS, because as shown in Proposition 1, wR/Zp and $(1+\tau)/\beta R$ are constant. But an increase in τ will decrease the RHS, w/Zp. As a result, inflation distorts the payoffs from different occupations. The only way to equate the LHS and RHS is to increase n.

When $n = \bar{n}$, we need to show that an increase in τ would not induce the existing entrepreneurs to become workers. If we fix n, we know from Proposition 4 and Corollary 5 that inflation will shift wealth from workers to entrepreneurs, so any further increase in τ will not change n.

From Proposition 6, we know that an increase in the growth rate of the money supply can increase the number of entrepreneurs. Because of $G'(n) > 0$, economic growth can be enhanced by higher rate of growth of money supply. Also notice from

Proposition 4, when $n = \bar{n}$, any further increase in the growth rate of the money supply will not affect economic growth. So this channel only works for low inflation rates.

5 Alternative Models

In this section, I present two variation of the model to address two issues: (1) the robustness of the prediction on alternative CIA assumptions and (2) how to generate a negative channel from inflation to growth. The discussion of the first issue is necessary because we have learned from monetary economics that sometimes the results sensitively depend on where do we put the CIA constraint. The second issue might be of more importance because as we see in the introduction, the empirical evidence on the relationship of inflation and growth is positive when the inflation rate is low and negative when the inflation is high. Although Schumpeter (1934) stressed the importance of entrepreneurship in promoting growth, he also noted the R&D efforts conducted by big corporations are another genuine force on economic growth. So we will investigate the R&D channel in this section.

5.1 CIA for Consumption Good

In this section we consider adding a cash-in-advance constraint for consumption goods. Specifically, assume that entrepreneurs start with the period with some inventory stock and production technology. They sell inventory stock to the consumers demanding cash payment, and on the other hand hire workers to produce inventory which they can sell to consumers in the next period. At the end of the period, agents can trade one period bond and entrepreneurs pays workers in cash, which can be used to purchase goods in the following period. To simplify our analysis, assume entrepreneurs receives no productivity shocks and the entrepreneurs have production function: $Zf(\ell)$

$$\begin{aligned} V^i(m,b,s;M) &= \max_{x,y,h,\ell} log(x) + y + \beta E V^i(m',b',s';M') \\ s.t. &\quad xp + yp_y + b' + m' = ZYp_y + m + Rb + W^i(s;M) + T \\ &\quad xp \leq m \\ &\quad s = Zf(\ell) \end{aligned}$$

where $s^w = 0$, $W^w = w$ and $W^e = ps - wR\ell$.

Assuming binding CIA constraint, then the Euler equations for money and bond are

$$\frac{1}{p_y} = \beta \frac{1}{p'} \frac{1}{x'} \tag{40}$$

$$\frac{1}{p_y} = \beta R \frac{1}{p'_y} \tag{41}$$

Since $xp = m$ and $m = M$ because of money market clearing, so xp grows at the constant rate,τ, as with the money supply. Because of (40), this implies p_y will grow at rate of τ. Then from (41) we know $1 + \tau = \beta R$. The envelope condition for s is $V_s = \frac{p}{p_y}$ and the first order condition for ℓ is

$$\frac{1}{p_y} wR = Zf_\ell(\ell) \beta V_s. \tag{42}$$

So the optimal ℓ satisfy $wR/p_y = Zp'f_\ell(\ell)/p'_y$. When Z is constant, $p'/p = p'_y/p_y = 1+\tau$. So $\ell = f_\ell^{-1}(wR/Zp)$. The labor market clearing requires: $nf_\ell^{-1}(wR/Zp) = 1-n$. The free entry condition requires that $W^w = W^e$.

Proposition 7. *There exists a unique steady state equilibrium and n and wR/Zp are increasing in τ.*

Proof: In steady state, we have three unknowns: n, R and wR/Zp. R is given by $1+\tau = \beta R$. The market clearing condition can be rewritten as

$$f_\ell^{-1}(wR/Zp) = \frac{1-n}{n} \tag{43}$$

The LHS is decreasing in wR/Zp and the RHS is decreasing in n so wR/Zp is monotonically increasing in n. Next the free entry condition in steady state is $w = pZf(\ell) - wR\ell$. This can be rearranged as

$$\frac{1}{R} = \frac{pZ}{wR} f[f_\ell^{-1}(wR/Zp)] - f_\ell^{-1}(wR/Zp) \tag{44}$$

The LHS is decreasing in R while the RHS is decreasing in wR/Zp so the equilibrium exist and is unique. Further, an increase in τ will increase R (from Euler equation) and thus wR/Zp. An increase in wR/Zp will induce an increase in n from the market clearing condition (43). QED.

From Proposition 6 we know that as long as the working capital requirement is there, adding CIA for consumption good will not change the prediction that higher inflation encourages entrepreneurship.

5.2 R & D by Corporations

Here we will deviate from Section 5.1 and extends upon Section 3.2. Assume ξ firms have the production function $s(\ell_{d,t})Z_{t+1}f(\ell_{c,t+1})$ can hire ℓ_d workers on a R&D process to increase next periods productivity. Specifically, $s(\ell_d)$ is a concave function.

The current nominal cost of R&D is $wR\ell_{d,t}$, the next period's nominal operational profit is $s(\ell_{d,t})Z_{t+1}p_{t+1}f(\ell_{c,t+1}) - w_{t+1}R_{t+1}\ell_{c,t+1}$. The first order condition for ℓ_d is

$$\frac{w_tR_t}{x_tp_t} = \beta\frac{1}{x_{t+1}p_{t+1}}s_\ell(\ell_{d,t})Z_{t+1}p_{t+1}f(\ell_{c,t})$$

Notice in steady state $\frac{x_tZ_{t+1}}{x_{t+1}Z_t} = 1$, and $sZpf_\ell(\ell_c) = wR$, so that

$$s_\ell(\ell_d) = \frac{1}{f(\ell_c)}\frac{wR}{Zp}$$

where then $\ell_c = f_\ell^{-1}(\frac{wR}{sZp})$. We would like ℓ_c and ℓ_d to be both decreasing in wR/Zp.

For example, if $f(\ell) = \ell^\gamma$ and $s(\ell) = \ell^\theta$, $\gamma\ell_c^{\gamma-1} = \frac{wR}{sZp}$, so $\ell_c = (\frac{wR}{sZp\gamma})^{\frac{1}{\gamma-1}}$ and $f(\ell_c) = (\frac{wR}{sZp\gamma})^{\frac{\gamma}{\gamma-1}}$. We can show that

$$\theta\ell_d^{\frac{\theta+\gamma-1}{1-\gamma}} = (\frac{wR}{Zp})^{\frac{1}{1-\gamma}}\gamma^{\frac{\gamma}{\gamma-1}}$$

In order for ℓ_d to be decreasing in wR/Zp , we need $\theta < 1-\gamma$. Next we can have

$$\ell_c = (\frac{wR}{Zp})^{\frac{1}{\theta+\gamma-1}}\gamma^{\frac{1}{1-\gamma}}(\gamma^{\frac{\gamma}{\gamma-1}}\frac{1}{\theta})^{\frac{\theta}{\theta+\gamma-1}}$$

When $\theta < 1-\gamma$, ℓ_c is decreasing in $\frac{wR}{Zp}$.

The market clearing condition is

$$n[\sigma f_l^{-1}(\frac{wR}{\bar{\eta}Zp}) + (1-\sigma)f_l^{-1}(\frac{wR}{Zp})] + \xi(\ell_c + \ell_d) + n = 1$$

LHS is increasing in n but decreasing in wR/Zp. Further, free entry conditions requires:

$$\frac{Zp}{wR}\{\sigma[\bar{\eta}f(\ell_H) - \frac{wR}{Zp}\ell_H] + (1-\sigma)[f(\ell_L) - \frac{wR}{Zp}\ell_L]\} = \frac{1}{R} = \frac{\beta}{1+\tau}$$

LHS is decreasing in wR/Zp and RHS is decreasing in R. So an increase in R will increase wR/Zp. So we have two competing forces for growth: an increase in money growth rate, will induce more entrepreneur but less R&D by corporate sector.

6 Concluding Remarks

In this paper, I examine how inflation affects agents' occupational choices of whether to become an entrepreneur or a worker. As long as we have the working capital requirement for entrepreneurs, a higher inflation rate will induce more people to become entrepreneurs instead of workers. This is because inflation effectively act as a tax that redistribute income from workers to entrepreneurs. More entrepreneurs generate more innovation and thus higher growth. This mechanism is in line with the literature relating growth to entrepreneurship and is novel compared to the existing literature studying the effects of long-run inflation on economic growth. In addition, this mechanism could help explain why long-run inflation and growth can be positively correlated when inflation levels are low. I will discuss two possible extensions/limitations of the model in the rest of this section.

In the current version of the model, this mechanism will only be effective at low inflation rate because I assume that there is a fixed number of identical potential entrepreneurs. It is worthwhile to explore a more realistic case: what will happen if potential entrepreneurs differ in their abilities to come up with innovations? Future versions of this paper will address this question explicitly. Here I give an informal argument. Note that the way inflation drives more people into entrepreneurship, in this paper, is by lowering real wages. When inflation is low, wages for workers are still high. The lower ability potential entrepreneurs would rather work for wages and the marginal agent must have high ability. The effect of inflation on growth would therefore be big. As inflation increases and the real wage decreases, the ability of the marginal potential entrepreneur will decrease as well. So the effect of this channel will be decreasing in inflation rates. This gives us hope that when inflation is high the corporate R&D channel will dominate so the net effect of inflation on growth could be negative.

Another potential issue concerns the inelastic labor supply assumed in the model. If workers supply labor inelastically, all the inflation tax is borne by the workers, while the expected payoff for an entrepreneur from market activity remains the same. In general, if individual labor supply is elasticity, then both entrepreneurs and workers will suffer from the inflation tax and which group suffers more depends on preferences and technologies. However, there exists a broad literature about hours constraints i.e. workers cannot easily adjust their hours worked. Hours constraints may comes from a fixed cost per worker faced by employers, implicit contracts, agency problems and firm-specific human capital. See Martinez-Granado (2005) for a survey of the theoretical and empirical studies that support the existence of hours constraint. Another simple reason for hours constraints not discussed in this literature could be that employers across countries are required to pay higher hourly wages for overtime work, so individual labor supply might be constrained from above when inflation is low and real wages are still high. According to this explanation, if enforcement of labor law is weaker in emerging economies and thus individual labor supply is not in its corner solution, then inflation would hurt entrepreneurs as much. Inflation in this case would not enhance growth as much as when we have corner solutions for individual labor supply. This might even help explain why Lopez-Villavicencio and Mignon (2011) did not find a positive correlation of inflation and economic growth in these economies.

Nevertheless, it is interesting to know the quantitative implications of elastic labor supply on the mechanism proposed in this paper. For example, it is unrealistic to think that workers are willing to supply the same labor when real wages are extremely low, which happens when inflation is extremely high in the model. Labor supply elasticity will be studied more in future versions of this paper.

Part II

Housing and Liquidity

This part is joint work with Randall Wright[11] and Yu Zhu from University of Wisconsin-Madison.

7 Introduction

In this chapter, we study economies in which housing plays two roles. First, houses provide utility, either directly as durable consumption goods or indirectly as inputs into household production. Second, houses are assets that can facilitate transactions when credit markets are imperfect. In the presence of limited commitment/enforcement, it can be difficult to get unsecured consumer loans, and this generates a role for home equity as collateral. This implies equilibrium house prices can bear a liquidity premium, since people are willing to pay more than the fundamental price, as defined below, because home ownership provides security in the event that one needs a loan. Given this, we show how equilibrium house prices can display a variety of interesting dynamic paths, some of which look like bubbles. Intuitively, liquidity is at least to some extent a self-fulfilling prophecy, which means that the price of a liquid asset is to some extent a matter of beliefs. In this sense houses are similar, although not the same as, money – e.g., both ameliorate trading frictions, but houses generate direct utility and can be produced by the private sector.

Our goal is to make these ideas precise and study their implications. This seems

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[11] Professor Wright is also affiliated with NBER, FRB Minneapolis and FRB Chicago

interesting for several reasons, primarily because it is consistent with experience since the turn of the millennium. It is commonly heard that there was a bubble in house prices during this period, which eventually burst, leading to all kinds of economic problems. And it has been noted that there was over the period a huge increase in home equity loans. Reinhart and Rogoff (2009b) contend developments in financial markets allowed consumers "to turn their previously illiquid housing assets into ATM machines."[12] Ferguson (2008) also says this "allowed borrowers to treat their homes as cash machines," and reports that between 1997 and 2006 "US consumers withdrew an estimated $9 trillion in cash from the equity in their homes." Mian and Sufi (2011), estimate that homeowners extracted 25 cents for every dollar increase in home equity, and these loans added $1.25 trillion to household debt between 2002 and 2008. These authors also emphasize that the borrowed funds were used by households for consumption, rather than, e.g., paying off credit card debt or purchasing financial assets.[13]

Figure 2 shows some data for the US over the relevant period (exact data definitions and sources are given below; all figures are at the end of the paper). Housing prices are deflated in two ways. One divides by the CPI to correct for the impact

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[12] The financial development they have in mind is *securitization*. As **?** put it, "In the runup to the subprime crisis, securitization of mortgages played a major role. ... Securitization, by making [previously] nontradable mortgages tradable, led to a dramatic growth in the US volume of mortgages, home equity loans, and mortgage-backed securities in 2000 to 2008, partly in response to increased globabl demand for savings instruments."

[13] Mian and Sufi (2011) also find that home equity loans were used more by younger households and those with lower credit scores, consistent with our approach. In related work, Greenspan and Kennedy (2007) find that home equity withdrawal financed about 3% of personal consumption from 2001 to 2005, and Disney and Gathergood (2011) find one-fifth of the recent growth in household indebtedness can be explained by rising house prices. Also related is the evidence reported in Ferraris and Watanabe (2008) that in 2004, 47.9 percent of the US households had home-secured debt (and they point out that real estate is also used to collateralize commercial and industrial bank loans accounting for 46.9 percent of the total value in the US, but the emphasis here is on consumption loans).

of inflation. The other divides by an index of rental rates, to correct for inflation plus changes in the demand for shelter relative to other goods and services, yielding the inverse of the rent-price ratio. Even before we define terms precisely, this illustrates what people have in mind when they talk about a bubble: a dramatic run up followed by collapse in housing prices. Also shown are measures of home equity loans, this time normalized in three ways. The first again uses the CPI, showing a huge increase in real home equity loans over the period. The second normalizes by all bank loans, and still shows a big increase, to establish that the increase in home equity loans is not merely part of an overall increase in lending. The third normalizes by all real estate loans, to make it clear that the increase in home equity loans is not an artifact of an increase in the value of real estate generally – e.g., it is not merely the case that housing prices, mortgage loans and home equity loans all go up by the same factor. Finally, we show a measure of housing investment, normalized by GDP, to get an idea about what was happening to supply.[14]

The message we take away from all this is: coinciding with the start of the boom in house prices, there is a large increase in the real value of home equity loans and a moderate increase in housing investment; later, prices fall, while home equity borrowing stays high and investment drops. This suggests to us that the role of home equity as collateral is potentially important for trying to understand the episode. If

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[14] Definitions of the series used in to construct Figure 2 are as follows: Home Prices uses the FHFA Purchase Only price index. To turn it into a real variable, it is divided by the CPI, or by the BLS Rent index, with these real series then normalized to 1 in 1991. Loan data are from the Federal Reserve and are for all commercial banks in the US. Home Equity Loans are similarly divided by nominal GDP, by Bank Loans and Leases, and by Real Estate Loans, with the resulting three series normalized to 0.3 in 1991. Bank Loans and Leases includes Commercial and Industrial Loans, Real Estate Loans, Consumer Loans and Other Loans and Leases. Real Estate Loans contains all loans secured by real estate, including Revolving Home Equity Loans, Closed-end Residential Loans and Commercial Real Estate Loans. Finally, the Residential Fixed Investment Index is from BEA; it is divided by real GDP and then normalized to 0.7 in 1991.

one considers a house only as a durable consumption good, with its value determined by the utility it provides, the ratio of rent (which should measure the utility flow) to the house price should be roughly the sum of the discount and depreciation rates. There can be other costs and benefits of owning, including tax implications, but while these may affect the level of the rent-price ratio, as long as they are approximately constant, this should not generate the time series in Figure 2.[15] Our position, following the authors quoted above, is that financial development led to a bigger role for home equity in the credit market, this fueled an increase in the housing demand, and that led to an increase in price in the short run and quantity in the long run. We show that once one takes account of the liquidity role for home equity, it is not hard to generate equilibria consistent with this position.

