Tracking down fraction acquisition:
Investigating neural and behavioral signatures of a neurocognitive tool for early fractions learning

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Abstract

The acquisition of symbolic fraction knowledge is critical for mathematical development. However, many young children have struggled to learn the concept of symbolic fractions than to learn the concept of whole numbers. One emerging approach to investigating this challenging domain of math is to understand neurocognitive mechanisms of symbolic fraction knowledge that can be leveraged to help support young learners. Throughout development, these underlying mechanisms interact consistently with different cortical networks and also with educational environments. Understanding developmental interactions between neural networks and developing competence during symbolic fraction acquisition may inform design principles for more effective approaches to instruction.

With developmental cognitive neuroscience perspectives, the present dissertation aimed to explore developmental changes in functional specialization of the brains as children learn fraction instructions. In particular, this dissertation focuses on a recently proposed neurocognitive tool, a Ratio Processing System (RPS), a primitive ability to process nonsymbolic ratio magnitudes (e.g., ratios instantiated by juxtaposing two line-segments) that might be used to help build students' understanding of symbolic fractions.

With Study 1, I describe a cross-sectional functional MRI study comparing functional engagement for processing nonsymbolic ratio magnitudes and symbolic fractions. Study 1 compared groups of children prior to formal fractions instruction and after a few years of the instruction. The study reveals that shared functional substrates for nonsymbolic ratios and symbolic ratios emerge during the early years of fractions instruction. Second, using a cross-sequential approach, Study 2 investigates developmental changes of individual differences in microstructures that relates to ratio and fraction processing abilities during early years of fraction instructions. Next, Study 3 investigates how nonsymbolic ratio processing ability can contribute to early fraction instructions compared to other cognitive abilities important for fractions with a special focus on learning. Tracking down fraction acquisition from the children's brain to behavior, this dissertation puts forward the potential use of nonsymbolic ratios as perceptual tool that might benefit children's fraction learning.

Chapter 1: General Introduction and Background

Important but challenging math domain – fractions

Developing students' mathematical competence benefits individuals themselves and society via the cultivation of human capital for Science, Technology, Engineering, and Mathematics (STEM) fields (Daempfle, 2003). So far, numerous developmental, cognitive, and neuroscience studies have focused on investigating whole number knowledge to enhance students' overall mathematical competence (Geary, 2007; Leslie, Gelman, & Gallistel, 2008). However, recently fractions knowledge has been more emphasized as a critical factor for supporting mathematical development and mastering advanced mathematics like algebra, which can be a key to accessing higher education (Bailey, Hoard, Nugent, & Geary, 2012; Booth & Newton, 2012; Dewolf, Bassok, & Holyoak, 2015; Siegler et al., 2012a). For instance, researchers have found that early fractions ability in fifth and sixth grades was uniquely predictive of advanced mathematical achievement in tenth grade (Siegler et al., 2012a).

Despite the importance of fractions knowledge, it has been remained one of the most complicated mathematical topics to grasp for children (e.g., Lesh, Post, & Behr, 1987; Ni & Zhou, 2005; Vamvakoussi & Vosniadou, 2004, 2010). Even high school and college students are often confused by the concept of fractions (Carpenter, Corbitt, & National Council of Teachers of Mathematics., 1981; Lortie-Forgues et al., 2015; Schneider & Siegler, 2010). Carpenter et al. (1981) provided one famous example of this: As a part of the National Assessment of Education Progress (NAEP), 8th graders and high schoolers were asked to choose the nearest number to the sum of 12/13 + 7/8. Amongst options of 1, 2, 19, 21 and "I don't know", the most common answer was "19," whereas the correct answer was

"2". Only 24 % of the students chose of 8^{th} graders the right answer. In the test, most students added numerators from the given fractions ("12" from 12/13 and "7" from 7/8), instead of considering the holistic magnitude of each fraction (both fractions are about "1"). However, these struggles are not similarly prevalent with natural numbers (Lortie-Forgues et al., 2015). Why are fractions more challenging to learn compared to whole numbers?

The current dissertation has been motivated by explaining this discrepancy. The aims of this chapter are to critically review existing perspectives and studies about fraction learning and to explore what we do not know yet. Another aim of this chapter is to lay out potential theoretical approach to better understand fraction learning and the experimental studies in this dissertation. The aims of the experimental studies in this dissertation will be introduced at the end of the chapter.

The innate constraints account

Why fractions are hard is one of the long-standing questions in children's mathematical development (Lortie-Forgues et al., 2015; Ni & Zhou, 2005). Some prior work answered this question by suggesting that humans lack a perceptual ground to utilize for fraction acquisition as opposed to whole number acquisition. Most prominent existing theories about the foundations of number knowledge argued that the human brain is equipped with an innate ability to estimate and manipulate large numerosities approximately, *the approximate number system* (ANS) (Dehaene, Dehaene-Lambertz, & Cohen, 1998; Halberda & Feigenson, 2008). They suggested this preexisting system provides a neurocognitive foundation for learning the natural number system, a culturally derived invention (Dehaene & Cohen, 2007; Feigenson, Dehaene, & Spelke, 2004; Piazza, 2011). Piazza (2011) dubbed this system as a *neurocognitive startup tool* for symbolic numbers.

Numerosity representation, a number of discrete, individual elements can be approximately mapped onto a single symbolic number. Due to this characteristic, nonsymbolic numerosity representation is more limited for other types of numbers, such as rational numbers (Feigenson et al., 2004; Gallistel & Gelman, 1992; Wynn, 1995). Therefore, these theorists attribute children's struggles with fractions to a lack of cognitive primitive that provides intuitive access to fractions magnitude – which is an *"innate constraint account"* (Ni & Zhou, 2005). This innate constraint account has been argued by many ANS theorists along with several experimental evidence that strengthen the ANS theory.

The ANS theory posits logarithmically spaced, approximate analog representations of discrete magnitudes with a fixed amount of noise around each numerosity representation (Izard & Dehaene, 2008; Nieder & Dehaene, 2009; Piazza, Izard, Pinel, Le Bihan, & Dehaene, 2004). Thus, the discrimination between numerosities follows Weber's law, i.e., discriminability decreases as the ratio between magnitudes approaches 1 (Cantrell, Boyer, Cordes, & Smith, 2015; Odic, 2018; Odic & Starr, 2018; Xu & Spelke, 2000). In other words, the performance on numerosity discrimination is likely to show *a distance effect* (Buckley & Gillman, 1974). As two numerosities are farther away from each other, the sizes of numbers can be compared more rapidly and accurately. For example, 3 dots vs. 8 dots is much easier than 3 dots vs. 2 dots since the distance between 3 and 8 is farther than 3 and 2. The minimum ratio or distance between magnitudes that an individual can discriminate can indicate one's acuity for the ANS (Pica, Lemer, Izard, & Dehaene, 2004).

Moreover, there are numerous supporting results indicating that the ANS is a phylogenetically conserved system as hypothesized. For example, even newborn infants (e.g., Izard, Sann, Spelke, & Streri, 2009) and nonhuman primates (e.g., Nieder & Miller, 2004) can discriminate numerosities. Also, ANS acuity sharpens over the course of development. ANS acuity was reported to be 1:3 in newborns (Izard et al., 2009), 1:2 in 6-month old infants (Xu & Spelke, 2000), 3:4~5:6 in preschoolers (Halberda & Feigenson, 2008; Odic, 2018; Odic, Libertus, Feigenson, & Halberda, 2013), 6:7 at 6 years (Halberda & Feigenson, 2008; Odic, 2018; Odic et al., 2013) and 9:10~10:11 in adults (Halberda, Ly, Wilmer, Naiman, & Germine, 2012; Halberda, Mazzocco, & Feigenson, 2008; Odic et al., 2013).

In addition to this, individual differences in the ANS acuity have measured its concurrent or predictive relationships with formal math achievement have documented (for review, see Chen & Li, 2014; but see De Smedt, Noël, Gilmore, & Ansari, 2013). Even a retrospective predictive relationship has been found by Halberda et al. (2008). They found that the ANS acuities measured at 14 years-old retrospectively predicted children's past math achievement scores extending all the way back to kindergarten (Halberda et al., 2008). Beyond predictions, a few studies showed that the ANS training result in improved math achievement (Kim, Jang, & Cho, 2018; Park & Brannon, 2013, 2014; But see; Bugden, Szkudlarek, & Brannon, 2021; Szkudlarek, Park, & Brannon, 2021).

This ANS theory has also been supported by neural evidence. Several functional neuroimaging and electrophysiological studies supported the hypothesis on the preexisting neural system for numbers. Functional neuroimaging studies with humans revealed that both nonsymbolic and symbolic number processing recruit a fronto-parietal network, especially the intraparietal Sulcus (IPS) (Ansari, 2008; Cantlon et al., 2011; Dehaene, 2011; Dehaene & Cohen, 2007; Nieder & Dehaene, 2009; Piazza, 2011; Piazza & Izard, 2009; Piazza et al., 2004). Even the data from diffusion tensor imaging (DTI) suggested that the microstructures underneath fronto-parietal areas are related to whole number competence (Matejko & Ansari, 2015; Matejko, Price, Mazzocco, & Ansari, 2013; Rykhlevskaia, Uddin, Kondos, & Menon, 2009; van Eimeren, Niogi, McCandliss, Holloway, & Ansari, 2008). Not just with humans, even single-neuron recordings in monkeys found that individual neurons are tuned to specific numerosity (Brannon, 1998; Nieder & Dehaene, 2009).

With these various strands of supporting evidence, the ANS has been theorized and serves as a good explanation that this innate system exists only for natural numbers (for review, Dehaene, 2011; Piazza, 2011). In other words, symbolic number acquisition can be privileged from representations of numerosities, but rational numbers cannot be advantaged by nonsymbolic magnitude representations. This innate constraint hypothesis has served as a good explanation of the question why fraction is hard.

Possible neurocognitive startup tool: Ratio Processing System (RPS)

In contrast to the innate constraints account, a new perspective on a potential cognitive foundation for fractions knowledge has emerged. Lewis, Matthews and Hubbard (2015) proposed that a neurocognitive foundation exists for fractions, one they dubbed the Ratio Processing System (RPS). This RPS is mainly dedicated to processing *nonsymbolic ratios*, such as ratios instantiated by juxtaposing two line-segments as Figure 1. This nonsymbolic ratio magnitude is classified differently from other nonsymbolic magnitudes such as numerosity that represents the ANS. While the raw amount of substance (e.g., line-lengths, numerosity, or area) can be classified as *simple* magnitudes, nonsymbolic ratio magnitudes can be classified as *relational* magnitudes, which emerge from comparative relations between two simple magnitudes.

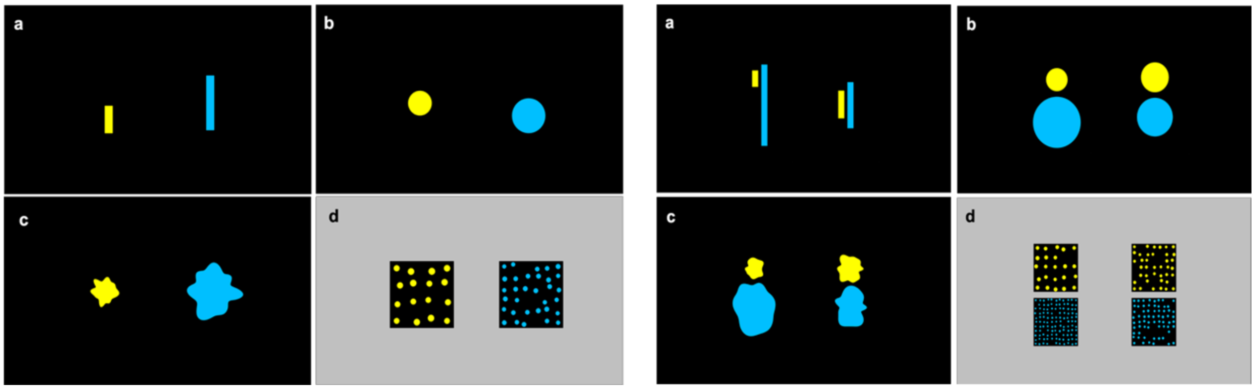

Figure 1. The example stimuli of simple (left panel) and ratio magnitudes (right panel): (A) line, (B) circle, (C) blob, and (D) dot formats. In each example, the right side of each stimulus indicates a larger magnitude (Park et al., 2020).

With these relationally defined magnitudes, the RPS theory hypothesizes that the RPS enables a perceptual mapping between nonsymbolic ratios and symbolic fractions and eventually helps learners acquire understanding of symbolic fractions by representing them as a form of holistic magnitudes (see Figure 2). Even though nonsymbolic ratios are constructed from two simple magnitudes, the RPS helps to process it as an analog magnitude.

Based on the arguments of the RPS hypothesis, fractions can be challenging largely due to children's lack of experience in using nonsymbolic ratio magnitudes with the specific intent of grounding symbolic representations. In turn, enough experience with nonsymbolic ratios may facilitate the RPS to function and may eventually help children learn fractions. A specific intent of using nonsymbolic ratio magnitude as a perceptual tool to represent symbolic fraction as analog magnitudes might aid children's fraction acquisition.

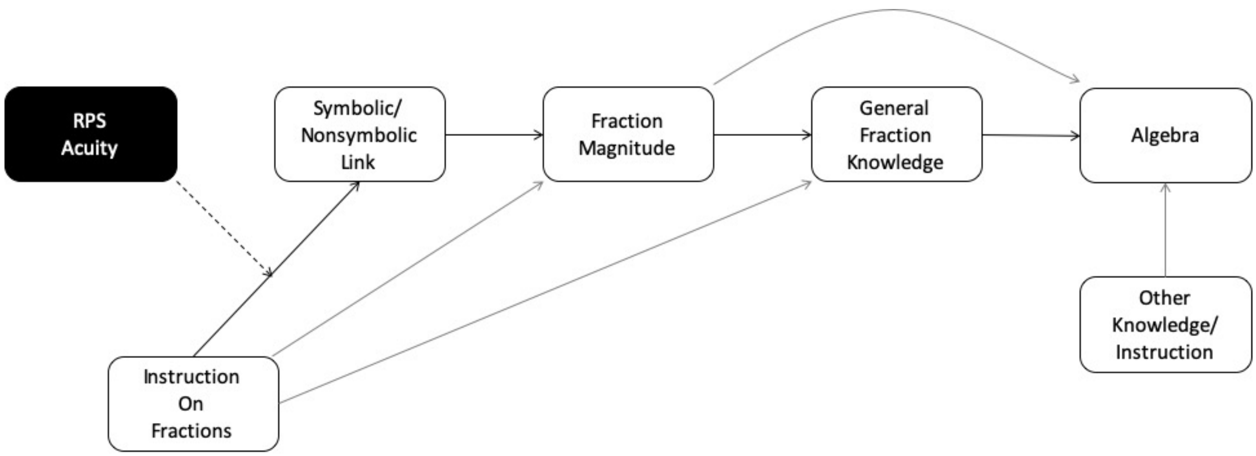

Figure 2. A conceptual model of how the RPS moderates the link between nonsymbolic and symbolic fractions link and how it eventually influences algebra (Lewis, Matthews, & Hubbard, 2016).

Several studies have supported the existence of the RPS. Evidence suggests the RPS exists early in development and across different species. Studies have observed the ability to discriminate ratio magnitudes in 6 months infants (McCrink & Wynn, 2007), non-human as monkeys (Drucker, Rossa, & Brannon, 2016; Vallentin & Nieder, 2008), and even parrots (Bastos & Taylor, 2020). Furthermore, RPS acuity sharpens with development (Park, Viegut, & Matthews, 2020). My colleagues and I investigated the discriminability of simple and ratio magnitudes in multiple formats among preschoolers, 2^{nd} graders, 5^{th} graders, and adults (see Figure 1). In the study, we conducted magnitude discrimination tasks in four different nonsymbolic formats: numerosities, line-lengths, circle-areas, and irregular blob-areas. Measured accuracy patterns showed the RPS acuity improves with age regardless of formats and the growth rate of accuracy was similar to that of simple magnitude acuity. Furthermore, the performance largely depends on the types of magnitude (simple vs. ratio) rather than nonsymbolic formats. These results support the RPS as an independent processing system that can be separate from the simple magnitude processing.

The RPS and symbolic fractions processing

Given the evidence supporting the existence of the RPS, researchers have sought to investigate if symbolic fractions can be grounded on the RPS. Some behavioral studies substantiate this possibility by showing co-processing of symbolic and nonsymbolic ratios through a common system by conducting cross-format fraction comparison tasks with children (Kalra, Binzak, Matthews, & Hubbard, 2020) or with adults (Binzak, Matthews, & Hubbard, 2019; Matthews & Chesney, 2015). For example, Matthews & Chesney (2015) had participants compare which of two ratios (made from pairs of circles, dot arrays, or number symbols) was larger (Matthews & Chesney, 2015). They observed the distance effect similar to the findings with whole numbers and numerosities (Buckley & Gillman, 1974; De Smedt et al., 2013; Halberda & Feigenson, 2008). Also, the reaction time patterns suggested that participants completed cross-format comparisons in 1406ms on average which is ~300ms shorter than the simple numerosity comparison task (Halberda & Feigenson, 2008). This reaction time was rapid enough to preclude the possibility that they first converted nonsymbolic ratios into symbolic form and to suggest the use of perceptual pathway to access the fractional value of nonsymbolic ratio representation. This finding implies that both nonsymbolic and symbolic ratios were compared through a common system.

Furthermore, similar to the ANS, studies have generated findings on the individual differences in RPS acuity and its relation to symbolic fractions competence (Hansen et al., 2015; Matthews, Lewis, & Hubbard, 2016; Möhring, Newcombe, Levine, & Frick, 2015). Matthews et al.(2016) revealed that RPS acuity could predict fractions knowledge and even algebra achievement in college students. This novel result was successfully replicated by Park and Matthews (accepted). Similar results have also been found with children (Hansen et al., 2015; Möhring et al., 2015). These associations imply a link between RPS and symbolic fraction understanding, consistent with the hypothesis that the RPS may serve as a neurocognitive foundation for symbolic fraction acquisition.

Shared neural substrates between the RPS and symbolic fractions processing

In addition to behavioral evidence, emerging findings suggest there is shared neural circuity between nonsymbolic and symbolic ratio processing (Jacob, Vallentin, & Nieder, 2012; Lewis et al., 2016; Mock et al., 2018). These results have converged to display that both symbolic and nonsymbolic ratio processing engage the IPS and a broader parietal-prefrontal network, where adults are believed to process analog magnitude (ex. numerosity or single line-length). Functional neuroimaging studies have used various paradigms to investigate the neural underpinnings of ratio processing. Jacob & Nieder (2009b) used an adaptation paradigm to investigate the neural distance effect for ratios, in which activation increases as the distance between the habituated and deviant ratio increases. They observed that both the IPS and the PFC regions were sensitive to the distance between nonsymbolic ratio magnitudes (represented by line and dot ratios). The neural distance effects in the IPS and the PFC regions have also been found in magnitude comparison and adaptation paradigms using symbolic fractions (Ischebeck, Schocke, & Delazer, 2009, Cui et al., 2020, Worth et al.,2020). These results demonstrate that adults might have overlapping brain regions that process both nonsymbolic ratios and symbolic fractions. However, except for Mock et al. (2018), most previous studies have not confirmed the overlaps between nonsymbolic ratios and symbolic fractions within individuals.

To confirm the overlaps across nonsymbolic ratios and symbolic fraction processing, two studies have used a within-subjects design comparing symbolic and non-symbolic ratio processing in the same participants (Mock et al., 2018; Binzak et al., submitted). Binzak et al. (submitted) investigated neural substrates for comparing nonsymbolic ratios and symbolic ratios with a within-subject design in adults. They used a ratio comparison task that included symbolic fractions, nonsymbolic ratios, and also cross-notation comparisons. The neural data showed overlaps across the neural distance effects in different notations, particularly in the bilateral parietal regions, including the IPS and bilateral prefrontal regions. Along with the capability of cross-notation comparisons, the neural distance effects shown in the same regions for all notation comparisons indicate the possibility of a common system across nonsymbolic ratios and symbolic fractions at the neural level.

Understanding development of functional specialization of fractions processing

According to the evidence reviewed above, we now know that there are common brain regions involved in nonsymbolic and symbolic fractions processing. Given that nonsymbolic ratio processing is a phylogenetically conserved ability, it is interesting that the brain comes to use similar neural regions where also process nonsymbolic ratio to process symbolic fractions, which is one of culturally driven inventions. A question of how the neural circuits specialized for processing symbolic fractions emerge still remains as a question. Given that we acquire symbolic fractions later in development it is possible that functional specialization for symbolic fractions processing may be supported by the RPS. One way we can explore this possibility is with the perspective of developmental neurocognitive theories that can smoothly link several pieces of evidence. Looking into existing neurocognitive developmental theories may provide an explanation on how neural functionalization for symbolic fraction emerges and develops.

Neuronal Recycling

One theory that can be compatible with the RPS account is *the neuronal recycling hypothesis.* This theory seeks to provide an explanation of how fairly recent cultural inventions, such as symbolic number or reading, will be processed via specialized neural circuits (Dehaene & Cohen, 2007). It suggests that ancient cortical systems originally dedicated to a certain function which is close enough to new information can be reoriented to support a novel usage. Thus, a partial alteration of cortical systems eventually results in functional specialization for novel symbolic inputs.

For instance, functional specification in posterior parietal cortex (PPC) for symbolic numbers, one of the recently invented concept could be explained by the neuronal recycling hypothesis (Piazza, 2011). In the human brain, the PPC region has been known to be genetically tuned to process spatial information, including nonsymbolic magnitudes. Numerosity, the number of individuals (e.g., dot array) can naturally correspond to a certain natural number exactly or approximately. Thus, as humans experience novel symbolic numbers, the PPC can be reoriented to process symbolic numbers which is new but relevant to numerosity information. This cortical recycling can be the case of fractions knowledge as proposed by Lewis et al. (2016). As such, the cortical substrates for nonsymbolic ratios can be recycled and reutilized for processing symbolic fractions (Lewis et al., 2016).

Interactive Specialization

Another developmental theory that can account for functional specialization for symbolic fractions is *an interactive specialization* theory (IS) advanced by Johnson (2001). (Johnson, 2001, 2011; Johnson, Grossmann, & Kadosh, 2009). The IS theory takes a domain-general framework for human functional brain development. According to Johnson (2011), the IS framework complements the shortcomings of two other developmental cognitive neuroscience theories (for review, Johnson, 2001) – the *maturation* and *skill learning* perspectives. One is termed as the *Maturation Viewpoint,* which attempts to interpret human brain development simply as biological maturation of the brain. Since this view seeks to explain all development solely based on intrinsic genetic and biochemical factors, it cannot explain the influence of environment, such as experience-dependent brain plasticity. Another viewpoint is termed as *Skill Learning*, which suggests that the onset of new perceptual input and newly acquired motor skills will result in neural changes during development. However, it also cannot explain genetically determined cortical displacement, such as the occipital lobe dedicated to visual processing in the first place.

By integrating these two incompatible views, the IS framework suggests the integrative effects of the biological maturation and environment to human brain development. It also assumes that the changes in connectivity between cortical regions will lead to interactive brain networks which will eventually contribute to the emergence of functional specialization. Given that fractions are one of the recent cultural inventions that we need to acquire from the outside environment, both preexisting ability and external influence from the environment will contribute to functional specialization for symbolic fractions.

Biased connectivity hypothesis

If these constant developmental interactions between neural circuits and experience assist functional specialization for symbolic fractions processing, it is possible that structural organization of the brain contributes to the changes of the brain. Especially, white matter connectivity that links different cortical regions with axon fibers is likely to be involved to functional brain changes as the previous studies have shown (Damoiseaux & Greicius, 2009; Johansen-Berg & Rushworth, 2009; Zimmermann, Griffiths, & McIntosh, 2018). Furthermore, some researchers have suggested that not just functional cortices, but white matter microstructure might have initial biases toward a certain stimuli or information. Hannagan et al. (2015) proposed the *biased connectivity hypothesis*, suggesting white matter microstructure may also have preexisting biases that eventually contribute to a particular functional specialization. The theory proposes that higher structural connectivity will be present between the specialized areas critical to a target task.

In the case of whole number processing, the neural circuits involved in symbolic numbers are not limited to a single region of the brain. As reviewed above, a part of the PPC area, the IPS, is specialized for numerosity and symbolic numbers. Adding on the PPC, it has been revealed that a part of the temporal cortex, the number form area (NFA) is also recruited for visual processing of symbolic numbers (Hannagan, Amedi, Cohen, Dehaene-Lambertz, & Dehaene, 2015; Yeo, Wilkey, & Price, 2017). In turn, the IPS and the NFA placed in the different regions of the brain communicate each other to process symbolic fractions. Previous work has demonstrated the underlying white matter between these two regions for number processing (Abboud, Maidenbaum, Dehaene, & Amedi, 2015). The biased connectivity theory holds the potential to explain how particular interactions among different brain regions have evolved to process symbolic numbers such as a parietal-temporal network between the NFA and the IPS (Abboud et al., 2015).

Even though this biased connectivity theory has been proposed to explain how the NFA and the visual word form area (VWFA) for language processing emerge, it is worth taking into account one of the possible scenarios of the brain's functional specialization for fraction processing. A very recent study reported that functional connectivity from the left medial temporal cortex to the IPS was more robust when adults process symbolic fractions compared to when they process whole numbers (Cui, Li, Li, Siegler, & Zhou, 2020). This result may reflect a possible scenario of biased structural connectivity between the regions of the brains recruited for fractions processing.

Taken together, as reviewed above, current theories on neural circuits for number symbols take a neuroconstructivist viewpoint on functional specialization. Ontogeny plays a crucial role in how the brain gradually becomes specialized during development, but an interplay with outside inputs is also critical. Thus, theories regarding the emergence of numerical cognition may benefit from adopting Karmiloff-Smith's (2010, 2015) *domain-relevant framework*. This framework suggests that the early brain is equipped with biases relevant to processing certain information and becomes progressively specialized through neuronal competition throughout development.

This framework may take pieces of the neuronal recycling, the IS, and the biased connectivity hypothesis in terms of postulating innate biases in the primitive brain organization and allowing the inputs from the environment. The RPS hypothesis also very much fits well with these frameworks. The neural circuit for nonsymbolic ratio processing may exist before fractions instruction and have a bias of connectivity with other brain regions. With years of educational instruction, the RPS and interactions with other brain areas are reoriented toward symbolic relational magnitudes, fractions.

This dissertation takes the domain-relevant theoretical approach to explore how the functional specialization for fractions processing forms in the early years and how the structural and functional brain change over throughout educational experiences.

Main questions and summary of current studies

With the domain-relevant framework, this dissertation explores how the functional specialization for fractions processing forms in the early years and how it changes throughout educational experiences. Furthermore, this dissertation attempts to test to what extent the RPS is influential to early fractions ability relative to other cognitive skills.

In Chapter 2, I examined developmental differences in the functional activations for the RPS and symbolic fractions processing before and after formal fractions instruction. I utilized functional magnetic resonance imaging (fMRI) and investigated 2nd graders who have not yet received formal fractions instructions and 5th graders who have received two to three years of fractions education. Using whole-brain analyses, I identifies brain regions that were sensitive to ratio or fraction magnitudes in 2nd and 5th graders and evaluated the developmental differences between 2nd and 5th graders. Furthermore, with a region of interest (ROI) approach, I focused on the IPS regions sensitive to magnitude information. I tested whether the IPS was recruited for ratio or fraction magnitude processing and compared the engagement of the IPS between 2nd and 5th graders.

Chapter 3 further explored developmental changes in structural connectivity that relate to nonsymbolic and symbolic fraction processing. With the same cohorts of 2nd and 5th graders as Chapter 2, I employed diffusion tensor imaging (DTI) to investigate the differences in the relationships between white matter connectivity and nonsymbolic and symbolic fraction processing abilities between children with and without formal fraction instruction. First, with tract-based spatial statistics (TBSS), I explored the regions of white matter that related to nonsymbolic and symbolic fraction processing abilities. Moreover, I examined how these relationships have changed a year later.

The possibility that symbolic fractions might be grounded on the RPS was supported by the neural investigations reported in Chapters 2 and 3. This motivated me to investigate this possibility with a behavioral approach. In Chapter 4, I attempted to test how influential the RPS, a possible cognitive start-up tool for fractions, was on early fraction ability compared to other cognitive skills. Among various cognitive skills, I focus on linguistic knowledge, which has been frequently reported as a critical factor for children's fraction abilities. The Chapter 4 aimed to investigate how the RPS uniquely contributes to early fraction ability.

Chapter 5 summarizes the findings provided by Chapters 2-4: the neural changes for nonsymbolic and symbolic fraction processing and the possible efficacy of using nonsymbolic representations in fraction instructions. From the brain to behavior, the studies in this dissertation attempt to understand the early development of neural representations for ratios and fractions with various approaches and evaluate a possible neurocognitive tool for early fractions learning. Furthermore, this dissertation aims to understand this neurocognitive tool in the context of the domain-relevant framework to explain a flexible approach in promoting symbolic fractions learning. By tracking down fractions acquisition, this dissertation can serve as an essential piece to provide the scientific explanation in promoting nonsymbolic representations in formal fraction instructions.

Chapter 2: Developmental Changes in Nonsymbolic and Symbolic Fractions Processing: A Cross-Sectional fMRI study

Introduction

As Chapter 1 has introduced, fractions knowledge is considered to be a critical building block of future mathematical competence (Siegler et al., 2012b; Siegler, Fazio, Bailey, & Zhou, 2013; Siegler, Thompson, & Schneider, 2011). Fractions knowledge is fundamental to understanding higher mathematics and is essential for gaining competence in science, technology, engineering and mathematics (STEM) fields. Even though the importance of fractions knowledge is well documented, there remains a dearth of knowledge about how symbolic fractions are processed and how this processing develops over time.

Recent studies suggest that there may be a “neurocognitive startup tool” that may play a crucial role in understanding fractional magnitudes and key concepts of fractions (Jacob, Vallentin, & Nieder, 2012; Lewis, Matthews, & Hubbard, 2016; Matthews, Lewis, & Hubbard, 2016; compare with Piazza, 2011). Previous studies showed the existence of a primitive ability to represent fractions. Specifically, both humans and nonhuman primates have the ability to process nonsymbolic ratio magnitudes early in development (Jacob & Nieder, 2009a, 2009b; McCrink & Wynn, 2007; Vallentin & Nieder, 2010). Lewis, Matthews & Hubbard (2016) proposed that this primitive ability largely depends on an ancient system for symbolic fraction learning and dubbed this system as Ratio Processing System (RPS) (Jacob et al., 2012; Lewis et al., 2016; Matthews et al., 2016).

The RPS account can be compatible with current cognitive developmental theories. For example, according to Dehaene & Cohen (2007)’s neuronal recycling hypothesis, human brains are equipped with some evolutionary-ancient systems that were originally dedicated to certain functions with functionality that is close enough to novel external inputs that they respond (perhaps initially poorly) to those novel inputs. Related cortical regions for this system may partially change during development and reorient to process novel information. This partial alteration of cortical systems may eventually help the acquisition of recent cultural inventions, such as symbolic numbers or language. Therefore, the RPS might represent an ancient system for fractions processing, and it may be well-suited to be "recycled" for symbolic fraction processing.

So far, the RPS account has supported by several behavioral studies. One line of research has shown co-processing of symbolic, and nonsymbolic ratios through the same system by using cross-format tasks (Kalra, Binzak, et al., 2020; Matthews & Chesney, 2015). Kalra et al. (2020) conducted cross-format fraction comparison tasks, whereby participants compared which of two ratios (comparing line ratios to symbolic ratios) was larger. They observed a classic *distance effect* in which performance improved as the distance between ratios compared increased (Buckley & Gillman, 1974). The cross-format distance effect substantiates holistic magnitude processing for both nonsymbolic and symbolic ratio. Additionally, reaction times were rapid enough (mean = 2043ms in 2^{nd} graders) to suggest that children were able to compare nonsymbolic line ratios and symbolic fractions without first converting nonsymbolic to symbolic or vice versa. This finding implies the possibility that nonsymbolic and symbolic ratios were compared through a common system.

Moreover, this RPS processing of nonsymbolic ratios seems to automatically affect the processing of symbolic fractions. Matthews and Lewis (2017) showed a Stroop-like interference effect demonstrating the interference effect between symbolic fractions and a physical size of fractions by using a simple symbolic fraction comparison task (See Figure 2.1). In the task, they created numerical stimuli for which font sizes were manipulated such that numerical fractions simultaneously represent congruent or incongruent nonsymbolic ratio magnitudes (i.e., the font sizes of a numerically larger fraction present larger nonsybolic ratio magnitude). They found a ratio size congruity effect, such that the congruity of nonsymbolic ratio features (font size) influenced the performance of symbolic fraction comparison task.

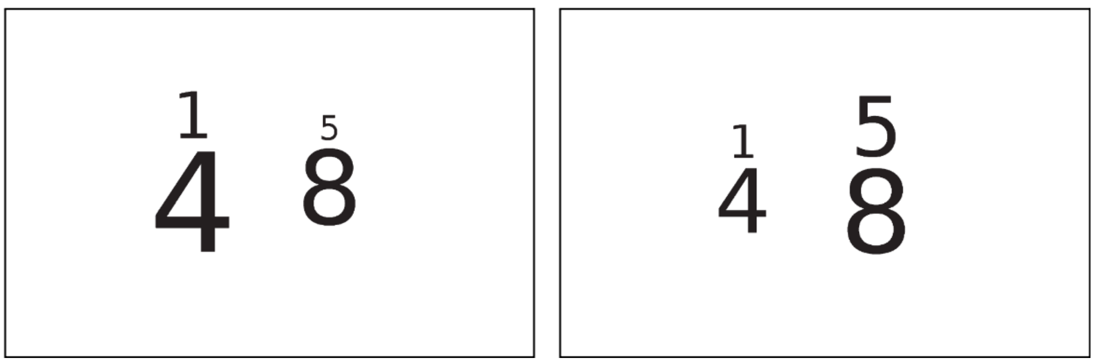

Figure 2.1. Example stimuli of Matthews & Lewis (2017). Both font sizes and nonsymbolic ratios that physical sizes of fraction represent are incongruent with fraction values (Left) and are congruent with fraction values (Right).

In addition to these relations, there have been several other findings suggesting a correlation between RPS acuity and competence with fractions knowledge and other symbolic mathematics domains (Hansen et al., 2015; Matthews et al., 2016; Möhring et al., 2015). For instance, Matthews et al. (2016) revealed that participant's acuity in nonsymbolic ratio comparison tasks can predict fractions knowledge and even algebra achievement in college students (see also Park & Matthews, accepted). Similar results were also reported with children (Hansen et al., 2015; Möhring et al., 2015). These associations imply a link between the RPS and symbolic fraction understanding, consistent with the claim that the RPS serves as a neurocognitive start-up tool for symbolic fraction acquisition.

Neuroimaging studies have also yielded findings consistent with the RPS account. Emerging findings suggest a similar substrate between nonsymbolic and symbolic ratios processing (Ischebeck et al., 2009; Jacob & Nieder, 2009b, 2009a; Mock et al., 2018). Although most of these studies looked into only nonsymbolic or symbolic ratios specifically, these results have converged to show that both symbolic and nonsymbolic ratio processing engage the intraparietal sulcus (IPS) and a broader parietal-prefrontal network, where adults are believed to process analog magnitude (e.g., numerosity or single line-length) (Dehaene et al., 1998; Nieder & Dehaene, 2009; Piazza et al., 2004).

Using single-neuron recordings in rhesus macaques, Vallentin & Nieder (2010) identified neurons in the IPS that were tuned to nonsymbolic ratio information. Jacob & Nieder (2009b) used a functional MRI adaptation paradigm to investigate the neural distance effect for nonsymbolic ratios, in which activation increases as the distance between the habituated ratio and deviant ratio increases. Extending their findings in macaques, they observed that both IPS and prefrontal cortex (PFC) regions exhibited the distance effects for nonsymbolic ratio magnitudes (represented by line and dot ratios).

Neural distance effects in IPS and PFC regions have also been found in magnitude comparison and adaptation paradigms using symbolic fractions (Ischebeck et al., 2009; Jacob & Nieder, 2009a; Mock et al., 2018). Early studies examined only nonsymbolic ratios or nonsymbolic fractions, so functional overlap was only inferred based on similar locations. A more recent within-subjects design comparing symbolic and non-symbolic ratio processing in the same participants has provided some support for a shared processing system. Mock et al. (2018) investigated neural substrates for nonsymbolic ratios (instantiated by dots and pie charts) and symbolic ratios (fractions and decimals) in the same individual. Consistent with the prior work reviewed above, they observed a shared system for ratio processing in the bilateral IPS. In addition to the shared substrate, they also found differential neural substrates for nonsymbolic and symbolic ratios; whereas nonsymbolic ratios elicited greater activation in the right ventral visual stream, insula, and superior frontal gyrus compared to symbolic ratios, symbolic ratios showed greater activation in the left frontal gyrus, the angular gyrus, and the middle occipital gyrus.

However, the results of Mock et al. (2018) need to be carefully considered due to potential confounds in the methodology. According to work by Dewolf and colleagues (2016), neural engagement is different for fractions and decimals. By conducting a symbolic comparison task using integers, fractions and decimals in fMRI, DeWolf et al. found differences in processing fractions vs. decimals and fractions vs. integers, but not in decimals vs. integers (DeWolf, Chiang, Bassok, Holyoak, & Monti, 2016). However, in Mock et al. (2018), the "nonsymbolic ratio" contrast included *both* dots and pie charts, and the "symbolic ratios" contrast included *both* fractions and decimals. This is a problem if fraction and decimals are indeed distinctively processed in the brain. Therefore, Mock et al.'s results likely represent both fraction and decimal processing, which are different. Considering that previous studies specifically studied symbolic fractions, it is hard to say Mock et al. (2018)'s results are comparable to other studies.

Recently, Binzak et al. (submitted) addressed this issue by investigating fractional magnitudes, specifically. They observed how nonsymbolic, symbolic and cross-notation ratio comparisons were processed in the same adults' brain by conducting a ratio comparison task. They found overlapping activation across different notation conditions, particularly in the bilateral parietal regions including the IPS and the bilateral prefrontal regions. Along with the capability of cross-notation comparisons, the neural distance effect shown in the same regions for cross-notation conditions suggests co-processing between nonsymbolic and symbolic at the neural level.

With behavioral and neural evidence, one remaining question would be how these similar substrates between nonsymbolic and symbolic fractions have developed and emerged. Studying exclusively adult participants cannot unravel whether a cortical system for nonsymbolic ratio magnitude is the basis for symbolic fractions. To answer the question of how shared substrates for nonsymbolic ratios and symbolic ratios emerge requires a developmental approach. Thus far, there have been no studies looking into the RPS in young children or how neural activations for nonsymbolic and symbolic ratios change as a function of educational experience with fractions.

We hypothesized that children have a similar, early neural architecture for ratio processing - even among those who have received little instruction on symbolic fractions. Secondly, if the RPS plays a role as a foundation for symbolic fractions, symbolic fraction processing will recruit the same regions engaged in nonsymbolic ratio processing after fraction instruction. Therefore, we predicted that older children exposed to years of fractions instruction would be likely to show overlapping activation for both symbolic and nonsymbolic ratios, whereas younger children would be likely to show activation only for nonsymbolic ratios.

The aim of current study was two-fold: 1) to explore similarities and differences between nonsymbolic and symbolic ratio processing in young children at both the behavioral and neural levels, and 2) to explore how behavioral and neural signatures of ratio processing change from 2nd grade (prior to formal fractions instruction) to 5th grade (after receiving fractions instruction). To this end, we used fMRI to investigate neural activation during ratio comparison in multiple formats in 2nd and 5th grade primary school children.

Methods

Participants

We recruited 47 2nd- (M_{Age} = 7.68, SD_{Age} =.43) and 45 5th-grade (M_{Age} = 10.68, SD_{Age} =.47) children from several public schools in a mid-sized Midwestern city. All participants were right handed, native English-speakers with normal vision. Parents or guardians gave written consent, and children gave verbal assent. All protocols were approved by the biomedical research ethics committee of the University of Wisconsin – Madison (IRB #2013-1346) . Participants received monetary compensation and small gifts for their participation. One 2nd grade child was excluded due to an ADHD diagnosis, and two children failed to complete the scan due to excessive movement (1 2nd grade and 1 5th grade). Another two 5th grade children were excluded due to technical issues with scan during acquisition. In addition to this, we also excluded children with head movement greater than 2.5 mm (15 2nd grade and 8 5th grade) and children who performed at chance level behavioral performance in the scanner (2 2nd grade and 1 5th grade). Participants were excluded if these occurred in three or more of the six functional runs. Therefore, in the final analysis, 28 2nd graders and 33 5th graders were included.

The Ratio Comparison Task

Participants completed a ratio comparison task in the MRI scanner (Binzak et al., submitted). Stimuli were presented by E-prime software (Psychology Software Tools, Shapsburg, PA). On each trial, participants compared two ratios made either from juxtaposed pairs of nonsymbolic ratios made from line segments or symbolic fractions (see Figure 2, below). Comparison pairs were presented in each of three types: 1) Fraction vs. Fraction (Frac-Frac), 2) Line ratio vs. Fraction (Line-Frac), and 3) Line ratio vs. Line ratio (Line-Line) comparisons (Figure 2). In all comparisons, stimuli were presented side-by-side in a light gray color on a black background. Participants indicated their judgments by pressing a button box with either their index (indicating the left ratio was judged larger) or middle finger (indicating the right). Children were trained outside the scanner before the real scan.

To manipulate the difficulty of the tasks, we varied the numerical distance between comparison stimuli among trials. Here, we defined difficulty as the difference between the magnitude of the compared ratios [fraction A – fraction B]. Although the distance manipulation was designed continuously, pairs were organized into three distance bins for the purposes of analysis (Jacob & Nieder, 2009a; Kalra, Binzak, et al., 2020): near (.048-.233), medium (.262-.446), and far (.514-.750) (See Table 1). We expected near comparisons to be the hardest behaviorally and to elicit the greatest neural activation. In contrast, we expected the far condition to be the easiest and to elicit lower activation.

Each participant completed six runs of 36 trials per run for a total of 216 trials. Each individual run included an equal number of trials from each notation condition (Line-Line,

Line-Frac, and Frac-Frac), and the trials in each notation were evenly distributed among the distance bins (near, medium and far). Each run was split into two blocks of 18 trials depending on the way that component line segment lengths of line ratios were controlled (see *Stimuli* section below). All notations and distances were counterbalanced across the six runs. Within each run, stimulus presentation order was random for each participant.

Each trial began with a fixation cross, presented for 1250 – 1750 ms (with the range corresponding to randomly applied jitter, 1500ms ± 250ms) followed by presentation of ratio stimuli. Participants could respond upon stimulus onset, and stimuli remained on-screen until participants answered or until the trial timed out after 4,000ms (Figure 2). The program did not proceed to the next trial until participants respond.

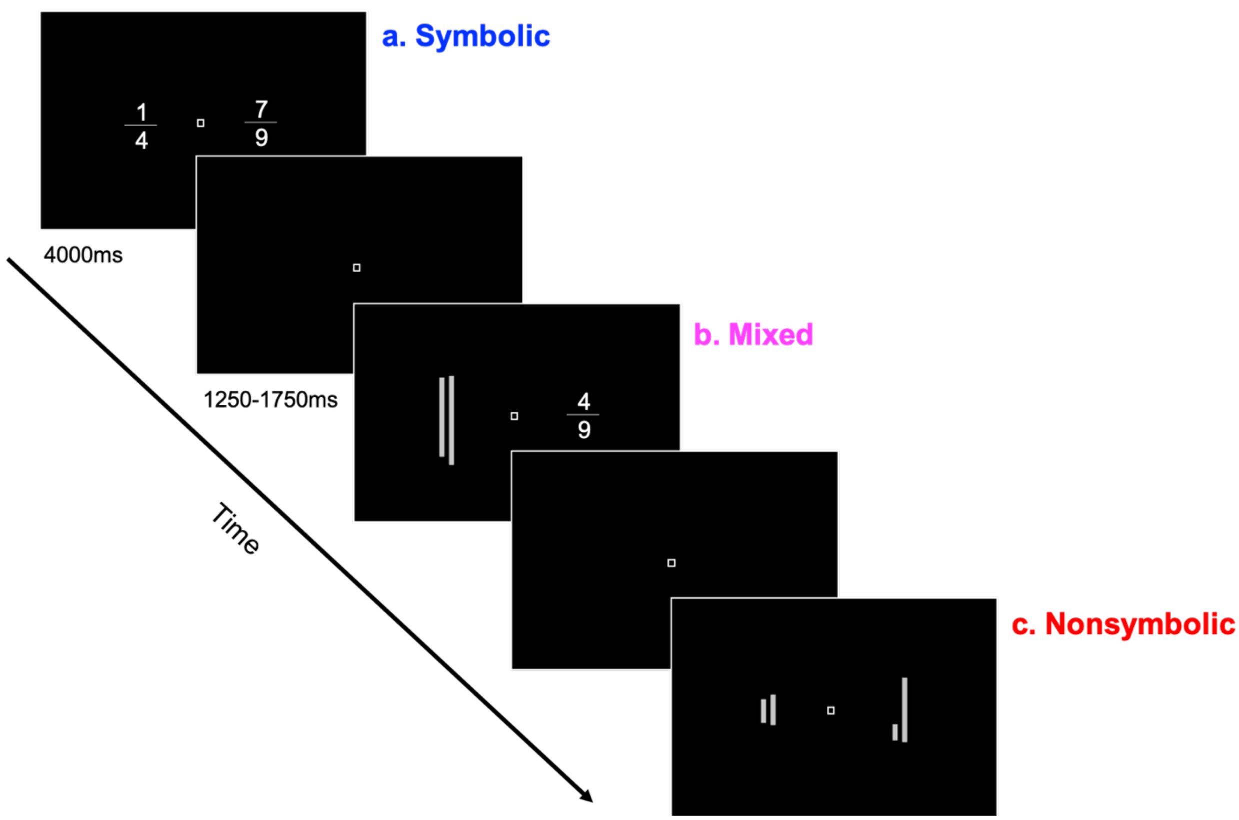

Figure 2.2 Sample sequence of three trials for the comparison task. Trials include a) the symbolic condition comparing fractions, b) the mixed condition comparing fraction and line ratio, and c) the nonsymbolic condition comparing line ratios.

Table 2.1. Description of Distance bins in the ratio comparison task.

Distance Bin	Mean	SD	Min	Max
Near	.144	.054	.048	.233
Medium	.341	.063	.262	.446
Far	.613	.069	.514	.750

Stimuli

Symbolic Fractions. Symbolic fractions were selected from the set of the 27 possible irreducible fractions composed from single digit numerators and denominators of value 1 – 9. These selected pairs were identical to Binzak et al. (submitted). We selected 36 pairs composed from those fractions, guided by the following considerations for the variations of fractions involved. The first consideration was the distance between fractions. Thus, 12 pairs of fractions were selected from each distance bin. Second, we wanted to reduce the likelihood that participants might compare fractions based on numerator or denominator component value instead of on the holistic magnitudes of fractions. To discourage the use of such componential strategies, the fraction pairs were equally selected from 1) fraction pairs that had a common denominator, 2) pairs where the numerically larger fraction had a larger numerator and a smaller denominator than the smaller fraction, 3) pairs where the numerically larger fraction had a larger numerator and a larger denominator, and 4) pairs where the larger fraction had smaller numerator and denominator than the smaller fraction. However, in the far distance bin, we did not include the incongruent numerator pairs because no qualifying pair existed with distance greater than .306 given the set of fractions to be used.

Nonsymbolic Ratios. Nonsymbolic ratios were composed of pairs of juxtaposed gray lines. The line stimuli were created with the intent to minimize the probability that participants use each line-length (i.e., the numerator or denominator component) as a cue to make the comparison decision. Thus, we created two sets of line ratios corresponding to the magnitudes of the symbolic ratios described above. One set was controlled to minimize the correlation between the length of numerator component and overall ratio magnitude (Numerator controlled, see Table 2.2). For this set, the length of the numerator was first randomly generated to be between 33-336 pixels. The length of denominator was then determined based on numerator line length. The other set was controlled to minimize the correlation between the length of denominator component and overall ratio magnitude (Denominator controlled, see Table 2.2). For this set, the length of denominator was randomly generated to be between 130-300 pixels. Correlations between line-lengths of components and ratio value are described in Table 2.2.

Table 2.2. Correlations between line-lengths of components and overall ratio values

	Correlation coefficient (r) with overall ratio values	
	Numerator controlled	Denominator controlled
Numerator line-lengths	.34	.83
Denominator line-lengths	-.68	-.22
Summed line-lengths	-.42	.38

Fraction Instruction

Similar to Karla et al. (2020), all children received a brief PowerPoint lesson introducing the concept of ratios/fractions prior to experimental runs because our sample included children who (a) had no prior formal fraction instructions and (2) were unfamiliar with nonsymbolic ratio representations. For example, this included instruction on the notion that fractions get larger as numerator sizes increase, as denominator sizes decrease, and as the two components become closer to the same value. In order to introduce nonsymbolic and symbolic ratio in a kid-friendly manner, we used cartoon characters to depict how height comparisons can make a ratio. For instance, children were instructed that "Joey is half as tall as Sara. When we think Sara's and Joey's heights together, we can call it a RATIO. And we use numbers to talk about ratios. These numbers are called Fractions." Cartoon characters were eventually replaced by lines, so that children could gain some familiarity with the types of line ratios that would be presented as stimuli. When we introduced line ratios, the corresponding fractions were also presented simultaneously.

Data Acquisition

Participants were scanned in a General Electric 3-Tesla scanner (GE Medical Systems, Waukesha, WI) equipped with a 32-channel array head coil (Nova Medical) at Waisman Center of University of Wisconsin-Madison. Foam padding was used to limit head motion. Structural images were collected by using motion-corrected 3D T1-weighted (T1w) MPnRAGE with 1mm isotropic resolution (TR = 4.876ms, TE = 1.82ms, Flip angle = 4°, FOV = 224mm X 224mm, in plane resolution: 256 X 256 pixels, the number of axial slices = 176) (Kecskemeti et al., 2016). Functional images were acquired with a 3D T2-weighted (T2w) echo-planar imaging sequence (TR = 2000ms, TE = 22ms, Slice thickness = 3mm, Flip angle

= 75°, FOV = 224mm X 224mm, 128X128 matrix). Each volume consisted of 38 slices (1.75mm X 1.75mm voxel size) with a 52ms inter slice interval. The first 5 volumes of each functional run, during which participants waited for the task to begin, were also collected to allow for T2 equilibrium effects. In total, 120 volumes were acquired for each functional run.

Imaging analysis

All images were analyzed using Brain Voyager QX 2.8.2 (Brain Innovation, Maastricht, Netherlands). Each individual's data were preprocessed using the following procedure. The first five volumes of each functional run were discarded to account for the stabilization of magnetic saturation. Functional images were corrected for differences in slice time acquisition by using sinc interpolation with ascending and interleaved order and 3D motion by using trilinear sinc interpolation, followed by high-pass temporal filtering (GLM-Fourier with a cut-off of 2 sines/cosines per cycle). The preprocessed functional images were co-registered to the T1 anatomical images through initial alignment followed by fine tuning. These co-registered data were transformed into Talairach Space (Talairach & Tournoux, 1988). These co-registration and Talairach transformation processes were visually inspected by the analysis team and problematic cases were rerun and resolved by the team. Functional images were smoothed by applying an 8mm full width at half maximum (FWHM) Gaussian kernel. A hemodynamic response function was used to model the expected BOLD signal for each distance condition and notation (near, medium, and far/LL, LF, and FF).

We then performed a random effects GLM for each 2nd and 5th grader groups. Whole brain contrast was thresholded at an uncorrected p-value of .01, then corrected for multiple-comparisons using BV's cluster level statistical threshold estimator which resulted in a false positive rate (α) below 0.05 (Goebel, Esposito, & Formisano, 2006). The procedure uses Monte Carlo Simulations to identify the minimum cluster threshold size (Forman et al., 1995; Goebel et al., 2006), considering spatial smoothness and spatial correlations of the data (see detailed mathematical explanation in Forman et al., 1995). Critically, this approach avoids false positive results due to invalid cluster inferences (Eklund, Nichols, & Knutsson, 2016).

We first investigated the brain regions that were sensitive to the changes in the holistic distance between compared magnitudes. We contrasted near and far distance conditions to identify the regions of the brain showing greater activity in near distances relative to far distances – that is, those exhibiting *neural distance effects*. Next, to isolate the regions involved in each notation, we conducted the same random effects analysis contrasting near and far distances for nonsymbolic, mixed, and symbolic notations. To identify the overlapping regions across the neural distance effects in different notations, we then conducted conjunction analysis on the brain across different notations in each grade, for example, [(FF notation with near distance – FF notation with far distance) $\cap$ (LL notation with near distance – LL notation with far distance)].

Additionally, we conducted a two factor random effects ANOVA (1 between: grade levels, 1 within: near vs. far) to identify the regions showing significant interactions between the distance effects in 2nd vs. 5th graders. Significant regions were identified by whole brain analysis, and then the mean beta values for each subject were extracted from each cluster. Next, we confirmed the clusters where 5th graders showed larger distance effects than 2nd graders did and vice versa. To unpack the results, we ran the same analysis for each notation (LL, LF, and FF).

Moreover, we performed a priori regions of interest (ROI) analysis on the bilateral IPS, the region most consistently implicated in processing numerical magnitudes in the extant literature (e.g., Dehaene, Piazza, Pinel, & Cohen, 2003; Piazza, Izard, Pinel, Le Bihan, & Dehaene, 2004). We used a coordinate set based on Houdé et al.'s (2010) meta-analysis of number processing in children. We centered a 10 mm X 10 mm X 10 mm cube around each IPS coordinate identified by Houdé et al. (2010) and extracted the average beta values for each subject at each cross section of 3 notations (FF, LF, LL) X 3 distance bins (near, medium, far). With these extracted beta values, we conducted a mixed-effects regression in order to evaluate the distance effects in the IPS.

Results

Behavioral analysis

Behavioral distance effects and format effects

Children in both grades were capable of discriminating ratio magnitude accurately (2nd: M_{err} = .11, SD_{err} =.31; 5th: M_{err} = .08, SD_{err} =.27) and rapidly (2nd: M_{rt} = 1619.07 ms, SD_{rt} = 636.23; 5th: M_{rt} = 1515.32 ms, SD_{rt} =623.53) across all notations (See Table 2.3 for each notation). We conducted mixed effects logistic regression models to account for within-subject correlation among trials using the 'gImer' function of lme4 package in R software (Bates, Mächler, Bolker, & Walker, 2015). We regressed error rate (0 or 1) against notations (3 levels, FF = 0, LF = 1, LL = 2), age groups (2 levels, 2nd graders = 0, 5th graders = 1), and absolute distance (Table 2.4; Figure 2.3). Since we had hypotheses regarding the change of performance between levels (i.e., 2nd graders < 5th graders and FF < LF < LL)

(Kalra, Binzak, et al., 2020), to facilitate analysis, we used a backward difference coding scheme so that we could compare adjacent levels of variables (each level minus prior level). The fixed effect results showed a significant distance effect; as absolute distance between two fractions increased, error rate decreased (OR = .002, p < .001). A main effect of grade showed that 5th graders were .793 times less likely to make errors than 2nd graders (p = .040). As we expected, the liklihood of making an error in nonsymbolic comparsons was lower than that of mixed notation (OR = .408 , p<.001). However, the liklihood of making an error in symbolic notation was not statistically higher than that of mixed notation (OR = .901 , p =.152).

For reaction times, we conducted linear mixed effects regression models to account for within-subject correlation among trials using the 'lmer' function of lme4 package in R software (Bates et al., 2015). As with the analysis for error rate, we regressed reaction times against notations (3 levels, FF = 0, LF = 1, LL = 2), age groups (2 levels, 2nd graders = 0, 5th graders = 1), and absolute distance (Table 2.5; Figure 2.4). We used a backward difference coding scheme so that we could compare adjacent levels of variables. The results were consistent with the analyses of error rate. There was a significant distance effect (β = -791.37, p<.001). Also, 5th-graders responded significantly faster than 2nd-graders (β = -167.34, p <.005). In terms of notations, children reponded faster on nonsymbolic comparisons than on mixed ones (β = -265.69, p <.001), and reponded faster on mixed comparisons than on fractions comparisons (β = -217.40, p <.001).

Table 2.3. Descriptive statistics for the ratio comparison task

grade	2nd graders						5th graders					
notation	FF		FL		LL		FF		FL		LL	
	err	RT	err	RT	err	RT	err	RT	err	RT	err	RT
mean	.14	1907	.13	1630	.06	1347	.10	1717	.09	1535	.04	1306
Sd	.35	662	.34	606	.23	509	.30	637	.29	623	.21	540

Table 2.4. Logistic mixed effects that regressed err against notations, distance, and age groups.

	β	*Odds Ratio*	*p*
Intercept	-.676	.509	<.001**
FL - FF	-.105	.901	.152
LL -FL	-.914	.408	<.001**
5th – 2nd graders	-.231	.793	.040*
distance	-6.471	.002	<.001**

Table 2.5. Linear mixed effects that regressed RT against notations, distance, and age groups.