As we said, many people seem to think that the data indicate bubbles, although it is not always obvious what they mean. As Case and Shiller (2003) put it, “The term ‘bubble’ is widely used but rarely clearly defined. We believe that in its widespread use the term refers to a situation in which excessive public expectations of future price increases cause prices to be temporarily elevated.” Shiller (2011) more recently says “In my view, bubbles are social epidemics, fostered by a sort of interpersonal contagion. A bubble forms when the contagion rate goes up for ideas that support a bubble. But contagion rates depend on patterns of thinking, which are difficult to judge.” Phenomena like “excessive public expectations, social epidemics, and interpersonal contagion” are nothing if not bewitching, but here we emphasize a more pedestrian idea of liquidity, a bubble is simply an asset price different from its fun-

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[15] Others have considered the data discussed above. Harding et al. (2007), e.g., estimate the depreciation rate on houses to be around 2.5 percent, so if the discount rate is around 3 percent, the rent-price ratio should be around 5. In Campbell et al. (2009), from 1975 to 1995, this ratio is indeed around 5, but then declines to around 3.7 percent in 2007.

damental value, given by the present value of holding the asset (this is made precise below). This is consistent with, e.g., Stiglitz (1990), who says that "if the reason that the price is high today is only because investors believe that the selling price is high tomorrow – when 'fundamental' factors do not seem to justify such a price – then a bubble exists." We show that house prices can differ from fundamental values due to a liquidity role for home equity.

In emphasizing credit frictions, we follow a large literature summarized in Gertler and Kiyotaki (2010) and Holmström and Tirole (2011), as well as a related literature surveyed in Nosal and Rocheteau (2011) and Williamson and Wright (2010a, 2010b) that endeavors to be relatively explicit about the process of exchange by going into detail concerning how agents trade (bilateral, multilateral, intermediated etc.), using which instruments (barter, money, credit etc.) and at what terms (price taking, posting, bargaining etc.). A direct antecedent to our approach is the body of work emanating from Kiyotaki and Moore (1997, 2005). In terms of technical details, the model is similar to Rocheteau and Wright (2005, 2010), although differences arise because we study housing as opposed to some generic asset. To illustrate, a typical asset generates a dividend stream that enters your budget equation; housing does this, too, but additionally enters your utility function, and this matters for results. Thus, it can change the circumstances under which bubbles exist from one of low supply to one of either low or high supply depending on preferences. Also, it means welfare might decrease with an exogenous increase in the stock of houses, which does not happen with financial assets. And it implies demand for housing does not go to 0 when the (accounting) return is less than available elsewhere, as would be the case for a generic financial asset.

Our approach is related to a body of work on bubbles and liquidity, in general, which is too large to survey here, so we refer readers to Farhi and Tirole (2011) for references. Again, there are several interesting differences between houses and generic assets that come up in our analysis. In terms of research on housing markets, there are several other papers that also try to take seriously the precautionary or collateral function of home equity. A technical difference from some of this work is that we focus on fully rational agents, with homogenous beliefs, and indeed we can generate bubble-like equilibria under perfect foresight. We can also do this with an endogenous supply of housing, which seems relevant since it has been suggested by, e.g., Shiller (2011), that "The housing-price boom of the 2000's was little more than a construction-supply bottleneck, an inability to satisfy investment demand fast enough, and was (or in some places will be) eliminated with massive increases in supply" (see also Glaeser et al. (2008)). The housing literature more generally is sufficiently voluminous that, at the risk of neglecting some relevant contributions, we can only cite a few example that influenced our thinking on the issues.[16]

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[16] Carroll et al. (2003) study the impact of a precautionary (against job loss) motive on the demand for housing, and find evidence for it at moderate and higher income. Hurst and Stafford (2004) find that unemployment shocks and low asset positions increase households' likelihood of using home-equity loans. Campbell and Hercowitz (2005) study a growth model where demand for housing increases over time, and existing houses alleviate borrowing constraints. Arce and Lopez-Salido (2011) study a life-cycle model where some agents hold houses purely as a saving vehicle. Other work emphasizing the role of asset shortages includes Caballero and Krishnamurthy (2006) and Fostel and Geanakoplos (2008). Brady and Stimel (2011) find that household's response to house price shocks has shifted since 1998, consistent with our timing assumptions. Aruoba et al. (2011a) study the interaction between housing markets and inflation, as we do below, and give citations to previous work along these lines. Other recent work on housing market dynamics includes Burnside et al. (2011), Coulson and Fisher (2009), Ngai and Tenreyro (2009) , Novy-Marx (2009), Piazzesi and Schneider (2009b,a) and Jaccard (2011).

We do highlight a recent paper by Liu et al. (2011) that independently develops a complementary model. They also assume real estate can be used as collateral, but by producers, as in the usual application of Kiyotaki and Moore (1997, 2005), not by consumers, They present calibration results, while we emphasize more theoretical results, in part because for their parameter values equilibrium exhibits saddle-path stability. If one knew for sure that their exact specification and parameters were correct, one might argue that in the empirically relevant case exotic dynamics of the type we highlight cannot happen. We think this would be hasty, and in any case one should want to know how models behave, more generally, and to see just what it takes to generate bubbles in housing markets, in

The rest of the paper is organized as follows. Section 8 lays out the basic environment. Section 9 discusses steady state equilibrium. Section 10 discusses dynamics, presenting explicit examples to show how bubble-like outcomes may arise. Section 11 endogenizes the supply of housing, and presents examples that look somewhat like recent experience. Section 12 presents a monetary version of the model to study interplay between housing and inflation. Section 13 concludes. [17]

8 The Basic Environment

Each period in discrete time agents interact in two distinct markets. First, they participate a decentralized market, labeled DM, with explicit frictions detailed below; then they trade in a frictionless centralized market, labeled CM. At each date t, in addition to labor ℓ_t, there are two nonstorable consumption goods x_t and y_t, plus housing h_t. We assume ℓ_t, x_t and h_t are traded in the CM, while y_t is traded in the DM. The utility of a household is given by

$$\lim_{T\to\infty} \mathbb{E} \sum_{t=0}^{T} \beta^t \left[\mathcal{U}(x_t, y_t, h_t) - A\ell_t \right], \tag{45}$$

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particular. We also mention work by Miao and Wang (2011) and Liu and Wang (2011) that discuss bubbles or cycles in economies with credit constraints, but again, on firms and not on households. We are interested in applying theories of credit frictions not to producers using generic capital goods to finance investment, but to households using home equity to finance consumption.

[17] To close these introductory remarks, we emphasize that this paper is *not* about imperfect housing markets: houses are traded in frictionless markets, the way other forms of capital are traded in standard growth theory. This is not because we think imperfect housing markets are uninteresting, but because we want to focus on other issues, especially the role of home equity in imperfect credit markets. Housing market models with search go back to Wheaton (1990). See Albrecht et al. (2007a) or Head et al. (2010) for more recent examples and citations to the literature.

where $\beta \in (0,1)$.[18] Much tractability comes from quasi-linear utility, although this can be replaced with indivisible labor such as in Rogerson (1987), which also generates unemployment (Rocheteau et al. (2008)). To ease the presentation, we assume $\mathcal{U}(x_t, y_t, h_t) = U(x_t, h_t) + u(y_t)$, where $U(\cdot)$ and $u(\cdot)$ satisfy the usual assumptions, including $u(0) = 0$.

For now there is a fixed stock of housing H. In terms of CM goods, ℓ_t can be converted one-for-one into x_t (the framework is easily extended to more general production functions). In terms of DM goods, some agents can produce y_t using a technology summarized by the cost function $v(y_t)$. In many related models, households produce for each other in the DM, and $v(y_t)$ is interpreted as a direct disutility; in other models, DM producers are retail firms. Although it does not matter much for the results, in this paper, we follow the latter approach, with households buying y_t from DM retailers. Our retail technology works as follows: by investing at $t-1$ a fixed amount, normalized to 1, of the CM numeraire x_{t-1}, a retailer can at t convert it into any amount $y_t \leq 1$ of the DM good and some amount $x_t = F(1-y_t)$ of the CM good. The profit from this activity, conditional on selling y_t in the DM at t for revenue R_t, measured in period t numeraire, is $R_t + F(1-y_t) - (1+r)$, given the initial investment at $t-1$ is repaid in the CM at t at an interest rate $1+r = 1/\beta$.

Not all retailers earn the same payoff, since not all trade, in the DM. Let α_f be the probability a retailer trades in the DM, and symmetrically let α_h be the probability a household trades. Also, assume $y \leq 1$ is not binding, as must be the case, e.g., if

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[18] We assume here the limit in (45) exists; if not, one can use more advanced optimization techniques (see the discussion and citations in Rocheteau and Wright (2010)).

$F'(0) = \infty$. Then expected profit is

$$\begin{aligned} \Pi_t &= \alpha_f \left[R_t + F(1-y_t) \right] + (1-\alpha_f) F\left(1\right) - (1+r) \\ &= \alpha_f \left[R_t - v(y_t) \right] + F(1) - (1+r), \end{aligned} \tag{46}$$

where $v(y_t) \equiv F\left(1\right) - F(1-y_t)$ as the opportunity cost of selling y_t in the DM. As in standard search theory, if there is a $[0,1]$ continuum of households and a $[0,N]$ continuum of retail firms, the trading probabilities can be endogenized by $\alpha_f = \alpha(n)/n$ and $\alpha_h = \alpha(n)$, where $\alpha(\cdot)$ comes from a standard matching technology and $n \leq N$ is the measure of firms in the DM. Firms may have to pay a DM participation cost, in addition to their initial investment in goods, and n can be determined by the usual free-entry condition. To make our main points, however, we assume the cost is small and $1+r < F(1)$, so that $n = N$, and α_h and α_f are fixed constants.

In fact, it is not necessary to invoke search or matching at all.[19] An alternative story that is equivalent for our purposes is that households sometimes realize a demand for y_t due to preference or opportunity shocks. Nice examples include the possibility that one has an occasion to throw a party, or an opportunity to buy a boat at a good price; not-so-nice examples include the possibility that one has a medical emergency, or one's boat breaks down. In any case, the probability of such an event is α_h. Then we can assume that the DM clears in various ways, and could involve bilateral or multilateral matching. We consider various options, but to simplify notation we always assume the same number of agents trade on each side of the market,

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[19] In terms of the literature discussed in the Introduction, it is common to have search, and with it bargaining, in the papers surveyed by Nosal and Rocheteau (2011) or Williamson and Wright (2010a,b), but not those surveyed by Gertler and Kiyotaki (2010) or **?**. A side product of this project is to help integrate the two approaches – i.e., we provide a search-and-bargaining version of Kiyotaki and Moore (1997, 2005).

so that $\alpha_h = N\alpha_f$. More significantly, in the DM, households use *credit*, since they have nothing to offer by way of quid pro quo: y_t is acquired in exchange for a debt obligation d_t to be retired in the next CM.[20]

Credit is limited, however, by lack of commitment/enforcement: households are free to default, albeit at the risk of some punishment. At one extreme, punishment can be so severe that credit is effectively perfect. At the other extreme is no punishment, not even exclusion from future credit, as in Kehoe and Levine (2001) or Alvarez and Jermann (2000), say, because borrowers are anonymous. This would completely rule out unsecured credit. In general, we use a debt limit $d \leq D = D(e_t)$ with $e_t = \psi_t h_t$, where ψ_t is the price of h_t in terms of x_t. We focus on the linear case $D(e_t) = D_0 + D_1 e_t$ with $D_0, D_1 \geq 0$. If D_0 is big, the limit never binds, and unsecured lending works well; if D_0 is small, the limit binds with insufficient home equity. A natural specification is $D(e_t) = e_t$, but it may also make sense to consider $D(e_t) < e_t$ if we can seize only part of their assets when debtors default(e.g., we get the house but they run off with the appliances), or if some value is lost due to litigation or other transaction costs. And $D(e_t) > e_t$ is interesting if we have punishments beyond confiscating collateral. For now, we take $D(e_t)$ to be exogenous, but in the appendix we show how to endogenize it. We do this both for the case where punishment for default is taking away future unsecured credit, and where it is taking way all future credit.

Let $W_t(d_t, h_t)$ be a household's value function entering the CM at t, with debt d_t and house h_t brought in from $t-1$. Since d_t is paid off each period in the CM, households start debt free in next period's DM, where $V_{t+1}(h_{t+1})$ is the value func-

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[20] One period debt is imposed, but this is usually without loss of generality, due to quasi-linearity. It is possible that agents may want to roll over debt for strategic reasons for some pricing mechanisms, but not for the ones we focus mainly on here, including Walrasian pricing and Kalai bargaining.

tion. The CM problem is

$$W_t(d_t, h_t) = \max_{x_t, \ell_t, h_{t+1}} \{U(x_t, h_t) - A\ell_t + \beta V_{t+1}(h_{t+1})\} \tag{47}$$
$$\text{st } x_t + \psi_t h_{t+1} = \ell_t + \psi_t h_t + T_t - d_t \text{ and } \ell_t \in [0, \bar{\ell}]$$

where T_t is other wealth, including government transfers, returns from investments etc., but since wealth does not affect anything except leisure, with our quasi-linear utility function, this is all left implicit.[21]

Before proceeding, we take a brief digression into the theory of household production.[22] According to this theory, households value two types of goods: those acquired on the market x_t^1, and those produced in the home x_t^2. They also engage in two types of labor: work in the market ℓ_t^1, and work in the home ℓ_t^1. Let $x_t^2 = G_t\left(\ell_t^2, h_t\right)$ be the household production function, mapping home time and home capital – i.e., housing, broadly defined to include structures, appliances etc. – into home goods. The generalized version of (47) is

$$W_t(d_t, h_t) = \max_{x_t^1, x_t^2, \ell_t^1, \ell_t^2, h_{t+1}} \left\{U(x_t^1, x_t^2) - A_1\ell_t^1 - A_2\ell_t^2 + \beta V_{t+1}(h_{t+1})\right\} \tag{48}$$
$$\text{st } x_t^1 + \psi_t h_{t+1} = \ell_t^1 + \psi_t h_t + T_t - d_t,\ x_t^2 = G_t\left(\ell_t^2, h_t\right) \text{ and } \ell_t^1 + \ell_t^2 \in [0, \bar{\ell}].$$

In this specification h_t does not enter $U(\cdot)$ directly, but indirectly as an input to $G_t(\cdot)$, although one could generalize this, by disaggregating home capital into a part that produces goods or services (e.g., vacuum cleaners) and a part that generates utility directly (e.g., televisions). It is well known that one can substitute out

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[21] This requires that wealth *other than home equity* cannot be used to collateralize loans, perhaps because it is hard to seize; in the language of **?**, h_t is and $T_t - d_t$ is not *pledgeable*.