	β	*t*	*p*
Intercept	1885.11	46.852	<.001**
FL - FF	-217.40	-17.896	<.001**
LL -FL	-265.69	-22.266	<.001**
5^{th} – 2^{nd} graders	-167.34	-2.948	<.005**
distance	-791.37	-22.664	<.001**

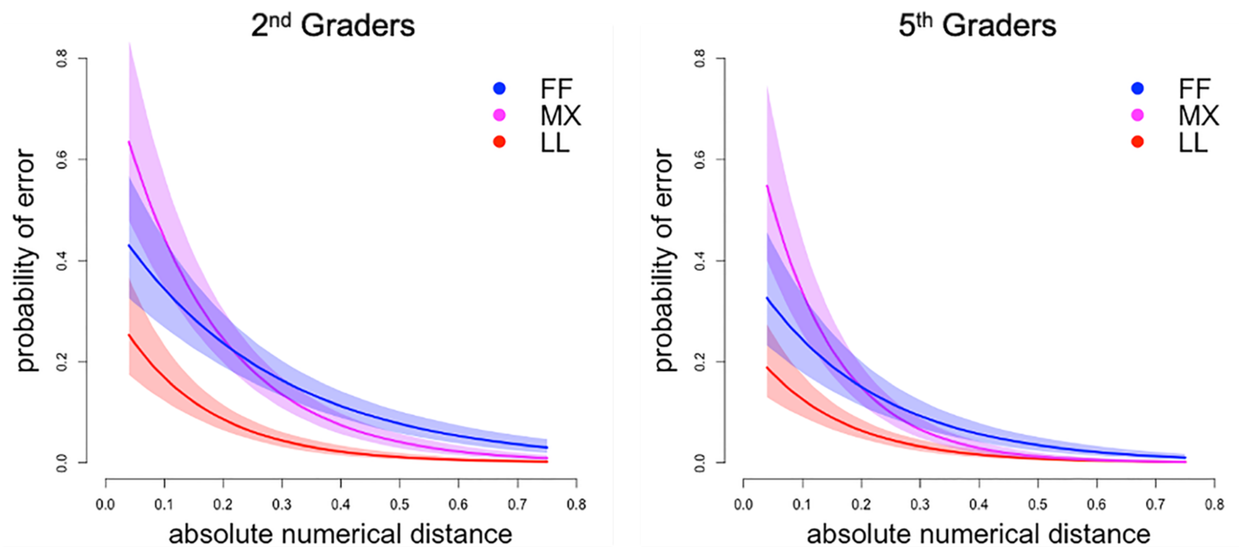

Figure 2.3. Describing logistic mixed effects of error of 2nd graders (left) and 5th grader (right)'s performance.

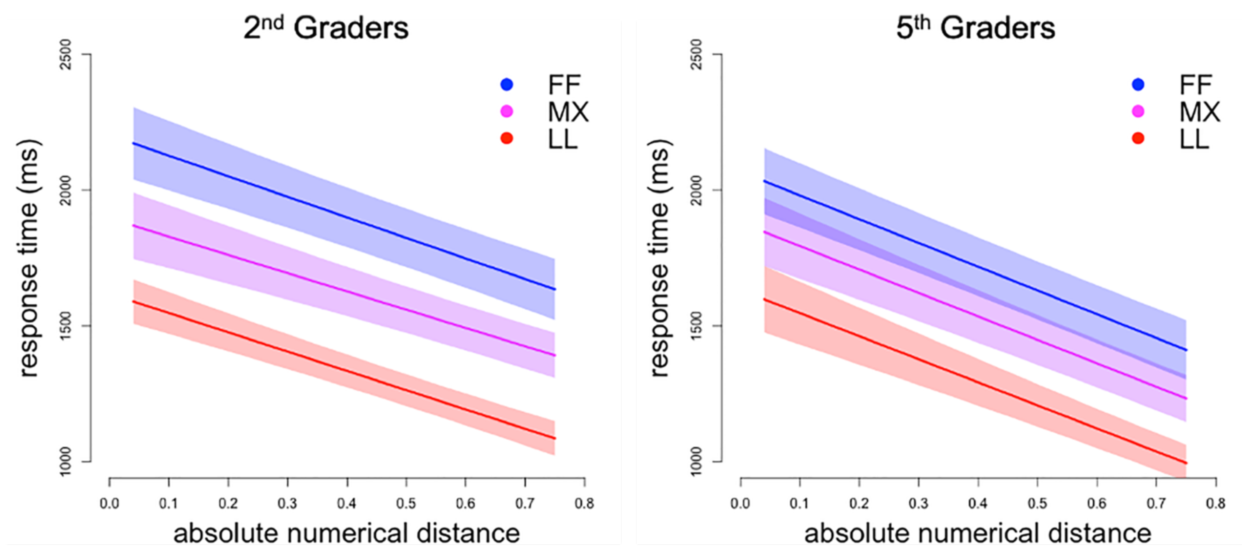

Figure 2.4. Describing linear mixed effects of reaction times of 2nd graders (left) and 5th grader's (right) performance.

The Effects of Gap strategy in Symbolic and Nonsymbolic Fractions Comparisons

Previous studies suggested the possible use of heuristic strategies while comparing the magnitudes. One such incorrect strategy that recently has been highlighted is the *gap strategy* (Morales, Dartnell, & Gómez, 2020). This strategy is based on an assumption that a larger fraction has a smaller difference between its numerator and denominator (Denominator – Numerator = Gap) (Morales et al., 2020). Take, for example, the comparison 2/9 vs. 3/5. 3/5 has a gap of 2 and 2/9 has a gap of 7, so a participant using the gap strategy would choose 3/5 because of the smaller gap. This strategy often yields the right answer but there are many cases in which it is invalid, such as 2/3 vs. 6/8. Still, it is important to exclude the possibility of this gap strategy usage to validate the distance effect as an evidence of holistic processing of ratio or fraction information.

To evaluate the gap strategy effects, we calculated the gap distances between each fraction pair (|fraction 1's gap – fraction 2' gap|) and conducted linear mixed effects regression examining fixed effects by regressing reaction times against the distance and the gap distance. Thus, we evaluated whether the distance effect observed in RTs depends on the gap distance. We observed significant distance effects in both 2nd and 5th graders even after controlling for the gap distances (See Figure 2.5, Table 2.6). The data showed that 2nd graders' reaction times were significantly explained only by the holistic distance between compared fractions (β = -127.97, p < .001), but not by the gap distance (β = -15.73p = .324). On the contrary, 5th graders' reaction times were significantly explained by both holistic distance (β = -204.09, p < .001) and gap distance (β = 53.72, p < .001). These results clearly showed that, children did process holistic distances to choose a larger ratio even though 5th graders also used gap strategies.

We also evaluated the gap distance effects in the nonsymbolic ratio comparisons. In parallel to the analysis with the symbolic fraction comparisons, we calculated the distances between two line-lengths of each line ratio pair (line ratio 1's gap –line ratio 2' gap in pixels). Our line ratio stimuli included the pairs with incongruent (negative) gap distance (n=3) in which a larger ratio had a larger gap, and the pairs with positive gap distances (n=69, max 0-275 pixels). With these line gap distances, we performed linear mixed effects regression examining fixed effects by regressing reaction times against the distance and the line gap distance. Similar to the analysis with symbolic fractions, we observed significant distance effects in both 2^{nd} and 5^{th} graders even after controlling for the gap distances (See Figure 2.6, Table 2.6). The holistic distance was a significant predictor of reaction times in both 2^{nd} (β=-724.25, $p < .001$) and 5^{th} graders (β=-883.37, $p < .001$). Moreover, the gap distance was not a significant predictor for reaction times for either 2^{nd} ($p = .669$) or 5^{th} graders ($p = .862$). These results clearly stated that when it comes to comparing nonsymbolic line ratios, children did not rely on the gap distance to choose a larger ratio.

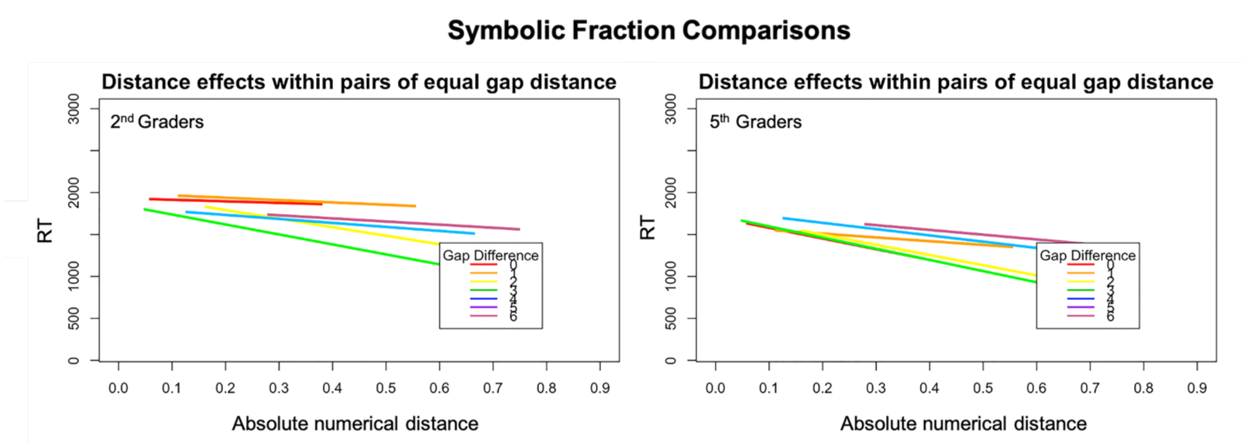

Figure 2.5. Distance effect slopes across gap distance for symbolic fractions in 2^{nd} (left panel) and 5^{th} graders (right panel).

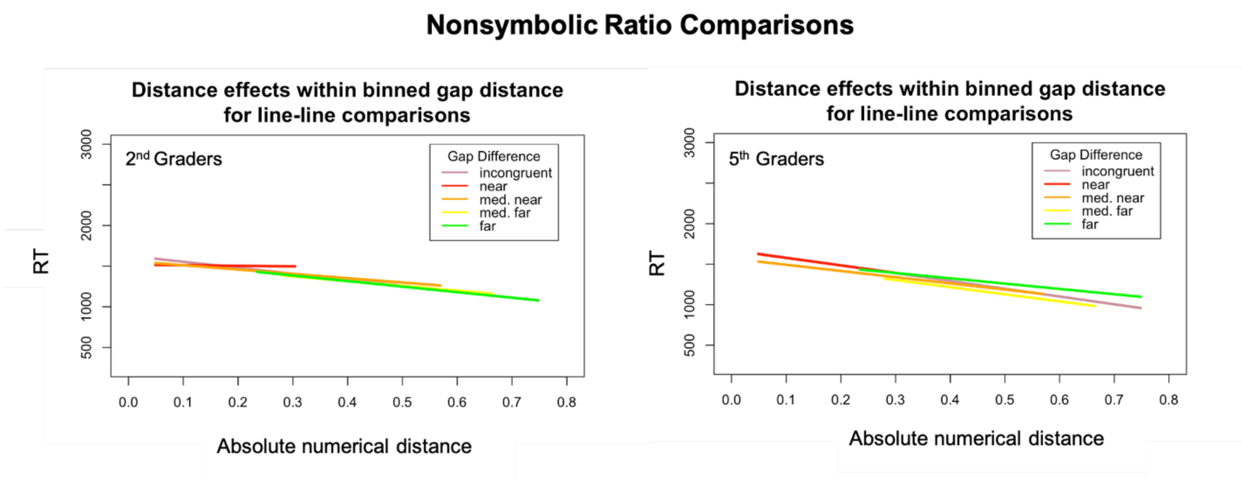

Figure 2.6. Distance effect slopes across gap distance for nonsymbolic ratios in 2nd (left panel) and 5th graders (right panel). To make parallel graph as Figure 4, Line ratio stimuli were classified into the pairs with negative gap distance in which a larger ratio had a larger gap and the pairs with near (0-50 pixels), medium-near (50-100 pixels), far-medium (100-150) and far (>150) distances.

Table 2.6. *Linear mixed effects that regressed RTs for symbolic fraction or nonsymbolic ratio comparisons against holistic distance and gap distance.*

Symbolic fractions		2nd graders			5th graders		
		β	*t*	*p*	*β*	*t*	*p*
	intercept	1920.82	58.30	<.001	1736.48	30.087	<.001
	Distance	-127.97	16.20	<.001	-204.09	-15.211	<.001
	Gap distance	-15.74	15.95	.324	53.72	4.043	<.001
	Distance*Gap distance	-8.61	14.30	.547	13.79	1.153	.249
Nonsymbolic Ratios		**2nd graders**			**5th graders**		
		β	*t*	*p*	*β*	*t*	*p*
	intercept	1356.99	41.618	<.001	1323.953	28.152	<.001
	Distance	-724.252	-13.565	<.001	-883.37	-18.734	<.001
	Gap distance	.055	.428	.669	-.020	-.174	.862
	Distance*Gap distance	-.416	-.778	.437	.418	.880	.379

Neuroimaging analysis

Neural distance effects in whole-brain analysis.

To explore the regions of the brain that are sensitive to the holistic distance between compared stimuli in each grade, we performed a random effects GLM contrasting near and far distances for 2^{nd}-graders vs. 5^{th}-graders. Results of whole brain analysis showed the brain regions with significant neural distance effects when collapsing across all notations in both 2^{nd} and 5^{th} graders. The contrast near > far in 2^{nd}-graders revealed a several set of regions that included bilateral superior parietal lobules including the IPS, the medial frontal cortex, the insula, the precentral gyrus and left lingual gyrus ($p < .05$; see Figure 2.7 and Table 2.7). Similar regions were found in 5th graders' brains. The regions showing the neural distance effects in 5th graders included bilateral superior and inferior parietal lobules, medial prefrontal cortex, insula, and other frontal regions ($p < .05$; see Figure 2.7 and Table 2.7).

As a follow-up test, to see the neural distance effects for each notation, we used the contrast near > far within each notation. We found that 2nd graders' neural distance effects were mainly from nonsymbolic and mixed notations (LL and LF) but not from symbolic fraction (FF) notation ($p < .05$; see Figure 2.8 and Table 2.8). For the neural distance effect for symbolic fraction comparison, we found a small cluster in the inferior frontal gyrus (IFG) is engaged only with uncorrected statistical analysis at a threshold $p < .05$ level. Neural distance effects in mixed notation were found in broader regions relative to the nonsymbolic notation, including more frontal and parietal regions of the brain. On the other hand, we found overlapping regions across the neural distance effects from all notations in case of 5th graders ($p < .05$; see Figure 2.8 and Table 2.9). All notations recruited both fronto-parietal regions, although LL and LF comparisons recruited broader areas. As these results show, there is a developmental difference between 2nd and 5th graders that functional engagement of symbolic fraction starts after 2nd year of the primary school years.

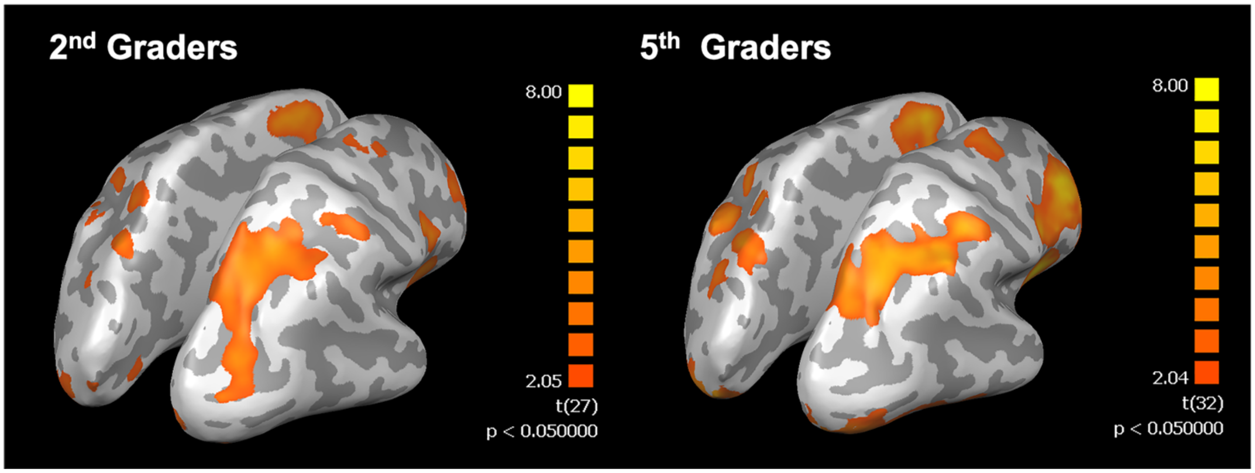

Figure 2.7. Significant neural distance effect across all notations in 2nd graders (left) and 5th grader (right)' brains. The brains are inflated to allow visualization of activations in the sulci (dark gray) and gyri (light gray). Each brain is viewed from the back right, with hot colors indicating the strength of activation (see color bar).

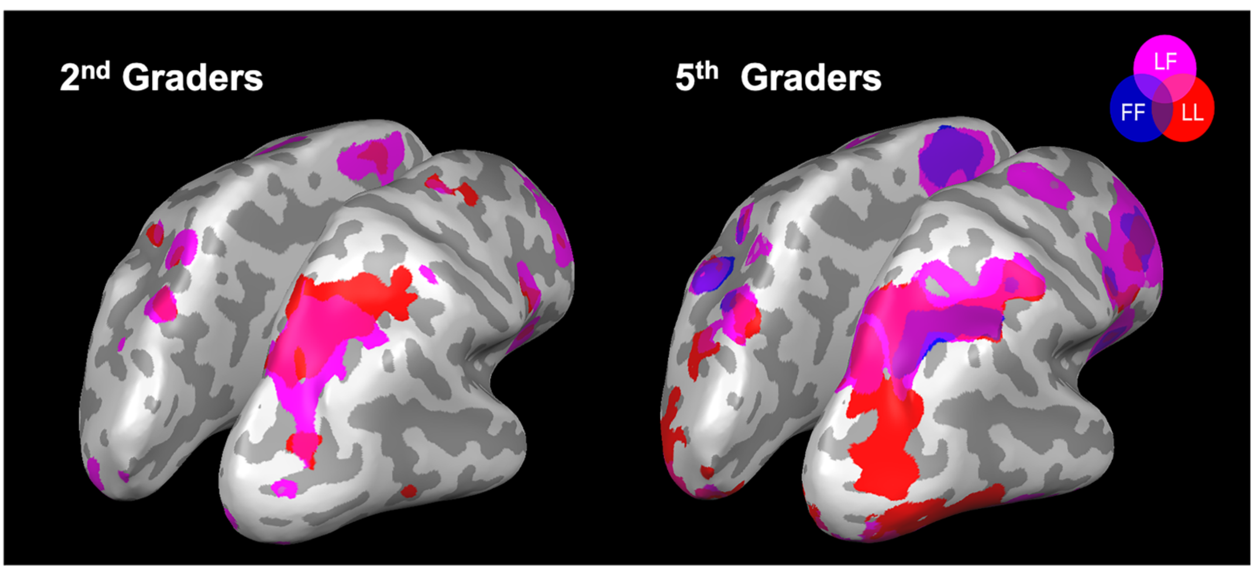

Figure 2.8. Significant neural distance effect in each notation in 2nd graders (left) and 5th grader (right)' brains. The regions colored by red indicate the neural distance effects in nonsymbolic (LL) notation, the regions colored by magenta indicate the neural distance

effects in mixed (LF), and the regions colored by blue indicate the neural distance effects in symbolic (FF) notations. The brains are inflated to allow visualization of activations in the sulci (dark gray) and gyri (light gray).

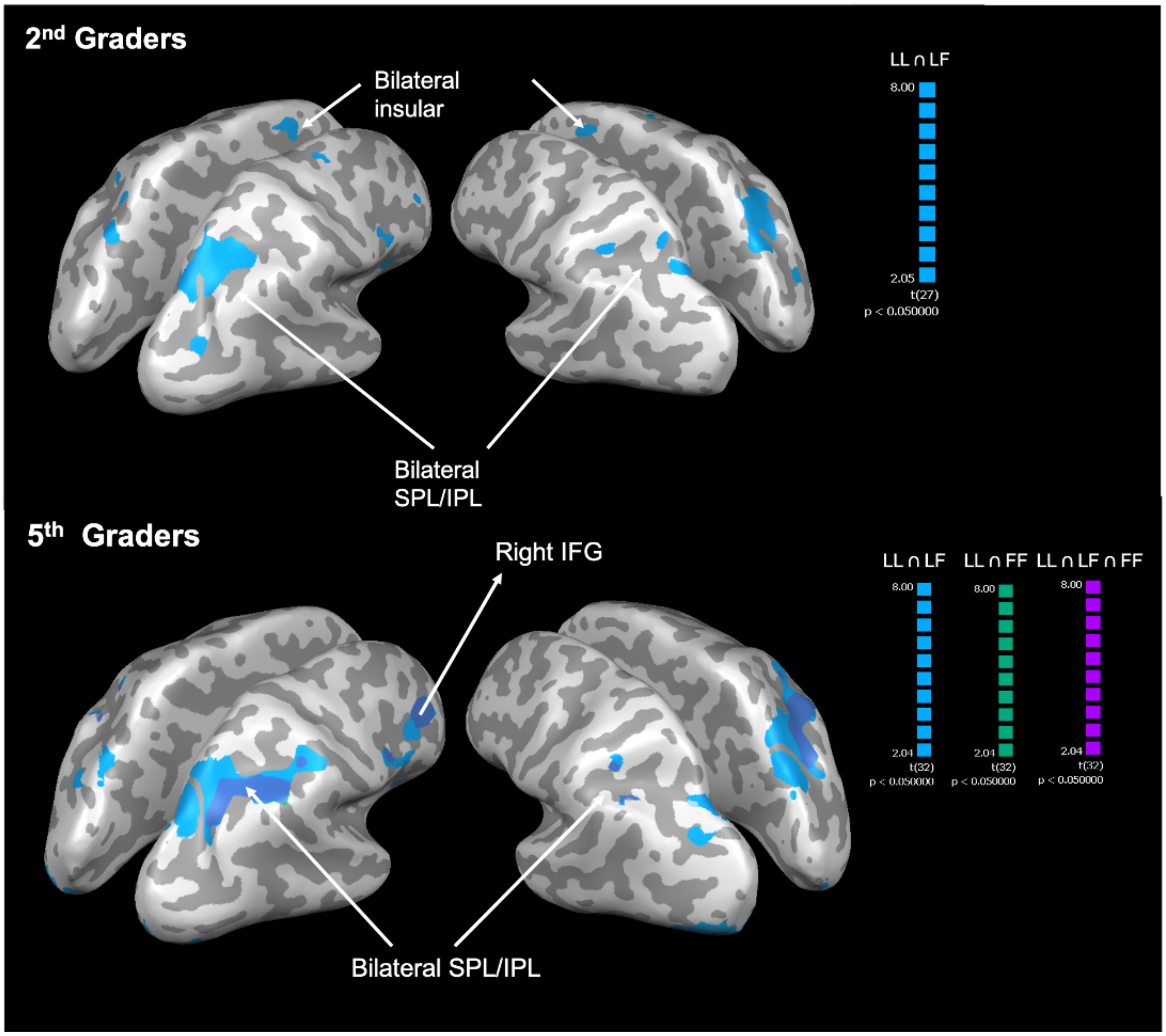

Figure 2.9. Conjunction analysis across the neural distance effect in each notation in 2nd graders (upper) and 5th grader (below)' brains. The regions colored by light blue indicate conjunction of the distance effects across nonsymbolic (LL) and mixed (LF) notations, the regions colored by green indicate conjunction of the distance effects across mixed and symbolic (FF) notations, and the regions colored by purple indicate conjunction

of the distance effects across all three notations. Due to overlaps, green color is mixed with other colors and is indicated with dark blue color.

Conjunction analyses across the neural distance effects in different notation

To examine the regions that overlapped across the neural distance effects from different notations, we performed conjunction analyses. The conjunction analyses on the distance effects between LL and LF in 2nd graders identified bilateral parietal lobules, bilateral insula, right precentral and lingual gyrus, and medial frontal gyrus. In 5th graders, the same conjunction analysis on the distance effects between LL and LF identified similar regions including bilateral parietal lobules, right insula, left supplementary motor area, and a few parts of right frontal gyrus ($p < .05$; see Figure 2.9 and Table 2.10). As an extension, when FF was added to conjunction analysis, we still identified bilateral parietal lobules, right insula, right precentral gyrus and a part of frontal gyrus ($p < .05$; see Table 2.9). These results show overlaps of the distance effects across different notations of ratios similar to the results with adults (Mock et al., 2018; Binzak et al., submitted).

Table 2.7. Distance effect collapsing across all notations

Anatomical Region	TAL coordinates (x,y,z)			Mean t-score	Number of Voxels
2nd Graders: Near > Far					
Right precentral gyrus	44	4	30	4.647	4331
Right medial frontal cortex	35	43	12	3.945	6512
Right superior occipital cortex	23	-59	42	4.570	26044
Right insula	29	16	12	4.075	3517
Right medial frontal cortex	23	58	-15	2.280	334
Right superior frontal gyrus	23	-5	51	2.928	897
Right hippocampus	23	-26	-3	2.738	1903
Left supplementary motor area	-1	7	51	5.026	12596
Left superior parietal gyrus	-25	-65	42	3.824	8194
Left lingual gyrus	-22	-92	-9	3.070	4495
Left parahippocampal gyrus	-19	-35	-3	2.694	341
Left medial frontal cortex	-22	46	-15	4.165	2196
Left superior occipital gyrus	-25	-71	24	2.554	293
Left insula	-31	16	12	5.089	4262
Left precentral gyrus	-40	-2	30	2.985	903
5th Graders: Near > Far					
Right insula	29	16	9	7.641	39312
Right posterior cerebellum	14	-95	-18	4.576	13828
Right inferior parietal cortex	41	-47	48	5.342	26891
Right medial frontal cortex	29	55	-15	5.983	6969
Right superior frontal gyrus	23	-5	51	3.771	2878
Left supplementary motor area	2	16	45	6.946	25151
Left posterior cingulum	-1	-29	21	2.757	663
Left lingual gyrus	-13	-98	-15	5.359	11286
Left inferior parietal cortex	-28	-62	42	4.755	18456
Left medial frontal cortex	-31	59	-15	4.682	2510
Left insula	-34	16	6	6.141	18248
Left inferior frontal gyrus, triangular part	-43	37	9	3.454	2227

Table 2.8. Distance effect in each notation in 2nd graders

Anatomical Region	TAL coordinates (x,y,z)			Mean t-score	Number of Voxels
nonsymbolic notations Near > Far					
Right precentral gyrus	44	4	27	3.568	1871
Left inferior frontal gyrus, triangular part	53	34	15	2.689	796
Right inferior temporal gyrus	44	-53	-9	2.635	606
Right superior/inferior lobule	21	-66	44	4.801	15150
Right fusiform gyrus	29	-74	6	2.467	631
Right insula	29	16	15	3.161	393
Right lingual gyrus	23	-86	-6	2.523	519
Right superior frontal gyrus	23	-5	51	2.686	1025
Right supplementary motor area	5	16	45	3.030	2526
Left medial occipital gyrus	-22	-89	3	2.262	291
Left superior parietal lobule/IPS	-22	-65	42	3.028	2325
Left insula	-34	13	12	3.353	476
Left inferior parietal lobule	-37	-41	39	3.161	1516
cross-notations: Near > Far					
Right precentral gyrus	38	1	30	3.141	3073

Right middle frontal gyrus	38	43	6	3.657	14562
Right superior parietal lobule	44	-41	55	2.471	267
Right inferior parietal lobule/superior occipital lobule	27	-61	37	2.675	11884
Right medial frontal cortex	17	46	-18	2.833	329
Right cerebellum	32	-53	-33	2.604	434
Right lingual gyrus	20	-93	-12	3.497	2774
Right precentral gyrus	23	-8	51	2.437	434
Right thalamus	8	-14	9	2.856	1787
Left supplementary motor area	-1	22	42	3.342	11301
Left cerebellum	-7	-26	-37	2.953	1366
Left thalamus	-13	-20	0	2.530	547
Left superior parietal lobule	-25	-65	42	3.747	6846
Left lingual gyrus	-22	-92	-9	3.220	2582
Left medial frontal cortex	-21	46	-18	3.886	1883
Left insula	-28	22	3	3.662	4839
Left precentral gyrus	-46	1	33	2.902	2744
Left inferior parietal lobule	-56	-38	51	2.671	1466
Left inferior frontal gyrus, triangular part	-40	37	6	2.562	420
Left Inferior frontal gyrus, opercular part	-64	13	10	3.026	479

Table 2.9. Distance effect in each notation in 5th graders

Anatomical Region	TAL coordinates (x,y,z)	Mean t-score	Number of Voxels	Anatomical Region	TAL coordinates (x,y,z)
nonsymbolic notations Near > Far					
Right inferior frontal gyrus, triangular part	50	28	30	4.784	12229
Right superior/inferior parietal lobule	14	-77	48	5.340	85575
Right medial frontal cortex	23	52	-12	3.785	3186
Right insula	32	19	9	2.489	415
Left supplementary motor area	-1	22	48	2.481	340
Left medial frontal cortex	-22	49	-19	2.871	1326
Left superior/inferior parietal lobule	-46	-44	57	2.658	1646
cross notations Near > Far					
Left supplementary motor area	2	19	45	6.694	93351
Right superior/inferior parietal lobule	25	-66	49	3.753	27866
Right medial frontal cortex	17	40	-18	3.455	910
Left lingual gyrus	-13	-98	-12	3.645	6130
Left caudate nucleus	-13	4	9	3.126	718
Left superior/inferior parietal lobule	-31	-65	42	4.507	18699
Left superior frontal cortex	-25	-5	54	2.657	660
Left precentral gyrus	-46	1	33	5.550	32010
symbolic notations Near > Far					
Left inferior frontal gyrus, triangular part	35	25	27	3.426	7736
Right inferior parietal lobule/superior occipital lobule	30	-60	37	3.388	7024
Right insula	29	19	9	2.903	1774
Right medial frontal cortex	23	49	-15	3.214	2806
Left supplementary motor area	-4	4	51	5.403	9646

Left medial frontal cortex	-25	37	-22	3.837	925
Left angular gyrus/inferior parietal lobule	-34	-47	33	3.476	6736
Left insula	-31	13	15	3.630	2297
Left precentral gyrus	-46	1	33	4.068	5380

Table 2.10. Conjunction analyses across distance effect for different notations

Anatomical Region	TAL coordinates (x,y,z)			Mean t-score	Number of Voxels
2nd Grader: (Near > Far in LL) ∩ (Near > Far in LF)					
Right medial frontal gyrus	47	31	24	2.422	386
Right precentral gyrus	41	1	30	2.958	1038
Right superior/inferior parietal lobule	23	-61	42	2.732	6343
Right insula	29	16	15	2.924	374
Right medial occipital gyrus	29	-74	12	2.249	261
Right lingual gyrus	23	-86	-6	2.523	471
Left supplementary motor area	-4	13	48	2.837	2427
Left superior parietal lobule	-22	-65	42	3.028	2185
Left insula	-31	16	12	2.749	211
Left inferior parietal lobule	-37	-44	36	2.496	576
5th Grader: (Near > Far in LL) ∩ (Near > Far in LF)					
Right inferior frontal gyrus, triangular part	44	25	30	4.563	10506
Right superior/inferior parietal lobule/IPS	17	-74	45	4.736	20002
Right insula	32	19	9	2.489	415
Left supplementary motor area	-1	22	48	2.481	340
Left superior parietal lobule/IPS	-25	-71	48	3.274	4681
Left inferior parietal lobule	-46	-44	57	2.658	1581
5th Grader (Near > Far in LL) ∩ (Near > Far in LF) ∩ (Near > Far in FF)					
Right inferior frontal gyrus, triangular part	44	28	30	3.392	3397
Right precentral gyrus	41	-2	30	2.852	692
Right superior/inferior parietal lobule	32	-58	44	2.177	5316
Right insula	32	19	9	2.489	378
Left supplementary motor area	-1	22	48	2.481	272
Left superior parietal lobule/IPS	-30	-69	42	2.039	346
Left inferior parietal lobule/IPS	-37	-47	36	2.598	831

The difference in the neural distance effects between 2nd vs. 5th graders.

To examine the regions showing a difference between 2nd and 5th graders in terms of the neural distance effects, we conducted a two-way random effects ANOVA (1 within: near vs. far distances and 1 between: 2nd vs. 5th) on the whole brain. Next, we extracted the average beta values for each distance bin individually from each significant cluster. By doing so, we identified the clusters where 5th graders showed larger distance effects than 2nd graders and vice versa. 5th graders showed larger distance effects in bilateral frontal cortex including right IFG, right inferior parietal lobule, and left precentral gyrus (Table 2.11). The regions where 2nd graders showed larger distance effects included clusters in right rolandic operculum, left fusiform gyrus and left inferior frontal gyrus (Table 2.10). When we unpack the results by notation, 5th graders showed larger distance effects in the symbolic notations, especially in the right inferior parietal lobule and left precentral gyrus, whereas the interaction found in the frontal cortex was mostly from the nonsymbolic and cross notations (Table 2.10). Also, the frontal area where 2nd graders showed larger distance effect was mainly from symbolic fraction notations (Table 2.11).

Table 2.11. The regions showing significant interactions between 2nd and 5th graders distance effects.

Notation	Anatomical Region	x	y	z	Mean t-score	Number of Voxels
	5th > 2nd distance effects					
collapsing across all notations	Right Inferior frontal gyrus, opercular part	44	16	33	12.46	3821
	Right inferior parietal lobule	41	-44	45	8.18	1046
	Right superior frontal gyrus	14	46	-12	5.44	97
	left anterior cingulum	-4	16	21	6.15	157
	left mid frontal gyrus	-50	46	3	9.06	3770
	left precentral gyrus	-55	2	42	7.83	2636
symbolic fractions (FF)	Right precentral gyrus	54	13	39	7.36	319
	Right inferior parietal cortex	43	-47	45	5.43	385
	Right superior occipital cortex	29	-62	33	5.58	222
	Right mid frontal gyrus	26	37	-24	7.01	293
	Left supplementary motor cortex	-7	1	50	6.58	308
	Left mid frontal gyrus	-25	37	-22	8.34	2537
	Left anterior cingulum	-10	1	27	5.22	264
	Left pallidum	-18	-2	6	5.31	95
	Left superior frontal cortex	-22	25	-9	6.10	578
	Left angular gyrus	-34	-47	33	7.19	627
	Left mid occipital gyrus	-34	-69	36	5.21	247
	Left precentral gyrus	-49	1	33	8.95	3054
cross notations (LF)	Right inferior frontal gyrus, triangular part	38	19	27	4.83	98
	Right superior occipital gyrus	14	-89	39	6.38	128
	Right mid cingulum	2	4	39	9.15	811
	Left anterior cingulum	-1	16	21	9.23	447
	Left superior frontal cortex	-22	49	-12	7.83	724
Nonsymbolic notation (LL)	Right inferior frontal gyrus, triangular part	32	16	27	7.79	1144
	Left lingual gyrus	-24	-92	-9	5.17	305
	Left mid frontal cortex	-31	31	-21	6.72	110
	Left inferior frontal gyrus, triangular part	-50	43	3	10.46	5027
	2nd > 5th distance effects					
collapsing across all notations	Right ronlandic operculum	41	-17	21	6.71	435
	Left fusiform gyrus	-22	-68	-6	5.65	218
	Left inferior frontal gyrus	-64	14	9	8.51	1247
symbolic fractions (FF)	right inferior frontal cortex	32	28	-21	7.33	534
	Left inferior temporal cortex	-61	-65	-15	12.75	847
Nonsymbolic notation (LL)	Left insula	-37	1	12	9.32	1045

ROI analysis.

To examine whether our a priori region of interest – the IPS – is also sensitive to the ratio and fraction magnitudes, we tested the neural distance effects in the IPS coordinates from Houde et al. (2010) sing three distance bins in each notation. We first conducted a linear mixed-effect regression examining fixed effects by regressing extracted mean beta values against grade (2nd = 0, 5th = 1), hemisphere (left = 0, right = 1), the distance bins (far = 0, medium = 1, near = 2) and the notations (Nonsymbolic-LL=0, Mixed-LF =1, Symbolic-FF=2). Based on the performance, we had hypotheses regarding the change of brain engagement between levels (i.e., 2nd graders > 5th graders, FF > LF > LL, and near > med > far). To facilitate analysis, we used a backward difference coding scheme so that we could compare adjacent levels of variables. The fixed effects results showed that the beta values increased as the distance bins becomes nearer, especially between far and medium (p<.001; Table 2.12). We also found that engagement of the IPS increased with notation in the order predicted although LL < LF shows marginal significance (LL < LF; p = .055, LF < FF; p = .014). However, there were no significant differences due to hemisphere or to grade level

To better understand the results, we tested the main effect of distance in the IPS at each notation, hemisphere, and grade. We used the 'anova' function from the 'car' package in R that performs the Wald chisquare test which shows the main effect across different levels of distance (near, med, and far) (see Figure 2.10). We found that in 2nd graders, only the right IPS showed significant distance effects in LL (χ^2 =7.27, p=.026) and LF (χ^2 =7.27, p=.026) notations. In 5th graders, with the exception of the left IPS, which did not show significant distance effects in LL notation (χ^2 =2.88, p=.236), bilateral IPS showed significant distance effects for all notations (left: χ^2 =8.21, p=.017 for LF, χ^2 =8.20, p=.017 for LF; right: χ^2 =6.91, p=.031 for LL, χ^2 =17.80, p<.001 for LF, χ^2 =14.31, p<.001 for FF).

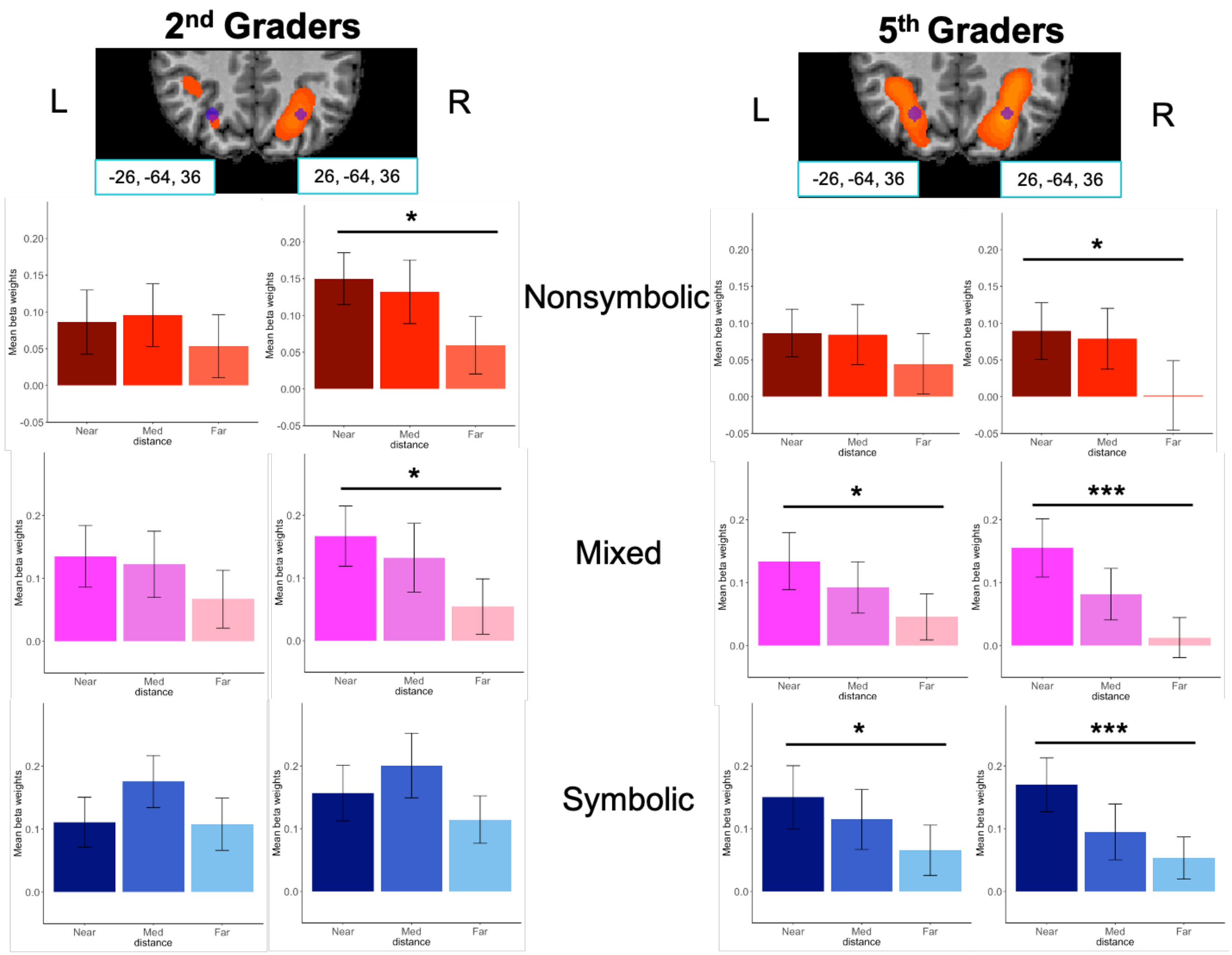

Figure 2.10. Mean beta values from the left/right IPS of 2nd (left panel) and 5th (right panel) graders. *p<.05, **p<.001. Nonsymbolic (LL), mixed (LF), and symbolic (FF) notations were presented.

Table 2.12. Fixed effects that regressed beta values against grade, hemisphere, notations, and distance bins.

	β	*t*	*p*
Intercept	.117	3.300	.002**
Distance: medium-far	.060	5.700	<.001***
Distance: near-medium	.017	1.660	.097
Notation: LL -LF	.020	1.921	.055+
Notation: LF -FF	.026	2.450	.014*
Hemisphere: Right-Left	.006	.325	.746
Grade: 5th – 2nd graders	-.036	-.785	.436

Discussion

With a growing interest in fractions learning, we aimed to understand developmental changes in the underlying neural mechanism for fraction processing. Specifically, the present study put the RPS hypothesis to the test to unravel the possible use of primitive cognitive architecture for supporting acquisition of symbolic fractions competence. By taking cross-sectional approach, we compared the neural activations between the children who have not yet received formal fractions instruction (2nd graders) and children who have received a few years of fractions instruction (5th graders). The current study demonstrated the similarities and differences in both behavioral and neural activity when processing ratio magnitudes between 2nd and 5th graders.

Behaviorally, as expected, 5th graders were more accurate and faster than 2nd graders in all notations as found in the previous work using an identical task in a purely behavioral experiment (Kalra, Binzak, et al., 2020). However, we found similar behavioral trends between 2nd and 5th graders. Both groups of children were capable of comparing rational number magnitudes accurately and rapidly with nonsymbolic ratios, symbolic fractions and even cross-notation representations. Consistent with the previous reports with children and adults, our results also showed the behavioral distance effects in all notations in both 2nd and 5th graders. (Kalra, Binzak, et al., 2020; Matthews & Chesney, 2015), even after controlling for children's use of a gap strategy. These results indicate that children can process ratios and fractions as holistic magnitudes.

Additionally, we found significant differences across different notations. Within-notation comparison with nonsymbolic ratio magnitudes was the most accurate and the fastest, and the comparison with symbolic fractions was the least accurate and the slowest for both groups. The fact that nonsymbolic ratio comparison was the easiest is well-aligned with the RPS theory implicating nonsymbolic ratio processing as a cognitive primitive. In addition to this, the findings showing the comparison of mixed notation ratios is easier than that of symbolic fractions substantiate the suggestion of co-processing across nonsymbolic ratios and symbolic fractions (Kalra, Binzak, et al., 2020; Matthews & Chesney, 2015)

Consistent with the behavioral results, the whole-brain analysis contrasting near and far distances in 2nd and 5th graders showed similarities and differences. As we hypothesized, both groups showed significant distance effects in a bilateral frontal-parietal network including the IPS, the medial frontal gyrus, the left precentral gyrus and the bilateral insula. The frontal-parietal network including the IPS and medial frontal gyrus were similar to previous studies with nonsymbolic and symbolic fraction processing and number processing (e.g., Houdé et al., 2010; Ischebeck et al., 2009; Jacob & Nieder, 2009b, 2009a; Mock et al., 2018; Wortha et al., 2020). The precentral gyrus and insula were also frequently reported to be related magnitude processing generally (e.g., Ansari, Garcia, Lucas, Hamon, & Dhitalm, 2005; Mock et al., 2018; Pinel, Dehaene, Rivière, & LeBihan, 2001; Rosenberg-Lee, Barth, & Menon, 2011). These results support the RPS hypothesis that children could process nonsymbolic ratio and symbolic fractions as holistic magnitude.

On the other hand, neural results between 2nd and 5th graders were different in two aspects. First, broader regions of parietal-prefrontal areas showed neural distance effects in 5th graders compared to 2nd graders. Whereas the neural distance effect in 2nd graders was found more right lateralized fronto-parietal lobule, the neural distance effect in 5th graders was more balanced bilaterally. Secondly, the patterns of the neural distance effect by each notation were different for 2nd and 5th graders. The neural distance effect in 2nd graders was mainly due to nonsymbolic ratio and cross-notation comparison, but the distance effect in 5th graders were observed in all notations. Given that 2nd graders have not received formal fraction instructions, it is reasonable that 2nd graders showed no effect on symbolic fractions. Critically, these results are consistent with the RPS theory predicting the neurocognitive architecture for nonsymbolic ratio likely exists prior to exposure to fractions instruction.

However, it should still be noted that 2nd graders were able to complete the task accurately and rapidly for all notations. It should be considered that we observed the small neural distance effect for symbolic fraction comparison in the uncorrected data in the IFG, where relates to orientation toward stimuli, inhibition, and other higher order cognition (for review see Aron, Robbins, & Poldrack, 2004; for meta-nalaysis see Levy & Wagner, 2011). Furthermore, the between group ANOVA test revealed that 2nd grader showed a greater distance effect in the right IFG for processing symbolic fractions. Even though we did not find any clusters that remained statistically significant after correction for multiple comparisons in 2nd graders, the aforementioned findings indicate the possibility of neural engagement of the frontal gyrus. If 2nd graders recruited the frontal gyrus for symbolic fractions, the results would have been well aligned with the previous work investigating symbolic number processing with children and adults (Ansari & Dhital, 2006; Ansari et al., 2005; Cantlon et al., 2009; Rivera, Reiss, Eckert, & Menon, 2005). In their result, children showed greater BOLD response in the IFG, while adults showed greater BOLD response in the bilateral parietal cortex. As the previous work proposed for symbolic number learning, the functional neuroanatomy underlying symbolic fractions processing undergoes an ontogenetic shift toward greater parietal engagement starting from frontal lobe and less reliance to frontal areas.

Compared to 2nd graders, 5th graders' neural distance effect for each notation overlapped in right prefrontal and bilateral parietal lobule including bilateral IPS. These results from 5th graders are in line with the previous reports on adults' nonsymbolic ratio processing (Jacob & Nieder, 2009b, 2009a; Mock et al., 2018), and symbolic fraction processing (Cui et al., 2020; Ischebeck et al., 2009; Mock et al., 2018; Wortha et al., 2020). Moreover, adding to these observable differences, statistical differences between 2nd and 5th graders exhibited interesting developmental trends. For symbolic fraction processing, older children presented increased BOLD signal in the right parietal lobule and left precentral gyrus, regions critical for magnitude processing (for meta-analysis, Sokolowski, Fias, Bosah Ononye, & Ansari, 2017). In turn, it demonstrates older children can process symbolic fractions as magnitude. Additionally, older children exhibited a greater engagement in the frontal gyrus for nonsymbolic and cross notations, which indicates a more adult-like fronto-parietal network for ratio magnitudes (Cui, Li, Li, Siegler, & Zhou, 2020; Ischebec k et al., 2009; Mock et al., 2018; Wortha et al., 2020). Taken together, our results demonstrate that the neurocognitive architecture for ratio magnitude becomes more adult-like as children progress through the primary school years followed by years of exposure to symbolic fractions.

Subsequent region of interest (ROI) analysis further revealed the differences in neural activations in response to ratio tasks between 2nd and 5th graders. A significant distance effect in the bilateral IPS was observed in 5th graders (except for left IPS for nonsymbolic ratios), but in 2nd graders, only right IPS was found to be engaged for nonsymbolic and cross-notation ratio comparisons. Thus, the ROI analysis indicates that the right IPS is more sensitive to ratios compared to the left IPS. These differential activations between hemispheres might imply a right-to-bilateral developmental model that has previously been suggested by Ansari (Ansari, 2016; Ansari & Dhital, 2006; Ansari et al., 2005; Holloway & Ansari, 2010). Prior to fractions instruction, the neurocognitive architecture for ratio processing may be tuned only to nonsymbolic ratio, and it may initially localize to the right hemisphere, especially the right IPS. However, years of instruction on symbolic fractions may help develop the RPS sensitivity toward symbolic fractions as well. That is, consistent practice on processing ratio magnitudes may lead to a developmental shift whereby the IPS plays an increasingly important role in ratio processing.

In conclusion, the present study offers the first evidence demonstrating the existence of primitive cognitive architecture for ratio processing in young children at neural level. Furthermore, by taking cross-sectional approach, our study suggests the developmental shift in ratio processing during the early stages of fraction instructions. Future study should work on how this ratio processing ability can support fraction acquisition with longitudinal or training designs.

Chapter 3: Developmental Changes in the Relationships between White Matter Integrity and Nonsymbolic and Symbolic Fractions Processing

Introduction

Chapter 2 showed how functional specialization for symbolic fractions changes during early years of fractions instruction and substantiated the possibility of grounding symbolic fractions in the RPS. However, this developmental change is not simply a matter of functional recruitment of the brain. The functional engagement can also be depend on changes in underlying white matter microstructure that connects different cortical regions (Damoiseaux & Greicius, 2009; Johansen-Berg & Rushworth, 2009; Zimmermann et al., 2018). Thus, it is possible that white matter microstructure may also relate to fraction processing. In turn, if regions of the fronto-parietal network are engaged for processing fraction magnitude information, it is likely that the degree of structural connectivity linking these different regions of the brain are also involved in fraction processing. Moreover, studying its early development may contribute to understanding the development of neural circuits for fraction processing.

The same logic has already prompted investigations into the relations between whole number competence and white matter microstructures. By employing a diffusion tensor imaging (DTI) (for review, Matejko & Ansari, 2015) that measures diffusion of water in the white matter (Alexander, Lee, Lazar, & Field, 2007; Beaulieu, 2002), the studies have measured fractional anisotropy (FA), a measure of white matter microstructure derived from DTI. The previous studies have found that FA values were related to various numerical skills in different age ranges. Individual differences in FA were associated with nonsymbolic and symbolic number processing in 6-year-old (Cantlon et al., 2011), whole number operations in ages 7-15 years (Jolles et al., 2016; Tsang, Dougherty, Deutsch, Wandell, & Ben-Shachar, 2009; Van Beek, Ghesquière, Lagae, & De Smedt, 2014; van Eimeren et al., 2010, 2008), and even advanced math achievement as Preliminary Scholastic Aptitude Test (PSAT) performance in 17-18 years-old (Matejko et al., 2013).

These previous studies reported the involvement of various tracts, but consistently highlighted frontoparietal and temporal white matter tracts as being associated with nonsymbolic and symbolic number competence, compatible with functional studies (e.g., Dehaene et al., 2004; Piazza et al., 2004; Peters & De Smedt, 2017 for review). The reported tracts included the corpus callosum (CC) (Cantlon et al., 2011; Hu et al., 2011; Till et al., 2011), the inferior fronto-occipital fasciculus (IFOF) (Li, Wang, Hu, Liang, & Chen, 2013; Rykhlevskaia et al., 2009), the superior longitudinal fasciculus (SLF) (Kucian et al., 2014; Rykhlevskaia et al., 2009; van Eimeren et al., 2010), and the inferior longitudinal fasciculus (ILF) (Li et al., 2013; Tsang et al., 2009; van Eimeren et al., 2008).

Similar to the case with whole numbers, it is possible that the white matter tracts underneath parietal and frontal lobe are also related to fractions processing. Considering the similarities in functional engagement between whole number and fraction processing, the regions in the SLF, the ILF, the SCR, or the IFOF may relate to the performance of the fraction/ratio comparison tasks. Furthermore, the relation between white matter and fraction processing may change over the course of development similar to the case with cortical functioning as revealed in Chapter 2. Yet, little is known about the relationship between the microstructure and ratio processing ability as investigations into the neural underpinnings of fractions processing have only recently emerged.

Therefore, the present study of Chapter 3 aimed to investigate white matter microstructure and its relation to fractions processing in children with a cross-sequential approach. Furthermore, to capture developmental changes in the relationship, we investigated the same cohorts of children as in Chapter 2; 2nd graders with little symbolic fractions knowledge and 5th graders who have learned symbolic fractions in school. Chapter 3's study was divided into Experiment 1 and 2. In Experiment 1, we conducted a whole brain analysis using tract-based spatial statistics (TBSS) to explore regions correlated with fractions processing ability. As an extension of the cross-sectional approach, Experiment 2 additionally analyzed the data one year after the first scan, when 2nd and 5th graders in Experiment 1 became 3rd and 6th graders, and looked at the longitudinal changes between grades. In Experiment 2, since we narrowed down the regions of interest in Experiment 1, we employed a region of interest (ROI) analysis to investigate each region's development separately. Thus, we explored differential development of each tract and how its change related to the development of ratio and fraction processing ability.

Experiment 1

Specific Introduction

Unlike fraction processing, the relation between white matter microstructure and whole number processing ability has been extensively studied. However, even in the realm of whole number processing, few studies have investigated developmental differences. Only Cantlon et al. (2011) compared children's and adults' structural differences and reported different individual differences in FA in the CC. In their results, while children's FA from left isthmus of the CC was correlated with number processing ability, adult's FA from the CC did not show any correlation with their abilities.

Furthermore, even though FA is the most representative diffusion parameter measuring microstructure, but it is also important to look at other features of microstructure such as the degree of myelination or axonal coherence to better grasp the results with FA. Therefore, other diffusion parameters should be looked into, including 1) mean diffusivity (MD), an average of water diffusion, which indicates the microstructure properties, such as tissue organization (e.g., closely arrayed cellular structures in tissues assumed to have lower MD) (Beaulieu, 2002; Takeuchi et al., 2015; Winklewski et al., 2018), 2) radial diffusivity (RD), a magnitude of water diffusion that is perpendicular to axons which indicates the degree of myelination (e.g., demyelinated white matter regions assumed to have lower RD), and 3) axial diffusivity (AD), a magnitude of water diffusion that is parallel to axons which indicates the degree of axonal propagation (e.g., higher axonal integrity assumed to have higher AD) (Beaulieu, 2002; Song et al., 2002, 2005; Winklewski et al., 2018).

Especially, given that each aspect of white matter microstructures differentially develops (Tamnes et al., 2010), other diffusion parameters should be investigated to better understand developmental differences. To date, few studies have looked at which white matter structural characteristic drives the relation by using DTI parameters beyond FA (e.g., Hu et al., 2011; Kucian et al., 2014; Matejko et al., 2013). Matejko et al. (2013) investigated whether white matter microstructure was correlated with the math subtest in preliminary scholastic aptitude test (PSAT) in adults through a tract based spatial statistics (TBSS) approach (Matejko, Price, Mazzocco, & Ansari, 2013). They found that FA and RD were correlated with the PSAT score in the intersection of the left superior corona radiata (SCR), the corticospinal tract (CST), and the SLF underneath left parietal lobule. These observations of overlaps between FA and RD enables researchers to suggest that the degree of myelination of the left parietal cortex contributes individual differences in connectivity and it may relate to advanced math performance.

In the present study, we investigated children's white matter microstructure in relation to their nonsymbolic and symbolic fraction processing. Unlike whole number instruction, which begins from preschool period, formal fractions instruction starts in 2nd grade. Thus, we aimed to investigate two cohorts of children, 2nd graders who have not received formal fraction instructions and 5th graders who have received a few years of fraction instructions, so that we could look into a developmental difference in the relation between microstructure and nonsymbolic and symbolic fraction competence. Considering the fact that the ability to process nonsymbolic ratios emerges earlier than symbolic fractions knowledge (Y. Park et al., 2020), the white matter microstructure may suggest a developmental change in the relation with nonsymbolic and symbolic fractions processing. Based on the findings of Chapter 2 with fMRI, we expect that 2nd graders' white matter connectivity may relate only to nonsymbolic ratio processing abilities while 5th graders' white matter connectivity may relate to both nonsymbolic ratio and symbolic fractions.

We utilized a whole brain approach, TBSS, to explore the association between white matter microstructure and nonsymbolic ratio and symbolic fractions processing ability. In addition to FA, we investigated DTI parameters of AD, RD, and MD to better understand observed associations with the white matter microstructure. Ratio processing ability was measured using nonsymbolic ratio and symbolic fraction comparison tasks conducted during our functional MRI scan. Since this was a computer-based task, children's abilities to identify and process information efficiently may influence their task performance. Thus, we additionally measure children's processing speed as controls.

Methods and Measures

Participants

The same participants from Chapter 2 also took part in the diffusion imaging. Forty-seven 2nd graders and forty-five 5th graders were recruited from several public schools in Madison, WI. All participants were right-handed native English-speakers with normal vision. Parents or guardians gave written consent, and children gave verbal assent. All protocols were approved by the research ethics committee of the University. Participants received monetary compensation and small gifts for their participation. Among the participants, data acquisition for one 2nd grader and one 5th grader was stopped due to excessive movements, 3 children were excluded due to movement artifacts in diffusion weighted images (DWI), and 1 child was excluded due to an ADHD diagnosis. Overall, 44 2nd graders and 42 5th graders were included in the final analysis. Two 2nd graders showing below chance performance in the behavioral task in the scanner were excluded from the correlation analyses.