[22] See Aruoba et al. (2011a) for a detailed analysis and up-to-date list of references.

$x_t^2 = G_t\left(\ell_t^2, h_t\right)$ and maximize out ℓ_t^2, for any given values of x_t^1, ℓ_t^1 and h_{t+1}, to derive a reduced-form model that only depends on market variables, as in our baseline specification. Still, in general, there are several reasons for being explicit about household production, especially for quantitative work. In the interests of space, however, we proceed using as a benchmark model the version where home production is not made explicit. We merely wanted to make the point that our general approach is consistent with the more detailed work on home production in both the micro and macro literatures. We now return to the baseline specification, without an explicit home production sector.

Returning to the baseline specification, and assuming $\ell_t \in [0, \bar{\ell}]$ does not bind, we eliminate ℓ_t using the budget equation to write

$$W_t(d_t, h_t) = \psi_t h_t + T_t - d_t + \max_{x_t}\left\{U(x_t, h_t) - x_t\right\} + \max_{h_{t+1}}\left\{\beta V_{t+1}\left(h_{t+1}\right) - \psi_t h_{t+1}\right\}, \tag{49}$$

where we have normalized the utility parameter $A = 1$. Immediately (49) implies that choices at t, and in particular h_{t+1}, are independent of (d_t, h_t), which simplifies the analysis because we do not have to keep track of distributions across agents.[23] The FOC for the maximization in (49) are

$$U_1(x_t, h_t) = 1 \text{ and } \psi_t = \beta \frac{\partial V_{t+1}}{\partial h_{t+1}}. \tag{50}$$

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[23] These results hold if the constraint $\ell_t \in [0, \bar{\ell}]$ is slack. More generally, people with very low or high net worth may be unable to set ℓ_t high or low enough to settle all debt or get to their preferred h_{t+1} in a given period, and, in this case, they would need to borrow or save between CM meetings using frictionless (mortgage) credit. But if we start with a distribution of h_t and d_t that is not too disperse, relative to $[0, \bar{\ell}]$, households can settle all their debt and choose the same housing in each CM without borrowing or lending. If one were to pursue these issues in greater depth, we think it would be preferable to use a life-cycle model, rather than a inifinit-lived-agent model.

Also, we have

$$\frac{\partial W_t}{\partial d_t} = -1 \text{ and } \frac{\partial W_t}{\partial h_t} = U_2(x_t, h_t) + \psi_t, \tag{51}$$

so that, in particular, W is linear in debt (more generally, in net worth), but not housing.

We now describe what happens in the DM in a trading opportunity, which we recall arises with probability α_h for a household and α_f for a retail firm each period. In such an opportunity, firms (or sellers) produce y_t, while households (or buyers) consume y_t, in return for which the latter issue a promise of payment $d_t \leq D(e_t)$ in the CM. The terms of trade (y_t, d_t) can be determined in many ways in this type of model, as discussed in the surveys mentioned in the Introduction. We begin by describing competitive Walrasian pricing, where, to motivate this, we assume agents with opportunities to trade meet in large groups, rather than bilaterally.[24] First, note that buyers' trade surplus is $S_{bt} = u(y_t) + W(d_t, h_t) - W(h_t) = u(y_t) - d_t$, since $W(\cdot)$ is linear in d_t by (51). Similarly, for sellers, $S_{st} = d_t - v(y_t)$. Then sellers and buyers solve

$$\begin{aligned}(y_t, d_t) &= \arg\max S_{st} \text{ st } d_t = p_t y_t \\ (y_t, d_t) &= \arg\max S_{bt} \text{ st } d_t = p_t y_t \leq D_t,\end{aligned}$$

taking as given the price p_t and debt limit D_t.

To solve this model, as a preliminary step, first note that equilibrium without the debt constraint is given by $y_t = y^*$, where $u'(y^*) = v'(y^*)$, and $p_t = p^* = v'(y^*)$. Define $d^* = p^* y^* = v'(y^*) y^*$. If $d^* \leq D_t$ then the DM equilibrium at t is what we

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[24] By analogy, our multilateral DM is like a Lucas-Prescott labor market, while our bilateral DM is like Mortenson-Pissarides.

found ignoring the constraint. But if $d^* > D_t$ then equilibrium is instead given by $p_t = v'(y_t)$, from the sellers' FOC, and $y_t = D_t/p_t$, from the buyers' constraint. In other words, when $d^* > D_t$ we get a constrained equilibrium with $d_t = D_t$, and y_t is the solution to $v'(y_t)\, y_t = D_t$. For future reference, define $g(y_t) = v'(y_t)\, y_t$, so that when $d^* > D_t$ the equilibrium output can be written $y_t = g^{-1}(D_t) < y^*$. Summarizing:

Proposition 8. *Let* y^* *and* $p^* = v'(y^*)$ *be the equilibrium ignoring the constraint* $d_t \leq D_t$*, and let* $d^* = g(y^*)$*. Then, as shown in Figure 3, equilibrium in the DM at t is given by:*

$$y_t = \begin{cases} g^{-1}(D_t) & \textit{if } D_t < d^* \\ y^* & \textit{if } D_t \geq d^* \end{cases} \quad \textit{and } d_t = \begin{cases} D_t & \textit{if } D_t < d^* \\ d^* & \textit{if } D_t \geq d^* \end{cases} \tag{52}$$

As an alternative trading mechanism, suppose we pair off buyers and sellers and have them bargain bilaterally. One approach is to use the generalized Nash solution

$$(y_t, d_t) = \arg\max S_{bt}^{\theta} S_{st}^{1-\theta} \text{ st } d_t \leq D_t.$$

One show that the outcome is the same as (52), except that instead of $g(y) = v'(y)\, y$, we redefine

$$g(y) = \frac{\theta v(y)\, u'(y) + (1-\theta)\, u(y)\, v'(y)}{\theta u'(y) + (1-\theta)\, v'(y)} \tag{53}$$

(see Lagos and Wright (2005b)). In particular, with Nash we have $d^* = g(y^*) = \theta v(y^*) + (1-\theta)\, u(y^*)$, while with Walras we have $d^* = v'(y^*)\, y^*$, but in either case Proposition 8 holds exactly as stated. Alternatively, one can use the proportional bargaining solution in Kalai (1977a),

$$(y_t, d_t) = \arg\max S_{bt} \text{ st } S_{bt} = \theta\left[u(y_t) - v(y_t)\right] \text{ and } d_t \le D_t,$$

which has some advantages in this class of models (see Aruoba et al. (2007)). Kalai gives the same outcome as Nash iff the constraint is slack, but in any case, the outcome still satisfies (52), except now we redefine

$$g\left(y\right) = \theta v\left(y\right) + \left(1-\theta\right) u\left(y\right). \tag{54}$$

Finally, in the appendix we also provide a simple strategic bargaining model that works well in this environment.

We consider all the above trading mechanisms to show the results are robust. In each case, the equilibrium outcome is given by (52) for a particular choice of $g\left(\cdot\right)$. Then the DM value function can be written

$$V_t(h_t) = W(0, h_t) + \alpha_h \left\{u\left[y(\psi_t h_t)\right] - d(\psi_t h_t)\right\}, \tag{55}$$

where it is understood that $y(\psi_t h_t)$ and $d(\psi_t h_t)$ are given by (52) with $D_t = D\left(\psi_t h_t\right)$. Using (51), we have

$$\frac{\partial V_t}{\partial h_t} = U_2(x_t, h_t) + \psi_t + \alpha_h \psi_t \left[u'\left(y\right) y'(\psi_t h_t) - d'(\psi_t h_t)\right].$$

Differentiating y and d using (52), we reduce this to

$$\frac{\partial V_t}{\partial h_t} = U_2(x_t, h_t) + \psi_t + \alpha_h \psi_t D_1 L\left[y\left(\psi_t h_t\right)\right], \tag{56}$$

where

$$L(y) \equiv \begin{cases} u'(y)/g'(y) - 1 & \text{if } y < y^* \\ 0 & \text{otherwise} \end{cases}. \tag{57}$$

The function $L(y)$, represents a *liquidity premium*, is common and can be shown to be proportional to the Lagrangian multiplier on the credit constraint. Inserting these results into the FOC for h_{t+1} in (50), and using $1+r = 1/\beta$, we get the Euler equation for housing

$$(1+r)\psi_t = U_2(x_{t+1}, h_{t+1}) + \psi_{t+1} + \alpha_h \psi_{t+1} D_1 L[y(\psi_t h_t)]. \tag{58}$$

The three terms on the RHS describe the three benefits of owing a bigger house: 1) it yields more utility; 2) it makes you wealthier; and 3) it allows you to borrow more. Setting $h_t = H$, and using the FOC $U_1(x_t, H) = 1$ to define $x = X(H)$, (58) defines a difference equation in the price of housing, $\psi_t = \Psi(\psi_{t+1})$. An *equilibrium* is any sequence $\{\psi_t\}$ solving $\psi_t = \Psi(\psi_{t+1})$ that is non-negative and bounded, boundedness being required to satisfy a standard transversality condition (see Rocheteau and Wright (2010)). Given $\{\psi_t\}$, we can recover the other variables, $e_t = \psi_t H$, $D_t = D(e_t)$, $y_t = y(e_t)$ etc.

9 Steady State Equilibrium

A stationary equilibrium, which here is equivalent to a steady state, is a constant solution to the difference equation $\psi = \Psi(\psi)$. In steady state there are no capital gains, and (58) becomes

$$r\psi = U_2\left[X(h), h\right] + \alpha_h \psi D_1 L\left[y\left(e\right)\right]. \tag{59}$$

One can interpret this as the long-run demand for housing. The slope is

$$\frac{\partial h}{\partial \psi} = \frac{U_{11}\left(U_2 - \alpha_h \psi^2 h L' y'\right)}{\left(U_{11}U_{22} - U_{12}^2 + U_{11}\alpha_h \psi^2 L' y'\right)\psi} < 0, \tag{60}$$

after inserting $X'(h) = -U_{12}/U_{11}$ and using (58) to eliminate r. In this paper we assume $L'\left(y\right) < 0$, which immediately implies that demand slopes downward, we do *not* actually need this, as one can show all of the same results without $L'\left(y\right) < 0$ using the method in Wright (2010). Here we prefer to avoid these technicalities, especially since $L'\left(y\right) < 0$ follows automatically for many of the mechanisms used here anyway, including Walras and Kalai pricing. Since demand is downward sloping we have,

Proposition 9. *Given fixed supply H, steady state equilibrium is unique.*[25]

If in steady state $e = \psi H > e^*$, then $L(e) = 0$ and (59) implies $\psi = \psi^* \equiv U_2\left[X(H), H\right]/r$, where ψ^* is *the fundamental price*, defined as the present value of the marginal utility of living in house H forever. In this case, households have enough home equity that they are never liquidity constrained, and houses bear no premium. But if $e < e^*$ then $L(y) > 0$ and (59) implies $\psi > \psi^*$. In this case, home equity is scarce and houses bear a liquidity premium, which means price is above the fundamental value. As discussed in the Introduction, we call this a *bubble*. The idea behind

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[25] This highlights another big difference between housing and fiat currency: in any decent monetary model, whenever there exists a steady state where currency is valued there exists another where it is not. This is not true when money is replaced by home equity as a way to facilitate transactions.

it this: if at the fundamental price $e = H\psi^* < e^*$, there will be excess demand, because agents not only get a utility flow from shelter, they also use it to collateralize loans. If the credit constraint binds, agents would pay a premium for assets to relax it. The exact outcome depends on the mechanism – e.g., Walras versus Kalai pricing – but a bubble emerges whenever $e < e^*$.

There are related results in similar models with different assets, including not only fiat currency, but Lucas trees and neoclassical capital, all of which can bear liquidity premia in some circumstances (Geromichalos et al. (2007a); Lagos and Rocheteau (2008a); Lester et al. (2011)). The economics is different with houses. With trees, there is a liquidity premium iff the exogenous supply is low; and with capital there is a liquidity premium iff the endogenous supply is low, but the endogenous supply without liquidity considerations is itself pinned down by productivity. With housing, there can be a liquidity premium iff e is low, but e can be low either when H is low or when H is high, depending on the elasticity of demand. In particular,

$$\frac{de}{dH} = \psi + H\frac{d\psi}{dH} = \frac{\psi H \left(U_{22}U_{11} - U_{21}^2\right) + \psi U_2 U_{11}}{U_{11}\left(U_2 - \alpha\psi^2 H L'\right)} \simeq -H\left(U_{22}U_{11} - U_{21}^2\right) - U_2 U_{11},$$

where the notation $A \simeq B$ means that A and B have the same sign.[26]Also, this means that welfare W might fall as H increases, which is typically not the case with financial assets in these models. If H increases here, even though CM utility must rise, one can construct explicit examples where ψH falls DM utility might fall by enough to reduce total utility.

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[26] Consider the example

$$U(x,h) = \tilde{U}(x) + \frac{h^{1-\sigma}}{1-\sigma}.$$

For $\sigma < 1$ housing bears a liquidity premium when H is low, while for $\sigma > 1$ this happens when H is high.

Proposition 10. *Suppose $h = H$ is fixed. If $e > e^*$ in steady state we get the fundamental price $\psi^* = U_2\left[X(H), H\right]/r$; if $e < e^*$ we get a premium, or bubble, $\psi > \psi^*$. We can have $e < e^*$ and hence $\psi > \psi^*$ either when H is low or when H is high, depending on utility. It is possible to have $\partial W/\partial H < 0$.*

If α_h increases, home equity is used more often as collateral and the housing price increases. It is less clear how it changes with D_1. On the one hand, higher D_1 makes home equity more useful as collateral. On the other, liquidity is less scarce as D_1 increases. The former effect increases price while the latter effect decreases it.

Proposition 11. *Steady state housing prices are increasing in α_h, but can be increasing or decreasing in D_1.*

10 Dynamics: Cyclic, Chaotic and Stochastic Equilibria

Consider first deterministic equilibria, given by non-negative and bounded solutions to

$$(1+r)\,\psi_t = \Psi\left(\psi_{t+1}\right) = U_2\left[X\left(H\right), H\right] + \psi_{t+1} + \alpha_h \psi_{t+1} D_1 L\left[y(\psi_{t+1} H)\right]. \qquad (61)$$

The first observation is that any interesting dynamics must emerge from liquidity considerations, which show up in the nonlinear term $L\left[y(\psi_{t+1}H)\right]$. To see this, set α_h or D_1 to 0. Then (61) is a linear difference equation, which can be rearranged as

$$\psi_{t+1} = -U_2\left[X\left(H\right), H\right] + (1+r)\,\psi_t.$$

This has a unique steady state at the fundamental price $\psi = \psi^*$, which is also the unique equilibrium, since any solution other than $\psi_t = \psi^* \ \forall t$ is unbounded or becomes negative. There are no dynamics unless the liquidity motive is operative.

When $\alpha_h > 0$, as long as $H\psi_{t+1} < e^*$ we have $L\left[y(\psi_{t+1}H)\right] > 0$, and the nonlinear part of (61) kicks in. We analyze this in (ψ_{t+1}, ψ_t) space, where it is natural to think of ψ_t as a function of ψ_{t+1}, because for any ψ_{t+1} there is a unique individual demand h_t given ψ_t, so market clearing pins down ψ_t. However, as usual, there can be multiple values of ψ_{t+1} for which this mapping yields the same ψ_t, so that the inverse $\psi_{t+1} = \Psi^{-1}(\psi_t)$ can be a correspondence. Of course, Ψ and Ψ^{-1} cross on the 45^o line in (ψ_{t+1}, ψ_t) space at the unique steady state. Textbook methods (e.g., Azariadis (1993)) tell us that whenever Ψ^{-1} and Ψ cross off the 45^o line there exists a cycle of period 2 – i.e., a solution $\left(\psi^1, \psi^2\right)$ to $\psi^2 = \Psi\left(\psi^1\right)$ and $\psi^1 = \Psi\left(\psi^2\right)$ that is non-degenerate, in the sense that $\psi^1 \neq \psi^2$ – and that this happens whenever Ψ has a slope less than -1 on the 45^o line. In this 2-cycle equilibrium, even though fundamentals are constant, ψ oscillates between ψ^1 and ψ^2 as a self-fulfilling prophecy. This is a non-stationary bubble.

Before discussing the economic intuition, consider higher-order n-cycles – i.e., non-degenerate solutions to $\psi = \Psi^n(\psi)$. To reduce notation, normalize $H = 1$, and consider by way of example $v(y) = y$, $D(e) = e$, and[27]

$$U(x,h) = \tilde{U}(x) + \kappa\frac{h^{1-\sigma}}{1-\sigma} \text{ and } u(y) = \eta\frac{(y+\varepsilon)^{1-\gamma} - \varepsilon^{1-\gamma}}{1-\gamma}.$$

To show our results are robust to the choice of mechanism used to determine the terms of trade in the DM, we consider Walrasian pricing, Kalai bargaining and the

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[27] The form of $\tilde{U}$ is irrelevant for all the results. The role of ε in $u(y)$ is simply to force $u(0) = 0$. Also, the value of σ is irrelevant, since it vanishes from $\partial U/\partial h$, given $h = H = 1$.

extensive-form game specified in the appendix. These choices and the parameter values across examples are shown in Table 1.

	Example 1	Example 2	Example 3	Example 4	Example 5,6
Mechanism	Walras	Walras	Kalai	Game	Walras
α_h	0.5	0.5	0.9	0.5	0.99
β	0.8	0.6	0.6	0.8	0.95
θ	n/a	n/a	0.9	0.6	n/a
κ	0.125	0.3333	0.1	0.125	0.1
σ	n/a	n/a	n/a	n/a	10
η	1.5125	3.2479	0.5882	3.0368	1.043
γ	2	7	9	8	16
ε	0.1	0.1	0.5	0.1	0.001

Table 1: Parameter Values for the Examples

Example 1, with Walrasian pricing, is shown in Figure 4, where the unique equilibrium is the steady state. Example 2, also with Walrasian pricing, but different parameters, is shown in Figure 4, which exhibits a 2-cycle. In this case, the unconstrained value of y is $y^* = 1.0833$, the constraint binds iff $\psi < y^*$, and this happens in alternate periods. Figures 5 show similar results for Examples 3 and 4, using Kalai and strategic bargaining, to show the effects do not require a particular pricing mechanism but from general liquidity considerations.[28] It is not hard to

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[28] This is relevant because in some related models, such as the model of endogenous credit limits in Gu and Wright (2010), cycles emerge for some mechanisms (Walras and Nash) but not the others (Kalai).

generate higher order cycles. Example 2 also has a 3-cycle, with $\psi^1 = 0.8680 < y^*$, $\psi^2 = 1.5223 > y^*$, $\psi^3 = 1.1134 > y^*$. When a 3-cycle exists, by the Sarkovskii theorem and the Li-Yorke theorem (again see Azariadis (1993)), there exist cycles of all orders and chaotic dynamics. Chaos is defined in the context of our model as a non-negative and bounded solution $\{\psi_t\}$ to (61) with the property that $\psi_s \neq \psi_t$ $\forall s \neq t$.

We conclude once we take into account the collateral role of home equity, we can generate equilibria where house prices display complicated dynamics, even though fundamentals are stationary, and agents have perfect foresight. House prices can fluctuate simply as a self-fulfilling prophecy. This is because houses have liquidity value in imperfect credit markets, and, liquidity is at least partially a matter of beliefs. [29]

The point of the above analysis was to show that housing markets can in principle generate complicated dynamics. However, at least simple examples of equilibria with cycles or chaos do not resemble recent housing market experience all that well – prices tend to go up and down rather too regularly, and do not display the stereotypical bubble pattern of a prolonged boom followed by collapse. But we have to work a little harder to construct housing price dynamics which resemble the data. Example 5 gives an example of such equilibrium which is illustrated in Figure 6. The red curve depicts ψ_{t+1} as a correspondence of ψ_t and the intersection of the red line and the 45^o line corresponds to a steady state. Notice, however, the steady

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[29] Heuristically, in a 2-cycle, if at t one believes ψ_{t+1} and hence e_{t+1} will be high, then liquidity will be plentiful at $t+1$, which lowers the price people are willing to pay for it today. Therefore low ψ_t is consistent with high ψ_{t+1}, and by the same logic high ψ_{t+1} is consistent with low ψ_{t+2}, and so on. One might wonder why rational agents are willing to pay a lot for an asset at $t+1$ when they know its price is about to drop at $t+2$? It is precisely *because* the price is about to drop that liquidity will soon become scarce, and thus liquidity is currently in high demand. A cycle of order $n > 2$, or a chaotic path, is a more complicated self-fulfilling prophecy, but the intuition is similar.

state price ψ^* corresponds to two ψ_{t+1}(There is another ψ_{t+1} can give the steady state price today, but it is not depicted in the graph). One is $\psi_{t+1} = \psi^*$, which means price stays at the steady state level. The other is $\psi_1 > \psi^*$. If $\psi_{t+1} = \psi_1$, the prices will set on an increasing price following the dashed line. The price keeps increasing at a rate slightly less than $r = 1/\beta - 1$ to ψ_2. After reaching ψ_2, there is no equilibrium where the price continues to increase but it could drop next period to ψ_3 and then converge back to the steady state level ψ^*. This equilibrium produces price dynamics with booms and collapse. This example shows that housing prices can exhibit bubble-like dynamics even under perfect foresight in this simple model.

Additionally, following standard methods, we can construct sunspot equilibria, or stochastic cycles. In such an equilibrium, e.g., the price can be ψ^1 and jump to $\psi^2 > \psi^1$ with some probability λ^1 each period; then when it is ψ^2 it can fall back to ψ^1 with some probability λ^2 each period.[30] Agents have rational expectation, they all know the stochastic structure of the equilibrium – in this example, λ^1 and λ^2 – yet we can still have random price fluctuations even if fundamentals are deterministic.[31] There also exist sunspot equilibria where ψ takes on many values and nonstationary sunspot equilibria. In Example 6 there is one such equilibrium. Suppose before period 4, price stays at steady state $\psi^* = 0.5255$. From period 4 to period 8, every period it has some probability to get into a deterministic path starting from $\psi = 0.5350$ and converging to ψ^* and some probability to go to a higher ψ_{t+1}. The probability of getting to the deterministic path is changing every period and agents

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[30] There are different methods for constructing sunspot equilibria. Azariadis and Guesnerie (1986) note that in the limit when $\lambda^1 = \lambda^2 = 1$ the sunspot equilibrium described in the text reduces to a 2-cycle, which exists under conditions described above, and appeal to continuity. This method applies in our model.

[31] In equilibrium, demand for liquid assets and hence ψ^2 will be high not because the price is about to drop to ψ^1 for sure, as in a 2-cycle, but because it drops with some probability; still, the basic economic intuition is similar.

are fully aware of the probabilities. After period 8, everything is deterministic again. One possible equilibrium path in this economy is that ψ increase at 9% per year for 5 years. And then collapse at year 9 and then oscillates back to ψ^*. In this example $\beta = 0.95$, indicating that $r = 1/\beta - 1 \approx 0.053$. We have the housing price increase at a rate much higher than interest rate for a long period. However, this path happens only with around 12% probability, e.g., every century this economy should expect about one decade of bubbles.

Of course this is related to some standard results in monetary theory, where it is well know how to generate similar dynamics (again Azariadis (1993) is a textbook reference). But H is different from M. Any reasonable model of fiat currency has at least two steady states, one where M is valued and one where it is not, while our housing model has a unique steady state. Obviously, $\psi = 0$ cannot be an equilibrium when H has fundamental value as shelter. Part of our motivation for this project came from an example of Kocherlakota, where something he called housing could exhibit somewhat interesting dynamics – not as interesting as described here, but at least an equilibrium where price jumps stochastically from $\psi > 0$ to an absorbing state with $\psi = 0$ (a special case of the sunspot equilibria discussed above, special because once ψ goes down it never comes back). But what he calls housing is in fact a fiat object, since he endows it with fundamental utility value 0. Giving H intrinsic value as in our model not only rules out equilibria where $\psi = 0$, it also rules out equilibria where $\psi_t \to 0$, either stochastically or deterministically. Hence, the dynamics here are not merely reinterpretations of simple results in monetary theory.

11 Endogenous Housing Supply

All examples provides in the previous section relies on fixed H. To see whether these results are robust to endogenous housing supply. In the next Section, we now show we can get similar results when we allow houses to be produced by the private sector. Assume that in the CM, in addition to the technology for converting ℓ_t into x_t, we introduce a technology for the construction of H, with cost function $x_t = c(\Delta h_t)$. Thus, Δh_t units of new housing require an input of $c(\Delta h_t)$ units of numeraire. Construction, like other CM activity, is perfectly competitive. Hence, profit maximization implies

$$\psi_t = c' \left[h_{t+1} - (1-\delta)h_t\right], \tag{62}$$

equating price to the marginal cost of augmenting the supply from $(1-\delta)h_t$ to h_{t+1}, where δ is the depreciation rate. The households' CM problem is unchanged, except now $e_t = \psi_t \left(1-\delta\right) h_t$, and the Euler equation becomes

$$(1+r)\,\psi_t = U_2\left[X\left(h_{t+1}\right), h_{t+1}\right] + \psi_{t+1}\left(1-\delta\right) + \alpha_h\left(1-\delta\right)\psi_{t+1} D_1 L\left[y\left(e_{t+1}\right)\right]. \tag{63}$$

In steady state, (62)-(63) can be written

$$\begin{aligned} (r+\delta)\,\psi &= U_2\left[X\left(h\right), h\right] + \alpha_h\left(1-\delta\right)\psi D_1 L\left[y\left(e\right)\right] && (64) \\ \psi &= c'\left(\delta h\right), && (65) \end{aligned}$$

where (64) is the straightforward generalization of long-run demand (59), while (65) is a long-run supply relation. It is easy to show they intersect uniquely. As in Section 10, we can get a liquidity premium when the (now endogenous) supply of housing is high or when it is low, depending on elasticities. Therefore, we have:

Proposition 12. *The results in Propositions 8-11 all hold with H endogenous.*

Moving beyond steady states, equilibrium generally is defined by a path for $\{h_t, \psi_t\}$ satisfying the dynamical system (62)-(63). One should anticipate the existence of interesting dynamics in this system, given the results in Section 10, even if the methods are somewhat different for the bivariate system. Instead of an exhaustive analysis of the system, we use it to organize a particular narrative concerning recent events. As the story goes, at the start of the episode in question, financial innovation gave households easier access to home equity loans: this is what it means to say developments allowed households "to turn their previously illiquid housing assets into ATM machines." We now show that this can lead to dramatic increase in house prices followed by a crash.

We present two ways to construct housing price path that resembles the data without appealing to stochastic equilibria. One way is that the housing price is purely driven by self-fulfilling prophecy. Under some parameters, there can be many transition paths after one time unexpected change in financial sector. Households' different expectations about future prices give different transition paths. Some of them look qualitatively like the data. The other way does not relying on self-fulfilling prophecy. instead, we simply take the credit limit from the data and show that a sequence of technological changes, some expected and some unexpected, that relax the credit limit of home equity loans can generate the data. In this case, there is a unique transition path. This allows for a more serious calibration.

11.1 Self-fulfilling Prophecy

Suppose at $t = 1$ the economy is at the unique steady state, where housing is hard to use as collateral, say $D(e) = D_1 e$ with D_1 small. To make the point stark, in the CM at $t = 2$, suppose an unexpected once-and-for-all financial innovation occurs that we formalize by increasing D_1. The resulting transition path depends on parameters. For some parameter values, we get the analog of saddle-path stability: there is a unique equilibrium where the price ψ jumps with the change in D_1, and monotonically declines to its new steady state value as constructing raised the housing stock to its new steady state value. For other parameters, the system displays a classic indeterminacy associated with a stable steady state: in this case, there are many transition paths from the initial to the new steady state. This means we can let ψ jump after the change in D_1 to any value in some range before beginning the transition, giving us some freedom to pick a path that we find reminiscent of recent experience.

A particular transition is shown in Figure 7, constructed under Walrasian pricing, using parameters such that the constraint $y \leq D(e)$ is binding, and verifying numerically that both eigenvalues are real and less than 1 at the new steady state. The equilibrium path in Figure 7 looks like the data in Figure 2, starting around 2000, in the following sense. We do not mean they look *exactly* the same, which would be more than one should expect (primarily because the actual data were presumably generated by something other than a one-time surprise increase in D_1, given that it may take time for financial developments to evolve, for agents to understand and take advantage of them, etc.). All we mean is that the paths look qualitatively similar, in that housing prices first soar then tumble, whether we measure them by the relative price ψ_t or the price-rent ratio. Also, home equity loans go up, and stay up, as households take advantage of financial innovation. Construction also rises, then drops as we approach the new steady state. Note home equity loans rise quickly even though the housing stock increase takes time because $e_t = p_t h_t$ rises before h_t.

What is perhaps more subtle is that welfare increases over the period, as also shown in the Figure 7. Financial development, formalized as an increase in D_1, is good because it relaxes credit constraints, even though it can set us off on a path that resembles a bubble, complete with the ultimate collapse.

11.2 Determinate Transition Path

This method described in last section can generate booms and bursts in housing price. But it is not satisfactory in two ways. First, it relies on unreasonable parameters. In particular, in the example, β is set to be 0.6, which is very low. Second, it relies on the multiplicity which makes calibration very difficult. Therefore, it is very hard to take the theory to data. However, in last example, there is only one time change in financial sector when D_1 changes from 0 to 1. But we can imagine that the financial innovation is not a one period change but happens in several periods. The households might be aware of some changes while other changes might be surprise to the households. If that is the case, we can construct price path resembling the data without appealing to the multiplicity. It is fairly clear that the housing price can increase due to the news that house will be more valuable as a collateral. But what is less intuitive is that according to Proposition 11, D_1 can decrease the housing price. As a result, if there is an unexpected financial change driving D_1 to a comparatively high level, the housing price plumbs. This is the key to construct the housing price dynamics.

We first take the ratio of revolving home equity line of credit and total residential fixed asset from 1996 to 2010 and then divide it by α_h to construct D_1. We set $\alpha_h = 0.1$, which is approximately the proportion of home owners with home equity line of credit. The constructed D_1 has three major change periods. The first period is from 2000 to 2003, when D_1 changes gradually from around 0.11 to 0.20. The second period is 2004, when D_1 jumps from 0.2 to 0.26. And the last change is in the year 2008 when D_1 suddenly increases from 0.26 to around 0.33. Now suppose in 1996, households expect D_1 will go up in the future. They predict that D_1 stays constant at 0.11 until the year 2000. And then it starts to increase to gradually to 0.2 in 2003 and stay at 0.2 for several years before reaching the new steady state 0.26. When the year 2003 comes, they find that the innovation is more than what they thought and then believe D_1 will jump to 0.26 and stays there forever. After four years, they find the innovation is still going on, and next year they can borrow up to 33% of their home equity and they believe this is the new steady state. But a year later, in 2008, they find D_1 will keep 0.33 for two years and then jump to 0.8 and stay there forever. The transition is displayed Figure 8. The residential investment and home equity loan index series are normalized to fit into the graph. In this example, housing price peaks in 2007 and is about 11% higher than the price in 1996. This is about a quarter of the change in the data. Price-rent ratio has a similar change. After 2007, the house price drops suddenly by around 7%. The amount of home equity loan triples during this period which is almost the same as in data.