The multiple notation ratio comparison task

We used the same data collected from the ratio comparison task in Chapter 2. Mean accuracy (ACC) and reaction times (RT) of each notation were used as outcome variables.

Processing speed measure

We used the pair cancellation subtest from the WISC-IV to measure children's global processing speed (Kaufman, Flanagan, Alfonso, & Mascolo, 2006; Vaughn-Blount et al., 2011). This test is a paper and pencil test. Children were given a single standardized response sheet with a sequence of pictures (a mix of the ball, the dog, or a cup pictures in a row) and were required to circle as many target pairs (a dog followed by a ball) as they could in three minutes. Each item pair was scored as 1 point if the child circled both items when they appeared in the correct order (dog-ball, but not ball-dog). The total score was 69 (item pairs).

Diffusion MRI acquisition and analysis

Participants were scanned on a 3T GE Discovery MR750 MRI scanner using a 32-channel array head coil (Nova Medical). Foam padding was placed around the participants' heads to prevent head motion. Diffusion weighted images (DWIs) were acquired as a part of a larger, comprehensive scan protocol with the following scan acquisition parameters: TR= 6000ms, TE = 68.8ms, multiple b-values (b = 0 s/mm^2 with 6 directions, 500 s/mm^2 with 24 directions, 1500 s/mm^2 with 24 directions), and 2.5mm isotropic voxel resolution. Additional b = 0 s/mm^2 images were acquired with reverse phase encoding directions. The total acquisition time was 8 min.

All DWIs images were manually inspected for motion artifacts, and images determined to contain artifacts were removed prior to processing. DWI processing was performed using an in-house processing pipeline that utilizes the DIPY toolkit (Garyfallidis et al., 2014), MRtrix (Tournier, Calamante, & Connelly, 2012), FMRIB Software Library (FSL) (Jenkinson, Beckmann, Behrens, Woolrich, & Smith, 2012), and Analysis of Functional

Neuroimages (AFNI) (Cox, 1996) software packages. Following the manual inspection, images were corrected for Rician noise (Veraart et al., 2016) and Gibbs ringing (Kellner, Dhital, Kiselev, & Reisert, 2016). Artifacts from field inhomogeneities and eddy currents were corrected using TOPUP and Eddy tools from FSL (Andersson & Sotiropoulos, 2016; Smith et al., 2004). Non-parenchyma signals were removed using the 3dSkullStrip tool from AFNI, and diffusion tensors were estimated using the robust estimation of tensors by outlier rejection (RESTORE) (Chang, Jones, & Pierpaoli, 2005) algorithm implemented in the DIPY toolkit (Garyfallidis et al., 2014). Maps of fractional anisotropy (FA) were subsequently constructed from the diffusion tensors (Basser & Pierpaoli, 1996). A population-specific FA template was generated separately for 2nd and 5th graders using the buildtemplateparallel.sh from Advanced Normalization Tools (ANTs) (Avants et al., 2014), which utilizes iterative diffeomorphic registration methods. A single, overall template was then constructed from the 2nd and 5th grade templates. Each individual's FA map was then nonlinearly aligned to the final overall population template using ANTs. Along with FA, the computed ANTs transformations were applied to the axial diffusivity (AD), radial diffusivity (RD), and mean diffusivity (MD) maps to align these complementary measures to the template.

Tract based spatial statistics (TBSS) was employed for statistical analysis (Smith et al., 2006). An FA skeleton was first created from the mean FA map of all participants, and the FA of each subject was projected onto this skeleton. This FA skeleton map was reused for testing MD, AD, and RD maps. General linear models were constructed to examine the differences between 2nd and 5th graders and the correlations between ratio processing ability and DTI parameters in each grade. The model tested each voxel within the FA skeleton using nonparametric permutation testing (FSL randomise tool; (Winkler, Ridgway, Webster, Smith, & Nichols, 2014) and 5000 permutations. Age in months, gender of participants and a total motion index calculated from the eddy-current correction were controlled for in the analyses. Results were corrected for multiple comparisons using Threshold-Free Cluster Enhancement (TFCE) (Smith & Nichols, 2009), which is similar to cluster-extent thresholding, but does not require an initial arbitrary setting for cluster-forming threshold. All correlation results are reported at alpha levels of $p < .05$, TFCE-corrected. Regions found to be significant were identified using the JHU ICBM-DTI White Matter Atlas and JHU White Matter Tractography Atlas.

Results

Behavioral differences between 2nd and 5th graders

Children in both grades were capable of discriminating ratio magnitude accurately (2nd: M_{acc} = .88, SD =.33; 5th: M_{acc} = .92, SD =.27) and rapidly (2nd: M_{rt} = 1642ms, SD = 655; 5th: M_{rt} = 1480ms, SD =629). We conducted mixed effects logistic regression and mixed effects linear regression models for the accuracies and reaction times (RTs) analysis to account for within-subject correlation among trials using the 'lmer' function of the lme4 package in R software (Bates et al., 2015). To analyze accuracies and RTs, we regressed each dependent variable against notations (3 levels, FF = 0, LF = 1, LL = 2), age groups (2 levels, 2nd graders = 0, 5th graders = 1), and their interactions. We used a backward difference coding scheme so that we could compare adjacent levels of variables (i.e., each level minus prior level). We found that 5th graders offered significantly more accurate (β = .502, $p < .001$) and rapid (β = -171.54, p = .003) responses than did 2nd graders (Table 3.1

and 3.2; Figure 3.1). Furthermore, we also found significant notation effects. Children reponded more accurately (β = .788, p <. 001) and more rapidly (β = -250.8, p < .001) in nonsymbolic trials than on mixed trials, and more accurately (β = .143, p = .013) and more rapidly (β = -238.88, p < .001) on mixed trials than on trials involving only symbolic fractions. Significant interactions were found between notations and grades with RT; 2nd and 5th graders showed larger differences in symbolic fraction trials than mixed trials (β = 43.6, p = .046), while the differences between graders were larger in mixed trials compared to nonsymbolic trials (β = 56.01, p = .009).

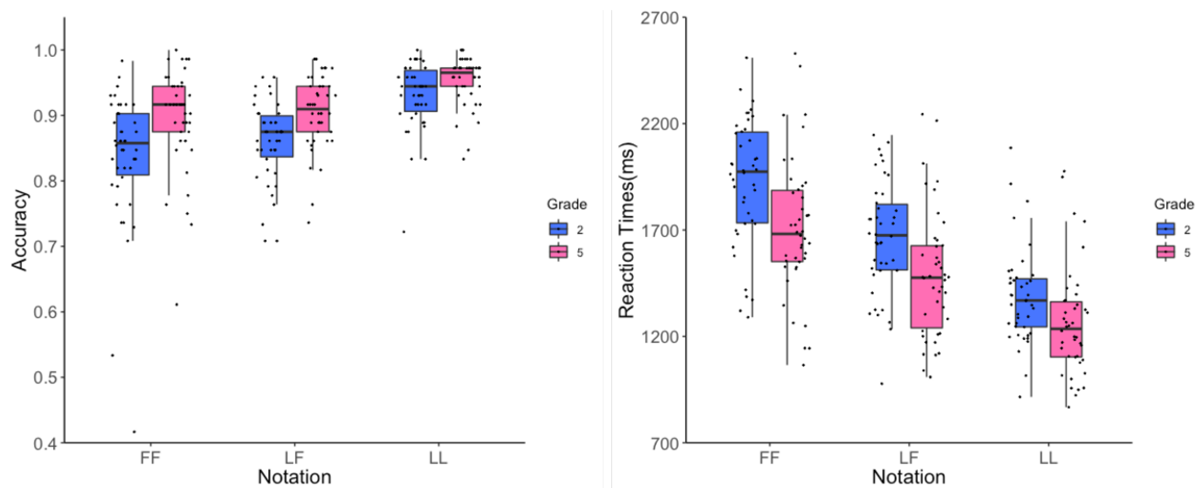

Figure 3.1. Box plots with individual values for mean accuracy (left) and for mean reaction times (right) for the comparison task in each grade.

Table 3.1. Descriptive statistics

notation\grade	2nd graders				5th graders			
	accuracy		RT		accuracy		RT	
	m	sd	m	sd	m	sd	m	sd
FF	.84	.37	1921	673	.90	.30	1701	652
LF	.86	.34	1656	633	.91	.29	1486	622
LL	.93	.25	1379	544	.95	.21	1266	535

Table 3.2. Linear mixed effects, regressing ACC and *RT* against notation and grade.

	ACC			*RT*		
	β	*Odds Ratio*	*p*	β	*t*	*p*
Intercept	2.348	10.465	<.001***	1571.640	56.345	<.001***
FL - FF	.143	1.154	.013*	-238.880	-21.842	<.001***
LL -FL	.788	2.199	<.001***	-250.800	-23.417	<.001***
5th graders-2nd graders	.502	1.651	<.001***	-171.540	-3.075	.003**
|FL – FF|*|5th graders-2nd graders|	-.091	0.913	.426	43.600	1.993	.046*
|LL – FL|*|5th graders-2nd graders|	.012	1.012	.932	56.010	2.615	.009**

Note, p<.05, p<.01**, p<.001****

Developmental differences between 2nd vs. 5th graders with FA

To see developmental difference in white matter connectivity, we first tested the group difference in FA between 2nd and 5th graders. In this analysis, 5th graders showed significantly higher FA compared to 2nd graders in broad white matter regions when controlling for gender and head motions ($p < .01$, TFCE corrected; Figure 3.

2). The regions included the bilateral splenium of the corpus callosum, the IFOF and the ILF in the occipital cortex, the posterior and superior corona radiata, the CST, the frontal and the temporal part of the SLF, the forceps major, and the cingulum. The right temporal part of the IFOF and the ILF were also found to be significantly different between

5th and 2nd graders. Except for the frontal area of the brain, most regions showed greater white matter connectivity in 5th graders compared with 2nd graders. On the other hand, the opposite contrasts (2nd graders > 5th graders) did not yield any significant results.

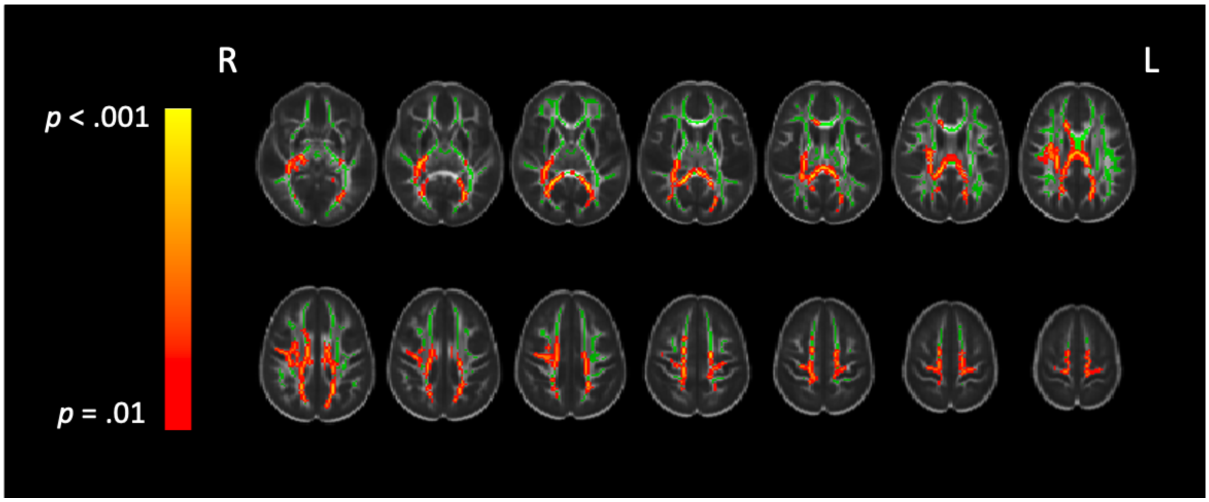

Figure 3.2. Differences between 2nd and 5th graders in FA. The statistical map was superimposed onto the mean of FA skeleton (green) and mean FA map registered from a population-based template (gray scale). The red-yellow colored statistical map indicates higher value in 5th graders than 2nd graders. No significant difference was found in the opposite contrasts (2nd graders > 5th graders). L, left and R, right.

In subsequent analyses we explored which white matter regions were correlated with nonsymbolic ratio and symbolic fraction processing abilities. We used both mean accuracy and mean reaction times (RT) in each notations. (FF, LF, and LL) as explanatory factors. In all correlational analyses, we controlled for age in months, gender, head motion and children's processing speed. We first tested correlations with FA maps. With 2nd graders, we did not find any significant correlations with accuracy or RT. However, for 5th graders, we found a significant negative correlation between mean RT in FF and FA in the bilateral parietal-temporal regions of the brain when controlling for age, gender, head motion, and processing speed (*p* < .05, TFCE corrected; Figure 3.3). In other words, children with better processing abilities, indicated by faster response times, showed higher FA. Voxels in these clusters spanned bilateral sagittal stratum including the ILF and IFOF and left SLF including its temporal part.

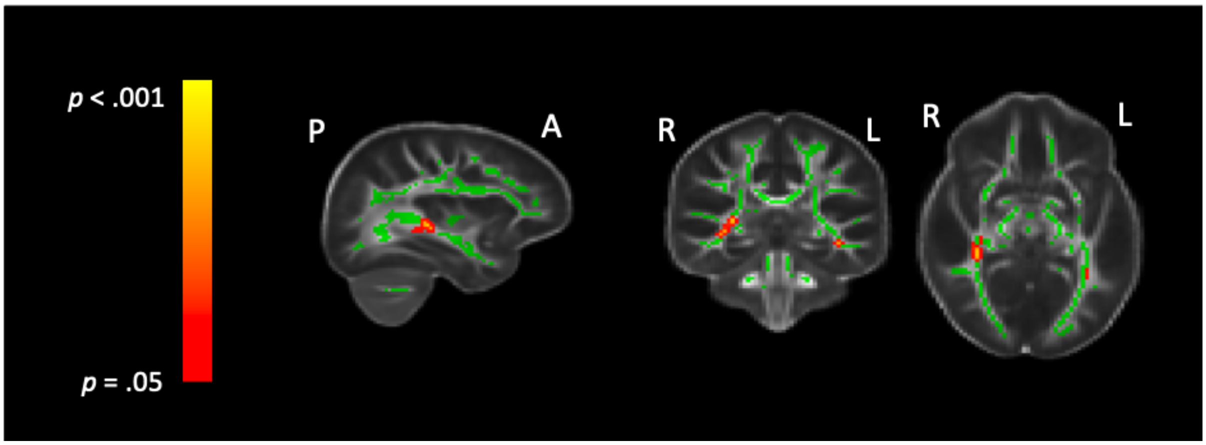

Figure 3.3. Correlations in 5th graders between FA values and mean comparison RT in symbolic fraction notation (FF) when age, gender, head motion and processing speed were corrected. Higher FA values were associated with faster participant responses.*
The statistical map was superimposed onto the mean of FA skeleton (green) and mean FA map registered from a population-based template (gray scale). The red-yellow colored statistical map indicates a significant correlation (p <.05). P, posterior and A, anterior. L, left, and R, right.

*Note, because faster response indicates better fraction processing ability, we indicate the regions with red-yellow color.

Developmental differences between 2nd vs. 5th graders with other diffusion parameters

Next, to better understand FA results, we also looked into other diffusion parameters (MD, RD, and AD). When regard to MD and RD, 2nd graders showed significantly higher MD and RD compared to 5th graders in broad white matter regions (p <.01 TFCE corrected; Figure 3.4). The regions included most of the IFOF and the ILF connecting the frontal-temporal regions. Also, the region included a part of the SLF connecting the frontal-parietal and parietal-temporal regions. Additionally, we found that a part of corticospinal tracts and body of corpus callosum were also included. Since we did not find any significant differences with AD, the significant differences found in MD and RD suggests increased density of cellular structure and the degree of demyelination may have mainly contributed to developmental differences in structural connectivity between 2nd and 5th graders.

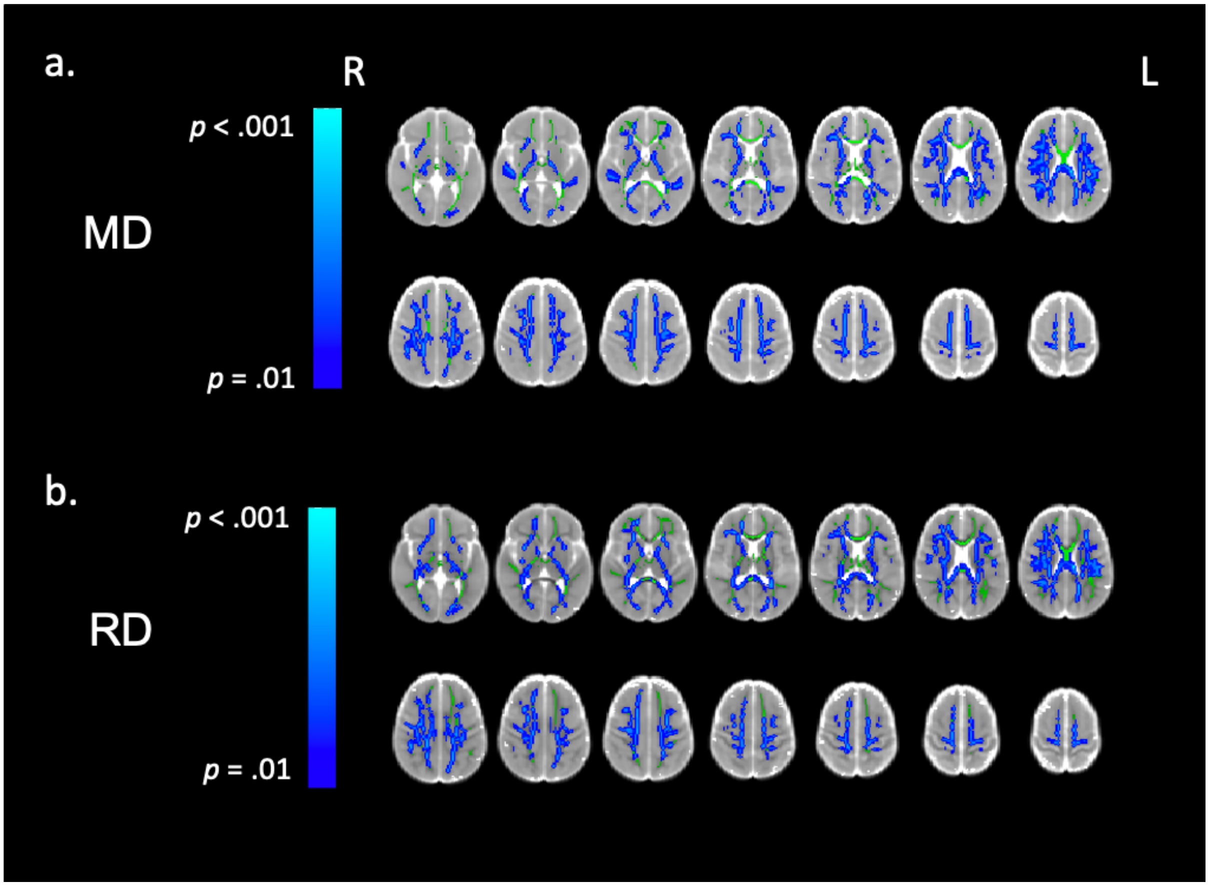

Figure 3.4. Differences between 2nd and 5th graders in a) MD, and b) RD. The statistical map was superimposed onto the mean of FA skeleton (green) and each mean MD and RD maps registered from a population-based template (gray scale). The blue-light colored statistical map indicates higher value in 2nd graders than 5th graders (*p* <.01). P, posterior and A, anterior. L, left and R, right

To examine the questions of which white matter characteristics were particularly related to performance with nonsymbolic or symbolic fractions, we explored correlations between accuracies and RTs and other diffusion parameters. We did not find any significant correlations between diffusion parameters and either accuracy or RT among 2nd graders. However, with 5th graders, we found that MD and RD parameters were positively correlated with RT measures of symbolic fraction processing ability when accounting for age in months, gender, head motion and processing speed ($p < .05$, TFCE corrected; Figures 3.5 and 3.6). We also found that 5th graders' AD parameter was positively correlated with accuracy of nonsymbolic ratio processing ability when age in months, gender, head motion and processing speed were accounted for ($p < .05$, TFCE corrected; Figure 3.7).

Diffusion parameters that relate to the cellular structure of white matter, MD, and the degree of myelination of axons, RD, showed significant correlations with symbolic fraction processing in broader regions compared to the correlations between FA and that processing (Beaulieu, 2002; Sen & Basser, 2005; Song et al., 2002, 2005). With MD, children with faster RTs in FF showed lower MD mostly in the right hemisphere ($p < .05$, TFCE corrected; Figure 3.6). The significant regions included right IFOF spanning from frontal-posterior regions, right ILF, right SLF which also includes its temporal part, the frontal part of right anterior thalamic radiation and forceps minor. Adding to this, we found bilateral CST and anterior thalamic radiation. The results with MD showed 5th graders with higher density of cellular structure in these white matter regions had higher symbolic fraction processing abilities.

With RD, children with faster RT in FF and across all notations showed lower RD ($p < .05$, TFCE corrected; Figure 3.7). Whereas mean RT across all notations showed significant correlations only in the right superior CST, mean RT in FF was significantly correlated with broad regions. The regions included bilateral SLF including temporal part, the IFOF, the ILF, the CST, and anterior thalamic radiation. However, the frontal part and mid-superior part of the SLF, right under the fronto-parietal regions, were only found in the right hemisphere. That means, the degree of demyelination in the major tracts in the right hemisphere is related with higher symbolic fraction processing ability.

Lastly, diffusion parameters that relate to the degree of axonal propagation showed significant correlation only with nonsymbolic ratio processing, but not with symbolic fractions processing (Kumar, Nguyen, Macey, Woo, & Harper, 2012; Song et al., 2003, 2005). Children with higher accuracy in LL showed higher AD mostly in the left hemisphere which was contrary to the results of MD and RD ($p < .05$, TFCE corrected; Figure 3.8). The significant regions included a large portion of left SLF in the parietal and temporal lobes, left body of corpus callosum, left ILF and IFOF in both occipital and temporal lobes, and left CST. Children with higher axonal propagation in these tracts in the left hemisphere seemed to have higher nonsymbolic ratio processing ability.

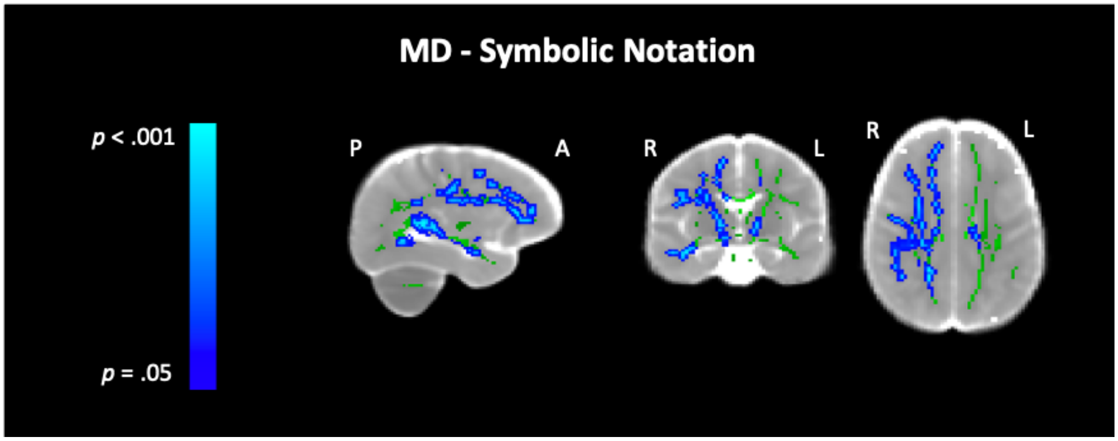

Figure 3.5. Positive correlations between MD values and mean RT in symbolic fraction notation (FF) when age, gender, head motion, and processing speed were corrected. As MD values were lower, participants' responses were faster. The statistical map was superimposed onto the mean of FA skeleton (green) and mean MD map registered from a population-based template (gray scale) for visualization purpose. The blue-light colored statistical map indicates a significant correlation (p<.05). P, posterior and A, anterior. L, left and R, right.

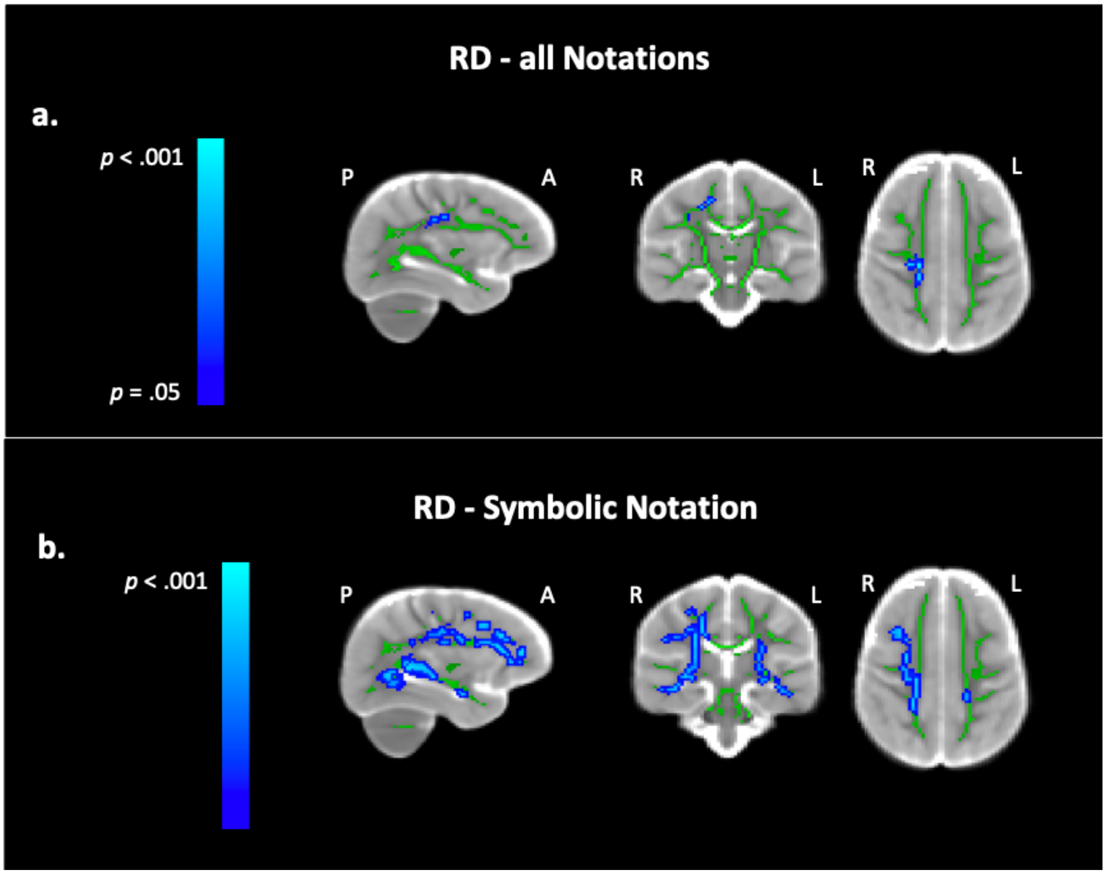

Figure 3.6. Positive correlations between RD values and a) mean RT across all notations and b) mean RT in symbolic fraction notation (FF) when age, gender, head motion, and processing speed were corrected. As RD values were lower, participants' responses were faster. The statistical map was superimposed onto the mean of FA skeleton (green) and mean RD map registered from a population-based template (gray scale) for visualization purpose. The blue-light colored statistical map indicates a significant correlation ($p<.05$). P, posterior and A, anterior. L, left and R, right.