We can conclude that in this simple example, we use D_1 recovered from data and find that our model can generate housing price series very similar to the data. We cannot generate very big housing price changes in this example but it still accounts for a quarter of the change in the data. In addition, we can generate the big drop in housing price after 2007.

12 Intermediated Collateralized Lending: Banks

In the model, so far, households put up home equity as collateral for consumption loans. This is consistent in a stylized way with experience, but in reality, more typically, households use home equity to borrow cash from a bank or related institution, then use the cash to buy consumption goods. Here we model this explicitly, not only for the sake of realism, but to investigate the interaction between monetary policy and housing.[32] We assume for the sake of this discussion that money is the only means of payment accepted in the DM due to the fact that buyers and sellers trade anonymously (as is standard in modern monetary economics; see the references in the Introduction). Intuitively, if a seller in the DM does not know the identity of individual buyers, and if this seller were to offer a buyer a consumption loan collateralized by equity in a house, the latter could put up a claim on a nonexistent house, or the house of a stranger, or one that is under water, etc. Nevertheless, buyers here have relationships with their bankers, who keep records and know their identities. Hence, bank loans are possible, while pure consumption loans from retailers are not.

To ease the presentation we fix the housing stock at H and set $\delta = 0$. Also, we are more explicit about preference/opportunity shocks. At the beginning of each period, with probability α_h a household wants to consume the DM good, and with

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[32] Here we are motivated by Ferraris and Watanabe (2008) and Li and Li (2010), who also provide detailed models where real assets are used to secure money loans. Intuitively, in such models, money and other assets can be complements, while usually they are substitutes, in a Tobinesque kind of way, because they provide different ways to store wealth and facilitate trade (see, e.g., Lester et al. (2011) for a recent discussion and more references).

probability $1 - \alpha_h$ does not. Conditional on wanting to consume, for simplicity, the household trades in the DM with probability 1. At the start of each period, before the DM opens, households not only have access to any cash brought in from the previous period, they can also access a financial market, FM. We describe FM in terms of intermediaries that we call banks and that work as follows. Households that want to consume in the DM (borrowers) may withdraw cash from banks to increase their purchasing power, while those that do not (depositors) keep their money in the banks. The FM operates as a competitive market, with settlement in the CM, but we maintain the assumption of limited commitment: households can renege on bank loans, if they like, but then banks can seize their houses. It is not important, and cannot be determined, who carries money out of the CM into the next period (see below). Therefore, we assume all currency is put into banks at the end of each CM, and those that want to consume in the DM withdraw – generally, more than their deposits – while the rest leave it in the bank until the next CM.[33]

The generalized CM value function is $W_t(d_t, h_t, m_t)$, where a household's portfolio now consists of real debt d_t, housing h_t and money in the bank m_t. The FM value function next period is $J_{t+1}(h_{t+1}, m_{t+1})$, given that all debt is paid off in the

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[33] This setup is basically the same as the model of banking in Berentsen et al. (2007), except they assume households carry cash between periods, and all intermediaries do is reallocate it after the shocks are realized. We assume banks hold the cash between periods to highlight the similarity at a general level to the literature following Diamond and Dybvig (1983), where again the role of intermediation is to allocate liquidity. The key feature in our model is that home equity is required to secure bank loans. The reason it does not matter if one's cash is kept in one's pocket or one's bank account between periods is that there is no interest paid on idle balances across periods, only those kept in the bank between the FM and CM, while thee CM is open, since these can be used to make loans.

CM, without loss of generality, as in the baseline model. The CM problem is

$$W_t\left(d_t, h_t, m_t\right) = \max_{x_t, \ell_t, h_{t+1}, m_{t+1}} \left\{U\left(x_t, h_t\right) - \ell_t + \beta J_{t+1}\left(h_{t+1}, m_{t+1}\right)\right\} \tag{66}$$

$$\text{st } x_t + \psi_t h_{t+1} + \phi_t m_{t+1} = \ell_t + \psi_t h_t + T_t + \phi_t m_t - d_t$$

where ϕ_t is the value of a dollar in terms of the CM numeraire (the inverse of the nominal price level). As in the baseline model we can eliminate ℓ_t and derive the FOC

$$U_1\left(x_t, h_t\right) = 1, \psi_t = \beta\frac{\partial J_{t+1}}{\partial h_{t+1}} \text{ and } \phi_t = \beta\frac{\partial J_{t+1}}{\partial m_{t+1}}, \tag{67}$$

showing $\left(x_t, h_{t+1}, m_{t+1}\right)$ is independent of $\left(d_t, h_t, m_t\right)$, and as usual W_t is linear in wealth.

The FM value function satisfies

$$J_t\left(h_t, m_t\right) = \alpha_h \max_{\hat{m}_t} V_t\left[\left(1+\rho_t\right)\left(\hat{m}_t - m_t\right)\phi_t, h_t, \hat{m}_t\right] + \left(1-\alpha_h\right) W_t\left[-\left(1+\rho_t\right) m_t \phi_t, h_t, 0\right] \tag{68}$$

$$\text{st } \left(1+\rho_t\right)\left(\hat{m}_t - m_t\right)\phi_t \leq D\left(\psi_t h_t\right), \tag{69}$$

where ρ_t is the interest rate and $D\left(\psi_t h_t\right)$ the borrowing limit on bank loans. Thus, with probability α_h the household increases money balances from m_t to $\hat{m}_t$, takes it all to the DM, and has a real debt obligation in the next CM of $\left(1+\rho_t\right)\left(\hat{m}_t - m_t\right)\phi_t$. And with probability $1-\alpha_h$ the household leaves all money in the bank, skips the DM, and goes straight to the next CM with a negative real liability (i.e., a credit) $-\left(1+\rho_t\right) m_t \phi_t$. Notice that V_t is now the DM value function conditional on wanting to being able to consume, and it depends on debt and cash, as well as housing (in the baseline model there was no debt entering the DM and there was no cash). It satisfies

$$V_t\left(d_t, h_t, m_t\right) = u\left(y_t\right) - \phi_t \hat{m}_t + W_t\left(d_t, h_t, m_t\right) \tag{70}$$

where, following Proposition 8 in the baseline model, y_t is determined by some mechanism according to $g\left(y_t\right) = \phi_t \hat{m}_t$.

Equilibrium can be divided into two cases according to whether the FM borrowing constraint, $\left(1+\rho_t\right)\left(\hat{m}_t - m_t\right)\phi_t \leq D\left(\psi_t h_t\right)$, is binding; and if it is binding, there are two subcases, as discussed below. In case 1, where it is slack, from (68) we get the FOC for $\hat{m}$

$$-\left(1+\rho_t\right)\phi_t + \frac{\partial V_t}{\partial \hat{m}_t} = 0, \tag{71}$$

using the fact that the derivative of V_t wrt debt is simply -1, since debt in the DM is simply carried over into the CM. Using (70), it is easy to show this reduces to $L\left(y_t\right) = \rho_t$. This says the amount of cash one brings to the DM, inclusive of loans, sets the marginal benefit of liquidity $L\left(y_t\right)$ to the marginal cost, which here is the loan rate ρ_t.

In terms of the choice of cash deposits coming out of the CM, as opposed to the choice coming out of the FM $\hat{m}$, the FOC for m_{t+1} in (67) reduces to the Euler equation

$$(1+r)\,\phi_t = \left(1+\rho_{t+1}\right)\phi_{t+1}. \tag{72}$$

To understand this, let i_t denote the nominal interest rate agents would require to do the following deal: give up a dollar in the CM at t and get back $1+i_t$ dollars in the CM at $t+1$.[34] The standard Fisher equation says $1+i_t = \phi_t/\beta\phi_{t+1}$, since $\phi_t/\phi_{t+1} =$

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[34] Note that we are *not* thinking of a tangible nominal bond here – the loan of a dollar is simply a book entry on some balance sheet, and cannot be traded in the DM. We refer to such a asset (this

$1+\pi_t$ is inflation and $1/\beta = 1+r_t$ is the real interest rate. Hence (72) simply says $\rho_{t+1} = i$. This is equivalent to our earlier assertion that agents are indifferent about carrying cash deposits out of the CM, since they can always adjust $\hat{m}$ in the FM, depending on whether or not they want to consume in the DM. Similarly, the Euler equation for housing in case 1 is

$$(1+r)\,\psi_t = \psi_{t+1} + U_2\left(x_{t+1}, h_{t+1}\right).$$

As in the baseline model, this says that housing must be priced fundamentally when the borrowing constraint is slack, the only difference being that now the borrowing constraint applies to cash loans in the FM rather than consumption loans in the DM.

We now turn to case 2, where the borrowing constraint is binding. Then households who want to consume in the DM borrow to their limit in the FM, and

$$J_t\left(h_t, m_t\right) = \alpha_h V_t\left[D\left(\psi_t h_t\right), h_t, m_t + \frac{D\left(\psi_t h_t\right)}{\left(1+\rho_t\right)\phi_t}\right] + \left(1-\alpha_h\right) W_t\left[-\left(1+\rho_t\right)\phi_t m_t, h_t, 0\right] \tag{73}$$

In this case, the Euler equation for money is

$$(1+r)\,\phi_t = \alpha_h\left[L\left(y_{t+1}\right)+1\right]\phi_{t+1} + \left(1-\alpha_h\right)\rho_{t+1}\phi_{t+1}, \tag{74}$$

which is very similar to what one sees in related models along the lines of Berentsen et al. (2007). Intuitively, with probability α_h you desire DM goods, and an extra dollar in the FM relaxes your constraint, while with probability $1-\alpha_h$ you do not desire DM goods, so you keep your money in the bank. The Euler equation for housing in

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claim on a dollar) as illiquid.

this case is

$$(1+r)\,\psi_t = U_2\left(x_{t+1},h_{t+1}\right) + \psi_{t+1} + \frac{\alpha_h D_1 \psi_{t+1}}{1+\rho_{t+1}}\left[L\left(y_{t+1}\right) - \rho_{t+1}\right]. \tag{75}$$

This is similar to (58), except in the final term, the marginal value of liquidity depends upon $L\left(y_{t+1}\right)$ (because more housing relaxes your borrowing constraint) minus ρ_{t+1} (because you have to pay it back).

We now distinguish two subcases. In case 2a there is no idle cash in the FM at $t+1$, in the sense that borrowing exhausts the deposits, so we need $\rho_{t+1} > 0$ to clear the market. This means

$$\alpha_h D\left(\psi_{t+1}h_{t+1}\right) = \left(1-\alpha_h\right)\phi_{t+1}m_{t+1},$$

which together with $\phi_{t+1}m_{t+1} = g\left(y_{t+1}\right)$ yields

$$g\left(y_{t+1}\right) = \frac{1}{1-\alpha_h}D\left(\psi_{t+1}h_{t+1}\right).$$

Second, in case 2b, there may be idle cash in the FM at $t+1$, in the sense that when all the borrowers borrow up to the constraint, total demand for loans does not exhaust the cash on deposit, which means the $\rho_{t+1} = 0$. In case 2b, the Euler equation for housing is

$$(1+r)\,\psi_t = \psi_{t+1} + U_2\left(x_{t+1},h_{t+1}\right) + D_1\psi_{t+1}i.$$

At this point we focus on steady state. To determine which case obtains, two conditions are relevant. One is the individual debt limit: can individuals in FM borrow as much as they want? If $D\left(e\right)$ is low, borrowers are constrained in the FM and housing bears a liquidity premium. The other relevant condition is the aggregate debt limit: are total deposits more than constrained borrowers can borrow? If so, there is idle cash, and $\rho = 0$. There are three possibilities: 1) The aggregate and individual debt limits both slack; 2a) the individual debt limit binds but the aggregate is slack; and 2b) both are binding (the other combination, where the individual constraint is slack and the aggregate binds, cannot happen). Since the outcome depends on $D(\psi_t h_t) = D_0 + D_1 \psi_t h_t$, naturally, we partition (D_0, D_1) space into regions according to which case obtains.

To this end, let $\tilde{y}$ and $\bar{y}$ satisfy $L(\tilde{y}) = i/\alpha_h$ and $L(\bar{y}) = i$, and define

$$\begin{aligned} B_1(D_0) &= \begin{cases} \dfrac{r\left[g(\tilde{y})(1-\alpha_h) - D_0\right]}{ig(\tilde{y})(1-\alpha_h) - iD_0 + HU_2} & \text{if } D_0 < g(\tilde{y})(1-\alpha_h) \\ 0 & \text{if } D_0 > g(\tilde{y})(1-\alpha_h) \end{cases} \\ B_2(D_0) &= \max\left\{ \frac{r\left[g(\bar{y})(1-\alpha_h)(1+i) - D_0\right]}{HU_2}, 0 \right\}. \end{aligned}$$

For large D_0 and D_1, we get case 1, where deposits are plentiful and borrowers are unconstrained. As D_0 and D_1 decrease, we move to case 2a, where all deposits are borrowed, and borrowers are constrained in the sense that they do not bring enough cash to the DM to buy y^*. In this case, $\rho \in (0, i)$, so deposits pay, and loans charge, a positive interest rate, but less than the rate on illiquid nominal assets, i. As D_0 and D_1 decrease further, we move to case 2b, where there are idle deposits and $\rho = 0$.

Suppose the central bank increases the nominal interest rate i.[35] What happens to house prices? In case 1, housing is priced fundamentally, households can borrow

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[35] For this exercise it does not matter *how* they increase i – by Fisher Equation they can do so by increasing the inflation rate, and by the Quantity Equation they can do that by increasing the rate of monetary expansion M_{t+1}/M_t (either via lump sum transfers or purchases of x_t in the CM).

as they like given ρ in FM, and $\rho = i$ makes them indifferent between taking holding cash or deposits coming out of the CM. In this case, monetary policy does not affect real home values, since $\psi = \psi^*$ is the fundamental price. In case 2a, where $\rho \in (0, i)$, the results are ambiguous: real house prices may go up or down, depending on parameters, although they must fall under the condition given in the Proposition below (which is analogous to a condition in Li and Li (2010), although they only have financial assets). In case 2b, where $\rho = 0$, one can show real house prices rise. Intuitively, in this case a higher nominal interest-cum-inflation rate makes money less valuable, but because there are idle deposits ρ does not adjust. With the same amount of home equity, borrowers can borrow the same amount of real balances, so they want to hold in their portfolios more housing and less cash, leading to higher real house prices.

Details of the calculations are provided in Appendix C. Here, we summarize some key results as follows.

Proposition 13. *There is a unique steady state equilibrium, and it satisfies:*

1) if $B_2(D_0) < D_1$ then $\rho = i$, $\psi = U_2/r$ and $\partial\psi/\partial i = 0$.

2a) if $B_1(D_0) < D_1 < B_2(D_0)$ then $\rho \in (0, i)$ and $\partial\psi/\partial i$ is ambiguous: $\partial\psi/\partial i < 0$ iff $g(y)[L(y) + 1]$ is increasing in y.

2b) if $B_1(D_0) > D_1$ then $\rho = 0$, $\psi = U_2/(r - iD_1)$ and $\partial\psi/\partial i > 0$.

The main economic finding in terms of monetary policy is that higher nominal interest-cum-inflation rates have ambiguous effects on the housing market, but we provide precise conditions concerning how these effects depend on the tightness of home-equity borrowing constraints and other parameters. Other results can be derived – e.g., one can show that y decreases with inflation in all cases, one can study the model with an endogenous housing supply, one can consider optimal policy, and so on. We leave further exploration of this to future work, since the current project was more about nonlinear dynamics and housing bubbles. The purpose of this Section is mainly to show that once we have a liquidity-based theory of the housing market we can naturally analyze the effects of monetary policy in an interesting new light.

13 Conclusion

This goal of this project was to study economies where housing, in addition to providing utility as shelter, can be used as collateral to facilitate credit transactions. We showed that in equilibrium a house can bear a liquidity premium, and can hence be priced above its fundamental value (the discounted stream of marginal utility from living in it). Intuitively, this follows from the idea that liquidity is to some extent a self-fulfilling prophecy, as is evidently the case with fiat money. However, houses are also very different from fiat currency, because they can be produced by the private sector, and because they either generate direct utility or are inputs into household production, which means they can never have a price of 0, the way money can. In a sense, all we did is formalize some ideas discussed in the Introduction about financial developments allowing consumers to "treat their homes as cash machines," but we think such formalization is useful. We found that equilibrium house prices can display a variety of dynamic equilibrium paths, some of which look like bubbles, and some of which are qualitatively consistent with the data on prices, home equity loans and housing investment. Perhaps future work can investigate whether this kind of theory can do well quantitatively.

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Appendix A: Proof of Proposition 6.

There are three equations characterizing the equilibrium. Euler Equation for money holdings:

$$\Gamma^1(R, \frac{w}{Zp}, n) = \sigma\bar{\eta}\frac{Zp}{w}f_l(\frac{1-n}{n} + \frac{\bar{b}Zp}{wR}) + (1-\sigma)R - \frac{(1+\tau)}{\beta} = 0 \tag{76}$$

Inputs market clearing:

$$\Gamma^2(R, \frac{w}{Zp}, n) = \sigma\frac{\bar{b}Zp}{wR} + (1-\sigma)f_l^{-1}(\frac{wR}{Zp}) - (1-\sigma)\frac{1-n}{n} = 0 \tag{77}$$

Free entry for entrepreneurship:

$$\Gamma^3(R, \frac{w}{Zp}, n) = \sigma\bar{\eta}f(\frac{1-n}{n} + \frac{\bar{b}Zp}{wR}) + (1-\sigma)f[f_l^{-1}(\frac{wR}{Zp})] - \frac{1+\tau}{\beta}\frac{1-n}{n}\frac{w}{Zp} - \frac{w}{Zp} \geq 0 \tag{78}$$

To show existence and uniqueness. From (77), we know wR/Zp is increasing in n. Condition (76) can be rewritten as

$$\frac{(1+\tau)}{\beta}\frac{w}{Zp} = \sigma\bar{\eta}f_l(\frac{1-n}{n} + \frac{\bar{b}Zp}{wR}) + (1-\sigma)\frac{wR}{Zp}. \tag{79}$$

The RHS is increasing in n, there fore w/Zp is increasing in n as well. From (77) we have

$$f_l^{-1}(\frac{wR}{Zp}) = \frac{1-n}{n} - \frac{\sigma}{1-\sigma}\frac{\bar{b}Zp}{wR}.$$

Now the derivative w.r.t. n of Γ^3 becomes

$$\begin{aligned}\partial\Gamma^3/\partial n &= -\frac{1}{n^2}\sigma\bar{\eta}f_l(\frac{1-n}{n}+\frac{\bar{b}Zp}{wR})+\sigma\bar{\eta}f_l(\frac{1-n}{n}+\frac{\bar{b}Zp}{wR})\partial\frac{\bar{b}Zp}{wR}/\partial n \\ &\quad +(1-\sigma)f_l(\frac{1-n}{n}-\frac{\sigma}{1-\sigma}\frac{\bar{b}Zp}{wR})(-\frac{1}{n^2}-\frac{\sigma}{1-\sigma}\partial\frac{\bar{b}Zp}{wR}/\partial n) \\ &\quad +\frac{1}{n^2}\frac{1+\tau}{\beta}\frac{w}{Zp}-\frac{1+\tau}{\beta}\frac{1-n}{n}\partial\frac{w}{Zp}/\partial n-\partial\frac{w}{Zp}/\partial n\end{aligned}$$

Using (79), we have

$$\begin{aligned}\partial\Gamma^3/\partial n &= \sigma[\bar{\eta}f_l(\frac{1-n}{n}+\frac{\bar{b}Zp}{wR})-\frac{wR}{Zp}]\partial\frac{\bar{b}Zp}{wR}/\partial n \\ &\quad -\frac{1+\tau}{\beta}\frac{1-n}{n}\partial\frac{w}{Zp}/\partial n-\partial\frac{w}{Zp}/\partial n \\ &< 0\end{aligned}$$

From 77 we know that if $n \to 0$, then $wR/Zp \to 0$. From 79 we know that if $n \to 0$, then $w/Zp \to 0$. The first three terms in 78 is the surplus of being entrepreneur, which is positive even if $n \to 0$ (the analogy of an environment with finite agent model is the case with one entrepreneur having monopoly power over production), while the last term in 78, w/Zp goes to zero, so we know $\lim_{n\to 0}\Gamma^3 > 0$. On the other hand, when $n \to 1$, the first two terms of 78 converge to zero, while the last two terms can be grouped together as

$$-(\frac{1+\tau}{\beta}\frac{1-n}{n}+1)\frac{w}{Zp},$$

where the terms in the bracket is positive and w/Zp converges to infinity. So we know $\lim_{n\to 1}\Gamma^3 < 0$. So if $\bar{n} = 1$, we must have a unique equilibrium. If $\bar{n} < 1$, when $\Gamma^3(\bar{n}) < 0$, we must have an unique equilibrium with$n < \bar{n}$; when $\Gamma^3(\bar{n}) > 0$, then we must have an unique equilibrium with $n = \bar{n}$.

Derivatives (when $n < \bar{n}$):

Total differentiation of (76), (77) and (78), we have the differential equations system:

$$\left\{ \begin{matrix} \frac{1}{\beta} d\tau \\ \\ 0 \\ \\ \frac{1-n}{n} \frac{w}{Zp} \frac{1}{\beta} d\tau \end{matrix} \right\} = \left\{ \begin{matrix} R_1 & {\frac{w}{Zp}}_2 & n_7 \\ \\ R_3 & {\frac{w}{Zp}}_4 & n_8 \\ \\ R_5 & {\frac{w}{Zp}}_6 & n_9 \end{matrix} \right\} \left\{ \begin{matrix} dR \\ \\ d(\frac{w}{Zp}) \\ \\ dn \end{matrix} \right\} = Q \left\{ \begin{matrix} dR \\ \\ d(\frac{w}{Zp}) \\ \\ dn \end{matrix} \right\},$$

where

$R_1 = \Gamma_1^1 = (1-\sigma) - \sigma \frac{Zp}{w} \bar{\eta} f_{ll}(\bar{l}) \frac{\bar{b} Zp}{wR} \frac{1}{R} > 0$

${\frac{w}{Zp}}_2 = \Gamma_2^1 = -\sigma (\frac{Zp}{w})^2 [\bar{\eta} f_l(\bar{l}) + \bar{\eta} f_{ll}(\bar{l}) \frac{\bar{b} Zp}{wR}] = [-\sigma (\frac{Zp}{w})^2 \bar{\eta} f_l(\bar{l}) - \sigma (\frac{Zp}{w})^2 \bar{\eta} f_{ll}(\bar{l}) \frac{\bar{b} Zp}{wR}]$

$R_3 = \Gamma_1^2 = -\sigma \frac{\bar{b} Zp}{wR} \frac{1}{R} + (1-\sigma) \frac{1}{f_{ll}(l)} \frac{w}{Zp} < 0$

${\frac{w}{Zp}}_4 = \Gamma_2^2 = -\sigma \frac{\bar{b} Zp}{wR} \frac{Zp}{w} + (1-\sigma) \frac{1}{f_{ll}(l)} R < 0$

$R_5 = \Gamma_1^3 = -\sigma \bar{\eta} f_l(\bar{l}) \frac{\bar{b} Zp}{wR} \frac{1}{R} + (1-\sigma) \frac{1}{f_{ll}(l)} \frac{w}{Zp} \frac{wR}{Zp} < 0$

${\frac{w}{Zp}}_6 = \Gamma_2^3 = -\sigma \bar{\eta} f_l(\bar{l}) \frac{\bar{b} Zp}{wR} \frac{Zp}{w} + (1-\sigma) \frac{1}{f_{ll}(l)} R^2 \frac{w}{Zp} - 1 - \frac{1+\tau}{\beta} \frac{1-n}{n} < 0$

$n_7 = \Gamma_3^1 = -\sigma \frac{Zp}{w} \bar{\eta} f_{ll}(\bar{l}) \frac{1}{n^2} > 0$

$n_8 = \Gamma_3^2 = (1-\sigma) \frac{1}{n^2} > 0$

$n_9 = \Gamma_3^3 = (1-\sigma) \frac{1}{n^2} \frac{wR}{Zp} > 0$

To show $\det Q > 0$**:**

$$\begin{aligned}
\det Q &= n_7[R_3\frac{w}{Zp_6} - \frac{w}{Zp_4}R_5] + n_8[R_5\frac{w}{Zp_2} - \frac{w}{Zp_6}R_1] + n_9[R_1\frac{w}{Zp_4} - \frac{w}{Zp_2}R_3] \\
&= \frac{1}{n^2}\{-\sigma\frac{Zp}{w}\bar{\eta} f_{ll}(\bar{l})[R_3\frac{w}{Zp_6} - \frac{w}{Zp_4}R_5] + (1-\sigma)[R_5\frac{w}{Zp_2} - \frac{w}{Zp_6}R_1] \\
&\quad +(1-\sigma)\frac{wR}{Zp}[R_1\frac{w}{Zp_4} - \frac{w}{Zp_2}R_3]\},
\end{aligned}$$

where

$$\begin{aligned}
R_3\frac{w}{Zp_6} - \frac{w}{Zp_4}R_5 &= \frac{1}{R}[\sigma\frac{\bar{b}Zp}{wR} - (1-\sigma)\frac{1}{f_{ll}(l)}\frac{wR}{Zp}] \\
&\quad +\frac{1}{R}[\sigma\frac{\bar{b}Zp}{wR} - (1-\sigma)\frac{1}{f_{ll}(l)}\frac{wR}{Zp}]\frac{1+\tau}{\beta}\frac{1-n}{n}
\end{aligned}$$

$$\begin{aligned}
R_5\frac{w}{Zp_2} - \frac{w}{Zp_6}R_1 &= \sigma\bar{\eta} f_l(\bar{l})\frac{\bar{b}Zp}{\theta wR}\frac{Zp}{w}\frac{1}{R}\frac{1+\tau}{\beta} - (1-\sigma)\frac{1}{f_{ll}(l)}\frac{wR}{Zp}]\frac{1+\tau}{\beta} \\
&\quad +[(1-\sigma) - \sigma\frac{Zp}{w}\bar{\eta} f_{ll}(\bar{l})\frac{\bar{b}Zp}{wR}\frac{1}{R}] \\
&\quad +[(1-\sigma) - \sigma\frac{Zp}{w}\bar{\eta} f_{ll}(\bar{l})\frac{\bar{b}Zp}{wR}\frac{1}{R}]\frac{1+\tau}{\beta}\frac{1-n}{n}
\end{aligned}$$

and

$$R_1\frac{w}{Zp_4} - \frac{w}{Zp_2}R_3 = \frac{1+\tau}{\beta}[(1-\sigma)\frac{1}{f_{ll}(l)} - \sigma\frac{\bar{b}Zp}{wR}\frac{Zp}{wR}].$$

Therefore

$$
\begin{aligned}
\det Q \;=\;& \frac{1}{n^2}\{-\sigma\frac{Zp}{wR}\bar{\eta}f_{ll}(\bar{l})[\sigma\frac{\bar{b}Zp}{wR}-(1-\sigma)\frac{1}{f_{ll}(l)}\frac{wR}{Zp}] \\
& -\sigma\frac{Zp}{wR}\bar{\eta}f_{ll}(\bar{l})[\sigma\frac{\bar{b}Zp}{wR}-(1-\sigma)\frac{1}{f_{ll}(l)}\frac{wR}{Zp}]\frac{1+\tau}{\beta}\frac{1-n}{n} \\
& +(1-\sigma)[(1-\sigma)-\sigma\frac{Zp}{w}\bar{\eta}f_{ll}(\bar{l})\frac{\bar{b}Zp}{wR}\frac{1}{R}] \\
& +(1-\sigma)[(1-\sigma)-\sigma\frac{Zp}{w}\bar{\eta}f_{ll}(\bar{l})\frac{\bar{b}Zp}{wR}\frac{1}{R}]\frac{1+\tau}{\beta}\frac{1-n}{n} \\
& +\sigma(1-\sigma)\frac{1+\tau}{\beta}\frac{\bar{b}Zp}{wR}[\bar{\eta}f_l(\bar{l})\frac{Zp}{wR}-1]\}
\end{aligned}
$$

Notice $\bar{\eta}f_l(\bar{l})\frac{Zp}{wR}-1>0$ is from (26). Now all of the terms are positive, therefore $\det Q>0$.

To find the derivatives. First for $dR/d\tau$

$$
\begin{aligned}
dR/d\tau \;=\;& \frac{1}{\det Q}\frac{1}{\beta}\det\left\{\begin{array}{ccc} 1 & \frac{w}{Zp}_2 & n_7 \\ 0 & \frac{w}{Zp}_4 & n_8 \\ \frac{1-n}{n}\frac{w}{Zp} & \frac{w}{Zp}_6 & n_9 \end{array}\right\} \\
=\;& \frac{1}{\det Q}\frac{1}{\beta}\{(n_9\frac{w}{Zp}_4-n_8\frac{w}{Zp}_6)+\frac{1-n}{n}\frac{w}{Zp}(\frac{w}{Zp}_2n_8-\frac{w}{Zp}_4n_7)\}
\end{aligned}
$$

where

$$\begin{aligned}
n_9\frac{w}{Zp_4}-n_8\frac{w}{Zp_6} &= (1-\sigma)\frac{1}{n^2}\frac{wR}{Zp}[-\sigma\frac{\bar{b}Zp}{wR}\frac{Zp}{w}+(1-\sigma)\frac{1}{f_{ll}(l)}R] \\
&\quad -(1-\sigma)\frac{1}{n^2}[-\sigma\bar{\eta}f_l(\bar{l})\frac{\bar{b}Zp}{wR}\frac{Zp}{w} \\
&\quad +(1-\sigma)\frac{1}{f_{ll}(l)}R^2\frac{w}{Zp}-1-\frac{1+\tau}{\beta}\frac{1-n}{n}] \\
&= (1-\sigma)\frac{1}{n^2}\sigma\frac{\bar{b}Zp}{wR}[\frac{Zp}{wR}\bar{\eta}f_l(\bar{l})-1]+(1-\sigma)\frac{1}{n^2}[1+\frac{1+\tau}{\beta}\frac{1-n}{n}]
\end{aligned}$$

and

$$\begin{aligned}
\frac{w}{Zp_2}n_8-\frac{w}{Zp_4}n_7 &= [-\sigma(\frac{Zp}{w})^2\bar{\eta}f_l(\bar{l})-\sigma(\frac{Zp}{w})^2\bar{\eta}f_{ll}(\bar{l})\frac{\bar{b}Zp}{wR}](1-\sigma)\frac{1}{n^2} \\
&\quad -[\sigma(\frac{Zp}{w})^2\bar{\eta}f_l(\bar{l})+\sigma(\frac{Zp}{w})^2\bar{\eta}f_{ll}(\bar{l})\frac{\bar{b}Zp}{wR}]\sigma\frac{Zp}{w}\bar{\eta}f_{ll}(\bar{l})\frac{1}{n^2} \\
&= -\sigma(1-\sigma)(\frac{Zp}{w})^2\bar{\eta}f_l(\bar{l})\frac{1}{n^2}+\sigma(1-\sigma)\frac{\bar{\eta}f_{ll}(\bar{l})}{f_{ll}(l)}R\frac{Zp}{w}\frac{1}{n^2} \\
&\quad -\sigma\frac{\bar{b}Zp}{wR}\frac{Zp}{w}\frac{Zp}{w}\bar{\eta}f_{ll}(\bar{l})\frac{1}{n^2}.
\end{aligned}$$

Therefore

$$\begin{aligned}
dR/d\tau &= \frac{1}{\det Q}\frac{1}{\beta}\{(1-\sigma)\frac{1-n}{n}\frac{1}{n^2}[\frac{1+\tau}{\beta}-\sigma\frac{Zp}{w}\bar{\eta}f_l(\bar{l})] \\
&\quad +(1-\sigma)\frac{1}{n^2}\sigma\frac{\bar{b}Zp}{wR}[\frac{Zp}{wR}\bar{\eta}f_l(\bar{l})-1]+(1-\sigma)\frac{1}{n^2} \\
&\quad +\sigma(1-\sigma)\frac{\bar{\eta}f_{ll}(\bar{l})}{f_{ll}(l)}R\frac{1}{n^2}\frac{1-n}{n}-\sigma\frac{\bar{b}Zp}{wR}\frac{Zp}{w}\bar{\eta}f_{ll}(\bar{l})\frac{1}{n^2}\frac{1-n}{n}\}
\end{aligned}$$

Notice from Euler equation $\frac{1+\tau}{\beta}-\sigma\frac{Zp}{w}\bar{\eta}f_l(\bar{l})=(1-\sigma)R>0$. So every term is positive thus $dR/d\tau>0$.