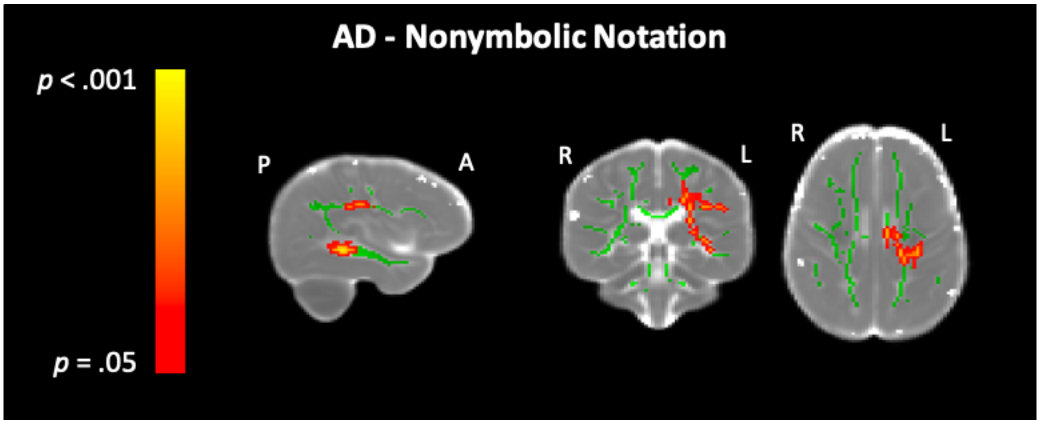

Figure 3.7. Positive correlations between AD values and mean ACC in nonsymbolic fraction notation (LL) when age, gender, head motion, and processing speed were corrected. As AD values were higher, participants' responses were more accurate. The statistical map was superimposed onto the mean of FA skeleton (green) and mean AD map registered from a population-based template (gray scale) for visualization purpose. The red-yellow colored statistical map indicates a significant correlation (p<.05). P, posterior and A, anterior. L, left and R, right.

Discussion

The current experiment explored neural signatures for nonsymbolic ratio and symbolic fraction processing abilities in relation to white matter microstructure. By testing 2nd and 5th graders, our results revealed developmental differences in the relations between white matter and nonsymbolic ratio and symbolic fraction processing abilities. Also, our results demonstrate differential relations between diffusion parameters and notations (nonsymbolic ratio or symbolic fractions).

We found significant developmental differences at a group-level in both behavioral and neural analyses. Behaviorally, 5th graders showed significantly better nonsymbolic ratio and symbolic fraction processing abilities compared to 2nd graders, but with the similar notation effects, exhibiting the best performance in nonsymbolic notation. These results are consistent with those of a prior study using the identical task (Kalra, Binzak, et al., 2020). In the neural analysis, 5th graders showed significantly increased FA, decreased RD and MD in broad regions of the brain even when controlling for gender and head motion, consistent with previous developmental DTI studies (e.g., Barnea-Goraly et al., 2005; Kumar, Nguyen, Macey, Woo, & Harper, 2012; Mukherjee et al., 2001, 2002; Qiu, Tan, Zhou, & Khong, 2008). The significant differences in RD and MD between 2nd and 5th graders are potentially associated with an increase of myelin or axonal density during primary school years in most of the white matter regions (Beaulieu, 2002; Sen & Basser, 2005; Song et al., 2002, 2005). The regions showing higher FA in 5th graders were limited to the parietal-temporal regions, but the regions found in FA overlapped with the regions showed differences in RD and MD. Given that we did not find differences in AD corresponding to diffusion parallel to the microstructure, but did observe significant differences in RD indicative of perpendicular diffusion (Kumar et al., 2012; Song et al., 2003, 2005), the significant differences in FA may be associated with increased myelination.

Along with the group-level differences, we observed developmental differences in white matter microstructure at a subject-level. The significant relations between diffusion parameters and nonsymbolic ratio and symbolic fraction processing abilities were only found in 5th graders, but not in 2nd graders. Contrary to our prediction, 2nd graders' white matter did not show significant correlations with nonsymbolic ratio processing ability. Since nonsymbolic ratio processing has been revealed as a primitive ability that even monkeys and infants have (Jacob et al., 2012; McCrink & Wynn, 2007), we expected for 2nd graders' white matter microstructure to relate to their nonsymbolic ratio processing under fronto-parietal areas even without any formal instructions.

This may be because the period that individual differences in white matter reflect individual differences in nonsymbolic processing may need more consistent experience as other training studies have showed (Huber, Donnelly, Rokem, & Yeatman, 2018; Jolles et al., 2016). It should be noted that Chapter 2 explored a group-level differences not their individual differences. Furthermore, even though there are some previous studies that argue that functional engagement can be associated with changes in underlying white matter microstructure (Damoiseaux & Greicius, 2009; Johansen-Berg & Rushworth, 2009; Zimmermann et al., 2018), it is also true that it is not always the case (Damoiseaux, 2017; Hirsiger et al., 2016; Zimmermann et al., 2016). To better understand these results, following studies should investigate individual differences in functional engagement and connectivity between the fronto-parietal regions and how those individual differences correlate to ratio or symbolic fraction processing.

Unlike with 2nd graders, for 5th graders, we found that each diffusion parameter was differentially correlated with nonsymbolic vs. symbolic fractions processing. FA, RD, and MD were particularly correlated with symbolic fraction ability even when controlling for age, gender, head motion, and global processing speed. On the other hand, AD was correlated with nonsymbolic ratio processing ability when controlling for the same covariates. These results may suggest that different aspects of white matter microstructure differentially relate to nonsymbolic ratio and symbolic fraction processing.

In terms of correlations with symbolic fractions, overlapping regions across the results with FA, MD, and RD were localized, especially in the occipito-temporal areas of white matter. The regions spanned a small part of bilateral ILF and IFOF, and part of left SLF, tracts whose integrities has previously reported to correlate with whole numbers (for review, see Matejko & Ansari, 2015; Moeller, Willmes, & Klein, 2015; Peters & De Smedt, 2018). The occipito-temporal cortex has not been frequently reported in earlier fMRI studies with ratio and fractions (Ischebeck, Schocke, & Delazer, 2009; Jacob & Nieder, 2009; Mock et al., 2018; but see Cui et al., 2020), but it has been emphasized well enough with symbolic numbers in regards to its visual or verbal representations (for review, Yeo, Wilkey, & Price, 2017). Our results may indicate the importance of processing visual or verbal features of symbolic fractions.

Additionally, we found that RD and MD were correlated with symbolic fractions processing ability in broader regions than was FA. The regions covered most of the tracts linking parietal-frontal regions of the brain that we predicted but also covered tracts linking the temporal-occipital regions, as well as the IFOF that links the frontal-occipital regions, which has also been reported in previous studies (e.g., Hu et al., 2011; Rykhlevskaia et al., 2009). This result may demonstrate that the degree of myelination (Song et al., 2002, 2005) and density of axonal membranes (Beaulieu, 2002; Sen & Basser, 2005) are particularly sensitive to children's fraction processing abilities in larger regions. Given the developmental differences are shown in larger areas in MD and RD between 2nd and 5th graders compared to FA, the increased axonal membranes density and demyelination may contribute to a broader sensitivity to symbolic fractions compared to the case of FA.

On the contrary, regions where AD was significantly correlated with nonsymbolic ratios in left hemisphere included the fronto-parietal tracts in line with previous studies (e.g., Cantlon et al., 2011; Matejko, Price, Mazzocco, & Ansari, 2013; van Eimeren et al., 2008). Even though we did not find developmental differences in AD between 2nd and 5th graders in these regions at the group level, the axonal organizations and orientations in white matter may have changed at the individual-level and contributed to the correlations with nonsymbolic ratios (Kumar et al., 2012; Song et al., 2003, 2005). Future studies are needed to chart the developmental trajectories between 2nd and 5th grade to better understand the white matter microstructural changes contributing to the relationship with nonsymbolic ratio and symbolic fraction processing.

One interesting aspect of these results is that different lateralization was observed depending on the diffusion parameter of interest. Whereas significant associations in FA and MD were mostly bilateral, RD and AD associations were found right or left lateralized. Previous DTI studies investigating the relations with number processing and its operation have not provided consistent results regarding this lateralization (Moeller et al., 2015). Some studies with children have reported bilateral or right-hemispheric results (e.g., Hu et al., 2011; Klein, Moeller, & Willmes, 2013; Rykhlevskaia et al., 2009), while a number of studies have also observed left lateralized results (e.g., Cantlon et al., 2011; Van Beek, Ghesquière, Lagae, & De Smedt, 2014; van Eimeren et al., 2008). Developmental fMRI studies have suggested that asymmetrical activations for numerical processing in early years eventually become bilateral during maturation. Some showed left lateralized activations in children (Ansari & Dhital, 2006; Ansari et al., 2005; Holloway & Ansari, 2010) and some showed right lateralization (Cantlon, Brannon, Carter, & Pelphrey, 2006; Cantlon et al., 2009; Rivera et al., 2005).

However, regardless of the direction of lateralization, this asymmetrical hemispheric involvement may simply be due to immature numerical processing in early years or immature interhemispheric connections (Cantlon et al., 2011). Even though 5th graders have received formal fractions instruction for 2-3 years, children often have a hard time learning about fractions (e.g. Lesh, Post, & Behr, 1987; Ni & Zhou, 2005; Vamvakoussi & Vosniadou, 2004, 2010). It is also possible that immature interhemispheric connections that are largely controlled via the corpus callosum might play a role. Even though 5th graders showed higher FA in the corpus callosum, immature integrity in corpus callosum in early years of development may cause asymmetric involvement of the brain. Future work should investigate how the proficiency matters for bilateral engagement by comparing different age ranges such as children vs. adults. It may further examine the role of corpus callosum to better understand the lateralization of structural and functional brain.

Experiment 2

Specific Introduction

Experiment 1 showed the association between white matter microstructure and nonsymbolic ratio and symbolic fraction processing abilities and how the relations change during early fraction instructions using a cross-sectional design. In Experiment 1, we identified multiple tracts that related with ratio and fraction processing. However, since we took a whole-brain approach, we could not narrow down the results into a few specific tracts. Also, use of a cross-sectional design limited our understanding on developmental changes.

Therefore, as an extension of the cross-sectional approach, Experiment 2 aimed to investigate the changes in the microstructures one year after the first scan and how each tract differentially relates to the changes of ratio and fraction processing. We specifically examined the changes in white matter connectivity in the bilateral SLF, the bilateral ILF, and each part of the CC (genu, splenium, and body). These tracts have been frequently reported as related to numerical processing (Matejko & Ansari, 2015; Peters & De Smedt, 2018).

By looking into differential development of white matter connectivity in different tracts and how it relates to development of fraction processing, Experiment 2 aimed to help understand how the brain changes during early years of fraction instruction.

Methods and Measures

Participants

We analyzed DWI images acquired a year after the first scan. For year 2 scans, 41 (of 47) 2nd graders and 38 (of 45) 5th graders returned to the lab and took part in the same scan procedure one year after their initial scan.

The multiple notation ratio comparison task

We used the identical test as Experiment 1.

Diffusion MRI acquisition and analysis

Participants were scanned on the same scanner as Experiment 1 using the same protocol. DWI acquisition parameters were also identical to those of the first time point (Experiment 1). As in Experiment 1, all DWI images at both time points were preprocessed by using the identical in-house processing pipeline (see Experiment 1's methods). After constructing FA maps from the diffusion tensors (Basser & Pierpaoli, 1996), a population-specific FA template was generated for longitudinal analysis using the buildtemplateparallel.sh from ANTs (Avants et al., 2014). First, an individual template was generated from the FA maps of both time points, and then overall templates for each younger (2^{nd}-3^{rd} grades) and older (5^{th}-6^{th} grades) child groups were constructed. Lastly, a single, final template was then generated from the younger and older children templates. Each individual's FA map at each time point was then nonlinearly aligned to the final overall population template using ANTs.

To examine the changes in different regions of white matter, we employed an atlas-based ROI approach. We used JHU ICBM-DTI White Matter Atlas map to identify the ROIs: the SLF, the ILF, and three parts of the CC (body, splenium, and genu), major tracts underneath fronto-parietal and temporal-occipital regions of the brain that have frequently been reported in the field of numerical processing (Matejko & Ansari, 2015; Peters & De Smedt, 2018). Moreover, these tracts were found to have significant correlations with fractions processing in Experiment 1. The ROI from the atlas was registered to the template space. Once registered, we extracted individual mean FA from each ROI (Figure 3.8).

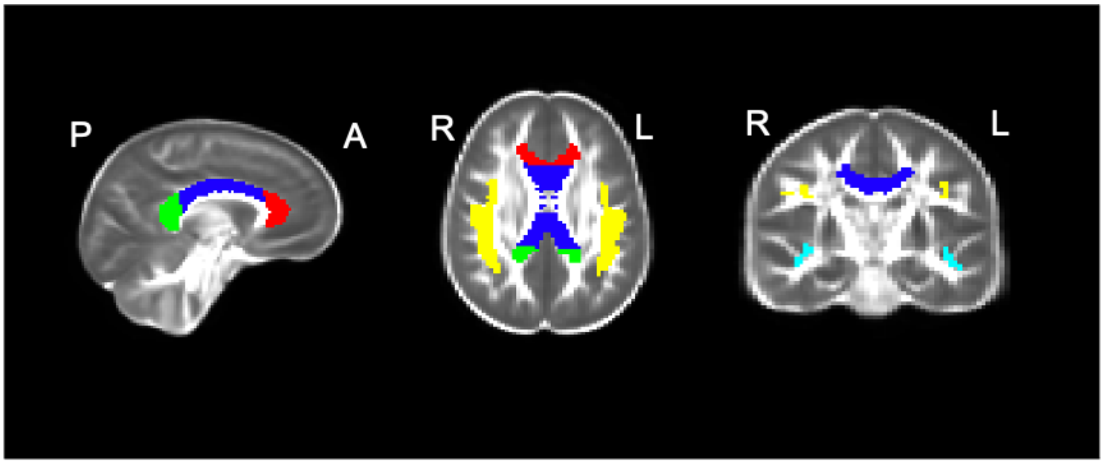

Figure 3.8. ROIs registered from a population-based template. Mean FA map registered from a population-based template (gray scale) for visualization purpose. Red: the Genu of the CC, Green: the splenium of the CC, Blue: the body of the CC, Yellow: the SLFs, Light blue: the ILFs. P, posterior and A, anterior. L, left and R, right.

Results

Behavioral analysis

Since Experiment 2 used longitudinal data investigating Experiment 1's participants a year later, we divided the participants into a Younger group who transitioned to 3rd grade from 2nd grade and an Older group who transitioned to 6th grade from 5th grade. Also, we named each time point as Time 1 for the first scan and Time 2 for the second scan a year later. To explore behavioral changes, we conducted mixed effects logistic and linear regression models for accuracy and reaction time (RT) analysis to account for within-subject correlation among trials using the 'lmer' function of the lme4 package in R software (Bates et al., 2015). To analyze accuracy and RT, we regressed each dependent variable against notations (3 levels, FF = 0, LF = 1, LL = 2), age groups (2 levels, Younger (2nd -3rd) =

0, Older (5^{th} – 6^{th}) = 1), time points (2 levels, Time 1 = 0, Time 2= 1) and their interactions including all three way interactions. We used a backward difference coding scheme so that we could compare adjacent levels of variables (i.e., each level minus prior level). For accuracy, as in Experiment 1, we found that the older group responded significantly more accurately (β = .46, p < .001) and rapidly (β = -164.77, p < .001) than the younger group (Table 3.3; Figure 3.9). Also, we found significant notation effects similar to the results of Experiment 1. Children were more accurate (β = .15, p <. 001) and faster (β = -245.06, p < .001) in nonsymbolic trials than on mixed trials, and more accurate (β = .78, p = <.001) and faster (β = -245.11, p < .001) on mixed trials than on trials involving only symbolic fractions. A year later, children did not show improvement in accuracy (β = .083, p = .438), but their reponse times became more rapid (β = -.90.678, p =.024). In addition to this, we found significant interactions between notations and age groups with RT: Younger group showed larger differences between mixed and nonsymbolic trials (β =-50.17, p = .002) than Older group. Also, Younger group showed larger difference between mixed and symbolic fraction trials (β = = - 45.7, p = .003) than Older group. Other than these interactions, we did not find any signifiicant interactitons with time points.

Table 3.3. Mixed effects, regressing ACC and *RT* against notation, age group, and time point.

<table>
<tr><th></th><th></th><th>ACC</th><th></th><th></th><th>RT</th><th></th></tr>
<tr><th></th><th>β</th><th>Odds Ratio</th><th>p</th><th>β</th><th>t</th><th>p</th></tr>
<tr><td>Intercept</td><td>2.401</td><td>11.032</td><td><.001***</td><td>1526.296</td><td>76.726</td><td><.001***</td></tr>
<tr><td>FL - FF</td><td>.146</td><td>1.157</td><td>.001</td><td>-245.063</td><td>-30.774</td><td><.001***</td></tr>
<tr><td>LL -FL</td><td>.776</td><td>2.172</td><td><.001***</td><td>-245.109</td><td>-31.415</td><td><.001***</td></tr>
<tr><td>Older -Younger</td><td>-.460</td><td>.631</td><td><.001***</td><td>164.765</td><td>4.141</td><td><.001***</td></tr>
<tr><td>Time 2 – Time 1</td><td>.083</td><td>1.086</td><td>.438</td><td>-90.675</td><td>-2.279</td><td>.024*</td></tr>
<tr><td>|FL – FF|*| Older -Younger |</td><td>.155</td><td>1.167</td><td>.070</td><td>-50.170</td><td>-3.150</td><td>.002**</td></tr>
<tr><td>|LL – FL|*| Older -Younger |</td><td>-.001</td><td>.999</td><td>.989</td><td>-45.699</td><td>-2.929</td><td>.003**</td></tr>
<tr><td>|FL – FF|*| Time 2-Time1 |</td><td>.005</td><td>1.005</td><td>.957</td><td>-12.372</td><td>-.777</td><td>.437</td></tr>
<tr><td>|LL – FL|*| Time 2-Time1 |</td><td>-.028</td><td>.973</td><td>.789</td><td>11.373</td><td>.729</td><td>.466</td></tr>
<tr><td>| Older -Younger |*| Time 2-Time1 |</td><td>.099</td><td>1.104</td><td>.644</td><td>-13.526</td><td>-.170</td><td>.865</td></tr>
<tr><td>|FL – FF|*|*| Older -Younger |*| Time 2-Time1 |</td><td>.126</td><td>1.135</td><td>.460</td><td>-13.129</td><td>-.412</td><td>.680</td></tr>
<tr><td>|LL – FL|*|*| Older -Younger |*| Time 2-Time1 |</td><td>.022</td><td>1.022</td><td>.917</td><td>20.631</td><td>.661</td><td>.509</td></tr>
</table>

Note, $p<.05^{*}$, $p<.01^{**}$, $p<.001^{**}$

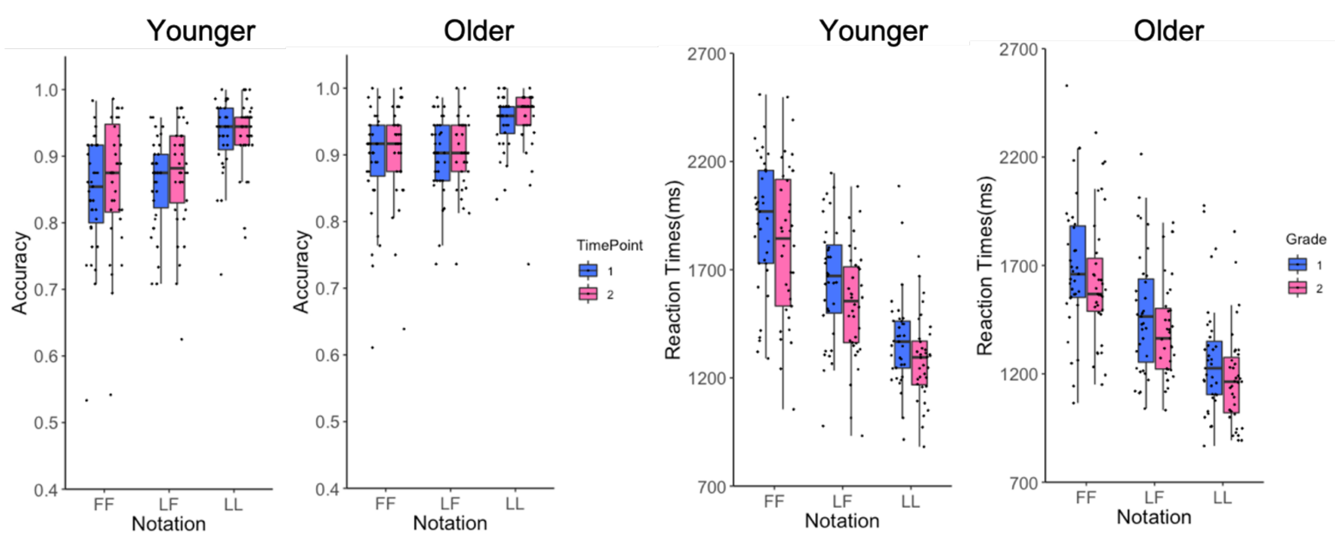

Figure 3.9. Box plots with individual values for mean accuracies (left panel) and for mean reaction times (right panel) for the comparison task in each age group.

ROI analysis

First, we analyzed differences across different age groups and also the changes within each age group over time from Time 1 to Time 2. To test for these changes, we conducted linear mixed effects regressions for FA in each ROI by using the 'lmer' function of the lme4 package in R software (Bates et al., 2015). To analyze changes in FA, we regressed FA against age groups (2 levels, Younger = 0, 5th Older = 1) and time points (2 levels, Time 1 = 0, Time 2= 1), and their interactions. We used a backward difference coding scheme so that we could compare adjacent levels of variables (i.e., each level minus prior level). We found significant increases in FA in the bilateral SLF (left: β = .002, . p <.001, right: β = .006, .p <.001), bilatteral ILF (left: β = .009, p <.001, right: β = .011, . p <.001) and genu of the CC (β = .007, . p <.001) (see Table 3.4 and 3.5). We did not find a significnat increase in FA of the splenium of the CC (β = .001, p =.618). In the body of the CC, we rather found a significant decrease in FA (β =- .002, p =.009). In terms of age groups, the older group showed significnantly higher FA in the body (β = .009, p <.004) and the splenium (β = .011, p <.005) of the CC, the right SLF (β = .007, p <.001) and the right ILF (β = .008, p <.004). We did not find any significant differences in the left SLF and ILF, and the genu of the CC between the age groups. Except for the body of CC, more than 50% of children showed increases in FA in other ROIs (the bilateral ILF, SLF and splenium and genu of CC) (Figure 3.10). See Figure 3.11 and 3.12 for the changes of FA by age in months.

Table 3.4. Mixed effects, regressing FA in the SLF and the ILF against age group and time point.

	R SLF			*L SLF*			*R ILF*			*L ILF*		
	β	*t*	*p*	β	*t*	*p*	β	*t*	*p*	β	*t*	*p*
Intercept	.282	313.332	<.001	.253	225.533	<.001	.272	234.542	<.001	.245	214.614	<.001
Older - Younger	.007	4.009	<.001	.002	.956	.343	.008	3.509	.001	.004	1.808	.075
Time 2 – Time 1	.006	5.232	<.001	.006	5.012	<.001	.011	6.594	<.001	.009	5.851	<.001
|Older - Younger| *|Time 2 – Time 1|	.001	.493	.624	.001	0.242	.809	.002	.582	0.562	.003	.984	.328

Table 3.5. Mixed effects, regressing FA in the CC against age group and time point.

	GCC			*BCC*			*SCC*		
	β	*t*	*p*	β	*t*	*p*	β	*t*	*p*
Intercept	.294	217.420	<.001	.355	227.934	<.001	.480	258.516	<.001
Older - Younger	-.001	-.523	.602	.009	3.020	.004	.011	2.928	.005
Time 2 – Time 1	.007	4.669	<.001	-.002	-2.658	.010	.001	0.501	.618
|Older - Younger| *|Time 2 – Time 1|	.003	1.197	.236	.001	.742	.460	.000	-0.211	.834

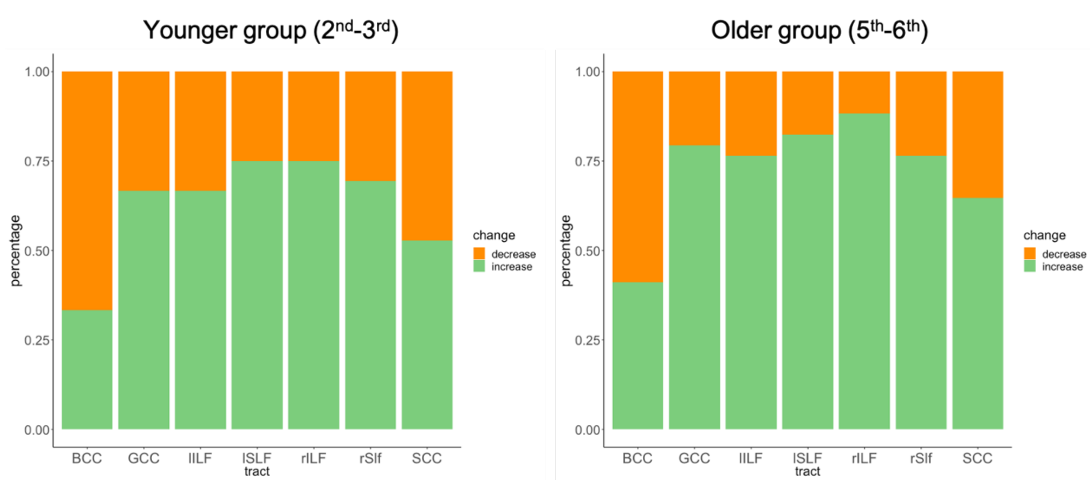

Figure 3.10. Percentage of children who showed increased (green) and decreased (orange) FA in each ROI.

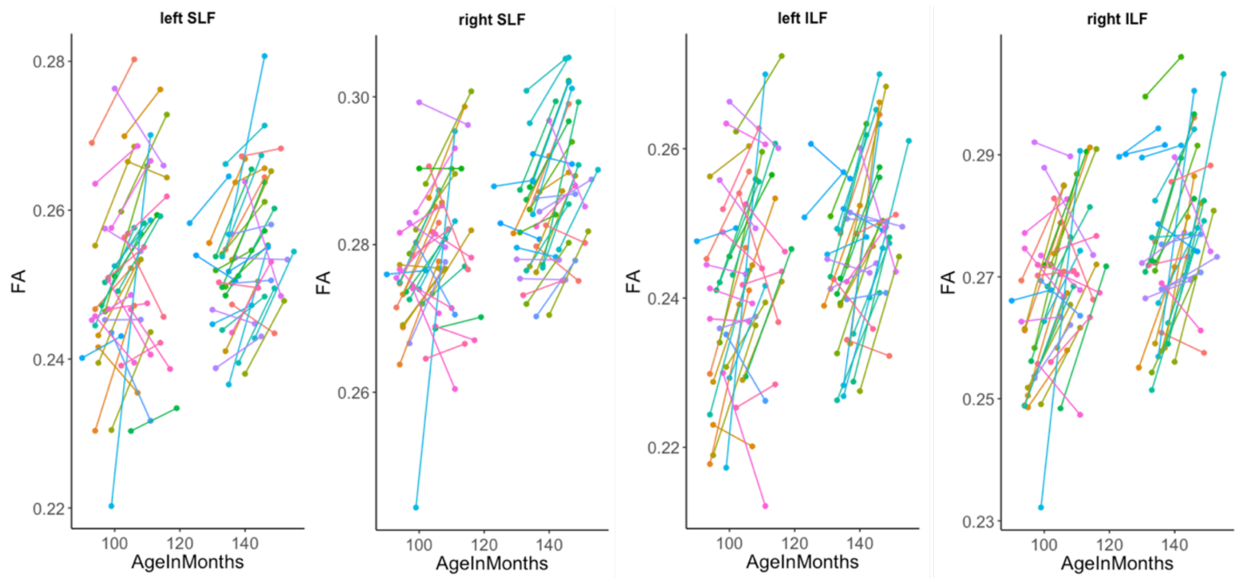

Figure 3.11. Individual changes in FA from the SLF and the ILF by age in months.

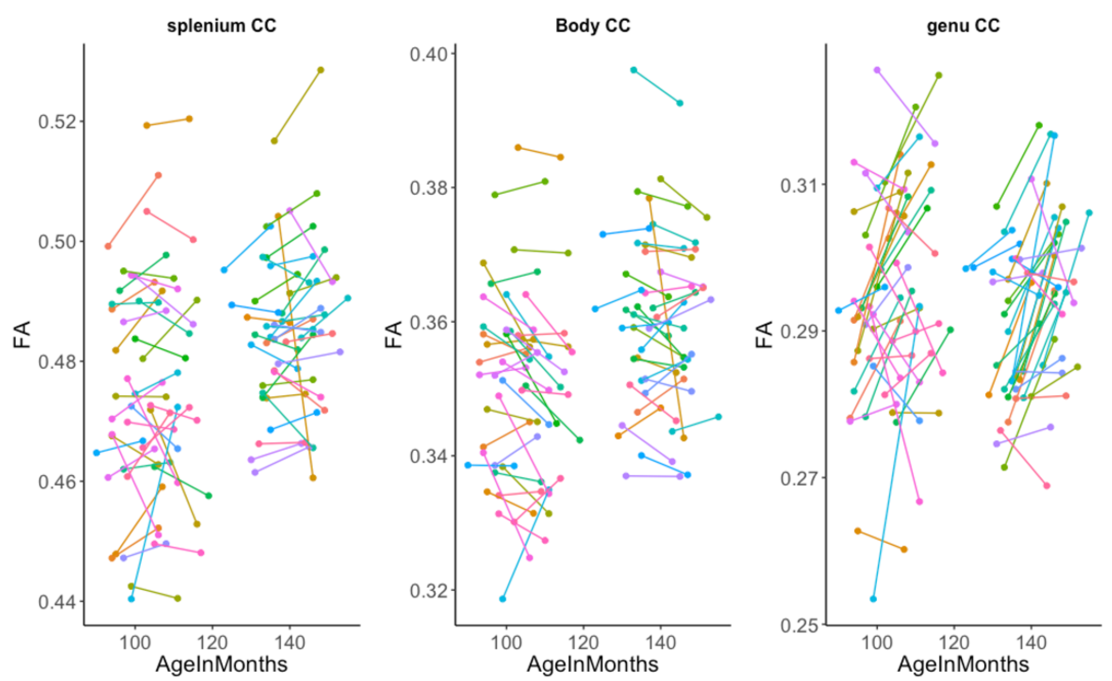

Figure 3.12. Individual changes in FA from the CC (splenium, body and genu) by age in months.

Next, we explored the relations across the changes of white matter connectivity within different ROIs and the changes of children's ratio and fraction processing ability. The changes of FA and ratio and fraction processing abilities were normalized by dividing the difference between the values from two timepoints by the value from Time 1 ((Time 2-Time 1)/Time 1). The change of ratio and fraction processing ability was calculated by using accuracy (ACC) and reaction time (RT) of each individual at each timepoint and each notation (LL, LF, and FF). Furthermore, to see the developmental differences between the younger and the older groups, we conducted bivariate correlation analyses separately.

We found that changes in FA in each ROI were differentially related with different notations. Also, there were differences in those relations between the two age groups. In the younger group, we found the changes in the performance of all three notations were correlated with most ROIs (See Table 3.6 and Figure 3.13). The changes in accuracy in LL notation were positively correlated with the changes in FA in the bilateral ILFs (L: $r = .436$, $p = .011$, R: $r = .409$, $p = .018$) and the genu of the CC ($r = .417$, $p = .016$). In terms of LF notation, the changes in the performance were positively correlated with the changes in FA in the splenium of the CC (ACC: $r = .357$, $p = .041$, RT: $r = -.418$, $p = .016$) and the left ILF (ACC: $r = .343$, $p = .050$). Lastly, the changes in RT of FF notation were negatively correlated with the changes in FA in the splenium of the CC ($r = -.350$, $p = .046$).

On the other hand, in the older group, significant correlations were found with the changes in the performance with LF and LL notations, but positive and also negative correlations unlike the younger group (See Table 3.7 and Figure 3.14). We found the RT changes in LF notation were negatively correlation with FA changes in the body of the CC and (RT: $r = -.390$, $p = .023$), while the RT changes of LL notation were positively correlated with changes in FA in the bilateral ILFs and the left SLF (L ILF r =.411., p = .016; R ILF: r =.358, p = .038; L SLF: r =.351, p = .042).

Table 3.6. Correlations across the changes of FA from the ROIs and efficiency score in the younger group (n=34).

	LF (ACC)	LL (ACC)	FF (RT)	LF (RT)	LL (RT)	L SLF	R SLF	L ILF	R ILF	BCC	SCC	GCC
FF (ACC)	.457**	.201	-.076	.079	-.054	.060	.135	.123	.015	.156	.227	.142
LF (ACC)		.220	-.214	-.137	-.168	.255	.312^{+}	.343*	.295^{+}	.183	.357*	.316^{+}
LL (ACC)			.161	.093	.017	.293^{+}	.276	.436*	.409*	.179	.141	.417*
FF (RT)				.777***	.805***	-.238	-.034	-.146	-.242	-.078	-.350*	-.225
LF (RT)					.852***	-.231	-.065	-.088	-.243	-.184	-.418*	-.222
LL (RT)						-.171	.047	-.071	-.192	-.040	-.288	-.144
L SLF							.879***	.891***	.885***	.535***	.662***	.837***
R SLF								.877***	.854***	.652***	.746***	.806***
L ILF									.924***	.494**	.562***	.913***
R ILF										.460**	.591***	.869***
BCC											.732***	.590***
SCC												.631***

*each column value indicates the change of FA and the fraction/ratio processing ability; LL = Line vs. Line, LF = line vs. fraction, FF = fraction vs. fraction, ACC =accuracies, RT = reaction times, BCC = Body of the CC, SCC= Splenium of the CC, GCC = Genu of the CC. $p<.1^{+}$, $p<.05^{*}$, $p<.001^{***}$

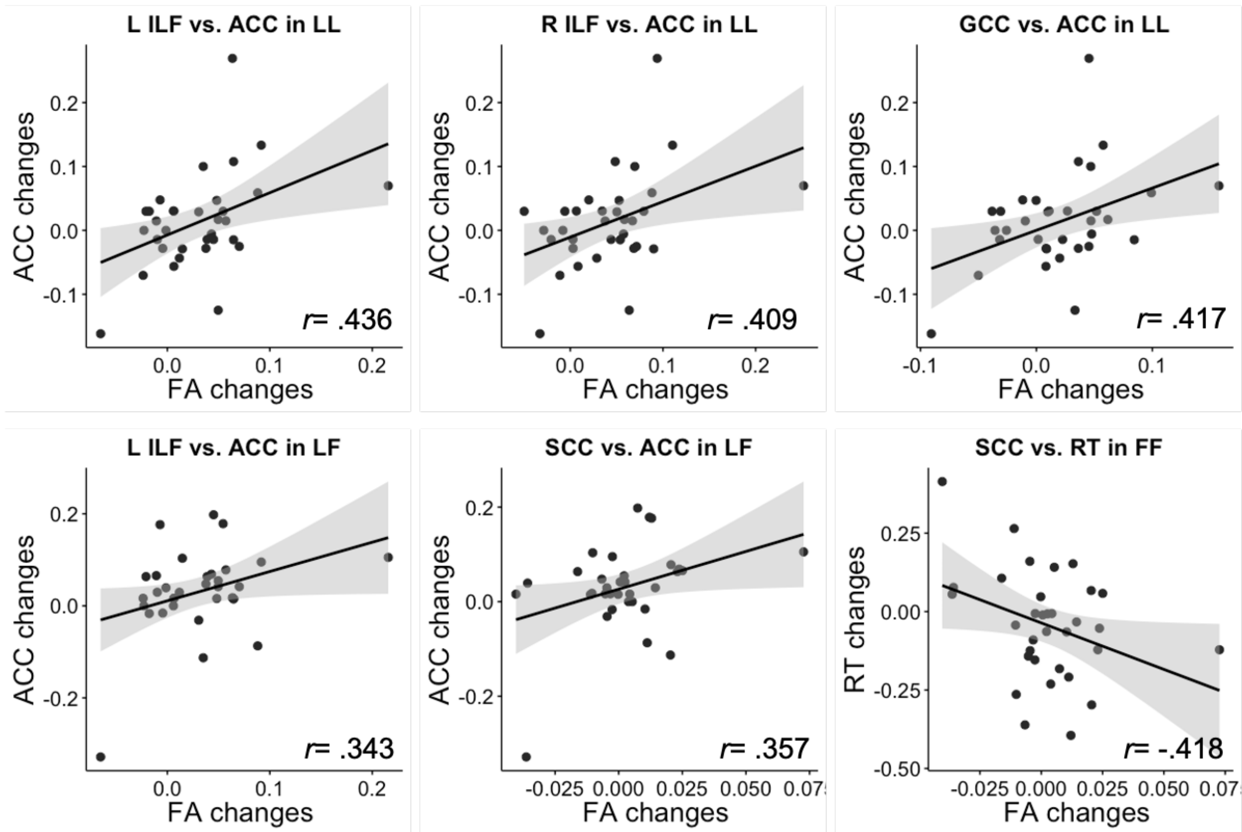

Figure 3.13. Correlations between the changes of FA in ROIs and the changes in the performance in each notation in the younger group.

Table 3.7. Correlations across the changes of FA from the ROIs and efficiency score in the older group (n=35).

	LF (ACC)	LL (ACC)	FF (RT)	LF (RT)	LL (RT)	L SLF	R SLF	L ILF	R ILF	BCC	SCC	GCC
FF (ACC)	.462**	-.069	.200	.217	.379*	.407*	.210	.167	.191	.261	.184	.091
LF (ACC)		.114	.016	.030	.036	.094	.069	-.011	.078	.272	.205	-.029
LL (ACC)			.196	.068	-.025	-.210+	-.231	-.119	-.038	-.094	-.028	-.215
FF (RT)				.718***	.663***	.302	.175	.265	.277	-.093	.047	.149
LF (RT)					.829***	.183	.050	.332+	.323+	-.390*	-.318+	.094
LL (RT)						.351*	.218	.411*	.358*	-.315+	-.132	.234
L SLF							.788***	.787***	.678***	.182	.355*	.747***
R SLF								.736***	.726***	.137	.392*	.766***
L ILF									.847***	-.225	-.047	.877***
R ILF										-.115	.046	.829***
BCC											.735***	.009
SCC												.162

*each column value indicates the change of FA and the fraction/ratio processing ability; LL = Line vs. Line, LF = line vs. fraction, FF = fraction vs. fraction, ACC =accuracies, RT = reaction times, BCC = Body of the CC, SCC= Splenium of the CC, GCC = Genu of the CC. $p<.1^{+}$, $p<.05^{*}$, $p<.01^{**}$, $p<.001^{***}$

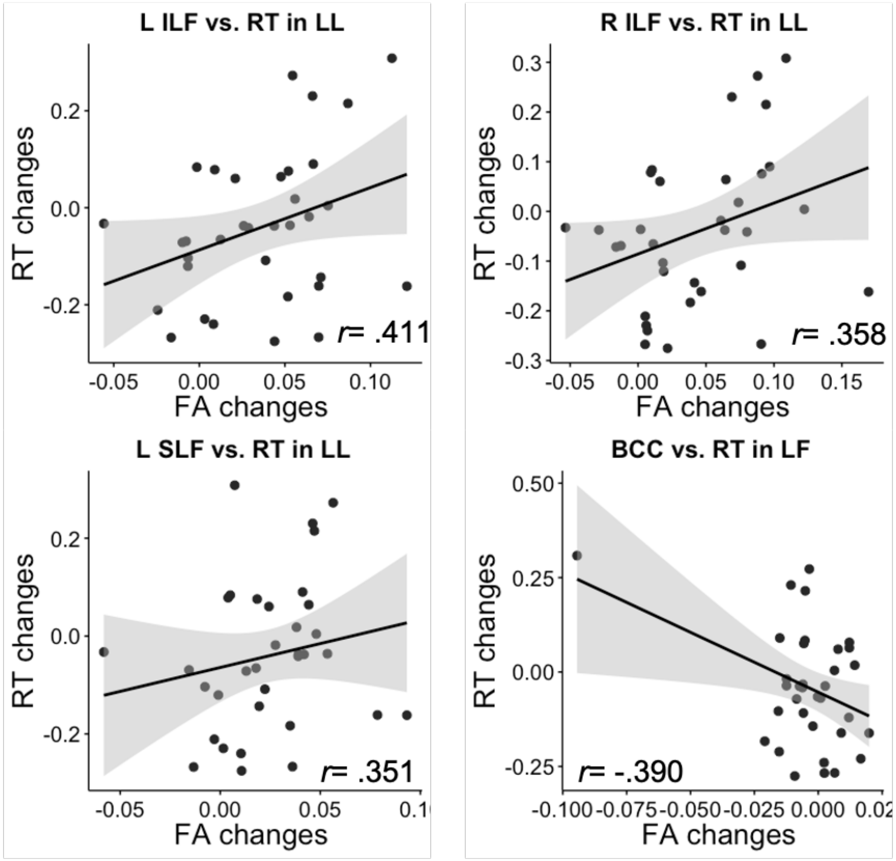

Figure 3.14. Correlations between the changes of FA in ROIs and the changes in the performance in each notation in the older group.

Discussion

With a cross-sequential approach, Experiment 2 explored how each ROI changes over a year and how those changes are related with the changes in nonsymbolic ratio and symbolic fraction processing ability. The results revealed different changes of connectivity in the bilateral SLF and ILF, and the CC and the changes of each ROI was differentially related with the improvement of ratio and fraction processing.

First, we observed both consistent and inconsistent changes in white matter connectivity with our two age groups. Similar to the previous studies, we found significant increases in FA in the bilateral SLF and ILF, and the genu of the CC in both the younger and the older groups (for review, Hermoye et al., 2006; Lebel & Deoni, 2018). These increases were not shown in every child, but rather a main effect emerged from a mix of individuals who showed increase in FA and who showed decrease in FA in each ROI as in the previous work (Catherine Lebel & Beaulieu, 2011).

Most regions showed the main effect showing increased FA, only with the exception of the body of the CC (Barnea-Goraly et al., 2005; Bonekamp et al., 2007). This is somewhat different from the prevalent notion that FA increases as age increases, especially in children (Bonekamp et al., 2007; Lebel et al., 2012; Lebel & Beaulieu, 2011). However, a similar report showing a negative correlation between age and FA in the isthmus of the CC (9-24 years-old) also exists (Muetzel et al., 2008). One possible account for these results can be underestimated FA due to a partial volume effect (Alexander, Hasan, Lazar, Tsuruda, & Parker, 2001; Alexander et al., 2007; Assaf & Pasternak, 2008). The partial volume effects happens when some voxels do not exhibit Gaussian diffusion behavior typical of DTI models, due to factors such as crossing fibers or cerebrospinal fluid (CSF) contamination. Since DTI measures the diffusion of water, around CSF, the outer white matter voxel with its surrounding tissue for the genu and splenium of the CC which consists mostly of CSF can easily show the partial volume effect. Studies have consistently showed underestimated diffusion parameters in the corpus callosum (Oouchi et al., 2007; Pfefferbaum & Sullivan, 2003). Considering the intricacy of the CC (Aboitiz, Scheibel, Fisher, & Zaidel, 1992b, 1992a), follow-up investigation using some techniques such as a free-water estimation to remove the CSF contamination during tensor estimation would be necessary to better understand these discrepancies (Hoy, Kecskemeti, & Alexander, 2015; Hoy, Koay, Kecskemeti, & Alexander, 2014; Pasternak, Sochen, Gur, Intrator, & Assaf, 2009; Pierpaoli & Jones, 2004).

We also observed different relationships between longitudinal changes in the ROIs and ratio and fraction processing abilities and also developmental differences between the younger and the older groups. In the younger group, the changes in processing ability each measured with different notation (symbolic, mixed, and nonsymbolic) were differentially related with the changes in white matter connectivity in each ROI. As expected, the changes of connectivity in ILFs were associated with changes of comparing nonsymbolic ratios and mixed notations. Within the CC, the genu and splenium parts were related with all notations. Considering we did not find any correlations between white matter connectivity and nonsymbolic or symbolic ratio processing in 2nd graders at Experiment 1, it might be possible that the associations may start between 2nd – 3rd years of the primary school.

On the other hand, in the older group, we found fewer associations between microstructures and ratio processing ability. One positive association we found was between changes of the body of the CC and ratio processing ability with the mixed notations. We rather found that the changes of microstructures were negatively associated with the changes in nonsymbolic ratio processing ability, especially in the ILF and the SLF.

One possibility may be a decreased involvement of the tracts with ratio processing. Considering that these regions connect the fronto-parietal and the temporal-occipital areas, these associations may imply decreasing importance of connections between the areas. As Chapter 2 mentioned, several fMRI studies indicate a frontal to parietal shift happens as children gain experience with symbolic number processing (Ansari & Dhital, 2006; Ansari et al., 2005). Similar to this logic, a functional response shift toward greater parietal engagement to process nonsymbolic ratios might occur and might result in less reliance to frontal areas. If this is the case, it may imply decreased involvement of white matter connectivity in processing nonsymbolic ratios.

General Discussion

A number of studies have investigated the relations between white matter connectivity and numerical competence with different assessments, but these previous studies have been largely limited to whole number knowledge and its operation (but see Matejko & Ansari, 2013 with higher-level mathematics). With the growing interest in fraction learning, the present study explored neural signatures for nonsymbolic ratio and symbolic fraction processing abilities in relation to white matter microstructure. By taking a cross-sequential approach, testing 2nd and 5th graders, we further explored how the relation changes during the ages that mark the early stages of fractions instruction.

Experiment 1's results revealed the developmental differences in the relations between white matter and nonsymbolic ratio and symbolic fraction processing abilities with 2nd and 5th graders. The results also demonstrated that different aspects of microstructure were differently related with nonsymbolic ratio and symbolic fraction processing, especially in 5th graders. As an extension, Experiment 2 demonstrated how different regions of white matter microstructures develop longitudinally and how those changes that happen over the course of one year are associated with ratio and fraction processing, especially in 2nd graders. With these results, the current study indicates that white matter microstructure may start to involve with ratio and fraction processing between 2nd and 3rd grade period and may eventually reflect individual differences of fraction processing later in primary school.

Furthermore, our results highlight the tracts under temporal lobe, the ILF, may be important for symbolic fractions processing ability. As discussed in Experiment 1, our results may simply indicate the importance of processing visual or verbal features of symbolic fractions. In the field of number cognition, a handful of numerical cognition studies have already repeatedly suggested the importance of the ventral occipital temporal cortex (vOTC), with a proposal of a triple-code model (Dehaene, 1992; Dehaene & Cohen, 1995) that suggests three distinct but necessary components for representing symbolic numbers. In addition to the IPS, which represents analog magnitude (for review, Ansari, 2008; Arsalidou & Taylor, 2011; Houdé, Rossi, Lubin, & Joliot, 2010; Jacob et al., 2012; Lewis, Matthews, & Hubbard, 2016), the model proposed a verbal word frame and a visual Arabic number form. The verbal word frame represents number words and has been suggested to lie in the left perisylvian language areas and angular gyrus (for review, Moeller et al., 2015; Zamarian, Ischebeck, & Delazer, 2009).

This visual Arabic number form can be represented internally, and it is believed to be processed at a 'number form area' in the inferior temporal gyrus (ITG) (for review, Yeo et al., 2017) Given these two components, it is likely that white matter tracts that propagate to language or number form areas (e.g. ILF), are involved with fractions processing. Children may engage verbally naming each fraction or visually processing the form of fraction symbols during the task more, resulting in the involvement of the occipto-temporal regions.

On the other hand, another interpretation would be possible. As a recent work of Cui et al. (2020) noted, fraction processing may require more involvement of temporal regions of the brain since it requires more semantic knowledge than whole numbers (Cui et al., 2020; Liu et al., 2019; Zhou et al., 2018). However, since this account assumes that understanding fraction magnitude is harder than understanding whole number magnitudes, further behavioral and neural understanding on fractions vs. whole number processing would be needed.

Conclusion

The current study is the first to examine the association between white matter microstructure and nonsymbolic ratio and symbolic fraction processing abilities and how the relations change during early fraction instructions. Our results demonstrate that different aspects of microstructure were differentially related with nonsymbolic ratio and symbolic fraction processing. Yet, these correlations may depend on the amount of experience with nonsymbolic and symbolic fractions. Future studies should explore the mechanism under the relations between white matter structures and educational experiences.

Chapter 4: The Effect of Nonsymbolic Ratio Processing on Early Fraction Ability- Investigating the Effect of Linguistic vs. Non-linguistic Contributors to Fractions Ability

Introduction

Chapter 2 and Chapter 3 helped understand neural development of fraction processing and suggested the possibility that nonsymbolic ratio processing may be used as a potential neurocognitive tool for acquisition of symbolic fraction knowledge (Matthews et al., 2016). Building on the understanding of the RPS with neural investigations, the study of Chapter 4 takes a behavioral approach to confirm the associations between the RPS and actual symbolic fraction abilities. Chapter 4 investigates how much the RPS uniquely contributes to early fraction abilities compared other cognitive abilities such as language ability which has been emphasized to be important to learn fraction (e.g., Miura, Okamoto, Vlahovic-Stetic, Kim, & Han, 1999; Seethaler, Fuchs, Star, & Bryant, 2011).

A possible link between nonsymbolic ratio processing and higher mathematics has been shown by a few previous studies in both adults (Matthews, Lewis, & Hubbard, 2016; Park & Matthews, accepted) and children (Hansen et al., 2015; Möhring, Newcombe, Levine, & Frick, 2015; but see Bhatia et al., 2020). However, unlike studies with adults, the studies with children did not measure RPS acuity explicitly. Their tasks were not designed for measuring acuity of participants. Also, the stimuli presented were a mix of discretized and continuous lines so the task cannot preclude the possibility of counting (Figure 4.1). Thus, even though Hansen et al (2015) measured several cognitive abilities including working memory, attention, reading fluency, and multiplication fact fluency, it is hard to conclude that these previous studies showed the influence of the RPS to early fraction ability over other cognitive abilities.

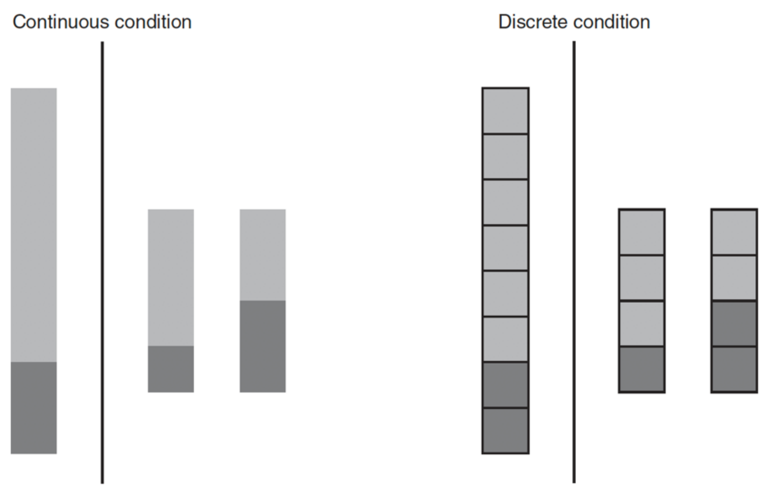

Figure 4.1. The example of a proportional reasoning match-to-sample task with continuous magnitude (left) and the discretized line (right) (Newcombe, Levine, & Mix, 2015).

Therefore, the main aim of current study was to examine the relative contributions of the RPS and other cognitive abilities. Among several cognitive abilities that have been investigated as relatable to fractions knowledge, Chapter 4 focused on the linguistic abilities. Language ability has been consistently reported as a factor that contributes to early fraction learning (Fuchs et al., 2013; Miura et al., 1999; Namkung & Fuchs, 2016; Seethaler et al., 2011) among several possible cognitive factors including relational reasoning, verbal ability, or working memory (e.g., Hansen et al., 2015; Kalra, Hubbard, & Matthews, 2020; Paik & Mix, 2003; Ye et al., 2016).

Being proficient in naming fractions is crucial for understanding fractions concepts (Mix, Levine, & Huttenlocher, 1999; Paik & Mix, 2003). Because fractions are constructed from concatenated whole number symbols, they require more complex naming compared to whole numbers. How quickly children learn fraction-specific words such as *numerator* or *denominator* may also depend on their language ability. Indeed, several behavioral studies have investigated whether development of whole number vs. fraction knowledge would rely on the same or distinct cognitive abilities put emphasis on language competence (Namkung & Fuchs, 2016; Paik & Mix, 2003). Some demonstrated a significance of language especially in fractions by showing that cultural difference in naming fractions affect fraction ability (Miura et al., 1999; Paik & Mix, 2003).

Recent neuroimaging data also indirectly support this argument. In the results of Study 2 above, a strong correlation with fraction processing ability was found in the bilateral ILF linking the temporal-parietal regions. Given that prior work studying language ability has also consistently reported the involvement of ILF (Wandell, Rauschecker, & Yeatman, 2012; Yeatman, Dougherty, Ben-Shachar, & Wandell, 2012), the findings are consistent with the hypothesis that linguistic ability plays an important role on fraction processing. Furthermore, a recent fMRI study revealed that the medial temporal lobe was particularly engaged for fractions processing as opposed to whole numbers (Cui et al., 2020). Considering these reports, linguistic factors might explain why the ILF is differentially related to fractions and whole numbers. However, this question has yet to be investigated behaviorally.

Another aim of the current study was to unravel the mechanisms underlying the relationship between RPS acuity and fraction ability, which still remains in question. Although previous work concluded the existence of this perceptual ability that may affect symbolic mathematics, it is not clear which part of mathematical ability the RPS contributes. Fractions knowledge tests so far included several aspects of knowledge such as fractions arithmetic, conceptual understanding, and algebraic thinking (Matthews et al., 2016; Möhring et al., 2015). However, no study has investigated which aspect of fractions knowledge is that the RPS specifically associates with.

In the domains of whole number learning, the same question about the relationship between nonsymbolic magnitude and symbolic math was raised for the ANS acuity (Dehaene, 2011; Feigenson, Libertus, & Halberda, 2013). One possible account was that nonsymbolic magnitude processing may contribute to the ability to detect errors in symbolic arithmetic and that eventually contributes to the link between nonsymbolic magnitude and symbolic mathematics (Feigenson et al., 2013; Lourenco, Bonny, Fernandez, & Rao, 2012; Szkudlarek & Brannon, 2017; Wong & Odic, 2020). For example, when children solve a symbolic equation (e.g., 11 + 17 = 42), they can map each operand (11 and 17) onto a number line and perform addition approximately and compare with the answer they calculated (see Figure 4.2). If there is a difference between expected and observed answers, they can go back to the problem and try to solve it again. Thus, by helping this error detection process, the nonsymbolic magnitude processing ability can be linked to symbolic math knowledge according to the account.

To explore this hypothesis, Wong & Odic (2020) developed a novel symbolic equation error detection task. The task presents a whole number equation (85 + 64 = 128) with an incorrect answer, and participants determine whether the given incorrect answer is larger or smaller than it should be. Wong and Odic showed that ANS acuity significantly predicted the error detection task performance, consistent with the error detection hypothesis. Similar to this logic, it might be possible that the RPS acuity may contribute to the error detection of fraction arithmetic and eventually elicit the link with formal fraction knowledge.

With these aims, Chapter 4's study examined the relative contributions of linguistic ability and the RPS to early symbolic fraction ability. We also measured the ANS as control. We recruited 5th and 6th graders who have recently started to receive formal arithmetic training on fractions. Investigating these age ranges is likely to capture individual differences in early fraction processing and fraction arithmetic ability.

First of all, we measured the RPS acuity by using a nonsymbolic ratio comparison task. We chose to use line ratios over other formats for comparison because in our prior work, lines yielded less noisy data compared to other nonsymbolic ratio formats such as dots or circles (Park et al., 2020). Also, recent work has shown that the line ratio format is the most predictive of symbolic math achievement in adults (Park & Matthews, accepted). Additionally, the ANS was measured by using a nonsymbolic dot comparison tasks that have used in Park et al. (2020).

Another independent variable, linguistic ability, was assessed by whole number and fraction transcoding tasks as well as a math vocabulary test. Children's ability to transcode from number words to symbolic numbers or fractions has been deemed as an important ability for formal math achievement (Barrouillet, Thevenot, & Fayol, 2010; Lopes-Silva, Moura, Júlio-Costa, Haase, & Wood, 2014; Lopes-Silva et al., 2016; Seron & Fayol, 1994) and also can be used as a screening tool for mathematics learning difficulty (Moura et al., 2015, 2013). The ability to flexibly translate number words to symbolic numbers may eventually contribute to symbolic fraction acquisition given that fractions require more complex naming skills than whole numbers. For assessing children's math vocabulary knowledge, we used Powell (2015)'s *Mathematics Vocabulary Grades 3 and 5***.** The test was developed to assess children's understanding of particular math vocabulary spanning from simple arithmetic to geometry (e.g., addend, isosceles triangle, or improper fraction). Some researchers suggested that children who show difficulty learning new concepts and corresponding vocabularies are likely to show low performance on formal math test (Forsyth & Powell, 2017; Purpura, Hume, Sims, & Lonigan, 2011; Purpura & Logan, 2015).

Next, to explore a mechanism underlying the relation between the RPS and formal fractions knowledge, we measured symbolic fraction processing ability by using symbolic fraction comparison tasks similar to Chapter 2 and 3 and newly developed symbolic fraction equation error detection task (adpted from Wong & Odic, 2020).

Lastly, since working memory capacity can influence performance on the outcomes of the transcoding test or other computerized tasks, we used digit span to control children's working memory per Wong & Odic (2020).