Second for $d(w/Zp)/d\tau$

$$\begin{aligned} d(w/Zp)/d\tau &= \frac{1}{\det Q}\frac{1}{\beta}\det\left\{\begin{array}{ccc} R_1 & 1 & n_7 \\ R_3 & 0 & n_8 \\ R_5 & \frac{1-n}{n}\frac{w}{Zp} & n_9 \end{array}\right\} \\ &= \frac{1}{\det Q}\frac{1}{\beta}\{(n_8R_5 - R_3n_9) + \frac{1-n}{n}\frac{w}{Zp}(R_3n_7 - R_1n_8)\}, \end{aligned}$$

where

$$n_8R_5 - R_3n_9 = -\sigma(1-\sigma)\frac{1}{n^2}\frac{\bar{b}Zp}{wR}\frac{1}{R}[\bar{\eta}f_l(\bar{l}) - \frac{wR}{Zp}],$$

and

$$\begin{aligned} R_3n_7 - R_1n_8 &= \sigma\frac{Zp}{w}\bar{\eta}f_{ll}(\bar{l})\frac{\bar{b}Zp}{wR}\frac{1}{R}\frac{1}{n^2} - (1-\sigma)(1-\sigma)\frac{1}{n^2} \\ &\quad -\sigma(1-\sigma)\frac{1}{n^2}\frac{\bar{\eta}f_{ll}(\bar{l})}{f_{ll}(l)} \end{aligned}$$

Since $n_8R_5 - R_3n_9 < 0$ (from (26)) and $R_3n_7 - R_1n_8 < 0$, so $d(w/Zp)/d\tau < 0$.

Lastly for $dn/d\tau$

$$\begin{aligned} dn/d\tau &= \frac{1}{\det Q_2}\frac{1}{\beta}\det\left\{\begin{array}{ccc} R_1 & {\frac{w}{Zp}}_2 & 1 \\ R_3 & {\frac{w}{Zp}}_4 & 0 \\ R_5 & {\frac{w}{Zp}}_6 & \frac{1-n}{n}\frac{w}{Zp} \end{array}\right\} \\ &= \frac{1}{\det Q}\frac{1}{\beta}\{\frac{1-n}{n}\frac{w}{Zp}[R_1{\frac{w}{Zp}}_4 - {\frac{w}{Zp}}_2R_3] + [R_3{\frac{w}{Zp}}_6 - R_5{\frac{w}{Zp}}_4]\} \end{aligned}$$

where both $R_1{\frac{w}{Zp}}_4 - {\frac{w}{Zp}}_2R_3$ and $R_3{\frac{w}{Zp}}_6 - R_5{\frac{w}{Zp}}_4$ have been calculated before.

Therefore

$$\begin{aligned}
dn/d\tau &= \frac{1}{\det Q}\frac{1}{\beta}\{-\frac{1+\tau}{\beta}\frac{1}{R}[\sigma\frac{\bar{b}Zp}{wR}-(1-\sigma)\frac{1}{f_{ll}(l)}\frac{wR}{Zp}]\frac{1-n}{n} \\
&\quad +\frac{1}{R}[\sigma\frac{\bar{b}Zp}{wR}-(1-\sigma)\frac{1}{f_{ll}(l)}\frac{wR}{Zp}] \\
&\quad +\frac{1}{R}[\sigma\frac{\bar{b}Zp}{wR}-(1-\sigma)\frac{1}{f_{ll}(l)}\frac{wR}{Zp}]\frac{1+\tau}{\beta}\frac{1-n}{n}\} \\
&= \frac{1}{\det Q}\frac{1}{\beta}\frac{1}{R}[\sigma\frac{\bar{b}Zp}{wR}-(1-\sigma)\frac{1}{f_{ll}(l)}\frac{wR}{Zp}] > 0.
\end{aligned}$$

■

Appendix B

Consider the following extensive-form bargaining game:[36]

Stage 1: The seller offers (y_t, d_t).

Stage 2: The buyer responds by accepting or rejecting, where:

- accept implies trade at these terms;
- reject implies they go to stage 3.

Stage 3: Nature moves (a coin toss) with the property that:

- with probability θ, the buyer makes a take-it-or-leave-it offer;
- with probability $1-\theta$, the seller makes a take-it-or-leave-it offer.

---

[36] Wong and Wright (2011) proved a recent discussion of why one might want to use an explicit noncooperative game, rather than some axiomatic solution, in dynamic search-and-bargaining models, especially with nonlinear utility, along with a discussion of the particular game used here, and references to the literature.

Any offer must satisfy $d_t \leq D_t$. In Appendix A we show that there is a unique SPE, characterized by acceptance of the initial Stage 1 offer, given by

$$(y_t, d_t) = \arg\max S_{st} \text{ st } S_{bt} = \theta \left[u(\bar{y}_t) - \bar{d}_t\right] \text{ and } d \leq D_t, \tag{80}$$

where $\left(\bar{y}_t, \bar{d}_t\right)$ is the offer a buyer would make if (off the equilibrium path) we were to reach Stage 3.

To solve this game, the first observation is that if (off the equilibrium path) bargaining were to go to Stage 3 and the buyer gets to make the final take-it-or-leave-it offer, he would offer $\left(\bar{y}, \bar{d}\right)$ where:

$$\bar{y} = \begin{cases} v^{-1}(D_t) & \text{if } D_t < v\left(y^*\right) \\ y^* & \text{if } D_t \geq v\left(y^*\right) \end{cases} \text{ and } \bar{d} = \begin{cases} D_t & \text{if } D_t < v\left(y^*\right) \\ v\left(y^*\right) & \text{if } D_t \geq v\left(y^*\right) \end{cases}$$

Now there are four possible cases: 1) the constraint $d \leq D$ is slack at the initial and the final offer stage; 2) it binds in the initial but not the final offer stage; 3) it binds in both; and 4) it binds in the final but not the initial offer stage. It is easy to check that case 4 cannot arise, so we are left with three.

Case 1: In the final offer stage, if the buyer proposes, his problem is

$$\max_{y,d} \left\{u\left(y\right) - d\right\} \text{ st } d = v\left(y\right),$$

with solution $y = y^*$ and $d = v\left(y^*\right)$. If the seller proposes the buyer gets no surplus, so the buyer's expected surplus before the coin flip is $\theta\left[u\left(y^*\right) - v\left(y^*\right)\right]$. Therefore, in the initial offer stage, the seller's problem is

$$\max_{y,d} \{d - v(y)\} \text{ st } u(y) - d = \theta [u(y^*) - v(y^*)],$$

with solution $y = y^*$ and $d = d^* = (1-\theta) u(y^*) + \theta v(y^*)$. Since $d^* > v(y^*)$, this case occurs iff $D > d^*$.

Case 2: The buyer's expected payoff before the coin flip is again $\theta [u(y^*) - v(y^*)]$, but at the initial offer stage the constraint binds, so the seller solves

$$\max_{y} \{D - v(y)\} \text{ st } u(y) - D = \theta [u(y^*) - v(y^*)].$$

The solution satisfies $u(y) = D + \theta [u(y^*) - v(y^*)]$ and $d = D$. This case occurs iff $v(y^*) < D < d^*$.

Case 3: In the final offer stage, if the buyer proposes, his problem is

$$\max_{y} \{u(y) - D\} \text{ st } D = v(y).$$

This implies $y = v^{-1}(D)$, and his expected surplus before the coin flip is $\theta \left[u \circ v^{-1}(D) - D\right]$. At the initial offer stage, the seller's problem is

$$\max_{y} \{D - v(y)\} \text{ st } u(y) - D = \theta \left[u \circ v^{-1}(D) - D\right].$$

The solution satisfies $u(y) = \theta u \circ v^{-1}(D) + (1-\theta) D$ and $d = D$. This case occurs iff $D < v(y^*)$ and $D < u(y^*) - \theta u \circ v^{-1}(D) + \theta D$, the last inequality coming from the observation that, at the first stage, if the constraint is slack, the buyer pays $u(y^*) - \theta u \circ v^{-1}(D) + \theta D$ to get y^*. This last inequality is equivalent to $(1-\theta) D <$

$u(y^*) - \theta u \circ v^{-1}(D)$, which always holds if $D < v(y^*)$.

To sum up, $d = D$ if $D < d^*$, and otherwise $d = d^*$; and y is given by

$$y = \begin{cases} u^{-1}\left[\theta u \circ v^{-1}(D) + (1-\theta) D\right] & \text{if } D < v(y^*) \\ u^{-1}\left[D + \theta\left[u(y^*) - v(y^*)\right]\right] & \text{if } v(y^*) < D < d^* \\ y^* & \text{if } D > d^* \end{cases} \cdot$$

If we look at the derivative dy/dD, we have

$$\frac{dy}{dD} = \begin{cases} \frac{\theta u'\left[v^{-1}(D)\right] + (1-\theta) v'\left[v^{-1}(D)\right]}{u'(y) v'\left(v^{-1}(D)\right)} & \text{if } D < v(y^*) \\ \frac{1}{u'(y)} & \text{if } v(y^*) < D < d^* \\ 0 & \text{if } D > d^* \end{cases} \cdot$$

Thus we get $y = g^{-1}(D)$ as a differentiable and strictly increasing function of D for $D < d^*$. ■

Appendix C

Here we verify Proposition 8 by considering each case in turn.

Case 1: The borrowing constraint is not binding. In steady state, we have

$$\begin{aligned} i &= L(y), \rho = i \\ r\psi &= U_2\left[X(H), H\right] \\ g(y) &< \frac{D_0 + D_1 \psi H}{(1+\rho)(1-\alpha_h)}. \end{aligned}$$

The last condition comes from two observations: when $\rho > 0$, to clear the market we must have $g(y) = \phi_t M_t / \alpha_h$, as borrowing exhausts deposits; and when the borrowing constraint is not binding, $(1 - \alpha_h) \phi_t M_t < \alpha_h (D_0 + D_1 \psi H) / (1 + \rho)$ .We can easily see that this equilibrium exists iff

$$g(\bar{y}) < \frac{D_0 + D_1 \psi H}{(1 - \alpha_h)(1 + \rho)}$$

with $\psi = U_2 / r$, or $D_1 > g(\bar{y})(1 + i)(1 - \alpha_h) - D_0$. Uniqueness follows immediately. Furthermore, $\partial \psi / \partial i = 0$ and $\partial y / \partial i < 0$.

Case 2: The borrowing constraint is binding. In steady state,

$$\begin{aligned} i &= \alpha_h L(y) + (1 - \alpha_h) \rho & (81) \\ r\psi &= \alpha_h [L(y) - \rho] \frac{\psi D_1}{1 + \rho} + U_2 [X(H), H] & (82) \\ g(y) &= \phi_t M_t + \frac{D_0 + D_1 \psi H}{1 + \rho}. & (83) \end{aligned}$$

We now consider the subcases separately.

Case 2a: If $\rho > 0$, market clearing and a binding borrowing constraint imply

$$\phi_t M_t = \frac{\alpha_h (D_0 + D_1 \psi h)}{(1 - \alpha_h)(1 + \rho)}. \tag{84}$$

Using (81), we get $\rho = (i - \alpha_h L) / (1 - \alpha_h)$. This, (84) and (83) yield

$$\psi = \frac{g(y)[1 + i - 1 - \alpha_h L(y)] - D_0}{D_1 H}.$$

Substituting these into (82), we get

$$\frac{r}{D_1} = \frac{\alpha_h L(y) - i}{1 + i - \pi [1 + \alpha_h L(y)]} + \frac{H U_2 [X(H), H]}{g(y) [1 + i - \pi - \pi \alpha_h L(y)] - D_0} = \Phi(y). \quad (85)$$

The RHS is decreasing in y, so there is at most one solution. Note in this subcase $\rho < L(y)$, implying $0 < \rho < i$. This and (81) imply $\alpha_h i < L(y) < i$. Consequently, this equilibrium exists iff (85) has a solution in $(\tilde{y}, \bar{y})$, where $L(\tilde{y}) = i/\alpha_h$ and $L(\bar{y}) = i$. This requires $\Phi(\tilde{y}) > r/D_1$ and $\Phi(\bar{y}) < r/D_1$, or $B_1(D_0) < D_1 < B_2(D_0)$. One can derive

$$\begin{aligned}
\frac{\partial y}{\partial i} &\sim -\frac{g D_1 (L(y) + 1) \alpha_h}{\psi (1 + \rho)^2} - \frac{g U_h}{(1 + \rho) \psi^2} < 0, \\
\frac{\partial \rho}{\partial i} &\sim -\frac{\alpha_h L' D_1 g}{(1 + \rho) \psi} + \frac{U_2 (X(H), H) g'}{\psi^2} > 0, \\
\frac{\partial \psi}{\partial i} &\sim -D_1 \alpha_h \left[g L' + g (L(y) + 1) \right] \sim -\frac{d}{dy} [L(y) + 1] g(y).
\end{aligned}$$

Therefore, $\partial \psi / \partial i < 0$ iff $[L(y) + 1] g(y)$ is increasing.