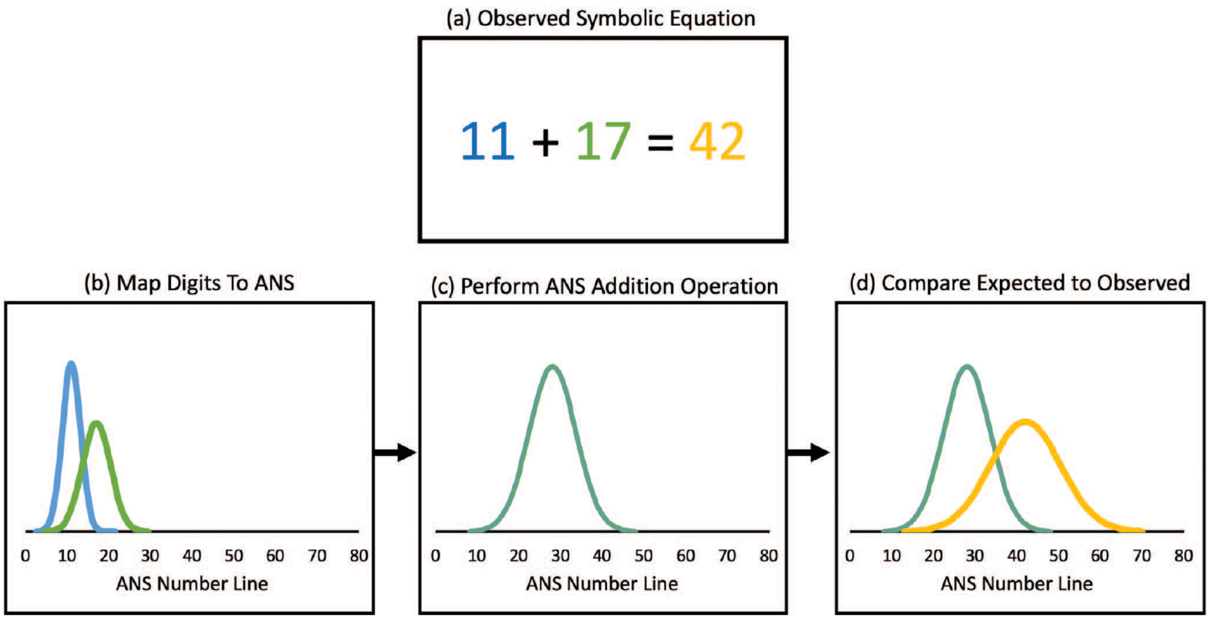

Figure 4.2. An example diagram that shows theorized process of error detection from Wong & Odic (2020). The process allows participants to use their ANS to determine the direction and magnitude of the error. (a) Participants observe the equation that presents incorrect answer (28, not 42). (b) the participants map each operand (11 and 17) to the ANS internally and each digit is represented with noisy Gaussian curve, centered on the number that corresponds to each operand (11, 17) with a noise. The noise distribution is proportioned to individual's ANS acuity (c) the participants perform approximate addition. (d) the participants can compare the expected answer generated by approximate addition (approximately 28) to observed answer (42).

Method

Preregistration

The experiment was fully preregistered at the Open Science Framework (OSF, https://osf.io/6hs5u). We planned to regress the fractions abilities against four variables (the

RPS, language ability as IVs, WM and the ANS as covariates). To detect the medium effect size .15 with an $\alpha = .05$ and power of .80, an analysis using the 'pwr' package in R showed that we need 83 participants at minimum.

Participants

As of March 14, 2021, 54 5th and 6th grade children ($mean_{age}$ = 10.89, sd_{age}= .78) were recruited via contacting families in our existing database and campus mass emails sent to faculty and staff at a major university in a medium sized Midwestern city.

Non-linguistic cognitive factors

Nonsymbolic ratio comparison task

In the ratio comparison task, trials began with a fixation cross appearing for 200ms which was followed by a 4000ms presentation of comparison stimuli (line-ratios) which disappeared leaving a blank screen. Participants compared pairs of line ratios, indicating their judgments via key press – pressing "d" if the larger ratio was on the left or pressing "k" if the larger ratio was on the right. Stimulus display time was 6000ms as in Park et al. (2020).

Line-ratios were constructed by juxtaposing yellow and blue line-segments. The yellow line (numerator) always appeared on the left, and the blue line (denominator) appeared on the right (see Figures 4.2). The yellow line was given a random vertical jitter relative to the blue line. The numerator segment ranged from approximately 35 to 228 pixels in length, and the denominator segment ranged from 50 to 304 pixels. To control for the possibility that participants might make their judgments based on the summed lengths of components for each stimulus, the larger ratio had greater summed length in half of all trials, and the larger ratio had lesser summed length in the other half. The ratio of two summed lengths was always approximately 1.3.

Task difficulty was determined by varying the ratio between each pair of magnitudes – in this case indicating a ratio of ratios. Each pair fell into one of five distance bins: 3:1, 2:1, 2:3, 3:4, and 5:6, with difficulty increasing as the distance became closer to 1. The magnitude of individual ratio stimuli was distributed in the range from 0.2 to 0.8. The small pair always included 0.2, the large pair always included 0.8, and the medium pair was always midway between the other two other conditions. For example, for the 2:1 ratio bin, 0.4 vs. 0.8, and 0.4 vs. 0.8 were representative comparisons for the small and large pairs, respectively

Each pair was presented twice, once with the larger stimulus on the left, and once with the larger on the right. Children completed 60 trials (5 ratio distance x 3 size x 2 congruity x 2 left/right, see stimuli descriptions below). These trials were presented in random order within each block. Participants were prompted by the software to take a break after 30 trials.

Numerosity comparison task (the ANS task)

On each trial, a fixation cross appeared for 200ms followed by comparison stimuli (dot arrays) which remained visible for a 1500ms before disappearing, leaving a blank screen. Participants determined which of two stimuli had the larger number of dots. Stimulus display time was adjusted to 1500ms as in Park et al. (2020) (Halberda & Feigenson, 2008; Park et al., 2020). The computer did not proceed to the next trial until participants made a choice. Participants indicated their judgment of which stimulus was larger via keystroke – "k" for right and "d" for left.

For the comparison stimuli, boxes of yellow and blue dot arrays were presented against gray backgrounds on each side of the screen (see Figure 4.3). Dots were randomly placed so that they evenly covered the area of each box. The number of dots in each array ranged from 16 to 118. Task difficulty was determined by varying the *ratio distance* between each pair of numerosity, defined as the ratio between compared numerosities (M_1:M_2 where M_1 is the smaller of the two magnitudes). We varied difficulty using five ratio distance bins based on Odic (2017): 1:2, 2:3, 5:6, 7:8 and 15:16, with difficulty increasing as ratio distance approached 1. For example, the 15:16 bin (e.g., 30 vs. 32 dots) was expected to be more difficult than the 1:2 bin (e.g., 16 vs. 32 dots).

Each distance bin contained three classes of arrays to be compared: small, medium, and large, which used three different numerosity ranges, spanning arrays as small as 16 and as large as 118. For example, for the 1:2 distance bin, 16 vs. 32, 37 vs. 74, and 59 vs. 118 represented small, medium, and large pairs, respectively.

We also controlled for the possibility that participants might choose based solely on the summed area or average size of dots throughout the task (Fazio, Bailey, Thompson, & Siegler, 2014; Halberda et al., 2008; Libertus, Feigenson, & Halberda, 2011). In half of all dot trials, the summed area of dots was equivalent across the two arrays on any given trial, meaning average dot size decreased with numerosity. In the other half of trials, average dot size was the same across the two arrays compared, meaning that the summed area of an array increased with numerosity. The size of each dot in a given array was allowed to vary from 80% - 120% of the average size of dots in that array.

Each stimulus pair was presented twice, once with the larger stimulus on the left, and once with the larger on the right. Participants completed a total of 60 trials (5 distance bins x 3 pairs per bin x 2 control conditions x 2 sides for presentation of the correct response). Trials were presented in random order. Participants were prompted by the software to take a break after 30 trials.

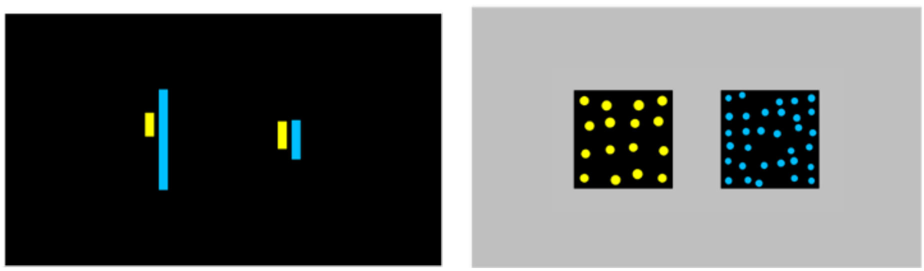

Figure 4.3. Example stimuli for nonsymbolic ratio comparison (left panel) with line ratios and nonsymbolic numerosity comparison (right panel) tasks. Both figures present a larger magnitude on the right side.

Linguistic cognitive factors

Whole Number transcoding ability

To evaluate whole number transcoding, the experimenter read numbers aloud, and children were instructed to write the corresponding symbolic numerical representations (Barrouillet et al., 2010; Lopes-Silva et al., 2014; Moura et al., 2013). The internal consistency of this task has been reported to be as high as 0.96 (KR-20) (Lopes-Silva et al., 2014, 2016; Moura et al., 2013). This task consisted of 40 items including 3 one-digit numbers, 9 two-digit numbers, 10 three-digit numbers and 18 four-digit numbers (Barrouillet et al., 2010). The sample of whole numbers is in Appendix A.

Fraction transcoding ability

Along with whole number transcoding, children were instructed to transcode symbolic fractions. This task consisted of 40 items, including 16 irreducible fractions, 12 improper fractions, and 12 mixed fractions. The sampled fractions ranged from 0 to 5 based on the previous studies using the fractions number (e.g., Hansen et al., 2015). Their number lines were bounded from 0 to 5, so we used fractions within this range. There were 16 fractions ranged 0-1, 9 fractions ranged 1-2, 5 fractions in each 2-3, 3-4 and 4-5 ranges. The full sample of fractions used is in Appendix A.

Math Vocabulary Test

To measure children's math specific vocabulary, we used *Mathematics Vocabulary Grades 3 and 5* (Powell, 2015; Powell et al., 2017). The math vocabulary terms on the test were selected from three common 3rd- and 5th-grade mathematics textbooks and textbook glossaries. Those included the student-level glossaries of *enVisonMATH* by Pearson (Charles et al., 2014a, 2014b), *Everyday Math* by McGraw-Hill (Bell et al., 2015) and *Go Math!* by Houghton Mifflin Harcourt (Dixon, Larson, Burger, & Sandoval-Martinez, 2014b, 2014a), which were largely used across the United States. In addition to this, math vocabulary terms from the Common Core State Standards (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) were also selected. The questions included naming parts of various equations (e.g., *augend* in an addition equation), as well as geometry (e.g., *regular polygon or intersecting lines*), or place-value terms (e.g., *negative integer*). Assessment items took various formats, including selecting a correct term by using multiple-choice selection, drawing a figure, or labeling parts of the given figure (see Powell et al., 2017 for the examples). Children had 30 min to answer as many questions as possible. Each problem scored 1 point with a total raw score of 129.

Symbolic fraction ability

Symbolic-Fraction Comparison

The symbolic-fraction comparison task required participants to decide which of two fractions was larger. We used the fraction stimuli selected by Morales et al. (2020) (Morales et al., 2020). Among the stimuli, we excluded pairs with common components (Appendix B) to minimize the dependency on componential strategies (e.g., judgment based on numerator or denominator comparisons rather than overall value). Selected pairs were consisted of irreducible double-digit fractions with varied numerical distance between two fractions. Numerical distance was divided into small (about 0.1), medium (about 0.17) and large (about 0.24). The total of 72 fraction pairs was presented in a random order.

On each trial, a fixation cross appeared for 500ms followed by 10s presentation of comparisons as Morales et al. (2020). Even after the stimuli disappeared after 10s, the computer did not proceed to the next trial until participants made a choice. Participants indicated their judgment of which stimulus was larger via keystroke – "k" for right and "d" for left. The side of the larger fraction (left/right) was counterbalanced across trials. Participants were prompted by the software to take a break every 20 trials.

Symbolic Fraction Equation Error detection

On each trial, participants were presented with solved fraction arithmetic equations in the center of the screen (e.g., $\frac{3}{8}+\frac{2}{5}=\frac{35}{48}$) featuring incorrect answers. Participants determined whether the given answer was smaller or larger than the correct answer.

Participants indicated their choices via key press – pressing "d" if the given answer is smaller than the correct answer and pressing "k" if the given answer is larger than the correct answer.

To vary the difficulty of the error detection task, we varied the ratio between the given answer and the correct answer using a method corresponding to that of the nonsymbolic ratio comparison task. Thus, the ratio difficulty fell into one of 5 distance bins: 3:1 ($\frac{3}{8} + \frac{2}{5} = \frac{1}{4}$, where the actual correct answer is $\frac{31}{40}$), 2:1 ($\frac{2}{5} - \frac{3}{8} = \frac{1}{80}$, where the actual correct answer is $\frac{1}{40}$), 2:3, 3:4, and 5:6. Participants completed a total of 100 trials (5 ratio bins X 2 possible answers (lower/higher) X 2 operations (addition/subtraction)) X 5 different trials within the same condition. Participants were prompted by the software to take a break every 10 trials.

Trials began with a fixation cross appearing for 500ms followed by the equation presentations on the center of the screen. The given equation remained on screen until participants responded, consistent with prior administrations of the arithmetic verification task with children (e.g., Rousselle & Noël, 2008). First two terms of equation ($\frac{3}{8} + \frac{2}{5}$) were presented with single digit or double digits proper fractions and the last terms of equation (i.e., given answer) were sometimes presented with fractions with single digit or fractions with double digits. The equation did not involve common numerators or denominators.

General Cognitive Measures

Digit span

To measure verbal working memory, we used the digit span subtest from the WISC -IV (Kaufman, Flanagan, Alfonso, & Mascolo, 2006; Wechsler, 2003). The test was divided into two subsections: 1) Digit Span Forward, in which participants were asked to repeat numbers in the same order as presented aloud by the experimenter, and 2) Digit Span Backward, in which participants were asked numbers in the reverse order of that presented aloud by the experimenter. Each test consisted of eight items, and each item had two trials. For each trial, the experimenter scored 1 point for a correct response or 0 point for an incorrect response or no response. The experimenter discontinued the test after participant scores of 0 on both trials of an item. The total possible score combining both Digit Span Forward and Backward subtests was 32 points.

Experimental procedures

Due to COVID-19, the data collection for the current study has been delayed. Therefore, Chapter 4 only presented the hypotheses and future significance. The data collection is still in progress and all data will be uploaded to the OSF (https://osf.io/6hs5u). The experimental sessions were conducted in a video-chat environment via Zoom (https://zoom.us). The experiment was divided into two sessions, each on a different day. In session 1, children completed the numerosity and ratio comparisons, the symbolic fraction comparison, and the fraction equation tasks. The computerized tasks including nonsymbolic comparisons, symbolic fraction comparison, and the fraction equation task were presented via Gorilla (https://gorilla.sc). Each task had its own link. The experimenter gave the link to children so that they could open the link on their laptop or desktop. After the experiment started, children shared their screen so that the experimenter could observe and monitor how children performed the task.

In session 2, children completed the whole number and fraction transcoding tests and the math vocabulary test. For the vocabulary test, we used an online platform called JotForm (https://jotform.com) to present the assessment. This online platform allows children to draw and write their answers by using mouse click, trackpad or keyboard press.

Hypotheses

Hypothesis1: As previous studies have shown (Matthews et al., 2016), RPS acuity will be associated with symbolic fractions comparison task performance even after controlling for ANS acuity and verbal WM.

Hypothesis 2: If the error detection theory with the ANS is applicable to the link between the RPS acuity and fraction competence, RPS acuity will be associated with the ability to detect errors in fractions arithmetic even after controlling for the ANS acuity and verbal WM.

Hypothesis 3: If language ability is important for fraction learning, language competence might also be associated with the fraction comparisons or the ability to detect error in fractions arithmetic or both after controlling for verbal WM.

Exploration: How influential is the RPS acuity to fraction comparison performance and the ability to detect error in fractions arithmetic compared to the language ability?

Future significance

The described study examines how the RPS contributes to early fraction ability. Starting from the finding of Chapter 3 which implied the association of language ability and fractions processing, we also investigate the association between language ability specific to mathematics and early fraction knowledge with a behavioral approach. Furthermore, the present study will put the error detection hypothesis to the test to better understand mechanisms underlying the relation between the RPS and early fraction ability. I believe that these studies will broaden our understanding on cognitive factors contributing to early fraction ability.

Chapter 5: Conclusion

Summary of Findings and significance

From the brain to human behavior, the studies in this dissertation aimed to better understand the nature of fraction acquisition by exploring both behavioral and neural developmental factors that may contribute. Findings of these studies suggested the ability to process nonsymbolic ratios (the RPS) might serve as a neurocognitive startup tool for fraction acquisition during the period of early fraction instructions. Starting from the question of why fractions are hard, this dissertation ponders over the under-researched neurocognitive startup tool that can possibly make fraction learning less challenging.

Chapter 2's study broadened our understanding of how shared functional substrates for nonsymbolic ratios and symbolic ratios emerge by investigating differences between groups of children prior to formal fractions instruction (2^{nd} graders) and after a few years of instruction (5^{th} graders). The results showed that after a few years of formal instruction, the substrates associated with processing symbolic fractions emerge in fronto-parietal cortex that overlaps with the substrate for the RPS. In turn, regions of the brain that were sensitive to nonsymbolic ratios also become sensitive to symbolic fractions with increasing age.

Adding to the functional brain investigation, Chapter 3's study explored how individual differences in the structural connectivity relate to nonsymbolic ratio and symbolic fraction processing ability in the same cohorts of children as Chapter 2. The results of Chapter 3's study showed that the relation between white matter connectivity and nonsymbolic ratio and symbolic fraction processing ability start to emerge during the period of early fraction instructions. By taking a cross-sequential approach, the study in Chapter 3 marked the first year of fraction instruction, from 2nd grade to 3rd grade, as when structural connectivity starts to reflect the ratio and fraction processing abilities. Especially, this study emphasized the potential importance of visuo- or verbal- processing for symbolic fraction processing.

Lastly, the behavioral study in Chapter 4 tested how influential the measured RPS sensitivity is to early fraction ability. To confirm the results better, we compared the contributions of RPS acuity and other factors contributing to fraction ability. We especially compared the RPS and language ability because it is one of the most consistently reported abilities associated with fraction learning. Also, it was the ability Chapter 3's study indirectly implicated by showing the importance of the ILF, which is known to be integrally involved with verbal- and visuo- processing(e.g., Yeatman, Dougherty, Ben-Shachar, & Wandell, 2012). The results of this experimental study will provide another path forward to advance children's fraction knowledge by suggesting an easily accessible perceptual tool that can be leveraged.

By investigating the development of fraction acquisition in early years, I believe that these studies stand to broaden our understanding of early fraction learning by looking at the interaction between brain and behavior during development. In the long run, this interplay of neural and behavioral studies on fraction acquisition may provide insight into how to efficiently leverage fraction learning by emphasizing neurocognitive factors that are particularly associated early fraction ability. Future studies might put the RPS's effectiveness to test regarding whether it is usable as an educational tool with training and longitudinal approaches. As such, I believe this dissertation's findings can appeal to a broad audience in numerical cognition encompassing researchers and practitioners in the fields of neuroscience, psychology, and education.

Taking a domain-relevant approach to investigate fraction learning

Ever since Dehaene's proposal of the triple-code model (Dehaene, 1992; Dehaene & Cohen, 1995), developmental cognitive neuroscience theories have become more flexible (Dehaene & Cohen, 2007; Hannagan et al., 2015; Johnson, 2011; Kanwisher, 2010) in terms of understanding neural development of reading and mathematics. Compared to strong modularity views that argue a specific region of the brain is highly specialized for a certain cognitive process alone, current theories have evolved to recognize the interactive network across different brain regions for processing a certain sort of cognitive information, and to recognized the active interactions between biological traits of the brain and external environment.

This dissertation took a domain-relevant approach which postulates the existence of biases in the early brain and following changes in certain regions that become sensitive to particular inputs (Karmiloff-Smith, 2015). This theory also hypothesizes that progressive functional specialization and specialized connectivity are shaped by continuous interactions between biological maturation and the external environment. Thus, the domain-relevant framework can incorporate the neuronal recycling hypothesis (Dehaene & Cohen, 2007), the biased connectivity hypothesis (Hannagan et al., 2015) and even the interactive specialization hypotheses (Johnson, 2011).

The first two neuroimaging studies in this dissertation focused on the period of early fraction instruction and attempted to capture progressive functional and structural brain changes as children go through fractions instruction. Results from Chapter 2 revealed that functional biases for nonsymbolic ratio, the RPS, exist even before fraction instruction and that functional specialization for symbolic fractions develops later on in overlapping substrates. Along with the changes in functional activations of the brain, individual differences in white matter connectivity start to reflect ratio and fraction processing ability as children experience fractions instruction. Symbolic fractions are one set of cultural inventions that humans need to acquire from external environments, but nonsymbolic ratios magnitudes can be processed early in development (McCrink & Wynn, 2007). Considering these differences, the neural findings of this dissertation suggest that fraction instructions might orient the regions that originally process nonsymbolic ratios. Therefore, the findings of this dissertation support the domain-relevant framework that inputs from outside can orient the cortical regions where its original functionality is similar to the novel output to new information.

In terms of connectivity, Chapter 3's study did not find existing biases of structural connectivity for nonsymbolic ratio magnitudes before fraction instruction. This result is somewhat different from the biased connectivity hypothesis that postulates that there might be functional and structural connectivity initially biased toward a certain stimuli or information. However, our findings are limited to structural connectivity. It should be noted that prior work has observed functional coactivity among distant brain regions such as the temporal-parietal-frontal areas of the brain (e.g., the ITG-the IPS-the IFG) to process fractions (Cui, Li, Li, Siegler, & Zhou, 2020; DeWolf, Chiang, Bassok, Holyoak, & Monti, 2016). To better understand how this cooperation happens and when it starts, we should look into functional connectivity in addition to structural connectivity and how both functional and structural connectivity are coupled. Future neuroimaging studies should additionally investigate resting state or dynamic functional connectivity if there is any preexisting connectivity among the regions that involve with processing symbolic fractions or function. As such, the domain-relevant framework can still be useful to understand these brain networks that progressively become co-active and cooperative to process symbolic fraction processing. Furthermore, investigating functional specialization, functional reorganization of the brain as a function of learning with intervention designs could offer new insight into brain plasticity's role in fraction learning.

Tapping into perceptual routes to fractions

As introduced in Chapter 1, learning fractions is challenging compared to learning whole numbers. Considering the RPS-based argument, learning fractions is hard likely due to a lack of experience with nonsymbolic ratio magnitudes. For whole number learning, current curriculum starts from mapping symbolic number to approximately corresponding number of objects, visual representations helpful to imbue whole number with nonsymbolic analog magnitudes. In case of fractions, however, formal curricula sometimes use these visual representations such as a certain location of a number line that ranges 0-1 or some shaded areas in a circle (Figure 5.1) without specific intent of helping children represent fractions as analog magnitude. This limited experience with nonsymbolic ratio magnitude that imbue fractions with analog magnitudes may interrupt the use of the RPS for fractions representations.

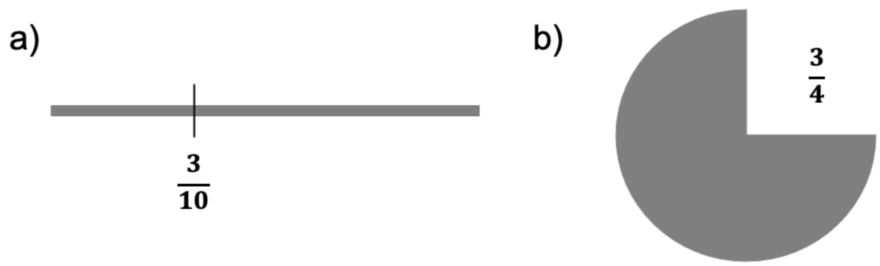

Figure 5.1. Representations of fractions with (a) number line and (b) circle.

Given Goldstone & Barsalou's (1998) suggestion that perceptual representations can be helpful to acquire symbol system, the best way to support children's understanding on symbolic fractions might be using various perceptual representations. For example, with a part-part visual representation, children might have better concept on different types of fractions including improper and proper fractions (Figure 2). One of the most interesting predictions of RPS-based theories is that nonsymbolic ratio processing ability might be effectively leveraged to improve intuitions about symbolic fractions, thereby improving math performance. Although this dissertation attempted to confirm an association between perceptual-based ratio processing abilities and symbolic fraction ability, the designs of the studies were not appropriate for testing the efficacy of the use of perceptual ratio representations for instructional purposes. Therefore, future studies need to explore RPS-based interventions to see its practical potential to enhance children's developing mathematical competence. Also, examining following neural changes with a multi-modal imaging will substantiate the prediction of the RPS.

From developmental cognitive neuroscience to education.

As Bruer (1997) wrote, the fields of neuroscience and education might still be a bridge too far (Bruer, 1997). I also agree that the neural level of understanding is hard to be directly linked to human's behavioral and cognition, and, definitely, to educational outcomes given current technology. However, Bruer's argument never indicated that neuroscience and education ***cannot*** be directly bridged. Developmental cognitive neuroscience has enough potential to make better educational environment.

First, it can broaden our understanding of neural mechanisms underlying children's learning, such as neural representations that underpin language and math (Gabrieli, 2016). Also, it helps identify neural markers of learning disabilities by studying children at educational risk. For example, researchers can study dyscalculia and can identify causal drivers of cognitive development such as the impaired IPS. Researchers may be able to find early neural markers for dyscalculia and intervene early in development. Lastly, these endeavors can lead to intervention studies that might help children with learning difficulty. Moreover, the studies can compare the neural effects of different educational cognitive trainings and may eventually recommend the most effective training to children.

A recent review by Goswami (2020) supported these potential impact of developmental cognitive neuroscience studies to early educations and pointed out to improve experimental design – i.e., longitudinal and intervention designs (Goswami, 2020). Beyond simply looking at the pattern of brain activations and behavior that correlates with the brain activations, developmental cognitive neuroscience can provide better understanding on the interactions between children's behavior and biological changes in different developmental stages. Furthermore, the field can show how those interactions will affect children's educational outcomes. By pursuing longitudinal and intervention designs (Rosenberg-Lee, 2018), the field can offer knowledge about how children learn and may suggest the most effective way to enhance children's educational outcomes appropriate to their developmental stage.

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Appendix

Appendix A. Transcoding stimuli

Item	Number to be dictated	Fraction to be dictated
1	4	4
2	7	7
3	1	1
4	11	11
5	40	40
6	16	16
7	30	30
8	73	73
9	13	13
10	68	68
11	80	80
12	25	25
13	200	200
14	109	109
15	150	150
16	101	101
17	700	7/6
18	643	6/5
19	8000	4/3
20	190	11/7
21	1002	13/8
22	951	1 3/4
23	1015	1 4/5
24	2609	1 5/6
25	1300	1 11/12
26	3791	13/6
27	1060	7/3
28	4701	1 9/17
29	1100	2 3/5
30	215	2 17/20
31	2140	22/7
32	1107	13/4
33	902	10/3

34	7013	3 17/26
35	3112	3 47/50
36	5147	4 3/13
37	6513	4 3/7
38	7105	9/2
39	4870	93/20
40	8844	4 3/4

Appendix B. Fraction stimuli

Fractions compared			
34/65	57/74	13/18	45/76
49/65	76/87	23/94	16/33
59/70	23/39	43/56	51/94
68/81	37/55	53/65	71/99
11/31	43/92	28/39	48/85
43/95	20/67	62/91	57/73
11/43	38/77	23/36	42/79
18/55	41/99	45/58	51/77
45/64	75/94	43/54	59/93
11/72	37/98	31/44	61/99
11/36	22/47	13/18	45/88
17/32	46/61	35/99	24/53
17/87	29/92	35/58	46/85
51/76	83/98	52/99	36/47
23/59	11/53	37/50	56/97
64/91	76/97	31/42	47/93
16/31	59/79	40/51	46/83
36/77	19/53	15/43	26/97
73/94	17/32	49/95	32/43
47/95	15/56	27/73	17/36
26/79	40/93	23/74	18/37
72/97	32/57	62/97	55/69
35/68	50/83	42/79	26/37
74/95	26/47	26/43	49/94
83/96	56/89	25/88	18/41
86/99	62/81	53/98	37/48
67/95	45/83	55/98	31/43
80/93	41/66	35/46	41/79
51/71	22/35	28/41	55/94
63/94	24/47	41/57	46/81

33/73	13/46	46/91	30/41
21/32	73/89	62/99	45/56
21/68	43/90	52/93	47/58
46/97	16/67	21/79	15/34
73/89	39/55	58/85	39/50
29/46	62/79	51/98	34/45