Case 2b: If $\rho = 0$, steady state is characterized by

$$\begin{aligned}
i/\alpha_h &= L(y) && (86) \\
r\psi &= i \psi D_1 + U_2 [X(H), H] && (87) \\
g(y) &> \frac{D(\psi H)}{1 - \alpha_h}. && (88)
\end{aligned}$$

In this subcase, (86) determines y and (87) determines ψ. This equilibrium exists iff (88) holds given y and ψ, which leads to $B_1(D_0) > D_1$. It is obvious in this case that $\partial y / \partial i < 0$ and $\partial \psi / \partial i > 0$. ■

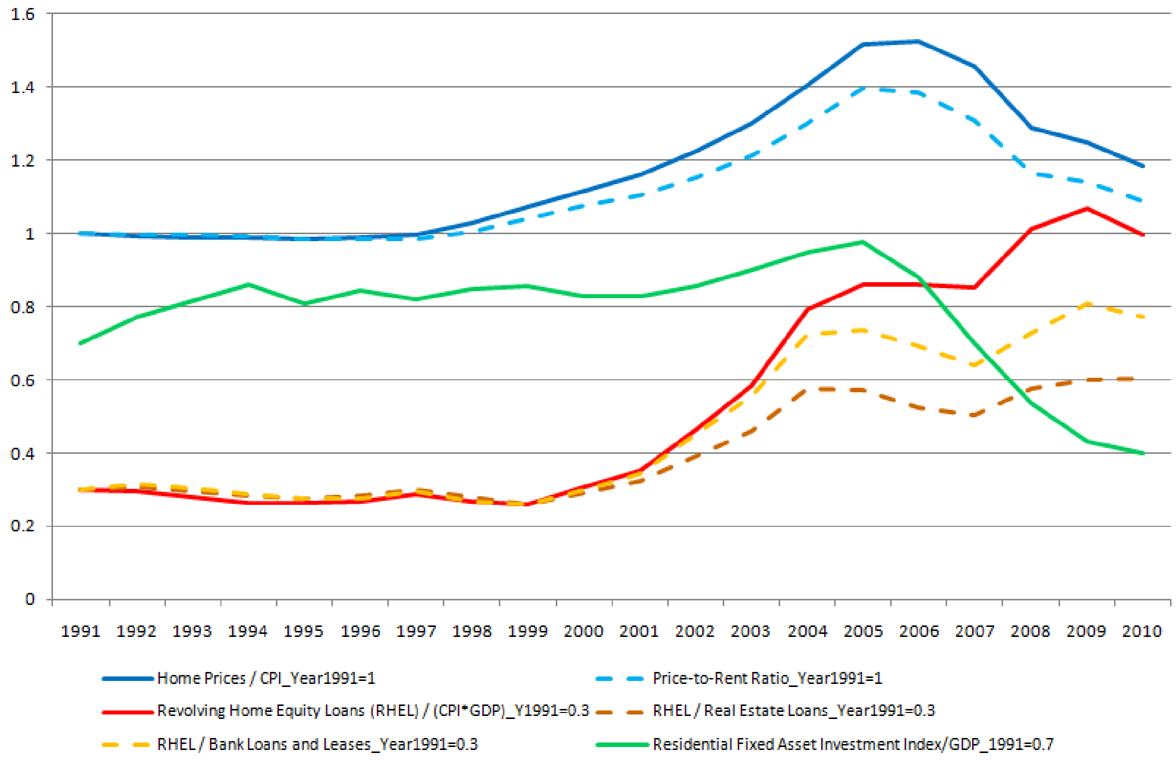


Figure 2: Housing Sector and Home Equity Loans

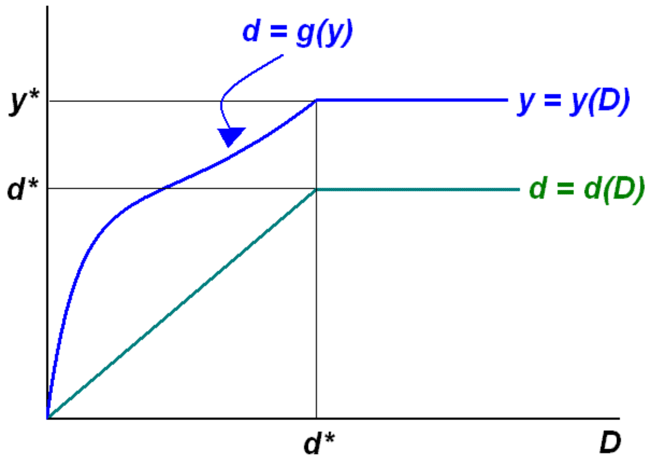
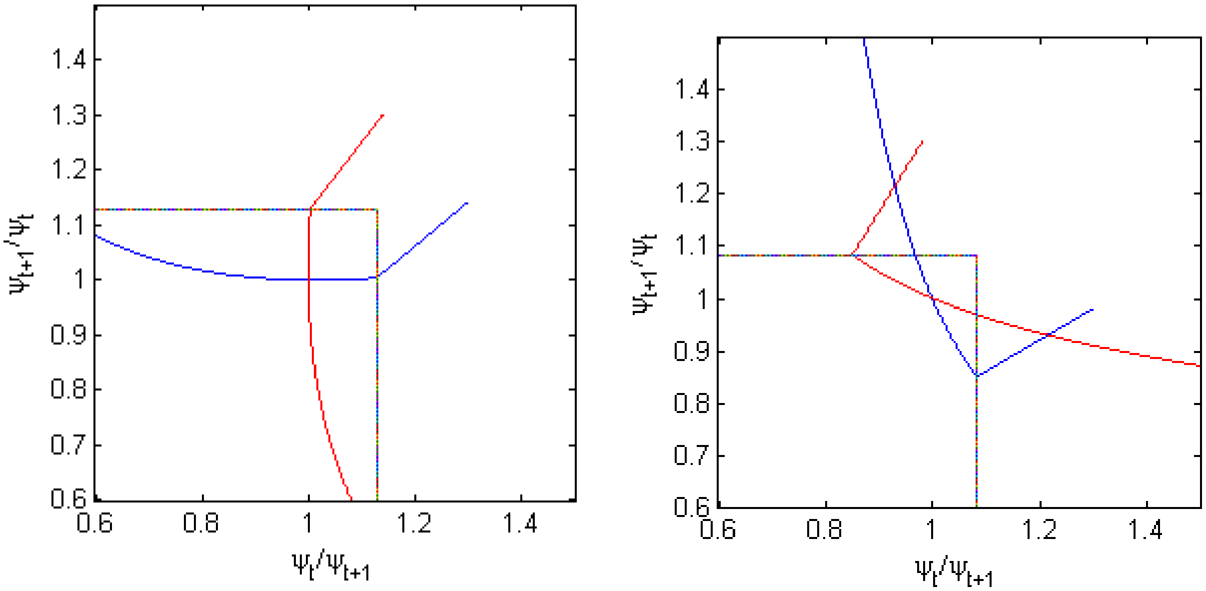
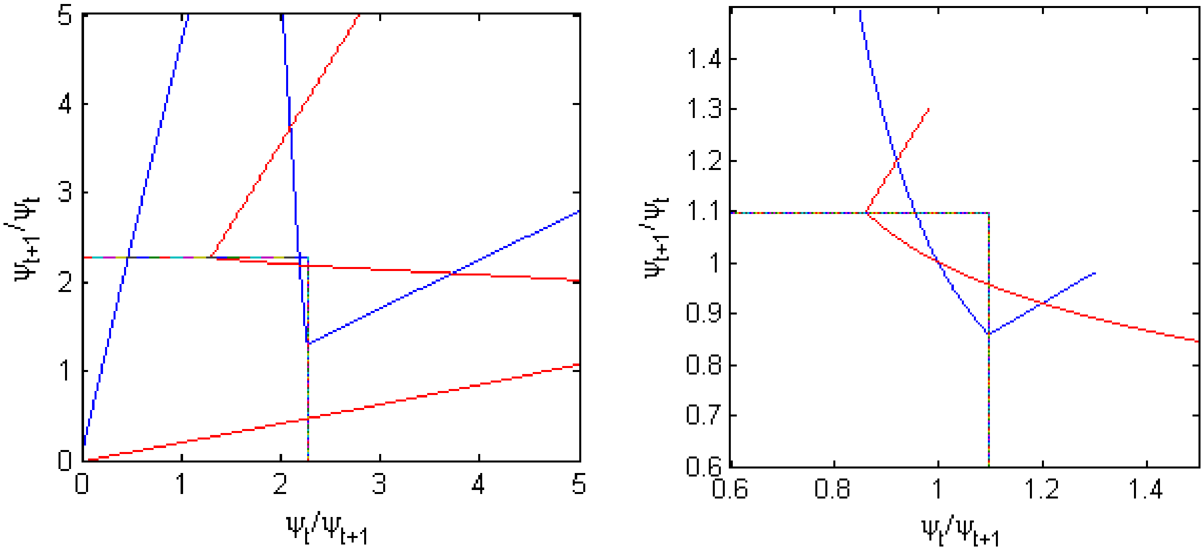


Figure 3: Term of Trade



Left: Walras Pricing no cycles. Right: Two Cycle under Walras Pricing.

Figure 4: Housing Price Dynamics



Left: Two cycles under Proportional Bargaining. Right: Two cycles under the Pricing Game.

Figure 5: Housing Price Dynamics(Continued)

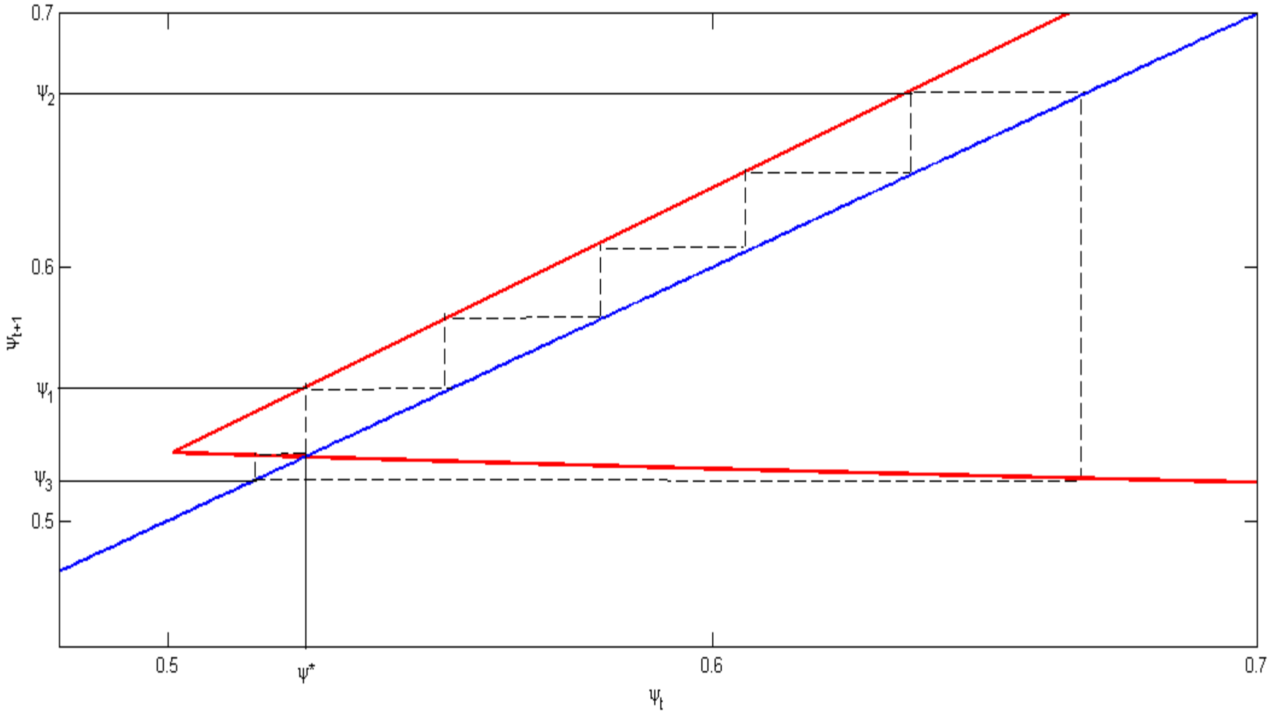


Figure 6: Price Dynamics

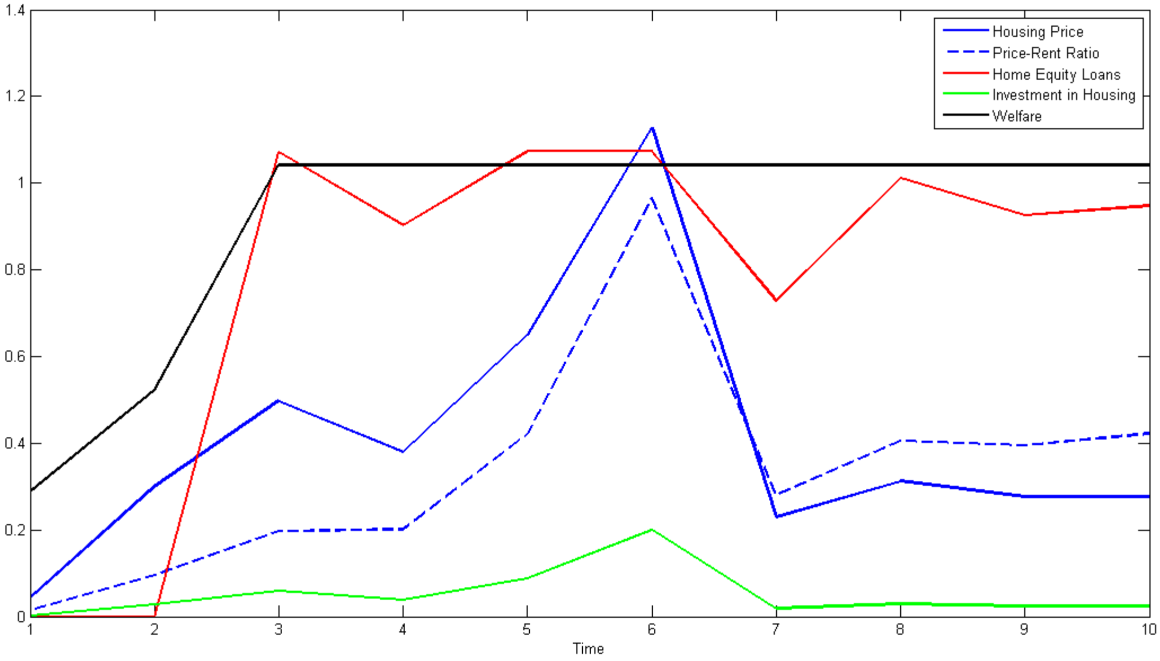


Figure 7: Housing Price Dynamics: One Period Unexpected Change in Financial Market

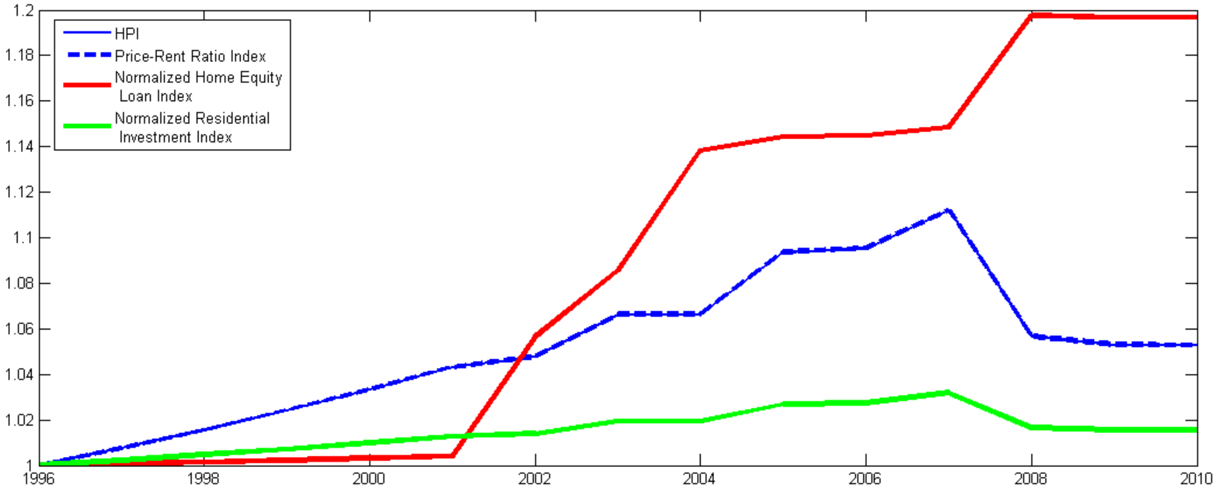


Figure 8: Housing Price Dynamics: Transition Path Triggered by Gradual Change in Financial Market